# LIFE CYCLE COST FOR REHABILITATION OF PUBLIC INFRASTRUCTURES: <br> <br> APPLICATION TO MONTREAL METRO SYSTEM 

 <br> <br> APPLICATION TO MONTREAL METRO SYSTEM}
by

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#### Abstract

Life Cycle Cost For Rehabilitation Of Public Infrastructures: Application to Montreal Metro System

Mazen Farran

According to the National Guide to Sustainable Municipal Infrastructure (InfraGuide), Canadian municipalities spend $\$ 12$ to $\$ 15$ billion annually on infrastructure; however, this does not seem to be enough to maintain ageing infrastructures and rehabilitate them to higher safety standards. The solution according to InfraGuide is "to change the way we plan, design, and manage infrastructures".

Several rehabilitation planning methods are reported in the literature for public infrastructures, such as bridges, pavements, sewers, or others. These methods, however, are limited to specific types of infrastructure. In this research, a novel method for Maintenance and Rehabilitation Planning for Public Infrastructure (M\&RPPI) is developed. One that is generic for any type of public infrastructure. The method aims at determining the optimal rehabilitation profile over a desired analysis period. Specifically, it will determine the best type of rehabilitation intervention, and its optimal timing. The M\&RPPI method is based on life-cycle costing (LCC) with probabilistic and continuous rating approach for condition states. The M\&RPPI also uses a new approach of "dynamic" Markov chain to represent the deterioration mechanism of an infrastructure and the impact of rehabilitation interventions on such infrastructure.


As an optimization technique, genetic algorithm (GA) is used in conjunction with Markov chains in order to find the optimal or quasi-optimal rehabilitation profile. The
way GA communicates with the transition probability matrices (TPM) is described. In addition, a new directed-GA approach was developed in order to guide the optimization process toward the final solution. Finally a computer program using Excel and VBA macros is developed in order to prove workability of the developed method.

The developed M\&RPPI methodology is applied to the deterioration problem of Montreal Metro system. In order to validate the performance of the proposed methodology, three different types of analysis are performed using: (1) traditional Markov decision process (MDP) that uses a discrete rating scale, (2) continuous rating method, and (3) the proposed M\&RPPI method with GA optimization technique. Results show the benefits of using continuous rating in contrast with discrete method. They also demonstrate the superiority of GA compared to other optimization methods. In addition, the proposed M\&RPPI method provides a complete M\&R Plan over a required study period, not only a stationary decision policy. Finally, the M\&RPPI is a major step towards a broader infrastructure management system, addressing network-level problems.

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## 1 INTRODUCTION

### 1.1 Background and problem statement

Public infrastructures such as sanitary and water systems, roadways, bridges as well as mass transit systems (e.g. metro systems or subways), or any other type of infrastructure, require proper management in order to maintain their operational and safety performance status. According to the National Guide to Sustainable Municipal Infrastructure (InfraGuide), Canadian municipalities spend $\$ 12$ - $\$ 15$ billion annually on infrastructures. However, this is not sufficient in maintaining ageing infrastructures and rehabilitating them to higher safety standards. In fact, these infrastructures are subject to deterioration and failure during their lifetime. Deterioration could occur because of two major categories; first due to time-invariant property of the structures, which depends on the normal loading history and the aging of the infrastructure and its operational environment. Second, deteriorations could occur due to time-variant loading such as earthquakes or other natural hazards (Cho et al., 2004). Negligence and mismanagement of public infrastructures could lead to catastrophic failures and costly and disruptive replacements (Wirahadikusumah, 1999). The limitations of financial resources, however, make the continuous rehabilitation of these infrastructures difficult, and the need of a decision support system becomes crucial in order to optimize the usage of available funds through cost effective solutions. In fact, the solution according to InfraGuide is "to change the way we plan, design, and manage infrastructures".

Several rehabilitation planning methods are reported in the literature for public infrastructures, such as bridges, pavements, sewers or others, however these methods are limited to a specific type of infrastructure. For instance, De Brito and Franco (1998) studied concrete bridge repair problem. Wirahadikusumah (1999) studied the problem of sewer management. Lee (2002) worked on performance of bridge deck expansion joints. Jawad (2003) studied the pavements rehabilitation problem. Shahata (2006) analysed water mains. In most of these studies, life-cycle costing (LCC) is found to be crucial to rehabilitation planning, and to the analysis of infrastructure life-cycle performance. In fact, LCC became a paradigm for most engineering decision problems in practice (Cho et al. 2004) and is used as a tool in the decision making process in a multitude of engineering fields. Actually, it is more effective to make decisions based on the life cycle costs and the total future expenditures, rather than totally relying on the initial construction costs. Thus, LCC has significant potential to reduce total capital expenditures (Wirahadikusumah, 1999). Consequently, an automated LCC procedure becomes essential in order to compare different scenarios for rehabilitating existing infrastructures. Also, in previous studies, several infrastructure rehabilitation models are developed based on different deterioration process representations. Some of these representations are based on a deterministic approach using regression methods, and some have a probabilistic approach.

A probabilistic modeling method for infrastructure performance is the Markov Chain, which is extensively used in its traditional mode in conjunction with dynamic programming in order to obtain the optimal decision policy (Zayed, 1999 \& 2002; Wirahadikusumah, 1999; Madanat, 1995b). This traditional technique is known as the

Markov Decision Process (MDP) and it allows obtaining the optimal decision schema according to both the infrastructure condition as well as the cost of each maintenance and rehabilitation alternative. This method has several limitations. First, it is based on a finite discrete number of condition states, then it assumes a Markovian behaviour (or memoryless) of the infrastructure, which might not be always totally true. Also, the outcome of this method is simply a set of optimal decisions, where an optimal intervention is found for each specific state, without providing clear impression concerning the life-cycle performance of the infrastructure. This might offer limited support to decision-makers and as a result a more realistic method becomes essential. The required methodology should have a probabilistic formulation that reflects the most deterioration process of infrastructure. It should also consider some uncertain factors that could influence each rehabilitation scenario and the uncertainties related to infrastructure deterioration behaviour. In addition, the different type of costs, such as user and agency costs, should be included.

### 1.2 Objectives and Scope of the Dissertation

The main objective of this research is to develop a new LCC-based method for maintenance and rehabilitation planning of public infrastructure M\&RPPI in order to obtain the optimal or quasi-optimal maintenance and rehabilitation (M\&R) profile over the desired analysis period. More precisely, the M\&RPPI will determine the best type of intervention and its proper timing as well as the corresponding infrastructure performance overview. An application to the Metro system will also be undertaken in order to
understand how the method could work on real systems. In order to attain these goals, several sub-goals will be included as follows:

- Develop a novel LCC-based method for M\&R planning of public infrastructure (M\&RPPI), using the Markov chain deterioration modelling and genetic algorithms optimization techniques.
- Develop a new model for the $M \& R$ planning of Metro stations using different methods including the traditional Markov decision process (MDP) and the proposed M\&RPPI method.
- Develop a computer program using Excel and VBA in order to implement the proposed M\&RPPI method and prove its workability.


### 1.3 Research Methodology

The proposed M\&RPPI method is LCC-based, and uses probabilistic formulation approach for the deterioration process during the infrastructure life-cycle. In fact, it is based on a modified-Markovian behaviour, which differs from the traditional approach by allowing the usage of multiple transition probability matrices (TPM) or a dynamic Markov chain. This will provide the proposed method with a much flexible way of addressing the problem at hand, especially with regard to modeling of infrastructure deterioration process. Also, the proposed method is based on a continuous condition rating and not a discrete one, which will empower it with more practicality and robustness over the traditional MDP approach.

The procedure includes an optimization technique based on genetic algorithms methodology, and which is adapted in this research for infrastructure M\&R problems. However as a limitation, the proposed methodology considers only a segment-specific problem and not a network-level analysis. The methodology tackles only a certain deteriorating area and comes up with the best possible rehabilitation profile for this specific problem. A complete network-level solution is however essential for real case infrastructures. But despite this limitation, the proposed M\&RPPI methodology is a key point for a broader infrastructure management system. It allows two major concepts, traditionally used separately, namely the Markov chain and the genetic algorithms, to be linked together in order to achieve an efficient performance. An application of this method is applied to a metro system rehabilitation problem, where a model is developed for this purpose. Beside public infrastructures the proposed methodology could also be applied for M\&R of any other type of deteriorating element.

### 1.4 Research Organization

Following this introductory chapter, chapter two presents the literature background where four basic topics will be discussed. First, the concepts of maintenance and rehabilitation of public infrastructures are presented, then the explanation of deterioration process and its modeling methods, with an emphasis on the Markov chain. Afterward, in the same chapter, the available rehabilitation methods are presented and a new diagram for the rehabilitation of infrastructure is introduced. Also, in the literature section, the economic principles used in comparing different rehabilitation scenarios are presented, and the foundations of life-cycle costing (LCC) is explained. The LCC includes the different type
of costs related to a project, namely the agency and user costs. Finally, the background and the theory of genetic algorithms (GA) are presented as optimization technique.

In chapter three, the proposed M\&RPPI method is described. This method is based on a combination of LCC concepts; the Markov chain for deterioration modeling and GA optimization technique. An explanation about the process of integrating the transition probability matrices to genetic algorithms is done, and the way GA could be adapted for the specific problem of $M \& R$ of infrastructure is also explained.

In order to demonstrate the workability of the proposed method, a computer program using Excel and VBA macros is developed for the purpose of this research. The explanation of the program and its possible improvement are discussed in chapter four.

In chapter five the Metro system is presented with some data collection from Montreal Metro.

Chapter six presents a Metro model using the proposed method, as well as using other traditional methods such as Markov Decision Process (MDP). Also in chapter six, an analysis of the results is performed as well as a comparison between the different methods.

Finally, chapter seven contains the conclusions of this thesis with some recommendations for possible future works.

## 2 LITERATURE REVIEW

### 2.1 Introduction

In this literature review chapter several concepts will be studied. Some concepts are related to infrastructure management, and some are related to the genetic algorithms optimization techniques. These concepts are as follows:

1. Deterioration models for public infrastructures.
2. Maintenance and rehabilitation of infrastructure.
3. Engineering economic concepts and life-cycle cost (LCC) analysis of public infrastructure.
4. Genetic algorithms optimization techniques.

Each one of these concepts is a field of research in itself, and a brief overview will be presented.

### 2.2 Modeling of Infrastructure Deterioration Mechanism

Infrastructure systems are normally constructed to serve for an intended service-life within a certain level of operational performance. In order to perform proper management to the infrastructure, and to evaluate the expected life-cycle maintenance and repair cost for a rehabilitation strategy, a performance modeling of the infrastructure should be established. Hudson et al. (1997) defines performance as "the degree to which a facility
serves its users and fulfills the purpose for which it was built or acquired, as measured by the accumulated quality and length of service that it provides to its users." Figure 2-1 illustrates several forms of performance behaviour, where a quality measurement such as condition index (CI) is plotted over-time on a scale of 100. A value of 100 represents the best possible measured quality and a value of 0 represents the worst state or totally unacceptable condition.


Figure 2-1 Illustration of several forms of performance curves (Hudson, et al., 1997) As it is shown in Figure 2-1, the overall operational performance of infrastructures tends to diminish with time, and rehabilitation interventions become necessary to maintain an acceptable level of service quality. An optimum time could be determined in order to perform a maintenance or a rehabilitation intervention with the minimum costs. It could be observed in Figure 2-1 that a minimum acceptable level of 40 is chosen as a threshold value where a rehabilitation intervention becomes mandatory. This threshold normally depends on the infrastructure type, its importance, or other factors such as the amount of safety involved or other economic issues.

Deterioration or performance trends could vary based on several factors that affect the infrastructure, such as material aging, loading conditions, environment conditions or other mechanisms. Also, lack of regular inspections and maintenance actions affect the behaviour of the infrastructure. For instance, curve "A" represents a case where the facility demonstrates a good performance and a high service level for most of its intended service life, whereas curve "C" presents a poorer performing infrastructure with a faster deteriorating rate. Curves " $B$ " and " $E$ " describe other types of performance behaviour. A proper rehabilitation intervention improves the quality condition as represented by the sudden increase of quality level in curve D , and an automatic extension of the infrastructure service-life.

It is unfortunate that it is rather difficult to accurately predict a deterioration process of an infrastructure (Yokota et al., 2004). Even though extensive studies have been undertaken to observe material such as concrete, and to analyze, for example, the chloride ion profile and the corrosion rate of the rebar, an adequate prediction model still presents a challenge to researchers. Prediction models may vary from one method to another. One method for predicting the deterioration process that is extensively used for infrastructure systems involves a mathematical principle called Markov-Chain. The next section presents an overview of this method.

### 2.2.1 Deterioration process by Markov-chain

Markov-chain models are commonly used to represent performance evolution and to predict future conditions of infrastructures. Transition probability matrices (TPM) are established based on previous deterioration data allowing future conditions to be
determined. Many researches used the Markov chains in various infrastructure management fields. For example, Morcous (2006), Lee (2002), and Jiang (1990) used Markov chains for bridges management systems. Wirahadikusumah (1999) and Baik et al. (2006) used it for sewer management. Butt et al. (1987), Yang et al. (2005), Ortiz-García et al. (2006), Hong and Prozzi (2006) used it for pavement deterioration modelling.

In a Markov model, the deterioration profile of the infrastructure is considered to be a stochastic process that evolves in time in a probabilistic manner. The stochastic process is represented by an indexed collection of random variables $\{\mathrm{X}(\mathrm{t}), \mathrm{t} \in \mathrm{T}\}$, and it describes the deterioration degree or state $\mathrm{X}_{1}, \mathrm{X}_{2}, \mathrm{X}_{\mathrm{m}}$ at a particular points of time $t$ of the deteriorating element service life (Hillier and Lieberman, 2005). For a stochastic process to be considered as a Markov-chain, it should have a key property called the Markovian property (also described as the "lack-of-memory property"), where the probability of any future "event" given any past "event" and the present "state" $\mathrm{X}_{\mathrm{t}}=\mathrm{i}$ is independent of the past event and only depends upon the present state. This is expressed as follows:
$P\left\{X_{t+1}=j \mid X_{0}=k_{0}, X_{1}=k_{1}, \ldots, X_{t-1}=k_{t-1}, X_{t}=i\right\}=P\left\{X_{t+1}=j \mid X_{t}=i\right\}$, for $t=0,1, \ldots$

## Equation 2-1

The conditional probabilities $P\left\{X_{t+1}=j \mid X_{t}=i\right\}$ also denoted $P_{i j}{ }^{(n)}$, are designated as the transition probabilities for $n$ periods, which are also called $n$-step transition probabilities. Thus $\mathrm{P}_{\mathrm{ij}}{ }^{(\mathrm{n})}$ is the conditional probability that the state $X$ (random variable), will switch from state (i) to state (j) after exactly n steps (in time units). For deteriorating infrastructures it represents the probability of shifting from a certain condition state of deterioration to another condition state at a future point in time. A notation to represent n step transition probabilities in a matrix form is as follows (Hillier and Lieberman, 2005):

Where $\quad P_{i j}{ }^{(n)} \geq 0$ for all $i$ and $j$, and $n=0,1,2 \ldots$
Equation 2-2

$$
\sum_{j=0}^{M} P_{i j}{ }^{(n)}=1 \text { for all i, and } \mathrm{n}=0,1,2, \ldots
$$

and $\quad 0 \leqslant P_{i j} \leqslant 1$

These transition probabilities represent the transition from the row state to the column state after $n$ steps. If the transition probabilities do not change in time thus $P\left\{X_{t+1}=j \mid\right.$ $\left.X_{t}=\mathrm{i}\right\}=P\left\{X_{1}=j \mid X_{0}=\mathrm{i}\right\}$ then they are said to be stationary (one step transition), and are denoted $P_{i j}{ }^{(0)}$ or simply $P_{i j}$. It should be noted that if the value of $P_{n n}$ has a value of 1 , then this is called the absorption state, where the state cannot be vacated once it is reached. This implies that the deterioration will stop at a certain level, usually the worst state, or at an unacceptable state and will remain in as such unless a rehabilitation action is performed or becomes mandatory.

In the majority of references, such as Lee (2002), Yokota et al. (2004), and Abraham (1998), the deterioration models using Markov chains use a discrete time (e.g. yearly interval) rather than continuous time, also a fixed transition matrix (one-state $\mathrm{P}_{\mathrm{ij}}$ ). Knowing the value of the initial state $X(0)$ and the value of the transition probability
matrix, the state vector $X(t)$ at any future time (after $n$ periods) can be calculated using the following equation:
$\mathrm{X}(\mathrm{t})=\mathrm{X}(0) * \mathrm{P}_{\mathrm{ij}} * \mathrm{P}_{\mathrm{ij}} * \mathrm{P}_{\mathrm{ij}} \ldots=\mathrm{X}(0) * \mathrm{P}_{\mathrm{ij}}{ }^{\mathrm{T}}$
Equation 2-3
where $P_{i j}{ }^{T}$ is the $t^{\text {th }}$ power of the transition matrix $P_{i j}{ }^{(n)}$

This equation is derived from the Chapman-Kolmogorov equations which provides a method for computing the $n$-step transition probability $\mathrm{P}_{\mathrm{ij}}{ }^{(\mathrm{n})}$ knowing the one-step transition probability (Hillier and Lieberman, 2005).

The most important step in a Markov chain is to determine the value of the transition probability. Finding the most representative transition probability for a specific infrastructure is not an easy task. In fact, the deterioration progress is affected by numerous factors such as material quality, infrastructure environment, the applied loads or other surrounding factors. In general the transition probability is generated from site investigation and degradation data, or from expert opinion and engineering probability theories. Several methods are available to estimate the transition probability matrix. The regression function, which is also named the expected value method, is one of the most common methods used in literature. Other methods used are the frequency method and the Poisson method that are a slight modification of the regression method (Madanat, 1995a). In these methods, infrastructure condition is observed and compared between two consecutive inspections. However, these methods could present some disadvantages and may lead to erroneous results (Lee, 2002).

Once a suitable transition probability is found, the future infrastructure state can be forecasted and the life-cycle cost related to rehabilitation interventions could be fairly estimated. The cost information depends on the $M \& R$ solution undertaken and on the degree of degradation. Any selected $M \& R$ intervention directly affects the transition probability as well as the related life cycle cost. Thus the infrastructure management system is a dynamic process that depends on the decisions undertaken during the lifetime of the infrastructure and on the behaviour of the infrastructure. In order to find the optimal $M \& R$ action, a method such as the dynamic programming will be required in order to solve sequential decision problems where decisions should be made at each period to select the M\&R action from the different available options. This is known as the Markov Decision Process (MDP), and is extensively used in the literature for public infrastructures. For example, Zayed and Fricker (1999), and Zayed et al. (2002) used the MDP concept to analyse the steel bridge paint system problem. Wirahadikusuma (1999) used the MDP to study the sewer management problem. The explanation of the MDP methodology as well as the LCC will be further explained later in this document.

### 2.2.2 Alternate performance modeling

Kong and Frangopol, (2003), (2004a), and (2004b) state that in order to perform a realistic life-cycle analysis of deteriorating structures under different maintenance scenarios, a method based on a reliability concept should be used. In fact, the reliability of an infrastructure is defined as "the probability that a component or a system will satisfactory perform its specified function for the specified or required period of time under given or predicted operation condition" (Hudson, et al., 1997).

Also, Kong \& Frangopol (2004b) stated that the structural management systems should be developed based on time-varying loads instead of considering only the structural deterioration. In their proposed method, the authors present a reliability index profile approach that considers simultaneously the time-varying effects of both loads and resistance deterioration. In addition, the reliability-based framework considers the effect of maintenance and rehabilitation interventions that are expected to occur in the future. Kong and Frangopol, (2003), (2004a), and (2004b) provide a more in-depth explanation about this topic as well as the methods of deriving the cost and probability functions.

### 2.3 Maintenance and Rehabilitation of Infrastructures

The next phase following the construction of the infrastructure is the operation phase. The length of the operation phase corresponds to the service period of the infrastructure during which degradation occurs. Preservation should be then undertaken in order to prevent any major failures, and to extend the infrastructure service-life while maintaining a minimum acceptable level of operational service. Preservation actions could be divided into both, maintenance as well as rehabilitation actions. The dividing line between maintenance and rehabilitation is sometimes not well understood, and there is a wide divergence in defining each type of action (Hudson, W. et al., 1997). Maintenance is sometimes defined as repair work done to infrastructure with local funds (e.g. maintenance department) while rehabilitation describes work done from external funding such as governmental sources (municipal, provincial, or federal). Also, maintenance and rehabilitation are sometime differentiated by the way the work is accomplished, and the amount of money invested in the activity. For instance, maintenance could define the
work done by in-house forces whereas rehabilitation is the work done by contract. Rehabilitation could also be utilized when regular maintenance work is not adequate to solve a certain deficiency and involves a larger scale of work. Thus differentiating between these two terms is often "policy-and-rule-dependent" (Hudson, W. et al., 1997).

Both maintenance and rehabilitation are expenditures incurred during the service life of the infrastructure in order to maintain its operational performance, to prevent major failures and to extend the operational period. Hudson et al. (1997) differentiates these two types of interventions as follows: Maintenance is "the action of keeping in effective condition, in working order, in repair". Some subgroups of maintenance exist. First, regular or routine maintenance are interventions performed at periodic intervals that are essential to the normal operation of the infrastructure. An analogy would be to oil and filter changes for cars which are essential for maintaining normal operation. Secondly, preventive maintenance, or proactive maintenance $(\mathrm{PM})$ is a maintenance action that is done in an effort to "forestall trouble" and prevent further deterioration. Preventive maintenance could be also undertaken on periodic intervals and if omitted, higher regular maintenance costs might be encountered in the future. But normally it may be harder to justify the PM funds versus regular maintenance (Frangopol et al., 2004). On-conditionMaintenance ( OCM ) is a type of preventive maintenance performed after a condition monitoring of the infrastructure is performed. The maintenance action will depend on the condition or deterioration state. Finally, corrective maintenance is a reactive action performed to repair damage that has become apparent, or to restore an infrastructure to a satisfactory level of operation. Corrective maintenance is an "after-the-fact-activity", and
critical maintenance is the maintenance that must be done immediately to prevent imminent collapse of the infrastructure.

On the other hand, Hudson et al. (1997) defines rehabilitation as "the act or process of making possible a compatible use of a property through repair, alterations, and additions, while preserving those portions or features that convey its historical, cultural or architectural values". It can be noticed in the definition the importance of the historical, cultural and architectural values beside the monetary values. An example of the importance of preserving cultural and architectural values is the rehabilitation project on the telecommunication network of the Montreal Metro called Réno-Systèmes. These two factors were predominant criteria in order to decide the location of new telecommunication rooms. Some easy-to-implement solutions were disregarded due to their effect on the architectural appearance of the station and more expensive solutions were considered instead.

From the aforementioned definitions of maintenance and rehabilitation ( $M \& R$ ), the diagram in Figure 2-2 is proposed for visualizing the different categories of M\&R. It will be possible to combine these categories in order to establish different solutions. However, in the context of this thesis the rehabilitation will be used to describe repair work or interventions that are aimed to correct deficiencies requiring considerable amount of finance. Rehabilitations also include modernizations or changes in order to comply with new codes and regulations. On the other hand, the maintenance works include small repair work or corrective maintenance, as well as preventive actions. In Figure 2-2 both
rehabilitation and maintenance are incorporated into a single envelope that includes three sub-categories as follows:

- Regular / preventive maintenance (PM)
- Corrective maintenance / Repair Work (CM)
- Major rehabilitation / Replacement (R)

In each category different actions could be taken. For instance, CM1, CM2, etc., would be possible corrective maintenance actions that could be considered. The dashed line between the sub-groups implies that some interaction exists between these categories.

When confronted with a specific deficiency, and in order to find the optimal solution, different scenarios or strategies could be found, and each alternative scenario could combine more than one action from different categories. For instance a specific problem could entail a solution that requires specific repair work (RP1) and a regular maintenance action (RM1), or it could require a major rehabilitation (R1) of a certain component and a few other minor repairs (RM2, RM3) etc. It is important to note that all rehabilitation scenarios are not equivalent. Some scenarios require more then one intervention at different periods of time while others are less efficient and are merely temporary solutions. Historical data regarding repairs and periodic or preventive maintenance could be used to forecast future maintenance and repair costs. Also these costs could be derived from prediction models or from expert opinions.


Figure 2-2 Proposed maintenance and rehabilitation (M\&R) diagram

### 2.4 Life-Cycle Cost Analysis for Public Infrastructures

Life-Cycle Cost (LCC) Analysis is widely used for bridges, pavement or other types of infrastructure rehabilitation planning. For any given item, LCC is defined as "the sum of
all expenditures associated with the item during its entire service life" (White et al., 1998). In our case, the term item represents the infrastructure under consideration, but it could also be interpreted as a machine, a project, a building, or a system. Also LCC is used as a systematic process for economical evaluation and comparison of competing projects over the same desired service life. LCC takes into account all the expected costs incurred during the project life cycle practice, and typically includes installation or initial costs, operation, maintenance and rehabilitation costs as well as salvage value. In the context of this work, LCC will be applied to evaluate different rehabilitation scenarios for any public infrastructure system, more specifically for metro systems, in order to determine the optimal rehabilitation solution. A metro system has the particularity of enclosing the tunnelling element between the different stations, as well as the building component at its entrance hall. An application to the Montreal Metro will be presented later in this research.

In order to have a common base of comparison, LCC of each alternative is converted to either the Present Value (PV), the Future Value (FV) or to an equivalent annual value referred to as Equivalent Uniform Annual Cost (EUAC) (Grant et al., 1990). The LCC process uses an economic technique known as "discounting"; the process of summing up the discounted monetary (explained later) equivalency of all benefits and costs that are expected to be incurred in each rehabilitation alternative (Ozbay et al., 2003).

