CP Phases in the Chargino Sector of the Left Right Supersymmetric Model

Artorix de la Cruz de Oña

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Abstract

We investigate the chargino sector of the left-right supersymmetric model. We consider the most general mass and mixing parameters, including all CP-violating phase. We present a consistent procedure for the calculation of the analytical expressions for the chargino mass eigenstates and eigenvectors.

We then follow with the analysis of the cross section for chargino pair production $e^+e^- \longrightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-$ and its dependence of the CP-violating phases was done. We explore also different possible experimental scenarios and signatures.

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Chapter 1

Introduction

Symmetries, or invariances, are very important in modern physics, since they give a simple and consistent way to construct Lagrangians from which the equations of motion can be found for the fields or particles of interest.

The Standard Model[1]-[3] has been the accepted theoretical picture of fundamental particles and forces during the last century. Using this model, the scientific community has succeeded in explaining most experimental phenomena about the elementary particles. The Standard Model has certain problems which prevent it from being the definitive theory of particle physics; going to higher energies new physics must emerge.

One of the most interesting uses of supersymmetry in present-day physics is as a means of extending the Standard Model. The Supersymmetry theories[4] predict a number of hypothetical particles that might exist in the energy range accessible at the New Linear Collider NLC(under construction).

The discovery of SUSY particles could be even more important than finding a Higgs boson, which would only confirm existing ideas about the Standard Model and not extend them.

1.1 The Standard Model

The Standard Model of Strong and Electroweak Interactions[5]-[6] includes all available present knowledge of the fundamental constituents of matter and their interactions.

The Standard Model is based on relativistic quantum gauge field theory. The relativistic quantum field theory applied to Maxwell equations opened new possibilities in physics. The Maxwell equations possess a special local symmetry called gauge invariance: under a local transformation ($\psi(x) \rightarrow e^{i\alpha(x)}\psi(x)$) whereby the photon field(vector potential), transforms as

$$A_\mu(x) \rightarrow A_\mu(x) - \partial_\mu \alpha(x). \tag{1.1}$$

For field strength we have

$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x), \tag{1.2}$$

the action

$$S = -\frac{1}{4}\int d^4x F^{\mu\nu}(x) F_{\mu\nu}(x), \tag{1.3}$$

and the physical observable are not changed.

The symmetry group of the Standard Model is $SU(3)_C \times SU(2)_L \times U(1)_Y$[7, 8]. The $SU(3)_C$ symmetry is that of the strong (colour) interaction; electromagnetism and the weak nuclear force (or electroweak sector) are mixed together in an overall $SU(2)_L \times U(1)_Y$ gauge symmetry[1, 4]. We believe that $SU(3)_C$ of colour is unbroken, but the symmetry of the electroweak sector is broken, resulting in the physical massive $W^{\pm}$ and Z bosons and massless photon.

The model considers the elementary particles grouped into bosons with spin that is either 0, 1 or 2 (particles that transmit forces) and fermions with spin 1/2 (particles that make up matter).

The fermions are three generations of leptons and three generations of quarks. All those particles interact by means of the exchange of virtual spin-one bosons. An explanation for why there are three generations of particles that make up matter has not been found yet. Perhaps string theory could answer it.

Since the weak nuclear force is a short range force, behaving as if the gauge bosons are heavy, in order to make a gauge invariant theory work for the weak nuclear force, theorists had to come up with a way to make heavy gauge bosons in a way that wouldn't destroy the consistency of the quantum theory. The method they came up with is called spontaneous symmetry breaking, where massless gauge bosons acquire mass by interacting with a scalar field called the Higgs field. The resulting theory has massive gauge bosons but still retains the nice properties of a fully gauge invariant theory where the gauge bosons would normally be massless.

Spontaneous symmetry breaking is invoked to explain massive vector bosons and the massless photon. The prediction of the $W^{\pm}$ and Z bosons sprang from symmetry considerations. The discovery of these bosons was one of the greatest triumphs of modern particle physics and it was achieved in 1983 by experiments at the particle accelerator in Geneva. The three weak interaction gauge bosons have the following masses in GeV: $(W^+, W^-, Z^0) = (80, 80, 91)$.

$\Longrightarrow$ The particles of the Standard Model come in 3 families:

• Leptons: 1-) electron and electron neutrino, 2-) muon and muon neutrino, 3-) tau and tau neutrino.

• Quarks: 1-) d (down) and u (up), 2-) s (strange), c (charm) and 3-)b (bottom), t (top).

The matter is made of protons (each a u-u-d quark triplet), neutrons (each a u-d-d quark triplet), and electrons. Quarks cannot exist singly (or so it appears), so the particles created in accelerator collisions include mesons (combinations of a quark and an anti-quark), baryons (combinations of three quarks), and leptons.

• The intermediate vector bosons are: 1-) gluon (nuclear force), 2-) photon (electromagnetic force) and 3-) W and Z bosons (weak force).

The Standard Model has been a partial triumph from the point of view experimental data. Unfortunately, the SM is not completely confirmed, leaving important interrogant in general about the validity of the theory. For example the detection of the

Higgs sector which contains scalar particles with spin-0 is missing.

1.2 Mathematics of the Standard Model

The first implication of the Abelian $U(1)$ gauge symmetry is that the massless vector bosons (photons) mediating electromagnetic interactions are a necessary consequence of the theory. A local $U(1)$ symmetry means that the field equations involved, i.e. of the Lagrangian, are invariant under the local transformations shown above.

In order for the Dirac Lagrangian of a free electron

$$\mathcal{L} = \bar{\psi}(x)(i\gamma^{\mu}\partial_{\mu} - m)\psi(x), \tag{1.4}$$

to be invariant under the $U(1)$ transformation, we must take into account a gauge field A_{μ} which transforms as Eq.(1.1).

The $SU(2)$ symmetry of the Standard Model is a non-Abelian gauge theory. To study this theory[9, 10], let us first look at the internal symmetry transformations.

The internal symmetry transforms states interpreted as different particles, i.e. the transformation changes label of the particle without transforming the coordinate system. An example is the isospin group of transformations taking up-quarks into down-quarks. Isospin is represented by the $SU(2)$ group of 2-dimensional unitary matrices with $det = +1$ acting on the two-vector representations of the up-and down quarks:

$$\mathbf{u} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad and \qquad \mathbf{d} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{1.5}$$

Under a local $SU(2)$ transformation, the a doublet $\psi(x)$ transform as follows:

$$\begin{aligned} \psi(x) \rightarrow \psi'(x) &= e^{-i\tau\cdot\theta(x)}\psi(x) \\ &= U(\theta)\psi(x), \end{aligned} \tag{1.6}$$

where τ are the Pauli matrices, such as

$$[\frac{\tau_i}{2}, \frac{\tau_j}{2}] = i\epsilon_{ijk}\frac{\tau_k}{2}, \qquad i, j, k = 1, 2, 3, \tag{1.7}$$

and $\theta(x)$ are the $SU(2)$ transformation parameters.

Since we need the free particle Lagrangian Eq.(1.4) to be invariant under local $SU(2)$ transformation, we must introduce the gauge fields $\mathbf{A}_\mu = A^i_\mu, (i = 1, 2, 3)$ and taking into account the form of the covariant derivative $D_\mu = \partial_\mu - ig(\tau \cdot \mathbf{A}_\mu)/2$, we arrive to expression of the field strength

$$\begin{aligned} \frac{\tau \cdot \mathbf{A}_{\mu\nu}}{2} &= \frac{1}{ig}(D_\mu D_\nu - D_\nu D_\mu) \\ &= \partial_\mu \frac{\tau \cdot \mathbf{F}_{\mu\nu}}{2} - \partial_\nu \frac{\tau \cdot \mathbf{A}_\mu}{2} - ig[\frac{\tau \cdot \mathbf{A}_\mu}{2}, \frac{\tau \cdot \mathbf{A}_\mu}{2}], \end{aligned} \tag{1.8}$$

where we have used the following transformation property for $\mathbf{A}_\mu$,

$$\frac{\tau \cdot \mathbf{A'}_\mu}{2} = U(\theta)\frac{\tau \cdot \mathbf{A}_\mu}{2}U^{-1}(\theta) - \frac{1}{g}[\partial_\mu U(\theta)]U^{-1}(\theta), \tag{1.9}$$

transforming like

$$\tau \cdot \mathbf{F'}_{\mu\nu} = U(\theta)(\tau \cdot \mathbf{F}_{\mu\nu})U^{-1}(\theta). \tag{1.10}$$

The kinetic invariant energy can be written as

$$\mathcal{L}_{KE} = -\frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu}. \tag{1.11}$$

1.2.1 Unbroken symmetry theory

The $SU(2) \times U(1)$ (for example see [7, 8]) is broken through a mechanism called the Higgs phenomenon, giving rise to the masses of the $W^{\pm}$ and Z bosons as well as the mass splitting between the left-handed electron state and the neutrino, in contrast to the massless bosons and fermions of the unbroken theory.

The left-handed fields, and the Higgs field, are ordered in $SU(2)$ doublets, while the right-handed fields are $SU(2)$ singlets. This is done because in the Standard Model, the neutrino only exists in the left-handed form.

The unbroken $SU(2)_L \times U(1)_Y$ theory has the following particle content:

- Field: $q_L = (u_L\ d_L)^T$, Spin: (1/2), Mass: 0.
- Field: u_R, Spin: (1/2), Mass: 0.
- Field: d_R, Spin: (1/2), Mass: 0.
- Field: e_R, Spin: (1/2), Mass: 0.
- Field: $l_L = (\nu_{eL}\ e_L)^T$, Spin: (1/2), Mass: 0.
- Field: $\mathbf{\Phi} = (\phi^+\ \phi^0)^T$, Spin: 0, Mass: Finite.

The gauge bosons of the unbroken $SU(2)_L \times U(1)_Y$ theory are given by

- Field: A^i_μ, Spin: 1, Mass:0.
- Field: B_μ, Spin: 1, Mass:0. (This is the Higgs boson)

The Lagrangian density of the unbroken theory can be written as

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4. \tag{1.12}$$

The kinetic energy of the vector gauge fields are

$$\mathcal{L}_1 = -\frac{1}{4}F^i_{\mu\nu}F^{i\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu}, \tag{1.13}$$

where $F^i_{\mu\nu} = \partial_\mu A^i_\mu - \partial_\nu A^i_\nu + g\epsilon^{ijk}A^j_\mu A^k_\nu$, $(i = 1, 2, 3)$ and $G_{\mu\nu}\partial_\mu B_\nu - \partial_\nu B_\mu$.

On the other hand,

$$\mathcal{L}_2 = \bar{\psi} i\gamma^\mu D_\mu \psi, \tag{1.14}$$

represents the fermionic kinetic energy, where $D_\mu\psi = (\partial_\mu - ig\mathbf{T}\cdot\mathbf{A}_\mu - ig'(Y/2)B_\mu)\psi$ is the relevant covariant derivative, $\mathbf{T}$ are the $SU(2)$ generators, and $\psi = l_L, e_R, q_L, u_R, d_R$.

The Lagrangian which represents the Higgs bosons is

$$\mathcal{L}_3 = (D_\mu\mathbf{\Phi})^\dagger(D^\mu\mathbf{\Phi}) - V(\mathbf{\Phi}), \tag{1.15}$$

where $V(\mathbf{\Phi}) = -\mu\Phi^\dagger\Phi + \lambda(\mathbf{\Phi}^\dagger\mathbf{\Phi})^2$. Finally,

$$\mathcal{L}_4 = f^{(e)}\bar{l}_L\mathbf{\Phi}e_R + f^{(u)}\bar{q}_L\tilde{\mathbf{\Phi}}u_R + f^{(d)}\bar{q}_L\mathbf{\Phi}d_R + h.c., \tag{1.16}$$

where $f^{(a)}$, $a = e, u, d$, are called the Yukawa couplings and $\tilde{\mathbf{\Phi}} = i\tau_2\Phi^*$ is the Higgs isodoublet which has hypercharge $Y(\tilde{\mathbf{\Phi}}) = -1$.

1.2.2 Broken symmetry theory

To study the symmetry breaking, let us start with a $U(1)$ invariant Klein-Gordon Lagrangian for a complex scalar field ϕ with the the self-interaction potential $U(\phi) =$

$\lambda(\phi^\dagger\phi)^2$, under local transformation of $\psi(x)$ and $A_\mu(x)$,

$$\mathcal{L} = -(D_\mu\phi)^\dagger(D^\mu\phi) + \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \tag{1.17}$$

Assuming $\mu^2 > 0$ we have a minimum in the potential energy for

$$V(\phi) = -\mu^2|\phi|^2 + \lambda|\phi|^4. \tag{1.18}$$

Solving the equation for ϕ, we get that,

$$|\phi| = \frac{v}{\sqrt{2}}, \qquad \textit{and} \qquad v = \sqrt{\frac{\mu^2}{\lambda}}. \tag{1.19}$$

The non-zero vacuum expectation value (VEV) of the field ϕ is $<\phi>_0 = v/\sqrt{2}$. The symmetry is broken when a certain direction is chosen for the VEV. The complex field in its real and imaginary parts can be written as

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2), \tag{1.20}$$

and then let

$$<\phi_1>_0 = v, \qquad \textit{and} \qquad <\phi_2>_0 = 0. \tag{1.21}$$

Let us shift the field into new fields having VEVs equal to zero,

$$\phi_1' = \phi_1 - v, \qquad \textit{and} \qquad \phi_2' = \phi_2, \tag{1.22}$$

arriving to

$$\phi = \frac{1}{\sqrt{2}}(v + \phi_1' + i\phi_2'). \tag{1.23}$$

After this shift of the vacuum the symmetry is broken. The kinetic part of the Lagrangian becomes

$$
\begin{aligned}
|D_\mu \phi|^2 &= |(\partial_\mu - igA_\mu)\phi|^2 \\
&= \frac{1}{2}(\partial_\mu \phi_1' + gA_\mu \phi_2')^2 + \frac{1}{2}(\partial_\mu \phi_2' + gA_\mu \phi_1')^2 \\
&- gA^\mu v \frac{1}{2}(\partial_\mu \phi_2' - gA_\mu \phi_1')^2 + \frac{g^2 v^2}{2} A^\mu A_\mu. \qquad (1.24)
\end{aligned}
$$

We have a gauge field with mass: the last term of the expression. To analyze the rest of the terms we must make a different choice for the direction of the VEV. Comparing with Eq.(1.20), we can express ϕ as

$$
\begin{aligned}
\phi &= \frac{1}{\sqrt{2}}(v + \eta(x))e^{i\xi(x)/v} \\
&= \frac{1}{\sqrt{2}}(v + \eta(x) + i\xi(x) + \ldots), \qquad (1.25)
\end{aligned}
$$

where the real scalar fields $\eta(x)$ and $\xi(x)$ have vanishing VEVs and from the last expression we can associate both scalar fields with ϕ_1' and ϕ_2', respectively.

Taking into account the form of the gauge local transformation shown before, we can fix the gauge to the unitary gauge, then

$$
\begin{aligned}
\phi^u(x) &= e^{-i\xi(x)/v}\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x)), \qquad (1.26) \\
B_\mu(x) &= A_\mu(x) - \frac{1}{gv}\partial_\mu \xi(x). \qquad (1.27)
\end{aligned}
$$

Then,

$$
\begin{aligned}
D_\mu \phi &= e^{-\xi/v}(\partial_\mu \phi^u - igB_\mu \phi^u) \\
&= \frac{1}{\sqrt{2}} e^{-i\xi/v}(\partial_\mu \eta - igB_\mu(v + \eta(x)) \qquad (1.28)
\end{aligned}
$$

The Lagrangian reads

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}|\partial_\mu \eta - igB_\mu(v+\eta)|^2 + \frac{\mu^2}{2}(v+\eta)^2 - \frac{\lambda}{4}(v+\eta)^4 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &= \mathcal{L}_0 + \mathcal{L}_1, \end{aligned} \tag{1.29}$$

where $F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ and

$$\begin{aligned}\mathcal{L}_0 &= \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \mu^2\eta^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(gv)^2 B_\mu B^\mu \\ \mathcal{L}_1 &= \frac{1}{2}g^2 B_\mu B^\mu \eta(2v+\eta) - \lambda v^2\eta^3 - \frac{1}{4}\lambda\eta^4. \end{aligned} \tag{1.30}$$

$\mathcal{L}_0$ is the Lagrangian (density) of a real scalar field η and a massive vector field B_μ with mass gv.

To break the symmetry of a non-Abelian group $SU(2)$, let us write the Lagrangian

$$\mathcal{L} = (D_\mu \mathbf{\Phi})^\dagger(D^\mu \mathbf{\Phi}) + \mu^2 \mathbf{\Phi}^\dagger \mathbf{\Phi} - \lambda(\mathbf{\Phi}^\dagger \mathbf{\Phi})^2 - \frac{1}{4}F^i_{\mu\nu}F^{i\mu\nu}, \tag{1.31}$$

where $\mathbf{\Phi} = (\phi_1\, \phi_2)^T$. For $\mu^2 > 0$ we obtain the minimum of the potential at $\mathbf{\Phi}^\dagger\mathbf{\Phi} = v^2/2$ and the *vev* for the field:$< \mathbf{\Phi}^\dagger\mathbf{\Phi} >_0 = v^2/2$ and $v = \sqrt{\mu^2}/\lambda$.

We must choose a certain direction for the VEV, thereby breaking the gauge symmetry. Since the vacuum must be neutral, we give the neutral component of the doublet a non-vanishing VEV, keeping the expectation value of the charged component zero:

$$< \mathbf{\Phi} >_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{1.32}$$

We can write $\mathbf{\Phi}(x)$ as

$$\mathbf{\Phi}(x) = U^{-1}(\zeta)\begin{pmatrix} 0 \\ (v+\eta(x))/\sqrt{2} \end{pmatrix}, \qquad U(\zeta) = e^{i\zeta(x)\cdot\tau/v}, \tag{1.33}$$

and the vector $\zeta = (\zeta_1, \zeta_2, \zeta_3)$ as well as η has vanishing VEVs. Assuming the unitary gauge,

$$\begin{aligned} \mathbf{\Phi}'(x) &= U(\zeta)\mathbf{\Phi} \\ &= \begin{pmatrix} 0 \\ (v+\eta(x))/\sqrt{2} \end{pmatrix}. \end{aligned} \tag{1.34}$$

Since all fields of the Standard Model are of the same gauge, we have to transform the fermionic $SU(2)$ doublet fields and the $SU(2)$ gauge bosons as well as the Higgs doublet, giving

$$\begin{aligned} l'_L &= U(\zeta)l_L \\ q'_L &= U(\zeta)q_L \\ \frac{\tau\cdot\mathbf{A}'_\mu}{2} &= U(\zeta)\frac{\tau\cdot\mathbf{A}_\mu}{2}U^{-1}(\zeta) - \frac{1}{g}[\partial_\mu U(\zeta)]U^{-1}(\zeta), \end{aligned} \tag{1.35}$$

while the $SU(2)$ singlets. i.e. the right-handed fields, and the U(1) vector field B_μ are left unchanged by the transformation. Notice that the Lagrangians $\mathcal{L}_1$ and $\mathcal{L}_2$ do not change, since they are both gauge-invariant and do not contain any VEVs. On the other hand, $\mathcal{L}_3$ becomes as follows

$$\mathcal{L}_3 = (D'_\mu\mathbf{\Phi}')^\dagger(D'^\mu\mathbf{\Phi}') - (\mu\eta)^2 + \lambda v\eta^3 + \frac{\lambda}{4}\eta^4, \tag{1.36}$$

where $D'_\mu \Phi' = (\partial_\mu - \frac{1}{2} ig\tau \cdot \mathbf{A'}_\mu - \frac{1}{2} ig' \mathbf{B'}_\mu) \begin{pmatrix} 0 \\ (v + \eta(x))/\sqrt{2} \end{pmatrix}$.

and $\mathcal{L}_4$ takes the form,

$$\begin{aligned} \mathcal{L}_4 &= \frac{\eta(x)}{\sqrt{2}} [f^{(e)} \bar{e}_L e_R + f^{(u)} \bar{u}_L u_R + f^{(d)} \bar{d}_L d_R] \\ &+ \frac{v}{\sqrt{2}} [f^{(e)} \bar{e}_L e_R + f^{(u)} \bar{u}_L u_R + f^{(d)} \bar{d}_L d_R] + h.c. \end{aligned} \tag{1.37}$$

We can read off the mass terms for the Higgs bosons and the fermion fields,

$$\begin{aligned} m_\eta &= \sqrt{2}\mu, \\ m_e &= f^{(e)} v/\sqrt{2}, \\ m_u &= f^{(u)} v/\sqrt{2}, \\ m_d &= f^{(d)} v/\sqrt{2}. \end{aligned} \tag{1.38}$$

The mass of the e and d fields are obtained through the interaction with $\mathbf{\Phi}$ and the mass term for u can be obtained by the interaction of $\tilde{\mathbf{\Phi}}$ (charge-conjugate isodoublet).

To study the mass term of the gauge vectors fields, we must use the Eq.(1.36), then

$$\begin{aligned} \mathcal{L}_3 &= \frac{v^2}{2} (0\ 1) (\frac{g}{2} \tau \cdot \mathbf{A'}_\mu + \frac{g'}{2} B'_\mu) (\frac{g}{2} \tau \cdot \mathbf{A'}^\mu + \frac{g'}{2} B'^\mu) (0\ 1)^T \\ &= M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu, \end{aligned} \tag{1.39}$$

from where we have the following associations $W_\mu^\pm = (\mathbf{A'}^1_\mu \mp i \mathbf{A'}^2_\mu)/\sqrt{2}$ and $M_W^2 =$

$g^2v^2/4$. On the other hand, we have for the neutral vector bosons,

$$\frac{v^2}{8}(g\mathbf{A}'^3_\mu - g'B'_\mu)^2 \;=\; \frac{1}{2}(Z_\mu\, A_\mu)\begin{pmatrix} M_Z^2 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix} = \frac{1}{2}M_Z^2 Z_\mu Z^\mu, \quad (1.40)$$

where the A_μ gauge boson remains massless.

To diagonalize the mass matrix in the last equation, we have used the orthogonal transformation

$$\begin{aligned} Z_\mu &= \cos\theta_W A'^{\,3}_\mu - \sin\theta_W B'_\mu \\ A_\mu &= \sin\theta_W A'^{\,3}_\mu - \cos\theta_W B'_\mu, \end{aligned} \quad (1.41)$$

here the mass of the Z boson is $M_Z^2 = v^2(g^2 + g'^2)/4$ and the angle θ_W(Weinberg angle) is defined by $\tan\theta_W = g'/g$.

The broken $SU(2)_L$ symmetric Standard Model has the following particle content

- Field: u, Spin: (1/2), Mass: m_u,
- Field: d, Spin: (1/2), Mass: m_d,
- Field: e, Spin: (1/2), Mass: m_e,
- Field: ν_e, Spin: (1/2), Mass:0,
- Field: η, Spin: 0, Mass: m_η.

