

# An Iterative Concatenated Multiuser Detector for Coded CDMA Systems

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## ABSTRACT

### An Iterative Concatenated Multiuser Detector for Coded CDMA Systems

Vahid Mohammad Giahi

Multiuser detection deals with demodulation of mutually interfering digital streams in digital communications. Multiple Access Interference (MAI) presents a major obstacle to reliable communications in Code Division Multiple Access (CDMA) systems. This fact results in a user capacity limit for CDMA systems in the sense that there exists a maximum number of users that can simultaneously communicate over the channel for specified level of performance. Over the years, researchers explored different methods of Multiuser Detection (MUD) to overcome the channel capacity limitation.

In this thesis, we have investigated through various techniques of Iterative (Turbo) MUD that employ channel encoder/decoders, with different performance/complexity trade-off. We have developed the idea of concatenating two effective techniques, which are Partial Parallel Interference Cancellation (PPIC) and updated Decorrelating Detector (DD). By deriving appropriate formulas, we have been able to achieve a better performance/complexity trade-off in comparison to the other competing methods such as Serial Interference Cancellation (SIC), Parallel interference Cancellation (PIC), and single DD. Our proposed algorithm uses tentative soft-decisions at both of the multiple PPIC stages and input of the updated DD, to produce the most reliably estimated received data for the MAI cancellation and data detection. Simulation results are given for a multitude of different situations to demonstrate the performance of our proposed algorithm over the other traditional methods. Moreover, at medium

to high Signal to Noise Ratio (SNR), the destructive effects of MAI can almost be overcome by iterative processing, and single-user performance can be achieved for the channels with medium cross-correlation.

*Dedicated to my beloved family*

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# Contents

<b>List of Figures</b>	<b>x</b>
<b>List of Tables</b>	<b>xiii</b>
<b>List of Acronyms</b>	<b>xv</b>
<b>1 Introduction</b>	<b>1</b>
1.1 A Brief History of Wireless Telecommunication . . . . .	1
1.2 Multiple Access Techniques . . . . .	3
1.3 Introductory Steps Towards CDMA . . . . .	5
1.4 Introduction to Multiuser Detection . . . . .	7
1.5 The Objective and Contribution of This Thesis . . . . .	9
<b>2 Spread Spectrum Techniques</b>	<b>11</b>
2.1 CDMA Concepts . . . . .	11
2.2 Direct Sequence CDMA . . . . .	12
2.2.1 General Capabilities . . . . .	15
2.2.2 Specific properties . . . . .	15
2.3 Frequency-Hopping CDMA . . . . .	16
2.3.1 General Capabilities . . . . .	17

2.4	Time-Hopping CDMA . . . . .	19
2.4.1	General Capabilities . . . . .	19
2.5	Hybrid Techniques . . . . .	21
2.6	Summary . . . . .	21
<b>3</b>	<b>Multuser Detection Techniques</b>	<b>24</b>
3.1	Signal Construction for a General DS-CDMA Model . . . . .	24
3.1.1	Synchronous Case . . . . .	24
3.1.2	Asynchronous Case . . . . .	26
3.2	Optimum Multuser Detection . . . . .	26
3.3	Linear Multuser Detection . . . . .	29
3.3.1	Decorrelating Detector . . . . .	29
3.3.2	MMSE Detector . . . . .	30
3.4	Subtractive Multuser Detection . . . . .	32
3.4.1	Serial Interference Cancellation (SIC) . . . . .	32
3.4.2	Parallel Interference Cancellation (PIC) . . . . .	34
3.5	Turbo (Iterative) Multuser Detection . . . . .	35
3.5.1	Optimum (ML) Iterative MUD . . . . .	36
3.5.2	Soft Instantaneous MMSE Interference Cancellation . . . . .	41
3.5.3	Turbo Multistage PIC . . . . .	42
3.6	Summary . . . . .	44
<b>4</b>	<b>Concatenation of PPIC and Decorrelating Detector</b>	<b>45</b>
4.1	General System Description . . . . .	46
4.1.1	CDMA Channel . . . . .	46
4.1.2	Convolutional Channel Encoder/Decoder . . . . .	47



4.1.3	Turbo Channel Encoder/Decoder . . . . .	51
4.2	Iterative (Turbo) PPIC Derivation . . . . .	53
4.2.1	Introduction to Partial PIC . . . . .	53
4.2.2	Iterative (Turbo) PPIC formula derivation . . . . .	54
4.3	Improved Iterative Decorrelating Detector . . . . .	64
4.4	Concatenation of the Modified PPIC and the New Decorrelating Detector	66
4.5	Simulation results . . . . .	69
4.6	Summary . . . . .	77
<b>5</b>	<b>Contributions and Suggestions for Future Work</b>	<b>78</b>

# List of Figures

1.1	A typical “Multiple Access” system. . . . .	4
1.2	Two Orthogonal waveforms in the time domain. . . . .	5
1.3	Signal constellation of different multiple access techniques. . . . .	6
2.1	Categorization of various CDMA techniques. . . . .	13
2.2	Basic structure of the Transmitter/Receiver of a DS-CDMA system. . .	13
2.3	Modulation process of a BPSK signal using DS-CDMA technique. (a) BPSK signal. (b) $N = 12$ signature sequence. (c) Spread BPSK. . . .	14
2.4	Comparison between FH and DS signals in terms of Time/Frequency occupancy. . . . .	17
2.5	Basic structure of the Transmitter/Receiver of a typical FH-CDMA system. . . . .	17
2.6	Time/Frequency occupancy of TH-CDMA signal. . . . .	19
2.7	Basic structure of the Transmitter/Receiver of a typical TH-CDMA system. . . . .	20
2.8	DS/FH-CDMA transmitter. . . . .	22
2.9	DS/TH-CDMA transmitter. . . . .	22
2.10	Time/Frequency occupancy of CDMA/TDMA system. . . . .	22
2.11	MC-CDMA transmitter for one user. . . . .	23

3.1	Offset modeling asynchronism. . . . .	26
3.2	Decorrelating Detector (DD) block diagram. . . . .	31
3.3	Successive cancellation for a two-user system. . . . .	33
3.4	PIC receiver block diagram. . . . .	34
3.5	A coded CDMA system and Turbo (Iterative) MUD block diagram. . . . .	37
3.6	The performance of a full-complexity SISO MUD as a function of the number of iterations, $K = 4$ users, $\rho_{ij} = 0.7$ , $L = 128$ , and all the users have equal power. . . . .	39
3.7	Performance of soft instantaneous MMSE SISO MUD. $K = 4$ users, $\rho_{ij} = 0.7$ , $L = 128$ , and all the users have equal power. . . . .	43
4.1	A general coded CDMA transmitter/receiver block diagram. The receiver is implemented in an iterative (turbo) fashion. . . . .	48
4.2	Realization of the convolutional encoder for the rate $R_k = \frac{1}{2}$ code with memory 6. . . . .	49
4.3	Realization of a systematic convolutional encoder with feedback for the rate $R_k = \frac{1}{2}$ code. . . . .	52
4.4	Soft In/Soft Out decoder. . . . .	52
4.5	Iterative decoding procedure with two “Soft In/Soft Out” decoders for turbo-coded information. . . . .	52
4.6	PPIC detector block diagram. . . . .	54
4.7	Probability of bit error performance of the modified turbo PPIC detector as a function of the number of iterations, with cross-correlation $\rho = 0.5$ , 4 equal power users, for different number of iterations. . . . .	62

4.8	Probability of bit error performance of the modified turbo PPIC detector as a function of the number of iterations, with cross-correlation $\rho = 0.7$ , 4 equal power users, for different number of iterations. . . . .	63
4.9	Performance comparison between soft interference cancellation (SIC) and SIC followed by the newly proposed decorrelator for $K = 5$ equal power users, cross-correlation $\rho = 0.5$ , and different number of iterations.	67
4.10	Performance comparison between soft interference cancellation (SIC) and SIC followed by the newly proposed decorrelator for $K = 5$ equal power users, cross-correlation $\rho = 0.7$ , and different number of iterations.	68
4.11	Performance comparison among PPIC and modified-PPIC followed by newly proposed decorrelator for $K = 4$ users, cross-correlation $\rho = 0.5$ , equal power users, and different number of iterations. . . . .	71
4.12	Performance comparison among SIC followed by the decorrelating detector and modified PPIC followed by the decorrelating detector for $K = 5$ users, cross-correlation $\rho = 0.5$ , equal power users, and different number of iterations. . . . .	72
4.13	Performance comparison between SIC followed by the decorrelating detector and modified PPIC followed by the decorrelating detector for $K = 5$ users, cross-correlation $\rho = 0.7$ , equal power users, and different number of iterations. . . . .	73
4.14	Performance comparison of graphs A, B, and C (all using modified PPIC followed by the decorrelating detector) as a function of the number of iterations, for $K = 4$ equal power users, cross-correlation $\rho = 0.5$ , turbo channel encoders, and different number of iterations. . . . .	75

4.15 Performance comparison of graphs A, B, and C (all using modified PPIC followed by the decorrelating detector) as a function of the number of iterations, for  $K = 5$  equal power users, cross-correlation  $\rho = 0.5$ , simple convolutional channel encoders, and different number of iterations. 76

# List of Tables

4.1	Different cancellation weight sets for $m = 2$ (Graphs A and B) and $m = 3$ ( Graph C) iterations. . . . .	74
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# List of Acronyms

3GPP	3rd Generation Partnership Project
AWGN	Additive White Gaussian Noise
BPSK	Binary Phase Shift Keying
CDMA	Code Divison Multiple Access
DD	Decorrelating Detector
DS-CDMA	Direct Sequence Code Division Multiple Access
ETSI	European Telecommunication Standards Institute
FDMA	Frequency Division Multiple Access
FEC	Forward Error Correcting
FH-SS	Frequency Hopping Spread Spectrum
FM	Frequency Modulation
GSM	Global System of Mobile Communications
IC	Interference Cancellation

ITU	International Telecommunication Union
LAN	Local Area Networks
LLR	Log-Likelihood Ratio
MAI	Multiple Access Interference
MF	Matched Filter
MMSE	Minimum Mean-Squared Error
ML	Maximum Likelihood
MUD	Multiuser Detection
PIC	Parallel Interference Cancellation
PM	Phase Modulation
PPIC	Partial Parallel Interference Cancellation
RSC	Recursive Systematic Code
SIC	Serial Interference Cancellation
SNR	Signal to Noise Ratio
TDMA	Time Division Multiple Access
TH-SS	Time Hopping Spread Spectrum
UMTS	Universal Mobile Telecommunication Systems
US-TDMA	IS-136, one of the 2nd generation systems mainly in USA



WCDMA Wide-band CDMA

ZF Zero Forcing

# Chapter 1

## Introduction

Flashing advances of technology and the increasing demand for high bit rate information transmission are fueling research and development in the area of digital and wireless communication. Over the past decade, researchers have strived to achieve the theoretical channel capacity, described by Shannon [1] which is the goal of wireless and digital communication. In order to achieve this ambition, scientists have focused on power consumption optimization and efficient use of bandwidth when the number of users requiring data is raising exponentially.

### 1.1 A Brief History of Wireless Telecommunication

Data transmission from different base stations to a satellite, communication between mobile phones and base stations, Local Area Networks (LAN), all are just a few examples of multi-access communication systems. Basically, multi-access communication is the technique of sharing one channel between different users. The advantages and powerful principals of this method motivated the researchers to benefit from multi-

access communication in satellite and mobile communications.

Analog cellular telephony is commonly referred to as the first generation of mobile communication systems [2]. The analog characteristics of this system prohibited the efficient use of power and channel bandwidth which are the fundamental factors in terms of system design. These limitations aroused an interest which brought forth what is known as the second generation.

The digital systems currently in use, such as Global System of Mobile Communication (GSM), North American CDMA (IS-95), and US-TDMA (IS-136) are referred to as second generation systems. Digital characteristics of the later generation facilitated communication problems in terms of power consumption, transmission quality, and bandwidth efficiency. These improvements were crucial because of the fast growing market demands and exponentially increasing applications.

Modern life and technology advancement called for the third generation which would empower multimedia communication. Future world needs personal communication with high quality images and videos, access to all daily information on public and private networks. Being reachable at any time and anywhere, all with high data rate and without boundaries through a handset is the goal of modern telecommunications. This vast and global advancement could not be done without new standardization of applied systems and among these new research achievements, Wide-band Code Division Multiple Access (WCDMA) has become the most widely adopted technique as the third generation.

The third generation is introduced as the Universal Mobile Telecommunication Systems (UMTS) and the standardization specifications are promoted by the 3rd Generation Partnership Project (3GPP) which is a joint project of standardization bodies from Europe, Japan, Korea, the USA, and China [3].

## 1.2 Multiple Access Techniques

The principal issue of a wireless or an air interface system design is how to share the channel between multiple users. The multi-access communication system basically utilizes one channel for several users. The diagram depicted in Figure 1.1 shows the basic idea of multiuser systems. Different users transmit their information using the same channel and the challenge is to receive and detect each users' information correctly while the received signal is embedded in the other users' information.

At the start of the twentieth century, linear modulation enabled radio transmission. Later on, the idea was employed to design first generation mobile communications which made possible the transmission by using different carrier frequencies. In Frequency Division Multiple Access (FDMA), each user is assigned a different carrier frequency resulting in signal separation in the frequency domain. On the receiver side, each user should use a bandpass filter fixed on the specific carrier frequency of the source.

The second generation systems benefited from the Time Division Multiple Access (TDMA) method. In TDMA, the system assigns each user a different time slot rather than a frequency slot and the information related to different users are separated in the time domain.

An important feature of both FDMA and TDMA systems is that the channel is divided into different non-overlapping slots. This means that the signals transmitted by different users are mutually orthogonal. This concept will be explained in the following chapter.

On the other hand, the two above-mentioned systems have limitations in terms of number of users accessing the channel simultaneously. The capacity of these channels is equal to the number of frequency/time slots that would fit into the frequency/time

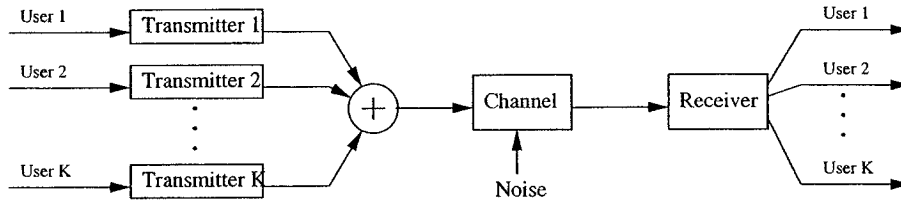


Figure 1.1: A typical “Multiple Access” system.

frame available to the system. Therefore, just a certain number of users are allowed to use the channel which results in prevention of system enhancement. In order to solve these limitations, Code Division Multiple Access (CDMA) system was introduced to improve the performance and capabilities of the second generation which led into the third generation systems.

In CDMA systems, each user is allocated a specific codeword which, ideally, should be orthogonal to the other users’ codeword. The user, utilizes the specified codeword to modulate transmitting data. The principal importance of codeword allocation is the cross-correlation between them. The mathematical definition of cross-correlation between users’ signals, e.g.  $S_1$  and  $S_2$ , is determined as

$$\rho_{1,2} = \langle S_1, S_2 \rangle = \int_0^T S_1(t)S_2(t)dt, \quad S_1(t), S_2(t) \in [0, T]. \quad (1.1)$$

Figure 1.2 shows an example of two time-limited signals. These signals overlap in both the frequency and time domain, but the cross-correlation between these two is zero meaning that they are orthogonal to each other. Therefore, in a two-user system, each of these signals can be employed as a codeword for data modulation. At the receiver end, each user correlates the received signal with the same waveform as it has used for data modulation to recover the transmitted data. In the synchronized case, if the codewords are orthogonal, each user’s filter eliminates the

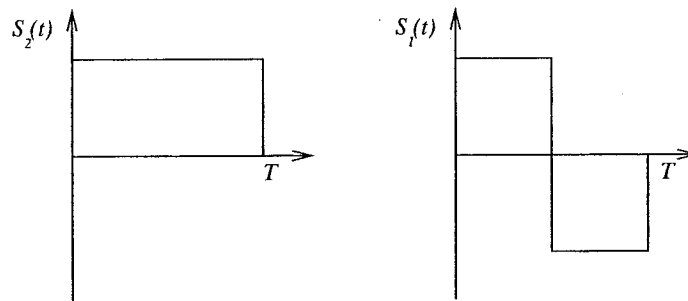


Figure 1.2: Two Orthogonal waveforms in the time domain.

other users' information and it just passes the signal that matches its own waveform.

The frequency and time division of FDMA, TDMA, and CDMA are shown in Figure 1.3.

### 1.3 Introductory Steps Towards CDMA

The need for higher data rates and the efficient use of bandwidth were the fundamental necessities which pushed the system design towards this third generation innovation. It is worth noting that CDMA was initially used as one of the standards in second generation system in North America (IS-95). The International Telecommunications Union (ITU) and Europe have been developing the standardization of the third generation networks since late 1980's which is referred to as IMT-2000 and UMTS, respectively [3].

The Main regional bodies of the ITU had already decided in favor of IMT-2000, but fast development happened due to participation of Association for Radio Industry and Business (ARIB), a standardization body responsible for Japan's radio organization which proceeded with detailed standardization of wide-band CDMA [2]. In 1998 the European Telecommunications Standards Institute (ETSI) decided upon WCDMA

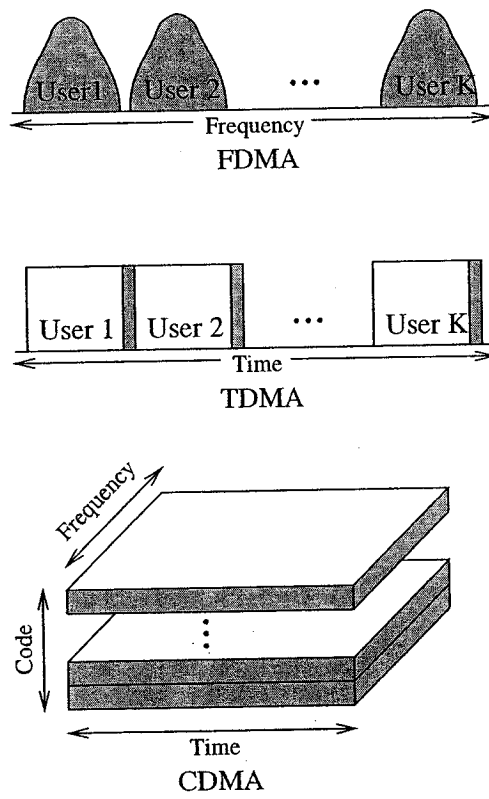


Figure 1.3: Signal constellation of different multiple access techniques.

as the third generation interface. Detailed work has been carried out as a part of the 3GPP standardization process. Finally, the first full specification was completed at the end of 1999.

CDMA can be classified based on several criteria. The most common classification is dependent on the modulation method used to obtain the wide-band signal. The most popular types of modulations are: Direct Sequence (DS), Frequency Hopping (FH), and Time Hopping (TH).

In DS-CDMA, the information is modulated by a pseudo-noise sequence resulting in a wide-band signal. In FH-CDMA, the pseudo-noise sequence identifies the instantaneous transmission frequency. Frequency Hopping can be either fast or slow depending on the number of hops over a symbol. Fast hopping happens in the case of waving several hops over one symbol and slow hopping is the result of transmitting several symbols during one hop. In TH-CDMA, the pseudo-noise sequence allocates each user a specified transmission moment to send the data. Different combinations of the above mentioned methods known as “Hybrid Techniques” will be discussed in the upcoming chapter.

## 1.4 Introduction to Multiuser Detection

CDMA systems are basically interference-limited from both the capacity and receiver point of view. From a capacity perspective, the number of users accessing the channel is limited by the signal-to-interference-plus-noise ratio. This means that the interference from other users’ information results in additive noise which degrades the performance. From the receiver angle, large number of users prevents the system improvement despite the Signal to Noise Ratio (SNR) increase.



