A Study of the Effects of the Coefficients of Generalized Bilinear Transformations in Design of Two-Dimensional Variable Recursive Digital Filters

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ABSTRACT

A Study of the Effects of the Coefficients of Generalized Bilinear Transformations in Design of Two-Dimensional Variable Recursive Digital Filters

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Two-dimensional variable recursive digital filters are applied in signal processing and communication systems where the frequency-domain characteristics of digital filters are required to be adjustable.

The main objective of this thesis is to propose a new technique of designing 2-D recursive digital filters with variable characteristics. From a 1-D second order Butterworth low-pass analog ladder structure, 2-D low-pass and high-pass digital filters can be obtained through the application of double generalized bilinear transformations when the coefficients of the transformations are chosen in their specified ranges. And when one or more these coefficients are changing, the resulting 2-D low-pass and high-pass filters possess variable magnitude responses. Another two important types of 2-D digital filters, 2-D band-pass and band-elimination filters, can also be obtained by properly combining a 2-D low-pass filter and a 2-D high-pass filter. When the coefficients used to obtain the 2-D low-pass and high-pass filters are changeable, the resulting 2-D band-pass and band-elimination filters also possess variable magnitude characteristics. The manner how each coefficient of generalized bilinear transformation affects each type of desiring 2-D recursive digital filters is investigated in detail.

Stability is always an important issue in 2-D recursive digital filter design. The stability conditions of generalized bilinear transformation and the stability conditions of the 2-D digital filters having a denominator with single degree of each variable are discussed in detail here.

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To

my dear daughter Carol Jialu Deng

and my baby expected in June 2004

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List of Important Symbols

z_1, z_2	Z-domain parameter in first and second dimensions
s_1, s_2	Laplace domain parameter in first and second dimensions
ω_1, ω_2	Frequencies in radians in the discrete domain in first and second dimensions
k_1, k_2	Band-effect coefficients of the generalized bilinear transformations in the first and second dimensions (also k_3 and k_4 for high-pass filters in Chapters 4 and 5)
α_{01}, α_{02}	Gain-effect coefficients of the generalized bilinear transformations in the first and second dimensions (also α_{03} and α_{04} for high-pass filters in Chapters 4 and 5)
β_{01}, β_{02}	Polarity-effect coefficients of the generalized bilinear transformations in the first and second dimensions (also β_{03} and β_{04} for high-pass filters in Chapters 4 and 5)
$H_d(z_1,z_2)$	Frequency responses of 2-D digital filters
$H_a(z_1,z_2)$	Frequency responses of 2-D analog filters
$N_d(z_1,z_2)$	Nominator of 2-D digital transfer functions
$D_d(z_1,z_2)$	Denominator of 2-D digital transfer functions
K	Adjustable Multiplier in the forward or backward paths
Σ	Summation

$\forall$	For all values
$[c_1, c_2]$	The range from c_1 to c_2 and includes the two end-points c_1 and c_2 (c_1 and c_2 are constants)
(c_1, c_2)	The range from c_1 to c_2 but does not include the two end-points c_1 and c_2 (c_1 and c_2 are constants)
$\prod$	Products of
$\in$	Belongs to

Chapter 1

Introduction

1.1 General

In recent years, considerable attention is being paid to two-dimensional (2-D) digital filter design and applications, as these are widely used in the telecommunication and radar systems, image processing and other applications where two-dimensional array data are used [1, 2, 3].

Just as in the case of 1-D digital filters, 2-D digital filters can be classified into two main groups, nonrecursive digital filters, also called FIR (Finite Impulse Response) filters, and recursive digital filters, also called IIR (Infinite Impulse Response) filters [2, 3, 4, 5, 6].

A nonrecursive digital filter is one for which the sample of the output can be computed as a linear combination of a finite number of samples of the input. The transfer function of a causal 2-D nonrecursive digital filter can be described as

$$H(z_1, z_2) = \sum_{i=0}^{N_1} \sum_{j=0}^{N_2} A_{ij} z_1^i z_2^j \tag{1.1}$$

where A_{ij}'s are real coefficients. As in the case of 1-D digital filters, the main properties of 2-D nonrecursive digital filter are its inherent stability and the linear phase features. It can be designed to approximate a required frequency response and can be modified by a linear phase term. It is also possible to have symmetries present in the impulse response [7, 8, 9].

On the other hand, for a 2-D recursive digital filter, the output of the filter is obtained by a suitable combination of a finite number of input samples and a number of past output samples. A typical 2-D recursive filter has the transfer function

$$H(z_1, z_2) = \frac{\sum_{m=0}^{M_1} \sum_{n=0}^{M_2} B_{mn} z_1^m z_2^n}{\sum_{i=0}^{N_1} \sum_{j=0}^{N_2} A_{ij} z_1^j z_2^j} \tag{1.2}$$

where $A_{00} = 1$, A_{ij} and B_{mn} are real coefficients.

A main issue in 2-D recursive filter design is its stability. A recursive digital system is said to be stable, if its output is well behaved for all bounded inputs. The most commonly used stability criterion is the bounded-input bounded-output (BIBO) rule. A system is stable in BIBO sense if for any bounded input sequence, the output sequence is bounded [10, 11, 12, 13].

Recently, 2-D filters with variable characteristics are widely applied to signal processing and communication systems where the frequency-domain characteristics of digital filters are required to be adjustable. Many researchers have started to study the properties of such filters. These could be variable magnitude response, phase response and group delay. To achieve the variable characteristics, some of the coefficients of the digital transfer functions should be changeable. However, the stability conditions have to be satisfied always [14].

Most of the existing design methods for 2-D filters with variable characteristics are based on frequency transformations [15, 16, 17, 18, 19, 20]. By such methods, 2-D low-pass, high-pass, band-pass and band-elimination filters with variable cut-off frequencies could be obtained. However, if more detailed and complicated specifications are given, these methods are not applicable due to the intrinsic constraints of frequency transformations.

Another popular method to design 2-D recursive digital filters having variable characteristics is adding adjustable multipliers to a stable analog filter to form a new filtering system. In this method, in connection with a stable filter $H_d(z_1, z_2)$, we can add a multiplier K in the feedback path as shown in Figure 1.1, or add the multiplier K in the forward path as shown in Figure 1.2 [21].

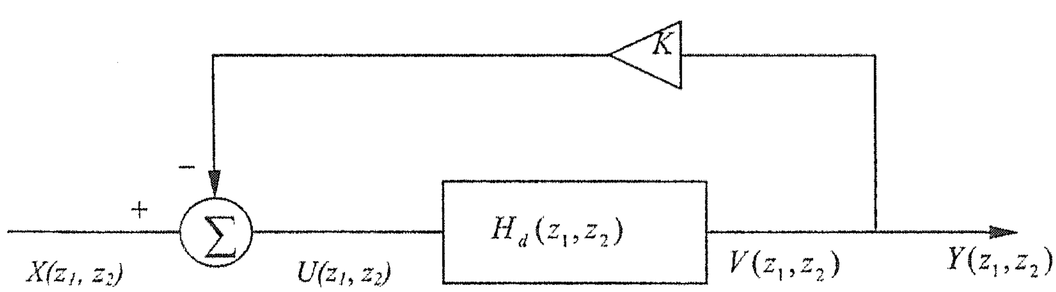


Figure 1.1 The multiplier K in the backward path (System I)

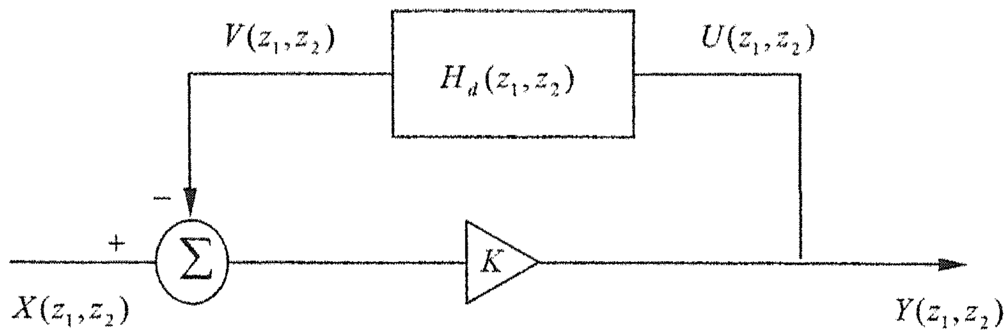


Figure 1.2 The Multiplier K in the forward path (System II)

From Figure 1.1, we can derive the transfer function for System I as

$$Y(z_1,z_2)=V(z_1,z_2) \tag{1.3a}$$

$$U(z_1,z_2)=X(z_1,z_2)-K\cdot V(z_1,z_2) \tag{1.3b}$$

$$V(z_1,z_2)=U(z_1,z_2)\cdot H_d(z_1,z_2) \tag{1.3c}$$

Therefore, the transfer function for System I is obtained as

$$H_1(z_1,z_2)=\frac{Y(z_1,z_2)}{X(z_1,z_2)}=\frac{H_d(z_1,z_2)}{1+KH_d(z_1,z_2)} \tag{1.4a}$$

Here, if the generating filter Hd(z1, z2) is expressed as

$$H_d(z_l,z_2)=\frac{N_d(z_l,z_2)}{D_d(z_l,z_2)}, \tag{1.4b}$$

then the transfer function for System I becomes

$$H_l(z_l,z_2)=\frac{N_d(z_l,z_2)}{D_d(z_l,z_2)+K\cdot N_d(z_l,z_2)} \tag{1.4c}$$

For this system, one can get variable frequency response by changing the value of the multiplier K, and this should be bounded by the stable conditions for the system.

In Figure 1.2, the following relationships can be obtained

$$Y(z_1,z_2)=U(z_1,z_2) \tag{1.5a}$$

$$V(z_1,z_2)=U(z_1,z_2)\cdot H_d(z_1,z_2) \tag{1.5b}$$

$$Y(z_1,z_2)=K\cdot[X(z_1,z_2)-V(z_1,z_2)] \tag{1.5c}$$

So the transfer function for the System II in Figure 1.2 is

$$H_2(z_1,z_2)=\frac{Y(z_1,z_2)}{X(z_1,z_2)}=\frac{K}{1+K\cdot H_d(z_1,z_2)} \tag{1.6}$$

Also assuming the generating filter $H_d(z_1,z_2)$ is given by (1.4b), the transfer function for System II can be written as

$$H_2(z_1,z_2)=\frac{K\cdot D_d(z_1,z_2)}{D_d(z_1,z_2)+K\cdot N_d(z_1,z_2)} \tag{1.7}$$

From this transfer function also, we can get variable characteristics for the system by changing the value of the multiplier *K*. The values of the *K* should be bounded by the stability conditions of the system.

However, for the systems in Figure 1.1 and 1.2, the drawbacks are obvious. Firstly, as the range of *K* is limited, sometimes it is impossible to get the desired characteristics by only adjusting one parameter. Secondly, the determination of the stable boundaries of the multiplier *K* becomes very complex as the degrees of the variables in $H_d(z_1, z_2)$ increase. Hence, it may be preferable to have a system in which several sections are cascaded, each section having unity degree in each variable [21]. Thirdly, the structures in Figure 1.1 and 1.2 can easily lead to delay-free loops if the coefficients are chosen improperly.

So, we need to investigate new methods, which can give 2-D digital filters having variable characteristics with more freedom.

One of the frequently used methods of designing a 2-D recursive digital filter is to start from a corresponding 2-D analog filter and then apply the well-known bilinear transformations [22]

$$s_i\rightarrow\frac{z_i-1}{z_i+1},\quad i=1,2 \tag{1.8}$$

This method could give rise to problems in stability as it has been demonstrated in

[23]. However, this can be overcome by a special class of 2-variable analog polynomials called Very Strict Hurwitz Polynomial (VSHP). In what follows, a brief review of such polynomials, and their properties and some methods of generation are discussed [22].

1.2 Overview of Very Strict Hurwitz Polynomial

1.2.1 Definition of Very Strict Hurwitz Polynomial

It is well known that a 1-D analog filter system with the transfer function

$$H_a(s) = \frac{N_a(s)}{D_a(s)} \tag{1.9}$$

is guaranteed to be stable, if the denominator of the transfer function $D_a(s)$ is a Strictly Hurwitz Polynomial (SHP), which contains all its zeros strictly in the left-half of s-plane.

However, for the 2-D analog filter system with the transfer function

$$H_a(s_1, s_2) = \frac{N_a(s_1, s_2)}{D_a(s_1, s_2)}, \tag{1.10}$$

where the denominator $D_a(s_1, s_2)$ is SHP, cannot always guarantee the stability, as it may contain the non-essential singularity of the second kind. That is, the denominator becomes zero at two points $s_1 = j\omega_{10}$ and $s_2 = j\omega_{20}$, but not in their neighborhoods [22, 23].

In fact, there are four types of 2-variable Hurwitz Polynomials, which are different from each other only in the region of analyticity [24].

Definition 1.1 A Polynomial $D_a(s_1, s_2)$ is said to be a Broad Sense Hurwitz Polynomial (BHP), if $\frac{1}{D_a(s_1, s_2)}$ has no singularities in the region

$$\{(s_1, s_2) \mid \mathrm{Re}(s_1) > 0, \mathrm{Re}(s_2) > 0, |s_1| < \infty, |s_2| < \infty\}.$$

Definition 1.2 A Polynomial $D_a(s_1, s_2)$ is said to be a Narrow Sense Hurwitz Polynomial (NHP), if $\frac{1}{D_a(s_1,s_2)}$ has no singularities in the region

$$\{(s_1,s_2) \mid \text{Re}(s_1)>0, \text{Re}(s_2)>0, |s_1|<\infty, |s_2|<\infty\}$$

$$\cup \{(s_1,s_2) \mid \text{Re}(s_1)=0, \text{Re}(s_2)>0, |s_1|\le\infty, |s_2|<\infty\}$$

$$\cup \{(s_1,s_2) \mid \text{Re}(s_1)>0, \text{Re}(s_2)=0, |s_1|<\infty, |s_2|\le\infty\}$$

Definition 1.3 A Polynomial $D_a(s_1, s_2)$ is a Strictly Hurwitz Polynomial (SHP), if it has no zeros in the regions

$$\{(s_1,s_2) \mid \text{Re}(s_1)\ge 0, \text{Re}(s_2)\ge 0, |s_1|<\infty, |s_2|<\infty\}.$$

Definition 1.4 A polynomial $D_a(s_1, s_2)$ is a Very Strictly Hurwitz Polynomial (VSHP), if the polynomial does not have zeros in the regions

$$\{(s_1,s_2) \mid \text{Re}(s_1)\ge 0, \text{Re}(s_2)\ge 0, |s_1|\le\infty, |s_2|\le\infty\}.$$

From these definitions, we can see that a VSHP is required to be necessarily a SHP. From the two-dimensional digital filter design experience, to get a guaranteed stable digital filter from the well-known bilinear transformations described in (1.8), the 2-D analog transfer function is required to have a 2-variable VSHP as its denominator.

1.2.2 Some Properties of VSHP [22, 24]

Our discussion for the properties of VSHP is based on the following definition of 2-D analog transfer function

$$H(s_1,s_2)=\frac{N(s_1,s_2)}{D(s_1,s_2)} \tag{1.11}$$

where

$$N(s_1,s_2)=\sum_{i=0}^{m}\sum_{j=0}^{n}B_{ij}s_1^is_2^j \tag{1.11a}$$

$$D(s_1,s_2)=\sum_{i=0}^{k}\sum_{j=0}^{l}A_{ij}s_1^is_2^j \tag{1.11b}$$

Property 1.1 $H(s_1,s_2)$ does not possess singularities in the closed right half of the (s_1,s_2) biplane defined as $\{(s_1,s_2)\,|\,\mathrm{Re}(s_1)\ge 0,\mathrm{Re}(s_2)\ge 0,|s_1|\le\infty,|s_2|\le\infty\}$, if and only if $D(s_1,s_2)$ is a VSHP.

This property is obtained easily from the definition of VSHP.

Property 1.2 If $D(s_1,s_2)=D_1(s_1,s_2)\cdot D_2(s_1,s_2)$ is a VSHP, the necessary and sufficient condition is both $D_1(s_1,s_2)$ and $D_2(s_1,s_2)$ are VSHPs.

Proof:

Sufficient condition:

As $D_1(s_1,s_2)$ and $D_2(s_1,s_2)$ are VSHPs, $D_1(s_1,s_2)$ and $D_2(s_1,s_2)$ have no singular points in the closed half plane of (s_1,s_2) -plane $\{(s_1,s_2)\,|\,\mathrm{Re}(s_1)\ge 0,\mathrm{Re}(s_2)\ge 0,|s_1|\le\infty,|s_2|\le\infty\}$. Also $D(s_1,s_2)$ is the product of $D_1(s_1,s_2)$ and $D_2(s_1,s_2)$, so $D(s_1,s_2)$ would not have any singular points in the closed right half plane of (s_1,s_2)-plane, i.e. $D(s_1,s_2)$ is a VSHP.

Necessary condition:

$D(s_1,s_2)$ is a VSHP.

Suppose that $D_1(s_1,s_2)$ is not a VSHP, then there exists some points in the closed right half plane of the (s_1,s_2)-plane $\{(s_1,s_2)\,|\,\mathrm{Re}(s_1)\geq 0,\mathrm{Re}(s_2)\geq 0,|s_1|\leq\infty,|s_2|\leq\infty\}$, such that $D_1(s_1,s_2)$ equals to zero. So $D(s_1,s_2)$ will be zero at those points too, it will contradict our assumption that $D(s_1,s_2)$ is a VSHP, so $D_1(s_1,s_2)$ must be a VSHP.

Similarly, we can prove that $D_2(s_1,s_2)$ must be a VSHP.

__Property 1.3__ If $D(s_1,s_2)$ is a VSHP, then $\frac{\partial D(s_1,s_2)}{\partial s_1}$ and $\frac{\partial D(s_1,s_2)}{\partial s_2}$ are also VSHPs.

Proof:

We can write $D(s_1,s_2)$ in the following form

$$D(s_1,s_2)=A_m(s_2)s_1^m+A_{m-1}(s_2)s_1^{m-1}+\ldots+A_1(s_2)s_1+A_0(s_2) \tag{1.12a}$$

and

$$D(s_1,s_2)=B_n(s_1)s_2^n+B_{n-1}(s_1)s_2^{n-1}+\cdots+B_1(s_1)s_2+B_0(s_1) \tag{1.12b}$$

For any point $\bar{s}_2$ in the open half of s_2-plane $\{s_2\,|\,\mathrm{Re}(s_2)\geq 0,|s_2|<\infty\}$, (1.12a) is an m^{th} order SHP with respect to s_1 including $|s_1|\leq\infty$. According to Lucas' theorem, $\frac{dD(s_1,\bar{s}_2)}{ds_1}$ is also a (m-1)th order SHP of s_1 including $|s_1|\leq\infty$. Now we need to check the behavior at $|s_2|=\infty$.

Differentiating (1.12b) with respect to s_1, we have

$$\frac{\partial D(s_1,s_2)}{\partial s_1}=B_n'(s_1)s_2^n+B_{n-1}'(s_1)s_2^{n-1}+\cdots+B_1'(s_1)s_2+B_0'(s_1) \tag{1.13}$$

Since (1.13) is known to be a SHP,

$$\left.\frac{\partial D(s_1,s_2)}{\partial s_1}\right|_{s_2\to\frac{1}{s_2}}$$ is also a SHP.

For $s_2 \to \infty$, we can get

$$\frac{\partial D(s_1,s_2)}{\partial s_1}\bigg|_{s_2\to\infty} = \frac{B_n'(s_1)}{0} \tag{1.14}$$

As $B_n(s_1)$ is a SHP of s_1, it has no zeros in the closed right-half of $s_1 - plane$ $\{s_1 \mid \mathrm{Re}(s_1) \geq 0, |s_1| \leq \infty\}$. So (1.14) will not be undeterminable. We can get the conclusion that $\frac{\partial D(s_1,s_2)}{\partial s_1}$ is VSHP.

Similarly, we can prove that $\frac{\partial D(s_1,s_2)}{\partial s_2}$ is also VSHP.

Property 1.4 When we express $D(s_1,s_2)$ as (1.12a) and (1.12b), then both $A_i(s_2), i = 0,1,2,\cdots m$ and $B_j(s_1), j = 0,1,2,\cdots,m$ are SHPs of s_2- and s_1-respectively.

Proof:

In (1.12a), since $D(s_1, s_2)$ is a VSHP, from Property 1.3, $A_0(s_2)$ is a SHP in s_2, which is obtained by setting $s_1 = 0$. Differentiating (1.12a) partially with respect to s_1 and putting $s_1 = 0$, we get $A_1(s_2)$ is a SHP in s_2, since, from Property 1.3, we know that $\frac{\partial D(s_1,s_2)}{\partial s_1}$ is a VSHP. By continuing this process, it is established that all the polynomials $A_1(s_2)$, $i = 0,1,2,\cdots\cdots,m$ are SHPs in s_2.

Using the same method, by successive differentiation with respect to s_2, and then putting $s_2 = 0$, it can be proven that all the polynomials $B_j(s_1)$, $j = 0,1,2,\cdots\cdots,n$ are SHPs in s_1.

Property 1.5 If a real 2-variable VSHP can be written as $D(s_1,s_2)=\sum_{i=0}^{m}\sum_{j=0}^{n}d_{ij}s_1^is_2^j$, then $d_{mn}d_{ij}>0$ for all i and j.

Proof:

The VSHP can be written in the compact form

$$D(s_1,s_2)=\sum_{i=0}^{m}A_i(s_2)s_1^i \tag{1.15a}$$

and
$$D(s_1,s_2)=\sum_{j=0}^{n}B_j(s_1)s_2^j \tag{1.15b}$$

From Property 1.4, both $A_i(s_1)$,$(i=0,1,\cdots\cdots,m)$ and $B_j(s_2)$, $(j=0,1,\cdots\cdots,n)$ are one-variable SHP's.

For the one-variable SHP $A_m(s_2)=\sum_{j=0}^{n}d_{mj}s_2^j$, the coefficients need to be positive or

$$d_{mn}d_{mj}>0\text{, for all } j=0,1,2,\cdots\cdots,n-1$$

And for the one-variable SHP $B_j(s_1)=\sum_{i=0}^{m}d_{ij}s_1^i$, the coefficients need to be positive or $d_{mj}d_{ij}>0$

So, $d_{mn}d_{ij}>0$, for all i and j.

Property 1.5 is proved.

Property 1.6 If we express $D(s_1,s_2)$ as (1.12a) and (1.12b), each of the functions $\frac{A_i(s_2)}{A_{i-1}(s_2)}$, $i=1,2,\cdots\cdots,m$, is a minimum reactive positive real function in s_2. Also each of the functions $\frac{B_j(s_1)}{B_{j-1}(s_1)}$, $j=1,2,\cdots\cdots,n$, is a minimum reactive positive real functions in s_1.

Proof:

Dividing both sides of (1.12a) by $A_m(s_2)$, we have

$$\frac{1}{A_m(s_2)}D(s_1,s_2) = s_1^m + \frac{A_{m-1}(s_2)}{A_m(s_2)}s_1^{m-1} + \cdots + \frac{A_1(s_2)}{A_m(s_2)}s_1 + \frac{A_0(s_2)}{A_m(s_2)} \qquad (1.16)$$

For any specified s_2 in the range of $Re(s_2) \geq 0$, (1.16) is a SHP in s_1, which means that it has all its zeros in the open left half of the s_1 plane. Let δ_i ($i = 1,2,3,\cdots\cdots,m$) be its zeros. Then we have

$$\frac{A_0(s_2)}{A_m(s_2)} = \prod_{i=1}^{m}(-\delta_i)$$

and $$\frac{A_1(s_2)}{A_m(s_2)} = \sum_{i=1}^{m}\left(\prod_{\substack{j=1\\ j\neq i}}^{m}(-\delta_j)\right).$$

So, we have

$$\frac{A_1(s_2)}{A_0(s_2)} = \sum_{i=1}^{m}(-\frac{1}{\delta_i})$$

As $\mathrm{Re}(\delta_i) < 0$, $\mathrm{Re}\left(\frac{A_1(s_2)}{A_0(s_2)}\right) > 0$ for all $\mathrm{Re}(s_2) \geq 0$. So we get the conclusion that $\frac{A_0(s_2)}{A_1(s_2)}$ is strict positive real function. In addition, from Property 1.5, $A_0(s_2)$ and $A_1(s_2)$ are SHPs without any missing coefficients. So $\frac{A_0(s_2)}{A_1(s_2)}$ is also minimum reactive function. In a similar manner, we can show that $\frac{A_i(s_2)}{A_{i-1}(s_2)}$ ($i = 1,2,\cdots\cdots,m$) are minimum reactive positive real functions in s_2.

Using the similar procedure, we can prove that $\frac{B_j(s_1)}{B_{j-1}(s_1)}$ ($j = 1,2,\cdots\cdots,n$) are minimum reactive positive real functions in s_1.

1.3 Review of Generation of VSHP

When VSHP is used in the denominator of a 2-D analog transfer function, it is guaranteed that the resulting 2-D digital bilinear transfer function obtained through the application of the well-known bilinear transformation is stable. Therefore, VSHP is highly useful in 2-D digital filter design. We can first generate a 2-variable Very Strictly Hurwitz Polynomial (VSHP) using its various properties, and assign the generated VSHP to the denominator of the 2-D analog transfer function, then obtain the digital transfer function through double bilinear transformations. Here we review some methods, which are used to generate VSHP.

1.3.1 Using Terminated n-Port Gyrator Networks [25]

For a n-port gyrator network, its ports are terminated by capacitances. In such a case, the overall admittance matrix will be

$$A = \begin{bmatrix} \mu_1 & g_{12} & g_{13} & \cdots & g_{1n} \\ -g_{12} & \mu_2 & g_{23} & \cdots & g_{2n} \\ -g_{13} & -g_{23} & \mu_3 & \cdots & g_{3n} \\ \vdots & & & \ddots & \\ -g_{1n} & -g_{2n} & -g_{3n} & \cdots & \mu_n \end{bmatrix} \tag{1.17}$$

The determinant of the matrix A can be expressed as

$$D_n = \sum_{1\le i\le n} \mu_i |A_i| + \sum_{1\le i_1<i_2<i_3<n} \mu_{i_1}\mu_{i_2}\mu_{i_3} |A_{i_1 i_2 i_3}| + \cdots + \mu_1\mu_2\mu_3\cdots\mu_n \quad (n \text{ is odd}) \tag{1.18a}$$

$$\text{or } D_n = |A_n| + \sum_{1\le i_1<i_2<n} \mu_{i_1}\mu_{i_2} |A_{i_1 i_2}| + \sum_{1\le i_1<i_2<i_3<i_4<n} \mu_{i_1}\mu_{i_2}\mu_{i_3}\mu_{i_4} |A_{i_1 i_2 i_3 i_4}| + \cdots$$

$$+\mu_1\mu_2\mu_3\cdots\mu_n \quad (n \text{ is even}) \qquad (1.18b)$$

where $|A_{i_1 i_2}|$ is the determinant of the sub-matrix of A obtained by deleting both i_1^{th} and i_2^{th} rows and columns; the same holds for $|A_{i_1 i_2 i_3}|$, $|A_{i_1 i_2 i_3 i_4}|$, etc.

By making some of the μ_i's equal to s_1, and some of μ_i's equal to s_2, under certain conditions, (1.18a) and (1.18b) will yield two-variable VSHPs.

1.3.2 Using the Properties of Positive Semi-definite Matrices [26]

In this case, we first define three $n \times n$ square matrices A, μ, and G as

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{12} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{1n} & a_{2n} & \cdots & a_{nn} \end{bmatrix} \qquad (1.19)$$

$$\mu = \begin{bmatrix} \mu_1 & & & & 0 \\ & \mu_2 & & & \\ & & \mu_3 & & \\ & & & \ddots & \\ 0 & & & & \mu_n \end{bmatrix} \qquad (1.20)$$

$$G = \begin{bmatrix} 0 & g_{12} & g_{13} & \cdots & g_{1n} \\ -g_{12} & 0 & g_{23} & \cdots & g_{2n} \\ -g_{13} & -g_{23} & 0 & \cdots & g_{3n} \\ \vdots & & & \ddots & \\ -g_{1n} & -g_{2n} & -g_{3n} & \cdots & 0 \end{bmatrix} \qquad (1.21)$$

where: A is a general symmetrical $n \times n$ square matrix

μ is an $n \times n$ diagonal matrix

G is an $n \times n$ skew-symmetric matrix

These matrices A, μ and G are physically realizable.

Now, we define a matrix C as:

$$C = A\mu A^T + G, \tag{1.22}$$

whose determinant is written as

$$M = \det(C) \tag{1.23}$$

Then, the polynomial M_n that is defined as

$$M_n = M + \sum_{j=1}^{n} k_j \frac{\partial M}{\partial \mu_i}, \tag{1.24}$$

is a two-variable VSHP when some of the μ_i's are properly made equal to s_1 and some of the μ_i's equal to s_2.

1.3.3 Using the Properties of the Derivative of Even or Odd Parts of Hurwitz Polynomial [27, 28]

Form (1.19) and (1.20), one can obtain an nth order polynomial M_n as

$$M_n = \det[\mu I + A] \tag{1.25}$$

From the diagonal expansion of the determinant of matrix, M_n can be written as (1.18a) and (1.18b). We can observe that M_n is the odd (even) part of a n-variable Hurwitz polynomial when n is odd (even), So $\frac{\partial M_n / \partial \mu_i}{M_n}$ is a reactance function. Therefore,

$$M = M_n + \sum_{j=1}^{n} K_j \left(\frac{\partial M_n}{\partial \mu_i}\right) \tag{1.26}$$

is a n-variable Hurwitz Polynomial.

Assigning some of μ_i's to s_1 and some to be s_2, and ensuring the conditions of two-variable VSHP, a two-variable VSHP could be generated from (1.26).

1.4 Review of symmetries of 2-D contours [8, 9, 10]

Two and high dimensional systems possess different types of symmetries. For analog s-domain systems and discrete z-domain systems, extensive studies have been made regarding the various symmetries in their respective two-dimensional polynomials and functions. The symmetry proprieties of 2-D contour are very important in 2-D digital filter design, as it can largely reduce the complexity and hence the computation efforts.

Mathematically, for a function $f(x,y)$ (where x and y being real coefficients) would have a unique value at each specified pair of values of x and y if the function is defined at the region. These may be represented by a three dimensional object having (x, y) plane as the base and the value of $f(x,y)$ at each point in the plane as the height.

Most of the applications require that 2-D digital filter shall have certain symmetry in their magnitude response. Therefore, they usually have some symmetric properties in their two-dimensional contour plots.

1.4.1 Reflection Symmetry about ω_1 or ω_2 axis

For the 2-D contour to have reflection symmetry about ω_1 or ω_2 axis, the two-variable transfer function $H(\omega_1, \omega_2)$ should satisfy the following conditions:

Reflection symmetry about ω_1 axis:

$$H(\omega_1,\omega_2) = H(\omega_1,-\omega_2) \qquad \forall(\omega_1,\omega_2) \tag{1.27}$$

Reflection symmetry about ω_2 axis:

$$H(\omega_1,\omega_2) = H(-\omega_1,\omega_2) \qquad \forall(\omega_1,\omega_2) \tag{1.28}$$

1.4.2 Reflection Symmetry about the Diagonal $\omega_1 = \omega_2$

In 2-D contour plot, in order that the contour plot has reflection symmetry about the diagonal $\omega_1=\omega_2$, the two variables transfer function $H(\omega_1, \omega_2)$ has to meet the following condition:

$$H(\omega_1,\omega_2) = H(\omega_2,\omega_1) \qquad \forall(\omega_1,\omega_2) \tag{1.29}$$

For the same reason, for the 2-D contour to have reflection symmetry about $\omega_1=-\omega_2$ diagonal, the transfer function shall have the relation

$$H(\omega_1,\omega_2) = H(-\omega_2,-\omega_1) \qquad \forall(\omega_1,\omega_2) \tag{1.30}$$

1.4.3 Centro Symmetry

For two-variable functions, twofold rotational symmetry (rotation by π radians) is called centro-symmetry. For the 2-D contour to have centro-symmetry, the two variables transfer function satisfies the following condition:

$$H(\omega_1,\omega_2) = H(-\omega_1,-\omega_2) \qquad \forall(\omega_1,\omega_2) \tag{1.31}$$

1.4.4 Quadrantal Symmetry

In 2-D digital filter systems, for the contour to have quadrantal symmetry, the system transfer function should meet the following condition:

$$H(\omega_1,\omega_2) = H(-\omega_1,\omega_2) = H(\omega_1,-\omega_2) = H(-\omega_1,-\omega_2) \qquad \forall(\omega_1,\omega_2) \tag{1.32}$$

For the system which possesses quadrantal symmetry, it is only necessary to consider the magnitude response in the first quadrant, because we can easily get the responses in the second, the third and fourth quadrants in (ω_1, ω_2)-plane using the symmetry property.

1.4.5 Diagonal Symmetry

In 2-D digital filter system, if the contour has a reflection symmetry with respect to the line of $\omega_1=\omega_2$ and $\omega_1=-\omega_2$ simultaneously, it is said the contour possesses Diagonal Symmetry (also called fourfold reflection symmetry). For the contour plot to possess

Diagonal Symmetry, the system transfer function has to meet the following condition:

$$H(\omega_1,\omega_2)=H(\omega_2,\omega_1)=H(-\omega_1,-\omega_2)=H(-\omega_2,\omega_1) \qquad \forall(\omega_1,\omega_2) \qquad (1.33)$$

1.4.6 Four-Fold Rotational Symmetry

The magnitude response of a 2-D filter system has the property of the four-fold rotational symmetry (Rotation 90°), the transfer function of the system satisfies the following relationships:

$$H(\omega_1,\omega_2)=H(-\omega_2,\omega_1)=H(-\omega_1,-\omega_2)=H(\omega_2,-\omega_1) \qquad \forall(\omega_1,\omega_2) \qquad (1.34)$$

1.4.7 Octagonal Symmetry

When the two-dimensional contour of the filter possesses quadrantal symmetry and diagonal symmetry simultaneously, we say the 2-D system has Octagonal Symmetry, The condition for Octagonal Symmetry for the two-dimensional filter transfer function is:

$$\begin{aligned}H(\omega_1,\omega_2)&=H(\omega_2,\omega_1)=H(-\omega_2,\omega_1)\\&=H(-\omega_1,\omega_2)=H(-\omega_1,-\omega_2)\\&=H(-\omega_1,\omega_2)=H(-\omega_1,-\omega_2)\\&=H(-\omega_2,-\omega_1)=H(\omega_2,-\omega_1)\\&=H(\omega_1.-\omega_2), \qquad \forall(\omega_1,\omega_2) \qquad (1.35)\end{aligned}$$

1.4.8 Circular Symmetry

If the 2-D contour satisfies the general equation of circle, we say the filter possesses circular symmetry. The condition of the circular symmetry is:

$$H(\omega_1,\omega_2)=\begin{cases} C_1, & \omega_1^2+\omega_2^2=R_1^2 \\ \vdots & \vdots \\ C_N, & \omega_1^2+\omega_2^2=R_N^2 \end{cases} \tag{1.36}$$

where C_i, $i = 1, 2, \ldots\ldots, n$ are constants, and constitute different magnitudes.

1.5 Review of generalized transformations

As has been mentioned earlier, one of familiar approaches to design a digital filter is to start from an analog prototype, get the transfer function in analog domain, then apply the bilinear transformation, which has one-to-one mapping relationship between analog and discrete domains. Thus, starting from the analog transfer function, one can get the discrete transfer function for the digital filter.

In general, we can use the well-known double bilinear transformation given in (1.8) to build the mapping relationship for 2-D filter systems. However, if one needs to design the 2-D digital filter having different characteristics, it is necessary to introduce changeable coefficients in the double bilinear transformations. A new generalized bilinear transformation is needed here.

1.5.1 Definition for Generalized Bilinear Transformation [29, 30]

To get variable characteristics in digital filters, we would like to have one or more changeable coefficients in the discrete transfer function. One of the methods is to change the coefficients in the bilinear transformations to get different mappings.

A generalized bilinear transformation of this type can be defined as

$$s_i \to k_i \frac{z_i+\alpha_{0i}}{z_i+\beta_{0i}}, \qquad i=1,2 \tag{1.37}$$

which can be applied to the analog prototype filter to get the discrete transfer function for the digital filters. It is important that the stability of the resulting discrete transfer function be ensured.

1.5.2 Stability Conditions for Generalized Bilinear Transformation [29]

We have to get first the stability conditions for the bilinear transformation (1.37), which we will employ. Here we first consider the first dimension only.

Theorem 1.1: When $k_1 > 0$, the condition for stability for the generalized bilinear transformation applied to an analog transfer function are:

$$(i) \quad |\alpha_{01}| \le 1 \tag{1.38a}$$

$$(ii) \quad |\beta_{01}| \le 1 \tag{1.38b}$$

$$(iii) \quad \alpha_{01} \cdot \beta_{01} < 0 \tag{1.38c}$$

Proof:

Letting: $s_1 = \sigma_1 + j\omega_1$ (1.39a)

$$z_1 = u_1 + jv_1 \tag{1.39b}$$

and substituting (1.39) into the generalized bilinear transformation (1.37), we can get:

$$\sigma_1 = k_1 \frac{(u_1^2 + v_1^2) + (\alpha_{01} + \beta_{01})u_1 + \alpha_{01}\beta_{01}}{(u_1 + \beta_{01})^2 + v_1^2} \tag{1.40a}$$

$$\omega_1 = k_1 \frac{v_1(\beta_{01} - \alpha_{01})}{(u_1 + \beta_{01})^2 + v_1^2} \tag{1.40b}$$

For the purpose of stability, it is required that the imaginary axis of s_1-plane or $\sigma_1=0$ be mapped to the inner or on the unity circle in the discrete z_1-dimension,

$$r_1^2 = u_1^2 + v_1^2 \le 1 \tag{1.41}$$

Letting: $u_1 = r_1 \cos\phi$ (1.42a)

$$v_1 = r_1 \sin\phi \tag{1.42b}$$

and substituting them into the equation (1.40a), for σ_1=0, we have

$$r_1^2 + (\alpha_{01} + \beta_{01})r_1 \cos\phi + \alpha_{01}\beta_{01} = 0 \tag{1.43}$$

The roots of equation (1.43) are given by

$$r_{11,12} = \frac{-(\alpha_{01} + \beta_{01})\cos\phi \pm \sqrt{(\alpha_{01} + \beta_{01})^2 \cos^2\phi - 4\alpha_{01}\beta_{01}}}{2} \tag{1.44}$$

The magnitude of the roots should not be greater than unity. The roots have their maximum values at $\phi = \pm\pi$, giving

$$r_{11} = \pm\alpha_{01} \tag{1.45a}$$

$$r_{12} = \pm\beta_{01} \tag{1.45b}$$

Thus, it is proved that

$$|\alpha_{01}| \le 1 \tag{1.46a}$$

$$|\beta_{01}| \le 1 \tag{1.46b}$$

Also for the stability purpose, the unity circle in the discrete domain should be mapped to the closed left of s_1-plane. That requires $\sigma_1 \le 0$ for $r_1 = 0$, hence from (1.40a), we can get

$$\alpha_{01}\beta_{01} \le 0 \qquad \text{for } k_1 > 0 \tag{1.46c}$$

$$\text{or} \quad \alpha_{01}\beta_{01} \ge 0 \qquad \text{for } k_1 < 0 \tag{1.46d}$$

Without loss of generality, we can assume k_1 to be positive, then α_{01} and β_{01} should be of opposite signs. Therefore, Theorem 1.1 is proved.

The results obtained here can be extended to the second dimension. The theorem can be applied in 1-D and 2-D cases.

1.5.3 The mapping relationship [29]

For the imaginary axis in s_1-plane or $\sigma_1 = 0$, from (1.40a), we have

$$u_1^2 + v_1^2 + (\alpha_{01} + \beta_{01})u_1 + \alpha_{01} \cdot \beta_{01} = 0 \tag{1.47}$$

Re-write (1.47) as

$$(u_1 + \frac{\alpha_{01} + \beta_{01}}{2})^2 + v_1^2 = (\frac{\alpha_{01} - \beta_{01}}{2})^2 \tag{1.48}$$

Equation (1.48) represents a circle centered at $(-\frac{\alpha_{01} + \beta_{01}}{2}, 0)$ with radius $R = \left|\frac{\alpha_{01}}{\beta_{01}}\right|$.

That is to say, the imaginary axis in s_1-plane is mapped to the circle in the z_1-plane, and the left-half plane of s_1-plane is mapped to the inner of the circle, and the right-half plane of s_1-plane is mapped to the outside of the circle.

The second dimension has the same mapping relationship.

1.6 Scope and Organization of The Thesis

The objective of the thesis is to develop a new approach for the design for a 2-D digital filter, which has variable magnitude characteristics in the frequency domain. This approach is based on the generalized bilinear transformation method. To get the digital filter whose characteristics are changeable, one or more of the coefficients of the digital transfer function should be variable. The generalized double bilinear transformation is one of the methods that can introduce variable coefficients into the transfer function of the resulting digital filters.

In Chapter 2, design methods for 2-D nonrecursive and recursive digital filter are first introduced. Then from the second order Butterworth low-pass ladder structure, the values of the inductor and the capacitor are calculated. Using these values, we build a first order doubly-unity-terminated low-pass ladder by setting the inductor as s_1-variable and the capacitor as the s_2-variable. The analog transfer function of the new circuit is obtained. Then the double generalized bilinear transformation is applied to the analog transfer function to get its digital counterpart. Also in this chapter, the stability condition for the 2-D digital transfer function with denominator of unity degree of z_1 and z_2 are considered. Using the links between the stability conditions and the coefficients of the double generalized bilinear transformations, we get the conditions for each of these coefficients, while the other coefficients are set to specified values. The effect of each coefficient on the resulting 2-D low pass digital filter's magnitude response is studied in detail.

The 2-D high-pass digital filters having variable magnitude characteristics are studied in Chapter 3. Starting from the same analog transfer function as in Chapter 2, different sets of coefficients in the double generalized bilinear transformation are chosen to obtain 2-D high-pass digital filter. The coefficients are constrained by the stability condition for digital filters introduced in Chapter 2. The manner in which coefficients of the double generalized bilinear transformation affects the digital filters' magnitude characteristics is also studied in detail.

In Chapter 4, the 2-D band-pass filter, which is formed by cascading a 2-D low-pass filter and a 2-D high-pass filter, is investigated. The low-pass and high-pass filter can be the ones we investigated in Chapters 2 and 3 respectively. When one or more coefficients of the bilinear transformations are changing, the resulting 2-D low-pass filter or high-pass filter has variable magnitude response, so does the resulting 2-D band-pass filter. Of course, all the coefficients should be constrained by the stability conditions. The relationship of the stability between the resulting 2-D band-pass filter and its two member filters are obtained here. The manner in which each coefficient affects the frequency response of the resulting band-pass filter is investigated in detail, and the simulation results are given.

In Chapter 5, another filter, 2-D band-elimination filter, is investigated. By connecting a 2-D low-pass filter and a 2-D high-pass filter in parallel, it is possible to obtain a 2-D band-elimination filter. When the low-pass and high-pass filters are designed by the double generalized bilinear transformations, and when one or more coefficients are changing, the resulting 2-D low-pass and high-pass filters have variable magnitude characteristics, and hence does the resulting 2-D band-elimination filter. How the stability of the two member filters affect the stability of the resulting 2-D band-elimination filter is discussed in Chapter 5, along with other behaviors of the resulting band-elimination filter.

The summary, conclusions and the directions for future research work are given in Chapter 6.

Chapter 2

Two-Dimensional Low-Pass Filters

Depending on the frequency requirements to be met, in general, filters are classified into five main groups: low-pass, high-pass, band-pass, band-elimination, and all-pass filters. We first study the 2-D low-pass filters in this chapter. In section 2.1, we give a brief definition for this type of filters, as well as its typical specifications in both mathematical and graphical forms. In section 2.2, we briefly review the well-known 2-D nonrecursive filter design methods, such as the Windows method, the frequency transformation method, and the linear programming method. In section 2.3, we review the 2-D recursive digital filter design based on the double bilinear transformations method. In sections 2.4 and 2.5, starting from a second order Butterworth low-pass ladder network, we obtain the values for the inductor and the capacitor, and then form a 2-D first order doubly-unity-terminated network by setting the inductor in the s_1-dimension and the capacitor in the s_2-dimension. The analog transfer function for the resulting 2-D circuit is also obtained here. The digital transfer function for 2-D low-pass filter obtained from its analog counterpart through the double generalized bilinear transformation is obtained in section 2.6, as well as the limits of each coefficient. The stability conditions for the digital transfer function with unity degree denominator are got in section 2.7. The manner how each coefficient affects the resulting filter's magnitude response is studied in this section 2.8. The summary and discussion are given in section 2.9.

2.1 Introduction

Low-pass filters stop the signal components at high frequencies, but pass the components at low frequencies. The description for a typical 2-D low- pass digital filter can be expressed as [2, 4]

$$H(\omega_1,\omega_2)=\begin{cases}1, & |\omega_i|\le\omega_{ip}\\ 0, & \omega_{is}<|\omega_i|\le\pi\end{cases} \tag{2.1}$$

where:

ω_{ip} ($i=1,2$) are the pass-band boundaries in z_1 and z_2-dimensions, respectively

ω_{is} ($i=1,2$) are the stop-band boundaries in z_1 and z_2-dimensions, respectively

The frequency ranges between ω_{ip} and ω_{is} is transition band.

The specification of a typical 2-D low-pass filter is illustrated in Figure 2.1.

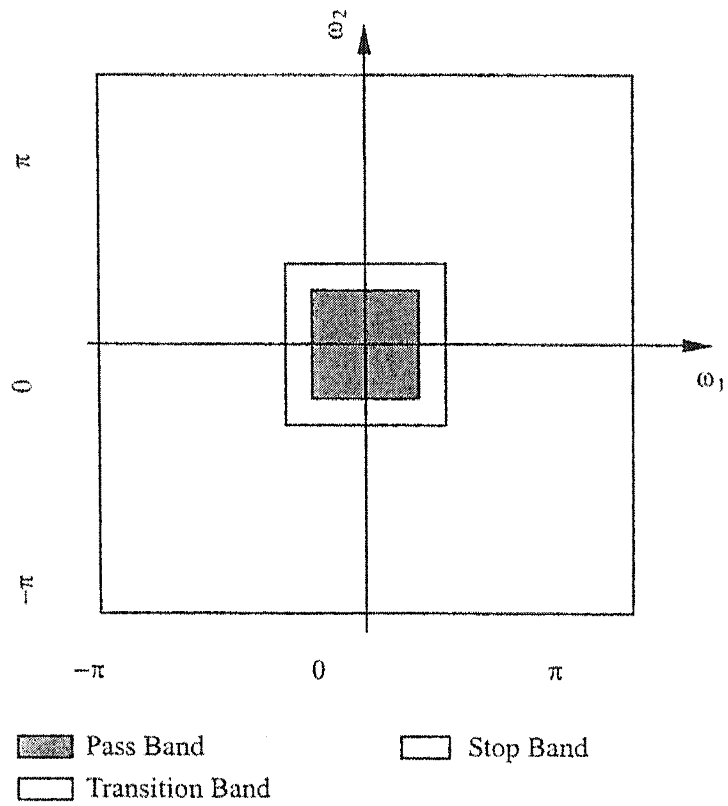


Figure 2.1 The specification for a typical 2-D low-pass filter

The methods used for 2-D nonrecursive filter design are different from those used for recursive digital filter design. We first give a brief review of some of the design methods for these two kinds of filters.

2.2 Design Methods for 2-D Nonrecursive Filters

Nonrecursive filters are also called Finite Impulse Response (FIR) Filters, which have their transfer function resulting from a finite-duration impulse response sequence. The output of a 2-D nonrecursive filter at point (m, n) can be computed as a linear combination of a finite number of input samples. The main properties of a 2-D nonrecursive filter are its inherent stability and linear phase feature [2, 4, 5].

There are several methods, such as Windows methods, frequency transformation, and linear programming, that can be used in 2-D nonrecursive filters design.

2.2.1 Windows Method [31]

The 2-D nonrecursive (FIR) filters could be designed using the extension of the 1-D technique. No essential modifications to the method need be made to accommodate the increase in dimensionality. In this method, an ideal impulse response is multiplied by a window function, which has finite support determined by the filter specifications. We can denote the ideal impulse response and the frequency response of the ideal filter as $i(m, n)$ and $I(\omega_1, \omega_2)$, respectively. The filter being designed would have impulses $h(m, n)$ and the frequency responses $H(\omega_1, \omega_2)$. Then, according to this method, the designed filter and the ideal filter have the following relationship

$$h(m,n) = i(m,n)w(m,n), \tag{2.2}$$

where $w(m, n)$ is the array of the window function, which has the same support region as the desired filter.

We can use 2-D convolution of the frequency response for the ideal filter $I(\omega_1, \omega_2)$ with the frequency transformation for the window function, $W(\omega_1, \omega_2)$, to get the 2-D digital filter's frequency response. Specifically,

$$H(\omega_1,\omega_2) = \frac{1}{(2\pi)^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} I(\alpha,\beta)W(\omega_1-\alpha,\omega_2-\beta)d\alpha d\beta \tag{2.3}$$

From the physical meaning of convolution, the desired filter frequency response is obtained just as the window frequency transformations W sweap the ideal frequency response.

There are many well-known 1-D window functions; it is very convenient to convert them to the 2-D case. There are two main ways to do this conversion depending on the region it supports. For circular support region, we can use the rotated formulation

$$w(m,n) = w(\sqrt{m^2 + n^2}) \tag{2.4}$$

or write in the form in the frequency domain

$$W(\omega_1, \omega_2) = W(\sqrt{\omega_1^2 + \omega_2^2}) \tag{2.5}$$

For a square or rectangular region, we can use the so-called Cartesian formulation to achieve the conversion

$$w(m,n) = w_1(m)w_2(n) \tag{2.6}$$

or in frequency domain:

$$W(\omega_1, \omega_2) = W_1(\omega_1)W_2(\omega_2), \tag{2.7}$$

where w_1 and w_2 are 1-D window functions, and W_1 and W_2 are their Fourier transformations.

2.2.2 Transformation Method [2, 4]

Another method to design 2-D nonrecursive filter is through the transformation of a 1-D design. This approach requires less computational work than window-method; it is especially useful in the design of high order 2-D zero phase nonrecursive filters.

For 1-D zero phase nonrecursive filter, the impulse response must be of odd length. The frequency response for the filter over the range of $[-N, N]$ can be written as

$$\begin{aligned} H(\omega) &= h(0) + \sum_{n=1}^{N} h(n)(e^{-j\omega n} + e^{j\omega n}) \\ &= \sum_{n=0}^{N} a(n)\cos(\omega n) \end{aligned} \tag{2.8}$$

where,

$$a(n) = \begin{cases} h(0), & n = 0 \\ 2h(n), & n > 0 \end{cases} \tag{2.9}$$

In the above equation, using the following transformation

$$\cos\omega = A\cos\omega_1 + B\cos\omega_2 + C\cos\omega_1\cos\omega_2 + D \tag{2.10}$$

where, A, B, C and D are free parameters, it is possible to get a 2-D transfer function as:

$$H(\omega_1,\omega_2) = \sum_{m=0}^{N}\sum_{n=0}^{N} a(m,n)\cos(m\omega_1)\cos(n\omega_2) \tag{2.11}$$

Equation (2.11) could be a transfer function for a 2-D nonrecursive digital filter.

2.2.3 Linear Programming Method [2, 4]

Another approach to design 2-D nonrecursive filter is using linear programming method. Some optimization techniques are employed in this method; the realizable 2-D filter is determined by comparing with the design specifications, and the one with minimum error is chosen.

This method requires heavy computational work compared with the window and transformation methods, and the result is not unique. But we can design a filter with any magnitude specifications.

2.3 Design Methods for 2-D Recursive Filters [2, 4, 6]

A recursive filter is one, which can be expressed in the form of a difference equation of the input and output samples with finite orders. Unlike the nonrecursive case, the design of 2-D recursive filter is more complex than the design of a 1-D one. The main reason is the stability consideration. Stability test gets more complex as the order of the filter increases.

To design a 2-D recursive digital filter expressed in the following equation

$$H(z_1, z_2) = \frac{\sum_{m=0}^{M_1}\sum_{n=0}^{M_2} B_{mn} z_1^m z_2^n}{\sum_{i=0}^{N_1}\sum_{j=0}^{N_2} A_{ij} z_1^i z_2^j} \tag{2.12}$$

where, $A_{00} = 1$, A_{ij} and B_{mn} are real coefficients.

The main work now is to choose the coefficients of A_{ij} and B_{mn} to approximate the frequency response of the desire one, and the coefficients should make the realizable filter stable.

A popular method to design a 2-D recursive digital filter is starting from an analog prototype filter, getting the analog transfer function, and then applying the double bilinear transformations to the analog transfer function to design the digital filter. If we assign a VSHP as the denominator of the analog transfer function, we can always obtain a stable digital transfer function, if the well-known bilinear transformation is used. However, when the double generalized bilinear transformations are applied to the analog transfer function with VSHP denominator, additional stability conditions need to be introduced to guarantee the stability of the resulting digital filter. Undoubtedly, the analog transfer function with a VSHP denominator is always necessary in the situations to obtain stable digital filters by the well-known bilinear transformation or generalized bilinear transformation.

2.4 Second Order Butterworth Low-Pass Ladder Network

Now we begin to design 2-D low-pass filter. First we need to choose the analog prototype. Here we consider a second order 1-D low-pass ladder network with an inductor in the series arm and a capacitor in the shunt arm, which is shown in Figure 2.2 [32].

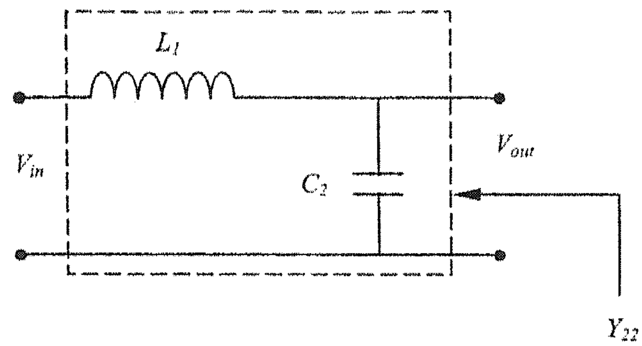


Figure 2.2 Second order low-pass ladder network

Form Figure 2.2, the short-circuit output admittance in a single-variable case is

$$Y_{22} = \frac{L_1C_2s^2 + 1}{C_2s} \tag{2.13}$$

If we need to make the ladder network as a filter with Butterworth response, the transfer function for the filter need to have the following relationship [32]

$$T(s)T(-s) = \frac{1}{1+(-1)^n s^{2n}} \tag{2.14}$$

The poles of (2.14) are the roots of

$$B_n(s)B_n(-s) = 1+(-1)^n s^{2n} = 0 \tag{2.15}$$

where $B_n(s)$ is said to be the Butterworth Polynomial.

The MATLAB® function *butterPolynomial.m* (All the M files can be found in the appendix) is used to calculate the coefficients for the nth order Butterworth Polynomials.

From *butterPolynomial.m*, we have the second Butterworh polynomial

$$B_2(s) = s^2 + 1.414s + 1 \tag{2.16}$$

so, the electronic elements, the inductor and the capacitor, in the ladder network have the following relationship

$$\begin{cases} L_1 C_2 = 1.0 \\ C_2 = 1.414 \end{cases} \tag{2.17}$$

Solving Equation (2.17), the values of the inductor and capacitor are:

$$\begin{cases} L_1 = 0.707 \\ C_2 = 1.414 \end{cases} \tag{2.18}$$

Now we can use the values to form the 2-D circuit network, and get the 2-D analog transfer function for our further investigation.

2.5 The 2-D Analog Transfer Function Obtained from Transformation of the 1-D Second Order Butterworth Prototype Filter

We can create a 2-D analog filter system by setting the inductor as the s_1-variable and the capacitor as variable s_2-variable, and specify them to be the values of the elements that we obtained in section 2.4 [33]. The resulting 2-D doubly-unity-terminated circuit is shown in Figure 2.3.

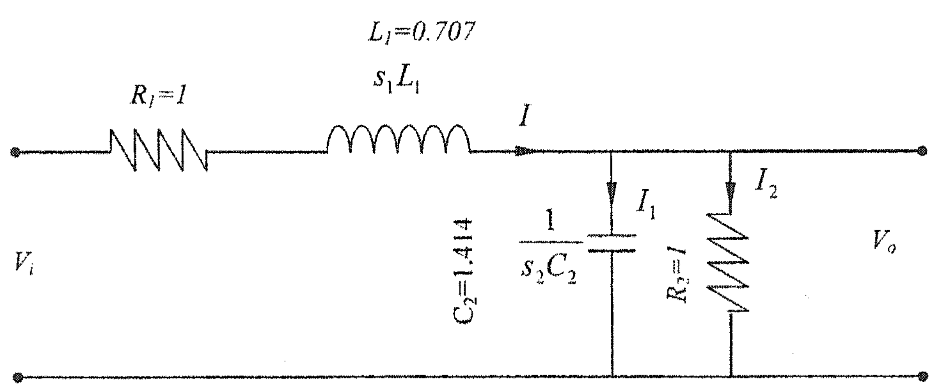


Figure 2.3 The 1st order 2-D doubly-terminated analog circuit with Butterworth elements

In Figure 2.3, the voltage and the currents are associated by the following equations

$$V_i = I(R_1 + s_1 L_1) + I_1 \frac{1}{s_2 C_2} \tag{2.19}$$

$$V_o = I_1 \frac{1}{s_2 C_2} = I_2 R_2 \tag{2.20}$$

$$I = I_1 + I_2 \tag{2.21}$$

As a result, we can obtain the voltage transfer function in continuous form for the 2-D circuit network in Figure 2.3 as

$$H_a(s_1, s_2) = \frac{V_o}{V_i} = \frac{1}{(1 + 0.707 s_1)(1.414 s_2 + 1) + 1}$$

$$= \frac{1}{s_1 s_2 + 0.707 s_1 + 1.414 s_2 + 2} \tag{2.22}$$

Obviously, the denominator

$$D_a(s_1, s_2) = s_1 s_2 + 0.707 s_1 + 1.414 s_2 + 2 . \tag{2.23}$$

is a VSHP of a single degree in each variable.

Till now, we have obtained the 2-D analog transfer function that represents a 2-D analog low-pass filter.

2.6 The Digital Transfer Function

2.6.1 The Low-Pass Limits for the Coefficients of the Generalized Bilinear Transformations

In the previous section, we have obtained the 2-D analog transfer function for the circuit in Figure 2.3. But under what conditions will it result in a 2-D low-pass digital filter by double generalized bilinear transformation? Here, we need to solve the problem first. As we mentioned before, when $k_i > 0$ (i=1, 2), it is possible to obtain stable digital transfer function when α_{0i} and β_{0i} (i=1, 2) are in the range of $|\alpha_{0i}| \leq 1.0$ and $|\beta_{0i}| \leq 1.0$. Here we investigate the situation in the first dimension..

For the n[th] order 1-D low-pass ladder network, which has n branches with one inductor in each series arm and one capacitor in each shunt arm, the voltage transfer function can be written as the following equation [32]

$$T(s) = \frac{K}{s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \cdots + a_1 s + a_0} \tag{2.24}$$

For $D(s) = \sum_{i=0}^{n} a_i s^i$ to be a VSHP, $a_i (i = 0,1,\cdots,n-1)$ should be nonnegative real coefficients, and $a_0 \neq 0$.

Applying the generalized bilinear transformation (1.37) of the first dimension to equation (2.24), we can get the digital transfer function as:

$$H(z_1) = \frac{(z_1 + \beta_{01})^n}{k_1^n (z_1 + \alpha_{01})^n + a_{n-1} k_1^{n-1} (z_1 + \alpha_{01})^{n-1} (z_1 + \beta_{01}) + \cdots + a_0 (z_1 + \beta_{01})^n} \tag{2.25}$$

The frequency response of (2.25) is

$$H(\omega_1) = \frac{(e^{-j\omega_1} + \beta_1)^n}{k_1^n (e^{-j\omega_1} + \alpha_{01})^n + a_{n-1} k_1^{n-1} (e^{-j\omega_1} + \alpha_{01})^{n-1} (e^{-j\omega_1} + \beta_{01}) + \cdots + a_0 (e^{-j\omega_1} + \beta_{01})^n} \tag{2.26}$$

The gains of low-pass filters at zero radians should be the highest, while the gains at π radians should be the lowest or zero.

In Equation (2.26), the magnitude responses have the mentioned properties as a low-pass filter, if and only if when β_{01} is positive or $0 \leq \beta_{01} \leq 1.0$. Also, as we mentioned before, to guarantee stable transformation, when $k_1 > 0$, α_{01} need to have opposite sign as β_{01}. That means that α_{01} should be negative or $-1.0 \leq \alpha_{01} \leq 0$.

The results are effective for the second dimension also.

So, to satisfy the conditions of obtaining 2-D low-pass digital filters, the coefficients of the double generalized bilinear transformations should meet the following requirements

$$(i)\ k_i > 0, \qquad i = 1,2 \tag{2.27a}$$

$$(ii)\ -1.0 \le \alpha_{0i} \le 0, \qquad i = 1,2 \tag{2.27b}$$

$$(iii)\ 0 \le \beta_{0i} \le 1.0, \qquad i = 1,2 \tag{2.27c}$$

With these constraints, we can obtain a 2-D low-pass digital filter from its analog counterpart by the application of the double generalized bilinear transformations.

2.6.2 2-D Low-Pass Digital Transfer Function from Double Generalized Bilinear Transformation

Applying double generalized bilinear transformation (1.37) with the low-pass coefficient constrains (2.27a – 2.27c) to the 2-D analog transfer function (2.22), the 2-D transfer function of a low-pass digital filter in discrete domain can be expressed as

$$H_d(z_1, z_2) = \frac{N_d(z_1, z_2)}{D_d(z_1, z_2)}, \tag{2.28}$$

where, $N_d(z_1, z_2) = z_1 z_2 + \beta_{02} z_1 + \beta_{01} z_2 + \beta_{01}\beta_{02}$ (2.29a)

$$\begin{aligned} D_d(z_1, z_2) = {} & (k_1 k_2 + 0.707k_1 + 1.414k_2 + 2)z_1 z_2 \\ & + (k_1 . k_2 \alpha_{02} + 0.707k_1\beta_{02} + 1.414k_2\alpha_{02} + 2\beta_{02})z_1 \\ & + (k_1 k_2 \alpha_{01} + 0.707k_1\alpha_{01} + 1.414k_2\beta_{01} + 2\beta_{01})z_2 \\ & + (k_1 k_2 \alpha_{01}\alpha_{02} + 0.707k_1\alpha_{01}\beta_{02} + 1.414k_2\alpha_{01}\beta_{01} \\ & + 2\beta_{01}\beta_{02} \end{aligned} \tag{2.29b}$$

We can express the denominator $D_d(z_1, z_2)$ in the general form of 2-variable polynomial with unity degree for each variable

$$D_d(z_1, z_2) = a_{11} z_1 z_2 + a_{10} z_1 + a_{01} z_2 + a_{00} \tag{2.30}$$

where, $a_{11} = k_1 k_2 + 0.707 k_1 + 1.414 k_2 + 2$ (2.31a)

$$a_{10} = k_1 k_2 \alpha_{02} + 0.707 k_1 \beta_{02} + 1.414 k_2 \alpha_{02} + 2\beta_{02} \quad (2.31b)$$

$$a_{01} = k_1 k_2 \alpha_{01} + 0.707 k_1 \alpha_{01} + 1.414 k_2 \beta_{01} + 2\beta_{01} \quad (2.31c)$$

$$a_{00} = k_1 k_2 \alpha_{01} \alpha_{02} + 0.707 k_1 \alpha_{01} \beta_{02} + 1.414 k_2 \alpha_{01} \beta_{01} + 2\beta_{01}\beta_{02} \quad (2.31d)$$

The MATLAB® function *lowPass.m* (refer to the APPENDIX) can be employed to obtain the contour and 3-D magnitude plots of the resulting 2-D low-pass digital filters with the transfer function (2.28). In this function, it is made certain that stability conditions are satisfied. As the changeable coefficients of the double generalized bilinear transformations can change the stability of the resulting 2-D digital filters, it is necessary to introduce additional stability constraints in the discrete domain.

2.7 The Stability Conditions of the 2-D Digital Filter with Unity Degree Denominator

Starting from a 2-D analog transfer function with a VHSP as its denominator, applying the well-known used bilinear transformation (1.8) will always result in 2-D stable digital filter. However, when the generalized bilinear transformation is applied, it has to be ensured that the resulting 2-D digital filter is always stable.

The two-variable polynomial with the form (2.30) is unity degree for each variable z_1 and z_2, while the overall term $z_1 z_2$ has a degree of 2.

The inverse generalized bilinear transformation (1.37) is

$$z_i \to \frac{\beta_{0i} s_i - k_i \alpha_{0i}}{k_i - s_i}, \quad i = 1,2 \quad (2.32)$$

Applying (2.32) to $\frac{1}{D_d(z_1,z_2)}$, where $D_d(z_1, z_2)$ is given by (2.30), we can obtain

$$\begin{aligned}D_a(s_1,s_2) &= (a_{11}\beta_{01}\beta_{02} - a_{10}\beta_{01} - a_{01}\beta_{02} + a_{00})s_1 s_2 \\ &+ (-a_{11}\alpha_{02}\beta_{01}k_2 + a_{10}k_2\beta_{01} + a_{01}k_2\alpha_{02} - a_{00}k_2)s_1 \\ &+ (-a_{11}\alpha_{01}\beta_{02}k_1 + a_{10}k_1\alpha_{01} + a_{01}k_1\beta_{02} - a_{00}k_1)s_2 \\ &+ (a_{11}\alpha_{01}\alpha_{02}k_1k_2 - a_{10}\alpha_{01}k_1k_2 - a_{01}\alpha_{02}k_1k_2 + a_{00}k_1k_2)\end{aligned} \tag{2.33}$$

In order that $D_a(s_1, s_2)$ is a Very Strictly Hurwitz Polynomial (VSHP), the necessary and sufficient condition is each polynomial coefficient needs to be positive [22, 29]. This gives

$$a_{11}\beta_{01}\beta_{02} - a_{10}\beta_{01} - a_{01}\beta_{02} + a_{00} > 0 \tag{2.34a}$$

$$-a_{11}\alpha_{02}\beta_{01}k_2 + a_{10}k_2\beta_{01} + a_{01}k_2\alpha_{02} - a_{00}k_2 > 0 \tag{2.35a}$$

$$-a_{11}\alpha_{01}\beta_{02}k_1 + a_{10}k_1\alpha_{01} + a_{01}k_1\beta_{02} - a_{00}k_1 > 0 \tag{2.36a}$$

$$a_{11}\alpha_{01}\alpha_{02}k_1k_2 - a_{10}\alpha_{01}k_1k_2 - a_{01}\alpha_{02}k_1k_2 + a_{00}k_1k_2 > 0 \tag{2.37a}$$

As a stable generalized bilinear transformation requires $k_1 > 0$ and $k_2 > 0$, the stability conditions in (2.34a)-(2.37a) become:

$$a_{11}\beta_{01}\beta_{02} - a_{10}\beta_{01} - a_{01}\beta_{02} + a_{00} > 0 \tag{2.34}$$

$$-a_{11}\alpha_{02}\beta_{01} + a_{10}\beta_{01} + a_{01}\alpha_{02} - a_{00} > 0 \tag{2.35}$$

$$-a_{11}\alpha_{01}\beta_{02} + a_{10}\alpha_{01} + a_{01}\beta_{02} - a_{00} > 0 \tag{2.36}$$

$$a_{11}\alpha_{01}\alpha_{02} - a_{10}\alpha_{01} - a_{01}\alpha_{02} + a_{00} > 0 \tag{2.37}$$

From these relationships, MATLAB® functions *k1LPRange.m*, *k2LPRange.m*, *A1LPRange.m*, *A2LPRange.m*, *B1LPRange.m* and *B2LPRange.m* are used to compute the ranges for k_1, k_2, α_{01}, α_{02}, β_{01} and β_{02}, from which can result in stable low-pass filter system when the other coefficients are specified. These MATLAB® functions take the coefficients of the double generalized bilinear transformations, except the one we are considering the range in the function, as their input arguments, and return the stable range of that coefficient which we intend to study.

2.8 Frequency Responses of the 2-D Low-Pass Filters

The MATLAB® function *lowPass.m* is used to obtain the contour and 3-D magnitude response plots of the resulting 2-D digital filters. With the input coefficients of the generalized bilinear transformations, we can obtain the contour and 3-D magnitude plots of the resulting 2-D digital filters. In this function, the stability problem is first treated to make sure that the 2-D low-pass filter is stable with these input arguments.

To investigate the manner in which each coefficient of generalized bilinear transformation affects the magnitude response of the resulting 2-D low-pass digital filters, we change the value of the deserving coefficient or coefficients while fixing the other coefficients to the specified values. That can separate the effects from the other coefficients. We know from the previous sections that it is possible to obtain 2-D low-pass filters when the coefficients are in their low-pass limits: $k_i>0$, $-1.0\leq\alpha_{0i}\leq 0$ and $0\leq\beta_{0i}\leq1.0$, (i = 1, 2). In the following, we study in detail the effect caused by each coefficient to the frequency responses of the resulting 2-D low-pass filter.

2.8.1 Frequency Response of the Resulting 2-D Low-Pass Filters with Variable k_1

To study the manner how k_1 affects the frequency response behaviours of the resulting 2-D low-pass filter and how the effect of k_1 is different from the effect of the other coefficients, we change the values of k_1, while fixing the other coefficients of the generalized bilinear transformation to be k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, and β_{02}=1.0. The MATLAB® function *k1LPRange.m* is used to compute the stable range of k_1 with the other specified coefficients. Figure 2.4 is the range obtained from the function, here we use k_1 = 1000 to simulate the situation of $k_1 \to \infty$.

