Solving Railway Routing and Scheduling Problems in an Intermodal Freight Transportation System

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Abstract

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The railway line haul is the terminal-to-terminal segment of a door-to-door intermodal transportation system. This research concentrates on the routing and scheduling of the railway line haul. Routing and scheduling is the most important portions of the planning activities performed by railway companies. In this research, we developed an integer programming model to determine optimal operations in minimizing the most significant cost figures involved in such operations. Although the intermodal transportation system combines several transportation modes, our model concentrates on rail segment operations because improving the on-time performance of the rail segment can increase the timeliness of the entire intermodal route. Given container demands differentiated by origin, destination, arrival date at origin, and due date, the objective is to determine a train schedule and container routing scheme to minimize operational costs while meeting on-time delivery requirements of the shipments. The model was extensively tested by example problems of practical backgrounds, derived from those available in literature.

Acknowledgments

I would like to express my gratitude to my supervisor, Dr. M. Chen, for his guidance and supervision throughout all stages of this thesis. I sincerely appreciate his valuable suggestions and thoughtful comments.

I am profoundly thankful to my parents, my husband and my son for their patience and encouragement.

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Chapter One

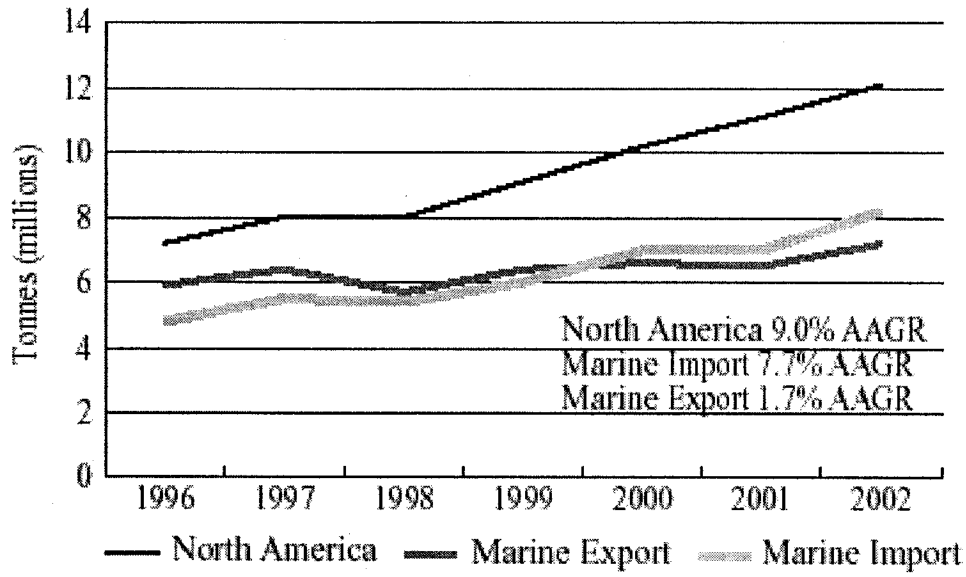
Introduction

1.1 Background

The Canadian railway industry played a leading role in North American freight transportation. Although Canada has just 9.9% of the North American population and 6.6% of its retail sales, it handles close to 15% of North American containerized trade with the world. And it has emerged as a transshipment hub for US-bound cargo. Between 1990 and 2000, the value of US-bound cargo transshipped through Canada increased 210%, amounting to over $28 billion per year (Viewpoint - August 2004).

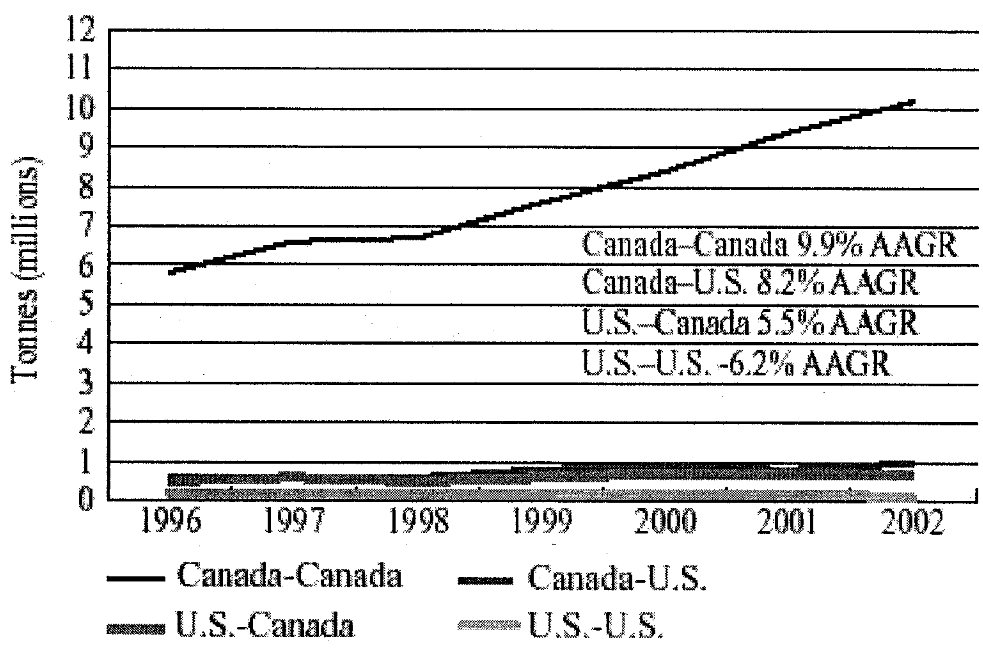
Figure 1.1 shows trends in intermodal traffic over the last several years. Between 1996 and 2002, CN and CPR intermodal tonnage grew by 9.2 million tonnes, an average annual growth rate of 7.1%.

As Figure 1.2 shows, volumes of Canadian origin–destination increased at an even higher average annual growth rate of almost 10% over this period.



Note: AAGR: average annual growth rate.

Figure 1.1 Rail Intermodal Traffic, 1996 – 2002



Note: AAGR: average annual growth rate.

Figure 1.2 Origin and Destination of Domestic Intermodal Traffic, 1996 - 2002

Growth in total rail intermodal volumes was significant between 2001 and 2002 (10 per cent) after only a small growth the year before. Volumes of North American traffic remained strong, increasing by 9% and accounting for 44% of total rail intermodal volumes (Transportation in Canada 2003 Annual Report).

Because of the increased amount of freight traffic, more stringent environmental requirements, as well as the long haul efficiency of trains results in savings in terminal costs and labour cost Where used for a distance of at least 500 miles, the growing importance of intermodalism as a transportation option is without question.

1.2 About Intermodal Transportation

The European Conference of Ministers of Transportation defines intermodal freight transportation as "the movement of goods in one and the same loading unit or vehicle, which uses successively various modes of transport (road, rail, water) without any handling of the goods themselves during transfers between modes." The major part of the journey is done by rail, inland waterway or sea, and any initial and/or final legs carried out by road are as short as possible. In this contribution, intermodal transportation is characterized by the combination of the advantages of rail and road, rail for long distances and large quantities, road for collecting and distributing over short or medium distances.

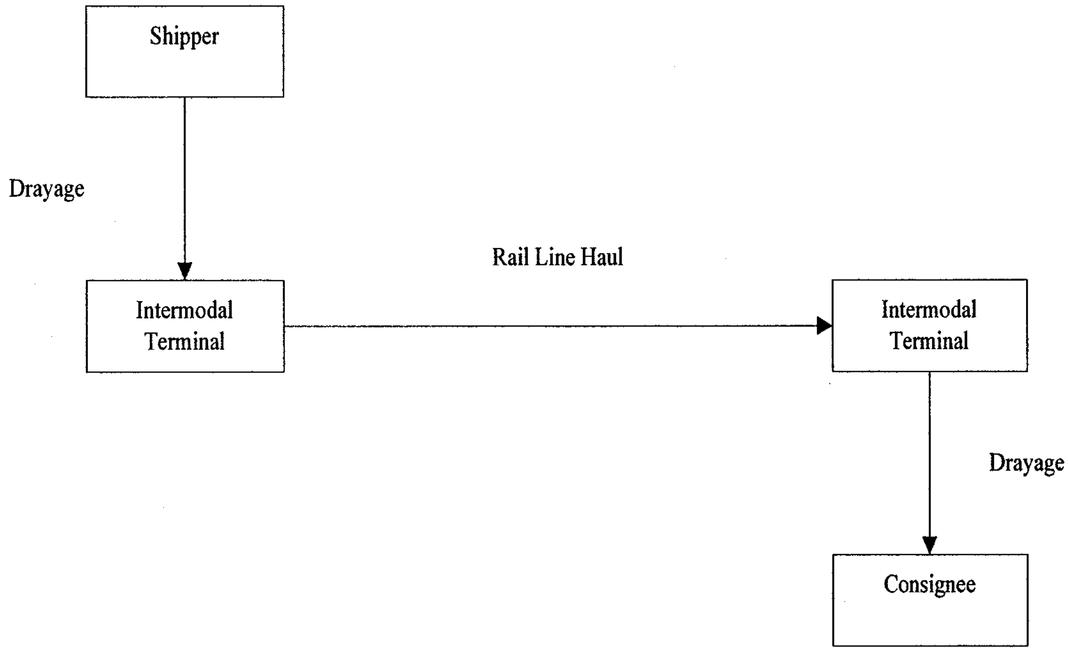


Figure 1.3 The Intermodal Journey

Figure 1.3 provides a simple depiction of intermodal freight transportation. Drayage operations take place by truck between a terminal and shippers or receivers. The Rail Line Haul is the terminal-to-terminal segment of the door-to-door intermodal trip.

This research concentrates on the routing and scheduling of "Rail Line Haul". Routing and scheduling represent the most important portion of the planning activities performed by railway companies.

Demand for freight transportation is usually expressed in terms of tonnage of certain commodities to be moved from an origin to destination. Given these demands, the railroad must establish a set of operating policies that will govern the route of trains and freight. For every origin-destination pair of traffic demand, the corresponding freight may be shipped either directly or indirectly. When demand is important enough, delivery delays are obviously minimized by using direct trains as opposed to sending the traffic through a sequence of links. To benefit from economies of scale, trains are thus often formed by grouping cars with various commodities and having different origins and destinations. These trains operate between particular nodes of the network, called classification yards. At these yards, cars are separated, sorted according to their final destination, and combined to form new outbound trains. However, because the classification process requires considerable resources, cars are not reclassified at every yard on their trip from origin to destination. Instead, cars with different final destinations but sharing some initial portion of their trips are assembled into blocks. Cars in the same block may then pass through a series of intermediate classification yards, being separated and reclassified only after they have reached the destination of the block. The blocking policy specifies what blocks should be built at each yard of the network and which cars should go into each block.

In each yard, blocks are built on classification tracks where they await the departure of an outbound train. The list of potential blocks that may go into each outbound train is specified by the makeup policy. Also, when a train passes through an intermediate classification yard, it may leave or pick up blocks of cars. A block left by an inbound train is either transferred to a different train or it is broken up and its cars are reclassified. Hence, although the origin and destination of a block may correspond to those of a train, a block may also switch trains several times before reaching its final destination.

The nature of intermodal transportation distinguishes itself from traditional rail haul networks in five areas. First, in intermodal transport, fixed schedules are used. While in traditional rail haul networks, trains run only when full and excessive classification at intermediate nodes takes place. Intermodal goods are not only more fragile, but they are also more time-sensitive. Therefore, these goods are treated with a greater sense of urgency, and timetables for running intermodal trains have been established. Second, intermodal traffic incurs fewer stops and reclassifications between its origin and destination than traditional rail system. Third, Fleet management issues in intermodal transport are more complex, because of the separation of the transport unit (rail flatcar) and the load unit (container/trailer). One aspect of the flatcar management problem is the tremendous variety of flatcars (including double stack cars), along with the variety of trailers and containers that the flatcars need to move. In contrast, in traditional rail transport, only boxcars (loaded and unloaded) are modelled. Fourth, because the transport unit can be separated from the load unit, rail–rail transhipment terminals can replace intermediate rail yards for classification. Fifth, location decisions for intermodal rail–road terminal are different from rail yards, as the former needs to connect two types of infrastructure.

1.3 Research in This Thesis

In this research, we developed an integer programming model to optimize operation costs of the rail segment of container intermodal transportation to decide when and how to route intermodal trains and which containers to send on each train. Although the intermodal transportation combines several transportation modes, our model concentrates on rail segment operations because improving the on-time performance of the rail segment can increase the timeliness of the entire intermodal route. Given container demands differentiated by origin, destination, arrival date at origin, and due date, the objective is to determine a train schedule and container routing scheme to minimize operational costs while meeting on-time delivery requirements of the shipments.

1.4 Thesis Organization

The remainder of this thesis is organized as following.

Chapter Two presents an overview of literatures. In Chapter Three we formulate a discrete-time integer programming model to achieve minimum operational costs of the rail segment of container intermodal transportation by improving daily train schedules and container routes. Chapter Four presents numerical example and analysis. Chapter Five summarizes the major conclusions of this thesis and conceives an objective in future research.

Chapter Two

Literature Review

Intermodal freight transportation research has emerged as a new transportation research application field while it still is in a pre-paradigmatic phase. Since 1990s a substantial number of analytical publications specifically addressing intermodal transport issues have appeared (Bontekoning, 2004). In this chapter, we will present the literature review about rail freight operations and intermodal freight operations. Based on the four main activities in intermodal transportation, the review about intermodal freight operations will be categorized to four areas: (1) drayage operations, (2) terminal operations, (3) network operations, and (4) intermodal operations.

2.1 Rail Freight Operations

Because our research focuses on the "rail line haul" portion of intermodal transportation, we give a brief overview of prior research on traditional rail freight operations.

Marin and Salmeron (1996) formulated an integer programming model to determine optimal train schedules for a traditional rail network. They used simulated annealing and tabu search methods to solve the model. For small problem instances with several hundred variables and constraints, they compared their results with exact solutions obtained from a specialized branch and bound algorithm. They concluded that for these smaller problem instances, their solution procedures provide results that are approximately 5% to 15% from the optimal solutions.

Cordeau et al. (1998) presented a review of optimization models for rail transportation problems. They proposed a classification of models and described their important characteristics by focusing on model structure and algorithmic aspects. The review mainly concentrated on routing and scheduling problems in rail transportation. Routing and scheduling represent the most important portion of the planning activities performed by railways. Routing models concern the operating policies for freight transportation and railcar fleet management. Scheduling models address the dispatching of trains and the assignment of locomotives and cars. They found that early models were usually built by linear programming or network optimization, whereas some recent models were solved with more sophisticated mathematical programming techniques. Others were solved using meta-heuristics which are very effective for several classes of discrete optimization problems.

Yano and Newman (2001) considered a problem of scheduling trains and containers (or trucks and pallets) between a depot and a destination. Goods arrive at the depot dynamically over time and have distinct due dates at the destination. There is a fixed

-charge transportation cost for each vehicle, and each vehicle has the same capacity. The cost of holding goods may be different at the depot and at the destination. The objective is to minimize the sum of transportation and holding costs. They extended several results of the single-item problem to correspond multi-item cases and showed that the optimal vehicle schedule can be obtained by solving a related single-item problem in which the item demands are aggregated in a particular way. The optimal assignment of vehicles can be found by solving a linear programming problem.

Forkenbrock (2001) used information of internal and external costs arising from intercity freight railroad operations. The external costs for four types of freight trains were estimated. For each type of freight trains, they estimated three general types of external costs were estimated and compared with internal costs experienced by railroad companies. The general types of external costs include: accidents (fatalities, injuries, and property damage); emissions (air pollution and greenhouse gases); and noise. Resulting internal and external costs were compared with those of freight trucking, estimated in an earlier article. Rail external costs are 0.24 cent to 0.25 cent (US) per ton-mile, well less than the 1.11 cent for freight trucking. That is, on a per-ton-mile basis, trucking generates over three times the external costs of any of the four types of freight trains considered in the analysis. External costs for rail transportation generally constitute a larger amount relative to internal costs. Because the internal cost (direct cost to the transportation provider) is much lower for rail, rail external costs often constitute larger amounts relative to internal costs than trucking.

Caprara et al. (2002) proposed a graph theory model to formulate a train scheduling problem. They used a directed multi-graph in which nodes correspond to departures/arrivals at a certain station at a given time instant. They considered a single, one-way track linking two major stations, with a number of intermediate stations in between. The train scheduling problem is to determine a periodic timetable for a set of trains that does not violate track capacities and satisfies certain operational constraints. The trains must run every period in a given time horizon. This formulation was used to derive an integer linear programming model solved by Lagrangian relaxation. A novel feature of the model is that the variables in the relaxed constraints are associated only with nodes (as opposed to arcs) of the graph. This allows a considerable speed-up in the solution of the relaxation. The relaxation was embedded within a heuristic algorithm which makes extensive use of the dual information associated with the Lagrangian multipliers.

Kraft (2002) proposed a method for managing a reservation based and capacity constrained car scheduling process for freight railroads shipments. The method was to compute a "real dollar" objective function for future locomotive and crew distribution systems, allowing a direct tradeoff between the values and the cost to provide extra capacity. The concept of scheduling appointment times directly follows patterned after current motor carrier industry practice, so that customers can plan for rail or truck deliveries in the same way. The author compared it with a widely used revenue management formulation and suggested a Lagrangian heuristic for obtaining a primal solution.

Lingaya et al. (2002) addressed an operational car assignment problem (OCAP) encountered at VIA Rail Canada (hereafter referred to as VIA) although the proposed model and solution methodology may also be applied to other passenger railways that rely on a similar planning process. They introduced a formulation and a solution approach for the complicated operational problem. They developed a modeling and solution methodology for the car assignment problem. This methodology considers both typical constraints such as maintenance requirements and more complex constraints such as minimum connection times. The problem was solved heuristically by a branch-and-bound method in which the linear relaxations were solved by column generation. Simulation experiments performed on realistic data show that the solution approach yields good quality solutions in very short computing time.

