ARE CANADIAN SMALL CAP STOCKS A SEPARATE ASSET CLASS? A MEAN-VARIANCE SPANNING APPROACH

Iain Scott

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ABSTRACT

Are Canadian Small Cap Stocks a Separate Asset Class? –A Mean-variance spanning approach

This paper introduces size based indices of Canadian markets using all firms listed on the Toronto Stock Exchange from December 31, 1969 to December 31, 2004 to assess the extent to which small-cap portfolios can enlarge the efficient frontier for investors. Both traditional and step-down spanning tests are performed. Furthermore, we evaluate the economic impact of adding Canadian small cap stocks by measuring the changes in the global minimum variance frontier and improvement in the Sharpe Ratio of the optimal portfolio. Canadian small-cap (as well as micro-cap) portfolios are shown to behave as separate classes, with performance enhancing effects for the entire period as well as various subperiods examined. The results are robust to the inclusion of alternative international indices to the benchmark portfolios, and policy constraints do not necessarily reduce the benefits from diversification.

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1.1 INTRODUCTION

The starting point for portfolio construction involves defining the asset classes that will constitute the portfolio and then determining the proportion of the available funds to be invested in each asset class. The allocation decision typically involves choosing between the following asset classes: domestic equities, foreign equities, corporate and government bonds, real estate, private equities, and the risk-free asset. However, portfolio managers also want to know if adding a non-traditional asset class enhances return-generating potential and mitigates portfolio risk. In this paper we employ both traditional and step-down mean-variance spanning tests to study the effects for Canadian investors of adding Canadian small capitalization stocks to various benchmark portfolios, and find out if Canadian small capitalization stocks should be considered a separate asset class when making the asset allocation decision.

Intuitively, adding a separate small cap asset class may enhance the mean-variance frontier of an existing portfolio for several reasons. Firstly, a substantial amount of work has been written ever since Banz (1981) and Reiganum (1981) first reported that small cap firms outperform large cap firms even after controlling for risk. These studies, using a wide range of methodologies, have covered many different time periods and data sets, but none have succeeded in explaining satisfactorily the 'small firm effect.' Even though none of the academic studies have led to a conclusive explanation, the extensive body of literature validates the hypothesis that size is an important variable in explaining the variation in the cross-section of expected returns. This evidence leads us to hypothesize that Canadian small capitalization stocks may provide measurable benefits. Secondly, interest in small-cap stocks and in size-related investing in general has motivated the creation of small-cap stock indexes around the world. The proliferation of index related products has substantially reduced transaction costs for investors interested in adding a small cap asset class to their existing portfolio; thereby removing a potential drag on profits for investors. Finally, empirical evidence from Reilly and Wright (2002), Eun et al. (2004), Petrella (2005) and Switzer and Fan (2006), show that the correlations between small-size portfolios and large-size portfolios are less than one and that the return generating mechanisms for large- and small cap stocks are quite different. This is important since the extent to which risk can be reduced is limited by the correlation between portfolio returns. Thus, small-cap stocks can potentially offer Canadian investors significant diversification benefits.

As we show in this paper, Canadian investors that included Canadian small cap firms as a separate asset class from 1970 to 2004 dramatically enhanced the performance of their portfolios. Over the same period, and during the subperiods examined, we also find that Canadian micro caps had a significant positive impact on the mean-variance frontier. Adding international indices to our study, confirmed that Canadian small cap stocks offer diversification benefits to investors already invested in the following indices: S&P 500, Russell 2000, FTSE 100 and MSCI EAFE. We also examine the issue from the perspective of a Canadian investor who has diversified internationally but with purely large cap indices. We find that there are additional gains for investors who choose diversify beyond the traditional large-cap indices and include Canadian small capitalization firms in their portfolios. For the cases in which mean-variance spanning is rejected we measure the ex-post diversification gains from expanding the investment opportunity set. In fact, this study clearly shows that adding various Canadian size based portfolios can enhance the global minimum variance portfolio and the 'optimal' portfolio, defined as that portfolio on the efficient frontier with the maximum Sharpe ratio (excess return divided by the standard deviation). Finally, the magnitude of these benefits is compared subject to different constraints, such as investors' ability to take short positions.

The rest of our paper is organized as follows. Section 2 briefly reviews some of the literature dealing with the small cap anomaly. Section 3 describes the data and the Huberman-Kandel regression based mean-variance spanning tests that measure the significance of adding a risky asset to an existing benchmark of risky assets. Section 4 reports the results of the LM tests, F-tests, and step-down spanning tests performed on each of the different cases. This section also measures the diversification benefits of each of the cases identified as having significantly enhanced mean-variance frontiers. Section 5 provides a summary and conclusions.

2.1 LITERATURE REVIEW

A substantial amount of work has been written about the 'small firm effect' over the past 25 years, since Banz (1981) and Reinganum (1981) first introduced investors to the possibility that over a long-term investment horizon the returns on small cap stocks can consistently outperform the returns on large cap stocks. That is, that average returns to small firms' stocks are substantially higher than any known capital asset pricing model predicts.

In his study, Banz examined the role of size of a stock and he found that smaller firms have had higher risk adjusted returns, on average, than larger firms. He documented that excess returns (alphas) would have been earned over the period 1936-1977 by holding low capitalization companies. The differential return from buying very small firms versus very large firms was 19.8% per year. The real payoff from holding small stocks came from holding the smallest 20% of firms in Banz's sample of NYSE firms while there is little difference in return between average sized and large firms. While the return from holding the smallest firms was large and statistically significant, on average there were periods where large firms outperformed small firms. The seminal paper by Banz provided the impetus for subsequent investigations into the small cap effect[1].

Once it was established that size may have a substantial role in explaining the variation of the cross-section of expected returns researchers attempted to explain the anomaly. Stoll and Whaley (1983) test whether the 'small firm effect' is a result of using gross returns rather than returns after transaction costs. They conclude that, during the period 1960 through 1979, after adjusting for both trading costs and market risk, small stocks earn lower returns than large stocks if held for two months or less. However, as the length of the investment horizon is increased, the effect is diminished; and the after-transaction-cost abnormal returns for the small firm portfolio became positive. Therefore, according to Stoll and Whaley, the existence of abnormal returns for small size-based portfolios is also contingent upon the length of the investment horizon.

Another hypothesis advanced to explain the small firm effect is the tax-loss-selling-pressure hypothesis. Keim (1983) finds that over the period from 1963-1979 about half the excess returns accruing to holders of small firms are earned in January. Furthermore, based on Keim's research, "more than twenty-six percent of the premium is attributable to large abnormal returns during the first week of January and almost eleven percent is attributable to the first trading day." According to the tax-loss-selling-pressure hypothesis, toward the end of the year investors sell stocks that have declined in price during the year. They do this to take advantage of the opportunity to write-off capital losses against ordinary income. In January, the selling pressure ceases and stocks prices rapidly rebound to their "equilibrium" levels. Berges et al. (1984) examine Canadian stock returns for a January effect and to determine whether the effect is concentrated among firms with small market values. They

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[1] A summary of the original debate on size anomaly can be found in the Special Issue "Symposium on size and stock returns, and other empirical regularities" of the Journal of Financial Economics (1983, vol. 12, no. 1).

find that there is a significant January effect in Canadian stock returns, and the effect is more pronounced for firms with smaller values. However, this pattern existed prior to the introduction of a capital gains tax in Canada; and the relationship between a measure of tax-loss-selling-pressure and January returns is statistically insignificant. The authors conclude that the evidence does not support the tax-loss-selling pressure hypothesis as the complete explanation of the "small firm" effect in either Canada or the U.S.

Chan and Chen (1988) propose that the size effect is an artifact of large measurement errors in betas that allow firm size to serve as a proxy for true beta. They report that when more accurate estimates of betas are employed, no size-related differences in average returns are observed. In contrast, Fama and French (1992, 1993, and 1996) find that two variables SIZE (stock price times number of shares outstanding) and BE/ME (book-to-market equity ratio), and not beta (β), explain the variation in the cross-section of portfolio returns.

In Canada, Elfakhani et al. (1998) test for a size effect in the Canadian stock market using stock returns of firms that traded on the TSE or on the MSE sometime during June 1975 through December 1992. Using cross-sectional regressions of returns on beta and firm size, they find a significant negative relation between firm market capitalization and average returns. More generally, this evidence suggests that size-based portfolios could, at least in principle, perform according to structurally different pattern and, thus form the basis for portfolio allocation.

The bulk of the early research into the 'size anomaly' focused on testing its robustness under different methodologies and using independent datasets. Unfortunately, none of this research supplies a satisfactory explanation of the size effect. Most recently, Dimson and Marsh (1999) conclude that from 1955 to 1989 the long-term outperformance of smaller U.K. companies over larger companies was approximately 6 percent per year. Over the subsequent decade, a reversal occurred and smaller companies underperformed larger companies by a similar margin. Horowitz et al. (2000) investigate the existence of a size effect for the period 1980-1996 using data from the NYSE, Amex and NASDAQ. Using three separate methodologies, they find no evidence that small firms have higher realized returns than do large firms. Reilly and Wright (2002) update the analysis of small-cap performance through the year 2000. They examine the annual returns of six different small-cap stock indexes for the period 1984-2000, and after adjusting for risk they find that small-cap stocks have underperformed. However, they also find that over time there have been two important trends: a negative trend in the correlations between small and large-cap stocks and lower interest rate sensitivity for small cap stocks. Both trends mean that small-caps are desirable asset for diversification purposes.

A number of authors believe that the contrasting evidence presented above means that the small firm effect is cyclical and driven by economic fundamentals. Reinganum (1992) argues that the relationship between market capitalization and the outperformance of smaller firms varies over time, and even exhibits predictable patterns. He recognized that accurately forecasting the performance of small-cap stocks and large-cap stocks could greatly enhance a portfolio's performance. To support his hypothesis Reinganum collects data on all common stocks on the NYSE over a long period (1926-1989). He examines the autocorrelations of the differential returns between the largest NYSE market cap decile portfolio and the other nine market cap portfolios over different investment horizons. The empirical results show that, over one-year and two-year horizons, the autocorrelations are positive but not significant, and become negative and significant for longer investment horizons. The five-year investment horizon's estimated autocorrelation is the lowest and has the highest significance. Reinganum concludes that based on the long-term evidence, periods in which large-cap stocks outperform small-cap ones tend to be followed by periods in which the relative performance is reversed.

Jensen, Johnson and Mercer (1997, 1998) examine the relationship between the small firm premium and monetary policy. They measure the small-firm premium across different time periods, that are broadly classified as either expansive (the month following a decrease in the Fed's discount rate) or restrictive (the month following an increase in the discount rate). The premium for small firms is positive and highly significant during periods of monetary expansion, but insignificant during periods of restrictive monetary policy.

In an attempt to explain why small and large firms have different responses to economic news, Chan and Chen (1991) look at structural differences between these two types of firms which may cause them to react differently to the same economic news. They point out that small firms are more likely to be inefficient producers. Furthermore small firms are more likely to be highly leveraged and have cash flow problems. They are also more likely to have limited access to capital markets, particularly during periods of tight credit conditions. A small cap portfolio will customarily hold a larger number of these types of firms than a large cap portfolio; therefore the returns will react differently to the same piece of economic news.

In addition, Christopherson, Ding and Greenwood (2002) look at small-cap excess returns and find that there is significant inverse relationship between small cap excess returns and both the level of assets under management and the annual increase in assets under management. They contend that as assets grow, so does the need for liquidity, therefore over time small cap portfolio managers will reduce their exposure to the smallest and generally least efficiently priced segment of the market, and increase their exposure to more liquid and more efficiently priced mid-cap stocks (with lower excess return potential).

These portfolio managers may be missing out on important return generating opportunities because new listings in the United States over the past twenty years have been dominated by smaller firms. This change in the fundamentals of new listings is carefully documented by Fama and French (2004) who examine the profile of new listings in the United States in the 1980s and 1990s. They document that the number of new listings in the United States explodes after 1979, from about 160 to near 550 per year. Typically, more than 95% of new lists are small (assets below the median for NYSE firms), so small firms dominate the cross sections of new list profitability. Overall, new listings are more negatively skewed in their profitability, more positively skewed in their growth, and have lower survival rates. They show that new lists, which are dominated by small firms, have riskier fundamentals thus portfolios of firms sorted according to market capitalization will be also be fundamentally different and may provide diversification benefits.

Eun et al. (2004) address the following question: Are there 'additional gains' from international diversification with small-cap stocks? Using data from the 20-year period (1980-1999), for ten developed countries with relatively open capital markets they create three market-capitalization-based funds, namely a large-cap, mid-cap, and small-cap fund for each country, in order to examine the potential international diversification benefits from size based investing for a U.S. (or dollar-based) investor. Using mean-variance spanning tests, Eun et al. find that international small-cap funds are not 'spanned' by country stock market indices. They also find that small-cap fund returns are driven primarily by local and idiosyncratic factors. In contrast, large-cap funds are made up of multinational firms whose returns are largely driven by common global factors. Thus, small-cap funds have relatively low correlations with large-cap funds and amongst each other. Eun et al. conclude that there are additional gains in implementing a small-cap diversification strategy; but caution that problems may arise for institutional investors wanting to make block trades since small-cap markets are much less liquid and these large investors must control transaction costs in order to obtain the benefits.

Based on the belief that size based portfolios' returns behave according to structurally different patterns Petrella (2005) investigates the hypothesis of euro area size-sorted portfolios as autonomous asset classes using mean-variance spanning tests. He constructs size-based portfolios according to three separate methodologies: threshold, quartiles and quintiles, and uses data from euro area stocks from December 31, 1998 through December 31, 2002. In his study the test for spanning of the smallest size portfolio relative to a set of benchmark assets (including international indices) strongly rejects the spanning hypothesis.

More recently, Switzer and Fan (2006) apply mean-variance spanning tests to replicable G-7 small cap portfolios and find that only the Japanese and Canadian small cap portfolios provide significant diversification benefits for U.S. investors. In this study we apply a similar methodology in order to determine whether Canadian small cap firms behave as a separate asset class when benchmarked against Canadian large and mid caps, and two international benchmarks. To my knowledge, this study is the first to investigate these effects using Canadian size-based portfolio data.

3.1 *DESCRIPTION OF THE DATA*

To investigate the effect on portfolio efficiency of including Canadian small cap stocks as a separate asset class, it is necessary to obtain time-series of returns on Canadian large cap, mid cap and small cap companies. These indices must be available using the same methodology, with no overlap between constituents, and in total return index form. These requirements were met by several publicly available indices on Canadian large and mid cap firms; however, existing small cap indices in Canada did not meet the necessary requirements, based on several dimensions.

Two small cap indices were considered and rejected, namely the BMO Nesbitt Burns Small Cap indices and S&P/TSX Small Cap Index. BMO Nesbitt Burns maintains two Canadian small cap indices: firstly, an unweighted or equally weighted index, which begins in 1969, and secondly a cap-weighted index that begins in December 1986. However, the BMO Nesbitt Burns Small-Cap indices which consist of 400 companies with an upper capitalization equal to 0.1% of the total market capitalization of the S&P/TSX Composite Index suffer serious shortcomings. They are not strictly a passive small-cap benchmark since there is some discretion in the addition/deletion of firms. And they have a "loser's bias" due to BMO Nesbitt Burns maintaining firms whose stock price has declined but is still in the 401 to 450th segment of firms. Most of all, the index lacks consistency in the timing of rebalancing. Prior to April 1, 1998 the Index was rebalanced monthly, from April 1, 1998 to October 1, 2000, rebalancing changed to semi-annually, and finally on October 1, 2000, to the present, BMO Nesbitt Burns switched to quarterly rebalancing. The only other Canadian small-cap index that was considered was the S&P/TSX Small Cap Index. This index is a subset of the S&P/TSX Composite Index and suffers from a "large company" tilt because it is simply the portion of the Composite Index that is neither in the S&P/TSX60 nor in the S&P/TSX MidCap Index. In addition, this index has only been in existence since 1999, and tracks the returns of only approximately 100 firms; whereas the BMO Nesbitt Burns Index includes 400 firms and the total number of TSX listed stocks is over 1,300. In constructing our indices, we sought to create a strictly passive benchmark which retains the benefits of cap weighting, truly represents the return characteristics of the aggregate equity market and is rebalanced annually throughout the sample period.

