A Study of Designing Recursive 2-D Digital Filter from an analog Bridged-T network

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ABSTRACT

A Study of Designing Recursive 2-D Digital Filter from an analog Bridged-T network

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Two-dimensional recursive digital filters are widely used in signal processing and image processing, as well as divers communication systems. The main objective of this thesis has been to propose a new technique of designing 2-D recursive digital filters from an analog Bridged-T network. Starting from transfer function of a Bridged-T network in the analog domain which is VSHP, 2-D recursive digital filters can be obtained through the application of the double generalized bilinear transformations with the coefficients in their specified ranges. The impedance values of the transfer function of the Bridged-T network are obtained with compare to the fourth order Butterworth polynomial. For different impedance values of the Bridged-T network we get different types of filter output - all pass filter, band pass filter, band stop filter and low pass filter. The manner how each coefficient of generalized bilinear transformation affects each kind of 2-D recursive digital filter is investigated in details.

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To my beloved mother Ayesha Saleha

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List of Important Symbols

z_1, z_2	z domain parameter in first and second dimensions
s_1, s_2	Laplace domain parameter in first and second dimensions
ω_1, ω_2	Frequencies in radians in the discrete domain in first and second dimensions
k_1, k_2	Coefficients of the generalized bilinear transformations in the first and second dimensions.
a_1, a_2	Coefficients of the generalized bilinear transformations in the first and second dimensions
b_1, b_2	Coefficients of the generalized bilinear transformations in the first and second dimensions
$H(z_1, z_2)$	Frequency responses of 2-D digital filters
$H_a(s_1, s_2)$	Frequency responses of 2-D analog filters
$N_d(s_1, s_2)$	Numerator of 2-D digital functions
$D_d(s_1, s_2)$	Denominator of 2-D digital functions
$N_a(s_1, s_2)$	Numerator of 2-D analog functions
$D_a(s_1, s_2)$	Denominator of 2-D analog functions
$H_a(s_1, s_2)$	Frequency responses of 2-D analog filters
$\sum$	Summation

$\prod$	Products of
Δ	Determinant
T1	Table 1
T2	Table 2
VSHP	Very Strict Hurwitz Polynomial
R_1	Source resistance of the Bridged-T network
R_2	Load resistance of the Bridged-T network
L, C	Inductance and capacitance of the Bridged-T network

Chapter 1

Introduction

1.1 General [21,24, 25, 26]

Two dimensional 2-D digital filters have been used extensively in recent years for the processing, enhancement and restoration of images. Their application encompasses many fields and includes tomography, seismic record processing, geophysical exploration, oil prospecting, radar, radio astronomy, to name a few. Two dimensional processing of an image is accomplished by scanning the image and then digitizing it by means of an analog to digital converter. The discrete data generated are then stored in the memory of a computer and, subsequently, they are processed by using a 2-D digital filter. It is possible to process such signals by means of 1-D digital filters. However, it is preferable to use 2-D techniques, because of some important inherent advantages in these techniques. On the other hand, computation time can be reduced and other 2-D systems have many more degrees of freedom which give a system designer flexibility not encountered in 1-D techniques. In addition, in 2-D techniques the rate at which a band limited signal is sampled can be adjusted and also the scanning of an image can be performed in several directions, whereas in 1-D techniques only the sampling rate can be adjusted. Two classes of digital filters can be identified depending on the nature of their impulse response, namely, infinite impulse response (IIR) and finite impulse response (FIR) filters. IIR filters are, in general, implemented recursively while FIR filters are, in general, implemented non-recursively.

FIR filters are always stable and can readily be designed to have constant group delay. However, in order to obtain high selectivity, the order of an FIR filter has to be much larger relative to that of an IIR filter having similar characteristics.

IIR filters, on the other hand, cannot be designed to have constant group delay and, consequently, their design entails the solution of a difficult approximation problem whereby amplitude and group delay specification must be satisfied simultaneously. A second problem associated with IIR filters is that their stability is not always assured as in the case of FIR filters and hence their design must incorporate stability tests in order to ensure that the filters obtained in the solution of the approximation problem are stable.

Much more attention has been devoted to IIR digital filters than to FIR filters due to their potential advantages and efficiency in processing large amounts of data. Recently, 2-D filters with variable characteristics are widely applied to signal processing processes and communication systems where the frequency-domain characteristics of digital filters are required to be adjustable. More researchers have started to study the properties of such filters. These could be variable magnitude response, phase response and group delay. To achieve the variable characteristics, one or more coefficients of the digital transfer functions should be changeable. However, the stability conditions have to be satisfied always.

There are various methods for designing 2-D digital filters. Among them the popular methods are based on frequency transformation, adding adjustable multipliers to filter analog circuits and bilinear transformation. The most frequently used methods for designing a 2-D recursive digital filter is to start from a corresponding 2-D analog filter and then apply the well-known bilinear transformation [23].

$$s_i = k_i \frac{z_i - a_i}{z_i + b_i}, \text{ where } i = 1, 2 \tag{1.1}$$

where k_i, a_i, b_i are positive constants, and $k_i>0$, $|a_i| \leq 1$ and $|b_i| \leq 1$, $i = 1,2$.

The main problem of this method is stability. This can be overcome by a special class of polynomial with 2-variable analog domain called Very Strict Hurwitz Polynomial. A brief review of such polynomials is given below.

1.2 Overview of Very Strict Hurwitz Polynomial

1.2.1 Definition of Very Strict Hurwitz Polynomial [1]

In one-dimensional systems (both analog and discrete), we can use suitably chosen transfer functions having no common factors between the numerator and denominator in order to design a filter required specifications. Specifically, let

$$H_a(s) = \frac{N_a(s)}{D_a(s)} \tag{1.2}$$

be a transfer function in the analog domain with $N_a(s)$ and $D_a(s)$ being relatively prime. In order that the function is stable, $D_a(s)$ should be a Strictly Hurwitz Polynomial (SHP), which contains all its zeros strictly in the left-half of s-plane.

However, for 2-D analog filter system with the transfer function

$$H_a(s_1,s_2) = \frac{N_a(s_1,s_2)}{D_a(s_1,s_2)} \tag{1.3}$$

the denominator $D_a(s_1,s_2)$ is a SHP cannot always guarantee stability, as it contains non-essential singularity of the second kind. That is, the numerator and denominator become zero at two points $s_1 = j\omega_{10}$ and $s_2 = j\omega_{20,}$ but not in its neighborhood.

We need to explain in short some definitions in order to better understanding of getting digital filter circuit.

1.2.2 Singularities [1]

Considering the case of 2-D analog system, it is quite possible that both the even and the odd parts of a polynomial may become simultaneously zero at specified sets of points, but not in their neighborhood. If this occurs in the denominator of the transfer function, it is called non-essential singularity of first kind. In addition, in 2-D transfer functions, both the numerator and denominator polynomials can become zero simultaneously at a given set of points. When this happens, it is known as non-essential singularity of second kind. Mathematically, for

$$H_a(s_1,s_2) = \frac{N_a(s_1,s_2)}{D_a(s_1,s_2)} \tag{1.4}$$

i) $D_a(s_{10},s_{20}) = 0$ and $N_a(s_{10},s_{20}) \neq 0$ constitute non-essential singularity of the first kind at (s_{10}, s_{20}).

ii) $D_a(s_{10},s_{20}) = 0$ and $N_a(s_{10},s_{20}) = 0$ constitute non-essential singularity of the second kind.

1.2.3 Stability [4,22]

As mentioned above, 2-D filters can be classified into two main categories namely the Finite Response Filters (FIR) and the Infinite Impulse Response Filters (IIR).

The Finite Response Filters have transfer functions resulting from a finite sequence and the Infinite Impulse Response Filters have transfer functions resulting from an infinite sequence.

One important issue concerning both the above types of filters is the stability of the filter. Now it is known that FIR filters are inherently stable, while IIR filters may or may not be stable depending upon the transfer function.

The most commonly used definition for stability is based on the bounded input bounded output (BIBO) criterion. This criterion states that a filter is stable if its response to a bounded input is also bounded. Mathematically, it is possible to show that for causal linear shift invariant systems, this corresponds to the condition that

$$\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\left|h(n_1,n_2)\right| < \infty \qquad (1.5)$$

where, $h(n_1, n_2)$ is the impulse response of the filter.

The above definition points out an important observation that the stability criterion is always verified if the number of terms of the impulse response is finite which is the case with FIR filters. However, the above condition does not prove feasible to the test of stability of IIR filters. In the 1-D case, it is possible to relate the BIBO stability conditions to the positions of Z-domain transfer function poles which have to be within the unit circle and it is possible to test the stability by determining the zeros of the denominator polynomial. Similarly, in the 2-D case, a theorem establishing the relationship between the stability of the filter and the zeros of the denominator polynomial can be formulated. This theorem states that [28], for causal quadrant filters, if $B(z_1, z_2)$ is a polynomial in z_1 and z_2, the expansion of $1/B(z_1, z_2)$ in the negative powers of z_1 and z_2 converges absolutely if and only if

$$B(z_1, z_2) \neq 0 \text{ for } \{|z_1| \geq 1,\ |z_2| \geq 1\} \tag{1.6}$$

The above theorem has the same form as in the 1-D case, i.e., it relates the stability of the filter to the singularities of the z-transform. However, in the 2-D case such a formulation for stability condition does not produce an efficient method for stability test, as in 1-D case, due to the lack of appropriate factorization theorem of algebra. Therefore, it is necessary in principle, to use an infinite number of steps to test the stability. Also, even if it is possible to find methods to test conditions equivalent to equation (1.6) in a finite number of steps [4], computationally it is not easy to incorporate them in a design method and there is a problem of stabilizing the filters which may become unstable.

From the point of view of stability tests, there can be two different approaches that can be considered, in designing in IIR filter. One method is to carry out the stability test in every stage of the filter design so that eventually the filter is stable. In the second method, stability is not considered as a part of the design and a magnitude squared function is first designed. Then a stable filter is obtained, by choosing the poles in the stability region. Such an approach is convenient, because squared magnitude functions can be in a simple form and it is easy to find the poles of the filter.

1.2.4 Bilinear Transformation [1, 6, 23]

One method of generating a stable 2-D digital function has been to apply generalized double bilinear transformation

$$s_i = k_i \frac{z_i - a_i}{z_i + b_i}, \text{ where } i = 1,2 \tag{1.7}$$

on a two-variable analog function with strict Hurwitz denominator. In some cases the 2-D digital function generated using double bilinear transformation may possess nonessential singularities of the second kind on the closed unit bi-disk of the (z_1, z_2) biplane. As the bilinear transformation maps the entire (s_1, s_2) biplane on the entire (z_1, z_2) biplane on a one-to-one basis, the behavior of the function is not altered by the application of the double bilinear transformation.

We first give the definitions of certain terms which we will use in the following sections. We use the notation Re (x) to mean real part of x.

Definition 1.1 A rational function f(s) with real coefficients such that $Re(f(s)) > 0$ for $Re(s) > 0$ is called a positive real function.

Definition 1.2 A positive real function f(s) is said to be a strict positive real function if $Re(f(s)) > 0$ for $Re(s) = 0$.

Definition 1.3 A positive real function f(s) is said to be minimum reactive, susceptive if it has neither poles nor zeros on the imaginary axis of the s plane.

Definition 1.4 A positive real function f(s) is called a reactance function if $Re(f(s)) = 0$ for $Re(s) = 0$.

Definition 1.5 A two-variable rational function $f(s_1,s_2)$ with real coefficients such that $Re(f(s_1,s_2)) \geq 0$ for $Re(s_1) > 0$, $Re(s_2) > 0$ is called a two-variable positive real function.

Definition 1.6 A two-variable positive real function $f(s_1,s_2)$ such that $f(s_1,s_2) = -f(-s_1,-s_2)$ is called a two-variable reactance function.

Definition 1.7 A two-variable polynomial $Q(s_1,s_2)$ is an even polynomial if $Q(s_1,s_2) = Q(-s_1,-s_2)$ and is an odd polynomial if $Q(s_1,s_2) = -Q(-s_1,-s_2)$.

1.2.4.1 Stability conditions for Generalized Bilinear Transformation [23]

We have to get the stability conditions for the bilinear transformation which we will employ. Here we first consider the one domain only.

Theorem 1.1 When $k_1 > 0$, the condition for stability for the generalized bilinear transformation applied to analog transfer function are:

i) $|a_1| \leq 1$ (1.8)

ii) $|b_1| \leq 1$ (1.9)

iii) $a_1 b_1 < 0$ (1.10)

Proof: Let,

$$s_1 = \sigma_1 + j\omega_1 \tag{1.11}$$

$$z_1 = u_1 + jv_1 \tag{1.12}$$

Substitute them into equation of the generalized bilinear transformation (1.7), we get,

$$\sigma_1 = k_1 \frac{(u_1^2 + v_1^2) + (a_1 + b_1)u_1 + a_1 b_1}{(u_1 + b_1)^2 + v_1^2} \tag{1.13}$$

$$\omega_1 = k_1 \frac{v_1 (b_1 - a_1)}{(u_1 + b_1)^2 + v_1^2} \tag{1.14}$$

For the purpose of stability, it is required the imaginary axis of s_1 plane or $\sigma_1 = 0$ need to be mapped to the inner or on the unity circle in the discrete z_1 domain.

$$r_1^2 = u_1^2 + v_1^2 \leq 1 \tag{1.15}$$

Let, $u_1 = r_1 \cos\varphi$ (1.16)

$v_1 = r_1 \sin\varphi$ (1.17)

Substituting them into the equation (1.13), and for $\sigma_1 = 0$, we can get the equation

$$r_1^2 + (a_1 + b_1) r_1 \cos\varphi + a_1 b_1 = 0 \tag{1.18}$$

The roots of equation (1.18) are

$$r_{1,2} = \frac{-(a_1+b_1)\cos\varphi \pm \sqrt{(a_1+b_1)^2\cos^2\varphi - 4a_1b_1}}{2} \tag{1.19}$$

The magnitude of the roots should not be greater than unity. The roots have their maximum values at $\varphi = \pm\pi$. The corresponding roots are

$$r_1 = \pm a_1 \tag{1.20}$$

$$r_2 = \pm b_1 \tag{1.21}$$

Thus it is proved that

$$|a_1| \le 1 \tag{1.22}$$

$$|b_1| \le 1 \tag{1.23}$$

Also for the stability, the unity circle in the discrete domain should be mapped to the closed left of s_1 plane. That requires $\sigma_1 \le 0$ for $r_1 = 0$, hence from (1.13) we can get

$$a_1b_1 \le 0 \text{ for } k_1 > 0 \tag{1.24}$$

or

$$a_1b_1 \ge 0 \text{ for } k_1 < 0 \tag{1.25}$$

Without any loss of generality, we can assume k_1 to be positive, then a_1 and b_1 should be of opposite signs. Thus Theorem 1.1 is proved. The results obtained here can be extended to the two domains. The theorem can be applied in 1-D and 2-D cases.

1.2.4.2 The mapping relationship [23]

For the imaginary axis in $s_1$plane or $\sigma_1 = 0$, from (1.13) we have

$$u_1^2 + v_1^2 + (a_1 + b_1)u_1 + a_1b_1 = 0 \tag{1.26}$$

$$\Rightarrow (u_1 + \frac{a_1+b_1}{2})^2 + v_1^2 = (\frac{a_1-b_1}{2})^2 \tag{1.27}$$

Equation (1.27) is a function of a circle, which the center is at ($-\frac{a_1+b_1}{2}$, 0), with radius = $\left|\frac{a_1}{b_1}\right|$. That is to say, the imaginary axis in s_1 plane is mapped to the circle in the z_1 plane, and the left half plane of s_1 plane is mapped to the inner of the circle, and the right-half plane of s_1 plane is mapped to the outside of the circle.

1.2.5 Two variable Hurwitz polynomial [1]

In a recent survey paper on the stability of multidimensional polynomials [2], Jury has discussed the existence of more than one type of two-variable Hurwitz polynomials. In the study of properties of two-variable reactance functions, Ansell [3] defined a two-variable Hurwitz polynomial in the narrow sense as against the two-variable Hurwitz polynomial in the broad sense, which is similar to the one-variable Hurwitz polynomial.

Finding this definition inadequate in the study of stability analysis, Huang [4] modified this definition so as to avoid the zeros on the imaginary axes of the (s_1, s_2) biplane and called the resulting polynomial a strict Hurwitz polynomial. As stated earlier, the nonessential singularities of the second kind were not considered, and this has caused the difficulty reported by Goodman [5]. As one is interested in the closed right half of the (s_1, s_2) biplane in the study of the stability of transfer functions, it is desirable to include the behavior of the polynomial at infinite distant points in the definition of Hurwitz polynomials. Here a modified definition so as to avoid the second-kind singularities at infinite distant points is proposed. To distinguish this class of polynomials from the earlier classes, these are called Very Strict Hurwitz (VSH) Polynomials. In all, then, there are four types of Hurwitz polynomials and their definitions are stated below in a slightly different form in terms of singularities rather than zeros as has been the common practice. This has been done so as to facilitate a uniform definition for all the four types of polynomials, differing only in the region of analyticity. In the following definitions, $D_a(s_1,s_2)$ is a polynomial in s_1 and s_2 and Re (s) refers to the real part of s.

Definition 1.8 $D_a(s_1,s_2)$ is a broad sense Hurwitz polynomial (BHP) if $\frac{1}{D_a(s_1,s_2)}$ does not possess any singularities in the region

$\{(s_1,s_2) \,|\mathrm{Re}(s_1) > 0, \mathrm{Re}(s_2) > 0,$

$|\, s_1| < \infty$, and $|\, s_2| < \infty \}$.

Definition 1.9 $D_a(s_1,s_2)$ is a narrow sense Hurwitz polynomial (NHP) if $\frac{1}{D_a(s_1,s_2)}$ does not possess any singularities in the region

$\{(s_1,s_2) \,|\mathrm{Re}(s_1) > 0, \mathrm{Re}(s_2) > 0,$

$|\, s_1| < \infty$, and $|\, s_2| < \infty \}$

$\cup \{(s_1,s_2) \,|\mathrm{Re}(s_1) = 0, \mathrm{Re}(s_2) > 0,$

$|\, s_1| \leq \infty$, and $|\, s_2| < \infty \}$

$\cup \{(s_1,s_2) \,|\mathrm{Re}(s_1) > 0, \mathrm{Re}(s_2) = 0,$

$|\, s_1| < \infty$, and $|s_2| \leq \infty \}$.

Definition 1.10 $D_a(s_1,s_2)$ is a strict Hurwitz polynomial (SHP) if $\frac{1}{D_a(s_1,s_2)}$ does not possess any singularities in the region

$\{(s_1,s_2) \,|\mathrm{Re}(s_1) \geq 0, \mathrm{Re}(s_2) \geq 0,$

$|s_1| < \infty$, and $|s_2| < \infty \}$.

Definition 1.11 $D_a(s_1,s_2)$ is a very strict Hurwitz polynomial (VSHP) if $\frac{1}{D_a(s_1,s_2)}$ does not possess any singularities in the region

$\{(s_1,s_2)\,|\text{Re}(s_1) \geq 0, \text{Re}(s_2) \geq 0,$

$|s_1| \leq \infty, \text{ and } |s_2| \leq \infty\}.$

From these definitions, one can see that a VSHP is required to be necessarily a SHP. From the 2-D digital filter design experience, to get a guaranteed stable digital filter from the well-known bilinear transformation, the 2-D analog transfer function is required to have a 2- variable VSHP as its denominator.

1.3 Design methods for 2-D Recursive Filters [20]

A recursive filter is one, which can be expressed in the form of a difference equation of the input and output samples with finite orders. Unlike in the non-recursive case, the design of 2-D recursive filter is more complex than the design of a 1-D one. The main reason is the stability consideration. Stability test is more complex as the order of the dimension increases.

To design a 2-D recursive digital filter expressed in the following equation

$$H(z_1, z_2) = \frac{\sum_{m=0}^{M_1}\sum_{n=0}^{M_2} B_{mn} z_1^m z_2^n}{\sum_{i-0}^{N_1}\sum_{j=0}^{N_2} A_{ij} z_1^i z_2^j} \tag{1.28}$$

where, $A_{00} = 1$, A_{ij} and B_{mn} are real coefficients.

The main work is now is to choose the coefficients of A_{ij} and B_{mn} to approximate the frequency response of the desire one, and the coefficients should make the realizable filter stable.

A popular method to design a 2-D recursive digital filter is to start from an analog prototype filer, getting the analog transfer function, and then applying the double bilinear transformations to the analog transfer function to design the digital filter. If we assign a VSHP as the denominator of the analog transfer function, we can always obtain a stable digital transfer function, if the well known bilinear transformation is used. However, when the double generalized bilinear transformations are applied to the analog transfer function with VSHP denominator, additional stability conditions need to be introduced to guarantee the stability of the resulting digital filter. It is obvious that, the analog transfer function with a VSHP denominator is always necessary in the situations to obtain stable digital filters by the well known bilinear transformation.

1.4 Methods of generation of VSHP [8]

From the various properties of VSHPs discussed so far, it is possible to generate such polynomials. Some of the methods used for this purpose are discussed below:

First Method :

In this method, the starting point is the generation of a SHP in 2- or n-variables first This is always possible, because the input impedance or admittance of a k-variable physically realizable network always represents an even or an odd part of a SHP in the corresponding number of variables. Some of the possibilities are briefly discussed below:

i) In [9], the starting network is an n-port gyrator terminated in n-variables reactances, each of degree unity. It has been shown that the determinant of the immitance matrix yields an even or an odd part of an n-variable Hurwitz polynomial. A SHP results by the addition of this determinant to its derivatives with respect to the n-variables. The resulting SHP in n-variables can be converted to a 2-variable VSHP. A large number of possibilities exist.

ii) In [10], it is shown that, instead of taking the derivatives of the determinant of the terminated n-port gyrator, one can make some of the n-variables positive real constants, VSHPs can be generated. The number of computations required can be considerably decreased.

iii) It is well known that a positive definite or positive semi definite matrix is physically realizable [11-14]. Consider

$$D_n = A\Psi A^t s_1 + B\Lambda B^t s_1 + R\Gamma R^t + G \tag{1.29}$$

where A, B and R are lower triangular matrices given by

$$A = \begin{bmatrix} a_{11} & 0 & 0 \dots & \dots & \dots & \dots & 0 \\ a_{12} & a_{22} & 0 \dots & \dots & \dots & \dots & 0 \\ a_{13} & a_{23} & a_{33} & \dots & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & .. \\ \dots & \dots & \dots & \dots & \dots & \dots & .. \\ \dots & \dots & \dots & \dots & \dots & \dots & .. \\ a_{1n} & a_{2n} & a_{3n} & \dots & \dots & \dots & a_{nn} \end{bmatrix} \tag{1.30}$$

$$B = \begin{bmatrix} b_{11} & 0 & 0 \dots & \dots & \dots & \dots & 0 \\ b_{12} & b_{22} & 0 \dots & \dots & \dots & \dots & 0 \\ b_{13} & b_{23} & b_{33} & \dots & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & .. \\ \dots & \dots & \dots & \dots & \dots & \dots & .. \\ \dots & \dots & \dots & \dots & \dots & \dots & .. \\ b_{1n} & b_{2n} & b_{3n} & \dots & \dots & \dots & b_{nn} \end{bmatrix} \tag{1.31}$$

$$R = \begin{bmatrix} r_{11} & 0 & 0\ldots & \ldots & \ldots & \ldots & 0 \\ r_{12} & r_{22} & 0\ldots & \ldots & \ldots & \ldots & 0 \\ r_{13} & r_{23} & r_{33} & \ldots & \ldots & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & .. \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & .. \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & .. \\ r_{1n} & r_{2n} & r_{3n} & \ldots & \ldots & \ldots & r_{nn} \end{bmatrix} \tag{1.32}$$

ψ, Λ and Γ are diagonal matrices given by

$$\Psi = \text{diag}\ [\psi_1, \psi_2, \ldots \quad \ldots \quad \ldots \psi_n] \tag{1.33}$$

$$\Lambda = \text{diag}\ [\lambda_1, \lambda_2, \ldots \quad \ldots \quad \ldots \lambda_n] \tag{1.34}$$

$$\Gamma = \text{diag}\ [\gamma_1, \gamma_2, \ldots \quad \ldots \quad \ldots \gamma_n] \tag{1.35}$$

And G is a skew-symmetric matrix given by

$$G = \begin{bmatrix} 0 & g_{12} & g_{13} & \ldots & \ldots & \ldots & g_{1n} \\ -g_{12} & 0 & g_{23}\ldots & \ldots & \ldots & \ldots & g_{2n} \\ -g_{13} & -g_{23} & 0 & \ldots & \ldots & \ldots & g_{3n} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & .. \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & .. \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & .. \\ -g_{1n} & -g_{2n} & -g_{3n} & \ldots & \ldots & \ldots & 0 \end{bmatrix} \tag{1.36}$$

The matrices A, B and R can be upper-triangular also. If all the ψ_i's, λ_i's and γ_i's are positive, A, B and R are positive-definite matrices, and they are physically realizable. Some of ψ_i's, λ_i's and γ_i's can be made equal to zero without affecting physical realizable, because in such cases A, B and R become positive semi-definite matrices respectively. If Γ is null matrix, determinant of D_n becomes a strictly even or strictly odd polynomial depending on whether n is even or odd. In such a case, by the use of derivatives, it has been shown that suitable VSHP can be obtained.

Second Method :

One of the simplest methods of generating a VSHP is to start from the VSHP

$$D_a(s_1, s_2) = a_{11}s_1s_2 + a_{10}s_1 + a_{01}s_2 + a_{00} \tag{1.37}$$

For the reactance function so obtained as

$$G_{a1}(s_1, s_2) = \frac{a_{11}s_1s_2 + a_{00}}{a_{10}s_1 + a_{01}s_2} \tag{1.38}$$

One can apply the transformation

$$s_1 \rightarrow \frac{b_{11}s_1s_2 + b_{00}}{b_{10}s_1 + b_{01}s_2} \tag{1.39}$$

where $b_{11} > 0$, $b_{10} > 0$, $b_{01} > 0$ and $b_{00} > 0$

which results in

$$G_{a2}(s_1, s_2) = \frac{P_{a2}(s_1, s_2)}{Q_{a2}(s_1, s_2)} \tag{1.40}$$

where

$$P_{a2}(s_1, s_2) = a_{11}b_{11}s_1s_2^2 + a_{00}b_{10}s_1 + (a_{11}b_{00} + a_{00}b_{01})s_2 \tag{1.41}$$

$$Q_{a2}(s_1, s_2) = a_{01}s_2^2 + (a_{10}b_{11} + a_{01}b_{10})s_1s_2 + a_{10}b_{00} \tag{1.42}$$

The polynomial $D_{a2}(s_1, s_2) = P_{a2}(s_1, s_2) + Q_{a2}(s_1, s_2)$ is a VSHP in which s_1 is of unity degree and s_2 is of second degree. When the transformations s_1 as in (1.39) and

$$s_2 \rightarrow \frac{c_{11}s_1s_2 + c_{00}}{c_{10}s_1 + c_{01}s_2} \tag{1.43}$$

where, $c_{11} > 0$, $c_{10} > 0$, $c_{01} > 0$ and $c_{00} > 0$

are applied simultaneously for $G_{a1}(s_1, s_2)$ in (1.38), the resulting VSHP contains s_1 and s_2 of second degree each. The resulting reactance function is given by

$$G_{a3}(s_1, s_2) = \frac{P_{a3}(s_1, s_2)}{Q_{a3}(s_1, s_2)} \tag{1.44}$$

where, $P_{a3}(s_1, s_2) = a_{11}b_{11}c_{11}s_1^2s_2^2 + a_{00}b_{00}c_{10}s_1^2 + (a_{00}b_{01}c_{01} + a_{00}b_{01}c_{01} + a_{11}b_{00}c_{11} + a_{11}b_{11}c_{00})s_1s_2 + a_{00}b_{01}c_{01}s_2^2 + a_{11}b_{00}c_{00}$ (1.45)

and $Q_{a3}(s_1, s_2) = (a_{10}b_{11}c_{10} + a_{01}b_{10}c_{11})s_1^2s_2 + (a_{10}b_{11}c_{01} + a_{01}b_{10}c_{11})s_1s_2^2 + (a_{10}b_{00}c_{10} + a_{01}b_{10}c_{00})s_1 + (a_{10}b_{00}c_{01} + a_{01}b_{01}c_{00})s_2$ (1.46)

If higher order VSHPs are desired, these transformations can be repeated.

Third Method :

In certain cases, product-separable denominators of the type $D_{a1}(s_1).D_{a2}(s_2)$ may be required. It is obvious that $D_{a1}(s_1)$ and $D_{a2}(s_2)$ shall be SHP in s_1 and s_2. Such denominator polynomials can be generated by

a) the substitution of $s_1 = s_2 = s$ in (1.29) or

b) either making $A_1 = 0$ or $B_1 = 0$ in (1.29).

Then the required polynomials are associated with $D_{a1}(s_1)$ and $D_{a2}(s_2)$.

Alternatively, one can generate Schur polynomials [27] directly in the discrete domains. In this method, it is shown that any discrete-domain polynomial can be decomposed as

$$D_d(z) = \sum_{i=1}^{q} d_i z^i = F_1(z) + F_2(z) \tag{1.47}$$

where, $F_1(z)$ is the mirror-image polynomial given by

$$\frac{1}{2}[D(z) + z^{q}D(z^{-1})] \tag{1.48}$$

and $F_2(z)$ is the mirror-image polynomial given by

$$\frac{1}{2}[D(z) - z^{q}D(z^{-1})] \tag{1.49}$$

It is further shown that D(z) will be Schur polynomial, if and only if the following conditions hold:

$$\text{i)} \left| \frac{d_0}{d_q} \right| < 1 \tag{1.50}$$

ii) When q is even:

$$F_1(z) = K_e \prod_{i=1}^{q/2} (z^2 - 2a_i z + 1) \tag{1.51}$$

and

$$F_2(z) = (z^2 - 1) \prod_{i=1}^{(q-2)/2} (z^2 - 2b_i z + 1) \tag{1.52}$$

with

$$1 > a_1 > b_1 > a_2 > b_2 > \ldots\ \ldots\ \ldots\ \ldots > b_{(q-2)/2} > a_{q/2} > -1 \tag{1.53}$$

iii) When q is odd

$$F_1(z) = K_o(z+1) \prod_{i=1}^{(q-1)/2} (z^2 - 2a_i z + 1) \tag{1.54}$$

and

$$F_2(z) = (z-1)\prod_{i=1}^{(q-2)/2}(z^2 - 2b_i z + 1) \tag{1.55}$$

with

$$1 > a_1 > b_1 > a_2 > b_2 > \dots\dots\dots\dots > a_{(q-1)/2} > b_{(q-1)/2} > -1 \tag{1.56}$$

Hence, $D_1(z_1)$ and $D_2(z_2)$ can be generated independently and can be used in the design of 2-D filters [15,16].

Now, depending on the filter desired, a suitable numerator is associated with the VSHP so generated. This gives the transfer function in the 2-D analog domain and the various coefficients have to be determined. By applying the bilinear transformations $s_i = (z_i - 1)/(z_i + 1)$, $i = 1, 2$, the transfer function in the discrete domain is obtained [17,18]. In the case when such polynomials are generated directly in the 2-D Z-polydomain, the bilinear transformations are not required and one can proceed with the designs directly.

1.5 Scope and Organization of the Thesis

The objective of the thesis is to get a new approach of the design of a 2-D recursive digital filter, which has variable magnitude characteristics in the frequency domain. To design a stable recursive digital filter, we start from a typical Bridged-T network. Researchers did not try before with Bridged-T network for the design of digital filter. To ensure stability we start with Very Strict Hurwitz Polynomial for designing doubly terminated Bridged-T network in the analog domain. To get the digital filter whose characteristics are changeable, one or more of the coefficients of the digital transfer function should be variable. The generalized double bilinear transformation is one of the processes that can introduce variable coefficient into the transfer function of the resulting digital filters.

In chapter 2, at first we introduce Bridged-T network circuit. Then we discuss thoroughly the procedure for testing whether a polynomial is VSHP or not. Then we test all the possibilities of the transfer function for VSHP. There are four impedance arms in the Bridged-T circuit. So there are many combinations for the four arms with inductances and capacitances as s_1 and s_2 variables. Here we arrange two tables for different combinations of impedances. In TABLE 2.1 we get nine cases which are VSHP and nine cases which are not VSHP. In TABLE 2.2 we get seven cases for the VSHP and eleven cases for not VSHP. In this chapter we give brief discussion about the testing method for VSHP and then compare with the fourth order Butterworth polynomial to the transfer function of the Bridged-T network, we find the impedance values of the transfer function those which are VSHP.

In chapter 3 we analyze and design the digital filter from VSHP transfer function. From the TABLE 2.1 and 2.2 we get total sixteen possibilities whose transfer functions contain VSHP. Then we compare them with the fourth order Butterworth polynomial to make the equations for finding impedance values. But unfortunately for the most of the cases there are not enough equations to solve the impedance values. Here we get total three cases from the TABLE 2.1 which we can find the impedance values. In this chapter at first we give the total design procedure of digital filter for the Case of 7 from TABLE 2.1. In this case we choose different values of source and load resistances (R_1,R_2) and get different values of impedances. Here we discuss the two different cases for the R_1 and R_2. At first we consider the $R_1 = 0.6$, $R_2 = 0.4$ and then $R_1 = 0$, $R_2 = 1$. To investigate the manner in which each coefficient of generalized bilinear transformation affects the magnitude response of the resulting 2-D digital filters, we change the value of the deserving coefficient or coefficients while fixing the other coefficients to the specified values. Then we discuss about the design procedure for the Case 13 from TABLE 2.1. From the Case 13, we get also different combinations of R_1 and R_2. We design filter for $R_1 = 0.6$, $R_2 = 0.4$ for comparing with other cases. And then we also design for the Case 17 from TABLE 2.1 with $R_1 = 0.6$, $R_2 = 0.4$.

In chapter 4 we compare the magnitude response curves of three different cases from TABLE 2.1 which we get from chapter 3. We compare the effect of each bilinear transformation coefficient on different impedance values of the transfer function of the Bridged-T network.

The conclusions and the directions for future work are given in chapter 5.

Chapter 2

Design procedure to make digital filter from Bridged-T Network

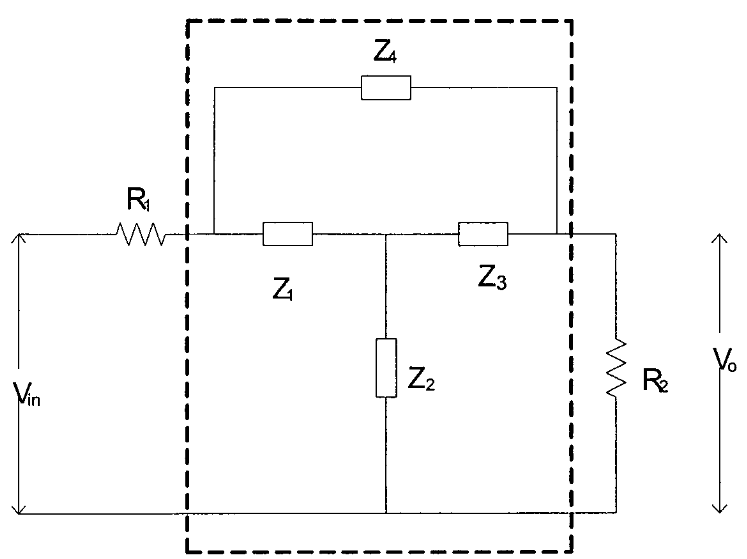


Figure 2.1 A typical Doubly Terminated Bridged-T Network

Networks having impedances (capacitances and inductances) will have a frequency and phase response, either or both of which may be advantageous to the engineer in circuit application for filter design. Doubly-terminated Bridged-T Network, as represented by figure 2.1 is advantageous in that they are the least sensitive to small variations in component values. In this circuit R_1 and R_2 are the source and load resistances respectively where the main bridge network is in the dotted portion.

We can create a 2-D analog filter system by setting the impedances values of Z_1, Z_2, Z_3, Z_4 as inductors and capacitors in the s_1 and s_2 variable.

After putting the values of Z_1, Z_2, Z_3, Z_4 we have to verify whether the transfer function is VSHP or not. The method of testing VSHP is described in this chapter. In our thesis we tried many possible ways by putting the different impedance values of Z_1, Z_2, Z_3, Z_4. Among them we got some transfer functions which are not VSHP and some are VSHP. Among the transfer functions which are VSHP, some are complicated to determine the value of impedance and resistance values of the transfer function R_1, R_2, L. C. For finding these values we have to compare our transfer function with fourth order Butterworth Polynomial. After finding the values of impedance variable we have to apply generalized bilinear transformation to get recursive digital filter.

With the help of node analysis we can find the equation of the transfer function of this circuit (Fig 2.1).

$$\frac{V_o}{V_{in}} = R_2T / [R_1(T+ Z_3Z_4) + R_2(T+Z_1Z_4) + R_1R_2(Z_1 + Z_3 + Z_4) + Z_4(T - Z_2Z_4)] \qquad (2.1)$$

where, $T = Z_2 Z_4 + Z_2 Z_3 + Z_1 Z_3 + Z_1Z_2$

So, applying the different values of Z_1, Z_2, Z_3 and Z_4 we test whether it is VSHP or not.

Now we have to explain how to determine whether a given polynomial is a VSHP or not.

2.1 Methods to determine whether a given polynomial is a VSHP or not [8]

In order to determine whether a given two variable polynomial $D_a(s_1,s_2)$ is a VSHP or not, we have to first determine whether it is SHP or not. For this purpose, the following procedure shall be adopted:

i) Determine that $D_a(s_1,1)$ is SHP in s_1.

ii) From the given polynomial $D_a(s_1,s_2)$, formulate

$$D_a(j\omega_1,j\omega_2) = [A_p(\omega_2)\omega_1^p + A_{p-1}(\omega_2)\omega_1^{p-1} + \ldots\ldots A_2(\omega_2)\omega_1^2 + A_1(\omega_2)\omega_1 + A_0(\omega_2)] + j[B_p(\omega_2)\omega_1^p + B_{p-1}(\omega_2)\omega_1^{p-1} + \ldots\ldots B_2(\omega_2)\omega_1^2 + B_1(\omega_2)\omega_1 + B_0(\omega_2)] \tag{2.2}$$

where, $A_i(\omega_2)$ and $B_i(\omega_2)$, $i = 0,1,2,\ldots..p$ are polynomials in ω_2.

iii) Now (2.2) shall be rearranged in the form of Inners as follows:

$$\left[\begin{array}{ccc:cccc:ccc}
B_p & B_{p-1} & B_{p-2} & --- & --- & --- & --- & 0 & 0 & 0 \\
0 & B_p & B_{p-1} & --- & --- & --- & --- & 0 & 0 & 0 \\
0 & 0 & B_p & --- & --- & --- & --- & 0 & 0 & 0 \\
\hdashline
0 & 0 & 0 & B_p & B_{p-1} & B_{p-2} & B_{p-3} & --- & --- & --- \\
0 & 0 & 0 & 0 & B_p & B_{p-1} & B_{p-2} & --- & --- & --- \\
0 & 0 & 0 & 0 & A_p & A_{p-1} & A_{p-2} & --- & --- & --- \\
0 & 0 & 0 & A_p & A_{p-1} & A_{p-2} & A_{p-3} & --- & --- & --- \\
\hdashline
0 & 0 & A_p & --- & --- & --- & --- & 0 & 0 & 0 \\
0 & A_p & A_{p-1} & --- & --- & --- & --- & 0 & 0 & 0 \\
A_p & A_{p-1} & A_{p-2} & --- & --- & --- & --- & 0 & 0 & 0
\end{array}\right]$$

iv) In order that $D_a(s_1,s_2)$ is a SHP, it is required that the inner determinants $\Delta_k > 0$, k = 1,2, ...p, for all ω_2.

v) If the certain conditions of $D_a(s_1,s_2)$ are satisfied then we can concluded that the given $D_a(s_1,s_2)$ is a VSHP. The conditions are :

$$D_a(s_1,\frac{1}{s_2}) \neq \frac{0}{0}, \text{ as } s_1 \rightarrow 0 \text{ and } s_2 \rightarrow 0$$

$$D_a(\frac{1}{s_1},s_2) \neq \frac{0}{0}, \text{ as } s_1 \rightarrow 0 \text{ and } s_2 \rightarrow 0$$

$$D_a(\frac{1}{s_1},\frac{1}{s_2}) \neq \frac{0}{0}, \text{ as } s_1 \rightarrow 0 \text{ and } s_2 \rightarrow 0$$

In this thesis we use the procedure (v) for testing the polynomial whether it is VSHP or not.

