A technique of designing Two-Dimensional Recursive Filters with Flexible Characteristics

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Abstract

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Different kinds of two-dimensional recursive filters are used in signal processing and pro-imagining processing process and communication systems where flexible frequency responses of the digital filters are necessitated.

The focal intention of this thesis has been to propose a technique of designing different kinds of 2-D recursive filters with adaptable properties. From the 2-variable existing VSHP (Very Strictly Hurwitz Polynomial), a 2-D digital transfer function has been obtained through the application of the double generalized bilinear transformations with some changeable coefficients. 2-D LPF and 2-D HPF are designed from this 2-D transfer function, and the frequency response of either of them changes when one or more of these coefficients change.

Other two important types of 2-D digital filters, 2-D band-pass filters and 2-D band-elimination filters could also be achieved by combining a 2-D LPF and a 2-D HPF. One of them, 2-D band-pass recursive filter has been designed in this thesis work.

The manner how each coefficient of the designed filters affects the frequency response is analyzed in detail with simulation results – 3-D amplitude plots and 2-D contour plots. We also derived required constrains for each case to obtain a stable response.

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Table of Contents

List of Figures

List of Important Abbreviations and Symbols

1-D	One-Dimensional
2-D	Two-Dimensional
z_1, z_2	Z-domain parameter in two-dimensions
S_1, S_2	Laplace domain parameter in two-dimensions
ω_1, ω_2	Frequencies in radians in the analog domain parameter in two-dimensions
$D(S_1, S_2)$	2-variable Analog transfer function
$D(z_1, z_2)$	Two-dimensional digital function
$H(z_1, z_2)$	Frequency response of 2-D digital filter
FIR	Finite Impulse Response
IIR	Infinite Impulse Response
SHP	Strictly Hurwitz Polynomial
VSHP	Very Strictly Hurwitz Polynomial
NFSK	Non-essential Singularity of First Kind
NSSK	Non-essential Singularity of Second Kind
BIBO	Bounded Input Bounded Output
LPF	Low Pass Filter
HPF	High Pass Filter
BPF	Band Pass Filter
Y_1, Y_2	Positive Constants

1. INTRODUCTION

1.1 General

2-D digital filters are increasingly being used in many applications in modern day devices, soft-wares for the processing of spatial data (e.g. radar arrays, image arrays, biomedical data arrays, geographical data arrays etc.) in order to fetch some desired information from a 2-D data array. This leads to the focusing of considerable research efforts in studying 2-D digital systems. [1-4]

Interest has been directed by researchers into the area of 2-D digital systems due to several reasons: high efficiency due to high speed computations, permitting high quality image processing and analysis, great application flexibility and adaptability, decreasing cost of software and hardware. The 2-D digital systems perform important operations which include: 2-D digital filtering, 2-D digital transformations, local space processing, data compression and pattern recognition. [2, 3]

Digital filtering and data transformations play important roles in preprocessing of images, performing smoothing, enhancement, and noise reduction. [2, 3] In many of the applications of 2-D filters, variable filter frequency response is required in terms of bandwidth and gain of the system; hence researchers have shown particular interest in 2-D digital filters with flexible characteristics.

1.2 2-D digital filter types

Similar to 1-D digital filters, 2-D digital filters can be classified into two distinct types: Finite Impulse Response (FIR) filters and Infinite Impulse Response (IIR) filters. FIR filters are also known as nonrecursive digital filters. On the other hand IIR filters are known as recursive digital filters. [1-3]

A nonrecursive digital filter is the one whose impulse response possesses only a finite number of nonzero samples. For such a filter, the impulse response is always absolutely summable; and therefore FIR filter is always stable. Formally, a 2-D FIR filter can be characterized in the z domain as $H(z_1, z_2)$, where

$$H(z_1, z_2) = \sum_{i=0}^{N_1} \sum_{j=0}^{N_2} X_{ij} z_1^i z_2^j, \tag{1.1}$$

where, X_{ij} is a real constant.

Similar to 1-D FIR filters, the main properties of 2-D digital filters are precisely linear phase and the advantage of being very well understood. FIR filters have zero-phase property hence, the design problem is simplified. Along with that, the symmetry constraint on the impulse response of the filter can be exploited in the implementation of the filter.

An IIR (Infinite Impulse Response), or recursive, filter is one whose input and output satisfy a multidimensional difference equation of finite order. These filters may or may not be stable, but in many cases they may be less complex to realize than equivalent FIR filters. Formally, a 2-D FIR filter can be characterized in the z domain as $H(z_1, z_2)$, where

$$H(z_1,z_2)=\frac{N(z_1,z_2)}{D(z_1,z_2)}=\frac{\sum_{i=0}^{M_1}\sum_{j=0}^{M_2}Y_{mn}z_1^i z_2^j}{\sum_{i=0}^{N_1}\sum_{j=0}^{N_2}X_{ij}z_1^i z_2^j} \tag{1.2}$$

where, $X_{00} = 1; X_{ij}$ and Y_{mn} are real constants.

1.2.1 2-D recursive filters

The 2-D IIR filter does not have the phase linearity feature of the 2-D FIR filter but it substitutes this limitation by providing an improved magnitude frequency response. Because of the feedback structure of the 2-D IIR filter, it allows passing the desired frequency domain behavior and attenuates the undesired one. 2-D Recursive structure has reduced computational complexity over that of the nonrecursive filter form, e.g. fewer arithmetic operations, smaller memory requirements etc, so it seems more suitable with small-scale hardware implementations. And for these reasons, 2-D IIR filters are widely used in communications, controls, biomedicine, geophysics, vibration analysis, radar, sonar and so on, where high performance frequency selectively is required. And this widespread application of recursive filters motivates more researchers to study the different properties, e.g. variable magnitude response, phase response and group delay, of such filters. Though, the stability testing is one of the prime issues in 2-D recursive filters. [2, 3, 4, 10]

From a practical viewpoint, a realizable 2-D recursive digital filter must produce a bounded output if stimulated by a bounded input. Similar to the case of a 1-D digital feedback filter, a 2-D recursive filter can be unstable. The output array can be unbounded, though we give a bounded input array. Hence, in order to assure the stability of the 2-D recursive filter, we need to put constraints on the coefficients of the 2-D recursive filter. The coefficient of the denominator $D(z_1, z_2)$ determines the stability of the 2-D recursive filter. The stability criterion for the filter is given by following theorem.

$$D(z_1, z_2) = \sum_{i=0}^{N_1} \sum_{j=0}^{N_2} X_{ij} z_1^i z_2^j \neq 0; \quad |z_1| \leq 1 \cap |z_2| \leq 1 \tag{1.3}$$

"In other words, if there are any values (real or complex) for z_1 and z_2, for which $D(z_1, z_2)$ is zero and for which z_1 and z_2 are simultaneously less than or equal to one in magnitude then the filter $1/D(z_1, z_2)$ will be unstable. If there are not any, then $1/D(z_1, z_2)$ will be stable." [6, 7, 8, 9]

In the early age of designing of 2-D recursive filter, one can divide design approaches into two noticeable groups, those involving spectral transformations [11-13] and parameter optimization. [14-16]. Usually, a spectral transformation must have the following characteristics.

- It must produce an IIR stable transfer function from a stable transfer function.
- It must transform a real rational function into a real rational function.
- It must preserve some basic characteristics of the magnitude response.

After keeping these characteristics in mind, one can design a 2-D recursive filter using 1-D analog or digital filters or 2-D analog or digital filters. These kinds of filters are also known as rotated filters because they are obtained by rotating 1-D or 2-D filters. Parameter optimization technique is also called as approximation technique. In this kind of approach, the first step consists of choosing an approximating function. Then the problem mathematically is formulated and the optimization technique is developed to the design filter. Most of the designs mentioned above have quite typical design methods and limited filter forms. This limitation in the frequency response opened a new era in development of more simplified and flexible designs.

Most of the existing design methods for 2-D recursive filters have variable frequency domain characteristics based on frequency transformations. [17-19]. And using these methods all kinds of filters, e.g. low pass, high pass, band pass and band elimination, can be designed with variable cutoff frequencies. But when more complicated and detailed variable specifications are given, these methods fail because of the intrinsic limitations of the frequency transformation.

Another general approach to design 2-D recursive filter is to start with a two-variable Strictly Hurwitz Polynomial (SHP). "$D_a(s_1, s_2)$ is known to be a SHP, if $1/D_a(s_1, s_2)$ does not possess any singularities in the region $\{(s_1, s_2) \mid \mathrm{Re}(s_1) \geq 0, \mathrm{Re}(s_2) \geq 0, where |s_1| < \infty \; and |s_2| < \infty\}$". SHP contains all its zeros strictly in the left-half of s-plane. After generating a SHP, apply that SHP to the denominator of a referenced 2-D analog filter wherein the SHP coefficients are used as the variables of optimization for a guarantied stable 2-D analog filter. Then, a stable 2-D recursive filter is obtained by applying the bilinear transformation $S \longrightarrow \left(\frac{1-z}{1+z}\right)$ to this 2-D analog filter. [20]. But in this designed 2-D recursive filter if non-essential singularities of the second kind[1] will show up on the closed unit circle of (z_1, z_2) biplane, then the filter may be unstable. [23]

And to overcome these stability problems, the analog transfer function must meet the following conditions.[26, 27]

- Transfer function must contain its zeroes strictly in the left half of the s-plane.
- Transfer function does not posses non-essential singularity of the first kind[2] because its occurrence always results in an unstable filter.
- Transfer function does not posses non-essential singularity of the second kind because its occurrence may cause instability.

This argument leads us to a new kind of polynomial called "Very Strict Hurwitz Polynomial" (VSHP). [26]

1.3 "Very Strict Hurwitz Polynomial" (VSHP) and 2-D recursive filter design [26]

"A polynomial $D_a(s_1, s_2)$ is known to be a VSHP, if $1/D_a(s_1, s_2)$ does not possess any singularities in the region of 2-D plane (S_1, S_2) such that, $\{(s_1, s_2) \mid \operatorname{Re}(s_1) \geq 0, \operatorname{Re}(s_2) \geq 0, where |s_1| \leq \infty \; and |s_2| \leq \infty\}$"

[1] Where the relatively prime numerator and denominator polynomials share a common zero, the transfer function is said to have a non-essential singularity of the second kind.
[2] Where the relatively prime numerator polynomial is non zero and denominator polynomial is zero, the transfer function is said to have a non-essential singularity of the first kind

After carefully reviewing this definition, one can say that a VSHP must be a SHP. So, one can generate a VSHP from a SHP after making sure of the absence of any kinds of singularities at points of infinity. The points of infinity are studied by taking into account the reciprocal of the variable. In a 2-D plane (S_1, S_2), considering two complex planes, S_1 and S_2, the points of infinity can be divided into three classes:

- S_1 = finite and S_2 = infinite (1.4a)
- S_1 = infinite and S_2 = finite (1.4b)
- S_1 = infinite and S_2 = infinite (1.4c)

1.3.1 Some properties of VSHP

In this section, we discuss some of the important properties of VSHP. We can express a 2-D analog transfer function $H_a(S_1, S_2)$ as,

$$H_a(S_1, S_2) = \frac{N_a(S_1, S_2)}{D_a(S_1, S_2)}., \tag{1.5a}$$

where,

$$N_a(S_1, S_2) = \sum_{i=0}^{m} \sum_{j=0}^{n} B_{ij} S_1^i S_2^j \tag{1.5b}$$

$$D_a(S_1, S_2) = \sum_{i=0}^{k} \sum_{j=0}^{l} A_{ij} S_1^i S_2^j \tag{1.5c}$$

By applying the transformation to the value of the function on each of the three points defined in (1.4a-1.4c), it can be shown that k ≥ m and l ≥ n. And if this is true then no kind of singularities exist at the set of points of infinity in the closed right-half of the (S_1, S_2) biplane. [23] So after assuming k ≥ m and l ≥ n, one can state the following properties.

1. The transfer function $H_a(S_1,S_2)$ does not possess any singularity in the closed right-half of the (S_1, S_2) – biplane, if and only if $D_a(S_1,S_2)$ is a VSHP where, the closed right-half biplane is $\{(s_1,s_2) \mid \mathrm{Re}(s_1) \geq 0, \mathrm{Re}(s_2) \geq 0, where |s_1| \leq \propto \ and |s_2| \leq \propto\}$

2. $D(S_1,S_2) = [D_1(S_1,S_2)] \cdot [D_2(S_1,S_2)]$ shall be a VSHP, if and only if $D_1(S_1,S_2)$ and $D_2(S_1,S_2)$ are individually VSHPs.

3. If $D_a(S_1,S_2)$ is a VSHP, $\dfrac{\partial \mathrm{D_a}(\mathrm{S_1},\mathrm{S_2})}{\partial S_1}$ and $\dfrac{\partial \mathrm{D_a}(\mathrm{S_1},\mathrm{S_2})}{\partial S_2}$ are also VSHPs.

   We can always write $D_a(S_1,S_2)$ as

   $$D_a(S_1,S_2) = E_p(S_2)S_1^p + E_{p-1}(S_2)S_1^{p-1} + \ldots + E_2(S_2)S_1^2 + E_1(S_2)S_1 + E_0(S_2) \quad (1.6a)$$

   or

   $$D_a(S_1,S_2) = F_q(S_1)S_2^q + F_{q-1}(S_1)S_2^{q-1} + \ldots\ldots + F_2(S_1)S_2^2 + F_1(S_1)S_2 + F_0(S_1) \quad (1.6b)$$

4. The polynomials $E_i(S_2)$, i = 0,1,2,….,p and $F_j(S_1)$, j = 0,1,2,….,q defined in above equations are SHPs in S_2 and S_1 respectively.

5. If a real two-variable VSHP can be written as $D_a(S_1,S_2) = \sum_{i=0}^{k}\sum_{j=0}^{l} A_{ij}S_1^i S_2^j$, then the coefficient $A_{kl}A_{ij} > 0$ for all i and j.

6. Each of the functions $\frac{E_i(S_2)}{E_{i-1}(S_2)}$, i = 1, 2, ..., p, is a minimum reactive positive real function[3] in S_2. Similarly, each of the function $\frac{F_j(S_1)}{F_{j-1}(S_1)}$, where j = 1, 2,....q is a minimum reactive positive real function in S_1. [26, 27]

After revising the previous techniques [21, 22], it is guaranteed that the polynomial in the denominator of the analog filter does not have any non-essential singularities of the second kind at all time, which means a stable 2-D analog filter. This can be achieved via the constraints for the denominator of the filter to be a VSHP as a consequence of the variables of optimization. Though, this guarantee bears a profound price in calculation. If we use the various properties of VSHP discussed above, then it is possible to generate such polynomials. [28-31] And once we have a stable 2-D analog filter, we can easily develop a ensured stable 2-D recursive filter by applying double bilinear transformations to the 2-D analog filter.

1.4 Generalized Bilinear Transformation

The bilinear transformation maps the entire "(S_1, S_2) biplane" (analog domain) on the entire "(z_1, z_2) biplane" (digital domain) on a one-to-one basis. As a result, by the application of the double bilinear transformation ($S_1 \rightarrow \frac{1-z_1}{1+z_1}$ and $S_2 \rightarrow \frac{1-z_2}{1+z_2}$) one can

---

[3] A rational function $F(S) = \frac{P(S)}{Q(S)}$ with real coefficients is called a minimum reactive positive real function, if and only if

- $P(S) + Q(S)$ is a SHP in S
- $\operatorname{Re} F(S) \geq 0$ for $\operatorname{Re}(S) \geq 0$
- It has no poles or zeroes on the imaginary axis of S.

change the domain (e.g. from analog to digital) with the same functional behavior. Usage of this principle is one of the most common methods to design a stable 2-D recursive filter from existing stable 2-D analog filter. However in order to design a 2-D recursive filter with variable characteristics and more flexibility, it would be better to introduce a new generalized bilinear transformation with changeable coefficient in the digital filter transfer function. [32, 33]

A generalized bilinear transformation can be defined as

$$S_i \to k_i \frac{z_i + a_{0i}}{z_i + b_{0i}}, \qquad \mathrm{i} = 1, 2 \tag{1.7}$$

After applying the above transformation, one can change the value of k_i, a_{0i} and b_{0i} in order to vary the characteristics of the designed 2-D stable recursive filter (e.g. low-pass or high-pass filter with different cutoff frequencies). However there are some restrictions as a penalty for the ensured stability in the designed filter. For $k_i > 0$, the condition for the generalized bilinear transformation to an analog transfer functions are:

$$|a_{0i}| \leq 1 \tag{1.8a}$$

$$|b_{0i}| \leq 1 \tag{1.8b}$$

$$a_{0i} b_{0i} < 0 \tag{1.8c}$$

After carefully reviewing this condition, without loss of any generality we can assume k_i to be positive and a_{0i} and b_{0i} are opposite in signs. As well, when we map from an analog domain to the digital domain, the imaginary axis in (S_1, S_2) biplane will be mapped to the unit circle in the (z_1, z_2) biplane, and the left-half of the (S_1, S_2) biplane will be mapped to the inner part of the unit circle, and the right-half of (S_1, S_2) biplane will lie outside of the unit circle of (z_1, z_2) biplane. [31]

1.5 Symmetry in the 2-D magnitude response

There might be different kind of symmetries present in the frequency responses of 2-D systems. Usually, the values of the desired filter response at the different points of the frequency domain are interrelated in a certain symmetrical way. One can use this interrelation feature of the 2-D system to reduce the complexity of the d esign and the implementation of 2-D recursive filter. In this section, we study the possible frequency domain symmetries that result in savings in the filter design and implementation complexity. [34 - 38]

1.5.1 Reflection symmetry about axis

Figure 1.1a and Figure 1.1b show reflection symmetry about axis. A 2-D magnitude response has reflection symmetry about axis, if the two-variable transfer function $H(\omega_1, \omega_2)$ satisfies the following conditions:

- Reflection symmetry about ω_1 axis:

$$H(\omega_1, \omega_2) = H(\omega_1, -\omega_2), \qquad \forall(\omega_1, \omega_2) \tag{1.9a}$$

- Reflection symmetry about ω_2 axis:

$$H(\omega_1, \omega_2) = H(-\omega_1, \omega_2), \qquad \forall(\omega_1, \omega_2) \tag{1.9b}$$

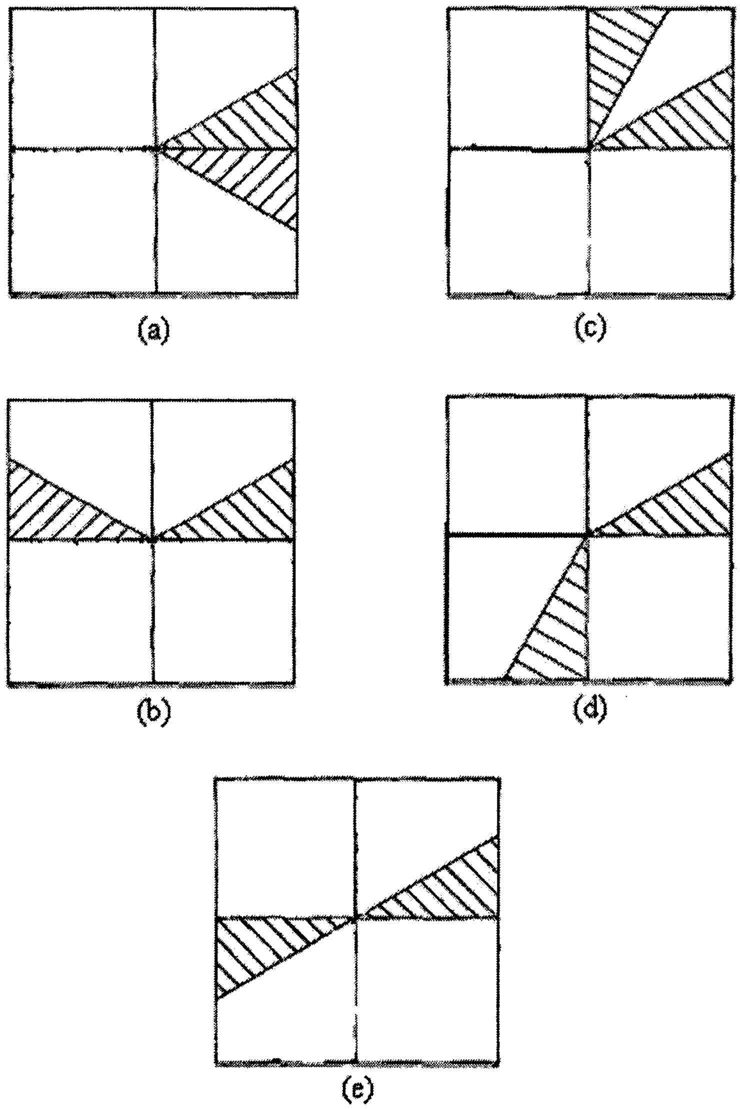

Figure 1.1: (a) Reflection symmetry about ω_1 axis, (b) Reflection symmetry about ω_2 axis (c) Reflection symmetry about $\omega_1 = \omega_2$ diagonal (d) Reflection symmetry about $\omega_1 = -\omega_2$ diagonal (e) Centro Symmetry.

1.5.2 Reflection Symmetry about Diagonal

Figure 1.1c and Figure 1.1d show reflection symmetry about $\omega_1 = \omega_2$ or $\omega_1 = -\omega_2$ diagonal. To posses this kind of symmetry, the two-variable transfer function $H(\omega_1, \omega_2)$ must satisfy the following conditions:

- Reflection symmetry about $\omega_1 = \omega_2$ diagonal

$$H(\omega_1,\omega_2) = H(\omega_2,\omega_1), \qquad \forall(\omega_1,\omega_2) \tag{1.10a}$$

- Reflection symmetry about $\omega_1 = -\omega_2$ diagonal

$$H(\omega_1,\omega_2) = H(-\omega_2,-\omega_1), \qquad \forall(\omega_1,\omega_2) \tag{1.10b}$$

1.5.3 Centro Symmetry

Figure 1.1e shows Centro symmetry characteristic. It is also known as twofold rotational symmetry (rotation by $180°$). To reflect this kind of symmetry in the counter response, the two-variable transfer function must meet the following conditions:

$$H(\omega_1,\omega_2) = H(-\omega_1,-\omega_2), \qquad \forall(\omega_1,\omega_2) \tag{1.11}$$

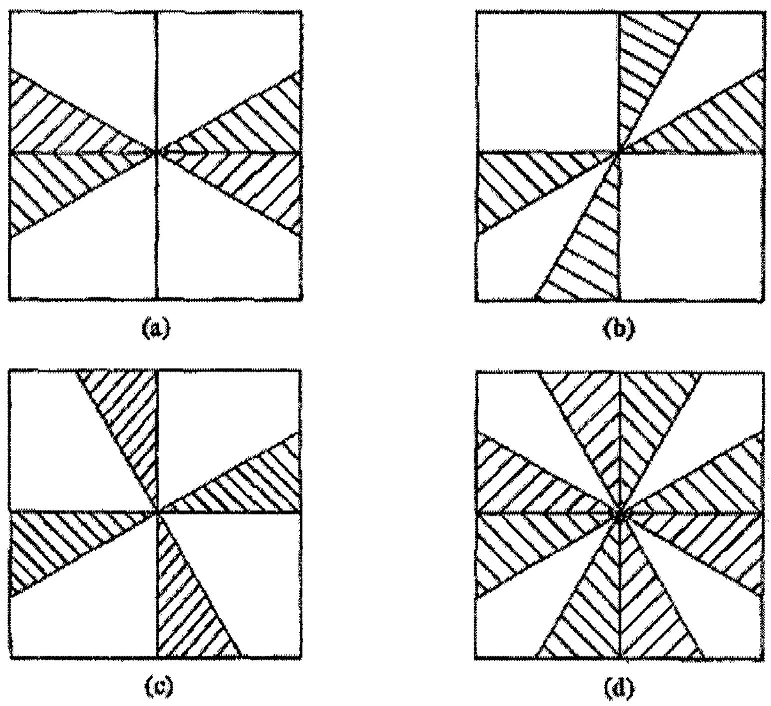

Figure 1.2: (a) Quadrantal symmetry, (b) Diagonal symmetry, (c) Four-fold rotational symmetry, (d) Octagonal symmetry. [37]

1.5.4 Quadrantal symmetry:

Quadrantal symmetry is common in 2-D recursive filter and is shown in Figure 1.2a. One can say that a magnitude response has this kind of symmetry if the system transfer function complies with the following condition

$$H(\omega_1,\omega_2) = H(-\omega_1,-\omega_2)=H(\omega_1,-\omega_2)=H(-\omega_1,\omega_2), \qquad \forall(\omega_1,\omega_2) \qquad (1.12)$$

In the designing of a 2-D recursive filter with this symmetry; the first step is to start with the magnitude response in the first quadrant. And then reflect the response in the second, third and the fourth quadrant using the symmetry techniques mentioned above.

1.5.5 Diagonal Symmetry

If the 2-D transfer function has reflection symmetry about $\omega_1=\omega_2$ and $\omega_1=-\omega_2$ diagonal with Centro symmetry, it becomes diagonal symmetry. The diagonal symmetry is shown in Figure 1.2b. The necessary condition to reflect this kind of symmetry is when the transfer function satisfies the below condition

$$H(\omega_1,\omega_2) = H(-\omega_1,-\omega_2)=H(\omega_2,-\omega_1)=H(-\omega_2,\omega_1), \qquad \forall(\omega_1,\omega_2) \qquad (1.13)$$

1.5.6 Four-fold rotational Symmetry

Figure 1.2c shows the four-fold $90°$ rotational symmetry. If the transfer function meets the following condition, then one can say "four-fold rotational symmetry" is present in the frequency response of 2-D recursive filter

$$H(\omega_1,\omega_2) = H(-\omega_1,-\omega_2)=H(-\omega_2,\omega_1)=H(\omega_2,-\omega_1), \qquad \forall(\omega_1,\omega_2) \qquad (1.14)$$

We can also say that Quadrantal symmetry is four-fold reflection symmetry about ω_1 and ω_2 axes, and Diagonal symmetry is four-fold reflection symmetry about the $\pm 45°$ lines in (ω_1, ω_2) plane.

1.5.7 Octagonal Symmetry

The characteristic of octagonal symmetry is shown in Figure1.2d. Octagonal symmetry possesses both quadrantal and diagonal symmetries. And, from whatever we discussed so far, one can say that octagonal symmetry also includes four-fold rotational symmetry. The necessary condition for appearance of octagonal symmetry in the magnitude response is on the next page

$$H(\omega_1,\omega_2) = H(-\omega_1,-\omega_2) = H(-\omega_2,-\omega_1) = H(\omega_2,\omega_1) = H(\omega_1,-\omega_2)$$
$$= H(-\omega_1,\omega_2) = H(-\omega_2,\omega_1) = H(\omega_2,-\omega_1), \qquad \forall(\omega_1,\omega_2) \qquad (1.15)$$

1.5.8 Circular Symmetry

In many applications of the 2-D recursive filter, e.g. image processing, the signal does not have any specific spatial direction, hence the transfer function of such filters possess circular symmetric frequency response characteristic. A four-fold rotational symmetry is an example of the 4th order circular symmetry.

1.6 Objective of this research

In this research, we will concentrate on a new method of generating 2-D stable recursive filter with variable magnitude response in the frequency domain. The two main objectives are:

- Design a stable 2-D digital filter whose transfer function has one or more changeable coefficients.
- Investigate how those coefficients change the filter type of and the frequency magnitude response of the filter.

Previous methods of designing 2-D recursive filters have many problems discussed in section 1.2. 2-D recursive filter has a widespread application and many real time applications need the flexibility in the frequency domain. As a result, researchers are still motivated to explore new methods to design 2-D recursive filter with more flexibility and variable characteristics. For this purpose, we use the generalized bilinear approach to develop the digital filter. Stability is one of the main problems in the recursive filter designing; hence we also study the stability conditions for the developed 2-D recursive filter.

1.7 Organization of the following chapters

Chapter 2 lists the previous studies done in designing of 2-D recursive filters, stability criteria, different methods of generating VSHPs and different kinds of symmetries in the magnitude response of the filters.

Chapter 3 first explains a method to get a two-variable analog transfer function using one of the methods of generating VSHPs. It also discusses how we get the 2-D digital transfer function with variable magnitude characteristics after applying the generalized bilinear transformation to the analog transfer function

In Chapter 4 using the 2-D digital transfer function derived in Chapter 3, we design a 2-D recursive low-pass filter with variable magnitude response. We also derive the stability conditions for this kind of filter. The manner in which each coefficient affects the magnitude response of the filter is studied in detail and simulation results are given.

Chapter 5 shows a designing method of a 2-D high-pass recursive filter with a flexible frequency domain response using the same 2-D digital transfer function derived from Chapter 3. The variable coefficients in the transfer function of the filter are constrained by the stability condition introduced in Chapter 3. The effect of each coefficient on the resulting 2-D high-pass digital filter's magnitude response is investigated in depth.

Chapter 6 shows the procedure of designing 2-D band-pass filter by combining double generalized bilinear transformation for LPF and HPF with stability criteria. We also determine the effects of some of the coefficients of this filter on the frequency response.

Chapter 7 reviews the study conducted in this research with our contribution and suggests what can be done as further study.

2. LITERATURE REVIEW

In this chapter, we review the literature in the field of two-dimensional recursive filter with special concentration in designing via bilinear transformation. And we also study some papers presented on some approaches of generating a 2-variable Very Strict Hurwitz Polynomial (VHSP) and its application.

Extensively used applications, which required use of two-dimensional signal processing, increased the demand for designing two-dimensional filters, and it compelled researchers to switch from one-dimensional filters to two-dimensional filters. In depth studies have been done in the field of designing two-dimensional filter since the early1970s.

Similarly for 1-D filters, scientists started studying on both recursive and nonrecursive two-dimensional filters. Hall [10] did the comparison between these two kinds of filters for calculations for spatial filtering. He suggested that recursive filters are best fitted in the spatial data applications, because of their reduced computations complexity over that of nonrecursive filters, e.g. fewer arithmetic operations, smaller memory requirements, etc. as well as, the recursive structure seems more suitable with the realization of small-scale hardware.

One of the prime issues with the 2-D recursive filter is the stability. Hence, much progress has been done in developing stability criterions in mid 70s. Shanks, Treitel and Justice [6] gave the general theorem for stability of multidimensional filter with examples. They have shown that two-dimensional recursive filter would be stable only if the denominator array of the filter has minimum phase[4] characteristic. They had also demonstrated how to make a non-minimum phase array stable with examples. Because Shanks' procedure is not finite, Huang [7] came out with an easier version of the stability theorem with a finite procedure, which requires bilinear transformation.

Anderson and Jury [8] developed a new stability technique for 2-D recursive filter, which is almost same as Huang's technique [7]. They replaced Hurwitz testing idea of Huang with the Schur-Cohn and related theory. Maria and Fahmy [9] presented an alternative stability test, which requires less computation with the case of Huang [7] because of the lower order of the determinants. Ramachandran and Ahmadi [41, 42] have also shown some stability tests, which can be performed using mirror-image and anti-mirror image polynomials.

Costa and Venetsanopoulos [11] discussed some basic concepts of 2-D recursive filter and used Shanks' method [6] to develop rotated filters[5] with stability conditions. They have shown few examples of designing 2-D recursive filters with a chosen cutoff frequency via circular symmetry. Chakrabati and Mitra [12] have shown two different

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[4] An array H is two-dimensionally minimum phase if and only if the polynomial $H(z_1, z_2)=0$ has no zeroes for which $|z_1| \leq 1$ and $|z_2| \leq 1$ simultaneously.

[5] Rotated filter is the one which is designed by mapping one-dimension into two-dimension with based on desirable 2-D frequency response.

design methods for 2-D digital filters, using spectral transformation. The first type is a rotation of 1-D filter into 2-D digital filter and second type shows the designing of 2-D digital filters using 2-D transfer functions. Ali [13] used the type I-A design approach of Chakrabati and Mitra [12] and designed 2-D recursive filter from 1-D analog filter with examples.

Maria and Fahmy [14] proposed a technique for designing a stable 2-D recursive filter via an optimization algorithm, which uses "minimize the p-error criterion"[6] for stability, with many solved examples. Bednar [15] used rational Chebyshev approximation which minimizes the difference between the desired function $F(z_1, z_2)$ and the rational function $A(z_1, z_2)/B(z_1, z_2)$, where $A(z_1, z_2)$, $B(z_1, z_2)$ are two-dimensional polynomials such that $1/B(z_1, z_2)$ is stable.

Ramamoorthy and Bruton [20] have suggested a general approach to design a 2-D recursive filter by applying a bilinear transformation to a stable 2-D analog filter. And they have also shown that we can design a stable 2-D analog filter using a 2-variable Strictly Hurwitz Polynomial (SHP). Prasad and Reddy [21] and Ahmadi et al. [22] have followed the same technique shown by Ramamoorthy et al [20] and designed 2-D recursive filters by applying double bilinear transformations to a stable 2-D analog filter.

Goodman [23] has shown that applying the double bilinear transformation to the stable 2-D analog filter will not always guarantee the stability of the designed filter, because of the presence of non-essential singularity of the second kind. He states that if the numerator of the analog filter transfer function has higher degree than the denominator then, the transfer function does not possess any kind of singularity and vice-

[6] Minimizing p-error criterion ensures that the chosen distance function ρ is the best approximation or the chosen approximation is the best fitted for the real-valued function, which gives minimum p-error.

versa. Bickart [24] gave two criteria to check the existence of the non-essential singularities of the second kind. As a result Rajan , Reddy et al [26] introduced a new kind of SHP which does not contain any kind of singularities, e.g. first kind or second kind, namely VSHP and showed that a 2-D analog filter always will be stable if it is implemented by a VSHP. And this 2-D analog filter will generate a guaranteed stable 2-D recursive filter via double bilinear transformation.

Ramachandran et al. [28-31] designed many stable 2-D recursive filters by generating VSHP using different methods. They showed that it is possible to design different kind of 2-D recursive filters by assuming value for some variables in the 2-D digital transfer funtion. Using the properties of 2-variable frequency independent n-port networks and the properties of the determinants of the immittance matrix, Ramachandran and Ahmadi [28] came up with a new method of designing a VSHP, which is called VSHP generated via terminated n-port gyrator networks. Using this VSHP, they designed a stable 2-D digital filter. The number of conditions shall be equal to the number of coefficients in the denominator polynomial.

Ramachandran and Ahmadi [29] designed another VSHP using the properties of the derivative of Even and Odd parts of Hurwitz Polynomials and designed a stable 2-D discrete filter using the VSHP. This approach does not necessitate any constraint optimization method and can be extended to multidimensional. Abiri, Ramachandran and Ahmadi [31] presented how to generate a VSHP using the matrix theory. They have also showed that it is possible to design a stable 2-D digital filter with desired magnitude response using this technique. This method is also expandable to multidimensional digital filters. This is one of the important researches for this thesis because we use the same matrix theory to generate a VSHP.

Till late 90's in the field of designing two-dimensional recursive filter, most studies were done in a very few specific manner to design a 2-D recursive filter without any flexibility in the frequency response. And this gave motivation to design a more simplified and flexible system.

Mitra, Neuvo and Roivainen [17] developed a quite effective method of designing 2-D recursive filters with desirable tunable characteristics, e.g. cutoff frequencies and central frequency, via single variable coefficient in the transfer function. And for frequency transformation, they used the Taylor series expansion theory. The simulation results show that this method is very efficient for elliptical filters. Gargour and Ramachandran [18] designed variable magnitude 2-D recursive filters without having delay-free loops. To keep the proposed design stable, the variable coefficients in the design are bounded in certain limits. They also noticed different kind of symmetries in the simulation results, which depend on the value and the sign of the variable coefficient of the system.

Ramachandran and Gaugour [27] derived the various properties of VSHP and proved them carefully. They also showed how these properties could help in generating VSHPs. There are very wide possibilities of generating VSHPs. They have also illustrated how to design different kinds of stable 2-D digital filters using any of these generated VSHPs, and explained the process in detail with many examples.

Gaugour and Ramachandran et al. [25] introduced one more technique of designing a very promising stable 2-D recursive filter with variable characteristics. By adding a multiplier K as a changeable variable either in the feedback path or in the forward path to the stable 2-D digital filter, one can get the variable characteristics of the filter. Because the stability criterion determines the range of K, sometimes it is impossible to achieve desired frequency portion.

The research work of Gaugour and Ramachandran et al. [32] is important in this research, since we also use the same methodology to design stable 2-D recursive filter with variable characteristics, while they have designed 1-D IIR discrete filter via the use of the general bilinear transformation to a starting analog filter transfer function. They have also given a theorem to determine certain conditions for the stability purpose and ensured a stable discrete filter. The magnitude characteristics can be varied in a large number of ways because the coefficients in generalized bilinear transformation are inequalities, which create infinite number of possibilities. They proved this by giving a numerical example of a Butterworth filter. I can say that my study is the extension of this research work.

Gaugour and Ramachandran et al. [33] modified 1-D filter response by generalized bilinear transformation and the inverse bilinear transformation. In this paper they have shown the conditions in the designed transfer function, those will generate different form (e.g., low-pass, high-pass, band-pass and band-eliminate). In this paper they designed band-pass and band-eliminate filters by combining low-pass and high-pass filter properly.

3. An alternative method of designing 2-D RECURSIVE FILTER

As we have seen so far, it is very important to design 2-D recursive filter having various magnitude characteristics. And in this chapter we will design one of these kinds of filters (with variable magnitude and bandwidth response) by following the steps given below:

- Generate a VSHP of a second order by any one of the methods of generating it.
- Apply the generalized bilinear transformation to obtain the corresponding 2-D polynomial discrete transfer function. (keeping stability conditions showed in section 1.8 in mind)
- Assume value for some variables to design different kind of 2-D filters.

In section 3.1, using the previous approaches of generating VHSP and the properties of VSHPs, we generate one two-variable VSHP. In section 3.2 we propose a 2-D recursive transfer function by applying the generalized bilinear transformation to the generated VSHP. In section 3.3, we determine the stability conditions for this designed 2-D discrete transfer function in terms of the range for those variable coefficients. In the next two chapters, we show how to design different forms of the 2-D recursive filter by changing the values of some variables.

3.1 Generation of a second order VSHP

Using the previous approaches of generating VHSP and the properties of VSHPs, we generate this kind of two-variable Polynomials. For this, we can start with the generation of a 2- or n- variables Strict Hurwitz Polynomial (SHP). We know that it is always possible to realize an even or an odd part of a SHP via the input immittance (impedance or admittance) of a k-variable physically realizable network and degree of the SHP depends on the corresponding number of variables. We discuss different possibilities of generating a VSHP below:

(a) In [29] it has been shown that the network of n-port gyrator results in n-variable reactances, each of degree unity. And the determinant of the immittance matrix yields in an even or an odd part of an n-variable Hurwitz polynomial. By discreet choices of some n-variables as positive real constants in this generated Hurwitz Polynomial, we can generate VSHPs. Using this method we can generate a VSHP with very few computational efforts.

(b) In [28], instead of playing with some constants, by the addition of this determinant of the terminated n-port gyrator to its derivatives with respect to the n-variables, we can generate a SHP. And using the different properties of VSHP, we can easily produce a VHSP from this developed SHP, which opens many directions

(c) It is well known that a positive definite or positive semi definite matrix is physically realizable and we can decompose this matrix A in the form of

$$A = B \cdot B^t \tag{3.1}$$

Where B is either an upper or lower triangular matrix and B′ is the transpose of B [31]. Using this decomposition technique and matrix theory we can write,

$$D = A\Psi A'.S_1 + B\Lambda B'.S_2 + R\Gamma R' + G \tag{3.2}$$

In equation 3.2, A, B and R are lower triangular matrices given by

$$A = \begin{bmatrix} a_{11} & 0 & 0 & . & . & . & . & 0 \\ a_{12} & a_{22} & 0 & . & . & . & . & 0 \\ a_{13} & a_{23} & a_{33} & . & . & . & . & 0 \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ a_{1n} & a_{2n} & a_{3n} & . & . & . & . & a_{nn} \end{bmatrix} \tag{3.3}$$

$$B = \begin{bmatrix} b_{11} & 0 & 0 & . & . & . & . & 0 \\ b_{12} & b_{22} & 0 & . & . & . & . & 0 \\ b_{13} & b_{23} & b_{33} & . & . & . & . & 0 \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ b_{1n} & b_{2n} & b_{3n} & . & . & . & . & b_{nn} \end{bmatrix} \tag{3.4}$$

$$R = \begin{bmatrix} r_{11} & 0 & 0 & . & . & . & . & 0 \\ r_{12} & r_{22} & 0 & . & . & . & . & 0 \\ r_{13} & r_{23} & r_{33} & . & . & . & . & 0 \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ r_{1n} & r_{2n} & r_{3n} & . & . & . & . & r_{nn} \end{bmatrix} \tag{3.5}$$

And Ψ, Γ and Λ are diagonal matrices given by

$$\Psi = \begin{bmatrix} \psi_1 & & & & 0 \\ & \psi_2 & & & \\ & & . & & \\ & & & . & \\ 0 & & & & \psi_n \end{bmatrix} \tag{3.6}$$

$$\Lambda = \begin{bmatrix} \lambda_1 & & & & 0 \\ & \lambda_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ 0 & & & & \lambda_n \end{bmatrix} \tag{3.7}$$

$$\Gamma = \begin{bmatrix} \gamma_1 & & & & 0 \\ & \gamma_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ 0 & & & & \gamma_n \end{bmatrix} \tag{3.8}$$

As well as, G is a skew-symmetric matrix given by

$$G = \begin{bmatrix} 0 & g_{12} & g_{13} & \cdot & \cdot & \cdot & g_{1n} \\ -g_{12} & 0 & g_{23} & \cdot & \cdot & \cdot & g_{2n} \\ -g_{13} & -g_{23} & 0 & \cdot & \cdot & \cdot & g_{3n} \\ -g_{14} & -g_{24} & -g_{34} & \cdot & \cdot & \cdot & g_{4n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ -g_{1n} & -g_{2n} & -g_{3n} & \cdot & \cdot & \cdot & 0 \end{bmatrix} \tag{3.9}$$

Here we use transpose of the matrices A, B and R; hence it does not make any difference if they are upper-triangular matrixes instead of lower-triangular matrixes. If all ψ_i's, λ_i's and γ_i's are positive, A, B and R are the positive-definite matrices, and they are physically realizable. If we make some of the ψ_i's, λ_i's and γ_i's to zero; A, B and R become positive semi-definite matrices respectively; hence it will not affect their physical reliability. If Γ is a null matrix, det D_n (determinant of D_n) becomes a strictly even or strictly odd polynomial depending on whether n is even or odd. By the use of derivatives, we can generate VSHPs from this even or odd HP. To generate second order VSHP, we have taken n=2 in above equation 3.2

For n=2, we can rewrite equation 3.2 as below

$$D_2 = \begin{matrix} \begin{bmatrix} a_{11} & 0 \\ a_{12} & a_{22} \end{bmatrix} \begin{bmatrix} \psi_1 & 0 \\ 0 & \psi_2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix} S_1 + \begin{bmatrix} b_{11} & 0 \\ b_{12} & b_{22} \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ 0 & b_{22} \end{bmatrix} S_2 + \\ \begin{bmatrix} c_{11} & 0 \\ c_{12} & c_{22} \end{bmatrix} \begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix} + \begin{bmatrix} 0 & g_{12} \\ -g_{12} & 0 \end{bmatrix} \end{matrix} \tag{3.10}$$

In above equation for simplicity we put $a_{11} = b_{11} = c_{11} = 1$; $a_{22} = b_{22} = c_{22} = 0$; $a_{12} = a$;

$b_{12} = b$; $c_{12} = c$; $g_{12} = g$ then we can rewrite it as below

$$D_2 = \begin{matrix} \begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix} \begin{bmatrix} \psi_1 & 0 \\ 0 & \psi_2 \end{bmatrix} \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} S_1 + \begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} S_2 + \\ \begin{bmatrix} 1 & 0 \\ c & 1 \end{bmatrix} \begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix} \begin{bmatrix} 1 & c \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} \end{matrix} \tag{311}$$

The above equation will result in S_1^2 term and we do not want this term in the final equation, so in order to get rid of it, we should make $\psi_2 = 0$. If ψ_1 is made equal to zero, the quantity 'a' will disappear in the determinant. Similarly, the above equation will also result in S_2^2 term and we neither want this term in the final equation. So in order to get rid of it, we should make $\lambda_2 = 0$. If λ_1 is made equal to zero, the quantity 'b' will disappear in the determinant. After substituting $\psi_2 = 0$ and $\lambda_2 = 0$, we can write D_2 as

$$D_2 = \begin{bmatrix} \psi_1 S_1 + \lambda_1 S_2 + \gamma_1 & a\psi_1 S_1 + b\lambda_1 S_2 + c\gamma_1 + g \\ a\psi_1 S_1 + b\lambda_1 S_2 + c\gamma_1 - g & a^2\psi_1 S_1 + b^2\lambda_1 S_2 + c^2\gamma_1 + \gamma_2 \end{bmatrix} \tag{312}$$

And from this, we can always write $|D_2|$ or determinant of D_2, as

$$\det D_2 = \psi_1\lambda_1(a-b)^2 S_1 S_2 + \psi_1\{\gamma_1(a-c)^2 + \gamma_2\}S_1 + \lambda_1\{\gamma_1(b-c)^2 + \gamma_2\}S_2 + (\gamma_1\gamma_2 + g^2), \tag{3.13}$$

After carefully reading the above equation and using the properties of VSHP, we can say that if $a \neq b$ and $\gamma_2 \neq 0$ then it can be a VSHP.

