# A Decentralized Markovian Jump $\mathcal{H}_\infty$ Control Routing Strategy for Mobile Multi-Agent Networked Systems

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***Abstract*— This paper presents a Markovian jump linear (MJL) system framework for developing routing algorithms in mobile ad hoc networks (MANETs) that encounter changes in the number of nodes and/or the number of destinations. A unified $\mathcal{H}_\infty$ control strategy is proposed by representing the dynamically changing destination nodes as singular switching control systems. A decentralized routing scheme is proposed and designed for the networked multi-agent system in presence of unknown time-varying delays. To solve the corresponding optimization problem the physical constraints are expressed as Linear Matrix Inequality (LMI) conditions. The resulting decentralized $\mathcal{H}_\infty$ routing control schemes for both regular and singular MJL systems are shown to formally achieve the desired performance specifications and requirements. Simulation results are presented to illustrate and demonstrate the effectiveness of our proposed novel routing control strategies.**

**Index Terms: Dynamic Routing, Ad hoc Mobile Networks, $\mathcal{H}_\infty$ Control, Singular and Regular Markovian Jump Linear (MJL) Systems, Decentralized Control, Switching Control Systems**

## I. Introduction

In recent years, the widespread availability and demand for autonomous mobile wireless networks in applications as diverse as space missions to intelligence, surveillance, and reconnaissance (ISR) of unmanned vehicles missions have stimulated active research on self-organizing networks such as *ad hoc* wireless networks [1], [2] that do not require a pre-established infrastructure. Contrary to cellular networks, where the nodes are restricted to communicate with a few strategically placed base stations, in mobile ad hoc networks (MANETs) they can directly communicate with one another. However, due to the nature of the wireless channels each node can effectively communicate with only certain finite nodes, typically those that lie in its vicinity or in its so-called neighboring set. Consequently, it becomes necessary that nodes cooperate with one another to forward data/messages to their final destinations. However, due to the restrictive physical requirements that are imposed on the network, performing routing in MANETs is not a trivial problem.

Routing problem, in general, deals with minimization of a certain objective function such as the shortest path, link congestion, end-to-end delay, or packet loss [1], [3], and [4]. For instance, an optimum route can be obtained such that the *total delay* is minimized. Indeed, the total delay minimization implies determining a route that messages have to take to reach their final destination in the *shortest time* (also known as the "fastest route") as opposed to the shortest distance. The problems of delays in routing and networked control systems have been recently investigated in the literature [5], [6], [7], [8], [9], [10], [11], [12], [13]. In [5] a routing-based admission control mechanism that considers an end-to-end delay for the IP traffic flows was introduced. In [11], a routing algorithm was proposed to minimize an average of the queueing delay by using capacity allocation. In [12], a set of paths between the source and the destination nodes are indexed based on their energy consumption in an increasing order of priority. An estimate of the end-to-end delay along each of the ordered paths is then obtained.

In [13], the dynamic routing problem was defined as a team optimization problem and an approximate solution based on neural networks was obtained. In [10], the authors have introduced robust centralized as well as decentralized routing control strategies for networks with a fixed topology based on minimization of the worst-case queueing length, which is related to the queueing delays. The routing problem is formulated as an $\mathcal{H}_\infty$ optimal control problem to achieve a robust routing performance in presence of *multiple and unknown fast time-varying network delays*. A Linear Matrix Inequality (LMI) constraint is obtained to design a delay-dependent $\mathcal{H}_\infty$ controller. The network physical constraints are expressed as LMI feasibility conditions.

In MANETs, the neighboring sets of nodes may change due to the mobility and variations in the network topology, left over energy resources, and increasing/decreasing the number of nodes. Therefore, the dynamics of the network characterizing the traffic flow will become time-varying. Towards this end, the approach introduced in [10] is generalized in this paper to ad hoc mobile multi-agent networks. To achieve this goal, the mobile network routing model is represented by a switching control system. Due to the fact that nodes mobilities, and therefore network topological changes, are not generally deterministic and involve random transitions, a Markovian jump process is an ideal candidate and a viable framework for modeling these network behaviors. Note that the network topologies and the selection of the new neighboring sets at each switching instant only depends on the existing neighboring sets.

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This research is supported in part by a Discovery grant from the Natural Sciences and Engineering Research Council of Canada (NSERC).

Recently, considerable attention has been devoted to Markovian jump systems and time-delays [14]- [22] (and references therein). In [15], a sufficient condition for exponential estimates of a class of Markovian jump systems with fixed state delay was introduced and by employing LMI techniques a state feedback stabilizing controller was presented. The authors then extended the controller to time-varying delays in [22]. In [23], a stabilizing control for the Markovian jump linear (MJL) systems with input delays was presented. However, a fixed gain $K$ was found corresponding to *all* the switching modes of the system. In other words the controller gain was not designed to switch corresponding to changes in the system mode. This, in general, would yield conservative conditions that can potentially reduce the possibility of obtaining a feasible solution.

A delay dependent stabilization of singularly perturbed Markovian jump systems with a fixed singular matrix was studied in [24]. The $\mathcal{H}_\infty$ control scheme for a singular system with time-delays developed in [25] was extended to Markovian jump systems in [26]. In [27], the MJL and singular systems were integrated into a new class known as stochastic singular systems with random abrupt changes where the control problem was investigated. However, the systems considered are not affected by the delay.

Research on decentralized control of descriptor systems has received considerable interest in the past few years. By using LMI techniques, a decentralized $\mathcal{H}_\infty$ control for a multi-channel descriptor system was introduced in [28]. In [29], LMI conditions were developed for decentralized $\mathcal{H}_\infty$ control of multi-channel descriptor systems with time-delays in states.

Our proposed state feedback control scheme for MJL systems with input delays has gains that are different corresponding to each switching mode. Therefore, it provides a more reliable performance according to the system specifications at each mode. It also increases the chances of obtaining a feasible solution under constraints. Moreover, the state feedback controller gains can be designed by solving the LMI conditions. For our considered traffic network dynamics, changing neighboring sets will result in interconnected terms (matrices) that also change at each switching mode. Therefore, standard $\mathcal{H}_\infty$ control schemes should be modified to handle MJL systems with mode-dependent interconnections.

In switching systems, the number of system states is usually assumed to be fixed. In other words, for linear systems generally the state matrix $A$ and the input matrix $B$ can change at each switching instant but not their dimensions. However, in certain circumstances the dimension of the states may also change. For instance, in wireless communication networks and sensor networks, the number of states may increase or decrease due to addition or deletion of nodes. Under these scenarios the system behavior can be expressed by a singular MJL system. Consequently, our proposed framework enables one to describe the overall system dynamics in a unified manner subject to variations in the number of states.

An important issue that is also addressed in the present work is the routing problem subject to the changes in the number of destination nodes. In other words, for some destination nodes no external traffic has to be routed in certain periods of time. However, due to the system dynamics and time-delays certain messages may still be present in the queues that should be routed to these destinations as quickly as possible. By simply eliminating the inactive destination states can actually lead to loss of integrity and consistency of the overall network. It also ignores the leftover messages that are kept in the eliminated queues. To cope with these problems, we propose to model such network behavior as singular MJL systems.

Our methodology is geared towards development of a decentralized routing algorithm for ad hoc mobile multi-agent networks. Hence, each node in the network requires only its own local information to route the received messages while ensuring that a global objective function is optimized. Consequently, our proposed routing algorithm can guarantee a minimum queueing delay for mobile multi-agent networks having a variable topology. Due to the fact that the routing strategy is obtained based on *local information*, the computational complexity of our proposed methodology should be lower when compared to other existing *centralized* algorithms in the literature.

To summarize, the main contributions of this paper are stated as follows:

1) Development of a *decentralized* $\mathcal{H}_\infty$ routing control strategy for mobile multi-agent MJL systems with *time-varying delays*. The proposed stabilizing state feedback control law designed for optimal traffic routing is changing at each switching mode corresponding to the Markovian transitions in order to cope with the variations in the system behavior due to each node's mobility.
2) Development of a *decentralized* $\mathcal{H}_\infty$ routing control scheme for singular MJL systems with *time-varying delays* due to changes in the number of destination nodes. Moreover, the singular matrix corresponding to each subsystem is assumed to be also changing at each switching mode corresponding to changes in the destination nodes.
3) The issue of mode-dependency of the MJL system matrices is addressed and analyzed for the first time in this paper. The introduced decentralized stabilizing control strategy can be applied to systems where the connections among the subsystems are changing due to either the physical characteristics of the system, faults and malfunctions in the communication, or increase or decrease in the number of subsystems, among others.

The remainder of the paper is organized as follows. Section II provides a description of the considered traffic model in terms of the queueing dynamics and the physical constraints. In Section III, a decentralized state feedback $\mathcal{H}_\infty$ routing control strategy for mobile multi-agent networks is designed to stabilize the MJL system with mode-dependent interconnections (matrices) and time-varying delays. Certain LMI conditions are stated to represent the corresponding physical constraints. In Section IV, the routing problem for a *variable number of destination* nodes is addressed by describing the network behavior as a singular MJL system. LMI conditions are derived to obtain a decentralized $\mathcal{H}_\infty$ control strategy for singular MJL systems with mode-dependent matrices and time-varying delays. The LMI conditions associated with the physical constraints are then modified for mobile multi-agent networks with *variable destinations*. Finally, in Section V the performance and capabilities of our proposed strategies are evaluated and compared with two popular routing algorithms, namely the AODV (Ad hoc On Demand Distance Vector) [30] and the OLSR (Optimized Link State Routing) [31] protocols and schemes.

## II. Problem Formulation

In certain applications, changing the system behavior can be represented by a switching or a transition that is based and triggered on a specific rule among certain dynamical models. On the other hand, if the system dynamics change randomly where the condition for switching is only dependent on the present state of the system, Markovian process can serve as a viable representation of the switching rule. Indeed, a Markovian jump linear (MJL) system can be considered as a hybrid system whose state vector has two components: $x(t)$ which is generally referred to as the state and $r_t$ which represents the modes or the configurations. MJL systems jump abruptly from one mode to another in a random manner and for this reason their switching is classified as stochastic switching. The switching among the modes (different system models) is governed by a continuous-time Markov process with discrete and finite state space, whereas at each mode the system evolves as a deterministic linear system.

A Markovian jump process is a credible framework for describing and modeling the multi-agent mobility behavior. The dynamics of our considered mobile multi-agent network is governed by the following Markovian jump linear (MJL) system

$$\begin{aligned}\dot{x}_i(t) &= B_i(r(t))u_i(t) + B_{w_i}(r(t))w_i(t) \\ &\quad + \sum_{j\in\wp_{r(t)}(i)} B_{dij}(r(t))u_j(t-\tau_{ji}(t)) \end{aligned} \tag{1}$$

where each node is considered as representing a subsystem that includes all the queues that are present in the node corresponding to multiple destination nodes. In other words, $x_i$ denotes the queue lengths in node $i$ for different destination nodes $d$, $d = 1, ..., \bar{d}$, where $\bar{d}$ denotes the number of destination nodes, $u_i(t)$ denotes the flows sent from node $i$, $\tau_{ji}(t)$ is an *unknown but bounded time-varying* total delay in transmitting, propagating, and processing messages at node $i$ from node $j$, $w_i(t)$ is the external input flow entering node $i$, and $B_i \in \Re^{n\times l}$ and $B_{dij} \in \Re^{n\times l}$ represent network connectivity matrices. In fact, each element of $B_i(B_{dij})$ is equal to $-1(1)$ if its corresponding flow is outgoing (incoming) flow to node $i$ and is zero otherwise, $B_{wi} = I_{n\times n}$, and $r(t)$ is a function that represents the rule for changing the neighboring sets $\wp_{r(t)}(i)$. Since the network topology changes are not known *a priori*, and given that changing the neighboring sets are independent of the current neighboring set, the network topology changes are modeled by the Markov process.