3.2 *PORTFOLIO FORMATION*

In order to create these indices, market data for all firms listed on the Toronto Stock Exchange from December 31, 1969 to December 31, 2004 was obtained from the TSE/Western database. The sample period was selected to cover a sufficiently long period and includes the effects of several business cycles. We obtained the monthly share prices (adjusted for dividends or stock splits), and the float adjusted number of shares outstanding from the database. Firms with missing market data were deleted from the sample and both of the dual class shares were included. The sample does not suffer from survivorship bias since delisted stocks have also been considered up until such time as they were delisted. For the analysis that follows, securities were grouped into portfolios on the basis of their total market value. The total market value of each stock was determined at the end of each month beginning with January 1970 using the following formula:

$$\# shs_{i_{t-1}} \times price_{i_t} = mktcap_{i_t}$$

Where, $\# shs_{i_{t-1}}$ is the number of shares outstanding at the end of the previous month for the i^{th} firm, $price_{i_t}$ and $mktcap_{i_t}$ are the monthly close on the Toronto Stock Exchange for the i^{th} firm and its market capitalization at the end of month t, respectively.

The stocks were then ranked according to their total market values at the end of each December and portfolios were created based on three different methodologies: threshold approach, size quartiles, and size quintiles. For the threshold approach, the portfolios were created using the following guidelines. The small cap portfolio included all stocks with a market capitalization between 0.00501% and 0.09355% of the total market capitalization, the mid cap portfolio included all stocks with a market capitalization greater than 0.09355% and less than or equal to 0.34699% of the total market capitalization, and the large cap portfolio included firms with market capitalization greater than 0.34699% of the total market capitalization. These bounds were chosen in order to approximate through time the following 2004 threshold values: the small cap bound is for stocks with market capitalization lower than or equal to one billion Canadian dollars, mid cap is for stocks with market capitalization greater than one billion Canadian dollars and lower than or equal to 3.75 billion Canadian dollars, and large cap stocks have market capitalization greater than 3.75 billion. These values are consistent with the divisions which separate the S&P/TSX Composite Index into the S&P/TSX 60, Mid-Cap, and Small Cap Indices in 2004, but enlarge the number of companies in the small cap portion.

In order to create the quartile- and quintile-based portfolios the following steps were necessary. First, all the firms in the sample are sorted in order of ascending market capitalization at the end of each calendar year (December 31 of year t). Then, we form quartile or quintile portfolios for the subsequent calendar year (t+1). Portfolios are rebalanced on a yearly basis.

3.3 *MEAN-VARIANCE SPANNING TESTS*

Whenever new assets are added to a portfolio, in general a shift will occur in the estimated mean-variance frontier. However, this shift may very well be the result of estimation error. In order to find out if the observed shift in the mean-variance frontier is statistically significant we must test for spanning. Spanning tests, first developed by Huberman and Kandel (1987, HK hereafter), reveal whether an asset offers additional diversification opportunities to assets already held in a portfolio. If the minimum-variance frontier of an initial set of K risky assets spans the minimum-variance frontier of a larger set of $N+K$ risky assets there is no net benefit of adding a small-cap portfolio to an existing benchmark portfolio.

HK propose a mean-variance spanning test based on regressions of the test assets on the benchmark assets:

$$R=XB+E \qquad [1]$$

where R is a $T \times 1$ vector of the returns on the test asset; X is a $T \times (K+1)$ matrix of the benchmark asset returns. The coefficient vector is denoted by $\mathbf{B}=[a, \beta]'$; and E is a $T \times 1$ vector of error terms ε_t. The null hypothesis of spanning, i.e. that the two minimum variance frontiers are identical is equivalent to the joint hypothesis:

$$H_0 : \alpha_j = 0 \text{, and } \sum_j \beta_j = 1 \quad [2]$$

The Lagrange multiplier (LM) and F-test for small samples (Kan and Zhou [2001]) are used to test H_0. We assume $T \geq K+2$ and $X'X$ is nonsingular. In order to obtain exact distributions of the test statistics, we also assume that the random vector E is uncorrelated with the returns of the benchmark assets and are independent and identically distributed as multivariate normal with mean zero and variance Σ.

If the null hypothesis is not rejected, and there is spanning, then for every test asset we can find a portfolio of the K benchmark assets that has the same mean but a lower variance than the test asset ($\underline{R}$ and $\underline{\varepsilon}$ are uncorrelated and Var[$\underline{\varepsilon}$]is positive definite). Hence, the addition of the test assets N can only add to the variance of portfolios of K benchmark assets, and not to the expected return. Therefore, mean-variance optimizing investors will not include such an asset in their portfolio. Adding the test asset will not offer diversification opportunities relative to those already included in the portfolio of benchmark assets K. This also implies that the Sharpe ratios are not statistically different (i.e. $S_{K+N} \approx S_K$). If the spanning test is rejected, then the inclusion of the test assets provides diversification advantages not available with the benchmark assets alone; thus these test assets should be included in any well-diversified portfolio.

3.4 *MEASURING THE EFFECTS OF DIVERSIFICATION* [2]

The first measure of the diversification benefit is obtained by comparing the global minimum variance (GMV) portfolio computed for the benchmark assets and the GMV portfolio computed for the benchmark and the additional asset(s). This will show if there are any benefits for investors from adding Canadian small cap stocks to their portfolio by measuring the reduction in portfolio risk.

Assume that the vector of the $K+N$ returns previously defined as ($\underline{R}$ and $\underline{r}$) has a multivariate normal distribution with mean $\mu_{K+N} = (\mu_1, \ldots, \mu_K, \ldots, \mu_{K+N})'$ and covariance matrix Ω_{K+N}. Let S be the set of all the real vectors $x = (x_1, \ldots, x_K, \ldots, x_{K+N})'$ such that $x_1 + \cdots x_N + \cdots x_{K+N} = 1$. The vector of weights in a portfolio is a point in S. A set of constraints on portfolio weights is represented by a closed convex subset C of S. For example, the case in which portfolio holdings are limited to the K benchmark assets can be represented by $C = \{x \in S : x_i = 0, K < i \leq K+N\}$. Given the above our first measure of diversification benefits from the GMV portfolios is the following:

$$\psi \equiv \min_x \left\{ \sqrt{\underline{x}_C{}' \Omega \underline{x}_C} \mid x_C \in C \right\} - \min_x \left\{ \sqrt{\underline{x}_S{}' \Omega \underline{x}_S} \mid x_S \in S \right\} \qquad [3]$$

Positive values of ψ indicate that adding the test asset improves the new GMV portfolio relative to the GMV limited to the benchmark assets.

A second measure of the diversification benefit is obtained by using the Sharpe ratio. The Sharpe Ratio measures the slope of the line from the riskless rate to the tangency

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[2] Li et al. (2003) describe in detail many of the existing diversification benefits measures.

portfolio on the efficient frontier. It gives the largest mean return per unit of standard-deviation risk attainable for the assets in question (Sharpe 1994). According to portfolio theory, when a risk-free asset exists and unlimited lending or borrowing is allowed, investors seek to maximize the Sharpe ratio. Thus, investors will want to know whether the tangency portfolio from using K benchmark assets is the same as the one from using all $N+K$ risky assets. In order to answer this question and measure the potential return enhancement investors can achieve per unit of risk by adding the additional N assets we calculate the following:

$$\lambda \equiv \max_{x}\left\{\frac{\underline{x}_S{}'\underline{\mu}}{\sqrt{\underline{x}_S{}'\Omega\underline{x}_S}} \mid x_S \in S\right\} - \max_{x}\left\{\frac{\underline{x}_C{}'\underline{\mu}}{\sqrt{\underline{x}_C{}'\Omega\underline{x}_C}} \mid x_C \in C\right\} \qquad [4]$$

Positive values of λ indicate an improvement in the Sharpe ratio from adding the additional N assets to the portfolio limited to the benchmark assets.

4.1 *PRELIMINARY CHARACTERISTICS OF THE PORTFOLIOS*

Summary statistics of the 12 portfolios, which are formed as described in Section 3, are presented in Table 1. We find that the threshold portfolios accurately reflect the fact that small capitalization stocks are, in terms of numbers, the largest group of stocks traded on the Toronto Stock exchange. Firstly, the number of firms in the large cap (58, 61, 56, 66, and 60) and mid cap (82, 84, 81, 86, and 83) portfolios remained relatively constant throughout the entire period, and are consistent with the number of firms in both the S&P/TSX60 and the S&P/TSX MidCap Indices. The S&P/TSX60 and the S&P/TSX MidCap are capitalization weighted indices that consist of the 60 largest firms and the next 60 largest firms on the Toronto Stock exchange. Secondly, the number of firms in the small cap portfolio (295, 373, 372, 608, and 500) varied considerably throughout the period, and included more firms than did the BMO Nesbitt Burns Small Cap Index. Therefore, the small cap portfolio we created is more representative of the aggregate small cap equity market than are the BMO indices. In sum, by looking at the number of firms in each of the threshold portfolios we find that they are consistent with other publicly available indices and accurately reflect the size-based divisions in the number of firms on the Toronto Stock exchange over the sample period.

In constructing our portfolios using three different approaches, we sought to compare results across a wide range of size based portfolios -including Canadian micro cap stocks; which are rarely considered by professional money managers because of liquidity and capacity constraints. Table 1 also reports the average, standard deviation, smallest, median, and largest estimates of market capitalization (in million CAD), and are obtained using monthly observations. The numbers in Table 1 show that the quartile and quintile portfolios include firms which are much smaller than the small cap portfolio created using the threshold approach. This confirms that our study will look at portfolios made up of the smallest listed firms in Canada. Looking more closely at average size of our small cap portfolio we find that in 1975 and 1982 it is larger than the two smallest quartiles and the three smallest quintiles. However, in 1989 the small cap portfolio's average size of $140.5 million is even larger than the 5Q4 portfolio (i.e. the average size of 80% of the firms on the Toronto Stock Exchange at that time was smaller than the average size of our small cap portfolio). By 1996, and in 2003, the average size of the small cap portfolio was larger than the three smallest quartile portfolios and the three smallest quintile portfolios. From examining the average size of the firms in each of the 12 different portfolios we see that numerically the Toronto Stock exchange is highly concentrated among small firms.

4.2 *ANALYSIS OF THE PORTFOLIO RETURNS*

Table 2 reports monthly return characteristics for each of the market value portfolios for the entire period 1970-2004 (420 months), and each of the subperiods (84 months). For every month, a portfolio's return is calculated as the sum of the market weighted returns of all the stocks in the portfolio based on the market weight of the previous month. Monthly returns are less susceptible to the biases from bid-ask effects and thin trading. The arithmetic and geometric average of monthly returns are calculated and will be used as a proxy for determining the future N-month portfolio return. The *raw* monthly portfolio returns provide mixed evidence of a size effect. For example, when we compare the arithmetic (geometric) average monthly returns of the threshold portfolios, it is the mid cap portfolio (1.03% (0.91%)) which outperforms the small cap portfolio (0.92% (0.79%)) and the large cap portfolio (0.87% (0.73%)) over the entire period. Over the same period, the arithmetic and geometric average of monthly returns for the quartile, and quintile, portfolios increases monotonically as the market capitalization the portfolios decline. This could be considered evidence of a small cap effect within the *raw* monthly returns. We also examine how the small cap premium has fared over time, since Dimson and Marsh (1999) show that the small firm premium may have reversed. Based upon the monthly threshold returns no discernable pattern emerges which would indicate a small firm effect, or a reversal. In contrast, the arithmetic and geometric average of quartile, and quintile monthly returns for the two final subperiods confirm a strong negative relationship when moving from the smallest to the largest size-based portfolio. Furthermore, based on the *raw* monthly returns the size premium has increased from the earliest subperiod to the latest subperiod.

Table 2 also presents the standard deviation of the portfolios return distributions. The standard deviation, as a proxy for risk among the size based portfolios, is only relevant for the quartile-and quintile-based portfolios. In the threshold approach to portfolio construction, there is a computational bias that implies an underestimation of risk for the smallest firm's portfolio since the number of constituents in that particular portfolio is much higher than that of the mid or large cap portfolios. Given that the dispersion of portfolio returns is inversely related to the number of stocks in a portfolio, it is meaningless to compare risk across portfolios based on the threshold approach. Subsequently, we turn our attention to the other two approaches to portfolio construction. Column 3 reports the standard deviation of the return distribution. We find that risk, as measured by the portfolio's standard deviation, is highest for the smallest size portfolio, and tends to decrease as size increases.

The returns for the smallest quartile, and quintile, portfolios exhibit positive skewness over the entire period. And skewness tends to decrease as market capitalization increases, whereas the opposite occurs with the threshold method. The third moment is incorporated in asset pricing models through the work of Kraus and Litzenberger (1976) and others. Positive skewness in returns can be described as the phenomenon where, after the returns have been standardized by subtracting the mean, positive returns of a given magnitude have higher probabilities than negative returns of the same magnitude or vice versa. Investors typically prefer positive skewness as this situation resembles a lottery payoff wherein a high payoff may be earned with low probability. If a financial asset's return distribution is positively skewed, then, all else constant, investors require a lower expected return to induce investment. This is not only logical but is also consistent with some empirical evidence that investors exhibit this preference.[3] A rationally functioning market only provides higher return expectations (i.e. an additional risk premium) if market returns are negatively skewed. In contrast, the positive skewness in the returns of the smallest quartile, and quintile portfolios should lead to lower return expectations and less of a risk

[3] Levy and Sarnat (1984) find a strong preference for positive skewness in the study of mutual funds. And Harvey and Siddique (2000) introduce an asset pricing model that incorporates conditional skewness, and show that an investor may be willing to accept a negative expected return in the presence of high positive skewness.

premium. The fourth moment has received little attention in asset pricing and behaves much like the second moment. A positive risk premium should be offered to investors as a reward for bearing higher kurtosis risk. Column 8 shows the kurtosis statistic for the return distributions of the market value portfolios. The kurtosis of the normal distribution is 3. The values from column 8 tend to be greater than three; therefore the return distributions are considered leptokurtic –a distribution that is "peaked," relative to the normal distribution. They appear to take less extreme values and exhibit a tendency towards the mean value. Our examination of the distribution of monthly portfolio returns also reveals strong evidence of non-normality, with Jarque-Bera tests rejecting the null hypothesis at 0.001 and beyond.[4]

4.3 *CORRELATION RESULTS*

Table 3 shows the correlation structure of returns. From this table we find that the correlations between small-size portfolios and large-size portfolios are all less than one. This is important since the extent to which risk can be reduced is limited by the correlation between portfolio returns. Also, diversification benefits vary inversely with the (average) correlation between portfolio returns. The table is divided into six panels. The first panel reports the correlation structure of the 12 portfolios over the entire sample period. The remaining panels cover each of the five subperiods. We compare each of the different periods in order to look for patterns of changes in correlations among the size-based portfolios. Panel A shows very significant correlation among the small cap portfolio 4Q2, 4Q3, 5Q3, and 5Q4 of between 0.91 and 0.97. This significant correlation is also confirmed in Panel B through F since the correlations remain between 0.86 and 0.99. Whereas the correlation among the small cap portfolio and the smallest portfolios constructed based on the quartile, and quintile, approach is much lower -between 0.74 and 0.76 over the entire

[4] The mean variance spanning approach ignores higher moments in the distribution. Expanding the analysis to account for other moments remains an issue for future investigation.

period. This would indicate that the smallest portfolios created using the quartile –quintile approach are different from the small cap threshold portfolio in terms of characteristics, and that the monthly rates of return do not tend to move together. The correlation coefficients of the smallest portfolios regardless of the construction method are never lower than 0.64 and the correlation coefficients between the largest portfolios tend to unity. Finally, the imperfect correlation between small and large cap returns has been declining over the sub-periods examined.

4.4 *RESULTS*

In order to provide a formal test of the effects Canadian small cap stocks have on portfolio efficiency we begin with mean-variance spanning tests on the size based portfolios using them as either test assets or benchmark assets. The results are summarized in Table 4, in which the first column indicates which portfolio is the test asset, and the second column shows which portfolios are the benchmark assets. Furthermore, column three and four report, respectively, the alpha and sum of the betas from the HK multivariate regression (equation 1) relating the returns on the N test assets as dependent variables and K benchmark assets as independent variables. The fifth column reports the adjusted-R^2 of the regression. Similar to Petrella (2005), we report the average Variance Inflation Factors (VIFs) of all the regressors in column six to detect possible collinearity, which may have a negative impact on the results of the LM test.