As we have discussed above, if we make Γ matrix null means in our case $\gamma_1 = \gamma_2 = 0$, we get an even polynomial in S_1 and S_2, which is:

$$M_a = \psi_1 \lambda_1 (a-b)^2 S_1 S_2 + g^2, \tag{3.14}$$

assuming $a \neq b$

And using this even polynomial we can easily obtain one VSHP by

$$D(S_1, S_2) = M_a + Y_1 \frac{\partial M_a}{\partial S_1} + Y_2 \frac{\partial M_a}{\partial S_2} \tag{3.15}$$

Where, Y_1 and Y_2 are constants.

$$\therefore\ D(S_1, S_2) = \psi_1 \lambda_1 (a-b)^2 S_1 S_2 + Y_1 \psi_1 \lambda_1 (a-b)^2 S_2 + Y_2 \psi_1 \lambda_1 (a-b)^2 S_1 + g^2 \tag{3.16}$$

which is a VSHP, if $a \neq b$

This treatment permits us to introduce four extra variables and these can be varied in order to change the magnitude characteristics. In this section, we have generated two VSHPs (equation 3.13 and 3.16), and for our future purpose we will consider the VSHP given in equation 3.16.

3.2 Designing of a 2-D recursive TF by applying the generalized bilinear transformation

If we consider a VSHP shown in equation 3.16 then,

$$\begin{aligned} D(S_1, S_2) &= \psi_1 \lambda_1 (a-b)^2 S_1 S_2 + Y_1 \psi_1 \lambda_1 (a-b)^2 S_2 + Y_2 \psi_1 \lambda_1 (a-b)^2 S_1 + g^2 \\ &= \psi_1 \lambda_1 (a-b)^2 \{S_1 S_2 + Y_2 S_1 + Y_1 S_2\} + g^2 \end{aligned} \tag{3.17}$$

By applying generalized bilinear transformation in above equation for S_1 and S_2 then, $S_1 = K_1\left(\dfrac{z_1 + a_{01}}{z_1 + b_{01}}\right)$ and $S_2 = K_2\left(\dfrac{z_2 + a_{02}}{z_2 + b_{02}}\right)$. And as discussed in 1.4 to make the system stable, we will follow the stability conditions in this case, which are:

$$\begin{aligned} &|\, a_{01} \,|\le 1; |\, b_{01} \,|\le 1; \\ &|\, a_{01} \,|\le 1; |\, b_{02} \,|\le 1; \\ &a_{01}b_{01} < 0; a_{02}b_{02} < 0; \\ &K_1 > 0; K_2 > 0 \end{aligned} \tag{3.18}$$

Now, by substituting the value of S_1 and S_2 in equation 3.17 then,

$$D(z_1, z_2) = \psi_1\lambda_1(a-b)^2 \begin{Bmatrix} K_1K_2\left(\dfrac{z_1 + a_{01}}{z_1 + b_{01}}\right)\left(\dfrac{z_2 + a_{02}}{z_2 + b_{02}}\right) + Y_2K_1\left(\dfrac{z_1 + a_{01}}{z_1 + b_{01}}\right) + \\ Y_1K_2\left(\dfrac{z_2 + a_{02}}{z_2 + b_{02}}\right) \end{Bmatrix} + g^2; \tag{3.19}$$

$$\because D(z_1, z_2) = \frac{\begin{Bmatrix} \psi_1\lambda_1(a-b)^2 \begin{bmatrix} K_1K_2[(z_1 + a_{01})(z_2 + a_{02})] + Y_2K_1\{(z_1 + a_{01})(z_2 + b_{02})\} \\ + Y_1K_2\{(z_2 + a_{02})(z_1 + b_{01})\} \end{bmatrix} + \\ g^2[(z_1 + b_{01})(z_2 + b_{02})] \end{Bmatrix}}{\{(z_1 + b_{01})(z_2 + b_{02})\}} \tag{3.20}$$

If we more simplify the equation then we will have

$$D(z_1, z_2) = \frac{\begin{Bmatrix} \psi_1\lambda_1(a-b)^2 \begin{bmatrix} K_1K_2[z_1z_2 + a_{02}z_1 + a_{01}z_2 + a_{01}a_{02}] + \\ Y_2K_1\{z_1z_2 + b_{02}z_1 + a_{01}z_2 + a_{01}b_{02}\} + \\ Y_1K_2\{z_1z_2 + a_{02}z_1 + b_{01}z_2 + a_{02}b_{01}\} \end{bmatrix} + \\ g^2[z_1z_2 + b_{02}z_1 + b_{01}z_2 + b_{01}b_{02}] \end{Bmatrix}}{\{z_1z_2 + b_{02}z_1 + b_{01}z_2 + b_{01}b_{02}\}} \tag{3.21}$$

and for the stability purpose we have to keep $a \neq b$ in above equation, we can also rewrite the above equation as below,

$$D(z_1, z_2) = \frac{\begin{array}{l} \left\{\psi_1\lambda_1(a-b)^2(K_1K_2 + Y_2K_1 + Y_1K_2) + g^2\right\}z_1z_2 + \\ \left\{\psi_1\lambda_1(a-b)^2(a_{02}K_1K_2 + b_{02}Y_2K_1 + a_{02}Y_1K_2) + b_{02}g^2\right\}z_1 + \\ \left\{\psi_1\lambda_1(a-b)^2(a_{01}K_1K_2 + a_{01}Y_2K_1 + b_{01}Y_1K_2) + b_{01}g^2\right\}z_2 + \\ \left\{\psi_1\lambda_1(a-b)^2(a_{01}a_{02}K_1K_2 + a_{01}b_{02}Y_2K_1 + a_{02}b_{01}Y_1K_2) + b_{01}b_{02}g^2\right\} \end{array}}{\left\{z_1z_2 + b_{02}z_1 + b_{01}z_2 + b_{01}b_{02}\right\}} \tag{3.22}$$

By applying this $D(z_1, z_2)$ to denominator of a unity numerator, we can realize a 2-D recursive filter. And the transfer function $H(z_1, z_2)$ can given as below,

$$H(z_1, z_2) = \frac{A(z_1, z_2)}{B(z_1, z_2)} = \frac{z_1z_2 + b_{02}z_1 + b_{01}z_2 + b_{01}b_{02}}{Q_{11}z_1z_2 + Q_{10}z_1 + Q_{01}z_2 + Q_{00}} \tag{3.23}$$

where, $Q_{11} = \psi_1\lambda_1(a-b)^2(K_1K_2 + Y_2K_1 + Y_1K_2) + g^2$; (3.23a)

$Q_{10} = \psi_1\lambda_1(a-b)^2(a_{02}K_1K_2 + b_{02}Y_2K_1 + a_{02}Y_1K_2) + b_{02}g^2$; (3.23b)

$Q_{01} = \psi_1\lambda_1(a-b)^2(a_{01}K_1K_2 + a_{01}Y_2K_1 + b_{01}Y_1K_2) + b_{01}g^2$; (3.23c)

$Q_{00} = \psi_1\lambda_1(a-b)^2(a_{01}a_{02}K_1K_2 + a_{01}b_{02}Y_2K_1 + a_{02}b_{01}Y_1K_2) + b_{01}b_{02}g^2$; (3.23d)

$\psi_1, \lambda_1 > 0; a \neq b$; (3.23e)

$$\left.\begin{array}{l} |a_{01}| \leq 1; |b_{01}| \leq 1; \\ |a_{01}| \leq 1; |b_{02}| \leq 1; \\ a_{01}b_{01} < 0; a_{02}b_{02} < 0; \\ K_1 > 0; K_2 > 0 \end{array}\right. \tag{3.23f}$$

By introducing the generalized bilinear transformations, one introduces more number of variables in the transfer function to change the magnitude characteristics.

3.3 Stability conditions of the designed 2-D digital filter

Stability is always a prime concern in designing 2-D digital filters. Intensive study has been done on the stability criteria of the 2-D filters. And using the properties of

VSHP we can say that, $D(S_1, S_2)$ will be Very Strict Hurwitz Polynomial only and only if all coefficients of this polynomial are positive.

Our designed transfer function is in (z_1, z_2)domain and to convert it to (S_1, S_2)domain we have to apply generalized inverse bilinear transformation to equation 3.23. Gargour et al [33], has proposed inverse bilinear transformation and using it we can define generalized inverse bilinear transformation as below.

$$z_i \rightarrow \frac{b_{oi} S_i - a_{oi} K_i}{K_i - S_i}, \quad \mathrm{i} = 1, 2 \tag{3.24}$$

After substituting this value of z1 and z2 from equation 3.24 into the denominator of equation 3.23, we can obtain the designed transfer function in the (S_1, S_2)domain and this can be expressed as

$$B(S_1, S_2) = \begin{bmatrix} (Q_{11} b_{01} b_{02} - Q_{10} b_{01} - Q_{01} b_{02} + Q_{00}) S_1 S_2 + \\ (-Q_{11} b_{01} a_{02} K_2 + Q_{10} b_{01} K_2 + Q_{01} a_{02} K_2 - Q_{00} K_2) S_1 + \\ (-Q_{11} b_{02} a_{01} K_1 + Q_{10} a_{01} K_1 + Q_{01} b_{02} K_1 - Q_{00} K_1) S_2 + \\ (Q_{11} a_{01} a_{02} K_1 K_2 - Q_{10} a_{01} K_1 K_2 - Q_{01} a_{02} K_1 K_2 + Q_{00} K_1 K_2) \end{bmatrix} \tag{3.25}$$

where, Q_{11}, Q_{10}, Q_{01} and Q_{00} are given in equation 3.23.

Now, as we have already discussed all the coefficient of the above equation should be positive for stability purpose and this guides us to

$$Q_{11} b_{01} b_{02} - Q_{10} b_{01} - Q_{01} b_{02} + Q_{00} > 0 \tag{3.26a}$$
$$-Q_{11} b_{01} a_{02} K_2 + Q_{10} b_{01} K_2 + Q_{01} a_{02} K_2 - Q_{00} K_2 > 0 \tag{3.26b}$$
$$-Q_{11} b_{02} a_{01} K_1 + Q_{10} a_{01} K_1 + Q_{01} b_{02} K_1 - Q_{00} K_1 > 0 \tag{3.26c}$$
$$Q_{11} a_{01} a_{02} K_1 K_2 - Q_{10} a_{01} K_1 K_2 - Q_{01} a_{02} K_1 K_2 + Q_{00} K_1 K_2 > 0 \tag{3.26d}$$

where, Q_{11}, Q_{10}, Q_{01} and Q_{00} are given in equation 3.23.

But the generalized bilinear transformation has already conditions of $K_1 > 0; K_2 > 0$ for stable result and using this fact we can reduce above equation to

$$Q_{11}b_{01}b_{02} - Q_{10}b_{01} - Q_{01}b_{02} + Q_{00} > 0 \tag{3.27a}$$
$$Q_{11}b_{01}a_{02} - Q_{10}b_{01} - Q_{01}a_{02} + Q_{00} > 0 \tag{3.27b}$$
$$Q_{11}b_{02}a_{01} - Q_{10}a_{01} - Q_{01}b_{02} + Q_{00} > 0 \tag{3.27c}$$
$$Q_{11}a_{01}a_{02} - Q_{10}a_{01} - Q_{01}a_{02} + Q_{00} > 0 \tag{3.27d}$$

And using the above relations we can find the stability condition on $\psi_1, \lambda_1, Y_1, Y_2$ and g.

Finally we can rewrite the transfer function of the designed 2-D recursive filter with all stability condition as below

$$H(z_1, z_2) = \frac{A(z_1, z_2)}{B(z_1, z_2)} = \frac{z_1 z_2 + b_{02} z_1 + b_{01} z_2 + b_{01} b_{02}}{Q_{11} z_1 z_2 + Q_{10} z_1 + Q_{01} z_2 + Q_{00}} \tag{3.28}$$

where, $Q_{11} = \psi_1 \lambda_1 (a-b)^2 (K_1 K_2 + Y_2 K_1 + Y_1 K_2) + g^2;$ (3.28a)

$$Q_{10} = \psi_1 \lambda_1 (a-b)^2 (a_{02} K_1 K_2 + b_{02} Y_2 K_1 + a_{02} Y_1 K_2) + b_{02} g^2; \tag{3.28b}$$
$$Q_{01} = \psi_1 \lambda_1 (a-b)^2 (a_{01} K_1 K_2 + a_{01} Y_2 K_1 + b_{01} Y_1 K_2) + b_{01} g^2; \tag{3.28c}$$
$$Q_{00} = \psi_1 \lambda_1 (a-b)^2 (a_{01} a_{02} K_1 K_2 + a_{01} b_{02} Y_2 K_1 + a_{02} b_{01} Y_1 K_2) + b_{01} b_{02} g^2; \tag{3.28d}$$
$$a \neq b; \tag{3.28e}$$
$$\left.\begin{array}{l} |a_{01}| \leq 1; |b_{01}| \leq 1; \\ |a_{01}| \leq 1; |b_{02}| \leq 1; \\ a_{01} b_{01} < 0; a_{02} b_{02} < 0; \\ K_1 > 0; K_2 > 0 \end{array}\right. \tag{3.28f}$$
$$Q_{11}b_{01}b_{02} - Q_{10}b_{01} - Q_{01}b_{02} + Q_{00} > 0 \tag{3.28g}$$
$$Q_{11}b_{01}a_{02} - Q_{10}b_{01} - Q_{01}a_{02} + Q_{00} > 0 \tag{3.28h}$$
$$Q_{11}b_{02}a_{01} - Q_{10}a_{01} - Q_{01}b_{02} + Q_{00} > 0 \tag{3.28i}$$
$$Q_{11}a_{01}a_{02} - Q_{10}a_{01} - Q_{01}a_{02} + Q_{00} > 0 \tag{3.28j}$$

3.4 Summary

In this chapter, we have shown one of the procedures to obtain 2-D recursive transfer function with the stability conditions. Using the previous techniques of generating VSHP and the properties of VSHP, we have formed a 2-D VSHP. The 2-D discrete transfer function has been derived from the analog transfer function by applying double generalized bilinear transformation. And the stability conditions have been obtained for the designed 2-D discrete transfer function with a unity degree denominator. By changing the values of the bilinear coefficients and four extra variables (Y_1, Y_2, g and **A**=$\psi_1\lambda_1(a-b)^2$), we can change the type and the frequency response of the 2-D digital filter. In the next few chapters we design different kinds of filters, e.g. low-pass, high-pass, band-pass, and we study the effects of different coefficients on the filter responses.

Thus, in this Thesis, this chapter is an important work towards the study of 2-D recursive filters with flexible characteristics.

4. 2-D LOW-PASS RECURSIVE FITLER

Filters are generally classified in terms of the filter function they perform. The basic filter functions are Low-pass, High-pass, Band-pass and Band-stop, hence, there are four basic kinds of filter. In this chapter we design a 2-D recursive low-pass filter with some examples.

Section 4.1 gives definition of 2-D low pass filters with transfer function and the graphical definition. In section 4.2, we define the necessary conditions to designe 2-D low-pass filter from the general 2-D recursive transfer function, derived in the previous chapter. Section 4.3 focuses briefly on the effects of different coefficients of the transfer function on the designed 2-D low-pass recursive filter. Section 4.4 shows how we can design a 2-D low-pass recursive filter with desirable characteristics. And we summarize results in section 4.5

4.1 Definition of 2-D Low-pass filters

Low-pass filter rejects the unnecessary high frequency components and passes only lower frequency components. Graphically we can show low-pass filter as showed in figure 4.1, where R_1 is the pass-band, R_2 is transition-band and R_3 is the stop-band of a 2-D low-pass filter.

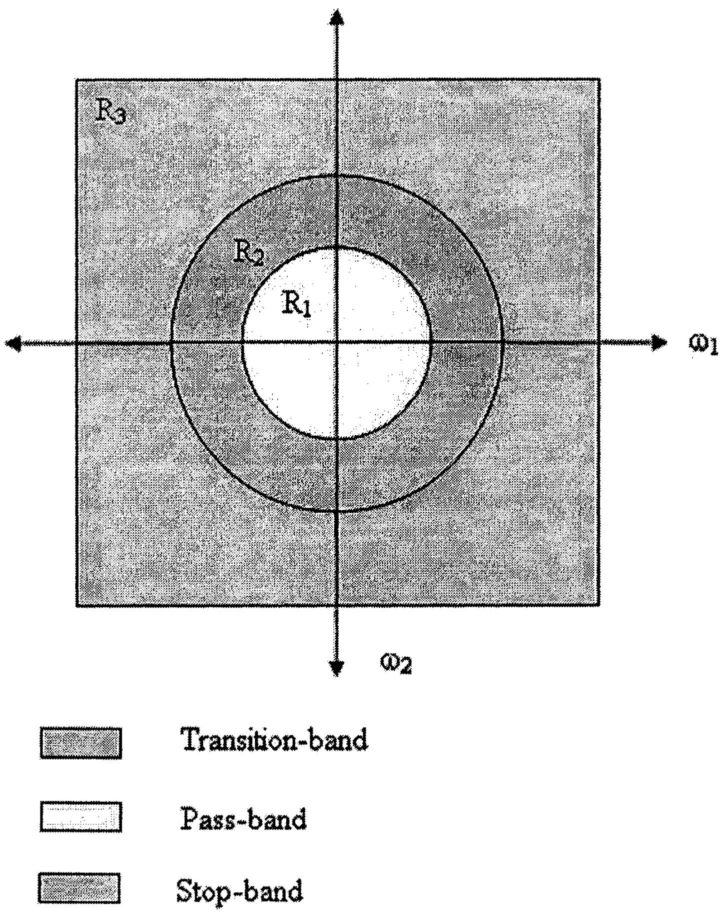


Figure 4.1: Graphical definition of a low-pass filter

Transfer function of a 2-D low-pass filter is given on the next page,

$$H(\omega_1, \omega_2) = 1 \qquad 0 \le \left(\omega_1^2 + \omega_2^2\right)^{\frac{1}{2}} \le \omega_{pi},\ \mathrm{i} = 1, 2 \qquad (4.1)$$

$$= 0 \qquad \left(\omega_1^2 + \omega_2^2\right)^{\frac{1}{2}} > \omega_{si},\ \mathrm{i} = 1, 2$$

Where : ω_{pi} is called pass-band radius

ω_{si} is called stop-band radius

And transition band is the region in-between ω_{pi} and ω_{si}

4.2 Defining the conditions for 2-D low-pass filter

In the previous chapter in section 3.2, we have already obtained transfer function of a 2-D recursive filter. But its very important to find out under what circumstances this designed transfer function will work as a 2-D low-pass recursive filter. In one of the published paper Gargour, Ramachandran et al. [32] have given conditions on the coefficients of the generalized bilinear transformation to design 1-D low-pass recursive filter form analog transfer function. They have shown that if we keep b_0 greater than or equal to zero then and then only the general transfer function will work as a low pass filter. As well as in Chapter 1, we mentioned that for guarantied stability, when K is greater than zero in the generalized bilinear transformation, a_0 must have the opposite sign than that of b_0. If we extend these conditions for two dimensions then we can define the conditions on the coefficients of the generalized bilinear transformation for low-pass response, which are as below for i=1, 2.

- $K_i > 0;$ (4.2a)
- $-1 \leq a_{0i} \leq 0;$ and $0 \geq b_{0i} \geq 1;$ (4.2b)

  where, $a_{0i}b_{0i} < 0$

We can get a 2-D low pass transfer function by applying these conditions on the designed transfer function showed in equation 3.28 of the previous chapter. In the next section we study how different coefficient of the 2-D low pass transfer function effects on the frequency response of the designed filter.

4.3 Frequency response of the designed 2-D low-pass recursive filter

To obtain the contour and 3-D magnitude response plots of the resulting 2-D low pass filter, we write a program for the designed transfer function of the low-pass filter using MATLAB ®. And after assigning value for each coefficient of the transfer function in the program "himthlp.m", we can get the contour and 3-D magnitude plots for the specified values.

There are so many coefficients to analyze in the designed low-pass transfer function. We divide these coefficients into two groups to make the scenario simple, coefficients introduced by the generalized bilinear transformation and the coefficient introduced in the transfer function while generating VSHP. We generated VSHP using matrix theory, so we call the second group as coefficients of matrix. While studying the effects of the coefficients of one group, we keep the coefficients of another group to some constant value for simplicity purpose.

4.3.1 2-D Low-pass recursive filter response with each different coefficient of the generalized bilinear transformation

In this section, we study how each different coefficient of the generalized bilinear transformation affects the designed filter response. In order to determine the effect of the coefficient, we keep changing the coefficient under study, while keeping the other coefficients to some constant value. Equation 4.2 shows the range of each coefficient of the generalized bilinear transformation for the low-pass filter. As well as, we also fix each coefficient of the matrix to some specific value. We have used, Y_1=0.9367, Y_2=0.7, g =2 and **A**=$\psi_1 \lambda_1 (a-b)^2$=6.1

Low Pass frequency response with different values of K_1

In order to separate the effects of the other coefficients beside K_1, we keep them to some constant value. Not to lose any generality, we have used $K_2 = b_{01} = b_{02} = 1$ and $a_{01} = a_{02} = -1$. We also have checked the range for this coefficient using MATLAB®, and it gave use stable result up to K_1=10,000, when we have fixed other generalized bilinear coefficients to unity with proper signs. Figure 4.2 shows the contour and the 3-D magnitude response plot for different values of K_1.

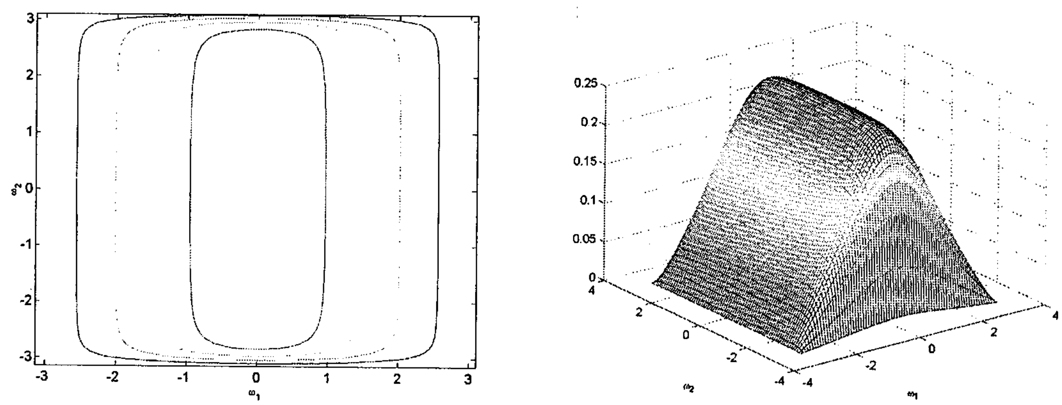


Figure 4.2(a): $K_1 = 0.1$

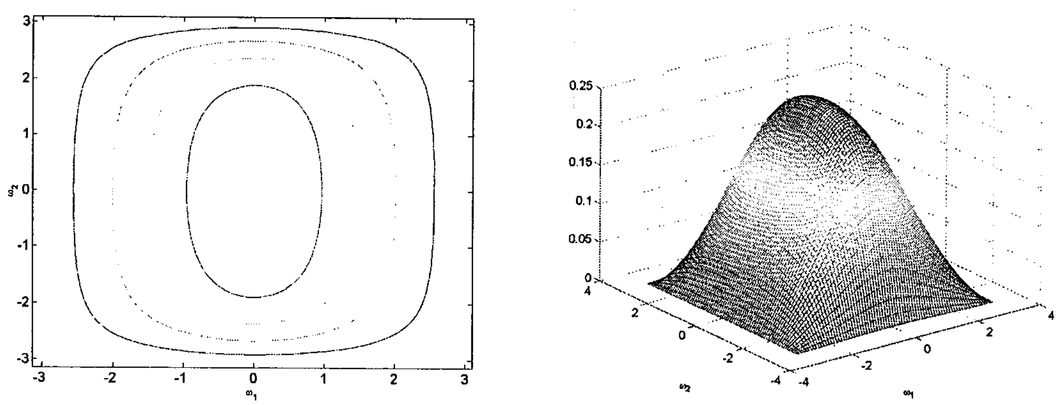


Figure 4.2(b): K_1=0.5

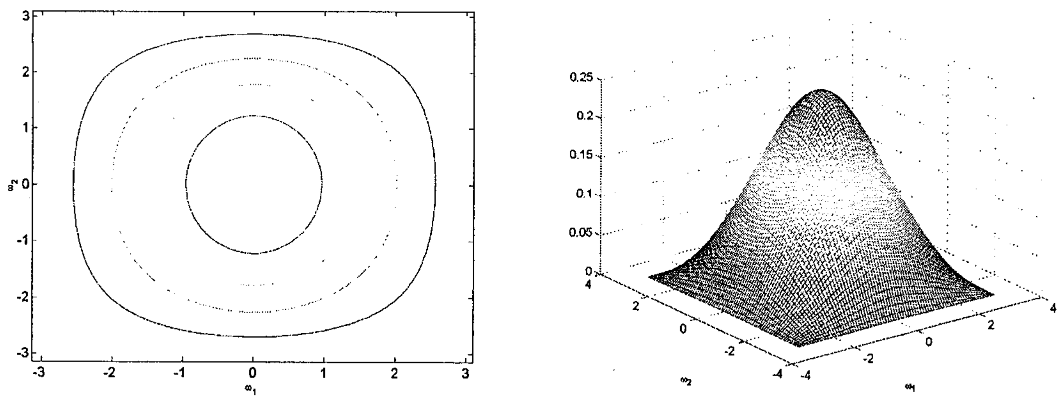


Figure 4.2(c): K_1 =1

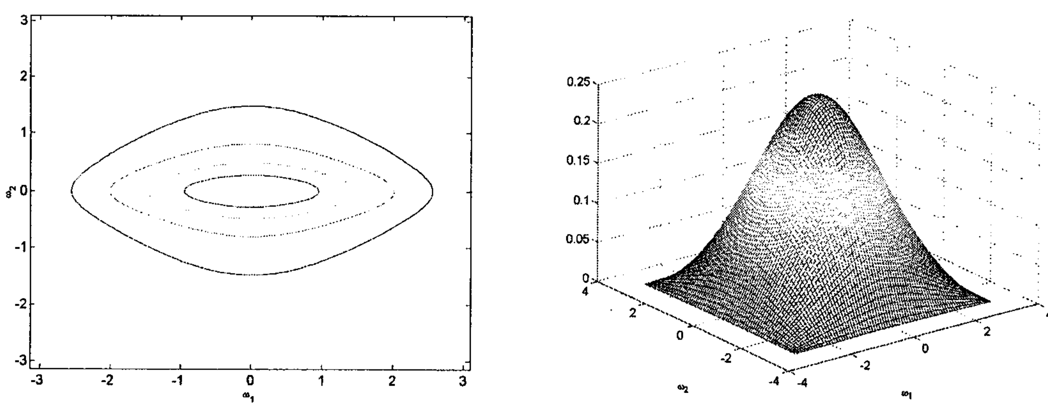


Figure 4.2(d): K_1 =5

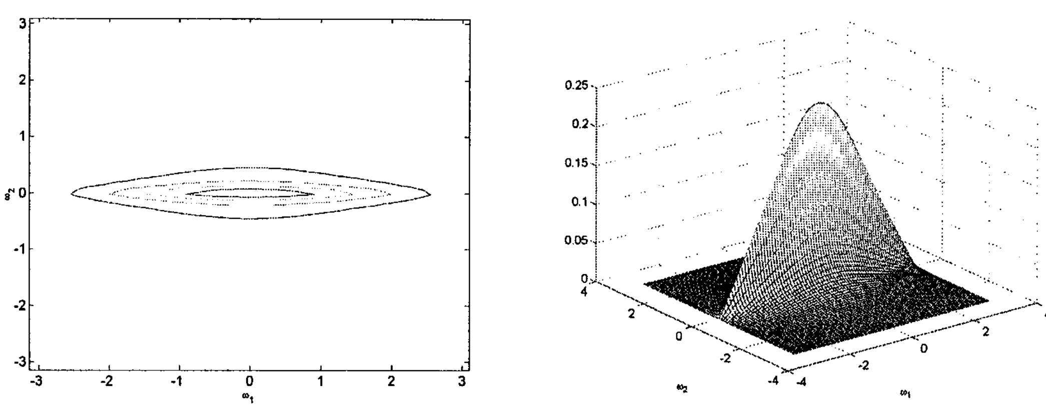


Figure 4.2(e): K_1 =20

Figure 4.2: The contour and 3-D magnitude plot of the resulting 2-D low pass filter for different values of K_1 with $K_2 = b_{01} = b_{02} = 1$ and $a_{01} = a_{02} = -1$.

This is clearly seen that the value of K_1 is inversely proportional with the bandwidth in ω_2 domain, and it does not make any difference in the bandwidth in ω_1 domain. As well as, it does not have any effects on the amplitude of the low-pass response as it remains constant approximately 0.17 for all values of K_1 .It is easily seen that the contour plot has Quadrantal symmetry for each case.

Low Pass frequency response with different values of K_2

K_2 also gives stable response up to 10,000 value, when other coefficients of the bilinear transfer function set to unity with proper signs, e.g. $K_1 = b_{01} = b_{02} = 1$ and $a_{01} = a_{02} = -1$. 3-D magnitude response and contour plot for K_2=1 are already showed

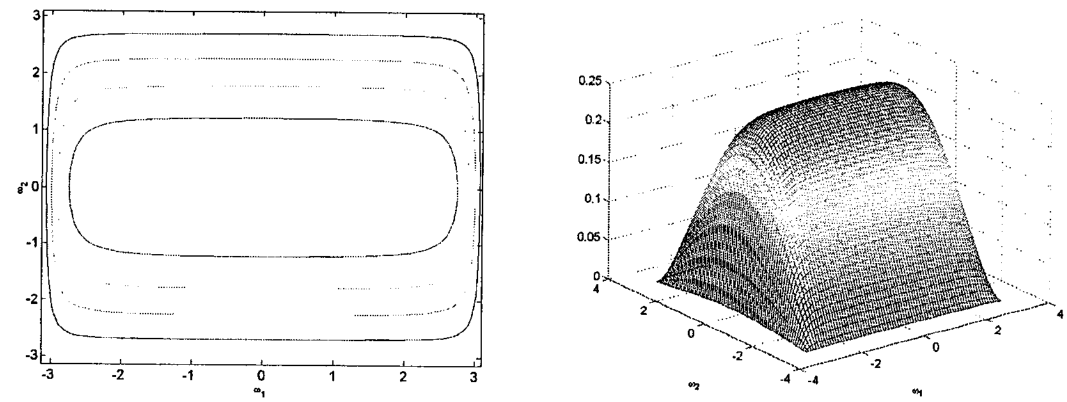


Figure 4.3(a): K_2=0.1

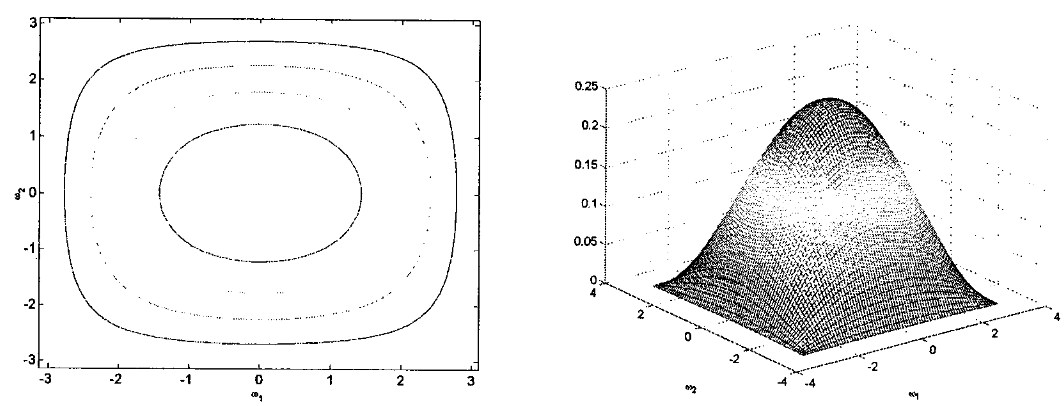


Figure 4.3(b): K_2=0.6

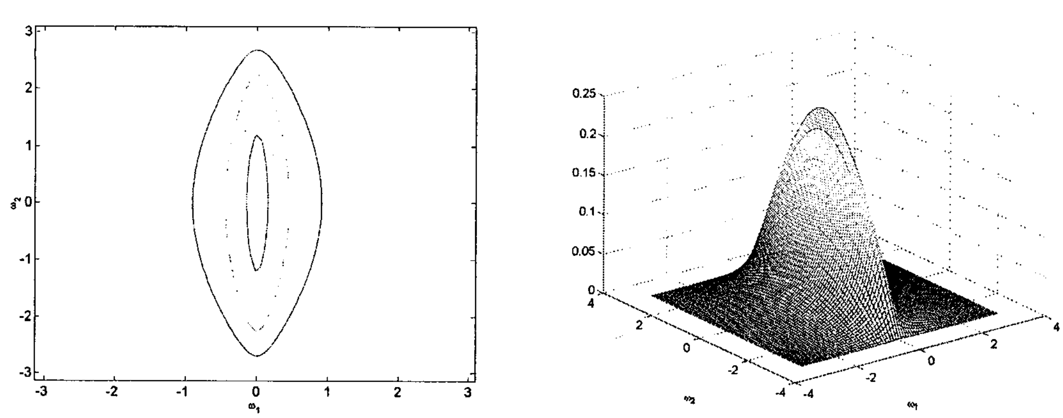


Figure 4.3(c): K_2=7

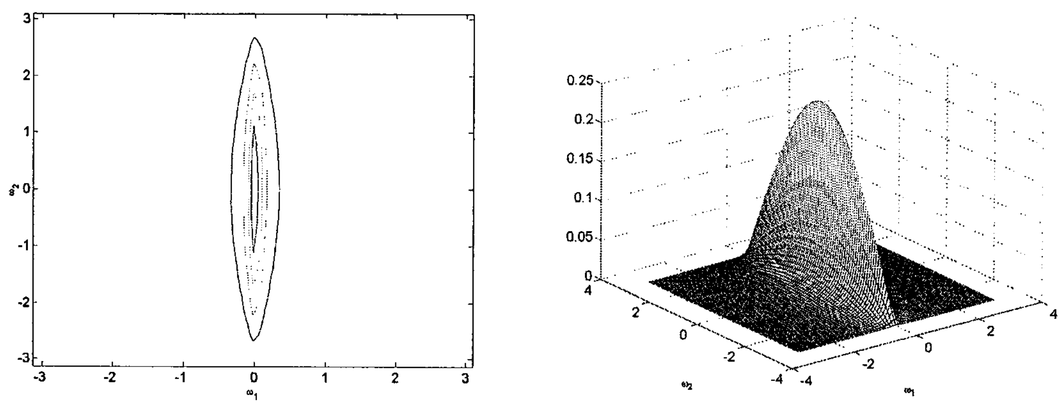


Figure 4.3(d): K_2=20

Figure 4.3: The contour and 3-D magnitude plot of the resulting 2-D low pass filter for different values of K_2 with $K_1 = b_{01} = b_{02} = 1$ and $a_{01} = a_{02} = -1$.

in Figure 4.2(c). Above Figure 4.3 shows few responses for various values of K_2.

After looking at the figure, one can say that value of K_2 is inversely proportional with the bandwidth of ω_1, while it does not make any difference in the bandwidth of ω_2. Same as the K_1 case, in this case too it does not make any effects on the gain of the low-pass response as amplitude remains constant approximately 0.17 throughout. In this case too, each contour plot has Quadrantal symmetry, which is observable.

From this discussion, we can say that K_1 and K_2 are **band-effective coefficients** and using this property of K_1 and K_2, we can design a 2-D low-pass filter with a desirable bandwidth. For an example, we want to design all pass filter and if we use the above relationship then we can say if we keep K_1 and K_2 very small then we can achieve this characteristic. And if we want to design really very narrow banded low-pass filter then in that case we have to keep K_1 and K_2 high. This is shown in the below figure.

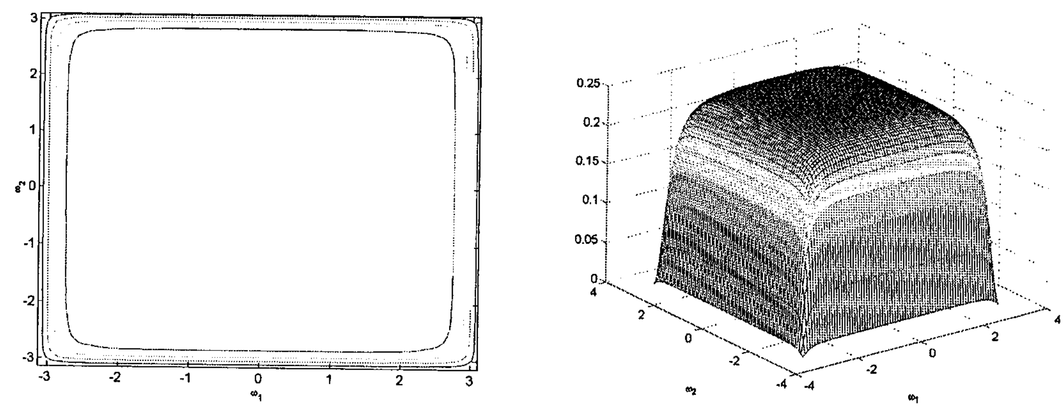


Figure 4.4(a): $K_1 = K_2 = 0.1$

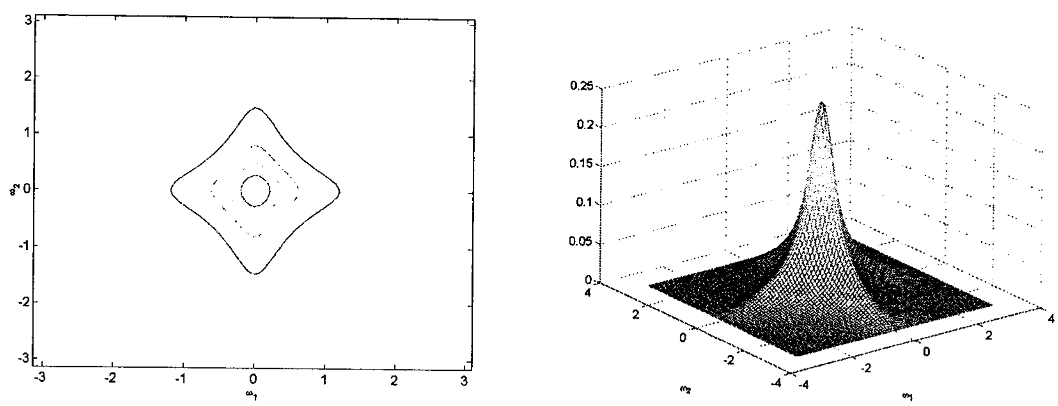


Figure 4.4(b): $K_1 = K_2 = 5$

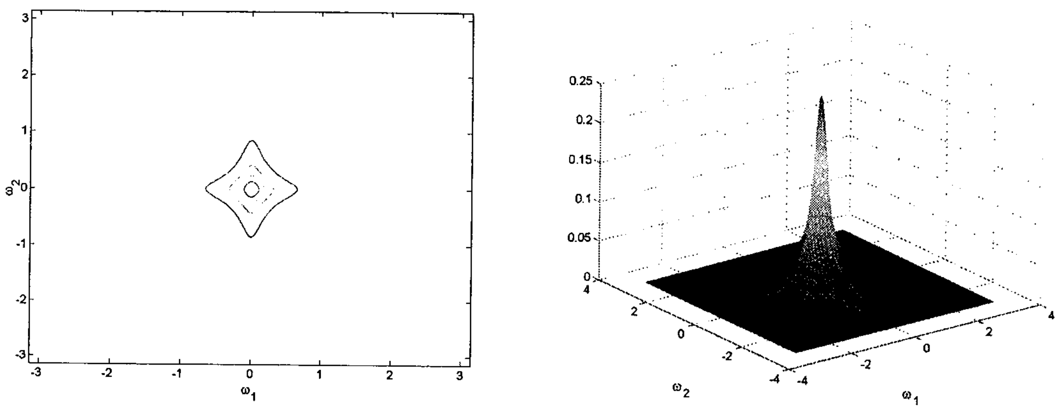


Figure 4.4(c): $K_1 = K_2 = 10$

Figure 4.4: The contour and 3-D magnitude plot of the resulting 2-D low pass filter for different values of K_1 and K_2 with $b_{01} = b_{02} = 1$ and $a_{01} = a_{02} = -1$.

Another noticeable thing in this case is the contour plot is having almost Octagonal symmetry. They are having Quadrantal symmetry but they are missing the Diagonal symmetry, hence they do not contain Octagonal symmetry.

Low Pass frequency response with different values of a_{01}

There are many combinations of the coefficients exit, but to keep the situation simple and not to lose any generality, we keep the other coefficients to unity with proper signs, e.g. $K_1 = K_2 = b_{01} = b_{02} = 1$ and $a_{02} = -1$. With this specification, we have

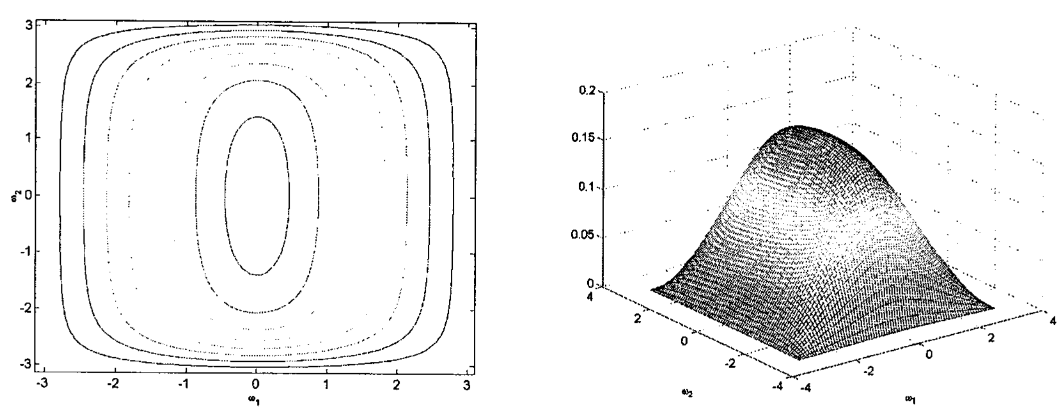


Figure 4.5(a): a_{01}=-0.1

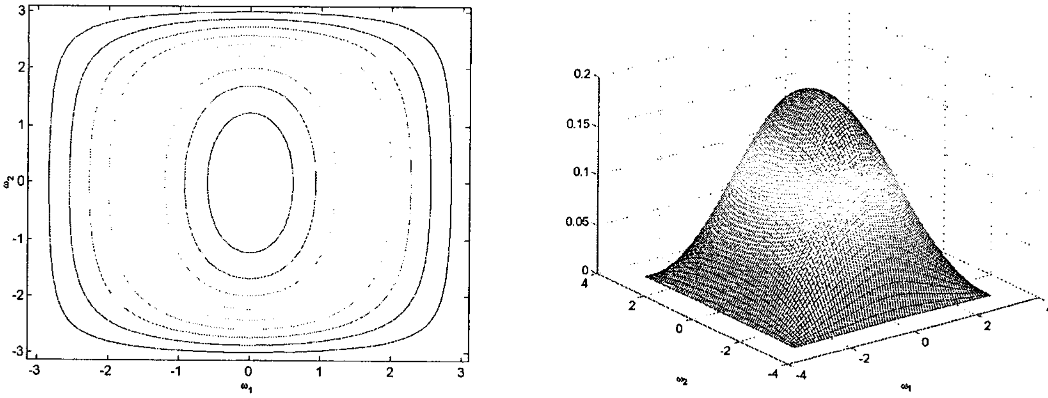


Figure 4.5(b): a_{01}=-0.5

Figure 4.5: The contour and 3-D magnitude plot of the resulting 2-D low pass filter for different values of a_{01} with $K_1 = K_2 = b_{01} = b_{02} = 1$ and $a_{02} = -1$.

checked that the designed low-pass recursive filter gives stable output for the complete range of a_{01} from -1 to 0. The contour and 3-D magnitude plots with different values of a_{01} are given in figure 4.5. As we have already presented the response for a_{01} = -1 in figure 4.2 (c), here we only give the results for a_{01} = -0.5 and a_{01} = -0.1.

This is evident that the main effect of a_{01} is on the gain of the designed filter. It is inversely proportional to amplitude of the low-pass filter. As the value decrease from -0.1 to -1, the amplitude approximately increases from 0.12 to 0.17. a_{01} also affects the bandwidth in ω_2 domain, it is obvious that as a_{01} decrease, the bandwidth in ω_2 domain also decrease. It is also important that the contour plots have Quadrantal symmetry in each case.

Low Pass frequency response with different values of a_{02}

To study the effect of a_{02} on the frequency response of the designed filter, we keep other coefficients of the bilinear transformation to unity in order not to lose any generality and to make the situation simple, e.g. with $K_1 = K_2 = b_{01} = b_{02} = 1$ and $a_{01} = -1$. Same as a case of a_{01}, a_{02} also gives stable response for the whole range, e.g. from -1 to 0. Below figure shows simulation results for different values of a_{02}. Simulation result for a_{02}= -1 is already shown in figure 4.2 (c), so we do not show it again in the below figure.

