A Performance Model of the IEEE 802.11 Distributed Coordination Function under Finite Load Condition

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ABSTRACT

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Modeling of the IEEE 802.11 wireless LAN MAC protocol has received considerable research attention recently. Most of the previous works have concerned with the derivation of system throughput for the Distributed Coordination Function (DCF) of the MAC protocol. There have been very few works investigating the delay performance of the DCF scheme. This thesis presents an analytical model of the DCF under un-saturated homogeneous and heterogeneous conditions. Our model allows stations to have either the same or different packet arrival and transmission rates. The arrival of packets is assumed to be according to a Poisson process. We model each station's MAC buffer as an M/G/1 queue and the service time of each station as a three dimensional Markov chain. The dependency between stations is taken into account through the backoff counter freezing and packet collision during transmission. We derive the probability generating function (PGF) of the packet service time distribution and obtain the closed form expression of the mean packet delay by applying the M/G/1 queuing result. We also investigate the impact of different contention window sizes on the delay performance. The accuracy of the model has been verified by extensive simulations.

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Dedicated to my dear parents…

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List of Symbols

Symbol	Definition
$b_{j,i,k}$	Stationary state distribution of the Markov Chain.
$\bar{D}$	Mean packet delay
$\bar{D}_f$	Mean packet delay of a fast STA
$\bar{D}_s$	Mean packet delay of a slow STA
i	Backoff stage number ($0 \ldots m$)
j_{fi}	Number of counter freezing due to only fast STAs transmitting in the i_{th} backoff stage
j_i	Number of counter freezing in the i_{th} backoff stage
$J_i(z)$	PGF of the number of counter freezing in the i_{th} backoff stage
j_{si}	Number of counter freezing due to only slow STAs transmitting in the i_{th} backoff stage of a slow STA
j_{sfi}	Number of counter freezing due to both slow and fast STAs transmitting in the i_{th} backoff stage of a slow STA
k_{fi}	Initial backoff counter value in the i_{th} backoff stage of a fast STA
$K_{fi}(z)$	PGF of the initial backoff counter value in the i_{th} backoff stage of a fast STA
k_i	Initial backoff counter value in the i_{th} backoff stage
$K_i(z)$	PGF of the initial value of backoff counter in the i_{th} backoff stage
k_{si}	Initial backoff counter value in the i_{th} backoff stage of a slow STA
$K_{si}(z)$	PGF of the initial backoff counter value in the i_{th} backoff stage of a slow STA
m	Maximum backoff stage
P_C	Probability that a transmitted packet encounters a collision

P_{Cf}	Probability that a transmitted fast STA packet encounters a collision
P_{CS}	Probability that a transmitted slow STA packet encounters a collision
P_{tr}	Probability that at least one transmission in a slot during the backoff stage of a STA
P_{trf1}	Probability that at least one transmission from the fast STAs during a slot that a fast STA is in a backoff stage
P_{trf2}	Probability that at least one transmission from the fast STAs in any slot
P_{trs1}	Probability that at least one transmission from the slow STAs during a slot that a fast STA is in a backoff stage
P_{trs2}	Probability that at least one transmission from the slow STAs in any slot
r	Packet size in slots
$R(z)$	PGF of the packet size
r_f	Packet size of a fast STA in slots
$R_f(z)$	PGF of the packet size of a fast STA
r_s	Packet size of a slow STA in slots
$R_s(z)$	PGF of the packet size of a slow STA
S	Packet service time in slots
$S(z)$	PGF of the packet service time
$\overline{S}$	Mean packet service time
$\overline{S^2}$	Second moment of the packet service time
S_f	Packet service time of a fast STA in slots
$S_f(z)$	PGF of the packet service time of a fast STA
$\overline{S}_f$	Mean packet service time of a fast STA
$\overline{S_f^2}$	Second moment of the packet service time of a fast STA
S_{fi}	Packet service time of a fast STA assuming i_{th} retransmission attempt

$S_{fi}(z)$	PGF of the packet service time of the fast STA assuming i_{th} retransmission attempt
S_i	Packet service time in slots assuming i_{th} retransmission attempt
$S_i(z)$	PGF of the packet service time assuming i_{th} retransmission attempt
$\overline{S}_s$	Mean packet service time of a slow STA
$\overline{S_s^2}$	Second moment of the packet service time of a slow STA
S_s	Packet service time of a slow STA in slots
$S_s(z)$	PGF of the packet service time of a slow STA
S_{si}	Packet service time of a slow STA assuming i_{th} retransmission attempt
$S_{si}(z)$	PGF of the packet service time of a slow STA assuming i_{th} retransmission attempt
W	Backoff period
$\overline{W}$	Average backoff period
W_0	Minimum contention window size
$\overline{W_f}$	Average backoff period of a fast STA
W_{f0}	Minimum contention window size of a fast STA
W_{fi}	Contention window size in the i_{th} backoff stage of a fast STA
W_i	Contention window size in the i_{th} backoff stage
$\overline{W_s}$	Average backoff period of a slow STA
W_{S0}	Minimum contention window size of a slow STA
W_{si}	Contention window size in the i_{th} backoff stage of a slow STA
x_{fi}	Duration of count down process in the i_{th} backoff stage of a fast STA
$X_{fi}(z)$	PGF of duration of count down process in the i_{th} backoff stage of a fast STA

x_i	Duration of the count down process in the i_{th} backoff stage
$X_i(z)$	PGF of the duration of the count down process in the i_{th} backoff stage
x_{fi}	Duration of the count down process in the i_{th} backoff stage of a fast STA
$X_{fi}(z)$	PGF of the duration of the count down process in the i_{th} backoff stage of a fast STA
x_{si}	Duration of count down process in the i_{th} backoff stage of a slow STA
$X_{si}(z)$	PGF of the duration of count down process in the i_{th} backoff stage of a slow STA
λ	Packet arrival rate (packets/slot)
λ_f	Packet arrival rate of a fast STA (packets/slot)
λ_s	Packet arrival rate of a slow STA (packets/slot)
ρ	Probability that a STA queue is busy
ρ_f	Probability that a fast STA queue is busy
ρ_s	Probability that a slow STA queue is busy
σ	Probability that no packet arrives to a STA's MAC buffer during a slot
τ	Probability that a STA transmit in a slot
τ_f	Probability that a fast STA transmit in a slot
τ_s	Probability that a slow STA transmit in a slot

List of Abbreviations

Abbreviation	Definition
AC	Access Category
ACK	Acknowledgement
AIFS	Arbitrary Inter-Frame Space
AP	Access Point
BER	Bit Error Rate
BSS	Basic Service Set
CCK	Complementary Code Keying
CSMA/CA	Carrier Sense Multiple Access/Collision Avoidance
CSMA/CD	Carrier Sense Multiple Access/Collision Detection
CTS	Clear To Sent
CW	Contention Window
CW_{max}	Maximum Contention Window
CW_{min}	Minimum Contention Window
DCF	Distributed Coordination Function
DIFS	DCF Inter-Frame Space
DS	Distributed System
DSSS	Direct Sequence Spread Spectrum
EDCA	Enhanced Distributed Channel Access
EIFS	Extended Inter-Frame Space
ESS	Extended Service Set
FHSS	Frequency Hopping Spread Spectrum
IBSS	Independent Basic Service Set

IFS	Inter-Frame Space
IR	Infra-Red
LAN	Local Area Network
LLC	Logical Link Control
MAC	Medium Access Control
MIB	Management Information Base
MPDU	MAC Protocol Data Unit
OFDM	Orthogonal Frequency Division Multiplexing
PCF	Point Coordination Function
PGF	Probability Generating Function
PHY	Physical
PIFS	PCF Inter-Frame Space
PLCP	Physical Layer Convergence Procedure
PMD	Physical Medium Dependent
PPDU	PLCP Protocol Data Unit
QoS	Quality of Service
RTS	Ready To Sent
SIFS	Short Inter-Frame Space
STA	Station
TxOP	Transmission Opportunity
WLAN	Wireless Local Area Network

CHAPTER 1

INTRODUCTION

1.1 Introduction

The world has become increasingly mobile in the past few decades. As a result, traditional wired network can no longer meet the demands of the mobile world. Wireless networks provide several advantages over fixed networks, such as mobility, ease of deployment and flexibility. Among the wireless technologies, Wireless Local Area Networks (WLANs) have been widely deployed during the recent years and have received considerable research attention.

Wireless LAN can offer performance nearly comparable to its wired counterpart, the Ethernet. As their price/performance continues to improve, wireless LAN has become a non-separable part of the business networks. Business organizations value the simplicity and scalability of wireless LAN and relative ease of integrating wireless access and the ability to roam with their network resources such as servers, printers and Internet access. There are basically two standards concerning wireless LANs, IEEE 802.11 and HiperLAN/2. Due to many reasons, the IEEE 802.11 standard has become the dominant one in the industry. The 802.11 WLAN works in the licence-free industrial, scientific and medical (ISM) band.

IEEE 802.11 specifications focus on the two lowest layers of the Open System Interconnection (OSI) model. It defines a uniform Medium Access Control (MAC) layer that rides over all the physical (PHY) layers with a slight difference with respect to each physical layer. Because the wireless environment is dynamically changing, unpredictable channel fading and high Bit Error Rate (BER) may occur. However, the system performance is not only affected by the PHY layer but also the MAC layer, which explains the considerable research attention paid to this layer. The focus of this thesis will be on the MAC layer.

The traditional Ethernet uses Carrier Sense Multiple Access with Collision Detection (CSMA/CD) as its MAC protocol. Collision detection is possible because the Ethernet devices can sense the channel by measuring the voltage or current in the cable. However, collision detection by carrier sensing in the wireless networks is extremely difficult due to channel fading and interference. In addition, a wireless station (STA) is unable to listen to the channel for collision while transmitting. Therefore, wireless LAN employs a different mechanism referred to as Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) in the MAC layer to control access to the wireless medium. The basic access mechanism in the wireless LAN is the Distributed Coordination Function (DCF). It lets all the STAs in the system to have equal opportunity to access the wireless medium. It basically avoids collision by letting the contending STAs to go into an exponential backoff period while the medium is busy. Only when its backoff period expires, then a STA can transmit the packet. If there is a collision the collided STA doubles the backoff counter and goes into backoff. Again when the counter reaches zero the packet is retransmitted. This cycle goes on if the packet transmission results in collisions until the maximum stage in reached and it stays in that stage until the maximum retry limit is reached.

1.2 IEEE 802.11 Standard

The standardization process of wireless LAN began in the late 1990s. In 1997 the IEEE adopted IEEE standard 802.11 [1], the first internationally sanctioned wireless LAN standard. Since then, several specifications in the 802.11 family have been proposed. We list some of the important ones as follows,

- 802.11–1997, the original WLAN standard which provides 1 or 2 Mbps transmission rate in the 2.4 GHz band using frequency hopping spread spectrum (FHSS), direct sequence spread spectrum (DSSS) or infrared (IR).
- 802.11a–1999, an extension to 802.11 that provides up to 54 Mbps PHY transmission rate in the 5GHz band. 802.11a uses an orthogonal frequency division multiplexing (OFDM) modulation scheme.
- 802.11b–1999, an extension to 802.11 that provides PHY transmission rate up to 11 Mbps in the 2.4 GHz band. 802.11b uses only DSSS scheme.
- 802.11g–2003, an extension to 802.11 that provides PHY transmission rates from 6Mbps to 54Mbps using OFDM modulation scheme in the 2.4 GHz band.
- 802.11e –2005, an enhancement of the MAC protocol for providing Quality of Service (QoS) support.

The majority of wireless LAN products in the market today follow the 802.11b/g standard. There are also some other types of WLAN standards such as High Performance

Radio Local Area Network–Type 2 (HiperLAN/2) and HomeRF but they have not received wide acceptance in the industry.

1.2.1 IEEE 802.11 Architecture

The IEEE 802.11 standard focuses on the two lowest layers of the OSI model (PHY and MAC layer). The MAC layer defines a set of rules of how to access the wireless medium, but details of transmission and reception are left to the PHY layer. The standard interfaces with the 802.2 Logical Link Control (LLC) layer. The protocol architecture is depicted in the Figure 1.1.

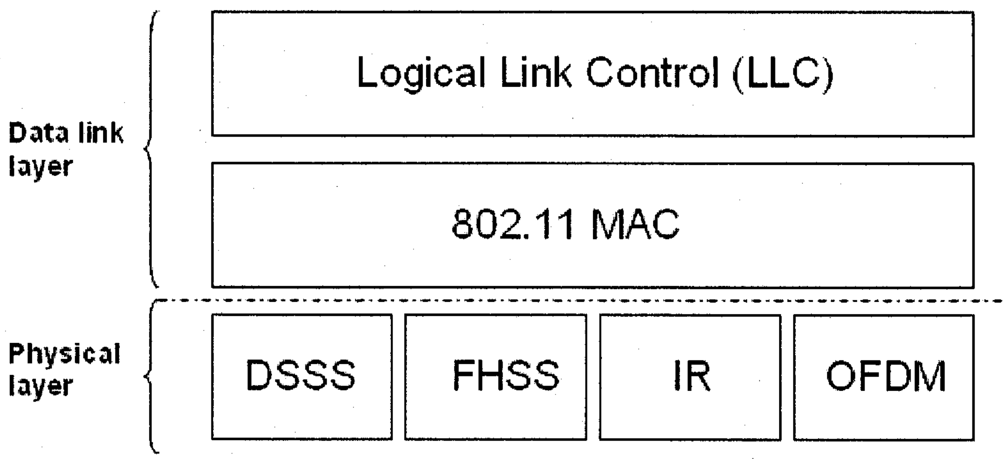

Figure 1.1 IEEE 802.11 Protocol Architecture

In the original 802.11 standard, there are three choices for the PHY layer,

- DSSS: Direct Sequence Spread Spectrum
- FHSS: Frequency Hopping Spread Spectrum
- IR: Infra-Red

Later on, the 802.11a/g standards defined Orthogonal Frequency Division Multiplexing (OFDM) modulation scheme which can offer PHY transmission rate up to 54Mbps. This has enabled WLANs to offer comparable performance to their wired counterpart, the Ethernet.

The 802.11 wireless LAN, in general, consists of the following physical entities.

- **Stations (STA)**: Mobile stations, which contains functionality of the 802.11 protocol.
- **Access Point (AP)**: Any entity that has station functionality and provides access to the distribution services, via the wireless medium for associated stations. The AP is analogous to the base station in the cellular networks.
- **Distribution System (DS)**: A particular access point that interconnects IEEE WLAN with wired 802.x LANs.

Two different topologies are defined in the standard: **Ad hoc** and **Infrastructure** based topologies. In Ad hoc mode, STAs can communicate with each other in a peer to peer manner without the mediation of a centralized AP. On the other hand, infrastructure based networks are established by using the APs and each STA in the network is controlled by one AP which can manage and bridge wireless communications for all the devices within range. AP also provides an interface to a distribution system which enables wireless users to access LAN and Internet resources.

The **Basic Service Set (BSS)** is the fundamental building block of the IEEE 802.11 architecture. A BSS is defined as a group of STAs that are under the direct control of a single coordination function. A STA can communicate within the BSS but it can not communicate with other STAs outside of the BSS. As in ad hoc mode, BSS without the control of AP is called Independent Basic Service Set (**IBSS**) which is shown in Figure 1.2.

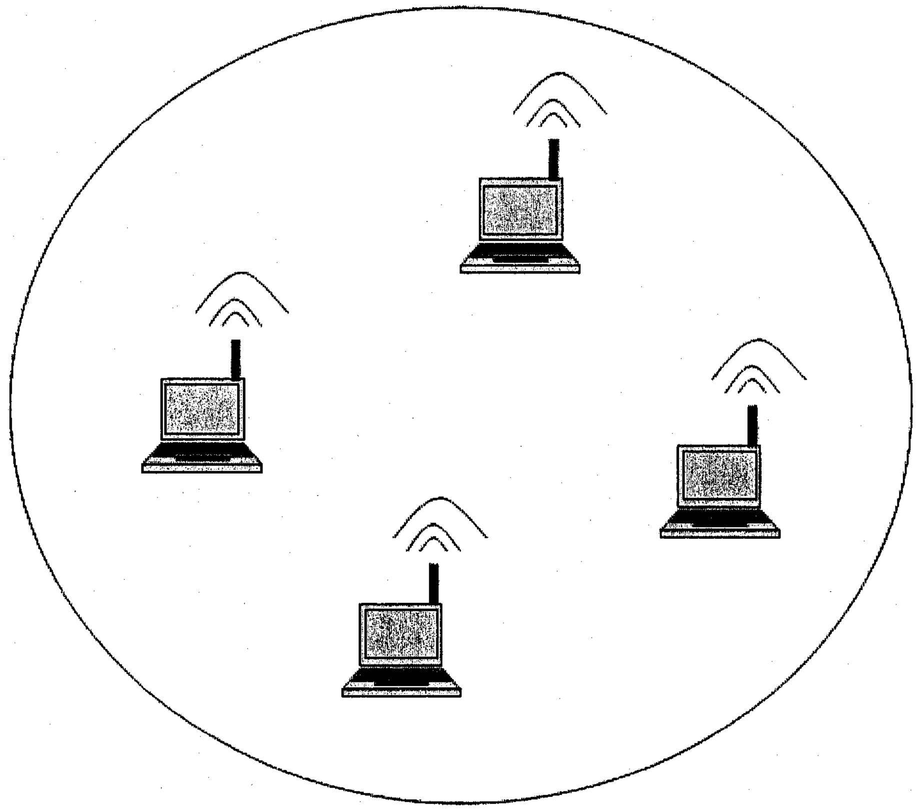

Figure 1.2 Independent Basic Service Set

In order to cover a larger area, several APs are deployed. Two or more BSSs connect to the same wired backbone network. Such a configuration of interconnected BSSs is referred to as an **Extended Service Set (ESS)** (shown as figure 1.3) and the associated wired backbone network is called the **Distribution System (DS)**. The ESS can provide wireless users with gateway access into wired networks. A portal is needed when connecting the 802.11 network with non-802.11 networks, such as the 802.2 network, at this time, the portal functions as a bridge. APs in the ESS are assigned different channels in order to minimize interference. When users roam between cells or BSSs, their mobile device will try to connect with the AP with the strongest signal and least amount of network traffic. In this way, a mobile device can seamlessly roam from one cell to another.

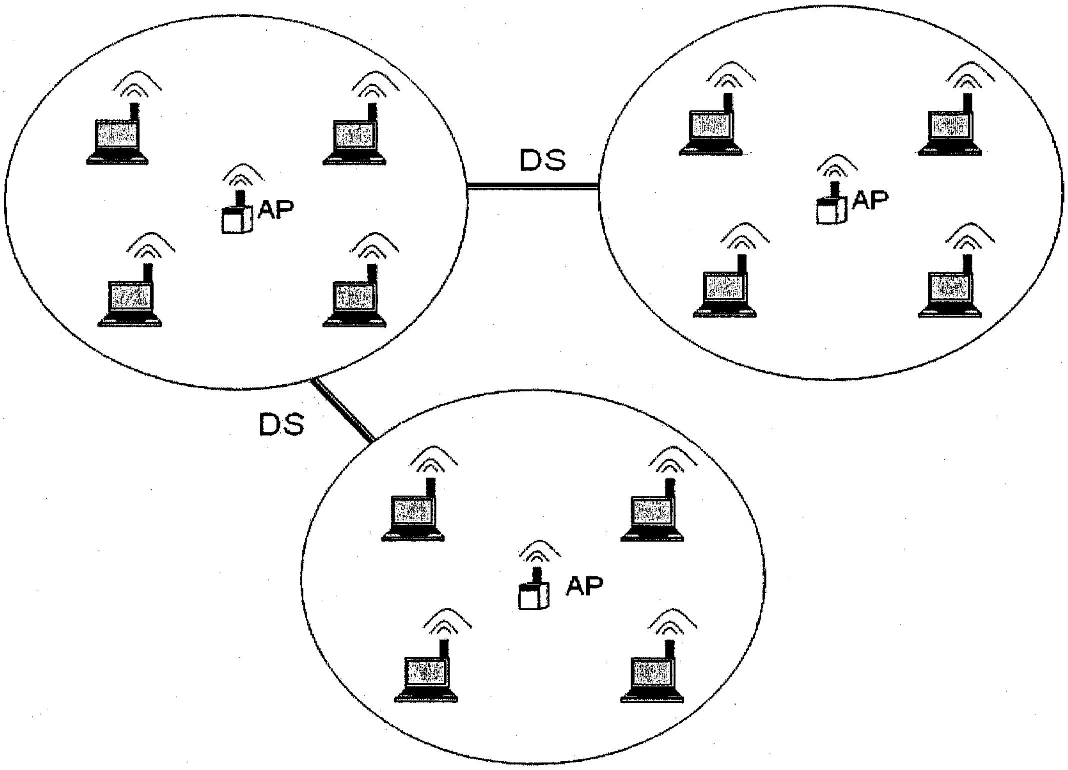

Figure 1.3 Extended Service Set

1.2.2 Physical Layer Overview

The IEEE 802.11 standard includes a common Medium Access Control (MAC) layer, on the other hand, it provides several alternative physical layers that specify the transmission and reception of 802.11 frames. Four different types of physical layers have been defined: Direct Sequence Spread Spectrum (DSSS), Frequency Hopping Spread

Spectrum (FHSS), Infra-Red(IR) and Orthogonal Frequency Division Multiplexing (OFDM).

The 802.11 standard defines a Physical Layer Convergence Procedure (PLCP) sublayer and a Physical Medium Dependent (PMD) sublayer (shown as Figure 1.4). The PLCP prepares 802.11 frames for transmission and directs the PMD to actually transmit signals, change radio channels, receive signals, etc.

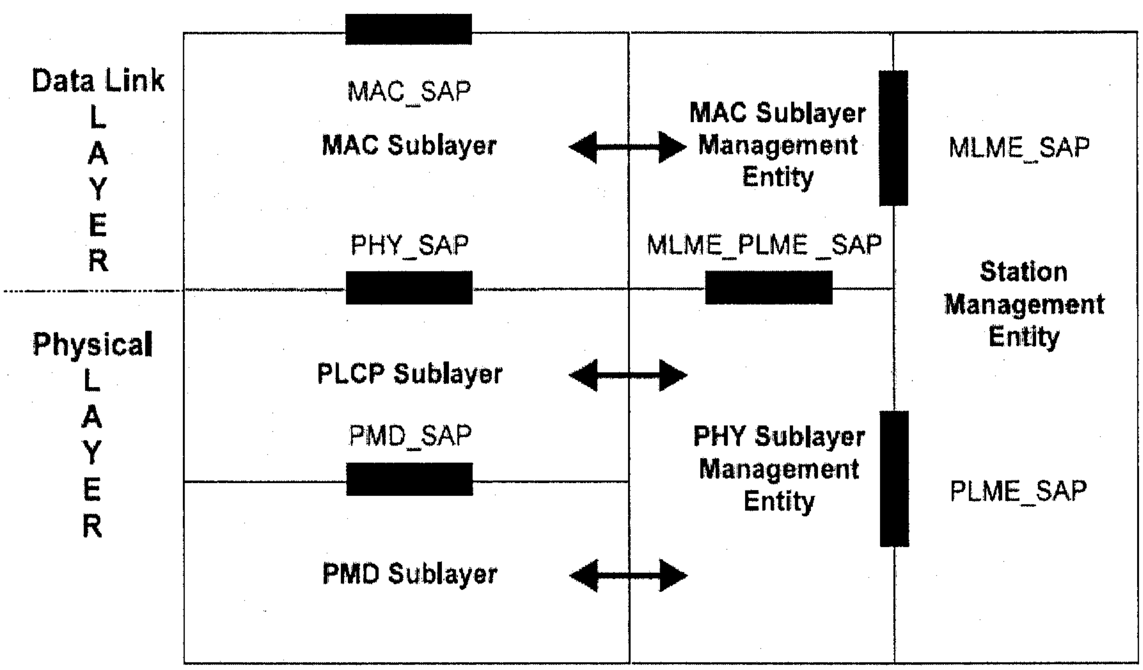

Figure 1.4 IEEE 802.11 Protocol Stack [1]

The PLCP takes each 802.11 frame that a STA wishes to transmit and forms what the 802.11 standard refers to as a PLCP protocol data unit (PPDU). Then the PMD sublayer transmits the PPDU with one of the following modulation schemes.

DSSS

Direct Sequence Spread Spectrum (DSSS) works by applying a chipping sequence to the data stream. 802.11b uses DSSS to disperse the data frame (PPDU) signal over a relatively wide channel (approximately 30MHz) in the 2.4GHz frequency band. This provides greater immunity against radio frequency (RF) interference compared to narrowband transmission. In order to actually spread the signal, an 802.11 transmitter combines the PPDU with a spreading sequence (chip) through the use of a modulo-2 adder. The spreading sequence is a binary code. For 1 and 2Mbps operations, the spreading code is the 11-chip Barker sequence. The binary adder effectively multiplies the length of the binary stream by the length of the sequence, which is 11. This increases the signal rate and makes the signal span a greater amount of bandwidth. 5.5 and 11Mbps operations of 802.11b do not use the Barker sequence. Instead, 802.11b uses complementary code keying (CCK) to provide the spreading sequences at these higher data rates. CCK derives a different spreading code based on fairly complex functions depending on the pattern of bits being sent. Direct sequence modulation trades bandwidth for throughput. It uses more bandwidth than the traditional narrow band modulation. However, due to the correlation process at the receiver side, it can provide greater immunity against the noise and coexist with other noise in its frequency band.

FHSS

Like the DSSS, the Frequency Hopping Spread Spectrum (FHSS) method also transmits the data frame (PPDU) signal over a very large spectrum. However, it uses rapidly changing, predetermined and pseudo-random carrier frequencies. The aim of constant frequency change is to overcome the narrow band interference, which makes signal interception extremely difficult. The frequency change is known to the receiver, that is to say, transmitter and receiver must be synchronized prior to data transmissions so that the receiver is always listening on the transmitter's frequency. The frequency band provides the transmitter with 79 different channels (North America and Europe) for changing the carrier frequency. The FHSS method is not easily susceptible to interference, as carrier frequencies with strong narrow-band interference sources can be left out and the data may be retransmitted with the aid of other carrier frequencies. The large number of carrier frequencies required reduces the transmission rates to between 1 and 2 Mbps.

