Sliding Mode Fault Tolerant Reconfigurable Control against Aircraft Control Surface Failures

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Abstract

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Operational failure of control surfaces is one of the main reasons leading to aircraft crash. Since the conventional control methodologies are not adequate to accommodate such failures, fault tolerant control (FTC) is required for safety critical system. The invariance property and unique synthesization procedure of sliding mode control (SMC) make it one of the most competitive candidates for FTC. In this thesis, SMC-based FTC methods for nonlinear systems are developed to handle both partial loss faults and total failures in the control surfaces. The first SMC-based FTC is developed to accommodate both modeling uncertainty and uncertainty incurred by the faults. Different design parameters are utilized to deal with the uncertainty incurred by fault and that due to modeling errors respectively in the SMC design. Direct adaptive control is combined into such a SMC to alleviate the requirement of the *a priori* knowledge of the uncertainty bounds. The second SMC-based FTC is developed to redistribute the control effort between faulty regular actuator and redundant actuator autonomously based on effectiveness of the regular actuators. The tolerability of the developed controller is characterized by the amount of fault that controller can deal with. It is used as the threshold to activate the redundant actuator when the regular actuator cannot accommodate the fault alone. In order to obtain the effectiveness of the actuator, special sensors or fault detection and diagnosis (FDD) schemes are required. Special sensors are costly and additional design of the system is required. Using FDD, during the period from the moment when fault occurred to that when the effectiveness information can be obtained, the system is under the danger of losing control. The third SMC-based FTC without a dedicated FDD is developed based on the absolute value quantity of switching surface. The control effort is redistributed to regular and redundant actuator autonomously by monitoring the absolute value of the sliding surface. The validity of the proposed algorithms is verified on a high fidelity model of Boeing 747-100/200.

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To my parents, my sister, my wife and my son.

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Nomenclature and Abbreviations

Nomenclature

p, q, r	roll, pitch and yaw rate (rad/sec)
V_{TAS}	true air speed (m/s)
α, β	angle of attack and sideslip angle (rad)
ϕ, θ, ψ	roll, pitch and yaw angle (rad)
h_e, x_e, y_e	altitude and geometric earth position along x and y axis (m)
δ	deflection of control surface
δ_e	deflection of elevator
σ	deflection of stabilizer
T	engine thrust
u	command signal
s	sliding mode switching function
R	field of real numbers
$\| \cdot \|$	Euclidean norm (vectors) or induced spectral norm (matrices)
Subscript	
$trim$	trim value
com	command signal
eq	equivalent
eil, eir, eol, eor	inner left, inner right, outer left and outer right elevator
ail, air, aol, aor	inner left, inner right, outer left and outer right aileron
rl, ru	lower and upper rudders
ih	stabilizer
sp	spoiler
r	redundant actuator
c	conventional or regular actuator
d	desired reference trajectory

Abbreviations

AFTC	Active Fault Tolerant Control)
CA	Control Allocation
CFIT	Control Flight Into Terrain
CG	Center of Gravity
DI	Dynamic Inversion
FDD	Fault Detection and Diagnosis
FTC	Fault Tolerant Control
IMM	Interactive Multiple Model
LFTC	Linear Fault Tolerant Control
LPV	Linear Parameter Varying
MMST	Multiple Model Switching and Tuning
MPC	Model Predictive Control
NFTC	Nonlinear Fault Tolerant Control
PFTC	Passive Fault Tolerant Control
PIM	Pseudo Inverse Method
RMS	Root Mean Square
RMSE	Root Mean Square Error
SMC	Sliding Mode Control
UAV	Unmanned Aerial Vehicle
UGV	Unmanned Ground Vehicle
UUV	Unmanned Underwater Vehicle

Chapter 1

Introduction

Fault tolerant control attracts more and more attention of researchers from academic community and industry recently, because of the increasing demand for safety critical system and complex autonomous system, such as, aircraft, nuclear reactor, satellite, autonomous unmanned aerial vehicle (UAV)/unmanned ground vehicle (UGV)/unmanned underwater vehicle (UUV), and etc. This thesis focuses on fault tolerant control of aircraft with faults in actuators. In order to clarify the concepts utilized in this thesis, the definition of faults and failures are cited here firstly. Then the motivation of this thesis is elaborated and the problem is stated. Finally the contribution is summarized and the structure of this thesis is outlined.

1.1 Definition of Faults and Failures

A standard of the terminology in fault detect and diagnosis (FDD) and fault tolerate control (FTC) fields has been drawn up by IFAC SAFEPROCESS technical committee to avoid ambiguity among researchers [Chen and Patton, 1999; Isermann and Balle, 1997]. Definitions of faults and failures are cited here in the following:

Fault: An unpermitted deviation of at least one characteristic property or parameter of the system from the acceptable/usual/standard condition.

Failure: A permanent interruption of a system's ability to perform a required function under specified operating conditions.

The above definitions make difference between faults and failures: a failure means complete breakdown of a function while a fault means partial loss of effectiveness of a function. As for a fault occurring on an actuator, the actuator can still contribute to the controlled system but may have a slower response or become less effective. But when a failure occurs on an actuator, a redundant actuator is needed to be able to generate the same desired effect.

1.2 Motivation

Along with the evolution of computer, sensor and control technology, it is possible not only to release human being from repeated mechanical labor, but also to free people from onerous brainwork, and even more to realize the dream of man for complicated tasks out of the reaching range of human's physical and mental limit. This stimulates the evolution from automatic machines to more and more intelligent and autonomous systems. In the control field, the theory and practice are evolved from not only to reject disturbance and suppress noise, but also to be robust to parameter uncertainty, and even more to be tolerant with changing dynamics due to contingent events, such as faults and failures in sensors, actuators or system structure.

On the other hand, avoidance of harm to human and damage of property is upmost for the safety critical systems, i.e. there must be some mechanism that can detect faults and failures and trigger the alarm. The faults and failures are kinds of contingent events in the system that mostly change the system dynamics and disable the normal controller, which may lead to catastrophe if they are not dealt with in time and properly. The industrial and academia have developed techniques to detect and isolate such contingent events in systems in the past 40 years. The information about these contingent events is used to activate an emergence response system. Such emergence response system mostly is monitored or processed by human being. To process these events in time and properly in complicated systems, such as aircrafts, satellite, nuclear power plants and robotic systems, is beyond the reaction capability of human being. In this kind of situation, considering these events in the controller design becomes more and more important, which is the newly emerging control architecture: fault tolerant control.

One of the main reasons leading to loss of control of aircraft is the operational failure in the actuators, the control surfaces, such as elevators, ailerons and rudders. On February 16, 2000, a McDonnell Douglas DC-8-71F lost its pitch control (elevator) on takeoff, resulting in a crash and destruction of the airplane and death of three flight crew members [FSS-2000, 2006]. Another air crash due to failure in elevator occurs on January 8, 2003, which killed all 19 passengers and 2 pilots aboard on an airplane Beechcraft 1900D operating as US Airways Express Flight 5481 [Wikepedia, 2006]. On September 8, 1994, a fault in the Boeing 737 rudder killed all 133 people on board of USAir Flight 427 [Wikipedia, 2012]. Flight simulation systems LLC made a list of faults and failures occurred in the flight control system from 1970 to 2006, many of which are due to faults and failures in control surfaces [FSS, 2006].

An ideal way of fault tolerant control is to combine the technologies developed in fault detection and identification fields (provide dynamics model of the faulty system), and the model-based control methods (reconfigure the controller online).

Similar to adaptive control method developed in process control system, the changing of the parameters (for FTC, the changing parameters induced by faults and failures) must be slow enough to let identification mechanism identify the system dynamics model (for FTC, rebuild the model of the faulty system), i.e., there must be abundant excitation to the changing dynamics of the system with updating so that a new controller can be synthesized online based on the identified system model and the stability of the system is guaranteed. This is why adaptive control is only suitable for the systems whose parameters are time invariant but unknown, or the changing of the parameters is very slow.

For the accidental events such as fault and failure, the change of the dynamics may be very fast and is unpredictable while the computation of the dedicated FDD is time consuming. Before such an FTC, which depends heavily on a reliable faulty system model generated by FDD, will be activated, the system may be in the danger of losing its stability. Fig. 1.1 shows the time history of such FTC [Zhang and Jiang, 2006], in which t_F stands for the time instant at which the fault occurs; t_D stands for time instant when the fault is detected; t_R stands for the time instant when the reconfigurable control has been synthesized; and t_C is the time instant after which all the transients due to the fault and the control system reconfiguration have settled down and a new steady-state has been reached, and the system enters the post-fault interval. After fault occurrence and before a reconfigured controller based on the faulty system model has been built, the system has been operated in a “bad” condition since the system is in a faulty condition but the feedback control designed for normal condition is still in action which provides inappropriate closed-loop control action to the system because of the system-controller mismatch. Performance and stability of the closed-loop system during this time period is mainly dependent on the severity of the fault and the fault tolerance of the nominal controller. The system may even become or tend to become unstable, as shown in the dash-line in Fig. 1.1. In other words, during the period of t_F and t_R the system is to some extent out of control, or lose of control. This is not tolerable even to FDD module, since if the system goes unstable, it is hard or impossible for the FDD module to collect correct information for generating a reliable faulty system model. Hence, FTC must have the capability to keep the faulty system stable at least before the faulty model can be identified and the better FTC based on the faulty model can be put in effect.

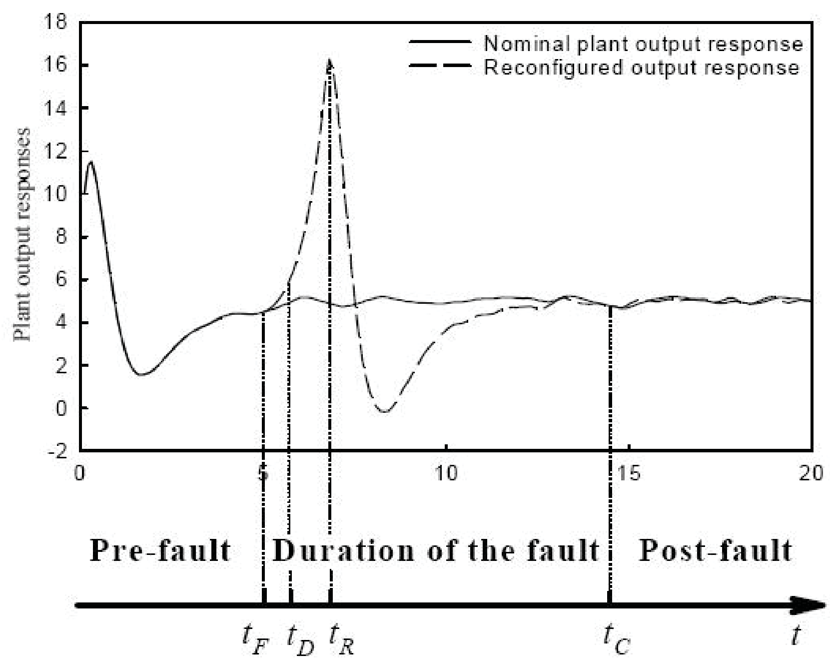

Figure 1.1: Three intervals in FTCS (adopted from [Zhang and Jiang, 2006])

Another method that can be borrowed here from the control field, is robust control which has gained more and more attention both in theory and practice recently. It is well-known that there is always difference or error between the model and the real physical system, i.e. there exists uncertainty in system model. The robust control considers this uncertainty in the synthesis of controller. A robust controller can be designed when the information of the uncertainty is available in statistical meaning. There is a trade-off with performance depending on how we simplify the model, i.e., how much we know the system or how much uncertainty the system will be assumed to have. In this sense, the uncertainty of system model is 'known' in statistical meaning. For some kinds of faults, like partial loss fault, robust control method can tolerate them in some extent and is called passive fault tolerant control (PFTC) compared with the above mentioned FDD-based FTC which is named as active FTC (AFTC). However, fault and failure in nature are contingent events for the system, and they are different from the system model uncertainty, i.e. it is 'unknown' even in statistical sense. These kinds of accidental events change the system dynamics greatly so that no *a priori* knowledge on the faults and failures will be available for the designer of the controller. It is clear here, fault should be dealt with differently from the normal system modeling uncertainty. In another aspect, the aircraft system is expected to operate normally all the time, while faults are unexpected events with small probability. One of the problems in PFTC is that it degrades the performance of normal healthy system significantly if trade-off is made to accommodate more faults. Hence, it is better to deal with faults and modeling error respectively.

From the above analysis, we can draw the conclusion that AFTC based on FDD has the problem of delay in building FTC based on new faulty model from FDD while PFTC based on robust control method has the problem of only dealing with partial loss fault in some extent. In this thesis, new FTCs that can deal with faults as well as failures without the delay of time in finding the faulty system model and without significant degradation of normal controller, are developed.

1.3 Statement of the Problem

From the above motivation, the objective of this thesis is to develop a controller that can stabilize the system when there are faults or failures in the actuators. In particular, this thesis focuses on developing fault tolerant controllers that can tolerate partial loss faults and total failures in real time without degrading the performance of normal controller.

The development of such controllers is carried out in three aspects. The first aspect is to investigate the method on how to separate the modeling uncertainty and faults in the controller design. This is based on the consideration of effectiveness of the controller where fault tolerance should not sacrifice the performance of the normal controller.

The second aspect is to define tolerability and use this information in developing reconfigurable controllers that can fully utilize the regular actuators in accommodating significant faults and total failures.

The third aspect is to develop fast responding FTC that can tolerate both faults and failures without delay of time in finding the faulty system model. To deal with the fast changing dynamics due to faults and failures, a feasible way is to extract the changing dynamic mode directly from the sensor data, e.g., model-free control [Han, 1994; Sipahi, 2012] and sensor-data driven control (like multi-scale wavelet control [Parvez, 2003; Cimino and Prabhakar, 2012]). The common problem of these methods is that it is hard to prove the stability in theory. In this thesis, change in sliding mode surface is used as the index to faults and failures occurrence, and a sliding mode reconfigurable fault tolerant controller is developed based on it. The stability of the proposed controller is proved.

1.4 Contributions

Although faults can be considered as a kind of uncertainty, it is different from modeling uncertainty. The modeling uncertainty is statistically known in the design period of the controller, i.e. it can be estimated *a priori*. It can also be interactive with the design of the system with consideration of the system performance and cost. Whereas, the fault is an contingent event which occurs in small probability. In order to tolerate such a fault, the normal control performance has to be sacrificed greatly which is ineffective and costly. So it is cost effective to deal with modeling uncertainty and fault separately. The first contribution of this thesis work is the separation of modeling uncertainty and fault in controller design. Extra design parameters are introduced in the sliding mode control (SMC), which make separation between the dealing with modeling uncertainties and faults naturally.

In the above partial loss fault tolerant control, the uncertainty bound of the fault must be assumed to be known in the controller design. This constrains the fault tolerant controller's tolerance to only the 'assigned' partial loss fault. The second contribution of this thesis lies in combining adaptive mechanism into the above SMC strategy that separates the modeling uncertainty and fault so that SMC has the capability to tolerate varied magnitude partial loss fault without sacrificing the normal healthy system performance [Wang et al., 2010a, 2012a].

In the sliding mode partial loss fault tolerant control, it is assumed that the actuator still can stabilize the system with acceptable performance when there are partial faults in the actuators. If the faults are more significant than what the regular actuators can tolerate, a functionally redundant actuator must be activated to work together with the regular actuator to stabilize the system. Here a key concept has to be defined: tolerability, i.e. how we know if the regular actuators can tolerate the faults by itself or not. In this thesis, tolerability is defined in the context of SMC design on regular actuator which comes to the third contribution of the thesis. Since it is difficult to get the analytical representation of tolerability, off-line simulation of tolerability has been implemented within the architecture of SMC in this thesis. Once the knowledge of the tolerability of SMC is obtained, i.e. how much fault the actuator can deal with solely without significant lose of performance, the design of reconfigurable control is quantitatively indexed. The control effort will be reconfigured among the faulty regular actuators and redundant actuators when the faulty regular actuators cannot tolerate the fault by themselves.

With the information of tolerability of the regular actuator under SMC, and the effectiveness of actuator obtained from special sensors or an FDD scheme, a reconfigurable controller is implemented as the fourth contribution of this thesis. With this method, the reconfiguration of the control effort is autonomous between the regular and redundant actuators when the regular actuators cannot accommodate the faults solely. The reconfiguration is not simply a switch between the regular and redundant actuators, but a seamless integration of them [Wang et al., 2010b, 2012b]. When the regular actuator cannot accommodate the fault but still can contribute to the control of the system, we use it to work with the redundant actuators together to stabilize the system instead of discarding it. This is a cost effective way of designing reconfigurable controller, since the redundant actuator is not designed to have the feature of regular actuator, for example, the stabilizer as the redundant actuator for elevator is slower than the elevator in response.

Although the above reconfigurable control is effective and economic, it is hard to get the information of effectiveness. There are two methods to get information of effectiveness: FDD or special sensors. For the special sensors, it will be costly to redesign the system with such special sensors. FDD is time consuming and may trigger wrong alarm because of the measurement noise and uncertainty in the dynamic model. Another problem with FDD is that it needs abundant excitation to extract the change in the dynamics of the system. Finally, FDD can only work out the change in a stable system dynamics while the fault or failure may lead to an irrational unstable system due to the inappropriately designed controller which is designed for normal situation. So before a better FTC based on the information from FDD can be built, some control must be put into action to stabilize the system. The sliding function (some literatures call switching function, or switching manifold) in SMC is a kind of index to the changes of the system dynamics. It can be used as the indicator of faults and failures. As the fifth contribution of this thesis, a new reconfigurable control method based on sliding function without a dedicated FDD is proposed [Wang et al., 2010c, 2012c]. Though there is some performance degradation, this method can stabilize the system. Because the sliding function is the combination of error signals which can be obtained in real time, the reconfiguration is also carried out in real time. The control structure does not change and there is no delay in 'finding' the faults and failures, so the reconfiguration is autonomous. As the redundant actuator is seamlessly integrated into the controller, this method can deal with not only partial loss fault but also total failures in regular actuators without redesigning the controller. The theoretical stability analysis is given and the simulation on FTLAB747 shows the effectiveness of this method.

In summary, the contributions of this dissertation are as the following:

1. Separate the modeling uncertainty and the fault in sliding mode controller design.

2. Develop an adaptive mechanism in the SMC strategy that separate the modeling uncertainty and fault.

3. Define tolerability in the context of SMC design on regular actuator.

4. Develop a reconfigurable controller using the information of the effectiveness of the regular actuators that can deal with not only partial loss fault but also failures in the regular actuators.

5. Develop a reconfigurable controller without a dedicated FDD mechanism.

1.5 Outline of the Thesis

Chapter 2 gives the literature review of the recent work in the field of FTC and specially in the sliding mode FTC. In this chapter, modeling of faults and failures and basic knowledge of FTC are introduced as well as the literature review on different methods of FTC. Also this chapter summarizes the features of SMC, how to design SMC for SISO system and MIMO system, and how it is used in FTC as well as the literature review of sliding mode FTC.

Chapter 3 introduces the simulation package FTLAB747 and three models of Boeing 747-100/200: one high fidelity model based on coefficients obtained from the wind tunnel test which is used in the simulation package FTLAB747; one nonlinear longitudinal model of Boeing 747-100/200 which is used in the derivation of the third model; one fitted nonlinear longitudinal model of Boeing 747-100/200 derived from the nonlinear longitudinal model is used for the controller design. Modeling of faults, failures and faulty system are also introduced in this chapter.

Chapter 4 compares uncertainty and fault. Two important features of control systems are investigated: i.e. the robustness dealing with uncertainty and the tolerability dealing with fault and failure. The tolerability of SMC is analyzed and simulated.

In Chapter 5, a sliding mode controller with two sets of design parameters that can deal with system modeling uncertainty and fault respectively is developed. Adaptive version of this kind of sliding fault tolerant control is also developed without using *a priori* knowledge of the system bound.

In Chapter 6, a reconfigurable FTC based on SMC using information of effectiveness of regular actuators is developed. The tolerability of SMC is used in the controller design to make the control system more energy efficient.

In Chapter 7, a sliding mode reconfigurable FTC utilizing sliding surface as the fault indicator is developed. A dedicated FDD is not required in this control strategy, which makes the system can respond to faults and failures instantly.

Finally, a conclusion and the future works are presented in Chapter 8.

Chapter 2

Literature Review

This chapter summarizes the basic knowledge of FTC and SMC, and reviews the literature on FTC and specially on sliding mode FTC. The first section of this chapter introduces the basic knowledge of FTC and reviews relevant works in this field published in the literature. The second section briefly introduces SMC in the following aspects: what are SMC and sliding surface, the reachability condition, the chattering problem of SMC, the design of SMC for affine SISO and MIMO nonlinear systems. In the third section, the works on FTC with SMC are reviewed and discussed. The last section gives a summary of literature review.

2.1 Fault Tolerant Control (FTC) Techniques

Fault tolerant control systems (FTCS) are control systems that can accommodate faults and failures in sensors, actuators or system struture automatically. They can maintain overall system stability and acceptable performance when there are faults or failures in the system. FTCS were also known as self-repairing, reconfigurable, restructurable, or self designing control systems [Zhang and Jiang, 2008].

There is a lot of literature on FTC. The works in [Stengel, 1991; Blanke et al., 1997; Patton, 1997; Jones, 2003; Zhang and Jiang, 2003b, 2008] are some widely referred surveys in this field. Some published books are [Mahmoud et al., 2003; Blanke et al., 2003; Ducard, 2009; Noura et al., 2009; Yang et al., 2010; Edwards et al., 2010b; Alwi et al., 2011]. In terms of the model used in the control design as linear or nonlinear, FTCS can be classified in two different groups, linear FTCS (LFTCS) dealing with linear models or nonlinear FTCS (NFTCS) dealing with nonlinear models. Most of the literatures deal with LFTCS [Zhang and Jiang, 2008]. A review on NFTCS can be found in [Benosman, 2010].

In general, FTCS can be classified into two types according to its synthesization method: passive FTCS (PFTCS) and active FTCS (AFTCS) [Patton, 1997; Blanke et al., 2003; Zhang and Jiang, 2008]. In passive fault tolerant control, controllers are fixed and are designed to be robust against a class of presumed faults and uncertainty [Eterno et al., 1985]. In contrast to passive fault tolerant control, active fault tolerant control reacts to the system faults actively by reconfiguring control actions based on the information from FDD scheme. A comprehensive review of AFTCS is presented in [Zhang and Jiang, 2008]. The paper gives various classification of AFTCS according to different criteria such as design methodologies and applications, and discusses open problems and current research topics in AFTCS. Figure 2.1 presents a general structure of AFTCS. The lightening arrows show where fault and failure may occur (actuators, sensors and system). The command governor block plans and manages the desired trajectory of the controlled outputs; the FDD block detects and identifies the faults and failures in the system; and reconfiguration mechanism block reconfigures new feedforward and feedback controller with the information from FDD.

AFTCS depends on online knowledge of faults from FDD. FDD utilizes analytical redundancy as a cheaper way in contrast to physical redundancy for fault tolerance. Analytical redundancy means an explicit mathematical model of the system is used for fault detection, identification and recovery/reconfiguration (FDIR). The faults are diagnosed by using the information included in the model and in the online measurements. However, due to the measurement noise, external disturbances and model uncertainties, FDD may falsely alarm. Another problem with AFTCS is the time delay in FDD and control reconfiguration. The FDD must search for a judge from the noisy measures affected by external disturbances and model uncertainties. Also, the controller redesign block needs time to design/search for a new controller according to the faults information.

PFTCS has the drawbacks that it is reliable only for the class of faults taken into account in the design of the PFTCS. Furthermore, the performance of the closed-loop system is not optimized for each fault scenario. However, it has the advantage to avoid the time delay due to online diagnosis of the faults and reconfiguration of the controller, as required in AFTCS [Zhang and Jiang, 2006].

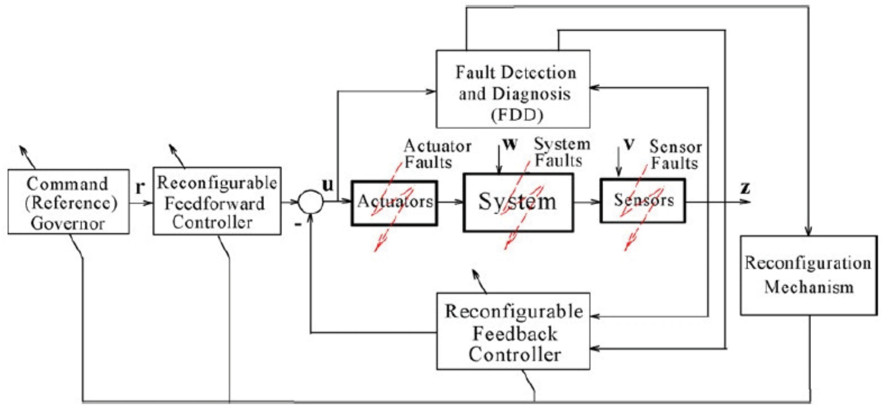

Figure 2.1: General structure of AFTCS (adopted from [Zhang and Jiang, 2008])

In practical applications, PFTCS is a complement of AFTCS. Indeed, PFTCS are necessary during the fault detection and estimation phases [Zhang et al., 2004], where PFTCS is used to ensure the stability of the faulty system, before switching to AFTCS. Another scenario where PFTCS is used as a complement of AFTCS is in the switching-based AFTCS, where the AFTCS switches between different PFTCS, each controller being designed off-line to cope with a finite number of expected faults and stored in a controller bank [Ingimundarson and Sáncheze Peña, 2008].

PFTCS is usually based on robust control ideas and therefore robustly handles faults/failures without requiring any information from any FDD scheme [Chen and Patton, 1999; Yang and Stoustrup, 2000]. AFTCS in general requires explicit information of the occurred faults/failures and therefore some mechanism of FDD is required. AFTCS can be divided into two sub-groups: projection type FTC and online reconfiguration/adaptation. In projection based FTC, the controller is designed for all possible faults/failures that might occur in the system. The projected controller will only be activated when certain fault/failure occurs. Projection based FTC is subdivided into three categories which are model switching or blending, scheduling and prediction. Online reconfiguration/adaptation AFTCS is based on reconfiguration (redistributing the control signals or reallocating control efforts) or adaption online. Some different FTC strategies are summarized in the following subsections.

2.1.1 Multiple Models Switching and Tuning (MMST) and Interactive Multiple Model (IMM)

The natural way of expanding linear control method to FTC is the using of multiple linear models. There are two ways to use multiple models. One is MMST [Bošković and Mehra, 1998; Gopinathan et al., 1998; Jones, 2003; Narendra and Balakrishnan, 1997; Narendra et al., 2003], the other is IMM. Multiple model schemes were initially proposed to deal with the changes in operating conditions and varying flight envelopes.

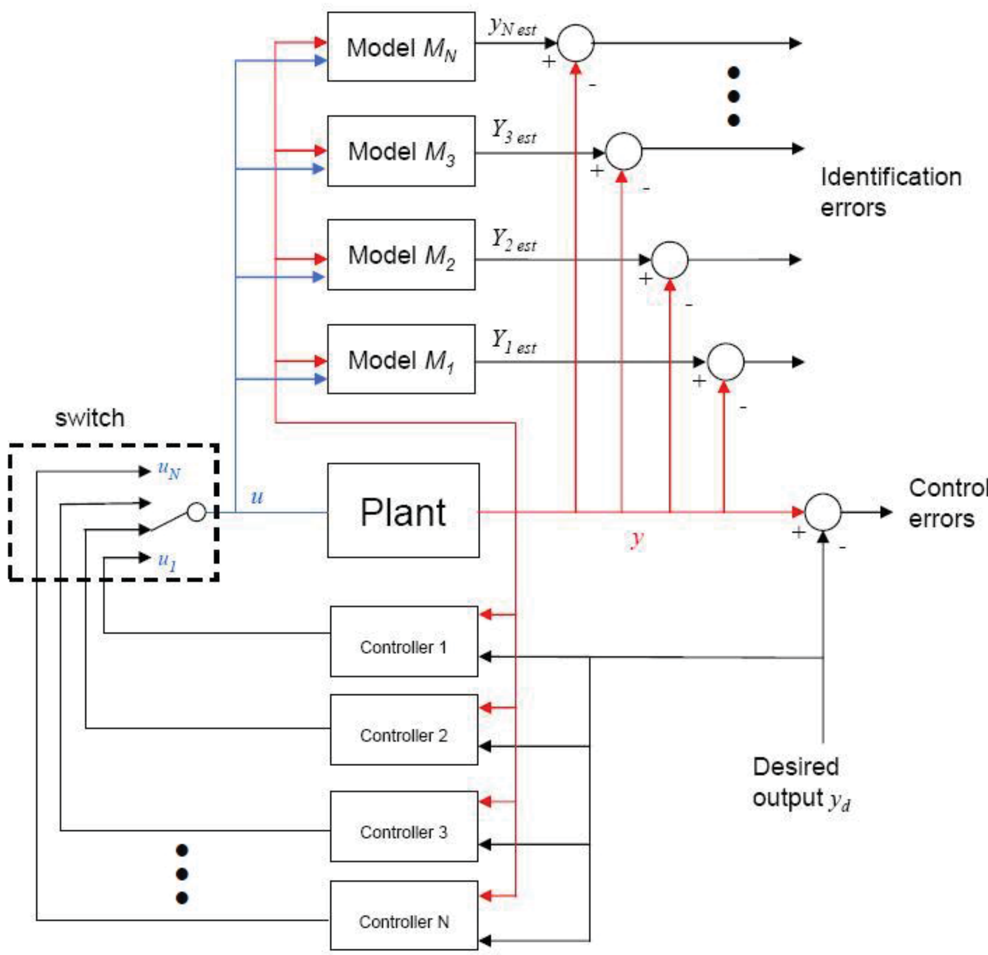

Figure 2.2: MMST control strategy (adopted from [Narendra et al., 2003])

For a chosen operating condition or a certain fault, as shown in Figure 2.2, a single model and controller will be chosen based on the error between the current system and the predesigned model in the MMST method. Although this method is based on well known linear control methods, it may be tedious to implement it. In order to deal with all possible types of faults and failures, enormous number of models and controllers are needed to be designed and tuned. The switching between models and controllers, sometimes may introduce undesired transients. Another disadvantage is that this method depends on the robustness of the FDD to identify the correct model. And it cannot deal with multiple faults/failures [Jones, 2003].

IMM method builds a few linear models based on a few carefully chosen flight conditions and design linear controllers at these selected operating conditions (or faults/failures). The estimated plant output or control input is obtained by blending the predetermined models when the operating conditions change or faults/failures occur as shown in Figure 2.3. In IMM, it is assumed that every possible flight condition including faults/failures can be modeled as a convex combination of the predetermined linear models. An IMM estimator detects and isolates the faults/failures by obtaining an estimate of the plant output from a blend of predefined linear models and provides a probability weight for the controller reconfiguration. The control signal is synthesized based on a blend of predefined controllers [Zhang and Jiang, 2001] or online control law calculations using the probability weight provided by the IMM estimator. One problem of IMM schemes is finding the right balance of blending/probability weights to get the best model match. IMM is also heavily dependent on the embedded IMM estimator based FDD scheme to correctly identify the faults/failures.