Multiuser Detectors deal with demodulation of mutually interfering information. The interference produced in CDMA systems is caused by the signature sequences not being perfectly orthogonal in a CDMA system. The first stage of the receiver end in Multiuser Detection systems (MUD) consists of a spreading code Matched Filter (MF) [4]. Multiple Access Interference (MAI) at the output of the MFs is due to the fact that in the real world, all the signature sequences used in one system are not always orthogonal. Therefore, additional noise caused by MAI is more destructive to the system performance than the noise caused by the channel.

Observing the nature of MAI that comes from the other users' information, Verdú ([5], [6], [7]) came up with an optimal multiuser detection algorithm. He proved that the interference limitation of CDMA is mainly due to the conventional detector's limitation. The technique used in the optimal multiuser detectors, which utilized maximum *a posteriori* or Viterbi algorithm [17], could combat the MAI. But one must remember that the complexity of the optimal multiuser detector is an exponential function of the number of users. Therefore, system implementation is impractical for moderate to high number of users. The complexity issue of this method drove researchers to invent various suboptimal algorithms.

Suboptimal methods are commonly classified into two major categories: linear MUDs and Interference Cancellation (non-linear) MUDs. Linear MUDs are based on MAI suppressive filters which are linearly applied to the output of the matched filters. The most well-known detectors in this category are Minimum Mean-Squared Error (MMSE), Decorrelating Detector (DD), and Zero Forcing (ZF) [9]. Such detectors are less complex in comparison to optimal detectors, but they still suffer from the complexity of cross-correlation matrix inversion calculation, which will be studied later. Furthermore, the existence of the inverse of the cross-correlation matrix makes them

more troublesome. Because of problematic properties of linear detectors, research has been extended to non-linear detectors.

Non-linear detectors, i.e. Interference Cancellation (IC) detectors, are based on interference subtraction. These receivers can be divided into two different classes of Serial Interference Cancellation (SIC) and Parallel Interference Cancellation (PIC) methods. In IC technique, each user's information is estimated and respread, then the interfering signal is constructed and subtracted from the received signal. Although IC receivers benefit from simple construction and low complexity, their performance degrades in comparison to that of linear detectors and consequently optimal detectors.

Both linear receivers and IC receivers have been studied during the last decade in terms of system performance and complexity. The conclusion of the studies shows that multiuser detection based on PIC is currently more suitable than the other methods for use in CDMA systems. Simplicity of implementation and low complexity are the two major characteristics of this method that make it dominant as compared to the other methods. The complexity issue as well as the system performance of the optimal and sub-optimal methods along with several simulation results will be discussed in the following chapters.

## **1.5 The Objective and Contribution of This Thesis**

The objective of this thesis is to improve the performance of the existing MUD methods with special consideration given to the complexity issue and the number of users sharing the channel simultaneously.

In the following chapter, various multiple access techniques are introduced and Code Division Multiple Access (CDMA) technique is studied in detail. Subsequently,

different methods of “Multiuser Detection” (MUD) are investigated and their properties are presented.

In this thesis, an effort has been put forth to improve the performance of the existing techniques. To accomplish our goal, we benefited from the advantages of two different techniques: Partial Parallel Interference Cancellation (PPIC) and Decorrelating Detector (DD). By modifying the algorithms in the two above-mentioned techniques, we derive appropriate mathematical formulas that enable us to concatenate PPIC and DD detectors in a turbo fashion. By investigating the simulation results, it is verified that the proposed algorithm improves bit error probability for moderate to high SNR, while special consideration is given to practical computational complexity.

## Chapter 2

# Spread Spectrum Techniques

In this chapter, the principles of CDMA systems and different methods of spread spectrum modulation are presented. The CDMA concept and the criteria that a transmitted signal should fulfill to be considered as a spread spectrum signal are explained. Different methods of spread spectrum techniques are mentioned and fundamental properties of various CDMA techniques (i.e. multiple access capability, protection against multi-path interference, privacy, interference rejection, anti-jamming capability, and Low Probability of Interception (LPI)) are discussed. In addition, both the advantages and disadvantages of these techniques are mentioned.

### 2.1 CDMA Concepts

Elemental theory in CDMA signals states that different code words or signature sequences are assigned to different users which are sharing the same channel simultaneously. At the receiver end, by knowing the signature sequence of each user, the detector can now identify and separate each user's information. Detection is possible

if the cross-correlation between signature sequences is small enough to recover data. Since the bandwidth of the codewords is much larger than that of the information containing signal, the spectrum will be spread which results in a spread spectrum signal. The ratio of a codeword's bandwidth to that of data bandwidth is called *Processing Gain* (i.e. the number of bits of the signature sequence per information bit). Each signature sequence consists of  $N$  bits which are called *chips* with duration  $t_s$ . Spread spectrum modulation must fulfill two criteria [2]:

1. The transmission bandwidth must be much larger than the information bandwidth.
2. The radio-frequency bandwidth is statistically independent of the information being transmitted excluding modulation techniques like Frequency Modulation (FM) and Phase Modulation (PM) cases.

Spread spectrum techniques are classified into two different categories: Pure CDMA and Hybrid CDMA. Under the pure CDMA class, Time-Hopping (TH), Frequency-Hopping (FH), and Direct-Sequence CDMA techniques are the most popular. Hybrid systems are just a combination of two or more of the above-mentioned techniques to benefit from their accumulated advantages. In Figure 2.1, a general categorization of CDMA spread spectrum techniques is shown [3].

The remainder of this chapter will discuss the most significant CDMA methods mentioned in Figure 2.1 and their specifications [10], [11], [12], [13].

## 2.2 Direct Sequence CDMA

In a Direct Sequence CDMA (DS-CDMA) system, data is directly modulated with the signature sequence meaning that each information bit is multiplied by all the

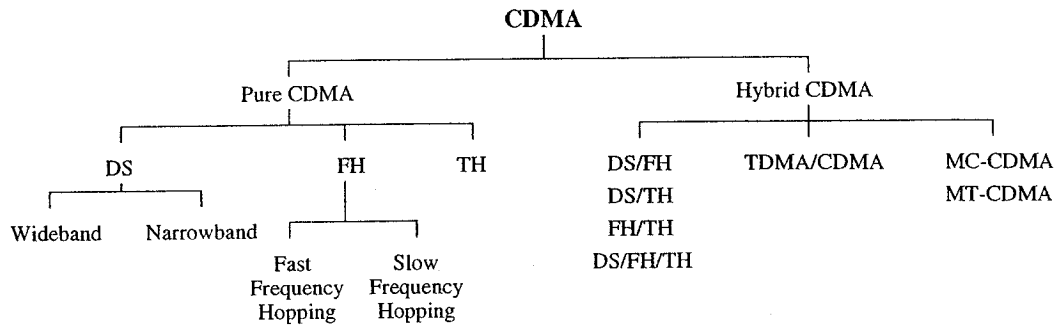


Figure 2.1: Categorization of various CDMA techniques.

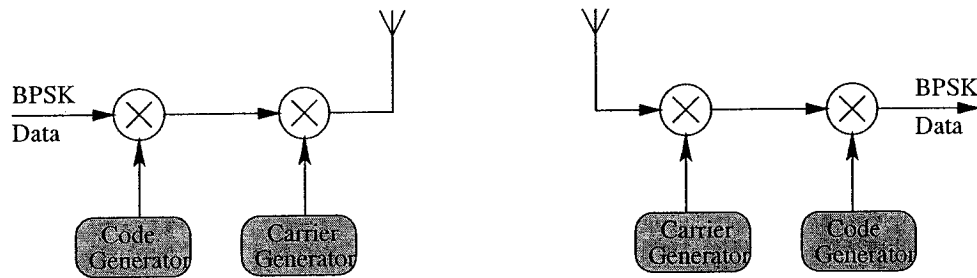


Figure 2.2: Basic structure of the Transmitter/Receiver of a DS-CDMA system.

codeword bits. Therefore, the bandwidth of the resultant signal is much larger than that of the information signal. The name Direct Sequence comes from the direct multiplication that is performed. As it is shown in Figure 2.2, after constructing the DS-CDMA signal, frequency translation is done to transmit the data. Figure 2.3 shows the process of modulating a Binary Phase Shift Keying (BPSK) signal using an  $N = 12$  (which is the number of chips per codeword) signature sequence.

Knowing the signature sequence of each user, the receiver can determine each user's data by passing the received signal through a bank of matched filters.

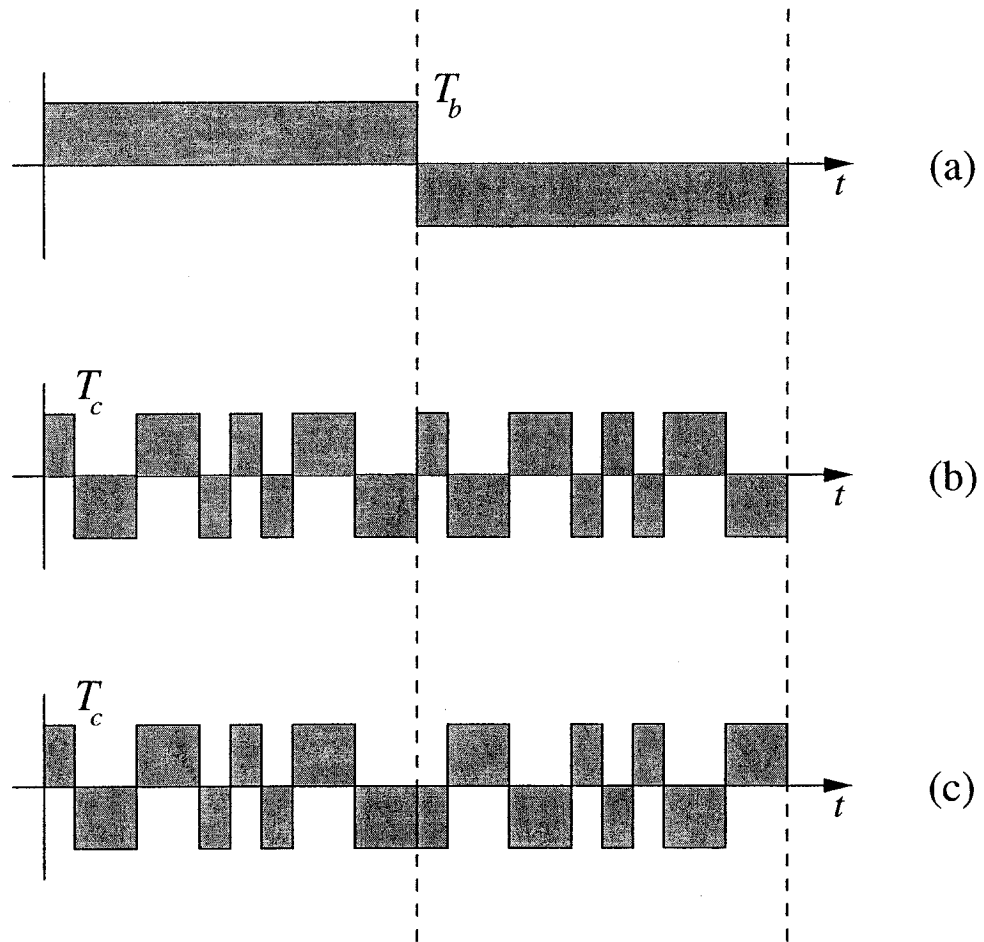


Figure 2.3: Modulation process of a BPSK signal using DS-SS-SSA technique. (a) BPSK signal. (b)  $N = 12$  signature sequence. (c) Spread BPSK.

## 2.2.1 General Capabilities

Here are some general properties of a DS-CDMA signal. These are further discussed in [3], [13], [14].

*Multiple access:* DS-CDMA signals overlap in both time and frequency domain, but detection is possible if the cross-correlation is low enough.

*Multipath interference:* Received signal is a compound of the transmitted signal traveling through different paths. So, the receiver can have a combination of constructive and destructive signals provided by one source and this can cause autocorrelation. “Despreading” considers the delayed versions as interfering signals and puts a small portion of the power of the interfering signals into the information bandwidth.

*Narrow-band interference:* Signal detection at the receiver end involves multiplication of the received signal by a locally generated code sequence. Consequently, multiplying a narrow-band signal by a wide-band code sequence spreads the spectrum of the narrow-band signal. So its power in the information bandwidth decreases by a factor equal to the processing gain.

*LPI:* Since the direct sequence signal uses the whole signal spectrum and its power is reduced by the processing gain, then signal detection is hard to implement. This feature prevents hearing the signal by the other users.

## 2.2.2 Specific properties

Here some exclusive advantages and disadvantages of the DS-CDMA technique are mentioned.

### *Advantages*

- Through a simple multiplication coded signal construction is possible.



- All users can use one single carrier frequency.
- Synchronization between different users is not necessary.
- Overloading the system affects the system performance, but unlike FDMA and TDMA, information will not be lost.

#### *Disadvantages*

- Synchronization between received codewords through different paths which originate from one transmitter, should be within a fraction of chip time duration that is hard to implement.
- Since users are not always synchronous, it is impossible to create codewords that are orthogonal. Therefore, the system suffers from MAI regularly.
- Each user has a different distance from the base station and the closer the user, the more powerful its signal can be which is destructive to the other users' signals. This issue is called *near-far problem* which can be solved by power control, but the system is practically hard to implement.

## 2.3 Frequency-Hopping CDMA

In the Frequency-Hopping Spread Spectrum technique (FH-SS), data modulation is done by changing the carrier frequency according to the signature sequences. Depending on each user's pseudo-random code, the modulator chooses a carrier frequency from a set of specific frequencies which is called *hop-set*.

According to Figure 2.4, FH-SS uses a fraction of the bandwidth at each time slot with high power while the DS-SS uses the whole bandwidth and the power is divided

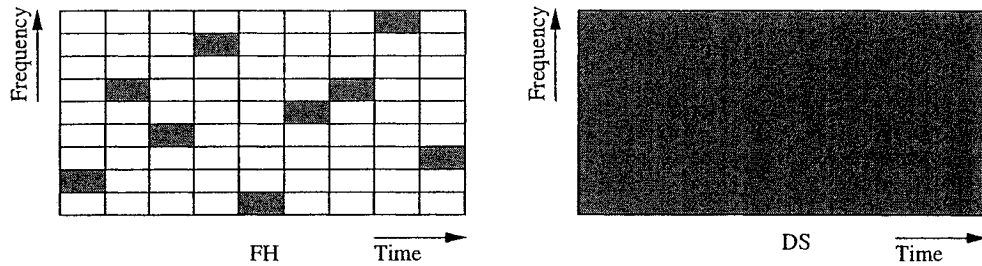


Figure 2.4: Comparison between FH and DS signals in terms of Time/Frequency occupancy.

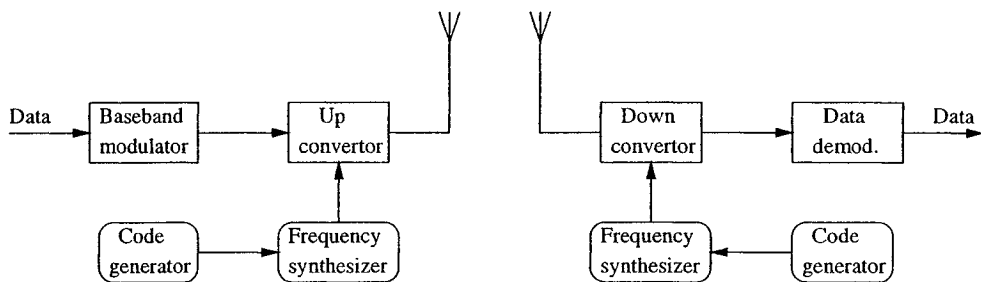


Figure 2.5: Basic structure of the Transmitter/Receiver of a typical FH-CDMA system.

between different users resulting in weaker signals. The transmitter and receiver of a typical FH-SS system are demonstrated in Figure 2.5.

The FH-SS technique is classified into two categories, fast FH and slow FH. The former happens when the hopping rate is much higher than the symbol rate. On the other hand, if the hopping rate is less than the symbol rate, it is called slow FH.

### 2.3.1 General Capabilities

General capabilities are discussed in the same way as it was done for DS-CDMA.

*Multiple access:* In fast FH, each symbol is transmitted over several frequency bands and since in each time slot just one powerful user is utilizing the channel,

interfering signals are small enough to make correct detection possible. On the other hand, in slow FH, several symbols are transmitted over one frequency band. In this case, if only one user is using the channel, then signal detection is possible; otherwise error correcting codes must be applied.

*Multipath interference:* In fast FH, one symbol is received over different carrier frequencies and multipath diversity does not have the same effect on different frequencies. So, the signal frequencies may be amplified or attenuated at different modulating carriers. By averaging over various hopping frequencies at the receiver, compensation of the multipath effect is possible, but it is not as efficient as DS-CDMA.

*Narrow-band interference:* If a narrow-band signal interferes one or some of hopping frequencies, still the other hopping frequencies are untouched which reduces the interference effect.

*LPI:* The frequency of the transmitted signal is unknown to the other users and changes over time. Therefore signal interception is a difficult task to perform.

#### *Advantages*

- Synchronization should be in a fraction of hop time which is much easier than that of DS-CDMA signal since hop time is much larger than chip duration.
- Since the probability of multiple users employing the same frequency band simultaneously is very small, Near-far problem is less likely to happen.
- Because of large bandwidth spectrum employment, narrow-band interference reduction is more applicable.

#### *Disadvantages*

- The multi-carrier characteristic of this system makes it difficult to implement.

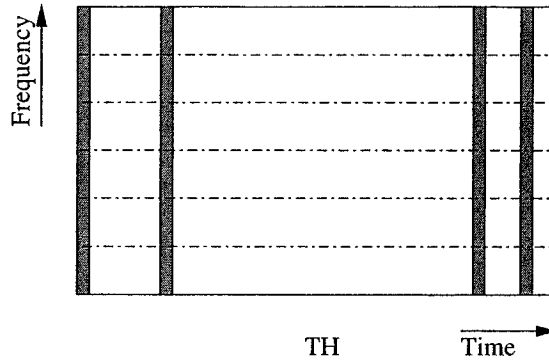


Figure 2.6: Time/Frequency occupancy of TH-CDMA signal.

- Carrier frequencies change frequently and abruptly which expands occupied bandwidth.

## 2.4 Time-Hopping CDMA

TH-CDMA assigns each user a short time slot based on its signature sequence. Each time frame is divided into several time slots which will be chosen by active users based on the codewords. Contrary to the technique that FH-CDMA utilizes, TH-CDMA provides each user the whole bandwidth at a specific time slot as can be seen in Figure 2.6. A general block diagram of a TH-CDMA transmitter and receiver is shown in Figure 2.7.

### 2.4.1 General Capabilities

Some general capabilities, advantages, and disadvantages of this method are discussed [3], [14], [15].

*Multiple access:* Since all users are transmitting over the same bandwidth, the probability of assigning one time slot to more than one user should be low enough

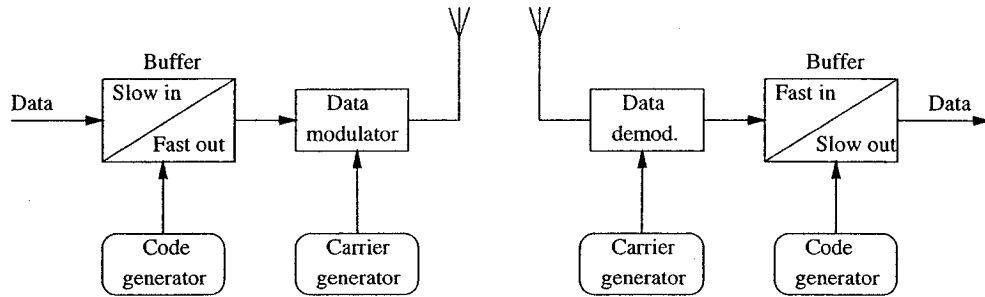


Figure 2.7: Basic structure of the Transmitter/Receiver of a typical TH-CDMA system.

to avoid MAI. Error correcting codes can ensure reliability of signal detection in the case of interference.

*Multipath interference:* TH-CDMA system uses short time slots for data transmission which results in a high signaling rate. Multipath reception of the signal will lead to overlap of adjacent bits which can be assumed as a drawback with respect to multipath interference rejection.