On the other hand, for the gauge bosons of the broken $SU(2)_L$ we have that,

- Field: $W^\pm_\mu$, Spin: 1, Mass: W_W.
- Field: Z_μ, Spin: 1, Mass: W_Z.
- Field: A_μ, Spin: 1, Mass: 0.

Until now, we have working on a simplification (so-called one-family approximation) of the fermionic structure. For the particle spectrum of the theory to correspond to the empirically found spectrum, the fermionic structure must in fact be tripled, making the following substitutions:

$$\begin{aligned} e \longrightarrow e'_A &= (e', \mu', \tau') \\ \nu_e \longrightarrow \nu'_A &= (\nu'_e, \nu'_\mu, \nu'_\tau) \\ u \longrightarrow p'_A &= (u', c', t') \\ d \longrightarrow n'_A &= (d', s', b'), \end{aligned} \tag{1.42}$$

where the μ, ν_μ, c and s particles are called the second-generation particles and τ, ν_τ, t and b are called the third-generation particles. The $SU(2)$ doublets can be written as

$$l_{AL} = \begin{pmatrix} \nu'_A \\ e'_A \end{pmatrix}_L, \quad q_{AL} = \begin{pmatrix} p'_A \\ n'_A \end{pmatrix}_L. \tag{1.43}$$

Neither the gauge field structure or the Higgs structure is affected by this change. The $\mathcal{L}_2$ is augmented by the new fields and the interaction Lagrangian $\mathcal{L}_4$ gains a family index matrix structure

$$\mathcal{L}_4 = f^{(e)}_{AB} \bar{l}_{AL} \mathbf{\Phi} e'_{BR} + f^{(p)}_{A} B \bar{q}_{AL} \tilde{\mathbf{\Phi}} p'_{BR} + f^{(n)}_{AB} \bar{q}_{AL} \mathbf{\Phi} n'_{BR} + h.c. \tag{1.44}$$

The Yukawa coupling constants must be replaced by coupling matrices in the family index space.

The fields l_{AB}, e'_{AR}, q_{AL}, p'_{AR} and n'_{AR} are gauge eigenfields (they transform as singlets or doublets under $SU(2)$ gauge transformations). The $\mathcal{L}_4$ takes the form

$$\begin{aligned}\mathcal{L}_4 &= \frac{\eta(x)}{\sqrt{2}}[f^{(e)}_{AB}\bar{e}_{AL}e'_{BR} + f^{(p)}_{AB}\bar{p}'_{AL}p'_{BR} + f^{(n)}_{AB}\bar{n}_{AL}n_{BR}] \\ &+ \frac{v}{\sqrt{2}}[f^{(e)}_{AB}\bar{e}_{AL}e'_{BR} + f^{(p)}_{AB}\bar{p}'_{AL}p'_{BR} + f^{(n)}_{AB}\bar{n}_{AL}n_{BR}] + h.c., \qquad (1.45)\end{aligned}$$

now we have a family index matrix of mass terms of the form $M^i_{AB} = -(1/\sqrt{2})vf^i_{AB}$, $(i = e, p, n)$.

1.3 The fine tuning problem

Most of the Standard Model[1]-[3] has been tested in detail by experiment; those quantities which have been measured to this date have been in excellent agreement with the existing data. There remain, however, some untested aspects of the model.

The spontaneous breaking of the electroweak symmetry by the Higgs mechanism is one of the remaining untested parts of the Standard Model. The Higgs scalar which is responsible for most of the 19 arbitrary parameters in the theory, is the most unsatisfactory part of the model[3, 4]. There is no definite experimental evidence of the Higgs particles to this date as was explained above.

The Large Hadron Collider (LHC) under construction at the European Centre for Nuclear Research (CERN) in Switzerland becomes operational in 2007 and will be the world's most powerful particle accelerator. If the Higgs boson exists, it will be detected at the LHC.

But the weakest part of the SM is its theoretical self-consistency.

In any quantum field theory involving interacting fundamental scalars, such as the Higgs bosons, one finds that radiative corrections to scalar mass δm_H produce a quadratic divergence in that mass.

This divergence is unphysical because the theory is not applicable for infinite momentum; it breaks down for momenta P that approach the mass scalar M_{scalar} at which new interactions or new particles, not in the model, become important.

The scale M_{scalar} is actually a cutoff, since physics not contained in the Standard Model becomes important above that scale. At least one such scale, namely, the Planck scale, $M_{scalar} = O(10^{19})\ GeV$, at which gravitation becomes relevant, must be present in any theory[4, 8].

Despite the presence of natural cutoff M_{scalar}, the quadratic divergence of the elementary scale mass still remains a problem. This problem arises because quadratically divergent radiative corrections try to push the Higgs mass to the Planck scale,

$$m_H^2 = -m_0^2 + g^2 M_{scalar}, \tag{1.46}$$

where m_0 is the bare Higgs mass parameter and g is a dimensionless coupling parameter. The value of m_H is known to be of the order of the electroweak scale, $O(10^2)$ GeV.

The above expression states that if M_{scalar} is as large as the Planck scale, and if g is of the order unity, then m_0 must be tuned to incredible precision in order to keep m_H of the correct order of magnitude. While this is not impossible, it is an $ad-hoc$ feature of the Standard Model. Furthermore, the tuning must be performed a new for each order in perturbation theory.

The most elegant solution to this problem (until now) is to introduce a new symmetry that forces the cancelation of divergences without fine-tuning.

Chapter 2

Supersymmetry

Supersymmetry(SUSY) is a symmetry that connects bosons and fermions. It introduces a fermionic counterpart for every boson (and vice-versa) identical in all others quantum numbers. The difference would be only in the spins (differs by half a unit) and in obeying different statistics. The connection between bosons and fermions is unique to supersymmetry[4, 5, 6]. Supersymmetry cannot be an unbroken symmetry in the real world. If SUSY were not broken, the partners would have the same mass, which contradicts experimental evidence.

One reason for introducing to supersymmetry is that it can protect the masses of elementary scalars from quadratic divergences through the association of a fermion with an elementary scalar. In an unbroken SUSY theory, for each diagram in any calculation (such as that of the mass of the Higgs) containing a scalar loop there is a corresponding diagram containing a fermion loop.

Because the couplings of a particle and its superpartner (sparticle) are related by supersymmetry, the boson and fermion contributions cancel exactly order by order in perturbation theory. No tuning parameter is required.

Supersymmetry express bosons and fermions into multiplets. This remove the distinction between matter and interaction. The particle and its superpartner can be considered to be carriers of force. The fermions must satisfy the Pauli's exclusion principle and as a result, cannot contribute to coherent potentials.

The distinction of matter from forces is phenomenological and it is due to the fact that no two identical fermions may have the same quantum numbers, then they manifest themselves as physical particles, while the classical fields which arise from superposition of bosons, yield an impression of the presence of forces. These particles were related by a symmetry transformation (Wess and Zumino renormalization model) [6], thus they can be placed in the same multiplet.

The generators of supersymmetry Q (operators that converts fermions into bosons and conversely) commute with the Hamiltonian of the system, then they are conserved quantities, and named as the so-called *supercharges*.

The supercharges with 1/2-spin are denoted as Q_α. Thus a left-handed Weyl spinor transforms as a $(1/2, 0)$ representation under Lorentz transformations. The operator which transforms as $(0, 1/2)$ is the Hermitian adjoint and will be denoted by $\bar{Q}_\beta$.

It is known that anticommutator of the generator Q_α of the supersymmetry transformation satisfies

$$\{Q_\alpha, \bar{Q}_\beta\} = 2\sigma^\mu_{\alpha\beta} P_\mu, \tag{2.1}$$

where P^μ are the generators of the Poincaré group (i.e. the energy momentum operator $P_\mu = i\partial^\mu$) and σ^μ are the Dirac matrices. The anti-commutator relation in Eq.(2.1) is a 2×2 matrix that transforms as $(1/2, 1/2)$ under Lorentz transformations.

In the case of $\mu = 0$, $[Q_\alpha, P_0]$ commutes. All non-zero energy states are paired by the action of Q and since Q is fermionic by definition, then all supersymmetric multiplets contain one degree of bosonic freedom for every degree of fermionic freedom. Thus, these partners must have equal masses.

If take into account the expression $\sigma^\mu_{\alpha\beta}\sigma^{\alpha\beta}_\nu = 2g^\mu_\nu$ on the commutation relation between Q_α and P_0 we have the following equation

$$H = P_0 = (1/4)(\bar{Q}_1 Q_1 + Q_1\bar{Q}_1 + \bar{Q}_2 Q_2 + Q_2\bar{Q}_2) \simeq QQ^\dagger. \tag{2.2}$$

Thus $H \geq 0$, the vacuum is well defined. For an unbroken SUSY model, the vacuum energy is given by

$$E_{vac} =< 0|H|0 >= 0. \tag{2.3}$$

On the other hand, if the theory is spontaneously broken $E_{vac} \neq 0$. We have that the fermionic state $Q|0 >= |\psi >= 0$, therefore there will be a particle with mass equal zero: the Goldstone fermion or Goldstino[11]-[14]. This particle is the candidate (must be found in the experiments) to couple to every particle and its superpartner. In supersymmetry, the fermions partners of the gauge bosons are called gauginos.

We have charged gauginos (the winos $\tilde{W}_L^{\pm}$) and correspondingly the neutral gauginos (the photino $\widetilde{\gamma}$ and the zinos $\widetilde{Z}_L$). These gauginos have the same quantum numbers as the fermionic partners of the Higgs bosons (the higgsinos), thus they mix.

2.1 The Minimal Supersymmetric Standard Model

Most searches for supersymmetry are performed in the context of the Minimal Supersymmetry Standard Model(MSSM)[7]. This model is minimal in the sense of based on the Standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ group symmetry, being the supersymmetric extension of the Standard Model which has the fewest particles added.

The MSSM is a low energy limit of several more fundamental theories, for instance supergravity Grand Unified Theories or String-inspired supersymmetric models. The MSSM adds additional Higgs scalars and the suerpsymmetric partners of all normal particles to the Standard Model spectrum[15, 16]. The gauge interactions of the Minimal Supersymmetric Standard Model Langragian allow for the definition of a new multiplicative quantum number called R-parity. R-parity distinguishes between normal particles and superparticles; R-parity may be chosen to be $+1$ for normal particles and -1 for supersymmetric particles.

It is also possible to define R-parity as

$$R = (-1)^{3(B-L)+2S}, \tag{2.4}$$

where B is the bayron number, L is the lepton number, and S is the spin of the particle. R-parity may or may not be conserved. Although it is possible to construct models in which R is not conserved, any R-violating terms in the Langragian also violate lepton or bayron number conservation and are thus severely restricted.

In the Minimal Supersymmetric Standard Model, R is a conserved quantum number, with the result that $R = -1$ particles (the sparticles) must always be a pair produced, and that the lightest supersymmetric particle (LSP) is absolutely stable.

Furthermore, the decay products of each supersymmetric particle must include an odd number of supersymmetric particles, and all decay chains must end with one LSP.

The LSP has to be electrically and colour neutral. Since the LSP may interact only weakly or gravitationally and therefore escape any detection, the basic signature for production of supersymmetric particles at high-energy colliders is missing energy carried by LSPs.

With the inclusion of only a few extra parameters, beyond those of the Standard Model, all processes in the model are calculated through standard perturbation theory. This includes masses, cross-sections for production, and the decay modes of the various particles. Thus supersymmetry provides us a plausible example of a theory with complex signatures and with calculable production and decay rates. Its predictions can be confronted with experiment.

2.2 Left-Right Symmetry

The Left-Right symmetric model[5] is based on the group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. In the model the right-handed and left-handed fermion components both transform as doublets under a right-handed group $SU(2)_R$ or $SU(2)_L$, respectively.

For the leptons and quarks, assuming the existence too of right-handed neutrinos, the doublets are

$$L_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L , \; L_R = \begin{pmatrix} \nu \\ e \end{pmatrix}_R , \; Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L , \; Q_R = \begin{pmatrix} \nu \\ e \end{pmatrix}_R . \tag{2.5}$$

The L-R symmety can be broken in three steps:

1-) Parity symmetry can be breaking the equality between g_L and g_R.

2-) Breaking $SU(2)_R$. This is chosen to break at the same scale as the parity symmetry.

3-)By the non-zero expectation of a Higgs bi-doublet.

This model has several attractive features, one of which is that by allowing for the existence of a right-handed neutrino.

In the SM as well the MSSM, left and right matter fields are treated differently. This rise immediate questions: why and how do we explain parity violations or charge-parity (CP) violation?

Chapter 3

Charge-Parity (CP) violation

3.1 Introduction

The Greek philosophers of the antiquity always had seen that the Nature would be regie by unchanging rules and independents of the man actions. This perception of the universe was due to the regularly behavior of the objects in our visual universe, where it seems that a systematic movement can be described by a Law and gives the possibility to predict by extrapolation part of the future behavior. The perception of the symmetry comes together with the concept of harmony, balance and periodicity.

Under certain transformations, the objects of the Physics preserve their proprieties, this is what we name symmetry.

Depending of the nature of these transformations we can define groups of symmetries, i.e., 1-) The continuous symmetries of the group of Poincare: the translations in the space-time, the rotations and the Lorentz transformations of the restricted relativity.

2-) Symmetries discrete: the conjugation of the charge, C, the parity, P, and the reversal of time, T.

3.2 Symmetries P, C, and T

The Parity P

We can express parity as the space inversion: the reflection in the origin of the space coordinates of a particle or particle system; i.e., the three space dimensions x, y, and z become, respectively, $-x$, $-y$, and $-z$. The operation P is called: the mirror transformation since it is equivalent to a π rotation for the coordinate axes.

Parity conservation means that right and left and down and up are impossible to differentiate in the way that the nucleus of an atom throws off decay products up as often as down and left as often as right. The parity transformation operator P is hermitic and unitary.

The Conjugation of the charge, C

The charge conjugation is a mathematical operation that transforms a particle into an antiparticle, i.e., changing the sign of the charge. This means that to a charged particle corresponds an oppositely charged antiparticle. The transformation operator

C is hermitic and unitary.

Inversion of time, T

The inversion of time T is to change the sign of the parameter t of the physics.

From the point of view the mathematical operation of the time inversion is easy but physically is impossible. In physics we consider two reciprocal reactions: $A+B \rightleftarrows E+F$ as a conjugates of T one respect to the other. The associated operator T is hermitic and anti-unitary.

3.2.1 C and P violation

The idea, according to which Nature would support neither right-hand side nor left was called into question in 1956 by Tsung-Dao Lee and Chen Ning Yang [17]. The inflection point came with the study of the nature of two particles with a quite singular behavior: The θ and τ.

The two strange particles of same mass, of same parities, but lifespan intrinsically different taking into account their respective decay products, it means two pions for θ (parity of +1) and three pions for τ (parity -1).

The authors arrived to the conclusion that it could be a question of only one particle, called Kaon. This particle should have two different ways of disintegration, in which case the parity will not be conserved during the weak interactions. The theory was confirmed experimentally by C.S. Wu [18]. She studied the direction of the electrons emissions during the β disintegration in a sample of Cobalt 60.

The spins in the sample were oriented by an external magnetic field at low temperatures. During the experiment it was showed that the emitted electrons were mainly in the opposite direction of the spin inside the Cobalt 60. The image in a mirror of this experiment shows that on the contrary the particles are emitted in the same direction as the moment magnetic of the nucleus, which is incompatible with the observation. Then the parity is not necessarily conserved by the weak interaction.

The C violation is the non conservation of the law associated with charge conjugation and on the other hand, P violation is, similarly to the case of the charge C, the no conservation of the law associated with the parity (P).

3.3 CP Violation

The violation of the combined the conservation laws associated with C and P in the weak nuclear force results in the so-called CP violation, i.e., the neutrino has an left helicity, it means that the particle spins in the opposite direction to its movement but the antineutrino has right helicity.

Now, if we apply the parity operation on the left-handed neutrino we have a right-handed neutrino (the spin is in the same direction of the movement) and this is something never seen in the nature. On the other hand for the charge conjugation operation applied on the left neutrino gives an impossible experimental result, the left-handed antineutrino. But for the product of those transformations (CP) we get a right-handed antineutrino(experimental fact). The transformation CP was considered as a fundamental symmetry until 1964.

Christensen, Cronin, Fitch et Turlay's experiment

The proof of the CP violation came in 1964 by Christensen, Cronin, Fitch et Turlay [19] in the kaons K neutrals system during the realization of an experiment which had as a primordial goal testing the CP invariance of the weak interaction. Ironically the result proved the converse was true.

The mathematical formalism to prove that the CP is a unitary transformation can be developed as follows: for the vectors: $|\alpha>$ and $|\beta>$, we have

$$CP|\alpha>=|\alpha_{CP}>, \qquad and \qquad CP|\beta>=|\beta_{CP}>. \tag{3.1}$$

They satisfy the relation

$$<\alpha_{CP}|\beta_{CP}>=<\alpha|\beta>, \tag{3.2}$$

i.e.

$$[<\alpha|(CP)^{\dagger}]CP|\beta>=<\alpha|[(CP)^{\dagger}CP|\beta>]\equiv<\alpha|\beta>, \tag{3.3}$$

from where we get that

$$(CP)^{\dagger}CP=1. \tag{3.4}$$

CP violation implied nonconservation of T, provided that the long-held CPT theorem was valid. In this theorem, regarded as one of the basic principles of quantum field theory, charge conjugation, parity, and time reversal are applied together.

As a combination, these symmetries constitute an exact symmetry of all types of fundamental interactions.

3.4 The Standard model and the CP violation

In the context of the Standard Model, the CP violation is introduced by taking into account a complex coupling in the matrix CKM (Cabibbo-Kobayashi-Maskawa)[20, 21]. This CKM quark mixing matrix connects the electroweak eigenvectors (d', s', b') of the quarks down, strange and beauty to eigenvalues de masse (d, s, b) by the following unitary transformation,

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \times \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \widehat{V}_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}. \tag{3.5}$$

The elements of the matrix describe the couplings of the charged currents.

The complex phases are currently assumed to be in the furthest off-diagonal elements V_{ub} and V_{td}. Three angles angles and a complex phase are necessary in order to parameterize the matrix CKM. The complex phase permits take into account the CP violation in the Standard Model.

There exist different forms of parametrization for the matrix. One of them is the Standard parametrization. This is the product of three successive rotations in the space states of the quarks down :

$$\widehat{V}_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \times \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\varphi} \\ 0 & 1 & 0 \\ -s_{13}e^{i\varphi} & 0 & c_{13} \end{pmatrix} \times \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\varphi} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\varphi} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\varphi} & s_{23}c_{13} \\ s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\varphi} & -c_{12}s_{23} - s_{12}s_{23}s_{13}e^{i\varphi} & c_{23}c_{13} \end{pmatrix}. \qquad (3.6)$$

or in more compact form

$$\sum_i V_{ij}^* V_{ik} = \delta_{jk} \qquad and \qquad \sum_j V_{ij}^* V_{kj} = \delta_{ik}, \qquad (3.7)$$

where $\delta_{(\dots)}$ is the Kroner delta function.

The parameter φ represents the phase, necessary to take into account the CP violation and it can take values from zero to 2π. But experimental measurements in the for the CP violation in the case of the K particles, limit the interval between zero and π. The $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ can be chosen to be positives.

3.4.1 Final remarks

No completely satisfactory explanation of CP violation has yet been devised. The size of the effect, only about two parts per thousand, has prompted a theory that invokes a new force, called the "superweak" force, to explain the phenomenon.

This force, much weaker than the nuclear weak force, is thought to be observable only in the K-meson system or in the neutron's electric dipole moment, which measures the average size and direction of the separation between charged constituents. Another theory, named the Kobayashi-Maskawa model after its inventors, ascribes certain quantum mechanical effects in the weak force between quarks as the cause of CP violation.

The attractive aspect of the superweak model is that it uses only one variable, the size of the force, to explain everything. Furthermore, the model is consistent with all measurements of CP violation and its properties. The Kobayashi-Maskawa model is more complicated, but it does explain CP violation in terms of known forces.

CP violation has important theoretical consequences. The violation of CP symmetry, taken as a kind of proof of the CPT theorem, enables physicists to make an absolute distinction between matter and antimatter. The distinction between matter and antimatter may have profound implications for cosmology.

One of the unsolved theoretical questions in physics is why the universe is made chiefly of matter. With a series of debatable but plausible assumptions, it can be demonstrated that the observed matter-antimatter ratio may have been produced by the occurrence of CP violation in the first seconds after the "big bang," the violent explosion that is thought to have resulted in the formation of the universe.

However, due to the large uncertainties in evaluating the effect of hadronic interactions for some processes and limited number of measurements of CP violation, we cannot exclude a possibility that physics beyond the Standard Model makes a sizable contribution to CP violation in elementary particle physics.

The current lower limit of the Higgs particle mass is already too high respects to the the small value predicted in the CP violation (Standard Model) to explain the observed matter-antimatter asymmetry in the universe[22]. This provides a strong motivation to search for effects of new physics in CP violation by introducing additional sources of CP violation.

Chapter 4

The Left-Right Supersymmetric Model (LRSUSY)

4.1 A brief description of the model

The motivation for the extension of the minimal supersymmetric standard model to include left-right symmetry was to investigate possible mechanisms for parity violation in weak interactions. This model provides a framerwork in which weak interactions obey all space-time symmetries, along with the strong, electromagnetic and gravitational interactions.

In the L-R supersymmetric model[8, 22, 25] which is described by the group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, where $B - L$ is a quantum number (bayron number minus lepton number). The triplet vector boson $(W^{\pm}, W^0)_{L,R}$ and their superpartners $(\lambda^{\pm}, \lambda^{0})_{L,R}$ are assigned to the gauge groups $SU(2)_{L,R}$; the singlet gauge boson V and its superpartner λ_V is assigned to the gauge group $U(1)_{B-L}$; g_L,g_R and g_V are the gauge coupling constants corresponding to the groups $SU(2)_L$, $SU(2)_R$ and $U(1)_{B-L}$ respectively.

The superpotential of the model can be written as

$$\begin{aligned} \mathbb{W} &= \mathbf{Y}_Q^{(i)} Q^T \tau_2 \Phi_i \tau_2 Q^c + \mathbf{Y}_L^{(i)} L^T \tau_2 \Phi_i \tau_2 L^c + i\mathbf{Y}_{LR}(L^T \tau_2 \Delta_L L + L^{cT} \tau_2 \Delta_R L^c) \\ &+ M_{LR}[Tr(\Delta_L \delta_L + \Delta_R \delta_R)] + \mu_{ij} Tr(\tau_2 \Phi_i^T \tau_2 \Phi_j) + \mathbb{W}_{NR}, \end{aligned} \tag{4.1}$$

where $\mathbb{W}_{NR}$ are the non-renormalizable terms coming from higher scale physics or Plank scale effects [26]. The $\mathbf{Y}_Q$ and $\mathbf{Y}_L$ are the Yukawa couplings for the quarks and leptons with bidoublets Higgs bosons, respectively, and $\mathbf{Y}_{LR}$ is the coupling for the leptons and triplet Higgs bosons.