2.1.2 Gain Scheduling and Linear Parameter Varying (LPV) Approaches

Gain scheduling is a kind of 'divide and conquer' design procedure [Leith and Leithead, 2000]. It decomposes the nonlinear system into a family of linear systems and design a linear controller for each one of them. MMST and IMM are particular types of gain scheduling according to this definition. Gain scheduling means scheduling of linear models and its associated controllers either by parameters or states to deal with nonlinear control problems resulting from a change in the operating conditions and flight envelope. Gain scheduling is also based on precalculated control laws. In some flight conditions, the controller structure does not need to be changed. Only the

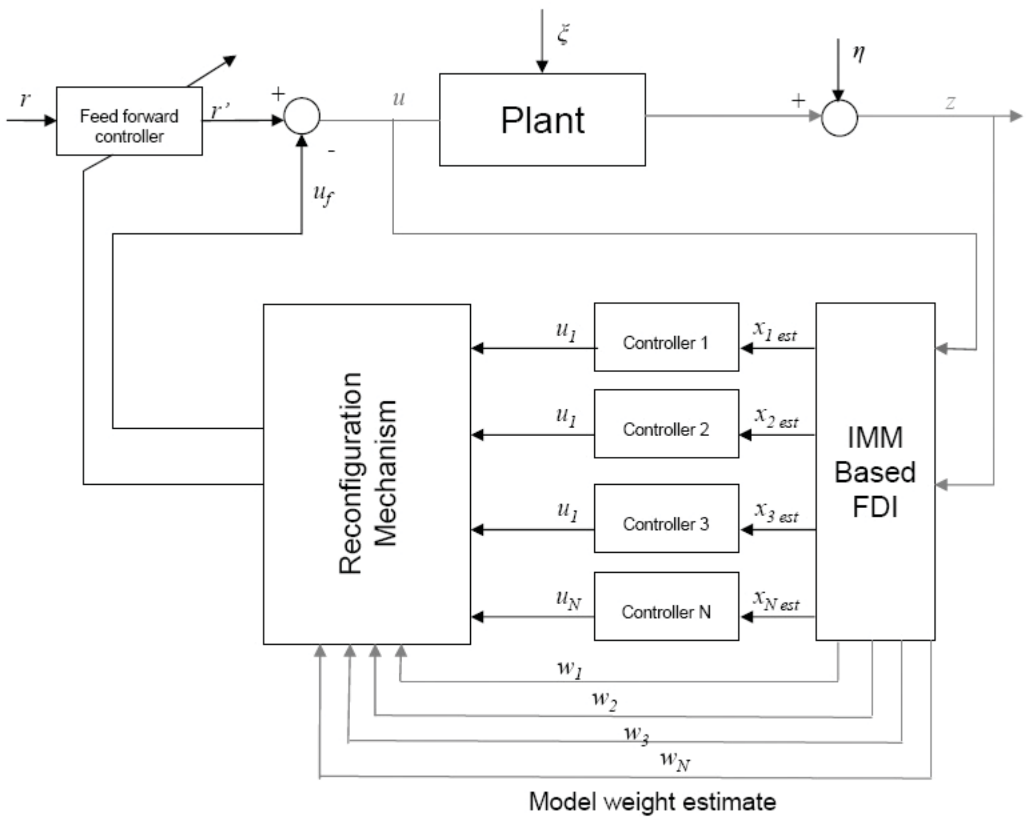

Figure 2.3: IMM control strategy (adopted from [Zhang and Jiang, 2001])

gains of the controller need to be changed according to the flight conditions or the faults/failures conditions. Predefined gains are chosen for specific flight conditions or specific parameters. This can be presented in the form of a simple logic switch between two gains, or more commonly through the use of lookup tables or curve fitting [Balas, 2002]. Gain scheduling is easily to be understood and implemented. However, when the faults/failures are significant, the structure of the nominal controllers may be incapable of coping with them. In this case, gain scheduling is insufficient and controller reconfiguration is required.

Another gain scheduling type of controller is linear parameter varying (LPV) control [Leith and Leithead, 2000]. In such a control strategy, LPV model is built as a smooth semi-linear model that varies with a parameter like altitude and/or speed. Instead of combining predefined linear models, the LPV model changes with some non-state parameters [Ganguli et al., 2002]:

$$\dot{x}(t) = A(p)x(t) + B(p)u(t) \tag{2.1}$$

$$y(t) = C(p)x(t) + D(p)u(t) \tag{2.2}$$

where p is the varying parameter e.g. speed or altitude. If p is a constant, then the LPV system becomes a linear time invariant (LTI) system [Ganguli et al., 2002]. LPV provides some guarantees of stability and performance when compared to classical gain scheduling. Controller synthesization for LPV model is unique and it is different from controller design for linear model and nonlinear model [Scherer, 2012]. Compared with linear model based methods, LPV-based controllers synthesization do not need to be designed on many models of different operation point, since LPV is a smooth continuous model instead of switch of multi-models. Some general literature on LPV are [Balas, 2002; Wu, 2001]. How to design the controller for LPV models is still a research topic [Scherer, 2012]. In the field of FTC, papers such as [Ganguli et al., 2002; Marcos et al., 2005; Rodrigues et al., 2007] represent some of the research work in this area.

Gain scheduling and LPV methods in FTC also depend heavily on reliable faulty system model from FDD.

2.1.3 Model Predictive Control (MPC)

Unlike many other control paradigms which came from the academic community, the development of predictive control/model predictive control (MPC) was initiated in the process industry. This is due to the fact that the concept and the mathematical description of MPC is easy to understand by most control engineers in industry. Therefore it is not surprise that MPC is the most widely applied method in the process control industry [Maciejowski, 2002] besides classical PID controller.

The original idea for MPC is to allow the production process to run as close as possible to the process limits without violating any of the limits, in order to maximize production and therefore profit. The main benefit of MPC is in the handling of limits and constraints. This is the main motivation for the study of MPC for flight control and especially FTC. Examples of MPC in the field of flight control and FTC can be found in [Magni et al., 1997; Maciejowski and Jones, 2003; Jones, 2003; Abdolhosseini et al., 2012]. Because of its synthesization method, MPC has the ability to handle the actuator limits by including these limits in the optimization process which is used to obtain the control signals.

Generally speaking, MPC is an iterative control algorithm based on optimal control. The iteration can be summarized as follows: at the current time, the current plant states are sampled and a cost minimizing strategy (using on-line optimal control and taking into account the system constraints) is computed for a relatively short time horizon into the future. The objective is to obtain predicted state trajectories in the future using the current states and the computed control signals. Only the first control signal from the optimization is applied to the real actuators. When new samples of system states are obtained, the calculations of the next controls are repeated. MPC is also known as receding horizon control [Maciejowski, 2002; Magni et al., 1997]. Figure 2.4 is the structure of MPC.

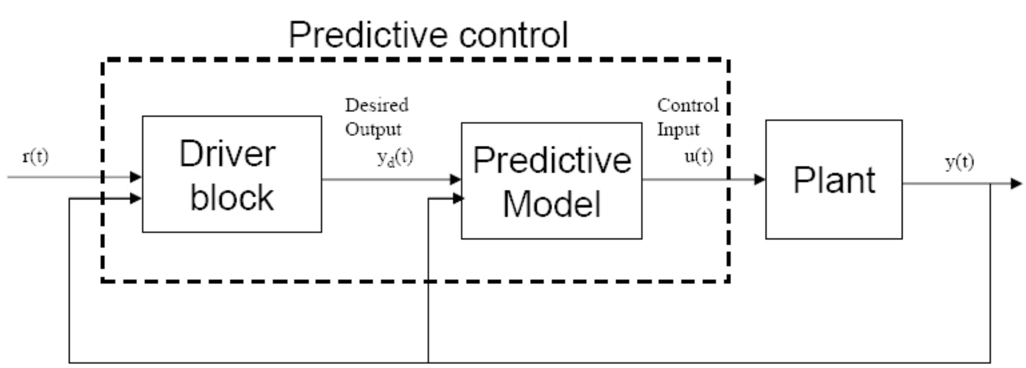

Figure 2.4: MPC control strategy (adopted from [Sánche and Rodellar, 1996])

The driver block/reference model generates the desired output based on the physical feasibility and desired dynamics. The predictive model block generates the control signals that force the output of the plant to follow the desired outputs using previous inputs and outputs of the plant [Sánche and Rodellar, 1996]. The optimization can be solved using quadratic programming or fast linear programming algorithms [Maciejowski, 2002]. Surely, MPC method in FTC also needs fault information from FDD in the optimization.

2.1.4 Adaptive FTC

One way of dealing with changes in the system (such as load variation, disturbance, accident events like faults/failures) is adaptive mechanism. Motivated by the design of autopilots for high performance aircraft in the 1950s, adaptive control was proposed as a way of dealing with a wide range of flight conditions [Slotine and Li, 1991]. Adaptive control is used in order to automatically adjust the controller parameters to keep the desired performance when the system changes in parameters or structure.

Adaptive control theory shows that it is efficient, stable and even robust for systems with slow varying parameters [Narendra and Annaswamy, 1989; Slotine and Li, 1991; Ioannou and Sun, 1995]. These assumptions of slow varying parameters are usually not met by the systems under the influence of faults and failures, which typically have a nonlinear behavior with sudden parameter or structure changes. So adaptive control alone does not have the capability to accommodate faults and failures.

In [Tao et al., 2004] adaptive control is studied for the systems with stuck actuator failure. In stuck actuator failure case, the actuator gets stuck on some fixed position which can be seen as some fixed unknown parameters that can be estimated online with adaptation mechanism. This is a method that parameterizes some fixed or slowly varying parameters. Faults and failures are random events which may occur abruptly, at unexpected location and without knowing which kind of fault or failure it is, i.e. how to know it is a stuck failure is a problem before the adaptive mechanism is activated. Even it is stuck failure, the system may go unstable before the adaptive mechanism is in effect. This means the adaptive mechanism has the opportunity to be in effect only if the normal controller can still stabilize the system before the adaptive mechanism can estimate the stuck failure. Another problem of this method is that it is designed for a fixed system structure, e.g., if it is not a stuck but a floating failure or partial loss fault, this method will not work.

Combined with sliding mode control (SMC), adaptive control is studied extensively in control with less *a priori* knowledge of model. In [Wheeler et al., 1998; Stepanenko et al., 1998], the uncertainty bounds are parameterized. The actuator effectiveness is transformed to uncertainty in [Shin et al., 2005] and the adaptive method is used to estimate the bound of this uncertainty. The gain of discontinuous control part in SMC is parameterized in [Alwi and Edwards, 2005, 2008a; Alwi et al., 2008]. Some SMC-based schemes have been proposed within MRAC (Model Reference Adaptive Control) frame such as [Leung et al., 1991; Chou and Cheng, 2003; Costa and Hsu, 1990; Hsu and Costa, 1989; Hsu et al., 1994, 1997, 2006; Alwi and Edwards, 2007b; Alwi et al., 2008]. These methods will be discussed in detail in the sliding mode fault tolerant control section.

2.1.5 Control Signal Redistribution

When faults and failures occur in actuators, one of the possible feature of FTC is to redistribute the control efforts to make them still can shape the system in the desired way. There are several ways of control signal redistribution: pseudo inverse (PIM) method [Gao and Antsaklis, 1989, 1991], control allocation [Bordignon, 1996; Hamayun et al., 2012] and dynamic inversion [Enns et al., 1994].

Pseudo Inverse Method (PIM)

Pseudo inverse is the minimum length solution of least squares problem of matrix [Lawson and Hanson, 1974]. The pseudo inverse method in FTC is to place the poles of the faulty system as close as possible to the nominal closed-loop poles. The following derivation gives insight into the pseudo inverse method. Consider a linear system given by

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{2.3}$$

where the state vector $x(t) \in R^n$, the control vector $u(t) \in R^m$, state matrix A has dimension $n \times n$ and input matrix B has dimension $n \times m$. Assume that a state feedback gain K of dimension $m \times n$ has been designed, and the control law is defined as

$$u(t) = Kx(t) \tag{2.4}$$

and therefore the closed-loop system is given by

$$\dot{x}(t) = (A + BK)x(t) \tag{2.5}$$

During faults/failures, the closed-loop faulty system can be represented by

$$\dot{x}_f(t) = (A_f + B_f K_f)x_f(t) \tag{2.6}$$

where $x_f \in R^n$ is the state vector of faulty system; A_f and B_f are the state matrix and input matrix of the faulty system. The idea is to obtain a K_f so that the faulty system closed-loop performance will be as close as possible to the nominal one (2.5). Since the objective is to obtain $x_f(t) = x(t)$, a necessary condition is to ensure

$$(A + BK) = (A_f + B_f K_f)$$

and therefore

$$K_f = B_f^{\dagger}(A - A_f + BK) \tag{2.7}$$

where $B_f^{\dagger}$ is the pseudo inverse of B_f. The plant matrices A and B and the gain K is assumed to be known a priori. The faulty system (A_f, B_f) can be obtained from FDD. So in theory, K_f can be obtained from (2.7). For a non-square B_f matrix, the pseudo inverse of B_f provides some degrees of freedom which can be used to redistribute the control effort to keep the desired performance [McLean and Aslam-Mir, 1991; Patton, 1997].

The main drawback of PIM is the theoretical analysis of its stability [Huzmezan and Maciejowski, 1997; Jones, 2003; Patton, 1997; Yang et al., 2007]. Other drawbacks are the assumption that the state measurements are always available [Patton, 1997; Yang et al., 2007] and lack of robustness [Yang et al., 2007].

Control Allocation (CA)

In most safety-critical systems such as aircraft system, there is actuator redundancy. This allows freedom to design fault tolerant control systems to maintain stability and acceptable performance when faults occur. When some control surfaces lose their effectiveness completely or the actuators saturate to the extent that stability cannot be attained, the control efforts must be reallocated to redundant actuators.

When all actuators have the same physical characteristics, for example they are segments of a multi-segment elevator or rudder for an aircraft, a reasonable design for the applied control inputs is the one with equal or proportional actuation for each actuator [Tao et al., 2004]. This is not the case all the time. Some redundant actuators have different dynamics, e.g. the stabilizer can be the redundant actuator for elevator. The natural way for control allocation is to initiate "back-up" controller using redundant actuator when regular actuator is found completely lost its effectiveness [Alwi and Edwards, 2006].

Early ideas of control allocation are discussed in [Patton, 1997]. In its early development, the idea of redistributing the control signals to the remaining healthy actuators was called 'restructuring' [Patton, 1997]. An early example is given in [Huber and McCulloch, 1984], where a 'restructuring controller' utilizing a 'control mixer concept' is used to redistribute the control signals. Control allocation attracts more and more interest in the FTC community partly because of the development of high performance, highly redundant aircraft [Bošković and Mehra, 2002; Buffington, 1997; Shtessel et al., 1999; Wells and Hess, 2003] and improvements in computational power (which is necessary in order to solve on-line optimization problems) [Beck, 2002; Bordignon and Durham, 1995; Durham, 1993; Enns, 1998].

PIM and CA seem to be identical since both employ a pseudo inverse which provides some design freedom, the major difference between CA and PIM is that in CA, the controller is designed based on a 'virtual control' signal and the CA will map the virtual control to the actual control demand to the actuators. The benefit here is that the controller design is independent of the CA unit: the virtual control is synthesized firstly and CA distributes the control signal into actuators. Papers such as [Härkegård and Glad, 2005; Shin et al., 2005; Zhang et al., 2007; Benosman et al., 2009; Alwi and Edwards, 2010; Zhou et al., 2010b,a; Hamayun et al., 2012] are some of the recent works in this area.

CA is based on separating the control law from the control allocation task (see Figure 2.5). This is done by designing a controller to provide a 'virtual control' which is mapped to the actual control signals sent to the actuators. Consider an overactuated system such as a passenger aircraft [Brière and Traverse, 1993] or modern fighter aircraft [Forssell and Nilsson, 2005] represented by a linear system

$$\dot{x}(t) = Ax(t) + B_u u(t) \tag{2.8}$$

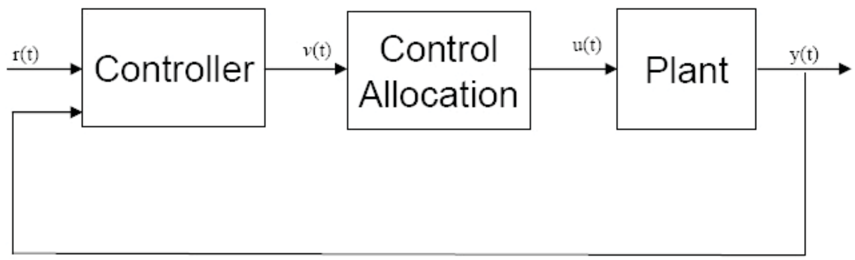

Figure 2.5: Control allocation strategy (adopted from [Jones, 2003])

where the state vector $x(t) \in R^n$, the control vector $u(t) \in R^m$, state matrix A has dimension $n \times n$ and input matrix B_u has dimension $n \times m$. B_u is assumed can be factorized such that

$$B_u = B_\nu B$$

Therefore, the linear system in (2.8) becomes

$$\dot{x}(t) = Ax(t) + B_\nu \nu(t)$$

where $\nu(t)$ is the 'virtual control' defined by

$$\nu(t) := Bu(t)$$

For a given $\nu(t)$ the control signal $u(t)$ is recovered as

$$u(t) = B^\dagger \nu(t)$$

where $B^\dagger = WB^T(BWB^T)^{-1}$ is a right pseudo inverse of B. The weight matrix W represents the design freedom which distributes the control signals to actuators according to the different contribution of each individual actuator.

In most of the literature, the weight $W = I$ [Shin et al., 2005] (i.e. equal control signal distribution among all actuators) is typically chosen. In some cases (such as finding the control signal distribution that reduces drag and fuel consumption), a different choice of weighting matrix W can be employed. In a constrained optimization problem, the weight W can be chosen to achieve the desired performance taking into consideration of actuator constraints [Enns, 1998].

The works in [Buffington et al., 1999; Davidson et al., 2001; Zhang et al., 2007] use CA as a means for FTC. The benefits of CA is that the controller structure needs not to be redesigned in the case of faults and it can deal directly with total actuator failures by automatically redistributing the control signals among the regular and redundant actuators. As in MPC, another major benefit of CA is that actuator limitations can be handled by including the actuator constraints in the optimization process.

One of the drawbacks of CA is that, for linear systems, the pure factorization $B_u = B_\nu B$ is a very strong requirement and therefore approximations $B_u \approx B_\nu B$ have been made [Buffington et al., 1999; Davidson et al., 2001; Härkegård and Glad, 2005; Hess and Wells, 2003]. Another drawback is online optimization like linear or quadratic programming is required. This is difficult even with nowadays high computational power computer to the optimization online and in real time.

Dynamic Inversion (DI)

DI has the ability to handle changes of operating condition naturally due to the modeling in the whole operating range. This capability has motivated researchers to consider DI for control of system with wide operating conditions like the space re-entry vehicle which flies from supersonic speed during re-entry and subsonic regions during the glide back to the runway [Ito et al., 2001, 2002].

The idea of DI can be shown by considering the following affine nonlinear system

$$\dot{x}(t) = f(x,t) + G(x,t)u(t)$$

where the state vector $x(t) \in R^n$, the control vector $u(t) \in R^m$, $f(x,t) \in R^n$, and $G(x,t) \in R^{n\times m}$; further, each entry in $f(x,t)$ and $G(x,t)$ is assumed to be continuous with continuous bounded derivative with respect to $x(t)$; $G(x,t) \neq 0\ \forall x$. By rearranging the equation with respect to $u(t)$, as in [Tandale and Valasek, 2005], the control law can be represented by

$$u(t) = G(x,t)^{-1}(\dot{x}_d(t) - f(x,t))$$

where $\dot{x}_d$ is the predetermined desired closed-loop reference demand. In [Ito et al., 2002], dynamic inversion is described as '... a control synthesis technique by which existing deficient, or undesirable dynamics are canceled and replaced by desirable dynamics. Cancelation and replacement are achieved through careful algebraic selection of the feedback function. For this reason, it is also called feedback linearization ...'.

Since a continuous nonlinear model which cover almost all the system operating range is used in DI, a fixed controller can be synthesized without model switching and gain scheduling. If the control input matrix $G(x,t)$ is precise, control allocation can be implemented naturally without an extra mechanism [Joosten et al., 2007].

DI requires a perfect model of the system dynamics, which is not realistic in practice. In [Ito et al., 2002] robust control methods such as H_∞ is used as outer loop control to minimize or suppress undesired behavior due to plant uncertainties which cause imperfect plant dynamic cancelation. Anyway, DI requires a deep understanding and knowledge of the plant in order to be able to cancel the plant dynamics perfectly. In reality, this is quite impractical.

Another drawback of dynamic inversion is the assumption of full-state feedback which is not an issue in modern aircraft, civil [Brière and Traverse, 1993] or advanced military aircraft [Forssell and Nilsson, 2005], but full state measurement is not always available for many other systems.

In [Fisher, 2004; Idan et al., 2001; Ito et al., 2002, 2001; Joosten et al., 2007] dynamic inversion is utilized in the implementation of FTC. Because of a perfect system dynamics model is required in the implementation of DI, for FTC it is the requirements of a perfect model of the faulty system, i.e. it depends heavily on the FDD mechanism [Lombaerts et al., 2007].

2.1.6 Robust Control (H_∞ Control)

Since there are always disturbances and uncertainties in the controlled system, robustness to disturbances and uncertainties is always the major concern in feedback control [Zhou and Doyle, 1999]. Robust control is a control method that makes trade-off between performance and robustness. H_∞ as the most developed robust control method has been researched and developed in many applications ranging from industrial process control to aircraft control problems, and it is robust control always referred to. Since partial loss fault can be seen as a kind of uncertainty, robust control method can be used to deal with it. FTC using robust control method doesn't require to get information of faults online and therefore works in normal situation as well as in faulty conditions. This is why it is called passive fault tolerant control in the literature. The capability to deal with faults depends on the predesigned controller which is based on minimizing the effect of uncertainty or disturbances on the system (robustness) [Magni et al., 1997], i.e. how much fault can be tolerated is predesigned. The design of H_∞ control is separated in two steps. The first step is to decide the type and structure of the uncertainty to be considered which is difficult and requires some insight into the plant [Magni et al., 1997]. The second step is to choose frequency dependent weights based on some performance specifications and then to solve an optimization problem. H_∞ mixed sensitivity, μ-synthesis and H_∞ loop shaping [Skogestad and Postlethewaite, 1996] are some of the mostly studied H_∞ controller design techniques.

One of the disadvantages of H_∞ is the controller is conservative in the normal conditions in order to guarantee the stability in the event of partial loss faults [Magni et al., 1997], and the performance in the normal condition is sacrificed for robustness. So H_∞ robust control can only tolerate the prescribed faults by sacrificing performance in the normal situation. Another drawback is that the final controller is usually of a higher order than the system. In the practice, model reduction is required to truncate the order of the controller [Magni et al., 1997] to make it implementable. The literatures [Marcos et al., 2005; Magni et al., 1997] describe some of the research results of H_∞ control in flight FTC.

Another kind of robust control method is SMC, which will be discussed in the next section.

Though in general, FTC is categorized into AFTC and PFTC, in the academia it is mostly referred to AFTC while PFTC is considered as robust control [Zhang and Jiang, 2008]. However, because of the delay of FDD and synthesization of reconfigurable control, which will not be a trivial time, the faulty system is posed in a situation during the period of this delay in which it is a system with fault and failure while it is controlled by the normal controller which surely is the 'wrong' controller. Furthermore, the system may lose its stability and disable the FDD mechanism. So in the normal controller there must be some control mechanism that can accommodate fault and failure before FDD can detect and identify the fault and failure and a better reconfigurable controller can be synthesized online.

2.2 Sliding Mode Control (SMC)

Originating from the 1950s in Union of Soviet Socialist Republics (USSR), sliding mode control has developed into a topic of great interest in control theory and practice in many applications. There are tons of publications dedicated to it, [Utkin, 1977; Young, 1978; Decarlo et al., 1988; Hung et al., 1993; Young et al., 1999] as some reviews and tutorials, [Gao, 1990; Slotine and Li, 1991; Utkin, 1992; Edwards and Spurgeon, 1998] as several books and [Perruquetti and Barbot, 2002; Liu, 2005; Edwards et al., 2006; Bartolini et al., 2008; Boiko, 2009; Bandyopadhyay et al., 2009; Fridman et al., 2011] as some books with recent advances in this field.

SMC is a nonlinear type of control strategy and is a special variable structure control (VSC). The design of SMC is unique compared to other strategies. The design is separated into two steps: first, a sliding surface is designed to assign the performance of the closed-loop system; second, the control law is designed to force the trajectory of the states towards the sliding surface and once reached, the states are forced to remain on the surface [Utkin, 1977].

SMC is a robust control methodology, and it is invariant to matched uncertainties which belongs to the range of the control input distribution matrix [Utkin, 1992; Edwards and Spurgeon, 1998]. With dynamic sliding mode [Shtessel, 1997] or high order sliding mode [Levant, 2001] or combined with backstepping approach [Khalil, 1992], even unmatched uncertainties can be tolerated in SMC. This robustness property of SMC, called invariance [Utkin, 1992], comes from the high-speed switching function that forces the state trajectory approaching the sliding surface and keeps on it.

The invariance property makes SMC a strong candidate for FTC when handling actuator faults. Because of the unique design, SMC can accommodate significant uncertainties without losing greatly of performance as other robust control methods such as H_∞. Moreover, since the system character is determined by the chosen sliding surface, this gives more freedom in the design of SMC which makes it easily of combining other methods and resorting to less *a priori* knowledge. The paper [Hess and Wells, 2003] argues that SMC has the potential to become an alternative to reconfigurable control and has the ability to maintain the required performance without requiring an FDD.

In this section, the basic concepts and principles of SMC are introduced. The SMC design with feedback linearization on SISO and MIMO system is introduced in the next section. Then the application of SMC in FTC is reviewed in the following section.

2.2.1 Brief Introduction to SMC

Consider affine nonlinear system, i.e., system has a state space model nonlinear in the the state vector and linear in the control vector of the form:

$$\dot{x}(t) = f(x,t) + G(x,t)u(t) \tag{2.9}$$

where the state vector $x(t) \in R^n$, the control vector $u(t) \in R^m$, $f(x,t) \in R^n$, and $G(x,t) \in R^{n\times m}$; further, each entry in $f(x,t)$ and $G(x,t)$ is assumed to be continuous with continuous bounded derivative with respect to $x(t)$.

In SMC each entry $u_i(t)$ of the switched control $u(t) \in R^m$ has the form

$$u_i(x,t) = \begin{cases} u_i^+(x,t) & with \quad s_i(x) > 0 \\ u_i^-(x,t) & with \quad s_i(x) < 0 \end{cases} \quad \forall i = 1, 2, \cdots, m \tag{2.10}$$

where $s_i(x) = 0$ is the ith sliding surface associated with the $(n-m)$-dimensional sliding surface

$$s(x) = [s_1(x), s_2(x), \cdots, s_m(x)]^T = 0 \tag{2.11}$$

The design of SMC is in two stages. First is the design of the sliding surface and second is the design of the control law that sliding mode is achieved and then maintained on the surface. Once the states are in sliding mode, the closed-loop system is robust to matched uncertainties and behaves as a reduced-order system with motion independent of the control. Matched uncertainty is the uncertainty within the range space of the input matrix $G(x,t)$. Consider uncertainty and disturbance in Eq.(2.9):

$$\dot{x} = f(x,t) + \Delta f(x,t) + G(x,t)u + d(t) \tag{2.12}$$

where $\Delta f(x,t)$ represents the modeling error and $d(t)$ the external disturbance. The matched uncertainty means there exist $\Delta\tilde{f}(x,t)$ and $\tilde{d}(t)$ such that

$$\Delta f(x,t) = G(x,t)\Delta\tilde{f}(x,t), \;\; d(t) = G(x,t)\tilde{d}(t) \tag{2.13}$$

The performance of the controlled system depends on the choice of the sliding surface. Typically, SMC consists of continuous and discontinuous components. The discontinuous component is designed to drive the states towards the sliding surface under modeling uncertainty and disturbance, and so it determines the robustness of SMC system. Once on the surface, the continuous component becomes more dominant than the discontinuous one and drives the system to the steady state.

2.2.2 Sliding Surface

The sliding surface

$$s(x) = 0 \tag{2.14}$$

is a $(n-m)$-dimensional manifold in R^n determined by the intersection of $m(n-1)$-dimensional sliding surfaces $s_i(x)$. The sliding surfaces are designed such that system response has a desired stability or tracking characteristics.

Mostly, for convenience and simplicity, linear sliding surface are prevalent, while nonlinear ones are possible, for example, in [Bandyopadhyay et al., 2009] nonlinear sliding surface is designed such that it changes the system's closed-loop damping ratio from its initial low value to a final high value. Initially, the system is lightly damped resulting in a quick response and as the system output approaches the set point, the system is made overdamped to avoid overshoot. In this method, the system behavior is fine tuned thanks to the uniqueness of SMC which separates the design of control law and the design of system performance. In this thesis linear sliding surface as following is used.

$$s(x) = Sx(t) \tag{2.15}$$

where S is an $m \times n$ matrix.

2.2.3 Regular Form and Reduced-Order Dynamics

Assuming the system Eq.(2.9) is completely linearizable in a reasonable domain, the system Eq.(2.9) can be transformed to a regular format [Slotine and Li, 1991]

$$\begin{aligned} \dot{x}_1 &= f_1(x,t) \\ \dot{x}_2 &= f_2(x,t) + G_2(x,t)u \end{aligned} \tag{2.16}$$

where $x_1 \in R^{n-m}$ and $x_2 \in R^m$; $G_2(x,t)$ is $m \times m$ nonsingular mapping. Assume a linear sliding surface of the form

$$s(x) = [S_1 \ S_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{2.17}$$

i.e.

$$S_1 x_1 + S_2 x_2 = 0$$

where without loss of generality S_2 is assumed to be nonsingular. So in a sliding mode

$$x_2 = -S_2^{-1} S_1 x_1 \tag{2.18}$$

and

$$\dot{x}_1 = f_1(x,t) = f_1(x_1, -S_2^{-1} S_1 x_1, t) \tag{2.19}$$

is the reduced-order dynamics which represents the performance of the closed-loop system. The design of the sliding surface $s(x)$ is the choice of S_1 and S_2 to make the reduced-order dynamics meets the desired performance.

2.2.4 Reachability Condition

The SMC control law is not designed to directly meet some desired closed-loop system performance, but to ensure the sliding surface is reached and motion on sliding mode is maintained which is called *reachability condition*. The reachability condition means the trajectory of the system states must always point towards the sliding surface. In the case of single input system, it is

$$\begin{aligned} \lim_{s \to 0^+} \dot{s} &< 0 \\ \lim_{s \to 0^-} \dot{s} &> 0 \end{aligned} \tag{2.20}$$

or in a compact method

$$s\dot{s} < 0 \tag{2.21}$$

around $s(t) = 0$. A more strict reachability condition that ensures the sliding surface is reached despite the presence of uncertainty and in finite time is given by

$$s\dot{s} \leq -\eta |s| \tag{2.22}$$

where η is a small positive design scalar. Eq.(2.22) is called 'η-reachability condition' [Slotine and Li, 1991].