***Assumption 1:*** Given the neighboring set $\wp_{r(t)}(i)$ corresponding to each node $i$ the underlying graph of the network is assumed to remain connected despite the arbitrary mobility of the nodes (agents) during the time interval of interest.

Note that although our proposed routing strategy assumes that *all considered network topologies* are *finite* and known *a priori*, the process and the time at which the network topology changes is *not known a priori*. In other words, we consider that the network topology is allowed to change within a given list of possible configurations arbitrarily and randomly. The above assumption is actually quite consistent with real networked multi-agent systems such as those found in a team of unmanned vehicles.

When the topology or the underlying graph of a network changes due to either (a) node mobility that results in creating or cutting links, or (b) addition of new nodes, the neighboring sets $\wp_{r(t)}(i)$ in model (1), and consequently the connectivity matrices $B_i$ and $B_{dij}$ will also change. On the other hand, the nature of node mobility is generally not deterministic and involves random transitions and switches. Furthermore, only the existing neighboring sets $\wp_{r(t)}(i)$ and connectivity matrices $B_i$ and $B_{dij}$ do affect the selection of new neighboring sets in the next transition step. In other words, changes in the *neighboring set* are independent from previous neighboring sets.

Let us consider $r(t)$ as a continuous-time Markov process taking values in a finite space $\mathcal{S} = \{1, ..., M\}$, where $M$ is the number of all possible neighboring sets (that is modes or network topologies and system models that are represented by $\wp_{r(t)}(i)$). The rate of switching among the $M$ topologies are described by the following probability transitions:

$$\mathbb{P}[r(t+h)=k|r(t)=l] = \left\{ \begin{array}{cc} \pi_{kl}h + o(h) & k \neq l \\ 1+\pi_{kk}h + o(h) & k = l \end{array} \right\}$$

where $\pi_{kl} > 0$ is the transition rate from mode (neighboring set) $k$ to mode $l$, $\pi_{kk} = -\sum_{l=1,l\neq k}^{M} \pi_{kl}$, and $o(h)$ is a function satisfying $\lim_{h\to 0} \frac{o(h)}{h} = 0$. To simplify the notation, we let $B_{ir}$ to represent $B_i(r(t))$ when $r(t) = r$. This notation is also applied to all the other applicable matrices.

***Remark 1:*** The transition rates $\pi_{ij}$ can be specified as functions of certain parameters that are responsible for changing the network topology. Some examples of these parameters are node mobility speed and rate of change of the activeness or inactiveness of the destination nodes, among others. When precise values of the transition rates are not available, one can incorporate uncertainty terms to the nominal values and then accordingly modify the analysis as shown in [32]. Moreover, it may be argued that the time-delays $\tau_{ij}$ can also be dependent on the Markovian jump modes, and this explicit information can be potentially used in our subsequent results. However, as stated at the outset of this work it is assumed that the time-delays are *unknown* but with a known upper bound. Therefore, our developed results corresponding to this scenario will still remain valid and applicable, despite the dependence of the time-delays on the modes.

Let us now consider certain physical characteristics of the network traffic that can impose specific constraints on the routing problem. A typical set of constraints can be expressed as

$$\begin{aligned} u_i(t) &\geq 0 & (2)\\ x_i(t) &\geq 0 & (3)\\ G_{k_i}u_i(t) &\leq c_{ki}(r(t)) \quad k_i = 1, ..., li, \quad i = 1, ..., n & (4)\\ Q_{dji}x_i(t) &\leq x_{max_{dji}} \quad d = 1, ..., \bar{d} & (5) \end{aligned}$$

where $li$ is the number of links in the subsystem $i$ and $\bar{d}$ is the number of destination nodes. Note that $li$ does depend and change subject to different selection of the neighboring set $\wp_{r(t)}(i)$, and the changes in the links $li$ result in the connectivity matrices $B_i$ and $B_{dij}$ to also change.

The first two constraints (i.e., (2) and (3)) are the so-called non-negativity constraints and are introduced for obvious physical reasons. The capacity constraint (4) states that the total flow in each link cannot exceed its capacity $c_{ki}(r(t))$ at each mode. The last condition, i.e., (5) indicates that to avoid packet loss the length of the queue should always remain smaller than a maximum value that is specified for the buffer as $x_{max_{dji}}$. Therefore, $G_{k_i}$ is defined such that by multiplying $G_{k_i}$ with $u_i$ one gets the total flow that goes through the link $k_i$, and $Q_{dji}$ is defined such that $Q_{dji}x_i$ leads to the queueing length of the buffer $dji$, for $d = 1, ..., \bar{d}, \ i, j = 1, ..., n$. We now state our assumption regarding the characteristics of the delay function.

***Assumption** 2:* The delays $\tau_{ji}(t)$ are unknown differentiable functions that for all $t \geq 0$ satisfy

$$0 \quad \leq \quad \max\{\tau_{ji}(t)\} \leq h_{ji}, \ \max\{|\dot{\tau}_{ji}(t)|\} \leq \bar{d}_{ji} < 1$$

In the above assumption, $\tau_{ji}$ is considered as the sum of the following delays, namely (i) transmitting delay: The time between starting and ending the transmission of a message from node $j$ to node $i$, (ii) propagating delay: The time for propagating a message on each link, and (iii) processing delay: The time that each message (from upstream nodes or outside of the network) should spend at each node to be received, identified by its destination, and inserted to the appropriate queue of node $i$. Even though the above delays are not known *a priori* and are time-varying, utilization of efficient processors will attempt to make them not to vary quickly when compared to the main source of the delay which is the queueing delay. Therefore, assuming that $|\dot{\tau}_{ji}(t)|$ is less than 1 $s$ is quite a realistic assumption in most real applications.

For simplicity, it is also assumed that the delay between any two nodes in both directions are the same, i.e. $\tau_{ji} = \tau_{ij}$. For more details refer to [33]. It should be stated that $\tau_{ji}(t)$ can also be defined in such a manner that it is dependent on the Markovian jump mode. Unfortunately, this problem is beyond the scope of this paper and not addressed any further.

The $\mathcal{H}_\infty$ robust optimal control design strategy is a suitable framework for dealing with system uncertainties and unknown time-delays. Therefore, at each node (subsystem or agent), the routing problem can be stated as that of finding an $\mathcal{H}_\infty$ state feedback control law governed by $u_i = K_i x_i$ such that it simultaneously, (a) guarantees stability of the overall network traffic in presence of time-varying delays, and (b) minimizes a global objective function which is considered as the *worst-case queueing length* due to external inputs. In other words, by designating the regulated output corresponding to system (1) as $z_i(t) = C_i x_i(t)$, where $C_i$ is a weight matrix that is full rank, the routing problem can be cast into the following optimization problem:

$$\min \gamma \quad s.t. \quad J(w) < 0,$$
$$J(w) = \int_0^\infty (z^T z - \gamma^2 w^T w)dt, \quad \gamma > 0 \qquad (6)$$

where $z(t) = vec\{z_i(t)\}$ and $w(t) = vec\{w_i(t)\}$.

In other words, by optimizing the objective function (6) the resulting state feedback strategy $u_i$ specifies and determines a portion of the associated queue length $x_i$ that corresponds to selected gain $K_i$. Therefore, the messages will be routed such that the network is simultaneously stabilized subject to the unknown transmitting, propagating, and processing delays $\tau_{ji}(t)$, and the queueing length, $x_i$, is minimized subject to the presence of the external input $w$.

The time-delays considered in model (1) can indeed be a major source of network instability. This instability arises due to deteriorations that can occur in the overall network quality of service requirements and difficulties that appear in delivering massages as optimally and as completely as desired. The notion of instability invoked here refers to the adverse consequences that one encounters in fulfilling the main network routing objectives. Specifically, one can end up with congestion and over flowing the nodes buffer that can eventually lead to significant packet losses. Classical control theory is not adequately capable of addressing and equipped to handle stability and performance issues of time-delayed systems. Complications do arise when there is no or very limited *a priori* knowledge about the time delays.

Let us now define the concept of stochastic stabilizability and $\mathcal{H}_\infty$ control of stochastic systems.

***Definition** 1:* [23] The free nominal Markovian jump linear system (1) is said to be *stochastically stabilizable* if there exists a linear state feedback $u_i = K_{ir}x_i$ such that for the closed-loop system when $w(t) \equiv 0$ for all $\phi \in L_2[-\tau, 0)$, and for an initial mode $r_0 \in \mathcal{S}$ there exists a constant $M(\phi(.), r_0) > 0$ such that $\mathbb{E}[\int_0^\infty x(t)^T x(t)|\phi(.), r_0] \leq M(\phi, r_0)$, where $\phi$ is an initial condition and $x(t) = vec\{x_i(t)\}$, for $i = 1, ..., n$.

***Definition** 2:* [23] Let $\gamma > 0$. System (1) is said to be stochastically stable with $\gamma$-disturbance attenuation if there exits a constant $M(\phi, r_0)$ with $M(0, r_0) = 0$ such that

$$\begin{aligned} \|z\|_{\mathbb{E}_2} &= \mathbb{E}\left\{\int_0^\infty z^T(s)z(s)ds\right\}^{1/2}\\ &\leq [\gamma^2\|w(t)\|_2 + M(\phi, r_0)]^{1/2} \qquad (7) \end{aligned}$$

Therefore, similar to the performance index that is defined for the fixed network topology in (6), the objective function (7) guarantees the boundedness and also stochastic $\mathcal{L}_2$ stability of the queueing length in presence of unknown delays and external input flow, $w(t)$ corresponding to a changing network topology.

The following lemma is used in our subsequent results whose proof can be found in [24].

***Lemma** 1:* [24] For any matrices $U, V \in \mathcal{R}^{n\times n}$ with $V > 0$, one has $UV^{-1}U^T \geq U + U^T - V$.

Note that the neighboring set $\wp_{r(t)}(i)$ corresponding to each node $i$ may vary. Therefore, each subsystem matrices and interconnection terms in model (1) is mode-dependent. This implies that the interconnected terms vary at each switching mode. In the next section, a decentralized $\mathcal{H}_\infty$ control for MJL systems with mode-dependent interconnected terms is proposed to provide an optimal routing solution that simultaneously guarantees internal stability of the traffic network and minimizes the worst case network queueing length. Appropriate LMIs are also provided to satisfy the associated traffic network physical constraints.

## III. A Markovian Jump $\mathcal{H}_\infty$ Control Strategy for Routing Problems in Mobile Multi-Agent Networks

Our first result in this section provides a stabilizing $\mathcal{H}_\infty$ control design strategy for the MJL system (1).

