Inspection Plan for Dependent Multi-Characteristic Components with Multi-Classifications

Jian Xiong Li

A Thesis

in

The Department

of

Mechanical and Industrial Engineering

Presented in Partial Fulfillment of the Requirements

for the Degree of Master of Applied Science (Industrial Engineering) at

Concordia University

Montreal, Quebec, Canada

April 2012

CONCORDIA UNIVERSITY

School of Graduate Studies

This is to certify that the thesis prepared

By: Jian Xiong Li

Entitled: "Inspection Plan for Dependent Multi-Characteristic Components with Multi-Classifications"

and submitted in partial fulfillment of the requirements for the degree of

Master of Applied Science (Industrial Engineering)

complies with the regulations of the University and meets the accepted standards with respect to originality and quality.

Signed by the final examining committee:

Dr. H. D. Ng Chair

Dr. Ivan Conteras Examiner

Dr. Anjali Awasthi Examiner

Dr. Mingyuan Chen Supervisor

Approved by Dr. W. Ahmed
Chair of Department or Graduate Program Director

Robin A.L. Drew
Dean of Faculty

Date April 10, 2012

Abstract

Inspection Plan for Dependent Multi-Characteristic Components with Multi-Classifications

Jian Xiong Li

In this research, a mathematical model is developed for inspecting multicharacteristic components with multi-classifications in a multistage production system. The characteristics' defective rates are statistically dependent. The output of the model is an optimized inspection plan. The plan minimizes the total cost per accepted component. A heuristic algorithm is proposed in solving the problem with optimized solutions. The developed model and proposed heuristic algorithm are demonstrated using an example from a medical equipment manufacturing system. The data used in the example are realistic but hypothetical. The model can be modified for solving similar problems in other applications.

Keywords: Quality improvement, inspection error, multistage production system, inspection plan.

Acknowledgements

I would like to express my sincere gratitude to my supervisor, Dr. Mingyuan Chen, for his encouragement, invaluable guidance and suggestions through all phases of my graduate studies and thesis preparation without his support the completion of this dissertation would not have been possible.

I would like to thank the faculty of Engineering and Computer Science, Concordia University, for the financial assistance to my research.

I would like to thank the members of the faculty and the staff of the Department of Mechanical and Industrial Engineering, Concordia University for their assistances throughout the course studies.

Finally, I would like to thank my wife and friends for their supports and encouragements throughout these years.

Table of Contents

List of Figures

List of Tables

Chapter One

Introduction

1.1 Motivation

Along with the popularity of Total Quality Management (TQM) philosophy, quality control and improvement of products and processes throughout organizations have become essential and indispensable in organizational strategic planning in various industries [23, 33 and 49]. Numerous tools have been developed and adopted in the past decades for product and process quality improvement. For example, robust design, cost of quality, Statistical Process Control (SPC), inspection strategy design, Six Sigma, etc., have been widely used [2, 5, 15, 20, 22, 25, 27, 30, 32, 37, 38, 42, 45, 51 and 52]. To survive and succeed in today's competitive environment, organizations often use combinations of these tools to achieve organizational objectives of profit and total customer satisfaction. The inspection oriented quality assurance strategy design, as one important category of the inspection strategy design, has been accepted as an effective solution methodology for achieving these objectives. This strategy design is to minimize the total system cost by optimizing the inspection parameters such as the number of inspection repetitions, inspection sequence and the allocation of inspections in order to ensure that customers receive high quality products [35, 39 and 46].

As a common practice in modern industries, a multistage system provides great opportunities for quality improvement and cost reduction. More specifically, in a multistage production system, the final products are manufactured with multicharacteristic components through the processing at multiple stations or stages. In such a system, inspection of the multicharacteristic products is broadly accepted and considered as necessary to be repeated in multiple times. One reason is that inspection is not perfect. Rejecting a conforming component or product by fault (Type I error) or failure to reject a defective component by fault (Type II error) can happen in practice and both errors bring in costs. Nonconforming products received by customers may cause injury or loss of life and bring in much higher cost in the manner of rework, penalty, judicial action and the loss of potential customers. Product inspections are performed in multiple times to reduce such errors. Inspecting different characteristics in different stations may cost differently. Inspection in an earlier station may cost much less than that in a later station due to cost accumulation along with the processing of products. To perform inspection in station with high defect rate first may contribute to more cost savings than that at a low defect rate station first.

Therefore, developing and applying economic models to improve the profit by minimizing the total cost per accepted component are very important. The total cost may include the costs associated with the two types of inspection error and the inspection cost. An optimized inspection plan with optimized solutions of inspection frequency and sequence will be essential for quality improvement to improve the average quality level (AQL) in a multistage production system.

1.2 Research Background

This research studies the problem of inspection frequency and inspection sequence optimization in a multistage production system. We extend the earlier work in Duffuaa and Nadeem (1994), which in turn, is an extension of the model in Raouf et al. (1983). In Duffuaa and Nadeem (1994), defective rates of the considered characteristics are assumed statistically dependent. Two inspection classifications, accept or reject, are assumed in the model in minimizing total expected cost per accepted component due to Type I and Type II error as well as the inspection cost. Another related work in Duffuaa and Khan (2002) also extends the model in Raouf et al. (1983) considering three inspection classifications, accept, reject and rework. However, the defective rates of the characteristics were assumed statistically independent. In this thesis, we extend these two earlier works by developing a new cost model. We consider the problem of multicharacteristic component inspection with three inspection classifications and dependent defective rates.

1.3 Objectives and Scope of the Thesis

This research is to design an inspection plan with optimized inspection frequency and inspection sequence so that the total cost per accepted component will be minimized. To avoid unnecessary complexity and redundant constraints in developing the model, the costs associated with Type I and Type II errors as well as the inspection are considered in the total cost formula. Including other costs may not significantly change the outcome of the model developed in this research.

The developed model applies to multistage production system with multicharacteristic components for inspection in a sequential manner. The characteristics' defective rates are assumed statistically dependent. The rework station is assumed as error-free. Inspection may be repeated more than once to improve the outgoing quality of final products. Inspection will be stopped when the minimized total cost per accepted product is achieved.

1.4 Research Methodology

In this research, we start with the development of a new mathematical model for the design of an inspection plan to be implemented in a multistage production system with multicharacteristic component inspection requirement. It is followed by developing a heuristic method to solve the model.

1.5 Contributions of the Thesis

This research generalizes and extends the earlier works of Duffuaa and Nadeem (1994) and Duffuaa and Khan (2002) in that a cycling inspection of the multicharacteristic components with three inspection classifications and defective rates are statistically dependent. The problem to optimize the cycling inspection frequency and inspection sequence is solved by solving the proposed new model. A heuristic approach is introduced in this research to efficiently solve the model. The main contributions of the research are two-folds:

- Design an inspection plan for the inspection of components with dependent defective rates with three inspection classifications,
- Implement a heuristic algorithm as the solution approach to efficiently solve problems of practical size.

1.6 Organization of the Thesis

Research literature in the field of multistage inspection plan optimization is reviewed in Chapter Two. Chapter Three presents the problem description and the formulated mathematical model along with the heuristic algorithm. One numerical example problem is solved and presented in Chapter Four with extensive analysis and discussions. Conclusions and future research directions in this area are discussed in Chapter Five.

Chapter Two

Literature Review

The problem of multistage system inspection strategy design with minimized overall cost has been studied by many researchers. Raouf et al. (1983) introduced a model to determine the optimal inspection sequence and inspection frequency for multicharacteristic components to minimize the total expected cost per accepted component due to Type I & Type II error costs and inspection cost. In their model, the inspection result was classified into two categories, accept and reject, and the defective rates of characteristics were assumed statistically independent. Duffuaa and Khan (2002) extended the model of Raouf et al. (1983) in that the inspection results were classified into three categories, accept, reject and rework.

Many researchers have discussed the issue of inspection strategy design in multistage systems. This chapter reviews a few of these works in this area. A summary of the reviewed articles is shown in Table 2.1.

Table 2.1: Classification of Literature

Heuristic and Meta-Heuristics	Ben-Daya and Rahim (2003), Duffuaa and Khan (2002), Duffuaa and Nadeem (1994), Duffuaa and Najjar (1997), Emmons and Rabinowitz (2002), Greenshtein and Rabinowitz (1997), Heredia-Langner et al. (2002), Kogan and Raz (2002), Lee and Unnikrishnan (1998), Mohib et al. (2009), Raouf et al. (1983), Rabinowitz and Emmons (1997), Rau et al. (2005), Shiau (2002), Yeh et al. (1999)
Dynamic Programming	Chen (1998), Chun (2010), Elshafei et al. (2006)
Other Methods	Chen and Lambrecht (1997), Chun (2009), Duffuaa and Khan (2005), Maleyeff et al. (2003), Veatch (2000)

2.1 Heuristic and Meta-Heuristics

Raouf et al. (1983): The authors introduced a model for multicharacteristic component repeated inspection with economic considerations. They considered a sequential and cycling inspection plan where inspection result for each characteristic was classified as accept or reject. The economical consideration was to minimize the total cost which included the cost of falsely rejecting the acceptable component and falsely accepting the rejected component and the inspection cost per final accepted component. They developed a computational procedure to solve the model. The optimal inspection sequence was to first inspect the characteristic having lowest ratio of inspection cost over rejection rate.

Duffuaa and Khan (2002): This paper was an extension of the earlier work done by Raouf et al. (1983). The authors developed a new mathematical model for multicharacteristic critical component inspection optimization. They considered 3 classifications of products, accept, reject and rework. This leads to 6 categories of misclassification probabilities. They assumed the rework station was error-free and the characteristics defective rates were statistically independent. A computational procedure to determine optimal inspection sequence and the number of inspection cycles was developed to minimize total quality related cost per final accepted component including misclassification costs and inspection cost.

Duffuaa and Nadeem (1994): This article is an extension of the earlier work done by Raouf et al. (1983). In this article, the authors developed a mathematical model for multicharacteristic critical component inspection optimization considering that the defective rates were statistically dependent. Similar as the assumption made in defective rate independent case, the joint probability mass function was given as known information. The inspection was sequential and cycling. The marginal mass function of non-inspected characteristics needed to be updated after the inspection of each characteristic due to their statistical dependency. A computational procedure was developed to search for optimal inspection frequency and inspection cycles.

Duffuaa and Al-Najjar (1997): In this article, the authors studied the multi-characteristic critical component inspection optimization problem. A model was developed to decentralize the inspection frequency performed on each characteristic based on different failure rates and inspection costs. The objective is to find optimal repeated inspection frequency on each characteristic so that total expected cost per accepted component would be minimized. The total cost includes Type I and Type II error costs as well as inspection cost. The defective rates between characteristics are assumed statistically independent. The steepest decent technique was used to solve the problem and to determine the optimal inspection plan.

Ben-Daya and Rahim (2003): In this article, the authors considered quality improvement and economical production quantity (EPQ) optimization problem in multistage production systems. They built a model for multistage process. The screening inspection of nonconforming products produced when the process is out-of-control can reduce total cost. This total cost was defined as the sum of the inventory cost, quality related cost and inspection error cost as well as inspection and restoration cost. They developed a pattern search technique to escape from the local optimal solution in searching for the global optimal solution. It demonstrated the contribution of inspection and restoration to the total cost.

Emmons and Rabinowitz (2002): The authors discussed an inspection scheduling problem in multistage production system. Solving the problem is to decide (1) the number of required inspection facility, (2) the assignment of the inspection facility and (3) the schedule of the inspection tasks. These decisions are mutually exclusive. The objective of the model is to minimize the work load imbalance related to these decisions. A heuristic solution procedure was developed to solve the problem.

Greenshtein and Rabinowitz (1997): The authors studied the problem for multi-attribute product inspection optimization. They presented applications of statistical-economical tools to achieve the objective of minimizing inspection cost and misclassification cost. A double-stage inspection system was proposed. They developed a stepwise algorithm based on multi-variable normal distribution and used conditional probability to decide which product needs to be inspected in the second stage after collecting data from the first-stage. The proposed system was evaluated in a real application. It concluded with good capability of identifying and classifying highly suspected components in the first stage so that the cost in the second stage can be reduced.

Heredia-Langner et al. (2002): The authors discussed a partial inspection option, such as rectifying inspection in a multistage inspection system. Binomial distribution was used in the developed model and the objective was to determine the optimum sample size and the threshold number since inspection is proportional to the average number of inspected items. They developed a procedure based on genetic algorithm to solve the multistage partial inspection problem.

Kogan and Raz (2002): In this article, the authors considered multistage multicharacteristic component inspection problem in continuous time. They used maximum principle and obtained several analytical results in identifying optimal inspection conditions, inspection sequence, optimal inspection timing between models, concurrent change over regime and consecutive change over regimes. They developed a computational method to solve the problem in minimizing inspection and penalty costs.

Lee and Unnikrishnan (1998): The authors developed a mathematical model for inspection station allocation and assignment in a multistage production system. They used inspection time as a constraint to the objective function which was to minimize the total cost. The considered problem has large combinations of different allocations. The authors developed 3 heuristic solution methods based on sequential plan selection method (SPS), time constrained solution method (TCS) and manufacturing cost and nonconforming probability selection method (CNS). The heuristic methods can find near optimal solution with less execution time and less computer memory required.

