A Multi-agent Methodology for Distributed and Cooperative Supervisory Estimation Subject to Unreliable Information

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***Abstract*— In this work, a novel multi-agent framework for cooperative supervisory estimation of linear time-invariant (LTI) systems is proposed. This framework is developed based on the notion of *sub-observers* and a discrete-event system (DES) supervisory control and is applicable to a large class of systems. We introduce a group of sub-observers where each sub-observer is estimating certain states that are conditioned on a given input, output, and state information. The cooperation among the sub-observers is managed by a DES supervisor. The supervisor makes decisions regarding the selection and configuration of a set of sub-observers to successfully estimate all the system states, while the feasibility of the overall integrated cooperative sub-observers is verified. When certain anomalies (faults) are present in the system, or the sensors and sub-observers become unreliable, the supervisor reconfigures the set of selected sub-observers so that the impacts of anomalies on the estimation performance are minimized to the extent that is possible. The application of our proposed methodology in a practical industrial process is demonstrated through numerical simulations.**

***Index Terms*— Cooperative supervisory estimation; Distributed observations; Discrete-event systems; Faults and anomalies in sensors and systems; Cooperative sub-observers**

I. Introduction

Estimation techniques have been extensively investigated in the literature for various applications ranging from tracking and navigation [1] to fault diagnosis and recovery [2], [3]. Due to the need for developing distributed estimation methodologies in applications such as industrial processes and formation flying satellites, decentralized filtering techniques are of most significance and importance in this domain. Considerable effort has already been made to distribute local estimation filters throughout a process. In [4], [5], the overall system model is partitioned into several subsystems according to its physical characteristics and considerations. A local Kalman filter is designed for state estimation in each subsystem. Among the local Kalman filters, the common observations are fused using bipartite fusion graphs and consensus averaging algorithms. The performance is shown to be acceptable when compared to a centralized Kalman filter, while their method offers less communication overhead.

In [6], decentralized estimation algorithms are surveyed and applied to the problem of state estimation of formation flying satellites. Their simulation results show that decentralized reduced-order filters result in near optimal estimates while simultaneously balancing the constraints on the communication cost and the computation resources among the satellites. Moreover, a hierarchical architecture is presented to embed the decentralized estimators while scaling the problem to large fleet of vehicles. In [7] a method of cascaded Kalman filters is proposed for estimation of multiple biases for applications to ground vehicles.

In [8], the estimation problem is addressed through a parallel operation of full-order observers by using local measurements. A necessary condition on the communication topology is derived to guarantee the stability of simultaneous parallel estimators and controllers.

Another method for distributed estimation among local filters is based on consensus filters. The consensus problems in coordination of multi-agent systems are surveyed in [9]. The results cover both the cases of time-varying and time-invariant information exchange topologies. In [10], [11], distribution of Kalman filters is performed in a consensus framework for the special case of static systems. Furthermore, it is pointed out that the case of dynamical systems is an open topic of research. In [12], the case of distributed Kalman filtering for dynamical systems is tackled by using micro-Kalman filters, and the resulting estimates are fused by using a consensus scheme. It is shown that the consensus error would be within a finite error bound whose radius depends on the variation rate of the measured states. In [13], a distributed Kalman filter is proposed for actuator fault estimation of deep space formation flying satellites, and the performance is shown to be acceptable when compared to a centralized Kalman filter.

Supervisory control framework was proposed by Ramadge and Wonham [14], in which a discrete-event system (DES) is modeled as a generator of a formal language, and which can be controlled by an external supervisor through enabling or disabling certain events (transitions). This enablement or disablement of events is carried out to restrict the system behavior in order to satisfy a variety of criteria. Safety specifications such as avoidance of prohibited regions of the state-space or observation of service priorities could be incorporated as examples of these criteria. The application of DES in a switched control system is also proposed in [15].

Considerable research has already been devoted to the problem of distributed estimation [1], [4], [6], [7], [9]–[12]. In these works, predefined levels of process and sensor uncertainties are assumed and incorporated in the estimation algorithms that can be classified as *passive* estimation techniques. These works have not considered the presence of probable process and/or sensor faults, an aspect that is of interest in the present paper.

In [16], the problem of hybrid estimation of complex systems is investigated by using the probabilistic interactive multiple model (IMM) approach. In [17] and [18], the authors have proposed a *centralized* scheme for fault tolerant estimation in the general case of nonlinear systems. The approach

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in [17] and [18] cannot be readily applied to the distributed estimation of a dynamic system due to the fact that these works are not based on a distributed estimation framework in which distributed estimation modules are appropriately defined and designed. In the distributed fault tolerant estimation problem the overall model is distributed among multiple local agents and the system information is cooperatively provided by local agents. This information is partially available to the local agents due to the communication constraints among them. Furthermore, in [17] and [18] the developed method is based on the "differential algebra" theory [19], in which multiple derivatives of the output sensor and input signals are required. However, sensor and measurement noise have significant negative impacts on the performance of the multiple differential operators, which is one of the main drawbacks of the differential algebra theory [20].

In this paper, we propose a novel multi-agent based methodology for supervisory cooperative estimation of linear time-invariant (LTI) systems in presence of permanent process and/or sensor faults and other unreliabilities in obtaining system information. In this cooperative framework, a group of sub-observers where each is estimating certain states given (conditioned on) the availability of certain inputs, outputs, and state estimates are designed and developed. The cooperation among the sub-observers is supervised by a DES. The DES supervisor makes decisions regarding the selection and configuration of a set of sub-observers to successfully estimate all the system states. Moreover, in case that certain anomalies, system faults, or unreliabilities in sensor measurements are present, the supervisor reconfigures the set of selected sub-observers so that the impacts of these anomalies on the estimation performance are minimized to the extent that is possible.

Our present methodology is different from the other work on multi-agent systems such as those in [21], [22]. Furthermore, since the main objective of our work is actually not to provide a fault detection and isolation (FDI) solution [21]–[24], any one of the available FDI methods in the literature can be utilized to determine the validity or invalidity of a sensor or an estimator. However, it should be noted that the results of our proposed estimation methodology can be further extended for use in designing observer-based FDI techniques. This problem is not addressed here and is left as a topic of our future work.

In case that our DES supervisory control framework is not adopted, one would require instead to use a large number of various look-up tables corresponding to each change in the system parameters or changes due to occurrence of system faults and unreliabilities in system information. It turns out that this is a nontrivial exercise specially when the system dimension is sufficiently large as in the case of large-scale systems. The differential operators (as in the differential algebra theory [19]) are avoided in our proposed estimation framework by implementing linear filtering techniques (such as Luenberger observers) to prevent and eliminate the negative impacts of sensor and measurement noise on the multiple differential operators. In summary, our proposed cooperative distributed estimation approach can be classified as an *active* and *reconfigurable* methodology as opposed to the conventional *passive* estimation techniques in the literature.

The remainder of the paper is organized as follows. Section II, provides the required preliminary definitions and concepts. The main results of the paper on supervisory cooperation of sub-observers are presented in Section III. The convergence property of the cooperative sub-observers is investigated in Section IV. Section V, presents a case study along with simulation results to justify and illustrate the advantages of our proposed methodology. The paper is concluded in Section VI.

II. Preliminaries

In this section, preliminary definitions and concepts that are required to develop our supervisory estimation framework are reviewed.

A. Discrete-Event Systems (DES)

Let Σ represent an alphabet and $L \subseteq \Sigma^*$ a language over Σ. Denote the *prefix-closure* of L as $\overline{L}$. L is called *prefix-closed* (or simply *closed*) if $L = \overline{L}$. For two languages $L, M \subseteq \Sigma^*$, L is called *M-closed* if $L = \overline{L} \cap M$ [14].

In the Ramadge-Wonham (RW) framework [14], it is assumed that the plant is modeled as a finite-state automaton $G = (Q, \Sigma, \delta, q_0, Q_m)$, where Q and Σ are the finite state and event sets, respectively, and $\delta : Q \times \Sigma \rightarrow Q$, q_0, and Q_m are the (partial) transition function, the initial state, and the set of marked states, respectively. It is assumed that the event set Σ can be partitioned into two blocks, namely the controllable events Σ_c and the uncontrollable events Σ_{uc}. Furthermore, the events that are observable are denoted by Σ_o and the unobservable events are denoted by Σ_{uo}. Let $L(G) := \{s | s \in \Sigma^*, \delta(q_0, s) \text{ is defined}\}$ and $L_m(G) := \{s | s \in L(G), \delta(q_0, s) \in Q_m\}$ be the closed and marked behaviors of G. The DES G is *nonblocking* if $\overline{L_m}(G) = L(G)$.

Let the legal (marked) behavior be denoted by $E \subseteq L_m(G)$. In the RW framework, the supervisor monitors the observable events that are generated by G and by disabling or enabling the controllable events in G, and ensures that the system under supervision that is denoted by (S/G) satisfies the following properties, namely (i) (S/G) satisfies the specification (i.e. $L_m(S/G) \subseteq E$), and (ii) (S/G) is nonblocking (i.e. $\overline{L}_m(S/G) = L(S/G)$), where $L_m(S/G) := L_m(G) \cap L(S/G)$.

Let $Pr : \Sigma^* \rightarrow \Sigma_o^*$ denote the natural projection. Since by assumption the supervisor can only monitor the observable events, for any $s, s' \in \Sigma^*$ with the same projection ($Pr(s) = Pr(s')$), the supervisor decision (i.e., the set of enabled events) should be the same. A supervisor satisfying this property is called *feasible*.

The solution to the problem of supervisory control can be described in terms of $L_m(G)$-closed, controllable, and observable sublanguages of E [14]. We may now state the following definitions.

Definition 1. ([14]) A language $K \subseteq \Sigma^*$ is *controllable* (with respect to G) if $\overline{K}\Sigma_{uc} \cap L(G) \subseteq \overline{K}$.

Definition 2. ([14]) A language $K \subseteq L(G)$ is called $(L(G), Pr)-$*observable* (or simply *observable*) if for all

$s, s' \in \Sigma^*$ such that $Pr(s) = Pr(s')$, we have

$$(\forall \sigma \in \Sigma)\ s\sigma \in \overline{K} \wedge s' \in \overline{K} \wedge s'\sigma \in L(G) \Rightarrow s'\sigma \in \overline{K}. \quad (1)$$

Theorem 1. ([14]) Suppose $K \neq \emptyset$ and $K \subseteq E \subseteq L_m(G)$. There exists a feasible nonblocking supervisory control S for G such that $L_m(S/G) = K$ if and only if (i) K is controllable (with respect to G), (ii) K is $(L(G), Pr)-$observable, and (iii) K is $L_m(G)$-closed.

Since the union of observable languages is not necessarily observable, the class of $L_m(G)$-closed, controllable, and observable sublanguages of a given language does not necessarily have a supremal element. Therefore, an optimal solution to the supervisory control problem may not exist in general. A subset of solutions can be obtained by replacing the observability with the stronger *normality* property. The normality property is closed under union, and therefore, the class of normal sublanguages of a given language has a supremal element.

Definition 3. ([14]) A language $K \subseteq L(G)$ is $(L(G), Pr)-$*normal* if $\overline{K} = L(G) \cap Pr^{-1}(Pr(\overline{K}))$.

In the special case when all the controllable events are observable, the controllable and observable languages are normal. Therefore, in such cases the control problem has an optimal solution given by the supremal element that is denoted by $E^{\uparrow}$.

Theorem 2. ([14]) Suppose all the controllable events are observable, $K \neq \emptyset$, and $K \subseteq E \subseteq L_m(G)$. Then, there exists a feasible nonblocking supervisor S such that $L_m(S/G) = K$ if and only if K is controllable, normal, and $L_m(G)-$closed.

In other words, $E^{\uparrow}$ (if nonempty) characterizes the optimal closed-loop behavior. Furthermore, when the legal behavior is a regular language, then the supremal controllable and normal $L_m(G)-$closed sublanguage will also be regular. In this case, an optimal supervisor can be realized in the form of a trim finite-state automaton (this is defined as a reachable and co-reachable finite-state automaton implying that any state of the state machine can be reached from the initial state, and moreover from any state in the state machine there exists a path to a marked state [14]) S such that the product $G \times S$ represents S/G, with $L_m(S/G) = E^{\uparrow}$ and $L(S/G) = \overline{E^{\uparrow}}$.

