Modeling and Comparative Analysis of a Stochastic Production Planning System with Demand Uncertainty

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Abstract

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Effective planning strategies are essential to minimize high costs of production and inventory. Uncertainty and seasonal variation in product demand is a major issue that contributes to a substantial share of production planning costs. Hence, it is important to consider the uncertain information while designing a production planning model. This thesis is aimed at presenting a comparative analysis of deterministic and stochastic approaches towards finding optimal solutions for demand uncertainty problems. The first model is a generic mixed-integer programming model to maximize total profit. Decision variables are identified and random values are substituted by their expected values considering uncertainty to obtain the expected value solutions. Second model is formulated as a stochastic programming model by adding scenarios and probabilities in the deterministic model to explicitly account for the uncertainties in the product demand. The models are programmed and solved by LINGO optimization solver based on data collected from a brewing company. Several test problems are solved by varying the input parameters, product demand and probability of existence of scenarios to study the sensitivity of the models. A statistical comparative analysis is conducted on all the example problems by measuring the Expected Value of Perfect Information (EVPI), Value of Stochastic Solution (VSS) and the results are discussed.

Keywords: Production planning, stochastic programming, demand uncertainty, scenarios, comparative analysis

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Chapter One
Introduction

1.1 Production Planning

In the past several decades, information systems, production management and other related technologies have made significant progress in many different manufacturing and processing industries. Competitiveness in the world market is compelling many firms to use their manufacturing and processing operations as a key factor to succeed in achieving an edge over competitors. Production automation techniques are widely used to achieve advantage in manufacturing. Other areas of improvement in operations and management include better utilization of facilities, lower inventory levels and shorter lead times (Buxey, 1993). Production planning and manufacturing management system is a vast field for implementing new philosophies and technologies in material procurement, shop floor scheduling, facility capacity planning, location and distribution optimization and inventory management (Seward et al., 1985). Material Requirement Planning (MRP), Capacity Requirement Planning (CRP), Just-in-time (JIT) and Hierarchical Production Planning (HPP) are constantly used as technological applications to improve the efficiency of manufacturing and processing operations. Products with seasonal demands face additional challenges in production and planning decisions.

1.2 Planning and Control Issues in Brewery Industry

Brewery industry is highly susceptible to seasonality. The most critical production planning and control issues include minimizing the costs due to uncertainty. Another control issue is scheduling for timely product delivery. Due to the seasonality and uncertainty of product demand, a highly systematic approach towards material procurement, production planning and inventory management is essential. Non-optimal decisions often lead to shortages or excessive inventories. In general, the key objective of most companies is "economize to survive" (Ware, 1992) and both shortages and excessive inventories should be minimized. The two common approaches for dealing with shortages due to uncertainty in customer demand and product completion times are to use safety stocks and safety lead times (Enns, 2002). In case of unexpectedly low demand leading to excessive inventory, stochastic optimization techniques may be applied to obtain better results. We shall first briefly discuss the brewery industry and its typical production planning and control functions.

1.3 The Company Studied in This Research

The company studied in this research is one of Canada's largest beer breweries and bottling companies. It often uses historical data for estimating demand for the subsequent planning year. The company has bottling and packaging plants all across the country. In one of its key plants, it processes roughly 2.2 million 330ml bottles of beer per day and 700 million bottles per year. However, the demand of the products is not distributed uniformly throughout the year. The company encounters extreme seasonal variation in demand which is difficult to predict. Shortages are covered by safety stocks to an extent and further higher demands lead to loss of sales. On the other hand, low demands lead to high inventories. To ensure sufficient inventory to meet high demands and to deal with low demands, substantial inventory costs are incurred.

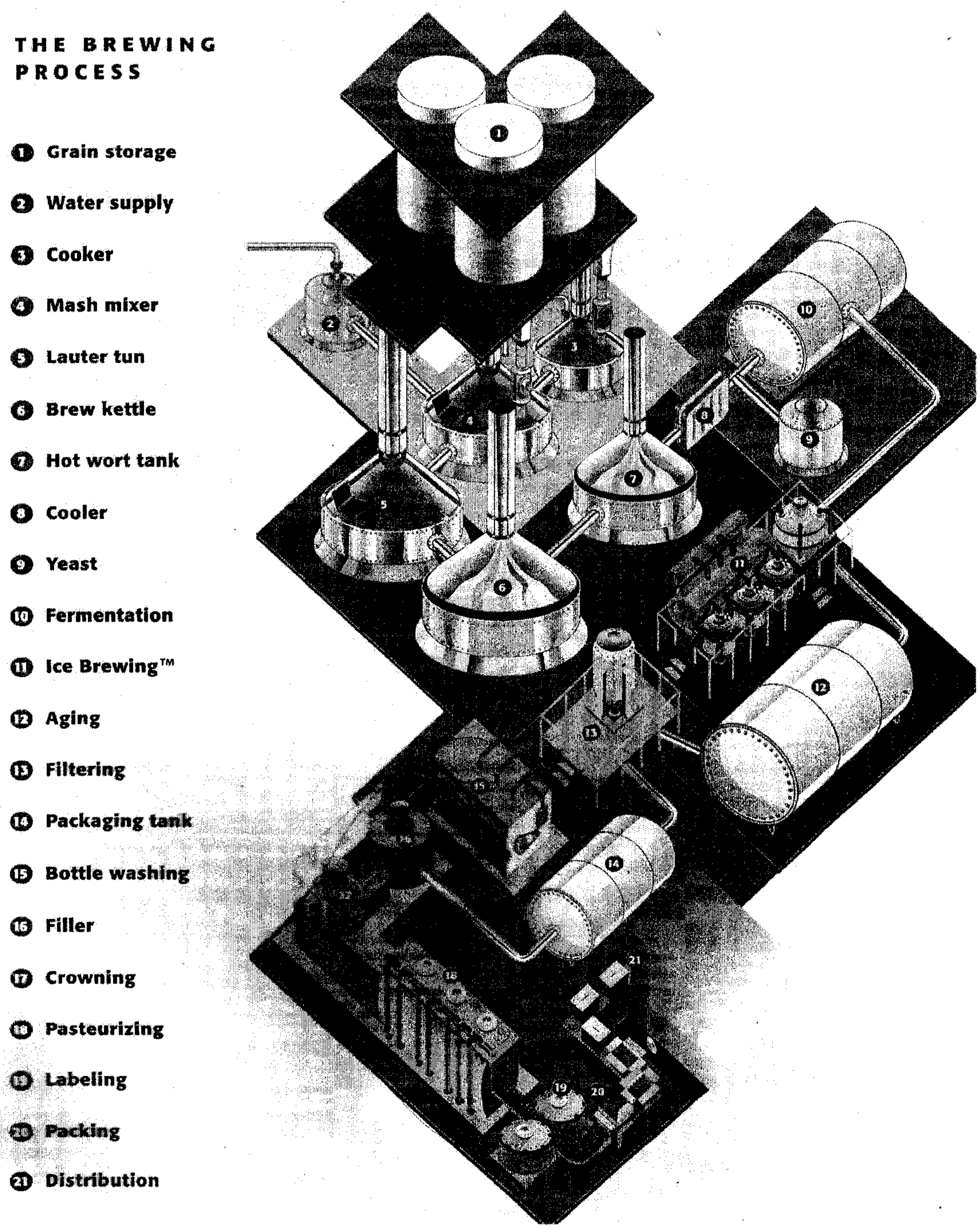


Figure 1.1 Brewing, Production and Packaging Process

Another key area of cost reduction is recovery of used bottles. The company has managed to be highly effective given that 98% of the bottles are returned for reuse. Costs and concerns revolve around efficient inventory management for raw materials and finished products to maximize output. Hence, it is essential to employ a good forecasting technique for predicting demand for the successive planning horizon.

The study begins with the analysis of the actual manufacturing environment for processing and packaging of bottles. Figure 1.1 gives a general schematic representation of brewing, production and packaging operations provided by the company sources. Processes 15 to 21 are studied in this thesis work. Bottles are purchased and retrieved from initial products inventory and filled with beer brewed from processes 1 to 14. Beer is home-brewed and there is no restriction on the quantity and supply of beer. Bottles, caps and labels as raw materials are ordered in equal quantity from the procurement department. Procurement department, in turn, orders the raw materials from different vendors for new and reused bottles. Hence, there is only one vendor for production department i.e., procurement or purchase department. Likewise, sales department receives the finished products and deliver them to the market. Hence, sales department is assumed to be the only distributor for production department.

1.4 Research Methodology

This section presents a framework of methodologies in the development of Mixed Integer Linear Programming (MILP) multiple-products, capacity-constrained production planning deterministic and stochastic models. General characteristics of production planning concepts and their relationship with each other are also discussed. It illustrates the methods involved in data collection, model formulation and optimization. Finally, the result analysis for the problems is presented.

1.4.1 Background

Fransoo et al. (1995) developed a multi-item, single-machine planning and scheduling linear programming model. Their model considered the case of higher demand compared to production capacity. The model has been extended to lower demand case and generalized on the basis of variation in demand from the forecasted values. This thesis also extends their work to multiple machines and increased the problem size and planning horizon substantially to observe the effects of seasonal variation. Stochastic model is compared with expected value solution of deterministic model for the identical input data to compare results under uncertainty. The models consider inventory both at raw material stage and finished product stage. Problem difficulty has been analyzed with respect to the size and tightness of constraints and data.

1.4.2 Data Collection

Data is collected mainly through historical records of the company for inventory, production, purchase of raw materials, sales trend and various processes followed. It was not feasible to collect all the data with utmost accuracy. Some of the information, mainly financial details, was not disclosed. The data used in this work is a blend of factual and assumed data. However, several runs of the model are executed for a large set of data to ensure the accuracy of the assumed data.

1.4.3 Model Formulation

A Mixed Integer Linear Programming (MILP) model is formulated for the data collected to maximize total profit. MILP is a classical mathematical approach that has been extensively utilized in solving production planning problems. MILP is a very powerful mathematical tool for problem solving. Application of Linear Programming (LP) is reported in Savsar and Cogun (1994) for the analysis and modeling of a production line in a corrugated box factory. Koksalan et al. (1995) used an MILP model for location and distribution problem of a beer company. Models in this thesis research are formulated, coded and solved in optimization software LINGO for all the variants of the problem.

1.4.4 Optimization Approach

Stochastic modeling approach is used to solve the MILP capacity-constrained production planning model. Stochastic programming and fuzzy logic are two most frequently and widely used programming tools when uncertainty is involved. Fuzzy logic is more appropriate in case of availability of limited input data. Also, it comes handy when the probability distributions of uncertain variables are unknown. In this technique, main emphasis is on experts' opinion to model uncertainty. For the models formulated in this thesis work, stochastic programming as solution approach is more pertinent. The model developed in this research and the problems will be solved directly by off-the-shelf optimization software due to less number of integer variables in the model.

1.4.5 Analysis of Results

The MILP capacity-constrained production planning models for deterministic and stochastic solutions are solved in commercial optimization software by branch and bound algorithm. The solution procedure is referred to as "stochastic programming with recourse" (Harrison and Van-Mieghem, 1999). The model is solved in two stages and results are presented in terms of two statistics, namely, Expected Value of Perfect Information (EVPI) and Value of Stochastic Solution (VSS). EVPI is the difference in the solutions obtained from deterministic model and stochastic model for identical set of input data. The difference is referred to as "the cost of perfect information". In other words, EVPI measures the cost or value of knowing the future with certainty (Dempster and Thompson, 1999). Calculation of EVPI concludes the first stage solution of stochastic programming with recourse. At this point, recourse actions are taken. The model is solved for the same input data as first stage by substituting the random variables by their expected value. It should be noted that, not all the random variables are replaced by their expected value. Only the variables for which recourse action is possible called "wait-and-see" variables are substituted. Variables for which no recourse action is possible are referred to as "here-and-now" variables. VSS is the difference between the stochastic solution and expected value solution which is obtained by substituting the random variables by their expected value in the deterministic model. It is also referred to as the cost of ignoring uncertainty while making a decision (Birge, 1995). This is the measure of superiority of approach between stochastic and deterministic while modeling uncertainty for the identical input data. Calculation of VSS concludes the second stage solution of stochastic programming with recourse.

1.5 Research Contribution

The work of Fransoo et al. (1995) is extended to multiple setups in this thesis work. They considered a single scenario where demand was higher than production capacity. The problem is extended to several scenarios of demand and a generic model is formulated for variation in demand. The contribution, along with modeling uncertainties, also includes a comparative analysis of two distinct techniques to deal with uncertain parameters. The comparison of expected value solution and stochastic solution for identical set of input data seems unique in the literature surveyed. Capacity constraints are included for purchase, production, inventory, etc. to make the models more practical. The two models formulated are extensively tested by several examples with majority of the data collected from the brewery company studied.

1.6 Objectives of the Thesis

The purpose of this thesis work is to outline a comparison between deterministic and stochastic modeling approaches towards finding an optimal solution. Assumptions are made for not including a few costs or variables because inclusion of such costs and variables would not change the course or outcome of the models. This is done to avoid unnecessary complexity and redundant constraints in the problem. Inventory cost contributes to a substantial share of production planning cost. High efficiency (minimization of waste) and effectiveness (supply to demand) is necessary to reduce this cost. Modern technology and automation tools have significantly improved the production process. Uncertainty is the main factor beyond control and effective planning strategies are essential to minimize inventory problems due to uncertainty. There are a host of issues that are taken care of in order to sketch a comparison in stochastic and deterministic programming to model uncertainty.

1.7 Organization of the Thesis

Following Chapter One, we shall review the literature on the earlier work done in the area of production planning and stochastic programming in Chapter Two, Chapter Three presents the problem description and model formulation for the system under study,

Chapter Four presents the numerical examples solved on the models and analysis of results. Scenario solution of the models to test their robustness in different situations is also presented,

Concluding remarks are presented in Chapter Five. Directions for future research work are also discussed within the limit of this thesis work.

Chapter Two

Literature Review

Significant research has gone into production planning for manufacturing and processing systems over the past few decades. A wide range of literature is dedicated in developing deterministic production planning models with capacity constraints. However, uncertainties in product demand and production lead time play a vital role in many production planning decision problems (Messina and Mitra, 1997). In such situations, stochastic programming and fuzzy logic approach are used to address these uncertainties. Stochastic programming models were first proposed in 1950s (Dantzig, 1955). In addition to production planning, stochastic programming has been applied to various areas in manufacturing system analysis including machine failure analysis (Cooke et al., 2005), supply chain management (Santoso et al., 2005), MRP (Grubbstrom and Wang, 2003), among others. Articles on importance of stochastic programming to tackle demand uncertainty and inventory control in manufacturing and processing plants will be reviewed in this chapter. Fuzzy logic approach to handle uncertainties in production planning has been reported in Aliev (1987). As discussed earlier in Chapter One, stochastic programming is an appropriate problem solving approach for the problem tackled in this thesis work.

This chapter is categorized into different sections based on the topics covered. Table 2.1 presents the summary of the topics covered in this study and related papers.

Table 2.1: Categorization of Literature

Index	Topics	Literatures
1	Brewing Industry Production Problems	Cooke et al (2005), Koksalan et al. (1995), Koksalan et al. (1999)
2	Deterministic Production Planning Models with Capacity Restrictions	Aggarwal et al. (1992), Chen and Wang (1997), Grubbstrom and Huynh (2006), Sana et al. (2004)
3	Characteristics of Uncertainty Models	Bhattacharjee and Ramesh (2000), Casey and Sen (2005), Enns (2002), Graves (1980), Iida (2002), Sox and Muckstadt (1996)
4	Stochastic Programming to Tackle Uncertainty	Fransoo et al. (1995), Harrison and Van-Mieghem (1999), Listes et al. (2003), Messina and Mitra (1997), Wilhelm and Som (1998), Yan (1995)
5	Other Papers on Deterministic and Stochastic Approaches	Arreola-Risa (1996), Grubbstrom and Wang (2003)

2.1 Brewing Industry Production Problems

Cooke et al. (2005): In this paper, the authors discussed the one way filling line of a brewery bottling unit. A similar bottling line is considered in the present thesis research. The authors of this paper constructed a simulation model of a mass balance production line based on constant machine rates, fixed finite buffer and stochastic failure and repair behavior. The system was tested for several runs to understand the statistical fluctuations of stochastic model for the same MTTF (Mean time to failure) and MTTR (Mean time to repair). In the end, stochastic fluctuations of line output for the different lengths of production runs were studied.

Koksalan et al. (1995): In this paper, the authors developed a mixed integer linear programming (MILP) model for location and distribution problems of a beer company. The model was constructed to minimize total location and transportation costs. The seasonal demand for beer of this company was forecasted by a separate study and was considered as an input to the model. Planning horizon of one year was considered in the study. The authors emphasized that the model was primarily used for the purpose of demonstrating the relative importance of the alternative sites rather than finding an optimal solution. In the end, best location alternatives amongst the options considered were selected.

Koksalan et al. (1999): In this paper, a medium-term model and a short-term model for forecasting and understanding the factors affecting beer demand were developed. To identify these factors, a survey was conducted among sales personnel in different regional sales departments and managers in the headquarters of the company. Twenty factors were considered initially and the individuals being surveyed were asked to add any additional factors they considered relevant. Linear regression models were developed to explain and forecast beer demand based on these factors. The authors proposed a procedure based on statistical process control (SPC) principles and techniques to detect non-random data points and to identify missing lurking variables using indicator variables. These lurking variables were integrated into the model. The modeling of the residual was conducted and shown to be useful for solving the key problem and two other randomly generated problems. The results indicated that short-term effects on beer consumption have been captured well and the model can be useful in forecasting short-term beer sales.

2.2 Deterministic Production Planning Models with Capacity Restrictions

Aggarwal et al. (1992): This paper presented a production planning and scheduling model to satisfy the demand of dried lumber at a minimum cost over a specified planning horizon. The model can be generalized to develop a number of manufacturing process solutions consisting of multiple products and multi-stage environment. The authors presented a MILP deterministic model to minimize production and inventory costs over the planning horizon. Deterministic model in the present thesis research is an extension of the model formulated in this paper with changes in the constraints pertinent to the industry. It was concluded that when machine processing times are relatively short in comparison to the length of the planning time period, aggregate production planning models can generate good solutions to scheduling problems. However, when processing times are longer, an integrated model over the complete planning horizon provides better solutions because it incorporates processing times explicitly in the formulation.

Chen and Wang (1997): This paper illustrated an integrated production-distribution planning problem for a system with a single semi-finishing central factory, multiple finishing factories and multiple customers. The flow of materials was from the supplier to the central factory, to the finishing factories and finally to the customers. A single period problem was formulated since the production and distribution planning decisions were to be taken simultaneously. A linear programming model was applied to a steel manufacturing industry and was solved using commercial optimization software. Computational results of the planning problem revealed that high benefits could be realized by integrating the functions discussed above.

Grubbstrom and Huynh (2006): A multi-level, multi-stage, capacity-constrained production-inventory problem was discussed in this paper. The authors considered the situation that lead times were non-zero constants and demands were deterministic. An analytical solution procedure based on dynamic programming was developed to solve the problem. The objective was to select the best production plan to maximize net present value (NPV) of the cash flow associated with production and demand. In the present thesis research, similar fundamentals are applied to the cash inflow and outflow with additional cost of purchasing as cash outflow. As an extension of a previous paper by the same author, combination of Laplace transforms and input-output analysis were used for solving the problem. The numerical examples showed that the solutions used up all available capacity and produced negligible inventory and backlogs. The NPV obtained behaved as expected and demonstrated a linear relation with available capacity.

Sana et al. (2004): A production-inventory model for a deteriorating item over a finite planning horizon with linear time-varying demand was developed in this paper. The authors considered four stages for the formulation of the model. In the first stage, inventory was accumulated as production continued after meeting demand and deterioration. Later, production was stopped and inventory shortages continued to accumulate. Finally, the production starts and shortages were cleared gradually after meeting the current demands. The model considered both increasing and decreasing demands. The periods of production and no-production in different cycles were optimally defined in the paper. In the end, the optimal number of production cycles that minimize the average system cost was determined. Sensitivity analysis of optimal solution was also carried out.

2.3 Characteristics of Uncertainty Models

Bhattacharjee and Ramesh (2000): The authors presented a single-product, multi-period inventory and pricing model where the product had fixed life perishability for a specified number of periods. This study investigated the dynamics of pricing and inventory policies at the retail end of the supply chain and developed an efficient decision model to determine pricing and inventory policies under known circumstances. The paper illustrated the problem of simultaneously deciding product prices and quantity to order from suppliers to maximize net profit. Two algorithms were developed to solve a multi-period pricing and ordering problem. The results obtained by Wagner-Whitin algorithm were compared with the results obtained by complete enumeration of solution space. The comparison demonstrated that their algorithm approach was robust, efficient and easy to implement. The authors concluded that the model and algorithms can be applied to solve planning and pricing problems and extended to multi-period pricing and ordering policies with stochastic demand scenarios.

Casey and Sen (2005): The authors developed a multi-stage stochastic linear programming (MSLP) optimization model. This paper presented a scenario generation algorithm for solving the optimization problem. The authors solved an approximation problem generated by either an aggregation or discretization of the probability model depending on the number of scenarios. The algorithm provided asymptotic convergence and measure of optimality of the decision. The algorithm also offered a sequence of policies for it to adapt to real-time applications.

Enns (2002): This paper investigated forecasting errors and their effects on a batch production system using MRP logic. The author formulated a model for handling uncertainty of demand in a make-to-stock environment where it is desirable to meet delivery performance objectives with minimum inventory. He discussed two common approaches to deal with uncertainty in customer demand and product completion times. These approaches are safety stock and safety lead time. The author suggested five policies or sets of decision variables that affect customer due date and delivery performance in a make-to-stock environment with varying but known demands. These include forecast, shipment, safety stock, lot-sizing and planned lead-time setting policies. It was concluded that delivery service levels improve with the increase in forecast-to-demand ratio. Moreover, increasing planned lead times and adding safety stocks are both effective in improving the customer delivery service level in MRP production environments.

Graves (1980): The author developed a heuristic for solving a multi-product production cycling problem (MPCP). The problem was to determine the production and inventory policies for a family of products, each of which requires processing on a single capacitated plant and had stochastic demands. Due to an inherent computational difficulty in solving a multi-product model as a Markov chain problem, a heuristic was developed to obtain desirable solutions. A single product problem was formulated over a two dimensional state space that consists of inventory level of products and machine status. The heuristic was tested against four other heuristics, based on traditional inventory theory.

Iida (2002): In this paper, the author considered a non stationary periodic review dynamic production-inventory model with uncertain production capacity and uncertain demand. The author developed upper and lower bounds of optimal policies for non-stationary production–inventory problems with uncertain capacity constraint. The objective function of the model was to minimize the total discounted expected cost including production, inventory holding and penalty costs. The production, inventory holding and penalty cost functions were assumed to be linear. Also, the demands and production capacity were assumed to be independent across time but may not be identically distributed. Furthermore, the demands and production capacities were assumed to be independent of each other. In the end, it was shown that the upper and lower bounds converged in calculating the results of the example problems solved in the paper.