2.2.5 Chattering

Invariance of SMC comes from infinite frequency switching of discontinuous finite control action instead of infinite high gains in the classical continuous control [Utkin, 1992]. However, infinite frequency switching is impractical as well as infinite high gain. In the practical mechanical or electrical system, there is always delay in the actuator, i.e. the switching of control is always in finite frequency. So SMC suffers from chattering in its originality. Figure 2.6 shows how chattering is caused by delay. A trajectory of state in the region $s > 0$ is heading toward the sliding manifold $s = 0$. It meets the manifold firstly at point a. In ideal SMC, the trajectory should start sliding on the manifold from this point a. In the real system, the delay between the changing of control signal and the sign change of the s causes the trajectory crosses the manifold into the region $s < 0$. When the control switches, the trajectory reverses its direction and goes again towards the manifold. Once again it crosses the manifold, and repeat of this process creates the oscillation shown in the figure, which is known as chattering. Chattering will decrease the control accuracy, lead to high heat losses in electrical power circuit, and wear the moving mechanical parts greatly. It also may excite unmodeled high-frequency dynamics, which degrades the performance of the system and may even lead to instability.

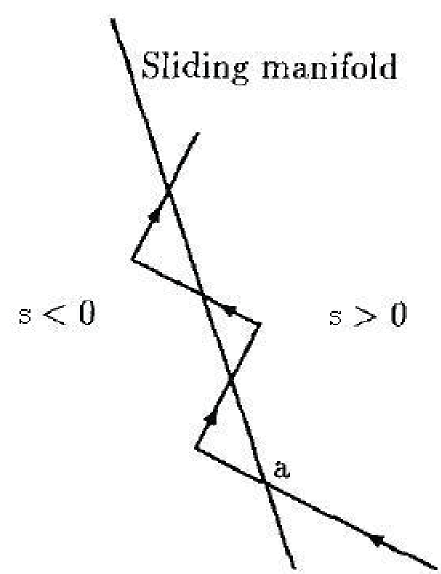

Figure 2.6: Chattering due to delay in control switching

Sliding mode system is equivalent to relay feedback system [Boiko and Fridman, 2005] and chattering can be considered as a limit cycle or as the existence of a fixed point of the Poincaré map [Boiko and Fridman, 2005]. Since chattering is unavoidable, controlling the magnitude of the limit cycle is one solution to make SMC practical in real system [Wang, 1990]. One way of attenuating the chattering magnitude is to use internal model, i.e. to use the *a priori* known system dynamics in the closed-loop control system such that the uncertainty can be decreased. Feedback linearization is such a method in nonlinear control field, which is the turn point work that makes SMC a practical control method [Slotine, 1984].

2.2.6 SMC Design with Feedback Linearization

Single Input Single Output (SISO) System

Consider a SISO nonlinear affine system:

$$\begin{cases} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{cases} \tag{2.23}$$

where $f(x)$ and $g(x)$ are smooth vector fields, and $h(x)$ is differentiable function with relative degree of n, $x \in R^n$ is the state vector, $y \in R$ is the output and the scalar u is the control input. The nonlinear system Eq.(2.23) can be transformed into following companion form by using feedback linearization technique [Utkin, 1977] as:

$$\begin{cases} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ &\vdots \\ \dot{z}_n &= A(x) + B(x)u \\ y &= z_1 \end{cases} \tag{2.24}$$

where

$$A(x) = L_f^n h(x), B(x) = L_g L_f^{n-1} h(x) \tag{2.25}$$

are the Lie derivatives of corresponding functions.

Define switching surface as:

$$s = e^{(n-1)} + \lambda_{n-2} e^{(n-2)} + \cdots + \lambda_1 \dot{e} + \lambda_0 e \tag{2.26}$$

where

$$e = y_d(t) - y(t) \tag{2.27}$$

y_d is the output reference profile, $\lambda_0 > 0, \lambda_1 > 0, \cdots, \lambda_{n-2} > 0$ are design coefficients chosen to provide desired sliding mode dynamics:

$$s = 0 \tag{2.28}$$

Derive this function we have:

$$\dot{s} = y_d^{(n)}(t) - A(x) - B(x)u + \lambda_{n-2}e^{(n-1)} + \cdot + \lambda_1 \ddot{e} + \lambda_0 \dot{e}$$

Choose Lyapunov function as:

$$V = \frac{1}{2}s^2 \tag{2.29}$$

Design the controller as:

$$u = B(x)^{-1}[y_d^{(n)}(t) - A(x) + \lambda_{n-2}e^{(n-1)} + \cdot + \lambda_1 \ddot{e} + \lambda_0 \dot{e} + \rho \text{sign}(s)] \tag{2.30}$$

The derivative of the Lyapunov function is:

$$\dot{V} = -\rho|s| \leq 0$$

According to the invariant set theory [Slotine and Li, 1991], the system converges to the origin.

Multi Input Multi Output (MIMO) System

This section extends the result of last section to MIMO system. Consider affine nonlinear system

$$\begin{cases} \dot{x} &= f(x) + G(x)u \\ y &= h(x) \end{cases} \tag{2.31}$$

where $x \in R^n$ is the state vector, $y \in R^m$ is the output vector, $u \in R^m$ is the control vector; $f(x) \in R^n$, $h(x) \in R^m$ are differentiable vector-functions, $G(x) \in R^{n \times m}$ is the control input distribution matrix:

$$G(x) = [g_1(x), g_2(x), \cdots, g_m(x)]$$

where $g_i(x) \in R^n (i = 1, 2, \cdots, m)$ are differentiable vector-functions. Assuming that the system is completely linearizable in a reasonable domain $x \in \Gamma$. The Eq. (2.31) can then be transformed into a companion form [Utkin, 1977] as:

$$\begin{bmatrix} y_1^{(r_1)} \\ y_2^{(r_2)} \\ \vdots \\ y_m^{(r_m)} \end{bmatrix} = \begin{bmatrix} L_f^{r_1} h_1(x) \\ L_f^{r_2} h_2(x) \\ \vdots \\ L_f^{r_m} h_m(x) \end{bmatrix} + E(x)u \tag{2.32}$$

where

$$E(x) = \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_1 & L_{g_2} L_f^{r_1-1} h_1 & \cdots & L_{g_m} L_f^{r_1-1} h_1 \\ L_{g_1} L_f^{r_2-1} h_2 & L_{g_2} L_f^{r_2-1} h_2 & \cdots & L_{g_m} L_f^{r_2-1} h_2 \\ \vdots & \vdots & \vdots & \vdots \\ L_{g_1} L_f^{r_m-1} h_m & L_{g_2} L_f^{r_m-1} h_m & \cdots & L_{g_m} L_f^{r_m-1} h_m \end{bmatrix} \tag{2.33}$$

$|E(x)| \neq 0 \ \forall \ x \in \Gamma$; $L_f^{r_i}$ and $L_{g_i} L_f^{r_i-1} h_i$ ($\forall \ i = 1, 2, \cdots, m$) are corresponding Lie derivatives.

Define sliding surface as:

$$s_i = e_i^{(r_i-1)} + \lambda_{i(r_i-2)} e_i^{(r_i-2)} + \cdots + \lambda_{i1} \dot{e}_i + \lambda_{i0} e_i, \ \forall \ i = 1, 2, \cdots, m \tag{2.34}$$

where

$$e_i = y_{d_i}(t) - y_i(t), \ e_i^{(j)} = \frac{d^j e_i}{dt^j}, \ \forall \ i = 1, 2, \cdots, m$$

$y_{d_i}(t)$ is the output reference, $\lambda_{i0} > 0, \ \lambda_{i1} > 0, \ \cdots, \ \lambda_{i(r_i-2)} > 0, \ \forall \ i = 1, 2, \cdots, m$ are design coefficients chosen to provide desired sliding mode dynamics:

$$s_i = 0, \ \forall \ i = 1, 2, \cdots, m \tag{2.35}$$

The dynamics of the system in s-subspace is derived as following:

$$\dot{s} = \Psi(x, t) - E(x)u \tag{2.36}$$

where

$$\Psi(x, t) = [\psi_1(x, t), \psi_2(x, t), \cdots, \psi_m(x, t)]^T \tag{2.37}$$

$$\psi_i(x, t) = y_{d_i}^{(r_i)} + \lambda_{i(r_i-2)} e_i^{(r_i-1)} + \cdots + \lambda_{i0} \dot{e}_i - L_f^{r_i} h_i(x), \ \forall \ i = 1, 2, \cdots, m \tag{2.38}$$

For convenience of derivation, define a new control output:

$$\mu = E(x)u \tag{2.39}$$

Eq.(2.36) can then be rewritten in scalar format as follows:

$$\dot{s}_i = \psi_i(x,t) - \mu_i, \ \forall \ i = 1, 2, \cdots, m \tag{2.40}$$

Choose a Lyapunov function as:

$$V = \frac{1}{2} s^T s = \frac{1}{2} \sum_{i=1}^{m} s_i^2 \tag{2.41}$$

Following theorem can be obtained.

Theorem 2.1 [Slotine and Li, 1991] *For the nonlinear system Eq.*(2.31)*, sliding surface Eq.*(2.34) *is asymptotically stable by employing the following feedback control:*

$$u = E(x)^{-1}(\Psi(x,t) + R \cdot \Sigma) \tag{2.42}$$

where

$$R = diag\{\rho_1, \rho_2, \cdots, \rho_m\}$$

$$\Sigma = [sign(s_1), sign(s_2), \cdots, sign(s_m)]^T, \ \forall \ i = 1, 2, \cdots, m$$

$\{\rho_i\}$ *are positive values representing the gains of the discontinuous control terms.*

Proof: Taking the derivative of the Lyapunov function Eq.(2.41) w.r.t. time, one can obtain following result:

$$\dot{V} = s^T \dot{s} = s^T[\Psi(x,t) - E(x)u] = -s^T R \Sigma = -\sum_{i=1}^{m} \rho_i |s_i| \leq 0$$

With invariant set theorem [Slotine and Li, 1991], Theorem 2.1 is proved. ■

2.3 Sliding Mode Fault Tolerant Control

These years, some researchers began to use sliding mode method on aircraft fault tolerant control [Shtessel et al., 1998; Alwi and Edwards, 2005; Hess and Wells, 2003]. From the control structure, sliding mode fault tolerant control can be considered as passive fault tolerant control since it does not need a unique block in the control system to do the collection of faults information [Zhang and Jiang, 2008]. It is a kind of robust control in this sense. But in contrast to regular robust control, which synthesizes the controller on some fixed performance index and does trade-off between performance and robustness to the uncertainty of the system dynamics, it can accommodate significant uncertainties without causing great degradation in performance. Combined with adaptive control, SMC can be synthesized with little or even no knowledge of uncertainties and faults [Hsu and Costa, 1989; Costa and Hsu, 1990; Leung et al., 1991; Hsu et al., 1994, 1997; Fisher, 2004; Tao et al., 2004; Alwi and Edwards, 2005; Hsu et al., 2006]. Despite the robustness in handling uncertainties due to actuator faults, SMC is a 'fixed' or 'unreconfigurable' or passive method for FTC, itself alone can only accommodate partial loss faults to some extents and cannot accommodate total failures and severe partial loss faults which saturates the actuator to the extent that the SMC cannot stabilize the system due to its position and rate limits.

Most of the FTC based on SMC is designed for partial loss of effectiveness in the actuators. Shtessel presented a decentralized pure SMC for aircraft in [Shtessel and Tournes, 1995] and developed finite-reaching-time continuous SMC [Shtessel and Buffington, 1998a,b]. These are the basics of SMC scheme of the following works of Shtessel on FTC dealing with partial loss control surface faults. A special continuous power function is used instead of discontinuous control when the states cross the switching manifold to smooth the discontinuous control and thus eliminate the chattering. In [Shtessel et al., 1998, 1999, 2000, 2002] multiple time scale reconfigurable sliding mode control concept, which partitions the system in different time scales, is presented. The control was synthesized based on the idea of backstepping control. In the most inner loop the discontinuous control is replaced with boundary layer continuous approximation. The boundary layer is chosen considering the integrator windup, actuator deflection limit and deflection rate limit. Though this method considered the position and rate limit of actuators, it cannot accommodate severe partial loss faults. This is because of the limit of position and rate in actuator deflection, SMC can only accommodate partial loss to some extent, with sacrifice performance of airplane in normal healthy condition without faults. In [Shtessel, 1995, 1997; Shtessel and Shkolnikov, 2003] a dynamic sliding mode control method that can not only accommodate matched uncertainties but also accommodate the unmatched uncertainties, is presented. All the methods can only deal with some kinds of partial loss faults.

In [Wells, 2002; Hess and Wells, 2003; Vetter et al., 2003] asymptotic observers are used to eliminate the effect of the actuator parasitic dynamics, i.e., high-frequency dynamics often neglected in control system design. SMC systems are vulnerable to the effect of these parasitic dynamics due to the chattering caused by high speed switching. In flight control applications, these neglected dynamics mostly are the high frequency dynamics of the actuators. This method also can only deal with partial loss actuator faults.

Works by [Corradini and Orlando, 2006; Corradini et al., 2006] present a sliding mode fault tolerant control method for linear model that detects fault by monitoring sliding surfaces and identifies the fault by applying particular test input to the plant. Before detecting and identifying the detailed faults, a conservative controller considering a very "pessimistic" worst case is designed. The occurrence of a fault is detected simply by monitoring the sliding surfaces: when the state leaves the sliding hyperplane, it means that a fault has occurred in one of the actuators components. The method uses specially designed test input to detect the faults from the sliding surface. When the test input has been utilized, if the absolute value of sliding surface increased, then this actuator stuck at some position. It is not easy to find such test input. After getting the knowledge of the faults the controller is reconfigured using the same method as that of the conservative controller design. This method can deal with stuck actuator failure. But the method assumes the redundant actuator is exact duplication of the regular actuator which is not available in most real systems. These results are the stimulation of this thesis in two aspects. Firstly, sliding surface can be used as the index or indicator of faults and failures; Secondly, some passive controller must be utilized to stabilize the system before the faults and failures can be detected and identified.

Combined with adaptive control, SMC can be synthesized with little and even without *a priori* knowledge of the uncertainties, this makes it more suitable for FTC [Wheeler et al., 1998; Stepanenko et al., 1998; Tao et al., 2004; Alwi and Edwards, 2005; Shin et al., 2005; Xiao et al., 2008].

[Alwi and Edwards, 2005, 2008a] proposed a novel adaptive gain in the nonlinear part of the control law of SMC which reacts to the occurrence of a fault and attempts to keep the switching function as close as possible to zero, thus maintaining tracking performance. When this gain reaches the maximum allowable set gain, a warning signal is sent to the pilot or an automatic change to the "back-up" controller could be initiated. This is a kind of way to detect the stuck fault in regular control surface, e.g., the elevator. The "back-up" redundant control surface, e.g., stabilizer is activated when the adaptive gain reaches some maximum value. Though this method can deal with total failure such as stuck as well as partial loss fault, it does not fully utilize the regular actuator. When the adaptive gain reaches its maximum value that stimulates the redundant actuator, it does not mean the regular actuator totally fails while this method discards the regular actuator completely. The faulty regular actuator may still contribute to the system, though itself alone cannot accommodate the 'severe' partial loss fault. Sometimes the redundant actuator may not work as well as the regular actuator, e.g., the stabilizer is slower than elevator in dynamics. The not totally failed faulty regular actuator can still work in some degree to help the redundant actuator to get better performance than only using redundant actuator.

In [Tao et al., 2004; Hu et al., 2011] adaptive SMCs are used to accommodate stuck failures. These methods need knowledge of the structure of the system when it is healthy and when it is faulty. This means for different structure of faulty system, e.g., stuck failure and partial loss fault, different controllers should be synthesized. There will be switching of controllers when the system structure changes.

In [Alwi and Edwards, 2006, 2007a,b, 2008b, 2010; Hamayun et al., 2012] SMC-based FTCs combined with control allocation that automatically distribute control efforts when there is fault or failure are proposed. These schemes use the effectiveness level of the actuators as the weight in distributing control efforts, and redistribute control to the remaining actuators when faults/failures occur. The effectiveness level of the actuators is assumed coming from FDD or special sensors. This method is synthesized based on linear model.

2.4 Summary

Generally, FTC is categorized into PFTC and AFTC. From the above literature review, it can be seen that most of the works on AFTC come together with the progress in FDD mechanisms. Theoretically, AFTC can deal with all kinds of faults and failures. In practice, AFTC will be costly because of the complicated architecture due to the combination with FDD and the reconfiguration of controller online. Besides, time delay from the faults occurrence to the detection and identification of faults and then to the reconfiguration of controller based on the faults information, is the main constraints of the application of AFTC. During the delay from faults occurrence to a reconfigured controller in execution, the system is in the danger of losing control due to the mismatch of controller and system dynamics. The controller-system mismatch may also disable the FDD which cannot get the right information for building the faulty system model if the system is out of control. So some kind of controller must be working to stabilize the system during the delay. This is the motivation of this thesis.

PFTC is mostly a kind of robust control that can accommodate preassigned faults, i.e., it can only deal with partial loss fault. H_∞ and SMC are two mostly researched robust control methods in FTC field. They both sacrifice the normal controller performance to get robustness to uncertainties in the system dynamics. However, SMC's unique design methodology, that separates the design procedure into two subsystems, makes it possible to be robust to uncertainties without sacrificing too much performance of normal controller. Even more, the reaching attractor in SMC is extended to sliding manifold (a dynamic subsystem) from the equilibrium in other methodologies. This means there is a dynamic subsystem that can sense the dynamic variation due to disturbances and faults. This is the reason this thesis uses SMC in the development of fast response reconfigurable FTC.

Chapter 3

Modeling of Boeing 747-100/200 and Faulty System

It is costy and time consuming in testing FTC on real systems, especially in aircraft system. There are available models in the open literature such as nonlinear F-16 model [Sonneveldt, 2006; Russell, 2003] and ADMIRE (Aero-Data Model In Research Environment) [Forssell and Nilsson, 2005] which comprise full order nonlinear equations of aircrafts. ADMIRE is a generic model of a small single-seat fighter aircraft with a delta-canard configuration, which is developed within the project GARTEUR Flight Clearance FM (AG11) in Europe. Both F-16 and ADMIRE are limited in redundancy, they are suitable for simulation of normal control and partial loss fault tolerant control. FTLAB747 is a simulation package of Boeing 747-100/200 with rich redundancy and is a good platform for FTC simulation test.

3.1 FTLAB747

The FTLAB747 software running under MATLAB/Simulink has been developed for the study of FTC and FDD schemes. It represents a 'real world' model of Boeing 747-100/200 aircraft, where the technical data and motion equations have been obtained from NASA [Hanke and Nordwall, 1970; Hanke, 1971]. The software evolved from DASMAT (Delft University Aircraft Simulation And Analysis Tool) [van der Linden, 1996] and Flight Lab 747 [Smaili, 1996]. Later it is enhanced to FTLAB747 V6.1/V6.6 for use in terms of fault detection and fault tolerant control [Marcos and Balas, 2003]. It was augmented with a classical autopilot and increased modularization which make it especially suitable for FTC under GARTEUR AG16 in Europe [Breeman, 2006]. Figure 3.1 shows the AG16 benchmark sketch and Figure 3.2 shows the main frame of AG16 benchmark.

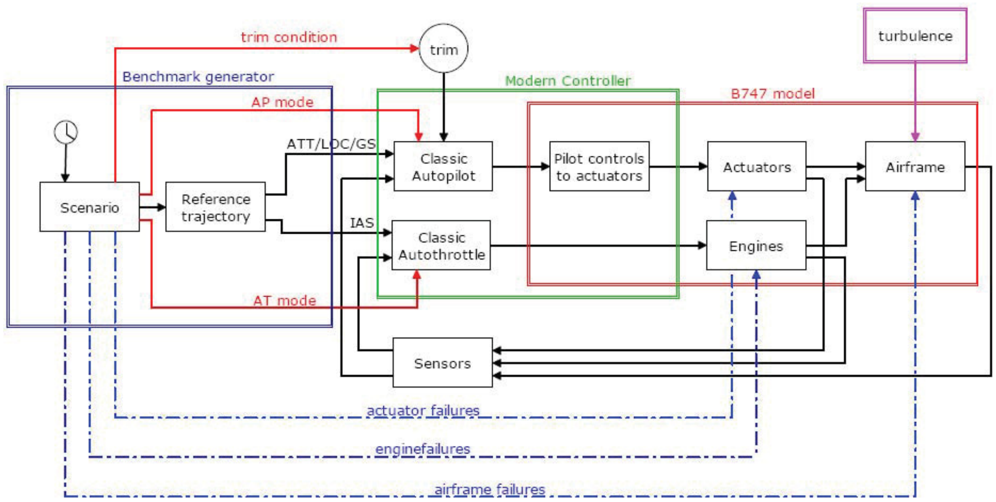

Figure 3.1: Sketch of AG16 benchmark model (Adopted from [Breeman, 2006])

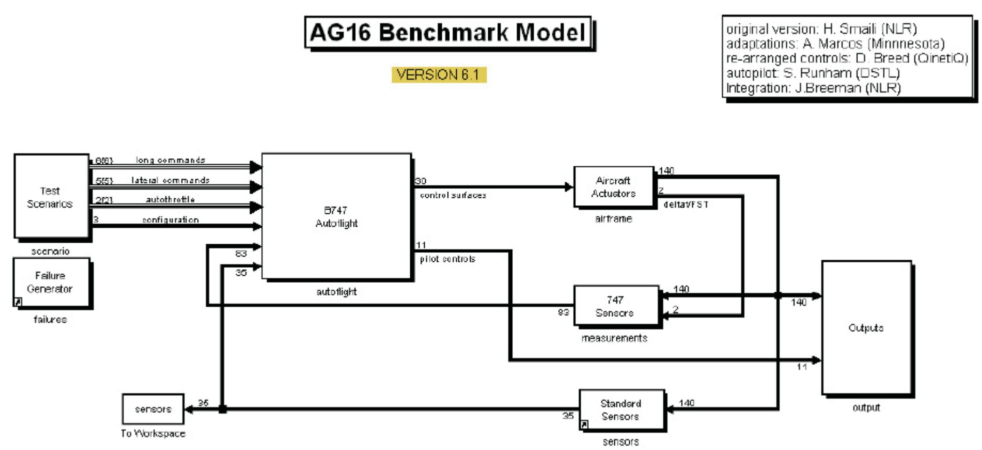

Figure 3.2: The main benchmark model of AG16 (Adopted from [Breeman, 2006])

The high fidelity nonlinear model of FTLAB747 has 77 states incorporating rigid body variables, sensors, actuators and aero-engine dynamics. All the control surfaces and engine dynamics are modeled with realistic position limits and rate limits. The specific aerodynamics coefficients are taken from [Hanke and Nordwall, 1970], which have been obtained from extensive wind tunnel experiments, simulations and test flights.

Till now, FTLAB747 is the only model in the open literature which replicates a real failure condition, and it was used in the investigation of the ELAL flight 1862 (Bijlmermeer incident) in 1992 [Smaili et al., 2006] using the real flight recorded data. Therefore, the FTLAB747 model represents a realistic test bed for the FTC schemes and it was chosen as the test bench for the FTC algorithms of this thesis.

3.2 High Fidelity Model of Boeing 747-100/200

Boeing 747-100/200 is an inter-continental wide-body transport airplane with range of 10,000 km, maximum level speed 975 km/hr and design ceiling of 137166 m [Hanke, 1971].

Boeing 747-100/200 aircraft has abundant actuators: four engines, a movable horizontal stabilizer (used for pitch trim purposes when the elevators work well, can provide pitch moment when elevators have faults or failures), four elevator segments (i.e. two inboards and two outboards) for the control of longitudinal axis motion; twelve spoilers (10 in-flight spoilers used symmetrically for speed brakes and used asymmetric to complement the directional control, 2 ground spoilers are only used during ground operations), two pairs of inboard and outboard flaps used for lateral control; a two-panel rudder for direction control [Marcos and Balas, 2003]. This makes it the perfect representative of commercial airplanes flying today, and thus an ideal benchmark to design and test FTC and FDD algorithms.

The general mathematical flight dynamics model of a rigid aircraft can be written as:

$$\dot{x} = f(x) + G(x)u(t) \tag{3.1}$$

$$y = h(x) \tag{3.2}$$

where $x \in R^n (n = 12)$ is the vector of states of the aircraft system:

$$x = [p \;\; q \;\; r \;\; V_{TAS} \;\; \alpha \;\; \beta \;\; \phi \;\; \theta \;\; \psi \;\; h_e \;\; x_e \;\; y_e]^T \tag{3.3}$$

$p,\ q,\ r$ are roll, pitch and yaw angular rates respectively; $V_{TAS},\ \alpha,\ \beta$ are the true air speed, angle of attack and sideslip angle respectively; $\phi,\ \theta,\ \psi$ are roll angle, pitch angle and yaw angle respectively; $h_e,\ x_e,\ y_e$ are the positions of the aircraft with respect to Earth (North-East-Down reference frame). $f(x) \in R^n (n = 12)$ is the system dynamics vector; $h(x) \in R^l$ is the output vector; $G(x) \in R^{n\times m}$ is the control input distribution matrix. u is the m-dimensional control vector; y is the l-dimensional output vector. For Boeing 747-100/200 with $m = 16$:

$$u = [\delta_{eil} \ \ \delta_{eir} \ \ \delta_{eol} \ \ \delta_{eor} \ \ \delta_{ail} \ \ \delta_{air} \ \ \delta_{aol} \ \delta_{aor} \ \ \delta_{rl} \ \ \delta_{ru} \ \ \delta_{ih} \ \ \delta_{sp} \ \ T_1 \ \ T_2 \ \ T_3 \ \ T_4] \tag{3.4}$$

where eil, eir, eol, eor represent inner left, inner right, outer left and outer right elevators; ail, air, aol, aor represent inner left, inner right, outer left and outer right ailerons; rl, ru represent lower and upper rudders; ih represents stabilizer; sp represents spoiler; δ is deflection of control surface; $T_i, \forall i = 1, 2, 3, 4$, is the thrust output of ith engine. The dynamics model can be detailed in the following four groups of differential equations [Marcos and Balas, 2003].

The force equations:

$$\dot{\alpha} = \frac{-F_x \sin\alpha + F_z \cos\alpha + m(-p\cos\alpha\sin\beta + q\cos\beta - r\sin\alpha\sin\beta)V_{TAS}}{mV_{TAS}\cos\beta + C_{L_{\dot{\alpha}}}\bar{q}S\frac{\bar{c}}{V_{TAS}}} \tag{3.5}$$

$$\dot{\beta} = \frac{-F_x\cos\alpha\sin\beta + F_y\cos\beta - F_z\sin\alpha\sin\beta + m(-p\sin\alpha + r\cos\alpha)V_{TAS}}{mV_{TAS}} \tag{3.6}$$

$$\dot{V}_{TAS} = \frac{1}{m}(F_x\cos\alpha\cos\beta + F_y\sin\beta + F_z\cos\beta\sin\alpha) \tag{3.7}$$

The moment equations:

$$\dot{p} = (c_1 r + c_2 p)q + c_3 M_x + c_4 M_z \tag{3.8}$$

$$\dot{q} = c_5 pr - c_6(p^2 - r^2) + c_7 M_y \tag{3.9}$$

$$\dot{r} = (c_8 p - c_2 r)q + c_4 M_x + c_9 M_z \tag{3.10}$$

The kinematics equations:

$$\dot{\phi} = p + \tan\theta(q\sin\phi + r\cos\phi) \tag{3.11}$$

$$\dot{\theta} = q\cos\phi - r\sin\phi \tag{3.12}$$

$$\dot{\psi} = (q\sin\phi + r\cos\phi)\frac{1}{\cos\theta} \tag{3.13}$$

The navigational equations:

$$\dot{h}_e = -(-u\sin\theta + v\cos\theta\sin\phi + w\cos\phi\cos\theta) \tag{3.14}$$

$$\begin{aligned}\dot{x}_e &= u\cos\psi\cos\theta + v(-\cos\phi\sin\psi + \cos\psi\sin\phi\sin\theta)\\ &\quad + w(\sin\phi\sin\psi + \cos\phi\cos\psi\sin\theta) \end{aligned} \tag{3.15}$$

$$\begin{aligned}\dot{y}_e &= u\sin\psi\cos\theta + v(\cos\phi\cos\psi + \sin\psi\sin\phi\sin\theta)\\ &\quad + w(-\cos\psi\sin\phi + \cos\phi\sin\psi\sin\theta) \end{aligned} \tag{3.16}$$

In these differential equations, the true airspeed $V_{TAS} = [u, v, w]$:

$$u = V_{TAS}\cos\alpha\cos\beta \tag{3.17}$$

$$v = V_{TAS}\sin\beta \tag{3.18}$$

$$w = V_{TAS}\sin\alpha\cos\beta \tag{3.19}$$

The products and moments of inertia coefficients are:

$$\begin{array}{llllll} c_1 & = & \frac{(I_{yy}-I_{zz})I_{zz}-I_{xz}^2}{\Gamma} & c_2 & = & \frac{(I_{xx}-I_{yy}+I_{zz})I_{xz}}{\Gamma} \\ c_3 & = & \frac{I_{zz}}{\Gamma} & c_4 & = & \frac{I_{xz}}{\Gamma} \\ c_5 & = & \frac{I_{zz}-I_{xx}}{I_{yy}} & c_6 & = & \frac{I_{xz}}{I_{yy}} \\ c_7 & = & \frac{1}{I_{yy}} & c_8 & = & \frac{(I_{xx}-I_{yy})I_{xx}+I_{xz}^2}{\Gamma} \\ c_9 & = & \frac{I_{xx}}{\Gamma} & \Gamma & = & I_{xx}I_{zz} - I_{xz}^2 \end{array} \tag{3.20}$$

The forces and moments in body-axes for the Boeing 747 are:

$$F_x = \bar{q}SC_{X_b} + \sum_{i=1}^{4} T_{n_i} - mg\sin\theta \tag{3.21}$$

$$F_y = \bar{q}SC_{Y_b} + 0.0349[T_{n_1} + T_{n_2} - (T_{n_3} + T_{n_4})] + mg\cos\theta\sin\phi \tag{3.22}$$

$$F_z = \bar{q}SC_{Z_b} - 0.0436\sum_{i=1}^{4} T_{n_i} + mg\cos\theta\cos\phi \tag{3.23}$$

$$\begin{aligned} M_x &= \bar{q}Sb\left[C_{l_b} + \frac{1}{b}(C_{Y_b}\bar{z}_{cg} - C_{Z_b}\bar{y}_{cg}) - \frac{\bar{c}\dot{\alpha}}{bV_{TAS}}C_{Z_{\dot{\alpha}_b}}\bar{y}_{cg}\right]\\ &\quad +0.0436[T_{n_1}yeng_1 + T_{n_2}yeng_2 - (T_{n_3}yeng_3 + T_{n_4}yeng_4)] \end{aligned} \tag{3.24}$$

$$\begin{aligned} M_y &= \bar{q}S\bar{c}\left[C_{m_b} + \frac{1}{\bar{c}}(C_{Z_b}\bar{x}_{cg} - C_{X_b}\bar{z}_{cg}) + \frac{\bar{c}\dot{\alpha}}{bV_{TAS}}\left(C_{m_{\dot{\alpha}_b}} + \frac{\bar{x}_{cg}}{\bar{c}}C_{Z_{\dot{\alpha}_b}}\right)\right]\\ &\quad + \sum_{i=1}^{4} T_{n_i}zeng_i \end{aligned} \tag{3.25}$$

$$\begin{aligned} M_z &= \bar{q}Sb\left[C_{n_b} + \frac{1}{b}(C_{X_b}\bar{y}_{cg} - C_{Y_b}\bar{x}_{cg}) + \frac{b\dot{\beta}}{V_{TAS}}C_{n_{\dot{\beta}_b}}\right]\\ &\quad +T_{n_1}yeng_1 + T_{n_2}yeng_2 - (T_{n_3}yeng_3 + T_{n_4}yeng_4) \end{aligned} \tag{3.26}$$