Mancuso and Reverberi (2003) used a transcendental logarithmic (Translog) short-run variable cost function for an Italian railway company with 1980–1995 data. A major implication of their findings is that the rail network is not used optimally, so that a cut in the frequency of trains or significant infrastructure investments may be needed. They also examined possible policy implications of the results obtained at the company level and estimated a variable cost function and thus performed a short-run analysis. They found that the presence of diseconomies of density does not preclude the possibility that some form of competition can be successful. It also revealed that joint or specialized production of passenger and freight carryings would be pursued, mainly depending on the size of the company. In particular, small companies are more likely to specialize to their economic activity than large ones.

2.2 Drayage Operations

Drayage operations involve the provision of an empty trailer or container to the shipper and the subsequent transportation of a full trailer or container to the terminal.

Morlok and Spasovic (1995) identified and discussed approaches for improving service quality and reducing cost in the highway portion of rail-truck intermodal transportation. In intermodal transportation, a load is moved between the origin and the destination in the same container in a coordinated manner using two or more transportation modes. The specific system of concern in the paper is how to use conjunction with rail-truck intermodal or piggyback service. In piggyback service, highway trailers or containers loaded on rail flat cars are hauled by train in line-haul service between the origin and the destination intermodal terminals, and locally picked up and delivered by trucks between the terminals and shippers, and terminals and receivers (termed consignees). Drayage costs are very high, and because these do not vary with the length of the intermodal haul, they preclude profitable intermodal service in the shorter domestic freight markets of less than 600 miles where the highest truck volumes are found. In addition, the inferior service quality precludes intermodal from competing for high quality premium traffic. The potential for overcoming these disadvantages through reorganization of the drayage operation and use of centralized drayage operations was discussed. Specific changes in the organizational structure of intermodal and in its operating procedures were outlined.

Choong et al. (2002) presented a computational analysis of the effect of planning horizon length on empty container management for intermodal transportation networks. The analysis was based on an integer programming that seeks to minimize total costs related to moving empty containers, subject to meeting requirements for moving loaded containers. Although the appropriate length of the planning horizon depends on the network under consideration, a longer planning horizon (used on a rolling basis) can give better empty container distribution plans for the earlier periods. The longer horizon allows better management of container outsourcing and encourages the use of slower and cheaper transportation modes. However, the advantages of using a long rolling horizon might be small for a system that has a sufficient number of container pools, since such a system has small end-of-horizon effects.

Taylor et al. (2002) developed two alternative methods of intermodal ramp assignment for minimizing total non-productive miles associated with circuity (out of route miles) and empty travel (brought about by imbalance and geographical separation between freight origins and destinations of intermodal freight movements). The authors compared two heuristic solutions to the intermodal ramp allocation problem to determine the robustness of the methods with respect to alternate ramp location assumptions and other pertinent parameters. Their results support the goal of determining how two heuristic solutions are performed under varying operational conditions and constraints.

2.3 Terminal Operations

Kozan (2000) discussed the major factors influencing the transfer efficiency of seaport container terminals. To increase the efficiency and speed of transportation, transportation companies should reduce the cost of maritime transport, mainly by reducing cargo handling and costs, and ships' time in port by speeding up handling operations. A network model is designed to analyse container progress in the system and applied to a seaport container terminal. The problem being investigated is the minimization of handling and traveling time of containers from the time the ship arrives at port until all the containers from that ship leave the port. This mathematical model can be used as a decision tool investment appraisal of multimodal transportation.

Trip and Bontekoning (2002) discussed the possibility of implementing innovative bundling models and new-generation terminals to integrate small flows, mainly from outside the economic core areas in intermodal transport systems. The integration of the small flows would increase the transport volume that is potentially suitable for intermodal transportation, and could therefore add to the modal shift from road to rail. Their conclusions indicated that it is possible to apply the concept of complex bundling with new generation terminal operations. The general theoretical advantages of such concepts can be shown in terms of a higher loading level and larger geographical coverage of the network.

Ballis and Golias (2002) evaluated technical and logistic developments to increased economic and technical efficiency of rail–road transport terminals. The main design parameters were identified (length and utilization of transshipment tracks, train and truck arrival patterns, type and number of handling equipment, mean stacking height in the storage area, terminal access system and procedures) and analysed. A comparative evaluation of selected conventional and advanced technologies was performed by an analysis tool developed for this purpose. The study consisted of a series of complementary parts designed to analyses requirements for integrated terminals and rolling stock in relation to market forces, transport modes, intermodal transport units, advanced intermodal terminal technology (including tests and demonstrations of pilot equipment), trunk haul production forms and Trans-European network effects. The analysis consisted of three modules (an expert system, a simulation model and a cost calculation module). The paper concluded with two groups of results: (a) A comparative evaluation of conventional and advanced technologies, (b) A critical assessment of terminal capacity issues. It is identified that the capacity limitations are imposed mainly by the transshipment track sub-system rather than by the handling equipment.

Rizzoli et al. (2002) presented a simulation model for the flow of intermodal terminal units (ITUs) within inland intermodal terminals. This module described the processes taking place in an intermodal rail/road terminal based on discrete-event simulation. The basic processes of the flow of the intermodal terminal units in the terminal had been considered in the model. The train arrivals are defined in a train timetable, while the patterns of truck arrivals for ITU delivery and pick-up can be either statistically modeled or given as a deterministic input. The simulation user can define the terminal structure and test alternative input scenarios to evaluate the impact of new technologies and infrastructures on existing terminals. The simulator can be used to simulate a single terminal or a rail network with two or more interconnected terminals. During the simulation, various statistics are gathered to assess the performance of the terminal equipment, the ITU residence time, and the terminal throughput.

2.4 Network Operations

The network operations face decision problems concerning infrastructure planning (strategic level), service schedules, pricing of services (tactical level) and daily operations of the services (operational level). Many of the studies related to intermodal infrastructure decisions deal with the interconnectivity of modes in order to achieve intermodal transport chains and the location of intermodal terminals.

Nozick and Morlok (1997) constructed a comprehensive model for an intermodal rail-truck system. The objective of their model was to minimize the cost of delivery such that the movements are physically feasible and the goods are delivered on time. They addressed the movement of freight from the shipper to the intermodal terminal, along the line haul portion of the trip to another intermodal terminal, and then to the consignee. Additionally, the model can be used for determining the optimal fleet size and the mix of equipment as well as the costs associated with providing various levels of service.

Gorman (1998a, 1998b) considered the problem of providing train service on a set of predetermined paths while simultaneously routing shipments to meet service requirements in the traditional rail boxcar operational setting. Other constraints, such as yard, line, and train capacity, are included. The formulation does not distinguish direct and indirect train service explicitly. Certain constraints, such as meeting the demand and adhering to train capacity constraints, may be violated with a penalty. Genetic and tabu-enhanced genetic searches were employed to determine a train schedule. Because the genetic search procedure does not necessarily perform well on larger problem instances, an enhanced tabu search procedure is adopted and incorporated in the genetic algorithm. This enhanced solution procedure provides results within 10% of the optimal solutions when compared with an exact solution procedure for smaller problem instances under the assumption of origin-destination independence. The procedure was then compared to current operational procedures of a railroad assuming origin-destination interdependence. A case study was performed on Santa Fe's intermodal operations using this algorithm. The result showed that great savings can be achieved.

Powell and Carvalho (1998) proposed a dynamic model for optimizing the flows of flatcars. The problem was formulated as a logistics queuing network that can handle a wide range of equipment types and complex operating rules. They formulated a global model to provide network information to local decision makers. The approach should be relatively easy to implement given current rail operations. Initial experiments suggested that a flatcar fleet that is managed locally, without the benefit of their network information. It can achieve the same demand coverage as a fleet that is 10% smaller, but is managed locally with their network information. The result shows that the gradient approximations provided by the Logistics Queuing Network approach can be used to improve the total contribution by 3.0% when compared to a series of myopic local problems solved over the entire horizon.

Newman and Yano (2000) addressed a problem to determine schedules for both direct and indirect (via a hub) trains and to allocate containers to these trains for the rail (line- haul) portion of the intermodal trip. The objective was to minimize operating costs, including a fixed-charge cost for the train, variable transportation and handling costs for each container and yard storage costs, while meeting on-time delivery requirements. They formulated the problem as an integer programming and developed a novel decomposition procedure to find near-optimal solutions. They also developed a method to provide relatively tight bounds on the objective function values. Finally, they compared their solutions to those obtained with heuristics designed to mimic current operations, and showed that savings between 5% and 20% can be gained from using their solution procedure.

Ziliaskopoulos and Wardell (2000) presented a time-dependent intermodal optimum path algorithm in multimodal transportation networks. This path algorithm arises from the development of intelligent transportation systems (ITS) and intermodal freight systems. A simple representation of the mode-to-mode switching options was introduced that results in a substantially improved design, with computational complexity independent of the number of modes. A preprocessor was designed that constructs the necessary input data from common transit timetables. The algorithm was coded, implemented, and tested on real size networks with promising results. Computational results indicated that the performance of the algorithm is substantially better than the worst case bound, suggesting almost linear computational time with the number of nodes and time intervals, and invariance to the number of modes.

Southworth and Peterson (2000) described the development of a large and detailed multimodal network, created and stored in digital form for use in a specific freight traffic routing study: the 1997 United States Commodity Flow Survey. The paper focused on the routing of the large number of intermodal freight movements. Routings involve different combinations of truck, rail and water transportation. Selection of appropriate intermodal routes requires procedures for linking freight origins and destinations to the transportation network, for modeling intermodal terminal transfers, and for generating multimodal impedance functions to reflect the relative costs of alternative routes.

2.5 Intermodal Operations

Intermodal operators organize transportation of shipments on behalf of shippers. Intermodal operators buy services from drayage, terminal and network operators. Decisions made by intermodal operators deal with route and service choices in existing intermodal networks.

Aggarwal et al. (1994) proposed a heuristic method for obtaining near solutions to the multicommodity maximum flow and minimum cost flow problem. The heuristic was based on identifying a solution that maximizes flow along the arcs for a single commodity. In this case, common arc capacities are allocated among individual arcs. The solutions were used as a solution for the multicommodity problem. Flows on the arcs were readjusted using dual information to reallocate arc capacity. The heuristic is shown to perform well on various instances.

Barnhart et al. (1998) presented a solution methodology for integer multicommodity flow problem with fixed capacity constraints. They transformed the traditional formulation into a path-based formulation in which paths from the source to the sink are explicitly enumerated. They applied the column generation procedure to the linear relaxation of this problem. This procedure constructs an initial set of paths. The problem was solved with a customized branch-and-price procedure. An improvement in these computational results can be realized from the use of a more specialized procedure and a branch-and-price-and-cut procedure.

Nozick and Morlok (1997) presented a thorough description of the nature of intermodal transportation and its differences from classical boxcar operations. Boxcar and intermodel traffic are handled in different ways. Many boxcar operations use the concept of blocking, in which groups of cars bound for destinations in close proximity, or along the same rail line, travel together on a train. At intermediate terminals, or hump yards, these blocks are regrouped into different trains according to their destinations. Because of the jarring motion caused by sending the boxcars over the reclassification humps, intermodal transportation is separated from boxcar traffic. In general, intermodal transportation has fewer stops and reclassifications between its origin and destination than boxcar transportation. Furthermore, intermodal transportation originates at and is bound for intermodal terminals, as opposed to boxcar operations. The nature of the goods shipped intermodally also differs from those sent in boxcars. Intermodal goods are not only more fragile (e.g., finished as opposed to raw materials such as coal, gravel, or sheet metal), but they are also more time-sensitive.

Macharis and Bontekoning (2004) reviewed operations research techniques applied to solving intermodal transport problems. They believed that intermodal freight transportation research is still in a pre-paradigmatic phase and needs different types of models from those for uni-modal transport.

Bontekoning et al. (2004) reviewed 92 publications in the area of the intermodal transportation research. They concluded that the problems in intermodal transportation are complex and require new knowledge to solve them.

2.6 Summary

The literature review in this chapter shows that intermodal transportation is an importation research area for solving practical transportation problems in today's global economy. It also shows that different math models and solution procedures have been developed to solve different types of intermodal transportation problem. In this research, we will present our integer programming for the optimal minimum cost of "rail line haul" part of intermodal transportation.

Chapter Three
Mathematical Formulation and Problem Structure

In this chapter, we will present our integer, discrete-time scheduling program used to optimize the rail segment of intermodal transportation.

3.1 Problem Description

Although the intermodal transportation combines several transportation modes, our model concentrates on rail operations. We choose this as our focus, because of the greater potential for increasing timeliness of the entire intermodal journey by improving the on-time performance of the rail segment.

The railroad offers several speeds of service and charges a premium for faster delivery. As shown in Figure 3.1, containers can be sent directly from an origin intermodal terminal O_m to a destination terminal D_n without stopping at a hub H providing the fast available service. Alternatively, trains carrying containers bound for several destinations may be sent to a hub H, where containers with different origins but for a common destination are consolidated onto a train outbound from the hub. This consolidation activity can lead to a delay of up to several days. Part of the delay is due to the need to transfer containers or to

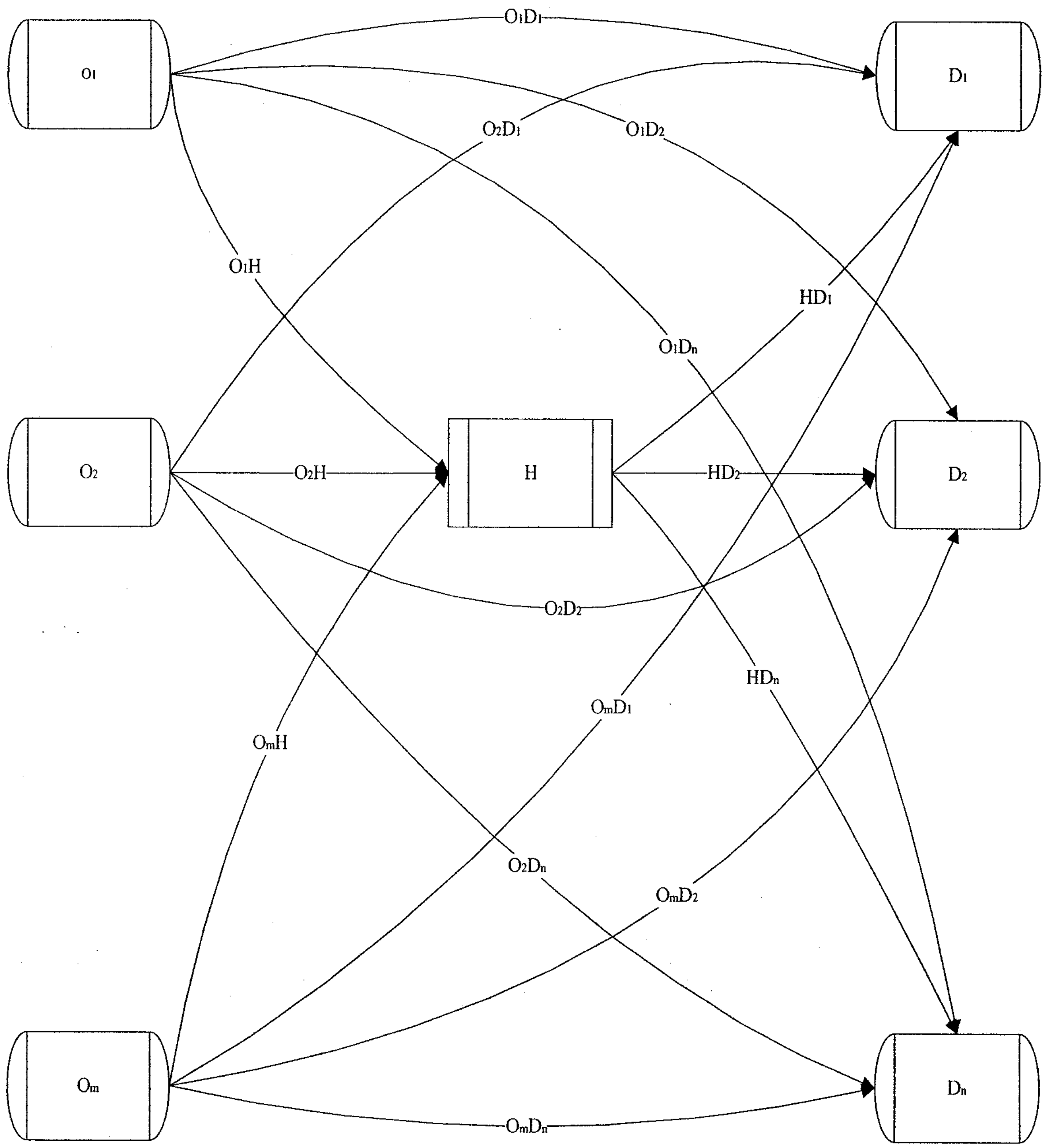


Figure 3.1 Figure of Network

the need to transfer containers or to reposition rail cars between trains. Further delays also occur when inbound and outbound schedules are not coordinated.

Each train has a limited capacity, where the capacity depends upon the power of the locomotives and the terrain over which the train will travel. Typically, for each transportation segment, locomotive capacity is determined in advance on the basis of demand forecasts, and from this, the train capacity is derived, expressed in terms of the number of containers to carry.

Yard storage space for containers waiting to be shipped, awaiting a transfer at the hub, or waiting to be picked up, is limited at all terminals. As the number of containers in storage increases, containers are stacked higher and more densely. This increases the time required to retrieve a container and places a further burden on material handling equipment which may already be a bottleneck.

In this research, we develop an integer programming model for the line-haul portion of the intermodal transportation. Given container demands differentiated by origin, destination, arrival date at origin, and due date, the objective is to determine a train schedule and shipment plan to minimize operational costs while meeting on-time delivery requirements and adhering to train capacity restrictions.

We refer to a time period as a "day". We assume that transit and hub delay times are deterministic and constant across the time horizon. Transit times and delays at the hub are expressed in terms of days. We also assume that the demand, arrival date at the origin, and the due date at the destination are known in advance or can be forecasted over the time horizon for planning purposes.