2.2 Testing Results of the Transfer Function of our circuit

In the Bridged-T network we have four impedances and two variables (s_1, s_2). So there are many ways to put the different values. In TABLE 2.1, for the cases of 1-16 we put s_1 variable in Z_1, Z_3 and s_2 variable in Z_2, Z_4. In case 17, it is similar to the case 7, just interchange the s variable in Z_4 arm to observe if there is significant change or not. In case 18, it is also similar to the case 8, just interchange the s_1, s_2 variables in Z_3, Z_4 arms of figure 2.1.

In TABLE 2.2, for the cases of 1-16 we keep the inductances and capacitances remain same position with their respective arms with respect to TABLE 2.1, just change the values of s_1 and s_2. In case 17, it is similar to the case 7, just interchange the s variable in Z_4 arm to observe if there is significant change or not. In case 18, it is also similar to the case 8, just interchange the s_1, s_2 variables in Z_3, Z_4 arms of figure 2.1.

TABLE 2.1

Case	Z_1	Z_2	Z_3	Z_4	VSHP or Not
1.	s_1L_1	s_2L_2	s_1L_3	s_2L_4	Not VSHP
2.	s_1L_1	s_2L_2	s_1L_3	$1/(s_2C_4)$	Not VSHP
3.	s_1L_1	s_2L_2	$1/(s_1C_3)$	s_2L_4	VSHP
4.	s_1L_1	s_2L_2	$1/(s_1C_3)$	$1/(s_2C_4)$	VSHP
5.	s_1L_1	$1/(s_2C_2)$	s_1L_3	s_2L_4	Not VSHP
6.	s_1L_1	$1/(s_2C_2)$	$1/(s_2C_3)$	$1/(s_1C_4)$	Not VSHP
7.	s_1L_1	$1/(s_2C_2)$	$1/(s_1C_3)$	s_2L_4	VSHP
8.	s_1L_1	$1/(s_2C_2)$	s_1L_3	$1/(s_2C_4)$	Not VSHP
9.	$1/(s_1C_1)$	s_2L_2	s_1L_3	s_2L_4	VSHP
10.	$1/(s_1C_1)$	s_2L_2	s_1L_3	$1/(s_2C_4)$	VSHP
11.	$1/(s_1C_1)$	s_2L_2	$1/(s_1C_3)$	s_2L_4	Not VSHP
12.	$1/(s_1C_1)$	s_2L_2	$1/(s_1C_3)$	$1/(s_2C_4)$	Not VSHP
13.	$1/(s_1C_1)$	$1/(s_2C_2)$	s_1L_3	s_2L_4	VSHP
14.	$1/(s_1C_1)$	$1/(s_2C_2)$	s_1L_3	$1/(s_2C_4)$	VSHP
15.	$1/(s_1C_1)$	$1/(s_2C_2)$	$1/(s_1C_3)$	s_2L_4	Not VSHP
16.	$1/(s_1C_1)$	$1/(s_2C_2)$	$1/(s_1C_3)$	$1/(s_2C_4)$	Not VSHP
17.	s_1L_1	$1/(s_2C_2)$	$1/(s_1C_3)$	s_1L_4	VSHP
18.	s_1L_1	$1/(s_2C_2)$	s_2L_3	$1/(s_1C_4)$	VSHP

TABLE 2.2

Case	Z_1	Z_2	Z_3	Z_4	VSHP or Not
1.	s_2L_1	s_2L_2	s_1L_3	s_1L_4	Not VSHP
2.	s_2L_1	s_2L_2	s_1L_3	$1/(s_1C_4)$	Not VSHP
3.	s_2L_1	s_2L_2	$1/(s_1C_3)$	s_1L_4	Not VSHP
4.	s_2L_1	s_2L_2	$1/(s_1C_3)$	$1/(s_1C_4)$	Not VSHP
5.	s_2L_1	$1/(s_2C_2)$	s_1L_3	s_1L_4	VSHP
6.	s_2L_1	$1/(s_2C_2)$	$1/(s_2C_3)$	$1/(s_1C_4)$	VSHP
7.	s_2L_1	$1/(s_2C_2)$	$1/(s_1C_3)$	s_1L_4	VSHP
8.	s_2L_1	$1/(s_2C_2)$	s_1L_3	$1/(s_1C_4)$	VSHP
9.	$1/(s_2C_1)$	s_2L_2	s_1L_3	s_1L_4	Not VSHP
10.	$1/(s_2C_1)$	s_2L_2	s_1L_3	$1/(s_1C_4)$	VSHP
11.	$1/(s_2C_1)$	s_2L_2	$1/(s_1C_3)$	s_1L_4	VSHP
12.	$1/(s_2C_1)$	s_2L_2	$1/(s_1C_3)$	$1/(s_1C_4)$	Not VSHP
13.	$1/(s_2C_1)$	$1/(s_2C_2)$	s_1L_3	s_1L_4	Not VSHP
14.	$1/(s_2C_1)$	$1/(s_2C_2)$	s_1L_3	$1/(s_1C_4)$	Not VSHP
15.	$1/(s_2C_1)$	$1/(s_2C_2)$	$1/(s_1C_3)$	s_1L_4	VSHP
16.	$1/(s_2C_1)$	$1/(s_2C_2)$	$1/(s_1C_3)$	$1/(s_1C_4)$	Not VSHP
17.	s_2L_1	$1/(s_2C_2)$	$1/(s_1C_3)$	s_2L_4	Not VSHP
18.	s_2L_1	$1/(s_2C_2)$	s_1L_3	$1/(s_2C_4)$	Not VSHP

The required details of these tables are given in section 2.4.

2.3 Fourth order Butterworth Low Pass Filter Network

Now we begin to design 2-D filter. For this we have to find the values of input, output resistances and capacitances/inductances of these four impedances. Here we consider a fourth order 1-D low pass Butterworth polynomial and compare to our transfer function equation to find the values of C, L and R.

We know, the transfer function of fourth order Butterworth low pass filter (analog) is

$$T_{lp}(s) = 1/(s^4 + 2.6131259s^3 + 3.4142136s^2 + 2.6131259s + 1) \quad (2.3)$$

From TABLE 2.1 we get nine values which transfer function is VSHP and nine values which are not.

Now we will discuss all the results of TABLE 2.1.

2.4 Transfer function of the Bridged-T network which is not VSHP (case T1-1)

Here in the Bridged-T network we put inductances in Z_1, Z_2, Z_3 and Z_4 in equation (2.1).

We get,

$$\frac{V_o(s_1, s_2)}{V_{in}(s_1, s_2)} = (L_2L_4R_2s_2^2 + L_1L_2R_2s_1s_2 + R_2L_1L_3s_1^2 + R_2L_2L_3s_1s_2) / (R_1L_2L_4s_2^2 + R_2L_2L_4s_2^2 + R_1L_1L_3s_1^2 + R_2L_1L_3s_1^2 + R_1L_2L_3s_1s_2 + R_1L_1L_2s_1s_2 + R_1L_3L_4s_1s_2 + R_2L_2L_3s_1s_2 + R_2L_1L_2s_1s_2 + R_2L_1L_4s_1s_2 + R_1R_2L_1s_1 + R_1R_2L_3s_1 + R_1R_2L_4s_2 + L_2L_3L_4s_1s_2^2 + L_1L_2L_4s_1s_2^2 + L_1L_3L_4s_1^2) \quad (2.4)$$

Now we apply the procedure (v) of article (2.1) then we see

$$\frac{V_o(s_1,\frac{1}{s_2})}{V_{in}(s_1,\frac{1}{s_2})} = \frac{0}{0}, \text{ as } s_1 \to 0 \text{ and } s_2 \to 0$$

So, $\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)}$ is not VSHP.

2.5 Transfer function of the Bridged-T network which is not VSHP (case T1-2)

Here in the Bridged-T network we put inductances in Z_1, Z_2, Z_3 and capacitance in Z_4 in equation (2.1).

We get,

$$\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)} = (R_2L_2s_2 + R_2L_2L_3C_4s_1^2s_2 + R_2L_1L_3C_4s_1^3 + R_2L_1L_2C_4s_1^2s_2) / (R_1L_2s_2 + R_2L_2s_2 + R_1L_2L_3C_4s_1^2s_2 + R_1L_1L_2C_4s_1^2s_2 + R_2L_2L_3C_4s_1^2s_2 + R_2L_1L_2C_4s_1^2s_2 + R_1L_1L_3C_4s_1^3 + R_2L_1L_3C_4s_1^3 + R_1R_2 + R_1R_2L_1C_4s_1^2 + R_1R_2L_3C_4s_1^2 + L_1L_3s_1^2 + L_2L_3s_1s_2 + L_1L_2s_1s_2 + R_1L_3s_1 + R_2L_1s_1) \qquad (2.5)$$

Now we apply the procedure (v) of article (2.1) then we see

$$\frac{V_o(s_1,\frac{1}{s_2})}{V_{in}(s_1,\frac{1}{s_2})} = \frac{0}{0}, \text{ as } s_1 \to 0 \text{ and } s_2 \to 0$$

So, $\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)}$ is not VSHP.

2.6 Transfer function of the Bridged-T network which is VSHP (case T1-3)

Here in the Bridged-T network we put inductances in Z_1, Z_2, Z_4 and capacitance in Z_3.

Putting the value of Z_1, Z_2, Z_3 and Z_4 in the equation (2.1) and make it 1-D, i.e., put $s_1 = s_2 = s$ and then we find

$$\frac{V_o(s)}{V_{in}(s)} = (s^3(R_2L_2L_4C_3 + R_2L_1L_2C_3) + s(R_2L_2 + R_2L_1)) \,/\, (s^4L_1L_2L_4C_3 + s^3(R_1L_2L_4C_3 + R_2L_2L_4C_3 + R_1L_1L_2C_3 + R_2L_1L_2C_3 + R_2L_1L_4C_3) + s^2(R_1R_2L_1C_3 + R_1R_2L_4C_3 + L_1L_4 + L_2L_4) + s(R_1L_2 + R_1L_4 + R_2L_2 + R_1L_1 + R_2L_1) + R_1R_2) \quad (2.6)$$

If we equate the coefficient of s in the denominator of equation (2.3) and (2.6) then we get five equations.

$L_1L_2L_4C_3 = 1$ --- --- --- (2.6.1)

$R_1L_2L_4C_3 + R_2L_2L_4C_3 + R_1L_1L_2C_3 +$

$R_2L_1L_2C_3 + R_2L_1L_4C_3 = 2.6131259$ --- --- --- (2.6.2)

$R_1R_2L_1C_3 + R_1R_2L_4C_3 + L_1L_4 + L_2L_4 = 3.4142136$ --- (2.6.3)

$R_1L_2 + R_1L_4 + R_2L_2 + R_1L_1 + R_2L_1 = 2.6131259$ --- --- (2.6.4)

$R_1R_2 = 1$ --- --- --- --- --- (2.6.5)

and if we equate the coefficient of s in the numerator of equation (2.3) and (2.6) then we get one equation.

$L_2L_4C_3 + L_1L_2C_3 = L_1 + L_2$ --- --- --- (2.6.6)

It is noted that for ease of the calculation here we put the value of angular frequency $\omega = 1$.

Now using these six equations we can't solve for the values of R_1, R_2, L_1, L_2, L_4, C_3 and hence is not considered.

2.7 Transfer function of the Bridged-T network which is VSHP (case T1-4)

Here in the Bridged-T network we put inductances in Z_1, Z_2 and capacitances in Z_3, Z_4.

Putting the value of Z_1, Z_2, Z_3 and Z_4 in the equation (2.1) and make it 1-D, i.e., put $s_1 = s_2 = s$ and then we find

$$\frac{V_o(s)}{V_{in}(s)} = (s^2(R_2L_2C_4 + C_3 R_2 L_2 + C_4R_2L_1) + s^4C_3C_4R_2L_1L_2)/ (s^4(C_3C_4R_1L_1L_2 + C_3C_4R_2L_1L_2) + s^3(C_3C_4R_1R_2L_1 + C_3L_1L_2) + s^2(C_3R_2L_1+ C_4R_1L_2+ C_4R_2L_2+ C_3R_1R_2+ C_4R_1L_1+ C_3R_2L_2 + C_4R_2L_1) + s(C_4R_1R_2 + L_2 + L_1 + R_1R_2C_3)+ R_1) \quad (2.7)$$

If we equate the coefficient of s in the denominator of equation (2.3) and (2.7) then we get five equations.

$C_3C_4R_1L_1L_2 + C_3C_4R_2L_1L_2 = 1$ --- --- --- (2.7.1)

$C_3C_4R_1R_2L_1 + C_3L_1L_2 = 2.6131259$ --- --- --- (2.7.2)

$C_3R_2L_1+ C_4R_1L_2+ C_4R_2L_2+ C_3R_1R_2+ C_4R_1L_1+ C_3R_2L_2 + C_4R_2L_1 = 3.4142136$ --- --- (2.7.3)

$C_4R_1R_2 + L_2 + L_1 + R_1R_2C_3 = 2.6131259$ --- --- --- (2.7.4)

$R_1 = 1$ --- --- --- --- --- (2.7.5)

and if we equate the coefficient of s in the numerator of equation (2.3) and (2.7) then we get one equation.

$C_3C_4L_1L_2 = L_2C_4 + C_3 L_2 + C_4L_1$ --- --- --- (2.7.6)

It is noted that for ease of the calculation here we put the value of angular frequency $\omega = 1$.

Now using these six equations we can't solve the value of R_2, L_1, L_2, C_3, C_4 and hence is not considered.

2.8 Transfer function of the Bridged-T network which is not VSHP (case T1-5)

Here in the Bridged-T network we put inductances in Z_1, Z_3, Z_4 and capacitance in Z_2 in equation (2.1).

We get,

$$\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)} = (R_2L_4s_2 + R_2L_1L_3C_2s_1^2s_2 + R_2L_3s_1 + R_2L_1s_1) \; / \; (L_1L_3L_4C_2s_1^2s_2^2 + R_2C_2L_1L_3s_1^2s_2 + R_1L_3s_1 + R_1L_1s_1 + R_2L_3s_1 + R_2L_1s_1 + C_2R_1L_1L_3s_1^2s_2 + C_2R_2L_1L_4s_1s_2^2 + C_2R_1L_3L_4s_1s_2^2 + C_2R_1R_2L_1s_1s_2 + C_2R_1R_2L_3s_1s_2 + L_3L_4s_1s_2 + L_1L_4s_1s_2 + C_2R_1R_2L_4s_2^2 + R_1L_4s_2 + R_2L_4s_2) \qquad (2.8)$$

Now we apply the procedure (v) of article (2.1) then we see

$$\frac{V_o(s_1,\frac{1}{s_2})}{V_{in}(s_1,\frac{1}{s_2})} = \frac{0}{0}, \text{ as } s_1 \to 0 \text{ and } s_2 \to 0$$

So, $\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)}$ is not VSHP.

2.9 Transfer function of the Bridged-T network which is not VSHP (case T1-6)

Here in the Bridged-T network we put inductance in Z_1 and capacitances in Z_2, Z_3 and Z_4 in equation (2.1).

We get,

$$\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)} = (C_3R_2s_2 + C_4R_2s_1 + R_2L_1C_2C_4s_1^2s_2 + R_2L_1C_3C_4s_1^2s_2) / (C_2C_3C_4R_1R_2L_1s_1^2s_2^2$$

$$+ C_2C_4R_1L_1s_1^2s_2 + C_3C_4R_1L_1s_1^2s_2 + C_2C_4R_2L_1s_1^2s_2 + C_3C_4R_2L_1s_1^2s_2 + R_1C_4s_1 + R_2C_4s_1 +$$

$$C_2C_3R_2L_1s_1s_2^2 + C_2C_4R_1R_2s_1s_2 + C_2L_1s_1s_2 + C_3L_1s_1s_2 + C_2C_3R_1R_2s_2^2 + 1 + C_3R_1s_2 +$$

$$C_3R_2s_2 + C_2R_1s_2) \qquad (2.9)$$

Now we apply the procedure (v) of article (2.1) then we see

$$\frac{V_o(s_1,\frac{1}{s_2})}{V_{in}(s_1,\frac{1}{s_2})} = \frac{0}{0}, \text{ as } s_1 \rightarrow 0 \text{ and } s_2 \rightarrow 0$$

So, $\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)}$ is not VSHP.

2.10 Transfer function of the Bridged-T network which is VSHP (case T1-7)

Here in the Bridged-T network we put inductances in Z_1, Z_4 and capacitances in Z_2, Z_3.

Putting the value of Z_1, Z_2, Z_3 and Z_4 in the equation (2.1) and make it 1-D, i.e., put $s_1 = s_2 = s$ and then we find

$$\frac{V_o(s)}{V_{in}(s)} = (R_2 + s^2(C_3R_2L_1 + C_3R_2L_4 + C_2R_2L_1)) / (s^4C_2C_3R_2L_1L_4 + s^3(C_2C_3R_1R_2L_1 + C_3L_1L_4 + C_2C_3R_1R_2L_4 + C_2L_1L_4) + s^2(C_3R_1L_1 + C_3R_2L_1 + C_2R_1L_4 + C_3R_1L_4 + C_2R_1L_1 + C_3R_2L_4 + C_2R_2L_1) + s(C_2R_1R_2 + L_4) + R_1 + R_2) \quad (2.10)$$

If we equate the coefficient of s in the denominator of equation (2.3) and (2.10) then we get five equations.

$C_2C_3R_2L_1L_4 = 1$ --- --- --- --- (2.10.1)

$C_2C_3R_1R_2L_1 + C_3L_1L_4 + C_2C_3R_1R_2L_4 + C_2L_1L_4 = 2.6131259$ (2.10.2)

$C_3R_1L_1 + C_3R_2L_1 + C_2R_1L_4 + C_3R_1L_4 +$

$C_2R_1L_1 + C_3R_2L_4 + C_2R_2L_1 = 3.4142136$ -- --- (2.10.3)

$C_2R_1R_2 + L_4 = 2.6131259$ --- --- --- --- (2.10.4)

$R_1 + R_2 = 1$ --- --- --- --- (2.10.5)

and if we equate the coefficient of s in the numerator of equation (2.3) and (2.10) then we get one equation.

$C_3 L_1 + C_3 L_4 + C_2 L_1 = 1$ --- --- --- --- (2.10.6)

It is noted that for ease of the calculation here we put the value of angular frequency ω = 1.

From these six equation for different values of R_1, R_2 we get the values of inductances and capacitances.

Thus we make a TABLE 2.3 for finding the different impedance variables (For the case 7).

TABLE 2.3

Case	R_1	R_2	L_1	L_4	C_2	C_3
1)	0	1	0.6340	2.6131	0.6532	0.9238
2)	0.3	0.7	0.0462	1.1756	6.8454	3.8389
3)	0.4	0.6	0.1708	0.7981	7.5628	1.6170
4)	0.5	0.5	0.2977	0.5995	8.0546	1.3914
5)	0.6	0.4	0.5807	0.4455	9.0316	1.0699
6)	0.7	0.3	7.9596	0.3152	10.9425	0.1231

After finding the impedance value then we can design the digital filter. The detailed design procedures for digital filter are described in the next chapter.

2.11 Transfer function of the Bridged-T network which is not VSHP (case T1-8)

Here in the Bridged-T network we put inductances in Z_1, Z_3 and capacitances in Z_2, Z_4 in equation (2.1).

We get,

$$\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)} = (R_2 + C_2C_4R_2L_1L_3s_1^2s_2^2 + C_4R_2L_1s_1s_2^2 + C_4R_2L_3s_1s_2) \,/\, (C_2C_4R_1L_1L_3s_1^2s_2^2$$

$$+C_2L_1L_3s_1^2s_2 + C_2R_1R_2s_2 + L_1s_1 + L_3s_1 + R_1 + R_2 + C_2C_4R_2L_1L_3s_1^2s_2^2 + C_2C_4R_1R_2L_1s_1s_2^2$$

$$+ C_2C_4R_1R_2L_3s_1s_2^2 + C_4R_1L_3s_1s_2 + C_4R_1L_1s_1s_2 + C_2R_1L_3s_1s_2 + C_4R_2L_3s_1s_2 + C_4R_2L_1s_1s_2$$

$$+ C_2R_2L_1s_1s_2) \qquad (2.11)$$

Now we apply the procedure (v) of article (2.1) then we see

$$\frac{V_o(s_1, \frac{1}{s_2})}{V_{in}(s_1, \frac{1}{s_2})} = \frac{0}{0}, \text{ as } s_1 \to 0 \text{ and } s_2 \to 0$$

So, $\frac{V_o(s_1, s_2)}{V_{in}(s_1, s_2)}$ is not VSHP.

2.12 Transfer function of the Bridged-T network which is VSHP (case T1-9)

Here in the Bridged-T network we put inductances in Z_2, Z_3, Z_4 and capacitance in Z_1.

Putting the value of Z_1, Z_2, Z_3 and Z_4 in the equation (2.1) and make it 1-D, i.e., put $s_1 = s_2 = s$ and then we find

$\frac{V_o(s)}{V_{in}(s)} = (s^3(R_2L_2L_4C_1 + R_2L_2L_3C_1) + s(R_2L_3 + R_2L_2)) / (s^4L_2L_3L_4C_1 + s^3(R_1L_2L_4C_1 + R_2L_2L_4C_1 + R_1L_2L_3C_1 + R_1L_3L_4C_1 + R_2L_2L_3C_1) + s^2(R_1R_2L_3C_1 + R_1R_2L_4C_1 + L_3L_4 + L_2L_4) + s(R_1L_2 + R_2L_2 + R_2L_4 + R_1L_3 + R_2L_3) + R_1R_2)$ (2.12)

If we equate the coefficient of s in the denominator of equation (2.3) and (2.12) then we get five equations.

$L_2L_3L_4C_1 = 1$ --- --- --- (2.12.1)

$R_1L_2L_4C_1 + R_2L_2L_4C_1 + R_1L_2L_3C_1 +$

$R_1L_3L_4C_1 + R_2L_2L_3C_1 = 2.6131259$ --- --- --- (2.12.2)

$R_1R_2L_3C_1 + R_1R_2L_4C_1 + L_3L_4 + L_2L_4 = 3.4142136$ --- (2.12.3)

$R_1L_2 + R_2L_2 + R_2L_4 + R_1L_3 + R_2L_3 = 2.6131259$ --- --- (2.12.4)

$R_1R_2 = 1$ --- --- --- --- --- (2.12.5) and if we equate the coefficient of s in the numerator of equation (2.3) and (2.12) then we get one equation.

$L_2L_4C_1 + L_2L_3C_1 = L_2 + L_3$ --- --- --- (2.12.6)

It is noted that for ease of the calculation here we put the value of angular frequency ω = 1.

Now using these six equations we can't solve for the values of R_1, R_2, L_2, L_3, L_4, C_1 and hence is not considered further.

2.13 Transfer function of the Bridged-T network which is VSHP (case T1-10)

Here in the bridge network we put inductances in Z_2, Z_3 and capacitances in Z_1, Z_4.

Putting the value of Z_1, Z_2, Z_3 and Z_4 in the equation (2.1) and make it 1-D, i.e., put $s_1 = s_2 = s$ and then we find

$$\frac{V_o(s)}{V_{in}(s)} = (s^2(R_2L_2C_4 + C_1R_2L_2 + C_4R_2L_3) + s^4\, C_1C_4R_2L_2L_3) / (s^4(C_1C_4R_1L_2L_3 + C_1C_4R_2L_2L_3) + s^3(C_1C_4R_1R_2L_3 + C_1L_2L_3) + s^2(C_1R_1L_3 + C_4R_1L_2 + C_4R_2L_2 + C_1R_1L_2 + C_4R_1L_3 + C_1R_2L_2 + C_4R_2L_3) + s(C_4R_1R_2 + L_2 + L_3 + R_1R_2C_1) + R_2) \qquad (2.13)$$

If we equate the coefficient of s in the denominator of equation (2.3) and (2.13) then we get five equations.

$C_1C_4R_1L_2L_3 + C_1C_4R_2L_2L_3 = 1$ --- --- --- (2.13.1)

$C_1C_4R_1R_2L_3 + C_1L_2L_3 = 2.6131259$ --- --- --- (2.13.2)

$C_1R_1L_3 + C_4R_1L_2 + C_4R_2L_2 + C_1R_1L_2 + C_4R_1L_3 + C_1R_2L_2 +$

$C_4R_2L_3 = 3.4142136$ --- --- (2.13.3)

$C_4R_1R_2 + L_2 + L_3 + R_1R_2C_1 = 2.6131259$ --- --- --- (2.13.4)

$R_2 = 1$ --- --- --- --- --- (2.13.5)

and if we equate the coefficient of s in the numerator of equation (2.3) and (2.13) then we get one equation.

$C_1C_4L_2L_3 = L_2C_4 + C_1L_2 + C_4L_3$ --- --- --- (2.13.6)

It is noted that for ease of the calculation here we put the value of angular frequency ω = 1.

Now using these six equations we can't solve the value of R_1, L_1, L_2, C_1, C_4 and hence is not considered.

2.14 Transfer function of the Bridged-T network which is not VSHP (case T1-11)

Here in the Bridged-T network we put inductances in Z_2, Z_4 and capacitances in Z_1, Z_3 in equation (2.1).

We get,

$$\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)} = (R_2 + L_2C_3R_2s_1s_2 + R_2L_2L_4C_1C_3s_1^2s_2^2 + R_2L_2C_1s_1s_2) / (C_1C_3R_1L_2L_4s_1^2s_2^2 + C_1C_3R_2L_2L_4s_1^2s_2^2 + C_1C_3R_1R_2L_4s_1^2s_2 + C_3L_2L_4s_1s_2^2 + C_1R_1L_2s_1s_2 + R_1R_2C_3s_1 + R_1R_2C_1s_1 + C_1L_2L_4s_1s_2^2 + C_3R_1L_2s_1s_2 + C_1L_4R_1s_1s_2 + C_1R_2L_2s_1s_2 + C_3R_2L_2s_1s_2 + R_2L_4C_3s_1s_2 + R_1 + R_2 + L_4s_2) \quad (2.14)$$

Now we apply the procedure (v) of article (2.1) then we see

$$\frac{V_o(s_1,\frac{1}{s_2})}{V_{in}(s_1,\frac{1}{s_2})} = \frac{0}{0}, \text{ as } s_1 \rightarrow 0 \text{ and } s_2 \rightarrow 0$$

So, $\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)}$ is not VSHP.

2.15 Transfer function of the Bridged-T network which is not VSHP (case T1-12)

Here in the Bridged-T network we put inductance in Z_2 and capacitances in Z_1, Z_3, Z_4 in equation (2.1).

We get,

$\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)}$ = $(R_2L_2C_1C_3s_1^2s_2 + R_2L_2C_1C_4s_1s_2^2 + R_2C_4s_2 + R_2L_2C_3C_4s_1s_2^2)$ / $(R_1C_4s_2 + R_1L_2C_1C_4s_1s_2^2 + R_1L_2C_3C_4s_1s_2^2 + R_2L_2C_1C_4s_1s_2^2 + R_2L_2C_3C_4s_1s_2^2 + R_2C_4s_2 + R_1C_1s_1 + R_2C_3s_1 + R_1R_2C_3C_4s_1s_2 + R_2C_3s_1 + R_1R_2C_1C_4s_1s_2 + L_2C_1s_1s_2 + L_2C_3s_1s_2 + R_1R_2C_1C_3s_1^2 + 1 + R_1L_2C_1C_3s_1^2s_2)$ (2.15)

Now we apply the procedure (v) of article (2.1) then we see

$$\frac{V_o(s_1,\frac{1}{s_2})}{V_{in}(s_1,\frac{1}{s_2})} = \frac{0}{0}, \text{ as } s_1 \rightarrow 0 \text{ and } s_2 \rightarrow 0$$

So, $\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)}$ is not VSHP.

2.16 Transfer function of the Bridged-T network which is VSHP (case T1-13)

Here in the Bridged-T network we put inductances in Z_3, Z_4 and capacitances in Z_1, Z_2.

Putting the value of Z_1, Z_2, Z_3 and Z_4 in the equation (2.1) and make it 1-D, i.e., put $s_1 = s_2 = s$ and then we find

We have,

$$\frac{V_o(s)}{V_{in}(s)} = (R_2 + s^2(C_1R_2L_3 + C_1R_2L_4 + C_2R_2L_3)) / (s^4C_1C_2R_1L_3L_4 + s^3(C_1C_2R_1R_2L_3 + C_1L_3L_4 + C_1C_2R_1R_2L_4 + C_2L_3L_4) + s^2(C_1R_1L_3 + C_1R_2L_3 + C_2R_2L_4 + C_1R_1L_4 + C_2R_1L_3 + C_1R_2L_4 + C_2R_2L_3) + s(C_2R_1R_2 + L_4) + R_1 + R_2) \quad (2.16)$$

If we equate the coefficient of s in the denominator of equation (2.3) and (2.16) then we get five equations.

$C_1C_2R_1L_3L_4 = 1$ --- --- --- --- (2.16.1)

$C_1C_2R_1R_2L_3 + C_1L_3L_4 + C_1C_2R_1R_2L_4 + C_2L_3L_4 = 2.6131259$ (2.16.2)

$C_1R_1L_3 + C_1R_2L_3 + C_2R_2L_4 + C_1R_1L_4 +$

$C_2R_1L_3 + C_1R_2L_4 + C_2R_2L_3 = 3.4142136$ --- --- (2.16.3)

$C_2R_1R_2 + L_4 = 2.6131259$ --- --- --- --- (2.16.4)

$R_1 + R_2 = 1$ --- --- --- --- --- (2.16.5)

and if we equate the coefficient of s in the numerator of equation (2.3) and (2.16) then we get one equation.

$C_1L_3 + C_1L_4 + C_2L_3 = 1$ --- --- --- (2.16.6)

It is noted that for ease of the calculation here we put the value of angular frequency ω = 1.

From these six equation for different values of R_1, R_2 we get the values of inductances and capacitances.

Thus we make a TABLE 2.4 for finding the different impedance variables (For the case 13).

TABLE 2.4

Case	R_1	R_2	L_3	L_4	C_1	C_2
1)	0.3	0.7	7.8305	0.3152	0.1233	10.9425
2)	0.4	0.6	0.3496	0.7225	1.2566	7.8762
3)	0.5	0.5	0.2977	0.5995	8.0546	1.3914
4)	0.6	0.4	0.1705	0.7981	1.6190	7.5626
5)	0.7	0.3	0.0462	1.1756	3.8406	6.8451

After finding the impedance value then we can design the digital filter. The detailed design procedures for digital filter are described in the next chapter.

2.17 Transfer function of the Bridged-T network which is VSHP (case T1-14)

Here in the Bridged-T network we put inductance in Z_3 and capacitances in Z_1, Z_2, Z_4.

Putting the value of Z_1, Z_2, Z_3 and Z_4 in the equation (2.1) and make it 1-D, i.e., put $s_1 = s_2 = s$ and then we find

$$\frac{V_o(s)}{V_{in}(s)} = (s^3(R_2L_3C_1C_4 + R_2L_3C_2C_4) + s(R_2C_1 + R_2C_4)) / (s^4R_1R_2L_3C_1C_2C_4 + s^3(R_1L_3C_1C_4 + R_1L_3C_1C_2 + R_2L_3C_1C_4 + R_1L_3C_2C_4 + R_2L_3C_2C_4) + s^2(R_1R_2C_2C_4 + R_1R_2C_1C_2 + L_3C_2 + L_3C_1) + s(R_1C_1 + R_2C_1 + R_1C_4 + R_2C_4 + R_2C_2) + 1) \quad (2.17)$$

If we equate the coefficient of s in the denominator of equation (2.3) and (2.17) then we get four equations.

$R_1R_2L_3C_1C_2C_4 = 1$ --- --- --- (2.17.1)

$R_1L_3C_1C_4 + R_1L_3C_1C_2 + R_2L_3C_1C_4 +$

$R_1L_3C_2C_4 + R_2L_3C_2C_4 = 2.6131259$ --- --- --- (2.17.2)

$R_1R_2C_2C_4 + R_1R_2C_1C_2 + L_3C_2 + L_3C_1 = 3.4142136$ --- (2.17.3)

$R_1C_1 + R_2C_1 + R_1C_4 + R_2C_4 + R_2C_2 = 2.6131259$ --- --- (2.17.4)

and if we equate the coefficient of s in the numerator of equation (2.3) and (2.17) then we get one equation.

$L_3C_1C_4 + L_3C_2C_4 = C_1 + C_4$ --- --- --- (2.17.5)

It is noted that for ease of the calculation here we put the value of angular frequency ω = 1.

Now using these five equations we can't solve for the values of R_1, R_2, L_3, C_1, C_2, C_4 and hence is not considered.

2.18 Transfer function of the Bridged-T network which is not VSHP (case T1-15)

Here in the Bridged-T network we put inductance in Z_4 and capacitances in Z_1, Z_2, Z_3 in equation (2.1).

We get,

$$\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)} = (R_2L_4C_1C_3s_1^2s_2 + R_2C_1s_1 + R_2C_2s_2 + R_2C_3s_1) / (R_2C_1s_1 + R_1C_3s_1 + R_2C_1s_1 + R_2C_3s_1 + R_1C_2s_2 + R_2C_2s_2 + R_1L_4C_1C_2s_1s_2^2 + R_2L_4C_2C_3s_1s_2^2 + R_1R_2C_2C_3s_1s_2 + R_1R_2C_1C_2s_1s_2 + L_4C_1s_1s_2 + L_4C_3s_1s_2 + R_1R_2L_4C_1C_2C_3s_1^2s_2^2 + R_2L_4C_1C_3s_1^2s_2 + L_4C_2s_2^2 + R_1L_4C_1C_3s_1^2s_2) \quad (2.18)$$

Now we apply the procedure (v) of article (2.1) then we see

$$\frac{V_o(s_1,\frac{1}{s_2})}{V_{in}(s_1,\frac{1}{s_2})} = \frac{0}{0}, \text{ as } s_1 \rightarrow 0 \text{ and } s_2 \rightarrow 0$$

So, $\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)}$ is not VSHP.

2.19 Transfer function of the Bridged-T network which is not VSHP (case T1-16)

Here in the Bridged-T network we put capacitances in Z_1, Z_2, Z_3, Z_4 in equation (2.1).

We get,

$\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)}$ = $(R_2C_1C_3s_1^2 + R_2C_1C_4s_1s_2 + R_2C_2C_4s_2^2 + R_2C_3C_4s_1s_2\ )\ /\ (R_1C_1C_3s_1^2 +$ $R_2C_1C_3s_1^2 + R_1C_1C_4s_1s_2 + R_2C_1C_2s_1s_2 + R_2C_1C_4s_1s_2 + R_2C_3C_4s_1s_2 + R_2C_2C_3s_1s_2 +$ $R_1C_3C_4s_1s_2 + R_1C_2C_4s_2^2 + R_2C_2C_4s_2^2 + R_1R_2C_1C_2C_3s_1^2s_2 + R_1R_2C_2C_3C_4s_1s_2^2 +$ $R_1R_2C_1C_2C_4s_1s_2^2 + C_1s_1 + C_3s_1 + C_2s_2)$ (2.19)

Now we apply the procedure (v) of article (2.1) then we see

$$\frac{V_o(s_1,\frac{1}{s_2})}{V_{in}(s_1,\frac{1}{s_2})} = \frac{0}{0}, \text{ as } s_1 \rightarrow 0 \text{ and } s_2 \rightarrow 0$$

So, $\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)}$ is not VSHP.

2.20 Transfer function of the Bridged-T network which is VSHP (case T1-17)

Here in the Bridged-T network we put inductances in Z_1, Z_4 and capacitances in Z_2, Z_3.

Putting the value of Z_1, Z_2, Z_3 and Z_4 in the equation (2.1) and make it 1-D, i.e., put $s_1 = s_2 = s$.

We have,

$$\frac{V_o(s)}{V_{in}(s)} = (R_2 + s^2(C_3R_2L_4 + C_2R_2L_1 + C_3R_2L_1)) \,/\, (s^4C_2C_3R_2L_1L_4 + s^3(C_2C_3R_1R_2L_1 + C_3L_1L_4 + C_2C_3R_1R_2L_4 + C_2L_1L_4) + s^2(C_3R_1L_1 + C_3R_2L_1 + C_2R_1L_4 + C_3R_1L_4 + C_2R_1L_1 + C_3R_2L_4 + C_2R_2L_1) + s(C_2R_1R_2 + L_4) + R_1 + R_2) \qquad (2.20)$$

If we equate the coefficient of s in the denominator of equation (2.3) and (2.20) then we get five equations.

$C_2C_3R_2L_1L_4 = 1$ --- --- --- --- (2.20.1)

$C_2C_3R_1R_2L_1 + C_3L_1L_4 + C_2C_3R_1R_2L_4 + C_2L_1L_4 = 2.6131259$ (2.20.2)

$C_3R_1L_1 + C_3R_2L_1 + C_2R_1L_4 + C_3R_1L_4 +$

$C_2R_1L_1 + C_3R_2L_4 + C_2R_2L_1 = 3.4142136$ --- --- (2.20.3)

$C_2R_1R_2 + L_4 = 2.6131259$ --- --- --- --- (2.20.4)

$R_1 + R_2 = 1$ --- --- --- --- --- (2.20.5)

and if we equate the coefficient of s in the numerator of equation (2.3) and (2.20) then we get one equation.

$C_3 L_1 + C_3 L_4 + C_2 L_1 = 1$ --- --- --- (2.20.6)

It is noted that for ease of the calculation here we put the value of angular frequency $\omega = 1$.

From these six equations for different values of R_1, R_2 we get the values of inductances and capacitances.

Thus we make a TABLE 2.5 for finding the different impedance variables (For the case 17).

TABLE 2.5

Case	R_1	R_2	L_1	L_4	C_2	C_3
1)	0.3	0.7	0.8064	1.4375	5.5982	0.2201
2)	0.4	0.6	1.6139	1.8150	3.3253	0.1711
3)	0.5	0.5	0.3478	2.0137	2.3978	1.1908
4)	0.6	0.4	0.2568	2.1676	1.8563	2.4195
5)	0.7	0.3	0.0259	2.2979	1.5009	37.33

After finding the impedance value then we can design the digital filter. The detailed design procedures for digital filter are described in the next chapter.

2.21 Transfer function of the Bridged-T network which is VSHP (case T1-18)

Here in the Bridged-T network we put inductances in Z_1, Z_3 and capacitances in Z_2, Z_4.

Putting the value of Z_1, Z_2, Z_3 and Z_4 in the equation (2.1) and make it 1-D, i.e., put $s_1 = s_2 = s$ and then we find

$$\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)} = (R_2 + s^2(C_4R_2L_1 + C_4R_2L_3) + s^4C_2C_4R_2L_1L_3)/\ (s^4(C_2C_4R_1L_1L_3 + C_2C_4R_2L_1L_3) + s^3(C_2C_4R_1R_2L_1 + C_2L_1L_3 + C_2C_4R_1R_2L_3) + s^2(C_4R_1L_1 + C_4R_2L_1 + C_2R_1L_3 + C_4R_1L_3 + C_4R_2L_3 + C_2R_2L_1) + s(C_2R_1R_2 + L_3 + L_1) + R_1 + R_2) \qquad (2.21)$$

If we equate the coefficient of s in the denominator of equation (2.3) and (2.21) then we get five equations.

$C_2C_4R_1L_1L_3 + C_2C_4R_2L_1L_3 = 1$ --- --- --- (2.21.1)

$C_2C_4R_1R_2L_1 + C_2L_1L_3 + C_2C_4R_1R_2L_3 = 2.6131259$ --- (2.21.2)

$C_4R_1L_1 + C_4R_2L_1 + C_2R_1L_3 + C_4R_1L_3$

$+ C_4R_2L_3 + C_2R_2L_1 = 3.4142136$ --- --- (2.21.3)

$C_2R_1R_2 + L_3 + L_1 = 2.6131259$ --- --- --- (2.21.4)

$R_1 + R_2 = 1$ --- --- --- --- --- (2.21.5)

and if we equate the coefficient of s in the numerator of equation (2.3) and (2.21) then we get one equation.

$1 + C_2C_4L_1L_3 = C_4L_1 + C_4L_3$ --- --- --- --- (2.21.6)

It is noted that for ease of the calculation here we put the value of angular frequency $\omega = 1$.

Now using these six equations we can't solve for the values of R_1, R_2, L_1, L_3, C_2, C_4 and hence is not considered.