Transformation of aerodynamic coefficients in stability reference frame to body reference frame are:

$$\begin{aligned}
C_{X_b} &= -C_D\cos\alpha + C_L\sin\alpha & C_{Y_b} &= C_Y \\
C_{Z_b} &= -C_D\sin\alpha - C_L\cos\alpha & C_{l_b} &= C_l\cos\alpha - C_n\sin\alpha \\
C_{m_b} &= C_m & C_{n_b} &= C_l\cos\alpha + C_n\cos\alpha
\end{aligned} \tag{3.27}$$

The aerodynamic coefficients are [Hanke and Nordwall, 1970]:

$$\begin{aligned}
C_L = {} & C_{L_{basic}} + (\Delta C_L)_{\alpha_{wdp=0}} + \Delta(\tfrac{dC_L}{d\alpha})_{\alpha_{wdp=0}} + \tfrac{dC_L}{d\dot{\alpha}}\tfrac{\dot{\alpha}\bar{c}}{2V_{TAS}} + \tfrac{dC_L}{dq}\tfrac{q_s\bar{c}}{2V_{TAS}} \\
& + \tfrac{dC_L}{dn_Z}n_Z + K_\alpha\left[\tfrac{dC_L}{d\delta_{ih}}\delta_{ih_{frl}} + \tfrac{dC_L}{d\delta_{E_I}}\delta_{E_I} + \tfrac{dC_L}{d\delta_{E_O}}\delta_{E_O}\right] + \Delta C_{L_{spoilers}} \\
& + \Delta C_{L_{outboard\ ailerons}} + \Delta C_{L_{landing\ gear}} + \Delta C_{L_{ground\ effect}} + \Delta C_{L_{flap\ failure}}
\end{aligned} \tag{3.28}$$

$$\begin{aligned}
C_D = {} & K\left[C_{D_{basic}} + \tfrac{dC_D}{\delta_{ih}}\delta_{ih_{frl}}\right] + (1-K)C_{D_{Mach}} + \Delta C_{D_{spoilers}} + \Delta C_{D_{landing\ gear}} \\
& + \Delta C_{D_{ground\ effect}} + \Delta C_{D_{sideslip}} + \Delta C_{D_{rudders}} + \Delta C_{D_{flap\ failure}}
\end{aligned} \tag{3.29}$$

$$\begin{aligned}
C_Y = {} & \tfrac{dC_Y}{d\beta}\beta + \tfrac{dC_Y}{dp}\tfrac{p_s b}{2V_{TAS}} + \tfrac{dC_Y}{dr}\tfrac{r_s b}{2V_{TAS}} + \Delta C_{Y_{spoilers}} + \Delta C_{Y_{rudders}} + \Delta C_{Y_{flap\ failure}} \\
& + \Delta C_{Y_{le\ flap\ failure}}
\end{aligned} \tag{3.30}$$

$$\begin{aligned}
C_m = {} & C_{m_{basic}} + (\Delta C_{m0.25})_{\alpha_{wdp=0}} + \Delta(\tfrac{dC_{m0.25}}{d\alpha})_{\alpha_{wdp=0}} + C_L(CG-0.25) + \tfrac{dC_m}{d\dot{\alpha}}\tfrac{\dot{\alpha}\bar{c}}{2V_{TAS}} \\
& + \tfrac{dC_{m0.25}}{dq}\tfrac{q_s\bar{c}}{2V_{TAS}} + \tfrac{dC_m 0.25}{dn_Z}n_Z + K_\alpha\left[\tfrac{dC_{m0.25}}{d\delta_{ih}}\delta_{ih_{frl}} + \tfrac{dC_{m0.25}}{d\delta_{E_I}}\delta_{E_I} + \tfrac{dC_{m0.25}}{d\delta_{E_O}}\delta_{E_O}\right] \\
& + \Delta C_{m0.25_{spoilers}} + \Delta C_{m0.25_{inboard\ ailerons}} + \Delta C_{m0.25_{outboard\ ailerons}} \\
& + \Delta C_{m0.25_{landing\ gear}} + \Delta C_{m0.25_{ground\ effect}} + \Delta C_{m0.25_{sideslip}} + \Delta C_{m0.25_{rudder}} \\
& + \Delta C_{m0.25_{flap\ failure}}
\end{aligned} \tag{3.31}$$

$$\begin{aligned}
C_l = {} & \tfrac{dC_l}{d\beta}\beta + \tfrac{dC_l}{dp}\tfrac{p_s b}{2V_{TAS}} + \tfrac{dC_l}{dr}\tfrac{r_s b}{2V_{TAS}} + \Delta C_{l_{spoilers}} + \Delta C_{l_{rudders}} + \Delta C_{l_{inboard\ ailerons}} \\
& + \Delta C_{l_{outboard\ ailerons}} + \Delta C_{l_{flap\ failure}} + \Delta C_{l_{le\ flap\ failure}}
\end{aligned} \tag{3.32}$$

$$\begin{aligned}
C_n = {} & \tfrac{dC_n}{d\beta}\beta + \tfrac{dC_n}{d\dot{\beta}}\tfrac{\dot{\beta}b}{2V_{TAS}} + \tfrac{dC_n}{dp}\tfrac{p_s b}{2V_{TAS}} + \tfrac{dC_n}{dr}\tfrac{r_s b}{2V_{TAS}} + \Delta C_{n_{spoilers}} + \Delta C_{n_{inboard\ ailerons}} \\
& + \Delta C_{n_{outboard\ ailerons}} + \Delta C_{n_{rudders}} + \Delta C_{n_{flap\ failure}} + \Delta C_{n_{le\ flap\ failure}}
\end{aligned} \tag{3.33}$$

The meaning of the variables and parameters in these aero-dynamic coefficients can be found in [Hanke and Nordwall, 1970].

The transformation of angular rate in body-axes to stability-axes is:

$$p_s = p\cos\alpha + r\sin\alpha \tag{3.34}$$

$$q_s = q \tag{3.35}$$

$$r_s = -p\sin\alpha + r\cos\alpha \tag{3.36}$$

Wing design plane angle of attack (angle between the airflow and the wing root chord line) has the following relationship with fuselage reference line angle of attack:

$$\alpha_{wdp} = \alpha_{frl} + 2\pi/180 \tag{3.37}$$

where 2° is the incidence angle, i_w.

The above model is the high fidelity nonlinear model used in the FTLAB747 simulation platform.

In [Marcos, 2001], the aerodynamic coefficients are reduced in complexity using analytical and simulation methods. The following is the reduced aerodynamic coefficients using simulation method:

$$\begin{aligned}
C_L &= C_{L_{basic}} + \frac{dC_L}{dq}\frac{q_s\bar{c}}{2V_{TAS}} + \Delta C_{L_{spoilers}} & (3.38)\\
C_D &= KC_{D_{basic}} + (1-K)C_{D_{Mach}} + \Delta C_{D_{spoilers}} + \Delta C_{D_{sideslip}} & (3.39)\\
C_Y &= \frac{dC_Y}{d\beta}\beta + \frac{dC_Y}{dp}\frac{p_s b}{2V_{TAS}} + \Delta C_{Y_{rudders}} & (3.40)\\
C_m &= C_{m_{basic}} + \frac{dC_m}{d\dot{\alpha}}\frac{\dot{\alpha}\bar{c}}{2V_{TAS}} + \frac{dC_{m0.25}}{dq}\frac{q_s\bar{c}}{2V_{TAS}} & (3.41)\\
C_l &= \frac{dC_l}{d\beta}\beta + \frac{dC_l}{dp}\frac{p_s b}{2V_{TAS}} + \frac{dC_l}{dr}\frac{r_s b}{2V_{TAS}} + \Delta C_{l_{rudders}} + \Delta C_{l_{ailerons}} & (3.42)\\
C_n &= \frac{dC_n}{d\beta}\beta + \frac{dC_n}{dp}\frac{p_s b}{2V_{TAS}} + \frac{dC_n}{dr}\frac{r_s b}{2V_{TAS}} + \Delta C_{n_{rudders}} & (3.43)
\end{aligned}$$

These are the aerodynamic coefficients used in the following longitudinal model.

3.3 Nonlinear Longitudinal Model of Boeing 747-100/200

A full nonlinear equations of the Boeing 747 longitudinal motion are taken from [Marcos, 2001; Szászi et al., 2005] over the up-and-away flight regime: altitude $h_e \in [3000, 12000]m$, angle of attack $\alpha \in [-2°, 8°]$ and total airspeed $V \in [150, 280]m/s$. The detailed nonlinear equations of motion are:

$$\begin{aligned}
\dot{\alpha} = &\left[1 - \frac{\bar{q}S\bar{c}}{2mV_{TAS}^2}(1.45 - 1.8x_{cg})\frac{dC_L}{dq}\right]q + \frac{g\cos(\alpha-\theta)}{V_{TAS}} - \frac{\bar{q}S}{mV_{TAS}}C_{L_{basic}}\\
&-\left[\frac{\bar{q}S}{mV_{TAS}}K_\alpha\frac{dC_L}{d\delta_{ei}}\right]\delta_{ei} - \left[\frac{\bar{q}S}{mV_{TAS}}K_\alpha\frac{dC_L}{d\delta_{eo}}\right]\delta_{eo}\\
&-\frac{0.0436\cos\alpha + \sin\alpha}{mV_{TAS}}\sum_{i=1}^{4}Tn_i
\end{aligned} \tag{3.44}$$

$$
\begin{aligned}
\dot{q} &= \frac{c_7 \bar{q} S \bar{c}}{2V_{TAS}} \left[\bar{c}\frac{dC_m}{dq} - (1.45 - 1.8x_{cg})(\bar{x}_{cg}\cos\alpha + \bar{z}_{cg}\sin\alpha) \right] q + c_7 \bar{q} S \bar{c} C_{m_{basic}} \\
&+ c_7 \bar{q} S \left[C_{D_{Mach}}(\bar{z}_{cg}\cos\alpha - \bar{x}_{cg}\sin\alpha) - C_{L_{basic}}(\bar{z}_{cg}\sin\alpha + \bar{x}_{cg}\cos\alpha) \right] \\
&+ c_7 \bar{q} S K_\alpha \left[\bar{c}\frac{dC_m}{d\delta_{ei}} - (\bar{x}_{cg}\cos\alpha + \bar{z}_{cg}\sin\alpha)\frac{dC_L}{d\delta_{ei}} \right] \delta_{ei} \\
&+ c_7 \bar{q} S K_\alpha \left[\bar{c}\frac{dC_m}{d\delta_{eo}} - (\bar{x}_{cg}\cos\alpha + \bar{z}_{cg}\sin\alpha)\frac{dC_L}{d\delta_{eo}} \right] \delta_{eo} \\
&+ c_7 \bar{q} S K_\alpha \bar{c}\frac{dC_m}{d\sigma}\sigma + c_7 \sum_{i=1}^{4} zeng_i Tn_i \qquad (3.45)
\end{aligned}
$$

$$\dot{V}_{TAS} = g\sin(\alpha - \theta) - \frac{\bar{q}S}{m} C_{D_{Mach}} + \frac{\cos\alpha - 0.0436\sin\alpha}{m} \sum_{i=1}^{4} Tn_i \qquad (3.46)$$

$$\dot{\theta} = q \qquad (3.47)$$

$$\dot{h}_e = -\sin(\alpha - \theta) V_{TAS} \qquad (3.48)$$

The aerodynamic coefficients and their derivatives are calculated from the look-up table described in [Hanke and Nordwall, 1970].

3.4 Fitted Nonlinear Longitudinal Model of Boeing 747-100/200

An approximate fitted nonlinear longitudinal model of Boeing 747-100/200 is obtained from fitted aerodynamic coefficients as polynomial functions of angle of attack and velocity for level flight over the flight envelope [Shin et al., 2006]. The thrust generated by four engines is described by "4T" using one variable and also the four elevators are described as one variable δ_e for simplicity. These mean:

$$Tn_1 = Tn_2 = Tn_3 = Tn_4 = T$$

$$\delta_{ei} = \delta_{eo} = \delta_e$$

$$\frac{dC_L}{d\delta_{ei}} = \frac{dC_L}{d\delta_{eo}} = \frac{dC_L}{2d\delta_e}$$

$$\frac{dC_m}{d\delta_{ei}} = \frac{dC_m}{d\delta_{eo}} = \frac{dC_m}{2d\delta_e}$$

The aerodynamic coefficients are approximated as:

$$C_{D_{Mach}} = \kappa_{20}\alpha^2 + \kappa_{10}\alpha + \kappa_{01}V_{TAS} + \kappa_{00} \qquad (3.49)$$

$$\frac{dC_L}{d\delta_e} = \tau_{02}V_{TAS}^2 + \tau_{01}V_{TAS} + \tau_{00} \quad (per\ degree) \qquad (3.50)$$

$$
\begin{aligned}
C_{L_{basic}} &= \eta_{10}\alpha + \eta_{01}V_{TAS} + \eta_{00} && (3.51)\\
C_{m_{basic}} &= \xi_{20}\alpha^2 + \xi_{10}\alpha + \xi_{01}V_{TAS} + \xi_{00} && (3.52)\\
\frac{dC_m}{d\delta_e} &= \zeta_{02}V_{TAS}^2 + \zeta_{01}V_{TAS} + \zeta_{00} \;\; (per\ degree) && (3.53)
\end{aligned}
$$

where the constant coefficients are:

$$
\begin{array}{llll}
\kappa_{20} = 3.27, & \kappa_{10} = 3.48\times10^{-2}, & \kappa_{01} = 4.45\times10^{-5}, & \kappa_{00} = 9.92\times10^{-3},\\
\tau_{02} = -1.44\times10^{-7}, & \tau_{01} = 4.26\times10^{-5}, & \tau_{00} = 3.21\times10^{-3}, & \\
\eta_{10} = 5.15, & \eta_{01} = 1.21\times10^{-3}, & \eta_{00} = 6.15\times10^{-3}, & \\
\xi_{20} = 2.39, & \xi_{10} = -1.46, & \xi_{01} = -3.20\times10^{-4}, & \xi_{00} = 0.12,\\
\zeta_{02} = 4.35\times10^{-7}, & \zeta_{01} = -1.16\times10^{-4}, & \zeta_{00} = -1.76\times10^{-2}. &
\end{array}
$$

The other derivatives are:

$$
\begin{aligned}
K_\alpha &= 1, && (3.54)\\
\frac{dC_L}{dq} &= 5.1707, && (3.55)\\
\frac{dC_m}{dq} &= -20.8073, && (3.56)\\
\frac{dC_m}{d\sigma} &= -2.8374\ (per\ rad). && (3.57)
\end{aligned}
$$

The other parameters: $x_{cg} = 0.25, \bar{x}_{cg} = -(x_{cg} - 0.25)\bar{c} = 0, \bar{z}_{cg} = z_{cgref} - z_{cg} = 0, m = 3\times10^5 kg, c_7 = 1/I_{yy}, I_{yy} = 4.5278\times10^7 kg\cdot m^2, g = 9.7851 m/s^2, \bar{c} = 8.324m, S = 511m^2, \rho = 0.59kg/m^3, zeng_1 = 0.94m, zeng_2 = 2.53m, zeng_3 = 2.53m, zeng_4 = 0.94m$. The trim point is: $\alpha_{trim} = 0.0162rad, q_{trim} = 0rad/s, V_{TAStrim} = 230m/s, \theta_{trim} = 0.0162rad, h_{trim} = 7000m, \delta_{etrim} = 0deg, \sigma_{trim} = 0.0128rad, T_{trim} = 41631N$.

The ultimate approximate nonlinear longitudinal model over up-and-away flight rigime of Boeing 747-100/200, which is utilized in the synthesization of some of the algorithms in this proposal, is shown as following:

$$
\begin{aligned}
\dot{\alpha} = {} & 0.989186q + \frac{9.7851\cos(\alpha-\theta)}{V_{TAS}} - 0.000502483(0.00615 + 5.15\alpha\\
& +0.00121V_{TAS})V_{TAS} - 0.000502483V_{TAS}(0.00321 + 0.0000426V_{TAS}\\
& -1.44\times10^{-7}V_{TAS}^2)\delta_e - \frac{0.0436\cos\alpha + \sin\alpha}{75000V_{TAS}}T \qquad (3.58)
\end{aligned}
$$

$$
\begin{aligned}
\dot{q} &= -0.00239997qV_{TAS} + 0.0000277133(0.12 - 1.46\alpha + 2.39\alpha^2 \\
&\quad -0.00032V_{TAS})V_{TAS}^2 + 0.0000277133V_{TAS}^2(-0.0176 - 0.000116V_{TAS} \\
&\quad +4.35 \times 10^{-7}V_{TAS}^2)\delta_e - 0.0000786336V_{TAS}^2\sigma + 1.53275 \times 10^{-7}T && (3.59) \\
\dot{V}_{TAS} &= 9.7851\sin(\alpha - \theta) - 0.000502483(0.00992 + 0.0348\alpha + 3.27\alpha^2 \\
&\quad +0.0000445V_{TAS})V_{TAS}^2 + \frac{\cos\alpha - 0.0436\sin\alpha}{75000}T && (3.60) \\
\dot{\theta} &= q && (3.61) \\
\dot{h}_e &= -\sin(\alpha - \theta)V_{TAS} && (3.62)
\end{aligned}
$$

3.5 Open-Loop Longitudinal Response of Boeing 747-100/200

Here is the simulation results of the open-loop response of longitudinal axis of Boeing 747-100/200 on FTLAB747 (AG16) with high fidelity nonlinear model, the reduced aerodynamic coefficients based nonlinear longitudinal model and the fitted approximate nonlinear longitudinal model.

The elevator changes from 0 degree to -2 degree at 25 second, then changes to 2 degree at 50 second and at last changes to 0 degree at 75 second. The mass of the aircraft is 30000kg. The trim point is $\alpha_{trim} = 0.029rad$, $q_{trim} = 0rad/s$, $V_{TAStrim} = 230m/s$, $\theta_{trim} = 0.029rad$, $h_{trim} = 7000m$, $\delta_{etrim} = 0deg$, $\sigma_{trim} = 0.061rad$, $T_{trim} = 41375N$. The simulation result is shown in Figure 3.3. It can be seen that the inclination is similar though the difference is not small on the FTLAB747 to the other two platforms. This is due to the longitudinal dynamics on the other two platforms are reduced by aerodynamic coefficient complexity in simulation similarity, refer to [Marcos, 2001]. The difference between the other two platforms comes from the fitted aerodynamic coefficients and also the approximation of some coefficients and variables to be constants.

3.6 Modeling of Faults and Failures

3.6.1 Types of Actuator Faults and Failures

Faults and failures may occur at any part of the system, such as sensors, controllers, actuators or the plant components. This thesis focuses on faults and failures occurring at actuators on aircrafts. Actuator faults and failures on the aircraft can be classified as: partial loss faults, total failure such

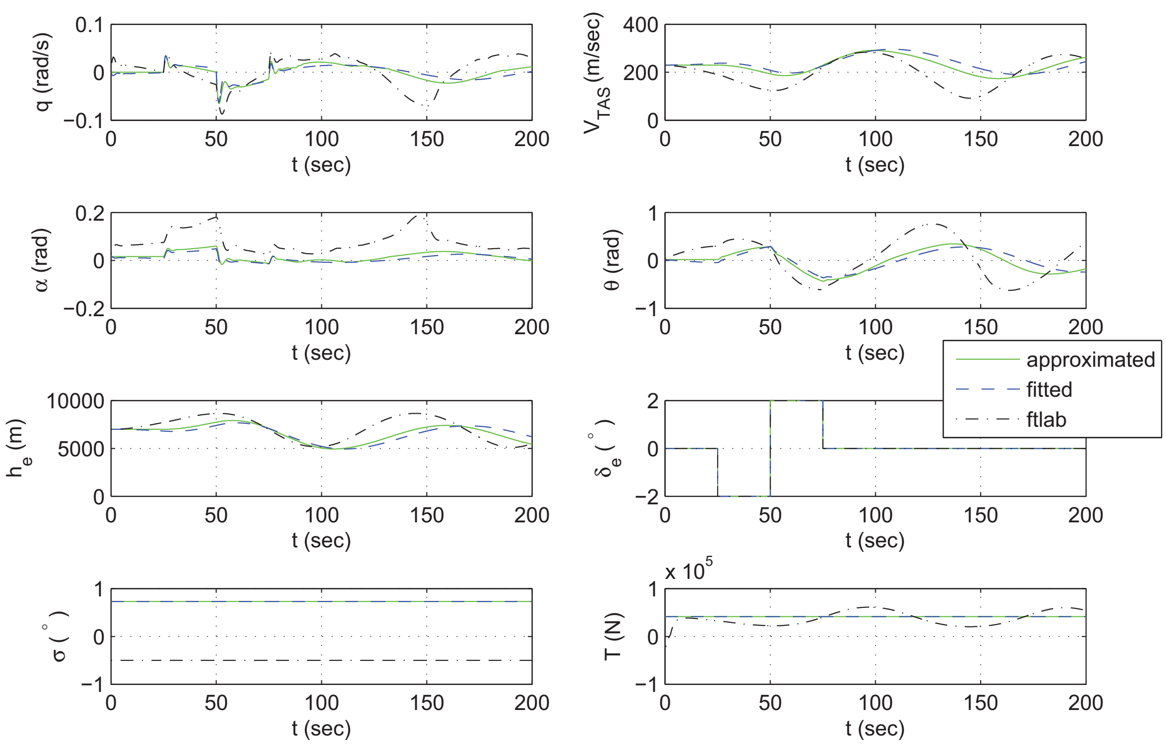

Figure 3.3: Open loop response to elevator doublet on three platforms

as stuck, hard-over and floating.

Partial loss actuator fault means decreasing in the actuator's effectiveness. This is the most common fault situation in aircraft system which may be caused by partial loss of a control surface, or pressure reduction in hydraulic lines [Fisher, 2004; Zhang and Jiang, 2002]. According to the IFAC SAFEPROCESS definition of fault in Chapter 1, fault always means partial loss fault. Total failure means the actuator cannot exert expected designed efforts any more. In aircraft systems three most commonly occurred actuator failures are stuck, hardover and floating.

Stuck, or lock-in-place failure, is a failure condition when an actuator is stuck at some fixed position and immovable. This might be caused by a mechanical jam, due to lack of lubrication for example. This type of failure is studied in [Chen and Jiang, 2005; Fisher, 2004; Ganguli et al., 2002; Gopinathan et al., 1998; Zhang and Jiang, 2003a].

Float failure is a failure condition when the control surface moves freely without providing any moment to the aircraft. An example of a float failure is the loss of mechanical link in the elevator's actuator causing it to move freely in the direction of angle of attack and therefore not producing any effective moment in the pitch axis. This situation is considered in [Burcham et al., 1998; Fisher, 2004; Ganguli et al., 2002].

Hardover, or runaway failure is the most catastrophic types of failure where the control surface moves at its maximum rate limit until it reaches its maximum position limit or its blowdown limit which is the aerodynamic limit of the control surface deflection at a specified speed which overpowers the movement of the actuator. It might not be the maximum physical deflection of the control surface. Any deflection above the blowdown limit can cause structural damage) [Stengel, 2004]. For example, a rudder runaway failure can occur when there is an electronic component failure which causes a wrong large signal to be sent to the actuators leading the rudder to be deflected at its maximum rate to its maximum deflection at low speed (or its blowdown limit at high speed). Hardover can be seen as a special stuck failure at the maximum/minimum limit position. This type of failure is studied in [Smaili et al., 2006].

The above faults and failures can be graphically shown in Figure 3.4.

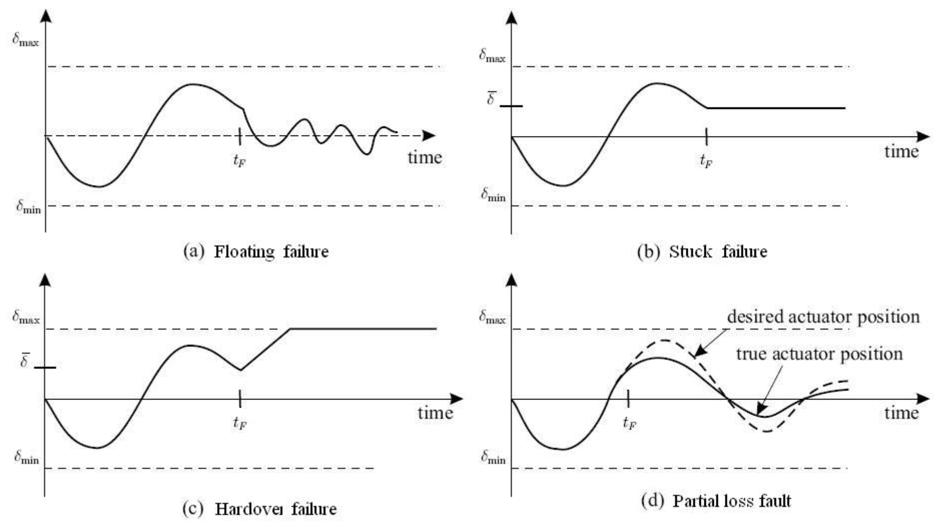

Figure 3.4: Types of actuator faults and failures on aircraft (adopted from [Ducard, 2009])

3.6.2 Modeling of Faults and Failures

The faults and failures defined in the afore section can be modeled as [Zhang, 2006]:

$$u_a(t) = L_a(t)u(t) + (I - L_a(t))f_a(t) \tag{3.63}$$

where $u_a(t) = [u_a^1(t), u_a^2(t), \cdots, u_a^m(t)]^T \in R^m$ is the actual control output vector, $u(t) = [u^1(t), u^2(t), \cdots, u^m(t)]^T \in R^m$ is the synthesized control output vector, $L_a = diag\{l_a^1, l_a^2, \cdots, l_a^m\}$ is a diagonal matrix represents the operational effectiveness of the actuators, I is $m \times m$ identity matrix and $f_a(t) = [f_a^1(t), f_a^2(t), \cdots, f_a^m(t)]^T \in R^m$ is the vector of stuck or floating value of the actuators. For different types of actuator fault and failure the above model can be specified in detail by ($\forall\ i = 1, 2, \cdots, m$):

$$u_a^i = \begin{cases} u^i(t) & l_a^i = 1; f_a^i = 0 & for\ all\ t \geq t_F & Fault\ and\ failure\ free \\ l_a^i u^i(t) & 0 < l_a^i < 1; f_a^i = 0 & for\ all\ t \geq t_F & Partial\ loss\ fault \\ f_a^i = u^i(t_F) & l_a^i = 0 & for\ all\ t \geq t_F & Stuck\ failure \\ f_a^i = \bar{u}^i\ or\ \underline{u}^i & l_a^i = 0 & for\ all\ t \geq t_F & Hardover\ failure \\ f_a^i = f^i(t) & l_a^i = 0 & for\ all\ t \geq t_F & Floating\ failure \end{cases} \tag{3.64}$$

where $\bar{u}^i$ is the maximum control output of ith actuator and $\underline{u}^i$ is the minimum control output of ith actuator, t_F is the occurrence time of fault or failure, $f^i(t)$ is unknown bounded function out of control.

3.6.3 Modeling of Faulty Systems

Consider affine nonlinear system:

$$\dot{x}(t) = f(x,t) + G(x,t)u(t) \tag{3.65}$$

$$y(t) = h(x,t) \tag{3.66}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ the control vector, $y \in R^l$; $f(x,t) \in R^n$ the output vector, $G(x,t) \in R^{n\times m}$ and $h(x,t) \in R^l$; further, each entry in $f(x,t)$, $G(x,t)$ and $h(x,t)$ is assumed to be continuous with continuous bounded derivative with respect to $x(t)$. With actuator faults (3.63) and (3.64), the affine nonlinear system can be expressed as:

$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & f(x,t) + G(x,t)u_a \\ & = & f(x,t) + G(x,t)L_a(t)u(t) + G(x,t)(I - L_a(t))f_a(t) \\ & = & f'(x,t) + G'(x,t)u(t) \\ y(t) & = & h(x,t) \end{array} \right. \tag{3.67}$$

where

$$f'(x,t) = f(x,t) + G(x,t)(I - L_a(t))f_a(t) \tag{3.68}$$

and

$$G'(x,t) = G(x,t)L_a(t) \tag{3.69}$$

It can be seen from (3.67) that the partial loss actuator faults will affect the control input distribution $G(x)$, while the total failures (stuck, hardover and floating) will change the system dynamics by $f_a(t)$ through the input matrix $G(x)$. For total failures (stuck, hard-over and floating), the system must have redundant actuator to apply/replace the control action that would be generated by the regular actuators which are totally failed.

3.7 Summary

This chapter introduces the high fidelity model of Boeing 747-100/200 in the simulation and two approximated longitudinal models used in the design of control algorithms in this thesis. Open-loop responses of the three models show how the model used in the design of controller approximates the high fidelity model. The types and modeling of actuator faults and failures are introduced as well as the modeling of system with faults and failures in actuators in this chapter.

Chapter 4

Robustness vs. Tolerability

The emergence of closed-loop feedback control is due to the fact that there is always something uncertain, e.g. disturbance, modeling uncertainty, unmodeling dynamics and measurement noise. In order to evaluate how the system can deal with these uncertain things, some characteristic of feedback control system are defined, such as stability, sensitivity and robustness. For the modern complicated control system like autonomous vehicles, satellite, aircraft and nuclear power plant, another uncertainty must be considered: the accidental events like load changing, fault and failure. Robustness can be a very general characteristic that indexes how the feedback control can deal with all the uncertainties, such as disturbance, measurement noise, modeling uncertainty, unmodeling dynamics and even more accidental events like fault and failure. Technically, robustness is referred to how the system can deal with the perturbations of the system model, while leaves disturbance and measurement noise to other characteristic like stability and sensitivity. For the most uncertain things, the accidental events like fault and failure, a new characteristic is needed since the different physical meaning between modeling uncertainty and fault/failure. In this chapter, firstly modeling uncertainty and fault/failure are compared and distinguished. Then robustness and tolerability are defined. Lastly, tolerability of SMC with regular actuator is defined and tested on the simulation model of Boeing 747-100/200.