The operation costs consist of both fixed cost and variable cost. Fixed cost includes locomotive requirements costs and operators' wages. We assume that equipment is available where we need. We also assume that the capacity of a train on each transportation segment and the fixed cost of each train on a specific segment are known. We assign the fixed cost for a train to be approximately equal to the transportation cost per container multiplied by the capacity of a train. We estimate the fixed costs at origin and the hub to be commensurate to the distance traveled between locations.

The variable cost of each container consists of three parts:

(1) Transportation costs, such as fuel, oil, and track maintenance. We assume that the transportation costs per container are constant over time and they depend only on the origin-destination pair.

(2) Handling costs caused for moving containers on and off the rail cars and repositioning the rail cars at an intermediate terminal. Handling cost per container is based on an hourly wage of yard operators and the approximate time needed to load a container

onto a stack car. They mainly depend on the equipment used for such operations.

(3) Inventory costs consist primarily of yard storage costs. Inventory costs per container per day are assigned a low value as a slight deterrent to occupying yard space with containers. In actuality, the opportunity cost of capital, which constitutes most of the holding cost, is incurred by the shipper or consignee, and not the railroad. We assume that customers will accept early deliveries, so no inventory is held at the destinations. Inventory cost only occurs at origins and the hub. We set that there is no inventory before the first day, and there is no inventory in the last day.

The relative values of the parameters change between problem instances in order to reflect particular physical settings. For example, handling costs that are lower at the hub than at the origin and destination reflect greater handling requirements at the initial and terminal nodes. This could result from the need to move containers on and off the rail cars at the origin and destination, respectively, only to re-order rail cars at the hub. Conversely, higher handling costs at the hub represent instances in which the containers must be transferred between cars. Similarly, if the fixed cost of an indirect train is higher at the origin than at the hub, it reflects a longer travel time between the origin and the hub, than between the hub and the destination. A reverse relationship in the relative magnitude of these fixed costs would reflect the scenario in which the origin and the hub were closer than the hub and the destination. Although the indirect fixed costs at the origin and the hub are each less than the direct fixed cost at the origin, the total indirect fixed cost per train is always more than the direct fixed cost.

To summarize, the object of our problem is to minimize the operational costs by choosing train schedules and routes of containers. It consists of a fixed charge per train (FCOST) with a given capacity, a transportation cost per container (TCOST), both of which dependent on the origin and destination, handling cost per container (HCOST) dependent upon the location, and inventory holding cost for containers (ICOST) held at origins and hub.

3.2 Mathematical Formulation

Base on the above discussion and a careful study of the problem, integer programming is chosen as the problem solving approach due to its simplicity and reliability because it is straightforward to handle alternate analysis. The following sub-sections will present the model.

3.2.1 Notation and Variable Definition

In presenting the mathematical programming model, the following notation and variables are defined.

The parameters in the model are as follows:

M	Total number of origins
N	Total number of destinations
H	The only hub
T	Total time span in days
J	Total number of container sets arriving to all origins
K	Total number of container sets sent from the hub
O_m	Origin, $m=1, 2...M$
D_n	Destination, $n=1, 2...N$
t	Time (day), t= $1, 2...T$
j	Container set, $j = 1, 2...J$
k	Container set, $k = 1, 2...K$

- Time

α_{mn}	Direct shipping time from origin m to destination n
β_{mH}	Shipping time from origin m to the hub
β_{Hn}	Shipping time from the hub to destination n
δ	Delay time at the hub
β_{mn}	Indirect shipping time from origin m to destination n
γ_m	Holding time at origin m
γ_H	Holding time at the hub H
v_j	Due date of container set j

- Cost

$CFOD_{mn}$ Fixed cost of running a train directly from origin m to destination n

$CFOH_m$ Fixed cost of running a train from origin m to the hub

$CFHD_n$ Fixed cost of running a train from the hub to destination n

$CTOD_{mn}$ Unit cost of transporting a container from origin m to destination n

$CTOH_m$ Unit cost of transporting a container from origin m to the hub

$CTHD_n$ Unit cost of transporting a container from the hub to destination n

CHO_m Handling cost of placing a container on the train at origin m

CHH Handling cost of rearranging a container at the hub

CHD_n Handling cost of removing a container from the train at destination n

CIO_m Inventory cost of holding a container at origin m per day

CIH Inventory cost of holding a container at the hub per day

- Capacity

NTO_m Maximum number of trains sent from origin m each day

NTH Maximum number of trains sent from the hub each day

NOD_{mn} Capacity of a train (number of containers) directly from origin m to destination n

NOH_m Capacity of a train (number of containers) from origin m to the hub

NHD_n Capacity of a train (number of containers) from the hub to destination n

NIO_m Maximum number of containers in inventory at origin m

NIH Maximum number of containers in inventory at the hub

- Transportation demand

$NAOD_{jmn}^{tv_j}$ Number of containers in container set j at origin m on day t to be transported to destination n with a due date of v_j

<u>The decision variables in the model are as follows:</u>

- Number of Containers Shipped

$NSOD_{jmn}^{tv_j}$ Number of containers in container set j shipped from origin m directly to destination n at day t, which are required to be at destination n by day v_j

$NSOD_{mn}^{t}$ Total number of containers sent directly from origin m to destination n at day t

$NSOH_{jmn}^{tv_j}$ Number of containers in container set j shipped from origin m at day t to the hub, which are required to be at destination n by day v_j

$NSHD_{kn}^{t}$ Number of containers in container set k shipped from the hub at day t to destination n

$NSOH_{m}^{t}$ Total number of containers sent from origin m to the hub at day t:

$NSHD_{n}^{tv}$ Number of containers shipped from the hub at day t, which are required to be at destination n by day v

$NSHD_{n}^{t}$ Total number of containers sent from the hub to destination n at day t

$NAHD_{n}^{tv_j}$ Number of containers arriving at the hub on day t bound for destination n with a due date of day v_j

- Number of Trains sent

$NTOD_{mn}^{t}$ Number of trains sent directly from origin m to destination n at day t

$NTOH_{m}^{t}$ Number of trains sent from origin m to the hub at day t

$NTHD_{n}^{t}$ Number of trains sent from the hub to destination n at day t

- Inventory Number of Containers

NIO_{m}^{t} Inventory number of containers at origin m at day t

NIH^{t} Inventory number of containers at the hub at day t

3.2.2 Constraints

In this section, the constraints of the integer programming model are presented in the form of mathematical equations with explanations relevant to the planning procedure.

- **Constraints about the Inventory Balance at Origin** O_m

As shown in Figure 3.2, each set of containers arriving at origin m will be sent from origin m either directly to destination n or passing the hub in the following days, and they must be sent to destination n before the due date. This can be expressed by:

$$NAOD_{jmn}^{\tau v_j} = \sum_{t=\tau}^{v_j-\alpha_{mn}} NSOD_{jmn}^{\tau v_j} + \sum_{t=\tau}^{v_j-\beta_{mn}} NSOH_{jmn}^{\tau v_j} \qquad \forall m,n,j,t \qquad (3.1)$$

The number of containers shipped directly from origin m to destination n at day t should be equal to the total number of containers in all container sets shipped directly from origin

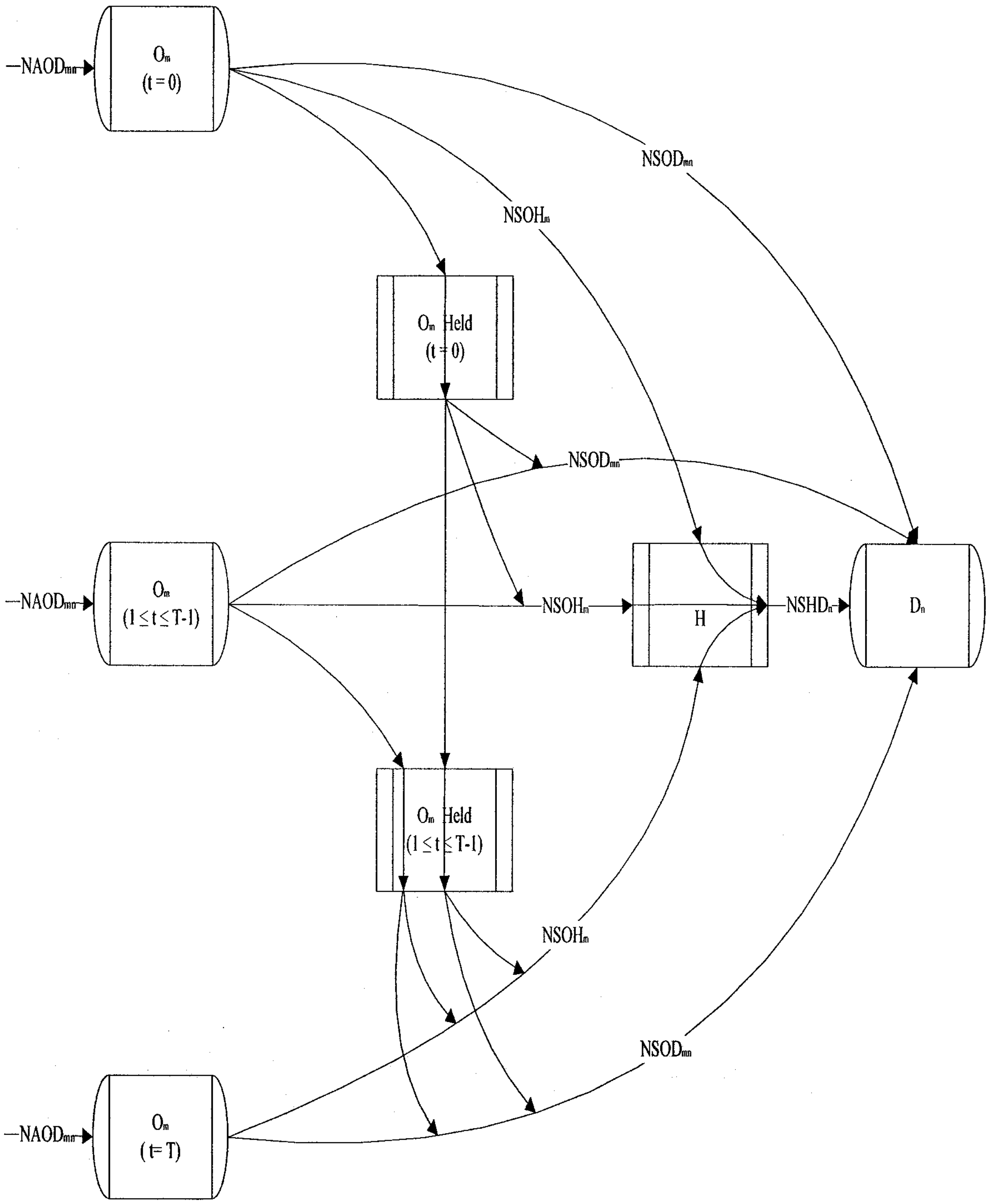


Figure 3.2 Network of Origins

m to destination n at day t.

$$NSOD_{mn}^{t} = \sum_{j=1}^{J} NSOD_{jmn}^{tv_j} \qquad \forall m,n,t \qquad (3.2)$$

The number of containers shipped from origin m to the hub at day t should be equal to the total number of containers in all container sets shipped from origin m to the hub at day t.

$$NSOH_{m}^{t} = \sum_{j=1}^{J} \sum_{n=1}^{N} NSOH_{jmn}^{tv_j} \qquad \forall m,t \qquad (3.3)$$

The inventory number of containers at origin m at day t should be equal to the total numbers of containers of all sets arriving at origin m at day t and shipped from origin m directly to destination n or passing the hub after day t.

$$NIO_{m}^{t} = \sum_{t=0}^{\tau} \sum_{j=1}^{J} NAOD_{jmn}^{\tau v_j} - \sum_{t=0}^{\tau} \sum_{j=1}^{J} \sum_{n=1}^{N} (NSOD_{jmn}^{\tau v_j} + NSOH_{jmn}^{\tau v_j}) \qquad \forall m, \quad t \leq T-1 \qquad (3.4)$$

The total inventory number of containers at origin m at day t should be less or equal to the capacity of inventory at origin m.

$$NIO_{m}^{t} \leq NIO_{m} \qquad \forall m, \quad t \leq T-1 \qquad (3.5)$$

There is no inventory before the first day.

$$NIO_{m}^{t-1} = 0 \qquad \forall m, t = 0 \qquad (3.6)$$

- **Constraints about the Inventory Balance at the Hub**

As shown in Figure 3.3, the number of containers arriving at the hub at time t bound for destination n should be equal to the number of containers in all container sets shipped from all origins at day (t- β_{mH}).

$$NAHD_{n}^{t} = \sum_{m=1}^{M} \sum_{j=1}^{J} NSOH_{jmn}^{(t-\beta_{mH})v_j} \qquad \forall n, \quad \beta_{mH} \leq t \leq T + \beta_{mH} \qquad (3.7)$$

The number of containers arriving at the hub at day t bound for destination n should be

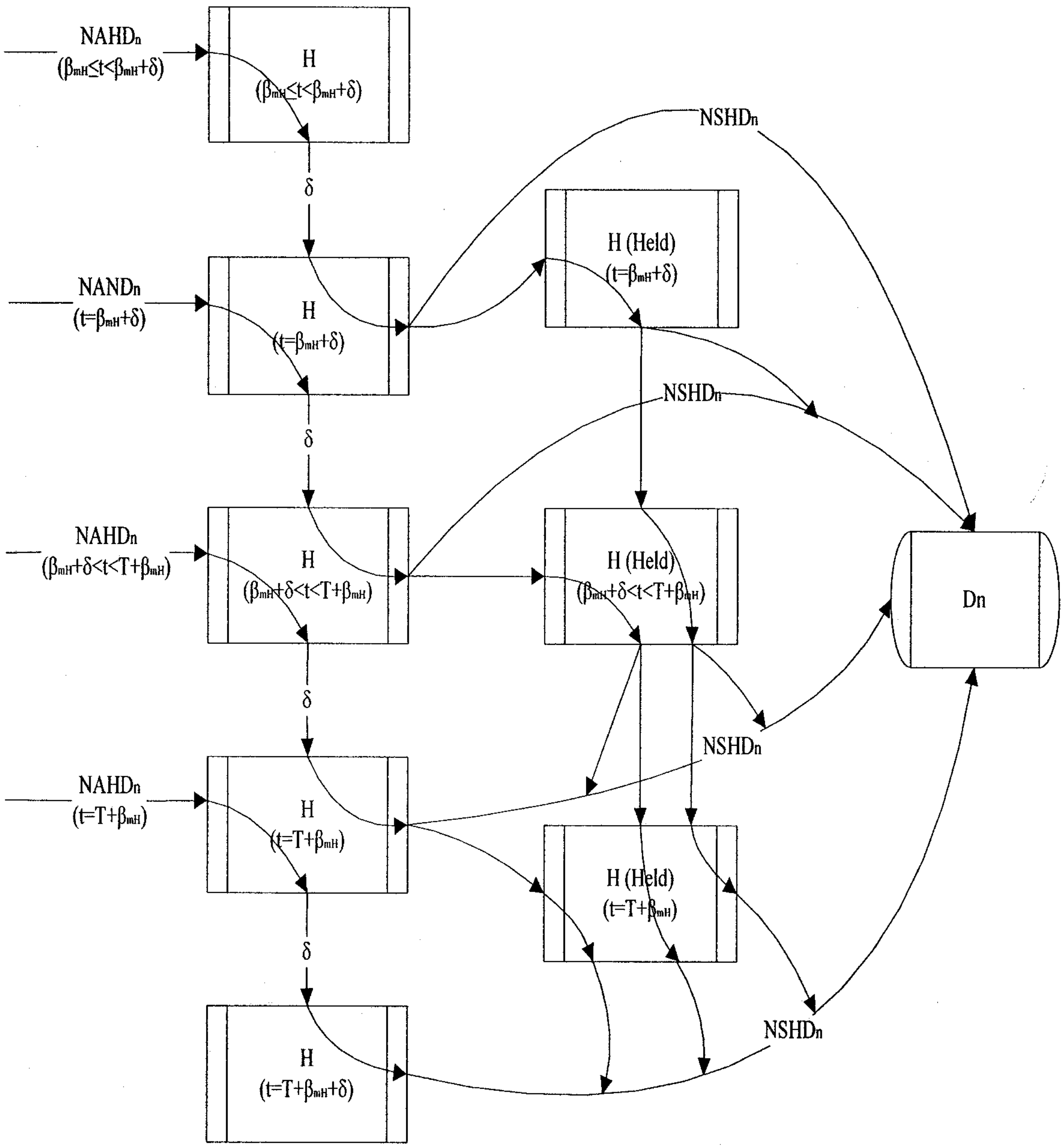


Figure 3.3 Network of Hub

shipped from the hub to destination n at day ($t+\delta$) till day (v_j - β_{Hn}).

$$NAHD_n^t = \sum_{t=\tau+\delta}^{v_j-\beta_{Hn}} NSHD_{kn}^{\tau+\delta} \qquad \forall n,\ \beta_{mH} \le t \le T + \beta_{mH} \qquad (3.8)$$

The number of containers shipped from the hub to destination n at day t should be equal to the total number of containers in all sets shipped from the hub to destination n at day t.

$$NSHD_n^t = \sum_{k=1}^{K} NSHD_{kn}^t \qquad \forall n,\ \beta_{mH} + \delta \le t \le T + \beta_{mH} + \delta \qquad (3.9)$$

The inventory number of containers at the hub at day t should be equal to total numbers of containers of all sets arriving at the hub before day ($t+\delta$) and shipped from the hub to destination n after day t.

$$NIH^t = \sum_{t=\beta_{mH}}^{\tau-\delta} NAHD_n^\tau - \sum_{t=\beta_{mH}+\delta}^{\tau} \sum_{k=1}^{K} \sum_{n=1}^{N} NSHD_{kn}^\tau \qquad \beta_{mH} + \delta \le t \le T + \beta_{mH} \qquad (3.10)$$