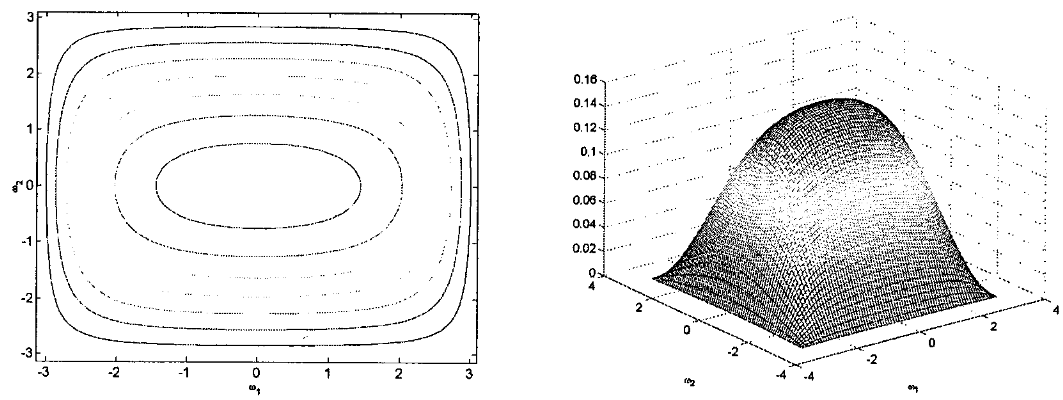


Figure 4.6(a): a_{02}=-0.1

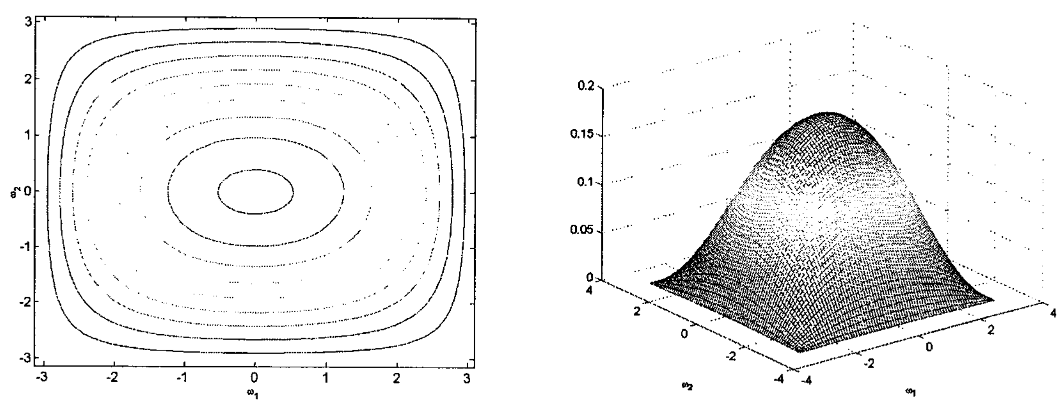


Figure 4.6(b): a_{02}=-0.5

Figure 4.6: The contour and 3-D magnitude plot of the resulting 2-D low pass filter for different values of a_{01} with $K_1 = K_2 = b_{01} = b_{02} = 1$ and $a_{02} = -1$.

Same as the a_{01} case, here too the value of a_{02} is inversely proportional to the magnitude of the filter and directly proportional to the bandwidth of ω_1. And it can be seen that, the response in this case is approximately 90 degree rotated with respect to the relative case of a_{01}.

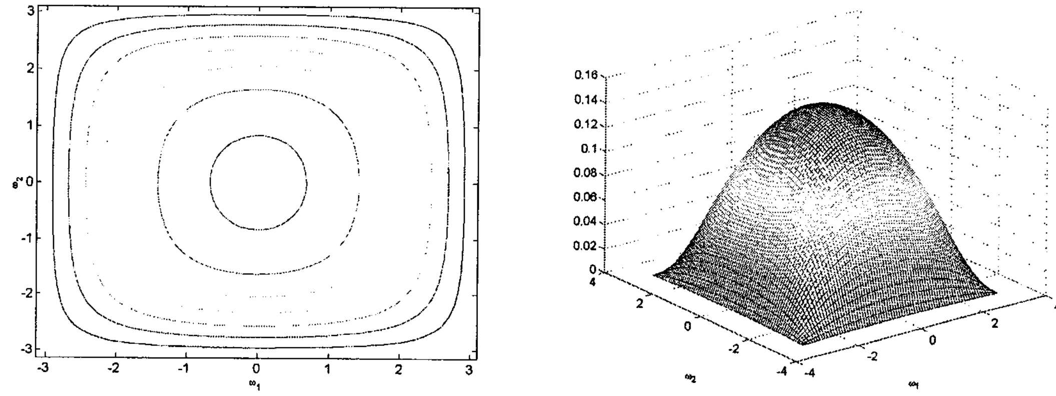


Figure 4.7(a): $a_{01}=a_{02}$=-0.5

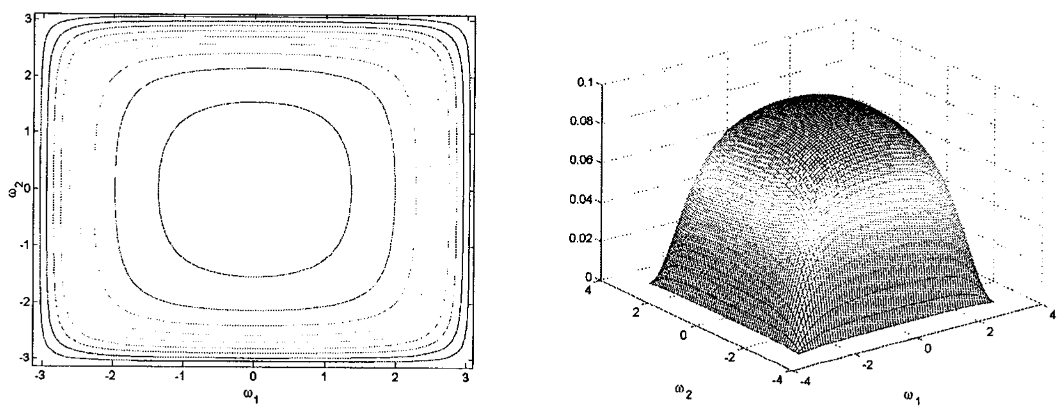


Figure 4.7(b): $a_{01}=a_{02}$=0

Figure 4.7: The contour and 3-D magnitude plot of the resulting 2-D low pass filter for different values of a_{01} and a_{02} with $K_1 = K_2 = b_{01} = b_{02} = 1$.

It is obvious that the effect of a_{01} and a_{02} on bandwidth is limited in comparison of the effect of K_1 and K_2 is limited. They are mainly gain affected coefficient. So we call them **gain-effective coefficients**. And using this knowledge we can say that if we reduce both a_{01} and a_{02} simultaneously then the magnitude value will drop more, which is shown in above figure.

It is noticeable that when we keep a_{01} equal to a_{02}, the contour plot tries to obtain Octagonal symmetry same as we have seen earlier for the $K_1 = K_2$ case. The magnitude

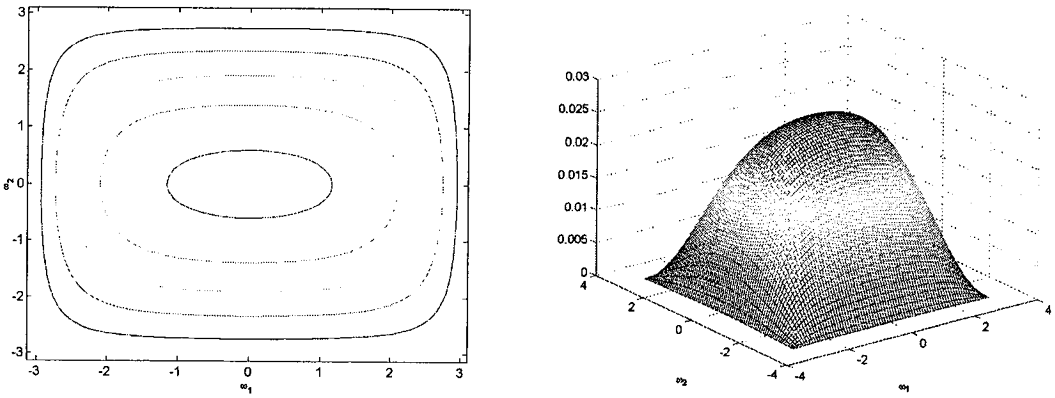


Figure 4.8(a): K_1=10; K_2=1; $a_{01}=a_{02}$=-0.1

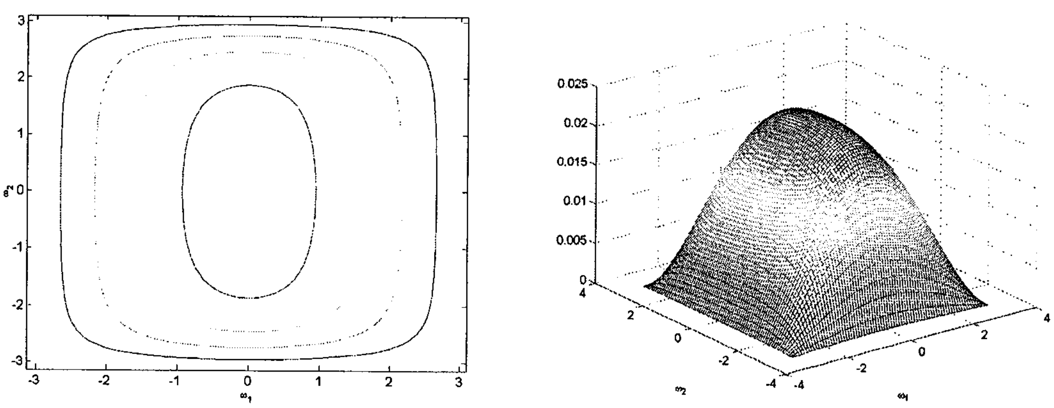


Figure 4.8(b): K_1=1; K_2=10; $a_{01}=a_{02}$=-0.1

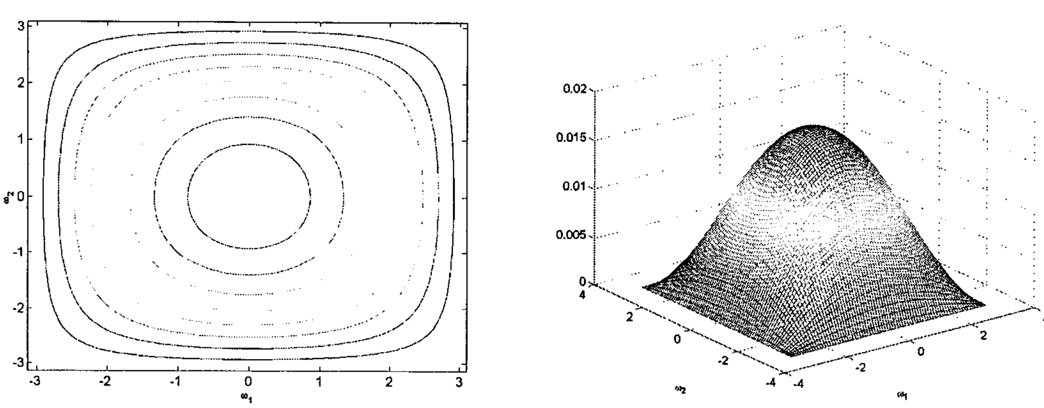


Figure 4.8(c): $K_1=K_2$=5; $a_{01}=a_{02}$=-0.1

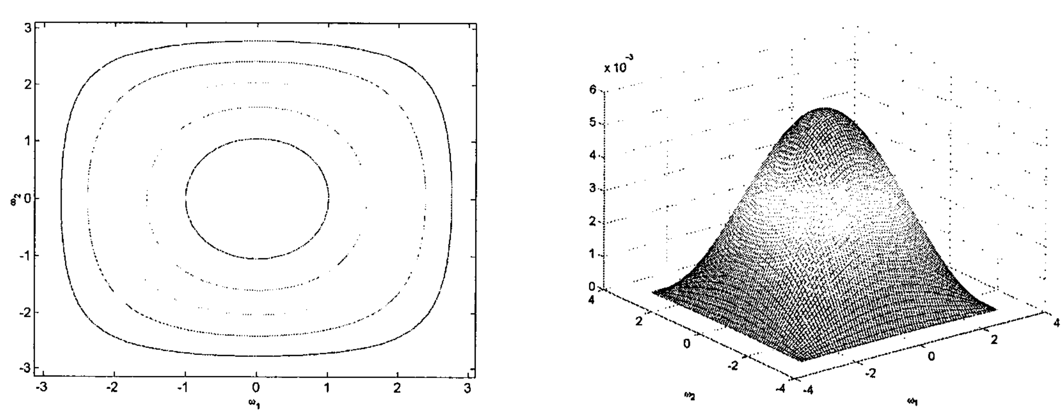


Figure 4.8(d): $K_1 = K_2$=10; $a_{01} = a_{02}$=-0.1

Figure 4.8: The contour and 3-D magnitude plot of the resulting 2-D low pass filter for different values of K_1, K_2, a_{01} and a_{02} with $b_{01} = b_{02} = 1$.

changes from 0.07 to 0.17 when a_{01} and a_{02} change from upper boundary to lower boundary. We could change the magnitude value dramatically as shown in above figure, when we combine the effect of a_{0i} with bigger K_i.

And it can be seen that, the response in case of K_1=10 and K_2=1 is approximately 90 degree rotated with respect to the case of K_1=1 and K_2=10. And in this case the magnitude value does not change a lot with comparing other two cases of K_1= K_2=5 and K_1= K_2=10. It is also visible that the bandwidth of pass band of the response decrease with higher values of K_1 and K_2.

Low Pass frequency response with different values of b_{01}

For analyzing the consequences of b_{01} on the frequency response of the designed filter, we maintain other coefficients of the bilinear transformation to unity in order not to lose any generality and to make the condition simple, e.g. with $K_1 = K_2 = b_{02} = 1$ and $a_{01} = a_{02} = -1$. We have verified that b_{01} gives stable response for the entire scale, e.g.

from 0 to 1. Below figure shows simulation results for different values of b_{01}. Simulation result for b_{01}= 1 is already shown in figure 4.2 (c), so we do not show it again in the below figure.

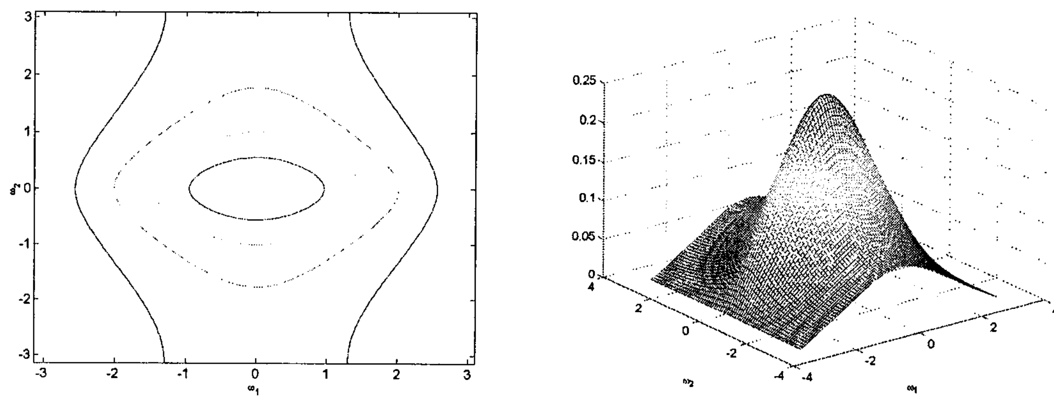


Figure 4.9(a): b_{01}=0.1

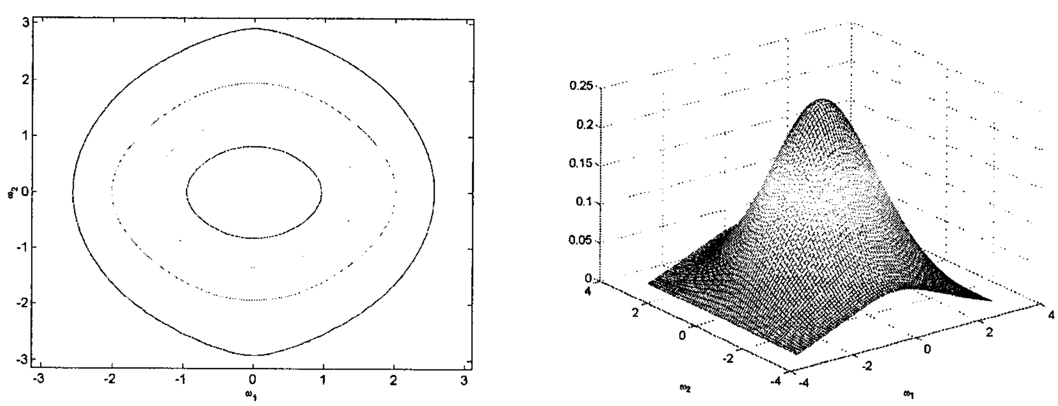


Figure 4.9(b): b_{01}=0.5

Figure 4.9: The contour and 3-D magnitude plot of the resulting 2-D low pass filter for different values of b_{01} with $K_1 = K_2 = b_{02} = 1$ and $a_{01} = a_{02} = -1$.

It is evident in the above result that if b_{01} is not equal to one will result in non-zero gain in the stop-band portion. And it also little bit affects the bandwidth of the pass-band of the filter in ω_2 domain. It is also observable that each contour plot has Quadrantal symmetry and the magnitude value remains same in each case.

Low Pass frequency response with different values of b_{02}

In many ways we can combine the coefficients of the bilinear transformation, but to maintain the scenario simple and not to lose any generality, we hold the other coefficients to unity with proper signs, e.g. $K_1 = K_2 = b_{01} = 1$ and $a_{01} = a_{02} = -1$. With this condition, we have checked that the designed low-pass recursive filter gives stable output for the full range of b_{02} from 0 to 1. Below figure shows the contour and 3-D magnitude plots with different values of b_{02}. As we have already studied the response for

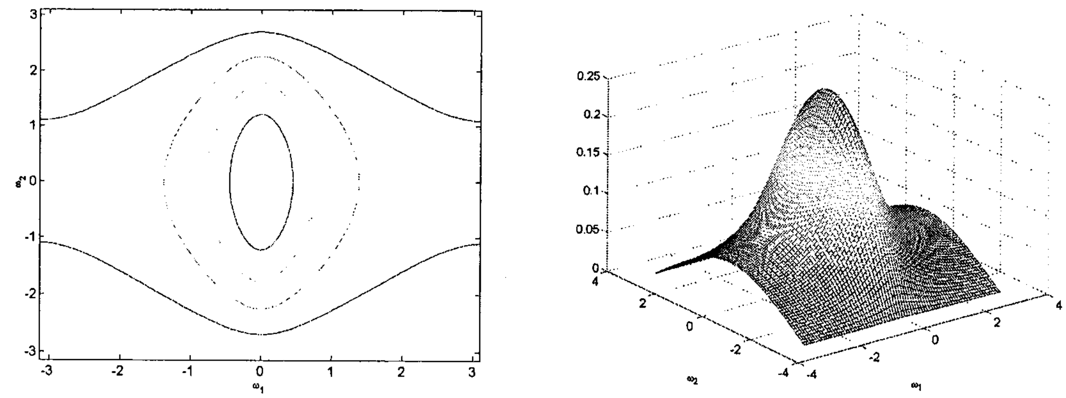


Figure 4.10(a): b_{02}=0.1

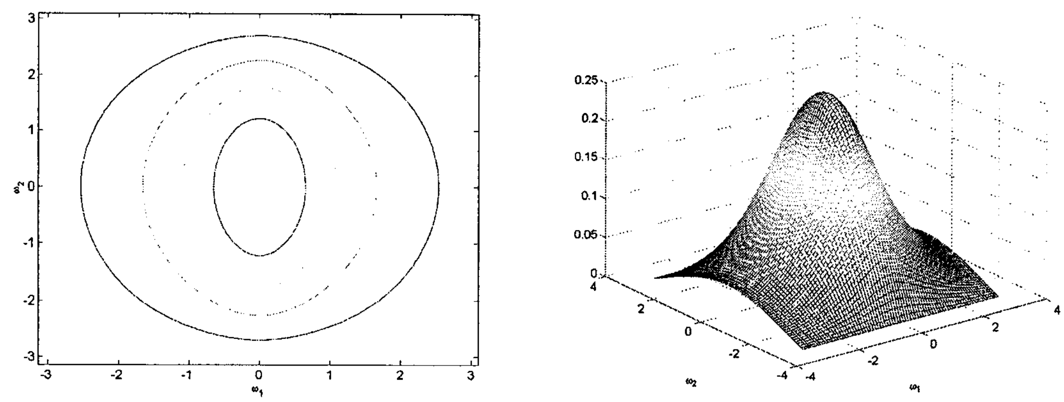


Figure 4.10(b): b_{02}=0.5

Figure 4.10: The contour and 3-D magnitude plot of the resulting 2-D low pass filter for different values of b_{02} with $K_1 = K_2 = b_{01} = 1$ and $a_{01} = a_{02} = -1$.

b_{02} = 1 in figure 4.2 (c), hence in the above figure we only give the results for b_{02} = 0.5 and b_{02} = 0.1

It is visible that the response with b_{02} is approximately 90 degree rotated with respect to the response of b_{01}. In this configuration too the non-zero gain in stop-band region appears and that because of the non unity value of b_{02}. It is observable that as the value of b_{02} increase the non-zero gain in the stop-band region decrease. Along with this b_{02} does not make any changes on the magnitude of the response, and it has little bit effect on the pass band of the response in ω_1 region. Presence of the Quadrantal symmetry is clear in the contour plots.

Non-zero gain in the stop band of the filter response might change the filter polarity from low-pass to high-pass, so we call b_{01} and b_{02} as **polarity-effective coefficients**. And to nullify this problem we can use the band-effective coefficients K_1 and K_2. Presence of both non unity b_{01} and b_{02} coefficients will produce unwanted non-zero gain at the higher frequency in both the domains, e.g. ω_1 and ω_2 domains.

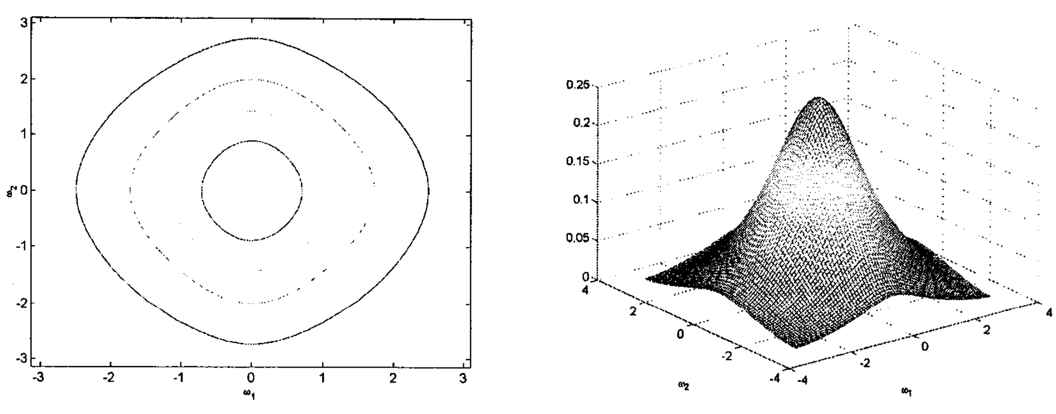


Figure 4.11(a): K_1=K_2=1; b_{01}=b_{02}=0.6

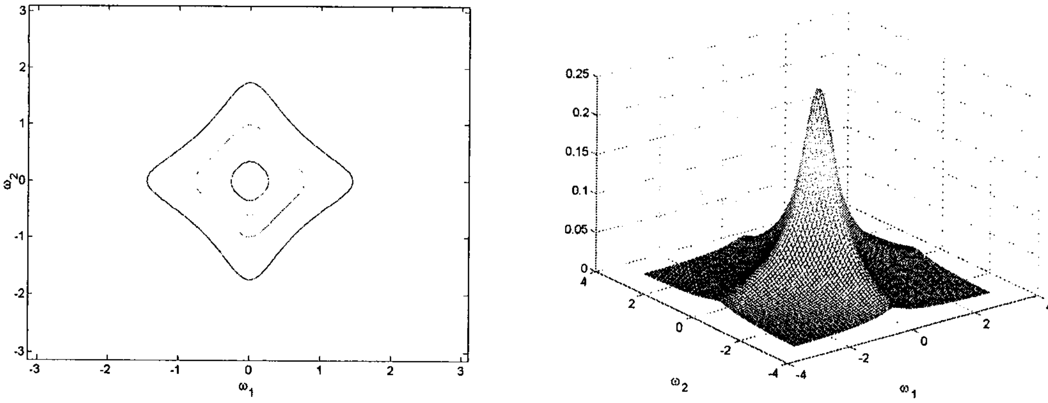


Figure 4.11(b): $K_1 = K_2 = 3$; $b_{01} = b_{02} = 0.6$

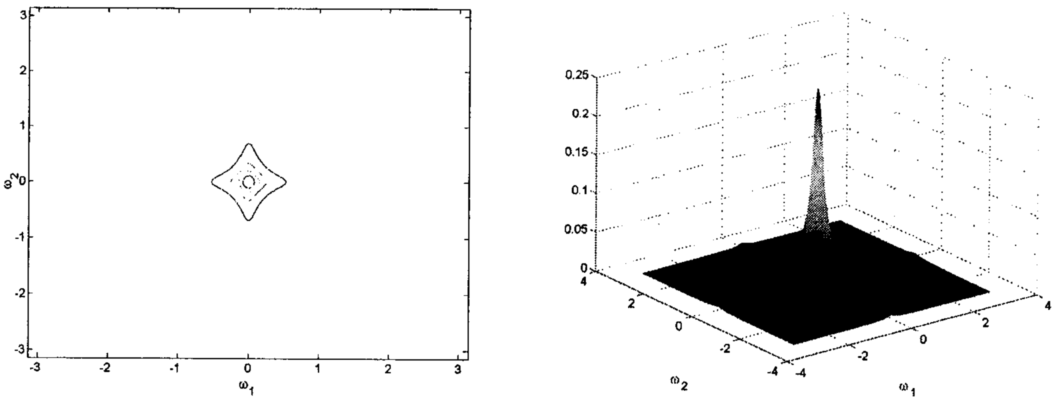


Figure 4.11(c): $K_1 = K_2 = 10$; $b_{01} = b_{02} = 0.6$

Figure 4.11: The contour and 3-D magnitude plot of the resulting 2-D low pass filter for different values of K_1, K_2, b_{01} and b_{02} with $a_{01} = a_{02} = -1$.

It indicates that with bigger value of K_i, we can reduce the non-zero gain in the stop-band region, but as a cost we obtain smaller bandwidth in the pass-band of the response. So we have to trade-off in between these two parameters, non-zero gain in the stop-band region and bandwidth of the pass-band, for the particular application.

4.3.2 2-D Low-pass recursive filter response with each different coefficient of the matrix

In this section, we investigate how each different coefficient of the matrix influences the designed filter response. In order to establish the effect of the coefficient on the response, we change the coefficient under the examination continuously, while keeping the others to some specific value. As we mentioned early, we should keep each coefficient of the generalized bilinear transformation to some specific value. Not to lose any generality, we should keep $K_1 = K_2 = b_{01} = b_{02} = 1$ and $a_{01} = a_{02} = -1$ for the simplicity.

If we plug in these values of the bilinear transformation in the Equation 3.27 then we can get conditions on the matrix coefficients, which are as below.

- Equation 3.27a gives condition, $\psi_1\lambda_1 > 0$ (4.3a)
- Equation 3.27b gives condition, $Y_2 > 0$ (4.3b)
- Equation 3.27c gives condition, $Y_1 > 0$ (4.3c)
- Equation 3.27d gives condition, $g \neq 0$. (4.3d)

For the quadrantal symmetry, transfer function $H(z_1, z_2)$ must be expressible as

$$H(z_1, z_2) = H_1(z_1 + z_1^{-1}, z_2) . H_2(z_1, z_2 + z_2^{-1}). \tag{4.4}$$

And after carefully studying the designed 2-D low pass transfer function, it is possible to make our transfer function as product of two different two-variable polynomials shown in above equation, if it follows the below condition

$$Y_1 = \frac{g^2}{\psi_1\lambda_1 Y_2 (a-b)^2} \tag{4.5}$$

One of the possible combinations for matrix coefficients satisfying the above condition is **A**= $\psi_1\lambda_1(a-b)^2$ =6.1, g=2, Y_1=0.937, Y_2 =0.7. We have used this combination for studying the effects of the coefficients of the generalized bilinear transformation on the filter response, and this is the reason we had Quadrantal symmetry in all the cases we have studied so far. With reference of these values, we analyze the effects of the matrix coefficients on the designed 2-D low-pass recursive filter in this section.

We have also investigated that the frequency response of the designed filter for a particular value of **A**=$\psi_1\lambda_1(a-b)^2$, using different combinations of ψ_1, a, b and λ_1, remains same. So instead of studying them individually we study them as a common variable **A**, where **A**=$\psi_1\lambda_1(a-b)^2$.

Low Pass frequency response with different values of A =$\psi_1\lambda_1(a-b)^2$

There are many possibilities exit for the different combinations of the matrix coefficients, but to maintain the scenario simple and not to lose any generality, we hold the other coefficients to the specific value showed above, e.g. g=2, Y_1=0.937, Y_2 =0.7 to study the effects of **A** on the designed filter. With this condition, we have checked that the designed 2-D recursive low-pass filter gives stable output for **A** equal to 1 to 1000. Below figure shows the contour and 3-D magnitude plots with different values of **A**. As we have already studied the response for **A** = 6.1 in figure 4.2(c), which is our reference.

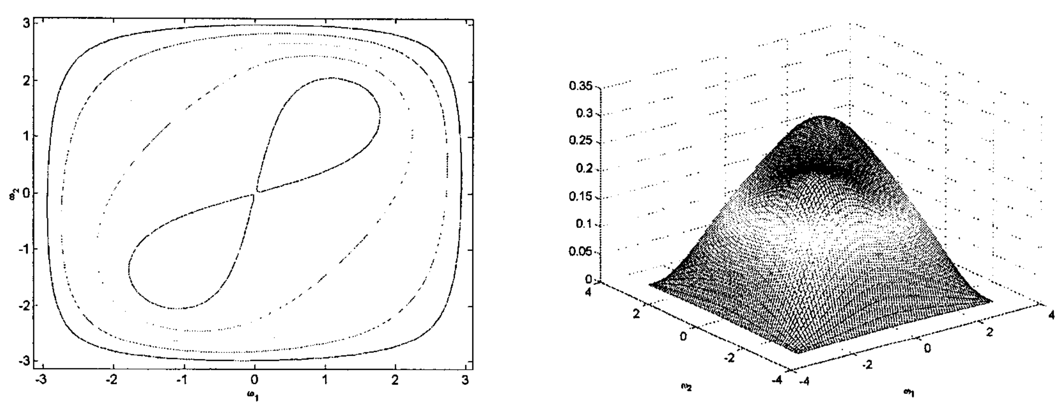


Figure 4.12(a): **A**=2

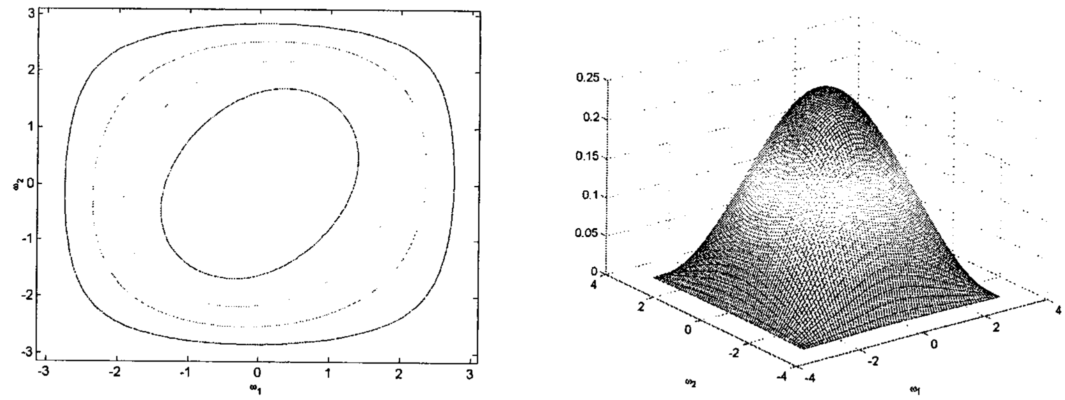


Figure 4.12(b): **A**=4

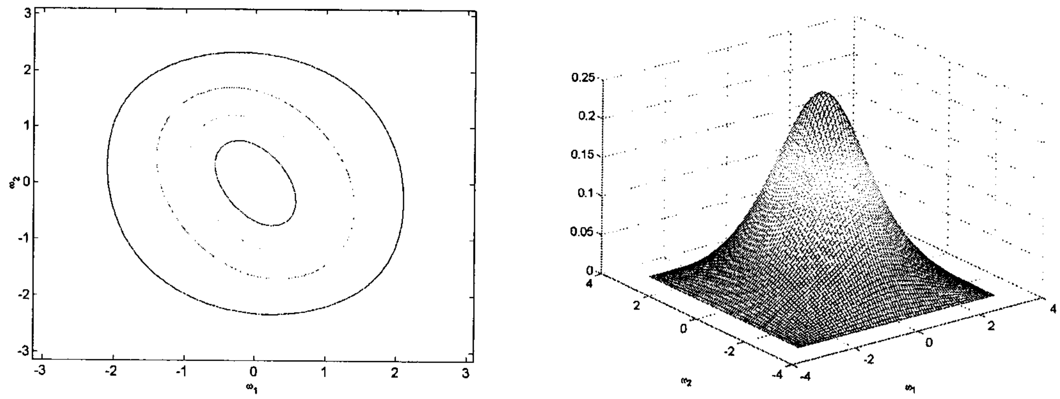


Figure 4.12(c): **A**=12

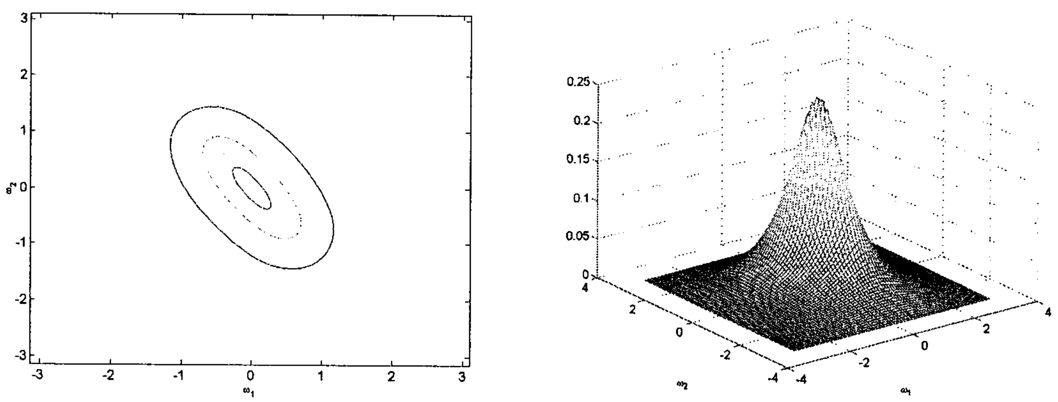


Figure 4.12(d): **A**=40

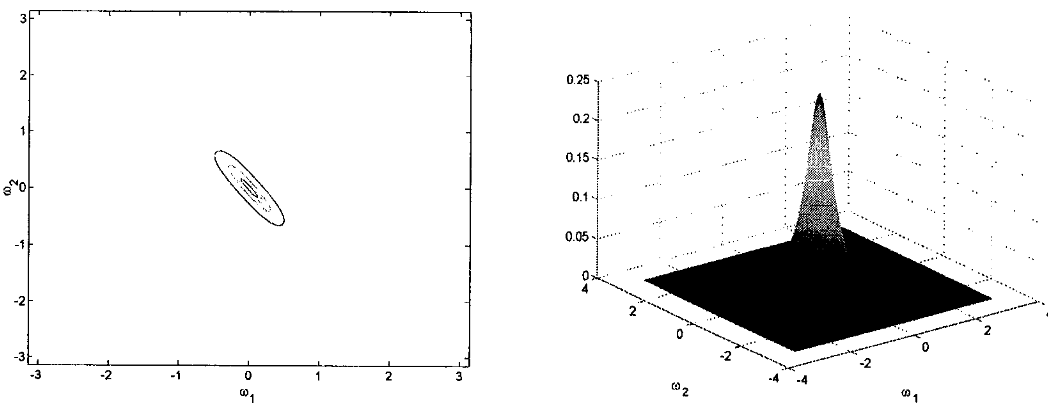


Figure 4.12(e): **A**=200

Figure 4.12: The contour and 3-D magnitude plot of the resulting 2-D low pass filter for different values of **A** $=\psi_1\lambda_1(a-b)^2$ with g=2, Y_1=0.937, Y_2 =0.7.

It is clear that the magnitude of the frequency response remains same for all the values of **A**. If we compare these different simulation results for different values of **A**, with the reference response for **A**=6.1 then we can say that if we decrease the value of **A** from the reference value 6.1 then the response rotates anti-clock direction and vice-versa. Along with this, it is also observable that value of **A** is inversely proportional to the bandwidth of the frequency response. Existence of the Centro symmetry in each contour plot is noticeable.

Low Pass frequency response with different values of g

To see the effect of g on the designed filter, we should keep the other matrix coefficients to some specific value. To maintain the situation simple, we keep each of them to the reference value, e.g. **A** $=\psi_1\lambda_1(a-b)^2$=6.1, Y_1=0.937, Y_2 =0.7 and at the same time we change the value of g. The reference simulation result is given in figure 4.2(c), where value of g is 2.

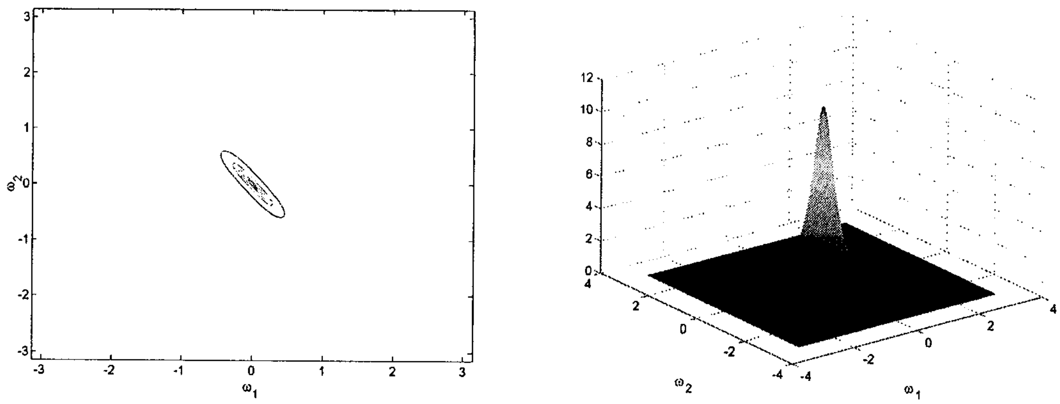


Figure 4.13(a): g=0.3

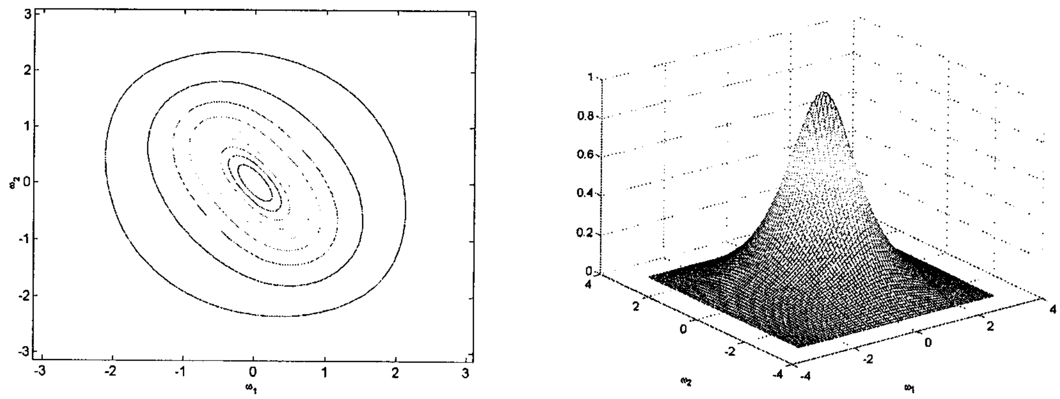


Figure 4.13(b): g=1

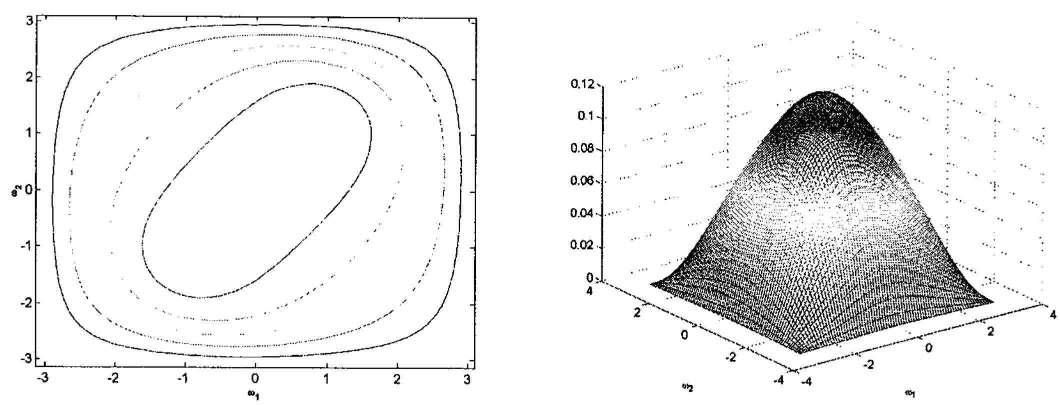


Figure 4.13(c): g=3

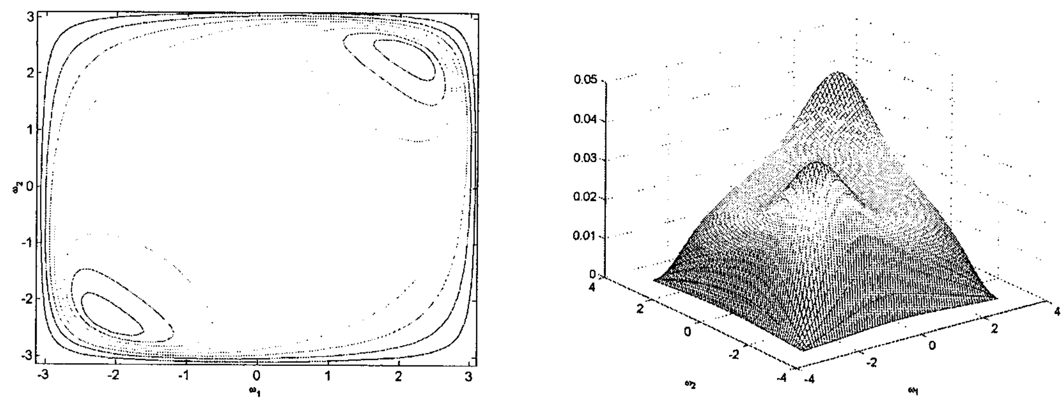


Figure 4.13(d): g=6

Figure 4.13: The contour and 3-D magnitude plot of the resulting 2-D low pass filter for different values of g with **A** $=\psi_1\lambda_1(a-b)^2$=6.1, Y_1=0.937, Y_2 =0.7.

It is visible that the magnitude of the frequency response changes noticeably with different the values of g, and we decease the value of g from the reference value then the magnitude increase and vice-versa. Also the frequency response rotates clock wise if we decrease the value of g from the reference frequency response and rotates anti-clock wise if we increase the value of g from the reference value. Beside this, it is also evident that g is directly proportional to the bandwidth of the frequency response, and each contour plot has Centro symmetry. Effect of g on the magnitude is very observable than the effect on the bandwidth. Because of this reason, we should consider g as **gain-effective coefficient**.

If we combine the value of **A** and value of g in some particular manner then we can change the gain of the frequency response significantly. Below figure shows some cases and after carefully observing these cases we can say that if we divide g by a factor “x” and **A** by the square of the factor “x”, then we can increase the gain by square of that factor "x", while contour plots remain same for all the cases. And this is very evident from the transfer function of the designed filter.