4.1 Modeling Uncertainty vs. Fault/Failure

Faults and failures can be treated as uncertainties and disturbances in some sense. In the faulty system model (3.67), consider uncertainties, i.e.,

$$f(x,t) = f_0(x,t) + \Delta f(x,t), G(x,t) = G_0(x,t) + \Delta G(x,t) \tag{4.1}$$

where $f_0(x,t)$ is the nominal system dynamics, $\Delta f(x,t)$ is the uncertainty of nominal system dynamics; $G_0(x,t)$ is the nominal control distribution matrix, $\Delta G(x,t)$ is the uncertainty of nomimal control distribution matrix. Then the model of the system with modeling uncertainty and faults or failures becomes:

$$\begin{aligned} \dot{x}(t) &= f_0(x,t) + \Delta f(x,t) + [G_0(x,t) + \Delta G(x,t)]u_a \\ &= f_0(x,t) + \Delta f(x,t) + [G_0(x,t) + \Delta G(x,t)]\{L_a(t)u(t) + [I - L_a(t)]f_a(t)\} \\ &= f_0(x,t) + \Delta' f(x,t) + [G_0(x,t) + \Delta' G(x,t)]u(t) \end{aligned} \tag{4.2}$$

where

$$\Delta' f(x,t) = \Delta f(x,t) + [G_0(x,t) + \Delta G(x,t)][I - L_a(t)]f_a(t) \tag{4.3}$$

and

$$G'(x,t) = \Delta G(x,t)L_a(t) - G_0(x,t)[I - L_a(t)] \tag{4.4}$$

From Eq.(4.2) it can be seen that partial loss faults go into the uncertainty of control distribution matrix and failures go into the uncertainty of system dynamics. Though in the mathematical model Eq.(4.2) faults and failures are represented the same as with modeling uncertainty, the nature of fault and failure is different from the modeling uncertainties. Modeling uncertainties are inevitable while faults are some occasional events. For example of aircraft, there will always be modeling uncertainty in the system dynamics while it will work almost all the time in normal healthy condition without fault and failure.

Modeling uncertainty can be negotiated with the control performance in the design period, i.e. in the design of the controller, when the performance is degraded too much that cannot meet the design specification or the control is too conservative such that demanding more powerful actuators, the system must be refined to be more precise to lessen the uncertainties. In other words, the statistical property of modeling uncertainty is 'fixed' after the system has been built (generally). However, faults and failures are contingent in their happening. Even more complicated, it is contingent in their occurring time, occurring place, magnitude and character (type of fault and failure).

In the design of controllers, modeling uncertainties can be estimated in their bounds *a priori* and robust control strategy can be resorted to deal with such kinds of uncertainties determinately without degrading the performance beyond the requirement. For a FTC system, it is hard to decide how much severity of the fault and failure and which kind of fault and failure it should deal with. For safety-critical system like aircraft, the FTC must be able to deal with all kinds of fault and failure since they are intolerable due to the big risk to the human beings on board.

With robust control method, faults can only be assumed to be in some bounds that only certain faults can be dealt with. So if we design FTC using robust control method, only certain preassigned faults can be accommodated. When we want to accommodate more severe faults, the performance of the closed-loop system will be sacrificed more. To some extent, the trade-off between the performance and robustness cannot lead to a working system even with degraded performance, e.g., system with failures or faults saturate the actuator to certain extent that the regular actuator cannot deal with alone.

Because of the different nature of modeling uncertainty and fault/failure, an effective FTC which can deal with both of them should deal with them respectively. In the character of FTC, an extra one should be defined besides robustness. In the following sections, robustness will be introduced firstly, and then tolerability, the ability against fault and failure, in the sense of SMC design will then be defined.

4.2 Robustness

Robustness can be a very general character that indexes how the feedback control can deal with all the uncertainties, such as disturbance, measurement noise, modeling uncertainty, unmodeling dynamics and fault/failure. Technically, robustness is referred to how the system can deal with the perturbations of the system model.

Rigid definition of robustness or robust stability can be found in many books, such as [Doyle et al., 1990; Zhou and Doyle, 1999]. The following shows the robustness of a SMC control system.

To assess the robustness of the sliding mode control, the vector-function $\Psi(x,t)$ and the control distribution matrix $E(x)$ in Eq. (2.36) are rewritten in the following format:

$$\Psi(x,t) = \Psi_0(x,t) + \Delta\Psi(x,t) \tag{4.5}$$

$$E(x) = E_0(x) + \Delta E(x) \tag{4.6}$$

where $\Psi_0(x,t)$ and $E_0(x)$ are the norminal value of $\Psi(x,t)$ and $E(x)$, while $\Delta\Psi(x,t)$ and $\Delta E(x)$ are bounded uncertain vector and matrix. $\Delta\Psi(x,t)$ is due to modeling error of system dynamics or failures and $\Delta E(x)$ is due to modeling error of control distribution or partial loss fault. Define:

$$\Delta M = \Delta E(x) \cdot E_0^{-1}(x) \tag{4.7}$$

$$\Phi(x,t) = \Delta\Psi(x,t) - \Delta M \cdot \Psi_0(x,t) \tag{4.8}$$

Assumption 4.1 *The nominal value of control gain distribution has the following properties:*

$$|E_0(x)| \neq 0 \tag{4.9}$$

Assumption 4.2 *The uncertainty of control gain distribution meets the following bounds:*

$$|(\Delta M)_{ii}| < D_i < 1, \ \forall\ i = 1, 2, \cdots, m \tag{4.10}$$

where $D_i, \ \forall\ i = 1, 2, \cdots, m$, *are some positive constants.*

Assumption 4.3 *The uncertainty of system dynamics meets the following bounds:*

$$|[\Phi(x,t)]_i| \leq L_i, \ \forall\ i = 1, 2, \cdots, m \tag{4.11}$$

where $L_i, \ \forall\ i = 1, 2, \cdots, m$ *are some positive constants.*

Substituting the control output Eq. (2.42) with nominal values

$$u = E_0(x)^{-1}(\Psi_0(x,t) + R \cdot \Sigma) \tag{4.12}$$

into Eq. (2.36) yields

$$\dot{s} = \Phi(x,t) - (I_m + \Delta M)R \cdot \Sigma$$

Choose Lyapunov function as:

$$V = \frac{1}{2}s^T s \tag{4.13}$$

Take the derivative of this function leading to:

$$\begin{aligned}
\dot{V} &= s^T \dot{s} \\
&= s^T[\Phi(x,t) - (I_m + \Delta M)R\Sigma] \\
&= \textstyle\sum_{i=1}^{m} (\Phi)_i s_i - \sum_{i=1}^{m} (1 + \Delta M_{ii})\rho_i |s_i| \\
&\leq \textstyle\sum_{i=1}^{m} [L_i |s_i| - (1 - |\Delta M_{ii}|)\rho_i |s_i|] \\
&\leq \textstyle\sum_{i=1}^{m} [L_i |s_i| - (1 - |D_i|)\rho_i |s_i|]
\end{aligned} \tag{4.14}$$

Considering the robustness of the control system, Theorem 2.1 becomes the following theorem.

Theorem 4.1 *For the nonlinear system Eq.* (2.31) *with bounded uncertainties Eq.* (4.11)*, sliding surface Eq.* (2.34) *is asymptotically stable by employing control Eq.* (4.12) *with design parameters as following:*

$$\rho_i > \frac{L_i}{1 - D_i}, \ \forall \ i = 1, 2, \cdots, m \tag{4.15}$$

Proof: From Eq. (4.14) and Eq. (4.15), following can be derived:

$$\dot{V} \leq 0$$

Theorem 4.1 is proved. ■

From Theorem 4.1, it can be seen that SMC is robust to model perturbation, i.e. SMC is a kind of robust control.

4.3 Tolerability

In SMC theory, if the control effort can be increased without limits, it can accommodate very large uncertainties. However, this is not the case in practice, where all the actuators have position and rate limits, e.g., the control surfaces in the aircraft system have physical position limits in their deflections and rate limits in the deflection rates. Table 4.1 shows the deflection position and rate limits of control surfaces of Boeing 747-100/200.

Position limit means that the physical position constraints and rate limit means the actuator has its own dynamics and it can only change with a limited rate. Partial loss fault in actuator can be treated as a kind of uncertainty in the control distribution matrix. Hence, SMC can accommodate

Table 4.1: Control surfaces deflection position and rate limits of Boeing 747-100/200 [Adopted from Breeman [2006]]

control surface	symbol	maximum displacement	normal operation (full boost) rate	one hydraulic system failure rate
		[deg]	*[deg*/s]	*[deg/s]*
elevators				
• inboard	δ_{EI}	+17/-23	+37/-37	+30/-26
• outboard	δ_{EO}	+17/-23	+37/-37	-
stabilizer	Δ_{FRL}	+3/-12	0.5→0.2	0.25→0.1
ailerons				
• inboard	δ_{AI}	+20/-20	+40/-45	+27/-35
• outboard	δ_{AO}	+15/-25	+45/-55	+22/-45
spoilers	δ_{SP}			
• inboard		+20/0	+75/-75	-
• midspan		+45/0	+75/-75	-
• outboard		+45/0	+75/-75	-
• ground		+20/0	+75/-75	-
flaps				
• inboard	δ_{FI}	+113/0	+1.83/-1.83	-
• outboard	δ_{FO}	+113/0	+1.83/-1.83	-
rudder				
• upper	δ_{RU}	+25/-25	+50/-50	+40/-40
• lower	δ_{RL}	+25/-25	+50/-50	+40/-40
yaw damper				
• upper	δ_{YU}	+3.6/-3.6	+15/-15	-
• lower	δ_{YL}	+3.6/-3.6	+15/-15	-

partial loss fault in some extent due to its inherent robustness as stated in the last section, i.e., there is a point of partial loss fault, where the faulty actuator cannot deal with the fault alone. In this scenario the redundant actuators will be activated to work together with the faulty regular actuator to keep the same/similar function of the healthy regular actuators.

This point can be found from the position limit and rate limit. Here position limit is used to show how to reach the point of the tolerability of system for regular actuator with SMC. Consider companion affine nonlinear system Eq. (2.32) and rewrite $E(x)$ as:

$$E(x) = E_0(x)W_c$$

$E_0(x)$ is the nominal input distribution matrix. W_c is effectiveness matrix of actuators:

$$W_c = diag\{w_{c1}, w_{c2}, \cdots, w_{cm}\} \tag{4.16}$$

where:

$$1 \geq w_{ci} \geq 0, \ \forall \ i = 1, 2, \cdots, m \tag{4.17}$$

$w_{ci} = 1$ means there is no fault in the regular actuator, $w_{ci} = 0$ means total loss of the regular actuator and the value between 1 and 0 means there is a partial loss fault in the regular actuator.

Choose the control law as:

$$u = R \cdot \Sigma \tag{4.18}$$

where R and Σ are the same as in Eq. (2.42). By choosing the same Lyapunov function as in Eq. (2.41) and taking derivative of this Lyapunov function, one obtains:

$$\dot{V} = s^T\dot{s} = s^T(\Psi - E_0W_cR\Sigma)$$

where $\Psi(x,t)$ is the same as in Eq. (2.37). We have:

$$s_i\dot{s}_i = \psi_i s_i - E_{0ii}w_{ci}\rho_i|s_i|, \ \forall \ i = 1, 2, \cdots, m$$

Utilizing η-reachability condition [Slotine and Li, 1991]:

$$\psi_i s_i - E_{0ii}w_{ci}\rho_i|s_i| \leq -\eta|s_i|$$

That is:

$$E_{0ii}w_{ci}\rho_i \geq \eta_i - |\psi_i|$$

Here ρ_i must have the same sign as E_{0ii}. So one has:

$$w_{ci} \geq \frac{\eta_i - |\psi_i|}{|E_{0ii}||\rho_i|} \tag{4.19}$$

Suppose ρ_i is limited in its magnitude by $|\rho_i|_{max}$ and ψ has a bound of F, then w_{ci} should be:

$$w_{ci} \quad \geq \quad \frac{1}{|\rho_i|_{max}} \frac{\eta_i - F}{|E_{0ii}|} = w_{c_{i_{tolerable}}} \tag{4.20}$$

This is the point of tolerability of the system for faults in the regular actuators with SMC, i.e., when the effectiveness of the actuator is less than this upper bound $w_{c_{itolerable}}$, the faulty actuator cannot accommodate the fault alone, some redundant actuators are needed. The tolerability is defined as:

Definition 4.1 *Tolerability of the system for faults in actuators with SMC is defined as the limited fault (uncertainty) that certain actuator can accommodate alone with SMC.*

Remark 4.1 *This definition is the starting point of activating redundant actuators, since from this point the regular actuators cannot accommodate the fault solely, redundant actuator(s) must be activated.* $w_{c_{itolerable}}$ *is the threshold of activating the redundant actuators.*

4.4 Tolerability of the Elevator on Boeing 747-100/200 with SMC

It is not easy to get the analytical tolerability because analytical model is not always obtainable, e.g., the aerodynamic coefficients are always given in lookup table. Since we have the high fidelity simulation model of Boeing 747 with lookup tables of aerodynamic coefficients, simulations have been done to estimate the tolerability of elevator with SMC. The simulations here are performed on the 3 platforms: longitudinal axis of Boeing 747-100/200 on FTLAB747 (AG16) with high fidelity nonlinear model, the reduced aerodynamic coefficients based nonlinear longitudinal model and the fitted approximate nonlinear longitudinal model.

Figure 4.1 and Figure 4.2 show the simulation results under SMC with maximum deflection of elevator at the full range of 17 °. This is the situation the effectiveness is 1, i.e., there is no fault in elevator.

The simulation results under SMC with maximum deflection of elevator at 10 ° (59% of effectiveness) are shown in Figure 4.3 and Figure 4.4.

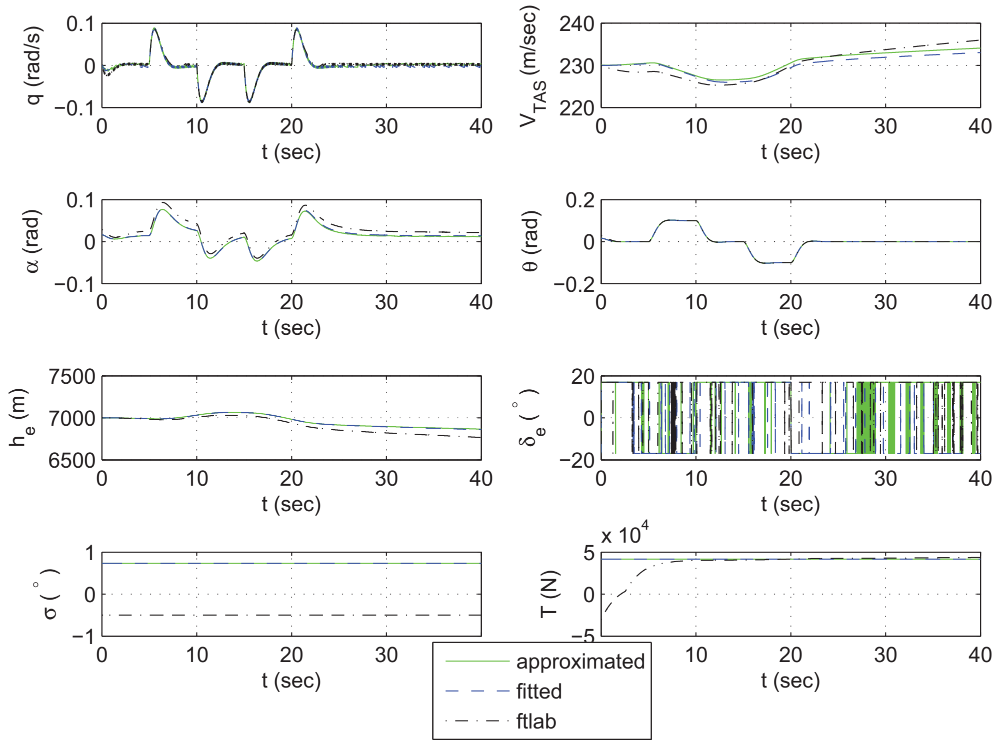

Figure 4.1: States and control of SMC controlled system with full effectiveness

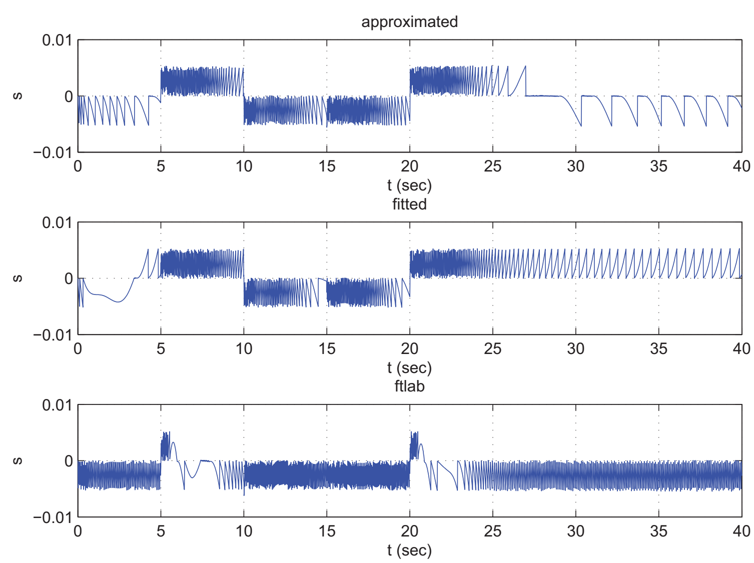

Figure 4.2: Sliding surface of SMC controlled system with full effectiveness

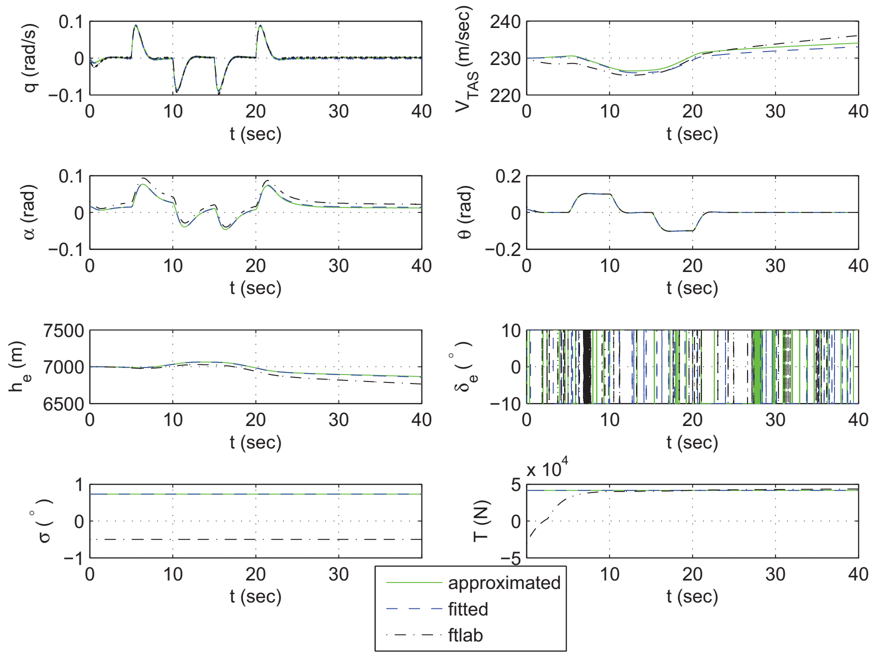

Figure 4.3: States and control of SMC controlled system with 59% effectiveness

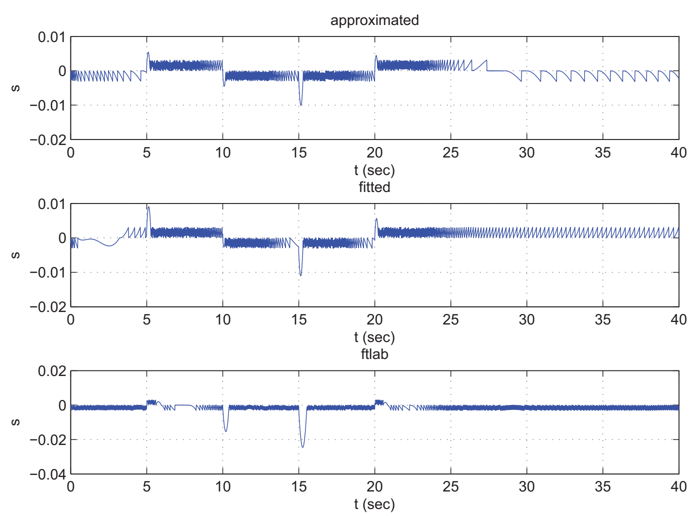

Figure 4.4: Sliding surface of SMC controlled system with 59% effectiveness

The simulation results under SMC with maximum deflection of elevator at 5° (29% of effectiveness) are shown in Figure 4.5 and Figure 4.6.

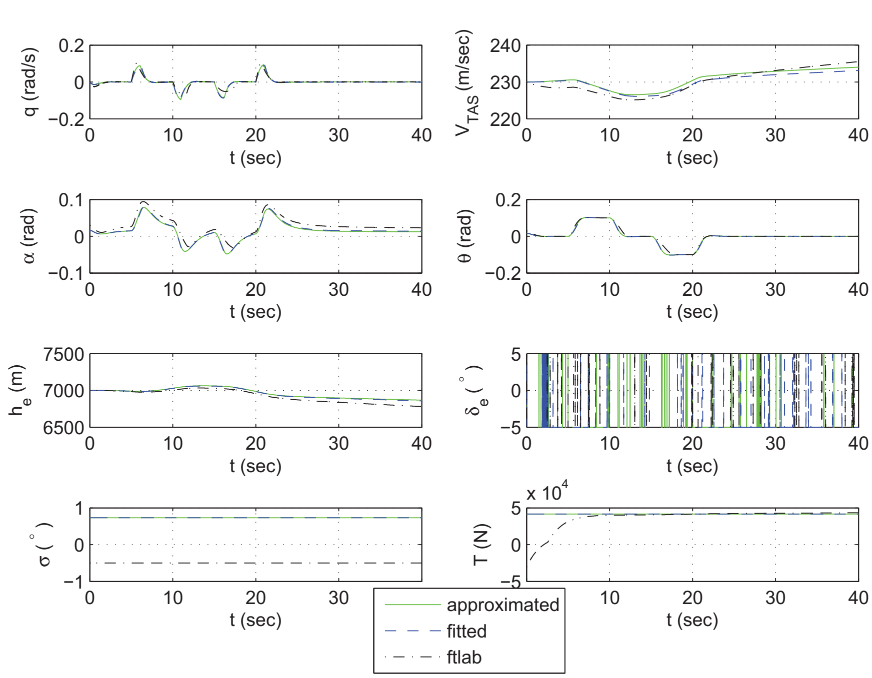

Figure 4.5: States and control of SMC controlled system with 29% effectiveness

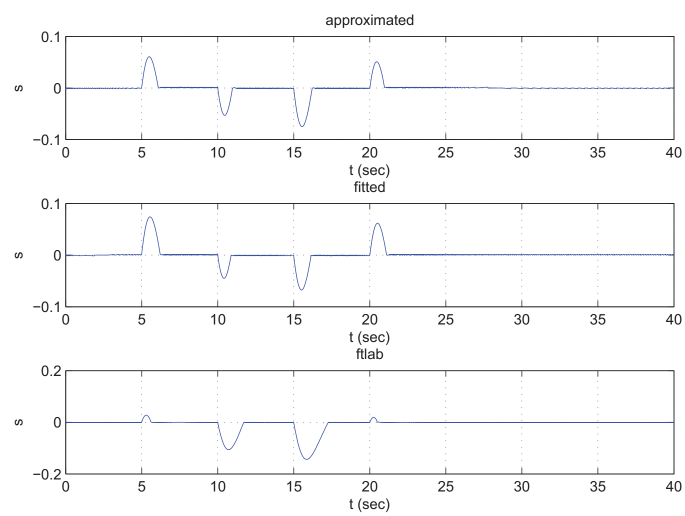

Figure 4.6: Sliding surface of SMC controlled system with 29% effectiveness

The simulation results under SMC with maximum deflection of elevator at 4° (24% of effectiveness) are shown in Figure 4.7 and Figure 4.8.

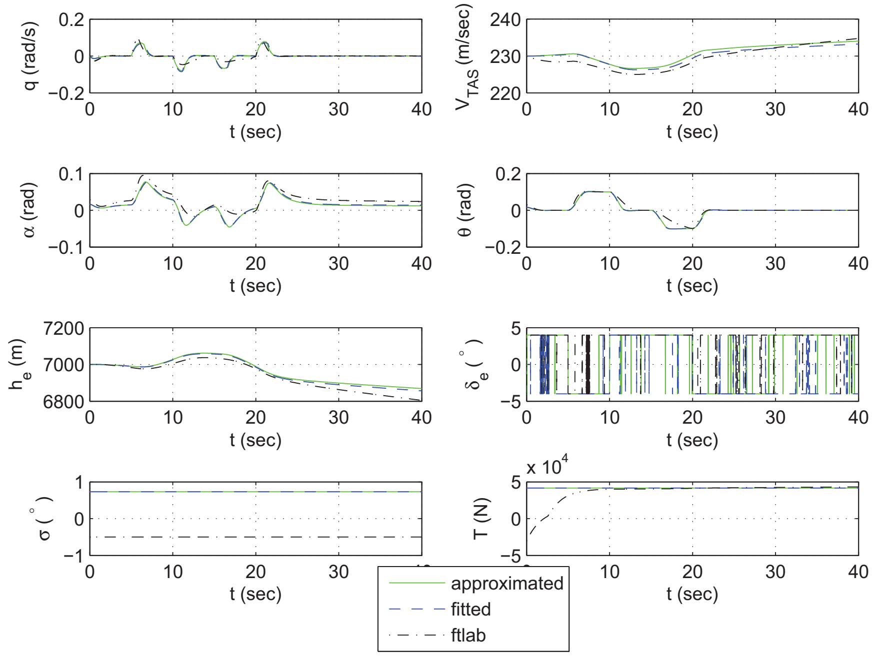

Figure 4.7: States and control of SMC controlled system with 24% effectiveness

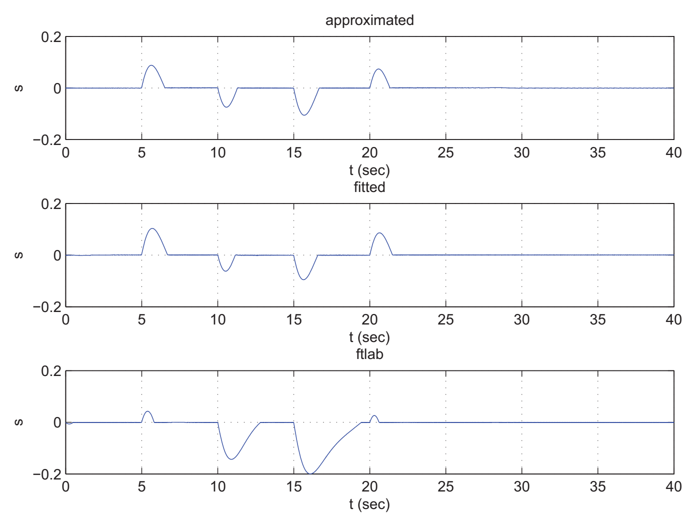

Figure 4.8: Sliding surface of SMC controlled system with 24% effectiveness

The simulation results under SMC with maximum deflection of elevator at 3° (18% of effectiveness) are shown in Figure 4.9 and Figure 4.10.

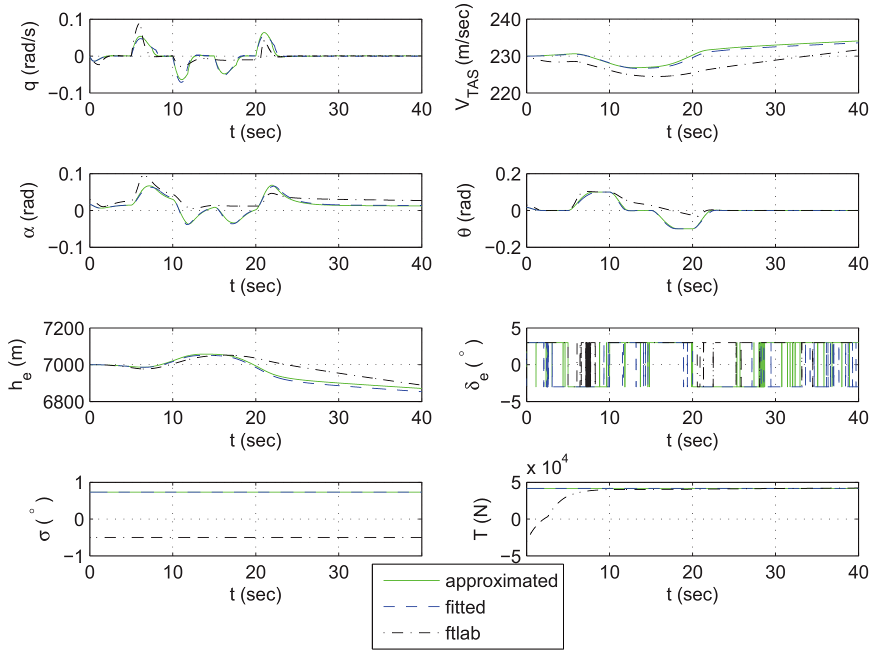

Figure 4.9: States and control of SMC controlled system with 18% effectiveness

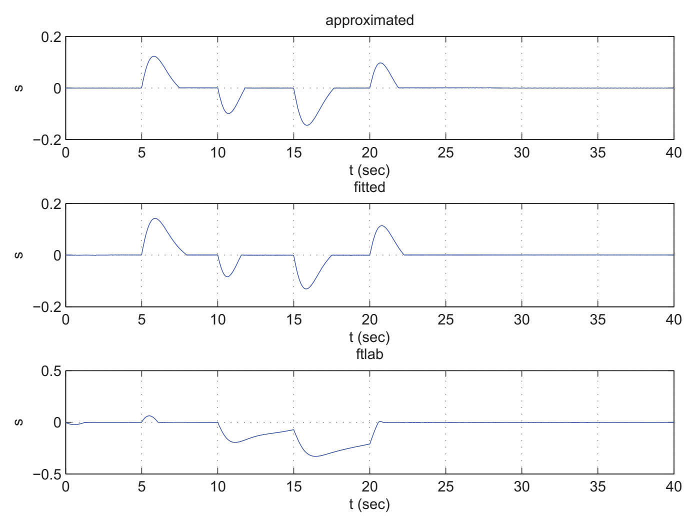

Figure 4.10: Sliding surface of SMC controlled system with 18% effectiveness

The simulation results under SMC with maximum deflection of elevator at 2° (12% of effectiveness) are shown in Figure 4.11 and Figure 4.12.

