A Nonlinear Control Design Technique for Formation Flight of a Constellation of Satellites

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ABSTRACT

A Nonlinear Control Design Technique

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Autonomous control greatly reduces the demand on ground-based resources to keep satellites in a constellation and working reliably, which would be nearly impossible to do without such automation. Linear feedback control is primarily used to counter unexpected small perturbations and to maintain generally sub-optimal trajectories. Linearized approximation of the actual non-linear satellite dynamics cannot predict the "non-local" behavior far from the operating point and certainly not the "global" behavior throughout the state space.

The objective of this thesis is development of a formation control and station keeping control laws for a constellation of satellites in a circular orbit. It is shown that by utilizing feedback linearization or nonlinear compensation techniques, an efficient nonlinear control design strategy may be implemented that leads to solutions valid for larger set point regulation, perturbations and station keeping problems. To assess the quality of the proposed control scheme, its performance is compared with that of linear pole placement controller and a nonlinear controller designed using potential function method in a number of simulation case studies. Advantages and disadvantages of the proposed nonlinear control technique are also presented for formation control and station keeping.

To the loving memory of my dear grandmother by birth and mother by deeds, ***Jiji***

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Chapter 1

Introduction

Man has been putting single satellites into space since October 4, 1957, when USSR launched the world's first artificial satellite, Sputnik 1. But often this lone satellite is only over part of the world at any time. It cannot cover the entire world at once. A number of satellites are needed for global or near-global coverage so that at least one satellite can be seen from every point on the earth at once. Low Earth Orbiting satellites (LEOs) are often deployed in constellations, because the coverage area provided by a single LEO satellite covers a small area, and the satellite travels at a high angular velocity to maintain its orbit. Multiple LEO satellites are needed to maintain consistent coverage. Apart from the field of telecommunication, interferometric imaging using multiple satellites in formation also has gained much interest in recent years. The advantages of such formations include the replacement of large monolithic telescopes and superior angular resolution.

Formally, spacecraft/satellite formation flying can be defined as the motion control of a group of spacecrafts/satellites where the space-vehicle positions relative to each other are of concern. In general, such satellite constellations in autonomous formation flight can decrease mission costs and increase performance, scientific gains, reliability, adaptability and survivability of space missions. However, the technological challenges presented by this technique have been difficult to overcome; fleet-wide communication and fault detection, collision avoidance, and planning and control while minimizing fuel usage are all active areas of exploration in this field. For orbital control, the most useful approaches consider a spacecraft perturbed from a reference orbit to develop theoretically stable formations of multiple satellites. Planning and control are then performed in two modes: formation keeping, where spacecraft remain in a stable formation to within a specified tolerance; and formation maneuvering, where spacecraft plan and execute thrust commands to move the cluster from one stable formation to another.

Some of the many examples of spacecraft formation flying are stated here. Gemini VI accomplished the first space rendezvous on December 15, 1965, and performed station-keeping with Gemini VII over three orbits at distances of 0.3 meter to 90 meters. Gemini VIII followed with the first docking of two spacecraft in orbit on March 16, 1966. The first automated rendezvous was performed by Soviet spacecraft Cosmos 186 and Cosmos 188 on October 27, 1967, when the two unmanned spacecraft performed a pre-programmed closure and docking manoeuvre. Since the 1960s, numerous manned and unmanned spacecraft have performed operations in close proximity, mostly for the purpose of docking for re-supply or crew transfer, and observation, as in the case of Shuttle flyarounds of the International Space Station.

Autonomous formation flight became a reality for spacecraft on May 17, 2001, when NASA-Goddard's Earth Observing-1 spacecraft performed its first autonomous manoeuvre to maintain a one minute in-track separation with Landsat7. Examples of satellite constellations include the Global Positioning System (GPS) and Global Navigation Satellite System (GLONASS) constellations for navigation and geodesy, the

Iridium and Globalstar satellite telephony services, the Orbcomm messaging service, Russian elliptical-orbit Molniva and Tundra constellations, and the Teledesic and Skybridge [1] broadband constellation proposals. These spacecraft as well as satellite formations have paved the way for using the formation flight and station keeping technology for use in a number of varied scientific, military, and satellite service operations.

1.1 Motivation

In the past twenty years spacecraft/satellite formation flying has emerged as a topic of great discussions and research interest. Traditionally spacecraft orbits have been controlled by engineers on the ground, who command the spacecraft to perform open-loop thrust profiles designed to force optimal maneuver trajectories. These are not only time consuming and inaccurate but also prone to human error and misjudgment causing a considerable cost overhead to ground segment and mission control. Autonomous control would greatly reduce the demand on ground-based resources to keep satellites in a cluster and working reliably, which would be nearly impossible to do without such automation. Linear feedback control is primarily used to counter unexpected perturbations, and maintain the optimal trajectory. Various work in literature [2, 3] have intended to explore the use of nonlinear techniques for controlling the orbits of the satellites in formation and the advantage earned thereof. Fully autonomous constellation of formation flying satellites is bound to become a reality and will set the stage for many more scientific discoveries.

1.2 Problem statement

This research work primarily aims to investigate and develop a novel nonlinear orbital control law, based on nonlinear compensation technique, for autonomous control of satellites in constellation. The developed nonlinear control law should be able to achieve the task of keeping a constellation of satellites in formation and fulfill the mission objectives thereby. It should be able to counter the perturbations caused by for instance atmospheric drag, J_2 drag and solar radiation pressure with high tolerance. It should be applicable to real satellites and therefore must work in the presence of constraints such as thrust magnitude.

1.3 Approach

We deal with the problem of formation control for a group of satellites in a circular orbit. Firstly we define the geometry of the formation and the specifics of the satellites used in the mission. We assume that the reference trajectory is uploaded into each satellite mission computer by the ground segment. To force the convergence of the formation flying satellites to the desired states we use controllers designed by the conventional pole placement technique, potential function method and the nonlinear compensation technique in separate simulations. We treat each maneuver as a rendezvous between a maneuvering spacecraft, and a passive, target orbit and compare the simulation results for the three control techniques used.

1.4 Thesis Outline

In Chapter 2, we present literature review and previous works on formation design, formation geometries and formation control methodologies. In Chapter 3, we present a review of spacecraft dynamics and concepts in orbital mechanics along with a discussion on orbital perturbations. In Chapter 4, we describe the concept of decentralized, relative; behavior based formation control and provide a critique and simulations on the work done by Collin McInnes [4]. In Chapter 5, we develop a linear orbital controller based on the conventional pole placement technique and provide simulations for its use on linear as well as nonlinear satellite dynamics in various mission simulations. In Chapter 6, we propose and develop a nonlinear orbital controller based on the nonlinear compensation technique and provide simulations for its use on nonlinear satellite dynamics in various scenarios. In Chapter 7, we present comparative discussions and conclusion on the methodologies presented in Chapters 4, 5 and 6. Towards the end of the 7th Chapter we provide some recommendations for future work as a direct or indirect result of this thesis work.

Chapter 2

Literature Review

In this chapter we present a literature review on satellite/spacecraft formation flying. We start with formation flying applications that have been defining the past and future research needs in this area with numerous examples of missions in various stages of completion. Then we go over some of the terminologies used in this field. Thereafter we discuss the formation design, formation control methodologies and formation geometries investigated and introduced by various researchers in this field.

2.1 Applications

Constellation of satellites in formation has many applications. It provides many new possibilities such as increased accuracy for the Global Positioning System, remote sensing, and communications systems. For some of these applications, especially remote sensing of both the Earth and deep space, these satellites that are in close formation should be able to provide the benefits of a lower life-cycle cost, more adaptability to changing mission goals, less susceptibility to the loss of individual satellites, and better performance in general. The Air Force Research Laboratory (AFRL) of U.S.A. is considering one such constellation of 35 low-orbit "virtual" satellites (and an additional 5 for spares), in which each "virtual" satellite is actually a cluster of 8 "micro" satellites, which would fly within 250 meters of each other. It is called the TechSat 21 program [5].

The ground control of such a system would create an almost impossible demand on the ground crew. This is where the autonomous, onboard management of the satellites comes in. It is proposed that by outfitting these "micro" satellites with a program such a GPS Enhanced Orbit Determination Experiment (GEODE) such a cluster of formation flying satellites is feasible and should be built. Some more examples of proposed formation flying missions in various stages of development are ALPHA [6], Aqua [5], Aura [1], Auroral Lites [7], Auroral Multiscale Midex (AMM) [7], CloudSat [8], ClusterII, GRACE [6], ION-F [9], Leonardo-BRDF [10], Magnetospheric Multiscale (MMS), MUSIC [6], Power Sail, ST3, Parasol, PICASSO, SMART-2, Starlight, Terrestrial Planet Finder (TPF), and TOPSAT [11]. Missions for these spacecraft include extra-solar planet studies (TPF), formation flying optical interferometry tests (Starlight), magnetosphere and solar wind studies (AMM and Cluster II), and a variety of other science studies and technology demonstrations.

2.2 Formation Design

Formation design can be broadly classified into Centralized and Decentralized formations as shown in the figure 2.1 below

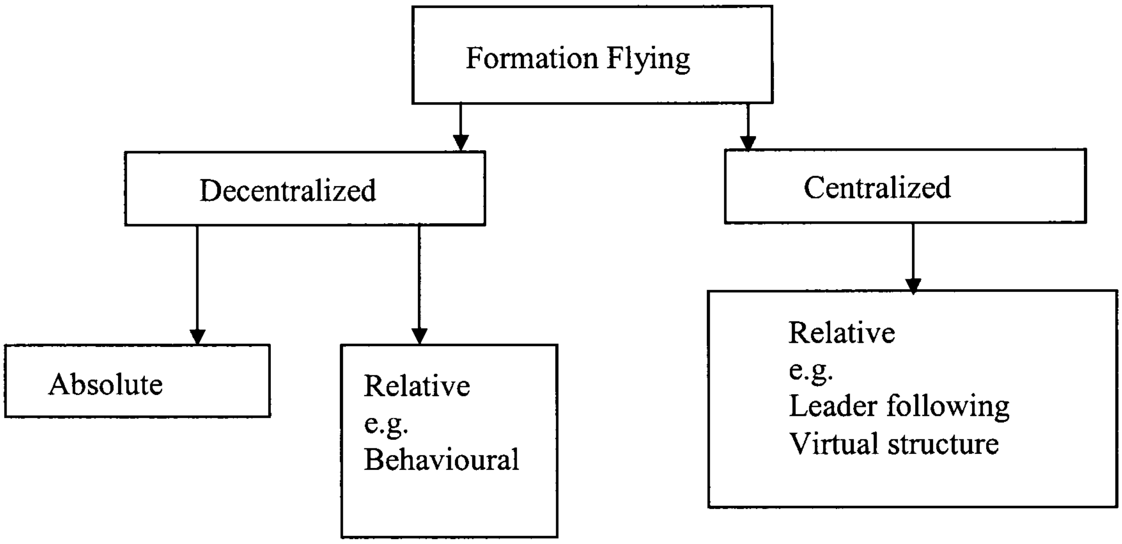


Fig. 2. 1 Formation flying classification

2.2.1 Centralized Approach

Previous work have dealt with the subject of formation control in two ways. Most formation flying control papers [12, 13] attack the problem of controlling the relative orbits of two or more spacecraft, one of which does not perform primary formation control and follows some reference trajectory, and the rest of which perform orbit control to maintain the relative formation. This is also referred to as the centralized formation flying since only one spacecraft in reality is controlling the entire formation. The controlled spacecraft appears in the literature as the follower, the slave, the deputy, the interceptor, the chase satellite, the free-flying satellite, the collector, the daughter, and so on. The uncontrolled spacecraft appears as the leader, the lead, the master, the chief, the target, the reference, the head, or virtual head, the control, the combiner, the mother, and so on. We find two approaches in the literature which can be classified under the centralized formation flying category, Leader following approach [10, 11, 12, 14] and Virtual structure approach [17, 18].

Leader Following

The basic idea here is that the leader regulates its position to some desired possibly time varying goal and the followers track the position and orientation of the leader to some prescribed possibly time varying offset. There are numerous variations on this theme including designating multiple leaders forming a chain and other tree topologies. Leader following generally requires that the followers feed forward the acceleration of the leader. Given the acceleration of the leader convergence of the spacecraft to the proper relative position is guaranteed. However this requires frequent communication of acceleration information from the leader to the followers. One of the first studies on leader following strategies reported by Wang [14] discusses formation control laws for mobile robots.

The application of these ideas to spacecraft formations was developed by Wang and Hadaegh [12] where explicit control laws for formation keeping and relative attitude alignment based on nearest neighbor tracking were derived. Several leader following techniques were discussed including leader tracking, nearest neighbor tracking, barycenter tracking and other tree topologies. Leader following has been applied to the coordination of spacecraft in earth orbit. There are two basic approaches to solve this problem. The first approach is to linearize the spacecraft equations about a nominal orbit. The resultant equations are known as the Clohessy-Wiltshire equations also known as the Hill equations. Kapila [15] discretized the dynamics and derived a pulse based control law. On the other hand Leonard Hollister and Bergman [16] used differential drag combined with switching curves such that the follower spacecraft tracks the motion of the leader. Yan de Queiroz [2] derived adaptive control laws to coordinate the motion of the spacecraft based on the full nonlinear equations.

Virtual Structure

A virtual structure model [16, 17, 18] resides onboard a satellite computer. This structure contains several states that correspond to the position, orientation and relative spacecraft separations of the formation. The states of the virtual structure are allowed to evolve over time to trace out the desired motion of the formation. This in turn translates to desired trajectories for every spacecraft in the formation. The virtual structure would simply trace out the position and orientation of some point in the formation. This implementation of the virtual structure is similar to the leader following approach where the leader only exists in the memory of a computer. Since leader following and the simple version of the virtual structure approach are identical the analysis to show convergence is identical as well.

The two approaches share the weakness that acceleration information must continually be communicated to the several spacecraft in the formation. Thus current implementations of the virtual structure approach are centralized. The trajectory of the virtual leader is generated in a central location and then communicated to the other members of the formation. This approach was applied to formations of mobile robots by Lewis and Tan [17]. The application to formations of spacecraft in free space is described in [18, 19].

As discussed above both the above approaches suffer from disadvantages created due to passing of information from the leader satellites to the slave satellites or from the virtual structure aboard a satellite computer to other satellites in formation. There are obvious lags between the slave spacecrafts and the leader spacecrafts and above all since there is no feedback from slave satellites to the master satellite, the formation becomes a single point failure system. This means that if one of the slaves is not tracking the required trajectory there will be a formation break-up without the master satellite even knowing about it. Current research activities in this field are trying to combine the concepts of virtual/leader-following formation with those of the decentralized formation flying trying to make it more robust and less susceptible to obvious disadvantages.

2.2.2 Decentralized Approach

The decentralized formation flying approach can be either absolute or relative depending on the way the formation is controlled.

Absolute Configuration

In the absolute formation flying approach, also called as "box method", each satellite is maintained within tight limit of its nominal state. It is similar to the approach used by Global Positioning System (GPS) constellation and by the method proposed for the Iridium constellation [20]. In this method each satellite is individually responsible for maintaining its own position in the constellation. Closed loop controllers onboard each satellite command thruster impulses for in-track and cross-track corrections for external perturbations. This approach though does not take into consideration the concept of information sharing between the satellites in formation.

Behaviour based

In behaviour based approach [2, 20, 22] several desired behaviours are uploaded by the ground segment for each satellite. The control action of each satellite is a weighted average of the relative positions of all the satellites in the formation so that expected behaviour is achieved. Possible behaviours include collision avoidance, obstacle avoidance, goal seeking and formation keeping. There are also numerous variations on the behaviour based approach to multi-vehicle coordination most of which are derived by novel weightings of the behaviours. Behaviour based control laws feed back the relative positions among spacecraft whereas leader following and virtual structure control laws feed forward the acceleration of the leader to achieve convergence. Behavior based strategies have the advantage that less information needs to be communicated among vehicles. However until now neither convergence analysis nor bounds on the formation keeping error have been established.

The behaviour based approach is applied to the problem of maintaining a constellation of satellites in an equally distributed ring formation in earth orbit [4]. Simple Lyapunov control functions are used to maintain distance and avoid collisions. The application of the behaviour based approach to aircraft flying in formation is described in [21] where the control strategies are derived to mimic the instinctive behavior of birds and fish. Balch and Arkin [22] describe the behaviour based approach to formation keeping for mobile robots where control strategies are derived by averaging several competing behaviors including goal seeking collision avoidance and formation maintenance. In [23] the behaviour based approach is used to cause a group of robots to create line and circle formations. These ideas are extended in [24] to the problem of controlling a formation of mobile robots to transport objects.

2.3 Formation Control Strategies

Colin McInnes [4] uses Lyapunov's direct method to derive control laws to maintain relative formation for a fleet of ten satellites in a circular orbit. An energy function including the kinetic terms and a periodic fourier term is considered. Linearized set of Hill's equation is used for simulation. No cross-track correction for radial perturbation is possible in the derived equations. It is observed that the resultant dynamics of the overall formation induce a complex nonlinear interaction between all the satellites and a loose formation is shown to emerge after iterations lasting several orbital periods. The work being done at European Space Agency (ESA) for formation flying missions is based on the work done by Colin McInnes.

Y. Ulybyshev [25] presents a control law using linear quadratic (LQ) regulator technique to implement relative formation keeping for low-earth orbiting satellites.

Leonard [16] controlled two spacecraft positions using the differential drag between them. The atmospheric density was assumed uniformly and the velocities and ballistic coefficients of the satellites were assumed to be initially equal. The differential drag between two satellites is the difference in drag per unit mass acting on each satellite. The equations of motions were derived and a coordinate transformation was made to reduce the formation-keeping problem. A main control law and the eccentricity-minimizing control scheme were derived.

The main control law was able to move the average position of the slave vehicles to the origin of the target reference coordinate system (i.e. zero) by reducing as much as eccentricity as possible whereas the eccentricity-minimizing control scheme activated for reducing the eccentricity when the average position of the slave vehicles was at the target (origin). The formation-keeping problem is formulated as the simultaneous solutions of both double integrator and harmonic oscillator. Solving both double integrator and harmonic oscillator obtained the position of slave vehicle to target.

Wang, and Hadaegh [14] considered the problem of coordination and control of multiple microspacecraft moving in formation in low Earth orbit (LEO). It is assumed that each microspacecraft was modeled by a fixed center of mass of a rigid body. Difference schemes for creating a formation pattern were discussed, and the explicit control laws for formation-keeping were derived. The discussions of deriving a control law, and the integration of the microspacecraft formation coordination and control system with a proposed inter-spacecraft communication or computing network were presented. The result shows that there are no collisions between the microspacecraft due to the small magnitude of the initial deviation from the desired state.

De Queiroz [2] used full nonlinear dynamics to develop a nonlinear adaptive control law for the relative position tracking of multiple spacecraft in formation flying. Performance was illustrated using various simulations.

A Folta-Quinn (FQ) Formation Flying Algorithm [26] uses Lambert's two point boundary value problem and the `C*' guidance and navigation matrix to maintain a formation of two spacecraft.

Carpenter, Folta and Quinn [27] use the FQ algorithm, in conjunction with a linear-quadratic-Gaussian (LQG) control to develop a decentralized control for autonomous formation flying for use in the New Millenium Program and the Earth Observing-1 (EO-1) mission. The formation model includes 2 spacecraft, one with real-time GPS and autonomous control (EO-1), and one with orbit determination and maneuver generation performed on the ground (Landsat-7). The authors conclude that autonomous, decentralized control is not only feasible, but beneficial over traditional orbit control procedures.