B. Structural Observability

In this section, the notion of a *linear structured system* is introduced and its *directed graph* is defined. The notion of a *structural observability* and its necessary/sufficient conditions will be investigated based on the concepts of *circuit* and *state-output path*.

An LTI system $\mathscr{S}$ is represented by the triplet matrices (A, B, C)

$$\mathscr{S} : \left\{ \begin{array}{l} \dot{X} = AX + BU + W + \Gamma^x F \\ Y = CX + V + \Gamma^y F \end{array} \right. \quad (2)$$

where

$$B = \begin{bmatrix} B_1 \\ \vdots \\ B_n \end{bmatrix}_{n\times r}, C = \begin{bmatrix} C_1 \\ \vdots \\ C_m \end{bmatrix}_{m\times n}, \Gamma^x = \begin{bmatrix} \Gamma_1^x \\ \vdots \\ \Gamma_n^x \end{bmatrix}_{n\times n_f},$$

$$\Gamma^y = \begin{bmatrix} \Gamma_1^y \\ \vdots \\ \Gamma_m^y \end{bmatrix}_{m\times n_f}$$

Moreover, $X = \begin{bmatrix} x_1, & \dots & ,x_n \end{bmatrix}^T \in \mathbb{R}^n$, $Y = \begin{bmatrix} y_1, & \dots & ,y_m \end{bmatrix}^T \in \mathbb{R}^m$, and $U = \begin{bmatrix} u_1, & \dots & ,u_r \end{bmatrix}^T \in \mathbb{R}^r$ are the state, output, and input vectors, respectively. In addition, $W = \begin{bmatrix} w_1, & \dots & ,w_n \end{bmatrix}^T \in \mathbb{R}^n$, $V = \begin{bmatrix} v_1, & \dots & ,v_m \end{bmatrix}^T \in \mathbb{R}^m$, and $F = \begin{bmatrix} f_1, & \dots & ,f_{n_f} \end{bmatrix}^T \in \mathbb{R}^{n_f}$ are the process disturbance, sensor uncertainty, and fault vectors, respectively. The system (A, B, C) is called a *linear structured system* if the entries of the three matrices A, B, and C are either fixed zeros or independent parameters (not related by algebraic equations). For such systems, one can apply graph theory to study generic properties that are true for almost all values of the system parameters [25]. In the following, a linear structured system will be represented by a directed graph that enables us to investigate the structural observability of the system. The following definitions are adopted from [26].

Definition 4. For a structured system $\mathscr{S}$, represented by the triplet matrices (A, B, C), the associated directed graph $\mathscr{G}(\mathscr{S}) = (\mathscr{V}, \mathscr{E})$ is defined as follows:

- The *vertex* set is $\mathscr{V} = set(U) \cup set(X) \cup set(Y)$ where U, X, and Y are the input, state, and output vectors, respectively, and $Set(\Psi)$ denotes the set of the elements of the vector Ψ.
- The *edge* set is defined as

$$\begin{aligned} \mathscr{E} &= \{(u_i, x_j) | b_{ji} \neq 0\} \cup \{(x_i, x_j) | a_{ji} \neq 0\} \quad (3) \\ &\cup \{(x_i, y_j) | c_{ji} \neq 0\} \end{aligned}$$

  where a_{ji}, b_{ji}, and c_{ji} are the corresponding elements of the matrices A, B, and C, respectively.

Definition 5. Consider a directed graph $\mathscr{G}$. A *directed path* $P = (v_{p_0}, v_{p_1}, ..., v_{p_{(l-1)}}, v_{p_l})$ from a vertex v_{p_0} to a vertex v_{p_l} is a sequence of edges (v_{p_0}, v_{p_1}), (v_{p_1}, v_{p_2}), ..., $(v_{p_{(l-2)}}, v_{p_{(l-1)}})$, $(v_{p_{(l-1)}}, v_{p_l})$ such that

- $v_{p_k} \in \mathscr{V}$ for $k = 0, 1, ..., l$, and
- $(v_{p_{(k-1)}}, v_{p_k}) \in E$ for $k = 1, 2, ..., l$.

Definition 6. A path with $v_{p_0} \in U$ and $v_{p_l} \in Y$ is called an *input-output path*. Similarly, a path with $v_{p_0} \in X$ and $v_{p_l} \in Y$ is called a *state-output path*. A path with $v_{p_0} = v_{p_l}$ is called a *circuit*. A set of paths with no common vertex is said to be *vertex disjoint*. The *length* of a path is the number of the consecutive edges required to complete the path.

The directed graph representation simplifies and visualizes the study of generic properties of a system. In this work, we are interested in *structural observability* among all the generic properties of a system. It should be noted that if a system is structurally unobservable, then it is unobservable for any arbitrary values of nonzero entries of the triplet matrices (A, B, C). But once a system is guaranteed to be structurally observable, its observability should be further validated by using the conventional Grammian theory [27]. Consequently, one can ensure that none of the system modes loses the observability property due to some pathological parameter matching circumstances. The Grammian theory provides a fundamental necessary/sufficient condition for observability of dynamical systems. In this work, although we introduce and use structural observability and graph-based analysis to demonstrate and visualize the feasibility of cooperative sub-observers, one needs to always verify the observability of the overall system (with respect to the integrated sub-observers) by using the Grammian theory.

Proposition 1. ([26], [28]) Let $\mathscr{S}$ be a linear structured system that is represented by the triplet (A, B, C) with its associated graph $\mathscr{G}(\mathscr{S})$. The system $\big($or equivalently, the pair $(C, A)\big)$ is *structurally observable* if and only if the following two conditions hold:

- There exists at least one state-output path originating from any state vertex in X, and
- There exists a set of vertex disjoint circuits and state-output paths which cover all state vertices. ■

Using the properties of directed graph and structural observability, the notion of *sub-observers* is defined in the next section that is utilized in the DES cooperative framework. Moreover, we present results to guarantee the cooperative convergence of the overall integrated sub-observers.

III. Supervisory Cooperation of Sub-Observers

In the following, the definition of *sub-observers* is first presented and the notions of directed graph and structural observability that have already been discussed in the previous section are then used to verify and demonstrate the feasibility of integrated sub-observers and to subsequently design them for a general class of LTI systems.

Definition 7. A *sub-observer* $\#i$ represented by $SO^{(i)}(R^{(i)}|U^{(i)}, Y^{(i)}, D^{(i)})$ is a filter with the following specifications. Namely, the *range* $R^{(i)}$ is the set of state estimates that are generated by $SO^{(i)}$. The *domain* $D^{(i)}$ is the set of state estimates that are not generated by $SO^{(i)}$ but are received from the ranges $R^{(j)}$ $(j \neq i)$ of the other sub-observers $SO^{(j)}$ as they directly affect the state estimates in $R^{(i)}$ through the dynamic equations of the system. The input set $U^{(i)} \subset Set(U)$ and the output set $Y^{(i)} \subset Set(Y)$ are those sets that are required by $SO^{(i)}$ in order to generate the state estimates in $R^{(i)}$. Given (conditioned on) availability of the information on $U^{(i)}$, $Y^{(i)}$, and $D^{(i)}$, all the states of $\mathscr{S}$ whose estimates belong to $R^{(i)}$ are observable by using the sub-observer $SO^{(i)}$, provided that the sub-observer exists.

If a state estimate $\hat{x}_p$, $p \in \{1, \ldots, n\}$ is in the range of $SO^{(j)}$, then we write $\hat{x}_p^{(j)} \in R^{(j)}$. Moreover, if $\hat{x}_p$ is in the domain of another sub-observer $SO^{(i)}$, then we write $\hat{x}_p^{(i)} \in D^{(i)}$. In addition, the members $u_s \in U^{(i)}$, $s \in \{1, \ldots, r\}$, and $y_q \in Y^{(i)}$, $q \in \{1, \ldots, m\}$ of the sub-observer $SO^{(i)}$ are members of the $Set(U)$ and $Set(Y)$ in (2), respectively.

The above definition implies that a sub-observer $SO^{(i)}$: $\big(U^{(i)}, Y^{(i)}, D^{(i)}\big) \rightarrow R^{(i)}$ is a map that cannot operate independently if $D^{(i)} \neq \emptyset$. In such a case, $SO^{(i)}$ requires the information about the state estimates $\hat{x}_m^{(i)} \in D^{(i)}$ from the other sub-observers $SO^{(j)}$ $(\hat{x}_m^{(j)} \in R^{(j)}; \hat{x}_m^{(i)} := \hat{x}_m^{(j)})$. This implies that one needs to design and develop a *cooperative* framework for the sub-observers in the sense that they share and exchange information about their state estimates. The procedure presented below provides a constructive mechanism for designing the sub-observers.

Procedure 1. In the general case of an LTI system with the triplet matrices (A, B, C) as described in Definition 4, the sub-observers can be designed by following the proposed two steps, namely

Step 1. The directed graph of the system is sketched according to the Definition 4.

Step 2. The sub-observer $SO^{(i)}$ is designed by choosing $U^{(i)} \subset Set(U)$, $Y^{(i)} \subset Set(Y)$, and two sets of vertices from the $Set(X)$ to form its domain $D^{(i)}$ and range $R^{(i)}$ such that the following two conditions are satisfied:

Condition 1. The states in $R^{(i)}$ and the output sensor measurements in $Y^{(i)}$ should satisfy the conditions in Proposition 1.

Condition 2. The set of input vertices in $U^{(i)}$ and the state vertices in $D^{(i)}$ should include all the vertices in $Set(U)$ and $Set(X)$, respectively, from which there exist incoming edges to the set of state vertices in $R^{(i)}$ and output vertices in $Y^{(i)}$, unless the sub-observer $SO^{(i)}$ represents a direct measurement of one state $\hat{x}_p^{(i)} \in R^{(i)}$, in which $R^{(i)} = \{\hat{x}_p^{(i)}\}$, $D^{(i)} = \{\}$, $U^{(i)} = \{\}$, and $Y^{(i)} = \{y_p\}$ ($y_p \in Set(Y)$ is a direct measurement of x_p).

In its simplest graphical representation, a sub-observer consists of a single/multiple circuit(s) or state-output path(s), such that the structural observability (as well as the observability) of the states in the corresponding circuits and state-output paths is guaranteed. These sub-observers and their corresponding sensors should be first validated before being used for the purpose of estimation. The *validity condition* of a sub-observer depends on the accuracy of its sub-model. Similarly, the *validity condition* of a sensor depends on the accuracy of its measurement. These notions are formally specified in the following definition.

Definition 8. A sensor $y_q \in Y^{(i)}$, $q \in \{1, \ldots, m\}$ is said to be uncertain or invalid (faulty) if $F_{y_q} \triangleq \Gamma_q^y F \neq 0$, otherwise $\big($for $F_{y_q} = 0\big)$ it is said to be valid. Moreover, the dynamic equation of a sub-observer $SO^{(i)}$ is said to be uncertain or invalid (faulty) if for some state estimates $\hat{x}_p^{(i)} \in R^{(i)}$, $p \in \{1, \ldots, n\}$, $F_{x_p} \triangleq \Gamma_p^x F \neq 0$ which represents the effects of a permanent process anomaly (failure), otherwise $\big($for $F_{x_p} = 0\big)$ the sub-observer is said to be valid.

As stated in the next assumption, implicit in the above definition is the fact that one has access to *a* diagnostic system for examining the validity conditions in Definition 8. For this purpose any suitable and capable fault diagnostic system can be used, for example such as those that are proposed in [23] and [24]. Depending on the properties of the fault diagnostic system that is employed, one should consider and impose a lower bound condition on the severity of the process fault F_{x_p} and the sensor fault F_{y_q} that can occur in the system in order for the diagnoser to be able to detect and isolate faults from the levels of the process disturbance W and the sensor noise V. It should be noted that a more detailed investigation on developing and designing these diagnosers are clearly beyond the scope of this work and are therefore not discussed here any further. We now formalize the above discussion with the following assumption.

Assumption 1. Without loss of generality, it is assumed that an effective fault detection and isolation (FDI) module has already been selected and is available. This module will be independent of the sub-observers that are designed to determine the validity or invalidity of a sensor or a sub-observer system (as formalized according to the conditions in Definition 8).