OFDM

The Orthogonal Frequency Division Multiplexing (OFDM) scheme divides the data into parallel data streams. Each data stream is transmitted with its own carrier frequency. Because the carrier frequencies are carefully designed to be orthogonal, they can affect each other very little. These carrier frequencies or subchannels are then multiplexed into a much larger combined channel. Because OFDM divides a high speed data transmission into several frequency channels and uses error-correcting transmission procedures, it is largely impervious to multipath propagation and narrow-band parasitic signals, making this method particularly interesting for industrial applications. In 802.11, OFDM can achieve transmission rate up to 54 Mbps in the 2.4 GHz or 5 GHz band.

1.3 Medium Access Control Layer

Unlike wireline networks, dynamic changes in the wireless channel always lead to unpredictable channel fading and high Bit Error Rates (BER). To make matters worse, wireless STAs can not detect a collision in the wireless medium. Typically, packet error or collision will cause costly retransmissions. Thus the throughput of wireless LANs is affected by the high BER and retransmissions. Under these circumstances, MAC protocol plays an essential part in the improvement of the wireless LAN performance.

1.3.1 MAC Overview

MAC can be considered as an engine to perform reliable, high speed data communications along with the PHY layer. From the functional point of view, MAC can be deemed as set of services composed of information exchange, power control, synchronization, procedure management. All MAC services are realized by defining an appropriate MAC frame and exchanging those frames. From the network management point of view, the MAC protocol uses Management Information Base (MIB) to store the network information.

The MAC architecture of IEEE 802.11 standard is shown in Figure 1.5. As may be seen, MAC protocol provides two access modes referred to as Point Coordination Function (PCF) and Distributed Coordination Function (DCF) respectively. The CSMA/CA is the basis of the DCF access mechanism. Like Ethernet, it first checks to see that the radio link is idle before transmitting. However, as the wireless STAs can not send and listen to the wireless medium at the same time, in order to avoid collision, wireless STAs use a random backoff procedure after each frame, with the first transmitter seizing the channel.

The Point Coordination Function (PCF) is a polling based channel access method that provides contention-free services. Special STAs called point coordinators are used to ensure that the medium is provided without contention. Point coordinators reside in access points, so the PCF is restricted to infrastructure networks. To gain priority over standard contention-based services, the PCF allows STAs to transmit frames after a shorter interval referred to as SIFS. PCF is provided through the services of DCF.

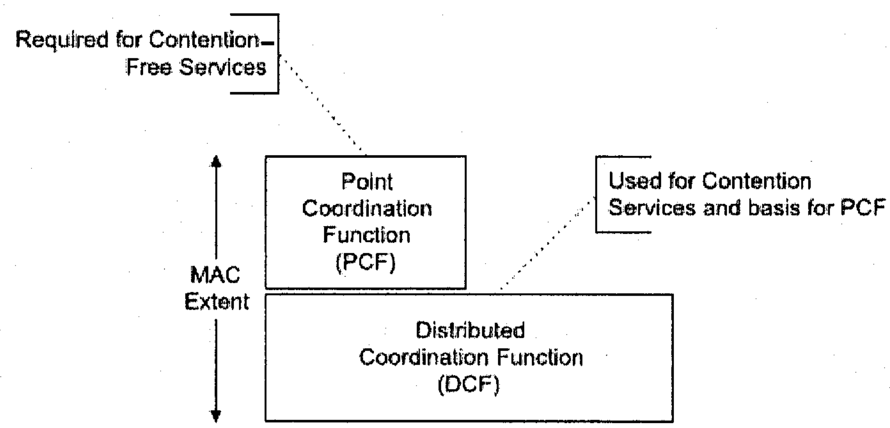

Figure 1.5 Coexistence of DCF and PCF [1]

1.3.2 Inter-frame Space

A time interval between the frames is called Inter-Frame Space (IFS). A STA should be able to determine whether the wireless medium is busy or idle during the IFS period through the carrier sensing function. Access priority of the wireless medium is controlled through the IFSs. SIFS, PIFS, DIFS and EIFS are four different IFSs defined in the standard to give access priority to the wireless medium (shown in Figure 1.6). These four IFSs have the following relative durations, SIFS < PIFS< DIFS< EIFS and they are described below,

- SIFS Short inter-frame space, it is used by highest priority frames, such as RTS/CTS frames and positive acknowledgments.

- PIFS PCF inter-frame space, it is used by the PCF during contention-free operation.
- DIFS DCF inter-frame space, it is the minimum medium idle time for contention-based operations.
- EIFS Extended inter-frame space, it is used when there is a transmission error.

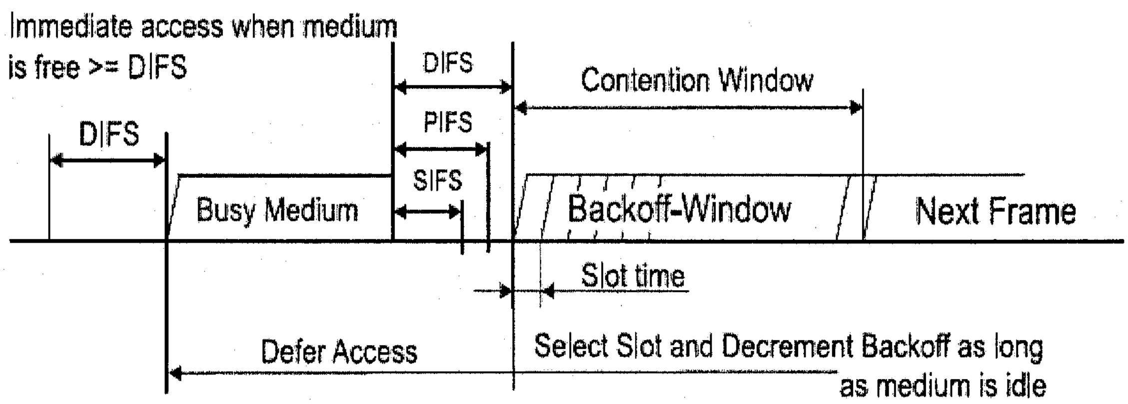

Figure 1.6 Different IFS [1]

High priority frames, such as control frames (ACK, RTS frame and etc), only wait for SIFS period before transmission. Since SIFS is shorter than DIFS, it has more opportunity to access the wireless medium.

1.3.3 Distributed Coordination Function

Access to the wireless medium is controlled by the Distributed Coordination Function (DCF). DCF uses CSMA/CA to regulate the access to the wireless medium. DCF allows several independent STAs to contend for the channel without a centralized control. It basically gives each STA equal opportunity to compete for one time slot, thus it can achieve fairness among STAs that should have the same long term access probability to the wireless medium. The CSMA/CA is designed to reduce the collision probability between multiple STAs. The highest probability of collision occurs right after the medium becomes idle following a busy medium. This is why a random backoff procedure is necessary to resolve the collision problem. There are essentially two DCF protocol defined in the standard – a basic access mode that uses two way handshaking (DATA-ACK) and a RTS/CTS mode that uses request-to-send and clear-to-send in a four way handshake (RTS-CTS-DATA-ACK).

In basic access mode, each STA with packets to transmit first uses carrier sensing mechanism to check whether the medium is idle. If the medium is idle for greater than or equal to a DIFS period or an EIFS period if the previous received frame was in error, the transmission may proceed. If under these conditions the medium is not idle, it waits for the medium to become idle plus a DIFS duration and then initiates a random backoff procedure to avoid collision by deferring the transmission (shown as Figure 1.7).

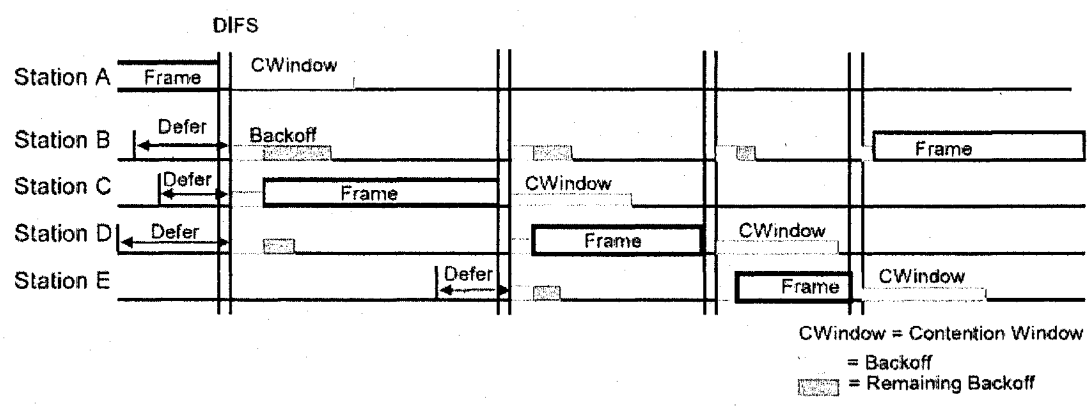

Figure 1.7 Backoff Procedure of Basic DCF Access Mode [1]

The backoff period follows a DIFS and it is chosen from backoff window or else referred to as contention window (CW). This contention window is slotted and the slot duration is dependent on the physical layer. For example, the slot duration is equal to $20\,\mu s$ for DSSS and $50\,\mu s$ for FHSS. Initially, the backoff counter value is uniformly chosen between 0 and minimum contention window CW_{min}. Then the backoff period is given by,

$$\text{Backoff Period} = \text{BackoffCounter} \times \text{SlotTime}$$

After the initial backoff period is chosen, the backoff counter decrements by one during each slot. If there are transmissions during the backoff period the STA freezes its counter and resumes the count decrement operation after other STAs complete transmissions plus a DIFS period. After the backoff period expires, the STA then transmits the packet. When several STAs contend for the channel simultaneously, the STA with the smallest contention window size usually wins the contest. If the transmission is successful, the STA receives Acknowledgement (ACK) frame from its destination.

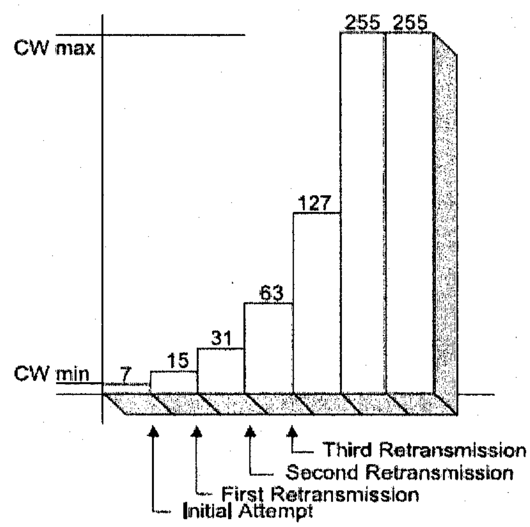

Figure 1.8 An Example of the Exponential Backoff Algorithm [1]

If more than one STA begin transmission at the same time slot, collision occurs. Unlike the Ethernet, a collision can not be detected by the wireless STA and thus the failed transmission can not be stopped. After corrupted frames are transmitted in full, a collided STA then chooses a new CW size and defers its transmission by going into backoff procedure again. After the backoff period plus a DIFS period has elapsed, the STA transmits the packet again. The new CW size is determined as follows,

$$CW_{new} = CW_{old} \times 2 + 1$$

The above procedure is repeated until either packet is successfully transmitted or CW reaches its maximum CW_{max} and it remains unchanged at CW_{max} until the retry limit is reached. An example of the exponential backoff algorithm is shown in Figure 1.8.

In RTS/CTS access mode, each STA with packets to transmit first send out a request-to-send (RTS) frame to the receiver to reserve the channel prior to data transmission. After the receiver receives the RTS frame, it responds to the sender with a clear-to-send (CTS) frame to indicate the channel is reserved for the transmission. Then the sender starts to transmit the data frame. Because the channel is reserved by the RTS/CTS mechanism, other STAs waits for the current transmission to end. Thus, collision can only happen on the initial RTS frames. Since RTS frame is much shorter than normal data frame, it reduces the time that the channel spends in collisions. This mechanism is very effective in terms of system performance, especially for large data packet. However, it may cause some communication overheads for smaller data packets. Therefore, RTS/CTS access mode should be employed when a data packet exceeds a certain threshold.

1.3.4 Quality of Service

Quality of Service (QoS) is becoming ever more important in the communication systems. QoS provision in a network basically concerns the establishment of a network resource sharing policy and then the enforcement of that policy. The 802.11e standard is the enhancement of the MAC layer and provides QoS support to the LAN applications. Legacy DCF can provide fairness among the STAs, however, it can not support time bounded services. 802.11e uses Enhanced Distributed Channel Access (EDCA) to support QoS. In contrast to the legacy DCF which uses the same DIFS, CW_{max} and CW_{min}, EDCA stations (QSTAs) support up to four access categories (ACs) to provide prioritized frame delivery for up to eight levels of user priorities (UP). Each AC has the following parameter set,

- AIFS[AC]—Arbitrary inter-frame space
- $CW_{min}[AC]$, $CW_{max}[AC]$—Minimum and maximum contention window
- TxOPlimit[AC]—Medium occupancy limit

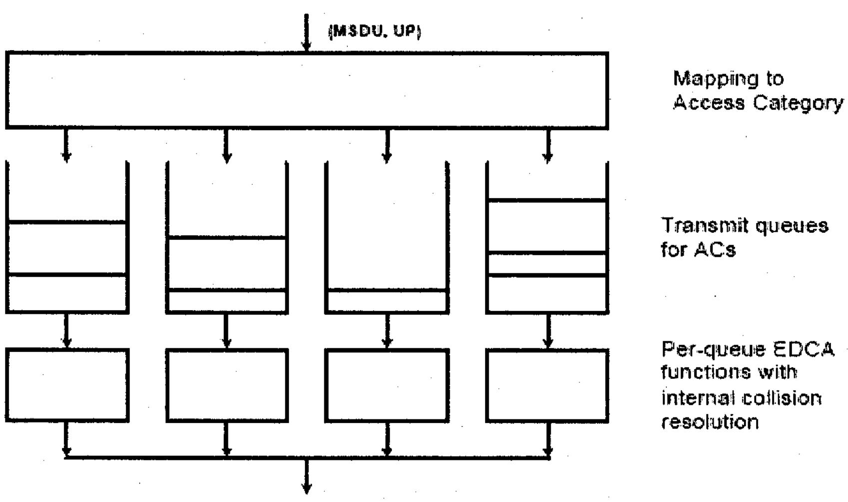

Figure 1.9 IEEE 802.11e Reference Implementation Model [11]

A model of the reference implementation is shown in Figure 1.9. The figure illustrates a mapping from frame type or user priorities (UP) to AC: the four transmit queues and the four independent EDCA functions, one for each queue.

In legacy DCF, all backoff procedures begin after a DIFS from the end of the busy medium, whereas EDCA backoff procedures begin after a differential AC specific AIFS period. The duration of AIFS for each AC is given by,

$$AIFS[AC] = AIFSN[AC] \times aSlotTime + aSIFSTime$$

where AIFS[AC], AIFSN[AC],aSlotTime and aSIFSTime are Management Information Base(MIB) attributes. High priority STAs have smaller value of AIFS, thus have a lower probability of collision because lower priority STAs are still waiting to transmit in the AIFS period. Figure 1.10 shows the AIFS and other IFS relations.

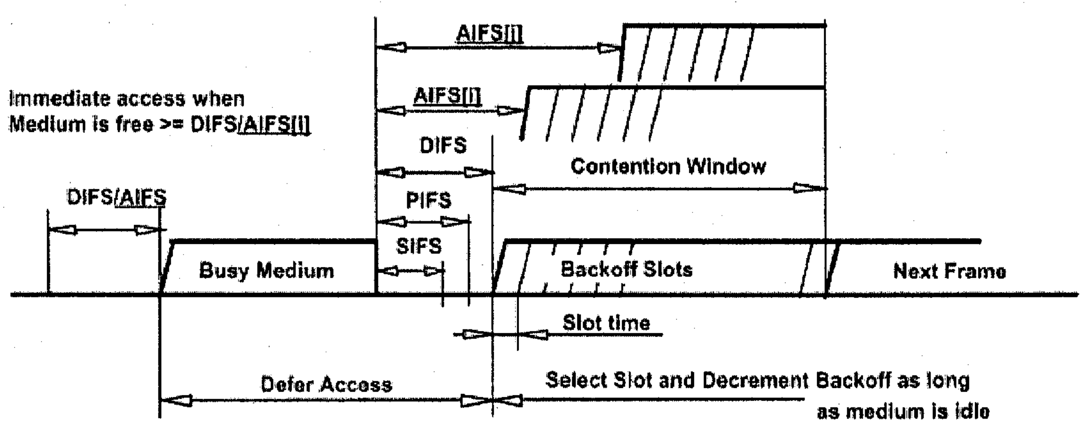

Figure 1.10 AIFS and other IFS Relations [11]

EDCA backoff counter values are uniformly selected from the interval [0,CW], where CW is a function of an AC specific $CW_{min}[AC]$ and $CW_{max}[AC]$ attributes. Higher priority STAs can receive superior service by means of smaller $CW_{min}[AC]$ and $CW_{max}[AC]$. This results in a smaller backoff period and reduced packet service time.

An important new feature of 802.11e EDCA is the transmission opportunity (TxOP). TxOP is the amount of time that a STA can use during each possession of the wireless medium. A QSTA using EDCA is allowed a transmission opportunity (TxOP) for a particular access category (AC) if the medium is idle at the AIFS[AC] slot boundary following a correctly received frame and the backoff period for that TC has elapsed.

1.4 Literature Survey

In this section, we present a literature survey regarding modeling of the IEEE 802.11 DCF protocol. Performance modeling of this protocol has been a hot research topic from the moment that the IEEE 802.11 standard was released.

In [2], Bianchi proposed a two dimensional Markov chain model for the Distributed Coordination Function (DCF) of IEEE 802.11. The main assumptions of this work are: 1) All STAs in the system are saturated, i.e., always have packets waiting to be transmitted. 2) The collision of a packet in the present transmission is independent of its previous collisions and occurs with a constant probability P_C. This is an accurate assumption as long as contention window size and number of STAs are large. Each STA is modeled by a two-dimensional Markov chain where a state is denoted by a pair of integers (i,k). The variable i denotes the backoff stage, it starts from 0 and is incremented by one after each transmission that ends up in a collision. The backoff counter value k is initially uniformly chosen between $(0,W_i-1)$ in stage i. When the wireless medium is idle, the counter is decremented in each slot and when it reaches to zero the packet is transmitted. The paper determines the equilibrium packet collision probability P_C and transmission probability τ of a STA in a generic slot by solving the Markov chain. Then these results were used to determine the network throughput.

Since Bianchi's original work, many researchers have tried to improve the model by relaxing its assumptions. The many assumptions chosen for relaxation are that the system is under saturation, i.e., all STAs in the system always have packets waiting to be transmitted, and homogeneous STAs, i.e., all STAs are identical which have the same packet arrival and transmission rates. However, as discussed earlier, IEEE 802.11b standard defines different PHY layer options which offer PHY layer transmission rate from 1Mbps to 11Mbps.

In [5], Cantieni, *et al.* have proposed a model for un-saturated sources and multirate traffic situation. They investigated the throughput degradation caused by heterogeneous traffic load and proposed two simple mechanisms to overcome the performance downgrading, but they have not provided numerical results or validated their analytical model.

In [4], Zaki, *et al.* have proposed a Markov chain model in which each state has a fixed real time length. They derived the system throughput under both finite and saturated load conditions. However, by virtue of its design it can not predict the pre-saturation peak in the throughput.

In [7], Malone, *et al.* have proposed a model to determine the system throughput. It allows STAs to have different traffic arrival rates, which enables them to address the question of fairness between competing flows. However, their model doesn't capture the effects of different physical layer transmission rates.

In [6], Tickoo, *et al.* have proposed a model where each STA is modeled as a discrete time G/G/1 queue. They derived the delay and queue length distributions of each STA. However, their derivation has been done in an ad-hoc manner. Further, their results only apply to the homogeneous traffic condition.

In [8], S.Ci proposed a model under fading channel condition. They considered retry limit, backoff suspension and fading channel errors and evaluated the saturation throughput of the 802.11 MAC protocol.

1.5 Major Contribution

As may be seen from the literature survey, while most of the previous works have concerned with the derivation of system throughput for the Distributed Coordination Function (DCF) of the MAC protocol, little attention has been paid to the delay performance of the DCF scheme. However, the packet service time and delay are very important metrics for 802.11 DCF, especially for real time applications, such as IP telephony or video conferencing.

In this thesis we also extend the analysis of Bianchi for the IEEE 802.11 DCF protocol. We extend the analysis to both homogeneous and heterogeneous sources under finite load condition. In the homogeneous case, all sources have the same arrival and transmission rate, on the other hand, heterogeneous case allows sources with different arrival and transmission rates. The arrival of packets to each STA's MAC buffer is assumed to be according to a Poisson process. We model each STA's MAC buffer as an M/G/1 queue. To derive the probability distribution of service time, we model the packet transmission of each STA as a three dimensional Markov chain. The dependency between STAs is taken into account through the freezing of the counter decrement operation and packet collision during packet transmission. We determine and numerically solve equations that involve collision probability, packet transmission probability in a slot and busy probability of a queue. Then we use these results to derive the PGF of the packet service time distribution. Following that, we determine the mean packet delay through the application of the M/G/1 queuing results. We also investigate the impact of the different contention window sizes on the delay performance. A discrete event C++ test bed has been developed to simulate 802.11 DCF protocol in homogeneous and heterogeneous environment. The accuracy of the analytical model is verified by extensive simulations.

1.6 Thesis Organization

This thesis is organized as follows.

Chapter 1 provides a general introduction to IEEE 802.11 Wireless LAN standard. It presents an overview of the physical (PHY) and Medium Access Control (MAC) layers of the standard. It also gives a literature review of previous modeling of the IEEE 802.11 DCF, discusses problem formulation and contributions of this thesis.

Chapter 2 models a STA in un-saturated homogeneous environment as an M/G/1 queue. The service time of a STA is modeled as a three dimensional Markov chain. We determine the packet collision probability and the packet service time distribution of each STA. Then the application of the M/G/1 queuing results gives the mean packet delay. The accuracy of the analytical model is proved through extensive simulations.

Chapter 3 derives a system model with an M/G/1 queue and a three dimensional Markov chain under un-saturated heterogeneous conditions. We analyze the system with two groups of STAs of different packet arrival and transmission rates (fast and slow). We determine the packet collision probability and the packet service time distributions of both types of STAs. Then we obtain the mean packet delay by using the M/G/1 queuing results. We also investigate the impact of different CW_{min} on the delay performance. The analytical model is verified by extensive simulations.

Chapter 4 gives a brief summary and conclusion along with some suggestions of future research considerations.

CHAPTER 2

PERFORMANCE MODELING OF 802.11 DCF WITH HOMOGENEOUS SOURCES

In this Chapter, we present a performance modeling of 802.11 DCF assuming STAs with homogeneous traffic under un-saturated condition. The STAs in the system have the same packet arrival and physical transmission rate but not necessarily always have packets to transmit. We model each STA as an M/G/1 queue, then we use a Markov chain model to derive the PGF of the service time distribution. Following that, we determine the mean packet delay and present some numerical results.

2.1 Model Assumptions

We assume that the wireless channel is shared by n STAs under ideal conditions. The channel's time axis is slotted with its slot duration defined in the 802.11 PHY layer. Propagation delay and hidden terminal problem are not considered. Each STA has an infinite buffer to store its packets. Basic DCF access mechanism is considered. It is assumed that the packets arrive to each STA according to a Poisson process with the same rate. When a packet reaches the head of its queue, its service time begins. The service time of a packet is governed by the DCF protocol. We make some simplifying assumptions in order to have mathematically tractable model of DCF protocol. For example, we drop the DIFS but our model captures the main functionality of the DCF protocol.

The packet service begins with initialization of backoff stage to zero and choice of a counter value. The counter value is decremented during each slot that the channel is idle, otherwise it is frozen. The counter decrement operation is unfrozen after the channel becomes idle again. When the counter value reaches to zero, the packet is transmitted. On the other hand, if the packet transmission is successful, then the service time of the packet is completed. If the packet transmission ends up in a collision, backoff stage is incremented by one, a new counter value is randomly chosen and the above process is repeated. At stage *i*, the counter value is chosen as a uniformly distributed number in the interval $[0, CW_i - 1]$ where CW_i is the contention window size at stage *i*. The contention window size is determined through the recursion $CW_{i+1} = 2CW_i$ with minimum and maximum window sizes $CW_{min} = W_0$ and $CW_{max} = 2^m W_0$ respectively. If a packet transmission in its maximum backoff stage ends up in a collision then that packet is discarded. It is assumed that packet collision probability is independent of the previous retransmissions. After a packet is transmitted successfully or its transmission is terminated following discarding, the STA begins transmission of a new packet if its buffer is not empty, otherwise it remains idle until a new packet arrives to its buffer.

We model each STA's buffer as an M/G/1 queue. The dependencies between STAs are taken into account through the freezing of counter decrement operation and packet collision during transmission. We need to determine the packet service time distribution to be able to use results available from M/G/1 queue.