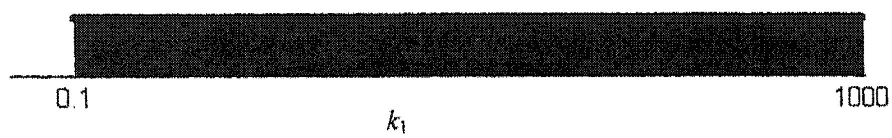
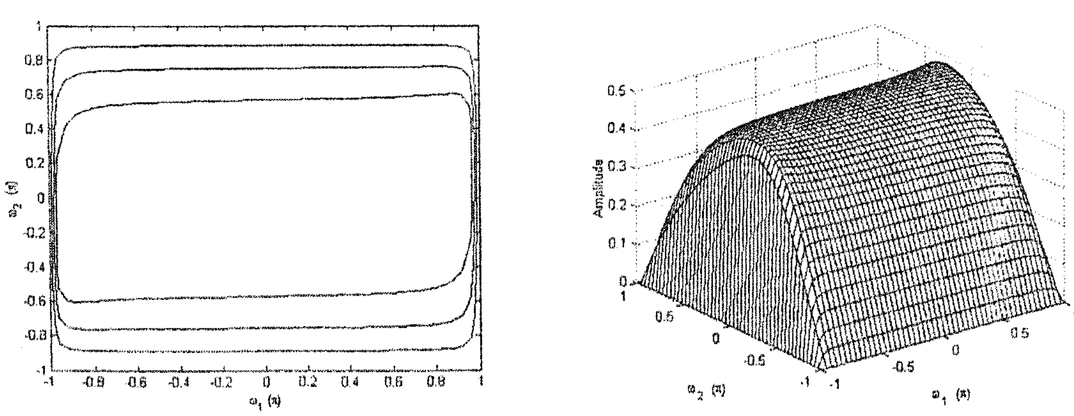
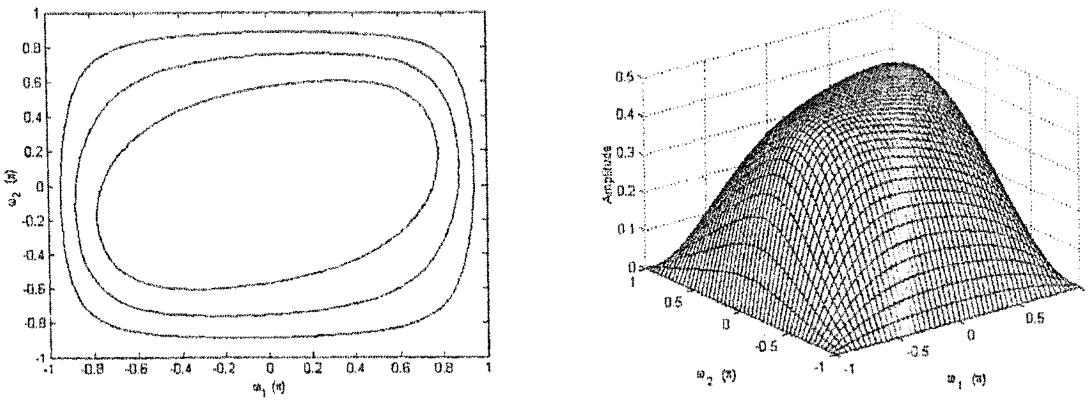


Figure 2.4 The range of k_1 when the other coefficients are specified to be k_2=1.0, α_{01}=−1.0. α_{02}=−1.0, β_{01}=1.0, and β_{02}=1.0.

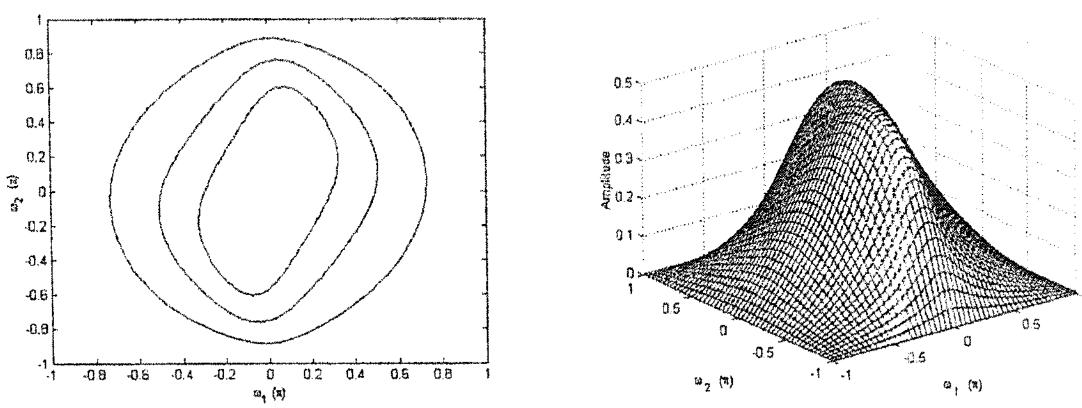
From Figure 2.4, we can see that any value of k_1 in the range of $k_1 > 0$ can result in a stable 2–D low-pass digital filter when the other ones are set to unity with proper signs. The contour and 3-D magnitude response plots for the filter with the representative values of k_1=0.1, k_1=1.0, k_1=5.0, k_1=10.0, and k_1=50.0 are given in Figure 2.5.



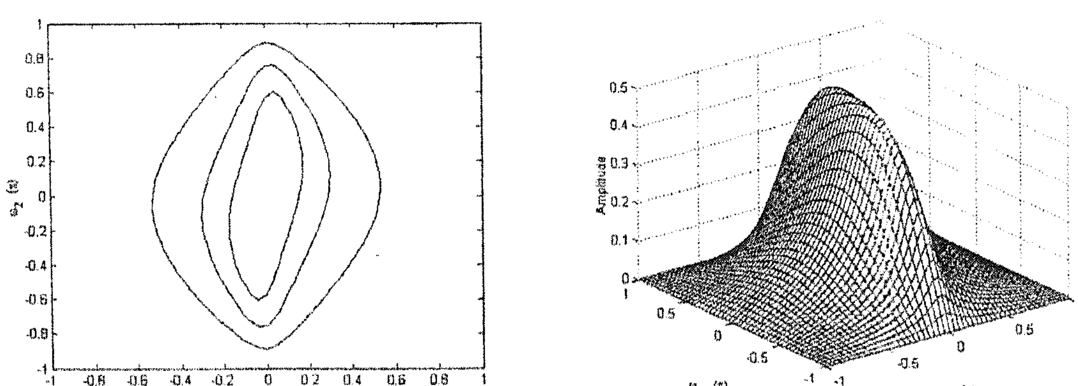
a. k_1 = 0.1



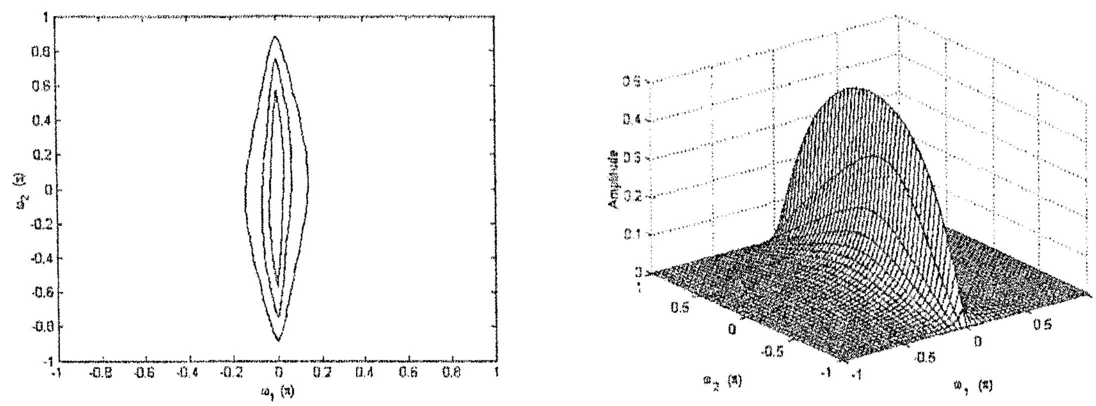
b. $k_1 = 1.0$



c. $k_1 = 5.0$



d. $k_1 = 10.0$



e. $k_1 = 50.0$

Figure 2.5 The contour and 3-D magnitude plots of the resulting 2-D low-pass filter with variable k_1 and the other coefficients fixed as k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01} 1.0 and β_{02}=1.0.

It can be seen that the coefficient k_1 mainly affects the bandwidth of the pass-band in ω_1-dimension. As k_1 increases, the pass-band in ω_1-dimension of the filter becomes compact, from almost all-pass in lower boundary of k_1 to almost all-stop in very large k_1. However, the pass band and transition band in ω_2-dimension remain unchanged. Another phenomena can be observed is that as k_1 varies from 0.1 to 50.0, the symmetric axis of ellipse-like magnitude contour curves rotates from $\theta = 0$ radians to almost $\theta = \pi/2$ *radians* with respect to ω_1-axis.

The coefficient k_1 has no effects on the gains in the pass-band and stop-band, and it does not affect the filters' polarity properties.

2.8.2 Frequency Response of the 2-D Low-Pass Filter with Variable k_2

Figure 2.6 is the result of the range of k_2 obtained from the MATLAB® function *k2LPRange.m*, while the other coefficients are set to be k_1=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, and β_{02}=1.0. Here we still use $k_2 = 1000$ to stimulate the situation of $k_2 \to \infty$.

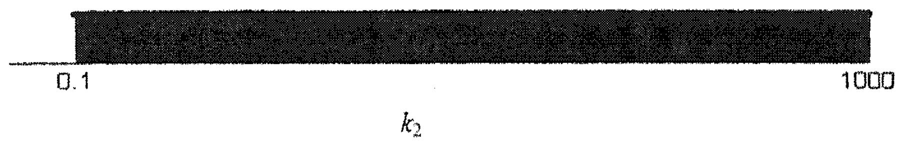
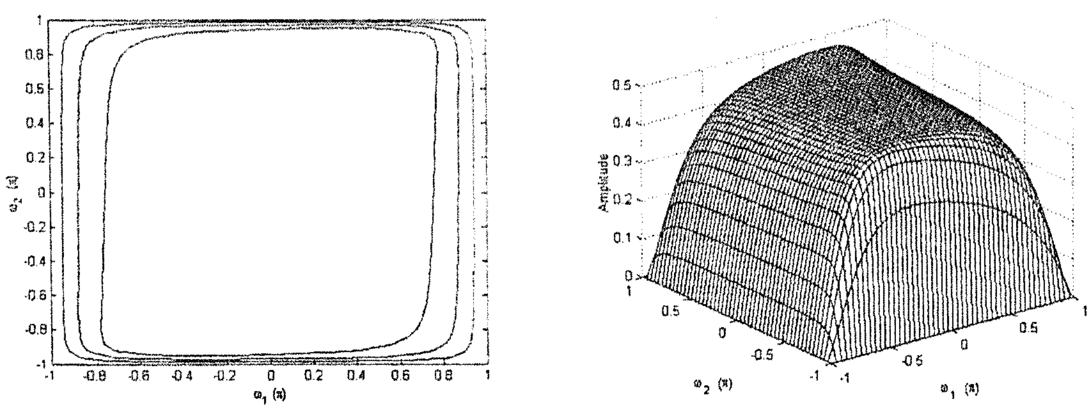


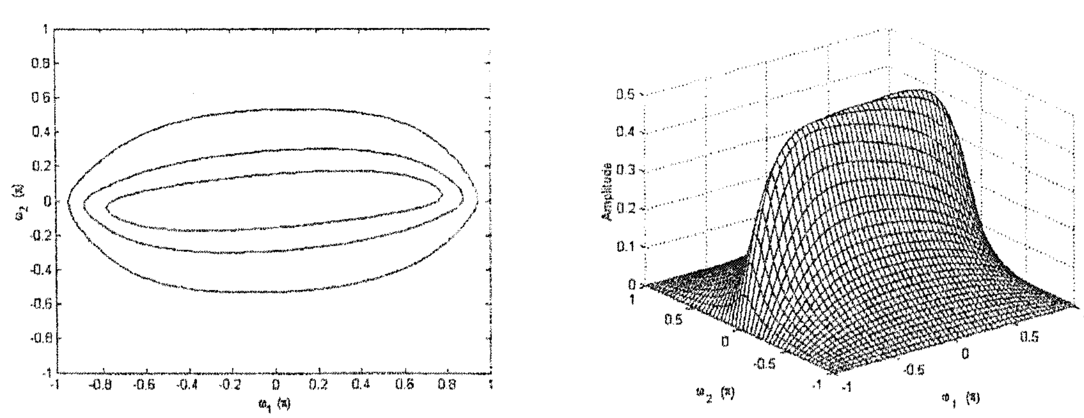
Figure 2.6 The range of k_2 when the other coefficients are specified to be k_1=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, and β_{02}=1.0.

Figure 2.6 indicates that any value of k_2 in the range of k_2 >0 can result in a stable 2-D low-pass filter, when the other coefficients were set to be unity with proper signs.

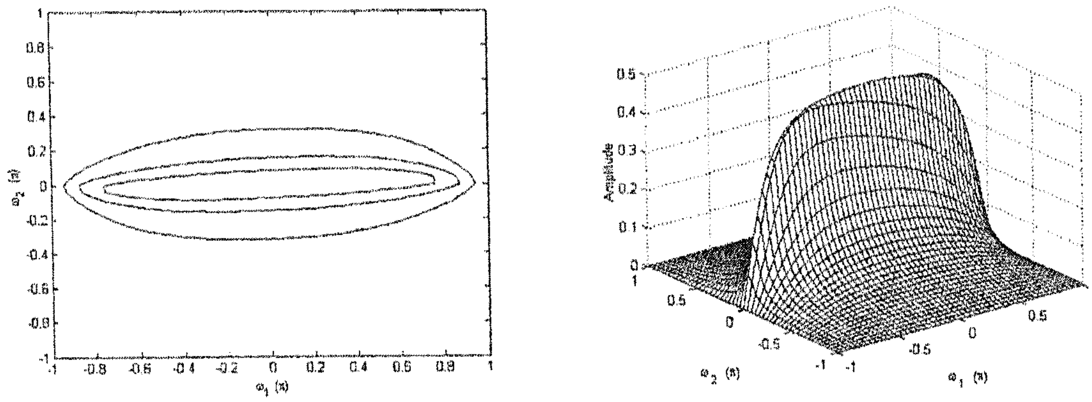
Figure 2.7 is the contour and 3-D magnitude plots for the 2-D low-pass filters with different k_2 and the other coefficients are k_1=1.0, α_{01}=−1.0 α_{02}=−1.0, β_{01}=1.0 and β_{02}=1.0.



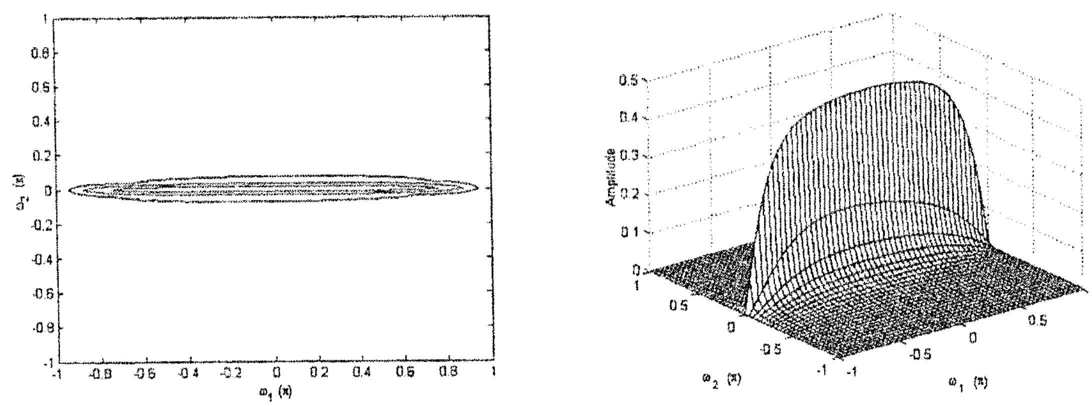
a. $k_2 = 0.1$



b. k_2=5.0



c. $k_2 = 10.0$



d. $k_2 = 50.0$

Figure 2.7 The contour and 3-D magnitude plots of the resulting 2-D low-pass filter with variable k_2 and the other coefficients fixed as k_1=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, and β_{02}=1.0

The coefficient k_2 mainly affects the magnitude characteristics in the ω_2-dimension. When k_2 increases from 0.1, the lower boundary, to 50.0, the pass-band in ω_2-dimension changes from almost all-pass to almost all-stop, while the pair of pass-band and stop-band in ω_1-dimension remains unchanged. The symmetrical axis for the ellipse-like contour curves rotates from the position of almost $\theta=\pi/2$ *radians* to the position of $\theta=0$ *radians*.

The coefficient k_2 has no effect on the gains of the pass-band of the 2-D low-pass filter, as well as its polarity.

2.8.3 Frequency Response of the Resulting 2-D Low-Pass Filter with Variable α_{01}

Using the MATLAB® function *a1LPRange.m*, we can get the stable range of α_{01} with the other specified coefficients of the double generalized bilinear transformation. Generally, there exist many combinations for the coefficients. In order to make the problem simple without loss of generality, we specify the other coefficients to be unity with proper signs, when studying the effect of α_{01}on the magnitude. Figure 2.8 is the range of α_{01} which make the system stable when the other coefficients are specified as k_1=1.0, k_2=1.0, α_{02}=−1.0, β_{01}=1.0, and β_{02}=1.0.

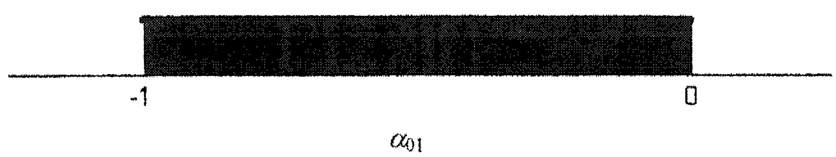
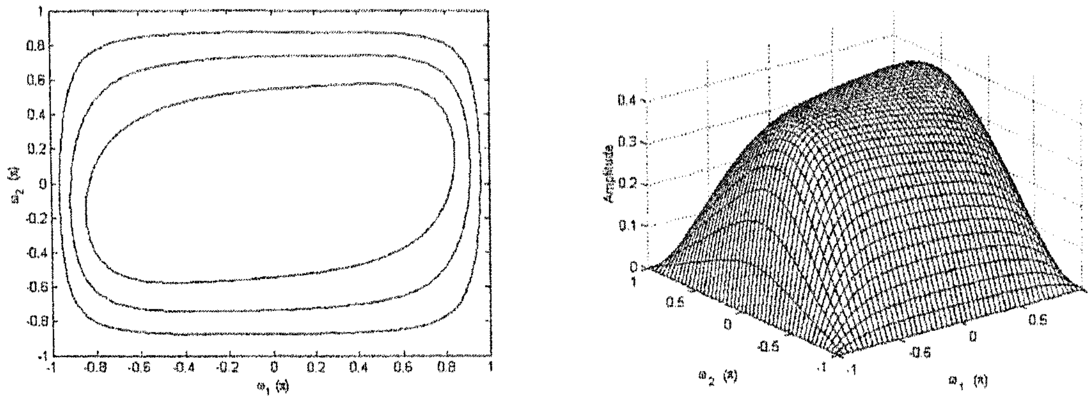
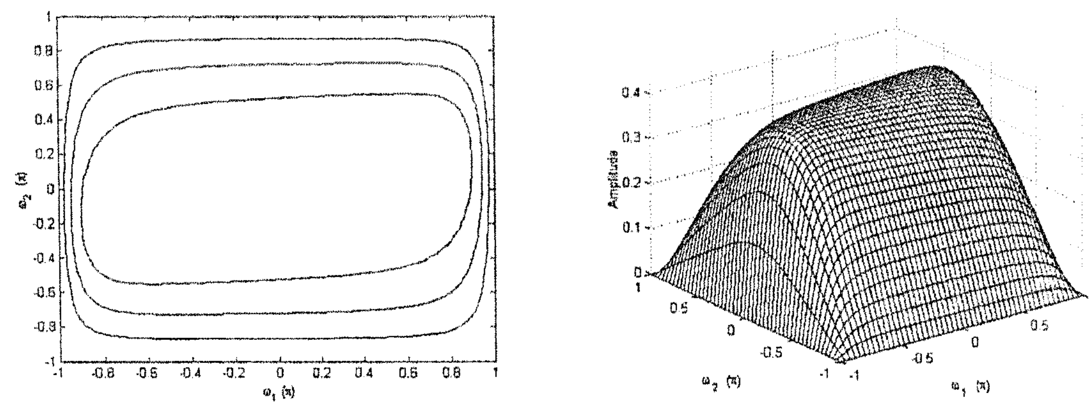


Figure 2.8 The range of α_{01} when the other coefficients are specified to be k_1=1.0, k_2=1.0, α_{02}=−1.0, β_{01}=1.0, and β_{02}=1.0.

Any value of α_{01} in the range of $-1.0 \le \alpha_{01} \le 0$ can result in a stable 2-D low-pass digital filter. The contour and 3-D magnitude response plots of the resulting 2-D low-pass filter with representative values of α_{01}'s are given in Figure 2.9. As we have obtained the frequency responses of the resulting 2-D low-pass filter with α_{01}=−1.0 and other unity coefficients in section 2.8.1, here we only give the results of the two cases for α_{01}=−0.5 and α_{01}=0.



a. $\alpha_{01} = -0.5$



b. $\alpha_{01} = 0$

Figure 2.9 The contour and 3-D magnitude plots of the resulting 2-D low-pass filter with variable α_{01} and the other coefficients fixed as k_1=1.0, k_2=1.0, α_{02}= −1.0, β_{01}=1.0, and β_{02}= 1.0

It can be seen that the coefficient α_{01} mainly affects the gain of the pass-band of the resulting 2 -D l ow-pass filter. S pecifying t he o ther c oefficients t o b e u nity w ith p roper signs, and when α_{01} changes from the lower boundary of −1.0 to the upper boundary of 0, the gain in the pass-band decreases from 0.5 to about 0.4.

The low-pass coefficient α_{01} can also affect the bandwidth of pass-band in ω_1-dimension. As α_{01} increases from the lower boundary of −1.0 to the upper boundary of

0, the pass-band with respect to ω_1 becomes slightly larger. For the symmetrical axis for the ellipse-like curves, the angle of the axis to the horizontal axis (ω_1 axis) gets smaller as α_{01} increases in the range of $-1.0 \leq \alpha_{01} \leq 0$.

2.8.4 Frequency Response of the Resulting 2-D Low-Pass Filter with Variable α_{02}

The MATLAB® function *a2LPRange.m* is employed to compute the range of α_{02}, which meets the stability requirements with the other specified coefficients. The other coefficients, which are in the range defined in (2.27), can have many possible combinations, and these different combinations may have different requirements for the coefficient α_{02} in order that the resulting condition is stable. The MATLAB® function *a2LPRange.m* takes the combinations of the other coefficients as its input arguments, and outputs the stable range of α_{02} in plot form. Here to make the problem simple without loss of generality, we set them to be k_1=1.0, k_2=1.0, α_{01}=−1.0, β_{01}=1.0 and β_{02}=1.0. It can be verified from the function *a2LPRange.m* that all the α_{02}'s in the range of $-1.0 \leq \alpha_{02} \leq 0$ could make the resulting filters stable. The range of α_{02} is given in Figure 2.10.

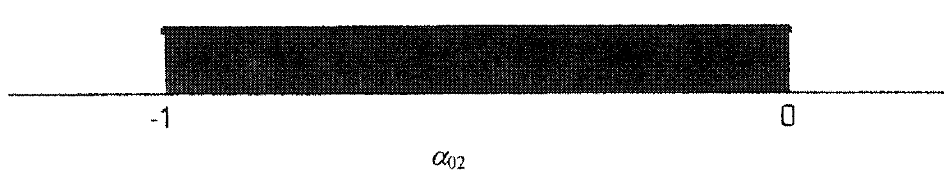
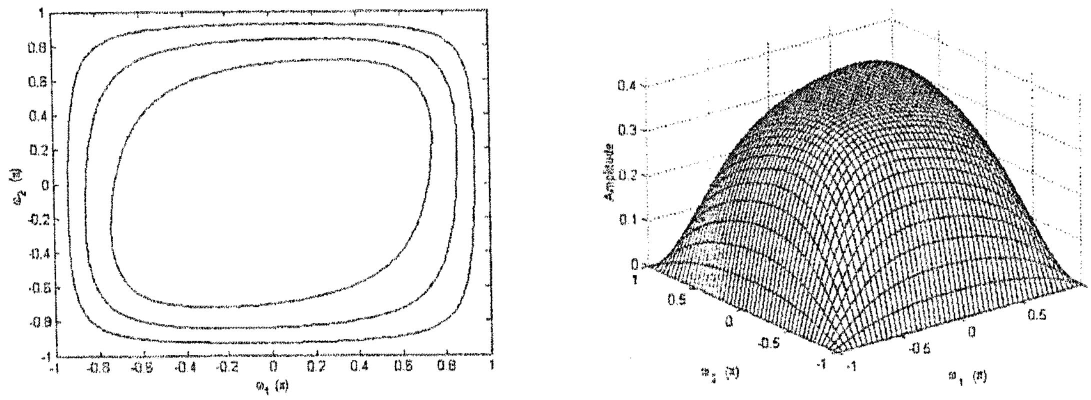
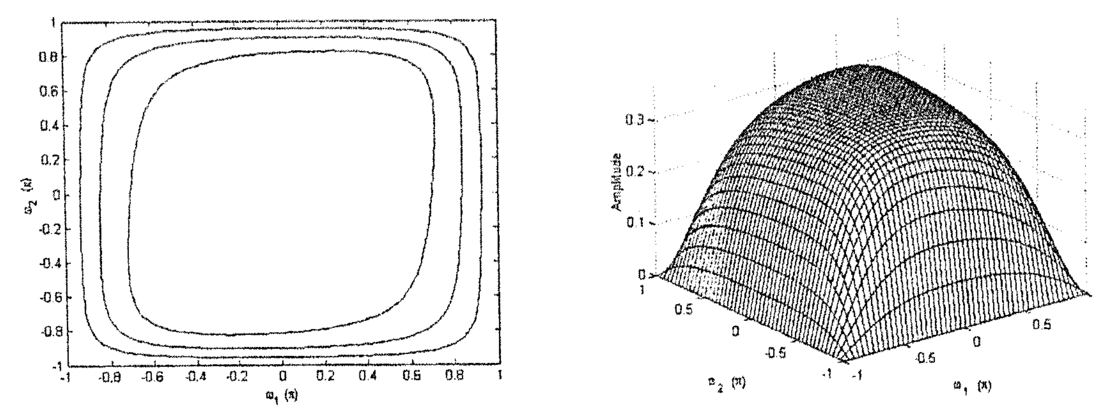


Figure 2.10 The range of α_{02} when the other coefficients are specified to be k_1=1.0, k_2=1.0, α_{01}=−1.0, β_{01} =1.0, and β_{02} =1.0.

Any α_{02} in the range of $-1.0 \leq \alpha_{02} \leq 0$ can result in a stable 2-D low-pass filter when the other coefficients are set to be k_1=1.0, k_2=1.0, α_{01}=−1.0, β_0=1.0 and β_{02}=1.0. The magnitude response behaviors of the resulting 2-D low-pass digital filter with the representative values of α_{02}'s are given in Figure 2.11. Here again, we do not give the results with α_{02}=−1.0.



a. $\alpha_{02} = -0.5$



b. $\alpha_{02} = 0$

Figure 2.11 The contour and 3-D magnitude plots of the resulting 2-D low-pass filter with variable α_{02} and the other coefficients fixed as k_1=1.0, k_2=1.0, α_{01}=−1.0, β_{01}=1.0 and β_{02}=1.0

The coefficient α_{02} also affects the gain in the pass-band of the resulting 2-D low-pass filter. When α_{02} changes from its lower boundary of −1.0 to the upper boundary of 0, the gain decreases from 0.5 to about 0.4.

And also α_{02} has little effect on the bandwidth of the pass-band in ω_2-dimension.

When α_{02} is increasing in the region of $-1.0 \le \alpha_{02} \le 0$, the pass-band in ω_2-dimension becomes larger. For the symmetrical axis, it rotates from a smaller angle to a bigger one.

Compared with the effects of k_1 and k_2 on the bandwidth of the pass-band, the effects of α_{01} and α_{02} are very small. But α_{01} and α_{02} play an important role in view of their effects on the gain of pass-band. Therefore we define α_{01} and α_{02} **gain-effect coefficients**, and k_1 and k_2 **band-effect coefficients.**

2.8.5 Frequency Response of the Resulting 2-D Low-Pass Filter with Equal Variables α_{01} and α_{02}

From the last two sections, both α_{01} and α_{02} affect the gain at the pass-band areas. Now we study the joint effect of α_{01} and α_{02}.

Still, the other coefficients are set to be k_1=1.0, k_2=1.0, β_{01}=1.0, and β_{02}=1.0, while the values of α_{01} and α_{02} are changing. Figure 2.12 shows the stable ranges of α_{01} and α_{02}.

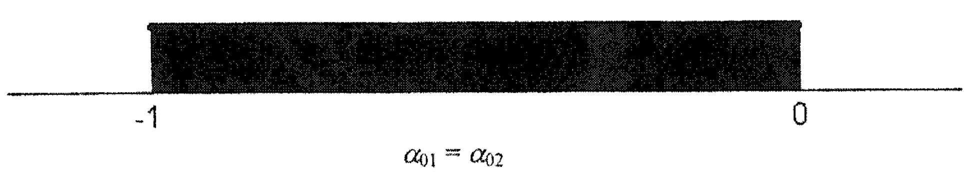
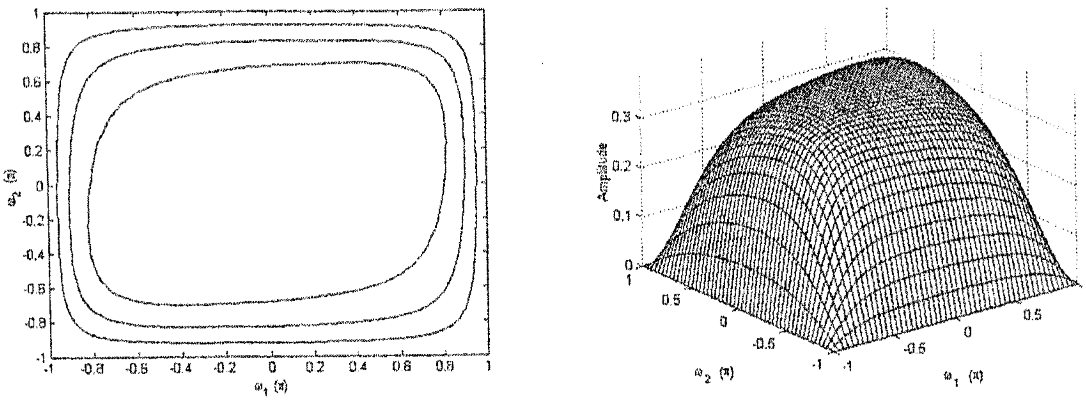
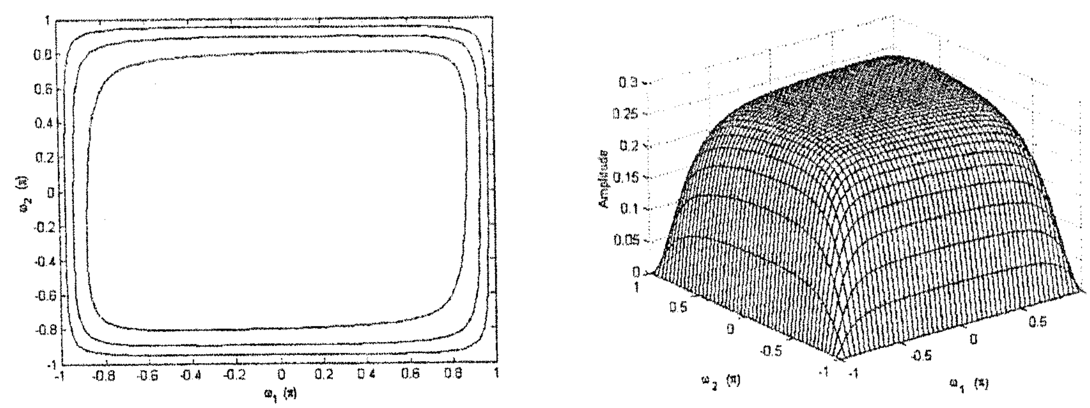


Figure 2.12 The range of equal α_{01} and α_{02} when the other coefficients are specified to be k_1=1.0, k_2=1.0, β_{01}=1.0, and β_{02}=1.0

When we choose equal values of α_{01} and α_{02} in the range of [−1.0, 0] and specify the other coefficients to be k_1 = 1.0, k_2 = 1.0, β_{01} = 1.0, and β_{02} = 1.0, the resulting 2-D low-pass filter is stable. Figure 2.13 is the contour and 3-D magnitude plots of the resulting 2-D low-pass filter at $\alpha_{01}=\alpha_{02}=-0.5$ and $\alpha_{01}=\alpha_{02}=0$.



a. $\alpha_{01} = \alpha_{02} = -0.5$



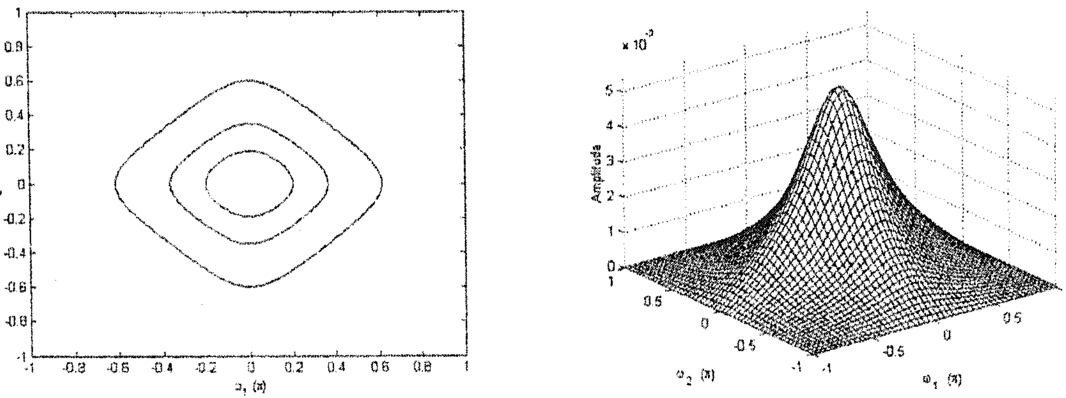
b. $\alpha_{01} = \alpha_{02} = 0$

Figure 2.13 The contour and 3-D magnitude plots of the resulting 2-D low-pass filter with equal variables α_{01} and α_{02} and the other fixed coefficient: k_1=1.0, k_2=1.0, β_{01}=1.0 and β_{02}=1.0.

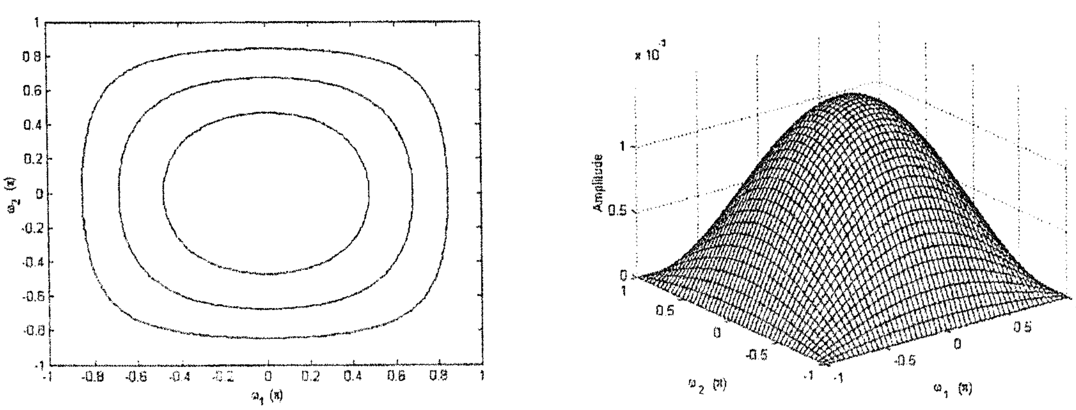
The effect on the gain of pass-band portions becomes more pronounced when we change the two coefficients, α_{01} and α_{02}, simultaneously than the effect caused by the individual variation of α_{01} or α_{02}. When the values of α_{01} and α_{02} change from their lower boundary to their upper boundary, the gain of the pass-bands decreases from 0.5 to about 0.3.

The pass-band area becomes enlarged in both the ω_1 and ω_2 dimensions, but the effect is very slight.

Both the effects on the gain and the bandwidth of the pass-band can be enlarged when we choose large values of k_i (i=1,2). Figure 2.14 shows the enlargement. Here we use 50.0 for k_1 and k_2 to illustrate the situation of larger values.



a. $\alpha_{01}=\alpha_{02}=-0.5$



b. $\alpha_{01}=\alpha_{02}=0$

Figure 2.14 The contour and 3-D magnitude plots of the resulting 2-D low-pass filter with equal variable equal α_{01} and α_{02} and the other coefficient fixed as k_1=5.0, k_2=5.0, β_{01}=1.0 and β_{02}=1.0

Compared with the situation when k_1=5.0, k_2=5.0, $\alpha_{01}=-1.0$, $\alpha_{02}=-1.0$, β_{01}=1.0 and

β_{02}=1.0, the filter in Figure 2.14 has very smaller gain, which decreases from 0.5 to about 5×10^{-3} and 1.5×10^{-4} when the values of α_{01} and α_{02} increase from −1.0 to −0.5 and 0, respectively.

And also, we can compare the filters in Figure 2.13 and 2.14. With the same values of α_{01} and α_{02}, the filter with bigger values of k_1 and k_2 has smaller gain and smaller bandwidth of pass-band. However, the effect on the gain of pass-band is more effective than the effect on bandwidth.

2.8.6 Frequency Response of the Resulting 2-D Low-Pass Filter with Variable β_{01}

The MATALB® function *b1LPRange.m* is used to calculate the stable range of β_{01} when the other coefficients are specified. There are many choices for these coefficients. Here, for the sake of convenience, while changing the value of β_{01}, we still set these coefficients to be unity with proper signs, specifically, k_1=1.0, k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, and β_{02}=1.0. We can check from Figure 2.15 that any value of β_{01} in the range of $0\le\beta_{01}\le1.0$ can result in stable 2-D low-pass filter.

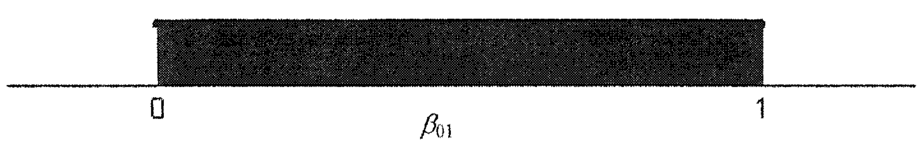
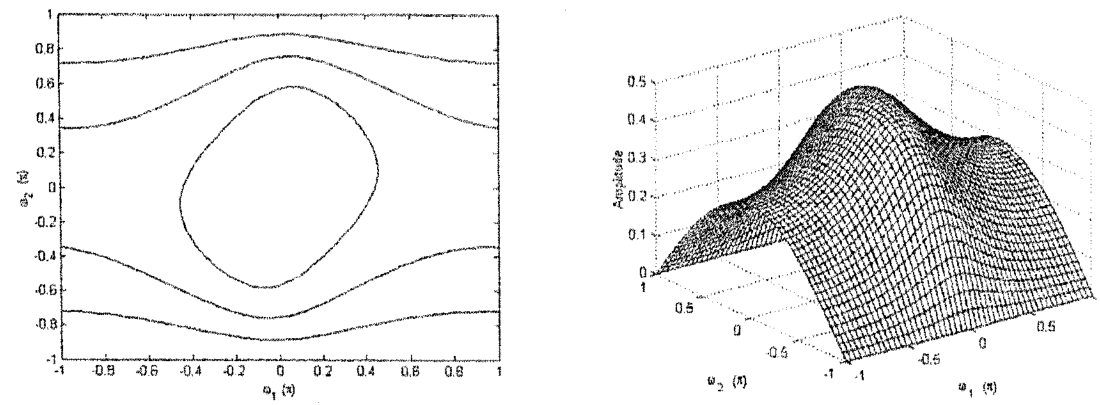
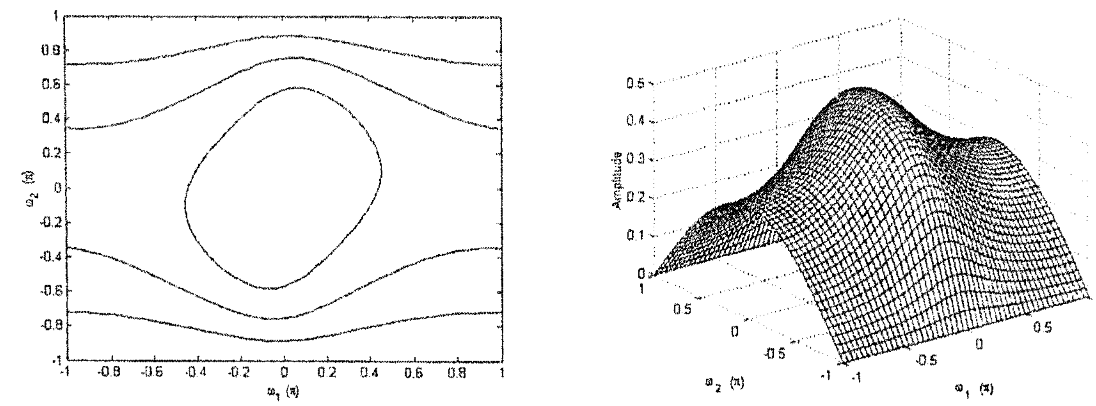


Figure 2.15 The range of β_{01} when the other coefficients are specified to be k_1=1.0, k_2 =1.0, α_{01}=−1.0, α_{02}=−1.0, and β_{02} =1.0.

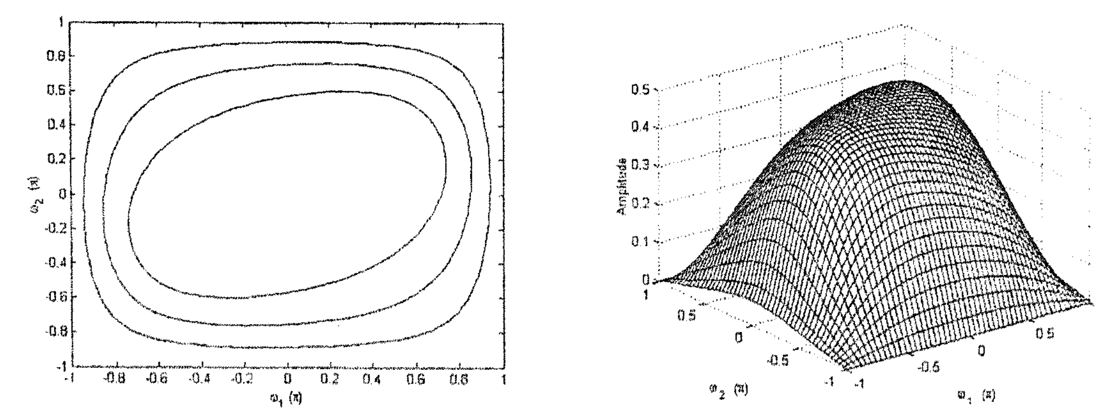
Figure 2.16 gives the contour and 3-D magnitude response plots of the resulting 2-D low-pass filter with different values of β_{01} when the other coefficients are set to be k_1=1.0, k_2=1.0, α_{01}=−1.0, α_{02} = −1.0, and β_{02} = 1.0.



a. $\beta_{01} = 0$



b. $\beta_{01} = 0.5$

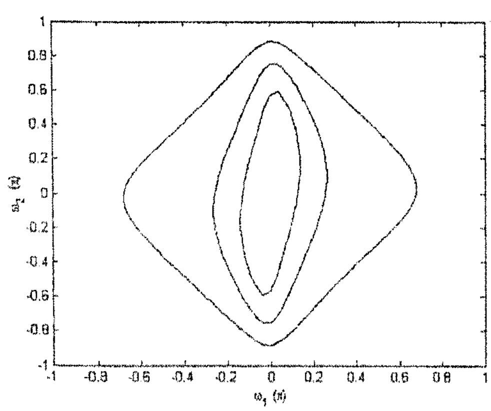
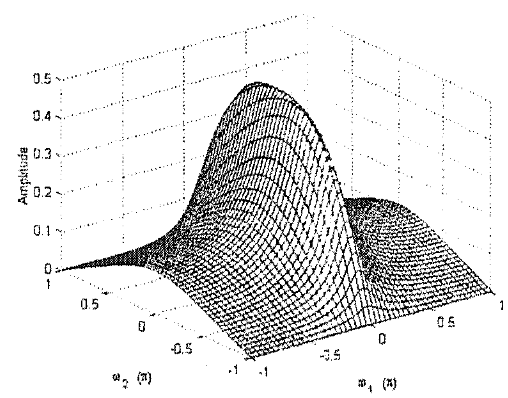


c. $\beta_{01} = 0.9$

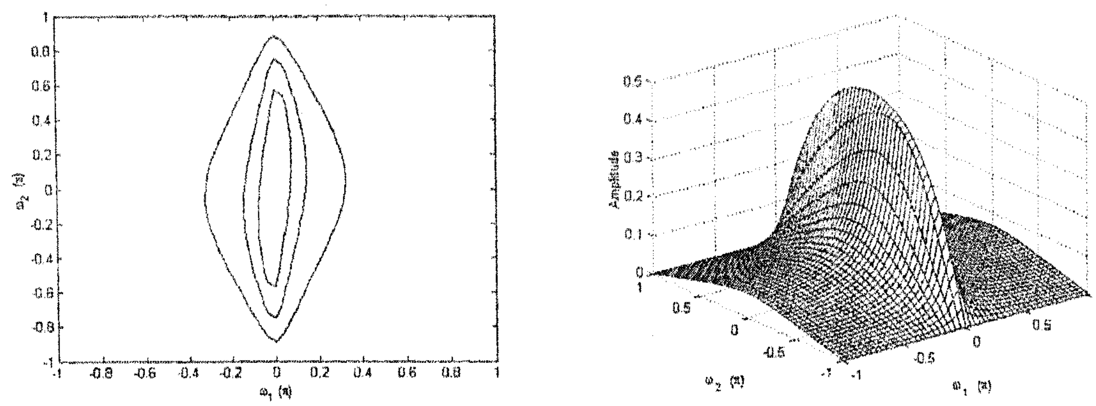
Figure 2.16 The contour and 3-D magnitude plots of the resulting 2-D low-pass filter with variable β_{01} and the other coefficients fixed as k_1=1.0, k_2=1.0, α_{01}=−1.0, α_{01}=−1.0, and β_{02} = 1.0

From the results, it can be seen that the main effect from β_{01} is that there exists a non-zero gain at the stop-band portions when β_{01} does not equal 1.0. In addition, it results in different pass-band ranges of the filter in ω_1-dimension and the symmetrical axis of the contour rotates from a large angle to a small one, when the value of β_{01} increases. In fact, the non-zero gain at stop-band of the 2-D low-pass filter will affect the filter's polarity, from low-pass filter to high-pass one, so we also define β_{01} and β_{02} **polarity-effect coefficients**.

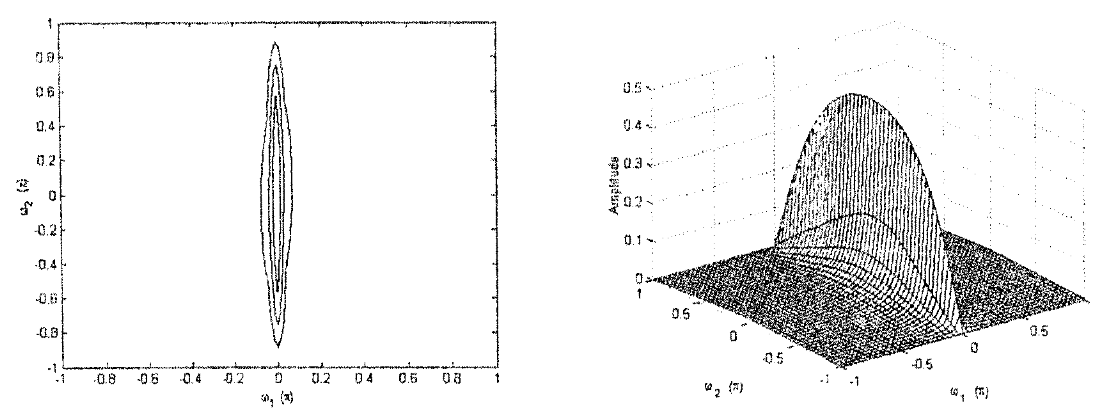
For the nonzero gain at the stop-band in high frequencies, it is possible to reduce it without changing the value of β_{01}. From the previous results, both the coefficients k_1 and α_{01} can affect the magnitude response in ω_1-dimension. As the effect of α_{01} is small, here we consider only the reduction of the non-zero gain by increasing the value of k_1. Figure 2.17 is the contour and 3-D magnitude plots of the resulting 2-D low-pass filter with different values of k_1 but other coefficients remaining the same setting as in Figure 2.16 (a), i.e.: k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=0, and β_{02}=1.0.

a. k_1=5.0



b. k_1=10.0



c. k_1=50.0

Figure 2.17 The contour and 3-D magnitude plots of the resulting 2-D low-pass filter with variable k_1 and the other coefficients fixed as k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=0, and β_{02}=1.0

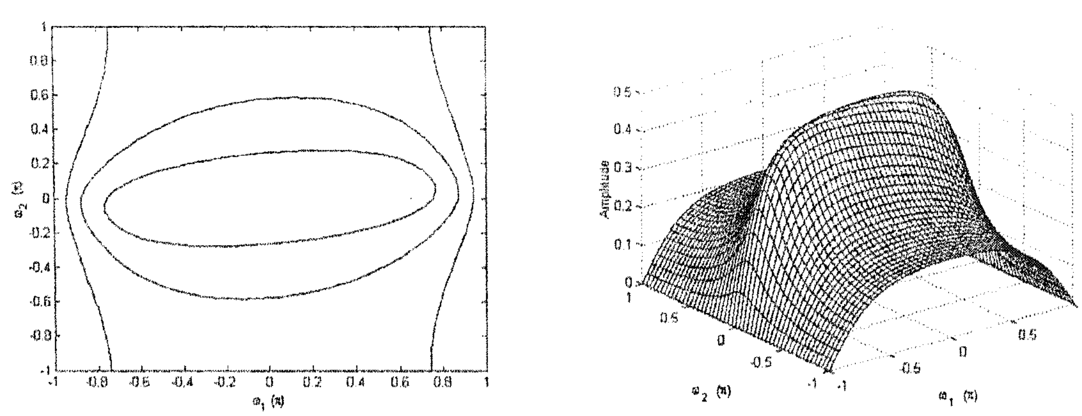
Figure 2.17 indicates that with a large value of k_1, we can have a small gain at stop-band in ω_1-domension, but the cost is to lose the bandwidth in the pass-band. But even with k_1 as large as 50.0, the gain at stop-band becomes almost zero, but it is still not absolutely zero. Some kinds of optimization techniques need to be introduced to balance the gain and loss, and to control the gain inside the design specifications.

2.8.7 Frequency Response of the Resulting 2-D Low-Pass Filter with Variable β_{02}

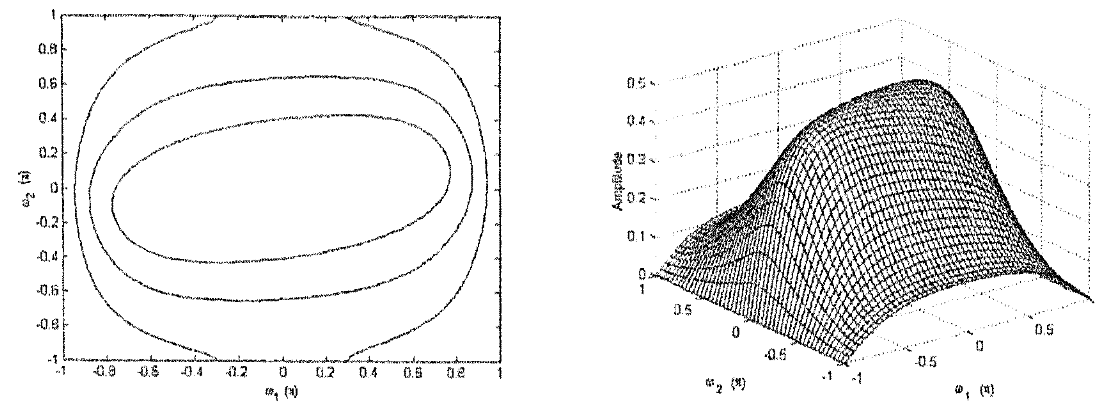
The MATLAB® function *b2LPRange.m* is used to compute the stable ranges for β_{02} with specified k_1, k_2, α_{01}, α_{02} and β_{01}. In this function, it takes the coefficients other than β_{02} at its input arguments, after checking the ranges for these coefficients, then scan the stable range for β_{02}, and plot the output. For the same reason mentioned before, here we still let k_1=1.0, k_2=1.0, α_{01}=−1.0, α_{02}=−1.0 and β_{01}=1.0, From the MATLAB® function *b2LPRange.m*, it can be checked that any value of β_{02} in the range of $0 \leq \beta_{02} \leq 1.0$ can result in a stable 2-D low-pass filter when the other coefficients are set to the specified values. The result is illustrated in Figure 2.18.

Figure 2.18 The range of β_{02} when the other coefficients are specified to be k_1=1.0, k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, and β_{01}=1.0.

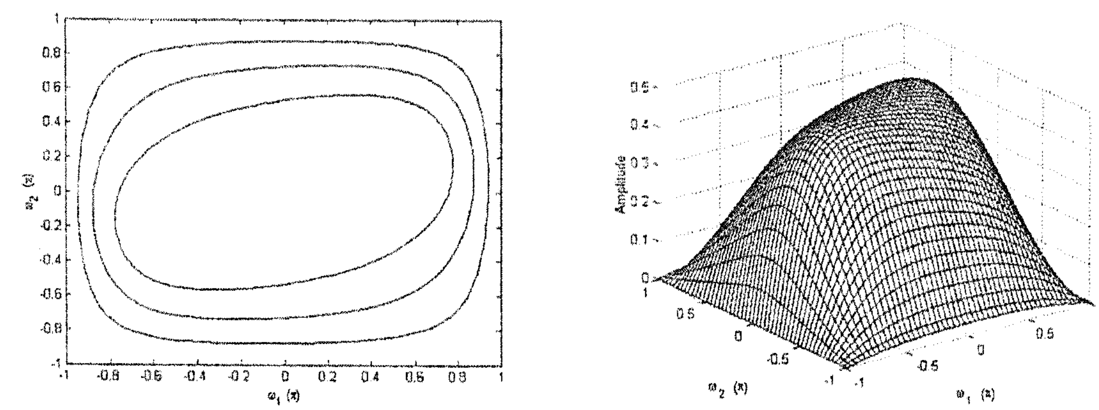
The various frequency responses of the resulting 2-D low-pass digital filters caused by different values of β_{02} are given in Figure 2.19.



a. $\beta_{02} = 0$



b. $\beta_{02} = 0.5$

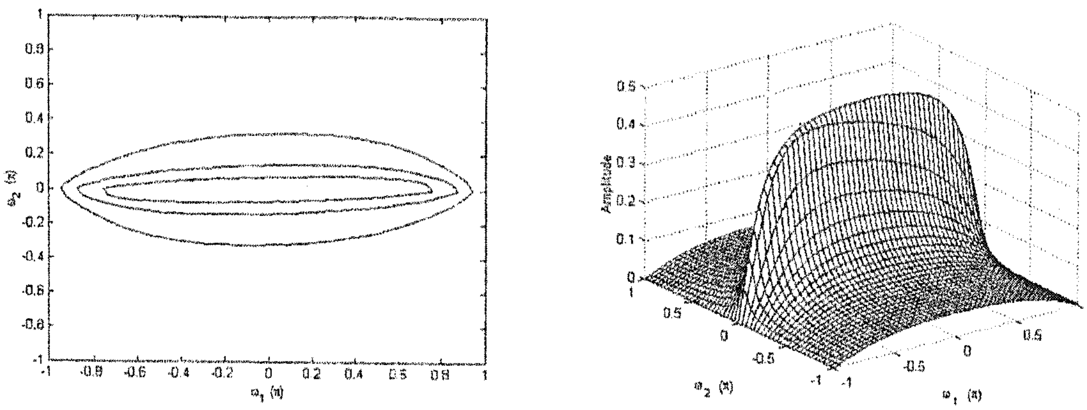


c. $\beta_{02} = 0.9$

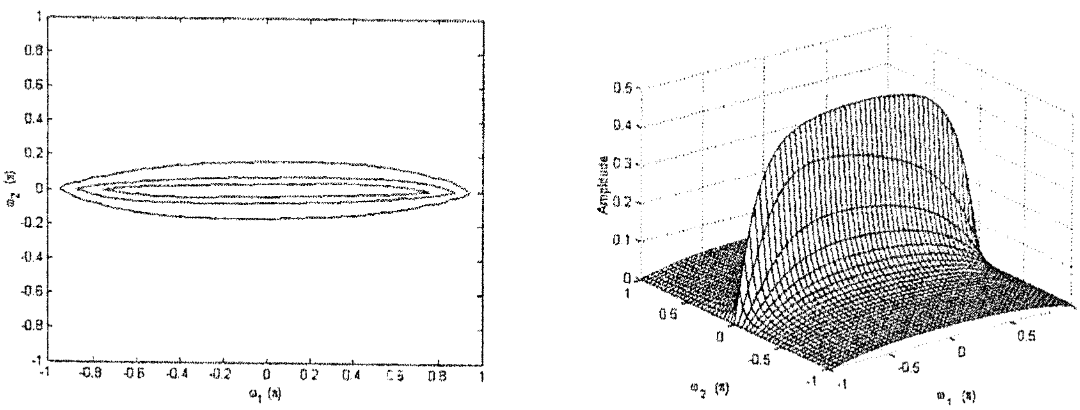
Figure 2.19 The contour and 3-D magnitude plots of the resulting 2-D low-pass Filter with variable β_{02} and the other coefficients fixed as k_1=1.0, k_2=1.0, α_0 =−1.0, α_{02}=−1.0 and β_{01}=1.0

The parameter β_{02} could lead to a 2-D low-pass filter with variable magnitude response. A bigger value of β_{02} produces a wider bandwidth in the pass-band of the filter in ω_2-dimension, and the symmetrical axis angle also changes in opposite direction as that caused by β_{01}.

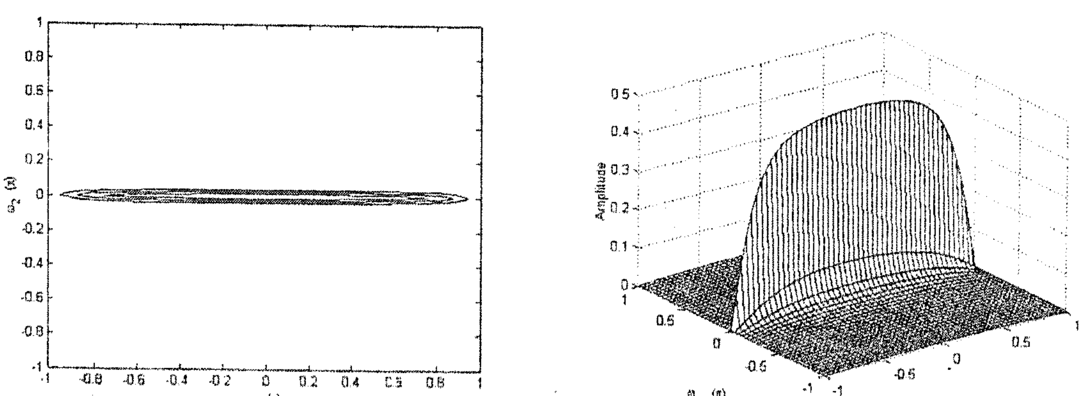
Nonzero gain at stop-band in ω_2-dimension exists when $\beta_{02} \neq 1.0$. The non-zero gain can be reduced by increasing the value of k_2. The simulation results are given in Figure 2.20 with different values of k_2.



a. $k_2 = 5.0$



b. $k_2 = 10.0$



c. $k_2 = 50.0$

Figure 2.20 The contour and 3-D magnitude plots of the resulting 2-D low-pass filter with variable k_2 and the other coefficients fixed as k_1=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, and β_{02}=0

The gain in the stop-band of the resulting 2-D low-pass filter is reduced by increasing the values of k_2 with the tradeoff considering the bandwidth of the pass-band in ω_2-dimension. How to balance the degree of gain reduction and the pass-band bandwidth loss should be determined by the design specifications and the optimization techniques. We can get optimum combination for k_2 and β_{02} to meet the design specifications, although the combination would not be unique.

2.9 Summary and Discussion

In this chapter, we have introduced the procedure for the design of 2-D low-pass discrete filters by double generalized bilinear transformations. The manner how each coefficient of the double generalized bilinear transformation affects the magnitude response behavior of the resulting 2-D low-pass filter has been studied in detail also.

From the second order 1-D Butterworth low-pass ladder network, we have formed a first-order 2-D doubly-unity-terminated circuit network. And then the 2-D analog transfer function is obtained. The 2-D discrete transfer function has been derived from the analog transfer function by double generalized bilinear transformations when the coefficients are in their specified ranges to get low-pass filters. When one or more coefficients of the double bilinear transformations are changing, the resulting 2-D low-pass filter has variable magnitude characteristics.

Stability is always the most important issue in 2-D recursive digital filter design. The stability conditions have been obtained for the resulting 2-D digital filter with a unity degree denominator for each variable. Using the link between the stability conditions and the coefficients of double generalized bilinear transformations, we have got the stable range for each coefficient when the other ones are specified. Also, whenever we try to obtain the magnitude response for a 2-D low-pass filter, these conditions need be satisfied.

The coefficients of k_1, α_{01} and β_{01} only affect the magnitude response in ω_1-dimension, while the coefficients of k_2, α_{02} and β_{02} only affect the behaviors of the magnitude response in ω_2-dimension. Depending on the main effects caused by each group of coefficients on the magnitude response, we define k_i's (i = 1, 2) as **band-effect coefficients**, α_{0i}'s (i = 1, 2) as **gain-effect coefficients**, and β_{0i}'s (i = 1, 2) as **polarity-effect coefficients**. Larger values of k_i's compact the bandwidth of the pass-band in their corresponding dimension. Although α_{0i} and β_{0i} also affect the bandwidths of the pass-bands, the main effects caused by α_{0i}'s are on the gain of the pass-bands and β_{0i}'s determine whether the resulting filter is either a low-pass filter or a high-pass filter when β_{0i}'s is in its negative half or in its positive half. The effect of β_{0i}'s appears in the 2-D low-pass filter as the non-zero gain of the stop-band when they have values other than –1.0. The larger the distance from –1.0, the bigger the non-zero gain. For α_{0i}'s, the larger absolute values can result in bigger gains in the pass-bands.

This chapter is an important work towards the study of the variable magnitude characteristics of 2-D recursive filters by double generalized bilinear transformations. Also, from the results presented in this chapter, using the symmetric prosperities of 2-D low-pass filters and the optimization techniques, it should be possible to design 2-D low-pass filters having variable magnitude characteristics.

Chapter 3

Two-Dimensional High-Pass Filters

In this chapter, another important type of 2-D filter, 2-D high-pass filter, is investigated. In section 3.1, we give a brief definition for 2-D high-pass filter, as well as the typical specifications in mathematical and plot forms. In section 3.2, the 2-D analog transfer function is obtained from the 1-D analog Butterworth low-pass prototype. The ranges of the coefficients, which can result in 2-D high-pass filter from the obtained 2-D analog transfer function by generalized bilinear transformation, are given in section 3.3. The digital transfer function of 2-D high-pass digital IIR filter is given in section 3.4. In section 3.5, we mainly consider the stability of the resulting 2-D high-pass filter with a unity degree denominator for each variable. In section 3.6, the frequency response of the resulting 2-D high-pass filter, as well as the manner how each coefficient of the generalized bilinear transformations affects the behaviour of the frequency response of the 2-D high-pass filter is studied. The summary and some useful conclusions are given in section 3.7.

3.1 Introduction

In contract to the low-pass filters, the high-pass filters pass the signal components with high frequencies, but attenuate the ones at low frequencies. High-pass filters have a bigger gain for the high frequency signals and a smaller gain for the low frequency signals. A 2-D high-pass filter can be described as [2, 4]

$$H(\omega_1,\omega_2)=\begin{cases}0, & 0\le|\omega_i|\le\omega_{is}\\ 1, & \omega_{ip}<|\omega_i|\le\pi\end{cases} \tag{3.1}$$

where:

ω_{ip} ($i=1,2$) are the pass bands in z_1 and z_2-dimensions, respectively

ω_{is} ($i=1,2$) are the stop bands in z_1 and z_2-dimensions, respectively

The frequency range between ω_{ip} and ω_{is} is the transition band.

The typical specifications of 2-D high-pass digital filters in the frequency domain are given in Figure 3.1.

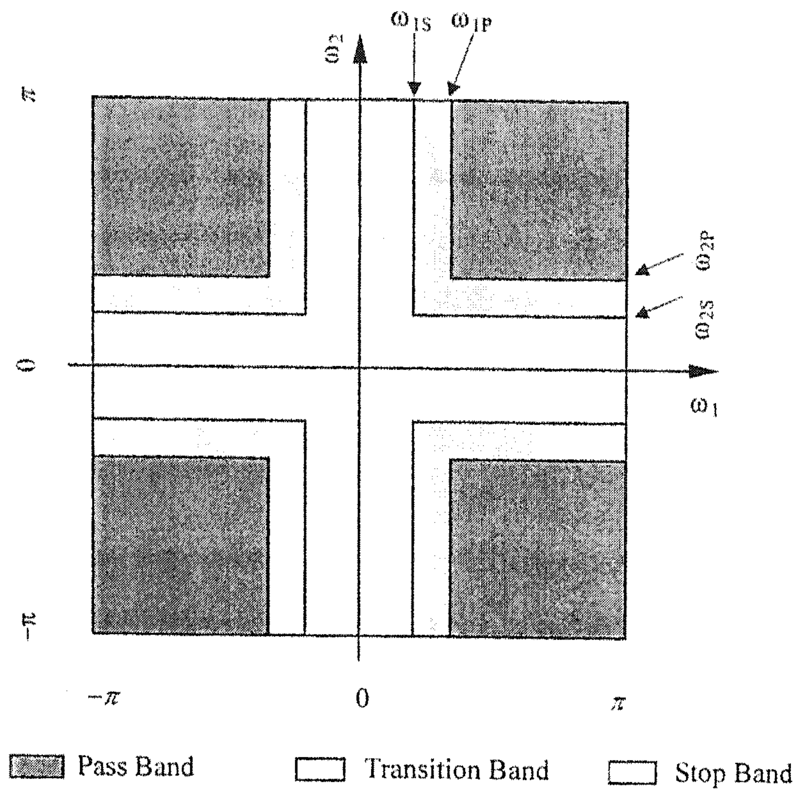


Figure 3.1 The Specifications of a typical 2-D digital high-pass filter in the frequency domain

3.2 2-D Analog Transfer Function

We have obtained in Chapter 2 the analog transfer function of the first order 2-D circuit network shown in Figure 2.3 as given by

$$H_a(s_1,s_2)=\frac{V_o}{V_i}=\frac{1}{(1+0.707s_1)(1.414s_2+1)+1}$$

$$=\frac{1}{s_1s_2+0.707s_1+1.414s_2+2} \tag{3.2}$$

The denominator of (3.2) is

$$D_a(s_1,s_2)=s_1s_2+0.707s_1+1.414s_2+2, \tag{3.3}$$

which is a Very Strictly Hurwitz Polynomial (VSHP) and constitutes a single degree in each variable obviously.

3.3 The High-Pass Limits of the Coefficients of the Generalized Bilinear Transformations

It is easy to prove that equation (3.2) is the transfer function of a 2-D low-pass analog filter.

To get the frequency response of the 2-D analog filter given by (3.2), let

$$s_1 = j\omega_1 \tag{3.4a}$$

$$s_2 = j\omega_2 . \tag{3.4b}$$

We obtain

$$H_a(j\omega_1, j\omega_2) = \frac{1}{-\omega_1\omega_2 + 0.707\omega_1 j + 1.414\omega_2 j + 2} \tag{3.5}$$

At some specified frequency points, we can get the magnitude of the filter as

$$H_a(j0, j\varpi_2) = C_1 \tag{3.6a}$$

$$H_a(j\omega_1, j0) = C_2 \tag{3.6b}$$

$$H_a(j0, j0) = C_3 \tag{3.6c}$$

$$H_a(j\infty, j\omega_2) = 0 \tag{3.6d}$$

$$H_a(j\omega_1, j\infty) = 0 \tag{3.6e}$$

$$H_a(j\infty, j\infty) = 0 \tag{3.6f}$$

where C_1, C_2, and C_3 are nonzero constants.

As we mentioned in Chapter 2, from a 2-D analog low-pass transfer function, we can obtain 2-D low-pass digital filter by double bilinear transformations with specified ranges for its coefficients. Now, we need to determine the ranges of the coefficients of double linear transformations to obtain 2-D digital high-pass filters from 2-D low-pass transfer functions.

3.3.1 Generalized Bilinear Transformation Coefficients

In order to get the general results, for the 2-D analog low-pass filter with a denominator with single degree for each variable, we have the following general form

$$H_a(s_1,s_2)=\frac{K}{a_{11}s_1s_2+a_{10}s_1+a_{01}s_2+a_{00}} \tag{3.7}$$

where K is a positive constant.

Here, we consider the transfer function with a denominator with unity degree of z_1 and z_2, and the overall degree is two with z_1z_2, so $a_{11}\neq 0$ in equation (3.7).

From the properties of VSHP, all the coefficients of the denominator, a_{11}, a_{10}, a_{01} and a_{00} must be positive.

We apply double generalized bilinear transformations (1.37) to (3.7) to obtain the digital transfer function. The resulting 2-D transfer function in the discrete domain can be expressed as

$$H_d(z_1,z_2)=\frac{N_d(z_1,z_2)}{D_d(z_1,z_2)} \tag{3.8}$$

where

$$N_d(z_1,z_2)=(z_1+\beta_{01})(z_2+\beta_{02}) \tag{3.9a}$$

$$D_d(z_1,z_2)=a_{11}k_1k_2(z_1+\alpha_{01})(z_2+\alpha_{02})+a_{10}k_1(z_1+\alpha_{01})(z_2+\beta_{02})$$

$$+\alpha_{01}k_2(z_2+\alpha_{02})(z_1+\beta_{01})+a_{00}(z_1+\beta_{01})(z_2+\beta_{02}) \tag{3.9b}$$

The stability conditions require that the denominator of (3.8) cannot be zero at any points in the s_1-s_2 plane including the origin and the infinity.