Mohib et al. (2009): The authors proposed a hybrid inspection plan for multicharacteristic component inspection. The component may have different geometric characteristics so that both contact inspection and non-contact inspection are applied. They used Travelling Salesman Problem (TSP) formulation to decide the optimal inspection sequence of the hybrid inspection tasks. The objective is to minimize total inspection time and cost.

Rabinowitz and Emmons (1997): In this article, the authors discussed the problem of scheduling multiple inspection tasks in a single inspection station. The inspection facility was used for detecting the processes malfunction rather than screening defective parts. A two-stage system model was developed to maximize the number of good items produced by the production system. The authors used 5 heuristic methods to find feasible solutions of the considered problem.

Rau et al. (2005): The authors studied multicharacteristic component inspection allocation problem in a re-entrant production system. They classified inspection results into accept, reject, repair and rework in their quality characteristic measurement model. The developed model is to maximize the total production profit. A heuristic method was developed to solve the problem.

Shiau (2002): The author discussed the inspection allocation problem in multistage production systems. The considered problem has finite inspection resources subject to inspection errors. An optimal inspection plan should be developed to respond quickly when a customer changes the tolerance requirements. The objective is to minimize the total manufacturing cost. The author used two heuristic methods, earliest stage assignment method and hybrid weighting assignment method, to solve the problem.

Yeh et al. (1999): The authors studied specific multicharacteristic component inspection problem considering carryover defect between characteristics. Due to carryover defect, the inspection of all characteristics in one component may not be realized every time and the inspection needs to be continued for the subsequent characteristic inspection with a different component. The authors proposed an inspection plan to minimize the required inspection. They used heuristic methods to solve the considered problem. The model was further extended to include minimum inspection time as an addition objective. The optimal inspection plan compromised solution considering these two objectives.

2.2 Dynamic Programming

Chen (1998): The author discussed inspection allocation problems in multistage production systems. The author used the cost of detecting and discarding a defective item as the intermediate objective function in the solution process. The author proposed two models, optimal allocation for defective penalty cost model and optimal allocation for a specified AOQ Level. The author used dynamic programming to solve the problem.

Chun (2010): The author studied problem of determining optimal inspection interval and stopping rules in a serial production system subject to random failure. The author used Bayesian model to estimate defect rate. A renewal-reward process model was introduced considering inspection cost, non-defective product market value and the salvage value of a discarded product. Stochastic dynamic programming was used to solve the problem.

Elshafei et al. (2006): The authors introduced a model to determine inspection sequence for multicharacteristic component inspection with repeated inspection on each characteristic. They classified the inspection results into three categories: accept, scrap and rework. The defective rate among each characteristic was assumed statistically independent. The objective is to minimize inspection error costs and inspection cost per accepted component. They developed an efficient beam search dynamic programming algorithm to solve the model. The solution is an inspection plan.

2.3 Other Methods

Chen and Lambrecht (1997): The authors developed a model to solve multicharacteristics inspection problems. The objective function of the model is to maximize final profit by reducing all inspection costs. The model was solved using branch-and-bound algorithm.

Chun (2009): The author used Bayesian model for sequential inspection plan and consider that inspections were positively related. Prior planning and posterior planning were considered. The author used the number of undetected non-conforming items and probability of undetected faults to make decision whether or not to perform another cycle of inspection for both planning stages. They used this approach to solve the problem.

Duffuaa and Khan (2005): The authors discussed economic aspects of inspecting multicharacteristic components. The inspection may have inspection errors related to average outgoing quality (AOQ), average total inspection (ATI) and expected total cost per accepted component (ETC). They developed a mathematical model to determine inspection plans where inspection results were classified into accept, reject and rework. Various inspection errors were studied in the paper. They also used sensitivity analysis to study the impact of inspection errors on inspection performances.

Maleyeff et al. (2003): The authors proposed a cost model for multicharacteric product inspection. The objective is to minimize cost by determining the number of required inspection characteristics.

Veatch (2000): The author developed a quality cost model for multistage inspection considering variable defect rate for different characteristics. Cost per accepted component was to be minimized. The author proposed to integrate quality and product configuration data in generating cost report.

2.4 Summary

Different models and solution methods for solving inspection optimization problems in multistage production systems have been developed by many researchers and practitioners. They include various heuristic and meta-heuristic methods in addition to many other solution methods based on dynamic programming.

In the next chapter, a mathematical model is presented to decide inspection plans for multicharacteristic components with dependent defective rates. This is an extension of the research in Duffuaa and Khan (2002) and Duffuaa and Nadeem (1994). The inspection result classifications are accept, reject and rework. A heuristic algorithm is developed to find optimal or near-optimal solution of the model.

Chapter Three

Model Formulation and Solution Approach

This chapter presents detailed problem description in this research and discusses the development of mathematical model for the considered problem. The following topics are organized to be presented in this chapter.

- Detailed description of the inspection frequency and sequence optimization problem
- Assumptions in the considered problem
- Notation
- Development of the mathematical model
- Heuristic solution approach

3.1 Problem Introduction

The considered problem is to establish an optimized inspection plan in a multi-stage production system by determining two inspection parameters: the cycling inspection frequency and the inspection sequence. The objective is to minimize the total cost per final accepted component. In the considered production system, inspection is conducted in stages corresponding to quality characteristics. Probability to detect the defect at each stage, probability of inspection errors at each stage, cost of inspection and cost of inspection errors are associated with the inspection process.

The multi-stage product inspection process is presented in Figure 3.1. The figure is adopted from that in Duffuaa and Khan (2002) assuming that there are a number of components with *N* characteristics of each component entering the inspection stations. Each inspection station performs the inspection one characteristic of the product or component. An inspector inspects one particular characteristic for each component entering the inspection process, and after inspection, classifies them into three categories: accept, reject and rework. All the accepted (good) components go to the second inspector. Those classified as "rework" will be sent to the rework station. They will be reworked on the inspected characteristic and become accepted for that characteristic. They will then be sent to the second inspector who inspects the second characteristic. This chain of inspection continues until all the characteristics are inspected. This completes one cycle of inspection. All accepted components, if necessary, go to the next cycle of inspection. This process will repeat *n* times before it stops where *n* is the optimized number of inspections to minimize the total expected cost per accepted component. In this research, the inspection process is the same as described in Duffuaa and Khan (2002). At each station, the components are 100% inspected subject to 6 types of inspection errors. These errors are associated with that the inspector falsely classifies:

- good components as rework,
- good components as scrap,
- rework components as good,

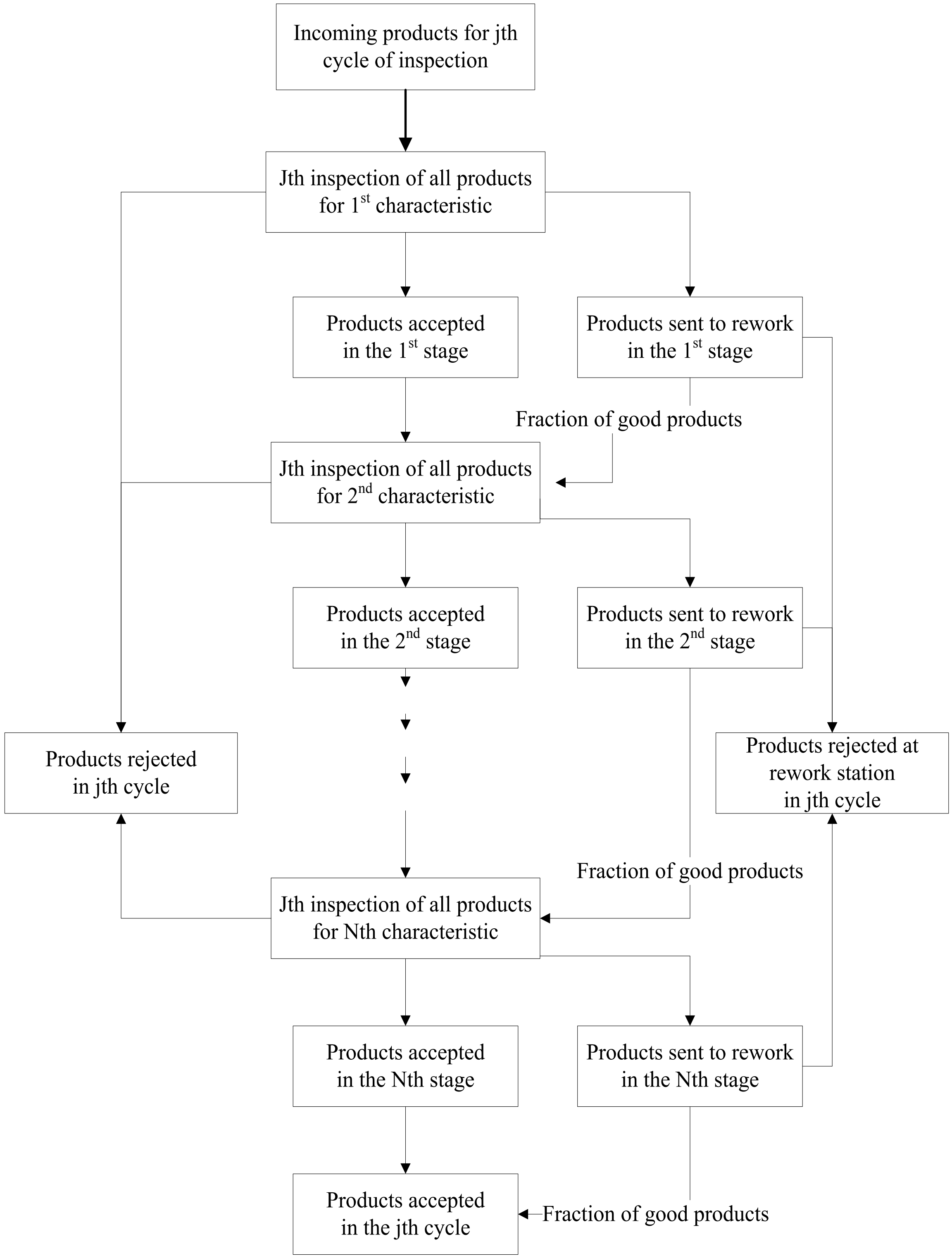

Figure 3.1: Multi-stage Product Inspection Process (Duffuaa and Khan,2002)

- rework components as scrap,
- scrap components as good and
- scrap components as rework.

These false classifications create two types of costs: false rejection cost and false acceptance cost. These two costs and the inspection cost will be considered as the expected total cost of inspection which is to be minimized in solving the considered problem.

3.2 Assumptions in the Considered Problem

1. The multi-stage system consists of N inspection stations, each station is to inspect one characteristic and the inspection is performed in sequence.
2. 100% inspection of component is assumed for each inspection station.
3. A component is accepted if all of its characteristics are accepted as good, A component is rejected if one of its characteristics is classified as defective, A to-be-reworked component is reworkable in the rework station.
4. All inspections have errors in inspecting each characteristic.
5. The rework station is assumed as error free.
6. The joint probability mass function of the multivariate random variable $X = (X_1, X_2, \ldots X_{\mathrm{N}})$ is assumed to be known or can be estimated empirically.
7. The probability of inspection errors of each characteristic is known.
8. Cost due to falsely rejecting a non-defective component, cost due to falsely accepting a defective component and the inspection cost per component are assumed to be known.

A mathematical model is developed to determine inspection frequency and inspection sequence in order to minimize the total expected cost per final accepted component. Before the mathematical model is presented, we first give the notation used in the mathematical model.

3.3 Notation

$N =$ Number of characteristics to be inspected for each component.

$n =$ Number of inspection cycles.

$M_{i,j} =$ Number of components entering the i-th stage of the j-th cycle of inspection.

$i = 1,2,\ldots,N. \quad j = 1,2,\ldots,N.$

X_i: a discrete random variable, $X_i = \begin{cases} 0, & \text{if characteristic } i \text{ is scrap,} \\ 0.5, & \text{if characteristic } i \text{ is rework,} \\ 1, & \text{if characteristic } i \text{ is good.} \end{cases}$

${}^{j}P(X_1, X_2, \ldots X_N) =$ Joint probability mass function of X_i for a component.

entering the j-th cycle of inspection.

${}^{k,j}P\ (X_1, X_2 \ldots, X_N) =$ Joint probability mass function of the random variable X_i for

a component entering the k-th stage of the j-th cycle of inspection.

${}^{j}P_i(X_i) =$ Marginal probability mass function of the i-th characteristic of the random

variable X_i, while entering the j-th cycle of inspection.

${}^{j}PG =$ Probability of a component being good entering the j-th cycle of inspection.

${}^{j}PS =$ Probability of a component being scrap entering the j-th cycle of inspection.

${}^{j}PR =$ Probability of a component being rework entering the j-th cycle of inspection.

$E_{igs} =$ Probability of classifying i-th good characteristic as scrap with inspection.

$E_{igr} =$ Probability of classifying i-th good characteristic as rework with inspection.

$E_{isg} =$ Probability of classifying i-th scrap characteristic as good with inspection.

$E_{isr} =$ Probability of classifying i-th scrap characteristic as rework with inspection.

$E_{irg} =$ Probability of classifying i-th rework characteristic as good with inspection.

$E_{irs} =$ Probability of classifying i-th rework characteristic as scrap with inspection.

$FGR_{i,j} =$ Number of falsely sending good components to rework station at the i-th stage of the j-th cycle of inspection.

$FS_{i,j} =$ Number of falsely send to scrap in i-th stage of j-th cycle of inspection.

$FA_{i,j} =$ Number of falsely accepted components in the i-th stage of the j-th cycle of inspection.