2.2 Packet Arrival Process

First, we describe the packet arrival process to a STA. It is assumed that the arrival of packets to each STA's MAC buffer is according to a Poisson process with rate λ packets/slot, thus,

$$\begin{aligned} P_k &= \Pr(k \text{ packets arrivals during a slot}) \\ &= \frac{e^{-\lambda}\lambda^k}{k!} \end{aligned} \tag{2.1}$$

Let σ denote the probability that no packet arrives to a STA's buffer in a time slot, then we have,

$$\sigma = e^{-\lambda} \tag{2.2}$$

Thus the interarrival time of the packets to a STA follows a geometric distribution, which is given by,

$$\Pr(\text{packet interarrival time is k slots}) = (1-\sigma)\sigma^{k-1}, \quad k=1,2,3... \tag{2.3}$$

2.3 Derivation of Packet Collision and Transmission Probabilities

We assume that the probability that a STA transmits in any slot is given by τ. Since the STA itself does not transmit during its own backoff period, the probability that there is at least one transmission during the backoff stage in any slot is given by,

$$P_{tr} = 1-(1-\tau)^{n-1} \tag{2.4}$$

We let P_C denote the probability that a transmitted packet encounters a collision. A packet collision occurs when more than one STA transmit in the same time slot. Clearly, P_C is given by,

$$P_C = P_{tr} \tag{2.5}$$

The major approximation of the above analysis is that the packet collision probability P_C and probability P_{tr} that a channel is in a busy state are constant regardless of its transmission history, i.e., how many retransmission the data packet has suffered. We will see later in the numerical results that the above approximation is accurate.

We now determine the packet transmission probability τ in any given time slot. First, we determine the average backoff period $\overline{W}$ [16]. We let random variable W to denote the backoff period of a STA. Since backoff period is uniformly distributed over [0,CW] in each stage, the average backoff period in each stage is $CW/2$. Each transmission ends up in a collision with probability P_C. A STA will keep retransmitting a packet until it receives an acknowledgement or its maximum retry limit is reached. Then the probability distribution of W is given by,

$$\Pr(W = CW) = \begin{cases} P_C^{\,k-1}(1-P_C), & CW = 2^{k-1}\dfrac{CW_{\min}}{2}, \quad k \in (1,m) \\ P_C^{\,m+1}, & CW = \dfrac{CW_{\max}}{2} \end{cases} \tag{2.6}$$

The average backoff period is then given by,

$$\begin{aligned} \overline{W} &= \sum_{k=1}^{m} P_C^{\,k-1}(1-P_C)2^{k-1}\frac{CW_{\min}}{2} + P_C^{\,m+1}\frac{CW_{\max}}{2} \\ &= \frac{1-P_C-P_C(2Pc)^m}{1-2P_C}\frac{W_0}{2} \end{aligned} \tag{2.7}$$

Consider that STA A and B are contending for the channel. Because A's backoff counter is frozen whenever B is transmitting, it appears to A that B's transmission occupies only one slot and B is only transmitting for every W slot. Assuming the number of STAs is large enough so that A and B's transmissions are not synchronized, then A could begin transmission in any slot of B's time line. Its probability of transmission in any slot is $1/W$. We use the average of this backoff period W to represent the probability of transmission in one slot.

$$\Pr(\text{a STA will transmit in a slot}) = \frac{1}{W} \tag{2.8}$$

Equation (2.8) determines the packet transmission probability in any slot under saturation condition. Assuming that each STA queue may be busy with probability ρ , the probability τ that a STA transmits in a randomly chosen time slot in un-saturated case is given by,

$$\begin{aligned} \tau &= \Pr(\text{a STA is busy}) \times \Pr(\text{a STA will transmit in a slot}) \\ &= \frac{\rho}{W} \end{aligned} \tag{2.9}$$

2.4 Markov Chain Model of a STA

In this section, we describe the Markov chain model of a STA that will enable us to determine packet service time distribution.

In [2], Bianchi has modeled state of a STA under saturation with a two dimensional Markov chain {s(t),c(t)},where,

s(t) denotes the value of the backoff stage at slot time t.

c(t) denotes the value of the backoff counter at slot time t.

A discrete time scale is adopted here: *t* and *t+1* correspond to the beginning of two consecutive time slots, and the backoff counter of each STA is decremented at the beginning of each slot time. We extend Bianchi's model of a STA with a three dimensional discrete time Markov chain {q(t),s(t),c(t)}where the third dimension **q(t)** denotes whether a STA has a packet to transmit at slot time t. Thus q(t) is a Bernoulli random variable with values 1 and 0 representing busy and idle queues respectively.

Next, we determine the state transition probability matrix of this Markov chain which is shown in Figure 2.1. We adopt the following short notation for transition probabilities.

$$p\{j_1,i_1,k_1 \,|\, j_0,i_0,k_0\} = p\{q(t+1)=j_1, s(t+1)=i_1, c(t+1)=k_1 \,|\, q(t)=j_0, s(t)=i_0, c(t)=k_0\}$$

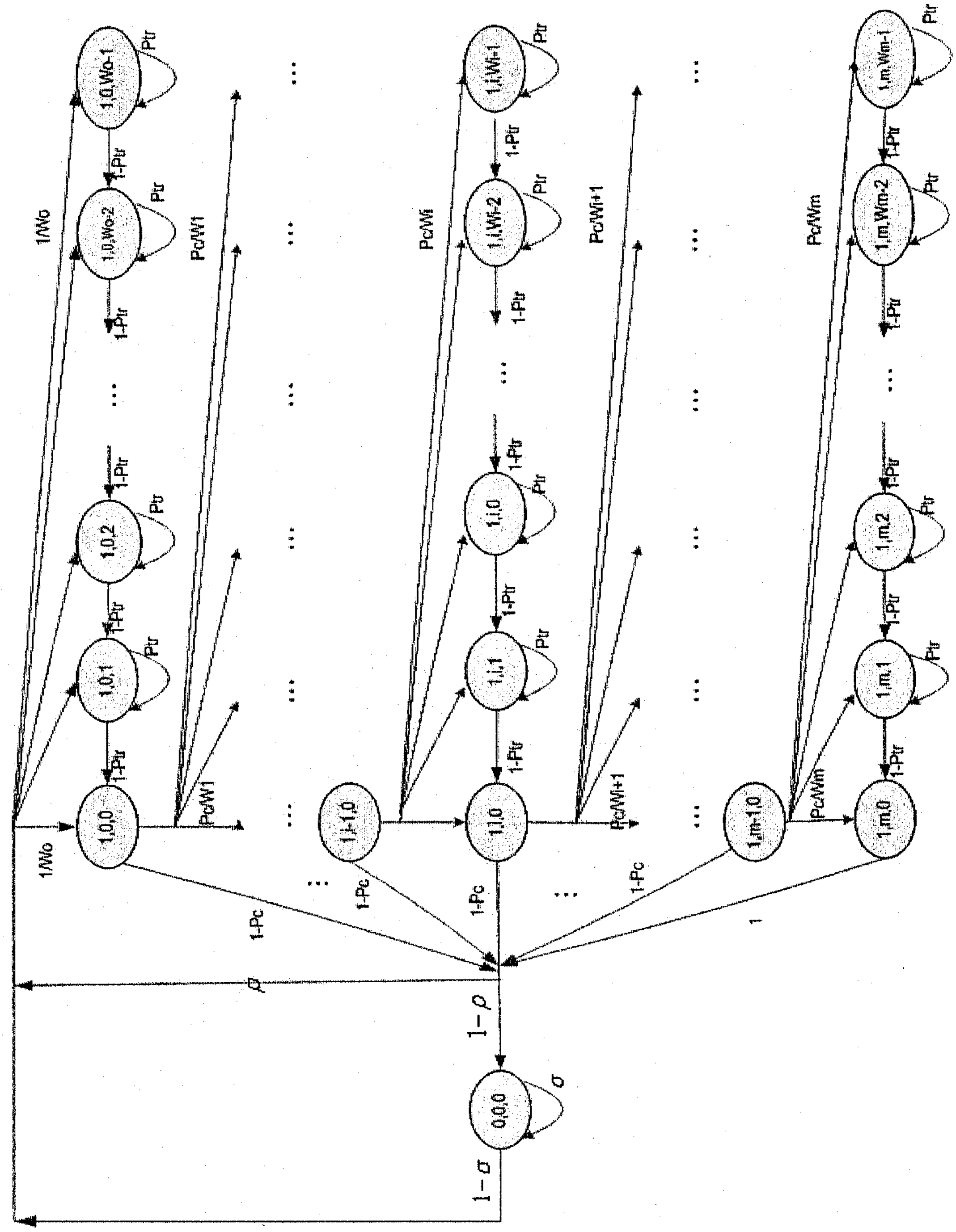

Figure 2.1 Markov Chain Model for IEEE 802.11 DCF under Finite Load

Many of the state transition probabilities are zero. The non-zero transition probabilities are given below. Assuming that an STA has a packet to transmit, then,

1) At the beginning of each time slot, the backoff counter is decremented when the STA senses that the channel is idle.

$$p\{1,i,k|1,i,k+1\} = 1 - P_{tr} \qquad k \in (0, W_i - 2), i \in (0,m) \tag{2.10}$$

2) The backoff counter is frozen when the STA senses that the channel is busy.

$$p\{1,i,k|1,i,k\} = P_{tr} \qquad k \in (0, W_i - 1), i \in (0,m) \tag{2.11}$$

3) When the backoff counter reaches zero at each backoff stage, the STA transmits the packet. If an unsuccessful transmission occurs, i.e., collision, the contention window size is doubled until it reaches its maximum.

$$p\{1,i,k|1,i-1,0\} = P_C / W_i \qquad k \in (0, W_i - 1), i \in (1,m) \tag{2.12}$$

4) Following a successful transmission, a new packet is available in the buffer with probability ρ. The new packet's transmission begins by choosing a counter value at backoff stage 0.

$$p\{1,0,k|1,i,0\} = (1 - P_C)\rho / W_0 \qquad k \in (0, W_0 - 1), i \in (0, m-1) \tag{2.13}$$

5) If there is no new packet available following a successful transmission, the system returns to a "sleeping" period or a vacation state.

$$p\{0,0,0|1,i,0\} = (1 - P_C)(1 - \rho) \qquad i \in (0, m-1) \tag{2.14}$$

6) At the maximum backoff stage, the packet will be transmitted; if it experiences a collision, the packet will be discarded. If a packet is available in the buffer the backoff stage returns to zero and a new backoff counter value is uniformly chosen.

$$p\{1,0,k \mid 1,m,0\} = \rho / W_0 \qquad k \in (0, W_0 - 1) \tag{2.15}$$

7) At the maximum backoff stage m, following successful transmission or discarding of a packet if no packet is available in the buffer, the STA goes into a "sleeping" period or a vacation state.

$$p\{0,0,0|1,m,0\} = (1-\rho) \tag{2.16}$$

Let $\mathbf{P}$ denote the transition probability matrix of this system, then it is given by,

$$\mathbf{P} = [p\{j_1, i_1, k_1 \mid j_0, i_0, k_0\}] \tag{2.17}$$

2.5 Steady State Probability Distribution of the Markov Chain

Next, we determine the steady state probability distribution of the above Markov chain. We let $b_{j,i,k}$ to denote the steady state probabilities.

$$\begin{aligned} b_{j,i,k} &= \lim_{t\to\infty} p\{q(t)=j, s(t)=i, c(t)=k\} \\ &= Pr(q=j, s=i, c=k) \end{aligned} \qquad j=0,1;\ i \in (0,m);\ k \in (0, W_i - 1) \tag{2.18}$$

Let $\mathbf{B}$ denote the vector of steady state probabilities, $\mathbf{B} = [b_{j,i,k}]$, then it may be determined by solving the $\mathbf{B} = \mathbf{BP}$. However, the steady state distribution may be determined from inspection of state transition probabilities given above.

Since collision probabilities of each retransmission is given by P_C, then we have,

$$b_{1,i,0} = P_C{}^i b_{1,0,0} \qquad i \in (1,m) \tag{2.19}$$

For the first backoff slot of each stage, $k=0$,

$$b_{1,i,0} = (1-P_{tr}) b_{1,i,1} + \frac{1}{W_i} \begin{cases} (1-\sigma) b_{0,0,0} + \rho[(1-P_C)\sum_{j=0}^{m-1} b_{1,j,0} + b_{1,m,0}], & i=0 \\ P_C b_{1,i-1,0}, & i \in (1,m) \end{cases} \tag{2.20}$$

For each $k \in (1, W_i - 2)$,

$$b_{1,i,k} = (1-P_{tr})b_{1,i,k+1} + P_{tr}b_{1,i,k} + \frac{1}{W_i}\begin{cases}(1-\sigma)b_{0,0,0} + \rho[(1-P_C)\sum_{j=0}^{m-1} b_{1,j,0} + b_{1,m,0}], & i=0 \\ P_C b_{1,i-1,0}, & i \in (1,m)\end{cases}$$

(2.21)

For the last backoff slot of each stage, $k = W_i - 1$,

$$b_{1,i,k} = P_{tr}b_{1,i,W_i-1} + \frac{1}{W_i}\begin{cases}(1-\sigma)b_{0,0,0} + \rho[(1-P_C)\sum_{j=0}^{m-1} b_{1,j,0} + b_{1,m,0}], & i=0 \\ P_C b_{1,i-1,0}, & i \in (1,m)\end{cases} \quad (2.22)$$

From (2.19), we have,

$$\begin{aligned}\sum_{j=0}^{m-1} b_{1,j,0} &= \sum_{j=0}^{m-1} P_C{}^j b_{1,0,0} \\ &= \frac{b_{1,0,0} - b_{1,m,0}}{1-P_C}\end{aligned} \quad (2.23)$$

Using (2.20-2.23), we may obtain,

$$\begin{aligned}b_{1,i,k} &= \frac{W_i - k}{W_i} P_C{}^i b_{1,0,0} \\ &= \frac{W_i - k}{W_i} b_{1,i,0}\end{aligned} \qquad i \in (1,m), k \in (0, W_i - 1) \quad (2.24)$$

and

$$\begin{aligned}b_{1,0,k} &= \frac{W_0 - k}{W_0}\{(1-\sigma)b_{0,0,0} + \rho[(1-P_C)\sum_{j=0}^{m-1} b_{1,j,0} + b_{1,m,0}]\} \\ &= \frac{W_0 - k}{W_0}\{(1-\sigma)b_{0,0,0} + \rho[(1-P_C)\sum_{j=0}^{m-1}\frac{b_{1,0,0} - b_{1,m,0}}{1-P_C} + b_{1,m,0}]\} \\ &= \frac{W_0 - k}{W_0}\{(1-\sigma)b_{0,0,0} + \rho b_{1,0,0}\}\end{aligned} \quad (2.25)$$

$k \in (0, W_0 - 1)$

We note that $b_{0,0,0}$ is related to the probability ρ that the buffer is non-empty.

$$b_{0,0,0} = 1 - \rho \tag{2.26}$$

Next, we apply the normalization condition that the sum of the state probabilities of all the states in a Markov chain equals to 1.

$$\sum_{k=0}^{W_0-1} b_{1,0,k} + \sum_{i=1}^{m}\sum_{k=0}^{W_i-1} b_{1,i,k} + b_{0,0,0} = 1 \tag{2.27}$$

Substituting (2.24-2.26) into (2.27), we obtain the stationary probability of state (1,0,0) which corresponds to busy STA with value of backoff stage and counter values being set to zero.

$$b_{1,0,0} = \frac{2\rho - (W_0+1)(1-\sigma)(1-\rho)}{(W_0+1)\rho + W_0 \frac{(2P_C)^{m+1} - 2P_C}{2P_C - 1} + \frac{P_C{}^{m+1} - P_C}{P_C - 1}} \tag{2.28}$$

Next, we determine the probability that a STA is busy that it has at least one packet in its buffer waiting for transmission.

$$\Pr(q=1) = \sum_{i=0}^{m}\sum_{k=0}^{W_i-1} b_{1,i,k} \tag{2.29}$$

$$\Pr(q=1) = \rho \tag{2.30}$$

2.6 Probability Distribution of Packet Service Time

Now, we are ready to determine the PGF of the probability distribution of packet service time. It is defined as the duration from the time when the data packet reaches to head of its queue to the time when it is successfully transmitted.

A packet transmission may involve several retransmissions due to collisions. Each attempt consists of a choice of backoff counter value, which is counted down to zero, and retransmission. However, the count down process maybe interrupted by the transmission of other STAs. Duration of each interruption equals to the packet transmission time and during this period the count down operation is frozen. Let us define the following random variables,

r packet size in slots

k_i initial value of the backoff counter in the i_{th} backoff stage

j_i number of interruptions of the count down process in the i_{th} backoff stage

x_i duration of count down process in the i_{th} backoff stage

S_i duration of packet service time given that there is an i_{th} retransmission attempt

S packet service time in slots

Also defining the following PGFs of these random variables,

$R(z)=E[z^r], K_i(z)=E[z^{k_i}], J_i(z)=E[z^{j_i}], X_i(z)=E[z^{x_i}], S_i(z)=E[z^{S_i}], S(z)=E[z^S]$

Then we have the following relations for the service time,

$$x_i = k_i + \sum_{\ell=1}^{j_i} r \tag{2.31}$$

and

$$S_{i+1} = S_i + x_i + r \tag{2.32}$$

From the above equations (2.31) and (2.32), we have,

$$X_i(z) = J_i(z)\Big|_{z=R(z)} K_i(z) \tag{2.33}$$

$$S_{i+1}(z) = S_i(z) X_i(z) R(z) \qquad i \in (0, m-1) \tag{2.34}$$

where $S_0(z) = X_0(z)R(z)$

Next, we will determine the unknown PGFs, $K_i(z), J_i(z), X_i(z)$.

Probability distribution of k_i follows a uniform distribution.

$$\Pr(k_i) = \frac{1}{W_i} \tag{2.35}$$

Since the PGF of k_i has been defined as,

$$K_i(z) = E[z^{k_i}] \tag{2.36}$$

Then we have,

$$\begin{aligned} K_i(z) &= \sum_{k_i=0}^{W_i-1} z^{k_i} \Pr(k_i) = \frac{1}{W_i} \sum_{k_i=0}^{W_i-1} z^{k_i} \\ &= \frac{1}{W_i} \frac{1-z^{W_i-1}}{1-z} \end{aligned} \tag{2.37}$$

Since the interruption of the count down process occurs independently with probability P_{tr} in any given slot, the probability distribution of number of interruptions during the i_{th} stage is given by the binomial distribution.

$$\Pr(j_i \mid k_i) = \binom{k_i}{j_i} P_{tr}^{\,j_i} (1-P_{tr})^{k_i - j_i} \tag{2.38}$$

$J_i(z \mid k_i)$,the conditional PGF of j_i, is given by ,

$$J_i(z \mid k_i) = (P_{tr}z + 1 - P_{tr})^{k_i} \tag{2.39}$$

The unknown PGF of j_i may be written as,

$$J_i(z) = \sum_{k_i=0}^{W_i-1} J_i(z|k_i)\Pr(k_i) \tag{2.40}$$

Substituting from (2.35) and (2.39)

$$J_i(z) = \frac{1}{W_i}\sum_{k_i=0}^{W_i-1}(P_{tr}z + 1 - P_{tr})^{k_i} \tag{2.41}$$

$$J_i(z) = \frac{1}{W_i}K_i(z)\Big|_{z=\alpha=P_{tr}z+1-P_{tr}} \tag{2.42}$$

$$J_i(z) = \frac{1}{W_i}\frac{1-\alpha^{W_i}}{1-\alpha}\Big|_{\alpha=P_{tr}z+1-P_{tr}} \tag{2.43}$$

Substituting $J_i(z)$ and $K_i(z)$ into (2.33)

$$X_i(z) = \frac{1}{W_i}\frac{1-(P_{tr}z+1-P_{tr})^{W_i}}{1-(P_{tr}z+1-P_{tr})}\Bigg|_{z=R(z)}\frac{1}{W_i}\frac{1-z^{W_i}}{1-z} \tag{2.44}$$

$$X_i(z) = \frac{1}{W_i^2}\frac{1-(P_{tr}R(z)+1-P_{tr})^{W_i}}{1-(P_{tr}R(z)+1-P_{tr})}\frac{1-z^{W_i}}{1-z} \tag{2.45}$$

The probability of successful packet transmission is given by,

$$\begin{aligned} P_{sum} &= \sum_{i=0}^{m-1}(1-P_C)b_{1,i,0} + b_{1,m,0} \\ &= (1-P_C)\sum_{i=1}^{n}P_C{}^{i}b_{1,0,0} + P_C{}^{m}b_{1,0,0} \\ &= (1-P_C)\frac{1-P_C{}^{m}}{1-P_C}b_{1,0,0} + P_C{}^{m}b_{1,0,0} \\ &= b_{1,0,0} \end{aligned} \tag{2.46}$$

Finally, the PGF of the total packet service time in slots, S(z), may be expressed as follows,

$$S(z)=\frac{1}{P_{sum}}\{(1-P_C)[b_{1,0,0}S_0(z)+b_{1,1,0}S_1(z)+...+b_{1,m-1,0}S_{m-1}(z)]+b_{1,m,0}S_m(z)\}$$
$$=(1-P_C)\{X_0(z)R(z)+X_0(z)X_1(z)R^2(z)P_C+...+[\prod_{i=0}^{m-1}X_i(z)]R^m(z)P_C^{\ m-1}\}+P_C^{\ m}R^{m+1}(z)\prod_{i=0}^{m}X_i(z)$$
$$=(1-P_C)\sum_{j=1}^{m}\{[\prod_{i=0}^{j-1}X_i(z)]R^j(z)P_C^{\ j-1}\}+[\prod_{i=0}^{m}X_i(z)]R^{m+1}(z)P_C^{\ m}$$

(2.47)

Let us define,

$$\varphi_j(z)=\prod_{i=0}^{j-1}X_i(z) \qquad j\in(1,m+1) \tag{2.48}$$

$$Y_j(z)=[R(z)]^j \qquad j\in(1,m+1) \tag{2.49}$$

$$V_j(z)=\varphi_j(z)Y(z)P_C^{\ j-1} \qquad j\in(1,m+1) \tag{2.50}$$

Then S(z) in (2.47) can be expressed as,

$$S(z)=(1-P_C)\sum_{j=1}^{m}V_j(z)+V_{m+1}(z) \tag{2.51}$$

The mean service time is given by,

$$\bar{S}=S'(1)=(1-P_C)\sum_{j=1}^{m}V_j'(1)+V_{m+1}'(1) \tag{2.52}$$

where

$$V_j'(z)=[\varphi_j'(z)Y(z)+\varphi_j(z)Y'(z)]P_C^{\ j-1} \tag{2.53}$$

Next, we determine the unknown derivatives in the above equation. By inspection we obtain,

$$\varphi_j'(1) = \sum_{k=0}^{j-1} X_k'(1) \tag{2.54}$$

$$\varphi_j(1) = \prod_{k=0}^{j-1} X_k(1) = 1 \tag{2.55}$$

$$\begin{aligned} Y_j'(z)|_{z=1} &= j[R(z)]^{j-1} R'(z)\big|_{z=1} \\ &= jR'(1) \end{aligned} \tag{2.56}$$

Substituting z=1 into (2.53) and using (2.54-2.56),we obtain,

$$V_j'(1) = [\sum_{k=0}^{j-1} X_k'(1)]P_C^{\ j-1} + jR'(1)P_C^{\ j-1} \tag{2.57}$$

Next, we will determine the first moment of duration of the counter decrement operation in the i_{th} backoff stage, $X_i'(1)$.

Using (2.45) and multiplying the denominator with both sides of the equation, we have,

$$W_i^2(1-z)[1-(P_{tr}z+1-P_{tr})]X_i(z) = [1-(P_{tr}M(z)+1-P_{tr})^{W_i}][1-z^{W_i}] \tag{2.58}$$

Let us define the left and right hand side of the above equation,

$$LHS = A(z)B(z)C(z) \tag{2.59}$$

$$RHS = E(z)F(z) \tag{2.60}$$

Let $\quad A(z) = W_i^2(1-z)$, $B(z) = 1-D(z)$, $C(z) = X_i(z)$,

$D(z) = P_{tr}R(z)+1-P_{tr}$, $E(z) = 1-D(z)^{W_i}$, $F(z) = 1-z^{W_i}$.

Taking third order derivatives of *LHS* and *RHS* and substituting z=1, we have the following equations,

$$(LHS)'''|_{z=1} = \{6A'(z)B'(z)C'(z) + 3A'(z)B''(z)C(z)\}|_{z=1} \tag{2.61}$$

$$(RHS)'''|_{z=1} = \{3[E''(z)F'(z) + E'(z)F''(z)]\}|_{z=1} \tag{2.62}$$

where

$$A'(z)\Big|_{z=1} = -W_i^2$$

$$A''(z)\Big|_{z=1} = A'''(z)\Big|_{z=1} = A''''(z)\,|_{z=1} = 0$$

$$B^{(j)}(z)\,|_{z=1} = -P_{tr}R'(1)$$

$$D^{(j)}(z)\,|_{z=1} = P_{tr}R^{(j)}(1)$$

$$E'(z)\,|_{z=1} = -W_i P_{tr} R'(1)$$

$$E''(z)\,|_{z=1} = -W_i(W_i - 1)[P_{tr}R'(1)]^2 - W_i P_{tr} R''(1)$$

$$\begin{aligned} E'''(z)\,|_{z=1} = &-W_i(W_i-1)(W_i-2)[P_{tr}R'(1)]^3 - 2W_i(W_i-1)[P_{tr}R'(1)][P_{tr}R'(1)] \\ &-W_i(W_i-1)[P_{tr}R''(1)] - W_i P_{tr} R'''(1) \end{aligned}$$

$$F'(z)\,|_{z=1} = -W_i$$

$$F''(z)\,|_{z=1} = -W_i(W_i - 1)$$

$$F'''(z)\,|_{z=1} = -W_i(W_i - 1)(W_i - 2)$$

$$F''''(z)\,|_{z=1} = -W_i(W_i - 1)(W_i - 2)(W_i - 3)$$

Equating left and right hand side of (2.61) and (2.62), we have,

$$\begin{aligned} &6(-W_i^2)[-P_{tr}R'(1)]X_i'(1) - 3W_i^2[(P_{tr}R'(1))^2 + P_{tr}R''(1)] \\ &= 3\{[-W_i(W_i-1)[P_{tr}R'(1)]^2 - W_iP_{tr}R''(1)](-W_i) + [-W_iP_{tr}R'(1)][-W_i(W_i-1)]\} \end{aligned} \tag{2.63}$$

Then we have,

$$X_i'(1) = \frac{1}{2}(W_i - 1)[P_{tr}R'(1) + 1] \tag{2.64}$$

This completes the derivation of mean packet service time which is given by equation (2.52) with (2.57) and (2.64).