The use of life-cycle cost analysis provides decision-makers with valuable information. It offers several advantages. The following is a list of reasons for using LCC and some related advantages:

- Perform an economical comparison in the presence of project alternatives that fulfill the same performance requirements, but differ with respect to initial and operating costs.
- Compare several alternatives following a systematic procedure in order to select one that maximizes net savings or provides most benefits.
- Evaluate investments over their lifetime, reduce overall cost, and obtain an economically efficient solution.
- Provide an efficient decision support system for management, and perform effective management of resources and assets especially with limited financial resources and budget tightening.

However according to a survey among municipalities in the U.S. only $40 \%$ are using LCC, and the criteria used in selecting the project seems to be arbitrary (Wirahadikusumah, 1999). In the following sections an explanation of the LCC procedure is provided with an overview of the available economical.

### 2.4.1 Economic evaluation

The main economic bases of comparison for evaluating different rehabilitation strategies of public infrastructures are:
a) Present Worth method or Net Present Value (PW, NPV)
b) Future Worth method (FW)
c) Annual Equivalent method or Equivalent Uniform Annual Cost method (EUAC)
d) Internal Rate of Return method (IRR)
e) Payback Period (PB)
f) Project Balance
g) Benefit to Cost Ratio
h) Cost-Effectiveness Method

Each method presents a different means and basis to compare between competing alternatives. They each present different types of information about the economical attractiveness of the investment. The choice the most appropriate method for each rehabilitation project depends on the management requirements and expectations from the project. For instance a present-worth method may be more suitable for public agencies, whereas for a particular private facility, an annual cost method could be more attractive (Hudson et al., 1997). In this research the adapted method is the PV which has been chosen for its simplicity and ease of use, especially given that the different scenarios being compared all have the same planning period. Thus it will be easy to discount all costs to the present and compare them to each other. The following is the explanation of the NPV method.

### 2.4.1.1 Present Worth Method

The present-worth method (PW) is an economical evaluation of a project that involves the discounting of all future sums to the present, using an appropriate discount rate. In fact, Thusen and Fabrycky (1993) define the present-worth method as "the net equivalent amount at the present that represents the difference between the equivalent disbursements and the equivalent receipts of an investment's cash flow for a selected
interest rate." Thus, the PW considers both costs and benefits of an alternative, and can be expressed as:

$$
P W(N P V)=\sum_{t=0}^{n} \frac{B_{t}-C_{t}}{(1+i)^{t}}
$$

Equation 2-4
$B_{t}=\quad$ Benefits occurring in period $t ;$
$C_{t}=\quad$ Costs incurred in period $t$;
$\mathrm{i}=\quad$ Discount rate;
$\mathrm{n}=\quad$ Length of the study period; (usually in years)

And for evaluating project alternatives that have equal benefits, Equation 2-4 becomes:

$$
N P V C=\sum_{t=0}^{n} \frac{C_{t}}{(1+i)^{t}}
$$

## Equation 2-5

Numerous factors make the present-worth method suitable as a basis of economical evaluation. This method considers the time value of money according to a selected discount rate. In addition the PW concentrates the total cash-flow of an alternative to a single present value making it easy to compare between other alternatives (Thuesen and Fabrycky,1993). The present-worth method is particularly suitable when comparing a project alternative that carries a high initial cost versus an alternative with lower initial but high future costs. This method is also useful in comparing a project alternative with a "do-nothing" alternative, the latter has no initial costs but encounters higher maintenance and repair costs.

For infrastructure rehabilitation projects, the present-worth methods could consider costs and benefits separately or together, the present-worth for costs only is expressed as (Hudson et al., 1997):

$$
T P W C_{x 1, n}=(I C C)_{x 1}+\sum_{t=0}^{n}\left\{p w f_{i, t}\left[(C C)_{x 1, t}+(M O)_{x 1, t}+(U C)_{x 1, t}\right]\right\}-p w f_{i, n}(S V)_{x 1, n}
$$

Equation 2-6

Where the factor for discounting either costs or benefits is calculated as follows:

$$
p w f_{i, n}=1 /(1+i)^{n}
$$

$p w f_{i, t} \quad=$ Present-worth factor for discount rate, $i$, for $t$ years;
$\mathrm{n} \quad=$ Number of years to when the sum will be expended or saved;
i $\quad=$ Discount rate;
$\mathrm{TPWC}_{\mathrm{x} 1, \mathrm{n}} \quad=$ Total present worth of costs for alternative $x_{1}$, for analyses period of $n$ years;
$(\text { ICC })_{\mathrm{x} 1, \mathrm{n}} \quad=$ Initial capital costs of construction, etc., for alternative $x_{1}$;
$(\mathrm{CC})_{\mathrm{x} 1, \mathrm{n}} \quad=$ Capital costs of construction, etc. for alternative $x_{1}$, in year $t$, where $t<n$;
$(\mathrm{MO})_{\mathrm{x} 1, \mathrm{n}} \quad=$ Maintenance plus operation costs for alternative $x_{I}$ in year $t$;
$(\mathrm{UC})_{\mathrm{x} 1, \mathrm{n}} \quad=$ User costs, if applicable for alternative $x_{I}$ in year $t$; and
$(\mathrm{SV})_{\times 1, \mathrm{n}} \quad=$ Salvage value, if any, for alternative $x_{I}$ at the end of the analyses period $n$ years.

The present-worth of benefits is expressed as:

$$
T P W B_{x 1, n}=\sum_{t=0}^{n} p w f_{i, t}\left[(D U B)_{x 1, t}+(I U B)_{x 1, t}+(N U B)_{x 1, t}\right]
$$

Equation 2-8
$\mathrm{TPWB}_{\mathrm{x} 1, \mathrm{n}} \quad=$ Total present worth of benefits for alternative $x_{l}$, for analyses period of $n$ years;
(DUB) $)_{\mathrm{x} 1, \mathrm{n}} . \quad=$ Direct user benefits accruing from alternative $x_{I}$ in year;
$(\mathrm{IUB})_{\mathrm{x} 1, \mathrm{n}} \quad=$ Indirect user benefits if applicable, accruing from alternative $x_{l}$, in year $t$; and
$(\mathrm{NUB})_{\mathrm{x} 1, \mathrm{n}} \quad=$ Nonuser benefits, if applicable, accruing from alternative $x_{I}$ in year $t ;$

The net-present value is then expressed as the total present worth of benefits minus the total present worth of costs:

$$
N P V_{x 1}=T P W B_{x 1, n}-T P W C_{x 1, n}
$$

## Equation 2-9

### 2.4.2 LCC analysis methodology

The principal behind LCC is to account for monetary equivalency of costs and benefits resulting from project implementation, and taking into account their respective times of occurrence. In this thesis, the focus of the LCC analysis will be used to evaluate and compare different feasible rehabilitation scenarios over the same analysis period for typical deteriorating infrastructure systems. A case study on the Montreal Metro will be presented and analyzed in the next chapter in order to choose the optimal rehabilitation strategy.

LCC analysis should be conducted as early in the project development cycle as possible, and the level of detail in the analysis should be consistent with the level of investment. Basically, for maintenance and rehabilitation projects the process involves the following steps (Ozbay et al., 2003):

1. Define rehabilitation and maintenance alternatives for the analysis period.
2. Decide on the approach: Probabilistic vs Deterministic.
3. Choose the economic inputs parameters: discount rate, study period, etc.
4. Estimate the agency and user costs as well as the expenditure stream.
5. Compute the present value for each alternative.
6. Analyze and compare the results using either a deterministic or probabilistic approach and perform a sensitivity analysis.
7. Re-evaluate strategies and develop new ones if needed.

### 2.4.2.1 Define project alternatives

Prior to beginning a LCC analysis, rehabilitation alternatives need to be established. In order to have a valuable analysis it is recommended that a minimum of three different alternatives should be incorporated into the LCC. These alternatives should be distinctly different and should also be viable solutions for the deteriorating element being addressed. The chosen alternative is to be the most reasonable and cost-effective solution to the problem.

### 2.4.2.2 Deterministic and Probabilistic Approach

Two distinct methods are available in order to perform a LCC analysis: deterministic vs probabilistic approach. In the deterministic approach, point estimates are used for all inputs, whether for the different types of expenditures and costs, their timing, the discount rate to be used, or for other variables. Inputs are based on historical data, on estimates from contractors' quotations, from expert opinion or personal judgment. In the case of construction cost for example, data might be easier to quantify especially in the case where drawings and specifications are clear, and deterministic methods may provide adequate results. But due to varying factors affecting the deterioration process of the infrastructures during their service life and factors influencing the possible rehabilitation scenarios, LCC analysis is characterized with uncertainties. For instance the times of
application of a M\&R action and the duration of these actions depend on various parameters and are considered as random variables. Thus the lack of assurance about future state of the infrastructure, or even fluctuation of interest rates makes the deterministic method unreliable. In fact, deterministic methods give false impression about the outcome, which results in a misleading decision support system. For this reason, a method that considers data uncertainty should be used. The Probabilistic Life-Cycle Cost Analysis (PLCCA) may be defined as a LCCA method where the input variables as well as the results are represented be probability distributions instead of discrete values. Stochastic simulations such as Monte Carlo are generally used during the analysis. In fact, the Monte Carlo techniques are widely utilized in different fields, and many researchers use it for public infrastructures. Ozbay et al. (2003) prepared a guidelines report for lifecycle cost analysis where Monte Carlo was used, with an application to the New Jersey Department of Transportation (NJDOT). Also Shahata (2006) proposed a stochastic modelling for new installation or rehabilitation of water mains using a web-based Monte Carlo simulator.

### 2.4.2.3 Choose the economic inputs parameters

## Study Period

The study period is important in order to evaluate the total investment required to construct and maintain the infrastructure. Defining the length of the service life of an infrastructure utility is the most important component of a life cycle cost analysis. For certain infrastructure such as bridges, careful consideration should be given to their eventual cultural, scenic, or historical value (Lee 2002). The service life is defined as:
"the period in years over which a building, component, or subsystem provides adequate performance; a technical parameters that depends on design, construction quality, operations and maintenance practices, use, and environmental factors; different from economic life" (Hudson et al., 1997). The Economic Life of an asset, which is referred to as the minimum replacement cost life or the optimum replacement interval, is actually: "the time interval that minimizes the asset's total equivalent annual cost or maximizes its equivalent annual net income" (Thusen and Fabrycky, 1993).

While performing a LCCA, a clear distinction should be made between the length of the intended service life of the infrastructure facility and the study or the analyses period. In fact, the study period is usually shorter than the intended life of the infrastructure and it reflects the investors' intentions. The service life of an infrastructure starts from the time of completion of the construction when it becomes functional, until the point where it cannot provide an acceptable level of service due to physical deterioration, poor performance, functional obsolescence, or even high operation costs (Hudson et al., 1997). On the other side, the study period is the period of time over which each rehabilitation scenario is to be evaluated. Since disruption is generally caused to society whenever an infrastructure is replaced, the intended service life of an infrastructure should be planned to be the longest possible (De Brito and Franco, 1998). For public infrastructures the intended life cycle can range from 40 to 120 years, depending on its importance.

## Discount rate and inflation rate

Since infrastructure rehabilitation life-cycle costs include future costs, a discount rate must be selected in order to combine future and present costs. In other words, the
discount rate could be defined as "the rate of interest reflecting the investor's time value of money". This is also known as the minimum attractive rate of return (MARR), hurdle rate, required rate of return, or return on investment (White et al., 1998). It basically represents the percent change in the value of the dollars per period of time that would make an investor indifferent as to whether he paid or received a specific amount in the present versus a greater amount at some point time in the future, and it is used to relate future costs and benefits to present worth terms.

Discount rate has significant implications in the design and rehabilitation of civil infrastructures and in evaluating different rehabilitation alternatives. Variation in discount rate within the infrastructure LC is difficult to forecast and the uncertainty in the discount rate could highly affect the costs of different maintenance and rehabilitation strategies, thus the final solution. In general, the discount rate is established by government agencies, and this deterministic value has to be used when computing expected life-cycle cost.

### 2.4.2.4 Estimate the costs components and the expenditure stream

In order to perform a LCCA, all expected costs should be included in the analysis, thus at this point the schedule of future maintenance and rehabilitation ( $M \& R$ ) interventions is developed and their associated costs are estimated. Costs include both agency costs and user costs. Agency costs consist of those costs directly paid by the organization or the agency (or the government) such as construction costs, initial costs or maintenance costs, while user costs are those related to the functional deficiencies of the infrastructure and are the indirect costs to the users. Once these costs are identified, an expenditure stream diagram can be drawn in order to assist the management or the analyst in visualizing the
timing and the amount of expenditures during the analysis period. Figure 2-3 adapted from James (1991) presents a typical cash-flow diagram or expenditure stream for a typical rehabilitation project.


Figure 2-3 Rehabilitation cash-flow diagram (James, 1991)

| $\mathrm{A}=$ | Replacement structure first cost; |
| :--- | :--- |
| $\mathrm{B}=$ | Salvage value of present structure; |
| $\mathrm{C}=$ | Annual periodic or regular maintenance cost; |
| $\mathrm{F}=$ | Single future expenditure or major repair; |
| $\mathrm{N}^{\prime}=$ | Service life of the structure; |
| $\mathrm{G}=$ | Annual increase in maintenance cost due to progressive deterioration; |
| $\mathrm{n}^{\prime}=$ | Time to single future expenditure or major repair; |
| $\mathrm{g}=$ | Time to beginning of increasing maintenance cost due to progressive |
| $\mathrm{h}=$ | deterioration; <br> $\mathrm{i}=$ |
|  | Duration of increasing maintenance cost due to progressive deterioration; |
|  | Interest rate; |

The following is a description of the various cost components that should be taken into account in order to perform a LCCA for the rehabilitation scenarios:

## Initial cost

Initial costs are those encountered during the planning/construction period. They include all the material and installation costs as well as the indirect costs paid by the organization such as the expenses of additional supervision from in-house personnel, or additional control measures, etc. The initial costs should include both the agency costs as well as the user costs. In order to simplify the LCC calculation this research will assume that all initial costs have been incurred during the starting year of the study. As such, all initial costs will be entered into the LCC at their full value, and the starting year is assumed as the base year of service life-cycle and the base year of discounting.

## Expected Repairs / Maintenance costs

These are expenditures that are required to prolong the life of the infrastructure without replacing the system. Maintenance costs are usually planned actions that could be incurred annually or less frequently. Repair costs, on the other hand, are unforeseen actions that could be very hard to predict when they will occur. Section 2.3 presents an in-depth explanation of these types of disbursement. All maintenance and repair costs are to be discounted to their present value prior to addition to the LCCA total. De Brito and Franco (1998) stipulated that maintenance costs could be predicted as a percentage of the construction cost, and the authors presented a percentage of $1 \%$ to $2 \%$ of the initial cost as a typical average annual value for maintenance. This might not be necessarily true when a bigger maintenance awareness and high quality material are considered in the
construction phase, inducing higher initial costs but reducing the future maintenance required at later stages.

## Salvage cost (Residual value)

The salvage cost or residual value is a future expense that could be used in economic analysis. Generally the salvage cost value is the net worth of the element under study at the end of the life-cycle study period. It is a tangible asset of the infrastructure which could be significant in some cases and should be included in the LCCA.

## Replacement Costs

Replacement costs are anticipated expenditures to major infrastructure system components that are required to maintain their operational status. All replacement costs are to be discounted to their present value prior to addition to the LCCA total.

## User Cost

User costs are defined as the functional costs (De Brito and Franco, 1998) and are associated with the reduction of the functionality of the infrastructure. User costs are nonagency costs that are incurred directly by the users. Generally they are non-monetary and are related to inconveniences to the public such as accidents or time delays. For public infrastructures such as bridges or roadways, user costs typically involve vehicle operating costs, driver delay and detour costs, as well as accident costs. In the case of rehabilitation projects for a metro system, such costs arise from the timing, and the duration of the construction and rehabilitation work areas. In fact, construction areas and barricades reduce the traffic zone which causes delays, detours and even some incidents to users. Also factors such as increased noise and dust generated from the construction work
induce disturbance to the users and increases the complaints about the system. This might even discourage some users from using the system, thus reducing the number of daily passengers and inducing losses to the organization.

Incorporating the user costs into an LCCA enhances the validity of the results, but it remains a challenging task since it is difficult to accurately assess the monetary value of user costs. In the case where user costs are similar among alternatives, it is possible to remove them from the analysis.

### 2.4.2.5 Compute the present value of each alternative

Knowing that the costs are incurred at different points in time, the expenditure stream must be converted to a common base such as the Present Value (PV), the Future Value (FV) or the Equivalent Uniform Annual Cost (EAUC). Once the expenditure stream is constructed, the net present value of each alternative is calculated. It is important to compute the agency cost and the user cost separately in order to better understand the contribution of each type of cost to the total cost (Ozbay et al., 2003).

### 2.4.2.6 Analyze the results and perform a sensitivity analysis

This step involves interpreting the results using either a deterministic or a probabilistic approach. Normally the alternative providing the lowest NPV is preferred. In the case where the difference between the NPV of some alternatives is less than $10 \%$, then they could be considered to be equal due to the uncertainty in estimating the expenditure stream (Ozbay et al., 2003). For the case of adopting a deterministic approach, the sensitivity analysis would be required in order to examine the effect of the variability in
some core input parameters such as the discount rate, some important costs, or the timing of the interventions. The sensitivity analysis will supply the decision-maker with additional information in order to enable the final decision.

### 2.4.2.7 Re-validate the alternatives

Finally, a review of the results is performed in order to determine if any adjustments to any of the proposed alternatives need be done in order to improve the final outcome and determine the best choice for the project.

### 2.5 Genetic Algorithms

Many engineering optimization problems, and in particular Infrastructure Maintenance and Rehabilitation (M\&R) planning problems, are complex in nature. Due to a multitude of uncertainties and a wide range of possible scenarios, it becomes tedious to solve the problem using conventional optimization techniques. Since the 1960s, there has been a growing interest in solving such difficult real-world problems by imitating and simulating the natural evolutionary process of human beings (Gen and Cheng 1997, 2000). Such techniques are known as evolutionary algorithms and among them genetic algorithms (GA) are the most widely known (Michalewicz, 1996). Genetic algorithms, originally termed genetic plans, were initiated in the 1970s by John Holland and his colleagues and students at the University of Michigan. GA have received considerable attention in a broad range of different applications and disciplines as a search and optimization methodology (Sakawa, 2002). In their book, Gen and Cheng (2000) list more than 200 PhD dissertations that have been published in different disciplines using GA ranging
from biology to engineering control optimization and design. In the field of infrastructure management such as bridges, pavements and sewer management, GA has also been used in order to determine the best $\mathrm{M} \& \mathrm{R}$ scenarios over the life cycle of the infrastructure. Some PhD dissertations in the field of infrastructure management system where GA are used are:

| Thesis Title | Author | University and Year |
| :--- | :--- | :--- |
| - Multi-Objective Optimization in |  |  |
| Pavement management using <br> Genetic Algorithms and Efficient <br> Surfaces | Charles Conway <br> Pilson | University of Texas at <br> Austin, May 1999 |
| -Life Cycle Cost Optimization for <br> Infrastructure facilities | Dima J. Jawad | Graduate School-New <br> Brunswick, New <br> Jersey, October 2003 |

Besides these PhD works, many articles have been published about the subject, references, Chunlu et al. (1997), Fwa (1996), Chan et al. (1994a) and (1994b), Furuta (2001), Dandy and Engelhard, (2001), Zhang (2005), are among others where genetic algorithms are used for rehabilitation of infrastructures. However in all these studies, the problem considered was usually specific to a certain type of infrastructure, such as bridges, pavements, or sewers, and a deterministic deterioration model was usually undertaken. The explanation of the basic functioning of the genetic algorithm follows.

### 2.5.1 Outline of genetic algorithms

The genetic algorithms (GA) are stochastic search techniques based on the Darwinian mechanisms of natural selection, and their processes are simple yet powerful and robust (Goldberg, 1989a). GA use a mixed vocabulary borrowed from natural genetics and machine language (Michalewicz, 1996). They start from an initial population of random
solutions called individuals, also known as "strings" or "chromosomes", where each chromosome consists of a sequence of units called "genes" (also termed features, characters, or decoders). Genes are arranged in linear succession and each gene encodes a particular feature of the chromosome or the solution. During the search or optimization process, the chromosomes continue to evolve through successive iterations called "generations", until the problem converges on the chromosome which best represents the optimal or approximate optimal solution to the problem (Gen and Cheng, 1997). At each generation of the GA a specific number of the chromosomes are selected from the current population through a selection criteria based on the "fitness" of the chromosomes. New chromosomes ("offspring") are then created using genetic operators and by altering the genetic characteristics of the "parent" chromosomes. The most common genetic operators used are reproduction, crossover and mutation; a more detailed explanation will follow about their functioning. Michalewicz (1996) in his book "Genetic Algorithms + Data Structures $=$ Evolution Programs" compared GA to the evolution of a population of rabbits in order to survive being eaten by foxes. Here is the story of rabbits as presented by (Michalewicz, 1996): "At any given time there is a population of rabbits. Some of them are faster and smarter than other rabbits. These faster, smarter rabbits are less likely to be eaten by foxes, and therefore more of them survive to do what rabbits do best: make more rabbits. Of course, some of the slower, dumber rabbits will survive just because they are lucky. This surviving population of rabbits starts breeding. The breeding results in a good mixture of rabbit genetic material: some slow rabbits breed with fast rabbits, some fast with fast, some smart rabbits with dumb rabbits, and so on. And on the top of that, nature throws in a "wild hare" every one in a while by mutating
some of the rabbit genetic material. The resulting baby rabbits will (on average) be faster and smarter that these in the original population because more faster, smarter parents survived the foxes." Michalewicz also commented "It is a good thing that the foxes are undergoing similar process - otherwise the rabbits might become too fast and smart for the foxes to catch any of them".

This metaphoric narration provides the basic idea behind GA. In fact, GA differs from conventional optimization and search techniques in the following ways (Goldberg, 1989a), (Sakawa, 2002):

- GA do not work with the solution of a problem but with a coding of a solution set in the form strings representation.
- GA search from a population of possible solutions and not a single solution or a single-point approach of some traditional optimization and search techniques.
- GA use fitness information or performance with regard to the objective function, and not derivation or other auxiliary knowledge.
- GA use probabilistic transformation rules to guide the exploration within the search space and not deterministic rules.


### 2.5.2 Procedure fundamentals of genetic algorithms

In order to understand the functioning of GA as an optimization or search technique the "space" where GA evolve should be first explained. Figure 2-4 is a modified illustration of figure 2.2 from Sakawa (2002) that presents the fundamental structure of genetic algorithms. The GA space is divided into two components: the "phenotype" or the
problem space or solution space, and the "genotype", the coding or the search space. Potential solutions to the problem at hand lay in the phenotype, and during the GA process these solutions are coded into chromosome-like-structures and the complete set of the chromosomes represents the genotype. The basic feature of genetic algorithms is that they work alternatively between the "coding space" (genotype) and the "solution space" or the phenotype (Sakawa, 2002), (Gen and Cheng, 1997).

In the figure's original form, Sakawa (2002) has placed the evaluation and the selection in the same group with the other genetic operators. In this modified illustration the genetic algorithm space corresponds to the whole search or optimization process, and it is divided into a genotype and a phenotype. The genetic operations are placed in the coding space (the genotype) whereas the evaluation and selection mechanism of each individual are located in the solution space (the phenotype). The reason for this modification is to illustrate the concept of exploration and exploitation that underlies the GA process. "The genetic operators explore the search space and the evaluation and selection mechanism exploit the best solutions in the problem space" (Gen and Cheng, 1997). In fact, the evaluation mechanism measures the "fitness" or the performance of the decoded solution with respect the objective function, and the fitness through the selection mechanism becomes the link between chromosomes in the genotype and the performance of the decoded solution in the phenotype. It should be noted that the balance between the exploration of the search space and the exploitation of the best solutions found in the problem space is a recurrent theme in G.A. theory. "The more exploitation that is made, the faster the progress of the algorithm, but the greater the possibility of the algorithm failing to finally locate the next global optimal solution" (Gen and Cheng, 1997).


Figure 2-4 Fundamental structure of genetic algorithms
An essential feature of applying genetic algorithms to solve real-world problems is the use of an appropriate coding representation for the prospective solutions to the problem at hand. Several encoding methods have been used in order to represent all possible solutions, and the most common method is the "Binary String" representation. Each solution is represented by a finite-length binary bit; a series of 1 's and 0 's that represent a single solution to the problem. The length of the binary string depends on the complexity of the problem and on the required precision of the solution. As an example, for $M \& R$ problems Chunlu Liu et al. (1997) used two-bit binary strings in order to represent the four available maintenance methods for a bridge deck. For instance routine maintenance was coded (00), repair as (01), rehabilitation as (10) and replacement as (11). In order to represent the consecutive maintenance method over the planning period ( T ), the coding
structure is designed in the form of consecutive two-bit binary strings representing the maintenance method (m) at year ( t ). The final string consists of many of these substrings representing all bridges $\mathrm{i}=1,2, \ldots \mathrm{~N}$ in a given order.

| $\mathrm{m}_{1}$ | $\mathrm{~m}_{2}$ | $\ldots$ | $\mathrm{~m}_{\mathrm{T}-1}$ | $\mathrm{~m}_{\mathrm{T}}$ |
| :---: | :---: | :---: | :---: | :---: |
| 11 | 00 | $\ldots$ | 01 | 10 |

Other encoding techniques have also been used for particular problems. For instance real number encoding was used for constrained optimization problems and integer coding for combinatorial optimization problems (Gen and Cheng, 1997). Also, alphabets or some symbols may be used to represent strings. For instance in the M\&RPPI computer program developed for this research, an alphabetical representation of the solution is used. Even though the length of a chromosome is normally fixed at a certain value, it is also possible to construct a GA with a chromosome having variable length if it is more convenient for the problem at hand (Sakawa, 2002). The length of the chromosome and thus the encoding method may vary from one problem to another and even the same problem could have more than a single way to represent its solutions. The encoding method, however, directly affects the GA efficiency in the subsequent steps and should be carefully chosen. Two important concepts originating from the encoding technique are the concepts of feasibility and legality of the encoded chromosomes. In fact, during the mapping between the phenotype and the genotype, some chromosomes could be lying outside the feasible region of the problem and will represent unfeasible solutions. This is the case with a constrained optimization problem represented by a system of equalities and inequalities. Many efficient penalty methods have been proposed to handle unfeasible chromosomes. The concept of illegality refers to chromosomes that cannot be
decoded into a real solution. The penalty approach cannot be applicable in this case and repair techniques could be adapted (Gen and Cheng, 2000). Once the problem encoding methodology is chosen the GA process could be performed as presented in the following flowchart (Figure 2-5).