Sox and Muckstadt (1996): In this paper, the authors presented a finite horizon capacitated production planning problem. The problem was formulated excluding the setup costs and setup times, but including the backorder costs. The authors utilized a sub-gradient optimization algorithm to solve the model. The solutions obtained were 1% off the lower bound obtained from the Lagrangian dual. The main advantage of this approach, according to the authors, was that realistic problem instances were solved quickly. Moreover, better solutions were obtained in reasonable amount of computing time although the optimal solutions to these problems were difficult to obtain.

2.4 Stochastic Programming to Tackle Uncertainty

Fransoo et al. (1995): In this paper, a two-level hierarchical planning and scheduling model was introduced for multi-item, single-machine production systems facing stochastic demand. The authors considered the situation where demand levels were high compared to the available production capacity. This is also the situation considered in the present thesis research. It was assumed that production is continuous and production lines cannot be stopped. The objective function of the model was to maximize profit with optimal production planning and control subject to service level requirements and capacity constraints. The authors presented the solution for a two-level planning and control problem. The top level was for medium-term capacity coordination to specify which products to produce and for how long. The bottom level was for short-term operational scheduling to achieve the medium-term targets set at the top level. Extensive simulation tests comparing the proposed approach to a cycle time variation policy indicated the superiority of the proposed approach in terms of total profit and rate of production. In the end, the results validated the internal consistency of the proposed two-level hierarchical approach in which the actual profit and production rate, from the simulation run, were very close to the estimated values.

Harrison and Van-Mieghem (1999): This paper considered a firm that marketed multiple products manufactured using multiple resources, thereby varying the capacity of its manufacturing process. Resources considered were several types of labor and capital.

Resource increase is achieved at a particular cost, while a decrease may generate revenue. This paper illustrated a multi-resource investment problem with demand uncertainty. The authors solved this problem using stochastic programming with recourse. This approach yielded an economical descriptive multi-dimensional generalized model. The model developed was agreeable to analytic solution and graphical interpretation but may be difficult to implement for practical decision support. In general, the optimal solution requires solving the characteristic equations simultaneously using multivariate demand distribution. Various numerical examples were solved in two stages varying the number of products and resources for dealing with demand uncertainty. In the end, the authors reached the conclusion that the model explained current practice and quantified the optimal operational hedge.

Listes et al. (2003): In this paper, the authors presented a stochastic programming based approach by which a large scale deterministic location model for product recovery network design was extended to explicitly account for uncertainties. They applied the stochastic model to a case study concerning recycling of sand from demolition waste. Previously, these cases were mainly handled by scenario analysis. The goal was to develop insights for twenty problems with real-world dimensions. The construction of stochastic models followed a rather simple technique. It may be extended and used to solve large location models. In such models, uncertainty was an issue and a relatively small set of realistic scenarios can be identified. The objective function of the MILP model was to maximize the net revenue subject to constraints. The authors considered high supply and low supply cases. They solved the model for location uncertainty of demand and additional uncertainty of supply. The stochastic model was solved in two stages. In the first stage the location variables were fixed and then in the second stage the model was solved for the optimal flow of materials. The mixed integer solver CPLEX 6.5 was used to solve the model for all variants of the problem.

Messina and Mitra (1997): In this paper, the authors discussed the difficulties in developing multi-period stochastic models. The authors emphasized that time and uncertainty are the most important factors in decision problems and stochastic programming models are well suited for capturing both these aspects. Scenario tree structure was used to represent the data dependencies among different time stages where scenarios represent the hierarchical relationship of decision variables over time. In this paper, the authors developed new and versatile techniques and software tools for modeling and analyzing dynamic problems under uncertainty. This paper discussed the development of a software environment combining MDDB (multi-dimensional database) structures, declarative modeling languages and procedural languages which supported the automatic generation of multi-period stochastic models. The authors generated different instances of the model in a software environment by varying the sets of data. They commented that the user can apply different optimization algorithms depending on the model structure and dimensions.

Wilhelm and Som (1998): The authors of this paper modeled a single-stage, single-product stochastic assembly system according to an MRP controlled ordering philosophy. The authors explicitly demonstrated an underlying stochastic process that described the end-product inventory position, enabling production lead times to be treated as independent and generally distributed random variables. They stated that deterministic assumption seemed unrealistic for MRP parameters since production takes place in a stochastic environment and demand for end-products is seldom completely predictable. They developed a model allowing uncertain parameters, production times and demand to be treated as independent random variables. The effect of part flows to initiate inventory on average end-product inventory was used to describe some operating characteristics of the stochastic assembly systems under MRP control.

Yan (1995): In this paper, manufacturing systems with stochastic demands and failure-prone machines were considered. The author emphasized that uncertainties such as machine failures, demand fluctuations, stochastic set-up times and random yields are the most significant characteristics of many contemporary manufacturing and processing systems. He considered a failure-prone manufacturing system with a single machine that produced a single product. An iterative algorithm for stochastic optimization was developed. The algorithm presented in this paper utilized perturbation analysis to carry out the gradient estimation and stochastic approximation to find the optimal number of circulating Kanbans for a manufacturing system with general machine breakdown and stochastic demand. Numerical results indicated that the algorithm provided good approximation results and convergence properties.

2.5 Other Papers on Deterministic and Stochastic Approaches

Arreola-Risa (1996): An integrated production-inventory system with stochastic demand was studied in this paper. The authors considered a multi-item model in which the items shared a capacitated production process and were produced under either deterministic or stochastic circumstances. The problem tackled in this paper was to estimate the base stock levels to minimize expected inventory costs. The connection between capacity utilization and variability in production systems was successfully modeled by queuing models. After solving the test problems, the authors concluded that some of the implications were intuitive, while others were due to the randomness of manufacturing environment. Furthermore, for deterministic and exponential manufacturing times, the authors derived equations leading to the base stock levels minimizing expected inventory costs per unit time. They presented their results based on more than 200 test problems.

Grubbstrom and Wang (2003): This paper presented a model of a multi-level capacity-constrained system when external demand is stochastic. Laplace transform with input-output analysis were employed to construct the model. It used Laplace transform in combination with matrix representations of product structures and capacity requirements. It also extended previous analytical results in the direction of capacity considerations combined with uncertainty in external demand. Dynamic programming was used to solve the multi-stage optimization problem. To correctly model practical situations, it also considered the stochastic nature of the environment. A model was constructed assuming demand as a single source of uncertainty and occurring with known distribution function. After solving the model, the authors observed deterministic and stochastic solutions coincide for low level capacity. It was concluded that stochastic programming provided better solution with high capacity utilization in comparison to the deterministic solution which uses safety stock to deal with demand fluctuations.

2.6 Literature Review Summary

The literature review presented above is based on sequential progress in optimization of multi-period, multi-product, capacity-constrained, stochastic production planning model considered in this thesis research. We started with the study of brewery industry production related problems. Later, we discussed the papers on general deterministic problems with capacity constraints followed by the papers studying problems with uncertainty parameters and different techniques to overcome it. This section provided an overview on production-inventory problems for products with seasonal demands. Industry related and problem specific techniques are applied to resolve uncertainty issues. Later papers were specific to stochastic programming to tackle uncertainty and its consequences. Finally, we presented a study of other literatures related to the model developed in this thesis work.

Chapter Three

Problem Description and Model Formulation

This chapter discusses in detail the critical issues involved in developing the model and the characteristics of generic production planning strategies. It further includes:

- Detailed description of the general characteristics of production planning concepts and their relationship with each other.
- Explanation of the various processes/stages considered in the model.
- Assumptions made during the different stages of production.
- Mixed Integer Linear Programming deterministic and stochastic models formulation.
- Uncertainty involved in the process and related variables.
- Comparison of deterministic and stochastic model to estimate "Expected Value of Perfect Information (EVPI)" and "Value of Stochastic Solution (VSS)".

3.1 Problem Introduction

Operational effectiveness and strategic positioning are very critical for the success of any manufacturing, processing or service industry. Operational effectiveness implies doing similar things as business competitors but in a better way. Best practice in terms of operational effectiveness includes, for example, better technologies, superior inputs, better-trained employees, more effective management structure, and clearly articulated operations strategies. Strategic positioning means doing things differently from business competitors in a way that delivers a unique value to customers (Koksalan et al., 1999).

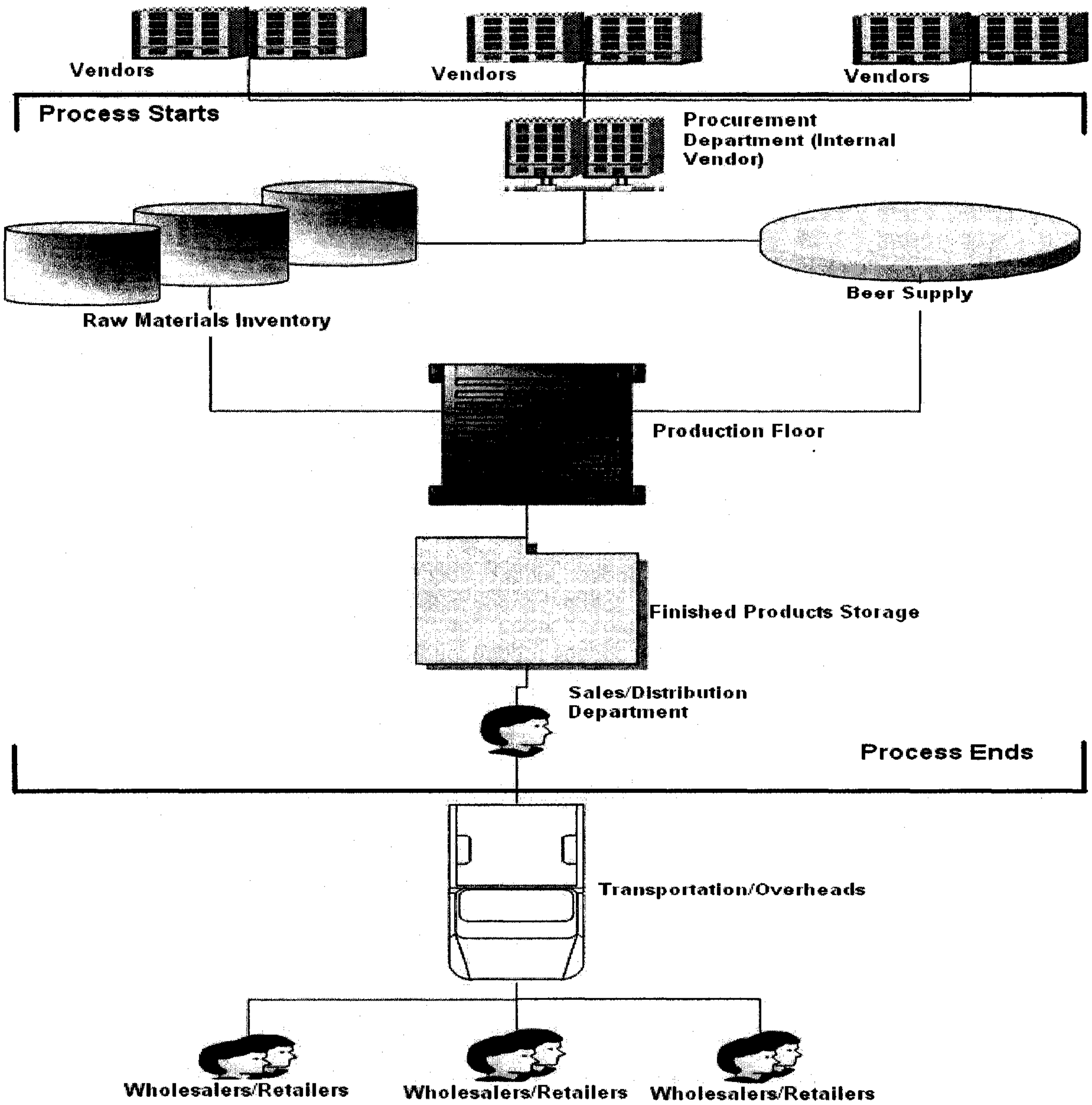


Figure 3.1 Brewery Production Planning System

3.1.1 General Characteristics of Brewery Production Planning System

The considered system assumes that the bottling plant is responsible for packing the beer received in big barrels into the bottles based on their concentrations and characteristics. The raw materials are supplied to the plant internally by the procurement department. Hence, we consider one supplier for all the raw materials. Figure 3.1 shows the schematic representation of the system considered in this research work.

3.2 Stages of Production

The filling and packaging process involves different stages of production. It includes processes starting from acquiring raw materials such as bottles, caps, labels and beer from the supplier. Raw materials are processed by different machines to obtain finished products. Finally, finished products are dispatched to the sales and distribution department or cold stored as finished products inventory. The various functions of the production system are:

- Acquisition/ Purchase
- Raw materials Inventory
- Production/ Processing
- Quality Control
- Finished Products Inventory
- Distribution/ Sales

These stages are further categorized into sub-stages as discussed below.

Acquisition/ Purchase

The raw materials involved in the production process are obtained from different sources. However, it is assumed that the purchase of different raw materials based on orders received from production department is handled by procurement department. Beer is the internal product of the company and is believed to be available in sufficient quantity. Hence, for production planning set-up, there is only one vendor for the raw materials supply. This leads to a significant reduction in the complexity of the problem.

Raw Materials Inventory

Ideal inventory is zero. However, we consider a more practical aspect of it. This study is based on an actual processing plant which has inventory at various stages of production. Raw materials are stored separately in the form of raw materials inventory for bottles, caps and labels. Each of these inventories is subjected to capacity, budget and utility restrictions. Based on the demand of the product, the inventory is consumed and subsequently replenished on reaching the minimum allowable level. The inventory cost consists of ordering cost and holding cost. The ordering cost has to be paid each time an order is placed and is fixed, i.e., it is independent of the quantity ordered. The holding cost reflects the cost per time unit of storage of the goods in the warehouse and is assumed to be linearly proportional to the quantity stored. For consistency of calculations, holding cost is considered at the end of each time period.

Production/ Processing

Processing of bottled beer takes place in different stages of a one way filling line (Cooke et al., 2005). A number of machines produce filled bottles of beer. Empty bottles arrive on pallets. The depalletizer takes the bottles from the pallets and puts them on a conveyor. The rinser washes the bottles and filler fills them with beer. Next, the pasteurizer heats the bottles to decontaminate them. The packer puts the caps on the bottles and the labeler applies the labels. Bottles, finally, assemble on crates and palletizer places the crates back on pallets. Between each machine there are conveyors. Conveyors have both transport function and buffer function. They transport bottles from one machine to the next and they store bottles so that a given machine may keep running even if the upstream or downstream machine is temporarily down. Each machine has a nominal rate of production, but can run faster or slower to compensate for breakdowns. The core machine is the machine with the slowest rate. Figure 3.2 illustrates the production process. In this study, a constant rate of production and no machine breakdown are assumed.

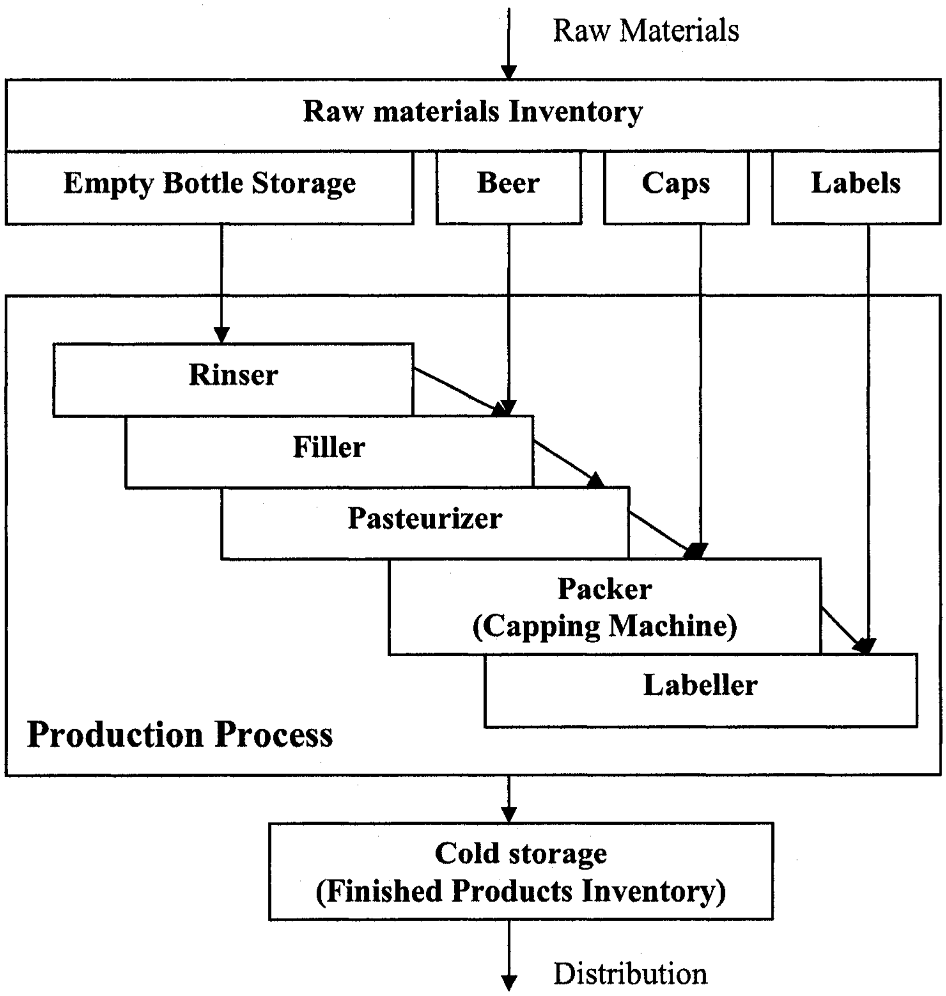


Figure 3.2 Production Process

Quality Control

In brewery bottling plant, quality control is extremely important at various stages of production. Quality control includes visual inspection, gauge measurement, laser testing, etc., for the random samples. Quality rejects, testing stations and man-power are few of the important factors constituting the cost of quality. The process of production is highly automated so the labor force required for quality control is minimal and mostly unskilled. Hence, quality and operations share the labor force. In addition, most of the quality processes take place during the process of production, making it an integral part of production. Therefore, cost of quality is expressed as a fraction of manufacturing cost.

Finished Products Inventory

After the processing of raw materials, finished products are cold stored as finished products inventory. However, in this stage of inventory, ordering cost is not encountered. The only cost associated with the finished products inventory is the inventory holding cost. Similar to the raw materials inventory, each of these inventories is subject to capacity, budget and utility restrictions.

Distribution/ Sales

Finished products from the final inventory or directly from the production floor are dispatched for sale. Similar to procurement, sales and distribution department in the company is assumed to supply the products to different buyers. Hence, there is only one buyer from the production department. The cost of the products charged to the sales department does not include overheads, delivery or transportation charges to wholesalers or retailers.

3.3 Multi Period Deterministic Model

We consider several concentrations of bottles being purchased, stored, processed (filled, crowned, pasteurized, labeled and tested), cold stored and dispatched. The planning horizon is divided into multiple time periods. The company has an ordering policy in which it has divided its operations into several time periods of precise length. The company brews beer followed by the bottling operation. The brewing process time for a single lot is twice as compared to the bottling process. The lot is dispatched after this. Hence, brewing is done in such a manner in multiple brewing plants that the bottling process is continuous.

Demand is estimated to be equal to a value based on historical data and production is done according to that estimated demand. Using the MILP model of the deterministic problem, we can observe the effects of demand on the decision variables and objective function for maximizing total profit.

3.3.1 Assumptions

1) Demand is forecasted based on historical data.
2) Shortages are allowed.
3) No backorder is permitted i.e., if any customer order is not fulfilled, then the order is lost.
4) Safety stock is not considered.
5) Machine failures are not considered.
6) Work in progress inventory on conveyer is considered.
7) No set-up time and cost as production operations do not require set-up.
8) Existence of one vendor and one distributor is assumed for reducing the complexity of the problem.
9) Production capacity is specified in terms of machine hours.
10) Production process includes different types and shapes of bottles having the same volumetric capacities to maintain the consistency of the model. Hence, the processing time for each product type is considered same.

11) Company purchases all the sub-parts of a unit in the same quantity. Therefore, the caps and labels purchased for a particular product are equal to the number of bottles purchased.

3.3.2 Modeling Approach

The solution methodology requires a practical approach for tackling issues, costs and constraints. Firstly, a deterministic MILP model for the problem is developed. The objective of this model is to maximize the profit based on costs incurred during or before the production or processing stage. Selling price mentioned is the internal price at which the finished products are received by the sales department. These processed goods are sold to the wholesalers and retailers by adding transportation cost and other overheads to this price. A deterministic MILP model is formulated on the basis of average expected demand which is also the forecasted demand based on historical data. Later, two more problem instances are considered for the variation of demand from the expected value. Hence, the deterministic production planning model for profit maximization is solved for three different problem instances of low, intermediate and high demand levels, reflecting demand fluctuation.

Cost Factors

The objective of the multi-period production planning model developed is to maximize the profit and minimize the costs in the system. The costs at different production stages in modeling include:

Material Purchasing Costs:

It includes the cost of purchasing raw materials from the vendor. The raw materials include beer and different types of bottles, caps and labels. As mentioned above, the production department purchases the raw materials from the procurement department which is considered as the internal vendor.

Production Costs:

Labor and Quality Costs – Labor cost and cost of quality are dependent on the units produced. More man-power is needed with an increase in the number of units required to be produced in a time period. Quality rejects percentage is assumed consistent for the planning periods independent of the units produced. However, the number of quality rejects increase with the increases in the number of units produced.

Unit Production Cost – In this model, unit production costs are assumed constant. They only vary depending on the types of the products.

Inventory Costs:

Holding Cost - It is a variable cost depending on the number of units in the inventory. Holding cost applies to both raw materials inventory and the finished products inventory. It includes depreciation of warehouse, electricity, heat, ventilation, etc.

Ordering Cost – This is a fixed cost irrespective of the number of units ordered. This cost is present whenever an order is placed during any time period.

3.3.3 MILP Deterministic Model Formulation

Based on the above discussion and thorough study of the problem, MILP model is formulated below to solve the deterministic problem. We will present the details of the model after we introduce the notations.