Kapila [15] developed a mathematically rigorous control design framework for linear control of spacecraft relative position dynamics with guaranteed closed-loop stability.

Schaub and Alfriend [28, 29] use mean orbital element differences to control the relative orbits of formation flying spacecraft with identical ballistic coeffcients. The authors' use of mean orbital element differences stresses long-term behavior over short-term deviation, by forcing the control to compensate for secular and long period drifts, while ignoring short period oscillations. The control law establishes the desired formation in a nearly fuel-optimal sense. The authors also develop a cartesian, Lyapanov-control law, and apply it using relative position and velocity tracking vectors calculated from mean orbital elements. A comparison of the two nonlinear feedback control laws shows that the cartesian form is effective for short time frame, high thrust manoeuvres, while the orbital element form performs best in multiple-orbit or low-thrust manoeuvres.

Ilgen [30] develops Lyapunov-optimal feedback control laws using Guass's form of Lagrange's planetary equations (LPE) in classical and equinoctial orbital-element forms. The author presents equations of motion for the equinoctial elements in terms of thrust components in the equinoctial coordinate axes. The paper provides an excellent starting point for the development of an elemental, Lyapunov-optimal, formation flying feedback control law.

Lau [31] described autonomous formation flying concept for extremely precise autonomous relative position and attitude determination for satellite formations. He uses autonomous formation flying concept in Global Positioning System (GPS).

Inalhan [32] investigated precise relative sensing and control via differential GPS for multiple spacecraft formation. He presented autonomous control architecture for formation flying, and a generalized closed-form solution of passive apertures for constellations with mean formation eccentricity.

Tan, Bainum [33] developed a strategy that is able to keep the separation distance in between four satellites in a coplanar elliptical orbit configuration constant. This strategy generated a small angular movement in the direction of the axis with respect to the axis of the combiner satellite using the force impulse. The separation distance between collectors was maintained within four percent of nominal separation distance for Keplerian orbits.

2.4 Formation Geometries

The goal of formation flying is to maintain a group of satellites/spacecrafts in a specific geometry to achieve scientific, commercial goals. Many researchers have focused their works to find various geometries suitable for various missions. They have worked on analytically proving the feasibility of such configurations while taking into consideration various real life constraints.

Sabol, Burns and McLaughlin [34] investigated the stability of four basic satellite formation flying designs: in-plane, in-track, circular and projected circular formation by applying realistic perturbations on these formations. The formations were derived from Hill's equations. The minimum amount of velocity impulse, Δv, is calculated to balance the $J2$ disturbance force and stabilize the satellite formations. The results show that circular and projected circular formations are very unstable when the perturbation is applied. However, the in-track formation is stable. The in-plane formation will require small, infrequent, along-track maneuvers to offset the effects of atmospheric drag. Therefore, the formation-keeping cost of circular and projected circular formations is 38 times higher than in-track and in-plane formations.

Folta and Quinn [26] describe three formation types for use in the EO-1 Landsat7 formation: the close formation, the ideal formation, and the dynamic formation. The close formation consists of spacecraft flying with a separation of less than one kilometer in the velocity direction, and is intended for missions making simultaneous, multi-point observations. This formation type has also been called the in-plane23 formation, and while it is often modeled with a transverse-direction separation only, inspection shows that there is also a small separation in the radial direction. For this reason, it is also called as the anomaly shift formation The ideal formation involves separation distances on the order of 1 to 100 kilometers, and is intended for missions making time-lapse observations of the same point. The ideal formation separation is mostly in the transverse direction, but also in the orbit-normal direction to compensate for the rotation of the earth, and to ensure that each formation flyer follows the same ground track.

The ideal formation is often called the same ground track, or in-track formation, and is usually modeled as a shift in true anomaly and right ascension. The dynamic formation is similar to the ideal formation, except that the maneuvering spacecraft is allowed to drift with respect to the non-maneuvering reference satellite as a result of differential atmospheric drag effects. In the case of EO-1 and Landsat7, differential drag effects are the largest source of error, as the two spacecraft are distinctly different in shape, size, and mass, and therefore have different ballistic coefficients. The formation is maintained by allowing EO-1 to drift from a carefully selected relative orbit until it either reaches the limit of its control box, or Landsat7 performs a manoeuvre, at which point EO-1 performs a manoeuvre to return itself to the predetermined relative orbit.

Schaub and Alfriend [29] develop an analytic method for defining $J2$ invariant relative orbits for spacecraft formations. The authors assume spacecraft with equal ballistic coefficients (and therefore negligible differential drag effects), and use mean orbit elements and Brouwer's artificial satellite theory without drag to find $J2$ invariant orbits. The method described matches the drift rates of two neighboring orbits up to the first order by constraining the values of $\delta a, \delta e$ and δi and provides the most signifficant fuel savings for non-circular, non-polar orbits.

Vadali and Vaddi [35] discuss the formation establishment problem, the objective of which is to maximize the inclination difference between two formation flying spacecraft while satisfying relative mean orbit elements and rate constraints. The authors find that satisfaction of rate constraints conflicts with the objective of maximizing the inclination difference.

2.5 Summary

In this chapter, we have broadly reviewed formation flying literature, enumerating various formation flying applications, and also summarizing previous works on formation design, formation control strategies and formation geometries. In the next chapter, we present a detailed review of orbital mechanics and spacecraft dynamics in cartesian coordinates as well as orbital element terms.

Chapter 3

Orbital Dynamics

The terms "orbital dynamics," "astrodynamics," "orbital mechanics," "spaceflight dynamics," and "astronautics" are often used interchangeably in the literature. Orbital dynamics is concerned with the orbital motion of space vehicles under the influence of gravitational and other external forces. The dynamics of space objects is most often described by two-body motion in a Keplerian gravitational field, with corrections to compensate for forces from perturbations such as non-spheroidal central body effects, atmospheric drag, solar and lunar gravitational effects, and solar radiation pressure. In this chapter we present a brief description and mathematical equations which govern the orbital dynamics of satellites in the form of two body relative equations of motion, classical orbital elements and equinoctial orbital elements. Further more, we describe some perturbations that affect formation dynamics.

3.1 Two-Body Relative Motion Equation

Here, we briefly describe the dynamic problem of two point masses under the influence of their mutual gravitational attraction. Such a problem is called the two-body problem in celestial mechanics. The following equations are derived using Newton's laws of motion and law of gravity. Kepler's three laws of planetary motion, provide the fundamental framework of motion of planets and celestial bodies. They describe the motion of planets around the sun, which is considered to be inertially fixed. As shown in the figure 3.1 consider two particles of masses m_1 and m_2 with position vectors

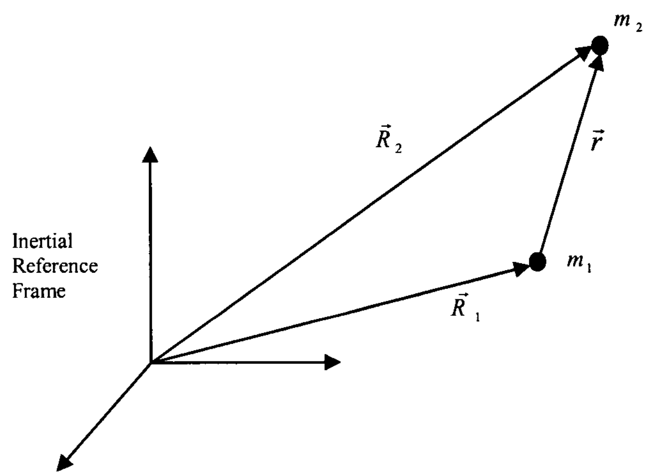


Fig. 3. 1 Two body dynamics in space

$\vec{R}_1$ and $\vec{R}_2$, respectively, in an inertial reference frame. Applying Newton's second law and his law of gravity to each mass, we write the equations of motion as

$$m_1 \ddot{\vec{R}}_1 = +\frac{Gm_1 m_2}{r^3}\vec{r} \tag{3.1}$$

$$m_2 \ddot{\vec{R}}_2 = -\frac{Gm_1 m_2}{r^3}\vec{r} \tag{3.2}$$

where $\vec{r} = \vec{R}_2 - \vec{R}_1$ is the position vector of m_2 from m_1, $r = \left|\vec{r}\right|$, $\ddot{\vec{R}}_i = \frac{d^2 \vec{R}_i}{dt^2}$ is the inertial acceleration of the $i\,th$ body, and $G = 6.6695 \times 10^{-11} N.m^2 / kg^2$ is the universal gravitational constant. Subtracting (3.1) times m_2 from (3.2) times m_1, we obtain

$$\ddot{\vec{r}} + \mu\frac{\vec{r}}{r^3} = 0 \tag{3.3}$$

where $\ddot{\vec{r}} = \frac{d^2 \vec{r}}{dt^2}$ is the inertial acceleration of m_2 with respect to m_1, $r = |\vec{r}|$, and $\mu = G(m_1 + m_2)$ is called the gravitational parameter of the two body system under consideration.

Equation (3.3) describes the motion of m_2 relative to m_1 in the inertial reference frame and is the fundamental equation in the two-body problem. In the practical case of satellite motion around earth, the mass of the primary body (earth) is much greater than that of the secondary body (satellite). Therefore, since $m_1 >> m_2$, it results in $\mu \approx Gm_1$. It is worth emphasizing here that the primary body is not inertially fixed in the two body problem. Thus we can see that the orbital dynamics of a satellite around the earth is a restricted two-body problem in which the primary body of mass m_1 is assumed to be inertially fixed.

The equations of motion in (3.3) depict the ideal Keplerian situation where the central body is a perfect sphere and all other disturbances are zero. In a more realistic scenario, equation (3.4) below includes perturbation accelerations $\vec{f}$ caused by non-spheroidal gravitational effects of the central body, atmospheric drag, third body effects and so on.

$$\ddot{\vec{r}} + \mu \frac{\vec{r}}{r^3} = \vec{f} \tag{3.4}$$

The above non-linear equation can also be represented in polar co-ordinates. A single satellite of mass m moves under the inverse law in the presence of a central gravitational body, earth of mass M as shown in the figure 3.2 below. With $m << M$ it is reasonable to assume that the central body is fixed in space and the equations of planar motion are derived from either Newton's second law or Lagrange as shown.

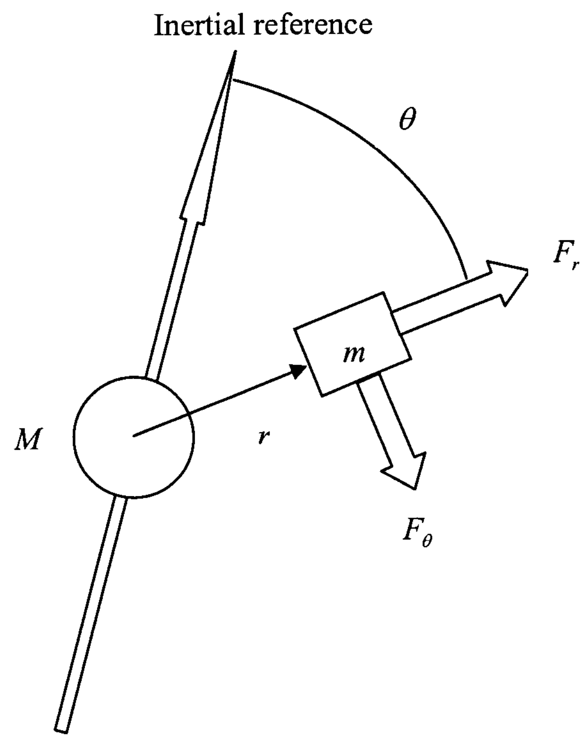


Fig. 3. 2 Satellite orbital dynamics

$$\left.\begin{aligned} \ddot{r} - r\dot{\theta}^2 &= -\frac{\mu}{r^2} + u_r \\ r\ddot{\theta} + 2\dot{r}\dot{\theta} &= u_\theta \end{aligned}\right\} \tag{3.5}$$

where $u_r = \frac{F_r}{m}$ and $u_\theta = \frac{F_\theta}{m}$ are the radial and transverse low-thrust control accelerations available from onboard thrusters, θ is the mean motion or orbital rate defined by

$$\theta = \sqrt{\frac{\mu}{r^3}} \tag{3.6}$$

and r is the orbital radius.

3.2 Classical Orbital Elements

We can also express the motion of a point-mass satellite around the earth in terms of orbital elements[36]. [28, 29] have shown that Lyapunov control based on orbital element feedback performs well in low-thrust, multi-orbit manoeuvres. Even though the second order vector differential equation in (3.3) is non-linear, the equation is capable of a completely general analytical solution by specific vector operations applied to the equation of motion re-written in the form

$$\frac{dv}{dt} = -\frac{\mu}{r^3} r \qquad (3.7)$$

In each case, the vector manipulations result in transformed versions of (3.7) which are perfect differentials and hence immediately integrable. The constants of integration called *integrals of motion* are also referred to as *orbital elements*.

In general, the above two body relative motion equation given in (3.3) has three degrees of freedom and the orbit is uniquely determined if the six initial conditions $\vec{r}$ and $\vec{v} \equiv \dot{\vec{r}}$ are specified. Such initial conditions can be considered as six possible classical orbital elements. Three of these six scalars specify the orientation of the orbit plane with respect to the *geocentric-equatorial* reference frame, which has it's origin at the center of the earth. This geocentric- equatorial reference frame has an inclination of 23.45° with respect to the heliocentric-eliptic reference frame that has its origin at the center of the sun.

A set of orthogonal unit vectors $\left\{\vec{I}, \vec{J}, \vec{K}\right\}$ is selected as basis vectors of the geocentric-equatorial reference frame with (X, Y, Z) co-ordinates as shown in the figure

3.3 below. The (X, Y) plane is the earth's equatorial plane, simply called the equator. The Z axis is along the earth's polar axis of rotation.

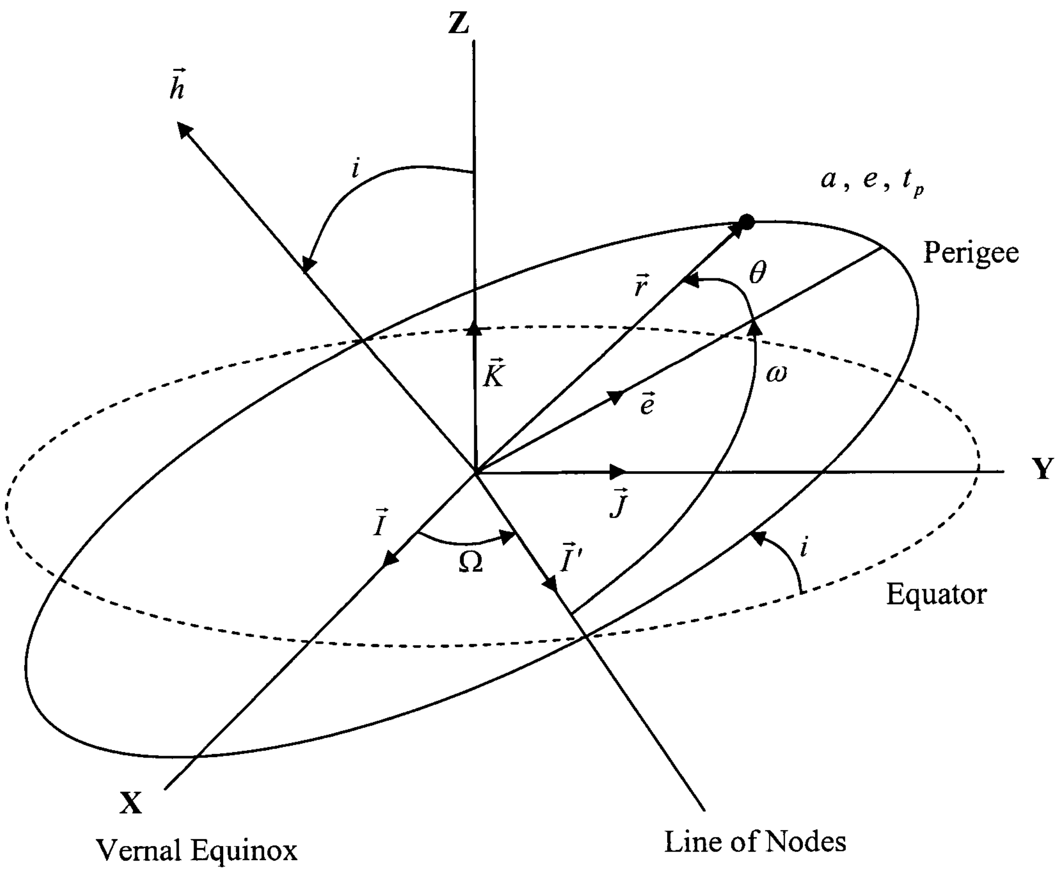


Fig. 3. 3 Orbit orientation using orbital elements

The six classical orbital elements consist of five independent quantities, which are sufficient to completely describe the size, shape and orientation of an orbit, and one quantity required to pinpoint the position of the satellite along the orbit at any particular time. The six classical orbital elements are:

a = Semi-major axis.

e = Eccentricity.

t_p = Time of perigee passage.

Ω = Right ascension longitude of the ascending node.

i = Inclination of the orbital plane.

ω = Argument of perigee.

The elements a and e determine the size and shape of the elliptic orbit, respectively, and t_p relates position in orbit to time. The angles Ω and i specify the orientation of the orbit plane with respect to the geocentric-equatorial reference frame. An orbit with i near 90° is called a polar orbit. An equatorial orbit has zero inclination. The angle ω specifies the orientation of the orbit in its plane. The equations of motion of a controlled spacecraft in terms of the classical orbital element set are given by Gauss's form of Lagrange's planetary equations, as follows:

$$\frac{d\Omega}{dt} = \frac{r \sin\theta}{h \sin i} u_h \tag{3.8}$$

$$\frac{di}{dt} = \frac{r \cos\theta}{h} u_h \tag{3.9}$$

$$\frac{d\omega}{dt} = \frac{1}{he}[-p \cos f\, u_r + (p+r) \sin f\, u_\theta] - \frac{r \sin\theta \cos i}{h \sin i} u_h \tag{3.10}$$

$$\frac{da}{dt} = \frac{2a^2}{h}(e \sin f\, u_r + \frac{p}{r} u_\theta) \tag{3.11}$$

$$\frac{de}{dt} = \frac{1}{h}\{p \sin f\, u_r + [(p+r)\cos f + r e]u_\theta\} \tag{3.12}$$

$$\frac{dM}{dt} = n + \frac{b}{a h e}[(p \cos f - 2 r e)u_r - (p+r)\sin f u_\theta \tag{3.13}$$

where u_r, u_θ and u_h are the radial, transverse, and orbit normal control acceleration components, M is the mean anomaly, $\theta = \omega + v$, is the argument of latitude, and b, h

and p are the semi-minor axis, the angular momentum, and the semi-latus rectum, respectively. The mean motion, n , is calculated from the gravitational parameter, μ and the semi-major axis, a as:

$$n = \sqrt{\frac{\mu}{a^3}} \tag{3.14}$$

which is equivalent to equation (3.6). In case of a circular orbit semi-major axis is equal to the semi-minor orbit. In the above equations it is also seen that the mean anomaly M or sometimes even the true anomaly θ replace the time of perigee passage t_p in the set of orbital elements and is only a case of convenience in computing and depends on the problem at hand.

3.3 Equinoctial Orbital Elements

Some researchers [37] use equinoctial orbital elements as further correction and accuracy to the classical orbital elements are added. [30] develops Lyapunov optimal feedback control techniques for low thrust orbit transfer between orbits of different semi-major axis, eccentricity, and inclination using both classical, and equinoctial orbital element sets. These equations have been stated here without much detail just for the sake of completeness.