Next, we express the probability that a STA is busy as,

$$\rho = \lambda \overline{S} \tag{2.65}$$

Finally, using equations (2.4),(2.5),(2.65) and (2.52), we have three equations and three unknown variables ($P_C(P_{tr}), \rho, \overline{S}$).

$$P_{tr} = P_C = 1-(1-\tau)^{n-1}$$

$$\rho = \lambda \overline{S}$$

$$\overline{S} = (1-P_C)\sum_{j=1}^{m} V_j\,'(1) + V_{m+1}\,'(1)$$

The above equation set may be solved numerically with MATLAB optimization toolbox. We note that variable ρ should be in the range (0,1). Since the non-linear equation set is governed by the Markov chain and the Markov chain can be uniquely solved by using **B=BP,** a unique solution exists for the non-linear equation set.

2.7 Mean Packet Delay

In this section, we derive the mean packet delay in the MAC layer. It is defined as the duration from the time when the MAC buffer receives the data packet to the time when the data packet is successfully transmitted. Since we model each node as an M/G/1 queue, the mean packet delay maybe determined from the M/G/1 queuing result [10].

$$\overline{D} = \overline{S} + \frac{\lambda \overline{S^2}}{2(1-\rho)} \tag{2.66}$$

As may be seen, we need to determine the second moment of packet service time in order to calculate mean packet delay.

As before, we let,

$$A(z) = W_i^2(1-z), B(z) = 1 - D(z),\ C(z) = X_i(z),$$

$$D(z) = P_{tr}R(z) + 1 - P_{tr},\ E(z) = 1 - D(z)^{W_i},\ F(z) = 1 - z^{W_i}.$$

Taking fourth order derivatives of both sides of the equation (2.58) and substituting z=1, we have,

$$\begin{aligned}&\{6A'(z)B'(z)C''(z) + 12A'(z)B''(z)C'(z) + 4A'(z)B'''(z)C(z)\}\big|_{z=1}\\ &= \{4E'''(z)F'(z) + 6E''(z)F''(z) + 4E'(z)F'''(z)\}\big|_{z=1}\end{aligned} \tag{2.67}$$

Then we obtain,

$$\begin{aligned}X_i''(1) &= \frac{2}{3}(W_i-1)(W_i-2)P_{tr}R'(1) + \frac{4}{3}(W_i-1)P_{tr}R''(1) + (W_i-1)^2 P_{tr}R'(1)\\ &+ \frac{2}{3}(W_i-1)(W_i-2) - \frac{R''(1)}{R'(1)}[(P_{tr}R'(1) - \frac{2}{3})(W_i-1)]\end{aligned} \tag{2.68}$$

Taking second order derivatives of (2.48) and (2.50) and substituting z=1, we obtain,

$$\varphi_j''(1) = \varphi_{j-1}''(1) + 2\varphi_{j-1}'(1)X_{j-1}'(1) + X_{j-1}''(1) \qquad j \in (1, m+1) \tag{2.69}$$

$$V_j''(1) = [\varphi_j''(1) + 2\varphi_j'(1)Y_j'(1) + Y_j''(1)]P_C^{\,j-1} \qquad j \in (1, m+1) \tag{2.70}$$

where

$$Y_j''(1) = jR''(1) + j(j-1)[R'(1)]^2 \tag{2.71}$$

The second moment of the total packet service time can be expressed as,

$$\overline{S^2} = S''(z)\big|_{z=1} + S'(z)\big|_{z=1} \tag{2.72}$$

where

$$\begin{aligned}&S''(z)\big|_{z=1}\\ &= (1-P_C)\sum_{j=1}^{m} V_j''(1) + V_{m+1}''(1)\\ &= (1-P_C)\sum_{j=1}^{m}[\varphi_j''(1) + 2\varphi_j'(1)Y_j'(1) + Y_j''(1)]P_C^{\,j-1} + [\varphi_{m+1}''(1) + 2\varphi_{m+1}'(1)Y_j'(1) + Y_j''(1)]P_C^{\,m}\end{aligned} \tag{2.73}$$

This completes the derivation of mean packet delay which is given by equation (2.66) with (2.72) and (2.52).

2.8 Numerical and Simulation Results

In this section, we present some numerical results for the analysis of this chapter and simulation results to verify them. As explained in the first chapter, IEEE 802.11 standard defines four physical layer specifications, Frequency Hopping Spread Spectrum (FHSS), Direct Sequence Spread Spectrum (DSSS), Infra-Red (IR) and Orthogonal Frequency Division Multiplexing (OFDM). We choose IEEE 802.11b DSSS as the physical layer for determining the numerical results. Some of the characteristics of DSSS are shown in Table 1. IEEE 802.11b DSSS can offer transmission rates of 1, 2, 5.5 and 11Mbps. The slot duration in DSSS is 20 μs. The minimum contention window size is 31.

Characteristics	**Value**
Transmission rate	1Mbps,2Mbps,5.5Mbps,11Mbps
Slot duration	20 μs
CW_{min}	31
CW_{max}	1023

Table 2.1 IEEE 802.11b DSSS PHY and MAC layer characteristics

Using (2.4),(2.5),(2.65) and (2.52), we have three equations and three unknown variables, $P_C(P_{tr}), \rho, \overline{S}$. We solve these equations simultaneously with MATLAB optimization toolbox. We note that variable ρ should be in the range (0,1). Since the non-linear equation set is governed by the Markov chain and the Markov chain can be uniquely solved by using **B=BP,** a unique solution exists for the non-linear equation set.

In the following section, we report our numerical results for different network situations. We plot the mean packet service time, mean delay in slots and collision probability as functions of packet arrival rate. Numerical results are obtained for different packet arrival rates from 0 to the saturation point. For simplicity, we assume that the packet sizes are constant and PHY and MAC layer overheads are included in the packet sizes. We vary the packet size from 15 to 170slots. For 1Mbps transmission rate, 60 slots equal to 1200bits and 170 slots equal to 3400bits. For 11Mbps transmission rate, 15 slots equal to 3300bits. We also change the number of STAs in the system from 5 to 20 and the minimum contention window size CW_{min} from 31 to 127 slots.

In order to prove the accuracy of the model, we have designed a discrete event C++ test bed that simulates the 802.11 DCF under homogeneous condition. The numerical results are verified by extensive simulations in different network situations.

2.8.1 Example 1

Number of STAs	PHY TX. Rate	Packet Size	CW_{min}	Max Stage
n=5	1 Mbps	r=60	W_0=31	m=5

Table 2.2 Network parameters for homogeneous example 1

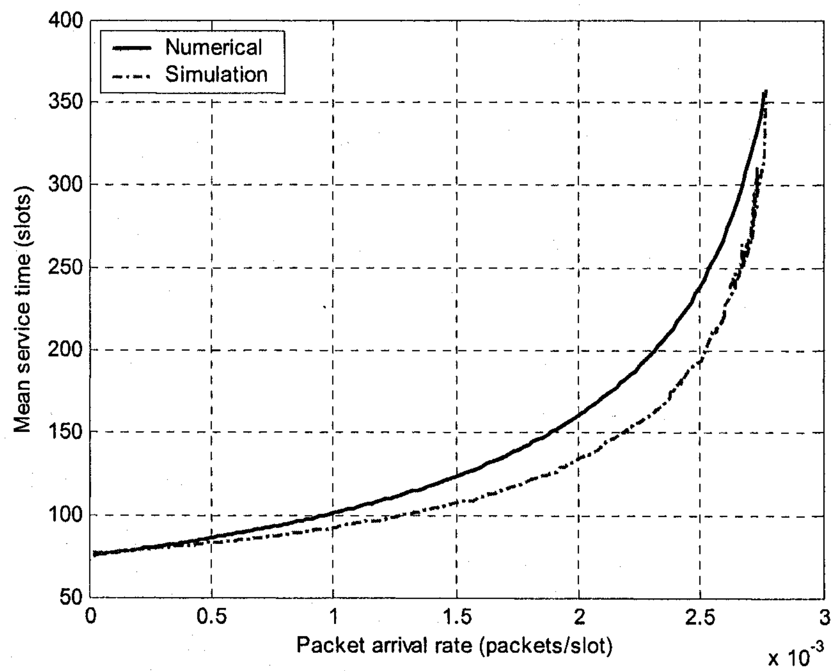

Figure 2.2 Mean service time (n=5, Packet size=60slots, W_0=31)

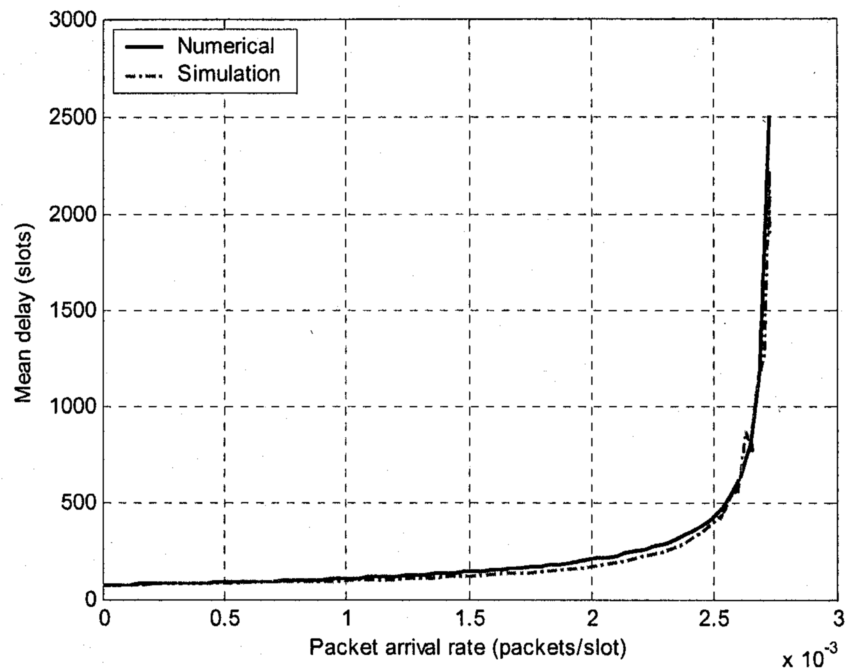

Figure 2.3 Mean delay (n=5, Packet size=60slots, W_0 =31)

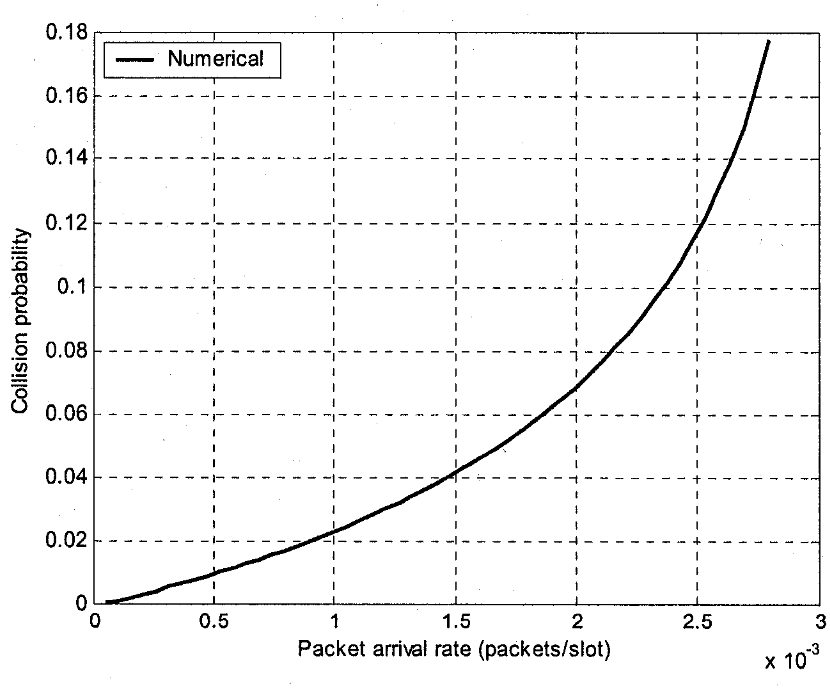

Figure 2.4 Collision probability (n=5, Packet size=60slots, W_0 =31)

2.8.2 Example 2

Number of STAs	PHY TX. Rate	Packet Size	CW_{min}	Max Stage
n =5	1 Mbps	r =60	W_0 =127	m =5

Table 2.3 Network parameters for homogeneous example 2

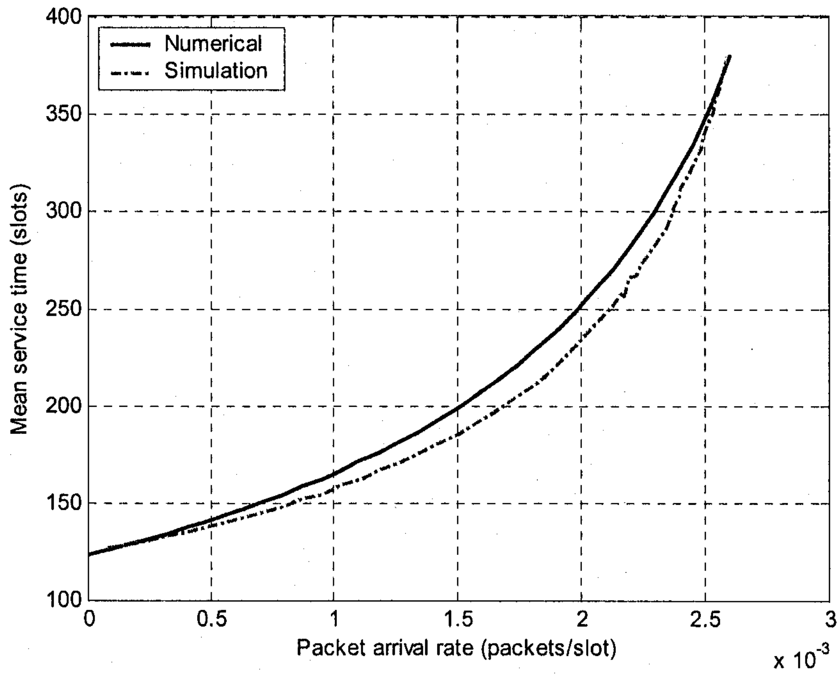

Figure 2.5 Mean service time (n=5, Packet size=60slots, W_0 =127)

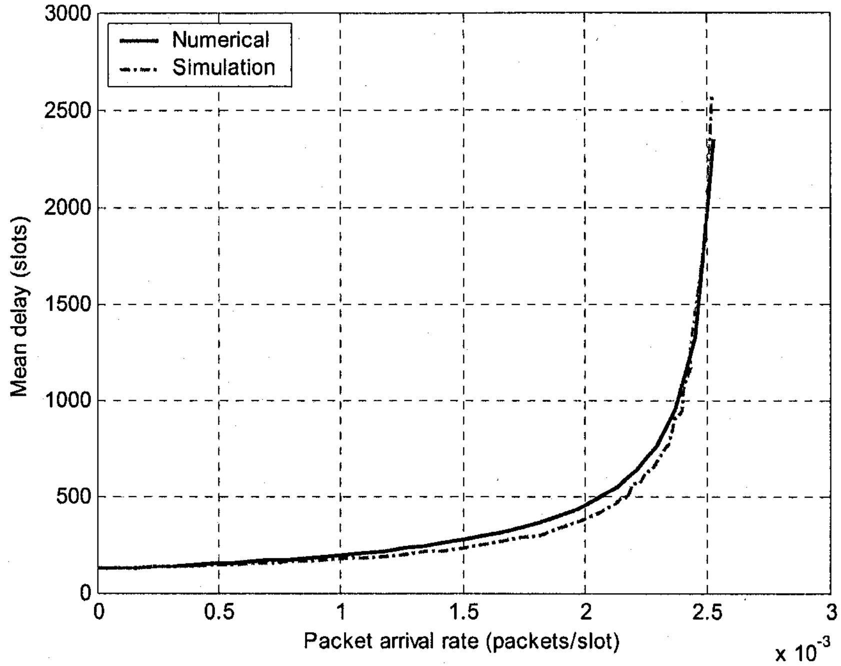

Figure 2.6 Mean delay (n=5, Packet size=60slots, W_0 =127)

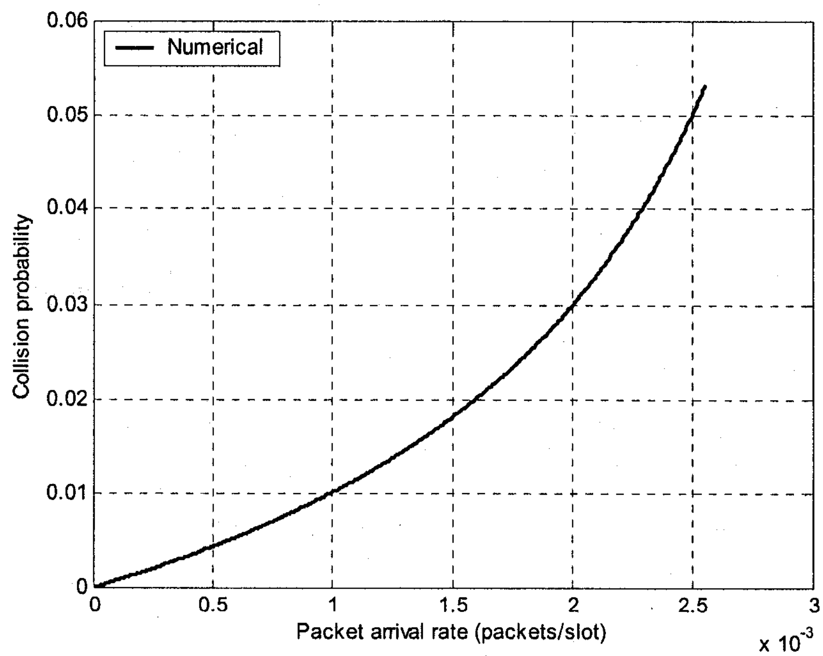

Figure 2.7 Collision probability (n=5, Packet size=60slots, W_0 =127)

2.8.3 Example 3

Number of STAs	PHY TX. Rate	Packet Size	CW_{min}	Max Stage
$n=5$	1 Mbps	$r=170$	$W_0=31$	$m=5$

Table 2.4 Network parameters for homogeneous example 3

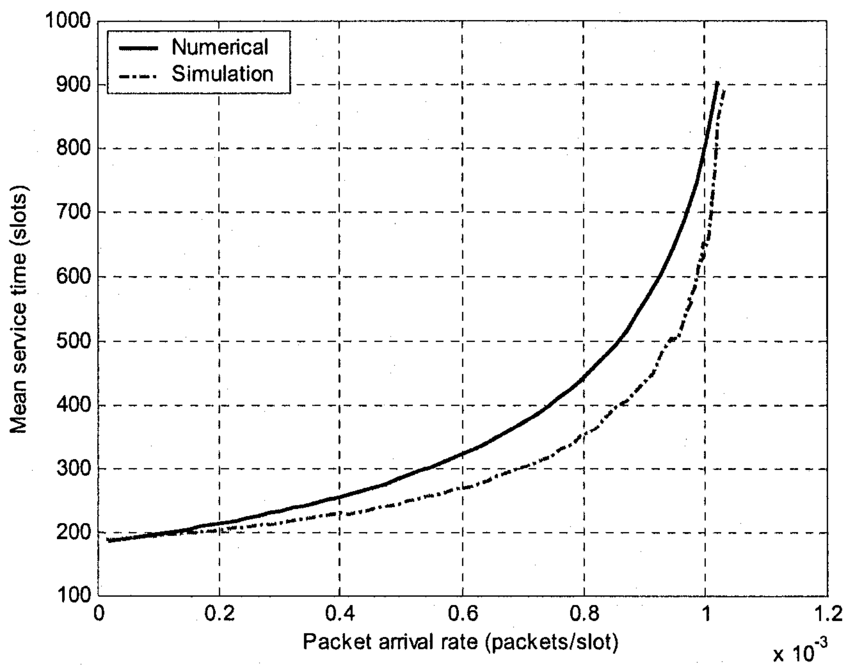

Figure 2.8 Mean service time (n=5, Packet size=170slots, W_0=31)

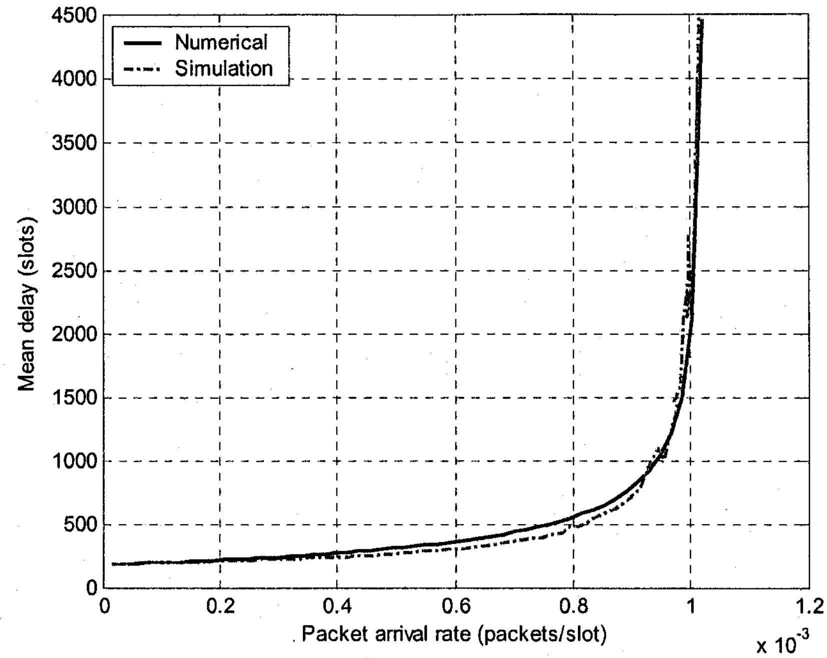

Figure 2.9 Mean delay (n=5, Packet size=170slots, W_0 =31)

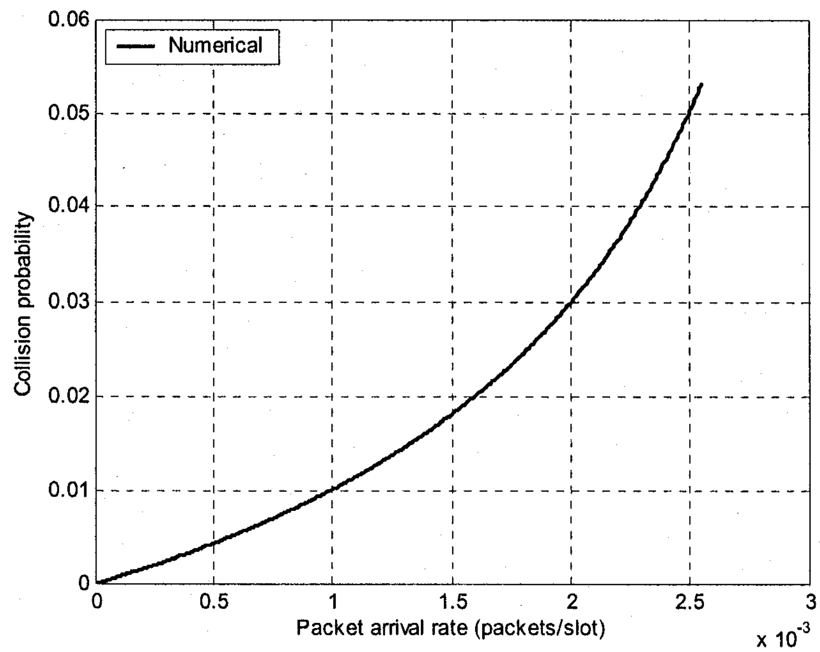

Figure 2.10 Collision probability (n=5, Packet size=170slots, W_0 =31)

2.8.4 Example 4

Number of STAs	PHY TX. Rate	Packet Size	CW_{min}	Max Stage
$n=10$	1 Mbps	$r=60$	$W_0=31$	$m=5$

Table 2.5 Network parameters for homogeneous example 4

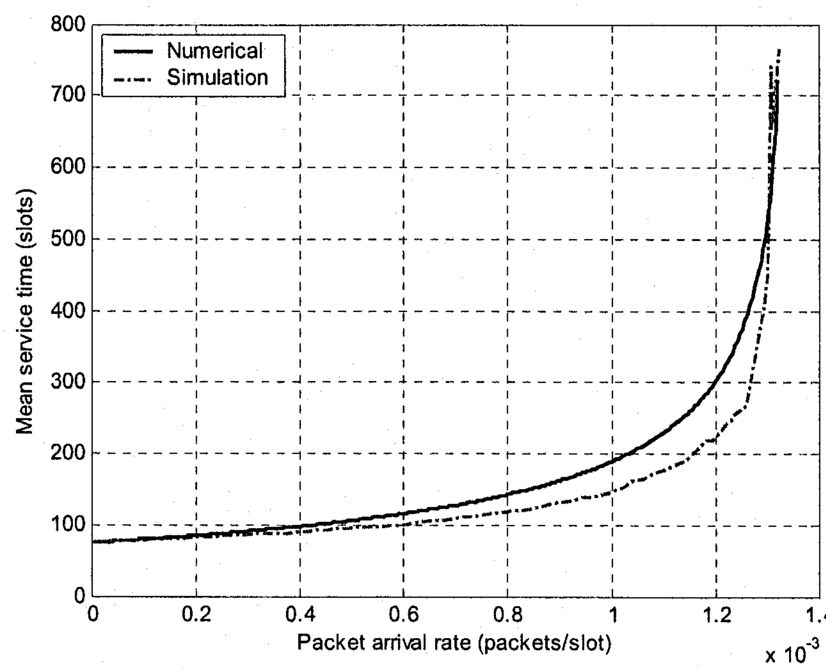

Figure 2.11 Mean service time (n=10, Packet size=60slots, W_0=31)