Indices

i = Type of products considered, $i = 1,2,....,N$

t = Time periods considered in the planning horizon, $t = 1,2,....,T$

Decision Variables

U_{it} = Units of product i sold at time t

X_{it} = Units of product i produced at time t

I_{it} = Units of finished products i carried from time t to $t+1$

B_{it} = Units of raw materials i carried from time t to $t+1$

P_{it} = Units of raw materials i purchased at time t

Z_t = Binary variable determining the status of order placed in time t

Parameters

D_{it} = Demand of product i during time t

S_{it} = Unit sales price of product i at time t

C_{it} = Unit cost of product i at time t (Combined cost of bottle, cap, label, liquid and processing)

V_{it} = Unit cost of bottles i purchased at the beginning of time t

O_t = Ordering cost at time t

M_{it} = Unit Production cost of product i at time t

Φ_t = Number of production lines available at time t

β_i = Inventory carrying/ holding cost as a fraction of unit cost of the initial product i

λ_i = Inventory carrying/ holding cost as a fraction of unit cost of finished product i

h = Number of hours each production line is available per day

∂ = Number of days in each time period t

W_t = Maximum investment on raw materials inventory at time t

J_{it} = Maximum finished products inventory allowed for product i at time t

T_t = Maximum investment on purchasing at time t

L_{it} = Maximum raw materials inventory allowed for product i at time t

b^{cap} = Purchasing cost of caps as a fraction of purchasing cost of bottles

b^{lab} = Purchasing cost of labels as a fraction of purchasing cost of bottles

ℓ = Labor cost as a fraction of production cost

c = Number of units of all products produced per hour

Q = Cost of quality as a fraction of production cost

$B^{(1)}$ = Raw materials inventory at the beginning of first time period

$I^{(1)}$ = Finished products inventory at the beginning of the first time period

Objective Function

The objective function of this model to maximize profit is expressed as:

Total profit = Sales (SP) – Production Cost (Processing + Labor + Quality) (PC) – Raw Materials Inventory Cost (RMIC) – Finished Products Inventory Cost (FPIC) – Purchasing Cost (PUC)

Sales

The production team applies a sales price tag on the finished goods and forwards them to the sales and distribution department. They, in turn, add overheads and transportation cost to the price and supply the products to the retailers and wholesalers. Sales refer to internal sales price of the products from production to the sales department.

$$SP = \sum_{i=1}^{N}\sum_{t=1}^{T} S_{it}U_{it} \tag{3.1}$$

Production Cost

The total cost of production for a planning horizon including the cost of machine hours, man hours, quality checks and rejects. Production cost is given by

$$PC=\sum_{i=1}^{N}\sum_{t=1}^{T} M_{it}X_{it}(1+\ell+Q) \tag{3.2}$$

Quality rejections and labor cost are specified as a fraction of manufacturing or processing cost which is expressed in terms of machine hours.

Raw Materials Inventory Cost

The raw materials purchased are stored in separate warehouses as raw materials inventory, each incurring a holding cost. The initial inventory holding cost is expressed as a fraction of the purchasing cost of the bottles. Ordering cost is added irrespective of the quantity ordered. Ordering cost is applicable as long as the raw materials order quantity is more than zero otherwise no ordering cost is incurred.

$$RMIC = \sum_{i=1}^{N}\sum_{t=1}^{T} \beta_i \{V_{it} \times B_{it}(1+b^{cap}+b^{lab})\} + \sum_{t=1}^{T} Z_t O_t \tag{3.3}$$

Finished Products Inventory Cost

Final inventory is the inventory of finished products cold stored and ready to be dispatched.

$$FPIC = \sum_{i=1}^{N}\sum_{t=1}^{T} \lambda_i C_{it} I_{it} \tag{3.4}$$

Purchasing Cost

Total purchasing cost of raw materials for each time period is expressed. Purchasing cost of caps and labels are expressed as a fraction of the purchasing cost of the bottles.

$$PUC = \sum_{i=1}^{N} \sum_{t=1}^{T} V_{it} \times P_{it} (1 + b^{cap} + b^{lab}) \tag{3.5}$$

Constraints

Sales Constraints

These are the logical constraints which govern the inventory, production and units sold. Equation 3.6 ensures that the sum of inventory carried from one time period to the next and units produced in that time period equals the sum of units sold for that period and inventory carried to the successive period. However, if the inventory carried forward from the previous period is zero then Equation 3.7 is applicable. It implies that units sold in a time period do not exceed the sum of units produced and the inventory carried to the successive time period.

$$I_{i,t-1} + X_{it} - I_{it} - U_{it} = 0 \qquad \forall i,\, t \tag{3.6}$$

and

$$U_{it} \le I_{it} + X_{it} \qquad \forall i,\, t \tag{3.7}$$

Units sold for a product in any time period are less than the demand for that period. This holds true for all products i and for each time period t.

$$U_{it} \le D_{it} \qquad \forall i,\, t \tag{3.8}$$

Production Constraints

The maximum number of productive hours in each time period is at least equal to the total hours used for production in that period.

Hence, the processing time does not exceed the maximum capacity of production lines.

$$\sum_{i=1}^{N} \frac{1}{c} X_{it} \leq \Phi_t \times h \times \partial \qquad \forall t \qquad (3.9)$$

Raw Materials Inventory Constraints

This constraint enforces that raw materials inventory cannot exceed the maximum capacity limit for each product in all the time periods. The maximum limit on the inventory is governed by storage space restriction.

$$B_{it} \leq L_{it} \qquad \forall i, t \qquad (3.10)$$

Equation 3.11 is a budget constraint on the raw materials inventory. The raw materials inventory carrying cost in each period for all the products should not exceed the maximum budget allocated to the raw materials inventory.

$$\sum_{i=1}^{N} \{V_{it} \times B_{it}(1 + b^{cap} + b^{lab})\} \leq W_t \qquad \forall t \qquad (3.11)$$

Initial inventory for raw materials is known for the first time period of the planning horizon.

$$B_{i1} = B^{(1)} \qquad \forall t \qquad (3.12)$$

Finished products Inventory Constraints

This constraint ensures finished products inventory for all the products and for each time period should be below the maximum permitted capacity.

$$I_{it} \leq J_{it} \qquad \forall i, t \qquad (3.13)$$

Finished products inventory for bottles is known for the first time period of the planning horizon.

$$I_{i1} = I^{(1)} \qquad \forall t \qquad (3.14)$$

Purchasing Constraints

This constraint enforces that the sum of raw materials purchasing price for bottles, caps and labels with ordering cost for each time period should not exceed the total fixed investment on purchasing for that period

$$\sum_{i=1}^{N} V_{it} \times P_{it}(1 + b^{cap} + b^{lab}) + Z_t O_t \le T_t \qquad \forall t \qquad (3.15)$$

Equation 3.16 is a logical constraint which ensures the sum of the units purchased and carried from the inventory of previous time period is equal to the sum of the units produced and carried to the successive period inventory.

$$P_{it} + B_{i,t-1} - X_{it} - B_{it} = 0 \qquad \forall i, t \qquad (3.16)$$

Equation 3.17 implies that ordering cost is encountered only if there is an order placed. If the purchase quantity for a particular period is zero, the ordering cost would be zero.

$$\sum_{i=1}^{N} P_{it} \le M \times Z_t \qquad \forall t \qquad (3.17)$$

Z_t is binary and

M is any large positive number

3.3.4 Summary Representation of Deterministic Model

Putting the objective function (Equations 3.1 to 3.5) and constraints (Equations 3.6 to 3.17) together, we get the MILP Production Planning Model with deterministic demand as:

Maximize Total Profit (Z)

$$\sum_{i=1}^{N}\sum_{t=1}^{T} S_{it}U_{it} - \sum_{i=1}^{N}\sum_{t=1}^{T} M_{it}X_{it}(1+\ell+Q) - \sum_{i=1}^{N}\sum_{t=1}^{T} \beta_i\{V_{it}\times B_{it}(1+b^{cap}+b^{lab})\} - \sum_{t=1}^{T} Z_t O_t$$
$$-\sum_{i=1}^{N}\sum_{t=1}^{T} \lambda_i C_{it} I_{it} - \sum_{i=1}^{N}\sum_{t=1}^{T} V_{it}\times P_{it}(1+b^{cap}+b^{lab})$$

Subject to constraints:

$$I_{i,t-1} + X_{it} - I_{it} - U_{it} = 0 \qquad \forall i, t$$

$$U_{it} \le I_{it} + X_{it} \qquad \forall i, t$$

$$U_{it} \le D_{it} \qquad \forall i, t$$

$$\sum_{i=1}^{N} \frac{1}{c} X_{it} \le \Phi_t \times h \times \partial \qquad \forall\ t$$

$$B_{it} \le L_{it} \qquad \forall i, t$$

$$\sum_{i=1}^{N}\{V_{it}\times B_{it}(1+b^{cap}+b^{lab})\} \le W_t \qquad \forall\ t$$

$$P_{it} + B_{i,t-1} - X_{it} - B_{it} = 0 \qquad \forall i, t$$

$$I_{it} \le J_{it} \qquad \forall i, t$$

$$\sum_{i=1}^{N} V_{it}\times P_{it}(1+b^{cap}+b^{lab}) + Z_t O_t \le T_t \qquad \forall\ t$$

$$\sum_{i=1}^{N} P_{it} \le M \times Z_t \qquad \forall\ t$$

Binary Variables

$$Z_t \in \{0,1\}$$

Continuous Variables

$0 \leq X_{it}, P_{it}, U_{it}, B_{it}, I_{it} \leq 1$

In this model, the calculations are based on the average forecasted demand which is referred to as intermediate demand instance. We solved several individual problem instances based on the variation in demand. The demand is variable but deterministic based on historical data. The model is solved for low, intermediate and high demand instances.

3.4 Stochastic Programming Model Formulation and Solution

Stochastic programming is a framework for modeling optimization problems that involve uncertainty. Whereas deterministic optimization problems are formulated with known parameters, real world problems almost invariably include some unknown parameters. We have shown the deterministic production planning model in Section 3.3. The demand is forecasted based on historical data. However, uncertainty phenomenon is more prominent for brewery industry (Koksalan et al., 1999). The demand may be lower or higher than this intermediate forecasted value.

3.4.1 Scenario Probability

The different cases of variation in demand are referred to as scenarios in this thesis. Each of these scenarios is associated with a corresponding probability of occurrence. In developing the stochastic programming model, we consider a limited number of scenarios. ρ^k is the probability that scenario k will happen. Hence, we have

$\rho^k \in P$, where $\sum_{k=1}^{K} \rho^k = 1$

Unlike deterministic solution, the stochastic solution tries to estimate the results considering all possible scenarios together. It is quite obvious that deterministic solution would present a higher value of profit due to its deterministic nature. Since it does not consider the uncertainty in its calculations, its output is based on the exact information on the occurrence of a particular event. Whereas, the stochastic model formulation takes into consideration the uncertainty while modeling a problem and the results obtained can be more realistic. The most widely applied and studied stochastic programming models are two-stage linear programs. The decision maker takes some action in the first stage, after which a random event occurs affecting the outcome of the first-stage decision. A recourse decision can then be taken in the second stage that compensates for any loss of sales or excessive inventory that has been experienced as a result of the first-stage decision.

3.4.2 EVPI and VSS Fundamentals

Stochastic solutions can be analyzed and interpreted in terms of the following two parameters.

Expected Value of Perfect Information (EVPI): It measures as to how much more one is expected to win if perfect information about the stochastic components of a problem is available (Dempster and Thompson, 1999). In other words, EVPI measures the cost or value of knowing the future with certainty. Hence, this is the maximum amount that may be spent to gather information about the uncertain elements. The difference in the profits obtained by the deterministic solution and the stochastic solution is referred to as EVPI. The solution of a deterministic problem is always equal to or better than the solution of a corresponding stochastic problem. This is due to the absence of uncertainty in the deterministic approach. Based on the outcome of the deterministic solutions, decision variables are identified.

Consequently, they are segregated into 2 different categories, "here-and-now" variables and "wait-and-see" variables depending on their nature of recourse. "Here-and-now" decisions are the ones which are taken before the beginning of a planning horizon and no recourse is possible later. "Wait-and-see" decisions may be modified later based on the outcome of uncertainty. This part of the solution is called the first stage solution of stochastic modeling. Figure 3.3 represents the estimation of EVPI from the deterministic and stochastic models.

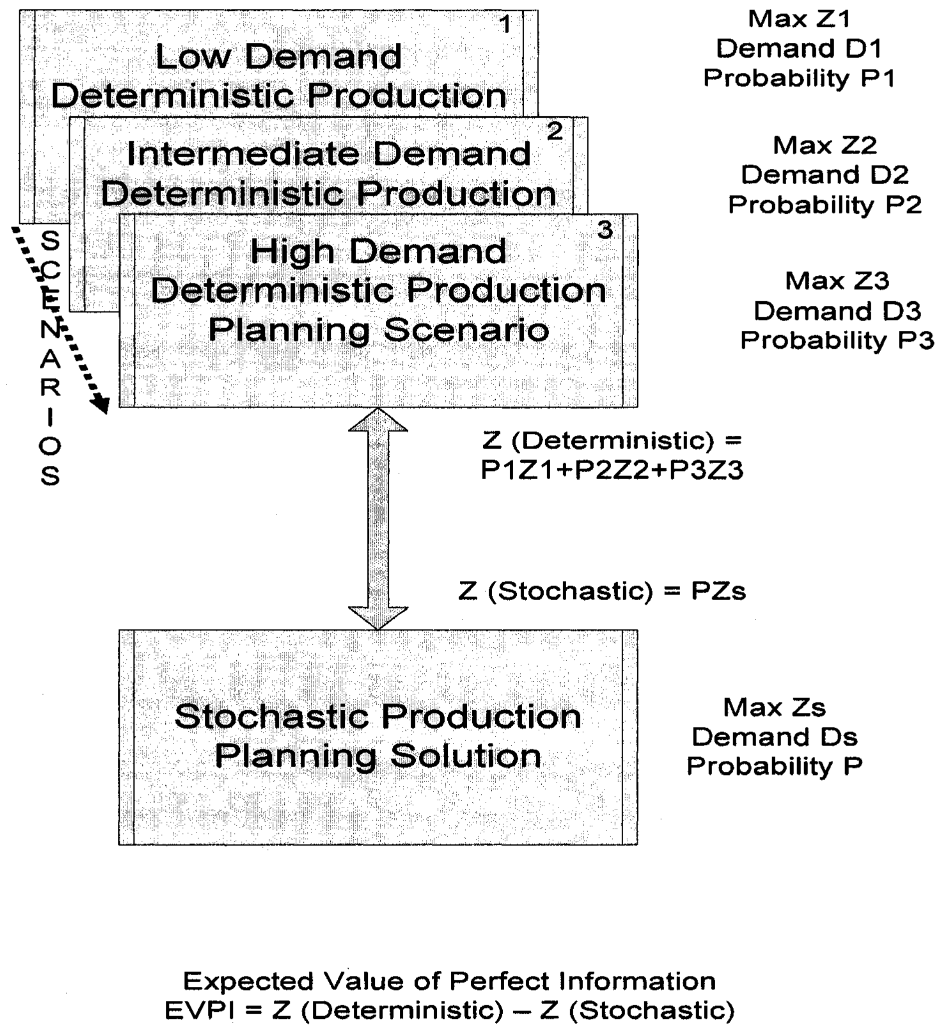


Figure 3.3 Estimation of EVPI

Value of Stochastic Solution (VSS): It is also referred to as the cost of ignoring uncertainty while making a decision (Birge, 1995). VSS is calculated in the second stage of the stochastic programming solution. In this stage, the random decision variables are substituted by their expected value. Moreover, the variables that are identified based on "here-and-now" decisions and "wait-and-see" decisions are segregated on the basis of effects due to uncertain parameters. "Wait-and-see" variables are assigned the respective probabilities on the basis of scenarios. The solution, thus, obtained is the Expectation of the Expected Value, EEV or Expected Value Solution, EVS. This solution is compared to the stochastic solution for the given scenarios. Higher values of VSS justify stochastic approach over EVS for the given problem. The collective solution is referred to as "stochastic solution with recourse" (Harrison and Van-Mieghem, 1999). The fundamentals of VSS are shown in Figure 3.4 as the second stage of stochastic programming. Only 3 scenarios are shown in Figures 3.3 and 3.4, however, it can represent any number of scenarios.

3.4.3 Modeling with Uncertainty

The MILP model presented in Section 3.3.4 can be extended to a stochastic programming model with the corresponding variables and parameters re-defined by incorporating different scenarios and their probabilities. We first present the modified notations for the new model. The same parameters and variables used in the deterministic model will not be re-defined.

Index

k = Number of different scenarios considered, $k = 1,2,\ldots.,K$

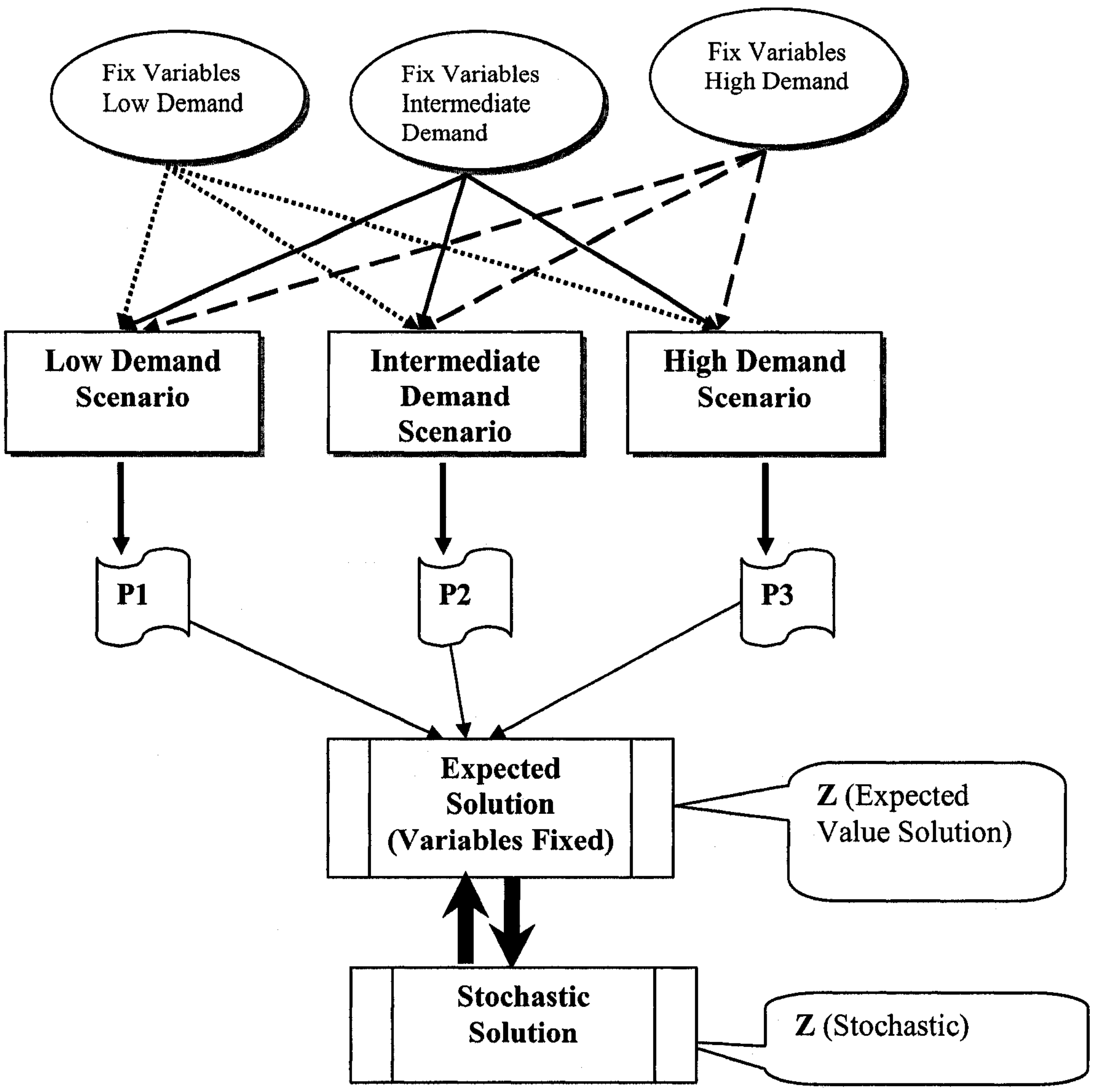
Decision Variables

U_{itk} = Units of product i sold at time t for scenario k

X_{itk} = Units of product i produced at time t for scenario k

I_{itk} = Units of finished products i carried from time t to $t+1$ for scenario k

B_{itk} = Units of raw materials i carried from time t to $t+1$ for scenario k



Value of Stochastic Solution,
VSS = Z (Stochastic) – Z (Expected Value Solution)

Figure 3.4 Estimation of VSS

Parameters

D_{itk} = Demand of product i during time t for scenario k

ρ^k = Probability of the occurrence of scenario k

Apart from the decision variables and parameters declared above, all the other variables and parameters are identical to the deterministic model.