The Left-Right symmetry requires all $\mathbf{Y}$-matrices to be Hermitian in the generation space and $\mathbf{Y}_{LR}$ matrix to be symmetric: $\mathbf{Y}^i_{Q,L} = \mathbf{Y}^{i\dagger}_{Q,L}$ and $\mathbf{Y}_{L,R} = \mathbf{Y}^i_{L,R}$.

Supersymmetry is responsible for doubling in the number of Higgs fields; Φ_u and Φ_d are needed in order to give masses to both the up and down quarks. This sector (Higgs) contains two bi-doublet fields,

$$\phi_{u,d} = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}_{u,d} \equiv \left(\tfrac{1}{2}, \tfrac{1}{2}, 0 \right). \tag{4.2}$$

In order to give masses to all quarks, the spontaneous breaking of $SU(2)_R \times U(1)_{B-L}$ to $U(1)_Y$ can be arranged by introducing the four Higgs triplet fields,

$$\Delta_{L,R} = \begin{pmatrix} \frac{1}{\sqrt{2}}\Delta^{+} & \Delta^{++} \\ \Delta^{0} & -\frac{1}{\sqrt{2}}\Delta^{+} \end{pmatrix}_{L,R} . \tag{4.3}$$

$$\delta_{L,R} = \begin{pmatrix} \frac{1}{\sqrt{2}}\delta^{-} & \delta^{0} \\ \delta^{--} & -\frac{1}{\sqrt{2}}\delta^{-} \end{pmatrix}_{L,R} . \tag{4.4}$$

The Higgs $\delta_{L,R}$ transform as $(1,0,2)$ and $(0,1,2)$ respectively. The triplet Higgs $\delta_{L,R}$ which transform as $(1,0,-2)$ and $(0,1,-2)$ respectively, are introduced to cancel anomalies in the fermionic sector that would otherwise happen.

The process of the breaking symmetry is done in three steps[23]

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P \longmapsto_{M_P} SU(2)_L \times SU(2)_R \times U(1)_{B-L},$$

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \longmapsto_{M_{W_R}} SU(2)_L \times U(1)_Y,$$

$$SU(2)_L \times U(1)_Y \longmapsto_{M_{W_L}} U(1)_{cm}. \tag{4.5}$$

During the first step, the parity symmetry is broken (M_P is the mass scale at which this breaking occurs; no gauge boson of that mass is produced). Then we have that $g_L \neq g_R$ and leaves $\mathbf{W}_{L,R}$ massless.

The second stage breaks the $SU(2)_R \times U(1)_{B-L}$ to $U(1)_Y$ into the gauge group $U(1)_{em}$. This is possible by considering $< \Delta_R > \neq 0$.

The Higgs multiplets can be chosen in such a way that the parity symmetry and $SU(2)_R$ are broken at the same scale, i.e., $M_P = M_{W_R}$[23]. The third and last stage of breaking can be done considering $< \Phi > \neq 0$ and $< \Delta_L > \neq 0$.

In order to maintain the $U(1)_{em}$ unbroken, we must consider the neutral Higgs fields with nonzero $VEV's$. These quantities take the values,

$$\langle\Delta_L\rangle = \langle\delta_{L,R}\rangle = 0\,,\langle\Delta_R\rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix},\langle\phi_u\rangle = \begin{pmatrix} k_u & 0 \\ 0 & 0 \end{pmatrix},\langle\phi_d\rangle = \begin{pmatrix} 0 & 0 \\ 0 & k_d \end{pmatrix} \quad (4.6)$$

Here $k_{u,d}$ are the VEV's of the doublet Higgs $\phi_{u,d}$ of order of the electroweak scale.

The full Lagrangian of the model can be written as follows

$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{matter} + \mathcal{L}_Y - V + \mathcal{L}_{soft}.$$

The $\mathcal{L}_{gauge}$ is given by

$$\begin{aligned}\mathcal{L}_{gauge} &= \frac{1}{4}\mathbf{W}^L_{\mu\nu}\cdot\mathbf{W}^{\mu\nu}_L + \frac{1}{2}\bar{\lambda}_L\bar{\sigma}_\mu D^L_\mu\lambda_L - \frac{1}{4}\mathbf{W}^R_{\mu\nu}\cdot\mathbf{W}^{\mu\nu}_R + \frac{1}{2}\bar{\lambda}_R\bar{\sigma}_\mu D^R_\mu\lambda_R - \frac{1}{4}V_{\mu\nu}V^{\mu\nu} \\ &+ \frac{1}{2}\bar{\lambda}_V\bar{\sigma}_\mu\partial_\mu\lambda_V. \qquad (4.7)\end{aligned}$$

This Lagrangian is related to the gauge fields and contains the kinetic energy and the self-interaction terms for the vector fields and the Dirac Lagrangian of the gaugino fields.

For the Lagrangian $\mathcal{L}_{matter}$ we have the expression

$$\begin{aligned}
\mathcal{L}_{matter} &= Q_L^\dagger \bar{\sigma}_\mu \left[\partial_\mu - \frac{ig_L}{2}\tau \cdot \mathbf{W}_\mu^L - \frac{ig_V}{6}V_\mu\right] Q_L + Q_R^\dagger \bar{\sigma}_\mu \left[\partial_\mu - \frac{ig_R}{2}\tau \cdot \mathbf{W}_\mu^R\right. \\
&- \left.\frac{ig_V}{6}V_\mu\right] Q_R + L_L^\dagger \bar{\sigma}_\mu \left[\partial_\mu - \frac{ig_L}{2}\tau \cdot \mathbf{W}_\mu^L - \frac{ig_V}{6}V_\mu\right] L_L + L_R^\dagger \bar{\sigma}_\mu \left[\partial_\mu - \frac{ig_R}{2}\right. \\
&\times \left.\tau \cdot \mathbf{W}_\mu^R - \frac{ig_V}{6}V_\mu\right] L_R + Tr\left[(\tau \cdot \tilde{\Delta}_L)^\dagger \bar{\sigma}_\mu [\partial_\mu - \frac{ig_L}{2}\tau \cdot \mathbf{W}_\mu^L - \frac{ig_V}{6}V_\mu]\right. \\
&\times \left.\tau \cdot \tilde{\Delta}_L\right] + Tr\left[(\tau \cdot \tilde{\delta}_L)^\dagger \bar{\sigma}_\mu [\partial_\mu - \frac{ig_L}{2}\tau \cdot \mathbf{W}_\mu^L - \frac{ig_V}{6}V_\mu]\tau \cdot \tilde{\delta}_L\right] \\
&+ Tr\left[(\tau \cdot \tilde{\Delta}_R)^\dagger \bar{\sigma}_\mu [\partial_\mu - \frac{ig_R}{2}\tau \cdot \mathbf{W}_\mu^R - \frac{ig_V}{6}V_\mu]\tau \cdot \tilde{\Delta}_R\right] \\
&+ Tr\left[(\tau \cdot \tilde{\delta}_R)^\dagger \bar{\sigma}_\mu [\partial_\mu - \frac{ig_R}{2}\tau \cdot \mathbf{W}_\mu^R - \frac{ig_V}{6}V_\mu]\tau \cdot \tilde{\delta}_R\right] \\
&+ Tr\left[\bar{\tilde{\mathbf{\Phi}}}_u \bar{\sigma}_\mu [\partial_\mu - \frac{ig_L}{2}\tau \cdot \mathbf{W}_\mu^L - \frac{ig_R}{2}\tau \cdot \mathbf{W}_\mu^R]\tilde{\mathbf{\Phi}}_u\right] \\
&+ Tr\left[\bar{\tilde{\mathbf{\Phi}}}_d \bar{\sigma}_\mu [\partial_\mu - \frac{ig_L}{2}\tau \cdot \mathbf{W}_\mu^L - \frac{ig_R}{2}\tau \cdot \mathbf{W}_\mu^R]\tilde{\mathbf{\Phi}}_d\right] \\
&+ \left|\left[\partial_\mu - \frac{ig_L}{2}\tau \cdot \mathbf{W}_\mu^L - \frac{ig_V}{6}V_\mu\right]\tilde{Q}_L\right|^2 + \left|\left[\partial_\mu - \frac{ig_R}{2}\tau \cdot \mathbf{W}_\mu^R - \frac{ig_V}{6}V_\mu\right]\tilde{Q}_R\right|^2 \\
&+ \left|\left[\partial_\mu - \frac{ig_L}{2}\tau \cdot \mathbf{W}_\mu^L - \frac{ig_V}{2}V_\mu\right]\tilde{L}_L\right|^2 + \left|\left[\partial_\mu - \frac{ig_R}{2}\tau \cdot \mathbf{W}_\mu^R - \frac{ig_V}{2}V_\mu\right]\tilde{L}_R\right|^2 \\
&+ Tr\left|\partial_\mu \mathbf{\Phi}_u - \frac{ig_L}{2}\tau \cdot \mathbf{W}_\mu^L \mathbf{\Phi}_u + \mathbf{\Phi}_u \frac{ig_R}{2}\tau \cdot \mathbf{W}_\mu^R\right|^2 \\
&+ Tr\left|\partial_\mu \mathbf{\Phi}_d - \frac{ig_L}{2}\tau \cdot \mathbf{W}_\mu^L \mathbf{\Phi}_d + \mathbf{\Phi}_d \frac{ig_R}{2}\tau \cdot \mathbf{W}_\mu^R\right|^2 \\
&+ Tr\left|\left[\partial_\mu - \frac{ig_L}{2}\tau \cdot \mathbf{W}_\mu^L - ig_V V_\mu\right]\tau \cdot \Delta_L\right|^2 + Tr\left|\left[\partial_\mu - \frac{ig_R}{2}\tau \cdot \mathbf{W}_\mu^R - ig_V V_\mu\right]\tau \cdot \delta_L\right|^2 \\
&+ Tr\left|\left[\partial_\mu - \frac{ig_R}{2}\tau \cdot \mathbf{W}_\mu^R - ig_V V_\mu\right]\tau \cdot \Delta_R\right|^2 + Tr\left|\left[\partial_\mu - \frac{ig_R}{2}\tau \cdot \mathbf{W}_\mu^R - ig_V V_\mu\right]\tau \cdot \delta_R\right|^2 \\
&+ i\tilde{Q}_L^\dagger [\frac{g_L}{\sqrt{2}}\tau \cdot \lambda_L + \frac{g_V}{3\sqrt{2}}\tau \cdot \lambda_V]Q_L + h.c. + i\tilde{Q}_R^\dagger [\frac{g_R}{\sqrt{2}}\tau \cdot \lambda_R + \frac{g_V}{3\sqrt{2}}\tau \cdot \lambda_V]Q_R + h.c. \\
&+ \frac{i}{\sqrt{2}}\tilde{L}_L^\dagger (g_L \tau \cdot \lambda_L - g_V \lambda_V)L_L + h.c. + \frac{i}{\sqrt{2}}\tilde{L}_R^\dagger (g_R \tau \cdot \lambda_R - g_V \lambda_V)L_R + h.c.
\end{aligned}$$

$$
\begin{aligned}
&+ \quad i\sqrt{2}Tr[(\tau\cdot\Delta_L)^\dagger(g_L\tau\cdot\lambda_L + 2g_V\lambda_V)\tau\cdot\tilde{\Delta}_L] + h.c. \\
&+ \quad i\sqrt{2}Tr[(\tau\cdot\delta_L)^\dagger(g_L\tau\cdot\lambda_L - 2g_V\lambda_V)\tau\cdot\tilde{\delta}_L] + h.c. \\
&+ \quad i\sqrt{2}Tr[(\tau\cdot\Delta_R)^\dagger(g_R\tau\cdot\lambda_R + 2g_V\lambda_V)\tau\cdot\tilde{\Delta}_R] + h.c. \\
&+ \quad i\sqrt{2}Tr[(\tau\cdot\delta_R)^\dagger(g_R\tau\cdot\lambda_R - 2g_V\lambda_V)\tau\cdot\tilde{\delta}_R] + h.c. \\
&+ \quad i\sqrt{2}Tr[\mathbf{\Phi}_u^\dagger(g_L\tau\cdot\lambda_L + g_R\tau\cdot\lambda_R)\tilde{\mathbf{\Phi}}_u] + h.c. \\
&+ \quad i\sqrt{2}Tr[\mathbf{\Phi}_d^\dagger(g_L\tau\cdot\lambda_L + g_R\tau\cdot\lambda_R)\tilde{\mathbf{\Phi}}_d] + h.c. \qquad (4.8)
\end{aligned}
$$

The Yukawa Lagrangian concerns the self-interaction of the matter multiplets,

$$
\begin{aligned}
\mathcal{L}_Y &= h_u^L(L_L^\dagger\Phi_u L_R) + h_d^L(L_L^\dagger\Phi_d L_R) + h_u^Q(Q_L^\dagger\Phi_u Q_R) + h_d^Q(Q_L^\dagger\Phi_d Q_R) \\
&+ h_u^L(\tilde{L}_L^\dagger\tilde{\Phi}_u L_R) + h_d^L(\tilde{L}_L^\dagger\tilde{\Phi}_d L_R) + h_u^Q(\tilde{Q}_L^\dagger\tilde{\Phi}_u Q_R) + h_d^Q(\tilde{Q}_L^\dagger\tilde{\Phi}_d Q_R) \\
&+ h_u^L(\tilde{L}_R^\dagger\tilde{\Phi}_u L_L) + h_d^L(\tilde{L}_R^\dagger\tilde{\Phi}_d L_L) + h_u^Q(\tilde{Q}_R^\dagger\tilde{\Phi}_u Q_L) + h_d^Q(\tilde{Q}_R^\dagger\tilde{\Phi}_d Q_L) \\
&+ Tr[\mu_1(\tau_1\tilde{\Phi}_u\tau_1)^T\tilde{\Phi}_d] + Tr[\mu_2(\tau\cdot\tilde{\Delta}_L)(\tau\cdot\tilde{\delta}_L)] + Tr[\mu_3(\tau\cdot\tilde{\Delta}_R)(\tau\cdot\tilde{\delta}_R)] \\
&+ h_{LR}(L_L^T\tau_1\tau\cdot\Delta_L L_L + L_R^T\tau_1\tau\cdot\Delta_R L_R) \\
&+ h_{LR}(\tilde{L}_L^T\tau_1\tau\cdot\tilde{\Delta}_L L_L + \tilde{L}_R^T\tau_1\tau\cdot\tilde{\Delta}_R L_R) + h.c. \qquad (4.9)
\end{aligned}
$$

The scalar potential can be written as follows

$$
V = |F|^2 + \frac{1}{2}|D|^2 + V_{soft}, \qquad (4.10)
$$

where

$$
\begin{aligned}
|F|^2 &= |h_u^Q\tilde{Q}_L\tilde{Q}_R + h_u^L\tilde{L}_L\tilde{L}_R|^2 + |h_d^Q\tilde{Q}_L\tilde{Q}_R + h_d^L\tilde{L}_L\tilde{L}_R|^2 + |h_u^Q\Phi_u\tilde{Q}_R h_d^Q\Phi_d\tilde{Q}_R|^2 \\
&+ |h_u^Q\Phi_u\tilde{Q}_L + h_d^Q\Phi_d\tilde{Q}_L|^2 + |h_u^L\Phi_u\tilde{L}_R + h_d^L\Phi_d\tilde{L}_R + 2h_{LR}\tau\cdot\Delta_L\tilde{L}_L|^2 \\
&+ |h_u^L\Phi_u\tilde{L}_L + h_d^L\Phi_d\tilde{L}_L + 2h_{LR}\tau\cdot\Delta_R\tilde{L}_R|^2 + h.c. \qquad (4.11)
\end{aligned}
$$

The term D is given by

$$|D|^2 = g_L \sum_L |\sum_A A^\dagger \tau_L A|^2 + g_R \sum_L |\sum_A A^\dagger \tau_R A|^2 + g_V \sum_L |\sum_A A^\dagger \mathcal{V} A|^2, \quad (4.12)$$

where $A = \tilde{Q}_L, \tilde{Q}_R, \tilde{L}_L, \tilde{L}_R, \Phi_u, \Phi_d, \Delta_L, \Delta_R, \delta_L,$ and $\delta_R,$ and $\tau_L, \tau_R,$ and $\mathcal{V}$ are the generators of the gauge groups:

$$\begin{aligned} V_{soft} &= m_s \Big(\{ h_u^Q \tilde{Q}_L^\dagger \Phi_u \tilde{Q}_R + h_d^Q \tilde{Q}_L^\dagger \Phi_d \tilde{Q}_R + h_u^L \tilde{L}_L^\dagger \Phi_u \tilde{L}_R + h_d^L \tilde{L}_L^\dagger \Phi_d \tilde{L}_R \\ &+ h_{LR} (\tilde{L}_L^T \tau_1 \tau \cdot \Delta_L \tilde{L}_L + \tilde{L}_R^T \tau_1 \tau \cdot \Delta_R \tilde{L}_R) + Tr[\mu_1 (\tau_1 \Phi_u \tau_1)^T \Phi_d] \} \\ &+ Tr[\mu_2 (\tau \cdot \Delta_L)(\tau \cdot \delta_L)] + Tr[\mu_3 (\tau \cdot \Delta_R)(\tau \cdot \delta_R)] + h.c. \Big) \\ &+ m_{QR}^2 \tilde{Q}_R^\dagger \tilde{Q}_R + m_{LL}^2 \tilde{L}_L^\dagger \tilde{L}_L + m_{LR}^2 \tilde{L}_R^\dagger \tilde{L}_R. \end{aligned} \quad (4.13)$$

Finally, the soft-breaking Lagrangian $\mathcal{L}_{soft}$ is given by

$$\mathcal{L}_{soft} = m_L (\lambda_L^a \lambda_L^a + \bar{\lambda}_L^a \bar{\lambda}_L^a) + m_R (\lambda_R^a \lambda_R^a + \bar{\lambda}_R^a \bar{\lambda}_R^a) + m_V (\lambda_V \lambda_V + \bar{\lambda}_V \bar{\lambda}_V). (4.14)$$

By means of $\mathcal{L}_{soft}$ it is possible to give Majorana mass to the gauginos.

In order to study the gauge boson-chargino cross section we must find the mass eigenstates from the diagonalization of the vector bosons associated mass matrix.

In the LR-SUSY model the generation of boson masses is split in two steps given the fact that the values of the VEV's chosen for the Higgs fields allows us to distinguish left and right breaking scales ($v_R >> k_u, k_d$ and $k'_u, k'_d = v_L = 0$).

The first breaking step has $\langle \Delta_R \rangle$ generating masses for $W_R^\pm, W_R^0$ and V. Subsequently, the mixing of the two neutral states W_R^0 and V give rise to the physical fields Z_R and B.

The next step involves $\Phi_{u,d}$, a field which couples to both left and right-handed components. This is responsible for mixing W_L and W_R, but to such a small degree that it warrants the treatment of right-handed fields as "effectively decoupled" at this energy.

Thus, it is only $W_L^{\pm}, W_L^0$ and B which acquire masses at this level. The neutral fields W_L^0 and B mix and give rise to the Z_L and A_μ bosons which are familiar from the Standard Model.

The Lagrangian to be considered in the first stage of boson mass generation is[11, 12]

$$+Tr|(-i\frac{g_R}{2}\tau \cdot \mathbf{W}_\mu^R - ig_V V)\,\tau \cdot \boldsymbol{\Delta}_R|^2 \tag{4.15}$$

from where we can get the fields

$$Z_R = \frac{g_R W_R^0 - 2g_V V}{(g_R^2 + 4g_V^2)^{1/2}} \tag{4.16}$$

with the mass

$$M_{Z_R} = \frac{1}{\sqrt{2}} V_R (g_R^2 + 4g_V^2)^{1/2}, \tag{4.17}$$

and

$$B = \frac{g_R V + 2g_V W_R^0}{(g_R^2 + 4g_V^2)^{1/2}} \tag{4.18}$$

with

$$M_B = 0. \tag{4.19}$$

The massless eigenstate B_μ is the gauge boson of the symmetry group $U(1)_Y$, which survives the breaking of $SU(2)_R \times U(1)_{B-L}$. The eigenstates $W_R^\pm$ and Z_R decouple from the low-energy theory leaving B_μ as the only field which will be taken into account in the next stage process of symmetry breaking [12].

For left-handed vectors bosons, the Lagrangian terms to be considered in the second stage of symmetry breaking are:

$$\begin{aligned} &+ \quad Tr|\partial_\mu \Phi_u - i\frac{g_L}{2}\tau \cdot \mathbf{W}_\mu^L \Phi_u + \Phi_u \frac{ig_R}{2}\tau \cdot \mathbf{W}_\mu^R|^2 \\ &+ \quad Tr|\partial_\mu \Phi_d - i\frac{g_L}{2}\tau \cdot \mathbf{W}_\mu^L \Phi_d + \Phi_d \frac{ig_R}{2}\tau \cdot \mathbf{W}_\mu^R|^2. \end{aligned} \tag{4.20}$$

Here the terms containing the charged gauge bosons $W_R^\pm$ and Z_R are neglected. We will consider only the neutral boson W_R^0, which is given by[10]

$$W_R^0 = \frac{g_R Z_R + 2g_V B}{(g_R^2 + 4g_V^2)^{1/2}}, \tag{4.21}$$

where the gauge coupling constant g' of $U(1)_Y$ can be written as

$$g' = \frac{g_R g_V}{(g_R^2 + 4g_V^2)^{1/2}}, \tag{4.22}$$

in order to get the neutral mass eigenstates as

$$Z_L = \frac{g_L W_L^0 + 2g' B}{(g_L^2 + 4g'^2)^{1/2}} \tag{4.23}$$

of mass

$$M_{Z_L} = \frac{1}{\sqrt{2}} \left[(k_u^2 + k_d^2)\,(g_L^2 + 4g'^2) \right]^{1/2}. \tag{4.24}$$

For the $W_L^\pm$ the expressions for the masses are given by:

$$M_{W_L} = \frac{1}{\sqrt{2}} g_L\, (k_u^2 + k_d^2)^{1/2}. \tag{4.25}$$

Finally the massless photon is

$$A_\mu = \frac{2g'W_L^0 + 2g'B}{(g_L^2 + 4g'^2)^{1/2}}. \tag{4.26}$$

Chapter 5

Charged Gauginos and Higgsinos (Charginos)

5.1 Introduction

In this chapter we obtain the chargino masses and the chargino mixing matrix in terms of analytic expressions. The method is developed in the context of the left-right supersymmetric model. We also provide a comparison with the numerical solutions previously published.