From TABLE 2.2 we get seven values which transfer function are VSHP and eleven values which are not.

Now we will discuss all the results of TABLE 2.2.

2.22 Transfer function of the Bridged-T network which is not VSHP (case T2-1)

Here in the Bridged-T network we put inductances in Z_1, Z_2, Z_3 and Z_4 in equation (2.1).

We get,

$$\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)} = (L_1L_2R_2s_2^2 + L_2L_4R_2s_1s_2 + R_2L_1L_3s_1s_2 + R_2L_2L_3s_1s_2) /(R_1L_2L_4s_1s_2 + R_1L_2L_3s_1s_2 + R_1L_1L_3s_1s_2 + R_2L_2L_4s_1s_2 + R_2L_2L_3s_1s_2 + R_2L_1L_3s_1s_2 + R_2L_1L_4s_1s_2 + R_1L_1L_2s_2^2 + R_2L_1L_2s_2^2 + R_1L_3L_4s_1^2 + R_1R_2L_1s_2 + R_1R_2L_3s_1 + R_1R_2L_4s_1 + L_1L_2L_4s_1s_2^2 + L_2L_{34}s_1^2s_2 + L_1L_3L_4s_1^2s_2) \qquad (2.22)$$

Now we apply the procedure (v) of article (2.1) then we see

$$\frac{V_o(s_1,\frac{1}{s_2})}{V_{in}(s_1,\frac{1}{s_2})} = \frac{0}{0}, \text{ as } s_1 \rightarrow 0 \text{ and } s_2 \rightarrow 0$$

So, $\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)}$ is not VSHP.

2.23 Transfer function of the Bridged-T network which is not VSHP (case T2-2)

Here in the Bridged-T network we put inductances in Z_1, Z_2, Z_3 and capacitance in Z_4 in equation (2.1).

We get,

$$\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)} = (R_2L_2s_2 + R_2L_2L_3C_4s_1^2s_2 + R_2L_1L_3C_4s_1^2s_2 + R_2L_1L_2C_4s_1s_2^2) / (R_1L_2s_2 + R_2L_2s_2 + R_1L_2L_3C_4s_1^2s_2 + R_1L_1L_3C_4s_1^2s_2 + R_2L_2L_3C_4s_1^2s_2 + R_2L_1L_3C_4s_1^2s_2 +$$

$R_1L_1L_2C_4s_1s_2^2 + R_2L_1L_2C_4s_1s_2^2 + R_1R_2 + R_1R_2L_1C_4s_1s_2 + R_1R_2L_3C_4s_1^2 + L_2L_3s_1s_2 + L_1L_3s_1s_2 + L_1L_2s_2^2 + R_1L_3s_1 + R_2L_1s_2)$ (2.23)

Now we apply the procedure (v) of article (2.1) then we see

$$\frac{V_o(s_1,\frac{1}{s_2})}{V_{in}(s_1,\frac{1}{s_2})} = \frac{0}{0}, \text{ as } s_1 \rightarrow 0 \text{ and } s_2 \rightarrow 0$$

So, $\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)}$ is not VSHP.

2.24 Transfer function of the Bridged-T network which is not VSHP (case T2-3)

Here in the Bridged-T network we put inductances in Z_1, Z_2, Z_4 and capacitance in Z_3 in equation (2.1).

We get,

$\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)} = (R_2L_2s_2 + R_2L_2L_3C_3s_1^2s_2 + R_2L_1s_2 + R_2L_1L_2C_3s_1s_2^2) / (R_1L_2s_2 + R_1L_1s_2 + R_1L_2L_4C_3s_1^2s_2 + R_2L_2L_4C_3s_1^2s_2 + R_2L_1L_4C_3s_1^2s_2 + L_1L_2L_4C_3s_1^2s_2^2 + R_1L_1L_2C_3s_1s_2^2 + R_2L_1L_2C_3s_1s_2^2 + R_1R_2 + R_1R_2L_1C_3s_1s_2 + R_1R_2L_4C_3s_1^2 + L_2L_4s_1s_2 + L_1L_4s_1s_2 + R_1L_2s_2 + R_1L_4s_1 + R_2L_1s_2)$ (2.24)

Now we apply the procedure (v) of article (2.1) then we see

$$\frac{V_o(s_1,\frac{1}{s_2})}{V_{in}(s_1,\frac{1}{s_2})} = \frac{0}{0}, \text{ as } s_1 \rightarrow 0 \text{ and } s_2 \rightarrow 0$$

So, $\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)}$ is not VSHP.

2.25 Transfer function of the Bridged-T network which is not VSHP (case T2-4)

Here in the Bridged-T network we put inductances in Z_1, Z_2 and capacitances in Z_3, Z_4 in equation (2.1).

We get,

$$\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)} = (R_2L_2C_3s_1s_2 + R_2L_1L_2C_3C_4s_1^2s_2^2 + R_2L_2C_4s_1s_2 + R_2L_1C_4s_1s_2) / (R_1L_2C_3s_1s_2 + R_1L_2C_4s_1s_2 + R_1L_1C_4s_1s_2 + R_2L_2C_3s_1s_2 + R_2L_2C_4s_1s_2 + R_2L_1C_4s_1s_2 + R_2L_1C_3s_1s_2 + R_1L_1L_2C_3C_4s_1^2s_2^2 + R_1 + R_2L_1L_2C_3C_4s_1^2s_2^2 + R_1R_2L_1C_3C_4s_1^2s_2 + R_1R_2C_4s_1 + R_1R_2C_3s_1 + L_1L_2C_3s_1s_2^2 + L_2s_2 + L_1s_2) \qquad (2.25)$$

Now we apply the procedure (v) of (2.1) then we see

$$\frac{V_o(s_1,\frac{1}{s_2})}{V_{in}(s_1,\frac{1}{s_2})} = \frac{0}{0}, \text{ as } s_1 \rightarrow 0 \text{ and } s_2 \rightarrow 0$$

So, $\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)}$ is not VSHP.

2.26 Transfer function of the Bridged-T network which is VSHP (case T2-5)

Here in the Bridged-T network we put inductances in Z_1, Z_3, Z_4 and capacitance in Z_2.

Putting the value of Z_1, Z_2, Z_3 and Z_4 in the equation (2.1) and make it 1-D, i.e., put $s_1 = s_2 = s$ and then we find

$$\frac{V_o(s)}{V_{in}(s)} = (s(R_2L_4 + R_2L_3 + R_2L_1) + s^3C_2R_2L_1L_3)/ (s^4C_2L_1L_3L_4 + s^3(C_2R_1L_1L_3 + C_2R_2L_1L_3 + C_2R_2L_1L_4 + C_2R_1L_3L_4) + s^2(C_2R_1R_2L_1 + C_2R_1R_2L_3 + C_2R_1R_2L_4 + L_1L_4 + L_3L_4) + s(L_4R_1 + L_3R_1 + L_4R_2 + L_3R_2 + L_1R_1 + L_1R_2) \quad (2.26)$$

If we equate the coefficient of s in the denominator of equation (2.3) and (2.26) then we get four equations.

$C_2L_1L_3L_4 = 1$ --- --- --- (2.26.1)

$C_2R_1L_1L_3 + C_2R_2L_1L_3 + C_2R_2L_1L_4 + C_2R_1L_3L_4 = 2.6131259$ --- --- (2.26.2)

$C_2R_1R_2L_1 + C_2R_1R_2L_3 + C_2R_1R_2L_4 + L_1L_4 + L_3L_4 = 3.4142136$ --- --- (2.26.3)

$L_4R_1 + L_3R_1 + L_4R_2 + L_3R_2 + L_1R_1 + L_1R_2 = 2.6131259$ --- --- --- (2.26.4)

and if we equate the coefficient of s in the numerator of equation (2.3) and (2.26) then we get one equation.

$C_2L_1L_3 = L_4 + L_3 + L_1$ --- --- --- (2.26.5)

It is noted that for ease of the calculation here we put the value of angular frequency ω = 1.

Now using these five equations we can't solve for the values of R_1, R_2, L_1, L_3, L_4, C_2 and hence is not considered.

2.27 Transfer function of the Bridged-T network which is not VSHP (case T2-6)

Here in the Bridged-T network we put inductance in Z_1 and capacitances in Z_2, Z_3, Z_4 in equation (2.1).

We get,

$$\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)} = (R_2C_3s_1 + R_2L_1C_3C_4s_1^2s_2 + R_2C_4s_1 + R_2L_1C_2C_4s_1s_2^2) / (R_1C_3s_1 + R_2C_4s_1 +$$

$$R_1L_1C_3C_4s_1^2s_2 + R_2L_1C_3C_4s_1^2s_2 + R_2L_1C_2C_4s_1s_2^2 + R_1R_2L_1C_2C_3C_4s_1^2s_2^2 + R_2L_1C_2C_3s_1s_2^2$$

$$+ R_1L_1C_2C_4s_1s_2^2 + 1 + R_1R_2C_2C_4s_1s_2 + L_1C_2s_2^2 + R_1R_2C_2C34s_1s_2 + L_1C_3s_1s_2 + R_1C_2s_2 +$$

$$R_2C_3s_1 + R_2C_4s_1) \qquad (2.27)$$

Now we apply the procedure (v) of article (2.1) then we see

$$\frac{V_o(s_1,\frac{1}{s_2})}{V_{in}(s_1,\frac{1}{s_2})} = \frac{0}{0}, \text{ as } s_1 \rightarrow 0 \text{ and } s_2 \rightarrow 0$$

So, $\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)}$ is not VSHP.

2.28 Transfer function of the Bridged-T network which is VSHP (case T2-7)

Here in the Bridged-T network we put inductances in Z_1, Z_4 and capacitances in Z_2, Z_3.

Putting the value of Z_1, Z_2, Z_3 and Z_4 in the equation (2.1) and make it 1-D, i.e., put $s_1 = s_2 = s$ and then we find

$$\frac{V_o(s)}{V_{in}(s)} = (R_2 + s^2(C_3R_2L_4 + C_3R_2L_1 + C_2R_2L_1)) / (s^4C_2C_3R_2L_1L_4 + s^3(C_2C_3R_1R_2L_1 +$$

$$C_3L_1L_4 + C_2C_3R_1R_2L_4 + C_2L_1L_4) + s^2(C_3R_1L_1 + C_3R_2L_1 + C_2R_1L_4 + C_3R_1L_4 + C_2R_1L_1 +$$

$$C_3R_2L_4 + C_2R_2L_1) + s(C_2R_1R_2 + L_4) + R_1 + R_2) \qquad (2.28)$$

Equation (2.28) is totally same to equation (2.10). So the rest of the procedure for finding impedance variables would be the same as that of section 2.10.

2.29 Transfer function of the Bridged-T network which is VSHP (case T2-8)

Here in the Bridged-T network we put inductances in Z_1, Z_3 and capacitances in Z_2, Z_4.

Putting the value of Z_1, Z_2, Z_3 and Z_4 in the equation (2.1) and make it 1-D, i.e., put $s_1 = s_2 = s$ and then we find

$$\frac{V_o(s)}{V_{in}(s)} = (R_2 + s^4R_2L_1L_3 + s^2(R_2L_3C_4 + R_2L_1L_3))/(s^4(R_1L_1L_3C_2C_4 + R_2L_1L_3C_2C_4) + s^3(R_1R_2L_1C_2C_4 + R_1R_2L_3C_2C_4 + C_2L_1L_3) + s^2(R_1L_3C_4 + R_2L_3C_4 + R_2L_1C_2 + R_1L_1C_4 + R_1L_3C_2 + R_2L_1C_4) + s(R_1R_2C_2 + L_3 + L_1) + R_1 + R_2) \quad (2.29)$$

If we equate the coefficient of s in the denominator of equation (2.3) and (2.29) then we get five equations.

$R_1L_1L_3C_2C_4 + R_2L_1L_3C_2C_4 = 1$ --- --- --- (2.29.1)

$R_1R_2L_1C_2C_4 + R_1R_2L_3C_2C_4 + C_2L_1L_3 = 2.6131259$ --- --- (2.29.2)

$R_1L_3C_4 + R_2L_3C_4 + R_2L_1C_2 + R_1L_1C_4 +$

$R_1L_3C_2 + R_2L_1C_4 = 3.4142136$ --- --- (2.29.3)

$R_1R_2C_2 + L_3 + L_1 = 2.6131259$ --- --- --- (2.29.4)

$R_1 + R_2 = 1$ --- --- --- (2.29.5)

and if we equate the coefficient of s in the numerator of equation (2.3) and (2.29) then we get one equation.

$1 + L_1L_3 = L_3C_4 + L_1L_3$ --- --- --- (2.29.6)

It is noted that for ease of the calculation here we put the value of angular frequency $\omega = 1$.

Now using these six equations we can't solve for the values of R_1, R_2, L_1, L_3, C_2, C_4 and hence is not considered.

2.30 Transfer function of the Bridged-T network which is not VSHP (case T2-9)

Here in the Bridged-T network we put inductances in Z_2, Z_3, Z_4 and capacitance in Z_1 in equation (2.1).

We get,

$$\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)} = (R_2L_2s_2 + R_2L_2L_4C_1s_1s_2^2 + R_2L_3s_1 + R_2L_2L_3C_1s_1s_2^2) / (R_1L_3s_1 + R_2L_3s_1 + R_1L_2L_4C_1s_1s_2^2 + R_1L_2L_3C_1s_1s_2^2 + R_2L_2L_4C_1s_1s_2^2 + R_2L_2L_3C_1s_1s_2^2 + R_1L_3L_4C_1s_1^2s_2 + L_2L_3L_4C_1s_1^2s_2^2 + R_1R_2 + R_1R_2L_3C_1s_1s_2 + L_3L_4s_1^2 + R_1R_2L_4C_1s_1s_2 + L_2L_4s_1s_2 + R_1L_2s_2 + R_2L_4s_1 + R_2L_4s_2) \quad (2.30)$$

Now we apply the procedure (v) of article (2.1) then we see

$$\frac{V_o(s_1,\frac{1}{s_2})}{V_{in}(s_1,\frac{1}{s_2})} = \frac{0}{0}, \text{ as } s_1 \rightarrow 0 \text{ and } s_2 \rightarrow 0$$

So, $\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)}$ is not VSHP.

2.31 Transfer function of the Bridged-T network which is VSHP (case T2-10)

Here in the Bridged-T network we put inductances in Z_2, Z_3 and capacitances in Z_1, Z_4.

Putting the value of Z_1, Z_2, Z_3 and Z_4 in the equation (2.1) and make it 1-D, i.e., put $s_1 = s_2 = s$ and then we find

$$\frac{V_o(s)}{V_{in}(s)} = (s^2(R_2L_1C_1 + C_4R_2L_3 + C_4R_2L_2) + s^4C_1C_4R_2L_2L_3)/ (s^4(C_1C_4R_1L_2L_3 + C_1C_4R_2L_2L_3) + s^3(C_1C_4R_1R_2L_3 + C_1L_2L_3) + s^2(C_1R_1L_3 + C_4R_1L_2 + C_4R_2L_2 + C_1R_1L_2 + C_4R_1L_3 + C_1R_2L_2 + C_4R_2L_3) + s(C_4R_1R_2 + L_2 + L_3 + R_1R_2C_1) + R_2) \quad (2.31)$$

Equation (2.31) is totally same to equation (2.13). So the rest of the procedure for finding impedance variables would be the same as that of section 2.13.

2.32 Transfer function of the Bridged-T network which is VSHP (case T2-11)

Here in the Bridged-T network we put inductances in Z_2, Z_3, Z_4 and capacitance in Z_1.

Putting the value of Z_1, Z_2, Z_3 and Z_4 in the equation (2.1) and make it 1-D, i.e., put $s_1 = s_2 = s$ and then we find

$$\frac{V_o(s)}{V_{in}(s)} = (R_2 + s^2(R_2L_2C_1 + C_3R_2L_2) + s^4C_1C_3R_2L_2L_4)/ (s^4(C_1C_3R_1L_2L_4 + C_1C_3R_2L_2L_3) + s^3(C_1C_3R_1R_2L_4 + C_1L_2L_4 + C_3L_2L_4) + s^2(C_1R_1L_2 + C_1R_2L_2 + C_3R_2L_4 + C_3R_1L_2 + C_1R_1L_4 + C_3R_2L_2) + s(C_3R_1R_2 + L_4 + R_1R_2C_1) + R_1 + R_2) \quad (2.32)$$

If we equate the coefficient of s in the denominator of equation (2.3) and (2.32) then we get five equations.

$C_1C_3R_1L_2L_4 + C_1C_3R_2L_2L_3 = 1$ --- --- --- (2.32.1)

$C_1C_3R_1R_2L_4 + C_1L_2L_4 + C_3L_2L_4 = 2.6131259$ --- --- (2.32.2)

$C_1R_1L_2 + C_1R_2L_2 + C_3R_2L_4 + C_3R_1L_2 +$

$C_1R_1L_4 + C_3R_2L_2 = 3.4142136$ --- --- (2.32.3)

$C_3R_1R_2 + L_4 + R_1R_2C_1 = 2.6131259$ --- --- --- (2.32.4)

$R_1 + R_2 = 1$ --- --- --- (2.32.5)

and if we equate the coefficient of s in the numerator of equation (2.3) and (2.32) then we get one equation.

$1 + C_1C_3L_2L_4 = L_2C_1 + C_3L_2$ --- --- --- (2.32.6)

It is noted that for ease of the calculation here we put the value of angular frequency ω = 1.

Now using these six equations we can't solve for the values of R_1, R_2, L_2, L_4, C_1, C_3 and hence is not considered.

2.33 Transfer function of the Bridged-T network which is not VSHP (case T2-12)

Here in the Bridged-T network we put inductance in Z_2 and capacitances in Z_1, Z_3, Z_4 in equation (2.1).

We get,

$$\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)} = (R_2L_2C_3C_4s_1^2s_2 + R_2L_2C_1C_4s_1s_2^2 + R_2C_4s_1 + R_2L_2C_3C_4s_1s_2^2) \,/\, (R_2C_1s_2 + R_1L_2C_1C_3s_1s_2^2 + R_1L_2C_1C_4s_1s_2^2 + R_2L_2C_1C_3s_1s_2^2 + R_2L_2C_1C_4s_1s_2^2 + R_1C_4s_1 + R_2C_4s_1 + R_1C_3s_1 + R_1R_2C_1C_4s_1s_2 + R_2C_3s_1 + R_1R_2C_1C_3s_1s_2 + L_2C_3s_1s_2 + L_2C_1s_2^2 + R_1R_2C_3C_4s_1^2 + 1 + R_1L_2C_3C_4s_1^2s_2) \quad (2.33)$$

Now we apply the procedure (v) of article (2.1) then we see

$$\frac{V_o(s_1,\frac{1}{s_2})}{V_{in}(s_1,\frac{1}{s_2})} = \frac{0}{0}, \text{ as } s_1 \to 0 \text{ and } s_2 \to 0$$

So, $\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)}$ is not VSHP.

2.34 Transfer function of the Bridged-T network which is not VSHP (case T2-13)

Here in the Bridged-T network we put inductances in Z_3, Z_4 and capacitances in Z_1, Z_2 in equation (2.1).

We get,

$$\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)} = (R_2L_4C_1s_1s_2 + R_2L_3C_1s_1s_2 + R_2 + R_2L_3C_2s_1s_2) / (R_1L_4C_1s_1s_2 + R_1L_3C_1s_1s_2 + R_1L_3C_2s_1s_2 + R_2L_4C_1s_1s_2 + R_2L_3C_1s_1s_2 + R_2L_3C_2s_1s_2 + R_2L_4C_2s_1s_2 + R_1R_2L_4C_1C_2s_1s_2^2 + R_1 + R_2 + R_1L_3L_4C_1C_2s_1^2s_2^2 + R_1R_2C_2s_2 + R_1R_2L_3C_1C_2s_1s_2^2 + L_3L_4C_1s_1^2s_2 + L_3L_4C_2s_1^2s_2 + L_4s_1) \quad (2.34)$$

Now we apply the procedure (v) of article (2.1) then we see

$$\frac{V_o(s_1,\frac{1}{s_2})}{V_{in}(s_1,\frac{1}{s_2})} = \frac{0}{0}, \text{ as } s_1 \rightarrow 0 \text{ and } s_2 \rightarrow 0$$

So, $\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)}$ is not VSHP.

2.35 Transfer function of the Bridged-T network which is not VSHP (case T2-14)

Here in the Bridged-T network we put inductance in Z_3 and capacitances in Z_1, Z_2, Z_4 in equation (2.1).

We get,

$\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)}$ = $(R_2C_1s_2 + R_2L_3C_1C_4s_1^2s_2 + R_2C_4s_1 + R_2L_3C_2C_4s_1^2s_2) / (R_1C_1s_2 + R_2C_1s_2 +$

$R_2C_2s_2 + R_1L_3C_1C_4s_1^2s_2 + R_1L_3C_2C_4s_1^2s_2 + R_2L_3C_1C_4s_1^2s_2 + R_2L_3C_2C_4s_1^2s_2 + R_1C_4s_1 +$

$R_2C_4s_1 + R_1R_2C_2C_4s_1s_2 + L_3C_1s_1s_2 + L_3C_2s_1s_2 + R_1R_2L_3C_1C_2C_4s_1^2s_2^2 + R_1R_2C_1C_2s_2^2 +$

$R_1L_3C_2s_1s_2^2 + 1)$ (2.35)

Now we apply the procedure (v) of article (2.1) then we see

$$\frac{V_o(s_1,\frac{1}{s_2})}{V_{in}(s_1,\frac{1}{s_2})} = \frac{0}{0}, \text{ as } s_1 \rightarrow 0 \text{ and } s_2 \rightarrow 0$$

So, $\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)}$ is not VSHP.

2.36 Transfer function of the Bridged-T network which is VSHP (case T2-15)

Here in the Bridged-T network we put inductance in Z_4 and capacitances in Z_1, Z_2, Z_3.

Putting the value of Z_1, Z_2, Z_3 and Z_4 in the equation (2.1) and make it 1-D, i.e., put $s_1 = s_2 = s$ and then we find

$\frac{V_o(s)}{V_{in}(s)}$ = $(s(R_2C_1 + R_2C_2 + R_2C_3) + s^3L_4R_2C_1C_3)/ (s^4R_1R_2L_4C_1C_2C_3 + s^3(L_4R_1C_1C_3 +$

$L_4R_2C_1C_3 + L_4R_2C_2C_3 + L_4R_1C_1C_2) + s^2(R_1R_2C_2C_3 + R_1R_2C_1C_2 + C_2R_1R_2L_4 + L_4C_2 +$

$L_4C_3) + s(R_1C_1 + R_2C_2 + R_2C_1 + R_2C_2 + R_1C_3 + R_2C_3)$ (2.36)

If we equate the coefficient of s in the denominator of equation (2.3) and (2.36) then we get four equations.

$R_1R_2L_4C_1C_2C_3 = 1$ --- --- --- (2.36.1)

$L_4R_1C_1C_3 + L_4R_2C_1C_3 + L_4R_2C_2C_3 + L_4R_1C_1C_2 = 2.6131259$ --- --- (2.36.2)

$R_1R_2C_2C_3 + R_1R_2C_1C_2 + C_2R_1R_2L_4 + L_4C_2 + L_4C_3 = 3.4142136$ --- --- (2.36.3)

$R_1C_1 + R_2C_2 + R_2C_1 + R_2C_2 + R_1C_3 + R_2C_3 = 2.6131259$ --- --- --- (2.36.4)

and if we equate the coefficient of s in the numerator of equation (2.3) and (2.36) then we get one equation.

$L_4C_1C_3 = C_1 + C_2 + C_3$ --- --- (2.36.5)

It is noted that for ease of the calculation here we put the value of angular frequency ω = 1.

Now using these five equations we can't solve for the values of R_1, R_2, L_4, C_1, C_2, C_3 and hence is not used further.

2.37 Transfer function of the Bridged-T network which is not VSHP (case T2-16)

Here in the Bridged-T network we put capacitances in Z_1, Z_2, Z_3, Z_4 in equation (2.1).

We get,

$$\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)} = (R_2C_1C_3s_1s_2 + R_2C_1C_4s_1s_2 + R_2C_3C_4s_1^2 + R_2C_2C_4s_1s_2) \,/\, (R_1C_1C_3s_1s_2 + R_1C_1C_4s_1s_2 + R_1C_2C_4s_1s_2 + R_2C_1C_3s_1s_2 + R_2C_1C_4s_1s_2 + R_2C_2C_4s_1s_2 + R_2C_2C_3s_1s_2 + R_1C_2C_4s_1^2 + R_2C_3C_4s_1^2 + R_1C_1C_2s_2^2 + R_1R_2C_2C_3C_4s_1^2s_2 + R_1R_2C_1C_2C_4s_1s_2^2 + R_1R_2C_1C_2C_3s_1s_2^2 + C_1s_2 + C_2s_2 + C_3s_1) \qquad (2.37)$$

Now we apply the procedure (v) of article (2.1) then we see

$$\frac{V_o(s_1, \frac{1}{s_2})}{V_{in}(s_1, \frac{1}{s_2})} = \frac{0}{0}, \text{ as } s_1 \rightarrow 0 \text{ and } s_2 \rightarrow 0$$

So, $\frac{V_o(s_1, s_2)}{V_{in}(s_1, s_2)}$ is not VSHP.

2.38 Transfer function of the Bridged-T network which is not VSHP (case T2-17)

Here in the Bridged-T network we put inductances in Z_1, Z_4 and capacitances in Z_2, Z_3 in equation (2.1)..

We get,

$$\frac{V_o(s_1, s_2)}{V_{in}(s_1, s_2)} = (R_2L_4C_3s_1s_2 + R_2 + R_2L_1C_2s_2^2 + R_2L_1C_3s_1s_2\ ) \ / \ (R_1 + R_2 + R_1L_1C_2s_2^2 + R_1L_4C_2s_2^2 + R_2L_1C_2s_2^2 + R_2L_1L_4C_2C_3s_1s_2^3 + R_1R_2L_1C_2C_3s_1s_2^2 + R_1R_2L_4C_2C_3s_1s_2^2 + L_1L_4C_3s_1s_2^2 + R_1R_2C_2s_2 + L_4s_2 + L_1L_4C_2s_2^3 + R_1L_4C_3s_1s_2 + R_1L_1C_3s_1s_2 + R_2L_4C_3s_1s_2 + R_2L_1C_3s_1s_2\ ) \quad (2.38)$$

Now we apply the procedure (v) of article (2.1) then we see

$$\frac{V_o(s_1, \frac{1}{s_2})}{V_{in}(s_1, \frac{1}{s_2})} = \frac{0}{0}, \text{ as } s_1 \rightarrow 0 \text{ and } s_2 \rightarrow 0$$

So, $\frac{V_o(s_1, s_2)}{V_{in}(s_1, s_2)}$ is not VSHP.

2.39 Transfer function of the Bridged-T network which is VSHP (case T2-18)

Here in the Bridged-T network we put inductances in Z_1, Z_3 and capacitances in Z_2, Z_4.

We get,

$$\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)} = (R_2L_3C_4s_1s_2 + R_2 + R_2L_1L_3C_2C_4s_1s_2^3 + R_2L_1C_4s_2^2) / (R_1 + R_2 + R_1L_3C_4s_1s_2 + R_2L_3C_4s_1s_2 + R_1L_3C_2s_1s_2 + R_2L_1L_3C_2C_4s_1s_2^3 + R_2L_1L_3C_2C_4s_1s_2^3 + R_1R_2L_1C_2C_4s_2^3 + R_1R_2L_3C_2C_4s_1s_2^2 + L_1L_3C_2C_4s_1s_2^2 + L_1s_2 + R_1R_2C_2s_2 + L_3s_1 + R_1L_1C_4s_2^2 + R_2L_1C_4s_2^2 + R_2L_1C_2s_2^2) \tag{2.39}$$

Now we apply the procedure (v) of article (2.1) then we see

$$\frac{V_o(s_1,\frac{1}{s_2})}{V_{in}(s_1,\frac{1}{s_2})} = \frac{0}{0}, \text{ as } s_1 \rightarrow 0 \text{ and } s_2 \rightarrow 0$$

So, $\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)}$ is not VSHP.

2.40 Summary and Discussion

In this chapter, we have introduced a doubly terminated Bridged-T Network. The main difficulty of designing a filter is to ensure stability. To maintain the stable filter output we use in this circuit a special class of polynomial with 2-variable analog domain called Very Strict Hurwitz Polynomial. The procedure of testing whether a given polynomial is VSHP or not is described in this chapter. Applying the procedure we get two tables which give a list of testing transfer function. In this chapter we give details description of the equations for TABLE 2.1 and 2.2. In TABLE 2.2, we keep the inductances and capacitances remain same position with their respective arms of the Bridged-T network with respect to TABLE 2.1, just change the values of s_1 and s_2. Among the 18 cases of TABLE 2.1 we get nine values which are VSHP and nine values which are not VSHP. In TABLE 2.2 we get seven values which are VSHP and eleven values which are not VSHP. Among the sixteen values from the two tables which are VSHP, we face difficulties for finding the impedance values from the derived equations. Though we have six equations for six variables, but the equations are non-linear and not possible to find the impedance values. Anyway there are three cases (7, 13, 17 from TABLE 2.1) which we can determine the value of impedance variable of the Bridged-T network. While computing the impedance variables we compare with fourth order Butterworth polynomial with the denominator of the transfer function of the Bridged-T network. The numerator of the Butterworth polynomial is considered as unity. For obtaining the equation from the numerator we let the angular frequency be unity for the ease of calculation.

This chapter mainly describes for choosing the transfer function which is VSHP from the different combinations of the impedance values of the Bridged-T network. Among total 36 combinations of transfer function we get only three combinations which are suited for designing 2-D digital filter. After finding the impedance values we design digital filter which will describe in chapter 3.

Chapter 3

Analyze and design of the digital filter from VSHP Transfer Function

A popular method to design a 2-D recursive digital filter is starting from an analog filter, getting the analog transfer function, and then applying the double bilinear transformations to the analog transfer function to design the digital filter. If we assign a VSHP in the denominator of the analog transfer function, we can obtain a stable digital transfer function, if the bilinear transformation is used. There are some stability conditions for coefficients of general bilinear transformation which we discussed in article 1.2.4.1.

The conditions are $k_1 > 0$, $k_2 > 0$, $|a_1| \leq 1$, $|b_1| \leq 1$, $a_1b_1 < 0$, $|a_2| \leq 1$, $|b_2| \leq 1$, $a_2b_2 < 0$. In our whole thesis we maintain these conditions of coefficients of bilinear transformations.

In the previous chapter from TABLE 2.1 we can get the impedance values for the case 7, 13 and 17. Now we have to design digital filter for case 7 which transfer function is VSHP.

3.1 Design procedure of Digital Filter for the case 7 from TABLE 2.1

Using the transfer function of the Bridged-T network of equation (2.10), if we compare with 1-D fourth order Butterworth Polynomial we get the impedance values of the Bridged-T network. After obtaining the values of impedance variables given in TABLE 2.3, we will make the transfer function in 2-D by putting the values of Z_1, Z_2, Z_3, Z_4 in (2.1) and then we get,

$$\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)} = (R_2 + C_3R_2L_1s_1^2 + C_3R_2L_4s_1s_2 + C_2R_2L_1s_1s_2) / (C_3R_1L_1s_1^2 + C_3R_2L_1s_1^2 + C_2R_1L_4s_2^2 + C_2C_3R_1R_2L_1s_1^2s_2 + C_3L_1L_4s_1^2s_2 + C_2R_1R_2s_2 + L_4s_2 + C_2R_1L_1s_1s_2 + R_1 + R_2 + C_2C_3R_2L_1L_4s_1^2s_2^2 + C_2C_3R_1R_2L_4s_1s_2^2 + C_2L_1L_4s_1\,s_2^2 + C_3R_1L_4s_1s_2 + C_3R_2L_4s_1s_2 + C_2R_2L_1s_1s_2) \tag{3.1}$$

Putting the value of R_1, R_2, L_1, L_4, C_2, C_3 in the above equation and apply generalized bilinear transformation which is given below:

$s_i = k_i\,(z_i - a_i) / (z_i + b_i)$ where $|a_i| \le 1$ and $|b_i| \le 1$

For stability we have to ensure, $k_1>0$, $k_2>0$, $|a_1| \le 1$, $|b_1| \le 1$, $a_1b_1<0$

and $|a_2| \le 1$, $|b_2| \le 1$, $a_2b_2<0$.

3.2 Frequency response of the 2-D recursive Digital Filters (When $R_1 = 0.6$, $R_2 = 0.4$)

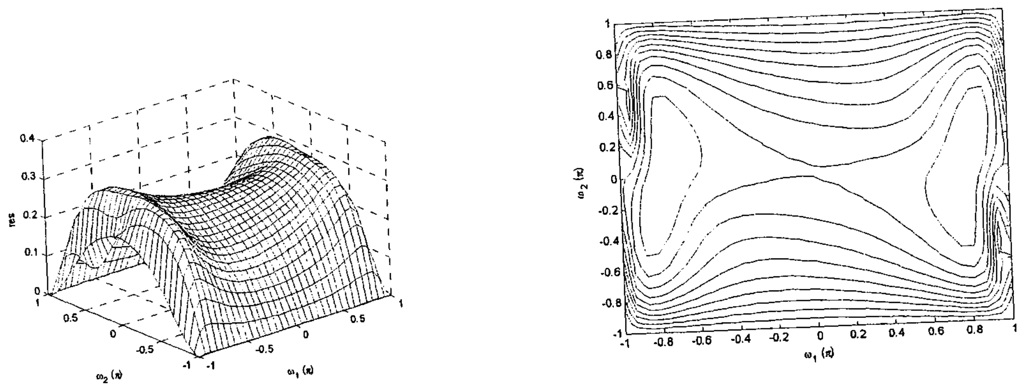
In this thesis we use MATLAB (The programs are given in Appendix) to obtain the contour and 3-D magnitude response plots of the resulting 2-D digital filters. With the input coefficients of the generalized bilinear transformations, we can obtain the contour and 3-D magnitude plots of the resulting 2-D digital filters. To investigate the manner in which each coefficient of generalized bilinear transformation affects the magnitude response of the resulting 2-D digital filters, we change the value of the deserving coefficients while keeping the other coefficients make constant. That can separate the effects from the other coefficients. Now we will observe the effect caused by each coefficient to the frequency responses of the resulting 2-D digital filter.

From TABLE 2.3 we analyze the data when $R_1 = 0.6$ and $R_2 = 0.4$.

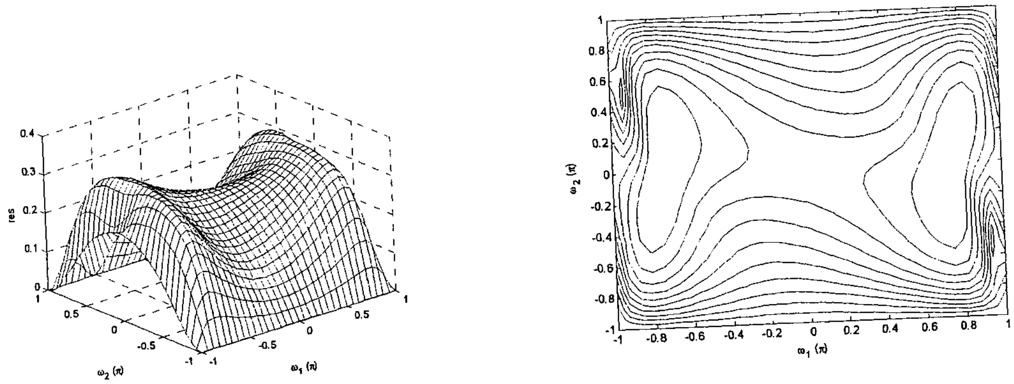
3.2.1 Frequency response for the 2-D Digital Filter with variable a_1

To study the manner how a_1 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of a_1, while fixing the other coefficients of the generalized bilinear transformation to be $a_2 = 0.5$, $b_1 = 1$, $b_2 = 1$, $k_1 = 1$, $k_2 = 1$.

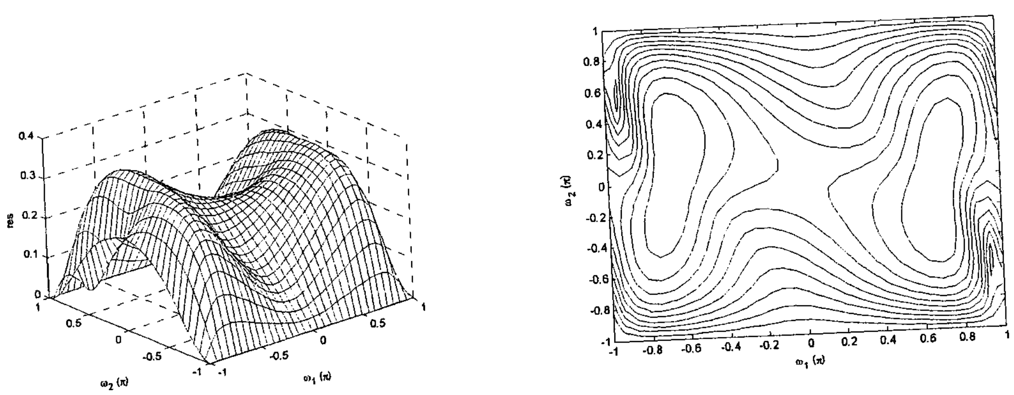
For ensuring the stability here we maintain conditions stated in (1.2.4.1).



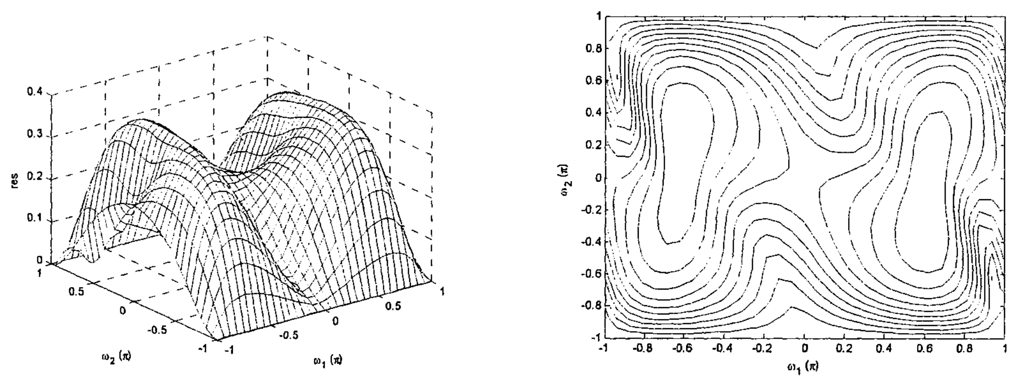
(a) $a_1 = 0$



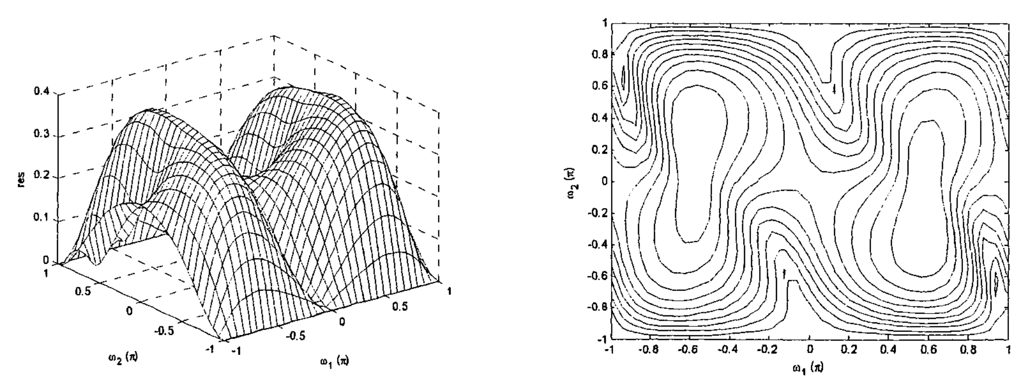
(b) $a_1 = 0.2$



(c) $a_1 = 0.5$



(d) $a_l = 0.8$



(e) $a_l = 1$

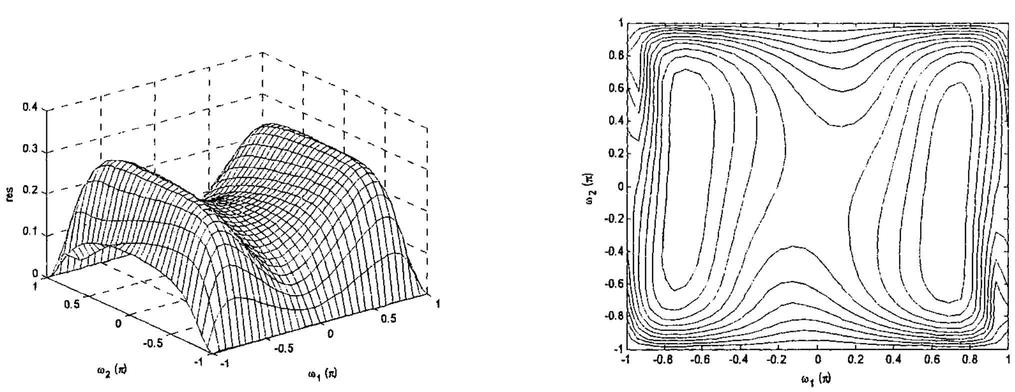
Figure 3.1 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable a_l and other coefficients constant.