Figure 2-5 Flowchart of fundamental procedures of genetic algorithms (Sakawa, 2002)

What follows is the detailed explanation of each of the above listed steps.

### 2.5.2.1 Initial Population Generation

The first step in the genetic algorithms process is to randomly generate an initial population of candidate chromosomes. An important factor at this step is to choose the population size, which represents the number of chromosomes found in a generation. The choice of the population size carries significant importance for the GA application, and may be critical in many applications. The population size directly affects the
effectiveness and the efficiency of GA, with a small population sample the genetic algorithms will be running with an insufficient range of solutions and may converge too quickly in finding the optimal solution. On the other hand, a large population size will provide a more representative range of solutions and prevent the algorithms from premature convergence, but could involve lengthy and expensive computational time. Thus, choosing an appropriate population size for genetic algorithms could be a difficult task for GA users. During the first ages of the genetic algorithms, the common belief was that an increase in the population size would improve the effectiveness of GA, and that a larger population will require increased processing time but will eventually achieve better results than smaller populations (Davis, 1991). Many researchers, De Jong (1975), Grefenstette (1986), Goldberg, (1989a) have investigated the optimal size of a population for genetic algorithms as function of problem complexity. Reeves (1993) described an approach to specifying a minimal population size for the application of GA. Robertson's study of population size machines found that performance increased monotonically with population size (Robertson (1988)). Goldberg showed that optimal population size increases exponentially with the problem size, and suggested the population size $\mathrm{N}=$ $1.65 * 2^{0.21 \ell}$ where $l$ is the length of the chromosome. Alander (1992) stated that a population size varying between the chromosomes length $l$ and $2 * l$ would render good results depending on the problem's complexity.

Finally some researches showed that a population size as low as 30 is sufficient for some problems, and a common population size found in the GA literature and which seems to work well for different problems is that of 50 individuals (Alander, 1992). Thus as stated
in Davis (1991), the most effective population size is dependent on the problem being solved, the encoding method and the genetic operators used.

### 2.5.2.2 Evaluation

The evaluation of the chromosomes in the current population involves the process of measuring the performance of each chromosome in the genotype with respect to the objective function of the problem in the phenotype in order to assign to it a fitness value. The fitness function is used as a measure by which to select an individual to reproduce for the next generation (Sakawa, 2002), and through the evaluation process the highly fitted strings will have high fitness values. As shown in Figure 2-5, the fitness is the link between the genetic algorithm space and the solution space, and it is the measure used for selecting the "parent" chromosomes of the next generation. Thus the fitness evaluation is the revolving center point of the GA where the population $\mathrm{P}(\mathrm{t})$ of a generation ( t$)$ evolves and forms the next population $\mathrm{P}(\mathrm{t}+1)$. It should be noted that the evaluation of an individual is done independently from other individuals in the population, while the fitness value does depend on other individuals. In general the fitness value has a nonnegative figure and for maximization type of problems, the fitness could be simply equal to the value of the objective function of the decoded solution. If the optimization problem is to minimize a function such as minimizing a cost function $g(x)$, then the solutions with the lowest objective function (lowest cost) become those with the highest fitness value. In this case, it will become necessary to map the objective function $g(x)$ to a fitness function f(x). Goldberg (1989a) proposed a "cost-to-fitness" transformation method for this type of minimization problem which works as follows:

A $C_{m a x}$ constant is introduced satisfying $C_{\max }-g(x) \geq 0$ and the fitness of each chromosome is calculated as:

| $f(x)$ | $=C_{\max }-g(x)$ |  | if $g(x)<C_{\text {max }}$ |
| :--- | :--- | ---: | :--- |
|  | $=0$ |  | Otherwise |

Equation 2-10

However the value of $\mathrm{C}_{\text {max }}$ is not known in advance, and it may be taken as an input coefficient, as the largest $g(x)$ value observed so far, as the largest $g(x)$ value in the current population or as the largest $\mathrm{g}(\mathrm{x})$ value of the last $(\mathrm{k})$ generations.

### 2.5.2.3 Selection

The basic principle that underlies genetic algorithms is essentially the Darwinian natural selection and the strive for survival. In fact, highly fitted individuals will have high chances to be selected and will reproduce more often compared to lower fitted ones. The selection process will depict how to select the "parent" chromosomes in order to create offspring for the next generation. Knowing that GA work by exploiting the best solutions and exploring the search space, then the selection mechanism works on the exploitation side and provides the driving force in a genetic algorithm by directing the solution towards promising region of the search space (Bäck, 1994). In fact the selection operator uses the fitness value of each chromosome in a way to favour highly fitted individuals to transfer their information to the next generation of the evolution process.

There are many different methods that genetic algorithms can use to select the "parents" for the next generation. The choice of the selection method for an optimization problem has a direct effect on the convergence of GA toward the optimal solution. A premature
(rapid) convergence is a common problem in genetic algorithms, and it is normally caused by "super-individuals" which will be selected more often compared to others. Such individuals will be rewarded with a larger number of offspring and will prevent other individuals from contributing to the reproduction process. After few generations some desirable alleles (information) found in other individuals may be eliminated (Baker, 1985) and GA may lead to a local optimum. The term selective pressure is used in order to characterize a strong (high selective pressure) respectively weak (low selective pressure) emphasis of selection on the best individuals (Bäck, 1994). As the selective pressure is increased, the search focuses on super-individuals and will decrease the diversity of the population, whereas a weak selective pressure can make the search ineffective (Michalewicz 1996). Thus the selection process should provide a balance between the exploration (diversity) of the search space and the exploitation of the individuals with higher fitness values. The following are a few methods that are commonly used in literature for the selection process. Some of these methods are mutually exclusive, but others can be combined:

## Roulette-Wheel Selection

Also known as fitness-proportional selection, is a method introduced to GA by Holland where each individual in the population has a selection probability according to the relative fitness of individuals. For each chromosome (i) in a population of ( N ) individuals, the selection probability is calculated as:

$$
P_{i}=\frac{f_{i}}{\sum_{j=1}^{N} f_{j}}
$$

A roulette-wheel is constructed where each individual gets a slice proportional to its selection probability. In order to select ( N ) "parents" chromosomes, the weighted wheel is spun ( N ) times and at each time a chromosome is selected depending on the section the wheel pointer lands on.

This method's shortcoming comes from the fact that highly fitted individuals will be selected more frequently, which leads to the problem of rapid convergence discussed at the beginning of the section.

## Elitist Selection

This selection method, first introduced by De Jong (1975), ensures that the "elite" or most fit individuals of each generation are selected for the next iteration process. It could be the single best chromosome, or a certain percentage of the top best individuals from each generation that are reproduced. This method is important in order to ensure preserving the best chromosomes since the fitness-proportional selection method does not guarantee the selection of any particular individual, including those with higher fitness. It should be noted that in some cases the loss of the best chromosomes could be advantageous by slowing the algorithm and allowing a higher exploration of the search space before convergence. But for many applications, the G.A. can be more efficient by not losing the best or elite members between generations.

Elitism requires not only that elite members be preserved for future generations, but the chromosomes should be protected from being disrupted by genetic operators.

## Tournament selection

This method consists of selecting subgroups of individuals each containing a user defined number " $k$ " (the so called tournament size) of individuals from the current population and selecting the best chromosome of this set of " $k$ " elements in order to pass it to the next generation. At each iteration this process is repeated " $N$ " times, equal to the population size, and a new set of " N " individuals are selected as parent chromosomes. Typically a value of $k=2$ is accepted for many applications (Michalewicz, 1996).

## Fitness Scaling

In the fitness scaling method, the actual fitness value of each individual is scaled back in a way to prevent the rapid convergence and the takeover of the population by some superindividuals (Goldberg, 1989a). On the other hand, later in the GA process when many individuals will be similar, there will be little to distinguish between individuals and the competition among population members diminishes. In this case a random behaviour will arise and the fitness value must be scaled up between individuals to continue rewarding the best performers (Goldberg, 1989a). For example, the linear fitness scaling works by pivoting the fitness of the individuals about the average fitness value or the population. This will allow approximately a constant proportion of copies of the best members to be selected compared with the average individuals.

Three categories of fitness scaling are found: linear scaling, sigma-truncation and power law scaling. In linear scaling the chromosome fitness is scaled as $\mathrm{fs}_{i}=a \mathrm{f}_{\mathrm{i}}+b$ where the parameters $a$ and $b$ are normally selected in such a manner so that the average fitness is mapped to itself and the best fitness is scaled up by a user-defined multiple of itself ( 2 to
3). This method does not work with negative fitness values and a sigma ( $\sigma$ ) truncation is introduced. The variance of the population is used as $f s=f_{i}(\bar{f}-c . \sigma)$ and $c$ is a multiple of the population standard deviation (between 1 and 3) (Goldberg, 1989a), and the negative results $\mathrm{f}^{\prime}<0$ are set to zero. Finally power law form of scaling is taken as some user defined power of the raw fitness value $\mathrm{f}^{\prime}=\mathrm{f}^{\mathrm{k}}$, (e.g. $k=1.005$ ).

## Ranking Selection Method

In the ranking selection method, each individual is assigned a rank from $\mathrm{i}=1 \ldots \mathrm{~N}$ based on its fitness within the current population, and the selection probability is based on the rank of the individual rather than its absolute fitness. This method prevents highly fit individuals from dominating the search space, at the expense of less fit ones and slows down the convergence rate of GA. Baker (1985) proposed a linear tanking method where individuals in the population are ranked in increasing order of their fitness and the selection probability of each individual is calculated as follows:

The best individual is given a fitness value MAX such that $1.0 \leq$ MAX $\leq 2.0$, and the worst individual a fitness of MIN $=2$-MAX. The constants MAX and MIN are respectively the maximum and minimum expected values of selection as shown in Figure 2-6.


## Figure 2-6 Baker ranking method

The fitness value $f(i)$ of the intermediate individuals ( $i$ ) will be calculated by interpolating between the MAX and MIN values as shown in Equation 2-12:

$$
f_{i}=M A X-\frac{2(M A X-1)}{n-1}(i-1)
$$

Equation 2-12

MAX is a user-defined value that determines the selective pressure of the genetic algorithms. The more MAX is close to 2.0 the higher the focus is on the best fit individuals. Baker (1985) suggests a value of $\mathrm{MAX}=1.1$.

As explained by Michalewicz (1996), such a ranking approach improves the algorithms but it shifts the responsibility to the users to decide the parameters involved, plus this method ignores information about the relative evaluation among the individuals and treats all cases uniformly.

## Other selection methods

Many other selection methods are available as references. Gen and Cheng (1997), and Michalewicz (1996), present some of them. Bäck (1994) presents a comparison of the
most important selection operators. Finally, Baker (1985) deals with inhibiting premature convergence by the use of adaptive selection methods.

### 2.5.2.4 Crossover operation

The crossover operation consists of randomly choosing two distinct chromosomes from the population and then swapping some portion of the string between them. The simplest crossover method used in order to exchange the chromosome information is the one-cutpoint method. For a chromosome that has a length ( $l$ ), a uniformly distributed random integer $(k)$ is chosen between 1 and $l-1$ as a cut-point, and two new strings are created by swapping all characters between position $\mathrm{k}+1$ and $l$ inclusively (Goldberg, 1989a). Thus the portion of the right part of the cut-point is exchanged between both chromosomes. For example chromosomes A \& B, each having ten bits, are considered as follows:

| Chromosome A | $\mathrm{a}_{1}$ | $\mathrm{a}_{2}$ | $\mathrm{a}_{3}$ | $\mathrm{a}_{4}$ | $\mathrm{a}_{5}$ | $\mathrm{a}_{6}$ | $\mathrm{a}_{7}$ | $\mathrm{a}_{8}$ | $\mathrm{a}_{9}$ | $\mathrm{a}_{10}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| Chromosome B | $\mathrm{b}_{1}$ | $\mathrm{~b}_{2}$ | $\mathrm{~b}_{3}$ | $\mathrm{~b}_{4}$ | $\mathrm{~b}_{5}$ | $\mathrm{~b}_{6}$ | $\mathrm{~b}_{7}$ | $\mathrm{~b}_{8}$ | $\mathrm{~b}_{9}$ | $\mathrm{~b}_{10}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

The cut-point is selected randomly as being $k=4$, and two new offspring offspring $\mathrm{A}^{\prime}$ and B ' are generated as follows:

| Chromosome $\mathrm{A}^{\prime}$ | $\mathrm{a}_{1}$ | $\mathrm{a}_{2}$ | $\mathrm{a}_{3}$ | $\mathrm{a}_{4}$ | $\mathrm{~b}_{5}$ | $\mathrm{~b}_{6}$ | $\mathrm{~b}_{7}$ | $\mathrm{~b}_{8}$ | $\mathrm{~b}_{9}$ | $\mathrm{~b}_{10}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

The

| Chromosome B' | $b_{1}$ | $b_{2}$ | $b_{3}$ | $b_{4}$ | $a_{5}$ | $a_{6}$ | $a_{7}$ | $a_{8}$ | $a_{9}$ | $a_{10}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | crossover is performed according to some probability denoted $p c$ which depicts the average probability that a crossover operation is performed at each iteration. For instance for a $p c=0.25$, it could be expected that at each iteration, an average of $25 \%$ of the chromosomes undergo crossover. The crossover probability $p c$ is normally a user-defined

value and is constant for all generations, but sometimes $p c$ could be also modified dynamically during the process in order to control the premature convergence of the GA and to reduce the loss of genetic materials.

Other forms of crossover may be found in literature. Gen and Cheng (1997) presented a few such as the two-cut-point and multi-cut-point, a derivation from the on-cut-point. The uniform crossover, random crossover, flat crossover and blend crossover are also listed in this reference. Also, Davis (1991) presented other interesting variability of crossovers, among these the position-based crossover.

Both reproduction and crossover are simple operations involving random number generation, string copies and some partial string exchange, yet the combined effect of reproduction and the randomized but structured information exchange of the crossover operation give GA much of their power (Goldberg, 1989a). Reproduction and crossover operate on the principle that the process of forming new innovative ideas is performed by the exchange of notions and the juxtaposition of things that have worked well in the past. In addition, during GA process "high-performance notions are repeatedly tested and exchanged in the search for better and better performance" (Goldberg, 1989a).

### 2.5.2.5 Mutation Operator

The mutation consists of randomly altering one or more genes of the chromosomes in order to create new offspring. For example in a binary encoding method of chromosomes a randomly selected bit is altered by changing its value from a " 0 " to a " 1 " or vice versa. For other encoding methods, a modification of the bit value is performed depending on
the available range of values. Therefore, mutation operator creates a new individual by making changes on a single parent individual, which differs from the crossover operator that creates two new individuals by combining parts of two parent chromosomes. The mutation usually plays a secondary role in GA and works as a background operator. Similar to the crossover, the mutation is performed according to a certain probability denoted $p m$. In general $p m$ has lower values than the $p c$. The crossover probability, as an example for a $p m=0.01$ in a population of 25 chromosomes having each 10 bits length, an average of $0.01 \times 25 \times 10=2.5$ mutation could be expected at each iteration of the GA.

The mutation maintains diversity in a population so that the other operators (crossover and reproduction) can continue to work while also being a search operator on its own (Davis, 1991). In fact, even though the crossover and reproduction have a principal role, during the GA iterations some potentially useful genetic material could be lost and the mutation could protect against such irrecoverable loss (Goldberg, 1989a). In other terms, the mutation prevents from pre-mature convergence of GA. It allows for a better exploration of the search space without being trapped in local minimum / maximum point.

### 2.5.2.6 Other GA operators

Another genetic operator could also be used in the search and optimization process. Holland, in his work, used the inversion operator which proceeds by inverting the order of the genes on the totality or on a randomly selected portion of the chromosome string. (Goldberg, 1989a) explained the mathematical foundation and the functioning of the inversion and other reordering operators in genetic algorithms. He also presented other
"micro-operators" or "low-level" operators that have been suggested for use in genetic adaptive searches.

### 2.5.2.7 Termination of the algorithms

The termination establishes the termination conditions for the algorithms. Several criteria could be used in order to determine when to stop the program:

- Number of generations: This is the simplest termination condition where the program will stop after reaching a predefined maximum number of generations. However this method relies on the user's knowledge of the characteristics of the function which may not be easy in many cases.
- Convergence of the program: There are two basic categories to determine if the program has converged. It could be either from the phenotype space by examining the fitness of a particular solution, or from the genotype space based on the chromosome structure. In the first approach, the program will stop if no improvement of the objective function is achieved after a certain consecutive number of generations or after a certain time frame. If such progress is smaller than a certain parameter, the search is terminated. In the second approach, the program checks the number of converged alleles, or those having a gene which at least $95 \%$ of the population has the same value as defined by De Jong (1975). Baker (1985) uses this approach to identify the convergence state of the program by calculating the bias or the average percent convergence of each gene.
- Time limit: The program will stop after a predefined amount of time, which prevents the algorithm from running for a long period of time. This method also
provides some limitations similar to the case of the number of generations. In addition, the time limit required to obtain an optimal solution may vary from one computer to another depending on processor speed.


### 2.5.3 Major advantages of genetic algorithms

Using genetic algorithms as optimization technique has several advantages mainly because GA do not have much mathematical requirements about the optimization problems. In fact, due to their evolutionary nature GA will search for solutions without regard to the specific inner workings of the problems. Thus GA can handle any kind of objective functions and any kind of constraints (i.e linear or non linear) defined or discrete, continuous, or mixed search spaces. In general the following advantages and uses could be pinpointed:

- GA are general purpose problem solving and optimization techniques.
- GA are theoretically and empirically proven to provide robust search in complex space (Goldberg, 1989a).
- GA are applicable as an optimization tool capable of overcoming combinatorial explosion for real-life rehabilitation and maintenance problems. For instance, Chan et al. (1994a), (1994b) used GA for a maintenance management problem of a total network of a road system.
- GA are computationally simple yet powerful in their search for improvement, and they are theoretically and empirically proven to provide robust search in complex space (Goldberg, 1989a).


# 3 PROPOSED MAINTENANCE AND REHABILITATION PLANNING METHOD FOR PUBLIC INFRASTRUCTURES 

### 3.1 Introduction of a New Maintenance and Rehabilitation Planning for Public Infrastructures Method

As discussed in the earlier sections of this thesis, a managerial tool and an automated methodology are required for cost-effectiveness comparison in order to generate leastcost solutions to infrastructure rehabilitation. The proposed methodology of this research is a new Maintenance and Rehabilitation Planning for Public Infrastructures (M\&RPPI) method that combines the concept of the Markov chains with genetic algorithm optimization methodology. The proposed method enables the determination of the optimal maintenance and rehabilitation ( $M \& R$ ) decisions to be taken over the life cycle of the infrastructure, or over a predefined planning period. The M\&R actions are those possible alternatives that allow the restoration or maintenance of the infrastructure, and could entail either corrective or preventive action, or no action at all. Of course each type of $M \& R$ action has a certain effect on the deterioration process of the infrastructure and will require a certain cost. The proposed M\&RPPI method will determine the type of M\&R action to be taken, and its timing, in order to maintain the infrastructure above a certain acceptable level of operability. The outcome of the proposed M\&RPPI method is a M\&R profile producing the lowest net present value of expenditures over the planning
period of the infrastructure. The influence of discounted costs is also taken into account during the optimization process.

In this approach, the deterioration process of an infrastructure is modeled as a "dynamic" Markov chain in a form of multiple transition probability matrices (TMP). Each TPM could be derived based on expert opinion, or on historical deterioration data (see section 2.2.1). The effect of any $M \& R$ action will also be modeled in a TPM, thus the future progress of deterioration can be predicted over the planning period with the effect of any M\&R action. In this proposed M\&RPPI method, the optimization process uses genetic algorithms in conjunction with the TPM (Markov chains), in order to find the optimal or quasi-optimal solution. In traditional methods the dynamic programming (DP) is used in conjunction with Markov chains in order to determine the optimal policy, which is known as the Markov decision process (MDP). The main difference between these two methods will be explained and their different outcomes discussed.

In order to demonstrate the workability of this methodology, a computer program is developed using an Excel spreadsheet and VBA macros. The structure of the Excel program is explained in chapter 4 . In order to use the proposed methodology, the M\&RPPI require few input data. Some assumptions are made regarding the infrastructure system. The following is the description of the proposed methodology as well as the description of the required data for the analysis.

### 3.2 Required Data for the Proposed Method

Prior to starting with the optimization process several data will be required. The data could be related to the infrastructure itself with the available $M \& R$ alternatives, to the management policy, or to the GA optimization process. Figure 3-1 presents the dataflow chart of the proposed method. The following is a list of the required data:

1. Infrastructure condition rating schema and the initial state.
2. Alternative M\&R actions.
3. Cost information of M\&R actions.
4. Decision criteria for undertaking each $M \& R$ action.
5. Impact of the $M \& R$ action on the deterioration curve (Transition Probability Matrix).
6. Planning period. (see section 2.4.2.3)
7. Population size. (see section 2.5.2.1)
8. Maximum number of generations. (see section 2.5.2.7)
9. Discount rate. (see section 2.4.2.3)

The description of items 1 to 5 will be explained in the following sections. The remaining elements have already been previously explained in the literature review chapter.


Figure 3-1 Data flowchart of the proposed M\&RPPI method

### 3.2.1.1 Infrastructure condition rating schema and the initial state

The condition rating of the infrastructure corresponds to its structural and functional performance. The structural performance corresponds to the ability of the infrastructure to carry its designed loads, whereas the functional performance relates to the serviceability performance of the infrastructure. The condition assessment method is not
in the scope of the present study. Several publications and studies are actually available about this specific subject, whereby many rating scales are proposed. In this proposed method, the infrastructure is ranked on a range of 1 to 5 , where 1 represents the best possible state and 5 the worst condition as illustrated in Figure 3-2.


Figure 3-2 Example of infrastructure condition rating scale

Table 3-1 could, for example, define the linguistic description of each state corresponding to each efficiency level or performance measure of the infrastructure. This table is for illustration purpose only and could be modified for another type of problem.

Table 3-1 Condition state description and corresponding efficiency level

| State Xn | State Description | Efficiency Level |
| :---: | :--- | :---: |
| 1 | Excellent state: like-new condition | $85-100 \%$ |
| 2 | Good state: presence of minor damages | $70-84 \%$ |
| 3 | Fair state: presence of moderate damages | $55-69 \%$ |
| 4 | Poor state: presence of major damages | $40-55 \%$ |
| 5 | Critical state: presence of extensive damages | $\leq 40 \%$ |

The initial or current state of the infrastructure is an input data that is required as a starting point for the optimization process. For a real-life infrastructure analysis problem, the initial state could be either determined by an expert opinion or by a certain availale methodology. In the context of this research, it is assumed that the initial state of the infrastructure is a known data. However it is important to clarify the possible values of the condition rating of the infrastructure. As previously stated the range of the condition
state is between 1 and 5 and could have either discrete or intermediate values. In the M\&RPPI Excel program, the initial state of the infrastructure is considered to have discrete values, which means that only integer values (not decimals) are accepted. Thus, the possible values of the initial state are $1,2,3,4$ or 5 corresponding to the description in Table 3-1. Subsequently during the optimization process, the infrastructure evolves following a continuous rating scale (or decimals values) giving a more real representation for the behaviour of the infrastructure.

Finally, an important feature added to the M\&RPPI is the possibility to consider a threshold or a lower limit value for the condition state that the infrastructure condition could not go below. Thus during the optimization process, the M\&RPPI forces an effective restoration action in order to move the infrastructure state beyond the threshold limit. For convenience purposes, the threshold limit is fixed to a value of 4.5 , but the program could be easily modified in order to make the threshold a user-defined data.

### 3.2.1.2 Alternative M\&R actions

The types of M\&R actions are those corrective or preventive solutions that are available for the specific problem at hand. For instance, for a certain type of damage occurring to the infrastructure, there might be several possible corrective or preventive scenarios available such as replacing the damaged section or patching and spot repairing. Also, a preventive maintenance program could be considered, or even no action at all. In the case of no action, the infrastructure is left as is, and without any repair for a certain period of time. A multitude of M\&R actions might be found, and it is important to define the
complete list of all these available solutions. The rehabilitation alternatives are symbolized (coded) by alphabetical letters, A, B, C, etc.

### 3.2.1.3 Cost information of M\&R actions

Each type of action undertaken requires a certain cost value; this value should include all direct and indirect costs generated by selecting a specific action. Direct costs are expenditures directly spent in order to perform the M\&R action, while indirect costs are those induced to the organization by selecting this specific M\&R action. Indirect costs could include the overhead or some indirect costs due to arrangements required in order to undergo the M\&R action. They could also represent user costs caused by the M\&R action. In the case study presented later, more explanation and an example will be presented about the direct and indirect costs. It is important to note that the cost might also depend on the actual state of the infrastructure, thus for the same type of rehabilitation action, e.g. "repair", the cost incurred is different for one state or another, (e.g state 3 or state 4). In the proposed methodology, as well as in the Excel M\&RPPI file, a cost table should be completed with all the costs for each rehabilitation alternative and each state.