***Theorem** 1:* Consider a mobile multi-agent traffic network whose dynamics is governed by (1) for which $w_i \in L_2[0,\infty)$ and satisfies Assumptions 1 and 2. The state feedback routing controller $u_i = K_{ir}x_i$ with the gain $K_{ir} = M_{ir}Y_{ir}^{-1}$ guarantees that the closed-loop system is stochastically stable and $J(w) < 0$, provided that there exist matrices $M_{ir}$, and symmetric positive definite matrices $Y_{ir}$, $\bar{R}_{ir}$, $\bar{Q}_i$ for $i = 1,...,n$, $r = 1,...,M$ such that the LMI conditions (8) and (9) are satisfied where $\theta_{ir1} = M_{ik}^T B_{ik}^T + B_{ik}M_{ik} + \pi_{kk}Y_{ik}, \theta_{ir2} = \tilde{B}_{dik}\tilde{R}_{ik}, \theta_{ir3} = (\bar{\pi}_k Y_{ik})^T, \theta_{ir4} = -(1-\bar{d}_{ji})\tilde{R}_{ik}, \theta_{ir5} = -diag\{Y_{i1},...,Y_{i(k-1)},Y_{i(k+1)},...,Y_{iM}\}, \theta_{ir6} = -m_{ik}\bar{R}_{ik}, \theta_{ir7} = -h_{ji}\bar{Q}_i, \bar{\pi}_k = [\sqrt{\pi_{k1}} \ \cdots \ \sqrt{\pi_{k(k-1)}} \ \sqrt{\pi_{k(k+1)}} \ \cdots \ \sqrt{\pi_{kM}}]^T, \tilde{\pi}_k = [\sqrt{m_{i1}\pi_{k1}} \ \cdots \ \sqrt{m_{i(k-1)}\pi_{k(k-1)}} \ \sqrt{m_{i(k+1)}\pi_{i(k+1)}} \ \cdots \ \sqrt{m_{iM}\pi_{kM}}]^T$, and $m_{ik}$ = the number of subsystems where subsystem $i$ belongs to their $\wp_k(.)$ in mode $k$, $\tilde{R}_{ik} = diag_{j\in\wp_k(i)}\{\bar{R}_{jk}\}$, $\tilde{M}_{jk} = diag_{j\in\wp_k(i)}\{M_{jk}\}$, and $\tilde{B}_{dik} = vec\{B_{dijk}\}, \quad for\ j \in \wp_k(i)$.

$$W_{ir1} = \begin{bmatrix} \theta_{ir1} & \theta_{ir2} & B_{wik} & Y_{ik}^T C_{ik}^T & \theta_{ir3} & m_{ik}M_{ik}^T & h_{ji}M_{ik}^T \\ * & \theta_{ir4} & 0 & 0 & 0 & 0 & 0 \\ * & * & -\gamma I & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & \theta_{ir5} & 0 & 0 \\ * & * & * & * & * & \theta_{ir6} & 0 \\ * & * & * & * & * & * & \theta_{ir7} \end{bmatrix} < 0 \quad (8)$$

$$W_{ir2} = \begin{bmatrix} 2(1-m_{ik}\pi_{kk})I - \bar{Q}_i + m_{ik}\pi_{kk}\bar{R}_{ik} & \tilde{\pi}_k \\ * & \tilde{R}_{ir} \end{bmatrix} \geq 0 \quad (9)$$

**Proof:** The proof is provided in Appendix A.

Under the circumstances input flow $w$ does not belong to the $\mathcal{L}_2$ space, i.e., $w \notin \mathcal{L}_2$, one needs to filter $w$ through a shaping filter before applying it to the network. The use of a shaping filter, however, might remove some information from the input signal. Therefore, one should employ decoding or interpolation techniques to recover the missing information at the destination nodes. It should be noted that if information loss cannot be tolerated the filtering should be avoided, in which case it is no longer possible to guarantee that the queueing lengths remain in $\mathcal{L}_2$. However, our proposed routing methodology can still be modified such that the boundedness of the queueing lengths are guaranteed for a bounded input flow $w$. Due to space limitations this issue is not investigated in this paper (for details cf. [34]).

The routing strategy that is given in Theorem 1 guarantees that the queueing lengths converge to zero in finite time in presence of unknown time-varying delays and $\mathcal{L}_2$ bounded external input flows. However, there is *no guarantee* that the capacity and the buffer constraints are also simultaneously satisfied and ensured. The LMI conditions presented in the next section are developed to satisfy these constraints.

### A. LMI Conditions for Incorporating the Physical Constraints

In this section, the physical constraints (2)-(5) are represented as LMI feasibility conditions. These constraints are taken into account for determining a complete solution to our robust dynamic traffic routing problem.

*1) Capacity Constraint:* The capacity constraint (4) for each subsystem is defined as $G_{k_i}u_i \leq c_{k_i}(r(t)), \quad k_i = 1,...,li, \quad i = 1,...,n$. Consider the following ellipsoid for a selected $\varrho_i > 0$, that is

$$\Sigma_i = \{x_i(t) | x_i^T(t)Y_{ir}^{-1}x_i(t) \leq \varrho_{ir}, \ Y_{ir} = Y_{ir}^T > 0\} \quad (10)$$

If the stability condition (8) is satisfied, then from the definition of the Lyapunov-Krasovskii functional $V(x_t, r_t)$ (refer to Appendix A - Proof of Theorem 3.1), it follows that $x_i^T(t)Y_{ir}^{-1}x_i(t) \leq V(x_t, r_t)$. Therefore, $\mathbb{E}[x_i^T(t)Y_{ir}^{-1}x_i(t)] \leq \mathbb{E}[V(x_t, r_t)]$.

On the other hand, by integrating $J_1 < 0$ defined in (44) from 0 to $t$ one gets

$$\begin{aligned} \mathbb{E}[V(x_t, r_t)] &\leq \mathbb{E}[-\int_0^t z^T(t)z(t)dt + \int_0^t \gamma w^T(t)w(t)dt] \\ &+ V(x_0, r_0) \leq \gamma L_1 + L_2 \end{aligned} \quad (11)$$

where $L_1 = \int_0^\infty w_i^T(t)w_i(t)dt$ is an upper bound on the energy of the external input $w_i(t)$, and $L_2 = V(x_0, r_0)$ . Therefore, $x_i(t)$ belongs to an invariant set $\Sigma_i$ if $\gamma L_1 + L_2 \leq \varrho_{ir}$.

Furthermore, from Theorem 1 the state feedback controller $u_i$ is given by $u_i = M_{ir}Y_{ir}^{-1}x_i$. Therefore, (4) can be rewritten as $G_{kir}M_{ir}Y_{ir}^{-1}x_i \leq c_{kir}$. Now, squaring this inequality yields $x_i^T(t)(G_{kir}M_{ir}Y_{ir}^{-1})^T G_{kir}M_{ir}Y_{ir}^{-1}x_i(t) \leq c_{kir}^2$. Furthermore, $x_i^T(t)Y_{ir}^{-1}x_i(t) \leq \varrho_{ir}$ implies that to satisfy the last inequality it suffices to show that

$$(G_{kir}M_{ir}Y_{ir}^{-1})^T(\varrho_{ir}/c_{kir}^2)G_{kir}M_{ir}Y_{ir}^{-1} \leq Y_{ir}^{-1} \quad (12)$$

Hence, by applying the Schur complement to (12), the capacity constraints for our mobile multi-agent network can be expressed as the LMI conditions

$$W_{c1ir} \triangleq \gamma \leq \max_{i,r}\{(\varrho_{ir} - L_2)/L_1\} \qquad r = 1,...,M \quad (13)$$

$$W_{c2irk_i} \triangleq \begin{bmatrix} Y_{ir} & M_{ir}^T G_{kir}^T \\ G_{kir}M_{ir} & c_{kir}^2/\varrho_{ir} \end{bmatrix} \geq 0 \qquad k_i = 1,...,li, i = 1,...,n \quad (14)$$

*2) Upper Bound on the Buffer Size:* The constraint on the queue buffer size (5) for each subsystem can be expressed as

$$Q_{di}x_i \leq x_{max_{di}}, \ d = 1,...,\bar{d}, \ i = 1,...,n \quad (15)$$

Following along the similar lines as those used for the capacity constraint and considering the ellipsoid (10), equation (15) can be satisfied through the following LMI conditions

$$W_{c3ir} \triangleq \begin{bmatrix} Y_{ir} & Y_{ir}^T Q_{di}^T \\ Q_{di} Y_{ir} & x^2_{max_{di}}/\varrho_{ir} \end{bmatrix} \geq 0,$$
$$d = 1, ..., \bar{d}, i = 1, ..., n, r = 1, ..., M \quad (16)$$

*3) Non-negative Orthant Stability:* The non-negativity constraint (3) can be expressed in terms of the non-negative orthant stability condition that is given by the following theorem.

***Theorem** 2:* [35] The linear time-delayed system $\dot{x} = Ax(t) + A_d x(t - \tau(t))$ is non-negative if and only if $A \in \mathcal{R}^{n\times n}$ is essentially nonnegative, i.e., its off-diagonal entries are non-negative and $A_d \in \mathcal{R}^{n\times n}$ is non-negative, i.e., all its elements are non-negative. ■

When the state feedback controller $u_i = K_{ir} x_i$ is substituted into the dynamical model (1) and Theorem 2 is invoked it follows that condition (3) is obtained if the off-diagonal entries of $B_{ir} K_{ir}$ and all entries of $B_{dijr} K_{ir}$ are non-negative. By selecting the matrix $Y_{ir}$ to be a diagonal matrix and by setting $K_{ir} = M_{ir} Y_{ir}^{-1}$ for subsystem $i$ the (essential) non-negativity of $(B_{dijr} K_{ir})$ $B_{ir} K_{ir}$, which ensures the non-negativity constraint (3), can be expressed as

$$W_{c4ir} \triangleq (B_{ir} M_{ir})_{sm} \geq 0$$
$$s \neq m, i = 1, ..., n \quad (17)$$
$$W_{c5ir} \triangleq (B_{dijr} M_{jr})_{sm} \geq 0$$
$$m, s = 1, ..., \bar{d}, r = 1, ..., M, j \in \wp_r(i) \quad (18)$$

Once the non-negativity condition $x_i \geq 0$ is satisfied, the second non-negativity condition $u_i \geq 0$ as given by (2), can be easily satisfied if we specify $K_{ijr} > 0$. Therefore, by noting that $Y_{ir}$ is a diagonal positive definite matrix, (2) is satisfied if the following LMI condition holds

$$W_{c6ir} \triangleq M_{ir(sm)} \geq 0, \;\; s, m = 1, ..., \bar{d}$$
$$i = 1, ..., n, r = 1, ..., M \quad (19)$$

Note that since the elements of $B_{ir}$ are either $-1$ or 0, satisfying condition (19) results in a square matrix $B_{ir} M_{ir}$ with negative or zero elements. On the other hand, satisfying $W_{c4ir}$ leads to a diagonal negative definite matrix $B_{ir} M_{ir}$. This is also validated by the fact that the queues at each node are decoupled from each other. Therefore, $B_{ir} K_{ir}$ should always be diagonal. Moreover, since the elements of $B_{dijr}$ are either 1 or 0, satisfying condition (19) results in a square matrix $B_{dijr} M_{ir}$ with positive or zero elements. Therefore, $W_{c5ir}$ is trivially satisfied.

It is worth emphasizing that by merely satisfying the LMI conditions provided above one cannot yield a proper routing strategy without considering the LMI conditions that are given in Theorem 1. We are now in a position to summarize the above results in the following theorem.

***Theorem** 3:* A decentralized $\mathcal{H}_\infty$ routing control scheme for a mobile multi-agent network governed by the MJL system (1) is obtained by solving the following optimization problem:

$$\min_{M_{ir}, Y_{ir}, \; \bar{R}_{ir}, \; \bar{Q}_i} \gamma \quad (20)$$

subject to the selection of the positive definite matrices $Y_{ir}$, $\bar{R}_{ir}$, $\bar{Q}_i$, and the LMI conditions for $W_{ir1}, W_{ir2}$, $W_{c1ir}$, $W_{c2irk_i}$, $W_{c3ir}, W_{c4ir}$, and $W_{c6ir}$ for $i = 1, ..., n$, $r = 1, ..., M$, as expressed by equations (8), (9), (13), (14), (16), (17), and (19), respectively.