It can be seen from Fig 3.1 that the coefficient a_l mainly affects the ω_1 domain of the filter. Here we vary the value of a_l within 0 to 1. As a_l increases the value, the output converges to band pass filter with respect to ω_1 domain. But with respect to ω_2 domain, it is seen from the figure that the filter output is a low pass filter. It is also seen from the figure that the pass band of the filter with respect to ω_1 becomes smaller from the lower boundary of 0 to the upper boundary of 1.

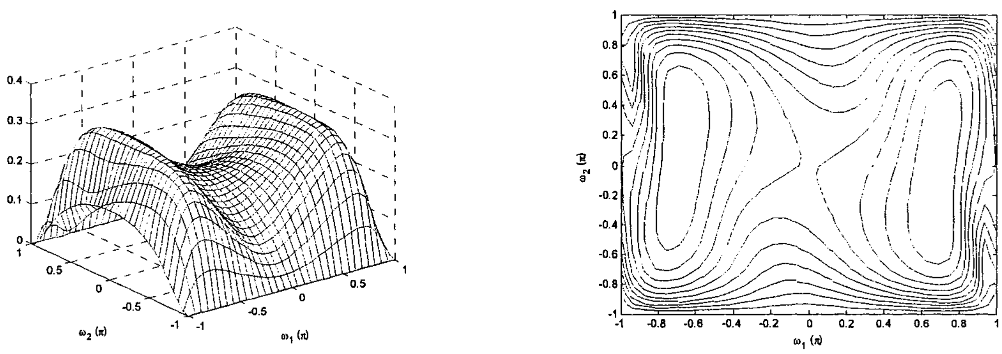
3.2.2 Frequency response for the 2-D Digital Filter with variable a_2

To study the manner how a_2 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of a_2, while fixing the other coefficients of the generalized bilinear transformation to be $a_1 = 0.5$, $b_1 = 1$, $b_2 = 1$, $k_1 = 1$, $k_2 = 1$.

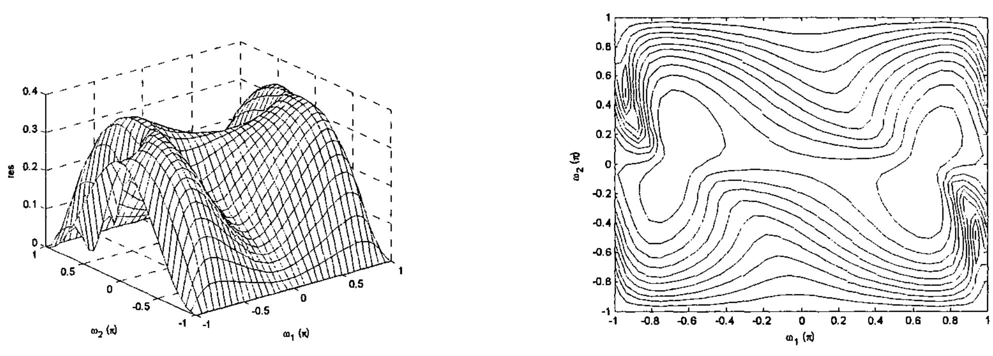
For ensuring the stability here we maintain conditions stated in (1.2.4.1).



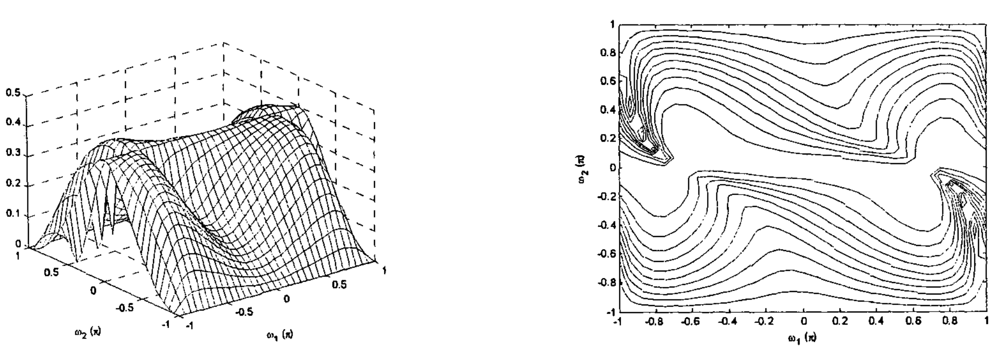
(a) $a_2 = 0$



(b) $a_2 = 0.2$



(c) $a_2 = 0.8$



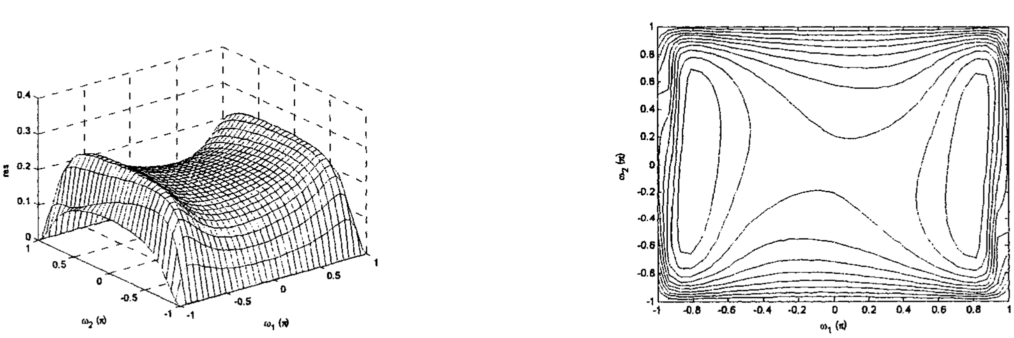
(d) $a_2 = 1$

Figure 3.2 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable a_2 and other coefficients constant

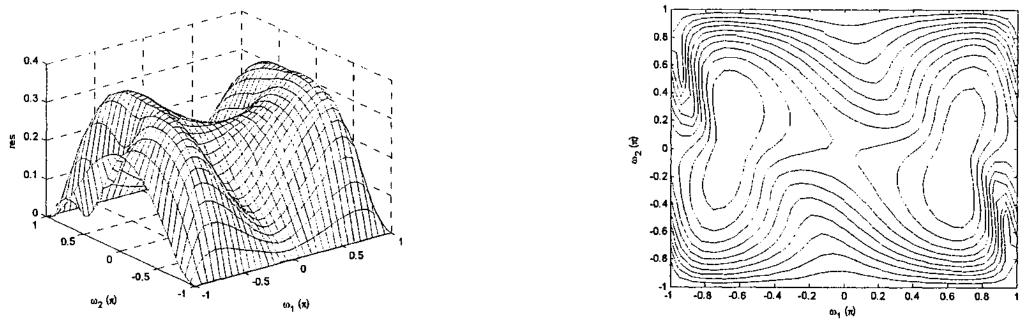
It can be seen from Fig 3.2 that the coefficient a_2 mainly affects the ω_2 domain of the filter. From the figure it is observed that a_2 varies the gain of the pass band of the resulting 2-D filter. Specifying the other coefficients to be with proper signs, and when a_2 changes from 0 to 1, the gain in the pass band increases from 0.25 to 0.4. It is noticeable that the changing of gain of the resulting filter at origin is opposite in the case of a_2 to the case of a_1. For the case of a_1 the gain at origin decreases from the value 0 to 1, while for the case of a_2 the gain at origin increases from the value of 0 to 1. At $a_2 = 0$ the filter characteristics correspond to band pass filter with respect to ω_1 domain and it is converging to all pass filter increasing the value a_2 from 0 to 1. But with respect to ω_2 domain, it is low pass filter.

3.2.3 Frequency response for the 2-D Digital Filter with equal a_1 and a_2

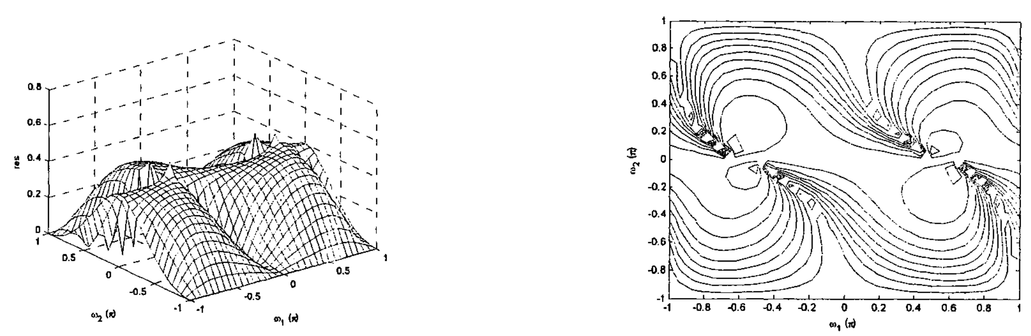
To study the manner how equal a_1 and a_2 affect the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of $a_1 = a_2$, while fixing the other coefficients of the generalized bilinear transformation to be $b_1 = 1$, $b_2 = 1$, $k_1 = 1$, $k_2 = 1$.



(a) $a_1 = a_2 = 0$



(b) $a_1 = a_2 = 0.6$



(c) $a_1 = a_2 = 1$

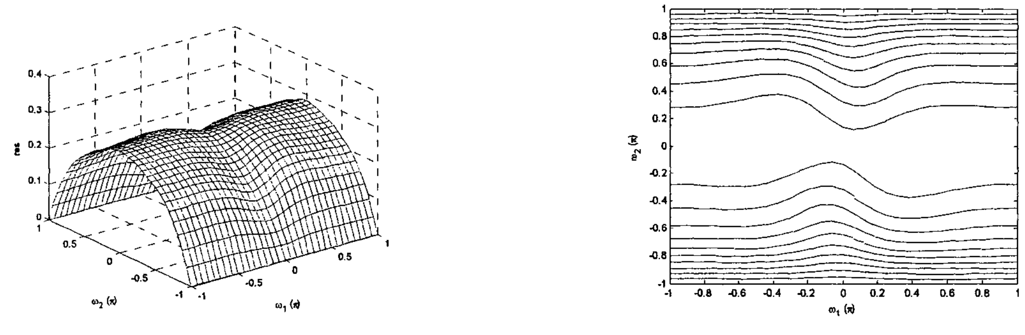
Figure 3.3 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable equal a_1 and a_2 and other coefficients constant.

From Fig 3.3, it is seen that the effect on the gain of pass band portions becomes more pronounced when we change the two coefficients, a_1 and a_2, simultaneously than the effect from the individual a_1 or a_2 only. When the values of a_1 and a_2 change from their lower boundary to their upper boundary, the gain of the pass band increases from 0.2 to 0.4.

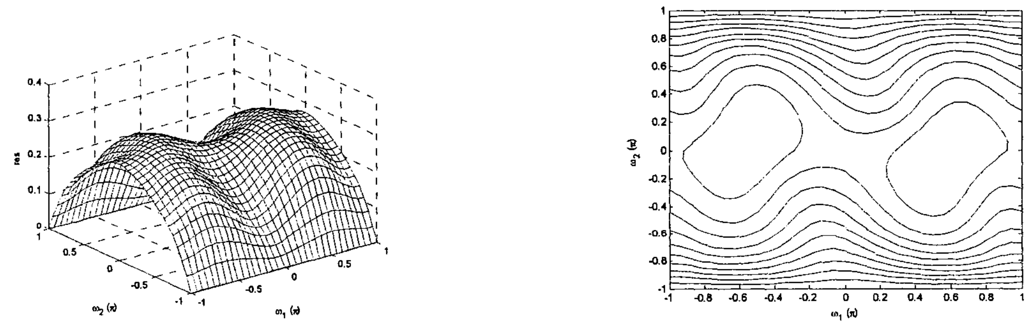
3.2.4 Frequency response for the 2-D Digital Filter with variable b_1

To study the manner how b_1 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of b_1, while fixing the other coefficients of the generalized bilinear transformation to be $a_1 = 0.5$, $a_2 = 0.5$, $b_2 = 1$, $k_1 = 1$, $k_2 = 1$.

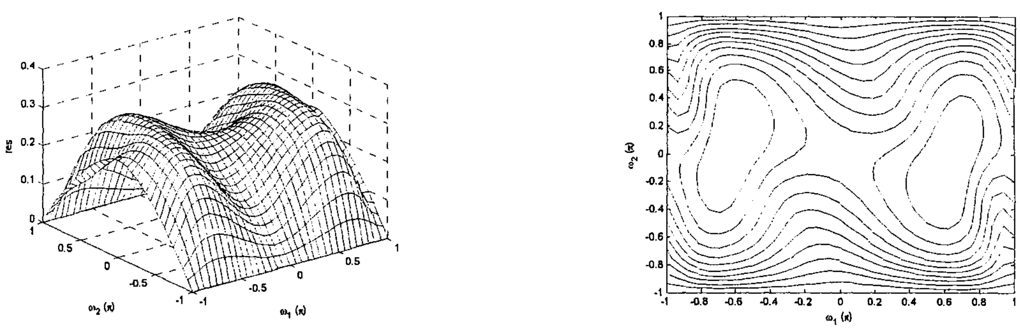
For ensuring the stability here we maintain conditions stated in (1.2.4.1).



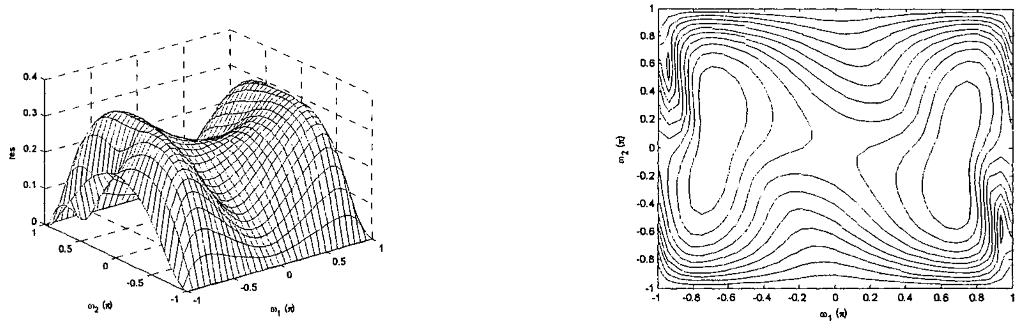
(a) $b_1 = 0$



(b) $b_1 = 0.5$



(c) $b_1 = 0.8$



(d) $b_1 = 1$

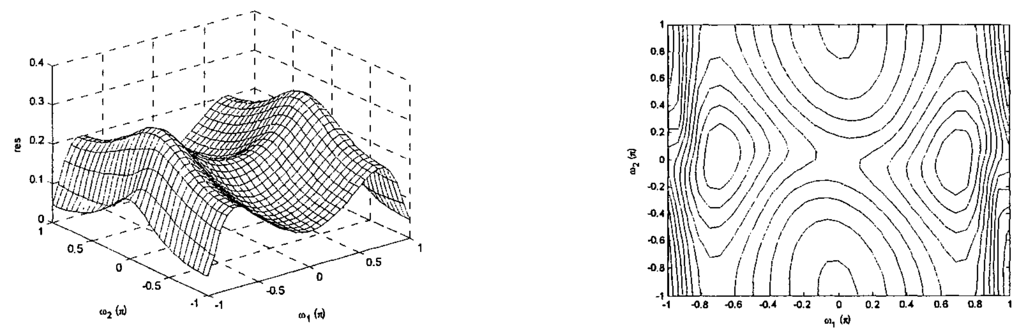
Figure 3.4 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable b_1 and other coefficients constant

From Fig 3.4, it is seen that the coefficient b_1 affects the filter characteristics. As we change the value from 0 to 1, we see that at $b_1 = 0$ it is almost all pass filter with respect to ω_1 domain. If we increase the value of b_1 the output characteristics changes to likely band pass filter. But with respect to ω_2 domain, the filter output is low pass filter. Here the effect on gain is not significant amount compare to the coefficients a_1 and a_2.

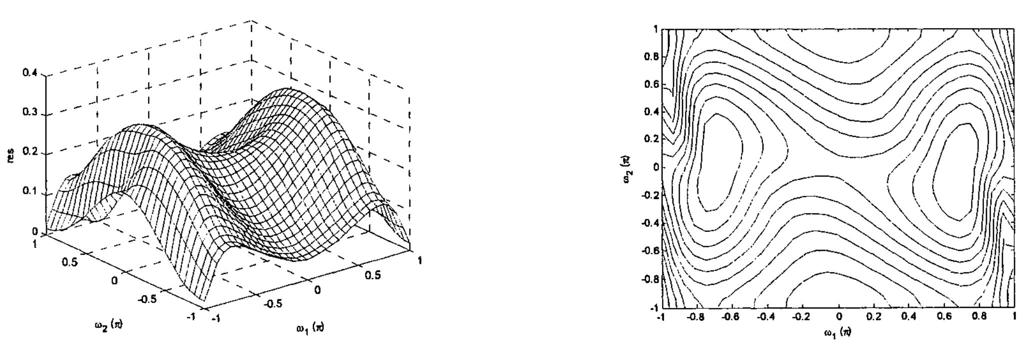
3.2.5 Frequency response for the 2-D Digital Filter with variable b_2

To study the manner how b_2 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of b_2, while fixing the other coefficients of the generalized bilinear transformation to be $a_1 = 0.5$, $a_2 = 0.5$, $b_1 = 1$, $k_1 = 1$, $k_2 = 1$.

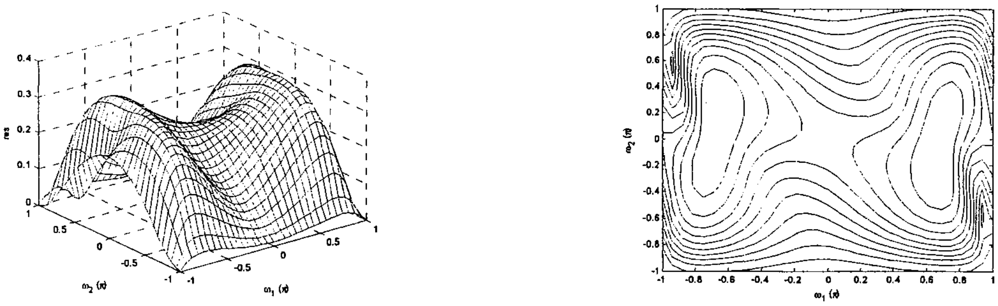
For ensuring the stability here we maintain conditions stated in (1.2.4.1).



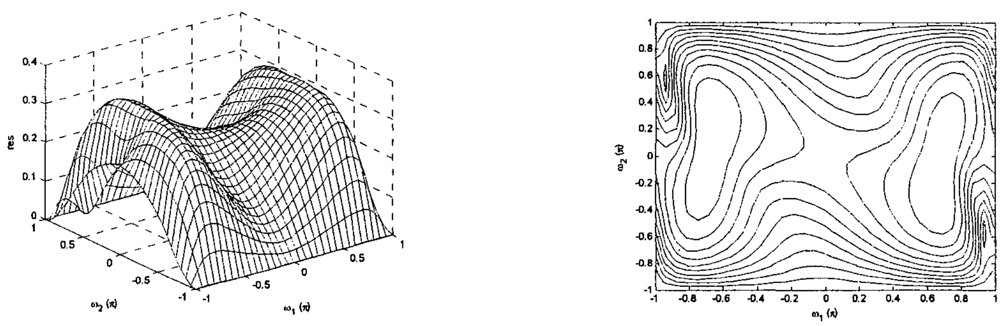
(a) $b_2 = 0$



(b) $b_2 = 0.5$



(c) $b_2 = 0.9$



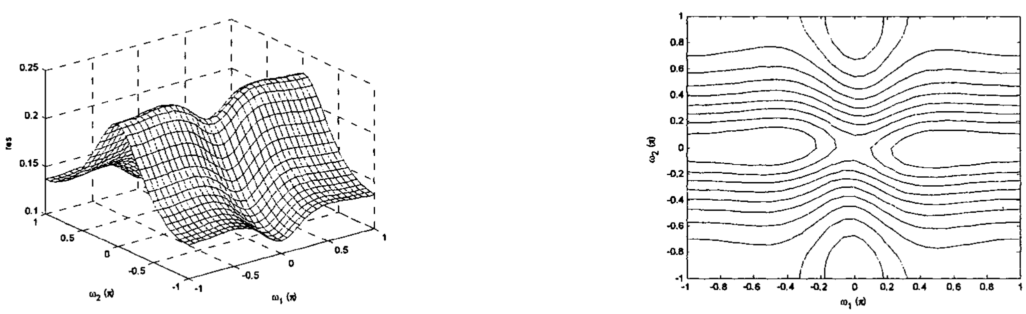
(d) $b_2 = 1$

Figure 3.5 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable b_2 and other coefficient constant

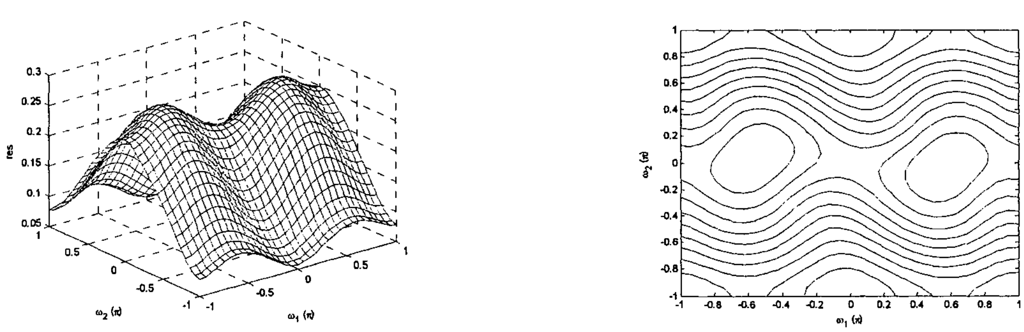
It is seen from the Fig 3.5 that b_2 also affects the filter characteristics. When $b_2 = 0$, its output is band pass filter with respect to ω_1 domain and low pass filter with respect to ω_2 domain. As we increase the value of b_2, it converges all pass filter in ω_1 domain and low pass in ω_2 domain. The b_2 coefficient has no significant effect on gain of the pass band filter.

3.2.6 Frequency response for the 2-D Digital Filter with equal b_1 and b_2

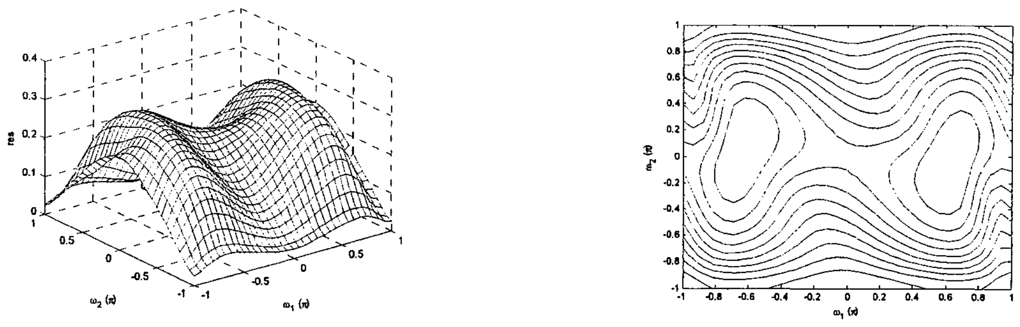
To study the manner how equal b_1 and b_2 affect the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of $b_1 = b_2$, while fixing the other coefficients of the generalized bilinear transformation to be $a_1 = 0.5$, $a_2 = 0.5$, $k_1 = 1$, $k_2 = 1$.



(a) $b_1 = b_2 = 0$



(b) $b_1 = b_2 = 0.5$



(d) $b_1 = b_2 = 0.8$

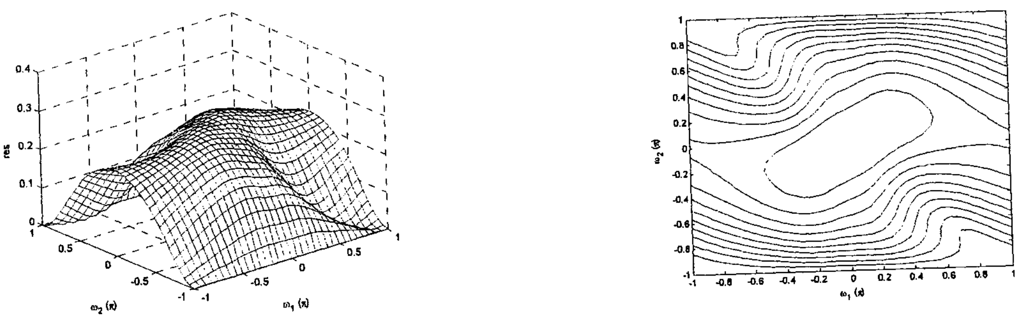
Figure 3.6 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable equal b_1 and b_2 and other coefficients constant

From Fig 3.6, it is seen that the effect on the filter characteristics is different when we change the two coefficients, b_1 and b_2, simultaneously than the effect from the individual b_1 or b_2 only. When we change the two coefficients the gain remains almost constant.

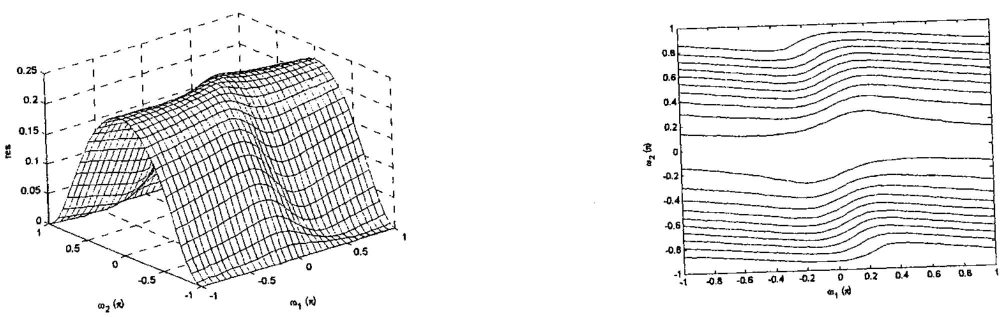
3.2.7 Frequency response for the 2-D Digital Filter with variable k_1

To study the manner how k_1 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of k_1, while fixing the other coefficients of the generalized bilinear transformation to be $a_1 = 0.5$, $a_2 = 0.5$, $b_1 = 1$, $b_2 = 1$, $k_2 = 1$.

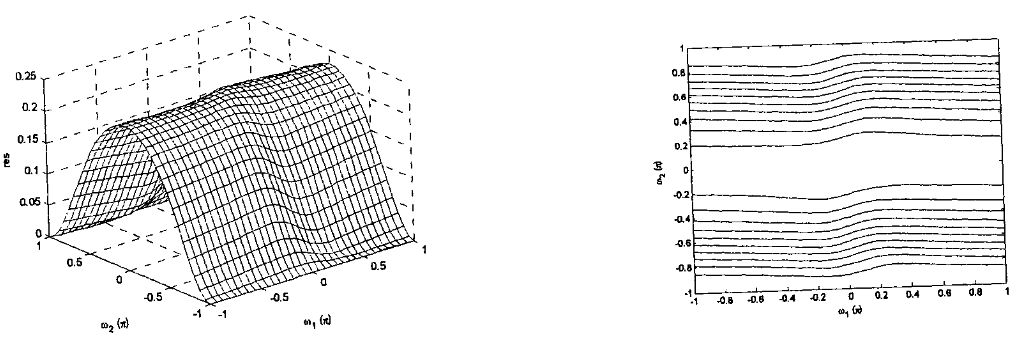
For ensuring the stability here we maintain conditions stated in (1.2.4.1).



(a) $k_1 = 10$



(b) $k_1 = 50$



(c) $k_1 = 100$

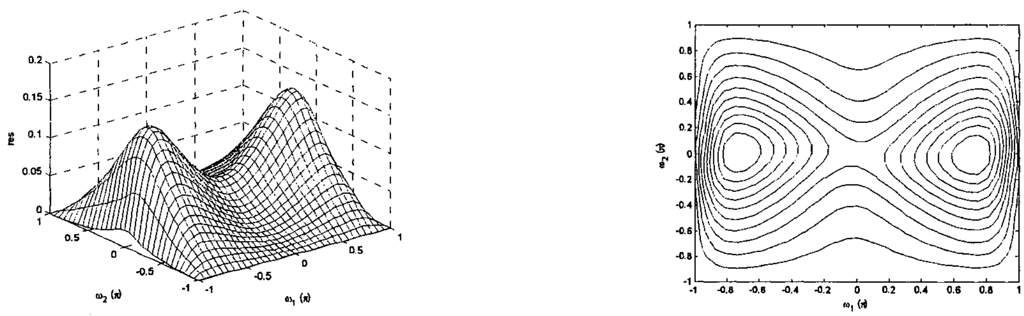
Figure 3.7 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable k_1 and other coefficients constant

From Fig 3.7, it is seen that the higher value of k_1 makes almost all pass filter with respect to ω_1 domain and low pass in ω_2 domain. The gain is not affected by the change of the k_1 variable.

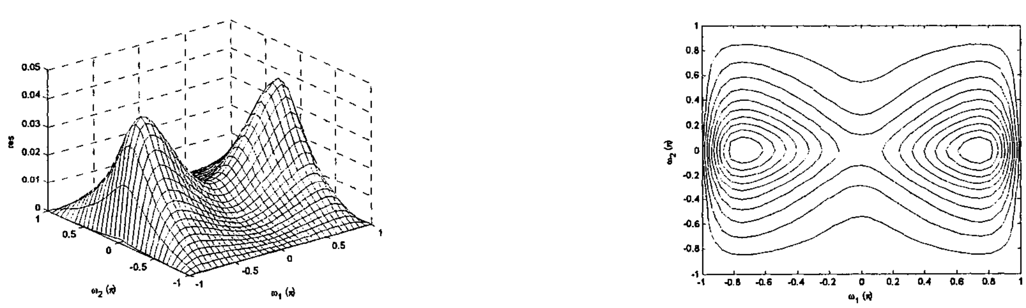
3.2.8 Frequency response for the 2-D Digital Filter with variable k_2

To study the manner how k_2 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of k_2, while fixing the other coefficients of the generalized bilinear transformation to be $a_1 = 0.5$, $a_2 = 0.5$, $b_1 = 1$, $b_2 = 1$, $k_1 = 1$.

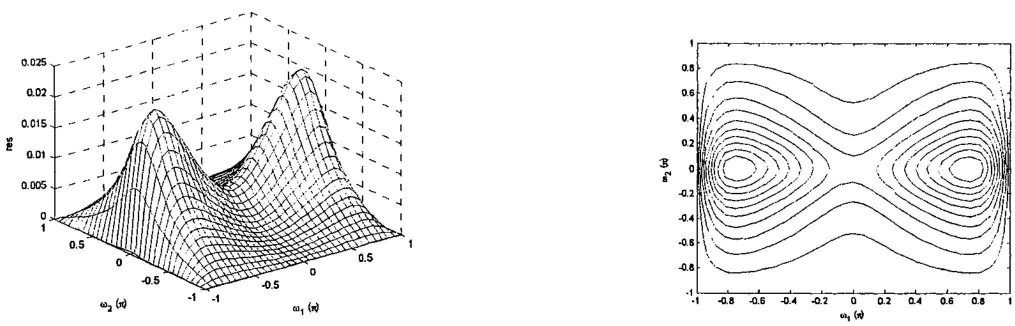
For ensuring the stability here we maintain conditions stated in (1.2.4.1).



(a) $k_2 = 10$



(b) $k_2 = 50$



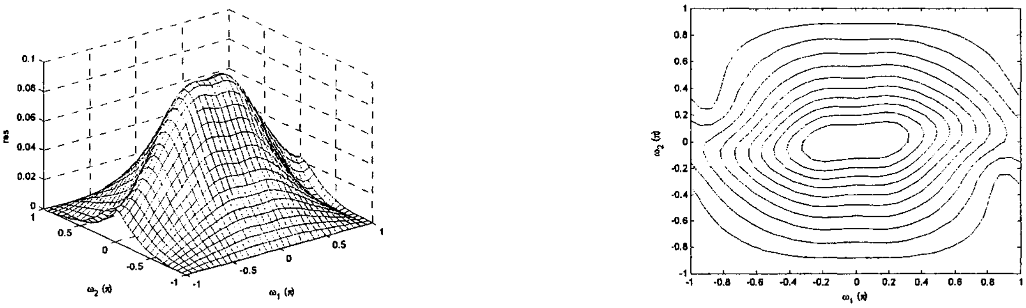
(c) $k_2 = 100$

Figure 3.8 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable k_2 and other coefficient constant.

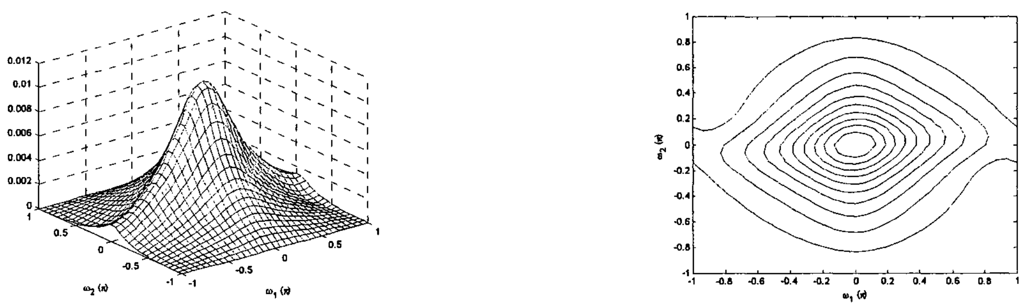
From Fig 3.8, it is seen that when we increase the value of k_2 variable it becomes band pass filter with respect to ω_1 domain and low pass in ω_2 domain. The gain of this filter is significantly changes while we increase the value of k_2. The gain decrease from 0.12 to 0.02 as we increase the value of k_2 from 10 to 100.

3.2.9 Frequency response for the 2-D Digital Filter with equal k_1 and k_2

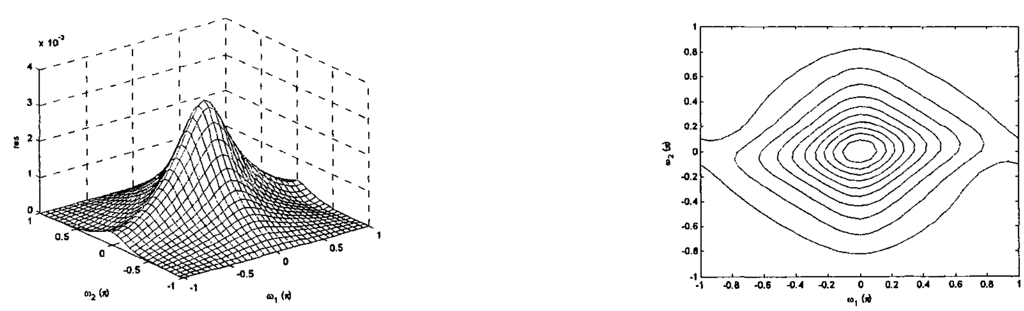
To study the manner how equal k_1 and k_2 affect the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of $k_1 = k_2$, while fixing the other coefficients of the generalized bilinear transformation to be $b_1 = 1$, $b_2 = 1$, $a_1 = 0.5$, $a_2 = 0.5$.



(a) $k_1 = k_2 = 10$



(c) $k_1 = k_2 = 50$



(c) $k_1 = k_2 = 100$

Figure 3.9 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable equal k_1 and k_2 and other coefficients constant.

From Fig 3.9, it is seen that the effect on the gain of pass band portions becomes more pronounced when we change the two coefficients, k_1 and k_2, simultaneously than the effect from the individual k_1 or k_2 only. When the values of k_1 and k_2 change from 10 to 100, the gain of pass band decreases from 0.06 to 0.002 The pass band area becomes enlarged in both ω_1 and ω_2 domains, but the effect is very slight. Here it is noticeable that if we make high both the value of k_1 and k_2 then we get low pass filter in both domain.

3.3 Frequency response of the 2-D recursive Digital Filters (When $R_1 = 0$, $R_2 = 1$)

To investigate the manner in which each coefficient of generalized bilinear transformation affects the magnitude response of the resulting 2-D digital filters, we change the value of the deserving coefficients while keeping the other coefficients make constant. That can separate the effects from the other coefficients.

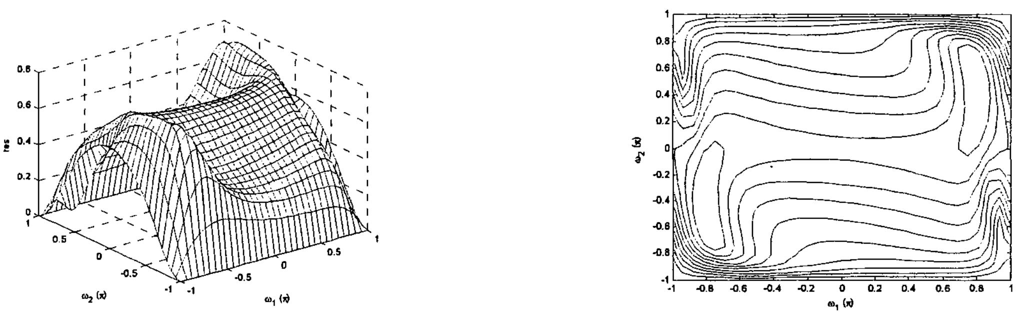
Now if we make the input resistance of the Bridged-T network zero then the effect of coefficient of the generalized bilinear transformation is quite different from the previous one.

Now we will analyze the data when $R_1 = 0$ and $R_2 = 1$

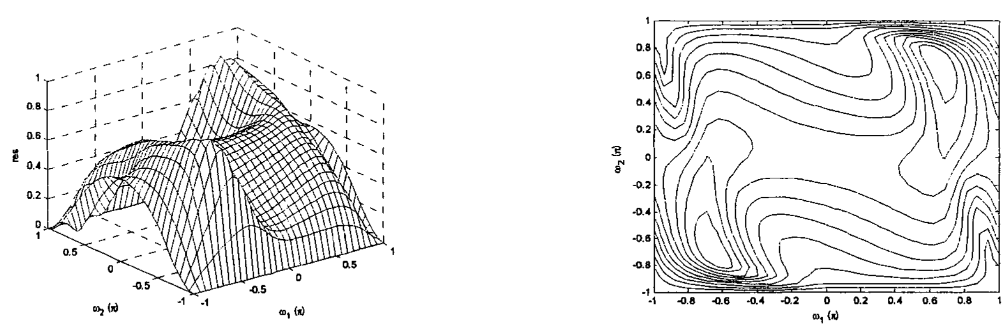
3.3.1 Frequency response for the 2-D Digital Filter with variable a_1

To study the manner how a_1 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of a_1, while fixing the other coefficients of the generalized bilinear transformation to be $a_2 = 0.5$, $b_1 = 1$, $b_2 = 1$, $k_1 = 1$, $k_2 = 1$.

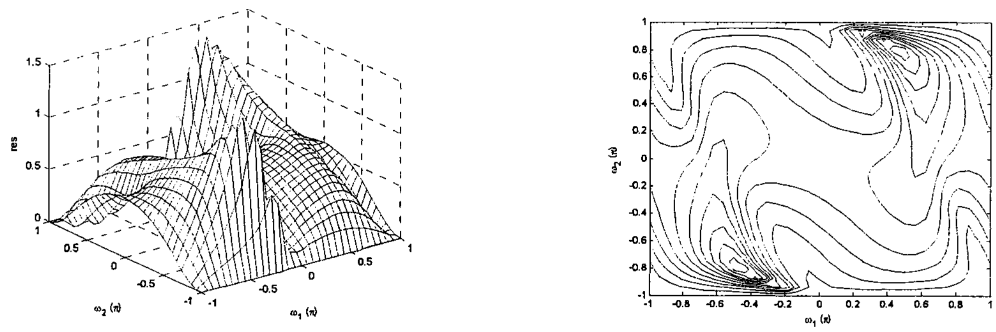
For ensuring the stability here we maintain conditions stated in (1.2.4.1).