The total inventory number of containers at the hub at day t should be less or equal to the capacity of inventory at the hub.

$$NIH^t \le NIH \qquad \beta_{mH} + \delta \le t \le T + \beta_{mH} \qquad (3.11)$$

There is no inventory after day $T + \beta_{mH}$.

$$NIH^t = 0 \qquad T + \beta_{mH} < t \le T + \beta_{mH} + \delta \qquad (3.12)$$

- **Constraints of Due Date**

All containers should arrive at their destinations before the due date.

$$\gamma_m + \alpha_{mn} \le v_j \qquad \forall m, n, j \qquad (3.13)$$

$$\gamma_m + \beta_{mn} + \gamma_H \le v_j \qquad \beta_{mn} = \beta_{mH} + \delta + \beta_{Hn}, \qquad \forall m, n, j \qquad (3.14)$$

- **Constraints of Freight Volume**

For all origins, destinations and time periods, the number of containers sent from origin m to destination n and the hub or from the hub to destination m at day t must not exceed the total capacity of the departure trains.

$$NTOD_{mn}^{t} \geq NSOD_{mn}^{t} / NOD_{mn} \qquad \forall m,n,t \qquad (3.15)$$

$$NTOH_{m}^{t} \geq NSOH_{m}^{t} / NOH_{m} \qquad \forall m,t \qquad (3.16)$$

$$NTHD_{n}^{t} \geq NSHD_{n}^{t} / NHD_{n} \qquad \forall n,t \qquad (3.17)$$

The number of trains sent from origin m or the hub at day t cannot exceed the maximum capacity of the railway system.

$$\sum_{n=1}^{N} NTOD_{mn}^{t} + NTOH_{m}^{t} \leq NTO_{m} \qquad (3.18)$$

$$\sum_{n=1}^{N} NTHD_{n}^{t} \leq NTH \qquad (3.19)$$

- **Constraints on decision variables**

The following constrains represent that all decision variables are nonnegative and integers.

$$NSOD_{mn}^{t} \geq 0 \quad \text{and integer} \qquad \forall m,n,t \qquad (3.20)$$

$$NSOH_{m}^{t} \geq 0 \quad \text{and integer} \qquad \forall m,t \qquad (3.21)$$

$$NSHD_{n}^{t} \geq 0 \quad \text{and integer} \qquad \forall n,t \qquad (3.22)$$

$$NTOD_{mn}^{t} \geq 0 \quad \text{and integer} \qquad \forall m,n,t \qquad (3.23)$$

$$NTOH_{m}^{t} \geq 0 \quad \text{and integer} \qquad \forall m,t \qquad (3.24)$$

$$NTHD_{n}^{t} \geq 0 \quad \text{and integer} \qquad \forall n,t \qquad (3.25)$$

$$NSOD_{mn}^{tv_j} \geq 0 \quad \text{and integer} \qquad \forall j,m,n,t \qquad (3.26)$$

$NSOH_m^{tv_j} \geq 0$ and integer $\forall j,m,t$ (3.27)

$NSHD_n^{tv_j} \geq 0$ and integer $\forall j,m,t$ (3.28)

$NAHD_n^{tv_j} \geq 0$ and integer $\forall j,m,t$ (3.29)

$NIO_m^t \geq 0$ and integer $\forall m,t$ (3.30)

$NIH^t \geq 0$ and integer $\forall t$ (3.31)

3.2.3 The Objective Function

The object function of this model is to minimize the total operation costs. The operation costs consist of both fixed costs and variable costs. Fixed costs include locomotive requirement cost and operators' wages. Fixed cost per train with a given capacity depends on the origin and the destination.

Fixed Cost:

$$\boldsymbol{FCOST} = \sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{n=1}^{N}(CFOD_{mn} \times NTOD_{mn}^t) + \sum_{t=1}^{T}\sum_{m=1}^{M}(CFOH_m \times NTOH_m^t) + \sum_{t=1}^{T}\sum_{n=1}^{N}(CFHD_n \times NTHD_n^t)$$

Variable costs of each container consist of three parts:

(1) Transportation costs, such as fuel, oil, and track maintenance. Transportation cost per container depends on the origin and the destination.

Transportation Cost:

$$TCOST = \sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{n=1}^{N}(CTOD_{mn} \times NSOD_{mn}^{t}) + \sum_{t=1}^{T}\sum_{m=1}^{M}(CTOH_{m} \times NSOH_{m}^{t}) + \sum_{t=1}^{T}\sum_{n=1}^{N}(CTHD_{n} \times NSHD_{n}^{t})$$

(2) Handling costs caused for moving containers on and off the rail cars and repositioning the rail cars at an intermediate terminal. They mainly depend on the equipment used for such operations.

Handling Cost:

$$HCOST = \sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{n=1}^{N}(CHO_{m} \times NAOD_{mn}^{t}) + \sum_{t=1}^{T}\sum_{n=1}^{N}(CHH \times NAHD_{n}) + \sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{n=1}^{N}(CHD_{n} \times NAOD_{mn}^{t})$$

(3) Inventory costs for containers held at origin and the hub consist primarily of yard storage costs.

Inventory Cost:

$$ICOST = \sum_{t=1}^{T}\sum_{m=1}^{M}(CIO_{m} \times NIO_{m}^{t}) + \sum_{t=1}^{T}(CIH \times NIH^{t})$$

The total cost considered in this model is the summation of the four cost items:

$$\textit{Total Operation Cost} = FCOST + TCOST + HCOST + ICOST$$

3.2.4 Mathematical Formulation

The complete mathematical formulation for minimizing total operation costs of the considered container transportation problem in summarized below.

$$Min\ \ Z = [\sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{n=1}^{N}(CFOD_{mn} \times NTOD_{mn}^{t}) + \sum_{t=1}^{T}\sum_{m=1}^{M}(CFOH_{m} \times NTOH_{m}^{t}) +$$

$$\sum_{t=1}^{T}\sum_{n=1}^{N}(CFHD_{n} \times NTHD_{n}^{t})] + [\sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{n=1}^{N}(CTOD_{mn} \times NSOD_{mn}^{t}) +$$

$$\sum_{t=1}^{T}\sum_{m=1}^{M}(CTOH_{m} \times NSOH_{m}^{t}) + \sum_{t=1}^{T}\sum_{n=1}^{N}(CTHD_{n} \times NSHD_{n}^{t})] +$$

$$[\sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{n=1}^{N}(CHO_{m} \times NAOD_{mn}^{t}) + \sum_{t=1}^{T}\sum_{n=1}^{N}(CHH \times NAHD_{n}^{t}) +$$

$$\sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{n=1}^{N}(CHD_{n} \times NAOD_{mn}^{t})] + [\sum_{t=1}^{T}\sum_{m=1}^{M}(CIO_{m} \times NIO_{m}^{t}) + \sum_{t=1}^{T}(CIH \times NIH^{t})]$$

Subject to

$$NAOD_{jmn}^{tv_j} = \sum_{t=\tau}^{v_j-\alpha_{mn}} NSOD_{jmn}^{\tau v_j} + \sum_{t=\tau}^{v_j-\beta_{mn}} NSOH_{jmn}^{\tau v_j} \qquad \forall m,n,j,t \qquad (3.1)$$

$$NSOD_{mn}^{t} = \sum_{j=1}^{J} NSOD_{jmn}^{tv_j} \qquad \forall m,n,t \qquad (3.2)$$

$$NSOH_{m}^{t} = \sum_{j=1}^{J}\sum_{n=1}^{N} NSOH_{jmn}^{tv_j} \qquad \forall m,t \qquad (3.3)$$

$$NIO_{m}^{t} = \sum_{t=0}^{\tau}\sum_{j=1}^{J} NAOD_{jmn}^{\tau v_j} - \sum_{t=0}^{\tau}\sum_{j=1}^{J}\sum_{n=1}^{N}(NSOD_{jmn}^{\tau v_j} + NSOH_{jmn}^{\tau v_j}) \qquad \forall m, t \le T-1 \qquad (3.4)$$

$$NIO_{m}^{t} \le NIO_{m} \qquad \forall m, t \le T-1 \qquad (3.5)$$

$$NIO_{m}^{t-1} = 0 \qquad \forall m\ ,\ t = 0 \qquad (3.6)$$

$$NAHD_n^t = \sum_{m=1}^{M}\sum_{j=1}^{J} NSOH_{jmn}^{(t-\beta_{mH})v_j} \qquad \forall n, \quad \beta_{mH} \le t \le T + \beta_{mH} \qquad (3.7)$$

$$NAHD_n^t = \sum_{t=\tau+\delta}^{v_j-\beta_{Hn}} NSHD_{kn}^{\tau+\delta} \qquad \forall n, \quad \beta_{mH} \le t \le T + \beta_{mH} \qquad (3.8)$$

$$NSHD_n^t = \sum_{k=1}^{K} NSHD_{kn}^t \qquad \forall n, \quad \beta_{mH} + \delta \le t \le T + \beta_{mH} + \delta \qquad (3.9)$$

$$NIH^t = \sum_{t=\beta_{mH}}^{\tau-\delta} NAHD_n^\tau - \sum_{t=\beta_{mH}+\delta}^{\tau}\sum_{k=1}^{K}\sum_{n=1}^{N} NSHD_{kn}^\tau \qquad \beta_{mH} + \delta \le t \le T + \beta_{mH} \qquad (3.10)$$

$$NIH^t \le NIH \qquad \beta_{mH} + \delta \le t \le T + \beta_{mH} \qquad (3.11)$$

$$NIH^t = 0 \qquad T + \beta_{mH} < t \le T + \beta_{mH} + \delta \qquad (3.12)$$

$$\gamma_m + \alpha_{mn} \le v_j \qquad \forall m, n, j \qquad (3.13)$$

$$\gamma_m + \beta_{mn} + \gamma_H \le v_j \qquad \beta_{mn} = \beta_{mH} + \delta + \beta_{Hn}, \ \forall m, n, j \qquad (3.14)$$

$$NTOD_{mn}^t \ge NSOD_{mn}^t / NOD_{mn} \qquad \forall m, n, t \qquad (3.15)$$

$$NTOH_m^t \ge NSOH_m^t / NOH_m \qquad \forall m, t \qquad (3.16)$$

$$NTHD_n^t \ge NSHD_n^t / NHD_n \qquad \forall n, t \qquad (3.17)$$

$$\sum_{n=1}^{N} NTOD_{mn}^t + NTOH_m^t \le HTO_m \qquad \forall m, t \qquad (3.18)$$

$$\sum_{n=1}^{N} NTHD_n^t \le NTH \qquad \forall t \qquad (3.19)$$

$$NSOD_{mn}^t \ge 0 \quad \text{and integer} \qquad \forall m, n, t \qquad (3.20)$$

$$NSOH_m^t \ge 0 \quad \text{and integer} \qquad \forall m, t \qquad (3.21)$$

$$NSHD_n^t \ge 0 \quad \text{and integer} \qquad \forall n, t \qquad (3.22)$$

$$NTOD_{mn}^t \ge 0 \quad \text{and integer} \qquad \forall m, n, t \qquad (3.23)$$

$$NTOH_m^t \ge 0 \quad \text{and integer} \qquad \forall m, t \qquad (3.24)$$

$$NTHD_n^t \ge 0 \quad \text{and integer} \qquad \forall n, t \qquad (3.25)$$

$$NSOD_{mn}^{tv_j} \geq 0 \quad \text{and integer} \quad \forall j,m,n,t \quad (3.26)$$

$$NSOH_{m}^{tv_j} \geq 0 \quad \text{and integer} \quad \forall j,m,t \quad (3.27)$$

$$NSHD_{n}^{tv_j} \geq 0 \quad \text{and integer} \quad \forall j,m,t \quad (3.28)$$

$$NAHD_{n}^{tv_j} \geq 0 \quad \text{and integer} \quad \forall j,m,t \quad (3.29)$$

$$NIO_{m}^{t} \geq 0 \quad \text{and integer} \quad \forall m,t \quad (3.30)$$

$$NIH^{t} \geq 0 \quad \text{and integer} \quad \forall t \quad (3.31)$$

The above integer programming model can be solved directly using off-shelf optimization software for problems considered in this research. If the size of the problems becomes much larger with large number of integer variables, efficient solution methods should be developed for solving such large problems.

In the next chapter, several numerical examples are provided to illustrate the model and its solutions.

Chapter Four
Numerical Examples and Analysis

This chapter presents several numerical examples and the analysis on the computational results. To demonstrate the developed model presented in Chapter Three, we consider a network with two origins O_1 and O_2, one hub H and two destinations D_1 and D_2 as shown in Figure 4.1. The example data are generated based on those given in Newman and Yano (2000) as shown in Table 4.1.

Table 4.1 Parameters for Test Problem Instances

Parameter	Range
Container arrival rate per day	0-65
Fixed cost at origin (direct train)($/train)	11000-15000
Fixed cost at origin (indirect train)($/train)	5000-8500
Fixed cost at hub ($/train)	6200-9800
Transportation cost ($/container)	40-100
Handling cost ($/container)	1.0-2.0
Inventory holding cost ($/container/day)	1.5-2

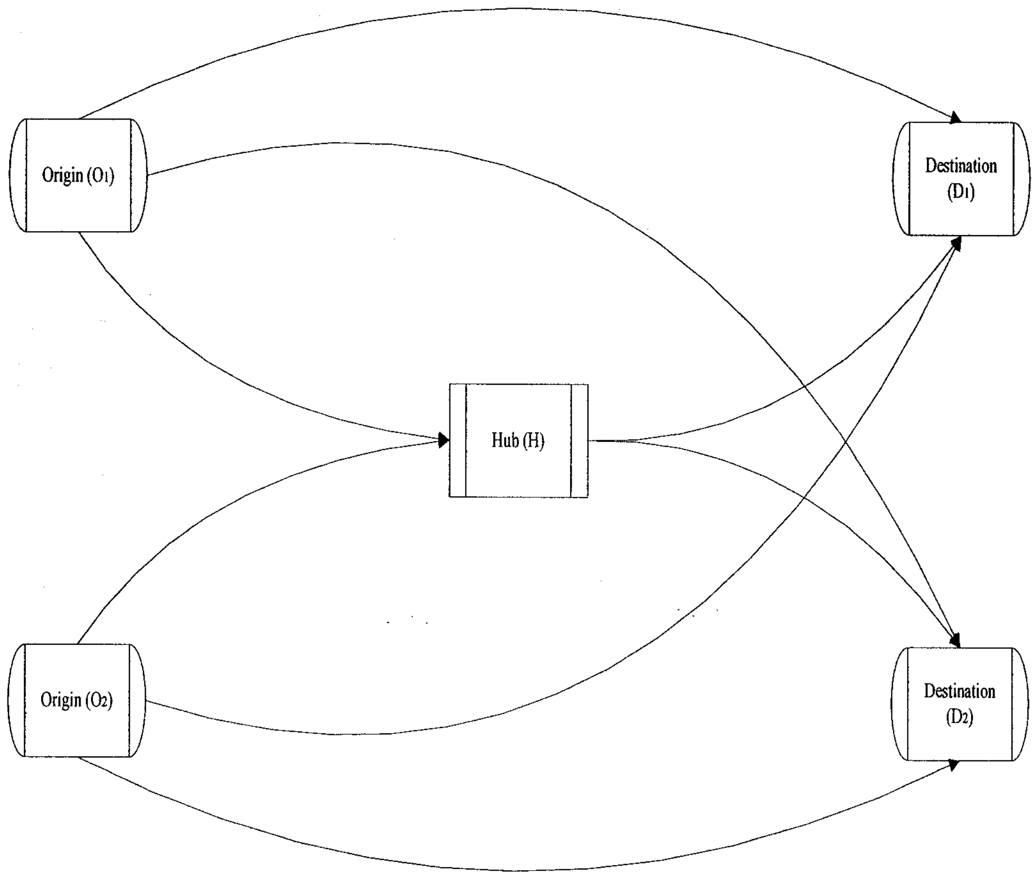


Figure 4.1 Network with Two Origins, One Hub and Two Destinations

4.1 Features of the Example Problems

In this example, containers can be sent directly from the two origins to the two destinations or indirectly passing the hub H. The direct travel time between any origin and any destination is four days, and indirect travel time is six days, which includes two days' travel time from the origin to the hub, three days' travel time from the hub to the destination, and one day of delay at the hub.

Figure 4.2 illustrates that containers are shipped directly from origins O_1 and O_2 to destinations D_1 and D_2 or indirectly passing the hub H. Data related to container transportation demands considered in this example are given in Table 4.2. It shows the time, the containers are ready to be transported, the size of the container sets, their origins and destinations of transportation and their due dates.