**Proof:** The proof follows along the constructive lines that are given in this section and is omitted due to space limitations. ■

***Remark** 2:* Note that the number of the LMI conditions in our proposed decentralized routing algorithm depends on the number of network modes. Therefore, by increasing the number of modes one also increases the number of the LMI conditions. However, since the dimension of the LMI conditions depends on the number of nodes, these dimensions do not increase at the same rate that the number of modes increases. This property of our proposed scheme makes the algorithm scalable in principle to large networks when compared to its centralized counterpart algorithms where the dimension of the LMI conditions depends also on the number of modes. Although, by increasing the number of possible network topologies ($M$) one needs to solve a larger number of LMI conditions, interestingly enough this does not however affect the number of the neighboring sets $M$ and the dimension of the LMI conditions.

***Remark** 3:* It should be noted that although each node implements its routing control strategy in a decentralized manner and based on local information from its neighboring set, agents nevertheless require a centralized communication mechanism for *only* broadcasting the changes that take place in their configurations or their topologies to the other agents. The amount of information that needs to be exchanged (which basically is the knowledge of the operating mode that is present at any given time) is minimal and impose no major restrictions in terms of practical considerations.

By representing the multi-agent network mobility by a MJL system will enable one to model the changes in the number of nodes resulting from *new* node additions or deletions. For instance, if in a given switching mode the number of nodes is increased by one, the LMI conditions corresponding to the new subsystem (node) is added and the corresponding connection matrices $B_{ir}$ and $B_{dijr}$ of all the nodes for which the new node is in their neighboring set will change accordingly. On the other hand, if the number of nodes (subsystem) is decreased by one, the corresponding LMI conditions of that node corresponding to the applicable mode is eliminated and the connection matrices $B_{ir}$ and $B_{dijr}$ of all the nodes for which the deleted node was in their neighboring set will change accordingly.

In the next section, our proposed mobile multi-agent network routing scheme is generalized to situations when the network is confronted with variable destinations.

## IV. $\mathcal{H}_\infty$ ROUTING CONTROL STRATEGY FOR MOBILE MULTI-AGENT NETWORKS WITH VARIABLE DESTINATION NODES

In multi-agent mobile networks occasionally the number of destination nodes may vary over time. In other words, certain destination nodes may have no external traffic at given periods of time. However, due to the network dynamics some messages may still be present in queues that should be routed to their destinations as quickly as possible. Moreover, in the dynamical model (1) the states are defined as queueing length at each node corresponding to a particular destination node. Therefore, the number of states depends on the active destination nodes. On the other hand, simply deleting the corresponding states associated with the inactive destinations can lead to loss of integrity and consistency of the overall network. This will also lead to dropping out of the leftover messages that are kept in the eliminated queues. To cope with these issues, we propose to model and represent the behavior of the network as a *singular MJL* system which is governed by the following expressions:

$$\begin{aligned} E(r(t))\dot{x}_i(t) &= B_i(r(t))u_i(t) + B_{w_i}(r(t))w_i(t) \\ &\quad + \sum_{j\in\wp_{r(t)}(i)} B_{dij}(r(t))u_j(t-\tau_{ji}(t)) \quad (21) \\ x_i(t) &= \phi_i(t), \quad t\in[-h_i,0], \quad h_i = max\{h_{ij}\} \\ z_i(t) &= C_i(r(t))x_i(t) \end{aligned}$$

where $E(r(t))$ is a diagonal matrix that is specified according to the following two scenarios:

(a) <u>Regular mobile networks:</u> In this case we have $E(r(t)) := E^I(r(t)) = I$; and

(b) <u>Varying number of destination nodes:</u> In this case some destination nodes become inactive. Therefore, $E(r(t)) := E^D(r(t)) = diag\{e_j(r(t))\}$ , where

$$e_j(r(t)) = \begin{cases} 1 & \text{if the queue is associated with an active destination node} \\ 0 & \text{if the queue is associated with an inactive destination node} \end{cases}$$

In other words, the activeness or inactiveness of a destination node can be characterized as a switching mode. Therefore, when a destination node becomes inactive (active), the network dynamics switches from regular to the singular (singular to the regular). On the other hand, in order to ensure the existence of a unique solution for the singular MJL system, regularity and impulse-free conditions should be investigated at each switching mode. Towards this end, the definitions of piecewise regularity and piecewise impulse-free conditions are now given below.

***Definition 3:*** [27], [25] The system $E(r(t))\dot{x}_i(t) = A_i(r(t))x_i(t) + B_i(r(t))u_i(t) + B_{w_i}(r(t))w_i(t)$ is said to be *piecewise regular* if the characteristic polynomial $det(sE_r - A_{ir})$ is not identically zero for $r = 1,...,M, i = 1,...,n$.

***Definition 4:*** [27], [36] The system $E(r(t))\dot{x}_i(t) = A_i(r(t))x_i(t) + B_i(r(t))u_i(t) + B_{w_i}(r(t))w_i(t)$ is said to be *piecewise impulse-free* if $deg(det(sE_r - A_{ir})) = rank(E_r)$ for $r = 1,...,M, i = 1,...,n$.

The following lemma provides a necessary and sufficient condition for satisfying the regularity and piecewise impulsive-free conditions.

***Lemma 2:*** [27] The MJL system (21) with the state feedback control law $u_i = K_{ir}x_i$ satisfies the piecewise regularity and piecewise impulse-free conditions if and only if $A_{cli} = B_{ir}K_{ir}$ and $A_{cli} + A_{dcli}$ is nonsingular where $A_{dcli} = \sum_{j\in\wp(i)} B_{dij}(r(t))K_{jr}$.

In the subsequent subsections in order to design a decentralized routing controller for the system (21), first the LMI constraints that simultaneously ensure robust stability and $\mathcal{H}_\infty$ performance of the closed-loop singular MJL system in the presence of time-varying delays are obtained. In addition, the physical constraints (2)-(5) are expressed as LMI feasibility conditions for the dynamical system (21).

### *A. A Decentralized $\mathcal{H}_\infty$ Control of Singular Time-Varying Delay System with Markovian Jump Dynamics*

Our first result below provides sufficient conditions for constructing a decentralized $\mathcal{H}_\infty$ state feedback routing controller of the form $u_i = K_{ir}x_i$ for the system (21).

***Theorem 4:*** The fluid flow model of a mobile multi-agent network governed by equation (21) with $w \in L_2[0,\infty)$ that satisfies Assumptions 1 and 2 is stochastically stabilizable, piecewise regular, and piecewise impulse-free if the decentralized state feedback routing control is designed as $u_i = K_{ir}x_i$ with an $L_2$-gain that is less than $\gamma$, and provided that there exist matrices $M_{ir}$, nonsingular matrices $Y_{ir}$ and symmetric positive definite matrices $\bar{R}_{ir}$, $\bar{Q}_i$ for $i = 1,...,n$, $r = 1,...,M$ such that the following LMI conditions hold

$$E_rY_{ir}^T = Y_{ir}E_r^T > 0$$

$$W_{ir1} = \begin{bmatrix} \theta_{ir1} & \theta_{ir2} & B_{w_{ir}} & Y_{ir}^TC_{ir}^T & \theta_{ir3} & m_{ir}M_{ir}^T & h_{ji}M_{ir}^T \\ * & \theta_{ir4} & 0 & 0 & 0 & 0 & 0 \\ * & * & -\gamma I & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & \theta_{ir5} & 0 & 0 \\ * & * & * & * & * & \theta_{ir6} & 0 \\ * & * & * & * & * & * & \theta_{ir7} \end{bmatrix} < 0 \quad (22)$$

$$W_{ir2} = \begin{bmatrix} M_{ir}^TB_{ir}^T + B_{ir}M_{ir} & \tilde{B}_{dir}\tilde{M}_{jr} & Y_{ir} \\ * & -2Y_{jr}+I & 0 \\ * & * & -I \end{bmatrix} < 0 \quad (23)$$

$$W_{ir3} = \begin{bmatrix} 2(1-m_{ir}\pi_{rr})I - \bar{Q}_i + m_{ir}\pi_{rr}\bar{R}_{ir} & \tilde{\pi}_r \\ * & \tilde{R}_{ir} \end{bmatrix} \geq 0 \quad (24)$$

where $\theta_{ir1} = M_{ir}^TB_{ir}^T + B_{ir}M_{ir} + \pi_{rr}E_rY_{ir}, \theta_{ir2} = \tilde{B}_{dir}\tilde{R}_{ir}, \theta_{ir3} = (\bar{\pi}_rY_{ir})^T, \theta_{ir4} = -(1-\bar{d}_{ji})\tilde{R}_{ir}, \theta_{ir5} = -diag\{Y_{i1},...,Y_{i(r-1)},Y_{i(r+1)},...,Y_{iM}\}, \theta_{ir6} = -m_{ik}\bar{R}_{ik}, \theta_{ir7} = -h_{ji}\bar{Q}_i, \bar{\pi}_r = [\sqrt{\pi_{r1}}E_1 \ ... \ \sqrt{\pi_{r(r-1)}}E_{(r-1)} \ \sqrt{\pi_{r(r+1)}}E_{(r+1)} \ ... \ \sqrt{\pi_{rM}}E_M]^T$, $\tilde{\pi}_r = [\sqrt{m_{i1}\pi_{r1}} \ ... \ \sqrt{m_{i(r-1)}\pi_{r(r-1)}} \ \sqrt{m_{i(r+1)}\pi_{i(r+1)}} \ ... \ \sqrt{m_{iM}\pi_{rM}}]^T$, and $m_{ir}$ = the number of subsystems that subsystem $i$ belongs to their $\wp_r(.)$ in mode $r$, $\tilde{R}_{ir} = diag_{j\in\wp_r(i)}\{\bar{R}_{jr}\}$, $\tilde{M}_{jr} = diag_{j\in\wp(i)}\{M_{jr}\}$, and $\tilde{B}_{dir} = vec\{B_{dijr}\}$, for $j \in \wp_r(i)$. The robust decentralized state feedback controller gain is given by $K_{ir} = M_{ir}Y_{ir}^{-1}$.

**Proof:** The proof is provided in Appendix B.

Note that the input signal $u_i$ at each node can always be defined such that the matrix $B_i(r(t))$ is block diagonal, i.e., $B_i(r(t)) = diag\{B_{1i}(r(t)),\ B_{2i}(r(t))\}$, where $B_{1i}(r(t))$ and $B_{2i}(r(t))$ correspond to the active and inactive destinations, respectively.

### B. LMI Conditions for Incorporating the Physical Constraints

The LMI conditions for guaranteeing the network physical constraints, as discussed in Section III-A, are now modified corresponding to the MJL system (21).