Spanning tests, as they are linear restrictions on the coefficients, can be performed using either the likelihood ratio (LR) the Wald (W) or the Lagrange multiplier (LM) test. Kan and Zhou (2001) explain that although LR, W, and LM all have an asymptotic X^2_{2N} distribution we must have $W \geq LR \geq LM$ in finite samples, therefore to be more conservative in the acceptance/rejection decision the LM test is used in this paper. Let U be the ratio of unconstrained and constrained maximum likelihood estimator of variance.

$$LM = T(1-U) \sim \chi_2^2 \qquad [5]$$

Results from the LM test are reported in column seven. For the entire period (Panel A) the test produces evidence against mean-variance spanning at the 5% level of significance for all of the combinations of assets based on the threshold approach to portfolio construction. The results of the test done on the size quartiles and quintiles, rejects the null hypothesis of spanning at 5% level, only when the smallest (i.e., 4Q1 and 5Q1) and largest (4Q4 and 5Q5) portfolios are used as test assets relative to the benchmark of the remaining portfolios. When looking at the five sub-periods (Panels B to F) we find that rejection of the null hypothesis is far more likely to be due to adding the largest portfolio to an investment set of the remaining portfolios than from adding the smallest portfolio. We reject the null hypothesis of spanning at the 5% level of significance each time the largest portfolio is the test asset from December 31, 1976 through December 31, 2004. However, as demonstrated by Gibbons, Ross, and Shanken (1989) these asymptotic tests could be grossly misleading in finite samples. Thus, in order to test the robustness of these results we use the finite distribution of the Wald F-test as written in Kan and Zhou (2001) when $N = 1$ to perform the following F-test for small samples:

$$\left[\left(\frac{1}{U}-1\right)\left(\frac{T-K-1}{2}\right)\right] \sim F_{2,T-K-1} \qquad [6]$$

The results are reported in column eight of Table 4 and are completely consistent with the LM test in terms of statistical significance. In sum, from the test results presented in Table 4 we find that adding the largest market capitalization portfolio to the benchmark assets has a significant impact on the mean-variance frontier over the entire period and in four of the five subperiods; whereas, a significant shift in the mean-variance frontier due to adding the small cap portfolios is only found when we test the entire period.

Adding international benchmark assets to the investment opportunity set should have an impact on the results obtained in Table 4. Two reasons lie behind this assumption; firstly, adding international assets will reduce the possibility that our results suffer from collinearity in the dependent variables, and secondly, including international assets is an important requirement for efficient portfolio diversification. We begin with a Canadian investor with a benchmark portfolio that consists of the S&P 500 for US large cap stocks, Russell 2000 for US small cap stocks, FTSE100 for UK large cap stocks and the MSCI EAFE portfolio.[5] We then test the effect of adding the Canadian market capitalization portfolios by adding the smallest portfolio from each approach to the international benchmark and if it is significant create a new benchmark which includes the significant test asset. Each time the spanning hypothesis is rejected, we form a new benchmark to which the next largest portfolio is added until there is no longer a significant shift in the mean-variance frontier. The results are shown in Table 5, each of the periods are divided into the three approaches to portfolio construction. We begin with the threshold approach for the entire period and find that adding any one of the three size based portfolios (S, M, or L) to the original benchmark is significant in the spanning test. Therefore we form new benchmark portfolios which include the international indices and one of the three different threshold portfolios against which we test adding the next largest portfolio. We find that adding the medium cap portfolio is not significant, whereas adding the small cap portfolio to the new benchmark is significant. Furthermore, during the first and second subperiods, we find that there is one case which rejects the spanning hypothesis. More specifically, the tests reject the

[5] Canadian dollar denominated returns are used to eliminate currency effects.

spanning hypothesis only when the small cap portfolio is added to the benchmark of international assets during the second subperiod. Our analysis continues with the size quartiles, and size quintiles, when we include the 4Q1, 4Q2, 5Q1 or 5Q2 returns as test assets, the tests reject spanning in 19 out of 26 cases which provides strong evidence that diversification benefits exist from adding Canadian micro cap stocks to an internationally diversified portfolio.[6]

To further assess the potential of Canadian small-cap stocks as a vehicle for international diversification we examine the issue from the perspective of a Canadian investor who has an internationally diversified portfolio of purely large-cap stocks. In order to accomplish this we modified the initial reference portfolio so that it is a large cap benchmark portfolio (LCB) made up of the following popular country indices: S&P500 (U.S.), Nikkei 300 (Japan), FTSE100 (U.K.), CAC40 (France), DAX30 (Germany), and the SPTSX (Canada).[7] We repeat the same procedure of adding the smallest Canadian size-based portfolio, from each portfolio construction approach, to the benchmark until the test asset no longer has a significant impact on the mean-variance frontier. The results are presented in Table 6 and the sample period covers December 31, 1987 through December 31, 2004. Looking at the results of the spanning tests on the threshold portfolios during the entire period, we reject the spanning hypothesis at the 5% level when the small-cap portfolio is the test asset and the LCB asset. Moreover, we also reject the spanning hypothesis at the 5% level when the small-cap is the test asset and the international large cap benchmark has had the Canadian mid-cap portfolio added to it. This provides strong evidence of diversification benefits for a Canadian investor who adds small-cap stocks to their portfolio. For the size

[6] We also performed the same spanning tests as those in Table 5 on benchmark assets which included the largest size based portfolio from each of the portfolio construction approaches and international indices. This is so that we could identify any additional cases which offer diversification benefits.

[7] Canadian dollar denominated returns.

quartile portfolios, we find that adding the 4Q1, 4Q2 and 4Q3 to the LCB led to rejecting the spanning hypothesis at the 5% level. Furthermore, adding the 4Q1 portfolio to a benchmark which included either the 4Q2 and 4Q3 portfolios with the international large caps also led to rejecting the spanning hypothesis. The spanning tests performed on the quintile portfolios confirm that Canadian micro caps have a significant impact on the efficient frontier. We reject the spanning hypothesis at the 5% level each time the 5Q1, 5Q2, 5Q3, or 5Q4 portfolio is added to LCB. This confirms that adding Canadian micro caps can be beneficial to an existing portfolio of popular large cap country indices.

Finally, because the test of spanning is a joint test of $\alpha_j = 0_N$ and $\sum_j \beta_j = 1$ and it weighs the estimates $\hat{\alpha}$ and $\hat{\delta}$ according to their statistical accuracy, the spanning test emphasizes $\hat{\delta}$ because it can be estimated more accurately than $\hat{\alpha}$. A small difference in the GMV portfolio as measured by the second part of the test ($\sum_j \beta_j = 1$) is not necessarily economically significant, may overshadow large changes in the Sharpe ratio which can be of great economic importance, but more difficult to detect statistically. Kan and Zhou (2001) propose an alternative step-down test to mitigate this problem. The step-down procedure is a sequential test. We first test $\alpha_j = 0_N$, and then test $\sum_j \beta_j = 1$ but conditional on the constraint $\alpha_j = 0_N$. To test $\alpha_j = 0_N$ we perform the following F-test:

$$F_1 = \left[\left(\frac{|\bar{\Sigma}|}{|\hat{\Sigma}|} - 1\right)\left(\frac{T-K-N}{N}\right)\right] \sim F_{N,T-K-N} \qquad [6]$$

Where $\hat{\Sigma}$ is the unconstrained estimate of Σ and $\bar{\Sigma}$ is the constrained estimate of Σ by imposing the only constraint of $\alpha_j = 0_N$. Now to test $\sum_j \beta_j = 1$ but conditional on the constraint $\alpha = 0_N$, we use the following F-test:

$$F_2 = \left[\left(\frac{|\tilde{\Sigma}|}{|\bar{\Sigma}|} - 1 \right) \left(\frac{T-K-N+1}{N} \right) \right] \sim F_{N,T-K-N+1} \qquad [7]$$

Where $\tilde{\Sigma}$ is the constrained estimate of Σ by imposing both the constraints of $\alpha_j = 0_N$ and $\sum_j \beta_j = 1$. Under the step-down procedure, we will accept the spanning hypothesis if we accept both tests. In other words if we cannot reject $\alpha_j = 0_N$ and $\sum_j \beta_j = 1$, then adding the test asset has no beneficial effect on the efficient frontier. Furthermore, the two-step test allows us to identify what is causing the rejection. If the rejection is due to the first test, it means that the two tangency portfolios are statistically very different. If the rejection is due to the second test, it means that the two global minimum-variance portfolios are statistically very different. Table 7 reports the results of the above tests on each of the cases identified in Table 5 that offer significant diversification benefits to investors, and Table 8 reports the results of the step-down test on each of the cases identified in Table 6.

By looking at the F1-test and the F2-test results in Table 7 we find that rejection of the spanning hypothesis is usually due to statistically different global-minimum variance portfolios. More specifically, in the entire period from 1986 to 2004 we find that the source of the spanning hypothesis rejection from adding Canadian size based portfolios is due to an enhancement of the global minimum variance portfolio in 21 of the 33 cases, rather than an enhancement of the tangency portfolio which occurs in 9 of the cases. Rejection of the spanning hypothesis occurs in the remaining cases due to a significant enhancement to both the GMV and tangency portfolio. During the first subperiod the results of the step-down tests show that only the portfolios formed by the smallest firms in our sample lead to significant enhancement of the tangency portfolio. During the second subperiod we find that the threshold small cap portfolio enhances the GMV portfolio and the 4Q1 portfolio enhances the tangency portfolio. Table 9 presents the results of the step-down spanning test used to identify the source of the diversification benefits from adding the Canadian size based portfolios to the international large cap benchmark. Overall we find that there are fewer cases that have significant improvements in either the GMV portfolio or the tangency portfolio. This is not surprising since the LCB includes the returns from the SPTSX, while the cases which do offer diversification benefits are concentrated within the Canadian small cap portfolios.

In order to confirm these results we measure the ex post diversification gains from expanding the investment opportunity set using the Ibbotson optimization software to compute ψ and λ from equations [3] and [4].[8] The difference of the standard deviation of the global minimum variance (GMV) portfolios and the changes in the Sharpe ratio of the tangency portfolio captures risk reduction effects and improvements in the risk-reward potential, respectively. Similar to Petrella (2005) we also examine the impact of policy constraints on the potential diversification benefits of Canadian market capitalization portfolios:

$$\sum_{i+1}^{K+N} x_i = 1 \text{ and } -1 \le x_i \le 1 \text{ (unconstrained)} \qquad [8a]$$

[8] The risk-free rate is assumed to be zero. According to Jorion (1985) the zero risk-free rate assumption with monthly returns reduces the estimation risk that a positive risk-free rate would have on the tangency portfolio.

$$\sum_{i+1}^{K+N} x_i = 1 \text{ and } 0 \le x_i \le 1 \text{ (no short sales)} \quad [8b]$$

$$\sum_{i+1}^{K+N} x_i = 1 \text{ and } 0 \le x_i \le 0.5 \text{ (upper bound)} \quad [8c]$$

The diversification benefits are reported in Table 10. In this table we compare each of the expanded portfolios with an initial portfolio, which consists of the international indices. Overall, we find that policy constraints reduce the gains of GMV portfolios and the optimal Sharpe ratio portfolios. Looking at the diversification benefits from the threshold approach to portfolio construction for the period January 1986 to December 2004, we find that the GMV portfolio for the investment opportunity set which includes the small cap portfolio in addition to the mid cap or the large cap portfolio, and international benchmarks has a lower variance regardless of the policy constraints in place. The threshold approach provides little evidence of an improvement in the Sharpe Ratio. From Table 8 we find that the largest improvements occur in the second subperiod from adding the mid cap portfolio to the investment opportunity set. This is also consistent with the results in Table 6 which showed that the rejection of the spanning hypothesis due to a significant change in the tangency portfolio occurred at the 5% level only once: when we added the mid cap portfolio to the benchmark assets of large cap and international indices in the second subperiod. This also shows that the significance tests of step-down spanning are reflected in the performance measures reported in Table 8. Secondly, we examine the potential diversification benefits from the quartile portfolios. Adding the 4Q1 and 4Q3 portfolio to the benchmark portfolio of international indices has the largest impact when compared with the Sharpe ratio of the large cap benchmark portfolio. Finally, among the different opportunity sets which include the quintile portfolios it is the smallest of the quintile portfolios which provides the best improvement to the optimal Sharpe ratio portfolio. The potential gains from adding Canadian micro cap stocks to an existing portfolio can be seen more clearly by examining Figure 1 & 2. Figure 1 shows the efficient frontier of the initial benchmark (S&P 500, Russell 2000, FTSE 100, and MSCI EAFE) and the new efficient frontier after the incorporation of Canadian micro caps (4Q1) as test assets. Figure 2 displays the results of the same comparison over the period January 1998 to December 2004.

5.1 *CONCLUSION*

Identifying sources of diversification benefits for investors is an important goal for financial academics and spanning tests are a useful tool in the search for those assets which may enhance the efficient frontier. We examine the diversification effects of adding various portfolios of Canadian firms grouped by market capitalization to find out if Canadian small cap stocks should be considered a separate asset class. The results contribute to the literature in three ways. First, using mean-variance spanning tests we demonstrate that adding Canadian small capitalization firms to an existing benchmark of Canadian firms during the December 31, 1969 through December 31, 2004 period does have a significant impact on the efficient frontier. Second, the benefits are robust to investors already diversified with international assets. Finally, the ex post diversification benefits remain despite constraints on investors.

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Table 1 – Summary statistics for Canadian size based portfolios

This table reports descriptive statistics for each of the size based portfolios created according to the threshold, quartile, or quintile approach; furthermore for comparison we also include the entire investment universe and the median portfolios. Column 2 reports the number of stocks in each of the size based portfolios. Columns 3 through 7 report the average size, standard deviation, smallest stock, median stock and largest stock respectively. Each panel (A through E) provides a chronological end-of-year snapshot of the portfolios throughout the sample period which is from December 31, 1969 through December 31, 2004. The data was obtained from the TSE/Western database and the market capitalization values of each firm is based on the number of shares outstanding at the end of the previous month multiplied by the monthly close on the Toronto Stock Exchange.

		Market capitalization (millions of dollars)				
Portfolio	N. of stocks	Average size	Standard deviation	Smallest stock	Median stock	Largest stock
Panel A: December 1975						
Investment Universe	581	107.6	542.3	0.0	11.5	11,186.6
Small cap	295	17.2	13.3	3.2	11.7	57.5
Mid cap	82	115.4	47.1	58.7	102.6	216.1
Large cap	58	823.4	1,548.0	220.3	488.8	11,186.6
4Q1 (Smallest)	146	1.4	0.8	0.0	1.3	3.1
4Q2	145	7.2	2.4	3.2	7.2	11.5
4Q3	145	25.9	11.2	11.6	24.2	55.0
4Q4 (Largest)	145	396.6	1,035.4	55.9	172.4	11,186.6
5Q1 (Smallest)	117	1.1	0.6	0.0	1.0	2.2
5Q2	116	4.8	1.8	2.2	4.7	8.1
5Q3	116	12.6	3.5	8.1	11.6	19.7
5Q4	116	40.8	16.1	20.0	37.2	75.0
5Q5 (Largest)	116	479.6	1,143.6	75.5	218.2	11,186.6
Median 1 (Smallest)	291	4.3	3.4	0.0	3.1	11.5
Median 2 (Largest)	290	211.3	754.2	11.6	55.4	11,186.6
Panel B: December 1982						
Investment Universe	1,065	547.1	2,912.3	0.1	25.6	63,152.4
Small cap	372	140.5	122.5	29.3	92.3	538.1
Mid cap	81	1,077.0	439.9	551.1	942.7	1,997.4
Large cap	56	7,827.4	10,259.7	2,030.2	4,232.1	63,152.4
4Q1 (Smallest)	180	2.6	1.5	0.2	2.3	5.7
4Q2	180	11.6	4.4	5.8	11.1	20.4
4Q3	179	47.7	20.2	20.6	44.9	88.4
4Q4 (Largest)	180	670.1	1,405.8	89.2	281.3	13,097.4
5Q1 (Smallest)	144	2.0	1.0	0.2	1.9	4.1
5Q2	144	7.6	2.3	4.1	7.4	12.5
5Q3	143	21.9	6.7	12.6	20.4	37.1
5Q4	144	72.1	24.0	37.3	69.5	128.0
5Q5 (Largest)	144	811.2	1,540.6	129.1	373.4	13,097.4
Median 1 (Smallest)	360	7.1	5.6	0.2	5.7	20.4
Median 2 (Largest)	359	359.8	1,041.8	20.6	89.2	13,097.4

Table 1 – Summary statistics for Canadian size based portfolios (continued)