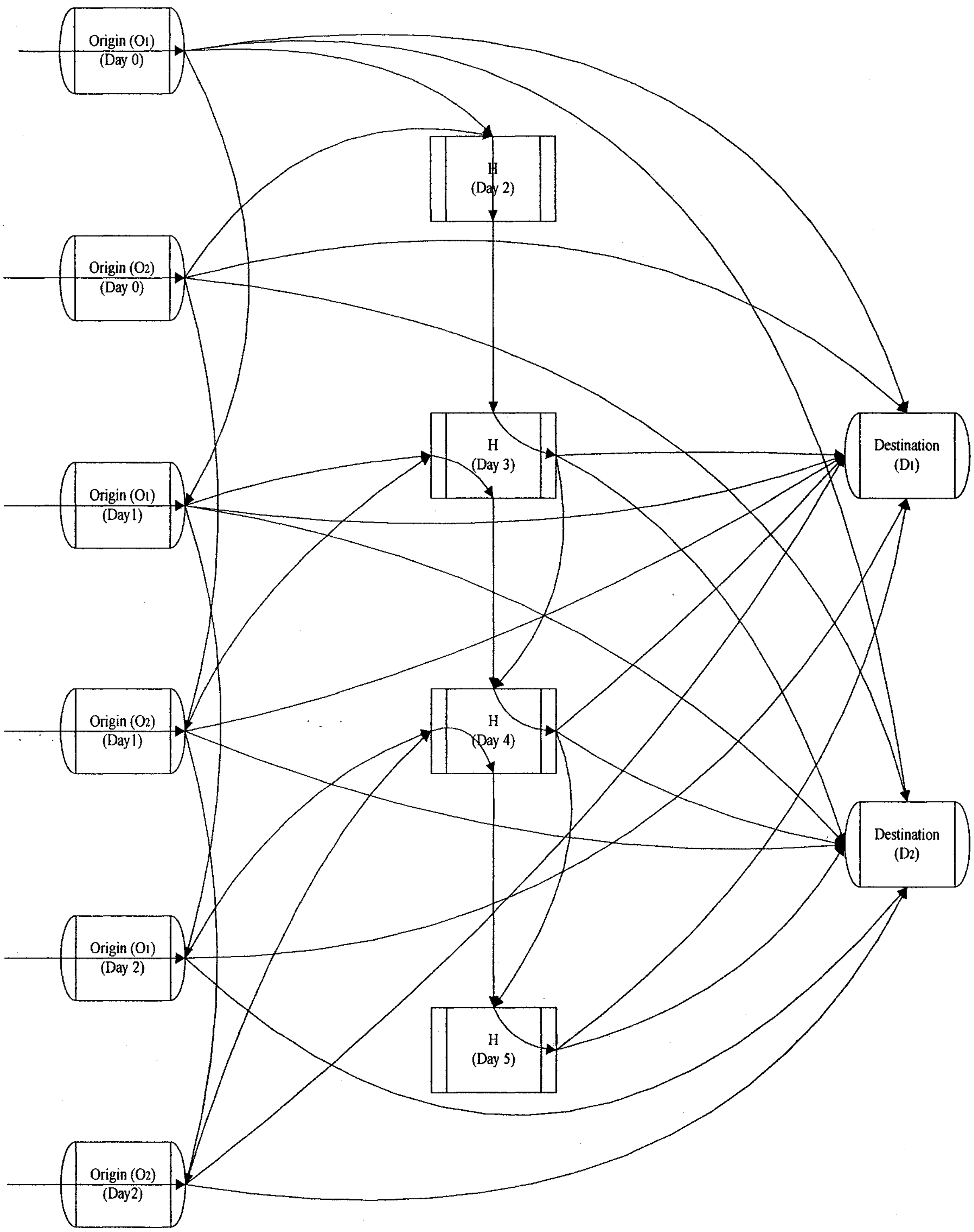


Figure 4.2 Figure of Network with Two Origins and Two Destinations

Table 4.2 Transportation Demand and Due Date

Available On	Container Set Number (j)	Number of Containers	Origin	Destination	Due Date (v_j)
Day 0	*1*	380	O_1	D_1	7
	2	220	O_1	D_1	5
	3	160	O_2	D_1	8
	4	150	O_1	D_2	6
	5	100	O_2	D_2	7
	6	160	O_2	D_2	4
Day 1	*7*	165	O_1	D_1	6
	8	100	O_1	D_1	5
	9	220	O_2	D_1	7
	10	160	O_1	D_2	8
	11	160	O_2	D_2	4
Day 2	*12*	130	O_1	D_1	5
	13	105	O_2	D_1	6
	14	120	O_1	D_2	4
	15	160	O_2	D_2	7

4.2 Determining Optimal Routing and Scheduling

Table 4.3 presents the data for the first example, Example I. In this example, we assume that there is no limit on the number of trains sent from the origins and the hub.

Example I

Table 4.3 Data for Example I

Train capacity (container/train)	100
Fixed cost from origin to destination ($/train)	11000
Fixed cost from origin to hub ($/train)	5500
Fixed cost from hub to destination ($/train)	6500
Transportation cost from origin to destination ($/container)	100
Transportation cost from origin to hub ($/container)	50
Transportation cost from hub to destination ($/container)	60
Handling cost at origin ($/container)	2
Handling cost at hub ($/container)	2
Handling cost at destination ($/container)	2
Inventory cost at origin ($/container/day)	1.5
Inventory cost at hub ($/container/day)	2

The integer programming model developed in Chapter Three was solved with the given data by LINGO 8.0 software on a PC computer. The optimal solution was obtained after 56 seconds of computation. The solution is presented in Table 4.4 - Table 4.6. The corresponding optimal objective function value is *540497.5*. Table 4.7 shows the routings of the containers considered in the Example.

Table 4.4 Optimal Solution for Example I: Day 0

From / Sent to	Origin O_1			Origin O_2		
	Set Number	Demand Transportation	Total Number	Set Number	Demand Transportation	Total Number
Destination D_1	$j = 1$	380	600	$j = 3$	100	100
	$j = 2$	220				
Destination D_2	$j = 4$	150	150	$j = 5$	40	200
				$J = 6$	160	
Inventory				$j = 3$	60	120
				$j = 5$	60	

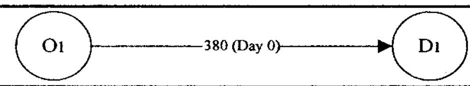
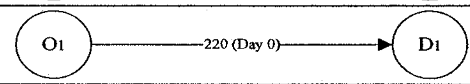
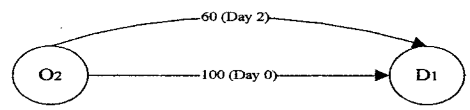
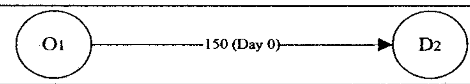
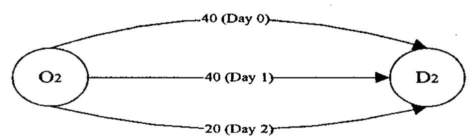
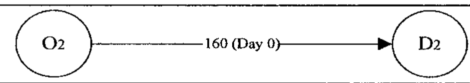
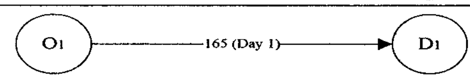
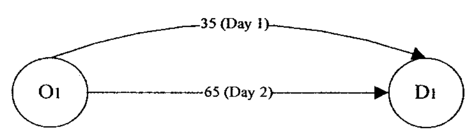
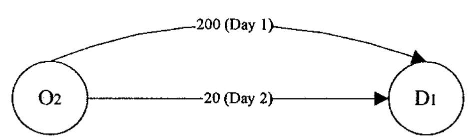
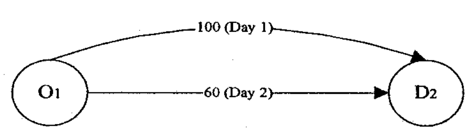
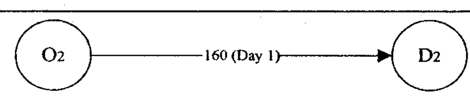
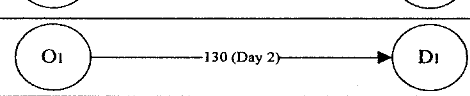
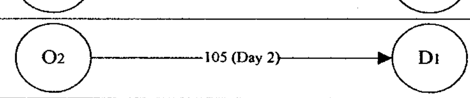
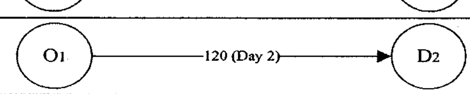
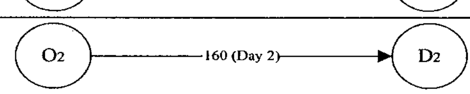
Table 4.5 Optimal Solution for Example I: Day 1

From / Sent to	Origin O_1			Origin O_2		
	Set Number	Demand Transportation	Total Number	Set Number	Demand Transportation	Total Number
Destination D_1	$j = 7$	165	200	$J = 9$	200	200
	$j = 8$	35				
Destination D_2	$j = 10$	100	100	$j = 5$	40	200
				$j = 11$	160	
Inventory	$j = 8$	65	125	$j = 3$	60	100
	$j = 10$	60		$j = 5$	20	
				$j = 9$	20	

Table 4.6 Optimal Solution for Example I: Day 2

From / Sent to	Origin O_1			Origin O_2		
	Set Number	Demand Transportation	Total Number	Set Number	Demand Transportation	Total Number
Destination D_1	$j = 8$	65	195	$j = 3$	60	185
	$j = 12$	130		$j = 9$	20	
				$j = 13$	105	
Destination D_2	$j = 10$	60	180	$j = 5$	20	180
	$j = 14$	120		$j = 15$	160	

Table 4.7 Routing and Scheduling of Example I

Container Set Number (*j*)	Route and Schedule	Transportation Time (Day)
1		2
2		2
3		4
4		2
5		4
6		2
7		2
8		3
9		3
10		3
11		2
12		2
13		2
14		2
15		2

From this result we can see that the total number of trains sent from origin O_1 is 15, the total number of trains sent from origin O_2 is 11, and there is no train sent from the Hub *H*. All containers arrive at their destinations before or on their due dates.

Example II

In this example, we investigate the situation when the fixed and variable costs are different from those in Example I. In particular, the fixed cost is reduced from $5500 and $6500. The new set of data is shown in Table 4.8.

With the information presented above used in the integer programming model, the optimal solution was found by LINGO 8.0. The optimal solution is presented in Table 4.9 - Table 4.14. Figure 4.3 is the sketch map. The corresponding optimal objective function value is *$472680.* Table 4.15 is the routing and scheduling of Example II.

From this result we can see that the total number of trains sent from origin O_1 is 15, the total number of trains sent from origin O_2 is 11, and the total number of trains sent from the Hub *H* is 15. So, the total number of trains used is 41. Comparing to the results in Table 4.7 of Example I, we can see that some of the containers are shipped indirectly.

Table 4.8 Data for Example II

The limited containers number of the train (container/train)	100
Fixed cost from origin to destination ($/train)	11000
Fixed cost from origin to hub ($/train)	2500
Fixed cost from hub to destination ($/train)	2800
Transportation cost from origin to destination ($/container)	50
Transportation cost from origin to hub ($/container)	25
Transportation cost from hub to destination ($/container)	28
Handling cost at origin ($/container)	25
Handling cost at hub ($/container)	22
Handling cost at destination ($/container)	20
Inventory cost at origin ($/container/day)	3
Inventory cost at hub ($/container/day)	2

Table 4.9 Optimal Solution for Example II: Day 0

From / Sent to	Origin O_1			Origin O_2		
	Set Number	Demand Transportation	Total Number	Set Number	Demand Transportation	Total Number
Destination D_1	$j = 2$	200	200			
Destination D_2				$j = 5$	40	200
				$j = 6$	160	
Hub H	$j = 1$	380	530	$j = 3$	160	180
	$j = 4$	150		$j = 5$	20	
Inventory	$j = 2$	20	20	$j = 5$	40	40

Table 4.10 Optimal Solution for Example II: Day 1

From / Sent to	Origin O_1			Origin O_2		
	Set Number	Demand Transportation	Total Number	Set Number	Demand Transportation	Total Number
Destination D_1	$j = 2$	20	100			
	$j = 7$	50				
	$j = 8$	30				
Destination D_2				$j = 5$	40	200
				$j = 11$	160	
Hub H	$j = 7$	115	195	$j = 9$	200	200
	$j = 10$	80				
Inventory	$j = 8$	70	150	$j = 9$	20	20
	$j = 10$	80				

Table 4.11 Optimal Solution for Example II: Day 2

From / Sent to	Origin O_1			Origin O_2		
	Set Number	Demand Transportation	Total Number	Set Number	Demand Transportation	Total Number
Destination D_1	$j = 8$	70	200			
	$j = 12$	130				
Destination D_2	$j = 10$	80	200			
	$j = 14$	120				
Hub H				$j = 9$	20	285
				$j = 13$	105	
				$j = 15$	160	

Table 4.12 Optimal Solution for Example II: Day 3

From / Sent to	Hub H		
	Set Number	Shipped Containers	Total Number
Destination D_1	$j = 1$	380	500
	$j = 3$	120	
Destination D_2	$j = 4$	150	270
	$j = 5$	120	
Inventory	$j = 3$	40	40

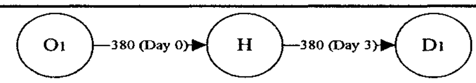
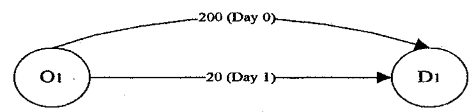
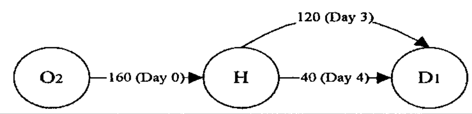
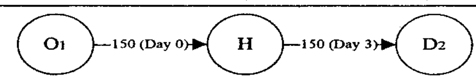
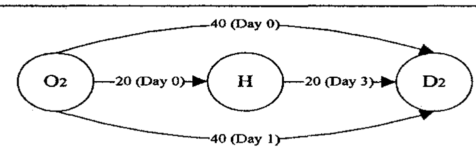
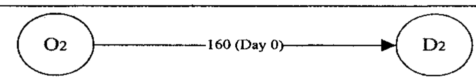
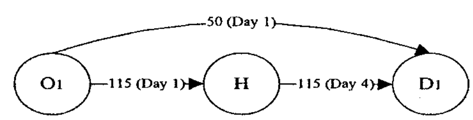
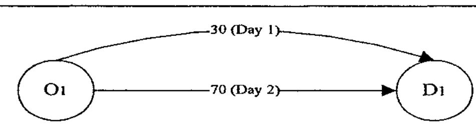
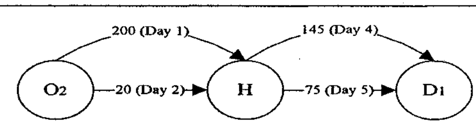
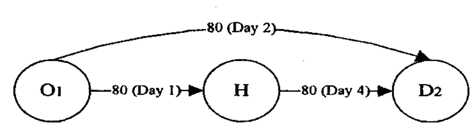
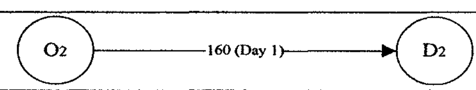
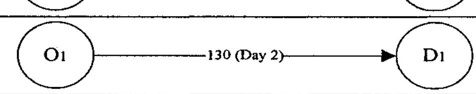
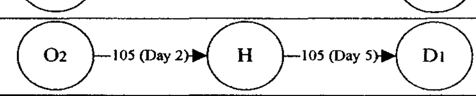
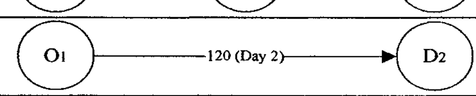
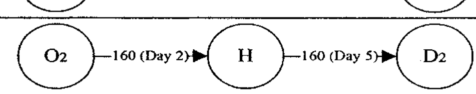
Table 4.13 Optimal Solution for Example II: Day 4

From / Sent to	Hub H		
	Set Number	Shipped Containers	Total Number
Destination D_1	$j = 3$	40	300
	$j = 7$	115	
	$j = 9$	145	
Destination D_2	$j = 10$	80	80
Inventory	$j = 9$	55	55

Table 4.14 Optimal Solution for Example II: Day 5

From / Sent to	Hub H		
	Set Number	Shipped Containers	Total Number
Destination D_1	$j = 9$	75	180
	$j = 13$	105	
Destination D_2	$j = 15$	160	160

Table 4.15 Routing and Scheduling of Example II

Container Set Number (*j*)	Route and Schedule	Transportation Time (Day)
1		6
2		3
3		7
4		6
5		6
6		2
7		6
8		3
9		7
10		6
11		2
12		2
13		6
14		2
15		6

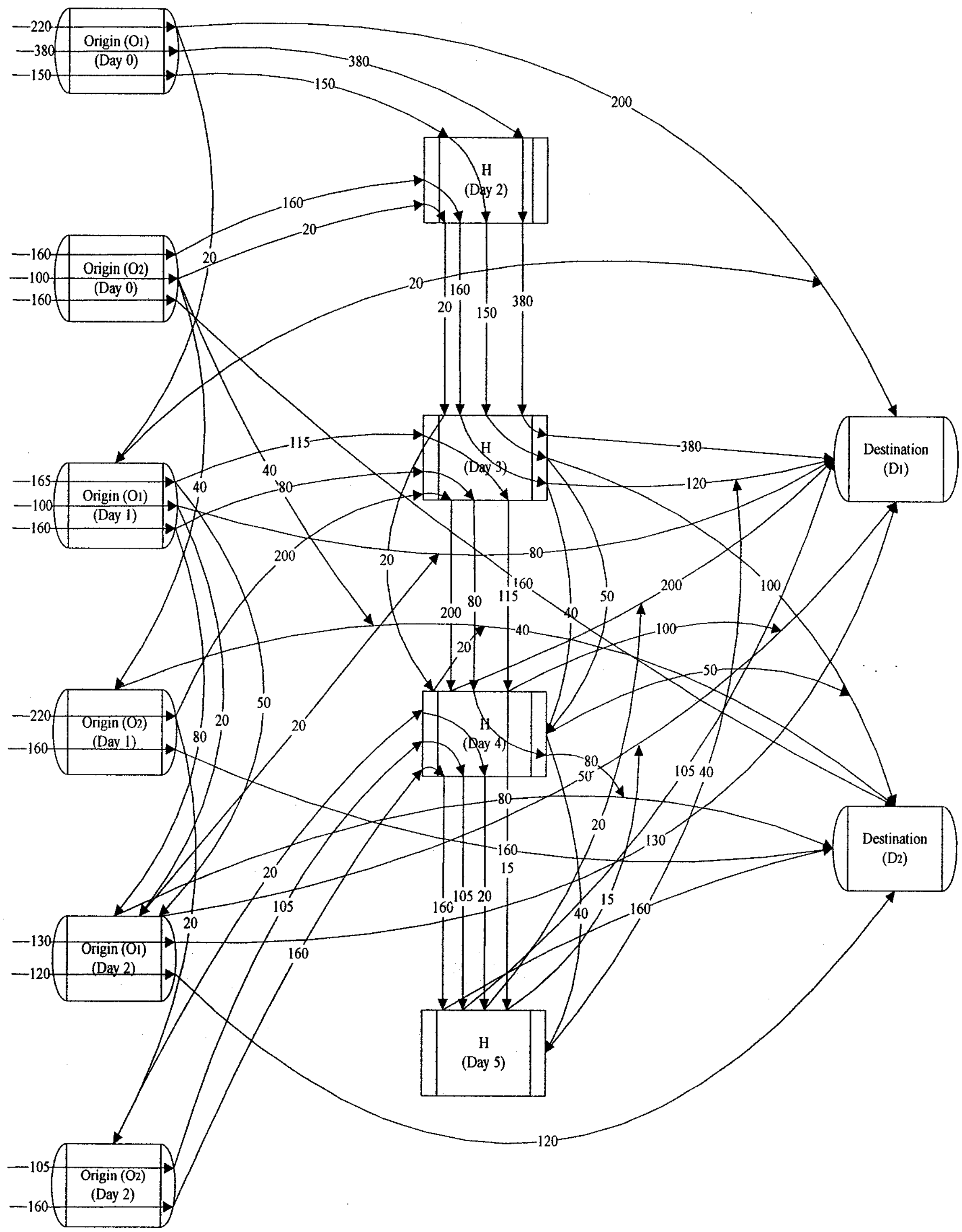


Figure 4.3 Sketch Map of Example II

4.3 Optimal Solution Analysis

As shown in Example I and Example II, the variation of the input data will change the optimal solution obtained from the model. For recognizing the critical factors on the economical performance, we will carry out the following experiments for the analysis on:

- ✧ Increase the number of available locomotives
- ✧ Increase the power of the locomotives
- ✧ Increase the inventory capacity at the origins and the hub

4.3.1 The Impact of the Total Number of trains

The number of locomotives is an important factor in railway transportation. In the following computational tests, we will investigate the impact of the number of locomotives on the quality of the solutions and the transportation costs.