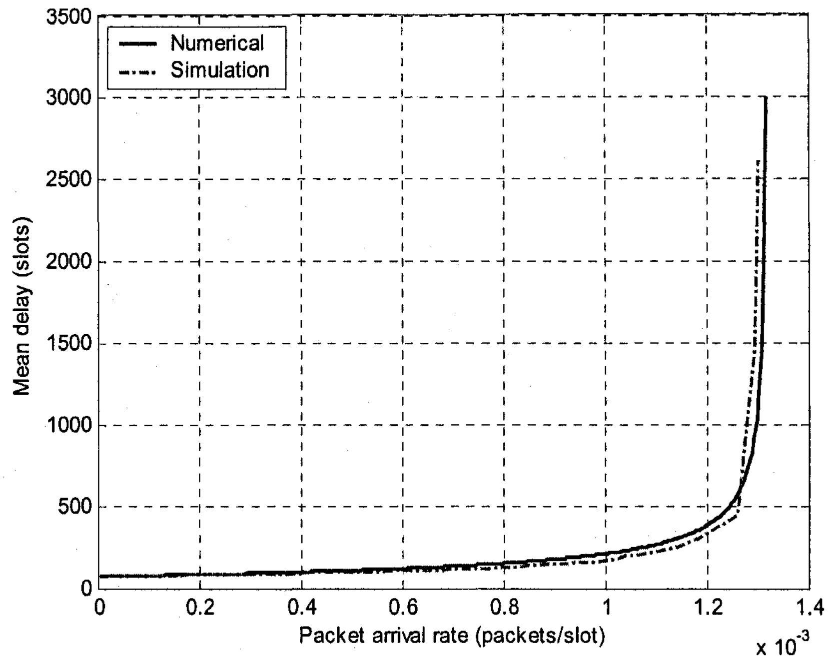

Figure 2.12 Mean delay (n=10, Packet size=60slots, W_0 =31)

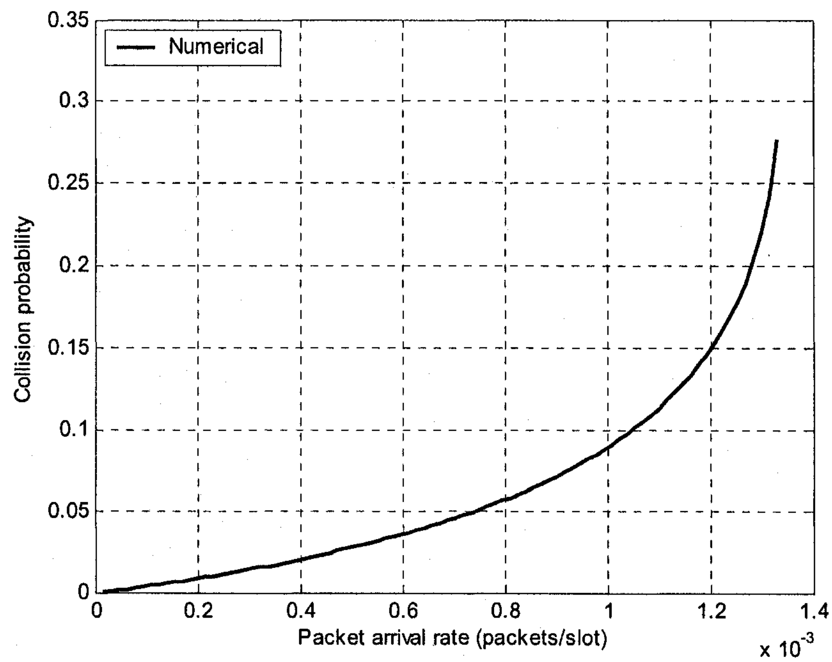

Figure 2.13 Collision probability (n=10, Packet size=60slots, W_0 =31)

2.8.5 Example 5

Number of STAs	PHY TX. Rate	Packet Size	CW_{min}	Max Stage
n=5	11 Mbps	r=15	W_0=31	m=5

Table 2.6 Network parameters for homogeneous example 5

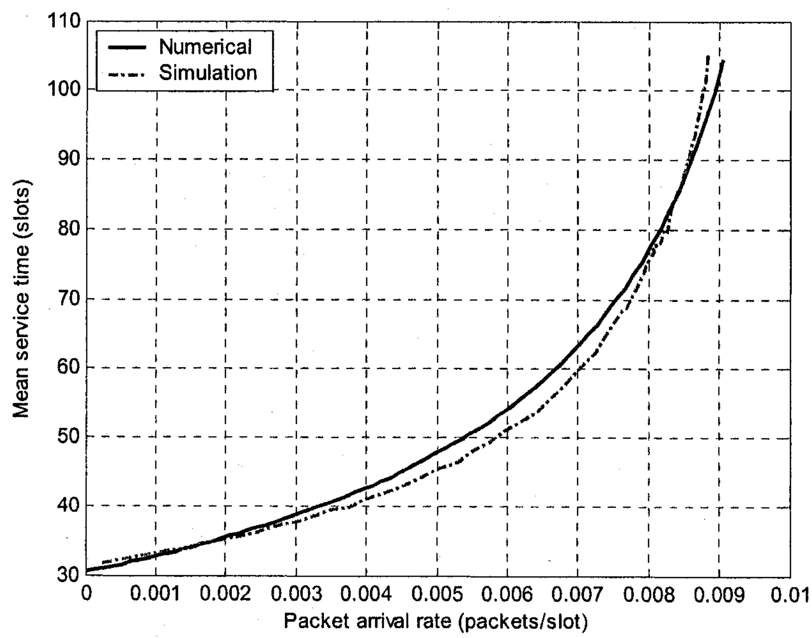

Figure 2.14 Mean service time (n=5, Packet size=15slots, W_0=31)

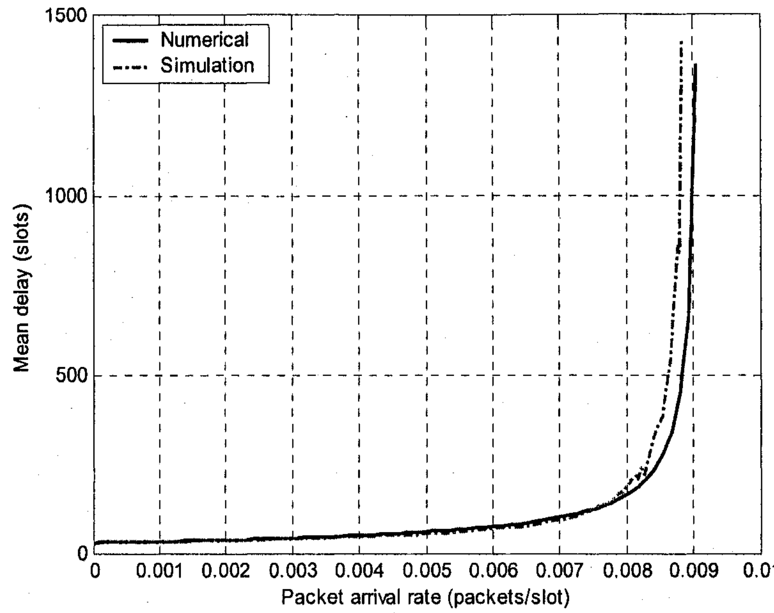

Figure 2.15 Mean delay (n=5, Packet size=15slots, W_0 =31)

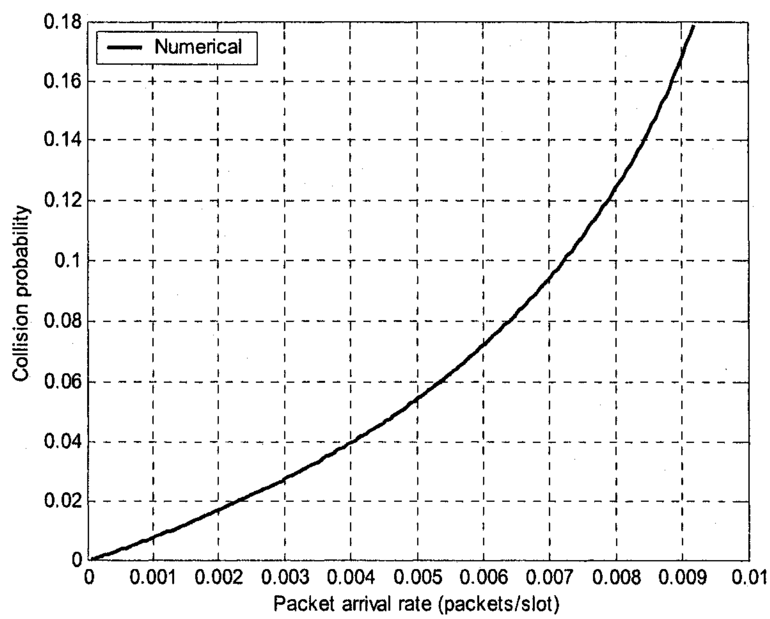

Figure 2.16 Collision probability (n=5, Packet size=15slots, W_0 =31)

2.8.6 Example 6

Number of STAs	PHY TX. Rate	Packet Size	CW_{min}	Max Stage
n =20	1 Mbps	r =60	W_0 =31	m =5

Table 2.7 Network parameters for homogeneous example 6

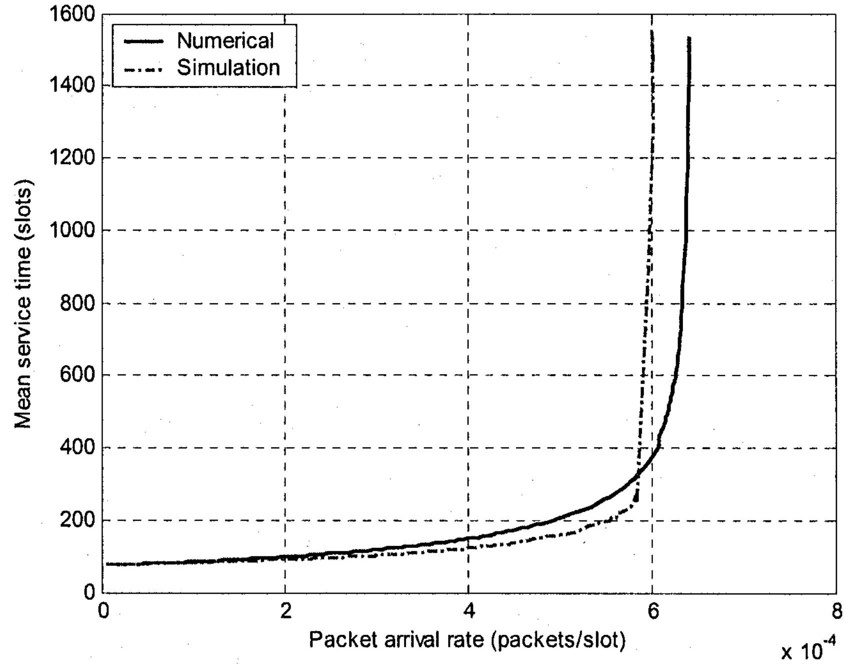

Figure 2.17 Mean service time (n=20, Packet size=60slots, W_0 =31)

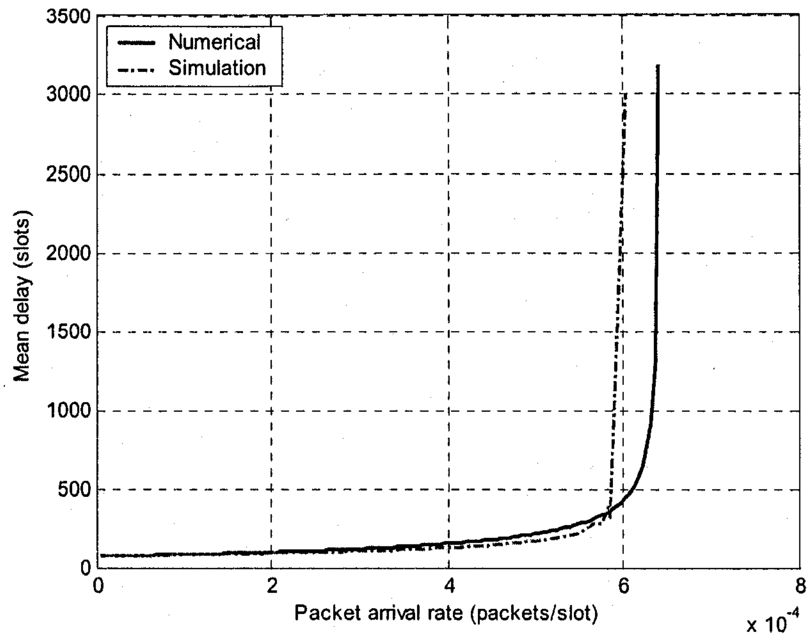

Figure 2.18 Mean delay (n=20, Packet size=60slots, W_0=31)

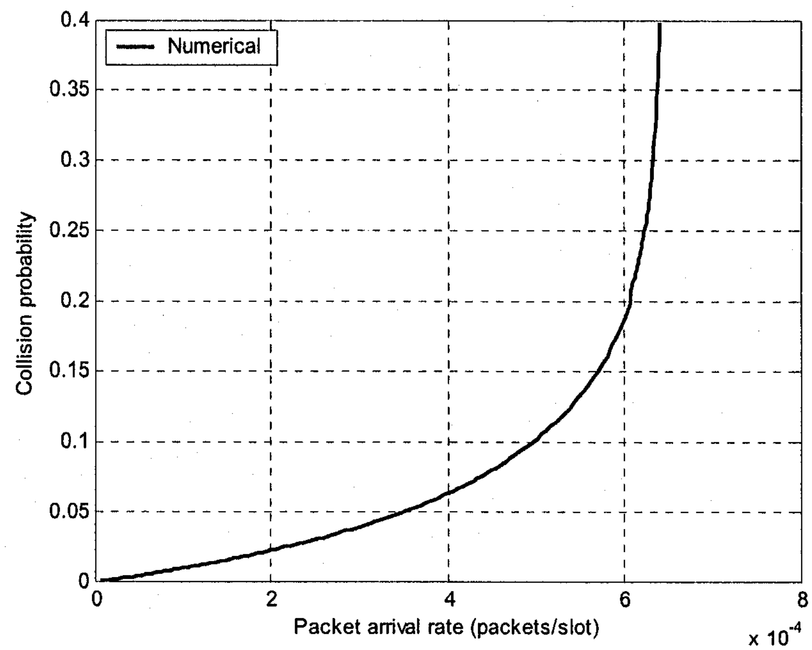

Figure 2.19 Collision probability (n=20, Packet size=60slots, W_0=31)

2.8.7 Conclusion

As may be seen from the above figures, the numerical results have a good match with the simulation results in most of the cases. However, due to the approximations in our analysis, there are some mismatches in some of the cases. Because 802.11 DCF is a contention based MAC protocol regulating the channel access, the probability of packet collision increases as the offered traffic load gets heavier. Ultimately this results in the increase of packet service time and delay. Although we assume fixed packets sizes when comparing the numerical and simulation results, our model also applies to arbitrary packet size distributions.

In examples 1,4 and 6, we observe the effect of number of STAs in the system on the delay performance. We change the number of STAs in the system from 5 to 10 and 20. We can see from the figures, as the number of STAs in the system increases, the STA saturates more quickly and the mean packet service time, mean delay and collision probability also increase. A system of 5 STAs saturates at almost half of the load of a system of 10 STAs, as expected. Similarly, a system of 10 STAs saturates at almost half of the load of a system of 20 STAs. Also, as the number of STAs increases, the curve of packet mean service time tends to turn more sharply near the saturation point.

In examples 1 and 2, we change the minimum contention window size CW_{min} from 31 to 127. We notice that the increase of the value of CW_{min} increases the mean packet service time and mean delay. However, the collision probability decreases as CW_{min} increases. This is because packet transmission probability τ in any slot is inversely proportional to CW_{min}. The value of CW_{min} is especially important for delay sensitive applications or real time applications, such as IP telephony and video conferencing. CW_{min} is later defined as a configurable parameter in the 802.11e EDCA for providing QoS support.

In examples 1, 3 and 5, we change the packet size from 15 to 60 and 170 slots. As may be seen, increase of the packet size results in a significant increase of packet mean service time. It implies that packet size is a very important factor affecting the packet service time and delay. It also implies a major drawback of the basic DCF access mode: the collision time and backoff counter freezing time are proportional to the size of the transmitted data packet. As the packet size increases, the collision time of each unsuccessful transmission and backoff counter freezing time due to busy medium also increase. One partial solution of this problem is to use RTS/CTS access mode of the DCF protocol. The RTS/CTS access mode can reserve the channel for the time needed to transfer the data packet, thus packet collision can only happen on RTS frames. Since RTS frames are much shorter than normal data frames, it has significantly reduced time spent in packet collisions. The RTS/CTS access mode is very effective in terms of system performance, particularly for large data packets, as it reduces the packet collision time. Therefore, the RTS/CTS access mode should be employed when packet size exceeds a certain threshold.

CHAPTER 3

PERFORMANCE MODELING OF IEEE 802.11 DCF WITH HETEROGENEOUS SOURCES

The analysis in the previous chapter has assumed homogeneous STAs. Thus, all STAs are identical which have the same packet arrival rate, transmission rate and minimum backoff window size. However, the 802.11b standard supports multirate adaptation and STAs are allowed to choose from transmission rates 1, 2, 5.5 and 11Mbps. Assuming the packet sizes are of identical length in bits, the packet transmission time in slots of higher speed STA is shorter. This underlies a fundamental unfairness among the STAs with different rates in the 802.11 network. As discussed in [18], in the multirate environment, saturation throughput of any STA in the system is equal to saturation throughput of the STA with the lowest transmission rate. This has significantly downgraded the system performance. In this chapter, we extend the previous performance model to determine the packet service time distribution and mean delay of STAs with different packet arrival rates, transmission rates and contention window sizes.

3.1 Model Assumptions

Although our model can be generalized to a larger number of groups of STAs of different transmission rates, for computational simplicity, we extend our model to only two groups of STAs with transmission rates S(Slow) and F(Fast). We model each type of STA as an M/G/1 queue and packet service time as a three dimensional Markov chain as in the previous chapter.

Let n_s and n_f denote the number of slow and fast STAs in the system respectively. Define τ_S and τ_f as the probabilities that a STA transmits in an arbitrary time slot in the case of slow and fast STAs respectively. Similar to homogeneous case analysis, we define the following probabilities,

$$\begin{aligned} P_{cs} &= \text{Pr(a slow STA's transmission experiences a collision)} \\ &= 1-(1-\tau_s)^{n_s-1}(1-\tau_f)^{n_f} \end{aligned} \tag{3.1}$$

$$\begin{aligned} P_{cf} &= \text{Pr(a fast STA's transmission experiences a collision)} \\ &= 1-(1-\tau_s)^{n_s}(1-\tau_f)^{n_f-1} \end{aligned} \tag{3.2}$$

$$\begin{aligned} P_{trs1} &= \text{Pr(at least one transmission from the slow group during a slot} \\ &\qquad \text{that a slow STA is in a backoff stage)} \\ &= 1-(1-\tau_s)^{n_s-1} \end{aligned} \tag{3.3}$$

$$\begin{aligned} P_{trf1} &= \text{Pr(at least one transmission from the fast group during a slot} \\ &\qquad \text{that a fast STA is in a backoff stage)} \\ &= 1-(1-\tau_f)^{n_f-1} \end{aligned} \tag{3.4}$$

$$\begin{aligned} P_{trs2} &= \text{Pr(at least one transmission from the slow group in any slot)} \\ &= 1-(1-\tau_s)^{n_s} \end{aligned} \tag{3.5}$$

$$P_{trf2} = \Pr(\text{at least one transmission from the fast group in any slot}) \\ = 1-(1-\tau_f)^{n_f} \tag{3.6}$$

The probabilities that queues of slow and fast STAs are not empty are, respectively, given by,

$$\Pr(q_s = 1) = \rho_s \tag{3.7}$$

$$\Pr(q_f = 1) = \rho_f \tag{3.8}$$

From the equations (2.7) and (2.9), we now express the probability τ_S and τ_f that a STA transmits in an arbitrary time slot in the case of slow and fast STAs respectively as,

$$\tau_s = \frac{\rho_s}{\overline{W_S}} \\ = \frac{\rho_S}{\frac{1-P_{CS}-P_{CS}(2P_{CS})^m}{1-2P_{CS}}\frac{W_{S0}}{2}} \tag{3.9}$$

and

$$\tau_f = \frac{\rho_f}{\overline{W_f}} \\ = \frac{\rho_f}{\frac{1-P_{Cf}-P_{Cf}(2P_{Cf})^m}{1-2P_{Cf}}\frac{W_{f0}}{2}} \tag{3.10}$$

where $\overline{W_S}$ and $\overline{W_f}$ are average backoff period and W_{S0} and W_{f0} are the minimum contention window sizes in the case of slow and fast STAs respectively.

3.2 Packet Service Time Distribution of a Slow STA

In this section, we derive the PGF of the packet service time distribution of a slow STA. A packet may involve in several retransmissions due to collisions. Each attempt consists of a choice of backoff counter value and retransmission. Count down process may be interrupted due to transmissions by other STAs. Duration of an interruption depends whether interruption is due to only slow, fast or mix of two types of STAs. Thus, the number of slots that a STA spends in the i_{th} backoff stage due to counter freezing is given by sum of the random number of counter freezing times where duration of each freezing time depends on its source.

Let us define the following random variables,

r_s packet size of a slow STA in slots

r_f packet size of a fast STA in slots

k_{si} backoff counter value of a slow STA in the i_{th} backoff stage

j_{si} number of interruptions due to only slow STAs transmitting in the i_{th} backoff stage

j_{fi} number of interruptions due to only fast STAs transmitting in the i_{th} backoff stage.

j_{sfi} number of interruptions due to both slow and fast STAs transmitting in the i_{th} backoff stage

j_i number of interruptions in the i_{th} backoff stage

x_{si} duration of count down process of a slow STA in the i_{th} backoff stage

S_{si} duration of packet service time of a slow STA given that there is an i_{th} retransmission attempt

S_s packet service time of a slow STA in slots

We make the assumption that $r_s > r_f$, therefore, if interruption is due to transmissions of only fast STAs then duration of interruption is r_f, otherwise it is r_s.

Also defining the following PGFs of these random variables,

$$R_s(z) = E[z^{r_s}], R_f(z) = E[z^{r_f}], K_{si}(z) = E[z^{k_{si}}], J_i(z) = E[z^{j_i}], J_{si}(z) = E[z^{j_{si}}]$$

$$J_{fi}(z) = E[z^{j_{fi}}], J_{sfi}(z) = E[z^{j_{sfi}}], X_{si}(z) = E[z^{x_{si}}], S_{si}(z) = E[z^{S_{si}}], S_s(z) = E[z^{S_s}]$$

Then we have the following expressions for the service time of a slow STA,

$$x_{si} = k_{si} + \sum_{\ell=1}^{j_{si}} r_s + \sum_{\ell=1}^{j_{fi}} r_f + \sum_{\ell=1}^{j_{sfi}} r_s \tag{3.11}$$

and

$$S_{s(i+1)} = S_{si} + x_{si} + r_s \tag{3.12}$$

Let $J_i(z_s, z_f, z_{sf})$ denote the joint distribution of $(j_{si}, j_{fi}, j_{sfi})$.

From equations (3.11) and (3.12), we have,

$$X_{si}(z) = J_i(z_s, z_f, z_{sf})\Big|_{z_s = R_s(z), z_f = R_f(z), z_{sf} = R_s(z)} K_{si}(z) \tag{3.13}$$

$$S_{s(i+1)}(z) = S_{si}(z) X_{si}(z) R_s(z) \qquad i \in (0, m-1) \tag{3.14}$$

where $S_{s0}(z) = X_{s0}(z) R_s(z)$

Next, we determine the unknown PGFs, $J_i(z_s, z_f, z_{sf})$ and $K_{si}(z)$.