$CA_{i,j} =$ Number of correctly accepted components in the i-th stage of the j-th cycle of inspection.

$R_{i,j} =$ Rate of rejection of components due to i-th characteristic entering the j-th cycle of inspection.

$R_{i,j,k} =$ Rate of rejection of components due to i-th characteristic entering the k-th stage of the j-th cycle of inspection.

$C_a =$ Cost of false acceptance of a scrap or rework component.

$C_r =$ Cost of false rejection of a good or rework component.

$C_i =$ Cost of inspection of a component.

$A\ (j) =$ Number of accepted components in j-th cycle.

$CFR\ (j) =$ Cost of false rejection in the j-th cycle.

$CFA\ (j) =$ Cost of false acceptance in the j-th cycle.

$CI\ (j) =$ Cost of inspection in the j-th cycle.

$TCFR =$ Total cost of false rejection.

$TCFA =$ Total cost of false acceptance.

TCI = Total cost of inspection.

TA = Total number of accepted components.

$EXP(tc)|j$ = Expected total cost per accepted component after j-th cycles of inspection.

$P(\,)$ = Probability.

3.4 Development of the Model

This model is developed for inspecting components among which the defective rates of characteristics are statistically dependent. We first establish certain important relations between different variables.

The probability ${}^{j}P_{\mathrm{i}}(X_{\mathrm{i}})$

Since the joint probability mass function (jpmf) of the random variable $X_i, i = 1,2,\ldots,N$ is considered known at the 1st stage of inspection, we may therefore obtain the individual marginal probability mass function (mpmf) based on the following equation.

$$P_{\mathrm{i}}(X_{\mathrm{i}}) = \sum_{x1} \sum_{x2} \sum_{x3} \ldots \sum_{xi-1} \sum_{xi+1} \ldots \sum_{xN} P(X_1, X_2, \ldots, X_N) \tag{1}$$

Since the characteristics are assumed as statistically dependent, the joint and marginal mass functions need to be updated for inspections among characteristics according to Bayes' Theorem.

The probability of a component goes into the next stage of inspection is:

$$P(\,) = P_{\mathrm{i}}(1)(1 - E_{\mathrm{igr}} - E_{\mathrm{igs}}) + P_{\mathrm{i}}(1)E_{\mathrm{igr}} + P_{\mathrm{i}}(0.5)E_{\mathrm{irg}} + P_{\mathrm{i}}(0)E_{\mathrm{isg}}$$

or $$P(\,) = P_{\mathrm{i}}(1)(1 - E_{\mathrm{igs}}) + P_{\mathrm{i}}(0.5)E_{\mathrm{irg}} + P_{\mathrm{i}}(0)E_{\mathrm{isg}}$$

The marginal probability mass function for the $i\text{-}th$ characteristic of cycle 2 being good is:

$$^{2}P_{\mathrm{i}}(1) = \frac{^{1}P_{\mathrm{i}}(1)(1-E_{\mathrm{igs}})}{^{1}P_{\mathrm{i}}(1)(1-E_{\mathrm{igs}}) + {}^{1}P_{\mathrm{i}}(0.5)E_{\mathrm{irg}} + {}^{1}P_{\mathrm{i}}(0)E_{\mathrm{isg}}} \tag{2}$$

The marginal probability mass function for the $i\text{-}th$ characteristic of cycle 2 being rework is:

$$^{2}P_{\mathrm{i}}(0.5) = \frac{^{1}P_{\mathrm{i}}(0.5)E_{\mathrm{irg}}}{^{1}P_{\mathrm{i}}(1)(1-E_{\mathrm{igs}}) + {}^{1}P_{\mathrm{i}}(0.5)E_{\mathrm{irg}} + {}^{1}P_{\mathrm{i}}(0)E_{\mathrm{isg}}} \tag{3}$$

The marginal probability mass function for the $i\text{-}th$ characteristic of cycle 2 being scrap is:

$$^{2}P_{\mathrm{i}}(0) = \frac{^{1}P_{\mathrm{i}}(0)E_{\mathrm{isg}}}{^{1}P_{\mathrm{i}}(1)(1-E_{\mathrm{igs}}) + {}^{1}P_{\mathrm{i}}(0.5)E_{\mathrm{irg}} + {}^{1}P_{\mathrm{i}}(0)E_{\mathrm{isg}}} \tag{4}$$

In general, the marginal probability mass function for $i\text{-}th$ characteristic of $j\text{-}th$ cycle being good can be written as:

$$^{\mathrm{j}}P_{\mathrm{i}}(1) = \frac{^{\mathrm{j-1}}P_{\mathrm{i}}(1)(1-E_{\mathrm{igs}})}{^{\mathrm{j-1}}P_{\mathrm{i}}(1)(1-E_{\mathrm{igs}}) + {}^{\mathrm{j-1}}P_{\mathrm{i}}(0.5)E_{\mathrm{irg}} + {}^{\mathrm{j-1}}P_{\mathrm{i}}(0)E_{\mathrm{isg}}} \tag{5}$$

The marginal probability mass function for $i\text{-}th$ characteristic of $j\text{-}th$ cycle being rework is:

$$^{\mathrm{j}}P_{\mathrm{i}}(0.5) = \frac{^{\mathrm{j-1}}P_{\mathrm{i}}(0.5)E_{\mathrm{irg}}}{^{\mathrm{j-1}}P_{\mathrm{i}}(1)(1-E_{\mathrm{igs}}) + {}^{\mathrm{j-1}}P_{\mathrm{i}}(0.5)E_{\mathrm{irg}} + {}^{\mathrm{j-1}}P_{\mathrm{i}}(0)E_{\mathrm{isg}}} \tag{6}$$

The marginal probability mass function for $i\text{-}th$ characteristic of $j\text{-}th$ cycle being scrap is:

$$^{\mathrm{j}}P_{\mathrm{i}}(0) = \frac{^{\mathrm{j-1}}P_{\mathrm{i}}(0)E_{\mathrm{isg}}}{^{\mathrm{j-1}}P_{\mathrm{i}}(1)(1-E_{\mathrm{igs}}) + {}^{\mathrm{j-1}}P_{\mathrm{i}}(0.5)E_{\mathrm{irg}} + {}^{\mathrm{j-1}}P_{\mathrm{i}}(0)E_{\mathrm{isg}}} \tag{7}$$

Based on the Eqs.(5), (6) and (7), the marginal probability mass function of the $i\text{-}th$ characteristic in the $j\text{-}th$ cycle of inspection can be obtained after the completion of the

$(j\text{-}1)th$ cycle of inspection. The marginal probability mass function of other characteristics must be updated prior to inspecting them. Eq. (1) shows that the mpmf can be obtained based on the jpmf. Therefore the jpmf must be updated first after the completion of the inspection of the $i\text{-}th$ characteristic. This update can be accomplished by:

$$^{\mathrm{k,j}}P(X_1, X_2 \ldots, X_{\mathrm{N}}) = {}^{\mathrm{k\text{-}1,j}}P(X_1, X_2 \ldots, X_{\mathrm{N}}) \frac{^{\mathrm{k,j}}P_{\mathrm{i}}(X_{\mathrm{i}})}{^{\mathrm{k\text{-}1,j}}P_{\mathrm{i}}(X_{\mathrm{i}})} \tag{8}$$

In the inspection process for dependent characteristics defective rates, the joint probability mass function for the next stage will be updated according to Eq. (8) after the $k\text{-}th$ stage of the $j\text{-}th$ cycle of inspection. The independent marginal probability mass function can be obtained using Eq. (1). It proceeds to the $(i\text{+}1)th$ stage of the $j\text{-}th$ cycle of inspection until it completed with all characteristics inspected.

<u>The Probability $PG_{\mathrm{N,j}}$</u>

The probability of a good component entering the N stage of the $j\text{-}th$ cycle is

$$PG_{\mathrm{N,j}} = {}^{\mathrm{N,j}}P(1,1,\ldots,1) \tag{9}$$

The expected total cost per accepted component after the $n\text{-}th$ cycle of inspection is

$$EXP(tc)|_{\mathrm{j=n}} = [TCFR(n) + TCFA(n) + TCI(n)] / \, TA(n) \tag{10}$$

When there is no inspection, the expected total cost per accepted component is the cost due to falsely accepting all defective components

$$EXP(tc)|_{\mathrm{j=0}} = C_{\mathrm{a}} \, (1 - PG) \tag{11}$$

The objective is to determine the optimized number of inspection cycles, n, so that the total expected cost per accepted component in Eq. (10) can be minimized. Therefore, we need to decide the inspection cycle and the value of $TCFR, TCFA, TCI$ and TA.

3.4.1 Analysis of the *j*-th Cycle of Inspection

Stage 1

Let M_{j} be the number of components entering the j-th cycle of inspection

$$M_{1,j} = M_{j} \tag{12}$$

Probabilities of a component being good, rework or scrap are

$$PG_{1,j} = PG_{N,j-1} = P(1, 1, \ldots, 1) \tag{13}$$

$$PR_{1,j} = {}^{N,\,j-1}PR$$

$$PS_{1,j} = {}^{N,j-1}PS$$

$FGR_{1,j}$, the number of good components falsely sent to rework in the j-th cycle of inspection, can be calculated by:

$$FGR_{1,j} = M_{1,j}\,PG_{1,j}{}^{N,j-1}P_1(1)E_{1gr} \tag{14}$$

$FS_{1,j}$, the number of components falsely sent to scrap in the j-th cycle of inspection, is calculated by:

$$FS_{1,j} = M_{1,j}\,(PG_{1,j}{}^{N,j-1}P_1(1)E_{1gs} + PR_{1,j}{}^{N,j-1}P_1(0.5)E_{1rs}) \tag{15}$$

$FA_{1,j}$, the number of components falsely accepted in the j-th cycle of inspection, is calculated by:

$$FA_{1,j} = M_{1,j}\,(PR_{1,j}{}^{N,j-1}P_1(0.5)E_{1rg} + PS_{1,j}{}^{N,j-1}P_1(0)E_{1sg}) \tag{16}$$

$CA_{1,j}$, the number of correctly accepted components, is calculated by:

$$CA_{1,j} = M_{1,j}\, PG_{1,j}\, {}^{N,j-1}P_1(1)\,(1 - E_{1gr} - E_{1gs}) \tag{17}$$

All accepted components in this stage proceed to the next stage to be inspected on other characteristics.

Stage 2

$$M_{2,j} = CA_{1,j} + FA_{1,j} + FGR_{1,j} \tag{18}$$

$$PG_{2,j} = {}^{2,j}P\,(1, 1, \dots, 1) \tag{19}$$

$$PR_{2,j} = {}^{2,j}PR$$

$$PS_{2,j} = {}^{2,j}PS$$

$$FGR_{2,j} = M_{2,j}\, PG_{2,j}\, {}^{2,j}P_2(1)E_{2gr} \tag{20}$$

$$FS_{2,j} = M_{2,j}\,(PG_{2,j}\, {}^{2,j}P_2(1)E_{2gs} + PR_{2,j}\, {}^{2,j}P_2(0.5)E_{2rs}) \tag{21}$$

$$FA_{2,j} = M_{2,j}\,(PR_{2,j}\, {}^{2,j}P_2(0.5)E_{2rg} + PS_{2,j}\, {}^{2,j}P_2(0)E_{2sg}) \tag{22}$$

$$CA_{2,j} = M_{2,j}\, PG_{2,j}\, {}^{2,j}P_2(1)\,(1 - E_{2gr} - E_{2gs}) \tag{23}$$

•
•
•

Stage N

$$M_{N,j} = CA_{N-1,j} + FA_{N-1,j} + FGR_{N-1,j} \tag{24}$$

$$PG_{N,j} = {}^{N,j}P\,(1, 1, \dots, 1) \tag{25}$$

$$PR_{N,j} = {}^{N,j}PR$$

$$PS_{N,j} = {}^{N,j}PS$$

$$FGR_{N,j} = M_{N,j}\, PG_{N,j}\, {}^{N,j}P_N(1)E_{Ngr} \tag{26}$$

$$FS_{N,j} = M_{N,j}\,(PG_{N,j}{}^{N,j}P_N(1)E_{Ngs} + PR_{N,j}{}^{N,j}P_N(0.5)E_{Nrs}) \tag{27}$$

$$FA_{N,j} = M_{N,j}\,(PR_{N,j}{}^{N,j}P_N(0.5)E_{Nrg} + PS_{N,j}{}^{N,j}P_N(0)E_{Nsg}) \tag{28}$$

$$CA_{N,j} = M_{N,j}\,PG_{N,j}{}^{N,j}P_N(1)\,(1 - E_{Ngr} - E_{Ngs}) \tag{29}$$

If the components will be inspected in the next cycle, then the number of components entering the first stage of $(j+1)th$ cycle is

$$M_{1,j+1} = CA_{N,j} + FA_{N,j} + FGR_{N,j} \tag{30}$$

The cost of falsely sending components to scrap in the $j\text{-}th$ cycle is

$$CFR\,(j) = C_r \sum_{i=1}^{N} FS_{i,j} \tag{31}$$

The cost of falsely accepting the components in the $j\text{-}th$ cycle is

$$CFA\,(j) = C_a \sum_{i=1}^{N} FA_{i,j} \tag{32}$$

The cost of inspection in the $j\text{-}th$ cycle is

$$CI\,(j) = C_i \sum_{i=1}^{N} M_{i,j} \tag{33}$$

3.4.2 Minimizing the Cost of Inspection in the *j*-th Cycle of Inspection

The cost of inspection in rejecting different characteristics can be different and defective rates of characteristics may vary from one to another. Inspection sequence may affect the total expected costs and thus needs to be considered in order to minimize the total expected cost. In general, we may want to first inspect those components associated with minimum inspection cost and highest defective rate to reduce overall cost. In this research, we propose two sequence rules.