Example III

Test I

In this test, we use the same data for Example II, except that the total number of trains sent from origin O_1, O_2 and H is 40. The optimal solution with this limit is presented in Table 4.16-Table 4.21. The corresponding optimal objective function value is *$475860.* Table 4.22 is the routing and scheduling of Test I.

Table 4.16 Optimal Solution for Test I: Day 0

From / Sent to	Origin O_1			Origin O_2		
	Set Number	Demand Transportation	Total Number	Set Number	Demand Transportation	Total Number
Destination D_1	$j = 1$	80	300			
	$j = 2$	220				
Destination D_2				$j = 5$	40	200
				$j = 6$	160	
Hub H	$j = 1$	300	450	$j = 3$	160	180
	$j = 4$	150		$j = 5$	20	
Inventory				$j = 5$	40	40

Table 4.17 Optimal Solution for Test I: Day 1

From / Sent to	Origin O_1			Origin O_2		
	Set Number	Demand Transportation	Total Number	Set Number	Demand Transportation	Total Number
Destination D_1	$j = 7$	70	100			
	$j = 8$	30				
Destination D_2				$j = 5$	40	200
				$j = 11$	160	
Hub H	$j = 7$	95	175	$j = 9$	200	200
	$j = 10$	80				
Inventory	$j = 8$	70	150	$j = 9$	20	20
	$j = 10$	80				

Table 4.18 Optimal Solution for Test I: Day 2

From / Sent to	Origin O_1			Origin O_2		
	Set Number	Demand Transportation	Total Number	Set Number	Demand Transportation	Total Number
Destination D_1	$j = 8$	70	200			
	$j = 12$	130				
Destination D_2	$j = 10$	80	200			
	$j = 14$	120				
Hub H				$j = 9$	20	285
				$j = 13$	105	
				$j = 15$	160	

Table 4.19 Optimal Solution for Test I: Day 3

From / Sent to	Hub H		
	Set Number	Shipped Containers	Total Number
Destination D_1	$j = 1$	300	400
	$j = 3$	100	
Destination D_2	$j = 4$	150	170
	$j = 5$	20	
Inventory	$j = 3$	60	60

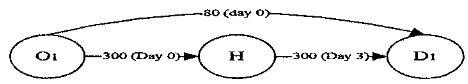
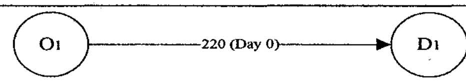
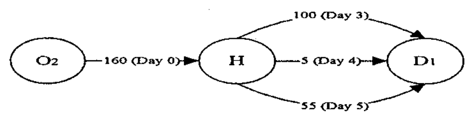
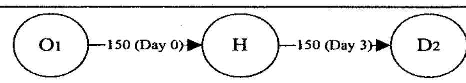
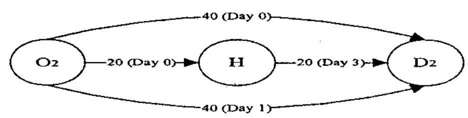
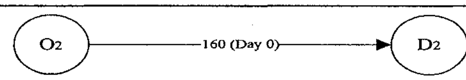
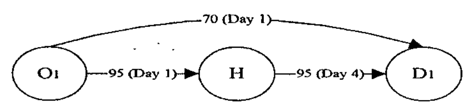
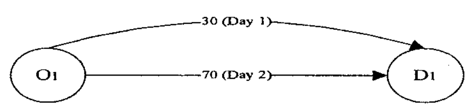
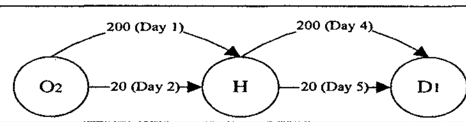
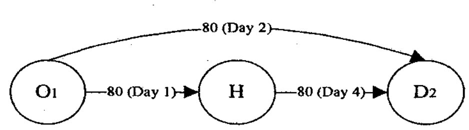
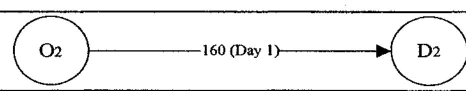
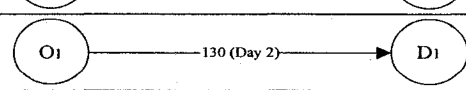
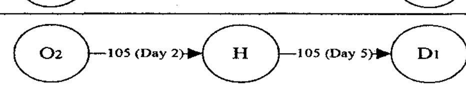
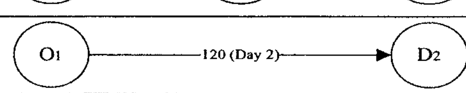
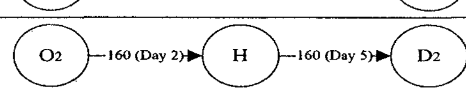
Table 4.20 Optimal Solution for Test I: Day 4

From / Sent to	Hub H		
	Set Number	Shipped Containers	Total Number
Destination D_1	$j = 3$	5	300
	$j = 7$	95	
	$j = 9$	200	
Destination D_2	$j = 10$	80	80
Inventory	$j = 3$	55	55

Table 4.21 Optimal Solution for Test I: Day 5

From / Sent to	Hub H		
	Set Number	Shipped Containers	Total Number
Destination D_1	$j = 3$	55	180
	$j = 9$	20	
	$j = 13$	105	
Destination D_2	$j = 15$	160	160

Table 4.22 Routing and Scheduling of Test I

Container Set Number (*j*)	Route and Schedule	Transportation Time (Day)
1		6
2		2
3		8
4		6
5		6
6		2
7		6
8		3
9		7
10		6
11		2
12		2
13		6
14		2
15		6

Test II

In this test run, the only difference from Test I is that the total number of trains sent from origin O_1, O_2 and H is 39. The optimal solution was found after 3 minutes and 34 seconds of computation. The optimal solution is presented in Table 4.23-Table 4.28. The corresponding optimal objective function value is *$479060*. Table 4.29 is the routing and scheduling of Test II.

Table 4.23 Optimal Solution for Test II: Day 0

From / Sent to	Origin O_1			Origin O_2		
	Set Number	Demand Transportation	Total Number	Set Number	Demand Transportation	Total Number
Destination D_1	$j = 1$	180	400			
	$j = 2$	220				
Destination D_2				$j = 5$	40	200
				$j = 6$	160	
Hub H	$j = 1$	200	350	$j = 3$	160	180
	$j = 4$	150		$j = 5$	20	
Inventory				$j = 5$	40	40

Table 4.24 Optimal Solution for Test II: Day 1

From / Sent to	Origin O_1			Origin O_2		
	Set Number	Demand Transportation	Total Number	Set Number	Demand Transportation	Total Number
Destination D_1	$j = 8$	100	100			
Destination D_2				$j = 5$	40	200
				$j = 11$	160	
Hub H	$j = 7$	95	175	$j = 9$	200	200
	$j = 10$	80				
Inventory	$j = 7$	70	150	$j = 9$	20	20
	$j = 10$	80				

Table 4.25 Optimal Solution for Test II: Day 2

From / Sent to	Origin O_1			Origin O_2		
	Set Number	Demand Transportation	Total Number	Set Number	Demand Transportation	Total Number
Destination D_1	$j = 7$	70	200			
	$j = 12$	130				
Destination D_2	$j = 10$	80	200			
	$j = 14$	120				
Hub H				$j = 9$	20	285
				$j = 13$	105	
				$j = 15$	160	

Table 4.26 Optimal Solution for Test II: Day 3

From / Sent to	Hub H		
	Set Number	Shipped Containers	Total Number
Destination D_1	$j = 1$	200	300
	$j = 3$	100	
Destination D_2	$j = 4$	150	170
	$j = 5$	20	
Inventory	$j = 3$	60	60

Table 4.27 Optimal Solution for Test II: Day 4

From / Sent to	Hub H		
	Set Number	Shipped Containers	Total Number
Destination D_1	$j = 3$	5	300
	$j = 7$	95	
	$j = 9$	200	
Destination D_2	$j = 10$	80	80
Inventory	$j = 3$	55	55

Table 4.28 Optimal Solution for Test II: Day 5

From / Sent to	Hub H		
	Set Number	Shipped Containers	Total Number
Destination D_1	$j = 3$	55	180
	$j = 8$	20	
	$j = 13$	105	
Destination D_2	$j = 15$	160	160

Table 4.29 Routing and Scheduling of Test II

Container Set Number (*j*)	Route and Schedule	Transportation Time (Day)
1	O_1 —180 (day 0)→ D_1; O_1 —200 (Day 0)→ H —200 (Day 3)→ D_1	6
2	O_1 —220 (Day 0)→ D_1	2
3	O_2 —160 (Day 0)→ H; H —100 (Day 3)→ D_1; H —5 (Day 4)→ D_1; H —55 (Day 5)→ D_1	8
4	O_1 —150 (Day 0)→ H —150 (Day 3)→ D_2	6
5	O_2 —40 (Day 0)→ D_2; O_2 —20 (Day 0)→ H —20 (Day 3)→ D_2; O_2 —40 (Day 1)→ D_2	6
6	O_2 —160 (Day 0)→ D_2	2
7	O_1 —70 (Day 1)→ D_1; O_1 —95 (Day 1)→ H —95 (Day 4)→ D_1	6
8	O_1 —100 (Day 1)→ D_1	2
9	O_2 —200 (Day 1)→ H —200 (Day 4)→ D_1; O_2 —20 (Day 2)→ H —20 (Day 5)→ D_1	7
10	O_1 —80 (Day 2)→ D_2; O_1 —80 (Day 1)→ H —80 (Day 4)→ D_2	6
11	O_2 —160 (Day 1)→ D_2	2
12	O_1 —130 (Day 2)→ D_1	2
13	O_2 —105 (Day 2)→ H —105 (Day 5)→ D_1	6
14	O_1 —120 (Day 2)→ D_2	2
15	O_2 —160 (Day 2)→ H —160 (Day 5)→ D_2	6

Other Tests

Several other test runs were conducted based on the same input data with the total number of trains sent from origin O_1, O_2 and H varying from 25 to 41. The optimal solutions for these run are presented in Table 4.30.

From Table 4.30 we can see that (1) when the total numbers of trains' decreases, the number of trains' from the hub decreases, the number of trains' from the origins do not change and the total cost increases as shown in Figure 4.4. We can increase investment of purchasing locomotive to decrease transportation cost. (2) Hub is very important. (3) We can increase handling efficiency of locomotives and transportation efficiency. (4) When the total number of trains is 26, all of containers are shipped directly from origins to destinations. Table 4.31 - Table 4.33 show the optimal solution in this case. (5) When the total number of trains is less than 26, the problem is infeasible. Table 4.34 is the routing and scheduling of the test when total number of trains is 26.

Table 4.30 Summary of Example III

Total Number of trains	Number of trains in O_1	Number of trains in O_2	Number of trains in H	Total Cost ($)	Calculation Time
41	15	11	15	472680	0:10
40	15	11	14	475860	1:37
39	15	11	13	479060	3:34
38	15	11	12	482260	10:58
37	15	11	11	485460	12:05
36	15	11	10	488660	11:49
35	15	11	9	491860	26:02
34	15	11	8	495060	26:52
33	15	11	7	498260	24:15
32	15	11	6	501460	13:34
31	15	11	5	504660	8:25
30	15	11	4	507920	2:01
29	15	11	3	511240	0:39
28	15	11	2	515055	0:10
27	15	11	1	518755	0:04
26	15	11	0	523585	0:01
25	Infeasible				

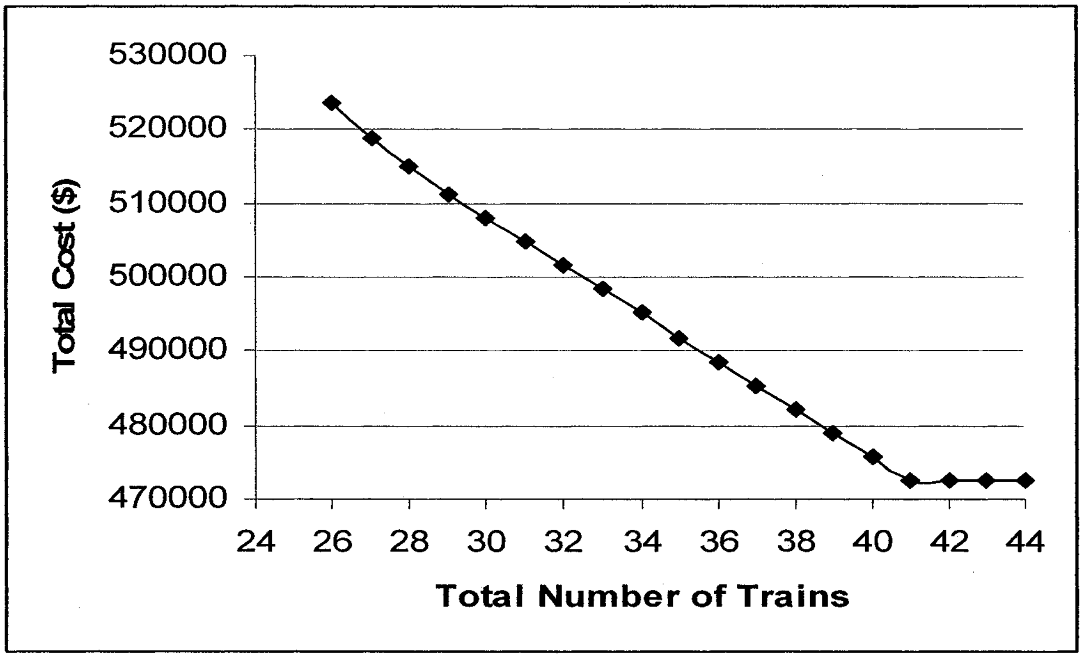


Figure 4.4 Total Cost vs. Total Number of Trains

Table 4.31 Optimal Solution When the Total Number of Trains is 26: Day 0

From / Sent to	Origin O_1			Origin O_2		
	Set Number	Demand Transportation	Total Number	Set Number	Demand Transportation	Total Number
Destination D_1	$j = 1$	380	600	$j = 3$	100	100
	$j = 2$	220				
Destination D_2	$j = 4$	100	100	$j = 5$	40	200
				$J = 6$	160	
Inventory	$j = 4$	50	50	$j = 3$	60	120
				$j = 5$	60	

Table 4.32 Optimal Solution When the Total Number of Trains is 26: Day 1

From / Sent to	Origin O_1			Origin O_2		
	Set Number	Demand Transportation	Total Number	Set Number	Demand Transportation	Total Number
Destination D_1	$j = 7$	165	200	$j = 3$	60	200
	$j = 8$	35		$j = 9$	140	
Destination D_2	$j = 4$	40	200	$j = 5$	40	200
	$j = 10$	160		$j = 11$	160	
Inventory	$j = 4$	10	75	$j = 5$	20	100
	$j = 8$	65		$j = 9$	80	

Table 4.33 Optimal Solution When the Total Number of Trains is 26: Day 2

From / Sent to	Origin O_1			Origin O_2		
	Set Number	Demand Transportation	Total Number	Set Number	Demand Transportation	Total Number
Destination D_1	$j = 8$	65	195	$j = 9$	80	185
	$j = 12$	130		$j = 13$	105	
Destination D_2	$j = 4$	10	130	$j = 5$	20	180
	$j = 14$	120		$j = 15$	160	

Table 4.34 Routing and Scheduling of Test When the Total Number of Trains is 26

Container Set Number (j)	Route and Schedule	Transportation Time (Day)
1	O1 → D1: 380 (Day 0)	5
2	O1 → D1: 220 (Day 0)	3
3	O2 → D1: 100 (Day 0); 60 (Day 1)	6
4	O1 → D2: 100 (Day 0); 40 (Day 1); 10 (Day 2)	5
5	O2 → D2: 40 (Day 0); 40 (Day 1); 20 (Day 2)	5
6	O2 → D2: 160 (Day 0)	2
7	O1 → D1: 165 (Day 1)	5
8	O1 → D1: 35 (Day 1); 65 (Day 2)	3
9	O2 → D1: 140 (Day 1); 80 (Day 2)	6
10	O1 → D2: 160 (Day 1)	5
11	O2 → D2: 160 (Day 1)	2
12	O1 → D1: 130 (Day 2)	2
13	O2 → D1: 105 (Day 2)	5
14	O1 → D2: 120 (Day 2)	2
15	O2 → D2: 160 (Day 2)	5

4.3.2 The Impact of the Power of the Locomotives

We use the next example, Example IV, to investigate the impact of the power of the locomotives on the problem solution. The power of a locomotive can vary in terms of the number of containers it can take. In this example, we use the same data as in example II except assuming that the locomotives can take 100 to 125 containers. The solutions are shown in Table 4.35. Figure 4.5 shows the relationship between the total operation cost and the number of containers taken by the trains. As we can see, when the capacity of train increases, the total operation cost decreases and the total number of trains decreases.