		Market capitalization (millions of dollars)				
	N. of stocks	Average size	Standard deviation	Smallest stock	Median stock	Largest stock
Panel C: December 1989						
Investment Universe	1,065	547.1	2,912.3	0.1	25.6	63,152.4
Small cap	372	140.5	122.5	29.3	92.3	538.1
Mid cap	81	1,077.0	439.9	551.1	942.7	1,997.4
Large cap	56	7,827.4	10,259.7	2,030.2	4,232.1	63,152.4
4Q1 (Smallest)	267	2.4	1.3	0.1	2.2	5.1
4Q2	266	13.2	5.8	5.1	12.2	25.6
4Q3	266	63.6	30.5	26.1	54.1	138.3
4Q4 (Largest)	266	2,111.1	5,547.7	139.5	567.9	63,152.4
5Q1 (Smallest)	213	1.9	0.9	0.1	1.8	3.5
5Q2	213	7.9	3.1	3.6	7.5	14.0
5Q3	213	27.1	9.1	14.1	25.6	46.0
5Q4	213	105.5	46.8	46.2	93.2	221.6
5Q5 (Largest)	213	2,592.9	6,107.4	223.8	797.0	63,152.4
Median 1 (Smallest)	533	7.8	6.9	0.1	5.1	25.6
Median 2 (Largest)	532	1,087.3	4,050.9	26.1	138.9	63,152.4
Panel D: December 1996						
Investment Universe	1,173	627.1	2,714.7	0.8	75.2	57,718.1
Small cap	608	186.2	158.3	36.9	126.6	683.2
Mid cap	86	1,269.8	508.2	691.3	1,163.6	2,446.7
Large cap	66	7,674.5	8,783.2	2,620.6	4,863.3	57,718.1
4Q1 (Smallest)	294	11.2	5.6	0.8	10.7	21.6
4Q2	293	42.2	14.5	21.7	40.0	75.2
4Q3	293	145.7	52.3	75.5	135.6	268.7
4Q4 (Largest)	293	2,311.5	5,076.7	278.0	720.4	57,718.1
5Q1 (Smallest)	235	9.1	4.2	0.8	9.2	16.9
5Q2	234	28.9	8.1	16.9	28.2	44.4
5Q3	235	76.6	22.8	44.5	75.2	121.0
5Q4	234	214.8	74.4	121.5	191.9	371.9
5Q5 (Largest)	235	2,801.7	5,562.5	373.8	1,044.4	57,718.1
Median 1 (Smallest)	587	26.6	19.0	0.8	21.6	75.2
Median 2 (Largest)	586	1,228.6	3,747.0	75.5	273.3	57,718.1

Table 1 – Summary statistics for Canadian size based portfolios (continued)

		Market capitalization (millions of dollars)				
	N. of stocks	Average size	Standard deviation	Smallest stock	Median stock	Largest stock
Panel E: December 2003						
Investment Universe	1,013	1,066.8	3,780.2	0.3	96.2	40,553.3
Small cap	500	260.0	238.3	54.1	155.9	993.2
Mid cap	83	2,037.2	777.6	1,011.0	1,827.0	3,749.8
Large cap	60	12,879.2	9,392.7	3,761.1	9,453.2	40,553.3
4Q1 (Smallest)	254	15.5	8.7	0.3	15.1	31.2
4Q2	253	58.6	18.1	31.4	56.6	96.2
4Q3	253	189.9	80.4	97.1	162.8	405.3
4Q4 (Largest)	253	4,007.3	6,767.0	408.4	1,310.0	40,553.3
5Q1 (Smallest)	203	18.4	7.3	6.6	17.7	31.2
5Q2	202	41.3	11.0	24.9	40.1	61.9
5Q3	203	98.7	24.6	62.4	96.2	148.2
5Q4	202	301.0	132.0	148.4	253.8	588.4
5Q5 (Largest)	203	4,871.7	7,302.5	594.6	1,827.0	40,553.3
Median 1 (Smallest)	507	37.0	25.8	0.3	31.2	96.2
Median 2 (Largest)	506	2,098.6	5,148.2	97.1	406.8	40,553.3

Table 2 – Portfolio Returns

This table reports monthly percentage returns for each of the size based portfolios, a portfolio's return is calculated as the sum of the market weighted returns of all stocks in the portfolio based on the market weight of the previous month. The table reports the number of observations, the time-series arithmetic and geometric average of the returns, the standard deviation, the minimum return, the median return, the maximum return, the skewness coefficient, kurtosis, and Jarque-Bera statistic for each portfolio. Panel A reports monthly returns for the entire period 1970-2004 (420 months), whereas Panels B through F cover each of the subperiods (84 months).

	N. of obs.	Arithmetic average	Geometric average	Standard deviation	Min	Median	Max	Skewness	Kurtosis	Jarque-Bera
Panel A: entire period 1970/1 - 2004/12										
Small cap	420	0.92	0.79	5.03	-25.98	1.12	24.61	-0.59	6.60	250.88
Mid cap	420	1.03	0.91	5.04	-23.54	1.15	20.18	-0.34	5.60	125.79
Large cap	420	0.87	0.73	5.35	-20.31	0.82	48.05	1.14	17.97	4010.59
4Q1(Smallest)	420	1.81	1.55	7.43	-28.32	1.31	46.23	0.77	7.64	417.84
4Q2	420	1.00	0.84	5.69	-27.43	1.10	28.12	-0.17	5.95	154.25
4Q3	420	0.94	0.80	5.32	-27.31	1.10	37.86	0.13	10.58	1007.07
4Q4(Largest)	420	0.88	0.77	4.67	-21.24	0.83	16.41	-0.47	5.36	112.74
5Q1(Smallest)	420	1.74	1.43	8.12	-27.71	1.50	53.06	1.09	8.87	686.80
5Q2	420	1.24	1.06	5.88	-27.61	1.31	33.71	0.13	6.77	249.52
5Q3	420	1.20	1.04	5.55	-28.38	1.24	23.45	-0.35	6.18	185.34
5Q4	420	0.98	0.84	5.21	-28.34	1.19	20.06	-0.59	6.42	228.68
5Q5(Largest)	420	0.84	0.72	4.76	-21.18	0.80	17.38	-0.35	5.30	100.71
Panel B: first subperiod 1970/1 - 1976/12										
Small cap	84	0.74	0.57	5.83	-14.05	0.74	24.61	0.39	5.78	29.01
Mid cap	84	0.56	0.38	5.98	-12.87	0.27	20.18	0.43	4.47	10.08
Large cap	84	0.95	0.70	7.48	-19.88	0.96	48.05	2.72	20.49	1174.34
4Q1(Smallest)	84	1.23	1.02	6.65	-16.18	1.36	30.85	0.87	7.27	74.25
4Q2	84	0.81	0.64	6.04	-12.35	1.00	28.12	0.81	7.04	66.16
4Q3	84	0.69	0.50	6.54	-13.25	0.50	37.86	2.07	14.47	520.96
4Q4(Largest)	84	0.50	0.39	4.81	-9.83	0.49	16.41	0.16	3.79	2.53
5Q1(Smallest)	84	1.28	1.06	6.83	-16.08	1.52	31.25	0.85	6.82	61.05
5Q2	84	0.99	0.80	6.37	-12.39	1.27	33.71	1.42	10.28	213.68
5Q3	84	1.02	0.83	6.26	-15.69	0.58	21.26	0.27	4.40	7.88
5Q4	84	0.70	0.56	5.44	-12.58	0.40	20.06	0.43	4.76	13.36
5Q5(Largest)	84	0.57	0.42	5.45	-10.01	0.36	17.38	0.60	4.09	8.98

Table 2 – Portfolio Returns (continued)

	N. of obs.	Arithmetic average	Geometric average	Standard deviation	Min	Median	Max	Skewness	Kurtosis	Jarque-Bera
Panel C: second subperiod 1977/1 - 1983/12										
Small cap	84	1.72	1.56	5.54	-15.48	2.46	12.42	-0.86	4.02	13.51
Mid cap	84	1.89	1.73	5.59	-18.59	2.43	13.77	-0.73	4.32	13.33
Large cap	84	1.53	1.40	5.24	-17.34	1.63	14.87	-0.53	4.32	9.78
4Q1(Smallest)	84	1.86	1.57	7.79	-18.79	0.56	29.70	0.69	5.22	23.60
4Q2	84	1.87	1.69	5.91	-16.34	2.93	14.86	-0.64	4.00	9.05
4Q3	84	1.98	1.83	5.40	-15.89	2.65	13.72	-0.75	3.86	10.27
4Q4(Largest)	84	1.59	1.45	5.27	-17.54	1.88	14.07	-0.63	4.36	11.82
5Q1(Smallest)	84	1.66	1.29	8.87	-20.42	0.24	34.96	1.00	5.60	37.18
5Q2	84	1.85	1.67	5.90	-15.87	2.44	16.58	-0.57	4.20	9.46
5Q3	84	1.93	1.77	5.60	-16.17	2.23	13.25	-0.73	4.05	11.11
5Q4	84	2.03	1.88	5.63	-17.48	2.34	14.37	-0.76	4.19	12.69
5Q5(Largest)	84	1.56	1.43	5.25	-17.40	1.90	13.99	-0.62	4.30	11.16
Panel D: third subperiod 1984/1 - 1990/12										
Small cap	84	0.43	0.32	4.60	-25.98	0.39	10.38	-2.16	14.37	515.24
Mid cap	84	0.72	0.60	4.65	-23.54	0.48	11.34	-1.36	10.61	227.59
Large cap	84	0.52	0.42	4.46	-20.31	0.57	14.27	-0.81	8.05	98.06
4Q1(Smallest)	84	0.52	0.31	6.47	-28.32	0.75	23.18	-0.83	8.49	114.99
4Q2	84	-0.26	-0.38	4.87	-27.43	-0.23	14.16	-1.78	13.54	431.55
4Q3	84	0.28	0.16	4.84	-27.31	0.43	11.00	-2.12	14.12	494.12
4Q4(Largest)	84	0.55	0.45	4.38	-21.24	0.59	13.44	-1.08	9.43	160.52
5Q1(Smallest)	84	0.61	0.35	7.06	-27.71	0.95	27.45	-0.56	8.08	94.65
5Q2	84	-0.04	-0.18	5.16	-27.61	-0.83	14.80	-1.35	11.78	294.45
5Q3	84	0.55	0.40	5.44	-28.38	0.49	23.45	-0.85	14.35	460.78
5Q4	84	0.27	0.14	4.93	-28.34	0.61	12.35	-2.19	15.14	580.97
5Q5(Largest)	84	0.55	0.45	4.39	-21.18	0.54	13.56	-1.05	9.32	154.79

Table 2 – Portfolio Returns (continued)

	N. of obs.	Arithmetic average	Geometric average	Standard deviation	Min	Median	Max	Skewness	Kurtosis	Jarque-Bera
				Panel E: forth subperiod 1991/1 - 1997/12						
Small cap	84	1.18	1.11	3.71	-9.32	1.15	11.00	-0.10	3.04	0.16
Mid cap	84	0.91	0.85	3.54	-7.49	1.13	9.03	-0.12	2.62	0.71
Large cap	84	1.01	0.95	3.42	-5.58	1.29	8.14	0.06	2.14	2.64
4Q1(Smallest)	84	2.94	2.78	5.84	-12.17	2.34	22.10	0.51	3.73	5.41
4Q2	84	1.43	1.31	5.05	-10.02	0.92	13.04	0.18	2.78	0.58
4Q3	84	1.19	1.11	3.82	-8.64	1.06	10.98	-0.15	2.83	0.41
4Q4(Largest)	84	1.05	0.99	3.34	-5.86	1.21	9.06	0.15	2.32	1.94
5Q1(Smallest)	84	2.96	2.75	6.81	-12.44	2.16	34.25	1.31	7.39	90.43
5Q2	84	2.04	1.91	5.21	-9.91	1.46	16.90	0.29	3.22	1.30
5Q3	84	1.44	1.33	4.65	-11.74	1.53	14.63	-0.02	3.10	0.04
5Q4	84	1.28	1.21	3.76	-8.88	1.40	10.89	-0.20	2.91	0.58
5Q5(Largest)	84	1.04	0.99	3.35	-5.85	1.24	9.12	0.16	2.32	1.99
				Panel F: fifth subperiod 1998/1 - 2004/12						
Small cap	84	0.52	0.38	5.19	-20.87	1.38	14.33	-0.75	5.23	25.29
Mid cap	84	1.10	0.97	5.11	-18.60	1.59	16.12	-0.40	4.96	15.75
Large cap	84	0.33	0.19	5.36	-19.02	0.63	10.58	-0.77	4.22	13.52
4Q1(Smallest)	84	2.50	2.07	9.70	-17.09	2.28	46.23	1.18	6.89	71.69
4Q2	84	1.16	0.96	6.35	-20.57	2.17	21.33	-0.24	4.48	8.47
4Q3	84	0.56	0.40	5.62	-21.82	1.23	17.60	-0.64	5.34	24.60
4Q4(Largest)	84	0.69	0.55	5.28	-19.38	0.93	14.45	-0.61	4.87	17.36
5Q1(Smallest)	84	2.20	1.72	10.39	-18.56	1.93	53.06	1.56	8.99	158.57
5Q2	84	1.34	1.13	6.55	-18.25	1.72	24.00	0.06	4.18	4.95
5Q3	84	1.05	0.89	5.73	-20.14	2.05	18.40	-0.52	4.65	13.18
5Q4	84	0.59	0.42	5.96	-21.23	1.29	17.24	-0.41	4.54	10.62
5Q5(Largest)	84	0.45	0.32	5.07	-19.37	0.78	11.21	-0.88	4.80	21.78

Table 3 – Returns correlation matrix

This table reports the contemporaneous correlation structure of size-based portfolio returns.

Panel A: entire period 1970/1 - 2004/12

	S	M	L	4Q1	4Q2	4Q3	4Q4	5Q1	5Q2	5Q3	5Q4	5Q5
S	1.00											
M	0.89	1.00										
L	0.74	0.78	1.00									
4Q1	0.76	0.65	0.52	1.00								
4Q2	0.92	0.81	0.67	0.81	1.00							
4Q3	0.95	0.86	0.70	0.76	0.89	1.00						
4Q4	0.85	0.89	0.89	0.63	0.78	0.81	1.00					
5Q1	0.74	0.64	0.51	0.95	0.81	0.74	0.61	1.00				
5Q2	0.89	0.78	0.63	0.84	0.96	0.87	0.75	0.80	1.00			
5Q3	0.91	0.79	0.66	0.78	0.92	0.90	0.76	0.77	0.87	1.00		
5Q4	0.97	0.89	0.74	0.77	0.91	0.96	0.86	0.76	0.89	0.89	1.00	
5Q5	0.83	0.88	0.90	0.61	0.76	0.82	0.97	0.59	0.72	0.75	0.84	1.00

Panel B: first subperiod 1970/1 - 1976/12

	S	M	L	4Q1	4Q2	4Q3	4Q4	5Q1	5Q2	5Q3	5Q4	5Q5
S	1.00											
M	0.89	1.00										
L	0.69	0.70	1.00									
4Q1	0.96	0.86	0.57	1.00								
4Q2	0.99	0.91	0.68	0.95	1.00							
4Q3	0.88	0.80	0.62	0.85	0.86	1.00						
4Q4	0.91	0.91	0.78	0.84	0.90	0.81	1.00					
5Q1	0.95	0.86	0.58	0.99	0.94	0.84	0.83	1.00				
5Q2	0.97	0.89	0.62	0.96	0.98	0.85	0.87	0.95	1.00			
5Q3	0.97	0.85	0.67	0.92	0.95	0.90	0.88	0.92	0.92	1.00		
5Q4	0.96	0.94	0.72	0.93	0.95	0.92	0.93	0.92	0.94	0.95	1.00	
5Q5	0.86	0.84	0.72	0.80	0.85	0.84	0.94	0.79	0.82	0.85	0.89	1.00

Panel C: second subperiod 1977/1 - 1983/12

	S	M	L	4Q1	4Q2	4Q3	4Q4	5Q1	5Q2	5Q3	5Q4	5Q5
S	1.00											
M	0.92	1.00										
L	0.85	0.93	1.00									
4Q1	0.79	0.72	0.64	1.00								
4Q2	0.96	0.89	0.82	0.81	1.00							
4Q3	0.98	0.93	0.86	0.77	0.94	1.00						
4Q4	0.88	0.96	1.00	0.67	0.85	0.89	1.00					
5Q1	0.75	0.69	0.62	0.97	0.80	0.74	0.64	1.00				
5Q2	0.96	0.89	0.82	0.86	0.97	0.94	0.85	0.82	1.00			
5Q3	0.94	0.87	0.80	0.79	0.96	0.93	0.82	0.77	0.92	1.00		
5Q4	0.98	0.95	0.89	0.77	0.94	0.98	0.91	0.73	0.94	0.91	1.00	
5Q5	0.87	0.96	1.00	0.66	0.84	0.88	0.99	0.64	0.84	0.81	0.90	1.00

Symbol	**description**
S, M, L	Small cap, mid cap, large cap portfolios
4Qx	x-th quartile portfolio
5Qx	x-th quintile portfolio

Table 3 – Returns correlation matrix (continued)