Rule 1. At the beginning of cycle j, we calculate

$C_{\mathrm{i}} / R_{\mathrm{i,j}}$ where $R_{\mathrm{i,j}} = {}^{\mathrm{i,j\text{-}1}}Pi(0)(1 - E_{\mathrm{isg}} - E_{\mathrm{isr}}) + (1 - {}^{\mathrm{i,j\text{-}1}}Pi(0))(E_{\mathrm{igs}} + E_{\mathrm{irs}})$ (34)

The characteristics i with the lowest ratio of $C_{\mathrm{i}} / R_{\mathrm{i,j}}$ will be inspected first.

Rule 2. At the $k\text{-}th$ stage of $j\text{-}th$ cycle, we calculate

$C_{\mathrm{i}} / R_{\mathrm{i,j,k}}$ where $R_{\mathrm{i,j,k}} = {}^{\mathrm{k\text{-}1,j}}Pi(0)(1 - E_{\mathrm{isg}} - E_{\mathrm{isr}}) + (1 - {}^{\mathrm{k\text{-}1,j}}Pi(0))(E_{\mathrm{igs}} + E_{\mathrm{irs}})$(35)

The characteristics with the lowest ratio of $C_{\mathrm{i}} / R_{\mathrm{i,j,k}}$ will be inspected first.

3.4.3 Expected Total Cost per Accepted Component

Based on the analysis of the $j\text{-}th$ cycle of inspection presented in section 3.4.1, we can calculate $EXP(tc)|_{\mathrm{j=n}}$, the total expected cost per accepted component after n cycles of inspection shown below.

$$TCFR\ (n) = \sum_{j=1}^{n} CFR(j) \tag{36}$$

$$TCFA\ (n) = CFA(n) \tag{37}$$

$$TCI\ (n) = \sum_{j=1}^{n}\ \sum_{i=1}^{N} M_{\mathrm{i,j}} \tag{38}$$

$$TA(n) = CA_{\mathrm{N,n}} + FA_{\mathrm{N,n}} + FGR_{\mathrm{N,n}} \tag{39}$$

We use the algorithm presented in the next section to determine the optimized number of inspections.

3.5 Heuristic Solution Approach

A simple algorithm is developed to calculate the probabilities and cost functions in the model discussed in Section 3.4. This algorithm as discussed previously progressively computes the variable values of the equations. The computing template is created in Excel 2010. The program requires to input the number of components (M), the inspection errors probability ($E_{igs}, E_{igr}, E_{isg}, E_{isr}, E_{irg}, E_{irs}$), the unit costs ($Ca, Cr, Ci$) and the joint probability $P(X_1, X_2, X_3)$. It will generate inspection cycle with minimized total inspection costs as discussed previously. The steps of the algorithm are given below.

Step 1: Calculate $EXP(tc)|_{j=0}$ using Eq. (11). Set j=1.

Step 2: Calculate ${}^{j}P_i(1), {}^{j}P_i(0.5), {}^{j}P_i(0)$ for i=1,2,..,N using Eq.(5) ~ (7). Select the $i\text{-}th$ characteristic which has the lowest ratio in Eq. (35), and continue the inspection until all the characteristics have been inspected.

Update ${}^{k,j}P(X_1, X_2 \dots, X_N)$ for each stage k of cycle j using Eq. (8) and calculate the marginal probability ${}^{k,j}P_i(X_i)$.

Step 3: Calculate $FGR_{i,j}, FS_{i,j}, FA_{i,j}, CA_{i,j}, M_{i,j}$ using Eqs. (26) ~ (30) for each i.

Step 4: Calculate $CFR\ (1), CFA\ (1),\ CI\ (1)$ using Eqs. (31) ~ (33).

Step 5: Calculate $EXP(tc)|_j$ using Eq.(10), set j=j+1.

Step 6: If $EXP(tc)|_j < EXP(tc)|_j$, Then go to step 2; if not, n=j-1.

Step 7: Calculate $TCFR\ (n), TCFA\ (n), TCI\ (n), TA(n)$ using Eqs. (36)~ (39).

Step 8: Search n with the minimum total expected cost per accepted unit $EXP(tc)|_j$.

We present several example problems to illustrate and validate the developed model in the next chapter.

Chapter Four

Numerical Example and Analysis

This chapter presents a numerical example to illustrate the developed mathematical model and the heuristic solution method presented in the previous chapter. The purpose of this research is to design an inspection plan by determining inspection sequence and inspection cycles in order to minimize total cost per accepted product. The example problem is based on the inspection practice in a medical device manufacturing company. The data used in the example are realistic but hypothetical. The inspection process is illustrated in Figure 4.1. The products are processed by the first manufacturing station. They will be inspected repeatedly through a 3-stage inspection. Accepted products then proceed to the next manufacturing station. The products may be inspected again after they are processed by the second manufacturing station.

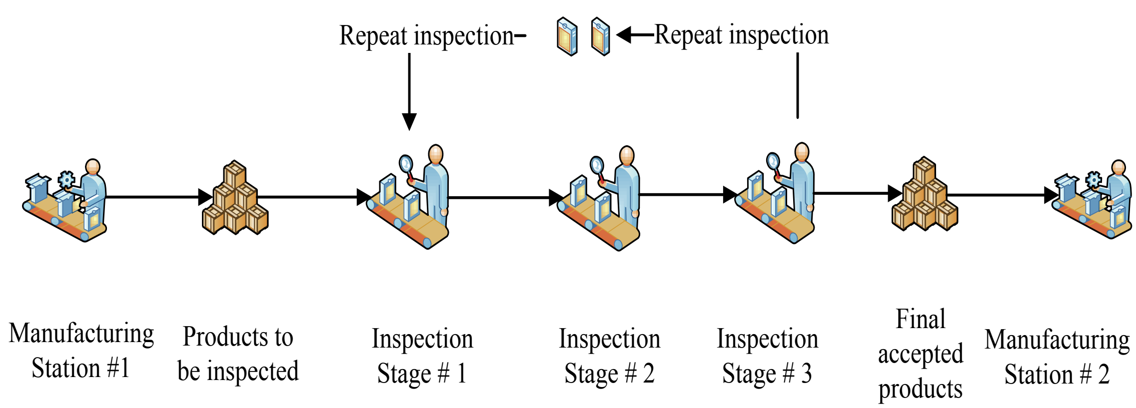

Figure 4.1 Multicharacteristic Products Inspection Process

The calculations of the model with different combinations of the parameter values were performed following the systematic design of experiment (DOE) approach. The details will be presented next.

4.1 Example Problem

The manufacturing system considered in this study produced different types of products. Since these products follow similar manufacturing and inspection processes, the parameters used in the model are for different products except the ranges of their values may vary. The developed model can be applied for different product types. The ranges of the parameters values are presented in Table 4.1. Due to the fact that the products are for medical use, falsely accepting a non-conforming product may cause very serious health or even safety problems, the cost of failure to reject non-conforming products is considered very high. This consideration is reflected in the ranges of such cost values presented in Table 4.1.

Table 4.1: Data Range of Inspection Process Parameters

Parameter	Description	Min	Max
E_{gs}	Probability of false classification of good product as scrap	0.03	0.08
E_{gr}	Probability of false classification of good product as rework	0.001	0.005
E_{sg}	Probability of false classification of scrap product as good	0.05	0.15
E_{sr}	Probability of false classification of scrap product as rework	0.001	0.005
E_{rg}	Probability of false classification of rework product as good	0.05	0.1
E_{rs}	Probability of false classification of rework product as scrap	0.02	0.05
Ca	Cost of false acceptance of a scrap or rework product	200000	1000000
Cr	Cost of false rejection of a good or rework product	4000	6000
Ci	Cost of inspection a product	150	300

Other data for the considered problem are given in Table 4.2. Inspection cost is the cost incurred for one product at k-th stage of j-th cycle of inspection.

If there is no inspection, the expected total cost per accepted product will be the cost due to false acceptance of all defective products. In this example, expected total cost per accepted product without inspection assumed to be 80,000.

Table 4.2: General Data of 3-Stages Inspection Problem

Number of inspection stations (N)	3
Number of products to be inspected (M)	200
Probability of false classification of good product as scrap (Egs)	0.05
Probability of false classification of good product as rework (Egr)	0.001
Probability of false classification of scrap product as good (Esg)	0.1
Probability of false classification of scrap product as rework (Esr)	0.002
Probability of false classification of rework product as good (Erg)	0.06
Probability of false classification of rework product as scrap (Ers)	0.02
Cost of false acceptance of a scrap or rework product (Ca)	200000
Cost of false rejection of a good or rework product (Cr)	5000
Cost of inspection a product (Ci)	200

4.1.1 Determine Inspection Sequence

Following the solution procedure explained previously, we first inspect those products with lowest inspection cost and highest rejection rate. The ratio of inspection cost to the defective rate is used to determine the inspection sequence. The characteristics that have the lowest ratio will be inspected first. The ratios for this problem are given in Table 4.3.

Table 4.3: Ratios of Inspection Cost to Rejection Rate

$Ci / R_{1,1,1}$	1180.92
$Ci / R_{2,1,2}$	2393.25
$Ci / R_{3,1,3}$	2552.96

The result indicates that the inspection of characteristic #1 should be conducted first, followed by inspecting characteristic #2 and #3 to minimize inspection cost.

4.1.2 Determine Inspection Cycles

Three characteristics are inspected in stages in one cycle of inspection. After inspection of one characteristic is completed, marginal probabilities of the characteristics are updated based on the new joint probability mass function. The inspection cycle repeats until the minimum total cost per accepted product is attained. This repeated inspection cycles are used for the designed inspection plan. We used Excel spreadsheet to perform the calculation in solving the problem. The results after the 1st cycle of inspection are shown in Table 4.4.

Table 4.4: 1st Cycle of Inspection Results

Stage 1		**Stage 2**		**Stage 3**	
$M_{1,1}$	200	$M_{2,1}$	86.37	$M_{3,1}$	55.710
$PG_{1,1}$	0.6	$PG_{2,1}$	0.78	$PG_{3,1}$	0.883
$PR_{1,1}$	0.11	$PR_{2,1}$	0.05	$PR_{3,1}$	0.020
$PS_{1,1}$	0.29	$PS_{2,1}$	0.18	$PS_{3,1}$	0.097
$FGR_{1,1}$	0.09	${}^{2,1}P_2(1)$	0.87	${}^{3,1}P_3(1)$	0.901
$FS_{1,1}$	4.56	${}^{2,1}P_2(0.5)$	0.05	${}^{3,1}P_3(0.5)$	0.020
$FA_{1,1}$	0.87	${}^{2,1}P_2(0)$	0.08	${}^{3,1}P_3(0)$	0.079
$CA_{1,1}$	85.41	$FGR_{2,1}$	0.06	$FGR_{3,1}$	0.044
		$FS_{2,1}$	2.93	$FS_{3,1}$	2.216
		$FA_{2,1}$	0.14	$FA_{3,1}$	0.044
		$CA_{2,1}$	55.51	$CA_{3,1}$	42.043

The updated marginal probabilities are non-negative and the sum adds to one, this shows the updated marginal probabilities are also the probability mass function.

Due to inspection errors and low probability of good products at the beginning of the inspection, only 86 out of 200 products (43%) are accepted as “good”. They will go to the 2^{nd} stage inspection for the 2^{nd} characteristic. After inspections in the 2^{nd} stage, 55 out of 86 products (64%) are accepted as “good” to continue for inspection in the 3^{rd} stage. An increasing percentage of products are accepted as “good” in the following stage. Probability of good products increases while probabilities of products being rework or scrap decrease with inspections due to the update of joint probability mass function. The accepted products have less probability of being defective after inspection.

After three stages of inspections are complete, total cost per accepted product is computed. The total cost per accepted product after 1^{st} cycle of inspection for this example is shown in Table 4.5.

Table 4.5: Total Cost after the 1^{st} Cycle of Inspection

TA(1)	42.13
CFR(1)	48505.58
CFA(1)	210168.36
CI(1)	68415.51
$\text{EXP}(tc)\vert_{j=1}$	7763.58
PG(1)	0.96930

As shown in Table 4.5, there are 42 out of 200 products (21%) are accepted as "good" at the end of the 1st cycle of inspection. The probability of the accepted products being "good" is 96.93%. The total cost per accepted product is 7764. Comparing with 80,000 calculated previously for the total cost per accepted product without inspection, there is 90% of saving.

We used inspection cycles to decide the total cost per accepted product. The results of 2nd cycle of inspection are shown in Table 4.6.

