The impact of direct instruction on quantitative representations of manipulatives in the context of first-graders' learning of place value concepts

Allyson Cooperman

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By: Allyson Cooperman

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Signed by the final Examining Committee:

______________________________________Chair
Dr. Claude Martel

______________________________________Examiner
Dr. Holly Recchia

______________________________________Examiner
Dr. Nadia Hardy

______________________________________Supervisor
Dr. Helena P. Osana

Approved by:

____
Chair of Department or Graduate Program Director

____________________ 2012 __________________________________
Dean of Faculty

ABSTRACT

The impact of direct instruction on quantitative representations of manipulatives in the context of first-graders' learning of place value concepts

By Allyson Cooperman

This study examines the impact of directly telling first grade students the quantitative meaning of manipulatives on their learning of place value. Fifty-three first-grade students and four second-grade students ($N = 57$) were randomly assigned to one of three conditions: Math Encoding, Free Play Encoding or Control. The students in the Math Encoding condition were explicitly told that a blue chip was worth one and a red chip was worth ten. The students in the Free Play Encoding condition encoded the chips as anything they wished through informal free play activities. The students in the third condition acted as a Control group. The primary objective was to determine whether it is essential for teachers to explicitly tell students what manipulatives represent before using them procedurally to learn place value concepts. In line with my predictions, it appears that being explicitly taught what mathematics manipulatives represent in the context in which they are being used results in correct quantitative representation of the manipulatives. Contrary to my predictions, it appears that having a correct quantitative representation of the manipulatives does not give students an advantage for acquiring place value knowledge. The results of this study will inform classroom practice involving manipulatives and conceptual understanding in mathematics.

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Chapter 1: Statement of the Problem

Research has shown that it is essential for children to acquire conceptual knowledge to learn mathematics with understanding (Fuson & Briars, 1990; Osana & Pitsolantis, 2011; Uttal, Scudder & DeLoache, 1997; Wearne & Hiebert, 1988). Conceptual knowledge can be defined as knowledge that links pieces of information together (Hiebert & Lefevre, 1986). Conceptual understanding is important for several reasons. On a theoretical level, the National Research Council (NRC, 2001) argued that mathematical proficiency is a combination of five separate strands: (a) conceptual understanding, (b) procedural fluency, (c) strategic competence, (d) adaptive reasoning, and (e) productive disposition. It argued that these five strands are interconnected and assist in the development of each other. Thus, conceptual knowledge aids in the progression of learning in these five areas. Moreover, the NRC highlighted that when children learn with understanding, their learning leads to improvement in retention, fluency, and transfer (see also Kilparick, Swafford, & Findell, 2001). This is because when children reach a level of understanding, they are better able to organize and link together the material they are learning (NRC, 2001).

There are several ways in which conceptual knowledge can be incorporated and taught in elementary mathematics classes. One tool that can be beneficial for children to use when learning mathematics is manipulatives (Uttal & O'Doherty, in press) and indeed, teachers report using the manipulatives regularly with their students (Moyer, 2001; Moyer & Jones, 2004). Manipulatives may convey abstract mathematical concepts in a concrete way and as such, are believed to be vehicles used to improve conceptual understanding (Uttal, 2003). Uttal et al. (1997) argued, however, that concrete objects by themselves do not inherently embed the concepts in the children's minds. They argued that the use of these concrete objects needs to be accompanied by additional instruction on what concepts they represent. It is important to link the manipulatives to the concepts that underlie the procedures used in their mathematics classes so that children will ultimately understand what they are doing and why they are doing it (Fuson & Briars, 1990; Resnick & Omanson, 1987; Uttal et al., 1997).

Ensuring that students attach conceptual meaning to manipulatives or symbols in elementary mathematics classes is a growing concern in educational research (Fuson & Briars, 1990; Osana & Pitsolantis, 2011; Resnick & Omanson, 1987; Wearne & Heibert, 1988). In mathematics, students need to learn how to use the standard mathematical notation, often referred to as symbols (Heibert, 1992). More generally, however, symbols are defined as characters that act as a representation of a concept or an action (DeLoache, 1995; Goldin, 2003). In line with this notion, then, concrete objects are also symbols because concrete objects, or manipulatives, are often used in mathematics instruction to represent concepts. Although written characters are most often associated with the term "symbol" (numerical symbols), manipulatives (concrete symbols) can also be used as symbols to represent the meaning of quantity. Therefore, as symbols, both manipulatives and numerical notation must be connected to relevant concepts to be understood as representations of something else.

Over the years, developmental researchers have shown that children require explicit instruction on the conceptual representation of concrete objects to internalize their meaning (e.g., DeLoache, 2000; Uttal, 2003). This has also been articulated by Uttal et al. (1997), but there is little evidence in the mathematics literature to support this contention. In the context of classroom mathematics, one can use these findings to argue that manipulatives must be introduced in such a way for children to encode them in the intended manner. The research seems to suggest that teachers must explicitly tell students what the manipulatives mean before using them during instruction (Fuson & Briars, 1990; Osana & Pistolantis, 2011, Resnick & Omanson, 1987).

Additionally, the environment and learning atmosphere in the classroom have a strong impact on the children's capacity to learn, including the ways in which children encode the manipulatives they are using. The environment within which children learn can affect their cognitive gains (Bruner, 1966, as cited in Martin, 2009), even with the use of manipulatives. Initial conceptual knowledge of what the manipulatives represent has been argued to enhance procedural knowledge (Rittle-Johnson, Seigler, & Alibali, 2001), and help children obtain conceptual understanding of the procedures they are using. Despite this, in elementary classrooms, students generally learn procedures void of conceptual understanding (e.g., Rittle-Johnson et al., 2001).

Research has uncovered the conditions under which manipulatives are effective for learning (Fuson & Briars, 1990; Moyer, 2001; Resnick & Omanson, 1987). One of these conditions involves how children understand or think about the manipulatives they are using (Bruner, 1966, as cited in Martin 2009; DeLoache, 2000; Uttal, 2003; Uttal, Liu, & DeLoache, 1999; Uttal et al., 1997). A problem, however, is that teachers often assume that children attach appropriate meanings to the manipulatives on their own, when in fact, it is not at all clear that they do (Barlow & McCrory, 2011; Buczynski, Gorsky, McGrath, & Myers, 2011; Lo & McCrory, 2010; Macken, 2011; Moyer, 2001; Mueller & Maher, 2010; Thom, 2011; Voza, 2011). For example, Thom (2011) described incorporating lessons that engage her students in activities with manipulatives to foster mathematical reasoning. Her lessons involved counting cubes to solve problems, but she did not discuss the relationship between the cube and the quantity it represents, thus assuming that her students understood this relationship prior to the activity. Several other examples can be found in the literature (e.g., Lo & McCrory, 2010). Therefore, it is important to see whether explicit instruction, which attaches a specific meaning to the manipulatives before students use them in instruction, would enhance the acquisition of conceptual knowledge.

The research appears to show that teachers may not be introducing manipulatives to children in such a way that emphasizes their conceptual meaning (e.g., Moyer, 2001). Therefore, the objective of the present study is to examine whether directly giving the quantitative meaning of manipulatives before receiving procedural instruction will positively impact their conceptual understanding of place value. This will provide evidence about whether students do in fact need to learn the conceptual basis of manipulatives prior to instruction to make the lessons meaningful, or if they can pick up the meaning of the manipulatives on their own during instruction. The results of the study speak to theory about lesson structure (i.e., concepts and procedures) and practice in the mathematics classroom.

Chapter 2: Literature Review

Definitions

A considerable portion of the existing literature on mathematical knowledge focuses on differentiating conceptual and procedural knowledge (e.g., Hiebert & Lefevre, 1986; Rittle-Johnson & Koedinger, 2008; Rittle-Johnson & Siegler, 1998; Rittle-Johnson et al., 2001; Star, 2005). *Conceptual knowledge* is characterized as knowledge that links pieces of information together (Hiebert & Lefevre, 1986). Conceptual knowledge of place value, for example, involves knowing a number of interconnected ideas, one of them being that the value of a digit in the decimal representation of a number is determined by its position or “place” in that representation. A second idea is that one cannot have more than nine groups in a denomination. Moreover, a third idea is knowledge of the additive principle -- that is, understanding that 325 is in fact three times 100, two times 10, and five times 1 added together (Heibert, 1992).

Procedural knowledge has been defined as a composition of two distinct but related types of knowledge. The first type is characterized by familiarity with the rules for writing symbols, such as the standard conventions for using mathematical notation (Hiebert, 1984). The second type of procedural knowledge is characterized by knowing the instructions or rules required to perform a mathematical task, such as executing an algorithm (Hiebert & Lefevre, 1986). Examples of rules in addition are placing only single digit numbers in each of the columns (e.g., ones, tens, hundreds) and knowing how to regroup when necessary (Hiebert, 1992).

In mathematics, students need to learn how to use the standard mathematical notation, often referred to as *symbols* (Heibert, 1992). More generally, however, symbols are defined as characters that act as a representation of a concept or an action (DeLoache, 1995; Goldin, 2003). Concrete objects are often used in mathematics classes to represent mathematical concepts and can therefore also be viewed as symbols.

Children use manipulatives, but too often they represent these objects solely as objects, not as objects that “stand for” something else, such as quantity. When using manipulatives for mathematics learning, however, children must be able to understand that a block can represent a plastic object and a quantity simultaneously. This notion is called “dual representation” (DeLoache, 2000). Dual representation is the ability to perceive something concrete as an object in and of itself and at the same time understand it as a representation of something more abstract, such as a quantity (Uttal et al., 1997). For example, possessing the ability to recognize colored chips as simply plastic chips and as representations of quantities demonstrates a dual representation of the objects.

Using Concepts to Support Procedures

Several researchers have argued that concepts and procedures develop iteratively (Rittle-Johnson & Koedinger, 2008; Rittle-Johnson et al., 2001). More specifically, it is believed that children’s knowledge of concepts benefits their knowledge of procedures, which leads to further knowledge of concepts, and so on. Educational psychologists have used these findings to design instruction that uses concepts to promote the understanding of procedures. For example, Rittle-Johnson and Koedinger (2008) compared students’ performance on decimal concepts and procedures after receiving “iterative lessons” or “concept first lessons” on the subject. One group of students received lessons on place value concepts followed by lessons on procedures, and the second group of students received the same number of lessons with the conceptual and procedural lessons given iteratively. Although the findings were mixed, the authors found that students in the iterative condition had higher scores in arithmetic knowledge compared to the students in the concepts first condition.

Researchers have conducted a number of studies to assist students to make links between concepts and procedures in the areas of decimals, fractions, and place value. For example, Osana and Pitsolantis (2011) conducted a study with fifth- and sixth-grade students to determine the effectiveness of an intervention on their knowledge of fractions. The intervention instruction, based on Hiebert (1984), focused on numeric and concrete symbol interpretation, procedural execution, and solution evaluation, all in the context of connecting concepts and symbols. The authors found that when students completed lessons on the conceptual underpinnings of the symbolic manipulations of standard algorithms, their performance on knowledge of concepts and links between concepts, and symbols, improved. This improvement was relative to those who received lessons on concepts and procedures separated from one another. Their results suggest that students cannot make the link between concepts and procedures on their own, highlighting the fact that this link needs to be explicitly made when teaching students.

Although Osana and Pitsolantis (2011) found that the treatment group was better able to link concepts to symbols, it is unclear whether this learning was dependent on the initial link made between the concepts and their physical representations or whether instruction on the links between concepts and procedures was sufficient for this learning to occur. In other words, the Osana and Pitsolantis study was not designed to check whether the students encoded the physical representations in the intended way before or during instruction. It is thus difficult to conclude from this study whether first linking manipulatives to their associated concepts is a necessary prerequisite to instruction. This is an important question, because if teachers need to decide when and how to introduce the manipulatives to their students to have them learn conceptual meaning, empirical evidence that determines the most critical period for students to learn this notion is important.

In another study, Wearne and Heibert (1988) examined fourth, fifth, and sixth grade children's understanding of decimal fractions with the use of base-10 blocks by explicitly linking concrete objects to mathematical concepts. They claimed that making connections -- that is, connections between numeric symbols and base-10 blocks -- and developing procedures with the use of those blocks are dependent on creating links between written numbers and meaningful concrete objects. In particular, they claimed that making the connection between manipulatives and procedures is dependent on the *initial* link between the manipulatives and the concepts they represent. After instruction, Wearne and Heibert found that children performed better on both connecting written numbers to manipulatives and executing procedures. Moreover, the students also increased their performance on transfer tasks. The authors assumed that they performed better on these tasks because they made the initial connection between the concepts and the blocks, thus speculating that the blocks were in fact meaningful to the students from the beginning (i.e., that the blocks by themselves represented concepts before instruction). It is unclear, however, whether the blocks could have become meaningful as a result of the instruction alone because Wearne and Heibert did not directly test this question. Once again, this notion of *when* the correct representation of the manipulatives should be made is critical. Although the quantitative meaning of the manipulatives may be very obvious to teachers, it is not evident that children also obtain this initial connection (Uttal et al., 1997).

Fuson and Briars (1990) implemented an intervention with first-and-second-graders to teach them concepts and procedures of place value using base-10 blocks. The first lesson linked various representations of quantities: English words, concrete materials, and written notation. More complex lessons on addition and subtraction involving the same relationships were built on from there. Children who learned the blocks earlier in the previous year were given the same lessons to ensure that they represented the manipulatives in the intended manner.

The authors' learning goal was for students to compute problems made up solely of written numbers. The conceptual understanding of the other embodiments, such as written words and concrete materials, was argued to be a critical aspect of the students' ability to learn the standard algorithm with meaning (Fuson & Briars, 1990). Thus, if the students did not understand which concepts the concrete materials represented, they would have more difficulty understanding the concepts and procedures involved in computing numbers. The authors did not, however, examine whether students had encoded an alternative understanding of the manipulatives, one that did not match that intended by the researchers, or the effects of alternate understandings. In other words, it is difficult to conclude whether the students actually held the correct representation of the manipulatives before instruction. It is essential to do so to determine whether they acquired this correct representation prior to learning place value concepts or whether they were able to make the connection during the lesson.

In sum, all of these studies (Fuson & Briars, 1990; Osana & Pitsolantis, 2011; Wearne & Heibert, 1988) suggest that when teaching children mathematics, the link between concepts and procedures is essential for optimal learning to occur. More specifically, it is beneficial to use conceptual understanding to foster a meaningful development of procedural knowledge (Rittle-Johnson et al., 2001). The results also suggest that concrete symbols can effectively embody a concept and can therefore be used to make the connection between concepts and procedures more visible for students. It follows then, that a critical feature of integrating concrete symbols to promote the link between concepts and procedures is to have a clear understanding of the particular concept the concrete symbol is intended to represent. Failure to do so makes the connection between the concrete symbol and the procedure less transparent (Uttal et al., 1997). The critical question is, however: when do children actually make this connection? The answer to this question is essential when trying to improve teaching that includes the use of manipulatives.

Linking Concepts to Concrete Symbols

The previous section discussed the importance of the link between concepts and procedures in mathematics classes. When manipulatives are used as a tool to facilitate the learning of this link, an additional link between the concepts and the manipulatives must be made. Duval (1999) argued that representation is the foundation for understanding the conceptual representation of manipulatives. Children's understanding of manipulatives as representations for concepts is critical when they are used to link concepts and procedures. Thus, the following section will discuss the specific link between concepts and the manipulatives.

Systems of representations theory. In addition to the empirical evidence supporting the use of concrete symbols to link concepts and procedures, Goldin's (2003) theory of systems of representation provides additional support. That is, he proposed that representation can be understood in the context of an internal psychological system and an external system. The internal psychological system refers to the mental representation or abstraction of an entity the individual holds in his or her mind. Conversely, the external system refers to a symbol or object that represents an entity, such as three blocks representing the quantity "three." Goldin suggested that the systems are independent and remain so unless explicit connections are promoted. In line with this, Duval (1999) argued that without representation, it is difficult to fully obtain the intended use of mathematics objects. The key element is that the *intended* representation becomes salient, and not the symbolic ones.

Put in the context of learning mathematics, Goldin's theory suggests that a student's implicit knowledge of a concrete symbol, such as a manipulative, may not match the concept intended by the teacher. It is therefore important for a teacher to link the internal system, or concept, to the external system, or concrete symbol, to promote understanding. Wearne and Heibert (1988), for instance, included an "encoding" phase where an explicit link between the concept and the concrete symbol was made prior to the intervention. Indeed, Wearne and Heibert reasoned that an encoding phase is a critical prerequisite to subsequent instruction on problem solving and computation because manipulatives can be perceived as everyday materials and their purpose needs to be made clear in the context in which they are being used.

Dual representation theory. Understanding that concrete objects can be thought of as two things (i.e., as representing an abstract concept and a physical object in its own right) is central to the theory of dual representation. Educators have stated that the benefit of having their students use manipulatives to learn mathematics is that they enable the students to think concretely so that they do not have to think abstractly (Piaget, 1970, as cited in Uttal et al., 1997). Uttal et al's. (1997) argument is that educators and researchers often make the assumption that the manipulatives used are the concepts themselves, but for students to represent these manipulatives as “stand ins” for their intended concepts, students in fact are required to think abstractly. Goldin’s (2003) theory of internal and external systems of representation is useful in interpreting Uttal et al.’s argument. For students to represent the manipulatives as concepts, they need to make the connections between the physical representation and the desired elements (i.e., quantity).

Uttal et al. get support for their claim from the developmental literature on children’s understanding of concepts and scale models. DeLoache (1987) examined two and a half and three year old children on their understanding of a scale model as a representation of a larger room. They examined the children’s ability to make the link between the model room and the actual room. The children were shown the model and witnessed the experimenter hide an object in it. Following this, the younger children were unable to find the object in the larger room. When shown the model again, however, they were able to identify where the object was hidden in the model. Thus, children did not have difficulty remembering where the object was hidden in the model. Rather, they had difficulty seeing the model as a representation of the larger room and could therefore not make the connection between them. According to DeLoache, the children did not demonstrate dual representation of the scale model.

Uttal et al. (1997) argued that if the young children in DeLoache's (1987) study had difficulty interpreting the scale model as a representation of a target referent, then teachers should not expect children in mathematics classes to naturally interpret manipulatives as representing specific abstract concepts. Furthermore, the authors pointed out that a concrete object cannot physically resemble something abstract. Therefore, children should not be expected to immediately make this connection without guidance.

Uttal and O'Doherty (in press) discussed students' understanding of the representational meaning of symbols. They cited evidence that undergraduate science students have trouble understanding that the "red dots or green circles" in the pictures in their textbooks were representations of various proteins. This evidence suggests that individuals tend to focus on the physical representations before thinking abstractly, and that in order for them to internalize the intended conceptual understanding, they must first make the link between the concepts and the symbols used to represent them. Uttal and O'Doherty pointed out that the experts and professors in the field understand the concrete representations and their link to the intended conceptual meaning with ease. Therefore, this can create a discrepancy between the experts' instruction and the students' knowledge.