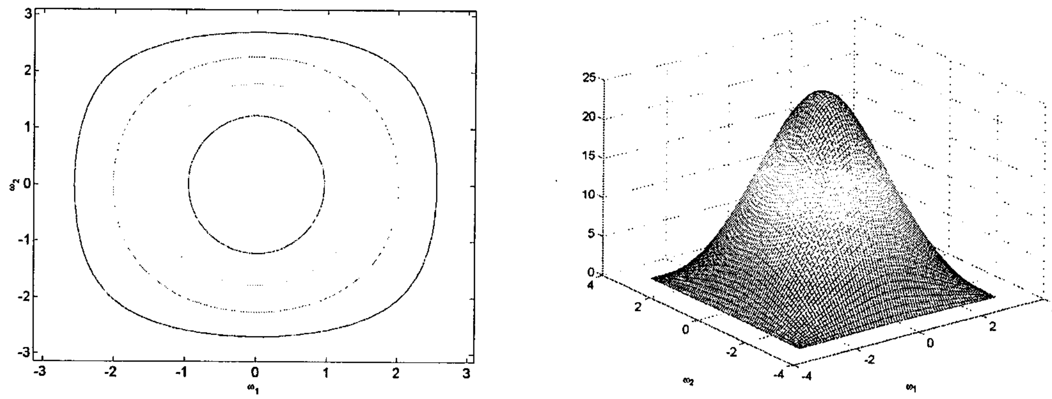


Figure 4.14(a): A=0.061 and g=0.2

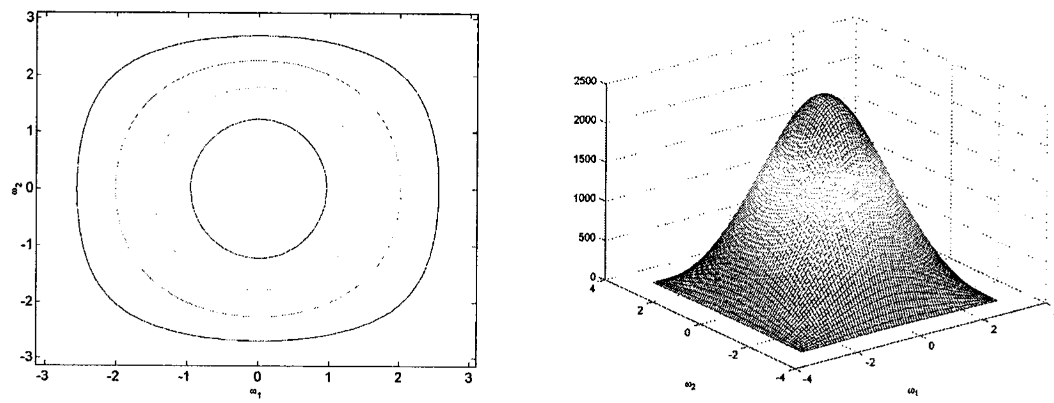


Figure 4.14(b): A=0.00061 and g=0.02

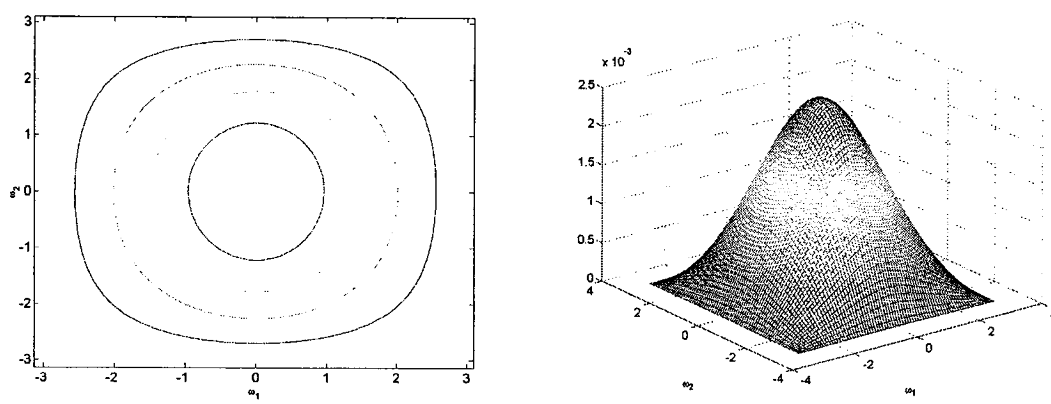


Figure 4.14(c): A=610 and g=20

Figure 4.14: The contour and 3-D magnitude plot of the resulting 2-D low pass filter for different values of $\mathbf{A} = \psi_1 \lambda_1 (a-b)^2$ and g with Y_1=0.937, Y_2 =0.7.

Low Pass frequency response with different values Y_1

We keep the other parameters of the matrix to the reference value e.g., Y_2=0.7, g =2, $\mathbf{A} = \psi_1 \lambda_1 (a-b)^2$=6.1, with the purpose of understanding the effect of Y_1 on the

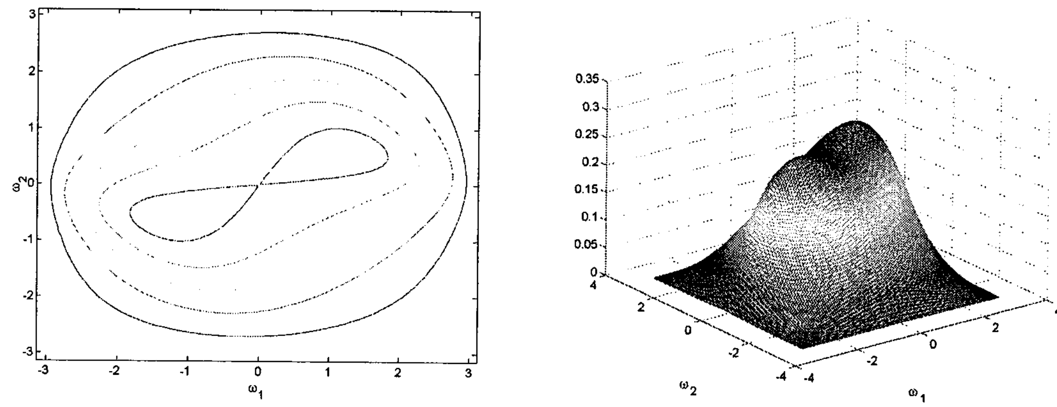


Figure 4.15(a): Y_1=0.2

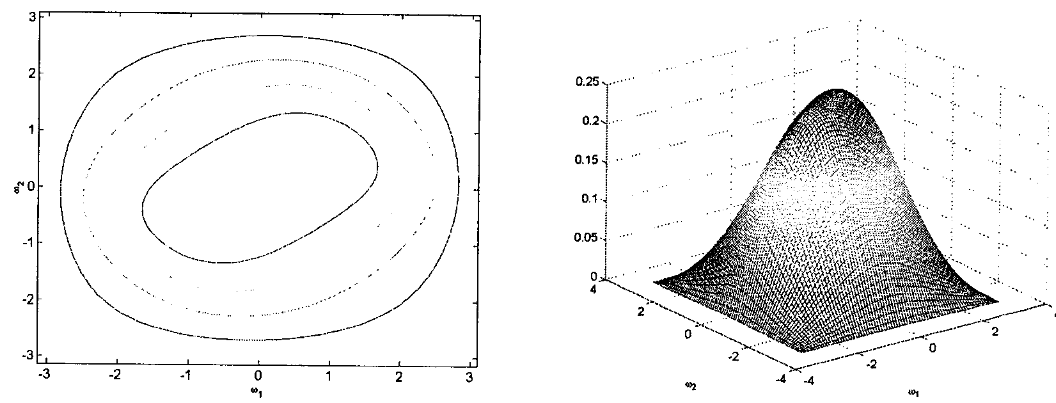


Figure 4.15(b): Y_1=0.5

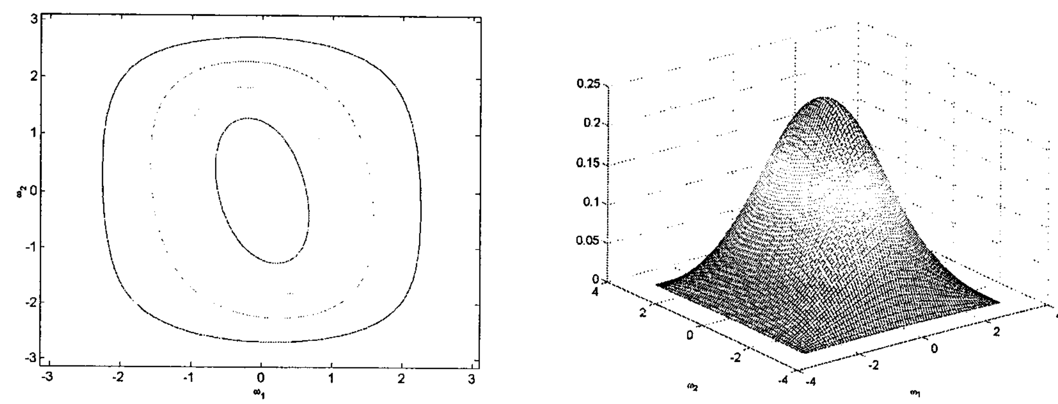
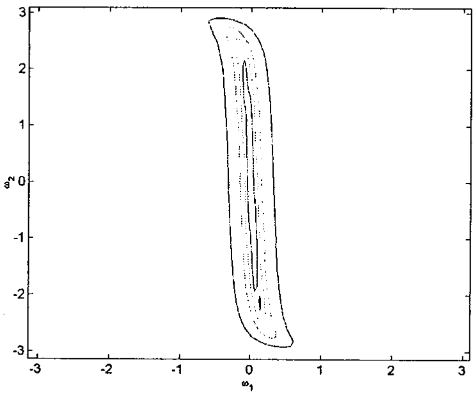


Figure 4.15(c): Y_1=1.5



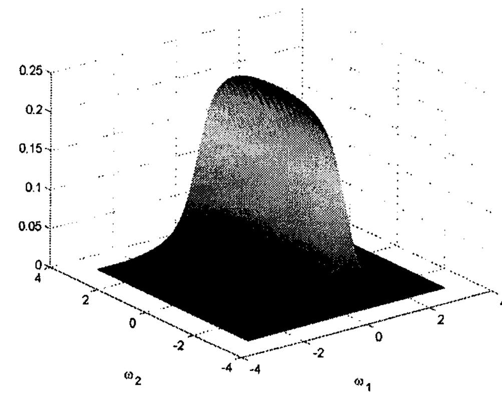


Figure 4.15(d): Y_1=20

Figure 4.15: The contour and 3-D magnitude plot of the resulting 2-D low pass filter for different values of Y_1 with $\mathbf{A} = \psi_1 \lambda_1 (a-b)^2$=6.1, g=2 and Y_2 =0.7

designed low pass filter. We have checked that Y_1 gives stable response up to 1000 with keeping other coefficients to the specified value as showed above. Figure 4.2(c) shows the reference response for Y_1=0.937 and we analyze the effect of Y_1 with this response.

After thoroughly investigating, we can say that the main effect of Y_1 is on the bandwidth in ω_1 domain, while there is no effect on the bandwidth in ω_2 domain. It is inversely proportional to the bandwidth in ω_1 domain. Another observable thing is Y_1 has no effects on the amplitude of the response. In this case too, presence of the Centro symmetry in the contour plots is evident.

Low Pass frequency response with different values Y_2

To realize the effect of Y_2 with the reference response showed in figure 4.2(c), we should keep Y_1=0.937, g =2 and $\mathbf{A}$ $= \psi_1 \lambda_1 (a-b)^2$=6.1. With this specification, for stability purpose we checked the frequency response of the designed filter with different values of Y_2, from 0.1 to 1,000. Above figure shows the response for different values of Y_2.

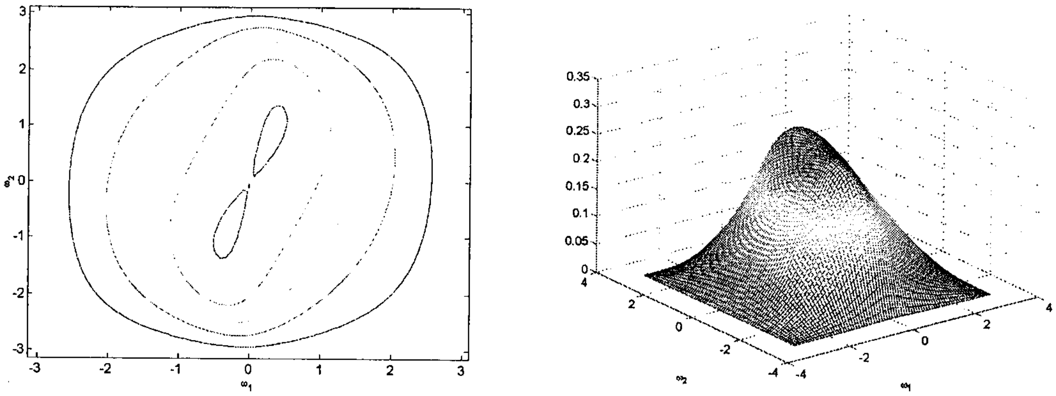


Figure 4.16(a): Y_2=0.3

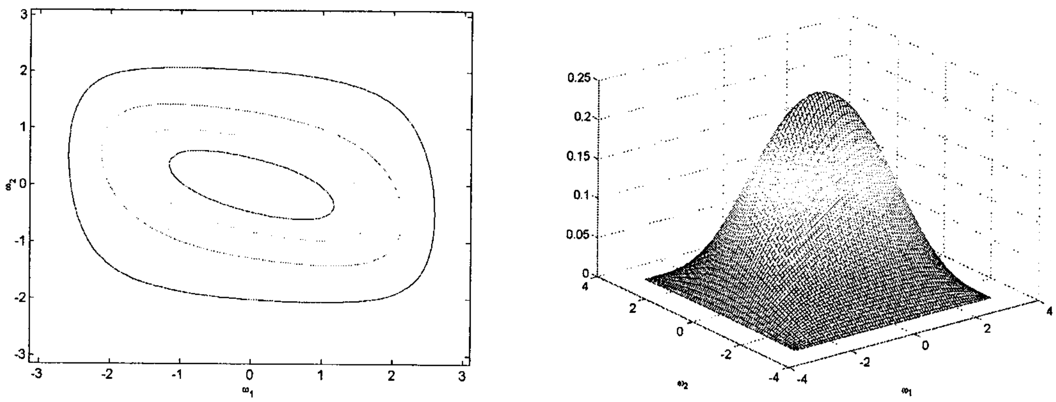


Figure 4.16(b): Y_2=2

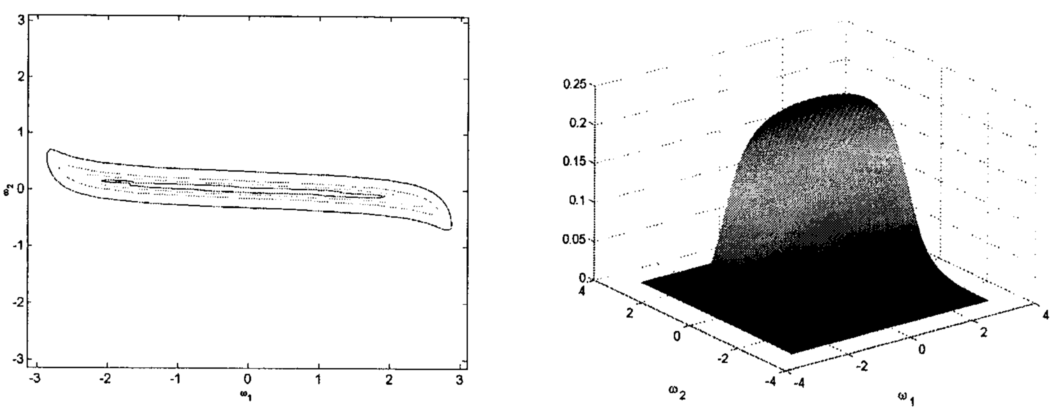


Figure 4.16(c): Y_2=20

Figure 4.16: The contour and 3-D magnitude plot of the resulting 2-D low pass filter for different values of Y_2 with A $=\psi_1\lambda_1(a-b)^2$=6.1, g=2 and Y_1 =0.937

If we explore the above results in details then we can disclose that the main effect of Y_2 is on the bandwidth in ω_2 region, while there is no effect on the bandwidth in ω_1 domain. There will be bigger bandwidth with smaller value of Y_2. It is apparent that each contour plot has Centro symmetry. Same as the case of Y_1, Y_2 is also not having any effects on the amplitude of the response. Because of this, we can consider them as **band-effective coefficients**. When we change Y_1 and Y_2 together, we were expecting the same results we have got for K_1 and K_2. But we do not get same results. Above figure shows some simulation results for different values of Y_1 and Y_2

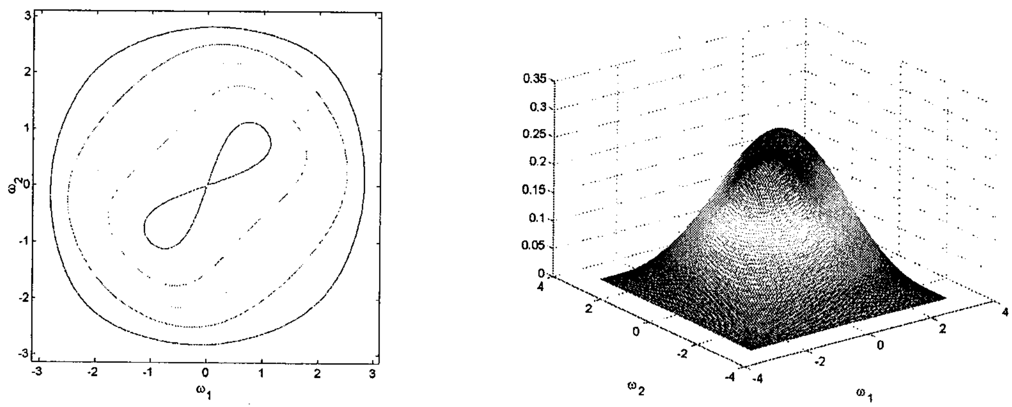


Figure 4.17(a): Y_1= Y_2=0.5

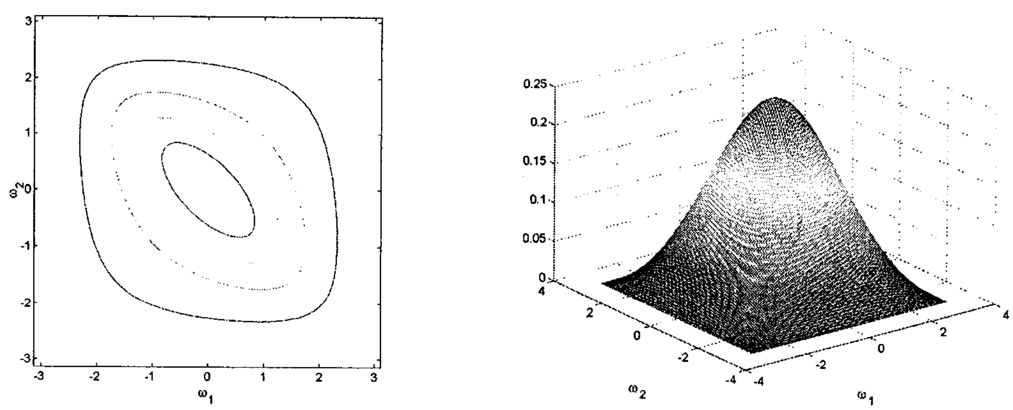


Figure 4.17(b): Y_1= Y_2=1.5

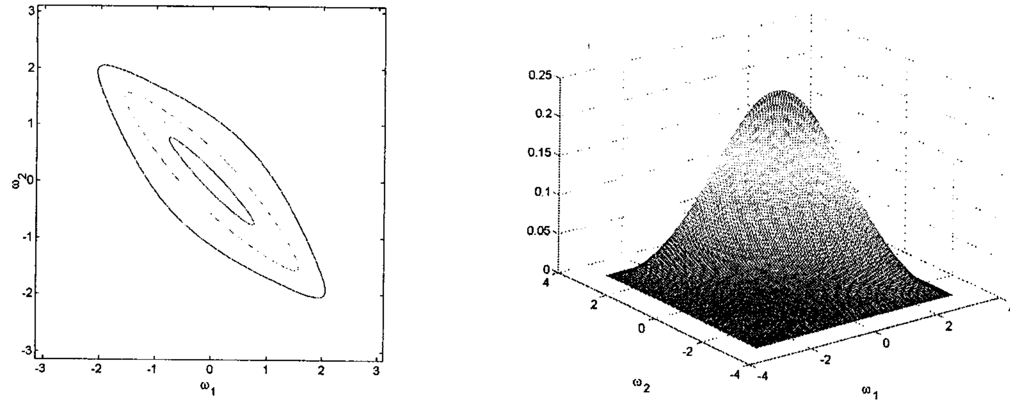


Figure 4.17(c): $Y_1 = Y_2 = 5$

Figure 4.17: The contour and 3-D magnitude plot of the resulting 2-D low pass filter for different values of Y_1 and Y_2 with $\mathbf{A} = \psi_1 \lambda_1 (a-b)^2 = 6.1$, g=2.

If we analysis the above results cautiously then we can say instead of increasing or decreasing the bandwidth in ω_1 and ω_2 region, it increases or decreases the bandwidth diagonally either $\omega_1 = \omega_2$ or $\omega_1 = -\omega_2$. Another remarkable thing is the presence of Diagonal symmetry in each contour plot. And this leads us to the idea of making the value of Y_1 equal to the value of Y_2 with fulfilling the equation 4.5, so that we can combine Diagonal and Quadrantal symmetry, which results in the Octagonal symmetry.

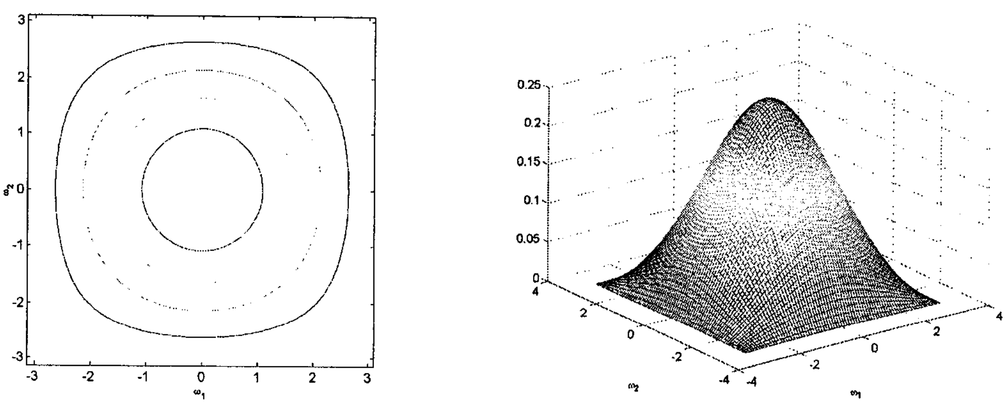


Figure 4.18: The contour and 3-D magnitude plot of the resulting 2-D low pass filter for $\mathbf{A} = \psi_1 \lambda_1 (a-b)^2 = 6.1$, g=2, $Y_1 = 0.81$ and $Y_2 = 0.81$

In order to obtain Diagonal and Quadrantal symmetry in the frequency response, we change the value of Y_1 and Y_2 from 0.937 and 0.7 to 0.81 and 0.81 in our reference case. Figure 4.18 shows the contour plot and magnitude plot for these new values. Still the combine effect of Y_1 and Y_2 on the bandwidth is not evident. In order to study further we should keep changing value of Y_1 and Y_2 with satisfying the equation 4.5 for g=2. Below figure shows some simulation results for this case.

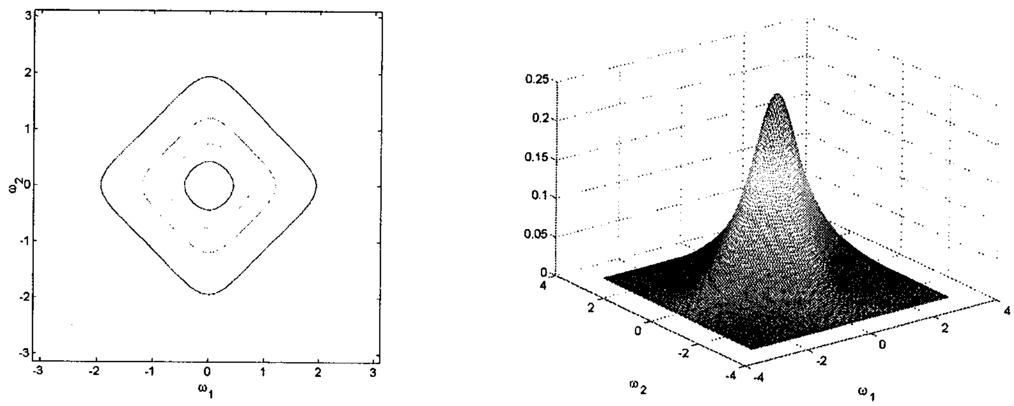


Figure 4.19(a): $Y_1 = Y_2 = 0.3$ and $\mathbf{A} = 44.44$

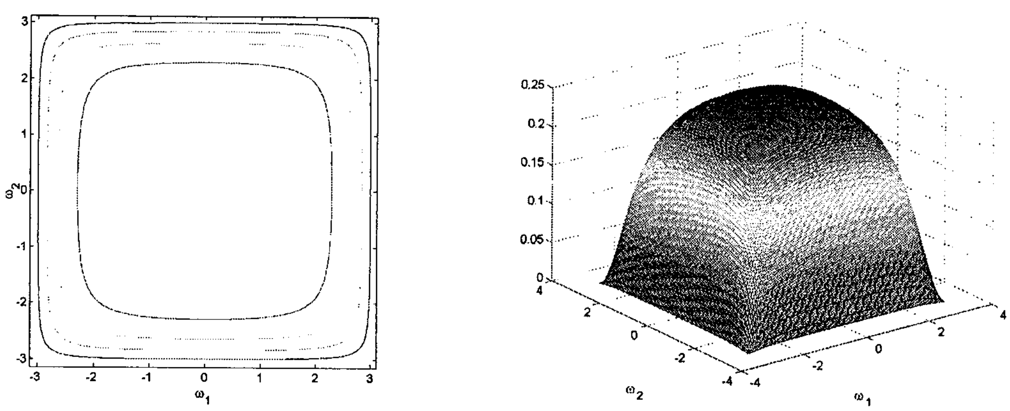


Figure 4.19(b): $Y_1 = Y_2 = 3$ and $\mathbf{A} = 0.44$

Figure 4.19: The contour and 3-D magnitude plot of the resulting 2-D low pass filter for different values of Y_1, Y_2 and $\mathbf{A} = \psi_1 \lambda_1 (a-b)^2$ with g=2.

From these results the effects of Y_1 and Y_2 on bandwidth are obvious. We can say that after satisfying equation 4.5, the effect of Y_1 and Y_2 on bandwidth is directly proportional with the value of Y_1 and Y_2. But the more effective and easy way to obtain the desirable bandwidth is by changing the value of K_1 and K_2, because in this case we do not have to fulfill any conditions.

4.4 Designing low-pass filter with a desirable response.

With this knowledge of various effects of different coefficients of the matrix and generalized bilinear transformation, now we can design a low-pass filter with a desirable response. For an example, we want to increase the gain and we want to keep the bandwidth of the response showed in figure 4.18. This we can achieve with decreasing the value of g, lets say 0.5, and maintaining the same value of $Y_1 = Y_2 = 0.81$ with satisfying the equation 4.5.

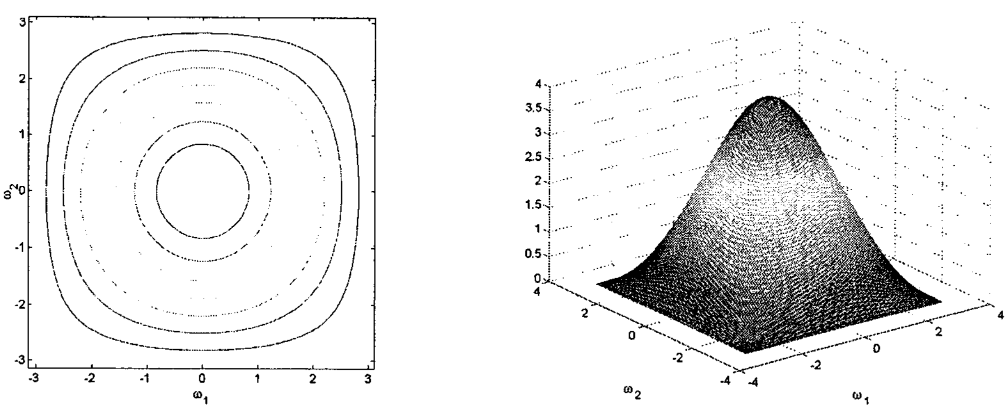


Figure 4.20: The contour and 3-D magnitude plot of the resulting 2-D low pass filter for $\mathbf{A} = \psi_1\lambda_1(a-b)^2 = 0.38$, g=0.5, $Y_1 = Y_2 = 0.81$, $K_1 = K_2 = b_{01} = b_{02} = 1$ and $a_{01} = a_{02} = -1$

It is evident that the above response has same bandwidth characteristics as figure 4.18, but the magnitude is almost 15 times more. Now, at this point if we want to make some minor changes in the gain then we can obtain this by changing the value of a_{01} and a_{02}, which will change the bandwidth little bit and that can be adjustable with either K_1 and K_2 or Y_1 and Y_2. Figure 4.21 shows the simulation results for this case and Figure 4.22 shows that we have achieved almost same bandwidth characteristics as that of the Figure 4.20.

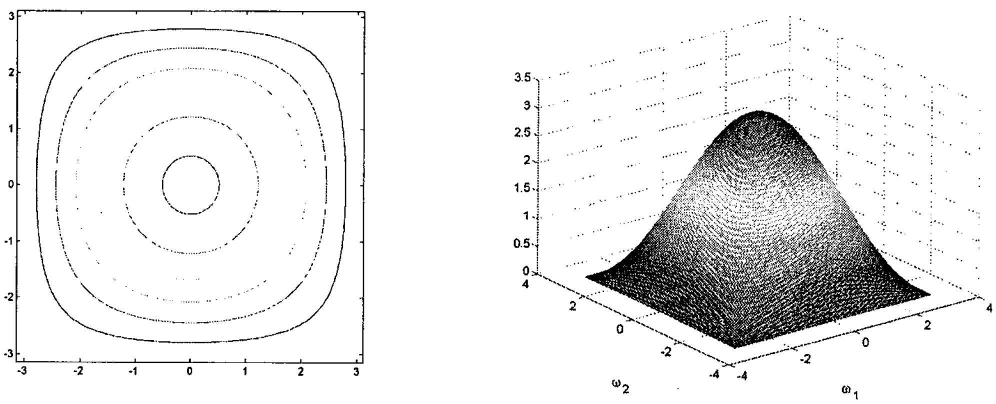


Figure 4.21: The contour and 3-D magnitude plot of the resulting 2-D low pass filter for $\mathbf{A} = \psi_1\lambda_1(a-b)^2$=0.44, g=0.5, Y_1=Y_2=0.75, $K_1 = K_2 = b_{01} = b_{02} = 1$ and $a_{01} = a_{02} = -0.8$

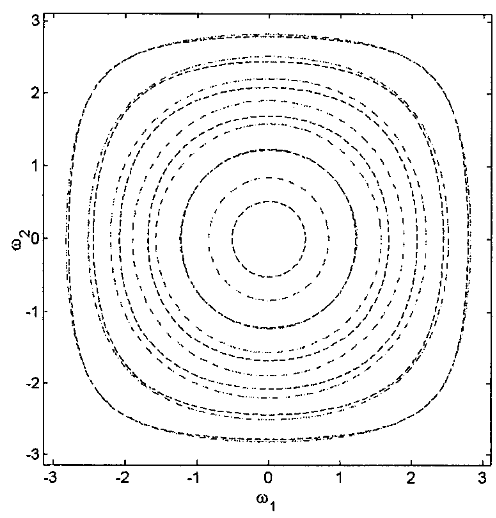


Figure 4.22: Overlapping of the contour plots showed in figure 4.20 and 4.21

And we want to keep the gain same but we want to decrease the bandwidth in the response shown in figure 4.18. This kind of response we can achieve by increasing the value of K_1 and K_2, while keeping all the matrix coefficients same, e.g. g=2, **A** = 6.1, and $Y_1 = Y_2$ =0.81, which is clearly showed in below figure.

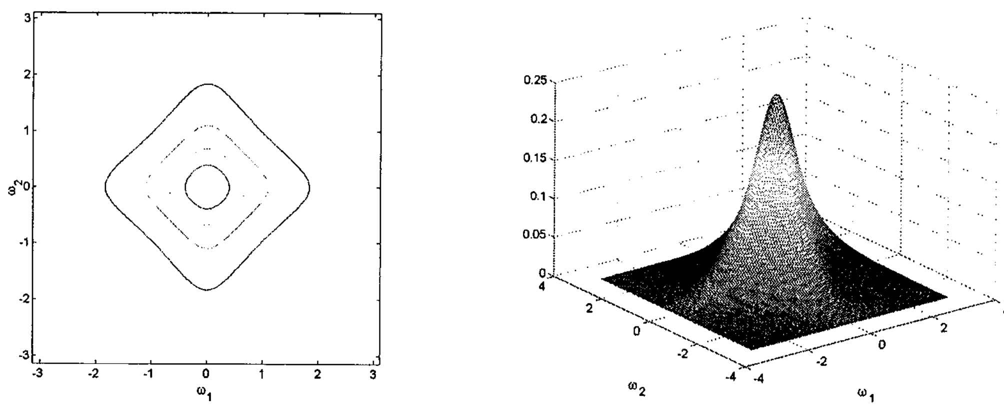


Figure 4.23: The contour and 3-D magnitude plot of the resulting 2-D low pass filter for $\mathbf{A} = \psi_1 \lambda_1 (a-b)^2$=6.1, g=2, $Y_1 = Y_2$=0.81, $b_{01} = b_{02} = 1$, $K_1 = K_2 = 3$ and $a_{01} = a_{02} = -1$

4.5 Summary and Discussion

In this chapter, we have introduced the procedure used to design 2-D low-pass recursive filter by double generalized bilinear transformations. The manner how each coefficient of the 2-D low-pass transfer function affects the magnitude response of the resulting 2-D low-pass filter has been studied in detail with simulation results.

Stability is always the most important problem in 2-D recursive filter design. The stability conditions have been obtained for the resulting 2-D digital filter with a unity degree denominator for each variable. Using the link between the stability conditions and the coefficients of the double generalized bilinear transformations, we have got the stable range for each coefficient of the 2-D low-pass transfer function when the others are specified. Also, every time we try to obtain the magnitude response for a 2-D low-pass recursive filter, these conditions need to be satisfied.

This chapter is an important work towards the study of the variable magnitude characteristics of 2-D Low-pass recursive filters by double generalized bilinear transformation. It is noted that by changing the values of the variables, the contour and symmetry characteristics can be varied. Also using the results presented in this chapter and some optimum techniques, it should be possible to design 2-D low-pass filter having desired magnitude characteristics.

5. 2-D HIGH-PASS RECURSIVE FITLER

In the previous chapter we discussed about 2-D low-pass recursive filter. And in this chapter, we study about the another basic form of the filters, 2-D High-pass recursive filter, with illustrated examples

In Section 5.1, we review some basic concepts of a 2-D high-pass filter with a graphical illustration. In section 5.2 we show the conditions on the coefficients of the generalized bilinear transformation to design 2-D high pass filter from the general 2-D transfer function derived in Chapter 3. In section 5.3 we determine the effects of the different coefficients of the 2-D high pass transfer function. And with the knowledge gained from section 5.3, in the section 5.4 we show how to design 2-D high pass recursive filter with a desirable characteristics. Section 5.5 is the summary of this chapter.

5.1 Definition of 2-D High-pass filters

From the name itself, it is clear that high-pass filter allows the high frequency components and filters out the lower frequency components. Figure 5.1 is the graphical representation of a 2-D high pass filter with pass-band, transition band and stop-band.

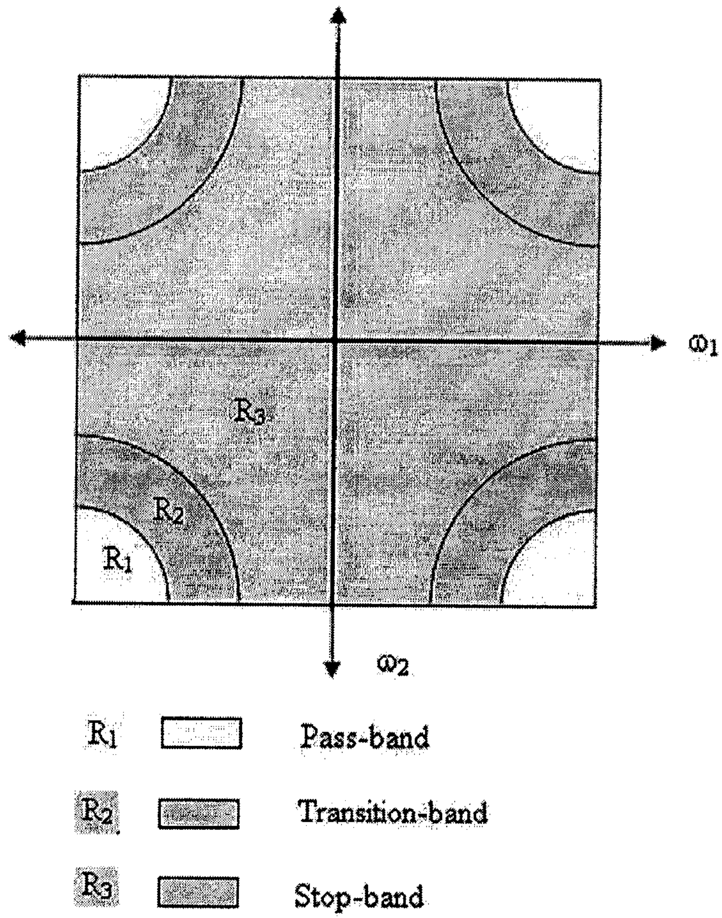


Figure 5.1: Graphical definition of a high-pass filter

Mathematically we can describe 2-D high-pass filter as below,

$$H(\omega_1, \omega_2) = 0 \qquad 0 \le \left(\omega_1^2 + \omega_2^2\right)^{\frac{1}{2}} \le \omega_{si},\ \mathrm{i} = 1, 2 \tag{5.1}$$

$$= 1 \qquad \omega_{pi} < \left(\omega_1^2 + \omega_2^2\right)^{\frac{1}{2}} < \pi,\ \mathrm{i} = 1, 2$$

Where : ω_{pi} is called pass-band radius

ω_{si} is called stop-band radius

And transition band is the region in-between ω_{pi} and ω_{si}

5.2 Defining the conditions for 2-D high-pass filter

It is possible to design a 2-D high-pass filter from the general 2-D transfer function under some conditions. We have already obtained a 2-D recursive transfer function (equation 3.28) in chapter 3 by applying the generalized bilinear transformation to the 2-D analog transfer function. So the next step is to find out the conditions to develop a 2-D high-pass recursive filter from the equation 3.28.

In one of the published papers, Gargour, Ramachandran et al. [32] have proposed condition on the coefficients of the generalized bilinear transformation to design 1-D high-pass recursive filter from analog transfer function. This says that the coefficient b_0 of the generalized bilinear transformation must be zero or negative to design a high pass filter. On the other hand in section 1.4, we already showed that for guaranteed stability, when K is greater than zero, a_0 must have the opposite sign then that of b_0. And we have checked that these conditions are also very true for two-dimensions so we can define necessary conditions to designed 2-D high-pass recursive filter as below.

- $K_i > 0;$ (5.2a)
- $-1 \le b_{0i} \le 0;\text{and } 0 \ge a_{0i} \ge 1;$ (5.2b)

  where, i=1, 2 and $a_{0i}b_{0i} < 0$

After putting these conditions in the general 2-D transfer function showed in equation 3.28, we can obtain 2-D high pass recursive transfer function. And we can express it as below

$$H(z_1, z_2) = \frac{A(z_1, z_2)}{B(z_1, z_2)} = \frac{z_1 z_2 + b_{02} z_1 + b_{01} z_2 + b_{01} b_{02}}{Q_{11} z_1 z_2 + Q_{10} z_1 + Q_{01} z_2 + Q_{00}} \tag{5.3}$$

$$\text{where, } Q_{11} = \psi_1\lambda_1(a-b)^2(K_1K_2 + Y_2K_1 + Y_1K_2) + g^2; \tag{5.3a}$$
$$Q_{10} = \psi_1\lambda_1(a-b)^2(a_{02}K_1K_2 + b_{02}Y_2K_1 + a_{02}Y_1K_2) + b_{02}g^2; \tag{5.3b}$$
$$Q_{01} = \psi_1\lambda_1(a-b)^2(a_{01}K_1K_2 + a_{01}Y_2K_1 + b_{01}Y_1K_2) + b_{01}g^2; \tag{5.3c}$$
$$Q_{00} = \psi_1\lambda_1(a-b)^2(a_{01}a_{02}K_1K_2 + a_{01}b_{02}Y_2K_1 + a_{02}b_{01}Y_1K_2) + b_{01}b_{02}g^2; \tag{5.3d}$$
$$a \neq b; \tag{5.3e}$$
$$-1 \leq b_{0i} \leq 0; \text{and } 0 \geq a_{0i} \geq 1; \quad \mathrm{i} = 1, 2 \tag{5.3f}$$
$$K_i > 0; \quad \mathrm{i} = 1, 2 \tag{5.3g}$$
$$Q_{11}b_{01}b_{02} - Q_{10}b_{01} - Q_{01}b_{02} + Q_{00} > 0 \tag{5.3h}$$
$$Q_{11}b_{01}a_{02} - Q_{10}b_{01} - Q_{01}a_{02} + Q_{00} > 0 \tag{5.3i}$$
$$Q_{11}b_{02}a_{01} - Q_{10}a_{01} - Q_{01}b_{02} + Q_{00} > 0 \tag{5.3j}$$
$$Q_{11}a_{01}a_{02} - Q_{10}a_{01} - Q_{01}a_{02} + Q_{00} > 0 \tag{5.3k}$$

In the next section, we study the effects of different coefficients shown in above equation on the high pass filter response

5.3 Frequency response of the designed 2-D high-pass recursive filter

We can simulate the above 2-D high pass transfer function in MATLAB ® and obtain 3-D magnitude plot and contour plot for a particular set of the coefficients. After writing a function "himthhp.m", we assign value to each and every coefficient of the high pass transfer function in the function "himthhp.m" to get the contour and 3-D magnitude plots for the specified values.

It is hard to analyze the effects of one particular coefficient on the high pass filter response because of the presence of many coefficients in the 2-D high pass transfer function. To assist this situation, we explore the coefficients into two groups

- coefficients offered by the generalized bilinear transformation
- coefficients offered by the 2-D analog transfer function

To make the situation simpler, while investigating the effects of the coefficients of one group, we keep the coefficients of another group to some content value. We have obtained 2-D analog transfer function using a VSHP, which was generated with the help of matrix theory, so we call the second group as "coefficients of matrix"

5.3.1 2-D high-pass recursive filter response with each different coefficient of the generalized bilinear transformation

In this section, we investigate how the effects of each different coefficient of the generalized bilinear transformation modify the designed filter response. In order to study the effects of the coefficient and to make the analyzing job easy, we keep changing the coefficient under the examination, while setting the others to some constant values. Equation 5.2 shows the range of each coefficient of the generalized bilinear transformation for the designed 2-D high-pass recursive filter. As we have already discussed, we also assign each coefficient of the matrix to some specific value. In this section (section 5.3.1) we use, $Y_1 = Y_2 = 0.81$, g = 2 and $\mathbf{A} = \psi_1 \lambda_1 (a-b)^2 = 6.1$

High Pass frequency response with different values of K_1

In order to distinguish the effects of the other coefficients beside K_1 and not to lose any generality, we keep other coefficients to some constant values. We have used $K_2 = a_{01} = a_{02} = 1$ and $b_{01} = b_{02} = -1$, with these values we have checked the range for

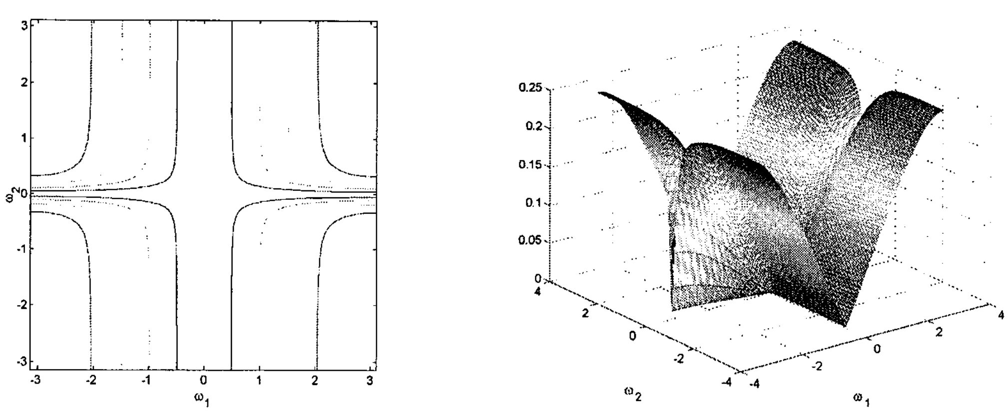


Figure 5.2(a): $K_1 = 0.1$

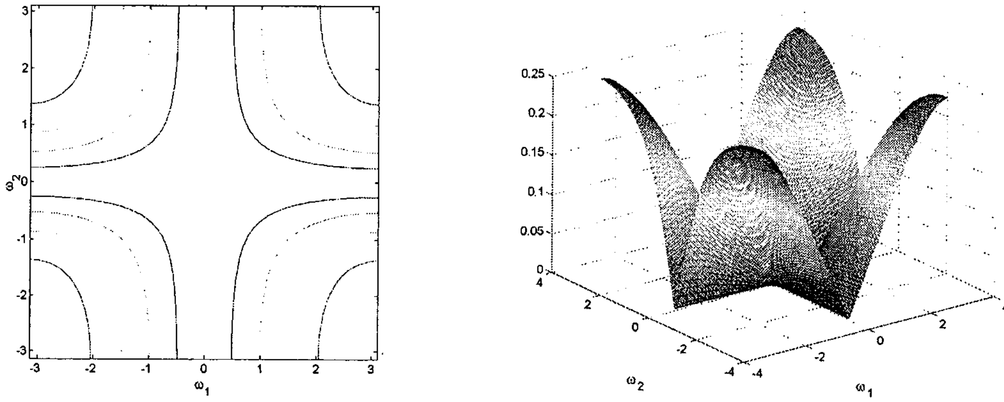


Figure 5.2(b): K_1 =0.5

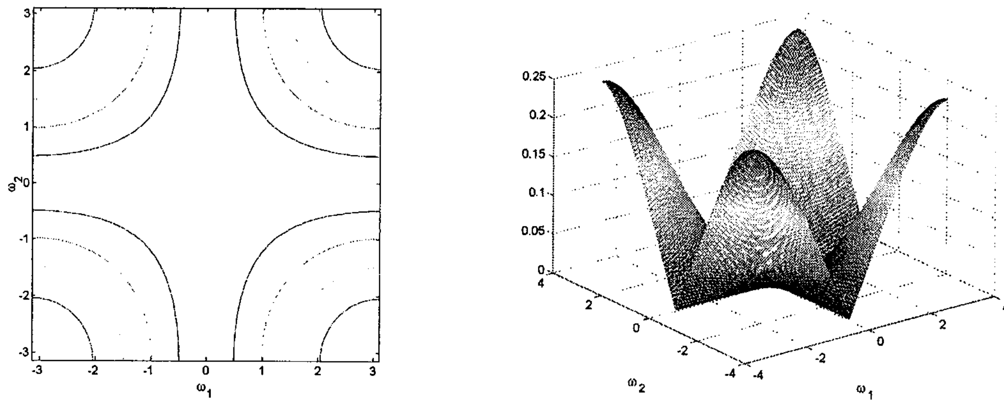


Figure 5.2(c): K_1 =1

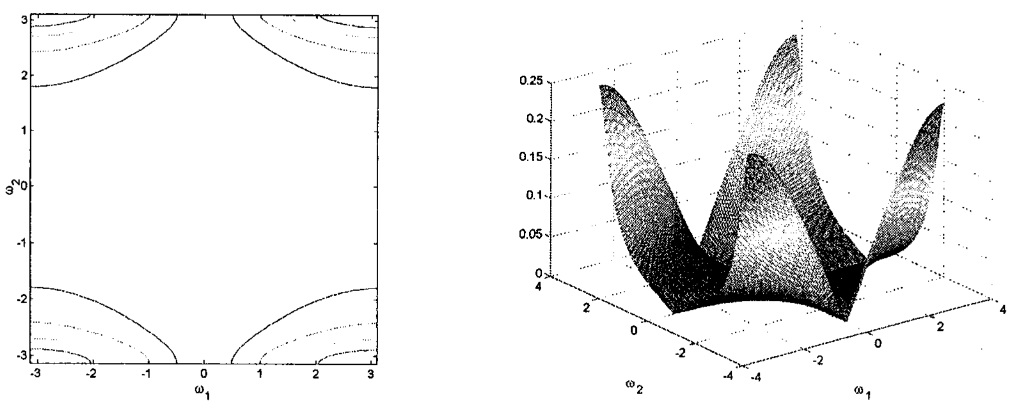


Figure 5.2(d): K_1 =5

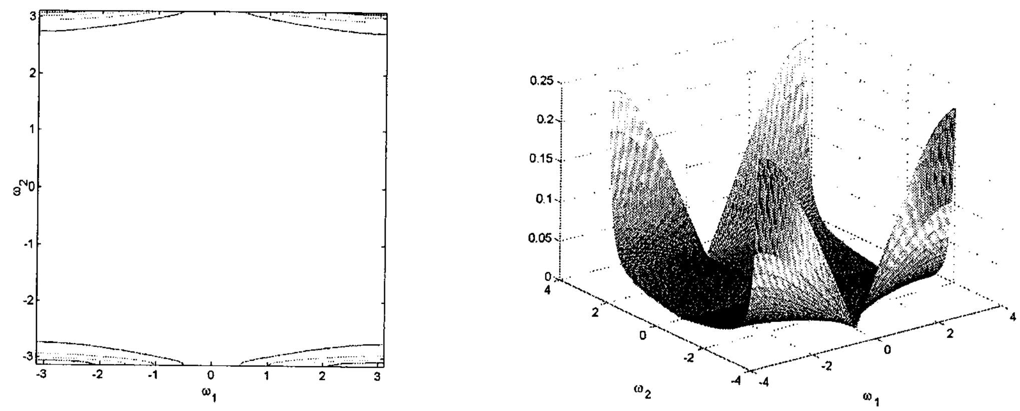


Figure 5.2(e): K_1=20

Figure 5.2: The contour and 3-D magnitude plot of the resulting 2-D high pass filter for different values of K_1 with $K_2 = a_{01} = a_{02} = 1$ and $b_{01} = b_{02} = -1$.