Probability distribution of k_{si} follows a uniform distribution,

$$\Pr(k_{si}) = \frac{1}{W_{si}} \quad (3.15)$$

Since the PGF of k_{si} has been defined above as,

$$K_{si}(z) = E[z^{k_{si}}] \quad (3.16)$$

Then we have,

$$\begin{aligned} K_{si}(z) &= \sum_{k_{si}=0}^{W_{si}-1} z^{k_{si}} \Pr(k_{si}) = \frac{1}{W_{si}} \sum_{k_{si}=0}^{W_{si}-1} z^{k_{si}} \\ &= \frac{1}{W_{si}} \frac{1-z^{W_{si}}}{1-z} \end{aligned} \quad (3.17)$$

We assume that three types of interruptions of the count down process occur independently with following probabilities in any slot,

Pr(counter interruption due to transmissions by only slow STAs)= $P_{trs1}(1-P_{trf2})$

Pr(counter interruption due to transmissions by only fast STAs)= $P_{trf2}(1-P_{trs1})$

Pr(counter interruption due to transmissions by both types of STAs)= $P_{trf2}P_{trs1}$

Thus the probability distribution of the number of interruptions during i_{th} stage is given by the multinomial distribution.

$$\begin{aligned} &\Pr(j_{si}, j_{fi}, j_{sfi} \mid k_{si}) \\ &= \frac{k_{si}!}{j_{si}!\, j_{fi}!\, j_{sfi}!\,(k_i - j_{si} - j_{fi} - j_{sfi})!} [P_{trs1}(1-P_{trf2})]^{j_{si}} [P_{trf2}(1-P_{trs1})]^{j_{fi}} \\ &\times [P_{trs1}P_{trf2}]^{j_{sfi}} [1 - P_{trs1}(1-P_{trf2}) - P_{trf2}(1-P_{trs1}) - P_{trs1}P_{trf2}]^{k_{si}-j_{si}-j_{fi}-j_{sfi}} \end{aligned} \quad (3.18)$$

$J_i(z_s, z_f, z_{sf} \mid k_{si})$, the conditional joint distribution of $(j_{si}, j_{fi}, j_{sfi})$, is given by,

$$\begin{aligned} &J_i(z_s, z_f, z_{sf} \mid k_{si}) \\ &= [P_{trs1}(1-P_{trf2})z_f + P_{trf2}(1-P_{trs1})z_s + P_{trs1}P_{trf2}z_{sf} + 1 - P_{trs1}(1-P_{trf2}) - P_{trf2}(1-P_{trs1}) - P_{trs1}P_{trf2}]^{k_{si}} \end{aligned} \quad (3.19)$$

The unknown PGF of j_i may be written as,

$$J_i(z_s, z_f, z_{sf}) = \sum_{k_{si}=0}^{W_{si}-1} J_i(z_s, z_f, z_{sf} \mid k_{si}) \Pr(k_{si}) \tag{3.20}$$

Substituting from (3.15) and (3.19), we have,

$$J_i(z_s, z_f, z_{sf}) \tag{3.21}$$

$$= \frac{1}{W_{si}} \sum_{k_i=0}^{W_{si}-1} [P_{trs1}(1-P_{trf2})z_f + P_{trf2}(1-P_{trs1})z_s + P_{trs1}P_{trf2}z_{sf} + 1 - P_{trs1}(1-P_{trf2}) - P_{trf2}(1-P_{trs1}) - P_{trs1}P_{trf2}]^{k_i}$$

$$J_i(z_s, z_f, z_{sf}) = \frac{1}{W_{si}} K_{si}(z) \Big|_{z=P_{trs1}(1-P_{trf2})z_f + P_{trf2}(1-P_{trs1})z_s + P_{trs1}P_{trf2}z_{sf} + 1 - P_{trs1}(1-P_{trf2}) - P_{trf2}(1-P_{trs1}) - P_{trs1}P_{trf2}} \tag{3.22}$$

$$J_i(z_s, z_f, z_{sf}) = \frac{1}{W_{si}} \frac{1-\alpha^{W_{si}}}{1-\alpha} \Big|_{\alpha=P_{trs1}(1-P_{trf2})z_f + P_{trf2}(1-P_{trs1})z_s + P_{trs1}P_{trf2}z_{sf} + 1 - P_{trs1}(1-P_{trf2}) - P_{trf2}(1-P_{trs1}) - P_{trs1}P_{trf2}} \tag{3.23}$$

Substituting $J_i(z_s, z_f, z_{sf})$ and $K_{si}(z)$ into (3.13), we have,

$$X_{si}(z) = \frac{1}{W_{si}} \frac{1-z^{W_{si}}}{1-z} \frac{1-\alpha^{W_{si}}}{1-\alpha} \Big|_{\alpha=P_{trf2}(1-P_{trs1})R_f(z) + P_{trs1}R_s(z) + 1 - P_{trf2}(1-P_{trs1}) - P_{trs1}} \tag{3.24}$$

The probability of successful packet transmission is given by,

$$\begin{aligned} P_{sum} &= \sum_{i=0}^{m-1} (1-P_{CS}) b_{s(1,i,0)} + b_{s(1,m,0)} \\ &= (1-P_{CS}) \sum_{i=1}^{n} P_{CS}{}^{j} b_{s(1,0,0)} + P_{CS}{}^{m} b_{s(1,0,0)} \\ &= (1-P_{CS}) \frac{1-P_{CS}{}^{m}}{1-P_{CS}} b_{s(1,0,0)} + P_{CS}{}^{m} b_{s(1,0,0)} \\ &= b_{s(1,0,0)} \end{aligned} \tag{3.25}$$

The PGF of the packet service time of a slow STA, $S_s(z)$, may be expressed as follows,

$$S_s(z) = \frac{1}{P_{SUM}}\{(1-P_{CS})[b_{s(1,0,0)}S_{s0}(z) + b_{s(1,1,0)}S_{s1}(z) + \ldots + b_{s(1,m-1,0)}S_{s(m-1)}(z)] + b_{s(1,m,0)}S_{sm}(z)\}$$
$$= (1-P_{CS})\{X_{s0}(z)R_s(z) + X_{s0}(z)X_{s1}(z)R_s^2(z)P_{CS} + \ldots + [\prod_{i=0}^{m-1} X_{si}(z)]R_s^m(z)P_{CS}^{m-1}\} + P_{CS}^m R_s^{m+1}(z)\prod_{i=0}^{m} X_{si}(z)$$
$$= (1-P_{CS})\sum_{j=1}^{m}\{[\prod_{i=0}^{j-1} X_{si}(z)]R_s^j(z)P_{CS}^{j-1}\} + [\prod_{i=0}^{m} X_{si}(z)]R_s^{m+1}(z)P_{CS}^m$$

(3.26)

Let us define,

$$\varphi_{sj}(z) = \prod_{i=0}^{j-1} X_{si}(z) \qquad j \in (1, m+1) \qquad (3.27)$$

$$Y_{sj}(z) = [R_s(z)]^j \qquad j \in (1, m+1) \qquad (3.28)$$

$$V_{sj}(z) = \varphi_{sj}(z)Y_{sj}(z)P_{CS}^{j-1} \qquad j \in (1, m+1) \qquad (3.29)$$

Thus $S_s(z)$ may be expressed as,

$$S_s(z) = (1-P_{CS})\sum_{j=1}^{m} V_{sj}(z) + V_{s(m+1)}(z) \qquad (3.30)$$

The mean packet service time can now be computed as $S_s'(z)|_{z=1}$.

$$\overline{S}_s = S_s'(1) = (1-P_{CS})\sum_{j=1}^{m} V_{sj}'(1) + V_{s(m+1)}'(1) \qquad (3.31)$$

where

$$V_{sj}'(z) = [\varphi_{sj}'(z)Y_{sj}(z) + \varphi_{sj}(z)Y_{sj}'(z)]P_{CS}^{j-1} \qquad (3.32)$$

Next, we determine the unknown derivatives in the above equation.

From (3.27-3.28), we obtain,

$$\varphi_{sj}'(1) = \sum_{k=0}^{j-1} X_{sk}'(1) \qquad (3.33)$$

$$\varphi_{sj}(1)=\prod_{k=0}^{j-1}X_{sk}(z)=1 \tag{3.34}$$

$$\begin{aligned}Y_{sj}'(1)&=j[R_s(z)]^{j-1}R_s'(z)\big|_{z=1}\\&=jR_s'(1)\end{aligned} \tag{3.35}$$

Substituting z=1 into (3.32) and using (3.33-3.35), we have,

$$V_{sj}'(1)=[\sum_{k=0}^{j-1}X_{sk}'(1)]P_{CS}^{\ j-1}+jR_s'(1)P_{CS}^{\ j-1} \tag{3.36}$$

Next, we will determine the first moment of the duration of the count down process in the i_{th} backoff stage, $X_{si}'(1)$.

Using (3.24) and multiplying denominator with both sides of the equation, we have,

$$\begin{aligned}&W_{si}^{\ 2}(1-z)\{1-[P_{trs1}R_s(z)+P_{trf2}(1-P_{trs1})R_f(z)+1-P_{trs1}-P_{trf2}(1-P_{trs1})]\}X_{si}(z)\\&=\{1-[P_{trs1}R_f(z)+P_{trf2}(1-P_{trs1})R_f(z)+1-P_{trs1}-P_{trf2}(1-P_{trs1})]^{W_{si}}\}[1-z^{W_{si}}]\end{aligned} \tag{3.37}$$

Let us define,

$$LHS=A(z)B(z)C(z) \tag{3.38}$$

$$RHS=E(z)F(z) \tag{3.39}$$

where $A(z)=W_{si}^{\ 2}(1-z)$, $B(z)=1-D(z)$, $C(z)=X_{si}(z)$,

$$D(z)=P_{trs1}R_s(z)+P_{trf2}(1-P_{trs1})R_f(z)+1-P_{trs1}-P_{trf2}(1-P_{trs1})$$

$$E(z)=1-D(z)^{W_{si}},\ F(z)=1-z^{W_{si}}.$$

Taking third order derivatives of *LHS* and *RHS* at z=1, we have,

$$LHS'''|_{z=1}=\{6A'(z)B'(z)C'(z)+3A'(z)B''(z)C(z)\}|_{z=1} \tag{3.40}$$

$$RHS'''\big|_{z=1}=\{3[E''(z)F'(z)+E'(z)F''(z)]\}|_{z=1} \tag{3.41}$$

where

$$A'(z)\Big|_{z=1} = -W_{si}^{\ 2}$$

$$A''(z)\Big|_{z=1} = A'''(z)\Big|_{z=1} = A''''(z)\,|_{z=1} = 0$$

$$B^{(j)}(z)\,|_{z=1} = -[P_{trf2}(1-P_{trs1})R^{(j)}{}_{f}(1) + P_{trs1}R^{(j)}{}_{s}(1)]$$

$$D^{(j)}(z)\,|_{z=1} = [P_{trf2}(1-P_{trs1})R^{(j)}{}_{f}(1) + P_{trs1}R^{(j)}{}_{s}(1)]$$

$$E'(z)\,|_{z=1} = -W_{si}[P_{trf}(1-P_{trs1})R'_f(1) + P_{trs1}R'_s(1)]$$

$$\begin{aligned}E''(z)\,|_{z=1} = &-W_{si}(W_{si}-1)[P_{trf2}(1-P_{trs1})R'_f(1) + P_{trs1}R'_s(1)]^2 \\ &-W_{si}[P_{trf2}(1-P_{trs1})R''_f(1) + P_{trs1}R''_s(1)]\end{aligned}$$

$$\begin{aligned}E'''(z)\,|_{z=1} = &-W_{si}(W_{si}-1)(W_{si}-2)[P_{trf2}(1-P_{trs1})R'_f(1) + P_{trs1}R'_s(1)]^2 \\ &-2W_{si}(W_{si}-1)[P_{trf2}(1-P_{trs1})R'_f(1) + P_{trs1}R'_s(1)][P_{trf2}(1-P_{trs1})R''_f(1) + P_{trs1}R''_s(1)] \\ &-W_{si}(W_{si}-1)[P_{trf2}(1-P_{trs1})R''_f(1) + P_{trs1}R''_s(1)] - W_i[P_{trf2}(1-P_{trs1})R'''_f(1) + P_{trs1}R'''_s(1)]\end{aligned}$$

$$F'(z)\,|_{z=1} = -W_{si}$$

$$F''(z)\,|_{z=1} = -W_{si}(W_{si}-1)$$

$$F'''(z)\,|_{z=1} = -W_{si}(W_{si}-1)(W_{si}-2)$$

$$F''''(z)\,|_{z=1} = -W_{si}(W_{si}-1)(W_{si}-2)(W_{si}-3)$$

Equating left and right hand sides of (3.40) and (3.41), we have,

$$X_{si}{}'(1) = \frac{1}{2}(W_{si}-1)\{[P_{trf2}(1-P_{trs1})R'_f(1) + P_{trs1}R'_s(1)] + 1\} \qquad (3.42)$$

This completes the mean packet service time derivation of the slow STA which is given by (3.31) with (3.42) and (3.36).

3.3 Packet Service Time Distribution of a Fast STA

The distribution of a fast STA packet service time follows the approach of the slow STA, however, there are some differences. Since a fast STA packet may collide with fast or slow STA packet in each stage, the duration of collision time may be equal to the slow or fast STA packet size respectively. On the other hand, the counter interruption process is similar to the slow STA. Each transmission attempt consists of a choice of backoff counter value and retransmission. Count down process may be interrupted due to transmissions by other STAs. Duration of an interruption depends whether interruption is due to only slow, fast or mix of two types of STAs. Thus, the number of slots that a STA spends in the i_{th} backoff stage due to counter freezing is given by sum of the random number of counter freezing times where duration of each freezing time depends on its source.

Let us define the following random variables,

- r_s packet size of a slow STA in slots
- r_f packet size of a fast STA in slots
- k_{fi} backoff counter value of a fast STA in the i_{th} backoff stage
- j_{si} number of interruptions due to only slow STAs transmitting in the i_{th} backoff stage
- j_{fi} number of interruptions due to only fast STAs transmitting in the i_{th} backoff stage
- j_{sfi} number of interruptions due to both slow and fast STAs transmitting in the i_{th} backoff stage

j_i number of interruptions in the i_{th} backoff stage

x_{fi} duration of count down process of a slow STA in the i_{th} backoff stage

r_{sf} packet collision time of a fast STA

r_{sfm} packet transmission time of a fast STA in the m_{th} backoff stage

S_{fi} duration of packet service time of a slow STA given that there is an i_{th} retransmission attempt

S_f packet service time of a fast STA in slots

We make the assumption that $r_s > r_f$, therefore, if packet collision is due to transmissions of only fast STAs then duration of collision time is r_f, otherwise it is r_s. Also, if counter interruption is due to transmissions of only fast STAs then duration of interruption is r_f, otherwise it is r_s.

Also defining the following PGFs of these random variables,

$R_s(z) = E[z^{r_s}], R_f(z) = E[z^{r_f}], K_{fi}(z) = E[z^{k_{fi}}], J_i(z) = E[z^{j_i}]$

$J_{fi}(z) = E[z^{j_{fi}}], J_{sfi}(z) = E[z^{j_{sfi}}], X_{fi}(z) = E[z^{x_{fi}}], S_{si}(z) = E[z^{S_{si}}]$

$R_{sfm}(z) = E[z^{r_{sfm}}], R_{sf}(z) = E[z^{r_{sf}}], J_{si}(z) = E[z^{j_{si}}], S_s(z) = E[z^{S_s}]$

Then we have the following expressions for service time of a fast STA,

$$x_{fi} = k_{fi} + \sum_{\ell=1}^{j_{si}} r_s + \sum_{\ell=1}^{j_{fi}} r_f + \sum_{\ell=1}^{j_{sfi}} r_s \tag{3.43}$$

And for the 0_{th} stage to $(m-1)_{th}$ stage, the service time for each stage is,

$$S_{fi} = \sum_{j=1}^{i-1} (x_{fj} + r_{sf}) + r_f \qquad i \in (1,m) \tag{3.44}$$

where $r_{sf} = \begin{cases} r_f & with\ prob.\ P_{trf1}(1-P_{trs2})/[P_{trf1}(1-P_{trs2})+P_{trs2}] \\ r_s & with\ prob.\ P_{trs2}/[P_{trf1}(1-P_{trs2})+P_{trs2}] \end{cases}$ (3.45)

Also, for the maximum stage m, the packet transmission may end up in a collision or a successful transmission. The service time for the m_{th} stage is given by,

$$S_{f(m+1)} = \sum_{j=0}^{m} x_{fj} + mr_{sf} + r_{sfm} \tag{3.46}$$

where $r_{sfm} = \begin{cases} r_f & with\ \ prob.\ \ 1\text{-}P_{Cf} \\ r_s & with\ \ prob.\ \ P_{Cf} \end{cases}$ (3.47)

From the above equations (3.43-3.47), we have the corresponding PGFs ,

$$S_{fi}(z) = R_f(z)R_{sf}^{\ i-1}(z)\prod_{j=0}^{i-1} X_{fj}(z) \qquad\qquad i \in (1,m) \tag{3.48}$$

$$S_{f(m+1)}(z) = R_{sfm}(z)R^{m}{}_{sf}(z)\prod_{j=0}^{m} X_{fj}(z) \tag{3.49}$$

where

$$R_{sf}(z) = \frac{P_{trf1}(1-P_{trs2})R_f(z) + P_{trs2}R_s(z)}{P_{trf1}(1-P_{trs2}) + P_{trs2}} \tag{3.50}$$

and

$$R_{sfm}(z) = (1-P_{Cf})R_f(z) + P_{Cf}R_{sf}(z) \tag{3.51}$$

Similar to the slow STA analysis, the packet service time distribution of a fast STA can be expressed as follows,

$$S_f(z) = (1-P_{Cf})\sum_{j=1}^{m}\{[\prod_{i=0}^{j-1} X_{fi}(z)]R_{sf}^{\ j-1}(z)R_f(z)P_{Cf}^{\ j-1}\} + [\prod_{i=0}^{m} X_{fi}(z)]R_{sf}^{\ m}(z)R_{sfm}(z)P_{Cf}^{\ m}$$

(3.52)

Let us define,

$$\varphi_{fj}(z)=\prod_{i=0}^{j-1}X_{fi}(z) \qquad j\in(1,m) \tag{3.53}$$

$$Y_{fj}(z)=[R_{sf}(z)]^{j-1}R_{f}(z) \qquad j\in(1,m) \tag{3.54}$$

$$V_{fj}(z)=\varphi_{fj}(z)Y_{fj}(z)P_{Cf}{}^{j-1} \qquad j\in(1,m) \tag{3.55}$$

and

$$\varphi_{f(m+1)}(z)=\prod_{i=0}^{m}X_{fi}(z) \tag{3.56}$$

$$Y_{f(m+1)}(z)=[R_{sf}(z)]^{m}R_{sfm}(z) \tag{3.57}$$

$$V_{f(m+1)}(z)=\varphi_{f(m+1)}(z)Y_{f(m+1)}(z)P_{Cf}{}^{m} \tag{3.58}$$

Then the PGF of the service time of a fast STA in (3.52) can be expressed as,

$$S_{f}(z)=(1-P_{Cf})\sum_{j=1}^{m}V_{fj}(z)+V_{f(m+1)}(z) \tag{3.59}$$

Taking the first order derivative of (3.59) and setting the z=1, we obtain the mean packet service time of a fast STA.

$$\begin{aligned}\overline{S}_{f}&=S_{f}'(z)|_{z=1}\\ &=(1-P_{Cf})\sum_{j=1}^{m}V_{fj}'(1)+V_{f(m+1)}'(1)\end{aligned} \tag{3.60}$$

where $V_{fj}'(1)=[\sum_{k=0}^{j-1}X_{fk}'(1)]P_{Cf}{}^{j-1}+[(j-1)R_{sf}{}'(1)+R_{f}{}'(1)]P_{Cf}{}^{j-1}$ (3.61)

and $V_{f(m+1)}'(1)=[\sum_{k=0}^{m}X_{fk}'(1)]P_{Cf}{}^{j-1}+[(j-1)R_{sf}'(1)+R_{sfm}'(1)]P_{Cf}{}^{j-1}$ (3.62)

Taking first order derivatives of (3.50) and (3.51), we have,

$$R_{sf}'(1)=\frac{P_{trf1}(1-P_{trs2})R_f'(1)+P_{trs2}R_s'(1)}{P_{trf1}(1-P_{trs2})+P_{trs2}} \tag{3.63}$$

$$R_{sfm}'(1)=(1-P_{Cf})R_f'(1)+P_{Cf}R_{sf}'(1) \tag{3.64}$$

Next, we derive the PGF of the duration of the count down process of a fast STA in the i_{th} backoff stage, $X_{fi}(z)$.

We assume that three types of interruptions of the count down process occur independently with following probabilities in any slot.

Pr(counter interruption due to transmissions by only slow STAs)=$P_{trs2}(1-P_{trf1})$

Pr(counter interruption due to transmissions by only fast STAs)=$P_{trf1}(1-P_{trs2})$

Pr(counter interruption due to transmissions by both types of STAs)=$P_{trf1}P_{trs2}$

Because the counter decrement operation is similar to the slow STA, from (3.24), we have,

$$X_{fi}(z)=\frac{1}{W_{fi}}\frac{1-z^{W_{fi}}}{1-z}\frac{1-\alpha^{W_{fi}}}{1-\alpha}\Big|_{\alpha=P_{trf1}(1-P_{trs2})R_f(z)+P_{trs2}R_s(z)+1-P_{trf1}(1-P_{trs2})-P_{trs2}} \tag{3.65}$$

Then following the same approach of the slow STA analysis, from (3.42) we obtain $X_{fi}'(1)$, the first moment of the duration of count down process of a fast STA in the i_{th} backoff stage.

$$X_{fi}'(1)=\frac{1}{2}(W_{fi}-1)\{[P_{trf1}(1-P_{trs2})R_f'(1)+P_{trs2}R_s'(1)]+1\} \tag{3.66}$$

This completes the derivation of the mean packet service time of a fast STA which is given by equation (3.60) with (3.63-3.64) and (3.66).

Next, we express the busy probabilities of both types of STAs are, respectively, given by,

$$\rho_s = \lambda_s \times \overline{S}_s \qquad (3.67)$$

$$\rho_f = \lambda_f \times \overline{S}_f \qquad (3.68)$$

Finally, equations (3.1-3.6, 3.31, 3.60, 3.67, 3.68) represent a non-linear system with 10 unknown variables ($P_{CS}, P_{Cf}, P_{trs1}, P_{trs2}, P_{trf1}, P_{trf2}, \overline{S_s}, \overline{S_f}, \rho_s, \rho_f$) which can be solved numerically using MATLAB optimization toolbox. As the equation set is governed by the Markov chain and the Markov chain can be solved by using **B=BP**, a unique solution exists for the non-linear equation set.

3.4 Mean Packet Delay of a Slow STA

In order to compute the mean packet delay of a slow STA using the M/G/1 results, we need the second moment of the packet service time. We take fourth order derivatives of both sides of the equation (3.37) and substitute z=1, then we have,

$$\begin{aligned}&\{6A'(z)B'(z)C''(z) + 12A'(z)B''(z)C'(z) + 4A'(z)B'''(z)C(z)\}|_{z=1} \\ &= \{4E'''(z)F'(z) + 6E''(z)F''(z) + 4E'(z)F'''(z)\}|_{z=1} \qquad (3.69)\end{aligned}$$

Thus we obtain,

$$\begin{aligned}X_{si}''(1) &= \frac{2}{3}(W_{si}-1)(W_{si}-2)[P_{trf2}(1-P_{trs1})R_f'(1)+P_{trs1}R_s'(1)] \\ &+\frac{4}{3}(W_{si}-1)[P_{trf2}(1-P_{trs1})R_f''(1)+P_{trs1}R_s''(1)] \\ &+(W_{si}-1)^2[P_{trf2}(1-P_{trs1})R_f'(1)+P_{trs1}R_s'(1)] \\ &+\frac{2}{3}(W_{si}-1)(W_{si}-2)-\frac{P_{trf2}(1-P_{trs1})R_f''(1)+P_{trs1}R_s''(1)}{P_{trf2}(1-P_{trs1})R_f'(1)+P_{trs1}R_s'(1)} \\ &\times[P_{trf2}(1-P_{trs1})R_f'(1)+P_{trs1}R_s'(1)-\frac{2}{3}](W_{si}-1)\end{aligned} \tag{3.70}$$

Taking second order derivatives of (3.27) and (3.29) and substituting z=1, we have,

$$\varphi_{s(j+1)}''(1)=\varphi_{sj}''(1)+2\varphi_{sj}'(1)X_{sj}'(1)+X_{sj}''(1) \qquad j\in(1,m) \tag{3.71}$$

$$V_{s(j+1)}''(1)=[\varphi_{sj}''(1)+2\varphi_{sj}'(1)Y_{sj}'(1)+Y_{sj}''(1)]P_{CS}^{\ j-1} \qquad j\in(1,m) \tag{3.72}$$

where $Y_{sj}''(1)=jR_s''(1)+j(j-1)[R_s'(1)]^2$ (3.73)

The second moment of the slow STA's packet service time can be expressed as,

$$\overline{S_s^{\ 2}}=S_s''(z)\big|_{z=1}+S_s'(z)\big|_{z=1} \tag{3.74}$$

where

$$\begin{aligned}S_s''(z)\big|_{z=1} &= (1-P_{CS})\sum_{j=1}^{m}V_{sj}''(1)+V_{s(m+1)}''(1) \\ &=(1-P_{CS})\sum_{j=1}^{m}[\varphi_{sj}''(1)+2\varphi_{sj}'(1)Y_{sj}'(1)+Y_{sj}''(1)]P_{CS}^{\ j-1}+[\varphi_{s(m+1)}''(1)+2\varphi_{s(m+1)}'(1)Y_{sj}'(1)+Y_{sj}''(1)]P_{CS}^{\ m}\end{aligned} \tag{3.75}$$

Using M/G/1 queuing result [10], the packet mean delay of a slow STA is given by,

$$\overline{D}_s=\overline{S}_s+\frac{\lambda_s\overline{S_s^{\ 2}}}{2(1-\rho_s)} \tag{3.76}$$

3.5 Mean Packet Delay of a Fast STA

Following the approach of the slow STA in chapter 3.4, from (3.70) we may obtain the second order derivative of the duration of count down process of a fast STA in the i_{th} backoff stage.

$$\begin{aligned} X_{fi}''(1) &= \frac{2}{3}(W_{fi}-1)(W_{fi}-2)[P_{trf1}(1-P_{trs2})R_f'(1)+P_{trs2}R_s'(1)] \\ &+\frac{4}{3}(W_{fi}-1)[P_{trf1}(1-P_{trs2})R_f''(1)+P_{trs2}R_s''(1)] \\ &+(W_{fi}-1)^2[P_{trf1}(1-P_{trs2})R_f'(1)+P_{trs2}R_s'(1)] \\ &+\frac{2}{3}(W_{fi}-1)(W_{fi}-2)-\frac{P_{trf1}(1-P_{trs2})R_f''(1)+P_{trs2}R_s''(1)}{P_{trf1}(1-P_{trs2})R_f'(1)+P_{trs2}R_s'(1)} \\ &\times[P_{trf1}(1-P_{trs2})R_f'(1)+P_{trs2}R_s'(1)-\frac{2}{3}](W_{fi}-1) \end{aligned} \tag{3.77}$$

Taking second order derivatives of (3.53) and (3.55) and substituting z=1, we have,

$$\varphi_{fj}''(1)=\varphi_{f(j-1)}''(1)+2\varphi_{f(j-1)}'(1)X_{f(j-1)}'(1)+X_{f(j-1)}''(1) \qquad j\in(1,m) \tag{3.78}$$

$$V_{fj}''(1)=[\varphi_{fj}''(1)+2\varphi_{fj}'(1)Y_{fj}'(1)+Y_{fj}''(1)]P_{Cf}^{\ j-1} \qquad j\in(1,m) \tag{3.79}$$

where

$$Y_{fj}''(1)=(j-1)\{R_{sf}''(1)+(j-2)[R_{sf}'(1)]^2+2R_{sf}'(1)R_f'(1)\}+R_f'(1) \tag{3.80}$$

Taking second order derivatives of (3.56) and (3.58) and substituting z=1, we have,

$$\varphi_{f(m+1)}''(1)=\varphi_{fm}''(1)+2\varphi_{fm}'(1)X_{fm}'(1)+X_{fm}''(1) \tag{3.81}$$

$$V_{f(m+1)}''(1)=[\varphi_{fm}''(1)+2\varphi_{fm}'(1)Y_{fm}'(1)+Y_{fm}''(1)]P_{Cf}^{\ m} \tag{3.82}$$

where

$$Y_{fm}''(1)=(m-1)\{R_{sf}''(1)+(m-2)[R_{sf}'(1)]^2+2R_{sf}'(1)R_{sfm}'(1)\}+R_{sfm}'(1) \tag{3.83}$$

The second moment of the packet service time of a fast STA can now be expressed as,

$$\overline{S_f^2} = S_f''(z)\big|_{z=1} + S_f'(z)\big|_{z=1} \tag{3.84}$$

where

$$S_f''(z)\big|_{z=1} = (1-P_{Cf})\sum_{j=1}^{m} V_{fj}''(1) + V_{f(m+1)}''(1)$$

$$= (1-P_{Cf})\sum_{j=1}^{m}[\varphi_{fj}''(1) + 2\varphi_{fj}'(1)Y_{fj}'(1) + Y_{fj}''(1)]P_{Cf}^{\,j-1} + [\varphi_{f(m+1)}''(1) + 2\varphi_{f(m+1)}'(1)Y_{fj}'(1) + Y_{fj}''(1)]P_{Cf}^{\,m} \tag{3.85}$$

Then we can apply the M/G/1 queuing result [10] and obtain the mean packet delay $\overline{D}_f$ of a fast STA.

$$\overline{D}_f = \overline{S_f} + \frac{\lambda_f \overline{S_f^2}}{2(1-\rho_f)} \tag{3.86}$$

3.6 Numerical and Simulation Results

In this section, we present some numerical results for the analysis in this chapter and simulation results to verify the assumptions. The IEEE 802.11 standard defines four physical layer specifications, Frequency hopping Spread Spectrum (FHSS), Direct Sequence Spread Spectrum (DSSS), Infra-Red (IR) and Orthogonal Frequency Division Multiplexing (OFDM). We choose IEEE 802.11b DSSS as the physical layer for determining the numerical results. Each STA in IEEE 802.11b network maintains a basic operational rate set which is composed of transmission rate 1, 2, 5.5 and 11Mbps.