Gauss's form of Lagrange's planetary equations contains singularities for circular or equatorial orbits. For orbits with near zero inclination or eccentricity, we can use the non-singular, equinoctial elements. We define the equinoctial orbital element set, using Battin's notation [38], as the semi-major axis, a, unnamed elements P_1, P_2, Q_1, Q_2 and the mean longitude, l.The non-singular equinoctial elements are defined in terms of the classical orbital elements as:

$$a = a,\ P_1 = e\sin\varpi,\ P_2 = e\cos\varpi,\ Q_1 = \tan\frac{1}{2}i\cos\Omega,\ Q_2 = \tan\frac{1}{2}i\sin\Omega,\ l = \varpi + M$$

where ϖ , the longitude of pericenter, is defined as :

$\varpi = \Omega + \omega$.

The true longitude, L, defined as :

$L = \varpi + v$

The reverse transformation back to the orbital elements is given by:

$$a = a \tag{3.15}$$

$$e = (P_1^2 + P_2^2)^{\frac{1}{2}} \tag{3.16}$$

$$i = 2\tan^{-1}(Q_1^2 + Q_2^2)^{\frac{1}{2}} \tag{3.17}$$

$$\Omega = \tan^{-1}\left(\frac{Q_1}{Q_2}\right) \tag{3.18}$$

$$\omega = \tan^{-1}\left(\frac{P_1}{P_2}\right) - \tan^{-1}\left(\frac{Q_1}{Q_2}\right) \tag{3.19}$$

$$M = l - \tan^{-1}\left(\frac{P_1}{P_2}\right) \tag{3.20}$$

3.4 Orbital Perturbations

In an ideal Keplerian orbit, the primary body which in our case is earth, has a spherically symmetric mass distribution and its orbital plane is fixed in space. Practically speaking we have to consider a non-keplerian orbit whose orbital plane is not fixed in space due to *asphericity* of earth. The small deviations from the ideal Keplerian orbital motion are called orbital perturbations. Here we describe the effects of earth's oblateness and atmospheric drag on the orbital motions of near-earth satellites.

3.4.1 Earth's Oblateness Effects

The earth is not a perfect sphere but it is an oblate spheroid of revolution; that is, the earth is flattened at the poles to produce geoid or ellipsoid of revolution. There are also minor harmonics of the earth's shape that produce a pear shape. This pear shape and the polar flattening cause perturbations to the satellite orbit. The equatorial bulge caused by the polar flattening is about twenty one kilometres and distorts the path of the satellite each time it passes either the ascending node or descending node. The attractive force from the bulge shifts the satellite path northward as the satellite approaches the equatorial plane from the south. As the satellite leaves the equatorial plane, it is shifted southward. The net result is the ascending node having shifted or regressed opposite the direction of satellite motion. Thus earth's oblateness not only causes motion of the orbital plane but also affects the position of satellites within the orbital plane.

3.4.2 Atmospheric Drag

Atmospheric drag is another very important perturbation for Low Earth Orbiting (LEO) satellites. The exact effect of atmospheric drag is difficult to predict due to uncertainties in the dynamics of the upper atmosphere. Air density is constantly changing in this region; there are diurnal variations because the sun heats up the air on day light side of the earth, causing the air to expand. This heating increases the number of molecules encountered by LEO satellites. There is a similar seasonal variation between summer and winter. There is also a 27-day cycle in atmospheric density, as well as an 11-year cycle [38]. Magnetic storms can heat the atmosphere as can solar flares. Major solar events emit charged particles that heat the outer atmosphere and produce significant changes in satellite orbits. Because the atmosphere drops off so rapidly with altitude, most drag is expected at perigee. The less time the satellite spends at near-perigee altitudes, the less total mechanical energy the satellite dissipates by air drag. A reduction in total energy produces a corresponding reduction in the length of the semi-major axis. Also, air friction causes the eccentricity of the orbit to diminish toward zero, making orbit more circular. Thus, it can be said that the apogee drops faster than the perigee in elliptical orbits. Orbital perturbations due to atmospheric drag are relevant to this thesis as the simulations consider a LEO constellation. Air density drops off so rapidly with increasing height that high-altitude satellites can essentially ignore air drag.

3.5 Summary

In this chapter, we have presented the orbital dynamics expressed in three different ways. In the following chapters we use the two body relative motion equation in polar coordinates for designing the control laws and simulations. Here, we have also discussed the factors responsible for satellite orbit perturbations. The nonlinear controllers designed in this thesis later on will be used to correct the perturbations caused by disturbing effects given above.

Chapter 4

Formation Flying Using Potential Function Method

In this chapter, we present a detailed critique of the decentralized relative formation flying method, introduced by Colin R. McInnes [4]. We start with the concept of Lyapunov's Direct Method since the potential function method used by McInnes is based on it. Later, the actual potential function method and derivation of the control laws is described for formation flying as devised by McInnes. We end the chapter with simulations and discussion on using potential function method for formation flying.

4.1 Lyapunov's Second Method

Lyapunov's Second method also known as Lyapunov's direct method is a well-known mechanism to study various qualitative properties such as stability, asymptotic stability, etc. of both linear and nonlinear systems. It is based on the concept of energy and the relation of stored energy with system stability. Consider a physical system described by the equation

$$\dot{x}(t) = f(x(t))$$

and let $x(x(t_0),t)$ be a solution. Further, let $V(x)$ be the total energy associated with the system.

If the derivative $\frac{dV(x)}{dt}$ is negative for all $x(x(t_0),t)$ except the equilibrium point, then it follows that energy of the system decreases as t increases and finally the system will reach the equilibrium point. This holds because energy is non-negative function of the system state which reaches a minimum only if the system motion stops. For ready reference with McInnes potential function method given later and without going into much detail, Lyapunov's method in its simplest form can be stated as follows:

For a system described by,

$\dot{x} = f(x)$; $f(0) = 0$

suppose that there exists a scalar function $V(x)$ which, for some real number $\varepsilon > 0$, satisfies the following properties for all x in the region $\|x\| \leq \varepsilon$:

- $V(x) > 0$; $x \neq 0$
- $V(0) = 0$, thus $V(x)$ is a positive definite scalar function
- $V(x)$ has continuous partial derivatives with respect to all components of x.
- $\frac{dV}{dt} < 0$ (i.e. $\frac{dV}{dt}$ is negative definite scalar function)

Then the system is asymptotically stable. It is intuitively obvious since a continuous V function, $V > 0$ except at $x = 0$, satisfies the condition $\frac{dV}{dt} < 0$, we expect that x will eventually approach the origin.

4.2 Potential Function Method

In this section we provide the background, concept and approach of deriving the potential function method used by McInnes for deriving the formation flying control laws.

4.2.1 Background

Lyapunov's Second Method was originally translated from Russian in the early part of the twentieth century. In the early sixties, two seminal papers by Kalman and Bertram [39, 40] evolved the technique into the *Potential Function Method,* which has been in widespread use in terrestrial application ever since. Professor Colin McInnes [4, 41-43] at the University of Glasgow began the process of applying the method to space based applications. Further work by McInnes has also applied the technique to multi-spacecraft operations. The control method was originally designed for the control of free flying robots to be used in space construction problems. However, the relative interaction and co-operative nature of the method has direct applications in formation flying. The function itself is based on the separation and positioning of the satellite relative to the constellation members. The main conclusion drawn from the work is that the Potential Function Method represents a promising method of relative control in complex systems.

4.2.2 Concept

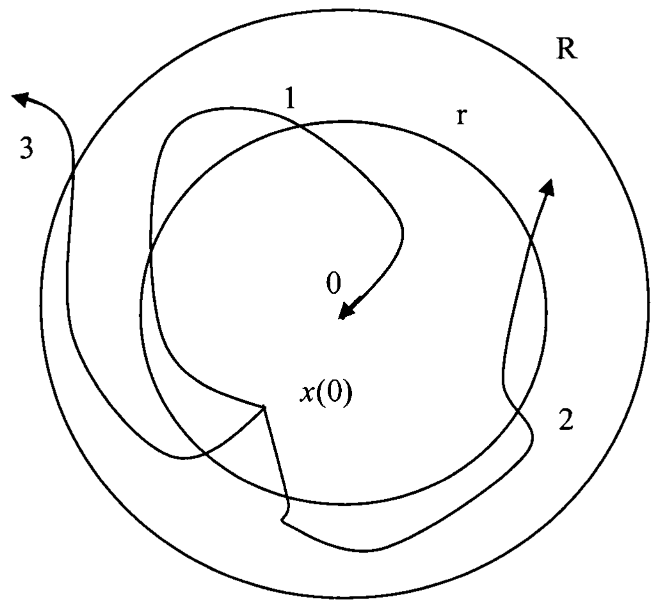
The method used by McInnes is the *Potential Function Method*, which, in turn, is based on *Lyapunov's Second Method.* The method utilises a *Potential Function* to measure the '*correctness*' of the formation. When the Potential Function evaluates to zero, the formation is correctly deployed. This means that for a constellation of N satellites, the desired configuration corresponds to zero energy. Then, if the rate of change of the energy is negative, i.e. the energy is continually decreasing, then eventually the energy will decrease to zero and the system will converge to the desired configuration. An analytical solution is obtained to manoeuvre the satellites in such a manner to reduce the potential and thus configure the formation. However, the method is not devoid of problems inasmuch as the Potential function itself is problem-dependent and relies too much on presumptions.

4.2.3 Approach

A *Potential Function* is analytically derived that describes the potential energy of the system. McInnes has then used a stability analysis approach by searching for a control function to fit the pre-defined potential function. We discuss the notion of Lyapunov stability in the next section.

4.3 Stability

The equilibrium state $x = 0$ is said to be stable if, for any $R > 0$, there exists $r > 0$, such that if $\|x(0)\| < r$, then $\|x(t)\| < R$ for all $t \geq 0$. Otherwise the equilibrium point is unstable. Essentially, stability also called as Lyapunov stability means that the system trajectory can be kept arbitrarily close to the origin by starting sufficiently close to it.



Curve 1 - Asymptotically stable

Curve 2 - Marginally stable

Curve 3 - Unstable

Fig. 4. 1 Stability and its definitions

In our application of formation control or absolute station keeping Lyapunov stability is not enough. When a satellite's position is disturbed from its desired position, we not only want the satellite to maintain its position in a range determined by the magnitude of the disturbance i.e. Lyapunov stability, but also require that the position gradually go back to its original value. This requirement is covered under asymptotic stability which is formalized next.

Definition: Equilibrium point 0 is asymptotically stable if it is stable and if in addition there exists some $r > 0$ such that $\|x(0)\| < r$ implies that $x(t) \to 0$ as $t \to \infty$.

4.3.1 Global Asymptotic Stability

The above definitions are formulated to characterize the local behavior of systems i.e. the evolution of states after starting near the equilibrium point. These properties say little about how the system will behave when the initial state is some distance away from the equilibrium. This is specified by the next definition.

Definition: If asymptotic stability holds for any initial states, the equilibrium point is said to be globally asymptotically stable.

The specifics of the mathematical model used and the control laws derived are given in the next section. We will investigate subsequently in this thesis the local and global asymptotic stability of our formation control model for the linear as well as the original nonlinear model.

4.4 Control Laws

The two body relative equation of satellite motion around the earth given in Chapter 3 used to represent the dynamical system. For a constellation of N satellites in orbit around the earth, the same equation can be rewritten as

$$\left.\begin{aligned} \ddot{r}_i - r_i\dot{\theta}_i^2 &= -\frac{\mu}{r_i^2} + u_{ri} \\ r_i\ddot{\theta}_i + 2\dot{r}_i\dot{\theta}_i &= u_{\theta i} \end{aligned}\right\} \quad (i = 1,\ldots,N) \tag{4.1}$$

where,

$u_{ri} = \dfrac{F_{ri}}{m} =$ Radial low thrust control acceleration.

$u_{\theta i} = \dfrac{F_{\theta i}}{m} =$ Transverse low thrust control acceleration.

It should be noted that the derivation and controller synthesis is based on linearized dynamics model of (4.1). It is linearized around the desired orbital operating point of(r,θ), where r is the desired radius and θ is the desired orbital rate. The linearized set of equations is given by

$$\left.\begin{array}{l} \ddot{l}_i - 2\omega r\dot{\phi}_i - 3\omega^2 l_i = u_{li} \\ r\ddot{\phi}_i + 2\omega\dot{l}_i = u_{\phi i} \end{array}\right\} \quad (i = 1,\ldots,N) \tag{4.2}$$

where, $\phi_i = \theta_i - \omega t$, $l_i = r_i - \tilde{r}$, ϕ is the actual angular position of the satellite with respect to the ring, l is the offset between the actual and operational radius of the ring and ω is the Keplerian angular velocity.

A *Potential Function* is analytically derived that describes the potential energy of the system. It is then used to drive the state vector of the system to the desired configuration.

$$\overline{V} = \frac{1}{2}\sum_{i=1}^{N}(\dot{\phi}_i^2 + \dot{l}_i^2) + \lambda_i \sum_{j=1, j\neq i}^{N}\sec^2(\phi_{ij}) \tag{4.3}$$

where, $\phi_{ij} = \frac{1}{2}[\phi_i - (\phi_j - \pi)]$

The first part of the potential function in (4.3) is the kinetic energy of the system, so that the satellite remains steady in the desired position once this term drops to zero. The second part is a periodic function such that $V(\phi + 2k\pi) = V(\phi)$ for $k = 1,2,\ldots,$ integer. This will ensure that the potential increases as any two satellites move close and decreases as they get repelled away from each other. The selection of (3) as the candidate potential function ensures the following:

- Satellite Separation

➢ Collision Avoidance

The lower the potential, the closer the geometry of the formation is to an operational configuration. For a minimum energy which is the desired configuration the rate of change of potential of the system should be negative definite.

$$\frac{d\overline{V}}{dt} = \sum_{i=1}^{N} \dot{l}_i \ddot{l}_i + \sum_{i=1}^{N} \dot{\phi}_i \left\{ \ddot{\phi}_i - 2\lambda_i \sum_{j=1, j\neq i}^{N} \sec^2(\phi_{ij}) \tan(\phi_{ij}) \right\} \tag{4.4}$$

In his methodology, McInnes follows the conventional stability analysis approach by searching for a control function to fit the pre-defined Lyapunov function. This way he ensures that he has a feasible control function that satisfies the stability criterion of Lyapunov function methodology. The control function selected is as follows:

$$\left. \begin{aligned} u_{li} &= -k_{li} \dot{l}_i - 2\omega r \dot{\phi}_i - 3\omega^2 l_i \\ u_{\phi i} &= -k_{2i} r \dot{\phi}_i + 2\omega \dot{l}_i + 2r\lambda_i \sum_{j=1, j\neq i}^{N} \sec^2(\phi_{ij}) \tan(\phi_{ij}) \end{aligned} \right\} \tag{4.5}$$

Substituting (5) in (2), we have,

$$\left. \begin{aligned} \ddot{l}_i &= -k_{li} \dot{l}_i \\ \ddot{\phi}_i &= -k_{2i} \dot{\phi}_i + 2\lambda_i \sum_{j=1, j\neq i}^{N} \sec^2(\phi_{ij}) \tan(\phi_{ij}) \end{aligned} \right\} \tag{4.6}$$

Therefore, the rate of change of potential (4) can be rewritten as,

$$\frac{d\overline{V}}{dt} = -\sum_{i=1}^{N} k_{1i} \dot{l}_i^2 - \sum_{i=1}^{N} k_{2i} \dot{\phi}_i^2 \tag{4.7}$$

To ensure that the solution is reached, the rate of change of the potential must be assured negative semi-definite. As seen from (4.7) the rate of change of potential function is made to be negative semi-definite by the choice of the specified control laws in (4.5). This may be expressed in simpler terms by considering that as the rate of change of potential is always nonpositive, the potential itself shall always be nonpositive. Therefore, the configuration shall converge to the solution at the potential equal to zero. In the next section, we will go over some simulations of formation flying using the control laws developed by McInnes.

4.5 Mission Simulations

In this section, we have designed two mission scenarios to test the performance of the formation control laws designed by McInnes. Towards this end, we consider a constellation of 3 satellites each of mass of $75Kg$ with $10N$ thrusters for radial and transverse control. The satellites are deployed in a relative formation at an altitude of $400Km$ and angular separation of 120°.

The objective of the formation controller is to track the required radius and maintain equal angular separation. In Table 4.1 we consider the formation variables in the absence of any external perturbation.

Table 4. 1 Simulation scenario - potential function method

Satellite 1	Satellite 2	Satellite 3
$r = 400Km$ $\delta r = 0Km$	$r = 400Km$ $\delta r = 0Km$	$r = 400Km$ $\delta r = 0Km$
$\phi = 0^\circ$ $\delta\phi = 0^\circ$	$\phi = 120^\circ$ $\delta\phi = 0^\circ$	$\phi = 240^\circ$ $\delta\phi = 0^\circ$

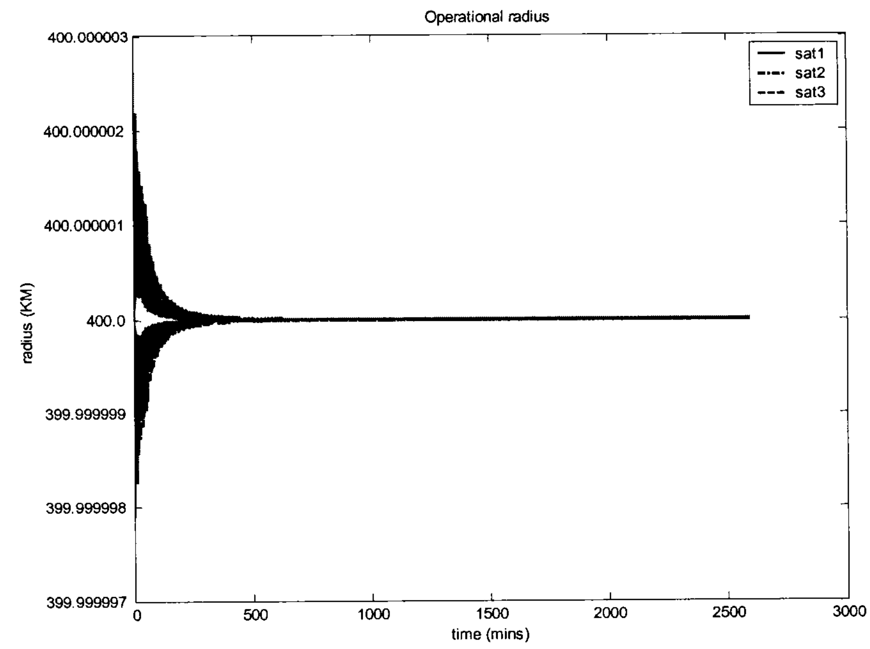


Fig. 4. 2 Operational radii of the 3 satellites in formation

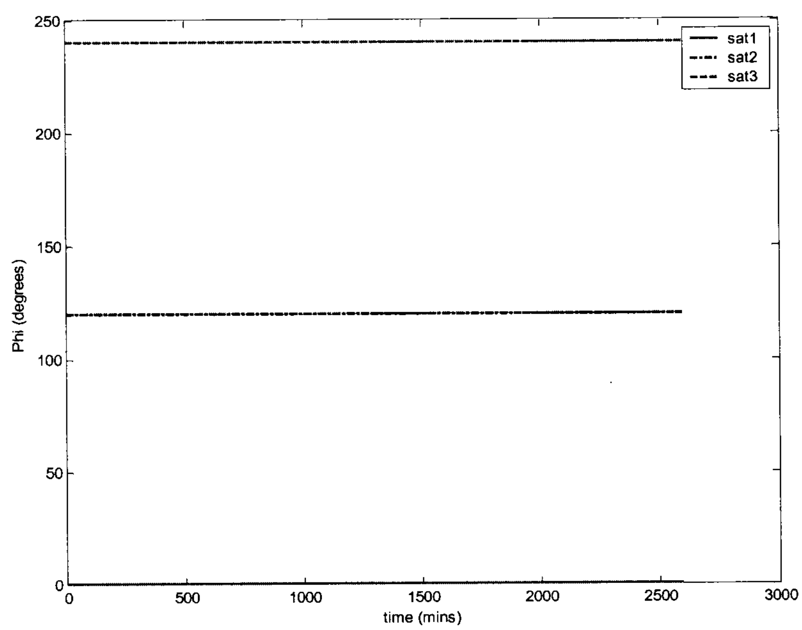


Fig. 4. 3 Operational phi of the 3 satellites in formation

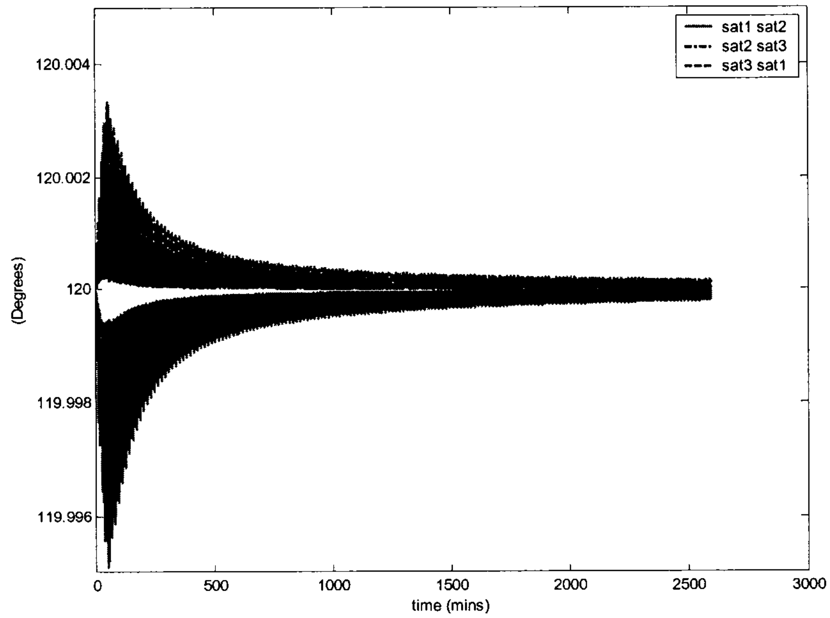


Fig. 4. 4 Inter-satellite spacing

In Table 4.2, we consider the formation variables with small perturbations in the azimuthal direction.