Example IV

Table 4.35 Optimal Solutions with Different Train Capacity

Maximum Containers Number of Train	Number of Trains at O_1	Number of Trains at O_2	Number of Trains at H	Total Number of Trains	Total Cost ($)	Calculation Time
100	15	11	15	41	472680	0:10
105	14	11	14	39	466470	0:16
110	14	10	13	37	461070	0:04
115	13	10	13	36	452620	0:01
120	12	10	13	35	442190	0:04
125	12	10	12	34	438660	0:05

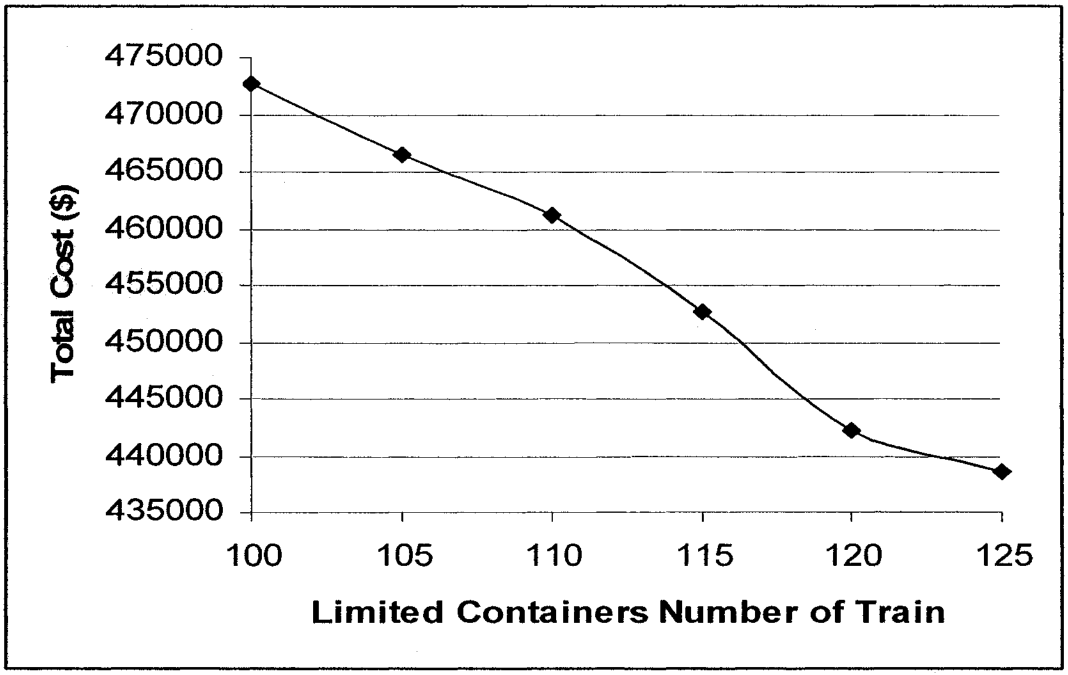


Figure 4.5 Total Cost vs. Limited Containers Number of Train

4.3.3 Increasing Inventor Capacity at an Origin or Hub

In the next example, Example V, we vary the capacity of the inventory level at an origin or at the hub. We assume that the number of containers that can be held at these locations varies from 90 to 200. The solutions are presented in Table 4.36. Figure 4.6 shows the relationship between the total operation cost and the capacities of inventory. As we can see, when the capacities of inventory increases, the total operation costs decrease and the total number of trains decreases. The cost deduction is less significant when the inventory level is more than 80 containers. When the inventory level is more than 150 containers, the total operation cost and the total number of trains do not change.

<u>Example V</u>

Table 4.36 Optimal Solutions with Different Capacities of Inventory

Capacity of Inventory at an Origin or the Hub	Number of Trains at O_1	Number of Trains at O_2	Number of Trains at H	Total Number of Trains	Total Cost ($)	Calculation Time
200	15	11	15	41	472680	0:18
190	15	11	15	41	472680	0:18
180	15	11	15	41	472680	0:20
170	15	11	15	41	472680	0:09
160	15	11	15	41	472680	0:10
150	15	11	15	41	472680	0:09
140	15	11	15	41	472930	0:05
130	15	11	15	41	473150	0:03
120	15	11	15	41	473390	0:02
110	15	11	15	41	473630	0:01
100	15	11	15	41	473870	0:01
90	15	11	15	41	474115	0:00
80	15	11	15	41	474430	0:01
70	16	11	15	42	477195	0:01
60	16	11	16	43	479900	0:02
50	16	11	16	43	480140	0:01
40	17	11	16	44	482885	0:02
30	17	11	17	45	486040	0:02
20	17	11	18	46	489190	0:02
10	17	13	17	47	497710	0:04
0	17	13	18	48	500950	0:01

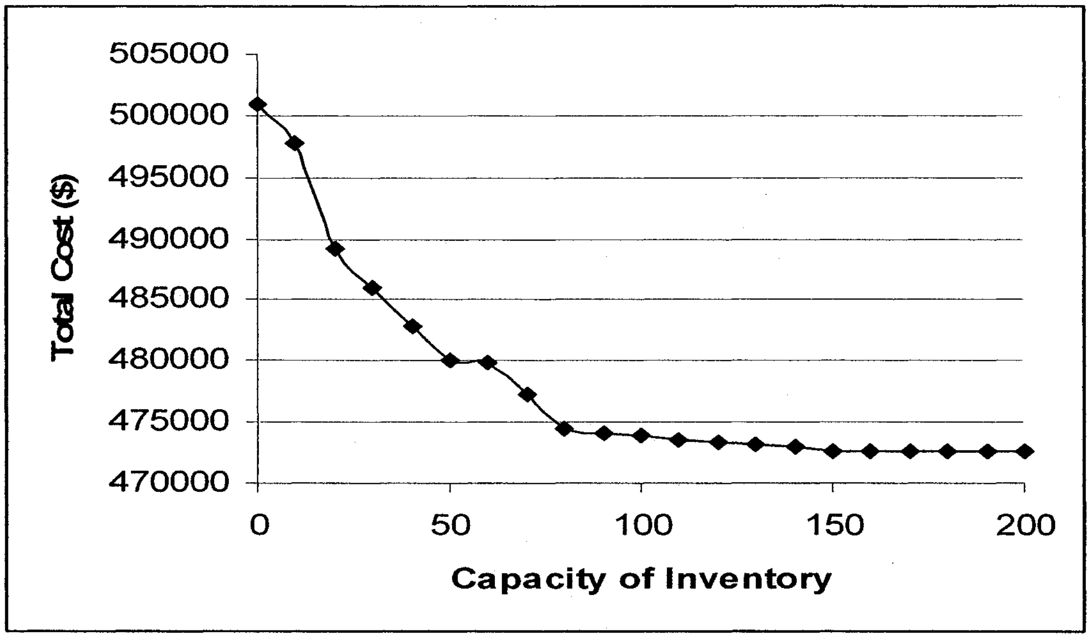


Figure 4.6 Total Cost vs. Capacities of Inventory

4.4 Optimal Combinations of the Tested Parameters

From the results of the previous examples, we selected a new set of the values of the parameters as shown as in Table 4.37. The optimal solution based on this set of data is presented in Table 4.38-Table 4.43. The corresponding optimal objective function value is *$474430*. Table 4.44 is the routing and scheduling for this example, Example VI.

Example VI

Table 4.37 Data for Example VI

The limited containers number of the train (container/train)	100
Total number of trains (train)	41
Capacity of inventory at an origin or the hub (container)	80
Fixed cost from origin to destination ($/train)	11000
Fixed cost from origin to hub ($/train)	2500
Fixed cost from hub to destination ($/train)	2800
Transportation cost from origin to destination ($/container)	50
Transportation cost from origin to hub ($/container)	25
Transportation cost from hub to destination ($/container)	28
Handling cost at origin ($/container)	25
Handling cost at hub ($/container)	22
Handling cost at destination ($/container)	20
Inventory cost at origin ($/container/day)	3
Inventory cost at hub ($/container/day)	2

Table 4.38 Optimal Solution for Example VI: Day 0

<table>
<tr><th rowspan="2">From / Sent to</th><th colspan="3">Origin O_1</th><th colspan="3">Origin O_2</th></tr>
<tr><th>Set Number</th><th>Demand Transportation</th><th>Total Number</th><th>Set Number</th><th>Demand Transportation</th><th>Total Number</th></tr>
<tr><td>Destination D_1</td><td>$j = 2$</td><td>200</td><td>200</td><td></td><td></td><td></td></tr>
<tr><td rowspan="2">Destination D_2</td><td></td><td></td><td rowspan="2"></td><td>$j = 5$</td><td>40</td><td rowspan="2">200</td></tr>
<tr><td></td><td></td><td>$j = 6$</td><td>160</td></tr>
<tr><td rowspan="2">Hub H</td><td>$j = 1$</td><td>360</td><td rowspan="2">500</td><td>$j = 3$</td><td>160</td><td rowspan="2">180</td></tr>
<tr><td>$j = 4$</td><td>140</td><td>$j = 5$</td><td>20</td></tr>
<tr><td rowspan="3">Inventory</td><td>$j = 1$</td><td>20</td><td rowspan="3">50</td><td>$j = 5$</td><td>40</td><td rowspan="3">40</td></tr>
<tr><td>$j = 2$</td><td>20</td><td></td><td></td></tr>
<tr><td>$j = 4$</td><td>10</td><td></td><td></td></tr>
</table>

Table 4.39 Optimal Solution for Example VI: Day 1

From / Sent to	Origin O_1			Origin O_2		
	Set Number	Demand Transportation	Total Number	Set Number	Demand Transportation	Total Number
Destination D_1	$j = 2$	20	100			
	$j = 8$	80				
Destination D_2				$j = 5$	40	200
				$j = 11$	160	
Hub H	$j = 7$	135	295	$j = 9$	200	200
	$j = 10$	160				
Inventory	$j = 1$	20	80	$j = 9$	20	20
	$j = 4$	10				
	$j = 7$	30				
	$j = 8$	20				

Table 4.40 Optimal Solution for Example VI: Day 2

From / Sent to	Origin O_1			Origin O_2		
	Set Number	Demand Transportation	Total Number	Set Number	Demand Transportation	Total Number
Destination D_1	$j = 1$	20	200			
	$j = 7$	30				
	$j = 8$	20				
	$j = 12$	130				
Destination D_2	$j = 4$	10	130			
	$j = 14$	120				
Hub H				$j = 9$	20	285
				$j = 13$	105	
				$j = 15$	160	

Table 4.41 Optimal Solution for Example VI: Day 3

From / Sent to	Hub H		
	Set Number	Shipped Containers	Total Number
Destination D_1	$j = 1$	360	500
	$j = 3$	140	
Destination D_2	$j = 4$	100	100
Inventory	$j = 3$	20	80
	$j = 4$	40	
	$j = 5$	20	

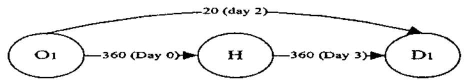
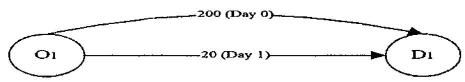
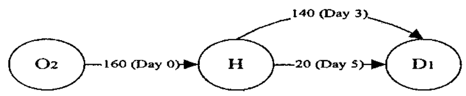
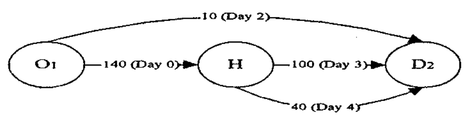
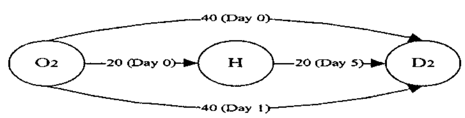
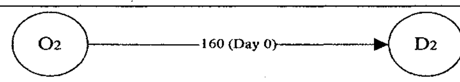
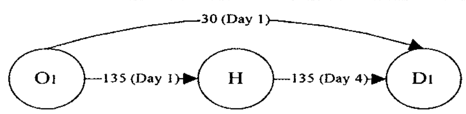
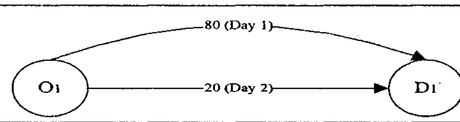
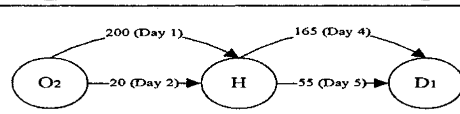
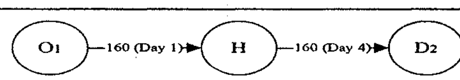
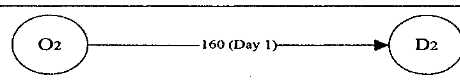
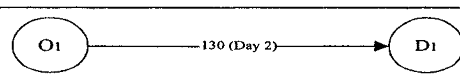
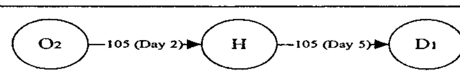
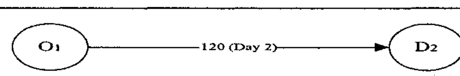
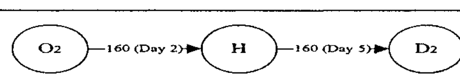
Table 4.42 Optimal Solution for Example VI: Day 4

Sent to \ From	Hub H		
	Set Number	Shipped Containers	Total Number
Destination D_1	$j = 7$	135	300
	$j = 9$	165	
Destination D_2	$j = 4$	40	200
	$j = 10$	160	
Inventory	$j = 3$	20	75
	$j = 5$	20	
	$j = 9$	35	

Table 4.43 Optimal Solution for Example VI: Day 5

Sent to \ From	Hub H		
	Set Number	Shipped Containers	Total Number
Destination D_1	$j = 3$	20	180
	$j = 9$	55	
	$j = 13$	105	
Destination D_2	$j = 5$	20	180
	$j = 15$	160	

Table 4.44 Routing and Scheduling of Example VI

Container Set Number (j)	Route and Schedule	Transportation Time (Day)
1		5
2		3
3		6
4		5
5		5
6		2
7		5
8		3
9		6
10		5
11		2
12		2
13		5
14		2
15		5

4.5 Summary

In this chapter, the integer programming model for the line-haul portion of intermodal transportation was tested by several realistic example problems using several data sets modified after published cases. Using a network with two origins, one hub and two destinations, we carried out several experiments with vary number of locomotives, locomotive power and inventor capacity at an origin or the hub. The main observations from these experiments are:

(1) Increasing the number of locomotives properly can decrease the total operation cost. When the total number of trains decreases, the number of trains in the hub is decreased, the number of trains in the origins does not change and total cost increases. The hub is very important in intermodal freight transportation. Operation cost will increase greatly without the hub. One should also consider more investment on locomotive to decrease total transportation cost.

(2) Increasing locomotive power is also effective to decrease the total cost. When the train capacity increases, the total operation cost decreases and the total number of trains decreases.

(3) When the inventory level increases, the total operation cost decreases and the total number of trains decreases. The cost reduction is less significant when the inventory

capacity exceeds certain level.

Chapter Five
Conclusions and Future Research

In this chapter, we first briefly summarize the research done in this thesis and then discuss some future work topics in this area.

5.1 Summary

In this thesis, an integer programming model was developed to provide optimal solutions to the problem of determining train scheduling and container routing decisions in an intermodal transportation system. The model can be solved to obtain optimal solutions of different problem settings of practical sizes within reasonable computation time on PC computers. The main contributions of this work include:

(1) The development of a generic mathematical model to find optimal solutions for a certain type of problems arising in supply chain management.

(2) Solving realistic problems with hypothetical data sets.

(3) Experimentation in investigating different features of the model and the impacts

of various factors on the optimal solutions. The investigated factors include capacity of railway network, power of trains and inventory capacity limits at the origin and the hub in the considered network.

To our knowledge, this is a new attempt in modeling this type of problems and there are no existing model and computational results in the current literature.

The developed model is not limited to applications within the railway industry. The model can be applied to solving problems in other transportation systems involving tradeoffs between direct transportation and indirect transportation through one or more hubs with demand being freight or passengers. For example, commercial airlines face decisions regarding the time-cost trade-off of sending their passengers on direct versus connecting flights. Express delivery companies may route their packages through hubs to save cost. The developed model could be extended without much difficulty to solving problems in those applications. Similar or more extensive analysis can be performed to obtain more conclusive results in solving such problems. This research also demonstrates that mathematical programming is a powerful and effective approach in obtain solutions for complicated optimization problems such as railway operation scheduling problems studied in this thesis. It requires careful data collection and data processing in conjunction with model developing. The optimal solutions reached by solving the validated model with correct data sets can provide the users with references and guidelines in practical operations.

5.2. Future Work

The model developed in this thesis is based on a realistic but rather simple network with a single hub. This is based on a typical railway segment of intermodal transportation. The model can be extended to include more hub locations in more general transportation systems including ocean, highway and air transportation system segments. As for the presented model itself, it may be further extended to allow certain flexibility of the train capacity. That is, the number of containers in a train is able to take may be within a certain limited range, rather than being rigid with a fixed number. Such flexibility normally exists in real world railway operations. Another realistic consideration may be built into the model is the flexibility of the due dates for container transportation. This can be imposed by associating penalty terms with the lateness of container deliveries at the destinations.