Table 4.6: 2nd Cycle of Inspection Results

Stage 1		**Stage 2**		**Stage 3**	
$M_{1,2}$	42.131274	$M_{2,2}$	38.296382	$M_{3,2}$	35.384387
$PG_{1,2}$	0.969299	$PG_{2,2}$	0.980844	$PG_{3,2}$	0.988348
$PR_{1,2}$	0.007340	$PR_{2,2}$	0.001225	$PR_{3,2}$	0.001072
$PS_{1,2}$	0.023361	$PS_{2,2}$	0.017932	$PS_{3,2}$	0.010580
${}^{3,1}P_1(1)$	0.987103	${}^{2,2}P_2(1)$	0.991572	${}^{3,2}P_3(1)$	0.990206
${}^{3,1}P_1(0.5)$	0.005474	${}^{2,2}P_2(0.5)$	0.001220	${}^{3,2}P_3(0.5)$	0.001072
${}^{3,1}P_1(0)$	0.007423	${}^{2,2}P_2(0)$	0.007203	${}^{3,2}P_3(0)$	0.008722
$FGR_{1,2}$	0.040311	$FGR_{2,2}$	0.037246	$FGR_{3,2}$	0.034630
$FS_{1,2}$	2.015589	$FS_{2,2}$	1.862311	$FS_{3,2}$	1.731479
$FA_{1,2}$	0.000832	$FA_{2,2}$	0.000498	$FA_{3,2}$	0.000329
$CA_{1,2}$	38.255239	$CA_{2,2}$	35.346643	$CA_{3,2}$	32.863454

The probability of good products in the accepted ones increases after each stage inspection. More products are accepted as good for next stage inspection. The total cost per accepted product after the 2nd cycle of inspection is shown in Table 4.7.

Table 4.7: Total Cost after the 2nd Cycle of Inspection

TA(2)	32.90
CFR(2)	76552.47
CFA(2)	331.85
CI(2)	91577.92
$\text{EXP}(tc)\|_{j=2}$	5120.68
PG(2)	0.99713

There are 32 out of 200 products(16%) are accepted as "good" at the end of the 2nd cycle of inspection and 99.713% of these products are assumed as good. Total cost per accepted product is reduced to 5120 after two cycles of inspection compared with 7764 with one cycle of inspection. 2644 dollars of saving per product is gained.

To possibly reduce the total cost per accepted product, the 3rd cycle of inspection is conducted. The results are shown in Table 4.8.

Table 4.8: 3rd Cycle of Inspection Results

Stage 1		**Stage 2**		**Stage 3**	
$M_{1,3}$	32.898413	$M_{2,3}$	31.130303	$M_{3,3}$	29.493916
$PG_{1,3}$	0.997131	$PG_{2,3}$	0.998105	$PG_{3,3}$	0.998830
$PR_{1,3}$	0.000465	$PR_{2,3}$	0.000067	$PR_{3,3}$	0.000067
$PS_{1,3}$	0.002404	$PS_{2,3}$	0.001827	$PS_{3,3}$	0.001103
${}^{3,2}P_1(1)$	0.998924	${}^{2,3}P_2(1)$	0.999192	${}^{3,3}P_3(1)$	0.999011
${}^{3,2}P_1(0.5)$	0.000333	${}^{2,3}P_2(0.5)$	0.000067	${}^{3,3}P_3(0.5)$	0.000067
${}^{3,2}P_1(0)$	0.000742	${}^{2,3}P_2(0)$	0.000740	${}^{3,3}P_3(0)$	0.000923
$FGR_{1,3}$	0.032769	$FGR_{2,3}$	0.031046	$FGR_{3,3}$	0.029430
$FS_{1,3}$	1.638437	$FS_{2,3}$	1.552311	$FS_{3,3}$	1.471513
$FA_{1,3}$	0.000006	$FA_{2,3}$	0.000004	$FA_{3,3}$	0.000003
$CA_{1,3}$	31.097528	$CA_{2,3}$	29.462865	$CA_{3,3}$	27.929320

The trend of the increasing of probability of good products and the decreasing of probabilities of rework and scrap products are the same as shown in the previous cycles. Total cost per accepted product after the 3rd cycle of inspection is shown in Table 4.9.

Table 4.9: Total Cost after the 3rd Cycle of Inspection

TA(3)	27.96
CFR(3)	99863.78
CFA(3)	2.68
CI(3)	110282.45
$\text{EXP}(tc)\vert_{j=3}$	7516.39
PG(3)	0.99972

There are additional 5 out of 200 products (2.5%) are scraped in the 3rd cycle of inspection. Compare with total cost in Table 4.7, the total cost per accepted product increases by 2396. Since a higher value of total cost per accepted product is observed after the 3rd cycle of inspection, we determine that 2 cycles of inspection for this example will lead to the minimum total cost per accepted product.

We stop the inspection once an optimized solution is found.

In analyzing this example problem, we conducted the 4th cycle of inspection with results shown in Table 4.10. The total cost per accepted product after the 4th cycle of inspection is shown in Table 4.11.

Table 4.10: 4th Cycle of Inspection Results

Stage 1		Stage 2		Stage 3	
$M_{1,4}$	27.958753	$M_{2,4}$	26.550699	$M_{3,4}$	25.216232
$PG_{1,4}$	0.999718	$PG_{2,4}$	0.999807	$PG_{3,4}$	0.999881
$PR_{1,4}$	0.000029	$PR_{2,4}$	0.000004	$PR_{3,4}$	0.000004
$PS_{1,4}$	0.000253	$PS_{2,4}$	0.000189	$PS_{3,4}$	0.000115
${}^{3,3}P_1(1)$	0.999901	${}^{2,4}P_2(1)$	0.999918	${}^{3,4}P_3(1)$	0.999899
${}^{3,3}P_1(0.5)$	0.000021	${}^{2,4}P_2(0.5)$	0.000004	${}^{3,4}P_3(0.5)$	0.000004
${}^{3,3}P_1(0)$	0.000078	${}^{2,4}P_2(0)$	0.000078	${}^{3,4}P_3(0)$	0.000097
$FGR_{1,4}$	0.027948	$FGR_{2,4}$	0.026543	$FGR_{3,4}$	0.025211
$FS_{1,4}$	1.397405	$FS_{2,4}$	1.327170	$FS_{3,4}$	1.260533
$FA_{1,4}$	0.000000	$FA_{2,4}$	0.000000	$FA_{3,4}$	0.000000
$CA_{1,4}$	26.522750	$CA_{2,4}$	25.189688	$CA_{3,4}$	23.924922

Table 4.11: Total Cost after the 4th Cycle of Inspection

TA(4)	23.95
CFR(4)	119789.32
CFA(4)	0.02
CI(4)	126227.58
$\text{EXP}(tc)\|_{j=4}$	10272.05
PG(4)	0.99997

The total cost per accepted product after the 4th cycle of inspection is 10272, higher than that after the 3rd cycle.

In summary, for the considered example problem,

- We should inspect the products in the sequence of characteristics #1, #2 then #3.
- The inspect will stop after 2 cycles

4.2 Experimental Design and Analysis

Design of Experiment (DOE) is a systematic approach to analyze a process or a system in evaluating the impact of process inputs (X_s) on the process output (Y). It will help to determine the target level of those inputs to achieve a desired output. Following a DOE approach, a series of structured tests are designed where planned changes are made to input factors of a process or a system. The effects of changes on the pre-defined output are then assessed. DOE is also referred to as experimental design.

In this study, experiments were conducted to evaluate various parameters and their interactions to total cost per accepted product. A two-level fractional factorial design was used to analyze the effect on total cost per accepted product and to determine the significant input factors.

The experimental design analysis was conducted using statistical software Minitab R16.

4.2.1 Effect on Total Cost per Accepted Product

The experiments based on a 2^{9-4} fractional factorial design were conducted for the effect of nine considered input factors on the total cost per accepted product. The experiments require 32 runs. These runs were conducted randomly to reduce the variations and biases caused by the runs. Table 4.12 presents the information on the design used for this example problem analysis

.

Table 4.12: 2^{9-4} Fractional Factorial Experimental Design

Factor	9
Runs	32
Resolution	IV
Fraction	1/16

Table 4.13 presents the detailed experimental design matrix and the response values of the calculations. As shown in Table 4.13, the input factors chosen for analyzing the effect on total cost per accepted product are:

- probability of false classification of good product as scrap (*Egs*)
- probability of false classification of good product as rework (*Egr*)
- probability of false classification of scrap product as good (*Esg*)
- probability of false classification of scrap product as rework (*Esr*)
- probability of false classification of rework product as good (*Erg*)
- probability of false classification of rework product as scrap (*Ers*)
- cost of false acceptance of a scrap or rework product (*Ca*)
- cost of false rejection of a good or rework product (*Cr*)
- cost of inspection a product (*Ci*)

The levels of input factors are shown in Table 4.14. They are based on historical and empirical information.

Table 4.13: Design of Experiment Matrix

Std. Order	Run Order	*Egs*	*Egr*	*Esg*	*Esr*	*Erg*	*Ers*	*Ca*	*Cr*	*Ci*	*EXP(tc)*
1	23	-	+	+	-	+	-	+	+	-	7023.53
2	24	+	+	+	-	+	-	-	-	+	13195.84
3	4	+	+	-	-	-	-	-	+	+	7865
4	9	-	-	-	+	-	-	-	-	+	5318.56
5	21	-	-	+	-	+	+	+	-	+	9165.59
6	18	+	-	-	-	+	-	+	+	+	14167.34
7	28	+	+	-	+	+	-	-	+	-	7529.62
8	29	-	-	+	+	+	-	-	+	+	8258.79
9	25	-	-	-	+	+	+	+	+	-	5211.25
10	16	+	+	+	+	-	-	-	-	-	8689.81
11	26	+	-	-	+	+	+	-	-	+	8065.25
12	13	-	-	+	+	-	+	+	-	-	6067.9
13	30	+	-	+	+	+	-	+	-	-	10130.8
14	1	-	-	-	-	-	+	+	+	+	7565.67
15	22	+	-	+	-	+	+	-	+	-	11445.74
16	10	+	-	-	+	-	-	+	+	-	10402.47

Table 4.13: Design of Experiment Matrix (Cont.)

Std. Order	Run Order	*Egs*	*Egr*	*Esg*	*Esr*	*Erg*	*Ers*	*Ca*	*Cr*	*Ci*	*EXP(tc)*
17	17	-	-	-	-	+	-	-	-	-	4100.37
18	5	-	-	+	-	-	-	-	+	-	5237.64
19	3	-	+	-	-	-	-	+	-	-	4331.84
20	2	+	-	-	-	-	+	-	-	-	5806.65
21	8	+	+	+	-	-	+	+	+	-	12787.65
22	19	-	+	-	-	+	+	-	+	+	4802.49
23	20	+	+	-	-	+	+	+	-	-	8520.53
24	12	+	+	-	+	-	+	+	-	+	11475.49
25	27	-	+	-	+	+	-	+	-	+	7145.29
26	6	+	-	+	-	-	-	+	-	+	13952.68
27	32	+	+	+	+	+	+	+	+	+	16963.37
28	31	-	+	+	+	+	+	-	-	-	4706.77
29	15	-	+	+	+	-	-	+	+	+	9505.65
30	14	+	-	+	+	-	+	-	+	+	12835.66
31	11	-	+	-	+	-	+	-	+	-	4460.22
32	7	-	+	+	-	-	+	-	-	+	7299.53

Table 4.14: The Levels of Factors

	Low (−)	High (+)
E_{gs}	0.03	0.08
E_{gr}	0.001	0.005
E_{sg}	0.05	0.15
E_{sr}	0.001	0.005
E_{rg}	0.05	0.1
E_{rs}	0.02	0.05
Ca	200000	1000000
Cr	4000	6000
Ci	150	300

Confounded patterns were not used in this analysis. Table 4.15 presents the estimated effects and the coefficients of the experiments. The coefficients with probability less than 0.05 are considered significant. Figure 4.2 presents normal probability plot of effects estimated from the experiments. It shows that the main effects *Egs, Esg, Ca* and *Ci* are significant to the total cost per accepted product.