Costs

If uncertainty is taken into consideration, Equation 3.1 for sales would be expressed as

$$SP = \sum_{k=1}^{K} \rho^k \times \sum_{i=1}^{N} \sum_{t=1}^{T} S_{it} U_{itk} \tag{3.18}$$

Equation 3.2 does not involve the terms of uncertainty. Hence, can be expressed as it is

$$PC = \sum_{i=1}^{N} \sum_{t=1}^{T} M_{it} X_{it} (1 + \ell + Q) \tag{3.19}$$

Introduction of the scenarios would change Equation 3.3 as

$$RMIC = \sum_{k=1}^{K} \rho^k \times \sum_{i=1}^{N} \sum_{t=1}^{T} \beta_i \{V_{it} \times B_{itk} (1 + b^{cap} + b^{lab})\} + \sum_{t=1}^{T} Z_t O_t \tag{3.20}$$

Equation 3.4 for finished products inventory cost may be expressed as

$$FPIC = \sum_{k=1}^{K} \rho^k \times \sum_{i=1}^{N} \sum_{t=1}^{T} \lambda_i C_{it} I_{itk} \tag{3.21}$$

Purchasing cost in Equation 3.5 does not change with the introduction of uncertainty and may be written again as

$$PUC = \sum_{i=1}^{N} \sum_{t=1}^{T} V_{it} \times P_{it} (1 + b^{cap} + b^{lab}) \tag{3.22}$$

Constraints

Sales constraint in Equation 3.6 for deterministic model becomes

$$I_{i,t-1,k} + X_{it} - I_{itk} - U_{itk} = 0 \text{ and} \qquad \forall i, t, k \qquad (3.23)$$

Equation 3.7 is rewritten with uncertainty as

$$U_{itk} \leq I_{itk} + X_{it} \qquad \forall i, t, k \qquad (3.24)$$

Units sold for a specific product in any time period are less than the demand for that period. This holds true for all the scenarios. Hence, Equation 3.8 is expressed as

$$U_{itk} \leq D_{itk} \qquad \forall i, t, k \qquad (3.25)$$

Equation 3.9 has no change and re-written as

$$\sum_{i=1}^{N} \frac{1}{c} X_{it} \leq \Phi_t \times h \times \partial \qquad \forall t \qquad (3.26)$$

Raw materials inventory constraint changes with uncertainty. Hence, Equation 3.10 is modified to

$$B_{itk} \leq L_{it} \qquad \forall i, t, k \qquad (3.27)$$

This is a budget constraint on the initial inventory. This constraint expressed earlier in Equation 3.11 is re-written as

$$\sum_{i=1}^{N} \{V_{it} \times B_{itk}(1 + b^{cap} + b^{lab})\} \leq W_t \qquad \forall t, k \qquad (3.28)$$

Initial declaration for raw materials inventory is

$$B_{i1k} = B^{(1)} \qquad \forall t, k \qquad (3.29)$$

Final inventory for all the products and for each time period should be below the maximum permitted capacity. With the introduction to scenarios, Equation 3.13 is expressed as

$$I_{itk} \leq J_{it} \qquad \forall i, t, k \qquad (3.30)$$

Initial declaration for finished products inventory is

$I_{i1k} = I^{(1)}$ $\quad \forall t, k \quad (3.31)$

Equation 3.15 shows the sum of the purchase price of all the raw materials should not exceed the total allotted investment on purchase for that time period. This holds true for all the scenarios.

$$\sum_{i=1}^{N} V_{it} \times P_{it}(1 + b^{cap} + b^{lab}) + Z_t O_t \leq T_t \qquad \forall t \qquad (3.32)$$

Logical constraint Equation 3.16 for initial inventory is expressed as

$$P_{it} + B_{i,t-1,k} - X_{it} - B_{itk} = 0 \qquad \forall i, t, k \qquad (3.33)$$

Equation 3.17 implies that ordering cost is encountered only if there is an order placed for a period. This holds true irrespective of the scenario.

$$\sum_{i=1}^{N} P_{it} \leq M \times Z_t \qquad \forall t \in T \qquad (3.34)$$

Z_t is binary

3.4.4 Summary of Stochastic Model Solution

This section presents the summary of model after rewriting and re-optimizing the deterministic problem with the introduction of stochastic modeling. The effect of probability is clearly evident when demand varies from the estimated range in the deterministic problem instances. Putting the objective function (Equations 3.18 to 3.22) and the constraints (Equations 3.23 to 3.34) together, we get the complete MILP Production Planning problem with uncertainty in demand as follows:

Maximize Total Profit (Z)

$$-\sum_{i=1}^{N}\sum_{t=1}^{T} M_{it} X_{it}(1+\ell+Q) - \sum_{i=1}^{N}\sum_{t=1}^{T} V_{it} \times P_{it}(1+b^{cap}+b^{lab}) +$$

$$\sum_{k=1}^{K} \rho^{k} \times \left(\sum_{i=1}^{N}\sum_{t=1}^{T} S_{it} U_{itk} - \sum_{i=1}^{N}\sum_{t=1}^{T} \beta_i \{V_{it} \times B_{itk}(1+b^{cap}+b^{lab})\} - \sum_{t=1}^{T} Z_t O_t - \sum_{i=1}^{N}\sum_{t=1}^{T} \lambda_i C_{it} I_{itk} \right)$$

Subject to constraints:

$I_{i,t-1,k} + X_{it} - I_{itk} - U_{itk} = 0$ $\quad \forall i, t, k$

$U_{itk} \le I_{itk} + X_{it}$ $\quad \forall i, t, k$

$U_{itk} \le D_{itk}$ $\quad \forall i, t, k$

$\sum_{i=1}^{N} \frac{1}{c} X_{it} \le \Phi_t \times h \times \partial$ $\quad \forall t$

$B_{itk} \le L_{it}$ $\quad \forall i, t, k$

$\sum_{i=1}^{N} \{V_{it} \times B_{itk}(1+b^{cap}+b^{lab})\} \le W_t$ $\quad \forall t, k$

$P_{it} + B_{i,t-1,k} - X_{it} - B_{itk} = 0$ $\quad \forall i, t, k$

$I_{itk} \le J_{it}$ $\quad \forall i, t, k$

$\sum_{i=1}^{N} V_{it} \times P_{it}(1+b^{cap}+b^{lab}) + Z_t O_t \le T_t$ $\quad \forall t$

$\sum_{i=1}^{N} P_{it} \le M \times Z_t$ $\quad \forall t$

Binary Variables

$Z_t \in \{0,1\}$

Continuous Variables

$0 \le X_{it}, P_{it}, U_{itk}, B_{itk}, I_{itk} \le 1$

The above model is a stochastic programming model with recourse. It can be solved explicitly following standard stochastic programming approach. The size of the problem, however, is much larger than the corresponding deterministic model, due to the scenarios considered in the stochastic approach. The deterministic and stochastic models are tested and compared through a number of test problems. The problem details and results are reported in the next chapter.

Chapter Four

Numerical Examples and Analysis

This chapter presents the numerical analysis of the model discussed in Chapter 3. The data used in the example problems represent the pertinent information over one planning horizon. The models developed are solved for various problem instances based on demand variation. These solutions with demand variations are referred to as individual scenario solutions. Parametric analysis is conducted by varying the inputs, scenarios and probabilities. The impact of these parameters on the decision variables and total profit is observed. Stochastic model is also formulated representing different scenarios with corresponding probabilities. Consequently, deterministic models are compared with the stochastic model for similar set of input data. This forms the first stage of solution for stochastic programming. Based on the results of deterministic problems, the decision variables are fixed and interchangeably substituted in all problem instances for the same set of data values to obtain "Expected Value Solution (EVS)" or "Expectation of the Expected Values (EEV)". For each set of probability values, 3 different scenarios are formulated to validate the existence of uncertainty. The expected value solutions with uncertainty are compared with the corresponding stochastic solution. This gives the second stage optimization of the stochastic programming problem calculations. Based on the historical data analysis, each of these scenarios is assigned the probability of its existence. Four examples are solved with different probabilities of existence of scenarios.

4.1 Example Problems

To demonstrate the applications of the linear programming model presented in Chapter 3, we consider a production planning problem with a planning horizon of one year. This planning horizon is further divided into 12 time periods of one month each. The general data of the examples are given in Table 4.1. There are 3 different products to be processed in 12 time periods. Hence, there are 36 values of demand for each scenario. Considering only 3 demand scenarios, there are $3^{36} = 1.5 \times 10^{17}$ combinations of possible demand scenarios.

Table 4.1 General Parametric Values

Number of Products *(N)*	3
Number of Time Periods *(T)*	12
Number of Working Days in each time period (∂)	22
Number of Hours/Day (h)	24
Number of Scenarios *(K)*	3
Ordering Cost (O_t)	5000

However, practically the probability of occurrence of most of these demand instances is very low. We consider the more likely 27 instances. Figure 4.1 represents the combination of demands values for 3 products. It is observed that if demand for a product shows an upwards or downwards trend for a particular time period, it may have the same trend for the other periods as well. It is possible that the demand for a product is less for one time period and more for the successive period. However, the probability for such a case is low and can be ignored. Figure 4.1 shows the 27 likely scenario combinations among all the possible permutations, which have higher probability of occurrence compared to the 1.49999×10^{17} alternatives.

Moreover, the 27 deterministic solutions in the first stage lead to 81 solutions in the second stage. To reduce the complexity of the problem and considering the solutions which are most relevant, 3 cases with highest probability of occurrence are solved. Figure 4.1 shows the set of demand instances for which the first stage solution is computed.

Product 1	Product 2	Product 3
High	High	High
High	High	Intermediate
High	High	Low
High	Intermediate	High
High	Intermediate	Intermediate
High	Intermediate	Low
High	Low	High
High	Low	Intermediate
High	Low	Low
Intermediate	High	High
Intermediate	High	Intermediate
Intermediate	High	Low
Intermediate	Intermediate	High
Intermediate	Intermediate	Intermediate
Intermediate	Intermediate	Low
Intermediate	Low	High
Intermediate	Low	Intermediate
Intermediate	Low	Low
Low	High	High
Low	High	Intermediate
Low	High	Low
Low	Intermediate	High
Low	Intermediate	Intermediate
Low	Intermediate	Low
Low	Low	High
Low	Low	Intermediate
Low	Low	Low

Most Likely Cases

Figure 4.1 Demand Scenarios with Highest Probability of Occurrence

To tackle uncertainty, in the second stage, we consider 9 sub-problems by assigning the corresponding probability to compare with the stochastic solution to estimate VSS.

In reality, demand may repeat its pattern each year with variations. This may be due to the growth or decline of the company sales, population increase of the region, rise or fall in the brand name of the product, improvement in the advertising, change in the product according to the customer demands, competition, etc. It is also experienced and observed that the trend is generally same for all the products of a company.

As shown in Table 4.2, for each set of examples, the value of demand varies. The scenarios are based on this variation in demand given in Table 4.2. The problem was initiated with intermediate demand values, $D_{it}^{(I)}$. It is assumed to have a standard deviation of 5% from this mean value. Hence, range of the intermediate demand is considered as $D_{it}^{(I)} \pm 5\%$. The demand data for the products in each time period and scenarios are shown in Table 4.2. Other data used in the example is given in Appendix 1. Furthermore, there is a probability that the demand is outside the limit of the intermediate demand. Considering the continuous blocks of demand domain and same value of standard deviation ($\pm 5\%$), the demand for all 3 scenarios are shown in Table 4.2. Demand is assumed to be normally distributed. For calculation purpose, the mean of the distribution is considered as the demand value. Most of the data used in the examples are based on those from the company studied. However, some of the remaining data are hypothetical. Various verifications and tests were carried out to validate the data and results.

Table 4.2 Demand Data Variation

Product	Time Period	Deterministic Models		
i	t	Low Demand, $0.9 \times D_{it}^{(I)} \pm 5\%$	Intermediate Demand, $D_{it}^{(I)} \pm 5\%$	High Demand, $1.1 \times D_{it}^{(I)} \pm 5\%$
1	1	167400	186000	204600
1	2	148050	164500	180950
1	3	135900	151000	166100
1	4	144000	160000	176000
1	5	168300	187000	205700
1	6	184500	205000	225500
1	7	176400	196000	215600
1	8	139950	155500	171050
1	9	127800	142000	156200
1	10	131850	146500	161150
1	11	142380	158200	174020
1	12	180450	200500	220550
2	1	152100	169000	185900
2	2	139950	155500	171050
2	3	127800	142000	156200
2	4	135900	151000	166100
2	5	160200	178000	195800
2	6	176400	196000	215600
2	7	168300	187000	205700
2	8	132300	147000	161700
2	9	119700	133000	146300
2	10	123750	137500	151250
2	11	134280	149200	164120
2	12	172350	191500	210650
3	1	176400	196000	215600
3	2	164250	182500	200750
3	3	152100	169000	185900
3	4	160200	178000	195800
3	5	184500	205000	225500
3	6	200700	223000	245300
3	7	192600	214000	235400
3	8	156150	173500	190850
3	9	144000	160000	176000
3	10	148050	164500	180950
3	11	158580	176200	193820
3	12	196650	218500	240350

The model is programmed and solved using LINGO optimization solver, version 8.0, for the optimal solution. The size of the stochastic linear programming problem can be estimated by the number of variables and constraints. In this case, the total number of variables are 628 including 12 integers and 721 constraints. Hence, the size of the constraint coefficient matrix is 628×721.

The two statistics used for stochastic optimization problems are:

- Expected value of perfect information, EVPI
- Value of stochastic solution, VSS

4.1.1 Calculation of EVPI

Expected Value of Perfect Information, EVPI, is the expected or average profit or return, in long run, if perfect information is available before the decision is made (Dempster and Thompson, 1999). In order to calculate EVPI, the objective function for each demand scenario is multiplied with corresponding probability of occurrence.

Hence, on the basis of the above explanation, we have:

$$\textbf{EVPI} = Z_{Det} - Z_{Stoc} \tag{4.1}$$

Deterministic Solution, Z_{Det}

Let Z_1, Z_2 *and* Z_3 denote the objective functions obtained from scenarios 1, 2 and 3, and $\rho^{(1)}, \rho^{(2)}$ *and* $\rho^{(3)}$ denote the probabilities of scenarios 1, 2 and 3, respectively.

Then, we have

$$Z_{Det} = (\rho^{(1)} \times Z_1) + (\rho^{(2)} \times Z_2) + (\rho^{(3)} \times Z_3) \tag{4.2}$$

Stochastic Solution, Z_{Stoc}

Without perfect information, the total maximum profit can be obtained by solving the recourse problem. Recourse model is the deterministic equivalent of the stochastic model. The probabilities are associated with the decision variables based on "wait-and-see" or "here-and-now" decisions.

Wait-And-See Solutions

Wait-and-see (WS) problems assume that the decision-maker waits until the uncertainty is resolved before implementing optimal decisions. This approach, therefore, relies upon perfect information about the future.

Here-And-Now Solutions

Unlike wait-and-see solutions, here-and-now (HN) solutions are taken before the uncertainty is resolved. These decisions are irrespective of the probability of occurrence of a scenario in the future. Moreover, recourse of these variables is not possible.

Considering the demand as the input, the model is solved for low, intermediate and high demand scenarios. Results are provided in Appendix 2. Produced units and purchased units are observed as two main decision variables. Figure 4.2 shows the trend in the produced units of the 3 products for the 12 time periods. Similarly, the second decision variable is the number of the purchased units. Figure 4.3 represents the variation in the purchased units for each product over the entire planning horizon.

4.1.2 Calculation of VSS

Value of Stochastic Solution, VSS is the difference between the objective function value of the stochastic problem and the objective function value of the deterministic problem computed with stochastic variables replaced by their expected values (Harrison and Van-Mieghem, 1999).

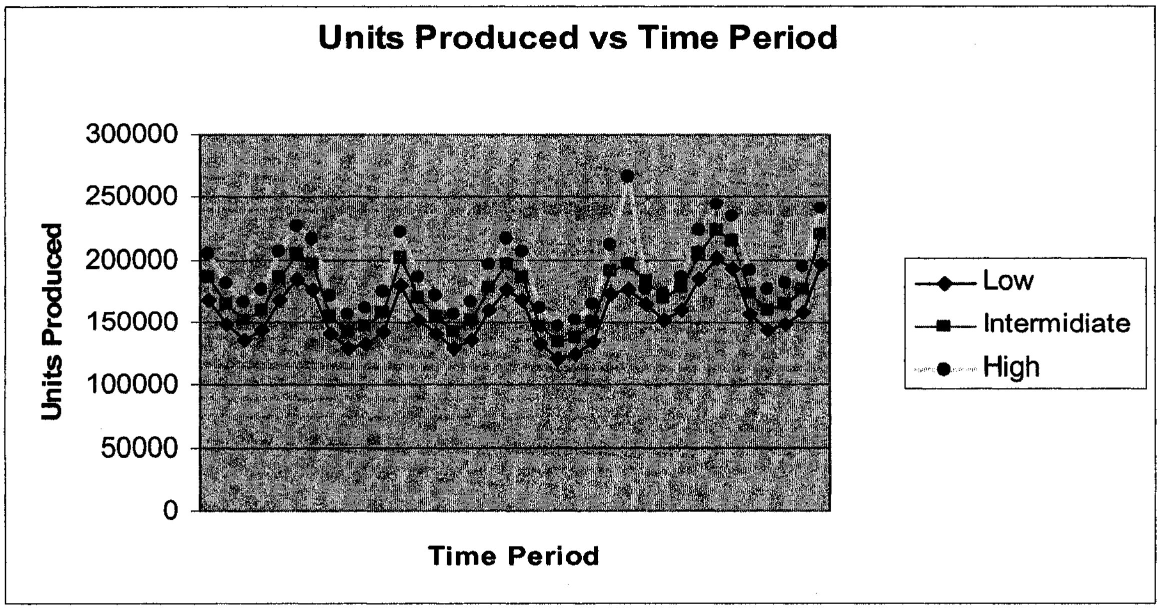


Figure 4.2 Trend of Decision Variable 1 w.r.t Time and Product Variation

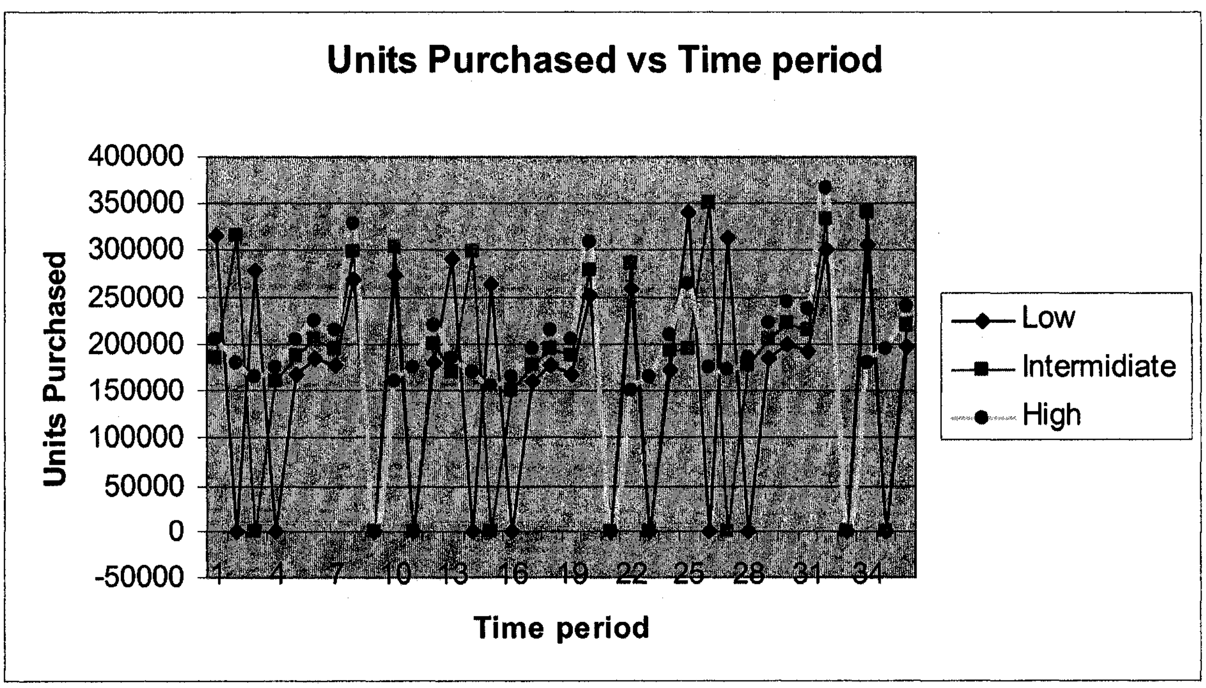


Figure 4.3 Trend of Decision Variable 2 w.r.t Time and Product Variation

In other words, it is the difference between stochastic solution value and expected value solution (EEV or EVS).

Therefore,

$$\text{VSS} = Z_{Stoc} - Z_{EEV} \tag{4.3}$$

where Z_{EEV} is computed by substituting the decision variables by their expected value in deterministic problem to obtain the second stage expected value solution.

It is not possible to find the "wait-and-see" solution if perfect information is not available. Easier way to solve is to replace all random variables by their expected values in a simpler problem to calculate EEV or EVS. Expectation of the expected value, EEV or EVS, is the parameter that measures how $\bar{x}(\bar{\xi})$ performs, allowing second stage decisions to be chosen optimally as functions of $\bar{x}(\bar{\xi})$ and ξ.

where $\bar{\xi} = E\ (\xi)$ denotes the expectation of the random variable ξ and $\bar{x}(\bar{\xi}) = \mathrm{E}\ (x(\bar{\xi}))$ denotes the expected value of the function $x(\bar{\xi})$ with $\bar{\xi}$ as the expectation of the random variable ξ

VSS is the statistical tool that measures how good a decision is, in terms of the deterministic equivalent of the stochastic solution. After the first stage solution, the decision variables are identified based on "here-and-now" or "wait-and-see" characteristics.

4.2 Variation in Probability of Scenarios

Example 1

Probability: 0.2, 0.2, 0.6

In example 1, the first set of probability of occurrence is assigned to the demand scenarios. Maximum profit for the deterministic problem is obtained from Equations 3.1 to 3.17 given in Chapter 3 for different scenarios. Based on the input data in Tables 4.1 and 4.2 and in Appendix 1, the first set of deterministic problems is solved to obtain the objective function for maximizing total profit shown in Table 4.3. Stochastic problem is formulated using Equations 3.18 to 3.34 and solved for the same set of data as the corresponding deterministic problem. The deterministic solution provides higher value of objective function compared to the stochastic value. This is due to the uncertainty in the stochastic solution.

- **Expected Value of Perfect Information, EVPI**

We compute EVPI for the data in Example 1. The profit values are shown in Table 4.3.

$Z_{Det} = (0.2 \times 5711869) + (0.2 \times 6350747) + (0.6 \times 6975182)$

Table 4.3 Example 1: Scenarios, Profit and Related Probabilities of Occurrence

Scenario	Demand Nature	Probability of Occurrence	Objective Function Total Maximum Profit
1	Low	0.2	5711869
2	Intermediate	0.2	6350747
3	High	0.6	6975182
Deterministic			**6597632**
Stochastic			**5857792**

$Z_{Det} = 6597632$

From the equations of stochastic solution,

$Z_{Stoc} = 5857792$

Calculating EVPI from Equation 4.2,

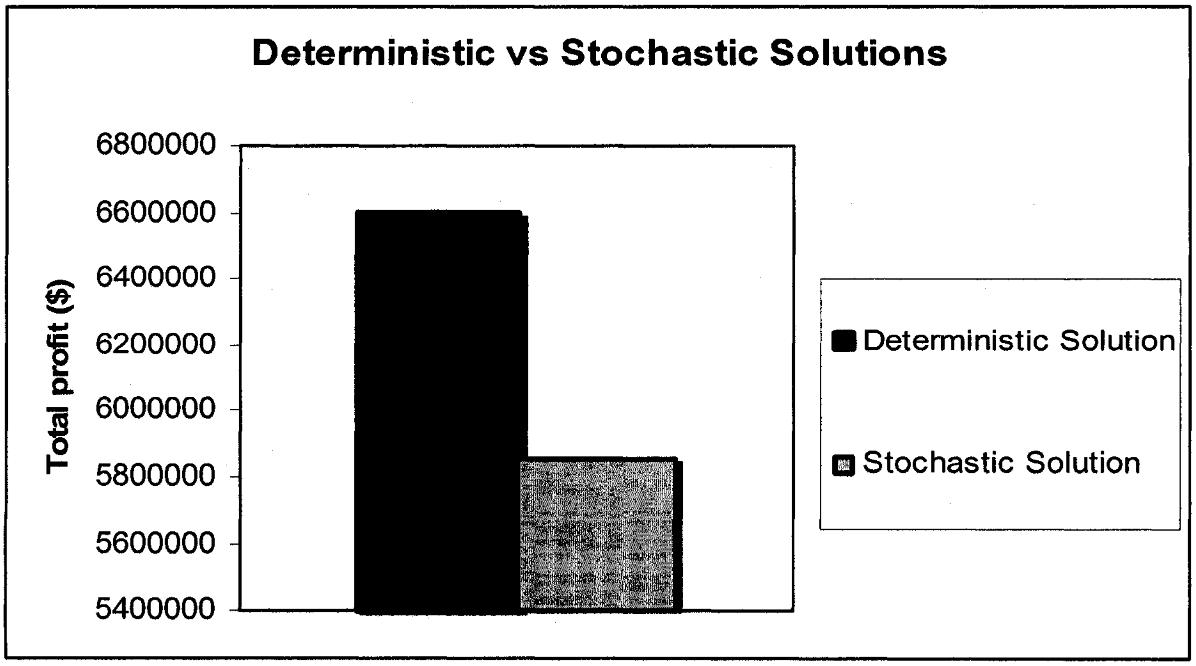


Figure 4.4 EVPI for Example 1

EVPI = 6597632 − 5857792 = 739840

Hence, it can be seen that the total profit would be **12.6%** more, if perfect information on the future is available. The difference in the maximum total profit obtained from deterministic and stochastic solutions is represented in Figure 4.4

- **Value of Stochastic Solution, VSS**

We compute Z_{EEV} by the optimal decision variables from the deterministic solution with corresponding probabilities to obtain EVS or EEV as follows

$Z^{(1)}_{EEV} = (\rho^{(1)} \times Z_{11}) + (\rho^{(2)} \times Z_{12}) + (\rho^{(3)} \times Z_{13})$, where $Z^{(1)}_{EEV}$ denotes the expectation of the expected value when random variables are substituted by their expected value for scenario 1.