Panel D: third subperiod 1984/1 - 1990/12												
	S	M	L	4Q1	4Q2	4Q3	4Q4	5Q1	5Q2	5Q3	5Q4	5Q5
S	1.00											
M	0.95	1.00										
L	0.83	0.87	1.00									
4Q1	0.79	0.73	0.64	1.00								
4Q2	0.91	0.81	0.71	0.71	1.00							
4Q3	0.99	0.92	0.80	0.79	0.92	1.00						
4Q4	0.88	0.92	0.99	0.68	0.76	0.86	1.00					
5Q1	0.75	0.69	0.60	0.98	0.67	0.75	0.64	1.00				
5Q2	0.85	0.76	0.67	0.71	0.96	0.88	0.72	0.65	1.00			
5Q3	0.80	0.70	0.62	0.73	0.77	0.82	0.66	0.70	0.75	1.00		
5Q4	0.99	0.93	0.83	0.80	0.91	0.99	0.88	0.76	0.86	0.82	1.00	
5Q5	0.88	0.92	0.99	0.68	0.76	0.85	1.00	0.64	0.71	0.66	0.87	1.00

Panel E: forth subperiod 1991/1 - 1997/12												
	S	M	L	4Q1	4Q2	4Q3	4Q4	5Q1	5Q2	5Q3	5Q4	5Q5
S	1.00											
M	0.80	1.00										
L	0.64	0.73	1.00									
4Q1	0.73	0.51	0.42	1.00								
4Q2	0.86	0.59	0.42	0.79	1.00							
4Q3	0.97	0.78	0.60	0.77	0.87	1.00						
4Q4	0.75	0.81	0.98	0.50	0.52	0.71	1.00					
5Q1	0.68	0.42	0.39	0.93	0.77	0.70	0.45	1.00				
5Q2	0.80	0.60	0.37	0.84	0.89	0.82	0.48	0.70	1.00			
5Q3	0.89	0.64	0.47	0.76	0.96	0.92	0.57	0.72	0.86	1.00		
5Q4	0.98	0.80	0.63	0.72	0.83	0.97	0.74	0.66	0.77	0.87	1.00	
5Q5	0.74	0.80	0.98	0.49	0.51	0.70	1.00	0.44	0.47	0.56	0.73	1.00

Panel F: fifth subperiod 1998/1 - 2004/12												
	S	M	L	4Q1	4Q2	4Q3	4Q4	5Q1	5Q2	5Q3	5Q4	5Q5
S	1.00											
M	0.87	1.00										
L	0.69	0.76	1.00									
4Q1	0.64	0.50	0.43	1.00								
4Q2	0.87	0.76	0.68	0.80	1.00							
4Q3	0.96	0.84	0.69	0.73	0.92	1.00						
4Q4	0.78	0.82	0.92	0.51	0.76	0.77	1.00					
5Q1	0.68	0.55	0.46	0.92	0.84	0.76	0.53	1.00				
5Q2	0.84	0.71	0.63	0.82	0.97	0.89	0.72	0.85	1.00			
5Q3	0.90	0.81	0.69	0.76	0.96	0.95	0.75	0.81	0.90	1.00		
5Q4	0.95	0.82	0.67	0.72	0.90	0.95	0.80	0.75	0.89	0.88	1.00	
5Q5	0.78	0.84	0.99	0.47	0.73	0.76	0.94	0.50	0.68	0.76	0.74	1.00

Symbol	**description**
S, M, L	Small cap, mid cap, large cap portfolios
4Qx	x-th quartile portfolio
5Qx	x-th quintile portfolio

Table 4 – Mean-Variance Spanning Tests for Canadian size based portfolios

The table presents the results of the Huberman-Kandel regression based mean-variance spanning tests on the returns of the Canadian size based portfolios, i.e., the small cap threshold portfolio (S), the mid cap threshold portfolio (M), the large cap threshold portfolio (L), the quartile portfolios (4Q1, 4Q2, 4Q3, 4Q4) and the quintile portfolios (5Q1, 5Q2, 5Q3, 5Q4, 5Q5) using them as either the test asset or the benchmark assets. Column 3 (alpha), column 4 (sum of beta) and column 4 (adj-R2) are from the estimation of equation R=XB+E. Mean VIF indicates the average variance inflation factor for all dependent variables. The last two columns of the table report the results of the Lagrange Multiplier (LM) test and F-test of H0: $\alpha = 0_N$ and $\sum \delta_i = 1$ which is equivalent to the joint hypothesis that alpha is equal to zero and the sum of the betas is equal to one. The reported p-values are exact under the normality assumption on the residuals. The results are presented for the entire sample period as well as for its five subperiods.

Test asset	Benchmark asset	Alpha	Sum of Beta	Adj-R2	Mean VIF	LM Test (p-value)	F Test (p-value)
		Panel A: entire period 1970/1 - 2004/12					
Threshold approach							
S	M + L	-0.01	0.91	0.80	2.60	17.78	9.22
						(0.00)	(0.00)
M	S + L	0.17	0.95	0.83	2.18	6.77	3.41
						(0.03)	(0.03)
L	S + M	0.01	0.85	0.62	4.98	20.07	10.46
						(0.00)	(0.00)
Size Quartiles							
4Q1	4Q2 + 4Q3 +4Q4	0.75	1.07	0.66	4.78	16.25	8.37
						(0.00)	(0.00)
4Q2	4Q1 + 4Q3 + 4Q4	-0.12	0.99	0.85	3.17	1.38	0.68
						(0.50)	(0.50)
4Q3	4Q1 + 4Q2 + 4Q4	-0.03	0.95	0.84	3.27	4.56	2.28
						(0.10)	(0.10)
4Q4	4Q1 + 4Q2 + 4Q3	0.21	0.72	0.67	4.77	95.98	61.61
						(0.00)	(0.00)
Size Quintiles							
5Q1	5Q2 + 5Q3 + 5Q4 + 5Q5	0.32	1.16	0.67	5.86	14.00	7.15
						(0.00)	(0.00)
5Q2	5Q1 + 5Q3 + 5Q4 + 5Q5	0.08	0.98	0.84	4.67	1.07	0.53
						(0.59)	(0.59)
5Q3	5Q1 + 5Q2 + 5Q4 + 5Q5	0.13	0.95	0.83	4.95	5.74	2.87
						(0.06)	(0.06)
5Q4	5Q1 + 5Q2 + 5Q3 + 5Q5	-0.10	0.98	0.89	3.98	2.65	1.32
						(0.27)	(0.27)
5Q5	5Q1 + 5Q2 + 5Q3 + 5Q4	0.11	0.78	0.70	5.37	66.52	39.05
						(0.00)	(0.00)

Table 4 – Mean-Variance Spanning Tests for Canadian size based portfolios (continued)

Test asset	Benchmark asset	Alpha	Sum of Beta	Adj-R2	Mean VIF	LM Test (p-value)	F Test (p-value)
			Panel B: first subperiod 1970/1 - 1976/12				
Threshold approach							
S	M + L	0.21	0.88	0.80	1.97	5.90	3.06
						(0.05)	(0.05)
M	S + L	-0.16	0.93	0.81	1.89	2.49	1.24
						(0.29)	(0.30)
L	S + M	0.36	0.93	0.50	4.94	0.82	0.40
						(0.66)	(0.67)
Size Quartiles							
4Q1	4Q2 + 4Q3 +4Q4	0.38	1.01	0.90	5.50	2.82	1.37
						(0.24)	(0.26)
4Q2	4Q1 + 4Q3 + 4Q4	-0.12	1.04	0.93	4.20	1.69	0.81
						(0.43)	(0.45)
4Q3	4Q1 + 4Q2 + 4Q4	-0.19	1.00	0.75	10.20	0.29	0.14
						(0.86)	(0.87)
4Q4	4Q1 + 4Q2 + 4Q3	-0.01	0.73	0.82	8.23	32.01	24.32
						(0.00)	(0.00)
Size Quintiles							
5Q1	5Q2 + 5Q3 + 5Q4 + 5Q5	0.23	1.05	0.91	10.95	2.50	1.21
						(0.29)	(0.30)
5Q2	5Q1 + 5Q3 + 5Q4 + 5Q5	-0.02	1.02	0.93	10.03	0.44	0.21
						(0.80)	(0.81)
5Q3	5Q1 + 5Q2 + 5Q4 + 5Q5	0.12	1.05	0.91	11.46	1.96	0.94
						(0.38)	(0.39)
5Q4	5Q1 + 5Q2 + 5Q3 + 5Q5	-0.11	0.91	0.94	9.24	12.70	7.03
						(0.00)	(0.00)
5Q5	5Q1 + 5Q2 + 5Q3 + 5Q4	-0.01	0.91	0.79	13.00	3.05	1.49
						(0.22)	(0.23)

Table 4 – Mean-Variance Spanning Tests for Canadian size based portfolios (continued)

Test asset	Benchmark asset	Alpha	Sum of Beta	Adj-R2	Mean VIF	LM Test (p-value)	F Test (p-value)
			Panel C: second subperiod 1977/1 - 1983/12				
Threshold approach							
S	M + L	-0.01	0.90	0.85	7.73	4.93	2.52
						(0.09)	(0.09)
M	S + L	0.20	1.04	0.93	3.60	4.52	2.30
						(0.10)	(0.11)
L	S + M	-0.12	0.87	0.87	6.59	12.86	7.32
						(0.00)	(0.00)
Size Quartiles							
4Q1	4Q2 + 4Q3 +4Q4	-0.19	1.06	0.65	8.48	0.40	0.19
						(0.82)	(0.83)
4Q2	4Q1 + 4Q3 + 4Q4	-0.11	1.02	0.90	4.53	0.49	0.23
						(0.78)	(0.79)
4Q3	4Q1 + 4Q2 + 4Q4	0.32	0.94	0.91	4.06	4.95	2.50
						(0.08)	(0.09)
4Q4	4Q1 + 4Q2 + 4Q3	-0.12	0.87	0.78	7.43	8.46	4.48
						(0.01)	(0.01)
Size Quintiles							
5Q1	5Q2 + 5Q3 + 5Q4 + 5Q5	-0.50	1.19	0.68	9.41	3.23	1.58
						(0.20)	(0.21)
5Q2	5Q1 + 5Q3 + 5Q4 + 5Q5	-0.05	0.98	0.92	6.58	0.52	0.24
						(0.77)	(0.78)
5Q3	5Q1 + 5Q2 + 5Q4 + 5Q5	0.17	0.90	0.86	8.47	5.11	2.56
						(0.08)	(0.08)
5Q4	5Q1 + 5Q2 + 5Q3 + 5Q5	0.21	1.03	0.93	5.91	3.63	1.79
						(0.16)	(0.17)
5Q5	5Q1 + 5Q2 + 5Q3 + 5Q4	-0.16	0.84	0.81	8.54	13.25	7.40
						(0.00)	(0.00)

Table 4 – Mean-Variance Spanning Tests for Canadian size based portfolios (continued)

Test asset	Benchmark asset	Alpha	Sum of Beta	Adj-R2	Mean VIF	LM Test (p-value)	F Test (p-value)
Panel D: third subperiod 1984/1 - 1990/12							
Threshold approach							
S	M + L	-0.24	0.94	0.89	4.03	4.91	2.51
						(0.09)	(0.09)
M	S + L	0.26	1.01	0.91	3.20	3.22	1.62
						(0.20)	(0.21)
L	S + M	-0.06	0.84	0.75	9.52	8.75	4.71
						(0.01)	(0.01)
Size Quartiles							
4Q1	4Q2 + 4Q3 +4Q4	0.16	1.05	0.61	6.82	0.40	0.19
						(0.82)	(0.83)
4Q2	4Q1 + 4Q3 + 4Q4	-0.48	0.90	0.84	3.93	9.20	4.92
						(0.01)	(0.01)
4Q3	4Q1 + 4Q2 + 4Q4	0.16	1.03	0.91	2.58	1.72	0.84
						(0.42)	(0.44)
4Q4	4Q1 + 4Q2 + 4Q3	0.26	0.77	0.73	5.80	17.32	10.39
						(0.00)	(0.00)
Size Quintiles							
5Q1	5Q2 + 5Q3 + 5Q4 + 5Q5	0.22	1.08	0.58	5.76	0.78	0.37
						(0.68)	(0.69)
5Q2	5Q1 + 5Q3 + 5Q4 + 5Q5	-0.24	0.88	0.75	4.71	4.53	2.25
						(0.10)	(0.11)
5Q3	5Q1 + 5Q2 + 5Q4 + 5Q5	0.40	0.85	0.69	5.30	4.75	2.37
						(0.09)	(0.10)
5Q4	5Q1 + 5Q2 + 5Q3 + 5Q5	-0.14	1.06	0.92	2.57	3.38	1.66
						(0.18)	(0.20)
5Q5	5Q1 + 5Q2 + 5Q3 + 5Q4	0.36	0.76	0.77	4.05	21.34	13.45
						(0.00)	(0.00)

Table 4 – Mean-Variance Spanning Tests for Canadian size based portfolios (continued)

Test asset	Benchmark asset	Alpha	Sum of Beta	Adj-R2	Mean VIF	LM Test (p-value)	F Test (p-value)
			Panel E: forth subperiod 1991/1 - 1997/12				
Threshold approach							
S	M + L	0.37	0.88	0.64	2.11	3.72	1.87
						(0.16)	(0.16)
M	S + L	-0.11	0.92	0.72	1.68	2.36	1.17
						(0.31)	(0.31)
L	S + M	0.32	0.72	0.52	2.83	12.07	6.80
						(0.00)	(0.00)
Size Quartiles							
4Q1	4Q2 + 4Q3 +4Q4	1.50	1.09	0.64	4.43	15.36	8.95
						(0.00)	(0.00)
4Q2	4Q1 + 4Q3 + 4Q4	-0.24	1.01	0.80	2.74	0.76	0.36
						(0.68)	(0.70)
4Q3	4Q1 + 4Q2 + 4Q4	-0.11	0.92	0.85	2.35	3.90	1.95
						(0.14)	(0.15)
4Q4	4Q1 + 4Q2 + 4Q3	0.31	0.66	0.53	4.07	19.84	12.37
						(0.00)	(0.00)
Size Quintiles							
5Q1	5Q2 + 5Q3 + 5Q4 + 5Q5	1.11	1.15	0.52	4.72	6.63	3.38
						(0.04)	(0.04)
5Q2	5Q1 + 5Q3 + 5Q4 + 5Q5	0.47	0.95	0.75	3.86	2.34	1.13
						(0.31)	(0.33)
5Q3	5Q1 + 5Q2 + 5Q4 + 5Q5	-0.27	1.00	0.84	3.02	1.66	0.80
						(0.44)	(0.45)
5Q4	5Q1 + 5Q2 + 5Q3 + 5Q5	0.00	0.94	0.84	3.10	1.33	0.63
						(0.52)	(0.53)
5Q5	5Q1 + 5Q2 + 5Q3 + 5Q4	0.27	0.67	0.54	4.20	20.21	12.52
						(0.00)	(0.00)

Table 4 – Mean-Variance Spanning Tests for Canadian size based portfolios (continued)

Test asset	Benchmark asset	Alpha	Sum of Beta	Adj-R2	Mean VIF	LM Test (p-value)	F Test (p-value)
			Panel F: fifth subperiod 1998/1 - 2004/12				
Threshold approach							
S	M + L	-0.42	0.90	0.76	2.35	5.83	3.02
						(0.05)	(0.05)
M	S + L	0.67	0.94	0.80	1.93	7.63	4.05
						(0.02)	(0.02)
L	S + M	-0.48	0.81	0.57	4.19	8.09	4.32
						(0.02)	(0.02)
Size Quartiles							
4Q1	4Q2 + 4Q3 +4Q4	1.12	1.08	0.64	5.47	3.57	1.77
						(0.17)	(0.18)
4Q2	4Q1 + 4Q3 + 4Q4	0.18	1.05	0.89	2.82	1.87	0.91
						(0.39)	(0.41)
4Q3	4Q1 + 4Q2 + 4Q4	-0.39	0.87	0.85	3.52	10.78	5.89
						(0.00)	(0.00)
4Q4	4Q1 + 4Q2 + 4Q3	0.25	0.74	0.62	5.86	14.27	8.19
						(0.00)	(0.00)
Size Quintiles							
5Q1	5Q2 + 5Q3 + 5Q4 + 5Q5	0.25	1.26	0.75	5.76	5.57	2.81
						(0.06)	(0.07)
5Q2	5Q1 + 5Q3 + 5Q4 + 5Q5	0.26	0.98	0.88	4.49	1.28	0.61
						(0.53)	(0.55)
5Q3	5Q1 + 5Q2 + 5Q4 + 5Q5	0.11	0.92	0.87	4.93	3.20	1.56
						(0.20)	(0.22)
5Q4	5Q1 + 5Q2 + 5Q3 + 5Q5	-0.43	0.97	0.82	5.46	2.91	1.42
						(0.23)	(0.25)
5Q5	5Q1 + 5Q2 + 5Q3 + 5Q4	-0.02	0.78	0.62	6.30	10.26	5.49
						(0.01)	(0.01)

Table 5 – Mean-Variance Spanning Tests with international benchmark assets

This table reports the results of two mean-variance spanning tests on the returns of the Canadian size based portfolios and an international benchmark. Whenever the test asset causes the null hypothesis of spanning to be rejected at the 5% level using an international benchmark (*) asset we include it in a new benchmark asset until the null hypothesis is no longer rejected. The * in the benchmark asset column indicates that in addition to the listed portfolio, all the tests include the CAD denominated returns on the following indices: S&P 500, Russell 2000, MSCI EAFE, and FTSE100. Column 3 (alpha), column 4 (sum of beta) and column 4 (adj-R2) are from the estimation of equation R=XB+E. Mean VIF indicates the average variance inflation factor for all dependent variables. The last two columns of the table report the results of the Lagrange Multiplier (LM) test and F-test of H_0: $\alpha_j = 0_N$ and $\sum \delta_j = 1$ which is equivalent to the joint hypothesis that alpha is equal to zero and the sum of the betas is equal to one. The reported p-values are exact under the normality assumption on the residuals. The results are presented for the entire sample period as well as for its two subperiods.