If the collider energy is sufficient to produce the two chargino states in pairs, the underlaying fundamental $LRSUSY$ parameters, such as $\mu, \tan\beta, M_L$ and M_R, can be extracted from the chargino masses $\tilde{\mathrm{M}}_{\chi_i^{\pm}}, i = 1, ..., 4$.

Finding the physical chargino states is a quite complicated task given the fact that it involves the diagonalization of 4×4 matrix, which determines the chagino masses and mixing, and thereby the various couplings.

An important step in the development of the LRSUSY model was in Ref.[27], where approximated analytical expressions and numerical solutions for chargino and neutralino masses were determined.

The goal of this chapter is to give the exact expressions for the chargino masses and their corresponding mixing matrix for the left-right supersymmetric ($LRSYSU$) model.

Higgsinos are the fermionic superpartners of the Higgs bosons while the gauginos are the fermionic superpartners of the gauge bosons $\gamma, W_L^\pm, W_R^\pm, Z_L$ and Z_R.

Since the higgsinos and gauginos have some identical quantum numbers, it is possible for these to mix. After gauge symmetries breaking, we obtain the physical states that, in the case of charged particles, are called charginos.

The terms of the Lagrangian $\mathcal{L}$ (see Eq.4.7) that are relevant to the mixing of chargino and higgsinos are

$$
\begin{aligned}
\mathcal{L}_{gh} &= i\sqrt{2}Tr\left[(\tau\cdot\Delta_L)^\dagger(g_L\tau\cdot\lambda_L+2g_V\lambda_V)\tau\cdot\tilde{\Delta_L}\right]+h.c \\
&+ i\sqrt{2}Tr\left[(\tau\cdot\Delta_R)(g_R\tau\cdot\lambda_R+2g_V\lambda_V)\tau\cdot\tilde{\Delta_R}\right]+h.c \\
&+ \frac{iTr}{\sqrt{2}}\left[\Phi_d(g_L\tau\cdot\lambda_L+2g_V\lambda_V)\tilde{\Phi}_d\right]+h.c \\
&+ \frac{iTr}{\sqrt{2}}\left[\Phi_u(g_L\tau\cdot\lambda_L+2g_V\lambda_V)\tilde{\Phi}_u\right]+h.c
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad M_L(\lambda_L^a\lambda_L^a + \bar{\lambda_L}^a\bar{\lambda_L}^a) + M_R(\lambda_R^a\lambda_R^a + \bar{\lambda_R}^a\bar{\lambda_R}^a) \\
&+ \quad M_V(\lambda_V\lambda_V + \bar{\lambda_V}\bar{\lambda_V}) + Tr(\mu_1[\tau_1\tilde{\phi}_u\tau_1]^T\tilde{\Phi}_d) \\
&+ \quad Tr\left[\mu_2(\tau\cdot\tilde{\Delta}_L)(\tau\cdot\tilde{\delta}_L)\right] + Tr\left[\mu_2(\tau\cdot\tilde{\Delta}_R)(\tau\cdot\tilde{\delta}_R)\right]. \qquad (5.1)
\end{aligned}
$$

In order to get the expressions for the charged gaugino-higgsino mixing Lagrangian we must substitute in Eq.(5.1) the VEVs given by Eq.(4.6), then it yields

$$
\begin{aligned}
\mathcal{L}_{c.m.} &= \Big[i\lambda_R^-(\sqrt{2}\, g_R\, \nu_R\, \bar{\Delta}_R^+ + g_R\, k_d\, \widetilde{\phi}_d^+) + i\, g_L\, k_d\, \widetilde{\phi}_d^+\, \lambda_L^- \\
&+ \quad i\, g_R\, k_u\, \widetilde{\phi}_u^-\, \lambda_R^+ + i\, g_L\, k_u\, \widetilde{\phi}_u^-\, \lambda_L^+ + M_L\, \lambda_L^+\, \lambda_L^- \\
&+ \quad M_R\, \lambda_R^+\, \lambda_R^- + \mu\, \widetilde{\phi}_u^+\, \widetilde{\phi}_d^- + \mu\, \widetilde{\phi}_u^-\, \widetilde{\phi}_d^+ \Big] + \text{H.c.} \qquad (5.2)
\end{aligned}
$$

Then the two stages of symmetry breaking are considered separately. First, charged fermions combine into four component Dirac spinors from the $\mathcal{L}_W$ part of the Lagrangian

$$
\mathcal{L}_W = (i\sqrt{2}g_R\nu_R)\lambda_R^-\tilde{\Delta}_R^+ + h.c, \qquad (5.3)
$$

At this stage supersymmetry is unbroken and the mass of $\tilde{W}_R^+$, $\sqrt{2}g_R\nu_R$ is the same as that of W_R^+. The particles produced at this stage are very massive and decouple from the low-energy theory.

At the next step the remaining terms in the Lagrangian will be analyzed in the next section.

5.2 Chargino masses

The mass eigenstates are identified by finding and diagonalizing the mass matrix M^C defined by

$$\mathcal{L}_C = -\frac{1}{2}\begin{pmatrix} \psi^+ \, \psi^- \end{pmatrix}\begin{pmatrix} 0 & M^{CT} \\ M^C & 0 \end{pmatrix}\begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} + \text{H.c.} \tag{5.4}$$

where

$$\begin{aligned} \psi^+ &= (-i\,\lambda_L^+, -i\,\lambda_R^+, \widetilde{\phi}_u^+, \widetilde{\phi}_d^+)^T, \\ \psi^- &= (-i\,\lambda_L^-, -i\,\lambda_R^-, \widetilde{\phi}_u^-, \widetilde{\phi}_d^-)^T. \end{aligned} \tag{5.5}$$

The mass matrix is read directly off the Langrangian and is given by

$$M^C = \begin{pmatrix} M_L & 0 & 0 & g_L\,k_d \\ 0 & M_R & 0 & g_R\,k_d \\ g_L\,k_u & g_R\,k_u & 0 & -\mu \\ 0 & 0 & -\mu & 0 \end{pmatrix}, \tag{5.6}$$

where we have taken, for simplification, $\mu_{ij} = \mu$. Since M^C is an asymmetric matrix, we require two unitary matrices U and V to diagonalize M^C, that can be expressed as $U^*\,M^C\,V^{-1} = M_D$. The diagonalizing procedure results in the physical chargino states given by

$$\chi_i^+ = V_{ij}\,\psi_j^+,\; \chi_i^- = U_{ij}\,\psi_j^-,\; (i,j = 1,\ldots,4). \tag{5.7}$$

whose masses are the positive square roots of

$$M_D^2 = V\,M^{C\dagger}\,M^C\,V^{-1} = U^*\,M^C\,M^{C\dagger}\,(U^*)^{-1}. \tag{5.8}$$

The values of V, U and M_D are unknown and have been determined applying, until now, a numerical or perturbative analysis by assuming values for the gauge boson masses, the couplings and through variation of the higgsino mass parameter μ shown in the next section.

5.3 Recent developments for Chargino masses

In Ref.[27], the authors have shown the numerical results of the chargino masses. The Fig.(5.1) shows the behavior of the four chargino masses as a function of μ.

On the other hand, the same paper, by means of a perturbative analysis, presents analytical expressions for the chargino masses by assuming for large M_R, M_L, μ, that $|M_R\mu| \gg M_W^2 \sin^2\theta_k, M_W^2 \cos^2\theta_k$ and $|M_L\mu| \gg M_w^2 \sin^2\theta_k, M_W^2 \cos^2\theta_k$. The physical masses are defined to be positive and by convention,

$$\tilde{\mathrm{M}}_{\chi_4^\pm} > \tilde{\mathrm{M}}_{\chi_3^\pm} > \tilde{\mathrm{M}}_{\chi_2^\pm} > \tilde{\mathrm{M}}_{\chi_1^\pm}. \tag{5.9}$$

The Fig.(5.2) shows the plot of the chargino masses following the analytical expressions obtained from Ref.[22, 27]. Notice the difference with the Fig.(5.1). The fact that the explicit formulation of the masses was obtained under asymptotic considerations is the explanation to the mismatch between both graphs.

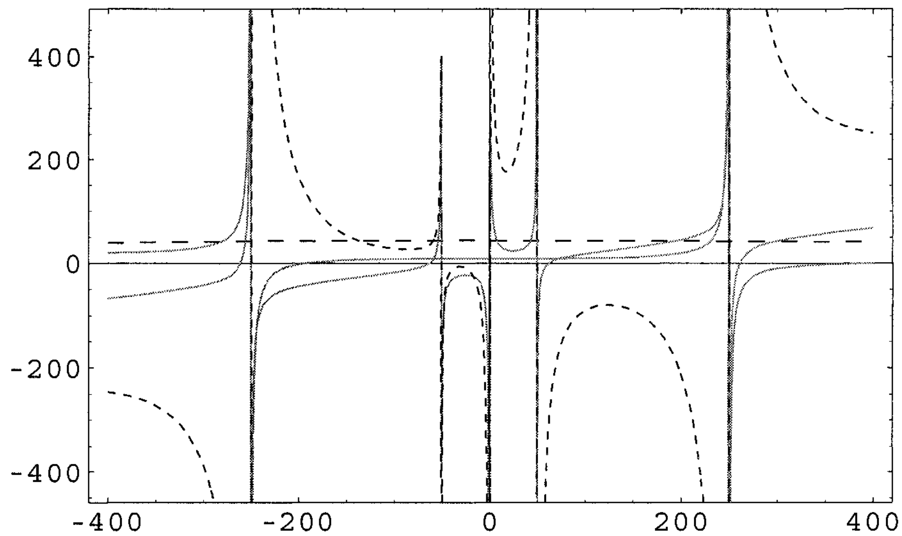
The chargino mass expressions obtained in Refs.[22, 27] were

$$
\begin{aligned}
M^{\pm}_{\chi_1} &\simeq M_L + \frac{D + M_L^2(M_L^2 A + B) - 2M_L^6(\mu^2 + 2M_W^2) - M_L^6 M_R^6}{2M_L(M_R^2 - M_L^2)(M_L^4 - \mu)}, \\
M^{\pm}_{\chi_2} &\simeq M_R - \frac{M_L^2 M_R^2 B + M_L^2 D + M_R^4(M_L^2 A - M_L^4(\mu^2 + 2M_W^2) - M_L^4 M_R^2)}{2M_L^2 M_R(M_R^2 - M_L^2)(M_L^4 - \mu^4)} \\
&- \frac{M_R^6 M_L^2(\mu^2 + 2M_W^2) - M_L^6 M_R^6}{2M_L^2 M_R(M_R^2 - M_L^2)(M_L^4 - \mu^4)}, \\
M^{\pm}_{\chi_3} &\simeq \mu + \frac{M_L^2\mu^6 + M_L^2 M_R^2\mu^2(\mu^2 + 2M_W^2) - D + 4\mu^4 M_W^2 + \mu^8 - \mu^2 B}{4\mu^3(M_L^2 - \mu^2)(M_R^2 - \mu^2)} \\
&- \frac{\mu^4(A + 2M_W^2 M_R^2)}{4\mu^3(M_L^2 - \mu^2)(M_R^2 - \mu^2)}, \\
M^{\pm}_{\chi_4} &\simeq \mu + \frac{M_L^2\mu^6 + M_L^2 M_R^2\mu^2(\mu^2 + 2M_W^2) + D + 4\mu^4 M_W^2 + 3\mu^8 - \mu^2 B}{4\mu^3(M_L^2 + \mu^2)(M_R^2 + \mu^2)} \\
&+ \frac{\mu^4(A + 2M_W^2 M_R^2) + 2M_R^2\mu^6}{4\mu^3(M_L^2 + \mu^2)(M_R^2 + \mu^2)},
\end{aligned} \tag{5.10}
$$

where

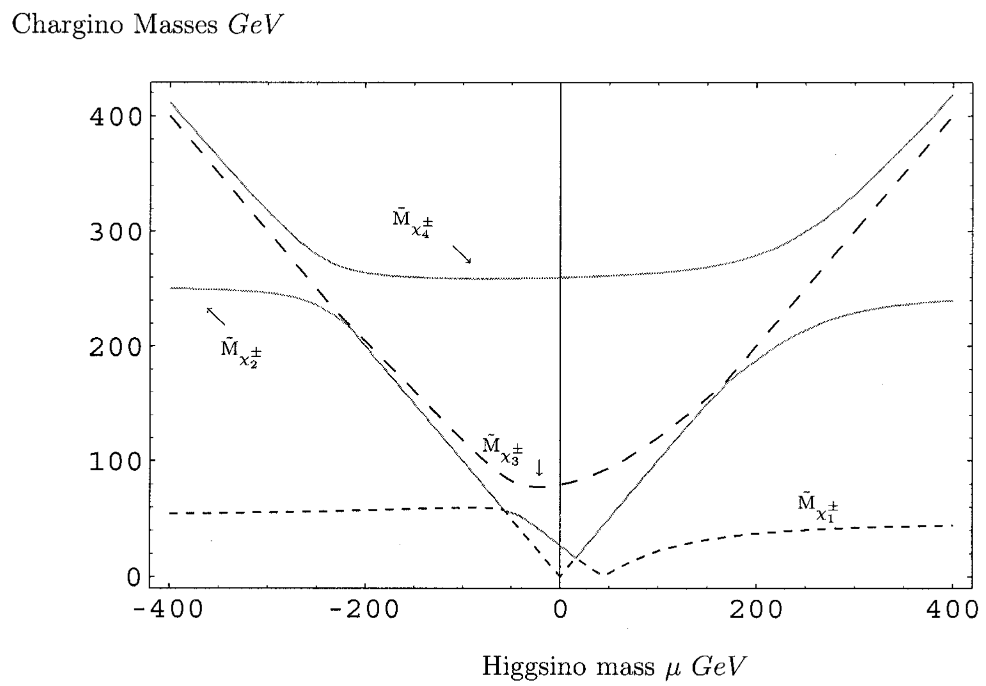
$$
\begin{aligned}
A &\equiv cd - a^2 + f^2 - ab + (e + \mu^2)(c + d) + \mu^2 e, \\
B &\equiv -(e + \mu^2)(cd + f^2) - e\mu^2(c + d) + a^2(\mu^2 + c) + ab(\mu^2 + d), \\
D &\equiv \mu^2 e(f^2 + cd) - a\mu^2(ac + bd),
\end{aligned} \tag{5.11}
$$

Chargino Masses GeV



Higgsino mass μ GeV

Figure 5.1: Chargino masses versus the higgsino mass parameter. The graph was obtained using the expressions of Ref.(27). $M_R = 300$ GeV, $M_L = 50$ GeV, $M_W = 80$ GeV and $\tan\theta_k = 1.6$.



Chargino masses versus the higgsino mass parameter. The graph was obtained using numerical computation. $M_R = 300$ GeV, $M_L = 50$ GeV, $M_W = 80$ GeV and $\tan\theta_k$=1.6

and

$$\begin{aligned}
a &= \sqrt{2}M_W(M_R\cos\theta_k - \mu\sin\theta_k), b = \sqrt{2}M_W(M_L\cos\theta_k - \mu\sin\theta_k),\\
c &= M_R^2 + 2M_W^2\sin^2\theta_k, d = M_L^2 + 2M_W^2\sin^2\theta_k,\\
e &= \mu^2 + 4M_W^2\cos^2\theta_k, f = 2M_W^2\sin^2\theta_k. \qquad (5.12)
\end{aligned}$$

5.4 Analytic expressions for Chargino masses

Here, we will present a method to obtain the analytic expressions for the masses mentioned above. Specifically, the equations for the chargino masses and the matrix V are given. To find the elements of the U the same procedure is valid.

Since the matrix V is unitary ($V = V^{-1}$), it is not difficult to see from Eq.(5.8) that the following relation holds

$$V\,(M^{C\dagger}\,M^{C}) - M_D^2\,V = 0. \qquad (5.13)$$

Equating the elements of the ith row of Eq.(5.13), it yields

$$\begin{aligned}
[M_{11} - (M_D^2)_{ii}]\,V_{i1} + M_{21}\,V_{i2} + M_{31}\,V_{i3} + M_{41}\,V_{i4} &= 0\,,\\
M_{12}\,V_{i1} + [M_{22} - (M_D^2)_{ii}]\,V_{i2} + M_{31}\,V_{i3} + M_{41}\,V_{i4} &= 0\,,\\
M_{13}\,V_{i1} + M_{21}\,V_{i2} + [M_{33} - (M_D^2)_{ii}]\,V_{i3} + M_{41}\,V_{i4} &= 0\,,\\
M_{14}\,V_{i1} + M_{21}\,V_{i2} + M_{31}\,V_{i3} + [M_{44} - (M_D^2)_{ii}]\,V_{i4} &= 0\,. \qquad (5.14)
\end{aligned}$$

The ith system of homogeneous linear equations contains only the ith row of V and one of the masses. The chargino masses can be determined by solving the eigenvalue equation

$$\begin{vmatrix} M_{11}-(M_D^2)_{ii} & M_{21} & M_{31} & M_{41} \\ M_{12} & M_{22}-(M_D^2)_{ii} & M_{32} & M_{42} \\ M_{13} & M_{23} & M_{33}-(M_D^2)_{ii} & M_{43} \\ M_{14} & M_{24} & M_{34} & M_{33}-(M_D^2)_{ii} \end{vmatrix} = 0. \qquad (5.15)$$

Substituting the corresponding expressions for the M_{ij} we get

$$(M_D^2)^4_{ii} - a\,(M_D^2)^3_{ii} + b\,(M_D^2)^2_{ii} + c\,(M_D^2)_{ii} + d = 0. \qquad (5.16)$$

In the general form we have the following expressions,

$$a \;=\; \mathrm{Tr}\,M, \qquad d = \mathrm{Det}\,M, \qquad (5.17)$$

$$\begin{aligned} b \;=\; & \begin{vmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{vmatrix} + \begin{vmatrix} M_{11} & M_{13} \\ M_{31} & M_{33} \end{vmatrix} + \begin{vmatrix} M_{11} & M_{14} \\ M_{41} & M_{44} \end{vmatrix} + \begin{vmatrix} M_{22} & M_{23} \\ M_{32} & M_{33} \end{vmatrix} + \begin{vmatrix} M_{22} & M_{24} \\ M_{42} & M_{44} \end{vmatrix} \\ + & \begin{vmatrix} M_{33} & M_{34} \\ M_{43} & M_{44} \end{vmatrix}, \end{aligned} \qquad (5.18)$$

$$
\begin{aligned}
c \;=\; & \begin{vmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{vmatrix} + \begin{vmatrix} M_{11} & M_{12} & M_{14} \\ M_{21} & M_{22} & M_{24} \\ M_{41} & M_{42} & M_{44} \end{vmatrix} + \begin{vmatrix} M_{11} & H_{13} & M_{14} \\ M_{31} & M_{33} & M_{34} \\ M_{41} & M_{43} & M_{44} \end{vmatrix} \\
+ & \begin{vmatrix} M_{22} & M_{23} & M_{24} \\ M_{32} & M_{33} & M_{34} \\ M_{42} & M_{43} & M_{44} \end{vmatrix}.
\end{aligned}
\tag{5.19}
$$

For our specific problem we have that,

$$
\begin{aligned}
a \;&\equiv\; (M_{22} + M_{33} + M_{44} + M_{11}), \\
b \;&\equiv\; \Big(M_{33}\, M_{44} + M_{33}\, M_{11} + M_{44}\, M_{11} - M_{12}\, M_{21} - M_{14}\, M_{41} - M_{24}\, M_{42} \\
&+\; M_{22}\, [M_{33} + M_{44} + M_{11}]\Big), \\
c \;&\equiv\; \Big(M_{24}\, M_{33}\, M_{42} + M_{14}\, [M_{22}\, M_{41} + M_{33}\, M_{41} - M_{21}\, M_{42}] - M_{22}\, M_{33}\, M_{44} \\
&+\; M_{12}\Big[-M_{24}\, M_{41} + M_{21}\, [M_{33} + M_{44}]\Big] \\
&-\; M_{22}\, M_{33}\, M_{11} + M_{24}\, M_{42}\, M_{11} - M_{22}\, M_{44}\, M_{11} - M_{33}\, M_{44}\, M_{11}\Big), \\
d \;&\equiv\; \Big(M_{33}\Big[M_{14}\, [-M_{22}\, M_{41} + M_{21}\, M_{42}] + M_{12}\, [M_{24} M_{41} - M_{21}\, M_{44}] \\
&+\; M_{11}\, [-M_{24}\, M_{42} + M_{22}\, M_{44}]\Big]\Big).
\end{aligned}
\tag{5.20}
$$

Solving la Eq.(5.16), the charginos exact masses analytic formulas are given by

$$\tilde{\mathrm{M}}_{\chi_1^\pm} = \Re\left[\frac{a}{4} - \frac{\alpha}{2} - \frac{1}{2}\sqrt{\xi - \varpi - \frac{\lambda}{4\alpha}}\right]^{\frac{1}{2}}, \tag{5.21}$$

$$\tilde{\mathrm{M}}_{\chi_2^\pm} = \Re\left[\frac{a}{4} - \frac{\alpha}{2} + \frac{1}{2}\sqrt{\xi - \varpi - \frac{\lambda}{4\alpha}}\right]^{\frac{1}{2}}, \tag{5.22}$$

$$\tilde{\mathrm{M}}_{\chi_3^\pm} = \Re\left[\frac{a}{4} + \frac{\alpha}{2} - \frac{1}{2}\sqrt{\xi - \varpi + \frac{\lambda}{4\alpha}}\right]^{\frac{1}{2}}, \tag{5.23}$$

$$\tilde{\mathrm{M}}_{\chi_4^\pm} = \Re\left[\frac{a}{4} + \frac{\alpha}{2} + \frac{1}{2}\sqrt{\xi - \varpi + \frac{\lambda}{4\alpha}}\right]^{\frac{1}{2}}, \tag{5.24}$$

where $\Re$ represents the value real value of the function and

$$\begin{aligned}
\alpha &\equiv \sqrt{(\beta + \nu + \varpi)}, \quad \beta \equiv \left[\frac{a^2}{4} - \frac{2b}{3}\right], \\
\gamma &\equiv (b^2 + 3\,a\,c + 12\,d),\ \delta \equiv (2\,b^3 + 9\,a\,b\,c + 27\,c^2 + 27\,a^2\,d - 72\,b\,d), \\
\epsilon &\equiv (\delta + \sqrt{\eta})^{\frac{1}{3}}, \qquad \eta \equiv (-4\,\gamma^3 + \delta^2), \\
\lambda &\equiv (a^3 - 4\,a\,b - 8\,c),\ \nu \equiv \frac{(2^{\frac{1}{3}}\,\gamma)}{3\,\epsilon}, \\
\xi &\equiv \left[\frac{a^2}{2} - \frac{4b}{3} - \nu\right], \quad S \equiv (\xi - \varpi - \lambda), \\
\varpi &\equiv \frac{\epsilon}{32^{\frac{1}{3}}}.
\end{aligned}$$

5.5 Analytic eigenvectors expressions (Matrices V and U^*)

To find the matrix V we must use the system of equations Eq.(5.14). Dividing the four equations by V_{i1}, where it is assumed that $V_{i1} \neq 0$, the system becomes

$$M_{21}\frac{V_{i2}}{V_{i1}} + M_{31}\frac{V_{i3}}{V_{i1}} + M_{41}\frac{V_{i4}}{V_{i1}} = -[M_{11} - (M_D)_{ii}],$$

$$[M_{22} - (M_D)_{ii}]\frac{V_{i2}}{V_{i1}} + M_{32}\frac{V_{i3}}{V_{i1}} + M_{42}\frac{V_{i4}}{V_{i1}} = -M_{12},$$

$$M_{23}\frac{V_{i2}}{V_{i1}} + [M_{33} - (M_D)_{ii}]\frac{V_{i3}}{V_{i1}} + M_{43}\frac{V_{i4}}{V_{i1}} = -M_{13},$$

$$M_{24}\frac{V_{i2}}{V_{i1}} + M_{34}\frac{V_{i3}}{V_{i1}} + [M_{44} - (M_D)_{ii}]\frac{V_{i4}}{V_{i1}} = -M_{14}. \tag{5.25}$$

Solving these system of equations and taking into account the unitary relation

$$V_{i1}^2 + V_{i2}^2 + V_{i3}^2 + V_{i4}^2 = 1, \tag{5.26}$$

where the V_{ij} matrix's component are given by

$$V_{1j} = \frac{|\Delta_j|}{\sqrt{|\Delta_j|^2 + |\Delta_{2j}|^2| + |\Delta_{3j}|^2| + |\Delta_{4j}|^2}}, \tag{5.27}$$

$$V_{ij} = -\frac{\Delta_{ij}\,|\Delta_j|}{\Delta_j\,\sqrt{|\Delta_j|^2 + |\Delta_{2j}|^2| + |\Delta_{3j}|^2| + |\Delta_{4j}|^2}}, \tag{5.28}$$

for $i = 2, 3, 4$, and

$$\Delta_j = \begin{vmatrix} M_{22} - (M_D^2)_{jj} & M_{23} & M_{24} \\ M_{32} & M_{33} - (M_D^2)_{jj} & M_{34} \\ M_{42} & M_{43} & M_{44} - (M_D^2)_{jj} \end{vmatrix}$$

and the $\Delta'_{ij}s$, $(i = 2, 3, 4)$ are formed from Δ_j by substituting the $(i-1)$th column for $\begin{pmatrix} M_{21} \\ M_{31} \\ M_{41} \end{pmatrix}$.