Table 4.15: Estimated Effects and Coefficients on Total Cost

Predictor	Coef.	SE Coef.	T	P
Constant	-1155	1404	-0.82	0.42
Egs	88290	7433	11.88	0.000
Egr	-69230	92912	-0.75	0.464
Esg	23437	3716	6.31	0.000
Esr	-54690	92912	-0.59	0.562
Erg	4789	7433	0.64	0.526
Ers	-5576	12388	-0.45	0.657
Ca	0.0024842	0.0004646	5.35	0.000
Cr	0.4716	0.1858	2.54	0.019
Ci	-0.08011	0.01249	-6.41	0.000

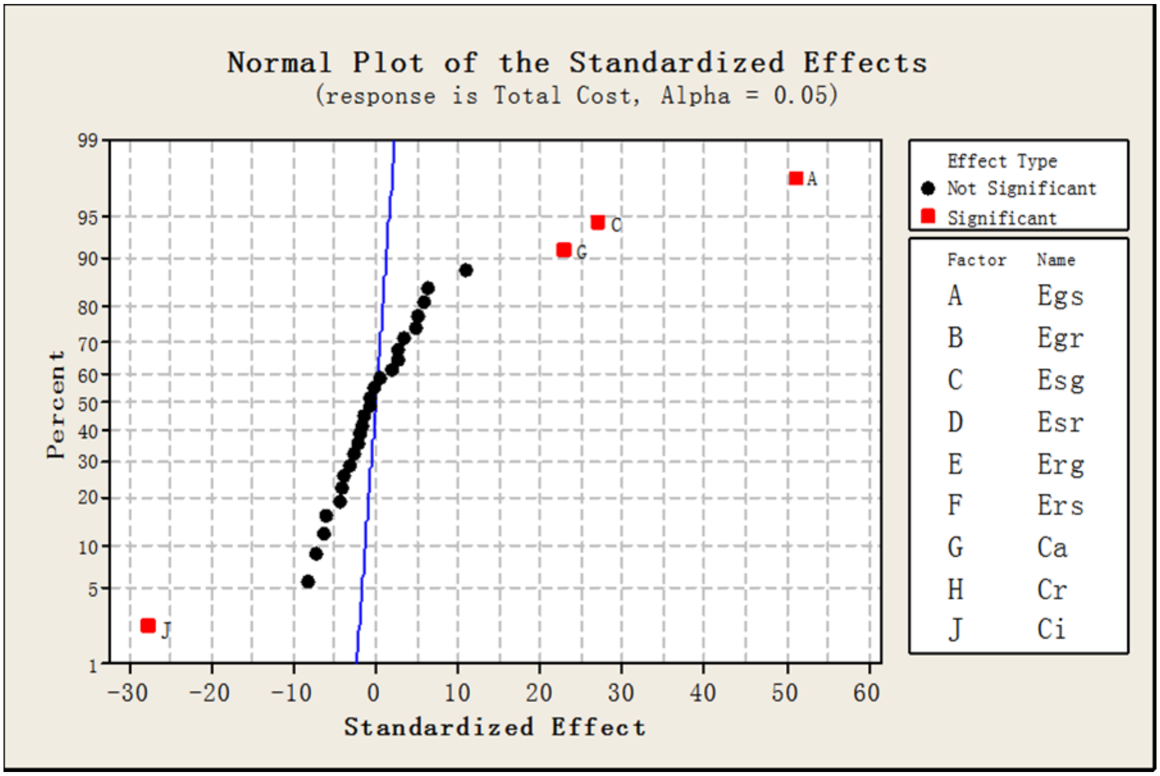

Figure 4.2: Normal Probability Plot of Effects

Based on the analysis, the linear regression equation of the coefficients can be generated and shown in Eq. (4.1). It can be used to estimate the total cost per accepted product in solving this example problem.

Total Cost per accepted product

$$= - 1155 + 88290\ Egs - 69230\ Egr + 23437\ Esg - 54690\ Esr + 4789\ Erg - 5576\ Ers + 0.00248\ Ca + 0.472\ Cr - 0.0801\ Ci \quad \text{.... (4.1)}$$

The analysis of variance (ANOVA) shown in Table 4.16 confirms the results presented in Figure 4.2. It also indicates that the main effects *Egs, Esg, Ca* and *Ci* are significant with probability values less than 0.05.

Table 4.16: Analysis of Variance on Total Cost per Accepted Product

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	9	285674423	285674423	31741603	532.06	0.034
Egs	1	155901643	155901643	155901643	2613.24	0.012
Egr	1	613480	613480	613480	10.28	0.192
Esg	1	43945102	43945102	43945102	736.61	0.023
Esr	1	382854	382854	382854	6.42	0.239
Erg	1	458678	458678	458678	7.69	0.22
Ers	1	223859	223859	223859	3.75	0.303
Ca	1	31597494	31597494	31597494	529.64	0.028
Cr	1	7116057	7116057	7116057	119.28	0.058
Ci	1	45435255	45435255	45435255	761.59	0.023
2-Way Interactions	21	24250019	24250019	1154763	19.36	0.178
Residual Error	1	59658	59658	59658		
Total	31	309984100				

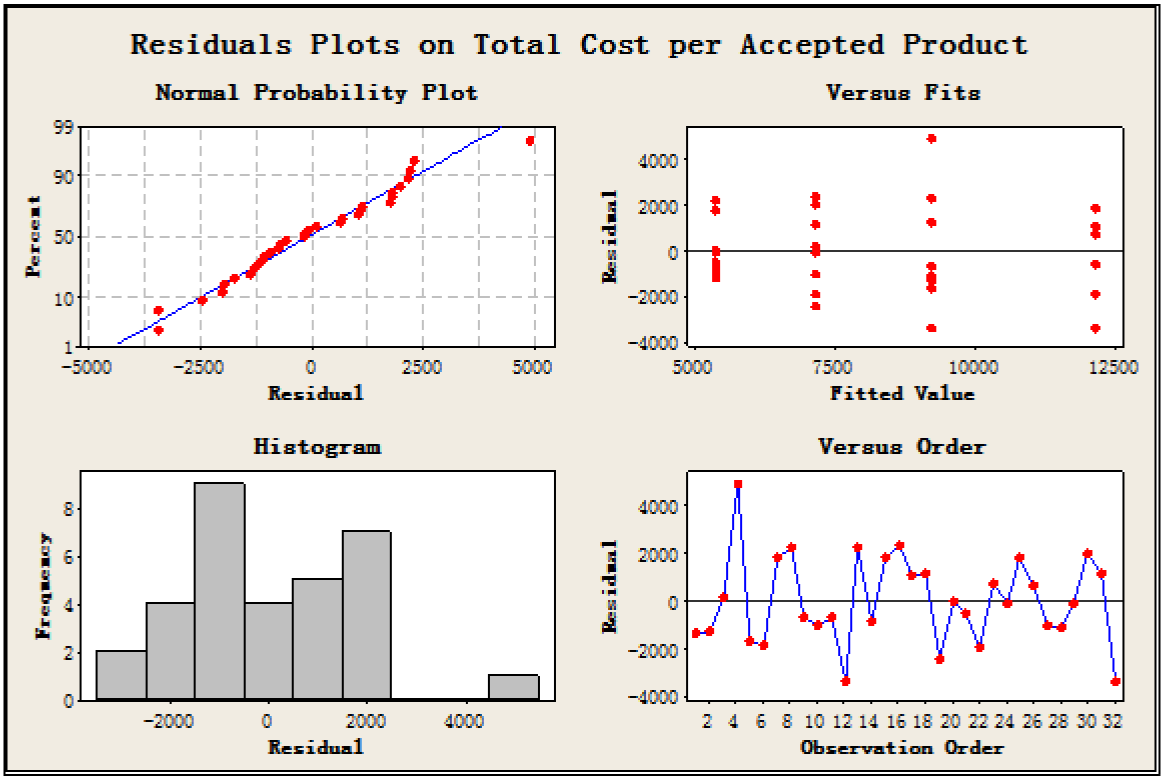

Figure 4.3: Plots of Residuals

As shown in Figure 4.3, the model was validated as adequate by analysis of residual plots for total cost per accepted product. As a diagnostic check, the normal probability plot of the residuals shows that the residuals are normally distributed.

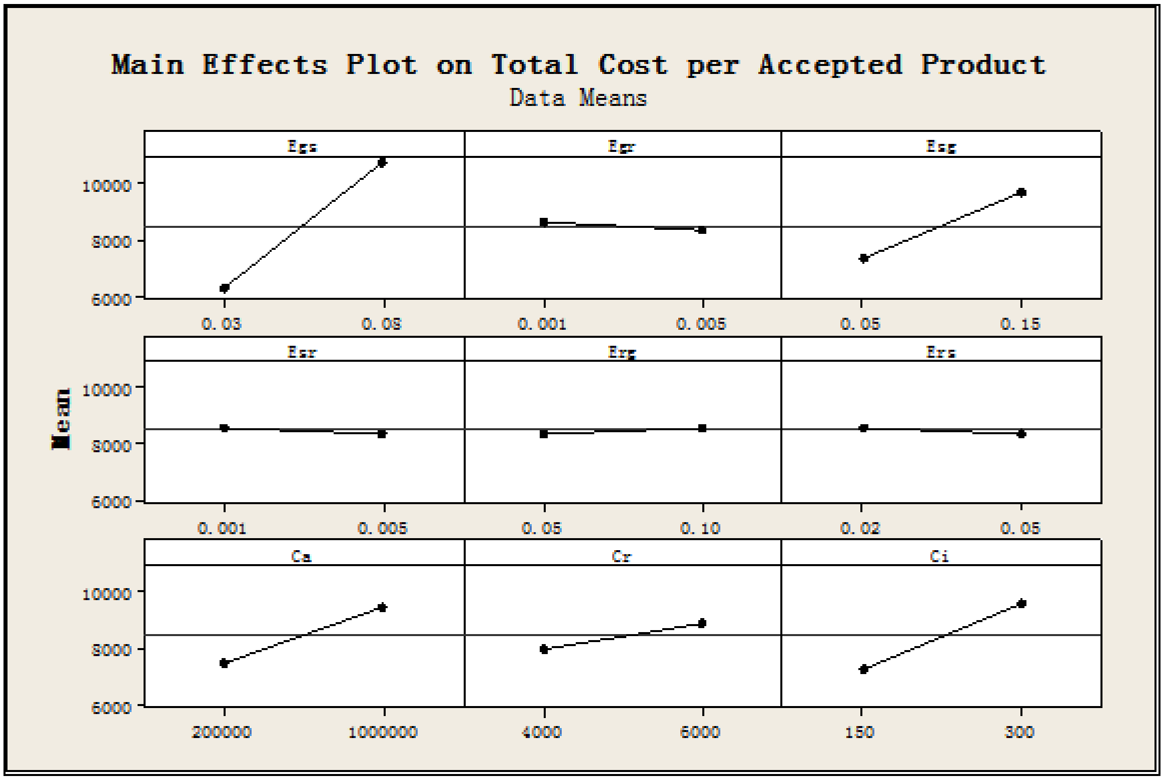

Figure 4.4: Main Effects Plot on Total Cost per Accepted Product

Figure 4.4 shows the main effects plot of the input factors on the total cost per accepted product. Inspection errors *Egs, Esg, Erg* and unit costs *Ca, Cr, Ci* have positive effects on the total cost per accepted product. Total cost per accepted product increases when these input values increase. Inspection errors *Egr Esr, Ers* have negative effect to the total cost per accepted product. These results are reasonable since lower probabilities of inspection errors and lower unit costs will reduce the total cost per accepted product.

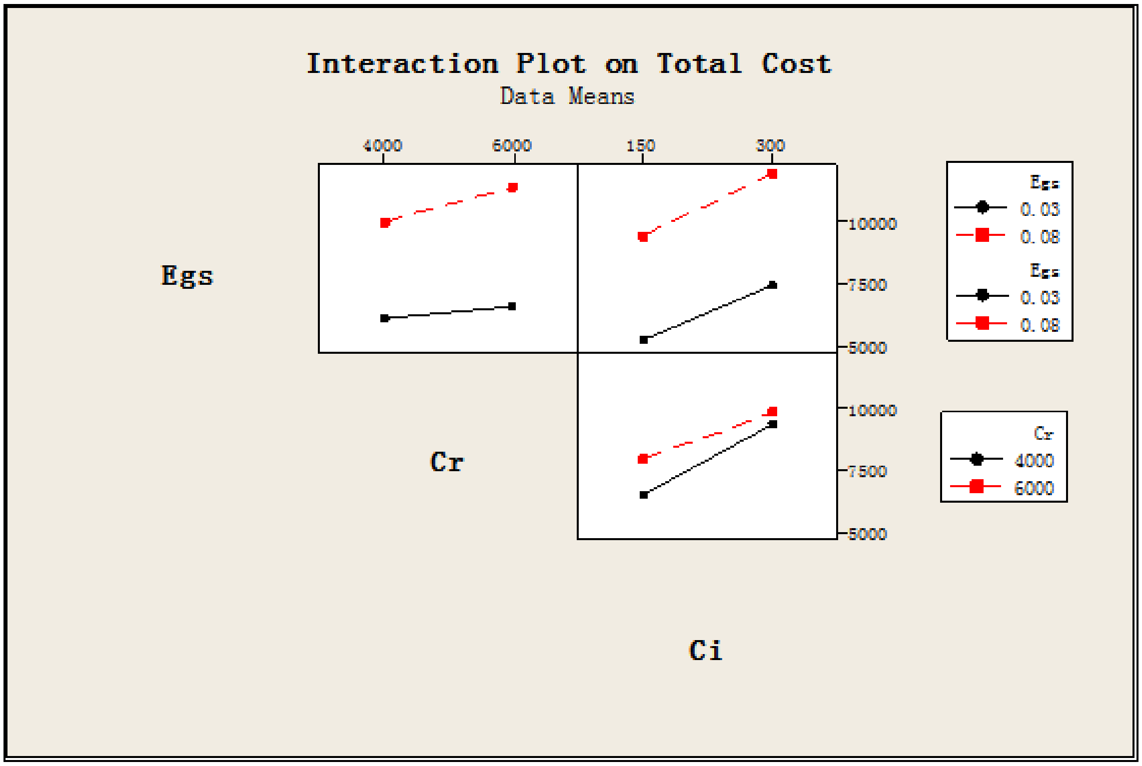

Figure 4.5: Interaction Plot (AHJ) on Total Cost per Accepted Product

Figure 4.5 shows the interaction plot (AHJ) of *Egs, Cr* and *Ci* on the total cost per accepted product. As shown in the figure, if *Egs* (factor A) increases, *Cr* (factor H) and *Ci* (factor J) also increase and the total cost per accepted product increase as well.