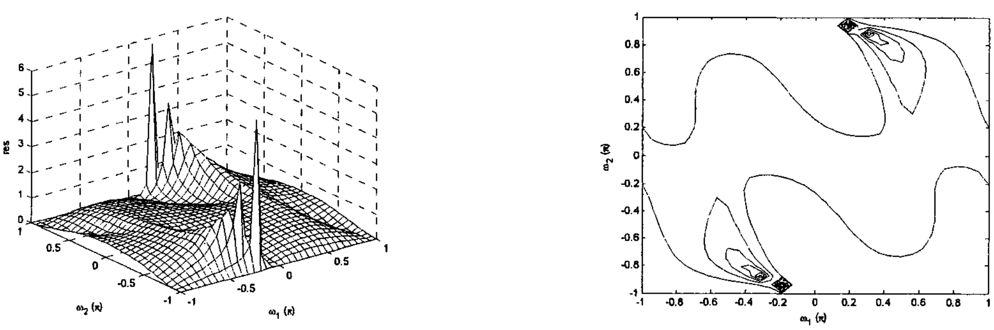
(a) $a_1 = 0$



(b) $a_1 = 0.4$



(c) $a_1 = 0.8$



(d) $a_1 = 1$

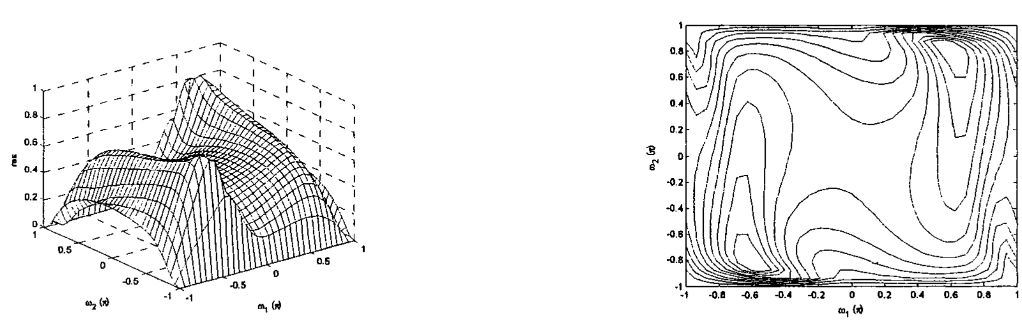
Figure 3.10 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable a_1 and other coefficient constant

From Fig 3.10, it is seen that the coefficient a_1 mainly affects the ω_1 domain of the filter. Here we vary the value of a_1 within 0 to 1.. It is seen from the figure that the gain of the filter becomes larger from the lower boundary of 0 to the upper boundary of 1. The shape of the filter curve is distorted and the region of the band pass is affected for $R_1 = 0$.

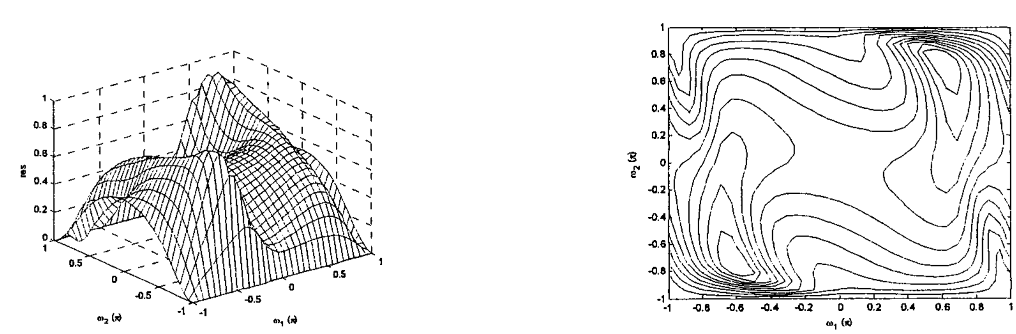
3.3.2 Frequency response for the 2-D Digital Filter with variable a_2

To study the manner how a_2 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of a_2, while fixing the other coefficients of the generalized bilinear transformation to be $a_1 = 0.5$, $b_1 = 1$, $b_2 = 1$, $k_1 = 1$, $k_2 = 1$.

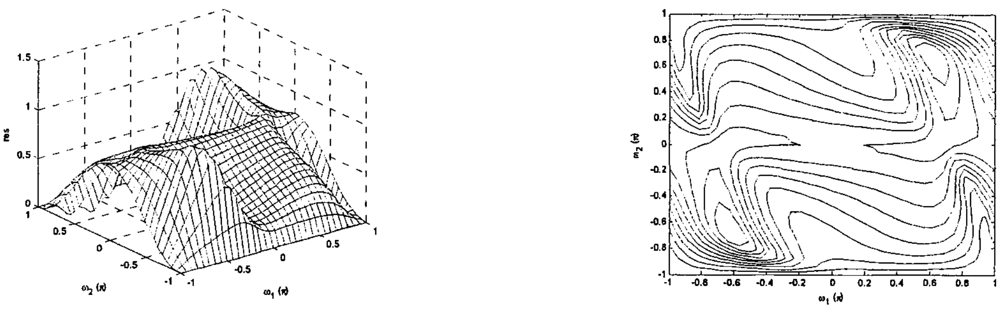
For ensuring the stability here we maintain conditions stated in (1.2.4.1).



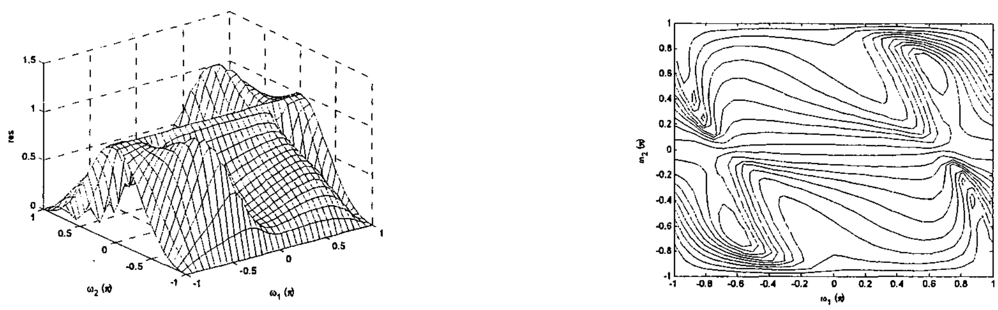
(a) $a_2 = 0$



(b) $a_2 = 0.4$



(c) $a_2 = 0.8$



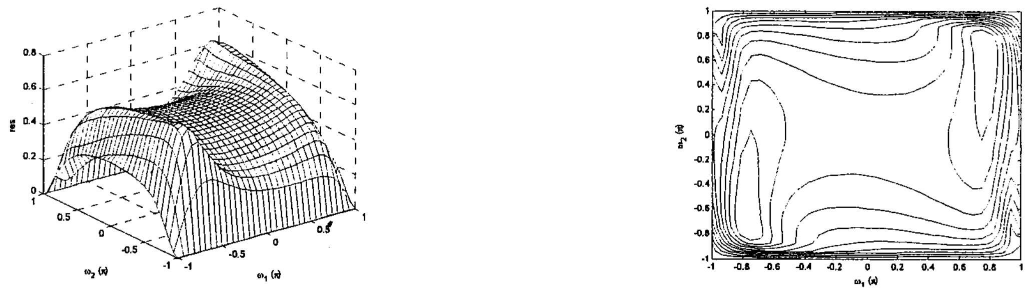
(d) $a_2 = 1$

Figure 3.11 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable a_2 and other coefficient constant.

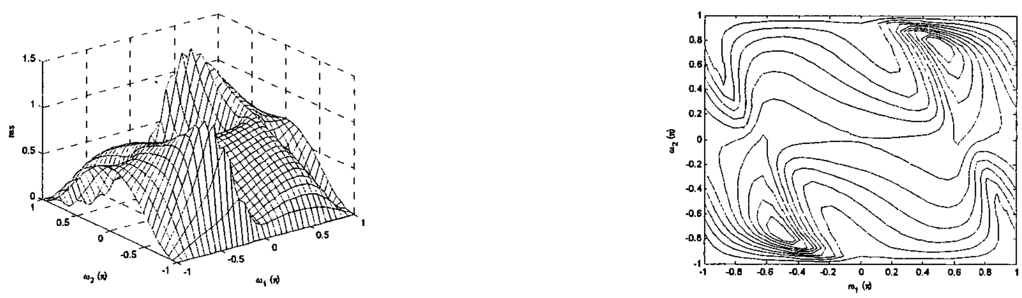
From Fig 3.11, it is seen that the shape of the magnitude response is also distorted when we change the coefficient a_2.

3.3.3 Frequency response for the 2-D Digital Filter with equal a_1 and a_2

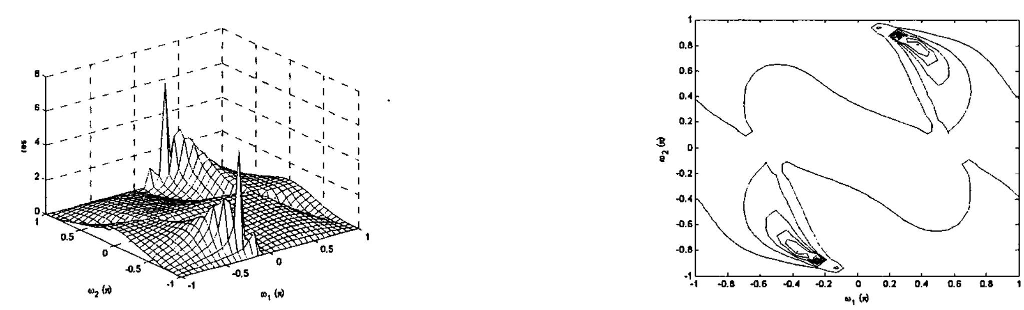
To study the manner how equal a_1 and a_2 affect the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of $a_1 = a_2$, while fixing the other coefficients of the generalized bilinear transformation to be $b_1 = 1$, $b_2 = 1$, $k_1 = 1$, $k_2 = 1$.



(a) $a_1 = a_2 = 0$



(c) $a_1 = a_2 = 0.8$



(c) $a_1 = a_2 = 1$

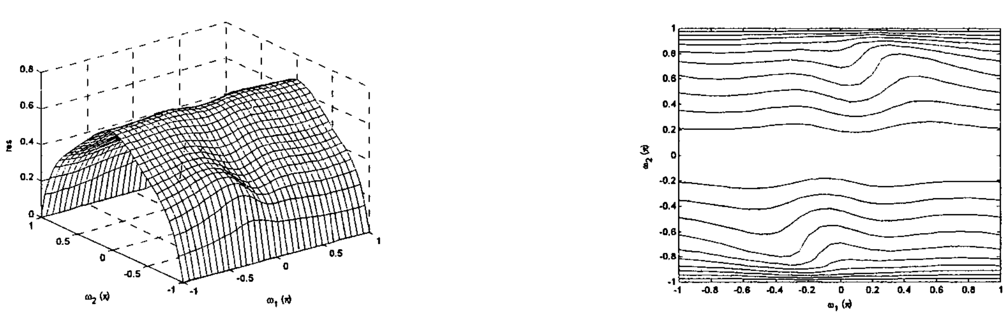
Figure 3.12 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable equal a_1 and a_2 and other coefficients constant.

From Fig 3.12, it is seen that the effect on the gain of pass band portions becomes more pronounced when we change the two coefficients, a_1 and a_2, simultaneously then the effect from the individual a_1 or a_2 only. But the output filter curve is still distorted.

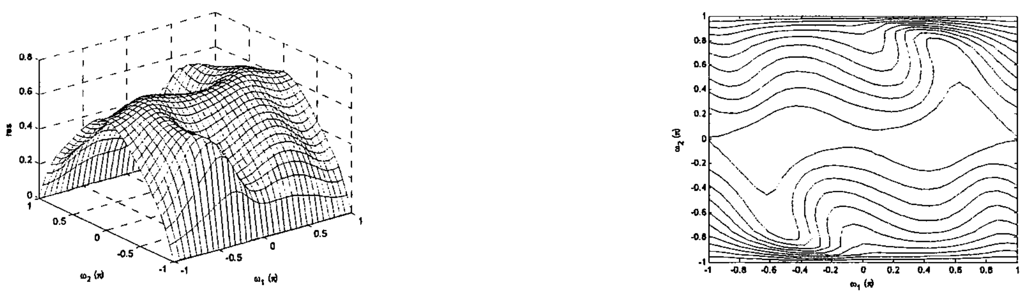
3.3.4 Frequency response for the 2-D Digital Filter with variable b_1

To study the manner how b_1 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of b_1, while fixing the other coefficients of the generalized bilinear transformation to be $a_1 = 0.5$, $a_2 = 0.5$, $b_2 = 1$, $k_1 = 1$, $k_2 = 1$.

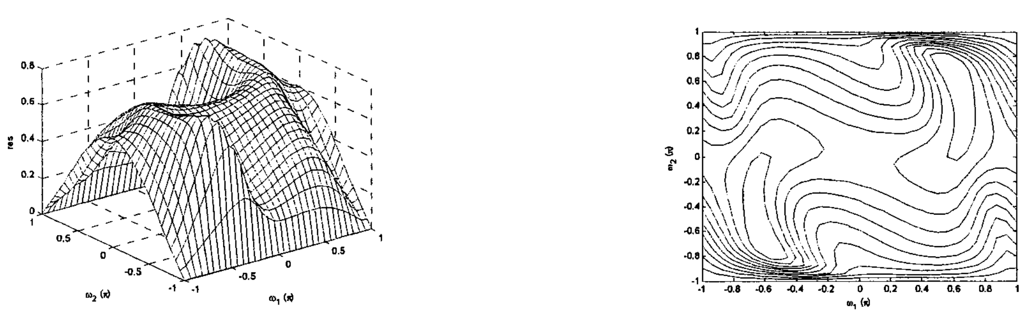
For ensuring the stability here we maintain conditions stated in (1.2.4.1).



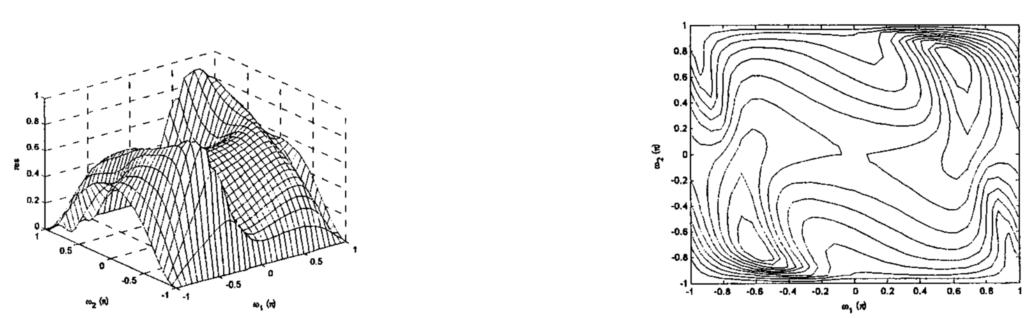
(a) $b_1 = 0$



(b) $b_1 = 0.5$



(c) $b_1 = 0.8$



(d) $b_1 = 1$

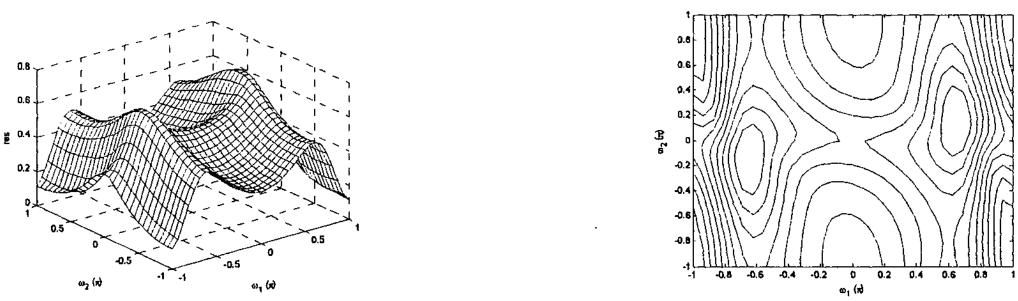
Figure 3.13 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable b_1 and other coefficient constant.

From Fig 3.13, it is seen that at $b_1 = 0$, the magnitude response is all pass filter with respect to ω_1 domain an low pass filter in ω_2 domain without any distortion. But when we increase the value b_2 from 0 to 1, it starts to distort the shape of the curve.

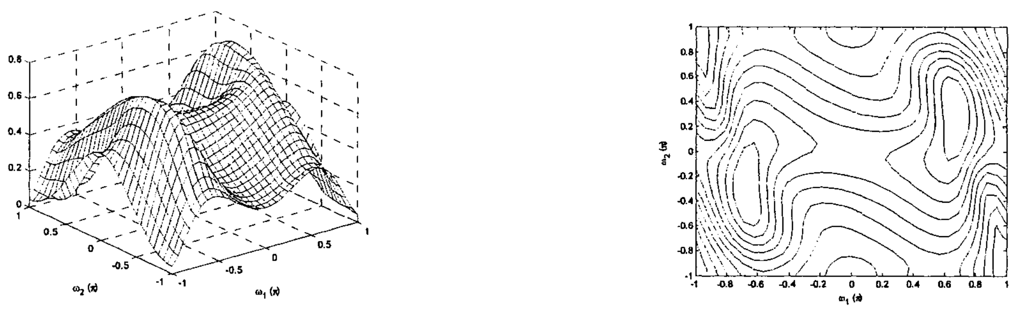
3.3.5 Frequency response for the 2-D Digital Filter with variable b_2

To study the manner how b_2 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of b_2, while fixing the other coefficients of the generalized bilinear transformation to be $a_1 = 0.5$, $a_2 = 0.5$, $b_1 = 1$, $k_1 = 1$, $k_2 = 1$.

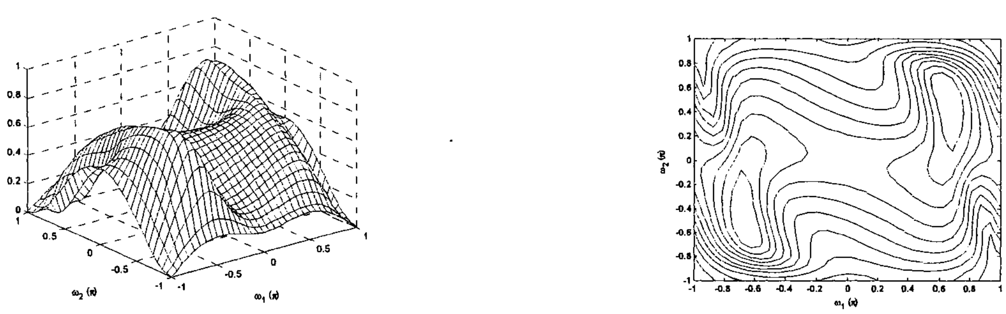
For ensuring the stability here we maintain conditions stated in (1.2.4.1).



(a) $b_2 = 0$



(d) $b_2 = 0.5$



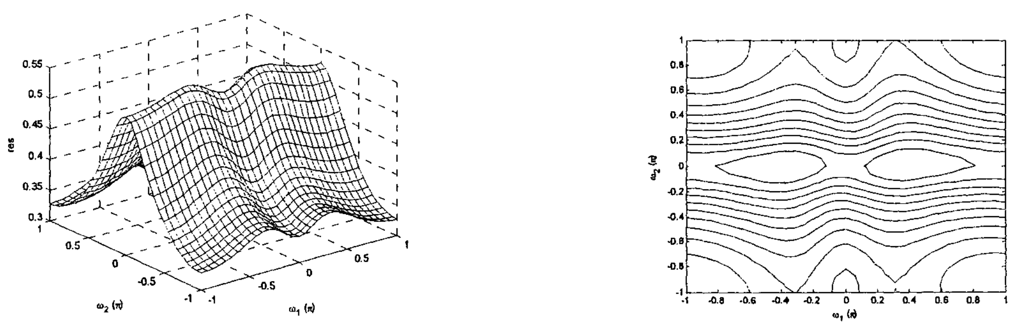
(d) $b_2 = 0.8$

Figure 3.14 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable b_2 and other coefficient constant.

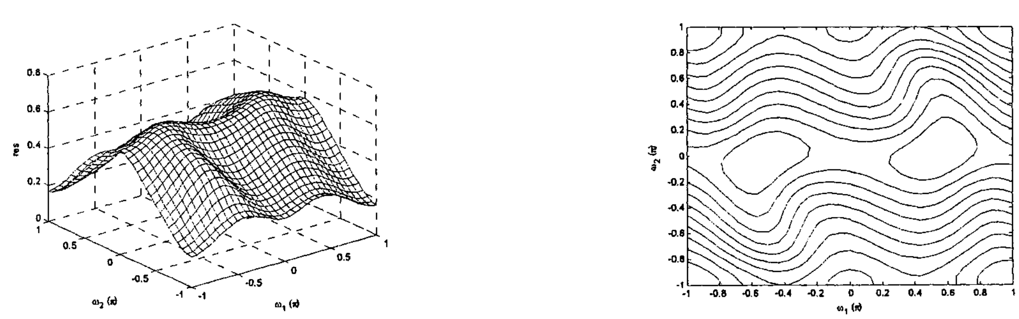
From Fig 3.14, it is seen that the gain of the pass band is not affected by changing the coefficient of bilinear transformation b_2. The distortion of the shape of the curve increases if we increase the value of b_2 from 0 to 1.

3.3.6 Frequency response for the 2-D Digital Filter with equal b_1 and b_2

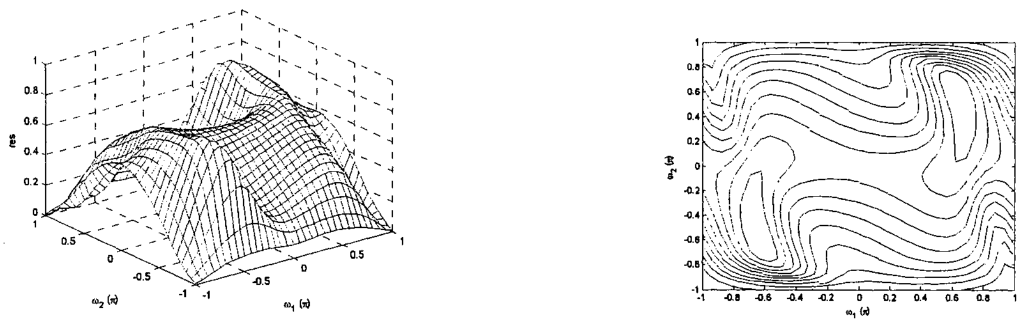
To study the manner how equal b_1 and b_2 affect the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of $b_1 = b_2$, while fixing the other coefficients of the generalized bilinear transformation to be $a_1 = 0.5$, $a_2 = 0.5$, $k_1 = 1$, $k_2 = 1$.



(a) $b_1 = b_2 = 0$



(b) $b_1 = b_2 = 0.5$



(c) $b_1 = b_2 = 0.9$

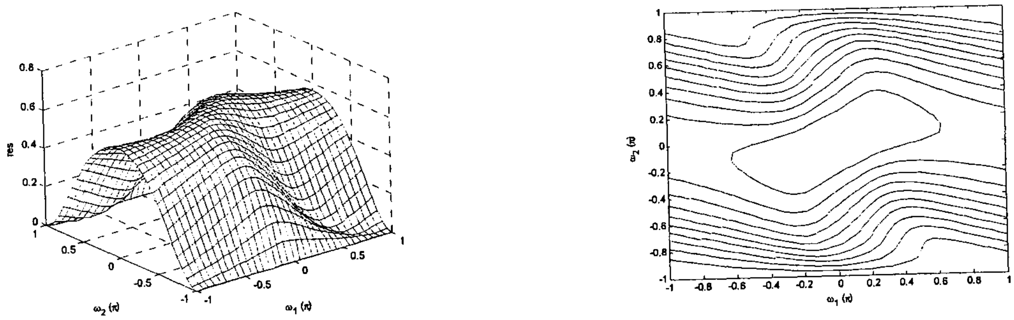
Figure 3.15 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable equal b_1 and b_2 and other coefficients constant.

From Fig 3.15, it is seen that the effect on the filter characteristics is different when we change the two coefficients, b_1 and b_2, simultaneously than the effect from the individual b_1 or b_2 only. When we change the two coefficients the gain remains almost constant.

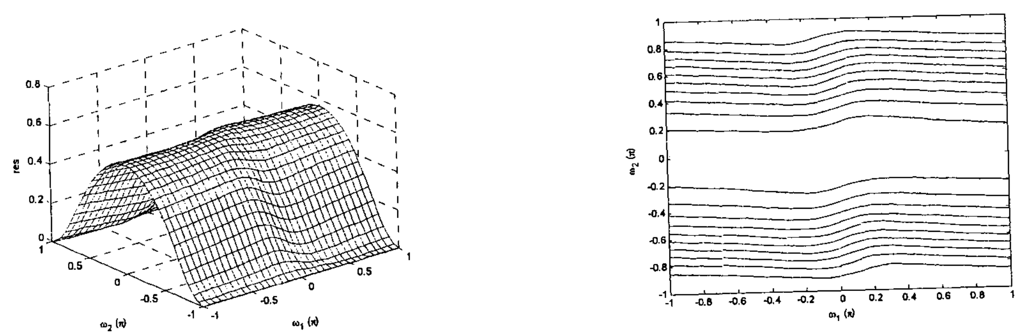
3.3.7 Frequency response for the 2-D Digital Filter with variable k_1

To study the manner how k_1 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of k_1, while fixing the other coefficients of the generalized bilinear transformation to be $a_1 = 0.5$, $a_2 = 0.5$, $b_1 = 1$, $b_2 = 1$, $k_2 = 1$.

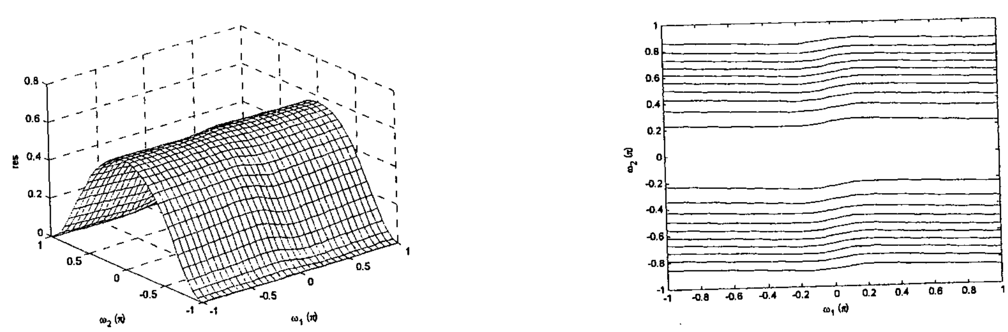
For ensuring the stability here we maintain conditions stated in (1.2.4.1).



(a) $k_1 = 10$



(b) $k_1 = 50$



(d) $k_1 = 100$

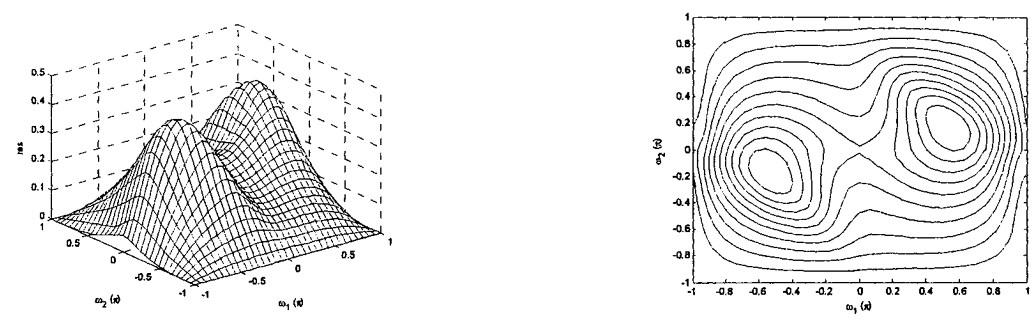
Figure 3.16 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable k_1 and other coefficient constant

From Fig 3.16, it is seen that the higher value of k_1 makes almost all pass filter with respect to ω_1 domain and low pass filter with respect to ω_2 domain. The gain is not affected by the change of the k_1 variable. It is seen from the figure that the higher value of k_1 makes the filter output distortion less.

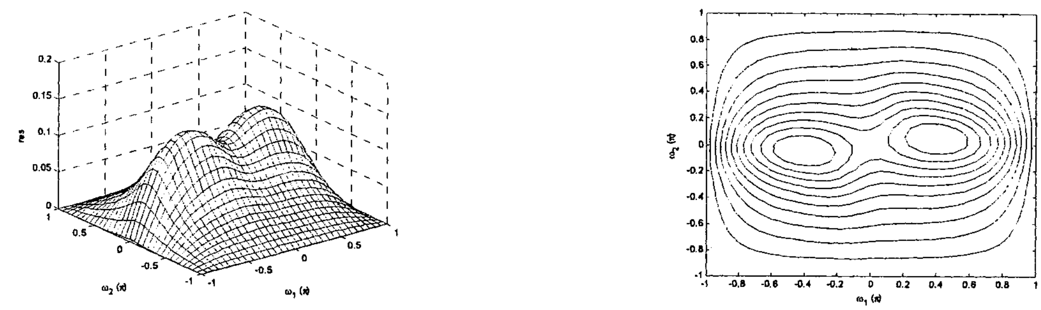
3.3.8 Frequency response for the 2-D Digital Filter with variable k_2

To study the manner how k_2 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of k_2, while fixing the other coefficients of the generalized bilinear transformation to be $a_1 = 0.5$, $a_2 = 0.5$, $b_1 = 1$, $b_2 = 1$, $k_1 = 1$.

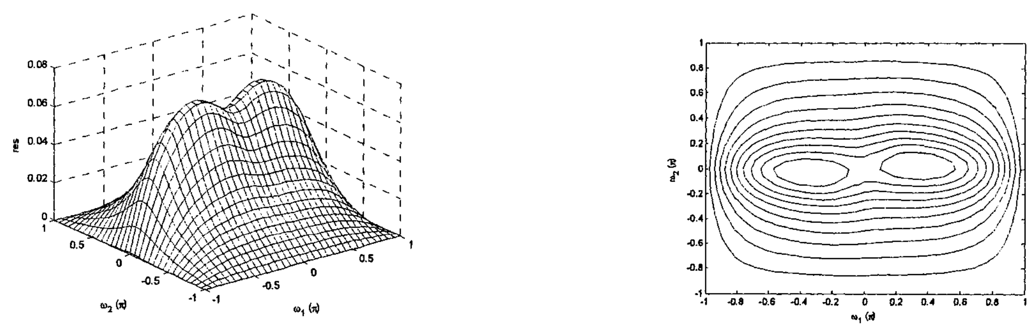
For ensuring the stability here we maintain conditions stated in (1.2.4.1).



(a) $k_2 = 10$



(b) $k_2 = 50$



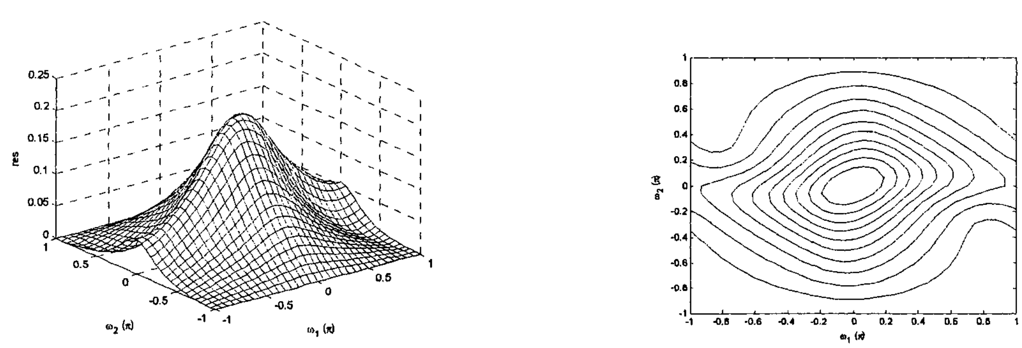
(c) $k_2 = 100$

Figure 3.17 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable k_2 and other coefficient constant

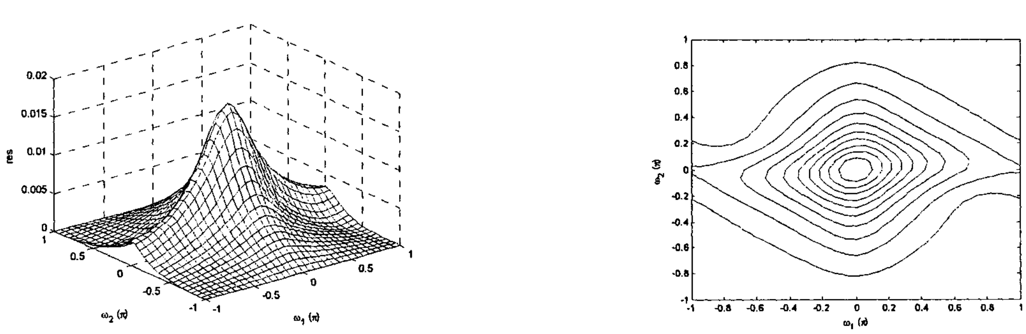
From Fig 3.17, it is seen that when we increase the value of k_2 variable it tends to low pass filter from band pass filter. The gain of this filter is significantly changed while we increase the value of k_2. The gain decreases from 0.3 to 0.05 as we increase the value of k_2 from 10 to 100. It is noticeable that for $R_1 = 0$, there are distortions when we change the bilinear transformation coefficients a_1, a_2, b_1, b_2 . But there is no distortion for the curve of k_1 and k_2.

3.3.9 Frequency response for the 2-D Digital Filter with equal k_1 and k_2

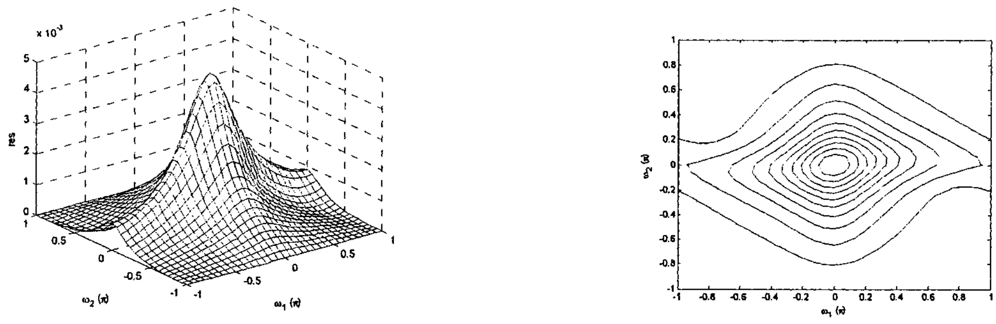
To study the manner how equal k_1 and k_2 affect the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of $k_1 = k_2$, while fixing the other coefficients of the generalized bilinear transformation to be $b_1 = 1$, $b_2 = 1$, $a_1 = 0.5$, $a_2 = 0.5$.



(a) $k_1 = k_2 = 10$



(b) $k_1 = k_2 = 50$



(c) $k_1 = k_2 = 100$

Figure 3.18 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable equal k_1 and k_2 and other coefficients constant.

From Fig 3.18, it is seen that the effect on the gain of pass band portions becomes more pronounced when we change the two coefficients, k_1 and k_2, simultaneously than the effect from the individual k_1 or k_2 only. When the values of k_1 and k_2 change from 10 to 100, the gain of pass band decreases from 0.15 to 0.003. The pass band area becomes enlarged in both ω_1 and ω_2 domains, but the effect is very slight. Higher the equal value of k_1 and k_2 makes the filter low pass filter in both domain.

Now we will design 2- D digital filter of the case 13 from TABLE 2.1.

3.4 Design procedure of Digital Filter for the case 13 from TABLE 2.1

Using the transfer function of the Bridged-T network of equation (2.16), if we compare with 1-D fourth order Butterworth Polynomial we get the impedance values of the Bridged-T network. After obtaining the values of impedance variables given in TABLE 2.4, we will make the transfer function in 2-D by putting the values of Z_1, Z_2, Z_3, Z_4 in (2.1) and then we get,

$$\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)} = (R_2 + C_1R_2L_3s_1^2 + C_1R_2L_4s_1s_2 + C_2R_2L_3s_1s_2) / (C_1R_1L_3s_1^2 + C_1R_2L_3s_1^2 + C_2R_2L_4s_2^2 + C_1C_2R_1R_2L_3s_1^2s_2 + C_1L_3L_4s_1^2s_2 + C_2R_1R_2s_2 + L_4s_2 + R_1 + R_2 + C_1C_2R_1L_3L_4s_1^2s_2^2 + C_1C_2R_1R_2L_4s_1s_2^2 + C_2L_3L_4s_1 s_2^2 + C_1R_1L_4s_1s_2 + C_2R_1L_3s_1s_2 + C_1R_2L_4s_1s_2 + C_2R_2L_3s_1s_2) \quad (3.2)$$

Putting the value of R_1, R_2, L_3, L_4, C_1, C_2 in the above equation and apply generalized bilinear transformation which is given below:

$s_i = k_i (z_i - a_i) / (z_i + b_i)$ where $|a_i| \leq 1$ and $|b_i| \leq 1$

For stability we have to ensure, $k_1>0$, $k_2>0$, $|a_1| \leq 1$, $|b_1| \leq 1$, $a_1b_1<0$

and $|a_2| \leq 1$, $|b_2| \leq 1$, $a_2b_2<0$.

3.5 Frequency response of the 2-D recursive Digital Filters (When $R_1 = 0.6$, $R_2 = 0.4$)

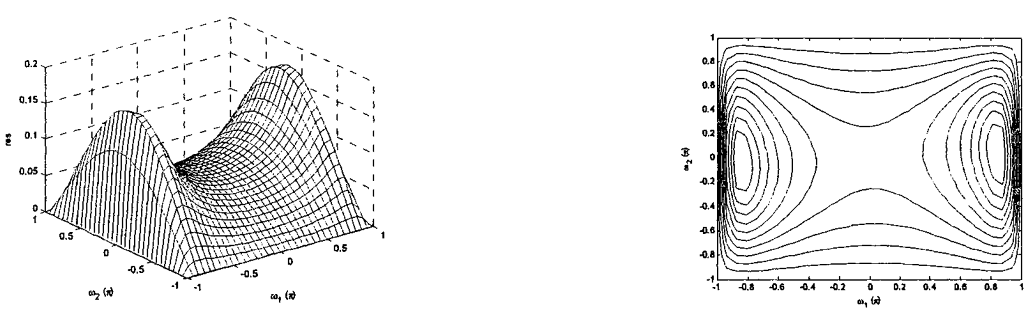
In this thesis we use MATLAB (The programs are given in Appendix) to obtain the contour and 3-D magnitude response plots of the resulting 2-D digital filters. With the input coefficients of the generalized bilinear transformations, we can obtain the contour and 3-D magnitude plots of the resulting 2-D digital filters. To investigate the manner in which each coefficient of generalized bilinear transformation affects the magnitude response of the resulting 2-D digital filters, we change the value of the deserving coefficients while keeping the other coefficients make constant. That can separate the effects from the other coefficients. Now we will observe the effect caused by each coefficient to the frequency responses of the resulting 2-D digital filter.

From TABLE 2.4 we analyze the data when $R_1 = 0.6$ and $R_2 = 0.4$

3.5.1 Frequency response for the 2-D Digital Filter with variable a_1

To study the manner how a_1 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of a_1, while fixing the other coefficients of the generalized bilinear transformation to be $a_2 = 0.5$, $b_1 = 1$, $b_2 = 1$, $k_1 = 1$, $k_2 = 1$.

For ensuring the stability here we maintain conditions stated in (1.2.4.1).