Test asset	Benchmark Asset	Alpha	Sum of Beta	Adj-R2	Mean VIF	LM Test (p-value)	F Test (p-value)
			Panel A: entire period 1986/1 - 2004/12				
S	*	0.02	0.69	0.62	2.46	40.86	24.35
						(0.00)	(0.00)
M	S + *	0.13	0.98	0.81	2.84	1.19	0.58
						(0.55)	(0.56)
M	*	0.14	0.77	0.63	2.46	25.69	14.16
						(0.00)	(0.00)
S	M + *	-0.09	0.86	0.81	2.73	18.20	9.63
						(0.00)	(0.00)
L	S + M + *	-0.33	0.99	0.74	3.76	4.94	2.45
						(0.08)	(0.09)
L	*	-0.29	0.87	0.67	2.46	15.67	8.23
						(0.00)	(0.00)
S	L + *	0.15	0.75	0.68	2.78	30.65	17.24
						(0.00)	(0.00)
M	S + L + *	0.18	0.98	0.82	3.76	2.04	1.00
						(0.36)	(0.37)
4Q1	*	1.64	0.60	0.32	2.46	22.07	11.95
						(0.00)	(0.00)
4Q2	4Q1 + *	-0.58	0.81	0.68	2.48	24.32	13.25
						(0.00)	(0.00)
4Q3	4Q1 + 4Q2 + *	-0.19	0.92	0.88	2.84	13.02	6.69
						(0.00)	(0.00)
4Q4	4Q1 + 4Q2 + 4Q3 + *	-0.14	0.97	0.77	4.22	1.97	0.96
						(0.37)	(0.38)
4Q2	*	0.13	0.64	0.44	2.46	26.46	14.64
						(0.00)	(0.00)
4Q1	4Q2 + *	1.50	0.96	0.62	2.54	19.88	10.60
						(0.00)	(0.00)
4Q3	4Q1 + 4Q2 + *	-0.19	0.92	0.88	2.84	13.02	6.69
						(0.00)	(0.00)
4Q4	4Q1 + 4Q2 + 4Q3 + *	-0.14	0.97	0.77	4.22	1.97	0.96
						(0.37)	(0.38)
4Q3	*	-0.02	0.70	0.61	2.46	34.87	20.13
						(0.00)	(0.00)
4Q1	4Q3 + *	1.66	0.99	0.58	2.87	22.48	12.14
						(0.00)	(0.00)

Table 5 – Mean-Variance Spanning Tests with international benchmark assets (continued)

Test asset	Benchmark asset	Alpha	Sum of Beta	Adj-R2	Mean VIF	LM Test (p-value)	F Test (p-value)
4Q4	*	-0.08	0.83	0.66	2.46	18.72	9.97
						(0.00)	(0.00)
4Q1	4Q4 + *	1.72	0.78	0.43	2.74	19.34	10.29
						(0.00)	(0.00)
4Q2	4Q1 + 4Q4 + *	-0.42	0.86	0.71	2.86	14.31	7.40
						(0.00)	(0.00)
4Q3	4Q1 + 4Q2 + 4Q4 + *	0.04	0.81	0.73	2.74	9.59	4.83
						(0.01)	(0.01)
5Q1	*	1.56	0.62	0.29	2.46	16.51	8.70
						(0.00)	(0.00)
5Q2	5Q1 + *	-0.11	0.79	0.66	2.45	15.09	7.86
						(0.00)	(0.00)
5Q3	5Q1 + 5Q2 + *	-0.33	0.99	0.74	2.77	4.94	2.45
						(0.08)	(0.09)
5Q2	*	0.53	0.64	0.42	2.46	23.68	12.92
						(0.00)	(0.00)
5Q1	5Q2 + *	1.01	0.99	0.59	2.53	7.61	3.83
						(0.02)	(0.02)
5Q3	5Q1 + 5Q2 + *	-0.33	0.99	0.74	2.77	4.94	2.45
						(0.08)	(0.09)
5Q3	*	0.45	0.61	0.41	2.46	30.88	17.47
						(0.00)	(0.00)
5Q1	5Q3 + *	1.07	1.04	0.57	2.51	9.03	4.58
						(0.01)	(0.01)
5Q2	5Q1 + 5Q3 + *	-0.04	0.94	0.76	2.71	1.86	0.91
						(0.39)	(0.40)
5Q4	*	0.01	0.71	0.60	2.46	29.42	16.52
						(0.00)	(0.00)
5Q1	5Q4 + *	1.55	1.01	0.56	2.84	16.45	8.63
						(0.00)	(0.00)
5Q2	5Q1 + 5Q4 + *	0.21	0.93	0.76	3.03	2.77	1.36
						(0.25)	(0.26)
5Q5	*	-0.20	0.85	0.71	2.46	20.14	10.80
						(0.00)	(0.00)
5Q1	5Q5 + *	1.82	0.82	0.42	2.88	16.65	8.75
						(0.00)	(0.00)
5Q2	5Q1 + 5Q5 + *	0.02	0.82	0.67	3.01	10.20	5.17
						(0.01)	(0.01)
5Q3	5Q1 + 5Q2 + 5Q5 + *	0.07	0.88	0.76	3.22	7.32	3.65
						(0.03)	(0.03)
5Q4	5Q1 + 5Q2 + 5Q3 + 5Q5 + *	-0.24	0.96	0.86	3.57	6.28	3.10
						(0.04)	(0.05)

Table 5 – Mean-Variance Spanning Tests with international benchmark assets (continued)

Test asset	Benchmark asset	Alpha	Sum of Beta	Adj-R2	Mean VIF	LM Test (p-value)	F Test (p-value)
			Panel B: first subperiod: 1991/1 - 1997/12				
S	*	0.28	0.68	0.36	1.60	5.48	2.76
						(0.06)	(0.07)
M	*	-0.19	0.81	0.38	1.60	4.18	14.16
						(0.12)	(0.13)
L	*	-0.21	0.89	0.49	1.60	2.76	1.34
						(0.25)	(0.27)
4Q1	*	2.18	0.64	0.23	1.60	10.41	5.59
						(0.01)	(0.01)
4Q2	4Q1 + *	-0.67	0.81	0.62	1.80	8.28	4.26
						(0.02)	(0.02)
4Q3	4Q1 + 4Q2 + *	-0.27	0.89	0.81	2.13	6.18	3.06
						(0.05)	(0.05)
4Q2	*	0.70	0.58	0.22	1.60	4.24	2.10
						(0.12)	(0.13)
4Q3	*	0.32	0.65	0.35	1.60	6.24	3.17
						(0.04)	(0.05)
4Q1	4Q3 + *	1.81	1.05	0.60	1.72	17.43	10.22
						(0.00)	(0.00)
4Q2	4Q1 + 4Q3 + *	-0.13	1.00	0.79	2.16	0.24	0.11
						(0.89)	(0.90)
4Q4	*	-0.15	0.88	0.51	1.60	2.83	1.38
						(0.24)	(0.26)
5Q1	*	2.37	0.65	0.21	1.60	8.95	4.71
						(0.01)	(0.01)
5Q2	5Q1 + *	0.10	0.73	0.55	1.66	3.24	1.57
						(0.20)	(0.22)
5Q2	*	1.23	0.56	0.24	1.60	5.89	2.98
						(0.05)	(0.06)
5Q3	*	0.69	0.58	0.25	1.60	5.23	2.62
						(0.07)	(0.08)
5Q4	*	0.35	0.70	0.38	1.60	4.88	2.44
						(0.09)	(0.09)
5Q5	*	-0.16	0.88	0.51	1.60	2.81	1.37
						(0.25)	(0.26)

Table 5 – Mean-Variance Spanning Tests with international benchmark assets (continued)

Test asset	Benchmark asset	Alpha	Sum of Beta	Adj-R2	Mean VIF	LM Test (p-value)	F Test (p-value)
			Panel C: second subperiod: 1998/1 - 2004/12				
S	*	-0.04	0.71	0.69	2.74	8.18	4.26
						(0.02)	(0.02)
M	S + *	0.57	1.02	0.81	3.36	5.45	2.71
						(0.07)	(0.07)
M	*	0.54	0.85	0.70	2.74	5.24	2.63
						(0.07)	(0.08)
L	*	-0.20	1.01	0.67	2.74	0.36	0.17
						(0.84)	(0.84)
4Q1	*	1.93	0.58	0.30	2.74	6.38	3.25
						(0.04)	(0.04)
4Q2	4Q1 + *	-0.19	1.05	0.80	2.72	0.56	0.26
						(0.76)	(0.77)
4Q2	*	0.56	0.88	0.55	2.74	2.00	0.96
						(0.37)	(0.39)
4Q3	*	-0.02	0.78	0.68	2.74	4.24	2.10
						(0.12)	(0.13)
4Q4	*	0.15	0.99	0.65	2.74	0.20	0.10
						(0.90)	(0.91)
5Q1	*	1.60	0.64	0.29	2.74	3.95	1.95
						(0.14)	(0.15)
5Q2	*	0.73	0.86	0.53	2.74	2.92	1.42
						(0.23)	(0.25)
5Q3	*	0.50	0.81	0.58	2.74	3.57	1.75
						(0.17)	(0.18)
5Q4	*	0.01	0.78	0.64	2.74	3.23	1.58
						(0.20)	(0.21)
5Q5	*	-0.08	0.96	0.73	2.74	0.31	0.15
						(0.86)	(0.86)
5Q5	*	-0.10	0.78	0.73	2.74	4.55	2.26
						(0.10)	(0.11)

Table 6 – Mean-Variance Spanning Tests with International Large Cap benchmark

This table reports the results of the Canadian size based portfolios used as test assets compared with an initial international large cap benchmark. Whenever the test asset causes the null hypothesis of spanning to be rejected at the 5% level using an international large cap benchmark (*LCB*) asset we include it in a new benchmark asset until the null hypothesis is no longer rejected. The *LCB* in the benchmark asset column indicates that in addition to the listed portfolio, all the tests include the CAD denominated returns on the following indices: S&P 500, Nikkei 300, CAC40, DAX30, SPTSX and FTSE100. Column 3 (alpha), column 4 (sum of beta) and column 4 (adj-R2) are from the estimation of equation R=XB+E. Mean VIF indicates the average variance inflation factor for all dependent variables. The last two columns of the table report the results of the Lagrange Multiplier (LM) test and F-test of H0: $\alpha_j = 0_N$ and $\sum \delta_j = 1$ which is equivalent to the joint hypothesis that alpha is equal to zero and the sum of the betas is equal to one. The reported p-values are exact under the normality assumption on the residuals. The results are presented for the entire sample period as well as for its two subperiods.

Test asset	Benchmark asset	Alpha	Sum of Beta	Adj-R2	Mean VIF	LM Test (p-value)	F Test (p-value)
			Panel A: entire period 1988/1 - 2004/12				
S	*LCB*	0.30	0.83	0.73	3.09	14.40	7.48
						(0.00)	(0.00)
M	S + *LCB*	0.28	0.97	0.83	3.77	4.90	2.41
						(0.09)	(0.09)
M	*LCB*	0.40	0.90	0.78	3.09	11.20	5.72
						(0.00)	(0.00)
S	M + *LCB*	0.10	0.88	0.78	3.83	8.20	4.10
						(0.02)	(0.02)
L	S + M + *LCB*	-0.04	0.98	0.83	4.31	0.56	0.27
						(0.76)	(0.77)
L	*LCB*	-0.13	1.02	0.82	3.09	1.01	0.49
						(0.60)	(0.61)
4Q1	*LCB*	2.27	0.73	0.33	3.09	23.18	12.63
						(0.00)	(0.00)
4Q2	4Q1 + *LCB*	-0.31	0.88	0.71	3.05	7.30	3.64
						(0.03)	(0.03)
4Q3	4Q1 + 4Q2 + *LCB*	-0.09	0.96	0.87	3.36	2.65	1.28
						(0.27)	(0.28)
4Q2	*LCB*	0.54	0.78	0.52	3.09	10.26	5.22
						(0.01)	(0.01)
4Q1	4Q2 + *LCB*	1.70	0.97	0.59	3.25	20.41	10.90
						(0.00)	(0.00)
4Q3	4Q1 + 4Q2 + *LCB*	-0.09	0.96	0.87	3.36	2.65	1.28
						(0.27)	(0.28)
4Q3	*LCB*	0.31	0.84	0.68	3.09	10.45	5.32
						(0.01)	(0.01)
4Q1	4Q3 + *LCB*	1.81	0.97	0.56	3.62	21.61	11.61
						(0.00)	(0.00)
4Q2	4Q1 + 4Q3 + *LCB*	-0.10	0.96	0.82	3.69	1.14	0.55
						(0.56)	(0.58)
4Q4	*LCB*	0.14	0.98	0.83	3.09	1.35	0.66
						(0.51)	(0.52)

Table 6 – Mean-Variance Spanning Tests with International Large Cap benchmark (continued)

Test asset	Benchmark asset	Alpha	Sum of Beta	Adj-R2	Mean VIF	LM Test (p-value)	F Test (p-value)
5Q1	*LCB*	2.22	0.80	0.35	3.09	18.56	9.86
						(0.00)	(0.00)
5Q2	5Q1 + *LCB*	0.11	0.83	0.66	3.09	6.44	3.20
						(0.04)	(0.04)
5Q3	5Q1 + 5Q2 + *LCB*	0.13	0.85	0.72	3.28	6.85	3.39
						(0.03)	(0.04)
5Q4	5Q1 + 5Q2 + 5Q3 + *LCB*	-0.18	0.97	0.84	3.49	2.44	1.17
						(0.30)	(0.31)
5Q2	*LCB*	0.93	0.75	0.47	3.09	14.48	7.52
						(0.00)	(0.00)
5Q1	5Q2 + *LCB*	1.27	1.05	0.59	3.17	10.70	5.42
						(0.00)	(0.01)
5Q3	5Q1 + 5Q2 + *LCB*	0.13	0.85	0.72	3.28	6.85	3.39
						(0.03)	(0.04)
5Q4	5Q1 + 5Q2 + 5Q3 + *LCB*	-0.18	0.97	0.84	3.49	2.44	1.17
						(0.30)	(0.31)
5Q3	*LCB*	0.87	0.71	0.48	3.09	19.07	10.16
						(0.00)	(0.00)
5Q1	5Q3 + *LCB*	1.31	1.10	0.56	3.22	11.56	5.89
						(0.00)	(0.00)
5Q2	5Q1 + 5Q3 + *LCB*	0.02	0.94	0.74	3.27	1.03	0.50
						(0.60)	(0.61)
5Q4	*LCB*	0.36	0.85	0.68	3.09	9.48	4.80
						(0.01)	(0.01)
5Q1	5Q4 + *LCB*	1.70	1.02	0.55	3.58	16.26	8.49
						(0.00)	(0.00)
5Q2	5Q1 + 5Q4 + *LCB*	0.20	0.91	0.74	3.63	2.86	1.39
						(0.24)	(0.25)
5Q5	*LCB*	-0.01	1.00	0.89	3.09	0.03	0.01
						(0.99)	(0.99)

Table 6 – Mean-Variance Spanning Tests with International Large Cap benchmark (continued)