$\rho^{(1)}, \rho^{(2)}$ *and* $\rho^{(3)}$ denote the probabilities of scenarios 1, 2 and 3, respectively.

Z_{11} = Objective function value of scenario 1 by setting the first stage decision variables of scenario 1

Z_{12} = Objective value of scenario 2 by setting the first stage decision variables of scenario 1

Z_{13} = Objective value of scenario 3 by setting the first stage decision variables of scenario 1

The second stage objective function values are shown in Table 4.4 on fixing the first stage decision variables.

$$Z_{EEV}^{(1)} = (0.2 \times 5711869) + (0.2 \times 5711172) + (0.6 \times 5711172)$$

$$Z_{EEV}^{(1)} = 5711312$$

Table 4.4 Second Stage Objective Fixing Scenario 1 Variables

Scenario	Demand Nature	Objective Value Maximum Profit
1	Low	5711869
2	Intermediate	5711172
3	High	5711172

Deterministic equivalent of stochastic solution, $Z_{Stoc} = 5857792$

VSS = Z_{Stoc} - Z_{EEV}

VSS = $5857792 - 5711312 = 146480$

Hence, it can be seen that stochastic solution is **2.56%** higher than $Z_{EEV}^{(1)}$.

Similarly, the first stage decision variables for scenario 2 are fixed and the objective function value for every scenario is calculated to obtain the total maximum profit as shown in Table 4.5. This provides the second stage of the stochastic solution. Uncertainty is involved and the random variables are substituted by their expected values. This solution is compared with the stochastic solution to find out which approach is better.

$$Z_{EEV}^{(2)} = (P^{(1)} \times Z_{21}) + (P^{(2)} \times Z_{22}) + (P^{(3)} \times Z_{23})$$

$$Z_{EEV}^{(2)} = (0.2 \times 3797669) + (0.2 \times 6350747) + (0.6 \times 6350747)$$

$$Z_{EEV}^{(2)} = 5840132$$

Deterministic equivalent of stochastic solution, $Z_{Stoc} = 5857792$

Therefore, **VSS** = 17660

Table 4.5 Second Stage Objective Fixing Scenario 2 Variables

Scenario	Demand Nature	Objective Value Maximum Profit
1	Low	3797669
2	Intermediate	6350747
3	High	6350747

Hence, the stochastic solution is **0.3%** higher than $Z_{EEV}^{(2)}$.

Lastly, the first stage decision variables for scenario 3 are fixed and the objective value for every scenario is calculated as represented in Table 4.6.

$$Z_{EEV}^{(3)} = (P^{(1)} \times Z_{31}) + (P^{(2)} \times Z_{32}) + (P^{(3)} \times Z_{33})$$

$$Z_{EEV}^{(3)} = (0.2 \times 1950251) + (0.2 \times 4503329) + (0.6 \times 6975182)$$

$$Z_{EEV}^{(3)} = 5475825$$

Deterministic equivalent of stochastic solution, $Z_{Stoc} = 5857792$

Table 4.6 Second Stage Objective Function Fixing Scenario 3 Variables

Scenario	Demand Nature	Objective Value Maximum Profit
1	Low	1950251
2	Intermediate	4503329
3	High	6975182

Therefore, **VSS** = 381967

Hence, the stochastic solution is **7%** higher than $Z_{EEV}^{(3)}$.

Comparing the stochastic solution with the individual scenario solutions, it is clearly evident that the stochastic solution gives higher value of total profit. Figure 4.5 shows the comparison of objective function values for total maximum profit.

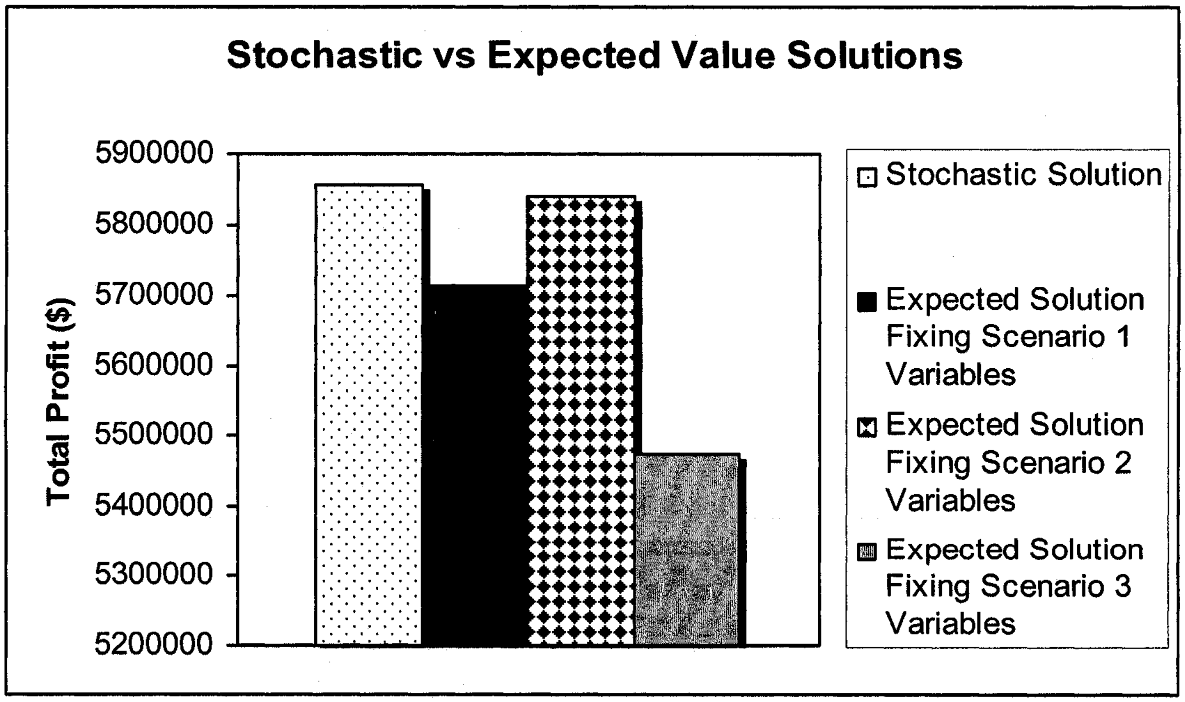


Figure 4.5 VSS for Example 1

Example 2

Probability: 0.7, 0.2, 0.1

In this example, the values of probability assigned to each scenario are changed. This is done to investigate the effect of the probability on the final solution. One particular scenario that has higher probability of occurrence is expected to play a key role in the final results. It would have a considerable impact on the values of the decision variables. Other data such as number of products, time periods in one cycle, costs and demand values remain constant. They are same as those presented in Tables 4.1 and 4.2. Other data used in the problem are shown in Appendix 1.

Table 4.7 Example 2: Scenarios, Profit and Related Probabilities of Occurrence

Scenario	**Demand Nature**	**Objective Function Total Maximum Profit**	**Probability of Occurrence**
1	Low	5711869	0.7
2	Intermediate	6350747	0.2
3	High	6975182	0.1

Table 4.7 shows the summary of the deterministic problem objective function values for each scenario and their related probabilities as assigned for Example 2.

$$Z_{Det} = (0.7 \times 5711869) + (0.2 \times 6350747) + (0.1 \times 6975182)$$

$$Z_{Det} = 5965976$$

$$Z_{Stoc} = 5711869$$

EVPI = $5965976 - 5711869 = 254107$

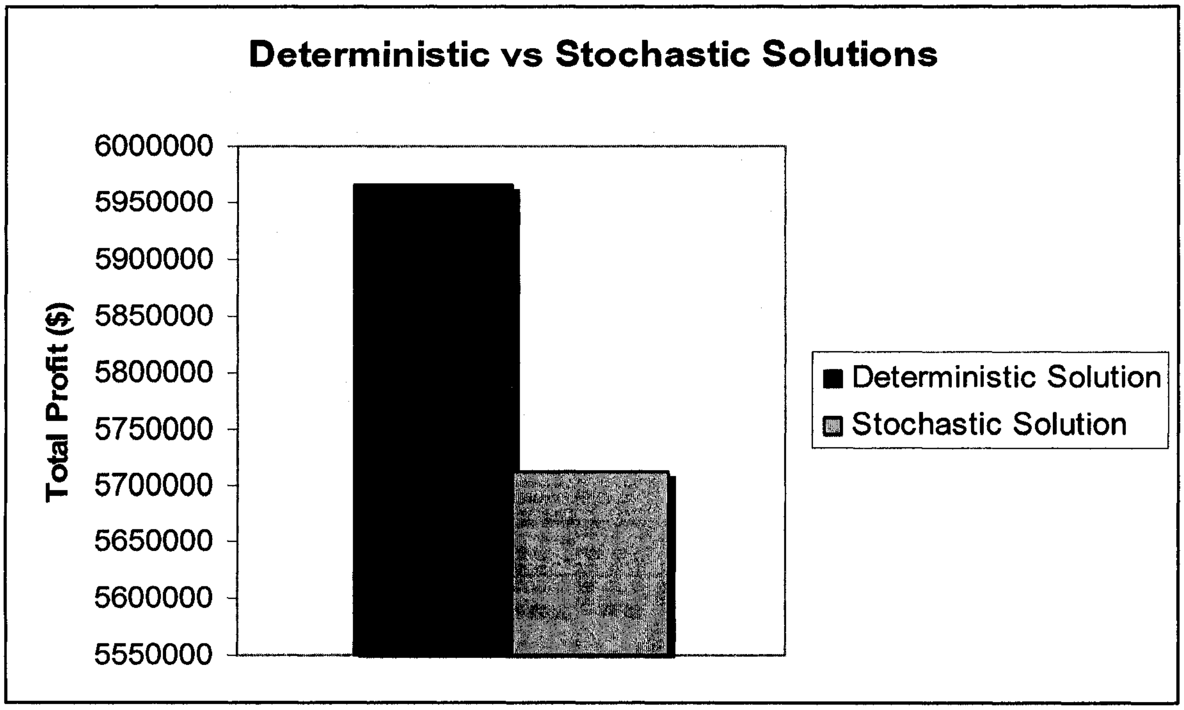


Figure 4.6 EVPI for Example 2

Hence, it can be seen that the maximum total profit would be **4.4%** more, if perfect information on the future is available. The difference in the maximum total profit obtained from the deterministic and stochastic solutions is presented in Figure 4.6

Value of Stochastic Solution, VSS

VSS = Z_{Stoc} - Z_{EEV}

The second stage objective function values are shown in Tables 4.4, 4.5 and 4.6 by fixing the first stage decision variables. Table 4.8 shows the summary of the objective function values obtained.

$$Z^{(1)}_{EEV} = (0.7 \times 5711869) + (0.2 \times 5711172) + (0.1 \times 5711172)$$

$$Z^{(1)}_{EEV} = 5711660$$

$$Z_{Stoc} = 5711869$$

VSS = 5711869 − 5711660 = 209

Table 4.8 Second Stage Objective Values Fixing Decision Variables

Scenario	Demand Nature	Objective Value Maximum Profit		
		Scenario 1 Variables Fixed	**Scenario 2 Variables Fixed**	**Scenario 3 Variables Fixed**
1	Low	5711869	3797669	1950251
2	Intermediate	5711172	6350747	4503329
3	High	5711172	6350747	6975182

Hence, it can be seen that the stochastic solution is higher than $Z_{EEV}^{(1)}$ by **$ 209**.

It is interesting to note that there is a negligible difference between the values obtained by stochastic solution and EVS fixing scenario 1 variables. This is due to the high probability of existence of scenario 1. The EVS is administered by scenario 1. Hence, when the random variables are substituted by their expected value, the substitution of scenario 1 variables resulted in nearly ideal solution.

$Z_{EEV}^{(2)} = (0.7 \times 3797669) + (0.2 \times 6350747) + (0.1 \times 6350747)$

$Z_{EEV}^{(2)} = 4563593$

$Z_{Stoc} = 5711869$

VSS = 1148276

Hence, the stochastic solution is **25.16%** higher than $Z_{EEV}^{(2)}$.

$Z_{EEV}^{(3)} = (0.7 \times 1950251) + (0.2 \times 4503329) + (0.1 \times 6975182)$

$Z_{EEV}^{(3)} = 2963360$

$Z_{Stoc} = 5711869$

Therefore, **VSS** = 2748509

Hence, $Z_{EEV}^{(3)}$ is **48%** lower than stochastic solution.

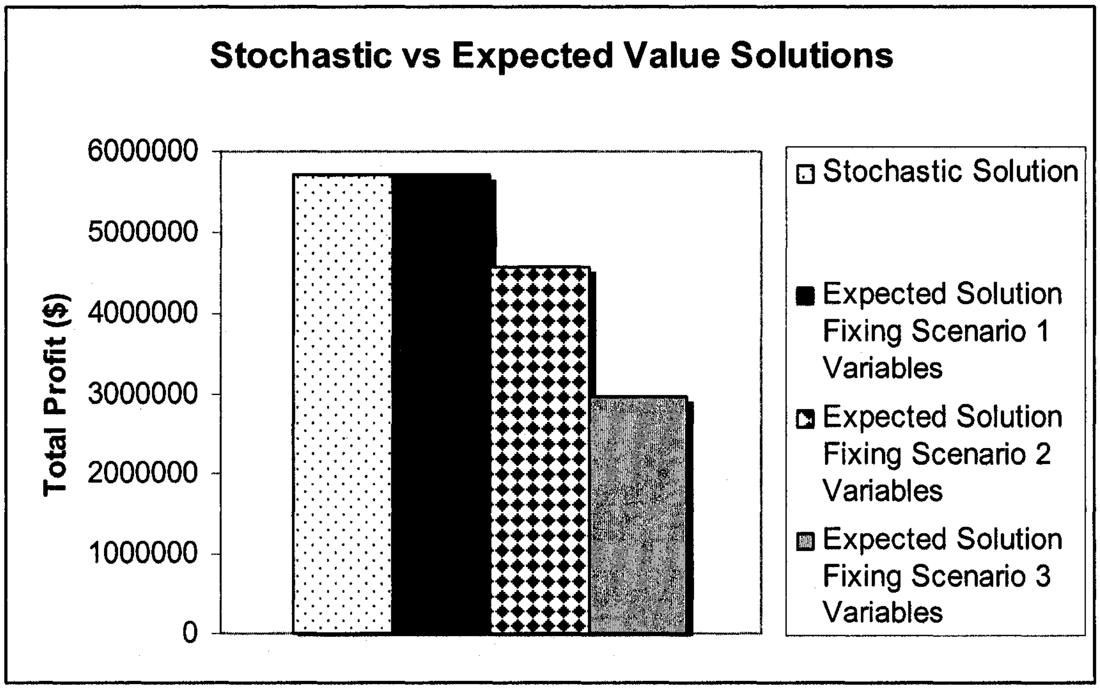


Figure 4.7 VSS for Example 2

Comparing the stochastic solution with the individual scenario solutions, it is clear that the stochastic solution gives a higher total profit. Figure 4.7 shows the comparison of the objective values.

Example 3

Probability: 0.3, 0.5, 0.2

Table 4.9 shows the summary of the objective function values of the deterministic model objective values for each scenario and their related probabilities as assigned for Example 3.

$Z_{Det} = (0.3 \times 5711869) + (0.5 \times 6350747) + (0.2 \times 6975182)$

$Z_{Det} = 6283971$

$Z_{Stoc} = 5747722$

EVPI = $6283971 - 5747722 = 536249$

Hence, it can be seen that the maximum total profit would be **9.3%** more, if perfect information on the future is available.

Table 4.9 Example 3: Scenarios, Profit and Related Probabilities of Occurrence

Scenario	Demand Nature	Objective Function Total Maximum Profit	Probability of Occurrence
1	Low	5711869	0.3
2	Intermediate	6350747	0.5
3	High	6975182	0.2

The difference in the maximum total profit obtained from the deterministic and stochastic solutions is shown in Figure 4.8.

Value of Stochastic Solution, VSS

VSS = Z_{Stoc} - Z_{EEV}

$Z_{EEV}^{(1)} = (0.3 \times 5711869) + (0.5 \times 5711172) + (0.2 \times 5711172)$

$Z_{EEV}^{(1)} = 5711381$

$Z_{Stoc} = 5747722$

VSS = $5747722 - 5711381 = 36341$

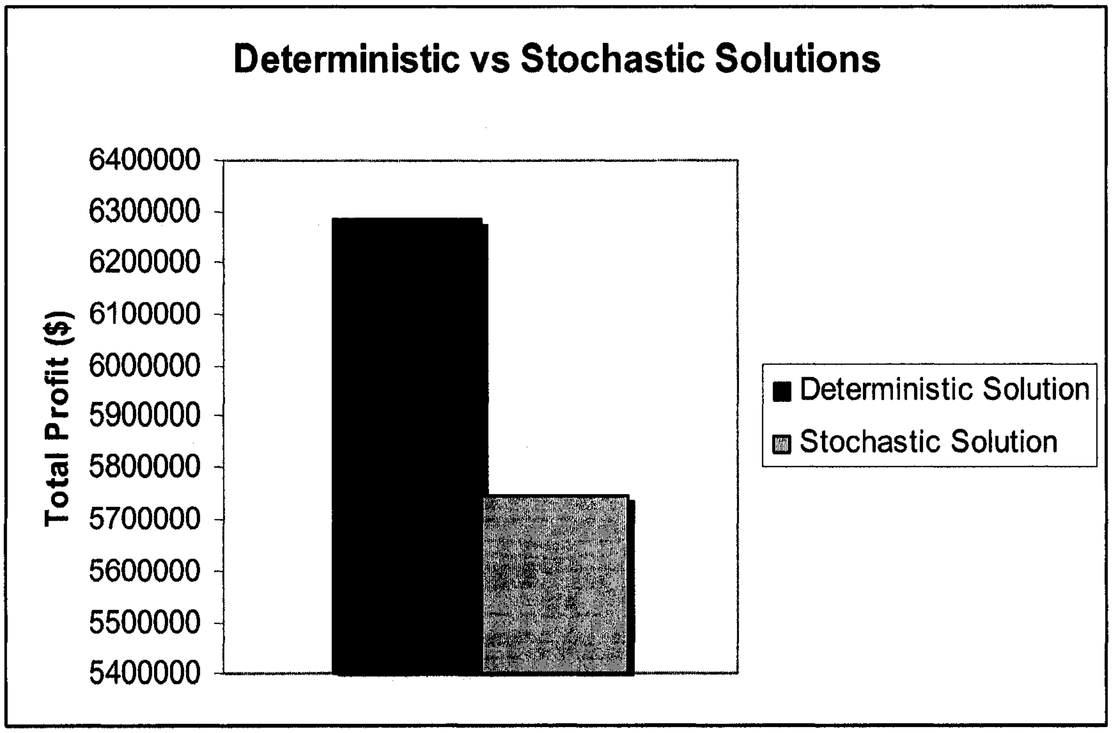


Figure 4.8 EVPI for Example 3

Hence, it can be concluded that stochastic solution is **0.63%** higher than $Z_{EEV}^{(1)}$.

$$Z_{EEV}^{(2)} = (0.3 \times 3797669) + (0.5 \times 6350747) + (0.2 \times 6350747)$$

$$Z_{EEV}^{(2)} = 5584824$$

$$Z_{Stoc} = 5747722$$

VSS = 162898

Hence, the stochastic solution is **2.9%** higher than $Z_{EEV}^{(2)}$.

$$Z_{EEV}^{(3)} = (0.3 \times 1950251) + (0.5 \times 4503329) + (0.2 \times 6975182)$$

$$Z_{EEV}^{(3)} = 4231776$$

$$Z_{Stoc} = 5747722$$

VSS = 1515946

Hence, $Z_{EEV}^{(3)}$ is **26.37%** lower than the stochastic solution.

Comparing the stochastic solution with the individual scenario solutions, it is clear that the stochastic solution gives a higher total profit. Figure 4.9 shows the comparison of the objective values.

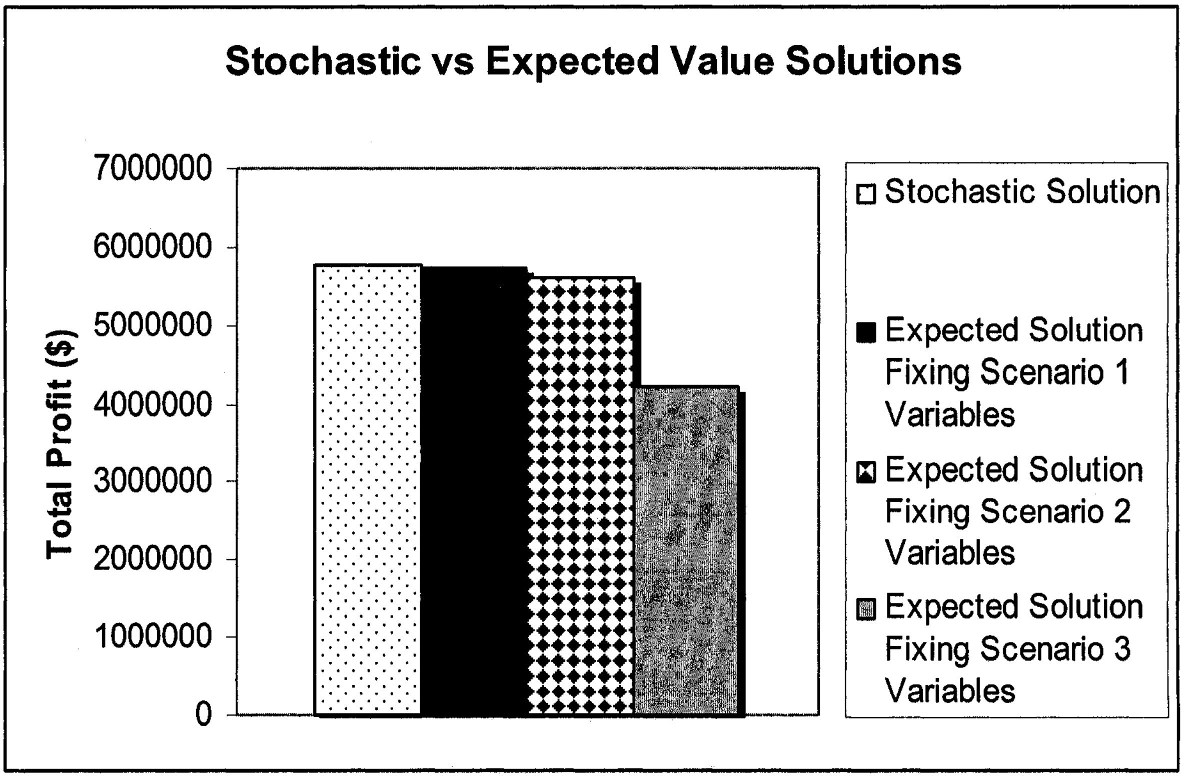


Figure 4.9 VSS for Example 3

Example 4

Probability: 0.333, 0.334, 0.333

Table 4.10 shows the summary of the deterministic model objective values for each scenario and their related probabilities as assigned for Example 4.

$$Z_{Det} = (0.333 \times 5711869) + (0.334 \times 6350747) + (0.333 \times 6975182)$$

$$Z_{Det} = 6339587$$

$Z_{Stoc} = 5720982$

EVPI $= 6339587 - 5720982 = 618605$

Table 4.10 Example 4, Scenarios, Profit and Related Probabilities of Occurrence

Scenario	Demand Nature	Objective Function Total Maximum Profit	Probability of Occurrence
1	Low	5711869	0.333
2	Intermediate	6350747	0.334
3	High	6975182	0.333

Hence, it can be seen that the maximum total profit would be **10.8%** more, if perfect information on the future is available. The difference in the maximum total profit obtained from the deterministic and stochastic solutions is shown in Figure 4.10.