*Narrow-band interference:* The transmission time slot is reduced by the processing gain factor. At the receiver, the interfering signal can just be viewed at the mentioned time slot. Therefore, just a fraction of the interfering signal can affect the desired data and MAI is weakened by the processing gain factor.

*LPI:* The time slot belonging to each user is very short and unknown. Therefore, it is difficult for an intercepting receiver to recognize each user's data and distinguish different time slots.

#### *Advantages*

- Because of single-carrier structure, implementation is simple.
- Near-far problem is less likely to happen since the probability of more than one user employing the channel at the same time is low enough.

### *Disadvantages*

- The dedicated time to synchronization is very short which makes it hard to be performed.
- In order to prevent data loss, employing error-correcting codes and interleavers will be necessary if interference and simultaneous transmission is probable to happen.

## 2.5 Hybrid Techniques

Hybrid spread spectrum techniques were invented to benefit from individual advantages of different pure CDMA techniques. By combining simple methods, techniques such as DS/FH, DS/TH, FH/TH, and DS/FH/TH are achievable. Typical transmitter diagrams of a DS/FH-CDMA system and a DS/TH-CDMA are presented in Figures 2.8 and 2.9. Besides these techniques, combination of CDMA with TDMA and Multi-Carrier modulation can result in CDMA/TDMA and MC-CDMA techniques, respectively. Undoubtedly, increased complexity in implementation is one of the disadvantages of this class [3]. Time/Frequency occupancy of a CDMA/TDMA system is presented in Figures 2.10, and Figure 2.11 shows a typical transmitter diagram of an MC-CDMA system.

## 2.6 Summary

In this chapter, various spread spectrum techniques were discussed and some of their specifications were mentioned. These techniques imply the signal construction of a CDMA system and they show the composition of a transmitted signal. The most

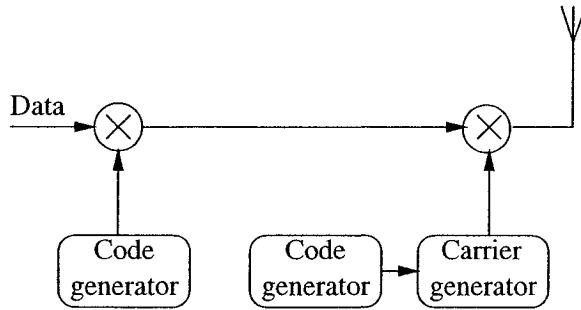


Figure 2.8: DS/FH-CDMA transmitter.

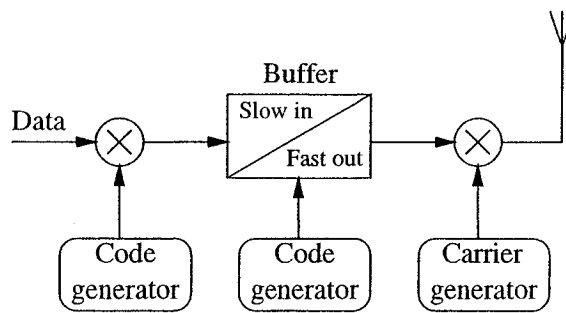


Figure 2.9: DS/TH-CDMA transmitter.

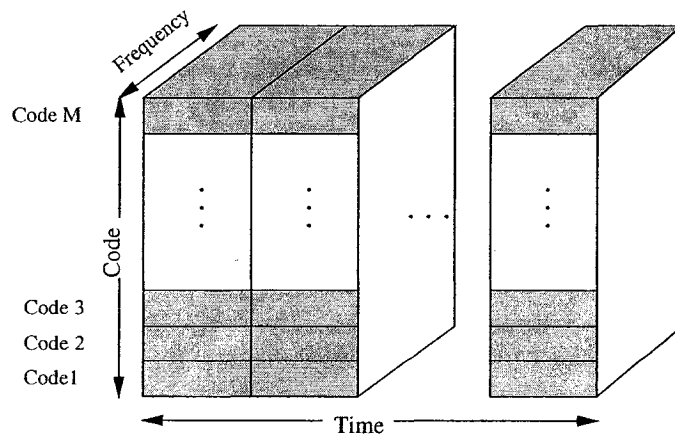


Figure 2.10: Time/Frequency occupancy of CDMA/TDMA system.

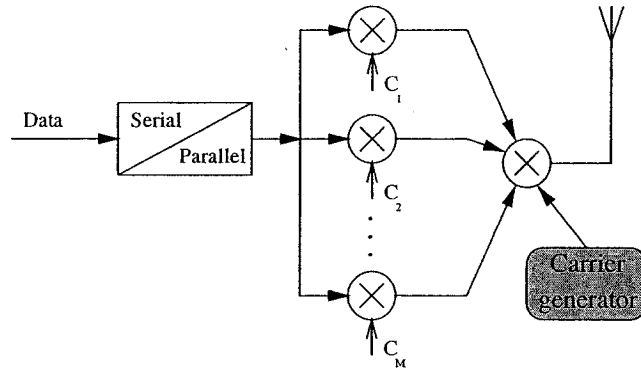


Figure 2.11: MC-CDMA transmitter for one user.

popular and adopted method is the DS-CDMA technique [12], [13]. In the next chapter, different methods of DS-CDMA signal detection are discussed.



# Chapter 3

## Multuser Detection Techniques

In this chapter, signal construction and a standard model of a general CDMA system will be reviewed. Subsequently, different methods of CDMA signal detection which is known as *Multuser Detection* (MUD) will be introduced. Finally, *Turbo Multuser Detection* based on Forward-Error-Correcting (FEC) codes and iterative multuser detection techniques will be discussed.

In the following, DS-CDMA signal model which is widely employed in most spread spectrum applications is explained.

### 3.1 Signal Construction for a General DS-CDMA Model

#### 3.1.1 Synchronous Case

In a general DS-CDMA synchronous channel, BPSK modulated data belonging to each user is spread by a pseudo-random code. It is assumed that the received bits from

different users are all synchronous and the received signal is embedded in Additive White Gaussian Noise (AWGN). At the input of the matched filters, the signals can be modeled as

$$y(t) = \sum_{k=1}^K A_k b_k S_k(t) + n(t), \quad t \in [0, T], \quad (3.1)$$

where

$K$  : number of users sharing the channel

$A_k$  : is the signal amplitude of  $k$ th user's received signal

$b_k \in [-1, +1]$  : is the  $k$ th user's transmitted bit

$S_k(t) \in [0, T]$  :  $k$ th user normalized pseudo – random signature waveform

$$\| S_k(t) \|^2 = \int_0^T S_k^2(t) dt = 1,$$

and  $n(t)$  is an AWGN with zero mean and double -sided power spectral density  $\sigma^2 = \frac{N\alpha}{2}$ .

The length of the signature waveform implies the processing gain  $N = T_b/T_c$  where  $T_b$  and  $T_c$  are the time duration of an information bit and a chip bit, respectively. Therefore, each signature sequence can be presented as

$$S_k(t) = \sum_{n=1}^N C_n P_{T_c}(t - nT_c), \quad C_n \in \left[-\frac{1}{\sqrt{N}}, +\frac{1}{\sqrt{N}}\right]. \quad (3.2)$$

The performance of a CDMA system depends on the correlation between of the signature waveforms which is measured by the cross-correlation between them defined as using Equation (3.3) below

$$\rho_{ij} = \langle S_i, S_j \rangle = \int_0^T S_i(t) S_j(t) dt. \quad (3.3)$$

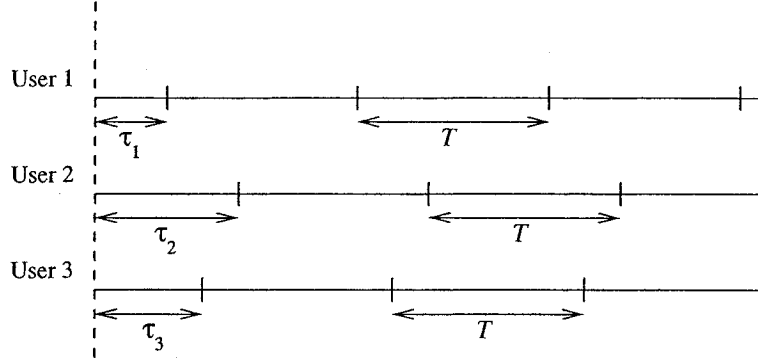


Figure 3.1: Offset modeling asynchronism.

### 3.1.2 Asynchronous Case

In reality, all the users' bit epochs are not aligned. Lack of alignment can be defined by time delay at the receiver:  $\tau_k \in [0, T)$  (Figure 3.1). The received signal embedded in noise can be modeled as following if we assume all the users transmit packets of length  $L$

$$y(t) = \sum_{k=1}^K \sum_{i=1}^L A_k b_k[i] S_k(t - iT - \tau_k) + n(t), \quad \tau_k \in [0, T). \quad (3.4)$$

The synchronous case is just a special case of asynchronous case where all the offsets are the same:  $\tau_1 = \tau_2 = \dots = \tau_K$ .

For the sake of simplicity, all the studies and simulations in this thesis is done for the synchronous case.

## 3.2 Optimum Multiuser Detection

The optimum multiuser detector is based on Maximum Likelihood (ML) criterion. In Section 3.1 it was mentioned that the received signal at the input of the detector

resembles Equation (3.1). In order to convert the received waveform to a discrete time signal, it is common to pass the received signal through a bank of matched filters which are matched to the signature waveforms of the users [4]. Therefore, the output of the matched filters will look like

$$\begin{aligned} y_1 &= \int_0^T y(t)S_1(t)dt, \\ &\vdots \\ y_K &= \int_0^T y(t)S_K(t)dt, \end{aligned} \tag{3.5}$$

and then each user's output can be written as

$$y_k = A_k b_k + \sum_{j \neq k} A_j b_j \rho_{jk} + n_k, \tag{3.6}$$

where

$$n_k = \int_0^T n(t)S_k(t)dt. \tag{3.7}$$

The vector representation of the matched filters' output can be modeled as,

$$\mathbf{y} = \mathbf{R}\mathbf{A}\mathbf{b} + \mathbf{n}, \tag{3.8}$$

where

$$\begin{aligned} \mathbf{y} &= [y_1, \dots, y_K]^T, \\ \mathbf{b} &= [b_1, \dots, b_K]^T, \\ \mathbf{A} &= \text{diag}\{A_1, \dots, A_K\}. \end{aligned}$$

$\mathbf{R}$  is the cross-correlation matrix with unit diagonal elements. The off-diagonal elements specify the cross-correlation between the users' codewords as defined in Equation 3.3 ( $\mathbf{R} = \{\rho_{ij}\}$ ).  $\mathbf{n}$  is a zero-mean Gaussian random vector with covariance matrix equal to

$$E[\mathbf{nn}^T] = \sigma^2 \mathbf{R}, \quad \sigma^2 = \frac{N_0}{2}. \quad (3.9)$$

In this method, in order to minimize the probability of bit error,  $\mathbf{b}$  should be chosen in favor of maximizing the joint *a posteriori* probability [4]

$$P[\mathbf{b} \mid \{y(t), 0 \leq t \leq T\}]. \quad (3.10)$$

Since all the bits from different users are equiprobable, it is shown that the optimum decision rule selects the vector  $\mathbf{b}$  that maximizes

$$f[\{y(t), 0 \leq t \leq T\} \mid \mathbf{b}] = \exp \left( -\frac{1}{2\sigma^2} \int_0^T \left[ y(t) - \sum_{k=1}^K b_k A_k S_k(t) \right]^2 dt \right), \quad (3.11)$$

or, equivalently maximizes

$$\begin{aligned} \Omega(\mathbf{b}) &= 2 \int_0^T \left[ \sum_{k=1}^K A_k b_k S_k(t) \right] y(t) dt - \int_0^T \left[ \sum_{k=1}^K A_k b_k S_k(t) \right]^2 dt \\ &= 2\mathbf{b}^T \mathbf{A} \mathbf{y} - \mathbf{b}^T \mathbf{H} \mathbf{b}, \end{aligned} \quad (3.12)$$

where

$$\mathbf{H} = \mathbf{A}\mathbf{R}\mathbf{A}. \quad (3.13)$$

One of the major factors in all multiuser detectors is computational complexity. Computational complexity is defined as the number of operations required to demodulate transmitted bits, divided by the number of detected bits. Since the second term in Equation (3.12) can be precalculated, in order to maximize it,  $\mathbf{b}$  should be chosen in a way to maximize the first term. According to the Viterbi algorithm [17],  $2^K$  operations are required to choose the optimum  $\mathbf{b}$ . Therefore, the computational complexity of the optimum MUD is an exponential function of the number of users ( $O(2^K)$ ). In spite of the optimal performance of this method, the complexity is the highest that makes the implementation impractical in reality for a moderate to large number of users. This fact encouraged the researchers to invent sub-optimal methods which can exhibit better complexity/performance trade-off.

Other methods can be classified into two major categories, linear MUD and interference cancellation MUD to which the remaining of this chapter is dedicated.

### 3.3 Linear Multiuser Detection

In this category of detectors, the MUD system applies a linear mapping to the outputs of the matched filters. Under this class, two leading detectors can be named, Decorrelating Detector (DD) and Minimum Mean-Square Error (MMSE) detector.

#### 3.3.1 Decorrelating Detector

In this technique, the linear mapping is applied in the sense of multiplying the outputs of the matched filters  $\mathbf{y}$ , by the inverse of the cross-correlation matrix [4],  $\mathbf{R}^{-1}$  (under

existence condition), as follows

$$\begin{aligned}\mathbf{R}^{-1}\mathbf{y} &= \mathbf{R}^{-1}(\mathbf{R}\mathbf{A}\mathbf{b} + \mathbf{n}) \\ &= \mathbf{A}\mathbf{b} + \mathbf{R}^{-1}\mathbf{n}.\end{aligned}\tag{3.14}$$

From Equation (3.14), it can be noticed that the decorrelating detector rejects the interference caused by the information bits, but the system suffers from the interference caused by the enhanced background noise. The block diagram of the receiver is shown in Figure 3.2.

From the implementation angle, the decorrelating detector benefits from the near-far resistance property since no knowledge of amplitude of the received bits is required. Another advantage of this method is that the computational complexity is a quadratic function of the number of users ( $O(K^2)$ ) [18], which is much less than that of the optimum MUD.

In addition to having a degradation in performance as compared to optimum MUD, existence condition of  $\mathbf{R}^{-1}$  is one of the drawbacks of this method [4].

In general, the decorrelating technique is not only a simple method, but it is also optimal in three different criteria: least-squares, near-far resistance, and maximum likelihood when the received amplitudes are not known [4].

### 3.3.2 MMSE Detector

In this approach,  $\mathbf{b}$  is estimated based on the minimization of the mean-square error between the transmitted data and a linear combination of the outputs of the matched filters [4]. The linear MMSE detector outputs a weighted combination of the matched

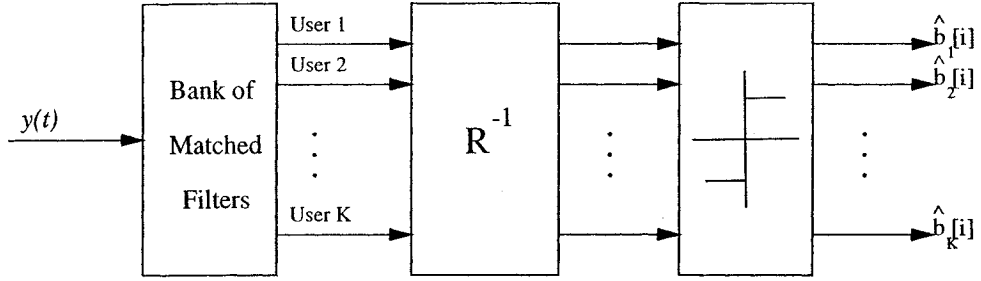


Figure 3.2: Decorrelating Detector (DD) block diagram.

filter outputs, namely choosing the  $K$ -vector  $\mathbf{m}_k$  that minimizes

$$E \left[ (b_k - \mathbf{m}_k^T \mathbf{y})^2 \right]. \quad (3.15)$$

In order to output the detected signals for  $K$  users, a  $K$  uncoupled optimization problems (one for each user) can be solved simultaneously by choosing  $K \times K$  matrix  $\mathbf{M}$  that achieves

$$\min_{\mathbf{M} \in \mathbb{R}^{K \times K}} E \left[ \|\mathbf{b} - \mathbf{M}\mathbf{y}\|^2 \right]. \quad (3.16)$$

The solution to (3.16) gives the following estimation for the information bits

$$\begin{aligned} \widehat{b}_k &= \text{sgn} \left( \frac{1}{A_k} \left( \left[ \mathbf{R} + \sigma^2 \mathbf{A}^{-2} \right]^{-1} \mathbf{y} \right)_k \right) \\ &= \text{sgn} \left( \left( \left[ \mathbf{R} + \sigma^2 \mathbf{A}^{-2} \right]^{-1} \mathbf{y} \right)_k \right). \end{aligned} \quad (3.17)$$

Therefore, the difference between this approach and decorrelation detector is that in a MMSE detector, instead of  $\mathbf{R}^{-1}$ , the transformation is applied by multiplying the matched filter outputs by  $[\mathbf{R} + \sigma^2 \mathbf{A}^{-2}]^{-1}$ .



A single-user matched filter is designed to decrease the effect of the background noise. The decorrelating detector tries to eliminate multiuser interference regardless of enhancing the background noise, but the MMSE detector is a compromise design which considers both MAI and background noise at the same time.

On the other hand, since this system requires knowledge of the received signal amplitude, it suffers from the near-far problem.

### **3.4 Subtractive Multiuser Detection**

The basic idea in these kinds of detectors can be explained as follows. The temporary estimation of information bits is done using matched filter outputs. This causes the MAI to be respread and then the interfering signal is constructed and subtracted from the received signal. Since this method, which is also called Interference Cancellation (IC), is based on the prior estimation of the received bits, reliability of the results is strongly controlled by the accuracy of the matched filter outputs. Cancellation can be done either in serial or parallel fashion which results in two cited methods, Serial IC (SIC) and Parallel IC (PIC).

#### **3.4.1 Serial Interference Cancellation (SIC)**

In such detectors, cancellation is done in a successive manner. It is assumed that the more powerful the received bit is, the more reliable it is. Therefore, the detector starts with the most powerful received bit. Being optimistic, the detector makes a tentative estimation on the mentioned bit, respreads the interfering signal, subtracts the MAI caused by the detected bit from the other received bits, and then the same procedure is applied to the next most powerful bit. Assuming that the decisions of

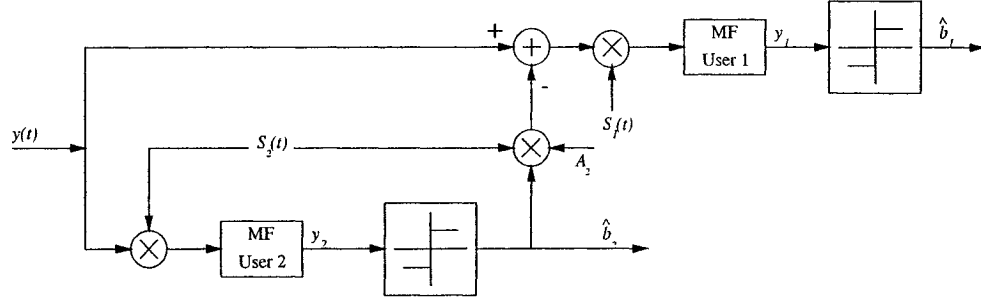


Figure 3.3: Successive cancellation for a two-user system.

users  $k + 1, \dots, K$  are already made, and signals for the users  $1, \dots, k - 1$  are weaker than that of the  $k$ th user, estimation of the  $k$ th user signal is done as following

$$\hat{b}_k = \text{sgn} \left( y_k - \sum_{j=k+1}^K A_j \rho_{jk} \hat{b}_j \right). \quad (3.18)$$

A simple block diagram of a two-user system in which the second user's bit is stronger than that of the first one is shown in Figure 3.3 [16].

There are some properties regarding the practical implementation of this system which are mentioned below:

- A delay of one bit is required for every single successive bit to be detected which makes the system impractical for large number of users.
- Any error in detection of one bit, increases the probability of error for the succeeding bits.
- When a user is changed, the process of reordering the users should be repeated.
- Users weaker than the desired user are neglected.