Test asset	Benchmark asset	Alpha	Sum of Beta	Adj-R2	Mean VIF	LM Test (p-value)	F Test (p-value)
		Panel B: first subperiod: 1991/1 - 1997/12					
S	*LCB*	0.59	0.85	0.70	2.48	6.66	3.32
						(0.04)	(0.04)
M	S + *LCB*	-0.06	0.99	0.82	3.14	0.20	0.09
						(0.90)	(0.91)
M	*LCB*	0.04	0.96	0.81	2.48	0.36	0.17
						(0.83)	(0.85)
L	*LCB*	-0.20	1.05	0.77	2.48	1.17	0.55
						(0.56)	(0.58)
4Q1	*LCB*	2.66	0.78	0.38	2.48	18.12	10.59
						(0.00)	(0.00)
4Q2	4Q1 + *LCB*	-0.28	0.88	0.65	2.55	2.48	1.15
						(0.29)	(0.32)
4Q2	*LCB*	1.14	0.76	0.42	2.48	6.08	3.00
						(0.05)	(0.06)
4Q3	*LCB*	0.61	0.85	0.63	2.48	5.40	2.64
						(0.07)	(0.08)
4Q4	*LCB*	-0.09	1.05	0.87	2.48	1.09	0.51
						(0.58)	(0.61)
5Q1	*LCB*	2.93	0.76	0.32	2.48	15.25	8.54
						(0.00)	(0.00)
5Q2	5Q1 + *LCB*	0.60	0.84	0.59	2.52	2.36	1.10
						(0.31)	(0.34)
5Q2	*LCB*	1.69	0.75	0.43	2.48	11.10	5.86
						(0.00)	(0.00)
5Q1	5Q2 + *LCB*	1.63	0.95	0.51	2.59	7.01	3.46
						(0.03)	(0.04)
5Q3	5Q1 + 5Q2 + *LCB*	-0.28	0.96	0.76	2.69	1.75	0.80
						(0.42)	(0.45)
5Q3	*LCB*	1.05	0.79	0.45	2.48	6.29	3.12
						(0.04)	(0.05)
5Q1	5Q3 + *LCB*	1.99	0.95	0.52	2.60	11.02	5.74
						(0.00)	(0.00)
5Q2	5Q1 + 5Q3 + *LCB*	0.56	0.94	0.75	2.70	2.59	1.19
						(0.27)	(0.31)
5Q4	*LCB*	0.62	0.87	0.68	2.48	5.92	2.92
						(0.05)	(0.06)
5Q5	*LCB*	-0.11	1.05	0.86	2.48	1.28	0.59
						(0.53)	(0.55)

Table 6 – Mean-Variance Spanning Tests with International Large Cap benchmark (continued)

Test asset	Benchmark asset	Alpha	Sum of Beta	Adj-R2	Mean VIF	LM Test (p-value)	F Test (p-value)
			Panel C: second subperiod: 1998/1 - 2004/12				
S	*LCB*	0.25	0.86	0.73	5.46	3.29	1.57
						(0.19)	(0.21)
M	*LCB*	0.83	0.91	0.76	5.46	9.43	4.87
						(0.01)	(0.01)
S	M + *LCB*	-0.26	0.91	0.81	5.91	2.81	1.31
						(0.25)	(0.27)
L	*LCB*	-0.17	0.98	0.85	5.46	0.69	0.32
						(0.71)	(0.73)
4Q1	*LCB*	2.40	0.52	0.31	5.46	9.71	5.03
						(0.01)	(0.01)
4Q2	4Q1 + *LCB*	-0.05	1.02	0.82	5.21	0.06	0.03
						(0.97)	(0.97)
4Q2	*LCB*	0.85	0.84	0.59	5.46	5.07	2.47
						(0.08)	(0.09)
4Q3	*LCB*	0.31	0.83	0.69	5.46	3.45	1.65
						(0.18)	(0.20)
4Q4	*LCB*	0.27	0.94	0.82	5.46	1.92	0.90
						(0.38)	(0.41)
5Q1	*LCB*	2.05	0.50	0.39	5.46	8.20	4.16
						(0.02)	(0.02)
5Q2	5Q1 + *LCB*	0.13	1.04	0.83	5.39	0.44	0.20
						(0.80)	(0.82)
5Q2	*LCB*	1.08	0.81	0.51	5.46	5.95	2.94
						(0.05)	(0.06)
5Q3	*LCB*	0.76	0.80	0.62	5.46	6.40	3.17
						(0.04)	(0.05)
5Q1	5Q3 + *LCB*	0.74	0.83	0.72	5.64	2.25	1.05
						(0.32)	(0.36)
5Q4	*LCB*	0.32	0.80	0.67	5.46	3.93	1.89
						(0.14)	(0.16)
5Q5	*LCB*	0.00	0.97	0.91	5.46	0.40	0.19
						(0.82)	(0.83)

Table 7 – Step-Down Mean-Variance Spanning Tests with International benchmark

The table presents the results of the step-down mean-variance spanning tests on all cases identified by the Lagrange Multiplier (LM) test and F-test for which the null hypothesis of spanning was rejected at the 5% level. The test asset is one of the Canadian size based portfolios, i.e., the small cap threshold portfolio (S), the mid cap threshold portfolio (M), the large cap threshold portfolio (L), the quartile portfolios (4Q1, 4Q2, 4Q3, 4Q4) and the quintile portfolios (5Q1, 5Q2, 5Q3, 5Q4, 5Q5). The * in the benchmark asset column indicates that in addition to the listed portfolio, all the tests include the CAD denominated returns on the following indices: S&P 500, Russell 2000, MSCI EAFE, and FTSE100. The test is a step-down test where F1 is an F-test of $\alpha_j = 0_N$, and F2 is an F-test of $\sum \delta_j = 1$ conditional on $\alpha_j = 0_N$. The two tests are performed on each of the cases and identify the impact of adding the test asset to an existing benchmark on 1) the tangency portfolio and 2) the global minimum variance portfolio on the efficient frontier. The tangency portfolio effect is captured in the F1-test and the global minimum variance effect is captured in the F2-test. The reported p-values are exact under the normality assumption on the residuals. The results are presented for the entire sample period as well as for its two subperiods.

Test	Benchmark	F1-test		F2-test	
asset	asset	Statistic	(p-value)	Statistic	(p-value)
	Panel A: entire period 1986/1 - 2004/12				
Threshold approach					
S	*	0.01	(0.93)	48.90	(0.00)
M	*	0.58	(0.45)	27.78	(0.00)
S	M + *	0.40	(0.53)	18.90	(0.00)
L	*	2.69	(0.10)	13.67	(0.00)
S	L + *	0.74	(0.39)	33.78	(0.00)
M	L + *	2.62	(0.11)	15.73	(0.00)
Size quartiles					
4Q1	*	14.08	(0.00)	9.28	(0.00)
4Q2	4Q1 + *	6.68	(0.01)	19.32	(0.00)
4Q3	4Q1 + 4Q2 + *	2.53	(0.11)	10.78	(0.00)
4Q4	*	0.22	(0.64)	19.78	(0.00)
4Q2	*	0.22	(0.64)	29.16	(0.00)
4Q1	4Q2 + *	20.87	(0.00)	0.30	(0.58)
4Q3	*	0.00	(0.96)	40.44	(0.00)
4Q1	4Q3 + *	23.39	(0.00)	0.81	(0.37)
4Q1	4Q4 + *	18.53	(0.00)	1.89	(0.17)
4Q2	4Q1 + 4Q4 + *	3.84	(0.05)	10.84	(0.00)
4Q2	4Q4 + *	0.65	(0.42)	12.31	(0.00)
4Q3	4Q4 + *	0.05	(0.82)	20.05	(0.00)
Size quintiles					
5Q1	*	10.19	(0.00)	6.92	(0.01)
5Q2	5Q1 + *	0.20	(0.65)	15.58	(0.00)
5Q2	*	3.01	(0.08)	22.63	(0.00)
5Q1	5Q2 + *	7.27	(0.01)	0.38	(0.54)
5Q3	*	2.52	(0.11)	32.20	(0.00)
5Q1	5Q3 + *	7.82	(0.01)	1.30	(0.26)
5Q4	*	0.00	(0.97)	33.19	(0.00)
5Q1	5Q4 + *	16.21	(0.00)	0.99	(0.32)
5Q5	*	1.50	(0.22)	20.05	(0.00)
5Q1	5Q5 + *	16.65	(0.00)	0.79	(0.37)
5Q2	5Q1 + 5Q5 + *	0.01	(0.93)	10.38	(0.00)
5Q3	5Q1 + 5Q2 + 5Q5 + *	0.11	(0.74)	7.20	(0.01)
5Q4	5Q1 + 5Q2 + 5Q3 + 5Q5 + *	3.21	(0.07)	2.96	(0.09)
5Q2	5Q5 + *	5.85	(0.02)	9.81	(0.00)
5Q3	5Q5 + *	6.13	(0.01)	14.96	(0.00)
5Q4	5Q5 + *	0.74	(0.39)	14.13	(0.00)

Table 7 – Step-Down Mean-Variance Spanning Tests with International benchmark (continued)

Test asset	Benchmark asset	F1-test Statistic	F1-test (p-value)	F2-test Statistic	F2-test (p-value)
Panel B: first subperiod: 1991/1 - 1997/12					
Size quartiles					
4Q1	*	11.17	(0.00)	0.01	(0.93)
4Q2	4Q1 + *	2.54	(0.11)	5.87	(0.02)
4Q3	*	0.68	(0.41)	5.68	(0.02)
4Q1	4Q3 + *	14.53	(0.00)	5.04	(0.03)
4Q1	4Q4 + *	15.77	(0.00)	0.62	(0.43)
Size quintiles					
5Q1	*	9.36	(0.00)	0.05	(0.82)
5Q1	5Q5 + *	13.12	(0.00)	0.79	(0.38)
5Q2	5Q5 + *	6.19	(0.01)	0.44	(0.51)
Panel C: second subperiod: 1998/1 - 2004/12					
Threshold approach					
S	*	0.01	(0.91)	8.62	(0.00)
S	L + *	0.01	(0.94)	9.55	(0.00)
M	L + *	4.27	(0.04)	2.31	(0.13)
Size quartiles					
4Q1	*	4.67	(0.03)	1.74	(0.19)
4Q1	4Q4 + *	15.77	(0.00)	0.62	(0.43)

Table 8 – Step-Down Mean-Variance Spanning Tests with Large Cap International benchmark

The table presents the results of the step-down mean-variance spanning tests on all cases identified by the Lagrange Multiplier (LM) test and F-test for which the null hypothesis of spanning was rejected at the 5% level. The test asset is one of the Canadian size based portfolios, i.e., the small cap threshold portfolio (S), the mid cap threshold portfolio (M), the large cap threshold portfolio (L), the quartile portfolios (4Q1, 4Q2, 4Q3, 4Q4) and the quintile portfolios (5Q1, 5Q2, 5Q3, 5Q4, 5Q5). The *LCB* in the benchmark asset column indicates that in addition to the listed portfolio, all the tests include the CAD denominated returns on the following indices: S&P 500, Nikkei 300, CAC40, DAX30, SPTSX and FTSE100. The test is a step-down test where F1 is an F-test of $\alpha_j = 0_N$, and F2 is an F-test of $\sum \delta_j = 1$ conditional on $\alpha_j = 0_N$. The two tests are performed on each of the cases and identify the impact of adding the test asset to an existing benchmark on 1) the tangency portfolio and 2) the global minimum variance portfolio on the efficient frontier. The tangency portfolio effect is captured in the F1-test and the global minimum variance effect is captured in the F2-test. The reported p-values are exact under the normality assumption on the residuals. The results are presented for the entire sample period as well as for its two subperiods.

Test asset	Benchmark asset	F1-test Statistic	F1-test p-value	F2-test Statistic	F2-test p-value
	Panel A: entire period 1988/1 - 2004/12				
Threshold approach					
S	*LCB*	3.40	(0.07)	11.43	(0.00)
M	*LCB*	7.74	(0.01)	3.59	(0.06)
S	M + *LCB*	0.42	(0.52)	7.81	(0.01)
Size quartiles					
4Q1	*LCB*	23.73	(0.00)	1.37	(0.24)
4Q2	4Q1 + *LCB*	1.81	(0.18)	5.44	(0.02)
4Q2	*LCB*	3.80	(0.05)	6.54	(0.01)
4Q1	4Q2 + *LCB*	21.44	(0.00)	0.33	(0.57)
4Q3	LCB	2.72	(0.10)	7.85	(0.01)
4Q1	4Q3 + *LCB*	22.83	(0.00)	0.34	(0.56)
Size quintiles					
5Q1	*LCB*	19.28	(0.00)	0.39	(0.53)
5Q2	5Q1 + *LCB*	0.20	(0.66)	6.21	(0.01)
5Q3	5Q1 + 5Q2 + *LCB*	0.40	(0.53)	6.34	(0.01)
5Q2	*LCB*	9.39	(0.00)	5.43	(0.02)
5Q1	5Q2 + *LCB*	9.61	(0.00)	1.18	(0.28)
5Q3	*LCB*	10.00	(0.00)	9.86	(0.00)
5Q1	5Q3 + *LCB*	9.42	(0.00)	2.25	(0.14)
5Q4	*LCB*	3.40	(0.07)	6.13	(0.01)
5Q1	5Q4 + *LCB*	16.06	(0.00)	0.85	(0.36)

Table 8 – Step-Down Mean-Variance Spanning Tests with Large Cap International benchmark (continued)

Test asset	Benchmark asset	F1-test Statistic	F1-test p-value	F2-test Statistic	F2-test p-value
Panel B: first subperiod: 1991/1 - 1997/12					
Threshold approach					
S	*LCB*	5.37	(0.02)	1.19	(0.28)
Size quartiles					
4Q1	*LCB*	20.90	(0.00)	0.22	(0.64)
Size quintiles					
5Q1	*LCB*	16.87	(0.00)	0.18	(0.68)
5Q2	*LCB*	11.60	(0.00)	0.11	(0.74)
5Q1	5Q2 + *LCB*	6.27	(0.01)	0.60	(0.44)
5Q3	*LCB*	5.70	(0.02)	0.50	(0.48)
5Q1	5Q3 + *LCB*	10.28	(0.00)	1.07	(0.30)
Panel C: second subperiod: 1998/1 - 2004/12					
Threshold approach					
M	*LCB*	8.88	(0.00)	0.78	(0.38)
Size quartiles					
4Q1	*LCB*	7.14	(0.01)	2.71	(0.10)
4Q1	4Q4 + *LCB*	6.05	(0.02)	2.22	(0.14)
Size quintiles					
5Q1	*LCB*	5.11	(0.03)	3.05	(0.08)
5Q3	*LCB*	3.74	(0.06)	2.52	(0.12)

Table 9 – Diversification Benefits with International benchmark

This table reports the ex-post measures of diversification benefits from expanding the investment opportunity set. For the definition of ψ and λ, respectively refer to equations [3] and [4] in the main body of the paper. GMV is the standard deviation of the global minimum variance portfolio on the efficient frontier and SR is the maximum Sharpe ratio (excess return divided by the standard deviation) on the efficient frontier. Results are separated into three sets of policy constraints: 1) investors are unconstrained and can take long or short positions up to 100% of total capital, 2) investors cannot take short positions, 3) investors are restrained by a 0.5 weight upper bound in any single asset. The * in the investment opportunity set column indicates that in addition to the listed portfolio, the optimization considers the CAD denominated returns on the following indices: S&P 500, Russell 2000, MSCI EAFE, and FTSE100. The results are presented for the entire sample period as well as for its two subperiods.