As discussed in this chapter, we have 10 equations (3.1-3.6, 3.31, 3.60, 3.67, 3.68) and 10 unknown variables ($P_{CS}, P_{Cf}, P_{trs1}, P_{trs2}, P_{trf1}, P_{trf2}, \overline{S_s}, \overline{S_f}, \rho_s, \rho_f$) which can be solved numerically using MATLAB optimization toolbox. We note that the unknown variables ρ_s and ρ_f should be in the ranges of (0,1). As the equation set is governed by the Markov chain and the Markov chain can be solved by using **B=BP**, a unique solution exists for the non-linear equation set.

In the following section, we report our numerical results in different network situations. We have two groups of STAs with different transmission rates (fast and slow). Assuming the packet sizes are of identical length in bits, the packet transmission time in slots of higher speed STAs will be shorter. For simplicity, we consider packet sizes are constant and the PHY and MAC overheads are included in the packet sizes. The slot duration in 802.11b DSSS is $20\,\mu s$. For 5.5 Mbps transmission rate, 120 slots equal to 13200bits. For 11 Mbps transmission rate, 60 slots equal to 13200bits. We plot the mean packet service time, mean delay and collision probability of one STA group as functions of packet arrival rate by fixing the other group's packet arrival rate. Numerical results are obtained for different packet arrival rates from 0 to the saturation point. We also vary the minimum contention window size CW_{min} to see its impact on the delay performance.

In order to prove the accuracy of the model, we have designed a discrete event C++ test bed that simulates the 802.11 DCF in heterogeneous environment. The numerical results are verified by extensive simulations in different network situations.

3.6.1 Example 1

	Number of STAs	PHY TX. Rate	Packet Size	CW_{min}	Max Stage
Fast STA	n_f =10	11 Mbps	r_f =60	W_{f0}=31	5
Slow STA	n_s =5	5.5 Mbps	r_s =120	W_{s0}=31	5

Table 3.1 Network parameters for heterogeneous example 1

We fix the slow STA's packet arrival rate to be $\lambda_s = 10^{-4}$ packets/slot and obtain the fast STA's results.

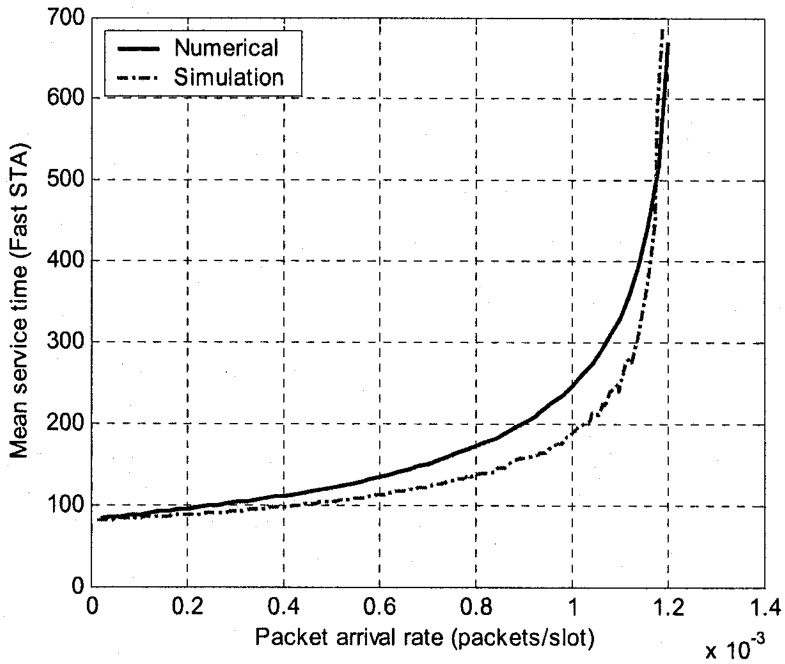

Figure 3.1 Mean service time of a fast STA ($n_s = 10, n_f = 5, r_s = 120, r_f = 60, \lambda_s = 10^{-4}$)

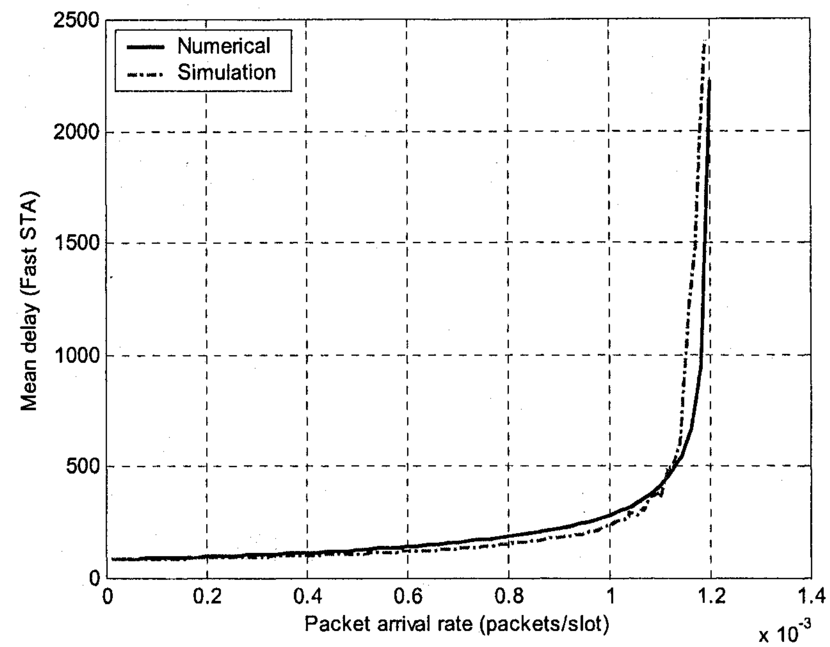

Figure 3.2 Mean delay of a fast STA ($n_s = 10, n_f = 5,\ r_s = 120, r_f = 60,\ \lambda_s = 10^{-4}$)

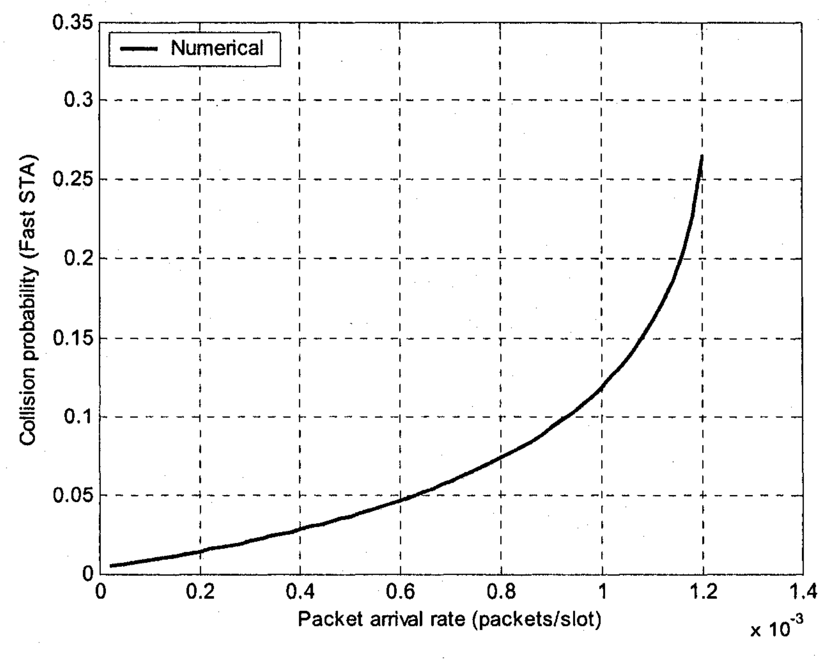

Figure 3.3 Collision probability of a fast STA
($n_s = 10, n_f = 5,\ r_s = 120, r_f = 60,\ \lambda_s = 10^{-4}$)

3.6.2 Example 2

	Number of STAs	PHY TX. Rate	Packet Size	CW_{min}	Max Stage
Fast STA	n_f =10	11 Mbps	r_f =60	W_{f0} =31	5
Slow STA	n_s =5	5.5 Mbps	r_s =120	W_{S0} =31	5

Table 3.2 Network parameters for heterogeneous example 2

We fix the slow STA's packet arrival rate to be $\lambda_s = 5\times10^{-4}$ packets/slot and obtain the fast STA's results.

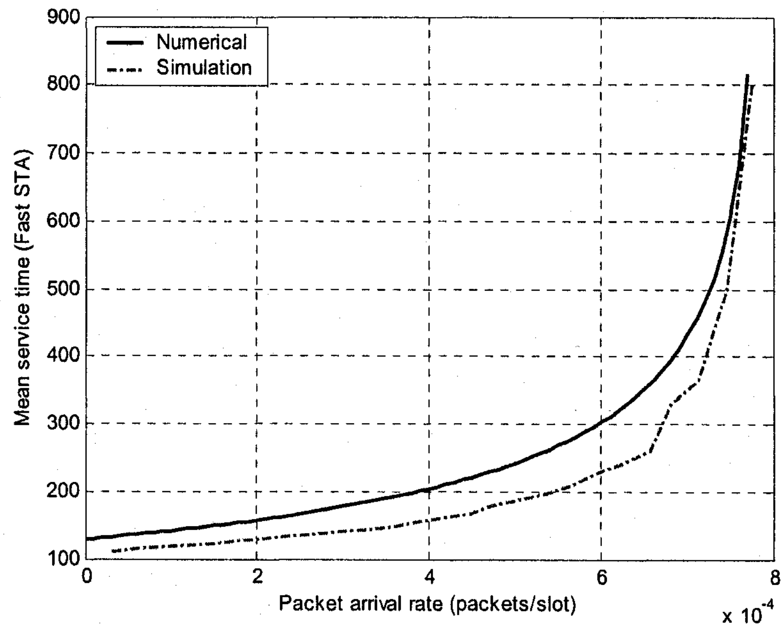

Figure 3.4 Mean service time of a fast STA
$(n_s = 10, n_f = 5,\ r_s = 120, r_f = 60,\ \lambda_s = 5\times10^{-4})$

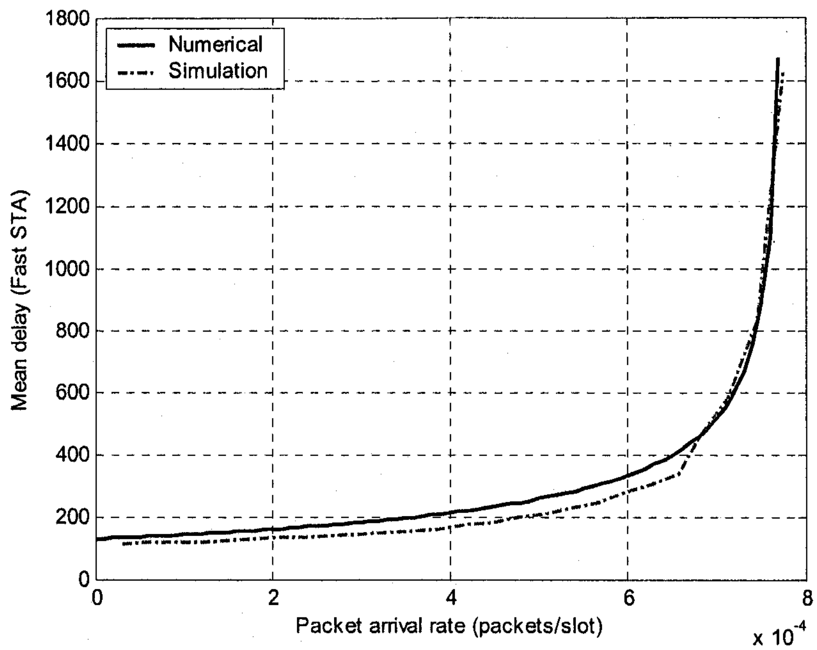

Figure 3.5 Mean delay of a fast STA ($n_s = 10, n_f = 5, r_s = 120, r_f = 60, \lambda_s = 5 \times 10^{-4}$)

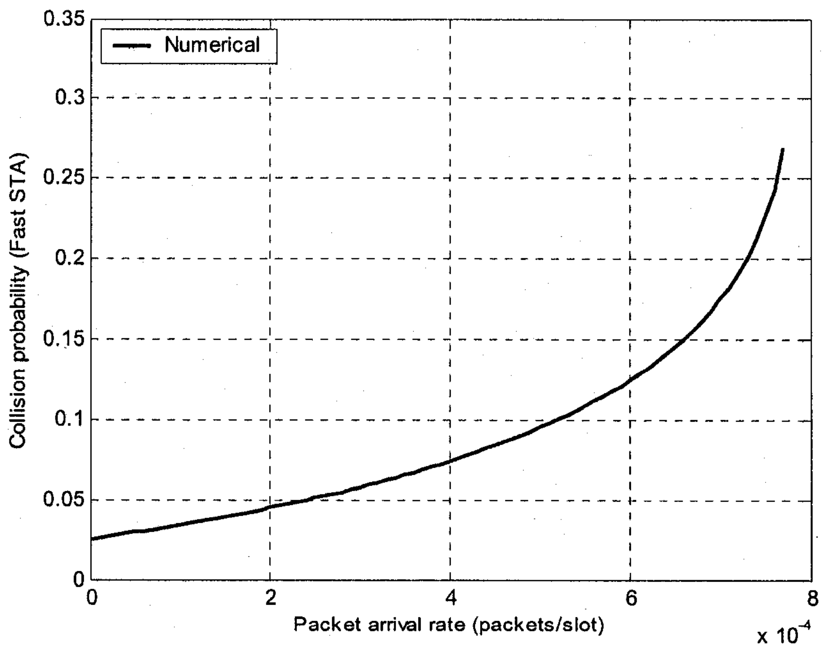

Figure 3.6 Collision probability of a fast STA
($n_s = 10, n_f = 5, r_s = 120, r_f = 60, \lambda_s = 5 \times 10^{-4}$)

3.6.3 Example 3

	Number of STAs	PHY TX. Rate	Packet Size	CW_{min}	Max Stage
Fast STA	n_f =10	11 Mbps	r_f =60	W_{f0} =31	5
Slow STA	n_s =5	5.5 Mbps	r_s =120	W_{s0} =31	5

Table 3.3 Network parameters for heterogeneous example 3

We fix the fast STA's packet arrival rate to be $\lambda_f = 2\times10^{-4}$ packets/slot and obtain the slow STA's results.

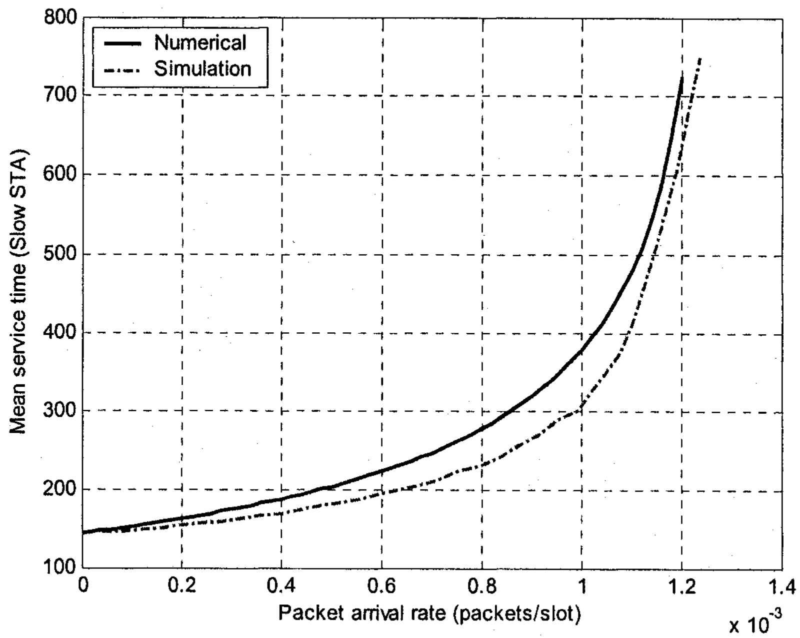

Figure 3.7 Mean service time of a slow STA
($n_s = 10, n_f = 5,\ r_s = 120, r_f = 60,\ \lambda_f = 2\times10^{-4}$)

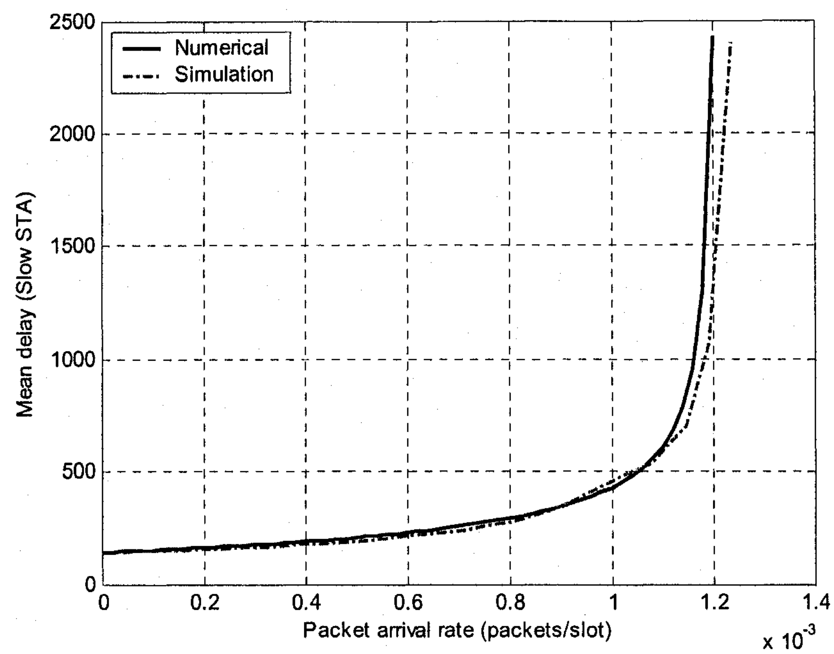

Figure 3.8 Mean delay of a slow STA ($n_s = 10, n_f = 5, r_s = 120, r_f = 60, \lambda_f = 2 \times 10^{-4}$)

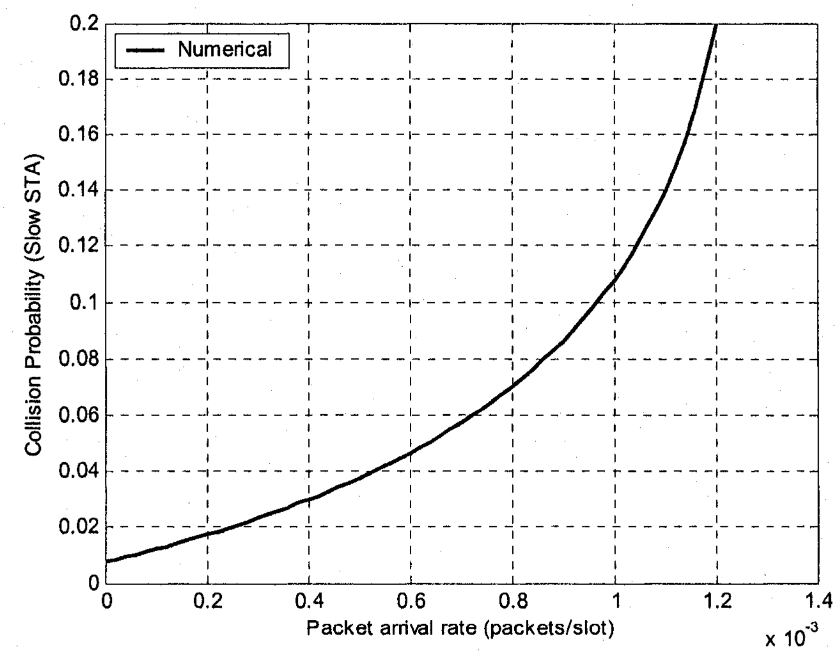

Figure 3.9 Collision probability of a slow STA
($n_s = 10, n_f = 5, r_s = 120, r_f = 60, \lambda_f = 2 \times 10^{-4}$)

3.6.4 Example 4

	Number of STAs	PHY TX. Rate	Packet Size	CW_{min}	Max Stage
Fast STA	n_f =10	11 Mbps	r_f =60	W_{f0} =31	5
Slow STA	n_s =5	5.5 Mbps	r_s =120	W_{s0} =31	5

Table 3.4 Network parameters for heterogeneous example 4

We fix the fast STA's packet arrival rate to be $\lambda_f = 5 \times 10^{-4}$ packets/slot and obtain the slow STA's results.

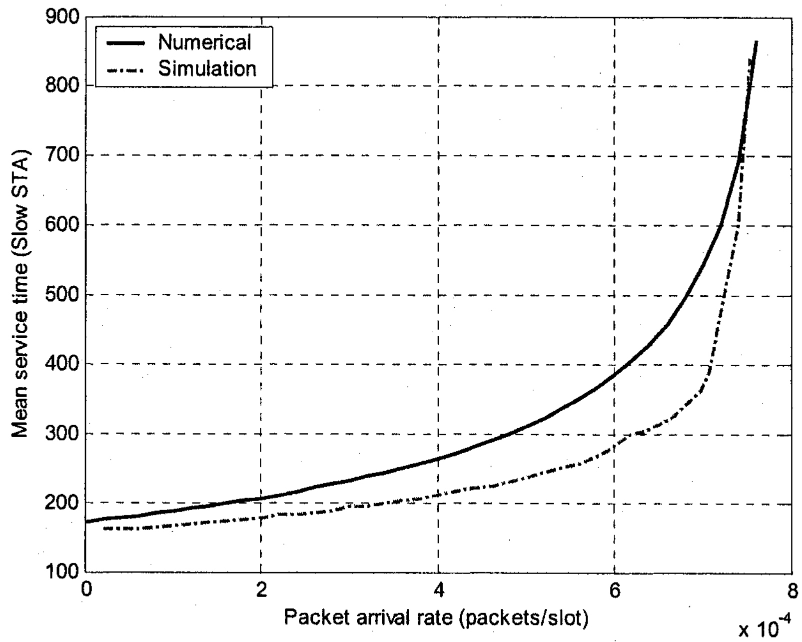

Figure 3.10 Mean service time of a slow STA
$(n_s = 10, n_f = 5,\ r_s = 120, r_f = 60,\ \lambda_f = 5 \times 10^{-4})$

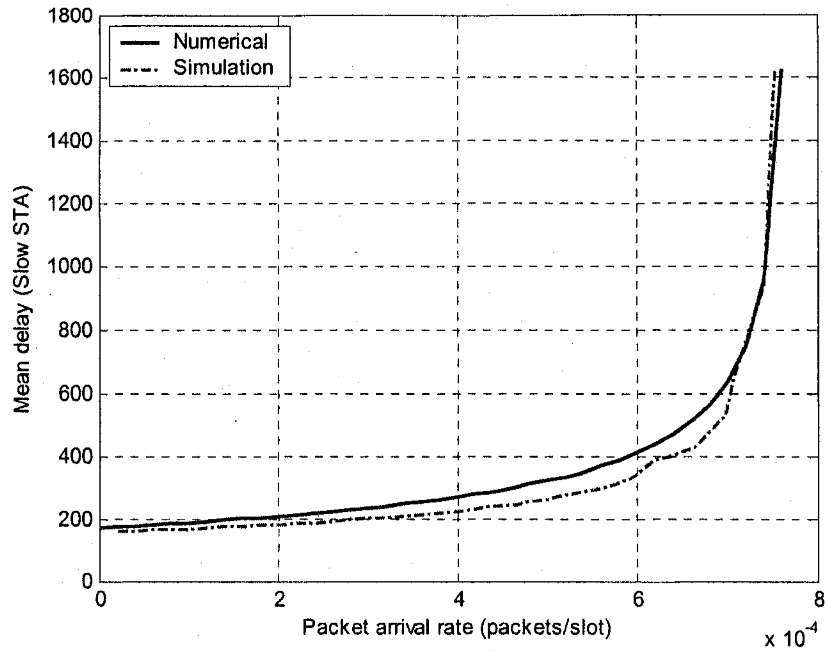

Figure 3.11 Mean delay of a slow STA ($n_s = 10, n_f = 5,\ r_s = 120, r_f = 60,\ \lambda_f = 5\times 10^{-4}$)

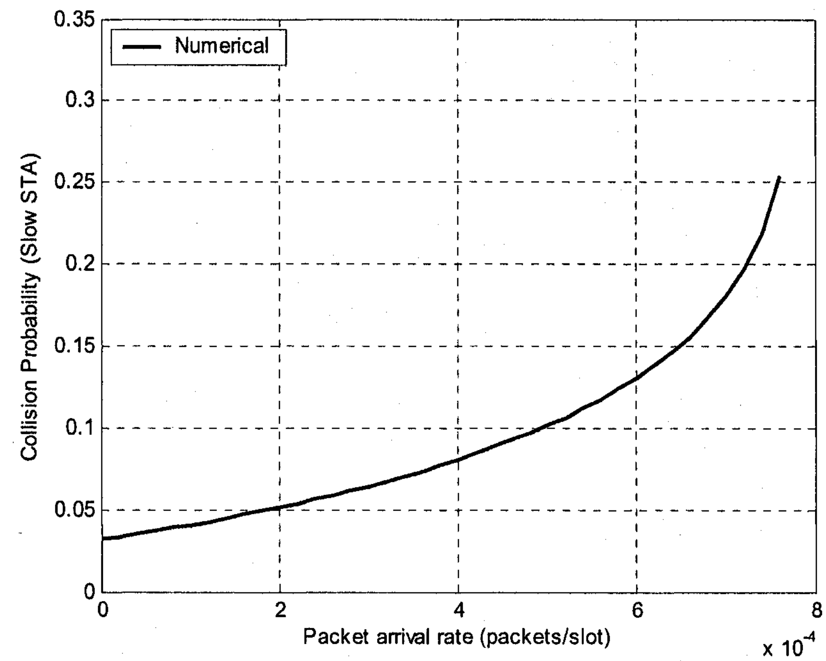

Figure 3.12 Collision probability of a slow STA
($n_s = 10, n_f = 5,\ r_s = 120, r_f = 60,\ \lambda_f = 5\times 10^{-4}$)

3.6.5 Example 5

	Number of STAs	PHY TX. Rate	Packet Size	CW_{min}	Max Stage
Fast STA	n_f =5	11 Mbps	r_f =60	W_{f0} =31	5
Slow STA	n_s =5	5.5 Mbps	r_s =120	W_{s0} =31	5

Table 3.5 Network parameters for heterogeneous example 5

We fix the fast STA's packet arrival rate to be $\lambda_f = 10^{-3}$ packets/slot and obtain the slow STA's results.