Value of Stochastic Solution, VSS

VSS = Z_{Stoc} - Z_{EEV}

$$Z_{EEV}^{(1)} = (0.333 \times 5711869) + (0.334 \times 5711172) + (0.333 \times 5711172)$$

$$Z_{EEV}^{(1)} = 5705693$$

$$Z_{Stoc} = 5720982$$

VSS $= 5720982 - 5705693 = 15289$

Hence, it can be seen that the stochastic solution is **$15289** higher than $Z_{EEV}^{(1)}$.

$$Z_{EEV}^{(2)} = (0.333 \times 3797669) + (0.334 \times 6350747) + (0.333 \times 6350747)$$

$Z_{EEV}^{(2)} = 5494221$

$Z_{Stoc} = 5720982$

VSS = 226761

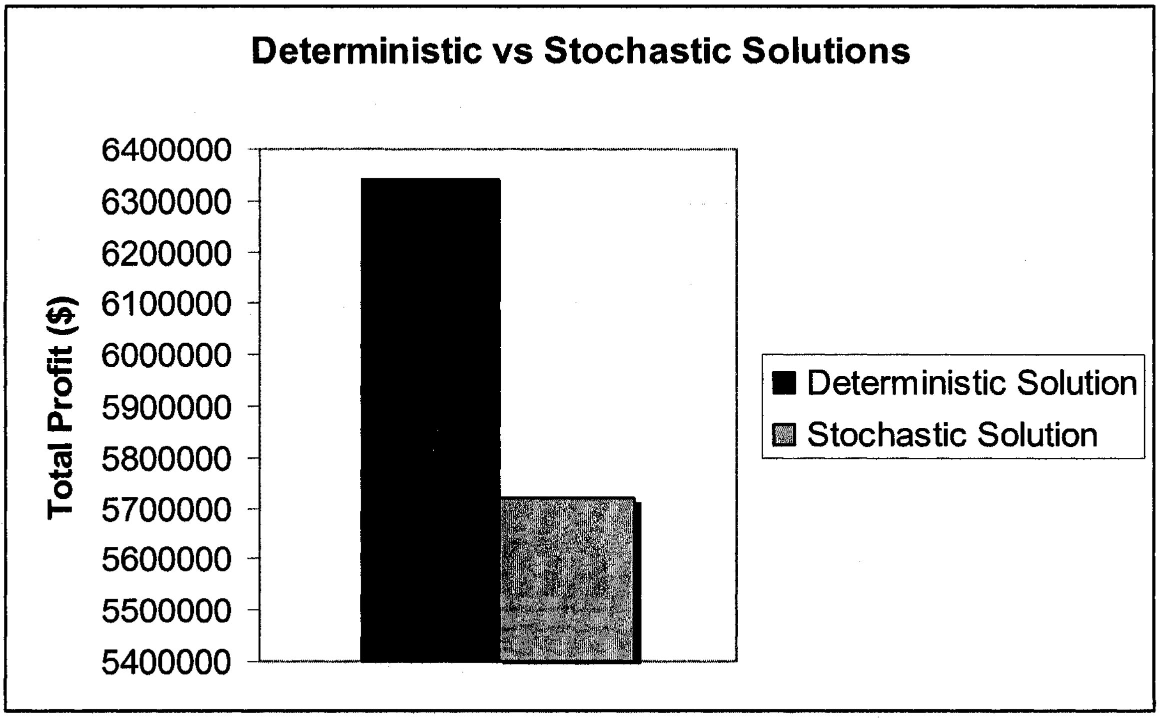


Figure 4.10 EVPI for Example 4

Hence, the stochastic solution is **4.1%** higher than $Z_{EEV}^{(2)}$.

$Z_{EEV}^{(3)} = (0.333 \times 1950251) + (0.334 \times 4503329) + (0.333 \times 6975182)$

$Z_{EEV}^{(3)} = 4471778$

$Z_{Stoc} = 5720982$

VSS = 1249204

Hence, $Z_{EEV}^{(3)}$ is **21.8%** lower than the stochastic solution.

Comparing the stochastic solution with the individual scenario solutions, it is clear that the stochastic solution gives a higher total profit. Figure 4.11 shows the comparison of the objective values.

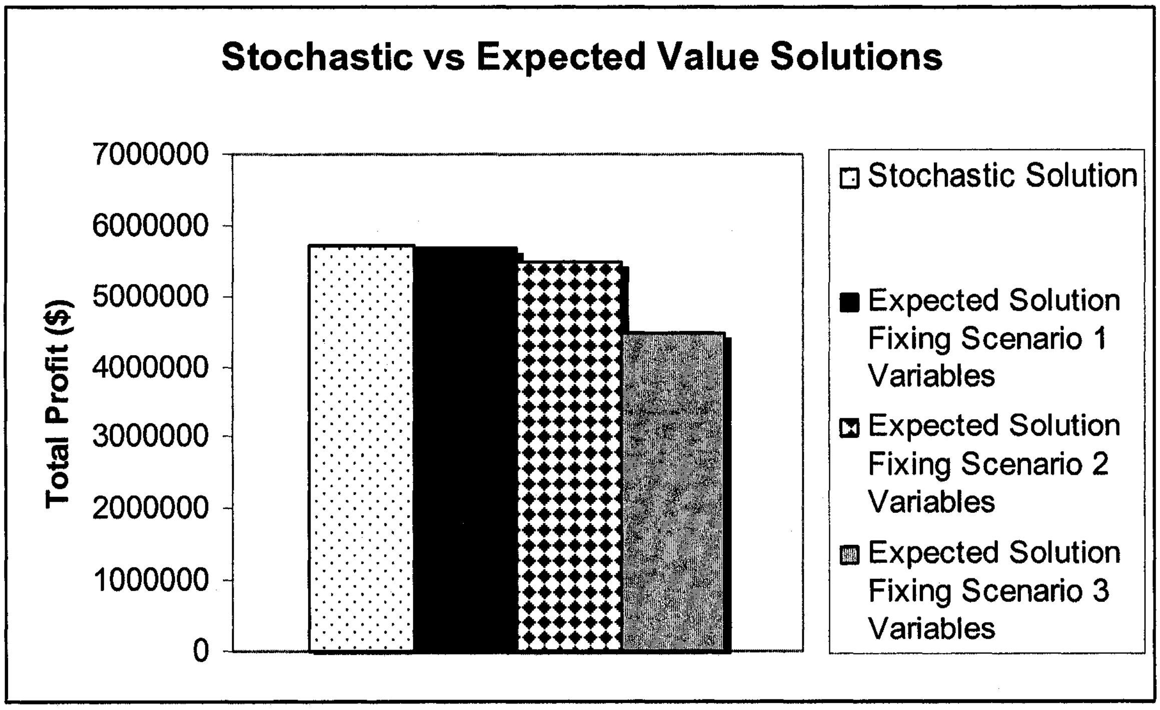


Figure 4.11 VSS for Example 4

4.3 Analysis of Results

The objective function values obtained in the examples vary with variation in the probability of occurrence of different scenarios. With change in probability, the solutions are governed by different scenarios or their combinations. The combinations of probabilities show that the stochastic solutions are more realistic. Table 4.11 shows the summary of the probabilities assigned to each scenario in the examples solved.

Table 4.11 Scenarios and Associated Probabilities

Scenario	Probability			
	Example 1	Example 2	Example 3	Example 4
1	0.2	0.7	0.3	0.333
2	0.2	0.2	0.5	0.334
3	0.6	0.1	0.2	0.333

Table 4.12 Deterministic Solution Summary

Scenario	Total Profit			
	Example 1	Example 2	Example 3	Example 4
1	1142374	3998308	1713561	1902052
2	1270149	1270149	3175374	2114799
3	4185109	697518	1395036	2322736
Total Profit (Deterministic)	**6597632**	**5965976**	**6283971**	**6339587**
Total Profit (Stochastic)	**5857792**	**5711869**	**5747722**	**5720982**
EVPI	**739840**	**254107**	**536249**	**618605**

Three deterministic problems are formulated for each example representing each of the low, intermediate and high demand scenarios. The summary of objective function values for maximum total profit for each problem is given in Table 4.12. It also represents the summary of the stochastic solution obtained from the examples and its comparison to the deterministic solution. Figure 4.12 shows the comparison of the first stage deterministic solution obtained from the problems solved above. It can be observed from Figure 4.12 that there is a considerable variation in the objective function values obtained by assigning different probabilities to each scenario. The reason for the variation is obvious. If high probability is associated with the high demand scenario, the profit is more.

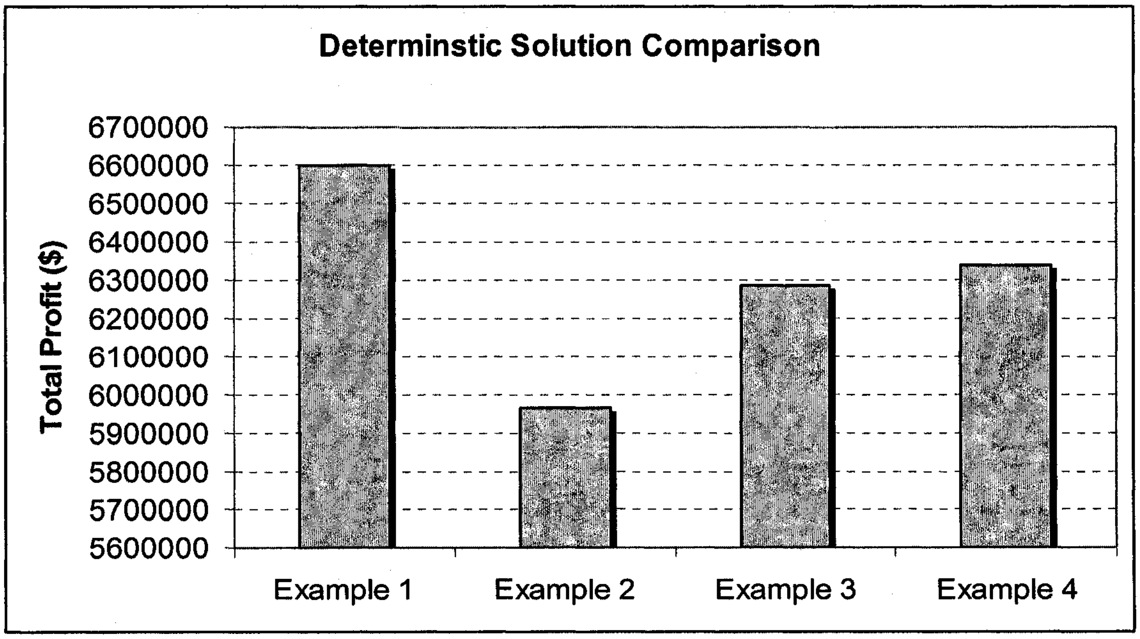


Figure 4.12 Deterministic Solution Comparisons

Since, there is a rise in the demand; the units sold would increase, leading to higher profit. Similarly, Figure 4.13 represents the comparison of the first stage stochastic solution obtained from the examples used in this thesis research.

Comparison of the values in Figures 4.12 and 4.13 is shown in Figure 4.14. The difference between the stochastic and the deterministic solution is EVPI.

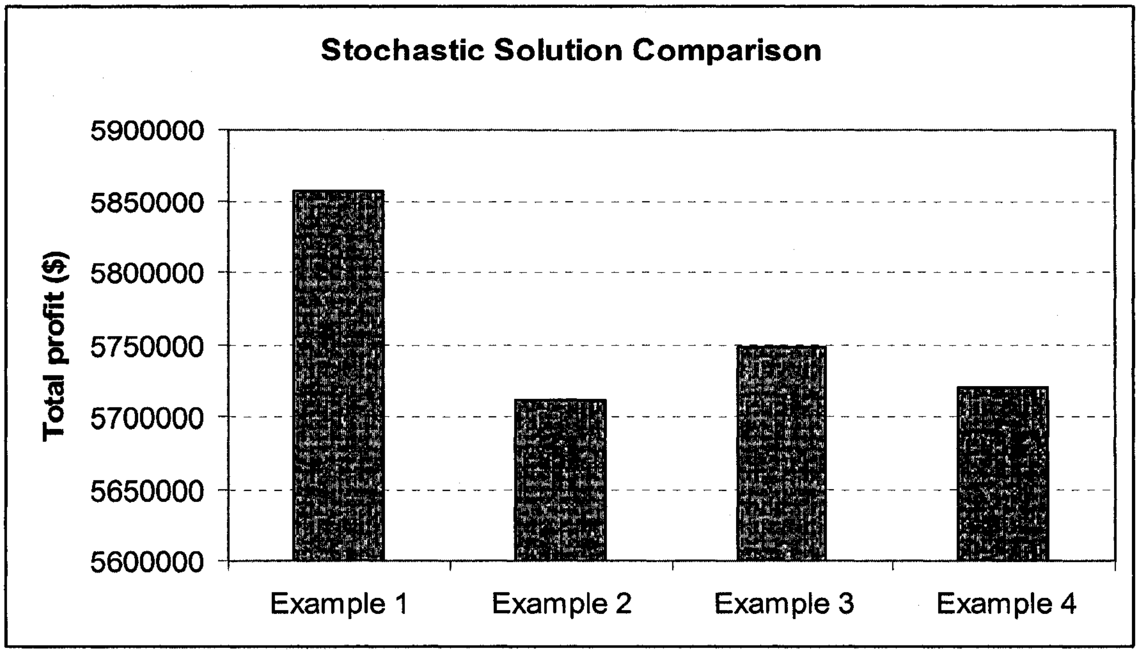


Figure 4.13 Stochastic Solution Comparisons

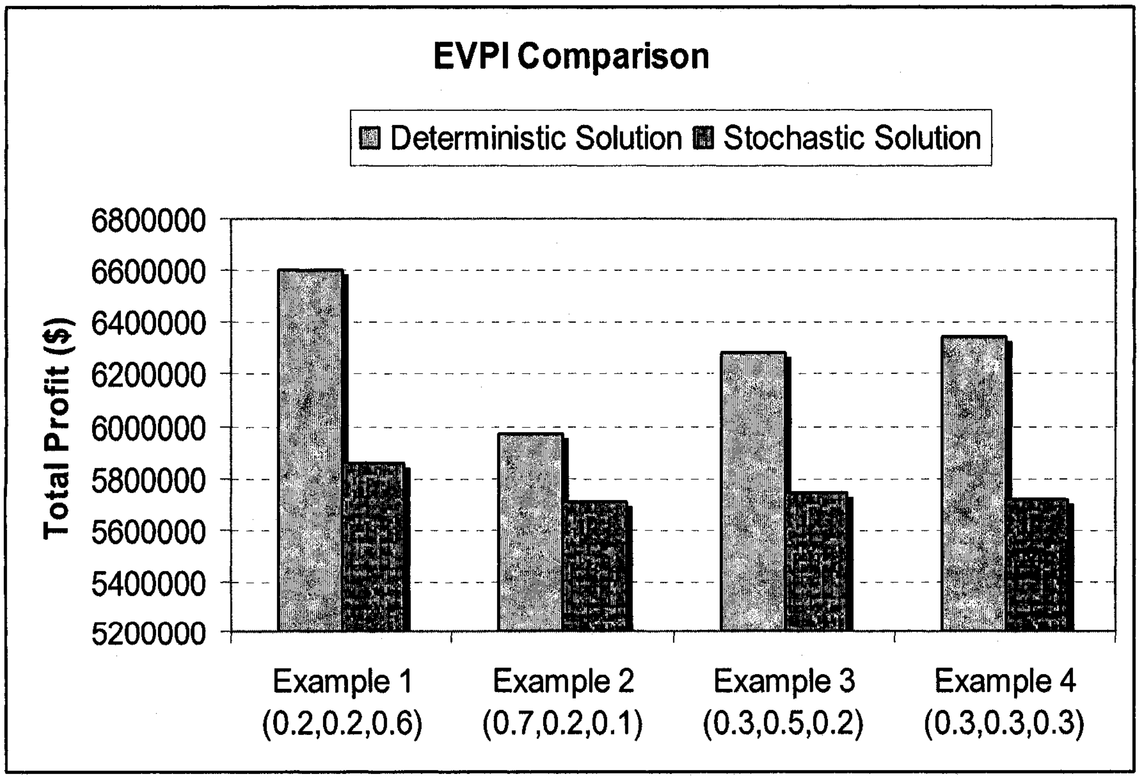


Figure 4.14 EVPI Comparison

"Wait-and-see" variables are assigned the related probabilities and "here-and-now" variables are fixed. Table 4.13 shows the summary of the second stage solution and the objective function values by fixing the variables to obtain EVS, Z_{EEV}. In this approach, the decision variables for scenarios 1, 2 and 3 are fixed one at a time and substituted for the random variables in the deterministic model. The second stage solution involves uncertainty in the existence of the scenarios. Based on uncertainty, the expected values of the decision variables are substituted. Moreover, Table 4.14 compares the numerical results of the expected value solutions and the stochastic solutions.

Table 4.13 Second Stage Stochastic Solution Summary

	Probability Set	Fixing Scenario	Total Profit		
			EVS	Stochastic Solution	VSS
Example 1	0.2	1	5711312	5857792	146480
	0.2	2	5840132		17660
	0.6	3	5475825		381967
Example 2	0.7	1	5711660	5711869	209
	0.2	2	4563593		1148276
	0.1	3	2963360		2748509
Example 3	0.3	1	5711381	5747722	36341
	0.5	2	5584824		162898
	0.2	3	4231776		1515946
Example 4	0.333	1	5705693	5720982	15289
	0.333	2	5494221		226761
	0.333	3	4471778		1249204

VSS obtained for each example problem and all the scenarios furnish higher value of the total profit which justifies the use of stochastic programming to model uncertainty. The comparison of the objective function values for stochastic solution and EVS is shown in Figure 4.15. The difference is the Value of Stochastic Solution, VSS.

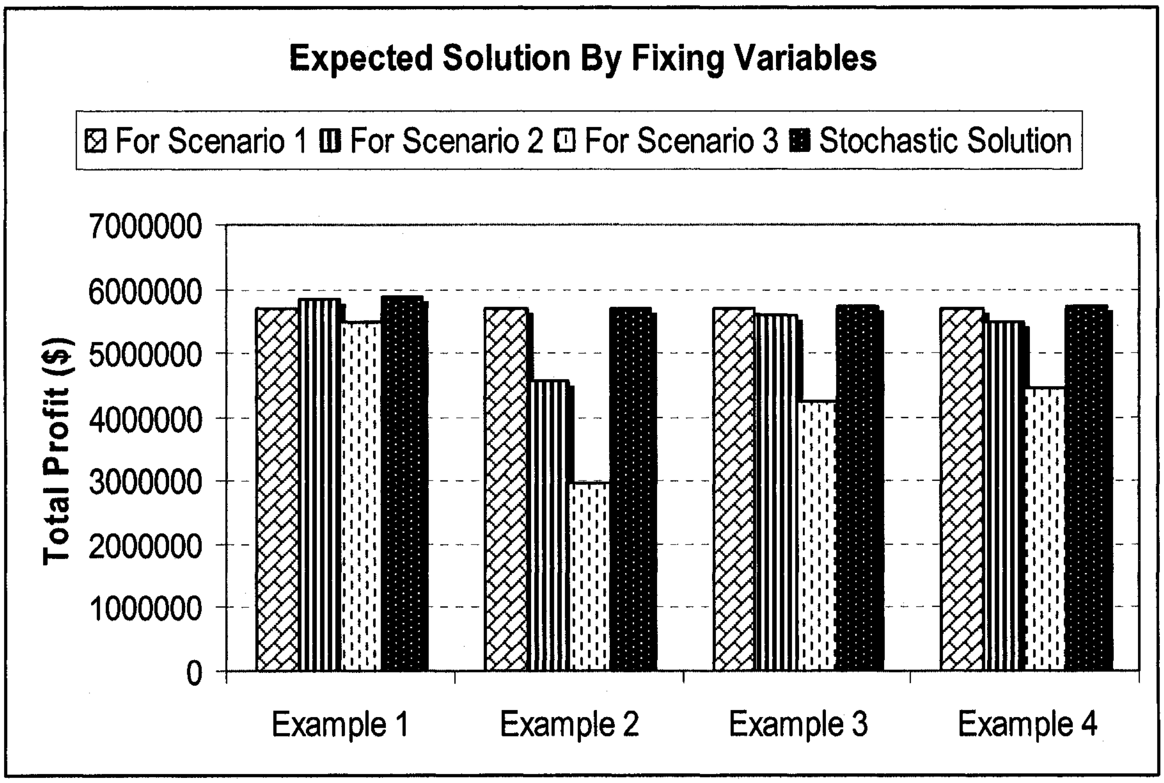


Figure 4.15 VSS Comparison

4.4 Summary

From the computational results of the example problems, one can see that the parameters that have the influence on the total profit, purchasing and production decisions are the probability of occurrence of each demand scenario. The uncertainty in the demand plays a major role in determining the value of maximum profit. However, there are other parameters which determine the outcome of a production planning model. The models in this research work are solved by varying the probability of occurrence of each scenario by keeping the number of scenarios constant. Other input data are maintained constant for the problems solved. The aim of this study is to draw a comparison between stochastic and deterministic solution methodology. The results obtained are reasonable for the variation in the most important parameter. Other parameters to be considered include introduction of more demand scenarios, variation of demand fluctuation rate, combination of different demand instances and assigning different probability sets to scenarios.

4.4.1 Stages of Solution

The solution of this type of stochastic model is obtained in 2 stages

First Stage

- Deterministic model is formed for maximizing total profit.
- Referring to the historical data, 3 most likely scenarios namely, low, intermediate and high, are obtained and assigned the respective values.
- Deterministic model is solved for each of the scenarios. Units purchased and units produced are the two decision variables found for 3 products and 12 time periods. Decision variables are identified on the basis of "here-and-now" decisions. Units sold, raw materials inventory and finished products inventory are identified as "wait-and-see" variables.
- Stochastic model is solved by assigning these sets of probabilities to the variables based on "wait-and-see" decisions. The objective function value for maximum profit is expressed as Z_{Stoc}.
- For deterministic model, objective function values are obtained assigning the probability sets and is expressed as Z_{Det}.