K_1 using "himthhp.m" in MATLAB®, and it gave us stable result response up to K_1=10,000. Figure 5.2 shows the contour and the 3-D magnitude response plot for different values of K_1.

This is detectable that the value of K_1 is inversely proportional with the bandwidth in ω_2 domain. Also, it does not modify the amplitude of the response and the bandwidth in ω_1 domain. We can see that the amplitude remains constant approximately 0.23 for all values of K_1 and each contour plot contains Quadrantal symmetry

High Pass frequency response with different values of K_2

K_2 also offers stable 2-D high pass response up to 10,000 value, when other coefficients of the generalized bilinear transfer function fix to unity with proper signs, e.g. $K_1 = a_{01} = a_{02} = 1$ and $b_{01} = b_{02} = -1$. Below figure shows few responses for different values of K_2. The frequency response for K_2 = 1 is already showed in figure 5.2(c), so we do not show it again here in the below figure.

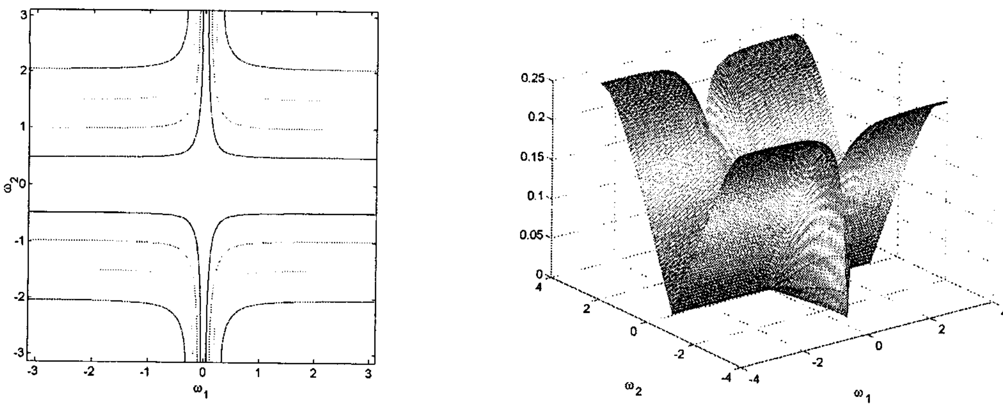


Figure 5.3(a): K_2=0.1

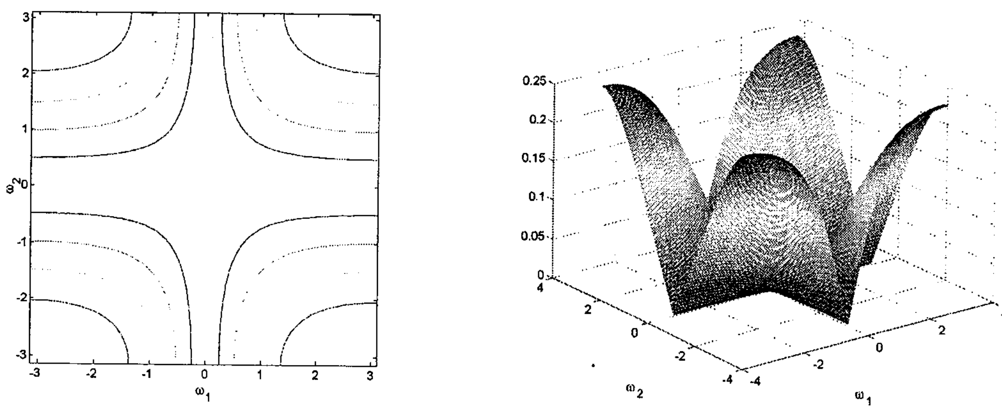


Figure 5.3(b): K_2=0.5

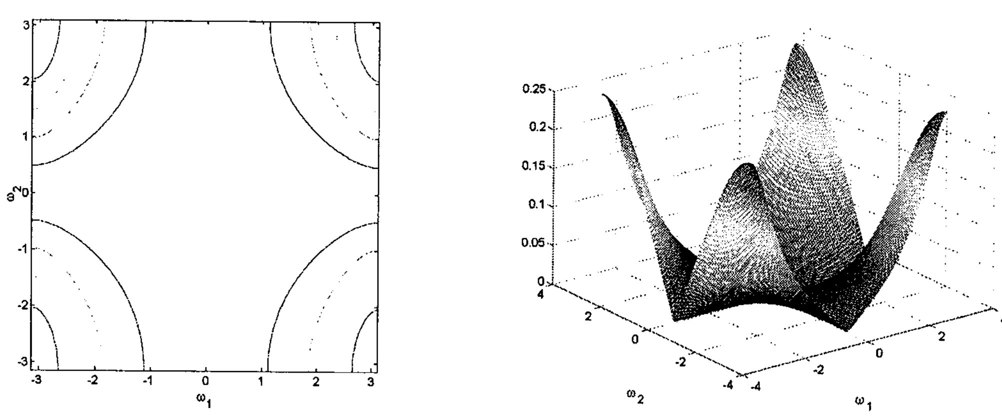
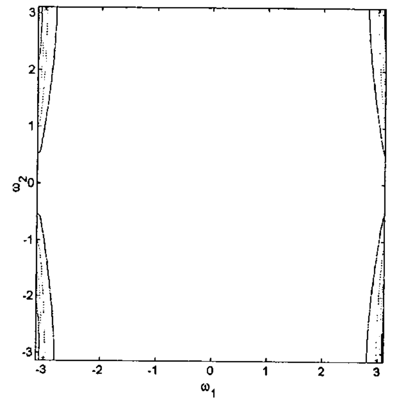


Figure 5.3(c): K_2=2.5



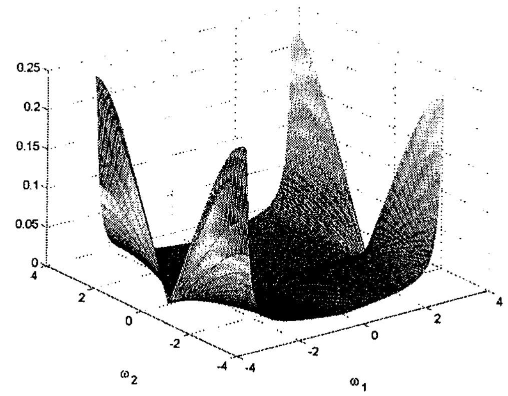


Figure 5.3(d): K_2=25

Figure 5.3: The contour and 3-D magnitude plot of the resulting 2-D high pass filter for different values of K_2 with $K_1 = a_{01} = a_{02} = 1$ and $b_{01} = b_{02} = -1$.

After reviewing the frequency response we can say that value of K_2 is inversely proportional with the bandwidth of ω_1 and it does not alter the bandwidth of ω_2. It is visible that it does not have any effects on the gain of the high-pass response as the amplitude value in the 3-D magnitude plot remains constant approximately 0.23 throughout, and each contour plot has Quadrantal symmetry.

The main effect of K_1 and K_2 is on the bandwidth of the designed high-pass frequency response, so we can call them as **band-effective coefficients.** And using this characteristic of K_1 and K_2, we can design the 2-D high-pass filter with a desirable bandwidth. For an example, we want to design a high-pass filter with bigger bandwidth response and if we utilize the above knowledge then we can say that by setting smaller value to K_1 and K_2 we can achieve the desirable response. And to design really very narrow banded high-pass filter, we have to assign higher values to K_1 and K_2. This is evident in the below figure.

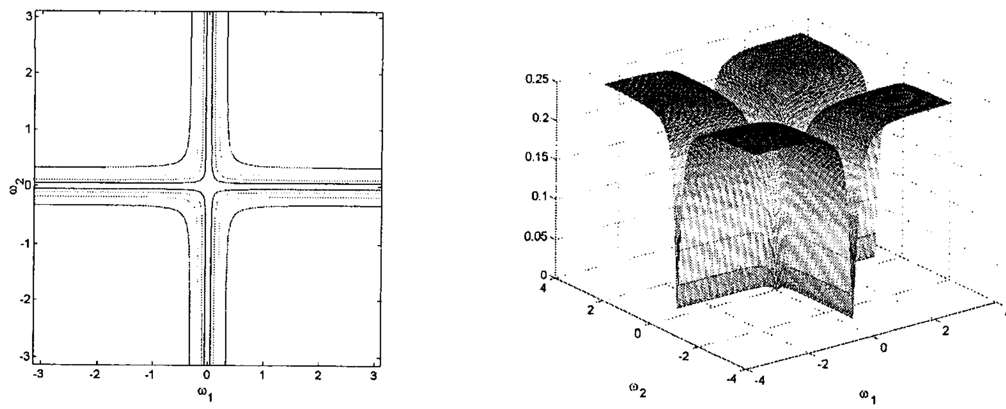


Figure 5.4(a): $K_1 = K_2 = 0.1$

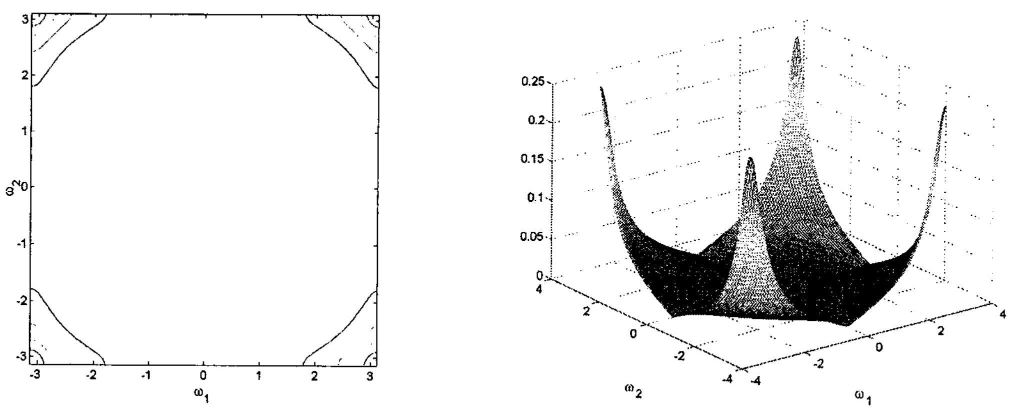


Figure 5.4(b): $K_1 = K_2 = 5$

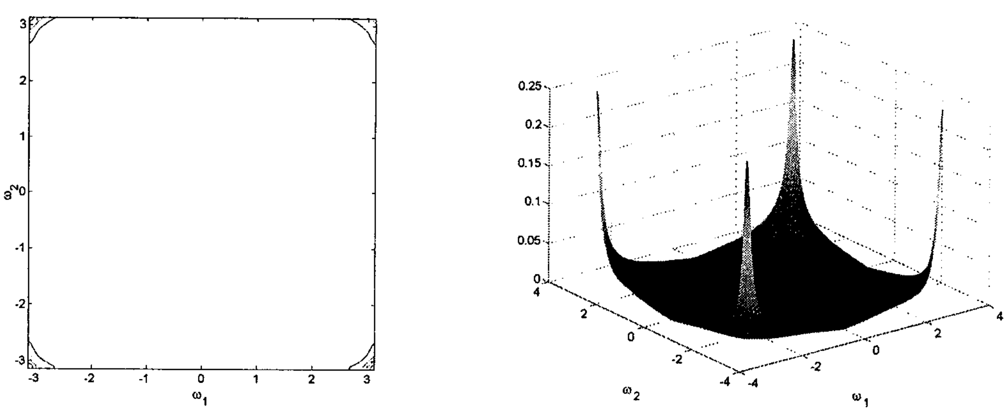


Figure 5.4(c): $K_1 = K_2 = 17$

Figure 5.4: The contour and 3-D magnitude plot of the resulting 2-D high pass filter for different values of K_1 and K_2 with $a_{01} = a_{02} = 1$ and $b_{01} = b_{02} = -1$.

Another noticeable thing in this case is each contour plot has Quadrantal symmetry, Diagonal Symmetry and Four-fold symmetry. In a single word each contour plot contains Octagonal symmetry.

High Pass frequency response with different values of a_{01}

To reduce the complications and to detect the consequences of a_{01} on the high pass filter response, we assign the other generalized bilinear coefficients to unity value with proper signs, e.g. $K_1 = K_2 = a_{02} = 1$ and. $b_{01} = b_{02} = -1$. With this setting, we have

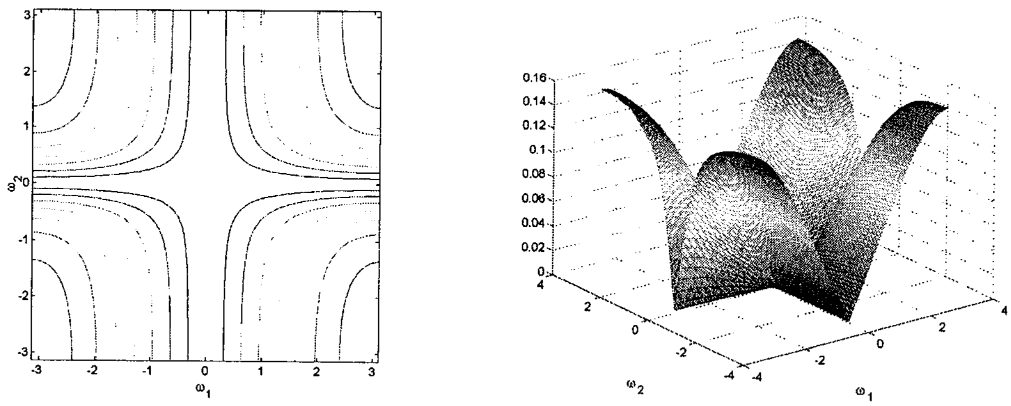


Figure 5.5(a): a_{01}=0

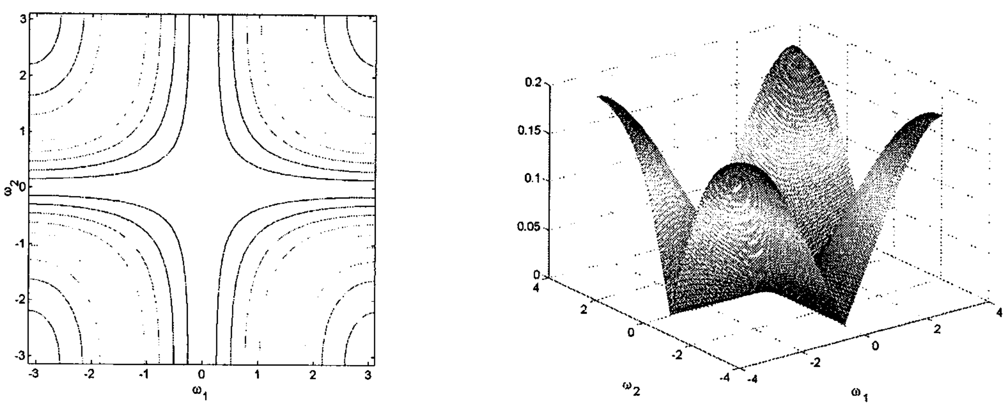


Figure 5.5(b): a_{01}=0.5

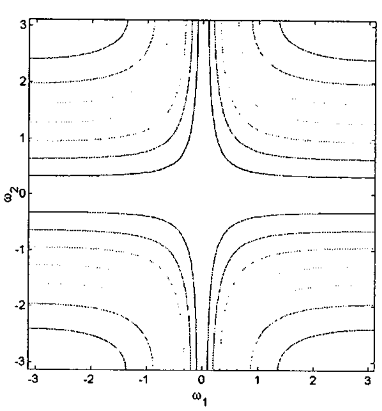
Figure 5.5: The contour and 3-D magnitude plot of the resulting 2-D high pass filter for different values of a_{01} with $K_1 = K_2 = a_{02} = 1$ and. $b_{01} = b_{02} = -1$

signs, e.g. $K_1 = K_2 = a_{02} = 1$ and. $b_{01} = b_{02} = -1$. With this setting, we have checked that the designed 2-D high-pass recursive filter gives stable results for the complete range of a_{01} from 0 to 1. The contour and 3-D magnitude plots with different values of a_{01} are given in figure 5.5. As we have already presented the response for a_{01} = 1 in figure 5.2(c), here we only show the results for $a_{01} = 0$ and $a_{01} = 0.5$.

After analyzing the above figure, we can state that the main effect of a_{01} is on the gain of the designed filter. It is directly proportional to amplitude of the high-pass filter. As the value increases from 0 to 1, the amplitude approximately increases from 0.14 to 0.23. a_{01} also affects the bandwidth in ω_2 domain, it is obvious that a_{01} is inversely proportional to the bandwidth in ω_2 domain and the contour plots have Quadrantal symmetry.

High Pass frequency response with different values of a_{02}

To understand the effects of a_{02} on the frequency response of the designed filter, we set other coefficients of the generalized bilinear transformation to unity with proper sign, e.g. $K_1 = K_2 = a_{01} = 1$ and $b_{01} = b_{02} = -1$. Same as the case of a_{01}, a_{02} also gives



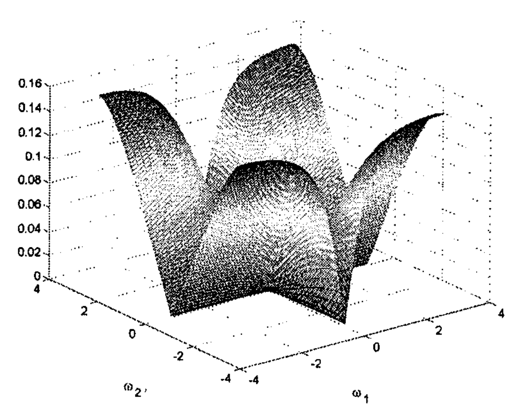


Figure 5.6(a): a_{02}=0

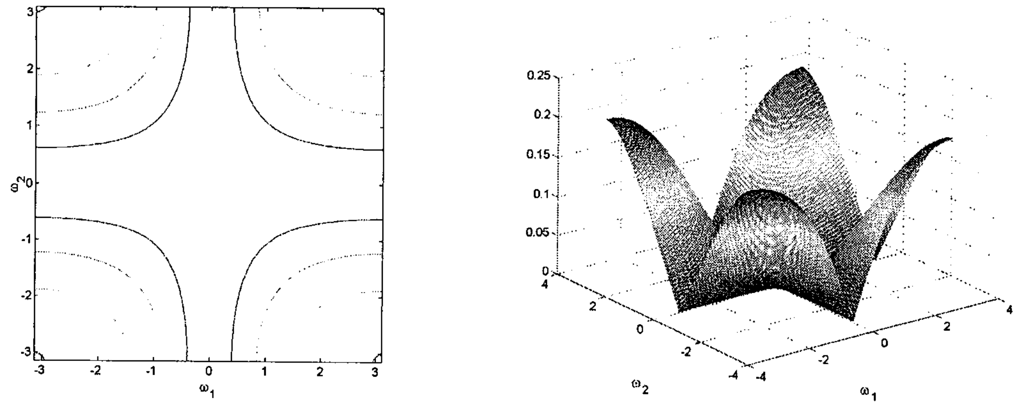


Figure 5.6(b): a_{02}=0.6

Figure 5.6: The contour and 3-D magnitude plot of the resulting 2-D high pass filter for different values of a_{01} with $K_1 = K_2 = a_{01} = 1$ and $b_{01} = b_{02} = -1$.

stable high pass filter for the entire range, e.g. from 0 to 1. Below figure shows simulation results for different values of a_{02}. Simulation result for a_{02}= 1 is already showed in figure 5.2 (c), so we do not show it again in the above figure.

It is evident that a_{02} is also directly proportional to the magnitude of the high pass filter and inversely proportional to the bandwidth in ω_1 region. Another interesting thing is the frequency response in this case is approximately 90 degree rotated with respect to the relative case of a_{01}.

It is clear that K_1 and K_2 has greater effects on the bandwidth of the response than the effects offered by a_{01} and a_{02}. a_{01} and a_{02} are mainly gain affected coefficient. So we call them **gain-effective coefficients**. In the below figure we show that we can reduce the gain of the designed filter response by setting lower values of a_{01} and a_{02} together.

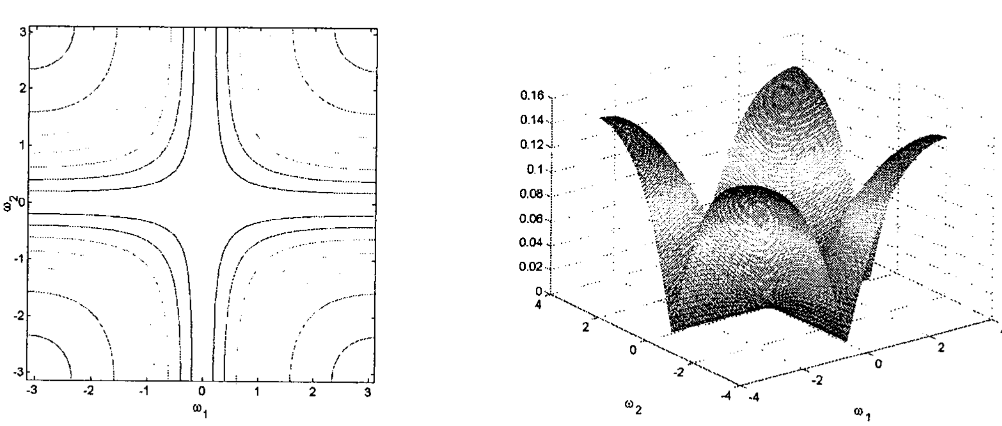


Figure 5.7(a): $a_{01} = a_{02} = 0.5$

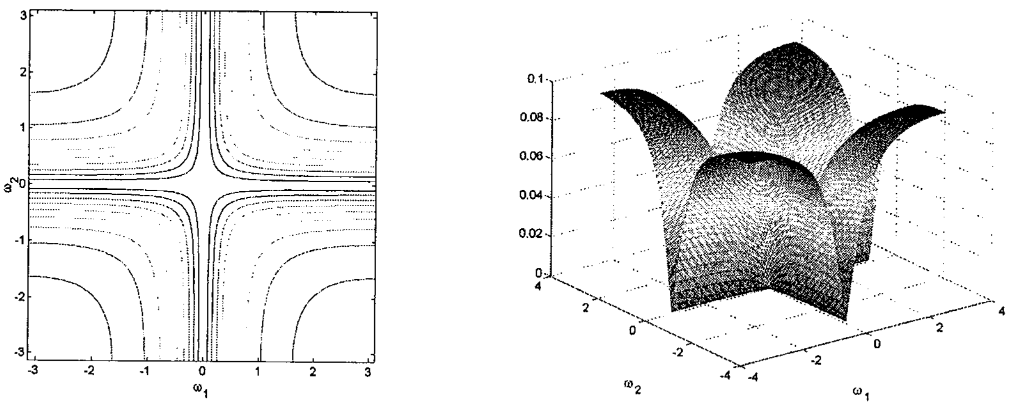


Figure 5.7(b): $a_{01} = a_{02} = 0$

Figure 5.7: The contour and 3-D magnitude plot of the resulting 2-D high pass filter for different values of a_{01} and a_{02} with $K_1 = K_2 = 1$ and $b_{01} = b_{02} = -1$

It is observable that when we assign equal values to a_{01} and a_{02}, the contour plot obtains Octagonal symmetry same as the case of $K_1 = K_2$. When a_{01} and a_{02} change from lower boundary to upper boundary, the magnitude of the response change from 0.09 to 0.23. We noticed that the gain of the designed filter reduce significantly when we combine the effect of a_{0i} with bigger K_i.

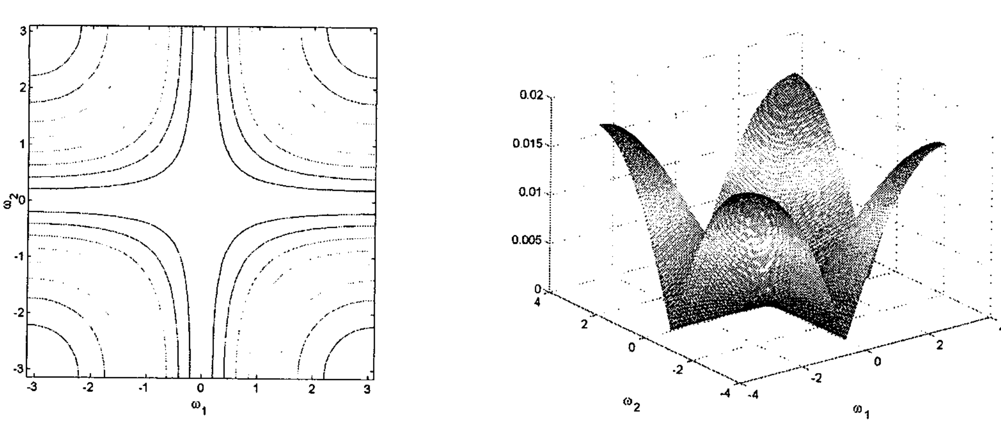


Figure 5.8(a): $K_1 = K_2 = 5$; $a_{01} = a_{02} = 0.1$

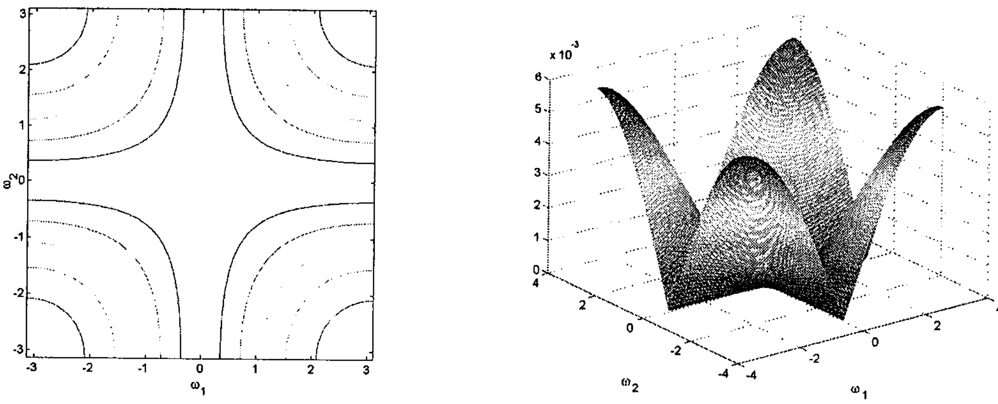


Figure 5.8(b): $K_1 = K_2 = 10$; $a_{01} = a_{02} = 0.1$

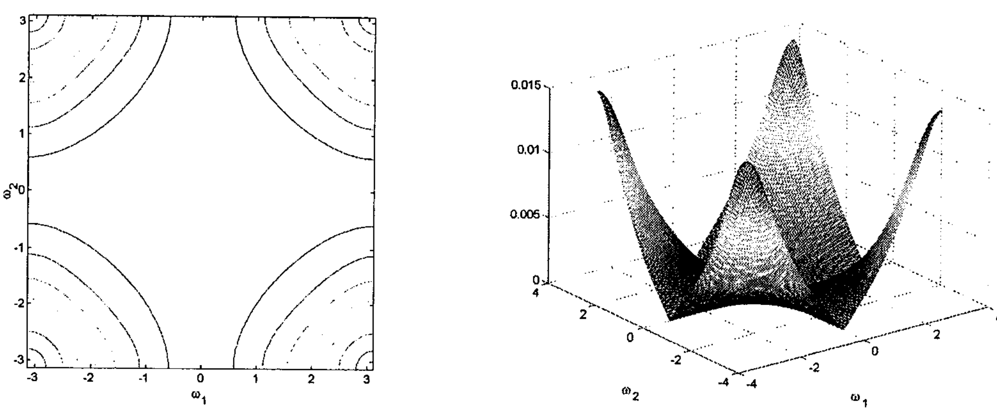


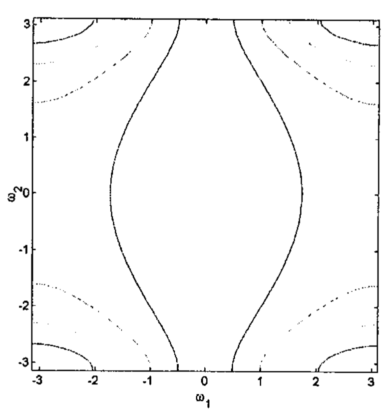
Figure 5.8(c): $K_1 = K_2 = 10$; $a_{01} = a_{02} = 0.5$

Figure 5.8: The contour and 3-D magnitude plot of the resulting 2-D high pass filter for different values of K_1, K_2, a_{01} and a_{02} with $b_{01} = b_{02} = -1$.

And it can be seen that, this combination gives us a wide range of the gain of the designed 2-D high pass filter from 0.23 to $5.3*10^{-3}$. But at the same time, we can only reduce the gain, we can not increase the gain and this inspires us to find another gain effective coefficient for the designed high pass filter to achieve desirable gain. It is also visible that the bandwidth of pass band of the response decrease with higher values of K_1 and K_2.

High Pass frequency response with different values of b_{01}

To analyze the consequences of b_{01} on the frequency response of the designed filter and to make the situation easy, we hold other coefficients of the generalized bilinear transformation to unity with appropriate sign, e.g. $K_1 = K_2 = a_{01} = a_{02} = 1$ and $b_{02} = -1$. We have verified that b_{01} gives stable output for the full range, e.g. from -1 to 0. Below figure shows simulation results for different values of b_{01}. Simulation result for b_{01} = -1 is already showed in figure 5.2 (c).



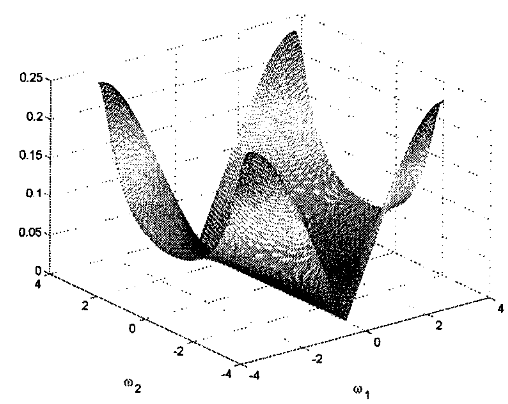
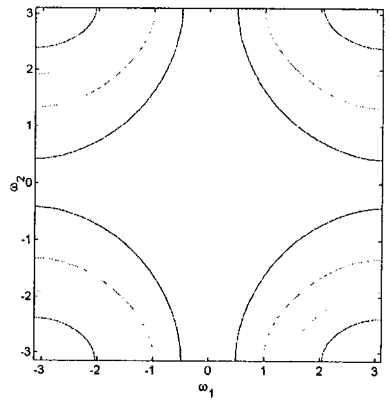


Figure 5.9(a): b_{01} =0



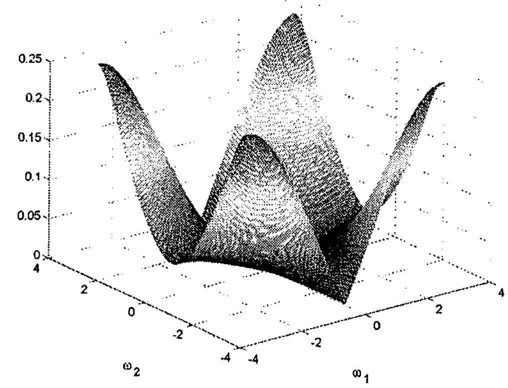


Figure 5.9(b): b_{01}=-0.5

Figure 5.9: The contour and 3-D magnitude plot of the resulting 2-D high pass filter for different values of b_{01} with $K_1 = K_2 = a_{01} = a_{02} = 1$ and $b_{02} = -1$.

It is visible in the above results that assigning non unity value to b_{01} is resulting in non-zero gain in the stop-band portion in ω_1 domain. And it is also inversely proportional to the bandwidth of the pass-band in ω_2 domain. It is also observable that each contour plot contains Quadrantal symmetry and the magnitude value remains same in each case.

High Pass frequency response with different values of b_{02}

In many ways we can combine the coefficients of the bilinear transformation, but to make the scenario simple and not to lose any generality, we hold the other coefficients to unity with proper signs, e.g. $K_1 = K_2 = a_{01} = a_{02} = 1$ and $b_{01} = -1$. With this condition, we have checked that the designed 2-D high-pass recursive filter gives stable output for the whole range of b_{02} from -1 to 0. Below figure shows the contour and 3-D magnitude plots with different values of b_{02}. As we have already studied the response for b_{02} = -1 in figure 5.2(c), here we only give the results for b_{02} = 0 and b_{02} = -0.4

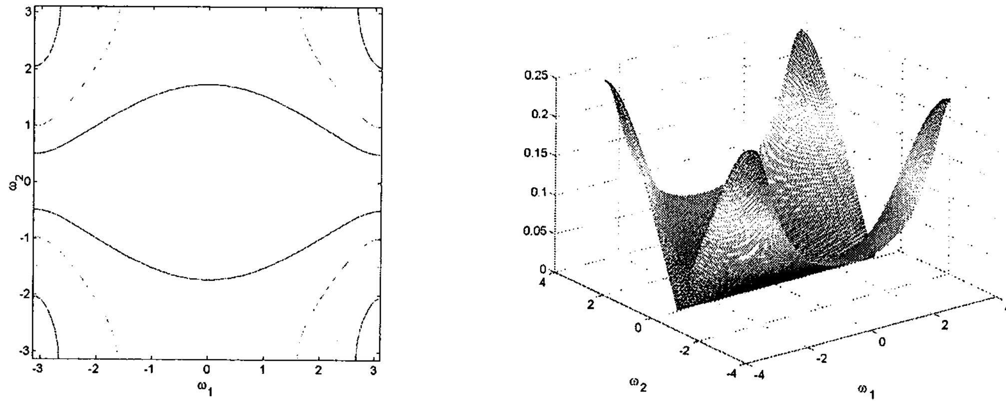


Figure 5.10(a): b_{02}=0

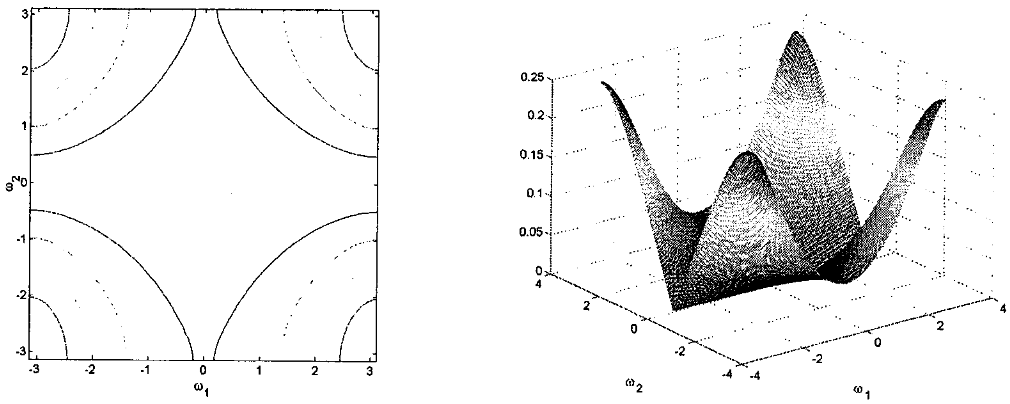


Figure 5.10(b): b_{02}=-0.4

Figure 5.10: The contour and 3-D magnitude plot of the resulting 2-D high pass filter for different values of b_{02} with $K_1 = K_2 = a_{01} = a_{02} = 1$ and $b_{01} = -1$.

It is obvious that the response with b_{02} is approximately 90 degree rotated with respect to the response of b_{01}. In this case too, we can see non-zero gain in the stop-band region when the non unity value to b_{02} is assigned. Non-zero gain in ω_2 region is directly proportional to the value of b_{02}. Another noticeable thing is that b_{02} does not modify the gain of the high pass response and presence of the Quadrantal symmetry in the contour plots.

Non-zero gain in the stop band of the filter response might be change the filter polarity from low-pass to high-pass, so we call b_{01} and b_{02} as **polarity-effective coefficients**. By changing the band-effective coefficients K_1 and K_2, we can rectify the problem of the non-zero gain in the stop band. If we set non unity values to both coefficients b_{01} and b_{02}, it will produce unwanted non-zero gain at stop-band in both ω_1 and ω_2 domains. Below figure shows the non-zero gain for $b_{01} = b_{02} = 0$ and the solution of this problem.

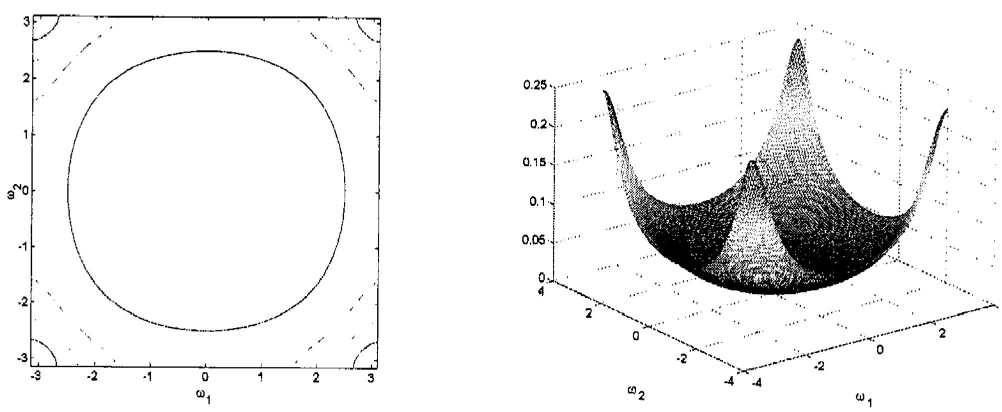


Figure 5.11(a): $K_1 = K_2 = 1$; $b_{01} = b_{02} = 0$

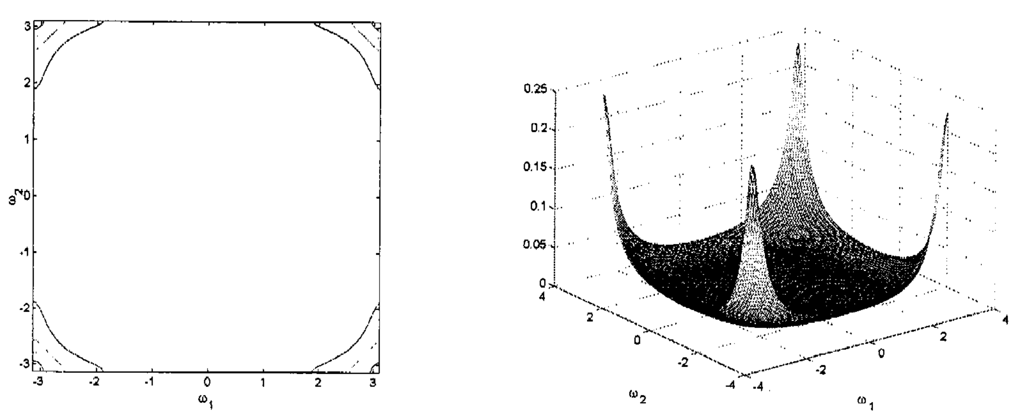


Figure 5.11(b): $K_1 = K_2 = 3$; $b_{01} = b_{02} = 0$

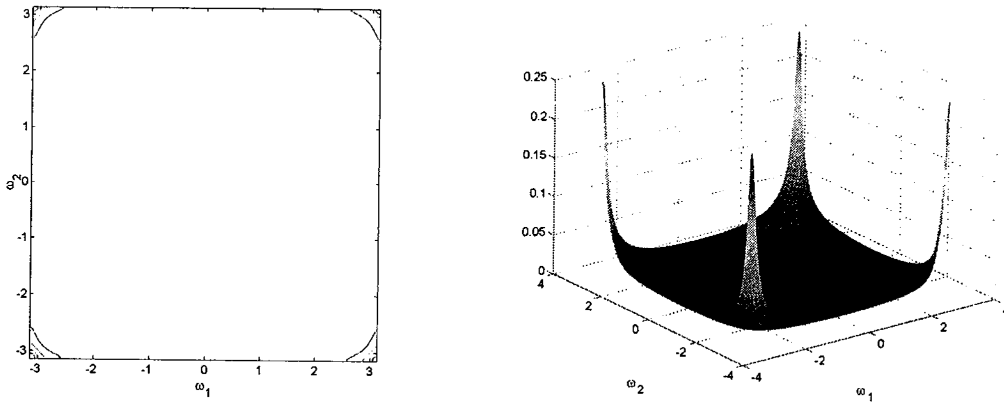


Figure 5.11(c): $K_1=K_2=7$; $b_{01}=b_{02}=0$

Figure 5.11: The contour and 3-D magnitude plot of the resulting 2-D high pass filter for different values of K_1, K_2, b_{01} and b_{02} with $a_{01}= a_{02}=1$.

It indicates that with bigger value of K_i, we can reduce the non-zero gain in the stop-band region, but on the other hand we get smaller bandwidth in the pass-band of the response. So we have to bargain in between these two parameters, non-zero gain in the stop-band region and bandwidth of the pass-band, for the particular application.

5.3.2 2-D High-pass recursive filter response with each different coefficient of the matrix

In this section, we analyze how each different coefficient of the matrix changes the designed high pass filter response. In order to explore the effect of the coefficient on the response, we keep changing the coefficient under the test, while holding the others to some specific values. To make the situation simple and to investigate the effects of the coefficients of the matrix, we should keep each coefficient of the generalized bilinear transformation to some specific values. We use $K_1 = K_2 = a_{01} = a_{02} = 1$ and $b_{01} = b_{02} = -1$ in the section 5.3.2 for simplicity.

If we plug in these values of the generalized bilinear transformation in the Equation 3.27 then we can derive conditions on the matrix coefficients for the high pass frequency response, which are as below.

- $\psi_1\lambda_1 > 0$ (5.4a)
- $Y_2 > 0$ (5.4b)
- $Y_1 > 0$ (5.4c)
- $g \neq 0.$ (5.4d)

Same as the case of 2-D low pass filter, in this case too if we follow the condition showed in equation 4.5, then we can rewrite the 2-D high pass transfer function showed in 5.3 as a product of two different two-variable polynomials showed in equation 4.4. This is the necessary condition to have Quadrantal symmetry in the response.

One of the possible combinations for matrix coefficients satisfying the condition showed in equation 4.5 is **A**=$\psi_1\lambda_1(a-b)^2$=6.1, g=2, Y_1= Y_2 =0.81. We have used this combination for studying the effects of the generalized bilinear transformation, and this is the reason we had Quadrantal symmetry in all the cases we have studied so far. In this section, with reference of these values of the matrix coefficients, we investigate the effects of the each matrix coefficient on the designed 2-D recursive filter.