The V_{ij} matrix's component can be written as

$$V_{i1} = \left[1 + \frac{g_R^2(g_L^2\, h_1 + (\mu k_u - k_d M_R)(M_L^2 - \tilde{\mathrm{M}}_{\chi_i^\pm}))^2 + h_i^2}{\Delta_{M_{i1}}}\right]^{-\frac{1}{2}}, \tag{5.29}$$

$$V_{i2} = \left[1 + \frac{g_L^2(g_R^2\, h_1 + (-\mu k_u + k_d M_L)(M_R^2 - \tilde{\mathrm{M}}_{\chi_i^\pm}))^2 + h_i^2}{\Delta_{M_{i2}}}\right]^{-\frac{1}{2}}, \tag{5.30}$$

$$V_{i4} = \left[1 + \frac{g_R^2(g_L^2\, h_1 + (\mu k_u - k_d M_R)(M_L^2 - \tilde{\mathrm{M}}_{\chi_i^\pm}))^2 + g_L^2 H_i^2}{\Delta_{M_{i4}}}\right]^{-\frac{1}{2}}, \tag{5.31}$$

where

$$\begin{aligned}
h_1 &= k_d k_u^2 (M_L - M_R), \\
h_i &= g_R^2 k_u^2(-M_L^2 + \tilde{\mathrm{M}}_{\chi_i^\pm}) + (g_L^2 k_u^2 + M_L^2 - \tilde{\mathrm{M}}_{\chi_i^\pm})(-M_R^2 + \tilde{\mathrm{M}}_{\chi_i^\pm}), \\
H_i &= [g_R^2\, h_1 + (-\mu\, k_u + k_d\, M_L)(M_R^2 - \tilde{\mathrm{M}}_{\chi_i^\pm})], \\
\Delta_{M_{i1}} &= g_L^2\, [g_R^2 h_1 + (-\mu k_u + k_d M_L)(-M_R^2 + \tilde{\mathrm{M}}_{\chi_i^\pm})]^2, \\
\Delta_{M_{i2}} &= g_R^2\, [g_L^2 h_1 + (\mu k_u - k_d M_R)(M_L^2 + \tilde{\mathrm{M}}_{\chi_i^\pm})]^2, \\
\Delta_{M_{i4}} &= g_R^2\, k_u^2[(-M_L^2 - \tilde{\mathrm{M}}_{\chi_i^\pm}) + (g_L^2 k_u^2 + M_L^2 - \tilde{\mathrm{M}}_{\chi_i^\pm})(-M_R^2 + \tilde{\mathrm{M}}_{\chi_i^\pm})]^2.
\end{aligned}$$

The components V_{i3} have been determined by means of the Eq.(5.26) as well as the elements of the matrix U^* but following the procedure used to find V.

The graph of the chargino masses using the exact expressions obtained in this work is identical to Fig.(5.2).

We have checked our results not only by the graphical way but computing numerically the matrices with experimental values and found they coincide. We have assumed $M_R = 250GeV$, $M_L = 50GeV$, $M_W = 80GeV$, $\tan\theta_k = 2$ and $\mu = 0$.

For V we have that

$$V \cdot V^{-1} = \begin{pmatrix} 1. & -1.52 \cdot 10^{-14} & -8.1878 \cdot 10^{-15} & -3.5882 \cdot 10^{-13} \\ 2.0605 \cdot 10^{-12} & 1. & -1.5071 \cdot 10^{-14} & -3.5882 \cdot 10^{-13} \\ 0 & 8.8817 \cdot 10^{-16} & 1. & 0 \\ 1.3642 \cdot 10^{-12} & 1.3722 \cdot 10^{-13} & -7.460 \cdot 10^{-14} & 1. \end{pmatrix} \tag{5.32}$$

and for the matrix U^* we have got

$$U^* \cdot U^{*-1} = \begin{pmatrix} 1. & 6.2196 \cdot 10^{-15} & 4.4408 \cdot 10^{-15} & -1.9378 \cdot 10^{-16} \\ -3.552 \cdot 10^{-14} & 1. & 1.3322 \cdot 10^{-14} & -2.1995 \cdot 10^{-17} \\ -1.59872 \cdot 10^{-14} & 1.71104 \cdot 10^{-14} & 1. & -2.8717 \cdot 10^{-16} \\ 0 & 0 & 0 & 1. \end{pmatrix} \tag{5.33}$$

Finally, it is important to say that we have eliminated in the process to find the chargino masses (diagonalization), the so-called mixed angles.

5.6 Study of the Phase contribution to chargino masses

The soft-supersymmety breaking part of the Lagrangian $\mathcal{L}$ (see Eq.4.7) involves a large number of arbitrary parameters[22]. Many of these parameters may be in general complex, adding new sources of CP violation with respect to the Standard Model.

It well known that supersymmetric theories contain many new sources of CP violation[28]-[35]. The effects of these new CP violations are expected to be probed in the near-future colliders.

Since M_L, M_R and μ are assumed complex, with nontrivial phases ξ_1, ξ_2 and θ_μ, respectively, then CP can be violated in the chargino sector. Diagonalizing the chargino mass matrix is more complicated than in the MSSM model, but it is possible, as will be shown through this section[36]-[40].

$$M_L \equiv |M_L| e^{i\xi_1}, \qquad M_R \equiv |M_R| e^{i\xi_2}, \qquad \mu \equiv |\mu| e^{i\theta_\mu}, \tag{5.34}$$

and recalling the chargino mass term in the LRSUSY Lagrangian, let us write down the chargino mass matrix M^c with the most general allowed set of CP violating phases

$$M^c = \begin{pmatrix} |M_L|e^{i\xi_1} & 0 & 0 & g_L\, k_d e^{-i\chi_2} \\ 0 & |M_R|e^{i\xi_2} & 0 & g_R\, k_d e^{i\chi_2} \\ g_L\, k_u e^{-i\chi_1} & g_R\, k_u e^{-i\chi_1} & 0 & -|\mu|e^{i\theta_\mu} \\ 0 & 0 & -|\mu|e^{i\theta_\mu} & 0 \end{pmatrix}, \tag{5.35}$$

where the phase angles (i.e. $|\cos\theta_\mu| \leq 1$) are in the interval $[0, 2\pi]$. The results in the $CP-$ conserving limit, may be obtained by simple taking the values 0 or π, in the before-mentioned matrix.

By using the transformation

$$M^c = P^T_{\chi^\pm} M^{c\pm} P_{\chi^\pm}, \tag{5.36}$$

where

$$P_{\chi^\pm} = \begin{pmatrix} e^{i\xi_1/2} & 0 & 0 & 0 \\ 0 & e^{i\xi_2/2} & 0 & 0 \\ 0 & 0 & e^{-i(\xi_1/2+\chi_1)} & 0 \\ 0 & 0 & 0 & e^{-i(\xi_2/2+\chi_1)} \end{pmatrix}, \tag{5.37}$$

Then the matrix $M^{c\pm}$ can be written as

$$M^{c\pm} = \begin{pmatrix} |M_L| & 0 & 0 & g_L\, k_d e^{-i(\Delta\xi_2)} \\ 0 & |M_R| & 0 & g_R\, k_d \\ g_L\, k_u & g_R\, k_u e^{i(\Delta\xi_2)} & 0 & -|\mu|e^{i\theta'} \\ 0 & 0 & -|\mu|e^{i\theta'} & 0 \end{pmatrix}, \tag{5.38}$$

where we have defined $\theta' = (\xi_1 + \xi_2)/2 + \theta_\mu + \chi_1 + \chi_2$ and $\Delta\xi = (\xi_1 - \xi_2)$. The linear dependence with respect two angles is now clear. The complex phase θ_μ and $\Delta\xi$ are the new source of CP violation, which can vary in the range $0 \le \theta_\mu, \Delta\xi \le 2\pi$.

Now for the matrix $M^{c\pm}$ we can write the following transformation

$$Y\left[(M^{c\pm})^\dagger M^{c\pm}\right]Y^{-1} = diag(\tilde{\mathrm{M}}_{\chi_1^\pm}, \tilde{\mathrm{M}}_{\chi_2^\pm}, \tilde{\mathrm{M}}_{\chi_3^\pm}, \tilde{\mathrm{M}}_{\chi_4^\pm}) \equiv M_D^2. \tag{5.39}$$

Notice that the transformation matrix Y is a function only of θ' and $\Delta\xi/2$. Eq.(5.39) can be rewritten as

$$Y\ \left[(M^{c\pm})^\dagger M^{c\pm}\right] - M_D^2\, Y = 0, \tag{5.40}$$

or

$$Y\,\tilde{M} - M_D^2\, Y = 0 \tag{5.41}$$

Equating the elements of the ith row of the above equation, we get

$$\begin{aligned}
[\tilde{M}_{11} - (M_D^2)_{ii}]\, Y_{i1} + \tilde{M}_{21}\, Y_{i2} + \tilde{M}_{31}\, Y_{i3} + \tilde{M}_{41}\, Y_{i4} &= 0\,, \\
\tilde{M}_{12}\, Y_{i1} + [\tilde{M}_{22} - (M_D^2)_{ii}]\, Y_{i2} + \tilde{M}_{31}\, Y_{i3} + \tilde{M}_{41}\, Y_{i4} &= 0\,, \\
\tilde{M}_{13}\, Y_{i1} + \tilde{M}_{21}\, Y_{i2} + [\tilde{M}_{33} - (M_D^2)_{ii}]\, Y_{i3} + \tilde{M}_{41}\, Y_{i4} &= 0\,, \\
\tilde{M}_{14}\, Y_{i1} + \tilde{M}_{21}\, Y_{i2} + \tilde{M}_{31}\, Y_{i3} + [\tilde{M}_{44} - (M_D^2)_{ii}]\, Y_{i4} &= 0\,,
\end{aligned} \tag{5.42}$$

where the Y_{ij}'s represent the components of Y. Actually, the explicit elements of the matrix Y here are not known and are not necessary in order to solve the mass problem.

The chargino masses can be determined by solving the eigenvalue equation

$$\begin{vmatrix} [\tilde{M}_{11}-(M_D^2)_{ii}] & \tilde{M}_{21} & \tilde{M}_{31} & \tilde{M}_{41} \\ \tilde{M}_{12} & [\tilde{M}_{22}-(M_D^2)_{ii}] & \tilde{M}_{31} & \tilde{M}_{41} \\ \tilde{M}_{13} & \tilde{M}_{21} & [\tilde{M}_{33}-(M_D^2)_{ii}] & \tilde{M}_{41} \\ \tilde{M}_{14} & \tilde{M}_{21} & \tilde{M}_{31} & [\tilde{M}_{44}-(M_D^2)_{ii}] \end{vmatrix} = 0. \quad (5.43)$$

Substituting the corresponding expressions for the $\tilde{M}_{ij}$ we get the exact analytic expressions for the chargino masses as functions of the CP angles, i.e.,

$$\tilde{\mathrm{M}}_{\chi_1^\pm} = \tilde{\mathrm{M}}_{\chi_1^\pm}(\theta', \Delta\xi/2), \quad (5.44)$$

$$\tilde{\mathrm{M}}_{\chi_2^\pm} = \tilde{\mathrm{M}}_{\chi_2^\pm}(\theta', \Delta\xi/2), \quad (5.45)$$

$$\tilde{\mathrm{M}}_{\chi_3^\pm} = \tilde{\mathrm{M}}_{\chi_3^\pm}(\theta', \Delta\xi/2), \quad (5.46)$$

$$\tilde{\mathrm{M}}_{\chi_4^\pm} = \tilde{\mathrm{M}}_{\chi_4^\pm}(\theta', \Delta\xi/2), \quad (5.47)$$

The orthogonal matrices Y and B^*, which are similar to V and U^* respectively, can be found by recalling the expression

$$Y\ \left[(M^{c\pm})^\dagger M^{c\pm}\right] - M_D^2 Y = 0, \quad (5.48)$$

as well as the expression

$$M_D^2 = B^*\left[(M^{c\pm})^\dagger M^{c\pm}\right](B^*)^{-1}, \quad (5.49)$$

for the matrix B^*.

The following graphs show the influence of the CP angles on the chargino masses. The most significant variation can be observed in the two lightest charginos ($\tilde{M}_{\chi_1^\pm}, \tilde{M}_{\chi_2^\pm}$), with a prominent and remarkable influence of the phase angles in the $\tilde{M}_{\chi_1^\pm}$.

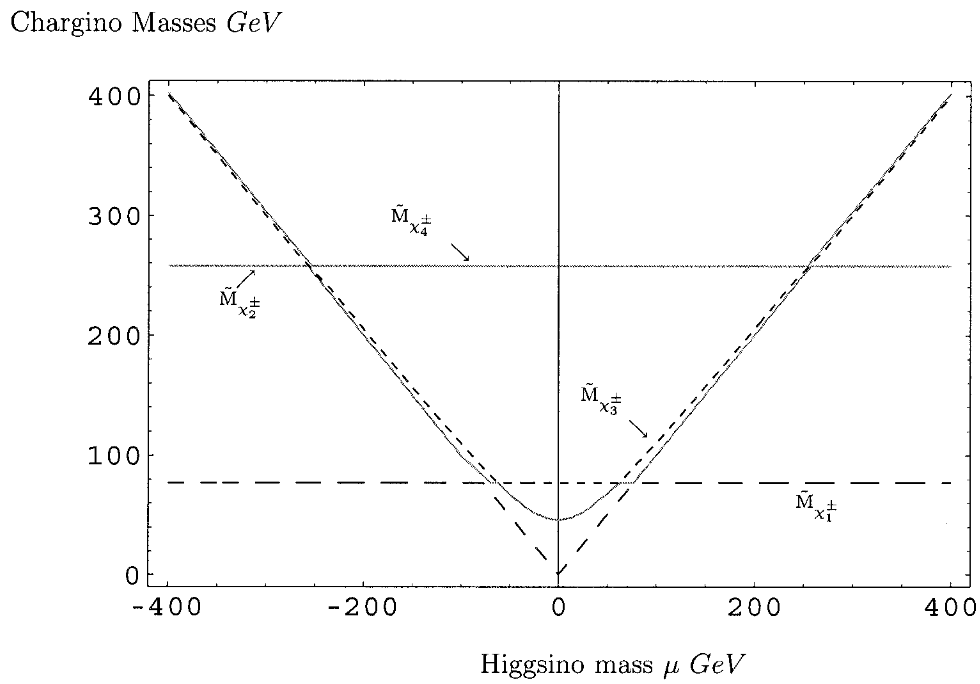
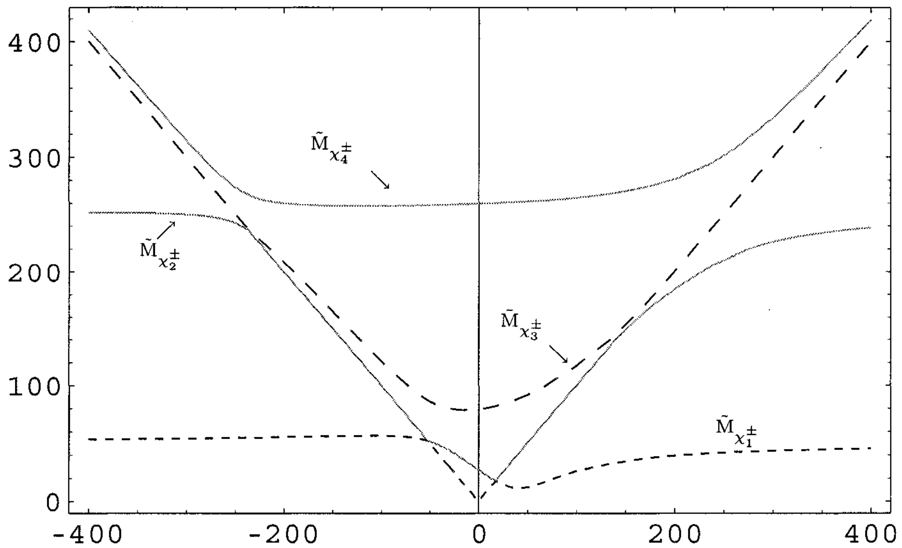


Figure 5.2: Chargino masses versus the higgsino mass parameter.The graph was obtained for the values of the CP phases $\theta' = 90$ degrees, $\Delta\xi = 90$ degrees. $M_R = 300$ GeV, $M_L = 50$ GeV, $M_W = 80$ GeV and $\tan\theta_k$=1.6

Chargino Masses GeV



Higgsino mass μ GeV

Figure 5.3: Chargino masses versus the higgsino mass parameter. The graph was obtained for the values of the CP phases $\theta' = 150$ degrees, $\Delta\xi = 50$ degrees. $M_R = 300\ GeV$, $M_L = 50\ GeV$, $M_W = 80\ GeV$ and $\tan\theta_k$=1.6

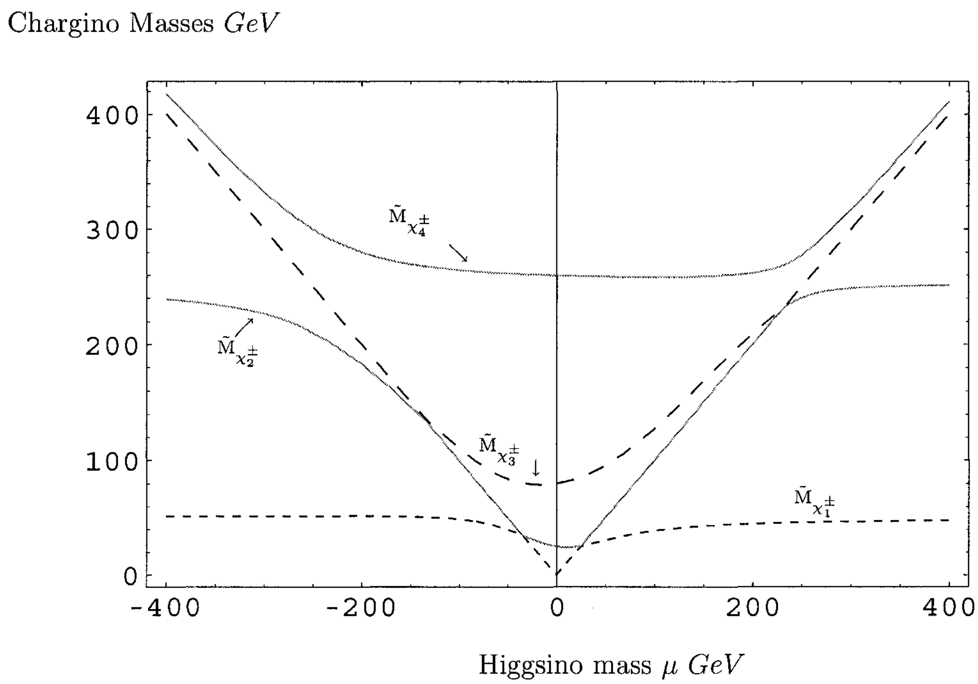
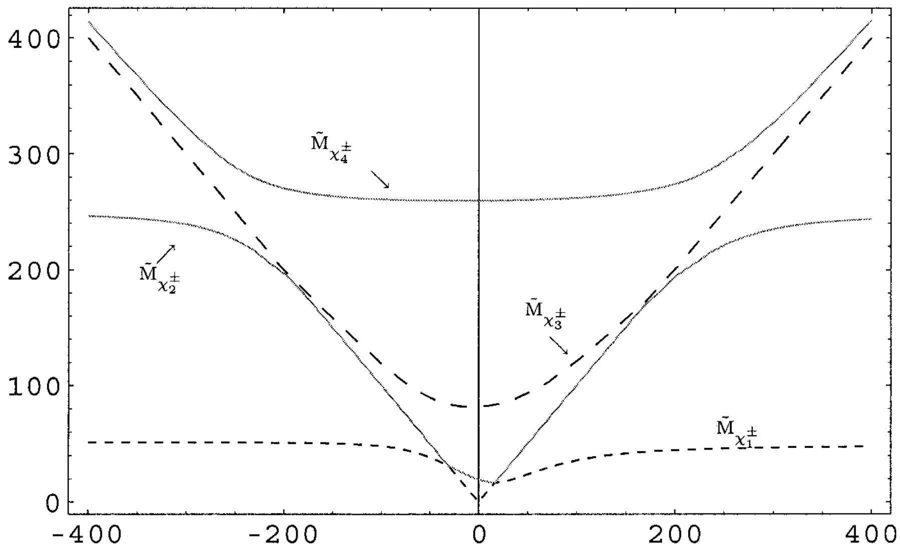


Figure 5.4: Chargino masses versus the higgsino mass parameter. The graph was obtained for the values of the CP phases $\theta' = 70$ degrees, $\Delta\xi = 80$ degrees. $M_R = 300$ GeV, $M_L = 50$ GeV, $M_W = 80$ GeV and $\tan\theta_k$=1.6

Chargino Masses GeV



Higgsino mass μ GeV

Figure 5.5: Chargino masses versus the higgsino mass parameter (μ). The graph was obtained for the values of the CP phases $\theta' = 225$ degrees, $\Delta\xi = 135$ degrees. $M_R = 300\ GeV$, $M_L = 50\ GeV$, $M_W = 80\ GeV$ and $\tan\theta_k$=1.6

Chapter 6

Total cross section for $e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_j^-$

6.1 Chargino-Gauge Field Interaction

One of the challenging problems in high energy physics is to predict and to understand the experimental results from the new generations of high-energy accelerators by the study of the cross section in the e^+e^- collosions.