Table 4. 2 Simulation scenario - potential function method

Satellite 1	Satellite 2	Satellite 3
$r = 400Km$ $\delta r = 0Km$	$r = 400Km$ $\delta r = 0Km$	$r = 400Km$ $\delta r = 0Km$
$\phi = 0.0001^\circ$ $\delta\phi = -0.0001^\circ$	$\phi = 119.999^\circ$ $\delta\phi = 0.0001^\circ$	$\phi = 239.999^\circ$ $\delta\phi = 0.0001^\circ$

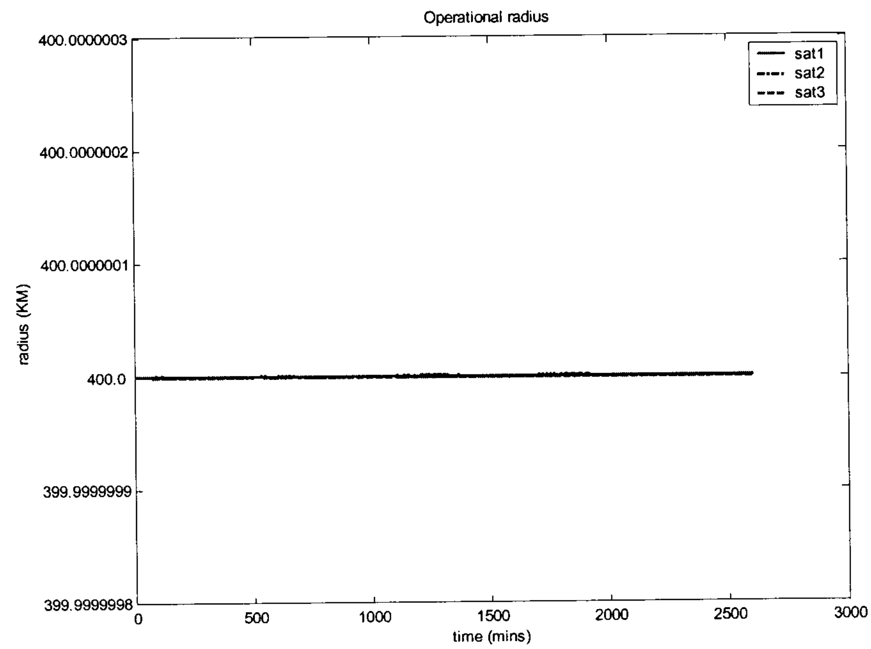


Fig. 4. 5 Operational radii of the 3 satellites in formation

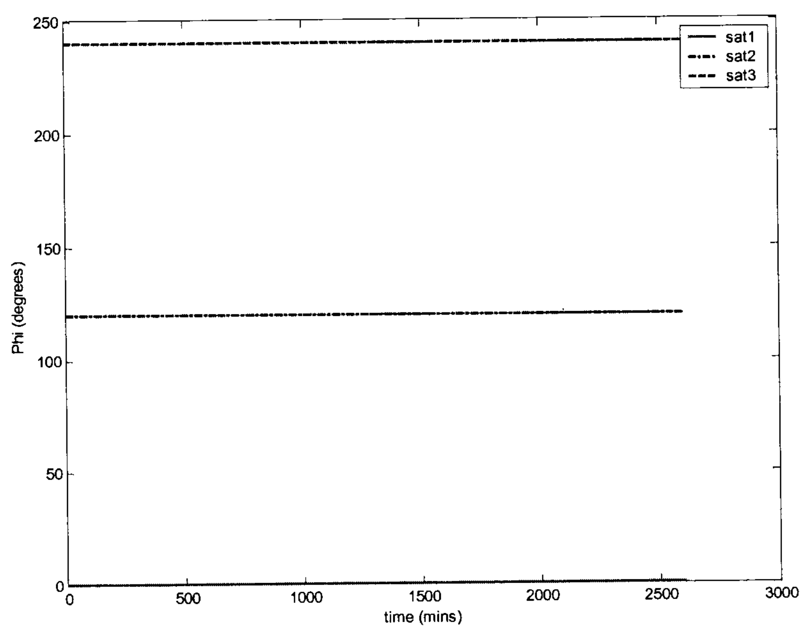


Fig. 4. 6 Operational phi of the 3 satellites in formation

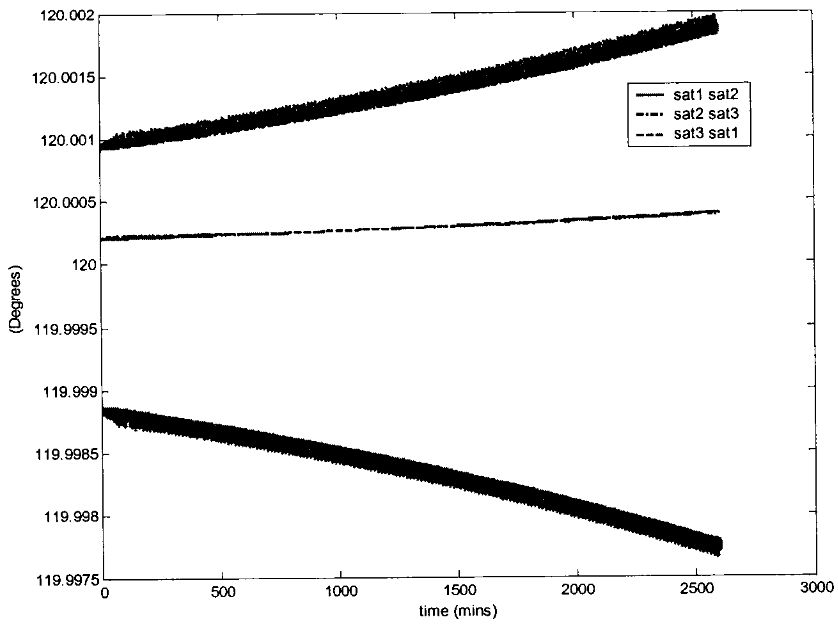


Fig. 4. 7 Inter-satellite spacing

4.6 Concluding Remarks

- From the simulation scenario in Table 4.1 and the figures 4.2, 4.3, 4.4 we can see that the designed formation control laws are able to hold the formation together. Equal Intersatellite spacing is maintained and satellites maintain the orbital radius they are launched at. There are no perturbations introduced.
- From the simulation in Table 4.2 and the figures 4.5, 4.6, 4.7, we can see that, the control law leads to a complex nonlinear interaction between the satellites. All the satellites are minimally perturbed in order to check the formation maneuvering performance of the controller. Although the inter-satellite spacing is not thrown into instability instantly, it shows a trend that would lead to a gradual drift and formation break-up if not corrected manually by the ground segment. It is evident

that although the control laws loosely hold the formation together they are unsuitable for large perturbation corrections i.e. the group cannot maintain formation very well during the manoeuvres.

- Mission scenarios with radial perturbation are not carried out since the designed control laws do not allow correction in the radial direction. This is one of the major limitations which is not addressed by McInnes in his methodology.
- This approach is not only hard to analyze mathematically but also has a limited ability for precise formation keeping.
- The design method is not devoid of problems inasmuch as the potential function itself is problem-dependent and relies too much on presumptions.

The potential function method shows promising results for formation flying and it can prove beneficial for long term formation flying missions which need intersatellite communications and some ground assistance. On the other hand, this might become one of its major disadvantages as it heavily relies on the availability of a communication channel and bandwidth for data passing between different satellites. Though McInnes has claimed that the formation control laws can control the formation in the event of orbital perturbations, it is clearly seen from the simulations that, it has a limited capability to do so. The methodology cannot be used where precise position control of the agents is of prime importance.

Chapter 5

Controller Using Linearization Technique

In this chapter, we design a formation controller using the linearization technique. The nonlinear two body relative equation of motion is first linearized around the operating point. State feedback using pole-placement technique is then used to develop an orbital controller for this linearized set of equations. Applying the appropriate definitions of variables used for linearization, the control law for the nonlinear state-space representation is then obtained. Further we present formation control mission simulations for the linear controller on the linear dynamics and for the linear controller on the nonlinear dynamics. We end the chapter with discussion and remarks on the formation control performance of the developed controller.

5.1 Linearization

A pole placement controller is developed for the linearized dynamics of the equation of satellite motion. Here, we discuss the Linearization theory briefly for the convenience of the reader and for the sake of completeness.

Qualitatively, a system is described as stable if starting the system somewhere near its desired operating point implies that it will stay around the point ever after. The linearization method draws conclusions about a non-linear systems local stability around an equilibrium point from the stability properties of its linear approximation. A state x^* is an equilibrium state (or equilibrium point) of the system if once $x(t)$ is equal to x^* for all future time.

Mathematically, this means that the constant vector x^* satisfies

$$0 = f(x^*)$$

Equilibrium points can be found by solving the above non-linear equation. A linear time-invariant system $\dot{x} = Ax$ has a single equilibrium point (the origin 0) if A is nonsingular. If A is singular, it has infinity of equilibrium points, which are contained in the null space of the matrix A, i.e. the subspace defined by $Ax = 0$.This implies that the equilibrium points are not isolated.

Practically all the physical systems are non-linear in nature. In the following sections we would be developing a state feedback orbital controller for the satellite model. Hence, we should linearize the non-linear satellite model around an operating point. The intuitive basis of linearization is that a smooth curve differs very little from its tangent line so long as the variable does not wander far from the point of tangency. Subsequently the region of operation of a linearized non-linear equation should be restricted to a narrow range.

The state equation $\dot{x} = f(x,u)$ of a general time-invariant system can be linearized for small variations about an equilibrium point (x_0, u_o).It is assumed that the system is in equilibrium under the conditions x_0 and u_0 , that is,

$$\dot{x} = f(x,u) = 0 \tag{5.1}$$

Since the derivatives of all the state variables are zero at the equilibrium point, the system continuous to lie at the equilibrium point unless otherwise disturbed. The state equation can be linearized about the operating point (x_0, u_o) by expanding it into Taylor's series and neglecting the terms of second and higher order. Thus for the i th state equation

$$\dot{x}_i = f_i(x_0, u_0) + \sum_{j=1}^{n} \left.\frac{\partial f_i(x,u)}{\partial x_i}\right|_{x=x_0 \& u=u_0} (x_j - x_{j0}) + \sum_{k=1}^{m} \left.\frac{\partial f_i(x,u)}{\partial u_k}\right|_{x=x_0 \& u=u_0} (u_k - u_{k0}) \qquad (5.2)$$

Recognizing that at the operating point $f_i(x_0, u_o) = 0$ and defining the variation about the operating point as

$$\begin{aligned} \tilde{x}_j &= x_j - x_{j0} \qquad \therefore \dot{\tilde{x}}_j = \dot{x}_j \\ \tilde{u}_k &= u_k - u_{k0} \end{aligned} \qquad (5.3)$$

the linearized i th state equation can be written as

$$\dot{\tilde{x}}_i = \sum_{j=1}^{n} \left.\frac{\partial f_i(x,u)}{\partial x_j}\right|_{x=x_0 \& u=u_0} \tilde{x}_j + \sum_{k=1}^{m} \left.\frac{\partial f_i(x,u)}{\partial u_k}\right|_{x=x_0 \& u=u_0} \tilde{u}_k \qquad (5.4)$$

The above linearized component equation can be written as the vector matrix equation

$\dot{\tilde{x}} = A\tilde{x} + B\tilde{u}$ where,

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} \frac{\partial f_1}{\partial x_2} \cdots\cdots \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} \frac{\partial f_2}{\partial x_2} \cdots\cdots \frac{\partial f_2}{\partial x_n} \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \frac{\partial f_n}{\partial x_1} \frac{\partial f_n}{\partial x_2} \cdots\cdots \frac{\partial f_n}{\partial x_n} \end{bmatrix}_{(x_0, u_0)} \qquad B = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} \frac{\partial f_1}{\partial u_2} \cdots\cdots \frac{\partial f_1}{\partial u_m} \\ \frac{\partial f_2}{\partial u_1} \frac{\partial f_2}{\partial u_2} \cdots\cdots \frac{\partial f_2}{\partial u_m} \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \frac{\partial f_n}{\partial u_1} \frac{\partial f_n}{\partial u_2} \cdots\cdots \frac{\partial f_n}{\partial u_m} \end{bmatrix}_{(x_0, u_0)} \qquad (5.5)$$

Matrices A and B are called the Jacobian matrices. All the partial derivatives in the matrices A and B defined above are evaluated at the equilibrium point (x_0, u_o).

In most systems governed by non-linear systems the exact analytic solution is not known and is rarely found analytically. But the dynamical equation we are considering here, the exact analytic solution is known to be a conic,

$$r = \frac{\rho}{1 - e.\cos(\theta - \delta)}, \qquad \text{giving} \qquad \begin{vmatrix} e = 0 - circle \\ 0 < e < 1 - ellipse \\ e = 1 - parabola \\ e > 1 - hyperbola \end{vmatrix} \tag{5.6}$$

where $p = \frac{(2.A)^2}{\mu}$ semi-latus rectum and

$$e = \sqrt{1 + p^2 (v_0{}^2 - \frac{2\mu}{r_0})} \quad , \quad v_0 = r_0 \dot{\theta}_0 \quad , \quad A = \frac{r_0{}^2 \dot{\theta}_0}{2}$$

Since the satellite path we are interested in here is circular, we select the operating point (r_0, θ_0) as a convenient point on the circular orbit. Thus, now we can linearize the satellite non-linear dynamical equation at the equilibrium point. The above non-linear equation can be converted into state-space representation as follows,

Let $x_1 = r, x_2 = \dot{r},\ x_3 = \theta,\ \ x_4 = \dot{\theta}$ and corresponding inputs are $u_1 = u_r$ and $u_2 = u_\theta$ then,

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_1 x_4{}^2 - \frac{\mu}{x_1{}^2} + u_1 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{-2x_2x_4}{x_1} + \frac{1}{x_1} u_2 \end{aligned}\right\} \tag{5.7}$$

To linearize, $F(x,u)$ can be expanded into Taylor's series around the operating point and higher order terms discarded.

The linearized set of equations of motion are given by (5.8)

$$\left.\begin{aligned} &\ddot{l}_i - 2\omega r\dot{\phi} - 3\omega^2 l_i = u_{li} \\ &r\ddot{\phi}_i + 2\omega\dot{l}_i = u_{\phi i} \end{aligned}\right\} \tag{5.8}$$

The above system can be represented in the state-space form as under:

$$x_1 = l_i,\ x_2 = \dot{l}_i, x_3 = \phi_i,\ x_4 = \dot{\phi}_i,\ u_1 = u_{li},\ u_2 = u_{\phi i}$$

$$\left.\begin{aligned} &\dot{x}_1 = x_2 \\ &\dot{x}_2 = 2\omega r x_4 + 3\omega^2 x_1 + u_1 \\ &\dot{x}_3 = x_4 \\ &\dot{x}_4 = -\frac{2\omega}{r}x_2 + \frac{1}{r}u_2 \end{aligned}\right\} \tag{5.9}$$

Pole placement controller would be developed for the above set of linear equation. Next section gives an overview on various conventional and modern techniques used for designing a linear controller. Later we discuss in detail the pole placement technique and give brief definitions for the Linear Quadratic and Linear Quadratic Gaussian Controllers.

5.2 Controller design

Lyapunov's linearization method discussed above is concerned with the local stability of a non-linear system. It is a formalization of the intuition that a non-linear system should behave similarly to its linearized approximation for small perturbations.

Since all physical systems are inherently nonlinear, it serves as the justification of using linear control techniques in practice.

Controllers vary widely in complexity and effectiveness. Simple controllers include the proportional (P), the proportional plus derivative (PD), the proportional plus integral (PI) and the proportional plus integral plus derivative (PID) controllers, which are widely and effectively used in many industries. More sophisticated controllers include the linear quadratic regulator (LQR), the estimated state feedback controller, and the linear Gaussian controller (LQG). Controllers are designed by many methods. Simple P or PI controllers have only a few parameters to specify, and these parameters might be adjusted empirically, while the control system is operating, using “tuning rules”. Controller design methods developed in the 1930’s through the 1950’s are often called as classical controller design methods. In 1960’s through the present time, state-space or “modern” controller design methods have been developed. These methods are based on the fact that the solutions to some optimal control problems can be expressed in the form of a feedback law or controller and the development of efficient computer methods to solve these optimal control problems.