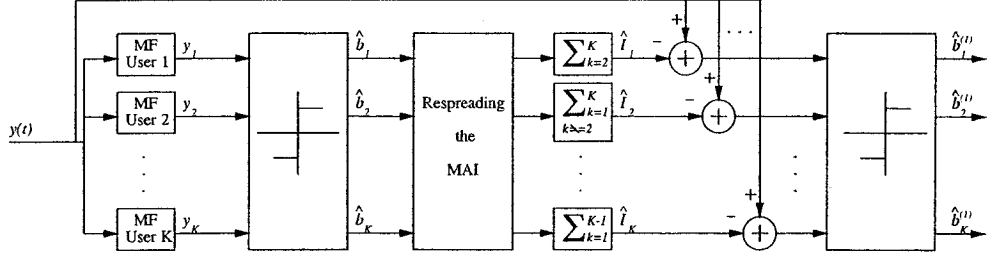


Figure 3.4: PIC receiver block diagram.

- SIC implementation benefits from a simple circuitry and it is the simplest method among the MUD techniques.
- The SIC can be applied to any spread spectrum system that suffers from additive superposition of the transmitted signals.

### 3.4.2 Parallel Interference Cancellation (PIC)

In the PIC scheme, the tentative decision of the bits is done all at the same time regardless of their power. Then, the MAI is respread and the interfering signal for all the users is cancelled at the same time. In this system, all the users are treated in the same way. The following formula shows the cancellation of the estimated MAI, affecting the output of the  $k$ th user's matched filter as

$$\widehat{b}_k = \text{sgn} \left( y_k - \sum_{j=1, j \neq k}^K A_j \rho_{jk} \widehat{b}_j \right). \quad (3.19)$$

Figure 3.4 shows a simple block diagram of a typical PIC detector.

One of the advantages of this system is that it takes approximately one bit duration delay to complete the operation in parallel for all the users. In addition, system implementation is simpler than that of DD and ML detectors. On the other hand, in

comparison to SIC, circuitry system implementation is much more complex and PIC is much more vulnerable to the near-far problem.

### 3.5 Turbo (Iterative) Multiuser Detection

Over the last decade, significant research has been dedicated to various methods of multiuser detection. Recently a new observation of CDMA signal construction has initiated new multiuser detection techniques that improved the performance dramatically. These methods are focused on CDMA systems in conjunction with forward error-control coding [18]-[24]. These systems are called *Iterative MUD* or *Turbo MUD*.

A CDMA signal is a linear transformation of the inputs to the channel. The outputs of the whitening MFs have special properties (see [25], [26]), which result in eligibility of viewing the transformed signal as a convolutionally coded signal. In practice, CDMA systems employ error-control coding and interleavers. Therefore, the CDMA signal can be viewed as a concatenation of two coding systems similar to turbo codes [27], [28]. Then, the so-called turbo-principal can be applied to detecting/decoding CDMA signals [29].

By benefiting from turbo properties, the recovery and demodulation of the information bits can be done in an iterative manner. In the CDMA case, the outer code is the error-control code that each user employs to encode the transmitted information, and the CDMA channel is assumed as an inner code. The receiver performs successive detecting and decoding, achieved by a multiuser detector and a bank of single-user decoders, respectively, in an iterative fashion. At each iteration, extrinsic information is extracted from the multiuser detector based on different methods mentioned earlier in this chapter. Using the extrinsic information as *a priori* information, the

single-user decoders produce extrinsic information (based on different methods, e.g., soft output Viterbi algorithm [17] or BCJR [30]) for both the information bits and the parity bits. Thereafter, the detector employs the extrinsic information delivered by the single-user decoders in the next iteration as *a priori* information to improve detecting channel values. The single-user channel encoder/decoder can be implemented in various fashions, e.g., simple convolutional codes or turbo codes. In the systems that apply turbo codes as error-control coding, at each detection/decoding iteration, turbo channel decoder performs specific numbers of iterations individually to provide the extrinsic information to be used by the multiuser detector in the next iteration. At the last detection/decoding iteration, single-user channel decoders decode the transmitted data (based on the technique employed to encode the data) and output the sign of the decoded data as final decision. A typical block diagram of a Turbo MUD is shown in Figure 3.5.

The combination of detecting and decoding has overcome some of the limitations of the old fashioned multiuser detectors. Various algorithms have been put forth to reach the performance of a single-user channel. These range from high complexity optimum detectors [19], [20] up to many sub-optimum algorithms which were mentioned earlier in this chapter [21], [22], [31]. In the following, some of the prominent and recent works are reviewed and an effort to improve the performance/complexity trade-off is done in the succeeding chapter.

### 3.5.1 Optimum (ML) Iterative MUD

It is shown in [32] that the output of the MFs,  $\mathbf{y}$  (as derived in Equation (3.5)), is a sufficient statistic for demodulating  $\mathbf{b}$ . In what follows, derivation of the exact soft input/soft output (SISO) MUD is shown briefly [18]. The SISO MUD calculates the

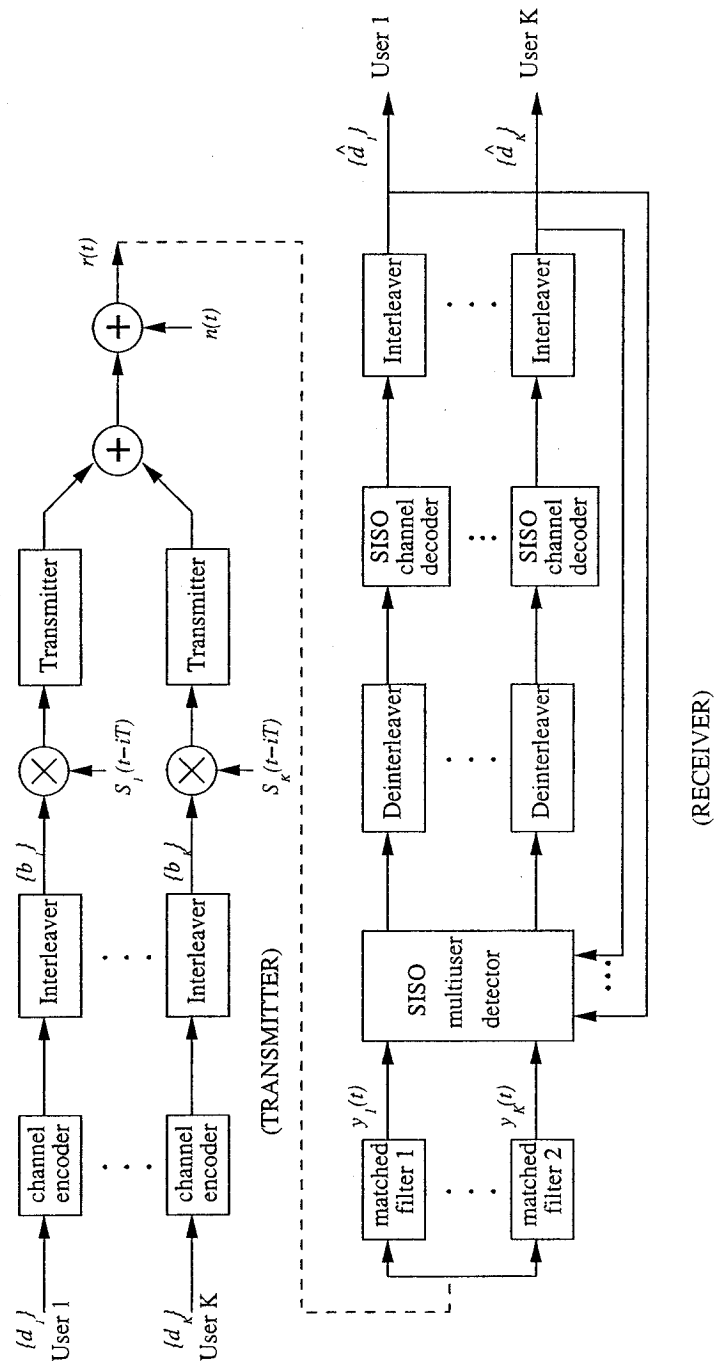


Figure 3.5: A coded CDMA system and Turbo (Iterative) MUD block diagram.

a *posteriori* log-likelihood ratio (LLR) of every transmitted bit of each user by

$$\Lambda_1[b_k(i)] = \log \frac{P[b_k(i) = +1 | r(t)]}{P[b_k(i) = -1 | r(t)]}, \quad i = 1, \dots, L, \quad (3.20)$$

where  $L$  is the length of the packet that each user transmits in each frame. Using Bayes' rule [33], (3.20) can be rewritten as

$$\Lambda_1[b_k(i)] = \underbrace{\log \frac{p[r(t) | b_k(i) = +1]}{p[r(t) | b_k(i) = -1]}}_{\lambda_1[b_k(i)]} + \underbrace{\log \frac{P[b_k(i) = +1]}{P[b_k(i) = -1]}}_{\lambda_2^p[b_k(i)]}. \quad (3.21)$$

The second term in (3.21) is delivered by the SISO channel decoders as *a priori* information from the previous ( $p$  stands for previous) iteration. The first term is the extrinsic information that the MUD outputs to be deinterleaved and then utilized by the channel decoders as *a priori* information. Denote

$$B_k^\pm \equiv \{(b_1, \dots, b_{k-1}, \pm 1, b_{k+1}, \dots, b_K) : b_j \in \{+1, -1\}, j \neq k\}, \quad (3.22)$$

and the extrinsic information to be delivered to the single-user decoders will be calculated by [18]

$$\lambda_1[b_k(i)] = \log \frac{\sum_{\mathbf{b} \in B_k^+} \exp \left[ -(\mathbf{y}(i) - \mathbf{R}\mathbf{A}\mathbf{b})^T \mathbf{R}^{-1} (\mathbf{y}(i) - \mathbf{R}\mathbf{A}\mathbf{b}) / (2\sigma^2) \right] \prod_{j \neq k} P[b_j]}{\sum_{\mathbf{b} \in B_k^-} \exp \left[ -(\mathbf{y}(i) - \mathbf{R}\mathbf{A}\mathbf{b})^T \mathbf{R}^{-1} (\mathbf{y}(i) - \mathbf{R}\mathbf{A}\mathbf{b}) / (2\sigma^2) \right] \prod_{j \neq k} P[b_j]}, \quad (3.23)$$

where  $P[b_j]$  is the *a priori* information of information bits which can be expressed in terms of their LLR's  $\lambda_2^p[b_k(i)]$  as follows

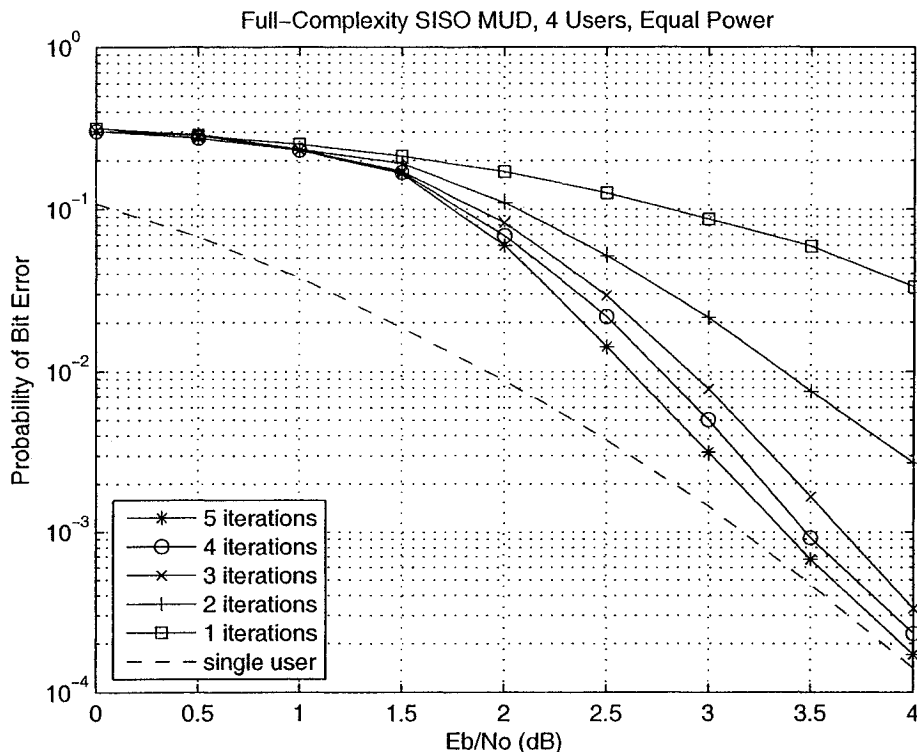


Figure 3.6: The performance of a full-complexity SISO MUD as a function of the number of iterations,  $K = 4$  users,  $\rho_{ij} = 0.7$ ,  $L = 128$ , and all the users have equal power.

$$P[b_j] = \frac{1}{2} \left[ 1 + b_j \tanh \left( \frac{1}{2} \lambda_2^p [b_k(i)] \right) \right]. \quad (3.24)$$

In Figure 3.6, simulation results of optimum MUD is demonstrated. The received signal is embedded in AWGN and BPSK modulation is applied for the data mapping. All the users utilize the same rate  $R_k = \frac{1}{2}$  convolutional code with constraint length  $v = 5$  (with generators 23 and 35 in octal notation). The simulation is done for  $K = 4$  users and it is assumed that the cross-correlation between users' signature waveform is constant ( $\rho = 0.7$ ) [18].



It can be noticed from Figure 3.6 that the more iterations are performed, the closer the results get to a single-user channel. It should be noted that after a certain number of iterations, the improvement will diminish as the extrinsic data that is exchanged between MUD and the single-user decoders become more and more correlated. As it was mentioned in Section 3.2, the computational complexity of this method is exponential in terms of the number of users  $K$  ( $O(2^K)$ ).

### 3.5.2 Soft Instantaneous MMSE Interference Cancellation

In this method, Poor and Wang [18] applied an instantaneous MMSE filter to the outputs of an interference cancellation system. From the *a priori* information from previous iterations, the following can be obtained

$$\begin{aligned}\tilde{\mathbf{b}}(i) &\equiv [\tilde{b}_1(i), \dots, \tilde{b}_K(i)], \quad \tilde{b}_j(i) = \tanh\left(\frac{1}{2}\lambda_2^p[b_j(i)]\right) \\ \tilde{\mathbf{b}}_k(i) &\equiv \tilde{\mathbf{b}}(i) - \tilde{b}_k(i)\mathbf{e}_k \\ &= [\tilde{b}_1(i), \dots, \tilde{b}_{k-1}(i), 0, \tilde{b}_{k+1}(i), \dots, \tilde{b}_K(i)]\end{aligned}\tag{3.25}$$

where  $\mathbf{e}_k$  is all-zero except the  $k$ th element which is one. Interference cancellation is done in parallel fashion as

$$\begin{aligned}\mathbf{y}_k(i) &\equiv \mathbf{y}(i) - \mathbf{R}\mathbf{A}\tilde{\mathbf{b}}_k(i) \\ &= \mathbf{R}\mathbf{A}[\mathbf{b}(i) - \tilde{\mathbf{b}}_k(i)] + \mathbf{n}, \quad k = 1, \dots, K, \quad E[\mathbf{n}\mathbf{n}^T] = \sigma^2\mathbf{R}.\end{aligned}\tag{3.26}$$

Furthermore, an instantaneous MMSE filter is applied to  $\mathbf{y}_k(i)$  with  $\mathbf{w}_k(i)$  weights

$$z_k(i) = \mathbf{w}_k(i)^T \mathbf{y}_k(i).\tag{3.27}$$

$\mathbf{w}_k(i)$  is chosen in order to minimize the mean-square error between  $b_k(i)$  and  $z_k(i)$  as

$$\begin{aligned}\mathbf{w}_k(i) &= \arg \min_{\mathbf{w} \in \mathbb{R}^K} E \left\{ [b_k(i) - \mathbf{w}^T \mathbf{y}_k(i)]^2 \right\} \\ &= \arg \min_{\mathbf{w} \in \mathbb{R}^K} E \left\{ \mathbf{y}_k(i) \mathbf{y}_k(i)^T \right\} \mathbf{w} - 2\mathbf{w}^T E \{ b_k(i) \mathbf{y}_k(i) \}.\end{aligned}\tag{3.28}$$

After some manipulation and calculation,  $z_k(i)$  can be written as

$$z_k(i) = A_k \mathbf{e}_k^T [\mathbf{V}_k(i) + \sigma^2 \mathbf{R}^{-1}]^{-1} [\mathbf{R}^{-1} \mathbf{y}(i) - \mathbf{A} \tilde{\mathbf{b}}_k(i)], \quad (3.29)$$

where

$$\mathbf{V}_k(i) = \sum_{j \neq k} A_j^2 [1 - \tilde{b}_j(i)^2] \mathbf{e}_j \mathbf{e}_j^T + A_k^2 \mathbf{e}_k \mathbf{e}_k^T. \quad (3.30)$$

The overall computational complexity of this method including single-user decoders is  $O(K^2 + 2^v)$  [18], where  $v$  is the constraint length of the single-user encoders. Performance simulation of this system is demonstrated and compared with that of full-complexity SISO MUD in Figure 3.7. With just a slight loss of performance compared to the optimum method, the complexity of this system is reduced dramatically [18].

### 3.5.3 Turbo Multistage PIC

The PIC technique, which was mentioned in Subsection 3.4.2, can be implemented in a multistage fashion and then employed in turbo MUD [24], [34]. After MAI cancellation the first time, the outputs can be used to make tentative decisions. Based on the temporary decisions, MAI can be constructed again and subtracted from the received signal. This process can be continued until no further improvement is achieved. Assuming  $\hat{\mathbf{b}}$  is the *a priori* information received from last iteration,

$$\mathbf{z} = \mathbf{y} - (\mathbf{R} - \mathbf{I}) \mathbf{A} \hat{\mathbf{b}} = \mathbf{A} \mathbf{b} + (\mathbf{R} - \mathbf{I}) \mathbf{A} (\mathbf{b} - \hat{\mathbf{b}}) + \mathbf{n}, \quad (3.31)$$

and then

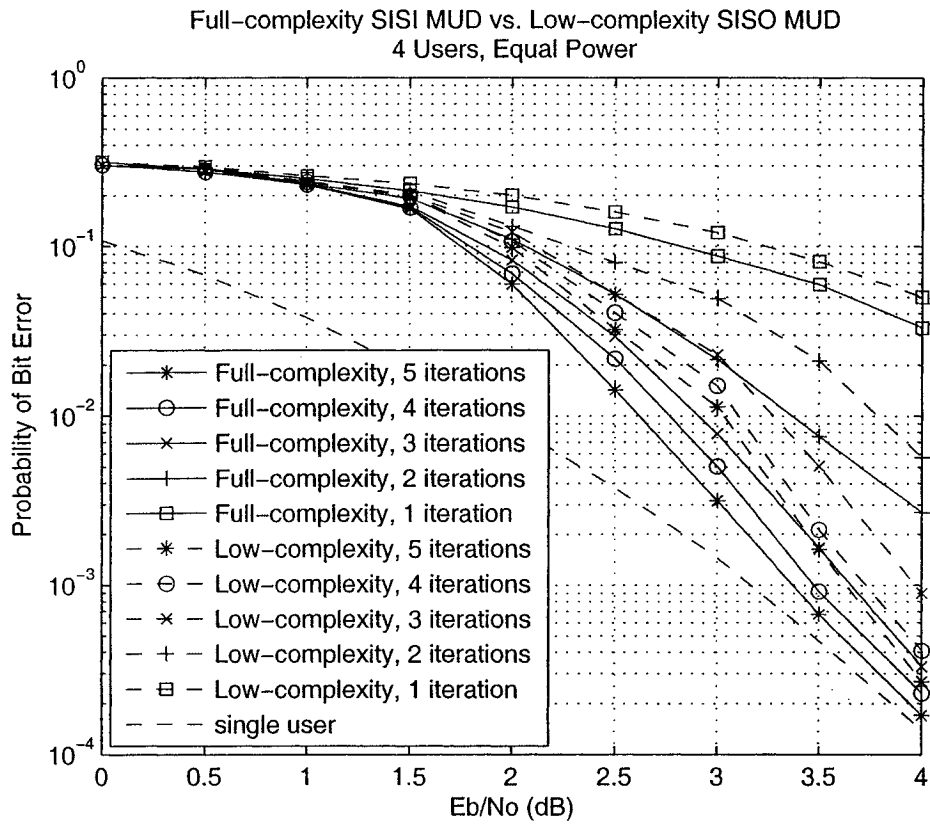


Figure 3.7: Performance of soft instantaneous MMSE SISO MUD.  $K = 4$  users,  $\rho_{ij} = 0.7$ ,  $L = 128$ , and all the users have equal power.

$$z_k = A_k b_k + \sum_{j=1, j \neq k}^K \rho_{jk} A_j (b - \hat{b}_j) + n_k. \quad (3.32)$$

Therefore, an estimation can be made on  $\mathbf{z}$  and the MAI be reconstructed, subsequently the same process as Equations (3.30) and (3.31) can be applied. The second term in Equation (3.31) which is residual MAI, can be approximated as a Gaussian random variable which is independent of  $n_k$ . Then the soft output of the multiuser detector can be calculated as follows

$$\lambda_1[b_k(i)] = \log \frac{P[z_k | b_k = +1]}{P[z_k | b_k = -1]} = \frac{2A_k z_k}{\sigma_k^2}, \quad (3.33)$$

where  $\sigma_k^2$  is the variance of the sum of residual interference and  $n_k$ , which can be statistically estimated by the multiuser detector [35]. Performance results of this algorithm is depicted along with other methods in the following chapter.