Uttal et al.'s (1997) theory of dual representation and the need for direct instruction is made several times in the developmental literature; there is little evidence, however, to support this theory in the domain of mathematics learning. One exception is the study by Resnick and Omanson (1987). In their study, they examined third graders' understanding of mathematical concepts involved in subtraction through written notation and concrete representations. Through instruction called "Mapping Instruction," they linked the blocks, written numbers, and concepts of number and subtraction. Overall, their students improved on tasks involving the manipulatives, but when asked to solve problems simply with written numbers, children had difficulty transferring the knowledge they learned with the manipulatives and were unable to solve symbolically-presented problems correctly. Therefore, the ability to manipulate the concrete representations procedurally did not transfer to being able to complete the subtraction algorithm correctly. This evidence highlights the fact that although students may be able to solve problems with manipulatives, one cannot make the inference that they fully understand the symbolic representation of those manipulatives, an important aspect of mathematical learning.

In sum, the literature suggests that when using concrete symbols to foster a connection between concepts and procedures, it is important to explicitly link the concept to the concrete symbol. Whether or not links between concepts and concrete symbols should be made before or during instruction remains to be seen; what is known to date is that making this connection is valuable for student learning (i.e., Wearne & Hiebert, 1988).

It is my contention, however, that the link between internal and external representations should be made *prior to* instruction. This view aligns with assumptions about critical periods for learning whereby intuitive or spontaneous conceptions of concrete symbols may create a "mental filter" (Bransford et al., 2006). That is, if the concrete symbol is not initially linked to the intended concept, students' mental filter may challenge the link between the concept and the procedure. The longer this misguided connection occurs, the more difficult it is to redirect the learning to what was intended. The "mental filter" theory suggests, then, that the initial link between the concrete symbol and its intended concept should be made clear as early as possible. More simply stated, if manipulatives are introduced to the children as an aid for making a mathematical concept clear, the children must first add to their initial perception of the manipulative as a plastic object in and of itself by making a connection to the manipulative's quantitative representation (i.e., concept of quantity). Once children have made this link, they can strengthen their understanding of the concept in the context of other representations, such as written symbols and procedures. Accordingly, children should acquire quantitative meaning of the concrete objects before any further instruction is provided to them.

Using Manipulatives in the Classroom

Currently, the literature suggests that there are ways that teachers can design classroom activities with manipulatives to increase the likelihood that students will acquire dual representation. One approach is to focus on the characteristics of the manipulatives (Martin, 2009). To examine this, Kaminski, Sloutsky, and Heckler (2005) conducted two studies examining the impact of relevant concreteness on the learning and transfer of undergraduate students. They examined the effects of symbols in the form of pictures in four conditions**:** relevant/perceptually dull, relevant/perceptually rich, irrelevant/perceptually dull, and irrelevant/perceptually rich. The dependent measure was students' learning of rules in a base domain and transfer performance. Relevant symbols were pictures of measuring cups and irrelevant symbols were pictures of shapes. Perceptually rich symbols were filled with patterns and perceptually dull symbols were solid black. Overall, they found that when students were exposed to relevant characteristics, they performed better on learning tasks than students who were exposed to irrelevant characteristics. Moreover, when examining the students' performance on transfer tasks, students who were introduced to relevant/perceptually dull symbols performed better compared to those who were introduced to relevant/perceptually rich symbols. Therefore, it can be concluded that symbols enhance learning the most when they are relevant to the tasks but are as perceptually dull as possible, relieving the learner from possible distractions (Kaminski et al., 2005).

In two separate experiements, McNeil, Uttal, Jarvin, and Sternberg (2009) examined the difference in children's performance on mathematical word problems using manipulatives that were either highly realistic or bland, and they compared their performance to a control group who did not use manipulatives during problem solving. Participants in their studies were between fourth- and sixth-grade. The highly realistic manipulatives were fake monetary bills that were designed to look like U.S. currency, thereby activating real world familiarity with U.S. money. The bland manipulatives were the same shape, but were white and only had the currency written on them in numeral form. The control group received no manipulatives to solve word problems related to money.

In Experiment 1, the authors compared the highly realistic condition to the control condition and found that students in the control condition outperformed the highly realistic condition. This suggests that manipulatives that are highly realistic hinder students' performance. Moreover, in Experiment 2, the authors compared the neutral experimental condition to the highly realistic experimental condition and the control group. Once again, students in the highly realistic experimental group did not perform as well as the other two conditions. The authors speculated that highly realistic manipulatives may in fact hinder students' learning and performance on mathematical tasks because they have to combat previous representations of those manipulatives. In other words, realistic manipulatives leave less room for thinking abstractly about the quantities they are meant to represent. Thus, although not directly addressed by McNeil and her colleagues (2009), students may need explicit instruction on the representation of manipulatives to go beyond their distracting features.

Another factor that may influence students' acquisition of dual representation is the way the manipulatives are introduced in the classroom, which can influence how they are encoded by the children (Uttal et al., 1997). Students may require assistance to understand the mathematical concepts behind the objects, particularly if they are difficult to identify (Kloos & Sloutsky, 2008). For example, Puchner, Taylor, O'Donnell, and Fick (2008) met with teachers during professional development sessions to discuss the use of manipulatives in mathematics lessons. The teachers in the study reported observing their students use manipulatives simply because they were told to use them rather than as an aid for solving problems. For example, one student said that he needed to know the answer to the problem before he could "solve" it with the manipulatives. As such, he began by solving the problems using a written algorithm and then attempted to show this same solution with the manipulatives. Thus, the student did not need the objects to find the solution to the problem, but used them because the teacher asked him to.

For students to link concepts and procedures, they need to acquire a correct problem representation of the task, including the manipulatives that are involved in it (Rittle-Johnson et al., 2001). Rittle-Johnson et al. (2001) maintained that a correct representation of the manipulatives is necessary to make the link between the two forms of knowledge. Furthermore, explicit instruction is argued to be a beneficial method to help children obtain that initial understanding of the mathematical concepts they are learning with manipulatives (Kloos & Sloutsky, 2008).

A similar conclusion was made by DeLoache (1989), who examined 3-year-old children's ability to determine where a toy was hidden in a regular room after watching it being hidden in a model room. She found that when the children were given direct explicit instructions on the relationship between the model room and the real room, they performed better compared to those who were not given direct instructions. In this context, explicit instruction on the link between a representation and an abstract concept facilitated learning, even when the model was a direct replica of its original form. Thus, it can be argued that mathematics manipulatives, which often do not physically resemble the abstractions they represent, would also need to be accompanied by explicit instruction on their symbolic representation (DeLoache, 1989).

Along the same lines, allowing children to play with manipulatives may hinder their ability to perceive them symbolically (DeLoache, 2000). For example, DeLoache (2000), using the same model scale test, examined whether enhancing the children's perspective of the model as an object, and not as a representation, would hinder their ability to acquire dual representation. In her study, all participants were placed in front of the model set while the experimenter completed the task with them, but the comparison group was able to play with the model on their own for 10 minutes prior to beginning the task. She found that the children who played with the model beforehand were less capable of making the link between the model room and the real room compared to the children who did not play with the model. Thus, allowing the children to play with the object appeared to hinder their ability to represent the model dually (DeLoache, 2000).

Similarly, enabling children to play with manipulatives in mathematics classes may facilitate the development of their own meaning of the manipulatives, but possibly not the intended one. It may also reinforce their understanding that the manipulatives are solely concrete objects (Uttal et al., 1997). The research reviewed have suggested, however, that neither of these is beneficial for the children's cognitive gains. Therefore, direct instruction, with a focus on what the concrete objects represent, may be required to ensure that children will encode the manipulatives appropriately (e.g., Fuson & Briars, 1990; Uttal et al., 1997).

Today's Classroom

Given the results of this research, and despite the fact that teachers frequently use manipulatives in the early grades (Clements, 1999; Uttal et al., 1997), there appears to be little evidence that teachers focus on ways to help students acquire dual representation. Research suggests that teachers are generally unaware that the superficial features of manipulatives and the way that they are introduced are important (Moyer, 2001; Moyer & Jones, 2004). Teachers often have a difficult time imagining that students are not aware of the correct representation of mathematics manipulatives because it is so clear to them (Puchner et al., 2008).

Most of the literature in the last few years described teachers' use of manipulatives in their classrooms in two ways. One way describes teachers who distribute the manipulatives and get their students to engage in small-group activities followed by class discussions (Barlow & McCrory, 2011; Buczynski et al., 2011; Lo & McCrory, 2010; Macken, 2011; Mueller & Maher, 2010; Voza, 2011). For example, Lo and McCrory (2010) described their own classroom activities in which they have children use manipulatives to first justify their problem solving strategies in groups followed by sharing those strategies with the class. The authors stated that they often push their students to be explicit in their explanations of how they used the manipulatives to solve the problems, but the authors never reported how these manipulatives could represent the concepts in the problem they were solving. Thus, it appears that Lo and McCroy assumed that their students already held a quantitative representation of the manipulatives prior to engaging in these activities.

The second group of studies describes a smaller portion of teachers who provide explanations for the importance of connecting concepts and procedures when using manipulatives, but do not illustrate further how they do so in their classrooms (Cain & Faulkner, 2011; Clarke, Downton, & Roche, 2011; Englard, 2011). For example, Cain and Faulkner (2011) discussed the importance of connecting the concept of quantities to multiple forms of manipulatives, such as colored tiles and Dienes blocks, but they did not describe how they would organize their lessons to ensure that the students actually made these connections. Therefore, although Uttal et al. (1997) argued that the conceptual representation of mathematics manipulatives must be made explicit to children before they engage in tasks with them, there appears to be little evidence that their argument has found its way to teaching practices in elementary classrooms.

Additionally, there is some evidence that teachers themselves have alternative perceptions of the manipulatives they use in their classes. In a qualitative study by Moyer (2001), for example, several teachers were interviewed on their beliefs about manipulatives concerning their purpose and use. Several teachers explained that manipulatives were a reward or privilege in the classroom; teachers reported that manipulatives acted as a way to change the pace and make mathematics more "fun." Additionally, few teachers described that the purpose of manipulatives was to deliver a concrete visual representation of concepts to their students; a greater number reported lessons focused on reviewing prior knowledge and using the manipulatives in a procedural way. Moreover, one teacher reported letting her students use manipulatives on Fridays during free time to take a constructivist view and allow the children to explore the blocks in any way they wished (Moyer, 2001).

In a similar study by Moyer and Jones (2004), teachers reported that the purpose of manipulatives in mathematics classes was to enhance the knowledge the students previously acquired. Moreover, most teachers reported using manipulatives in the context of problem solving. They explained that in their classes they would model how to use the manipulatives to solve a problem and then have their students repeat additional problems in the same manner (Moyer & Jones, 2004).

In a study by Puchner et al. (2008), researchers learned from working with teachers in a professional development setting that educators need to be taught how to use manipulatives in the mathematics classroom because they observed the teachers often misusing manipulatives during their lessons. For example, teachers often used the manipulatives in their classrooms as “the end to the lesson.” In other words, the goal of their lessons was often to show a mathematical concept with the use of manipulatives. Specifically, one of the teachers reported that he expected his students to represent a number with the manipulatives in three different ways. The teacher used the manipulatives to make sure that his students used manipulatives in a variety of ways, but did not use them to support students’ learning of place value concepts. Using manipulatives in this way is a problem because they should be used as an aid to learning a mathematical concept and not as a goal in itself. The authors concluded that teachers need to better understand student thinking when using manipulatives in their lessons. In other words, teachers should be aware of students’ cognitive representations of the manipulatives and whether they can apply these representations when the manipulatives are not present.

In sum, although many children in elementary mathematics classrooms hold representations of the manipulatives they are using, they are not necessarily the appropriate ones. Furthermore, teachers themselves often hold misguided representations of the tools they are using to teach their students. In such cases, two things may occur. First, the lessons designed with the use of manipulatives may deliver an incomplete representation of the manipulatives and second, it could explain the reason for students’ inability to represent the manipulatives as quantities. As a whole, it seems as though lessons need to instill the correct representation of the manipulatives for enhanced learning to occur.

Present Study

There is no consensus in the mathematics education literature on whether young children need to be given direct instruction regarding the conceptual link between concrete objects and what they represent (Resnick & Omanson, 1987; Uttal et al., 1997) or if they are able to grasp the true meaning behind the concrete objects on their own during instruction (e.g., Martin & Schwartz, 2005). The literature describes various ways in which concrete materials are introduced and used in elementary mathematics classes with the goal of fostering students' conceptual understanding. In fact, many teachers report viewing the manipulatives as play objects aimed to make learning mathematics more enjoyable (Moyer, 2001; Moyer & Jones, 2004). They also describe having their students engage in discussions of how to solve problems using manipulatives without discussing the significance of the manipulatives in the context they are being used (Barlow & McCrory, 2011; Buczynski et al., 2011; Lo & McCrory, 2010; Macken, 2011; Mueller & Maher, 2010; Voza, 2011).

Research has shown, however, that the manner in which young children encode these manipulatives is dependent on the way they are introduced in the classroom (Brown, McNeil, & Glenberg, 2009). Dual representation -- that is, the ability to see blocks as blocks *and* as representations of quantities -- is a concern at the early elementary grades. A lack of dual representation may hinder students' true numerical understanding of addition (Uttal et al., 1997). When children are introduced to an object in such a way that its main physical characteristics are highlighted – as opposed to the concept it targets -- it makes it more difficult for the children to then understand the abstract idea the object is meant to represent (DeLoache, 2000). In other words, allowing a child to play with an object in absence of instruction about its meaning may hinder his ability to understand its *intended* representational meaning. Therefore, children who do not receive direct instruction on the representation of symbols in mathematics may perform worse on tasks testing the understanding of these representations compared to children who do receive direct instruction. The goal of the present study is to investigate whether students need to have direct instruction on the quantitative representation of the manipulatives before instruction on addition or if the instruction on addition alone is enough to develop a conceptual understanding of the manipulatives and knowledge of place value.

In this study, first grade students were randomly assigned to three different groups. In two conditions, they received different encoding interventions, and the third acted as a control group. Group 1 encoded the manipulatives through direct instruction on the quantitative meaning (Math Encoding Condition) and Group 2 encoded the manipulatives through informal play activities (Free Play Encoding Condition). The third group acted as a Control group, where they were given activities that did not involve concrete objects or any other representation of quantity. After this encoding phase, instruction on addition was given. The instruction, based on Fuson and Briar (1990), involved only procedures using manipulatives. The literature would suggest that those who obtained a correct quantitative representation of the manipulatives during encoding (i.e., before instruction) would be better able to make the connection between the manipulatives and the symbols used during the adding procedure, which involved double digit numbers and regrouping. This connection would in turn result in improved place value understanding (e.g., Resnick & Omanson, 1987). The Free Play Encoding condition differed from the Control group in that it tested whether the students would have constructed the correct quantitative representations on their own during instruction. It also adds ecological validity to the study because teachers often use manipulatives in their classroom in a number of ways (e.g., Moyer, 2001).

Students' performance on two tasks that assess place value knowledge was measured before the manipulatives were encoded and again after instruction. These tests were administered in interview form on an individual basis and assessed place value understanding and recomposition concepts that underlie the adding procedure. I also administered a third task, called here the Cooperman Task, that served to measure students' quantitative representation of the materials both before and after instruction.

My specific research questions were: (a) Will the students in the Math Encoding condition outperform the students in both the Free Play Encoding and the Control conditions on their knowledge of the quantitative representation of the manipulatives?, (b) Will there be a difference between the performance of the students in the Free Play Encoding Condition and the Control group on their knowledge of the quantitative representation of the manipulatives?, (c) Will the students in the Math Encoding condition outperform the students in both the Free Play Encoding and Control conditions on measures of place value understanding?, and (d) Will there be a difference between the performance of the students in the Free Play Encoding condition and the students in the Control condition on measures of place value understanding?

I predicted that the Math Encoding condition would outperform both the Free Play Encoding condition and the Control group on their quantitative representation of the manipulatives after instruction because only the Math Encoding group would have obtained an initial conceptual understanding of the manipulatives before instruction on addition. I also predicted, however, that the Control condition would outperform the Free Play Encoding condition on their quantitative representation of the manipulatives after instruction because the students in the Free Play encoding condition would have already created their own representation of the chips making it more difficult for them to modify these representation. In contrast, the Control condition could potentially pick up the correct representation of the manipulatives from the instruction more readily as they would not have yet created a representation of them.

Moreover, because they would have obtained the conceptual understanding of the manipulatives before learning the procedure of addition with them, and would therefore have been better able to make the connections between the manipulatives and the written symbols, I predicted that students in the Math Encoding condition would outperform students in the Free Play Encoding and Control conditions as assessed by the Place Value tasks. Furthermore, I also predicted that the students in the Control condition would outperform the students in the Free Play Encoding condition on this measure because the students in the play condition would have a difficult time modifying the conceptions they themselves constructed during the play encoding, whereas the control group would not have constructed any prior conceptions.

The present study will add to the literature and help obtain a consensus on an effective way to enhance students' conceptual and procedural understanding of place value with the use of manipulatives. It will also inform teachers on how to introduce manipulatives to students so that they represent them appropriately in the context of mathematics.

Chapter 3: Method

Participants

This study is part of a larger study that involved 88 first-grade students and 4 second-grade students from three English and four French speaking classrooms in three different elementary schools in the Montreal area, 73 (N = 73) of which formed the sample for the present study. There was an overall 22% attrition rate, which entailed 16 students who left the study at various points during the data collection. Out of these 16 participants, 7 were in the Math Encoding condition, 5 were in the Free Play Encoding condition, and 4 were in the Control condition. This resulted in a final sample of 57 (N = 57) participants for the current study, with a mean age of 7 years and 2 months old. Because of an administrative error, this age was calculated based on 53 of the 57 participants' ages.

The current sample was 47% female and 53% male. The French classrooms were from each school's French Immersion program. The teachers from these 8 classrooms were recruited in collaboration with the principals and mathematics consultants in a local school board. All participants had received written consent from their parents to take part in the study, and had also been asked to personally give consent to participate.

Design

The present study consisted of a pretest-posttest experimental design, as the participants were randomly assigned to specific conditions within their classrooms. The entire study lasted approximately four months. There were five phases to this study, as shown in Figure 1, and described below.