5.2.1 Pole Placement Technique

A fundamental method of classical design consists of forcing the dominant close loop poles to be suitably located in the s- plane or z- plane to ensure satisfactory transient response. In the pole placement technique full state feedback is considered which gives us greater freedom to satisfy the classical performance indices. Using full state feedback permits us to place all the poles of the characteristic equation at any desired positions. Consider the single-input-single-output system with nth –order state model

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

The state variable feedback for this system is essentially a scalar function which takes the form

$$\sigma = Kx = [k_1 k_2 k_3 \ldots k_n]\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

The block diagram of the system with state variable feedback is shown below. We can see from this figure that

$u = -kx + r$ or $u = -kx$ if $r = 0$

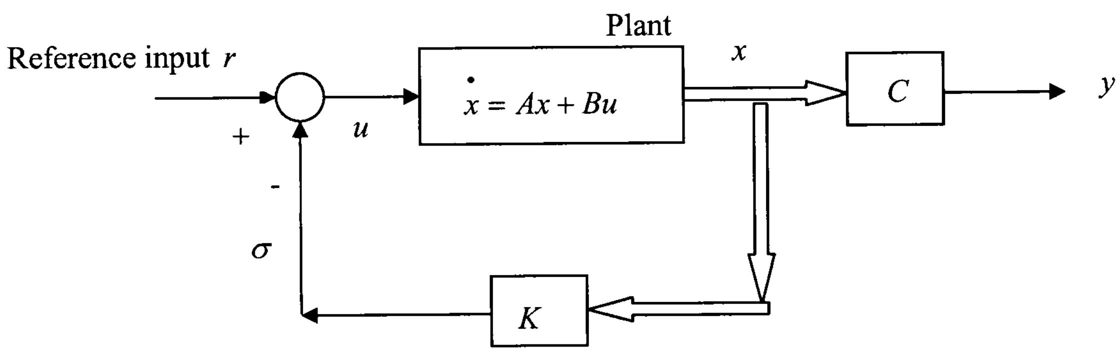


Fig. 5. 1 State variable feedback system

where u is the plant input and r is the reference input. The closed loop equation becomes

$$\dot{x} = (A - BK)x$$

It is already shown that if the pair (A, B) is controllable, then by state feedback equation above, the eigen-values of $(A - BK)$ can be arbitrarily assigned. The eigen-values of the closed loop state feedback system are roots of

$|\lambda I - A + BK| = 0$.

5.2.2 Linear Quadratic Regulator (LQR)

LQR falls under the realm of optimal control systems. The LQR design is an optimal pole placement technique which combines the linear quadratic control and the pole placement technique. Its design is based on a linear model of the system and a quadratic performance index that penalizes both the divergences of the output of the system with respect to the desired values and the control effort. The problem consists in finding a control that minimizes the quadratic performance index (i.e., both the terms under the integral sign and the function of the final event are quadratic functions), when the dynamic system is linear. The well known analytical solution for this problem makes reference to the solution of matrix Riccati equations. The result is a linear feedback of the state variable of the system multiplied by a set of gains. LQR has guaranteed gain and phase margins.

5.2.3 Linear Quadratic Gaussian Controller (LQG)

The LQG problem is a well-established approach to optimal control for stochastic linear systems. In order to find an optimal control law that minimizes a certain performance index (which is necessarily taken as an expected value), the Kalman filter is used to estimate the state. Thus, the optimal estimate for the state is taken such as to minimize the variance of the estimation error. The solution for the LQG problem is proved to be a controller that includes in its dynamics a Kalman filter. Mathematically, the designer is confronted with two differential (or algebraic, in the constant case) Riccati equations.

In this thesis we have used the pole placement technique for designing the linear controller. The next section covers the design of the full state feedback controller for the linearized set of dynamical equations.

5.3 Linear Controller by Pole Placement

In this section, we design a state feedback controller using pole placement technique for the linearized set of dynamical equation. Towards this end, we consider a constellation of 3 satellites each of mass of $75Kg$ with $10N$ thrusters for radial and transverse control. The satellites are deployed in an absolute formation at an altitude of $400Km$ and angular separation of 120°. The objective of the individual/decentralized satellite controllers is to track the required radius and maintain angular separation by tracking the angle specifications and requirements that are assigned by an upper level supervisory controller. For such a critical application, the control logic of the formation deployment must display the following properties:

- Closed loop performance guarantee.
- Smooth convergence to the solution.
- Scaleable (In capability and autonomy).
- Stablility.

We now show that the state feedback control laws given below can achieve the above objective by appropriately selecting the controller gains, namely we set

$$\left.\begin{array}{l} u_1 = g_2 x_2 + g_3 x_4 + g_1 x_1 \\ u_2 = g_4 x_4 + g_5 x_2 + g_6 x_3 \end{array}\right\} \quad (5.10)$$

Therefore, the closed loop state space representation of the system may be given as,

$$\left.\begin{array}{l} \dot{x}_1 = x_2 \\ \dot{x}_2 = g_2 x_2 + (3\omega^2 + g_1)x_1 + (2\omega r + g_3) + v_1 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \frac{g_4}{r}x_4 + \frac{g_6}{r}x_3 + (-\frac{2\omega}{r} + \frac{g_5}{r})x_2 + v_2 \end{array}\right\} \quad (5.11)$$

where the reference signals v_1 and v_2 are added for achieving set point regulation or tracking. With Keplerian angular velocity $\omega = 0.06\,\text{deg/sec}$ and orbital radius $r = 400Km$, open loop A and B matrices can be written as:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 3\omega^2 & 0 & 0 & 2\omega r \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{2\omega}{r} & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0.0108 & 0 & 0 & 48 \\ 0 & 0 & 0 & 1 \\ 0 & -0.003 & 0 & 0 \end{bmatrix} \quad (5.12)$$

$$B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1/r \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0.0025 \end{bmatrix} \quad (5.13)$$

Out of the four open loop poles for the linearized system dynamics two lie at the origin and the other two have their real parts at the origin and the imaginary part at $\pm 0.06i$. It is clear that the system in open loop is unstable. The closed loop representation of the satellite may then be expressed as follows:

$$A - BK = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \alpha_1 & \alpha_2 & 0 & \alpha_3 \\ 0 & 0 & 0 & 1 \\ 0 & \beta_1 & \beta_3 & \beta_2 \end{bmatrix} \quad (5.14)$$

where,

$$\left.\begin{aligned}
\alpha_1 &= 3\omega^2 + g_1 \\
\alpha_2 &= g_2 \\
\alpha_3 &= 2\omega r + g_3 \\
\beta_1 &= -\frac{2\omega}{r} + \frac{g_5}{r} \\
\beta_2 &= \frac{g_4}{r} \\
\beta_3 &= \frac{g_6}{r}
\end{aligned}\right\} \tag{5.15}$$

In order to design pole placement gains, the system has to be controllable. The pair A and B are controllable with full rank. Therefore, pole placement technique can be used. Placing the four poles at $\lambda_1 = -0.1, \lambda_2 = -1.0, \lambda_3 = -2.0, \lambda_4 = -3.0$ and solving for (5.11) and (5.14) the controller gains obtained in our case are,

$$\left.\begin{aligned}
g_1 &= --0.410 \\
g_2 &= -2.2 \\
g_3 &= -48 \\
g_4 &= -1600 \\
g_5 &= 0.12 \\
g_6 &= -1200
\end{aligned}\right\} \tag{5.16}$$

The external reference signals v_1 and v_2 that are to provide tracking commands for $x_1 = l_i$ and $x_3 = \phi_i$, respectively, are to be designed as follows,

$$\left.\begin{aligned}
v_1 &= -(3\omega^2 + g_1)x_1^d \\
v_2 &= -g_6 x_3^d
\end{aligned}\right\} \tag{5.17}$$

where x_1^d and x_3^d are the desired reference commands for x_1 and x_3, respectively. The designed controller will be tested on the linear as well as the non-linear dynamics of the satellite.

5.4 Linear Controller by Pole Placement for Nonlinear Model

In the previous section a linear orbital controller is designed for the linearized satellite dynamics. The above linear controller should actually be applied to the nonlinear system dynamics in order to evaluate how it will perform in more realistic conditions and in the presence of nonlinearities. Applying the appropriate definitions of variables used for linearization of, the control law for the nonlinear state-space representation is now obtained.

Since, $x_1 = r, x_2 = \dot{r}, x_3 = \theta, x_4 = \dot{\theta}$, we have,

$$\left.\begin{aligned} u_1 &= g_2 x_2 + g_3(x_4 - \omega) + g_1(x_1 - r) \\ u_2 &= g_4(x_4 - \omega) + g_5 x_2 + g_6(x_3 - \omega t) \end{aligned}\right\} \tag{5.18}$$

The external reference signals v_1 and v_2 for providing tracking for $x_1 = r_i$ and $x_3 = \theta_i$ respectively, can be assigned as follows,

$$\left.\begin{aligned} v_1 &= -(3\omega^2 + g_1)x_1^d \\ v_2 &= -g_6(x_3^d - \omega t) \end{aligned}\right\} \tag{5.19}$$

The controller designed for the linear dynamics as well as nonlinear satellite dynamics will be put to performance tests in the next section under two different simulation scenarios.

5.5 Formation Control Set-up and Simulations

In this section, we present mission simulations using the controllers developed above for the linear as well as nonlinear orbital dynamics. As discussed above, we consider a constellation of 3 satellites each of mass of $75Kg$ with $10N$ thrusters for radial and transverse control. The satellites are deployed in an absolute formation at an altitude of $400Km$ and angular separation of 120°. The objective of the individual/decentralized satellite controllers is to track the required radius and maintain angular separation by tracking the angle specifications and requirements that are assigned by an upper level supervisory controller. The results obtained are discussed subsequently.

5.5.1 Mission simulation (Linear Dynamics)

In Table 5.1 we consider the formation variables in equilibrium in the absence of any external perturbation. The following figures show the dynamical behaviour of the **linear formation dynamics** with the pole placement controller.

Table 5. 1 Simulation scenario - pole placement linear dynamics

Satellite 1	Satellite 2	Satellite 3
$r = 400Km$ $\delta r = 0Km$	$r = 400Km$ $\delta r = 0Km$	$r = 400Km$ $\delta r = 0Km$
$\phi = 0^\circ$ $\delta\phi = 0^\circ$	$\phi = 120^\circ$ $\delta\phi = 0^\circ$	$\phi = 240^\circ$ $\delta\phi = 0^\circ$

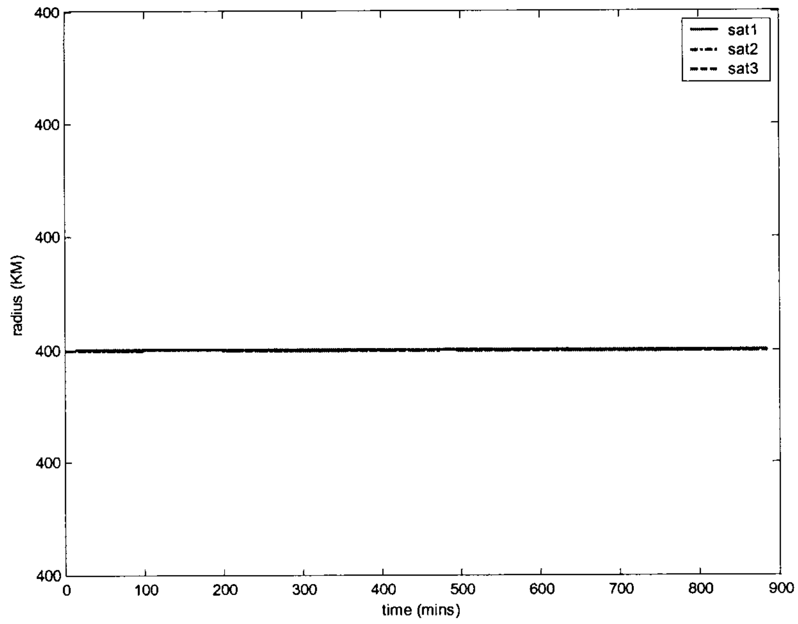


Fig. 5. 2 Operational radii of the 3 satellites in formation

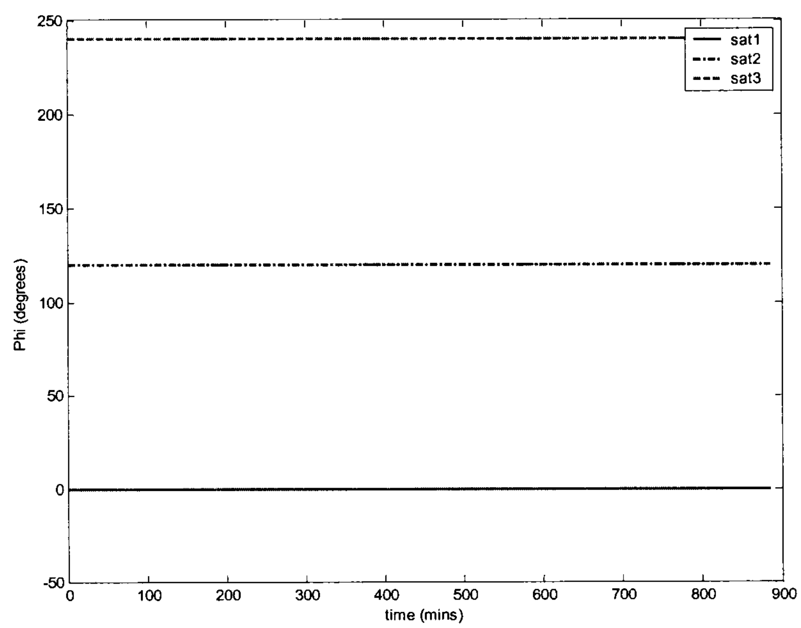


Fig. 5. 3 Operational phi of the 3 satellites in formation

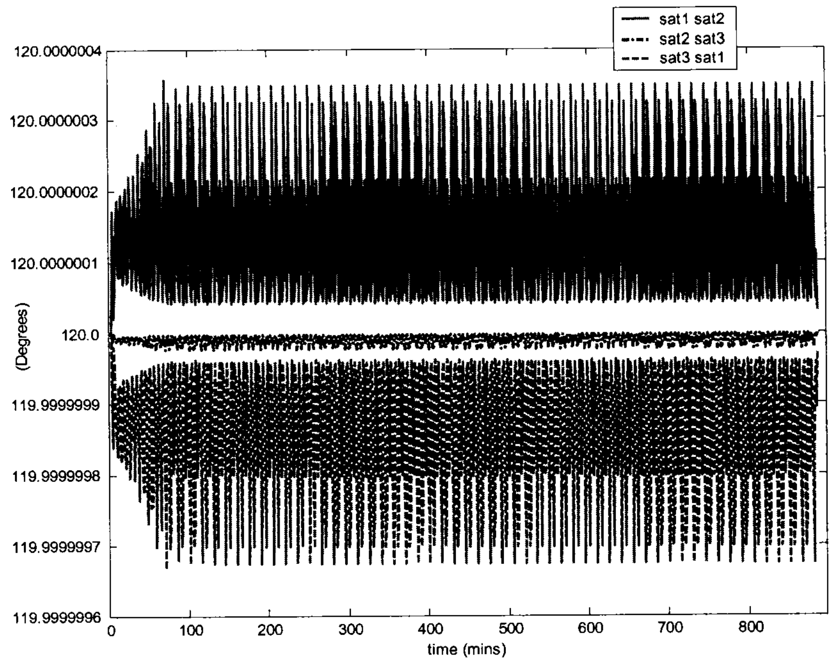


Fig. 5. 4 Intersatellite spacing

In Table 5.2, we consider the formation variables with perturbations in the azimuthal as well as radial direction. The control laws are expected to correct for these perturbations and track the desired positions. The following figures simulate the dynamical behaviour of the **linear formation dynamics** with the pole placement controller.

Table 5. 2 Simulation scenario - pole placement linear dynamics

Satellite 1	Satellite 2	Satellite 3
$r = 400Km$ $\delta r = 390Km$	$r = 400Km$ $\delta r = 410Km$	$r = 400Km$ $\delta r = 380Km$
$\phi = 1^\circ$ $\delta\phi = -1^\circ$	$\phi = 119^\circ$ $\delta\phi = 1^\circ$	$\phi = 238^\circ$ $\delta\phi = 2^\circ$

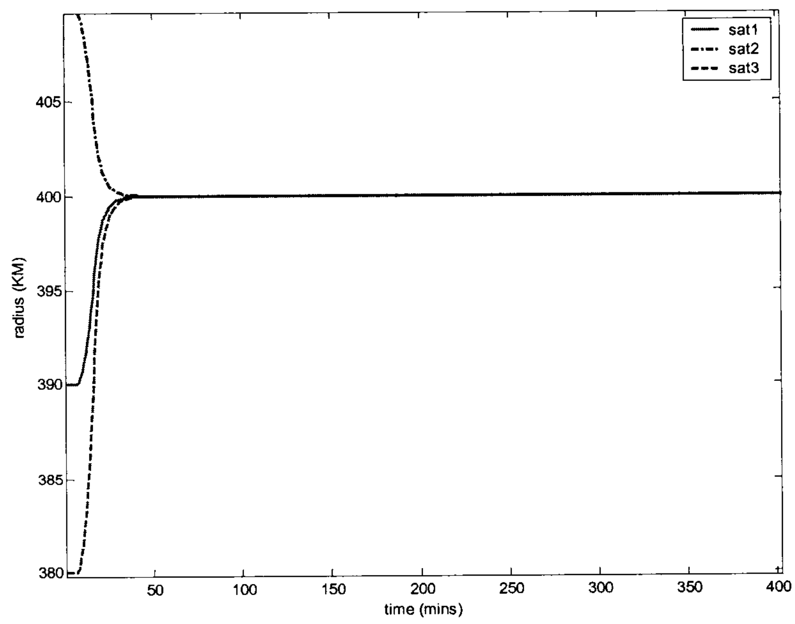


Fig. 5. 5 Operational radii of the 3 satellites in formation

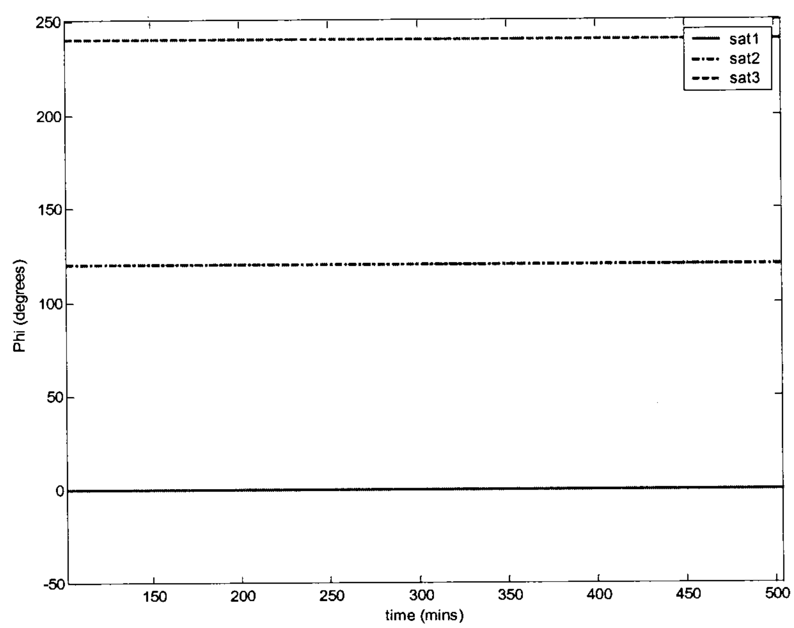


Fig. 5. 6 Operational phi of the 3 satellites in formation

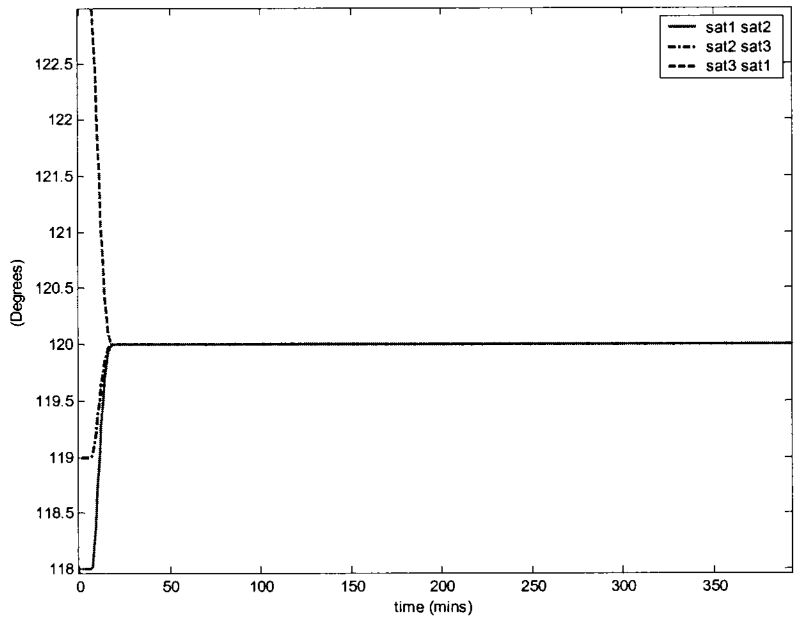


Fig. 5.7 Intersatellite spacing

5.5.2 Mission Simulation (Nonlinear Dynamics)

In Table 5.3 we consider the formation variables in equilibrium in the absence of any external perturbation. The following figures show the dynamical behaviour of the **nonlinear formation dynamics** with the pole placement controller.