## 3.6 Summary

In this chapter, different Multiuser Detection methods were reviewed and *Turbo Multiuser Detection* was studied. Then, formulization and performance simulation of some of these methods were presented. In the next chapter, an effort is put forth to combine two newly introduced iterative MUD techniques to achieve an improved scheme, with special consideration given to computational complexity. Then, a comparison between the proposed algorithm and some of the other competing methods, implementing simple convolutional and turbo encoders, is presented.

## Chapter 4

# Concatenation of PPIC and Decorrelating Detector

In this Chapter, we propose an improved algorithm which is the concatenation of the iterative Partial Parallel Interference Cancellation (PPIC) [37] followed by a Decorrelating Detector (DD) [35]. An improvement is made in terms of the bit error probability and these results are compared with that of the other competing MUD techniques. The system considered in this thesis, for the sake of simplicity, is a bit synchronous CDMA system and this idea can be applied to the asynchronous case. This chapter is organized as following.

First, a brief overview of the system and the channel which we will consider and use in our simulations, is presented. Second, the convolutional and turbo channel encoder/decoder that are utilized in the systems and simulations are studied. Third, improved PPIC is presented ([37]) and a slight modification is applied to achieve appropriate formulization to be utilized in our proposed algorithm. Subsequently, the Decorrelating Detector proposed by Zhang and D'Amours [35] is studied. Af-

terwards, our proposed “Multiuser Detection” technique which is a concatenation of the modified PPIC and the newly proposed Decorrelating Detector is introduced and simulations are performed to certify the excellence and improvements of our advised algorithm. Finally, in the last Section it is shown that we are capable of achieving lower probability of bit error in price of some additional complexity in different systems.

## 4.1 General System Description

### 4.1.1 CDMA Channel

In Figure 4.1, a general CDMA transmitter and receiver is depicted. The system considered in this thesis is a coded CDMA system, where  $K$  users access the channel at the same time, utilizing normalized modulation waveforms  $S_1(t), \dots, S_K(t)$ . The binary information data  $d_k(i)$  is encoded through single-user channel encoders by rate  $R_k$ . To avoid burst-error at the input of each channel decoder, each user applies a different random interleaver of length  $L$ , which is the size of the encoded information packet that each user transmits in each frame. The interleaved bits are BPSK modulated which results in  $b_k(i)$  of duration  $T$ , which is the  $i$ th information symbol of the  $k$ th user. Each symbol is modulated by the user’s signature waveform  $S_k(t)$  which consists of  $N$  (the processing gain) chips of duration  $T_c = T/N$ . A symbol synchronous system is assumed, meaning that all users transmit their signals with reference to a common clock. The received signal constellation embedded in AWGN at the input of the receiver looks like

$$r(t) = \sum_{k=1}^K A_k \sum_{i=1}^L b_k(i) S_k(t - iT) + n(t), \quad (4.1)$$

where

$K$  : The number of users sharing the channel

$A_k$  : The received signal amplitude of  $k$ th user

$b_k \in [-1, +1]$  : The  $k$ th user's transmitted bit

$S_k(t) \in [0, T]$  :  $k$ th user's normalized pseudo – random signature waveform

$n(t)$  : AWGN with variance  $\sigma^2 = \frac{N_0}{2}$

$L$  : The number of information bits that each user transmits.

The received signal is passed through a bank of matched filters and the output is sampled at the symbol rate. As mentioned in Equation (3.8) of the Section 3.2, the output of the MFs at each time interval can be written as follows

$$\mathbf{y} = \mathbf{R}\mathbf{A}\mathbf{b} + \mathbf{n}, \quad E[\mathbf{n}\mathbf{n}^T] = \sigma^2 \mathbf{R}. \quad (4.2)$$

In the two following subsections, two different types of channel encoder/decoder utilized in this thesis are introduced.

#### 4.1.2 Convolutional Channel Encoder/Decoder

The first encoder/decoder studied, is a simple convolutional encoder/decoder with a rate  $R_k = \frac{1}{2}$ , constraint length 7 with generators [1011011], [1111001]. The generator polynomials based on the mentioned generators are  $g_0(D) = 1 + D^2 + D^3 + D^5 + D^6$  and  $g_1(D) = 1 + D + D^2 + D^3 + D^6$ . Figure 4.2 shows a realization of the convolutional encoder with the above-mentioned polynomials.



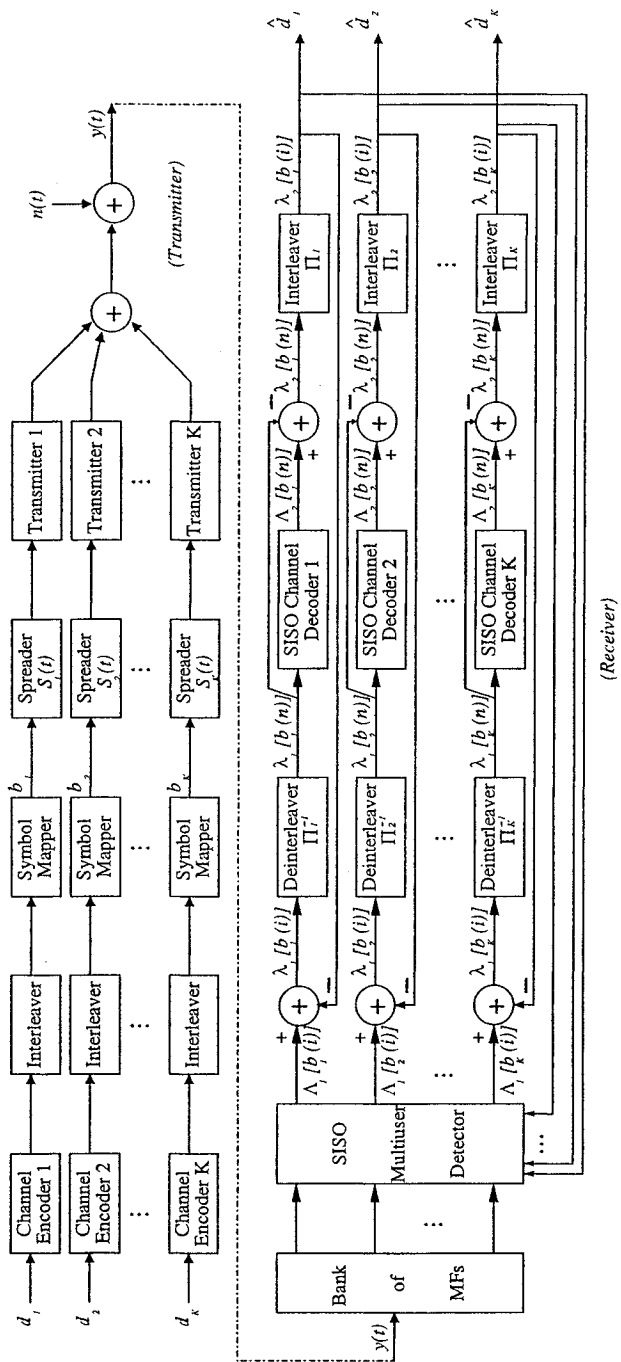


Figure 4.1: A general coded CDMA transmitter/receiver block diagram. The receiver is implemented in an iterative (turbo) fashion.

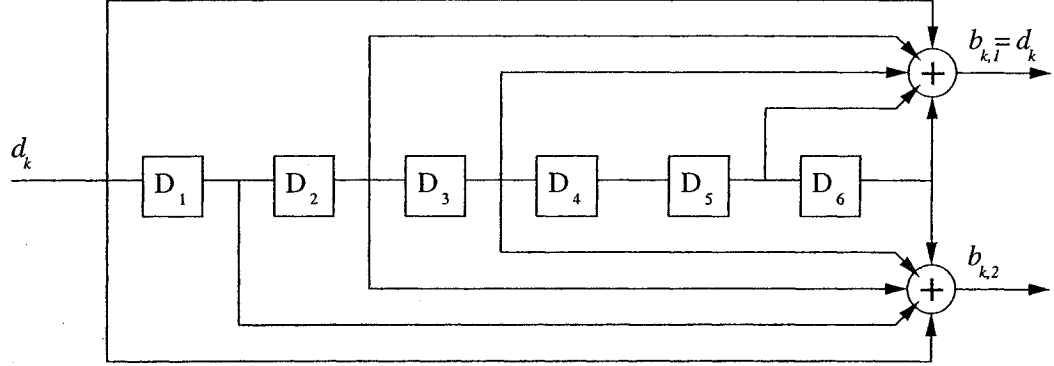


Figure 4.2: Realization of the convolutional encoder for the rate  $R_k = \frac{1}{2}$  code with memory 6.

At the receiver end, the input to the  $K$  single SISO channel decoders are the *a priori* LLRs of the information and parity bits. The decoders update the LLRs of the received bits for either employing them in the next iteration or making decision, using the full-complexity BCJR algorithm [30]. A brief review of decoding procedure along with the formulization are presented [18]. We will denote the information and parity bits that cause state transition from  $S_{t-1} = s'$  to  $S_t = s$  by  $\mathbf{b}(s', s)$  where the overall number of states is  $2^{v-1}$ . The forward and backward recursions can be defined as

$$\alpha_t(s) = \sum_{s'} \alpha_{t-1}(s') P[\mathbf{b}_t = \mathbf{b}(s', s)], \quad t = 1, 2, \dots, \tau \quad (4.3)$$

$$\beta_t(s) = \sum_{s'} \beta_{t+1}(s') P[\mathbf{b}_{t+1} = \mathbf{b}(s, s')], \quad t = \tau - 1, \tau - 2, \dots, 0 \quad (4.4)$$

where

$$P[\mathbf{b}_t(s', s)] = P[b_t^1(s', s)] P[b_t^2(s', s)] \cdots P[b_t^i(s', s)], \quad (4.5)$$

and

$$P [b_t^i(s', s)] = \frac{1}{2} \left[ 1 + b^i(s', s) \tanh \left( \frac{1}{2} \lambda_1 [b_t^i] \right) \right]. \quad (4.6)$$

In Equations (4.3) and (4.4),  $\tau = T_L + v$ , where  $T_L$  is the length of the uncoded data packet that each user transmits and  $v$  (constraint length) is the number of zeros that are fed into the encoder to end up in state zero ( $S_\tau = 0$ ). There are some boundary conditions that need to be stated as following

$$\begin{aligned} \alpha_0(s = 0) &= 1, & \alpha_0(s \neq 0) &= 0, \\ \beta_\tau(s = 0) &= 1, & \beta_\tau(s \neq 0) &= 0. \end{aligned} \quad (4.7)$$

Let  $S_j^+$  be the set of transitional pair  $(s', s)$  in which the  $j$ th bit of code symbol is  $+1$ . Similarly,  $S_j^-$  is defined such that the  $j$ th bit of code symbol is  $-1$ . The *a posteriori* LLR of each code bit at the output of the channel decoder can be derived as follows (see [18])

$$\begin{aligned} \Lambda_2[b_t^j] &\equiv \log \frac{P [b_t^j = +1 \mid \text{decoding}]}{P [b_t^j = -1 \mid \text{decoding}]} \\ &= \log \frac{\sum_{S_j^+} \alpha_{t-1}(s') \cdot \beta_t(s) \cdot \prod_{i=1}^{n_0} P [b_t^i(s', s)]}{\sum_{S_j^-} \alpha_{t-1}(s') \cdot \beta_t(s) \cdot \prod_{i=1}^{n_0} P [b_t^i(s', s)]} \\ &= \log \frac{\sum_{S_j^+} \alpha_{t-1}(s') \cdot \beta_t(s) \cdot \prod_{i \neq j} P [b_t^i(s', s)]}{\sum_{S_j^-} \alpha_{t-1}(s') \cdot \beta_t(s) \cdot \prod_{i \neq j} P [b_t^i(s', s)]} \\ &\quad \underbrace{\hspace{10em}}_{\lambda_2[b_t^j]} \\ &\quad + \log \underbrace{\frac{P [b_t^j = +1]}{P [b_t^j = -1]}}_{\lambda_1[b_t^j]}, \end{aligned} \quad (4.8)$$

where  $\lambda_2[b_t^j]$  is the extrinsic information produced by the SISO channel decoders and will be delivered to the MUD as *a priori* information for the next iteration.  $\lambda_1[b_t^j]$  is the extrinsic information provided by the SISO MUD which is employed as *a priori* information by SISO channel decoders. The LLRs of the information bits ( $\Lambda_2[d_t^j]$ ) are only calculated at the last iteration and the information bits will be decoded according to  $\widehat{d}_t^j = \text{sgn}(\Lambda_2[d_t^j])$ .

### 4.1.3 Turbo Channel Encoder/Decoder

The turbo channel encoder utilized in some of the simulations has a rate  $R_k = \frac{1}{2}$ , with two 4-state (constraint length  $v = 3$ , and octal generators 7 and 5). This encoder uses two Recursive Systematic Convolutional (RSC) codes. In Figure 4.3, a realization of the systematic convolutional encoder with feedback, memory 2, and generator polynomials  $g_0(D) = 1 + D + D^2$  and  $g_1(D) = 1 + D^2$  is depicted. The interleaver employed in between the two convolutional encoders, is a random interleaver of size  $L = 210$ . The parity part of the encoded information is punctured to achieve the desired rate, (i.e.  $R_k = \frac{1}{2}$ ).

At the receiver end, the decoder is based on the concatenation of two Soft-In/Soft-Out (SISO) decoders. The inputs and outputs of such a decoder are shown in Figure 4.4.

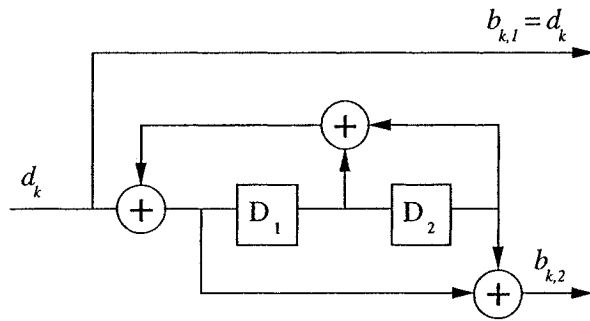


Figure 4.3: Realization of a systematic convolutional encoder with feedback for the rate  $R_k = \frac{1}{2}$  code.

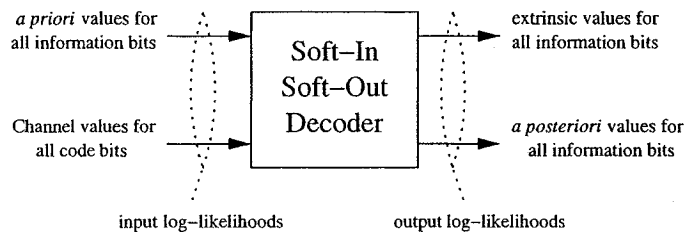


Figure 4.4: Soft In/Soft Out decoder.

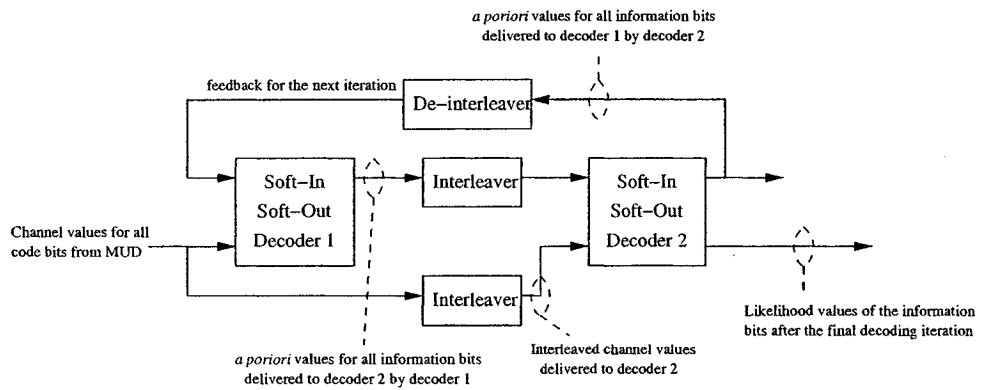


Figure 4.5: Iterative decoding procedure with two “Soft In/Soft Out” decoders for turbo-coded information.

As mentioned in Sub-section 4.1.2, a full-complexity SISO BCJR algorithm is engaged to decode the information bits in each one of the decoders [11], [12]. Decoder 1 uses channel values and *a priori* values as inputs to produce extrinsic values for information bits. Then, decoder 2 uses the extrinsic values produced by decoder 1 as *a priori* information to calculate extrinsic and *a posteriori* values for all information bits. The extrinsic values produced by decoder 2 can be utilized as *a priori* information by decoder 1 in the next iteration of decoding. Regardless of the number of iterations that the whole detecting/decoding procedure has to perform, the turbo channel decoder carries out numbers of iteration individually, between the two built-in SISO decoders to provide the extrinsic information for the next iteration of the MUD. At the final iteration of decoding, the *a posteriori* information produced by the second decoder can be employed to decode the information bits. A general turbo-code decoder diagram is presented in Figure 4.5.

## 4.2 Iterative (Turbo) PPIC Derivation

### 4.2.1 Introduction to Partial PIC

Divsalar *et al* [9], showed that in PIC, entire cancellation of MAI that affects each user at each stage is not necessarily the best approach. It has been shown that the performance improves slowly as the number of stages increases. Instead, Divsalar proved that partial cancellation of MAI at each stage improves the performance dramatically. Since in the early stages of interference cancellation, the tentative decisions are less reliable than they are in later stages, a better philosophy is that in early stages just a fraction of the interference be cancelled and as the iterations continue to the final data decisions, the cancellation factor be increased. This type of detector is called

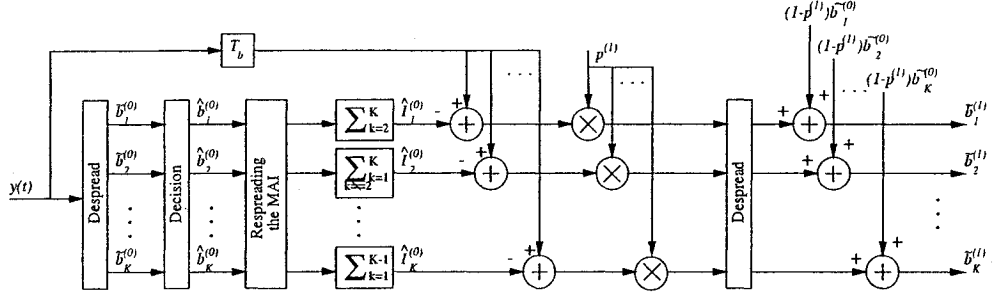


Figure 4.6: PPIC detector block diagram.