- The stochastic solutions for all 4 sets of probabilities are compared to the corresponding deterministic solution.
- It is observed that in each of the example solved above, the value of profit obtained from deterministic solution is more than the stochastic solution. The difference in the deterministic and stochastic solutions is the Expected Value of Perfect Information, EVPI.

EVPI is the amount a company would be willing to pay for the perfect information.

Second Stage

- The decision variables namely, units produced and units purchased identified in the first stage are fixed for the scenario 1 values.
- These decision variables are substituted for scenario 2 and scenario 3 values for the same set of input data. Hence, for fixing the variables for one scenario, three profit values are obtained in the second stage. The objective function values obtained are the Expectation of the Expected Values, EEV or Expected Value Solutions, EVS.
- The three objective function values obtained are assigned the respective probabilities to attain Z_{EEV} for first scenario.
- Similarly, decision variables are fixed for scenario 2 and scenario 3 to obtain 3 solutions of each scenario and assigned their corresponding probabilities.
- Hence, for Example 1, 3 EEV solutions are compared to the stochastic solution for the same example.
- It is observed that stochastic solution provides a higher objective function value in all three cases for maximum profit as compared to EVS.

- Similarly, the Z_{EEV} for example 2, 3 and 4 are calculated by varying the probabilities for each scenario and comparing with corresponding stochastic solution. Again, stochastic solution gives higher profit values than EEV or EVS.
- Stochastic solution is compared with several EEV solutions to demonstrate the efficiency of stochastic programming approach.
- The difference between the objective function values of Z_{Stoc} and Z_{EEV} is referred to as the Value of Stochastic Solution, VSS.

VSS is the value of profit obtained from stochastic solution over the expected value of deterministic solution considering uncertainty.

4.4.2 Effects of Variation in Data

The variations in the results calculated are dependent on various factors. Following are the factors considered in this study.

- **Variation in the demand**

With increase in demand for a product there is an increase in the units produced, units purchased, inventories and units sold. The increase in the profit with the increase of demand implies that the costs which are directly proportional to the units sold are lower than the profit.

- **Variation in the probability of scenarios with constant number of scenarios**

Total profit increases with the variation between the probabilities of occurrence of scenarios. EVPI decreases with the increase in variation of probability between scenarios. However, VSS increases with the increase in variation of probability between scenarios. There are other factors which have an effect on EVPI and VSS values. The variation of such other factors is not considered in this study.

Since, the results obtained are conducive with the variation of the factors considered; it must be conducive for the variation in other factors. Hence, to avoid the complexity of the model, following factors and their variations are not used for the calculations in this thesis work.

- **Variation in number of scenarios**

Total profit decreases with the increase in the number of scenarios. EVPI increases with the increase in the number of scenarios. VSS decreases with the increase in the number of scenarios.

- **Variation in demand for each scenario keeping the number of scenarios constant**

Increasing demand values for all the scenarios simultaneously: Total profit from the stochastic solution increases constantly. EVPI remains almost constant. VSS increases with the increase in the demand.

Increasing demand values for one scenario while keeping other 2 constant: Total profit from the stochastic solution increases at a constant rate. EVPI decreases with the increase in the demand of any one scenario. VSS increases sturdily with the increase of demand of one scenario keeping demand constant for other scenarios.

Chapter Five

Conclusions and Future Research

This chapter presents a summary of the research conducted in this thesis. It also presents several concluding remarks based on the problem modeling and results analysis. Future directions for research on this study are also discussed.

5.1 Concluding Summary

In this thesis, two MILP models for capacity-constrained production planning are proposed for modeling uncertainty in product demand. The first mathematical model is formulated as deterministic model for maximizing total profit. The profit obtained is based on known values of product demand. The same model is solved again with random variables substituted by their expected values to obtain Expected Value Solution (EVS) for each individual scenario. The second model is formulated as stochastic optimization model where demand is stochastic.

This research extends the work of Fransoo et al. (1995) to multiple machines. They developed a linear programming model for multi-item, single-machine planning and scheduling. In this thesis, a generic problem is formulated to accommodate several scenarios simultaneously. Problem size has been substantially increased due to the increase in planning horizon. Effects of seasonal variation in product demand on production and purchasing quantities subject to capacity constraints are discussed in this work. Based on the information of a brewery company, several example problems are solved and results are verified to ascertain the robustness of the two models.

It is important to note that the primary objective of this thesis work is to clearly outline a comparative analysis between deterministic and stochastic modeling approaches. The variations of the total profit are tabulated and represented graphically. Two statistics for stochastic optimization problems: Expected Value of Perfect Information (EVPI) and Value of Stochastic Solution (VSS) are measured and the variations are analyzed.

5.2 Future Research

In this research a stochastic production planning model is considered to cope with demand uncertainty. However, there are many other parameters for which uncertainty is not considered. These parameters include machine failures, setup costs, purchasing, and inventory.

The author would consider the following aspects for the future research of this study:

- Cost and time factors may be expressed as unique values rather than a fraction of other costs or time values. For example, Cost of Quality is expressed as a fraction of production cost in this thesis.
- Setup time and cost may be included in case of change over from one product to the other. Also, lead time and penalty on loss of sales may be taken into consideration.
- More robust stochastic models can be developed which are capable of handling more complex production planning problems with multiple vendors and multiple distributors.

- Meta-heuristics can be designed to obtain effective solutions for large instances of problems and complex data.
- Sensitivity analysis of the results can be conducted to analyze the competency of the solution.
- A generic algorithm can be formulated to account for uncertainty in any parameter without much change in model structure.
- Problem can be solved with more scenarios for real life applicability.

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Appendix 1

Tables Utilized in Solving the Model

Table A1: Purchasing Costs

Products / Months	Product 1	Product 2	Product 3
Jan	0.45	0.35	0.2
Feb	0.45	0.35	0.2
Mar	0.45	0.35	0.2
Apr	0.45	0.35	0.2
May	0.45	0.35	0.2
Jun	0.45	0.35	0.2
Jul	0.45	0.35	0.2
Aug	0.45	0.35	0.2
Sep	0.45	0.35	0.2
Oct	0.45	0.35	0.2
Nov	0.45	0.35	0.2
Dec	0.45	0.35	0.2

Table A2: Production Costs

Products / Months	Product 1	Product 2	Product 3
Jan	0.8	0.65	0.3
Feb	0.8	0.65	0.3
Mar	0.8	0.65	0.3
Apr	0.8	0.65	0.3
May	0.8	0.65	0.3
Jun	0.8	0.65	0.3
Jul	0.8	0.65	0.3
Aug	0.8	0.65	0.3
Sep	0.8	0.65	0.3
Oct	0.8	0.65	0.3
Nov	0.8	0.65	0.3
Dec	0.8	0.65	0.3

Table A3: Finished Product Costs

Products / Months	Product 1	Product 2	Product 3
Jan	2.2	1.8	0.85
Feb	2.2	1.8	0.85
Mar	2.2	1.8	0.85
Apr	2.2	1.8	0.85
May	2.2	1.8	0.85
Jun	2.2	1.8	0.85
Jul	2.2	1.8	0.85
Aug	2.2	1.8	0.85
Sep	2.2	1.8	0.85
Oct	2.2	1.8	0.85
Nov	2.2	1.8	0.85
Dec	2.2	1.8	0.85

Table A4: Selling Price

Products / Months	Product 1	Product 2	Product 3
Jan	2.9	2.4	1.1
Feb	2.9	2.4	1.1
Mar	2.9	2.4	1.1
Apr	2.9	2.4	1.1
May	2.9	2.4	1.1
Jun	2.9	2.4	1.1
Jul	2.9	2.4	1.1
Aug	2.9	2.4	1.1
Sep	2.9	2.4	1.1
Oct	2.9	2.4	1.1
Nov	2.9	2.4	1.1
Dec	2.9	2.4	1.1

Table A5: Raw Materials Inventory Capacity Restrictions

Products / Months	Product 1	Product 2	Product 3
Jan	200000	150000	250000
Feb	200000	150000	250000
Mar	200000	150000	250000
Apr	200000	150000	250000
May	200000	150000	250000
Jun	200000	150000	250000
Jul	200000	150000	250000
Aug	200000	150000	250000
Sep	200000	150000	250000
Oct	200000	150000	250000
Nov	200000	150000	250000
Dec	200000	150000	250000

Table A6: Finished Products Inventory Capacity Restrictions

Products / Months	Product 1	Product 2	Product 3
Jan	250000	180000	300000
Feb	250000	180000	300000
Mar	250000	180000	300000
Apr	250000	180000	300000
May	250000	180000	300000
Jun	250000	180000	300000
Jul	250000	180000	300000
Aug	250000	180000	300000
Sep	250000	180000	300000
Oct	250000	180000	300000
Nov	250000	180000	300000
Dec	250000	180000	300000

Table A7: Maximum Investment on Purchasing

Months	Products	Maximum Investment
Jan	Product 1 Product 2 Product 3	1200000
Feb	Product 1 Product 2 Product 3	1150000
Mar	Product 1 Product 2 Product 3	1100000
Apr	Product 1 Product 2 Product 3	1150000
May	Product 1 Product 2 Product 3	1200000
Jun	Product 1 Product 2 Product 3	1250000
Jul	Product 1 Product 2 Product 3	1180000
Aug	Product 1 Product 2 Product 3	1200000
Sept	Product 1 Product 2 Product 3	1150000
Oct	Product 1 Product 2 Product 3	1170000
Nov	Product 1 Product 2 Product 3	1180000
Dec	Product 1 Product 2 Product 3	1250000

Table A8: Maximum Investment on Raw Materials Inventory

Months	Products	Maximum Investment
Jan	Product 1	250000
	Product 2	
	Product 3	
Feb	Product 1	230000
	Product 2	
	Product 3	
Mar	Product 1	210000
	Product 2	
	Product 3	
Apr	Product 1	220000
	Product 2	
	Product 3	
May	Product 1	270000
	Product 2	
	Product 3	
Jun	Product 1	300000
	Product 2	
	Product 3	
Jul	Product 1	260000
	Product 2	
	Product 3	
Aug	Product 1	250000
	Product 2	
	Product 3	
Sept	Product 1	210000
	Product 2	
	Product 3	
Oct	Product 1	220000
	Product 2	
	Product 3	
Nov	Product 1	240000
	Product 2	
	Product 3	
Dec	Product 1	300000
	Product 2	
	Product 3	

Table A9: Product Demand

Months	Products	Demand		
		Scenario 1 (Low)	Scenario 2 (Intermediate)	Scenario 3 (High)
Jan	**Product 1**	**167400**	**186000**	**204600**
	Product 2	**152100**	**169000**	**185900**
	Product 3	**176400**	**196000**	**215600**
Feb	**Product 1**	**148050**	**164500**	**180950**
	Product 2	**139950**	**155500**	**171050**
	Product 3	**164250**	**182500**	**200750**
Mar	**Product 1**	**135900**	**151000**	**166100**
	Product 2	**127800**	**142000**	**156200**
	Product 3	**152100**	**169000**	**185900**
Apr	**Product 1**	**144000**	**160000**	**176000**
	Product 2	**135900**	**151000**	**166100**
	Product 3	**160200**	**178000**	**195800**
May	**Product 1**	**168300**	**187000**	**205700**
	Product 2	**160200**	**178000**	**195800**
	Product 3	**184500**	**205000**	**225500**
Jun	**Product 1**	**184500**	**205000**	**225500**
	Product 2	**176400**	**196000**	**215600**
	Product 3	**200700**	**223000**	**245300**
Jul	**Product 1**	**176400**	**196000**	**215600**
	Product 2	**168300**	**187000**	**205700**
	Product 3	**192600**	**214000**	**235400**
Aug	**Product 1**	**139950**	**155500**	**171050**
	Product 2	**132300**	**147000**	**161700**
	Product 3	**156150**	**173500**	**190850**
Sept	**Product 1**	**127800**	**142000**	**156200**
	Product 2	**119700**	**133000**	**146300**
	Product 3	**144000**	**160000**	**176000**
Oct	**Product 1**	**131850**	**146500**	**161150**
	Product 2	**123750**	**137500**	**151250**
	Product 3	**148050**	**164500**	**180950**
Nov	**Product 1**	**142380**	**158200**	**174020**
	Product 2	**134280**	**149200**	**164120**
	Product 3	**158580**	**176200**	**193820**
Dec	**Product 1**	**180450**	**200500**	**220550**
	Product 2	**172350**	**191500**	**210650**
	Product 3	**196650**	**218500**	**240350**

Table A10: Number of Production Lines

Months	No. of Production Lines
Jan	2
Feb	1
Mar	1
Apr	1
May	3
Jun	4
Jul	3
Aug	2
Sep	1
Oct	1
Nov	2
Dec	3

Table A11: Probabilities of Scenarios

Probability Set	Product 1	Product 2	Product 3
1	0.2	0.2	0.6
2	0.7	0.2	0.1
3	0.3	0.5	0.2
4	0.333	0.334	0.333

Table A12: Other Input Data

Ordering Cost	5000
Carrying Cost as Fraction of Unit Cost for Raw Materials	0.15
Carrying Cost as Fraction of Unit Cost for Finished Products	0.15
Labor Cost as Fraction of Production Cost	0.1
Cost of Quality as Fraction of Production Cost	0.02
Units Produced per Hour	1000

Appendix 2

Tables Utilized in Presenting the Discussion

Deterministic Solution for Different Demand Scenarios

Table A13: Units Produced vs. Time Period

Product	Month	Units Produced		
		Low Demand Scenario	Intermediate Avg. Demand Scenario	High Demand Scenario
1	Jan	167400	186000	204600
	Feb	148050	164500	180950
	Mar	135900	151000	166100
	Apr	144000	160000	176000
	May	168300	187000	205700
	Jun	184500	205000	225500
	Jul	176400	196000	215600
	Aug	139950	155500	171050
	Sep	127800	142000	156200
	Oct	131850	146500	161150
	Nov	142380	158200	174020
	Dec	180450	200500	220550
2	Jan	152100	169000	185900
	Feb	139950	155500	171050
	Mar	127800	142000	156200
	Apr	135900	151000	166100
	May	160200	178000	195800
	Jun	176400	196000	215600
	Jul	168300	187000	205700
	Aug	132300	147000	161700
	Sep	119700	133000	146300
	Oct	123750	137500	151250
	Nov	134280	149200	164120
	Dec	172350	191500	210650
3	Jan	176400	196000	265100
	Feb	164250	182500	176000
	Mar	152100	169000	173525
	Apr	160200	178000	185900
	May	184500	205000	222406
	Jun	200700	223000	243753
	Jul	192600	214000	234626
	Aug	156150	173500	190463
	Sep	144000	160000	175806
	Oct	148050	164500	180853
	Nov	158580	176200	193771
	Dec	196650	218500	240325

Table A14: Raw Materials Inventory vs. Time Period

Product	Month	Raw Materials Inventory		
		Low Demand Scenario	Intermediate Avg. Demand Scenario	High Demand Scenario
1	Jan	148050	0	0
	Feb	0	151000	0
	Mar	144000	0	0
	Apr	0	0	0
	May	0	0	0
	Jun	0	0	0
	Jul	0	0	0
	Aug	127800	142000	156200
	Sep	0	0	0
	Oct	142380	158200	0
	Nov	0	0	0
	Dec	0	0	0
2	Jan	139950	0	0
	Feb	0	142000	0
	Mar	135900	0	0
	Apr	0	0	0
	May	0	0	0
	Jun	0	0	0
	Jul	0	0	0
	Aug	119700	133000	146300
	Sep	0	0	0
	Oct	134280	149200	0
	Nov	0	0	0
	Dec	0	0	0
3	Jan	164250	0	0
	Feb	0	169000	0
	Mar	160200	0	0
	Apr	0	0	0
	May	0	0	0
	Jun	0	0	0
	Jul	0	0	1520
	Aug	144000	160000	175807
	Sep	0	0	0
	Oct	158580	176200	0
	Nov	0	0	0
	Dec	0	0	0

Table A15: Bottles Purchased vs. Time Period

Product	Month	Bottles Purchased		
		Low Demand Scenario	Intermediate Avg. Demand Scenario	High Demand Scenario
1	Jan	315450	186000	204600
	Feb	0	315500	180950
	Mar	279900	0	166100
	Apr	0	160000	176000
	May	168300	187000	205700
	Jun	184500	205000	225500
	Jul	176400	196000	215600
	Aug	267750	297500	327250
	Sep	0	0	0
	Oct	274230	304700	161150
	Nov	0	0	174020
	Dec	180450	200500	220550
2	Jan	292050	169000	185900
	Feb	0	297500	171050
	Mar	263700	0	156200
	Apr	0	151000	166100
	May	160200	178000	195800
	Jun	176400	196000	215600
	Jul	168300	187000	205700
	Aug	252000	280000	308000
	Sep	0	0	0
	Oct	258030	286700	151250
	Nov	0	0	164120
	Dec	172350	191500	210650
3	Jan	340650	196000	265100
	Feb	0	351500	176000
	Mar	312300	0	173525
	Apr	0	178000	185900
	May	184500	205000	222406
	Jun	200700	223000	243753
	Jul	192600	214000	236146
	Aug	300150	333500	364750
	Sep	0	0	0
	Oct	306630	340700	180853
	Nov	0	0	193771
	Dec	196650	218500	240325

Table A16: Units Sold vs. Time Period

Product	Month	Units Sold: Low Demand Scenario	Units Sold: Intermediate Avg. Demand Scenario	Units Sold: High Demand Scenario
1	Jan	167400	186000	204600
	Feb	148050	164500	180950
	Mar	135900	151000	166100
	Apr	144000	160000	176000
	May	168300	187000	205700
	Jun	184500	205000	225500
	Jul	176400	196000	215600
	Aug	139950	155500	171050
	Sep	127800	142000	156200
	Oct	131850	146500	161150
	Nov	142380	158200	174020
	Dec	180450	200500	220550
2	Jan	152100	169000	185900
	Feb	139950	155500	171050
	Mar	127800	142000	156200
	Apr	135900	151000	166100
	May	160200	178000	195800
	Jun	176400	196000	215600
	Jul	168300	187000	205700
	Aug	132300	147000	161700
	Sep	119700	133000	146300
	Oct	123750	137500	151250
	Nov	134280	149200	164120
	Dec	172350	191500	210650
3	Jan	176400	196000	215600
	Feb	164250	182500	200750
	Mar	152100	169000	185900
	Apr	160200	178000	192087
	May	184500	205000	225500
	Jun	200700	223000	245300
	Jul	192600	214000	235400
	Aug	156150	173500	190850
	Sep	144000	160000	176000
	Oct	148050	164500	180950
	Nov	158580	176200	193820
	Dec	196650	218500	240350

Table A17: Order Status vs. Months

Months	Order Placed		
	Low Demand Scenario	Intermediate Avg. Demand Scenario	High Demand Scenario
Jan	Yes	Yes	Yes
Feb	No	Yes	Yes
Mar	Yes	No	Yes
Apr	No	Yes	Yes
May	Yes	Yes	Yes
Jun	Yes	Yes	Yes
Jul	Yes	Yes	Yes
Aug	Yes	Yes	Yes
Sep	No	No	No
Oct	Yes	Yes	Yes
Nov	No	No	Yes
Dec	Yes	Yes	Yes

Stochastic Solutions

Table A18: Stochastic Order Status vs. Months

Months	Order Placed			
	Probability Set 1 (0.2,0.2,0.6)	Probability Set 2 (0.7,0.2,0.1)	Probability Set 3 (0.3,0.5,0.2)	Probability Set 4 (0.333,0.334,0.333)
Jan	Yes	Yes	Yes	Yes
Feb	No	No	No	No
Mar	Yes	Yes	Yes	Yes
Apr	No	No	No	No
May	Yes	Yes	Yes	Yes
Jun	Yes	Yes	Yes	Yes
Jul	Yes	Yes	Yes	Yes
Aug	No	No	No	No
Sep	Yes	Yes	Yes	Yes
Oct	No	No	No	No
Nov	Yes	Yes	Yes	Yes
Dec	Yes	Yes	Yes	Yes

Table A19: Stochastic Units Produced vs. Time Period

Product	Month	Units Produced			
		Probability Set 1	Probability Set 2	Probability Set 3	Probability Set 4
1	Jan	167400	167400	167400	167400
	Feb	148050	148050	148050	148050
	Mar	135900	135900	135900	135900
	Apr	144000	144000	144000	144000
	May	187000	168300	168300	168300
	Jun	205000	184500	184500	184500
	Jul	196000	176400	176400	176400
	Aug	155500	139950	139950	139950
	Sep	142000	127800	127800	127800
	Oct	146500	131850	146500	131850
	Nov	158200	142380	158200	158200
	Dec	220550	180450	200500	200500
2	Jan	152100	152100	152100	152100
	Feb	140580	139950	139950	139950
	Mar	142000	127800	127800	127800
	Apr	150000	135900	135900	135900
	May	178000	160200	160200	160200
	Jun	196000	176400	176400	176400
	Jul	187000	168300	168300	168300
	Aug	147000	132300	147000	132300
	Sep	133000	119700	133000	119700
	Oct	137500	123750	137500	137500
	Nov	149200	134280	149200	149200
	Dec	210650	172350	191500	191500
3	Jan	176400	176400	176400	176400
	Feb	164250	164250	164250	164250
	Mar	169000	152100	152100	152100
	Apr	178000	160200	160200	160200
	May	205000	184500	184500	184500
	Jun	223000	200700	200700	200700
	Jul	214000	192600	192600	192600
	Aug	173500	156150	156150	156150
	Sep	160000	144000	160000	144000
	Oct	164500	148050	164500	164500
	Nov	176200	158580	176200	176200
	Dec	240350	196650	218500	218500

Table A20: Stochastic Raw Material Inventory vs. Time Period

Product	Month	Raw Material Inventory			
		Probability Set 1	Probability Set 2	Probability Set 3	Probability Set 4
1	Jan	148050	148050	148050	148050
	Feb	0	0	0	0
	Mar	144000	144000	144000	144000
	Apr	0	0	0	0
	May	0	0	0	0
	Jun	0	0	0	0
	Jul	0	0	139950	139950
	Aug	142000	127800	0	0
	Sep	0	0	146500	131850
	Oct	158200	142380	0	0
	Nov	0	0	0	0
	Dec	0	0	0	0
2	Jan	140580	139950	139950	139950
	Feb	0	0	0	0
	Mar	150000	135900	135900	135900
	Apr	0	0	0	0
	May	0	0	0	0
	Jun	0	0	0	0
	Jul	0	0	147000	132300
	Aug	133000	119700	0	0
	Sep	0	0	137500	137500
	Oct	149200	134280	0	0
	Nov	0	0	0	0
	Dec	0	0	0	0
3	Jan	164250	164250	164250	164250
	Feb	0	0	0	0
	Mar	178000	160200	160200	160200
	Apr	0	0	0	0
	May	0	0	0	0
	Jun	0	0	0	0
	Jul	0	0	156150	156150
	Aug	160000	144000	0	0
	Sep	0	0	164500	164500
	Oct	176200	158580	0	0
	Nov	0	0	0	0
	Dec	0	0	0	0