*1) Capacity Constraint:* To guarantee the capacity constraint for each subsystem we require

$$G_{k_i} u_i \quad \leq \quad c_{k_i}(r(t)) \qquad k_i = 1, ..., li, \quad i = 1, ..., n$$

Let us consider the following ellipsoid for a selected $\varrho_i > 0$, namely

$$\begin{aligned} \Sigma_i \quad &= \quad \{x_i(t) | \int_{t-\tau_{ij}}^{t} x_i^T(s) K_{ir}^T \bar{R}_{ir}^{-1} K_{ir} x_i(s) ds \\ &\leq \quad \varrho_{ir}, \bar{R}_{ir} = \bar{R}_{ir}^T > 0\} \end{aligned} \tag{25}$$

Provided that the stability conditions (22)-(24) are satisfied, from the definition of $V(x_t, r_t)$ (refer to Appendix B), it follows that $\int_{t-\tau_{ij}}^{t} x_i^T(s) K_{ir}^T \bar{R}_{ir}^{-1} K_{ir} x_i(s) ds \leq V(x_t, r_t)$. Therefore, $\mathbb{E}[\int_{t-\tau_{ij}}^{t} x_i^T(s) K_{ir}^T \bar{R}_{ir}^{-1} K_{ir} x_i(s) ds] \leq \mathbb{E}[V(x_t, r_t)]$. On the other hand, by integrating $J_1 < 0$ as defined in (44) (refer to Appendix A) from 0 to $t$ one gets, $\mathbb{E}[V(x_t, r_t)] \leq \mathbb{E}[-\int_0^t z^T(t) z(t) dt + \int_0^t \gamma w^T(t) w(t) dt] + V(x_0, r_0) \leq \gamma L_1 + L_2$, where $L_1 = \int_0^\infty w_i^T(t) w_i(t) dt$ is an upper bound on the energy of the external input $w_i(t)$ and $L_2 = V(x_0, r_0)$. Therefore, $x_i(t)$ belongs to an invariant set $\Sigma_i$ if $\gamma L_1 + L_2 \leq \varrho_{ir}$. Furthermore, given the state feedback controller $u_i = K_{ir} x_i$ the capacity constraint can be re-written as $G_{kir} K_{ir} x_i < c_{kir}$. By squaring the last expression, given $h_i = \max\{h_{ij}\}$, and integrating both sides of the expression from $t - \tau_{ij}$ to $t$, one gets

$$\int_{t-\tau_{ij}}^{t} x_i^T(s) (G_{kir} K_{ir})^T G_{kir} K_{ir} x_i(s) ds < h_i c_{kir}^2 \tag{26}$$

Note that $\int_{t-\tau_{ij}}^{t} x_i^T(s) K_{ir}^T \bar{R}_{ir}^{-1} K_{ir} x_i(s) ds \leq \varrho_{ir}$, therefore (26) will be satisfied if

$$G_{kir}^T (\varrho_{ir} / (h_i c_{kir}^2)) G_{kir} < \bar{R}_{ir}^{-1} \tag{27}$$

By applying the Schur complement to inequality (27) and Lemma 1 the capacity constraints for the subsystem $i$ can be expressed by the following LMI conditions

$$\begin{aligned} W_{c1ir} \triangleq \gamma \leq \max_{i,r} \{(\varrho_{ir} - L_2)/L_1\} \\ r = 1, ..., M \end{aligned} \tag{28}$$

$$\begin{aligned} W_{c2irk_i} \triangleq \begin{bmatrix} 2I - \bar{R}_{ir} & G_{kir}^T \\ G_{kir} & h_i c_{kir}^2 / \varrho_{ir} \end{bmatrix} \geq 0 \\ k_i = 1, ..., li, i = 1, ..., n \end{aligned} \tag{29}$$

*2) Upper Bound on the Buffer Size:* For each subsystem the constraint on the queue buffer size is governed by

$$Q_{di} x_i \leq x_{max_{di}},\ d = 1, ..., \bar{d},\ i = 1, ..., n \tag{30}$$

Now, in view of $u_i = M_{ir} Y_{ir}^{-1} x_i$, for the selected matrices $\bar{M}_{ir}$ one can obtain $(\bar{M}_{ir} M_{ir} Y_{ir}^{-1})^{-1} \bar{M}_{ir} u_i = x_i$. Therefore, (30) can be expressed as

$$\begin{aligned} Q_{di} (\bar{M}_{ir} M_{ir} Y_{ir}^{-1})^{-1} \bar{M}_{ir} u_i \leq x_{max_{di}} \\ d = 1, ..., \bar{d}, i = 1, ..., n, r = 1, ..., M \end{aligned} \tag{31}$$

By squaring (31) and integrating both sides of the expression from $t - \tau_{ij}$ to $t$, we get

$$\begin{aligned} \int_{t-\tau_{ij}}^{t} u_i^T(s) (Q_{di} (\bar{M}_{ir} M_{ir} Y_{ir}^{-1})^{-1} \bar{M}_{ir})^T \\ Q_{di} (\bar{M}_{ir} M_{ir} Y_{ir}^{-1})^{-1} \bar{M}_{ir} u_i(s) ds < h_i x_{max_{di}}^2 \end{aligned} \tag{32}$$

Note that from the capacity constraint inequality we have $\int_{t-\tau_{ij}}^{t} u_i^T(s) \bar{R}_{ir}^{-1} u_i(s) ds \leq \varrho_{ir}$, therefore (32) will be satisfied if

$$\begin{aligned} (Q_{di} (\bar{M}_{ir} M_{ir} Y_{ir}^{-1})^{-1} \bar{M}_{ir})^T (\varrho_{ir} / (h_i x_{max_{di}}^2)) \\ (Q_{di} (\bar{M}_{ir} M_{ir} Y_{ir}^{-1})^{-1} \bar{M}_{ir}) < \bar{R}_{ir}^{-1} \end{aligned} \tag{33}$$

By applying the Schur complement to (33) and using Lemma 1, the constraint on the queue buffer size for each subsystem in (30) can be guaranteed through the following LMI conditions

$$\begin{aligned} W_{c3ir} \triangleq \begin{bmatrix} 4I - 2Y_{ir} - (Q_{di}^T Q_{di}) \varrho_{ir} / (h_i x_{max_{di}}^2) & (\bar{M}_{ir} M_{ir})^{-1} \bar{M}_{ir} \\ \bar{M}_{ir}^T (\bar{M}_{ir} M_{ir})^{-T} & 2I - \bar{R}_{ir} \end{bmatrix} \geq 0, \\ d = 1, ..., \bar{d}, i = 1, ..., n, r = 1, ..., M \end{aligned} \tag{34}$$

*3) Non-Negative Orthant Stability:* Corresponding to the switching modes when the matrix $E_{ir}$ is full rank (i.e., associated with the regular dynamics) the respective LMI conditions for guaranteeing non-negativeness of the states are defined similar to the conditions (17) and (18). However, when $E_{ir}$ is a singular matrix, the state $x_i$ is partitioned into $x_i = [x_{i1}^T\ x_{i2}^T]^T$, where $x_{i1}$ is the queue associated with the active destination nodes and $x_{i2}$ is the queue associated with the inactive destination nodes. We furthermore partition the gain into $K_{ir} = [K_{ir1}\ \ K_{ir2}]$ with appropriate dimensions for $K_{ir1}$ and $K_{ir2}$ corresponding to the states $x_{i1}$ and $x_{i2}$, respectively. Note that the queueing dynamics of the inactive destinations do not receive any external stimuli, i.e., $w_{i2} = 0$. Consequently, the closed-loop dynamics of (21) can be expressed as

$$\begin{aligned} \dot{x}_{i1} \quad &= \quad B_{ir1} K_{ir1} x_{i1}(t) + \sum_{i=1}^{n} B_{dijr1} K_{jr1} x_{j1}(t - \tau(t)) \\ &\quad + B_{iw1} w_{i1}(t) \end{aligned} \tag{35}$$

$$\begin{aligned} 0 \quad &= \quad B_{ir2} K_{ir2} x_{i2}(t) \\ &\quad + \sum_{i=1}^{n} B_{dijr2} K_{jr2} x_{j2}(t - \tau(t)) \end{aligned} \tag{36}$$

By applying Theorem 2 to equation (35), the off-diagonal entries of $B_{ir1} K_{ir1}$ and all entries of $B_{dijr1} K_{jr1}$ should be non-negative. By selecting a positive definite matrix $Y_{ir1}$ to be a diagonal matrix and by setting $K_{ir} = M_{ir} Y_{ir1}^{-1}$, the (essential) non-negativity of $B_{dijr1} K_{jr1}$ and $B_{ir1} K_{ir1}$ can be expressed as

$$W_{c4ir} \triangleq (B_{ir1}M_{ir1})_{sm} \geq 0$$
$$s \neq m, i = 1, ..., n \quad (37)$$
$$W_{c5ir} \triangleq (B_{dijr1}M_{jr1})_{sm} \geq 0$$
$$m, s = 1, ..., \bar{d}, r = 1, ..., M, j \in \wp_r(i) \quad (38)$$

The above conditions ensure that the non-negativity constraint (3) is satisfied for equation (35). The non-negative orthant condition of equation (36) is achieved by determining $K_{ir2}$ such that $B_{dijr2}K_{jr2}$ is non-negative and $B_{ir2}K_{ir2}$ is negative. Defining a diagonal positive definite matrix $Y_{ir1}$, these conditions are expressed as

$$W_{c6ir} \triangleq (B_{ir2}M_{ir2})_{sm} \leq 0$$
$$i = 1, ..., n, s, m = 1, ..., \bar{d}, r \in \bar{\mathcal{S}}, \quad (39)$$
$$W_{c7ir} \triangleq (B_{dijr2}M_{jr2})_{sm} \geq 0$$
$$i, s, m = 1, ..., \bar{d}, r = 1, ..., M, j \in \wp_r(i) \quad (40)$$

where $\bar{\mathcal{S}}$ is the set of modes in which $E_{ir}$ is singular. Therefore, $W_{c4ir}$, $W_{c5ir}$ and $W_{c7ir}$ are the same for both the regular and the singular modes. However, $W_{c6ir}$ changes from regular modes to the singular modes. Specifically, the $W_{c6ir}$ conditions for both the singular and the regular modes are defined as

$$W_{c6ir} = \begin{cases} (B_{ir2}M_{ir2})_{sm} \leq 0 \\ i = 1, ..., n, \quad s, m = 1, ..., \bar{d}, \quad r \in \bar{\mathcal{S}} \\ (B_{ir2}M_{ir2})_{sm} \geq 0, \quad i = 1, ..., n, \\ s, m = 1, ..., \bar{d}, \quad r \in \mathcal{S} - \bar{\mathcal{S}} \end{cases} \quad (41)$$

Provided that the non-negativity condition $x_i \geq 0$ is satisfied, $u_i \geq 0$ is guaranteed if the following LMI conditions hold

$$W_{c8ir} \triangleq M_{ir(sm)} \geq 0$$
$$s, m = 1, ..., \bar{d}, \; i = 1, ..., n, r = 1, \ldots, M \quad (42)$$

Since the elements of $B_{ir}$ are either $-1$ or 0, satisfying the condition (42) results in a square matrix for $B_{ir}M_{ir}$ having negative or zero elements. Therefore, $W_{c6ir}$ is trivially satisfied for the singular modes. On the other hand, satisfying $W_{c6ir}$ for the regular modes and $W_{c4ir}$ lead to a diagonal negative definite matrix $B_{ir}M_{ir}$. This is also validated by the fact that the dynamics of the queueing system is expressed in a decentralized framework and the queues at each node are decoupled from each other. Moreover, since the elements of $B_{dijr}$ are either 1 or 0, $W_{c8ir}$, trivially guarantees that the conditions $W_{c5ir}$ and $W_{c7ir}$ are satisfied.

To summarize, the following theorem states our robust $\mathcal{H}_\infty$ routing control strategy corresponding to the mobile multi-agent network (21) that satisfies the associated physical constraints (2)-(5).

***Theorem** 5:* A decentralized $\mathcal{H}_\infty$ routing control design for a mobile multi-agent network that is governed by the MJL system (21) is obtained by solving the following optimization problem:

$$\min_{M_{ir}, Y_{ir}, \; \bar{R}_{ir}, \; \bar{Q}_i} \gamma \quad (43)$$

subject to the selection of the positive definite matrices $\bar{R}_{ir}$, $\bar{Q}_i$, and the LMI conditions for $W_{ir1} - W_{ir3}$, $W_{c1ir}$, $W_{c2irk}$, $W_{c3ir}$, $W_{c4ir}$, $W_{c6ir}$ and $W_{c8ir}$ for $i = 1, ..., n$, $r = 1, ..., M$, as expressed by equations (22)-(24), (28), (29), (34), (37), (41), and (42), respectively.