Table 5. 3 Simulation scenario - pole placement nonlinear dynamics

Satellite 1	Satellite 2	Satellite 3
$r = 400Km$ $\delta r = 0Km$	$r = 400Km$ $\delta r = 0Km$	$r = 400Km$ $\delta r = 0Km$
$\phi = 0°$ $\delta\phi = 0°$	$\phi = 120°$ $\delta\phi = 0°$	$\phi = 240°$ $\delta\phi = 0°$

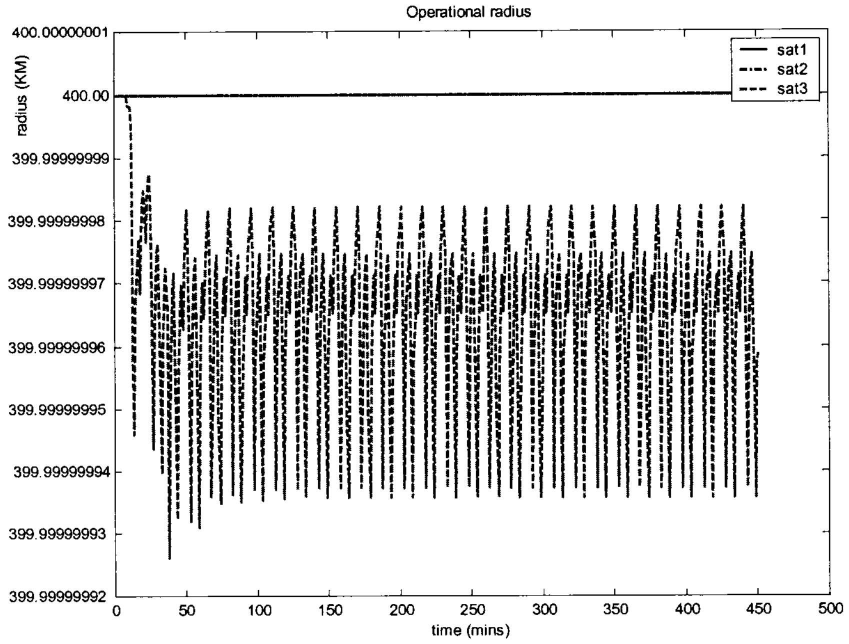


Fig. 5. 8 Operational radii of the 3 satellites in formation

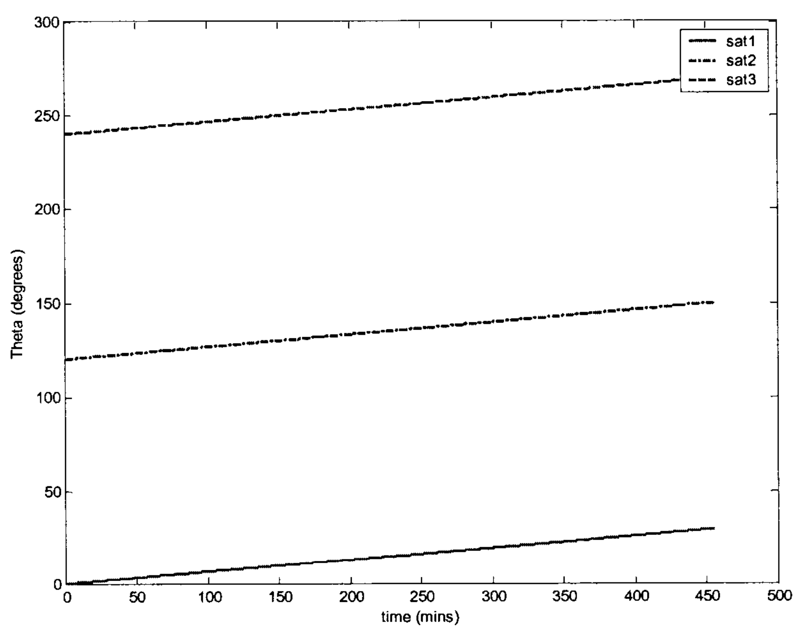


Fig. 5. 9 Operational theta of the 3 satellites in formation

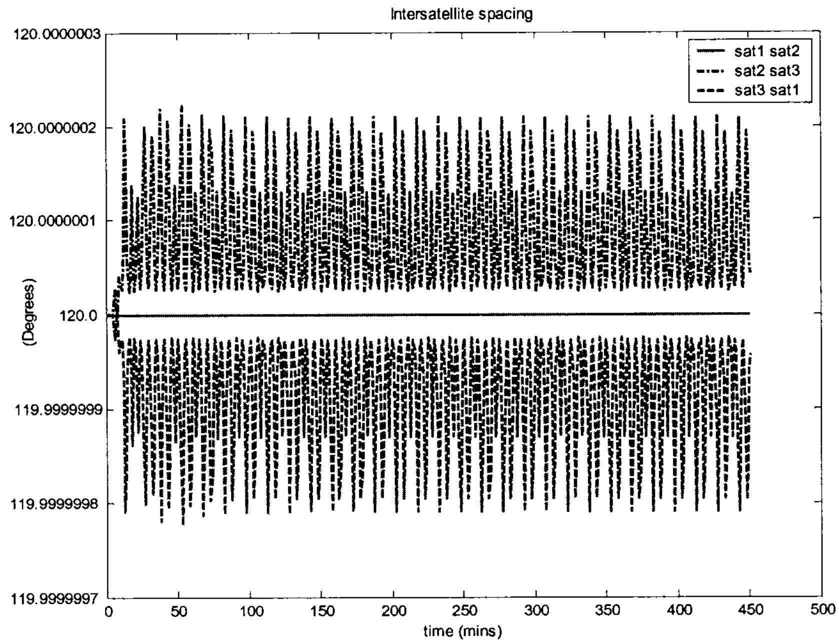


Fig. 5. 10 Intersatellite spacing

In Table 5.4, we consider the formation variables with perturbations in the azimuthal as well as radial direction. The control laws are expected to correct for these perturbations and track the desired positions. The following figures show the dynamical behaviour of the **nonlinear formation dynamics** with the pole placement controller. For this case we also show the control effort required for correcting the tracking error.

Table 5. 4 Simulation scenario - pole placement nonlinear dynamics

Satellite 1	Satellite 2	Satellite 3
$r = 399.95Km$ $\delta r = 0.05Km$	$r = 399.99Km$ $\delta r = 0.01Km$	$r = 399.98Km$ $\delta r = 0.02Km$
$\phi = 0.03^\circ$ $\delta\phi = -0.03^\circ$	$\phi = 120.01^\circ$ $\delta\phi = -0.01^\circ$	$\phi = 239.97^\circ$ $\delta\phi = 0.03^\circ$

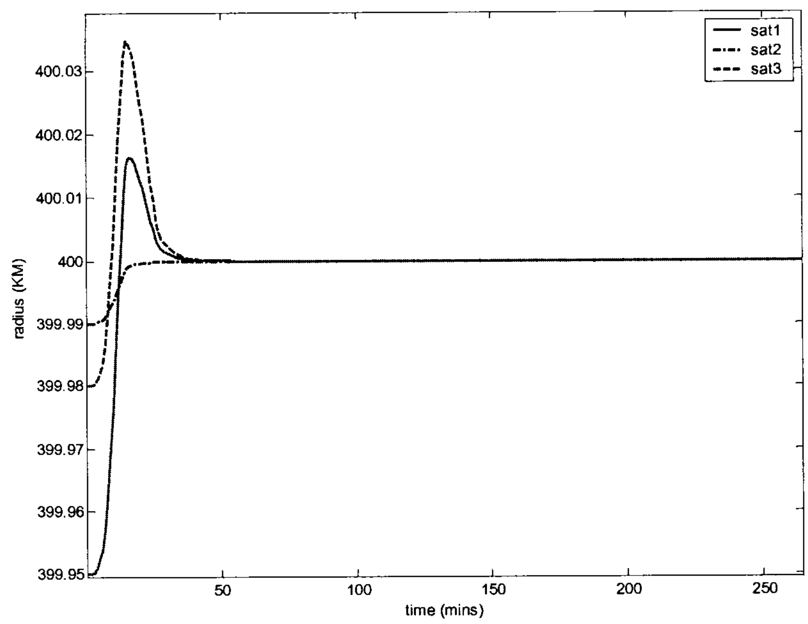


Fig. 5. 11 Operational radii of the 3 satellites in formation

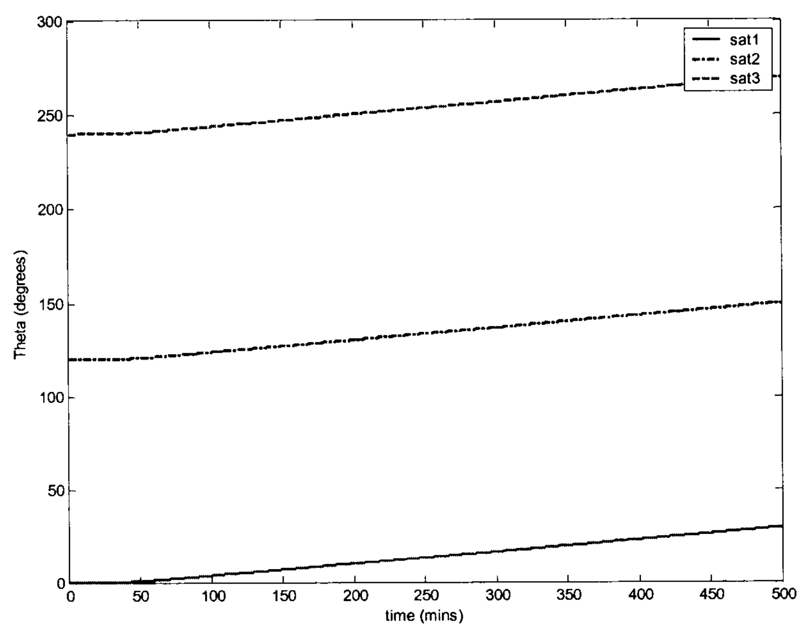


Fig. 5. 12 Operational theta of the 3 satellites in formation

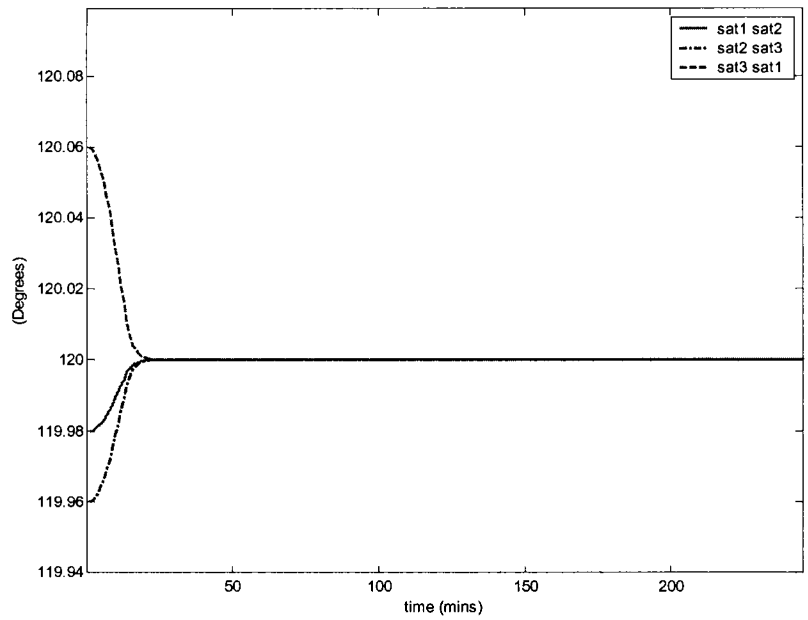


Fig. 5. 13 Intersatellite spacing

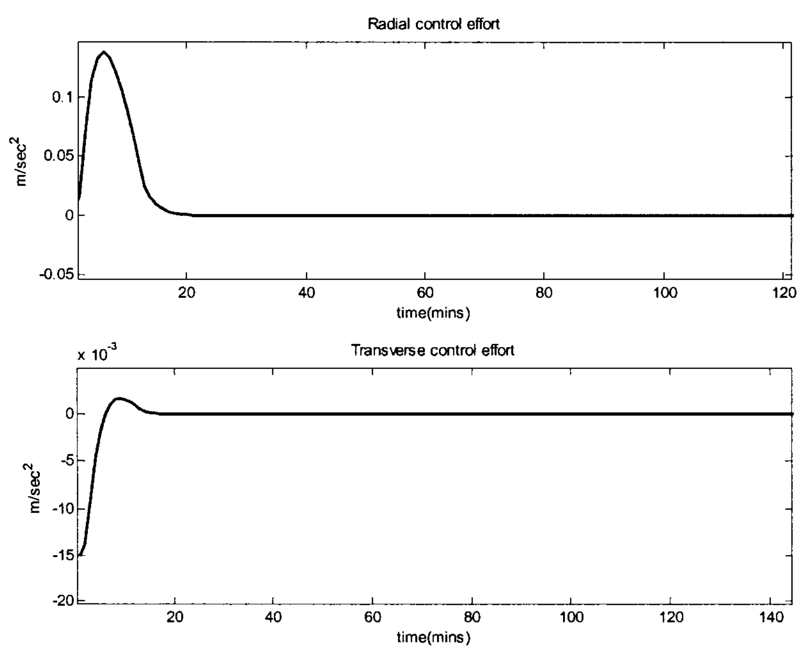


Fig. 5. 14 Control effort for satellite 1

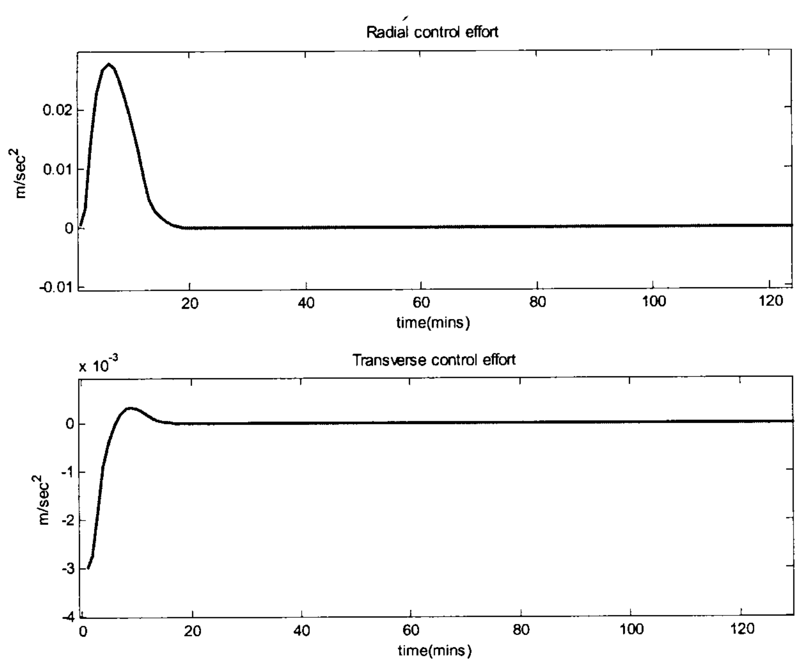


Fig. 5. 15 Control effort for satellite 2

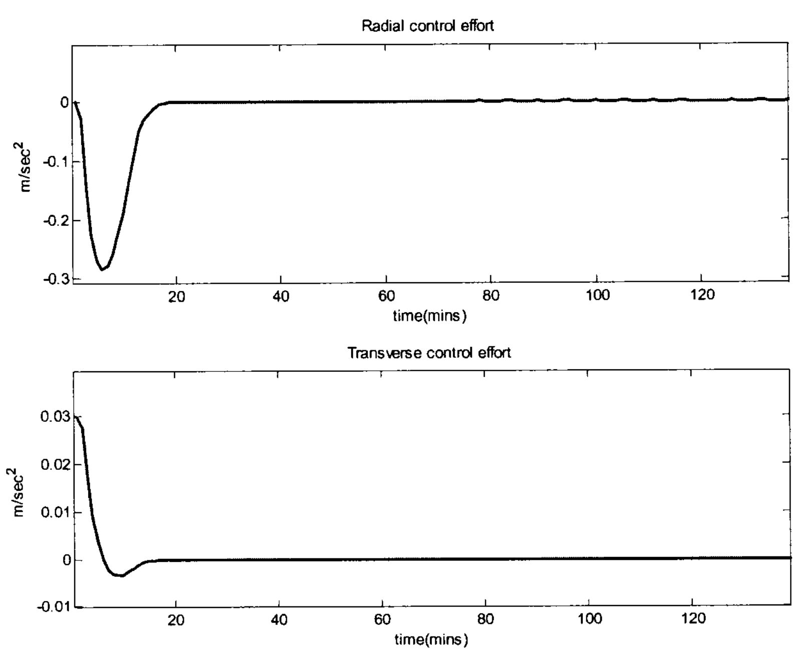


Fig. 5. 16 Control effort for satellite 3

5.6 Concluding Remarks

For the mission simulations for the linear dynamics (Fig. 5.2 – Fig. 5.7), we can see that the linear controller is globally asymptotically stable. The controller is able to maintain the formation in the desired state, in the absence of perturbations. In the presence of perturbations, all the satellites in the formation are able to manoeuvre back into the desired position. All the satellites in the formation maintain their position and do not drift.

In the case of nonlinear dynamics (Fig. 5.8 – Fig. 5.16), we can see that, the linear controller is locally asymptotically stable. It is able to maintain the formation very well in the absence of perturbations as well as in the presence of perturbations; the satellites smoothly manoeuvre back to their desired positions. It should be noted here though that, the perturbation tolerance range of the linear controller is very small. Beyond the specified ranges found by trial and error as given below, the system becomes unstable.

The closed-loop system is <u>stable</u> for <u>only</u> $399.2 < r < 400.3$ and $-0.09 < \delta\theta < 0.03$. Due to this shortcoming, it would be required to design a gain scheduling controller repeating the above design procedure for different operating conditions. This is a laborious procedure and again depends too much on assumptions and approximations. In order to improve the perturbation tolerance range for the formation controller and also to avoid the tedious process of linearization at different operating points, we will design an orbital controller in the next chapter using the feedback linearization or nonlinear compensation technique.