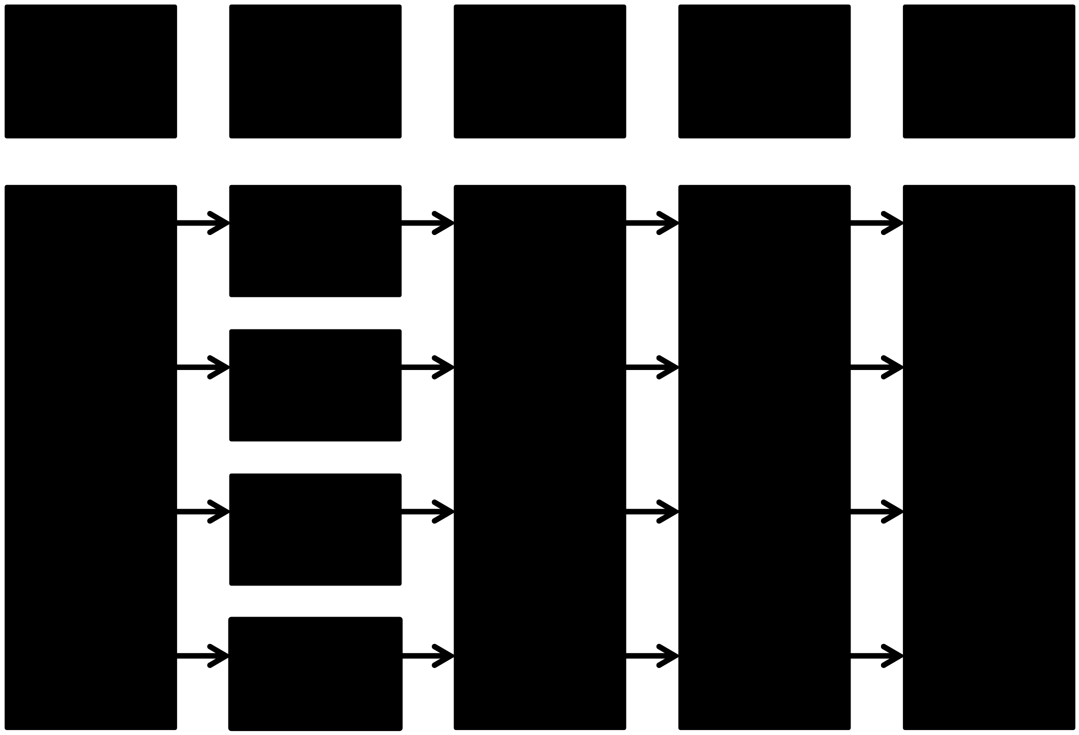

Figure 1. Study Design

Note. The black rectangle represents a fourth condition that is not included in the present study.

Phase I: Pretest. Students' place value understanding was assessed in an individual interview. It lasted approximately 15 to 25 minutes and was administered to all participants within four weeks. During the individual interviews, students were asked to complete two tasks, the Conventions of Place Value task (CPV) and the Word Problems task (WP).

Phase II: Encoding Intervention. Following the administration of the pretest, students were randomly assigned to one of four conditions: The Math Encoding, Free Play Encoding, Control, and a forth condition that was not part of the present study. Three of these conditions dictated specific ways for the students to encode blue and red plastic chips, which were the manipulatives used in the study. For the present research, I compared two groups, The Math Encoding and Free Play Encoding conditions, relative to the Control group. None of the participants had been introduced to the chips as mathematical tools in their classrooms prior to the study.

During the Encoding Intervention phase, the students in the two treatment groups, Math Encoding and Free Play Encoding, were introduced to the chips through different activities within one week, in small groups, during two 30-minute periods in separate areas of the school. The Math Encoding group received explicit instruction on the quantitative meaning behind the chips. For example, students in this condition were explicitly told that a blue chip represents the quantity "one," and a red chip represents the quantity "ten." The Free Play Encoding Group was given the colored chips without any instruction on what they represented. They were given the same amount of time with the manipulatives as the Math Encoding group, and were required to manipulate the chips on their own; the purpose was to encourage the participants to encode them in any way they wished through free play. The Control group was read the story of "Fancy Nancy and the Boy from Paris," and they engaged in different vocabulary activities that did not involve place value or concrete materials.

Phase III: Pre-Instruction. Following the Encoding Intervention Phase, all students were interviewed over the course of three weeks. During a 20- to 30-minute interview, they were asked to complete a task that I designed to assess their quantitative encoding of the chips (called here the Cooperman Task). There are two subscales to this measure: the Cooperman-Value subscale and the Cooperman-Comparison subscale. This task also acted as a treatment check to ensure the students in the Math Encoding group encoded the chips as intended and that the other groups did not.

Phase IV: Addition Instruction. In the Addition Instruction Phase, all participants were given two lessons in their classrooms on the addition algorithm with the use of manipulatives and written symbols. This instruction was designed to be procedural in nature and was given over two 40-to 55-minute mathematics periods within one week in the same small groups as the Encoding Intervention Phase.

Phase V: Posttest. All participants completed an isomorphic version of the CPV, WP tasks, and the Cooperman Task after instruction. Completed in an interview setting, the administration of these tasks lasted approximately 30 to 45 minutes and was completed within four weeks. This posttest assessed place value understanding and the students' quantitative representation of the chips after the instruction.

Description of the Interventions

Encoding. Trained research assistants delivered the Encoding Intervention to small groups of students in a quiet space outside their classrooms. They delivered the instruction in two sessions, each 30-minutes in duration, over a one-week period. Both treatment groups (Math Encoding and Free Play Encoding) engaged in separate activities with the colored chips and the Control group engaged in an activity not involving addition, place value, or manipulatives.

In an attempt to control for single participant group sizes, students participated in both encoding sessions even if they were absent for one of them. For example, if students were absent for the first day of encoding, they still participated in the second encoding session. Despite these attempts, group size varied between 1 and 5 participants. Table 1 illustrates the group sizes by class and condition during the encoding instruction. There were a total of five single-participant groups on one or both days of Encoding.

Table 1

Number of Participants and Small Group Sizes by Condition during Encoding

	Math Encoding			Free Play Encoding			Control		
Class	*n*	GS Day 1	GS Day 2	*n*	GS Day 1	GS Day 2	*n*	GS Day 1	GS Day 2
C1	4	4	4	4	5	4	3	3	4
C2	1	3	1	3	3	3	3	3	3
C3	2	4	2	3	4	3	3	3	3
C4	1	1	2	3	3	3	2	2	2
C5	1	1	2	2	4	4	3	3	3
C6	2	2	2	1	2	2	1	2	2
C7	3	4	4	2	2	2	2	3	3
C8	3	3	3	2	2	2	3	3	3
Total *n*	17			20			20		

Note. GS = Group Size; C = Class; *n* = number of participants included in present study

The colored chips were chosen for the present study for two reasons. First, non-proportional models were chosen to control for students' previous knowledge of proportional models of place value, such as base-10 blocks. Assigning arbitrary values to two different colored chips assured that the children had not seen this exact model previously. Second, using concrete symbols that are as perceptually dull as possible reduces distractions (Kaminski et al., 2005).

Math Encoding. The goal of the Math Encoding condition was to have the students encode the chips primarily as specified quantities, and the activities were designed to expose students to these quantitative representations. The activities were also designed to teach students to represent the same quantity in different ways using the chips. An example of representing the quantity 21 in two ways, for instance, would be to use 21 blue chips or 2 red chips and 1 blue chip.

The Math Encoding intervention was split across two days. On the first day, the manipulatives were introduced to the students. The instructor began by placing a white laminated mat in front of her and identical mats in front of each student. The instructor took out a container of 50 blue chips for herself and held up a blue chip and said, "Take a look. I have a blue circle. The blue circle is worth the same as 1. How much is this blue circle worth?" The children then responded, "one." The instructor then handed out a container of 50 blue chips to each student and repeated this same process, but allowed the students to manipulate the blue chips in their hands. Once the representation of a blue chip as "one" was illustrated, the instructor counted out loud a collection of 2 blue chips. The instructor pointed to each chip as she was counting them (e.g., "one," "two," "I have two") and asked the students to imitate her with their chips. The same procedure was followed for sets of 5 and 3 chips, in that order. Next, the instructor asked the students to count blue chips out loud on their own in unison for collections of size 6 and 4.

Following this, the instructor introduced the students to the quantitative representation of the red chips. She took out a container of red chips for herself and placed 10 blue chips on the mat in front of her; the students did the same. They pointed to the chips and counted them out loud. The instructor asked, "How much is this worth?" The students responded, "ten." She then showed how to recompose 10 blue chips with 1 red chip. Specifically, the instructor took 1 red chip out of the bin and said, "Now, take a look. I have a red circle. The red circle is worth the same as 10. How much is the red circle worth?," and the students responded, "ten." The instructor then directed the students' attention back to their mats and said, "Take a look at your mats. How much is that worth?" The children responded, "ten." The teacher followed by saying, "You're right, it's 10. One red circle [she held up one red chip] is worth the same as 10 blue circles [she pointed to the blue chips on the mat]." The instructor then moved the blue chips to the side and replaced them with a single red chip. The instructor handed out a container of 50 red chips to each student and then had them repeat the same process while manipulating the chips on their own.

Once the representation of the red chip as "10" was illustrated, the instructor counted out loud a collection of 2 red chips. The instructor pointed to each chip as she was counting them (e.g., "ten," "twenty," "I have twenty") and then had the students imitate her. The same procedure was followed for sets of size 7, 5, and 9, in that order. Next, the instructor had the students count out loud two additional collections of red chips, sets of size 3 and 6. Additional examples were illustrated using separate collections of blue and red chips. See Appendix A for details on the specific collections for each example. Day 1 of encoding was concluded by having the instructor place 1 blue chip on the mat and asked, “How much is the blue circle worth?” The process was repeated with 1 red chip.

On the second day of the encoding phase, a short review of the quantitative representation of each colored chip was given. The instructor first held up a blue chip and asked, “How much is this blue circle worth?” The children then responded, “one.” The instructor then said, “Just like last time, we are going to line up the circles in a straight line because it is easier to count them this way,” and placed 6 blue chips on the laminated mat and asked the children, “How much is this worth?” The children counted out loud with the instructor and responded, “six.” The students repeated the procedure for a set of 16 blue chips. The instructor then placed 10 blue chips on the mat and asked, “ How much is this worth?” The students responded, “ten.” The instructor then asked, “How else can we show 10?” The expected response was, “With a red circle,” after which the instructor responded, “You’re right! It’s still 10. You took a group of 10 blue circles and you traded it with 1 red circle. They’re the same, they’re both worth 10.” At the same time the instructor made a trade of 10 blue chips for 1 red chip. The instructor asked again, “How much is this worth?” The children responded again, “ten.”

Next, the instructor placed 4 red chips on the laminated mat and said, “Remember, when we count the red circles, we count by 10s. Let’s count together. How much is this worth?” The expected response was, “forty.” The instructor then asked the children to place 7 red chips on their mat and asked, “How much is this worth?” The children were expected to respond with, “seventy.” Last, the instructor placed 1 red chip on the mat and asked, “How much is this worth?” The given response was “10.” The instructor then asked, “How else can we show 10?” The given response was, “With 10 blue circles,” and the instructor showed the reverse trade of 1 red chip for 10 blue chips.

The instructor next illustrated the representation of quantities using both blue and red chips. The instructor pulled out 2 red chips and 3 blue chips and placed them on the mat in front of her. She modelled counting the red chips out loud first, “Ten, twenty,” then counted the blue chips, “one, two, three,” and finally counted them together, “twenty-one, twenty-two, twenty-three.” The instructor and the students then counted the chips once again together and the students were asked to repeat the steps on their own with the same set of chips. This procedure was repeated with a set of 1 red chip and 1 blue chip, but the instructor asked the students to construct their own sets: “I want to show 11 with the red and blue chips. What do I need?” Following this, the students repeated the same procedure on their own with sets of 4 red chips and 5 blue chips, followed by 2 red chips and 7 blue chips.

At this point, the instructor illustrated the recomposition of a set of blue chips to a set that has both red and blue chips. The instructor first demonstrated an example under 20 by asking, “I want to show 15 with blue circles. What do I need?” The expected response was, “15 blue circles.” The instructor then placed the 15 blue chips on the mat and asked, “How much is this worth?” The given response was, “15.” She then said, “I will show you another way you can show 15.” The instructor then counted out 10 blue chips and traded it in for 1 red chip [she took 10 blue chips and replaced them with one red chip]. The instructor asked again, “How much is this now?” They were then encouraged to count together, “10, 11, 12, 13, 14, 15. Fifteen.” The instructor then summarized their actions by saying, “We still have 15. We can show 10 in two different ways. We took 10 blue circles and traded them in for 1 red circle because they are worth the same.” The students then counted a set of 1 red chip and 2 blue chips on their own, followed by an instructor-led example with 2 red chips and 6 blue chips. The children then counted two more sets on their own (2 red chips and 2 blue chips; 1 red chip and 4 blue chips).

The final portion of the Math Encoding intervention entailed an illustration of the recomposition of both red and blue chips to a set with blue chips only. The instructor placed 1 red chip and 6 blue chips on the mat and asked, “How much is this? The expected response was, “Sixteen.” The instructor then said, “I’ll show you another way that you can have 16. I’m going to count out 1 red [she counted one red chip out] and trade it in for 10 blues [she swept one red chip away and replaced it with 10 blue chips].” The instructor then asked, “How much is this now?” The students responded, “Sixteen.” The instructor then summarized, “We still have sixteen. We can show 10 in two different ways. We took 1 red circle and traded it in for 10 blue circles because they are worth the same.”

The instructor then said, “I want you to show me 18 with the red and blue circles. What do you need?” The expected response was, “1 red circle and 8 blue circles.” She then asked, “How else can you show 18?” And the students were to respond with, “Take 1 red circle and trade it in for a group of 10 blue circles.” The same procedure was demonstrated by the instructor for a set valuing 22, and by the students for set valuing 33.

The instructor protocol for the Math Encoding condition is presented in Appendix A. The instructor corrected any errors the students made with immediate feedback. Examples of corrective statements were, “Remember the blue circle is the same as 1 and the red circle is the same as 10,” and “This is worth 10, so we need to count by 10s here, not ones.”

Free Play Encoding Condition. The activities in the Free Play Encoding condition were designed so that students represented the colored chips in any way they wished. The instructor was responsible for motivating the students to engage in any activity they wished with the chips as long as they used the chips during the full 60 minutes of encoding.

The instructor began by placing two bins on the table in front of each child. In one bin, there were 20 blue chips and in the other bin, there were 20 red chips. To begin, the instructor said, “Here are the materials we can play with in our group today.” They were encouraged by the instructor to play with the chips in any way they wished by being asked, “What can we do with these?” If the students stopped playing with the chips, the instructor prompted them by saying, “What are you doing with the circles?” On the second day of encoding, the same procedure was repeated, but if the students got off task, they were given the choice of using a pencil and paper with the chips. The instructor continuously encouraged the students to use the chips in a way determined only by the students and wrote down all ways that she saw the children use the chips. The instructor protocol for the Free Play Encoding condition is presented in Appendix B.

Control. The goal of the Control Encoding was to control for the effects of time spent with the instructor. Students in the Control group did not have access to the chips, in their small group. Instead, the instructor read the story, “Fancy Nancy and the boy from Paris” to the students, either in English or in French, depending on the language regularly used for their mathematics instruction. See Appendix C for the instructor protocol for the Control condition. The students were not introduced to the colored chips or to any concepts of number or quantity.

On Day 1 of encoding (English version), the Control group began with a pre-reading introductory activity where they were asked, “Do you know who Fancy Nancy is? Have you read any of her books?” The instructor then explained, “Fancy Nancy has many adventures. She is a girl who likes to dress up in fancy clothes. She also uses a lot of fancy words!” The instructor then continued to ask, “Do you know what the word *fancy* means?” Together the instructor and the students looked at each fancy word that appeared in the story together and discussed their meaning. These words included, “tardy,” “gorgeous,” “terrified,” “perplexed,” “bonjour,” “ami,” and “belle.” In the French version, the fancy words included, “À la traîne,” “Splendide,” “Terrifiés,” “Perplexe,” “Enchantée,” and “Pote.”

Next, the instructor read the story to the students while asking various questions such as, “How is Nancy feeling?” or “What do you think might/will happen next?” Afterwards, the instructor continued to discuss the story with the students by asking, for example, what their favorite part of the book was and what the difference was between Paris, France and Paris, Texas. Together the group then looked back at the fancy words and discussed their meaning again. Lastly, if there was extra time, the students were provided with paper and colored pencil to draw their favorite part of the book.

On Day 2 of encoding for the Control group, the instructor began by reviewing the book by asking, “Do you remember who Fancy Nancy is?” and “Do you remember what the word ‘fancy’ means?” Next, the instructor read through the story with the students while again asking questions such as, “What do you think might/will happen next?” Thirdly, the instructor went through the meaning of each of the fancy words that Nancy had used in the story. She then asked the students to match a fancy word with a regular word. Examples of such matchings were tardy - late, gorgeous - beautiful, terrified - scared, perplexed - mixed up, bonjour - hello, ami - friend, and belle - beautiful. They then continued to write out the fancy words on a worksheet in a “fancy way,” such as in bubble letters, and the “regular” words in standard print. Lastly, they were asked to color the picture of the Eiffel tower if there was time left over.

Addition instruction. The goal of the addition instruction was to have the students learn the addition procedure with no conceptual explanations of the procedure or the chips. All participants in all three conditions received the same instruction. The unit was modelled on the Teaching/Learning instructional activity designed by Fuson and Briars (1990), and was designed to teach children a procedure for how to add single-and double-digit numbers with the use of manipulatives. The instruction was administered over two 40-to 55-minute class periods in the same small groups as the encoding phase.

Once again, in an attempt to control for single participant group sizes, students participated in both addition instruction sessions even if they were absent for one of them. For example, if students were absent for the first day of addition instruction, they still participated in the second addition instruction session. In some cases, there were some students who did not participate in the study, but nevertheless received the instruction because it was part of the regular math curriculum. Despite these attempts, group size varied between 1 and 5 participants. Table 2 illustrates the group sizes by class and condition during the addition instruction. There were a total of three single-participant groups on one or both days of addition instruction.