**Proof:** Proof follows along the constructive lines that are derived earlier in this section. ■

## V. Simulation Results

In this section, simulation results are presented to evaluate the performance of our proposed $\mathcal{H}_\infty$ routing control strategy in mobile multi-agent networks. Agents in a mobile network generally move in teams. Furthermore, nodes within a team should exchange information among one another and also communicate with other specific nodes that are designated as commanders or supervisors of the entire network. In the first example considered below a mobile multi-agent network that consists of 50 nodes is investigated. By using the QualNet software environment, which is a commercially advanced scalable and high-fidelity emulator and simulator for network performance [37], the routing algorithm developed in Section III is now compared with two commonly used algorithms in the literature known as the Ad hoc On Demand Distance Vector (AODV) [30] and the Optimized Link State Routing Protocol (OLSR) [31] schemes. In the second example considered below we employ our routing algorithm developed in Section IV to present simulation results for a mobile multi-agent network consisting of 20 nodes where the number of the destination nodes are allowed to change.

**Example 1:** Let us consider a scenario of a network of unmanned systems having 50 mobile nodes that are partitioned into three teams covering an area of $8000m \times 12000m$. The first team $T_1$ which includes the nodes $1-10$ is stationary (fixed), the second team $T_2$ which includes the nodes $11-30$ moves towards the north-east direction, and the third team $T_3$ which contains the nodes $31-50$ moves towards the north direction. It is assumed according to Assumption 1 that the network graph *during the simulation period* of interest here remains connected. The nominal communication range for each node is considered to be 484 *m*, its capacity is 1 Mbps, and its maximum buffer size is 450 kbit. The transition mode is selected as $\pi_{rj} = 0.002$ for $r = j \pm 1$. The total simulation duration is selected as 700 $s$ for each run. The destination nodes are selected to be 7 and 10. Therefore, each node has two states: the first state is the queueing length associated with the destination node 10, and the second state is the queueing length associated with the destination node 7. Node 10 does not route any messages and is considered as a sink. Therefore, there are 49 subsystems and a total of 97 states (i.e. 49 queues corresponding to the destination node 10 and 48 queues corresponding to the destination node 7). Associated with each input flow the delay function is taken as a time-varying function specified by $\tau(t) = 3 + 0.8|sin(t)|s$. Note that as far as the controller is concerned the delay information is considered to be *unknown*. A total of 9 *switching modes* ($M = 9$) can be identified based on the changes that take place in the neighboring sets due to the mobility of the nodes.

The following three representative cases are considered for evaluating the performance of our robust $\mathcal{H}_\infty$ routing control scheme.

*Case A:* Messages Received and Lost Under Different Node Mobility

The traffic load for each node is based on the well-known Poisson distribution with the rate of $\lambda = 300$ bytes per second for 600 $s$. The total messages that are applied to the network is $139,680$ kbit. We assume that the maximum nodes speeds are 0, 10 $m/s$ and 20 $m/s$ for the agents in teams one, two and three, respectively. Simulations are repeated when the maximum nodes speeds are increased by factors of two and three times of the above values. The maximum speed of the second team is used as a benchmark for comparison studies. Fig. 1 depicts the total messages that are received at the destination nodes 10 and 7 as a function of the second team maximum speed by using (a) our proposed routing algorithm, (b) the AODV algorithm, and (c) the OLSR algorithm. The percentage of total messages that are lost corresponding to the $139,680$ kbit traffic load are shown in Table I. Theses results confirm that by increasing the speed of the nodes the proportion of the dropped messages is also increased in general. It also illustrates that our proposed scheme can route messages with fewer losses as compared to both the AODV and the OLSR methods.

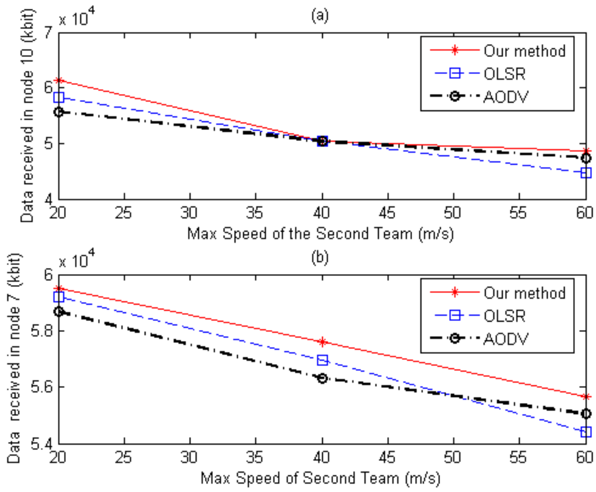


Fig. 1: The received messages at the destination nodes 10 and 7 (subplots (a) and (b), respectively) corresponding to different speeds and different routing algorithms in Example 1 (namely, our proposed robust $\mathcal{H}_\infty$ control routing algorithm (solid lines with star), the AODV algorithm (dashed-dot lines with circles) and the OLSR algorithm (dashed lines with squares)).
figure

TABLE I: The percentage of the messages that are lost corresponding to a $139,680$ kbit traffic load for different node speeds.
table

| Second team max speed (m/s) | 20 | 40 | 60 |
|---|---|---|---|
| % of lost data by using our proposed method | 13.49 | 22.75 | 25.32 |
| % of lost data by using the OLSR [31] method | 15.92 | 23.17 | 28.98 |
| % of lost data by using the AODV [30] method | 18.12 | 23.67 | 26.7 |

TABLE II: The percentage of the messages that are lost corresponding to different traffic loads ($l_1 = 4656$ $(kbit)$, $l_2 = 13968$ $(kbit)$, $l_3 = 46560$ $(kbit)$, $l_4 = 139680$ $(kbit)$, $l_5 = 279360$ $(kbit)$) at the maximum speed of 20 $m/s$ for the second team.
table

| Traffic load | $l_1$ | $l_2$ | $l_3$ | $l_4$ | $l_5$ |
|---|---|---|---|---|---|
| % of lost traffic by using our proposed method | 2.06 | 2.8 | 12.06 | 13.49 | 17.8 |
| % of lost traffic by using the AODV [30] method | 2.1 | 4.26 | 35.49 | 18.12 | 38.25 |
| % of lost traffic by using the OLSR [31] method | 4.64 | 4.55 | 38.9 | 15.92 | 23.93 |

*Case B:* Messages Received and Lost Under Different Traffic Loads

The performance of our proposed robust $\mathcal{H}_\infty$ routing control scheme is now evaluated subject to different traffic loads with rates of $\lambda = \{10 \quad 30 \quad 100 \quad 300 \quad 600\}$ bytes per second, when the nodes maximum speeds are set to $0,\ 10$, and 20 *m/s* for teams one, two and three, respectively. Fig. 2 depicts the total messages that are received at the destination nodes 10 and 7 as a function of different traffic loads by using (a) our proposed routing control algorithm, (b) the AODV algorithm, and (c) the OLSR algorithm. The percentage of lost messages are shown in Table II. Fig. 2 and Table II do indeed confirm the performance superiority of our proposed routing control algorithm in dealing with different traffic loads.

*Case C:* Maximum Queueing Length

The maximum queueing length of a node in a mobile multi-agent network can also be considered as an important issue for evaluating the performance of a routing algorithm. Fig. 3 depicts the maximum queueing lengths that are obtained by using our proposed $\mathcal{H}_\infty$ routing control scheme, the OLSR algorithm, and the AODV algorithm for the input rate of $\lambda = 300$ bytes per second when the maximum speed of the second team is set to 20 $m/s$.

As expected the AODV scheme, which is a reactive algorithm, keeps the messages longer in the queues for determining the optimal routes. However, the maximum queueing length obtained by using our proposed routing is comparable with the OLSR algorithm, in terms of both the distribution as well as the individual values. Moreover, the mean of the maximum queue obtained by using our proposed $\mathcal{H}_\infty$ routing control algorithm is $48.56$ kbit whereas it is $51.71$ kbit by using the OLSR algorithm and it is $127.41$ kbit by using the AODV algorithm. This confirms that overall our proposed routing control algorithm provides the shortest queues on average.

In closing, we would like to point out that indeed one could have explicitly incorporated in the originally considered performance index the packet losses as an attribute to be also

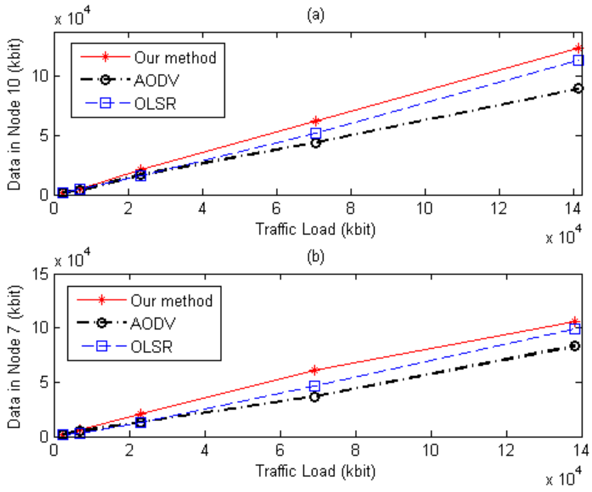


Fig. 2: The received messages at the destination nodes 10 and 7 (subplots (a) and (b), respectively) corresponding to different traffic loads and different routing algorithms for Example 1 (namely, our proposed robust $\mathcal{H}_\infty$ control routing algorithm (solid lines with star), the AODV algorithm (dashed-dot lines with circles), and the OLSR algorithm (dashed lines with squares)).
figure

simultaneously minimized. However, a formal development of this solution is not considered here as it is beyond the scope of this work. Notwithstanding this, it should be emphasized that the framework introduced in this paper enables one to generalize our results in various directions and aspects.

***Remark** 4:* The AODV scheme is implemented for networks where the topology can change randomly and possible topologies cannot be known in advance. Such an algorithm requires the communication to be postponed until a route to the destination is found. It is also assumed that during the routing process the topology does not change. Therefore, the messages may wait in the buffers longer as compared to pre-established routing algorithms. Moreover, there is a possibility that messages can be dropped if the topology is changed during the routing process. On the other hand, our proposed algorithm provides traffic routes that are based on knowledge of *all* possible topologies (although one *does not* need to know *a priori* when a given topology is going to change and which topology will be selected/chosen next, i.e., the choice of the network topology and the time of this change is assumed to be *unknown* and *random*). Therefore, the overhead for determining the routing solutions for messages is lower, and consequently the delays in delivering the messages to their destinations are less than that of the AODV algorithm. Furthermore, since our routing strategy guarantees that the messages are routed to their destinations for all random switchings among the topologies, the likelihood of dropping messages will also be lower. Therefore, by formally incorporating and taking advantage of this information, albeit that our approach is not applicable to all general types of wireless networks, one

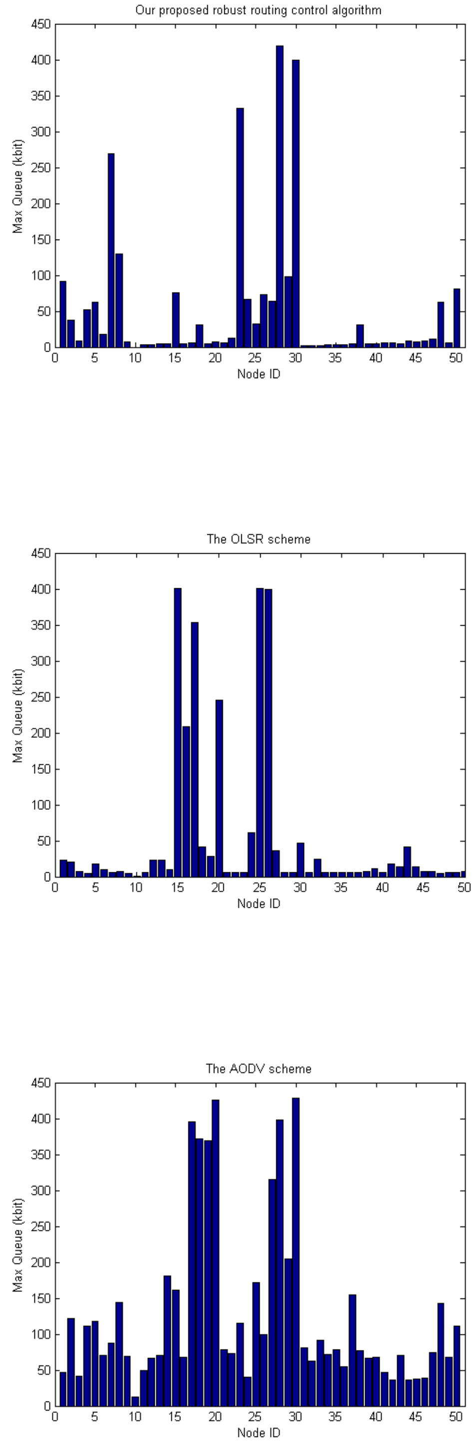


Fig. 3: The maximum queueing length corresponding to the input rate of $\lambda = 300$ bytes per second and the maximum speed of the second team selected as 20 $m/s$ as in Example 1. Shown from left to right are our proposed routing control scheme, the OLSR scheme, and the AODV scheme.
figure

can guarantee improved performance from using our approach.