Chapter 6

Controller Design Using Feedback Linearization

In the previous chapter an orbital controller was designed using the pole-placement technique. The dynamical equation of the satellite in a circular orbit was first linearized around the intended operating point. The mapping of the linear controller on the nonlinear dynamics showed local asymptotic stability but very low robustness to large perturbations around the operating point.

In this chapter, feedback linearization technique will be used for designing the orbital controller. The chapter ends with simulations showing absolute formation control and station keeping for a constellation of three satellites in a circular orbit. Intuitively, the use of nonlinear control as opposed to linear control should result in significant fuel savings. This savings should be particularly evident when feedback control is used to perform large maneuvers, where the nonlinear dynamics effects become pronounced.

6.1 Nonlinear Phenomena and Limitations of Linearization

As we saw in the previous chapters, an important concept in dealing with the state equation is the concept of an equilibrium point. A point $x = x^*$ in the state space is said to be an equilibrium point of $\dot{x} = f(t,x)$ if it has the property that whenever the state of the system starts at x^*, it will remain at x^* for all the future time. As we move from linear to non-linear systems, the situation becomes a bit more difficult. The superposition principle does not hold any longer and analysis tools involve more advanced mathematics.

Since linearization is an approximation of the non-linear system at the operating point, it shows the local behavior of the nonlinear system near the equilibrium point. It cannot predict the "non local" behavior far from the operating point and certainly not the "global" behavior throughout the state space. The dynamics of the non-linear system are much complex than the linearized version of the same equation. So there are special nonlinear phenomena that can occur only in the presence of nonlinearity and which cannot be described by or predicted by linear models. The linear versus non-linear phenomena are briefly enumerated in the table below.

Table 6. 1 Linear v/s nonlinear system characteristics

Characteristic	Linear time-invariant systems	Non-linear systems
A. Finite escape time	States can go to infinity as time approaches infinity	States can go to infinity in finite time.
B. Multiple isolated equilibria	Can have only a single equilibrium point to which the states converge irrespective of the initial system	May have multiple equilibrium points and state convergence depends on the initial system state.

	state.	
C. Limit cycles	Doesn't oscillate unless the system has a pair of eigen values on the imaginary axis	A stable oscillation must be produced by non-linear systems irrespective of the initial state.
D. Sub harmonic, harmonic or almost-periodic oscillations	A periodic input produces an output of the same frequency when the system is stable.	A periodic input may produce an output which is a multiple or sub multiple of the input frequency.
E. Chaos	Linear systems do not exhibit chaos	A nonlinear system may have a more complicated steady-state behavior that is not equilibrium, periodic oscillation or almost periodic oscillation termed as chaos.
F. Multiple modes of behavior	Non-existent in linear system	The same non-linear system may exhibit more than one mode of behavior, e.g. an unforced system may have more than one limit cycle.

6.2 Feedback Linearization

6.2.1 Concept

The main concept of feedback linearization is to find a state-feedback control

$$u = \alpha(x) + \beta(x)v$$

and a change of variables $z = T(x)$ if required, for a class of non-linear systems of the form

$$\dot{x} = f(x) + G(x)u$$
$$y = h(x)$$

which will transform the non-linear system into equivalent linear system. This linearization approach is different than the one used in the previous chapter. No approximation is used in this case; it is exact. We assume perfect knowledge of the state equation and use that to "cancel" the non-linearities in the system.

6.2.2 State-space Structure

In order to "cancel" the non-linearities the state-space equation should be in a specific form. In order to cancel a non-linear term $\alpha(x)$ by subtraction, the control u and the non-linearity $\alpha(x)$ must always appear as a sum $u + \alpha(x)$. To cancel a non-linear term $\gamma(x)$ by division, the control u and the non-linearity $\gamma(x)$ must always appear as a product $\gamma(x)u$. If the matrix $\gamma(x)$ is non-singular in the domain of interest, then it can be cancelled by $u = \beta(x)v$, where $\beta(x) = \gamma^{-1}(x)$ is the inverse of the matrix $\gamma(x)$.

Therefore, the ability to use feedback to convert a non-linear state equation into a controllable linear state equation by canceling non-linearities requires the non-linear state equation to have the structure

$$\dot{x} = Ax + B\gamma(x)[u - \alpha(x)]$$

where A is $n \times n$, B is $n \times p$, the pair (A,B) is controllable, the functions $\alpha : R^n \to R^p$ and $\gamma : R^n \to R^{pxp}$ are defined in a domain $D \subset R^n$ that contains the origin, and the matrix $\gamma(x)$ is non-singular for every $x \in D$. If the state equation takes the above form, then we can linearize it via the state feedback

$$u = \alpha(x) + \beta(x)v$$

where $\beta(x) = \gamma^{-1}(x)$, to obtain the linear state equation,

$$\dot{x} = Ax + Bv$$

For stabilization, we can then design $v = -kx$ such that $A - BK$ is Hurwitz or so that the roots of the characteristic polynomial equation fall in the left half s-plane. The overall non-linear stabilizing state feedback control is then

$$u = \alpha(x) - \beta(x)Kx$$

6.3 Controller Design

Below, we have the set of nonlinear state-space equation of the satellite motion around the earth as stated earlier. The nonlinear dynamical equations are repeated here just for the sake of convenience.

$$\left.\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= x_1 x_4{}^2 - \frac{\mu}{x_1{}^2} + u_1\\ \dot{x}_3 &= x_4\\ \dot{x}_4 &= \frac{-2x_2 x_4}{x_1} + \frac{1}{x_1} u_2\end{aligned}\right\} \tag{6.1}$$

The basic philosophy of feedback linearization is to cancel the non-linear terms. From the previous chapter we know that the linearized state-space of the above equation is controllable and stabilizable. Matching the non-linear state-space equation with the above general feedback linearizable equation it is clear that it is in the feedback linearizable form. The following controls are designed to cancel the non-linearities and convert it into a linear state-space equation.

$$\left.\begin{aligned}u_1 &= -x_1 x_4^2 + \frac{\mu}{x_1^2} + a_1 x_1 + a_2 x_1\\ u_2 &= 2x_2 x_4 + b_1 x_1 x_4 + b_{21} x_1 x_3\end{aligned}\right\} \tag{6.2}$$

Substituting equation (6.2) into the non-linear state-space (6.1) above we get the linearized closed loop equation as follows,

$$\left.\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= a_1 x_2 + a_2 x_1\\ \dot{x}_3 &= x_4\\ \dot{x}_4 &= b_1 x_4 + b_2 x_3\end{aligned}\right\} \tag{6.3}$$

Therefore,

$$[\ A - BK\ ] = \begin{bmatrix} 0 & 1 & 0 & 0\\ a_2 & a_1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & b_2 & b_1\end{bmatrix} \tag{6.4}$$

which is in the Jordan canonical form, by selecting appropriate values of a_1, a_2, b_1 and b_2, we can place the poles of the resultant linear system in the left half of the s plane.

Let us place the poles close to the imaginary axis to avoid peaking in the transient behavior and to avoid high gain values though this may mean slow stabilization. This is so that we may avoid overshoots in the system stabilizations.

Let $\lambda_1 = -0.1$ and $\lambda_2 = -0.15$ for the top left portion of the Jordan block. The resultant values of $a_1 = -0.25$ and $a_2 = -0.015$. Let $\lambda_1 = -0.2$ and $\lambda_2 = -0.25$ for the bottom right portion of the Jordan block. The resultant values of $b_1 = -0.45$ and $b_2 = -0.05$. The control laws designed above regulates all the variables. We need x_1 i.e. r to track a reference input equal to the desired orbital radius. Also x_3 i.e. $\theta = \phi + \omega t$ should track the desired angle from the inertial reference. The set points will be decided by an upper level planner which may be onboard one of the spacecraft or on the ground so that the absolute formation is maintained.

6.4 Tracking

Tracking can be achieved by adding the following tracking terms v_1 and v_2 to the resultant linearized state-space equation (6.3) as follows,

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= a_1 x_2 + a_2 x_1 + v_1 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= b_1 x_4 + b_2 x_3 + v_2 \end{aligned}\right\} \tag{6.5}$$

We require the variables to have the following tracking map,

$$\left.\begin{array}{l} x_1 \rightarrow R \\ x_2 \rightarrow 0 \\ x_3 \rightarrow \omega t \\ x_4 \rightarrow \omega \end{array}\right\} \tag{6.6}$$

where,

$R = x_1^d$ = Desired orbital radius.

$\omega t = x_3^d$ = Desired angle from the inertial reference.

$\omega = x_4^d$ = Desired angular velocity.

We can define the error terms as follows,

$$\left.\begin{array}{l} e_1 = x_1 - x_1^d \\ e_2 = x_2 - \dot{x}_1^d \\ e_3 = x_3 - x_3^d \\ e_4 = x_4 - \dot{x}_4^d \end{array}\right\} \tag{6.7}$$

Therefore the tracking error dynamics can be stated as,

$$\left.\begin{array}{l} \dot{e}_1 = \dot{x}_1 - \dot{x}_1^d = e_2 \\ \dot{e}_2 = a_1 x_2 + a_2 x_1 + v_1 - \ddot{x}_1^d \\ \dot{e}_3 = e_4 \\ \dot{e}_4 = b_1 x_4 + b_2 x_3 + v_2 - \ddot{x}_3^d \end{array}\right\} \tag{6.8}$$

Hence the tracking terms that need to be added to the resultant state-space in order to nullify the error between the reference and the actual variables are,

$$\left.\begin{array}{l} v_1 = -a_1 x_2 - a_2 x_1 + \ddot{x}_1^d \\ v_2 = -b_1 x_4 - b_2 x_3 + \ddot{x}_3^d \end{array}\right\} \tag{6.9}$$

Thus we now have an orbital controller. In the following section we have tested the controller on various formation keeping scenarios to gauge it's effectiveness in formation keeping and formation maneuvering to see if it satisfies our requirements.

6.5 Mission Simulation

In this section, we will test the performance of the designed formation controller in various scenarios. As in the previous chapters, the Table 6.2 tests the orbital controller in steady state and in the absence of external perturbations. In the table 6.3, all the satellites will be subjected to small radial as well as transverse perturbations equivalent to table 5.4 in chapter 5. In table 6.4, we test the orbital controller with larger perturbations. The results obtained are discussed subsequently.

The specific parameters, initial conditions and the situations considered are provided in the table bellow:

Table 6. 2 Simulation scenario - feedback linearized controller

Satellite 1	Satellite 2	Satellite 3
$r = 400Km$ $\delta r = 0Km$	$r = 400Km$ $\delta r = 0Km$	$r = 400Km$ $\delta r = 0Km$
$\phi = 0^\circ$ $\delta\phi = 0^\circ$	$\phi = 120^\circ$ $\delta\phi = 0^\circ$	$\phi = 240^\circ$ $\delta\phi = 0^\circ$

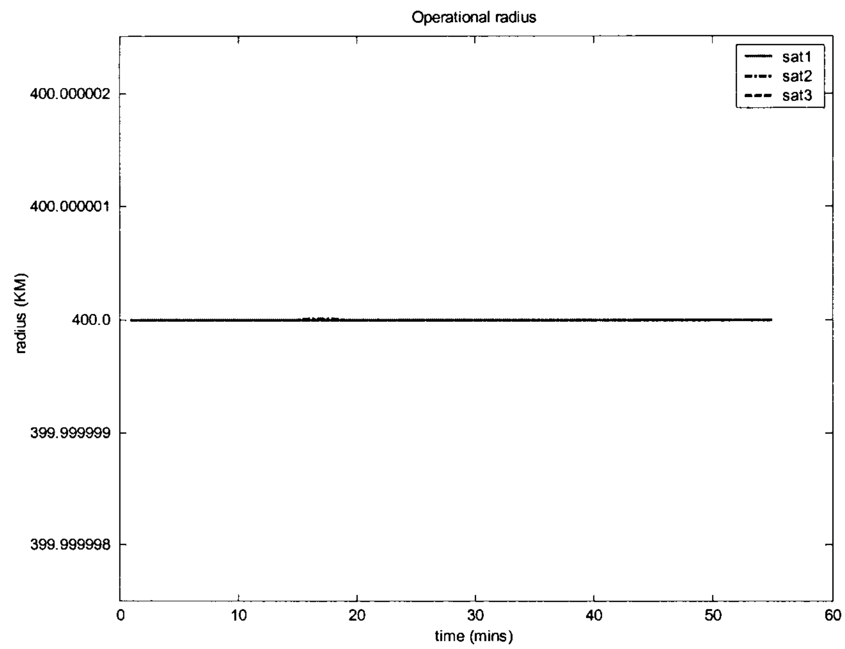


Fig. 6. 1 Operational radii of the 3 satellites

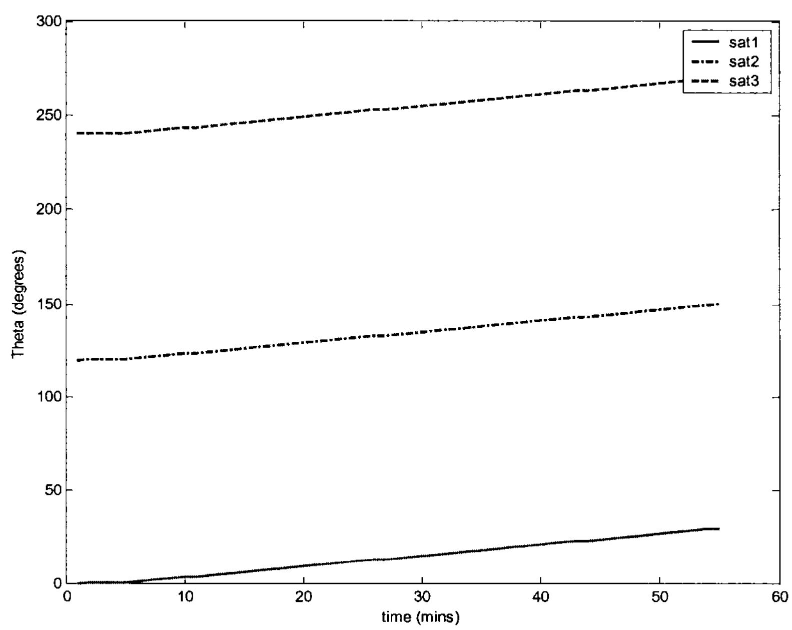


Fig. 6. 2 Operational theta of the 3 satellites

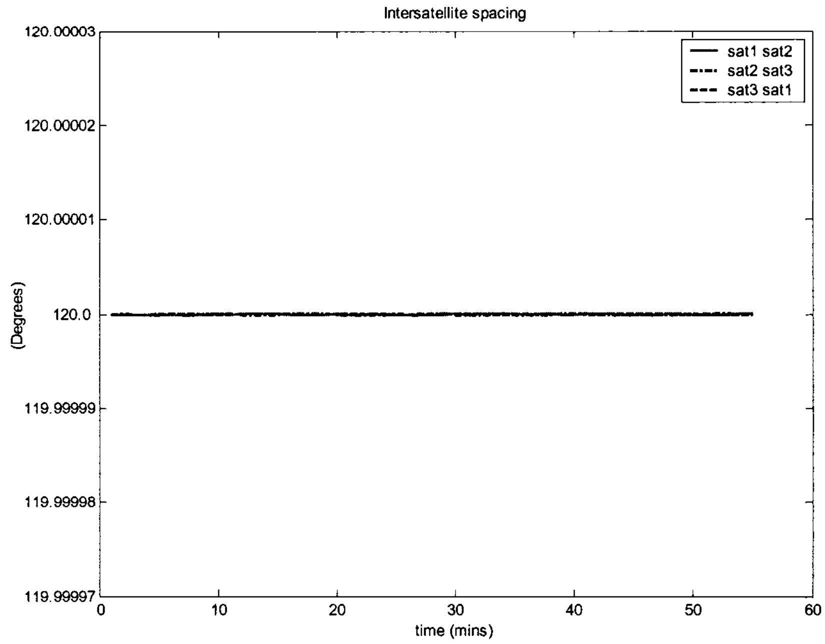


Fig. 6. 3 Intersatellite spacing

In Table 6.3 below same magnitudes of perturbations as applied to the pole placement controller in Table 5.4, are applied to the feedback linearized controller.

Table 6. 3 Simulation scenario - feedback linearized controller

Satellite 1	Satellite 2	Satellite 3
$r = 399.95Km$ $\delta r = 0.05Km$	$r = 399.99Km$ $\delta r = 0.01Km$	$r = 399.98Km$ $\delta r = 0.02Km$
$\phi = 0.03^\circ$ $\delta\phi = -0.03^\circ$	$\phi = 120.01^\circ$ $\delta\phi = -0.01^\circ$	$\phi = 239.97^\circ$ $\delta\phi = 0.03^\circ$