It should be noted that the functionality, operation and use of the FDI module is considered here to be completely independent from the estimation module (sub-observers) [23]. Consequently, the FDI module does not receive any feedback from the estimation module as depicted in Fig. 1. As shown in this figure the FDI module uses inputs and outputs of the system to detect and isolate any possible fault. Based on the knowledge of the detected fault(s) the DES supervisor determines the validity conditions of the sub-observers. The DES supervisor utilizes these validity conditions to propose and activate a set of sub-observers that can cooperatively estimate all the system states to be ultimately employed in the controller.

As pointed out above the problem of fault diagnosis in the FDI module can be investigated according to the techniques that are developed as in e.g. [24] and [29] by using the differential geometry approach. Moreover, the problem of fault diagnosis for switched and hybrid systems has also been investigated in e.g. [30] and [31].

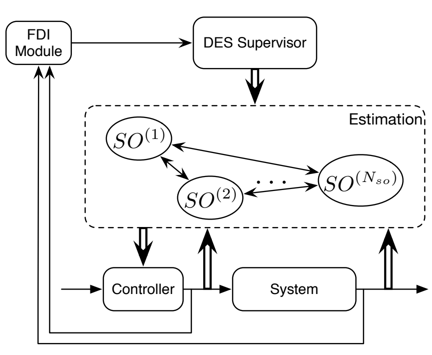

Fig. 1. Overview of the multi-agent based methodology for distributed and cooperative supervisory estimation of a system subject to unreliable information. The bus lines represent the possible information exchanges to/from the sub-observers of the estimation module.

We are now in the position to present our supervisory estimation strategy. In the following, the set of cooperative sub-observers along with their validity conditions are transformed into a DES framework. Moreover, a DES supervisor is designed to modify and update the set of sub-observers in presence of anomalies, faults, or unreliabilities in the sub-observers or sensors.

a) DES Event Set: The corresponding automaton event set contains the sensor measurements $Y^{(i)}$, inputs $U^{(i)}$, sub-observer $SO^{(i)}$, and states $R^{(i)}$ and $D^{(i)}$. Therefore, the set of events (Σ) is partitioned into the following four different categories, namely

1) Set of sensor measurement events Σ_Y and set of input events Σ_U: These events are observable but uncontrollable with
$$\Sigma_U \cup \Sigma_Y = \left\{\tilde{u}_1, \dots, \tilde{u}_r, \tilde{y}_1, \dots, \tilde{y}_m\right\} \tag{4}$$
where $\tilde{u}_i$ and $\tilde{y}_j$ denote the events of the input u_i and the measured output y_j, respectively.
2) Set of sub-observer events Σ_S: Each member of this set represents the utilization of an observer to estimate a set of states. These events are controllable and observable with
$$\Sigma_S = \left\{so^{(1)}, \dots, so^{(N_{so})}\right\} \tag{5}$$
where $so^{(i)}$ denotes the event of the usage of the sub-observer $SO^{(i)}$, and N_{so} denotes the total number of sub-observers.
3) Set of state estimation events Σ_X: These events are the outputs $R^{(i)}$ of the sub-observers $SO^{(i)}$, however they could possibly be the inputs $D^{(j)}$ to other sub-observers $SO^{(j)}$. These events are observable but uncontrollable with
$$\Sigma_X = \left\{\tilde{x}_1, \dots, \tilde{x}_n\right\} \tag{6}$$
where $\tilde{x}_i$ denotes the event of the state estimate $\hat{x}_i$.
4) Set of anomaly, fault, or unreliability events Σ_F: These events are the outputs of the FDI module (as per Assumption 1). These events model the occurrence of faults that affect the validity condition of the sensors or sub-observers. These events are observable but uncontrollable with
$$\Sigma_F = \left\{f_{x_1}, \dots, f_{x_n}, f_{y_1}, \dots, f_{y_m}\right\} \tag{7}$$
where f_{x_p} and f_{y_q} denote the DES events of the anomalies, faults, or unreliabilities F_{x_p} and F_{y_q}, respectively.

These events will be depicted in the DES schematic by using an arrow which is labeled by the name of the event that departs from one state to another state.

b) DES State Set: Each state of the DES model represents an estimation snapshot of the system. In each snapshot, a set of states is cooperatively estimated by using a set of sensor measurements and sub-observers. Since we would like to estimate all the states of the system, the DES states in which the set of used sub-observers can estimate all the states of the system are marked (these are known as *marked states*).

The DES states are depicted in the DES schematic by circles that sometimes they contain a number or name for referral. The initial state is recognized by an incoming arrow that does not exist from any other state. Moreover, the marked states are indicated by an outgoing arrow that does not lead to any other state.

c) Constructing the DES Plant Model: The complete DES plant model for the set of sub-observers cannot be designed in a single step due to the size of the DES model. Therefore, the DES model is constructed by using multiple DESs of lower complexities. Each sub-observer $SO^{(i)}$ can be modeled with two sets of DESs. The first set represents the relation amongst the required information on $U^{(i)}$, $Y^{(i)}$, and $D^{(i)}$ for the sub-observer $SO^{(i)}$. The second DES set models the relation amongst the sub-observer $SO^{(i)}$ and its state estimate $R^{(i)}$. The DES plant model G contains all the possible combinations that the set of sub-observers are utilized in. In other words, in each DES state some of the sub-observers are incorporated.

In the circumstances that certain sub-observers estimate the same common states (i.e. $\exists i, j$ s.t. $R^{(i)} \cap R^{(j)} \neq \emptyset$), the sub-observers might lead to conflicts or result in contradictions within the DES model, and consequently they may block the occurrence of certain events. One solution to remedy and address the blocking problem of these sub-observers automata with common events is to design the automata altogether [14]. However, this solution might result in a highly complicated outcome for a general class of large scale systems. Instead, a more efficient and a practical alternative solution is proposed and developed below. This procedure models a set of sub-observers in the DES framework.

Procedure 2. Consider a set of sub-observers $SO^{(i)}$, where $i \in \Omega$ and $\Omega = \{1, \ldots, N_{so}\}$ is the set of sub-observer indices. Furthermore, define $\Omega_{x_k} := \{i \in \Omega | \hat{x}_k \in R^{(i)}\}$. The system of sub-observers is now transformed into the DES framework according to the following procedure, namely

- $\forall i \in \Omega$, $SO^{(i)}$ is modeled along with all $\hat{x}_k \in R^{(i)} - \Cup_{j \in \Omega, j \neq i} R^{(j)}$, and
- $\forall i \in \Omega$, $\forall \hat{x}_k \in R^{(i)}$, if $n(\Omega_{x_k}) > 1$ ($n(.)$ denotes the cardinality of a set), then $\hat{x}_k$ is modeled with all $SO^{(j)} (j \in \Omega_{x_k})$.

d) Specifications: The fault information that is received from the FDI module (as per Assumption 1) is used as a specification in the DES supervisory control design. This specification contains the validity conditions of the sensors or sub-observers that are described in Definition 8. According to Theorem 2, if this specification is controllable, normal, and $L_m(G)-$closed, then there exists a feasible nonblocking supervisor such that the DES plant under supervision satisfies the given specification. The following proposition gives the requirements that a specification needs to contain in order to be controllable, normal and $L_m(G)-$closed for a set of sub-observers.

Proposition 2. The desired specification E, which includes all the possible faults (Σ_F) in the system (that is received from the FDI module), and which modifies (disables or enables) the utilization of the sub-observers (Σ_S), is $L_m(G)-$closed, controllable, and normal.

Proof:

- The $L_m(G)$-closeness can be shown from the definition of E. Since E is a closed language by the definition (it is marked by a finite state automaton), we have $E = \overline{E}$. On the other hand, $E \subseteq L_m(G)$. Therefore, $E = \overline{E} \cap L_m(G)$, or E is $L_m(G)$-closed.
- The controllability properly can be shown by verifying that $\overline{E}\Sigma_{uc} \cap L(G) \subseteq \overline{E}$. According to the event set which is described above, the following two cases may occur for the uncontrollable events ($\Sigma_{uc} = \Sigma_F \cup \Sigma_U \cup \Sigma_X \cup \Sigma_Y$), namely
  - $\sigma \in \Sigma_F$: Since all the fault signals which are received from the FDI module, are considered in the desired specification E, we therefore have

$$\begin{aligned} &\sigma \in \Sigma_F, \sigma \in L(G) \Rightarrow \sigma \in E \\ &\therefore \sigma \in \Sigma_F, \sigma \in \overline{E}\Sigma_{uc} \cap L(G) \Rightarrow \overline{E} \end{aligned} \tag{8}$$

  - $\sigma \in \Sigma_U \cup \Sigma_X \cup \Sigma_Y$: Since none of these events has any influence on the desired specification (all these events are self-looped at all the states of the desired specification), we therefore have

$$\begin{aligned} &\sigma \in \Sigma_U \cup \Sigma_X \cup \Sigma_Y, \sigma \in L(G) \Rightarrow \sigma \in E \\ &\therefore \sigma \in \Sigma_U \cup \Sigma_X \cup \Sigma_Y, \sigma \in \overline{E}\Sigma_{uc} \cap L(G) \Rightarrow \overline{E} \end{aligned} \tag{9}$$

  In view of equations (8) and (9), it now follows that $\overline{E}\Sigma_{uc} \cap L(G) \subseteq \overline{E}$. In other words, E is controllable.
- All the events (Σ) in the DES plant model G are observable and $E \subseteq L_m(G) \subseteq L(G)$, therefore, $Pr^{-1}(Pr(\bar{E})) = \bar{E} = L(G) \cap Pr^{-1}(Pr(\bar{E}))$. Hence, E is normal. ■

e) DES Supervisor Design: As discussed in Section II-A, the goal of the DES supervisor is to lead the DES plant model to reach the marked states through the available paths, while satisfying the desired specifications (validity conditions in our case). Such a DES supervisor can be designed according to Section II-A, if the desired specifications satisfy the necessary conditions of Theorem 2.

As discussed earlier, we would like to use a set of valid sub-observers and sensors to estimate all the states of the system. Therefore, the DES supervisor requires to incorporate and utilize only the valid sub-observers and sensors. According to the fault information that is received from the FDI module (as per Assumption 1), some of these sub-observers should not be used in the state estimation process. Therefore, the DES supervisor eliminates those paths along which these invalid sub-observers are employed. Consequently, some of the original marked states are made no longer reachable due to their reliance on the invalid sub-observers. In other words, the number of marked states is reduced.

It should be pointed out that in cases where there are multiple marked states (destinations) available, and where each satisfies the desired specifications one needs to consider other criteria to select the appropriate solution. For example, by taking advantage of DES optimization techniques that we have introduced in [32], the cooperation among the sub-observes can actually be optimized by further taking into account the sensor and the sub-observer costs and resources. These issues are beyond the scope of this work and have therefore not been addressed any further here.

In our proposed methodology, the DES supervisor takes advantage of the available redundancies in the system to propose a new set of sub-observers that overcomes the effects of faults (failures) and unreliability of sensors and sub-observers. In the case where there are insufficient redundancies available in the system, it is possible that the DES supervisor may not be able to determine a set of sub-observers for estimating all the system states (empty DES supervisor). This is due to the fact that there are no valid and certain (healthy, fault-free, or reliable) predesigned sub-observers that can estimate a certain set of states. Under these circumstances, the solution would be to either recover the faulty system hardware (e.g., by repairing the hardware or by using hardware redundancy) or add a new sensor that modifies the set of marked states and available paths in the DES model. These remedies are also beyond the scope of this work and are therefore not pursued further.

For sake of further clarity our proposed framework is demonstrated below through a numerical example. All the concepts that have been presented earlier in this section are discussed and illustrated in detail to provide a transparent and easy to follow transition to the verification procedure as well as the case study simulations that are presented in the next section.

Example 1. This example illustrates how the sub-observers are designed, how their DES models are constructed, and how the DES supervisor makes decisions on the proper set of sub-observers in the absence as well as in the presence of an anomaly. Consider the following two systems that are represented by the triplet (A, B, C) and $(A, B, \bar{C})$ that are both structurally observable, that is

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 0 & b_{22} \\ b_{31} & 0 \end{bmatrix}$$
$$C = \begin{bmatrix} c_{11} & 0 & 0 \\ 0 & 0 & c_{23} \end{bmatrix}, \bar{C} = \begin{bmatrix} c_{11} & 0 & 0 \end{bmatrix}$$

and which represent the model of a three tank system as shown in Fig. 2. In this figure o_{ij} represents the flow from tank i to tank j, o_i is the output flow of tank i, and u_1 and u_2 are the first and the second inputs to tanks 3 and 2, respectively. In the state space representation, y_1 and y_2 correspond to the sensor measurements of the output flows o_1 and o_3, respectively, and the state $x_i, i = 1, 2, 3$ represents the level of tank $i = 1, 2, 3$, respectively. The above system is actually a subsystem of the eight tank system that is analyzed in more detail in our case study simulation results that are provided in Section V.