	Unconstrained		No short sales		Upper bound	
Investment opportunity set	GMV (ψ)	SR (λ)	GMV (ψ)	SR (λ)	GMV (ψ)	SR (λ)
			Entire Period: 1986/1 - 2004/12			
Threshold approach						
*	4.14%	0.24	4.15%	0.24	4.15%	0.24
S + *	3.32%	0.25	3.90%	0.24	3.90%	0.24
	(0.82%)	(0.01)	(0.25%)	(0.00)	(0.25%)	(0.00)
M + *	3.91%	0.25	3.97%	0.25	3.97%	0.24
	(0.23%)	(0.01)	(0.18%)	(0.00)	(0.18%)	(0.01)
S + M + *	3.25%	0.25	3.90%	0.25	3.90%	0.24
	(0.89%)	(0.01)	(0.25%)	(0.00)	(0.25%)	(0.01)
L + *	4.05%	0.27	4.06%	0.24	4.06%	0.24
	(0.09%)	(0.03)	(0.09%)	(0.00)	(0.09%)	(0.00)
S + L + *	3.30%	0.29	3.90%	0.24	3.90%	0.24
	(0.84%)	(0.05)	(0.25%)	(0.00)	(0.25%)	(0.00)
Size Quartiles						
*	4.14%	0.24	4.15%	0.24	4.15%	0.24
4Q1 + *	4.01%	0.36	4.04%	0.33	4.04%	0.33
	(0.13%)	(0.12)	(0.10%)	(0.08)	(0.11%)	(0.09)
4Q2 + 4Q1 + *	3.90%	0.40	3.96%	0.33	3.96%	0.33
	(0.24%)	(0.16)	(0.19%)	(0.08)	(0.19%)	(0.09)
4Q3 + 4Q1 + 4Q2 + *	3.62%	0.42	3.78%	0.33	3.78%	0.33
	(0.52%)	(0.18)	(0.37%)	(0.08)	(0.37%)	(0.09)
4Q2 + *	3.91%	0.25	3.96%	0.24	3.96%	0.24
	(0.23%)	(0.01)	(0.19%)	(0.00)	(0.19%)	(0.00)
4Q3 + *	3.67%	0.25	3.78%	0.24	3.78%	0.24
	(0.47%)	(0.01)	(0.37%)	(0.00)	(0.37%)	(0.00)
4Q1 + 4Q3 + *	3.62%	0.41	3.78%	0.33	3.78%	0.33
	(0.52%)	(0.17)	(0.37%)	(0.08)	(0.37%)	(0.09)
4Q4 + *	3.99%	0.25	4.01%	0.24	4.01%	0.24
	(0.15%)	(0.01)	(0.14%)	(0.00)	(0.14%)	(0.00)
4Q1 + 4Q4 + *	3.95%	0.39	3.99%	0.33	3.99%	0.33
	(0.19%)	(0.15)	(0.16%)	(0.08)	(0.16%)	(0.09)
4Q2 + 4Q1 + 4Q4 + *	3.88%	0.41	3.94%	0.33	3.94%	0.33
	(0.26%)	(0.17)	(0.20%)	(0.08)	(0.21%)	(0.09)

Table 9 – Diversification Benefits with International benchmark (continued)

	Unconstrained		No short sales		Upper bound	
Investment opportunity set	GMV	SR	GMV	SR	GMV	SR
	(ψ)	(λ)	(ψ)	(λ)	(ψ)	(λ)
Size Quintiles						
*	4.14%	0.24	4.15%	0.24	4.15%	0.24
5Q1 + *	4.05%	0.33	4.07%	0.31	4.07%	0.31
	(0.09%)	(0.09)	(0.08%)	(0.06)	(0.08%)	(0.07)
5Q2 + 5Q1 + *	3.93%	0.33	3.98%	0.31	3.98%	0.31
	(0.21%)	(0.09)	(0.17%)	(0.06)	(0.17%)	(0.07)
5Q2 + *	3.93%	0.27	3.98%	0.26	3.98%	0.26
	(0.21%)	(0.03)	(0.17%)	(0.02)	(0.17%)	(0.02)
5Q3 + *	3.86%	0.27	3.92%	0.26	3.92%	0.26
	(0.28%)	(0.03)	(0.23%)	(0.01)	(0.23%)	(0.02)
5Q1 + 5Q3 + *	3.86%	0.33	3.92%	0.31	3.92%	0.31
	(0.28%)	(0.09)	(0.23%)	(0.06)	(0.23%)	(0.07)
5Q4 + *	3.88%	0.24	3.98%	0.24	3.98%	0.24
	(0.26%)	(0.00)	(0.17%)	(0.00)	(0.17%)	(0.00)
5Q1 + 5Q4 + *	3.88%	0.37	3.98%	0.31	3.98%	0.31
	(0.26%)	(0.13)	(0.17%)	(0.06)	(0.17%)	(0.07)
5Q5 + *	4.00%	0.26	4.02%	0.24	4.02%	0.24
	(0.14%)	(0.02)	(0.13%)	(0.00)	(0.13%)	(0.00)
5Q1 + 5Q5 + *	3.98%	0.38	4.01%	0.31	4.01%	0.31
	(0.16%)	(0.14)	(0.14%)	(0.06)	(0.14%)	(0.07)
5Q2 + 5Q1 + 5Q5 + *	3.89%	0.38	3.95%	0.31	3.95%	0.31
	(0.25%)	(0.14)	(0.20%)	(0.06)	(0.20%)	(0.07)
5Q3 + 5Q1 + 5Q2 + 5Q5 + *	3.83%	0.38	3.91%	0.31	3.91%	0.31
	(0.31%)	(0.14)	(0.24%)	(0.06)	(0.24%)	(0.07)

Table 9 – Diversification Benefits with International benchmark (continued)

Investment opportunity set	Unconstrained		No short sales		Upper bound	
	GMV (ψ)	SR (λ)	GMV (ψ)	SR (λ)	GMV (ψ)	SR (λ)
	First Subperiod: 1991/1 - 1997/12					
Size Quartiles						
*	2.37%	0.60	2.38%	0.60	2.38%	0.60
4Q1 + *	2.34%	0.74	2.36%	0.71	2.36%	0.71
	(0.03%)	(0.14)	(0.02%)	(0.11)	(0.02%)	(0.12)
4Q2 + 4Q1 + *	2.31%	0.78	2.34%	0.71	2.34%	0.71
	(0.06%)	(0.17)	(0.04%)	(0.11)	(0.04%)	(0.12)
4Q3 + 4Q1 + *	2.28%	0.79	2.32%	0.71	2.32%	0.71
	(0.09%)	(0.19)	(0.06%)	(0.11)	(0.06%)	(0.12)
4Q4 + 4Q1 + *	2.33%	0.80	2.35%	0.71	2.35%	0.71
	(0.04%)	(0.20)	(0.03%)	(0.11)	(0.03%)	(0.12)
Size Quintiles						
*	2.37%	0.60	2.38%	0.60	2.38%	0.60
5Q1 + *	2.35%	0.72	2.36%	0.69	2.36%	0.69
	(0.02%)	(0.12)	(0.02%)	(0.09)	(0.02%)	(0.10)
5Q1 + 5Q5 + *	2.34%	0.77	2.36%	0.69	2.36%	0.69
	(0.03%)	(0.17)	(0.02%)	(0.09)	(0.02%)	(0.10)
5Q2 + 5Q5 + *	2.31%	0.69	2.34%	0.65	2.34%	0.65
	(0.06%)	(0.09)	(0.04%)	(0.05)	(0.04%)	(0.06)
	Second Subperiod: 1998/1 - 2004/12					
Threshold approach						
*	3.19%	0.13	3.20%	0.13	3.22%	0.13
S + *	3.04%	0.13	3.13%	0.13	3.14%	0.13
	(0.16%)	(0.00)	(0.07%)	(0.00)	(0.08%)	(0.00)
M + L + *	3.13%	0.28	3.17%	0.22	3.19%	0.19
	(0.06%)	(0.14)	(0.02%)	(0.08)	(0.03%)	(0.06)
S + L + *	3.02%	0.15	3.13%	0.13	3.14%	0.13
	(0.18%)	(0.02)	(0.07%)	(0.00)	(0.08%)	(0.00)
Size Quartiles						
*	3.19%	0.13	3.20%	0.13	3.22%	0.13
4Q1 + *	3.15%	0.28	3.19%	0.26	3.17%	0.26
	(0.05%)	(0.15)	(0.01%)	(0.13)	(0.05%)	(0.13)
4Q1 + 4Q4 + *	3.14%	0.28	3.19%	0.26	3.17%	0.26
	(0.05%)	(0.15)	(0.01%)	(0.13)	(0.05%)	(0.13)

Table 10 – Diversification Benefits with Large Cap international Benchmark

This table reports the ex-post measures of diversification benefits from expanding the investment opportunity set. For the definition of ψ and λ, respectively refer to equations [3] and [4] in the main body of the paper. GMV is the standard deviation of the global minimum variance portfolio on the efficient frontier and SR is the maximum Sharpe ratio (excess return divided by the standard deviation) on the efficient frontier. Results are separated into three sets of policy constraints: 1) investors are unconstrained and can take long or short positions up to 100% of total capital, 2) investors cannot take short positions, 3) investors are restrained by a 0.5 weight upper bound in any single asset. The *LCB* in the investment opportunity set column indicates that in addition to the listed portfolio, the optimization considers the CAD denominated returns on the following large cap indices: S&P 500, Nikkei 300, CAC40, SPTSX, DAX30 and FTSE100. The results are presented for the entire sample period as well as for its two subperiods.

	Unconstrained		No short sales		Upper bound	
Investment opportunity set	GMV (ψ)	SR (λ)	GMV (ψ)	SR (λ)	GMV (ψ)	SR (λ)
Entire Period: 1988/1 - 2004/12						
Threshold approach						
LCB	3.41%	0.29	3.51%	0.26	3.51%	0.24
S + *LCB*	3.30%	0.32	3.41%	0.26	3.41%	0.24
	(0.11%)	(0.03)	(0.10%)	0.00	(0.10%)	(0.00)
M + *LCB*	3.37%	0.35	3.49%	0.26	3.49%	0.26
	(0.05%)	(0.06)	(0.02%)	0.01	(0.02%)	(0.02)
S + M + *LCB*	3.30%	0.35	3.41%	0.26	3.41%	0.26
	(0.12%)	(0.06)	(0.10%)	0.01	(0.10%)	(0.02)
Size Quartiles						
LCB	3.41%	0.29	3.51%	0.26	3.51%	0.24
4Q1 + *LCB*	3.38%	0.46	3.49%	0.36	3.49%	0.36
	(0.04%)	(0.17)	(0.02%)	0.10	(0.02%)	(0.12)
4Q2 + 4Q1 + *LCB*	3.34%	0.47	3.44%	0.36	3.44%	0.36
	(0.07%)	(0.19)	(0.07%)	0.10	(0.07%)	(0.12)
4Q2 + *LCB*	3.35%	0.32	3.44%	0.26	3.44%	0.26
	(0.07%)	(0.03)	(0.07%)	0.01	(0.07%)	(0.02)
4Q3 + *LCB*	3.34%	0.31	3.45%	0.26	3.45%	0.25
	(0.08%)	(0.02)	(0.07%)	0.00	(0.07%)	(0.01)
4Q1 + 4Q3 + *LCB*	3.34%	0.47	3.45%	0.36	3.45%	0.36
	(0.08%)	(0.19)	(0.07%)	0.10	(0.07%)	(0.12)

Table 10 – Diversification Benefits with Large Cap international Benchmark (continued)

Investment opportunity set	Unconstrained		No short sales		Upper bound	
	GMV (ψ)	SR (λ)	GMV (ψ)	SR (λ)	GMV (ψ)	SR (λ)
Size Quintiles						
*	3.41%	0.29	3.51%	0.26	3.51	0.24
5Q1 + *	3.40%	0.43	3.50%	0.33	3.50%	0.33
	(0.02%)	(0.14)	(0.01%)	(0.08)	(0.01%)	(0.10)
5Q2 + 5Q1 + *	3.34%	0.41	3.45%	0.33	3.45%	0.33
	(0.07%)	(0.12)	(0.06%)	(0.08)	(0.06%)	(0.10)
5Q2 + *	3.35%	0.36	3.45%	0.29	3.45%	0.29
	(0.07%)	(0.08)	(0.06%)	(0.03)	(0.06%)	(0.05)
5Q3 + *	3.30%	0.37	3.40%	0.29	3.40%	0.29
	(0.11%)	(0.08)	(0.11%)	(0.03)	(0.11%)	(0.05)
5Q1 + 5Q3 + *	3.30%	0.43	3.40%	0.33	3.40%	0.33
	(0.12%)	(0.15)	(0.11%)	(0.08)	(0.11%)	(0.10)
5Q4 + *	3.35%	0.32	3.46%	0.26	3.46%	0.25
	(0.06%)	(0.03)	(0.05%)	(0.00)	(0.05%)	(0.01)
5Q1 + 5Q4 + *	3.35%	0.44	3.46%	0.33	3.46%	0.25
	(0.06%)	(0.15)	(0.05%)	(0.08)	(0.05%)	(0.02)
5Q1 + 5Q2 + 5Q3 + *	3.29%	0.43	3.40%	0.33	3.40%	0.33
	(0.13%)	(0.15)	(0.11%)	(0.08)	(0.11%)	(0.10)

Table 10 – Diversification Benefits with Large Cap international Benchmark (continued)

Investment opportunity set	Unconstrained		No short sales		Upper bound	
	GMV (ψ)	SR (λ)	GMV (ψ)	SR (λ)	GMV (ψ)	SR (λ)
First Subperiod: 1991/1 - 1997/12						
Threshold approach						
LCB	2.79%	0.58	2.80%	0.50	2.80%	0.48
S + ***LCB***	2.72%	0.66	2.75%	0.51	2.75%	0.51
	(0.06%)	(0.07)	(0.05%)	(0.02)	(0.05%)	(0.03)
Size Quartiles						
LCB	2.79%	0.58	2.80%	0.50	2.80%	0.48
4Q1 + ***LCB***	2.76%	0.84	2.78%	0.66	2.78%	0.66
	(0.03%)	(0.25)	(0.03%)	(0.16)	(0.03%)	(0.18)
Size Quintiles						
LCB	2.79%	0.58	2.80%	0.50	2.80%	0.48
5Q1 + ***LCB***	2.76%	0.79	2.78%	0.64	2.78%	0.63
	(0.02%)	(0.21)	(0.02%)	(0.14)	(0.02%)	(0.15)
5Q1 + 5Q2 + ***LCB***	2.74%	0.82	2.76%	0.64	2.76%	0.63
	(0.05%)	(0.23)	(0.04%)	(0.14)	(0.04%)	(0.16)
5Q2 + ***LCB***	2.74%	0.74	2.76%	0.58	2.76%	0.58
	(0.04%)	(0.15)	(0.04%)	(0.09)	(0.04%)	(0.10)
5Q3 + ***LCB***	2.74%	0.66	2.76%	0.53	2.76%	0.53
	(0.04%)	(0.08)	(0.04%)	(0.03)	(0.04%)	(0.05)
Second Subperiod: 1998/1 - 2004/12						
Threshold approach						
LCB	3.34%	0.18	3.64%	0.11	3.73%	0.11
M + ***LCB***	3.31%	0.36	3.64%	0.22	3.72%	0.18
	(0.02%)	(0.19)	(0.00%)	(0.11)	(0.01%)	(0.07)
Size Quartiles						
LCB	3.34%	0.18	3.64%	0.11	3.73%	0.11
4Q1 + ***LCB***	3.27%	0.35	3.61%	0.26	3.69%	0.26
	(0.07%)	(0.18)	(0.04%)	(0.15)	(0.03%)	(0.15)
4Q1 + 4Q4 + ***LCB***	3.26%	0.36	3.61%	0.26	3.69%	0.26
	(0.07%)	(0.18)	(0.04%)	(0.15)	(0.03%)	(0.15)
Size Quintiles						
LCB	3.34%	0.18	3.64%	0.11	3.73%	0.11
5Q1 + ***LCB***	3.26%	0.31	3.60%	0.22	3.70%	0.22
	(0.07%)	(0.13)	(0.04%)	(0.11)	(0.03%)	(0.11)
5Q3 + ***LCB***	3.28%	0.28	3.59%	0.18	3.66%	0.17
	(0.06%)	(0.10)	(0.05%)	(0.08)	(0.06%)	(0.06)

FIGURE 1 –Diversification benefits from adding 4Q1 portfolio (1986-2004)

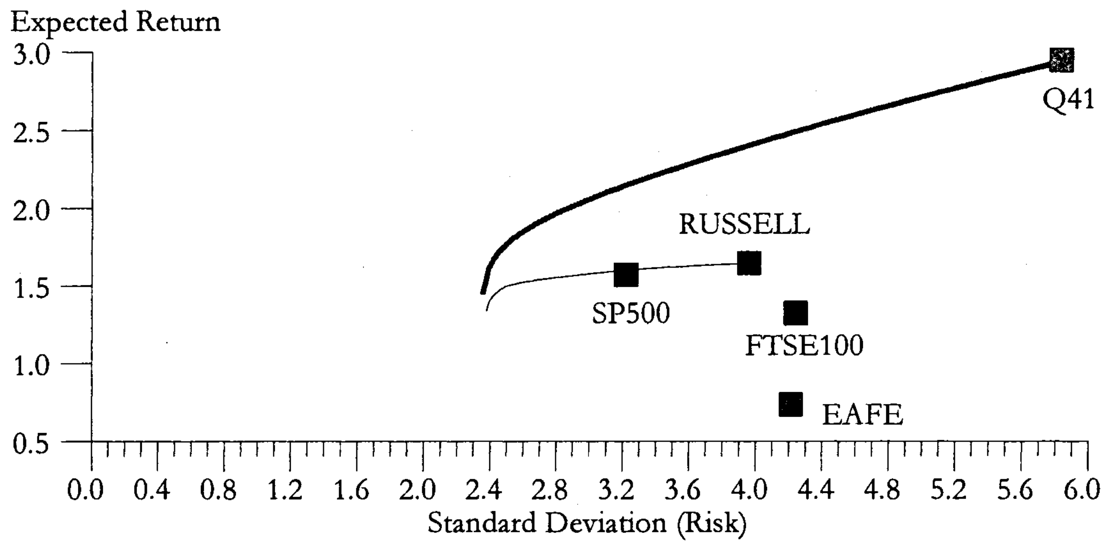

FIGURE 2–Diversification benefits from adding 4Q1 portfolio (1998-2004)