Table 2

Number of Participants and Small Group Sizes by Condition during Addition Instruction

	Math Encoding			Free Play Encoding			Control		
Class	*n*	GS Day 1	GS Day 2	*n*	GS Day 1	GS Day 2	*n*	GS Day 1	GS Day 2
C1	4	5	5	4	5	5	3	5	5
C2	1	4	4	3	5	5	3	5	5
C3	2	4	5	3	5	5	3	5	5
C4	1	5	5	3	5	5	2	5	5
C5	1	2	2	2	3	3	3	3	3
C6	2	2	2	1	1	1	1	2	1
C7	3	3	4	2	2	2	2	3	2
C8	3	3	3	2	2	2	3	3	3
Total *n*	17			20			20		

Note. GS = Group Size; C = Class; *n* = number of participants included in present study

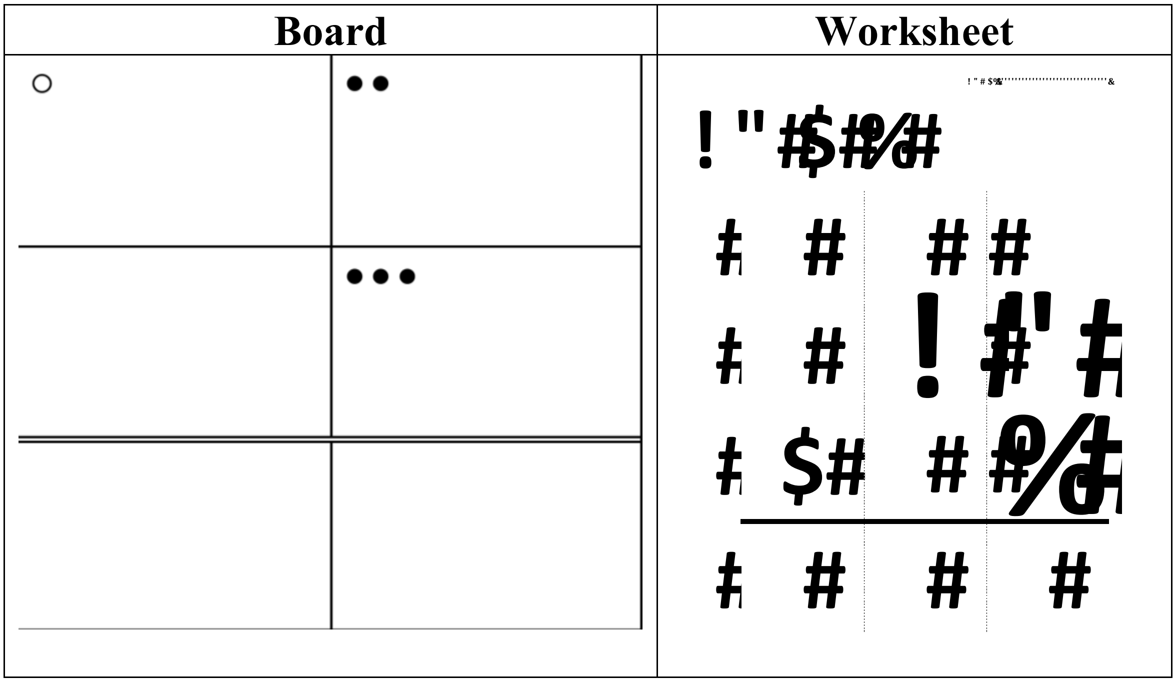

Figure 2. Addition instruction materials

The addition instruction covered 20 addition problems (10 on each day). The addition problems were in a workbook where the students were expected to record their work. There were four problem types: Adding single-digits, adding single digits with regrouping, adding double digits, and adding double digits with regrouping. The instructor introduced the participants to a laminated board designed for teaching place value, similar to that used by Fuson and Briars (1990). Each child and the instructor were given a laminated board and colored chips to use to solve the problems, as well as a workbook. The board and accompanied worksheet are presented in Figure 2.

The instruction involved two main steps: imitation and structured practice. The instructor began an example and the children imitated her step by step with the chips, after which the students engaged in structured practice activities where they completed several examples individually. The structured practice involved the instructor prompting the students to work through a series of addition problems step by step.

The imitation portion began with the instructor reading the equation at the top of the worksheet (the equation "3+2=_" will be used here for illustration purposes). The instructor then said, "Let's add three plus two." She pointed to the 3 in the vertical equation and said, "First, I am going to start with the 3." She then put three blue chips on the board in the top rightmost column and said, "That makes 3"; she paused for the children to imitated her. She continued, "Then, I am going to add the 2" (points to the 2 in the vertical equation). She then put 2 blue chips in the middle rightmost column and said, "That makes 2."

The instructor then said, "Now let's put all the blue circles together," and pulled them down to the last column on the board. Again, the instructor then paused for the children to imitate her. She then counted with the students, "one, two, three, four, five" and wrote "5" on the bottom rightmost column underneath the line in the vertical equation on the worksheet. She would wait for the children to imitate her actions. The instructor then pointed to the equation at the top of the worksheet and said with the children, "three plus two is five," and they each wrote "=5" next to the horizontal equation on the top of their individual worksheets.

The structured practice portion of the instruction began with the instructor reading the equation at the top of the worksheet and saying, "Let's add 5 plus 4." The instructor then asked, "What is the first thing that we do?" The students were expected to take 5 blue chips and place them on the board in the top rightmost column and write "5" in the top rightmost column on the worksheet. The instructor then asked, "What is the next thing we do?" The students were expected to take 4 blue chips and place them on the board in the middle rightmost column and write "4" in the middle rightmost column. The instructor then asked, “What do we do now?” The children were expected to move all the blue chips to the bottom rightmost column and begin to count them, “one, two, three, four, five, six, seven, eight, nine.” The instructor then asked, “Now what do we do?” and the students were expected to write “9” in the bottom rightmost column, under the line. The instructor then asked, “What is the last thing we do?” The students were expected to point to the equation at the top at the board and say, “five plus four is nine,” and write “=9” in the box.

The problems used in the imitation and practice portion of the instruction involved adding single digits with and without regrouping on Day 1 and adding double digits with and without regrouping on Day 2. Each type of question had an initial two imitation questions and one structured practice question. Half way through each instruction session, the students were responsible for writing all the written symbols on their worksheets when solving the problems with the chips.

The instructor protocol for the addition instruction is presented in Appendix D. If students asked why they were doing any of the procedures, the instructor would respond, “Because this is the way I’d like you to do it” or “Because this is the way we are going to learn it today.”

Instruments and Measures

Place value assessment. Trained research assistants and I individually interviewed students to examine their place value understanding. The interview lasted approximately 15 to 25 minutes and consisted of two tasks that assessed conceptual understanding of place value across two different contexts: symbolic and word problem contexts.

The first task, called the Conventions of Place Value task (CPV; based on Resnick & Omanson, 1987), included eight items that measured conceptual understanding of place value in a symbolic context. The interview protocol for the pretest version of the task is presented in Figure 3. The items were designed to measure students' understanding of the value of different places within a double-digit number. The students were asked to indicate how much a circled digit in the number was worth.

<table>
<tr><th colspan="2">Conventions of Place Value task</th></tr>
<tr><td>INITIAL SETUP:
Nothing in front of the child. Show the child one card at the time.
SAY:
"What number is this?" (Circle the whole number with your finger when asking the question)
1. If the child gives the wrong answer, say, "Well actually, the number is xxx."
"How many things is this worth?" (Circle the target digit with the eraser of a pencil)
"Why is it worth [child's response]?"
Turn the card over after the child has answered.
Repeat the same instructions for each one of the problems.
Don't forget to record the child's answers on the scoring sheet.</td><td>12 problems:
1. 68
2. 77
3. 53
4. 29
5. 33
6. 41
7. 25
8. 66
9. 42
10. 38
11. 11
12. 80</td></tr>
</table>

Figure 3. Protocol of the Conventions of Place Value task (Pretest; based on Resnick & Omanson, 1987). Underlined digits indicate those the interviewer circled in each item.

The interviewer showed the student a white index card with a double-digit number on it and asked, "What number is this?" If the student gave the wrong answer, the instructor said, "Well actually, the number is __." One digit in the number was then circled by the interviewer using her finger or the eraser of a pencil, either in the tens column or in the ones column (see Figure 3). The student was then asked, "How many things is this worth?" After the student responded, the interviewer asked, "Why is it [insert student response]?" to give him the opportunity to explain his response. Once the first item was completed, the interviewer removed the card and began the next item. The interviewer repeated the same process for eleven additional items. Only the answers to the questions and not the students' explanations were analyzed in the current study.

The second task, called here the Word Problem (WP) task, was based on a measure by Hiebert and Wearne (1996) that measured conceptual understanding of place value in the context of word problems. The task consisted of six Measurement Division problems. The protocol for the pretest version of the task is presented in Figure 4. The task was designed such that students who have place value understanding were able to respond without needing to compute the answer or model a solution strategy. In other words, students who have place value understanding would be able to respond quickly based on seeing the place of the digit in a number.

The Word Problem task	
1) The interviewer will place an index card with a large number printed on it in front of the child. 2) The interviewer will then read the following problems one by one to the child. 3) When done with one card, remove it before starting the next problem. FOR ALL WORD PROBLEMS: Let the child try to answer each problem without the use of paper/marker. If the child can't or asks to use materials, offer paper/marker. Make sure to remove the paper/marker from the child's view after each problem.	
A) 27	Take a look at the number on this card. This is how many donuts the store has left. To fill a box, Jane has to put 10 donuts in each box. How many full boxes can she make with the donuts the store has left? Why is it [child's response]?
B) 50	Take a look at the number on this card. This is how many chocolate bars are left in the cooler. A soccer coach wants to give each soccer player a box of 10 chocolate bars to take home. How many soccer players will take home a full box of chocolate bars? Why is it [child's response]?
C) 45	Take a look at the number on this card. This is how many Lego pieces are in the box. The teacher wants to give each child 10 pieces of Lego to build a tower. How many children will get 10 Lego pieces? Why is it [child's response]?
D) 36	Take a look at the number on this card. This is how many dolls Ann has. Ann wants to put her dolls in boxes. Each box can hold 10 dolls. How many boxes can Ann fill completely with the number of dolls she has? Why is it [child's response]?
E) 18	Take a look at the number on this card. This is how many hockey cards John has. John wants to put his hockey cards in packages. Each package can hold 10 hockey cards. How many packages can John fill completely with the number of hockey cards he has? Why is it [child's response]?
F) 52	Take a look at the number on this card. This is how many stickers Mary has. Mary pastes her stickers in her sticker book so that there are **10** stickers on each page. How many pages can she fill up **completely**? Why is it [child's response]?

Figure 4. Protocol of the Word Problem task (Pretest; based on Hiebert & Wearne, 1996)

The interviewer placed an index card with a double-digit number on it in front of the student. The interviewer then read out loud to the student a word problem that contained the number, and the student was then asked to solve it. To illustrate, for problem A in Figure 4, the instructor read, “Take a look at the number on this card [27]. This is how many donuts the store has left. To fill a box, Jane has to put 10 donuts in each box. How many full boxes can she make with the donuts the store has left? Why is it [child’s response]?” The interviewer did not recite the number verbally to prevent any linguistic cues about place value. The interviewer simply pointed to the card when reading the word problem out loud. The interviewer repeated the same procedure for five additional items. Paper and pencil were provided to the students if they needed them to solve the problems. Once again, only answers to the questions and not the students’ explanations were analyzed for the current study. Isomorphic versions of both the CPV and WP tasks were used at posttest during Phase V of the study (see Appendix E).

The Cooperman Task. The Cooperman Task consisted of eight items designed to assess students’ understanding of the quantitative representation of the chips (i.e., to assess whether the students encoded the colored chips in the intended manner). As arbitrary values had been given to the colored chips, the students could only respond correctly based on a conceptual understanding of the chips’ respective representations. The protocol of the Cooperman Task (pre-instruction version) are presented in Figure 5.

The Cooperman Task

1. The interviewer will place two cards with color circles on them on a mat in front of the child.
   a. Cards will match the mat by color and letter
2. The interviewer will ask the child:
   a. "How much is this worth?" *(Point to the left group of colored circles in a circular motion)*
   b. "How much is this worth?" *(Point to the right group of colored circles in a circular motion)*
   c. "Which is worth more *(point simultaneously to both cards of circles)* or are they worth the same?" *(Say the whole sentence without pausing)*
   d. "Why is [child's response] worth more?" OR "Why are they worth the same?"

Item 1	● ● ●		● ● ●
Item 2	● ● ● ● ●		○ ○ ○ ○ ○
Item 3	○		● ● ● ● ● ● ● ● ● ●
Item 4	● ● ● ● ● ● ●		○ ○ ○ ○
Item 5	○ ○ ○ ○ ○		○ ○ ○ ○ ○
Item 6	○ ○ ○ ○ ○ ○		● ● ● ● ● ●
Item 7	● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ●		○ ○
Item 8	○ ○		● ● ● ●

Figure 5. Protocol of the Cooperman Task (Pre-instruction)

Note. ● = blue chips, ○ = red chips

For each item, the student was presented with two index cards on which were glued red and blue chips. The two index cards were placed side by side on a laminated mat. There were two subscales to this task: the Cooperman-Value (CV) subscale and the Cooperman-Comparison (CC) subscale. For the Cooperman-Value portion of the task, the interviewer pointed to the chips on the left index card and asked the child: "How much is this?" After the child responded, the interviewer pointed to the chips on the card placed on the right and asked, "How much is this?" After the child responded, the interviewer pointed to both cards to administer the Cooperman-Comparison subscale portion of the task and asked, "Which is worth more or are they the same? Why is it [student response]?" Once again, only student responses and not students' explanations were analyzed. An isomorphic version of this task, which was used at posttest, is presented in Appendix F.

Procedure

Pretest. A team of research assistants and I went into each classroom and briefly introduced ourselves to the students. Students were then taken to a quiet area in the school to be interviewed. During the interview, the CPV and WP tasks were administered. Before the interview began, the interviewer explained the process to the child and informed him that he could withdraw from the interview at anytime and that his answers would not be shared with his teacher. The interviews lasted 15 to 25 minutes and were videotaped, where only the child's hands were visible. The interviewer filled out a coding sheet to record the child's responses while the interview was being conducted. The sheet is presented in Appendix G. The average amount of time between the administration of the pretest of the place value interview and the first day of the encoding intervention was 23.3 days.

Encoding intervention. A team of hired research assistants and I went into each classroom twice during a two-week period. Each research assistant took one small group of students outside the classroom to implement the encoding activities assigned to that group (i.e., Math Encoding, Free Play Encoding, and Control). Counterbalancing of the instructors for this phase was done separately for the English and French classrooms. All researchers were rotated through the conditions as much as possible to control for instructor effects.

Each session was conducted during two regularly scheduled mathematics classes and each lasted approximately 30 minutes. Each group was taken to a separate area in the school to engage in the encoding activities. The average amount of time between each encoding session was five days. The average amount of time between day two of the encoding intervention and the pre-instruction administration of the Cooperman Task was 12.3 days.

Pre-instruction (Cooperman Task). A team of graduate students and I individually administered the Cooperman Task to all students in the three conditions within two weeks of the second encoding session. Students were interviewed individually for approximately 25 minutes outside the classroom and were videotaped. The interviewers filled out a coding sheet to record the child's responses while the interview was being conducted. The coding sheets for the pre-instruction can be found in Appendix H. The average amount of time between the administration of the Cooperman Task and Day 1 of instruction was 12 days.

Addition instruction. Research assistants and I went into each classroom during two mathematics periods, approximately 40 to 55 minutes each time, to give the students the addition instruction. The children were divided into the same small groups as during the encoding phase. In some cases, students who did not participate in the study were nevertheless present in these groups because the content was part of the curriculum, and the teacher believed the instruction would benefit all students. The amount of time the instruction took varied, depending on the number of students in each group, which ranged from one and six students. Each lesson was identical in each group. The average amount of time between the two addition instruction sessions was 4.7 days. The average amount of time between the second instruction session and the posttest was 19.4 days.

Posttest (CPV, WP, and Cooperman Task). The same procedure was followed for the posttest of the CPV task, WP task, and the Cooperman Task as for the pretest administration of each of these measures. The coding sheets for the posttests of all measures can be found in Appendix I.

Coding and Scoring

The interviewers filled out a coding sheet of the child's responses for all tasks while the interview was being conducted. The coding sheets for the pretest, pre-instruction, and posttest for all measures can be found in Appendix G, H, and I, respectively.

Place Value Assessment. For the Conventions of Place Value task, each correct response on the coding sheet received a score of 1 and each incorrect response received a score of 0. The same process was used for the Word Problem task, but if the students received a score of 1 on the score sheet, the videotapes of the interviews were then coded to see whether the students were able to provide an answer within eight seconds. Eight seconds was the time limit chosen because during pilot testing, I noticed that children who took more time needed to model or compute a solution, which indicated a lack of place value understanding. Modelling or computing a solution would indicate lack of place value understanding because the students would not be able to ascertain the value of a digit by seeing its place within the number. In such cases, their responses were given a score of 0. In sum, the students were only awarded a score of 1 if they arrived at the correct response and answered within 8 seconds.

The Cooperman Task. For the Cooperman Task, students again received a score of 1 for each correct response and a score of 0 for each incorrect response. If students made a counting error, identified as two more or two less then the correct response, they still received a score of 1. For the CV subscale, total scores pertaining to the students' ability to identify the correct value of the chips were computed by summing the total number of points and converting the sum to percent. For the CC subscale, four items displayed the same quantities. For these items, a correct response received a score of 1 and an incorrect response received a score of 0. For the remaining four items, the student received 0 for indicating that the two collections of chips represented the same amount, 1 point for indicating the correct set was larger, and 0.5 for indicating that the smaller set was larger. Total scores for the subscale were computed by summing the total number of points and converting the sum to percents.

Chapter 4: Results

The following section will report the findings based on the statistical analyses. All data were grouped within conditions across all eight classrooms. Descriptive statistics are presented, as well as four 3 x 2 mixed ANOVAs with group (Math Encoding, Free Play Encoding, and Control) as the between-group factor and time as the within group factor. The two levels of the time factor differed by measure. Specifically, the levels for the CPV and WP tasks were pretest and posttest and the levels for the Cooperman Task were pre-instruction and posttest (see Figure 1).

Descriptive Statistics

The means and standard deviations for the Conventions of Place Value (CPV task), the Word Problem (WP task), and each subscale of the Cooperman Task [Cooperman-Value (CV) and Cooperman-Comparison (CC)] by condition at either pretest or pre-instruction and posttest can be viewed in Table 3.

Table 3

Means and Standard Deviations for the CPV Task, the WP Task, the CV Subscale, and the CC Subscale by Condition at either Pretest or Pre-instruction and Posttest (N = 57)

	Pretest		Pre-instruction		Posttest	
Variables	*M*	*SD*	*M*	*SD*	*M*	*SD*
Conventions of Place Value Task (CPV)						
Math (*n* = 17)	.55[a]	.25	--	--	.61	.19
Free Play (*n* = 20)	.54	.27	--	--	.60	.23
Control (*n* = 20)	.55	.22	--	--	.56	.21
Word Problem Task (WP)						
Math (*n* = 16)	.29	.39	--	--	.38	.42
Free Play (*n* = 20)	.26	.38	--	--	.38	.43
Control (*n* = 20)	.36	.34	--	--	.47	.44
Cooperman Task - Value Subscale (CV)						
Math (*n* = 16)	--	--	.92	.16	.88	.20
Free Play (*n* = 20)	--	--	.48	.04	.53	.13
Control (*n* = 20)	--	--	.49	.02	.57	.16
Cooperman Task - Comparison Subscale (CC)						
Math (*n* = 16)	--	--	.88	.20	.84	.23
Free Play (*n* = 20)	--	--	.38	.03	.43	.18
Control (*n* = 20)	--	--	.38	.00	.47	.21

Note. [a] All scores reported in percents

These data reveal similar performance (around 50 to 60%) on the CPV task for each condition at both pretest and posttest. The increase in mean scores on the Word Problem task was similar in each condition. The data also revealed that each condition's mean score on both subscales of the Cooperman Task (CV and CC) remained relatively constant from pre-instruction to posttest within each condition, but the Math Encoding condition's score was higher compared to both the Free Play Encoding and Control conditions at both time points.