Chargino production in high-energy e^+e^- collisions has already been studied in depth for the LRSUSY in Refs.[22]-[24]. We extended that calculation by including the effects of the allowed CP-violating phases. As an intermediated step in developing the expressions for the cross section for $e^+e^- \to \tilde{\chi}^+\tilde{\chi}^-$ calculation, in this chapter we present some results(see Ref.[24]) for the fundamental interactions of supersymmetric particles with other known particles. Notice the CPangles-implicit dependence of the quantities shown in the Chapter 3.

The first step in developing cross section calculations for chargino production in electron-positron collisions is to determine the contributions to the chargino gauge field Lagrangian L_{CNZ}. It should be noted that these fields will contribute directly to these interactions only after mixing.

The Lagrangian terms are

$$
\begin{aligned}
L_{CNZ} &= Tr\left[\bar{\tilde{\Phi}}_u\bar{\sigma}_\mu(-\frac{i}{2}g_L\tau\cdot W_\mu^L - \frac{i}{2}g_R\tau\cdot W_\mu^R)\tilde{\Phi}_u\right] + h.c. \\
&+ Tr\left[\bar{\tilde{\Phi}}_d\bar{\sigma}_\mu(-\frac{i}{2}g_L\tau\cdot W_\mu^L - \frac{i}{2}g_R\tau\cdot W_\mu^R)\tilde{\Phi}_d\right] + h.c. \\
&+ \frac{i}{2}g_L\bar{\lambda}_L\bar{\sigma}_L T_a G_\mu^a\lambda_L + \frac{i}{2}g_R\bar{\lambda}_R\bar{\sigma}_L T_a G_\mu^a\lambda_R. \qquad (6.1)
\end{aligned}
$$

The expansion of each term with its Hermitian conjugate yields explicit two-spinor expressions for the Lagrangian.

Here we develop these contributions, one term at a time. The expression

$$
+Tr\left[\bar{\tilde{\Phi}}_u\bar{\sigma}_\mu(-\frac{i}{2}g_L\tau\cdot W_\mu^L - \frac{i}{2}g_R\tau\cdot W_\mu^R)\tilde{\Phi}_u\right] + h.c. \qquad (6.2)
$$

where

$$
\bar{\tilde{\Phi}} = \begin{pmatrix} \bar{\tilde{\phi}}_{1u}^0 & \bar{\tilde{\phi}}_{2u}^- \\ \bar{\tilde{\phi}}_{1u}^+ & \bar{\tilde{\phi}}_{2u}^0 \end{pmatrix}, \qquad (6.3)
$$

can be written in an expanded form after some algebra:

$$
\begin{aligned}
&- \frac{i}{2}g_L\left[W_{\mu L}^0(-\bar{\tilde{\phi}}_{2u}^+\bar{\sigma}_\mu\tilde{\phi}_{2u}^+ + \bar{\tilde{\phi}}_{2u}^-\bar{\sigma}_\mu\tilde{\phi}_{2u}^+\right. \\
&+ \left.\bar{\tilde{\phi}}_{1u}^+\bar{\sigma}_\mu\tilde{\phi}_{1u}^+ + \bar{\tilde{\phi}}_{1u}^-\bar{\sigma}_\mu\tilde{\phi}_{1u}^-)\right], \qquad (6.4)
\end{aligned}
$$

the Hermitian conjugate term having been evaluated in the same manner, and neglecting those in W^+ and W^-, since they couple to neutralinos and charginos only.

In the same way, the expression

$$+Tr\left[\bar{\tilde{\Phi}}_d\bar{\sigma}_\mu(-\frac{i}{2}g_L\tau W^L_\mu - \frac{i}{2}g_R\tau W^R_\mu)\tilde{\Phi}_d\right] + h.c. \tag{6.5}$$

gives rise to similar contributions, with $u \rightarrow d$.

Next, the left-hand gauge field contribution is considered

$$\frac{i}{2}g_L\bar{\lambda}_L\bar{\sigma}_L T_a G^a_\mu\lambda_L, \tag{6.6}$$

where the T_a are $(1/2)$ times the regular Pauli matrices τ_a.

The expanded form yields the two-spinor terms

$$+\frac{i}{2}g_L W^0_{\mu L}(\bar{\lambda}^+_L\bar{\sigma}_\mu\lambda^+_L - \bar{\lambda}^-_L\bar{\sigma}_\mu\lambda^-_L). \tag{6.7}$$

were the terms in W^+ and W^- have been ignored as before. We obtain a similar term for λ_R. The conversion to chargino mass states requires the conversion to four-component notation and the recognition that weak interaction eigenstates can be represented in terms of the mass eigenstates

$$\begin{aligned}\tilde{F}_{1u} &\equiv \begin{pmatrix}\tilde{\phi}^+_{1u}\\ \bar{\tilde{\phi}}^-_{1u}\end{pmatrix}, \tilde{F}_{2u} \equiv \begin{pmatrix}\tilde{\phi}^+_{2u}\\ \bar{\tilde{\phi}}^-_{2u}\end{pmatrix}, \tilde{W}_L \equiv \begin{pmatrix}\lambda^+_L\\ \bar{\lambda}^-_L\end{pmatrix},\\ \tilde{F}_{1d} &\equiv \begin{pmatrix}\tilde{\phi}^+_{1d}\\ \bar{\tilde{\phi}}^-_{1d}\end{pmatrix}, \tilde{F}_{2d} \equiv \begin{pmatrix}\tilde{\phi}^+_{2d}\\ \bar{\tilde{\phi}}^-_{2d}\end{pmatrix},\end{aligned} \tag{6.8}$$

Then we can write (Ref.[12])

$$
\begin{aligned}
(\bar{\lambda}_L^+ \bar{\sigma}_\mu \lambda_L^+ - \bar{\lambda}_L^- \bar{\sigma}_\mu \lambda_L^-) &= \bar{\tilde{W}}_L \gamma^\mu P_L \tilde{W}_L + \bar{\tilde{W}}_L \gamma^\mu P_R \tilde{W}_L, \\
(\bar{\phi}_{2u}^+ \bar{\sigma}_\mu \phi_{2u}^+ + \bar{\phi}_{2u}^- \bar{\sigma}_\mu \phi_{2u}^-) &= \bar{\tilde{F}}_{2u} \gamma^\mu P_L \tilde{F}_{2u} - \bar{\tilde{F}}_{2u} \gamma^\mu P_R \tilde{F}_{2u}, \\
(\bar{\phi}_{1u}^+ \bar{\sigma}_\mu \phi_{1u}^+ + \bar{\phi}_{1u}^- \bar{\sigma}_\mu \phi_{1u}^-) &= \bar{\tilde{F}}_{1u} \gamma^\mu P_L \tilde{F}_{1u} - \bar{\tilde{F}}_{1u} \gamma^\mu P_R \tilde{F}_{1u}.
\end{aligned} \tag{6.9}
$$

Then the expression for the chargino interaction Lagrangian is given by

$$
\begin{aligned}
L_C &= -i\Bigg[(g_L Z_L^\mu + eA_\mu + \frac{g_R g'}{g_V} Z_R) \\
&\times \ (-\bar{\tilde{F}}_{1u} \gamma^\mu P_L \tilde{F}_{1u} + \bar{\tilde{F}}_{1u} \gamma^\mu P_R \tilde{F}_{1u} + \bar{\tilde{F}}_{2u} \gamma^\mu P_L \tilde{F}_{2u} - \bar{\tilde{F}}_{2u} \gamma^\mu P_R \tilde{F}_{2u} \\
&- \ \bar{\tilde{F}}_{1d} \gamma^\mu P_L \tilde{F}_{1d} + \bar{\tilde{F}}_{1d} \gamma^\mu P_R \tilde{F}_{1d} + \bar{\tilde{F}}_{2d} \gamma^\mu P_L \tilde{F}_{2d} - \bar{\tilde{F}}_{2d} \gamma^\mu P_R \tilde{F}_{2d}) \\
&+ \ eA_\mu (\bar{\tilde{W}}_L \gamma^\mu P_L \tilde{W}_L + \bar{\tilde{W}}_L \gamma^\mu P_R \tilde{W}_L + \bar{\tilde{W}}_R \gamma^\mu P_L \tilde{W}_R + \bar{\tilde{W}}_R \gamma^\mu P_R \tilde{W}_R) \\
&+ \ \Big(g_L cos\theta_W (\bar{\tilde{W}}_L \gamma^\mu P_L \tilde{W}_L + \bar{\tilde{W}}_L \gamma^\mu P_R \tilde{W}_L) - g' sin\theta_W (\bar{\tilde{W}}_R \gamma^\mu P_L \tilde{W}_R + \bar{\tilde{W}}_R \gamma^\mu P_R \tilde{W}_R)\Big) \\
&+ \ Z_R \frac{g_R g'}{g_V} (\bar{\tilde{W}}_R \gamma^\mu P_L \tilde{W}_R + \bar{\tilde{W}}_R \gamma^\mu P_R \tilde{W}_R)\Bigg]
\end{aligned} \tag{6.10}
$$

Now, taking into account the expressions

$$
\begin{aligned}
\chi_i^+ &= -iV_{i1}\lambda_L^+ - iV_{i2}\lambda_R^+ + V_{i3}\tilde{\phi}_u^+ + V_{i4}\tilde{\phi}_d^+, \\
\chi_i^- &= -iU_{i1}\lambda_L^- - iU_{i2}\lambda_R^- + U_{i3}\tilde{\phi}_u^- + U_{i4}\tilde{\phi}_d^-,
\end{aligned} \tag{6.11}
$$

where $i = 1, ..., 4$.

We can express the right-hand projection of $\tilde{W}_L$ as

$$
P_R \tilde{W}_L = P_R (U_{11}\tilde{\chi}_1 + U_{21}\tilde{\chi}_2 + U_{31}\tilde{\chi}_{31} + U_{41}\tilde{\chi}_4). \tag{6.12}
$$

In a similar way, the expressions of the Lagrangian contribution can be written as

(Ref.[11]).

$$
\begin{aligned}
\bar{\tilde{W}}_L\gamma^\mu P_L &= (V_{11}\bar{\tilde{\chi}}_1 + V_{12}\bar{\tilde{\chi}}_2 + V_{13}\bar{\tilde{\chi}}_3 + V_{14}\bar{\tilde{\chi}}_4)\gamma^\mu P_L \\
P_L\tilde{W}_L &= P_L(V^*_{11}\tilde{\chi}_1 + V^*_{21}\tilde{\chi}_2 + V^*_{31}\tilde{\chi}_3 + V^*_{41}\tilde{\chi}_4) \\
\bar{\tilde{W}}_L\gamma^\mu P_R &= (U^*_{11}\bar{\tilde{\chi}}_1 + U^*_{12}\bar{\tilde{\chi}}_2 + U^*_{13}\bar{\tilde{\chi}}_3 + U^*_{14}\bar{\tilde{\chi}}_4)\gamma^\mu P_R \\
P_R\bar{\tilde{W}}_R &= P_R(U_{12}\tilde{\chi}_1 + U_{22}\tilde{\chi}_2 + U_{32}\tilde{\chi}_3 + U_{42}\tilde{\chi}_4) \\
\bar{\tilde{W}}_R\gamma^\mu P_L &= (V_{21}\bar{\tilde{\chi}}_1 + V_{22}\bar{\tilde{\chi}}_2 + V_{23}\bar{\tilde{\chi}}_3 + V_{24}\bar{\tilde{\chi}}_4)\gamma^\mu P_L \\
P_L\tilde{W}_R &= P_L(V^*_{12}\tilde{\chi}_1 + V^*_{22}\tilde{\chi}_2 + V^*_{32}\tilde{\chi}_3 + V^*_{42}\tilde{\chi}_4) \\
\bar{\tilde{W}}_R\gamma^\mu P_R &= (U^*_{21}\bar{\tilde{\chi}}_1 + U^*_{22}\bar{\tilde{\chi}}_2 + U^*_{23}\bar{\tilde{\chi}}_3 + U^*_{24}\bar{\tilde{\chi}}_4)\gamma^\mu P_R \\
\bar{\tilde{F}}_{1u}\gamma^\mu P_R &= (U^*_{31}\bar{\tilde{\chi}}_1 + U^*_{32}\bar{\tilde{\chi}}_2 + U^*_{33}\bar{\tilde{\chi}}_3 + U^*_{34}\bar{\tilde{\chi}}_4)\gamma^\mu P_R \\
P_L\tilde{F}_{1u} &= P_L(V^*_{13}\tilde{\chi}_1 + V^*_{32}\tilde{\chi}_2 + V^*_{33}\tilde{\chi}_3 + V^*_{43}\tilde{\chi}_4) \\
\bar{\tilde{F}}_{1u}\gamma^\mu P_L &= (V_{31}\bar{\tilde{\chi}}_1 + V_{32}\bar{\tilde{\chi}}_2 + V_{33}\bar{\tilde{\chi}}_3 + V_{34}\bar{\tilde{\chi}}_4)\gamma^\mu P_L \\
P_R\tilde{F}_{1u} &= P_R(U_{31}\tilde{\chi}_1 + U_{32}\tilde{\chi}_2 + U_{33}\tilde{\chi}_3 + U_{34}\tilde{\chi}_4) \\
\bar{\tilde{F}}_{2d}\gamma^\mu P_R &= (U^*_{41}\bar{\tilde{\chi}}_1 + U^*_{42}\bar{\tilde{\chi}}_2 + U^*_{43}\bar{\tilde{\chi}}_3 + U^*_{44}\bar{\tilde{\chi}}_4)\gamma^\mu P_R \\
P_L\tilde{F}_{2d} &= P_L(V^*_{14}\tilde{\chi}_1 + V^*_{24}\tilde{\chi}_2 + V^*_{34}\tilde{\chi}_3 + V^*_{44}\tilde{\chi}_4) \\
\bar{\tilde{F}}_{2d}\gamma^\mu P_L &= (V_{41}\bar{\tilde{\chi}}_1 + V_{42}\bar{\tilde{\chi}}_2 + V_{43}\bar{\tilde{\chi}}_3 + V_{44}\bar{\tilde{\chi}}_4)\gamma^\mu P_L \\
P_R\tilde{F}_{2d} &= P_R(U_{14}\tilde{\chi}_1 + U_{24}\tilde{\chi}_2 + U_{34}\tilde{\chi}_3 + U_{44}\tilde{\chi}_4). \qquad (6.13)
\end{aligned}
$$

Finally,

$$
\begin{aligned}
L_C &= Z_L^\mu \Big\{ \frac{g_L \cos 2\theta_W}{\cos\theta_W} \bar{\tilde{\chi}}_i^+ \gamma^\mu \Big[(V_{i3}V_{j3}^* + V_{i4}V_{j4}^*) P_L + (U_{i3}U_{j3}^* + U_{i4}U_{j4}^*) P_R \Big] \tilde{\chi}_j^+ \\
&+ g_L \cos\theta_W Z_L^\mu \gamma_\mu \bar{\tilde{\chi}}_i^+ (V_{i1}V_{j1}^* P_L + U_{i1}U_{j1}^* P_R) \tilde{\chi}_j^+ - g' \sin\theta_W Z_L^\mu \gamma_\mu \bar{\tilde{\chi}}_i^+ \\
&\times (V_{i2}V_{j2}^* P_L + U_{i2}U_{j2}^* P_R) \tilde{\chi}_j^+ \Big\} + Z_R^\mu \Big\{ \frac{g_R g'}{g_V} \bar{\tilde{\chi}}_i^+ \gamma^\mu \Big[(V_{i2}V_{j2}^* + V_{i3}V_{j3}^* + V_{i4}V_{j4}^*) P_L \\
&+ (U_{i2}U_{j2}^* + U_{i3}U_{j3}^* + U_{i4}U_{j4}^*) P_R \Big] \tilde{\chi}_i^+ \Big\} + A^\mu \Big\{ e\delta_{ij} \bar{\tilde{\chi}}_i^+ \gamma^\mu \Big[(V_{i1}V_{j1}^* + V_{i2}V_{j2}^* \\
&+ V_{i3}V_{j3}^* + V_{i4}V_{j4}^*) P_L + (U_{i1}U_{j1}^* + U_{i2}U_{j2}^* + U_{i3}U_{j3}^* + U_{i4}U_{j4}^*) P_R \tilde{\chi}_j^+ \Big] \Big\}. \qquad (6.14)
\end{aligned}
$$

We can define the following matrices

$$
\begin{aligned}
O_{ij}^{\prime L} &= \cos^2\theta_W V_{i1}V_{j1}^* - \frac{g'}{2g_L} \sin 2\theta_W V_{i2}V_{j2}^* + \cos 2\theta_W (V_{i3}V_{j3}^* + V_{i4}V_{j4}^*), \\
O_{ij}^{\prime R} &= \cos^2\theta_W U_{i1}U_{j1}^* - \frac{g'}{2g_L} \sin 2\theta_W U_{i2}U_{j2}^* + \cos 2\theta_W (U_{i3}U_{j3}^* + U_{i4}U_{j4}^*), \\
Q_{ij}^L &= V_{i2}V_{j2}^* + V_{i3}V_{j3}^* + V_{i4}V_{j4}^*, \\
Q_{ij}^R &= U_{i2}U_{j2}^* + U_{i3}U_{j3}^* + U_{i4}U_{j4}^*, \\
O_{ij}^{\prime\prime L} &= -\frac{1}{2} B_{i3}B_{i3}^* + \frac{1}{2} B_{i4}B_{i4}^*, \\
O_{ij}^{\prime\prime R} &= -O_{ij}^{\prime\prime L *}. \qquad (6.15)
\end{aligned}
$$

Taking into account the above expressions we can write the Lagrangian

$$
L_C^{Z_L} = \left(\frac{g_L}{\cos\theta_W} \right) Z_L^\mu \Big[\bar{\tilde{\chi}}^+ \gamma (O_{ij}^{\prime L} P_L + O_{ij}^{\prime R} P_R) \tilde{\chi}_i^+ \Big], \qquad (6.16)
$$

and

$$
L_C^{Z_R} = \left(\frac{\cos 2\theta_W}{\cos\theta_W} \right) g_R Z_R^\mu \Big[\bar{\tilde{\chi}}^+ \gamma (Q_{ij}^L P_L + O_{ij}^R P_R) \tilde{\chi}_i^+ \Big]. \qquad (6.17)
$$

6.2 Total cross section

At the future Linear Collinder one will be able to determine the masses of charginos and the pair production cross section to high accuracies.

In this part, we recalculate and illustrate the chargino pair production in the complex framework with non-trivial phases. The case without CP-violation phases was analyzed in the past in order to get the total cross section(see Ref.[22] and [27]).

Scattering experiments involve directing a beam of particles at a target and observing how the incident particles become scattered by the target. The quantity measured in such experiments is called the scattering cross section and has units of area. The most likely means of gaining experimental evidence of supersymmetry will be through scattering experiments.

Since the quantum mechanics theory is based on a probabilistic approach, the quantum mechanical cross section is calculated by evaluating the transition probability per particle into final states within the range of momentum uncertainty of the measuring apparatus.

We have reproduced the graphs of the cross section obtained in Ref.[22] and [27] using the analytic expressions for the chargino masses and the respective eigenvectors.

On the other hand, the CP-violation effects on the total cross section σ_T is studied by taking into account the implicit phase dependences of those quantities. This is an important remark since the mathematical equations for σ_T were taken from Ref.[27] in which the authors published the most general expressions (valid for CP conservation and CP violation phase) in order to compute the Cross section.

Let us start by assuming the calculation be performed in the center of mass frame, the expression for the cross section is given by

$$\frac{d\sigma}{d\Omega_{CM}} = \frac{1}{32\,\pi\,s}\sqrt{\frac{\lambda(s,m_3^2,m_4^2)}{\lambda(s,m_1^2,m_2^2)}}\,|M(s,t)|^2, \tag{6.18}$$

where $\lambda(s,m_i^2,m_j^2) \equiv [s-(m_i+m_j)^2][s-(m_i-m_j)^2]$ denotes the normalization volume, m_i is the mass of the particles in general and $M(s,t)$ is the invariant amplitude of the collition process. We have introduced the following variables

$$\begin{aligned} s &= (q_1+q_2)^2, \\ t &= (q_1-p_1)^2, \\ u &= (q_1-p_2)^2, \end{aligned} \tag{6.19}$$

where the momenta of the incoming particles are represented by q_1, q_2 and the momenta of the outgoing particles by p_1, p_2. We define $k^2 = (q_1+q_2)^2 = (p_1+p_2)^2$.

Here we have followed the mathematical treatment developed by[24] for the specific process $e^+e^- \to Z_R \to \tilde{\chi}_i^+\tilde{\chi}_j^-$ in order to show how to go through the cross section computation. The left-handed intermediate-state equivalent will be omitted since it only requires minor modifications. The other intermediate state process given by

$$e^+e^- \to \tilde{\nu}_{L,R} \to \tilde{\chi}_i^+\tilde{\chi}_j^- \;\;,\;\; e^+e^- \to \gamma \to \tilde{\chi}_i^+\tilde{\chi}_j^-, \tag{6.20}$$

will be taken into account by their final expressions of the respective Cross sections.

The vertices are defined for $e^+e^- \to Z_R \to \tilde{\chi}_i^+\tilde{\chi}_j^-$ by the following expressions

$$-i\frac{g_R g'}{g_V}\gamma^\mu(Q_{ij}^L P_L + Q_{ij}^R P_R), \tag{6.21}$$

for the $Z_R - \tilde{\chi}_i^+\tilde{\chi}_j^-$ vertex and

$$-i\frac{g_R}{\cos\theta_W}\gamma^\nu(c_V - c_A\gamma_5), \tag{6.22}$$

for the $e^+e^- - Z_R$ vertex. In the above expressions $c_V = -16 + 15\sin^2\theta_W$ and $c_A = -3\sin^2\theta_W$.