$D_d(z_1,z_2)\neq 0$, for all z_1 and z_2. (3.10)

To get the frequency response for the resulting digital filter, letting

$$z_1 = e^{-j\omega_1} \tag{3.11a}$$

$$z_2 = e^{-j\omega_2} \tag{3.12b}$$

the digital transfer function (3.8) becomes

$$H_d(\omega_1,\omega_2) = \frac{N_d(\omega_1,\omega_2)}{D_d(\omega_1,\omega_2)} \tag{3.13}$$

where:

$$N_d(\omega_1,\omega_2) = (e^{-j\omega_1} + \beta_{01})(e^{-j\omega_2} + \beta_{02}) \tag{3.14a}$$

$$\begin{aligned} D_d(\omega_1,\omega_2) &= a_{11}k_1k_2(e^{-j\omega_1} + \alpha_{01})(e^{-j\omega_2} + \alpha_{02}) \\ &+ a_{10}k_1(e^{-j\omega_1} + \alpha_{01})(e^{-j\omega_2} + \beta_{02}) \\ &+ a_{01}k_2(e^{-j\omega_1} + \beta_{01})(e^{-j\omega_2} + \alpha_{02}) \\ &+ a_{00}(e^{-j\omega_1} + \beta_{01})(e^{-j\omega_2} + \beta_{02}) \end{aligned} \tag{3.14b}$$

To determine the ranges of the coefficients of the double generalized bilinear transfer functions, we investigate the different ranges of that β_{01} and β_{02}.

i) $\beta_{01} = -1$ or $\beta_{02} = -1$

When we choose the coefficient $\beta_{01}=-1$ or $\beta_{02}=-1$ or both equal to -1.0, we have the following relationship from (3.14a)

$$N_d(0,\omega_2) = N_d(\omega_1,0) = N_d(0,0) = 0 \tag{3.15}$$

As a result, the magnitudes of the digital filter are zero at the points $(0,\omega_2)$, $(\omega_1,0)$, and $(0,0)$ in ω_1-ω_2 plane, since the denominator is not zero at any time. So, we have

$$|H_d(0,\omega_2)| = |H_d(\omega_1,0)| = |H_d(0,0)| = 0 \tag{3.16}$$

It is easy to confirm that the filter will have the maximum magnitude at $\omega_1 = \omega_2 = \pm\pi$ (Radians)

So we can obtain 2-D high-pass filter when $\beta_{01}=\beta_{02}=-1$. And when $k_i>0$ ($i=1, 2$), the stability condition requires that α_{0i} ($i=1, 2$) need to have opposite signs to β_{0i}. It is possible to obtain 2-D high-pass digital filter from 2-D analog low-pass transfer function by generalized bilinear transformations with the constraints on the coefficients.

$$(i)\ k_i > 0, \qquad i = 1,2 \tag{3.17a}$$

$$(ii)\ 0 \le \alpha_{0i} \le 1.0, \qquad i = 1,2 \tag{3.17b}$$

$$(iii)\ \beta_{0i} = -1.0, \qquad i = 1,2 \tag{3.17c}$$

ii) β_{01} and β_{02} are in their negative parts

When we choose $-1.0 \le \beta_{01} < 0$ and $-1.0 \le \beta_{02} < 0$, then α_{01} and α_{02} would be nonnegative to meet the stability conditions. And the filter will have maximum magnitude at $\omega_1 = \pm\pi$ (*radians*) and $\omega_2 = \pm\pi$ (*radians*), and the filter will not have zero magnitude at $\omega_1=0$ (*radians*) or $\omega_2=0$ (*radians*). But it is still possible to obtain high-pass filter with nonzero and small gain in low frequency region, if the values of β_{01} and β_{02}, and other coefficients are properly chosen.

iii) β_{01} and β_{02} are in their positive parts

We have known from Chapter 2 that when we choose the values of β_{01} and β_{02} in their positive parts, 2-D low-pass filters will be achieved.

It can be concluded that to obtain 2-D recursive digital high-pass filters from 2-D analog low-pass transfer functions, the coefficients of double generalized bilinear transformations should satisfy

$$(i)\quad k_i > 0, \qquad i = 1,2 \tag{3.18a}$$

$$(ii)\quad 0 \le \alpha_{0i} \le 1.0, \qquad i = 1,2 \tag{3.18b}$$

$$(iii)\quad -1.0 \le \beta_{0i} < 0, \qquad i = 1,2 \tag{3.18c}$$

3.4 The Digital Transfer Function of 2-D High-Pass Filter

Applying the double generalized bilinear transformations (1.37) to the 2-D analog low-pass transfer function in (3.2), the digital transfer function for the resulting 2-D high-pass filter can be obtained. Specifically,

$$H_d(z_1,z_2)=\frac{N_d(z_1,z_2)}{D_d(z_1,z_2)} \quad (3.19)$$

where,

$$N_d(z_1,z_2)=z_1z_2+\beta_{02}z_1+\beta_{01}z_2+\beta_{01}\beta_{02} \quad (3.20a)$$

$$\begin{aligned}D_d(z_1,z_2)=&(k_1k_2+0.707k_1+1.414k_2+2)z_1z_2\\&+(k_1.k_2\alpha_{02}+0.707k_1\beta_{02}+1.414k_2\alpha_{02}+2\beta_{02})z_1\\&+(k_1k_2\alpha_{01}+0.707k_1\alpha_{01}+1.414k_2\beta_{01}+2\beta_{01})z_2\\&+(k_1k_2\alpha_{01}\alpha_{02}+0.707k_1\alpha_{01}\beta_{02}+1.414k_2\alpha_{01}\beta_{01}\\&+2\beta_{01}\beta_{02})\end{aligned} \quad (3.20b)$$

For convenience, $D_d(z_1, z_2)$ could be written in the general form of 2-variable polynomial with single degree in each variable

$$D_a(z_1,z_2)=a_{11}z_1z_2+a_{10}z_1+a_{01}z_2+a_{00} \quad (3.21)$$

where,

$$a_{11}=k_1k_2+0.707k_1+1.414k_2+2 \quad (3.22a)$$

$$a_{10}=k_1.k_2\alpha_{02}+0.707k_1\beta_{02}+1.414k_2\alpha_{02}+2\beta_{02} \quad (3.22b)$$

$$a_{01}=k_1k_2\alpha_{01}+0.707k_1\alpha_{01}+1.414k_2\beta_{01}+2\beta_{01} \quad (3.22c)$$

$$a_{00}=k_1k_2\alpha_{01}\alpha_{02}+0.707k_1\alpha_{01}\beta_{02}+1.414k_2\alpha_{01}\beta_{01}+2\beta_{01}\beta_{02} \quad (3.22d)$$

When we choose the coefficients of the double generalized bilinear transformations in the ranges defined in Equation (3.18a) – (3.18c), the transfer function (3.19) represents a 2-D high-pass digital filter. And when one or more coefficients are changing, the resulting 2-D high-pass digital filter has various magnitude characteristics. The MATLAB® function *highPass.m* (see Appendix) is employed to obtain the contour and 3-D magnitude response plots for the resulting 2-D high-pass filters. In the function, we take the values of all the coefficients as the input arguments, and the function returns the contour and 3-D magnitude response plots for the 2-D high-pass filter with the specified coefficients. Of course, the stability is first tested for each combination of these coefficients in the function.

As the changeable coefficients may affect the stability of the resulting 2-D high-pass filter, additional stability test criteria need to be introduced to test the stability of the resulting 2-D high-pass filter in discrete domain.

3.5 The Stability Conditions of 2-D Digital Filter with a Single Degree Denominator for Each Variable

In Chapter 2, we have already got the stability conditions for 2-D recursive digital transfer function with the unity degree denominators with the general form

$$D_d(z_1,z_2) = a_{11}z_1z_2 + a_{10}z_1 + a_{01}z_2 + a_{00} \tag{3.23}$$

The stability conditions are

$$a_{11}\beta_{01}\beta_{02} - a_{10}\beta_{01} - a_{01}\beta_{02} + a_{00} > 0 \tag{3.24}$$

$$-a_{11}\alpha_{02}\beta_{01} + a_{10}\beta_{01} + a_{01}\alpha_{02} - a_{00} > 0 \tag{3.25}$$

$$-a_{11}\alpha_{01}\beta_{02} + a_{10}\alpha_{01} + a_{01}\beta_{02} - a_{00} > 0 \tag{3.26}$$

$$a_{11}\alpha_{01}\alpha_{02} - a_{10}\alpha_{01} - a_{01}\alpha_{02} + a_{00} > 0 \tag{3.27}$$

where a_{11}, a_{10}, a_{01} and a_{00} should be evaluated in the ranges specified in Equations (3.24)-(3.27) to meet the stability requirements. And also the coefficients of the double generalized bilinear transformations should first meet the conditions given in Equations (3.17 a)- (3.17c).

The MATLAB® functions *k1HPRange.m*, *k2HPRange.m*, *a1HPRange.m*, *a2HPRange.m*, *b1HPRange.m* and *b2HPRange.m* are used to determine the ranges of k_1, k_2, α_{01}, α_{02}, β_{01} and β_{02}, respectively, when the other coefficients are specified.

3.6 Frequency Response of the 2-D High-Pass Digital Filters

The MATLAB® function *highPass.m* is employed to plot the magnitude contour and 3-D magnitude response of the resulting 2-D high-pass.

3.6.1 Frequency Response of the Resulting 2-D High-Pass Filter with Variable k_1

When the other coefficients are fixed, that is k_2=1.0, α_{01}=1.0, α_{02}=1.0, β_{01}=−1.0 and β_{02}=−1.0, the range of k_1 can be determined using the MATLAB® function *k1HPRange.m*. The result is shown in Figure 3.2. Here we still use 1000 to represent the infinite value of k_1.

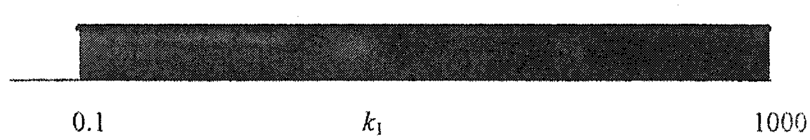
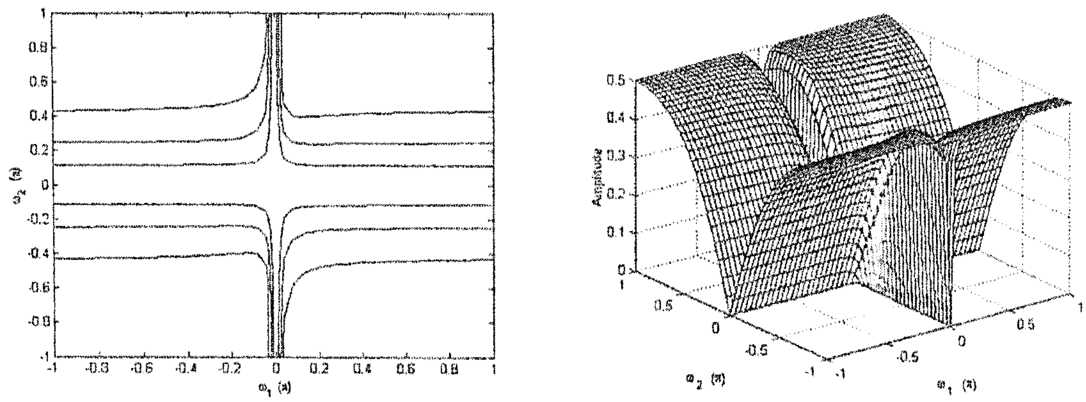


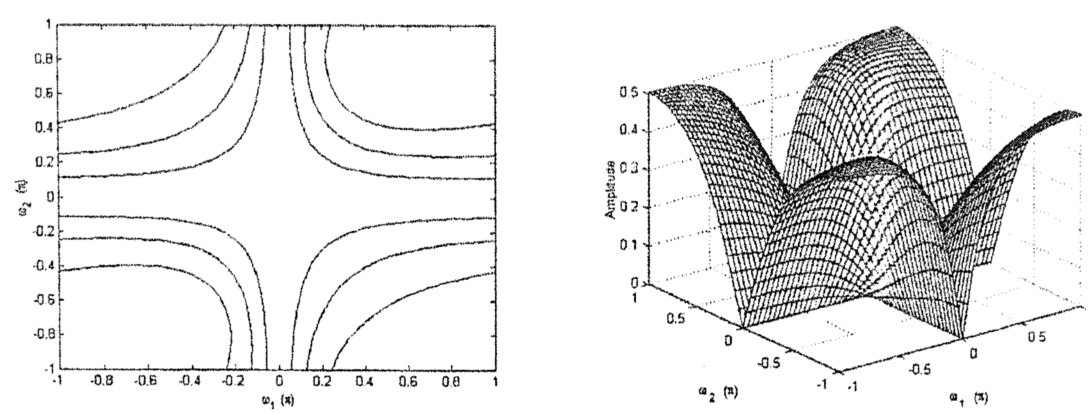
Figure 3.2. The range of k_1 when the other coefficients are set to k_2=1.0, α_{01}=1.0, α_{02}=1.0, β_{01}=−1.0 and β_{02}=−1.0

From Figure 3.2, it is obvious that any value of k_1 in the range of (0, +∞) could result in stable 2-D high-pass digital filters when the other coefficients are set to be

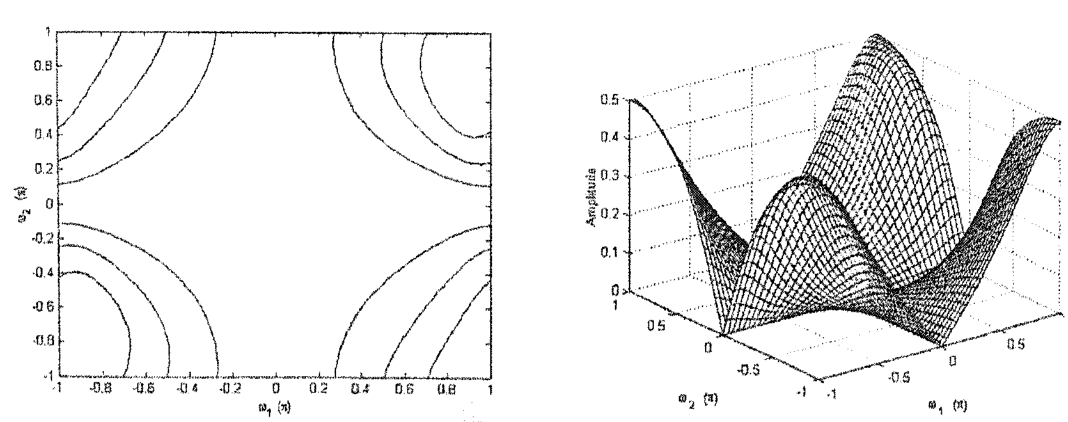
k_2=1.0, α_{01}=1.0, α_{02}=1.0, β_{01}=−1.0 and β_{02}=−1.0. The contour and 3-D magnitude plots of the resulting 2-D high-pass filters with different values of k_1 are given in Figure 3.3.



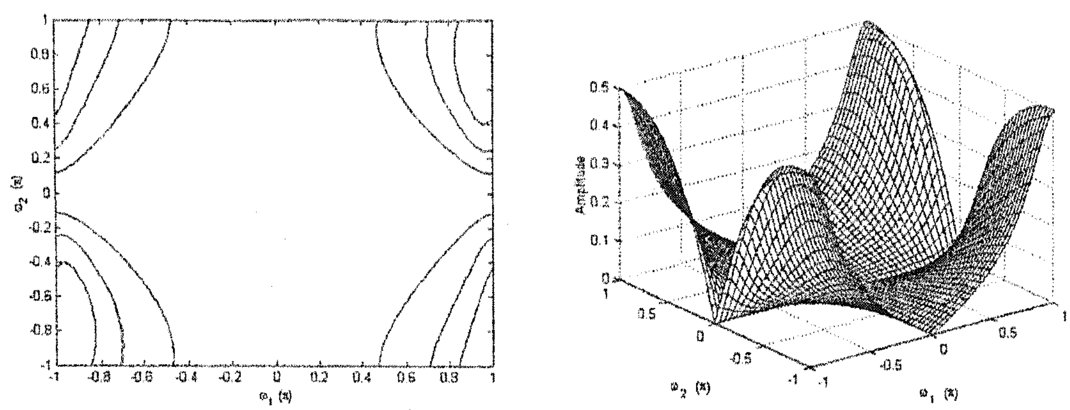
a. $k_1 = 0.1$



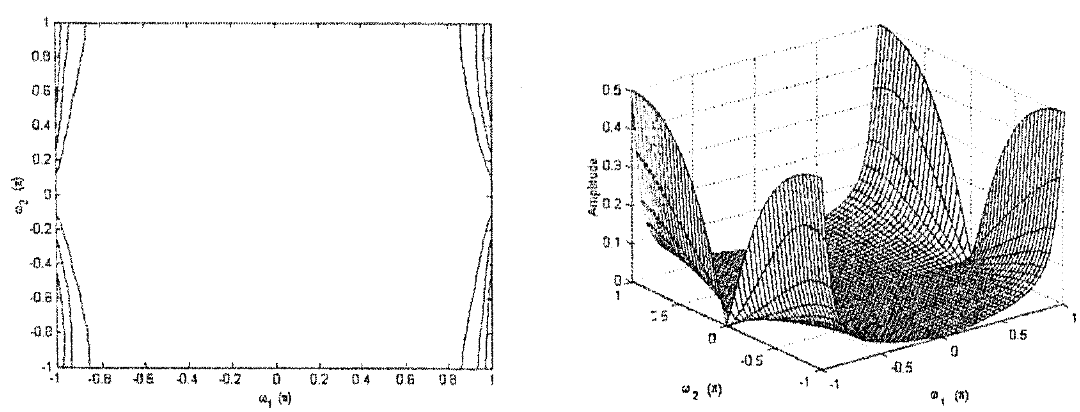
b. $k_1 = 1.0$



c. $k_1 = 5.0$



d. $k_1 = 10.0$



e. $k_1 = 50.0$

Figure 3.3 The contour and 3-D magnitude plots of the resulting 2-D high-pass filter with variable k_1 and the other coefficients fixed as k_2=1.0, α_{01}=1.0, α_{02}=1.0, β_{01}=−1.0 and β_{02}=−1.0

When the value of k_1 changes, the resulting 2-D high-pass filter has variable magnitude responses. The coefficient k_1 mainly affects the bandwidth in the pass-band of the resulting filter in ω_1-dimension. When k_1 increases from the lower boundary of slightly greater than zero to a higher value, the pass-band in ω_1-dimension becomes smaller, and the stop-band becomes wider. The value of k_1 does not affect the bandwidth of the pass-band in ω_2-dimension.

In general, the first- and the third-quadrants have bigger pass-band areas than the second- and the fourth- quadrants.

3.6.2 Frequency Responses of the Resulting 2-D High-Pass filter with Variable k_2

When we set $k_1 = 1.0$, $\alpha_{01} = 1.0$, $\alpha_{02} = 1.0$, $\beta_{01} = -1.0$ and $\beta_{02} = -1.0$, the range of k_2 can be computed by the MATLAB® function *k2HPRange.m*. The output of the function *k2HPRange.m* with the mentioned specified coefficients is shown in Figure 3.4.

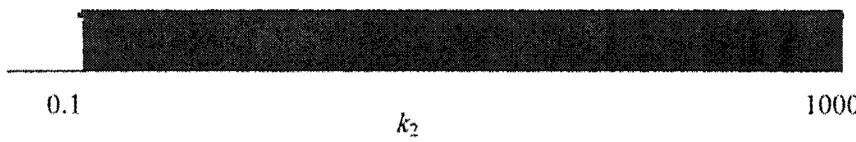
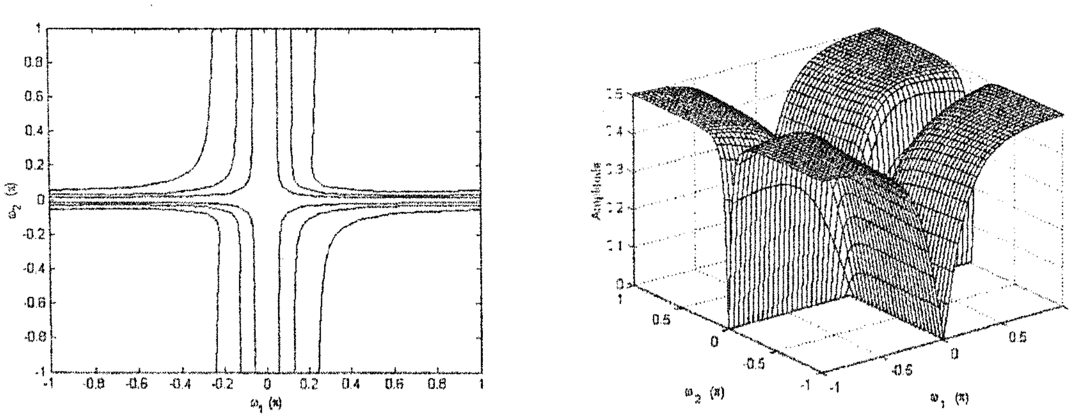


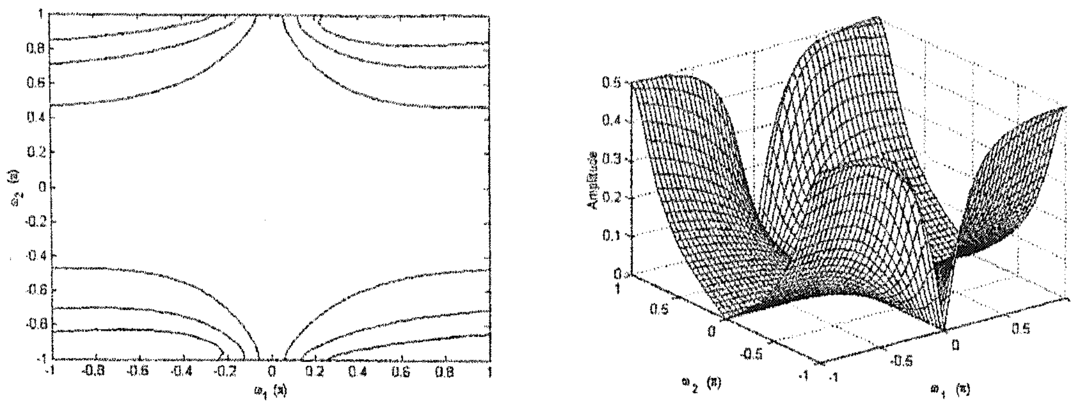
Figure 3.4 The range of k_2 when the other coefficients are set to be k_1=1.0, α_{01}=1.0, α_{02}=1.0, β_{01}=−1.0 and β_{02}=−1.0

Obviously, from Figure 3.4, any value of k_2 in the range of $(0, +\infty)$ can result in a stable 2-D high-pass digital filter when the other coefficients of bilinear transformations are fixed to be unity with proper signs.

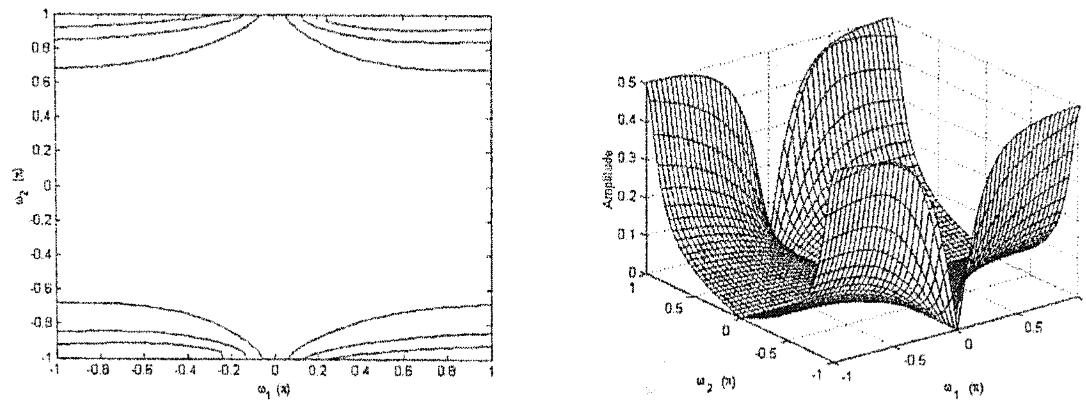
The contour and 3-D magnitude response plots of the resulting 2-D high-pass filter with different values of k_2 are given in Figure 3.5.



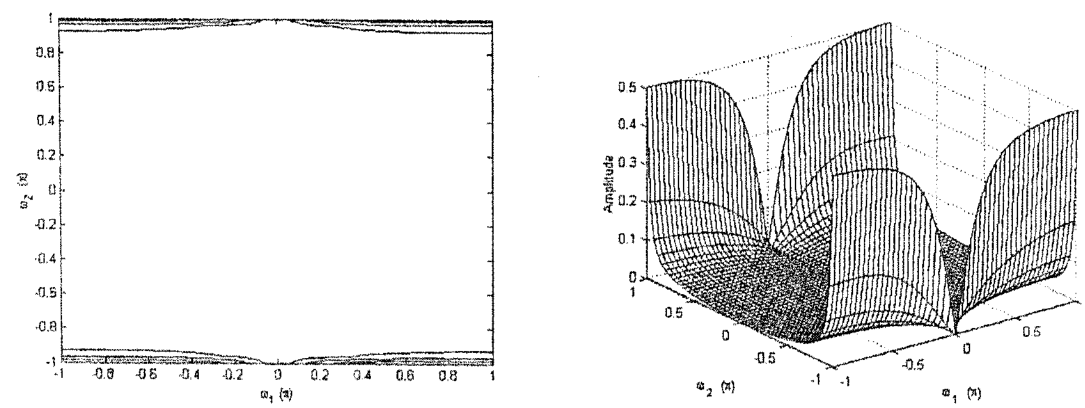
a. $k_2 = 0.1$



b. $k_2 = 5.0$



c. $k_2 = 10.0$



d. $k_2 = 50.0$

Figure 3.5 The contour and 3-D magnitude plots of the resulting 2-D high-pass filter with variable k_2 and the other coefficients fixed as k_1=1.0, α_{01}=1.0, α_{02}=1.0, β_{01}=−1.0 and β_{02}=−1.0

Changing the value of k_2, we can obtain different magnitude responses of the resulting 2-D high-pass filter in ω_2 dimension. When k_2 changes from the lower boundary of slightly greater than 0 to higher values, the pass-band of the filter in ω_2 dimension become smaller, while the stop band in ω_2 dimension becomes wide.

On the other hand, the changing values of k_2 have no any effect on the behaviours of the resulting 2-D high-pass filter in ω_1 dimension.

3.6.3 Frequency Response of the Resulting 2-D High-Pass Filter with Variable α_{01}

When the other coefficients are set to the specified values, the range for α_{01} can be computed using the MATLAB® function *a1HPRange.m*. As in the previous sections, we still fixed the other coefficients other than α_{01} to the specified values, say k_1=1.0, k_2=1.0, α_{02}=1.0, β_{01}=−1.0 and β_{02}=−1.0. The range of α_{01} is indicated in Figure 3.6.

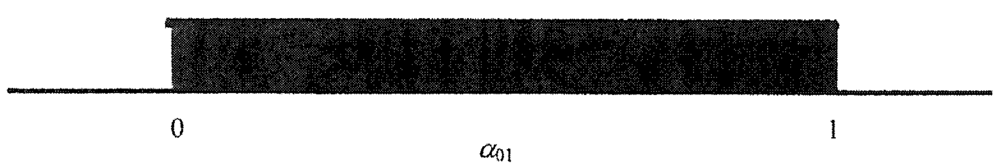
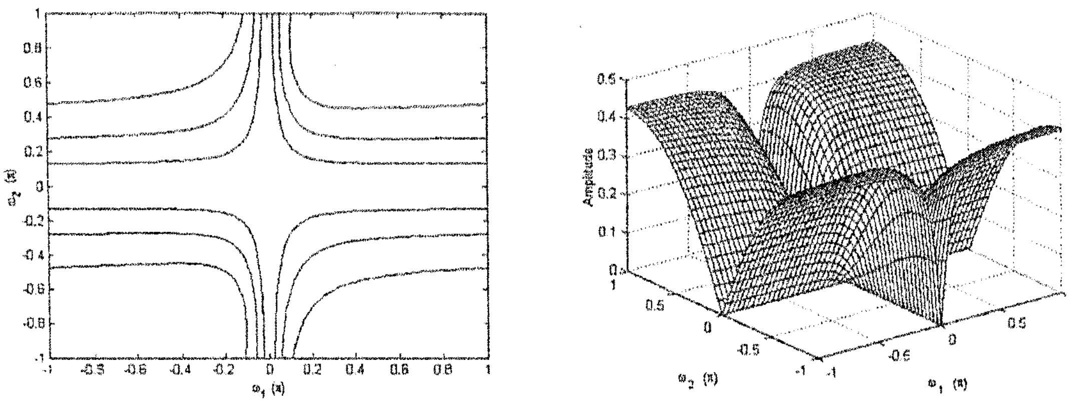


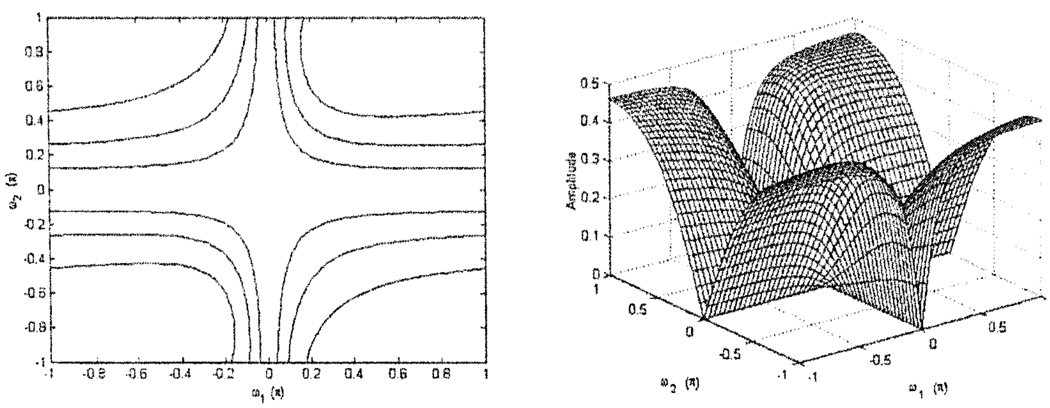
Figure 3.6 The range of α_{01} when the other coefficients are fixed as k_1=1.0, k_2=1.0, α_{02}=1.0, β_{01}=−1.0 and β_{02}=−1.0

Any value of α_{01} in the range of [0, 1.0] can produce a stable 2-D high-pass filter when the other coefficients are set to be k_1=1.0, k_2=1.0, α_{02}=1.0, β_{01}= −1.0 and β_{02}=−1.0.

The contour and 3-D magnitude plots of the resulting 2-D high-pass filter with α_{01}=0 and α_{01}=0.5 are given in Figure 3.7 a and b, respectively.



a. $\alpha_{01} = 0$



b. $\alpha_{01} = 0.5$

Figure 3.7 The contour and 3-D magnitude plots of the resulting 2-D high-pass filter with variable α_{01} and the other coefficients fixed as k_1=1.0, k_2=1.0, α_{02}=1.0, β_{01}=−1.0 and β_{02}=−1.0

From figure 3.7, it can be observed that α_{01} only affects the behaviors of the resulting filter in ω_1 dimension. When the value of α_{01} changes from the lower boundary of 0 to the upper boundary of 1.0, the pass-band in ω_1 dimension becomes slight smaller, but the stop-band remains almost unchanged; as a result, the transition band becomes wider with the increase of the value of α_{01}.

As the allowable variation range of α_{01} is limited, the effect of α_{01} on the behavior of the resulting 2-D recursive digital filter is small. In practice, α_{01} could be used to adjust the pass-band in ω_1 dimension slightly.

However, the main effect of α_{01} lies in the gain in the pass-band of the resulting 2-D high-pass filters. The bigger the value of α_{01}, the larger the gain is. So, similar to the case of 2-D low-pass filter, we define α_{01} as the **gain-effect coefficient**.

3.6.4 Frequency Response of the Resulting 2-D High-Pass filter with Variable α_{02}

When we change the value of α_{02} to demonstrate the variable magnitude response for the resulting 2-D high-pass filter, we can let the other coefficients be unity with proper signs, specifically k_1=1.0, k_2=1.0, α_{01}=1.0, β_{01}=−1.0 and β_{02}=−1.0. The range of α_{02} is shown in Figure 3.8.

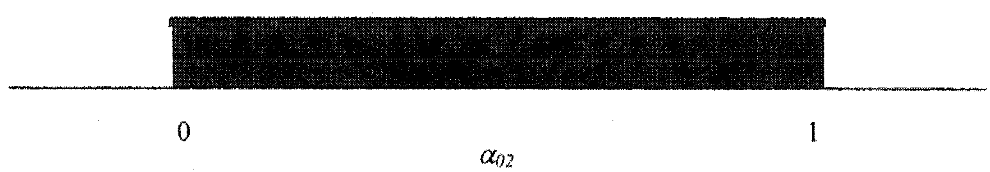
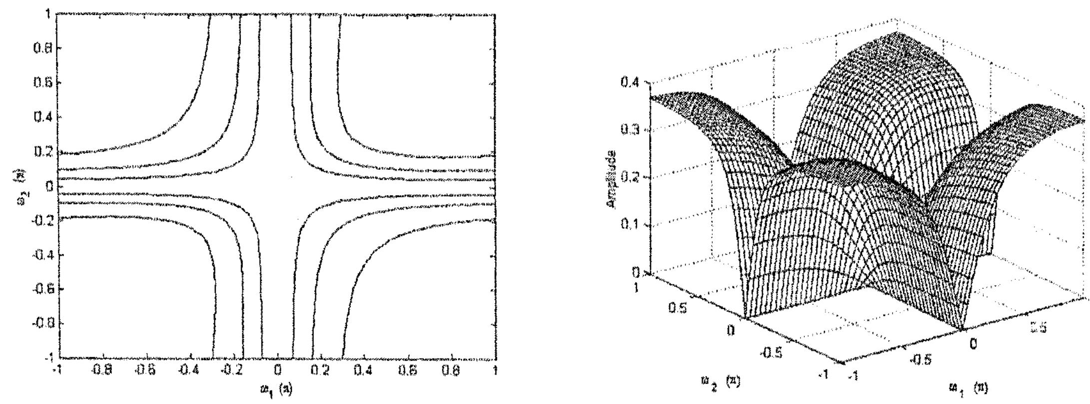
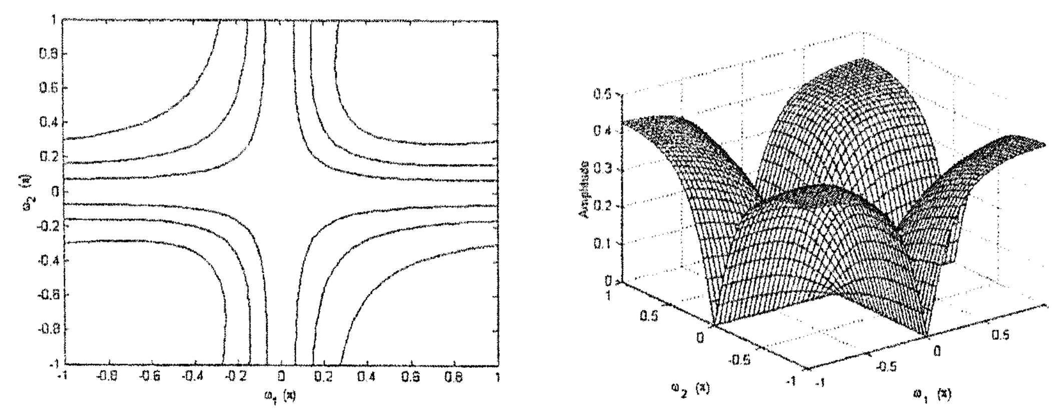


Figure 3.8 The range of α_{02} when the other coefficients are set to be k_1=1.0, k_2=1.0, α_{01}=1.0, β_{01}=−1.0 and β_{02}=−1.0

From Figure 3.8, we can observe that any value of α_{02} in the range of [0, 1.0], with other coefficients set to unity with proper signs, could result in a stable 2-D high-pass filter. The contour and 3-D magnitude response plots of the resulting 2-D high-pass filter with α_{02}=0 and α_{02}=0.5 are given in Figure 3.9.



a. $\alpha_{02} = 0$



b. $\alpha_{02} = 0.5$

Figure 3.9 The contour and 3-D magnitude plots of the resulting 2-D high-pass filter with variable α_{02} and the other coefficients fixed as k_1=1.0, k_2=1.0, α_{01}=1.0, β_{01}=−1.0 and β_{02}=−1.0

It is obvious that α_{02} mainly affects the gain in the pass-band of the resulting 2-D high-pass filter. When we choose the value of α_{02} as the lower bound zero, the gain in the pass-band of the filter is about 0.38, while the gain is 0.5 when α_{02} reaches the biggest value 1.0.

The changing value of α_{02} also affects the bandwidth of the pass-band in ω_2 dimension. As the value of α_{02} increases from zero to 1.0, the bandwidth of the pass-band in ω_2 dimension becomes smaller. However, as the range of α_{02} is limited, the effect on the bandwidth of the pass-band is small.

3.6.5 Frequency Response of the Resulting 2-D High-Pass Filter with Equal Variables α_{01} and α_{02}

In section 3.6.3 and 3.6.4, we investigated the manner in which how α_{01} and α_{02} individually affect the magnitude response of the resulting 2-D high-pass filter respectively. Both the coefficients mainly affect the gains of the pass-bands of the resulting filter, although they also have slight effect on the bandwidth of the pass-bands. In this section, we will investigate the combined effect from the two coefficients.

As in the previous subsection, the other coefficients other than α_{01} and α_{02}, which have changing values, are fixed to the specified values, say k_1=1.0, k_2=1.0, β_{01}=−1.0, and β_{02}=−1.0. The values of α_{01} and α_{02} are kept to have equal values while changing their values. Figure 3.10 is the range of α_{01} and α_{02} when the other ones are set to the specified values.

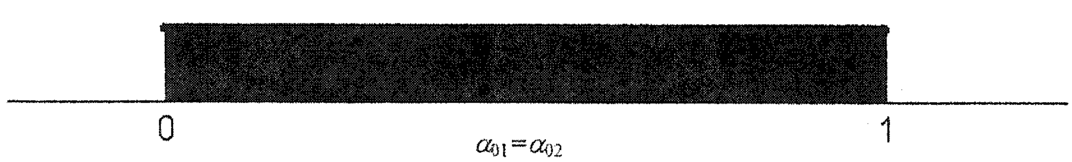
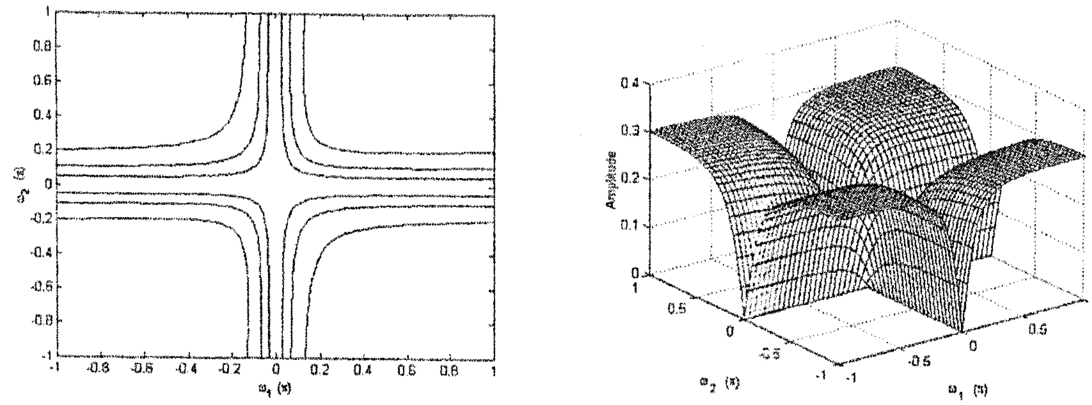


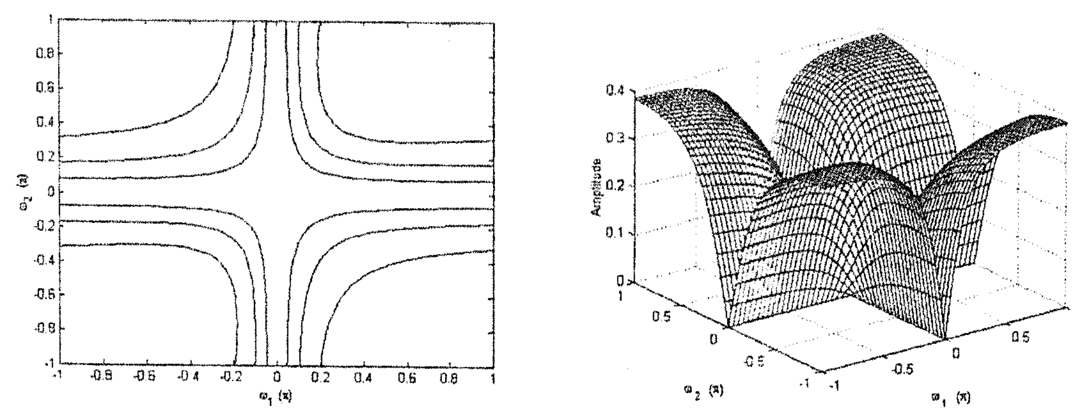
Figure 3.10 The range of equal α01 and α02 when the other coefficients are set to be k_1=1.0, k_2=1.0, β_{01}=−1.0 and β_{02}=−1.0

Any value of equal α_{01} and α_{02} in the range of [0, 1.0] can result in a stable 2-D high-pass filter when the other coefficients are set to be k_1=1.0, k_2=1.0, β_{01}=−1.0, and β_{02}=−1.0. The contour and 3-D magnitude plots of the resulting 2-D high-pass filter with

$\alpha_{01}=\alpha_{02}=0$ and $\alpha_{01}=\alpha_{02}=0.5$ are given in Figure 3.11.



a. $\alpha_{01} = \alpha_{02} = 0$



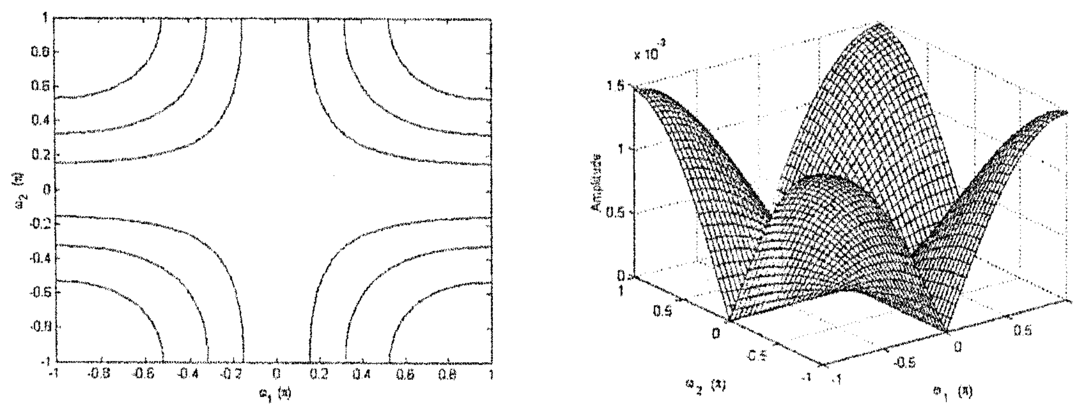
b. $\alpha_{01} = \alpha_{02} = 0.5$

Figure 3.11 The contour and 3-D magnitude plots of the resulting 2-D high-pass filter with equal variables α_{01} and α_{02} and the other coefficients fixed as $k_1=1.0$, $k_2=1.0$, $\beta_{01}=-1.0$ and $\beta_{02}=-1.0$

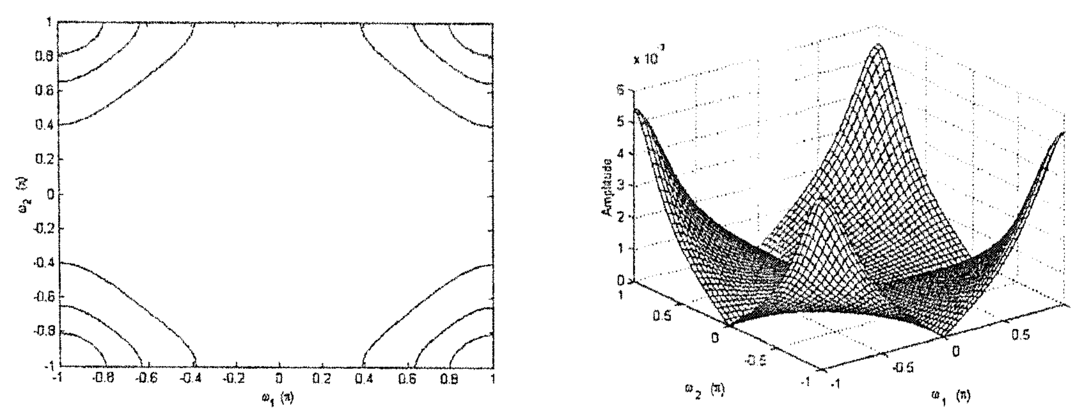
The effect on the gain in the pass-band of the resulting 2-D high-pass filter becomes more pronounced, when the two coefficients, α_{01} and α_{02}, are changing simultaneously. When their values change from the lower bound 0 to the upper one 1.0, the gain in the pass-bands increases from 0.3 to 0.5, while when we only change one of the coefficients, the minimum value of the gain is about 0.4. As shown above, changing α_{01} and α_{02}

implies changing the bandwidth of the pass-band in ω_1 and ω_2 dimensions, respectively.

The effects of α_{01} and α_{02} on the gain and bandwidth of the pass-band could be amplified by increasing the values of k_i's (i=1,2). The simulation results are given in Figure 3.12.



a. $\alpha_{01}=\alpha_{02}=0$



b. $\alpha_{01}=\alpha_{02}=0.5$

Figure 3.12 The contour and 3-D magnitude plots of the resulting 2-D high-pass filter with equal variables α_{01} and α_{02} and the other coefficients fixed as k_1=50.0, k_2=50.0, β_{01}=–1.0 and β_{02}=–1.0

Comparing the two filters in Figure 3.11 and Figure 3.12, we can find that the filter with bigger k_i's has the smaller gain in the pass-band even if the values of α_{01} and α_{02} are the same.

The effects of α_{01} and α_{02} on the bandwidth are clear when we compare the plots of the filter with $\alpha_{01}=\alpha_{02}=1.0$, $\alpha_{01}=\alpha_{02}=0.5$ and those with $\alpha_{01}=\alpha_{02}=0$ at $k_1=k_2=50.0$.

3.6.6 Frequency Response of the Resulting 2-D High-Pass Filters with Variable β_{01}

The MATLAB® function *b1HPRange.m* is employed to obtain the range of β_{01}, when the other coefficients are fixed. Here, we still set the other coefficients to be $k_1=1.0$, $k_2=1.0$, $\alpha_{01}=1.0$, $\alpha_{02}=1.0$ and $\beta_{02}=-1.0$, when the value of β_{01} is changing. The range of β_{01} is given in Figure 3.13.

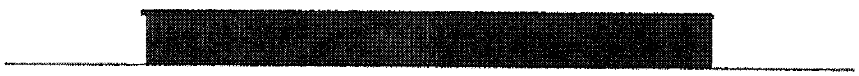
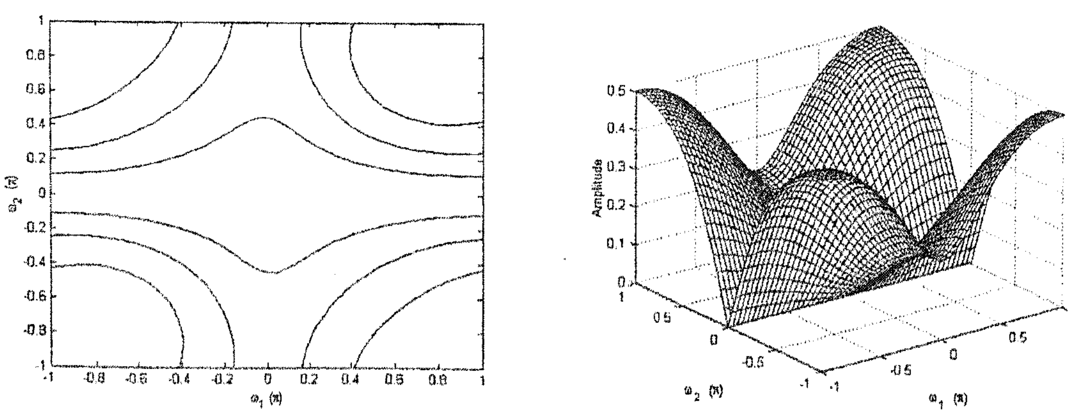
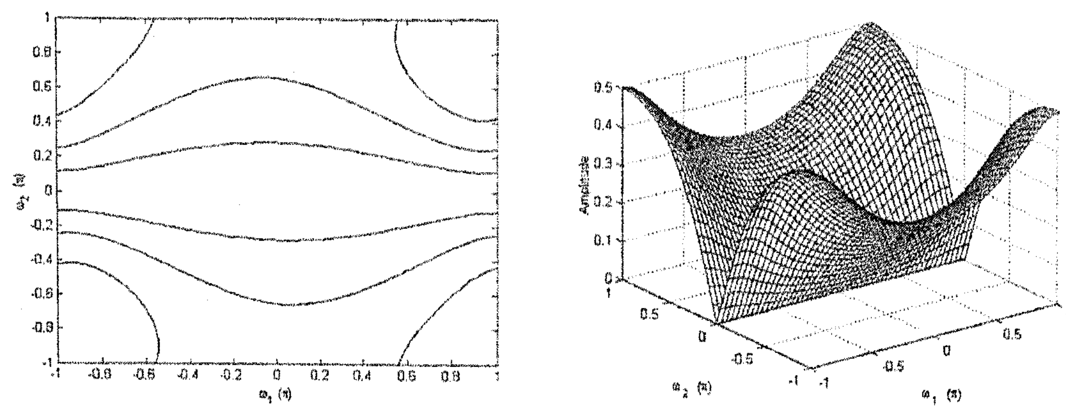


Figure 3.13 The range of β_{01} when the other coefficients are set to be $k_1=1.0$, $k_2=1.0$, $\alpha_{01}=1.0$, $\alpha_{02}=1.0$ and $\beta_{02}=-1.0$

From Figure 3.13, we can see that any value of β_{01} in the range of [−1.0, 0] is possible in designing a 2-D high-pass filter, when the other coefficients are set to be unity with proper signs. The contour and 3-D magnitude response plots are illustrated in Figure 3.14.



a. $\beta_{01}=-0.5$

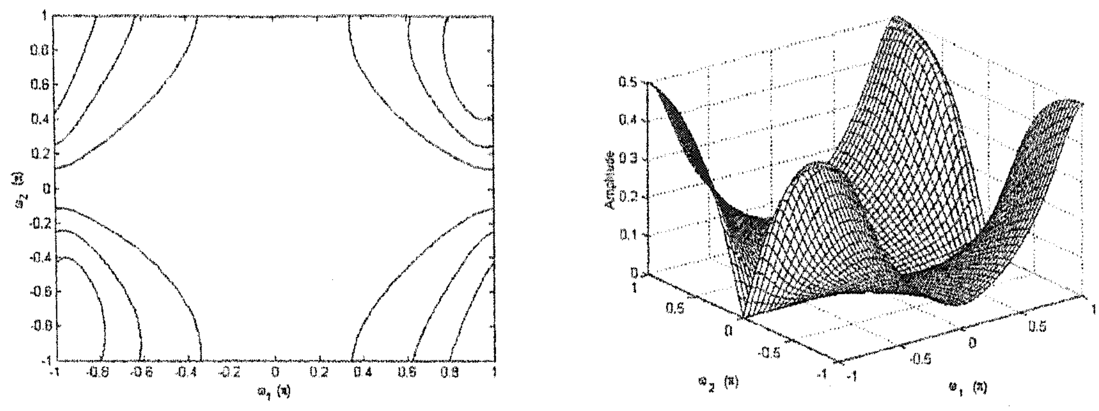


b. $\beta_{01} = 0$

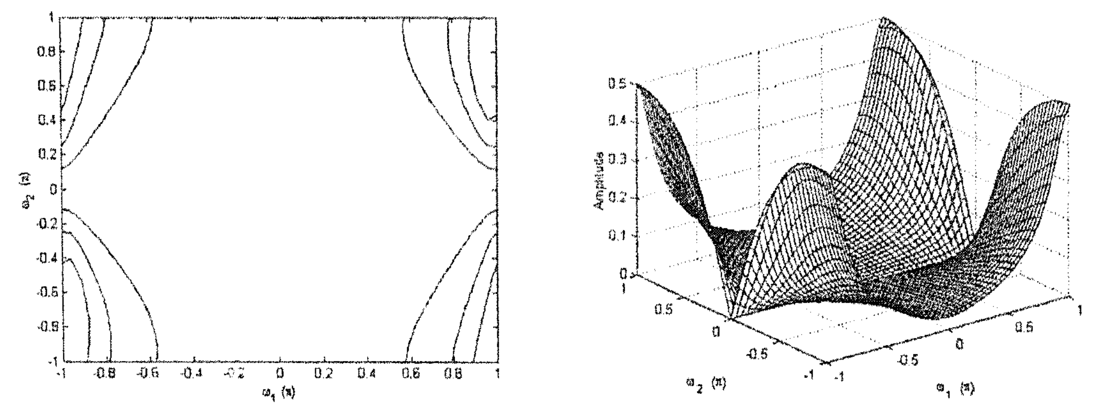
Figure 3.14 The contour and 3-D magnitude plots of the resulting 2-D high-pass filter with variable β_{01} and the other coefficients fixed as k_1=1.0, k_2=1.0, α_{01}=1.0, α_{02}=1.0 and β_{02}=−1.0

From Figure 3.14, we can clearly observe that when $\beta_{01} \neq -1.0$, the gain of the stop-band of the resulting 2-D high-pass filter in ω_1 dimension is nonzero, and even this gain can be half of the one of the pass-band when $\beta_{01} \rightarrow 0$. From Chapter 2, we know that when the value of β_{01} is in its positive part, the resulting filter will become a low-pass one. That is why we get non-zero gain at the stop-band when β_{01} is chosen as a value other than −1.0. Because of the polarity change, we define β_{01} and β_{02} as **polarity-effect coefficients**.

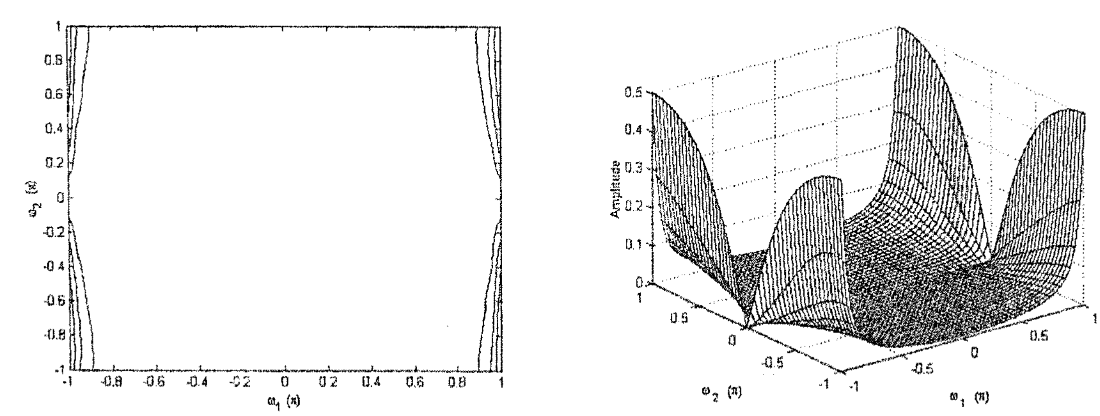
Like the case of the low-pass filter, we can deal with the non-zero gain caused by β_{01} in the stop-band of ω_1 dimension properly. Although from the mathematical point of view, we cannot obtain absolute zero gain at $\omega_1 = 0$ radians unless we choose $\beta_{01} = -1$, it is still possible to make the gain small enough to meet the design specifications by increasing the value of k_1. Figure 3.15 shows the reduced gain as k_1 increases. In Figure 3.15, we have used the same values as in Figure 3.14 (b) except that k_1 is changed to 5.0, 10.0, and 50.0.



a. $k_1 = 5.0$



b. $k_1 = 10.0$



c. $k_l = 50.0$

Figure 3.15 The contour and 3-D magnitude plots of the resulting 2-D high-pass filter with variable k_1 and the other coefficients fixed as k_2=1.0, α_{01}=1.0, α_{02}=1.0, β_{01}=−0.5 and β_{02}=−1.0

By increasing the value of k_1, it is possible to reduce the nonzero gain in stop-band of the resulting 2-D high-pass filter caused by β_{01}. The price is the sacrifice of bandwidth of the pass-band in ω_1-dimension. The proper values of β_{01} and k_1 should be determined by the design specifications and optimization techniques, and the combination of β_{01} and k_1 could not be unique to meet the design specifications.

3.6.7 Frequency Response for the Resulting 2-D High-Pass Filter with Variable β_{02}

Using the same procedure in section 3.6.6, we can investigate the variable magnitude behavior of the resulting 2-D high-pass filter caused by variable β_{02}. The stability is always the most important issue in 2-D recursive filter design. The MATLAB® function *b2HPRange.m* is used to obtain the range of β_{02}, when the other coefficients are specified. Figure 3.16 indicates the range of β_{02}, when the other coefficients are chosen to be unity with proper signs.

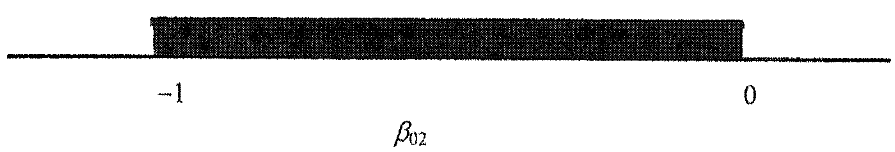
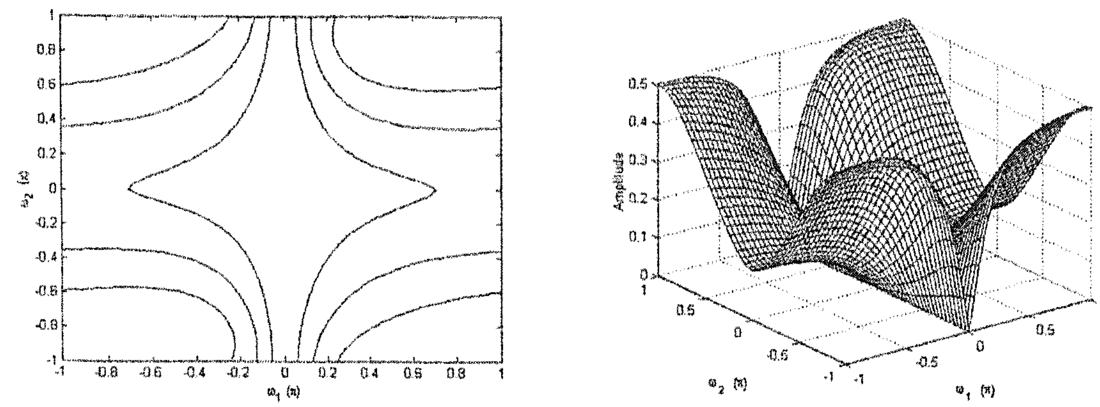


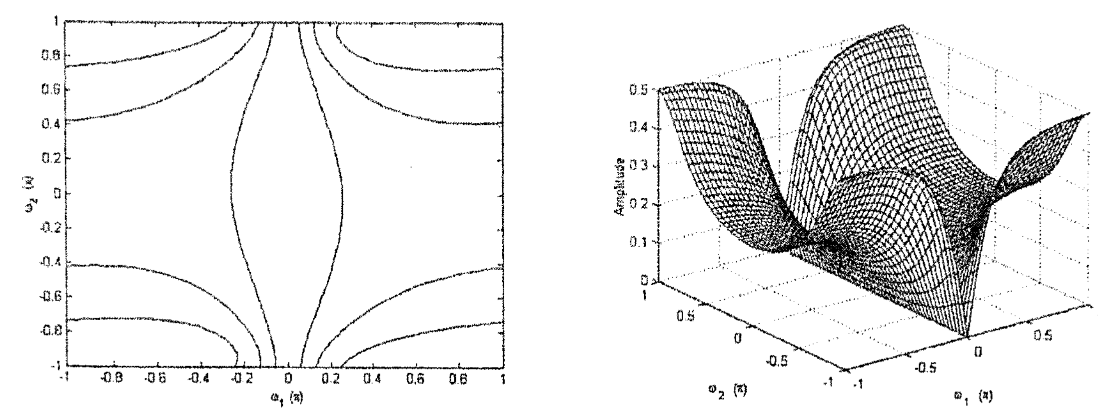
Figure 3.16 The range of β_{02} when the other coefficients set to be k_1=1.0, k_2=1.0, α_{01}=1.0, α_{02}=1.0 and β_{01}=−1.0

From Figure 3.16, any value of β_{02} in the range of [−1.0, 0] can make the resulting 2-D high-pass filter system stable, when the other coefficients are set to be k_1=1.0, k_2=1.0, α_{01}=1.0, α_{02}=1.0 and β_{01}=−1.0.

The contour and 3-D magnitude response plots of the resulting 2-D high-pass filter with different values of β_{02} are illustrated in Figure 3.17.



a. $\beta_{02} = -0.5$

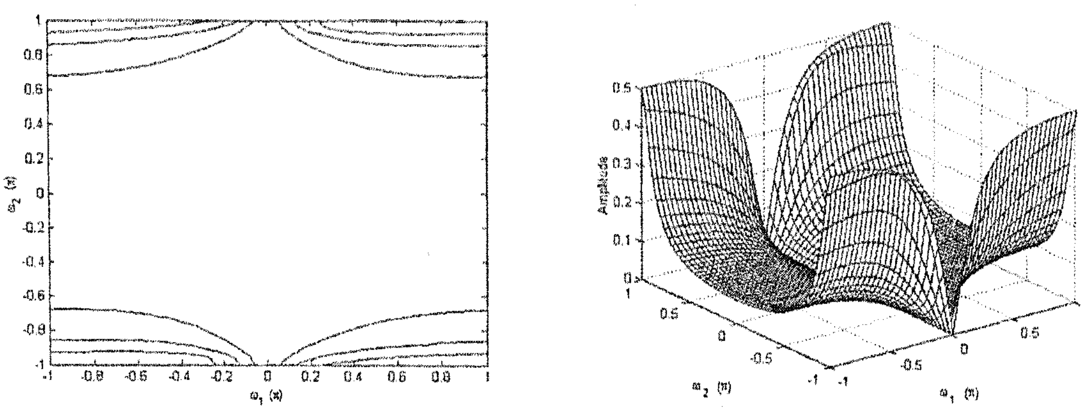


b. $\beta_{02} = 0$

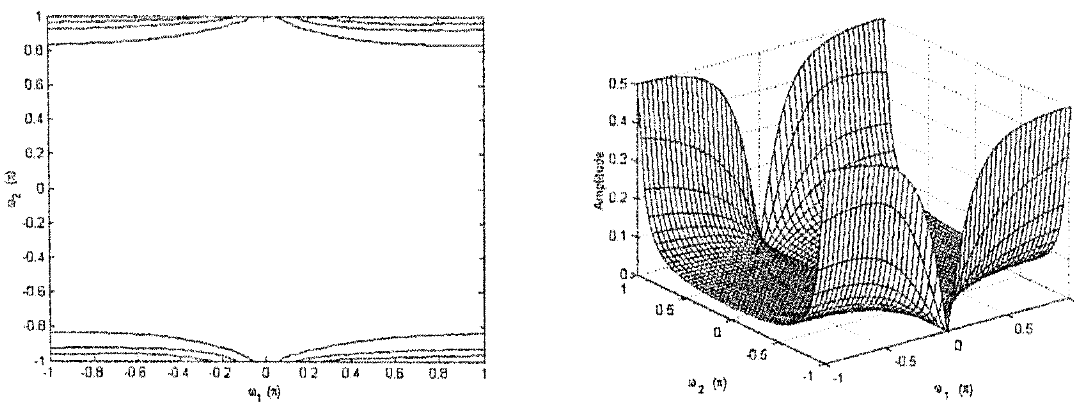
Figure 3.17 The contour and 3-D magnitude plots of the resulting 2-D high-pass filter with variable β_{02} and the other coefficients fixed as k_1=1.0, k_2=1.0, α_{01}=1.0, α_{02}=1.0 and β_{01}=−1.0

The same phenomena as in the previous section are found here. When β_{02} is chosen to be a value other than −1.0, the gain of the stop-band in ω_2-dimension will not be zero.

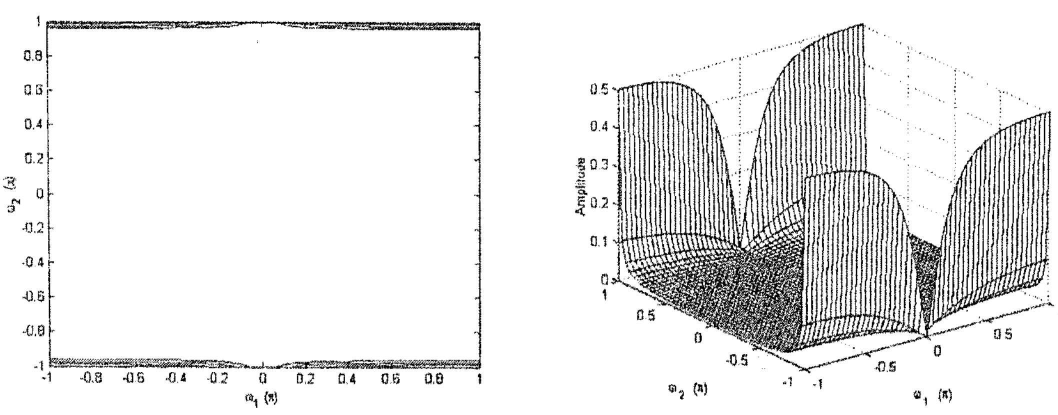
To reduce the non-zero gain, we can increase the value of k_2. Although we cannot get a zero gain, we still can make the value small enough to meet the design specifications. The contour and 3-D magnitude response plots with k_2 =5.0, 10.0 and 50.0 are given in Figure 3.18.



a. $k_2 = 5.0$



b. $k_2 = 10.0$



c. $k_2 = 50.0$

Figure 3.18 The contour and 3-D magnitude plots of the resulting 2-D high-pass filters with variable k_2 and the other coefficients fixed as k_1=1.0, α_{01}=1.0, α_{02}=1.0, β_{01}=−1.0 and β_{02}=−0.5

From Figure 3.18, increasing the value of k_2 could really reduce the nonzero gain in stop-band of the resulting 2-D high-pass filter when β_{02} is not chosen as -1.0. However, increasing the value of k_2 reduces the bandwidth of the pass-band in the second dimension. How to balance the gain and the loss needs to use optimization techniques and should subject to the constraints required by the design specifications.

3.6.8 Frequency Response of the Resulting 2-D High Pass Filters with Equal Variables β_{01} and β_{02}

The coefficients β_{01} and β_{02} mainly affect the gains of the stop-bands in their corresponding dimensions. The resulting 2-D high-pass filter always has a zero gain at the origin (0,0) in ω_1-ω_2 plane, when one of the two coefficients is -1.0.

Now we intend to investigate the combined effect from the two coefficients. As in the previous subsections, we keep the other coefficients unchanged when we change β_{01} and β_{02} simultaneously to obtain variable magnitude responses.

Figure 3.19 is the range of the equally β_{0i}'s when the other coefficients are specified to be unity with proper signs.

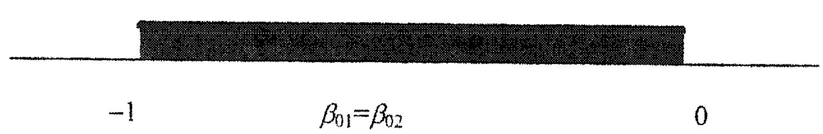
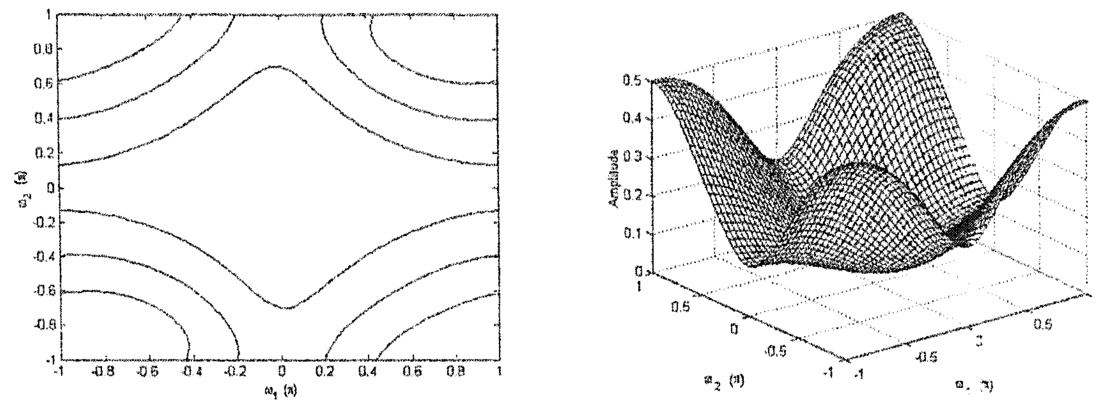
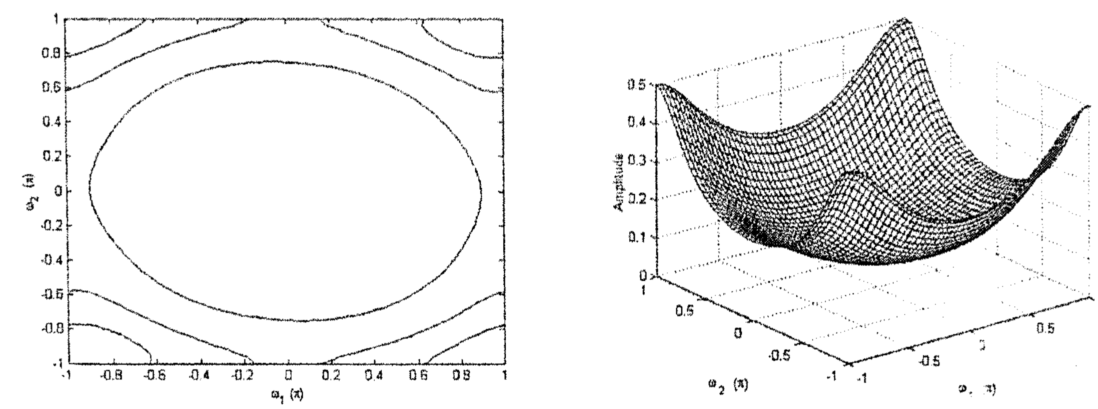


Figure 3.19 The range of equal β_{01} and β_{02} when the other coefficients are set to be k_1=1.0, k_2=1.0, α_{01}=1.0 and α_{02}=1.0

Figure 3.20 are the contour and 3-D magnitude plots of the resulting 2-D high-pass filter with equal variables β_{0i}'s, which are in the range specified in Figure 3.19.