The Effects of Encoding on Place Value Knowledge

The Conventions of Place Value task. A 3 x 2 mixed design ANOVA was conducted with group (Math Encoding, Free Play Encoding, Control) as the between-group factor and Time (pretest, posttest) as the within-group factor, with the CPV task as the dependent measure. Results displayed no main effect of time, $F(1, 54) = 1.28$, $p > .05$, or group, $F(2, 54) = 0.09$ $p > .05$. Furthermore, no significant time x group interaction was found. Therefore, it appears that overall, the students did not improve on this measure, and the students in the Math Encoding condition had no advantage over the students in the other conditions on their performance from pretest to posttest.

The Word Problem task. Figure 6 illustrates the mean scores by condition at pretest and posttest on the WP Task. To investigate learning of place value using the WP task, a 3 x 2 mixed design ANOVA was conducted with group (Math Encoding, Free Play Encoding, Control) as the between-group factor and time (pretest, posttest) as the within-group factor, with the WP task as the dependent measure. Results revealed a main effect of time, $F(1, 53) = 6.18$, $p = .016$. Across all conditions, the mean score at posttest ($M = 0.41$, $SD = .43$) was significantly higher than the mean score at pretest ($M = 0.30$, *SD* = .37). In other words, regardless of encoding type, the instruction caused the students to perform better on the WP task, implying that all students benefited from the instruction regarding their place value knowledge. The analysis of variance revealed no main effect of group, $F(2, 53) = 0.38$, $p > .05$ and no significant time x group interaction, $F(2, 53) = 0.06$, $p > .05$.

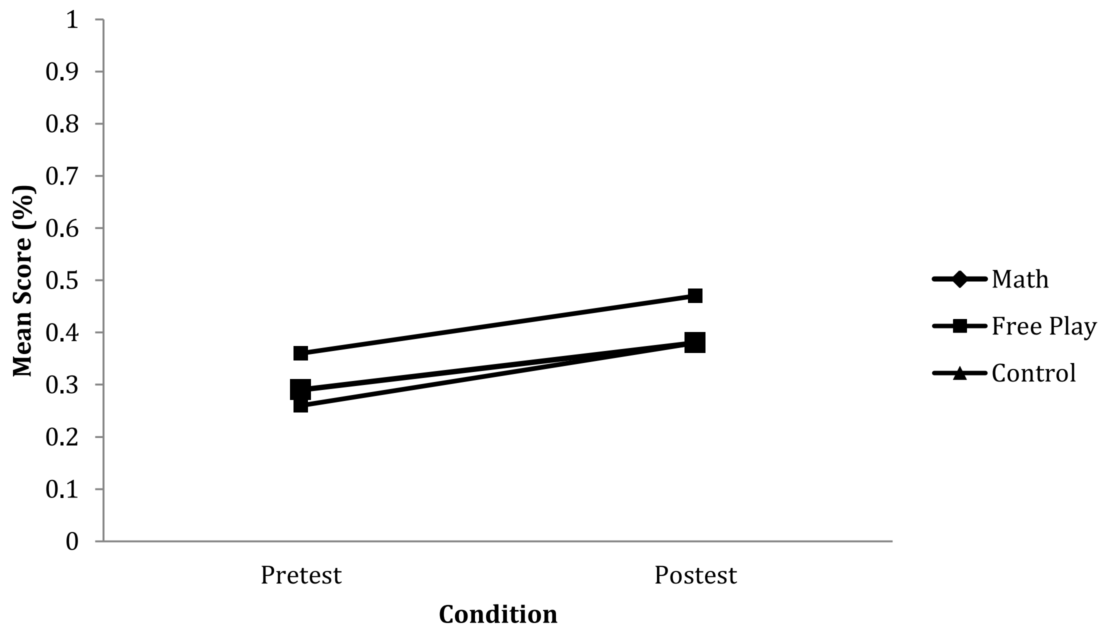

Figure 6. Mean scores for the Word Problem task

The Effects of Encoding on Correct Representation of Manipulatives

The Cooperman – Value subscale. Figure 7 illustrates the mean scores by condition at pre-instruction and posttest on the CV Subscale. To investigate students' knowledge of the value of the manipulatives, a 3 x 2 mixed design ANOVA was conducted with group (Math Encoding, Free Play Encoding, Control) as the between-group factor and time (pre-instruction, posttest) as the within-group factor. The CV subscale was used as the dependent measure in this analysis. The data revealed no main effect of time from pre-instruction to posttest, $F(1, 53) = 2.15$, $p > .05$. A significant difference was found between the mean scores for the Math Encoding Condition, the Free Play Encoding Condition, and the Control group regardless of time, resulting in a main effect of condition, $F(2, 53) = 71.75$, $p < .001$. Bonferroni post hoc analyses revealed a significant difference between the Math Encoding and Control conditions ($p < .001$) and between the Math Encoding and Free Play conditions ($p < .001$). No difference was found between the Free Play and Control conditions ($p > .05$). Moreover, no time x group interaction was found, $F(2, 53) = 2.85$, $p > .05$.

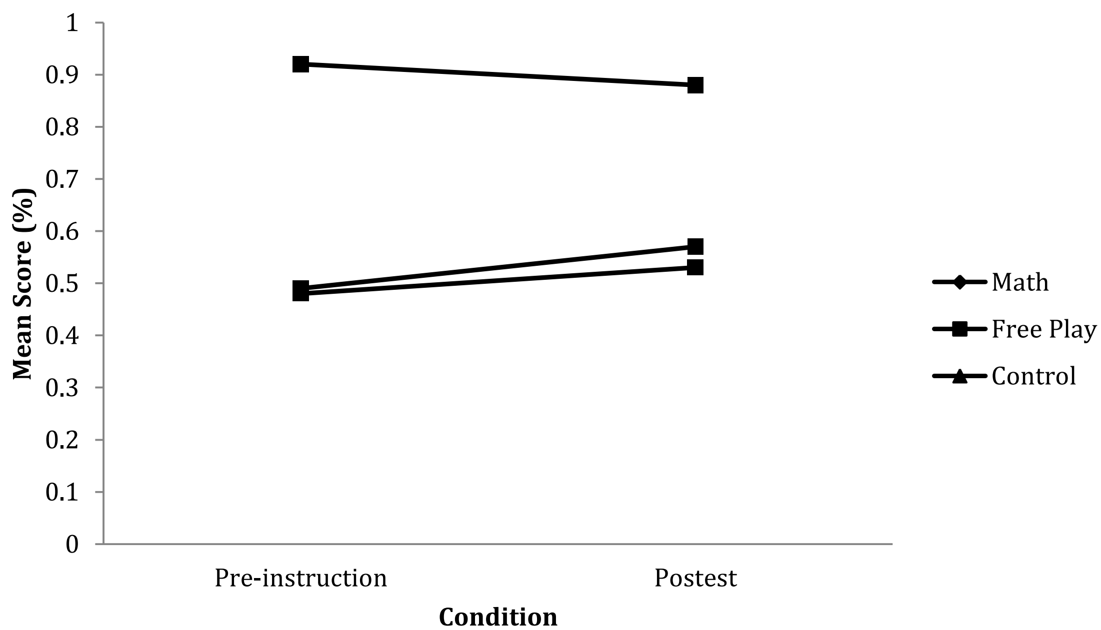

Figure 7. Mean scores for the Cooperman – Value Subscale

Simple effects analyses revealed a significant difference between the three conditions at pre-instruction, $F(2, 53) = 138.52$, $p < .001$. Post hoc comparisons with a Bonferroni correction revealed that the Math Encoding group significantly outperformed the Free Play group at pre-instruction ($p < .001$) as well as the Control group ($p < .001$). The Free Play and Control groups did not differ significantly on the CV subscale at pre-instruction ($p > .05$).

Similar effects were found at posttest. Significant differences were found between the groups, $F(2, 53) = 23.19$, $p < .001$, with Bonferroni post hoc comparisons demonstrating that the Math Encoding group outperforming the Free Play group ($p < .001$) and the Control group ($p < .001$). The Free Play and the Control group did not differ significantly from each other ($p > .05$).

Together, the results suggest that the addition instruction did not result in better knowledge of the manipulatives for any of the conditions. Nevertheless, students in the Math Encoding group had significantly greater knowledge of the manipulatives after encoding (at pre-instruction) and maintained this difference throughout instruction.

The Cooperman – Comparison Subscale. Figure 8 illustrates the mean scores by condition at pre-instruction and posttest on the CC Subscale. The CC Subscale of the Cooperman task was used to further investigate students' knowledge of the value of the manipulatives. Using the CC Subscale scores as the dependent measure, a 3 x 2 mixed design ANOVA was conducted with group (Math Encoding, Free Play Encoding, Control) as the between-group factor and time (pre-instruction, posttest) as the within-group factor. The data revealed no main effect of time, $F(1, 53) = 2.31$, $p > .05$, but a main effect of condition was found, $F(2, 53) = 56.54$, $p < .001$. Furthermore, Bonferroni post hoc analyses revealed a significant difference between the Math Encoding and Control conditions ($p < .001$), and the Math Encoding and Free Play conditions ($p < .001$). No significant difference was found between the Free Play and the Control conditions ($p > .05$). Finally, no time x group interaction was found, $F(2, 53) = 2.48$, $p > .05$.

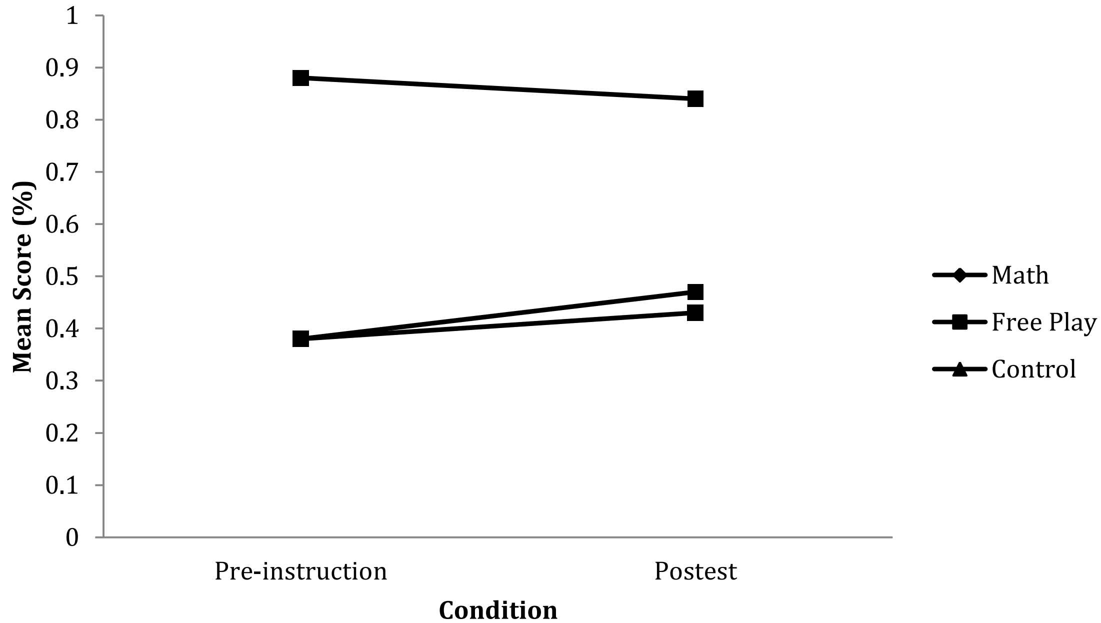

Figure 8. Mean scores for the Cooperman – Comparison Subscale

Simple effects analyses revealed a significant difference between the three conditions at pre-instruction, $F(2, 53) = 123.11$, $p < .001$. Post hoc comparisons with a Bonferroni correction revealed that the Math Encoding group significantly outperformed the Free Play group at pre-instruction ($p < .001$) as well as the Control group ($p < .001$). The Free Play and Control groups did not differ significantly on the CC subscale at pre-instruction ($p > .05$).

At posttest, significant differences were found between the groups, $F(2, 53) = 19.84$, $p < .001$, with the Math Encoding group outperforming the Free Play group ($p < .001$) as well as the Control group ($p < .001$). The Free Play and the Control group did not differ significantly from each other ($p > .05$). In sum, the results suggest that students' ability to compare two collections of manipulatives did not improve as a result of instruction, regardless of condition. Moreover, they suggest that the Math Encoding group acquired better knowledge of the manipulatives after encoding, which was maintained throughout instruction.

Chapter 5: Discussion

The present study examined the impact of directly telling first grade students the quantitative meaning of manipulatives on their learning of place value. The design of the study was such that one group of students was explicitly instructed on the correct quantitative representation of mathematics manipulatives, another group was given the opportunity to create their own representation of the manipulatives, and a third group was not given any exposure to the manipulatives prior to procedural instruction on double-digit addition. The primary objective was to determine whether it is essential for teachers to explicitly tell students what manipulatives represent before using them procedurally to learn place value concepts. A second objective was to test whether students would be able to learn the quantitative meaning of the manipulatives through instruction alone.

In line with my predictions, it appears that being explicitly taught what mathematics manipulatives represent in the context in which they are being used results in the correct quantitative representation of the manipulatives compared to those who either create a representation on their own or who are not introduced to the manipulatives prior to instruction with them. Otherwise said, instruction alone did not help the groups of students who were not explicitly taught what the mathematics manipulatives represent acquire this knowledge.

Contrary to my predictions, however, it appears that having a correct quantitative representation of the manipulatives does not give students an advantage for learning place value knowledge through instruction. In fact, the procedural instruction improved all students' performance on place value understanding, as assessed by the Word Problem task, regardless of how they represented the manipulatives before instruction. These results are inconsistent with the data on the CPV task, however, on which no significant effects were found.

The results of the study suggest that students need to be given explicit instruction on what mathematics manipulatives represent for them to understand their conceptual meaning. In other words, students who do not understand the correct representation of the manipulatives prior to using them in procedural instruction do not pick up their correct representation either before instruction, when freely manipulating them, or as a result of instruction, when they are being used to represent numbers and the regrouping process. Additionally, it has been argued that using mathematics manipulatives procedurally without conceptual understanding can hinder students' use of them because they get "stuck" on their initial conceptions (Ambrose, 2002). In other words, once students represent a manipulative in a specific way, it becomes more difficult for them to alter this representation (Bransford et al., 2006). This suggests that if students play with the manipulatives prior to using them in a mathematical context, the representation they generate on their own will be difficult to modify. This has been shown by the results of the present study, as students who played with the manipulatives did not learn their correct representation from the procedural instruction. It is, of course, possible that the students in the play condition constructed their own quantitative representation of the chips, but the data show that whatever representation they constructed, it was not the correct one.

Furthermore, the results revealed that there was no significant difference between the students who played with the manipulatives and the students who did not engage in any activities with the manipulatives prior to instruction. It is possible that the students in the Control group came up with their own representation when they were first introduced to them at the beginning of instruction, and that these representations were no more beneficial to them than those constructed by the students who played with the manipulatives beforehand. This would explain for the lack of difference between these two groups.

With regard to place value learning, the results indicated that all students learned. Nevertheless, the prediction that the Math Encoding group would be at an advantage was not borne out. It is possible that this occurred because of the nature of the instruction. I speculate that a quantitative representation of the manipulatives would have been more meaningful to the students during instruction had the lessons depended less on their prior knowledge and focused more on new material with the use of manipulatives as supports for learning. After implementing the addition instruction in the study, I was concerned that the students were not required to focus on both the manipulatives *and* the written symbols. That is, the task appeared not to be novel to the students, and as such, they used their own strategies to solve the problems instead of using the one that was taught to them which involved both manipulatives and symbols. This may explain why the Math Encoding condition did not improve more than the other groups in making the connection between the manipulatives and the written symbols, which was required on the place value measures.

Another possible explanation for the lack of predicted place value learning is related to what the students may have been attending to during instruction. I speculate that the students in the Math Encoding group were focused more on the manipulatives during instruction than their peers in the other conditions. It could be that only the students who were explicitly told the correct quantitative representation were focused on the manipulatives during instruction, as they were the only group of students for which the manipulatives were made meaningful. Perhaps during the addition instruction, the students who understood the correct quantitative meaning of the manipulatives were primarily focused on the manipulatives because they made sense to them, and the students in the other two conditions were focused on the written symbols, as to them, the manipulatives were not meaningful.

I claim, therefore, that the instruction did not require the students to make connections between concepts and procedures, and this may have reduced the relative benefit of the Math Encoding group. All students improved on their place value understanding because regardless of condition, they all focused on at least one aspect of the instruction: the concepts as represented by manipulatives or as presented by the written symbols (Rittle-Johnson & Koedinger, 2008; Rittle-Johnson et al., 2001). Had the instruction focused on the links between concepts (i.e., manipulatives) and procedures (i.e., the algorithm with written symbols), I suspect that the Math Encoding group would have been in a better position to learn the conceptual underpinnings of the algorithm than the other two groups.

Moreover, this explanation can be supported by Goldin's (2003) theory. Students who were explicitly taught the correct representation of the manipulatives were the only ones expected to make the link between their external system (the manipulatives) and the intended internal system (the concepts for which they represent). The results revealed that they, in fact, did do this. The findings also show that these students were not better than students in the other condition at making similar connections when the manipulatives were not present (i.e., during the Word Problem task). The fact that they did not make the link between the concepts (internal system) and the written symbols (external system) on this task further supports my contention that the Math Encoding condition did not make the link during instruction between the written symbols and the manipulatives. The Word Problem task required more cognitive effort because the items on this measure were not part of the instruction (i.e., it required transfer). As such, the students were not able to extract the conceptual meaning from the manipulatives and apply it to the written symbols because they were not required to make such connections during instruction (Goldin, 2003).

The results of this study will add to the literature by connecting developmental theory and education. More specifically, the findings support Uttal et al.'s (1997) argument regarding the explicit instruction of symbols in scale studies in a mathematics context. Specifically, Uttal et al. (1997) found, by conducting studies with younger children, that for children to hold the correct representation of a symbol, they must be explicitly told what that representation means in the context the symbol is being used. They subsequently argued that when teachers give students manipulatives as an aid for learning mathematics concepts, they first need to understand the relationship between the manipulatives and the their meaning in the context in which they are being used. Until the present study, no study had directly tested the theory in the context of the mathematics classroom.