(a) $a_1 = 0$

(b) $a_1 = 0.5$

(c) $a_1 = 0.8$

(d) $a_1 = 1$

Figure 3.19 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable a_1 and other coefficient constant

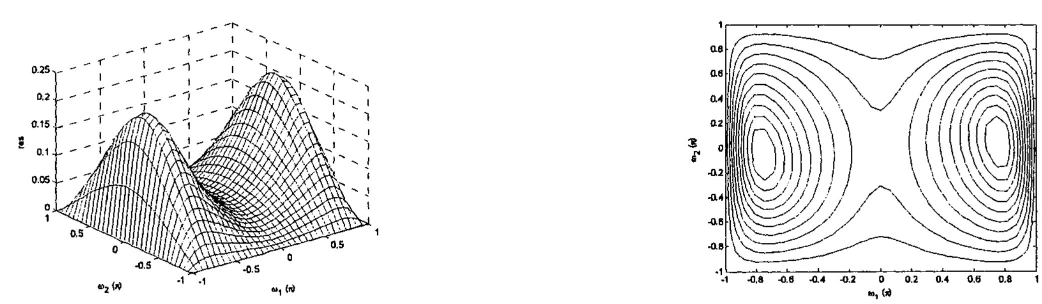
From Fig 3.19, it is seen that the coefficient a_1 mainly affects the ω_1 domain of the filter. Here we vary the value of a_1 within 0 to 1. As a_1 increases the value, the output converges to band pass filter with respect to ω_1 domain and low pass filter in ω_2 domain. It is also seen from the figure that the pass band of the filter with respect to ω_1 becomes smaller from the lower boundary of 0 to the upper boundary of 1.

3.5.2 Frequency response for the 2-D Digital Filter with variable a_2

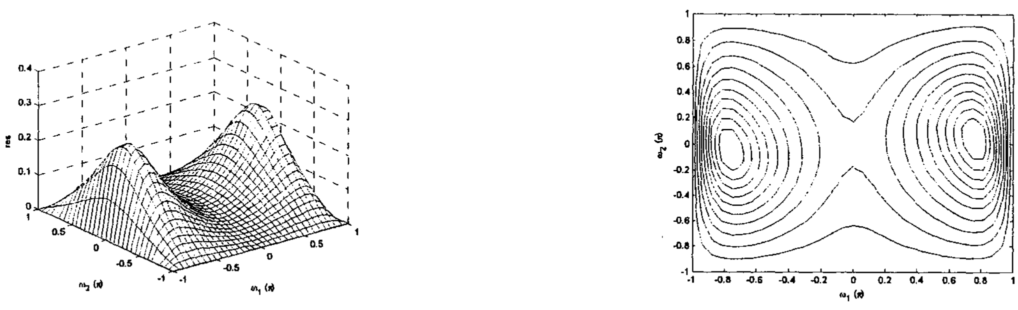
To study the manner how a_2 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of a_2, while fixing the other coefficients of the generalized bilinear transformation to be $a_1 = 0.5$, $b_1 = 1$, $b_2 = 1$, $k_1 = 1$, $k_2 = 1$.

For ensuring the stability here we maintain conditions stated in (1.2.4.1).

(a) $a_2 = 0$



(b) $a_2 = 0.6$



(c) $a_2 = 0.8$

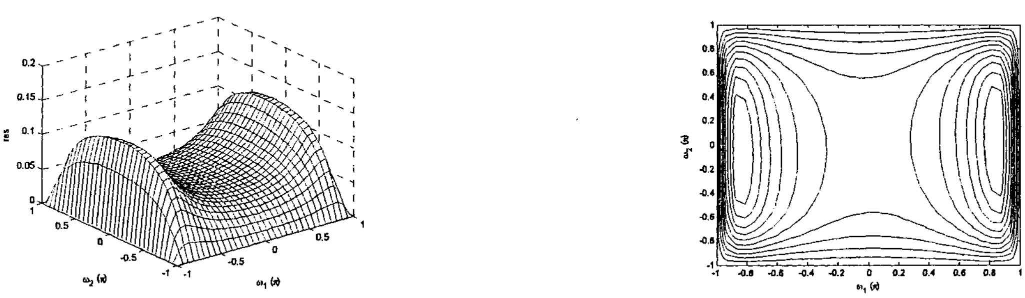
(d) $a_2 = 1$

Figure 3.20 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable a_2 and other coefficient constant

From Fig 3.20, it is seen that the coefficient a_2 mainly affects the ω_2 domain of the filter. From the figure it is observed that a_2 varies the gain of the pass band of the resulting 2-D filter. Specifying the other coefficients to be with proper signs, and when a_2 changes from 0 to 1, the gain in the pass band increases from 0.10 to 0.4. It is noticeable that the changing of gain of the resulting filter at origin is opposite in the case of a_2 to the case of a_1. For the case of a_1 the gain at origin decreases from the value 0 to 1, while for the case of a_2 the gain at origin increases from the value of 0 to 1.

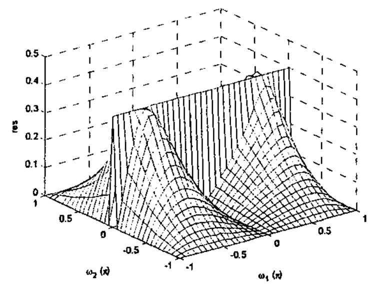
3.5.3 Frequency response for the 2-D Digital Filter with equal a_1 and a_2

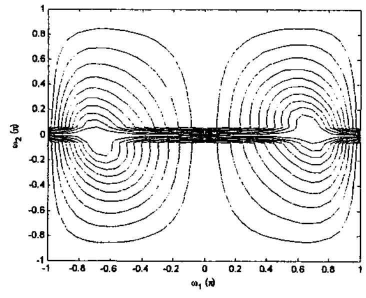
To study the manner how equal a_1 and a_2 affect the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of $a_1 = a_2$, while fixing the other coefficients of the generalized bilinear transformation to be $b_1 = 1$, $b_2 = 1$, $k_1 = 1$, $k_2 = 1$.



(a) $a_1 = a_2 = 0$

(b) $a_1 = a_2 = 0.6$





(c) $a_1 = a_2 = 1$

Figure 3.21 The contour and 3-D magnitude plot of the resulting 2-D digital filter with variable equal a_1 and a_2 and other coefficients constant.

From Fig 3.21, it is seen that the effect on the gain of pass band portions becomes more pronounced when we change the two coefficients, a_1 and a_2, simultaneously then the effect from the individual a_1 or a_2 only. When the values of a_1 and a_2 change from their lower boundary to their upper boundary, the gain of the pass band increases from 0.08 to 0.3.

3.5.4 Frequency response for the 2-D Digital Filter with variable b_1

To study the manner how b_1 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of b_1, while fixing the other coefficients of the generalized bilinear transformation to be $a_1 = 0.5$, $a_2 = 0.5$, $b_2 = 1$, $k_1 = 1$, $k_2 = 1$.

For ensuring the stability here we maintain conditions stated in (1.2.4.1).

(a) $b_1 = 0$

(b) $b_1 = 0.5$

(c) $b_1 = 0.8$

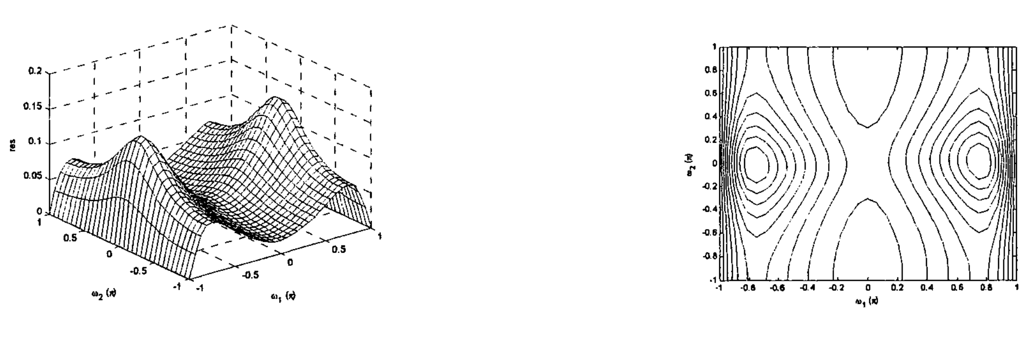
Figure 3.22 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable b_1 and other coefficient constant.

It is seen from the Fig 3.22 that the coefficient b_1 affects the filter characteristics. As we change the value from 0 to 1, we see that at $b_1 = 0$ it is almost all pass filter. If we increase the value of b_1 the output characteristics is likely to band pass filter with respect to ω_1 domain and low pass filter in ω_2 domain . Here the effect on gain is not significant amount compare to the coefficients a_1 and a_2.

3.5.5 Frequency response for the 2-D Digital Filter with variable b_2

To study the manner how b_2 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of b_2, while fixing the other coefficients of the generalized bilinear transformation to be $a_1 = 0.5$, $a_2 = 0.5$, $b_1 = 1$, $k_1 = 1$, $k_2 = 1$.

For ensuring the stability here we maintain conditions stated in (1.2.4.1).



(a) $b_2 = 0$

(b) $b_2 = 0.5$

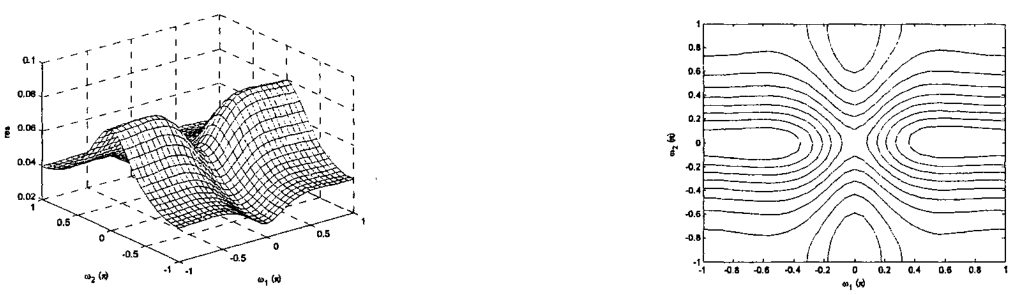
(c) $b_2 = 0.9$

Figure 3.23 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable b_2 and other coefficient constant

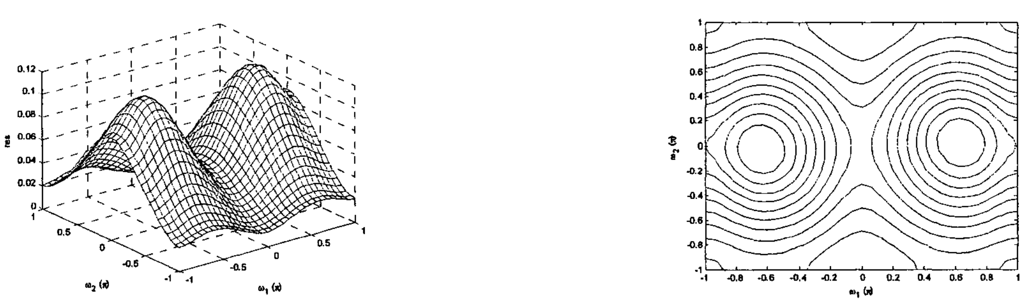
It is seen from the Fig 3.23 that b_2 also affects the filter characteristics. The shape of the magnitude response is band pass filter with respect to ω_1 domain and low pass filter with respect to ω_2 domain. The b_2 coefficient has little effect on gain. As we increase the value of b_2 the gain increases from 0.1 to 0.16.

3.5.6 Frequency response for the 2-D Digital Filter with equal b_1 and b_2

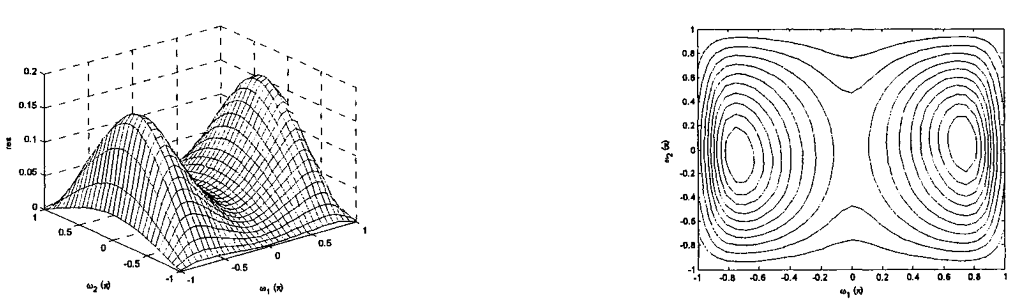
To study the manner how equal b_1 and b_2 affect the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of $b_1 = b_2$, while fixing the other coefficients of the generalized bilinear transformation to be $a_1 = 0.5$, $a_2 = 0.5$, $k_1 = 1$, $k_2 = 1$.



(a) $b_1 = b_2 = 0$



(b) $b_1 = b_2 = 0.5$



(c) $b_1 = b_2 = 0.9$

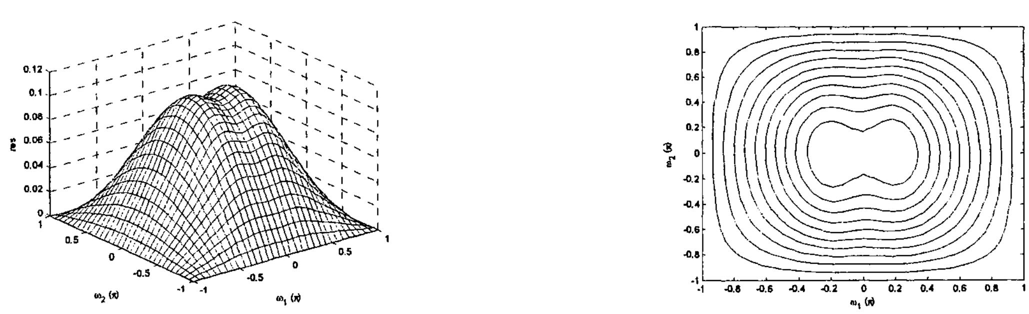
Figure 3.24 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable equal b_1 and b_2 and other coefficients constant.

From Fig 3.24, it is seen that the effect on the filter characteristics is different when we change the two coefficients, b_1 and b_2, simultaneously than the effect from the individual b_1 or b_2 only. When we change the two coefficients the gain remains almost constant. The magnitude response is still band pass filter with respect to ω_1 domain and low pass filter with respect to ω_2 domain.

3.5.7 Frequency response for the 2-D Digital Filter with variable k_1

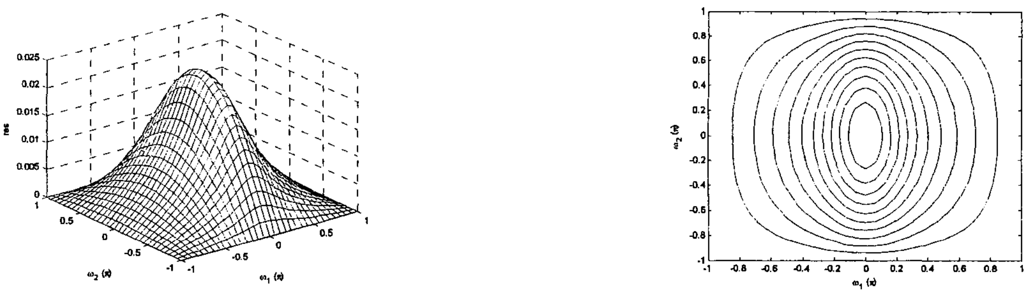
To study the manner how k_1 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of k_1, while fixing the other coefficients of the generalized bilinear transformation to be $a_1 = 0.5$, $a_2 = 0.5$, $b_1 = 1$, $b_2 = 1$, $k_2 = 1$.

For ensuring the stability here we maintain conditions stated in (1.2.4.1).



(a) $k_1 = 10$

(b) $k_1 = 50$



(c) $k_1 = 100$

Figure 3.25 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable k_1 and other coefficient constant

It is seen from the Fig 3.25 that the higher value of k_1 makes low pass filter in both ω_1 and ω_2 domain. The gain is not affected by the change of the k_1 variable.

3.5.8 Frequency response for the 2-D Digital Filter with variable k_2

To study the manner how k_2 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of k_2, while fixing the other coefficients of the generalized bilinear transformation to be $a_1 = 0.5$, $a_2 = 0.5$, $b_1 = 1$, $b_2 = 1$, $k_1 = 1$.

For ensuring the stability here we maintain conditions stated in (1.2.4.1).

(a) $k_2 = 10$

(b) $k_2 = 50$

(c) $k_2 = 100$

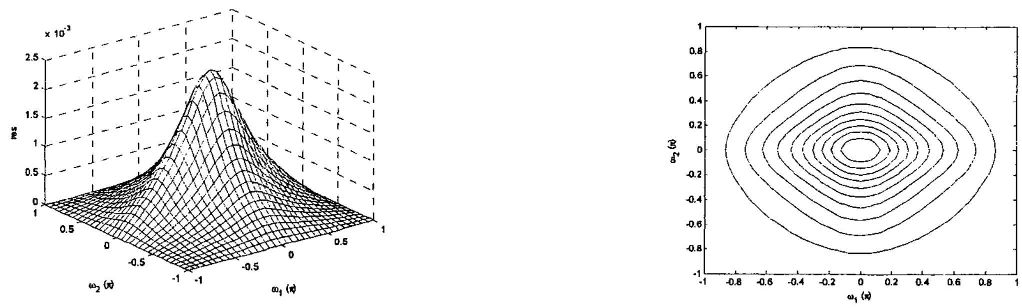
Figure 3.26 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable k_2 and other coefficient constant.

From Fig 3.26, it is seen that when we increase the value of k_2 variable it becomes band pass filter in ω_1 domain. The gain of this filter is significantly changed while we increase the value of k_2. The gain decrease from 0.04 to 0.004 as we increase the value of k_2 from 10 to 100.

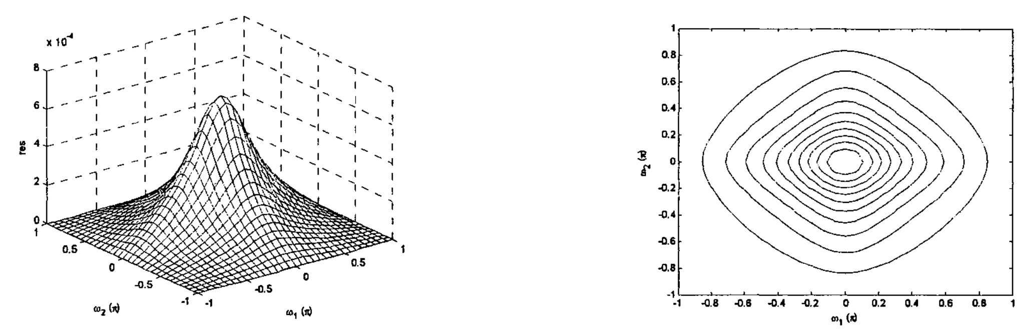
3.5.9 Frequency response for the 2-D Digital Filter with equal k_1 and k_2

To study the manner how equal k_1 and k_2 affect the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of $k_1 = k_2$, while fixing the other coefficients of the generalized bilinear transformation to be $b_1 = 1$, $b_2 = 1$, $a_1 = 0.5$, $a_2 = 0.5$.

(a) $k_1 = k_2 = 10$



(b) $k_1 = k_2 = 50$



(c) $k_1 = k_2 = 100$

Figure 3.27 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable equal k_1 and k_2 and other coefficients constant.

It is seen from the Fig 3.27 that the effect on the gain of pass band portions becomes more pronounced when we change the two coefficients, k_1 and k_2, simultaneously than the effect from the individual k_1 or k_2 only. When the values of k_1 and k_2 change from 10 to 100, the gain of pass band decreases from 0.06 to 0.006. The pass band area becomes enlarged in both ω_1 and ω_2 domains, but the effect is very slight.

Now we will design 2- D digital filter of the case 17 from TABLE 2.1.

3.6 Design procedure of Digital Filter for the case 17 from TABLE 2.1

Using the transfer function of the Bridged-T network of equation (2.20), if we compare with 1-D fourth order Butterworth Polynomial we get the impedance values of the Bridged-T network. After obtaining the values of impedance variables given in TABLE 2.5, we will make the transfer function in 2-D by putting the values of Z_1, Z_2, Z_3, Z_4 in (2.1) and then we get,

$$\frac{V_o(s_1,s_2)}{V_{in}(s_1,s_2)} = (R_2 + C_3R_2L_4s_1^2 + C_3R_2L_1s_1^2 + C_2R_2L_1s_1s_2) / (C_3R_1L_4s_1^2 + C_3R_1L_1s_1^2 + C_3R_2L_4s_1^2 + C_3R_2L_1s_1^2 + C_2C_3R_1R_2L_1s_1^2s_2 + C_2C_3R_1R_2L_4s_1^2s_2 + C_2L_1L_4s_1^2s_2 + R_1 + R_2 + C_2R_1R_2s_2 + L_4s_1 + C_2C_3R_2L_1L_4s_1^3s_2 + C_3L_1L_4s_1^3 + C_2R_1L_1s_1s_2 + C_2R_1L_4s_1s_2 + C_2R_2L_1s_1s_2) \qquad (3.3)$$

Putting the value of R_1, R_2, L_1, L_4, C_2, C_3 in the above equation and apply generalized bilinear transformation which is given below:

$s_i = k_i (z_i - a_i) / (z_i + b_i)$ where $|a_i| \le 1$ and $|b_i| \le 1$

For stability we have to ensure, $k_1>0$, $k_2>0$, $|a_1| \le 1$, $|b_1| \le 1$, $a_1b_1<0$

and $|a_2| \le 1$, $|b_2| \le 1$, $a_2b_2<0$.

3.7 Frequency response of the 2-D recursive Digital Filter (When $R_1 = 0.6$, $R_2 = 0.4$)

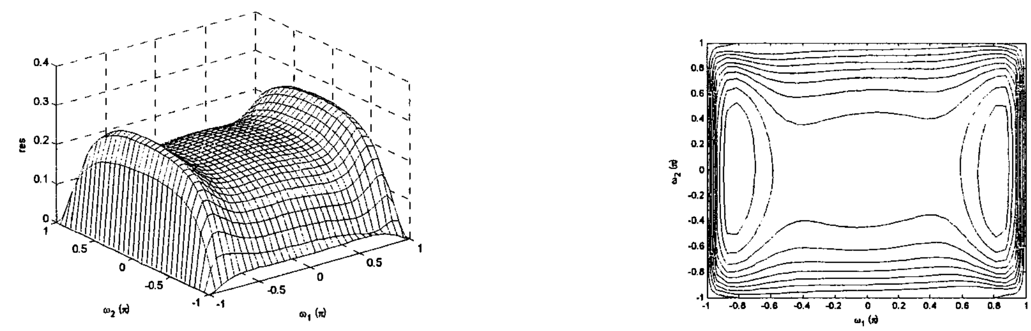
In this thesis we use MATLAB (The programs are given in Appendix) to obtain the contour and 3-D magnitude response plots of the resulting 2-D digital filters. With the input coefficients of the generalized bilinear transformations, we can obtain the contour and 3-D magnitude plots of the resulting 2-D digital filters. To investigate the manner in which each coefficient of generalized bilinear transformation affects the magnitude response of the resulting 2-D digital filters, we change the value of the deserving coefficients while keeping the other coefficients make constant. That can separate the effects from the other coefficients. Now we will observe the effect caused by each coefficient to the frequency responses of the resulting 2-D digital filter.

From TABLE 2.5 we analyze the data when $R_1 = 0.6$ and $R_2 = 0.4$

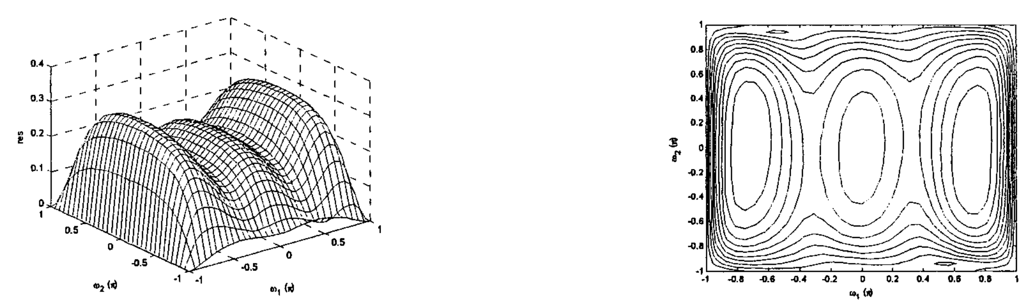
3.7.1 Frequency response for the 2-D Digital Filter with variable a_1

To study the manner how a_1 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of a_1, while fixing the other coefficients of the generalized bilinear transformation to be $a_2 = 0.5$, $b_1 = 1$, $b_2 = 1$, $k_1 = 1$, $k_2 = 1$.

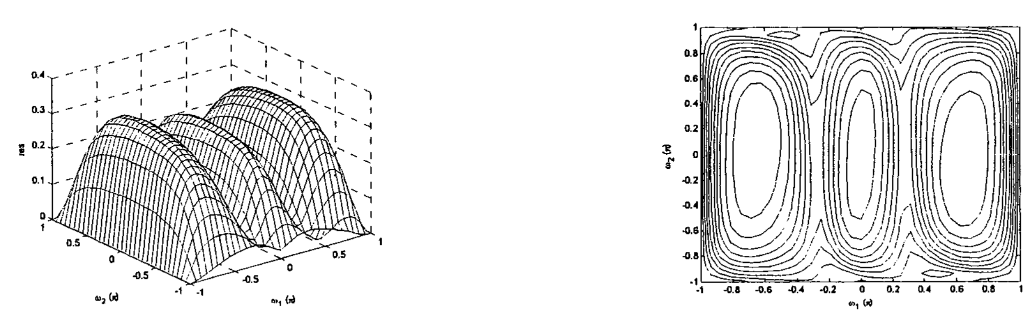
For ensuring the stability here we maintain conditions stated in (1.2.4.1).



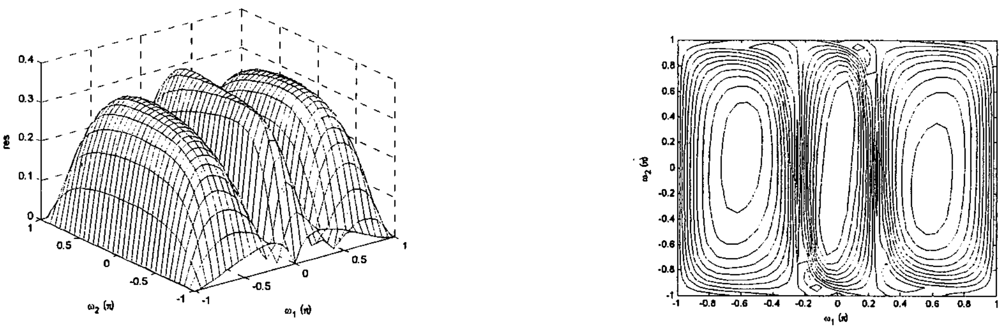
(a) $a_1 = 0$



(b) $a_1 = 0.5$



(c) $a_1 = 0.8$



(d) $a_1 = 1$

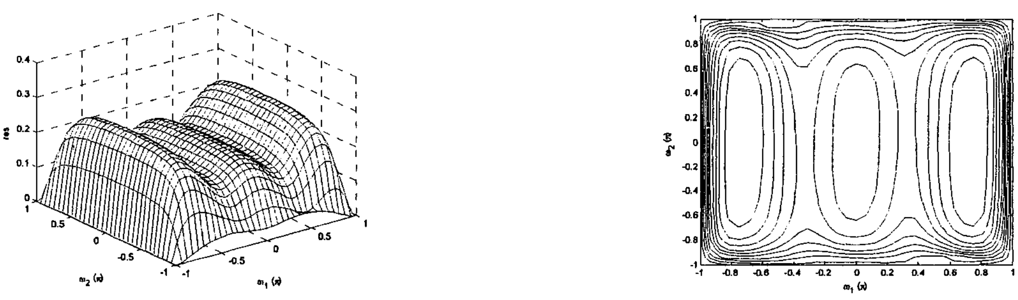
Figure 3.28 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable a_1 and other coefficients constant.

It can be seen from Fig 3.28 that the coefficient a_1 mainly affects the ω_1 domain of the filter. Here we vary the value of a_1 within 0 to 1. As a_1 increases the value, the output converges to stop band filter. It is also seen from the figure that the pass band of the filter with respect to ω_1 becomes smaller from the lower boundary of 0 to the upper boundary of 1.

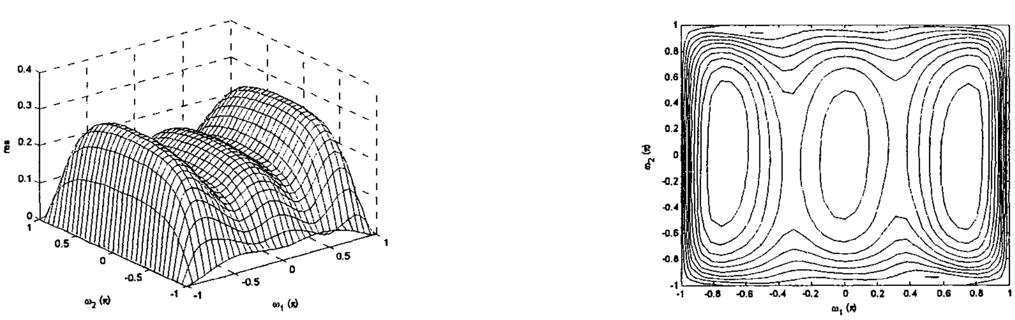
3.7.2 Frequency response for the 2-D Digital Filter with variable a_2

To study the manner how a_2 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of a_2, while fixing the other coefficients of the generalized bilinear transformation to be $a_1 = 0.5$, $b_1 = 1$, $b_2 = 1$, $k_1 = 1$, $k_2 = 1$.

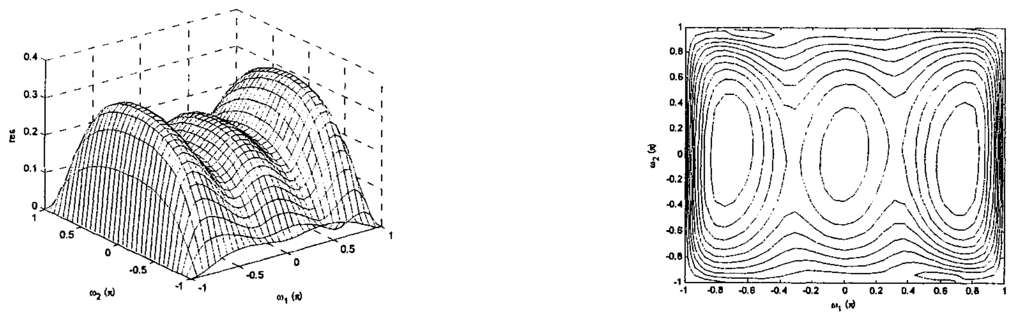
For ensuring the stability here we maintain conditions stated in (1.2.4.1).



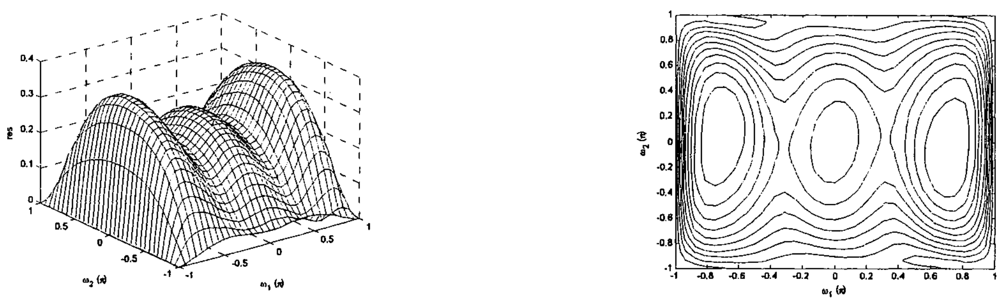
(a) $a_2 = 0$



(b) $a_2 = 0.4$



(c) $a_2 = 0.8$



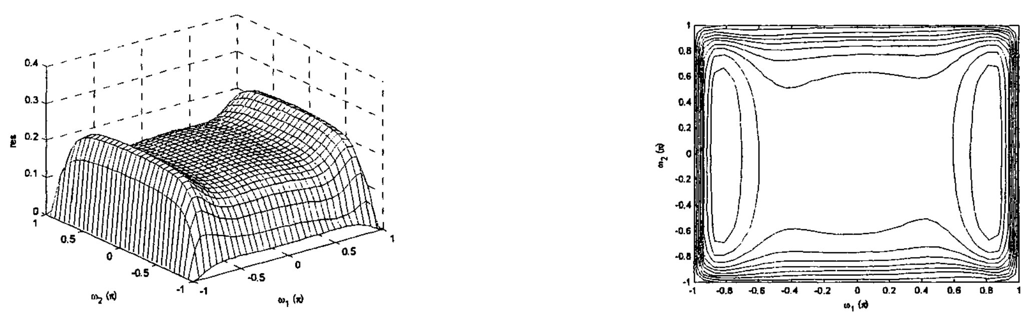
(d) $a_2 = 0.8$

Figure 3.29 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable a_2 and other coefficients constant.

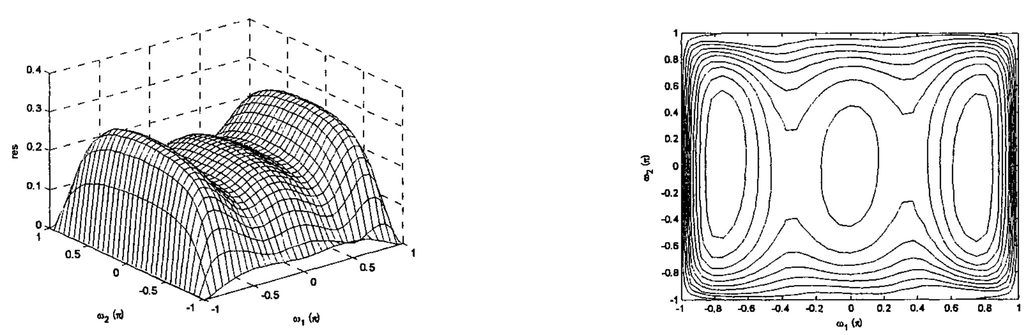
It is seen from Fig 3.29 that the coefficient a_2 mainly affects the ω_2 domain of the filter. From the figure it is observed that a_2 varies the gain of the pass band of the resulting 2-D band stop filter. Specifying the other coefficients to be with proper signs, and when a_2 changes from 0 to 1, the gain in the pass band increases from 0.25 to 0.4. It is noticeable that the changing of gain of the resulting filter at origin is opposite in the case of a_2 to the case of a_1. For the case of a_1 the gain at origin decreases from the value 0 to 1, while for the case of a_2 the gain at origin increases from the value of 0 to 1.

3.7.3 Frequency response for the 2-D Digital Filter with equal a_1 and a_2

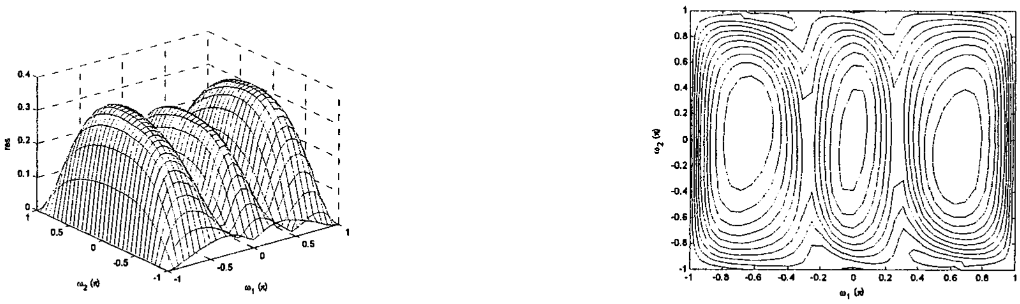
To study the manner how equal a_1 and a_2 affect the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of $a_1 = a_2$, while fixing the other coefficients of the generalized bilinear transformation to be $b_1 = 1$, $b_2 = 1$, $k_1 = 1$, $k_2 = 1$.



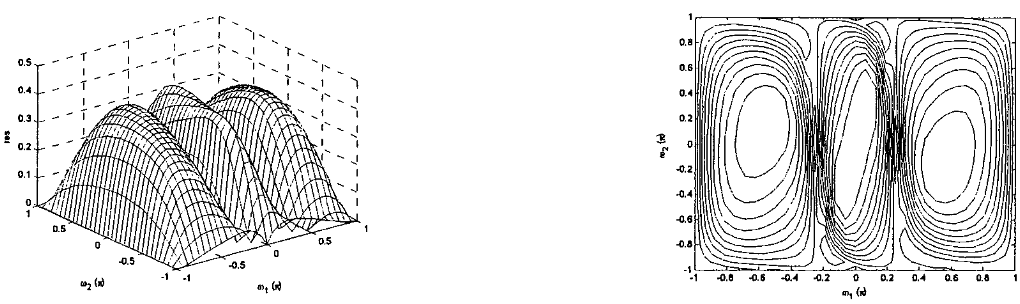
(a) $a_1 = a_2 = 0$



(b) $a_1 = a_2 = 0.4$



(c) $a_1 = a_2 = 0.8$



(d) $a_1 = a_2 = 1$

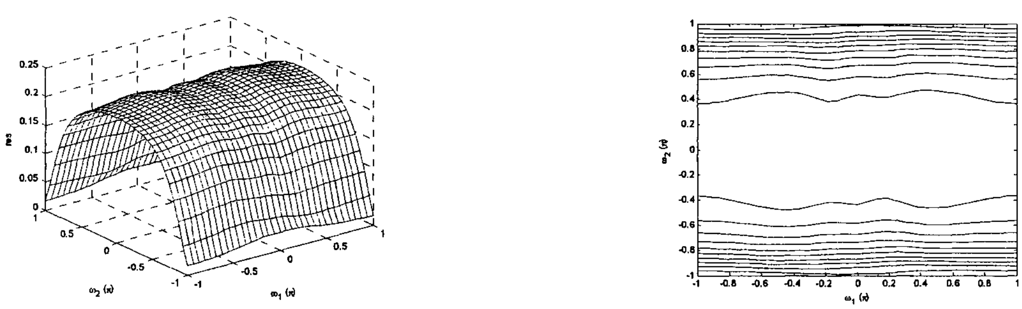
Figure 3.30 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable equal a_1 and a_2 and other coefficients constant.

From Fig 3.30, it is seen that the effect on the gain of pass band portions becomes more pronounced when we change the two coefficients, a_1 and a_2, simultaneously than the effect from the individual a_1 or a_2 only. When the values of a_1 and a_2 change from their lower boundary to their upper boundary, the gain of the pass band increases from 0.2 to 0.3.

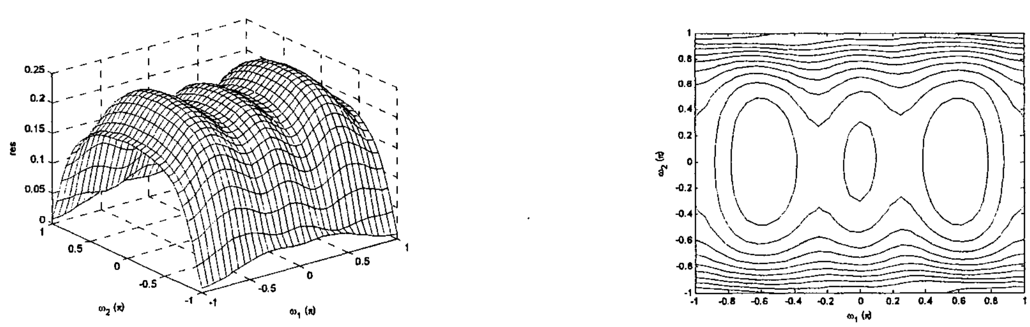
3.7.4 Frequency response for the 2-D Digital Filter with variable b_1

To study the manner how b_1 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of b_1, while fixing the other coefficients of the generalized bilinear transformation to be $a_1 = 0.5$, $a_2 = 0.5$, $b_2 = 1$, $k_1 = 1$, $k_2 = 1$.

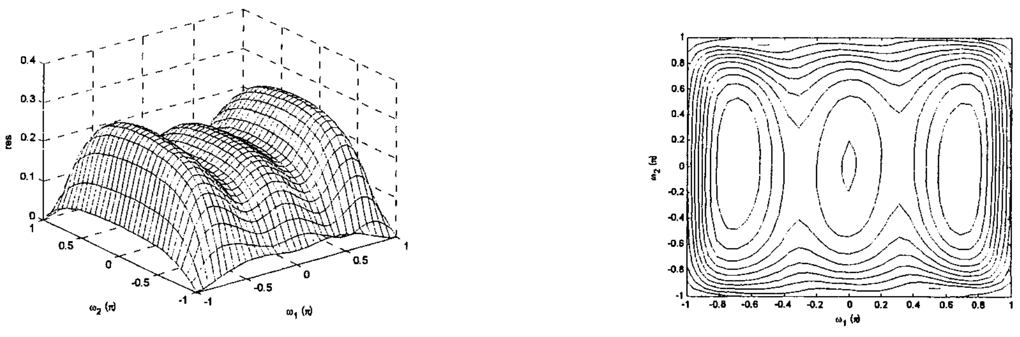
For ensuring the stability here we maintain conditions stated in (1.2.4.1).