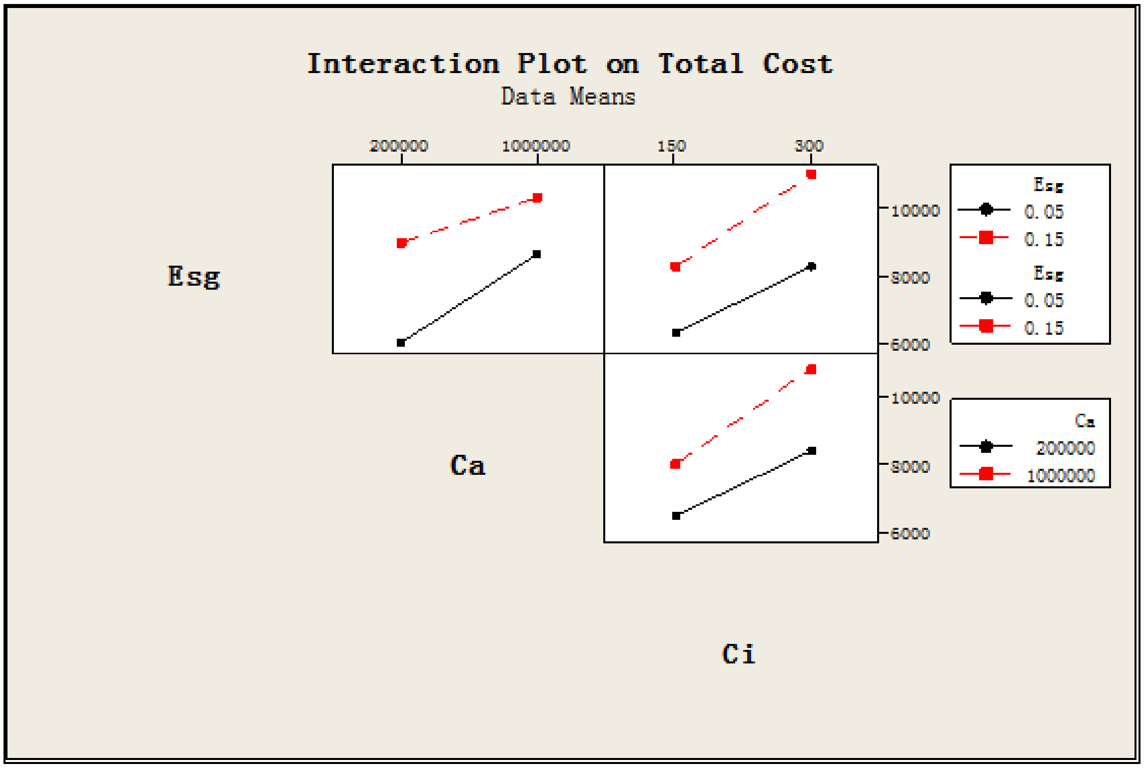

Figure 4.6: Interaction Plot (CGJ) on Total Cost per Accepted Product

Figure 4.6 shows the interaction plot (CGJ) of *Esg, Ca* and *Ci* on the total cost per accepted product. As shown in the figure, if *Esg* (factor C) increases, *Ca* (factor G) and *Ci* (factor J) also increase and the total cost per accepted product increase as well.

Inspection errors *Egs* and *Esg* play import roles in reducing the total cost per accepted product. A further analysis of *Egs* and *Esg* to the total cost per accepted product is presented in section 4.3.

The optimized number of inspections in the example problem is insensitive to the change of *Egs* and *Esg*.

Figure 4.7 shows optimization plot for total cost per accepted product for the model. We used Minitab and adjusted the value of each input factor in square bracket then the response value of the total cost per accepted product will be calculated directly. As shown in Figure 4.7, the model reaches the minimum cost per accepted product when all input factors were at their lower level limit.

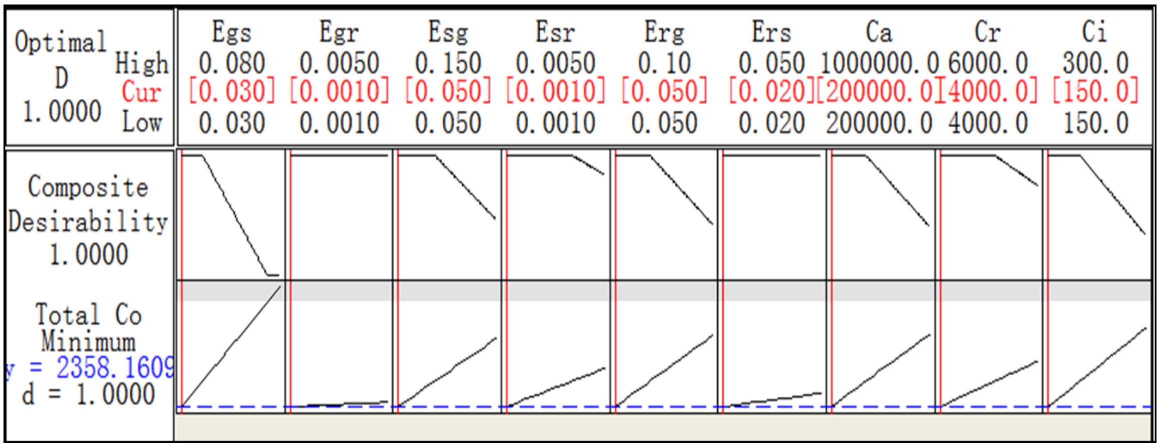

Figure 4.7: Optimization Plot of Total Cost per Accepted Product

4.3 Impact of Inspection Errors on Total Cost per Accepted Product

Inspection errors *Egs* and *Esg* are significant factors on the total cost per accepted product. We herein conducted further analysis using Minitab and the results are shown in Figure 4.8.

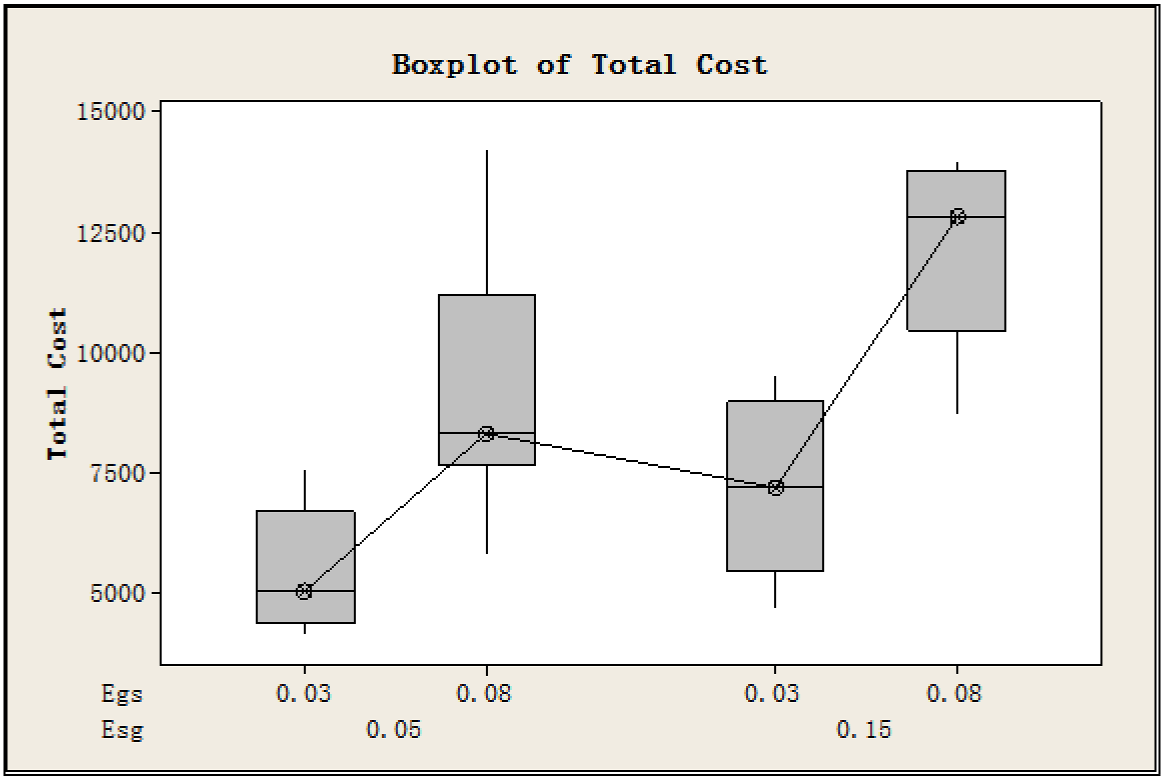

Figure 4.8: Boxplot of Inspection Errors on Total Cost per Accepted Product

To increase *Egs* or *Esg* will lead to the increase of the total cost per accepted product. However, the increase of *Esg* may result in a higher total cost per accepted product than the increase of *Egs*. This indicates that failure to reject a bad product will lead to much higher cost.

4.4 Impacts of Probability of Good Products

In this research, we design an inspection plan for products with characteristics' defective rates statistically dependent. Consider that probability of good products is of the most interest in industrial practice, we investigate its impact on the total cost per accepted product. Probability of good products at the beginning of inspection is assumed to be known.

4.4.1 Characteristics' Defective Rates Independence Verification

The characteristics' independence is verified by comparing the value of joint probabilities and the corresponding value of the multiplications of marginal probabilities. If the joint probability is equal to the multiplication of marginal probabilities, the characteristics are assumed statistically independent, Otherwise, they are considered statistically dependent. The independence of the characteristics considered in this example problem is verified as shown in Table 4.17.

Table 4.17: Characteristics' Defective Rates Independence Verification

No.	X_1	X_2	X_3	$P(X_1,X_2,X_3)$	$P(X_1).P(X_2).P(X_3)$	Independence Verification
1	0	0	0	0.01	0.001872	Characteristics' Defective Rates are Statistically Dependent
2	0	0	0.5	0.01	0.001296	
3	0	0	1	0.01	0.011232	
4	0	0.5	0	0.01	0.001404	
5	0	0.5	0.5	0.01	0.000972	
6	0	0.5	1	0.01	0.008424	
7	0	1	0	0.01	0.012324	
8	0	1	0.5	0.01	0.008532	
9	0	1	1	0.04	0.073944	
10	0.5	0	0	0.01	0.002028	
11	0.5	0	0.5	0.01	0.001404	
12	0.5	0	1	0.01	0.012168	
13	0.5	0.5	0	0.01	0.001521	
14	0.5	1	0	0.01	0.013351	
15	1	0	0	0.01	0.0117	
16	1	0	0.5	0.01	0.0081	
17	1	0	1	0.04	0.0702	
18	1	0.5	0	0.01	0.008775	
19	1	1	0	0.05	0.077025	

4.4.2 Impact of the Probability of Good Products on Total Cost per Accepted Product

Figure 4.9 shows the impact of probability of good products at the beginning of inspection on the total cost per accepted product. Higher probabilities of good products reduce significantly the total cost per accepted product. In practice, if the quality of manufactured products is very good, the probabilities of inspection errors may be reduced and consequently the total cost per accepted product is reduced.

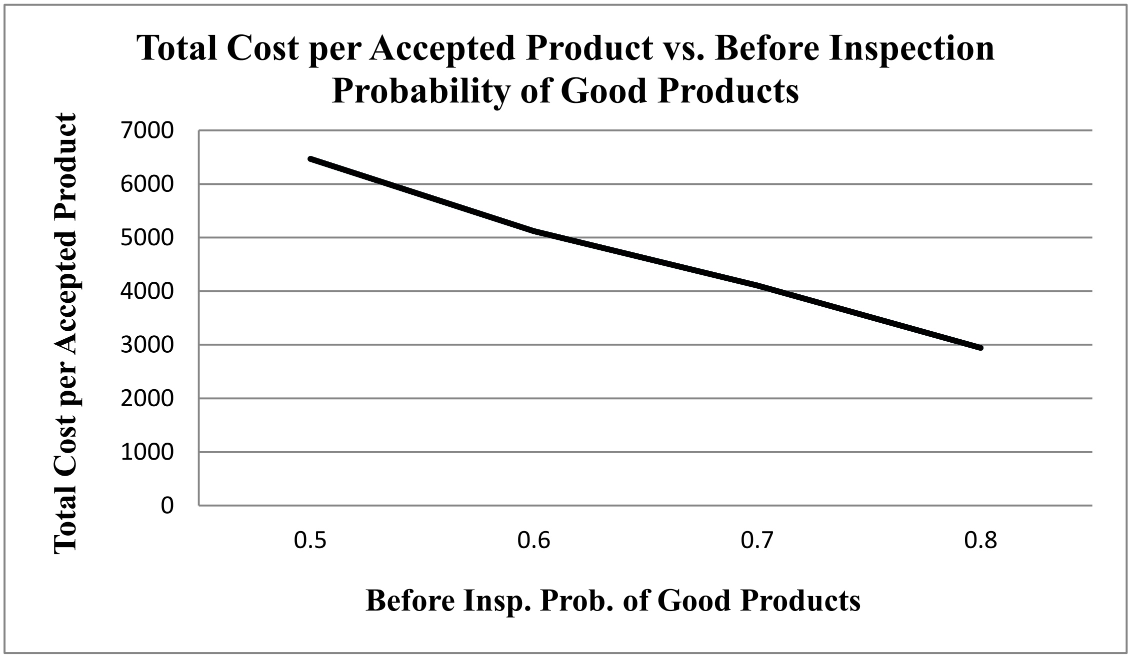

Figure 4.9: Impact of Good Products Probability on Total Cost per Accepted Product

4.4.3 Impact of the Probability of Good Products on Optimized Inspection Cycle

The effect of probability of good products before inspections on the optimized inspection cycles is illustrated in Figure 4.10. It shows that the probability of good products before inspections will affect the number of inspection cycles. A higher probability will reduce the required inspections. This result matches with industrial practice in that products of higher quality need less inspection.