In addition, the results of this study add to the literature concerning the use of play in contexts involving young children. Some authors argue that children learn a tremendous amount of mathematical knowledge from play (Ginsburg, 2006), but, as shown by this study, play in and of itself is not enough when using manipulatives to teach mathematical concepts in the classroom (Lee & Ginsburg, 2009). Lee and Ginsburg (2009) argued that children's environments, whether in preschool or early elementary settings, provide a foundation to promote learning, but does not result in learning on its own. Rather, Ginsburg, Lee and Boyd (2008) stated that play leads to teachable moments, which are points in time when a teacher observes children in play and highlights incidents that occur where children can benefit and learn through targeted interaction. The results of this study support their contention because the children who engaged in play with the manipulatives could not pick up their correct quantitative representation from procedural instruction alone. As such, something was missing for them -- perhaps a teachable moment -- on the correct quantitative representation of the manipulatives.

Conclusions and Implications

There are several limitations to the present study. First, the study was conducted in a school setting, and because of various time and space constraints, more time elapsed than expected between the addition instruction and the posttest. Perhaps having less time between these two phases would help the students maintain the connections between the meaning of the manipulatives and the procedures after instruction on addition, including the Math Encoding group, who had learned the correct quantitative representation beforehand.

Second, the procedural instruction appeared not to be appropriate for the purposes of this study because the students' ability to complete the procedure was not sufficiently dependent on the manipulatives. The instructional addition tasks appeared not to be novel to the students and this led them to solve the problems using what they already knew. For example, informal observations illustrated that students often attempted to compute the problems either by using mental strategies or their fingers as supports for counting. Therefore, a replication of this study with instruction on less familiar content, reducing the likelihood of prior knowledge activation, may result in greater support for my hypotheses. The procedure introduced during such instruction would need to focus on the manipulatives to complete the instructional tasks. In this sense, knowing the correct representation of the manipulatives would then be essential for the completion of the procedural tasks.

Thirdly, the place value assessments used in this study had no known psychometric properties. A difficulty was that manipulatives could not be used in these assessments because the intervention was entirely dependant on children's representations of them. In addition, it was difficult to assess whether the students actually understood the task on the Conventions of Place Value. I modified it several times during pilot testing, but still found no satisfactory way to formulate the question. I was unsatisfied with how the questions on the CPV task were formulated based on my informal observations of the children's reactions to the CPV task compared to the WP task. When presented with the CPV task, children's often looked unsure of what was being asked. In contrast, when presented with the WP task, children were more confident about completing the task as required. Fourthly, although the minimum sample size was achieved, replication with a larger sample may add more power to the analyses. Finally, more consistent counterbalancing of conditions in both the encoding and addition instruction phases would strengthen the design of the study.

It may be worthwhile to examine the effects of quantitative encoding on learning as a result of instruction that incorporates conceptual understanding of the addition algorithm. For example, an extension of this study may be to compare instruction that has been found to be effective in previous research (Rittle-Johnson & Koedinger, 2008; Rittle-Johnson et al., 2001) to the instruction used in this study. Perhaps allowing the students to receive the conceptual and procedural meaning behind the manipulatives iteratively (Rittle-Johnson & Koedinger, 2008) or simultaneously (Clements, 1999) would strengthen the learning of place value concepts for students who have the correct quantitative representations beforehand.

There are several educational implications that emerge from this study. The results could help teachers design elementary mathematics lessons with the use of manipulatives so that their students understand their quantitative representation. The findings suggest that for students to understand the correct quantitative representations of mathematics manipulatives, teachers must explicitly tell them their quantitative meaning in the context of use. This coincides with the position presented by Uttal et al. (1997), who suggested that manipulatives are not inherently meaningful to students until they are made meaningful through instruction. This also supports the argument that teachers themselves need to have the correct representation of the manipulatives to be better able to clearly pass that information on to their students (Moyer 2001; Puchner, 2008). Additionally, teachers sometimes assume that children understand the quantitative meaning of the manipulatives because it is so obvious to them and thus do not explicitly make it clear to their students (Puchner et al., 2008). Without obtaining the correct quantitative representation, students can be prevented from acquiring their meaning during instruction, which appears to be counterintuitive for many teachers (Moyer, 2001).

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Appendix A

Math Encoding: Instructor Protocol

Math Encoding: Instructor Protocol

DAY 1

Materials:

1. 1 container of blue circles for each child
2. 1 container of red circles for each child
3. 1 mat for each child
4. 1 mat for the instructor
5. As much as possible, please seat children facing you (in a semi circle or straight line) during the intervention.

1. Encoding: Blue circle	
Instructions	**Error Correction**
T: We are going to do some math activities together. I would like for you to look and listen to what I do and follow me when I say it's your turn.	
Teacher takes out container of blue circles for themselves. T: *Pull out a blue circle and hold it up for everyone to see.* Take a look. I have a blue circle. The blue circle is worth the same as one *(Pause)* T: How much is the blue circle worth? C: One	
Give a container of blue circles to each child. T: Please take out one blue circle from the container and hold it up. C: *Child takes out one blue circle* T: How much is the blue circle worth? C: One	*T: No. The blue circle is worth the same as one. (Pause) How much is the blue circle worth?* *C: One*

2. Represent 1-10: Blue circles	
T: Pull out 2 blue circles and place them on the mat in front of you. T: Let's count the blue circles. Watch me first. *T models by touching left to right.* T: One-two. I have two. Your turn, please take out two blue circles and place them on your mat. C: *take out circles*. T: Let's count. *(T counts with kids 1-2. Two)* T: How much is this worth? C: Two	*T: No. Remember, the blue circle is worth the same as one. (Pause) Let's count again. One-two. (pause) Two.* *How much is this worth?* *C: Two* ***Clear circles every time***
T: Pull out 5 blue circles and place them on the mat in front of you, lining them up in a fairly straight line. T**:* I'm putting the circles in a straight line so that it's easier to count them.** T: Let's count the blue circles. Watch me first. *T models by touching left to right.* T: One-two-three-four-five. I have five. Your turn, please take out five blue circles and place them on your mat. C: *take out circles*. T: Let's count. *(T counts with kids 1-2-3-4-5. Five)* T: How much is this worth? C: Five	** Emphasis on lining up circles in a straight line* *If children's array becomes too messy, restate " Remember, it's easier to count when the circles are in a straight line"*
Repeat for 3 blue circles	
T: Now it's your turn, please take out six blue circles and place them on your mat. (*Pause)* Ok, now count them. C: *Count one-two-three-four-five-six* T: How much is this worth? C: Six	*T: No. Remember, the blue circle is worth the same as one. (Pause)* *Let's count again.* *T: point as child counts.* *One- two-three-four-five-six.* *How much is this worth?* *C: Six*
Repeat for 6 and 4 blue circles	

3. Encoding: Red circle & Connect 10 Blue circles same as 1 Red	
T: Place 10 blue circles on the mat in front of you. *T: Also place 10 blue circles on your mat* *C: Take out 10 blue circles from the container and place them on their mats* T: Lets count the blue circles. C: *Count/touch the circles* T: How much is this worth? C: 10	*Provide necessary feedback*
T: *Pull out a red circle and hold it up for everyone to see.* Now, take a look. I have a red circle. The red circle is worth the same as 10 blue circles. *(Pause)* T: How much is the red circle worth? C: 10	*T: The red circle is worth the same as ten. (Pause) How much is the red circle worth?* *C: Ten*
T: Take a look at your mats. How much is that worth? C: 10 T: You're right, it's 10. One red circle *(hold up red circle)* is worth the same as 10 blue circles (*point to the blue circles on the mat*) T: They are the same *Sweep the blue circles to the side and replace them with a single red circle.*	*If children refer to the colour & / circles "10 blue/ 10 blue circles.." etc.* *T: "The blue circle is worth the same as one. You have ten blue circles, so all together you have ten."*
Give a container of red circles to each child. T: Now it's your turn. Take out a red circle... *C: take out a red circle* T: and trade with the 10 blue circles you have on your mat. *C: Sweep blue circles off the mat and replace with a single red circle.* T: Now, how much is this worth? C: 10 T: You're right! It's still 10. You took 10 blue circles and you traded with 1 red circle. They're the same, they're both worth 10.	*If C say "One red/ red circle.." etc, T: "The red circle is worth the same as ten. So you have ten"*

4. Representation 10-90: Red circles	
T: Pull out 2 red circles and place them on the mat in front of you. Let's count the red circles. Watch me first. *T models by touching left to right.* Ten-twenty. I have twenty. Your turn, please take out two red circles and place them on your mat. *C: Place two red circles on their mats.* T: Let's count together. *(T counts with kids; both T and C touch)* 10-20. How much do you have in front of you? C: Twenty	*T: Remember, the red circle is worth the same as ten. (Pause) Let's count again. Ten- Twenty. (pause) Twenty.* *How much is this worth?* *C: Twenty*
Repeat for 7, 5 and 9 red circles	
T: Now it's your turn, please take out three red circles and place them on your mat. *C: Take out circles and place them on their mat.* T: Count them? *C: Count ten-twenty-thirty* T: How much do you have in front of you? C: Thirty	*T: Remember, the red circle is worth the same as ten. (Pause) Let's count again. Ten- Twenty-Thirty. (pause) Thirty.* *How much is this worth?* *C: Thirty*
Repeat for 6 red circles	
Ensure when taking out larger quantities, T models lining them up in a way that will make counting easier (not necessarily a perfectly straight line, but close!)	

5. Additional Examples & Final Review	
T: *Places a blue circle on the mat and asks:* How much is the blue circle worth? C: One T: I'm going to place these on the mat. *T places 8 blues on the mat.* T: How much is this worth? C: (*count out independently or together*) eight. T: *provide feedback*	*T: Remember, the blue circle is worth the same as one. (Pause) Let's count again. One-two-….8. (pause) Eight. How much is this worth?* *C: Eight.*
Repeat with 14 blue circles.	
T: *Places a red circle on the mat and asks:* How much is the red circle worth? C: Ten T: I'm going to place these on the mat. *T places 6 reds on the mat.* T: How much is this worth? C: (*count out independently or together*). 10-20... 60. Sixty T: *provide feedback*	*T: Remember, the red circle is worth the same as ten. (Pause) Let's count again. Ten-Twenty-Thirty-Forty-Fifty-Sixty. (pause) Sixty. How much is this worth?* *C: Sixty*
Repeat with9 blue circles; 4 blue circles; 5 red circles; 8 red circles; and 2 blue circles	
Sum the lesson up by: T: *Places a blue circle on the mat and asks:* How much is the blue circle worth? C: One *And* T: *Places a red circle on the mat and asks:* How much is the red circle worth? C: Ten	
End of Math Encoding Day 1	

Math Encoding: Instructor Protocol

DAY 2

Materials:

6. 1 container of blue circles for each child
7. 1 container of red circles for each child
8. 1 mat for each child
9. 1 mat for the instructor
10. Place children as far apart as possible during the intervention.

Day 2	
1. Review	
Instructions	**Error Correction**
T: We had used these (*pointing to the circles)* last time we were together. This is very important. Just like last time, please do not take any of these out of the box, until I tell you. Watch me first and I will let you know when it is your turn. Let's go over what we did last week!	
T: *Pull out a blue circle and hold it up for everyone to see.* T: Take a look. T: How much is the blue circle worth? C: One	*T: No. The blue circle is worth the same as one. (Pause) How much is this worth?* *C: One*
T: *Place 6 blue on a mat, lining up in a straight line* T: Just like last time, we're going to line up the circles in a straight line because it's easier to count them this way. T: Let's count together. How much is this worth? C: Count with T, while T points to each circle. One, two three...six. T: Yes, this is worth six.	
T: Now it's your turn, please take out 16 blue circles and place them on your mat. Can you count them? C: Count 1-2-3…16. T: How much is this worth? C: Sixteen.	

T: *Place 10 blue on a mat.* T: How much is this worth? C: Count with T, while T points to each circle. One, two three...ten. T: Yes, this is ten. How else can we show ten? C: With a red circle. T: You're right; a group of 10 blue circles is worth the same as one red circle. *Sweep 10 blue circles off the mat and replace with one red.* T. How much is this worth? (*pointing to the red)* *C:* Ten T: You're right! It's still 10. You took a group of 10 blue circles and you traded with 1 red circle. They're the same; they're both worth 10.	*T: No. The red circle is worth the same as ten. (Pause) How much is this worth?* *C: Ten*
T: *Place 4 red on a mat.* T: Remember, when we count the red circles, we count by 10s. Let's count together. How much is this worth? C: count with T, while T points to each circle. Ten, twenty, thirty, forty. T: Yes, this is 40.	
T: Now it's your turn. Show me 70 with the red circles C: take out 7 reds and count them out. 10-20-30…70. T: How much is this worth? C: Seventy	
T: *Place 1 red on a mat. Ask:* T: How much is this worth? C: Ten. T: Yes, this is ten. How else can we show ten? C: With blue circles circle. T: You're right, 1 red circle is worth the same as a group of 10 blue circles. *Sweep 1 red off the mat and replace with 10 blues.* How much is this worth? (*pointing to the blue)* *C:* Ten You're right! It's still 10. You took 1 red circle and you traded with a group of 10 blue circles. They're the same, they're both worth 10.	*T: No. The blue circle is worth the same as one. (Pause) Let's count how much this is.(Count 1-10)* *C: Ten*

2. Representation of quantities using blue circles and red circles	
T: *Pull out 2 red circles and 3 blue circles and place them on the mat in front of you.* Remember how we count the red circles? Watch me first. *T models by touching the red circle* Ten-twenty. Now, remember how we count the blue circles? Watch me first. *T models by touching the blue circle* One-two-three. Let's count the blue and red circles together. We always count the red circles first. *T models by touching the circles* Ten-twenty *(pointing to the red circles)* 21-22-23. *(Pointing to the blue circles)*. Twenty-three. T: Your turn. Please take out two red circles and three blue circles and place them on your mat. T: Let's count. *(T counts with children)* T: How much is this worth? C: Twenty-Three.	*T: Remember the blue circles are the same as 1 and the red circles are the same as 10.* *(T counts the chips again).*
T: Let's do a different example. T: I want to show 11 with red and blue circles. What do I need? C: 1 red and 1 blue circle. T: *Take out chips and place them on the mat. Count out the chips with the children.* Ten *(pointing to the red circles)* 11 *(pointing to the blue circles)*. Eleven.	
T: Your turn. I want to show 45 with red and blue circles. What do I need? C: 4 red and 5 blue circles. T: Please take out four red circles and five blue circles and place them on your mat. T: ***** Individually ask children to count the circles (depending on time)*** Can you count them? *C:* Ten-twenty-thirty-forty *(pointing to the red circles*) 41-42-43-44-45. Forty five *(pointing to the blue circles).* T: How much is this worth? C: Forty-Five.	*T: Remember the blue circles are the same as 1 and the red circles are the same as 10.* *(T counts the chips again).* *T: We always count the red circles first.*
T: Let's do another example. Please take out two red circles and seven blue circles and place them on your mat. T: ***** Individually ask children to count the circles (depending on time)*** Can you count them? *C:* Ten-twenty *(pointing to the red circles*) 21-22-23-24...-27 *(pointing to the blue circles).* T: How much is this worth? *C:* Twenty-Seven.	*We always count the red circles first.*

3. Recomposition Over 10: All B to B & R	
T: Let's do a different example. T: I want to show 15 with blue circles. What do I need? C: 15 blue circles. T: *takes out 15 blue circles and places them on the mat.* T: How much is this worth? Let's count C: Count. 1, 2, 3, 4... 15. Fifteen. T: I'll show you another way you can show 15. Remember that a group of 10 blue circles are worth the same as 1 red circle. I trade a group of 10 blue circles (count out) for one red circle (sweep 10 blues, place them back in the container, and replace with one red) and I leave the rest on the mat. T: How much is this now? Let's count. C: T count with C: 10, 11, 12, 13, 14, 15. Fifteen. T: We still have fifteen. We took a group of 10 blue circles and traded it in for 1 red circle because they are worth the same.	*We always count the red circles first.*
T: Now it's your turn. Show me 12 with blue circles. *C:* Place 12 blue circles on the mats. T: How much is this worth? C: (Children count 1-2-3..12) 12 T: How can we show 12 in another way? C: trade a group of 10 blue circles for 1 red circle. T: ok, let's count (T with C) 1,2,3,4…10. And trade for one red circle. C: trade: place the group of 10 blue circles back to the container and take out one red circle and place it on their mats. T: Now how much do you have? ** ***Individually ask children to count the circles (depending on time)*** C: Count 10, 11, 12. Twelve. T: Yes, we still have 12!	

T: Let's try another one. *Place 26 blue circles on mat* T: Let's count C: count 1-2-3…26. T: How much is this worth? C: 26 T: How can we show 26 in another way? C: trade a group of 10 blue circles for 1 red circle. T: ok, let's count. 1,2,3,4…10. And trade for one red circle. T: Now how much do we have? C: count 10, 11, 12…26. Twenty six T: Yes, we still have 26! T: How many blue circles do we have left? C: 1,2,3,4,5,6-16. T: Can we trade 10 blue circles for another red? C: yes T: ok, let's count. 1,2,3,4…10. And trade for one red circle. T: Now how much do we have? C: count 10, 20,21,22,23,24,25,26. Twenty six T: Yes, we still have 26!	
T: Now it's your turn. Show me 22 with blue circles. *C:* Place 22 blue circles on the mats. T: How much is this worth? *(point to 1-2 children's mats)* C: 22 T: How can we show 22 in another way? C: trade a group of 10 blue circles for 1 red. T: ok, let's count (T with C) 1,2,3,4…10. And trade for one red circle. C: *trade by placing the group of 10 blue circles to the side and take out one red circle and place it on their mats.* T: Now how much do we have? C: Count 10, 11, 12,13….22. Twelve. T: Yes, we still have 22! T: How many blue circles do we have? C: 1,2,3,4,5,6-12. T: Can we trade a group of 10 blue circles for another red? C: yes T: ok, let's count. 1,2,3,4…10. And trade for one red circle. T: Now how much do we have? C: count 10, 20,21,22 Twenty-Two T: Yes, we still have 22!	

T: Now it's your turn again. Show me 14 with blue circles. What do you need? C: 14 blue circles. *C:* Place 14 blue circles on the mats. T: How much is this worth? C: Count out. 1-2-3-…14. Fourteen. T: How can we show 14 in another way? C: trade a group of 10 blue circles for 1 red. T: ok, let's count (T with C) 1,2,3,4…10. And trade for one red circle. C: *trade by placing the group of 10 blue circles to the side and take out one red circle and place it on their mats.* T: Now how much do we have? C: Count 10, 11, 12,…14. Fourteen. T: Yes, we still have 14! T: How many blue circles do we have? C: 1,2,3,4. T: Can we trade a group of 10 blue circles for another red? C: No T: Why can't we trade 10 blue circles for another red? C: Because we need a group of 10 blue circles to change it to 1 red circle. T: Good!	