(a) $b_1 = 0$



(b) $b_1 = 0.5$



(c) $b_1 = 0.9$

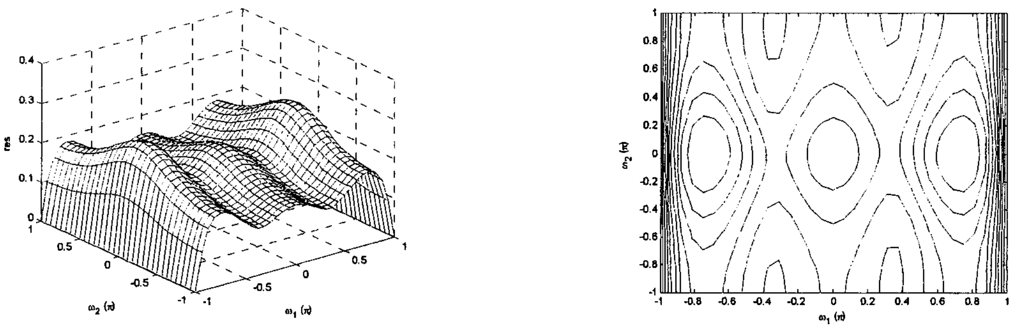
Figure 3.31 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable b_1 and other coefficients constant.

It is seen from the Fig 3.31 that the coefficient b_1 affects the filter characteristics. As we change the value from 0 to 1, we see that at $b_1 = 0$ it is almost all pass filter with respect to ω_1 domain and low pass filter with respect to ω_2 domain . If we increase the value of b_1 the output characteristics changes into stop band filter. Here the effect on gain is not significant amount compare to the coefficients a_1 and a_2.

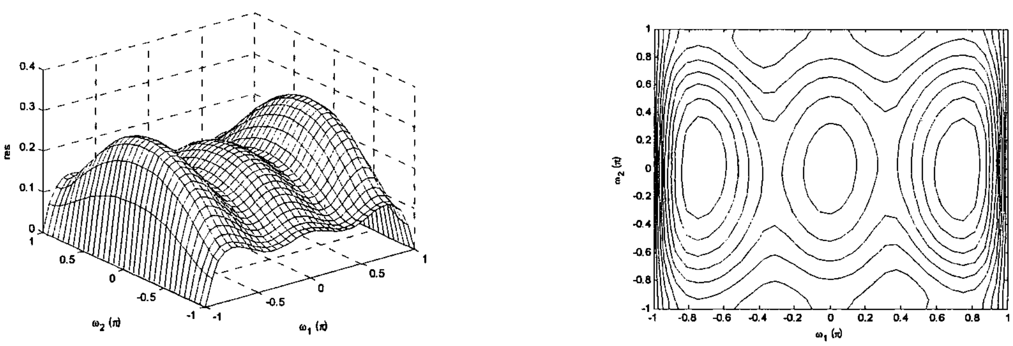
3.7.5 Frequency response for the 2-D Digital Filter with variable b_2

To study the manner how b_2 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of b_2, while fixing the other coefficients of the generalized bilinear transformation to be $a_1 = 0.5$, $a_2 = 0.5$, $b_1 = 1$, $k_1 = 1$, $k_2 = 1$.

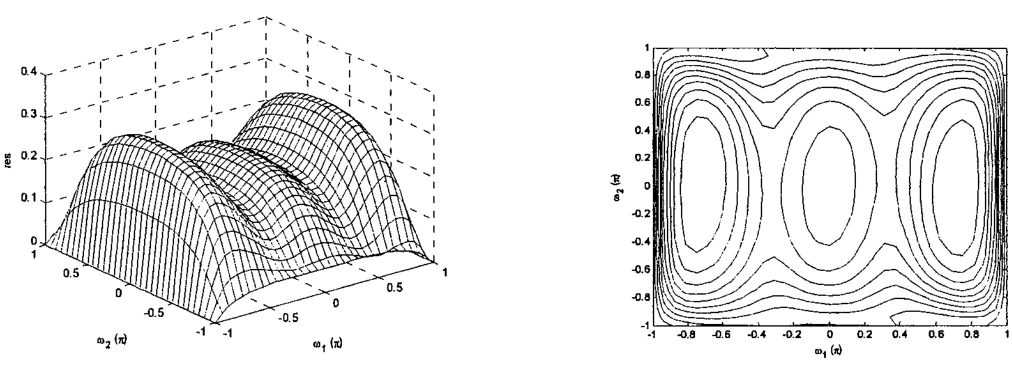
For ensuring the stability here we maintain conditions stated in (1.2.4.1).



(a) $b_2 = 0$



(b) $b_2 = 0.5$



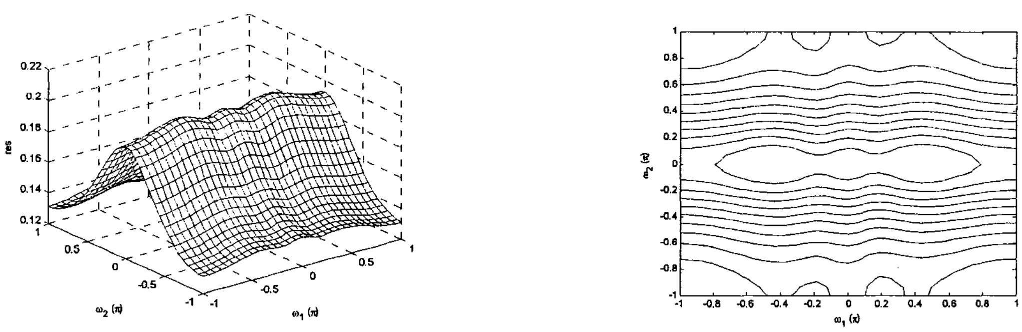
(c) $b_2 = 0.9$

Figure 3.32 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable b_2 and other coefficients constant.

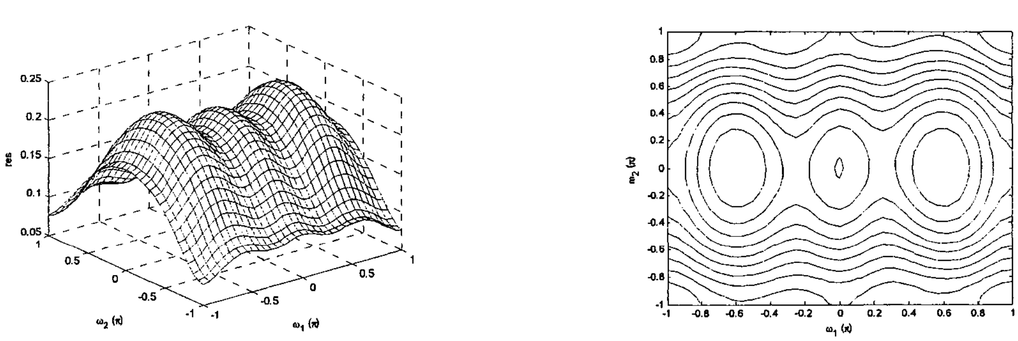
It is seen from the Fig 3.32 that b_2 also affects the filter characteristics. As we increase the value of b_2, it becomes stop band filter with respect to ω_1 domain. The b_2 coefficient has no significant effect on gain.

3.7.6 Frequency response for the 2-D Digital Filter with equal b_1 and b_2

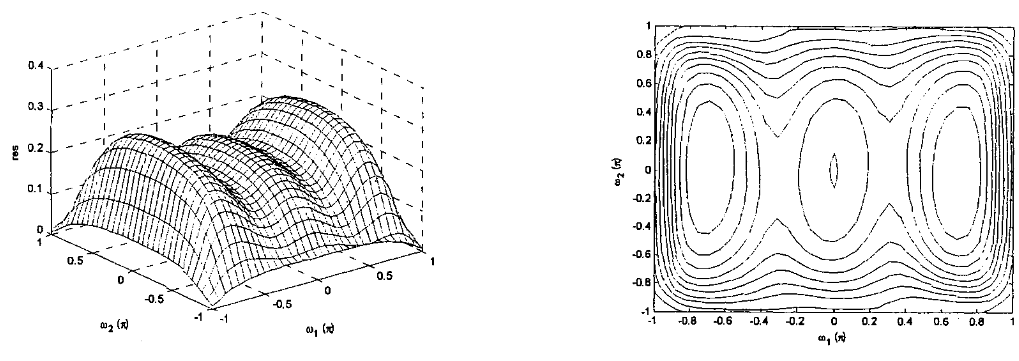
To study the manner how equal b_1 and b_2 affect the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of $b_1 = b_2$, while fixing the other coefficients of the generalized bilinear transformation to be $a_1 = 0.5$, $a_2 = 0.5$, $k_1 = 1$, $k_2 = 1$.



(a) $b_1 = b_2 = 0$



(b) $b_1 = b_2 = 0.5$



(c) $b_1 = b_2 = 0.9$

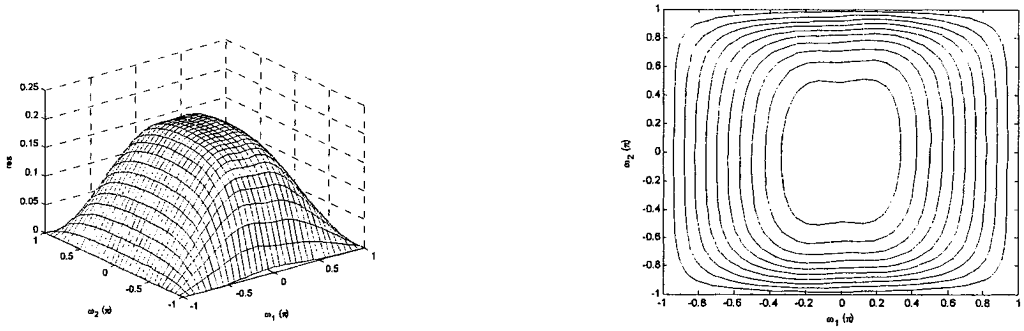
Figure 3.33 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable equal b_1 and b_2 and other coefficients constant.

From Fig 3.33, it is seen that the effect on the filter characteristics is different when we change the two coefficients, b_1 and b_2, simultaneously than the effect from the individual b_1 or b_2 only. When we change the two coefficients the gain remains almost constant.

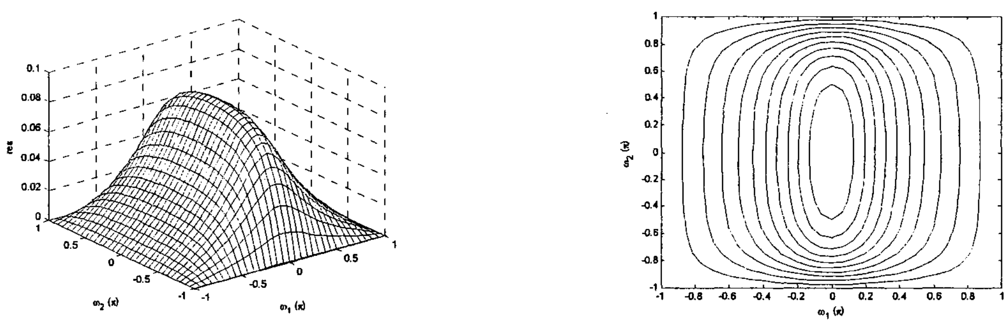
3.7.7 Frequency response for the 2-D Digital Filter with variable k_1

To study the manner how k_1 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of k_1, while fixing the other coefficients of the generalized bilinear transformation to be $a_1 = 0.5$, $a_2 = 0.5$, $b_1 = 1$, $b_2 = 1$, $k_2 = 1$.

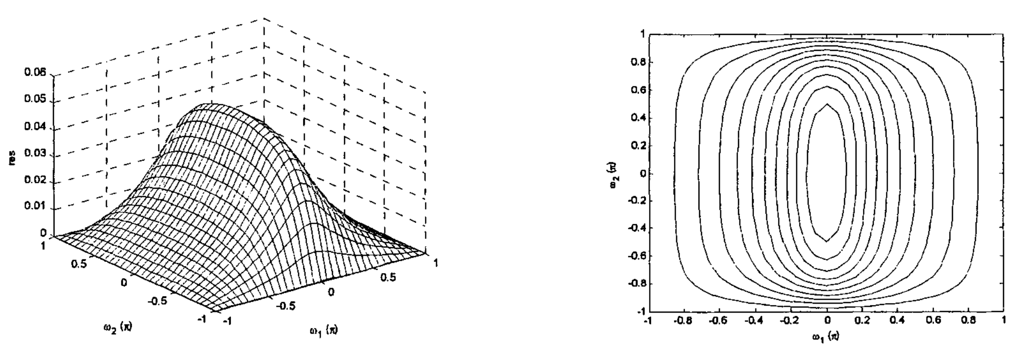
For ensuring the stability here we maintain conditions stated in (1.2.4.1).



(a) $k_1 = 10$



(b) $k_1 = 50$



(c) $k_1 = 100$

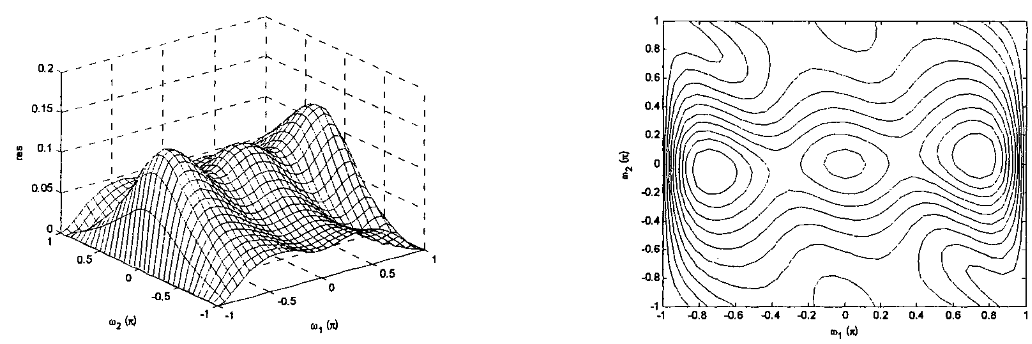
Figure 3.34 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable k_1 and other coefficients constant.

From Fig 3.34, it is seen that the higher value of k_1 makes low pass filter in both ω_1 and ω_2 domain. The gain is significantly decreased with the increasing the value of k_1.

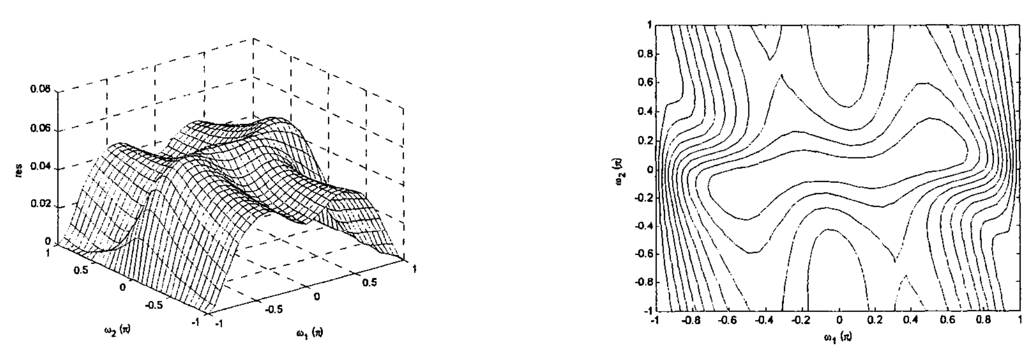
3.7.8 Frequency response for the 2-D Digital Filter with variable k_2

To study the manner how k_2 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of k_2, while fixing the other coefficients of the generalized bilinear transformation to be $a_1 = 0.5$, $a_2 = 0.5$, $b_1 = 1$, $b_2 = 1$, $k_1 = 1$.

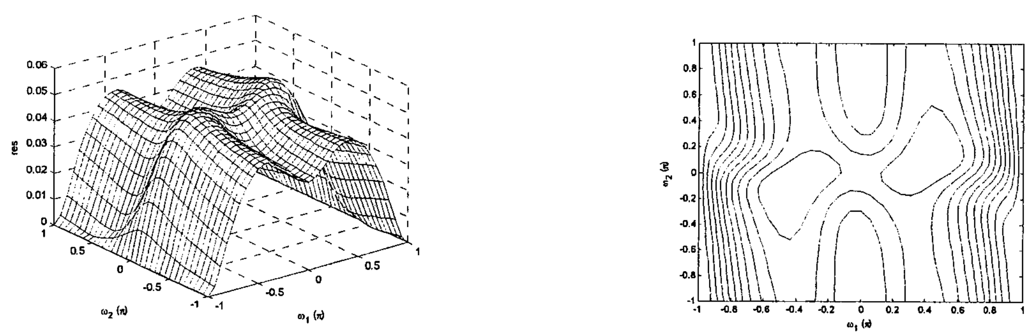
For ensuring the stability here we maintain conditions stated in (1.2.4.1).



(a) $k_2 = 10$



(b) $k_2 = 50$



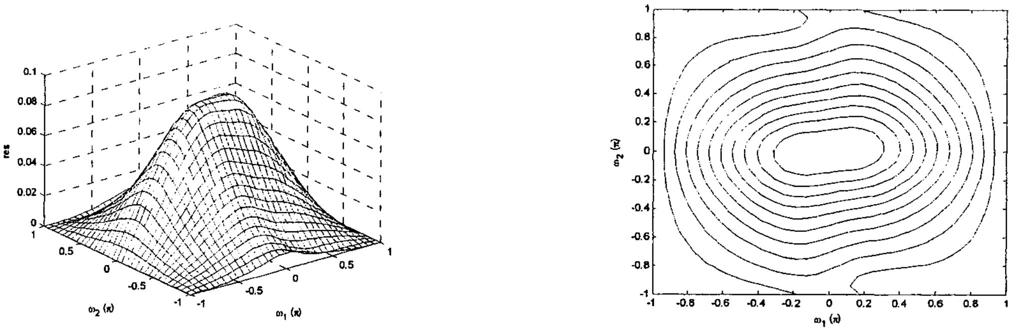
(c) $k_2 = 100$

Figure 3.35 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable k_2 and other coefficients constant

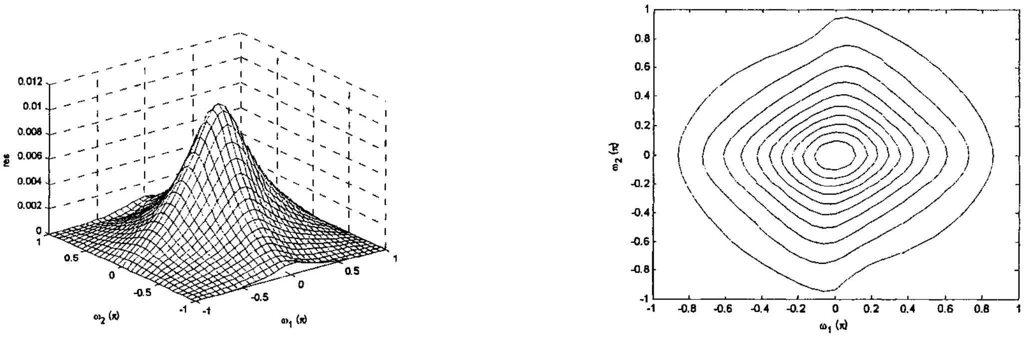
From Fig 3.35, it is seen that higher the value of k_2 makes the band stop filter distorted. The gain of the pass band region decreases significantly when we increase the value of k_2.

3.7.9 Frequency response for the 2-D Digital Filter with equal k_1 and k_2

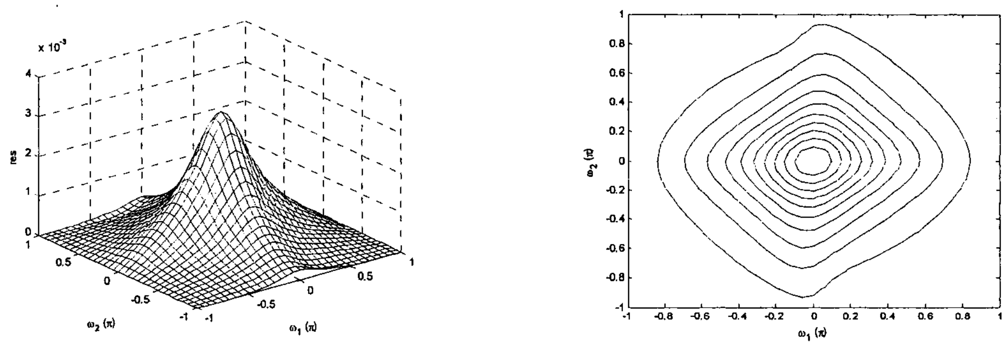
To study the manner how equal k_1 and k_2 affect the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of $k_1 = k_2$, while fixing the other coefficients of the generalized bilinear transformation to be $b_1 = 1$, $b_2 = 1$, $a_1 = 0.5$, $a_2 = 0.5$.



(a) $k_1 = k_2 = 10$



(b) $k_1 = k_2 = 50$



(c) $k_1 = k_2 = 100$

Figure 3.36 The contour and 3-D magnitude plots of the resulting 2-D digital filter with variable equal k_1 and k_2 and other coefficients constant.

From Fig 3.36, it is seen that the effect on the gain of pass band portions becomes more pronounced when we change the two coefficients, k_1 and k_2, simultaneously than the effect from the individual k_1 or k_2 only. When the values of k_1 and k_2 change from 10 to 100, the gain of pass band decreases from 0.06 to 0.002. The pass band area becomes enlarged in both ω_1 and ω_2 domains, but the effect is very slight. For higher value of k_1 and k_2, the filter output is low pass filter in both domain.

3.8 Summary and Discussion

In this chapter, we have introduced the procedure used to design 2-D recursive digital filters by generalized bilinear transformation method. The manner how each coefficient of the generalized bilinear transformation affects the magnitude response behavior of the resulting 2-D filter has been studied in detail also.

With the transfer function of the Bridged-T network we compare to the fourth order 1-D low pass Butterworth polynomial. Then we get 2-D analog transfer function. The 2-D discrete transfer function has been derived from the analog transfer function by double generalized bilinear transformations with stability constraints. When one or more coefficients of the double bilinear transformations are changing, the resulting 2-D filter has variable magnitude characteristics.

The coefficients of k_1, a_1 and b_1 affect the magnitude response in ω_1-domain, while the coefficients of k_2, a_2 and b_2 affect the behaviors of the magnitude response in ω_2-domain.

Here at first we design digital filter for the case of 7 from TABLE 2.1. The stability conditions of 2-D recursive digital filter with single degree for each variable are still effective. For ensure the stability we vary the bilinear transformation coefficient a_1, a_2, b_1, b_2 from 0 to 1. For stability we have to maintain $a_1b_1<0$, $a_2b_2<0$, $k_1>0$ and $k_2>0$. As we use negative sign before a_1 and a_2 in equation (1.7), so we take all positive values of a_i and b_i where i = 1, 2. While we change the coefficient a_i and b_i, we put the value $k_1 = k_2 = 1$. When we observe the effect of k_1 and k_2, we vary these value between 10 to 100 to see the affect of higher values of k. For the case 7, we have 5 cases (TABLE 2.3) for different values of $R_1$and R_2. We design digital filter for $R_1 = 0.6$, $R_2 = 0.4$ and $R_1 = 0$, $R_2 = 1$. For the second case, i.e., when the source resistance $R_1 = 0$ the magnitude response curve is significantly changed. So for the later cases(case 13 and 17) we do not design for $R_1 = 0$. We analyze and compare the filter for the case of $R_1 = 0.6$, $R_2 = 0.4$. It is noticeable that the effects of bilinear transformation coefficient a_1, a_2, b_1, b_2 on the transfer function for the case of 7 make the all pass filter or near to all pass filter having some exceptions (for example, figure 3.5 a) with respect to ω_1 domain. But the effects of k_1 and k_2 are different from other. For the case of k_2, the output is band pass filter. We also study the manner how equal coefficients i.e., $a_1 = a_2$, $b_1 = b_2$ and $k_1 = k_2$ affect the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value one equal coefficient while fixing the other coefficients of the generalized bilinear transformation. The effect on the filter characteristics is more pronounced when we change the two equal coefficients simultaneously than the effect from the individual coefficient only.

The details comparisons are given in the next chapter.

Then we design digital filter for the case of 13 from TABLE 2.1. Here we also maintain the stability conditions mentioning for the case 7. For this transfer function we have also got five cases (TABLE 2.4) for different values of $R_1$and R_2. We design digital filter for $R_1 = 0.6$, $R_2 = 0.4$ for making comparison with the case before which is described in the next chapter. It is noticeable that the effects of bilinear transformation coefficient a_1, a_2, b_1, b_2 on the transfer function for the case of 13 make the band pass filter or near to band pass filter having some exceptions (for example, figure 3.23 a) with respect to ω_1 domain. But the effects of k_1 and k_2 are different from each other. For the higher value of k_1, the filter output is a low-pass filter and for higher value of k_2 the filter output is that of band-pass. In this case we also study the manner how equal coefficients i.e., $a_1 = a_2$, $b_1 = b_2$ and $k_1 = k_2$ affect the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value one equal coefficient while fixing the other coefficients of the generalized bilinear transformation. The effect on the filter characteristics is more pronounced when we change the two equal coefficients simultaneously than the effect from the individual coefficient only.

And then we design digital filter for the case of 17 from TABLE 2.1. Here we also maintain the stability conditions mentioning for the case 7 and 13. For this transfer function we have also got five cases (TABLE 2.5) for different values of $R_1$and R_2. We design digital filter for $R_1 = 0.6$, $R_2 = 0.4$ for making comparison with the case before which is described in the next chapter. It is noticeable that the effects of bilinear transformation coefficient a_1, a_2, b_1, b_2 on the transfer function for the case of 17 make the band stop filter or near to band stop filter having some exceptions (for example, figure 3.31 a) wither respect to ω_1 domain. But the effects of k_1 and k_2 are different from other. We also study the manner how equal coefficients i.e., $a_1 = a_2$, $b_1 = b_2$ and $k_1 = k_2$ affect the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value one equal coefficient while fixing the other coefficients of the generalized bilinear transformation. The effect on the filter characteristics is more pronounced when we change the two equal coefficients simultaneously than the effect from the individual coefficient only.

This chapter could be useful in analyzing and designing 2-D recursive digital filters with variable magnitude response characteristics from Bridged-T network. The bilinear transformation coefficients contribute the significant role for the Bridged- T network filter output.

Chapter 4

Compare the resulting magnitude response for different filter characteristics

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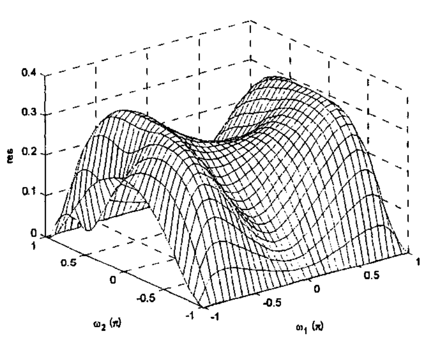
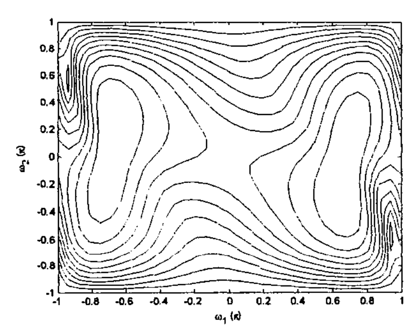
In this chapter we will compare the resulting magnitude response for digital filter design which we have obtained from the previous chapter. As we noticed earlier we test the transfer function of the Bridged-T network that VSHP or not. Among the 36 cases we get 16 cases that are VSHP. But finally we get only 3 cases which we can completely solve the value of impedance values. In the chapter 3, we discussed all of the 3 cases for designing digital filter. In this thesis for each case we give brief discussion for the value of $R_1 = 0.6$ and $R_2 = 0.4$. We also discuss the values of $R_1 = 0$ and $R_2 = 1$ for the case 7(TABLE 2.1) only. It is noticeable from the observation that when we put the source resistance $(R_1) = 0$, the magnitude response of the digital filter is distorted by changing the bilinear transformation coefficient a_1, a_2, b_1, b_2. But the magnitude response is not distorted by changing k_1, k_2. In the previous chapter, to investigate the manner in which each coefficient of generalized bilinear transformation affects the magnitude response of the resulting 2-D digital filters, we change the value of the deserving coefficients while fixing the other coefficients to the specified values. Now we will investigate how the changing of each coefficient affects the magnitude response for three different cases of impedance values. Here we will observe for the value of $R_1 = 0.6$ and $R_2 = 0.4$ of figure 2.1 for the case of 7, 13 and 17 from TABLE 2.1.

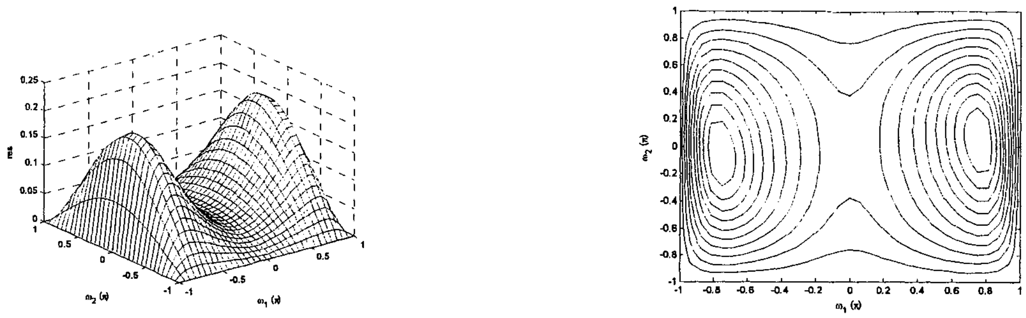
4.1 Frequency response of 2-D digital filter while changing a_1 coefficient

To study the manner how a_1 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of a_1, while fixing the other coefficients of the generalized bilinear transformation to be $a_2 = 0.5$, $b_1 = 1$, $b_2 = 1$, $k_1 = 1$, $k_2 = 1$. To compare the magnitude response for the three cases here we give the value of $a_1 = 0.5$.

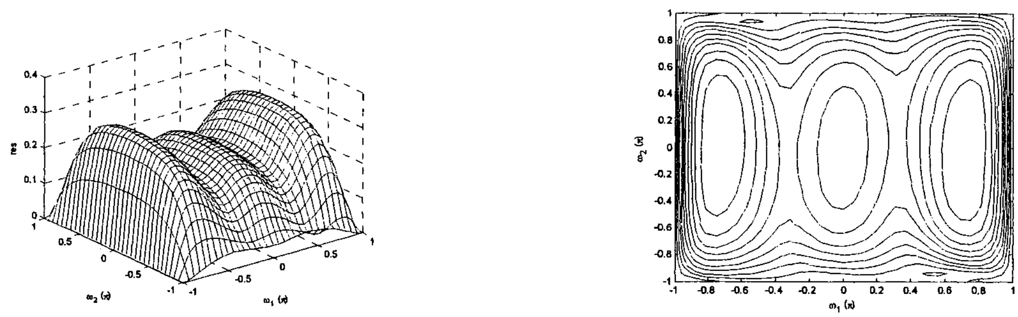




(a) For case 7



(b) For case 13



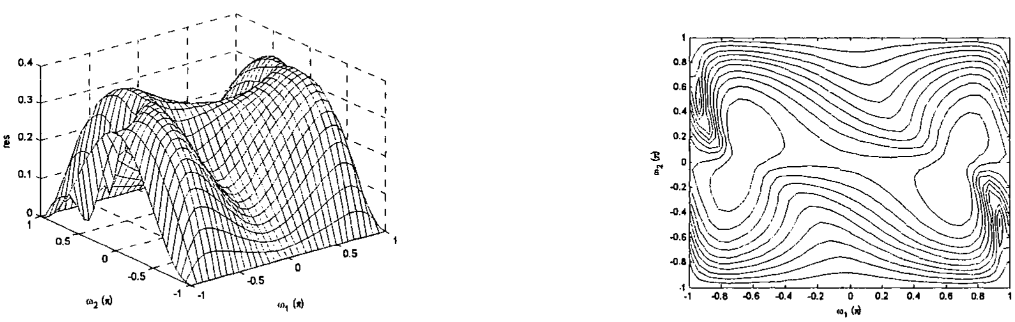
(c) For case 17

Figure 4.1 The contour and 3-D magnitude plots of the resulting 2-D digital filter with $a_1 = 0.5$, $a_2 = 0.5$, $b_1 = 1$, $b_2 = 1$, $k_1 = 1$, $k_2 = 1$ for $R_1 = 0.6$ and $R_2 = 0.4$.

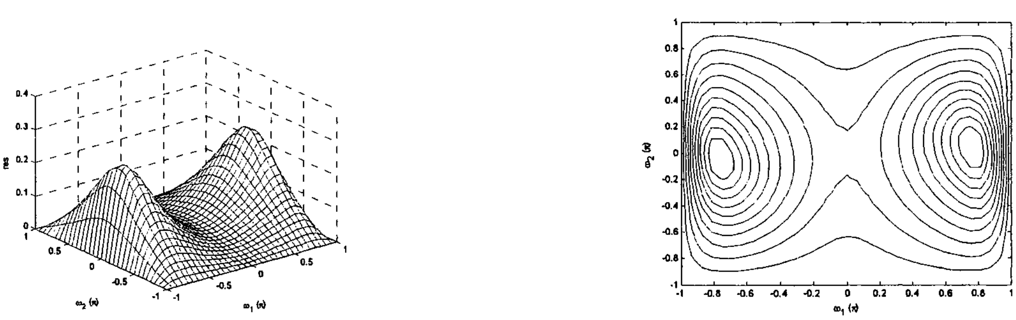
It is seen from the Fig 4.1 that the filter characteristics vary with their different impedance values. The magnitude response of the Bridged-T network with respect to ω_1 domain is an all pass filter for the case 7, a band pass filter for the case of 13 and a band stop filter for the case 17 from TABLE 2.1. These observations are taken while we change the bilinear transformation coefficient a_1. The gain is also varied for the three different impedance values of the transfer function.

4.2 Frequency response of 2-D digital filter while changing a_2 coefficient

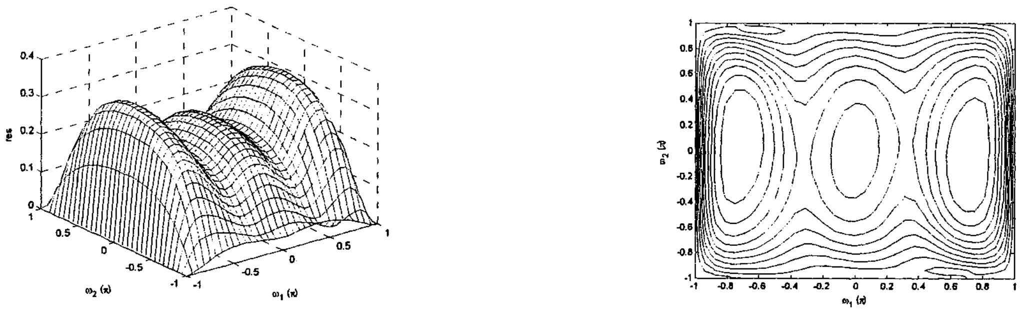
To study the manner how a_2 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of a_2, while fixing the other coefficients of the generalized bilinear transformation to be $a_1 = 0.5$, $b_1 = 1$, $b_2 = 1$, $k_1 = 1$, $k_2 = 1$. To compare the magnitude response for the three cases here we give the value of $a_2 = 0.8$.



(a) For case 7



(b) For case 13



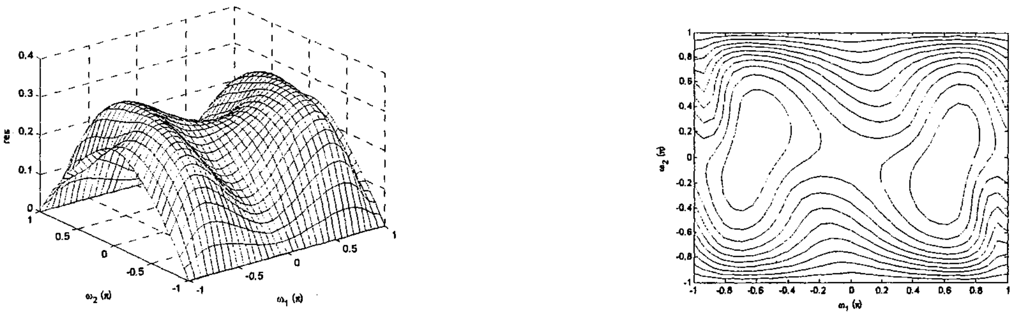
(c) For case 17

Figure 4.2 The contour and 3-D magnitude plots of the resulting 2-D digital filter with $a_1 = 0.5$, $a_2 = 0.8$, $b_1 = 1$, $b_2 = 1$, $k_1 = 1$, $k_2 = 1$ for $R_1 = 0.6$ and $R_2 = 0.4$.

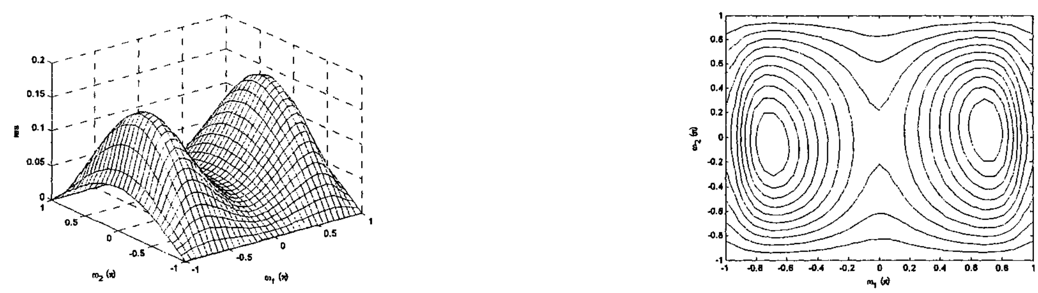
It is seen from the Fig 4.2 that the filter characteristics vary with their different impedance values. The magnitude response of the Bridged-T network is an all pass filter for the case 7, a band pass filter for the case of 13 and a band stop filter for the case 17 from TABLE 2.1. These observations are taken while we change the bilinear transformation coefficient a_2. The gain is also varied for the three different impedance values of the transfer function.

4.3 Frequency response of 2-D digital filter while changing b_1 coefficient

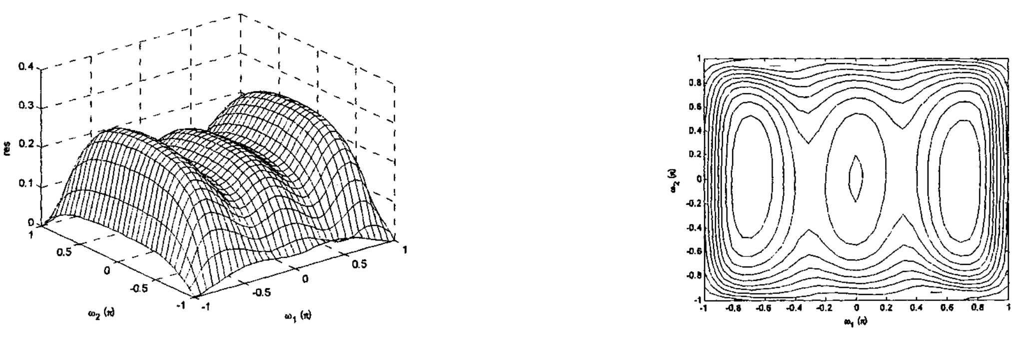
To study the manner how b_1 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of b_1, while fixing the other coefficients of the generalized bilinear transformation to be $a_1 = 0.5$, $a_2 = 0.5$, $b_2 = 1$, $k_1 = 1$, $k_2 = 1$. To compare the magnitude response for the three cases here we give the value of $b_1 = 0.8$



(a) For case 7



(b) For case 13



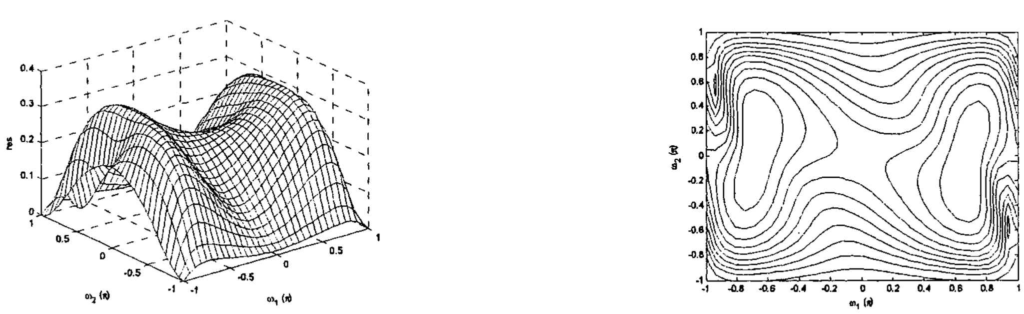
(c) For case 17

Figure 4.3 The contour and 3-D magnitude plots of the resulting 2-D digital filter with $a_1 = 0.5$, $a_2 = 0.5$, $b_1 = 0.8$, $b_2 = 1$, $k_1 = 1$, $k_2 = 1$ for $R_1 = 0.6$ and $R_2 = 0.4$.