We have observed that the frequency response of the designed filter for a value **A**=$\psi_1\lambda_1(a-b)^2$, using different combinations of ψ_1, a, b and λ_1, remains same. So instead of studying them individually we study them as a common variable **A,** where **A**=$\psi_1\lambda_1(a-b)^2$.

High Pass frequency response with different values of A =$\psi_1\lambda_1(a-b)^2$

To study the effects of **A** on the frequency response and to make the job easy, we hold the other matrix coefficients to the specific value showed above, e.g. g=2, Y_1= Y_2

=0.81. With this specification, we have checked that the designed 2-D high-pass recursive filter gives stable output for **A** equal to 1 to 1000. Below figure shows the contour and 3-D magnitude plots with different values of **A**.

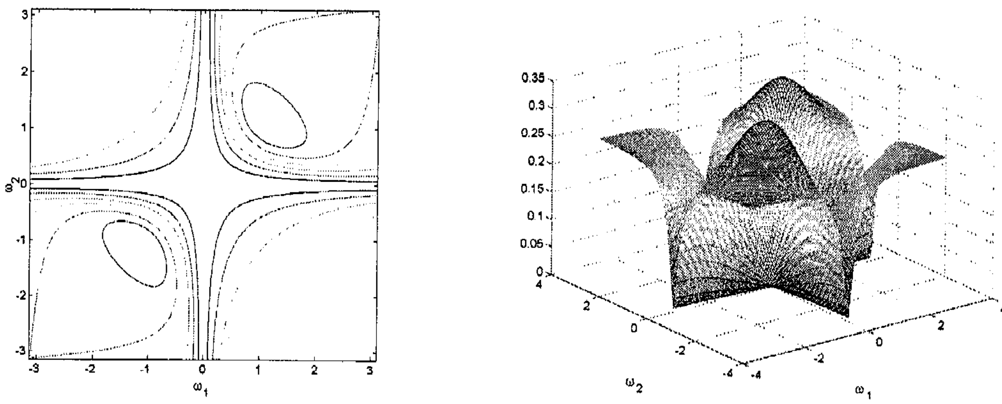


Figure 5.12(a): **A**=1

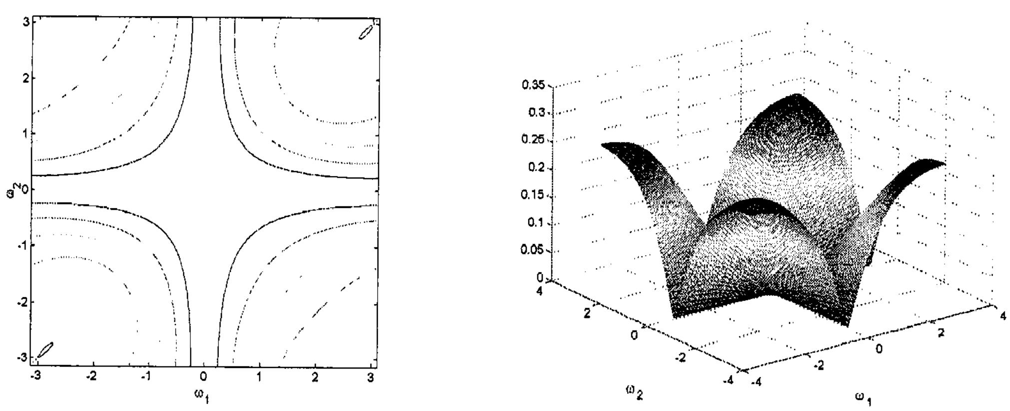


Figure 5.12(b): **A**=3

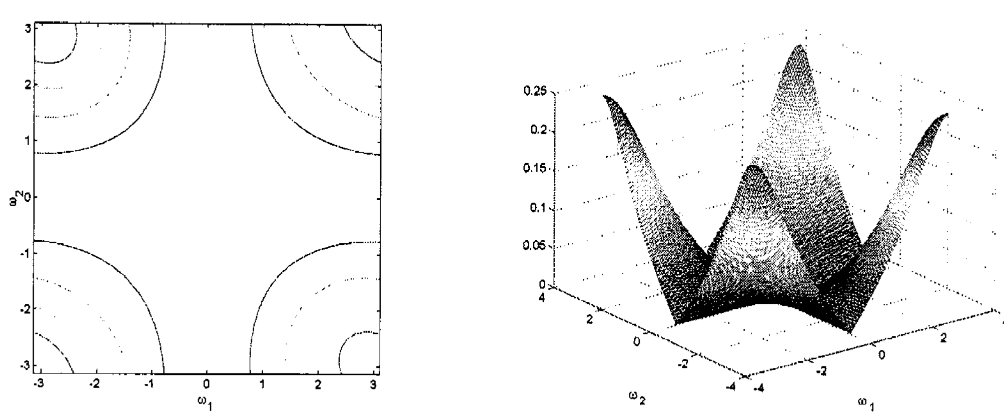


Figure 5.12(c): **A**=10

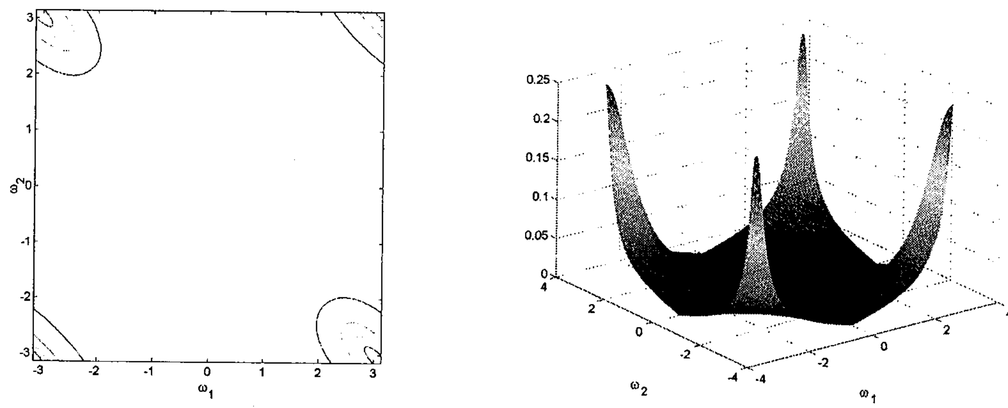


Figure 5.12(d): **A**=50

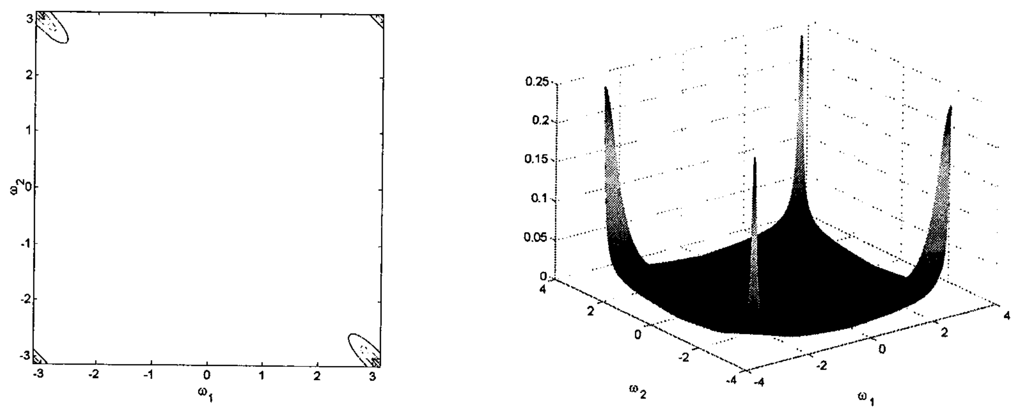


Figure 5.12(e): **A**=200

Figure 5.12: The contour and 3-D magnitude plot of the resulting 2-D high pass filter for different values of $\mathbf{A} = \psi_1 \lambda_1 (a-b)^2$ with $g = 2$, $Y_1 = Y_2 = 0.81$.

It is obvious that **A** does not change the gain of the frequency response. After comparing these simulation results to the reference response with **A** =6.1, we can say that if we decrease the value of **A** from the reference value 6.1 then in the frequency response, first and third quadrant obtain more bandwidth than the second and forth quadrant and vice-versa. In general value of **A** is inversely proportional to the bandwidth of the frequency response. In this case we did not follow the condition showed in equation 4.5, hence we did not obtain Quadrantal symmetry in the response. Though, presence of Diagonal symmetry and Centro symmetry are noticeable.

High Pass frequency response with different values of g

For exploring the consequences of g on the designed filter, we should keep the other matrix coefficients to some specific value. To make the situation simple we set each of them to the reference value, e.g. **A** $=\psi_1\lambda_1(a-b)^2$=6.1, Y_1= Y_2 =0.81 and we keep changing the value of g. Another thing we have noticed that absolute value of g makes difference in the response not the real value of g, means g = 2 and g = -2 give the same response. So for simplicity we study effects of g with all positive values. Figure 5.2(c) is the reference frequency response with value of g = 2.

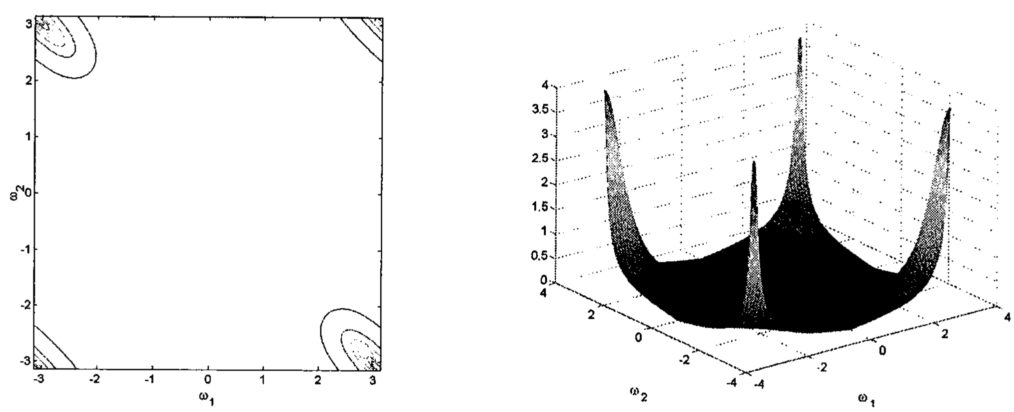


Figure 5.13(a): g=0.5

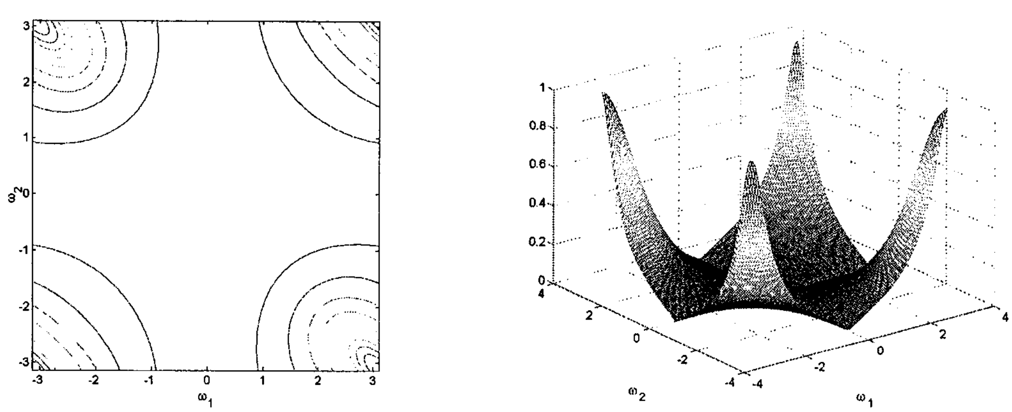


Figure 5.13(b): g=1

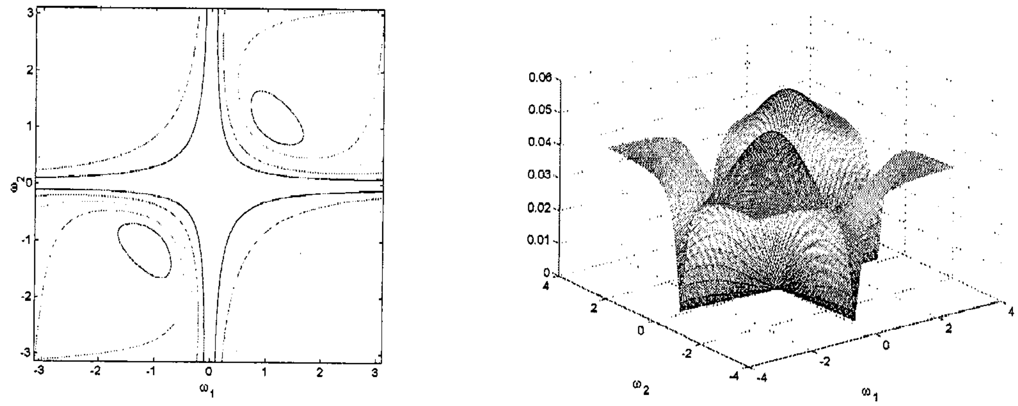


Figure 5.13(c): g=3

Figure 5.13: The contour and 3-D magnitude plot of the resulting 2-D high pass filter for different values of g with $\mathbf{A} = \psi_1 \lambda_1 (a-b)^2 = 6.1$, $Y_1 = Y_2 = 0.81$.

It is evident that g has significant effects on the magnitude of the frequency response and g is inversely proportional to the magnitude value of the designed high-pass response. Also in the frequency response, first and third quadrant obtain more bandwidth than the second and forth quadrant when we increase the value of g from the reference value and vice-versa. Beside this, it is also evident that g is directly proportional to the bandwidth of the frequency response and this effect is almost counter part of the effect of **A** on the bandwidth. We can see that each contour plot has Diagonal symmetry and Centro symmetry. Because of the significant effect of g on the magnitude of the high pass filter, we should consider g as **gain-effective coefficient.**

If we combine the value of **A** and g in some particular manner then we can change the gain of the frequency response drastically. Below figure shows some simulation results for different values of **A** and g, and after observing these cases in details we can say that if we divide g by a factor "x" and **A** by the square of the factor "x", then we can increase the gain by square of that factor "x", while there are not any effects on the bandwidth of the frequency response. And this is very evident from the transfer function of the designed filter.

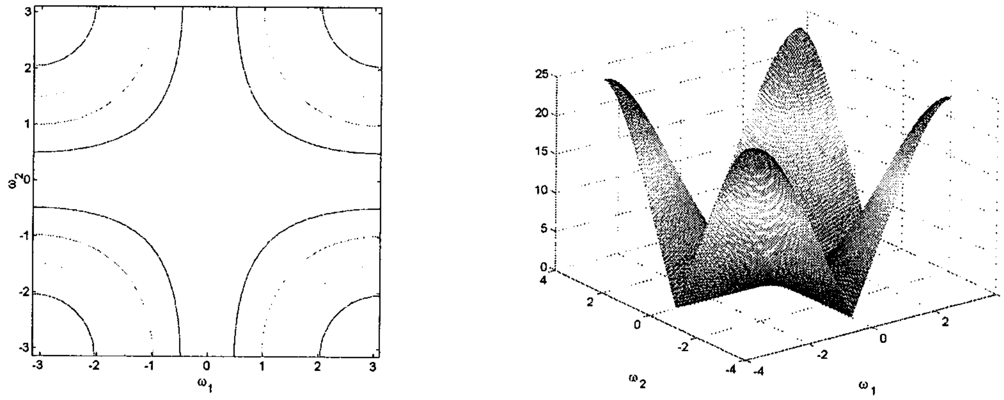


Figure 5.14(a): A=0.061 and g=0.2

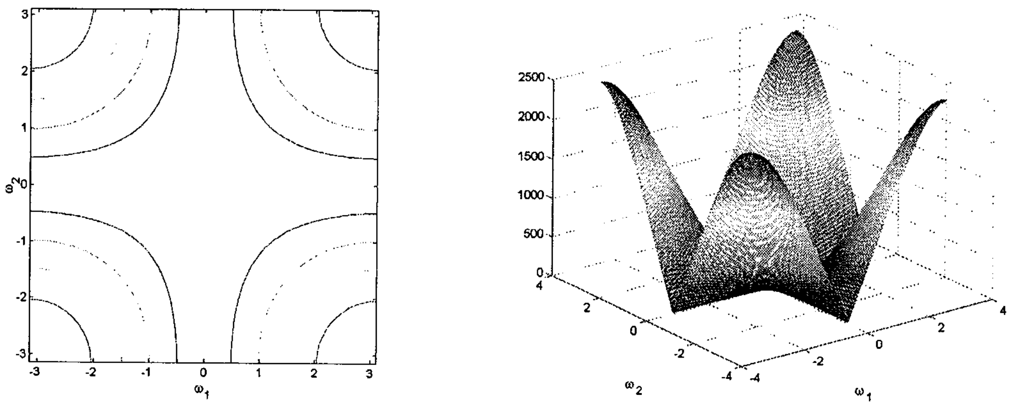


Figure 5.14(b): A=0.00061 and g=0.02

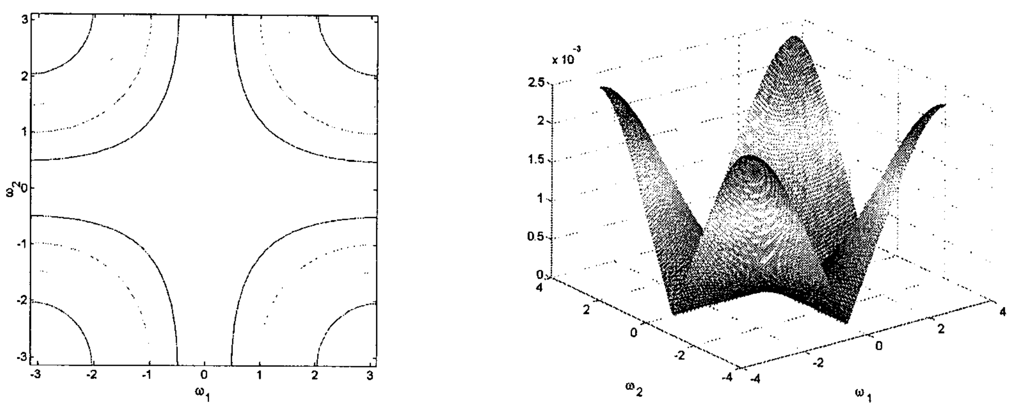


Figure 5.14(c): A=610 and g=20

Figure 5.14: The contour and 3-D magnitude plot of the resulting 2-D high pass filter for different values of **A** $=\psi_1\lambda_1(a-b)^2$ and g with $Y_1=Y_2$ =0.81.

High Pass frequency response with different values Y_1

With the purpose of realizing the effect of Y_1 on the designed high pass filter, we set the other parameters of the matrix to the reference value e.g., Y_2=0.81, g =2, **A** $=\psi_1\lambda_1(a-b)^2$=6.1. With these values, we have checked that Y_1 gives stable response up to 1000. Figure 5.2(c) shows the reference response for Y_1=0.81 and we analyze the effect of Y_1 with this response.

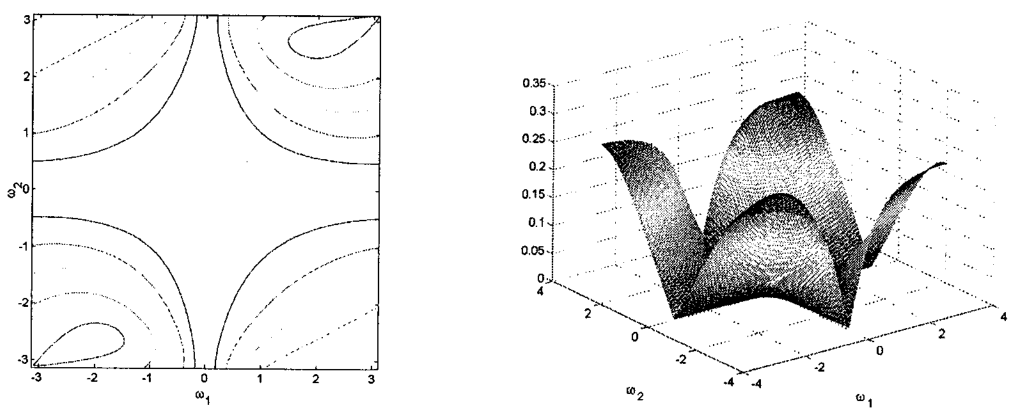


Figure 5.15(a): Y_1=0.3

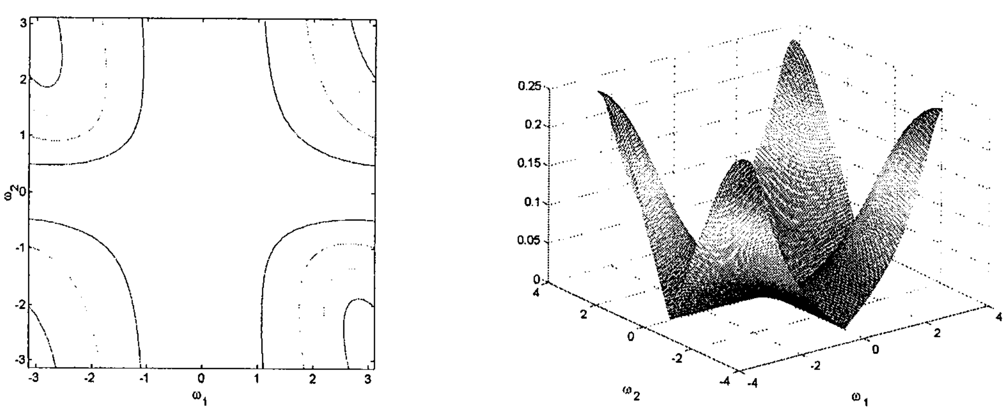
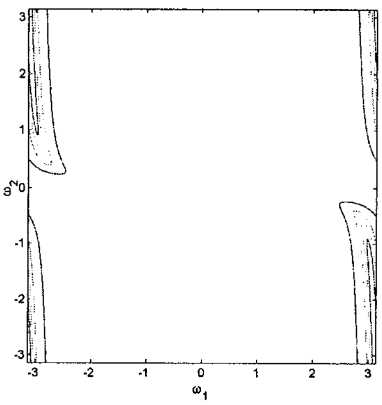


Figure 5.15(b): Y_1=2



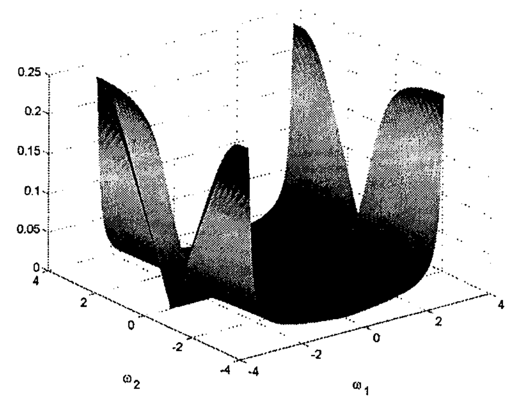


Figure 5.15(c): Y_1=20

Figure 5.15: The contour and 3-D magnitude plot of the resulting 2-D high pass filter for different values of Y_1 with $\mathbf{A} = \psi_1 \lambda_1 (a-b)^2$=6.1, g=2 and Y_2 =0.81

After thoroughly investigating, we can say that the main effect of Y_1 is on the bandwidth in ω_1 domain, while there is no effect on the bandwidth of the stop-band in ω_2 domain. It is inversely proportional to the bandwidth in ω_1 domain and directly proportional to the bandwidth of pass-band in ω_2 domain. Another noticeable thing is that Y_1 has no effects on the amplitude of the response, and the presence of the Centro symmetry in the contour plots.

High Pass frequency response with different values Y_2

We should keep Y_1=0.81, g =2 and $\mathbf{A} = \psi_1 \lambda_1 (a-b)^2$=6.1 in order to understand the effects of Y_2 on the high pass filter response with the reference high pass response showed in figure 5.2(c). With this arrangement, we have checked that Y_2 gives stable response up to 1,000. Below figure shows few high pass responses for different values of Y_2.

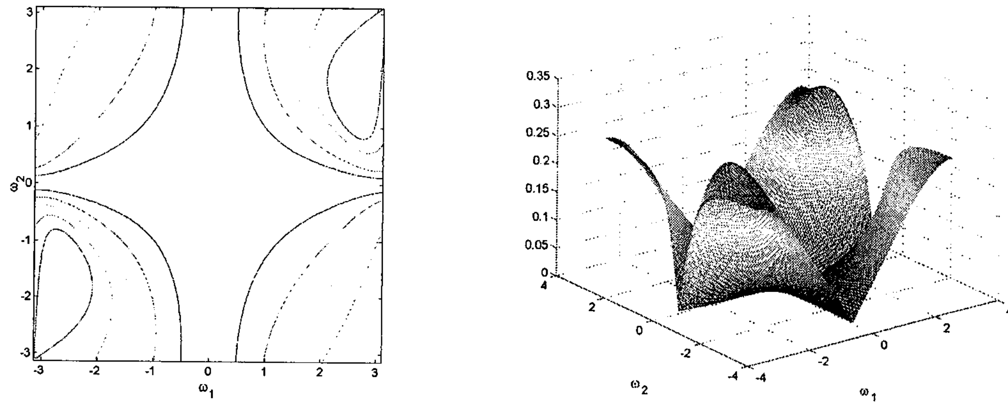


Figure 5.16(a): Y_2=0.2

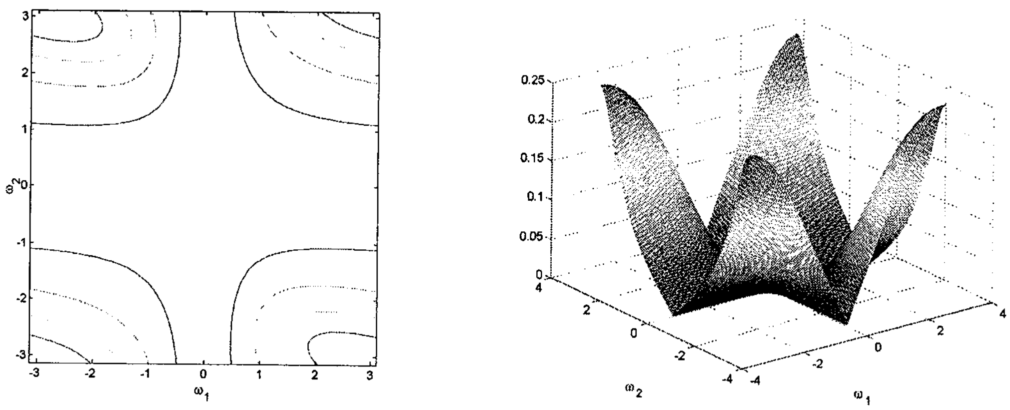


Figure 5.16(b): Y_2=2

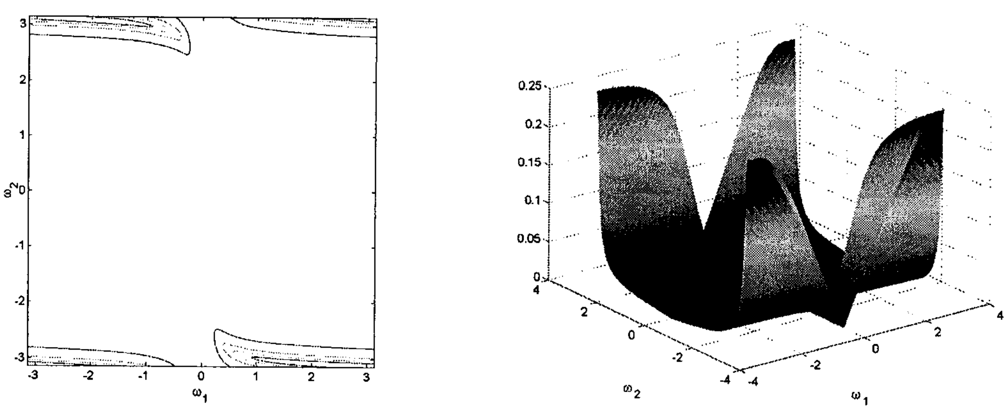


Figure 5.16(c): Y_2=20

Figure 5.16: The contour and 3-D magnitude plot of the resulting 2-D high pass filter for different values of Y_2 with A $=\psi_1\lambda_1(a-b)^2$=6.1, g = -2 and Y_1 =0.81

If we observe the above results in detail then we can disclose that the main effect of Y_2 is on the bandwidth in ω_2 region, while there is no effect on the bandwidth of stop-band in ω_1 domain. Y_2 is inversely proportional to the bandwidth in ω_2 region. It is apparent that each contour plot has Centro symmetry. Same as the case of Y_1, Y_2 also doest not modify the amplitude of the 2-D high pass response. Hence we can consider them as **band-effective coefficients**.

When we change Y_1 and Y_2 together, we were expecting the same results we have got for K_1 and K_2, But we do not get same results. Below figure shows some simulation results for different values of Y_1 and Y_2

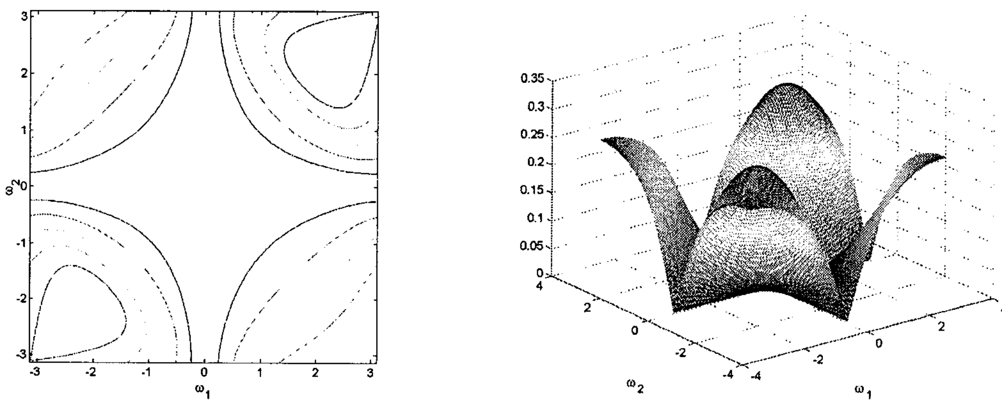


Figure 5.17(a): $Y_1 = Y_2 = 0.4$

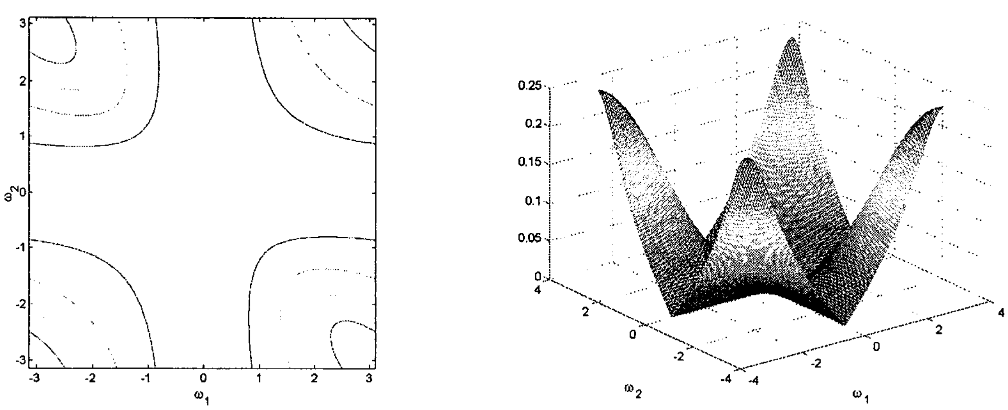
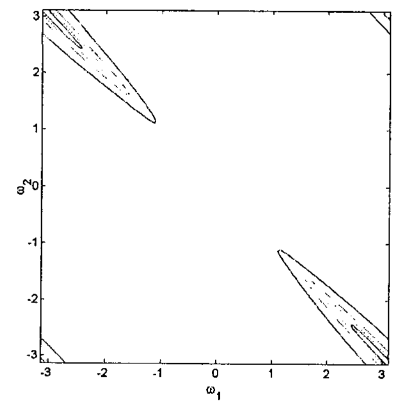


Figure 5.17(b): $Y_1 = Y_2 = 1.5$



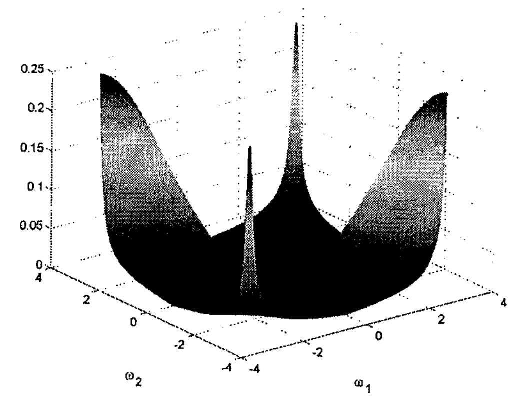


Figure 5.17(c): $Y_1 = Y_2 = 15$

Figure 5.17: The contour and 3-D magnitude plot of the resulting 2-D high pass filter for different values of Y_1 and Y_2 with $\mathbf{A} = \psi_1 \lambda_1 (a-b)^2 = 6.1$, g = 2.

After investigating the above simulation results carefully, we can say that the bandwidth of the frequency response changes diagonally rather than horizontally or vertically. Another observable thing is the presence of Diagonal symmetry in each contour plot. And this leads us to the idea of making the value of Y_1 equal to the value of Y_2 with fulfilling the equation 4.5, so that we can combine Diagonal and Quadrantal symmetry. In our reference case we have already choused equal values to Y_1 and Y_2 with fulfilling the equation 4.5 and this is the reason sometimes we have seen Diagonal and Quadrantal symmetry in the high pass response.

Still the combine effect of Y_1 and Y_2 on the bandwidth is not known. In order to determine this effect in details we should keep changing value of Y_1 and Y_2 with satisfying the equation 4.5 for g=2. Below figure shows some simulation results for this case.

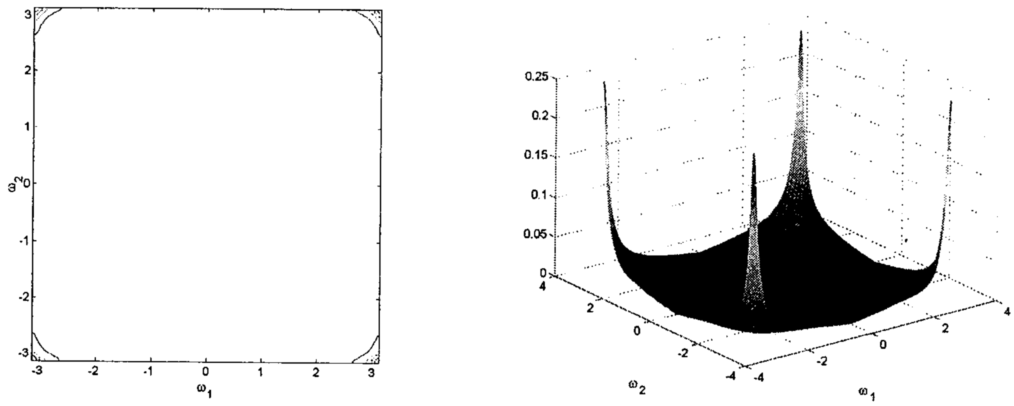


Figure 5.18(a): $Y_1 = Y_2$=0.05 and **A**=1600

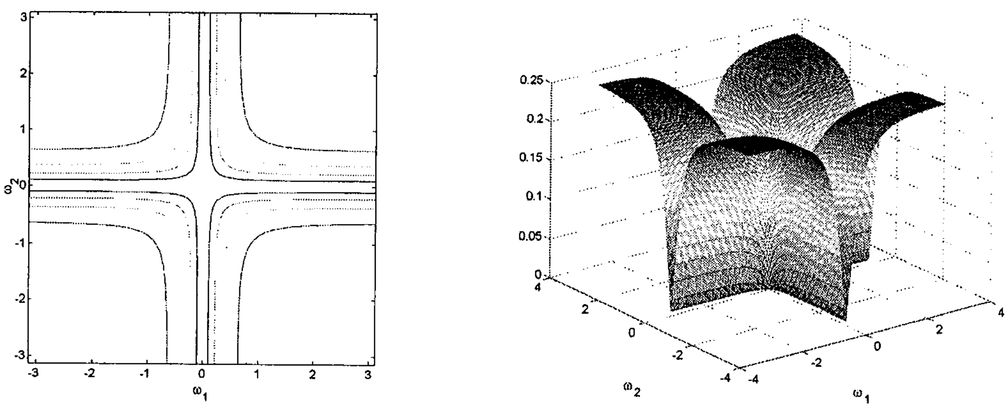


Figure 5.18(b): $Y_1 = Y_2$=4 and **A** = 0.25

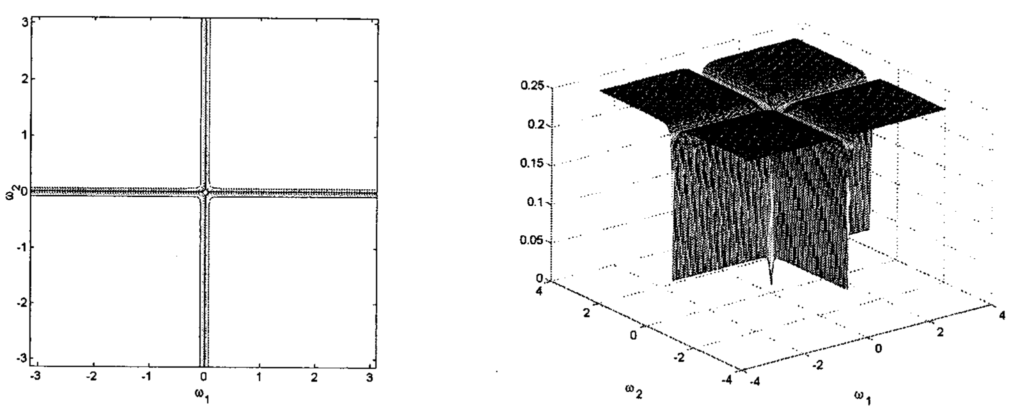


Figure 5.18(c): $Y_1 = Y_2$=40 and **A** = 0.0025

Figure 5.18: The contour and 3-D magnitude plot of the resulting 2-D high pass filter for different values of Y_1, Y_2 and $\mathbf{A} = \psi_1 \lambda_1 (a-b)^2$ with g = -2.

From this simulation results we can easily understand the effects of Y_1 and Y_2 on bandwidth of the frequency response. When we assign equal value to Y_1 and Y_2 with satisfying the equation 4.5, value of Y_1 and Y_2 is directly proportional to the bandwidth of the frequency response. It is visible that each contour plot is having Octagonal symmetry in above figure. But the more effective and easy way to obtain the desirable bandwidth is by changing the value of K_1 and K_2, because we do not have to satisfy any conditions in this case.

5.4 Designing High-pass filter with a desirable response.

With this knowledge of various effects of different coefficients of the matrix and generalized bilinear transformation on the behavior of the 2-D high pass filter, now we can design a 2-D high-pass filter with a desirable response. For an example, we want to increase the gain and we want to keep the bandwidth of the response same as the reference case showed in figure 5.2(c). We can achieve this by reducing the value of g,

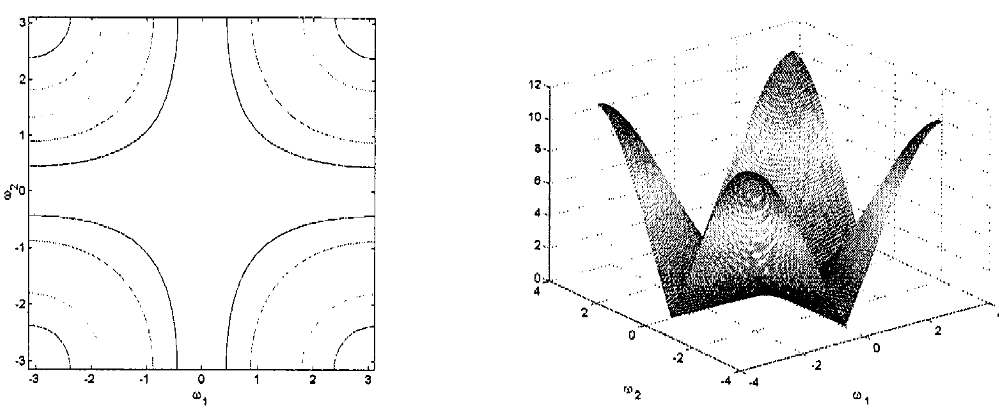
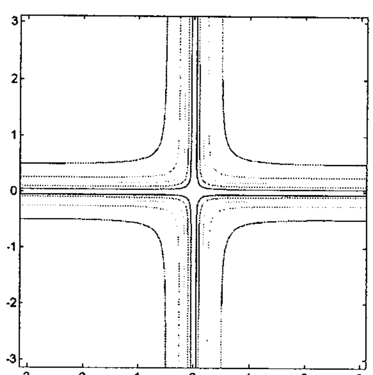


Figure 5.19: The contour and 3-D magnitude plot of the resulting 2-D high pass filter for **A** $=\psi_1\lambda_1(a-b)^2$=0.14, g = 0.3, Y_1=Y_2=0.81, $K_1=K_2=1$, $b_{01}=b_{02}=-1$ and $a_{01}=a_{02}=1$

lets say 0.3, and keeping the same value of $Y_1 = Y_2 = 0.81$. Now, for these values of Y_1, Y_2 and g , we calculate value of **A** from the equation 4.5, which comes 0.14. As well as, we also keep the generalized bilinear coefficients to unity value with proper sign. Above figure shows the simulation results of a 2-D high pass filter with these values.

Now for an example, let's say we want to obtain a 2-D high pass filter with high gain and greater bandwidth. To build this kind of high pass filter we start exactly in the same manner as above and we achieve the high gain in the response. And at this point now we can always alter the bandwidth of the response by changing the values of band-effective coefficients K_1 and K_2 or Y_1 and Y_2. But for the simplicity purpose we use K_1 and K_2 to achieve the desire bandwidth. And we know that K_1 and K_2 are inversely proportional to the bandwidth, so by assigning lower values to them we can obtain a response with greater bandwidth. Below figure shows the simulation results for this example.



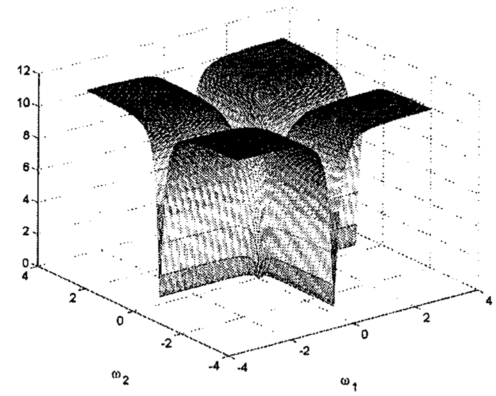


Figure 5.20: The contour and 3-D magnitude plot of the resulting 2-D high pass filter for **A** $=\psi_1\lambda_1(a-b)^2=0.14$, g = 0.3, $Y_1 = Y_2 = 0.81$, $K_1 = K_2 = 0.1$, $b_{01} = b_{02} = -1$ and $a_{01} = a_{02} = 1$

Now, let's say bandwidth of the frequency response is fine but for some reasons we have to reduce the gain of the response little bit. And the minor changes in the gain of the response can be achieved by changing the values a_{01} and a_{02}. But we also know that a_{01} and a_{02} also have some minor effect on the bandwidth and we can readjust the bandwidth of the response with Y_1, Y_2 and **A**. We have obtained the desirable response by setting $a_{01} = a_{02}$=0.5, $Y_1 = Y_2$=0.9 and **A**=0.11, while keeping all other parameters to the same value. Below figure shows simulation result for this example

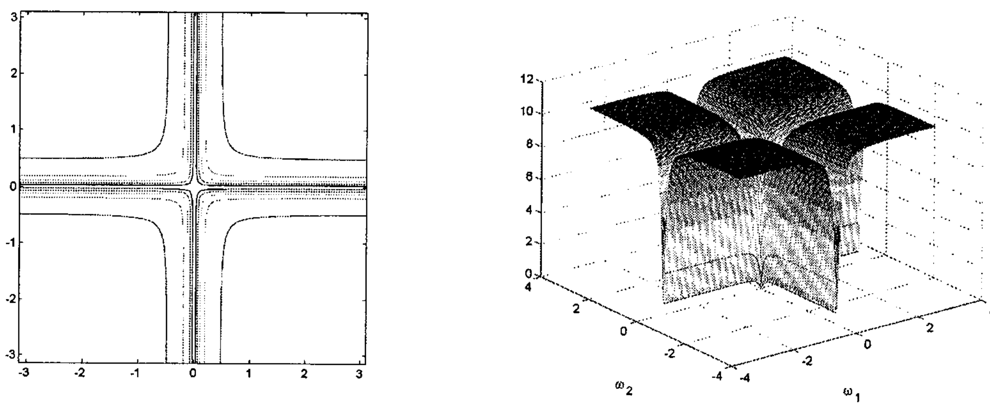


Figure 5.21: The contour and 3-D magnitude plot of the resulting 2-D high pass filter for **A** $=\psi_1\lambda_1(a-b)^2$=0.11, g = 0.3, $Y_1=Y_2$=0.9, $K_1 = K_2 = 0.1$, $b_{01} = b_{02} = -1$ and $a_{01} = a_{02} = 0.5$

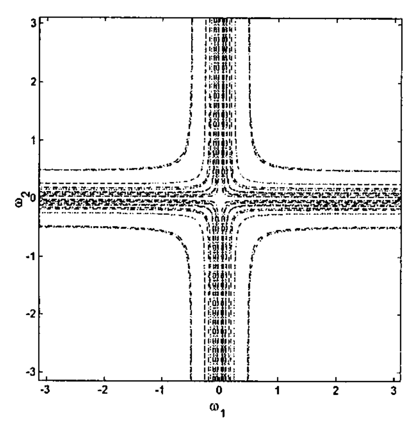


Figure 5.22: Overlapping of the contour plots showed in figure 5.20 and 5.21

5.5 Summary

It is amply demonstrated that the designed 2-D high-pass recursive filter response can be modified by changing one or more coefficients of the 2-D high-pass transfer function. In this chapter, a large family of responses of 2-D high-pass recursive filter is analyzed in detail with 2-D contour plots and 3-D magnitude plots. Effects of each coefficient of the proposed 2-D high-pass digital filter on the filter behavior are investigated with simulation results. It is evident that in addition to contour characteristics, symmetry of the 2-D high-pass digital filter can also be changed.