### 3.2.1.4 Decision criteria for undertaking each M\&R action

The decision criteria table depicts the management policy about the infrastructure. It represents the allowed rehabilitation actions that could be undertaken giving specific condition states. In other words, the rehabilitation policy corresponds to a set of decisions that are allowed, or are feasible for a given state. As an example, Table 3-2 represents
possible rehabilitation actions $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D allowed for each condition state. Thus for a state between 1.0 and 2.0 , only action C and D are allowed, but for a state between 4.0 and 5.0 all four actions are allowed etc.

Table 3-2 Example of a rehabilitation policy

| Condition State Xn | Allowed Rehabilitation <br>  <br>  <br>  $\mathbf{A c t i o n s}$ |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | No | No | Yes | Yes |
| $2.0 \leq \mathrm{Xn}<3.0$ | No | No | Yes | Yes |
| $3.0 \leq \mathrm{Xn}<4.0$ | No | Yes | Yes | Yes |
| $4.0 \leq \mathrm{Xn}<5.0$ | Yes | Yes | Yes | Yes |
| $5.0 \leq \mathrm{Xn}$ | Yes | Yes | No | No |

### 3.2.1.5 Impact of the M\&R action on the deterioration curve (transition probability matrices)

The proposed M\&RPPI method assumes that the infrastructure deteriorates over its service life according to a stochastic and time-dependant model with a dynamic Markovian behaviour. Thus, it is important for the functioning of the proposed M\&RPPI method to have previously determined the TPM that reflect the actual deterioration process of the infrastructure. The final TPM has a major impact on the result of the proposed method, thus they should be as close as possible to the reality. The term "dynamic" is used here to illustrate two possibilities. First the TPM is directly dependant on the undertaken $M \& R$ action, and it is modified at each state according to the $M \& R$ action. On the other hand, the TPM could be time-dependant, and is modified from one stage to another depending on the age of the infrastructure. For example an analysis period of 60 years could be divided into six groups of ten years, and at each time-segment, a different TPM is defined for the same M\&R action. This feature is extremely beneficial
in the case where the deterioration process modeling is better represented with multiple TPM. The purpose of this study is not to conduct a methodology about determining these TPM, and it is assumed that this information is available, either from expert-opinions or from any other method. Usually the TPM is developed from field performance data collected from successive inspections. In the literature review chapter of this research, a brief explanation about this topic was presented. The interested readers in the topic of determining the TPM are referred to the referenced articles (Wirahadikusumah, 1999), (Madanat, 1995a, 1995b, 1995c). For the purpose of this proposed methodology it is important to understand that for each type of M\&R action performed during the planning period, the condition state of the infrastructure changes either in a deterministic manner or according to a TPM. For instance, it could be assumed that by taking a certain M\&R action, the state of the infrastructure is going to be known with certainty. For example, if a major repair and a replacement of the majority of the structure is performed, then it will be assumed that the condition state is going to be as new, which is equal to state " 1 ". In Table 3-3, an example of the new condition state corresponding to each M\&R action is presented. $\mathrm{MC}_{\mathrm{C}}$ or $\mathrm{MC}_{\mathrm{D}}$ corresponds to a specific Markov chain or TPM that is going to dictate the new condition state of the infrastructure after applying a $M \& R$ action of type "C" or "D". In the case where an integer number is inscribed, it means that the new condition state switches to the specified state after performing the M\&R action. Finally N/A (not allowed) is inscribed in the case where there exists a restriction about applying a certain M\&R action according to Table 3-2.

The assumption considered in Table 3-3 is that the $M \& R$ action is done at the beginning of the period and the state calculated is at the end of the same planning period. This
assumption is important for calculating the deterioration process in section 3.3.1.1 and for the cost calculation in section 3.2.1.3. In section 3.3.1.1, the deterioration process is explained in further details according to each $\mathrm{M} \& \mathrm{R}$ action, and the corresponding TPM.

Table 3-3 Condition states corresponding to M\&R action

| Rehabilitation | New Condition State (Xn) |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | $\mathbf{D}$ |
| $\mathbf{1 . 0} \leq \mathbf{X i}<\mathbf{2 . 0}$ |  |  |  |  |
| $\mathbf{2 . 0} \leq \operatorname{Xin}<\mathbf{3 . 0}$ | $\mathrm{N} / \mathrm{A}^{*}$ | $\mathrm{~N} / \mathrm{A}$ | $\mathrm{MC}_{\mathrm{C}}$ | $\mathrm{MC}_{\mathrm{D}}$ |
| $\mathbf{3 . 0} \leq \operatorname{Xin}<\mathbf{4 . 0}$ | $\mathrm{N} / \mathrm{A}$ | $\mathrm{N} / \mathrm{A}$ | $\mathrm{MC}_{\mathrm{C}}$ | $\mathrm{MC}_{\mathrm{D}}$ |
| $\mathbf{4 . 0} \leq \operatorname{Xin}<\mathbf{5 . 0}$ | $\mathrm{N} / \mathrm{A}$ | 2 | $\mathrm{MC}_{\mathrm{C}}$ | $\mathrm{MC}_{\mathrm{D}}$ |
| $\mathbf{5 . 0} \leq \operatorname{Xin}$ | 1 | 2 | $\mathrm{MC}_{\mathrm{C}}$ | $\mathrm{MC}_{\mathrm{D}}$ |

( ${ }^{*} \mathrm{~N} / \mathrm{A}:$ Not allowed action at this state)

### 3.3 Description of the Proposed Methodology

The proposed M\&RPPI methodology is based on a dynamic Markov chain principal for the infrastructure deterioration process, and it uses GA optimization techniques. First, the integration of the deterioration mechanism into the optimization process is performed through some matrix manipulation. Second, some customizations are made to the traditional GA in order to adapt it to an infrastructure management system and ensure a proper convergence of the program. These two concepts will be explained in the following sections.

### 3.3.1.1 Deterioration mechanism through optimization process

The deterioration process of the infrastructure is modeled as a dynamic Markov chain starting from an initial state and evolving through its life-cycle depending on the performed $M \& R$ actions and the corresponding TPM. For each type of M\&R intervention including the "no-action", a TPM is set to describe the deterioration process. The
probability value $\mathrm{p}_{\mathrm{ij}}$ for the infrastructure to move from a state (i) to a state ( j ) could be either equal to or less than $1\left(0 \leq \mathrm{p}_{\mathrm{ij}} \leq 1\right)$. The specific case where $\mathrm{p}_{\mathrm{ij}}$ equals to one (1) implies that by performing a specific $\mathrm{M} \& \mathrm{R}$ action, it is certain that the state will go to another specific state (j), as in the case of a replacement. The possible values of the infrastructure condition states $(\mathrm{Xn})$ correspond to the predefined condition rating scale, and in this case, a condition rating scale from 1 to 5 is used, where 1 corresponds to the best possible value and 5 to the worst case. The current condition state at any time could be then represented by a state vector having ( $\mathrm{m}=5$ ) states (cells) and a probability $\mathrm{p}_{\mathrm{i}}$ for each state as follows:

State Vector:

| $p_{1}$ | $p_{2}$ | $p_{3}$ | $p_{4}$ | $p_{5}$ | $\Sigma p_{i}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |

And:

$$
\sum_{i=1}^{m} p_{i}=1
$$

Equation 3-1

Thus the expected current state of the infrastructure at time $(\mathrm{n})$ is equal to:

$$
E(X n)=\sum_{i=1}^{m} p_{i}^{*}(i) \text { for } i=1 . . m
$$

Equation 3-2

Based on Equation 3-2, it can be noted that the expected condition state of the infrastructure could have decimal values and not merely integer values. This feature is utilized further in the proposed methodology in order to perform a continuous condition rating prediction, and decision-making. Thus, in order to determine or to predict the condition state for the infrastructure at the next period (at time $n+1$ ), the current state vector is simply multiplied by the TPM corresponding to the M\&R action taken during
the year. The TPM required for the multiplication operation is dynamically modified throughout the optimization process. For simplification purposes the M\&R is considered to be taken at the beginning of the year and the condition evolvement occurs at the end of the year. Figure 3-3 presents a diagram of the condition state evolution during the optimization process.


Figure 3-3 Condition state evolution with respect to the transition probability
matrices

### 3.3.1.2 Description of the adapted genetic algorithms

In this proposed M\&R method, the genetic algorithm is adapted to the specific problem in a way so as to ensure the convergence toward the final solution. In fact, after each generation of the process, parent chromosomes are replaced by their offspring soon after they give birth. Since genetic operators are blind in nature, there will be no guarantee that the offspring are better than their parents, and by a strategy of replacing each parent with his offspring directly, some fitter chromosomes will be lost from the evolutionary process. In order to overcome this behaviour, the following two approaches are considered:

1. Elitist selection, which ensures that the best chromosome is automatically selected and passed onto the next generation.
2. Directed operation, where the problem-specific knowledge is introduced into genetic operation in order to produce improved offspring.

In fact, the developed process ensures that at each generation, the offspring created are better than their parents. The following figure illustrates the basic flow chart of the proposed methodology:


Figure 3-4 Flowchart of the proposed M\&RPPI method

## Generation of the initial population

First of all an initial population consisting of ( N ) user-defined number of individuals is randomly generated. Each individual represents a potential solution and consists of a sequence of $(T) M \& R$ action where (T) represents the user-defined planning period. Each type of $M \& R$ action is encoded into an alphabetical letter (A, B, C, D etc...), and the complete sequence of $M \& R$ actions are randomly generated based on the input data previously discussed. Figure 3-5 illustrates a population set for N chromosomes.


## Figure 3-5 Population of (N) chromosomes

The generation of any individual of the initial population starts from the same initial state, and it is generated in accordance with the rehabilitation policy (e.g. Table 3-2). The acceptable M\&R actions are identified, and one action is randomly selected among the available possibilities. Then the new condition state is calculated based on the TPM of the selected M\&R action and the initial state as described in Figure 3-3. The same process is then continued until the end of the planning period, where at each period the new state is determined and the possible alternatives identified and a single M\&R action is randomly selected. Figure 3-6 provides a flow chart of the procedure for generating the initial population.


Figure 3-6 Flow-chart for initial population generation
Both the planning period $(\mathrm{T})$ and the number of individuals $(\mathrm{N})$ in the population are user-defined values. The former data is related to the required analysis period and is normally dictated by the management. In the Excel program, the maximum planning period is limited to 240 years, as a result of a restriction on the number of columns Excel can consider. This value is adequate for analyzing most of the infrastructures since long planning periods ( $>100$ years) might not be seen as essential. An advantage of such lengthy periods is the possibility to illustrate the long-term profile of the infrastructure behaviour under the different $M \& R$ actions. On the other hand, the number of individuals
$(\mathrm{N})$ is directly related to the optimization process of the GA and should be chosen with care. This topic has been previously discussed in section 2.5.2.1.

It should be noted here, that by forcing only allowed M\&R actions to be selected, all generated individuals are feasible solutions. The concept of feasibility and legality have been discussed in section 2.5.2, and in the Excel program, the rule undertaken is to only generate feasible solutions, without having to care about corrective measures or applying penalties.

## Evaluation of the population

In section 2.5.2.2 the concept of evaluation was defined and explained in detail. For the infrastructure's M\&R program, the objective function is basically a minimization of the discounted cost over the life-cycle of the infrastructure. Thus, in order to evaluate the performance index of each individual with respect to the objective function, either a "cost-to-fitness" mapping could be performed as shown in Equation 2-10, or simply the inverse of the discounted cost $\left(1 / d_{\text {cost }}\right)$ is calculated for each individual, and the problem is then transformed into a maximization function. In both cases, the sum of all performance values is calculated, and the fitness of each individual is equivalent to the portion of the each individual performance with respect to the sum of all performance values (Equation 2-11).

## Selection of the parents' chromosomes

The major selection methods that are normally used are discussed in section 2.5.2.3, and the method that is adopted in this thesis is the well-known roulette-wheel method.

However in this proposed M\&RPPI, a slight modification to is implemented in order to control the selection of the parents' chromosomes, and in order to get a balance between the exploration of the search space and the exploitation of the best solutions. In fact, the two main modifications that are added to the roulette-wheel selection mechanism are the concept of "elitism" for the best chromosome, and restricting the selection amount of each chromosome in order to prevent super-individuals from dominating the population. This is done as follows:

1. Elitism: this topic was discussed in section 2.5.2.3 and consists of ensuring that the "elite" or most fit individuals of each generation are selected to the next iteration process. In this proposed method, the best chromosome is automatically selected as a parent chromosome.
2. Restriction of the selection amount of the chromosomes: this is performed by restricting to two times the maximum number of selection for each chromosome in order to allow a wider range of selection and to prevent super-individuals from being selected often and dominating the remaining population.

In this way, the concept of the roulette-wheel is preserved; nevertheless it would be applied differently from the Baker's ranking approach (section 2.5.2.3) which relies on the individual performance, or the NPV, during the selection process. This will ensure that no domination of super-individuals will occur. In addition this method does not focus solely on the above average individuals but rather gives a chance to all individuals depending on their fitness. This improves the population diversity and could positively contribute to finding the optimal or near-optimal solution.

The fact of having a rehabilitation policy (Table 3-2) controls the process of creation of chromosomes of the initial population, and prevent the emergence of super-individuals within the population. In fact, the creation of each chromosome is constrained within certain feasibility boundaries, and all individuals are within the feasible space. In the case where an individual obtained a NPV much lower than the average, then the choice of a high number of individuals within the population (e.g. 100 and higher) will "dilute" the effect of this super-individual compared to the total population. For this reason the selection behaviour of the population is almost random, which leads to this proposed method of selection that limits the amount of times an individual is chosen. Finally, the proposed method also ensures that the same individual is not selected twice in a row which allows pairing up all consecutive individual and mating them for further genetic operations. This improves the efficiency of the computer program by allowing two operations at the same time. After the selection process is completed two values are calculated: the contribution percentage (Equation 3-3) and the fitness share of the selected chromosomes (Equation 3-4).

> Contribution percentage $=$ Number of chromosomes selected $/$ Total number of chromosomes
> Fitness Share $=$
> Fitness of selected chromosomes $/$ Sum of all fitness

## Equation 3-3

Equation 3-4

The contribution or involvement percentage allows evaluating the proportion of selected chromosomes at each iteration, whereas the fitness share corresponds to the fitness percentage of the selected parents with respect to the sum of the total fitness of the whole population. Comparing both figures reveals whether super-individuals are dominating the population. For instance, if a low contribution percentage is observed with a high fitness
share, then this implies that only a small number of chromosomes are being selected. In the same time those selected chromosomes are much fitter than the remaining ones, thus the selection diversity is jeopardized. Other figures that are calculated are the average value of the offspring or the On-line Performances, and the average value of the selected chromosomes or the offline performance (Baker, 1985). These figures illustrate the convergence rate along the simulation process.

## Perform a directed crossover operation

The crossover operation proposed in this method corresponds to the one-cut-point method discussed in section 2.5.2.4, where the pair of chromosomes of length (l) exchanges the information found on the right part of a randomly chosen point along their length. The crossover operation is controlled by a probability denoted $p c$ which depicts the average probability that a crossover operation is performed at each iteration. The value of $p c$ directly controls the convergence of GA, and could be either a user-defined value or prefixed to a certain value. Since GA is considered to be blind in nature, a directed approach is applied to the crossover operation, helping the program converge and arrive at a better feasible solution. The directed approach consists of ensuring that only valid chromosomes are created, and at each generation, only better solutions are generated. The feasibility check of the offspring is performed by ensuring that all the bits beyond the crossover cut-point obey the rehabilitation policy according to their state (since those before the crossover cut-point have been previously validated). Also a minimum threshold for the condition state (if defined) should be respected for all bits beyond the crossover point. In addition the directed approach ensures that only better (lower discounted NPV) offspring are generated. If not, the solution is discarded.

The directed-crossover operation is performed in a pair of chromosomes. Where two offspring are created, each new child is validated and its NPV is compared to the parent. The discounted NPV of each child is compared to the parent-chromosome that supplied him with his left side of the crossover point. Thus in the example of section 2.5.2.4, chromosome $\mathrm{A}^{\prime}$ is compared to chromosome A and chromosome $\mathrm{B}^{\prime}$ is compared to chromosome B. Only feasible offspring and those possessing lower NPV are kept, and in the case where an invalid offspring (one or two) is found, two new offspring are regenerated. In this directed-based approach the concept of "brother-offspring" is introduced, and it is employed in the case where only one child is found to be valid (feasible and has a lower NPV) while the other one is not. In this case, the valid child is stored as a brother-offspring and two new children are regenerated from the original parent-chromosomes. This time, one of the children is compared to his "older-brother" instead of comparing him to the parent, whereas the other child is compared to his parent. If the new child is feasible and better than his brother, the new child replaces the brother, if not the brother is kept. The process is continued until two new offspring are found. If this is not achieved after a fixed number of iterations, the crossover operation is halted, and the parent-chromosomes are reproduced as is, or, depending on the situation, one brother-chromosomes and one of the parents chromosomes are reproduced. Figure 3-7 presents a flowchart of the proposed crossover process.

${ }^{*}$ ) Note: if the answer is NO, then the condition are not fulfilled
Figure 3-7 Directed-crossover flowchart

## Perform directed mutation operation

The mutation operation discussed in section 2.5.2.5 consists of replacing the action of a randomly chosen period ( $t$ ) with another possible action that is possible with the condition rate of period $t-1$. As previously mentioned, only valid solutions are created, thus a validation process is performed for all the actions beyond period $t$ in order to ensure that all the actions still correspond to the new condition states and their corresponding allowable action from the rehabilitation policy table (e.g. Table 3-2). If at any period beyond $t$, the action does not correspond any more to the rehabilitation policy, the mutation process is repeated again, until a feasible solution is found. If after a certain number of iterations, no feasible solution is found, the mutation operation is abandoned and the original solution is kept as is.

Differently from the crossover process, the proposed mutation operation does not ensure that only better solutions (having a lower NPV) are generated for the following reasons:

- The mutation probability is normally lower than the crossover ( $p m \leq 15 \%$ ) which generally induces little effect on the overall algorithm.
- The mutation operation is normally used as a secondary operator and helps to maintain the diversity in a population so that the other operators (crossover and reproduction) can continue to work properly.
- The mutation operation can possibly bring back some lost alleles that may be important for the optimization process.

Figure 3-8 presents the general structure of the proposed crossover and mutation directedprocess.


Figure 3-8 Directed-based genetic operation structure

## Verify the termination test

The termination of the program is achieved after attaining a certain user-defined number of iterations, or at the convergence to the solution. In the first case, the user defines the maximum numbers of generations, and the program is stopped after that. This could be advantageous for controlling the program running, but the efficiency of the output is not guaranteed, especially with a low number of iterations. On the other hand in the case of the convergence method, the test of the solution is the mean fitness / max fitness, and this ratio must approach one in order to have a convergence. In other words, the individuals have a similar genetic code (e.g. 95\%), which represents a similar point in the phenotype (the problem space).

### 3.4 Summary

In this chapter the research methodology is explained, and a M\&RPPI method is proposed. Both GA optimization technique and Markov chain are combined together in order to determine the best maintenance and rehabilitation profile over the life-cycle of the infrastructure. The transition probability matrices normally used for deterioration modeling were found to be highly beneficial for the method, and the integration path for the TPM to GA is presented. The directed approach for GA is also described in order to guide the optimization process toward the best solution.

## 4 M\&RPPI COMPUTER PROGRAM

### 4.1 M\&RPPI Excel Program Overview

A computer program using Excel and VBA macros is specifically developed for the purpose of this research in order to validate the workability of the proposed methodology. The choice of Excel and VBA is mainly because of the advantage of having cells and addresses within spreadsheets that could best represent the structure of a chromosome, and which could be used to store the input and output data. An additional incentive to use Excel is the straightforward means for computations available during the optimization process. The integrated possibility of creating charts in Excel helps produce life-cycle profiles or any other chart with the data at hand. By staying within the Excel environment, the user could easily analyze the output data with greater flexibility by performing different statistical calculations that are not necessarily pre-programmed. Another advantage is that the M\&RPPI program does not require special installation and could be used from any computer. On the other hand, this program contains some limitations that are either applied for simplification purposes, or the result of the limitations of the Excel spreadsheets. For instance the maximum number of $M \& R$ actions is limited to four, numbered as A, B, C and D, however with additional programming effort this could be modified since the proposed method does not limit the maximum number of allowed actions. Also some improvements may be added in order to make the Excel program more user-friendly and more flexible. In fact, the proposed M\&RPPI method could be
programmed in any other languages, such as Visual C++, with an enhanced graphical user interface and additional features. GA operators used in the program are the crossover and the mutation, and their occurrence during the optimization process is monitored with a certain probability predefined as $p c=75 \%$ and $p m=15 \%$ respectively. This decision is due to the fact that the user might not be familiar with the concept of genetic algorithms, in addition to the fact that the developed directed approach of the program makes little impact on the variations of these two variables. But also in this case, this feature could be easily modified in the program in order to make both variables user-defined. It should be noted that the purpose of the present study is not to develop a complete rehabilitation and optimization computer program, but rather to come up with a valid and easy-toimplement methodology for infrastructure planning. The description of the Excel program is presented in the following section.

### 4.2 M\&RPPI Computer Program Description

In this section, the description of the M\&RPPI computer program is presented as is the manipulation of the required data with an overview about the type of results. The M\&RPPI Excel file has five major sheets including one dedicated for an output graph. The sheets are divided as follow:

- "Deterioration Model": where the results data for the graph are stored, and the final solution results are found.
- "GA Input": where some input data of the program is entered.
- " $G A$ ": where the algorithm process takes place.
- "GA output": where the best chromosomes at each iteration are stored with some additional statistical result data.
- "Rehabilitation Graphic": where the output graph of the program is plotted.

The following is a description of the different features of the program:

### 4.2.1 Data input and storage

The data input in the M\&RPPI program is done through multiple user screens, and the information is either stored in the memory of the computer or in the "GA Input" Worksheet. When the program is lunched a welcome screen appears as shown in (Figure 4-1) including the main menu with four options. The user can either enter the criteria and cost tables, or the transition probability matrices (TPM), or start the GA simulation, or exit the program. A button is dedicated to each option and a new user input screen appears for each.

Prior to executing the simulation, all input data should be entered. The data is divided into a managerial components and infrastructure-specific components. The user screen related to the first type of data is shown in Figure 4-2. In the upper part of this form, the cost information is entered for each rehabilitation alternative depending on the condition state (see section 3.2-3). The lower part of the input screen is dedicated to the managerial decision criteria summarized in a form of a table with the allowable rehabilitation actions for each condition state (see section 3.2-4). In this table, a selection list is available for each condition state with only the allowable rehabilitation action code (A, B, C or D).


Figure 4-1 Welcome screen and main menu


Figure 4-2 Input screen for criteria and cost tables

On the other hand, the data related to the infrastructure deterioration mechanism, and more precisely the TPM is shown in Figure 4-3, where four tables are found related to the four available rehabilitation actions. The percentages values are entered in each case, and the sum of each row should be equal to one (1). The last box indicates the sum of the data in each row, and if its values differ from one, the box color turns red indicating an error in that line. It should be noted that each one of the above mentioned boxes in both input forms is directly linked to a specific cell in the "GA input" sheet where the data is stored. At the launch of the program, the background of the "GA input" worksheets is normally hidden to the user, but if the user desires, all the data could be revealed by pressing "Show Background". Figure 4-6 shows a print screen of the normally hidden background in "GA input" worksheets.


Figure 4-3 Input screen of transition probability matrices


Figure 4-4 Background screen of the "GA Input" worksheet
Even though some basic errors validation are implemented in the VBA macros, the user is still required to verify the consistency of the data and ensure that it is coherent. When the simulation starts running, all data is loaded into the computer memory in order to minimize the interaction with Excel spreadsheet.

Once the different type of data is entered, the user could load the GA form where the simulation data is entered and the simulation is executed. Figure $4-5$ presents the user simulation form, which is divided into two sections. The left part is used to enter the simulation parameters, while the right part is for the simulation execution. First, the simulation data such as the planning horizon, the initial condition state, the population size, the maximum number of chromosomes, or the discount rate will be directly entered in their corresponding textboxes. Each textbox has a default initial value and some basic
data validation in order to decrease possible errors. Once the program starts running, this data is also stored in the memory of the computer and is printed on the result spreadsheet "GA Output".


## Figure 4-5 M\&RPPI simulation form

A check box is available if the user desires to ensure that a minimal state level is maintained at all times in the final solution. This concept was previously discussed in section 3.3.1.2 and will be further explained in chapter 5 , when the case study example is presented. The threshold is fixed within the program to 4.5 but this could be easily modified.

Once the input data is entered, the simulation is started; first the "Generate Initial Population" button is pressed in order to generate the first population according to the entered data. Once this task is completed, the "Continue Algorithm" button is automatically enabled and should be pressed in order to continue the simulation. During the simulation process, the status bar on the bottom of the Excel Workbook will be
updated to indicate the generation number at which the program is located, and the estimated time remaining. Time estimate is calculated according to the time elapsed to perform the previous generations.

### 4.2.2 Chromosomes representation

The chromosome representation is done in a similar manner to data storage. In fact some chromosomes are stored within the Excel spreadsheet themselves and some are directly stored in computer memory in form of arrays. In the first case, such as for the initial population, each chromosome is stored in a separate row of the Excel sheet, where the first column (column "A") represents the identification number of the chromosome and each subsequent column holds the $M \& R$ action at a specific year. The end columns are used to store other information about the chromosome such as the total cost and the total discounted cost (see Figure 4-6). Also some columns at the end of each row were used during the optimization process to store relative information about the chromosome such as its fitness, its roulette-wheel portion, or other information. While the algorithm is running, the storing of the chromosome population is done in the Excel sheet, whether the first population, or the selected population, or the offspring population, they are all recorded in the Excel "GA" Sheet. In the case of the initial population, the information is kept in the Excel file for the entire process, whereas for the selection population and the offspring, the information is replaced by a new one after each generation. Since the maximum number of columns in Excel is 256, only 240 planning periods are allowed in the program, leaving 16 columns for other information. However, a 240 years planning
period is far beyond any practically required analysis period, and does not represent a practical limitation of the M\&RPPI program.