In fact, the linear model above is an approximation to the nonlinear model of a tank [33], namely for tank i we have

$$\dot{x}_i = -\delta_{ii}\sqrt{x_i} + \sum_{j\neq i}\delta_{ij}\sqrt{x_j} + \sum_{k=1}^{r}\lambda_{ik}u_k \tag{10}$$

where x_i is the level of the tank i, δ_{ii} and δ_{ij} are non-negative coefficients, $\delta_{ii} > 0$ indicates that there is an output flow from the tank i, $\delta_{ij} > 0$ indicates that there is a flow from the tank j to the tank i, u_k $(k = 1, ..., r)$ is an external input flow, and λ_{ik} is the u_k-corresponding input scalar for tank i. By using the first order Taylor series expansion around the nominal point x_{i0} we obtain

$$\sqrt{x_i} = \sqrt{x_{i0}} + \frac{1}{2\sqrt{x_{i0}}}(x_i - x_{i0}) = \frac{\sqrt{x_{i0}}}{2} + \frac{1}{2\sqrt{x_{i0}}}x_i$$

Consequently, the nonlinear system (10) can be linearized as follows

$$\dot{x}_i = -\delta_{ii}\left(\frac{\sqrt{x_{i0}}}{2} + \frac{1}{2\sqrt{x_{i0}}}x_i\right) + \sum_{j\neq i}\delta_{ij}\left(\frac{\sqrt{x_{j0}}}{2} + \frac{1}{2\sqrt{x_{j0}}}x_j\right) + \sum_{k=1}^{r}\lambda_{ik}u_k$$

or equivalently

$$\dot{x}_i = \underbrace{\frac{-\delta_{ii}}{2\sqrt{x_{i0}}}}_{a_{ii}}x_i + \sum_{j\neq i}\underbrace{\frac{\delta_{ij}}{2\sqrt{x_{j0}}}}_{a_{ij}}x_j + \underbrace{\frac{-\delta_{ii}\sqrt{x_{i0}}}{2} + \sum_{j\neq i}\frac{\delta_{ij}\sqrt{x_{j0}}}{2}}_{u_i^{vir}} + \sum_{k=1}^{r}\lambda_{ik}u_k$$

Therefore, we have

$$\dot{x}_i = a_{ii}x_i + \sum_{j\neq i}a_{ij}x_j + \sum_{k=1}^{r}\lambda_{ik}u_k + u_i^{vir}$$

where u_i^{vir} is the *virtual input* of tank i. In general, the overall linearized dynamics of an n-tank system becomes

$$\dot{X} = AX + BU + B^{vir}U^{vir}$$

where $X = [x_1 \ ... \ x_n]^T$ is the overall state vector, and the matrices A, B, U, B^{vir}, and U^{vir} are specified according to

$$A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \dots & a_{nn} \end{bmatrix}$$
$$B = \begin{bmatrix} \lambda_{11} & \dots & \lambda_{1r} \\ \vdots & \ddots & \vdots \\ \lambda_{r1} & \dots & \lambda_{rr} \end{bmatrix}, \ U = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}$$
$$B^{vir} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \ddots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}, \ U^{vir} = \begin{bmatrix} u_1^{vir} \\ \vdots \\ u_n^{vir} \end{bmatrix}$$

Without lost of generality, and due to the fact that the objective of this paper is to present a novel methodology for an estimation problem, the linear model above is deemed sufficient to be utilized in this example and in the subsequent examples in order to not complicate the analysis with difficulties associated with nonlinear models. Moreover, the dynamic equations are linearized around the nominal point $x_{i0} = 0$, $i = 1, ..., n$, which results in $u_i^{vir} = 0$.

Consider the following two systems, namely (a) the system

(A, B, C) with two sensor measurements $\{y_1, y_2\}$, and (b) the system $(A, B, \bar{C})$ with one sensor measurement $\{y_1\}$. The directed graphs of the corresponding systems are shown in Fig. 3-(a) and (b), respectively. In Fig. 3-(a), the two vertex disjoint state-output paths (x_2, x_1, y_1) and (x_3, y_2) cover all the states, and hence, guarantee the structural observability of the system (Proposition 1). Similarly, in Fig. 3-(b) the set of vertex disjoint state-output path (x_2, x_1, y_1) and circuit x_3 exist that cover all the states, implying that the system is structurally observable. For further analysis we only consider the directed graph of the system (A, B, C).

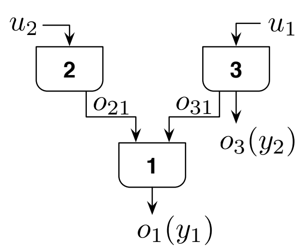

Fig. 2. A three tank system.

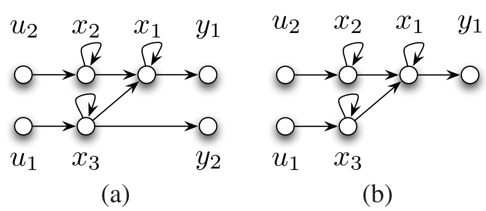

Fig. 3. The directed graphs of the two systems (a) (A, B, C) and (b) $(A, B, \bar{C})$.

Let us now investigate the application of the Procedure 1 to the structurally observable system (A, B, C). In step 1 of the Procedure 1, the directed graph of the system (A, B, C) is constructed as shown in Fig. 3-(a). According to the directed graph of the system it turns out that at most *nine* sub-observers, which are shown in Fig. 4, can be designed in step 2 of Procedure 1 that *simultaneously* satisfy both Conditions 1 and 2.

In Fig. 4, the dashed lines represent the information that the sub-observer $SO^{(i)}$ requires to receive regarding certain inputs $u_s \in U^{(i)}$ $\big(s \in \{1, \dots, r\}\big)$, outputs $y_q \in Y^{(i)}$ $\big(q \in \{1, \dots, m\}\big)$, and state estimates $\hat{x}_p^{(i)} \in D^{(i)}$ $\big(p \in \{1, \dots, n\}\big)$ from the other sub-observers $SO^{(j)}$ $\big($such that $\hat{x}_p^{(j)} \in R^{(j)}$ and $\hat{x}_p^{(i)} := \hat{x}_p^{(j)}\big)$. The solid lines represent the dynamic relations among the states $\hat{x}_p^{(i)} \in R^{(i)}$ of the sub-observer $SO^{(i)}$. Using the Proposition 1, for each sub-observer $SO^{(i)}$, it is easy to verify that the graph representing $SO^{(i)}$ satisfies both conditions of the Procedure 1.

Following the Procedure 1, the only *available* sub-observers

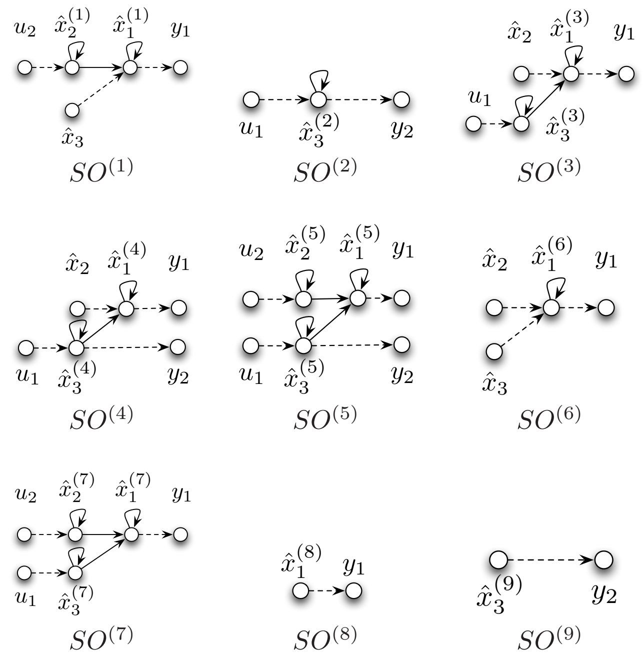

Fig. 4. The directed graphs of the nine sub-observers for the system (A, B, C).

are formally represented as follows:

$$(i) \quad SO^{(1)}(R^{(1)}|U^{(1)}, Y^{(1)}, D^{(1)}), R^{(1)} = \{\hat{x}_1^{(1)}, \hat{x}_2^{(1)}\}, \\ U^{(1)} = \{u_2\}, Y^{(1)} = \{y_1\}, D^{(1)} = \{\hat{x}_3^{(1)}\}$$

$$(ii) \quad SO^{(2)}(R^{(2)}|U^{(2)}, Y^{(2)}, D^{(2)}), R^{(2)} = \{\hat{x}_3^{(2)}\}, \\ U^{(2)} = \{u_1\}, Y^{(2)} = \{y_2\}, D^{(2)} = \{\}$$

$$(iii) \quad SO^{(3)}(R^{(3)}|U^{(3)}, Y^{(3)}, D^{(3)}), R^{(3)} = \{\hat{x}_1^{(3)}, \hat{x}_3^{(3)}\}, \\ U^{(3)} = \{u_1\}, Y^{(3)} = \{y_1\}, D^{(3)} = \{\hat{x}_2^{(3)}\}$$

$$(iv) \quad SO^{(4)}(R^{(4)}|U^{(4)}, Y^{(4)}, D^{(4)}), R^{(4)} = \{\hat{x}_1^{(4)}, \hat{x}_3^{(4)}\}, \\ U^{(4)} = \{u_1\}, Y^{(4)} = \{y_1, y_2\}, D^{(4)} = \{\hat{x}_2^{(4)}\}$$

$$(v) \quad SO^{(5)}(R^{(5)}|U^{(5)}, Y^{(5)}, D^{(5)}), R^{(5)} = \{\hat{x}_1^{(5)}, \hat{x}_2^{(5)}, \hat{x}_3^{(5)}\}, \\ U^{(5)} = \{u_1, u_2\}, Y^{(5)} = \{y_1, y_2\}, D^{(5)} = \{\}$$

$$(vi) \quad SO^{(6)}(R^{(6)}|U^{(6)}, Y^{(6)}, D^{(6)}), R^{(6)} = \{\hat{x}_1^{(6)}\}, \\ U^{(6)} = \{\}, Y^{(6)} = \{y_1\}, D^{(6)} = \{\hat{x}_2^{(6)}, \hat{x}_3^{(6)}\}$$

$$(vii) \quad SO^{(7)}(R^{(7)}|U^{(7)}, Y^{(7)}, D^{(7)}), R^{(7)} = \{\hat{x}_1^{(7)}, \hat{x}_2^{(7)}, \hat{x}_3^{(7)}\}, \\ U^{(7)} = \{u_1, u_2\}, Y^{(7)} = \{y_1\}, D^{(7)} = \{\}$$

$$(viii) \quad SO^{(8)}(R^{(8)}|U^{(8)}, Y^{(8)}, D^{(8)}), R^{(8)} = \{\hat{x}_1^{(8)}\}, \\ U^{(8)} = \{\}, Y^{(8)} = \{y_1\}, D^{(8)} = \{\}$$

$$(ix) \quad SO^{(9)}(R^{(9)}|U^{(9)}, Y^{(9)}, D^{(9)}), R^{(9)} = \{\hat{x}_3^{(9)}\}, \\ U^{(9)} = \{\}, Y^{(9)} = \{y_2\}, D^{(9)} = \{\}$$

For each of the sub-observers $SO^{(8)}$ and $SO^{(9)}$, there is only one sensor that is utilized in the observation resulting in having no specific dynamics associated with them. Consequently, $SO^{(8)}$ and $SO^{(9)}$ should be interpreted as direct measurements of the states.