Table A21: Stochastic Bottles Purchased vs. Time Period

Product	Month	Bottles Purchased			
		Probability Set 1	Probability Set 2	Probability Set 3	Probability Set 4
1	Jan	315450	279900	279900	315450
	Feb	0	0	0	0
	Mar	279900	168300	168300	279900
	Apr	0	184500	184500	0
	May	187000	176400	316350	168300
	Jun	205000	267750	0	184500
	Jul	196000	0	274300	316350
	Aug	297500	274230	0	0
	Sep	0	0	158200	259650
	Oct	304700	180450	200500	0
	Nov	0	292050	292050	158200
	Dec	220550	0	0	200500
2	Jan	292680	263700	263700	292050
	Feb	0	0	0	0
	Mar	292000	160200	160200	263700
	Apr	0	176400	176400	0
	May	178000	168300	315300	160200
	Jun	196000	252000	0	176400
	Jul	187000	0	270500	300600
	Aug	280000	258030	0	0
	Sep	0	0	149200	257200
	Oct	286700	172350	191500	0
	Nov	0	340650	340650	149200
	Dec	210650	0	0	191500
3	Jan	340650	312300	312300	340650
	Feb	0	0	0	0
	Mar	347000	184500	184500	312300
	Apr	0	200700	200700	0
	May	205000	192600	348750	184500
	Jun	223000	300150	0	200700
	Jul	214000	0	324500	348750
	Aug	333500	306630	0	0
	Sep	0	0	176200	308500
	Oct	340700	196650	218500	0
	Nov	0	279900	279900	176200
	Dec	240350	0	0	218500

Table A22: Stochastic Units Sold Inventory vs. Time Period

Product	Month	Units Sold			
		Probability Set 1	Probability Set 2	Probability Set 3	Probability Set 4
1	Jan	167400	167400	167400	167400
	Feb	148050	148050	148050	148050
	Mar	135900	135900	135900	135900
	Apr	144000	144000	144000	144000
	May	187000	168300	168300	168300
	Jun	205000	184500	184500	184500
	Jul	196000	176400	176400	176400
	Aug	155500	139950	139950	139950
	Sep	142000	127800	127800	127800
	Oct	146500	131850	146500	131850
	Nov	158200	142380	158200	158200
	Dec	200500	180450	200500	200500
2	Jan	152100	152100	152100	152100
	Feb	140580	139950	139950	139950
	Mar	142000	127800	127800	127800
	Apr	150000	135900	135900	135900
	May	178000	160200	160200	160200
	Jun	196000	176400	176400	176400
	Jul	187000	168300	168300	168300
	Aug	147000	132300	147000	132300
	Sep	133000	119700	133000	119700
	Oct	137500	123750	137500	137500
	Nov	149200	134280	149200	149200
	Dec	191500	172350	191500	191500
3	Jan	176400	176400	176400	176400
	Feb	164250	164250	164250	164250
	Mar	169000	152100	152100	152100
	Apr	178000	160200	160200	160200
	May	205000	184500	184500	184500
	Jun	223000	200700	200700	200700
	Jul	214000	192600	192600	192600
	Aug	173500	156150	156150	156150
	Sep	160000	144000	160000	144000
	Oct	164500	148050	164500	164500
	Nov	176200	158580	176200	176200
	Dec	218500	196650	218500	218500

Appendix 3

LINGO program for Deterministic Production Planning Model

```
MODEL:

! This model calculates and maximize the profit, under given constraints,
of a brewing company which Processes and packs the beer in 3 different
types and concentrations of bottles;

SETS:

! INITIALIZING VARIABLES;

BOTTLES / 1,2,3/:CARRYING_COST_INI, CARRYING_COST_FINAL,PROPORTION;

MONTHS / JAN FEB MAR APR MAY JUN JUL AUG SEP OCT NOV DEC/:ORDER_COST,
TOTAL_SETUP, MAX_INI_INVEST, MAX_FINAL_INVEST, TOTAL_INVESTMENT, Z;

LINKS (BOTTLES, MONTHS):SALES_COST, PRODUCTION_COST, UNITS_PRODUCED,
PURCHASE_COST, BOTTLE_INI_INVENTORY, FINAL_COST, FINAL_INVENTORY,
BOTTLE_PURCHASED, DEMAND, MAX_INI_INVENTORY, MIN_INI_INVENTORY,
MAX_FINAL_INVENTORY, UNITS_SOLD;

ENDSETS

! -------------------OBJECTIVE FUNCTION-----------------------;
```

! Maximize Total profit = Total Sales - Production Cost – Raw Materials Inventory Cost – Finished Products Inventory Cost - Purchase Cost;

```
Max =

@SUM (LINKS(i,t): SALES_COST(i,t)*UNITS_SOLD(i,t)) -
@SUM (LINKS(i,t): PRODUCTION_COST(i,t)*UNITS_PRODUCED(i,t)*(1 + 0.1 +
      .02))-
@SUM (LINKS(i,t): CARRYING_COST_INI(i)*
      PURCHASE_COST(i,t)*BOTTLE_INI_INVENTORY(i,t)*(1 + 0.15 + 0.15)) -
@SUM (MONTHS(t): Z(t)*5000) -
@SUM (LINKS(i,t): CARRYING_COST_FINAL(i)*
      FINAL_COST(i,t)*FINAL_INVENTORY(i,t)) -
@SUM (LINKS(i,t): PURCHASE_COST(i,t)*BOTTLE_PURCHASED(i,t)*(1 + 0.15 +
      0.15));

!---------------------CONSTRAINTS--------------------------;

! SALES CONSTRAINTS;

! This constraint implies that the total unit manufactured will be equal
to the total units sold(+ or - inventory) of all the items for each month;

@FOR (BOTTLES(i):
```

```
    UNITS_PRODUCED(i,1) - FINAL_INVENTORY(i,1) - UNITS_SOLD(i,1) = 0);
@FOR (BOTTLES(i):
@FOR (MONTHS(t)|t #GT# 1:
    FINAL_INVENTORY(i,t-1) + UNITS_PRODUCED(i,t) - FINAL_INVENTORY(i,t)
    - UNITS_SOLD(i,t) = 0));

@FOR (BOTTLES(i):
@FOR (MONTHS(t):
    UNITS_SOLD(i,t) <= FINAL_INVENTORY(i,t) + UNITS_PRODUCED(i,t)));


@FOR (BOTTLES(i):
@FOR (MONTHS(t):
    UNITS_SOLD(i,t) <= DEMAND(i,t)));

!------------------------------------------------------------;

! PRODUCTION CONSTRAINTS;

@FOR (MONTHS(t):
@SUM (BOTTLES(i):
    0.001*(UNITS_PRODUCED(i,t)))<= TOTAL_SETUP(t)*24*22);

!------------------------------------------------------------;

! RAW MATERIALS INVENTORY CONSTRAINTS;

! The inventory for the empty bottles should not be less than the minimum
inventory allowed as the safety stock;

@FOR (BOTTLES(i):
@FOR (MONTHS(t):
    BOTTLE_INI_INVENTORY(i,t) <= MAX_INI_INVENTORY(i,t)));

! Total investment on initial inventory should be less than the maximum
budget for the initial inventory;

@FOR (MONTHS(t):
@SUM (BOTTLES(i):
    CARRYING_COST_INI(i)*PURCHASE_COST(i,t)*BOTTLE_INI_INVENTORY(i,t)*(1
    + 0.15 + 0.15)) <= MAX_INI_INVEST(t));

!------------------------------------------------------------;

! FINISHED PRODUCTS INVENTORY CONSTRAINTS;

! The inventory for the final bottles should not exceed the maximum
inventory possible in the given space for the final product;

@FOR (BOTTLES(i):
@FOR (MONTHS(t):
    FINAL_INVENTORY(i,t) <= MAX_FINAL_INVENTORY(i,t)));
```

```
!-------------------------------------------------------------;

! PURCHASE CONSTRAINTS;

! Total amount spent on purchasing the raw materials should be less than
the maximum total investment allowable;

@FOR (MONTHS(t):
@SUM (BOTTLES(i):
      PURCHASE_COST(i,t)*BOTTLE_PURCHASED(i,t)*(1 + 0.15 + 0.15)+
      (Z(t)*5000)) <= TOTAL_INVESTMENT(t));

! The amount of bottles purchased and carried forward from the previous
month inventory should be equal to the sum of bottles produced and carried
over to the next month's inventory;

@FOR (BOTTLES(i):
      BOTTLE_PURCHASED(i,1) - UNITS_PRODUCED(i,1) -
      BOTTLE_INI_INVENTORY(i,1) = 0);

@FOR (BOTTLES(i):
@FOR (MONTHS(t)| t #GT# 1:
      BOTTLE_PURCHASED(i,t) + BOTTLE_INI_INVENTORY(i,t-1) -
      UNITS_PRODUCED(i,t) - BOTTLE_INI_INVENTORY(i,t) = 0 ));

@FOR (MONTHS(t):
@SUM (BOTTLES(i):
      BOTTLE_PURCHASED(i,t)) <= 1000000*Z(t));

@FOR (MONTHS(t):
@BIN (Z(t)));


DATA:

CARRYING_COST_INI =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data1A.XLS', 'CARRYING_COST_INI');

CARRYING_COST_FINAL =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data1A.XLS', 'CARRYING_COST_FINAL');

ORDER_COST =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data1A.XLS', 'ORDER_COST');

TOTAL_SETUP =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data1A.XLS', 'TOTAL_SETUP');

MAX_INI_INVEST =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data1A.XLS', 'MAX_INI_INVEST');

MAX_FINAL_INVEST =
```

```
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data1A.XLS', 'MAX_FINAL_INVEST');

TOTAL_INVESTMENT =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data1A.XLS', 'TOTAL_INVESTMENT');

PROPORTION =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data1A.XLS', 'PROPORTION');

SALES_COST =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data1A.XLS', 'SALES_COST');

PRODUCTION_COST =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data1A.XLS', 'PRODUCTION_COST');

PURCHASE_COST =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data1A.XLS', 'PURCHASE_COST');

FINAL_COST =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data1A.XLS', 'FINAL_COST');

DEMAND =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data1A.XLS', 'DEMAND');

MAX_INI_INVENTORY =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data1A.XLS', 'MAX_INI_INVENTORY');

MIN_INI_INVENTORY =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data1A.XLS', 'MIN_INI_INVENTORY');

MAX_FINAL_INVENTORY =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data1A.XLS', 'MAX_FINAL_INVENTORY');

ENDDATA

END
```

LINGO program for Stochastic Production Planning Model

```
MODEL:

SETS:

! INITIALIZING VARIABLES;

BOTTLES / 1,2,3/:CARRYING_COST_INI, CARRYING_COST_FINAL,PROPORTION;

MONTHS / JAN FEB MAR APR MAY JUN JUL AUG SEP OCT NOV DEC/:ORDER_COST,
TOTAL_SETUP, MAX_INI_INVEST, MAX_FINAL_INVEST, TOTAL_INVESTMENT, Z;

LINKS (BOTTLES, MONTHS):SALES_COST, PRODUCTION_COST, UNITS_PRODUCED,
UNITS_PRODUCED_1, UNITS_PRODUCED_2, PURCHASE_COST, BOTTLE_INI_INVENTORY,
BOTTLE_INI_INVENTORY_11, BOTTLE_INI_INVENTORY_22, FINAL_COST,
FINAL_INVENTORY, FINAL_INVENTORY_11, FINAL_INVENTORY_22, BOTTLE_PURCHASED,
BOTTLE_PURCHASED_1, BOTTLE_PURCHASED_2, DEMAND, MAX_INI_INVENTORY,
MIN_INI_INVENTORY, MAX_FINAL_INVENTORY, DEMAND1, DEMAND2, DEMAND3,
UNITS_SOLD1, UNITS_SOLD, UNITS_SOLD2;

ENDSETS


!------------OBJECTIVE FUNCTION--------------------------------;
```

! Maximize Total profit = Total Sales(TC) - Production Cost(PC) - Raw Materials Inventory Cost(RMIC) - Finished Products Inventory Cost(FPIC) - Purchase Cost(PUC);

```
Max = - 1 *
@SUM (LINKS(i,t): PRODUCTION_COST(i,t)*UNITS_PRODUCED(i,t)*(1 + 0.1 +
      .02)) -
@SUM (LINKS(i,t): PURCHASE_COST(i,t)*BOTTLE_PURCHASED(i,t)*(1 + 0.15 +
      0.15)) + (
@SUM (LINKS(i,t): SALES_COST(i,t)*UNITS_SOLD1(i,t)))*0.2 - (
@SUM (LINKS(i,t): CARRYING_COST_INI(i)*
      PURCHASE_COST(i,t)*BOTTLE_INI_INVENTORY_11(i,t)*(1 + 0.15 + 0.15)) +
@SUM (MONTHS(t): Z(t)*5000) +
@SUM (LINKS(i,t): CARRYING_COST_FINAL(i)*
      FINAL_COST(i,t)*FINAL_INVENTORY_11(i,t)))*0.2 + (
@SUM (LINKS(i,t): SALES_COST(i,t)*UNITS_SOLD(i,t)))*0.2 - (
@SUM (LINKS(i,t): CARRYING_COST_INI(i)*
      PURCHASE_COST(i,t)*BOTTLE_INI_INVENTORY(i,t)*(1 + 0.15 + 0.15)) +
@SUM (MONTHS(t): Z(t)*5000) +
@SUM (LINKS(i,t):CARRYING_COST_FINAL(i)*FINAL_COST(i,t)*FINAL_INVENTORY
      (i,t)))*0.2 + (
@SUM (LINKS(i,t): SALES_COST(i,t)*UNITS_SOLD2(i,t)))*0.6 - (
@SUM (LINKS(i,t): CARRYING_COST_INI(i)*
      PURCHASE_COST(i,t)*BOTTLE_INI_INVENTORY_22(i,t)*(1 + 0.15 + 0.15)) +
@SUM (MONTHS(t): Z(t)*5000) +
@SUM (LINKS(i,t): CARRYING_COST_FINAL(i)*
      FINAL_COST(i,t)*FINAL_INVENTORY_22(i,t)))*0.6;
```

```
!--------------------------CONSTRAINTS-----------------------;

! SALES CONSTRAINTS;

! This constraint implies that the total unit manufactured will be equal
to the total units sold(+ or - inventory) of all the items for each month.
This holds true for all the scenarios.;

@FOR (BOTTLES(i):
      UNITS_PRODUCED(i,1) - FINAL_INVENTORY_11(i,1) - UNITS_SOLD1(i,1) =
      0);

@FOR (BOTTLES(i):
@FOR (MONTHS(t)|t #GT# 1:
      FINAL_INVENTORY_11(i,t-1) + UNITS_PRODUCED(i,t) -
      FINAL_INVENTORY_11(i,t) - UNITS_SOLD1(i,t) = 0));


@FOR (BOTTLES(i):
      UNITS_PRODUCED(i,1) - FINAL_INVENTORY(i,1) - UNITS_SOLD(i,1) = 0);

@FOR (BOTTLES(i):
@FOR (MONTHS(t)|t #GT# 1:
      FINAL_INVENTORY(i,t-1) + UNITS_PRODUCED(i,t) - FINAL_INVENTORY(i,t)
      - UNITS_SOLD(i,t) = 0));

@FOR (BOTTLES(i):
      UNITS_PRODUCED(i,1) - FINAL_INVENTORY_22(i,1) - UNITS_SOLD2(i,1) =
      0);

@FOR (BOTTLES(i):
@FOR (MONTHS(t)|t #GT# 1:
      FINAL_INVENTORY_22(i,t-1) + UNITS_PRODUCED(i,t) -
      FINAL_INVENTORY_22(i,t) - UNITS_SOLD2(i,t) = 0));

! Units sold do not exceed the sum of the number of units produced and the
units available in the inventory at the end of the  month;

@FOR (BOTTLES(i):
@FOR (MONTHS(t):
      UNITS_SOLD1(i,t) <= FINAL_INVENTORY_11(i,t) + UNITS_PRODUCED(i,t)));

@FOR (BOTTLES(i):
@FOR (MONTHS(t):
      UNITS_SOLD(i,t) <= FINAL_INVENTORY(i,t) + UNITS_PRODUCED(i,t)));

@FOR (BOTTLES(i):
@FOR (MONTHS(t):
      UNITS_SOLD2(i,t) <= FINAL_INVENTORY_22(i,t) + UNITS_PRODUCED(i,t)));

! Units sold do not exceed the demand of the products;

@FOR (BOTTLES(i):
@FOR (MONTHS(t):
      UNITS_SOLD1(i,t) <= DEMAND1(i,t)));

@FOR (BOTTLES(i):
```

```
@FOR (MONTHS(t):
      UNITS_SOLD(i,t) <= DEMAND2(i,t)));

@FOR (BOTTLES(i):
@FOR (MONTHS(t):
      UNITS_SOLD2(i,t) <= DEMAND3(i,t)));

!-------------------------------------------------------;

! PRODUCTION CONSTRAINTS;

! Units produced are less than or equal to the capacity of the production
set-up;

@FOR (MONTHS(t):
@SUM (BOTTLES(i):
      0.001*(UNITS_PRODUCED(i,t)))<= TOTAL_SETUP(t)*24*22);

! This is used to assign zero value to a binary variable which represents
the Ordering Cost when there is no purchase;

@FOR (MONTHS(t):
@SUM (BOTTLES(i):
      BOTTLE_PURCHASED(i,t)) <= 1000000*Z(t));

!-------------------------------------------------------;

! RAW MATERIALS INVENTORY CONSTRAINTS;

! Bottle initial inventory cannot exceed the maximum capacity;

@FOR (BOTTLES(i):
@FOR (MONTHS(t):
      BOTTLE_INI_INVENTORY_11(i,t) <= MAX_INI_INVENTORY(i,t)));

@FOR (BOTTLES(i):
@FOR (MONTHS(t):
      BOTTLE_INI_INVENTORY(i,t) <= MAX_INI_INVENTORY(i,t)));

@FOR (BOTTLES(i):
@FOR (MONTHS(t):
      BOTTLE_INI_INVENTORY_22(i,t) <= MAX_INI_INVENTORY(i,t)));


! Total investment on initial inventory should be less than the maximum
budget for the initial inventory;

@FOR (MONTHS(t):
@SUM (BOTTLES(i):
      CARRYING_COST_INI(i)*PURCHASE_COST(i,t)*BOTTLE_INI_INVENTORY_11
      (i,t)*(1 + 0.15 + 0.15)) <= MAX_INI_INVEST(t));

@FOR (MONTHS(t):
@SUM (BOTTLES(i):
      CARRYING_COST_INI(i)*PURCHASE_COST(i,t)*BOTTLE_INI_INVENTORY(i,t)*(1
      + 0.15 + 0.15)) <= MAX_INI_INVEST(t));
```

```
@FOR (MONTHS(t):
@SUM (BOTTLES(i):
      CARRYING_COST_INI(i)*PURCHASE_COST(i,t)*BOTTLE_INI_INVENTORY_22
      (i,t)*(1 + 0.15 + 0.15)) <= MAX_INI_INVEST      (t));

! The amount of bottles purchased and carried forward from the previous
month inventory should be equal to the sum of bottles produced and carried
over to the next month's inventory;

@FOR (BOTTLES(i):
      BOTTLE_PURCHASED(i,1) - UNITS_PRODUCED(i,1) -
      BOTTLE_INI_INVENTORY_11(i,1) = 0);

@FOR (BOTTLES(i):
@FOR (MONTHS(t)| t #GT# 1:
      BOTTLE_PURCHASED(i,t) + BOTTLE_INI_INVENTORY_11(i,t-1) -
      UNITS_PRODUCED(i,t) -   BOTTLE_INI_INVENTORY_11(i,t) = 0 ));

@FOR (BOTTLES(i):
      BOTTLE_PURCHASED(i,1) - UNITS_PRODUCED(i,1) -
      BOTTLE_INI_INVENTORY(i,1) = 0);

@FOR (BOTTLES(i):
@FOR (MONTHS(t)| t #GT# 1:
      BOTTLE_PURCHASED(i,t) + BOTTLE_INI_INVENTORY(i,t-1) -
      UNITS_PRODUCED(i,t) - BOTTLE_INI_INVENTORY(i,t) = 0 ));

@FOR (BOTTLES(i):
      BOTTLE_PURCHASED(i,1) - UNITS_PRODUCED(i,1) -
      BOTTLE_INI_INVENTORY_22(i,1) = 0);

@FOR (BOTTLES(i):
@FOR (MONTHS(t)| t #GT# 1:
      BOTTLE_PURCHASED(i,t) + BOTTLE_INI_INVENTORY_22(i,t-1) -
      UNITS_PRODUCED(i,t) - BOTTLE_INI_INVENTORY_22(i,t) = 0 ));

!--------------------------------------------------------;

! FINISHED PRODUCTS INVENTORY CONSTRAINTS;

! The inventory for the finished products should not exceed the maximum
inventory possible in the given space for the final product;

@FOR (BOTTLES(i):
@FOR (MONTHS(t):
      FINAL_INVENTORY_11(i,t) <= MAX_FINAL_INVENTORY(i,t)));

@FOR (BOTTLES(i):
@FOR (MONTHS(t):
      FINAL_INVENTORY(i,t) <= MAX_FINAL_INVENTORY(i,t)));

@FOR (BOTTLES(i):
@FOR (MONTHS(t):
      FINAL_INVENTORY_22(i,t) <= MAX_FINAL_INVENTORY(i,t)));
```

```
!-------------------------------------------------------------;

! PURCHASE CONSTRAINTS;

! Total amount spent on purchasing the raw materials should be less than
the maximum total investment allowable;

@FOR (MONTHS(t):
@SUM (BOTTLES(i):
      PURCHASE_COST(i,t)*BOTTLE_PURCHASED(i,t)*(1 + 0.15 + 0.15) +
      (Z(t)*5000)) <= TOTAL_INVESTMENT(t));

@FOR (MONTHS(t):
@BIN (Z(t)));



DATA:

! GIVEN DATA;

CARRYING_COST_INI =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data1A.XLS', 'CARRYING_COST_INI');

CARRYING_COST_FINAL =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data.XLS', 'CARRYING_COST_FINAL');

ORDER_COST =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data1A.XLS', 'ORDER_COST');

TOTAL_SETUP =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data.XLS', 'TOTAL_SETUP');

MAX_INI_INVEST =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data.XLS', 'MAX_INI_INVEST');

MAX_FINAL_INVEST =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data.XLS', 'MAX_FINAL_INVEST');

TOTAL_INVESTMENT =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data.XLS', 'TOTAL_INVESTMENT');

PROPORTION =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data.XLS', 'PROPORTION');

SALES_COST =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data.XLS', 'SALES_COST');
```

```
PRODUCTION_COST =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data.XLS', 'PRODUCTION_COST');

PURCHASE_COST =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data.XLS', 'PURCHASE_COST');

FINAL_COST =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data.XLS', 'FINAL_COST');

DEMAND1 =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data.XLS', 'DEMAND1');

DEMAND2 =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data.XLS', 'DEMAND2');

DEMAND3 =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data.XLS', 'DEMAND3');

MAX_INI_INVENTORY =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data.XLS', 'MAX_INI_INVENTORY');

MAX_FINAL_INVENTORY =
@OLE ('C:\Documents and Settings\Vibhor\My
Documents\Research\Vibhor\Data.XLS', 'MAX_FINAL_INVENTORY');


ENDDATA

END
```