Figure 4-6 View of "GA" worksheet
On the other hand, in order to represent the chromosome in the computer memory, an array having four rows and $N+1$ columns was used, where $N$ is the planning horizon. The array representation is as follows:

Column number

| Row description | 1 | 2 | 3 | $\ldots$ | $\mathrm{~N}-1$ | N | $\mathrm{~N}+1$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :--- |
| M\&R actions <br> Condition State <br> Cost | $\mathrm{X}_{1}$ | $\mathrm{X}_{2}$ | $\mathrm{X}_{3}$ | $\ldots$ | $\mathrm{X}_{\mathrm{N}-1}$ | $\mathrm{X}_{\mathrm{n}}$ | Empty cell |
| $\mathrm{S}_{1}$ | $\mathrm{~S}_{2}$ | $\mathrm{~S}_{3}$ | $\ldots$ | $\mathrm{~S}_{\mathrm{N}-1}$ | $\mathrm{~S}_{\mathrm{n}}$ | Empty Cell |  |
| Discounted Cost | $\mathrm{C}_{1}$ | $\mathrm{C}_{2}$ | $\mathrm{C}_{3}$ | $\ldots$ | $\mathrm{C}_{\mathrm{n}+1}$ | $\mathrm{C}_{\mathrm{n}}$ | Total Cost |
|  | $\mathrm{dC}_{2}$ | $\mathrm{dC}_{3}$ | $\ldots$ | $\mathrm{dC}_{\mathrm{n}+1}$ | $\mathrm{dC}_{\mathrm{n}}$ | Total discounted Cost |  |

Figure 4-7 Chromosome structure in Excel file

The "ReDim" feature of the VBA in Excel allows redefining the length of the array for each optimization problem, which prevents allocating more than the required memory space, and enables efficient use of the computer's processing time. In addition, this type of array representation makes data manipulation much easier than using "pointers" in other object-oriented programming languages.

### 4.2.3 Condition state representation

In the developed Excel program, the condition state is represented in an " $m$ " cells vector where " $m$ " is the number of possible conditions of the system. In our case $m=5$ and each cell of the condition state vector holds the proportion or the probability that the infrastructure condition corresponds to that state. Figure 4-8 shows the condition state vector used in the M\&RPPI program where columns $i=1$ to 5 correspond to the condition states 1 to 5 and $p_{i}$ is the probability of being in each state.

| Cell identification / State (i) | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Probability $(p i)$ | $p_{1}$ | $p_{2}$ | $p_{3}$ | $p_{4}$ | $p_{5}$ |

## Figure 4-8 Condition state vector representation

As previously shown in Equation 3-1 and Equation 3-2, the sum of the probabilities should equal one, and the expected current state of the infrastructure is $E(X n)=\sum_{i=1}^{m} p_{i}{ }^{*}(i)$.

First, the initial state of the infrastructure is entered in the user form of Figure 4-5 as an integer value between 1 and 5 . Even tough this might be considered a limitation to the
program since no decimal values are originally entered, but this approach simplifies the user input and makes it closer to most condition assessment methods used for infrastructure systems, namely, a numerical discrete rating. However the limitation of the initial state format could be overcome by modifying the program structure in a way to request the user directly input a state vector instead of a discrete value.

Once the initial state is entered, the program converts the discrete value into a state vector with a value of one " $p_{i}=1$ " in the box corresponding to the state value and zeros " 0 " elsewhere. During the subsequent iterations, the discrete value is modified by multiplying the state vector by the TPM corresponding to the M\&R decision ( $\delta$ ) as previously explained in Figure 3-3. The TPM $\left(\mathrm{P}_{\mathrm{ijj}}\right)$ is a dynamic array and could be different at each stage of the analysis period, depending on the M\&R action.

$$
P_{\delta j=}\left|\begin{array}{lllll}
p_{\delta 11} & p_{\delta 12} & p_{\delta 13} & p_{\delta 14} & p_{\delta 15} \\
p_{\delta 21} & p_{\delta 22} & p_{\delta 23} & p_{\delta 24} & p_{\delta 25} \\
p_{\delta 31} & p_{\delta 32} & p_{\delta 33} & p_{\delta 34} & p_{\delta 35} \\
p_{\delta 41} & p_{\delta 42} & p_{\delta 43} & p_{\delta 44} & p_{\delta 45} \\
p_{\delta 51} & p_{\delta 52} & p_{\delta 53} & p_{\delta 54} & p_{\delta 55}
\end{array}\right|
$$

## Equation 4-1

Another measure that is used in the Excel M\&RPPI file is when the state reaches 4.99 , it is automatically converted into state 5.0 , which does not significantly affect the results. This measure was taken in order to overcome the asymptotic behaviour of multiplying the state vector by the TPM in Figure 3-3 when it approaches state 5.

Another feature of the condition state is its relation with the criteria table where decisions about possible M\&R actions are made. In fact, depending on the condition state of the infrastructure, only some allowed M\&R actions are possible (see Table 3-2). In the
criteria table discrete values are used for the condition state, but given that the M\&RPPI method allows decimal values for the condition state, two possibilities are allowed in the Excel M\&RPPI program. First the decision could be based on any state between two consecutive states. For instance, the criterion of state 1 is applied to any value between 1 and 2 , and the criterion of state 2 for values between 2 and 3 , and so forth. Or the decision could be based on the rounded value of the current state, e.g. the criterion of state 1 is applied for states 1 to 1.5 , the criterion of condition 2 is applied for states 1.5 to 2.5, and so forth. Either method could be used. The choice is irrelevant to the concept of the proposed methodology; it depends rather on the management politics.

### 4.3 General Comments about the M\&RPPI Excel Program

Excel's built-in functions are a big programming relief. For example the Min, Max, and Sort functions, as well as other functions are directly used in the VBA code without the need to program them again. Nevertheless, other mathematical operations such as array multiplications are programmed within the VBA code in order to decrease the interaction with Excel itself. Also, the use of the spreadsheet cells is a big help for storing and manipulating the different data. However, it is found that heavy use of the Excel cells during the optimization increases computer processing efforts and reduces the program's efficiency. In addition, the back and forth interaction between the computer memory and Excel cells also increase the use of the processor time. For this reason, the direct use of the memory, for instance in form of arrays or variables, reduces the dependency on Excel cells, and improves the performance of the program, thereby reducing processing time.

The M\&RPPI program developed might present some limitations, especially with regards to the lack of some flexibility. Nonetheless, the GA engine that is developed is very advanced, and performs extremely well. Of course, several improvements could be performed to the program, but this becomes mainly a programming effort rather than a research effort.

The following two chapters present a metro system case study and an application of the proposed M\&RPPI methodology. It will be seen that the objectives set for developing the M\&RPPI program are successfully met, mainly in proving the workability of the proposed method.

## 5 METRO SYSTEM CASE STUDY AND DATA COLLECTION

### 5.1 Overview of the Montreal Metro System

The Montreal Metro is the second subway system to be built in Canada, and was inaugurated in 1966 during the tenure of Mayor Jean Drapeau, twelve years after the Toronto subway. According to a document prepared by the STM in 2004, the Montreal Metro is now considered as a main form of public transportation within the city of Montreal transporting more than 780000 passengers daily, and participating in $17 \%$ of the total trips in the Montreal metropolitan area, and $47 \%$ of the downtown trips.

The Metro is operated by the Société de Transport de Montréal (STM), and its initial network consisted of 26 stations on three separate lines, with a central hub station named "Berri-UQUAM" common to the three lines. After some extensions in 1976 and 1988 the Montreal Metro now includes 65 stations on four lines and 70 km of tunnels, serving the north, east and centre of Montreal island with a connection to Longueuil on the south shore of the St. Lawrence River. In addition, an extension to the City of Laval is scheduled to be inaugurated in 2007 and has been under construction since 2002 along with three new stations and a total of 5.5 km . Figure $5-1$ shows the Montreal Metro network with its four lines each identified by a color: Green (line 1), Orange (line 2), Yellow (line 4) and Blue (line 5). Line 3, originally intended to be a surface train partly running through the existing railway tracks, was never built and it now exists as a commuter train line.


Figure 5-1 Montreal Metro map
The Montreal Metro design was influenced by the severe winter conditions and is completely underground except for the connecting tracks between stations Parc and Sauvé. It was the first metro in the world to run completely on rubber tires because of their exceptionally quiet functioning and low vibration (from "History of the Montreal Metro" web page).

The added value of a Metro station for a neighbourhood is very positive in the development of the area, and it was important that the architecture of each station be unique in order to ensure a proper identity for each station, distinct from any other one. In addition the entrance buildings are usually designed in a way to respect the urban texture
of the neighbourhood and in harmony with the urban amenity. In fact some entrance buildings are considered to be a continuation or a renewal to the surrounding area.

The total real estate inventory of the Montreal Metro network consists of more than 400 sites grouped in several major categories: underground garages, repair shops, Metro stations, entrance buildings, administrative buildings, tunnels, and auxiliary structures attached to the Metro network between the stations (ventilation posts, emergency exits from the tunnels, cables galleries, electrical rectification posts, etc.). In addition to these properties, the Metro network holds vital elements for its functioning; namely the fixed equipment such as the control center, the telecommunication network, the energy exploitation center, train control system, railing and other lane control equipment, perception equipment, the public address system, etc.

The aging of the infrastructure and the building components of the real estate inventory as well as the fixed equipment provoked an increasing number of corrective and repair actions. The impact of the Metro network deterioration was shown on the exploitation of the Metro and on the service quality. In fact certain building components exceeded their service life and for others it is the environment that influenced the premature aging of the infrastructure. The total extends of the restoration works was increasing with time affecting the exploitation of the system.

In recent years, several incidents have caused evacuation of passengers and several service interruptions causing enormous delays in the service and inconveniences to Metro readership. For example on Friday, June 17, 2005 a Metro derailment took place on the Green Line, but luckily there were no injuries and the driver was the only one on board.

However, the regular passenger platform has been closed for a while after the incident. This incident was described by the Canadian Press as: "Metro derailment to cost thousands". The following week, on Tuesday, June $21^{\text {st }}, 2005$ a fire from an electrical cable induced the evacuation of 27 of the 65 stations normally used by 200,000 to 250,000 passengers daily. In the Montreal Gazette the headline appeared as: "Cable fire brings metro to a halt". Other service interruptions due to equipment or infrastructure defects are becoming more closely watched which has led to several important rehabilitation programs in the Metro system since 1997. As a matter of fact, two principal projects were undertaken for the rehabilitation of the Metro stations and its equipment. First, there was the Réno-Station program (Phase I) from 1997-1999 for the rehabilitation of the 26 Metro stations of the initial network with a budget of $\$ 60 \mathrm{M}$. Then, another program for the maintenance of the fixed equipment (Réno-Systèmes) evaluated at $\$ 311 \mathrm{M}$ for phase I for the replacement of all technological equipment that had attained their service limits. Now the phase II of both programs are under study. It should be stated that according to a document prepared in 2004 by the STM, the total replacement cost of the Montreal Metro Infrastructure is estimated to be $\$ 10$ billion (CAN), where $\$ 5.5$ is for tunnels and stations.

With the increasing problem of congestion in metropolitan of Montreal, the Metro should remain a reliable solution for mass public transportation, with minimal service interruptions. Consequently, the need of an effective asset management system becomes mandatory in order to optimize the $M \& R$ costs over the life-cycle of the Metro, while ensuring that the operations are within an acceptable level of performance. The previously discussed (M\&RPPI) model could be considered an interesting starting point
that allows optimizing the best $M \& R$ profile for such infrastructures. In the following section, a case-study is presented for a specific type of problem encountered in the Metro system, and the application of the proposed M\&RPPI method will be explained and discussed.

### 5.2 Problem Definition

Given Montreal's winter conditions, nearly all Metro station entrances are completely enclosed, usually in small, separate buildings with revolving doors that mitigate the wind caused by train movements (piston effect) that can make the doors difficult to open. In total Montreal's Metro network encloses 124 Metro entrance buildings. Usually the halls at the entrance buildings serve as waiting areas, where the Metro passengers could see the buses or the waiting car, and bay windows and benches are installed for this purpose.

Normally, and especially during winter periods, Metro entrances are exposed to harsh weather and to high temperature changes when the entrance building doors are constantly opened during the day. The slab structure usually consists of a 6 inche reinforced concrete slab over concrete beams, on top of which a waterproofing membrane is applied, and a finish that consists of 1 inch of mortar and 2 inches of granite tiles. With age the joints between the tiles open up and the waterproofing membrane dries out and becomes punctured, letting water and debris reach the concrete slab. In addition, de-icing salt carried by passenger boots reaches the slab and increases the chloride concentration in the concrete, which provokes the corrosion of the reinforcing steel. Once the corrosion is started, the rust generated can occupy several times the volume of the original steel and the resultant pressures cause tensile forces to develop, which leads to spalling or cracking
of the concrete cover, along with delamination and further corrosion and cracking damages (see Figure 5-2). The corrosion of reinforcing steel in concrete can decrease the serviceability of concrete structures by inducing additional cracks and reducing the overall strength and stiffness of the concrete structure. Also, corrosion products are highly porous and decrease the bond strength between the reinforcement and concrete, and reduces the cross-sectional area of reinforcing steel, decreasing the overall ductility of the structure (Yoon et al., 2000).


Figure 5-2 Concrete spalling formation - adapted from PCI (1997)
In recent years, the STM engineering department requested inspection reports from several engineering firms about the overall condition state of the Metro system. The reports described the overall condition state of the stations and the entrances buildings, and included some proposed solutions and estimated restoration costs. The inspections undertaken for the deteriorating concrete slabs start with a visual inspection in order to verify the external condition of the structure, and to verify the amount of cracks and their spread. Usually underneath the entrance hall slab, a mechanical room called "upper machinery" is situated, where the escalator's bases and engines are located. The underside of the concrete slab is thus apparent and accessible from these spaces and the visual inspection usually starts from these locations. The surveys revealed that an
important amount of cracks were observed, jointly with an accumulation of efflorescent material, which is mainly a residue of salt. This also indicates that the waterproofing membrane has been compromised leading to water infiltration underneath the slab. An auscultation of the slab is then performed by hammering its surface in order to verify any evidence of delamination. If the condition reveals signs of high corrosion or major cracks then corrosion potential surveys are performed according to the ASTM C-876 norm HalfCell potentials of uncoated reinforcing steel in concrete. The method consists of dividing the entrance slab into a grid of $1 \mathrm{~m}^{2}$, and at each square a measurement of the electrical potentials is performed using a half-cell $\mathrm{Cu}-\mathrm{CuSO} 4$. The principle of the readings is derived from the notion of a closed electrical circuit, where a portion of the slab rebar is cleared at two locations and a measurement of the electrical resistance between these two points is taken using an ohmmeter. Once the electrical continuity is verified, the cleared rebar serve to plug one side of a voltmeter for the remaining survey. In order to validate the potential readings, some additional core samples of the slab are extracted in order to determine the chloride concentration in the concrete slab. The interpretation of the potential readings is as follows. If the half-cell potential reading is more positive than 0.20 V CSE (copper-copper sulphate electrode), there is a greater than $90 \%$ probability that no reinforcing steel corrosion is occurring in the area at the time of measurement. If the potential reading is in the range of -0.20 to -0.35 V CSE, corrosion activity of the reinforcing steel is assumed to be uncertain. If the potential reading is more negative than 0.35 V CSE, it is assumed that a greater than $90 \%$ probability of the reinforcing steel corrosion is occurring. In Table 5-1 the descriptive explanation of the ASTM C876 rebar potential readings is provided. It should be noted that the last column is an appendage to
the original ASTM C876 table, and corresponds to the condition state of the concrete slab related to each of these potential readings. The reason behind this new column is for illustrative purposes, in order to utilize the same states classification (1 to 5 ) that are previously used in Figure 3-2 as well as in the proposed methodology.

Table 5-1 Potentials of reinforcing steel in concrete (ASTM C876)

| Rebar Potential (mV Cu/CuSo4) | Probability of Reinforcing Steel Corrosion | Corresponding Condition State ${ }^{(*)}$ |
| :---: | :---: | :---: |
| $>-200 \mathrm{mV}$ | Greater than $90 \%$ probability that no reinforcing steel corrosion is occurring | State 1 |
| - 200 mV @ - 350 mV | Uncertain corrosion activity | State 2: -200mV @ -300mV <br> State 3: -300mV @ -350mV |
| $<-350 \mathrm{mV}$ | Greater than $90 \%$ probability that a reinforcing steel corrosion is occurring | State 4 |
| $<-500 \mathrm{mV}$ | Greater than $90 \%$ probability that a reinforcing steel corrosion is occurring | State 5 |

In Figure 5-3 and Figure 5-4 an example of potential readings is presented taken from the top and bottom surfaces of the entrance slab with a coloured legend for the condition states corresponding to Table 5-1.

The proposed M\&RPPI method will be used to analyse the problem of a deteriorating problem of concrete slabs at the Metro entrances. First, four different M\&R strategies will be described for the problem at hand, then their respective costs will be calculated, and the management decision policy will be identified. Afterward the deterioration models will be estimated for the concrete slab in the Metro as well as the TPM of each rehabilitation alternative. Finally this data will be entered in the Excel M\&RPPI file in
order to obtain the optimal rehabilitation profile over the desired planning period. The planning period that will be considered is 100 years. This will allow the visualization of long-term behaviour of the infrastructure. The different results will be then compared to another model that uses dynamic programming, and more precisely the traditional Markov decision process. The outcomes of these methods will be compared to each other.

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### 5.2.1 Proposed maintenance and rehabilitation alternatives

### 5.2.1.1 Alternative A: Replace the concrete slab (or a major portion)

This alternative is undertaken when the deterioration of the structure is very advanced, and the corrosion damages attain a major portion of the entrance slab. The patching and the spot repairs of the slab become ineffective and the slab as well as the top finishing surface may become hazardous. It should be noted that corrosion damage is generally confined to the near-surface regions of a structure and surveyors must guard against overestimation of damage (Yoon et al., 2000). In the case where very advanced damage is assessed, the entire concrete slab is demolished except for the supporting beams which are conserved, and a new slab is reconstructed (Figure 5-5). Usually the supporting beams, except for their upper part, are better preserved then the slabs, and at some locations only a patching of their surface may be required, even though in some situations it is possible that the totality of the slab including the beams would be replaced. Finally, a new waterproofing membrane is applied to the concrete surface, and all the granite tiles on top are replaced along with a new application of protective sealant.

Even though this solution ensures the best rehabilitation option in terms of final quality results, it remains costly and requires the complete shut-down of the station. A temporary shuttle service (by special buses service) will be required to ensure the transportation of passengers from the previous station to the station under rehabilitation and to the following stations. Of course, this provokes some disturbance to the passengers and some delays during the reconstruction period. Among the inconveniences (indirect-costs) encountered from this solutions are:

- Additional costs due to shuttle services.
- Additional costs due to signalisations required to reroute or advise passengers.
- Additional costs for inspectors and supervisors.
- Additional overhead costs for the coordination of the work.
- Additional disturbance to local traffic due to external passenger transportation.
- Additional disturbance due to noise and dust from construction site.


Figure 5-5 Slab replace detail
Table 5-2 Replacement cost of the entrance building slab (damaged section)

| Cost description | Quantity | Unit price (*) | Total cost |
| :--- | :---: | :---: | :---: |
| Direct costs: | Lump sum | $\$ 25,000$ | $\$ 25,000$ |
| -Temporary protections (Site <br> preparation) | $100 \mathrm{~m}^{2}$ | $\$ 500$ | $\$ 50,000$ |
| - Replace of the granite tiles | $85 \mathrm{~m}^{2}$ | $\$ 1,500$ | $\$ 127,500$ |
| -Slab demolition and replacement for <br> the advanced damaged portion | $25 \mathrm{~m}^{2}$ | $\$ 700$ | $\$ 17,500$ |
| - Patching of concrete beams |  |  |  |


| Cost description | Quantity | Unit price (*) | Total cost |
| :---: | :---: | :---: | :---: |
| - Waterproofing membrane reinstallation | $100 \mathrm{~m}^{2}$ | \$ 100 | \$ 10,000 |
| - Cracks injection using urethane | 30 m | \$ 275 | \$ 8,250 |
| - Tiles joints repair of non removed section | 50 m | \$20 | \$ 1,000 |
| - Granite tiles surface cleaning and sealing | $175 \mathrm{~m}^{2}$ | \$ 15 | \$2,625 |
| - Electrical and mechanical service relocation | Lump Sum | \$20,000 | \$20,000 |
| Total Direct Cost |  |  | \$ 261,875 |
| Indirect Costs (internal expenses to the STM): |  |  |  |
| - Special shuttle service | Lump Sum | \$100,000 | \$ 100,000 |
| - Additional supervision and relocation | Lump Sum | \$50,000 | \$50,000 |
| Total Indirect Cost |  |  | \$ 150,000 |
|  |  | Total Cost | \$ 411,875 |
| Contingencies (10\%) |  |  | \$ 41,200 |
| Overhead (3\%) |  |  | \$ 13,600 |
| Total Costs |  |  | \$ 466,675 |

(*) Note: The cost elements are derived from different documents provided by STM, and prepared by engineering firms.

### 5.2.1.2 Alternative B: Repair only the most defective portions

The repair methods depend on the deterioration level of the slab and the probability of the corrosion activity. In fact in the regions with high corrosion probability, a complete replacement of the slab is recommended, while maintaining the existing reinforcement as shown in Figure 5-6. Once the concrete demolition work is completed, the uncovered rebar should be cleaned, however if more than $25 \%$ of the rebar cross-section is found to
be lost, whether from corrosion or from the demolition works, additional rebar with the same diameter as the existing should be added.


Figure 5-6 Typical concrete repair section
On the other hand, in the region with lower corrosion potential is found, a patching of the delaminated concrete could be performed, the loose concrete is removed, the rebar cleaned and a cementations mortar is applied. Figure 5-7 present a typical repair section.


Figure 5-7 Typical concrete patching detail

Finally for the section where minimal damages are found, only concrete patching could be done, and some crack injection work could also be considered using some polyutherane materials. In addition to the repair work of the concrete, the granite tiles are removed and the waterproofing membrane on the top surface should be replaced, then the granite tiles in good condition are reinstalled, and the total surface is sealed. Among the inconveniences encountered from this solutions are:

- Additional costs due to temporary protection.
- Additional costs due to night shifts work and overtime for work during the weekends especially for the demolition work.
- Additional costs due to restricted construction work schedules during the day (especially for interruptions during rush hours).
- Additional overhead costs due to longer construction period.
- Additional costs due to signalisations required to reroute or advise passengers.
- Additional costs for inspectors and supervisors.
- Additional overhead costs for the coordination of the work.
- Additional costs due to constant relocation of the ticketing cabin (when applied).
- Additional costs due to additional ticket controller when the turnstiles are not functioning (when applied).
- Disturbance due to partly shutting the entrance building.
- Limited accesses and disturbance to passengers.
- Additional disturbance due to noise and dust from construction.

The above mentioned inconveniences and costs are usually in addition to the construction cost of the contractors, and represent the indirect costs, as well as users costs incurred. Even though some data could be collected for some indirect costs, the monetary assessment of the other inconveniences is much more difficult. For example the monetary conversion for the disturbance due to noise or dust is not an easy task. The repair cost for a station could be estimated as follows:

Table 5-3 Repair cost of the entrance building slab

| Cost description | Quantity | Unit price (*) | Total cost |
| :---: | :---: | :---: | :---: |
| Direct Costs: |  |  |  |
| - Temporary protections (site preparation) | Lump sum | \$25,000 | \$ 25,000 |
| - Removal and reinstallation of granite tiles | $100 \mathrm{~m}^{2}$ | 150\$ | \$ 15,000 |
| - Tiles joints repair of non removed section | 150 m | \$20 | \$ 3,000 |
| - Granite tiles surface cleaning and sealing | $175 \mathrm{~m}^{2}$ | \$ 15 | \$ 2,625 |
| - Concrete patching from the underside of the slab | $75 \mathrm{~m}^{2}$ | \$ 750 | \$ 63,750 |
| - Patching of concrete beams | 25 m | \$ 800 | \$ 20,000 |
| - Cracks injection using urethane | 30 m | \$ 275 | \$ 8,250 |
| - Waterproofing membrane | $100 \mathrm{~m}^{2}$ | \$ 100 | \$ 10,000 |
| - Electrical and mechanical service relocation | Lump Sum | \$20,000 | \$ 20,000 |
| Total direct costs |  |  | \$ 147,625 |
| Indirect Costs: |  |  |  |
| $-\quad$ $\begin{array}{l}\text { Relocation and additional } \\ \text { supervision costs (estimated) }\end{array}$ | Lump Sum | \$50,000 | \$ 50,000 |
| Total Indirect costs |  |  | \$ 50,000 |
|  |  | Total costs | \$ 197,625 |
| Contingencies (10\%) |  |  | \$ 19,760 |


| Cost description | Quantity | Unit price (*) | Total cost |
| ---: | ---: | ---: | ---: |
|  | Overhead (3\%) | $\$ 6,520$ |  |
|  |  |  |  |

(*) Note: The cost elements are derived from different documents provided by the STM, and prepared by engineering firms.