Now we begin by transforming the set of sub-observers into the DES framework. According to the discussions in Section III-a, the DES event sets are as follows:

$$\Sigma_U = \{\tilde{u}_1, \tilde{u}_2\}, \Sigma_Y = \{\tilde{y}_1, \tilde{y}_2\}, \Sigma_S = \{so^{(1)}, \dots, so^{(9)}\},$$

$$\Sigma_X = \{\tilde{x}_1, \tilde{x}_2, \tilde{x}_3\}, \Sigma_F = \{f_{x_3}\}$$

To construct the DES state set and the DES plant model as described in Sections III-b and III-c, respectively, let us consider the two sub-observers $SO^{(1)}$ and $SO^{(2)}$ as shown in Fig. 4. The sub-observer $SO^{(2)}$ only needs the information of sensor 2 (measurement y_2 or the DES event $\tilde{y}_2$) and the input u_1 (DES event $\tilde{u}_1$) as shown in the DES model of Fig. 5. The sub-observer $SO^{(1)}$ needs the information of sensor 1 (measurement y_1 or the DES event $\tilde{y}_1$), the input u_2 (DES event $\tilde{u}_2$), and the the state estimate x_3 (DES event $\tilde{x}_3$) which is estimated by the sub-observer $SO^{(2)}$. These DES models are constructed by applying the steps that are outlined in Procedure 2 in Section III and are shown in Fig. 6.

The DES model of the set of sub-observers $SO^{(1)}$ and $SO^{(2)}$ is now generated by the *sync* product of the individual models $SO^{(1)}$ and $SO^{(2)}$. The sync product [14] or the parallel composition of two generators is used to combine the two models in order to construct a single model for the entire system. Similarly, the modeling of all the other sub-observers in the healthy (fault-free or reliable) condition in the DES framework can be developed. These details are not included due to space limitations.

A differential pressure transmitter can be used to measure input and output flows of the tanks in this application. For example, the XYR differential pressure transmitter [34] can be used for flowmeter purposes. However, since this sensor can communicate with up to 20 other instruments at the frequency of 1 (Hz) and the fact that the total number of instruments in this example is less than 20, there are no practical limitations on utilizing this instrument in this application. This sensor is capable of communicating securely with other field units within a 500' distance. Consequently, it is preferable to use sensors/sub-observers that have shorter distance among them. On the other hand, it is also preferable to use sub-observers that invoke more information of the system in their estimation tasks. For example, from the two possible sets of sub-observers $\{SO^{(1)}, SO^{(2)}\}$ and $\{SO^{(1)}, SO^{(9)}\}$, the first set is preferable since $SO^{(2)}$ utilizes more information (u_1) than $SO^{(9)}$.

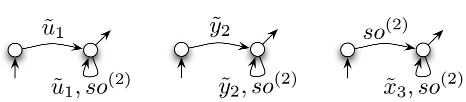

Fig. 5. The DES model of the sub-observer $SO^{(2)}$.

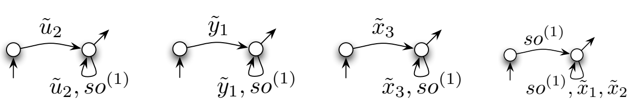

Fig. 6. The DES model of the sub-observer $SO^{(1)}$.

Let us consider that an actuator fault occurs in the input valve of tank 3 (that is, $F_{x_3} = \Gamma_3^x F = -\gamma B_3 U, |\gamma| \leq 1$) [35]. According to Section III-d, the validity condition of $SO^{(2)}$, which is described in Definition 8, is modeled as the specification, $Spec_{f_{x_3}}$, in the DES that is shown in Fig. 7. This model implies that if an uncertainty (anomaly, fault, or unreliability) F_{x_3} (the DES event f_{x_3}) occurs, $SO^{(2)}$ becomes invalid, and consequently it should not be used any further for constructing the state estimates. It can be observed from this figure that in the DES initial state the event $so^{(2)}$ can occur, however, in the other state this event cannot be present.

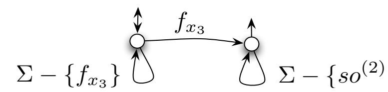

Fig. 7. The DES model corresponding to the validity condition of $SO^{(2)}$.

Although for system (A, B, C), there are nine possible sub-observers, however, certain subsets of them can be used to cooperatively estimate all the system states. For instance, in the fault free scenario the DES supervisor suggests the four initial sets of sub-observers $\{SO^{(1)}, SO^{(2)}\}$, $\{SO^{(1)}, SO^{(9)}\}$, $\{SO^{(5)}\}$, and $\{SO^{(7)}\}$, where each initial set can estimate all the system states. In the faulty (F_{x_3}) scenario, according to Section III-e, a supervisor can be designed by applying the results of Theorem 2 to modify G such that $Spec_{f_{x_3}}$ is satisfied. Consequently, this supervisor eliminates certain controllable paths in which the invalid sub-observer $SO^{(2)}$ is employed, so that the system can operate and function under the uncertain (faulty) situation F_{x_3}. As can be observed from Fig. 8, which shows a simplified (excluding the DES events Σ_U and Σ_X) version of the DES plant under supervision, the DES supervisor prevents the occurrence of the event $so^{(2)}$ so that $SO^{(2)}$ is not utilized after the uncertainty (anomaly, fault, or unreliability) F_{x_3} (the DES event f_{x_3}) occurs. In this figure a sample of a possible trajectory is highlighted in bold.

In Fig. 8, the selected bold trajectory corresponds to the DES string $\tilde{u}_1 f_{x_3} \tilde{y}_2 f_{x_3} \tilde{y}_1 f_{x_3} so^{(9)} f_{x_3} so^{(1)}$. Corresponding to this trajectory, after the input flow of tank 3 is measured (u_1 or DES event $\tilde{u}_1$), and the fault F_{x_3} (the DES event f_{x_3}) has occurred, the DES supervisor then disables $so^{(2)}$ associated with all the succeeding states. Next, the outputs y_2 and y_1 are measured, and consequently the two sub-observers $SO^{(9)}$ and $SO^{(1)}$ are utilized. For this trajectory the event f_{x_3} occurs after other events, indicating that the external FDI module is sending updates regarding the existence of faults, and the fault F_{x_3} is a permanent fault.

By using all the nine sub-observers one can conclude that after the fault F_{x_3} occurs, the supervisor proposes the reconfigured set of sub-observers $\{SO^{(1)}, SO^{(9)}\}$ to estimate all the states of the system. Therefore, the selected path in the state machine (the supervised DES model) to the marked states has the $so^{(9)}$ event implying that $SO^{(9)}$ is utilized. In other words, the initial valid sub-observer set $\{SO^{(1)}, \ldots, SO^{(9)}\}$ is divided into the valid sub-observer set $\{SO^{(1)}, SO^{(6)}, SO^{(8)}, SO^{(9)}\}$ and the invalid sub-observer set $\{SO^{(2)}, SO^{(3)}, SO^{(4)}, SO^{(5)}, SO^{(7)}\}$. Consequently, this in return implies that the DES supervisor selects the modified set of sub-observers $\{SO^{(1)}, SO^{(9)}\}$, which does not require information from the input u_1.

Note that one still needs to verify the convergence of the overall observation goal that is achieved by the DES-selected initial set of sub-observers $\{SO^{(1)}, SO^{(2)}\}$ in the fault free scenario, and the DES-reconfigured set of sub-

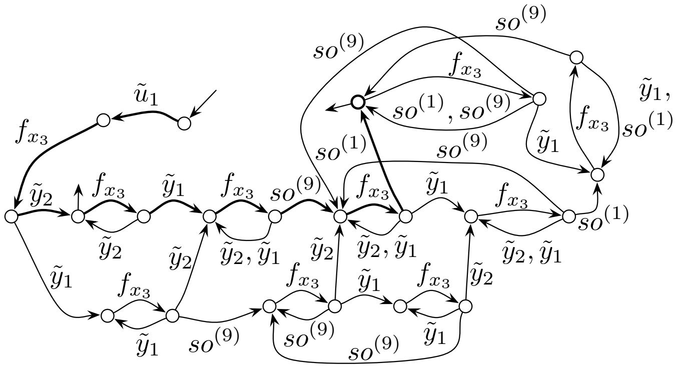

Fig. 8. A simplified diagram of the supervised DES in presence of the uncertainty (anomaly, fault, or unreliability) F_{x_3} (DES event f_{x_3}).

observers $\{SO^{(1)}, SO^{(9)}\}$ in the faulty scenario. Subsequently, in Section IV we will provide guidelines on how to transform the overall cooperative sub-observers scheme into an equivalent centralized estimation scheme having an observer gain matrix $\bar{K}$ with a certain block-diagonal structure (as illustrated in Example 2). Moreover, we will also provide a condition to verify the feasibility of the equivalent centralized estimation scheme having a block-diagonal observer gain matrix $\bar{K}$ (as illustrated in Example 3). □

To summarize, in this section we have proposed a DES supervisory methodology that selects a set of valid sub-observers for cooperatively estimate all the system states in presence of uncertainties, anomalies, and unreliabilities. It should be noted that it is actually possible to characterize the sub-observers based on only the Grammian observability theory. However, the advantage of using the structural observability is due to its graphical representation that provides a more intuitive understanding and interpretation behind our proposed concepts. If a system is structurally unobservable then it also implies (this is sufficient) that it is unobservable for all the values of the system parameters. However, if the system is structurally observable, it does not guarantee that the system is observable for all the values of the system parameters. Therefore, in the latter case, the Grammian theory should be employed to verify the observability of the system. On the other hand, the objective of this paper is actually to use and incorporate both the structural (graph based) and the Grammian approaches simultaneously as follows: First, the graph based approach is used to simplify the sub-observer design by visualizing the interactions among the states. Next, the Grammian approach is used to verify the observability of the overall cooperative sub-observer scheme as presented and discussed in the next section.

We are now in the position to formally verify that the overall integration of the cooperative sub-observers in the selected set will lead to an overall stable (convergent) system. Towards this end, convergence of our proposed cooperative estimation scheme is investigated in the next section.

IV. Convergence Analysis of Cooperative Valid Sub-Observers

In this section, our goal is to verify convergence of the cooperative estimation scheme that is constructed from the set of DES selected valid sub-observers. One needs to ensure that the estimation error dynamics is stable so that the state estimate $\hat{x}_i$ converges within a bound of the actual state x_i. Let us first provide further description to the problem of a given state that is being estimated by multiple sub-observers.

Each state $x_j,\ j = 1, \ldots, n$ could be estimated by one (multiple) sub-observer(s) $SO^{(k)}$ (i.e., $\hat{x}_j^{(k)} \in R^{(k)}$). In case that the estimate of x_j is required by the sub-observer $SO^{(l)}$ (i.e., $\hat{x}_j^{(l)} \in D^{(l)}$), a weighted average over all the sub-observers $SO^{(k)}$ which estimate x_j is made. In other words,

$$\forall l \in \Omega, \forall j \in \{1, \ldots, n\}; \hat{x}_j^{(l)} \in D^{(l)},$$

$$\hat{x}_j^{(l)} = \sum_{\forall k \in \{1,\ldots,N_{so}\}; \hat{x}_j^{(k)} \in R^{(k)}} \alpha_j^{(lk)} \hat{x}_j^{(k)} \tag{11}$$

where $0 \leq \alpha_j^{(lk)} \leq 1$ are the weights that satisfy $\sum_{\forall k \in \{1,\ldots,N_{so}\}; \hat{x}_j^{(k)} \in R^{(k)}} \alpha_j^{(lk)} = 1$. The equation above corresponds to a general form of information fusion that can be represented by techniques such as estimation versions ranking [18], weighted decision method (voting technique), and Bayesian inference [36]. Since developing an optimal fusion technique is beyond the scope of this paper, this discussion is not pursued any further here.

The convergence of a set of cooperative sub-observers is verified by investigating the properties of its equivalent centralized estimation scheme. This is accomplished through the following two stages:

1) Lemma 1 below provides guidelines on how to transform the overall cooperative sub-observers scheme into an equivalent centralized estimation scheme for sake of *only* analysis. The observer gain matrix $\bar{K}$ has a certain block-diagonal structure $\bar{\mathbf{K}}$, i.e., $\bar{K} \in \bar{\mathbf{K}}$.
2) Lemmas 2 and 3 below provide conditions on verifying the feasibility of the equivalent centralized estimation scheme that has a block-diagonal observer gain matrix $\bar{K} \in \bar{\mathbf{K}}$ when sensor measurements and system dynamics are uncertain, respectively.