In Refs.[22]and [24] the invariant amplitude M for $e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_j^-$ is written as

$$M = \mathcal{M}_1 D_{Z_R}\mathcal{M}_3, \tag{6.23}$$

where

$$\begin{aligned}
\mathcal{M}_1 &\equiv \left[i\frac{g_R}{\cos\theta_W}\bar{\tilde{\chi}}_j\gamma^\mu(Q^L_{ij}P_L + Q^R_{ij}P_R)\tilde{\chi}_i\right], \\
D_{Z_R} &\equiv \left(\frac{-i}{k^2 - M^2_{Z'} + i\varepsilon}\right)[g_{\mu\nu} - \frac{k_\mu k_\nu}{M^2_{Z'}(k^2 - M^2_{Z'} + i\varepsilon)}], \\
\mathcal{M}_3 &\equiv \left[i\frac{g_R}{\cos\theta_W}\bar{u}\gamma^\nu(c_V + c_A\gamma_5)u\right],
\end{aligned} \tag{6.24}$$

where D_{Z_R} is the so-called Z_R-propagator[].

The terms in $k_\mu k_\nu \to 0$ given that $k_\mu = p_{2\mu} - p_{1\mu}$ and using the Dirac equation

$$p_{2\mu}\bar{u} = 0, \quad and \quad p_{1\mu}\bar{u} = 0.$$

The Eq.(6.23) can be rewritten as

$$M = M_1 M_2, \tag{6.25}$$

where

$$
\begin{aligned}
M_1 &\equiv \left(i\frac{g' g_R^2}{g_V \cos\theta_W}\right)|D_{Z_R}(k^2)|\bar{\tilde{\chi}}_j(q_2, s_2)\gamma^\mu Q_{ij}^L P_L \tilde{\chi}_i(q_1, s_1)\bar{u} \\
&\times \gamma^\nu(c_V + c_A\gamma_5)\bar{u}, \\
M_2 &\equiv \left(i\frac{g' g_R^2}{g_V \cos\theta_W}\right)|D_{Z_R}(k^2)|\bar{\tilde{\chi}}_j(q_2, s_2)\gamma^\mu Q_{ij}^R P_R \tilde{\chi}_i(q_1, s_1)\bar{u} \\
&\times \gamma^\nu(c_V + c_A\gamma_5)\bar{u}. \qquad (6.26)
\end{aligned}
$$

Knowing that $|M|^2 = M\,M^+$, the average square of the invariant amplitude is defined by

$$
|M|^2_{average} = (1/4)\sum_{spins}[|M_1|^2 + |M_1 M_2^+| + |M_2 M_1^+| + |M_2|^2], \qquad (6.27)
$$

where

The expression for $|M_1|^2, |M_2|^2, |M_1 M_2^+|^2$ and $|M_2 M_1^+|^2$ are given by

$$
\begin{aligned}
|M_1|^2 &= 32\left(\frac{g_R^4 g'^2}{g_V^2 \cos^2\theta_W}\right)|Q_{ij}^L|^2|D_{Z_R}(k^2)|^2 \\
&\times \Big\{(c_R^2 + c_L^2)[(q_2\cdot p_2)(q_1\cdot p_1) + (q_2\cdot p_1)(q_1\cdot p_2)] \\
&+ (c_R^2 - c_L^2)[(q_2\cdot p_2)(q_1\cdot p_1) - (q_2\cdot p_1)(q_1\cdot p_2)]\Big\}, \\
|M_2|^2 &= 32\left(\frac{g_R^4 g'^2}{g_V^2 \cos^2\theta_W}\right)|Q_{ij}^R|^2|D_{Z_R}(k^2)|^2 \\
&\times \Big\{(c_R^2 + c_L^2)[(q_2\cdot p_2)(q_1\cdot p_1) + (q_2\cdot p_1)(q_1\cdot p_2)] \\
&- (c_R^2 - c_L^2)[(q_2\cdot p_2)(q_1\cdot p_1) - (q_2\cdot p_1)(q_1\cdot p_2)]\Big\}, \\
|M_1 M_2^+|^2 &= 32\left(\frac{g_R^4 g'^2}{g_V^2 \cos^2\theta_W}\right)|Q_{ij}^L Q_{ij}^{R*}|^2|D_{Z_R}(k^2)|^2(c_R^2 + c_L^2)(p_2\cdot p_1), \\
|M_2 M_1^+|^2 &= 32\left(\frac{g_R^4 g'^2}{g_V^2 \cos^2\theta_W}\right)|Q_{ij}^{L*} Q_{ij}^R|^2|D_{Z_R}(k^2)|^2(c_R^2 + c_L^2)(p_2\cdot p_1). \qquad (6.28)
\end{aligned}
$$

where we have defined $c_R \equiv c_V - c_A$ and $c_L \equiv c_V - c_A$.

Then, the expression of $|M|^2_{average}$ for a process evaluated in the center-of -mass frame reference yields,

$$
\begin{aligned}
|M|^2_{average} &= \left(\frac{32 g_R^4 g'^2}{g_V^2 \cos^2\theta_W}\right) \frac{\left(c_R^2 + c_L^2\right)}{\left(s - M_{Z_R}^2 + i\varepsilon\right)^2} \left\{ \left(|Q_{ij}^L|^2 + |Q_{ij}^R|^2 \right) \right. \\
&\times \frac{1}{4}\Big[2M^2_{\tilde{\chi}_j^-} M^2_{\tilde{\chi}_i^+} (M^2_{\tilde{\chi}_j^-} + M^2_{\tilde{\chi}_i^+} - s)(M^2_{\tilde{\chi}_j^-} + M^2_{\tilde{\chi}_i^+}) + s^2 \\
&+ [s^2 - 2s(M^2_{\tilde{\chi}_j^-} + M^2_{\tilde{\chi}_i^+}) + (M^2_{\tilde{\chi}_j^-} + M^2_{\tilde{\chi}_i^+})^2] \cos^2\theta_{CM} \Big] \\
&+ \left. (O_{ij}^L O_{ij}^{R*} + O_{ij}^R O_{ij}^{L*})(s M^2_{\tilde{\chi}_j^-} M^2_{\tilde{\chi}_i^+}) \right\}. \qquad (6.29)
\end{aligned}
$$

The differential cross section for two body scattering of non-identical particles is given by

$$
\frac{d\sigma}{d\Omega_{CM}} = \frac{|q'|}{q} \frac{1}{64\pi^2 s} |M(s,t)|^2, \qquad (6.30)
$$

where

$$
\begin{aligned}
q^2 &= \frac{[s - (M_{e^+} + M_{e^-})^2][s - (M_{e^+} - M_{e^-})^2]}{4s} \\
q'^2 &= \frac{[s - (M_{\tilde{\chi}^+} + M_{\tilde{\chi}^-})^2][s - (M_{\tilde{\chi}^+} - M_{\tilde{\chi}_j^-})^2]}{4s},
\end{aligned} \qquad (6.31)
$$

and assuming the approximation $M_{e^+}, M_{e^-} \to 0$, we have that

$$
\sqrt{\frac{\lambda(s, M^2_{\tilde{\chi}^+}, M^2_{\tilde{\chi}^-})}{\lambda(s, M^2_{e^+}, M^2_{e^-})}} = [1 - \frac{2}{s}(M^2_{\tilde{\chi}^+} + M^2_{\tilde{\chi}^+}) + \frac{1}{s^2}(M^2_{\tilde{\chi}^+} - M^2_{\tilde{\chi}^+})^2]^{1/2} \qquad (6.32)
$$

Then, we can arrive to the partially integrated differential cross section as

$$\begin{aligned}
\frac{d\sigma}{d(\cos\theta_{CM})} &= \frac{1}{256\pi}\left(\frac{g_R^4 g'^2}{g_V^2\cos^2\theta_W}\right)\frac{(c_R^2+c_L^2)}{s(s-M_{Z_R}^2+i\varepsilon)^2}\Bigg[1-\frac{2}{s}(\,M_{\tilde{\chi}_j^-}^2+M_{\tilde{\chi}_i^+}^2) \\
&+ \frac{1}{s^2}(M_{\tilde{\chi}_j^-}^2-M_{\tilde{\chi}_i^+}^2)^2\Bigg]^{1/2} \\
&\times \Bigg\{(\,|Q_{ij}^L|^2+|Q_{ij}^R|^2)\,[s^2-(M_{\tilde{\chi}_j^-}^2-M_{\tilde{\chi}_i^+}^2)^2] \\
&+ [s^2-2s(\,M_{\tilde{\chi}_j^-}^2+M_{\tilde{\chi}_i^+}^2)+(M_{\tilde{\chi}_j^-}^2+M_{\tilde{\chi}_i^+}^2)^2]\cos^2\theta_{CM}] \\
&+ (O_{ij}^L O_{ij}^{R*}+O_{ij}^R O_{ij}^{L*})(\,4sM_{\tilde{\chi}_i^+}M_{\tilde{\chi}_j^-})\Bigg\}. \qquad (6.33)
\end{aligned}$$

This will yield σ_R, corresponding to Z_R intermediate state. The total cross section is obtained by integrating over the angle θ_{CM}. It yields

$$\sigma_{Total} = \sigma_\gamma + \sigma_{ZL,R} + \sigma_{\tilde{\nu}} + \sigma_{\gamma ZL,R} + \sigma_{\gamma\tilde{\nu}} + \sigma_{ZL,R\tilde{\nu}} + \sigma_{ZL,R}\,, \qquad (6.34)$$

where the partial cross sections are obtained in a similar way to σ_R.

$$\sigma_\gamma = \frac{e^4}{16\pi s}\delta_{ij}\left[1-(y-z)^2+\frac{1}{3}\lambda(1,y,z)+4X\right]\lambda^{1/2}(1,y,z), \qquad (6.35)$$

$$\begin{aligned}
\sigma_{\tilde{\nu}} &= \frac{g^4}{128\pi s}|U_{i1}|^2|U_{j1}|^2\Bigg\{\Bigg[\frac{1}{(a^2-b^2)}-\frac{(y-z)^2}{(a^2-b^2)}-4\lambda^{-1/2}(1,y,z) \\
&\times \left(\ln(\frac{a+b}{a-b})-\frac{2ab}{a^2-b^2}\right)+4\left(1-\frac{a}{b}\ln(\frac{a+b}{a-b})+\frac{a^2}{a^2-b^2}\right)\Bigg]\Bigg\}\lambda^{1/2}(1,y,z),
\end{aligned}$$

(6.36)

$$\begin{aligned}
\sigma_{\gamma Z_L} &= \frac{e^2g^2}{32\pi\cos^2\theta_W}\delta_{ij}Re[D_{ZL}(s)](c_L+c_R)(O_{ij}'^L+O_{ij}'^R) \\
&\times [1-(y-z)^2+\frac{1}{3}\lambda(1,y,z)+4X]\lambda^{1/2}(1,y,z), \qquad (6.37)
\end{aligned}$$

$$\begin{aligned}
\sigma_{\gamma Z_R} &= \frac{e^2g^2}{32\pi\cos^2\theta_W}\cos2\theta_W\delta_{ij}Re[D_{ZR}(s)](c_L+c_R)(O_{ij}'^L+O_{ij}'^R) \\
&\times [1-(y-z)^2+\frac{1}{3}\lambda(1,y,z)+4X]\lambda^{1/2}(1,y,z), \qquad (6.38)
\end{aligned}$$

$$
\begin{aligned}
\sigma_{\gamma\tilde{\nu}} &= \frac{-e^2g^2}{64\pi s}|U_{ij}|^2\delta_{ij}\Bigg[[1-(y-z)^2+4X]\frac{1}{b}-\ln(\frac{a+b}{a-b})-4(1+a)\\
&\times \left(2-\frac{a}{b}\ln(\frac{a+b}{a-b})\right)\Bigg]\lambda^{1/2}(1,y,z),
\end{aligned} \tag{6.39}
$$

$$
\begin{aligned}
\sigma_{Z_L\tilde{\nu}} &= \frac{-g^4}{64\pi\cos^2\theta_W}(U_{i1}U_{j1})Re(D_{ZL})\Bigg\{O'^L_{ij}\Bigg([1-(y-z)^2]\frac{1}{b}\ln(\frac{a+b}{a-b})-4(1+a)\\
&\times [2-\frac{a}{b}\ln(\frac{a+b}{a-b})]\Bigg)-O'^R_{ij}\frac{4X}{b}\ln(\frac{a+b}{a-b})\Bigg\}\lambda^{1/2}(1,y,z),
\end{aligned} \tag{6.40}
$$

$$
\begin{aligned}
\sigma_{Z_R\tilde{\nu}} &= \frac{-g^4}{64\pi\cos^2\theta_W}\cos 2\theta_W c_L(U_{i1}U_{j1})Re(D_{ZR})\Bigg\{Q^L_{ij}\Bigg([1-(y-z)^2]\\
&- \frac{1}{b}\ln(\frac{a+b}{a-b})-4(1+a)[2-\frac{a}{b}\ln(\frac{a+b}{a-b})]\Bigg)-Q^R_{ij}\frac{4X}{b}\\
&\times \ln(\frac{a+b}{a-b})\Bigg\}\lambda^{1/2}(1,y,z),
\end{aligned} \tag{6.41}
$$

$$
\begin{aligned}
\sigma_{Z_{LR}} &= \frac{g^4 s}{64\pi\cos^4\theta_W}(\cos 2\theta_W)Re[D_{ZL}(s)D_{ZR}(s)](c_L^2+c_R^2)\\
&\times \Bigg[(O'^L_{ij}Q^{*L}_{ij}+O'^R_{ij}Q^{*R}_{ij})+[1-(y-z)^2+\frac{1}{3}\lambda(1,y,z)]\\
&+ 4X(O'^L_{ij}Q^{*R}_{ij}+O'^{L*}_{ij}Q^R_{ij})\Bigg]\lambda^{1/2}(1,y,z),
\end{aligned} \tag{6.42}
$$

$$
\begin{aligned}
\sigma_{ZL} &= \frac{g^4 s}{64\pi\cos^4\theta_W}|D_{ZL}(s)|^2(c_L^2+c_R^2)(|O'^L_{ij}|^2+|O'^R_{ij}|^2)\\
&\times [1-(y-z)^2+\frac{1}{3}\lambda(1,y,z)]+8XO'^L_{ij}O'^R_{ij}\lambda^{1/2}(1,y,z),
\end{aligned}
$$

(6.44)

$$
\begin{aligned}
\sigma_{ZR} &= \frac{g^4 s\cos^2 2\theta_W}{64\pi\cos^2\theta_W}|D_{ZL}(s)|^2(c_L^2+c_R^2)\Bigg\{(|O^L_{ij}|^2+|O^R_{ij}|^2)\\
&\times [1-(y-z)^2+\frac{1}{3}\lambda(1,y,z)]+4XO^L_{ij}O^R_{ij}\Bigg\}\lambda^{1/2}(1,y,z),
\end{aligned} \tag{6.45}
$$

Here, we have taken $a = -(1-y-z)/2 - \tilde{M}_{\nu}^2/s$ and $b = (1/2)\lambda^{1/2}(1, y, z)$; $D_{ZL,R}(s) = (s - M_z^2 + iM_z\Gamma_{Zz})^{-1}_{L,R}$. The triangle function λ is given by

$$\lambda^{1/2}(1, y, z) = 1 + y^2 + z^2 - 2y - 2z - 2yz, \tag{6.46}$$

where the variables x, y, z are given by

$$\begin{aligned} x &= (\tilde{M}_{\chi_i^{\pm}} \tilde{M}_{\chi_j^{\pm}})/s, \\ y &= (\tilde{M}_{\chi_i^{\pm}})^2/s, \\ z &= (\tilde{M}_{\chi_j^{\pm}})^2/s. \end{aligned} \tag{6.47}$$

6.2.1 Numerical study

In this section we show the total cross section for charginos plotted as functions as a function of the center of mass energy $\sqrt{s} = 2$TeV. In order to show the accuracy of the method developed in this thesis we have used the cases of $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$, $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-$, $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_3^-$ and $e^+e^- \to \tilde{\chi}_2^+\tilde{\chi}_2^-$. The consideration of others different chargino combinations is straightforward. The CP conservation case showed in Ref.[27] is reproduced in the simulations and compared with the present results.

In Fig.(6.1) we show an example of the numerical study related to combinations of different processes, i.e. $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$, $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-$ and $e^+e^- \to \tilde{\chi}_2^+\tilde{\chi}_2^-$. As expected, there is a pick shift in the direction of the c.m. energy increment when the masses of the particles increase as well as the Cross section energy when the the process's particles masses diminish. A similar behavior is corroborated in Fig.(6.2) when we have changed some parameters in the simulation.

As mentioned in the introduction of this chapter, we have studied the cross section σ_T by taking into account the effects of the CP-violation phases. The graphs show a distinctive variation of σ_T when compared with the CP-conservation case.

From Figs. (6.3) to (6.8) the study of the influence of the CP angles have been done for the value of characteristic angles 0, 90 and 180 degrees. It is important to remark that for the 0 degree we reproduce the CP conservative case and even if we have obtained in all graphs a mayor value of the Cross section for 180 degrees it does not mean that this is the maximum condition (extremum). In order to study under which conditions (angle values) the Cross section rich its maximum we must apply the theory of variational calculus.

From the mathematical point of view we can explain these differences by recalling that the consideration of phases in the mass matrix it is in fact a multiplication by factors which take values between -1 to 1.

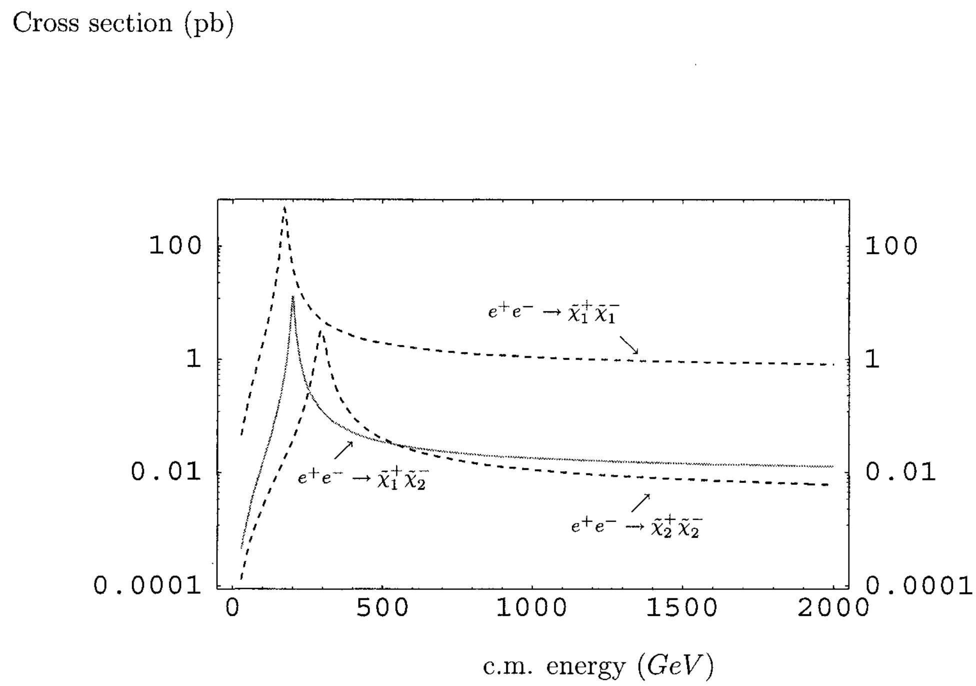


Figure 6.1: Total Cross section of the process $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$, $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-$ and $e^+e^- \to \tilde{\chi}_2^+\tilde{\chi}_2^-$ versus the center of mass energy $\sqrt{s}$. The graph was obtained for the values of the CP phases cero degrees, $M_R = 300\ GeV$, $M_L = 50\ GeV$, $M_V = 50\ GeV$, $\tan\theta_k$=1.6, $\mu = -100$, $\tilde{M}_{\chi_1^\pm}$=80 GeV, $\tilde{M}_{\chi_2^\pm}$=120 GeV and $\tilde{M}_{\chi_3^\pm}$=170 GeV

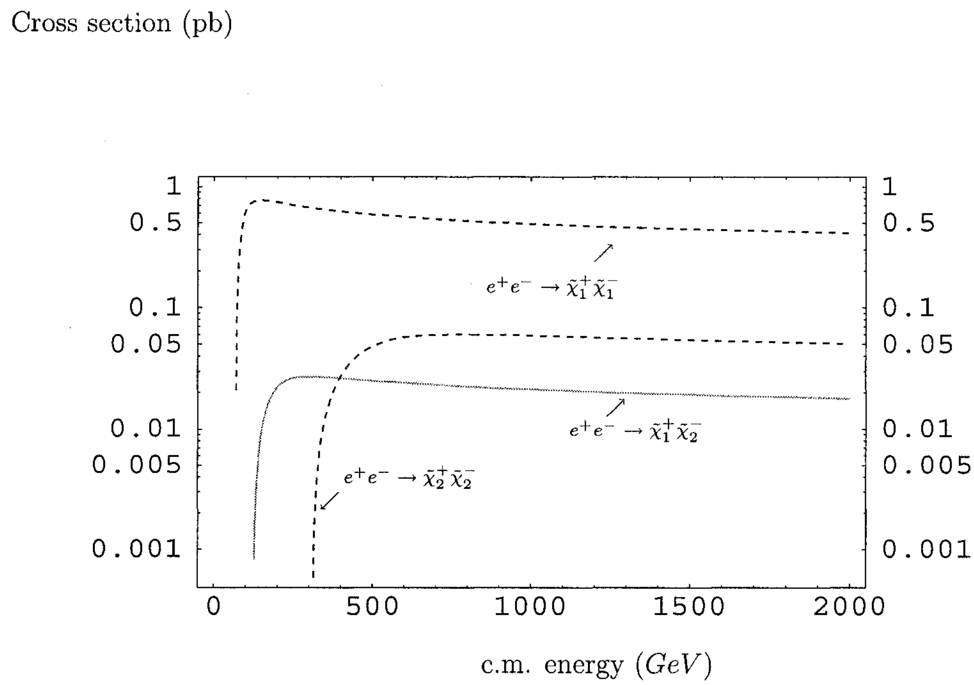
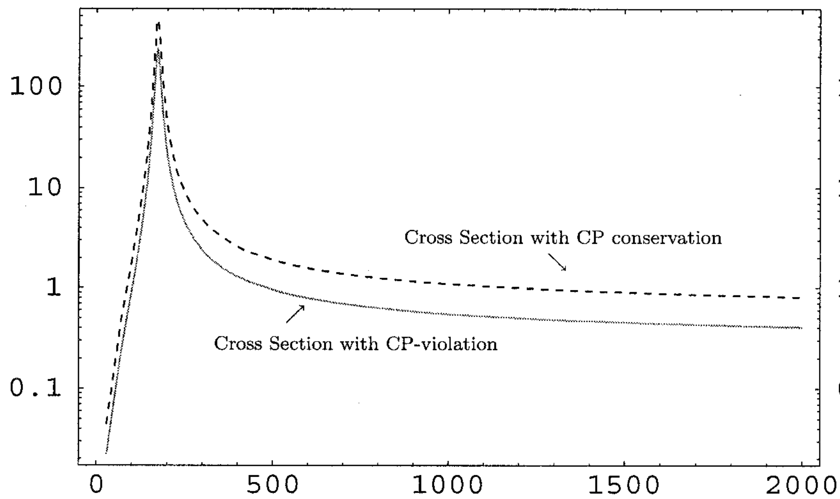


Figure 6.2: Total Cross section of the process $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$, $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-$ and $e^+e^- \to \tilde{\chi}_2^+\tilde{\chi}_2^-$ versus the center of mass energy $\sqrt{s}$. The graph was obtained for the values of the CP phases $\theta' = 70$ degrees, $\Delta\xi = 80$ degrees, $M_R = 300\ GeV$, $M_L = 50\ GeV$, $M_V = 80\ GeV$, $\tan\theta_k$=10, $\mu = -200$, $\tilde{M}_{\chi_1^\pm}$=180 GeV, $\tilde{M}_{\chi_2^\pm}$=240 GeV and $\tilde{M}_{\chi_3^\pm}$=370 GeV

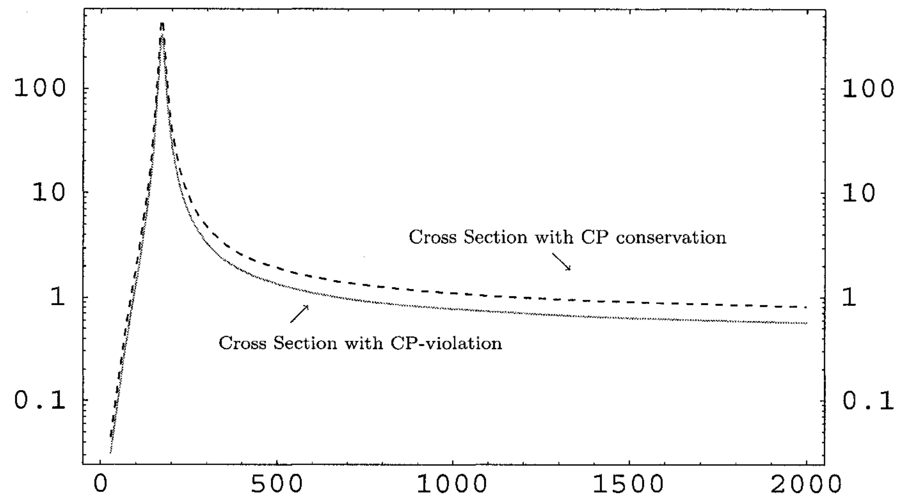
Cross section (pb)



c.m. energy (GeV)

Figure 6.3: Total Cross section of the process $e^+e^- \rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^-$ versus the center of mass energy $\sqrt{s}$. The graph was obtained for the values of the CP phases $\theta' = 180$ degrees, $\Delta\xi = 180$ degrees, $M_R = 300\ GeV$, $M_L = 50\ GeV$, $M_V = 50\ GeV$, $\tan\theta_k$=1.6, $\mu = -100$, $\tilde{M}_{\chi_1^\pm}$=80 GeV.