The above mentioned model modification and variations will lead to more complexity of the developed model. The increase of the complexity will be more significant if the model is modified to solving problems with multiple hubs in the network. In this research, we used an off-shelf optimization software to solve the example problems directly. The size of the model and the number of integer variables do not require excessive computational time using the direct solution approach to obtain optimal solutions for various problems with assumptions consistent with practical railway operations. However, if the above mentioned model modifications take place, effective and efficient solution methods must be developed to overcome the computational deficiencies resulted from the model complexity. Developing such methods based on proven and new heuristic algorithms to solve the more complicated models is also one of the tasks in future research in this area.

References

1. Aggarwal, C. C., Orlin, J. B. and Tai, R. P., 1994. Optimized crossover for the independent set problem. Research Report. Operations Research Center, MIT, Cambridge, MA.

2. Ballis, A. and Golias, J., 2002. Comparative evaluation of existing and innovative rail–road freight transport terminals. Transportation Research, Part A, No.36, pp. 593-611.

3. Barnhart, C., Hane, C. and Vance, P., 1998. Using branch-and-price to solve origin–destination integer multicommodity flow problem. Operations Research, Vol. 32, No. 3, pp.208-220.

4. Bixby, R. E. and Lee, E. K., 1998. Solving a truck dispatching scheduling problem using branch-and-cut. Operations Research, Vol. 46, No. 3, pp. 355-367.

5. Bontekoning, Y. M., Macharis, C. and Trip, J. J., 2004. Is a new applied transportation research field emerging? – A review of intermodal rail–truck freight transport literature. Transportation Research, Part A, No. 38, pp.1-34.

6. Caprara, A., Fischetti, M. and Toth, P., 2002. Modeling and solving the train timetabling problem. Operations Research, Vol. 50, No. 5, pp. 851-861.

7. Choong, S., Cole, M.H. and Kutanoglu, E., 2002. Empty container management for intermodal transportation networks. Transportation Research, Part E, Vol. 38, No. 6, pp. 423-438.

8. Cordeau, J. F., Toth, P. and Vigo, D., 1998. A survey of optimisation models for train

routing and scheduling. Transportation Science, Vol. 32, No. 4, pp. 380-404.

9. Forkenbrock, D. J., 2001. Comparison of external costs of rail and truck freight transportation. Transportation Research, Part A, No.35, pp. 321-337.
10. Geng, G. and Li, L. X., 2001. Scheduling railway freight cars. Knowledge-Based Systems, No.14, pp. 289-297.
11. Golob, T. F. and Regan, A. C., 2002. Trucking industry adoption of information technology: a multivariate discrete choice model. Transportation Research, Part C, No.10, pp. 205-228.
12. Golob, T. F. and Regan, A. C., 2000. Trucking industry perceptions of congestion problems and potential solutions in maritime intermodal operations in California. Transportation Research, Part A, No.34, pp. 587-605.
13. Golob, T. F. and Regan. A. C., 2000. The perceived usefulness of different sources of traffic information to trucking operations. Transportation Research, Part E, No.34, pp. 97-116.
14. Gorman, M. F., 1998a. An operating plan model improves service design at Santa Fe railway. Interfaces, Vol. 28, No. 4, pp. 1-12.
15. Gorman, M. F., 1998b. An Application of Genetic and Tabu Search to the Train Scheduling Problem. Annals of Operations Research, No. 78, pp. 51-69.
16. Holmberg, K., Joborn, M. and Lundgren, J. T., 1998. Improved empty freight car distribution. Transportation Science, Vol. 32, No. 2, pp. 163-173.
17. Kozan, E., 2000. Optimizing container transfers at multimodal terminals. Mathematical and computer modeling, No. 31, pp. 235-245.

18. Kraft, E. R., 2002. Scheduling railway freight delivery appointments using a bid price approach. Transport Management, No.36, pp.145-165.

19. Lingaya, N. and Cordeau, J. F., 2002. Operational car assignment at VIA Rail Canada. Transportation Research, Part B, Vol. 36, No.9, pp. 755-778.

20. Macharis, C. and Bontekoning, Y. M., 2004. Opportunities for OR in intermodal freight transport research: A review. European Journal of Operational Research, No.153, pp. 400-416.

21. Mancuso, P. and Reverberi, P., 2003. Operating costs and market organization in railway services. The case of Italy, 1980-1995. Transportation Research, Part B, No. 37, pp. 43-61.

22. Marin, A. and Salmeron, J., 1996. Tactical design of freight networks. Part I: Exact and heuristic methods. European Journal of Operational Research, Vol. 90, No. 1, pp. 26-44.

23. Morlok, E. K. and Spasovic, L. N., 1995. Approaches for improving drayage in rail truck. Research Report, University of Pennsylvania.

24. Morlok, E. K. and Chang, D. J., 2004. Measuring capacity flexibility of a transportation system. Transportation Research, Part A, No. 38, pp.405-420.

25. Newman, A. M. and Yano, C. A., 2000. Scheduling direct and indirect trains and containers in an intermodal setting. Transportation Science, Informs, Vol. 34, No.3, pp. 256-270.

26. Nozick, L. K. and Morlok, E. K., 1997. A model for medium-term operations planning in an intermodal rail-truck service. Transportation Research, Part A, Vol. 31,

No. 2, pp. 91-107.

27. Powell, W. B. and Carvalho, T. A., 1998. Real-time optimization of containers and flatcars for intermodal operations. Transportation Science, Vol. 32, No. 2, pp. 110-126.

28. Racunica, I. and Wynter, L., 2005. Optimal location of intermodal freight hubs. Transportation Research, Part B, No.39, pp. 453-477.

29. Rizzoli, A. E., Fornara, N. and Gambardella, L. M., 2002. A simulation tool for combined rail/road transport in intermodal terminals. Mathematics and Computers in Simulation, No.59, pp. 57-71.

30. Southworth, F. and Peterson, B. E., 2000. Intermodal and international freight network modeling. Transportation Research, Part C, No.8, pp. 147-166.

31. Taylor, G. D., Broadstreet, F., Meinert, T. S. and Usher, J. S., 2002. An analysis of intermodal ramp selection methods. Transportation Research, Part E, No.38, pp. 117-134.

32. Trip, J. J. and Bontekoning, Y., 2002. Integration of small freight flows in the intermodal transport system. Journal of Transport Geography, Vol. 10, No. 102, pp. 221-229.

33. Yano, C. A. and Newman, A. M., 2001. Scheduling trains and containers with due dates and dynamic arrivals. Transportation Science, Vol. 35, No. 2, pp. 110-126.

34. Ziliaskopoulos, A. and Wardell, W., 2000. An intermodal optimum path algorithm for multimodal networks with dynamic arc travel times and switching delays. European Journal of Operational Research, No.125, pp. 486-502.

Appendix

Code of an Integrated Methodology for Choosing Routing and Scheduling of Intermodal Freight Transportation

```
! The objective;
[TTL_COST]MIN = FCOST + TCOST + HCOST + ICOST;

! The demand constraints;
! FOR ORIGIN A, B;
! DAY1;
 NS1AC1 + NS1AH1C + NS1AC2 + NS1AH2C + NS1AC3 = 380;
 NS2AC1 + NS2AC2 = 220;
 NS6BC1 + NS6BH1C + NS6BC2 + NS6BH2C + NS6BC3 + NS6BH3C = 160;
 NS9AD1 + NS9AH1D + NS9AD2 + NS9AD3 = 150;
 NS10BD1 + NS10BH1D + NS10BD2 + NS10BH2D + NS10BD3 = 100;
 NS11BD1 = 160;
 - NS1AC1 - NS2AC1 + NSAC1 = 0;
 - NS9AD1 + NSAD1 = 0;
 - NS6BC1 + NSBC1 = 0;
 - NS10BD1 - NS11BD1 + NSBD1 = 0;
 - NS1AH1C - NS9AH1D + NSAH1 = 0;
 - NS6BH1C - NS10BH1D + NSBH1 = 0;
 - NS1AC2 - NS1AH2C - NS1AC3 - NS2AC2 - NS9AD2 - NS9AD3 + NIA1 = 0;
 - NS6BC2 - NS6BH2C - NS6BC3 - NS6BH3C - NS10BD2 - NS10BH2D - NS10BD3 + NIB1 = 0;
 NIA1 <= 150;
 NIB1 <= 150;
! DAY2;
 NS3AC2 + NS3AH2C + NS3AC3 = 165;
 NS4AC2 + NS4AC3 = 100;
 NS7BC2 + NS7BH2C + NS7BC3 + NS7BH3C = 220;
 NS12AD2 + NS12AH2D + NS12AD3 + NS12AH3D = 160;
 NS13BD2 = 160;
 - NS1AC2 - NS2AC2 - NS3AC2 - NS4AC2 + NSAC2 = 0;
 - NS9AD2 - NS12AD2 + NSAD2 = 0;
```

```
 - NS6BC2 - NS7BC2 + NSBC2 = 0;
 - NS10BD2 - NS13BD2 + NSBD2 = 0;
 - NS1AH2C - NS3AH2C - NS12AH2D + NSAH2 = 0;
 - NS6BH2C - NS10BH2D - NS7BH2C + NSBH2 = 0;
 - NS1AC3 - NS9AD3 - NS3AC3 - NS4AC3 - NS12AD3 - NS12AH3D + NIA2 = 0;
 - NS6BC3 - NS6BH3C - NS10BD3 - NS7BC3 - NS7BH3C + NIB2 = 0;
 NIA2 <= 150;
 NIB2 <= 150;
! DAY3;
 NS5AC3 = 130;
 NS8BC3 + NS8BH3C = 105;
 NS14AD3 = 120;
 NS15BD3 + NS15BH3D = 160;
 - NS1AC3 - NS3AC3 - NS4AC3 - NS5AC3 + NSAC3 = 0;
 - NS9AD3 - NS12AD3 - NS14AD3 + NSAD3 = 0;
 - NS6BC3 - NS7BC3 - NS8BC3 + NSBC3 = 0;
 - NS10BD3 - NS15BD3 + NSBD3 = 0;
 - NS12AH3D + NSAH3 = 0;
 - NS6BH3C - NS7BH3C - NS8BH3C - NS15BH3D + NSBH3 = 0;
 NIA3 = 0;
 NIB3 = 0;
! FOR HUB H;
! DAY3;
 - NS1AH1C - NS6BH1C + NAH3C = 0;
 - NS9AH1D - NS10BH1D + NAH3D = 0;
 NAH3C - NS1HC4 - NS1HC5 - NS1HC6 = 0;
 NAH3D - NS1HD4 - NS1HD5 - NS1HD6 = 0;
 - NAH3C - NAH3D + NAH3 = 0;
! DAY4;
 - NS1AH2C - NS6BH2C - NS3AH2C - NS7BH2C + NAH4C = 0;
 - NS10BH2D - NS12AH2D + NAH4D = 0;
 - NAH4C - NAH4D + NAH4 = 0;
 NAH4C - NS2HC5 - NS2HC6 = 0;
 NAH4D - NS2HD5 - NS2HD6 = 0;
 - NS1HC4 + NSHC4 = 0;
 - NS1HD4 + NSHD4 = 0;
 - NS1HC5 - NS1HC6 - NS1HD5 - NS1HD6 + NIH4 = 0;
 NIH4 <= 150;
! DAY5;
 - NS6BH3C - NS7BH3C - NS8BH3C + NAH5C = 0;
 - NS12AH3D - NS15BH3D + NAH5D = 0;
 - NAH5C - NAH5D + NAH5 = 0;
 NAH5C - NS3HC6 = 0;
 NAH5D - NS3HD6 = 0;
 - NS1HC5 - NS2HC5 + NSHC5 = 0;
```

```
- NS1HD5 - NS2HD5 + NSHD5 = 0;
- NS1HC6 - NS1HD6 - NS2HC6 - NS2HD6 + NIH5 = 0;
NIH5 <= 150;
! DAY6;
- NS1HC6 - NS2HC6 - NS3HC6 + NSHC6 = 0;
- NS1HD6 - NS2HD6 - NS3HD6 + NSHD6 = 0;
! The limited containers number of the train is 100;
100*NTAC1 - NSAC1 >= 0;
100*NTAC2 - NSAC2 >= 0;
100*NTAC3 - NSAC3 >= 0;
100* NTAD1 - NSAD1 >= 0;
100*NTAD2 - NSAD2 >= 0;
100*NTAD3 - NSAD3 >= 0;
100* NTAH1 - NSAH1 >= 0;
100*NTAH2 - NSAH2 >= 0;
100*NTAH3 - NSAH3 >= 0;
100*NTBC1 - NSBC1 >= 0;
100*NTBC2 - NSBC2 >= 0;
100*NTBC3 - NSBC3 >= 0;
100* NTBD1 - NSBD1 >= 0;
100*NTBD2 - NSBD2 >= 0;
100*NTBD3 - NSBD3 >= 0;
100*NTBH1 - NSBH1 >= 0;
100*NTBH2 - NSBH2 >= 0;
100*NTBH3 - NSBH3 >= 0;
100*NTHC4 - NSHC4 >= 0;
100*NTHC5 - NSHC5 >= 0;
100*NTHC6 - NSHC6 >= 0;
100*NTHD4 - NSHD4 >= 0;
100*NTHD5 - NSHD5 >= 0;
100* NTHD6 - NSHD6 >= 0;

- NTAC1 - NTAC2 - NTAC3 + NTAC = 0;
- NTBC1 - NTBC2 - NTBC3 + NTBC = 0;
- NTAD1 - NTAD2 - NTAD3 + NTAD = 0;
- NTBD1 - NTBD2 - NTBD3 + NTBD = 0;
- NTAH1 - NTAH2 - NTAH3 + NTAH = 0;
- NTBH1 - NTBH2 - NTBH3 + NTBH = 0;
- NTHC4 - NTHC5 - NTHC6 + NTHC = 0;
- NTHD4 - NTHD5 - NTHD6 + NTHD = 0;
- NSAC1 - NSAC2 - NSAC3 + NSAC = 0;
- NSBC1 - NSBC2 - NSBC3 + NSBC = 0;
- NSAD1 - NSAD2 - NSAD3 + NSAD = 0;
- NSBD1 - NSBD2 - NSBD3 + NSBD = 0;
- NSAH1 - NSAH2 - NSAH3 + NSAH = 0;
```

```
- NSBH1 - NSBH2 - NSBH3 + NSBH = 0;
- NSHC4 - NSHC5 - NSHC6 + NSHC = 0;
- NSHD4 - NSHD5 - NSHD6 + NSHD = 0;
- NAH3 - NAH4 - NAH5 + NAH = 0;

NTAC1 + NTAD1 + NTAH1 +NTBC1 + NTBD1 + NTBH1+NTHC4 + NTHD4 +NTAC2 + NTAD2 +
NTAH2+NTBC2 + NTBD2 + NTBH2 +NTHC5 + NTHD5+NTAC3 + NTAD3 + NTAH3 +NTBC3 + NTBD3 +
NTBH3 +NTHC6 + NTHD6 <=100;
NTAC1 + NTAD1 + NTAH1 +NTAC2 + NTAD2 + NTAH2+NTAC3 + NTAD3 + NTAH3 –NTA = 0;
NTBC1 + NTBD1 + NTBH1+NTBC2 + NTBD2 + NTBH2 +NTBC3 + NTBD3 + NTBH3 –NTB = 0;
NTHC4 + NTHD4 +NTHC5 + NTHD5+NTHC6 + NTHD6 –NTH = 0;
! Fixed cost of each train sent from origin A to destination C is 11000,
 Fixed cost of each train sent from origin to hub is 2500,
 Fixed cost of each train sent from hub to destination is 2800;
 FCOST - 11000 * NTAC - 11000 * NTBC - 11000 * NTAD - 11000 * NTBD - 2500 * NTAH - 2500 * NTBH - 2800 *
 NTHC - 2800 * NTHD = 0;
! Transportation cost is 50 from origin to destination,
 Transportation cost is 25 from origin to hub,
 Transportation cost is 28 from hub to destination;
 TCOST - 50 * NSAC - 50 * NSBC - 50 * NSAD - 50 * NSBD - 25 * NSAH - 25 * NSBH - 28 * NSHC - 28 * NSHD =
 0;
! Handling cost is 25 at origin,
 Handling cost is 22 at hub,
 Handling cost is 20 at destination,
 Handling Cost=25NAAC+20NAAC+22NAH;
 HCOST - 22 * NAH = 112050;
! Inventory cost is 3 at origin, is 2 at hub;
 ICOST - 3 * NIA1 - 3 * NIB1 - 3 * NIA2 - 3 *NIB2 - 2* NIH4 -2*NIH5 = 0;
 @GIN( NTAC1);
 @GIN( NTAC2);
 @GIN( NTAC3);
 @GIN( NTBC1);
 @GIN( NTBC2);
 @GIN( NTBC3);
 @GIN( NTAD1);
 @GIN( NTAD2);
 @GIN( NTAD3);
 @GIN( NTBD1);
 @GIN( NTBD2);
 @GIN( NTBD3);
 @GIN( NTAH1);
 @GIN( NTAH2);
 @GIN( NTAH3);
 @GIN( NTBH1);
 @GIN( NTBH2);
```

```
@GIN( NTBH3);
@GIN( NTHC4);
@GIN( NTHC5);
@GIN( NTHC6);
@GIN( NTHD4);
@GIN( NTHD5);
@GIN( NTHD6);
@GIN( NSAC1);
@GIN( NSAC2);
@GIN( NSAC3);
@GIN( NSBC1);
@GIN( NSBC2);
@GIN( NSBC3);
@GIN( NSAD1);
@GIN( NSAD2);
@GIN( NSAD3);
@GIN( NSBD1);
@GIN( NSBD2);
@GIN( NSBD3);
@GIN( NSAH1);
@GIN( NSAH2);
@GIN( NSAH3);
@GIN( NSBH1);
@GIN( NSBH2);
@GIN( NSBH3);
@GIN( NSHC4);
@GIN( NSHC5);
@GIN( NSHC6);
@GIN( NSHD4);
@GIN( NSHD5);
@GIN( NSHD6);
```