**Example 2:** In this example, the performance of our proposed $\mathcal{H}_\infty$ routing control scheme subject to a variable number of destination nodes is investigated. Towards this end, let us consider a network of unmanned vehicles consisting of 20 mobile nodes that are partitioned into two teams in an area of $8000m \times 12000m$. The first team includes the nodes 1 to 10 that moves towards the east direction with the speed of 5 $m/s$ and the second team includes the nodes 11 to 20 that moves towards the north-east direction with the speed of 25 $m/s$.

As in the previous example, and according to Assumption 1 the network graph remains connected during the simulation period of interest. The nominal communication range for each node is considered to be 450 $m$ and the channel capacity is set to 1 Mbps. The node's maximum buffer size is set to 450 kbit. The pause time is set to 200 $s$ and the total time for each simulation is considered to be 500 $s$.

The packet generation rate for each node is based on the well-known Poisson distribution with the rate of $\lambda = 200$ bytes per second for 400 $s$. The transition mode is selected as $\pi_{rj} = 0.002$ for $r = j \pm 1$. Nodes 7 and 10 are selected as the destination nodes. Therefore, each mode should have two states: the first state is the queue associated with the destination node 10 and the second state is the queueing length associated with the destination node 7. For each input flow the delay function is taken as a time-varying function specified by $\tau(t) = 1 + .1|sin(t)|$ $s$, which is clearly considered to be *unknown* to the controllers.

It is assumed that in certain time periods the destination node 7 is not active. Corresponding to the first three modes, the switching occurs as a result of nodes mobility and changes in the neighboring sets. In other words, according to the network model given by (21) we have $E_{i1} = E_{i2} = E_{i3} = I$. Corresponding to the 4th mode, it is assumed that the destination node 7 is inactive. Consequently, in the model (21) we have $E_{i4} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$. Our objective is to demonstrate that through modeling the inactive destination nodes by the singular Markovian jump dynamics and by utilizing our proposed $\mathcal{H}_\infty$ routing control algorithm one will be able to empty their associated queues more efficiently as compared to a control algorithm that does not explicitly take into account this characteristics. Specifically, the simulation results for our $\mathcal{H}_\infty$ routing controller that is designed according to Theorem 5 for a singular MJL system is now compared with the performance that is achieved by using our $\mathcal{H}_\infty$ routing controller that is designed based on Theorem 3 for a regular MJL system where the fact that node 7 is actually inactive for some periods of time is *completely ignored*. Fig. 4 depicts the queueing lengths of node 3 for the destination nodes 10 and 7 using our proposed algorithms based on the singular MJL dynamics (sub-figures (a)-(c)) and the regular MJL dynamics (sub-figures (d)-(f)).

Figs. 4-(a) and 4-(d) depict the queueing lengths for the destination node 10 and Figs. 4-(b) and 4-(e) depict the queueing lengths for the destination node 7 for the entire simulation period of 500 $s$. This confirms the stable behavior of our proposed $\mathcal{H}_\infty$ routing algorithms for both regular as well as singular MJL systems. Figs. 4-(c) and 4-(f) depict the behavior of our proposed routing algorithms when the destination node 7 becomes inactive at $t = 220$ $s$. It shows that the controller based on the singular MJL dynamics could empty the queue in 4 seconds. However, by applying the controller based on the regular MJL dynamics, it took 24 seconds to route the messages and empty the queues. On the other hand, due to the physical capacity constraints by decreasing the queueing length of one destination node the queueing length of other destination nodes will be increased. Therefore, application of the singular MJL dynamics will be more suitable when the other destination nodes have lower priorities in receiving the messages.

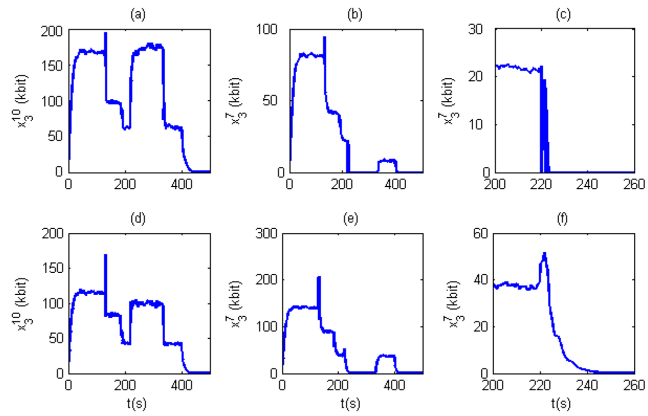


Fig. 4: The queueing lengths of node 3 for the destination nodes 10 and 7 in Example 2. The subplots (a)-(b) are obtained by using the singular MJL dynamics whereas the subplots (d)-(e) are obtained by using the regular MJL dynamics. Subplots (c) and (f) are the zoomed versions of subplots (b) and (e) around 220 seconds, respectively.
figure

## VI. Conclusion

In this paper, a Markovian jump linear (MJL) system subject to unknown and time-varying delays and mode-dependent interconnections (matrices) was introduced to represent the data traffic in mobile multi-agent networks. Our proposed decentralized $\mathcal{H}_\infty$ routing control strategy can simultaneously stabilize the mobile multi-agent network and provide a desired routing performance by minimizing a measure of the queueing length (namely, the *worst-case queueing length*) subject to a number of physical constraints. By taking advantage of the Markovian jump representation of the agents mobility one can stochastically handle the changes that occur in the network topologies and configurations due to both node mobility and random changes in the destination nodes. The LMI methodology is utilized for developing a unified multi-objective optimization framework where the multi-agent network routing problem can be represented and solved in a *decentralized* manner.

Although the proposed $\mathcal{H}_\infty$ routing strategy assumes that *all the possible network topologies* are *finite* and known *a priori*, the manner and the time at which the network topology changes is not known *a priori*. In other words, we assume in this work that the network topology changes are made arbitrarily and randomly within a given list of possible configurations. This assumption is actually quite consistent with real networked multi-agent systems such as a team of unmanned vehicles. Our proposed decentralized $\mathcal{H}_\infty$ control strategy could also be applied to systems where the number of the states can change as in the problem of formation control or cooperative control of unmanned systems where agents may be added or removed from a given team.

It should be noted that by performing some further analysis our proposed decentralized $\mathcal{H}_\infty$ control routing scheme can be extended to MJL systems which contain uncertain terms in their transition dynamics. Finally, it should be noted that other models have been proposed in the literature to model the mobility of nodes in ad hoc mobile networks, such as random way point and Gauss-Markov, among others. To further bring out the versatility of our proposed routing strategy, comparisons of the reported results and their performances with these mobility models will be investigated in our future work.

## VII. Acknowledgement

The authors would like to sincerely express their appreciations to the reviewers whose comments and suggestions have significantly improved the paper readability.

## Appendix I
## Proof of Theorem 1

Let us denote $\mathcal{C}[-h_{ji},0]$ as the space of continuous functions on the interval $[-h_{ji},0]$. Since the evolution of $x_i(t)$ in (1) depends on $x_i(s),\ t-\tau_{ij} \le s \le t$, it is not a Markov process. In order to cast this model into the framework of a Markov process, let us define a process in $\mathcal{C}[-h_{ji},0]$ by $x_{is}(t) = x_i(s+t), \quad t-\tau_{ij} \le s \le t$. The stability of the closed-loop system (1) with $u_i = K_{ir}x_i$, is investigated by considering the Lyapunov-Krasovskii functional candidate $V(x_t,r_t) = V_1+V_2+V_3$, where $V_1 = \sum_{i=1}^n x_i^T(t)P_{ir_t}x_i(t), V_2 = \sum_{i=1}^n \sum_{j\in\wp_{r_t}(i)} \int_{t-\tau_{ji}}^t u_j^T(s)R_{jr_t}u_j(s)ds$, and $V_3 = \sum_{i=1}^n \int_0^{h_{ij}}(h_{ij}-\sigma)u_i^T(t-\sigma)Q_iu_i(t-\sigma)d\sigma$. To achieve the $\mathcal{H}_\infty$ objective (7), one should show that

$$J_1 = \mathcal{A}V(x_t,r_t) + z^T(t)z(t) - \gamma w^T(t)w(t) < 0 \qquad (44)$$

where $\mathcal{A}$ is the infinitesimal generator of $\{(x_{it},r_t), t \ge 0\}$ [27]. Therefore, one can get $\mathcal{A}V(x_t,r_t) = \mathcal{A}V_1(x_t,r_t) + \mathcal{A}V_2(x_t,r_t) + \mathcal{A}V_3(x_t,r_t)$. Suppose $r_t = k \in S$, then $\mathcal{A}V_1(x_t,r_t) = \sum_{i=1}^n [x_i^T(t)((B_{ir_t}K_{ir_t})^TP_{ir_t} + P_{ir_t}(B_{ir_t}K_{ir_t}))x_i(t) + \sum_{j\in\wp_{r_t}(i)}[u_j^T(t-\tau_{ji})B_{dijr_t}^TP_{ir_t}x_i(t) + x_i^T(t)P_{ir_t}B_{dijr_t}u_j(t-\tau_{ji})] + x_i^T(t)P_{ir_t}B_{w_{ir_t}}w_i(t) + w_i^T(t)B_{w_{ir_t}}^TP_{ir_t}x_i(t) + x_i^T(t)\sum_{k=1}^M \pi_{r_tk}P_{ik}x_i(t)]$, $\mathcal{A}V_2(x_t,r_t) \le \sum_{i=1}^n \sum_{j\in\wp_{r_t}(i)}[u_j^T(t)R_{jr_t}u_j(t) - (1-\bar{d}_{ji})u_j^T(t-\tau_{ji})R_{jr_t}u_j(t-\tau_{ji})]$
$+ \sum_{i=1}^n\sum_{k=1}^M\sum_{j\in\wp_k(i)}\pi_{r_tk}\int_{t-\tau_{ji}(t)}^t u_j^T(s)R_{jk}u_j^T(s)ds$, and $\mathcal{A}V_3(x_t,r_t) \le \sum_{i=1}^n[h_{ij}u_i^T(t)Q_iu_i(t) - \int_{t-\tau_{ij}(t)}^t u_i^T(s)Q_iu_i(s)ds]$. Now assuming