Cross section (pb)



c.m. energy (GeV)

Figure 6.4: Total Cross section of the process $e^+e^- \rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^-$ versus the center of mass energy $\sqrt{s}$. The graph was obtained for the values of the CP phases $\theta' = 90$ degrees, $\Delta\xi = 90$ degrees, $M_R = 500\ GeV$, $M_L = 100\ GeV$, $M_V = 50\ GeV$, $\tan\theta_k$=2, $\mu = -100$ and $\tilde{M}_{\chi_1^\pm}$=80 GeV.

Cross section (pb)

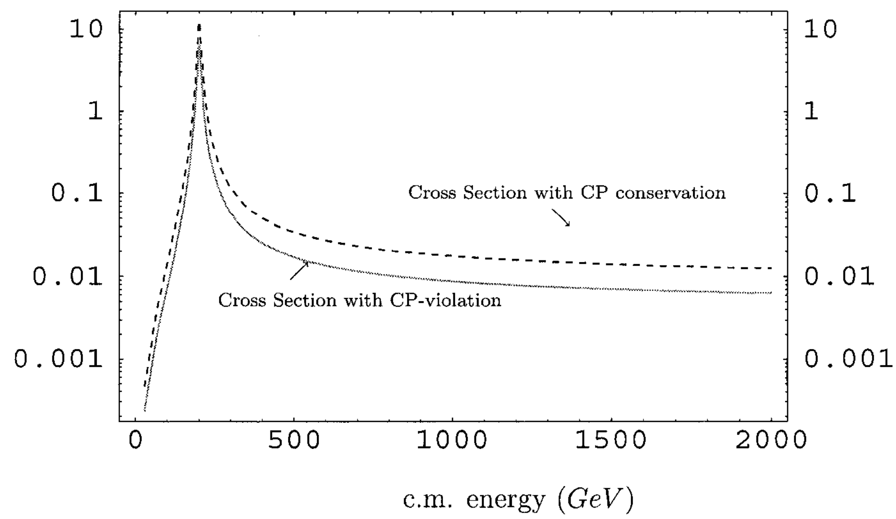
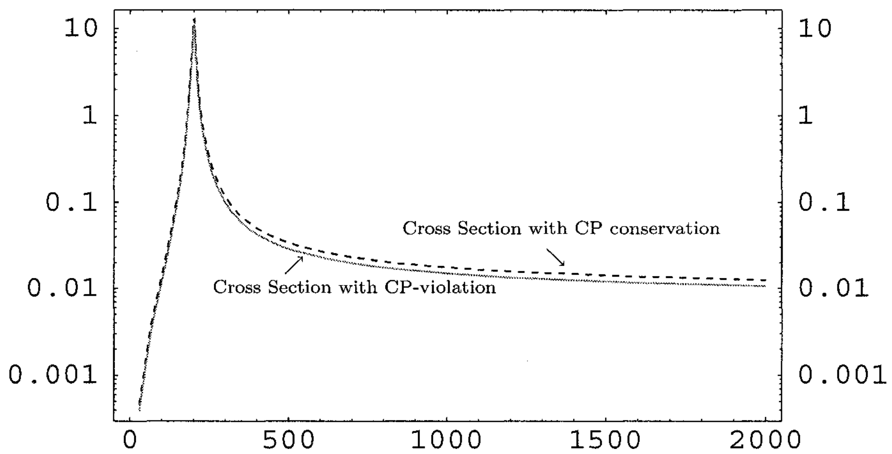


Figure 6.5: Total Cross section of the process $e^+e^- \rightarrow \tilde{\chi}_1^+\tilde{\chi}_2^-$ versus the center of mass energy $\sqrt{s}$. The graph was obtained for the values of the CP phases $\theta' = 180$ degrees, $\Delta\xi = 180$ degrees, $M_R = 500\ GeV$, $M_L = 100\ GeV$, $M_V = 50\ GeV$, $\tan\theta_k$=2, $\mu = -100$ and $\tilde{M}_{\chi_1^\pm}$=80 GeV and $\tilde{M}_{\chi_2^\pm}$=120 GeV.

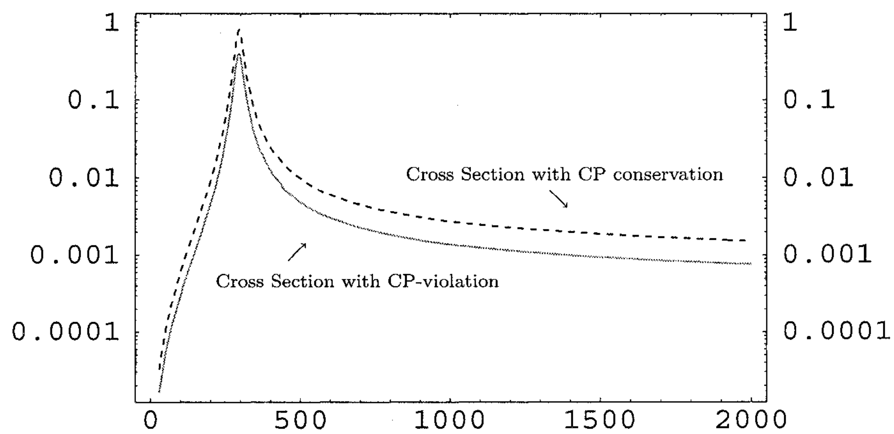
Cross section (pb)



c.m. energy (GeV)

Figure 6.6: Total Cross section of the process $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-$ versus the center of mass energy $\sqrt{s}$. The graph was obtained for the values of the CP phases $\theta' = 90$ degrees, $\Delta\xi = 90$ degrees, $M_R = 500\ GeV$, $M_L = 100\ GeV$, $M_V = 50\ GeV$, $\tan\theta_k$=2, $\mu = -100$, $\tilde{M}_{\chi_1^\pm}$=80 GeV and $\tilde{M}_{\chi_2^\pm}$=120 GeV.

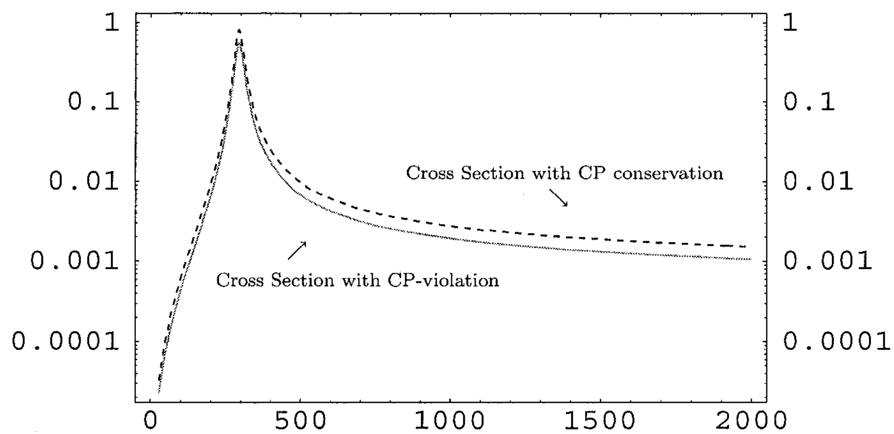
Cross section (pb)



c.m. energy (GeV)

Figure 6.7: Total Cross section of the process $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_3^-$ versus the center of mass energy $\sqrt{s}$. The graph was obtained for the values of the CP phases $\theta' = 180$ degrees, $\Delta\xi = 180$ degrees, $M_R = 500\ GeV$, $M_L = 100\ GeV$, $M_V = 50\ GeV$, $\tan\theta_k$=2, $\mu = -100$, $\tilde{M}_{\chi_1^\pm}$=80 GeV and $\tilde{M}_{\chi_3^\pm}$=170 GeV.

Cross section (pb)



c.m. energy (GeV)

Figure 6.8: Total Cross section of the process $e^+e^- \rightarrow \tilde{\chi}_1^+\tilde{\chi}_3^-$ versus the center of mass energy $\sqrt{s}$. The graph was obtained for the values of the CP phases $\theta' = 90$ degrees, $\Delta\xi = 90$ degrees, $M_R = 500\ GeV$, $M_L = 100\ GeV$, $M_V = 50\ GeV$, $\tan\theta_k$=2, $\mu = -100$, $\tilde{M}_{\chi_1^\pm}$=120 GeV and $\tilde{M}_{\chi_3^\pm}$=170 GeV.

Chapter 7

Conclusion

This thesis is concerned with the study of the process $e^+e^- \to \tilde{\chi}^+\tilde{\chi}^-$ in the left-right supersymmetric model (LRSUSY), in the presence of CP violating phases in the chargino sector.

In this thesis, we have obtained the chargino masses and the chargino mixing matrix in terms of analytic expressions. We also have proved the excellent agreement between our results with the numerical solutions previously published.

One step forward in the theory was achieved by considering the CP violation phase angle effects on the analytical expressions mentioned above. The chargino mass matrix was considered in the most general by taking the parameters as complex parameters. By means of the linear dependence property we have reduced to only two angles the CP violating phases to be take into account to describe our problem.

Of special importance is the study of the total cross section σ_T under the effect of the CP effects. We have performed the analysis for different values of the phase angles obtaining in all cases a variation in the cross section respect to the curve of the cross section with non CP phase angles.

The study of chargino production present an important step towards determining the parameters M_L, M_R, μ, $\tan\beta$ as well as the CP violating angles in supersymmetry. The impact of these angles is demonstrated here.

Appendix A

The metric is defined to be

$$g_{\mu\nu} = diag(1, -1, -1, -1). \tag{7.1}$$

The momentum four vector is $P^{\mu} = (E, \vec{P})$.

$$\sigma^{\mu} = (1, \vec{\sigma}); \sigma^{-\mu} = (1, -\vec{\sigma}), \tag{7.2}$$

denotes the Pauli matrices. The Dirac equation transform in two-component notation is

$$\begin{aligned} (\bar{\sigma}_{\mu}P^{\mu})^{\dot{\alpha}\beta}\xi_{\beta} &= m\bar{\eta}^{\dot{\alpha}} \\ (\sigma_{\mu}P^{\mu})_{\alpha\dot{\beta}}\bar{\eta}^{\dot{\beta}} &= m\bar{\eta}^{\dot{\alpha}}\xi_{\alpha}, \end{aligned} \tag{7.3}$$

where $\xi_{\alpha}, \xi_{\beta}$ are two-component spinors.

A four-component spinor satisfies

$$(\gamma_{\mu}P^{\mu} - m)\psi = o. \tag{7.4}$$

It follows that

$$\psi = \begin{pmatrix} \xi_\alpha \\ \bar{\eta}^{\dot{\alpha}} \end{pmatrix}_L , \qquad \gamma_\mu = \begin{pmatrix} 0 & \sigma_{\mu\nu\dot{\beta}} \\ \mu^{\sigma^{-\dot{\alpha}\beta}} & 0 \end{pmatrix}, \tag{7.5}$$

$$\begin{aligned} \gamma_5 &= i\gamma^0\gamma^1\gamma^2\gamma^3 \\ &\equiv \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \end{aligned} \tag{7.6}$$

where

$$\begin{aligned} \sigma_\alpha^{\mu\nu\beta} &= \frac{1}{4}(\sigma^\mu_{\alpha\dot{\alpha}}\sigma^{-\nu\dot{\mu}\beta} - \sigma^\nu_{\alpha\dot{\alpha}}\sigma^{-\mu\dot{\alpha}\beta}) \\ \sigma_{\dot{\beta}}^{-\mu\nu\dot{\alpha}} &= \frac{1}{4}(\sigma^{-\mu\dot{\alpha}\alpha}\sigma^\nu_{\alpha\dot{\beta}} - \sigma^{-\nu\dot{\mu}\beta} - \sigma^{-\nu\dot{\alpha}\alpha}\sigma^\mu_{\alpha\dot{\beta}}). \end{aligned} \tag{7.7}$$

This is called the chiral representation of the γ-matrices. The four-component spinors

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}, \tag{7.8}$$

where $\psi_{L,R} = P_{L,R}\psi$, and the left and right-handed projection operators are given by

$$P_{L,R} = \frac{1}{2}(1 \mp \gamma_5). \tag{7.9}$$

The charge conjugated spinor in terms of the charge conjugation operator C can be expressed as

$$\psi^C = C\bar{\psi}^T; \qquad C = -i\gamma^2\gamma^0. \tag{7.10}$$

The antisymmetric tensor ($\epsilon^{\alpha\beta} = -\epsilon^{\beta\alpha}$),

$$\begin{aligned} \epsilon^{\alpha\beta} &= -\epsilon^{\beta\alpha} \\ &= i\gamma^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \end{aligned} \tag{7.11}$$

By means of the tensor $\epsilon_{\alpha\beta}$ we can write

$$\xi^\alpha = \epsilon^{\alpha\beta}\xi_\beta, \qquad \xi^\beta = \epsilon_{\beta\alpha}\xi_\alpha. \tag{7.12}$$

In terms of the chiral representation we get, ($\epsilon^{\alpha\beta} = -\epsilon^{\beta\alpha}$),

$$C = -i\gamma^2\gamma^0 = \begin{pmatrix} \epsilon_{\beta\alpha} & 0 \\ 0 & \epsilon_{\dot\beta\dot\alpha} \end{pmatrix}, \tag{7.13}$$

and

$$\begin{aligned} \bar\psi^t &= \begin{pmatrix} \eta^\alpha \\ \bar\xi_{\dot\alpha} \end{pmatrix} \\ \psi^C &= \begin{pmatrix} \eta_\beta \\ \bar\xi^{\dot\beta} \end{pmatrix}, \end{aligned} \tag{7.14}$$

For a four-component Majorana spinor we get,

$$\psi_M = \begin{pmatrix} \xi^\alpha \\ \bar\xi^{\dot\alpha}. \end{pmatrix} \tag{7.15}$$

In the four-components framework we have

$$\begin{aligned} \bar\psi_1\psi_2 &= \eta_1\xi_2 + \bar\eta_2\xi_1 \\ \bar\psi_1\gamma_5\psi_2 &= -\eta_1\xi_2 + \bar\eta_2\xi_1 \\ \bar\psi_1\gamma^\mu\psi_2 &= \bar\xi_1\bar\sigma^\mu\xi_2 - \bar\eta_2\bar\sigma^\mu\eta_1 \\ \bar\psi_1\gamma^\mu\gamma_5\psi_2 &= -\bar\xi_1\bar\sigma^\mu\xi_2 - \bar\eta_2\bar\sigma^\mu\eta_1 \\ \frac{1}{2}i\bar\psi_1\sigma^{\mu\nu}\psi_2 &= \eta_1\sigma^{\mu\nu}\xi_2 - \bar\eta_2\sigma^{\mu\nu}\bar\xi_1, \end{aligned} \tag{7.16}$$

where the numbers 1 and 2 label two different four-component spinors.

Appendix B

In this appendix we give in a more explicit form the Particle content of the LRSUSY model.

Field	Comp. fields	Quant. number	Name
Matter			
Q_L	$(u\ d)_L^T$	$\frac{1}{2}\ 0\ \frac{1}{3}$	left-handed up(down) quark
Q_R	$(u\ d)_R^T$	$0\ \frac{1}{2}\ \frac{1}{3}$	right-handed up(down) quark
L_L	$(\nu\ e)_L^T$	$\frac{1}{2}\ 0\ -1$	left-handed neutrino(electron)
L_R	$(\nu\ e)_R^T$	$0\ \frac{1}{2}\ -1$	right-handed neutrino(electron)
$\tilde{Q}_L$	$(\tilde{u}\ \tilde{d})_L^T$	$\frac{1}{2}\ 0\ \frac{1}{3}$	left-handed up(down) quark
$\tilde{Q}_R$	$(\tilde{u}\ \tilde{d})_R^T$	$0\ \frac{1}{2}\ \frac{1}{3}$	right-handed up(down) quark
$\tilde{L}_L$	$(\tilde{\nu}\ \tilde{e})_L^T$	$\frac{1}{2}\ 0\ -1$	left-handed s neutrino(s electron)
$\tilde{L}_R$	$(\tilde{\nu}\ \tilde{e})_R^T$	$0\ \frac{1}{2}\ -1$	right-handed s neutrino(s electron)

Gauge

W_L	W_L^+, W_L^-, W_L^0	triplet	singlet	singlet	gauge boson
W_R	W_R^+, W_R^-, W_R^0	singlet	triplet	singlet	gauge boson
V	V	singlet	singlet	singlet	gauge boson
λ_L	$\lambda_L^+, \lambda_L^-, \lambda_L^0$	triplet	singlet	singlet	gaugino
λ_R	$\lambda_R^+, \lambda_R^-, \lambda_R^0$	singlet	triplet	singlet	gaugino
λ_V	λ_V	singlet	singlet	singlet	gaugino

<u>*Higgs*</u>

$\phi_{u,d}$	$\begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}_{u,d}$	$\frac{1}{2}$	$\frac{1}{2}$	0	Higgs boson
Δ_L	$\begin{pmatrix} \frac{1}{\sqrt{2}}\Delta^+ & \Delta^{++} \\ \Delta^0 & -\frac{1}{\sqrt{2}}\Delta^+ \end{pmatrix}_L$	1	0	2	Higgs boson
Δ_R	$\begin{pmatrix} \frac{1}{\sqrt{2}}\Delta^+ & \Delta^{++} \\ \Delta^0 & -\frac{1}{\sqrt{2}}\Delta^+ \end{pmatrix}_R$	0	1	2	Higgs boson
δ_L	$\begin{pmatrix} \frac{1}{\sqrt{2}}\delta^- & \delta^0 \\ \delta^{--} & -\frac{1}{\sqrt{2}}\delta^- \end{pmatrix}_L$	1	0	-2	Higgs boson

δ_R	$\begin{pmatrix} \frac{1}{\sqrt{2}}\,\delta^- & \delta^0 \\ \delta^{--} & -\frac{1}{\sqrt{2}}\,\delta^- \end{pmatrix}_R$	0	1	-2	Higgs boson
$\tilde{\phi}_{u,\,d}$	$\begin{pmatrix} \tilde{\phi}_1^0 & \tilde{\phi}_1^+ \\ \tilde{\phi}_2^- & \tilde{\phi}_2^0 \end{pmatrix}_{u,\,d}$	$\frac{1}{2}$	$\frac{1}{2}$	0	Higgs fermion
$\tilde{\Delta}_L$	$\begin{pmatrix} \frac{1}{\sqrt{2}}\,\tilde{\Delta}^+ & \tilde{\Delta}^{++} \\ \tilde{\Delta}^0 & -\frac{1}{\sqrt{2}}\,\tilde{\Delta}^+ \end{pmatrix}_L$	1	0	2	Higgsino
$\tilde{\Delta}_R$	$\begin{pmatrix} \frac{1}{\sqrt{2}}\,\tilde{\Delta}^+ & \tilde{\Delta}^{++} \\ \tilde{\Delta}^0 & -\frac{1}{\sqrt{2}}\,\tilde{\Delta}^+ \end{pmatrix}_R$	0	1	2	Higgsino
$\tilde{\delta}_L$	$\begin{pmatrix} \frac{1}{\sqrt{2}}\,\tilde{\delta}^- & \tilde{\delta}^0 \\ \tilde{\delta}^{--} & -\frac{1}{\sqrt{2}}\,\tilde{\delta}^- \end{pmatrix}_L$	1	0	-2	Higgsino
$\tilde{\delta}_R$	$\begin{pmatrix} \frac{1}{\sqrt{2}}\,\tilde{\delta}^- & \tilde{\delta}^0 \\ \tilde{\delta}^{--} & -\frac{1}{\sqrt{2}}\,\tilde{\delta}^- \end{pmatrix}_R$	0	1	-2	Higgsino

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