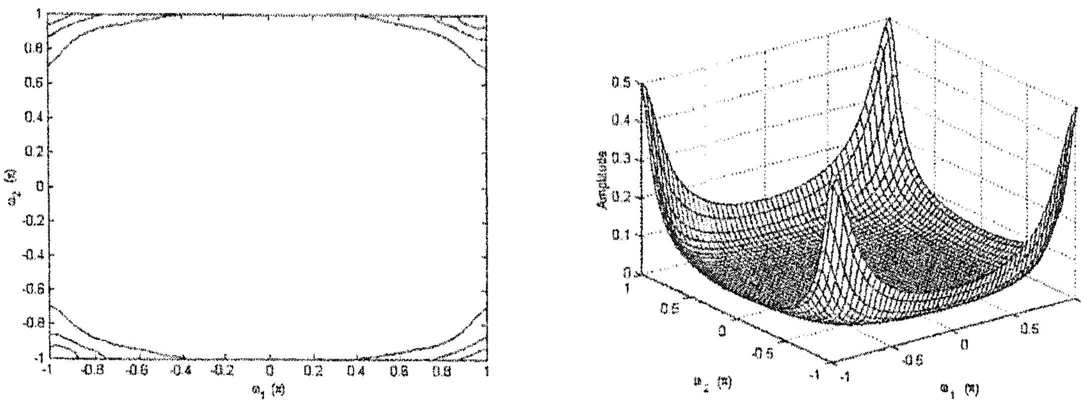
a. $\beta_{01} = \beta_{02} = -0.5$



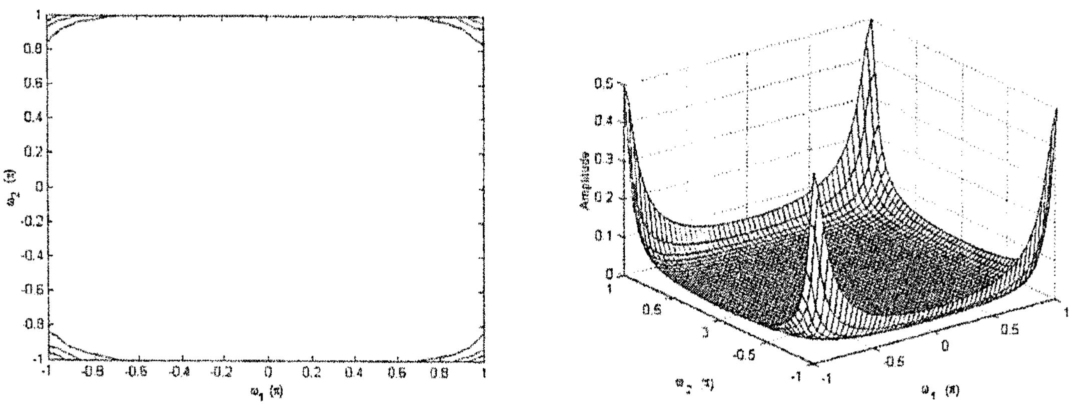
b. $\beta_{01} = \beta_{02} = 0$

Figure 3.20 The contour and 3-D magnitude plots of the resulting 2-D high-pass filter with equal variables β_{0i}'s and the other coefficients fixed as k_1=1.0, k_2=1.0, α_{01}=1.0 and α_{02}=1.0

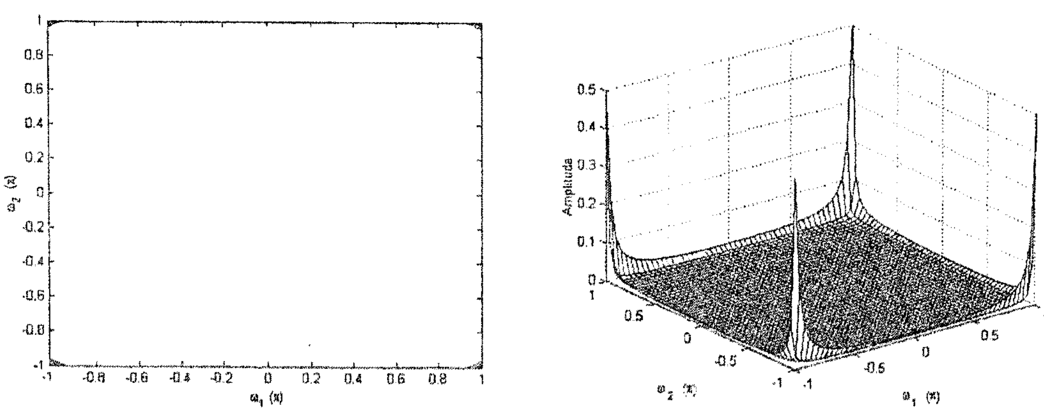
From Figure 3.20, it is observed that when the two coefficients have values other than −1.0, the gain at the point (0, 0) is no longer zero. The bigger the distance of the two coefficients from −1.0, the larger the gain is. To reduce the non-zero gain, as is the experience from the previous sections, we need to increase the values of k_i's at the same time. Figure 3.21 is the contour and magnitude response plots of the resulting filter with larger k_i's, while keeping the other coefficients the same in Figure 3.20.



a. $k_1 = k_2 = 5.0$



b. $k_1 = k_2 = 10.0$



c. $k_1 = k_2 = 50.0$

Figure 3.21 The contour and 3-D magnitude plots of the resulting 2-D high-pass filter with equal variables k_i's and the other coefficients fixed as α_{01}=1.0, α_{02}=1.0, β_{01}=0 and β_{02}=0

Increasing k_i's can really reduce the non-zero gain caused by the values of β_{0i}'s other than −1.0. The price is the loss of bandwidth of the pass-band portions. The optimum values of k_i's and β_{0i}'s could be determined by the design specification and proper optimization techniques.

3.7 Summary and Discussion

In this chapter, the variable magnitude responses of 2-D high-pass digital filters are investigated in detail.

Starting from the same point as in Chapter 2, and using the same 1-D Butterworth prototype analog filter, we obtain 2-D digital filters through the double generalized bilinear transformations with high-pass coefficients limits. When we choose the values of k_i's (i=1, 2) in the range of (0, +∞), α_{0i}'s (i=1, 2) in the range of [0, 1.0], and β_{0i}'s (i=1, 2) in the range of [-1.0, 0], it is possible to obtain 2-D high-pass filters from a 2-D low-pass Butterworth ladder structure through the double generalized bilinear transformations.

The stability conditions of 2-D recursive digital filter with single degree for each variable are still effective. We use these conditions to determine the range of each coefficient, and we also use them as stability test criterion in our 2-D high-pass filter design procedure.

As the generalized bilinear transformation coefficients are changeable, the characteristics of the resulting 2-D high-pass filter are variable. The manner how each coefficient affects the magnitude behavior of the resulting 2-D high-pass digital filter has been investigated in detail.

Just as in the case of 2-D low-pass filters, the coefficients k_i's (i = 1, 2) mainly affect the bandwidth of the pass-band portions in their corresponding dimensions. As the values of k_i's increase, the pass-bands of the resulting filter contract to their center frequency of $\pm\pi$ Radians. When k_i's are at their lower boundaries of slightly bigger than zero, the resulting 2-D high-pass filter can pass all the signal components except the ones with ω_1=0 radians or ω_2=0 radians. It is also clear that when k_i's are big enough, the resulting filter only passes the signal components with $\omega_1 = \pm\pi$ radians and $\omega_2 = \pm\pi$ radians, but block the others.

Although α_{0i}'s (i = 1, 2) also affect the bandwidth of the pass-bands of the resulting 2-D high-pass filter slightly, the main effect from these coefficients lies in the gain of the pass-band portions. The bigger the values of α_{0i}'s, the larger the gain is. The effect on the gain at the pass-band portions is cumulative. The gain reduces more considerably when the values of the two coefficients decrease simultaneously than only one of the two decreases. And the effects of α_{0i}'s can be enlarged by increasing the value of k_i's.

The effect of β_{0i}'s (i = 1, 2) are mainly on the gain of the stop-band of the resulting 2-D high-pass digital filters, although they also affect the bandwidth of the pass-band slightly. When β_{0i}'s have values other than the lower boundary of –1.0, there are non-zero gains at the stop-band portions. The non-zero gains increase as β_{0i}'s increase in their stability range, and the biggest non-zero gain happens at their upper boundary, zero.

This chapter is useful in designing 2-D high-pass filters with variable magnitude response characteristics. It can be served as the start points to design 2-D high-pass digital filters through the application of double generalized bilinear transformation.

Chapter 4

Two-Dimensional Band-Pass Filters

In this chapter, a class of 2-D band-pass filters that are based on the cascade connection of 2-D low-pass and high-pass filters, are investigated. In section 4.1, a brief introduction of this kind of filters, including the typical specifications in mathematical and graphic forms, are given. In section 4.2, from the analysis of the specifications, a 2-D band-pass filter could be obtained from a cascade connection of a 2-D low-pass filter and a 2-D high-pass filter. Based on the results from Chapters 2 and 3, the constituent 2-D low-pass and high-pass filters are treated in sections 4.3 and 4.4, respectively. The stability of 2-D band-pass filter obtained through filters cascading is consider in section 4.5. The manner in which how each coefficient and the combination of some coefficients affect the magnitude responses of the resulting 2-D band-pass filter is investigated in section 4.6 in detail. Summary and discussion are given in section 4.7.

4.1 Introduction

The 2-D band-pass digital filters are another widely used 2-D filters. These filters pass the signal components within a specified frequency range, but block the signals with frequencies higher and lower than the specified frequency ranges.

A typical 2-D band-pass recursive digital filter has the specifications in the frequency domain

$$H(\omega_1,\omega_2)=\begin{cases}0, & |\omega_i|\le\omega_{is1}\\ 1, & \omega_{ip1}\le|\omega_i|\le\omega_{ip2}\\ 0, & \omega_{is2}\le|\omega_i|\le\pi\end{cases} \tag{4.1}$$

where:

ω_{ip1} and ω_{ip2}, (i =1, 2), are pass-bands in z_1 and z_2-dimension, respectively

ω_{is1} and ω_{is2}, *(i =1, 2)*, are stop-bands in z_1 and z_2-dimension, respectively

The region between the pass-bands and stop-bands are transition bands

The specifications could be plotted as in Figure 4.1.

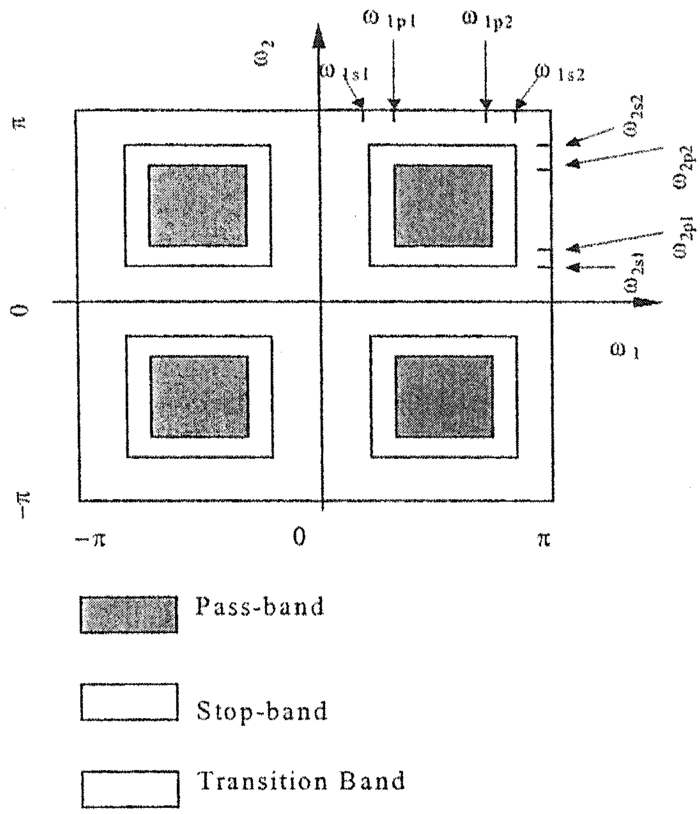


Figure 4.1 Frequency Response Specifications of a typical 2-D band-pass digital Filter

4.2 The Algorithms of Filters Cascading

Now, we consider a cascade-connection filter of the 2-D low-pass and high-pass filters obtained in Chapters 2 and 3, respectively, in order to achieve a 2-D band-pass filter.

To examine the frequency responses of these three types of 2-D filters clearly, the frequency domain specifications of these filters are plotted in Figure 4.2, in which the pass-bands of the low-pass and high-pass filters are shaded, and their intersection represents the pass-band of the resulting 2-D band-pass filter. The logical relations, which are listed in Table 4.1, of the frequency responses of the three types of filters can also be obtained from Figure 4.2.

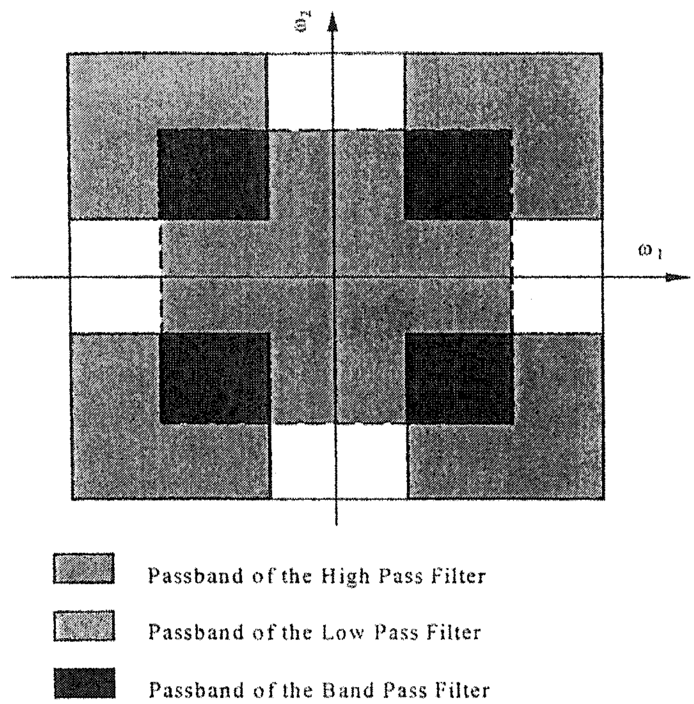


Figure 4.2 2-D band-pass filter from the cascade combination of 2-D low-pass and high-pass filters (in Frequency Domain)

Table 4.1 The logical relation of the frequency responses of the resulting 2-D band-ass filter with its Member Filters. (where, "1" represents the pass-band, "0" represents stop-band.)

2-D low-pass filter (A)	2-D high-pass filter (B)	The resulting 2-D band-pass filter (C)
1	1	1
1	0	0
0	1	0
0	0	0

The logical relations in Table 4.1 can be expressed as the following mathematical equation,

$$H_C(\omega_1,\omega_2) = H_A(\omega_1,\omega_2)H_B(\omega_1,\omega_2)\,. \tag{4.2}$$

Equation (4.2) can be realized by cascading a 2-D low-pass filter with a 2-D high-pass filter as shown in Figure 4.3, in which the signals may pass the 2-D low-pass filter first, and then the output goes through the high-pass filter as depicted in (a), or the signals pass the high-pass filter and then the low-pass one as shown in (b).

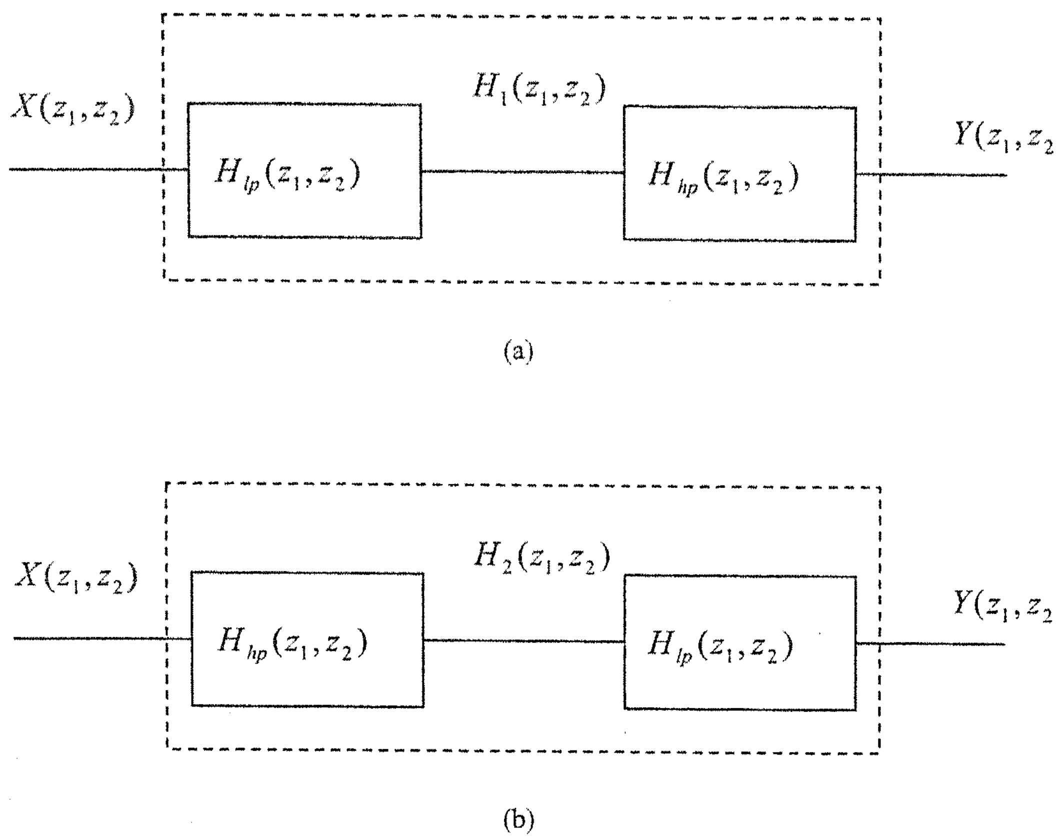


Figure 4.3 The cascade combinations of 2-D low-pass and high-pass filters

If the identical low-pass and high-pass filters are employed in both (a) and (b), it is easy to verify that the two systems in Figure 4.3 (a) and (b) give the identical results, as the order of the multiplication operation does not make different.

The low-pass and high-pas filters (herein called as member low-pass filter and member high-pass filter) in Figure 4.3 can be designed using the generalized bilinear transformation methods introduced in Chapters 2 and 3. When one or more coefficients of the double generalized bilinear transformations are changing, the resulting 2-D band-pass filter possesses variable magnitude characteristics.

The transfer functions of the two member filters are first given in the following sections.

4.3 The Member Low-Pass Filter

Starting from the same Butterworth analog ladder network discussed in Chapter 2, the transfer function of the 2-D analog filter in Figure 2.3 is given as

$$H_a(s_1,s_2)=\frac{V_o}{V_i}=\frac{1}{(1+0.707s_1)(1.414s_2+1)+1}$$

$$=\frac{1}{s_1s_2-0.707s_1+1.414s_2+2} \tag{4.3}$$

Applying the double generalized bilinear transformations (1.37) with proper low-pass coefficients; the digital transfer function of the resulting 2-D low-pass filter is obtained as

$$H_{lp}(z_1,z_2)=\frac{N_{lp}(z_1,z_2)}{D_{lp}(z_1,z_2)} \tag{4.4}$$

where: $$N_{lp}(z_1,z_2)=z_1z_2+\beta_{02}z_1+\beta_{01}z_2+\beta_{01}\beta_{02} \tag{4.5a}$$

$$\begin{aligned}D_{lp}(z_1,z_2)&=(k_1k_2+0.707k_1+1.414k_2+2)z_1z_2\\&+(k_1k_2\alpha_{02}+0.707k_1\beta_{02}+1.414k_2\alpha_{02}+2\beta_{02})z_1\\&+(k_1k_2\alpha_{01}+0.707k_1\alpha_{01}+1.414k_2\beta_{01}+2\beta_{01})z_2\\&+(k_1k_2\alpha_{01}\alpha_{02}+0.707k_1\alpha_{01}\beta_{02}+1.414k_2\alpha_{01}\beta_{01}\\&+2\beta_{01}\beta_{02})\end{aligned} \tag{4.5b}$$

And the coefficients are constrained by the following inequalities,

$$(i) \qquad k_i > 0, \; i = 1,2 \tag{4.6a}$$

$$(ii) \qquad -1.0 \le \alpha_{0i} \le 0, \; i = 1,2 \tag{4.6b}$$

$$(iii) \qquad 0 \le \beta_{0i} \le 1.0, \; i = 1,2 \tag{4.6c}$$

The denominator (4.5b) of the digital transfer function can be written in the general form of two-variable polynomial which has single degree for each variable and degree two for the multiplication of z_1z_2, specifically,

$$D_{lp}(z_1,z_2) = a_{11}z_1z_2 + a_{10}z_1 + a_{01}z_2 + a_{00} \tag{4.7}$$

The member filter should also meet the stability conditions introduced in Chapter 2.

4.4 The Member High-Pass Filter

From the same 2-D analog low-pass ladder network introduced in Chapter 2, the 2-D high-pass filters can be obtained by double generalized bilinear transformation (1.37) with proper high-pass limits. The detailed procedures have been given in Chapter 3. To identify from the low-pass ones, we use the subscription "3" and "4" to replace "1" and "2" which we used in Chapter 3. The digital transfer function of the member high-pass filter can be expressed as

$$H_{hp}(z_1,z_2) = \frac{N_{hp}(z_1,z_2)}{D_{hp}(z_1,z_2)} \tag{4.8}$$

where, $$N_{hp}(z_1,z_2) = z_1z_2 + \beta_{04}z_1 + \beta_{03}z_2 + \beta_{03}\beta_{04} \tag{4.9a}$$

$$D_{hp}(z_1,z_2) = (k_3k_4 + 0.707k_3 + 1.414k_4 + 2)z_1z_2$$

$$+ (k_3k_4\alpha_{04} + 0.707k_3\beta_{04} + 1.414k_4\alpha_{04} + 2\beta_{04})z_1$$

$$+ (k_3k_4\alpha_{03} + 0.707k_3\alpha_{03} + 1.414k_4\beta_{03} + 2\beta_{03})z_2$$

$$+ (k_3k_4\alpha_{03}\alpha_{04} + 0.707k_3\alpha_{03}\beta_{04} + 1.414k_4\alpha_{03}\beta_{03}$$

$$+2\beta_{03}\beta_{04}), \tag{4.9b}$$

with the coefficients satisfying

$$\text{(i)} \quad k_i > 0, \; i = 3,4 \tag{4.10a}$$

$$\text{(ii)} \quad 0 \le \alpha_{0i} \le 1.0, \; i = 3,4 \tag{4.10b}$$

$$\text{(iii)} \quad -1.0 \le \beta_{01} \le 0, \; i = 3,4 \tag{4.10c}$$

The denominator (4.9b) can be written in the general form of two-variable polynomial, which has single degree for each variable and the overall degree is two.

$$D_{hp}(z_1, z_2) = b_{11}z_1z_2 + b_{10}z_1 + b_{01}z_2 + b_{00} \tag{4.11}$$

Of course, the 2-D high-pass filter with the transfer function (4.8) should meet the stability conditions introduced in Chapter 3.

4.5 Stability of the 2-D Band-Pass Filter

Stability is of critical importance in 2-D recursive digital filter design. As the 2-D band-pass filter that we investigate here is the result of cascading two other filters, the stability of the overall system is guaranteed by the stability of both member low-pass and high-pass filters. The consideration for the stability of the overall system is equivalent to the consideration for the stability of each subsystem.

The stability conditions of the member low-pass and high-pass filters have been presented in Chapters 2 and 3, respectively.

4.6 The Frequency Response of the Resulting 2-D Band-Pass Recursive Digital Filters

The MATAB® function *bandPass.m* (please refer to the APPENDIX for detail) is employed to obtain the contour and 3-D magnitude plots of the resulting 2-D band-pass filter, as well as the contour relations between the overall 2-D band-pass filter and its member filters.

As mentioned earlier, starting from the same 2-D analog low-pass ladder network, the member low-pass and high-pass filters can be obtained through the application of double generalized bilinear transformations. If one and more the coefficients are changing, the member low-pass or high-pass filters have variable characteristics, and so does the resulting 2-D band-pass filter. Each coefficient has different contribution to the magnitude characteristics of the resulting 2-D band-pass filter. Below we begin to investigate the effect on the magnitude responses from each coefficient or their combinations.

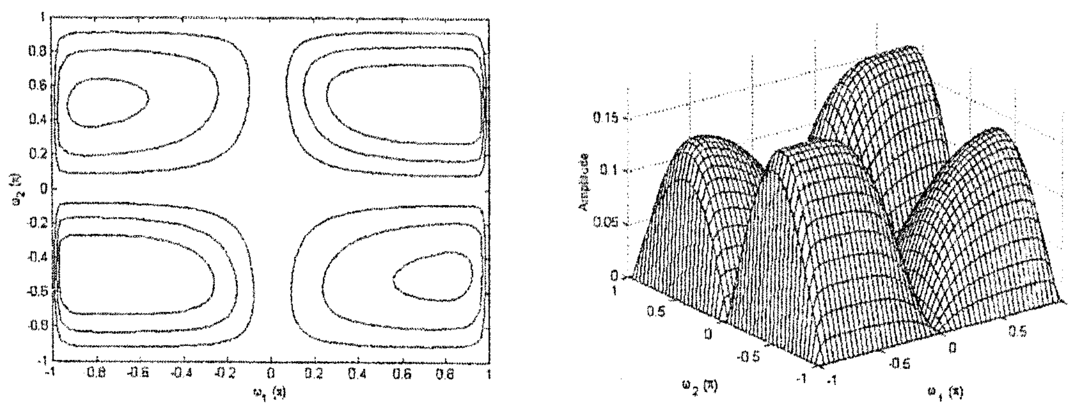
4.6.1 Frequency Response of the Resulting 2-D Band-Pass filter with Variable k_1

From Chapter 2, the coefficient k_1 mainly affects the bandwidth of the pass-band of the 2-D low-pass filter in ω_1-dimension. As the value of k_1 increases, the pass-band becomes small in ω_1-dimension, while the one in ω_2-dimension remains unchanged. As a result, the shape and the symmetric axis of the pass-band portions of the resulting 2-D low-pass filter also change. When the filter is cascaded with a 2-D high-pass filter to yield a 2-D band-pass filter, the overlapping areas of the pass-bands of the two member filters will also change, so we will obtain a 2-D band-pass filter having variable magnitude responses.

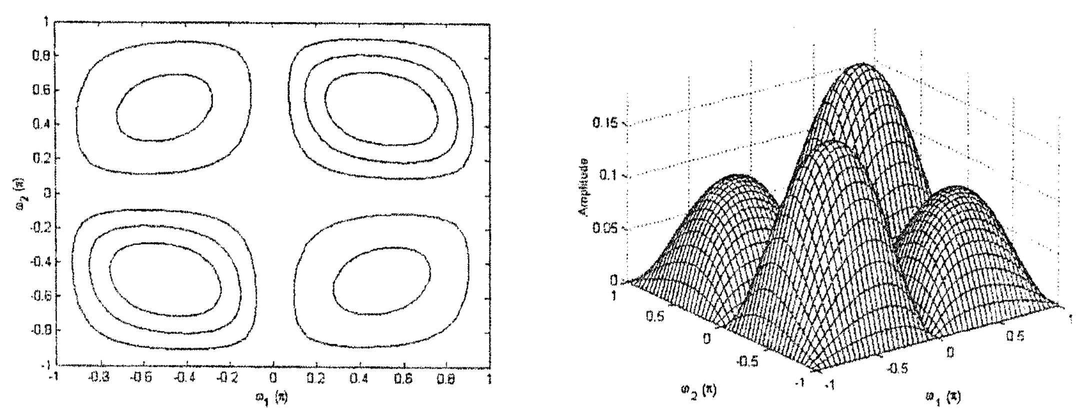
To make the problem simple without loss of generality, we set the other coefficients to be unity with proper signs, while changing the value of the k_1 to obtain variable magnitude responses.

With the specified coefficients, the high-pass filter is stable, and the low-pass filter is stable if k_1 has any value in the range of $(0, +\infty)$. Thus we obtain a stable 2-D band-pass filter.

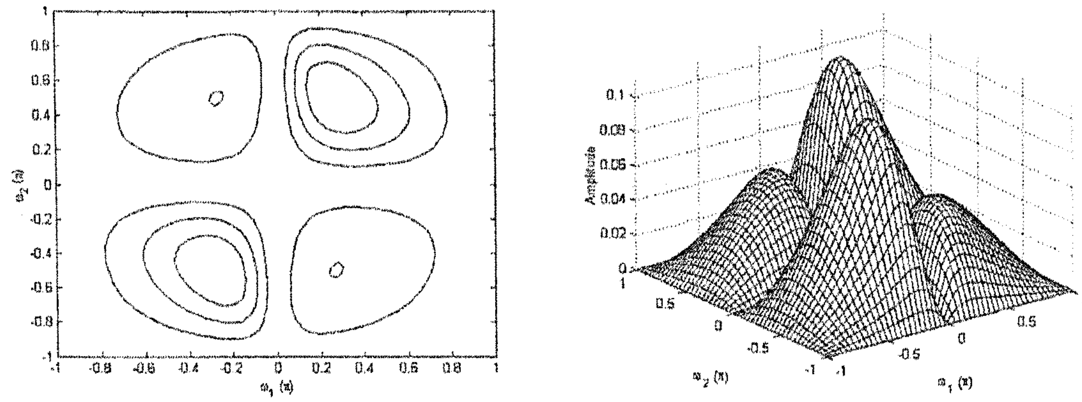
Figure 4.4 is the frequency response of the resulting 2-D band-pass filter with variable k_1. Figure 4.5 is the contour relations between the resulting 2-D band-pass filter and its member filters. We use solid lines to represent the iso-potential contours of the 2-D band-pass filter, and dot-dash-lines and dash-lines to represent the contours of the member high-pass filter and the member low-pass filter, respectively.



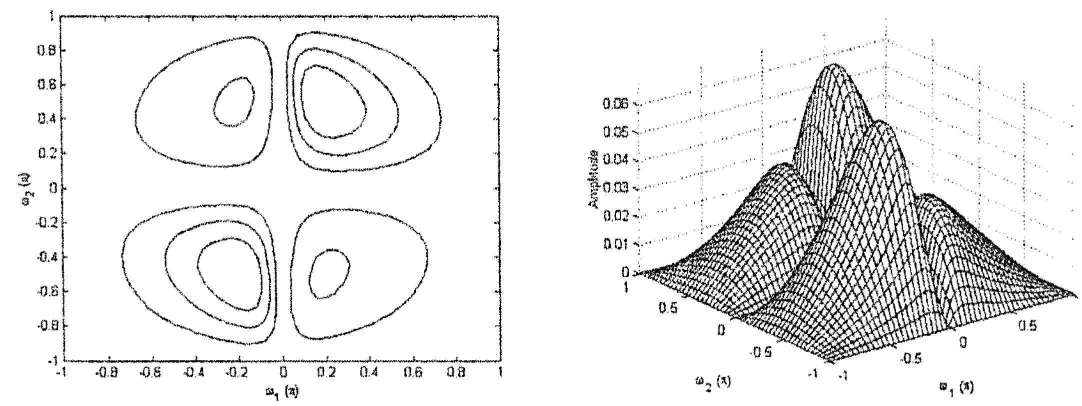
a. $k_1 = 0.1$



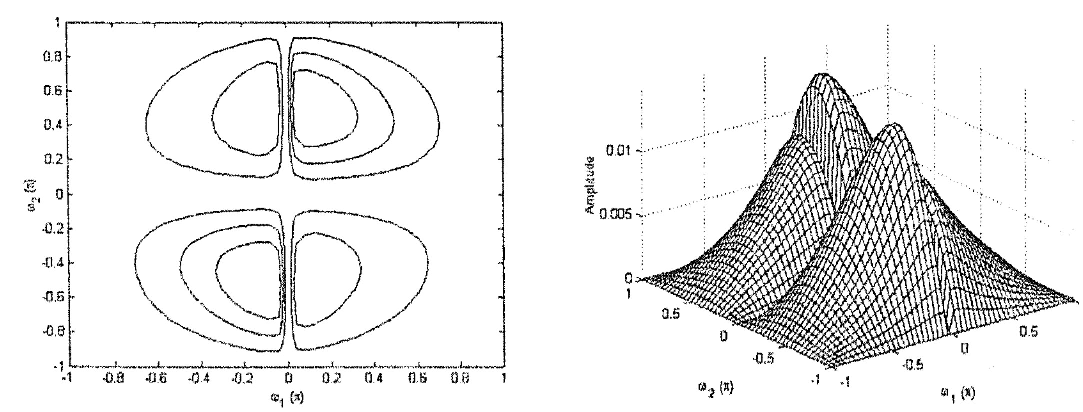
b. $k_1 = 1.0$



c. k_1=5.0

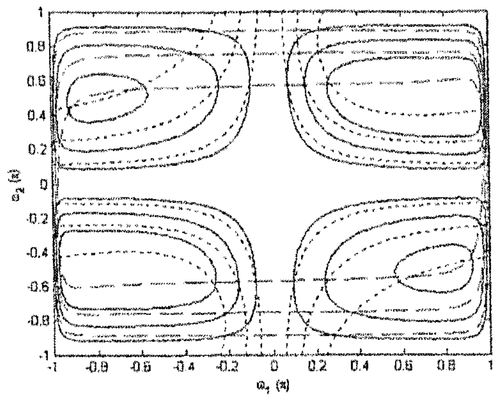


d. k_1=10.0

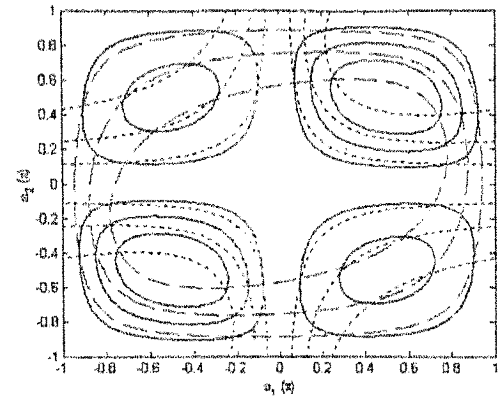


e. k_1=50.0

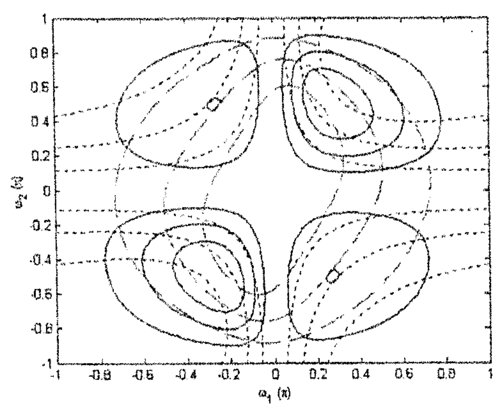
Figure 4.4 The contour and 3-D magnitude plots of the resulting 2-D band-pass filter with variable k_1 and the other coefficients fixed as k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=1.0, k_4=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0 and β_{04}=−1.0



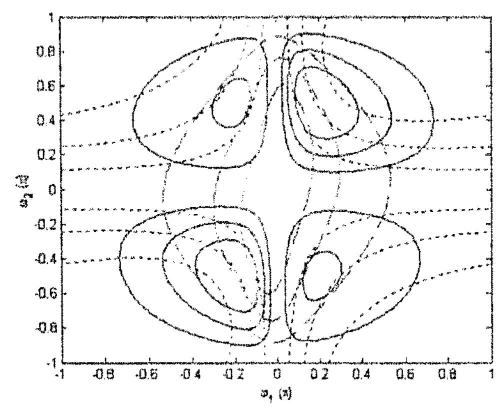
a. k_1=0.1



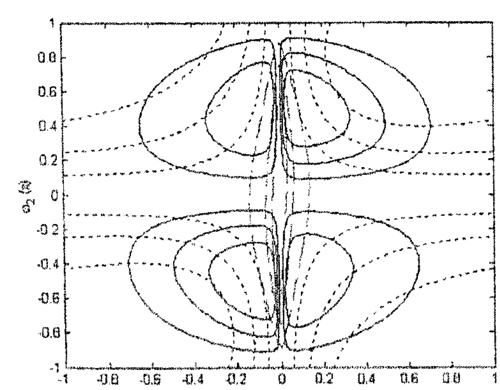
b. k_1=1.0



c. k_1 = 5.0



d. k_1 = 10.0



e. k_1 = 50.0

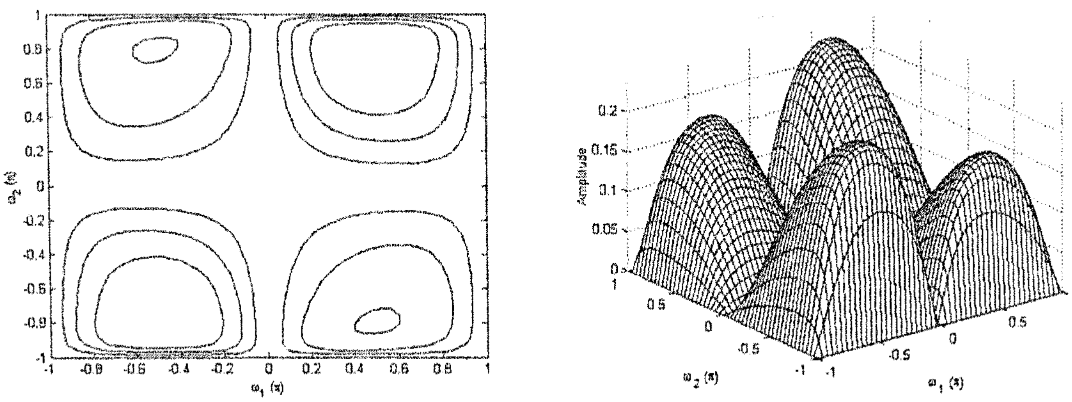
Figure 4.5 the contour relations between the 2-D band-pass filter and its member filters with variable k_1 and the other coefficients fixed as k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=1.0, k_4=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0 and β_{04}=−1.0

As the value of k_1 increases, the center frequency of the pass-band of the resulting 2-D band-pass filter moves from a higher frequency to a lower one in ω_1-dimension. The gain in the pass-bands decreases as the value of k_1 increases. That is because the changing value of k_1 caused the band-width of the pass-band of the member low-pass filter has variable magnitude responses, and in turn it causes the resulting 2-D band-pass filter has variable magnitude responses. When the value of k_1 increases while fixing the other coefficients, the pass-band of the member low-pass filter moves near the center frequency of zero *Radians*, and the magnitude responses of the member high-pas filter and the responses of the member low-pass filter in ω_2-dimension remain unchanged. As a result, the overlap areas of pass-bands of the two member filters moves from a high frequency to a low frequency. That causes the center frequency of the pass-bands of the resulting 2-D band-pass filter to move. At the same time, as the pass-band of the member low-pass filter contracts to the center frequency of zero Radian, the overlapping portions also become small, and even only have overlaps between the transition-bands of the two member filters. That is why the gains of the pass-bands of the resulting 2-D band-pass filter reduce as k_1 increases.

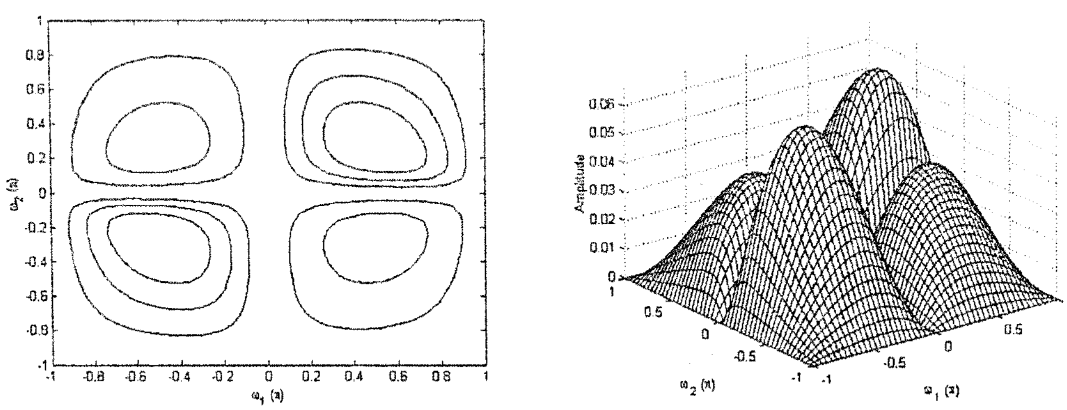
4.6.2 Frequency Response of the Resulting 2-D Band-Pass Filter with Variable k_2

From the results obtained in Chapter 2, the low-pass coefficient k_2 mainly affects the bandwidth of the pass-band of the resulting 2-D low-pass filter in ω_2-dimension. As the value of k_2 increases, the pass-band of the resulting 2-D low-pass filter becomes compact. When the 2-D low-pass filter with variable k_2 is cascaded with a 2-D high-pass filter with specified coefficients, the resulting 2-D band-pass filter possesses variable magnitude responses.

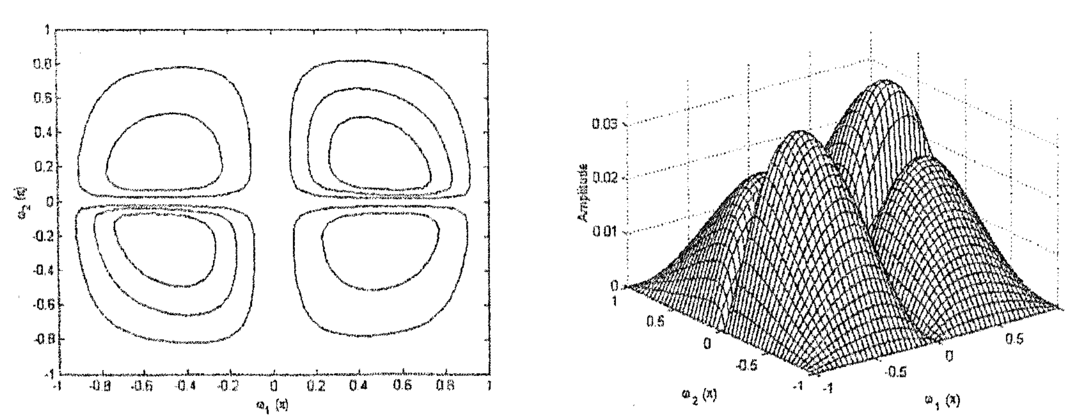
Similar to the previous discussions, when the value of k_2 is changing, we set the other coefficients to be unity with proper signs. Specifically, k_1=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=1.0, k_4=1.0, α_{03}=1.0, α_{04} =1.0, β_{03}=−1.0 and β_{04}=−1.0. The 2-D high-pass filter with these coefficients is stable, and the 2-D low-pass filter is also stable if the value of k_2 is positive. As a result, the 2-D band-pass filter is stable when k_2 is positive. The contour and 3-D magnitude plots of the 2-D band-pass filter with different values of k_2 are illustrated in Figure 4.6. In Figure 4.7, the contour relations between the overall 2-D band-pass filter with variable k_2 and its member filters are given.



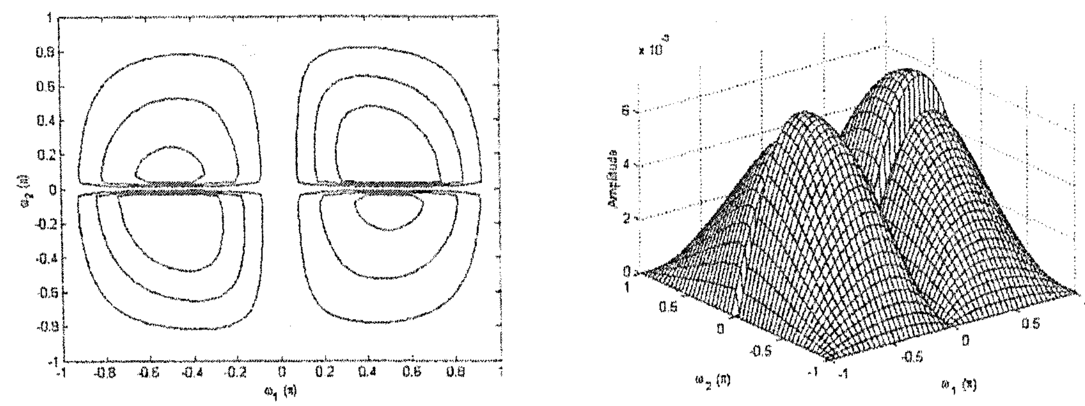
a. $k_2 = 0.1$



b. $k_2 = 5.0$

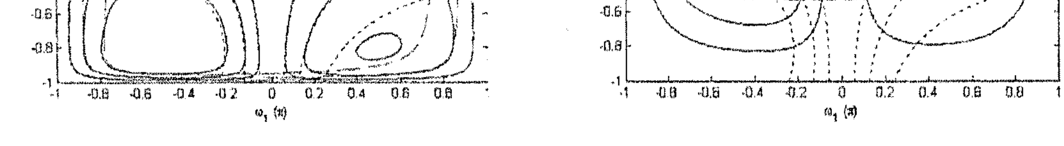


c. $k_2 = 10.0$

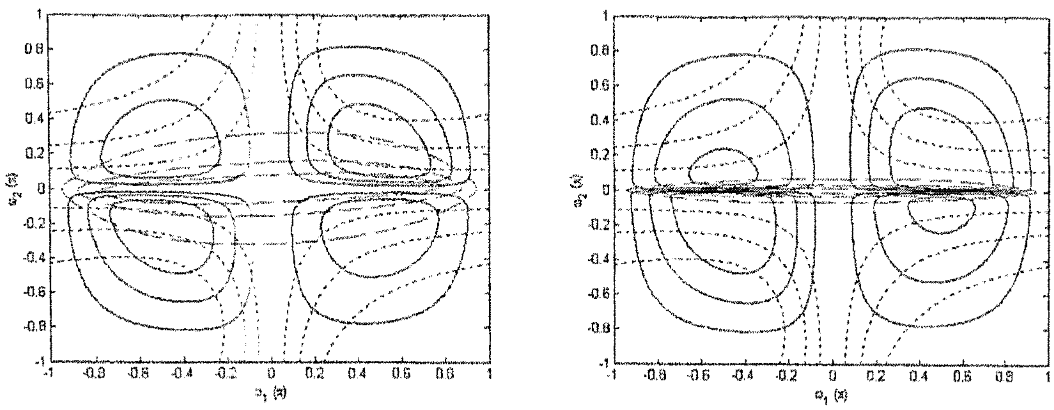


d. $k_2 = 50.0$

Figure 4.6 The contour and 3-D magnitude plots of the resulting 2-D band-pass filter with variable k_2 and the other coefficients fixed as k_1=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=1.0, k_4=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0 and β_{04}=−1.0



a. $k_2 = 0.1$ b. $k_2 = 5.0$



c. $k_2 = 10.0$ d. $k_2 = 50.0$

Figure 4.7 The contour relations between the 2-D band-pass filter and its member filters with different values of k_2 and the other coefficients fixed as k_1=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=1.0, k_4=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}= −1.0 and β_{04}= −1.0

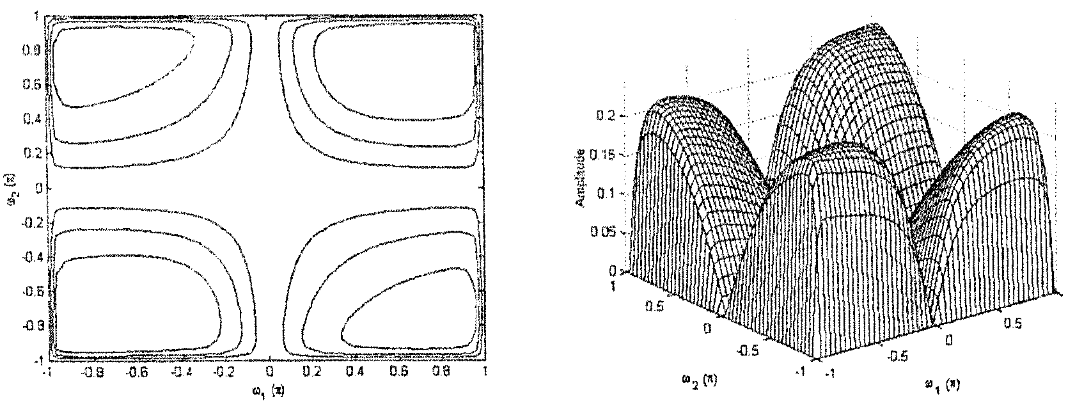
From Figure 4.7 and the plots that we have got for the situation of k_2=1.0 in the previous section, we can see that as k_2 increases, the center frequencies of the pass-bands move from high frequency to low ones in ω_2-dimension, and the gains of the pass-band portions decrease. It is noted that the pass-band of the member low-pass filter compacts as k_2 increases, while the pass-band of the member high-pass filter remains unchanged. That in turn decreases the overlap areas of the pass or transition bands of the two filters. As a result, the overall band-pass filter obtained from the cascade of the two member filters will have variable magnitude responses.

4.6.3 Frequency Response of the Resulting 2-D Band-Pass filter with Equal Variables k_1 and k_2

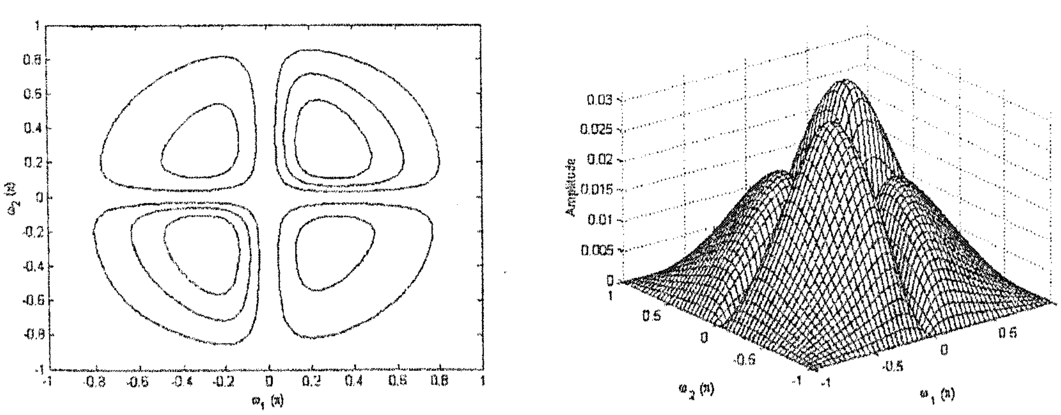
From Chapter 2, when the coefficients k_1 and k_2 change simultaneously but with equal values, the bandwidth of the pass-band change in both ω_1 and ω_2-dimensions, so the shape of the pass-band portion will also change.

To investigate the variation of the magnitude response caused by variables k_1 and k_2, we set the other coefficients to be unity with proper signs, letting α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=1.0, k_4=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0 and β_{04}=−1.0. The high-pass filter is stable, and the low-pass filter is also sable when k_1 and k_2 are positive, so is the overall 2-D band-pass filter.

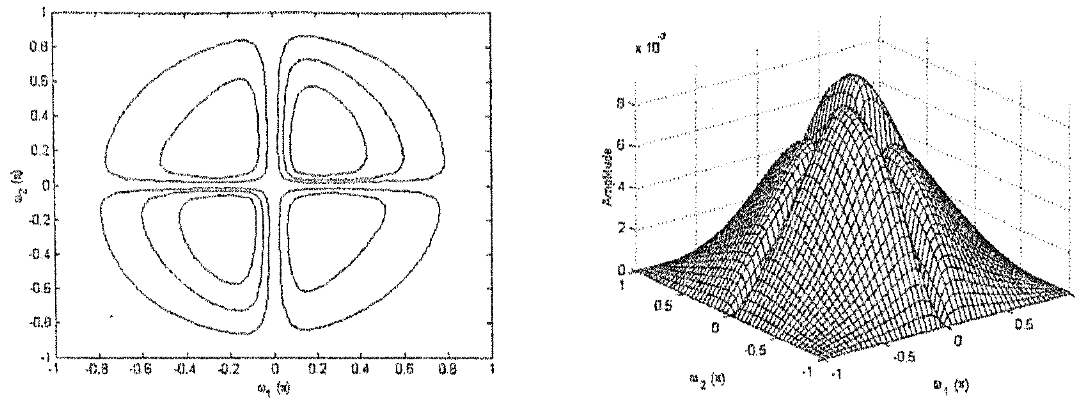
Figure 4.8 is the contour and 3-D magnitude response plots of the resulting 2-D band-pass filter with different values of equal k_1 and k_2. Figure 4.9 indicates the contour relations between the resulting 2-D band-pass filter and its member filters.



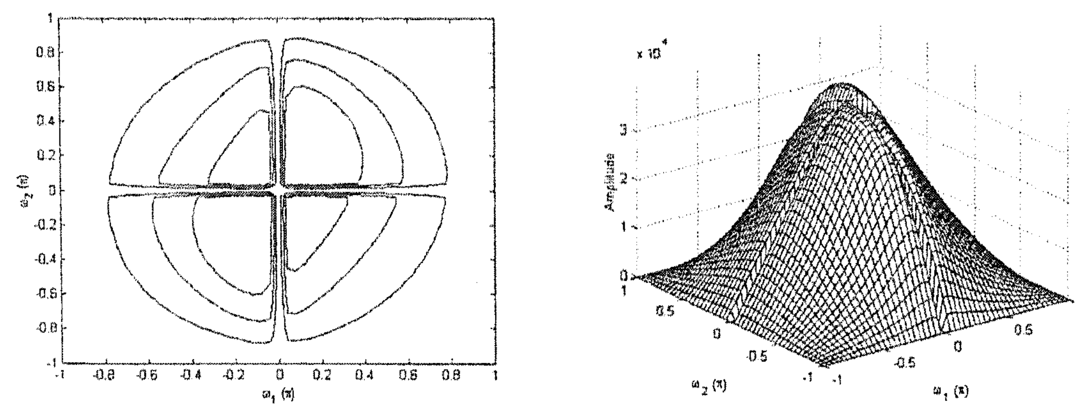
a. $k_1 = k_2 = 0.1$



b. $k_1 = k_2 = 5.0$

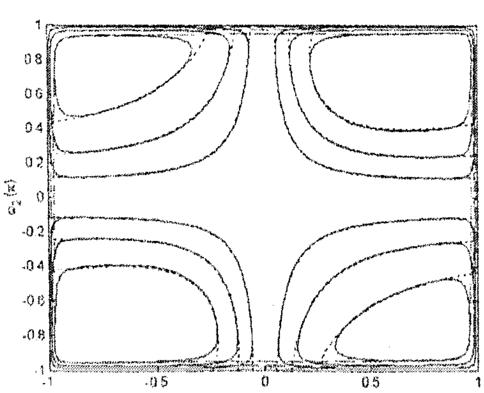


c. $k_1 = k_2 = 10.0$

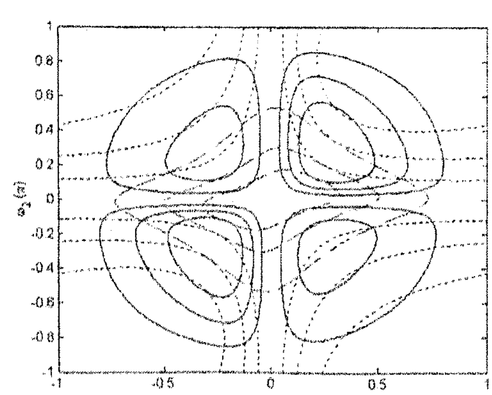


d. $k_1 = k_2 = 50.0$

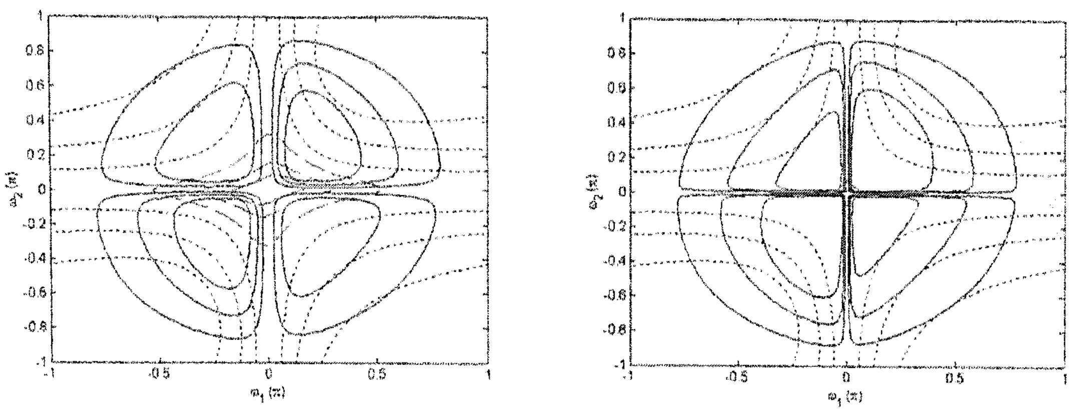
Figure 4.8 The contour and 3-D magnitude plots of the resulting 2-D band-pass filter with equal variables k_1 and k_2 and the other coefficients fixed as α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=1.0, k_4=1.0, α_{03}=1.0, α_{04}=.0, β_{03}=−1.0 and β_{04}=−1.0



a. $k_1 = k_2 = 0.1$



b. $k_1 = k_2 = 5.0$



c. $k_1 = k_2 = 10.0$ d. $k_1 = k_2 = 50.0$

Figure 4.9 The contour relations between the 2-D band-pass filter and its the member filters with variable equally k_1 and k_2 and other coefficients fixed as $\alpha_{01}=-1.0$, $\alpha_{02}=-1.0$, $\beta_{01}=1.0$, $\beta_{02}=1.0$; $k_3=1.0$, $k_4=1.0$, $\alpha_{03}=1.0$, $\alpha_{04}=1.0$, $\beta_{03}=-1.0$ and $\beta_{04}=-1.0$

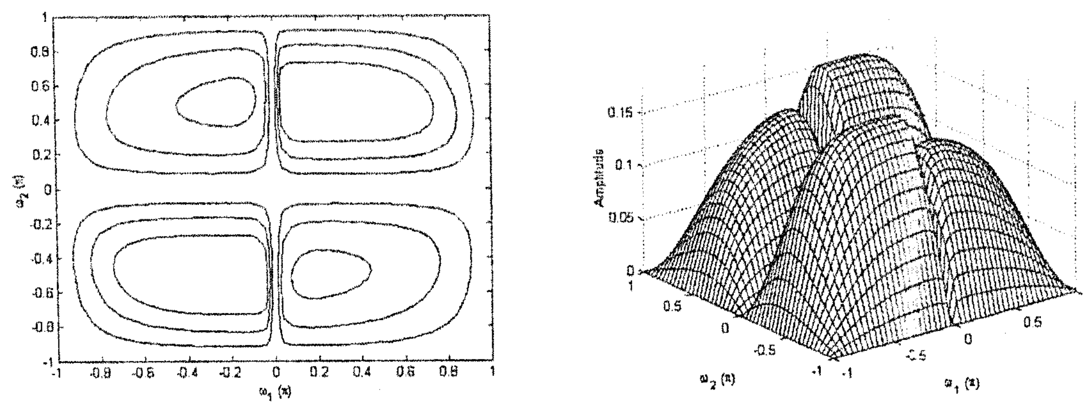
When k_1 and k_2 have equal values and increase simultaneously, the center frequency of the pass-band moves from a higher frequency to a lower one in both ω_1- and ω_2-dimensions, and the gains of the pass-bands also decrease. In fact, when k_1 and k_2 are big enough (bigger than 5.0), there is no overlap between the pass-bands of the two filters. However, the overall system still has the identical magnitude responses as the ones of 2-D band-pass filters, but the gains in pass-band portions decrease dramatically. When $k_1=k_2=5.0$, the highest gain is only 0.03, and when the values of k_1 and k_2 become greater than 50.0, the gains are in their 10^{-4}s, which is too small to be implemented as a filter. The reason that we still investigate big k_1 and k_2 is to obtain the behaviours of the magnitude responses.

4.6.4 Frequency Response of the Resulting 2-D Band-Pass Filter with Variable k_3

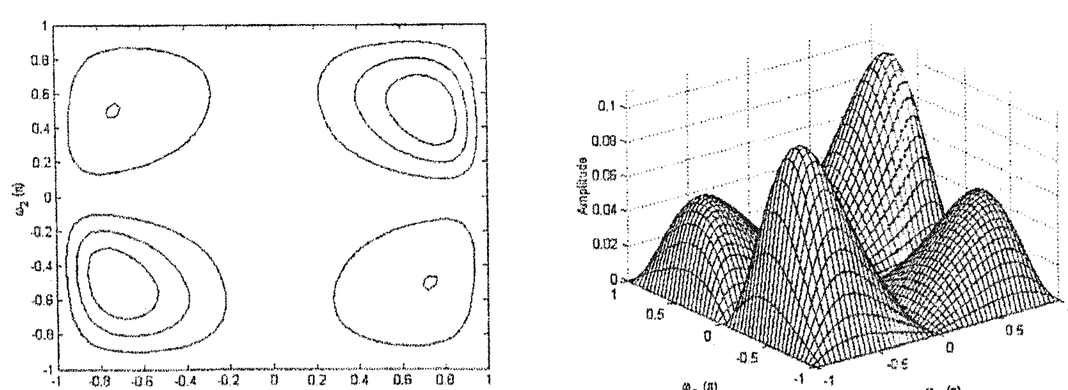
From Chapter 3, we know that the high-pass coefficient k_3 (known as k_1 in Chapter 3) mainly affects the bandwidth of the pass-band of the 2-D high-pass filter. When the value of k_3 increases, the pass-band is contracted to $\pm\pi$ Radians. If we employ the high-pass filter cascaded with a 2-D low-pass filter, the resulting 2-D band-pass filter possesses variable magnitude characteristics.

To investigate the variable behaviours of the resulting 2-D band-pass filter caused by the changing values of k_3, we set the other coefficients to be unity with proper signs. The 2-D low-pass filter with the specified coefficients is stable, and the 2-D high-pass filter is stable if k_3 is positive, so is the resulting overall 2-D band-pass filter.

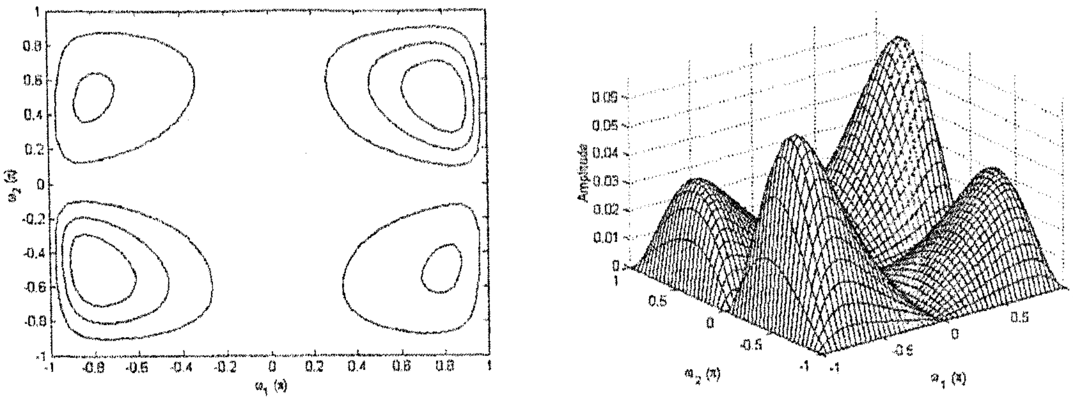
The contour and 3-D magnitude plots of the resulting 2-D band-pass filter with different values of k_3 are illustrated in Figure 4.10, and the contour relation between the resulting 2-D band-pass filter and its member filters is given in Figure 4.11.



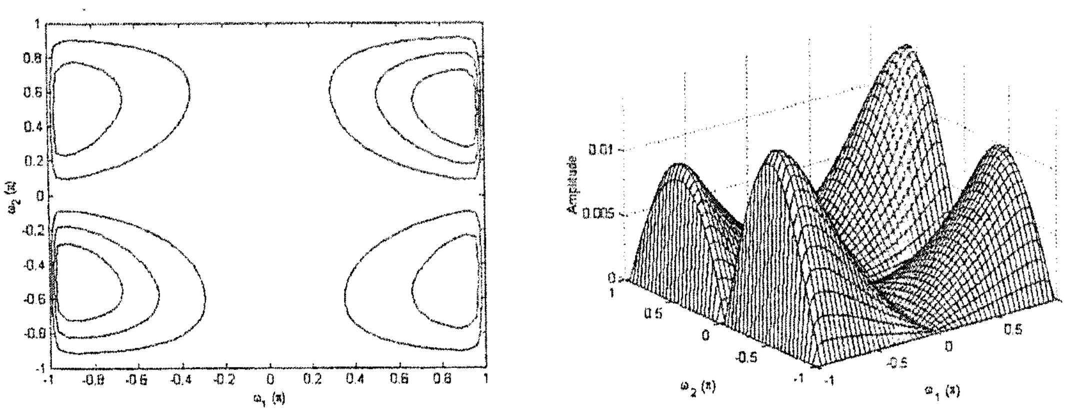
a. $k_3 = 0.1$



b. $k_3 = 5.0$

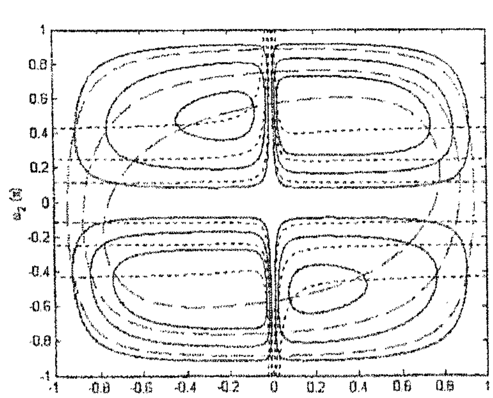


c. $k_3 = 10.0$

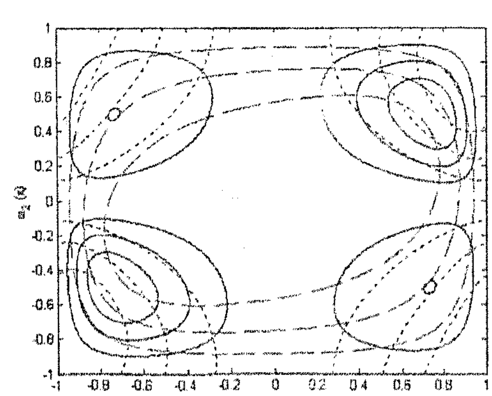


d. $k_3 = 50.0$

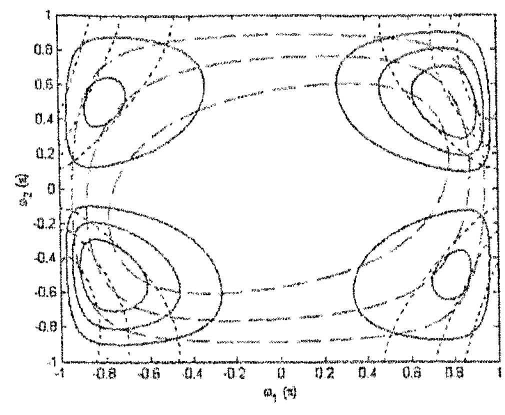
Figure 4.10 The contour and 3-D magnitude plots of the resulting 2-D band-pass filter with variable k_3 and other coefficients fixed as k_1=1.0, k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_4=1.0, α_{03}=1.0, α_{04}=.0, β_{03}=−1.0 and β_{04}=−1.0



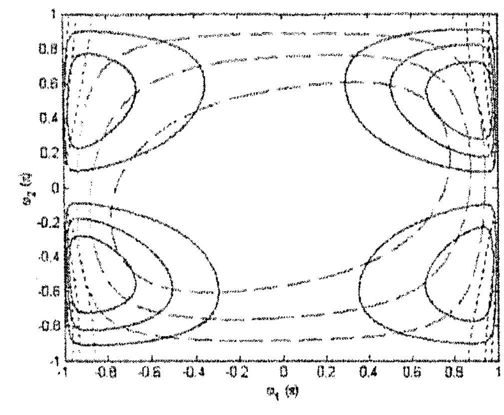
a. $k_3 = 0.1$



b. $k_3 = 5.0$



c. $k_3 = 10.0$



b. $k_3 = 50.0$

Figure 4.11 The contour relation between the 2-D band-pass filter and its member filters with different values of k_3 and the other coefficients fixed as k_1=1.0, k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_4=1.0, α_{03}=1.0. α_{04}=1.0, β_{03}=−1.0 and β_{04}=−1.0

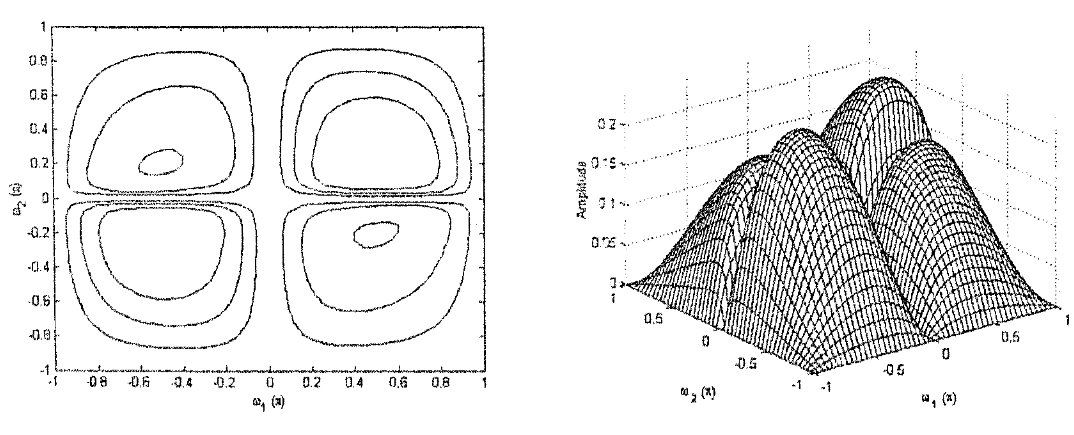
When the value of k_3 increases, the pass-bands of the high-pass filter contract to their center frequency of $\pm\pi$ radians in ω_1-dimension. The resulting 2-D band-pass filter obtained from the cascading of the high-pass filter and a low-pass filter with fixed coefficients has variable magnitude response. With the increase of k_3, the contracting bandwidth pushes the overlapping areas between the pass-bands of the two filters move from lower frequency to higher frequency, and the overlapping areas become smaller. As a result, the center frequencies of the pass-bands of the resulting 2-D band-pass filter move from lower frequency to higher ones in ω_1-dimension. At the same time, the gains of the pass-bands decrease due to the smaller or no overlap of the pass-bands of the two member filters. When there is no overlap occurring in the pass-bands of the two member filters, the pass-band of the 2-D band-pass filter is achieved from the overlapping of the transition bands of the two member filters.