To summarize, a set of cooperative sub-observers is convergent if and only if the equivalent centralized estimation scheme, which is obtained and constructed according to Lemma 1, is verified to be feasible by using Lemmas 2 and 3.

Lemma 1. For an observable multi-input multi-output (MIMO) system that is represented by the triplet (A, B, C), the feasibility of a cooperative estimation scheme as designed according to the Propositions 1 and 2 is equivalent to the feasibility of a centralized estimation problem having an observer gain matrix $\bar{K} \in \bar{\mathbf{K}}$ with a certain sparse structure $\bar{\mathbf{K}} = diag\big(K^{(1)}, \ldots, K^{(N_{SO})}\big)$, where $K^{(i)}$ is the gain of $SO^{(i)}$ and N_{SO} denotes the total number of sub-observers.

Proof: Consider a linear MIMO system that is given by (2) and the sub-observer $SO^{(l)}, l = 1, \ldots, N_{so}$ with its state vector $\hat{X}^{(l)} \in \mathbb{R}^{n^{(l)}}$ $\big(n^{(l)}$ is the dimension of $\hat{X}^{(l)}\big)$ that includes and concatenates all its states $\hat{x}_i^{(l)} \in R^{(l)}, i \in \{1,\ldots,n\}$. For all the states $\hat{x}_i^{(l)} \in R^{(l)}$, we have

$$\begin{aligned}\dot{\hat{x}}_i^{(l)} &= \sum_{\substack{\forall j\in\{1,\ldots,n\};\\ \hat{x}_j^{(l)}\in R^{(l)}}} a_{ij}\hat{x}_j^{(l)} \\ &+ \sum_{\substack{\forall j\in\{1,\ldots,n\};\\ \hat{x}_j^{(l)}\in D^{(l)}}} \underbrace{\sum_{\substack{\forall k\in\{1,\ldots,N_{so}\};\\ \hat{x}_j^{(k)}\in R^{(k)}}} a_{ij}\alpha_j^{(lk)}\hat{x}_j^{(k)}}_{\hat{x}_j^{(l)}} \\ &+ K_i^{(l)}\Big(Y^{(l)} - C^{(l)}\hat{X}^{(l)}\Big) + B_iU\end{aligned} \tag{12}$$

where a_{ij} is the ij^{th} element of A, $K_i^{(l)}$ is the x_i-corresponding element of the sub-observer matrix gain $K^{(l)}$, $\hat{X}^{(l)}$ is the estimate of $X^{(l)} \in \mathbb{R}^{n^{(l)}}$, which is the vector of all the states $\hat{x}_i^{(l)} \in R^{(l)}$, and $Y^{(l)} \in \mathbb{R}^{m^{(l)}}$ is the vector of all the local measurements, which are in terms of their corresponding uncertainties $V^{(l)}$ and estimated states in $R^{(l)}$ through the measurement matrix $C^{(l)}$, that is $Y^{(l)} = C^{(l)}X^{(l)} + V^{(l)}$.

Taking $e_i^{(l)} = \hat{x}_i^{(l)} - x_i$, the corresponding error dynamics becomes

$$\begin{aligned}\dot{e}_i^{(l)} &= \sum_{\substack{\forall j\in\{1,\ldots,n\};\\ \hat{x}_j^{(l)}\in R^{(l)}}} a_{ij}e_j^{(l)} \\ &+ \sum_{\substack{\forall j\in\{1,\ldots,n\};\\ \hat{x}_j^{(l)}\in D^{(l)}}} \sum_{\substack{\forall k\in\{1,\ldots,N_{so}\};\\ \hat{x}_j^{(k)}\in R^{(k)}}} a_{ij}\alpha_j^{(lk)}e_j^{(k)} \\ &- K_i^{(l)}C^{(l)}E^{(l)} + K_i^{(l)}V^{(l)} + w_i\end{aligned} \tag{13}$$

where $E^{(l)} = \hat{X}^{(l)} - X^{(l)}$ is the vector of all the estimation errors $e_i^{(l)}$. Now, augmenting all the error equations in (13) (for $i = 1,\ldots,n^{(l)}$ and $l = 1,\ldots,N_{SO}$) and *neglecting* the effects of uncertainties, disturbances, and unmodeled dynamics results in a compact form and representation of the estimation error dynamics, namely

$$\dot{E}^t = \big(\bar{A} - \bar{K}\bar{C}\big)E^t \tag{14}$$

where E^t is an $n_{N_{so}} \times 1$ vector, $\bar{A}$ is an $n_{N_{so}} \times n_{N_{so}}$ matrix, $\bar{K}$ is an $n_{N_{so}} \times m_{N_{so}}$ matrix, and $\bar{C}$ is an $m_{N_{so}} \times n_{N_{so}}$ matrix $\Big(n_{N_{so}} = \sum_{l=1}^{N_{SO}} n^{(l)}$ and $m_{N_{so}} = \sum_{l=1}^{N_{SO}} m^{(l)}\Big)$ that are defined as follows: $\forall l \in \Omega, \forall i'_l \in \mathbb{N}; 0 < i'_l < n^{(l)}$ and $\forall k \in \Omega, \forall j'_k \in \mathbb{N}; 0 < j'_k < n^{(k)}$; we have

$$E^t_{[n_{l-1}+i'_l]} = E^{(l)}_{[i'_l]} \tag{15}$$

$$\bar{A}_{[n_{l-1}+i'_l, n_{k-1}+j'_k]} = \begin{cases} a_{ij} & l = k, \exists\{x_i, x_j\} \subset Set(X) \\ & \hat{X}^{(l)}_{[i'_l]} = \hat{x}_i^{(l)}, \hat{X}^{(k)}_{[j'_k]} = \hat{x}_j^{(k)} \\ a_{ij}\alpha_j^{(lk)} & l \neq k, \exists\{x_i, x_j\} \subset Set(X) \\ & \hat{X}^{(l)}_{[i'_l]} = \hat{x}_i^{(l)}, \hat{X}^{(k)}_{[j'_k]} = \hat{x}_j^{(k)} \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

$$\bar{C} = diag\big(C^{(1)}, \ldots, C^{(N_{SO})}\big) \tag{17}$$

$$\bar{K} = diag\big(K^{(1)}, \ldots, K^{(N_{SO})}\big) \tag{18}$$

in which the index $[i]$ denotes the i^{th} element of a vector, the index $[i,j]$ denotes the ij^{th} element of a matrix, and $n_l = \sum_{s=1}^{l} n^{(s)}$ is the dimension of the cumulative state up to the $(l)^{th}$ sub-observer. Therefore, our proposed cooperative estimation scheme is equivalent to a centralized estimation problem for the pair $\big(\bar{A}, \bar{C}\big)$ with a block-diagonal observer gain matrix $\bar{K} \in \bar{\mathbf{K}}_{n_{N_{so}} \times m_{N_{so}}}$ where

$$\begin{aligned}\bar{\mathbf{K}}_{n_{N_{so}} \times m_{N_{so}}} = \Big\{&diag\big(K^{(1)}, \ldots, K^{(N_{SO})}\big) \\ &\big| K^{(1)} \in \mathbb{R}^{n^{(1)} \times m^{(1)}}, \ldots, K^{(N_{SO})} \in \mathbb{R}^{n^{(N_{SO})} \times m^{(N_{SO})}}\Big\}\end{aligned} \tag{19}$$

This completes the proof of the lemma. ■

The example below illustrates in detail the application of Lemma 1 where we have transformed the overall cooperative sub-observers scheme into an equivalent centralized estimation scheme with a block-diagonal observer gain matrix $\bar{K} \in \bar{\mathbf{K}}_{n_{N_{so}} \times m_{N_{so}}}$.

Example 2. Consider the system that is studied in Example 1 with the triplet matrices (A,B,C), and the initial set of sub-observers $\{SO^{(1)}, SO^{(2)}\}$ in the absence of any anomaly and the reconfigured set of sub-observers $\{SO^{(1)}, SO^{(9)}\}$ in the presence of an anomaly, which are both selected by the DES supervisor. Our goal is to find an equivalent centralized observer structure for each of these two sets of sub-observers. For the initial set of sub-observers $\{SO^{(1)}, SO^{(2)}\}$, we have

$$\begin{aligned}SO^{(1)}: &\frac{d}{dt}\begin{bmatrix}\hat{x}_1^{(1)} \\ \hat{x}_2^{(1)}\end{bmatrix} = \begin{bmatrix}a_{11} & a_{12} \\ 0 & a_{22}\end{bmatrix}\begin{bmatrix}\hat{x}_1^{(1)} \\ \hat{x}_2^{(1)}\end{bmatrix} + \begin{bmatrix}0 \\ b_{22}\end{bmatrix}u_2 \\ &+ \begin{bmatrix}a_{13} \\ 0\end{bmatrix}\hat{x}_3^{(2)} + \begin{bmatrix}k_{11} \\ k_{21}\end{bmatrix}(y_1 - c_{11}\hat{x}_1^{(1)}) \\ SO^{(2)}: &\frac{d}{dt}\hat{x}_3^{(2)} = a_{33}\hat{x}_3^{(2)} + b_{31}u_1 + k_{32}(y_2 - c_{23}\hat{x}_3^{(2)})\end{aligned} \tag{20}$$

Neglecting the effects of uncertainties, disturbances, and unmodeled dynamics, the corresponding estimation error dynamics are now governed by

$$\begin{aligned}SO^{(1)}: &\frac{d}{dt}\begin{bmatrix}e_1^{(1)} \\ e_2^{(1)}\end{bmatrix} = \begin{bmatrix}a_{11} & a_{12} \\ 0 & a_{22}\end{bmatrix}\begin{bmatrix}e_1^{(1)} \\ e_2^{(1)}\end{bmatrix} \\ &+ \begin{bmatrix}a_{13} \\ 0\end{bmatrix}e_3^{(2)} - \begin{bmatrix}k_{11} \\ k_{21}\end{bmatrix}c_{11}e_1^{(1)} \\ SO^{(2)}: &\frac{d}{dt}e_3^{(2)} = a_{33}e_3^{(2)} - k_{32}c_{23}e_3^{(2)}\end{aligned} \tag{21}$$

By augmenting the two error equations we obtain

$$\frac{d}{dt}\begin{bmatrix}e_1^{(1)} \\ e_2^{(1)} \\ e_3^{(2)}\end{bmatrix} = \left(\bar{A} - \underbrace{\begin{bmatrix}k_{11} & 0 \\ k_{21} & 0 \\ 0 & k_{32}\end{bmatrix}}_{\bar{K}}\bar{C}\right)\begin{bmatrix}e_1^{(1)} \\ e_2^{(1)} \\ e_3^{(2)}\end{bmatrix} \tag{22}$$

or equivalently, $\dot{e} = \big(\bar{A} - \bar{K}\bar{C}\big)e$, where $\bar{A} = A$ and $\bar{C} = C$. Therefore, according to Lemma 1, the feasibility of the cooperative estimation scheme with the use of the DES-selected initial set of sub-observers $\{SO^{(1)}, SO^{(2)}\}$ (as in equation (20)) is equivalent to the feasibility of the centralized estimation problem as given by equation (22) with an observer gain $\bar{K}$ having a sparse block-diagonal structure as follows

$$\bar{K} \in \bar{\mathbf{K}}_{3\times 2} = \left\{ diag\left(\begin{bmatrix} k_{11} \\ k_{21} \end{bmatrix}, k_{32} \right) \middle| \begin{bmatrix} k_{11} \\ k_{21} \end{bmatrix} \in \mathbb{R}^2, k_{32} \in \mathbb{R} \right\} \tag{23}$$

In this example, since each state is estimated by one and only one sub-observer, the pair $(\bar{A}, \bar{C})$ of the overall augmented error dynamics (22) is identical to the one obtained from the pair (A, C) of the original system.

Next, for the DES-reconfigured set of sub-observers $\{SO^{(1)}, SO^{(9)}\}$, it is easy to show that the resulting equivalent centralized observer has the same block-diagonal observer gain structure $\bar{K}$ as in equation (23). These details are not included due to space limitations. □

Up to this point, we have shown that the convergence properties of the cooperative set of sub-observers (that are selected by the DES supervisor) are equivalent to the centralized estimation scheme with a sparse block-diagonal gain $\bar{K}$. In the following, we investigate the convergence properties of a centralized estimation scheme having the gain matrix $\bar{K} \in \bar{\mathbf{K}}_{n_{N_{so}} \times m_{N_{so}}}$ given by equation (19).