<table>
<tr><th>4. Recomposition: Over 10: B&R to all B</th><th></th></tr>
<tr><td>T: Let’s do a different example.
T: Place 1 red and 6 blue circles on the mat.
T: Let’s count
T: Count out the chips with the children.
T: 10,11,12,13...16. Sixteen.
I’ll show you another way that you can have 16. I’m going to count out 1 red circle, which is worth 10, and trade it in for a group of ten blue circles (sweep one red and replace with 10 blues).
T: How much is this now? Let’s count.
C: T count with C: 1, 2, 3... 16. Sixteen.
T: We still have sixteen. We can show 10 in two different ways. We took one red circle and traded it in for a group of 10 blue circles because they are worth the same.</td><td>We always count the red circles first.</td></tr>
<tr><td>T: Show me 18 with red and blue circles. What do you need?
C: 1 red and 8 blues.
T: Ok, take out the circles and place them on your mat.
T: Count them out please.
C: Count.
T: How else can you show 18?
C: Take one red circle and trade it in for a group of 10 blue circles.
T: That’s right because one red is worth the same as a group of ten blue circles!
C: Trade.
T: Now let’s count. 1, 2,3, ...18.
We still have 18!</td><td></td></tr>
</table>

T: Your turn. Show me 22 with blue and red circles. C: Take out 2 red and 2 blue circles. T: Let's count C: Count: 10, 20, 21, 22. Twenty-two. T: How else can we show 22? C: Trade 1 red for a group of 10 blue circles T: ok go ahead. C: trade 1 red for a group of 10 blue circles T: Now let's count to see how much we have C: count 10, 11, 12... 22. Twenty-two T: We still have twenty-two. Do we have any red circles left? C: Yes, one. T: ok, let's do another trade. C: Trade 1 red for a group of 10 blue circles T: ok, let's count. ***** Individually ask children to count the circles (depending on time)*** C: count 1,2, 3, 4...22 T. We still have 22! We started off with 2 reds and 2 blue circles ended up with 22 blue circles. We traded the red circles for blue circles because 1 red circle is worth the same as a group of 10 blue circles.	

T: Now it's your turn. Show me 33 with red and blue circles. What do you need? C: 3 red and 3 blue circles *C:* Place the circles on their mats. T: How much is this worth? C: Count out. 10-20-30-31-32-33. Thirty-three. T: How can we show 33 in another way? C: trade 1 red circle for a group of 10 blue circles. T: ok, let's take 1 red (T with C) and trade for a group of 10 blue circles. C: count out 10 circles from bin. T: Now how much do we have? C: Count 10-20-21-22-23…33.Thirty-three. T: Yes, we still have 33! Can we make any more trades? C: Yes we have another red so we can trade it for a group of 10 blue T: ok, let's take 1 red (T with C) and trade for a group of 10 blue circles. C: count out 10 blue circles from bin. T: Now how much do we have? C: Count 10-11-12-13…33.Thirty-three. T: Yes, we still have 33! Can we make any more trades? C: Yes we have another red circle so we can trade it for a group of 10 blue circles. T: Ok, let's take 1 red circle (T and C) and trade for a group of 10 blue circles C: *Count out 10 blue circles from bin.* T: Now how much do we have? C: Count 1-2-3-4-5-6…33.Thirty-three. T: Yes, we still have 33! Can we make any more trades? C: No because we don't have any more reds. T: Good. We started off with 3 red circles and 3 blue circles and we ended up with 33 blue circles. We traded the red for blue circles because 1 red circle is worth the same as a group of 10 blue circles.	
End of Math Encoding Day 2	

Appendix B

Free Play Encoding: Instructor Protocol

Free Play Encoding: Instructor Protocol

DAY 1

Materials:

11. 1 container of blue circles for each child
12. 1 container of red circles for each child

Instructions:

1. The instructor will say, “Here are the materials we can play with in our group today.”
2. The instructor will then pass the containers of circles out to each child.
3. The instructor will encourage the students to play with the circles in any way they wish.

1. Give individual attention to each child to help keep it them on task.
2. It is ok if the children talk with each other but they should remain seated during the activity.
3. Example of probe: “What can we do with these?” or “What are you doing with the circles?”

4. **<u>Important:</u> do not give the children any ideas of what to do with the circles, make sure all activities with the circles are child-lead.**

Please write down on a separate sheet what the children are doing with the circles during the activity. Don’t forget to indicate your name, the school, the children’s teacher’s name, and the date.

Free Play Encoding: Instructor Protocol

DAY 2

Materials:

4. 1 container of blue circles for each child
5. 1 container of red circles for each child
6. Once children seem to be losing interest:
    1. Pencils
    2. Colour pencils
    3. Paper

Instructions:

1. The instructor will say, "Here are the materials we can play with in our group today."
2. The instructor will then pass the containers of circles out to each child.
3. The instructor will encourage the students to play with the circles in any way they wish.

7. Give individual attention to each child to help keep it them on task.
8. It is ok if the children talk with each other but they should remain seated during the activity.
9. Example of probe: "What can we do with these?" or "What are you doing with the circles?"

4. Once children as a group begin to lose interest, the paper and pencils can be handed out to EACH child at the same time. The instructor will continuously encourage the students who are using the paper and pencils to also use the circles. (**<u>Important:</u> do not give the children any ideas of what to do with the circles, make sure all activities with the circles are child-lead.)**

Please write down on a separate sheet what the children are doing with the circles during the activity. Don't forget to indicate your name, the school, the children's teacher's name, and the date.

Appendix C

Control: Instructor Protocol

Control: Instructor Protocol
Day 1

Materials:

1. Book**: Fancy Nancy and the Boy from Paris**
2. *Fancy* words
3. Sticky tack
4. Bristol board
5. Plain paper 1 x student
6. Markers/crayons

<u>**Throughout:**</u> ***Monitor that children are <u>not making any references</u> to quantities, digits, shapes, etc. If any mathematical ideas arise, redirect to the task at hand (words, different parts of pictures, etc.).***

1. <u>**Pre-reading**</u>

1. Introduction:

1. Today we are going to read a story about Fancy Nancy.
    1. Do you know who Fancy Nancy is?
    2. Have you read any of her books? With who? (On own, with parents, grandparents, etc.)
2. Fancy Nancy has many adventures. She is a girl who likes to dress up in fancy clothes. She also uses a lot of fancy words!
    1. Do you know what the word *fancy* means?
3. We are going to look at the fancy words Fancy Nancy uses in this book, called Fancy Nancy and the Boy from Paris.
    1. *Using the printed word cards, read each "fancy" word that will be in the story and arrange it in on the Bristol board in front of the children (on table, etc).*
    2. *Ask different children what they think each word means.*
        1. Tardy
        2. Gorgeous
        3. Terrified
        4. Perplexed
        5. Bonjour
        6. Ami
        7. Belle

2. <u>Reading</u>

3. *Read the story* ***Fancy Nancy and the Boy from Paris*** *aloud.*

4. *Throughout the story, ask questions:*
    1. How is Nancy feeling?
    2. What do you think might/will happen next?
    3. Questions regarding the environments presented within the story (classroom, playground, home, etc)

5. <u>Post-reading</u>

1. **Questions:** *Ask the children what they thought of the book*

1. What was your favourite part of the story? Why?
2. *Discuss the difference between Paris, France and Paris, Texas; whether the children have ever been there, etc.*

2. Fancy words activity:

3. Ask the children what the words on the board mean (they should know more of them at this point). Tell them what the words mean – use the regular words from the last page of the book.

4. Ask the children if they can think of other fancy words and discuss their meaning.

5. **Drawing (extra activity):**

6. If there is extra time, provide the children with a piece of paper and crayons/markers. Ask them to draw a picture of their favourite part of the story. Collect their drawings afterwards (make sure to write their name on it).

Control: Instructor Protocol
Day 2

Materials:

7. Book**: Fancy Nancy and the Boy from Paris**
8. *Fancy* words + regular words
9. Sticky tack
10. Bristol board
11. Worksheets (1 x child and instructor)
12. Markers/crayons

<u>**Throughout:**</u> ***Monitor that children <u>are not making any references</u> to quantities, digits, shapes etc. If any mathematical ideas arise, redirect to the task at hand (colours, letters, worksheet, book, etc.).***

<u>**1. Pre-reading**</u>

13. **Review:**
    1. Today we are going to read the story about Fancy Nancy again.
        1. Do you remember who Fancy Nancy is?
    2. Last time I said that Fancy Nancy has many adventures. She is a girl who likes to dress up in fancy clothes, and she likes to use a lot of fancy words!
        1. Who remembers what the word *fancy* means?
    3. Great! I want you to keep your ears open and listen for the fancy words that Nancy uses. We are going to do an activity after the story ☺
14. <u>**Reading**</u>
15. *Read the story* ***Fancy Nancy and the Boy from Paris*** *aloud.*
16. Throughout the story, ask <u>a few</u> questions:
    1. How is Nancy feeling?
    2. What do you think might/will happen next?
    3. Questions regarding the environments presented within the story (classroom, playground, home, etc.)
17. <u>**Post-reading**</u>
18. Last time we had talked about the fancy words that Fancy Nancy uses in this book.
19. Let's look at them now.
    1. *Using the printed word cards, read a "fancy" word that was in the story and attach it to the Bristol board.*
    2. *Ask the children what the word means.*
    3. *After they answer, read off and attach the "regular word" next to the fancy word.(Talk about the fact that the last 3 words are in French)*
        1. Tardy- late
        2. Gorgeous- beautiful
        3. Terrified- scared
        4. Perplexed- mixed up

5. Bonjour- hello
6. Ami- friend
7. Belle- beautiful

20. We are going to do an activity. We are going to write out the fancy words Nancy uses (*point to the fancy words on the board)* and also what they mean (*point to the regular words on the board)*.
21. I have a worksheet here (*show children a worksheet)* and some markers.
22. This is where you will write the fancy words (*point to the left column)* and over here is where you will write the regular words.
    1. How can we write out the fancy words in a fancy way? (squiggly or bubble letters, underlines, etc.)
    2. Great! Let me show you what we are going to do.
    3. *Write out first fancy word in first row of "fancy word" column. Make it fancy!*
    4. *Write out the regular word in regular letters in the first row of the "regular word" column.*
    5. Take your time and write as many of the words as you can (*point to the board*).
    6. *Give each child a worksheet and some markers.*

Colouring (Extra activity):

1. If there's extra time, ask children to colour a picture of the Eiffel Tower on the back of the worksheet.

Name:________________

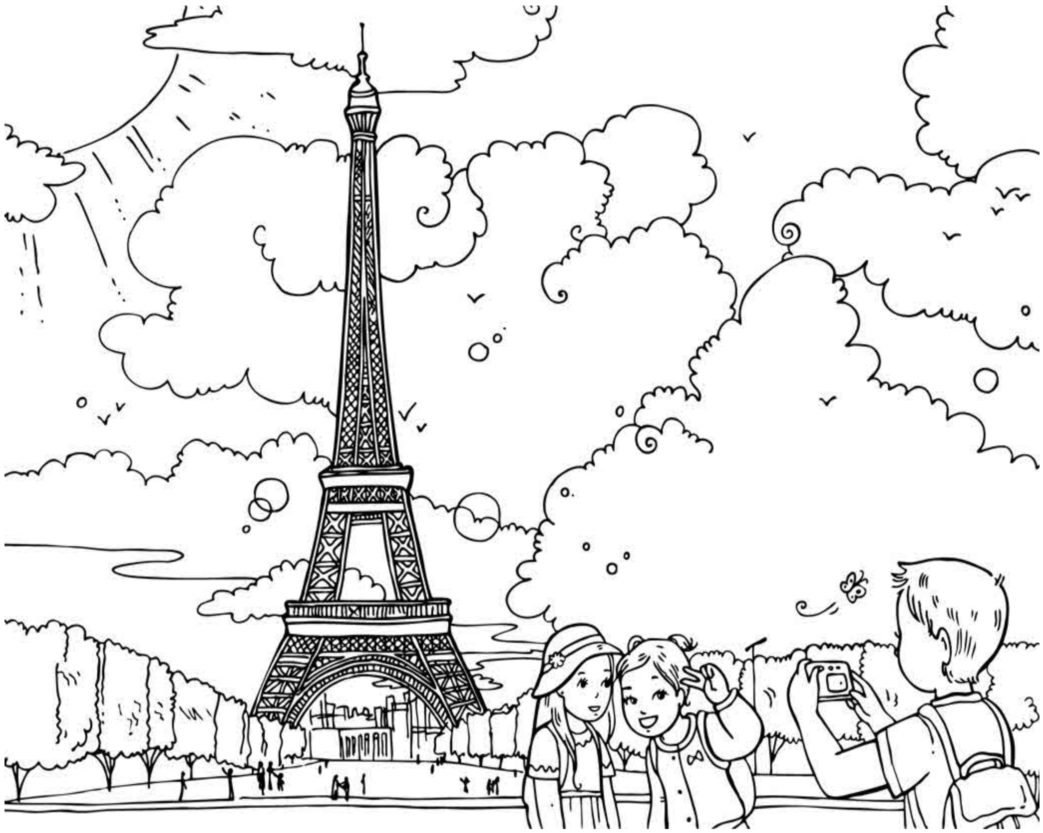
Grade: ________________

Fancy **NANCY**

and the

Boy from Paris

Fancy word	Regular word



Appendix D

Addition Instruction: Instructor Protocol

Addition Instruction: Instructor Protocol - Day 1

Materials (per child + instructor)
1. Small container of blue chips 2. Small container of red chips 3. Manipulatives board 4. Workbook 5. Pencil Eraser
Set up
1. Arrange desks facing the instructor as much as possible. 2. Place one set of materials per desk, per child.
Introduction to Board and Materials
1. We're going to do some addition today. You have some things in front of you. Please do not touch anything until I tell you. *2.* You have an addition board. We'll be adding with the red and blue circles on this board (*point to the manipulatives).* *3.* You also have a workbook (*point to the workbook)*. This is where we'll be writing down and adding numbers. Everyone also has a pencil and an eraser (*hold up the pencil and eraser).* *4.* You can write your name here *(point to the spot on the first sheet of the workbook).* *5.* We're going to do some math problems together and you'll get to work on some problems on your own! *6.* Let's get started! ☺

1 a) Adding Single Digits (Imitation)
T: Let’s start. I’m going to show you what to do so please look and listen. You’ll have a chance to do what I do when I say “it’s your turn.” Let’s start with the first page of your worksheet. Let's add three plus two(3+2) (*Point the horizontal equation at the top of the worksheet)* **T: First I'm going to start with the 3 *(point to the written 3 in the horizontal equation)*** T: I am going to put 3 blue circles here (top right circle column) T: That (*Circle all the chips with your finger)* makes 3 *(trace the written 3 in the vertical equation with your finger).* <u>T: Now it’s your turn (*Wait for students to complete the step)*</u> **T: Then, I am going to add the 2 *(Point to the written 2 in the horizontal equation)*** T: I am going to put 2 blue circles here (middle right circle column) T: That (*Circle all the chips with your finger)* makes 2 *(trace the written 2 in the vertical equation with your finger).* <u>T: Now it’s your turn (*Wait for students to complete the step)*</u> T: Now let's put all the blue circles together. *(Pull down all blue circles to the bottom)* T: Let’s count them 1,2,3,4,5. There are five. T: And I am going to write 5 over here (bottom right number column of vertical equation) <u>T: Now it’s your turn (*Wait for students to complete the step)*</u> T: *Point to the right number column* (3+2 is 5) T: *Point to the horizontal equation and say*, “ 3+2 is 5” and *write = 5 on the equation* <u>T: Now it’s your turn (*Wait for students to complete the step)*</u>
Repeat with 7+0

1 b) Adding Single Digits (Structured Practice)
<u>Feedback (error correction):</u> **1. "Remember, this is what we do next, we…"** **Each student may be at a different pace. Give individual feedback. Feedback should be procedural in nature. No concepts (i.e. We do this because…) should be given.** **2. If students ask why, the instructor should respond, "Because this is the way I would like for you to do it today."** **3. <u>Writing numbers on own:</u> emphasize correct placement on the worksheet. As applicable: "Line up the number __under/beside the number __." " Put the number___ over here *(point out which column)"*** **<u>Praise:</u> Praise effort "Good work, Keep it up."**
T: Let's add five plus four (5+4) (*Point the horizontal equation)* T: What do we do first? C: **Start with the 5** *(pointing to the 5 in the horizontal equation)* *C: Puts 5 blue circles in the top right circle column* *C:* That makes 5. *(Points to the written 5 in vertical equation)* T: Now what do we do? C: **Add the 4** *(pointing to the 4 in the horizontal equation)* *C: Puts 4 blue circles in the middle right circle column* *C:* That makes 4. *(Points to the written 4 in vertical equation)* T: Now what do we do? *C: Puts all the blue circles together.* *C: Count them 1,2,3,4,5,6,7,8,9. There are nine.* T: Now what do we do? *C: Writes 9 in the bottom right number column* T: *Point to the right number column on one student's board and say, "*Now what do we say?" C: *Says*, "5+4 is 9" T: Now what do we do? C: *Writes = 9 on the equation*
1. SEATED BODY BREAK (If approximately 25 minutes into instruction)

2 a) Adding single digits with sum greater than 9 (Imitation)
T: Let's add four plus seven (4+7) (*Point the horizontal equation).* T: **First I am going to add the 4** (pointing to the written 4) T: I am going to put 4 blue circles here (top right circle column) T: That *(Circle all the chips with your finger)* makes 4. So I am going to write 4 here (in the right number column) <u>T: Now it's your turn (*Wait for students to complete the step)*</u> T: **Then I am going to add the 7** (pointing to the written 7) T: I am going to put 7 blue circles here (middle right circle column) T: That *(Circle all the chips with your finger)* makes 7. So I am going to write 7 here (middle right number column) <u>T: Now it's your turn (*Wait for students to complete the step)*</u> T: Now let's put all the blue circles together. *(Pull down all blue circles to the bottom)* T: Let's count them 1,2,3,4,5,6,7,8,9,10,11. There are eleven. <u>T: Now it's your turn (*Wait for students to complete the step)*</u> T: Every time you have a group of 10 blue circles (*count out 10 blue circles)* you put the 10 blue circles back in the box (*place 10 blue circles in box)* and put a red circle here (*Put the red circle above the left circle column*) T: And now I am going to write a little 1 here (*Write 1 above the top left number column*) <u>T: Now it's your turn (*Wait for students to complete the step)*</u> T: Now I am going to count the blue circles, *Counts* "1. There is 1." T: I am going to write 1 here (bottom right number column) <u>T: Now it's your turn (*Wait for students to complete the step)*</u> T: Now let's put all the red circles together. (*Pull down all red circles to the bottom*) T: Let's count 1. There is 1. T: And I am going to write 1 over here (bottom left number column) <u>T: Now it's your turn (*Wait for students to complete the step)*</u> T: *Point to the number column* and says, "4+7 is 11" T: *Point to the equation and say*, " 4+7 is 11" and *write = 11 on the equation* <u>T: Now it's your turn (*Wait for students to complete the step)*</u>
Repeat with 5+5 1. T: This time we are going to write all of the numbers on the worksheet on our own. Take a look at your worksheet *(turn page to 5+5 equation).* 2. As you can see, there are thin lines here (*point to the vertical lines)*. These lines will help you to line up your numbers. When I say "it's your turn" please write the numbers in the same spots as I do. 3. **<u>** Writing numbers on own:</u> emphasize correct placement on the worksheet. As applicable: "Line up the number __under/beside the number __." " Put the number___ over here *(point out which column)"*** 4. Let's add five plus five (5+5). (*Point the horizontal equation)….*