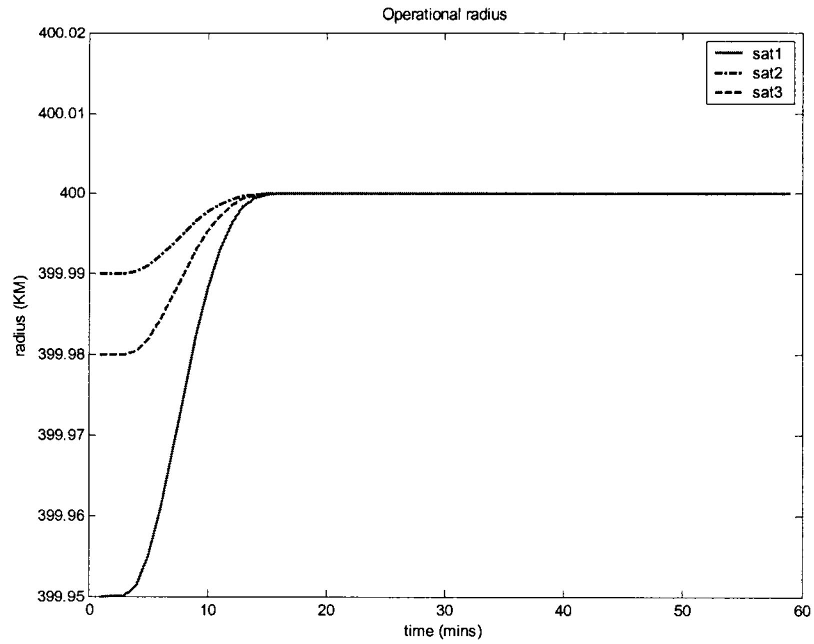


Fig. 6. 4 Operational radii of the 3 satellites

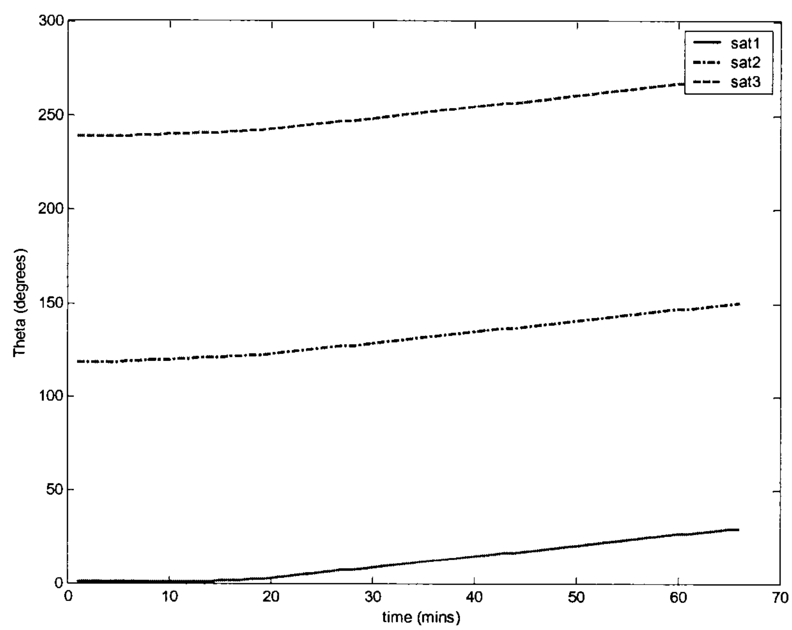


Fig. 6. 5 Operational theta of the 3 satellites

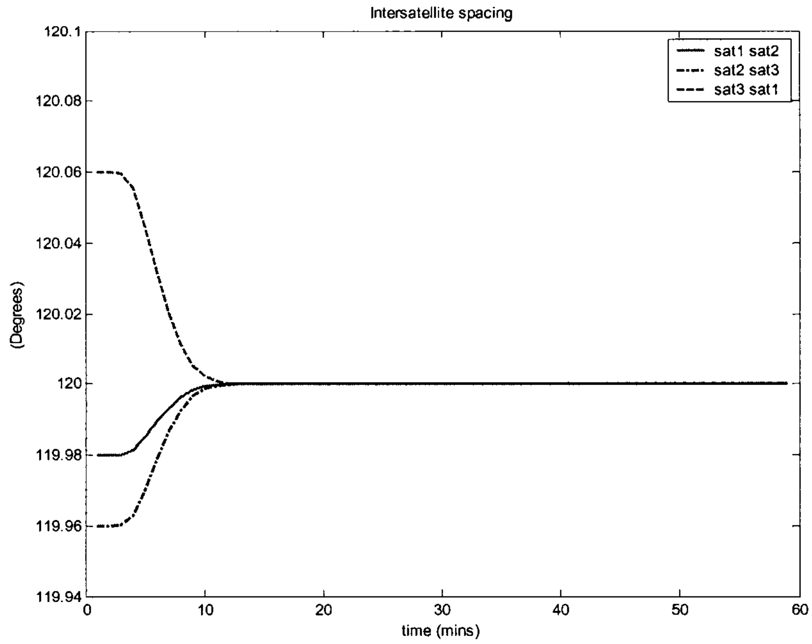


Fig. 6. 6 Intersatellite spacing

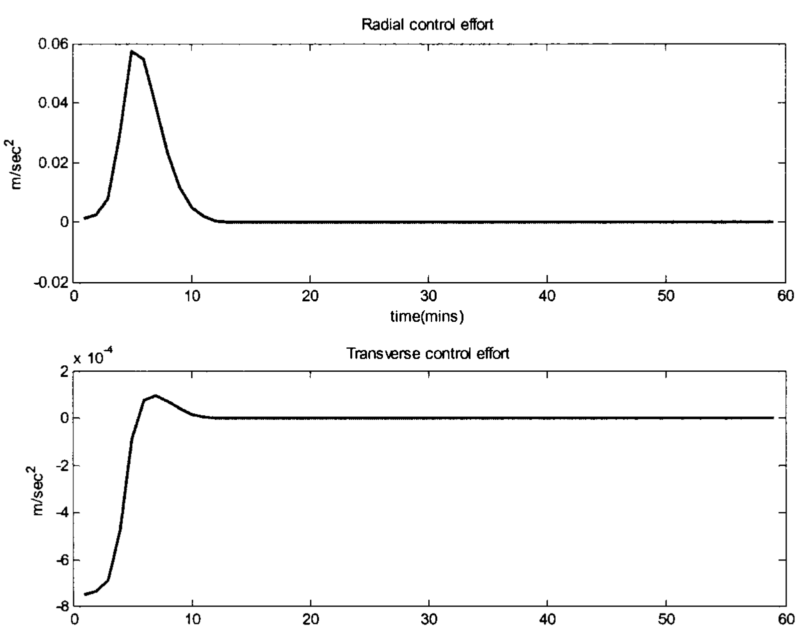


Fig. 6. 7 Control effort for satellite 1

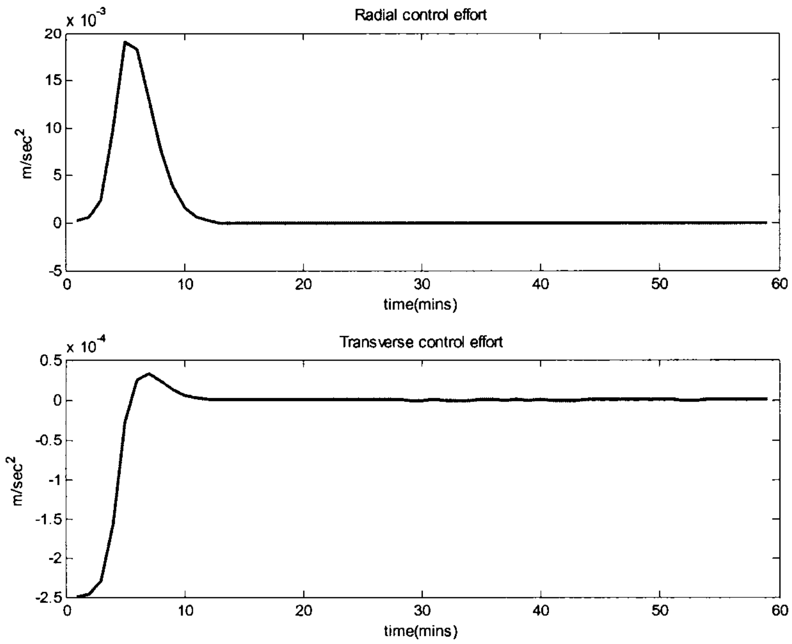


Fig. 6. 8 Control effort for satellite 2

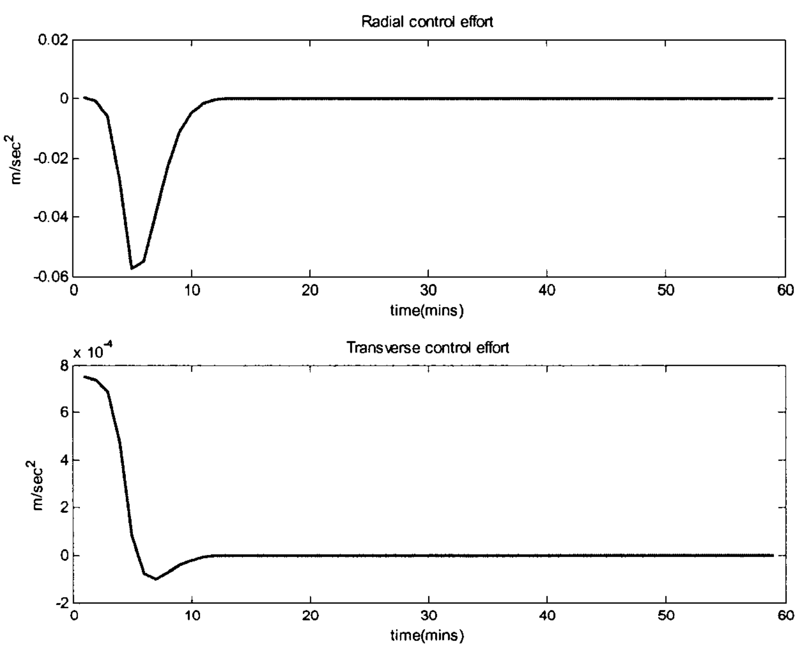


Fig. 6. 9 Control effort for satellite 3

In Table 6.4, we consider the formation variables with larger perturbations in the azimuthal as well as radial direction. The specific parameters, initial conditions and the situations considered are provided in the table bellow:

Table 6. 4 Simulation scenario - feedback linearized controller

Satellite 1	Satellite 2	Satellite 3
$r = 405Km$ $\delta r = -5Km$	$r = 403Km$ $\delta r = -3Km$	$r = 402Km$ $\delta r = -2Km$
$\phi = 1^\circ$ $\delta\phi = -1^\circ$	$\phi = 118.5^\circ$ $\delta\phi = 1.5^\circ$	$\phi = 239^\circ$ $\delta\phi = 1^\circ$

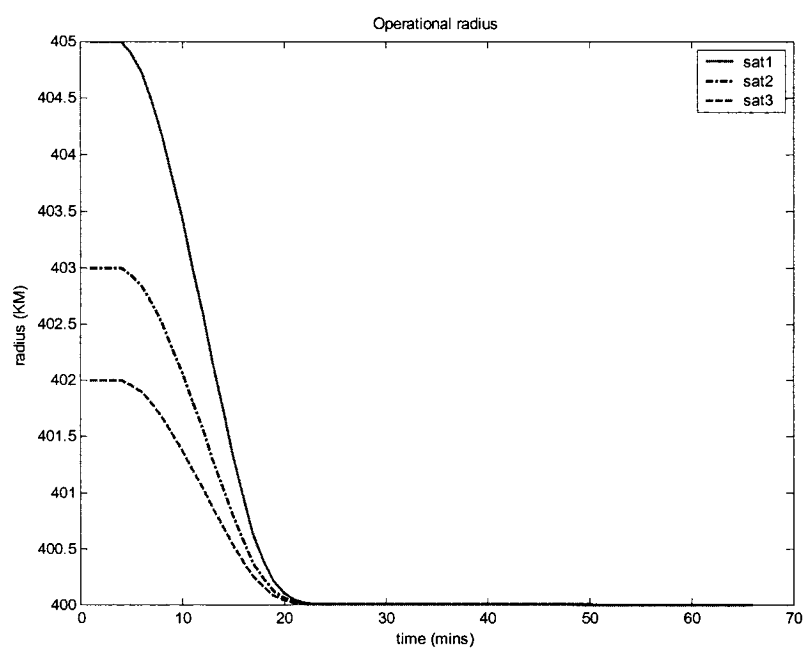


Fig. 6. 10 Operational radii of the 3 satellites

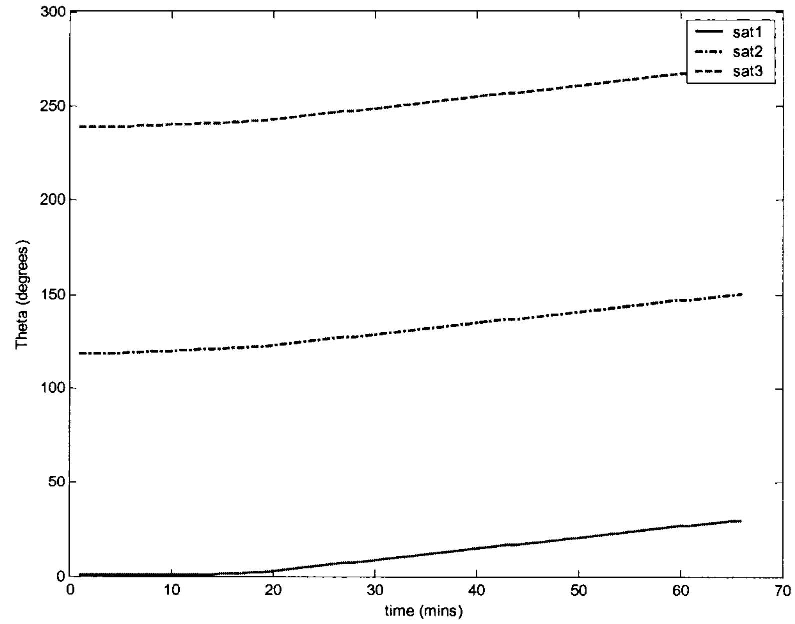


Fig. 6. 11 Operational theta of the 3 satellites

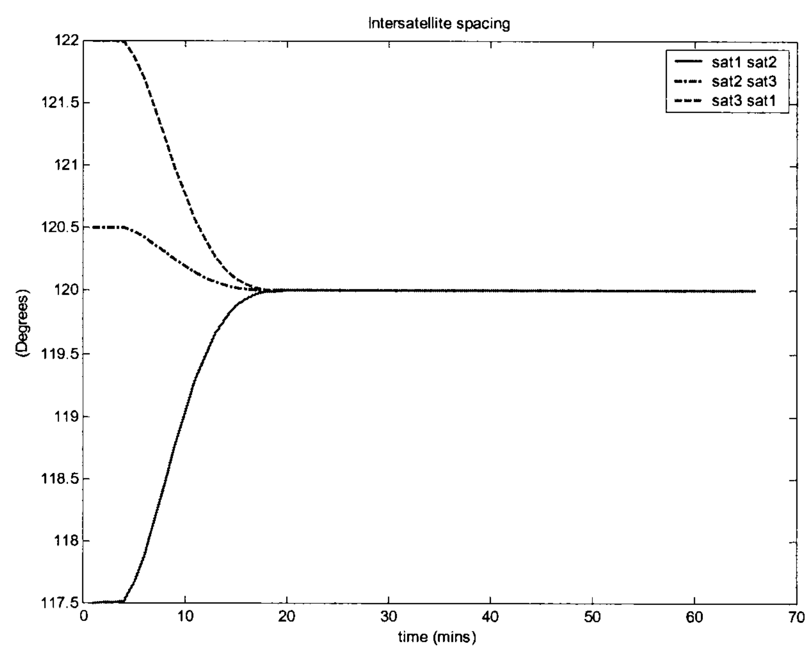


Fig. 6. 12 Intersatellite spacing

6.6 Remarks

From figures 6.1-6.3 it is clear that the designed formation controller holds the configuration in a tight formation. The satellites do not drift and their exact position can be known by the ground control at all times.

From figures 6.4-6.12, we can see that the formation controller corrects the position errors due to external perturbations. The tolerance to the perturbations is very good and the controller can handle high deviations from the desired values without throwing the system into instability. The resultant closed loop system can be considered to be globally stable, limited only by capability of thrusters to deliver the control effort required for correcting larger perturbations.

6.7 Summary

In this chapter, we have presented a novel formation controller design using feedback compensation. The design strategy has been explained in detail with the actual design. Towards the end, mission simulations have proved the formation controller successful as it satisfies our mission requirements with good formation maintenance and high tolerance to perturbations.

Chapter 7

Conclusion and Future Work

In this thesis we have presented and designed a novel nonlinear control law for formation flying of a group of satellites in a circular orbit. In chapter 4, we have presented a detailed critique on McInnes method [4] for formation control along with mission simulations. In chapter 5, we have developed a pole placement orbital controller and have provided various mission simulation scenarios to test its performance on nonlinear satellite dynamics. In chapter 6, we have designed the novel orbital controller using feedback linearization or feedback compensation technique. For the feedback linearized controller we have provided mission simulations to test its performance.

We have discussed in chapter 2 and also through the simulations in chapters 4, 5 and 6 we have seen that corrections against in-orbit perturbations are a vital function of the orbital controller. The mission success and performance is heavily dependant on the orbital controller and its ability to deal with external perturbations caused for instance by earth oblateness and atmospheric drag. We have essentially checked the controller performances in two real life situations,

- Formation keeping in absence of external perturbations.
- Formation keeping in the presence of varying magnitudes of external perturbations.

From the mission simulation results in chapters 4, 5 and 6 we can see that all the three controllers, namely, McInnes controller, pole placement controller and feedback linearization controller, are good in formation maintenance or station keeping in absence of external perturbations. The distinction between the controller performances is evident when the system is subject to external perturbations.

McInnes Controller (Relative Configuration):

- The controller developed by McInnes using the potential function method is applied to decentralized relative formation flying simulation scenarios. The control action of each satellite is a weighted average of the relative positions of all the satellites in the formation so that expected behaviour is achieved.
- The control law leads to a complex nonlinear interaction between the satellites to achieve the desired configuration. In the absence of external perturbations, the controller is able to hold the satellites together and hence maintain formation.
- Although the inter-satellite spacing is not thrown into instability instantly, in case of external perturbations, it shows a trend that would lead to a gradual drift and formation break-up if not corrected manually by the ground segment.
- It is evident that although the control laws loosely hold the formation together they are unsuitable for large perturbation corrections.
- This approach is not only hard to analyze mathematically but also has a limited ability for precise formation keeping, that is, the group cannot maintain formation very well during the manoeuvres.

Pole-placement Controller & Feedback Linearized Controller (Absolute Configuration):

- Pole-placement controller and feedback linearized controllers are applied to decentralized absolute formation flying simulation scenarios. The controllers onboard each satellite, command thruster impulses for in-track and cross-track corrections for external perturbations. This approach though does not take into consideration the concept of information sharing between the satellites in formation.

- Both the controllers perform well in formation keeping in the absence of external disturbances.
- When external perturbations are induced, both the controllers perform well in nullifying the tracking error.
- The most important advantage is that the feedback linearized controller has a more robust design. For the pole placement controller the system is stable for only $399.2 < r < 400.3$ and $-0.09 < \delta\theta < 0.03$.
- The resultant closed loop system for the feedback linearized controller can be considered to be globally stable, limited only by capability of thrusters to deliver the control effort required for correcting larger perturbations.

We have proposed and investigated the use of nonlinear compensation technique in comparison to a linear (pole-placement) and a nonlinear (Lyapunov) design technique, for orbital formation control of 3 satellites in constellation. From a performance point of view, the decision to use feedback to compensate for the nonlinear terms of a system model in reality is subject to robustness concerns. We have demonstrated the application and utility of the proposed methodology using a number of simulation scenarios and proved it to be better as compared to the methods available in the literature. The advantages and disadvantages of the proposed feedback linearization can be summed up as follows:

Advantages:

- Eliminates complex control functions and hence simplifies the commanding, reducing the risk of errors.
- Increased robustness to uncertain perturbations.
- Can be directly applied to nonlinear dynamics eliminating the cumbersome procedure of linearizing the nonlinear state space model, especially when designing a gain scheduling controller.
- In comparison with the potential function method, in this approach there is no need to find an energy function which in itself can be mathematically overwhelming.
- Significant fuel savings can be achieved, particularly when making large maneuvers, where the nonlinear dynamics effects become more pronounced.

Disadvantages:

- Exact mathematical cancellation of nonlinearities may be difficult to achieve due to parameter uncertainty, computational errors though it has been shown in that the stabilizing component obtained using such feedback achieves a certain degree of robustness to modeling uncertainty.
- Application is limited to dynamical systems that are feedback linearizable.
- The linearized model has to be controllable.

In this work we say that, significant fuel savings can be achieved during large manoeuvres since the design process uses the nonlinear dynamics directly instead of the approximated linear dynamics. Future studies should seek to find analytical methods for calculating fuel optimal manoeuvres. Another important consideration is the problem of non-uniform fuel consumption throughout the formation during both formation establishment and formation keeping manoeuvres. This is significant because it is virtually guaranteed that at some point in the mission, the satellites in the formation will have different levels of fuel left onboard and that will define the future of the entire constellation.

The formation controller as designed is directly applicable to absolute formation configurations. It would be worthwhile to apply the same feedback linearization technique to relative configuration taking into consideration the intersatellite communication.

Attitude and orbit dynamics and control coupling is an area requiring significant future work. As spacecraft pointing requirements and formation relative position requirements become stricter, the tasks of attitude and orbit determination and control subsystems become more and more coupled. We can no longer consider the dynamics and control of spacecraft in terms of orbital motion only. Rather, we must explore the concept of unified attitude and orbit dynamics and control.

Although absolute formation configuration can allow non-circular orbits, it will require further integration of collision avoidance measures to the control laws. Thus, another important work remaining in the development of these control laws is the modification to include non-circular orbits.

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