One of the prime issues with 2-D high-pass filters is stability. With the unity degree denominator, stable range for each coefficient for the proposed 2-D high-pass recursive filter is analyzed. We use it as stability test criteria for 2-D high-pass filter design procedure.

Designing of a 2-D high-pass recursive filter with the desirable magnitude, contour and symmetry characteristics is feasible using the technique presented in this chapter. This approach was demonstrated via several examples with guaranteed stability.

6. 2-D BAND-PASS RECURSIVE FITLER

Different combinations of a 2-D low-pass filter and a 2-D high-pass filter will result in different kind of filters, e.g. 2-D band-pass filter, 2-D band-elimination filter. In this chapter we study 2-D band-pass filter, one of these kinds of combination filter.

Section 6.1 is a brief introduction of a 2-D band-pass filter with a graphical illustration. In section 6.2 we show how to design 2-D high pass filter by applying the double generalized bilinear transformation to the 2-D analog transfer function derived in Chapter 3. In section 6.3 with the knowledge gained from Chapter 4 and Chapter5, we determine the effects of the different coefficients of the 2-D band pass transfer function on the filter response. Section 6.4 is the summery of this chapter.

6.1 Definition of 2-D Band-pass filters

A band-pass filter permits the signal components within a specified range, and attenuates the other signal components with a higher and a lower frequency than the specified frequency range. A 2-D band-pass filter can be plotted as shown below:

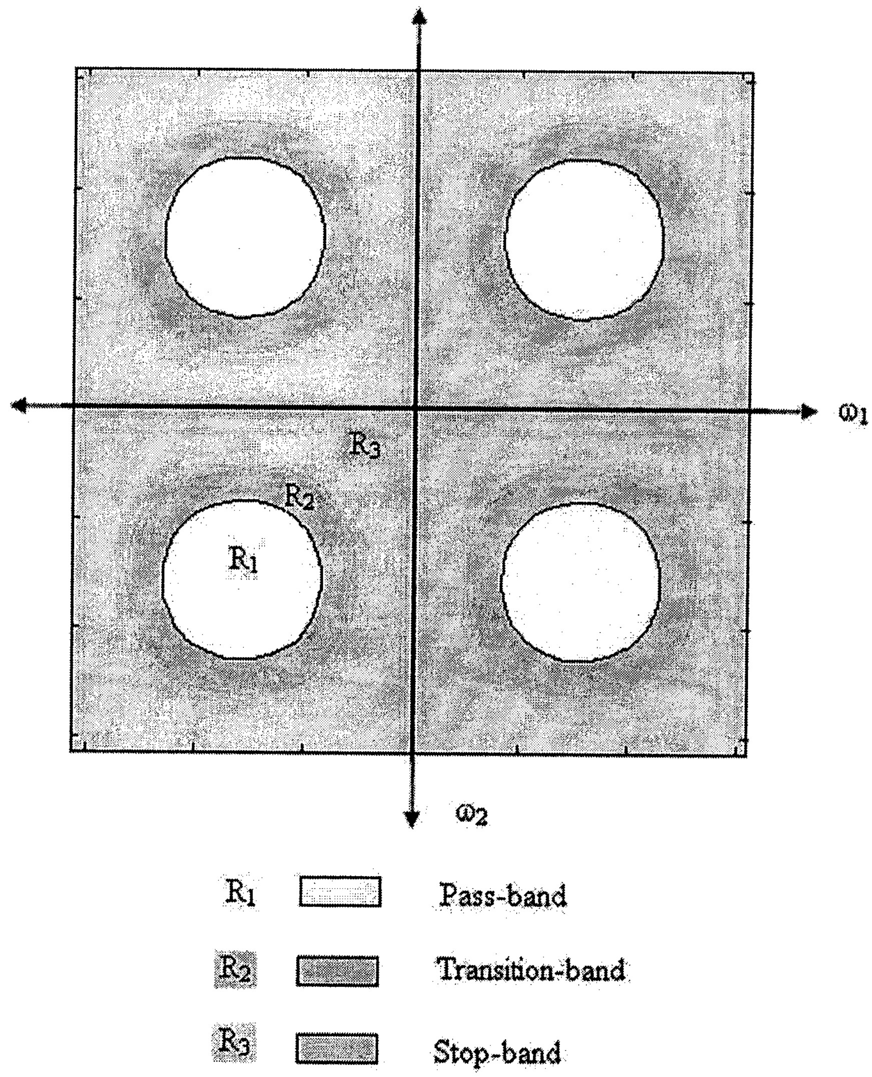


Figure 6.1: Graphical definition of a band-pass filter

A typical 2-D band-pass recursive filter has the specification in the frequency domain as below:

$$
\begin{aligned}
H(\omega_1,\omega_2) &= 0, && |\omega_i| \le \omega_{s1} \\
&= 1, && \omega_{p1} \le |\omega_i| \le \omega_{p2} \\
&= 0, && \omega_{s2} \le |\omega_i| \le \pi
\end{aligned}
\qquad (6.1)
$$

Where : ω_{pi} is called pass-band radius, i = 1, 2.

ω_{si} is called stop-band radius, i = 1, 2.

And transition band is the region in-between ω_{pi} and ω_{si}, i = 1, 2.

There are few possibilities to design 2-D band-pass filter from a 2-D LPF and a 2-D HPF. Some of them are as below:

- A cascade connection of a 2-D LPF and a 2-D HPF
- A cascade connection of a 2-D HPF and a 2-D LPF
- By implementing the double generalized bilinear transformation to a to the 2-D analog function, $S \longrightarrow K_1 \frac{z_1 - a_{01}}{z_1 + 1} + K_2 \frac{z_2 + a_{02}}{z_2 - 1}$, which is a combination of a low-pass filter and a high-pass filter. [33]

In this chapter, we design a 2-D band-pass filter by implementing the double generalized bilinear transformation.

6.2 Designing 2-D band-pass filter

If we consider a VSHP shown in equation 3.16 then,

$$D(S_1, S_2) = \psi_1\lambda_1(a-b)^2 S_1 S_2 + Y_1\psi_1\lambda_1(a-b)^2 S_2 + Y_2\psi_1\lambda_1(a-b)^2 S_1 + g^2$$

$$= \psi_1\lambda_1(a-b)^2\{S_1 S_2 + Y_2 S_1 + Y_1 S_2\} + g^2 \tag{6.2}$$

By applying the double generalized bilinear transformation in above equation for S_1 and S_2 then, $S_1 = K_1\left(\frac{z_1 - a_{01}}{z_1 + 1}\right) + K_3\left(\frac{z_1 + a_{03}}{z_1 - 1}\right)$ and $S_2 = K_2\left(\frac{z_2 - a_{02}}{z_2 + 1}\right) + K_4\left(\frac{z_2 + a_{04}}{z_2 - 1}\right)$. And as discussed in Section 1.4 to make the system stable, we will follow the stability conditions in this case, which are:

$$\begin{aligned} &0 < a_{0i} \le 1; \\ &K_i > 0; \end{aligned} \quad \text{where, i = 1, 2, 3, 4.} \tag{6.3}$$

Now, by substituting the value of S_1 and S_2 in equation 6.2 then,

$$D(z_1,z_2)=\psi_1\lambda_1(a-b)^2\left\{\begin{array}{l} K_1K_2\left(\frac{z_1-a_{01}}{z_1+1}\right)\left(\frac{z_2-a_{02}}{z_2+1}\right)+K_1K_4\left(\frac{z_1-a_{01}}{z_1+1}\right)\left(\frac{z_2+a_{04}}{z_2-1}\right)+ \\ K_3K_2\left(\frac{z_1+a_{03}}{z_1-1}\right)\left(\frac{z_2-a_{02}}{z_2+1}\right)+K_3K_4\left(\frac{z_1+a_{03}}{z_1-1}\right)\left(\frac{z_2+a_{04}}{z_2-1}\right)+ \\ Y_2K_1\left(\frac{z_1-a_{01}}{z_1+1}\right)+Y_2K_3\left(\frac{z_1+a_{03}}{z_1-1}\right)+Y_1K_2\left(\frac{z_2-a_{02}}{z_2+1}\right)+ \\ Y_1K_4\left(\frac{z_2+a_{04}}{z_2-1}\right) \end{array}\right\}+g^2$$

(6.4)

By applying this $D(z_1,z_2)$ to denominator of a unity numerator, we can realize a 2-D band-pass recursive filter. And the transfer function $H(z_1,z_2)$ can given as below,

$$H(z_1,z_2)=\frac{1}{D(z_1,z_2)}, \tag{6.5}$$

where, $D(z_1,z_2)$ is given in equation 6.4

$\psi_1,\lambda_1>0; a\neq b;$ (6.5a)

$0<a_{0i}\leq 1;$ $K_i>0;$, i = 1, 2, 3, 4. (6.5b)

In the next section, we study the effects of different coefficients shown in above equation on the band pass filter response

6.3 Frequency response of the designed 2-D band-pass recursive filter

First we write a program "himthbp.m" for the above 2-D band-pass transfer function in MATLAB ®. Then assign value to each and every coefficient of the band pass transfer function in the program to get the contour and 3-D magnitude plots for the specified values.

The designed 2-D band-pass filter is a combination of a 2-D LPF and a 2-D HPF. In Chapter 4 we briefly studied 2-D LPF and in Chapter 5 we studied 2-D HPF. So if we use the summary of those two chapters then we can predict the response of this designed 2-D band-pass filter.

In this chapter too we classify the coefficients in two groups same as last two chapters.

- coefficients offered by the generalized bilinear transformation
- coefficients offered by the 2-D analog transfer function or matrix coefficients

To make the situation simpler, while studying the effects of the coefficients of one group, we keep the coefficients of another group to some content value.

6.3.1 2-D band-pass recursive filter response with each different coefficient of the generalized bilinear transformation

Using the knowledge gained in last two chapters, in this section we study the effects of each different coefficient of the generalized bilinear transformation on the designed 2-D band-pass filter response. As usual, we keep changing the coefficient under the examination, while setting the others to some constant values. Equation 6.5b shows the range of each coefficient of the generalized bilinear transformation for the designed 2-D band-pass recursive filter. As we have already discussed, we also assign each coefficient of the matrix to some specific value. In this section we use, $Y_1 = Y_2 = 0.81$, $g = 2$ and $\mathbf{A} = \psi_1 * \lambda_1 * (a-b)^2 = 6.1$

High Pass frequency response with different values of K_is

To study the effects of K_is, we assign unity value to all the other generalized bilinear coefficients, e.g. $a_{01} = a_{02} = -1; a_{03} = a_{04} = 1$. From the previous two chapters, we can say that K_is only affect the bandwidth of the response. They do not have any effect on the magnitude of the response. Also we know that K_is related to S_1 affect bandwidth response only in ω_2 direction and K_is related to S_2 affect bandwidth response in ω_1 direction. Figure 6.2 shows the contour and the 3-D magnitude response plot for different values of K_is related to S_1, e.g. K_1 and K_3 and Figure 6.3 presents the simulation results for different values of K_is related to S_2, e.g. K_2 and K_4

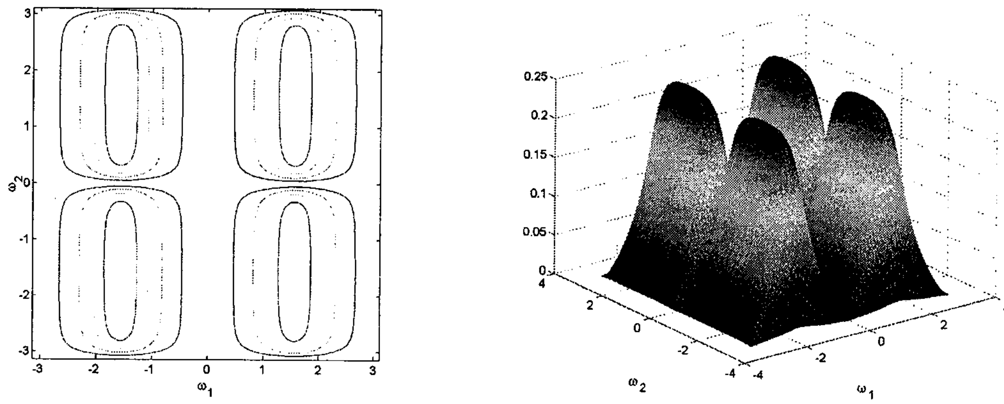


Figure 6.2(a): $K_1 = K_3 = 0.1$

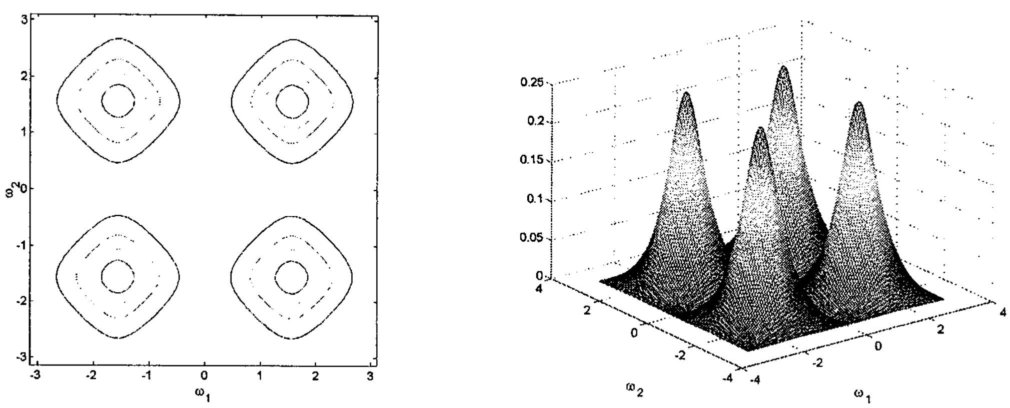


Figure 6.2(b): $K_1 = K_3 = 1$

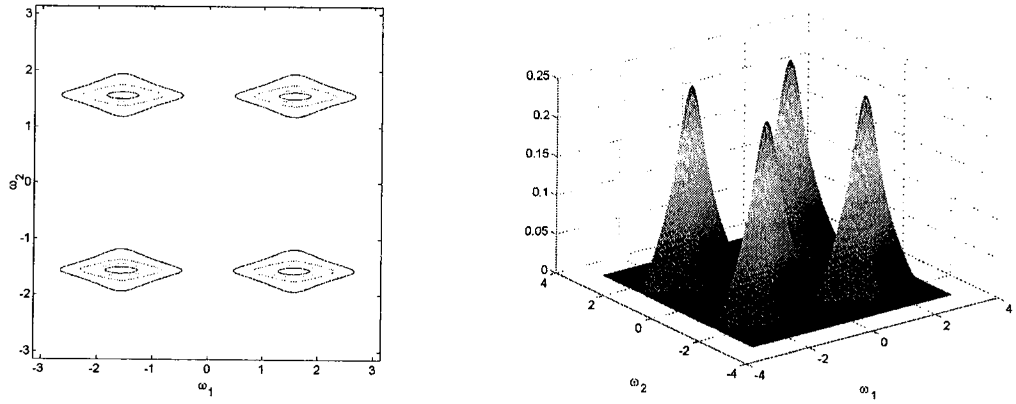


Figure 6.2(c): $K_1 = K_3 = 5$

Figure 6.2: The contour and 3-D magnitude plot of the resulting 2-D band pass filter for different values of K_1 and K_3 with $K_2 = K_4 = a_{03} = a_{04} = 1; a_{01} = a_{02} = -1$

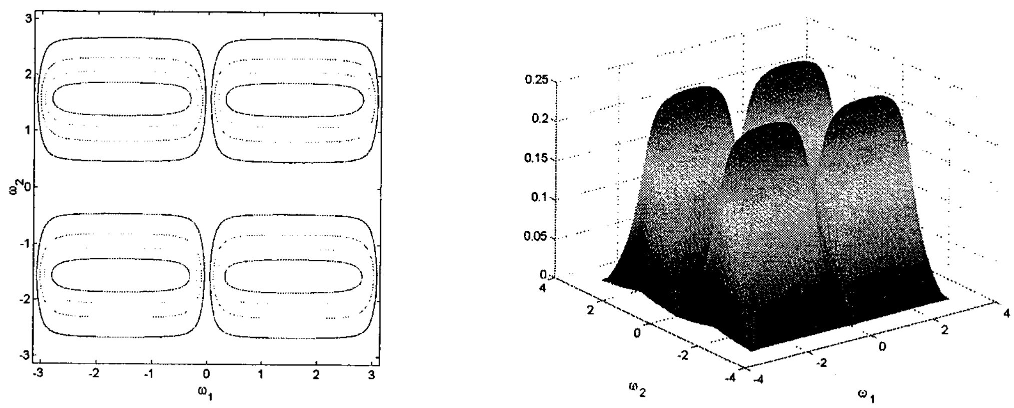


Figure 6.3(a): $K_2 = K_4 = 0.1$

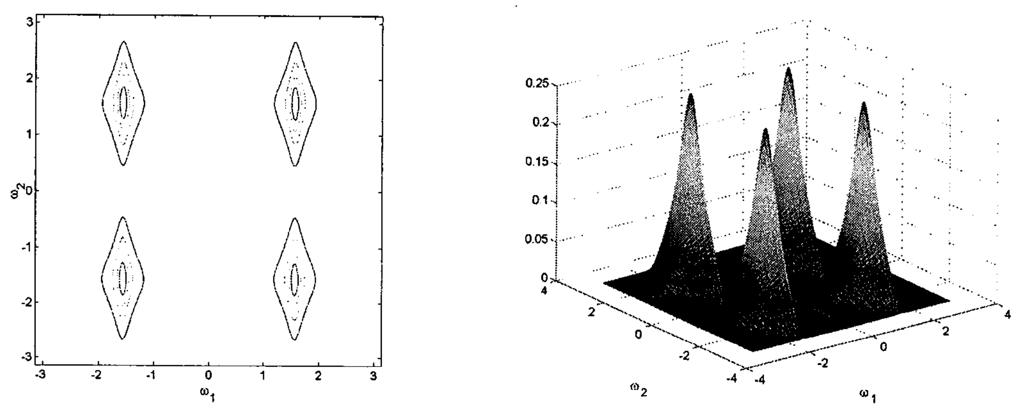


Figure 6.3(b): $K_2 = K_4 = 5$

Figure 6.3: The contour and 3-D magnitude plot of the resulting 2-D band pass filter for different values of K_2 and K_4 with $K_1 = K_3 = 1$; $a_{01} = a_{02} = -1; a_{03} = a_{04} = 1$.

After reviewing the Figure 6.2 and Figure 6.3, we can say that value of K_2 and K_4 together are inversely proportional with the bandwidth in ω_1 direction, while K_1 and K_3 together are inversely proportional with the bandwidth in ω_2 direction. It is clear that K_is have very minor effect on the gain of the band-pass response as the amplitude value in the 3-D magnitude plot remains almost constant approximately 0.2 throughout, and each contour plot has Quadrantal symmetry.

We can call K_is as **band-effective coefficients**, because they mostly affect the bandwidth of the designed band-pass frequency response. With this knowledge we can

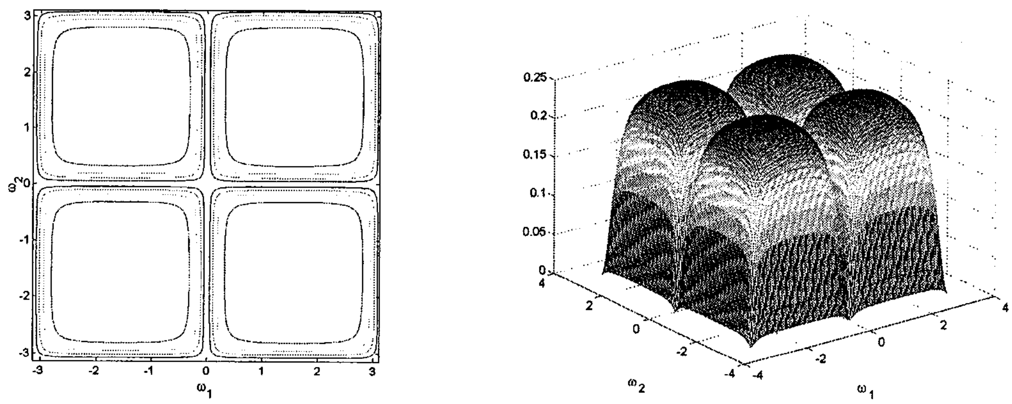


Figure 6.4(a): K_is =0.1.

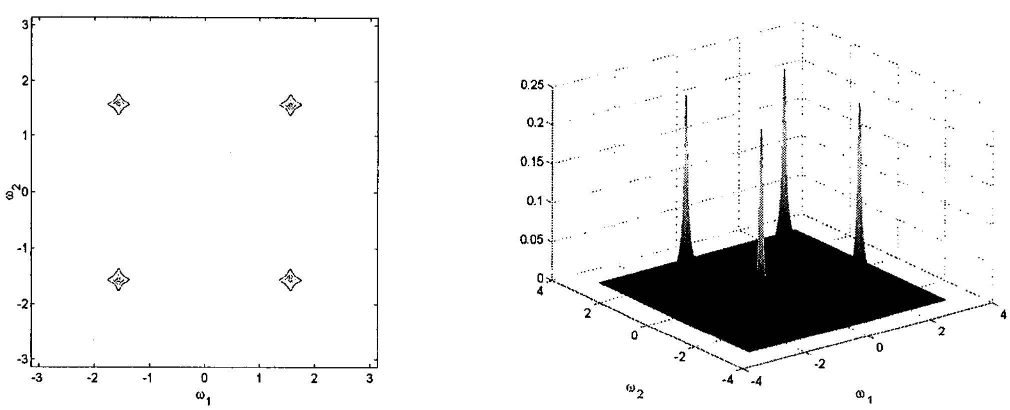


Figure 6.4(b): K_is = 10

Figure 6.4: The contour and 3-D magnitude plot of the resulting 2-D band pass filter for different values of K_is, i = 1, 2, 3, 4 with $a_{01} = a_{02} = -1; a_{03} = a_{04} = 1$.

design a 2-D band-pass filter with bigger pass-band region or with a smaller pass-band region as shown in the above figure.

Another noticeable thing in this case is both the contour plots have Quadrantal symmetry, Diagonal Symmetry and Four-fold symmetry. In a single word each contour plot contains Octagonal symmetry.

Band Pass frequency response with different values of a_{0i} s

With the information obtained from last two chapters, we can say that a_{0i} s mostly affect the gain in the pass-band of the designed filter. a_{0i} s related to LPF are inversely proportional to the gain of the response, while a_{0i} s related to LPF are directly proportional to the gain of the response. Figure 6.5 shows the results for a_{0i} s related to LPF, e.g. a_{01} and a_{02} and Figure 6.6 shows the simulation response for a_{0i} s related to HPF, e.g. a_{03} and a_{04}

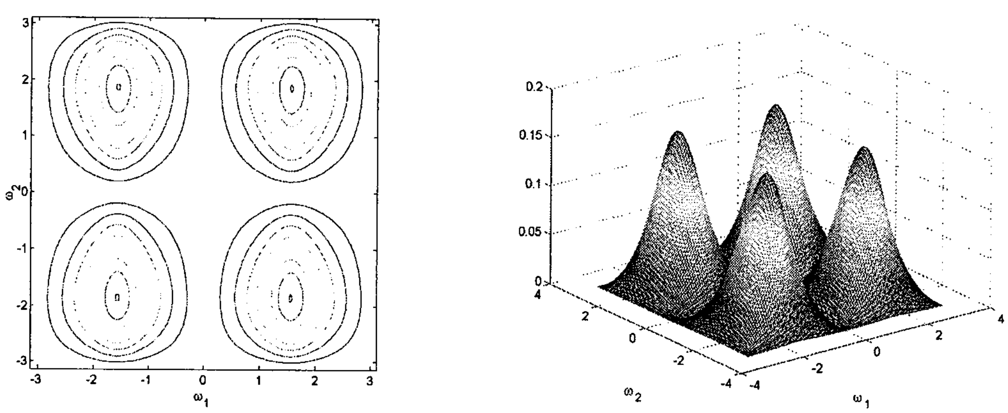


Figure 6.5(a): a_{01} = -0.1 and a_{02} = -1,

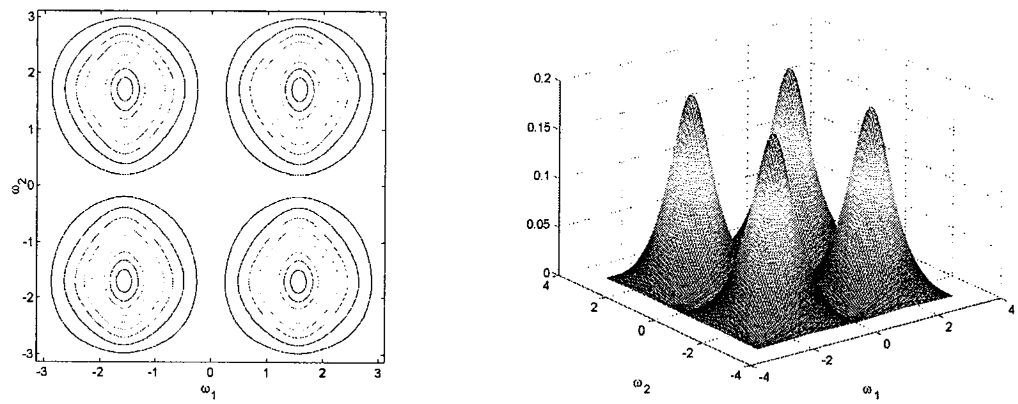


Figure 6.5(b): a_{01} = -0.5 and a_{02} = -1

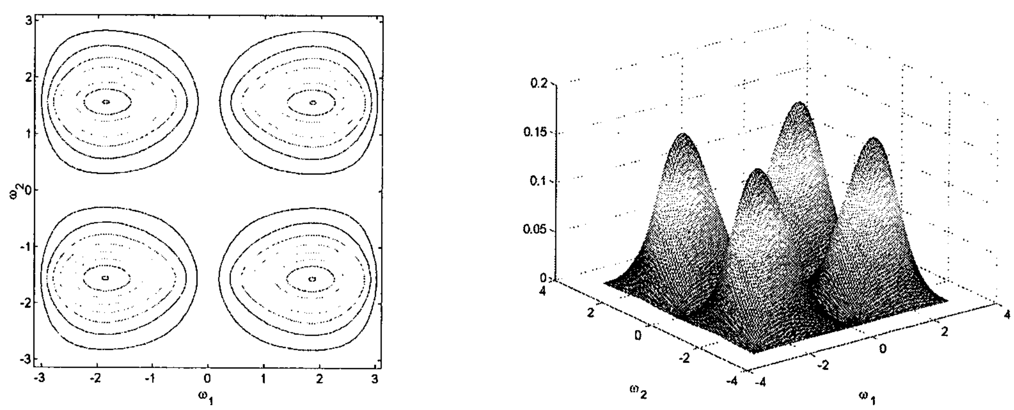


Figure 6.5(c): a_{02}=-0.1 and a_{01} = -1,

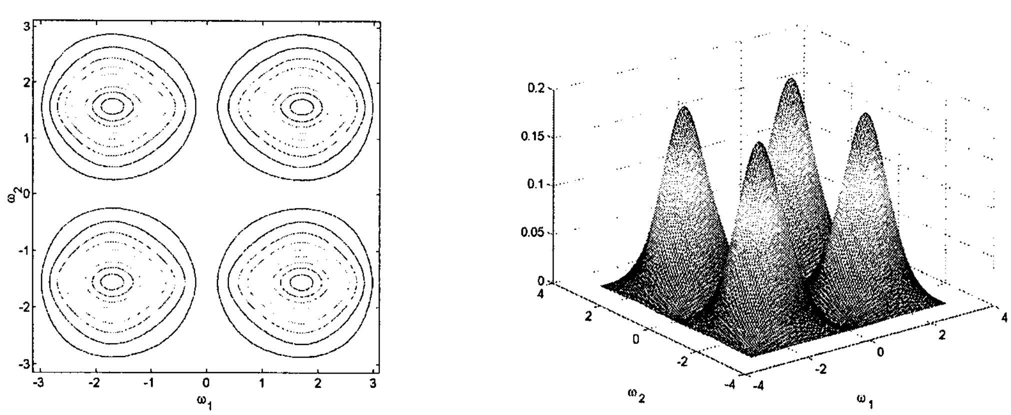


Figure 6.5(d): a_{02}=-0.5 and a_{01} = -1

Figure 6.5: The contour and 3-D magnitude plot of the resulting 2-D band pass filter for different values of a_{01} and a_{02} with $K_i =. a_{03} = a_{04} = 1$

It is evident in the above figure that a_{0i} related to LPF is inversely proportional to amplitude of the high-pass filter. As the value decreases from -0.1 to -1, the amplitude approximately increases from 0.12 to 0.2. a_{0i} s also has small effect on the bandwidth of the response and the contour plots have Quadrantal symmetry.

It is clear that a_{0i} related to HPF is also directly proportional to the magnitude of the band pass filter and affect the bandwidth of the response. Another interesting thing is in this case too, the a_{0i} s related to S_1, e.g. a_{01} and a_{03}, affect bandwidth response only in

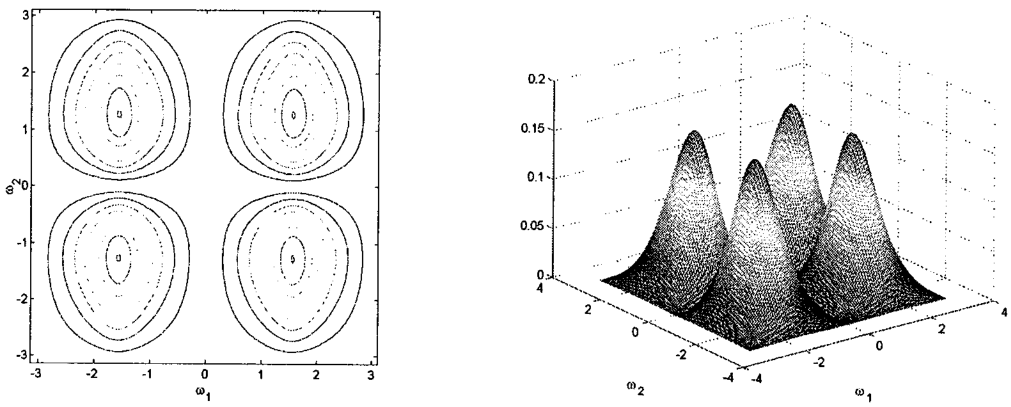


Figure 6.6(a): $a_{03} = 0.1$ and $a_{04} = 1$

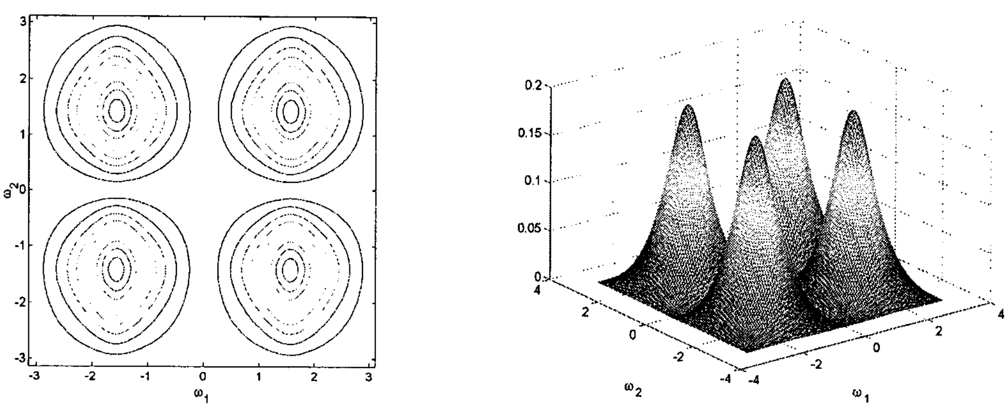


Figure 6.6(b): $a_{03} = 0.5$ and $a_{04} = 1$

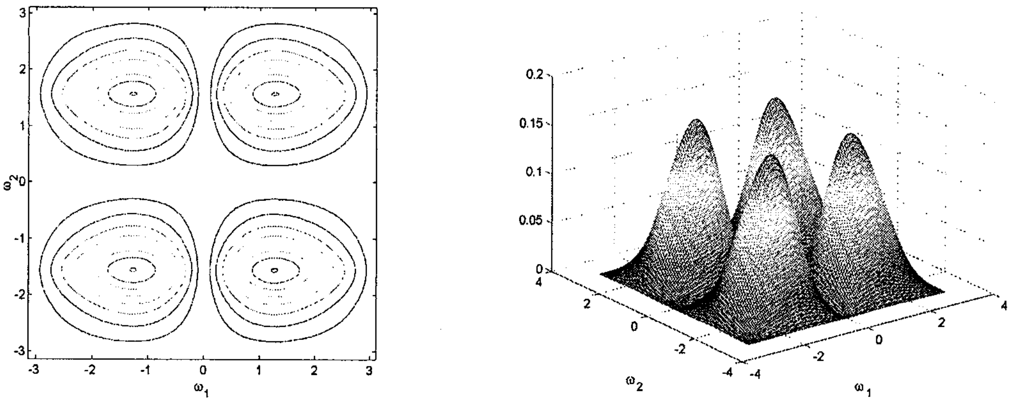


Figure 6.6(c): $a_{04} = 0.1$ and $a_{03} = 1$

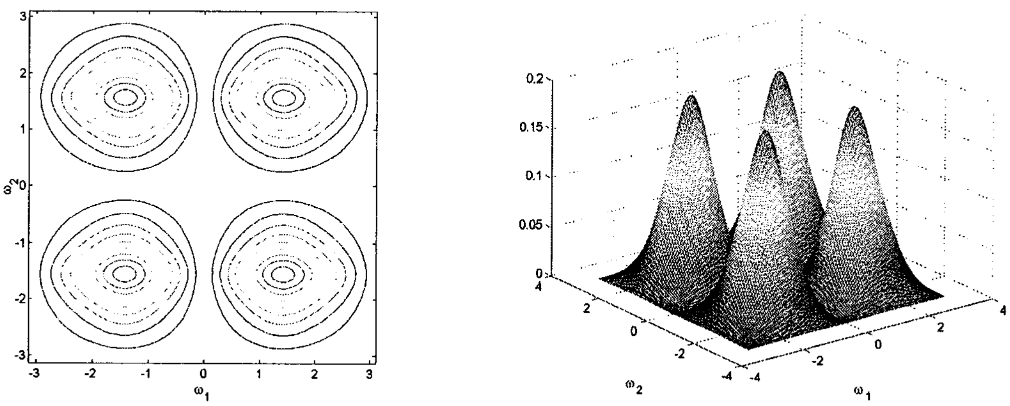


Figure 6.6(d): $a_{04} = 0.5$ and $a_{03} = 1$

Figure 6.6: The contour and 3-D magnitude plot of the resulting 2-D high band filter for different values of a_{03} and a_{04} with $K_i = 1$ and $a_{01} = a_{02} = -1$.

ω_2 direction and a_{0i}s related to S_2, e.g. a_{02} and a_{04} affect bandwidth response in ω_1 direction.

It is clear that K_is has greater effects on the bandwidth of the response than the effects offered by a_{0i}s. a_{0i}s are mainly gain affected coefficient. So we call them **gain-effective coefficients**. In the below figure we show that we can reduce the gain of the designed filter response by setting lower values to a_{01}, a_{02}, a_{03} and a_{04} together.

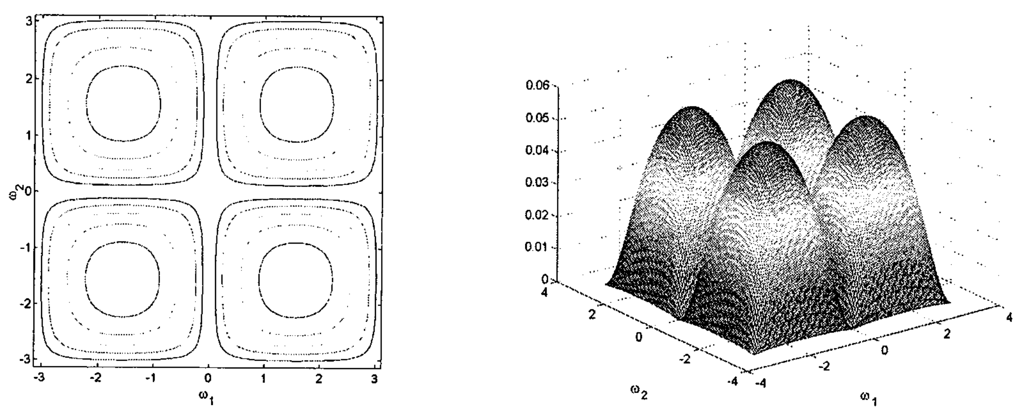


Figure 6.7(a): $|a_{0i}| = 0.1$

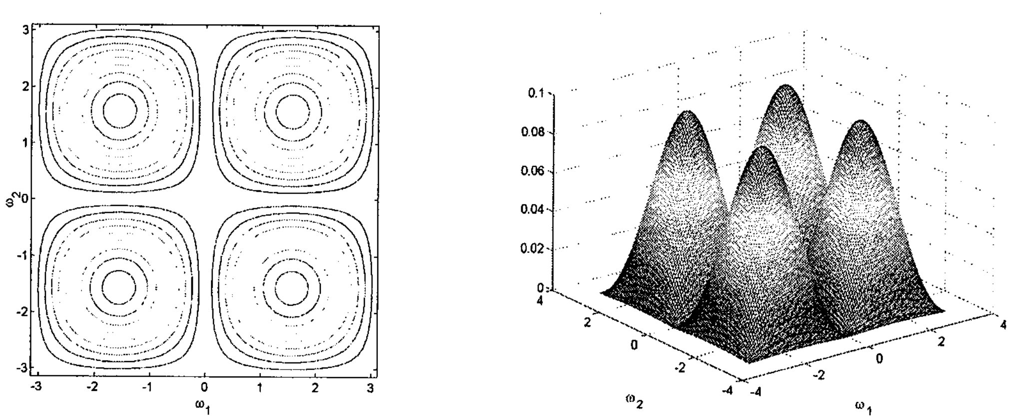


Figure 6.7(b): $|a_{0i}| = 0.5$

Figure 6.7: The contour and 3-D magnitude plot of the resulting 2-D band pass filter for different values of a_{0i}s with $K_i = 1$ and $a_{01} = a_{02} = -1$.

It is observable that when we assign equal values to a_{0i}s, the contour plot obtains Octagonal symmetry same as the case of K_is. When each a_{0i} changes from the lower boundary to the upper boundary, the magnitude of the response changes from 0.042 to 0.2 and the bandwidth of the response decreases noticeably. Another noticeable thing is using the a_{0i}s we can only reduce the magnitude of the response, we can not increase it more than 0.2. So we need to find some other gain-effective coefficient to increase the flexibility of the 2-D band-pass filter.

6.3.2 Effects of the Matrix coefficients on the 2-D band-pass recursive filter response

In this section, we analyze the effects of the matrix coefficients on the designed 2-D band pass filter response. As usual while studying the effects of the coefficient of this group, we keep the coefficients of another group, the double generalized bilinear coefficients, to some specific value. For simplicity we use $K_i s = a_{03} = a_{04} = 1$ and $a_{01} = a_{02} = -1$ in the section 6.3.2.

We can get the conditions on the matrix coefficients for the band pass frequency response after putting these values of the generalized bilinear transformation in the Equation 6.5, which is given below.

- $\psi_1 \lambda_1 > 0$ (6.6a)
- $Y_2 > 0$ (6.6b)
- $Y_1 > 0$ (6.6c)
- $g \neq 0$ and $a \neq b$ (6.6d)

Same as the case of 2-D LPF or 2-D HPF, in this case too if we follow the condition showed in equation 4.5, then we can rewrite the 2-D band pass transfer function, equation 6.5, as a product of two different two-variable polynomials showed in equation 4.4. This is the necessary condition to have Quadrantal symmetry in the response. A combination, A=$\psi_1 * \lambda_1 * (a-b)^2$=6.1, g=2, Y_1= Y_2 =0.81, satisfy this condition and we have used the same combination to study the effects of the coefficients of the generalized bilinear transformation, which resulted in the present of Quadrantal symmetry in the response. Below figure shows some simulation results for different values of the matrix parameters.