Depending on the condition state, and from the previous cost table, the repair action costs are as estimated in Table 5-4:

Table 5-4 Estimated repair cost per station

| Condition State | Repair cost per <br> year |
| :---: | :---: |
| 1,2 | Not allowed |
| 3 | $\$ 200,000.00$ |
| 4 | $\$ 225,000.00$ |
| 5 | $\$ 250,000.00$ |

### 5.2.1.3 Alternative C: Preventive Maintenance

As previously mentioned, a preventative maintenance program for an infrastructure could be seen as a cost-effective treatment applied at the proper time to preserve and extend the useful life of the infrastructure. The preventive maintenance could avoid larger scale repair work and will improve the durability of the structure. For the Metro entrance buildings it could consists of the following:

- Perform periodic inspection to the entrance building.
- Carry out cleaning activities, including periodical water flush of all the entrance from de-icing salts, especially in winter period, or from any dirt and debris that cause damage and premature wear to coatings and surface finishes.
- Apply a protective coating and sealant to the finishing tiles surfaces.
- Perform minor patching and repairs of the concrete slab from the underneath, where all cracked and delaminated concrete are removed and all corroded reinforcement are cleaned. (Figure 5-7)
- Perform minor repair to the waterproofing system underneath the tiles.
- Inspect and repair the drainage system.
- Perform crack repairs, epoxy injection, and sealing of joints between the tiles.

Such a program could be attractive due to its low costs and temporary aesthetic improvements. Some of its measures will definitely improve the performance of the entrance hall, and will enhance its appearance and prolong its useful life. But such measures have limited success against chloride-induced corrosion especially once the corrosion process is started. The maintenance cost (Alternative C) for a station could be estimated as follows:

## Costs assumptions:

- 3 cleaning program per year for each station
- Total of 66 stations and 124 entrance buildings
- A crew of 5 people would clean 3 stations per day
- 22 working days per month


## Cost calculation:

- Number of cleaning per year: $124 \times 3=372$ total cleanings
- Crew required per year: $5 \times 372 / 3=620$ workers - day per year
- 1 worker day cost $=\$ 60,000($ per year $) /(12 \times 22)=\$ 227$ per day $($ say $\$ 230)$

The following table shows the estimated preventive and corrective maintenance costs:

Table 5-5 Cost Breakdown for preventive maintenance (PM) intervention

| Cost description | Quantity | Unit price | Total cost |
| :---: | :---: | :---: | :---: |
| Cleaning cost: |  |  |  |
| - Man Power | 620 wk.day | \$ 230.00 | \$ 142,600.00 |
| - Material | 372 units | \$ 500.00 | \$ 186,000.00 |
| - Equipment | 372 units | \$ 250.00 | \$ 93,000.00 |
| Total cleaning Cost: |  |  | \$ 421,600.00 |
| - Floor tiles coating cost (one application per year): | $\begin{gathered} 200 \mathrm{~m}^{2} \mathrm{x} \\ 124 \\ \text { entrances } \end{gathered}$ | \$ 15.00 | \$ 372,000.00 |
| - Tile joints repair cost (yearly average): | 20 m x 124 entrances | \$ 250.00 | \$ 620,000.00 |
| - Cracks injection cost: | $\begin{gathered} 10 \mathrm{~m} \times 124 \\ \text { entrances } \\ \hline \end{gathered}$ | \$ 250.00 | \$310,000.00 |
| - Other patching and repair cost: | 124 entrances | \$ 2,500 | \$ 310,000.00 |
| Sub-total |  |  | \$ 1,612,000.00 |
| Contingencies (10\%) |  |  | \$ 161,200.00 |
| Overhead (3\%) |  |  | \$ 53,196.00 |
| Total Maintenance Costs |  |  | \$ 1,826,396.00 |

On average the preventive maintenance cost per entrance building is approximately $\$ 14,750.00$ per year. Depending on the condition state, the PM action cost will be estimated as follows:

Table 5-6 Estimated preventive maintenance cost per station

| Condition State | PM cost per year |
| :---: | :---: |
| 1 | $\$ 10,000.00$ |
| 2 | $\$ 12,000.00$ |
| 3 | $\$ 16,000.00$ |
| 4 | $\$ 18,000.00$ |
| 5 | Not Allowed |

### 5.2.1.4 Alternative D: Do-nothing

In this alternative, the concrete slab is kept as is without any rehabilitation or maintenance. Even though this alternative causes the slab to deteriorate more rapidly than other options, it remains appealing from a monetary standpoint since it incurs little no cost. In fact when leaving the slab as is, only some management overhead costs are incurred.

The different costs discussed are entered in the M\&RPPI Excel file as shown in Table 5-7.

Table 5-7 Summary of M\&R alternative actions

| Cost Table | Replace | Repair | Preventive <br> Maintenance | Do-Nothing |
| :---: | :---: | :---: | :---: | :---: |
| State $(\mathrm{l})$ | A | B | C | $\mathbf{D}$ |
| 1 |  |  | $\$ .10,000$ | $\$$ |
| 2 |  |  | $\$ .12,000$ | $\$$ |
| 3 |  | $\$ 200,000$ | $\$ 16,000$ | $\$$ |
| 4 | $\$ 470,000$ | $\$ 225,000$ | $\$ 18.000$ | $\$$ |
| 5 | $\$ 470,000$ | $\$ 250,000$ |  | $\$$ |

### 5.2.2 Deterioration models and transition probability matrices

As previously stated, the purpose of this study is not to perform deterioration modelling for public infrastructure, nevertheless this information is important for the M\&RPPI method. For this reason in this section some TPM will be estimated for each rehabilitation alternative. The condition states that will be considered are previously described in Figure 3-2 and Table 3-1, where the scaling system was from one to five, with one being the excellent state and 5 the critical state.

For the first two alternatives, replace and repair finding the TPM could be easy, in fact with the replace method the new condition state is going to go back to state 1 (as new) irregardless of the initial state. Thus the TPM for alternative A is:

## Table 5-8 TPM for alternative A: Replace



For the repair alternative, the TPM could be considered as follows. The portions with state 1 and 2 will go back to 1 , whereas the state portions 3 and 4 will become 2 . Finally state 5 will go to 3 as shown in the following table:

## Table 5-9 TPM for alternative B: Repair

|  | Next state |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 5 |
| ๕ | 1 | 1 | 0 | 0 | 0 | 0 |
| \% | 2 | 1 | 0 | 0 | 0 | 0 |
| \% | 3 | 0 | 1 | 0 | 0 | 0 |
| 家 | 4 | 0 | 1 | 0 | 0 | 0 |
| $\bigcirc$ | 5 | 0 | 0 | 1 | 0 | 0 |

The next two alternatives are more difficult to assess. In fact, their values are derived from personal judgement, and not directly compared with historical data. Nevertheless the Do-Nothing TPM is based on the fact that the Metro has been in operation for more than 40 years, and according to several reports provided by the STM the actual state for most of the entrance halls at the Metro stations is between fair (three) and poor state (four) revealing a high concentration of corrosion, cracks and damage. It will be assumed that
with the do-nothing option the slab will switch from one state to another approximately every ten years, thus a slab starting in state 1 will remain approximately ten years in this state before switching to state two, and so fourth for every other state. On the other hand, with a corrective and preventive maintenance option, the slab condition will be preserved for an additional five years in each state and will last for almost fifteen years at each state. In Table 5-10 and Table 5-11 the estimated TPM are shown for both alternatives. In addition, Figure 5-8 and Figure 5-9 show the deterioration profile for these alternatives according to the respective TPM over a 60 year period for five different initial states.

Table 5-10 TPM for alternative C: Preventive and corrective maintenance


Table 5-11 TPM for alternative D: Do-Nothing



Figure 5-8 Deterioration curve for do-nothing alternative

Deterioration Curve
(Preventive \& Corrective Maintenance Alternative)


Figure 5-9 Deterioration curve with preventive and corrective maintenance

## 6 RESULTS AND ANALYSIS

In this section several data processing and analyses will be performed according to the raw data previously presented for the case study. First the Markov Decision Process (MDP) will be explained and a Metro model will be developed based on the traditional MDP, thus using only discrete condition states. Afterward, a continuous MDP approach will be presented using the same optimal policy obtained with the discrete approach. Finally the proposed genetic algorithm method will be used with different optimization parameters in order to obtain the best rehabilitation profile. The results from the different methods will be compared together, and a discussion will be presented. Also the performance of the developed computer program will be illustrated.

### 6.1 Metro Model Using Markov Decision Process

As previously discussed, the traditional method utilized with the Markov chain uses the dynamic programming (DP) techniques and is called the Markov Decision Process (MDP). This technique allows finding the optimal decision to undertake at each condition state knowing all the possible alternatives, their costs and their transition probability matrix (TPM). In this section a brief discussion about this topic will be introduces and a model using the MDP will be developed, and some analysis will be undertaken. The purpose of this analysis is to compare the outcome of the proposed method of this thesis to the traditional way, and to assess the efficiency of the proposed method.

Dynamic programming is basically an optimization technique for making a sequence of interrelated decisions that works backward from the end of a problem toward the beginning, and beaks up a large, "unwieldy" problem into a series of smaller problem (Winston, 2004). DP is a method which is viable for LCCA of infrastructure management systems, because it provides a quick and systematic method for finding optimal maintenance and rehabilitation (M\&R) decisions at each stage throughout the analysis period. DP functions based on the deterioration schema of the infrastructure and the set of allowed M\&R alternatives for each state.

The MDP is a method used in infrastructure rehabilitation problems for selecting the optimal maintenance and rehabilitation ( $M \& R$ ) decision policy. It provides a set of rules that specifies how each period's decision is chosen from a set of allowed actions for each state by considering the initial and subsequent costs (Wirahadikusumah, 1999; Winston, 2004). MDP functions as an infinite horizon probabilistic dynamic programming (PDP) problems and the LCCA is performed on the basis of the long-term behaviour of the infrastructure (Winston, 2004). The MDP is described by four types of information:

1. State space: the state $i$ where the MDP is found at the beginning of each period. $i \in S=\{1,2, \ldots M\}$. (e.g Table 3-1).
2. Decision Set: For each state $i$, there is a finite set of allowable or feasible alternative and decisions $\mathrm{D}(i)$ that can be considered. (e.g. Table 3-2)
3. Transition Probabilities: The next period's state $j$ depends only on the current period's state $(i)$ and on the decision chosen during the current period $\mathrm{d} \in \mathrm{D}(i)$. This probability is denoted by $\mathrm{p}(j \mid i, d t)$.
4. Expected reward (or incurred costs): During a period in which the state is (i) and a decision $d \in D(i)$ is chosen, an expected reward ( $r_{i d}$ ) is received, or an expected costs $\mathrm{C}_{\mathrm{id}}$ is incurred. The set of rewards for the process may be described by a reward matrix R with $\mathrm{r}_{\mathrm{ij}}$ elements and could be expressed in dollars value.

On the other hand, a policy is defined as "a rule that specifies how each period's decision is chosen, a policy $\delta$ is a stationary policy if whenever the state is $(i)$, the policy chooses the same decision $\delta(i)$ independently of the period $(t)$ " (Winston, 2004).

In the case where the same M\&R rehabilitation action (decision $d$ ) is taken whenever the system is in state $(i)$ and independently of the time period $(t)$, then the decision policy $\delta(i)$ is said to be stationary policy. An optimal policy is the one that maximizes the total expected return for each ( $i$ ) or minimizes the total expected costs. The MDP calculates the minimum expected discounted cost that can be incurred during $(n)$ periods if the state at the beginning of the state of the current period (stage $t$ ) is $(i)$ (Zayed, 2002)

The average expected discounted cost is calculated as follows:

$$
E\left[C\left(X_{t}\right)\right]=\sum_{i=1}^{M} \pi_{i} C_{i} \quad \text { for state } \mathrm{i}=1,2 . . \mathrm{M}
$$

## Equation 6-1

Several methods can be used to determine an optimal stationary policy, such as:
a) Policy iteration or policy improvement technique.
b) Linear programming.
c) Value iteration or successive approximation.
d) Maximizing average reward per period.

Methods (b) will be used in this research. For additional information about other techniques, readers are referred to any operation research books such as (Winston, 2004) or (Hillier and Lieberman, 2005). For a minimization problem, the objective function to be solved using the linear programming (LP) is the following:

$$
\begin{aligned}
\operatorname{Max} \mathrm{z}=\mathrm{V}_{1}+\mathrm{V}_{2}+\ldots \mathrm{V}_{\mathrm{N}} & \text { For each state (i) and each } \mathrm{d} \in \mathrm{~d}(\mathrm{i}) \\
\text { s.t. } V i-\beta=\sum_{j=1}^{j=N} p(j \mid i, \delta(i)) \mathrm{V}_{\delta}(j) \leq C_{i d} & \text { Equation 6-2 }
\end{aligned}
$$

$\mathrm{V}_{\delta}(\mathrm{i})$ : Expected total discounted reward (or cost) incurred during an infinite number of periods given that at the beginning of period $(t)$, the state is $(i)$ and the stationary policy will be $\delta$.

The optimal solution to these LPs will have $\mathrm{V}_{\mathrm{i}}=\mathrm{r}_{\mathrm{i}}$. Also if a constraint for state (i) and decision (d) is binding (has no slack or excess) the decision (d) is optimal in state $(i)$.

For the case study at hand, the objective function and constraints are set as follows:

$$
\operatorname{Max} V_{1}+V_{2}+V_{3}+V_{4}+V_{5}
$$

Equation 6-3
s.t

|  |  | Action |
| :--- | :--- | :--- |
| 1. $V_{1}-\beta\left(0.95 \mathrm{~V}_{1}+0.05 \mathrm{~V}_{2}\right)$ | $\leq \$ 10000.00$ | (C) |
| 2. $\mathrm{V}_{1}-\beta\left(0.93 \mathrm{~V}_{1}+0.07 \mathrm{~V}_{2}\right)$ | $\leq \$ 0.00$ | (D) |
| 3. $\mathrm{V}_{2}-\beta\left(0.93 \mathrm{~V}_{2}+0.07 \mathrm{~V}_{3}\right)$ | $\leq \$ 12000.00$ | (C) |
| 4. $\mathrm{V}_{2}-\beta\left(0.90 \mathrm{~V}_{2}+0.10 \mathrm{~V}_{3}\right)$ | $\leq \$ 0.00$ | (D) |
| 5. $\mathrm{V}_{3}-\beta\left(\mathrm{V}_{2}\right)$ | $\leq \$ 200000.00$ | (B) |
| 6. $\mathrm{V}_{3}-\beta\left(0.90 \mathrm{~V}_{3}+0.10 \mathrm{~V}_{4}\right)$ | $\leq \$ 16000.00$ | (C) |

7. $V_{3}-\beta\left(0.85 V_{3}+0.15 V_{4}\right)$
$\leq \$ 0.00$
(D)
8. $\mathrm{V}_{4}-\beta\left(\mathrm{V}_{1}\right)$
$\leq \$ 470000.00$
(A)
9. $\mathrm{V}_{4}-\beta\left(\mathrm{V}_{2}\right)$
$\leq \$ 225000.00$
10. $\mathrm{V}_{4}-\beta\left(0.80 \mathrm{~V}_{4}+0.20 \mathrm{~V}_{5}\right)$
$\leq \$ 18000.00$
(C)
11. $\mathrm{V}_{4}-\beta\left(0.70 \mathrm{~V}_{4}+0.30 \mathrm{~V}_{5}\right)$
$\leq \$ \quad 0.00$
(D)
12. $\mathrm{V}_{5}-\beta\left(\mathrm{V}_{1}\right)$
$\leq \$ 470000.00$
(A)
13. $\mathrm{V}_{5}-\beta\left(\mathrm{V}_{3}\right)$
$\leq \$ 250000.00$
(B)

Solving the above system of linear equations with an interest rate of $5 \%$ ( $\beta=0.952$ ) gives:

| V1: | 125284.09 |
| :--- | :--- |
| V2: | 214772.73 |
| V3: | 322159.09 |
| V4: | 429545.45 |
| V5: | 556818.18 |

Where the constraints satisfy all the inequalities:

Table 6-1 Linear programming solution

|  | Constraints |  | Cik |  | Action Xi |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{V}_{1}$ | 1704.5 | $\leq$ | \$ | 10000.0 | C |
| $\mathrm{V}_{1}$ | 0.0 | $\leq$ | \$ | - | D |
| $\mathrm{V}_{2}$ | 3068.2 | $\leq$ | \$ | 12000.0 | C |
| $\mathbf{V}_{2}$ | 0.0 | $\leq$ | \$ | - | D |
| $\mathrm{V}_{3}$ | 117613.6 | $\leq$ | \$ | 200000.0 | B |
| $V_{3}$ | 5113.6 | $\leq$ | \$ | 16000.0 | C |
| $\mathrm{V}_{3}$ | 0.0 | $\leq$ | \$ | - | D |
| $\mathrm{V}_{4}$ | 310227.3 | $\leq$ | \$ | 470000.0 | A |
| $\mathrm{V}_{4}$ | 225000.0 | $\leq$ | \$ | 225000.0 | B |
| $\mathrm{V}_{4}$ | -3787.9 | $\leq$ | \$ | 18000.0 | C |
| $\mathrm{V}_{4}$ | -15909.1 | $\leq$ | \$ | - | D |
| $\mathrm{V}_{5}$ | 437500.0 | $\leq$ | \$ | 470000.0 | A |
| $\mathrm{V}_{5}$ | 250000.0 | $\leq$ | \$ | 250000.0 | B |

The following optimal policy is obtained from previous results:

Table 6-2 Optimal policy using MDP with linear programming

| State: | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Action: | D | D | D | B | B |

Basically the outcome of this method says do-nothing at state 1,2 and 3 and repair at states 4 and 5. These results are based on the TPM and cost estimates presented, but with small variation of the "repair" cost (increase of less than 5\%) at state 5, the results would change to "replace" for state 5 instead of repair. However, the other costs data are found to be affecting less the final solution. The combined TPM derived from this policy is as shown in Table 6-3 where the first 3 rows are those from the do-nothing TPM, and the last two rows are those from "Repair" TPM.

Table 6-3 Transition probability matrix of the optimal policy


Calculating the limiting probabilities (long-run) of this Markov chain gives

$$
\begin{aligned}
& \pi_{j}=\sum_{j=1}^{N} \pi_{i} P_{i j} \\
& \text { and } \sum_{j=1}^{N} \pi_{j}=1
\end{aligned}
$$

Solving for the problem at hand gives:

| $\pi_{1}=0.0000$ | $\pi_{2}=0.5660$ | $\pi_{3}=0.3774$ | $\pi_{4}=0.0566$ | $\pi_{5}=0.0000$ |
| :--- | :--- | :--- | :--- | :--- |

And from Equation 6-1, the expected average cost per station, and for one-year period is:
$\mathrm{E}[\mathrm{C}(\mathrm{Xt})]=0.00 * 0+0.566 * 0+0.3774 * 0+0.0566 * 225000+0.00 * 225000$
$\mathrm{E}[\mathrm{C}(\mathrm{Xt})]=\$ 12,735.72$

Thus the expected discounted cost for a 100 and a 200 year period at $5 \%$ interest rate will give $\operatorname{PV}\left(\mathbf{E C}\left(\mathbf{X}_{100}\right)\right]: \$ 252,777.46$, and $\mathbf{P V}\left(\mathbf{E C}\left(\mathbf{X}_{200}\right)\right]=\$ 254,699.71$ respectively, and it could be seen that beyond 100 years, little impact on the PV is found.

It should be noted that traditional MDP method assumes a discrete rating and no intermediate state values are allowed. Nevertheless in real situations, the switch from one state to another is usually attained in continuous transitions. For this reason, some results obtained with traditional Markov chain might not reflect reality and are less practical. In fact if the same model is run using the M\&RPPI Excel program, using only the optimal policy of Table 6-2, the graph plot of Figure 6-1 will be obtained and the total expected discounted present value becomes $\$ \mathbf{7 4 , 8 6 8 . 7 4}$ for a 100 years and $\$ \mathbf{7 6}, \mathbf{7 6 3 . 3 9}$ for a period of 200 years. Results with a continuous approach are much lower than the expected LCC estimation of the traditional approach. This is mainly because the calculations are not based on an average cost value per period. Instead they are calculated based on the expected timing of the rehabilitation activity. Of course the MPD is based on an infinite horizon planning-period, and the expected cost per period is calculated on this basis, however a 100 year period is relatively large enough to be considered as an infinite length.

The divergent results obtained from both methods are also due to the difference between the discrete and continuous approaches utilized. Thus, continuous rating approach provides better and more realistic results compared to traditional discrete approach.


Figure 6-1 Deterioration curve with the continuous rating approach
In the next section, the same input data will be analysed with a simulation using the developed M\&RPPI method and GA technique.

### 6.2 Analysis Using M\&RPPI Method with GA Simulation

The previous input data is entered into the M\&RPPI Excel file and the analysis period of 200 years was selected. For the simulation, a population size of 500 individuals is chosen with 50 iterations. Figure $6-2$ presents the rehabilitation profile obtained with results from the MDP and the GA methods, also the deterioration curve with a do-nothing option is included. It could be seen that according to the GA result, the first "major" rehabilitation action (Action B) is undertaken after 107 years, mean while some maintenance
intervention are proposed (Action C) at several stages (year 53, 73, 78...) see appendix A for complete set of results. These results suggest taking no-action until 53 years from the construction period, then to perform some corrective and preventive maintenance actions at different intervals, and then perform a repair action after 107 years. This solution is not the best one in terms of performance of the infrastructure, and for over a seventy year period the infrastructure is in poor to critical state. However the results comply with the objective function set by minimizing the NPV. The method results in the lowest NPV among the previous two methods with a value of $\mathbf{\$ 6 , 5 5 3 . 4 2}$ and undiscounted costs of $\$ 1,685,000.00$.

Deterioration Curve
(Comparaison between the MDP and GA results)


Figure 6-2 M\&RPPI output for 200 years period

The following table summarizes the previously obtained results:

Table 6-4 NPV results from different analysis methods

| Period length | Traditional <br> MDP | Continuous <br> MDP | GA - <br> M\&RPPI |
| :---: | :---: | :---: | :---: |
| 100 years | $\$ 252777.46$ | $\$ 74,868.74$ | $\$ 3,859.08$ |
| 200 years | $\$ 254,699.71$ | $\$ 76,763.39$ | $\$ 6,553.42$ |

First, it can be seen that the results from a continuous approach is much better than the traditional discrete approach. Second the results from a GA optimization are also more economical than the MDP, since GA method focuses on minimizing the NPV and not merely providing a stationary rehabilitation policy. The proposed GA method of this thesis will not result in a single decision policy, rather it will find a rehabilitation profile over the analysis period.

Nevertheless, in this output no concerns are made for the overall performance of the infrastructure, and the objective function focuses only on minimizing the NPV of the rehabilitation interventions of the infrastructure during the analysis period. In order to make the analysis more practical, an additional constraint will be added to the objective function from this point on. This constraint will ensure that, at all times, the infrastructure will not degrade below a certain acceptable threshold, which will be fixed to the state 4.5 . The possibility of adding this additional constraint is an interesting feature of the proposed methodology. It will allow finding a low NPV solution while being practical in terms of performance of the infrastructure.

In order to analyse the performance of the M\&RPPI computer program, several simulations are performed with different GA parameters, namely by varying the population size and the number of generations. First the population size is varied between seven values which are $100,250,500,1000,1500,2500$ and 5000 , and ten simulations are run for each population size, giving a total of 70 simulations. During these simulations the number of generations is kept fixed at 50. In general the shape of the rehabilitation profile obtained from all simulations is similar, with the main difference in the timing of repair action. In general, the final NPV results are very comparable for all simulations. Figure 6-3 presents the plot of the best results obtained for four population sizes; the $100,500,1500$ and 5000 chromosomes.


Figure 6-3 Rehabilitation profile for different population size after 50 generations

Among the simulations, some differences are also noticed with regard to the preventive maintenance action. In fact, in the optimal solution with the lowest NPV, the preventive maintenance is not utilized, the infrastructure is kept until it reaches state 4.5 and a repair action is then performed. However, other less economical solutions suggest performing preventive maintenance at different interval of the analysis period. The usage of such actions improves the overall performance of the infrastructure. Figure 6-4 shows the different profiles when preventive actions are/are not considered.


Figure 6-4 Effect of preventive maintenance on the infrastructure
The performance of the M\&RPP Excel program will be evaluated with regard to the processing time required for the simulation, and the accuracy and the repeatability of the results. Thus the average processing time, required to run each simulation are calculated, as well as the average NPV obtained with each population size. Figure $6-5$ presents the average processing time with a Pentium (M) 1.60 GHz processor and 512 Mo RAM.


Figure 6-5 Processing time with respect to the population size
Figure 6-6 shows the average NPV obtained with each population size. Appendix B contains a results summary from the 70 simulations.


Figure 6-6 Average NPV results with respect to population size
From the graphs in Figure 6-5 and Figure 6-6, it could be noticed that the processing time increases almost linearly with respect to a population size increase, whereas the average NPV drops significantly at population size of 500 and maintains a close value afterward.

Thus a value of 500 chromosomes could be recommended as a minimum value to use in order to perform such simulations.

The graph of Figure 6-6 is then converted into a generic convergence plot where the $100 \%$ is set to the best solution among all simulations, and which was found to be: $\$ 38,222.75$ with 5000 chromosomes and 50 iterations. Appendix C contains the full listing of this simulation. In order to create the generic plot, the average NPV for each population size is mapped into a percentage of this best solution by dividing its value with the best solution, giving the data in Table 6-5, as well as the graph of Figure 6-7.

Table 6-5 Percentage of the best solution for different population size

| Population size | \% of the best <br> solution |
| :---: | :---: |
| 100 | $65.6 \%$ |
| 250 | $80.1 \%$ |
| 500 | $92.2 \%$ |
| 1000 | $92.2 \%$ |
| 1500 | $93.4 \%$ |
| 2500 | $96.2 \%$ |
| 5000 | $97.6 \%$ |



Figure 6-7 Generic results convergence with respect to population size

In order to illustrate the variation of the average NPV values obtained, a plot of all the results is traced for population size $100,500,1500$ and 5000. This is shown in Figure 6-8 below.