$$\sum_{i=1}^n \int_{t-\tau_{ij}(t)}^t u_i^T(s)Q_iu_i(s)ds \ge \sum_{i=1}^n\sum_{k=1}^M \pi_{r_tk} \sum_{j\in\wp_k(i)} \int_{t-\tau_{ji}(t)}^T u_j^T(s)R_{ik}u_j^T(s)ds \qquad (45)$$

and using the fact that $\sum_{i=1}^n \sum_{j\in\wp_{r_t}(i)} u_j^T(t)R_{jr_t}u_j(t) = \sum_{i=1}^n m_{ir_t}u_i^T(t)R_{ir_t}u_i(t)$, and by applying the Schur complement, and substituting the result into (44), one gets $J_1 \le \sum_{i=1}^n X_i^T(t)\bar{L}_{ir_t}X_i(t)$, where

$$\bar{L}_{ik} = \begin{bmatrix} \Omega_{i1} & \Omega_{i2} & P_{ik}B_{w_{ik}} & C_{ik}^T & \Omega_{i3} \\ * & \Omega_{i4} & 0 & 0 & 0 \\ * & * & -\gamma I & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & \Omega_{i5} \end{bmatrix} < 0 \qquad (46)$$

and $X_i = [x_i^T(t) \quad U_j^T(t-\tau_i(t)) \quad w_i^T(t)]^T, U_j^T(t-\tau_i(t)) := vec\{u_j^T(t-\tau_{ji}(t))\}\ for\ j \in \wp_{r_t}(i), \Omega_{i1} = (B_{ik}K_{ik})^TP_{ik} + P_{ik}(B_{ik}K_{ik}) + K_{ik}^T[h_{ji}Q_i + m_{ik}R_{ik}]K_{ik} + \pi_{kk}P_{ik}, \Omega_{i2} = P_{ik}\tilde{B}_{dik}, \Omega_{i3} = [\sqrt{\pi_{k1}}P_{i1} \dots \sqrt{\pi_{k(k-1)}}P_{i(k-1)}\ \sqrt{\pi_{k(k+1)}}P_{i(k+1)} \dots \sqrt{\pi_{kM}}P_{iM}]^T$, $\Omega_{i4} = -diag_{j\in\wp_k(i)}\{(1-\bar{d}_{ji})R_{jk}\}, \Omega_{i5} = -diag\{P_{i1},...,P_{i(k-1)},P_{i(k+1)},...,P_{iM}\}$. Now, let $K_{ik} = M_{ik}Y_{ik}^{-1},\ P_{ik} = Y_{ik}^{-1},\ R_{ik} = \bar{R}_{ik}^{-1},\ Q_i = \bar{Q}_i^{-1},\ \bar{\bar{R}}_{ik} = vec\{\bar{R}_{jk}\}\ for\ j \in \wp_k(i),\ \bar{Y}_i = vec\{Y_{il}\}\ for\ l = 1,...,(p-1),(p+1),...,M$. By pre and post multiplying (46) by $\Delta_{ik} = diag\{Y_{ik},\ \bar{\bar{R}}_{ik},\ I,\ I,\ \bar{Y}_i\}$ and $\Delta_{ik}^T$ respectively, and applying the Schur complement one gets $L_{ik}$ in (8). Therefore, (8) guarantees negative definiteness of $J_1$ in (44). From Dynkin's formula [27], we have $J_T = \mathbb{E}[\int_0^T[J_1 - \mathcal{A}V(x_t,r_t)dt] \le \mathbb{E}\int_0^T\sum_{i=1}^n X_i^T(t)L_{ir_t}X_i(t)dt - \mathbb{E}[V(x_T,r_T) + V(x_0,r_0)]$. Using the fact that $L_{ir_t} < 0$ and $\mathbb{E}[V(x_T,r_T)] > 0$, yield $J_T \le V(x_0,r_0)$, and therefore $J_\infty \le V(x_0,r_0)$. In other words, the $\mathcal{H}_\infty$ objective (7) is satisfied according to $\|z\|_{\mathbb{E}_2} - \gamma^2\|w(t)\|_2 \le V(x_0,r_0)$.

To investigate the stability properties of the network states in absence of the external inputs $w_i$, let us eliminate the $(n+2)$th row and column (corresponding to the terms involving $w_i$) and the $(n+3)$th row and column of (8). Therefore, the following LMI condition is obtained

$$\hat{L}_{ik} = \begin{bmatrix} \theta_{ir1} & \theta_{ir2} & \theta_{ir3} & m_{ik}M_{ik}^T & h_{ji}M_{ik}^T \\ * & \theta_{ir4} & 0 & 0 & 0 \\ * & * & \theta_{ir5} & 0 & 0 \\ * & * & * & \theta_{ir6} & 0 \\ * & * & * & * & \theta_{ir7} \end{bmatrix} < 0$$

which implies that $\mathcal{A}V(x,r,t) < \sum_{i=1}^n \bar{X}_i^T(t)\hat{L}_{ir1}(h_{ji},\bar{d}_{ji})\bar{X}_i(t) < 0$. Hence, we have $\mathcal{A}V(x,r,t) \le -\alpha\sum_{i=1}^n\|\bar{X}_i\|^2 \le -\alpha\|x_t\|^2$, where $\alpha = \min_{i,r}\{\lambda_{min}(-\hat{L}_{ir1})\} > 0$. By applying the Dynkin's formula, one can get $\mathbb{E}[V(x(t),r(t))] - \mathbb{E}[V(x_0,r_0)] = \mathbb{E}\int_0^t[\mathcal{A}V(x,r_t)]ds \le -\alpha\mathbb{E}\int_0^t x^T(s)x(s)ds$. Since $\mathbb{E}[V(x(t),r(t))] \ge 0$, the above equation implies that

$\mathbb{E}\int_0^t x^T(s)x(s)ds \leq \alpha^{-1}\mathbb{E}[V(x_0, r_0)]$. This proves the stochastic stability of the unforced system (1).

In view of the above results condition (45) should now be expressed according to the new LMI parameters $\bar{Q}_i$ and $\bar{R}_{il}$. Substituting $Q_i$ and $R_{il}$ by $\bar{Q}_i$ and $\bar{R}_{il}$ in (45) yields

$$\sum_{i=1}^{n}\int_{t-\tau_{ij}(t)}^{t} u_i^T(s)\bar{Q}_i^{-1}u_i(s)ds \geq \sum_{i=1}^{n}\sum_{k=1}^{M}\pi_{r_tk}\sum_{j\in\wp_k(i)} \int_{t-\tau_{ji}(t)}^{T} u_j^T(s)\bar{R}_{il}^{-1}u_j^T(s)ds$$

Using the fact that $\pi_{kk} = -\sum_{l=1,l\neq k}^{N}\pi_{kl}$, where $\pi_{kl} > 0$, and noting that $\sum_{i=1}^{n}\int_{t-\tau_{ij}(t)}^{t} u_i^T(s)\sum_{k=1}^{M}\pi_{r_tk}m_{ik}R_{ik}u_i(s)ds = \sum_{i=1}^{n}\sum_{k=1}^{M}\pi_{r_tk}\sum_{j\in\wp_{r_t}(i)}\int_{t-\tau_{ji}(t)}^{t} u_j^T(s)R_{jk}u_j(s)ds$, and applying the Schur complement we get

$$\begin{bmatrix} \bar{Q}_i^{-1} - \pi_{kk}\bar{R}_{ik}^{-1} & \bar{\pi}_k \\ * & \bar{R}_i \end{bmatrix} > 0 \tag{47}$$

Furthermore, using Lemma 1 and considering the fact that $\pi_{kk}\bar{R}_{ik} < 0$, lead to $\bar{Q}_i^{-1} - \pi_{kk}\bar{R}_{ik}^{-1} > 2(1-\pi_{kk})I - \bar{Q}_i + \pi_{kk}\bar{R}_{ik}$. Therefore, to guarantee (47), it suffices to satisfy the LMI condition (9). This completes the proof of the theorem. ■

## Appendix II
## Proof of Theorem 4

To achieve the $\mathcal{H}_\infty$ objective function (7), it suffices to establish the inequality (44) where $V(x_t, r_t)$ is now selected as $V(x_t, r_t) = V_1 + V_2 + V_3$, where $V_1 = \sum_{i=1}^{n} x_i^T(t)E_{r_t}P_{ir_t}x_i(t), V_2 = \sum_{i=1}^{n}\sum_{j\in\wp_{r_t}(i)}\int_{t-\tau_{ji}}^{t} u_j^T(s)R_{jr_t}u_j(s)ds$, and $V_3 = \sum_{i=1}^{n}\int_0^{h_{ij}}(h_{ij}-\sigma)u_i^T(t-\sigma)Q_iu_i(t-\sigma)d\sigma$, where $R_{jr_t}$ and $Q_i$ are positive definite matrices, and $E_kP_{ik}^T = P_{ik}E_k^T > 0$. Therefore, $\mathcal{A}V(x_t, r_t) = \mathcal{A}V_1(x_t, r_t) + \mathcal{A}V_2(x_t, r_t) + \mathcal{A}V_3(x_t, r_t)$, where $\mathcal{A}V_2(x_t, r_t)$ and $\mathcal{A}V_3(x_t, r_t)$ are obtained as in Appendix A and $\mathcal{A}V_1(x_t, r_t)$ is obtained as $\mathcal{A}V_1(x_t, r_t) = \sum_{i=1}^{n}[x_i^T(t)((B_{ir_t}K_{ir_t})^TP_{ir_t} + P_{ir_t}(B_{ir_t}K_{ir_t}))x_i(t) + \sum_{j\in\wp_{r_t}(i)}[u_j^T(t-\tau)B_{dijr_t}^TP_{ir_t}x_i(t) + x_i^T(t)P_{ir_t}B_{dijr_t}u_j(t-\tau)] + x_i^T(t)\sum_{k=1}^{M}\pi_{r_tk}E_kP_{ik}x_i(t)]$. By substituting the above into (7), and by following along the similar lines as those given in proof of Theorem 1, one can show that the LMI condition (22) can guarantee negative definiteness of $J$ in (7). The convergence of the network states in the absence of the external input $w_i$ can also be shown by eliminating the $(n+2)$th row and column (corresponding to the terms involving $w_i$) and the $(n+3)$th row and column of matrix (22) and by following along the constructive lines that were invoked in the proof of Theorem 1.

Assume that the condition below is satisfied

$$\begin{bmatrix} (B_{ik}K_{ik})^TP_{ik} + P_{ik}(B_{ik}K_{ik}) + I & P_{ik}\tilde{B}_{dik}\tilde{K}_{jk} \\ * & -I \end{bmatrix} < 0 \tag{48}$$

where $\tilde{K}_{jk} = diag_{j\in\wp_k(i)}\{K_{jk}\}$. Condition (48) implies that $(B_{ik}K_{ik})^TP_{ik} + P_{ik}(B_{ik}K_{ik}) < 0$. Therefore, $A_{cli}$ is nonsingular. Furthermore, by applying the Schur compliment it follows that $A_{cli} + A_{dcli}$ is nonsingular. Therefore, the closed-loop system satisfies the piecewise regularity and the piecewise impulsive mode free conditions. Now, by substituting $P_{ik} = Y_{ik}^{-1}$ and pre and post multiplying (48) by $diag\{Y_{ik}, \bar{\bar{Y}}_{ik}\}$ and its transpose, respectively, where $\bar{\bar{Y}}_{ik} = vec\{\bar{Y}_{jk}\}\ for\ j \in \wp_k(i)$, one can obtain (23). This completes the proof of the theorem. ■

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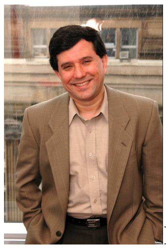

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