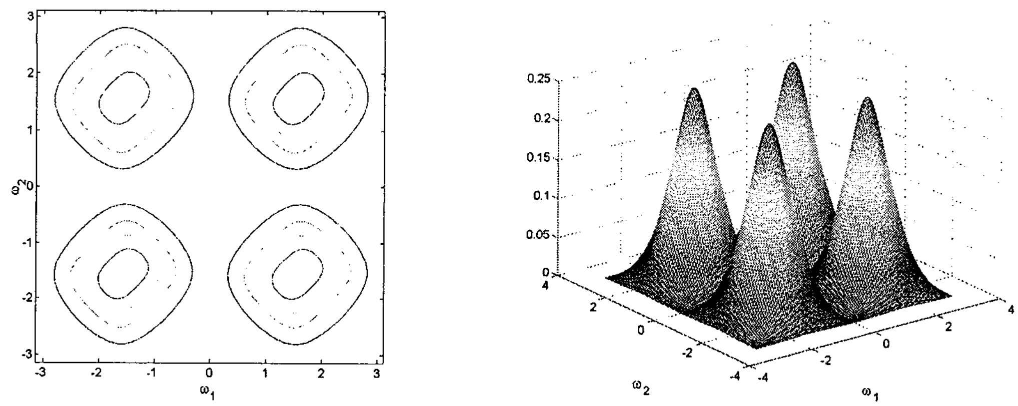


Figure 6.8(a): **A** = 1, g = 2, $Y_1 = Y_2 = 0.81$

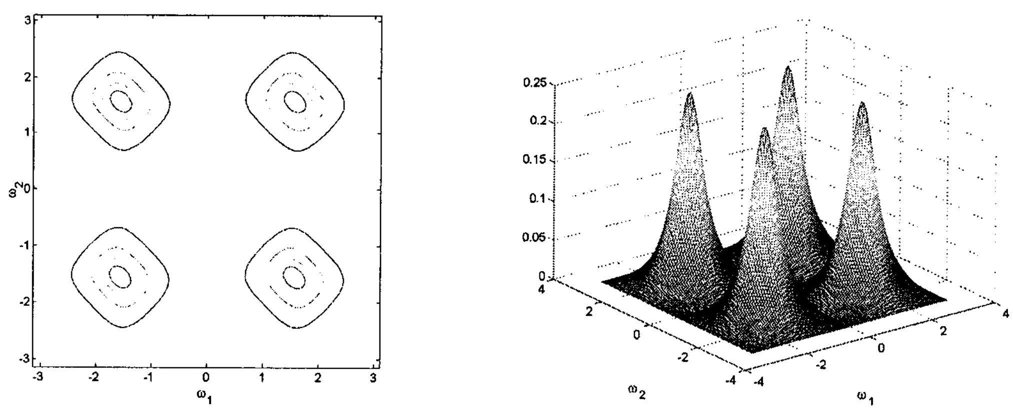


Figure 6.8(b): **A** = 10, g = 2, $Y_1 = Y_2 = 0.81$

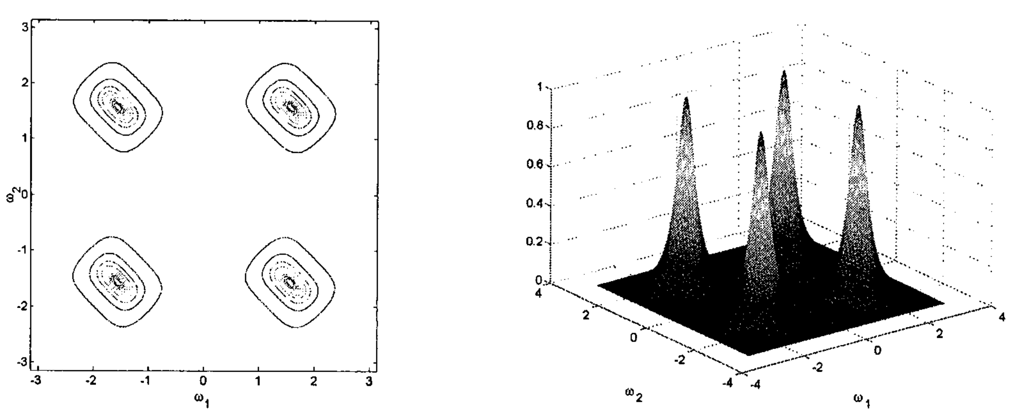


Figure 6.8(c): **A** = 6.1, g = 1, $Y_1 = Y_2 = 0.81$

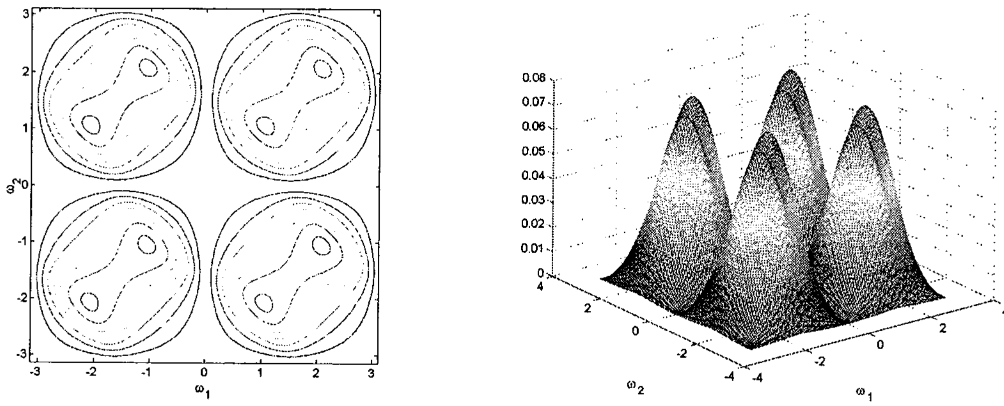


Figure 6.8(d): **A** = 6.1, g = 4, $Y_1 = Y_2 = 0.81$

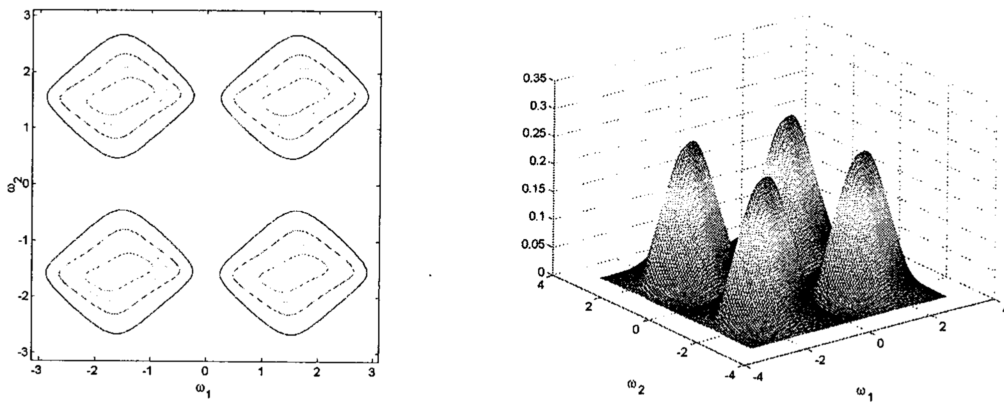


Figure 6.8(e): **A** = 6.1, g = 2, $Y_1 = 0.4$ and $Y_2 = 0.81$

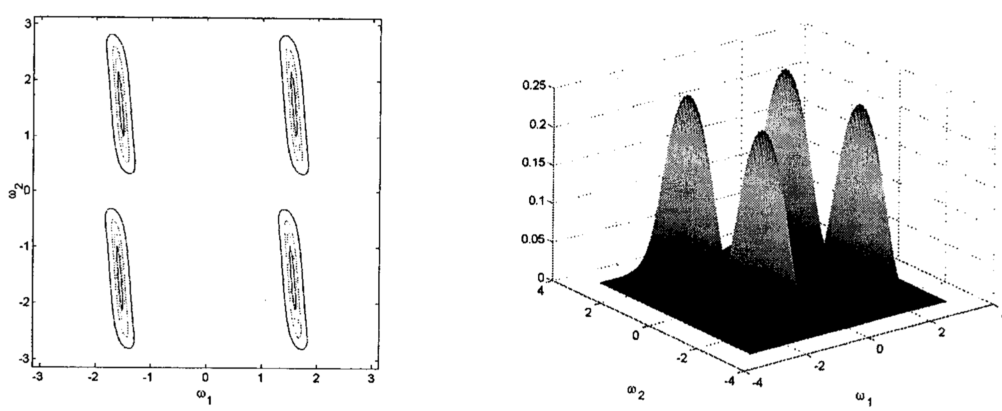


Figure 6.8(f): **A** = 6.1, g = 2, $Y_1 = 9$ and $Y_2 = 0.81$

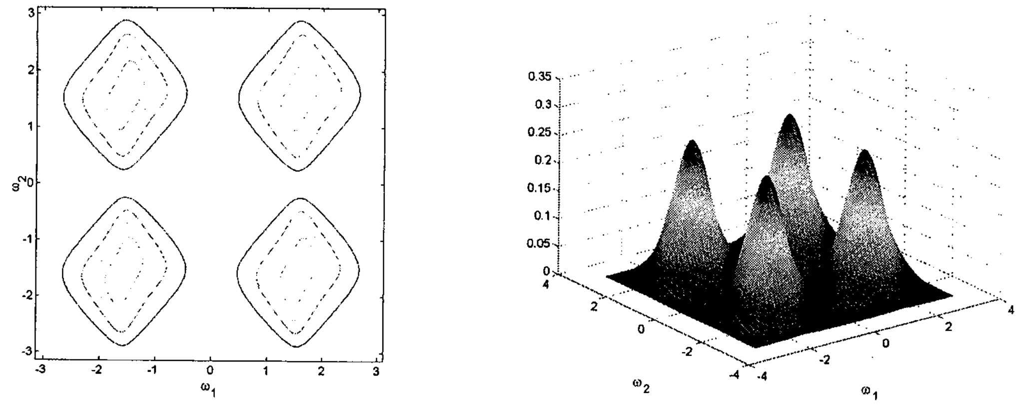


Figure 6.8(g): $\mathbf{A} = 6.1$, $g = 2$, $Y_1 = 0.81$ and $Y_2 = 0.4$

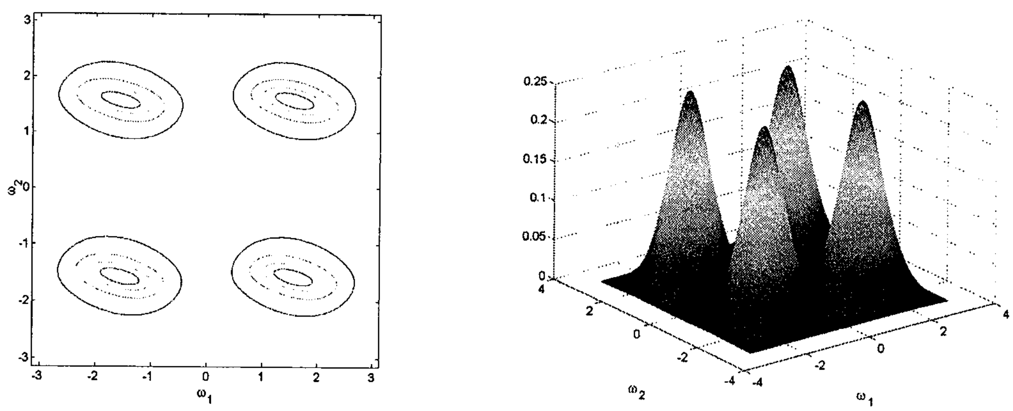


Figure 6.8(h): $\mathbf{A} = 6.1$, $g = 2$, $Y_1 = 0.81$ and $Y_2 = 2$

Figure 6.8: The contour and 3-D magnitude plot of the resulting 2-D band pass filter for different values of **A**, g, Y_1 and Y_2.

After exploring the above figure in details, one can say that **A,** Y_1 and Y_2.do not have any effects on the magnitude of the response individually. g is inversely proportional with the magnitude of the 2-D band-pass filter response. Y_1 is inversely proportional with the bandwidth in the ω_1 domain of the designed filter and Y_2 is inversely proportional with the bandwidth in the ω_2 domain of the band-pass response. And if you recall the effects of these coefficients on the 2-D LPF then you can see that the effects of each matrix coefficient on the 2-D band-pass filter response are exactly same as the effects of that coefficient on the 2-D low-pass filter. Hence rather than analyzing each matrix coefficient individually, we only study gain-effective matrix coefficients **A** and g together and band-effective coefficients Y_1 , Y_2 and **A** together.

High Pass frequency response with different values A and g **together**

Same as the case of 2-D LPF or 2-D HPF, **A** and g together in some particular manner change the gain of the frequency response drastically in the case of 2-D band pass filter too. Below figure shows some simulation results for different values of **A** and g.

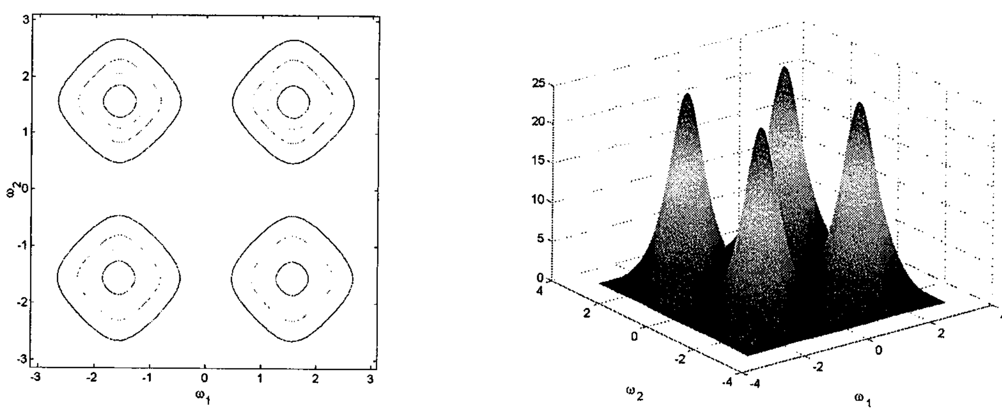


Figure 6.9(a): A=0.061 and g=0.2

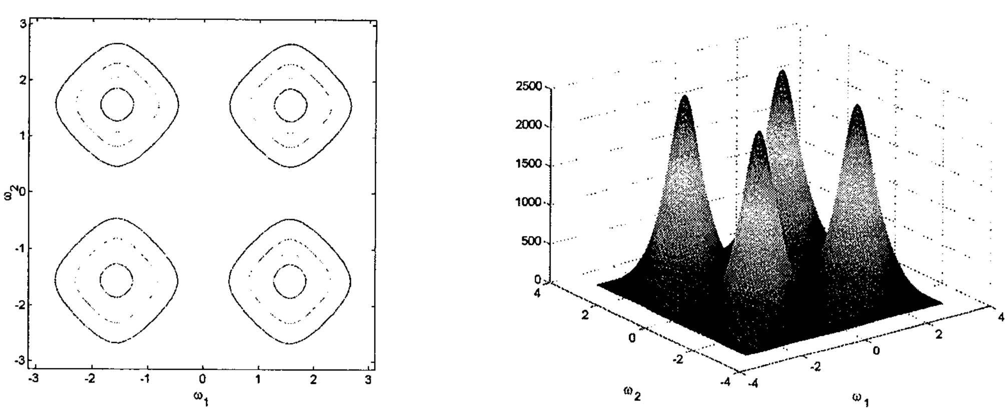


Figure 6.9(b): A=0.00061 and g=0.02

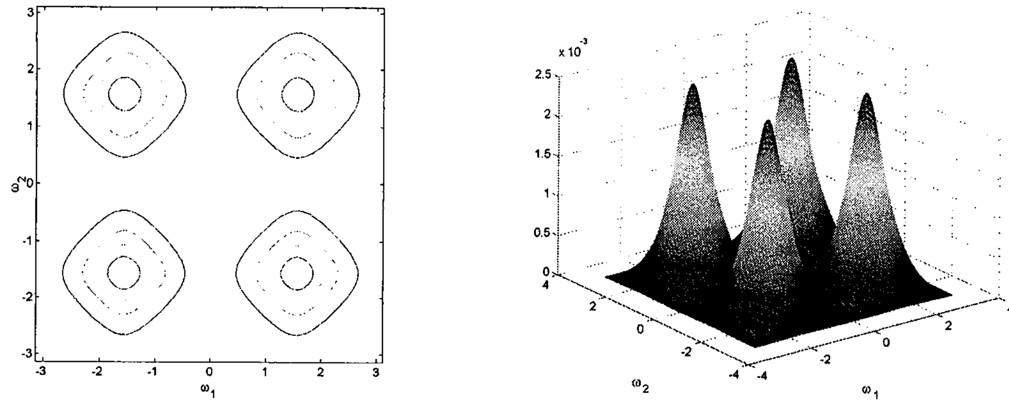


Figure 6.9(c): A=610 and g=20

Figure 6.9: The contour and 3-D magnitude plot of the resulting 2-D band pass filter for different values of **A** $=\psi_1 * \lambda_1 * (a-b)^2$ and g with $Y_1 = Y_2$ =0.81.

After observing these cases carefully, we can say that if we divide the reference value of g by a factor "x" and the reference value of **A** by the square of the factor "x", then we can increase the reference gain by square of that factor "x", while there are not any effects on the bandwidth of the frequency response. This is also evident from the transfer function of the designed 2-D band-pass filter.

High Pass frequency response with different values A, Y_1 and Y_2

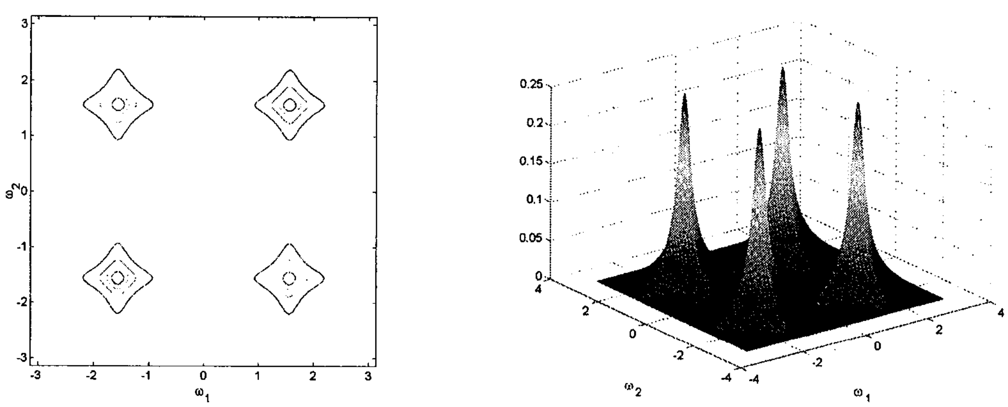


Figure 6.10(a): $Y_1 = Y_2 = 0.3$ and **A**=1600

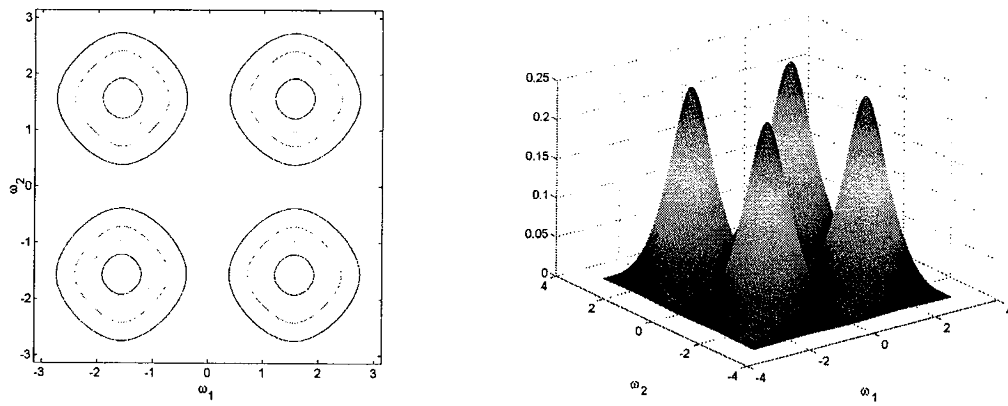


Figure 6.10(b): $Y_1 = Y_2 = 1$ and $\mathbf{A} = 4$

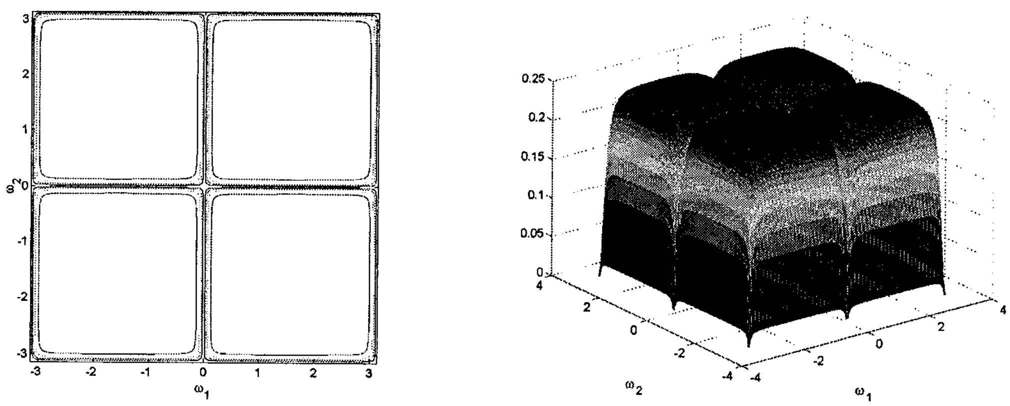


Figure 6.10(c): $Y_1 = Y_2 = 20$ and $\mathbf{A} = 0.01$

Figure 6.10: The contour and 3-D magnitude plot of the resulting 2-D band pass filter for different values of Y_1, Y_2 and $\mathbf{A} = \psi_1 * \lambda_1 * (a-b)^2$ with g = -2.

In last two chapters we have seen that behavior of Y_1 and Y_2 on the bandwidth is understandable, if we change the values of Y_1 and Y_2 with satisfying the equation 4.5 and keeping the value of g = 2. Above figure shows some simulation results for this case.

From this simulation results we can say that when we assign equal value to Y_1 and Y_2 with satisfying the equation 4.5, the values of Y_1 and Y_2 are directly proportional to the bandwidth of the frequency response of the 2-D band-pass filter. It is visible that each contour plot is having Octagonal symmetry in above figure. But the more effective and easy way to obtain the desirable bandwidth is by changing the value of $K_i s$, because we do not have to satisfy any conditions in this case.

6.4 Summary

In summary, this chapter has shown a new technique of designing 2-D band-pass recursive filter with variable properties by implementing double generalized bilinear transformations to 2-variable analog function. We have dealt with one of important combination filters, 2-D band-pass recursive filter. The purpose of this chapter has been to study the effects of different coefficients of the 2-D band-pass transfer function on the frequency response. In this case also, contour and symmetry properties can be varied by assigning different values to the variables in the 2-D transfer function.

In general from the results obtained in this chapter, it is realistic to develop a 2-D band-pass filter with required bandwidth and amplitude properties.

There are many combination filters are possible, and this chapter is one of the first stages towards the study of combination filters. These combination filters use in the modern image processing to enhance image quality.

7. CONCLUSIONS

In the modern Image Processing world, 2-D systems are widely being used and 2-D digital filters are one of the main uses in them. Different kinds of 2-D filters are used for many Image processing purposes e.g. smoothening, sharpen, edge detection, etc. In all the above applications data used is mainly 2-D. Hence necessitates the study of various 2-D digital filters with flexible characteristics, which motivated us to work on this thesis. In this chapter, we conclude our wok with the contribution to this thesis and suggestions for future work

7.1 Summary

It is amply demonstrated that a 2-D recursive filter response can be modified by the application of a suitable double generalized bilinear transformation, providing the designer with the modified 2-D digital transfer function. The cases considered are the 2-D low pass recursive filter, 2-D high-pass digital filter and 2-D band-pass recursive filter.

7.1.1 2-D Low-pass recursive filter

The double generalized bilinear coefficients $K_i s$ (i = 1, 2) and matrix coefficients $Y_i s$ (i = 1, 2) mainly affect the bandwidth characteristics of the 2-D low-pass recursive filter. The coefficients $Y_i s$ also has minor effect on the magnitude response, so we suggest to use the double generalized bilinear coefficients $K_i s$ for any c hanges in the bandwidth of the 2-D low-pass recursive filter because it has no effects on the magnitude response of the designed filter.

Double generalized bilinear coefficients $a_{oi} s$ and Matrix coefficients **A** and g together are inversely proportional to the magnitude response of the designed low-pass filter. Because of $a_{oi} s$ also affect the bandwidth characteristics of the 2-D low-pass recursive filter, for big changes in magnitude we put forward to use matrix coefficients **A** and g together.

The coefficients $b_{oi} s$ act as polarity-effect coefficients. $b_{oi} s$ determine whether the resulting filter is either a low-pass filter or a high-pass filter. The effect of $b_{oi} s$ results in non-zero gain of the stop-band of the designed filter. If one keep $K_1 = K_2$, $b_{01} = b_{02}$ and $a_{01} = a_{02}$, satisfying equation 4.5 will result in the Quadrantal symmetry in the response of the filter. As well as, by keeping Y_1 equal to Y_2, we can obtain Diagonal symmetry. And by combining the both conditions, one can get a 2-D low-pass filter response with an Octagonal symmetry.

7.1.2 2-D high-pass recursive filter

The double generalized bilinear coefficients $K_i s$ (i = 1, 2) are inversely proportional to the bandwidth of the 2-D high-pass filter and have no effects on magnitude response. Matrix coefficients $Y_i s$ (i = 1, 2) also affect the bandwidth characteristics of the 2-D high-pass recursive filter. To understand the effects of $Y_i s$ on the magnitude response, one has to satisfy some conditions as discussed in section 5.3.2. Hence, we propose to use the double generalized bilinear coefficients $K_i s$ for any changes in the bandwidth of the 2-D high-pass recursive filter.

Double generalized bilinear coefficients $a_{oi} s$ are directly proportional to the magnitude response of the designed high-pass filter, while Matrix coefficients **A** and g together are inversely proportional to the magnitude response of the filter. As $a_{oi} s$ have minor effects on the bandwidth characteristics of the response, hence for big changes in magnitude we suggest to use matrix coefficients **A** and g together.

The coefficients $b_{oi} s$ act as polarity-effect coefficients. The effect of $b_{oi} s$ results in non-zero gain of the stop-band of the designed filter, which might be change the polarity of the filter. One of the solutions for this problem is change in $K_i s$ as shown in section 5.3.1. If we satisfy the symmetry conditions discussed in Chapter 4.3.2, 2-D high-pass recursive filter also hold all the symmetries shown in the previous chapter "2-D low-pass recursive filter"

7.1.3 Combination filter, 2-D band-pass recursive filter

The double generalized bilinear coefficients $K_i s$ (i = 1, 2, 3, 4) are inversely proportional to the bandwidth of the 2-D high-pass filter and also have small effects on magnitude response. Double generalized bilinear coefficients $a_{oi} s$ related to HPF are directly proportional to the magnitude response of the designed filter, while $a_{oi} s$ related to LPF are inversely proportional to the magnitude response of the 2-D band-pass filter. $a_{oi} s$ also affect the bandwidth of the frequency response. $a_{oi} s$ and $K_i s$ related to S_1 have effects on the bandwidth in ω_2 domain and $a_{oi} s$ and $K_i s$ related to S_2 affect the bandwidth in ω_1 region.

Matrix coefficients **A** and g together are inversely proportional to the magnitude response of the 2-D band-pass recursive filter Matrix coefficients $Y_i s$ (i = 1, 2) with satisfying Equation 4.5 and g=2, are inversely proportional to the bandwidth of the 2-D band-pass recursive filter.

We propose to use the double generalized bilinear coefficients $K_i s$ for any changes in the bandwidth of the 2-D band-pass recursive filter, because in this case we do not have to satisfy any conditions.. As $a_{oi} s$ have minor effects on the bandwidth characteristics of the response, hence for big changes in magnitude we suggest to use matrix coefficients **A** and g together. 2-D band-pass recursive filter also hold all the symmetries shown in Chapter 4 and Chapter 5, if we fulfill the symmetry conditions discussed in the Section 4.3.2.

7.2 Contribution

This thesis is one of the initial steps towards the study of "Different kinds of 2-D recursive filters with flexible characteristics". We have obtained 2-D digital transfer function with few variable coefficients from an existing 2-variable VSHP by implementing the Double generalized bilinear transformation.

We have also shown under some conditions the designed 2-D transfer function works as 2-D low-pass recursive filter or 2-D high-pass recursive filter with stability conditions. Then we investigated the effects of each coefficient on the frequency response of 2-D LPF or 2-D HPF with simulation results – 3-D magnitude plots and 2-D contour plots. This study helps to design a 2-D LPF or a 2-D HPF with desirable characteristics in terms of magnitude, bandwidth and symmetry.

It is possible to develop new kinds of 2-D filters by combining a 2-D LPF and a 2-D HPF. Keeping this idea in mind, we designed 2-D band-pass filter with some variables. We determined the effects of some effective variables on the frequency response with simulation results. 2-D LPF and 2-D HPF with desirable characteristics can be designed with the help of this study; one can design a 2-D band-pass digital filter with expected properties such as magnitude, bandwidth and symmetry.

7.3 Suggestions for future work

- In this thesis we have studied one combination filter – 2-D band-pass filter. The work can be extended for different combinations e.g.. band-elimination filter, all stop filter..

- We came up with the conditions to obtain Quadrantal symmetry, Diagonal symmetry and Octagonal symmetry in the frequency response of the designed 2-D filters. One can further investigate the response of these designed filters to find constrains for other kinds of symmetries.
- This study may lead to the design of 2-D all-pass filters by appropriately associating the numerator polynomial to the transfer function.
- In Chapter 3, we have formulated two 2-variable VSHPs. We chose one of them and designed different kinds of 2-D recursive filters with flexible characteristics. One can always start with another 2-variable VSHP (Equation 3.13) and design 2-D recursive filters by applying the double generalized bilinear transformation to Equation 3.13.

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Appendix

A1. Program for Chapter 4: 2-D Low-Pass Filter

```
%-.-.-.-.-.-  2-D Low-Pass Recursive Filter Designing  -.-.-.-.-.-%

clear all;
clear workspace;

% sai1 = -6.1;
% lam1 = -1;
% a = 1;
% b = 2;
% g = 2;
% y1= 0.81;
% y2= 0.81;

answ = [-6.1 -1 1 2 2 0.81 0.81];

prompt={'Enter the value for sai1 ',
        'Enter the value for lam1',
        'Enter the value for a',
        'Enter the value for b other than a',
        'Enter the value for g'
        'Enter the value for y1',
        'Enter the value for y2'};
def={'-6.1','-1','1','2','2','0.81','0.81'};
dlgTitle='Matrix Coefficients for 2-D Low Pass Filter';
lineNo=1;
answer=inputdlg(prompt,dlgTitle,lineNo,def);

abc = char(answer);
e=size(abc);
for n = 1:e(1)
  S = abc(n,1);
  for m = 2:e(2)
```

```
      S = [S abc(n,m)];
    end
    answ(n)= sscanf(S,'%f');
  end

     sai1 = answ(1);
     lam1 = answ(2);
     a = answ(3);
     b = answ(4);
     g = answ(5);
     y1 = answ(6);
     y2 = answ(7);

% k1 = 1;
% k2 = 1;
% a01 = -1; a02 = -1; b01 = 1; b02 = 1;
answ = [1 1 -1 -1 1 1];

prompt={'Enter the value for k1 ',
        'Enter the value for k2',
        'Enter the value for a01 inbetween -1 to 0',
        'Enter the value for a02 inbetween -1 to 0',
        'Enter the value for b01 inbetween 0 to 1'
        'Enter the value for b02 inbetween 0 to 1'};
def={'1','1','-1','-1','1','1'};
dlgTitle='Double Generalized Bilinear Coefficients for 2-D Low Pass
Filter';
lineNo=1;
answer=inputdlg(prompt,dlgTitle,lineNo,def);

abc = char(answer);
e=size(abc);
for n = 1:e(1)
  S = abc(n,1);
  for m = 2:e(2)
     S = [S abc(n,m)];
  end
  answ(n)= sscanf(S,'%f');
end

   k1 = answ(1);
   k2 = answ(2);
   a01 = answ(3);
   a02 = answ(4);
   b01 = answ(5);
   b02 = answ(6);

t1 = 1;
t2 = 1;

con1 = 1;
[X,Y] = meshgrid(-3.14:0.05:3.14);

for w1 = -3.14:0.05:3.14
    con2 = 1;
    for w2 = -3.14:0.05:3.14
         z1 = 2.73^(w1*j*t1);
```

```
          z2 = 2.73^(w2*j*t2);
          ans1 =(sai1*lam1*k1*k2*[(a-b)^2] *
[(z1*z2)+(a02*z1)+(a01*z2)+(a01*a02)]);
          ans2 =(sai1*lam1*y2*k1*[(a-b)^2]*
[(z1*z2)+(a01*z2)+(b02*z1)+(a01*b02)]);
          ans3=(lam1*sai1*k2*y1*[(a-b)^2]*
[(z1*z2)+(b01*z2)+(a02*z1)+(a02*b01)]);
          ans4=([g^2]*[(z1*z2)+(b01*z2)+(b02*z1)+(b01*b02)]);

          ans5 = ans1 + ans2 + ans3 + ans4 ;

          ans6 = [(z1*z2)+(b01*z2)+(b02*z1)+(b01*b02)];

          an = ans6 / ans5 ;
          An(con1,con2)= abs(an);
          con2 = con2 +1;
      end;
      con1 = con1+ 1;
 end;

%-----------------------------------------------------------------
%  sai1 = -0.444;
%  g = 0.5;
%  y1 = 0.75;
%  y2 = 0.75;
%  a01 = -0.8; a02 = -0.8; b01 = 1; b02 = 1;
%
%  con1 = 1;
%
%  for w1 = -3.14:0.05:3.14
%      con2 = 1;
%      for w2 = -3.14:0.05:3.14
%            z1 = 2.73^(w1*j*t1);
%            z2 = 2.73^(w2*j*t2);
%            ans1 =(sai1*lam1*k1*k2*[(a-
b)^2]*[(z1*z2)+(a02*z1)+(a01*z2)+(a01*a02)])+
%                  (sai1*lam1*y2*k1*[(a-
b)^2]*[(z1*z2)+(a01*z2)+(b02*z1)+(a01*b02)])+
%                  (lam1*sai1*k2*y1*[(a-
b)^2]*[(z1*z2)+(b01*z2)+(a02*z1)+(a02*b01)])+
%                  ([g^2]*[(z1*z2)+(b01*z2)+(b02*z1)+(b01*b02)]);
%           ans2 = [(z1*z2)+(b01*z2)+(b02*z1)+(b01*b02)];
%           an = ans2 / ans1;
%           Anb(con1,con2)= abs(an);
%           con2 = con2 +1;
%       end;
%       con1 = con1+ 1;
%  end;
%-----------------------------------------------------------------

 mesh(X, Y, An);%,'FontSize',12);
 set(gca,'FontSize',12);
 xlabel('\omega_1','FontSize',14);
 ylabel('\omega_2','FontSize',14);
%  title('a01 = -0.1; a02 = 1; b01 = 1; b02 = -0.1','FontSize',12);
 figure;
 contour(X,Y,An);
```

```
set(gca,'DataAspectRatio',[1 1 1]);

%-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-
% figure;
% contour(X,Y,Anb);
% set(gca,'DataAspectRatio',[1 1 1]);
% figure;
% contour(X,Y,Anb,'b--');
% hold on,
% contour(X,Y,An,'r-.')
% set(gca,'DataAspectRatio',[1 1 1]);
% set(gca,'FontSize',12);
% xlabel('\omega_1','FontSize',14);
% ylabel('\omega_2','FontSize',14);
% title('a01 = -0.1; a02 = 1; b01 = 1; b02 = -0.1','FontSize',12);
%-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-
```

A2. Program for Chapter 5: 2-D High-Pass Filter

```
%-.-.-.-.-.-  2-D High-Pass Recursive Filter Designing  -.-.-.-.-.-%

clear all;
clear workspace;

% sail = -6.1;
% lam1 = -1;
% a = 1;
% b = 2;
% g = 2;
% y1= 0.81;
% y2= 0.81;

answ = [-6.1 -1 1 2 2 0.81 0.81]

prompt={'Enter the value for sail ',
        'Enter the value for lam1',
        'Enter the value for a',
        'Enter the value for b other than a',
        'Enter the value for g'
        'Enter the value for y1',
        'Enter the value for y2'};
def={'-6.1','-1','1','2','2','0.81','0.81'};
dlgTitle='Matrix Coefficients for 2-D High Pass Filter';
lineNo=1;
answer=inputdlg(prompt,dlgTitle,lineNo,def);

abc = char(answer);
e=size(abc);
for n = 1:e(1)
  S = abc(n,1);
  for m = 2:e(2)
     S = [S abc(n,m)];
  end
  answ(n)= sscanf(S,'%f');
end

sail = answ(1);
lam1 = answ(2);
a = answ(3);
b = answ(4);
g = answ(5);
y1 = answ(6);
y2 = answ(7);

% k1 = 1;
% k2 = 1;
% a01 = 1; a02 = 1; b01 = -1.0; b02 = -1;

answ = [1 1 1 1 -1 -1];

prompt={'Enter the value for k1 ',
```

```
        'Enter the value for k2',
        'Enter the value for a01 inbetween 0 to 1',
        'Enter the value for a02 inbetween 0 to 1',
        'Enter the value for b01 inbetween -1 to 0'
        'Enter the value for b02 inbetween -1 to 0'};
def={'1','1','1','1','-1','-1'};
dlgTitle='Double Generalized Bilinear Coefficients for 2-D HPF';
lineNo=1;
answer=inputdlg(prompt,dlgTitle,lineNo,def);

abc = char(answer);
e=size(abc);
for n = 1:e(1)
  S = abc(n,1);
  for m = 2:e(2)
     S = [S abc(n,m)];
  end
  answ(n)= sscanf(S,'%f');
end

   k1 = answ(1);
   k2 = answ(2);
   a01 = answ(3);
   a02 = answ(4);
   b01 = answ(5);
   b02 = answ(6);

t1 = 1;
t2 = 1;

con1 = 1;
[X,Y] = meshgrid(-3.14:0.05:3.14);

for w1 = -3.14:0.05:3.14
    con2 = 1;
    for w2 = -3.14:0.05:3.14
         z1 = 2.73^(w1*j*t1);
         z2 = 2.73^(w2*j*t2);
         ans01 = (sai1*lam1*k1*k2*[(a-b)^2]*
     [(z1*z2)+(a02*z1)+(a01*z2)+(a01*a02)]);
         ans02 = (sai1*lam1*y2*k1*[(a-b)^2]*
     [(z1*z2)+(a01*z2)+(b02*z1)+(a01*b02)]);
         ans03 = (lam1*sai1*k2*y1*[(a-b)^2]*
     [(z1*z2)+(b01*z2)+(a02*z1)+(a02*b01)]);
         ans04 = ([g^2]*[(z1*z2)+(b01*z2)+(b02*z1)+(b01*b02)]);

         ans1 = ans01 + ans02 + ans03 + ans04;
         ans2 = [(z1*z2)+(b01*z2)+(b02*z1)+(b01*b02)];

         an = ans2 / ans1;
         Anhp(con1,con2)= abs(an);
         con2 = con2 +1;
     end;
     con1 = con1+ 1;
 end;

%-----------------------------------------------------------------------
```

```
%  sai1 = 0.137;
%  g = 0.3;
%  y1 = 0.81;
%  y2 = 0.81;
%  a01 = 1; a02 = 1; b01 = -1; b02 = -1;
% k1 = 0.1;
% k2 = 0.1;
%  con1 = 1;
%
%  for w1 = -3.14:0.05:3.14
%     con2 = 1;
%     for w2 = -3.14:0.05:3.14
%           z1 = 2.73^(w1*j*t1);
%           z2 = 2.73^(w2*j*t2);
%           ans1 =(sai1*lam1*k1*k2*[(a-b)^2]*
[(z1*z2)+(a02*z1)+(a01*z2)+(a01*a02)])+
%                 (sai1*lam1*y2*k1*[(a-b)^2]*
[(z1*z2)+(a01*z2)+(b02*z1)+(a01*b02)])+
%                 (lam1*sai1*k2*y1*[(a-b)^2]*
[(z1*z2)+(b01*z2)+(a02*z1)+(a02*b01)])+
%                 ([g^2]*[(z1*z2)+(b01*z2)+(b02*z1)+(b01*b02)]);
%           ans2 = [(z1*z2)+(b01*z2)+(b02*z1)+(b01*b02)];
%           an = ans2 / ans1;
%           Anb(con1,con2)= abs(an);
%           con2 = con2 +1;
%      end;
%      con1 = con1+ 1;
%  end;
%-----------------------------------------------------------------------

 mesh(X, Y, Anhp);
 set(gca,'FontSize',12);

 xlabel('\omega_1','FontSize',14);
 ylabel('\omega_2','FontSize',14);
% title('a01 = -0.1; a02 = 1; b01 = 1; b02 = -0.1','FontSize',12);
 figure;
 contour(X,Y,Anhp);
 %title('a01 = -0.1; a02 = 1; b01 = 1; b02 = -0.1','FontSize',12);
 set(gca,'DataAspectRatio',[1 1 1]);
 set(gca,'FontSize',12);

%-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-
%  figure;
%  contour(X,Y,Anb);
%  set(gca,'DataAspectRatio',[1 1 1]);
%  figure;
%  contour(X,Y,Anb,'r--');
%  hold on,
%  contour(X,Y,Anhp,'k-.')
%  set(gca,'DataAspectRatio',[1 1 1]);
%  xlabel('\omega_1','FontSize',14);
%  ylabel('\omega_2','FontSize',14);
%-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-
```

A3. Program for Chapter 6: 2-D Band-Pass Filter

```
%-.-.-.-.-.-  2-D Band-Pass Recursive Filter Designing  -.-.-.-.-.-%

clear all;
clear workspace;

% sail = -6.1;
% lam1 = -1;
% a = 1;
% b = 2;
% g = 2;
% y1= 0.81;
% y2= 0.81;
answ = [-6.1 -1 1 2 2 0.81 0.81];

prompt={'Enter the value for sail ',
        'Enter the value for lam1',
        'Enter the value for a',
        'Enter the value for b other than a',
        'Enter the value for g'
        'Enter the value for y1',
        'Enter the value for y2'};
def={'-6.1','-1','1','2','2','0.81','0.81'};
dlgTitle='Matrix Coefficients for 2-D Band Pass Filter';
lineNo=1;
answer=inputdlg(prompt,dlgTitle,lineNo,def);

abc = char(answer);
e=size(abc);
for n = 1:e(1)
  S = abc(n,1);
  for m = 2:e(2)
     S = [S abc(n,m)];
  end
  answ(n)= sscanf(S,'%f');
end

   sail = answ(1);
   lam1 = answ(2);
   a = answ(3);
   b = answ(4);
   g = answ(5);
   y1 = answ(6);
   y2 = answ(7);

% low pass double generalized bilinear coefficients for 2-D Bandpass
filter
% k1 = 1;
% k3 = 1;
% a01 = -1; a02 = -1; b01 = 1; b02 = 1;
answ = [1 1 -1 -1 1 1];

prompt={'Enter the value for k1 ',
```

```
        'Enter the value for k2',
        'Enter the value for a01 inbetween -1 to 0',
        'Enter the value for a02 inbetween -1 to 0',
        'Enter the value for b01 inbetween 0 to 1'
        'Enter the value for b02 inbetween 0 to 1'};
def={'1','1','-1','-1','1','1'};
dlgTitle='Lowpass D.G.B. Coefficients for 2-D Band Pass Filter';
lineNo=1;
answer=inputdlg(prompt,dlgTitle,lineNo,def);

abc = char(answer);
e=size(abc);
for n = 1:e(1)
  S = abc(n,1);
  for m = 2:e(2)
     S = [S abc(n,m)];
  end
  answ(n)= sscanf(S,'%f');
end

   k1 = answ(1);
   k3 = answ(2);
   a01 = answ(3);
   a02 = answ(4);
   b01 = answ(5);
   b02 = answ(6);


% high-pass double generalized bilinear coefficients for Band-pass
filter
% k2 = 1;
% k4 = 1;
% a03 = 1; a04 = 1; b03 = -1.0; b04 = -1;
answ = [1 1 1 1 -1 -1];

prompt={'Enter the value for k1 ',
        'Enter the value for k2',
        'Enter the value for a01 inbetween 0 to 1',
        'Enter the value for a02 inbetween 0 to 1',
        'Enter the value for b01 inbetween -1 to 0'
        'Enter the value for b02 inbetween -1 to 0'};
def={'1','1','1','1','-1','-1'};
dlgTitle='Highpass D.G.B. Coefficients for 2-D Band Pass Filter';
lineNo=1;
answer=inputdlg(prompt,dlgTitle,lineNo,def);

abc = char(answer);
e=size(abc);
for n = 1:e(1)
  S = abc(n,1);
  for m = 2:e(2)
     S = [S abc(n,m)];
  end
  answ(n)= sscanf(S,'%f');
end

   k2 = answ(1);
```

```
    k4 = answ(2);
    a03 = answ(3);
    a04= answ(4);
    b03 = answ(5);
    b04 = answ(6);

t1 = 1;
t2 = 1;

con1 = 1;
[X,Y] = meshgrid(-3.14:0.02:3.14);

for w1 = -3.14:0.02:3.14
    con2 = 1;
    for w2 = -3.14:0.02:3.14
         z1 = 2.73^(w1*j*t1);
         z2 = 2.73^(w2*j*t2);

         ans1 = sai1*lam1*k1*k2*[(a-b)^2]*[(z1+a01)*(z2+a02)] /
[(z1+b01)*(z2+b02)];
         ans2 = sai1*lam1*k1*k4*[(a-b)^2]*[(z1+a01)*(z2+a04)] /
[(z1+b01)*(z2+b04)];
         ans3 = sai1*lam1*k3*k2*[(a-b)^2]*[(z1+a03)*(z2+a02)] /
[(z1+b03)*(z2+b02)];
         ans4 = sai1*lam1*k3*k4*[(a-b)^2]*[(z1+a03)*(z2+a04)] /
[(z1+b03)*(z2+b04)];
         ans5 = sai1*lam1*y2*k1*[(a-b)^2]*[(z1+a01)] / [(z1+b01)];
         ans6 = sai1*lam1*y2*k3*[(a-b)^2]*[(z1+a03)] / [(z1+b03)];
         ans7 = sai1*lam1*y1*k2*[(a-b)^2]*[(z2+a02)] / [(z2+b02)];
         ans8 = sai1*lam1*y1*k4*[(a-b)^2]*[(z2+a04)] / [(z2+b04)];
         ans9 = g^2;

         an = 1 / (ans1 + ans2 + ans3 + ans4 + ans5 + ans6 + ans7 +
ans8 + ans9);

         Anbp(con1,con2)= abs(an);
         con2 = con2 +1;
     end;
     con1 = con1+ 1;
 end;

%---------------------- Cascading Method ------------------------ %
% con1 = 1;
% [X,Y] = meshgrid(-3.14:0.05:3.14);
%
% for w1 = -3.14:0.05:3.14
%     con2 = 1;
%     for w2 = -3.14:0.05:3.14
%          z1 = 2.73^(w1*j*t1);
%          z2 = 2.73^(w2*j*t2);
%          ans1 =(sai1*lam1*k1*k2*[(a-b)^2]*
[(z1*z2)+(a02*z1)+(a01*z2)+(a01*a02)])+
%                (sai1*lam1*y2*k1*[(a-b)^2]*
[(z1*z2)+(a01*z2)+(b02*z1)+(a01*b02)])+
%                (lam1*sai1*k2*y1*[(a-b)^2]*
[(z1*z2)+(b01*z2)+(a02*z1)+(a02*b01)])+
%                ([g^2]*[(z1*z2)+(b01*z2)+(b02*z1)+(b01*b02)]);
```

```
%           ans2 = [(z1*z2)+(b01*z2)+(b02*z1)+(b01*b02)];
%           an = ans2 / ans1;
%           Anlp(con1,con2)= abs(an);
%           con2 = con2 +1;
%       end;
%       con1 = con1+ 1;
%   end;
%
% con1 = 1;
% [X,Y] = meshgrid(-3.14:0.05:3.14);
%
% for w1 = -3.14:0.05:3.14
%     con2 = 1;
%     for w2 = -3.14:0.05:3.14
%           z1 = 2.73^(w1*j*t1);
%           z2 = 2.73^(w2*j*t2);
%           ans1 =(sai1*lam1*k1*k2*[(a-b)^2]*
[(z1*z2)+(a02*z1)+(a01*z2)+(a01*a02)])+
%                 (sai1*lam1*y2*k1*[(a-b)^2]*
[(z1*z2)+(a01*z2)+(b02*z1)+(a01*b02)])+
%                 (lam1*sai1*k2*y1*[(a-b)^2]*
[(z1*z2)+(b01*z2)+(a02*z1)+(a02*b01)])+
%                 ([g^2]*[(z1*z2)+(b01*z2)+(b02*z1)+(b01*b02)]);
%           ans2 = [(z1*z2)+(b03*z2)+(b04*z1)+(b03*b04)];
%           an = ans2 / ans1;
%           Anhp(con1,con2)= abs(an);
%           con2 = con2 +1;
%       end;
%       con1 = con1+ 1;
%   end;
%
%   Anbp = Anlp + Anhp;
%-----------------------------------------------------------------------

 mesh(X, Y, Anbp);
 set(gca,'FontSize',12);
 xlabel('\omega_1','FontSize',14);
 ylabel('\omega_2','FontSize',14);
 figure;
 contour(X,Y,Anbp);
 set(gca,'DataAspectRatio',[1 1 1]);
 set(gca,'FontSize',12);
 xlabel('\omega_1','FontSize',14);
 ylabel('\omega_2','FontSize',14);

%-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-
%  figure;
%  contour(X,Y,Anbp,'r.');
%  hold on,
%  contour(X,Y,Anlp,3,'b--');
%  contour(X,Y,Anhp,3,'k:');
%  set(gca,'DataAspectRatio',[1 1 1]);
%  set(gca,'FontSize',12);
%  xlabel('\omega_1','FontSize',14);
%  ylabel('\omega_2','FontSize',14);
%  hold off;
%-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-.-
```