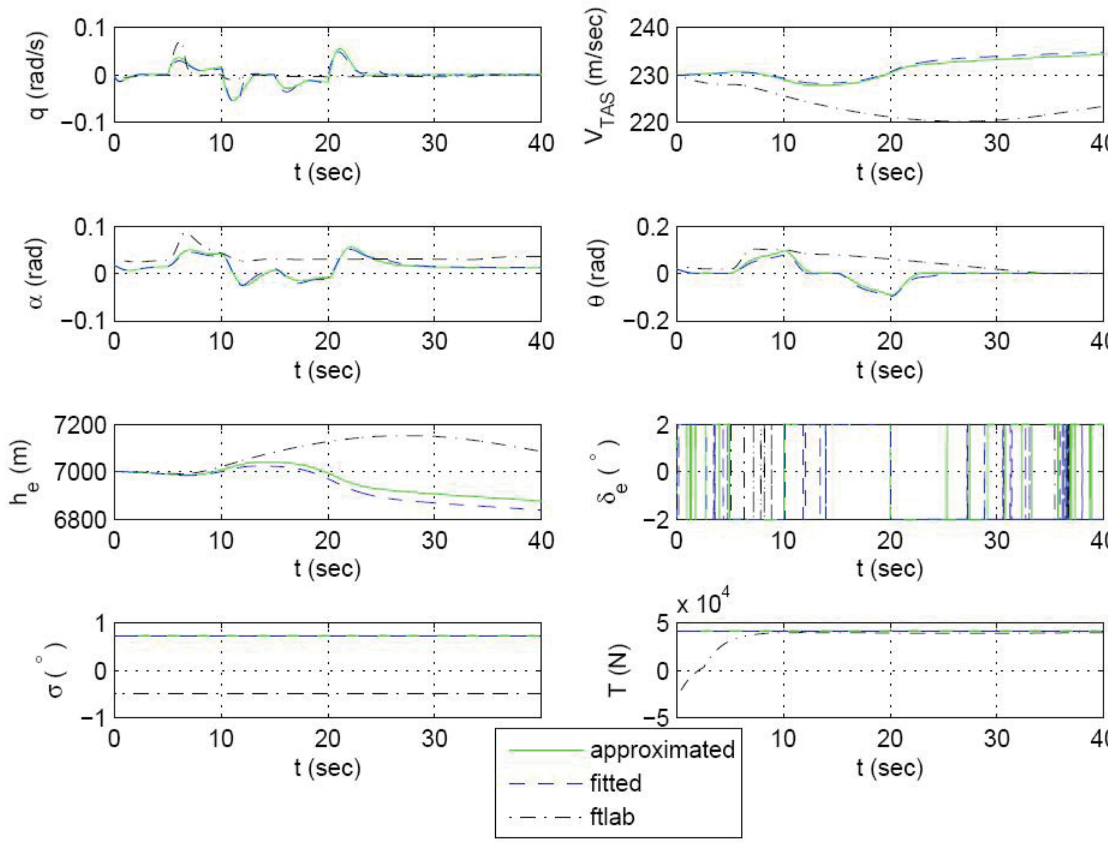

Figure 4.11: States and control of SMC controlled system with 12% effectiveness

From the above simulation results we can see that for the FTLAB747 when the elevator has maximum deflection of only 4° the elevator cannot accommodate the fault itself, while for the reduced nonlinear models this is 2°. This will be used in the reconfigurable control design.

4.5 Summary

The chapter investigates several characteristics of FTC systems. In the context of FTC, the capability to accommodate fault and failure is one extra characteristic of the control system. In this chapter, faults and failures are compared with modeling uncertainty firstly. Then the robustness of the controller against modeling uncertainty and the tolerability of the controller accommodating fault and failure are introduced and defined on the design of SMC. The tolerability will be used in

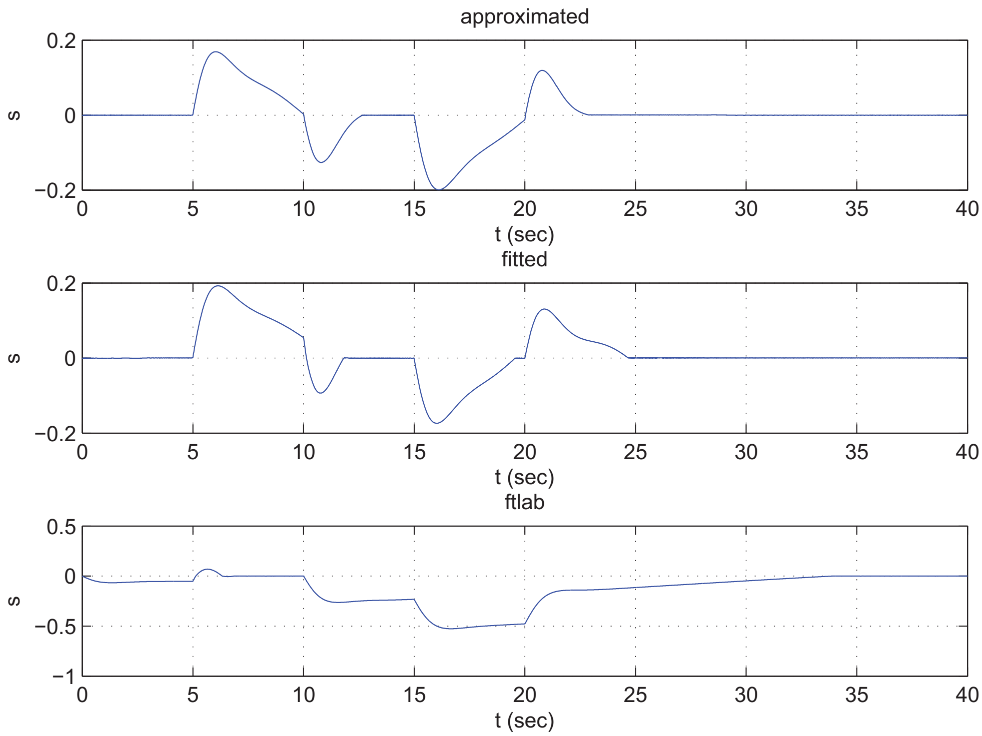

Figure 4.12: Sliding surface of SMC controlled system with 12% effectiveness

the reconfigurable control design in Chapter 6.

Chapter 5

Sliding Mode Fault Tolerant Control Dealing with Modeling Uncertainties and Actuator Faults Separately

In Chapter 4, the difference between modeling uncertainty and actuator faults have been discussed and it is necessary to deal with these two kind of uncertainties separately in an effective FTC design. In this chapter, two sliding mode control algorithms are developed for nonlinear systems with both modeling uncertainties and actuator faults. The first algorithm is developed under an assumption that the uncertainty bounds are known. Different design parameters are utilized to deal with modeling uncertainties and actuator faults respectively. The second algorithm is an adaptive version of the first one, which is developed to accommodate the uncertainties and faults without utilizing the exact bounds information. The stability of the overall control systems is proved by using Lyapunov function. The effectiveness of the developed algorithms have been verified on the nonlinear longitudinal model of Boeing 747-100/200.

5.1 Introduction

Partial loss fault in actuator is a very common fault in aircraft systems. This kind of fault can be treated as uncertainty added to the control distribution gain and incorporated in the controller design. Robust control techniques can then be resorted to accommodate this kind of fault. Traditional robust control, e.g. H_∞, makes trade-off between performance and robustness [Blanke et al., 2003]. Actuator fault may cause large changes in the control distribution gain, i.e., a large uncertainty may be caused by actuator fault which may occur uncertainly in terms of time, location and magnitude. In the normal situation, such uncertainty does not affect the system and only pose effects on the system when there is a fault. If this occasional large uncertainty is taken into consideration in the traditional robust control system design, it will lead to unacceptable system performance in normal situation due to the big trade-off for considering such great uncertainty. As discussed in Chapter 2.2, SMC is a suitable candidate because of its insensitivity and robustness.

Since faults do not occur all the time (normally real system will mostly work under conditions without faults), uncertainties due to faults are not always in effect. In general, SMC was synthesized with one design parameter in the discontinuous part of the control considering the uncertainty in the system dynamics and the uncertainty caused by actuator faults together [Shtessel et al., 1998; Wheeler et al., 1998; Alwi and Edwards, 2005; Huang and Way, 2001; Hess and Wells, 2003]. This leads to significant control effort even there is no actuator fault. In [Slotine and Coetsee, 1986] the uncertainties of system dynamics and control gain are separated. However, both lower and upper bounds of the uncertainty of control distribution gain should be known, because these uncertainties are still dealt with by only one design parameter.

For some applications, the uncertainty bound of the system dynamics is hard to obtain. In FTC system, faults in fact occur at unknown time and with unknown magnitude. The uncertainty incurred by such faults in control distribution gain may be significant and come into play at unknown time with unknown magnitude. This stimulates a new control strategy, which incorporates adaptive strategies in the SMC to accommodate unknown uncertainty bound [Wheeler et al., 1998; Alwi and Edwards, 2005; Alwi et al., 2010; Edwards et al., 2010a]. Adaptive mechanism was used on the uncertainty bound in [Wheeler et al., 1998] and on the discontinuous control term in [Alwi and Edwards, 2005; Alwi et al., 2010; Edwards et al., 2010a]. However, the estimated parameters are concerned with both uncertainties in system dynamics and those caused by actuator faults.

This chapter develops firstly a SMC algorithm that deals with the uncertainties in system dynamics and control distribution gain separately with only the information on upper bounds of the uncertainties. An extra design parameter, compared with traditional SMC, is introduced in the discontinuous component of the control to deal with the uncertainty caused by actuator faults. Then an adaptive SMC method is synthesized with different estimated parameters which deals with the uncertainties of system dynamics and actuator faults separately. This method does not need the exact values of the uncertainty bounds. The gain of discontinuous control term that deals with the uncertainty in system dynamics and the gain of discontinuous control term that deals with uncertainty incurred by actuator faults are estimated respectively.

The rest of this chapter is organized as follows. In Section 5.2 a SMC is derived with separated uncertainties in system dynamics and control distribution gain for affine nonlinear systems using Lyapunov method. Derivation of an adaptive SMC and proof of its stability are provided in Section 5.3. In Section 5.4, the simulation results on the nonlinear longitudinal model of Boeing 747-100/200 are given to show the effectiveness of the developed algorithms. Finally, a summary is presented in Section 5.5.

5.2 Sliding Mode Fault Tolerant Control with Separated Uncertainty Bounds

Considering the SISO affine nonlinear system Eq.(2.23) with modeling uncertainty and faults, the system dynamics and control distribution gain can be rewritten as:

$$\begin{aligned} A(x) &= A_0(x) + \Delta A(x) \\ B(x) &= B_0(x) + \Delta B(x) \end{aligned} \tag{5.1}$$

where $A_0(x)$ is the nominal system dynamics, $B_0(x)$ is the nominal control distribution gain; $\Delta A(x)$ is the modeling uncertainty on system dynamics and $\Delta B(x)$ is the uncertainty in control distribution gain incurred by a fault in actuator. In this chapter only partial loss fault in actuator is considered, which means that faulty actuator will not lose its effectiveness completely. Thus we have the following assumption.

Assumption 5.1 *The control distribution gain has the following properties:*

$$B_0(x) \neq 0, B(x) \neq 0 \tag{5.2}$$

We assume that the uncertainties in system dynamics and control distribution gain are limited to certain constants.

Assumption 5.2 *The uncertainty of system dynamics and the uncertainty caused by actuator fault are bounded and satisfy:*

$$\begin{aligned} &||\Delta A(x)|| < F < +\infty \\ &|\Delta B(x)B_0^{-1}(x)| < L < 1 \end{aligned} \tag{5.3}$$

where $F > 0, L > 0$ are positive numbers representing the upper bound of uncertainty in the system dynamics and upper bound of the uncertainty caused by actuator fault.

The derivative of the sliding manifold Eq.(2.26) becomes:

$$\begin{aligned} \dot{s} &= \lambda_0\dot{e} + \lambda_1\ddot{e} + \cdots + \lambda_{n-2}e^{(n-1)} + y_d^{(n)}(t) - y^{(n)}(t) \\ &= \psi_0(x) - \Delta A(x) - (B_0(x) + \Delta B(x))u \end{aligned}$$

where

$$\psi_0(x) = \lambda_0\dot{e} + \lambda_1\ddot{e} + \cdots + \lambda_{n-2}e^{(n-1)} + y_d^{(n)}(t) - A_0(x) \tag{5.4}$$

In traditional SMC-based FTC, one parameter ρ is designed as the gain of the discontinuous control term to deal with uncertainty in system dynamics and uncertainty caused by partial loss fault together. In the following theorem, two parameters (ρ, γ) are designed in the discontinuous control term to deal with uncertainty in system dynamics and uncertainty caused by fault separately, in the presence of both system uncertainties and faults.

Theorem 5.1 *For the nonlinear system Eq.*(2.23) *with bounded uncertainties Eq.*(5.3)*, sliding manifold Eq.*(2.26) *is asymptotically stable by employing following feedback control:*

$$u = u_n + u_{ss} + u_{sg} \tag{5.5}$$

where

$$u_n = B_0^{-1}\psi_0 \tag{5.6}$$

is the nominal control with knowledge of a priori nominal model and

$$u_{ss} = B_0^{-1}\rho\,\mathrm{sign}(s) \tag{5.7}$$

$$u_{sg} = B_0^{-1}\gamma|\psi_0|\mathrm{sign}(s) \tag{5.8}$$

are the discontinuous control terms. The design parameters ρ and γ are chosen as following:

$$\rho > \frac{F}{1-L} \tag{5.9}$$

$$\gamma > \frac{L}{1-L} \tag{5.10}$$

Proof:

Define a Lyapunov function as:

$$V = \frac{1}{2}s^2 \tag{5.11}$$

The derivative of the above function is:

$$\begin{aligned} \dot{V} &= -s\Delta A - s\Delta B B_0^{-1}\psi_0 - \rho|s| - \Delta B B_0^{-1}\rho|s| \\ &\quad -\gamma|\psi_0||s| - \Delta B B_0^{-1}\gamma|\psi_0||s| \end{aligned} \tag{5.12}$$

Using the uncertainty bound assumption Eq.(5.3) obtains:

$$\dot{V} < -[\rho(1-L) - F]|s| - [\gamma(1-L) - L]|\psi_0||s| \tag{5.13}$$

If the two design parameters ρ and γ are selected as in Eq.(5.9) and Eq.(5.10), it can be concluded that:

$$\dot{V} < 0$$

which shows the sliding manifold Eq.(2.26) is asymptotically stable. Thus the system can asymptotically track the desired reference $y_d(t)$. ■

Remark 5.1 *Here we use two design parameters* (ρ, γ) *to deal with the uncertainties in system dynamics and control distribution gain separately with only the information of the upper bounds of the uncertainties* (F, L). *Compared with traditional SMC, an extra design parameter* γ *is introduced into the discontinuous control term to accommodate the uncertainty caused by actuator faults separately from the uncertainty in system dynamics.*

5.3 Adaptive Sliding Mode Fault Tolerant Control

For some applications, not only the precise system model is hard to obtain, but also the uncertainty bound is hard to know in advance. This is obvious in FTC system. The faults may occur at uncertain time and with unknown magnitudes. Hence for partial loss fault in actuator, the change in control distribution gain of $\Delta B(x)$ is not available in advance. Adaptive method can be introduced into SMC to accommodate the unknown uncertainty bound of the system dynamics, and also partial loss fault in actuators. The uncertainty bounds in system dynamics are considered to be nominal, so the adaptive method can be used. In FTC system, partial loss fault in actuator occurs at certain unknown time but the fault magnitude will be kept invariable after the occurrence of the fault. Hence, the uncertainty in the control distribution gain caused by partial loss fault is bounded by a constant after the fault occurs. An adaptive SMC synthesized with two design parameters in the discontinuous control term is proposed for the nonlinear system with system dynamic uncertainties and actuator faults. Two adaptive laws are designed to estimate separately the uncertainty bounds of system dynamics and control distribution gain. This method avoids significant control effort in the methods that the combined uncertainty bound is used. This is the design philosophy of efficient FTC: when no fault occurs, no extra control effort will be exerted.

The adaptive SMC synthesized with two design parameters in the discontinuous control term is summarized as follows.

Theorem 5.2 *For the nonlinear system Eq.*(2.23) *under control of Eq.*(5.5)*, sliding manifold Eq.*(2.26) *is asymptotically stable utilizing the following adaptive laws:*

$$\begin{aligned} \dot{\rho} &= a_\rho |s| \\ \dot{\gamma} &= a_\gamma |s||\psi_0| \end{aligned} \tag{5.14}$$

where a_ρ and a_γ are adaptive rates.

Proof:

Define parameter errors as:

$$\begin{aligned} \tilde{\rho} &= \rho_b - \rho \\ \tilde{\gamma} &= \gamma_b - \gamma \end{aligned} \tag{5.15}$$

where

$$\rho_b = \frac{F}{1-L}, \gamma_b = \frac{L}{1-L} \tag{5.16}$$

$F > 0$ and $L > 0$ are defined as in Eq.(5.3).

Choose a Lyapunov function as:

$$V = \frac{1}{2}s^2 + \frac{1}{2}\frac{1-L}{a_\rho}\tilde{\rho}^2 + \frac{1}{2}\frac{1-L}{a_\gamma}\tilde{\gamma}^2 \tag{5.17}$$

The derivative of this Lyapunov function with respect to time is:

$$
\begin{aligned}
\dot{V} &= -s\Delta A - s\Delta BB_0^{-1}\psi_0 - \rho|s| - \Delta BB_0^{-1}\rho|s| \\
&\quad -\gamma|\psi_0||s| - \Delta BB_0^{-1}\gamma|\psi_0||s| \\
&\quad -(\rho_b - \rho)|s|(1-L) - (\gamma_b - \gamma)|\psi_0||s|(1-L)
\end{aligned}
\tag{5.18}
$$

All the symbols except the adaptive rates used here are as defined in Section 5.2. With Eq.(5.3), Eq.(5.18) can be rewritten as:

$$
\begin{aligned}
\dot{V} &< |s|F + L|s||\psi_0| - (1-L)\rho|s| - (1-L)\gamma|\psi_0||s| \\
&\quad -(\rho_b - \rho)|s|(1-L) - (\gamma_b - \gamma)|\psi_0||s|(1-L)
\end{aligned}
$$

Based on Eq.(5.16):

$$\dot{V} < 0$$

which shows the sliding manifold Eq.(2.26) with the adaptive sliding mode algorithm is asymptotically stable. Thus the system can asymptotically track the desired reference $y_d(t)$. ■

Remark 5.2 *The adaptive SMC algorithm proposed here is synthesized with two adaptive laws to estimate two parameters* (ρ, γ)*, which clearly separated the uncertainties in system dynamics and in control distribution gain. Thus the synthesization is clearly aimed and the control effort is greatly reduced when there is small uncertainty in control distribution gain. The control effort reduction is more significant in the FTC system where we can deal separately with uncertainty of system dynamics and the uncertainty caused by the partial loss fault in actuator.*

5.4 Simulation and Evaluation

The longitudinal motion considered is to track a pitch angle command θ_d in the presence of both partial loss fault of elevator and system dynamics uncertainties. The control was synthesized with the fitted nonlinear longitudinal model of Boeing 747-100/200 and the simulation was carried out on the nonlinear longitudinal model of Boeing 747-100/200 in Chapter 3.

The reference signals come from following prefilter:

$$\ddot{\theta}_d + 3\dot{\theta}_d + 4\theta_d = 4\theta^\star \tag{5.19}$$

where $\theta^{\star}$ changes from 0 to 0.1 rad at 5 second, and goes back to 0 at 10 second, then changes to – 0.1 rad at 15 seconds, and goes back to 0 at 20 seconds and last for 5 seconds. This pattern will repeat in the next 25 seconds. Partial loss of effectiveness in elevator occurs at 25 seconds and there is certain uncertainties in system dynamics $A(x)$. To make it simple, multiplicative uncertainty is used in the simulation, e.g., 10% uncertainty in system dynamics means that the nominal system dynamics used in the control synthesization is $(1 - 10\%)A$. Three cases are simulated. In the first case there is 10% uncertainty in system dynamics and 50% partial loss fault will occur at 25 seconds. In the second case there is 10% uncertainty in system dynamics and 70% partial loss fault will occur at 25 seconds. In the third case there is 20% uncertainty in system dynamics and 50% partial loss fault will occur at 25 seconds.

Case 1: 10% uncertainties in system dynamics, 50% partial loss fault

Choose $\lambda = 1$, $F = 0.02$, $L = 0.5$, $\rho = 0.04$, $\gamma = 1$ in the proposed sliding mode algorithm. For comparison, the traditional SMC with only one design parameter in the discontinuous control term had been implemented with $\rho = 0.166$.

Figure 5.1 shows the tracking performance and the control output with the proposed SMC (PSMC) and the traditional SMC (TSMC) simulated on the nonlinear longitudinal model of Boeing 747-100/200. The solid line is the desired pitch angle profile, the dash dot line is the pitch angle output of the system with the proposed SMC and the dotted line is the pitch angle output of the system with the traditional SMC. The bottom plot in the figure shows the control effort of the elevator. The solid line is the control output with the proposed SMC and the dotted line is the control output with the traditional SMC.

The simulation results shows that, with both the proposed a SMC method and the traditional SMC, the aircraft can track the pitch angle command profile with a small tracking error even when there is uncertainty in the system dynamics and partial loss fault in the elevator. The tracking performance is quantified by Root Mean Square Error (RMSE) of the tracking error of pitch angle shown in Table 5.1. Compared with the traditional SMC method, the control effort is greatly reduced in proposed SMC, which can be observed in Figure 5.1 and is quantified by the Root Mean Square (RMS) of the control effort as in Table 5.1. In the figure, θ_T and δ_{eT} mean the pitch angle and control output of the system under control of traditional SMC.

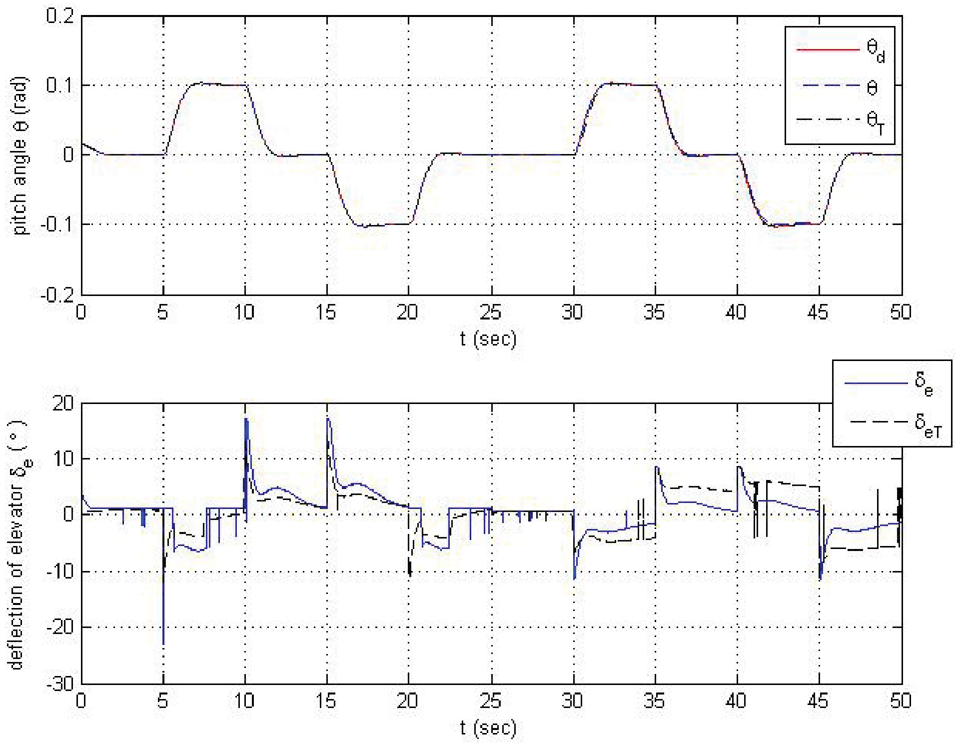

Figure 5.1: Tracking performance using SMCs with 10% uncertainty in system dynamics and 50% loss of effectiveness in elevator

In the proposed adaptive SMC method, $\lambda = 3$, $a_\rho = 0.3$, $a_\gamma = 5$ were chosen. For comparison, the traditional adaptive SMC with only one estimated parameter in the discontinuous control term had been implemented with adaptive rates as $a_\rho = 1$.

The tracking performance with the adaptive SMC methods is shown on top plot of Figure 5.2 and the control effort of the elevator is shown on the bottom plot of Figure 5.2. The adaptive parameter ρ and γ are illustrated in Figure 5.3. ρ_T means the variation of ρ in the simulation with the traditional adaptive SMC. The simulation results show that the adaptive SMC algorithms can still track the desired command without using the exact uncertainty bounds. Compared with the traditional adaptive SMC method, the control effort is reduced in the proposed adaptive SMC, which can be observed in Figure 5.2 and is quantified in Table 5.1. Compared with the nonadaptive algorithm, the control effort is less, which can be observed from Figure 5.1 and Figure 5.2 and the RMS value is shown in Table 5.1. The estimated parameters converge to some values.

Case 2: 10% **uncertainties in system dynamics,** 70% **partial loss fault**

It is shown in Figure 5.4, with the nonadaptive SMC, when the partial loss fault was increased to 70% loss of effectiveness in the elevator, i.e., only 30% of the control surface is in effective, the system cannot track the desired trajectory. However, the adaptive SMC can still track the desired trajectory although there is certain tracking errors in the initial stage after fault occurrence at 25 sec. In the figure, θ_a and δ_{ea} mean the pitch angle and control output of the system under control of adaptive SMC. Figure 5.5 shows the variation of the estimated parameters in adaptive SMC which converge to some values.

Case 3: 20% **uncertainties in system dynamics,** 50% **partial loss fault**

It is shown in Figure 5.6, with the nonadaptive SMC, when the uncertainty in system dynamics was increased to 20%, the tracking performance is bad. However, with the adaptive SMC, the system can still track the desired trajectory with acceptable performance. The estimated parameters in the adaptive SMC, which are shown in Figure 5.7, converge to some values.

All the three cases show that the proposed algorithms can accommodate partial loss fault in the actuator and uncertainty in the system dynamics without losing significant performance in normal

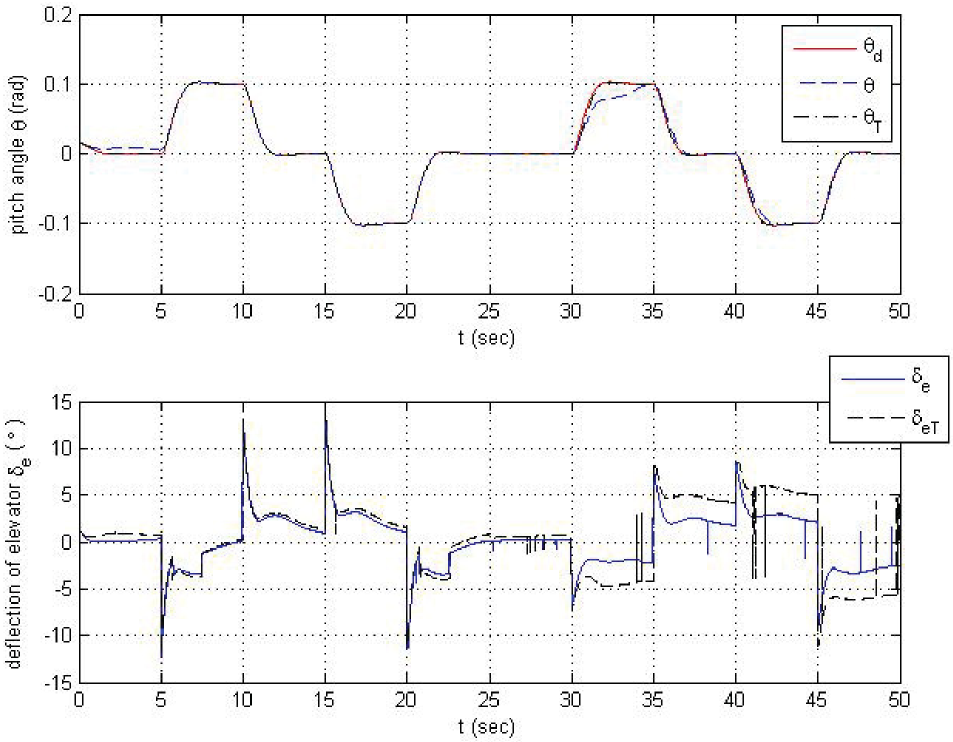

Figure 5.2: Tracking performance using adaptive SMCs with 10% uncertainty in system dynamics and 50% loss of effectiveness in elevator

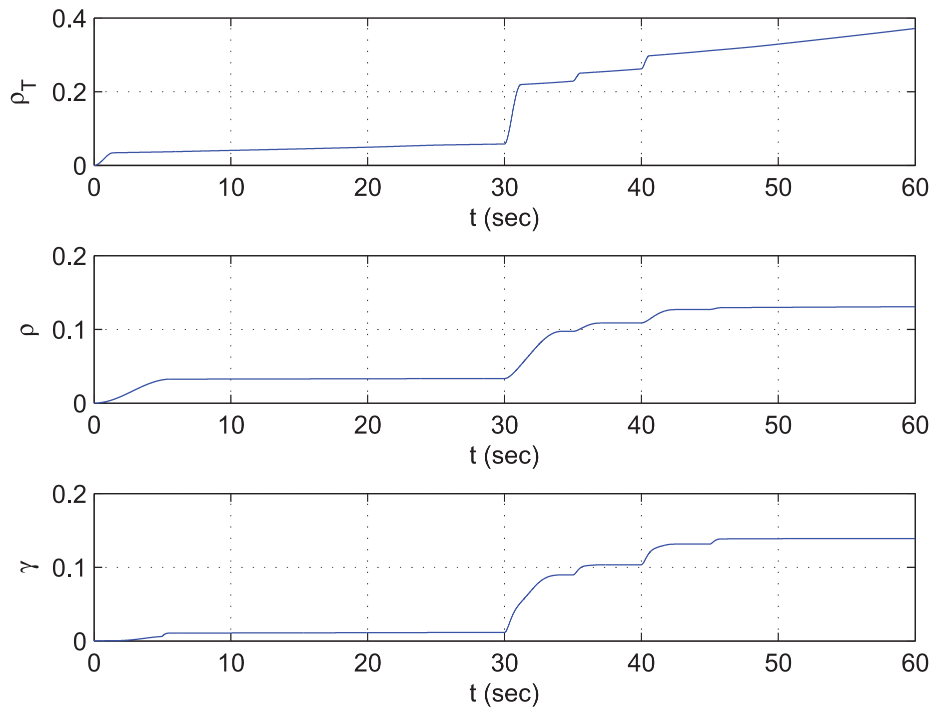

Figure 5.3: Adaptive parameters ρ and γ used in adaptive SMCs with 10% uncertainty in system dynamics and 50% loss of effectiveness in elevator

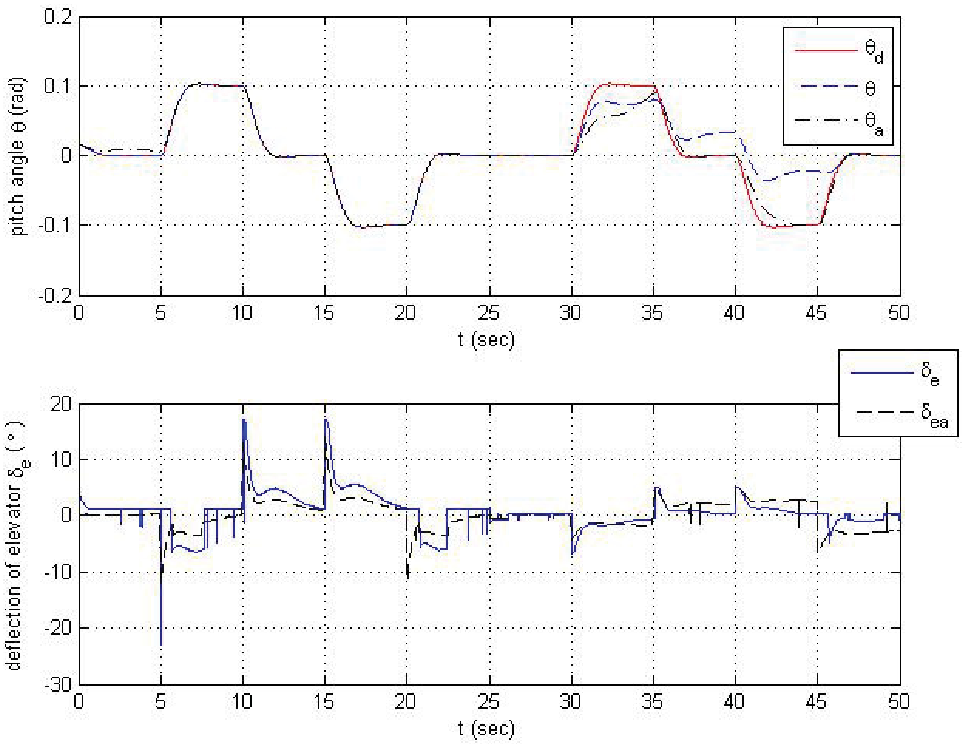

Figure 5.4: Tracking performance using proposed SMCs with 10% uncertainty in system dynamics and 70% loss of effectiveness in elevator