Figure 6-8 Variation of the NPV results for 10 simulations
It could be noticed that beyond a population size of 500 chromosomes, the results obtained provide a minimum of $92.2 \%$ of the best solution. On the other hand, comparable variation in the solution is found for all population size beyond 500 individuals. This variation is within a range of $-6.1 \%$ and $+5.6 \%$, except for one single simulation where it was $-8.1 \%$. Thus depending on the accuracy of the entry data, any population size beyond 500 would give acceptable results, with a minimal difference in the timing of the major repair interventions. Given the quasi-linear proportion of processing time with respect to the population size, it will be acceptable to use a population size of 500 individuals for this type of simulation, especially given that there are no guarantees that the best solution will be obtained with higher population size. Figure 6-9 presents the plot of the profiles obtained with a population size of 500 and

5000 , and it could be noticed that the main difference is found in the timing of the second repair, whether to perform it at 64 years or at 65 years. Of course, in this case this difference is really insignificant for decision-making, and both results could be considered equal. However the user could decide to go beyond a population size of 500, such as using 5000 individuals, or even using a 100 iterations in order to obtain more precise results, but the processing time becomes very long (more than 16 hours). Also the user could perform a similar type of analysis as the one presented here, but it should always be clear that the results are derived from simulations and will always hold a random behaviour, thus the interpretation of the final results should be performed with care.


Figure 6-9 Comparison between the results obtained with 500 and 5000 chromosomes

In the previous simulations the maximum number of generations was set to 50 . However if this number is increased to 100 , and the population size is set to 500 , then the progress of best NPV obtained after each generation is such as the plot presented in Figure 6-10.

In this figure, the progress of the average NPV of the whole population is plotted, which corresponds to the online performance of the program. Also the plot of the average NPV of the selected chromosomes at each generation is illustrated, which corresponds to the offline performance. These features are previously explained in the "Selection of the parents' chromosomes" in section 3.3.1.2.


Figure 6-10 NPV Results with respect to the number of iterations
It could be seen that the NPV decrease significantly during the first iterations, then the improvement rate diminishes. Actually after 40 iterations little improvements are observed, and consequently the recommended number of iterations would be 50 .

On the other hand the divergence between the best chromosome and the average of the population (the online performance) is calculated as follows:

$$
\text { Population Divergence Rate }=\frac{\left(\mathrm{Pop}_{\text {avg }}-\text { BestChrom }\right)}{\text { Pop }_{\text {avg }}}
$$

And the result is plotted in Figure 6-11 below where the same conclusion could be derived concerning the optimal population size. After 50 iterations, little divergence (4.78\%) is observed between the population average NPV and the best chromosome NPV.


Figure 6-11 Population divergence rate with respect to the number of iterations
In order to derive a generic results graph for the M\&RPPI, the data obtained from the simulation having 100 iterations and 500 chromosomes are mapped similarly as before, where the best result was set to $\$ 38,222.75$, and the remaining NPV are converted into a percentage of this results. The mapping is done by dividing the value of $\$ 38,222.75$, by the NPV obtained at each iteration. This is done for both the best chromosome and for the average of the population.


Figure 6-12 Solution convergence rate with respect to the number of iterations
Also the population convergence rate was calculated and plotted in Figure 6-13 using the following equation:

Population Convergence rate $=100 \%$ - Divergence rate
Equation 6-6


Figure 6-13 Population convergence rate with respect to the number of iterations

The previous results and conclusions are dependant on the input data and the PTM used. Results may vary if cost data or deterioration rates are different. For instance, if the PTM of preventive maintenance is modified as per Table 6-6 and the cost data as per the Table 6-7, then, the final M\&R profile becomes as per the graph shown in Figure 6-14. In this graph, the first major action is shifted forward in time to year 57 and preventive maintenance action are used as temporary remedies for improving the performance and appearance of the infrastructure.

Table 6-6 Alternative TPM for alternative C

|  | Next state |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 5 |
| $\pm$ | 1 | 0.98 | 0.02 | 0.00 | 0.00 | 0.00 |
|  | 2 | 0.00 | 0.95 | 0.05 | 0.00 | 0.00 |
| ] | 3 | 0.00 | 0.20 | 0.70 | 0.10 | 0.00 |
| . | 4 | 0.00 | 0.00 | 0.10 | 0.70 | 0.20 |
| $\bigcirc$ | 5 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 |

Table 6-7 Alternate preventive maintenance cost per station

| Condition State | PM cost per year |
| :---: | :---: |
| 1 | $\$ 5000.00$ |
| 2 | $\$ 7500.00$ |
| 3 | $\$ 15000.00$ |
| 4 | $\$ 30000.00$ |
| 5 | Not Allowed |



Figure 6-14 M\&R results with different input data

### 6.3 Further Analysis and Interpretation of the Results

Following are some observations from the above-presented analysis and results.

- The results obtained for the Metro case study depend on the input data and the objective function set. The input data is based on collected data from the Montreal metro system and on engineering estimates. On the other hand, the objective function is set to minimize the expenditures' NPV and to ensure a minimum level of performance over the analysis period. The final results suggest doing nothing until state (4.5) and then, performing a repair action. This might not; however, offer the best option in terms of aesthetical point of view, thus personal judgment and the problem knowledge should be used when taking the final decision.
- The minimum performance threshold depends on the infrastructure type and importance, the management policy, as well as other safety and economic issues. Also the choice threshold could be selected based on the age of the infrastructure. In the proposed M\&RPPI, the threshold value is set at state 4.5 . For real-life situation, more analysis should be undertaken for determining the proper value to be used. Also, in the Excel computer program, this value could be modified to accommodate a user-input parameter. This feature illustrates the flexibility of the GA over the traditional MDP in allowing more than a single one objective function, and could be a starting point to a multi-objective model.
- Maintenance actions are necessary elements for improving the appearance of infrastructures and prolonging their useful life. Maintenance are also useful as temporary solution and to delay major interventions. In some cases maintenance actions increase overall expenditures over the life-cycle of the infrastructure.
- The do-nothing approach is found a lot in the final solution, most probably because of its "Nil" costs. Even though the infrastructure will degrade during periods where no rehabilitation intervention is provided, nevertheless on a larger scale the funds available for the rehabilitation could be allocated to other segments of the network. This is an interesting point for capital budgeting type of problems. Even though this topic is not in the scope of this research, but it would be an interesting expansion of the present work.
- The effect of crossover probability $p_{c}$ and mutation probability $p_{m}$ are not explicitly analysed, initial values are set in the Excel program to $75 \%$ and $15 \%$ respectively for these two variables. These values are chosen according to several
recommendations from different references such as (Goldberg, 1989a), (Michalewicz, 1996), and (De Jong, 1975). However the fact that the directed approach is used, the effect of the $p_{c}$ and $p_{m}$ becomes secondary in the overall performance of the program.
- The fact that the selection of each chromosome was limited to two times, irrespectively of its fitness, provides at each iteration a broader range of selected individuals. The results show that the involvement percentage is between $65 \%$ and $70 \%$, and that at each generation the involvement percentage is extremely close to the selection fitness percentage (less than $0.5 \%$ difference). This demonstrates that no super-individuals are controlling the population, and diversity is always maintained.
- The planning period could be set for any period that the user might select. This feature does not exist in the traditional Markov decision process method, which demonstrates the flexibility of the proposed method over traditional methods. Nevertheless a longer period provides broader look about the long-term performance of the infrastructure, and it is recommended to select a larger period than the desired one when performing a real simulation for decision purposes.
- It is shown that using a higher population size does not necessarily guarantee a good solution, but does however increase the processing time. Also, a very low population size would not neither achieve acceptable results. A minimal population size of 500 is found to render good results to the problem at hand, and giving the precision required. This figure is higher than the one recommended by Alander (1992) which is between $l(100)$ and $2 l(200)$.

Finally the reader should differentiate between the limitations of the M\&RPPI Excel file and those of the proposed methodology. The developed Excel program aims mainly to prove the workability of the developed methodology, nevertheless it contains some limitations that could be overcome with additional programming efforts.

## 7 CONCLUSIONS AND RECOMMENDATIONS

### 7.1 Summary of the Thesis and Conclusions

The main objectives of this research are successfully achieved by developing a new maintenance and rehabilitation planning method for public infrastructure. The developed method determines the optimal rehabilitation profile over a certain desired analysis period based on life-cycle costing. Also, the developed method implementation to Metro system allowed for better understanding of such infrastructures. The proposed M\&RPPI method could be used for any type of deteriorating element not only for public infrastructures. In general, the following observations are noted:

- M\&R are key elements in order to preserve public infrastructures in their operational and safety performance level. The amount of $\mathrm{M} \& \mathrm{R}$ interventions and their related costs are a function of the deterioration degree of the infrastructure.
- Deterioration models are necessary elements for decision-makers because the determination of cost-effective $\mathrm{M} \& \mathrm{R}$ plans requires information about the current conditions of the infrastructure as well as the predicted future state.
- The Markov chain-based model is presented as one of the most commonly used models in infrastructure deterioration modeling. It is an excellent candidate and an underlying "mathematical tool" for infrastructure decision support system (Lee 2002).
- Using genetic algorithms as an optimization technique has several advantages. In fact, GA are blind in nature and do not have much mathematical requirements regarding optimization problems. Due to their evolutionary nature, GA will search for solutions without regard to the specific inner workings of the problems. GA can therefore handle any kind of objective functions with any kind of constraints. Consequently GA could be adapted to any type of optimization problem.
- GA are efficient and practical optimization techniques for infrastructure management type of problems; they overcome the computational difficulties in using Probabilistic Dynamic Programming (PDP). Whether with the mathematical formulation difficulties or the processing time required for large optimization problems, GA could be a viable and easy solutions. In fact, according to several previous research, Goldberg (1989a), Gen and Cheng (1997), Michalewicz (1996), Pilson (1999), etc., GA could deal with the explosive combinatorial type of problems such as network-level problems.
- The newly developed directed-approach with the concept of "brother-offspring" allows to guide the model toward the final solution, and to take advantage of the search space at each iteration of the optimization process. In addition, the proposed selection method together with the directed GA approach ensure a balance between the exploration (diversity) of the population and exploitation of the solution.
- The outcome of the developed M\&RPPI methodology surpasses the results of the traditional discrete Markov decision process. It provides a rehabilitation profile with the recommended interventions at different period of times during the
analysis period. The traditional MDP method uses a discrete number of condition states and results in an optimal stationary decision policy. The MDP solution is not necessarily the optimal one when it comes to minimizing the expenditures NPV over the life-cycle of the infrastructure.
- The use of Markovian models for the deterioration process is practical from the perspective of applying the M\&RPPI methodology to a wide variety of infrastructures. In fact, GA has been previously used for rehabilitation process, nevertheless they are used to specific types of infrastructures. In each case some deterioration curves should be developed for the infrastructure under study. The use of the Markov chain enables a generic technique for M\&RPPI, where only the TPM are modified in the model for each problem, while maintaining the same methodology.
- The use of multiple TPM overcomes some drawbacks of the traditional MDP. The analysis period could be divided into smaller segments, and the behaviour of the infrastructure could be modeled for each time segment. Thus the critics about the "memory-less" concept of infrastructure are addressed, giving a better timerepresentation of the infrastructure behaviour.
- The use of a continuous condition rating scale provides the methods with a more realistic feature. Also, it gives better results in terms of NPV when compared to the traditional discrete approach. In fact, the condition state vector could represent the portions of the infrastructure that are observed at each state, and provide and average value for the entire infrastructure. The TPM depicts the behaviour of the infrastructure from one period to another.
- The proposed methodology can be applied to specific planning periods, not only for infinite horizon. However, it is recommended to use a larger period for the simulation to better interpret the results for the required period. In this way, a broader view could be achieved for the problem at hand.

In general, this research develops a new approach for infrastructure rehabilitation planning. It constitutes a starting point in developing key concepts for several future works. It mainly allows linking a modified Markov chain approach to genetic algorithm optimization technique.

Nevertheless, an important aspect that should not be neglected in performing any type of planning is the importance of personal judgment, particularly in the interpretation of the results. In fact, the validity of the results of the proposed methodology is dependant on the accuracy of the data gathered, and the degree of uncertainties. The results obtained represent the expected values, which could be used to support the decision maker in his final solution. The results present some random behaviour like any other type of simulations (e.g. Monte Carlo analysis). Thus the comprehension of the problem under investigation is crucial in interpreting the results and taking the final decision.

### 7.2 Contributions of this Research

The novelties of this thesis and the main contributions are as follows:

- Build a novel LCC-based methodology for maintenance and rehabilitation planning of public infrastructure (M\&RPPI) with a probabilistic and a continuous rating approach for condition states.
- Integrate genetic algorithms and "dynamic" Markov chains to find the optimal or quasi-optimal rehabilitation profile over the desired planning period.
- Develop a new directed-GA approach in order to guide the model towards the optimal solution. This consists of a novel concept of "brother-offspring" and a special selection method.
- Develop a computer program using Excel and VBA macros in order to prove the workability of the developed LCC model.
- Implement the developed LCC model to Montreal Metro system where new Metro rehabilitation models are developed using (1) the MDP method, (2) continuous rating approach, and (3) the developed M\&RPPI technique.

In addition some secondary contributions are made as follows:

- Propose a maintenance and rehabilitation diagram (Figure 2-2) in order to describe the different type of rehabilitation that could be performed to public infrastructures.
- Present a new fundamental structure of genetic algorithms (Figure 2-4) in order to explain the genetic algorithm space.
- Study several rehabilitation alternatives for the Metro model, including engineering details. Analyse the different advantages and disadvantages. List and estimate costs elements including indirect costs.


### 7.3 Recommendations for Future Works

Given some limitations of the developed methodology, and time constraints to complete this thesis, several future research and improvements to the method are recommended as follows:

- Extend the developed methodology to address a network-level problem, and examine the problem of resource allocation and capital budgeting at a macro level. The ideas developed in this research, namely the adaptation of the genetic algorithms and the dynamic Markov chain, constitute key elements for such improvements.
- Enhance and optimize the M\&RPPI Excel program, or rewrite the program in a different language. Multiple TPM or higher number of rehabilitation alternatives constitute additional features that could be incorporated.
- Extend the study to consider a multi-objective optimization aspect, such as minimizing the cost, and maximizing the benefits and efficiency of the system. The present work handles both the NPV and a minimum performance threshold. An improvement is therefore to include additional multi-optimization techniques.
- Study the deterioration process of different infrastructures and derive more accurate transition probability matrices. In the case of metro systems, additional research about the deteriorating concrete slabs or tunnels will make the data and the results more accurate.


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## Appendices

Appendix A: Simulation output for 200 Years Planning period using 500 chromosomes and 50 iterations.

## Appendix B: Summary results of all simulations.

Appendix C: Simulation output for a 100 Years Planning period using 5000 chromosomes and 50 iterations.

## APPENDIX A

Simulation output for 200 Years Planning period using 500 chromosomes and 50 iterations

OUTPUT DATA FOR 200 YEARS

|  |  | DDDBB Policy |  |  |  |  |  | GA solution |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year | DoNothing | M\&R <br> Action | Det. Curve |  | Action cost |  | scounted Cost | M\&R Action | Det. Curve |  | Action cost |  | iscounted Cost |
| 0 | 1.00 |  | 1.00 |  |  |  |  |  | 1.00 |  |  |  |  |
| 1 | 1.07 | D | 1.07 | \$ | - | \$ | - | D | 1.07 | \$ | - | \$ | - |
| 2 | 1.14 | D | 1.14 | \$ | - | \$ | - | D | 1.14 | \$ | - | \$ | - |
| 3 | 1.22 | D | 1.22 | \$ | - | \$ | - | D | 1.22 | \$ | - | \$ | - |
| 4 | 1.29 | D | 1.29 | \$ | - | \$ | - | D | 1.29 | \$ | - | \$ | - |
| 5 | 1.37 | D | 1.37 | \$ | - | \$ | - | D | 1.37 | \$ | - | \$ | - |
| 6 | 1.46 | D | 1.46 | \$ | - | \$ | - | D | 1.46 | \$ | - | \$ | - |
| 7 | 1.54 | D | 1.54 | \$ | - | \$ | - | D | 1.54 | \$ | - | \$ | - |
| 8 | 1.63 | D | 1.63 | \$ | - | \$ | - | D | 1.63 | \$ | - | \$ | - |
| 9 | 1.72 | D | 1.72 | \$ | - | \$ | - | D | 1.72 | \$ | - | \$ | - |
| 10 | 1.82 | D | 1.82 | \$ | - | \$ | - | D | 1.82 | \$ | - | \$ | - |
| 11 | 1.92 | D | 1.92 | \$ | - | \$ | - | D | 1.92 | \$ | - | \$ | - |
| 12 | 2.01 | D | 2.01 | \$ | - | \$ | - | D | 2.01 | \$ | - | \$ | - |
| 13 | 2.11 | D | 2.11 | \$ | - | \$ | - | D | 2.11 | \$ | - | \$ | - |
| 14 | 2.22 | D | 2.22 | \$ | - | \$ | - | D | 2.22 | \$ | - | \$ | - |
| 15 | 2.32 | D | 2.32 | \$ | - | \$ | - | D | 2.32 | \$ | - | \$ | - |
| 16 | 2.42 | D | 2.42 | \$ | - | \$ | - | D | 2.42 | \$ | - | \$ | - |
| 17 | 2.52 | D | 2.52 | \$ | - | \$ | - | D | 2.52 | \$ | - | \$ | - |
| 18 | 2.62 | D | 2.62 | \$ | - | \$ | - | D | 2.62 | \$ | - | \$ | - |
| 19 | 2.72 | D | 2.72 | \$ | - | \$ | - | D | 2.72 | \$ | - | \$ | - |
| 20 | 2.82 | D | 2.82 | \$ | - | \$ | - | D | 2.82 | \$ | - | \$ | - |
| 21 | 2.92 | D | 2.92 | \$ | - | \$ | - | D | 2.92 | \$ | - | \$ | - |
| 22 | 3.01 | D | 3.01 | \$ | - | \$ | - | D | 3.01 | \$ | - | \$ | - |
| 23 | 3.10 | D | 3.10 | \$ | - | \$ | - | D | 3.10 | \$ | - | \$ | - |
| 24 | 3.19 | D | 3.19 | \$ | - | \$ | - | D | 3.19 | \$ | - | \$ | - |
| 25 | 3.28 | D | 3.28 | \$ | - | \$ | - | D | 3.28 | \$ | - | \$ | - |
| 26 | 3.37 | D | 3.37 | \$ | - | \$ | - | D | 3.37 | \$ | - | \$ | - |
| 27 | 3.45 | D | 3.45 | \$ | - | \$ | - | D | 3.45 | \$ | - | \$ | - |
| 28 | 3.53 | D | 3.53 | \$ | - | \$ | - | D | 3.53 | \$ | - | \$ | - |
| 29 | 3.60 | D | 3.60 | \$ | - | \$ | - | D | 3.60 | \$ | - | \$ | - |
| 30 | 3.68 | D | 3.68 | \$ | - | \$ | - | D | 3.68 | \$ | - | \$ | - |
| 31 | 3.75 | D | 3.75 | \$ | - | \$ | - | D | 3.75 | \$ | - | \$ | - |
| 32 | 3.82 | D | 3.82 | \$ | - | \$ | - | D | 3.82 | \$ | - | \$ | - |
| 33 | 3.88 | D | 3.88 | \$ | - | \$ | - | D | 3.88 | \$ | - | \$ | - |
| 34 | 3.94 | D | 3.94 | \$ | - | \$ | - | D | 3.94 | \$ | - | \$ | - |
| 35 | 4.00 | D | 4.00 | \$ | - | S | - | D | 4.00 | \$ | - | \$ | - |
| 36 | 4.06 | B | 2.40 | \$ | 225,000.00 | \$ | 40,790.31 | D | 4.06 | \$ | - | \$ | - |
| 37 | 4.11 | D | 2.53 | \$ | - | \$ | - | D | 4.11 | \$ | - | \$ | - |
| 38 | 4.16 | D | 2.67 | \$ | - | \$ | - | D | 4.16 | \$ | - | \$ | - |
| 39 | 4.21 | D | 2.81 | \$ | - | \$ | - | D | 4.21 | \$ | - | \$ | - |

## OUTPUT DATA FOR 200 YEARS

|  |  | DDDBB Policy |  |  |  |  |  | GA solution |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year | DoNothing | $\begin{gathered} \text { M\&R } \\ \text { Action } \end{gathered}$ | Det. Curve | Action cost |  | Discounted Cost |  | M\&R Action | Det. Curve |  | Action cost |  | $\begin{aligned} & \text { counted } \\ & \text { Cost } \end{aligned}$ |
| 40 | 4.26 | D | 2.96 | \$ | - | \$ | - | D | 4.26 | \$ | - | \$ | - |
| 41 | 4.30 | D | 3.09 | \$ | - | \$ | - | D | 4.30 | \$ | - | \$ | - |
| 42 | 4.34 | D | 3.23 | \$ | - | \$ | - | D | 4.34 | \$ | - | \$ | - |
| 43 | 4.38 | D | 3.35 | \$ | - | \$ | - | D | 4.38 | \$ | - | \$ | - |
| 44 | 4.42 | D | 3.46 | \$ | - | \$ | - | D | 4.42 | \$ | - | \$ | - |
| 45 | 4.45 | D | 3.57 | \$ | - | \$ | - | D | 4.45 | \$ | - | \$ | - |
| 46 | 4.49 | D | 3.67 | \$ | - | \$ | - | D | 4.49 | \$ | - | \$ | - |
| 47 | 4.52 | D | 3.76 | \$ | - | \$ | - | D | 4.52 | \$ | - | \$ | - |
| 48 | 4.55 | D | 3.85 | \$ | - | \$ | - | D | 4.55 | \$ | - | \$ | - |
| 49 | 4.58 | D | 3.93 | \$ | - | \$ | - | D | 4.58 | \$ | - | \$ | - |
| 50 | 4.60 | D | 4.00 | \$ | - | \$ | - | D | 4.60 | \$ | - | \$ | - |
| 51 | 4.63 | D | 4.06 | \$ | - | \$ | - | D | 4.63 | \$ | - | \$ | - |
| 52 | 4.65 | B | 2.44 | \$ | 225,000.00 | \$ | 18,686.51 | D | 4.65 | \$ | - | \$ | - |
| 53 | 4.67 | D | 2.56 | \$ | - | \$ | - | C | 4.67 | \$ | 18,000.00 | \$ | 1,423.73 |
| 54 | 4.69 | D | 2.70 | \$ | - | \$ | - | D | 4.69 | \$ | - | \$ | - |
| 55 | 4.71 | D | 2.85 | \$ | - | \$ | - | D | 4.71 | \$ | - | \$ | - |
| 56 | 4.73 | D | 2.99 | \$ | - | \$ | - | D | 4.73 | \$ | - | \$ | - |
| 57 | 4.75 | D | 3.13 | \$ | - | \$ | - | D | 4.74 | \$ | - | \$ | - |
| 58 | 4.77 | D | 3.27 | \$ | - | \$ | - | D | 4.76 | \$ | - | \$ | - |
| 59 | 4.78 | D | 3.39 | \$ | - | \$ | - | D | 4.78 | \$ | - | \$ | - |
| 60 | 4.79 | D | 3.51 | \$ | - | \$ | - | D | 4.79 | \$ | - | \$ | - |
| 61 | 4.81 | D | 3.62 | \$ | - | \$ | - | D | 4.80 | \$ | - | \$ | - |
| 62 | 4.82 | D | 3.72 | \$ | - | \$ | - | D | 4.82 | \$ | - | \$ | - |
| 63 | 4.83 | D | 3.81 | \$ | - | \$ | - | D | 4.83 | \$ | - | \$ | - |
| 64 | 4.84 | D | 3.89 | \$ | - | \$ | - | D | 4.84 | \$ | - | \$ | - |
| 65 | 4.85 | D | 3.97 | \$ | - | \$ | - | D | 4.85 | \$ | - | \$ | - |
| 66 | 4.86 | D | 4.05 | \$ | - | \$ | - | D | 4.86 | \$ | - | \$ | - |
| 67 | 4.87 | B | 2.42 | \$ | 225,000.00 | \$ | 8,988.53 | D | 4.87 | \$ | - | \$ | - |
| 68 | 4.88 | D | 2.54 | \$ | - | \$ | - | D | 4.88 | \$ | - | \$ | - |
| 69 | 4.89 | D | 2.68 | \$ | - | \$ | - | D | 4.89 | \$ | - | \$ | - |
| 70 | 4.90 | D | 2.83 | \$ | - | \$ | - | D | 4.89 | \$ | - | \$ | - |
| 71 | 4.90 | D | 2.97 | \$ | - | \$ | - | D | 4.90 | \$ | - | \$ | - |
| 72 | 4.91 | D | 3.11 | \$ | - | \$ | - | D | 4.91 | \$ | $\cdots$ | \$ | - |
| 73 | 4.92 | D | 3.24 | \$ | \$ - | \$ | - | C | 4.91 | \$ | 18,000.00 | \$ | 536.59 |
| 74 | 4.92 | D | 3.37 | \$ | \$ | \$ | - | D | 4.92 | \$ | \$ - | \$ | - |
| 75 | 4.93 | D | 3.49 | \$ | \$ - | \$ | - | D | 4.92 | \$ | - | \$ |  |
| 76 | 4.93 | D | 3.60 | \$ | \$ | \$ | - | D | 4.93 | \$ | - | \$ | - |
| 77 | 4.94 | D | 3.70 | \$ | 5 - | \$ | - | D | 4.93 | \$ | - - | \$ | - |
| 78 | 4.94 | D | 3.79 | \$ | - | \$ | - | C | 4.94 | \$ | 18,000.00 | \$ | 420.43 |
| 79 | 4.94 | D | 3.88 | \$ | - | \$ | - | D | 4.94 | \$ | - | \$ | - |
| 80 | 4.95 | D | 3.96 |  | \$ | \$ | - | D | 4.94 | \$ | - - | \$ |  |
| 81 | 4.95 | D | 4.03 |  | \$ - | \$ | - | C | 4.95 | \$ | 18,000.00 | \$ | 363.19 |
| 82 | 4.95 | B | 2.41 | \$ | 225,000.00 | \$ | 4,323.64 | c | 4.95 | \$ | 18,000.00 | \$ | 345.89 |