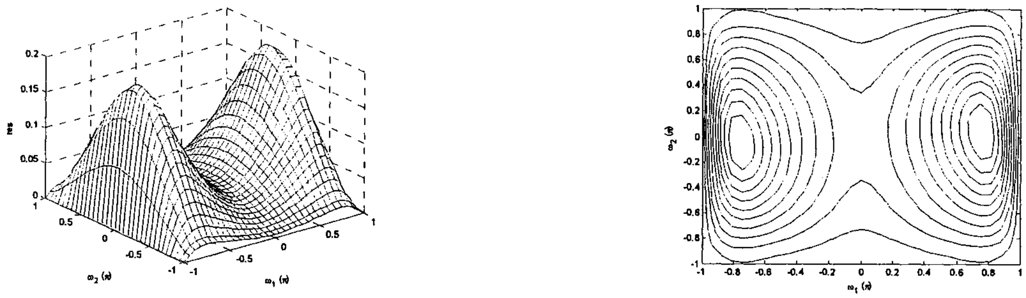
It is seen from the Fig 4.3 that the filter characteristics vary with their different impedance values. The magnitude response of the Bridged-T network is an all pass filter for the case 7, band pass filter for the case of 13 and band stop filter for the case 17 from TABLE 2.1. These observations are taken while we change the bilinear transformation coefficient b_1. The gain is also varied for the three different impedance values of the transfer function.

4.4 Frequency response of 2-D digital filter while changing b_2 coefficient

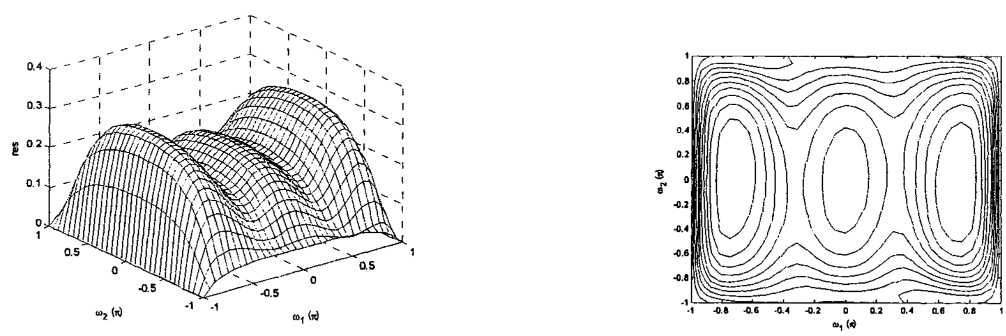
To study the manner how b_2 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of b_2, while fixing the other coefficients of the generalized bilinear transformation to be $a_1 = 0.5$, $a_2 = 0.5$, $b_1 = 1$, $k_1 = 1$, $k_2 = 1$. To compare the magnitude response for the three cases here we give the value of $b_2 = 0.9$.



(a) For case 7



(b) For case 13



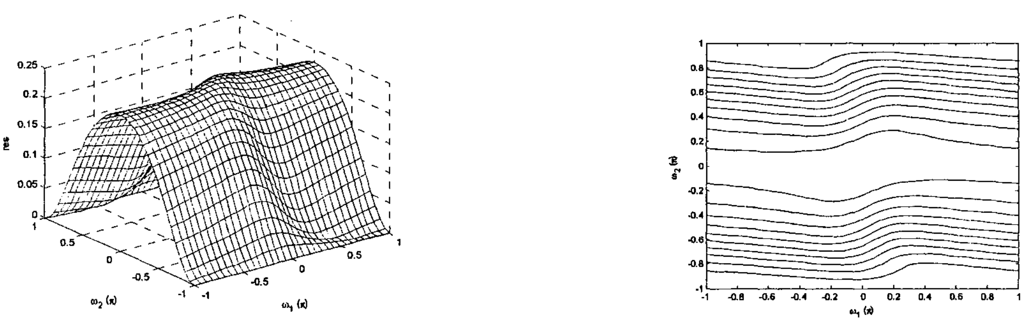
(c) For case 17

Figure 4.4 The contour and 3-D magnitude plots of the resulting 2-D digital filter with $a_1 = 0.5$, $a_2 = 0.5$, $b_1 = 1$, $b_2 = 0.8$, $k_1 = 1$, $k_2 = 1$ for $R_1 = 0.6$ and $R_2 = 0.4$

It is seen from the Fig 4.4 that the filter characteristics vary with their different impedance values. The magnitude response of the Bridged-T network is an all pass filter for the case 7, a band pass filter for the case of 13 and a band stop filter for the case 17 from TABLE 2.1. These observations are taken while we change the bilinear transformation coefficient b_2. The gain is also varied for the three different impedance values of the transfer function.

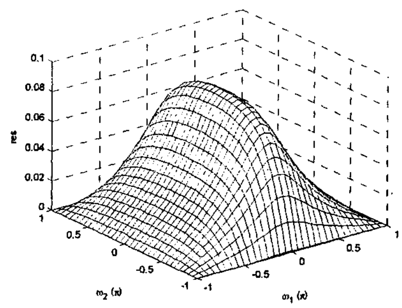
4.5 Frequency response of 2-D digital filter while changing k_1 coefficient

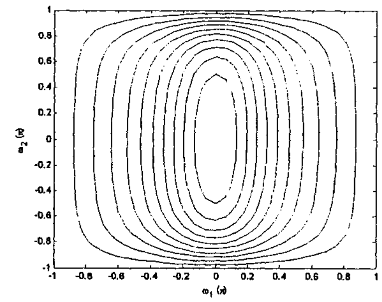
To study the manner how k_1 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of k_1, while fixing the other coefficients of the generalized bilinear transformation to be $a_1 = 0.5$, $a_2 = 0.5$, $b_1 = 1$, $b_2 = 1$, $k_2 = 1$. To compare the magnitude response for the three cases here we give the value of $k_1 = 50$.



(a) For case 7

(b) For case 13





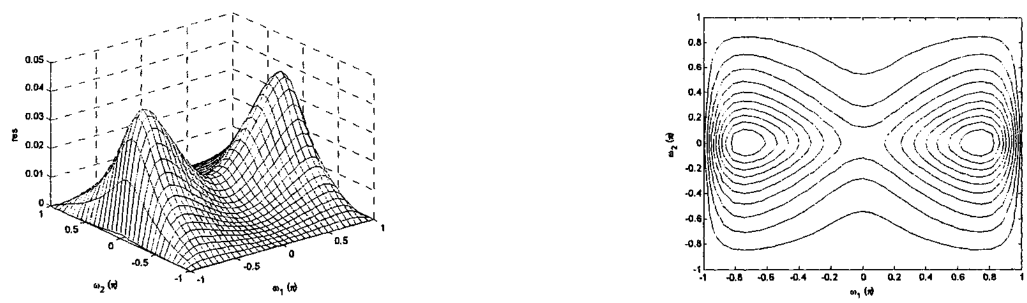
(c) For case 17

Figure 4.5 The contour and 3-D magnitude plots of the resulting 2-D digital filter with $a_1 = 0.5$, $a_2 = 0.5$, $b_1 = 1$, $b_2 = 1$, $k_1 = 50$, $k_2 = 1$ for $R_1 = 0.6$ and $R_2 = 0.4$.

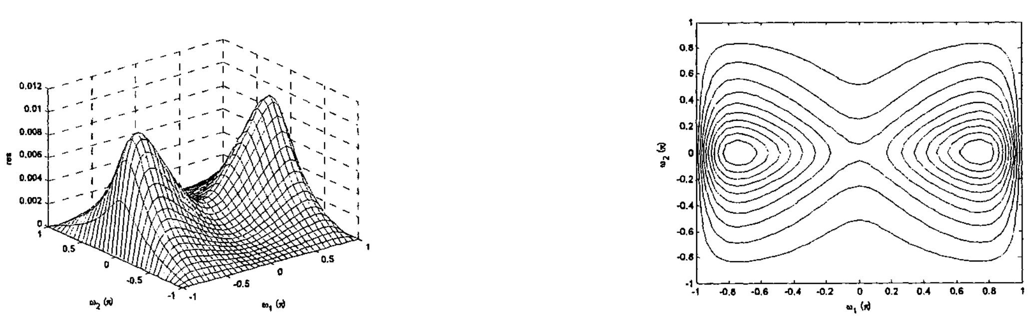
From Fig 4.5, it is seen that the effect on filter characteristics of bilinear transformation coefficient k_1 is different from the coefficient a_1, a_2, b_1, b_2. The magnitude response of the Bridged-T network is an all pass filter for the case 7 and low pass filter for the case of 13 and 17 from TABLE 2.1. It is seen that the higher value of k_1 make the transfer function low pass filter. It is also seen that the magnitude curves of the case 13 and 17 are almost same. These observations are taken while we change the bilinear transformation coefficient k_1. The gain is also varied for the three different impedance values of the transfer function.

4.6 Frequency response of 2-D digital filter while changing k_2 coefficient

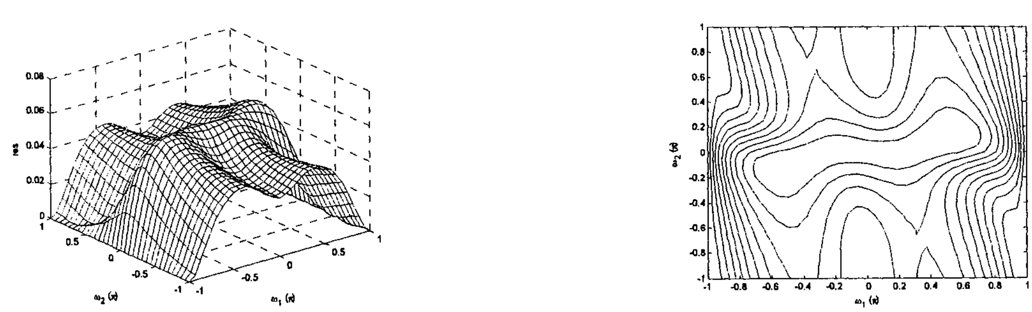
To study the manner how k_2 affects the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value of k_2, while fixing the other coefficients of the generalized bilinear transformation to be $a_1 = 0.5$, $a_2 = 0.5$, $b_1 = 1$, $b_2 = 1$, $k_1 = 1$. To compare the magnitude response for the three cases here we give the value of $k_2 = 50$.



(a) For case 7



(b) For case 13



(c) For case 17

Figure 4.6 The contour and 3-D magnitude plots of the resulting 2-D digital filter with $a_1 = 0.5$, $a_2 = 0.5$, $b_1 = 1$, $b_2 = 1$, $k_1 = 1$, $k_2 = 50$ for $R_1 = 0.6$ and $R_2 = 0.4$.

From Fig 4.6, it is seen that the effect on filter characteristics of bilinear transformation coefficient k_2 is completely different from the other. It is seen from the figure that, the filter characteristics vary with their different impedance values. The magnitude response of the Bridged-T network is band pass filter for the case 7 and 13. The shape of the magnitude response curve is distorted for the case of 17 from TABLE 2.1. These observations are taken while we change the bilinear transformation coefficient k_2. The gain is also varied for the three different impedance values of the transfer function.

4.7 Summary and Discussion

The comparison of magnitude response of the transfer function of the Bridged-T network between three cases from TABLE 2.1 for the value of $R_1 = 0.6$ and $R_2 = 0.4$ are discussed in this chapter. From the obtained magnitude response curves we can say that changing the impedance variables change the filter characteristics of the transfer function of Bridged-T network.

When we change the bilinear transformation coefficient a_1 for different impedance variable of the transfer function of Bridged-T network, we observe that the filter characteristic is an all pass filter for the case 7, a band pass filter for the case 13 and a band stop filter for the case 17 from TABLE 2.1. The gain is also varied for the different impedance variables of the transfer function.

As we change the bilinear transformation coefficient a_2 for different impedance variable of the transfer function of Bridged-T network, we observe that the filter characteristic is that of an all pass filter for the case 7, a band pass filter for the case 13 and a band stop filter for the case 17 from TABLE 2.1. Here the gain is not significantly changed for different values of impedance variables.

When we change bilinear transformation coefficient b_1 for different impedance variable of the transfer function of Bridged-T network, we observe that the filter characteristic is an all pass filter for the case 7, a band pass filter for the case 13 and a band stop filter for the case 17 from TABLE 2.1. Here the gain is not significantly changed for different values of impedance variables.

When we change bilinear transformation coefficient b_2 for different impedance variable of the transfer function of Bridged-T network, we observe that the filter characteristic is an all pass filter for the case 7, a band pass filter for the case 13 and a band stop filter for the case 17 from TABLE 2.1. The gain is also varied for the different impedance variables of the transfer function.

The behavior of the bilinear transformation coefficient k_1 on variable impedance values of the transfer function is quite different from a_1, a_2, b_1, b_2. For the case 7, the filter output is an all pass filter, but for the case 13 and 17 the filter output is a low pass filter. The gain is also quite low for the case 13 and 17 compare to case 7.

When we change bilinear transformation coefficient k_2 for different impedance variable of the transfer function of Bridged-T network, we observe that the filter characteristic is a band pass filter for the case 7, 13 and a distorted low pass filter for the case 17 from TABLE 2.1.

This chapter could be useful for understanding how different impedance variables change filter characteristics of a Bridged-T network. According to the required filter applications, the impedance variable of the four arms of Bridged-T network can be chosen.

Chapter 5

Conclusions and Directions for Future Research

5.1 Conclusions:

In this thesis, a technique in designing 2-D recursive digital filters having variable magnitude characteristics from a Bridged-T network has been proposed. Starting from the transfer function of this network which is VSHP in the analog domain, we apply generalized bilinear transformations and obtain a 2-D recursive digital filter. If one or more coefficients of the generalized bilinear transformations are changing, the resulting 2-D digital filters have variable frequency responses. Through the different combinations of the impedance values of the four arms of the Bridged-T network, we get all pass filter, band pass filter, band stop filter and in some cases low pass filter. The manner in which how each bilinear transformation coefficient and value of the impedance variable of the Bridged-T network affect the magnitude response of each 2-D digital filter has been investigated in this thesis.

In chapter 2, a doubly terminated Bridged-T Network has been introduced. In the transfer function of the Bridged-T network circuit, VSHP polynomials are used for the impedance values. The procedure of testing whether a given polynomial is VSHP or not is described in this chapter. Applying the procedure we get two tables which give a list of testing transfer function. In this chapter we give details description of the equations for TABLE 2.1 and 2.2. In TABLE 2.2, we keep the inductances and capacitances remain same position with their respective arms of the Bridged-T network with respect to TABLE 2.1, just change the values of s_1 and s_2. Among the 36 cases of TABLE 2.1 and TABLE 2.2, we get sixteen values which are VSHP and sixteen values which are not VSHP. Among the sixteen values which are VSHP, we face difficulties for finding the impedance values from the derived equation. Though we have six equations for six variables, but the equations are non-linear and not possible to find the impedance values. But there are three cases (7, 13, 17 from TABLE 2.1) from which we can determine the value of impedance variables of Bridged-T network. While computing the impedance variables we compare with fourth order Butterworth polynomial with the denominator of the transfer function of the Bridged-T network. The numerator of the Butterworth polynomial is considered as unity. For obtaining the equation from the numerator we let the angular frequency be unity for the ease of calculation.

This chapter mainly describes for choosing the transfer function which is VSHP from the different combinations of the impedance value of the Bridged-T network. Among total 36 combinations of transfer function, we get only three combinations which are suited for designing 2-D digital filter.

In chapter 3, we have introduced the procedure used to design 2-D digital filters by generalized bilinear transformation functions. The manner how each coefficient of the generalized bilinear transformation affects the magnitude response behavior of the resulting 2-D filter has been studied in detail also.

With the transfer function of the Bridged-T network we compare to the fourth order 1-D low pass Butterworth polynomial. Then we get 2-D analog transfer function. The 2-D discrete transfer function has been derived from the analog transfer function by double generalized bilinear transformations with stability constraints. When one or more coefficients of the double bilinear transformations are changing, the resulting 2-D filter has variable magnitude characteristics.

The coefficients of k_1, a_1 and b_1 affect the magnitude response in ω_1-domain, while the coefficients of k_2, a_2 and b_2 affect the behaviors of the magnitude response in ω_2-domain.

Here at first we design digital filter for the case of 7 from TABLE 2.1. For ensure the stability we vary the bilinear transformation coefficient a_1, a_2, b_1, b_2 from 0 to 1. For stability we have to maintain $a_1b_1<0$, $a_2b_2<0$, $k_1>0$ and $k_2>0$. While we change the coefficient a_i and b_i, we put the value $k_1 = k_2 = 1$. When we observe the effect of k_1 and k_2, we vary these value between 10 to 100 to see the effect of higher values of k. For the case 7, we have 5 cases (TABLE 2.3) for different values of R_1 and R_2. We design digital filter for $R_1 = 0.6$, $R_2 = 0.4$ and $R_1 = 0$, $R_2 = 1$. For the second case, i.e., the magnitude response curve is significantly changed. It is noticeable that the effect of bilinear transformation coefficient a_1, a_2, b_1, b_2 on the transfer function for the case of 7 makes the all pass filter or near to all pass filter having some exceptions (for example, figure 3.5 a) with respect to ω_1 domain. But the effect of k_1 and k_2 are different from other. We also study the manner how equal coefficients i.e., $a_1 = a_2$, $b_1 = b_2$ and $k_1 = k_2$ affect the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value one equal coefficient while fixing the other coefficients of the generalized bilinear transformation. The effect on the filter characteristics is more pronounced when we change the two equal coefficients simultaneously than the effect from the individual coefficient only.

Then we design digital filter for the case of 13 from TABLE 2.1. Here we also maintain the stability conditions mentioning for the case 7. For this transfer function we have also got 5 cases (TABLE 2.4) for different values of $R_1$and R_2. We design digital filter for $R_1 = 0.6$, $R_2 = 0.4$ for making comparison with the case before which is described in the next chapter. It is noticeable that the effect of bilinear transformation coefficient a_1, a_2, b_1, b_2 on the transfer function for the case of 13 makes the band pass filter or near to band pass filter having some exceptions (for example, figure 3.23 a). But the effect of k_1 and k_2 are different from other. In this case we also study the manner how equal coefficients i.e., $a_1 = a_2$, $b_1 = b_2$ and $k_1 = k_2$ affect the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value one equal coefficient while fixing the other coefficients of the generalized bilinear transformation. The effect on the filter characteristics is more pronounced when we change the two equal coefficients simultaneously than the effect from the individual coefficient only.

Then we design the digital filter for the case of 17 from TABLE 2.1. Here we also maintain the stability conditions mentioning for the case 7 and 13. For this transfer function, we have also got 5 cases (TABLE 2.5) for different values of $R_1$and R_2. We design digital filter for $R_1 = 0.6$, $R_2 = 0.4$ for making comparison with the case before which is described in the chapter 4. It is noticeable that the effect of bilinear transformation coefficient a_1, a_2, b_1, b_2 on the transfer function for the case of 17 makes the band stop filter or near to band stop filter having some exceptions (for example, figure 3.31 a). But the effect of k_1 and k_2 are different from other. We also study the manner how equal coefficients i.e., $a_1 = a_2$, $b_1 = b_2$ and $k_1 = k_2$ affect the frequency response behavior of the resulting 2-D filter and to separate the effect of the other coefficients, we change the value one equal coefficient while fixing the other coefficients of the generalized bilinear transformation. The effect on the filter characteristics is more pronounced when we change the two equal coefficients simultaneously than the effect from the individual coefficient only.

In chapter 4, the comparison of magnitude response of the transfer function of the Bridged-T network between three cases from TABLE 2.1 for the value of $R_1 = 0.6$ and $R_2 = 0.4$ are discussed. From the obtained magnitude response curves we can say that changing the impedance variables change the filter characteristics of the transfer function of Bridged-T network.

When we change the bilinear transformation coefficient a_1 for different impedance variable of the transfer function of Bridged-T network, we observe that the filter characteristic is an all pass filter for the case 7, a band pass filter for the case 13 and a band stop filter for the case 17 from TABLE 2.1. The gain is also varied for the different impedance variables of the transfer function.

As we change the bilinear transformation coefficient a_2 for different impedance variable of the transfer function of Bridged-T network, we observe that the filter characteristic is an all pass filter for the case 7, a band pass filter for the case 13 and a band stop filter for the case 17 from TABLE 2.1. Here the gain is not significantly changed for different values of impedance variables.

When we change bilinear transformation coefficient b_1 for different impedance variable of the transfer function of Bridged-T network, we observe that the filter characteristic is an all pass filter for the case 7, a band pass filter for the case 13 and a band stop filter for the case 17 from TABLE 2.1. Here the gain is not significantly changed for different values of impedance variables.

When we change bilinear transformation coefficient b_2 for different impedance variable of the transfer function of Bridged-T network, we observe that the filter characteristic is an all pass filter for the case 7, a band pass filter for the case 13 and a band stop filter for the case 17 from TABLE 2.1. The gain is also varied for the different impedance variables of the transfer function.

The behavior of the bilinear transformation coefficient k_1 on variable impedance values of the transfer function is quite different from a_1, a_2, b_1, b_2. For the case 7, the filter output is an all pass filter, but for the case 13 and 17 the filter output is low pass filter. The gain is also quite low for the case 13 and 17 compare to case 7.

When we change bilinear transformation coefficient k_2 for different impedance variable of the transfer function of Bridged-T network, we observe that the filter characteristic is band pass filter for the case 7, 13 and a distorted low pass filter for the case 17 from TABLE 2.1.

From the comparison and analysis of the results of magnitude characteristics of the filter output, we see that it is possible to get various type of filter output from a Bridged-T network by changing bilinear transformation coefficients or changing impedance variables.

5.2 The possible directions for future research:

In this thesis, we compare the denominator of transfer function of the Bridged-T network with fourth order Butterworth polynomial for finding the impedance variable of this circuit. Butterworth filter is a frequently used analog prototype filter, which has the maximally flat magnitude. Besides Butterworth polynomial, one can use other polynomials, e.g., Chebyshev. But the Butterworth is one of the popular polynomial and widely used an arena of filter design. In future research, we can use chebychev polynomial to compare the transfer function of the Bridged-T network.

In this thesis, we have investigated the manner how each coefficient of the generalized bilinear transformation affects the magnitude response behavior of the resulting 2-D filter. In every case we consider only the magnitude response. The phase response is not discussed in this thesis. For different coefficients of the generalized bilinear transformation, the phase response should have important role on the filter output. The consideration of phase response is suggestion for future work.

In this thesis, we design for digital filter for the case of 7,13 and 17 from TABLE 2.1. For each of these cases, we get different combinations for different values of R_1 and R_2. Here we handle the value of $R_1 = 0.6$ and $R_2 = 0.4$ for designing digital filter. Because this value of R_1 and R_2 has distinct filter output characteristics compare to other values. The other values for each case have different filter characteristics which could be analyzed for possible future research direction.

Also, required computer programs can be developed for testing the stability, the impedance values and the various designs.

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Appendix

Program Listing

5.1 Programs for Chapter 3: For the case of 7 from TABLE 2.1

```
% For the case of 7 from TABLE 2.1, when R1 = 0.6 and R2 = 0.4

% The magnitude plot

clear

%We change the different values of coefficients of bilinear transformations for observing

% the effects.

a1 = 0.5;

a2 = 0.5;

b1 = 1;

b2 = 1;

k1 =1;

k2 = 1;

R1 = 0.6; R2 = 0.4;

C2 = 9.0316; C3 = 1.0699;

L1 = 0.5807; L4 = 0.4455;
```

```
w1=-pi:pi/16:pi;
w2=-pi:pi/16:pi;
Z1=exp(-j*w1);
Z2=exp(-j*w2);
[z1,z2] = meshgrid(Z1,Z2);
% Applying bilinear transformation for making digital filter from analog domain
s1=k1.*(z1-a1)./(z1+b1);
s2=k2.*(z2-a2)./(z2+b2);
% The transfer function of the Bridged-T network
output = ( R2 + C3.*R2.*L1.*s1.^2 + C3.*R2.*L4.*s1.*s2 + C2.*R2.*L1.*s1.*s2)./
(C3.*R1.*L1.*s1.^2 + C3.*R2.*L1.*s1.^2 + C2.*R1.*L4.*s2.^2 + C3.*R1.*L4.*s1.*s2 +
C2.*C3.*R1.*R2.*L1.*s1.^2.*s2 + C3.*L1.*L4.*s1.^2.*s2 + C2.*R1.*R2.*s2 + R1 + R2
+ C2*C3*R2*L1*L4*s1.^2.*s2.^2+ + C2.*R1.*L4.*s1.*s2 +C2*L1*L4*s1.*s2.^2 +
C3.*R2.*L4.*s1.*s2 + C2.*R2.*L1.*s1.*s2 +L4*s2 + C2*C3*R1*R2*L4*s1.*s2.^2);
res = abs(output);
% The 3-D magnitude plot
mesh(w1/pi,w2/pi,res);
xlabel('\omega_1 (\pi)'),ylabel('\omega_2 (\pi)'),zlabel('res');
colormap cool
end
```

5.2 Programs for Chapter 3: For the case of 7 from TABLE 2.1

```
% For the case of 7 from TABLE 2.1, when R1 = 0.6 and R2 = 0.4
```

```
% The contour plot

clear

%We change the different values of coefficients of  bilinear transformations for

 % observing the effects.

a1 = 0.5;

a2 = 0.5;

b1 = 1;

b2 = 1;

k1 =1;

k2 = 1;

R1 = 0.6; R2 = 0.4;

C2 = 9.0316; C3 = 1.0699;

L1 = 0.5807; L4 = 0.4455;

w1=-pi:pi/16:pi;

w2=-pi:pi/16:pi;

Z1=exp(-j*w1);

Z2=exp(-j*w2);

[z1,z2] = meshgrid(Z1,Z2);

% Applying bilinear transformation for making digital filter from analog domain

s1=k1.*(z1-a1)./(z1+b1);

s2=k2.*(z2-a2)./(z2+b2);

% The transfer function of the Bridged-T network
```

```
output = ( R2 + C3.*R2.*L1.*s1.^2 + C3.*R2.*L4.*s1.*s2 + C2.*R2.*L1.*s1.*s2)./
(C3.*R1.*L1.*s1.^2 + C3.*R2.*L1.*s1.^2 + C2.*R1.*L4.*s2.^2 + C2.*R2.*L1.*s1.*s2 +
C2.*C3.*R1.*R2.*L1.*s1.^2.*s2 + C3.*L1.*L4.*s1.^2.*s2 + C2.*R1.*R2.*s2 + L4*s2 +
C2*C3*R2*L1*L4*s1.^2.*s2.^2 + C2*C3*R1*R2*L4*s1.*s2.^2 + C3.*R1.*L4.*s1.*s2
+ C2*L1*L4*s1.*s2.^2 + C2.*R1.*L4.*s1.*s2 + C3.*R2.*L4.*s1.*s2 + R1 + R2 );
 res = abs(output);
contour(w1/pi,w2/pi,res,10);
 xlabel('\omega_1 (\pi)'),ylabel('\omega_2 (\pi)'),zlabel('res');
 colormap cool
end
```

5.3 Programs for Chapter 3: For the case of 7 from TABLE 2.1

```
% For the case of 7 from TABLE 2.1, when R1 = 0 and R2 = 1
% The magnitude plot
clear
%We change the different values of coefficients of bilinear transformations for observing
% the effects.
a1 = 0.5;
a2 = 0.5;
b1 = 1;
b2 = 1;
k1 =1;
k2 = 1;
```

```
R1 = 0; R2 = 1;
C2 = 0.6532; C3 = 0.9238;
L1 = 0.6340; L4 = 2.6131;
w1=-pi:pi/16:pi;
w2=-pi:pi/16:pi;
Z1=exp(-j*w1);
Z2=exp(-j*w2);
[z1,z2] = meshgrid(Z1,Z2);
% Applying bilinear transformation for making digital filter from analog domain
s1=k1.*(z1-a1)./(z1+b1);
s2=k2.*(z2-a2)./(z2+b2);
% The transfer function of the Bridged-T network
output = ( R2 + C3.*R2.*L1.*s1.^2 + C3.*R2.*L4.*s1.*s2 + C2.*R2.*L1.*s1.*s2)./
(C3.*R1.*L1.*s1.^2 + C3.*R2.*L1.*s1.^2 + C2.*R1.*L4.*s2.^2 + C2.*R1.*L4.*s1.*s2 +
C2.*C3.*R1.*R2.*L1.*s1.^2.*s2 + C3.*L1.*L4.*s1.^2.*s2 + C2.*R1.*R2.*s2 + L4*s2 +
C2*C3*R2*L1*L4*s1.^2.*s2.^2 + C2*C3*R1*R2*L4*s1.*s2.^2 + C3.*R2.*L4.*s1.*s2
+ C2*L1*L4*s1.*s2.^2 + C3.*R1.*L4.*s1.*s2 + + C2.*R2.*L1.*s1.*s2 + R1 + R2 );
res = abs(output);
% The 3-D magnitude plot
mesh(w1/pi,w2/pi,res);
xlabel('\omega_1 (\pi)'),ylabel('\omega_2 (\pi)'),zlabel('res');
colormap cool
end
```

5.4 Programs for Chapter 3: For the case of 7 from TABLE 2.1

```
% For the case of 7 from TABLE 2.1, when R1 = 0 and R2 = 1

% The contour plot

clear

%We change the different values of coefficients of  bilinear transformations for

 % observing the effects.

a1 = 0.5;

a2 = 0.5;

b1 = 1;

b2 = 1;

k1 =1;

k2 = 1;

R1 = 0; R2 = 1;

C2 = 0.6532; C3 = 0.9238;

L1 = 0.6340; L4 = 2.6131;

w1=-pi:pi/16:pi;

w2=-pi:pi/16:pi;

Z1=exp(-j*w1);

Z2=exp(-j*w2);

[z1,z2] = meshgrid(Z1,Z2);

% Applying bilinear transformation for making digital filter from analog domain

s1=k1.*(z1-a1)./(z1+b1);
```

```
s2=k2.*(z2-a2)./(z2+b2);
% The transfer function of the Bridged-T network
output = ( R2 + C3.*R2.*L1.*s1.^2 + C3.*R2.*L4.*s1.*s2 + C2.*R2.*L1.*s1.*s2)./
(C3.*R1.*L1.*s1.^2 + C3.*R2.*L1.*s1.^2 + C2.*R1.*L4.*s2.^2 + C3.*R1.*L4.*s1.*s2 +
C2.*C3.*R1.*R2.*L1.*s1.^2.*s2 + C3.*L1.*L4.*s1.^2.*s2 + C2.*R1.*R2.*s2 + L4*s2 +
C2*C3*R2*L1*L4*s1.^2.*s2.^2 + C2*C3*R1*R2*L4*s1.*s2.^2 + C2*L1*L4*s1.*s2.^2
+ C2.*R1.*L4.*s1.*s2 + C3.*R2.*L4.*s1.*s2 + C2.*R2.*L1.*s1.*s2 + R1 + R2);
 res = abs(output);
contour(w1/pi,w2/pi,res,10);
 xlabel('\omega_1 (\pi)'),ylabel('\omega_2 (\pi)'),zlabel('res');
 colormap cool
end
```

5.5 Programs for Chapter 3: For the case of 13 from TABLE 2.1

```
% For the case of 13 from TABLE 2.1, when R1 = 0.6 and R2 = 0.4
% The magnitude plot
clear
%We change the different values of coefficients of bilinear transformations for observing
% the effects.
a1 = 0.5;
a2 = 0.5;
b1 = 1;
b2 = 1;
```

```
k1 =1;

k2 = 1;

R1 = 0.6; R2 = 0.4;

C1 = 1.6190; C2 = 705626;

L3 = 0.1705; L4 = 0.7981;

w1=-pi:pi/16:pi;

w2=-pi:pi/16:pi;

Z1=exp(-j*w1);

Z2=exp(-j*w2);

[z1,z2] = meshgrid(Z1,Z2);

% Applying bilinear transformation for making digital filter from analog domain

s1=k1.*(z1-a1)./(z1+b1);

s2=k2.*(z2-a2)./(z2+b2);

% The transfer function of the Bridged-T network

output = ( R2 + C1.*R2.*L3.*s1.^2 + C1.*R2.*L4.*s1.*s2 + C2.*R2.*L3.*s1.*s2)./

(C1.*R1.*L3.*s1.^2 + C1.*R2.*L3.*s1.^2 + C2.*R2.*L4.*s2.^2 + C1.*R1.*L4.*s1.*s2 +

C1.*C2.*R1.*R2.*L3.*s1.^2.*s2 + C1.*L3.*L4.*s1.^2.*s2 + C2.*R1.*R2.*s2 + L4*s2 +

C1*C2*R1*L3*L4*s1.^2.*s2.^2 + C1*C2*R1*R2*L4*s1.*s2.^2 + C2*L3*L4*s1.*s2.^2

+ C2.*R1.*L3.*s1.*s2 + C1.*R2.*L4.*s1.*s2 + C2.*R2.*L3.*s1.*s2 + R1 + R2 );

res = abs(output);

% The 3-D magnitude plot

mesh(w1/pi,w2/pi,res);

xlabel('\omega_1 (\pi)'),ylabel('\omega_2 (\pi)'),zlabel('res');
```

```
colormap cool

end
```

5.6 Programs for Chapter 3: For the case of 13 from TABLE 2.1

```
% For the case of 13 from TABLE 2.1, when R1 = 0.6 and R2 = 0.4

% The contour plot

clear

%We change the different values of coefficients of  bilinear transformations for

 % observing the effects.

a1 = 0.5;

a2 = 0.5;

b1 = 1;

b2 = 1;

k1 =1;

k2 = 1;

R1 = 0.6; R2 = 0.4;

C1 = 1.6190; C2 = 705626;

L3 = 0.1705; L4 = 0.7981;

w1=-pi:pi/16:pi;

w2=-pi:pi/16:pi;

Z1=exp(-j*w1);

Z2=exp(-j*w2);
```

```
[z1,z2] = meshgrid(Z1,Z2);
% Applying bilinear transformation for making digital filter from analog domain
s1=k1.*(z1-a1)./(z1+b1);
s2=k2.*(z2-a2)./(z2+b2);
% The transfer function of the Bridged-T network
output = ( R2 + C1.*R2.*L3.*s1.^2 + C1.*R2.*L4.*s1.*s2 + C2.*R2.*L3.*s1.*s2)./
(C1.*R1.*L3.*s1.^2 + C1.*R2.*L3.*s1.^2 + C2.*R2.*L4.*s2.^2 + C1.*R1.*L4.*s1.*s2 +
C1.*C2.*R1.*R2.*L3.*s1.^2.*s2 + C1.*L3.*L4.*s1.^2.*s2 + C2.*R1.*R2.*s2 + L4*s2 +
C1*C2*R1*L3*L4*s1.^2.*s2.^2 + C1*C2*R1*R2*L4*s1.*s2.^2 + C2*L3*L4*s1.*s2.^2
+ C2.*R1.*L3.*s1.*s2 + C1.*R2.*L4.*s1.*s2 + C2.*R2.*L3.*s1.*s2 + R1 + R2 );
 res = abs(output);
contour(w1/pi,w2/pi,res,10);
 xlabel('\omega_1 (\pi)'),ylabel('\omega_2 (\pi)'),zlabel('res');
 colormap cool
end
```

5.7 Programs for Chapter 3: For the case of 17 from TABLE 2.1

```
% For the case of 17 from TABLE 2.1, when R1 = 0.6 and R2 = 0.4
% The magnitude plot
clear
%We change the different values of coefficients of bilinear transformations for observing
% the effects.
a1 = 0.5;
```

```
a2 = 0.5;
b1 = 1;
b2 = 1;
k1 =1;
k2 = 1;
R1 = 0.6; R2 = 0.4;
C2 = 1.8563; C3 = 2.4195;
L1 = 0.2568; L4 = 2.1676;
w1=-pi:pi/16:pi;
w2=-pi:pi/16:pi;
Z1=exp(-j*w1);
Z2=exp(-j*w2);
[z1,z2] = meshgrid(Z1,Z2);
% Applying bilinear transformation for making digital filter from analog domain
s1=k1.*(z1-a1)./(z1+b1);
s2=k2.*(z2-a2)./(z2+b2);
% The transfer function of the Bridged-T network
output = ( R2 + C3.*R2.*L4.*s1.^2 + C2.*R2.*L1.*s1.*s2 + C3.*R2.*L1.*s1.^2)./
(C3.*R1.*L4.*s1.^2 + C3.*R1.*L1.*s1.^2 + C3.*R2.*L4.*s1.^2 + C3.*R2.*L1.*s1.^2 +
C2.*C3.*R1.*R2.*L1.*s1.^2.*s2 + C2.*L1.*L4.*s1.^2.*s2 + C2.*R1.*R2.*s2 + L4.*s1
+ R1 + R2 + C2.*C3.*R1.*R2.*L4.*s1.^2.*s2 + C2.*C3.*R2.*L1.*L4.*s1.^3.*s2 +
C3.*L1.*L4.*s1.^3+ C2.*R1.*L1.*s1.*s2 + C2.*R1.*L4.*s1.*s2 + C2.*R2.*L1.*s1.*s2);
res = abs(output);
```

```
% The 3-D magnitude plot
mesh(w1/pi,w2/pi,res);
xlabel('\omega_1 (\pi)'),ylabel('\omega_2 (\pi)'),zlabel('res');
colormap cool
end
```

5.8 Programs for Chapter 3: For the case of 17 from TABLE 2.1

```
% For the case of 17 from TABLE 2.1, when R1 = 0.6 and R2 = 0.4
% The contour plot
Clear
%We change the different values of coefficients of  bilinear transformations for
 % observing the effects.
a1 = 0.5;
a2 = 0.5;
b1 = 1;
b2 = 1;
k1 =1;
k2 = 1;
R1 = 0.6; R2 = 0.4;
C2 = 1.8563; C3 = 2.4195;
L1 = 0.2568; L4 = 2.1676;
w1=-pi:pi/16:pi;
w2=-pi:pi/16:pi;
```

```
Z1=exp(-j*w1);
Z2=exp(-j*w2);
[z1,z2] = meshgrid(Z1,Z2);
% Applying bilinear transformation for making digital filter from analog domain
s1=k1.*(z1-a1)./(z1+b1);
s2=k2.*(z2-a2)./(z2+b2);
% The transfer function of the Bridged-T network
output = ( R2 + C3.*R2.*L4.*s1.^2 + C2.*R2.*L1.*s1.*s2 + C3.*R2.*L1.*s1.^2)./
(C3.*R1.*L4.*s1.^2 + C3.*R1.*L1.*s1.^2 + C3.*R2.*L4.*s1.^2 +C3.*R2.*L1.*s1.^2 +
C2.*C3.*R1.*R2.*L1.*s1.^2.*s2 + C2.*L1.*L4.*s1.^2.*s2 + C2.*R1.*R2.*s2 + L4.*s1
+ R1 + R2 + C2.*C3.*R1.*R2.*L4.*s1.^2.*s2 + C2.*C3.*R2.*L1.*L4.*s1.^3.*s2 +
C3.*L1.*L4.*s1.^3 + C2.*R1.*L1.*s1.*s2 + C2.*R1.*L4.*s1.*s2+ C2.*R2.*L1.*s1.*s2);
res = abs(output);
contour(w1/pi,w2/pi,res,10);
 xlabel('\omega_1 (\pi)'),ylabel('\omega_2 (\pi)'),zlabel('res');
 colormap cool
end
```