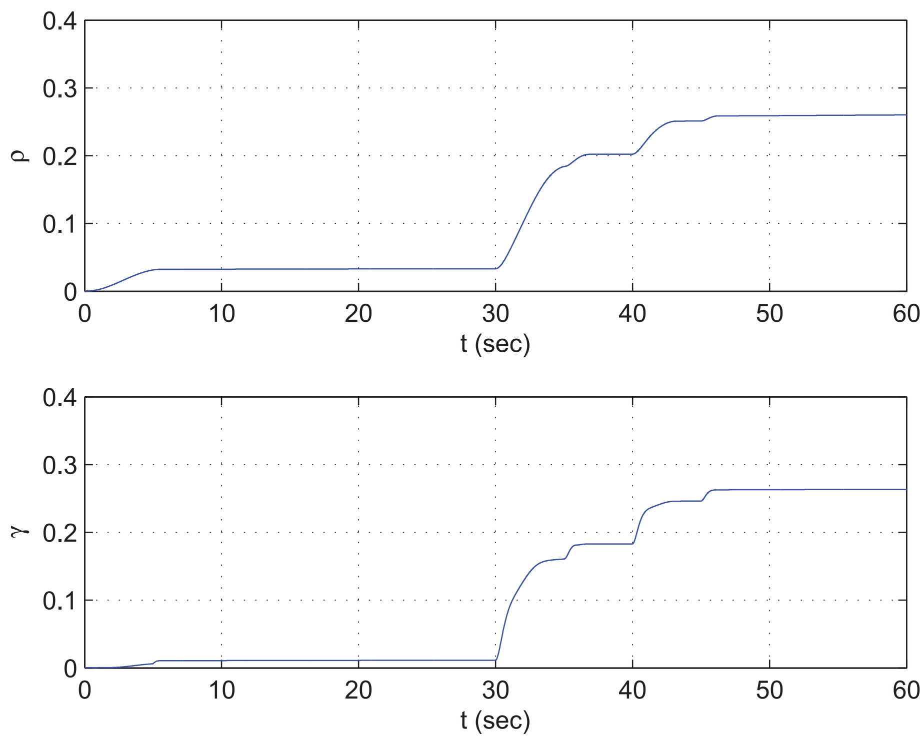

Figure 5.5: Adaptive parameters ρ and γ used in proposed adaptive SMC with 10% uncertainty in system dynamics and 70% loss of effectiveness in elevator

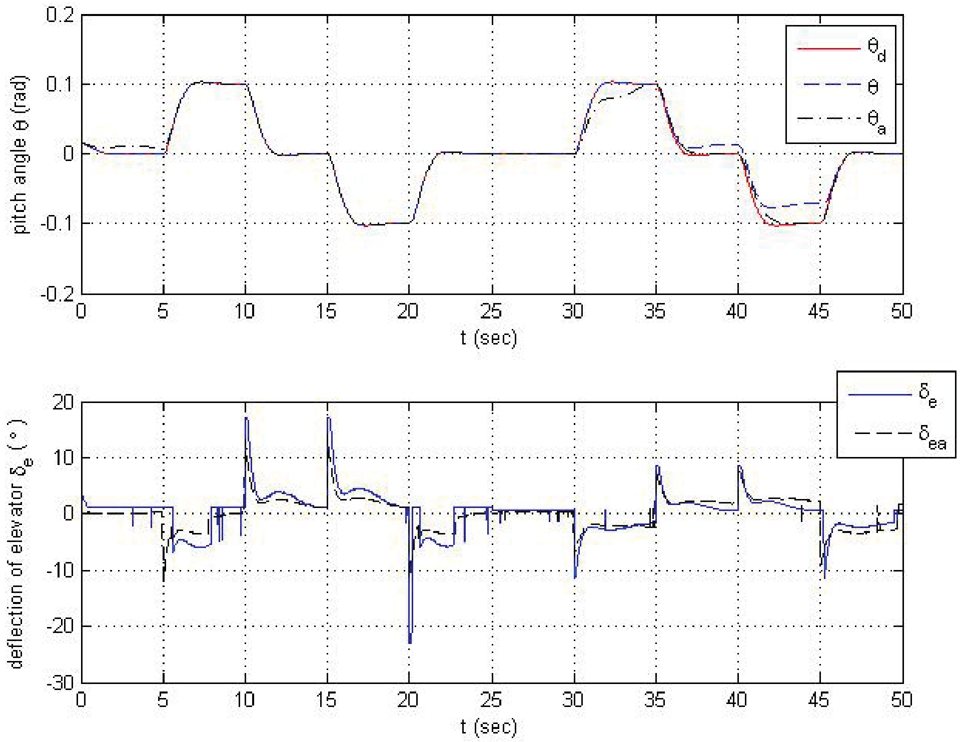

Figure 5.6: Tracking performance using proposed SMCs with 20% uncertainty in system dynamics and 50% loss of effectiveness in elevator

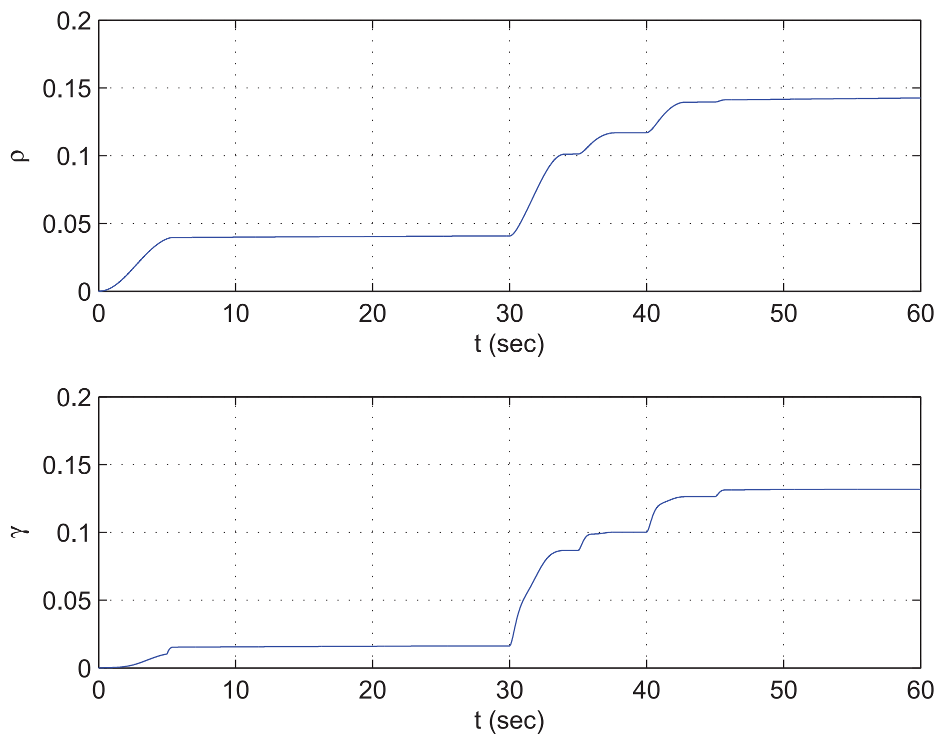

Figure 5.7: Adaptive parameters ρ and γ used in proposed adaptive SMC with 20% uncertainty in system dynamics and 50% loss of effectiveness in elevator

Table 5.1: RMSE of pitch angle's tracking error and RMS of control effort (PA: pitch angle, TE: tracking error, AF: after fault, CE: control effort)

Control algorithm			RMSE of PA's TE (rad)	RMSE of PA's TE AF (rad)	RMS of CE (°)
Case 1	nonadaptive	TSMC	0.0014	0.0019	5.1482
		PSMC	0.0013	0.0019	3.7238
	adaptive	TSMC	0.0061	0.0081	4.3172
		PSMC	0.0062	0.0081	2.6909
Case 2	nonadaptive	PSMC	0.0250	0.0354	3.2993
	adaptive	PSMC	0.0125	0.0174	2.6358
Case 3	nonadaptive	PSMC	0.0086	0.0122	3.6753
	adaptive	PSMC	0.0064	0.0080	2.7149

situation. The simulation results of the first case show that the proposed SMC can reduce the control effort without sacrificing the tracking performance compared to the traditional SMC with one design parameter in the discontinuous control term. The adaptive SMC can still track the command profile with little degradation of the tracking performance, without the information of the bound of the uncertainty. The second and third cases show that the proposed adaptive SMC can still track the command profile even when the partial loss fault or the uncertainty in system dynamics increases.

5.5 Summary

In this chapter, SMC algorithm is developed with introduction of an extra design parameter in the discontinuous control term to accommodate the uncertainty caused by actuator faults separately from the uncertainty in system dynamics for affine nonlinear system. The controller can deal with these two uncertainties respectively. In addition, an adaptive SMC algorithm, with two estimated parameters concerning the uncertainties of system dynamics and control distribution gain (the fault), is developed without using exact bound values of the uncertainties. Simulations on the nonlinear longitudinal Boeing 747-100/200 airplane show the effectiveness of both algorithms.

Chapter 6

Sliding Mode Reconfigurable Control Using Information of Control Effectiveness of Actuators

In this chapter, a sliding mode reconfigurable control algorithm is developed to deal with nonlinear aircraft system with partial loss fault or total loss failure of actuators. Sliding mode controllers for redundant actuators are combined with those for regular actuators to reconfigure the control system autonomously with the information of effectiveness of the regular actuators. The tolerability of the sliding mode control system for faults is utilized in the reconfigurable control to improve the efficiency of the controller. The stability of the control algorithm is proved with Lyapunov method. The effectiveness of the developed control system has been validated by simulation of the longitudinal control of Boeing 747-100/200 on FTLAB747.

6.1 Introduction

The paper [Alwi and Edwards, 2008a] developed an adaptive sliding mode control method to deal with partial loss faults and stuck failures in actuator system of an aircraft based on linear system model. This is a controller that can only work around the trim points of the aircraft. The redundant actuators are activated when the online estimated discontinuous control magnitude exceeds a certain limit. The regular actuators are discarded completely when the redundant actuators are activated, though the regular actuators may still contribute to the control of the airplane, e.g., when the regular actuators suffer from a severe partial loss faults that saturate the actuators. In this situation, the regular actuator cannot accommodate the faults solely, but they can still contribute to the control of aircraft. [Zhang et al., 2007; Alwi and Edwards, 2008b; Hamayun et al., 2010a,b, 2011] developed control allocation algorithms that reallocates the control efforts with health information of actuators. This method will redistribute the control efforts even under the situations where the regular actuators can still accommodate the partial loss faults, i.e., the reallocation will start even though the regular actuator can deal with the fault solely.

In the last chapter, a sliding mode fault tolerate control is developed to deal with fault and modeling uncertainty separately. The method can only deal with partial loss fault the below some level. As shown in Figure 6.1, when the partial loss fault increase to 90% the controller can not deal with it. And surely, this method cannot deal with failures. In this chapter, a sliding mode reconfigurable control law is developed to accommodate all levels of partial loss fault and total loss failure in the regular actuators without redesigning the controller, with benefits of simple and reliable control system design since the baseline controller does not need to be changed on-line for the concern to stability, safety as well as verification & certification in practical engineering practices. The control effort is reconfigured autonomously between the regular actuator and the redundant actuator when the regular actuator with fault cannot accommodate the fault solely. The difference between this method and the work of [Alwi and Edwards, 2008a] is that the regular actuator will still contribute to the fault tolerant control when the regular actuator does not totally fail, but loses its effectiveness partially. The redundant actuator is not designed for having the same features or capabilities as the regular actuator, such as stabilizer and elevator, since stabilizer is normally used for airplane trimming and its moving rate is much slower than that of the elevator, as shown in Table 4.1. It is better to use the regular actuator when it still can contribute to the control of the airplane since the highest priority of an aircraft is to try all possible solutions and available control resources for keeping the aircraft to be controlled and landed safely. The method of this thesis work is different from [Alwi and Edwards, 2008b; Hamayun et al., 2010a,b, 2011] in that it uses the tolerability of the system for regular actuator with sliding mode control to determine when the reconfiguration of the control effort starts, i.e., the redundant actuators are only started when the regular actuators cannot deal with faults solely. Faults information (effectiveness of certain actuators) is needed in the development of the fault tolerant controller. This information can be obtained from certain sensors installed on actuators [Alwi and Edwards, 2008b] or by using certain fault magnitude estimation scheme. The stability of the designed controller is proved by using Lyapunov method. The effectiveness of the control algorithm is simulated and validated on longitudinal control of Boeing 747 under platform FTLAB747.

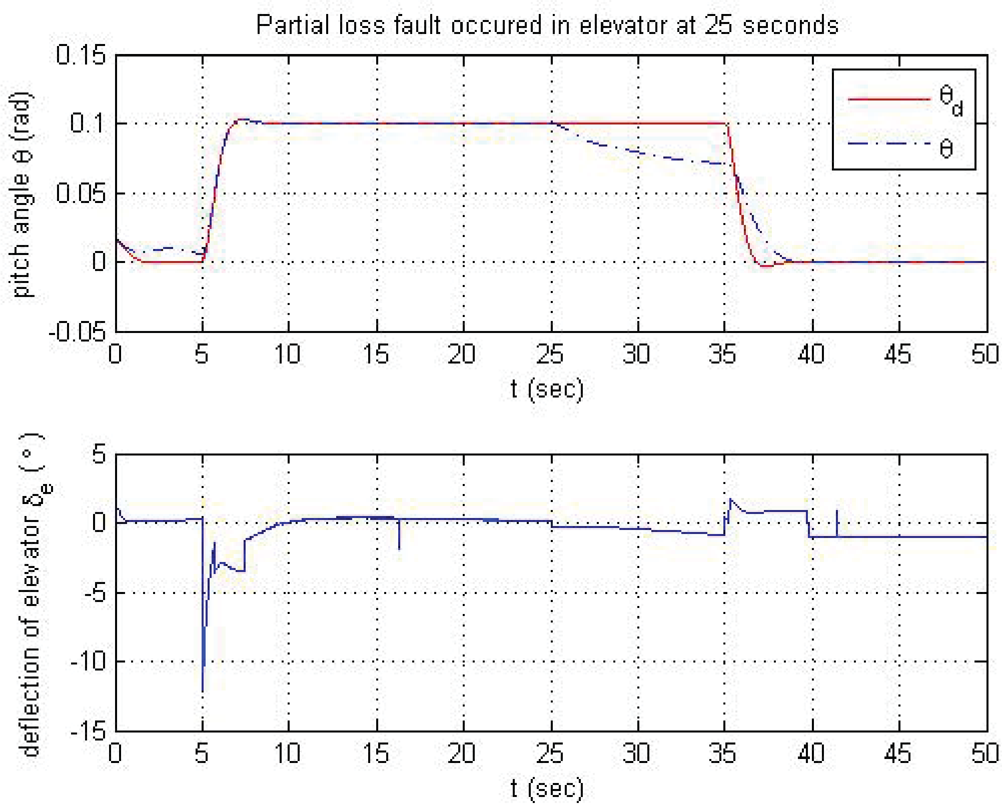

Figure 6.1: Control performance of 90% partial loss fault in elevator

The rest of this chapter is organized as follows. Firstly, the problem this chapter will deal with is formulated. Secondly, a sliding mode reconfigurable control is derived for affine nonlinear systems using Lyapunov method. Thirdly, the simulation results of the longitudinal control of Boeing 747-100/200 on FTLAB747 are given. Summary is given finally.

6.2 Problem Formulation

The flight dynamics of a rigid aircraft can be modeled as affine nonlinear system in Eq.(2.31) where $x \in R^{12}$ is the state vector of the aircraft:

$$x = [p \;\; q \;\; r \;\; V_{TAS} \;\; \alpha \;\; \beta \;\; \phi \;\; \theta \;\; \psi \; h_e \; x_e \; y_e]^T$$

where p, q, r are roll, pitch and yaw angular rates respectively; V_{TAS}, α, β are the true air speed, angle of attack and sideslip angle respectively; ϕ, θ, ψ are roll angle, pitch angle and yaw angle respectively.

Remark 6.1 *As one important family of nonlinear systems, the above affine nonlinear system representation in Eq.* (2.31) *is widely used for airplane modeling. Due to inherent nonlinearity of airplanes, the dynamics of airplane should generally be represented by nonlinear function $f(x)$ with respect to the states x, while the control actions can be approximated as linear addition to the system as $G(x)u$ (although $G(x)$ is nonlinear function with respect to the states x).*

Considering the modeling uncertainty and the control redundancy, system Eq. (2.31) can be reformulated as:

$$\begin{cases} \dot{x} &= f_0(x) + \Delta f(x) + (G_c(x) + \Delta G_c(x))u_c + (G_r(x) + \Delta G_r(x))u_r \\ y &= h(x) \end{cases} \tag{6.1}$$

where $f_0(x)$ is the nominal form of $f(x)$; $G_c(x) \in R^{n \times m}$ is the control input distribution matrix of regular actuators; $G_r(x) \in R^{n \times m}$ is the control input distribution matrix of redundant actuators. $\Delta f(x)$, $\Delta G_c(x)$, $\Delta G_r(x)$ are unknown bounded perturbations which may be caused by modeling uncertainties and faults. u_c is the control effort of regular actuators and u_r is the control effort of redundant actuators.

Remark 6.2 *When there are no redundant actuators for certain regular actuators, the output of redundant actuators will be zero. If there are more than one redundant actuators for one regular actuator, then allocation algorithms are needed. In this situation, the control output for these redundant actuators can still be synthesized as 'one' control output.*

Problem 6.1 *Given a real-time command reference profile $y_d(t)$, a SMC for aircraft system Eq.* (6.1) *is designed so that the system can track the command reference profile, even when there are faults/failures in the actuators, with tracking errors asymptotically converged to zero:*

$$\lim_{t\to\infty} |y_{d_i} - y_i| = 0, \ \forall \ i = 1, 2, \cdots, m \tag{6.2}$$

6.3 Sliding Mode Reconfigurable Control

From the robustness analysis of the method in Chapter 4, it can be seen that sliding mode control can accommodate modeling errors and certain partial loss faults in one regular actuator. When there is severe partial loss faults that saturate the regular actuators significantly, the regular actuator can not deal with them solely. Furthermore, when there is a total failure such as stuck or floating in the regular actuator, it has no capability to deal with it. From the tolerability analysis in Chapter 4, there is a point where the regular actuator with SMC cannot accommodate severe partial loss fault alone. For stuck and floating faults, the actuators lost their effectiveness totally, and even worse the outputs of the regular actuators become constant or time-varying disturbances added to the system. In the first situation, the regular actuator can still contribute to the fault tolerant control of the aircraft, and it can work together with redundant actuators to accommodate the faults. In the second situation, the redundant actuators will replace the regular actuators completely. In this section, a method that can reconfigure the controller autonomously when these two kinds of faults occur in the system by utilizing the effectiveness information of the regular actuators will be developed.

Rewrite the companion format Eq. (2.32) into the format of Eq. (6.1), i.e., rewrite $E(x)u$:

$$E(x)u = E_c(x)u_c + E_r(x)u_r = [E_{c0}(x) + \Delta E_c(x)]u_c + [E_{r0}(x) + \Delta E_r(x)]u_r \tag{6.3}$$

where $E_c(x)$, $E_r(x)$ are the control input distribution of regular actuators and redundant actuators; $E_{c0}(x)$, $E_{r0}(x)$ are the nominal value of control input distribution of regular actuators and redundant actuators; $\Delta E_c(x)$, $\Delta E_r(x)$ are bounded uncertain matrix of the nominal control input distribution of regular actuators and redundant actuators.

The control effectiveness of the regular actuators is defined in Eq. (4.16). It can be obtained from fault detection and diagnosis block or from certain sensors installed on the control surfaces.

With this effectiveness information of the regular actuators, Eq. (6.3) can be rewritten as:

$$E(x)u = [E_{c0}(x) + \Delta E_c(x)]W_c u_c + [E_{r0}(x) + \Delta E_r(x)]u_r \tag{6.4}$$

Following theorem can be obtained for the sliding mode reconfigurable controller that can accommodate not only partial loss but also total loss of regular actuators.

Theorem 6.1 *For the nonlinear system Eq.* (2.32) *and Eq.* (6.4)*, sliding surface Eq.* (2.34) *is asymptotically stable by employing the following feedback control law:*

$$u_c = E_{c0}(x)^{-1}(\Psi_0(x,t) + R\cdot\Sigma) \tag{6.5}$$

$$u_r = \begin{cases} u_{rtrim} & if\ \ w_{ci} \geq w_{c_{itolerable}} \\ E_{r0}(x)^{-1}(I_m - W_c)(\Psi_0(x,t) + R\cdot\Sigma) & if\ \ w_{ci} < w_{c_{itolerable}} \end{cases} \tag{6.6}$$

even when there are partial loss fault or total failure in regular actuators. The uncertainty of control input distribution meets the following bounds:

$$|(\Delta E_c E_{c0}^{-1})_{ii}| < D_i < 1,\ \ |(\Delta E_r E_{r0}^{-1})_{ii}| < D_i < 1,\ \ \forall\ i = 1, 2, \cdots, m \tag{6.7}$$

Proof: If $w_{ci} \geq w_{ci_{tolerable}}$, it is evident from Theorem 4.1.

If $w_{ci} < w_{ci_{tolerable}}$, choose a Lyapunov function as:

$$V = \frac{1}{2}s^T s \tag{6.8}$$

The time derivative of the Lyapunov function Eq. (6.8) can be obtained as following:

$$\begin{aligned}
\dot{V} &= s^T\dot{s} \\
&= s^T[\Psi_0 + \Delta\Psi - (I_m + \Delta E_c E_{c0}^{-1})W_c(\Psi_0 + R\cdot\Sigma) - (I_m + \Delta E_r E_{r0}^{-1})(I_m - W_c)(\Psi_0 + R\cdot\Sigma)] \\
&= s^T[\Phi_c - (I_m + \Delta E_c E_{c0}^{-1})W_c R\cdot\Sigma - (I_m + \Delta E_r E_{r0}^{-1})(I_m - W_c)R\cdot\Sigma] \\
&= \textstyle\sum_{i=1}^m\{(\Phi_c)_i s_i - [1 + (\Delta E_c E_{c0}^{-1})_{ii}]w_{ci}\rho_i|s_i| - [1 + (\Delta E_r E_{r0}^{-1})_{ii}](1 - w_{ci})\rho_i|s_i|\}
\end{aligned}$$

where $\Phi_c = \Delta\Psi - \Delta E_c E_{c0}^{-1}W_c\Psi_0 - \Delta E_r E_{r0}^{-1}(I_m - W_c)\Psi_0$.

Assume that the uncertainty of system dynamics meets the following bounds:

$$|[\Phi_c(x,t)]_i| \leq L_{ci},\ \ \forall\ i = 1, 2, \cdots, m \tag{6.9}$$

where $L_{ci},\ \forall\ i = 1, 2, \cdots, m$ are some positive constants.

Choose the design parameter as:

$$\rho_i > \frac{L_{ci}}{1 - D_i}, \ \forall\ i = 1, 2, \cdots, m \tag{6.10}$$

Then the derivative of V meets:

$$\begin{aligned}
\dot{V} &\leq \textstyle\sum_{i=1}^{m}\{L_i|s_i| - [1 - |(\Delta E_c E_{c0}^{-1})_{ii}|]w_{ci}\rho_i|s_i| - [1 - |(\Delta E_r E_{r0}^{-1})_{ii}|](1 - w_{ci})\rho_i|s_i|\} \\
&\leq \textstyle\sum_{i=1}^{m}[L_i|s_i| - (1 - D_i)w_{ci}\rho_i|s_i| - (1 - D_i)(1 - w_{ci})\rho_i|s_i|] \\
&= \textstyle\sum_{i=1}^{m}[L_{ci} - (1 - D_i)\rho_i]|s_i| \\
&\leq 0
\end{aligned}$$

The sliding surface Eq. (2.34) is then asymptotically stable. ■

6.4 Simulation Results

The longitudinal motion is considered to track a pitch angle command θ_d even in the presence of partial loss or total loss of effectiveness of elevator. The control law was synthesized with the fitted approximate longitudinal model of Boeing 747-100/200 and the simulation was carried out on FTLAB747.

The reference signals come from following prefilter:

$$\ddot{\theta}_d + 3\dot{\theta}_d + 4\theta_d = 4\theta^\star \tag{6.11}$$

where $\theta^\star$ changes from 0 to 0.1 rad at 5 second, and goes back to 0 at 10 second, then changes to -0.1 rad at 15 seconds, and then goes back to 0 at 20 seconds and lasts for 5 seconds. This pattern will repeat every 25 seconds.

The sliding surface is chosen as:

$$s = \dot{\theta}_d - q + \lambda(\theta_d - \theta) \tag{6.12}$$

Assume $\Delta E = 0.4E_0$, then $\Delta M = 0.4$ and $D = 1 + \Delta M = 1.4$. The focus of this chapter is on how the faults and failures affect the control system, therefore it is assumed that there is no uncertainty in $f(x)$. From the trim values, it can be calculated that $|\Phi| < 0.7073$, so $L = 0.5052$.

Choose $\lambda = 3$, $\rho = 0.53$ and $w_{c_{tolerable}} = 0.4$ and three sliding mode reconfigurable control algorithms are simulated: the one proposed in this chapter, the one with control allocation [Alwi and Edwards, 2008b] and the one proposed in [Alwi and Edwards, 2008a]. Simulation results are shown in Fig. 6.2, Fig. 6.3 and Fig. 6.4. In the figures, subscript CA means the related variable in control allocation sliding mode control algorithm and subscript $switch$ means the related variable in the adaptive sliding mode control proposed in [Alwi and Edwards, 2008a]. Six situations with different types and levels of elevator faults are simulated: 1) normal, 2) 50% partial loss fault, 3) 90% partial loss fault, 4) total failure floating with angle of attack α, 5) total failure stuck at 17° and 6) total failure stuck at $-1°$.