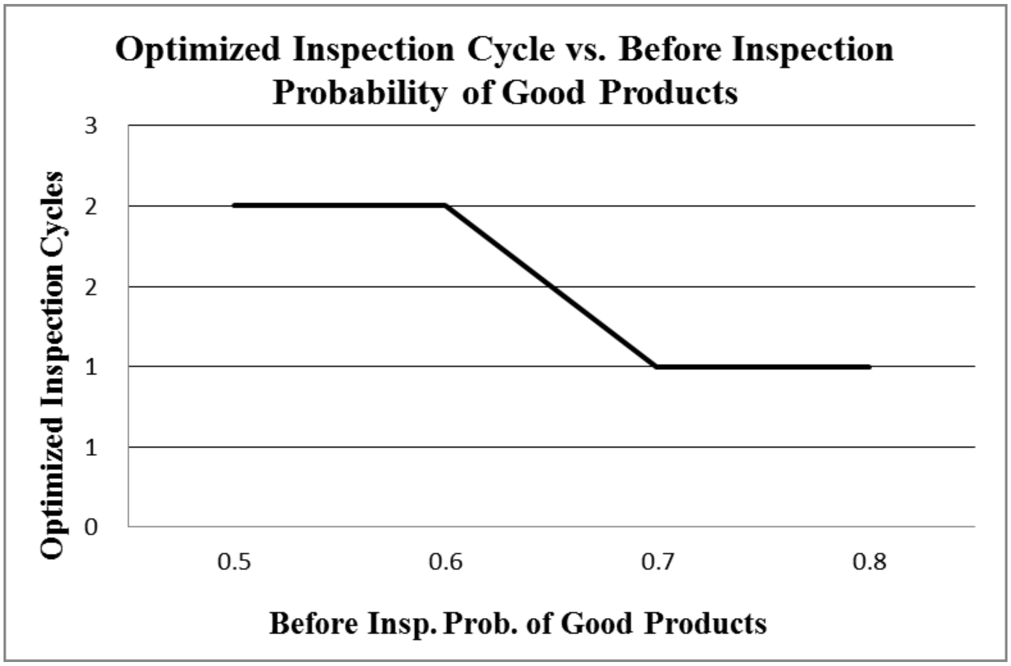

Figure 4.10: Impact of Good Products Probability on Optimized Inspection Cycle

4.5 Summary

The developed model to design an inspection plan with minimum total cost per accepted product is solved by using a heuristic algorithm. The methodology is tested by an example problem from a real manufacturing system. Data used for the tested problem are realistic but hypothetical. The obtained results are illustrative for variations of many important parameters. The heuristic solution approach is efficient and effective in handling different problem scenarios.

A better insight is obtained into the effects of adjusting model input factors such as inspection errors and cost parameters with different values. Design of Experiment (DOE) is conducted to identify significant input factors and to search for an optimized result. Inspection errors, *Esg* and *Egs*, and cost parameters, *Ca* and *Ci* are significant factors to total cost per accepted product. The interaction among significant factors shows that the reduction of inspection errors, especially *Esg* and *Egs* will reduce total cost per accepted product as a result.

The analysis of impact of probability of good products before inspection on total cost per accepted products and on the number of inspection cycles clearly shows that improved product quality will reduce total cost per accepted product as well as required inspection.

Chapter Five

Conclusions and Future Research

This chapter presents a summary of the research conducted in this thesis. Several concluding remarks based on the developed model and computational result analysis are also presented. Future directions for research are discussed at the end.

5.1 Concluding Summary

This research generalizes the work presented in Duffuaa and Nadeem (1994) and Duffuaa and Khan (2002). A new model is developed for inspecting critical characteristics of products with defective rates dependent. Bayes' theorem is employed in the development of the model. The output of the model is an optimized inspection plan for quality control on critical characteristics of products. The inspection plan is to minimize the total cost per accepted product. The developed model and proposed heuristic solution approach are illustrated using an example from a medical equipment manufacturing system. The model can be modified without much difficulty for solving similar problem in other applications.

5.2 Future Directions for Research

Some of possible extensions to this work include:

- Extending the model to cases in which to optimize inspection sequence in large scale inspections.
- Extending the model to cases where there are constraints on inspection time and inspection budget.

References

1. Ben-Daya, M. and Rahim, A. (2003), "Optimal lot-sizing, quality improvement and inspection errors for multistage production systems", International Journal of Production Research, Vol. 41, pp. 65-79.
2. Cai, D., Xie, M. and Goh, T. (2001), "SPC in an automated manufacturing environment", International Journal of Computer Integrated Manufacturing, Vol.14, pp.206-211.
3. Chen, A. (1998), "An alternative dynamic programming approach to allocating inspection points in multistage production systems", Quality Engineering, Vol.11, pp.197-205.
4. Chen, S. and Lambercht, M. (1997), "The optimal frequency and sequencing of tests in the inspection of multicharacteristic components", IIE Transactions, Vol. 29, pp.1039-1049.
5. Chimay, J. and Nosa, F. (1997), "Concurrent engineering in design build projects", Construction Management and Economics, Vol.15, pp.271-281.
6. Chun, Y. (2009), "Improving product quality by multiple inspections: prior and posterior planning of serial inspection procedures", IIE Transactions, Vol. 41, pp.831-842.
7. Chun, Y. (2010), "Bayesian inspection model for the production process subject to a random failure", IIE Transactions, Vol. 42, pp.304-316.

8. Duffuaa, S. and Al-Najjar, H. (1997), “A general inspection plan for critical multicharacteristic components”, International Journal of Production Research, Vol.35, pp.2723-2736.
9. Duffuaa, S. and Khan, M. (2002), “An optimal repeat inspection plan with several classifications”, Journal of the Operation Research Society, Vol. 53, pp.1016-1026.
10. Duffuaa, S. and Khan, M. (2005), “Impact of inspection errors on the performance measures of a general repeat inspection plan”, International Journal of Production Research, Vol.43, pp.4945-4967.
11. Duffuaa, S. and Nadeem, I. (1994), “A complete inspection plan for dependent multicharacteristic critical components”, International Journal of Production Research, Vol.32, pp.1897-1907.
12. Duffuaa, S. and Raouf, A. (1990), “An optimal sequence in multicharacteristics inspection”, Journal of Optimization Theory and Application, Vol. 67, pp.79–86.
13. Elshafei, M., Khan, M. and Duffuaa, S. (2006), “Repeat inspection planning using dynamic programming”, International Journal of Production Research, Vol. 44, pp.257-270.
14. Emmons, H. and Rabinowitz, G. (2002), “Inspection allocation for multistage deteriorating production systems”, IIE Transactions, Vol. 34, pp.1031-1041.
15. Ghiya, K., Terry Bahill, A. and Chapman, W. (1999), “QFD: validating robustness”, Quality Engineering, Vol.11, pp.593-611.
16. Greenshtein, E. and Rabinowitz, G. (1997), “Double-stage inspection for screening

multicharacteristic items”, IIE Transactions, Vol. 29, pp.1057-1061.

17. Heredia-Langner, A., Montgomery, D. and Carlyle, W. (2002), “Solving a multistage partial inspection problem using genetic algorithms”, International Journal of Production Research, Vol.40, pp.1923-1940.
18. Huang, Y., Lo, H. and Ho, J. (2008), “Design of effective inspection schema for imperfect production systems”, International Journal of Production Research, Vol.46, pp.4537-4551.
19. Jiao, Y. and Djurdjanovic, D. (2010), “Joint allocation of measurement points and controllable tooling machines in multistage manufacturing processes”, IIE Transactions, Vol.42, pp.703-720.
20. Jin, M. and Tsung, F. (2009), “A chart allocation strategy for multistage processes”, IIE Transactions, Vol. 41, pp.790-803.
21. Jin, M., Li, Y. and Tsung, F. (2010), “Chart allocation strategy for serial-parallel multistage manufacturing processes”, IIE Transactions, Vol.42, pp.577-588.
22. Jones, E., Parast, M. and Adams, S. (2010), “A framework for effective six sigma implementation”, Total Quality Management and Business Excellence, Vol.21, pp.415-424.
23. Ju, T., Lin, B., Lin, C. and Kuo, H. (2006), “TQM critical factors and km value chain activities”, Total Quality Management and Business Excellence, Vol.17, pp.373-393.
24. Kim, I. and Suh, H. (1998), “Optimal operation grouping and sequencing technique for multistage machining systems”, International Journal of Production Research,

Vol.36, pp.2061-2081.

25. Kim, Y. and Kim, S. (2011), "Cost analysis of information technology assisted quality inspection using activity based costing", Construction Management and Economics, Vol. 29, pp.163-172.
26. Kogan, K. and Raz, T. (2002), "Optimal allocation of inspection effort over a finite planning horizon", IIE Transactions, Vol.34, pp.515-527.
27. Kouvelis, P. and Mukhopadhyay, S. (1999), "Modeling the design quality competition for durable products", IIE Transactions, Vol.31, pp.865-880.
28. Lee, H. (1988), "On the optimality of a simplified multicharacteristic component inspection model", IIE Transactions, Vol. 20, pp.392–398.
29. Lee, J. and Unnikrishnan, S. (1998), "Planning quality inspection operations in multistage manufacturing systems with inspection errors", International Journal of Production Research, Vol. 36, pp.141-155.
30. Li, J., Blumenfeld, D. and Samuel, M. (2008), "Production system design for quality robustness", IIE Transactions, Vol. 40, pp.162-176.
31. Li, Y. and Tsung, F. (2011), "Detecting and diagnosing covariance matrix changes in multistage processes", IIE Transactions, Vol. 43, pp.259-274.
32. Linn, R., Au, E. and Tsung, F. (2002), "Process capability improvement for multistage processes", Quality Engineering, Vol. 15, pp.281-292.
33. Maleyeff, J., Kaminsky, F. and Farris, J. (2003), "Minimum cost 100% inspection systems with inspection error", Quality Engineering, Vol. 15, pp.557-563.

34. Mann, R. (2008), "Revisiting a TQM research project: the quality improvement activities of TQM", Total Quality Management and Business Excellence, Vol. 19, pp.751-761.

35. Meerkov, S. and Zhang, L. (2010), "Product quality inspection in bernoulli lines: analysis, bottlenecks, and design", International Journal of Production Research, Vol.48, pp.4745-4766

36. Mohib, A., Azab, A. and Elmaraghy, H. (2009), "Feature-based hybrid inspection planning: a mathematical programming approach", International Journal of Computer Integrated Manufacturing, Vol. 22, pp.13-29.

37. Montgomery, D. (2009), Introduction to Statistical Quality Control, John Wiley, 6th Ed., New Jersey, USA.

38. Moosa, K. and Sajid, A. (2010), "Critical analysis of six sigma implementation", Total Quality Management and Business Excellence, Vol. 21, pp.745-759.

39. Perng, D., Lee, S. and Chou, C. (2010), "Automated bonding position inspection on multi-layered wire IC using machine vision", International Journal of Production Research, Vol. 48, pp.6977-7001.

40. Rabinowitz, G. and Emmons, H. (1997), "Optimal and heuristic inspection schedules for multistage production systems", IIE Transactions, Vol. 29, pp.1063-1071.

41. Raouf, A., Jain, J. and Sathe, P. (1983), "A cost minimization model for multicharacteristic component inspection", IIE Transactions, Vol. 15, pp.187-194.

42. Rapley, C. (1999), "Quality costing: a study of manufacturing organizations. part 1:

case studies and survey”, Total Quality Management, Vol.10, pp.85-93.

43. Rau, H., Chu, Y. and Cho, K. (2005), “Layer modelling for the inspection allocation problem in re-entrant production systems”, International Journal of Production Research, Vol. 43, pp.3633-3655.

44. Roberts, M. (1998), “Inspection times and the selection task: are they relevant?”, The Quarterly Journal of Experimental Psychology, Vol. 51, pp.781-810.

45. Scibilia, B., Kobi, A., Barreau, A. and Chassagnon, R. (2003), “Robust designs for quality improvement”, IIE Transactions, Vol. 35, pp.487-492.

46. Shi, J. and Zhou, S. (2009), “Quality control and improvement for multistage systems: a survey”, IIE Transactions, Vol. 41, pp.744-753.

47. Shiau, Y. (2002), “Inspection resource assignment in a multistage manufacturing system with an inspection error model”, International Journal of Production Research, Vol. 40, pp.1787-1806.

48. Veatch, M. (2000), “Inspection strategies for multistage production systems with time-varying quality”, International Journal of Production Research, Vol. 38, pp.837-853.

49. Vining, G. (2011), “Technical advice: essential elements for quality improvement programs”, Quality Engineering, Vol. 23, pp.395-397.

50. Yeh, R., Tseng, S. and Ho, W. (1999), “Carryover effects in multi-attribute inspection”, International Journal of Production Research, Vol. 37, pp.2915-2925.

51. Zhang, Y., Yang, M. and Zhang, Y. (2006), “Concurrent design for process quality,

statistical tolerance, and SPC”, Communications in Statistics Theory and Methods, Vol. 35, pp.1869-1882.

52. Zhu, Y., You, J., Alard, R. and Schönsleben, P. (2009), “Design quality: a key to improve product quality in international production network”, Production Planning and Control, Vol. 20, pp.168-177.