## OUTPUT DATA FOR 200 YEARS

|  |  | DDDBB Policy |  |  |  |  |  | GA solution |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year | Do- <br> Nothing | M\&R Action | Det. Curve | Action cost |  | Discounted Cost |  | $\begin{gathered} \hline \text { M\&R } \\ \text { Action } \end{gathered}$ | Det. Curve | Action cost |  | Discounted Cost |  |
| 83 | 4.96 | D | 2.53 | \$ | \$ | \$ | - | D | 4.95 | \$ | - | \$ | - |
| 84 | 4.96 | D | 2.67 | \$ | \$ | \$ | - | D | 4.96 | \$ | - | \$ | - |
| 85 | 4.96 | D | 2.82 | \$ | \$ | \$ | - | C | 4.96 | \$ | 18,000.00 | \$ | 298.79 |
| 86 | 4.97 | D | 2.96 | \$ | - | \$ | - | D | 4.96 | \$ | - | \$ | - |
| 87 | 4.97 | D | 3.10 | \$ | - | \$ | - | D | 4.96 | \$ | - | \$ | - |
| 88 | 4.97 | D | 3.23 | \$ | \$ | \$ | - | C | 4.97 | \$ | 18,000.00 | \$ | 258.11 |
| 89 | 4.97 | D | 3.36 | \$ | \$ | \$ | - | D | 4.97 | \$ | - | \$ | - |
| 90 | 4.97 | D | 3.47 | \$ | \$ | \$ | - | D | 4.97 | \$ | - | \$ | - |
| 91 | 4.98 | D | 3.58 | \$ | \$ | \$ | - | D | 4.97 | \$ | - | \$ | - |
| 92 | 4.98 | D | 3.68 | \$ | \$ | \$ | - | C | 4.97 | \$ | 18,000.00 | \$ | 212.35 |
| 93 | 4.98 | D | 3.78 | \$ | \$ | \$ | - | D | 4.98 | \$ | - | \$ | - |
| 94 | 4.98 | D | 3.87 | \$ | - | \$ | - | D | 4.98 | \$ | - | \$ | - |
| 95 | 4.98 | D | 3.95 | \$ | - | \$ | - | D | 4.98 | \$ | - | \$ | - |
| 96 | 4.98 | D | 4.02 | \$ | \$ | \$ | - | D | 4.98 | \$ | - | \$ | - |
| 97 | 4.98 | B | 2.40 | \$ | 225,000.00 | \$ | 2,079.74 | D | 4.98 | \$ | - | \$ | - |
| 98 | 4.99 | D | 2.53 | \$ | \$ | \$ | - | D | 4.98 | \$ | - | \$ | - |
| 99 | 4.99 | D | 2.67 | \$ | \$ | \$ | - | C | 4.98 | \$ | 18,000.00 | \$ | 150.91 |
| 100 | 4.99 | D | 2.81 | \$ | \$ | \$ | - | C | 4.98 | \$ | 18,000.00 | \$ | 143.72 |
| 101 | 4.99 | D | 2.95 | \$ | \$ | \$ | - | D | 4.99 | \$ | - | \$ | - |
| 102 | 4.99 | D | 3.09 | \$ | - | \$ | - | D | 4.99 | \$ | - | \$ | - |
| 103 | 4.99 | D | 3.22 | \$ | - | \$ | - | D | 4.99 | \$ | - | \$ | - |
| 104 | 5.00 | D | 3.35 | \$ | \$ | \$ | - | D | 4.99 | \$ |  | \$ | - |
| 105 | 5.00 | D | 3.47 | \$ | \$ | \$ | - | D | 4.99 | \$ | - | \$ | - |
| 106 | 5.00 | D | 3.57 | \$ | \$ | \$ | - | D | 4.99 | \$ | - | \$ | - |
| 107 | 5.00 | D | 3.68 | \$ | \$ | \$ | - | B | 2.99 | \$ | 225,000.00 | \$ | 1,276.78 |
| 108 | 5.00 | D | 3.77 | \$ | \$ | \$ | - | D | 3.14 | \$ | - | \$ | - |
| 109 | 5.00 | D | 3.86 | \$ | \$ | \$ | - | D | 3.32 | \$ | - | \$ | - |
| 110 | 5.00 | D | 3.94 | \$ | - | \$ | - | D | 3.49 | \$ | - | \$ | - |
| 111 | 5.00 | D | 4.01 | \$ | - | \$ | - | C | 3.61 | \$ | 16,000.00 | \$ | 74.70 |
| 112 | 5.00 | B | 2.40 | \$ | 225,000.00 | \$ | 1,000.39 | C | 3.72 | \$ | 16,000.00 | \$ | 71.14 |
| 113 | 5.00 | D | 2.52 | \$ | \$ | \$ | - | C | 3.82 | \$ | 16,000.00 | \$ | 67.75 |
| 114 | 5.00 | D | 2.66 | \$ | \$ | \$ | - | D | 3.97 | \$ | - | \$ | - |
| 115 | 5.00 | D | 2.80 | \$ | \$ | \$ | - | C | 4.06 | \$ | 16,000.00 | \$ | 61.45 |
| 116 | 5.00 | D | 2.95 | \$ | \$ | \$ | - | D | 4.19 | \$ |  | \$ | - |
| 117 | 5.00 | D | 3.09 | \$ | \$ | \$ | - | D | 4.30 | \$ | - | \$ | - |
| 118 | 5.00 | D | 3.22 | \$ | - | \$ | - | D | 4.39 | \$ | - | \$ | - |
| 119 | 5.00 | D | 3.34 | \$ | - | \$ | - | C | 4.45 | \$ | 18,000.00 | \$ | 56.88 |
| 120 | 5.00 | D | 3.46 |  | \$ | \$ | - | D | 4.53 | \$ | - | \$ | - |
| 121 | 5.00 | D | 3.57 |  | \$ | \$ | - | C | 4.57 | \$ | 18,000.00 | \$ | 51.59 |
| 122 | 5.00 | D | 3.67 | \$ | \$ | \$ | - | c | 4.61 | \$ | 18,000.00 | \$ | 49.13 |

## OUTPUT DATA FOR 200 YEARS

|  |  | DDDBB Policy |  |  |  |  |  | GA solution |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year | DoNothing | M\&R <br> Action | Det. Curve | Action cost |  | $\begin{gathered} \hline \text { Discounted } \\ \text { Cost } \\ \hline \end{gathered}$ |  | M\&R Action | Det. Curve | Action cost |  | Discounted Cost |  |
| 123 | 5.00 | D | 3.76 | \$ | - | \$ | - | D | 4.67 | \$ | - | \$ | - |
| 124 | 5.00 | D | 3.85 | \$ | - | \$ | - | C | 4.70 | \$ | 18,000.00 | \$ | 44.56 |
| 125 | 5.00 | D | 3.93 | \$ | - - | \$ | - | D | 4.74 | \$ | - | \$ | - |
| 126 | 5.00 | D | 4.01 | \$ | - | \$ | - | D | 4.78 | \$ | - | \$ | - |
| 127 | 5.00 | B | 2.40 | \$ | 225,000.00 | \$ | 481.21 | C | 4.80 | \$ | 18,000.00 | \$ | 38.50 |
| 128 | 5.00 | D | 2.52 | \$ | - - | \$ | - | C | 4.82 | \$ | 18,000.00 | \$ | 36.66 |
| 129 | 5.00 | D | 2.66 | \$ | - | \$ | - | D | 4.85 | \$ | - | \$ | - |
| 130 | 5.00 | D | 2.80 | \$ | \$ - | \$ | - | D | 4.87 | \$ | - | \$ | - |
| 131 | 5.00 | D | 2.94 | \$ | - - | \$ | - | D | 4.89 | \$ | - | \$ | - |
| 132 | 5.00 | D | 3.08 | \$ | \$ - | \$ | - | C | 4.90 | \$ | 18,000.00 | \$ | 30.16 |
| 133 | 5.00 | D | 3.21 | \$ | - | \$ | - | D | 4.91 | \$ | - | \$ | - |
| 134 | 5.00 | D | 3.34 | \$ | - | \$ | - | C | 4.92 | \$ | 18,000.00 | \$ | 27.36 |
| 135 | 5.00 | D | 3.46 | \$ | - | \$ | - | C | 4.93 | \$ | 18,000.00 | \$ | 26.06 |
| 136 | 5.00 | D | 3.56 | \$ | \$ - | \$ | - | C | 4.94 | \$ | 18,000.00 | \$ | 24.82 |
| 137 | 5.00 | D | 3.67 | \$ | \$ - | \$ | - | D | 4.95 | \$ | - | \$ | - |
| 138 | 5.00 | D | 3.76 | \$ | - - | \$ | - | C | 4.95 | \$ | 18,000.00 | \$ | 22.51 |
| 139 | 5.00 | D | 3.85 | \$ | \$ - | \$ | - | c | 4.96 | \$ | 18,000.00 | \$ | 21.44 |
| 140 | 5.00 | D | 3.93 | \$ | - | \$ | - | C | 4.96 | \$ | 18,000.00 | \$ | 20.42 |
| 141 | 5.00 | D | 4.00 | \$ | - | \$ | - | D | 4.97 | \$ | - | \$ | - |
| 142 | 5.00 | B | 2.39 | \$ | 225,000.00 | \$ | 231.47 | C | 4.97 | \$ | 18,000.00 | \$ | 18.52 |
| 143 | 5.00 | D | 2.52 | \$ | - | \$ | - | C | 4.97 | \$ | 18,000.00 | \$ | 17.64 |
| 144 | 5.00 | D | 2.65 | \$ | - | \$ | - | C | 4.97 | \$ | 18,000.00 | \$ | 16.80 |
| 145 | 5.00 | D | 2.80 | \$ | - | \$ | - | C | 4.98 | \$ | 18,000.00 | \$ | 16.00 |
| 146 | 5.00 | D | 2.94 | \$ | - | \$ | - | c | 4.98 | \$ | 18,000.00 | \$ | 15.23 |
| 147 | 5.00 | D | 3.08 | \$ | \$ - | \$ | - | D | 4.98 | \$ | - | \$ |  |
| 148 | 5.00 | D | 3.21 | \$ | S | \$ | - | D | 4.98 | \$ | - | \$ | - |
| 149 | 5.00 | D | 3.34 | \$ | - | \$ | - | C | 4.99 | \$ | 18,000.00 | \$ | 13.16 |
| 150 | 5.00 | D | 3.45 | \$ | - | \$ | - | D | 4.99 | \$ | - | \$ | - |
| 151 | 5.00 | D | 3.56 | \$ | - | \$ | - | C | 4.99 | \$ | 18,000.00 | \$ | 11.94 |
| 152 | 5.00 | D | 3.66 | \$ | \$ | \$ | - | C | 4.99 | \$ | 18,000.00 | \$ | 11.37 |
| 153 | 5.00 | D | 3.76 | \$ | \$ - | \$ | - | B | 2.99 | \$ | 225,000.00 | \$ | 135.33 |
| 154 | 5.00 | D | 3.84 | \$ | \$ - | \$ | - | D | 3.14 | \$ | - | \$ |  |
| 155 | 5.00 | D | 3.93 | \$ | \$ | \$ | - | C | 3.26 | \$ | 16,000.00 | \$ | 8.73 |
| 156 | 5.00 | D | 4.00 | \$ | \$ | \$ | - | D | 3.43 | \$ | - | \$ | - |
| 157 | 5.00 | D | 4.07 | \$ | \$ | \$ | - | D | 3.61 | \$ | - | \$ | - |
| 158 | 5.00 | B | 2.43 | \$ | 225,000.00 | \$ | 106.04 | D | 3.77 | \$ | - | \$ | - |
| 159 | 5.00 | D | 2.56 | \$ | - | \$ | - | C | 3.88 | \$ | 16,000.00 | \$ | 7.18 |
| 160 | 5.00 | D | 2.70 | \$ | - | \$ | - | C | 3.97 | \$ | 16,000.00 | \$ | 6.84 |
| 161 | 5.00 | D | 2.84 | \$ | - | \$ | - | D | 4.11 | \$ | - | \$ | - |
| 162 | 5.00 | D | 2.99 | \$ | - | \$ | - | C | 4.19 | \$ | 18,000.00 | \$ | 6.98 |
| 163 | 5.00 | D | 3.13 | \$ | - | \$ | - | D | 4.30 | \$ | - | \$ | - |
| 164 | 5.00 | D | 3.26 | \$ | - | \$ | - | D | 4.39 | \$ | - | \$ | - |

## OUTPUT DATA FOR 200 YEARS

|  |  | DDDBB Policy |  |  |  |  |  | GA solution |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year | DoNothing | M\&R Action | Det. Curve | Action cost |  | Discounted Cost |  | M\&R Action | Det. Curve | Action cost |  | $\begin{gathered} \hline \text { Discounted } \\ \text { Cost } \\ \hline \end{gathered}$ |  |
| 165 | 5.00 | D | 3.39 | \$ | - | \$ | \$ | C | 4.45 | \$ | 18,000.00 | \$ | 6.03 |
| 166 | 5.00 | D | 3.50 | \$ | \$ - | \$ | \$ | c | 4.50 | \$ | 18,000.00 | \$ | 5.74 |
| 167 | 5.00 | D | 3.61 | \$ | \$ - | \$ | \$ | D | 4.57 | \$ | - | \$ | - |
| 168 | 5.00 | D | 3.71 | \$ | - | \$ | \$ | C | 4.61 | \$ | 18,000.00 | \$ | 5.21 |
| 169 | 5.00 | D | 3.81 | \$ | \$ - | \$ | \$ | D | 4.67 | \$ | - | \$ | - |
| 170 | 5.00 | D | 3.89 | \$ | \$ - | \$ | \$ | D | 4.72 | \$ | - | \$ | - |
| 171 | 5.00 | D | 3.97 | \$ | \$ | \$ | \$ | B | 2.81 | \$ | 225,000.00 | \$ | 56.23 |
| 172 | 5.00 | D | 4.05 | \$ | \$ | \$ | \$ | D | 2.95 | \$ | - | \$ | - |
| 173 | 5.00 | B | 2.42 | \$ | \$ 225,000.00 | \$ | \$ 51.01 | C | 3.06 | \$ | 12,000.00 | \$ | 2.72 |
| 174 | 5.00 | D | 2.54 | \$ | \$ |  | \$ | C | 3.17 | \$ | 16,000.00 | \$ | 3.45 |
| 175 | 5.00 | D | 2.68 | \$ | \$ |  | \$ | c | 3.28 | \$ | 16,000.00 | \$ | 3.29 |
| 176 | 5.00 | D | 2.83 | \$ | \$ |  | \$ | C | 3.39 | \$ | 16,000.00 | \$ | 3.13 |
| 177 | 5.00 | D | 2.97 | \$ | \$ |  | \$ | D | 3.55 | \$ | - | \$ | - |
| 178 | 5.00 | D | 3.11 | \$ | \$ - |  | \$ | C | 3.65 | \$ | 16,000.00 | \$ | 2.84 |
| 179 | 5.00 | D | 3.24 | \$ | \$ |  | \$ | C | 3.74 | \$ | 16,000.00 | \$ | 2.71 |
| 180 | 5.00 | D | 3.37 | \$ | \$ - |  | \$ | C | 3.83 | \$ | 16,000.00 | \$ | 2.58 |
| 181 | 5.00 | D | 3.49 | \$ | \$ |  | \$ | C | 3.91 | \$ | 16,000.00 | \$ | 2.45 |
| 182 | 5.00 | D | 3.60 | \$ | \$ |  | \$ | D | 4.03 | \$ | - | \$ | - |
| 183 | 5.00 | D | 3.70 | \$ | \$ |  | \$ | C | 4.10 | \$ | 18,000.00 | \$ | 2.51 |
| 184 | 5.00 | D | 3.79 | \$ | \$ | \$ | \$ | c | 4.17 | \$ | 18,000.00 | \$ | 2.39 |
| 185 | 5.00 | D | 3.88 | \$ | \$ |  | \$ | c | 4.23 | \$ | 18,000.00 | \$ | 2.27 |
| 186 | 5.00 | D | 3.96 | \$ | \$ |  | \$ | C | 4.29 | \$ | 18,000.00 | \$ | 2.16 |
| 187 | 5.00 | D | 4.03 | \$ | \$ |  | \$ | D | 4.37 | \$ | - | \$ | - |
| 188 | 5.00 | B | 2.41 | \$ | \$ 225,000.00 |  | \$ 24.53 | C | 4.41 | \$ | 18,000.00 | \$ | 1.96 |
| 189 | 5.00 | D | 2.53 | \$ | \$ |  | \$ | C | 4.46 | \$ | 18,000.00 | \$ | 1.87 |
| 190 | 5.00 | D | 2.67 | \$ | \$ |  | \$ | D | 4.52 | \$ | - | \$ | - |
| 191 | 5.00 | D | 2.82 | \$ | \$ |  | \$ | D | 4.57 | \$ | - | \$ | - |
| 192 | 5.00 | D | 2.96 |  | \$ |  | \$ | D | 4.62 | \$ | - | \$ | - |
| 193 | 5.00 | D | 3.10 |  | \$ |  | \$ | D | 4.66 | \$ | - | \$ | - |
| 194 | 5.00 | D | 3.23 |  | \$ |  | \$ | D | 4.70 | \$ | - | \$ | - |
| 195 | 5.00 | D | 3.36 |  | \$ |  | \$ | C | 4.72 | \$ | 18,000.00 | \$ | 1.39 |
| 196 | 5.00 | D | 3.47 |  | \$ |  | \$ | D | 4.75 | \$ | - | \$ | - |
| 197 | 5.00 | D | 3.58 |  | \$ |  | \$ | D | 4.78 | \$ | - | \$ | - |
| 198 | 5.00 | D | 3.68 |  | \$ |  | \$ | D | 4.81 | \$ | - | \$ | - |
| 199 | 5.00 | D | 3.78 |  | \$ |  | \$ | C | 4.82 | \$ | 18,000.00 | \$ | 1.15 |
| 200 | 5.00 | D | 3.86 | \$ | \$ |  | \$ | D | 4.84 | \$ | - | \$ | - |
|  |  |  | Total |  | \$ 2,475,000.00 |  | \$ 76,763.39 |  |  |  | \$ 1,685,000.00 | \$ | 6,553.42 |

## APPENDIX B

Summary results of all simulations

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Population size Average time




## APPENDIX C

Simulation output for a 100 Years Planning period using 5000 chromosomes and 50 iterations

## OUTPUT DATA

Planning period: $\quad 100$ years
Number of individulas: 5000 chromosomes
Number of iterations: 100

| GA solution |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Year | M\&R Action | Det. Curve |  | Action cost | Discounted Cost |
| 0 |  | 1.00 |  |  |  |
| 1 | D | 1.07 | \$ | - | \$ |
| 2 | D | 1.14 | \$ | - | \$ |
| 3 | D | 1.22 | \$ | - | \$ |
| 4 | D | 1.29 | \$ | - | \$ |
| 5 | D | 1.37 | \$ | - | \$ |
| 6 | D | 1.46 | \$ | - | \$ |
| 7 | D | 1.54 | \$ | - | \$ |
| 8 | D | 1.63 | \$ | - | \$ |
| 9 | D | 1.72 | \$ | - | \$ |
| 10 | D | 1.82 | \$ | - | \$ |
| 11 | D | 1.92 | \$ | - | \$ |
| 12 | D | 2.01 | \$ | - | \$ |
| 13 | D | 2.11 | \$ | - | \$ |
| 14 | D | 2.22 | \$ | - | \$ |
| 15 | D | 2.32 | \$ | - | \$ |
| 16 | D | 2.42 | \$ | - | \$ |
| 17 | D | 2.52 | \$ | - | \$ |
| 18 | D | 2.62 | \$ | - | \$ |
| 19 | D | 2.72 | \$ | - | \$ |
| 20 | D | 2.82 | \$ | - | \$ |
| 21 | D | 2.92 | \$ | - | \$ |
| 22 | D | 3.01 | \$ | - | \$ |
| 23 | D | 3.10 | \$ | - | \$ |
| 24 | D | 3.19 | \$ | - | \$ |
| 25 | D | 3.28 | \$ | - | \$ |
| 26 | D | 3.37 | \$ | - | \$ |
| 27 | D | 3.45 | \$ | - | \$ |
| 28 | D | 3.53 | \$ | - | \$ |
| 29 | D | 3.60 | \$ | - | \$ |
| 30 | D | 3.68 | \$ | - | \$ |
| 31 | D | 3.75 | \$ | - | \$ |
| 32 | D | 3.82 | \$ | - | \$ |
| 33 | D | 3.88 | \$ | - | \$ |
| 34 | D | 3.94 | \$ | - | \$ |
| 35 | D | 4.00 | \$ | - | \$ |
| 36 | D | 4.06 | \$ | - | \$ |

## OUTPUT DATA

Planning period: 100 years
Number of individulas: 5000 chromosomes
Number of iterations: 100

| GA solution |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year | M\&R Action | Det. Curve |  | Action cost |  | Inted Cost |
| 37 | D | 4.11 | \$ | - | \$ | - |
| 38 | D | 4.16 | \$ | - | \$ | - |
| 39 | D | 4.21 | \$ | - | \$ | - |
| 40 | D | 4.26 | \$ | - | \$ | - |
| 41 | D | 4.30 | \$ | - | \$ | - |
| 42 | D | 4.34 | \$ | - | \$ | - |
| 43 | D | 4.38 | \$ | - | \$ | - |
| 44 | D | 4.42 | \$ | - | \$ | - |
| 45 | D | 4.45 | \$ | - | \$ | - |
| 46 | D | 4.49 | \$ | - | \$ | - |
| 47 | B | 2.69 | \$ | 225,000.00 | \$ | 23,849.25 |
| 48 | D | 2.83 | \$ | - | \$ | - |
| 49 | D | 2.98 | \$ | - | \$ | - |
| 50 | D | 3.14 | \$ | - | \$ | - |
| 51 | D | 3.30 | \$ | - | \$ | - |
| 52 | D | 3.45 | \$ | - | \$ | - |
| 53 | D | 3.59 | \$ | - | \$ | - |
| 54 | D | 3.72 | \$ | - | \$ | - |
| 55 | D | 3.84 | \$ | - | \$ | - |
| 56 | D | 3.95 | \$ | - | \$ | - |
| 57 | D | 4.04 | \$ | - | \$ | - |
| 58 | D | 4.13 | \$ | - | \$ | - |
| 59 | D | 4.21 | \$ | - | \$ | - |
| 60 | D | 4.28 | \$ | - | \$ | - |
| 61 | D | 4.34 | \$ | - | \$ | - |
| 62 | D | 4.39 | \$ | - | \$ | - |
| 63 | D | 4.45 | \$ | - | \$ | - |
| 64 | D | 4.49 | \$ | - | \$ | - |
| 65 | B | 2.69 | \$ | 225,000.00 | \$ | 9,909.86 |
| 66 | D | 2.82 | \$ | - | \$ | - |
| 67 | D | 2.98 | \$ | - | \$ | - |
| 68 | D | 3.14 | \$ | - | \$ | - |
| 69 | D | 3.29 | \$ | - | \$ | - |
| 70 | D | 3.44 | \$ | - | \$ | - |
| 71 | D | 3.58 | \$ | - | \$ | - |
| 72 | D | 3.71 | \$ | - | \$ | - |
| 73 | D | 3.83 | \$ | - | \$ | - |
| 74 | D | 3.94 | \$ | - | \$ | - |
| 75 | D | 4.04 | \$ | - | \$ | - |
| 76 | D | 4.13 | \$ | - | \$ | - |

## OUTPUT DATA

Planning period: 100 years
Number of individulas: 5000 chromosomes
Number of iterations: 100

| GA solution |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | ---: |
| Year | M\&R Action | Det. Curve | Action cost | Discounted Cost |  |  |
| 77 | D | 4.21 | $\$$ | - | $\$$ | - |
| 78 | D | 4.28 | $\$$ | - | $\$$ | - |
| 79 | D | 4.34 | $\$$ | - | $\$$ | - |
| 80 | D | 4.40 | $\$$ | - | $\$$ | - |
| 81 | D | 4.45 | $\$$ | - | $\$$ | - |
| 82 | C | 4.49 | $\$$ | $18,000.00$ | $\$$ | 345.89 |
| 83 | B | 2.68 | $\$$ | $225,000.00$ | $\$$ | $4,117.75$ |
| 84 | D | 2.82 | $\$$ | - | $\$$ | - |
| 85 | D | 2.97 | $\$$ | - | $\$$ | - |
| 86 | D | 3.13 | $\$$ | - | $\$$ | - |
| 87 | D | 3.28 | $\$$ | - | $\$$ | - |
| 88 | D | 3.43 | $\$$ | - | $\$$ | - |
| 89 | D | 3.58 | $\$$ | - | $\$$ | - |
| 90 | D | 3.71 | $\$$ | - | $\$$ | - |
| 91 | D | 3.83 | $\$$ | - | $\$$ | - |
| 92 | D | 3.93 | $\$$ | - | $\$$ | - |
| 93 | D | 4.03 | $\$$ | - | $\$$ | - |
| 94 | D | 4.12 | $\$$ | - | $\$$ | - |
| 95 | D | 4.20 | $\$$ | - | $\$$ | - |
| 96 | D | 4.27 | $\$$ | - | $\$$ | - |
| 97 | D | 4.34 | $\$$ | - | $\$$ | - |
| 98 | D | 4.40 | $\$$ | - | $\$$ | - |
| 99 | D | 4.45 | $\$$ | - | $\$$ | - |
| 100 | D | 4.50 | $\$$ | - | $\$$ | - |
|  |  |  | $\$ 693,000.00$ | $\$$ | $38,222.75$ |  |