2 b) Adding single digits with sum greater than 9 (Structured Practice)
<u>**Feedback (error correction):**</u> 1. **"Remember, this is what we do next, we…"** **Each student may be at a different pace. Give individual feedback. Feedback should be procedural in nature. No concepts (i.e. We do this because…) should be given.** 2. **If students ask why, the instructor should respond, "Because this is the way I would like for you to do it today."** 3. **<u>Writing numbers on own:</u> emphasize correct placement on the worksheet. As applicable: "Line up the number __under/beside the number __." " Put the number___ over here *(point out which column)"*** <u>**Praise:**</u> **Praise effort "Good work, Keep it up."**
T: Let's add eight plus nine (8+9) (*Point the horizontal equation).* T: What do we do first? **C: Start with the 8 (pointing to the written 8)** *C: Put 8 blue circles here (top right circle column)* *C: Writes 8 on the board (in the right number column)* T: Now what do we do? **C: Add the 9 (pointing to the 9)** *C: Put 9 blue circles here (middle right circle column)* *C: Writes 9 on the board (middle right number column)* T: Now what do we do? *C: Put all the blue circles together. (Pulls down all blue circles to the bottom)* *C: Counts them 1,2,3,4,5,6,7,8,9,10,11, 12, 13, 14, 15, 16, 17. There are seventeen.* T: Now what do we do? *C: Takes 10 blue circles and puts them back in the box.* *C: Takes 1 red circle and puts it above the left circle column* *C: Writes 1 above the top left number column* T: Now what do we do? *C: Count the blue circles and write 7 here (bottom right number column)* T: Now what do we do? *C: Put all the red circles together. (Pulls down all red circles to the bottom)* *C: Counts 1. There is 1.* *C: Write 1 over here (bottom left number column)* T: Now what do we do? *C: Points to the number columns (8+9 is 17)* *C: Points to the equation and says*, " 8+9 is 17" and *writes = 17 on the equation*
Additional examples: **4+4 6+8 0+3 7+5**
End of Day 1 Instruction on Addition

Addition Instruction: Instructor Protocol - Day 2

1 a) Adding Double Digits (Imitation)
T: Let's do the first addition equation together. T: Let's add twelve plus six *(point to the horizontal equation at the top of the worksheet)* **T: First I am going to start with the 12 *(point to the written 12 in horizontal equation)*** T: I am going to put 1 red circle here (top left circle column) T: Then I am going to put 2 blue circles here (top right circle column) T: That (*Circle all the chips with your finger)* makes 12 (P*oint to written 12 in vertical equation*) T: Now it's your turn. (*Wait for students to complete the step)* **T: Then I am going to add the 6 *(point to the written 6 in equation)*** T: I am going to put 6 blue circles here (middle right circle column) T: That (*Circle all the chips with your finger)* makes 6 *(Point to written 6 in vertical equation)* T: Now it's your turn. (*Wait for students to complete the step)* T: Now let's put all the blue circles together. *(Pull down all blue circles to the bottom)* T: Now it's your turn (*Wait for students to complete the step)* T: Let's count the blue circles 1,2,3,4,5,6,7, 8. There are eight (*Circle all the chips with your finger).* T: So I am going to write 8 here (bottom right column). T: Now it's your turn (*Wait for students to complete the step)* T: Now let's put all the red circles together. *(Pull down all red circles to the bottom)* T: Let's count them 1. There is 1. T: And I am going to write 1 over here (bottom left number column) T: Now it's your turn (*Wait for students to complete the step)* T: *Point to the number column* and says, "*12 + 6* is 18" T: *Point to the equation and say*, "*12 + 6* is 18" and *write = 18 on the equation* T: Now it's your turn (*Wait for students to complete the step)*
Repeat with 44 + 13

1 b) Adding Double Digits (Structured Practice)
<u>Feedback (error correction):</u> 1. **“Remember, this is what we do next, we…”** **Each student may be at a different pace. Give individual feedback. Feedback should be procedural in nature. No concepts (i.e. We do this because…) should be given.** 2. **If students ask why, the instructor should respond, “Because this is the way I would like for you to do it today.”** 3. **<u>Writing numbers on own:</u> emphasize correct placement on the worksheet. As applicable: “Line up the number _under/beside the number _.” “ Put the number__ over here *(point out which column)”*** **<u>Praise:</u> Praise effort “Good work, Keep it up.”**
T: Let's add eleven plus four. **This time you’re going to write the numbers yourselves!** Please remember to line up the numbers just like last time when you wrote numbers on your own. T: What do we do first? **C: Start with the 11 *(points to the written 11 in the horizontal equation)*** C: *Puts 1 red circle in top left circle column* *C: Puts 1 blue circle in the top right circle column* *C: Writes 1 on in the right number column* *C: Writes 1 in the left number column* T: Now what do we do? **Add the 4 *(points to the written 4 in the horizontal equation)*** *C: Puts 4 blue circles here middle right circle column* *C: Writes 4 under the 1 (right number column)* T: Now what do we do? *C: Put all the blue circles together. (Pulls down all blue circles to the bottom)* *C: Counts them 1,2,3,4,5. There are five.* *C: Writes 5 (bottom right number column)* T: Now what do we do? *C: Put all the red circles together. (Pulls down all red circles to the bottom)* *C: Counts 1. There is 1.* *C: Write 1 over here (bottom left number column)* T: Now what do we do? *C: Points to the number columns (11+4 is 15)* *C: Points to the equation and says*, “11+4 is 15” and *writes = 15 on the equation*
SEATED BODY BREAK (if approximately 25 minutes into the instruction)

2 a) Adding Double Digits with sum in singles is greater than 9 (Imitation)
T: Let's add nineteen plus six *(point to the equation at the top of the worksheet)* **T: First I am going to start with the 19 *(point to the written 19 in the horizontal equation)*** T: I am going to put 1 red circles here (top left circle column) T: I am going to put 9 blue circles here (top right circle column) T: That *(Circle all the chips with your finger)* makes 19. I am going to write 1 here (in the top left number column) and I am going to write 9 here (in the right number column) <u>T: Now it's your turn (*Wait for students to complete the step)*</u> **T: Then I am going to add the 6 *(point to the written 6 in the horizontal equation)*** T: I am going to put 6 blue circles here (middle right circle column) T: That *(Circle all the chips with your finger)* makes 6. I am going to write 6 here, under the 9 (middle right number column) <u>T: Now it's your turn (*Wait for students to complete the step)*</u> T: Now let's put all the blue circles together. *(Pull down all blue circles to the bottom)* <u>T: Now it's your turn (*Wait for students to complete the step)*</u> T: Let's count them 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15. There are fifteen. T: Every time you have a group of 10 blue circles (*count out 10 blue circles)* you put the 10 blue circles back in the box (*place 10 blue circles in box)* and put a red circle here (*Put the red circle above the left circle column*) T: And now I am going to write a little 1 here (*Write 1 above the top left number column*) <u>T: Now it's your turn (*Wait for students to complete the step)*</u> T: Now I am going to count the blue circles, "Counts 1,2,3,4,5. There are 5." T: I am going to write 5 here (bottom right number column) <u>T: Now it's your turn (*Wait for students to complete the step)*</u> T: Now let's put all the red circles together. (*Pull down all red circles to the bottom*) <u>T: Now it's your turn (*Wait for students to complete the step)*</u> T: Now I am going to count the red circles, "Counts 1,2. There are 2." T: And I am going to write 2 over here (bottom left number column) <u>T: Now it's your turn (*Wait for students to complete the step)*</u> T: *Point to the number columns and say*, "(19+6 is 25)" T: *Point to the equation and say*, " 19+6 is 25" and *write = 25 on the equation* T: <u>Now it's your turn (*Wait for students to complete the step)*</u>
Repeat with 14 + 8=

2 b) Adding Double Digits with sum in singles is greater than 9 (Structured Practice)

Feedback (error correction):

1. **"Remember, this is what we do next, we…"**
   Each student may be at a different pace. Give individual feedback. Feedback should be procedural in nature. No concepts (i.e. We do this because…) should be given.
2. **If students ask why, the instructor should respond, "Because this is the way I would like for you to do it today."**
3. **Writing numbers on own: emphasize correct placement on the worksheet. As applicable: "Line up the number __under/beside the number __." " Put the number___ over here *(point out which column)"***

Praise: Praise effort "Good work, Keep it up."

T: Let's add twenty five plus nine (25+9) *(point to the horizontal equation at the top of the board)*
T: What do we do first?
C: Start with the 25 *(points to the written 25 in the horizontal equation)*
C: *Puts 2 red circles in top left circle column*
C: Puts 5 blue circles in top right circle column
C: *Writes 2 on the worksheet in the top left number column*
C: Writes 5 on the worksheet in the top right number column
T: Now what do we do?
C: Add the 9 *(points to the written 9 in the horizontal equation)*
C: Puts 9 blue circles in middle right circle column
C: Writes 9 on the worksheet in middle right number column
T: Now what do we do?
C: Put all the blue circles together. *(Pulls down all blue circles to the bottom)*
C: Counts them 1,2,3,4,5,6,7,8,9,10,11,12,13,14. There are fourteen.
T: Now what do we do?
C: Takes 10 blue circles and puts them back in the box.
C: Takes 1 red circle and puts it above the left circle column
C: Writes little 1 above the top left number column
T: Now what do we do?
C: Counts the blue circles 1,2,3,4 and write 4 here (bottom right number column)
T: Now what do we do?
C: Put all the red circles together. (Pulls down all red circles to the bottom)
C: Counts 1, 2, 3. There are 3.
C: Write 3 over here (bottom left number column)
T: Now what do we do?
C: Points to the number columns (25+9 is 34)
C: Points to the equation and says, "25+9 is 34" and *writes = 34 on the equation*

Additional examples: **34+24** **16+25** **46+1** **13+7**

End of Day 2 Instruction on Addition

Appendix E

Place Value Assessment (Posttest): Instructor Protocol

Conventions of Place Value Task: Instructor Protocol (Posttest)

INITIAL SETUP:
Nothing in front of the child. Show the child one card at the time.

SAY:

- "What number is this?" *(Circle the whole number with your finger when asking the question)*
    - If the child gives the wrong answer, say, "Well actually, the number is xxx."
- "How many things is this worth?" (*Circle the target digit with the eraser of a pencil)*
- "Why is it worth [child's response]?" **ONLY ASK THIS QUESTION FOR THE FIRST 6 PROBLEMS. Don't ask it for the last 6.**

- Turn the card over after the child has answered.
- Repeat the same instructions for each one of the problems.
- Don't forget to record the child's answers on the scoring sheet.

12 problems:
1) 2**7**
2) **4**4
3) 6**3**
4) **1**8
5) 8**8**
6) **9**0
7) 1**6**
8) **5**5
9) 6**4**
10) **3**5
11) 2**2**
12) **7**1

The Word Problem Task: Instructor Protocol (Posttest)	
1) The interviewer will place an index card with a large number printed on it in front of the child. 2) The interviewer will then read the following problems one by one to the child. 3) When done with one card, remove it before starting the next problem. <u>FOR ALL WORD PROBLEMS</u>: Let the child try to answer each problem <u>without</u> the use of paper/marker. If the child can't or asks to use materials, offer paper/marker. Make sure to remove the paper/marker from the child's view after each problem.	
A) 34	You see the number on this card? This is how many lollypops the teacher has left. The teacher wants to give each kid at the party a bag of **10** lollypops. How many kids will get a **full** bag of lollypops? Why is it (child's response)?
B) 60	You see the number on this card? This is how many toothbrushes the dentist has. The dentist puts the toothbrushes in packages of **10**. How many **full** packages of toothbrushes can he make with the toothbrushes he has? Why is it (child's response)?
C) 53	You see the number on this card? This is how many books Paul has. He puts them away in his bookcase so that there are **10** books on each shelf. How many shelves can he **fill** up completely with the books he has? Why is it (child's response)?
D) 25	You see the number on this card? This is how many toy cars Jason has. He puts them away in buckets so that there are **10** cars in each bucket. How many buckets can he fill up **completely** with the cars he has? Why is it (child's response)?
E) 47	You see the number on this card? This is how many tennis balls Jessica has. She puts them away in bags so that there are **10** tennis balls in each bag. How many bags can she fill up **completely** with the tennis balls she has? Why is it (child's response)?
F) 18	You see the number on this card? This is how many cupcakes the store has left. Bobby wants to put **10** cupcakes in each box. How many boxes can he fill up **completely** with the cupcakes the store has left? Why is it (child's response)?

Appendix F

Cooperman Task (Posttest): Instructor Protocol

The Cooperman Task: Instructor Protocol (Posttest)

1. The interviewer will place two cards with color circles on them on a mat in front of the child.
   a. Cards will match the mat by color and letter
2. The interviewer will ask the child:
   a. "How much is this worth?" *(Point to the left group of colored circles in a circular motion)*
   b. "How much is this worth?" *(Point to the right group of colored circles in a circular motion)*
   c. "Which is worth more *(point simultaneously to both cards of circles)* or are they worth the same?" *(Say the whole sentence without pausing)*
   d. "Why is [child's response] worth more?" OR "Why are they worth the same?"

Item 1	● ● ● ●		● ● ● ●
Item 2	● ● ●		○ ○ ○
Item 3	○		● ● ● ● ● ● ● ● ● ●
Item 4	● ● ● ● ●		○ ○ ○ ○
Item 5	○ ○ ○ ○ ○ ○ ○		○ ○ ○ ○ ○ ○ ○
Item 6	○ ○ ○ ○		● ● ● ●
Item 7	● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ●		○ ○
Item 8	○ ○ ○		● ● ● ● ●

Note. ● = blue circles, ○ = red circles.

Appendix G

Scoring Sheet for Pretest Testing

GRADE 1 – Scoring Sheet

(Pretest)

Name: ___________________________________

School: __________________________________

Teacher: _________________________________

Language most often spoken at home: ____________________

Gender: ☐Male ☐Female

Meeting #1

Interviewer: ____________

Date: ____________

Time: ____________

☐Conventions of Place Value

☐Word Problems (gr.1)

⇒ *Was paper/marker used? Y N*

Comments:

____

____

____

____

MEETING #1

1. Conventions of Place Value

For each number write down what the child answered to the question, "How many things is this?"

13) 6**8**	
14) **7**7	
15) 5**3**	
16) **2**9	
17) 3**3**	
18) **4**1	
19) 2**5**	
20) **6**6	
21) 4**2**	
22) **3**8	
23) 1**1**	
24) **8**0	

2) Word problems

Record the child's answers. If the child used the pad of paper and marker to help him/her find the answer, please indicate it next to the problem.

A. (Jane – 27): _____ boxes of donuts

B. (Soccer – 50): _____ soccer players

C. (Lego – 45): _____ children who will get Lego pieces

D. (Ann – 36): _____ boxes of dolls

E. (John – 18): _____ packages of hockey cards

F. (Mary – 52): _____ pages of her sticker book

END OF MEETING #1

Appendix H

Scoring Sheet for Pre-instruction Testing

1) Cooperman Task – VERSION A

Write down next to each array how much the child said it was worth.

Circle the array the child said was worth more. If the child said they were worth the same, write an "=" under the corresponding letter.

		How much is this worth? ←		*How much is this worth?* ←
A	● ● ●		● ● ●	
B	● ● ● ● ●		○ ○ ○ ○ ○	
C	○		● ● ● ● ● ● ● ● ● ●	
D	● ● ● ● ● ● ●		○ ○ ○ ○	
E	○ ○ ○ ○ ○		○ ○ ○ ○ ○	
F	○ ○ ○ ○ ○ ○		● ● ● ● ● ●	
G	● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ●		○ ○	
H	○ ○		● ● ● ●	

Appendix I

Scoring Sheet for Posttest Testing

GRADE 1 – **VERSION A**

POSTTEST

Name: ____

School: _______________________________________

Teacher: ______________________________________

Check on class list if instruction was done ☐

Interviewer: ______________

Date: ______________

Time: ______________

☐Conventions of Place Value

☐Word Problems (gr.1)

⇒ *Was paper/marker used? Y N*

☐Cooperman Task

Comments:

____

____

____

____

2. Conventions of Place Value

For each number write down what the child answered to the question, "How many things is this worth?"

25) 2**<u>7</u>**	
26) **<u>4</u>**4	
27) 6**<u>3</u>**	
28) **<u>1</u>**8	
29) 8**<u>8</u>**	
30) **<u>9</u>**0	
31) 1**<u>6</u>**	
32) **<u>5</u>**5	
33) 6**<u>4</u>**	
34) **<u>3</u>**5	
35) 2**<u>2</u>**	
36) **<u>7</u>**1	

3) Word problems

Record the child's answers. If the child used the pad of paper and marker to help him/her find the answer, please indicate it next to the problem.

A. (Teacher – 34): _____ bags of lollypops

B. (Dentist – 60): _____ packages of toothbrushes

C. (Paul – 53): _____ shelves

D. (Jason – 25): _____ buckets of toy cars

E. (Jessica – 47): _____ bags of tennis balls

F. (Bobby – 15): _____ boxes of cupcakes

2) Cooperman Task – VERSION A

Write down next to each array how much the child said it was worth.

Circle the array the child said was worth more. If the child said they were worth the same, write an "=" under the corresponding letter.

		How much is this worth? ←		*How much is this worth?* ←
A	● ● ● ●		● ● ● ●	
B	● ● ●		○ ○ ○	
C	○		● ● ● ● ● ● ● ● ● ●	
D	● ● ● ● ●		○ ○ ○ ○	
E	○ ○ ○ ○ ○ ○ ○		○ ○ ○ ○ ○ ○ ○	
F	○ ○ ○ ○		● ● ● ●	
G	● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ●		○ ○	
H	○ ○ ○		● ● ● ● ●	