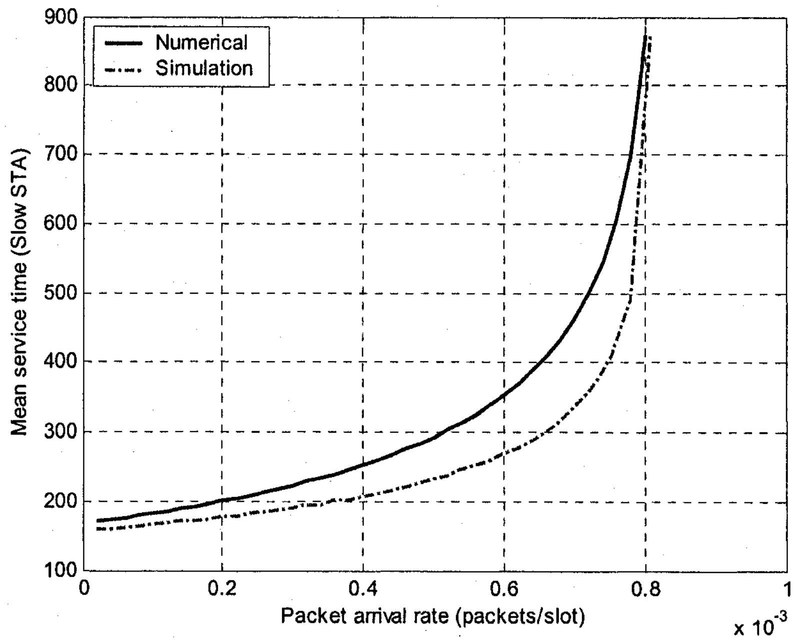

Figure 3.13 Mean service time of a slow STA
($n_s = 5, n_f = 5, r_s = 120, r_f = 60, \lambda_f = 10^{-3}$)

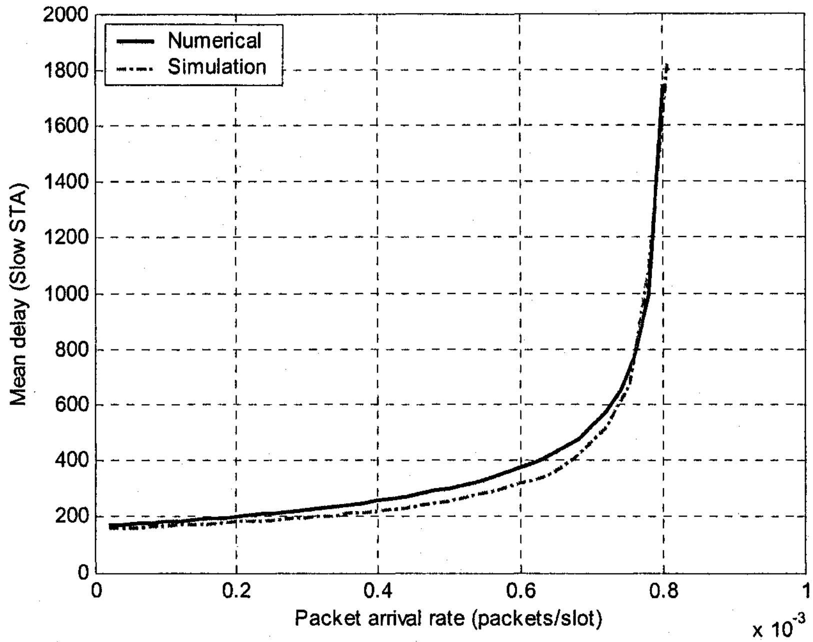

Figure 3.14 Mean delay of a slow STA ($n_s = 5, n_f = 5,\ r_s = 120, r_f = 60,\ \lambda_f = 10^{-3}$)

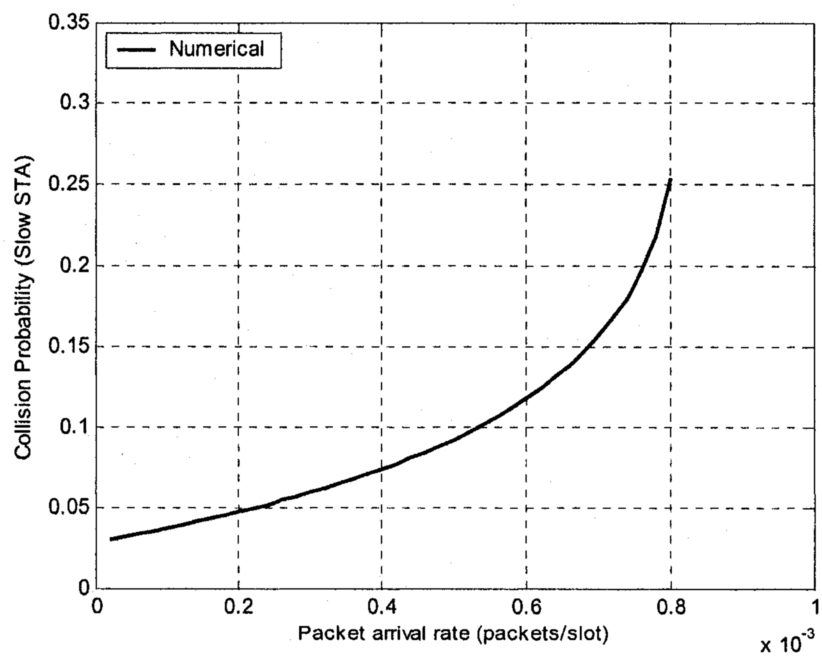

Figure 3.15 Collision probability of a slow STA
($n_s = 5, n_f = 5,\ r_s = 120, r_f = 60,\ \lambda_f = 10^{-3}$)

3.6.6 Example 6

	Number of STAs	PHY TX. Rate	Packet Size	CW_{min}	Max Stage
Fast STA	$n_f = 5$	11 Mbps	r_f=60	W_{f0}=31	5
Slow STA	$n_s = 5$	5.5 Mbps	r_s=120	W_{S0}=127	5

Table 3.6 Network parameters for heterogeneous example 6

We fix the slow STA's packet arrival rate to be $\lambda_s = 5\times10^{-4}$ packets/slot and obtain the fast STA's results.

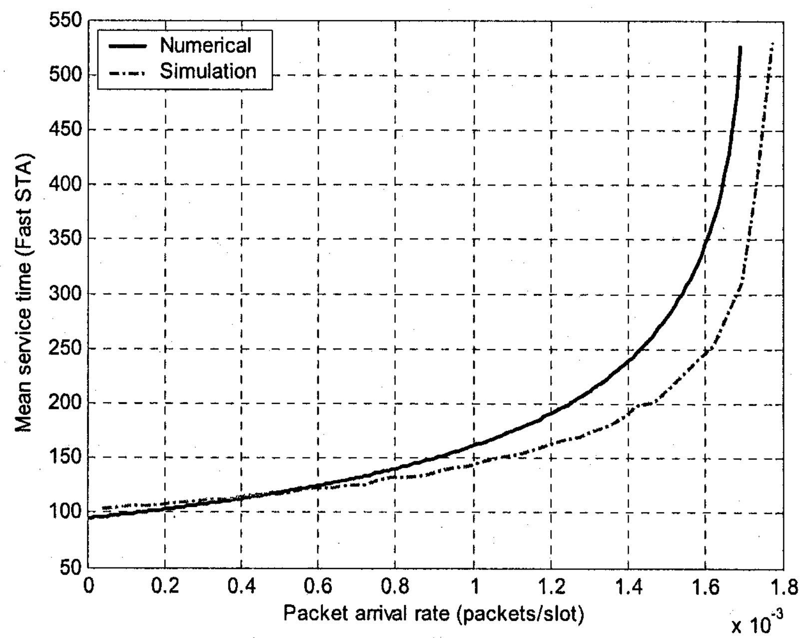

Figure 3.16 Mean service time of a fast STA
$(n_s = 5, n_f = 5, r_s = 120, r_f = 60, W_{S0} = 127, W_{f0} = 31, \lambda_s = 5\times10^{-4})$

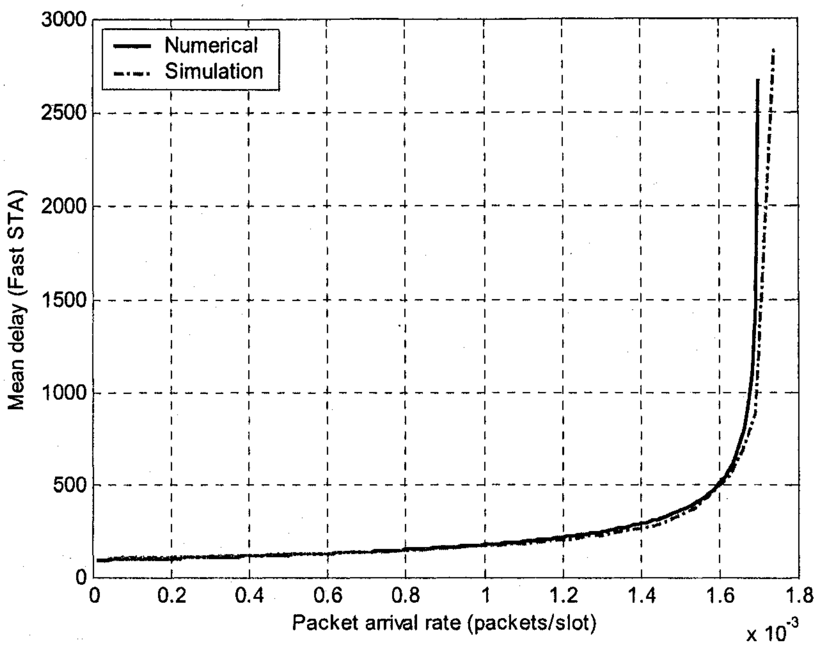

Figure 3.17 Mean delay of a fast STA

$(n_s = 5, n_f = 5, r_s = 120, r_f = 60, W_{S0} = 127, W_{f0} = 31, \lambda_s = 5\times 10^{-4})$

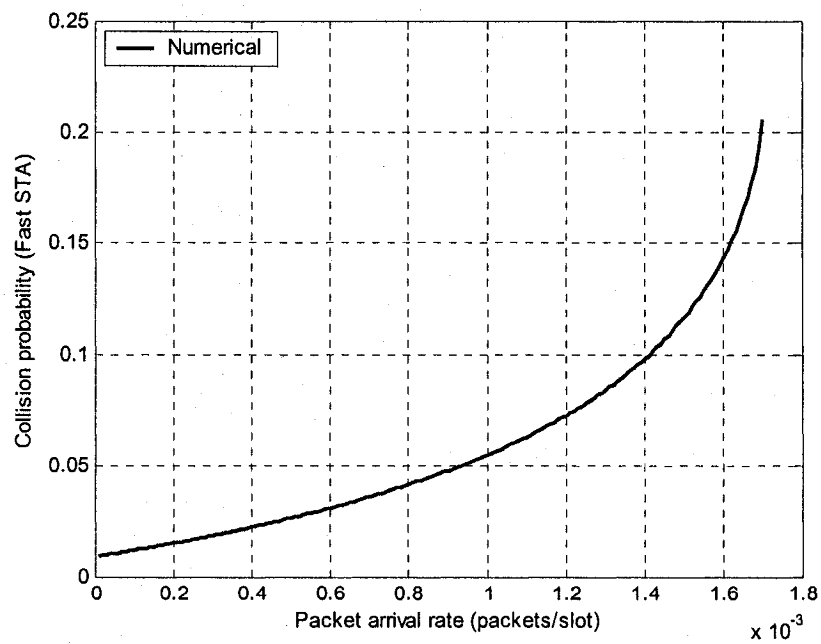

Figure 3.18 Collision probability of a fast STA

$(n_s = 5, n_f = 5, r_s = 120, r_f = 60, W_{S0} = 127, W_{f0} = 31, \lambda_s = 5\times 10^{-4})$

3.6.7 Example 7

	Number of STAs	PHY TX. Rate	Packet Size	CW_{min}	Max Stage
Fast STA	$n_f = 5$	11 Mbps	r_f=60	W_{f0}=127	5
Slow STA	$n_s = 5$	5.5 Mbps	r_s=120	W_{s0}=127	5

Table 3.7 Network parameters for heterogeneous example 7

We fix the slow STA's packet arrival rate to be $\lambda_s = 5\times10^{-4}$ packets/slot and obtain the fast STA's results.

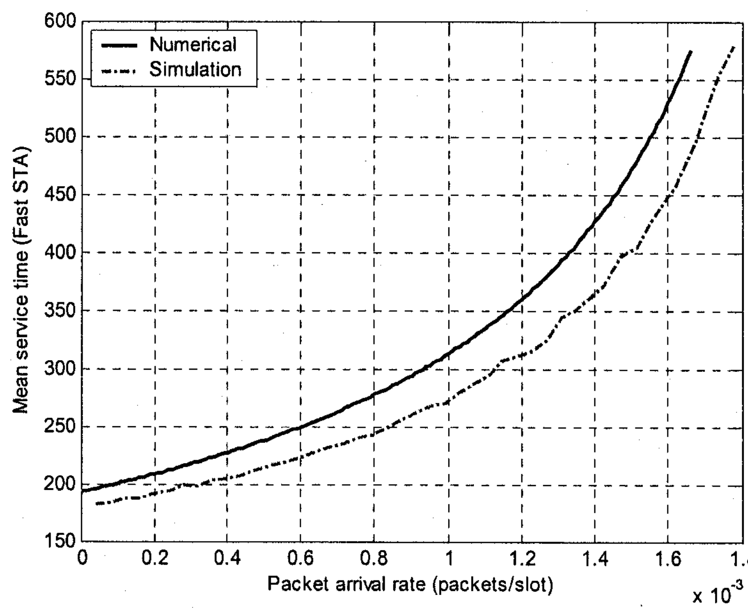

Figure 3.19 Mean service time of a fast STA
$(n_s = 5, n_f = 5, r_s = 120, r_f = 60, W_{S0} = 127, W_{f0} = 127, \lambda_s = 5\times10^{-4})$

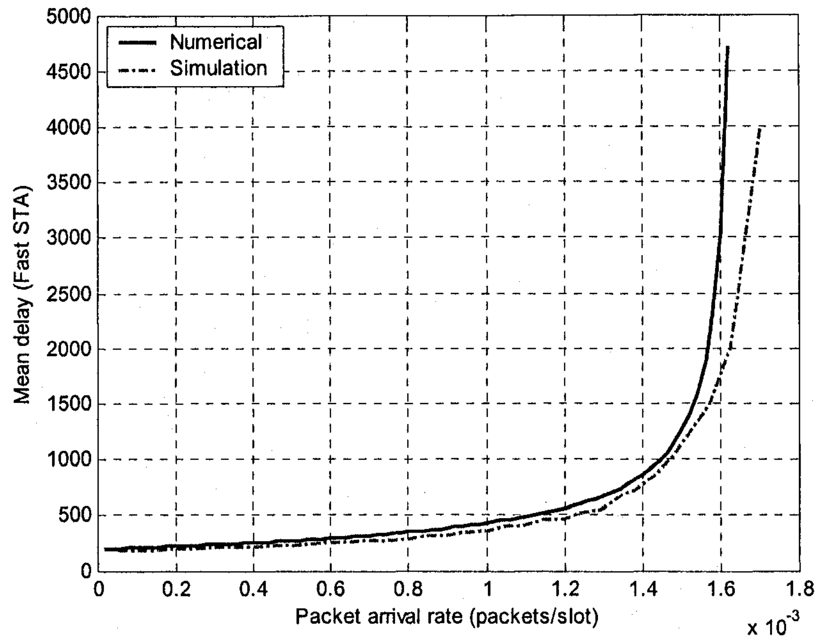

Figure 3.20 Mean delay of a fast STA
$(n_s = 5, n_f = 5, r_s = 120, r_f = 60, W_{S0} = 127, W_{f0} = 127, \lambda_s = 5 \times 10^{-4})$

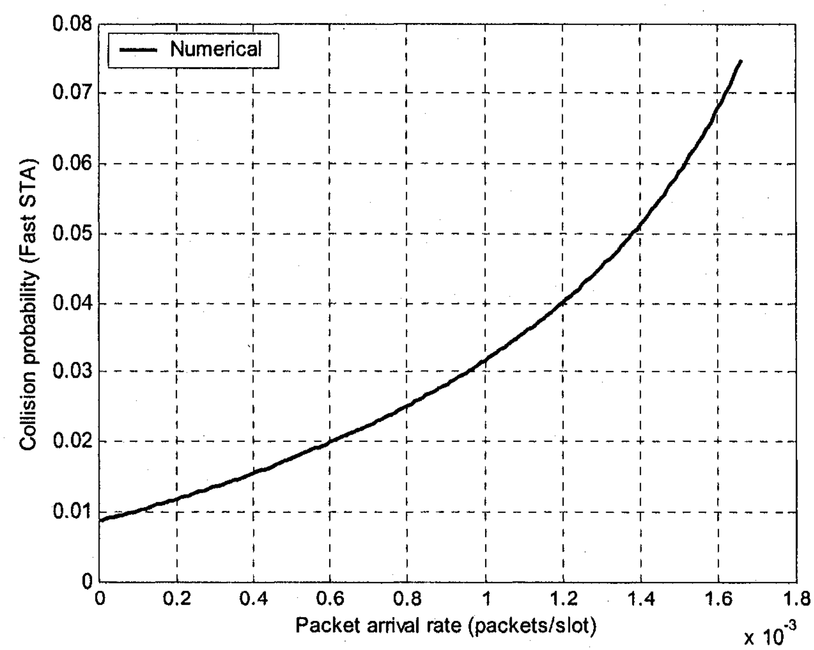

Figure 3.21 Collision probability of a fast STA
$(n_s = 5, n_f = 5, r_s = 120, r_f = 60, W_{S0} = 127, W_{f0} = 127, \lambda_s = 5 \times 10^{-4})$

3.6.8 Conclusion

As may be seen from the above figures, the numerical results have a good match with the simulation results in most of the cases. However, there are some mismatches in some of the cases due to the approximations that we have adopted in the analysis. Although we assume fixed packets sizes when comparing the numerical and simulation results, our model also applies to arbitrary packet size distributions.

In examples 1-4, we have two groups of STAs of different sizes with $n_f = 10$ and $n_s = 5$ respectively. The packet sizes of slow and fast STA in slots are 120 slots and 60 slots respectively. Since the transmission rate of the slow STA is half of the fast STA's, the actual packet sizes of both types of STAs are equal in bits. In examples 1 and 2, we fix the slow STA's packet arrival rate and vary the fast STA's packet arrival rate. In example 3 and 4, we fix the fast STA's packet arrival rate and vary the slow STA's packet arrival rate. Under various traffic load of one group of STAs, the numerical results of the other group have a good match with the simulation results. We can see from the figures that as the offered traffic load of one STA group gets heavier, the STAs of the other group saturate more quickly and there are significant increases of the packet service time, delay and collision probability of the other group. The fast STA group gets penalized in terms of packet service time and delay, as the packet collision time of a transmitted fast STA packet is much longer if a fast STA packet collides with the slow STA's packet and the backoff counter freezing time of a fast STA due to the busy medium is also much longer. This has significantly downgraded the delay performance of the fast STAs. One way to partially solve this problem is to use RTS/CTS access mode of the DCF protocol. The RTS/CTS access mode can reserve the channel for the time needed to transfer the data packet, thus packet collision can only happen on RTS frames. Since RTS frames are much shorter than normal data frames, it has significantly reduced time spent in packet collisions. The RTS/CTS access mode is very effective in terms of system performance, especially for large data packets, as it reduces the packet collision time. Therefore, the RTS/CTS access mode should be employed when packet size exceeds a certain threshold.

In examples 6 and 7, we observe the effect of different minimum contention window size CW_{min} on the delay performance. We let CW_{min} of the slow STAs to be 127 and vary CW_{min} of the fast STAs from 31 to 127. As CW_{min} increases, the packet service time and delay also increase. Thus, higher priority STAs enjoy superior services by means of smaller CW_{min}. However, the collision probability decreases as CW_{min} increases. This is because packet transmission probability τ in any slot is inversely proportional to CW_{min}. CW_{min} is a very important parameter for delay sensitive applications or real time applications, such as IP telephony or video conferencing. It is later defined as a configurable parameter in the 802.11e standard to provide QoS provisioning within the wireless LAN environment.

CHAPTER 4

CONCLUSION AND FUTURE WORK

4.1 Conclusion

In this thesis, we develop an analytical model of IEEE 802.11 Distributed Coordination Function (DCF) under finite load condition. While most of the previous models have been concentrated on system throughput, little attention has been paid to the delay performance of the DCF protocol. However, the packet service time and delay are very important metrics of the IEEE 802.11 DCF, especially for real time applications. We focus our interest on the packet service time and delay. We assume a finite number of un-saturated STAs and ideal channel condition. Basic DCF channel access mechanism is considered. We analyze the system with homogeneous and heterogeneous sources under finite load condition. STAs in the system are allowed to have either the same or different packet arrival and transmission rates. The arrival of the packets is assumed to be according to a Poisson process. The dependency between the STAs is taken into account through counter freezing operation and packet collision during transmission. We derive the packet service time distribution and packet collision probability using a three-dimensional Markov chain model. Then we determine the mean packet delay by applying the M/G/1 queuing result.

A discrete event C++ test bed has been developed to compare the numerical results from the analytical model with that of the simulation. Although we assume fixed packet sizes in the numerical results, our model also applies to arbitrary packet size distributions. We also investigate the impact of different sizes of CW_{min} on the delay performance. CW_{min} is an important parameter for the delay sensitive applications and is later defined as an configurable parameter in the 802.11e standard for providing QoS support. It is shown from the simulation results that our analytical model is quite accurate in most of the cases. This model provides a helpful tool to study the performance of DCF protocol and may further lead us to investigate the performance of 802.11e EDCA protocol.

4.2 Future Work

A number of future research topics exist based on this thesis. These research topics are listed as follows,

- **Modeling of the IEEE 802.11e EDCA with QoS support**

  One major drawback of current IEEE 802.11 wireless LANs is that it can not provide time bounded services such as voice or video. IEEE 802.11e with Quality of Service extension has recently been introduced to give differential services within a LAN environment. Current performance analyses of IEEE 802.11e EDCA in the literature do not have an accurate model for the packet service time distribution and delay. A new model based on ours may be developed to analyze the system performance with QoS constraints. By tuning the EDCA parameters in 802.11e, we may give performance analysis according to each QoS level through the analytical model.

- **Accurate traffic model**

  In this thesis, we assume the packet arrival is according to a Poisson process. However, in real networks, Internet traffic is dynamic and some times bursty. Hence, Internet traffic can not be accurately modeled by the Poisson process. We may extend our model to general packet arrival process and capture the effects of the general Internet traffic model.

- **Cross-layer modeling**

  This thesis analyzes the delay performance of 802.11 DCF protocol under ideal channel condition. As channel error caused by multipath fading and interference is inevitable in the wireless environment, packet transmissions are severely corrupted by the channel errors under adverse channel conditions. Thus, the probability of unsuccessful packet transmission is determined not only by the contention in the MAC layer but also by the channel errors in the PHY layer. A cross-layer performance modeling of the 802.11 MAC and PHY layers may be extended from our current model to determine the delay and throughput of the 802.11 DCF under non-ideal channel condition.

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