4.6.5 Frequency Response of the Resulting 2-D Band-Pass Filter with Variable k_4

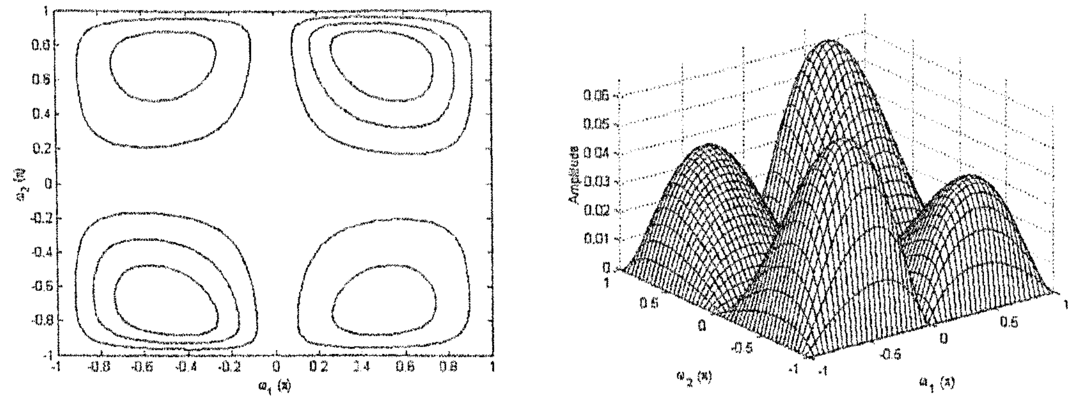
The high-pass coefficient k_4 (known as k_2 in Chapter 3) mainly affects the magnitude response of the 2-D high-pass filter in ω_2-dimension. The resulting 2-D band-pass filter, which has the response of the cascade of a 2-D high-pass filter with variable k_4 and a low-pass filter, has variable magnitude responses also.

In order to investigate the manner how the variable k_4 affects the magnitude responses of the resulting 2-D band-pass filter, the other coefficients than k_4, which has variable value, are fixed to be unity with proper signs using the same methods as discussed before, specifically, k_1=1.0, k_2=1.0, k_3=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0, and β_{04}=−1.0. It is easy to verify that the resulting 2-D band-pass filter is table if k_4 is chosen to be positive.

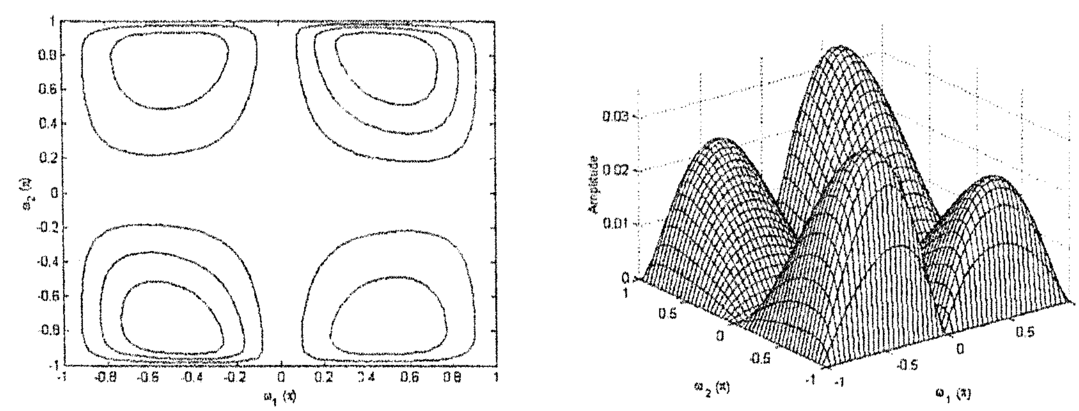
The contour and 3-D magnitude plots are given in Figure 4.12, and the contour relations between the resulting 2-D band-pass filter and its member filters is indicated in Figure 4.13.



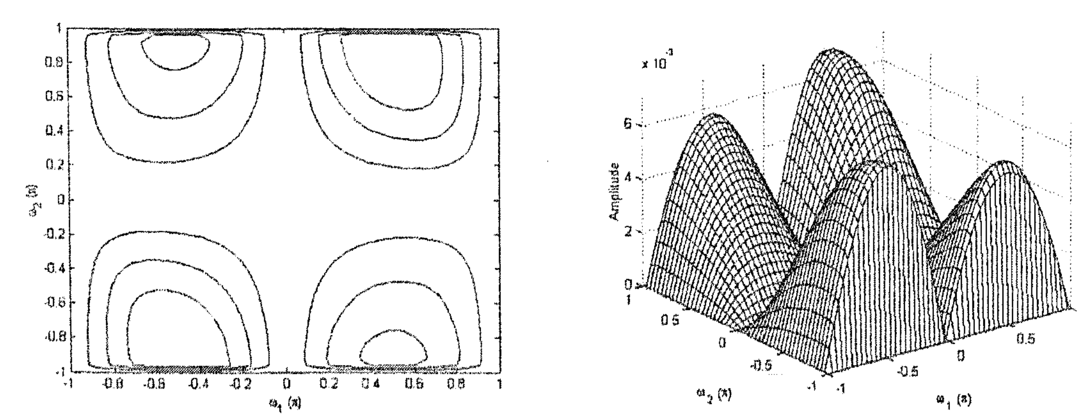
a. $k_4 = 0.1$



b. $k_4 = 5.0$

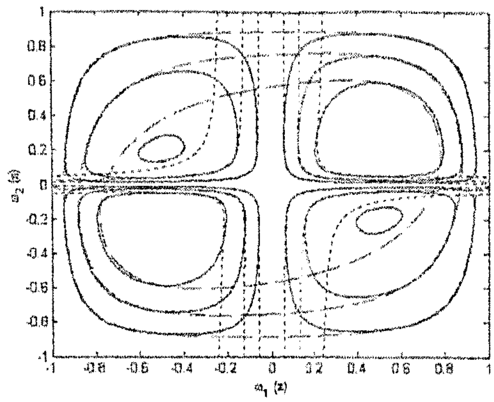


c. $k_4 = 10.0$

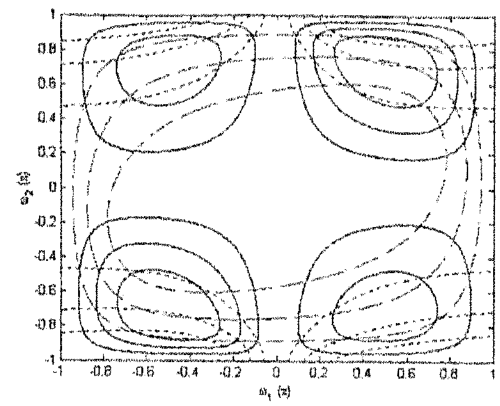


d. $k_4 = 50.0$

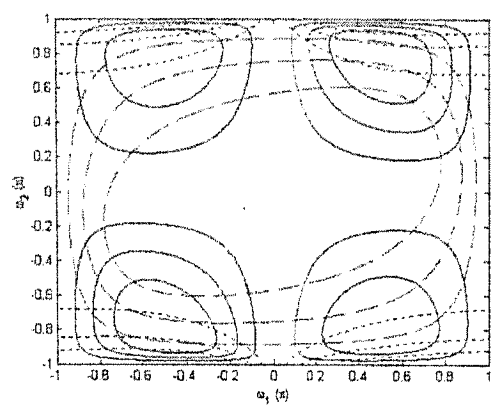
Figure 4.12 The contour and 3-D magnitude plots of the resulting 2-D band-pass filters with variable k_4 and the other coefficients fixed as k_1=1.0, k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0 and β_{04}=−1.0



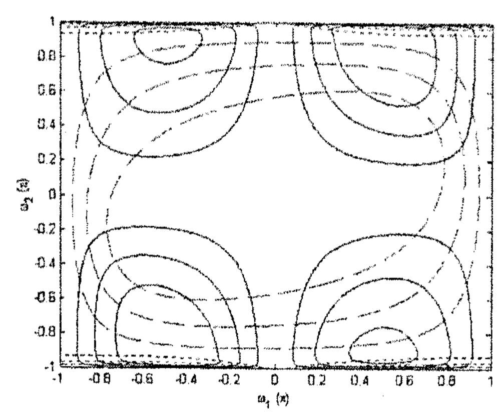
a. $k_4 = 0.1$



b. $k_4 = 5.0$



c. $k_4 = 10.0$



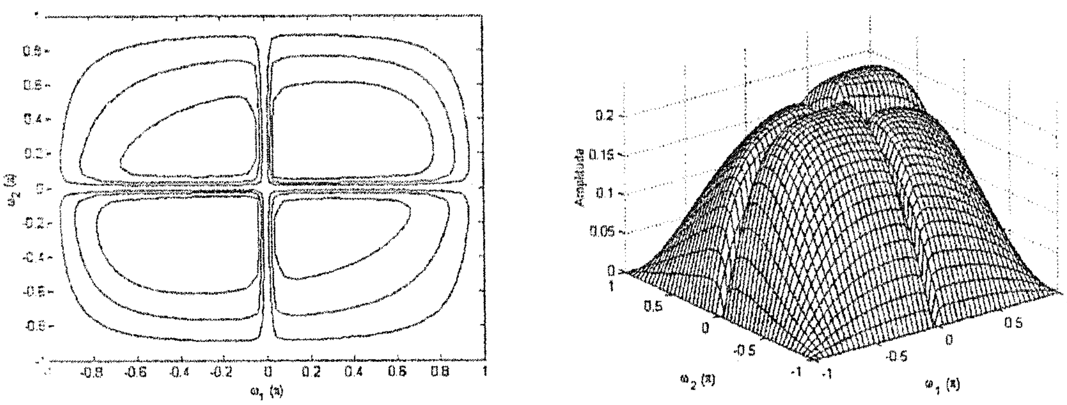
d. $k_4 = 50.0$

Figure 4.13 The contour relation between the 2-D band-pass filter and its member filters with different values of k_4 and the other coefficients fixed as $k_1 = 1.0$, $k_2 = 1.0$, $\alpha_{01} = -1.0$, $\alpha_{02} = -1.0$, $\beta_{01}=1.0$, $\beta_{02} = 1.0$; $k_3 = 1.0$, $\alpha_{03} = 1.0$, $\alpha_{04} = 1.0$, $\beta_{03} = -1.0$ and $\beta_{04} = -1.0$

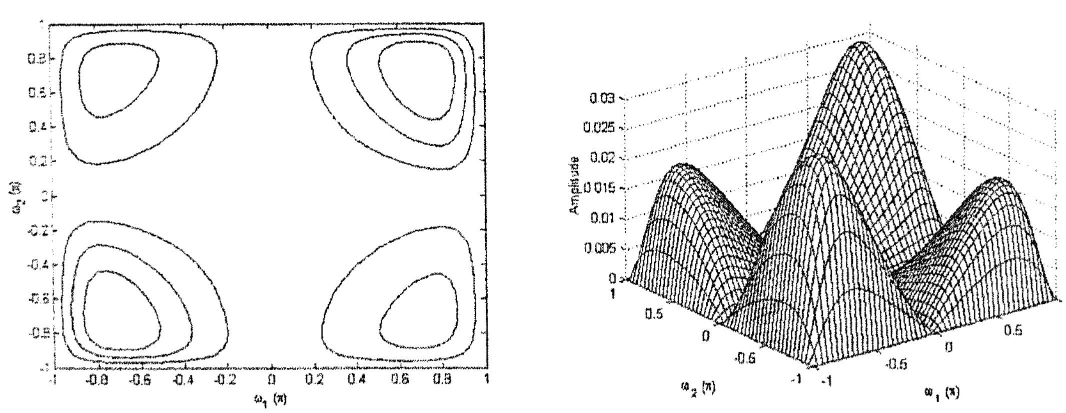
Changing value of k_4 makes the move of the center frequency of the pass-band in ω_2-dimension, and reduces the gains of the pass-bands of the resulting 2-D band-pass filter. The bigger the value of k_4, the higher the center frequency in ω_2-dimension, and the smaller the gain.

4.6.6 Frequency Responses of the Resulting 2-D Band-Pass Filter with Equal Variables k_3 and k_4

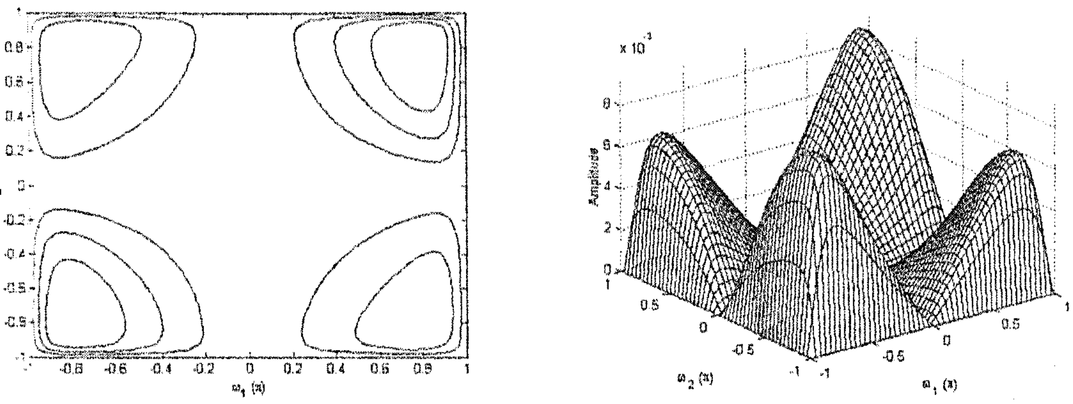
Figure 4.14 and Figure 4.15 shown the simulation results of the 2-D band-pass filter with equal variables k_3 and k_4, which have the same values at any time, and the other fixed coefficients: k_1=1.0, k_2=1.0, α_{01}=–1.0, α_{02}=–1.0, β_{01}=1.0, β_{02}=1.0; α_{03}=1.0, α_{04}=1.0, β_{03}=–1.0, β_{04}=–1.0. The 2-D band-pass filter is stable, if k_3 and k_4 are positive.



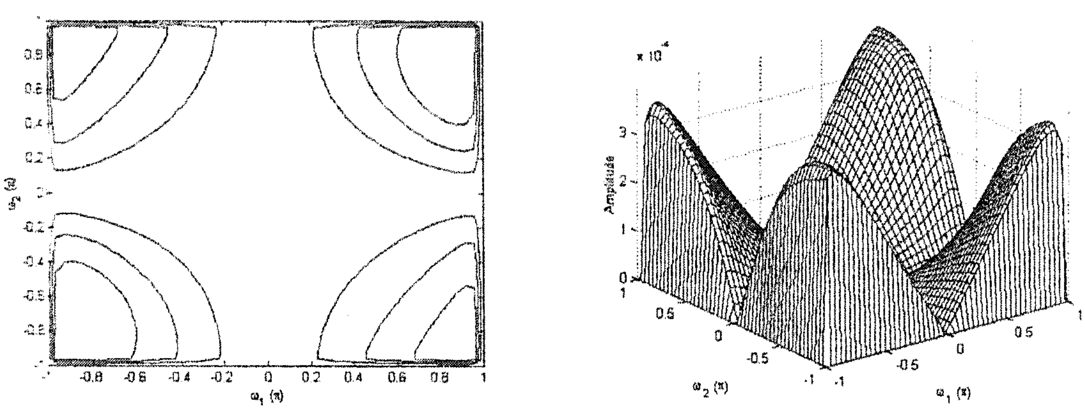
a. k_3=k_4=0.1



b. k_3=k_4=5.0

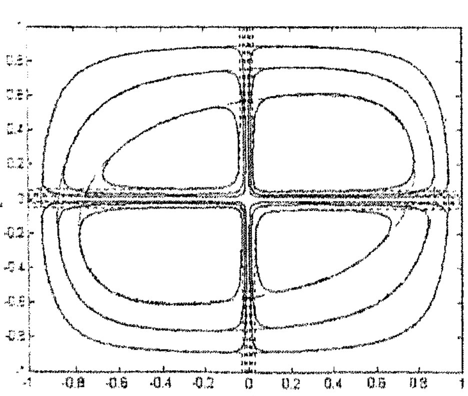


c. $k_3=k_4=10.0$

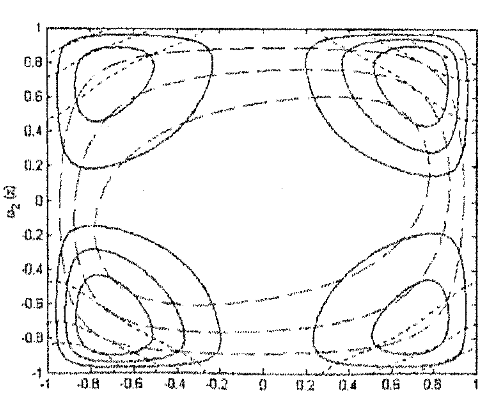


d. $k_3=k_4=50.0$

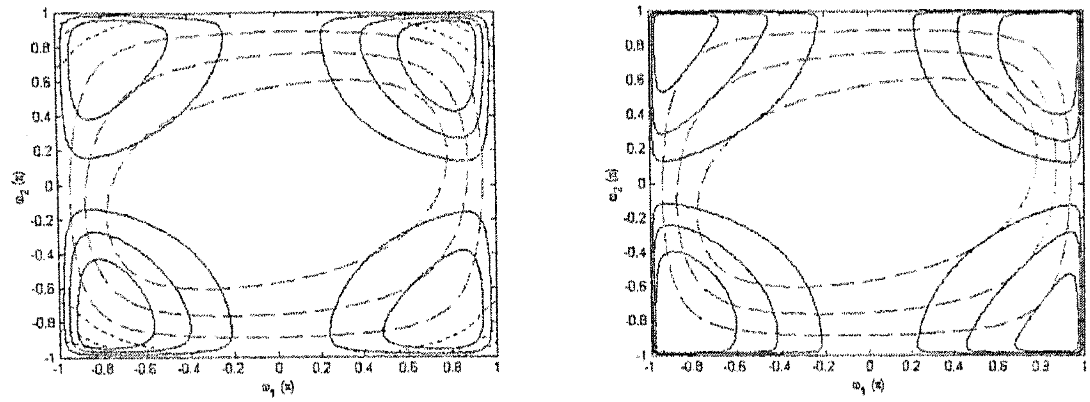
Figure 4.14 The contour and 3-D magnitude plots of the resulting 2-D band-pass filters with equal variables k_3 and k_4, and the other coefficients fixed as $k_1=1.0$, $k_2=1.0$, $\alpha_{01}=-1.0$, $\alpha_{02}=-1.0$, $\beta_{01}=1.0$, $\beta_{02}=1.0$; $\alpha_{03}=1.0$, $\alpha_{04}=1.0$, $\beta_{03}=-1.0$ and $\beta_{04}=-1.0$



a. $k_3=k_4=0.1$



b. $k_3=k_4=5.0$



c. k_3=k_4=10.0 d. k_3=k_4=50.0

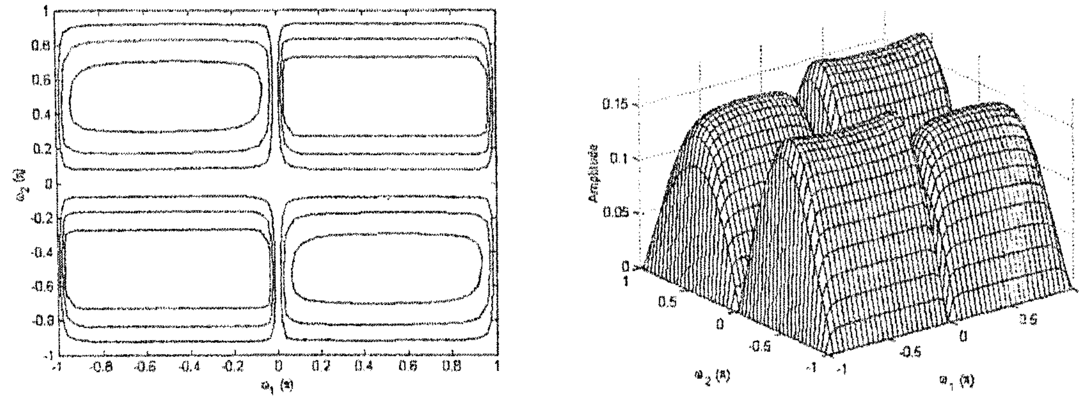
Figure 4.15 The contour relation between the 2-D band-pass filter and its member filters with different choices of k_3 and k_4, and other coefficients fixed as k_1=1.0, k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; α_{03}=1.0, α_{04}=1.0, β_{03}= −1.0 and β_{04}=−1.0

4.6.7 Frequency Responses of the Resulting 2-D Band-Pass Filter with Equal Variable k_1 and k_3

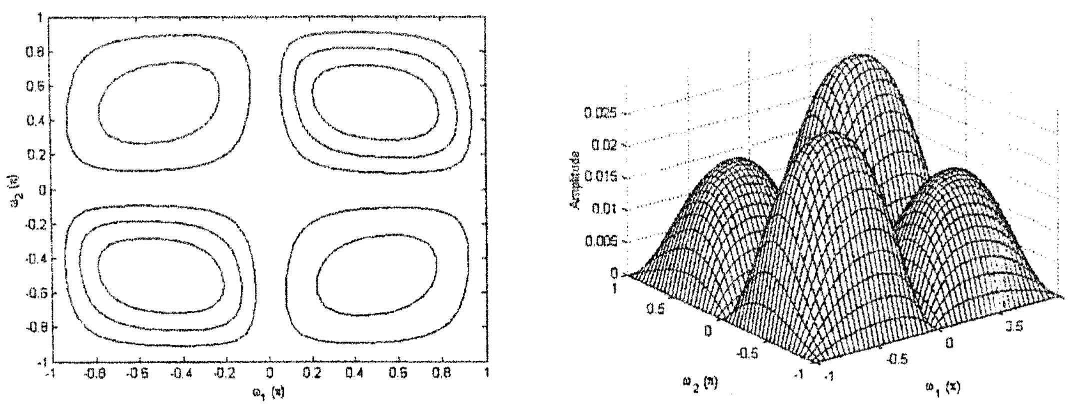
From the previous sections, we know that both k_1 and k_3 affect the center frequency of the pass-band of the resulting 2-D band-pass filter in ω_1-dimension, as well as its gain. When we increase the values of k_1 and k_3, the movements of the direction of the center frequencies are reversed. Now we want to investigate the joint effects of the two coefficients.

The other coefficients are set to be unity with proper signs as before, specifically, k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_4=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0, and β_{04}=−1.0. The resulting 2-D band-pass is stable when k_1 and k_3 are positive.

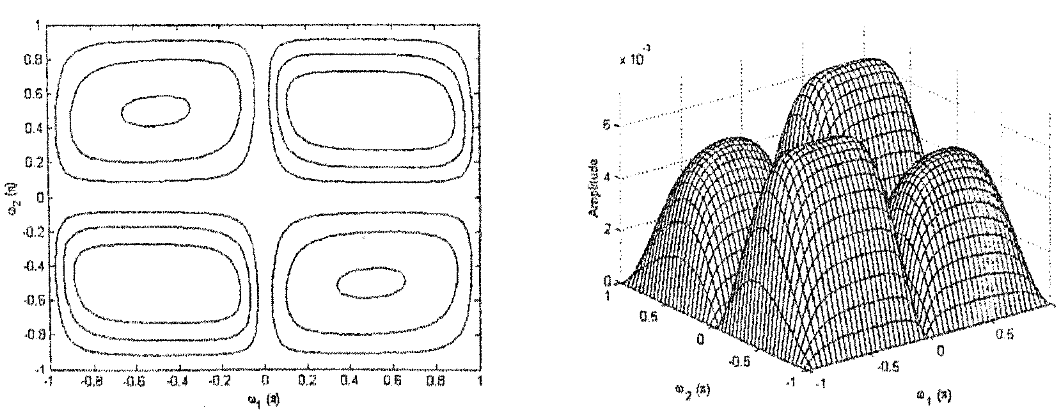
Figure 4.16 is the frequency response plots of the resulting 2-D band-pass filter, and Figure 4.17 is the contour relations between the overall 2-D band-pass filter and its member filters.



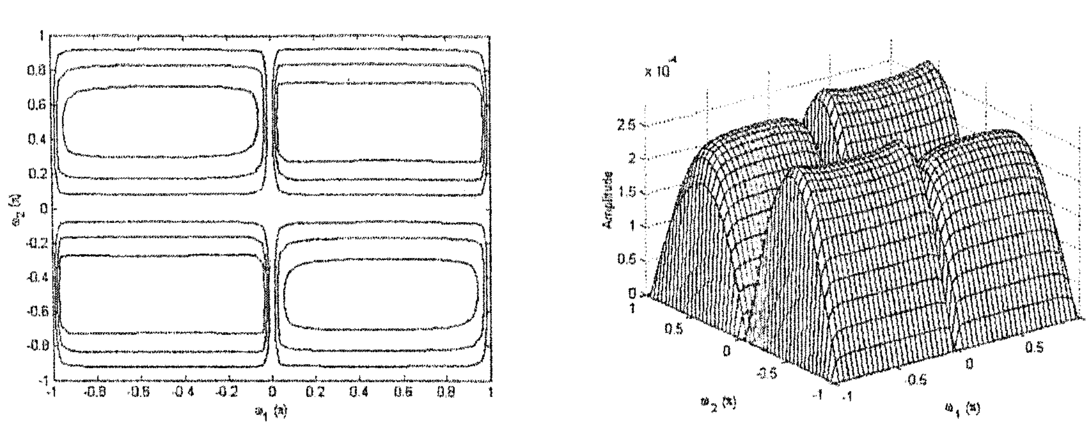
a. $k_1 = k_3 = 0.1$



b. $k_1 = k_3 = 5.0$

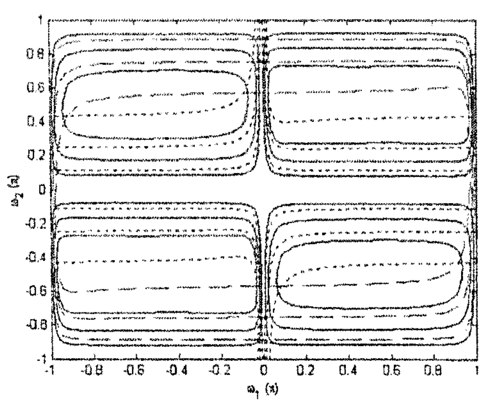


c. $k_1 = k_3 = 10.0$

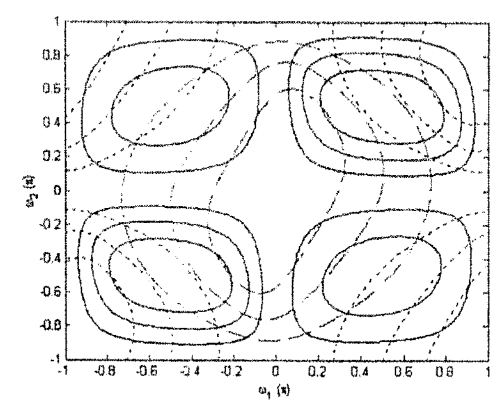


d. k_3=k_4=50.0

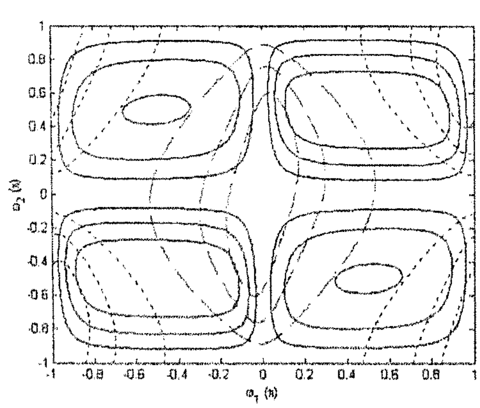
Figure 4.16 The contour and 3-D magnitude plots of the resulting 2-D band-pass filter with equal variables k_1 and k_3 and the other coefficients fixed as k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_4=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}= −1.0 and β_{04}=−1.0



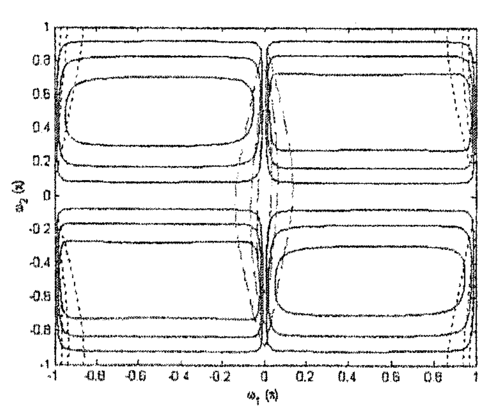
a. k_1=k_3 = 0.1



b. k_1=k_3 = 5.0



c. k_1=k_3 = 10.0



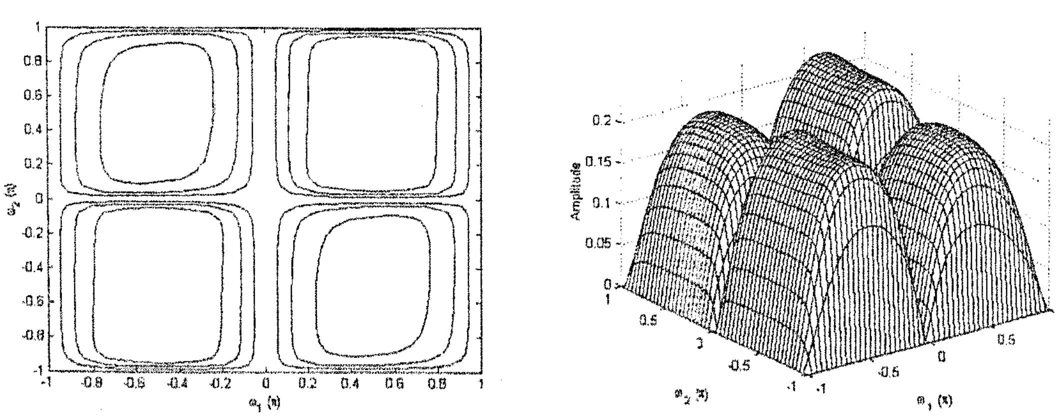
d. k_1=k_3 = 50.0

Figure 4.17 The contour relation of the resulting band-pass filter and its member filters with equal variables k_1 and k_3 and the other coefficients fixed as k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_4=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0 and β_{04}=−1.0

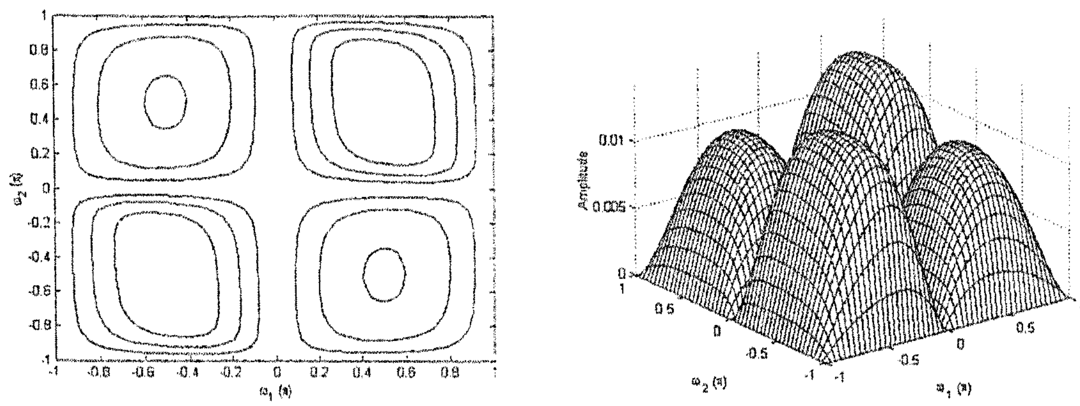
When k_1 and k_3 have equal values and change simultaneously, the center frequencies of the pass-band of the resulting 2-D band-pass filter remain unchanged. As the values of k_1 and k_3 increase, the bandwidths of the pass-bands contract to their center frequencies first, and then enlarge when the values become large enough. The reason is that when k_1 and k_3 are small, the pass-band of the resulting overall 2-D band-pass filter is the overlap of the pass-bands of the two member filters, while the pass-bands become the overlapping of the transition bands when the values of k_1 and k_3 are big enough.

The gain in the pass-band decreases dramatically with the increase of k_1 and k_3. The filter with big k_1 and k_3 is difficult to implement, as the gain is too small. In fact, the band-pass filter resulting from the overlapping of the transition bands of the member filters cannot be used in practical application due to the fact that the transition bands cannot be determined easily.

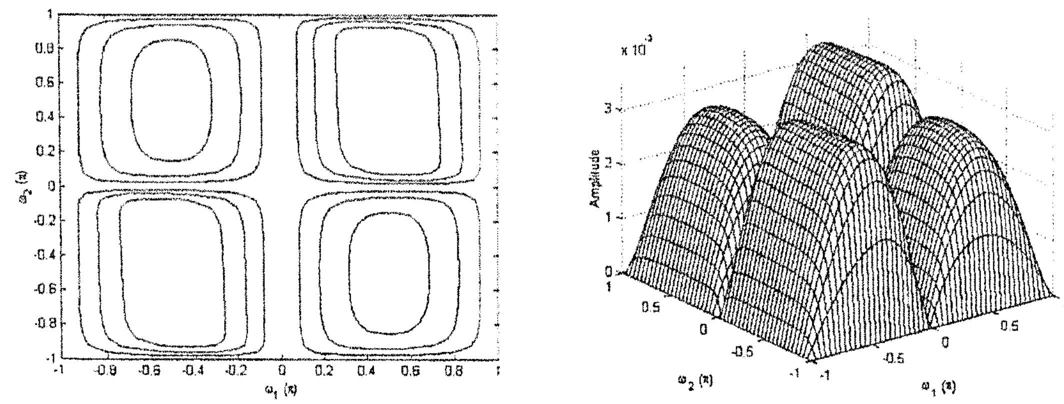
It is readily seen that the effect of k_2 and k_4 is the same as the effect caused by k_1 and k_3, except that the effect of k_2 and k_4 is in ω_2-dimension. The simulation results are given in Figure 4.18 and 4.19.



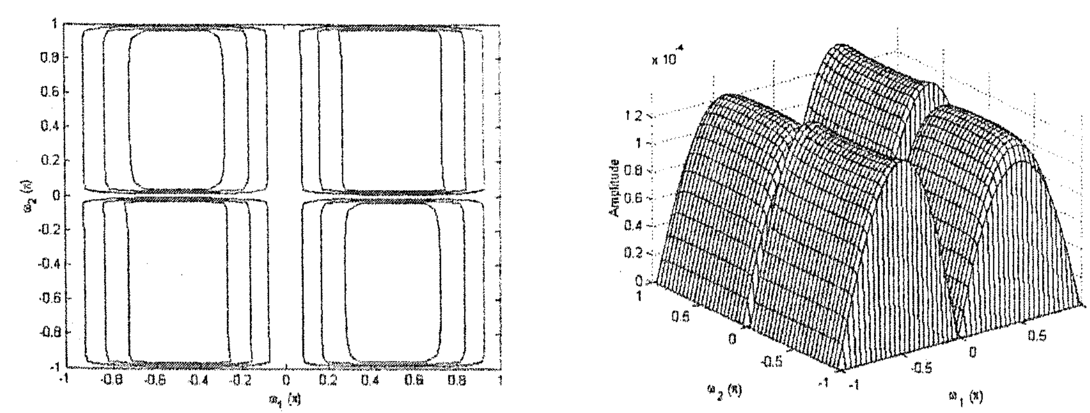
a. k_2=k_4=0.1



b. k_2=k_4=5.0

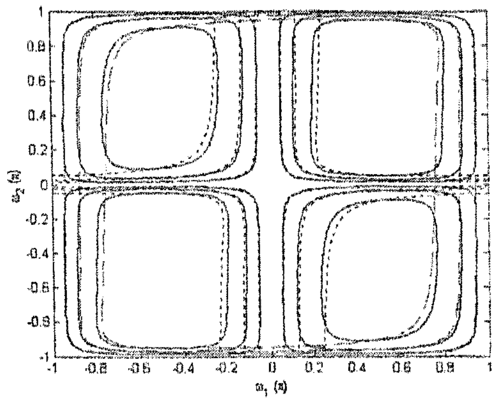


c. k_2=k_4=10.0

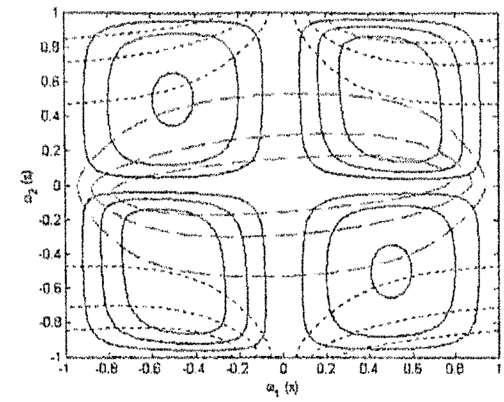


d. k_2=k_4=50.0

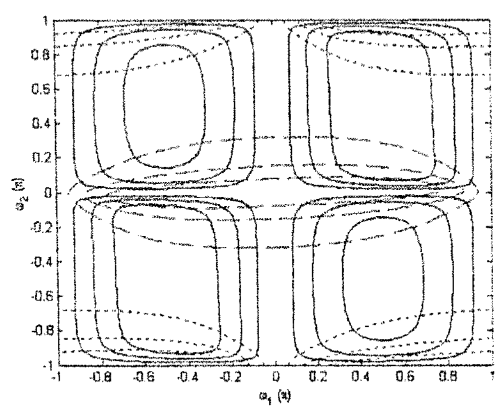
Figure 4.18 The contour and 3-D magnitude plots of the resulting 2-D band-pass filters with equal variables $k_2$and k_4 and the other coefficients fixed as k_1=1.0, α_{01}=–1.0, α_{02}=–1.0, β_{01}=1.0, β_{02}=1.0; k_3=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}= –1.0 and β_{04}=–1.0



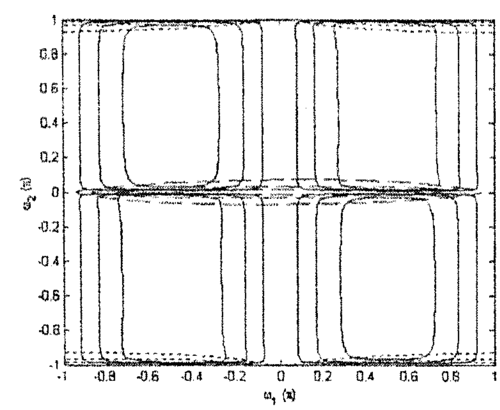
a. k_2=k_4=0.1



b. k_2=k_4=5.0



c. k_2=k_4=10.0



d. k_2=k_4=50.0

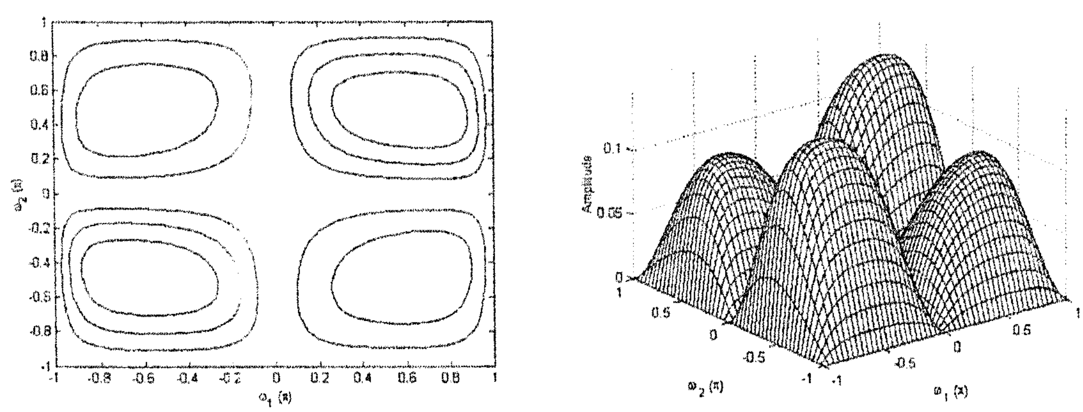
Figure 4.19 The contour relation between the resulting 2-D band-pass filter and its member filters with variable equal k_2 and k_4 and the other coefficients fixed as k_1=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0 and β_{04} = −1.0

4.6.8 Frequency Response of the Resulting 2-D Band-Pass filter with Variable α_{01}

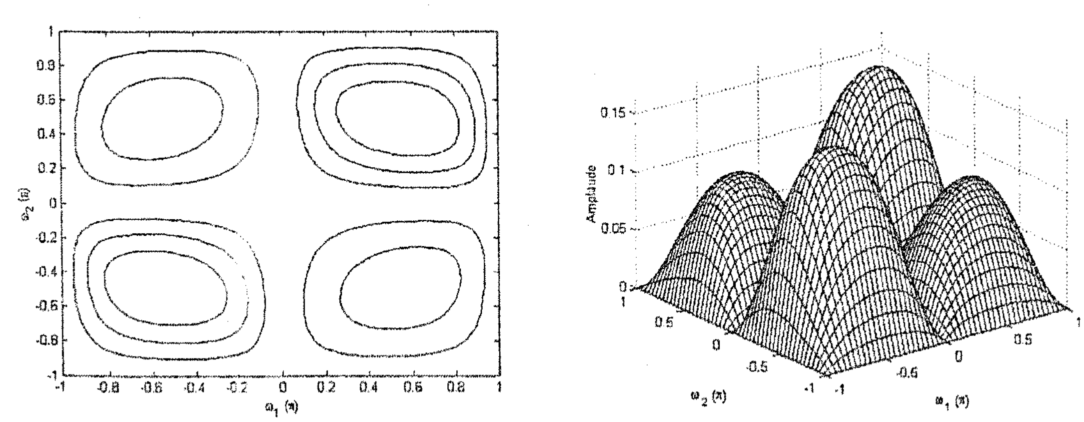
The low-pass coefficient α_{01} mainly affects the gain in the pass-band of the resulting low-pass filter, and slightly affects the bandwidth of the pass-band. When the low-pass filter with variable α_{01} is employed to obtain the 2-D band-pass filter with a high–pass filter, the resulting 2-D band-pass filter will also possess variable magnitude responses.

To study the variable magnitude behaviour, we change the values of α_{01} in its stable range, while keeping the other coefficients to be unity with proper signs, specifically, k_1=1.0, k_2=1.0, k_3=1.0, k_4=1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0, and β_{04}=−1.0. The overall band-pass filter is verified to be stable when α_{01} has any value in the range of [−1.0, 0].

Figure 4.20 is the contour and 3-D magnitude response plots for the resulting 2-D band-pass filters with different values of α_{01}. Figure 4.21 shows the contour relation between the resulting 2-D band-pass filter and its member filters.

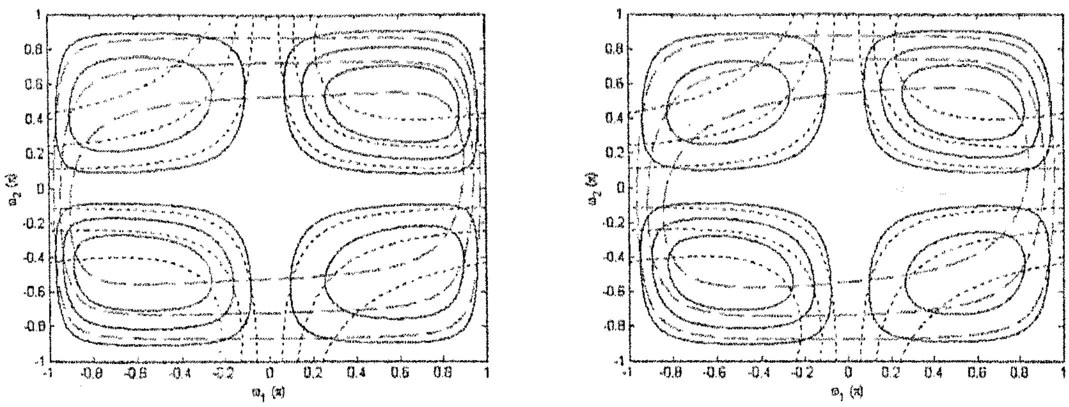


a. $\alpha_{01} = 0$



b. $\alpha_{01} = -0.5$

Figure 4.20 The contour and 3-D magnitude plots of the resulting 2-D band-pass filter with variable α_{01} and the other coefficients fixed as k_1=1.0, k_2=1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0;k_3=1.0, k_4=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0 and β_{04}=−1.0



a. $\alpha_{01} = 0$ b. $\alpha_{01} = -0.5$

Figure 4.21 The contour relation between the resulting 2-D band-pass filter and its member filters with variable α_{01} and the other coefficients fixed as k_1=1.0, k_2=1.0, α_{02}= −1.0, β_{01}=1.0, β_{02}=1.0; k_3=1.0, k_4=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}= −1.0 and β_{04}= −1.0

When the coefficient α_{01} increases from its lower boundary of −1.0 to the upper boundary of one, the pass-band of the resulting 2-D band-pass filter is enlarging. That is because the band-pass of the low-pass filter enlarges, so overlapping areas becomes larger.

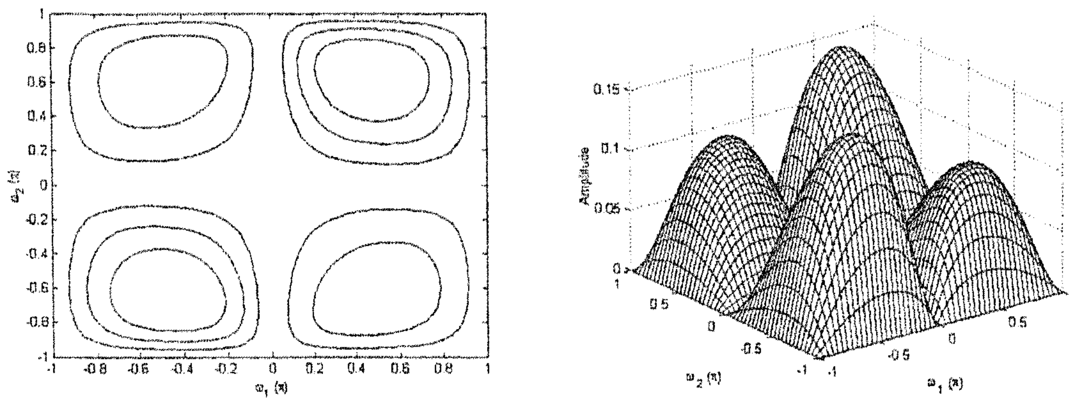
The gain in the resulting 2-D band-pass almost remains unchanged. The reason is that the gain in the resulting 2-D band-pass filter is determined by the overlapping of the pass-bands or the transition bands of its member filters.

4.6.9 Frequency Response of the Resulting 2-D Band-Pass Filter with Variable α_{02}

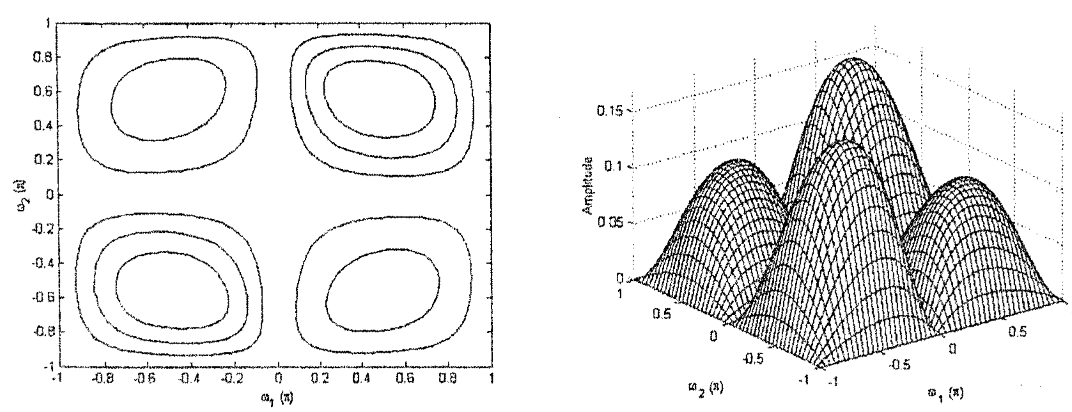
The low-pass coefficient α_{02} affects the gain in the pass-band of the resulting 2-D low-pass filter too, and it also slightly affects the bandwidth of the pass-band. As a result, the overall band-pass filter, resulting from the cascade of the low-pass filter and a high-pass filter, will also have variable magnitude characteristics.

The other coefficients than α_{02} are still set to be k_1=1.0, k_2=1.0, k_3=1.0, k_4=1.0, α_{01}=−1.0, β_{01}=1.0, β_{02}=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0, and β_{04}=−1.0 as in the previous subsections. From the results of the previous chapters and the stability conditions in this chapter, the overall 2-D band-pass filter is stable when α_{02} is in the range of $[-1.0, 0]$.

Figure 4.22 illustrates the contour and 3-D magnitude plots of the resulting 2-D band-pass digital filter with variable α_{02}. Figure 4.23 shows the contour relation between the resulting 2-D band pass filter with variable α_{02} and its member filters.

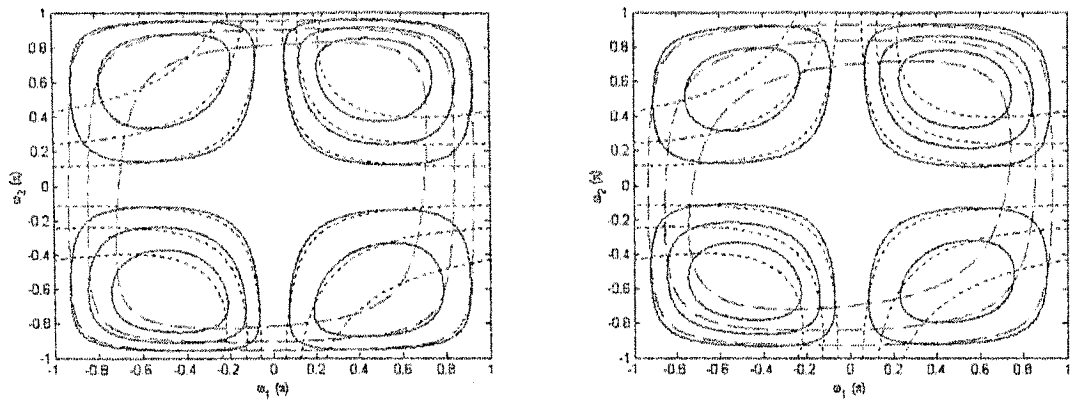


a. α_{02}=0



b. α_{02}=−0.5

Figure 4.22 The contour and 3-D magnitude plots of the resulting 2-D band-pass filter with variable α_{02} and the other coefficients fixed as k_1=1.0, k_2=1.0, α_{01}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=1.0, k_4=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0 and β_{04}=−1.0



a. $\alpha_{02} = 0$ b. $\alpha_{02} = -0.5$

Figure 4.23 The contour relation between the 2-D band-pass filter and its member filters with variable α_{02} and the other coefficients fixed as k_1=1.0, k_2=1.0, α_{01}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=1.0, k_4=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0 and β_{04}=−1.0

Changing the value of α_{02} affects the bandwidth of the pass-bands of the resulting 2-D band-pass filter. When α_{02} has a small value, the bandwidth is compact. That is because the bandwidth of the member low-pass filter has a narrow bandwidth with a small α_{02} in ω_2-dimension. That, in turn, affects the overlapping of the pass-bands of the two member filters.

Changing the value of α_{02} does not affect the gains of the resulting 2-D band-pass filter, which is different from the effect in the case of 2-D low-pass filter.

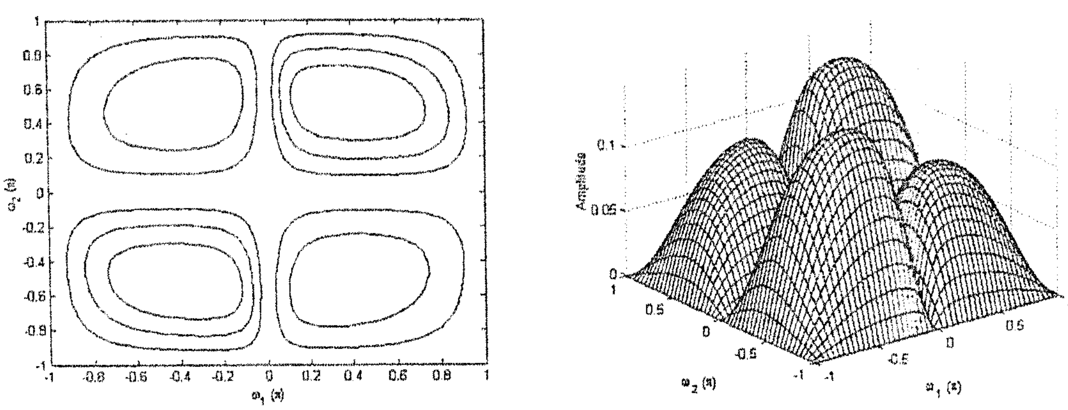
4.6.10 Frequency Response of the Resulting 2-D Band-Pass filter with variable α_{03}

The high-pass coefficient α_{03} (known as α_{01} in Chapter 3) mainly affects the gain in the pass-band of the resulting 2-D high-pass filter, and also it slightly affects the bandwidth in the pass-band of the 2-D high-pass filter in ω_1-dimension. When the high-pass filter with variable α_{03} is cascaded with a low-pass filter, no matter whether the filter possesses fixed or variable magnitude responses, to form a 2-D band-pass filter, the overall 2-D band-pass filter has variable magnitude behaviours.

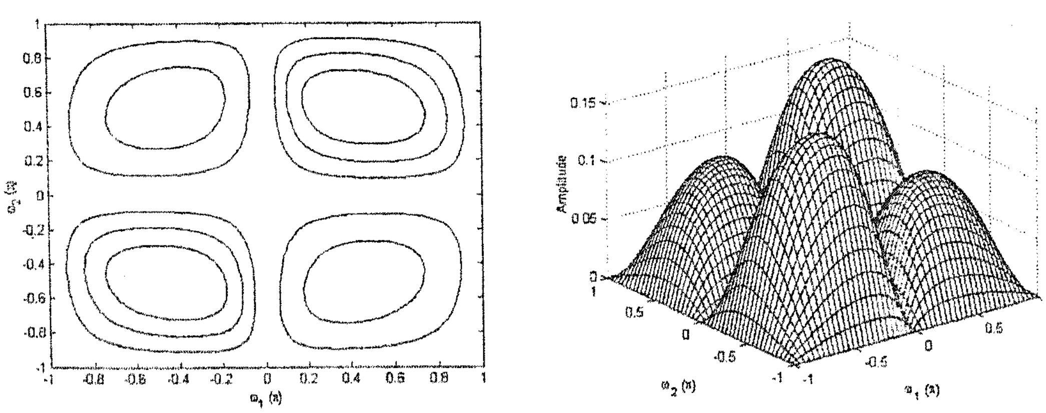
To investigate the manner how α_{03} affects the magnitude behaviour of the resulting

2-D band-pass filter, we set the other coefficients to the specified values, letting k_1=1.0, k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=1.0, k_4=1.0, α_{04}=1.0, β_{03}=−1.0, and β_{04}=−1.0. It is easy to verify that the resulting 2-D band-pass filter is stable with α_{03} in the range of [0, 1.0].

Figure 4.24 is the contour and 3-D magnitude response plots of the resulting 2-D band-pass filter with different values of α_{03}. Figure 4.25 is the contour relation between the resulting 2-D band-pass filter and its member filters.

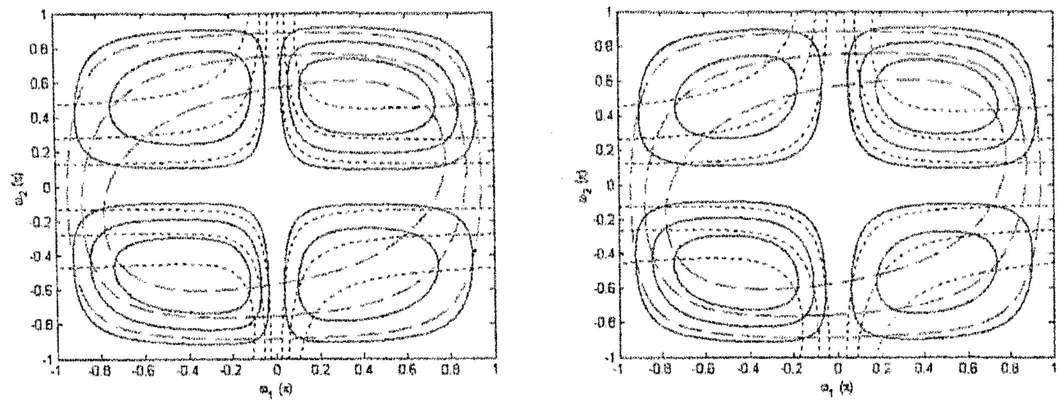


a. α_{03}=0



b. α_{03}=0.5

Figure 4.24 The contour and 3-D magnitude plots of the resulting 2-D band-pass filter with variable α_{03} and the other coefficients fixed as k_1=1.0, k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0;k_3=1.0, k_4=1.0, α_{04}=1.0, β_{03}=−1.0 and β_{04}=−1.0



a. $\alpha_{03} = 0$ b. $\alpha_{03} = 0.5$

Figure 4.25 The contour relation between the 2-D band-pass filter and its member filters with variable α_{03} and the other coefficients fixed as k_1=1.0, k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=1.0, k_4=1.0, α_{04}=1.0, β_{03}=−1.0 and β_{04}=−1.0

When the high-pass coefficient α_{03} increases from its lower boundary, 0, to its upper boundary, 1.0, the pass-band of the high-pass filter becomes compact in ω_1. As a result, the resulting 2-D band-pass filter, the combination of the high-pass filter and a low-pass one, has compacting pass-bands, and the angle of their symmetry axis also becomes bigger.

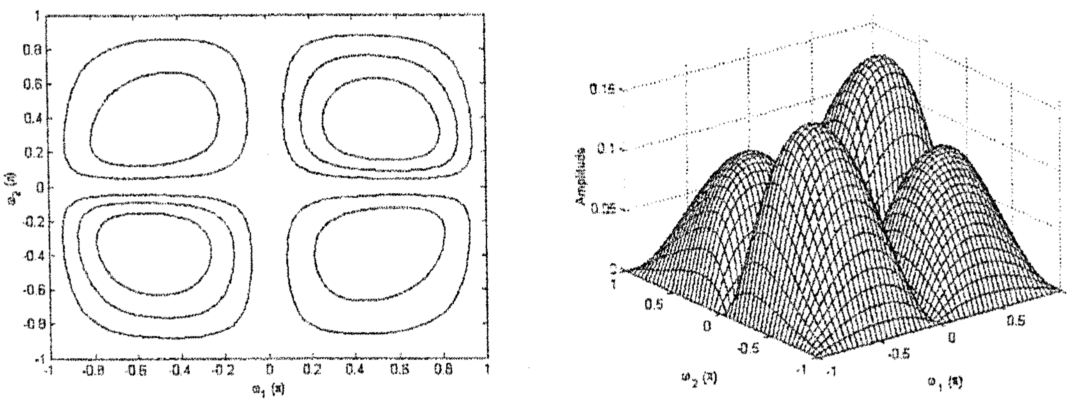
The effect on the gains of the resulting 2-D band-pass filter is small.

4.6.11 Frequency Response of the Resulting 2-D Band-Pass Filter with Variable α_{04}

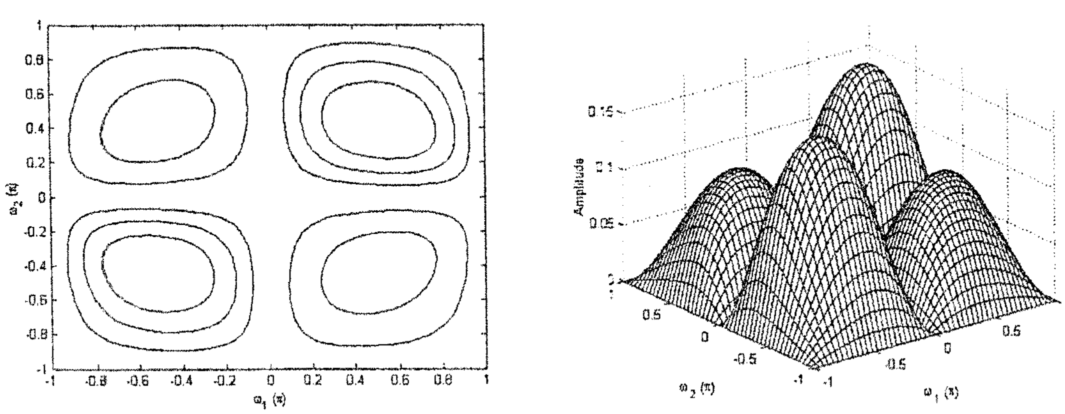
The effect of the high-pass coefficient α_{04} is mainly on the gains of the resulting high-pass filter. When the high-pass filter is cascaded with a low-pass one, the overall system is a 2-D band-pass filter with variable magnitude response, which is caused by the variation of α_{04}.

We still fix the other coefficients to be unity with proper signs, that is, k_1=1.0, k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=1.0, k_4=1.0, α_{03}=1.0, β_{03}=−1.0, and β_{04}=−1.0. The resulting 2-D band-pass filter is stable if α_{04} takes a value in [0, 1.0].

Figure 4.26 shows the simulation results for the contour and 3-D magnitude response of the resulting 2-D band-pass filter with different values of α_{04}. Figure 4.27 is the contour relation of the resulting 2-D band-pass filter and its member filters.

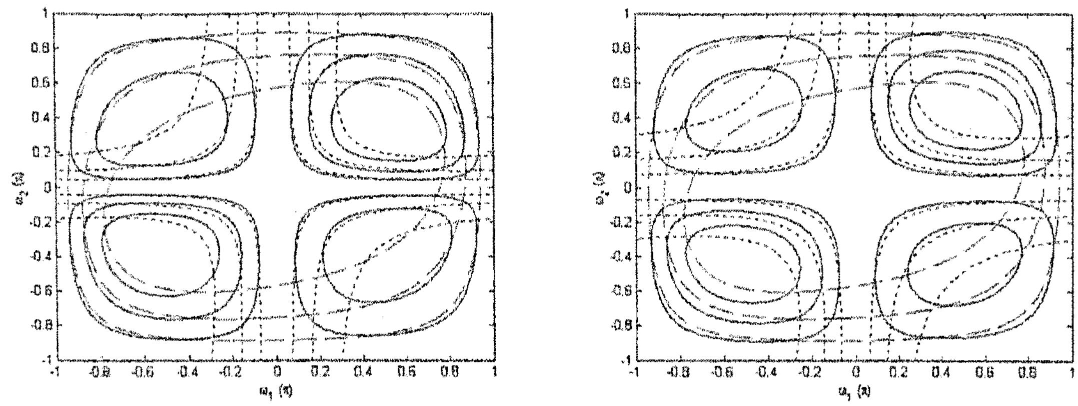


a. $\alpha_{04} = 0$



b. $\alpha_{04} = 0.5$

Figure 4.26 The contour and 3-D magnitude plots of the resulting 2-D band-pass filter with variable α_{04} and the other coefficients fixed as k_1=1.0, k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=−1.0, β_{02}=−1.0; k_3=1.0, k_4=1.0, α_{03}=1.0, β_{03}=−1.0 and β_{04}=−1.0



a. $\alpha_{04} = 0$ b. $\alpha_{04} = 0.5$

Figure 4.27 The contour relation between the 2-D band-pass filter and its member filters with different values of α_{04} and the other coefficients fixed as k_1=1.0, k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=1.0, k_4=1.0, α_{03}=1.0, β_{03}=−1.0 and β_{04}=−1.0

The high-pass coefficient α_{04} affects the bandwidth of the pass-band of the resulting 2-D band-pass filter slightly. It has very slight effect on the gain of the pass-bands as it is in the case of 2-D high-pass filter. The main reason is that the gains in the pass-band of the band-pass filter mainly depend on the overlapping of the pass-bands of the two member filters.

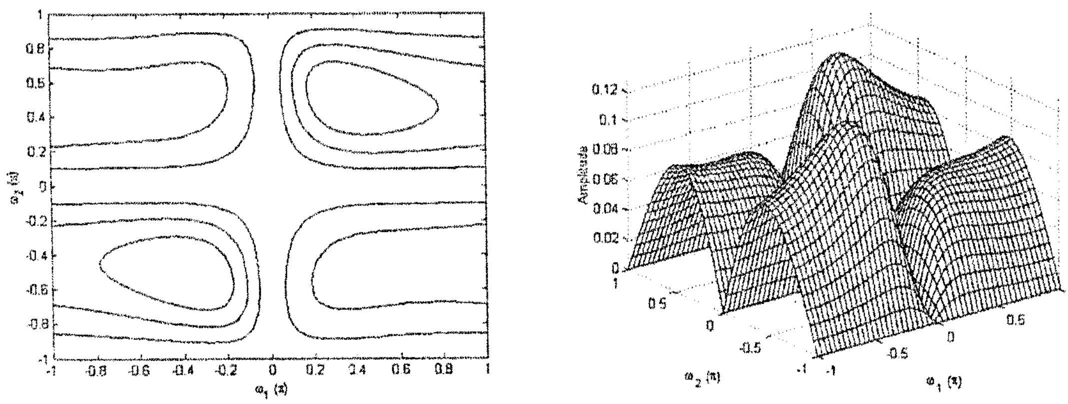
4.6.12 Frequency Response of the Resulting 2-D Band-Pass Filter with Variable β_{01}

The effect of low-pass coefficient β_{01} is mainly on the gain of the stop-band in ω_1-dimension. When β_{01} has values other than 1.0, the resulting 2-D low-pass filter has a non-zero gain in the stop-band, and the value of the gain depends on the distance of the value of β_{01} from 1.0. The larger the distance, the bigger the nonzero gain is. Of course, the value of β_{01} also affects the bandwidth of the pass-band, but the effect is very limited.

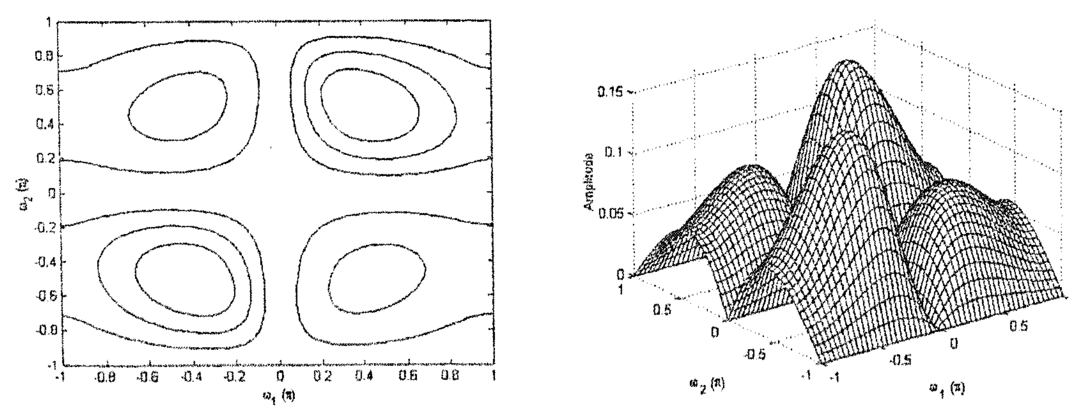
When the low-pass filter with variable β_{01} is cascaded with a high-pass filter to form a 2-D band-pass filter, the resulting filter will also have variable magnitude responses.

The same procedures are used here to investigate the effect of the variable β_{01} as which used in the investigations in the previous subsections. The other coefficients than β_{01} are still set to be unity with proper signs. Form Chapters 2 and 3, the high-pass filter is stable, and the low-pass filter is stable for any $\beta_{01} \in [0, 1.0]$, and therefore, the resulting 2-D band-pass filter is stable.

Figure 4.28 shows the result of the frequency responses of the resulting 2-D band-pass filter with different representative values of β_{01}. Figure 4.29 depicts the contour relation between the resulting 2-D band-pass filter and its member filters.