Definition 9. For a given matrix $M \in \mathbb{R}^{n\times m}$ if one eliminates the rows $r_1, \ldots, r_i$ $(1 \leq r_1 < \ldots < r_i \leq n)$ and the columns $c_1, \ldots, c_j$ $(1 \leq c_1 < \ldots < c_j \leq m)$, the resulting matrix is represented and denoted by

$$M^{(-\{r_1,\ldots,r_i\},-\{c_1,\ldots,c_j\})} = \begin{bmatrix} M_{[1:r_1-1,1:c_1-1]} & M_{[1:r_1-1,c_1+1:c_2-1]} & \cdots & M_{[1:r_1-1,c_j+1:m]} \\ M_{[r_1+1:r_2-1,1:c_1-1]} & M_{[r_1+1:r_2-1,c_1+1:c_2-1]} & \cdots & M_{[r_1+1:r_2-1,c_j+1:m]} \\ \vdots & \vdots & \ddots & \vdots \\ M_{[r_i+1:n,1:c_1-1]} & M_{[r_i+1:n,c_1+1:c_2-1]} & \cdots & M_{[r_i+1:n,c_j+1:m]} \end{bmatrix} \in \mathbb{R}^{(n-i)\times(m-j)}$$

where $M_{[r_i:r_j,c_i:c_j]}$ denotes the submatrix of M which includes the elements up to and including the rows r_i and r_j, and the columns c_i and c_j.

According to [37], for a given MIMO linear system that is represented by the triplet $(\bar{A}, \bar{B}, \bar{C})$, a centralized estimation design that is obtained by using an observer gain matrix $\bar{K}$ having a certain sparse structure ($\bar{K} \in \bar{\mathbf{K}}_{n_{N_{so}} \times m_{N_{so}}}$) is feasible if and only if the system does not have any fixed modes with respect to the structured gain matrix $\bar{K} \in \bar{\mathbf{K}}_{n_{N_{so}} \times m_{N_{so}}}$, that is

$$\bigcap_{\bar{K} \in \bar{\mathbf{K}}_{n_{N_{so}} \times m_{N_{so}}}} eigs(\bar{A} - \bar{K}\bar{C}) = \emptyset \tag{24}$$

where $eigs(M)$ denotes the set of all eigenvalues of the matrix M. Moreover, in case of sensor and dynamic (actuator/structural) uncertainties or invalidity (faults, anomalies, or unreliabilities as per Definition 8), the above necessary and sufficient condition is complemented by the following two lemmas.

Lemma 2. Given that the j^{th} sensor measurement $y_j = Y_{[j]}, j \in \{1, \ldots, m\}$ is uncertain, a centralized estimation scheme by using an observer gain matrix $\bar{K}$ with a certain sparse block-diagonal structure ($\bar{K} \in \bar{\mathbf{K}}_{n_{N_{so}} \times m_{N_{so}}}$) is feasible if and only if

$$\bigcap_{\bar{K}^{sen} \in \bar{\mathbf{K}}^{sen}_{n_{N_{so}} \times m'_{N_{so}}}} eigs\big(\bar{A} - \bar{K}^{sen}\bar{C}^{sen}\big) = \emptyset$$

where $\bar{C}^{sen} = \bar{C}^{(-T^{sen},-\{\})}$, and $\bar{\mathbf{K}}^{sen}_{n_{N_{so}} \times m'_{N_{so}}} = \left\{ \bar{K}^{(-\{\},-T^{sen})} \middle| \bar{K} \in \bar{\mathbf{K}}_{n_{N_{so}} \times m_{N_{so}}} \right\}$, in which $T^{sen} = \big\{k \in \{1, ..., m\} | \exists l \in \Omega; Y^{(l)}_{[k]} = y_j\big\}$, and $m'_{N_{so}} = m_{N_{so}} - n(T^{sen})$, with $n(.)$ denoting the *cardinality* of a set.

Proof: Follows from the results in [37] and those in Lemma 1. ■

Lemma 3. Given that the i^{th} dynamical equation

$$\dot{x}_i = \dot{X}_{[i]} = A_{[i,:]}X + B_iU + w_i + F_{x_i}$$

is uncertain (invalid as per Definition 8), where the index $[i,:]$ represents the i^{th} row of the matrix, a centralized estimation scheme that is obtained by using an observer gain matrix $\bar{K}$ with a certain sparse block-diagonal structure ($\bar{K} \in \bar{\mathbf{K}}_{n_{N_{so}} \times m_{N_{so}}}$) is feasible if and only if

1) The state x_i is directly measured by a sensor, namely y_q, that is $\exists q \in \{1, \ldots, m\}; y_q = C_qX + v_q = x_i + v_q$.
2) The following eigenvalue condition holds

$$\bigcap_{\bar{K}^{eq} \in \bar{\mathbf{K}}^{eq}_{n'_{N_{so}} \times m'_{N_{so}}}} eigs(\bar{A}^{eq} - \bar{K}^{eq}\bar{C}^{eq}) = \emptyset$$

where $\bar{A}^{eq} = \bar{A}^{(-S^{eq},-S^{eq})}$, $\bar{C}^{eq} = \bar{C}^{(-T^{eq},-S^{eq})}$, and $\bar{\mathbf{K}}^{eq}_{n'_{N_{so}} \times m'_{N_{so}}} = \left\{ \bar{K}^{(-S^{eq},-T^{eq})} \middle| \bar{K} \in \bar{\mathbf{K}}_{n_{N_{so}} \times m_{N_{so}}} \right\}$, in which $S^{eq} = \big\{k \in \{1, \ldots, n\} | \exists l \in \Omega; E_{[k]} = e^{(l)}_i\big\}$, $T^{eq} = \big\{k \in \{1, \ldots, m\} | \exists l \in \Omega; Y^{(l)}_{[k]} = y_q\big\}$, $n'_{N_{so}} = n_{N_{so}} - n(S^{eq})$, and $m'_{N_{so}} = m_{N_{so}} - n(T^{eq})$, with $n(.)$ denoting the *cardinality* of a set.

Proof: Follows from the results in [37] and those in Lemma 1. ■

For the j^{th} sensor uncertainty, Lemma 2 is motivated by the fact that the uncertain sensor should be disregarded in the estimation process, that is the y_j-corresponding columns should be eliminated from the matrix $\bar{K}$ to yield the modified matrix $\bar{K}^{sen}$, and the y_j-corresponding rows should be eliminated from the matrix $\bar{C}$ to yield the modified matrix $\bar{C}^{sen}$, which should subsequently be used to verify the eigenvalue condition (24). Similarly, for the i^{th} uncertain dynamical equation, Lemma 3 is motivated by the fact that the uncertain state x_i should be directly measured by a sensor, namely y_q, since the uncertain dynamical equation should not be incorporated in the estimation of x_i. Therefore, the estimation process proceeds with the remaining system states excluding x_i and y_q. In other words, the x_i-corresponding columns and rows should be eliminated from the matrix $\bar{A}$ to yield $\bar{A}^{eq}$, the x_i-corresponding columns and y_q-corresponding rows should be eliminated from the matrix $\bar{C}$ to yield $\bar{C}^{eq}$, and the x_i-corresponding rows and y_q-corresponding columns should be eliminated from the matrix $\bar{K}$ to yield $\bar{K}^{eq}$. Consequently, the matrices $\bar{A}^{eq}$, $\bar{C}^{eq}$, and $\bar{K}^{eq}$ should be used to subsequently verify the eigenvalue condition (24).

The example below illustrates how Lemmas 2 and 3 enable one to determine whether a system with sensor and dynamic (actuator/structural) faults, anomalies, and unreliabilities is observable with respect to an observer gain matrix $\bar{K}$ having a certain block-diagonal structure $\bar{K} \in \bar{\mathbf{K}}_{n_{N_{so}} \times m_{N_{so}}}$.

Example 3. Consider the system that is defined in Example 1, whose state space representation matrices A and C are given by

$$A = \begin{bmatrix} -0.1 & 0.1 & 0.1 \\ 0 & -0.2 & 0 \\ 0 & 0 & -0.2 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The DES-selected initial set of sub-observers are $\{SO^{(1)}, SO^{(2)}\}$ in the absence of any anomaly and the DES-reconfigured set of sub-observers are $\{SO^{(1)}, SO^{(9)}\}$ in the presence of an anomaly. In Example 2, equivalent centralized observer structure of these two sets of sub-observers are provided in equation (22) with the sparse block-diagonal gain structure $\bar{\mathbf{K}}_{3\times 2}$ given by equation (23). In this example, this equivalent centralized observer structure is used to study the convergence properties of the two sets of sub-observers.

For the initial set of sub-observers $\{SO^{(1)}, SO^{(2)}\}$ we have

$$\bigcap_{\bar{K} \in \bar{\mathbf{K}}} eigs(\bar{A} - \bar{K}\bar{C}) = \emptyset$$

Therefore, the system is observable by using the DES-selected initial set of sub-observers in the absence of any fault. For the DES-reconfigured set of sub-observers $\{SO^{(1)}, SO^{(9)}\}$, $S^{eq} = \{3\}$ corresponds to the fault in the dynamical equation of x_3, and $T^{eq} = \{2\}$ corresponds to the sensor y_2 which directly measures x_3. We have

$$\bar{A}^{eq} = \begin{bmatrix} -0.1 & 0.1 \\ 0 & -0.2 \end{bmatrix}, \quad \bar{C}^{eq} = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

$$\bar{\mathbf{K}}^{eq} = \left\{ \bar{K}^{eq} = \begin{bmatrix} k_{11} \\ k_{21} \end{bmatrix} \middle| k_{11}, k_{21} \in R \right\},$$

$$\bigcap_{\bar{K}^{eq} \in \bar{\mathbf{K}}^{eq}} eigs(\bar{A}^{eq} - \bar{K}^{eq}\bar{C}^{eq}) = \emptyset$$

Therefore, the system is observable by using the DES-reconfigured set of sub-observers $\{SO^{(1)}, SO^{(9)}\}$ in the presence of an actuator fault F_{x_3}. □

In Examples 1-3, we have shown and illustrated how to systematically implement the Procedures 1 and 2, and Lemmas 1-3 in our proposed DES supervisory cooperative estimation framework. In the next section, the effectiveness of our proposed DES supervisory cooperative estimation scheme will be verified and illustrated through a case study and simulation investigations.

V. Case Study Simulation Results

Let us now consider a set of connected tanks that is shown in Fig. 9 as a case study and as a practical industrial application to demonstrate the utility and effectiveness of our proposed methodologies. It is desired to estimate all the tank levels by using a set of distributed sub-observers.

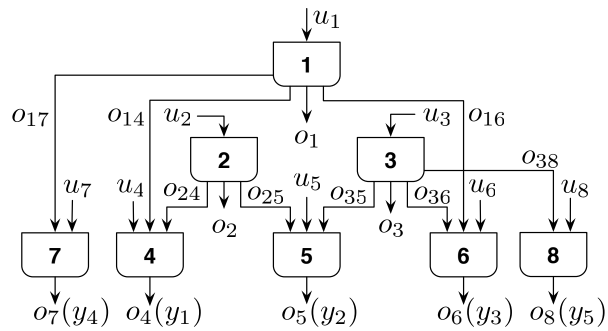

Fig. 9. An industrial process consisting of eight (8) tanks.

In Fig. 9, o_i and u_i represent the output and the input flows of tank i, respectively. Moreover, o_{ij} represents the flow from tank i to tank j. The linearized state space representation of the eight tank system that is derived from the fully nonlinear model of a single tank according to equation (10) is given by the triplet (A, B, C) where we have

$$A = \begin{bmatrix} -0.3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.2 & 0 & 0 & 0 & 0 & 0 \\ 0.1 & 0.1 & 0 & -0.1 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0.1 & 0 & -0.1 & 0 & 0 & 0 \\ 0.1 & 0 & 0.1 & 0 & 0 & -0.1 & 0 & 0 \\ 0.1 & 0 & 0 & 0 & 0 & 0 & -0.1 & 0 \\ 0 & 0 & 0.1 & 0 & 0 & 0 & 0 & -0.1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \; C = \begin{bmatrix} 0 & 0 & 0 & 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1 \end{bmatrix}$$

In the above state space representation, u_i corresponds to the input flow of tank $i = 1, 2, 3$, y_i corresponds to the sensor measurement of the output flow $o_j, j = 4, 5, \ldots, 8$, and the state x_k represents the level of tank $k = 1, \ldots, 8$, respectively.