*Partial Parallel Interference Cancellation (PPIC)*. In this technique, the output of the  $m$ th stage is based on the weighted sum of the output of the  $(m - 1)$ st stage ( $\tilde{d}_k^{(m-1)}$ ) and the interference cancelled version of the MF's output at the  $(m - 1)$ st stage [9]. Therefore, the output of the  $m$ th stage of PPIC for the  $k$ th user will be [9]

$$\tilde{b}_k^{(m)} = p^{(m)} [\tilde{b}_k^{(0)} - \hat{I}_k^{(m-1)}] + (1 - p^{(m)}) \tilde{b}_k^{(m-1)}, \quad (4.9)$$

where

- $p^m$  : partial cancellation coefficient at the  $m$ th stage
- $\tilde{b}_k^{(0)}$  : MF's output
- $\hat{I}_k^{(m-1)}$  : estimated interference affecting  $k$ th user at the  $m$ th stage
- $\tilde{b}_k^{(m-1)}$  : output of the PPIC at  $(m - 1)$ th stage

In figure 4.6 a general block diagram of the PPIC technique is depicted.

#### 4.2.2 Iterative (Turbo) PPIC formula derivation

The PPIC technique can be applied in a turbo multiuser detector [36]. In order to employ this technique, an alternative likelihood calculation must be done. The  $a$

*posteriori* log-likelihood calculation of  $b_k$  is needed, based on observing  $y_k$ ,  $\hat{I}_k^{(m)}$ , and  $\tilde{b}_k^{(m-1)}$  which are:

- $y_k$  : the output of the matched filter of user  $k$
- $\tilde{b}_k^{(m-1)}$  : the output of PPIC at the  $(m - 1)$ th stage
- $\hat{I}_k^{(m)}$  : the estimated MAI affecting the  $k$ th user at the end of the  $(m - 1)$ st stage  

$$\left( = \sum_{j=1, j \neq k}^K A_j \hat{b}_j^{(m-1)} \rho_{jk} \right).$$

In the following, derivation of  $p(b_k | y_k, \hat{I}_k^{(m)}, \tilde{b}_k^{(m-1)})$  is explained in details.

Without loss of generality, it is assumed that the extrinsic information of the  $k$ th user's signal is desired. For the sake of simplicity, the index number of the bit is omitted. Therefore, the output of the  $k$ th user's MF at a specific time interval will look like

$$\begin{aligned} y_k &= A_k b_k + \sum_{j=1, j \neq k}^K A_j b_j \rho_{jk} + n_k \\ &= A_k b_k + I_k + n_k, \end{aligned} \tag{4.10}$$

where  $I_k$  is the total interference affecting the  $k$ th user's bit. Using the outputs of the interference canceler at the  $(m - 1)$ st iteration, the PPIC MUD estimates the interference experienced by user  $k$  at  $m$ th iteration ( $\hat{I}_k^{(m)}$ ). Therefore, Equation (4.10) can be rewritten as



$$\begin{aligned}
y_k &\equiv A_k b_k + \hat{I}_k^{(m)} + W_k^{(m)}; \\
W_k^{(m)} &= I_k - \hat{I}_k^{(m)} + n_k,
\end{aligned} \tag{4.11}$$

where  $I_k - \hat{I}_k^{(m)}$  denotes the real residual (uncanceled) interference that affects the  $k$ th user's data at the end of  $m$ th iteration. For the purpose of what follows, the residual interference can be modeled as a Gaussian Random Variable (RV). The combination of residual interference and  $n_k$ , results in a zero-mean Gaussian RV  $W_k^{(m)}$  with variance  $E \{W_k^{2(m)}\} = \sigma_m^2$ . It is worth to mention that  $\sigma_m^2$  depends on the iteration stage  $m$ . For the sake of formula derivation we shall assume that it is possible to approximate  $\tilde{b}_k^{(m-1)}$  from previous stage, which is a conditional Gaussian RV (based on  $b_k$ ) as following

$$\tilde{b}_k^{(m-1)} = A_k b_k + W_k^{(m-1)}, \tag{4.12}$$

where,

$$\begin{aligned}
E \{W_k^{(m-1)}\} &= 0, \\
E \{W_k^{2(m-1)}\} &= \sigma_{m-1}^2.
\end{aligned} \tag{4.13}$$

Since both  $W_k^{(m)}$  and  $W_k^{(m-1)}$  contain the thermal noise  $n_k$ , they are correlated and the correlation can be defined as  $E \{W_k^{(m)} W_k^{(m-1)}\} \equiv \rho_m \sigma_m \sigma_{m-1}$  [37]. It will be mentioned soon that it is not necessary to be able to approximate  $\sigma_m$ ,  $\sigma_{m-1}$ , and

$\rho_m$ . Instead, a specific combination of these parameters will define  $p^{(m)}$  which specifies the amount of interference being cancelled at each stage.

Now, we look for recursively defining  $\tilde{b}_k^{(m)}$  as a LLR for  $b_k$ , based on observing  $\tilde{b}_k^{(m-1)}$ ,  $y_k$ , and  $\tilde{I}_k^{(m)}$ . According to [37], it will be advantageous if the conditional average of the LLR is normalized to  $A_k b_k$ . From Equation (4.12) and (4.13), the joint conditional probability density function (pdf) of  $y_k$  and  $\tilde{b}_k^{(m-1)}$ , given  $b_k$  and  $\tilde{I}_k^{(m)}$  can expressed as ([37])

$$p \left[ y_k, \tilde{b}_k^{(m-1)} \mid b_k, \tilde{I}_k^{(m)} \right] = \frac{1}{2\pi\sigma_m\sigma_{m-1}\sqrt{1-\rho_m^2}} \cdot \exp \left( - \left\{ \frac{\sigma_{m-1}^2 \left[ y_k - A_k b_k - \tilde{I}_k^{(m)} \right]^2 + \sigma_m^2 \left[ \tilde{b}_k^{(m-1)} - A_k b_k \right]^2}{2\sigma_m^2 \sigma_{m-1}^2 (1-\rho_m^2)} - \frac{2\rho_m \sigma_m \sigma_{m-1} \left[ y_k - A_k b_k - \tilde{I}_k^{(m)} \right] \left[ \tilde{b}_k^{(m-1)} - A_k b_k \right]}{2\sigma_m^2 \sigma_{m-1}^2 (1-\rho_m^2)} \right\} \right), \quad (4.14)$$

which under simplifications, can be rewritten as

$$p \left[ y_k, \tilde{b}_k^{(m-1)} \mid b_k, \tilde{I}_k^{(m)} \right] = C \exp \left( A_k b_k \left\{ \frac{(\sigma_{m-1}^2 - \rho_m \sigma_m \sigma_{m-1}) \left[ y_k - \tilde{I}_k^{(m)} \right]}{\sigma_m^2 \sigma_{m-1}^2 (1 - \rho_m^2)} + \frac{(\sigma_m^2 - \rho_m \sigma_m \sigma_{m-1}) \tilde{b}_k^{(m-1)}}{\sigma_m^2 \sigma_{m-1}^2 (1 - \rho_m^2)} \right\} \right), \quad (4.15)$$

where the constant  $C$  is independent of  $b_k$ . At this point, a normalization factor is introduced to make the coefficients of  $\left[ y_k - \tilde{I}_k^{(m)} \right]$ , and  $\tilde{b}_k^{(m-1)}$  to unity [37]. This factor is recommended as

$$\begin{aligned}
p^{(m)} &\equiv \frac{\sigma_{m-1}^2 - \rho_m \sigma_m \sigma_{m-1}}{\sigma_m^2 + \sigma_{m-1}^2 - 2\rho_m \sigma_m \sigma_{m-1}}, \\
1 - p^{(m)} &= \frac{\sigma_m^2 - \rho_m \sigma_m \sigma_{m-1}}{\sigma_m^2 + \sigma_{m-1}^2 - 2\rho_m \sigma_m \sigma_{m-1}}.
\end{aligned} \tag{4.16}$$

$p^{(m)}$  is called the partial interference cancellation factor which is the amount of interference canceled at each iteration. Intuitively,  $p^{(m)}$  lies between 0 and 1. As the reliability of the information improves, this factor may be increased.

The *a posteriori* log-likelihood ratio (LLR) of  $b_k$  conditioned on observing  $y_k, \hat{I}_k^{(m)}$ , and  $\tilde{b}_k^{(m-1)}$  can be evaluated by

$$\Lambda_1^{(m)} [b_k] = \log \frac{p(b_k = +1 | y_k, \hat{I}_k^{(m)}, \tilde{b}_k^{(m-1)})}{p(b_k = -1 | y_k, \hat{I}_k^{(m)}, \tilde{b}_k^{(m-1)})}. \tag{4.17}$$

By multiplying both the numerator and denominator of Equation (4.17) by  $p(y_k, \hat{I}_k^{(m)}, \tilde{b}_k^{(m-1)})$ , and employing the Bayes' rule, (4.17) can be rewritten as

$$\begin{aligned}
\Lambda_1^{(m)} [b_k] &= \log \frac{p(b_k = +1 | y_k, \hat{I}_k^{(m)}, \tilde{b}_k^{(m-1)}) \cdot p(y_k, \hat{I}_k^{(m)}, \tilde{b}_k^{(m-1)})}{p(b_k = -1 | y_k, \hat{I}_k^{(m)}, \tilde{b}_k^{(m-1)}) \cdot p(y_k, \hat{I}_k^{(m)}, \tilde{b}_k^{(m-1)})} \\
&= \log \frac{p(b_k = +1, y_k, \hat{I}_k^{(m)}, \tilde{b}_k^{(m-1)})}{p(b_k = -1, y_k, \hat{I}_k^{(m)}, \tilde{b}_k^{(m-1)})} \\
&= \underbrace{\log \frac{P(b_k = +1)}{P(b_k = -1)}}_{\lambda_{2,in}^{(m)} [b_k]} + \underbrace{\log \frac{p(y_k, \hat{I}_k^{(m)}, \tilde{b}_k^{(m-1)} | b_k = +1)}{p(y_k, \hat{I}_k^{(m)}, \tilde{b}_k^{(m-1)} | b_k = -1)}}_{\lambda_{1,ex}^{(m)} [b_k]}. \tag{4.18}
\end{aligned}$$

The first term of Equation (4.18)  $\lambda_{2,in}^{(m)} [b_k]$ , implies the *a priori* (intrinsic) information available to the  $m$ th stage of the interference cancellation. The second term of  $\lambda_{1,ex}^{(m)} [b_k]$ , is the extrinsic information delivered by the  $m$ th stage of the

interference cancellation. The extrinsic information produced by the MUD at  $m$ th stage, along with the other extrinsic information that comes from the single-user decoders, are employed as *a priori* values for the  $(m + 1)$ th stage of the interference cancellation ( $\lambda_{2,in}^{(m+1)}[b_k]$ ). In the following, derivation of extrinsic information ( $\lambda_{1,ex}^{(m)}[b_k]$ ) from channel values is explained. Using the Bayes' rule,  $\lambda_{1,ex}^{(m)}[b_k]$  can be expressed as

$$\begin{aligned}\lambda_{1,ex}^{(m)}[b_k] &= \log \frac{p(y_k, \tilde{I}_k^{(m)}, \tilde{b}_k^{(m-1)} | b_k = +1)}{p(y_k, \tilde{I}_k^{(m)}, \tilde{b}_k^{(m-1)} | b_k = -1)} \\ &= \log \frac{p(y_k, \tilde{b}_k^{(m-1)} | b_k = +1, \tilde{I}_k^{(m)}) p(\tilde{I}_k^{(m)} | b_k = +1)}{p(y_k, \tilde{b}_k^{(m-1)} | b_k = -1, \tilde{I}_k^{(m)}) p(\tilde{I}_k^{(m)} | b_k = -1)}.\end{aligned}\quad (4.19)$$

Since ideally the interference affecting the  $k$ th user ( $\tilde{I}_k^{(m)}$ ) is independent of  $b_k$ , then it can be assumed that  $p(\tilde{I}_k^{(m)} | b_k) \simeq p(\tilde{I}_k^{(m)})$ . Therefore,  $p(\tilde{I}_k^{(m)})$  can be omitted from both the numerator and the denominator of Equation (4.19). Consequently, using Equations (4.15) and (4.19), the extrinsic information can be rewritten as

$$\begin{aligned}\lambda_{1,ex}^{(m)}[b_k] &= \log \frac{p(y_k, \tilde{b}_k^{(m-1)} | b_k = +1, \tilde{I}_k^{(m)})}{p(y_k, \tilde{b}_k^{(m-1)} | b_k = -1, \tilde{I}_k^{(m)})} \\ &= 2A_k \left\{ \frac{(\sigma_{m-1}^2 - \rho_m \sigma_m \sigma_{m-1}) [y_k - \tilde{I}_k^{(m)}]}{\sigma_m^2 \sigma_{m-1}^2 (1 - \rho_m^2)} \right. \\ &\quad \left. + \frac{(\sigma_m^2 - \rho_m \sigma_m \sigma_{m-1}) \tilde{b}_k^{(m-1)}}{\sigma_m^2 \sigma_{m-1}^2 (1 - \rho_m^2)} \right\}.\end{aligned}\quad (4.20)$$

From Equations (4.11) and (4.13), with just a slight loss of performance in exchange of complexity, it can be approximated that  $\sigma_m^2 \approx \sigma_{m-1}^2$ . Since  $\rho_m < 1$ , an

assumption is used that is  $\frac{1-\rho_m}{1-\rho_m^2} \approx 1$ . Using the mentioned suppositions, Equation (4.20) can be approximated as

$$\lambda_{1,ex}^{(m)}[b_k] \approx \frac{2A_k [y_k - \tilde{I}_k^{(m)}]}{\sigma_m^2} + \frac{2A_k \tilde{b}_k^{(m-1)}}{\sigma_{m-1}^2}. \quad (4.21)$$

Substituting Equation (4.16) in (4.21), the extrinsic information can be expressed as

$$\lambda_{1,ex}^{(m)}[b_k] \approx 2A_k \frac{\tilde{b}_k^{(m)}}{\sigma_m^2 p^{(m)}}, \quad (4.22)$$

where from (4.9)

$$\tilde{b}_k^{(m)} = p^{(m)} [y_k - \tilde{I}_k^{(m-1)}] + (1 - p^{(m)}) \tilde{b}_k^{(m-1)}. \quad (4.23)$$

As a counterpart to [36], instead of estimating  $p^{(m)}$  at each stage, in order to reduce the complexity,  $p^{(m)}$  can be pre-assigned which results in a sub-optimal method. Furthermore,  $\sigma_m^2$  can be estimated through various complex methods [38]. For the sake of simplicity,  $\sigma_m^2$  can be approximated by the variance of the ambient AWGN ( $\sigma^2$ ) which affects the channel, with a very little loss of performance, in calculating Equation (4.22) [36]. Subsequently, the process of modified and improved estimation of the LLR of  $b_k$ , at the  $m$ th stage of partial interference cancellation, can be addressed as follows [36]:

1. Interference to be cancelled as stage  $m$  is approximated from the soft estimations of the previous iteration. At this stage a modification has been applied that in order to avoid loss of data, soft-decision is utilized instead of hard-decision, to

calculate the log-likelihood ratios from previous iteration. This soft-decision is defined in step 5. Interference calculation can be performed as

$$\tilde{I}_k^{(m-1)} = \sum_{j=1, j \neq k}^K A_j \tilde{b}_j^{(m-1)} \rho_{jk} \quad (4.24)$$

2. The soft estimation of  $b_k$  at the  $m$ th stage is achieved according to Equation (4.23) as

$$\tilde{b}_k^{(m)} = p^{(m)} [y_k - \tilde{I}_k^{(m-1)}] + (1 - p^{(m)}) \tilde{b}_k^{(m-1)} \quad (4.25)$$

3.  $\lambda_{2,in}^{(m)}[b_k]$  is calculated based on extrinsic information from the previous iteration of interference cancellation ( $\lambda_{1,ex}^{(m-1)}[b_k]$ ) and  $\lambda_{k,decoder} = \lambda_2[b_k]$  (Figure 4.1) which is the *a priori* information delivered by the single-user decoder as

$$\lambda_{2,in}^{(m)}[b_k] = \lambda_{1,ex}^{(m-1)}[b_k] + \lambda_{k,decoder} \quad (4.26)$$

4. Extrinsic information is calculated at stage  $m$  based on Equation (4.21)

$$\lambda_{1,ex}^{(m)}[b_k] = 2A_k \frac{\tilde{b}_k^{(m)}}{\sigma_{mD}^2 p^{(m)}} \quad (4.27)$$

5. Soft estimation of  $\tilde{b}_k^{(m)}$  is performed as following

$$\tilde{b}_k^{(m)} = \tanh \left( \frac{\lambda_{PPIC}^E [b_k]}{2} \right) = \tanh \left\{ \frac{(\lambda_{2,in}^{(m)}[b_k] + \lambda_{1,ex}^{(m)}[b_k])}{2} \right\}. \quad (4.28)$$

The simulation results demonstrating the performance of the new PPIC detector are presented. In figures 4.7 and 4.8, performance results of a turbo PPIC detector, using a turbo channel encoders (as mentioned in Subsection 4.1.3) are depicted. In

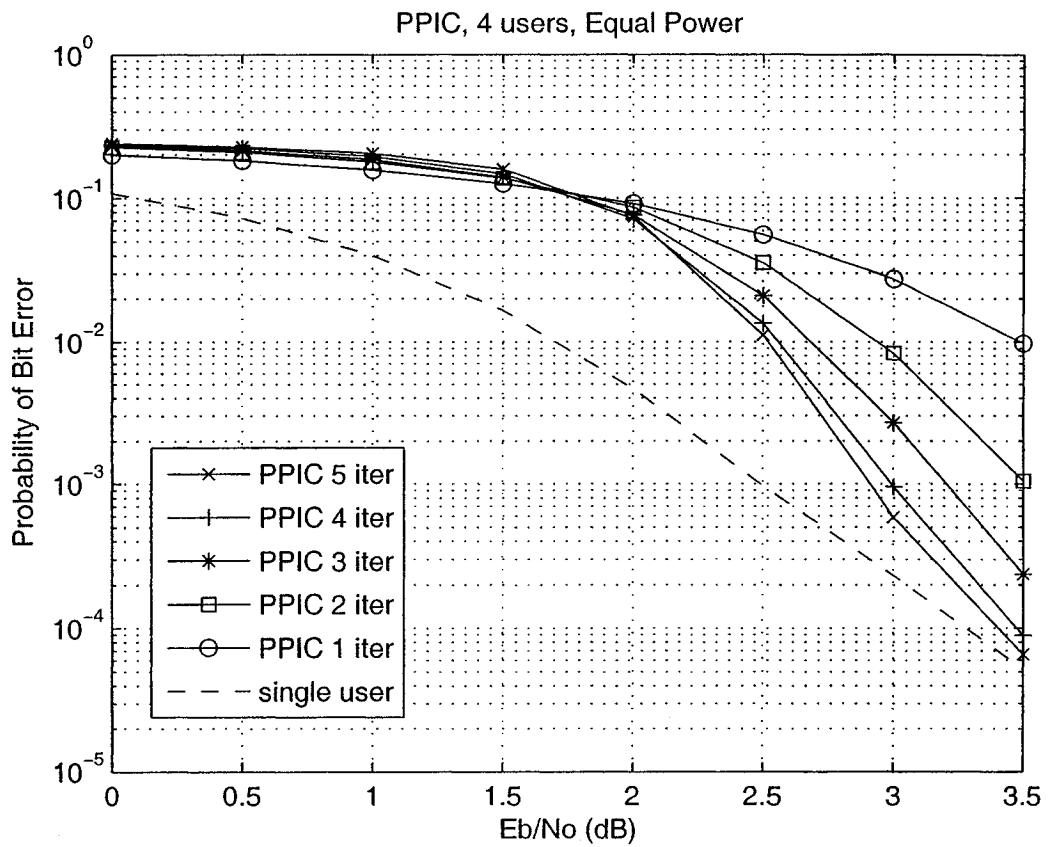


Figure 4.7: Probability of bit error performance of the modified turbo PPIC detector as a function of the number of iterations, with cross-correlation  $\rho = 0.5$ , 4 equal power users, for different number of iterations.