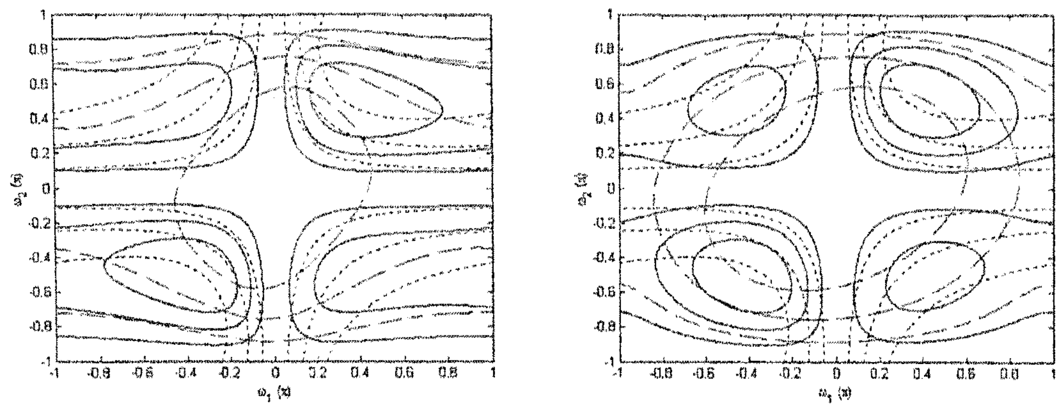


a. $\beta_{01} = 0$



b. $\beta_{01} = 0.5$

Figure 4.28 The contour and 3-D magnitude plots of the resulting 2-D band-pass filter with variable β_{01} and the other coefficients fixed as k_1=1.0, k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{02}=1.0; k_3=1.0, k_4=1.0, α_{03}=1.0, α_{04} =1.0, β_{03} = −1.0 and β_{04} = −1.0

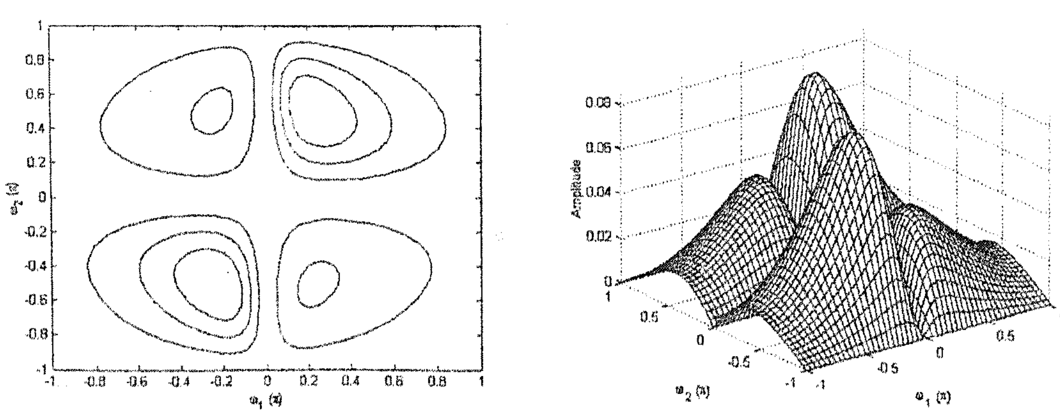


a. $\beta_{01} = 0$ b. $\beta_{01} = 0.5$

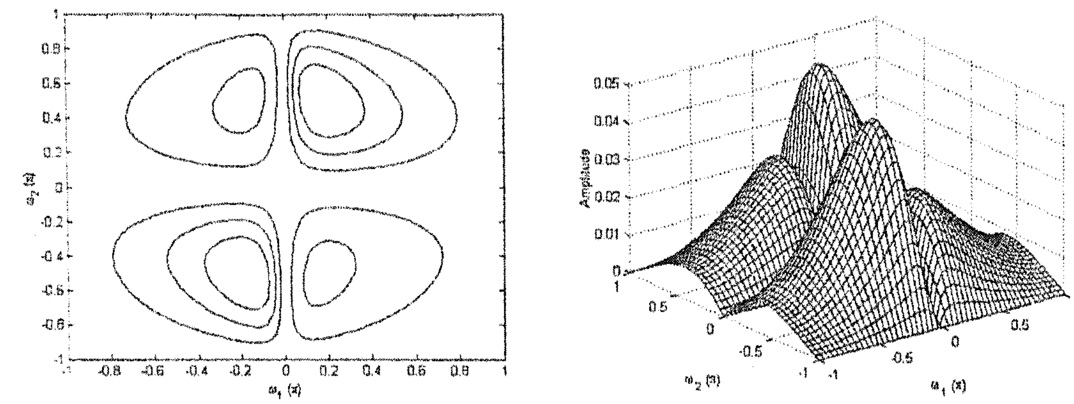
Figure 4.29 The contour relation between the 2-D band-pass filter and its member filters with variable β_{01} and the other coefficients fixed as k_1=1.0, k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{02}=1.0; k_3=1.0, k_4=1.0, α_{03}=1.0, α_{04} =1.0, β_{03}=−1.0 and β_{04}=−1.0

The low-pass coefficient β_{01} mainly affects the gain in the stop-bands of the resulting 2-D band-pass filter at high frequencies in ω_1-dimension. The resulting 2-D band-pass has zero gains in the stop-band of high-frequency range only when β_{01}=1.0.

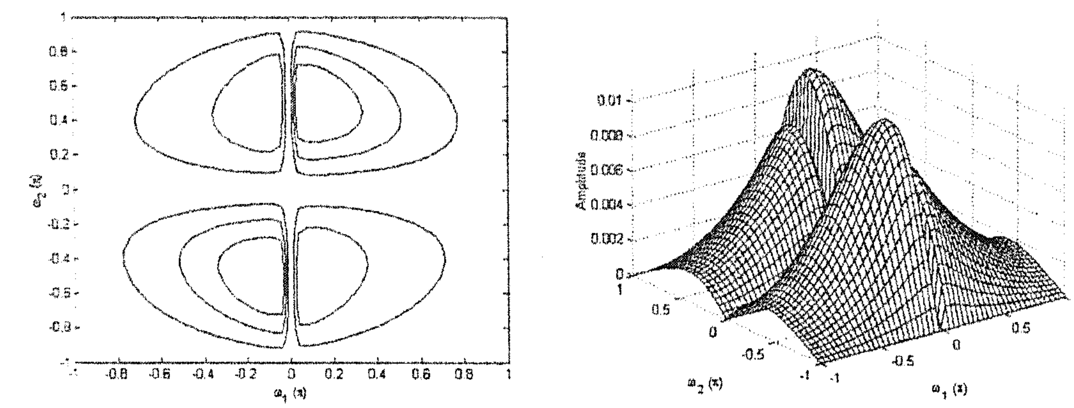
From Chapter 2, increasing k_1 can reduce the non-zero gains caused by β_{01} in the case of $\beta_{01}\neq-1.0$. Figure 4.30 and Figure 4.31 indicate the reduction of the non-zero gains in stop-bands in high frequencies of the resulting 2-D band-pass filter.



a. $k_1 = 5.0$

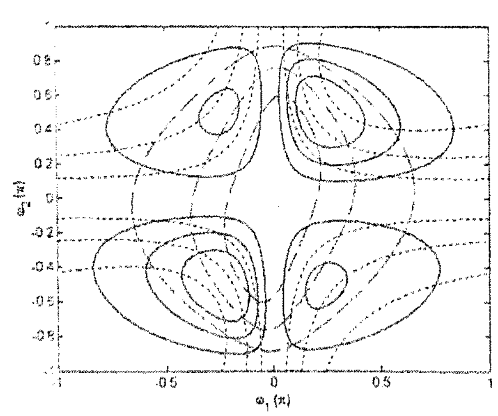


b. $k_1 = 10.0$

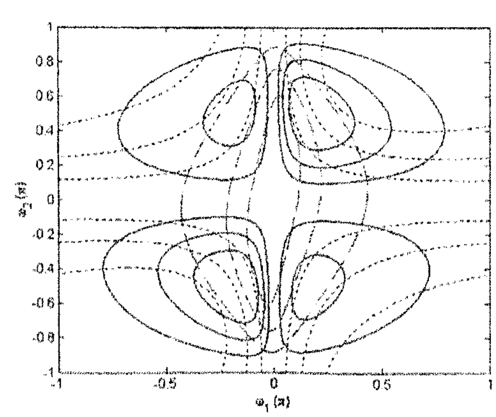


c. $k_1 = 50.0$

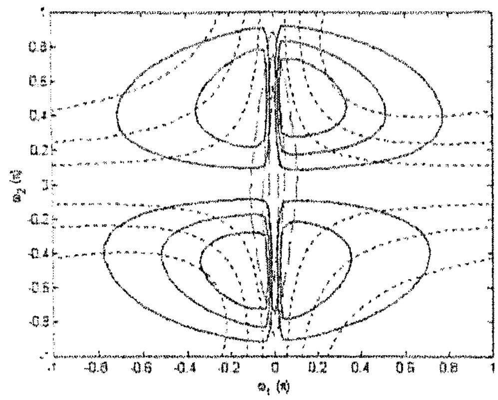
Figure 4.30 The contour and 3-D magnitude plots of the resulting 2-D band-pass filter with variable k_1 and the other coefficients fixed as k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=0.5, β_{02}=1.0; k_3=1.0, k_4=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0 and β_{04}= −1.0



a. $k_1 = 5.0$



b. $k_1 = 10.0$



c. $k_1 = 50.0$

Figure 4.31 The contour relation between the resulting 2-D band-pass filter and its member filters with variable k_1 and the other coefficients fixed as k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=0.5, β_{02}=1.0; k_3=1.0, k_4=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0 and β_{04}=−1.0

Increasing k_1 can also reduce the non-zero gains in the stop-bands in the high frequencies in ω_1-dimension, but the effect is very small.

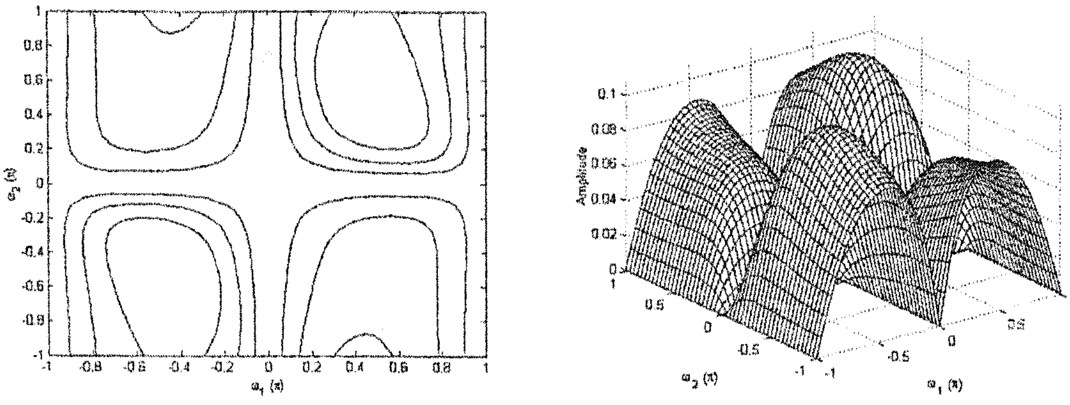
4.6.13 Frequency Response of the Resulting 2-D Band-Pass Filter with Variable β_{02}

The low-pass coefficient β_{02} mainly affects the gains of stop-bands of the member low-pass filter in ω_2-dimension. Only when β_{02} is chosen as 1.0, the upper boundary, the resulting low-pass filter has zero-gain at stop-bands.

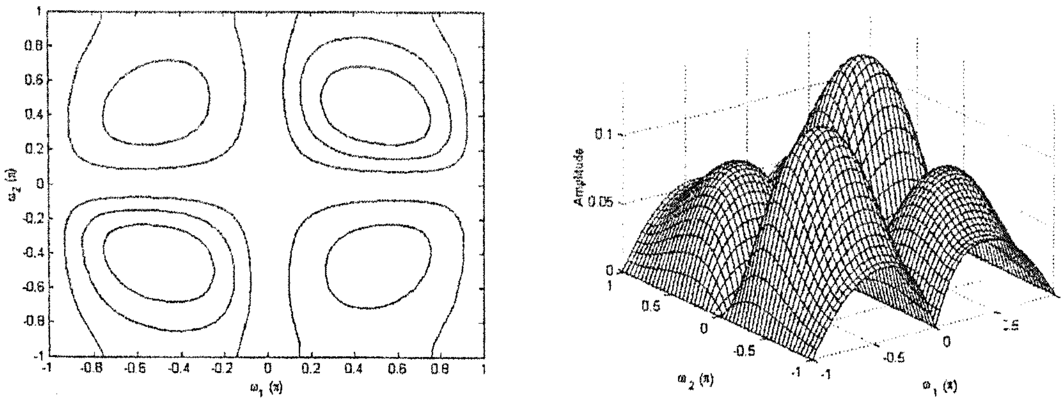
When the low-pass filter is employed with a fixed high-pass filter to form a 2-D system, which has the same frequency response as a 2-D band-pass filter, the overall system has variable magnitude responses.

We specify the other coefficients to be unity with proper signs, i.e. k_1=1.0, k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0; k_3=1.0, k_4=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0, and β_{04}=−1.0. It is easy to check that the 2-D band-pass filter is stable when β_{02} is chosen between 0 and 1.0.

The contour and 3-D magnitude plots of the resulting 2-D band-pass filter with different values of β_{02} are given in Figure 4.32. Figure 4.33 illustrated the contour relation between the 2-D resulting band-pass filter and its member filters.

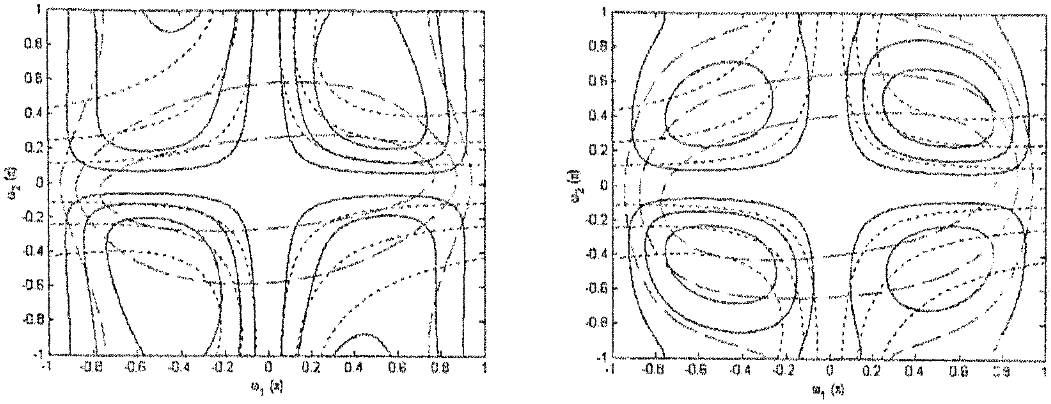


a. β_{02}=0



b. β_{02}=0.5

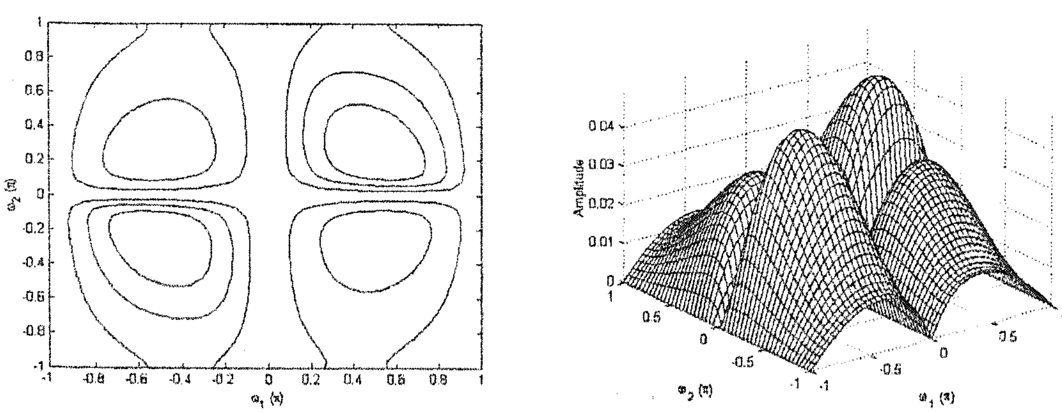
Figure 4.32 The contour and 3-D magnitude plots of the resulting 2-D band-pass filter with variable β_{02} and the other coefficients fixed as k_1=1.0, k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0; k_3=1.0, k_4=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0 and β_{04}=−1.0



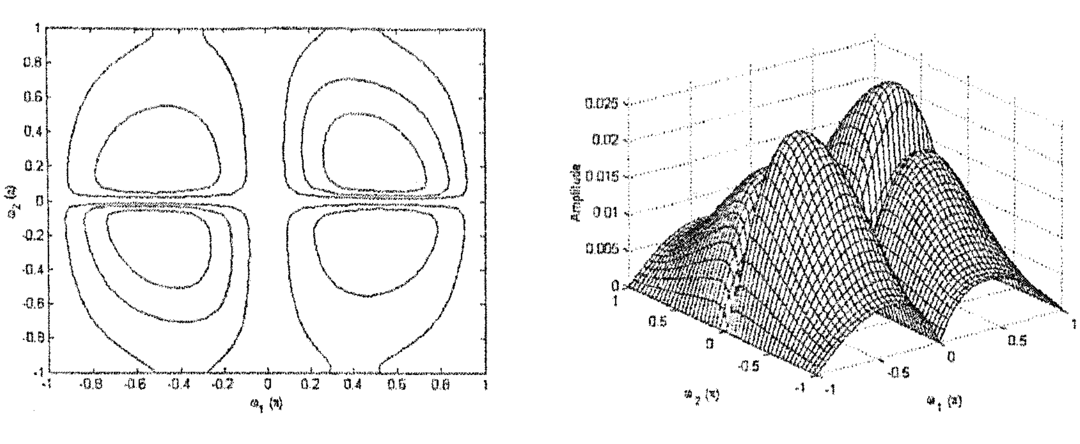
a. $\beta_{02} = 0$ b. $\beta_{02} = 0.5$

Figure 4.33 The contour relation between the 2-D band-pass filter and its member filters with variable β_{02} and the other coefficients fixed as k_1=1.0, k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0; k_3=1.0, k_4=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0 and β_{04}=−1.0

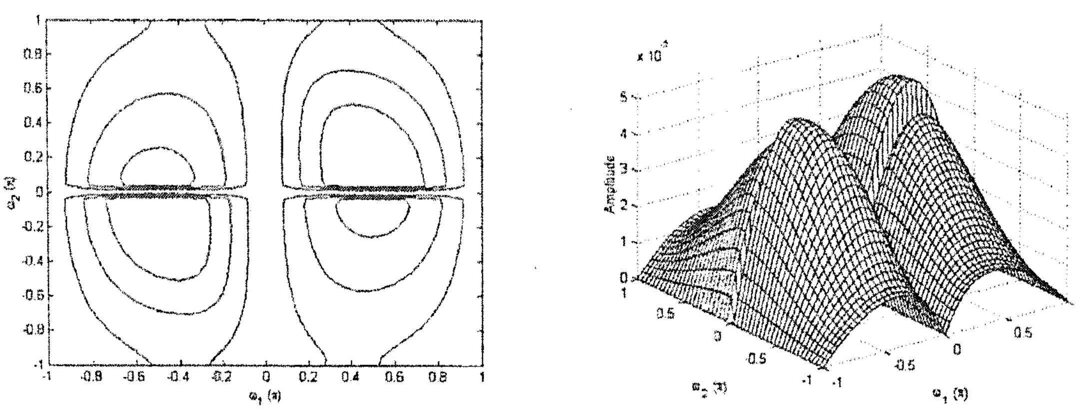
The low-pass coefficient β_{02} also causes the resulting 2-D band-pass filter to have non-zero gains in the stop-bands in high frequency regions in ω_2-dimension, if k_2 does not have the value 1.0. Increasing k_2 can reduce the non-zero gains. Figure 4.34 and Figure 4.35 illustrate the reduction.



a. $k_2 = 5.0$

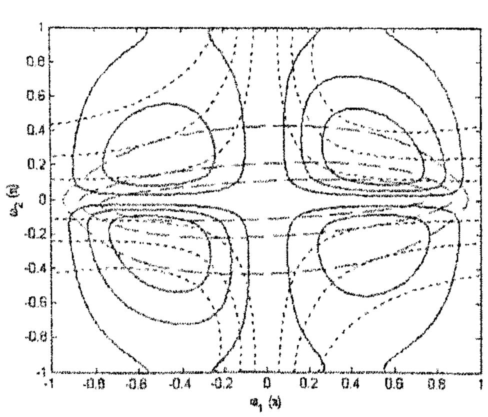


b. $k_2 = 10.0$

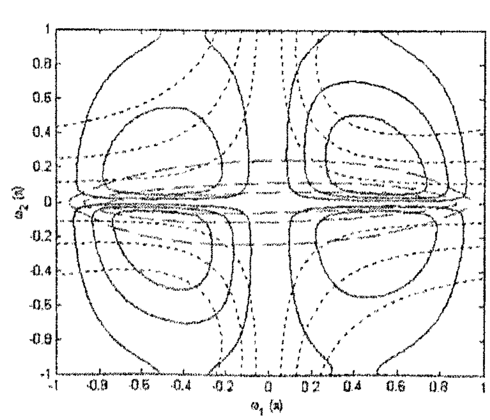


c. $k_2 = 50.0$

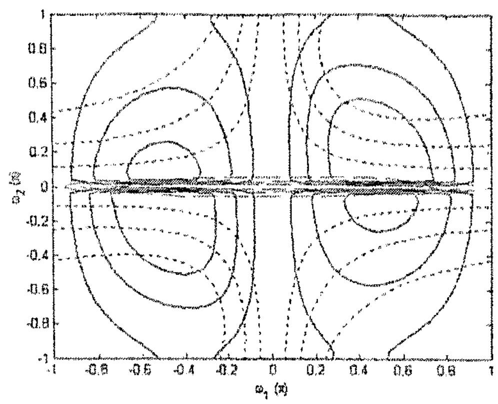
Figure 4.34 The contour and 3-D magnitude plots of the resulting 2-D band-pass filter with variable k_2 and the other coefficients fixed as k_1=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=0.5; k_3=1.0, k_4=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0 and β_{04}=−1.0



a. k_2 =5. 0



b. k_2 = 10.0



c. k_2 = 50.0

Figure 4.35 The contour relation between the resulting 2-D band-pass filter and its member filters with variable k_2 and the other coefficients fixed as k_1=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=0.5; k_3=1.0, k_4=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}= −1.0 and β_{04}= −1.0

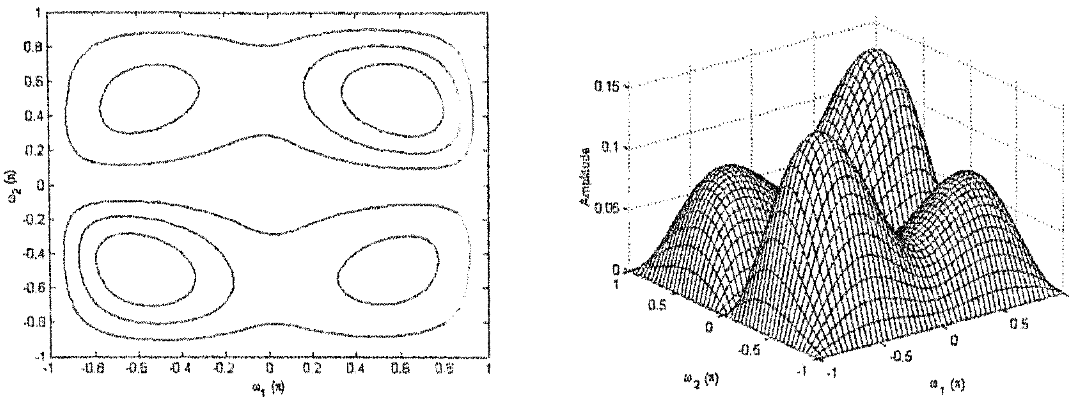
Increasing k_2 can really reduce the non-zero gains caused by in the case of $\beta_{02}\neq 1.0$. However, the reduction is non-remarkable.

4.6.14 Frequency Response of the Resulting 2-D Band Pass Filter with Variable β_{03}

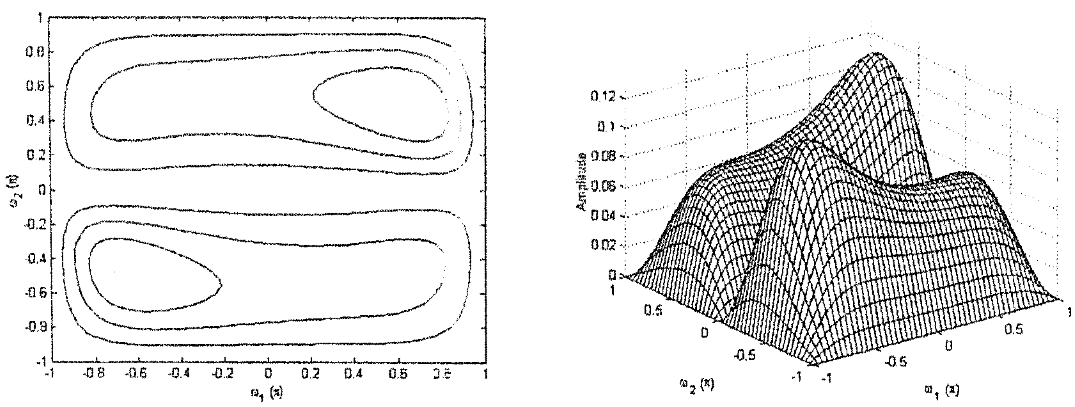
The high-pass coefficient β_{03} (known as β_{01} in Chapter 3) causes the resulting 2-D high-pass filter to have non-zero gains at stop-bands in ω_1-dimension. The high-pass filter can be employed as a member filter to form a 2-D band-pass filter. It is clear that the resulting 2-D band-pass filter has variable magnitude responses if the member high-pass filter has variable β_{03}.

The other coefficients other than β_{03} are still set to be unity with proper signs, that is, k_1=1.0, k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0; k_3=1.0, k_4 = 1.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0, and β_{04}=−1.0. Choosing any value in [−1.0, 0] for β_{03}, the resulting 2-D band-pass filter is stable.

The contour and 3-D magnitude plots of the resulting 2-D band-pass filter are illustrated in Figure 4.36. The contour relation between the resulting 2-D band-pass filter and its member filters is shown in Figure 4.37.

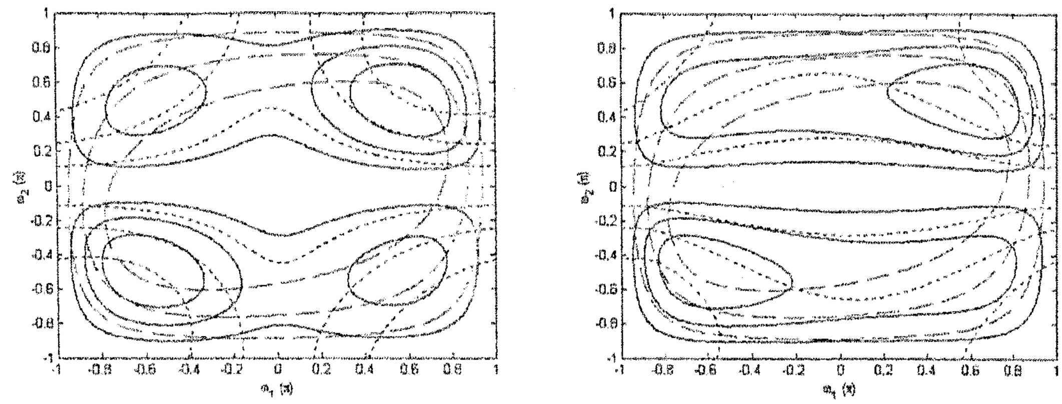


a. $\beta_{03} = -0.5$



b. $\beta_{03} = 0$

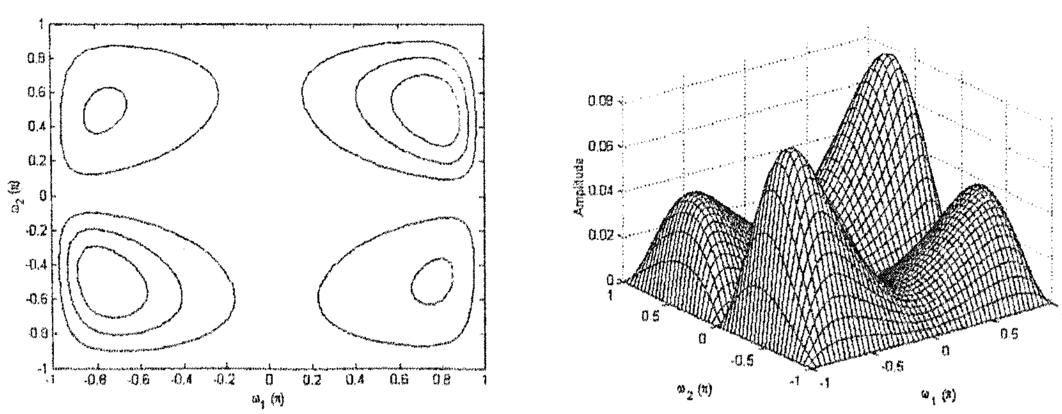
Figure 4.36 The contour and 3-D magnitude plots of the resulting 2-D band-pass filter with variable β_{03} and the other coefficients fixed as k_1=1.0, k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=1.0, k_4=1.0, α_{03}=1.0, α_{04}=1.0, and β_{04}=−1.0



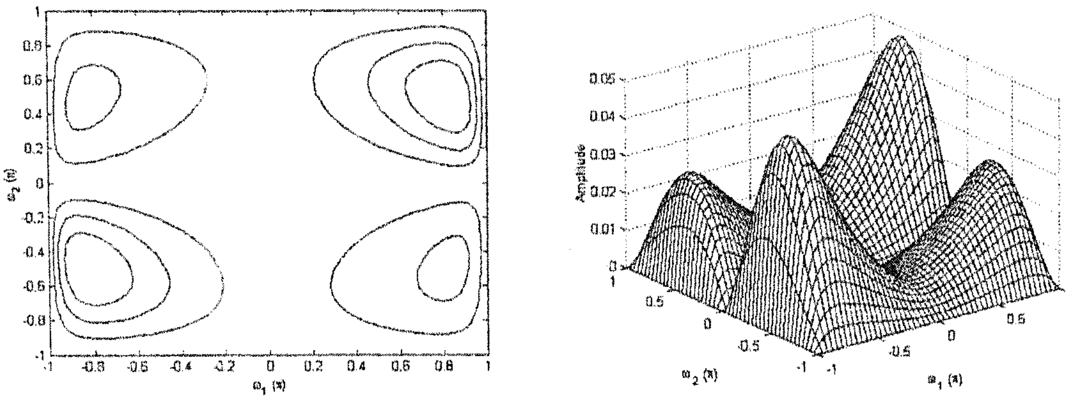
a. $\beta_{03} = -0.5$ b. $\beta_{03} = 0$

Figure 4.37 The contour relation of the resulting 2-D band-pass filter and its member filters with variable β_{03} and the other coefficients fixed as k_1=1.0, k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=1.0, k_4=1.0, α_{03}=1.0, α_{04}=1.0, and β_{04} = −1.0

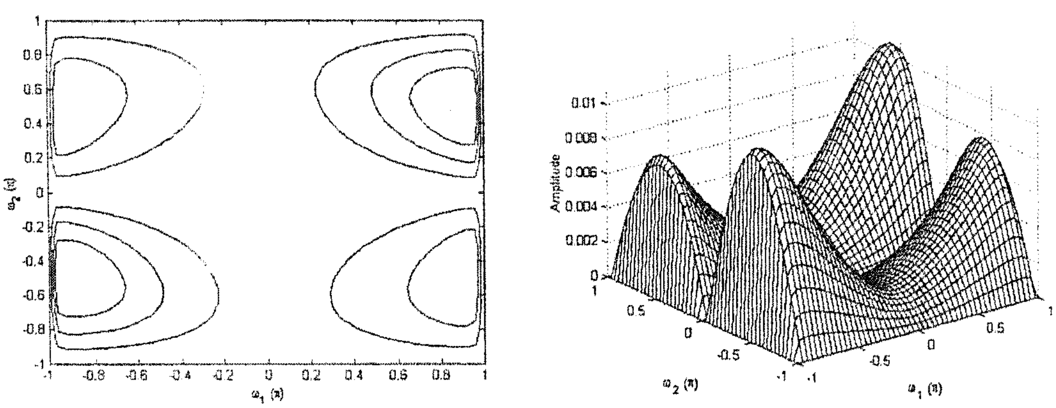
The resulting 2-D band-pass filter, which is formed by the cascading of fixed-characteristic low-pass filter and the high-pass filter with variable β_{03}, has non-zero gain in the low frequency stop-bands, unless β_{03} is equal to −1.0. In Chapter 3, we found that increasing k_3 can reduce the non-zero gains at stop-band of the high-pass filter. Here, we also need to investigate the effect on the gains of the stop-band by increasing the value of k_3. The simulation results are illustrated in Figure 4.38 and Figure 4.39.



a. k_3=5.0

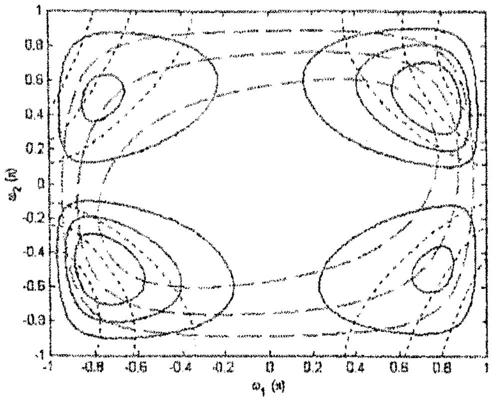


b. $k_3 = 10.0$

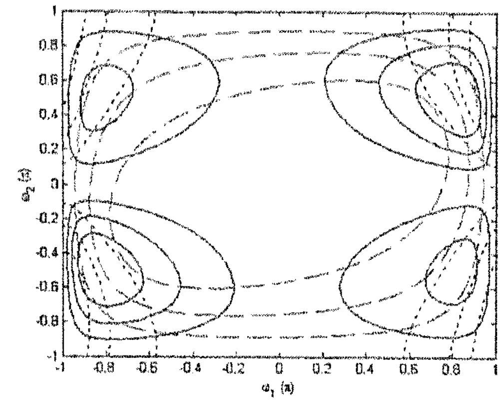


c. $k_3 = 50.0$

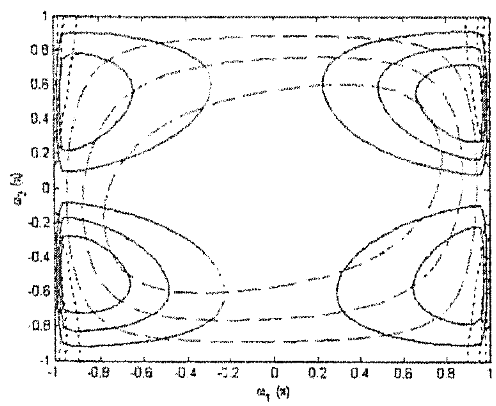
Figure 4.38 The contour and 3-D magnitude plots of the resulting 2-D band-pass filter with variable k_3 and the other coefficients fixed as k_1=1.0, k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_4=1.0, α_{03}=1.0, α_{04}=1.0, β_{03} = −0.5 and β_{04} = −1.0



a. k_3 =5. 0



b. k_3 = 10.0



b. k_3 = 10.0

Figure 4.39 The contour relation of the 2-D band-pass filter with its member filters with different values of k_3 and the other coefficients fixed as k_1=1.0, k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_4=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−0.5 and β_{04}=−1.0

Although increasing k_3 can reduce the non-zero gains of the stop-band in the low frequency parts in ω_1-dimension, the effect is limited.

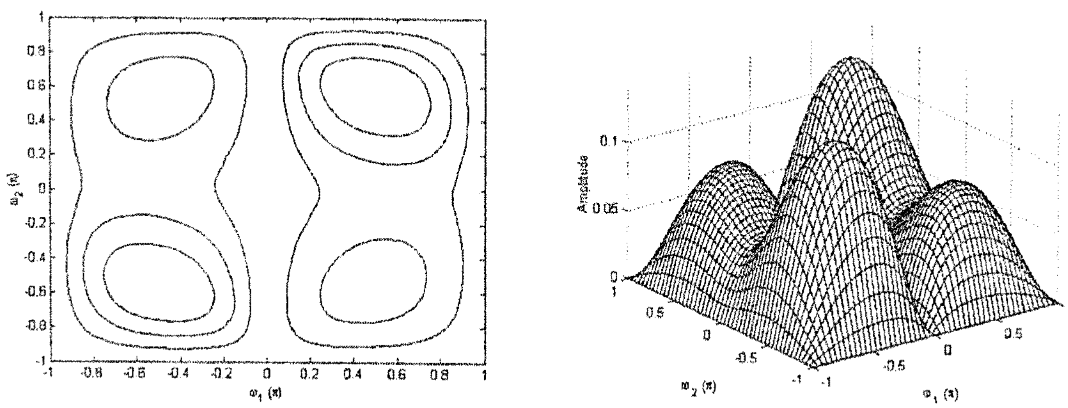
4.6.15 Frequency Response of the Resulting 2-D Band-Pass Filter with Variable β_{04}

The high-pass coefficient β_{04} (known as β_{02} in Chapter 3) mainly affects the gains at the stop-bands of the resulting 2-D high-pass filter in ω_2-dimension. As a result, the 2-D band-pass filter, formed by cascading the high-pass filter with a low-pass filter, will have variable frequency responses also.

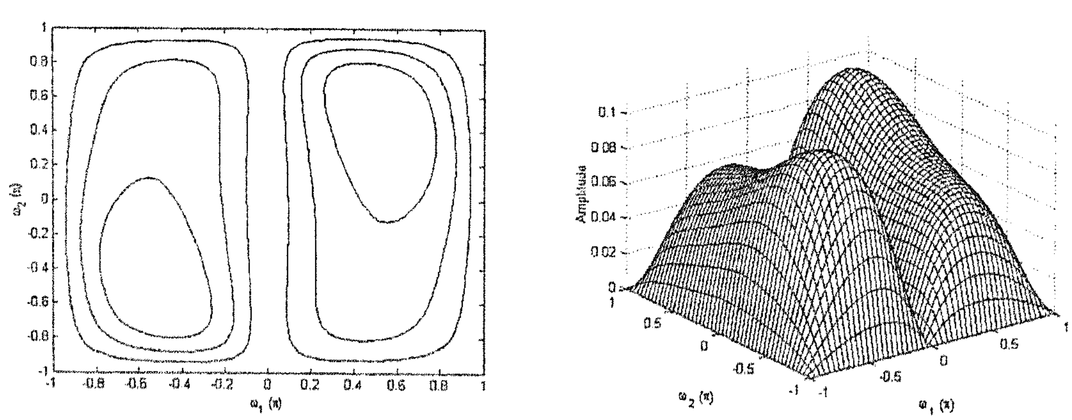
Similarly, the other coefficients are fixed to be unity with proper signs, i. e., k_1=1.0,

k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=1.0, k_4=1.0, α_{03}=1.0, α_{04}=1.0, and β_{03}=−1.0. The 2-D band-pass filter is stable if the value for β_{04} is chosen between −1.0 and 0.

Figure 4.40 gives the contour and 3-D magnitude plots of the resulting 2-D band-pass filter with β_{04}=−0.5 and 0. Figure 4.41 is the contour relation between the resulting 2-D band-pass filter and its member filters.

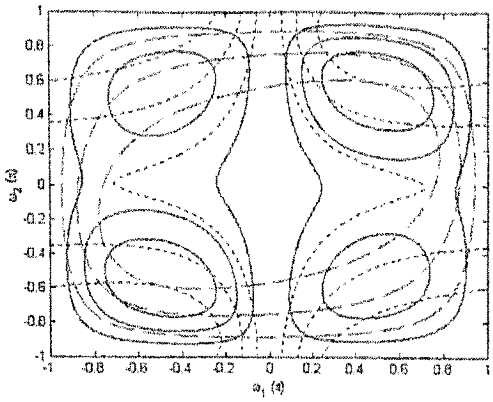


a. $\beta_{04} = -0.5$

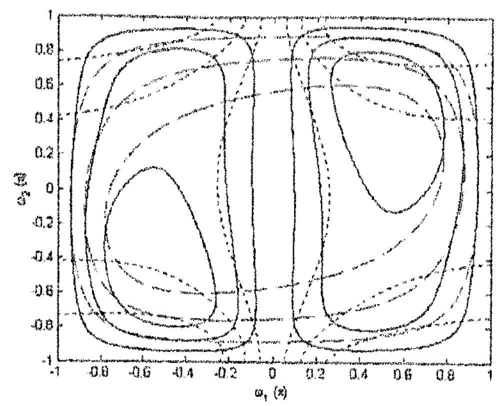


b. β_{04}=0

Figure 4.40 The contour and 3-D magnitude plots of the resulting 2-D band-pass filter with variable β_{04} and the other coefficient fixed as k_1=1.0, k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=1.0, k_4=1.0, α_{03}=1.0, α_{04} =1.0, and β_{03} = −1.0



a. $\beta_{04}=-0.5$

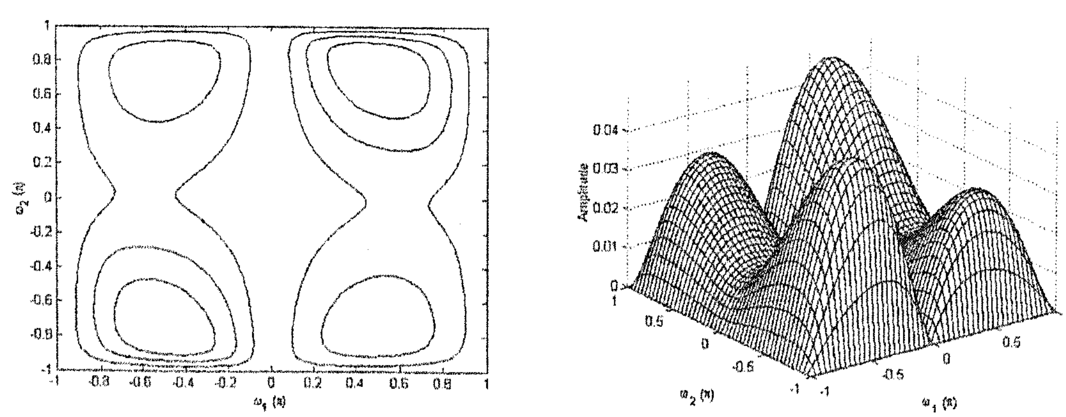


b. $\beta_{04}=0$

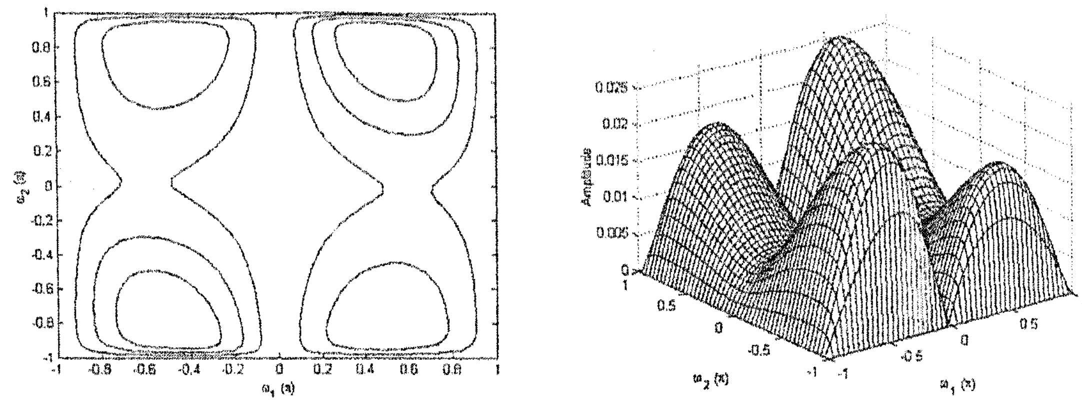
Figure 4.41 The contour relation of the 2-D band-pass filter and its member filters with different values of β_{04} and the other coefficients fixed as $k_1=1.0$, $k_2=1.0$, $\alpha_{01}=-1.0$, $\alpha_{02}=-1.0$, $\beta_{01}=1.0$, $\beta_{02}=1.0$; $k_3=1.0$, $k_4=1.0$, $\alpha_{03}=1.0$, $\alpha_{04}=1.0$, and β_{03} -1.0

The gain in the low-frequency stop-band in ω_2-dimension is non-zero, if β_{04} has a value other than -1.0, and the value of the non-zero gain depends on the distance of the actual value of k_4 from -1.0. The larger the distance, the bigger the non-zero gain is.

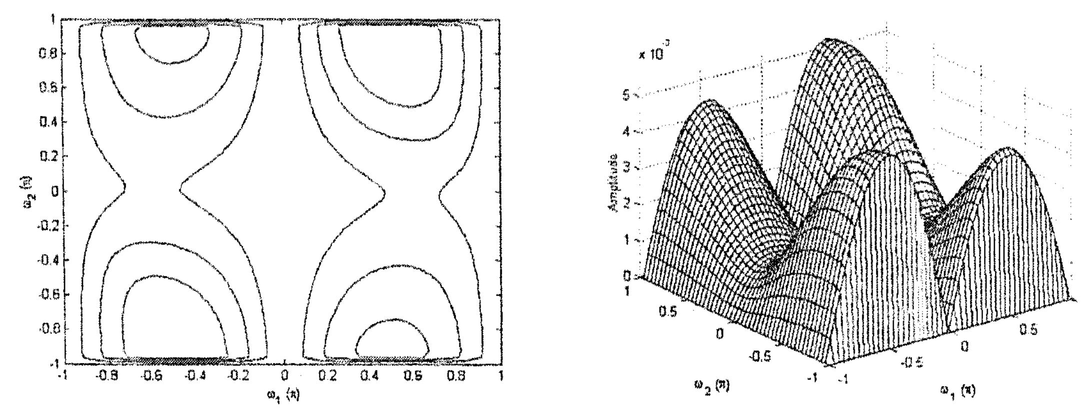
Figure 4.42 and Figure 4.43 show the gain reduction caused by increasing the values of k_4 when β_{04} is fixed at -0.5.



a. $k_4=5.0$

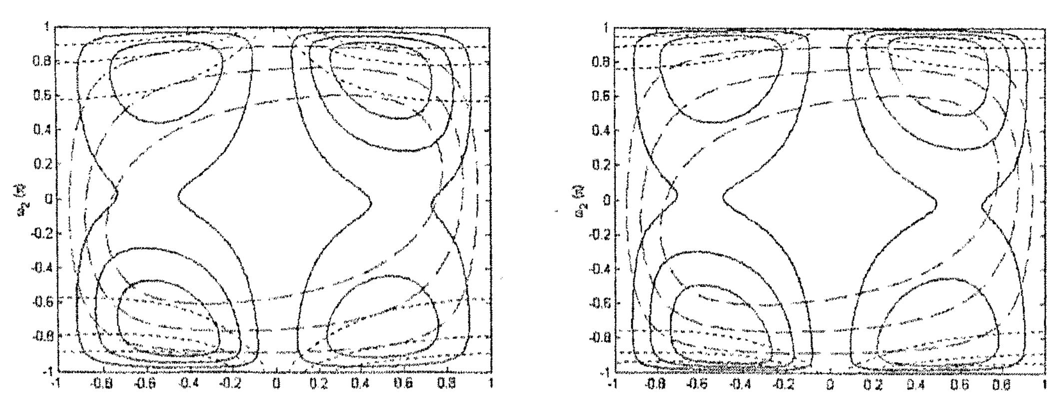


b. $k_4 = 10.0$

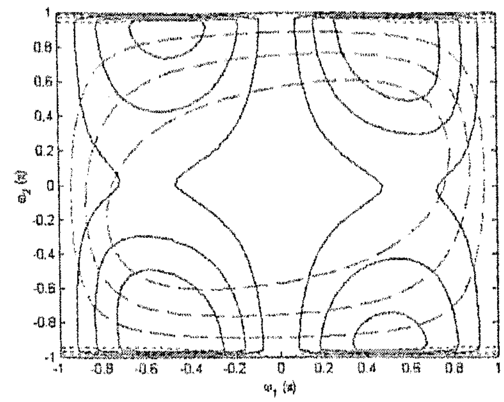


c. $k_4 = 50.0$

Figure 4.42 The contour and 3-D magnitude plots of the resulting 2-D band-pass filter with variable k_4 and other coefficients fixed as k_1=1.0, k_2=1.0, α_{01}=–1.0, α_{02}=–1.0, β_{01}=1.0, β_{02}=1.0; k_3=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}=–1.0 and β_{04}=–0.5



a. k_4 =5. 0 b. k_4 = 10.0



c. k_4=50.0

Figure 4.43 The contour relation of the 2-D band-pass and its member filters with different values of k_4 and the other coefficients fixed as k_1 = 1.0, k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0 and β_{04}=−0.5

Increasing the value of k_4 can reduce the non-zero gain caused by β_{04} in the case of $\beta_{04}\neq-1.0$, but the effect is limited.

4.7 Summary and Discussion

In summary, the cascade of a 2-D low-pass filter and a 2-D high-pass filter could obtain the identical frequency responses as a 2-D band-pass filter. When the 2-D low-pass and high-pass filters are designed from their analog prototype filters through the application of the double generalized bilinear transformations, changing coefficients causes the resulting 2-D low-pass and high-pass filters to have variable magnitude response, which we have investigated in Chapters 2 and 3 in detail, and therefore, lead to 2-D band-pass filters with variable magnitude characteristics.

The cascade combination enhances the stability of the overall system. The stability of each subsystem guaranteed the stability of the overall 2-D band-pass filtering system. As such the stability of the 2-D band-pass filter is equivalent to the stability for each individual subsystem.

As the characteristics of the resulting 2-D band-pass filter is mainly determined by the overlapping areas of the pass-bands of the two member filters, the coefficients k_i's

(i=1, 2, 3, 4) play the most important role in 2-D band-pass filter design. They not only affect the bandwidth of the pass-bands as they did in the cases of low-pass and high-pass filters, but also move the center frequency of the pass-bands. Briefly speaking, k_1 and k_3 move the center frequencies in ω_1-dimension in opposite directions, whereas k_2 and k_4 move the center frequencies in ω_2-dimension in the same manners. And also, k_i's determine the gains of the pass-bands of the resulting 2-D band-pass filter.

The coefficients α_{0i}'s (i=1, 2, 3, 4) no longer affect the gains of pass-bands as they did in the cases of 2-D low-pass and high-pass filters. The coefficients β_{0i}'s (i=1, 2, 3, 4) still affect the gains of the stop-bands and increasing k_i's (i=1, 2, 3, 4) do not effectively reduce the non-zero gains.

This chapter is just the first step towards the study of the 2-D band-pass filter formed by a cascade combination of 2-D low-pass and high-pass filters. Modern signal processing technology needs to use this feature to obtain user-specific variable magnitude response in frequency domain to enhance signal quality.

Chapter 5

Two-Dimensional Band-Elimination Filters

In this chapter, another type of combination-based filter, 2-D band-elimination filter, is studied. In section 5.1, a brief definition of 2-D band-elimination filter is given in both mathematical and graphical forms. The algorithm of parallel combination of a 2-D low-pass filter and a 2-D high-pass filter, which could result in the same frequency response of a 2-D band-elimination filter, is studied in section 5.2. The transfer functions of the member low-pass and high-pass filters, which are obtained from the same analog prototype by double generalized bilinear transformations, are given in sections 5.3 and 5.4, respectively. The extended stability conditions of the resulting 2-D band-elimination are studied in section 5.5. The frequency response of the 2-D band-elimination filter, as well as the manner how each coefficient affects the magnitude response is given in section 5.6. The summary and discussion are given in section 5.7.

5.1 Introduction

The band-elimination filters stop the signal components within a specified range of frequencies, while passing the signal components within lower and higher frequency regions. That is to say, the signals with low and high frequencies have a very high gain, but the signal components in the specified frequency band or range have a very low gain. A typical 2-D band-eliminating digital filter has a specification in the form as[2, 4]

$$H(\omega_1,\omega_2)=\begin{cases}0, & |\omega_i|\le\omega_{is1}\\ 1, & \omega_{ip1}\le|\omega_i|\le\omega_{ip2}\\ 0, & \omega_{is2}\le|\omega_i|\le\pi\end{cases} \tag{5.1}$$

where:

ω_{ip1} and ω_{ip2}, (i =1, 2), are pass bands in z_1 and z_2-dimension, respectively

ω_{is1} and ω_{is2}, (i =1, 2), are stop bands in z_1 and z_2-dimension, respectively

The specification of the typical 2-D band-elimination filter in frequency domain is given in Figure 5.1.

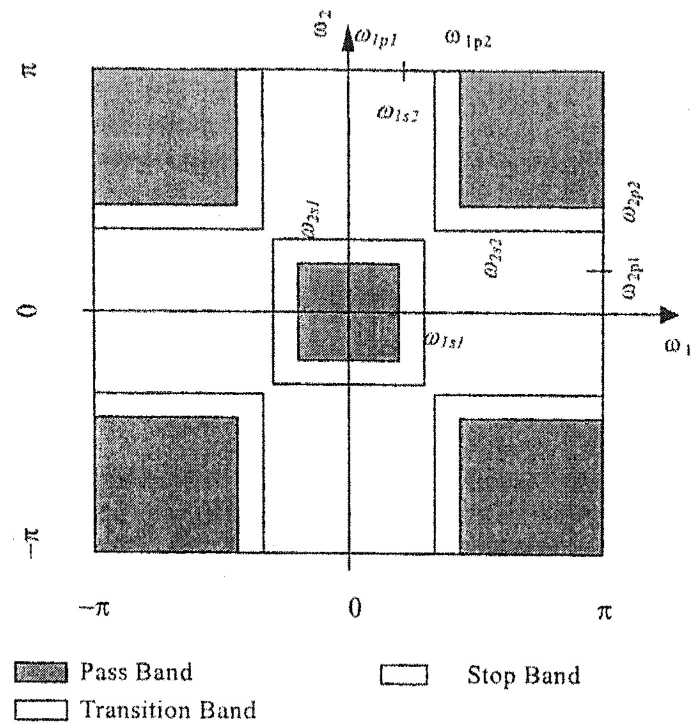


Figure 5.1 The specification for a typical 2-D band-elimination digital filter

5.2 The Algorithm of Parallel Combination of Two Filters

From Figure 5.1, it is obvious that a typical 2-D band-elimination digital filter has the pass-band of a typical 2-D low-pass filter illustrated in Figure 2.1 and the pass-band of a typical 2-D high-pass digital filter illustrated in Figure 3.1 simultaneously.

From Figure 5.1, we can also build the logical relations, which are listed in Table 5.1, between the frequency response of a typical 2-D band-elimination filter and the frequency response of a typical 2-D low-pass filter and a typical 2-D high-pass filter.

Table 5.1 The logical relation of the Frequency response between the 2-D band-eliminating digital filter and the frequency responses of the 2-D low-pass and 2-D high-pass filters. (where, "1" represents the pass-band, "0" represents stop-band.)

2-D low-pass filter (A)	2-D high-pass filter (B)	The resulting 2-D band-elimination filter (C)
1	0	1
0	1	1
1	1	1
0	0	0

The frequency response of the 2-D band-elimination filter (C) is the sum of frequency responses of the two member filters, i.e.,

$$H_C(\omega_1,\omega_2) = H_A(\omega_1,\omega_2) + H_B(\omega_1,\omega_2) \tag{5.2}$$

Equation (5.2) can be achieved by a parallel combination of a 2-D low-pass filter and a 2-D high-pass filter as shown in Figure 5.2.

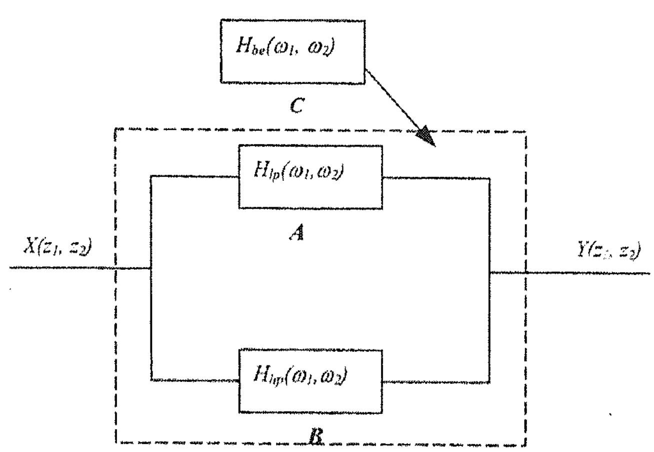


Figure 5.2 The possible 2-D band-elimination filter by parallel combination of a low-pass and a high-pass filter

From the above discussion, we can draw the conclusion that through parallel combination of a 2-D low-pass filter and a 2-D high-pass filter, the overall system has the identical frequency response of a 2-D band-elimination filter. We call the 2-D low-pass and high-pass filters as member low-pass filter and member high-pass filter, respectively. These member filters can be designed from the same analog prototype through the application of the double generalized bilinear transformations by the methods introduced in Chapters 2 and 3. When one or more of coefficients of the double generalized bilinear transformations are changing, the member filters have variable magnitude responses, and , in turn, the resulting 2-D band-elimination filter may possesses variable magnitude characteristics.

Below, we first briefly introduce the transfer functions of the two member filters.

5.3 The Member Low-Pass Filter

From Chapter 2, the 2-D analog transfer function of the circuit network in Figure 2.3 can be written as

$$H_a(s_1,s_2)=\frac{N_a(s_1,s_2)}{D_a(s_1,s_2)}=\frac{1}{(1+0.707s_1)(1.414s_2+1)+1}$$

$$=\frac{1}{s_1s_2+0.707s_1+1.414s_2+2} \tag{5.3}$$

The denominator

$$D_a(s_1,s_2)=s_1s_2+0.707s_1+1.414s_2+2 \tag{5.4}$$

is a VSHP and has a single degree for each variable.

Applying the double generalized bilinear transformation (1.37) to Equation (5.4), and constraining all the coefficients to low-pass limits, we can obtain the transfer function of the resulting 2-D low-pass digital filter as

$$H_{lp}(z_1,z_2)=\frac{N_{lp}(z_1,z_2)}{D_{lp}(z_1,z_2)} \tag{5.5}$$

where, $$N_{lp}(z_1,z_2)=z_1z_2+\beta_{02}z_1+\beta_{01}z_2+\beta_{01}\beta_{02} \tag{5.6a}$$

$$\begin{aligned}D_{lp}(z_1,z_2)=&(k_1k_2+0.707k_1+1.414k_2+2)z_1z_2\\&+(k_1k_2\alpha_{02}+0.707k_1\beta_{02}+1.414k_2\alpha_{02}+2\beta_{02})z_1\\&+(k_1k_2\alpha_{01}+0.707k_1\alpha_{01}+1.414k_2\beta_{01}+2\beta_{01})z_2\\&+(k_1k_2\alpha_{01}\alpha_{02}+0.707k_1\alpha_{01}\beta_{02}+1.414k_2\alpha_{01}\beta_{01}\\&+2\beta_{01}\beta_{02})\end{aligned} \tag{5.6b}$$

It is evident that the denominator (5.7) is a 2-variable polynomial, which has single degree for each variable, but the overall degree is 2. we can write it in the general form,

$$D_{lp}(z_1,z_2)=a_{11}z_1z_2+a_{10}z_1+a_{01}z_2+a_{00} \tag{5.7}$$

The 2-D low-pass filter can be obtained when the coefficients of the double bilinear transformation meet the low-pass limits

$$k_i>0,\quad i=1,2 \tag{5.8a}$$

$$-1.0\le\alpha_{0i}\le 0,\quad i=1,2 \tag{5.8b}$$

$$0\le\beta_{0i}\le 1.0,\quad i=1,2 \tag{5.8c}$$

The ranges of the coefficients should also be constrained by the stability conditions of the 2-D digital filter with denominator of single degree for each variable.

5.4 The Member High-Pass filter

Applying double bilinear transformation (1.37) with high-pass limits to the analog transfer function (5.6), the discrete transfer function of 2-D high-pass filter can be obtained,

$$H_{hp}(z_1,z_2)=\frac{N_{hp}(z_1,z_2)}{D_{hp}(z_1,z_2)} \tag{5.9}$$

where:

$$N_{hp}(z_1,z_2)=z_1z_2+\beta_{04}z_1+\beta_{03}z_2+\beta_{03}\beta_{04} \tag{5.10a}$$

$$\begin{aligned}D_{hp}(z_1,z_2)=&(k_3k_4+0.707k_3+1.414k_4+2)z_1z_2\\&+(k_3k_4\alpha_{04}+0.707k_3\beta_{04}+1.414k_4\alpha_{04}+2\beta_{04})z_1\\&+(k_3k_4\alpha_{03}+0.707k_3\alpha_{03}+1.414k_4\beta_{03}+2\beta_{03})z_2\\&+(k_3k_4\alpha_{03}\alpha_{04}+0.707k_3\alpha_{03}\beta_{04}+1.414k_4\alpha_{03}\beta_{03}\\&+2\beta_{03}\beta_{04})\end{aligned} \tag{5.10b}$$

The denominator (5.11b) is a 2-variable polynomial, which has a single degree for each variable and the overall degree is 2. It can be rewritten as the general form:

$$D_{hp}(z_1,z_2)=b_{11}z_1z_2+b_{10}z_1+b_{01}z_2+b_{00} \tag{5.11}$$

To identify the high-pass coefficients from the low-pass ones, we use the subscriptions "3" and "4" to replace "1" and "2", and the coefficients should meet the high-pass limits

$$k_i>0,\ \ i=3,4 \tag{5.12a}$$

$$0\le\alpha_{0i}\le1.0,\ \ i=3,4 \tag{5.12b}$$

$$-1.0\le\beta_{0i}\le0,\ \ i=3,4 \tag{5.12c}$$

The coefficients should also satisfy the requirements of the stability conditions for 2-D digital filter with a denominator of 2-variable that has single degree for each variable z_1 and z_2 and the overall degree of the term z_1z_2 is 2.

5.5 Stability of the 2-D Band-Elimination Filters

Stability is always an important problem in 2-D recursive digital filter design. The most common stability criterion is so called the Bounded-Input Bounded-Out (BIBO) criterion. A system is table in BIBO sense if every bounded input sequence produces a bounded output sequence.

As the 2-D band-elimination filter system is a parallel combination of a 2-D low-pass filter and a 2-D high-pass filter, we need to examine the stability issue in terms of the relationship between the resulting 2-D band-elimination filter and the two member filters.

Assuming that both the low-pass and high-pass filters are stable in BIBO sense, then the following relations exist:

$$\sum_{n_1=-\infty}^{\infty}\sum_{n_2=-\infty}^{\infty}\left|h_A(n_1,n_2)\right| = S_A < \infty \tag{5.13}$$

$$\sum_{n_1=-\infty}^{\infty}\sum_{n_2=-\infty}^{\infty}\left|h_B(n_1,n_2)\right| = S_B < \infty \tag{5.14}$$

The impulse response of the overall 2-D band-elimination system can be expressed as

$$\begin{aligned} S_C = \sum_{n_1=-\infty}^{\infty}\sum_{n_2=-\infty}^{\infty}\left|h_C(n_1,n_2)\right| &= \sum_{n_1=-\infty}^{\infty}\sum_{n_2=-\infty}^{\infty}\left|h_A(n_1,n_2)+h_B(n_1,n_2)\right| \\ &\le \sum_{n_1=-\infty}^{\infty}\sum_{n_2=-\infty}^{\infty}\left|h_A(n_1,n_2)\right| + \sum_{n_1=-\infty}^{\infty}\sum_{n_2=-\infty}^{\infty}\left|h_B(n_1,n_2)\right| \\ &= S_A + S_B < \infty \end{aligned} \tag{5.15}$$

It is evident that the overall 2-D band-elimination filter system is stable if the two subsystems are stable. The stability of the two individual subsystems shall be the sufficient conditions of the stability of the overall system. So the consideration of the stability for the overall system is equivalent to considering the stability for the two subsystems.

The stability of 2-D low-pass and high-pass recursive digital filters should be determined not only by the selection of the coefficients used in the generalized bilinear transformation, but also by the stability condition for the digital filters.

The stability conditions of 2-D low-pass and high-pass filters with denominators in which each variable has single degree and their mutilation degree is two are given in Chapters 2 and 3, respectively.

After solving the stability problem of the overall 2-D band-elimination filter, we need to examine the frequency response of the resulting 2-D band-elimination filter, as well as the effect of each coefficient on the magnitude characteristics.

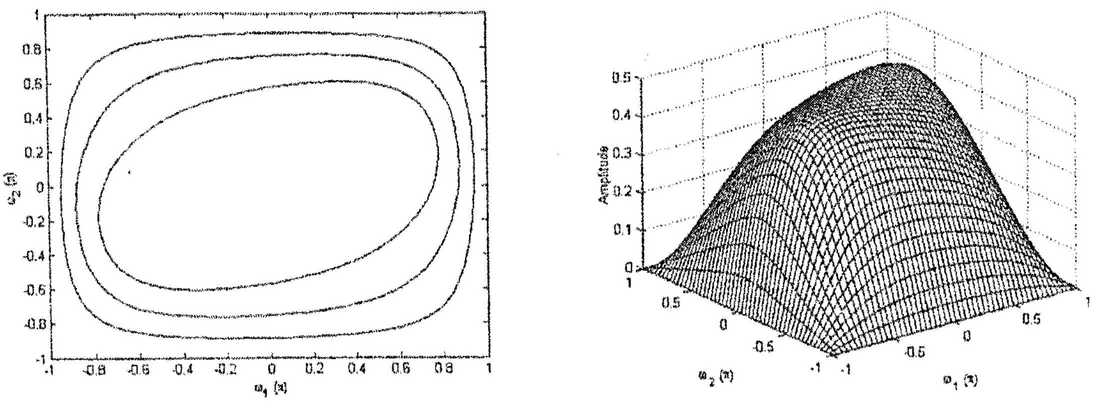
5.6 The Frequency Response of 2-D Band-Elimination Filters

The MATLAB® function *bandElimination.m* is employed to plot the frequency response of the 2-D band-elimination filter resulting from the parallel combination of a 2-D low-pass filter and a 2-D high-pass filter.

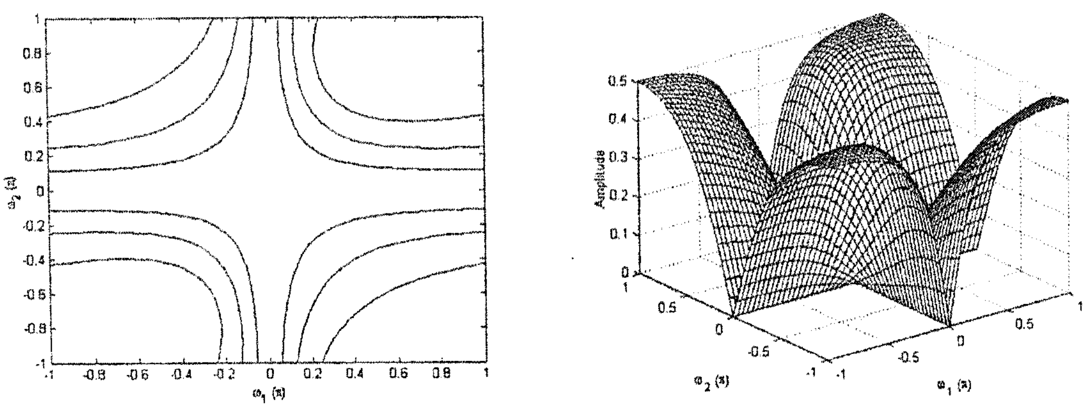
5.6.1 Frequency Response of the Resulting 2-D Band-Elimination Filter with Variable Coefficient k_i's

We first consider the situation when all the coefficients of the double bilinear transformation are set to be unity with proper signs, specifically, k_1=1.0, k_2 =1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=1.0, k_4=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0 and β_{04}=−1.0. From Chapters 2 and 3, both the low-pass and high-pass filters with the specified coefficients are stable, so the resulting 2-D band-elimination filter is also stable.

Figure 5.3 shows the contour and 3-D magnitude plots of the member low-pass and member high-pass filters. The contour and 3-D magnitude plots of the resulting 2-D band-elimination filter, as well as the contour relation between it and its member filters are illustrated in Figure 5.4.

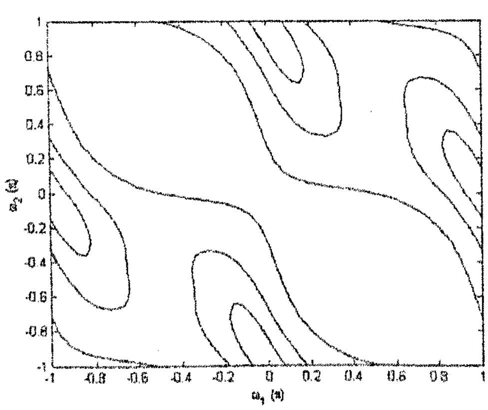


a. Member low- pass filter

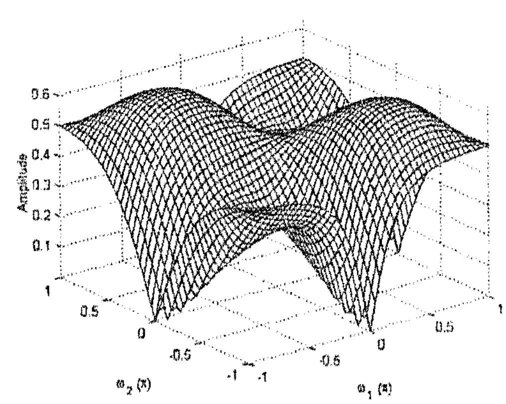


b. Member high-pass filter

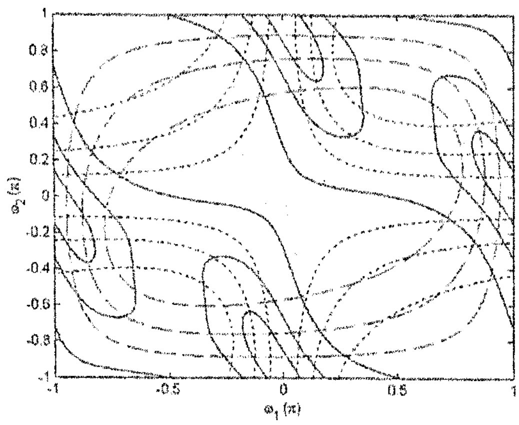
Figure 5.3 The frequency responses of the member filters with coefficients k_1=1.0, k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=1.0, k_4=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0 and β_{04}=−1.0.



a. Contour plot



b. 3-D magnitude response



c. Contour relation

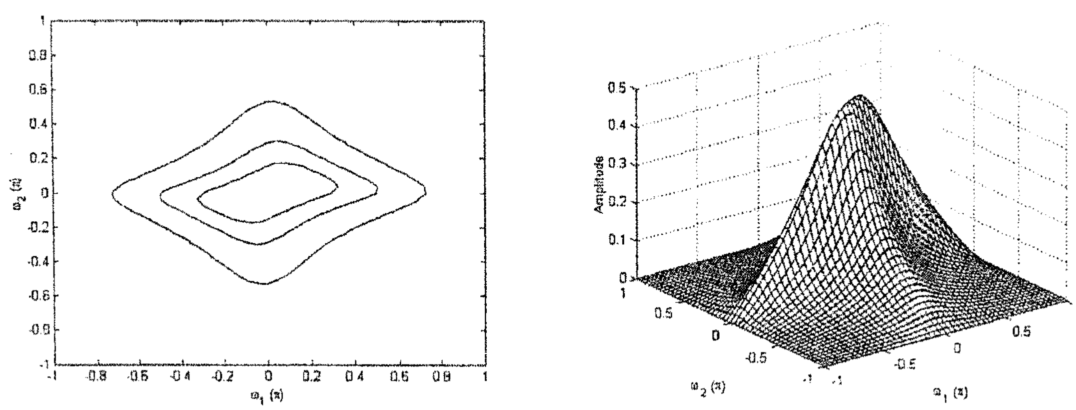
Figure 5.4 The frequency response of the resulting 2-D band-elimination filter with the coefficients fixed as k_1=1.0, k_2=1.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=1.0, k_4=1.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0 and β_{04}=−1.0.