For purpose of conducting simulations we assume that $o_1 = o_2 = o_3 = u_4 = u_5 = u_6 = u_7 = u_8 = 0$ in order to better demonstrate the effects of an uncertainty (anomaly, fault, or unreliability) on the system. Using the guidelines in Procedure 1 we design a set of *ten* sub-observers as follows: $SO^{(1)}$ $(\{\hat{x}_1, \hat{x}_4\} | \{u_1\}, \{y_4\}, \{\hat{x}_2\})$, $SO^{(2)}$ $(\{\hat{x}_2, \hat{x}_5\} | \{u_2\}, \{y_5\}, \{\hat{x}_3\})$, $SO^{(3)}$ $(\{\hat{x}_3, \hat{x}_6\} | \{u_3\}, \{y_6\}, \{\hat{x}_1\})$, $SO^{(4)}$ $(\{\hat{x}_7\} | \{\}, \{y_7\}, \{\hat{x}_1\})$, $SO^{(5)}$ $(\{\hat{x}_8\} | \{\}, \{y_8\}, \{\hat{x}_3\})$, $SO^{(6)}$ $(\{\hat{x}_4\} | \{\}, \{y_4\}, \{\})$, $SO^{(7)}$ $(\{\hat{x}_6\} | \{\}, \{y_6\}, \{\})$, $SO^{(8)}$ $(\{\hat{x}_1, \hat{x}_7\} | \{u_1\}, \{y_7\}, \{\})$, $SO^{(9)}$ $(\{\hat{x}_3, \hat{x}_8\} | \{u_3\}, \{y_8\}, \{\})$, and $SO^{(10)}$ $(\{\hat{x}_5\} | \{\}, \{y_5\}, \{\})$, where y_i $(i = 4, \ldots, 8)$ is the measurement of the output flow o_i.

The DES models of the sub-observers can be obtained by using the Procedure 2. As an example, the DES models of

$SO^{(1)}$ is shown in Fig. 10, where Fig. 10-(a), (b), and (c) models the requirements of u_1, $\hat{x}_2$, and y_4, respectively. The states $\hat{x}_1$ (with $\Omega_{\hat{x}_1} = \{1, 8\}$) and $\hat{x}_4$ (with $\Omega_{\hat{x}_4} = \{1, 6\}$) are modeled in Fig. 10 (d) and (e), respectively, with their corresponding sub-observers.

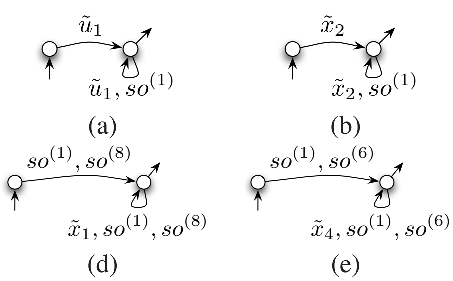

Fig. 10. The DES automata of $SO^{(1)}$ with its components (a) u_1, (b) $\hat{x}_2$, (c) y_4, (d) $\hat{x}_1$, and (e) $\hat{x}_4$.

The DES plant model G, which includes all the ten sub-observers and their estimated values, can be generated by using the *sync* product of all the constructed automata using the TCT software package [14]. The resulting plant model G contains 4080 states and 65264 transitions. The size of G should provide a clear and convincing justification as to why modeling the set of sub-observers cannot be accomplished in a single-step and through a single centralized DES design process. Moreover, in case that the problem was tackled by using a lookup table, the size of this table will be $4^{10} = 1,048,576$ given the 4 possible scenarios for each sub-observer (valid, invalid, used, not used) and the fact that we employ 10 sub-observers.

The simulation time lines are shown in Figs. 11 and 12. According to Figs. 11 and 12, in the time interval $[0-100](s)$, there is no fault in the system, and the DES supervisor selects the initial set of sub-observers as

$$Set_0 = \{SO^{(1)}, SO^{(2)}, SO^{(3)}, SO^{(4)}, SO^{(5)}\}$$

At time $t = 100(s)$, the fault $F_{x_4} = -0.05x_1$, which represents a 50% loss of effectiveness flow in the pipe connecting the tank 1 to tank 4 occurs. Therefore, the sub-observer $SO^{(1)}$ is no longer accurate and its validity condition is violated (as per Definition 8). The fault F_{x_4} is modeled as a specification in the DES framework as shown in Fig. 13 below. Therefore, in case of violation of the validity condition of $SO^{(1)}$, the DES supervisor prevents utilization of the invalid sub-observer $SO^{(1)}$. It can be shown that the DES plant under supervision has now 4080 states and 62800 transitions. Comparing this to the original plant model, the reduction in the size of the DES plant model is due to the elimination of all the paths where $SO^{(1)}$ is used. The estimation process is continued with the initial DES-selected set of sub-observers Set_0 until the time $t = 150(s)$, when the DES supervisor modifies the set of sub-observers as shown below

$$Set_1 = \{SO^{(2)}, SO^{(3)}, SO^{(5)}, SO^{(6)}, SO^{(8)}\}$$

which is triggered to compensate for the adverse effects of the fault F_{x_4} on the cooperative estimation performance.

At time $t = 250(s)$, the fault $F_{x_6} = -0.05x_1$, which represents a 50% loss of effectiveness flow in the pipe connecting the tank 1 to tank 6 occurs. The estimation process continues with the DES-reconfigured set of sub-observers Set_1 until the time $t = 300(s)$, when the DES supervisor modifies the set of sub-observers as shown below

$$Set_2 = \{SO^{(2)}, SO^{(6)}, SO^{(7)}, SO^{(8)}, SO^{(9)}\}$$

which is triggered to compensate for the adverse effects of the faults F_{x_4} and F_{x_6} on the cooperative estimation performance.

At time $t = 400(s)$, the fault $F_{x_5} = -0.05x_3$, which represents a 50% loss of effectiveness flow in the pipe connecting the tank 3 to tank 5 occurs. The estimation process continues with the DES-reconfigured set of sub-observers Set_2 until the time $t = 450(s)$, and the DES supervisor does not propose any new set of sub-observers that can compensate for the adverse effects of the simultaneous permanent faults F_{x_4},

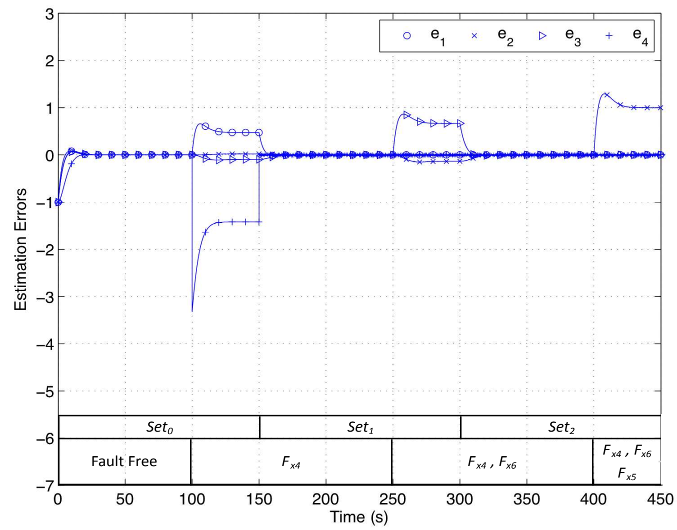

Fig. 11. Estimation errors (1 to 4) corresponding to different faults, sets of sub-observers, and simulation time intervals.

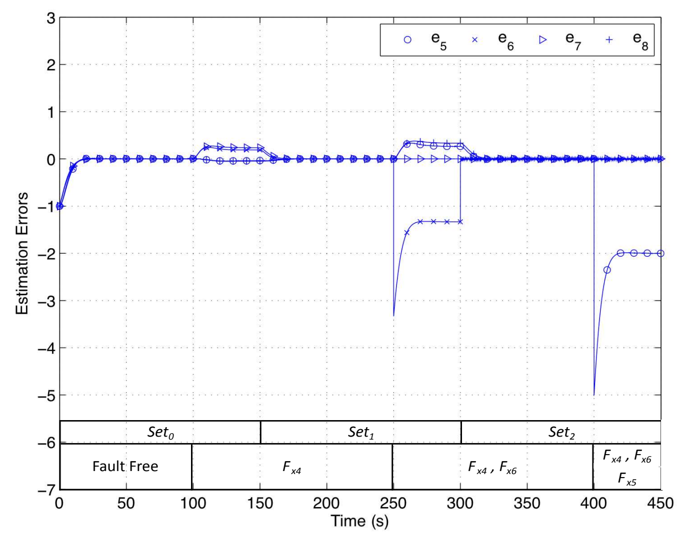

Fig. 12. Estimation errors (5 to 8) corresponding to different faults, sets of sub-observers, and simulation time intervals.

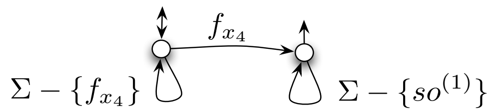

Fig. 13. The DES specification of the validity condition of $SO^{(1)}$.

F_{x_5}, and F_{x_6}. This is due to the fact that the sub-observers $SO^{(1)}, \ldots, SO^{(10)}$ are not sufficient and adequate for the DES supervisory cooperative estimation in the presence of these three concurrent faults.

The estimation performances of the DES-reconfigured sets of sub-observers Set_0, Set_1, and Set_2 are shown in Figs. 11 and 12 and Table I. These results clearly demonstrate that the estimation errors are considerably improved at each stage, when the DES supervisor modifies the set of sub-observers to compensate for the adverse effects of a new fault. It should be noted that according to Assumption 1, the existing FDI module provides the exact fault information *independent* of the sub-observers. Therefore, the estimation errors are not available to the DES supervisor nor to the external FDI module to arrive at any conclusions and decisions.

Time		$0^{(sec)}$	$100^{(sec)}$	$150^{(sec)}$	$250^{(sec)}$ $300^{(sec)}$		$400^{(sec)}$
		Set_0		Set_1		Set_2	
		Fault Free	F_{x4}		F_{x4} , F_{x6}		F_{x4} , F_{x6} , F_{x5}
Estimation Error	$\vert e_1 \vert$	< 0.01	**0.47**	< 0.01	< 0.01	< 0.01	< 0.01
	$\vert e_2 \vert$	< 0.01	**0.02**	< 0.01	**0.13**	< 0.01	**1.00**
	$\vert e_3 \vert$	< 0.01	**0.09**	< 0.01	**0.67**	< 0.01	< 0.01
	$\vert e_4 \vert$	< 0.01	**1.42**	< 0.01	< 0.01	< 0.01	< 0.01
	$\vert e_5 \vert$	< 0.01	**0.04**	< 0.01	**0.26**	< 0.01	**2.00**
	$\vert e_6 \vert$	< 0.01	**0.19**	< 0.01	**1.33**	< 0.01	< 0.01
	$\vert e_7 \vert$	< 0.01	**0.24**	< 0.01	< 0.01	< 0.01	< 0.01
	$\vert e_8 \vert$	< 0.01	**0.05**	< 0.01	**0.33**	< 0.01	< 0.01

TABLE I

ESTIMATION STEADY STATE ERRORS CORRESPONDING TO DIFFERENT FAULTS, SETS OF SUB-OBSERVERS, AND SIMULATION TIME INTERVALS.

VI. CONCLUSION

In this work, a novel multi-agent hybrid framework for supervisory cooperative estimation of LTI systems is proposed. The notion of sub-observers are first introduced and subsequently they are modeled in the DES framework. Moreover, a DES supervisor is designed to manage the cooperation amongst the sub-observers. The DES supervisor selects and configures a set of sub-observers to successfully estimate all the system states. Whenever certain anomalies, uncertainties, or unreliabilities are present in the system, the supervisor is capable of reconfiguring and reselecting the set of sub-observers such that the impacts of anomalies on the obtained state estimation performance is maximally reduced. In addition, the feasibility and convergence of the overall integrated sub-observers are validated formally and the application of our proposed methodology to a practical industrial process is demonstrated through simulations. Future work will involve investigation of other classes of estimation filters and their cooperative performance evaluations.

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