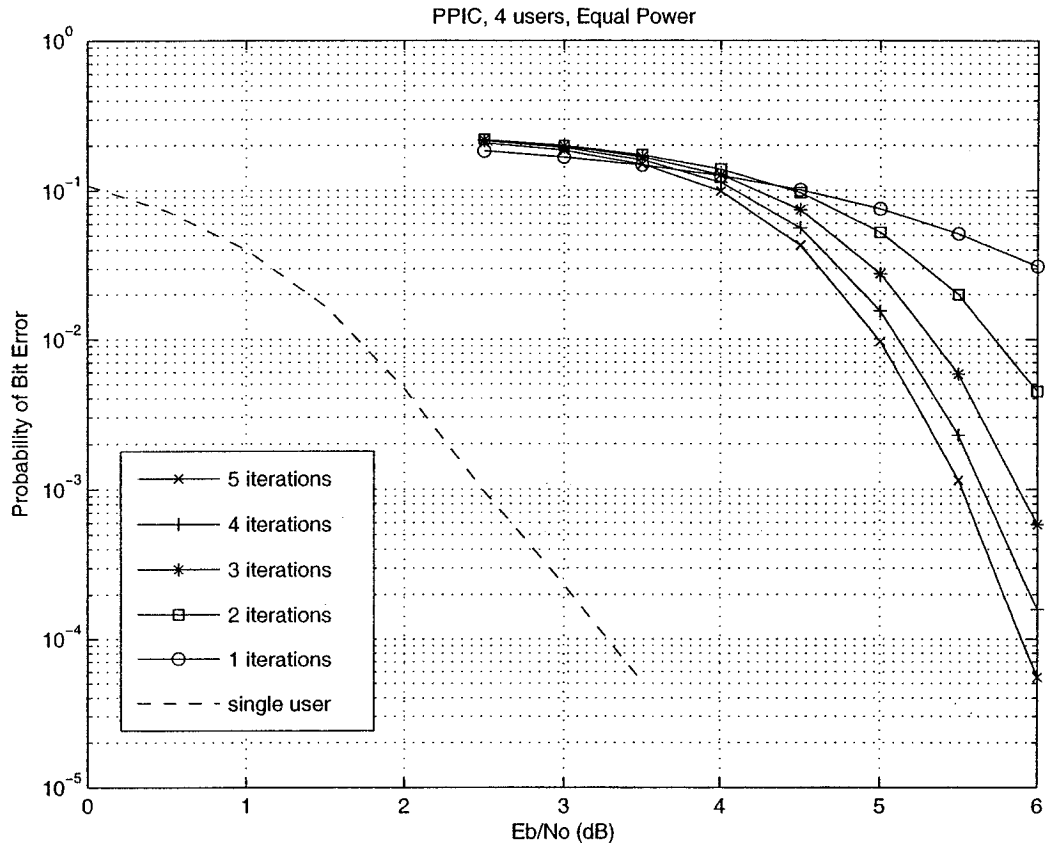


Figure 4.8: Probability of bit error performance of the modified turbo PPIC detector as a function of the number of iterations, with cross-correlation  $\rho = 0.7$ , 4 equal power users, for different number of iterations.



both of the mentioned graphs, the users employ different random interleavers. The Turbo code decoder has performed two iterations to decode and provide the extrinsic information for the next iteration of MUD. The length of the information bits sent by each user is  $L = 210$  at each transmission. The simulation is run for 4 detecting/decoding procedure iterations. For each complete iteration,  $m = 2$  iterations has been performed for the PPIC detector. Therefore, two partial interference cancellation coefficients are associated with each iteration. The chosen cancellation weight set  $[\rho^{(1)} \rho^{(2)}]$  in these simulations are assigned as  $[0.5 \ 0.55]$  for the first iteration,  $[0.6 \ 0.65]$  for the second iteration,  $[0.7 \ 0.75]$  for the third iteration, and  $[0.8 \ 0.85]$  for the final iteration. The simulation results show that for medium cross-correlation ( $\rho = 0.5$ ), after 3 detecting/decoding iterations the bit error probability gets close to that of the single-user system for medium to high SNR. In the systems that suffer from high cross-correlation ( $\rho = 0.7$ ), after 5 detecting/decoding iterations the performance is still more than 2.5 dB away from that of single-user performance. On the other hand, complexity of the PPIC system is linear in terms of the number of users ( $O(K)$ ). Later on, the simulation results in Figures 4.7 and 4.8 will be compared with that of our proposed algorithm.

### 4.3 Improved Iterative Decorrelating Detector

Recently, a new decorrelating method has been introduced and applied to the output of IC-MUD techniques [35]. The output of a traditional IC detector still suffers from residual interference which can be modeled by a new equivalent cross-correlating matrix  $\mathbf{R}_u$ . Therefore, this output can be expressed as

$$\mathbf{z} = \mathbf{y} - (\mathbf{R} - \mathbf{I})\mathbf{A}\hat{\mathbf{b}} = \mathbf{R}_u\mathbf{A}\mathbf{b} + \mathbf{n}, \quad (4.29)$$

where  $\mathbf{R}_u = \mathbf{R} \cdot \mathbf{C} + \mathbf{D}$ , in which  $\mathbf{C}$  and  $\mathbf{D}$  are both diagonal matrices as following

$$\mathbf{C} = \begin{bmatrix} \frac{b_1 - \hat{b}_1}{b_1} & & & \\ & \frac{b_2 - \hat{b}_2}{b_2} & & \\ & & \ddots & \\ & & & \frac{b_k - \hat{b}_k}{b_k} \end{bmatrix}_{K \times K}, \text{ and } \mathbf{D} = \begin{bmatrix} \frac{\hat{b}_1}{b_1} & & & \\ & \frac{\hat{b}_2}{b_2} & & \\ & & \ddots & \\ & & & \frac{\hat{b}_k}{b_k} \end{bmatrix}_{K \times K}. \quad (4.30)$$

Since in BPSK modulated systems  $b_k \in \{-1, +1\}$ , If the hard-decision based on  $\hat{b}_k$  is correct, then  $b_k = \text{sgn}(\hat{b}_k)$  and we have

$$\hat{b}_k / b_k = \hat{b}_k \cdot b_k = |\hat{b}_k|, \quad k = 1, 2, \dots, K. \quad (4.31)$$

Based on the estimated  $\mathbf{R}_u$ , the residual interference at the output of the IC multiuser detector can be reduced by decorrelating  $\mathbf{z}$  by  $\mathbf{R}_u^{-1}$ . Therefore, decorrelating Equation (4.29) by  $\mathbf{R}_u^{-1}$  will result in

$$\mathbf{x} = \mathbf{R}_u^{-1} \cdot \mathbf{z} = \mathbf{A}\mathbf{b} + \mathbf{R}_u^{-1}\mathbf{n} = \mathbf{A}\mathbf{b} + \mathbf{n}_u, \quad (4.32)$$

where noise vector  $\mathbf{n}_u$  is still a Gaussian RV vector with covariance matrix

$$E[\mathbf{n}_u \mathbf{n}_u^T] = E[(\mathbf{R}_u^{-1}\mathbf{n})(\mathbf{R}_u^{-1}\mathbf{n})^T] = \sigma^2 \mathbf{R}_u^{-1} \mathbf{R} (\mathbf{R}_u^{-1})^T. \quad (4.33)$$

The noise variable  $n_{u,k}$  is a Gaussian variable with distribution  $N(0, \sigma_{u,k}^2)$ , in which  $\sigma_{u,k}$  is the  $k$ th diagonal element of the covariance matrix expressed in Equation (4.33). Therefore, the LLR of the detected bit at the output of the decorrelator is

$$\lambda_{1,ex}[b_k] = \log \frac{P[x_k | b_k = +1]}{P[x_k | b_k = -1]} = \frac{2A_k x_k}{\sigma_{u,k}^2}, \quad k = 1, \dots, K. \quad (4.34)$$

In the Figures 4.9 and 4.10, performance results of the soft-IC detector followed by the introduced decorrelating detector are depicted and compared with that of simple soft-IC detector. In these simulations, each user applies a rate  $R_k = \frac{1}{2}$  convolutional encoder with a constraint length of  $\nu = 7$  and generators [1011011] and [1111001]. The single-user decoders apply full complexity BCJR algorithm [30]. Each user employs a different random interleaver of length  $L = 210$  and the number of information bits simulated to achieve each point is 10 million bits.

## 4.4 Concatenation of the Modified PPIC and the New Decorrelating Detector

In this section, a concatenation of the PPIC detector and the new decorrelating detector is proposed. In this method, PPIC uses the *a priori* information delivered by single-user decoders ( $\lambda_{k,other}^{(m)}[b_k]$ ) and the received signal from the channel. Then, after  $m$  iterations of partial parallel interference cancellation, PPIC detector delivers soft interference cancelled signals ( $\tilde{b}_k^{(m)}$ ) to the decorrelating detector. Besides  $\tilde{b}_k^{(m)}$ , the PPIC detector also provides *a priori* information as follows

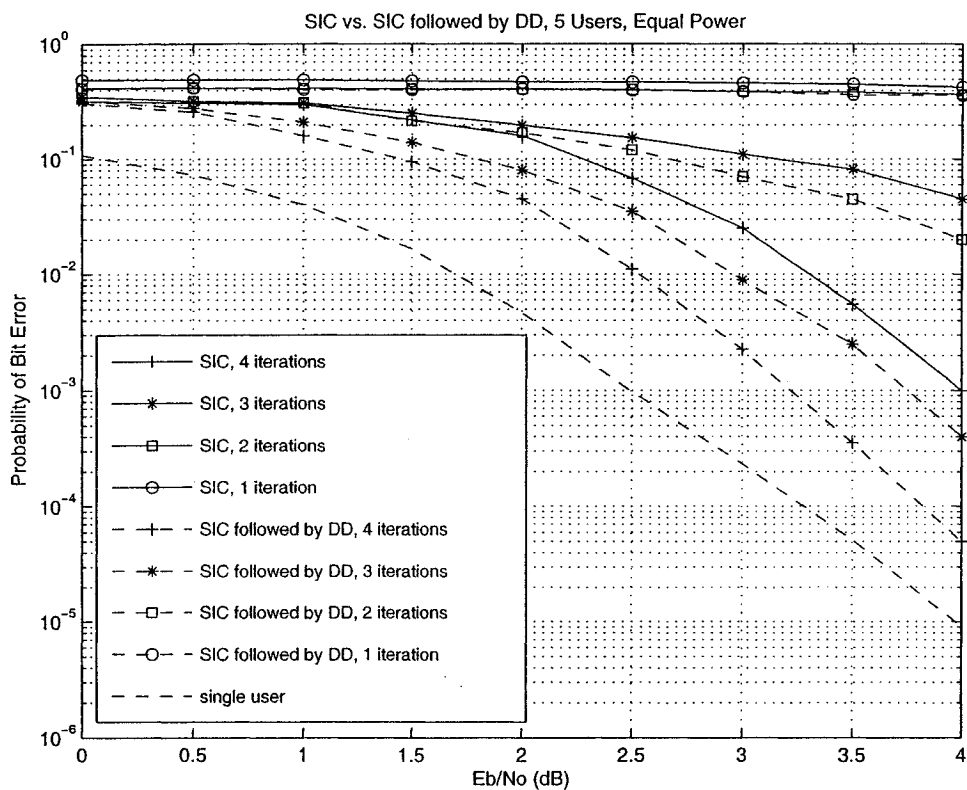


Figure 4.9: Performance comparison between soft interference cancellation (SIC) and SIC followed by the newly proposed decorrelator for  $K = 5$  equal power users, cross-correlation  $\rho = 0.5$ , and different number of iterations.

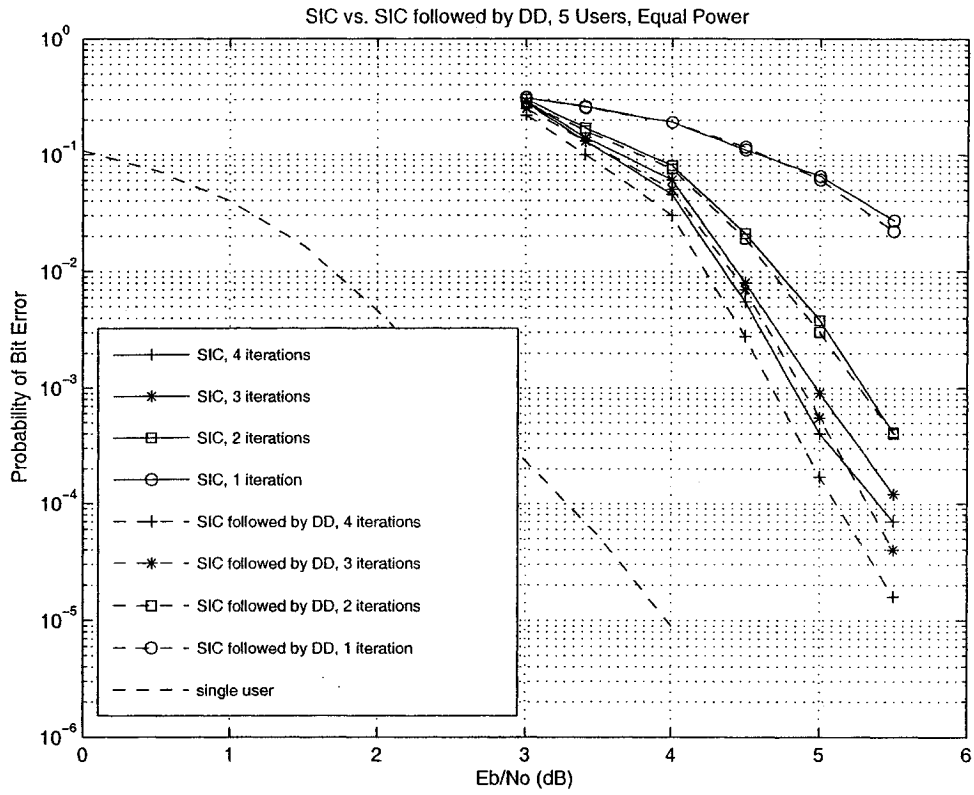


Figure 4.10: Performance comparison between soft interference cancellation (SIC) and SIC followed by the newly proposed decorrelator for  $K = 5$  equal power users, cross-correlation  $\rho = 0.7$ , and different number of iterations.

$$\begin{aligned}\lambda_{PPIC,ex} &= \lambda_{2,in}^{(m)}[b_k] + \lambda_{1,ex}^{(m)}[b_k] - \lambda_{k,decoder} \\ \Rightarrow \tilde{b}'_{k,PPIC} &= \tanh(\lambda_{PPIC,ex}/2).\end{aligned}\tag{4.35}$$

$\tilde{b}'_{k,PPIC}$  is employed in calculation of matrices  $\mathbf{C}$  and  $\mathbf{D}$  in Equation (4.30). Then,  $\mathbf{R}_u$  can be estimated based on  $\mathbf{C}$ ,  $\mathbf{D}$ , and  $\mathbf{R}$  ( which is defined in Equation (3.8)). As mentioned in Equation (4.32), after  $\mathbf{R}_u^{-1}$  calculation, we can apply the decorrelating process to  $\widetilde{\mathbf{b}}^{(m)}$  which is the output vector of the PPIC detector. Subsequently, by applying Equation (4.34), the extrinsic information to be delivered to the single-user decoders will be calculated by (  $\lambda_1 [b_k (i)]$  in Figure 4.1 )

$$\lambda_{1,ex}[b_k] = \frac{2A_k x_k}{\sigma_{u,k}^2} = \lambda_1 [b_k].\tag{4.36}$$

In the next Section simulation results and the probability of bit error of the mentioned multiuser detector techniques in this chapter will be depicted along with that of the proposed algorithm.

## 4.5 Simulation results

In this section, simulation results of the concatenation of the modified-PPIC and the newly introduced decorrelating detector are shown and compared with that of the other competing methods of “Turbo Multiuser Detection” techniques. The simulations have been performed for two different systems to compare the results with previously mentioned works. BPSK modulation is applied in all the simulations.

In the first system,  $K = 4$  users share an AWGN channel applying turbo channel encoders with rate  $R_k = \frac{1}{2}$ , constraint length  $\nu = 3$ , and generators in octal notations [111 101]. Without loss of generality, a random interleaver is employed between the

two convolutional encoders, in the turbo encoder structure. A power-balanced system is assumed and each user employs a different random interleaver of size  $L = 210$ . At the receiver end, each single-user turbo decoder performs  $N = 2$  number of iterations to decode information bits. The simulation is run for 4 complete iterations for the whole system. For each iteration,  $m = 2$  iterations have been performed for the PPIC detector and the soft outputs of the modified PPIC is fed into the decorrelating detector. Therefore, two partial interference cancellation coefficients are associated with each iteration of PPIC detector. The chosen cancellation weight sets ( $[\rho^{(1)} \rho^{(2)}]$ ) in these simulations are assigned as  $[0.5 \ 0.55]$  for the first iteration,  $[0.6 \ 0.65]$  for the second iteration,  $[0.7 \ 0.75]$  for the third iteration, and finally  $[0.8 \ 0.85]$  for the last iteration. These are just samples of cancellation weights and the simulation is run for more number of iterations of PPIC, but the improvement in price of additional complexity is not eye-catching. In Figure 4.11, performance comparison between turbo PPIC detector and turbo modified-PPIC followed by decorrelating detector is shown. Each point in the graph is achieved by simulating 10 million information bits.

Performance results in Figure 4.11 show that for the users applying turbo channel encoders, for medium SNR, 0.4 dB gain can be achieved after 4 complete system iterations if a decorrelating detector is applied to the soft outputs of PPIC detector for the channels that suffer from medium amount of cross-correlation.

In the second system,  $K = 5$  users share an AWGN channel applying 100 simple convolutional channel encoders with rate  $R_k = \frac{1}{2}$ , constraint length  $\nu = 7$ , and generators  $[1011011]$  and  $[1111001]$ . A power-balanced system is assumed and each user employs a different random interleaver of size  $L = 210$ . At the receiver end, each single-user decoder applies full-complexity BCJR algorithm [30] to decode information bits. Other factors such as number of iterations and cancellation weights

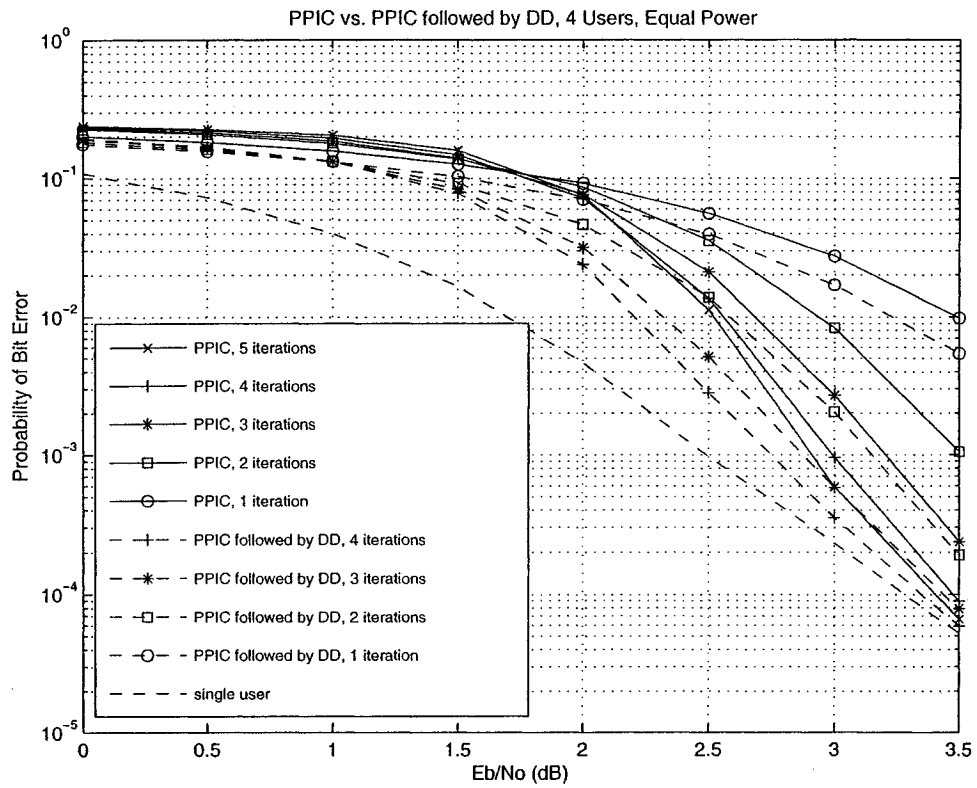


Figure 4.11: Performance comparison among PPIC and modified-PPIC followed by newly proposed decorrelator for  $K = 4$  users, cross-correlation  $\rho = 0.5$ , equal power users, and different number of iterations.