Form Figure 5.4, the parallel combination of the two member filters with unit coefficients with proper signs is difficult to be implemented as a 2-D band-elimination filter. The main reason is that there exist overlapping parts between the pass-bands or the transition-bands of the two member filters. As a result, when a series of signals pass the system, the signal components with very low frequencies will pass the system through the low-pass filter path, the components with very high frequencies will pass the system through the high-pass filter path, and the signals with the overlapping frequencies will pass both the two filters simultaneously. As a result, the signal components passing through the low-pass filter are amplified by the gain of the low-pass filter, the signals passing the high-pass filter have the gain of the high-pass filter, and the signal in the overlapping frequency ranges are amplified by the sum of the gains at the overlapping ranges of both the low-pass and high-pass filters. In 2-D band-elimination filters design through the parallel combination of a low-pass filter and a high-pass filter, the pass-band or transition-band overlapping is another important issue, which needs to be considered carefully.

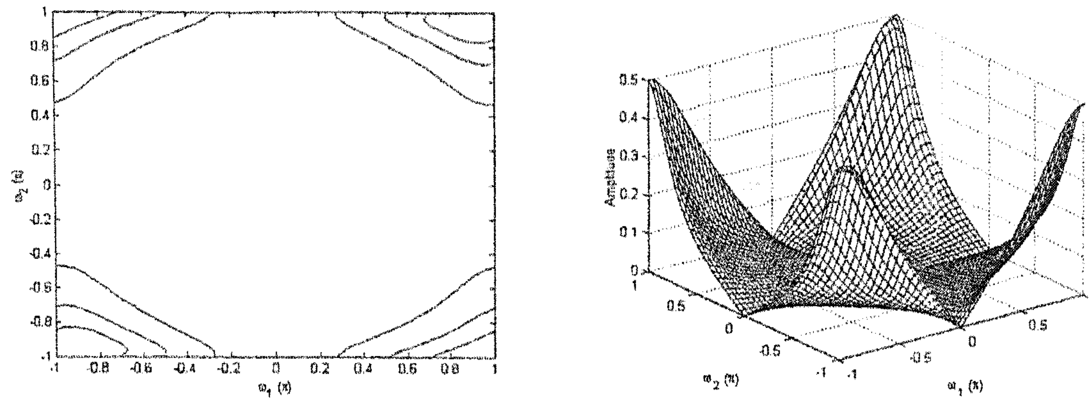
The results of the previous chapters show that increasing the values of coefficients of k_i's (i = 1, 2, 3, 4) makes the pass-bands contract to their center frequencies in their specified frequency domains. Increasing the values of k_1 and k_3 tunes the pass-bands and the transition bands of the two filters apart from each other in ω_1-dimension, and the bigger values of k_2 and k_4 make the pass-bands and the transition bands of the two filters apart from each other in ω_2-dimension.

In the following example, the values of k_1, k_2, k_3 and k_4 are increased from 1.0 to 5.0, while keeping the other coefficients unchanged, i.e., k_1=5.0, k_2=5.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=5.0, k_4=5.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0 and β_{04}=−1.0. From the previous chapters, we know that both the low-pass and high-pass filters are stable, and so is the resulting 2-D band-elimination filter.

The contour and 3-D magnitude plots of the low-pass and high-pass filters are given in Figure 5.5. Figure 5.6 is the frequency response of the resulting 2-D band-elimination filter.

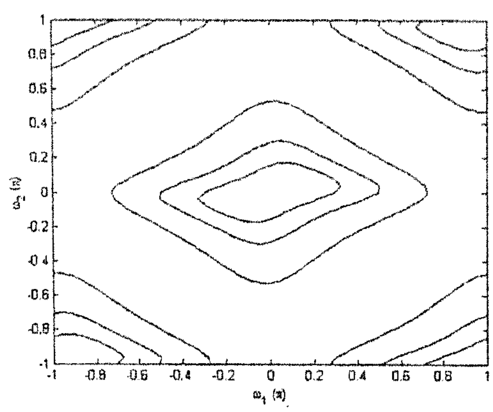


a. Member low-pass filter

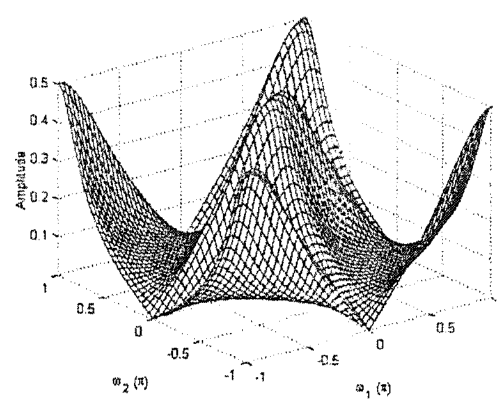


b. Member high-pass filter

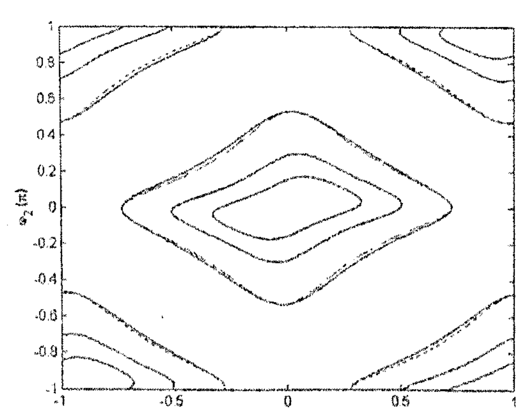
Figure 5.5 The frequency responses of the member filters with coefficients $k_1=5.0$, $k_2=5.0$, $\alpha_{01}=-1.0$, $\alpha_{02}=-1.0$, $\beta_{01}=1.0$, $\beta_{02}=1.0$; $k_3=5.0$, $k_4=5.0$, $\alpha_{03}=-1.0$, $\alpha_{04}=-1.0$, $\beta_{03}=1.0$, $\beta_{04}=1.0$



a. The Contour Plot



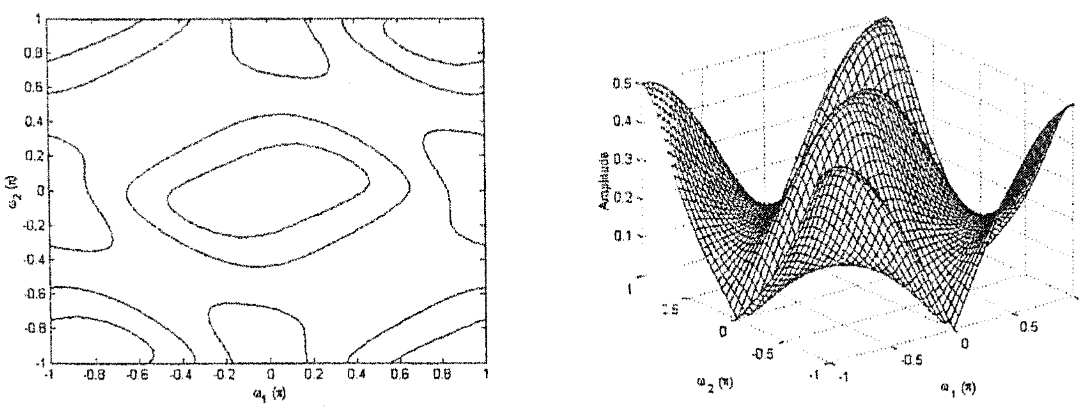
b. 3-D Magnitude Plot



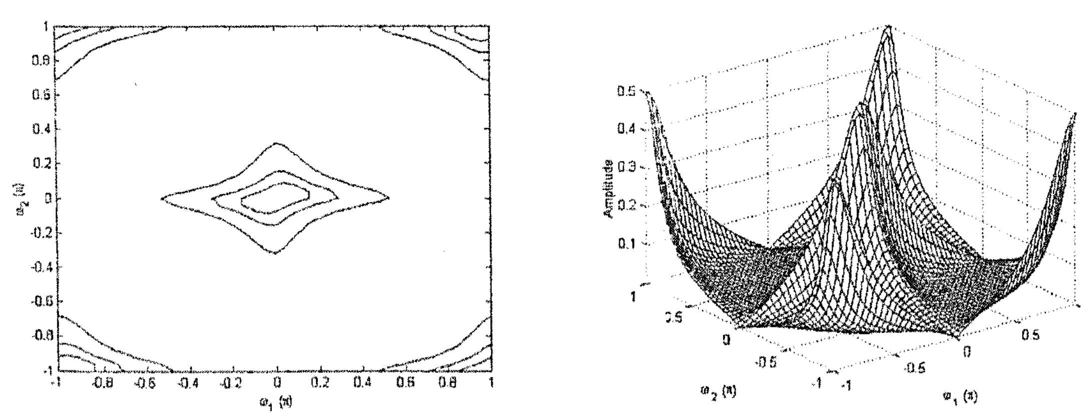
c. Contour Relation

Figure 5.6 Frequency response of the resulting 2-D band-elimination filter with coefficients fixed as $k_1=5.0$, $k_2=5.0$, $\alpha_{01}=-1.0$, $\alpha_{02}=-1.0$, $\beta_{01}=1.0$, $\beta_{02}=1.0$; $k_3=5.0$, $k_4=5.0$, $\alpha_{03}=1.0$, $\alpha_{04}=1.0$, $\beta_{03}=-1.0$ and $\beta_{04}=-1.0$

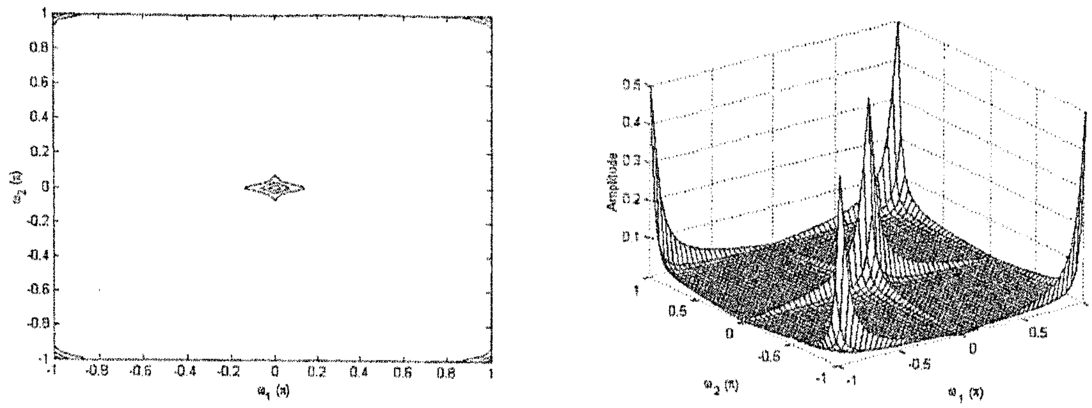
When the values of k_1, k_2, k_3, and k_4 are big enough to tune the pass-bands or transition bands of the two member filters apart from each other, by the parallel combination of the two systems, it is possible to implement the same frequency response as a 2-D band-elimination filter, which has both the pass-bands of the two member filter as its pass-bands, and the other portions as its stop-bands. These properties can be checked for other combination of k_i's. Figure 5.7 illustrates the frequency response of the resulting 2-D band-elimination filter with different equal k_i's. One can refer to Chapters 2 and 3 for the frequency response plots of the member filters. Figure 5.8 is the contour relations between the resulting 2-D band-elimination filter and its member filters.



a. $k_1=k_2=k_3=k_4=3.0$

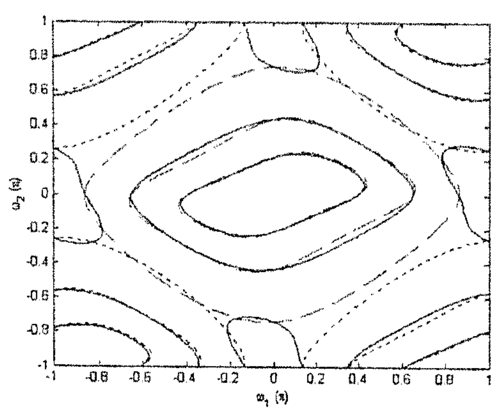


b. $k_1=k_2=k_3=k_4=10.0$

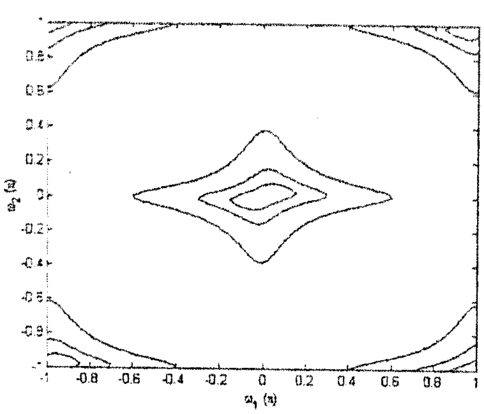


c. $k_1=k_2=k_3=k_4=50.0$

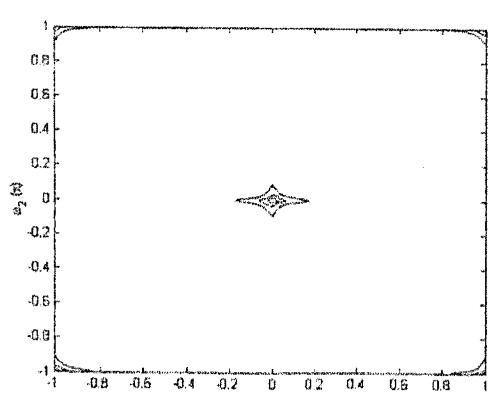
Figure 5.7 The contour and 3-D magnitude plots of the resulting 2-D band-elimination filter with equal variables k_i's (i=1, 2, 3, 4) and the other coefficients fixed as $\alpha_{01}=-1.0$, $\alpha_{02}=-1.0$, $\beta_{01}=1.0$, $\beta_{02}=1.0$; $\alpha_{03}=1.0$, $\alpha_{04}=1.0$, $\beta_{03}=-1.0$ and $\beta_{04}=-1.0$.



a. $k_1=k_2=k_3=k_4=3.0$



b. $k_1=k_2=k_3=k_4=10.0$



c. $k_1=k_2=k_3=k_4=50.0$

Figure 5.8 The contour relation between the resulting 2-D band-elimination filter and its member filters with equal variables k_i's (i=1, 2, 3, 4) and other coefficients fixed as $\alpha_{01}=-1.0$, $\alpha_{02}=-1.0$, $\beta_{01}=1.0$, β_{02} = 1.0; $\alpha_{03}=1.0$, $\alpha_{04}=1.0$, $\beta_{03}=-1.0$ and $\beta_{04}=-1.0$

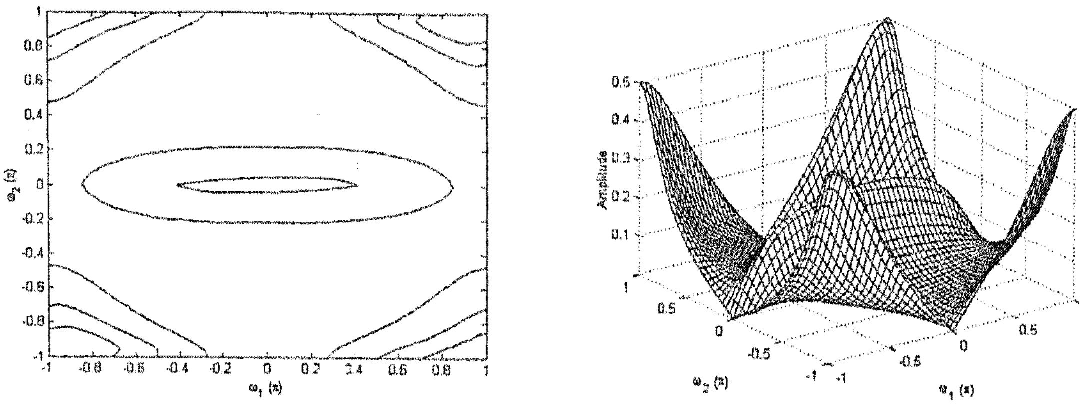
To avoid the interference from the overlapping of the two member filters' pass-bands or transition-bands, the remaining investigations are based on the basic coefficients setting as k_i=5.0 (i=1, 2, 3, 4), and the other coefficients are set to be unity with proper signs.

5.6.2 Frequency Response of the Resulting 2-D Band-Elimination Filter with Variable α_{01}

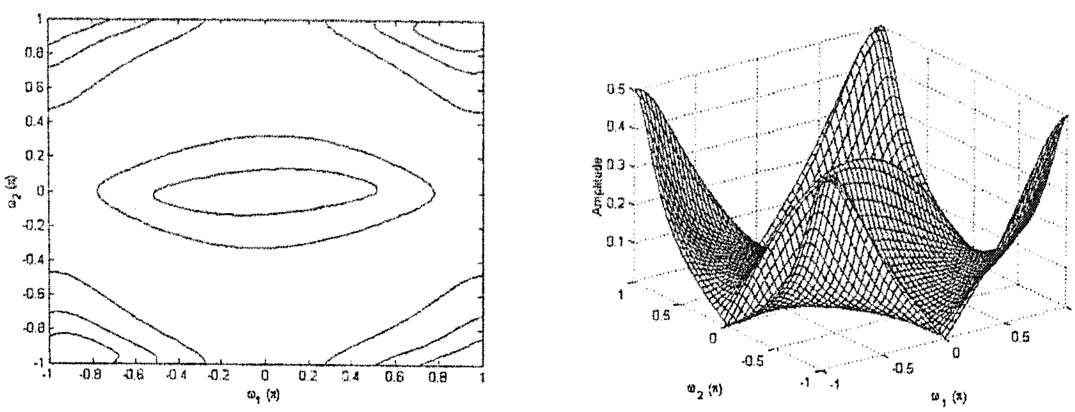
In Chapter 2, we obtained the results that the low-pass coefficient α_{01} mainly affects the pass-band gain of the resulting 2-D low-pass filter. When the low-pass filter with variable α_{01} is used as a member filter to form a 2-D band-elimination filter with another high-pass filter, the resulting 2-D band-elimination filter also possesses variable magnitude characteristics.

The other coefficients are set to the specified values k_1=5.0, k_2=5.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=5.0, k_4=5.0, α_{03}= 1.0, α_{04}=1.0, β_{03}=−1.0 and β_{04}=−1.0. Here to avoid the interference between the two member filters, we set k_i=5.0, (i=1, 2, 3, 4), which are different from the basic setting used in the other chapters. We have already known that the resulting high-pass filter is stable, and the resulting 2-D low-pass filter is stable when we choose α_{01} in [−1.0, 0]. So the resulting 2-D band-elimination filter is stable when α_{01} is chosen in [−1.0, 0]

The contour and 3-D magnitude plot of the resulting 2-D band-elimination filter α_{01} are illustrated in Figure 5.9. The contour relation between it and its member filters are shown in Figure 5.10.

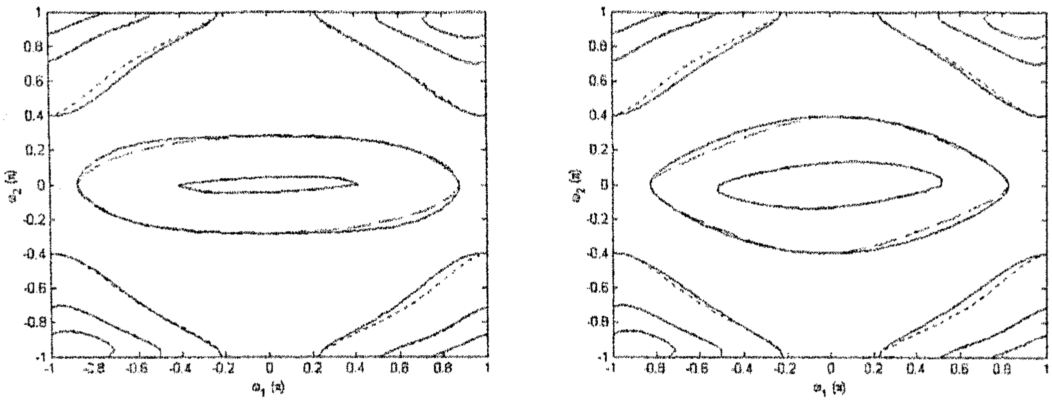


a. $\alpha_{01} = 0$



b. $\alpha_{01} = -0.5$

Figure 5.9 The contour and 3-D magnitude plots of the resulting 2-D band-elimination filter with variable α_{01} and the other coefficients fixed as k_1=5.0, k_2=5.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=5.0, k_4=5.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0 and β_{04}=−1.0



a. $\alpha_{01}=0$ b. $\alpha_{01}=-0.5$

Figure 5.10 The contour relation between the resulting 2-D band-elimination filter and its member filters with variable α_{01} and the other coefficients fixed as k_1=5.0, k_2=5.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=5.0, k_4=5.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0, β_{04}=−1.0

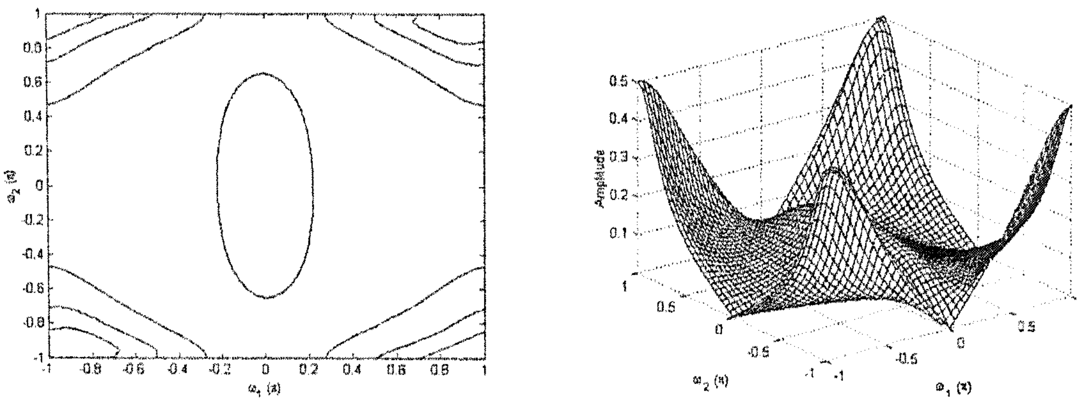
The low-pass coefficient α_{01} mainly affects the gain in the low-frequency parts of the pass-band of the resulting 2-D band-elimination filter, but has no any effect on the high frequency parts. As α_{01} changes from the lower boundary −1.0 to the upper boundary of zero, the gain becomes small. It also affects the bandwidth of the low frequency parts pass-band slightly.

5.6.3 Frequency Response of the Resulting 2-D Band-Elimination Filter with Variable α_{02}

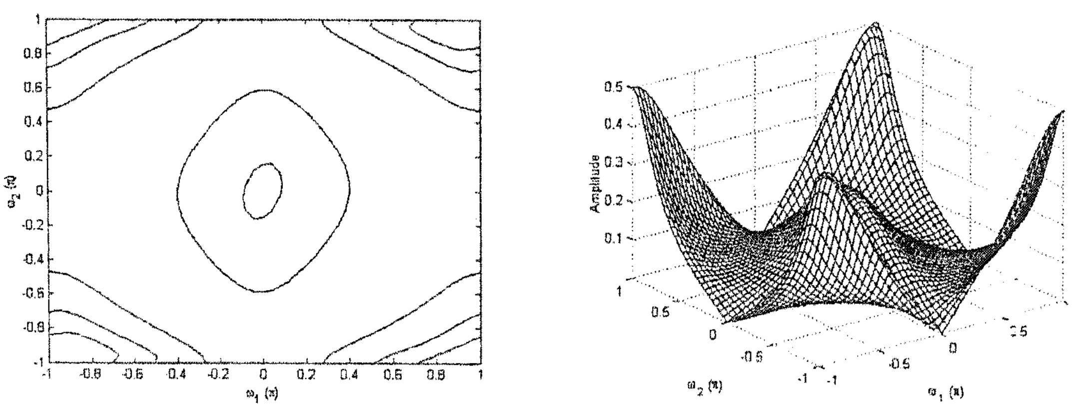
The low-pass coefficient α_{02} also affects the gain in the pass-band of the resulting 2-D low-pass filter, so it will affect the magnitude response of the 2-D band-elimination filter resulting from parallel combination of the low-pass and the high-pass filters.

Here we set the other coefficients to k_1=5.0, k_2=5.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=5.0, k_4=5.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0 and β_{04}=−1.0. It is easy to verify that the resulting 2-D band-elimination filter is stable when α_{02} is chosen as any value in the range of [−1.0, 0].

Figure 5.11 and 5.12 illustrate the simulation results of the resulting 2-D filter with the specified coefficients and variable α_{02} at $\alpha_{02} = -0.5$ and $\alpha_{02} = 0$.

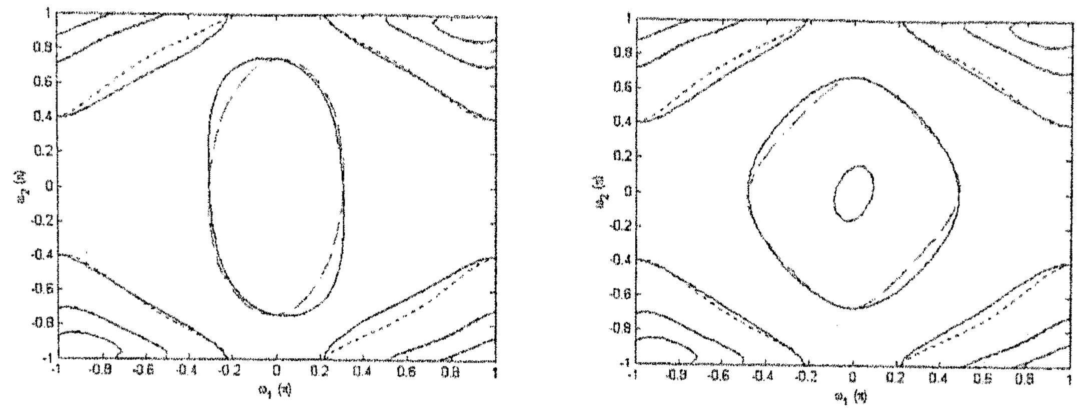


a. $\alpha_{02} = 0$



b. $\alpha_{02} = -0.5$

Figure 5.11 The contour and 3-D magnitude plots of the resulting 2-D band-elimination filter with variable α_{02} and the other coefficients fixed as k_1=5.0, k_2=5.0, α_{01}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=5.0, k_4=5.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0, β_{04}=−1.0



a. $\alpha_{02} = 0$ b. $\alpha_{02} = -0.5$

Figure 5.12 The contour relation between the resulting 2-D band-elimination filter and its member filters with variable α_{02} and the other coefficients fixed as k_1=5.0, k_2=5.0, α_{01}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=5.0, k_4=5.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0, β_{04}=−1.0

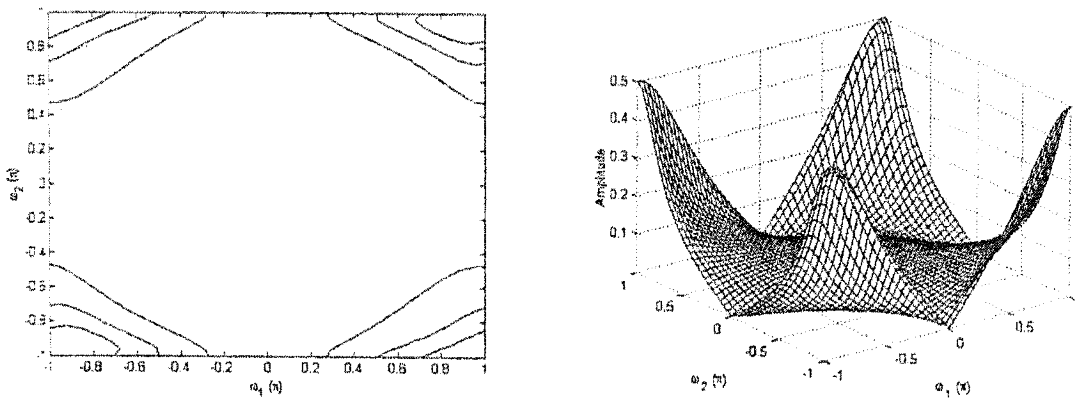
The low-pass coefficient α_{02} also affects the gain in the low-frequency parts of the pass-band of the resulting 2-D band-elimination filter. The smaller the value of α_{02}, the bigger the gain is. The coefficient α_{02} has no any effect on the gains of the high frequency part of the pass-bands. Also it affects the bandwidth of the low-frequency part of the pass-band in ω_2-dimension slightly.

5.6.4 Frequency Response of the Resulting 2-D Band-Elimination Filter with Equal Variable α_{01} and α_{02}

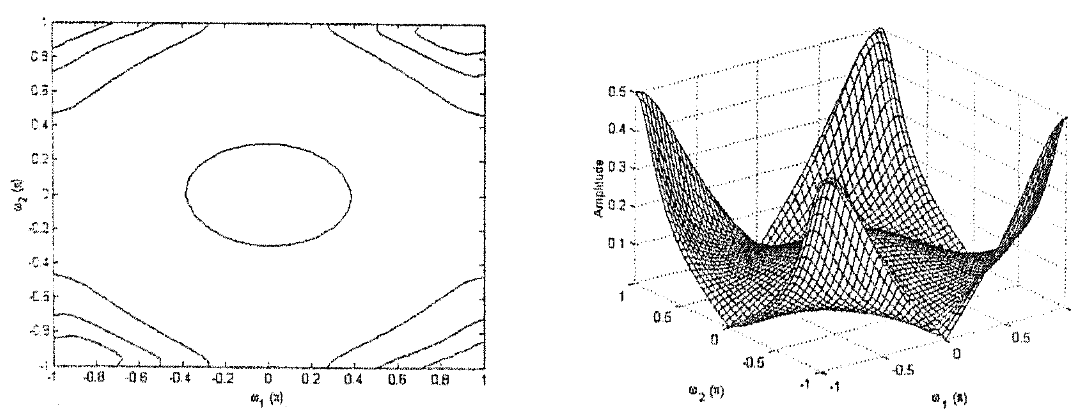
From sections 5.1.2 and 5.1.3, both the two coefficients α_{01} and α_{02} affect the gain in the low-frequency part of the pass-band of the resulting 2-D band-elimination filter. The gain has its maximum value only when the two coefficients have the values equal to −1.0. Now we want to investigate the joint effect of the two coefficients.

Changing the values of the two coefficients simultaneously and keeping then equally, the values of the other coefficients are fixed as unit with proper signs, i.e., k_1=5.0, k_2=5.0, β_{01}=1.0, β_{02}=1.0; k_3=5.0, k_4=5.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0 and β_{04}=−1.0. The resulting 2-D band-elimination filter is stable when the two coefficients are changing in the range of [−1.0, 0].

Figure 5.13 and 5.14 are the simulation results of the resulting 2-D band-elimination filter with different values of α_{01} and α_{02}.

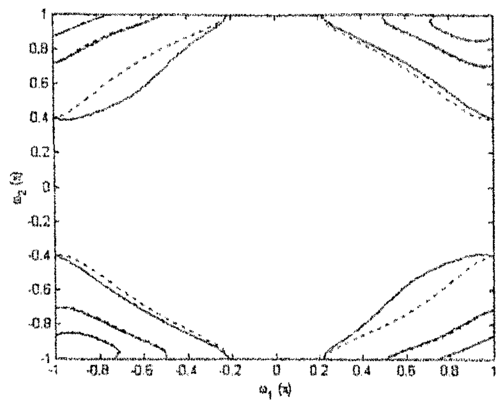


a. $\alpha_{01}=\alpha_{02}=0$

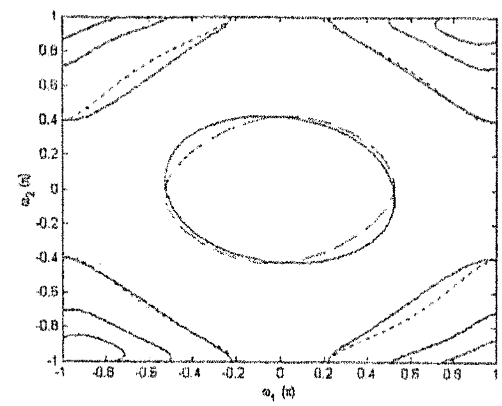


b. $\alpha_{01}=\alpha_{02}=-0.5$

Figure 5.13 The contour and 3-D magnitude plots of the resulting 2-D band-elimination filter with equal variables α_{01} and α_{02} and the other coefficients fixed as $k_1=5.0$, $k_2=5.0$, $\beta_{01}=1.0$, $\beta_{02}=1.0$; $k_3=5.0$, $k_4=5.0$, $\alpha_{03}=1.0$, $\alpha_{04}=1.0$, $\beta_{03}=-1.0$, $\beta_{04}=-1.0$.



a. $\alpha_{01}=\alpha_{02}=0$



b. $\alpha_{01}=\alpha_{02}=-0.5$

Figure 5.14 The contour relation between the resulting 2-D band-elimination filter and its member filters with equal variables α_{01} and α_{02} and the other coefficients fixed as k_1=5.0, k_2=5.0, β_{01}=1.0, β_{02}=1.0; k_3=5.0, k_4=5.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0, β_{04}=−1.0

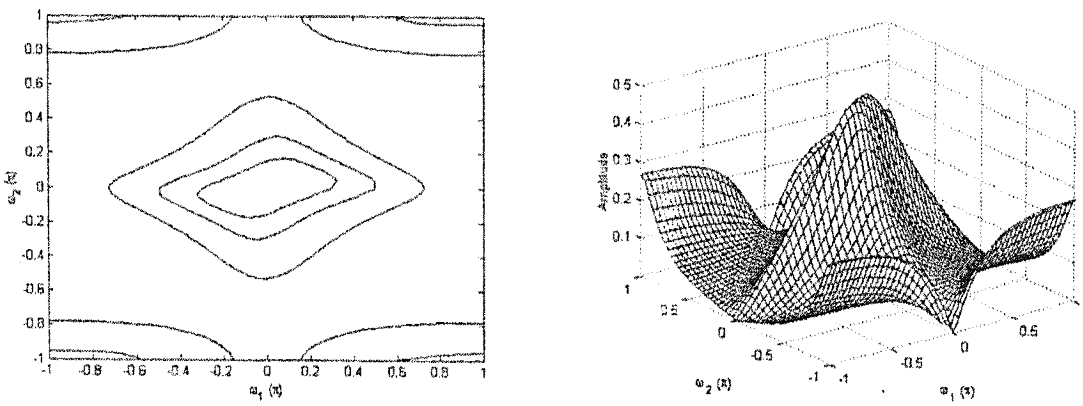
The effect on the gain in the low-frequency part of the pass-band of the resulting 2-D band-elimination filter becomes pronounced when the values of the two coefficients, α_{01} and α_{02}, are changing simultaneously.

5.6.5 Frequency Response of the Resulting 2-D Band-Elimination Filter with Variable α_{03}

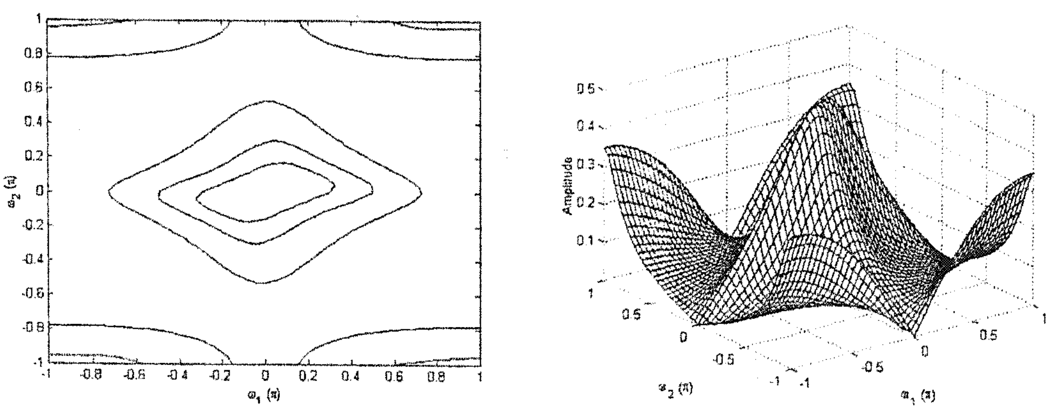
We have already known that the high-pass coefficient α_{03} (known as α_{01} in Chapter 3) affects the frequency response of the resulting 2-D high-pass filter, so it in turn affects the magnitude response of the 2-D band-elimination filter resulting from the parallel connection of the high-pass and low-pass filters.

Similarly, the other coefficients are still fixed to unit with proper signs, specifically, k_1=5.0, k_2=5.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=5.0, k_4=5.0, α_{04}=1.0, β_{03}=−1.0 and β_{04}=−1.0. The resulting 2-D band-elimination filter is stable when α_{03} varies in the range of [0, 1.0].

Figure 5.15 and 5.16 are the frequency response of the resulting 2-D band-elimination filter with α_{03}=0 and α_{03}=0.5.

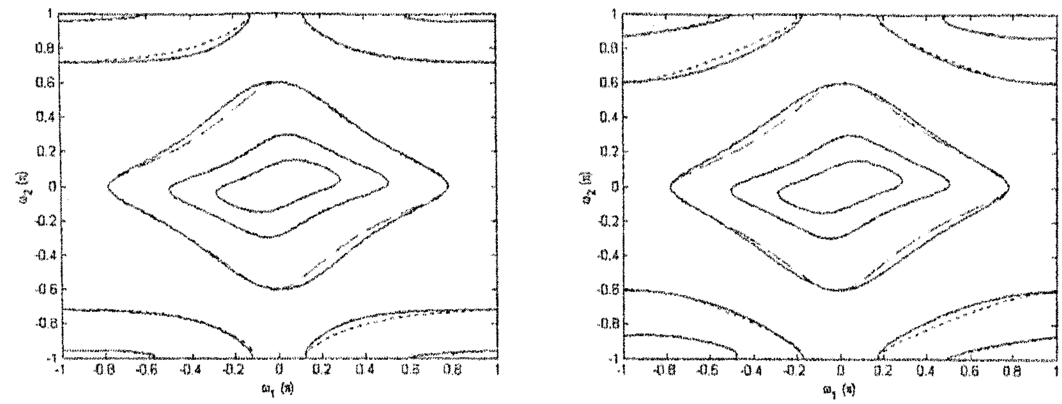


a. $\alpha_{03} = 0$



b. $\alpha_{03} = 0.5$

Figure 5.15 The contour and 3-D magnitude plots of the resulting 2-D band-elimination filter with variable α_{03} and the other coefficients fixed as k_1=5.0, k_2=5.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=5.0, k_4=5.0, α_{04}=1.0, β_{03}=−1.0, β_{04}=−1.0



a. α_{03}=0 b. α_{03}=0.5

Figure 5.16 The contour relation between the resulting 2-D band-elimination filter and its member filters with variable α_{03} and other coefficients fixed as k_1=5.0, k_2=5.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=5.0, k_4=5.0, α_{04}=1.0, β_{03}=−1.0, β_{04}=−1.0

The high-pass coefficient α_{03} mainly affects the gain in the high-frequency part of the pass-band of the resulting 2-D band-elimination filter. When α_{03} is changing in its stable range of [0, 1.0], the gain increases, and it has the biggest value at α_{03}=1.0.

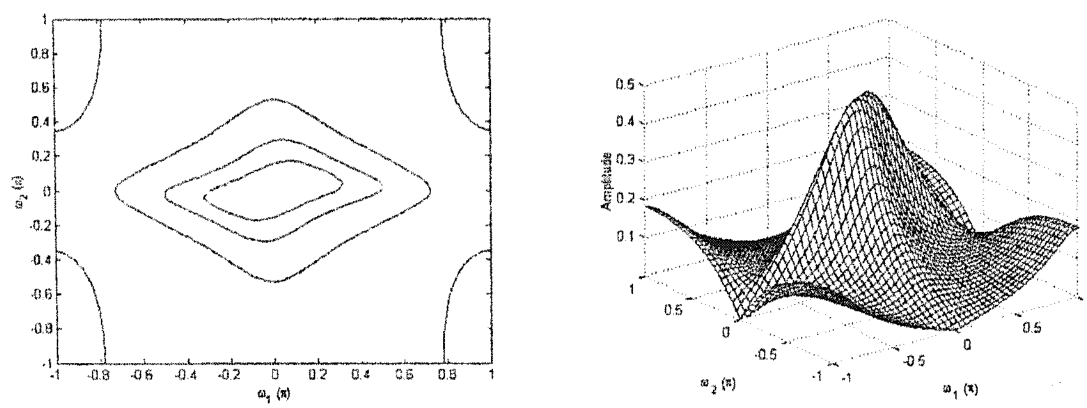
5.6.6 Frequency Response of the Resulting 2-D Band-Elimination Filter with Variable α_{04}

From Chapter 3, we know that the high-pass coefficient α_{04} (known as α_{02} in Chapter 3) mainly affects the magnitude response of the resulting 2-D high-pass filter. When the high-pass filter with variable α_{04} is parallel connected with a 2-D low-pass filter, the resulting system is a possible 2-D band-elimination filter system with variable magnitude response.

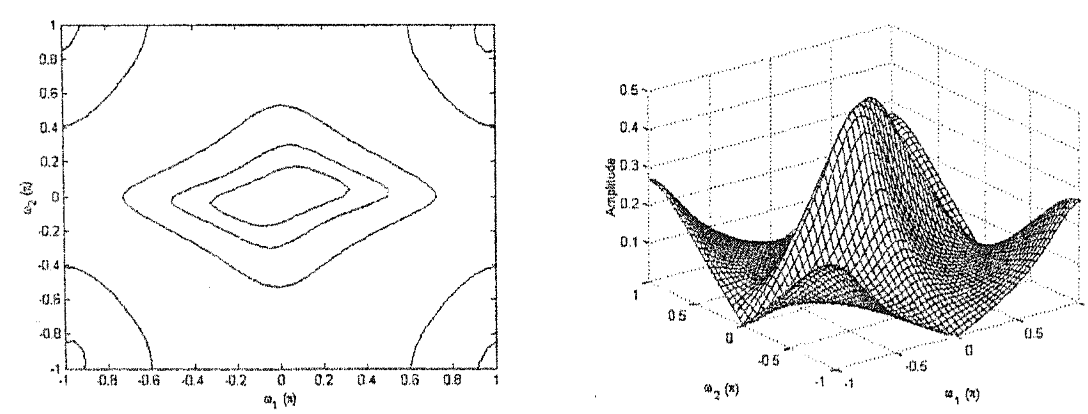
To study how α_{04} affects the magnitude response of the resulting 2-D band-elimination filter, the same methods as discussed in the previous section are used. The other coefficients are set to the specified values, say, k_1=5.0, k_2=5.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=5.0, k_4=5.0, α_{03}=1.0, β_{03}=−1.0, β_{04}=−1.0. The resulting

2-D band-elimination is guaranteed to be stable when α_{04} is chosen in [0, 1.0], as the member low-pass filter is stable and the member high-pass filter is stable with any α_{04} in [0, 1.0].

Figure 5.17 is the contour and 3-D magnitude plots of the resulting 2-D band-elimination filter with α_{04}=0 and α_{04}=0.5. Figure 5.18 shows the contour relation between the band-elimination filter and its two member filters.

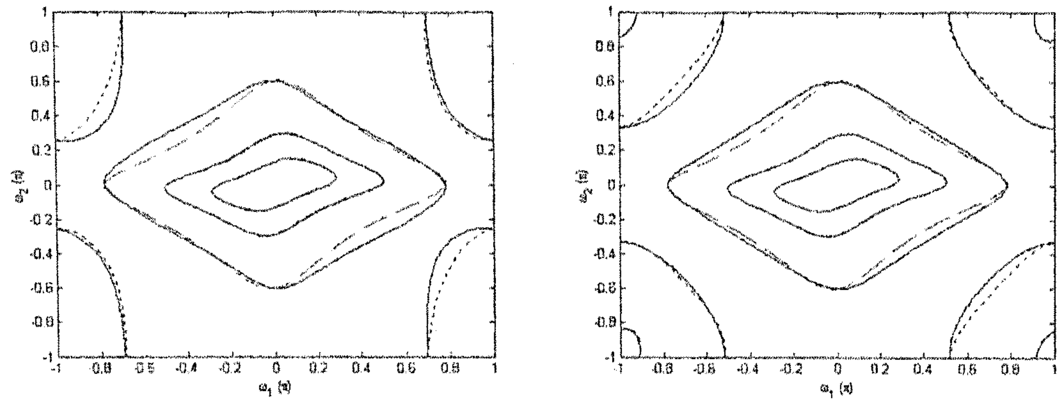


a. $\alpha_{04} = 0$



b. $\alpha_{04} = 0.5$

Figure 5.17 The contour and 3-D magnitude plots of the resulting 2-D band-elimination filter with variable α_{04} and the other coefficients fixed as k_1=5.0, k_2=5.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=5.0, k_4=5.0, α_{03}=1.0, β_{03}=−1.0, β_{04}=−1.0



a. $\alpha_{04} = 0$ b. $\alpha_{04} = 0.5$

Figure 5.18 The contour relation between the resulting 2-D band-elimination filter and its member filters with variable α_{04} and the other coefficients fixed as k_1=5.0, k_2=5.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=5.0, k_4=5.0, α_{03}=1.0, β_{03}=−1.0, β_{04}=−1.0

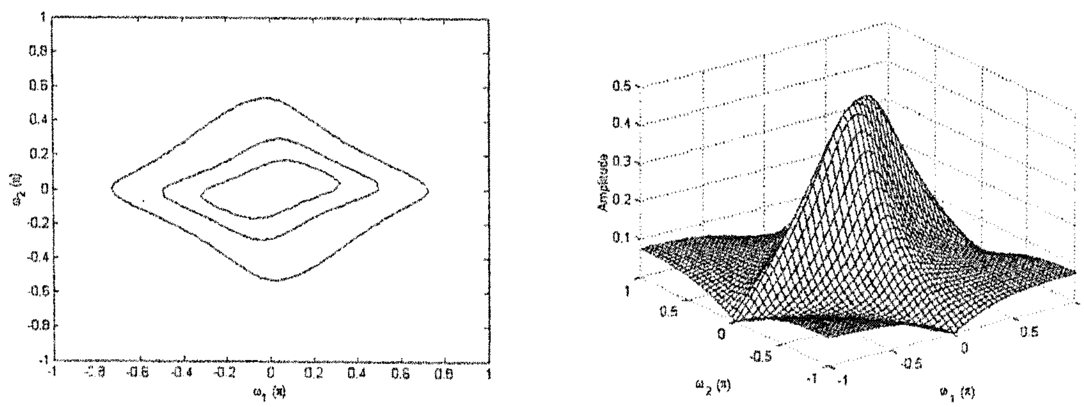
The high-pass coefficient α_{04} also affects the gain of the high-frequency part of the pass-bands of the resulting 2-D band-elimination filter. The filter has the biggest gain when α_{04} is at its upper bound, 1.0, and the smallest gain when α_{04} is at its lower bound, 0. And also it has slight effect on the bandwidth in the high-frequency part of the pass-band of the resulting 2-D band-elimination filter.

5.6.7 The Frequency Response of the Resulting 2-D Band-Elimination Filter with Variable α_{03} and α_{04}

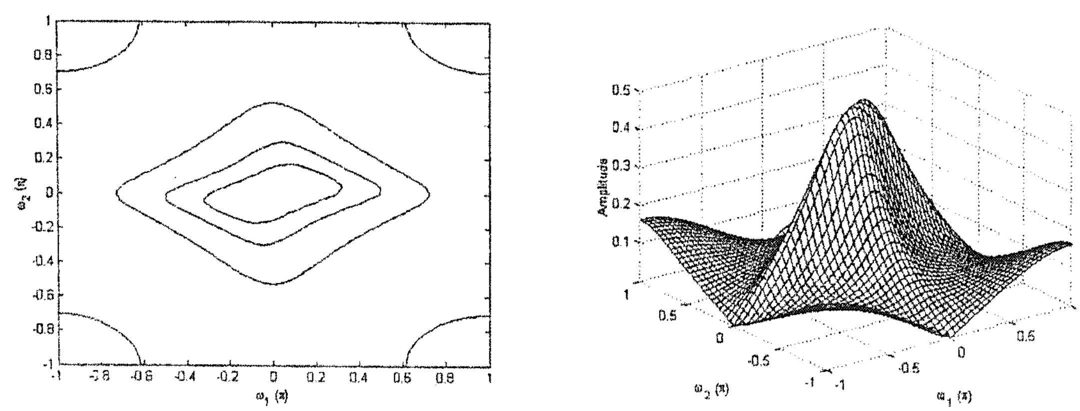
Both the coefficients α_{03} and α_{04} affect the gain in the high-frequency part of pass-band of the resulting 2-D band-elimination filter. Now we want to investigate the combined effects on the magnitude response from the two coefficients.

While setting the other coefficients to be k_1=5.0, k_2=5.0, α_{01}=−1.0, α_{02}=−1.0, β_{01}=1.0, β_{02}=1.0; k_3=5.0, k_4=5.0, β_{03}=−1.0, β_{04}=−1.0, we change the values of α_{03} and α_{04} but keep them equal. The resulting 2-D band-elimination filter is guaranteed to be stable when we choose the values of α_{03} and α_{04} in [0, 1.0].

Figure 5.19 is the contour and 3-D magnitude response of the resulting 2-D band-elimination filter, and Figure 5.20 is the contour relation between the band-elimination filter and its member filters.

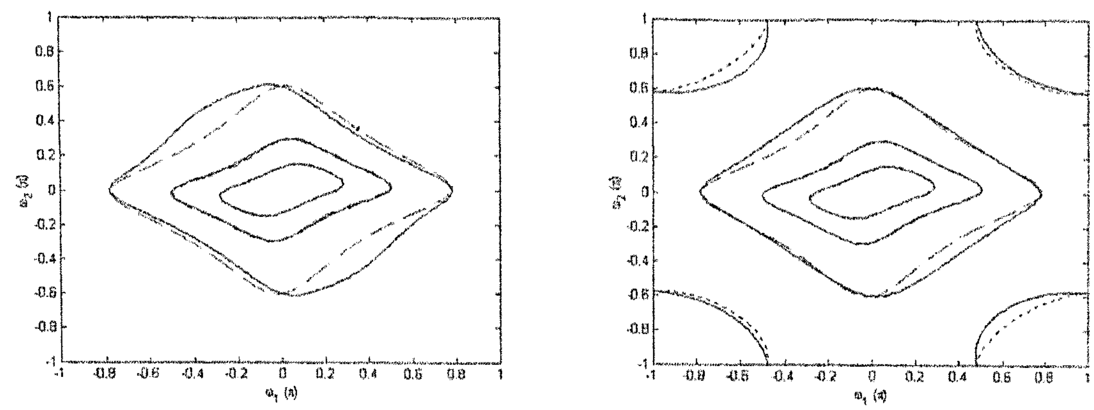


a. $\alpha_{03}=\alpha_{04}=0$



b. $\alpha_{03}=\alpha_{04}=0.5$

Figure 5.19 The contour and 3-D magnitude plots of the resulting 2-D band-elimination filter with equal variables α_{03} and α_{04} and the other coefficients fixed as $k_1=5.0$, $k_2=5.0$, $\alpha_{01}=-1.0$, $\alpha_{02}=-1.0$, $\beta_{01}=1.0$, $\beta_{02}=1.0$; $k_3=5.0$, $k_4=5.0$, $\beta_{03}=-1.0$, $\beta_{04}=-1.0$



a. $\alpha_{03}=\alpha_{04}=0$ b. $\alpha_{03}=\alpha_{04}=0.5$

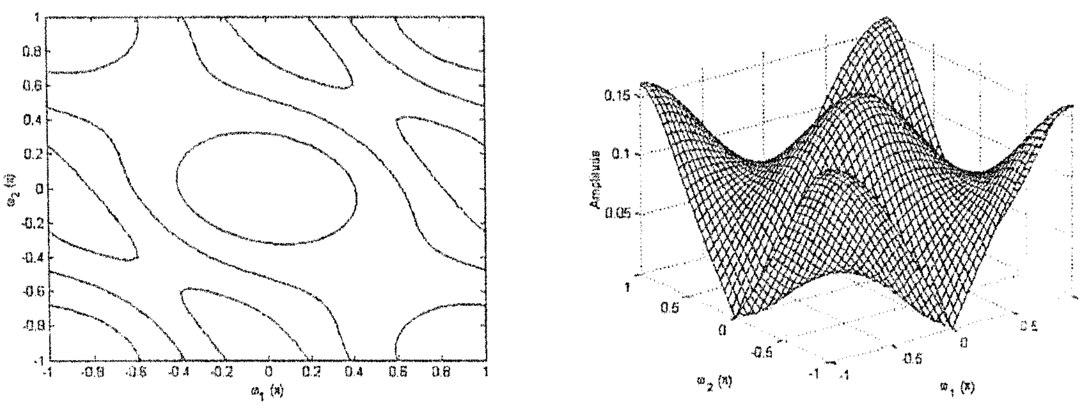
Figure 5.20 The contour relation between the resulting 2-D band-elimination filter and its member filters with equal variables α_{03} and α_{04} and the other coefficients fixed as $k_1=5.0$, $k_2=5.0$, $\alpha_{01}=-1.0$, $\alpha_{02}=-1.0$, $\beta_{01}=1.0$, $\beta_{02}=1.0$; $k_3=5.0$, $k_4=5.0$, $\beta_{03}=-1.0$, $\beta_{04}=-1.0$

The effect on the gains of the high-frequency parts of the pass-bands of the resulting 2-D band-elimination filter from the two high-pass coefficients α_{03} and α_{04} is pronounced.

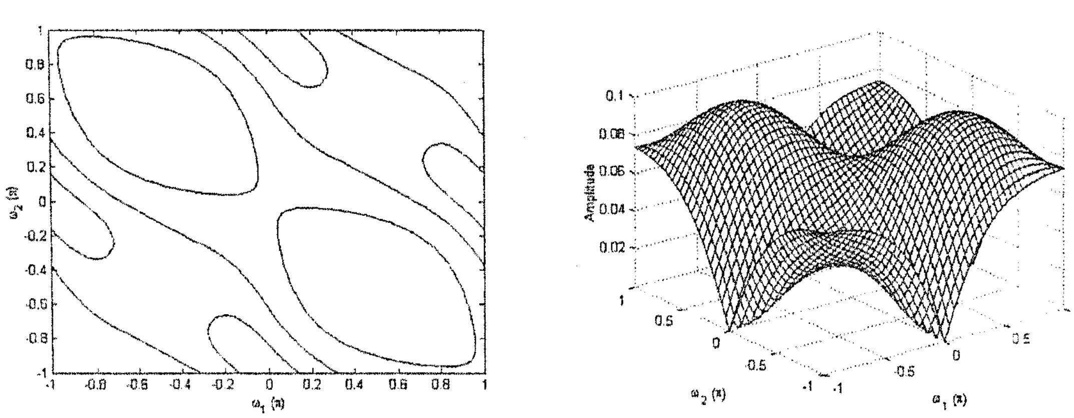
5.6.8 Frequency Response of the Resulting 2-D Band-Elimination Filter with Equal Absolute Values of α_{0i}'s

The coefficients α_{0i}'s (i=1, 2, 3, 4) mainly affect the gains in their corresponding part of the pass-bands. From the previous sections, it is easy to draw the conclusion that only when all the 4 coefficients have equal absolute values, the gains in both the low-frequency and high-frequency parts have the same values. Otherwise, if one or two low pass coefficients have smaller absolute values than the high-pass coefficients, the low-frequency part of the pass-band has smaller gain, and vice versa.

Also from the previous sections, we know that the coefficients α_{0i}'s (i=1, 2, 3, 4) also affect the bandwidth of their corresponding part of pass-band. When the effects of the coefficients enlarge the pass-bands of the two member filters simultaneously, it is possible to have overlapping between the pass-bands of the two filters. That, in turn, makes the implementation of the 2-D band-elimination filter difficult. Figure 5.21 and Figure 5.22 show these situations.

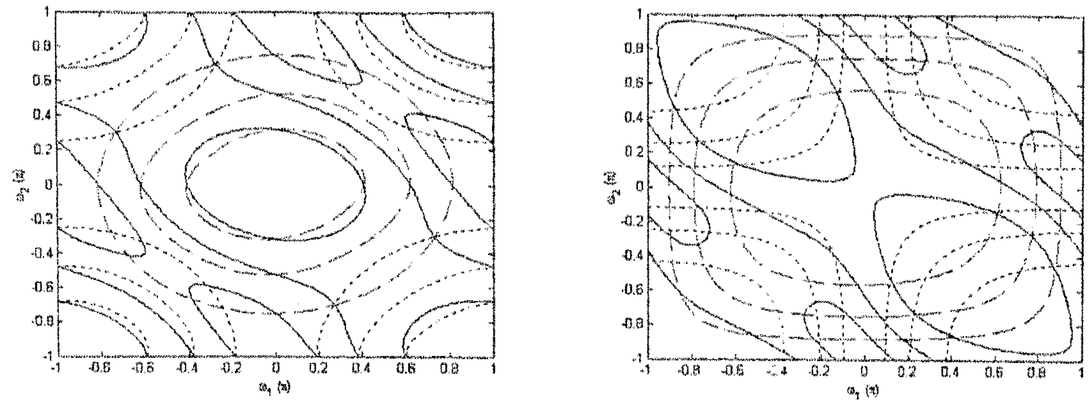


a. $\alpha_{01}=\alpha_{02}=-0.5$ and $\alpha_{03}=\alpha_{04}=0.5$



b. $\alpha_{01}=\alpha_{02}=\alpha_{03}=\alpha_{04}=0$

Figure 5.21 The contour and 3-D magnitude plots of the resulting 2-D band-elimination filter with equal absolute values of α_{0i}'s and other coefficients fixed as $k_1=5.0$, $k_2=5.0$, $\beta_{01}=1.0$, $\beta_{02}=1.0$; $k_3=5.0$, $k_4=5.0$, $\beta_{03}=-1.0$, $\beta_{04}=-1.0$



a. $\alpha_{01}=\alpha_{02}=-0.5$ and $\alpha_{03}=\alpha_{04}=0.5$ b. $\alpha_{01}=\alpha_{02}=\alpha_{03}=\alpha_{04}=0$

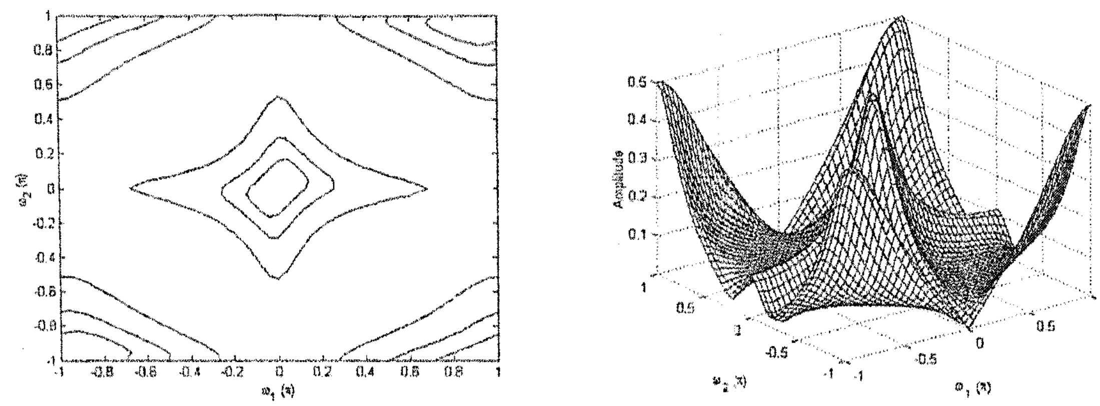
Figure 5.22 The contour relation between the resulting 2-D band-elimination filter and its member filters with equal absolute values of α_{0i}'s and the other coefficients fixed as k_1=5.0, k_2=5.0, β_{01}=1.0, β_{02}=1.0; k_3=5.0, k_4=5.0, β_{03}=−1.0, β_{04}=−1.0

5.6.9 Frequency Responses of the Resulting 2-D Band-Elimination Filter with Variable β_{0i}'s

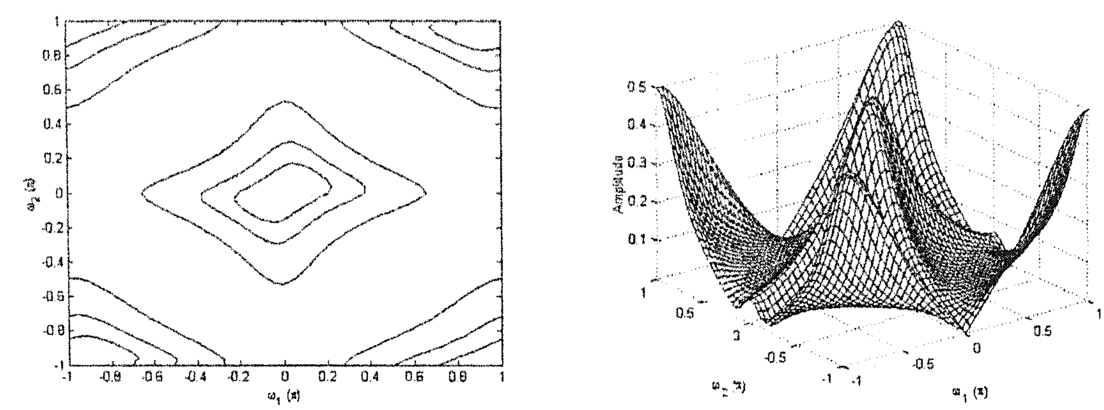
From Chapter 2, the low-pass coefficient β_{01} mainly affects the gain in the stop-band of the resulting 2-D low-pass filter in ω_1-dimension. Employing this low-pass filter with another high-pass filter to form a 2-D band-elimination filter, the resulting filter has variable magnitude response.

We specify the other coefficients as k_1=5.0, k_2=5.0, α_{01}=−1.0, α_{02}=−1.0, β_{02}=1.0; k_3=5.0, k_4=5.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0, β_{04}=−1.0. The resulting 2-D band-elimination filter is stable when β_{01} is changing in the range of [0, 1.0].

The contour and 3-D magnitude response of the resulting 2-D band-elimination filter are illustrated in Figure 5.23 and 5.24.

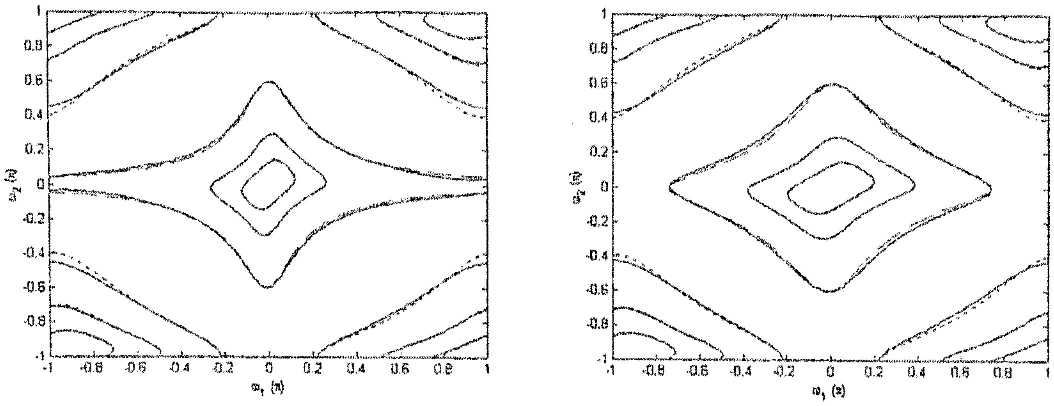


a. $\beta_{01} = 0$



b. $\beta_{01} = 0.5$

Figure 5.23 The contour and 3-D magnitude plots of the resulting 2-D band-elimination filter with variable β_{01} and the other coefficients fixed as k_1=5.0, k_2=5.0, α_{01}=−1.0, α_{02}=−1.0, β_{02}=1.0; k_3=5.0, k_4=5.0, α_{03}=−1.0, α_{04}=−1.0, β_{03}=1.0, β_{04}=1.0

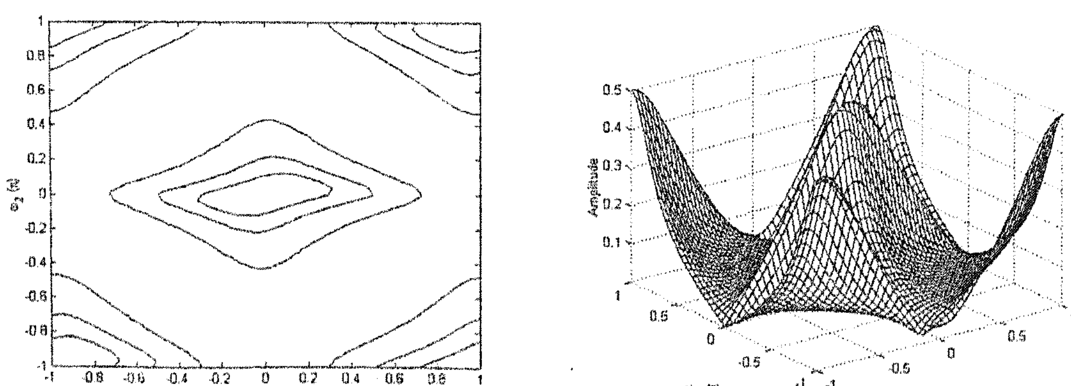


a. β_{01}= 0 b. β_{01} = 0.5

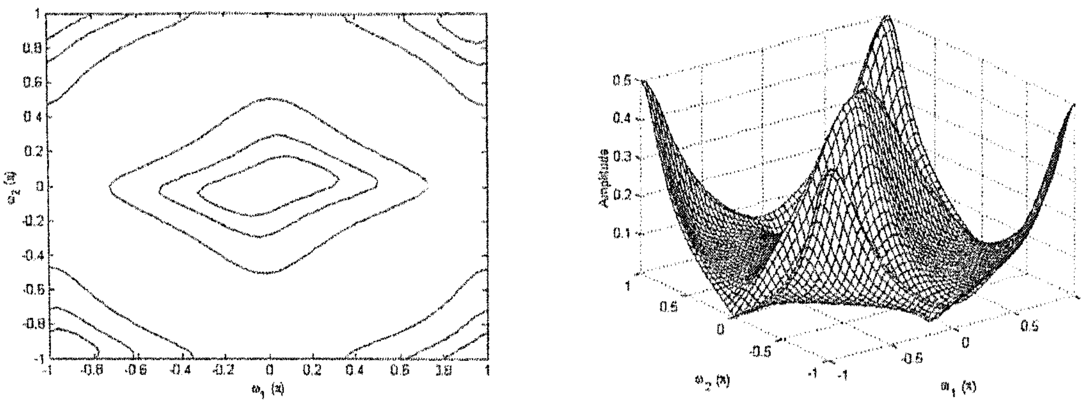
Figure 5.24 The contour relation between the resulting2-D band-elimination filter and its member filters with variable β_{01} and the other coefficients fixed as k_1=5.0, k_2=5.0, α_{01}=−1.0, α_{02}=−1.0, β_{02}=1.0; k_3=5.0, k_4=5.0, α_{03}=1.0, α_{04}=1.0, β_{03}=−1.0, β_{04}=−1.0

From Figures 5.23 and 5.24, when the coefficient β_{01} is changing, the frequency response of the resulting 2-D band-elimination is just the sum of the frequency response of its two member filters.

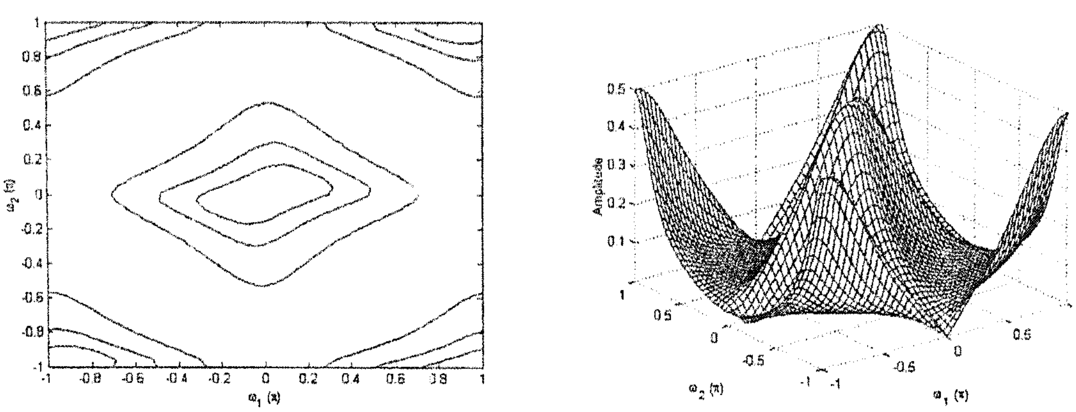
From the previous studies, it can be concluded that the other β_{0i}'s (i=2, 3, 4) will have the same results. Figures 5.25 and 5.26 are the frequency responses of the resulting 2-D band-elimination filter with variable β_{0i}'s (i=2, 3, 4). Here, we choose the absolute values of β_{0i}'s (i=2, 3, 4) to be 0.5.



a. β_{02} = 0.5

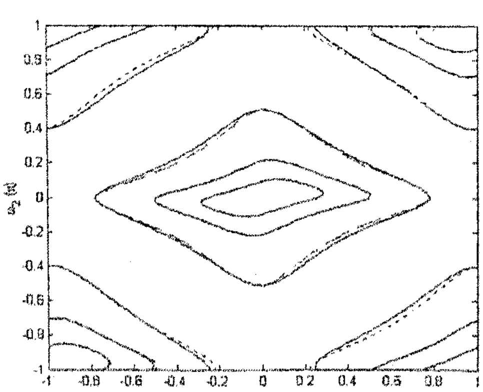


b. $\beta_{03}=-0.5$

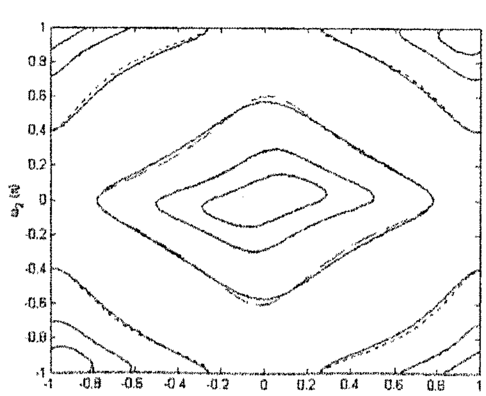


c. $\beta_{04}=-0.5$

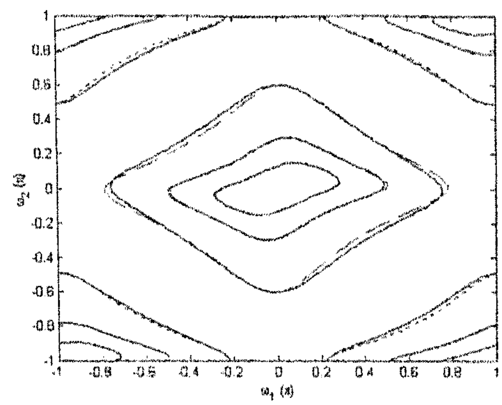
Figure 5.25 The contour and 3-D magnitude plots of the resulting 2-D band-elimination filter with variable β_{0i}'s (I=2, 3, 4) and other coefficients fixed to specified values



a. $\beta_{02}=0.5$



b. $\beta_{03}=-0.5$



c. β_{04}=−0.5

Figure 5.26 The contour relation between the resulting 2-D band-elimination filter and its member filters with variables β_{0i}'s (i=2, 3, 4) and other coefficients fixed to the specified values

5.7 Summary and Discussion

A 2-D band-elimination filter can be obtained from a parallel combination of a 2-D low-pass filter and a 2-D high-pass filter, both of which are obtained from the same analog prototype analog filter through double bilinear transformation using the methods demonstrated in Chapters 2 and 3. When one or more of the bilinear transformation coefficients are changing, the resulting 2-D band-elimination filter has variable magnitude responses.

The stability is always the most important issue in 2-D recursive digital filter design. For the 2-D band-elimination filter system resulting from the parallel combination of two member low-pass and high-pass filters, the overall band-elimination system is guaranteed to be stable when both the member filters are stable.

Another important problem in 2-D band-elimination filter is the intérface of the pass-bands of the two member filters. When the pass-bands of the two member filters have overlapping areas, sometimes even the overlapping areas only occur at the transition bands, the resulting 2-D band-elimination filter becomes difficult to implement.

The coefficients k_i's (i=1, 2, 3, 4) are the main factors that affects the bandwidth in the pass-bands of the member low-pass and high-pass filters. In the design of 2-D band-elimination filters, we should pay more attention in choosing the values of these coefficients. The coefficients α_{0i}'s (i=1, 2, 3, 4) and β_{0i}'s (i=1, 2, 3, 4) also affect the bandwidth of the pass-bands of the member filter slightly. Improperly choosing the values of the these coefficients also changes the overlapping relation between the pass-bands of the two member filters, and, it in turn, makes the implementation of the 2-D band-elimination filters difficult.

When the pass-bands or the transition-bands of the two member filters are tuned apart from each other successfully, the frequency response of the 2-D band-elimination filter is just the sum of the responses of the two member filters. The resulting 2-D band-elimination has both the pass-bands of the two member filters as its pass-band portions, and the other areas as its stop-band portions.

The manner each coefficient affects the magnitude response of the resulting 2-D band-elimination filter is just the same as the manner in which it affects the member low-pass or high-pass filter unless the coefficient causes the pass-bands interface between the two member filters.

Chapter 6

Conclusions and Directions of Future Research

6.1 Conclusions

In this thesis, a technique for designing 2-D digital filters having variable magnitude characteristics has been proposed. Through double generalized bilinear transformations, 2-D low-pass digital filter and high-pass digital filter can be achieved from the same prototype analog filter. If one or more coefficients of the double generalized bilinear transformations are changing, the resulting 2-D low-pass and high-pass filters have variable frequency responses. Through a proper combination of the 2-D low-pass and high-pass filters, 2-D band-pass and 2-D band-elimination filters with variable magnitude characteristics can also be obtained. The manner in which each coefficient affects the magnitude response of each 2-D digital filter has also been investigated in this thesis.

In Chapter 2, 2-D variable recursive low-pass digital filter has been investigated. Starting from a 2^{nd} order 1-D Butterworth low-pass analog ladder network, the value of each electronic element has been determined. By assigning the inductor as the s_1-variable and the capacitor as the s_2-variable, the doubly-unity-terminated 2-D 1^{st} order analog network could be formed, and then the analog transfer function of the 2-D analog network has been obtained. Through the application of double generalized bilinear transformations to the 2-D analog transfer function, the 2-D digital transfer function has been derived with the coefficients of the double generalized bilinear transformation in their low-pass stable ranges: $0 < k_i < \infty$, $-1.0 \leq \alpha_{0i} \leq 0$, $0 \leq \beta_{0i} \leq 1.0$ (i =1, 2), the 2-D low-pass digital filter with variable magnitude characteristics has been obtained. In addition, these coefficients also need to meet the stability conditions of 2-D digital filter with a single degree 2-variable denominator. The coefficient k_1 controls the pass-band width of the resulting 2-D low-pass filter in ω_1-dimension. When the value of k_1 is chosen near its lower boundary, the filter passes almost all the signal components of the frequency range in ω_1-dimension, except those with very high frequencies close to $\pm\pi$ rad. As the value of k_1 increases, the pass-band of the filter becomes narrower till only the signal components with the lowest frequency of 0 radian is passed. Changing the value of k_1, one can obtain almost all the required pass-band(s) width in ω_1-dimension determined by the design specifications. The same phenomenon is observed for the coefficient k_2, which controls the pass-band width for the 2-D low-pass filter in

ω_2-dimension. The pass-band gain is controlled by the coefficients α_{01} and α_{02}. The larger the absolute values of α_{01} and α_{02}, the bigger the gain is, and the effect from the two coefficients is additive. The stop-band gain is controlled by the values of the coefficients β_{01} and β_{02}. When the value of β_{01} or β_{02} is not equal to 1.0, the upper boundary of the coefficients, there is a non-zero gain in the stop-band in ω_1 or ω_2-dimension respectively. The non-zero gain in the stop-bands could be reduced by increasing the value k_1 or k_2, depending on the occurrence of the non-zero gain. The price of the reduction is losing pass-band width in its corresponding dimension. How to balance the gain reduction and the bandwidth loss should be determined by optimization techniques and the design specifications.

A 2-D high-pass filter with variable magnitude characteristics has been studied in Chapter 3. From the same analog prototype filter, 2-D high-pass filter could be obtained through the application of double generalized bilinear transformations with the coefficient ranges: k_i's > 0, $0 \leq \alpha_{0i}$'s≤ 1.0, $-1.0 \leq \beta_{0i}$'s≤ 0 (i=1, 2), and the coefficients should also satisfy the additional requirements for stable high-pass digital filter. When one or more coefficients are changing in their specified ranges determined by both high-pass limits and the stability conditions of 2-D digital filters, the resulting 2-D high-pass filter has variable magnitude responses. The manner in which how each coefficient affects the magnitude characteristics has been investigated in this chapter. The coefficients k_1 and k_2 control the pass-band width of the resulting 2-D high-pass filter in ω_1- and ω_2-dimensions, respectively. The larger the values of k_1 and k_2, the narrower the pass-band width is. The pass-band gain of the 2-D high-pass filter is controlled by the values of the coefficients α_{01} and α_{02}. The large values of α_{01} and α_{02} lead to big pass-band gain, and the effect of the two coefficients on the gains is additive. The coefficients β_{01} and β_{02} control the stop-band gain in their corresponding dimensions, ω_1 and ω_2. There are non-zero gains at the stop-bands in ω_1 and ω_2-dimensions unless β_{01} and β_{02} are equal to -1.0. These properties are useful in the design of 2-D variable high-pass digital filters.

From the results obtained in Chapters 2 and 3, we can classify the coefficients of the double generalized bilinear transformation as three groups: the **bandwidth-effect** coefficients k_i's (i =1, 2), the **pass-band gain-effect** or simply **gain-effect** coefficients α_{0i}'s (i = 1, 2), and the **polarity-effect** coefficients β_{0i}'s (i =1, 2). The polarity-effect property is reflected as non-zero stop-band gains when the two coefficients are changing in the low-pass or high-pass limits.

The 2-D band-pass filters have been investigated in Chapter 4. The cascade combination of a 2-D low-pass filter and a 2-D high-pass filter gives a 2-D band-pass filter. When the member low-pass and high-pass filters are designed from the same analog prototype filter by double generalized bilinear transformations with the procedures described in Chapters 2 and 3, and when one or more coefficients are changing, the resulting 2-D band-pass filter possesses variable magnitude characteristics. The stability of the member low-pass and high-pass filters guarantees the stability of the resulting 2-D band-pass filter. As the frequency responses of the resulting 2-D band-pass filter is obtained from the multiplication of the response of the member low-pass and high-pass filters in frequency domain, the pass-band overlapping is important in the implementations of the 2-D band-pass filter. The manner in which each coefficient affects the magnitude responses of the resulting 2-D band-pass filter has been studied in detail. The coefficients k_i's (i = 1, 2, 3, 4) mainly affect the location of the center frequencies of pass-bands in their specified dimensions, as well as the pass-band gains. When k_i's (i = 1, 2, 3, 4) are changing, the corresponding low-pass or high-pass filters have changing pass-band areas. That in turn makes the overlapping areas of the pass-bands of the two member filter changing. As a result, the coefficients k_1 and k_3 move the center frequencies of the pass-bands of the 2-D band-pass filter in ω_1-dimension in opposite directions, and the coefficients k_2 and k_4 move the center frequencies of the pass-bands of the resulting 2-D band-pass filter in ω_2-dimension in opposite directions. The gain-change is caused by the transition-band overlapping. When the overlapping area between the two member filters are changing to transition-bands, the gain in the transition-bands decrease dramatically, and so do the 2-D band-pass filter pass-band gains, which are the multiplication of the gains of the two member filters.

In Chapter 5, another type of combination-based filter, 2-D band-elimination filter, has been studied. By a parallel combination a 2-D low-pass filter and a 2-D high-pass filter, a 2-D band-elimination filter can be realized if the coefficients of the member filters are properly chosen. We can design the two member filters starting from the same analog prototype filter by the double generalized bilinear transformations, and the resulting 2-D band-elimination filter possesses variable characteristics when one or more the coefficients are changing. The stability of the overall 2-D band-elimination filter is guaranteed by the stability of the two member filters. An important issue in 2-D band-elimination filter design is the interference between the two member filters. To make the resulting 2-D band-elimination filter easily implementable, the pass-bands or transition-bands of the two member filters need to be tuned apart from each other. Otherwise, the implementation becomes difficult. From the results of Chapters 2 and 3, the pass-band width of the member filters are controlled by the coefficients k_i's (i = 1, 2, 3, 4). Increasing the values of k_i's can tune the pass-bands of the two member filters apart. Having avoided the interference problem, the resulting 2-D band-elimination filter has the pass-bands of the two member filters as its pass-bands, and the other portions as its stop-bands. The coefficients α_{0i}'s (i = 1, 2, 3, 4) mainly affect the gains of the low-frequency pass-bands or high-frequency pass-bands which they are corresponding to. Small absolute values of α_{01} or α_{02} lead to a small low-frequency pass-bands gains. Small values of α_{03} and α_{04} cause the high-frequency pass-bands to have small gains. The coefficients β_{0i}'s (i = 1, 2, 3, 4) have the same effects as they have in the low-pass and high-pass filter cases.

Through double generalized bilinear transformations, we can actually obtain 2-D digital filters with variable characteristics, and also we have introduced more changeable coefficients into our design. We thus have more freedom to design 2-D variable digital filters, including 2-D low-pass, 2-D high-pass, 2-D band-pass, and 2-D band-elimination filters, to meet the design specifications. Another benefit of double generalized bilinear transformation method is that we can design the 2-D low-pass and high-pass filters from the same analog prototype filter. That can largely simplify the design work.

6.2 Possible Directions of Future Research

In this thesis, we have used the Butterworth low-pass analog filter as our design staring point. Butterworth filter is a frequently-used analog prototype filter, which has the maximally flat magnitude. When the analog high-pass filters are used as our prototype filters, the coefficients of double generalized bilinear transformation should have different ranges as we used in this thesis to obtain 2-D low-pass and high-pass filters.

Beside Butterworth prototype analog filters, sometimes, we need to implement 2-D filters that have other properties. Then, other prototypes, such as Chebyshev and Inverse Chebyshev analog filters, need to be employed. These prototype analog filters have different value and combination requirements for the electronic elements. This may lead to other requirements for the coefficients of generalized bilinear transformations to obtain stable 2-D recursive digital filters.

The determination of the coefficients for stability requirement becomes complex as the order of the filters increases. However, we can still use other combination filters to achieve the specifications, which are more complex than its first counterpart, and at the same time it can reduce the work to determine the stable ranges. The different combination methods could be the possible future research direction.

In this thesis, we have investigated the effects of each coefficient on the magnitude response of the 2-D low-pass, high-pass, band-pass filter, and band-elimination filters.

These are the first step to design 2-D recursive digital filters with variable characteristics. In future research, we can use methods to determine the exact value for each coefficient to achieve specified design specifications. To meet the design specifications, the combination of all these coefficients might have many possibilities. How to determine the optimum coefficient combinations and how to balance the optimization and the computation loads are also the other possible research direction.

In the process of designing 2-D recursive digital filters, the techniques of optimization are always interesting, especially when we use the double generalized bilinear transformations, where many coefficients need to be treated. Effectively using the optimization techniques can not only save computer time, but also get more accurate results.

As introduced in Chapter 1, every 2-D digital filter has some kinds of symmetry in its contour plot. In design, one can use the symmetrical properties properly and effectively and thereby dramatically reduce and simplify the design work.

The above type of 2-D digital filters can be possible be used in noise suppression by cascading several sections suitably. This has to be explored future.

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APPENDIX

Program Listing

A1. Programs for Chapter 2: 2-D Low-Pass Filters

```
function Den = ButterPolynomial (n)
%       Den=ButterPolynomial (n) takes the order of the ButterWorth Polynomial and
%       returns the ButterWorth coefficients for the nth order ButterWorth Polynomial
%       in 1×(n+1) array
%
%       n -- The order of the Butterworth Polynomial
%
%             © Chen Bin Deng      May 2003
%             Last Revision:       July 2003

%   Check the order of the ButterWorth Polynomial is legal
    if n <=0
        error ('the order of the ButterWorth polynomial should greater than one');
    end

%   Calculate the Poles of nth order ButterWorth Polynomial B(n)*B(-n)
    all_poles=roots([(-1)^n, zeros(1,2*n-1),1]);
%   For system to be stable, choose all the poles in the left close half to be the pole for B(n)
    poles=all_poles(find(real(all_poles)<0));
%   Form the Denominator polynomial with all the chosen poles
    Den=poly(poles);

%   Output the coefficients of the nth order ButterWorth Polynomial
    fprintf('The cofficients for %d order ButterWorth polynomial are:', n);
    for i=1:1:n+1
        fprintf('\n a(%d)=%6.2f.',i-1,Den(i));
    end
```

```
%  Write the nth order ButterWorth Polynomail
   fprintf ('\n \n The ButterWorth Polynomial is:\n');
   fprintf ('Bn=%4f+',Den(1));
   for i=2:n
       fprintf ('%4f*s^%d+',Den(i),i-1);
   end
   fprintf ('%4f*s^%d',Den(n+1),n);

%  ==========  end of ButterPolynomial  ==================
```

```
function k1LPRange (alph01, belta01, k2, alph02, belta02)

%  function k1LPRange (alph01, belta01, k2, alph02, belta02) uses the coefficients of
%  the double generalized bilinear transformations, except k1, to get the stable range
%  for the coefficient k1 when the others are with specificed values.
%
%  Attention: all the inputs need to meet the reqirements to get a two-dimensional Low
%  Pass filter:
%                 -1.0 <= alph01 <= 0, 0 <= belta01 <= 1.0;
%                 -1.0 <= alph02 <= 0, 0 <= belta02 <= 1.0, k2 > 0
%  The function output the stable range of k1 in the form of plot. Here we use 1000 to
%  simulate infinite.
%
%  See also k2LPRange, a1LPRange. a2LPRange, b1LPRange, b2LPRange, k1HPRange
%
%                        © Chen Bin Deng, May 2003
%                        Last Revision:    August, 2003

% Check to see if the resulted filter is Low Pass one
if belta01 >0 & belta02 >0
   button = questdlg('Is the desire filter a low pass filter?', ...
         'Filter Type Dialog','Yes','No','No');
   switch button
   case 'Yes',
         case 'No',
             error('Please reinput the coefficients belta01& belta02 to get high pass one! ');
         end
     else
         error ('Low Pass filter requires both belta01 and belta02 to be positive! Try again');
     end

  % Make sure the coefficients could get stable double generalized bilinear transformation
     if k2 <= 0
         error('k2 could cause unstable transformation, please check it!');
     end

     if alph01>0 | alph01 < -1.0
         error('The value of alph01 will cause the transformation unstable, try again!');
     end

     if alph02>0 | alph02 < -1.0
```

```
        error('The value of alph02 will cause the transformation unstable, try again !');
    end

    if belta01 > 1.0
        error('The value of   belta01 will cause the transformation unstable, try again !');
    end

    if belta02 > 1.0
        error('The input of belta02 will cause the transformation unstable, try again !');
    end

%   compute the stable range of k1 from 0 to 100 in the step of 0.1
    k1=0;
    n=1;
    while k1 <= 100
        k1=k1+0.1;
        a = unityStable (k1, alph01, belta01, k2, alph02, belta02);
        if a == 1
            x(n)=k1;
            n=n+1;
        end
    end

% Plot the stable range of k1
    y=ones(1,length(x));
    figure (1), subplot(211),stem(x,y,'.'), axis ([-100,max(x)+0.5, 0, 5]);
    xlabel('k_1'),set(gca,'xtick',[min(x),max(x)], 'ytick',[0]); box off

% ==========    end of k1LPRange    ================
```

```
function a1LPRange (k1, belta01, k2, alph02, belta02)
%
%function a1LPRange (k1, belta01, k2, alph02, belta02) input the coefficients of
% the double generalized bilinear transformations, except k1, to get the stable range
%for the coefficient k1 when the others are with specificed values.
%
% Attention: all the inputs need to meet the reqirements to get a two-dimensional Low
% Pass filter:
%                 k1 > 0, 0<=belta01<=1.0;
%                 -1.0<=alph02<=0, 0<=belta02<=1.0; 0<k2
% The function output the stable range of k1 in the form of plot. Here we use 100 to
% simulate infinite.
%
% See also k1LPRange, k2LPRange, a2LPRange, b1LPRange, b2LPRange, lowPass
%
%                     © Chen Bin Deng, May 2003
%                     Last Revision:    August, 2003
%

% Confirm the desired two-dimensional filter is a Low Pass one
    if belta01 > 0 & belta02 > 0
        button = questdlg('Is the desire filter a low pass filter?', ...
            'Filter Type Dialog','Yes','No','No');
        switch button
        case 'Yes',
        case 'No',
            error('Please reinput the coefficients of belta01 and belta02');
        end
    else
        error ('Low Pass filter requires both belta01 and belta02 to be positive! Try again');
    end

%   Test if the coefficients could result in stable double generalized bilinear transformation
    if k1 <= 0
        error('k2 could cause unstable transformation, please check it!');
    end

    if k2 <= 0
        error('k2 could cause unstable transformation, please check it!');
    end
```

```
    if alph02>0 | alph02 < -1.0
        error('The input of alph02 could cause the transformation unstable, please check it !');
    end

    if belta01 > 1.0
        error('The input of belta01 could cause the transformation unstable, please check it !');
    end

    if belta02 > 1.0
        error('The input of belta02 could cause the transformation unstable, please check it !');
    end

%   initial alph01 and n
    alph01=-1.0;
    n=1;

%   Compute the stable range for alph01, scan from -1.0 to 0 with step of 0.001
    while alph01 <= 0.001
%   Use the criterion for the system with unity degree denominator to test the system stability
        a = unityStable(k1, alph01, belta01, k2, alph02, belta02);
        if a ==1
            x(n)=alph01;
            n=n+1;
        end
        alph01=alph01+0.001;
end

% Plot the stable range of alph01
    y=ones(1,length(x));
    figure (1), subplot (211), stem(x,y,'.'), axis ([min(x)-0.25,max(x)+0.25, 0, 5]);
    xlabel('\alpha_{01}'),set(gca,'xtick',[min(x),max(x)], 'ytick',[0]);box off

 %=======  end of a1LPRange  ==============
```

```
function b1LPRange (k1, alph01, k2,alph02, belta02)

% function b1LPRange (k1, alph01, k2,alph02, belta02) takes the coefficients of
% the double bilinear transformations, except belta01, as the inputs. and outputs
% the ranges for belta01, when the others take the specified values, with plot form.
%
% Call format: b1LPRange (k1, alph01, k2, alph02, belta02)
%
% First, the input coefficients should be in the ranges for getting stable bilinear
% transformation:
%            k1 > 0, -1.0 <= alph01 <= 0;
%            k2 > 0, -1.0 <= alph01 <= 0, 0 <= belta02 <=1.0
%
% Second, all the coefficients should meet the requirements for getting stable
% two-dimensional digital filters with unity degree denominator for z1 and z2.
%
% Also see: k1LPRange, a1LPRange, k2LPRange, a2LPRange, b2LPRange
%
%                       © Chen Bin Den,   April 2003
%                       Last Revision:    September 2003

% Test the stability of the double generalized bilinear transformation
   if k1 <= 0
      error('k2 will cause unstable transformation, please check it!');
   end

   if alph01>0 | alph01 < -1.0
      error('1alph01 will cause the transformation unstable, please check it !');
   end

   if k2 <= 0
      error('k2 will cause unstable transformation, please check it!');
   end

   if alph02>0 | alph02 < -1.0
      error('alph02 could cause the transformation unstable, please check it !');
   end

   if belta02 > 1.0 | belta02 < 0
      error('belta02 could cause the transformation unstable, please check it !');
   end
```

```
% end of bilinear transformation stability test

    belta01=0;
    n=1;

% Scan the stable range for belta01 from 0 to 1.0 with the step of 0.001
    while belta01 <= 1.0

% Calculate the coefficients for the denominator polynomial
        a11=k1*k2+0.707*k1+1.414*k2+2;
        a10=k1*k2*alph02+0.707*k1*belta02+1.414*k2*alph02+2*belta02;
        a01=k1*k2*alph01+0.707*k1*alph01+1.414*k2*belta01+2*belta01;
        a00=k1*k2*alph01*alph02+0.707*k1*alph01*belta02 ...
            +1.414*k2*alph02*belta01+2*belta01*belta02;

% To test the stability with the criterion for two-dimensional digital transfer function
% with unity degree denominator
        q1 = a11*belta01*belta01 - a10*belta01 - a01*belta02 + a00;
        q2 = -a11*alph02*belta01 + a10*belta01 + a01*alph02 - a00;
        q3 = -a11*alph01*belta02 + a10*alph01 + a01*belta02 - a00;
        q4 = a11*alph01*alph02 - a10*alph01 - a01*alph02 + a00;
        if q1 > 0 & q2 > 0 & q3 > 0 & q4 > 0
            x(n)=belta01;
            n=n+1;
        end
        belta01=belta01+0.001;
end

% Plot the stable range for belta01
    y=ones(1,length(x));
    figure (1), subplot(211),stem(x,y,'.'), axis ([min(x)-0.25,max(x)+0.25, 0, 5]);
xlabel('\beta_{01}'),set(gca,'xtick',[min(x),max(x)], 'ytick',[0]);box off;

% ====== end of b1LPRange ======
```

```
function k2LPRange (k1, alph01, belta01, alph02, belta02)
%
% function k2LPRange (alph01, belta01, k2, alph02, belta02) uses the coefficients of
% the double generalized bilinear transformations, except k2, to get the stable range
% for the coefficient k2 when the others are with specificed values.
% Attention: all the inputs need to meet the reqirements to get a two-dimensional Low
% Pass filter:
%               k1 > 0, -1.0<=alph01<=0, 0<=belta01<=1.0;
%               -1.0<=alph02<=0, 0<=belta02<=1.0
% The function output the stable range of k2 in the form of plot. Here we use 1000 to
% simulate infinite.
%
% See also k1LPRange, a1LPRange, a2LPRange, b1LPRange, b2LPRange, k1HPRange
%
%                       © Chen Bin Deng, May 2003
%                       Last Revision:    August, 2003

% Check to see if the resulted filter is Low Pass one
    if belta01 >0 & belta02 >0
        button = questdlg('Is the desire filter a low pass filter?', ...
            'Filter Type Dialog','Yes','No','No');
        switch button
        case 'Yes',
        case 'No',
            error('Please reinput the coefficients of belta01 and belta02');
        end
    else
        error ('Low Pass filter requires both belta01 and belta02 to be positive! Try again');
    end

 % Check the other coefficients to meet the stability conditions
    if k1 <= 0
        error('k1 could cause unstable transformation, please check it and try again!');
    end

    if alph01>0 | alph01 < -1.0
        error('The input of alph01 could cause the transformation unstable, please check it !');
    end

    if alph02>0 | alph02 < -1.0
        error('The input of alph02 could cause the transformation unstable, please check it !');
```

```
    end

    if belta01 > 1.0
        error('The input of belta01 could cause the transformation unstable, please check it !');
    end

    if belta02 > 1.0
        error('The input of belta02 could cause the transformation unstable, please check it !');
    end

% compute the stable range of k1 from 0 to 1000 in the step of 0.1
    k2=0;
    n=1;
    while k2 <= 1e+3
        k2=k2+0.1;

% calculate the coefficient of the denominator of the transf function with the form:
%     D(z1, z2)= a11*z1*z2 + a10*z1 + a01*z2 + a00
            a11=k1*k2+0.707*k1+1.414*k2+2;
            a10=k1*k2*alph02+0.707*k1*belta02+1.414*k2*alph02+2*belta02;
            a01=k1*k2*alph01+0.707*k1*alph01+1.414*k2*belta01+2*belta01;
            a00=k1*k2*alph01*alph02+0.707*k1*alph01*belta02 ...
                +1.414*k2*alph02*belta01+2*belta01*belta02;

% Use the stability criterion with unity z1 and z2 to determine the stability for all specified coe.
            q1 = a11*belta01*belta01 - a10*belta01 - a01*belta02+a00;
            q2 = -a11*alph02*belta01 + a10*belta01 + a01*alph02 - a00;
            q3 = - a11*alph01*belta02 - a10*alph01 + a01*belta02 -a00;
            q4 = a11*alph01*alph02 - a10*alph01 - a01*alph02 + a00;
            if q1 > 0 & q2 > 0 & q3 > 0 & q4 > 0
                x(n)=k2;
                n=n+1;
            end
    end

% Plot the stable range of k2
y=ones(1,length(x));
figure (1), subplot (211), stem(x,y,'.'), axis ([min(x)-100,max(x), 0, 5]);
xlabel('k_2'),set(gca,'xtick',[min(x),max(x)], 'ytick',[0]);box off

% ======== end of k2LPRange ===========
```

```
function a2LPRange (k1, alph01, belta01, k2, belta02)
%
% function a2LPRange (k1, alph01, belta01, k2, belta02) takes the coefficients, except
% alph02, of the double generalized bilinear transformations as its inputs, and determine
% the range of alph02 which can make the resulted two-dimensional digital filter to be a
% stable one when the others are given.
%
% To employ a stable double bilinear transformation, the coefficient should meet the
% following requirements:
%              k1 > 0, -1.0 <= alph01 <= 0, 0 <= belta01 <=1.0
%              k2 > 0, -1.0 <= alph02 <= 0, 0 <= belta02 <=1.0
% For the two-dimensional digital filter with transfer function having unity degree
% denominator
%              D(z1, z2)=a11*z1*z2 + a10*z1 + a01*z2 + a00
% The conditions are:
%              a11*belta01*belta01 - a10*belta01 - a01*belta02 + a0 > 0
%              -a11*alph02*belta01 + a10*belta01 + a01*alph02 - a00 > 0
%              -a11*alph01*belta02 + a10*alph01 + a01*belta02 - a00 > 0
%              a11*alph01*alph02 - a10*alph01 - a01*alph02 + a00 > 0
%
% Also see: a1LPRange, k1LPRange, b1LPRange, k2LPRange, b2LPRange
%
%                  © Chen Bin Deng.    March 2003
%                    Last Reversion:    September 2003

% Test the stability of the double generalized bilinear transformation
   if k1 <= 0
      error('k2 will cause unstable transformation, please check it!');
   end

   if belta01 > 1.0 | belta01 < 0
      error('belta01 could cause the transformation unstable, please check it !');
   end

   if alph01>0 | alph01 < -1.0
      error('1alph01 will cause the transformation unstable, please check it !');
   end

   if k2 <= 0
      error('k2 will cause unstable transformation, please check it!');
   end
```

```
    if belta02 > 1.0 | belta02 < 0
        error('belta02 could cause the transformation unstable, please check it !');
    end
%   end of bilinear transformation stability test

% initial k and n;
    alph02=-1.0;
    n=1;

%   Scan the stability ranges for alph02 from -1.0 to 0 with the step Of 0.001
    while alph02 <= 0.001

%   Compute the coefficients for the unity degree denominator polynomial
        a11=k1*k2+0.707*k1+1.414*k2+2;
        a10=k1*k2*alph02+0.707*k1*belta02+1.414*k2*alph02+2*belta02;
        a01=k1*k2*alph01+0.707*k1*alph01+1.414*k2*belta01+2*belta01;
        a00=k1*k2*alph01*alph02+0.707*k1*alph01*belta02 ...
            +1.414*k2*alph02*belta01+2*belta01*belta02;

%   Test the stability per the stability criterion for unity degree denominator
        q1=a11*belta01*belta01-a10*belta01-a01*belta02+a00;
        q2=-a11*alph02*belta01+a10*belta01+a01*alph02-a00;
        q3=-a11*alph01*belta02+a10*alph01+a01*belta02-a00;
        q4=a11*alph01*alph02-a10*alph01-a01*alph02+a00;
        if q1>0 & q2>0 & q3>0 & q4>0
            x(n)=alph02;
            n=n+1;
        end
        alph02=alph02+0.001
    end

%   Plot the stability ranges for alph02
    y=ones(1,length(x));
    figure (1), subplot(211),stem(x,y,'.'), axis ([min(x)-0.25,max(x)+0.25, 0, 5]);
    xlabel('\alpha_{02}'),set(gca,'xtick',[min(x),max(x)], 'ytick',[0]);box off;

%   ======== end of a2LPRange ======
```

```
function b2LPRange (k1, alph01, belta01, k2. alph02)

% function b2LPRange (k1, alph01,belta01, k2,alph02, ) takes the coefficients, except
% belta02, of the double bilinear transformations, except belta01, as the inputs.
% And outputs the ranges of   belta01, when the others take the specified values,
% with plot form.
%
% First, to get stabel double bilinear transformation, the coefficients should be bounded
% as:
%            k1 > 0, -1.0 <= alph01 <= 0, 0 <= belta01 <= 1.0
%            k2 > 0, -1.0 <= alph01 <= 0, 0 <= belta02 <= 1.0
%
% And for the two-dimensional digital filter with transfer function having unity degree
% denominator
%               D(z1, z2)=a11*z1*z2 + a10*z1 + a01*z2 + a00
% The conditions are:
%               a11*belta01*belta01 - a10*belta01 - a01*belta02 + a0 > 0
%               -a11*alph02*belta01 + a10*belta01 + a01*alph02 - a00 > 0
%               -a11*alph01*belta02 + a10*alph01 + a01*belta02 - a00 > 0
%               a11*alph01*alph02 - a10*alph01 - a01*alph02 + a00 > 0
%
% Also see: k1LPRange, a1LPRange, k2LPRange, a2LPRange, b1LPRange
%
%                      © Chen Bin Den, April 2003
%                      Last Revision:     September 2003

% Test the stability of the double generalized bilinear transformation
    if k1 <= 0
        error('k2 will cause unstable transformation, please check it!');
    end

    if alph01>0 | alph01 < -1.0
        error('1alph01 will cause the transformation unstable, please check it !');
    end

    if belta01 > 1.0 | belta01 < 0
        error('belta01 could cause the transformation unstable, please check it !');
    end

    if k2 <= 0
        error('k2 will cause unstable transformation, please check it!');
```

```
    end

    if alph02>0 | alph02 < -1.0
        error('alph02 could cause the transformation unstable, please check it !');
    end

%   end of bilinear transformation stability test

%   Scan the stability ranges for belta02 from 0 to 1.0 with the step 0f 0.001
    belta02=0;
    n=1;
    while belta02 <= 1.001
        a11=k1*k2+0.707*k1+1.414*k2+2;
        a10=k1*k2*alph02+0.707*k1*belta02+1.414*k2*alph02+2*belta02;
        a01=k1*k2*alph01+0.707*k1*alph01+1.414*k2*belta01+2*belta01;
        a00=k1*k2*alph01*alph02+0.707*k1*alph01*belta02 ...
            +1.414*k2*alph02*belta01+2*belta01*belta02;
        q1=a11*belta01*belta01-a10*belta01-a01*belta02+a00;
        q2=-a11*alph02*belta01+a10*belta01+a01*alph02-a00;
        q3=-a11*alph01*belta02+a10*alph01+a01*belta02-a00;
        q4=a11*alph01*alph02-a10*alph01-a01*alph02+a00;
        if q1>0 & q2>0 & q3>0 & q4>0
            x(n)=belta02;
            n=n+1;
        end
        belta02=belta02+0.001
    end

%   Plot the stable ranges of belta02
    y=ones(1,length(x));
    figure (1), subplot(211), stem(x,y,'.'), axis ([min(x)-0.25, max(x)+0.25, 0, 10]);
    xlabel('\beta_{02}'),set(gca,'xtick',[min(x),max(x)], 'ytick',[0]);box off

%   ========   end of b2LPRange   ========
```

```
function key = unityStable (k1, alph01, belta01, k2, alph02, belta02)
%  function key = unityStable (k1, alph01, belta01, k2, alph02, belta02) takes the coefficients
%  of the double generalized Bilinear Transformation as its inputs, and do the stability test
%  for the system with denominator of this form:
%         D(z1,z2)= a11*z1*z2 + a10*z1 + a01*z2 + a00
%
%  The stability criterion:
%         a11*belta01*belta01 - a10*belta01 - a01*belta02 + a00 > 0
%         -a11*alph02*belta01 + a10*belta01 + a01*alph02 - a00 > 0
%         -a11*alph01*belta02 + a10*alph01 + a01*belta02 - a00 > 0
%         a11*alph01*alph02 - a10*alph01 - a01*alph02 + a00      > 0
%
%  The inputs:
%         k1, alph01, belta01 -- coefficients of  s1 -->  z1
%         k2, alph02, belta02 -- coefficients of  s2 -->  z2
%  The outputs:
%         "1" -- the system is stable
%         "0" -- the system is unstable
%
%          © Chen Bin Deng, May 2003
%          Last Reversion:  July 2003

%  calculate the coefficients of the denominator of the transfer function
    a11=k1*k2+0.707*k1+1.414*k2+2;
    a10=k1*k2*alph02+0.707*k1*belta02+1.414*k2*alph02+2*belta02;
    a01=k1*k2*alph01+0.707*k1*alph01+1.414*k2*belta01+2*belta01;
    a00=k1*k2*alph01*alph02+0.707*k1*alph01*belta02 ...
        +1.414*k2*alph02*belta01+2*belta01*belta02;

%  Check the stability for the system
    q1=a11*belta01*belta01-a10*belta01-a01*belta02+a00;
    q2=-a11*alph02*belta01+a10*belta01+a01*alph02-a00;
    q3=-a11*alph01*belta02+a10*alph01+a01*belta02-a00;
    q4=a11*alph01*alph02-a10*alph01-a01*alph02+a00;
    if q1 > 0 & q2 > 0 & q3 > 0 & q4 > 0
        key=1;
    else
        key =0;
    end

%  =========== end of unityStable  =========================
```

```
function hlp = lowPass (c1, c2)

% function hlp = lowPass (c1, c2) takes the coefficients of the double Generalized Bilinear
% transformation as its input, after testing the stability for the resulted two-dimensional system,
% and then plots the contour curve and the 3-D magnitude for the resulted two-dimensional
Low %  Pass digital filter.
%
% Input arguments
%            c1 = [k1, alph01, belta01]
%            c2 = [k2, alph02, belta02]
% where, k1, alph01, belta01 are the coefficients of the bilinear transformation for the first
% dimension
%                               z1 + alph01
%         s1 ->   k1 --------------------
%                               z1 + belta01
% and k2, alph02, belta02 are the coefficients of the bilinear transformation for the second
% dimension
%                               z2 + alph02
%          s2   ->    k2-----------------
%                               z2 + belta02
%
% Also see: highPass, bandPass, bandStop, unityStable,
%             k1LPRange, a1LPRange, b1LPRange, k2LPRange, a2LPRange, b2LPRange
%
%                  © Chen Bin Deng, May 2003
%                  Last Reversion:    September 2003

% Define the girds
  [w1,w2]=meshgrid(-pi:2*pi/256:pi, -pi:2*pi/256:pi);

% Determine the polarity of the filter is a Low Pass one.
 if c1(3) > 0 & c2(3) > 0
     button = questdlg('Is the desire filter a low pass filter?', ...
         'Filter Type Dialog','Yes','No','No');
     switch button
     case 'Yes',
     case 'No',
         error('The result will be Low Pass filter! try again!');
     end
 else
     error ('Low Pass filter requires positive belta01 and belta02!');
```

```
    end

% Test the stability of the double generalized bilinear transformation
    if c1(1) <= 0
        error('k1 will cause unstable transformation, please check it!');
    end

    if c1(2)>0 | c1(2) < -1.0
        error('alph01 will cause the transformation unstable, please check it !');
    end

    if c1(3) > 1.0 | c1(3) < 0
        error('belta01 could cause the transformation unstable, please check it !');
    end

    if c2(1) <= 0
        error('k2 will cause unstable transformation, please check it!');
    end

    if c2(2)>0 | c2(2) < -1.0
        error('alph02 could cause the transformation unstable, please check it !');
    end

    if c2(3) > 1.0 | c2(3) < 0
        error('belta01 could cause the transformation unstable, please check it !');
    end

%  Call the function UNITYSTABLE to Check the stability for the system per the condition of the
%  digital filter with unity degree denominator
    a=unityStable (c1(1),c1(2),c1(3),c2(1),c2(2),c2(3));
    if a == 0
        error ('The system with the set of coefficients is unstable!');
    end

%  Compute frequency response for the desired two-dimensional Low Pass filter
    for n=1:length(w1)
        for m=1:length(w1)
            dd(n,m)=((exp(-j*w1(n,m))+c1(3))+c1(1)*0.707*(exp(-j*w1(n,m))+c1(2)))...
                *(c2(1)*1.414*(exp(-j*w2(n,m))+c2(2))+(exp(-j*w2(n,m))+c2(3)))...
                +(exp(-j*w1(n,m))+c1(3))*(exp(-j*w2(n,m))+c2(3));
          nd(n,m)=(exp(-j*w1(n,m))+c1(3))*(exp(-j*w2(n,m))+c2(3));
```

```
        end
    end
    hlp = nd./dd;

% Plot the contour and 3-D magnitude response for the two-dimensional digital filter
    Hlpw=abs(hlp);
    figure(1),contour(w1/pi,w2/pi,Hlpw,3);
    xlabel('\omega_1 (\pi)'),ylabel('\omega_2   (\pi)');
    figure(2),mesh(w1/pi,w2/pi,Hlpw);
    xlabel('\omega_1   (\pi)'),ylabel('\omega_2   (\pi)'),zlabel('Amplitude');

%   ============   end of lowPass   ==============
```

A2. Programs for Chapter 3: 2-D High-Pass Filters

```
function k1HPRange (alph01, belta01, k2, alph02, belta02)
%
%  function k1HPRange (alph01, belta01, k2, alph02, belta02) takes the coefficients, except k1,
%  of the double generalized bilinear transformations as the input arguments, and returns the
%  stable ranges of k1 that can make the system a high pass filter and stable with the inputted
%  coefficients.
%
%  To get stable double generalized bilinear transformations, for two-dimensional high pass
%  digital filter got from the analog low pass transfer function by bilinear transformation, the
%  coefficients are bounded as
%              k1 > 0, 0 <= alph0 <= 1.0, -1.0 <= belta01<= 0;
%              k2 > 0, 0 <= alph02<= 1.0, -1.0 <= belta02<= 0.
%
%  To guarantee the resulted system to be stable, the digital transfer function should meet the
%  stability requirements
%
%  For 2-D digital filters having transfer functions with unity degree denominators
%          D(z1,z2) = a11*z1*z2 + a10*z1 + a01*z2 + a00
%  Use the function UNITYSTABLE to test the stability.
%
%  Also see: a1HPRange, b1HPRange, k2HPRange, a2HPRange, b2HPRange, unityStable.m
%
%                    ©  Chen Bin Deng,    May 2003
%                       Last Reversion:   Sep. 2003

% Check the polarit of the desired two-dimensional filter
if belta01 <0 & belta02 <0
     button = questdlg('Is the desire filter a high pass filter?', ...
            'Filter Type Dialog','Yes','No','No');
     switch button
     case 'Yes',
     case 'No',
          error('This function cannot treat low pass filter problems!');
     end
else
     error ('High Pass filter requires negative belta01 and belta02');
```

```
end
% Test the stability of the bilinear transformations for two-dimensional High Pass filter
if k2 <= 0
    error('The input of k2 could cause unstable transformation, please check it!');
end

if alph01<0 | alph01 > 1.0
    error('The input of alph01 could cause unstable transformations, please check it !');
end

if alph02 <0 | alph02 > 1.0
    errordlg('The input of alph02 could cause unstable transformation, please check it !');
    error ('alph02 is not correct');
end

if belta01 < -1.0 | belta01 >0
    error('The input of belta01 could cause the transformation unstable, please check it !');
end

if belta02 < -1.0
    error('The input of belta02 could cause the transformation unstable, please check it !');
end

% scan the stable range for k1 from 0 to 1000 with step 0f 0.1
% here we sue 1000 to stimulate infinite
k1=0;
n=1;m=1;
while k1 <= 1e3
    k1=k1+0.1;
    a = unityStable (k1,alph01, belta01, k2, alph02, belta02);
    if a == 1
        x(n) = k1;
        n = n+1;
    end
end

% plot the stable range for k1
y=ones(1,length(x));
figure (1), subplot(211),stem(x,y,'.'), axis ([-100,max(x)+0.5, 0, 5]);
xlabel('k_1'),set(gca,'xtick',[min(x),max(x)], 'ytick',[0]); box off
```

```
% ========== end of k1HPRange ======
```

```
function a1HPRange (k1, belta01, k2, alph02, belta02)

% function a1HPRange (k1, belta01, k2, alph02, belta02) takes the coefficients, except alph01,
% of the double generalized bilinear transformations as the input arguments, and returns the
% stable ranges of alph01 that can make the system a high pass filter and stable with the
% inputted coefficients.
%
% To get stable double generalized bilinear transformations, of 2-D high pass digital filter
% obtained from the analog low pass transfer function through the application of
% double generalized bilinear transformation, the coefficients are should be bounded as
%            k1 > 0, 0 <= alph01 <= 1.0, -1.0 <= belta01 <= 0;
%            k2 > 0, 0 <= alph02 <= 1.0, -1.0 <= belta02 <= 0.
%
% To guarantee the resulted 2-D digital filter system to be stable, the digital transfer function
% should meet the stability requirements of the 2-Dl digital filter having  transfer functions
% with unity degree denominator
%            D(z1,z2) = a11*z1*z2 + a10*z1 + a01*z2 + a00
% the stable conditions are:
%            a11*belta01*belta01 - a10*belta01 - a01*belta02 + a00 > 0
%            -a11*alph02*belta01 + a10*belta01 + a01*alph02 - a00 > 0
%            -a11*alph01*belta02 + a10*alph01 + a01*belta02 - a00 > 0
%            a11*alph01*alph02 - a10*alph01 - a01*alph02 + a00 > 0
%
% See also: k1HPRange, b1HPRange, k2HPRange, a2HPRange, b2HPRange, unityStable
%
%                 © Chen Bin Deng, May 2003
%                 Last Reversion:     Sep. 2003
%
% Check the polarity of the desired two-dimensional filter
if belta01 <0 & belta02 < 0
    button = questdlg('Is the desire filter a high pass filter?', ...
        'Filter Type Dialog','Yes','No','No');
    switch button
    case 'Yes',
    case 'No',
        error('Can not use this function to treat low pass filter problems!');
    end
else
    error ('High Pass filter requires negative belta01 and belta02! Try again');
end
```

```
% Check the conditions forstable double bilinear transformation
if k1 <= 0
    error('The input of k1 could cause unstable transformation, please check it!');
end

if k2 <= 0
    error('The input of k2 could cause unstable transformation, please check it!');
end

if alph02 < 0 | alph02 > 1.0
    error('The input of alph02 could cause unstable transformation, please check it !');
end

if belta01 < -1.0 | belta01 > 0
    error('The input of belta01 could cause unstable transformation, please check it !');
end
if belta02 < -1.0 | belta02 > 0
    error('The input of belta02 could cause the transformation unstable. please check it !');
end

% Scan the stable range for alph01 from 0 to 1.0 with the step of 0.001
alph01=0;
n=1;
while alph01 <= 1.0
    a = unityStable (k1, alph01, belta01, k2, alph02, belta02);
    if a == 1
        x(n)=alph01;
        n=n+1;
    end
    alph01=alph01+0.001;
end

% plot the stale ranges for alph01
y=ones(1,length(x));
figure (1), subplot (2,1,1), stem(x,y,'.'), axis ([min(x)-0.25,max(x)+0.25, 0, 5]);
xlabel('\alpha_{01}'),set(gca,'xtick',[min(x),max(x)], 'ytick',[0]);box off

% ========== end of a1HPRange ==========
```

```
function b1HPRange (k1, alph01, k2, alph02, belta02)

%  function b1HPRange (k1, alph01, k2, alph02, belta02) takes the coefficients, except belta01,
%  of the double generalized bilinear transformations as the input arguments, and returns the
%  stable ranges of belta01 that can make the system a high pass filter and stable with the
%  inputted coefficients.
%
%  To get stable double generalized bilinear transformations, for two-dimensional high pass
%  digital filter got from the analog low pass transfer function by bilinear transformation, the
%  coefficients are bounded as
%               k1 > 0, 0 <= alph01 <= 1.0, -1.0 <= belta01 <= 0;
%               k2 > 0, 0 <= alph02 <= 1.0, -1.0 <= belta02 <= 0.
%
%  To guarantee the resulting system to be stable, the digital transfer function should meet the
%  stability requirements
%  For the two-dimensional digital transfer function with unity degree denominator
%               D(z1,z2) = a11*z1*z2 + a10*z1 + a01*z2 + a00
%  Use the function unityStable to test the stability of the resulted systems
%
%  See also: k1HPRange, a1HPRange, k2HPRange, a2HPRange, b2HPRange, unityStable
%
%                    ©  Chen Bin Deng, May 2003
%                    Last Reversion:       Sep. 2003

% Check the polarity of the desired two-dimensional filter
if belta02 <0
    button = questdlg('Is the desire filter a high pass filter?', ...
        'Filter Type Dialog','Yes','No','No');
    switch button
    case 'Yes',
    case 'No',
        error('This function annot treat low pass filter problems!');
    end
else
    error ('High Pass filter requires negative belta01 and belta02');

end

% Test the stability of the bilinear transformations for two-dimensional High Pass filter
if k1 <= 0
    error('The input of k1 could cause unstable transformation, please check it!');
```

```
end

if k2 <= 0
    error('The input of k2 could cause unstable transformation, please check it!');
end

if alph01<0 | alph01 > 1.0
    error('The input of alph01 could cause unstable transformations, please check it !');
end

if alph02 <0 | alph02 > 1.0
    errordlg('The input of alph02 could cause unstable transformation, please check it !');
    error ('alph02 is not correct');
end

if belta02 < -1.0 | belta02 > 0
    error('The input of belta02 could cause the transformation unstable, please check it !');
end

% Scan the stable ranges for belta02 from -1.0 to 0 with a step of 0.001
belta01= -1.0;
n=1;
while belta01 <= 1e-6
    a = unityStable (k1, alph01, belta01, k2, alph02, belta02);
    if a = = 1
        x(n)=belta01;
        n=n+1;
    end
    belta01=belta01+0.001;
end

% Plot the stable ranges for belta01

y=ones(1,length(x));
figure (1), subplot(2,1,1),stem(x,y,'.'), axis ([min(x) – 0.25,max(x)+0.25, 0, 5]);
xlabel('\beta_{01}'),set(gca,'xtick',[min(x),max(x)], 'ytick',[0]);box off;

% ========== end of b1HPRange ==========
```

```
function k2HPRange (k1, alph01, belta01, k2, alph02, belta02)

% function k2HPRange (k1, alph01, belta01, k2, alph02, belta02) takes the coefficients, except
% k2, of the double generalized bilinear transformations as the input arguments, and returns
the
% stable ranges of k2 that can make the system a stable high pass filter stable with the inputed
% coefficients.
%
% To get stable double generalized bilinear transformations, for a 2-D high pass digital filter
% obtained from the analog low pass transfer function by bilinear transformation,
% the coefficients are bounded as
%            k1 > 0, 0 <= alph01 <= 1.0, -1.0 <= belta01 <= 0;
%            k2 > 0, 0 <= alph02 <= 1.0, -1.0 <= belta02 <= 0.
% To guarantee the resulting system to be stable, the digital transfer function should meet the
% stability requirements. For the 2-D digital transfer function with unity degree denominator
%          D(z1,z2) = a11*z1*z2 + a10*z1 + a01*z2 + a00
% Use the function unityStable to test the stability of the resulted systems
%
% Also see: a1HPRange, b1HPRange, k2HPRange, a2HPRange, b2HPRange, unityStable
%
%                  © Chen Bin Deng, May 2003
%                  Last Reversion:      Sep. 2003
%
% Check the polarit of the desired two-dimensional filter

if belta01 <0 & belta02 <0
    button = questdlg('Is the desire filter a high pass filter?', ...
                            'Filter Type Dialog','Yes','No','No');
    switch button
    case 'Yes',
    case 'No',
        error('This function annot treat low pass filter problems!');
    end
else
    error ('High Pass filter requires negative belta01 and belta02');
end

% Check the input arguments
if k1 <= 0
    error('The input of k1 could cause unstable transformation, please check it!');
end
```

```
if alph01<0 | alph01 > 1.0
    error('The input of alph01 could cause unstable transformations, please check it !');
end

if belta01 < -1.0 | belta01 >0
    error('The input of belta01 could cause the transformation unstable, please check it !');
end

if alph02 <0 | alph02 > 1.0
    errordlg('The input of alph02 could cause unstable transformation, please check it !');
    error ('alph02 is not correct');
end

if belta02 < -1.0
    error('The input of belta02 could cause the transformation unstable, please check it !');
end

% scan the stable range for k2 from 0 to 1000 with step 0f 0.1, we sue 1000 to stimulate infinite
k2=0;
n=1;m=1;
while k2 <= 1e+3
    k2=k2+0.1;
    a = unityStable (k1, alph01, belta01, k2, alph02, belta02);
    if a == 1
        x(n)=k2;
        n=n+1;
    end
end

% plot the stable range of k2
y=ones(1,length(x));
figure (1), subplot (2,1,1), stem(x,y,'.'), axis ([min(x)-100,max(x), 0, 5]);
xlabel('k_2'),set(gca,'xtick',[min(x),max(x)], 'ytick',[0]);box off

% ========== end of k2HPRange ==========
```

```
function a2HPRange (k1, alph01, belta01, k2, belta02)

%  function k2HPRange (k1, alph01, belta01, k2, belta02) takes the coefficients, except alph02.
%  of the double generalized bilinear transformations as the input arguments, and returns the
%  stable ranges of alph02 that can make the system a high pass filter and stable with the
%  inputted    coefficients.
%
%  The conditions of stable double generalized bilinear transformation:
%              k1 > 0, 0 <= alph01 <= 1.0, -1.0 <= belta01 <= 0;
%              k2 > 0, 0 <= alph02 <= 1.0, -1.0 <= belta02 <= 0.
%
%  To guarantee the resulting system to be stable, the digital transfer function should meet the
%  stability requirements. For the two-dimensional digital transfer function with unity
%  degree denominator
%              D(z1,z2) = a11*z1*z2 + a10*z1 + a01*z2 + a00
%  Use the function unityStable to test the stability of the resulted systems
%
%  Also see: k1HPRange, a1HPRange, b1HPRange, k2HPRange, a2HPRange, b2HPRange,
%              unityStable
%
%                      ©   Chen Bin Deng, May 2003
%                      Last Reversion:      Sep. 2003

% Check the polarity of the desiring two-dimensional filter
if belta01 <0 & belta02 <0
    button = questdlg('Is the desire filter a high pass filter?', ...
                                      'Filter Type Dialog','Yes','No','No');
    switch button
    case 'Yes',
    case 'No',
        error('This function annot treat low pass filter problems!');
    end
else
    error ('High Pass filter requires negative belta01 and belta02');

end

% Check the input arguments
if k1 <= 0
    error('The input of k1 could cause unstable transformation, please check it!');
end
```

```
if alph01<0 | alph01 > 1.0
    error('The input of alph01 could cause unstable transformations, please check it !');
end

if belta01 < -1.0 | belta01 >0
    error('The input of belta01 could cause the transformation unstable, please check it !');
end

if k2 <= 0
    error('The input of k2 could cause unstable transformation, please check it!');
end

if belta02 < -1.0 | belta02 >0
    error('The input of belta02 could cause the transformation unstable, please check it !');
end

% scan the stable range for alph02 from 0 to 1.0 with a step of 0.001
alph02=0;
n=1;
while alph02 <= 1e0
    a = unityStable (k1, alph01, belta01, k2, alph02, belta02);
    if a == 1
        x(n)=alph02;
        n=n+1;
    end
    alph02=alph02+0.001;
end

% plot the stabel range for alph02
y = ones (1, length(x));
figure (1), subplot(211),stem(x,y,'.'), axis ([min(x)-0.25,max(x)+0.25, 0, 5]);
xlabel('\alpha_{02}'),set(gca,'xtick',[min(x),max(x)], 'ytick',[0]);box off;

% ==========    end of a2HPRange    ==========
```

```
function b2HPRange (k1, alph01, belta01,k2, alph02)

% function b2HPRange (k1, alph01, belta01,k2, alph02) takes the coefficients, except belta02,
% of the double generalized bilinear transformations as the input arguments, and returns the
% stable ranges of belta02 that can make the system a high pass filter and stable with
% the inputted coefficients.
%
% The conditions of a stable double generalized bilinear transformation of 2-D high-pass filter:
%           k1 > 0, 0 <= alph01 <= 1.0, -1.0 <= belta01 <= 0;
%           k2 > 0, 0 <= alph02 <= 1.0, -1.0 <= belta02 <= 0.
%
% To guarantee the resulted system to be stable, the digital transfer function should meet the
% stability requirements
% For the two-dimensional digital transfer function with unity degree denominator
%           D(z1,z2) = a11*z1*z2 + a10*z1 + a01*z2 + a00
% Use the function unityStable to test the stability of the resulted systems
%
% Also see: k1HPRange, a1HPRange, b2HPRange, k2HPRange, a2HPRange, unityStable
%
%                 ©   Chen Bin Deng, May 2003
%                 Last Reversion:      Sep. 2003

% Check the polarit of the desired two-dimensional filter
if belta01 <0
    button = questdlg('Is the desire filter a high pass filter?', ...
        'Filter Type Dialog','Yes','No','No');
    switch button
    case 'Yes',
    case 'No',
        error('This function annot treat low pass filter problems!');
    end
else
    error ('High Pass filter requires negative belta01 and belta02');
end

% Check the input arguments
if k1 <= 0
    error('The input of k1 could cause unstable transformation, please check it!');
end

if alph01<0 | alph01 > 1.0
```

```
    error('The input of alph01 could cause unstable transformations, please check it !');
end

if belta01 < -1.0 | belta01 > 0
    error('The input of belta01 could cause the transformation unstable, please check it !');
end

if k2 <= 0
    error('The input of k2 could cause unstable transformation, please check it!');
end

if alph02 <0 | alph02 > 1.0
    errordlg('The input of alph02 could cause unstable transformation, please check it !');
    error ('alph02 is not correct');
end

% Scan the stable range for belta02 from -1.0 to 0 with the step 0f 0.001
belta02 = -1.0;
n=1;
while belta02 <= 1e-6
    a = unityStable (k1, alph01, belta01, k2, alph02, belta02);
    if a == 1
        x(n)=belta02;
        n=n+1;
    end
    belta02=belta02+0.001;
end

% plot the stable range for belta02
y=ones(1,length(x));
figure (1), subplot(211), stem(x,y,'.'), axis ([min(x)-0.25, max(x)+0.25, 0, 10]);
xlabel('\beta_{02}'),set(gca,'xtick',[min(x),max(x)], 'ytick',[0]); box off

%  ==========    end of b2HPRange    ==========
```

```
function hhp = highPass (c1, c2)

% function hhp = highPass (c1, c2) takes the coefficients of the double Generalized
% Bilinear transformation as its input arguments, after testing the stability for the
% resulted two-dimensional system, and then plots the contour curve and the
% 3-D magnitude for the resulted two-dimensional High Pass digital filter.
%
% Input arguments
%          c1=[k1, alph01, belta01]
%          c2=[k2, alph02, belta02]
% where, k1, alph01, belta01 are the coefficients of the bilinear transformation
% for the first dimension
%                          z1 + alph01
%          s1 ----> k1---------------
%                          z1 + belta01
% and k2, alph02, belta02 are the coefficients of the bilinear transformation
% for the second dimension
%                          z2 + alph02
%          s2 ----> k2---------------
%                          z2 + belta02
%
% For high pass bilinear transformation, the coefficients should be bounded
%          k1 > 0, 0 <= alph01 <= 1.0, -1.0 <= belta01 <= 0;
%          k2 > 0, 0 <= alph02 <= 1.0, -1.0 <= belta02 <= 0;
%
% Also see: lowPass, bandPass, bandStop, unityStable,
%          k1HPRange, a1HPRange, b1HPRange, k2HPRange, a2HPRange, b2HPRange
%
%                   ©  Chen Bin Deng, May 2003
%                   Last Reversion:    September 2003

% Define the girds
[w1,w2]=meshgrid(-pi:2*pi/256:pi, -pi:2*pi/256:pi);

% Check the polarity of the 2-D filter
if c1(3) < 0 & c2(3) < 0
    button = questdlg('Is the desire filter a high pass filter?', ...
        'Filter Type Dialog','Yes','No','No');
    switch button
    case 'Yes',
    case 'No',
```

```
            error('This function only treat the problems of2-D high pass filter!');
        end
else
        error ('2-D High Pass filter requires negative belta01 and belta02!');
end

% Test the stability of high pass double generalized bilinear transformation
if c1(1) <= 0
        error('The input k1 will cause unstable transformation, please check it!');
end

if c1(2) < 0 | c1(2) > 1.0
        error('The input alph01 will cause the transformation unstable, please check it !');
end

if c1(3) < -1.0 | c1(3) > 0
        error('The input belta01 could cause the transformation unstable, please check it !');
end

if c2(1) <= 0
        error('The input k2 will cause unstable transformation, please check it!');
end

if c2(2) < 0 | c2(2) > 1.0
        error('The input alph02 could cause the transformation unstable, please check it !');
end

if c2(3) < -1.0 | c2(3) > 0
        error('The input belta01 could cause the transformation unstable, please check it !');
end

% Test the stability for the 2-D high pass filter with a unity degree denominator
a = unityStable (c1(1), c1(2), c1(3), c2(1), c2(2), c2(3));
if a == 0
        error ('The system with this set of coefficients is unstable!');
end

% Compute frequency response for the resulting 2-D high-pass filter
for n=1:length(w1)
        for m=1:length(w1)
                dd(n,m)=((exp(-j*w1(n,m))+c1(3))+c1(1)*0.707*(exp(-j*w1(n,m))+c1(2)))...
```

```
            *(c2(1)*1.414*(exp(-j*w2(n,m))+c2(2))+(exp(-j*w2(n,m))+c2(3)))...
            +(exp(-j*w1(n,m))+c1(3))*(exp(-j*w2(n,m))+c2(3));
        nd(n,m)=(exp(-j*w1(n,m))+c1(3))*(exp(-j*w2(n,m))+c2(3));
    end
end
hhp=nd./dd;
Hwhp=abs(hhp);

% plot the contour curve and 3-D magnitude response for the resulted 2-D high pass filter
figure(1),contour(w1/pi,w2/pi,Hwhp, 3),xlabel('\omega_1 (\pi)'),ylabel('\omega_2   (\pi)');
figure(2),mesh(w1/pi,w2/pi,Hwhp),   xlabel('\omega_1     (\pi)'),   ylabel('\omega_2     (\pi)'),
zlabel('Amplitude');

% ======    end of highPass.m ==========
```

A3. Programs for Chapter 4: 2-D Band-Pass Filters

```
function bandPass (lp, hp)
%
%   function bandPass (lp, hp) takes the 2-D low pass and 2-D high pass generalized
%   bilinear transformation coefficients as its input arguments, and cascading connect
%   these two member filter together, and resulted a 2-D band Stop filter. Plot the contour
%   and 3-D magnitude response for the resulted 2-D Band Pass filter.
%
%   Input arguments:
%           lp = [k1, alph01, belta01, k2, alph02, belta02]
%           hp = [k3, alph03, belta03, k4, alph04, belta04]
%   where the coefficients are the generalized bilinear transformation
%                              z+alph
%                   s ----> k -----------
%                              z+belta
%   and   the coefficients with subscript "1" is the low pass coefficients in 1st domian
%         the coefficients with subscript "2" is the low pass coefficients in 2nd dimension
%         the coefficients with subscript "3" is the high pass coefficients in 1st dimension
%         the coefficients with subscript "4" is the high pass coefficients in 2nd dimension
%
%   Also see: lowPass, highPass, k1LPRange, a1LPRagne, a2LPrange, k2LPRange, b1LPRange
%             b2LPRange, k1HPRange, k2HPRange, a1HPRange, a2HPRange, b1HPRange,
%             b2HPRange, unityStabel
%
%                  ©   Chen Bin Deng, June 2003
%                  Last Reversion:      Sep. 2003

% define the grid
[w1,w2]=meshgrid(-pi:2*pi/256:pi, -pi:2*pi/256:pi);

c1 = [lp(1), lp(2), lp(3)];
c2 = [lp(4), lp(5), lp(6)];
c3 = [hp(1), hp(2), hp(3)];
c4 = [hp(4), hp(5), hp(6)];

% call the function lowPass to form the member lpw pass filter filter
hlp = lowPass (c1,c2);
figure (1), contour (w1/pi, w2/pi, abs (hlp), 3); xlabel('\omega_1 (\pi)'),ylabel('\omega_2 (\pi)');
figure(2),mesh(w1/pi,w2/pi,abs (hlp));
```

```
xlabel('\omega_1 (\pi)'),ylabel('\omega_2 (\pi)'),zlabel('Amplitude');
button = questdlg('Is frequency response of the low pass filter correct?', ...
    'Member Low Pass filter Dialog','Yes','No','No');
switch button
case 'Yes',
case 'No',
    error('Modify the coefficients for the member low-pass filter and try again!');
end

%   call the function highPass to form the member high pass filter
hhp = highPass (c3,c4);
figure (3), contour (w1/pi, w2/pi, abs (hhp), 3); xlabel('\omega_1 (\pi)'),ylabel('\omega_2 (\pi)');
figure(4),mesh(w1/pi,w2/pi,abs(hhp));
xlabel('\omega_1 (\pi)'),ylabel('\omega_2 (\pi)'),zlabel('Amplitude');
button = questdlg('Is frequency response of the high pass filter correct?', ...
    'Member High Pass filter Dialog','Yes','No','No');
switch button
case 'Yes',
case 'No',
    error('Modify the coefficients for member high-pass filter and try again!');
end
%   Compute the frequency response for the resulted Band Pass filter
hbp = hlp.*hhp; Hwbp = abs (hbp);

%   Plot the contour curve and the 3-D magnitude response
figure(5),contour(w1/pi,w2/pi,Hwbp,3); xlabel('\omega_1 (\pi)'),ylabel('\omega_2 ( pi)');
figure(6),mesh(w1/pi,w2/pi,Hwbp);
xlabel('\omega_1 (\pi)'),ylabel('\omega_2 (\pi)'),zlabel('Amplitude');
figure(7),contour(w1/pi,w2/pi,abs(hlp),3,'r--');hold on,
contour(w1/pi,w2/pi,abs (hhp),3,'b:');
contour(w1/pi,w2/pi,Hwbp,3,'k-');
xlabel('\omega_1 (\pi)'),ylabel('\omega_2 (\pi)');hold off,

%   ==========   end of bandPass   ==========
```

A4. Programs for Chapter 5: 2-D Band-Elimination Filter

```
function bandElimination (lp, hp)
%
%   function bandElimination (lp, hp) takes the 2-D low-pass and 2-D high-pass generalized
%   bilinear transformation coefficients as its input arguments, and parallel combination of
%   these two member filter together, and resulting a 2-D band-elimination filter. Plot the
contour
%   and 3-D magnitude response plots for the resulting 2-D band-elimination filter.
%
%   Input arguments:
%           lp = [k1, alph01, belta01, k2, alph02, belta02]
%           hp = [k3, alph03, belta03, k4, alph04, belta04]
%   whenre the coefficients are the generalized bilinear transformation
%                       z+alph
%               s ----> k -----------
%                       z+belta
%   and the coefficients with subscript "1" is the low pass coefficients in 1st domian
%       the coefficients with subscript "2" is the low pass coefficients in 2nd dimension
%       the coefficients with subscript "3" is the high pass coefficients in 1st dimension
%       the coefficients with subscript "4" is the high pass coefficients in 2nd dimension
%
%   Also see: lowPass, highPass, bandPass k1LPRange, a1LPRagne, a2LPrange, k2LPRange,
%           b1LPRange, b2LPRange, k1HPRange, k2HPRange, a1HPRange, a2HPRange,
%           b1HPRange, b2HPRange, unityStabel
%
%               © Chen Bin Deng, June 2003
%               Last Reversion:     Sep. 2003

% define the grid
[w1,w2]=meshgrid(-pi:2*pi/256:pi, -pi:2*pi/256:pi);
c1 = [lp(1), lp(2), lp(3)];
c2 = [lp(4), lp(5), lp(6)];
c3 = [hp(1), hp(2), hp(3)];
c4 = [hp(4), hp(5), hp(6)];

% call the function lowPass to form the member lpw pass filter filter
hlp = lowPass (c1,c2);
figure (1), contour (w1/pi, w2/pi, abs (hlp), 3); xlabel('\omega_1 (\pi)'),ylabel('\omega_2 (\pi)');
figure(2),mesh(w1/pi,w2/pi,abs (hlp));
```

```
xlabel('\omega_1 (\pi)'),ylabel('\omega_2 (\pi)'),zlabel('Amplitude');
button = questdlg('Is frequency response of the low pass filter correct?', ...
    'Member Low Pass filter Dialog','Yes','No','No');
switch button
case 'Yes',
case 'No',
    error('Modify the coefficients for the member low-pass filter and try again!');
end

%   call the function highPass to form the member high pass filter
hhp = highPass (c3,c4);
figure (3), contour (w1/pi, w2/pi, abs (hhp), 3); xlabel('\omega_1 (\pi)'),ylabel('\omega_2 (\pi)');
figure(4),mesh(w1/pi,w2/pi,abs(hhp));
xlabel('\omega_1 (\pi)'),ylabel('\omega_2 (\pi)'),zlabel('Amplitude');
button = questdlg('Is frequency response of the high pass filter correct?', ...
    'Member High Pass filter Dialog','Yes','No','No');
switch button
case 'Yes',
case 'No',
    error('Modify the coefficients for member high-pass filter and try again!');
end

%   Compute the frequency response for the resulted Band Pass filter
hbp = hlp.+hhp; Hwbp = abs (hbp);

%   Plot the contour curve and the 3-D magnitude response
figure(5),contour(w1/pi,w2/pi,Hwbp,3);
xlabel('\omega_1 (\pi)'),ylabel('\omega_2 (\pi)');
figure(6),mesh(w1/pi,w2/pi,Hwbp);
xlabel('\omega_1 (\pi)'),ylabel('\omega_2 (\pi)'),zlabel('Amplitude');
figure(7),contour(w1/pi,w2/pi,abs(hlp),3,'r--');hold on,
contour(w1/pi,w2/pi,abs (hhp),3,'b:');
contour(w1/pi,w2/pi,Hwbp,3,'k-');
xlabel('\omega_1 (\pi)'),ylabel('\omega_2 (\pi)');hold off,

%   ==========   end of bandElimination   ==========
```