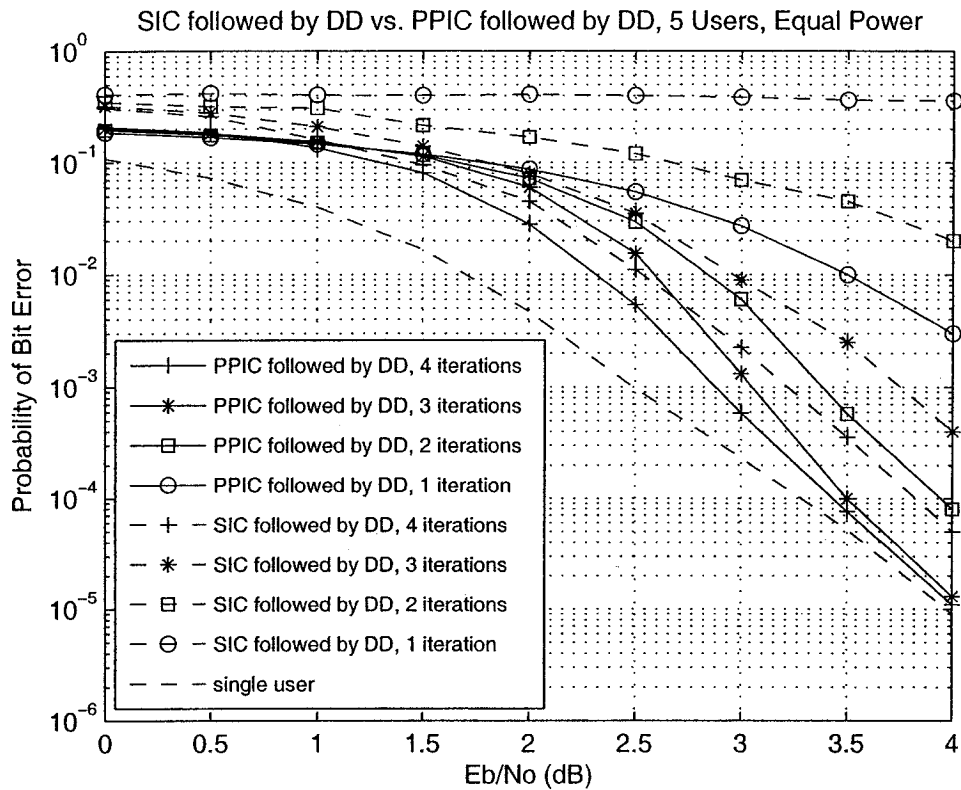


Figure 4.12: Performance comparison among SIC followed by the decorrelating detector and modified PPIC followed by the decorrelating detector for  $K = 5$  users, cross-correlation  $\rho = 0.5$ , equal power users, and different number of iterations.

are the same as mentioned for the previous system. In Figures 4.12 and 4.13, a performance comparison between the turbo SIC followed by decorrelating detector and the turbo modified PPIC followed by decorrelating detector is shown for different cross-correlations.

In Figures 4.12 and 4.13, the improvement of our newly proposed algorithm, which is the concatenation of modified PPIC followed by the decorrelating detector, is demonstrated. With just a slight increase in complexity, the performance improvement is obvious. For a moderate number of users that apply convolutional

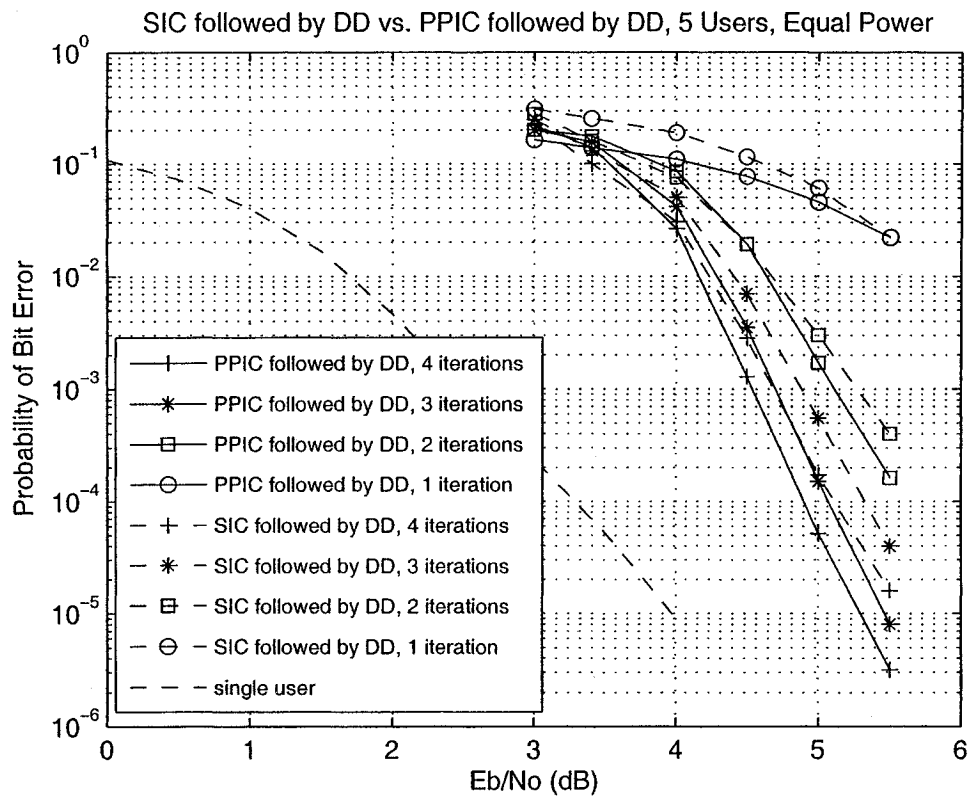


Figure 4.13: Performance comparison between SIC followed by the decorrelating detector and modified PPIC followed by the decorrelating detector for  $K = 5$  users, cross-correlation  $\rho = 0.7$ , equal power users, and different number of iterations.

	1st Iteration	2nd Iteration	3rd Iteration	4th Iteration
Graph A [ $\rho^{(1)}$ $\rho^{(2)}$ ]	[0.5 0.55]	[0.6 0.65]	[0.7 0.75]	[0.8 0.85]
Graph B [ $\rho^{(1)}$ $\rho^{(2)}$ ]	[0.3 0.4]	[0.5 0.6]	[0.7 0.8]	[0.9 0.95]
Graph C [ $\rho^{(1)}$ $\rho^{(2)}$ $\rho^{(3)}$ ]	[0.3 0.35 0.4]	[0.5 0.55 0.6]	[0.7 0.75 0.8]	[0.85 0.9 0.95]

Table 4.1: Different cancellation weight sets for  $m = 2$  (Graphs A and B) and  $m = 3$  (Graph C) iterations.

channel encoders, from moderate to high SNR, up to 0.3 dB gain can be achieved if multistage PPIC detector is applied instead of SIC detector, before the decorrelating detector in the channels that suffer from medium to high cross-correlation. In the proposed method, the additional complexity is a linear function of number of users  $K$  ( $O(K)$ ) which is negligible in price of the achieved performance gain.

By comparing the simulation results of our proposed algorithm in Figure 4.13 with that of the MMSE algorithm shown in Figure 3.7, one can notice the excellence of the MMSE algorithm. Although their computational complexity is both a cubic function of number of users, the MMSE algorithm has a better performance close to the single-user performance for highly correlated systems ( $\rho_{ij} = 0.7$ ).

In Figures 4.14 and 4.15, our proposed algorithm is simulated with different cancellation weight sets. Cancellation weight sets applied for each graph are mentioned in Table 4.1. Graphs A and B perform  $m = 2$  iterations for the PPIC detector and graph C performs  $m = 3$  number of iterations. In Figure 4.14, the graphs mentioned in Table 4.1 are simulated for  $K = 4$  users applying turbo channel encoders. In Figure 4.15, the same simulation is performed for  $K = 5$  users, applying simple convolutional

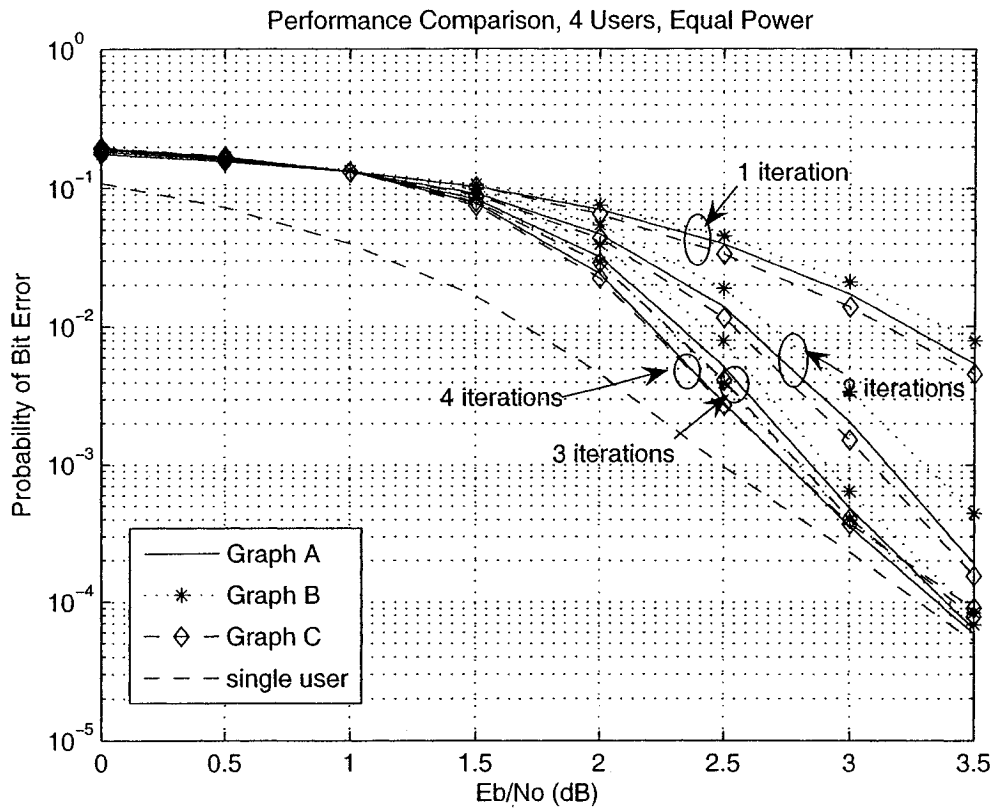


Figure 4.14: Performance comparison of graphs A, B, and C (all using modified PPIC followed by the decorrelating detector) as a function of the number of iterations, for  $K = 4$  equal power users, cross-correlation  $\rho = 0.5$ , turbo channel encoders, and different number of iterations.

encoders.

It can be seen that by adding one iteration to the PPIC part, not a significant improvement has been made in price of additional complexity. The simulation has been run for  $m = 5$  iterations also, but the performance does not improve considerably.

The computational complexity of the multistage PPIC is  $O(K)$  per stage which in overall comes to  $O(mK)$ , where  $m$  is the number of PPIC iterations. On the other hand, the decorrelating detector demands extra computations to calculate the inverse

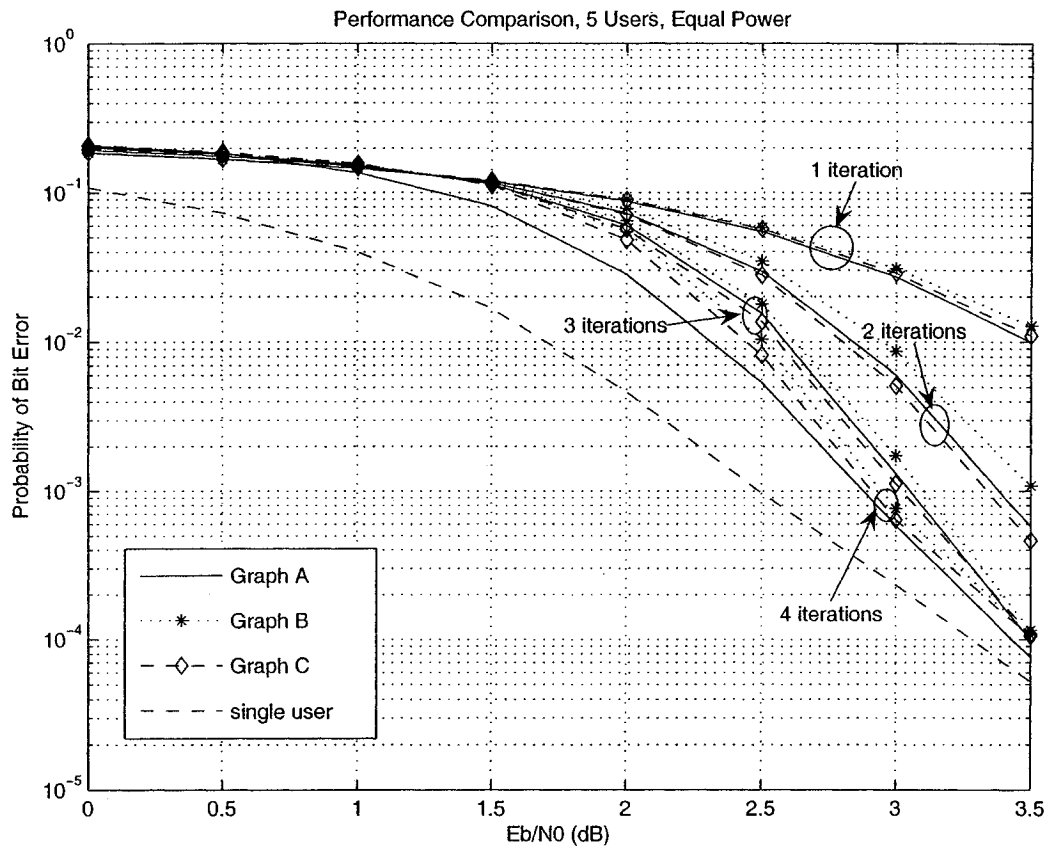


Figure 4.15: Performance comparison of graphs A, B, and C (all using modified PPIC followed by the decorrelating detector) as a function of the number of iterations, for  $K = 5$  equal power users, cross-correlation  $\rho = 0.5$ , simple convolutional channel encoders, and different number of iterations.

of Decorrelating Detector (DD) to achieve an excellent performance close to single-user performance, considering computational complexity. The detector is designed as a concatenation of a Multistage PPIC and an updating Decorrelating Detector. The computational complexity of the proposed method is  $O(mK + K^2)$  which is much less than that of the optimal multiuser detector ( $O(2^K)$ ). The first part of this method, which is a multistage PPIC, does not require any knowledge of users' parameters. This kind of detector is simple to implement compared to the other types of multiuser detectors. The second part of the proposed algorithm is a decorrelating detector which is not only a simple method, but it is also optimal in three different criteria: least-squares, near-far resistance, and maximum likelihood when received amplitudes are not known.

Simulation results demonstrated the excellence of our proposed method and that it can be considered as a suitable alternative considering the complexity and latency aspects. In Chapter 4, it has been shown that in moderate or highly correlated channels with certain number of users, up to 0.5 dB gain can be achieved in price of a slight increase in complexity.

Many parts of this thesis can be extended into further work such as

- Although neither the PPIC nor the Decorrelator receivers require any knowledge of the received amplitudes, this information is required for LLR calculation in the iterative scheme. A possible future work may involve assessing the effect of incomplete knowledge of the fading coefficients to derive simple estimation schemes to calculate them. This will remedy the short coming of this scheme and make it useful for applications in fading channels.
- It would be useful to investigate how these simulations could be adjusted to perform testing on various other communication channels besides AWGN such

of the updated cross-correlation matrix  $\mathbf{R}_u$ . To compute  $\mathbf{R}_u^{-1}$ , a recursive method has been applied which results in a computational complexity of  $O(K^2)$ . Therefore, the total complexity of the proposed turbo multiuser detector adds up to  $O(mK + K^2)$  which is a quadratic function of the number of users.

## 4.6 Summary

In this chapter, a new “Iterative (Turbo) Multiuser Detection” algorithm was introduced, studied, and simulated. The new method involves concatenation of modified Partial Parallel Interference Cancellation and a newly introduced Decorrelating Detector [35].

A Partial Parallel Interference Cancellation philosophy, in which the amount of cancellation increases as the reliability of the tentative decisions involved in forming interference estimates improves, is generally superior to the philosophy of entirely cancelling the interference at each stage. Using the hyperbolic tangent system to make the tentative decisions is superior to using a hard-decision limiter in a multi-stage PPIC as used in [36]. The modified PPIC can process the reliable information produced in the previous stages along with *a priori* information delivered by single-user decoders to produce extrinsic information for the next stage. This results in a more reliable likelihood iterating and hence improves the interference-cancelled soft outputs, delivered to the decorrelating detector.

The second part of the proposed algorithm includes a decorrelating detector which is updated at each system iteration. It makes use of soft information delivered by the PPIC detector and applies hyperbolic tangent for soft estimation to update the decorrelating matrix [35].

## Chapter 5

# Contributions and Suggestions for Future Work

In a CDMA system design, the complexity of the “Multiuser Detector” is the main obstacle to use optimal techniques. Using a powerful multiuser detector has the advantage that the probability of re-sending data because of a noisy message is lowered and it improves the capability of a channel to serve larger number of users in a limited bandwidth. By decreasing the number of retransmissions, we are effectively minimizing the time needed for reserving the available bandwidth for each transmission. Various research has been done over the last decade that result in different multiuser detection techniques. In the Chapter 3 of this thesis, several prominent works and techniques were mentioned and studied in terms of computational complexity and performance trade-off.

The major goal of this research was to develop a performant SISO Turbo Multiuser Detector, using single-user channel encoders. This method benefits from simplicity of Partial Parallel Interference Cancellation (PPIC) technique and the effectiveness



of Decorrelating Detector (DD) to achieve an excellent performance close to single-user performance, considering computational complexity. The detector is designed as a concatenation of a Multistage PPIC and an updating Decorrelating Detector. The computational complexity of the proposed method is  $O(mK + K^2)$  which is much less than that of the optimal multiuser detector ( $O(2^K)$ ). The first part of this method, which is a multistage PPIC, does not require any knowledge of users' parameters. This kind of detector is simple to implement compared to the other types of multiuser detectors. The second part of the proposed algorithm is a decorrelating detector which is not only a simple method, but it is also optimal in three different criteria: least-squares, near-far resistance, and maximum likelihood when received amplitudes are not known.

Simulation results demonstrated the excellence of our proposed method and that it can be considered as a suitable alternative considering the complexity and latency aspects. In Chapter 4, it has been shown that in moderate or highly correlated channels with certain number of users, up to 0.5 dB gain can be achieved in price of a slight increase in complexity.

Many parts of this thesis can be extended into further work such as

- Although neither the PPIC nor the Decorrelator receivers require any knowledge of the received amplitudes, this information is required for LLR calculation in the iterative scheme. A possible future work may involve assessing the effect of incomplete knowledge of the fading coefficients to derive simple estimation schemes to calculate them. This will remedy the short coming of this scheme and make it useful for applications in fading channels.
- It would be useful to investigate how these simulations could be adjusted to perform testing on various other communication channels besides AWGN such

as fading channels.

- Another interesting subject would applying this method to asynchronous CDMA systems to test the performance of the proposed algorithm for practical conditions in reality.

Finally, we hope that this research and studies can make a slight advancement in telecommunications for which I have pursued my studies.

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