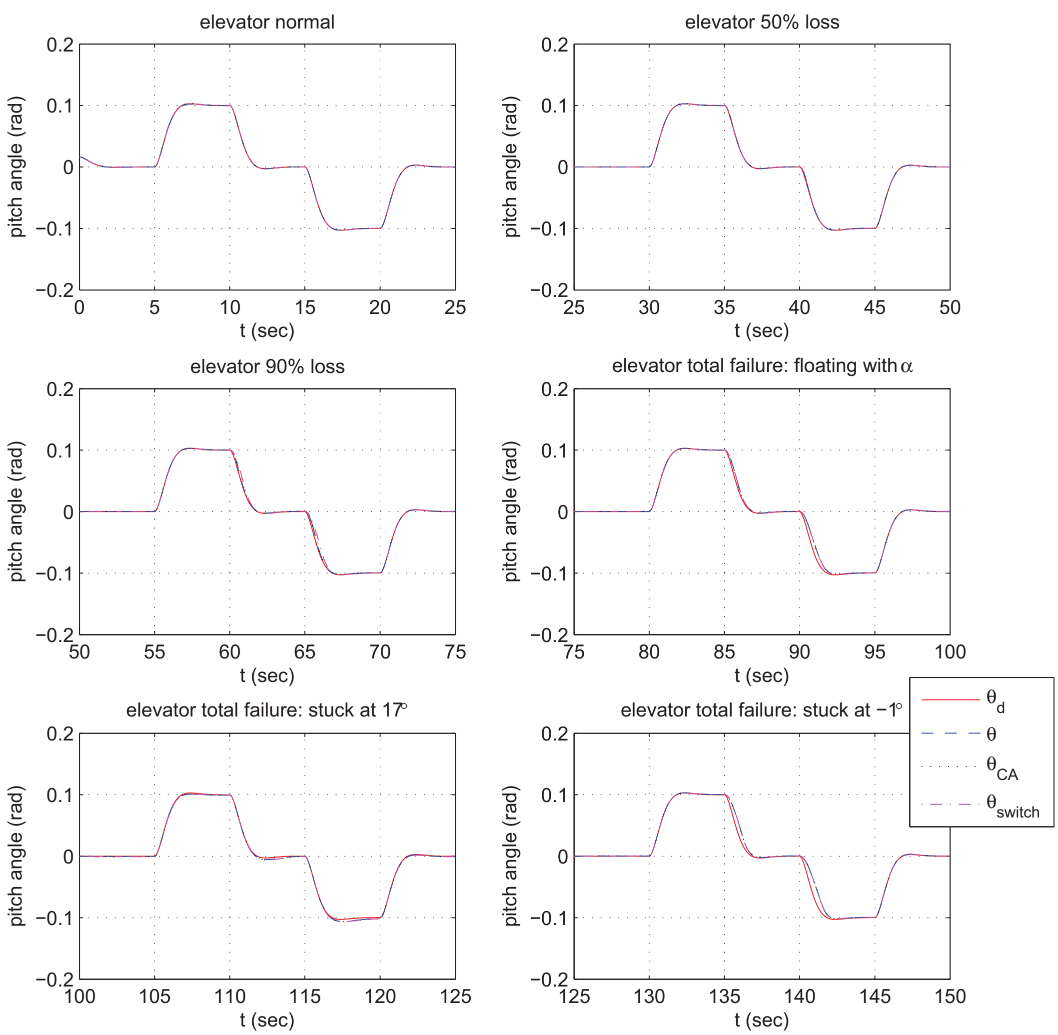

Figure 6.2: Pitch angle tracking under different testing scenarios

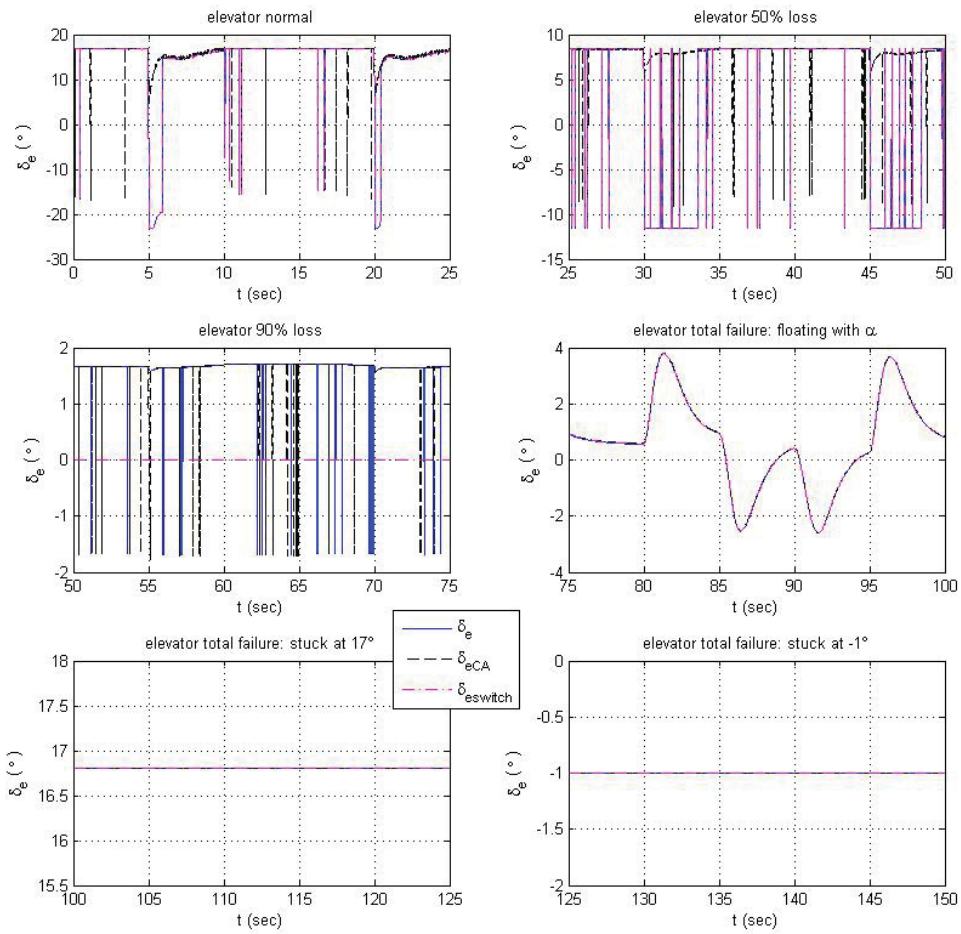

Figure 6.3: Elevator deflection under different testing scenarios

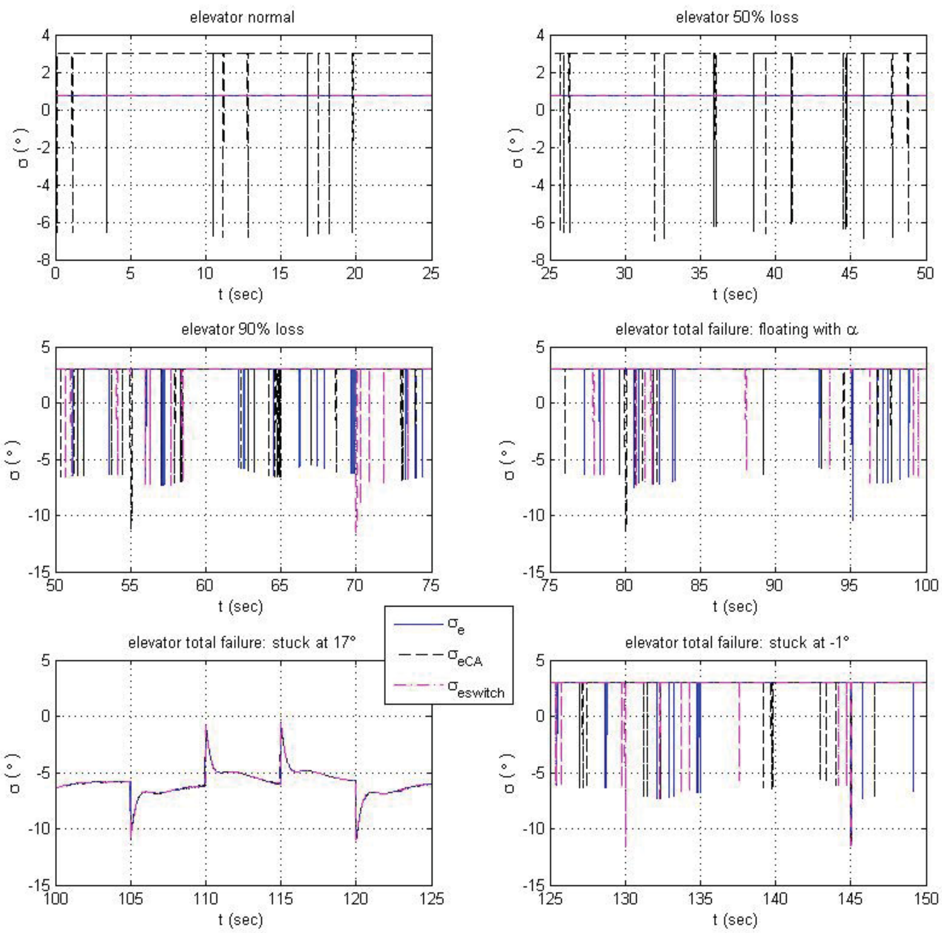

Figure 6.4: Stabilizer deflection under different testing scenarios

Fig. 6.2 shows the tracking performance with the three sliding mode reconfigurable control algorithms. The solid line is the reference profile θ_d. The dash line is the pitch angle simulated on the nonlinear longitudinal model Boeing 747-100/200 with the proposed algorithm in this chapter, the dot line is with the control allocation SMC and the dash-dot line is with the adaptive SMC proposed in [Alwi and Edwards, 2008a]. Fig. 6.3 shows the synthesized control output of elevator and Fig. 6.4 shows the synthesized control output of stabilizer. The dash line in these two figures is the control output with the SMC algorithm proposed in this chapter, the dot line with the control allocation SMC and the dash-dot line with the adaptive SMC presented in [Alwi and Edwards, 2008a].

The simulation results show that, with the three sliding mode reconfigurable controller, the aircraft can track the pitch angle command profile with small tracking error even when there is a partial loss fault or a total loss failure of elevator. With the proposed SMC in this chapter, the control effort is reconfigured autonomously between elevator and stabilizer with control effectiveness information of the elevator when there is a fault in the elevator which is beyond the tolerability of elevator. Contrast to the SMC with control allocation which will activate stabilizer whenever there is a fault, the stabilizer will only be activated beyond tolerability of the regular actuator with SMC. This can be found in Fig. 6.3 and Fig. 6.4. For the adaptive SMC proposed in [Alwi and Edwards, 2008a], when it is beyond the tolerability the stabilizer will be activated and the elevator will discarded completely even though the elevator still can contribute to fault accommodation. This can be observed in Fig. 6.3 and Fig. 6.4. In the design of the plane, stabilizer is not for control surface of pitch control under normal flight conditions, but mainly for trimming purpose. So practically, it is better only to activate stabilizer when the elevator cannot accommodate the fault soly and it is better to use elevator together with stabilizer when it is not completely failed.

6.5 Summary

In this chapter, a sliding mode reconfigurable control algorithm is developed to accommodate partial or total loss of control effectiveness of regular actuator under the assumption that the effectiveness of the regular actuator can be obtained from a fault detection and diagnosis scheme or certain sensors. The redundant actuator is activated autonomously when there is severe partial loss fault that saturates the regular actuator or the regular actuator fails totally. This algorithm is effective in normal situation, partial fault situation and even total failure situation. The stability of the reconfigurable control is proved using Lyapunov method. Simulation validation on the longitudinal control of Boeing 747-100/200 shows the effectiveness of the developed algorithm.

Chapter 7

Sliding Mode Reconfigurable Fault Tolerant Control for Nonlinear Aircraft Systems without FDD

In Chapter 6, a sliding mode reconfigurable controller using the information of actuator effectiveness from special sensors or an FDD scheme which are costly and are not always available, was proposed. In this chapter, a sliding mode reconfigurable control is developed to accommodate partial loss fault and total failure occurred in regular actuators without using explicit knowledge of the faults/failures. With the proposed reconfiguration control, the system even does not 'notice' the faults or failures. No fault detection and identification module (compared with the active fault tolerant control method [Song et al., 2003]) or special sensors is needed. The control is reconfigured autonomously by monitoring the switching function. The synthesis of sliding mode control on regular and redundant actuators are 'combined' or 'integrated' into one procedure. The redundant actuators will start to work together with the regular actuators when the regular actuators cannot suppress the tracking error to the defined boundary due to total failures or partial loss faults that saturate the regular actuators.

7.1 Sliding Mode Reconfigurable Control without Dedicated FDD

Considering the same Problem 6.1 in Chapter 6. With the nonlinear airplane Eq.(2.32), Eq.(6.1) and Eq.(6.3), we have the following theorem for the sliding mode reconfigurable controller that can accommodate partial loss and total failure of regular actuators such as stuck and floating without explicit knowledge of faults/failures.

Theorem 7.1 *For nonlinear system Eq.*(2.32)*, Eq.*(6.1) *and Eq.*(6.3)*, sliding manifold Eq.*(2.34) *is asymptotically stable by employing the following feedback control*

$$u_c = \frac{1}{2}E_{c0}(x)^{-1}(\Psi_0(x,t) + R\cdot\Sigma)(-P + I_m) \tag{7.1}$$

$$u_r = \frac{1}{2}E_{r0}(x)^{-1}(\Psi_0(x,t) + R\cdot\Sigma)(P + I_m) \tag{7.2}$$

where

$$P = diag\{sign(|s_1| - \epsilon_1), sign(|s_2| - \epsilon_2), \cdots, sign(|s_m| - \epsilon_m)\}$$

$\epsilon_1, \epsilon_2, \cdots, \epsilon_m$ *are small positive constants. This controller works in normal situation, under partial loss fault, and under total failure of regular actuators. The uncertainty of control input distribution meets the following bounds:*

$$|(\Delta E_c E_{c0}^{-1})_{ii}| < D_i < 1, \ \forall\ i = 1, 2, \cdots, m \tag{7.3}$$

$$|(\Delta E_r E_{r0}^{-1})_{ii}| < D_i < 1, \ \forall\ i = 1, 2, \cdots, m \tag{7.4}$$

where D_i*,* $\forall\ i = 1, 2, \cdots, m$*, are some positive constants.*

Proof: Choose a Lyapunov function as:

$$V = \frac{1}{2}s^T s \tag{7.5}$$

In the normal healthy situation or partial loss fault situation (which means the regular actuator can still contribute), taking the time derivative of Lyapunov function Eq.(7.5) and using control

Eq.(7.1)-Eq.(7.2), one obtains:

$$\begin{aligned}
\dot{V} &= s^T\dot{s} \\
&= s^T[\Psi_0 + \Delta\Psi - \frac{1}{2}(I_m + \Delta E_c E_{c0}^{-1})(\Psi_0 + R\cdot\Sigma)(-P+I_m) - \frac{1}{2}(I_m + \Delta E_r E_{r0}^{-1})(\Psi_0 + R\cdot\Sigma)(P+I_m)] \\
&= s^T[\Phi_c - \frac{1}{2}(I_m + \Delta E_c E_{c0}^{-1})R\cdot\Sigma(-P+I_m) - \frac{1}{2}(I_m + \Delta E_r E_{r0}^{-1})R\cdot\Sigma(P+I_m)] \\
&= \sum_{i=1}^{m}\{(\Phi_c)_i s_i - \frac{1}{2}[1 + (\Delta E_c E_{c0}^{-1})_{ii}]\rho_i|s_i|(-P_{ii}+1) - \frac{1}{2}[1 + (\Delta E_r E_{r0}^{-1})_{ii}]\rho_i|s_i|(P_{ii}+1)\} \qquad (7.6)
\end{aligned}$$

where

$$\Phi_c = \Delta\Psi - \frac{1}{2}\Delta E_c E_{c0}^{-1}\Psi_0(-P+I_m) - \frac{1}{2}\Delta E_r E_{r0}^{-1}\Psi_0(P+I_m)$$

It is assumed that the uncertainty of system dynamics meets the following bounds:

$$|[\Phi_c(x,t)]_i| \leq L_{ci},\ \forall\ i = 1,2,\cdots,m \qquad (7.7)$$

where $L_{ci},\ \forall\ i = 1,2,\cdots,m$, are some positive constants.

Choose the design parameter as:

$$\rho_i > \frac{L_{ci}}{1-D_i},\ \forall\ i = 1,2,\cdots,m \qquad (7.8)$$

Then the derivative of V becomes:

$$\begin{aligned}
\dot{V} &\leq \sum_{i=1}^{m}\{L_{ci}|s_i| - \frac{1}{2}[1 - |(\Delta E_c E_{c0}^{-1})_{ii}|]\rho_i|s_i|(-P_{ii}+1) - \frac{1}{2}[1 - |(\Delta E_r E_{r0}^{-1})_{ii}|]\rho_i|s_i|(P_{ii}+1)\} \\
&\leq \sum_{i=1}^{m}[L_{ci}|s_i| - \frac{1}{2}(1-D_i)\rho_i|s_i|(-P_{ii}+1) - \frac{1}{2}(1-D_i)\rho_i|s_i|(P_{ii}+1)] \\
&= \sum_{i=1}^{m}[L_{ci} - (1-D_i)\rho_i]|s_i| \\
&\leq 0 \qquad (7.9)
\end{aligned}$$

This means that the system is stable with the control laws Eq.(7.1)-Eq.(7.2).

When there is a total failure, u_c loses all its effectiveness and becomes disturbance added to the system, i.e., $u_c = U_c(x,t)$, where $U_c(x,t)$ is a bounded function, e.g., if it is a stuck failure, $U_c(x,t)$ will be a constant; if it is a floating failure, $U_c(x,t)$ will be a function of angle of attack which is limited. The control output u_r for regular actuator still keeps as in Eq.(7.2). We assume the bound Eq.(7.4) still keeps. When $|s_i| \geq \epsilon_i,\ \frac{1}{2}(P + I_m) = I_m,\ \frac{1}{2}(-P + I_m) = 0$. Choose a Lyapunov function

$$V = \frac{1}{2} s^T s \tag{7.10}$$

Taking the time derivative of the Lyaponov function and using control law Eq.(7.2), one can obtain:

$$\begin{aligned}
\dot{V} &= s^T \dot{s} \\
&= s^T[\Psi_0 + \Delta\Psi' - (I_m + \Delta E_r E_{r0}^{-1})(\Psi_0 + R \cdot \Sigma)] \\
&= s^T[\Phi_r - (I_m + \Delta E_r E_{r0}^{-1}) R \cdot \Sigma] \\
&= \sum_{i=1}^{m} \{(\Phi_r)_i s_i - [1 + (\Delta E_r E_{r0}^{-1})_{ii}]\rho_i |s_i|\}
\end{aligned} \tag{7.11}$$

where

$$\Phi_r = \Delta\Psi' - \Delta E_r E_{r0}^{-1} \Psi_0$$

$$\Delta\Psi' = \Delta\Psi - E_c(x) U_c(x,t)$$

The uncertainty of system dynamics is assumed to meet the following bounds:

$$|[\Phi_r(x,t)]_i| \leq L_{ri},\ \forall\ i = 1, 2, \cdots, m \tag{7.12}$$

where $L_{ri},\ \forall\ i = 1, 2, \cdots, m$, are some positive constants.

Choose the design parameter as:

$$\rho_i > \frac{L_{ri}}{1 - D_i},\ \forall\ i = 1, 2, \cdots, m \tag{7.13}$$

Then the derivative of V becomes:

$$\begin{aligned}
\dot{V} &\leq \sum_{i=1}^{m} \{L_i |s_i| - [1 - |(\Delta E_r E_{r0}^{-1})_{ii}|]\rho_i |s_i|\} \\
&\leq \sum_{i=1}^{m} [L_{ri} - (1 - D_i)\rho_i]|s_i| \\
&\leq 0
\end{aligned} \tag{7.14}$$

This means the system will converge to the boundary layer $|s_i| = \epsilon_i,\ \forall\ i = 1, 2, \cdots, m$.

We choose

$$\rho_i > max\{\frac{L_{ci}}{1 - D_i}, \frac{L_{ri}}{1 - D_i}\}, \tag{7.15}$$

With the control laws Eq.(7.1) and Eq.(7.2), the system can track the reference profile in all the situations: normal, partial loss fault and total failure. ■

Remark 7.1 *In this algorithm, control outputs are reconfigured autonomously to regular actuators and redundant actuators, using the variation of sliding surface, i.e., the combined state error signal of the system. When there is a big change in the state due to initial condition, big disturbance, big change in the desire state trajectory as well as fault or failures, the redundant actuators will be activated to help the regular actuator goes back to the steady state, or at least in a small boundary in the situation of totally failure in the regular actuators.*

7.2 Simulation Results

The longitudinal motion is considered to track a pitch angle command θ_d with partial loss fault and total failure of elevator. The control was synthesized with the fitted approximate longitudinal model of Boeing 747-100/200 and the simulation was done on FTLAB747. The redundant actuator is stabilizer.

The reference signals are generated by the following prefilter:

$$\ddot{\theta}_d + 3\dot{\theta}_d + 4\theta_d = 4\theta^\star \tag{7.16}$$

where $\theta^\star$ changes from 0 to 0.1 rad at 5 second, and goes back to 0 at 10 second, then changes to -0.1 rad at 15 seconds, and goes back to 0 at 20 seconds.

The switching surface is chosen as:

$$s = \dot{\theta}_d - q + \lambda(\theta_d - \theta) \tag{7.17}$$

Choose $\lambda = 3$, $\rho = 0.53$, $\epsilon = 0.01$ in the control algorithm. Simulation results are shown in Fig. 7.1 (the tracking performance with the sliding mode reconfigurable control), Fig. 7.2 (the synthesized control output of elevator), Fig. 7.3 (the synthesized control output of stabilizer) and Fig. 7.4 (the switching function). Six situations with respect to elevator operating condition are simulated: health, 50% partial loss fault, 90% partial loss fault, floating with angle of attack, lock at 17° and $-1°$. All the faults/failures occur at 10 second.

The simulation results shows that, with the designed sliding mode reconfigurable controller, the aircraft can track the pitch angle command profile with small tracking error even when there is partial loss and total failure on the regular actuator: elevator.

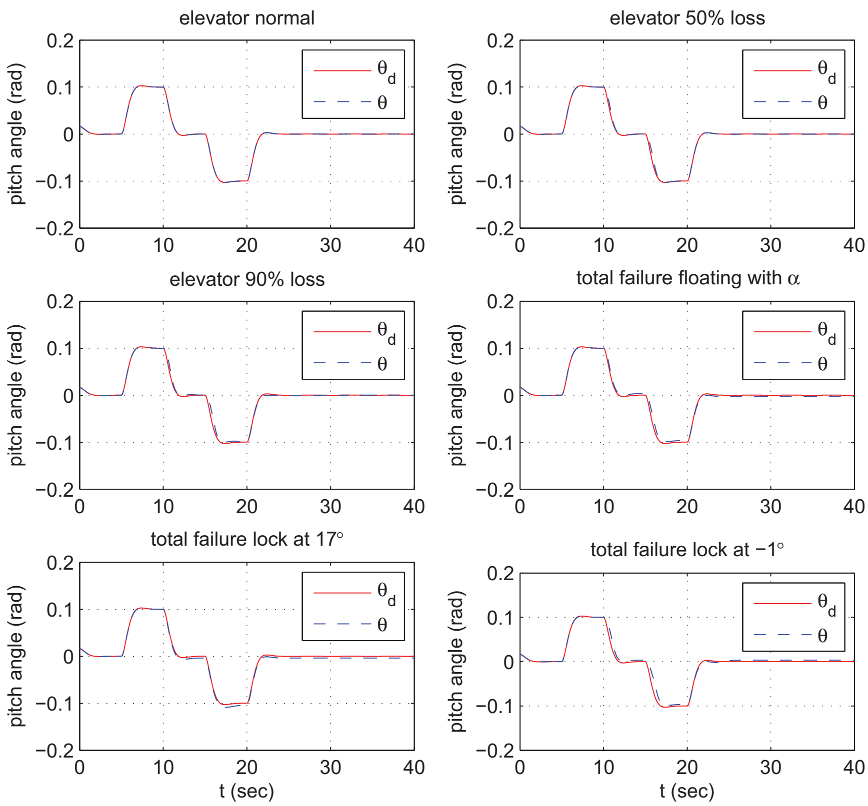

Figure 7.1: Pitch angle tracking

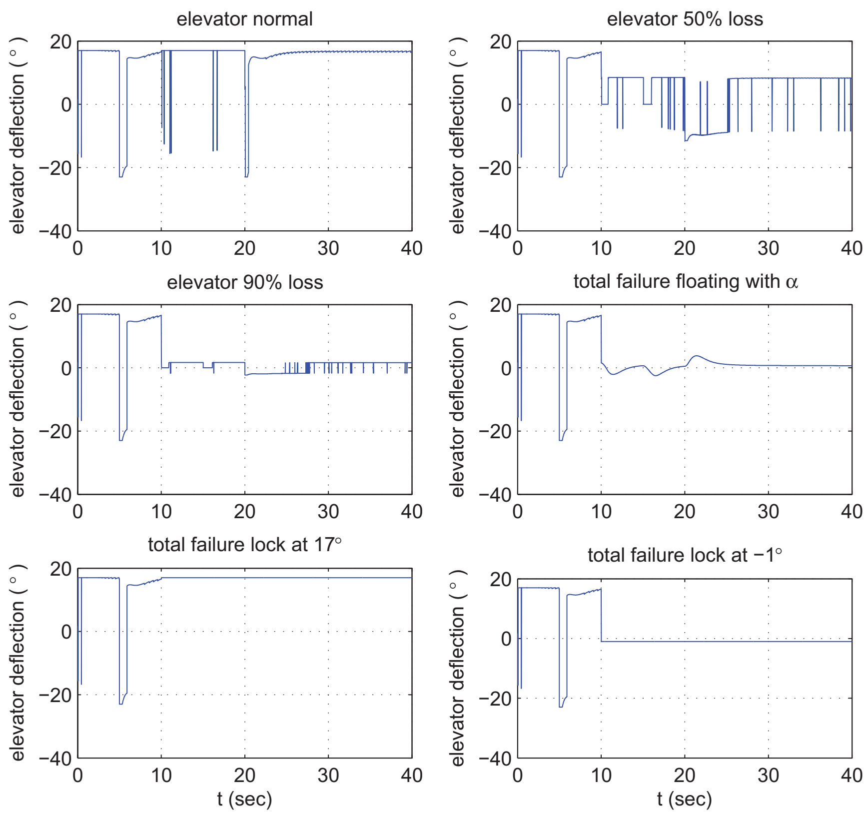

Figure 7.2: Elevator deflection

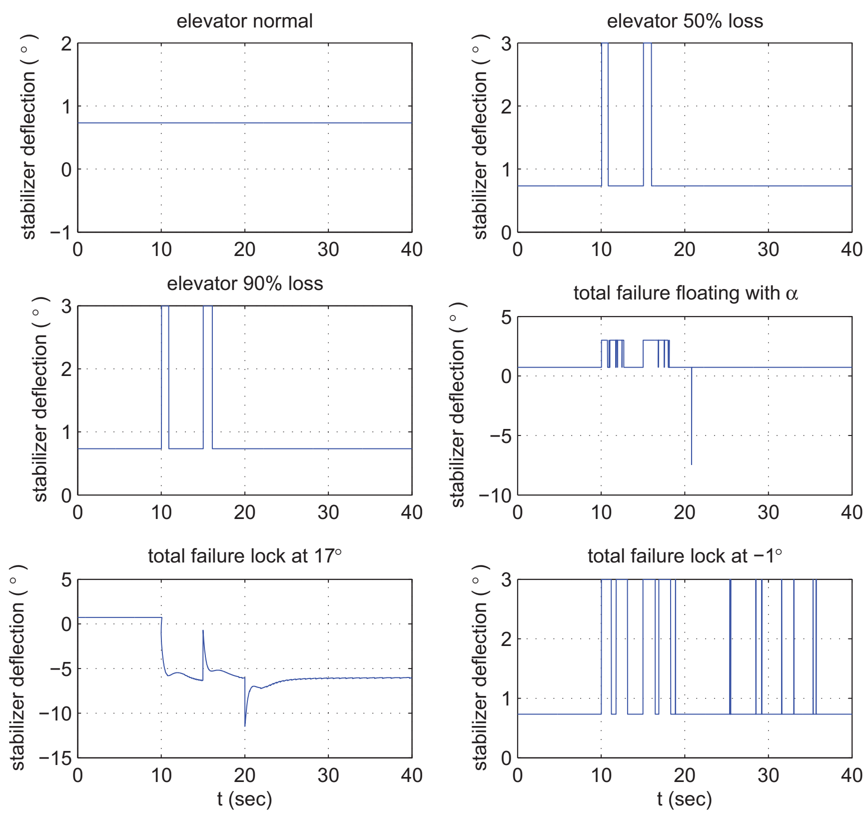

Figure 7.3: Stabilizer deflection

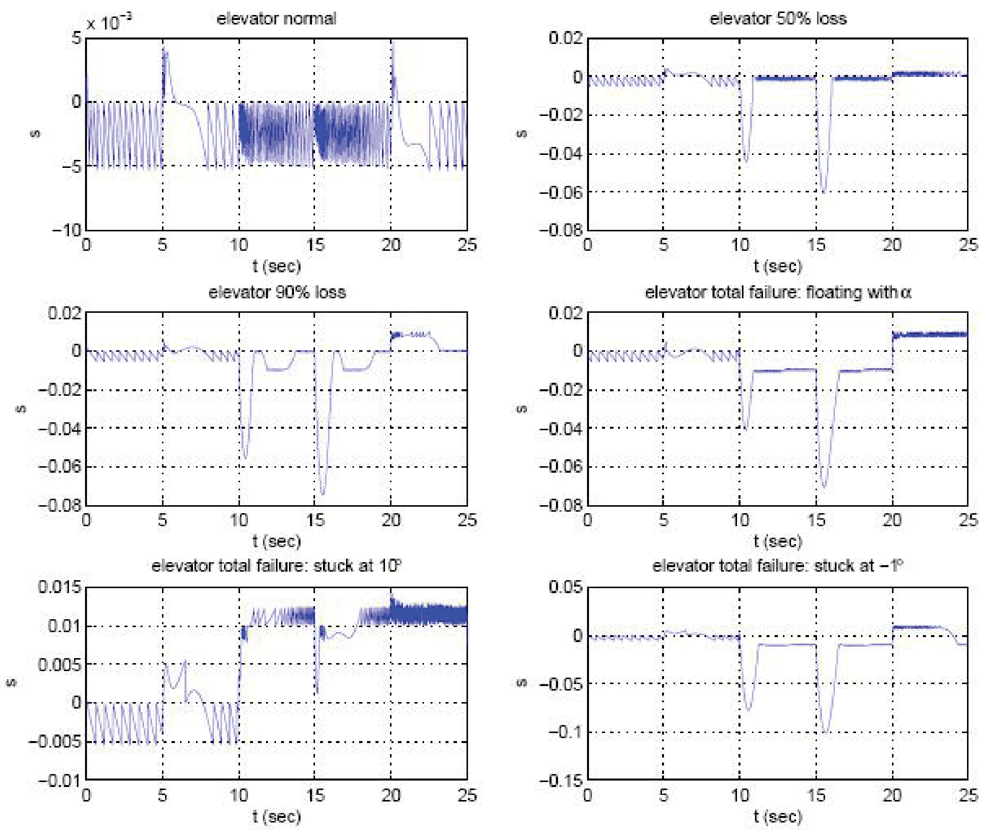

Figure 7.4: The switching function s

7.3 Summary

A sliding mode fault tolerant control algorithm is developed to accommodate partial loss fault and total failure in regular actuators with the help of redundant actuators. The fault and failure are detected by monitoring the sliding surface without using a dedicated fault and failure detection module. The controller integrates the regular actuators and the redundant actuators seamlessly. The stability of proposed controller is proved using Lyapunov method. Simulation results show the effectiveness of the proposed fault tolerant controller.

Chapter 8

Conclusions and Future Work

8.1 Conclusions

This thesis has developed several fault tolerant controllers based on sliding mode control. The research works focus on the aircraft with partial loss fault and totally failure in regular actuators.

In order to improve the control efficiency of fault tolerant control without sacrificing performance of normal controller, it is effective to deal with modeling uncertainty and fault separately in the controller design. The conceptual differentiation of modeling uncertainty and faults introduces an extra characteristic of fault tolerant controller other than normal or robust controller.

With extra design parameters in the sliding mode controller design, handling of fault can be separated from the handling of modeling uncertainty in the sliding mode controller naturally. This kind of fault tolerant control is efficient because of the separate dealing of faults and modeling uncertainty. The performance of the normal controller will not sacrifice much if adaptive mechanism is introduced in the controller.

If the effectiveness of the control surface can be obtained from special sensors or FDD scheme, an efficient reconfigurable controller can be synthesized to be tolerant with partial loss fault and total failure in regular actuator. The tolerability sets a point on which if the regular actuators can accommodate the fault solely. When the regular actuator cannot accommodate the fault solely, redundant actuators are activated to help the regular actuator to stabilize the system. The faulty regular actuator will still contribute even it cannot deal with the faults itself provided it is not in a failure situation. The reconfiguration of the control effort among the regular actuators and redundant actuators is autonomously and seamlessly.

Because of the cost of special sensors and the delay of FDD, the effectiveness information is not always available. A reconfigurable control that monitors the absolute value of the sliding function is developed to deal with faults as well as failures without dedicated fault and failure detection mechanism. This method can make sure the faulty system is stabilized all the time. This is also the requirement for a working fault detection and diagnosis system which will provide the fault and failure information of the system in the fault tolerant control with better performance.

The theoretical analysis with Lyapunov function and the simulation on the high fidelity Boeing 747-100/200 aircraft model showed the effective of all the algorithms developed in this thesis.

8.2 Future Works

Since there is always physical position and rate limit in the actuator, it is significant both in theory and practice in optimizing the SMC-based FTC considering these constraints. In the simulations of this thesis, although in the FTLAB747 and also in the nonlinear longitudinal model the physical position limit and rate limit are implemented in the simulation model, the effects of these limits have not been studied yet. This is one of the future works that expands the research of this thesis.

Chattering is an unavoidable problem in all SMC-based control algorithms. In the context of the research of this thesis, the immediate future work is the study on how the chattering will interact with the dynamics of the actuators. From the point of view in frequency domain, chattering is a limit cycle. The study on how this limit cycle will affect the system, especially how it will interact with faults in the actuators, is another future work.

SMC is chosen as an option for FTC in the thesis, it is not only because of its methodology but also the philosophy behind it: the dynamic behavior of the system can be partitioned through control. Since fault and failure are contingent event in the system, new methods that can partition the dynamic feature of the system, especially the faults and failures from the normal system is a challenging and interest future research direction.

In the Networked Autonomous Vehicles (NAV) Lab of Concordia, several platforms have been introduced in the research of FTC, such as the quadrotor and Airbus A380 unmanned aerial vehicle test-beds. In the near future, the algorithms of this thesis can be implemented on these platforms.

List of Publications

1. T. Wang, W. F. Xie and Y. M. Zhang, “Sliding Mode Fault Tolerant Control Dealing with Modeling Uncertainties and Actuator Faults,” *ISA Transactions*, vol. 51, no. 3, pp.386-392, 2012.

2. T. Wang, W. F. Xie and Y. M. Zhang, “Sliding Mode Reconfigurable Control Using Information of Effectiveness of Actuators,” *ASCE Journal of Aerospace Engineering*, Published on-line: 12 April, 2012; doi: 10.1061/(ASCE)AS.1943-5525.0000240.

3. T. Wang, W. F. Xie and Y. M. Zhang, “Sliding Mode Reconfigurable Fault Tolerant Control for Nonlinear Aircraft Systems,” submitted to *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering* (under review).

4. T. Wang, W. F. Xie and Y. M. Zhang, “Adaptive Sliding Mode Fault Tolerant Control of Civil Aircraft With Separated Uncertainties,” *Proceedings of 48th AIAA Aerospace Sciences Meeting Including the New Horizons Forum and Aerosapce Exposition*, Orlando, USA, Jan. 4-7, 2010.

5. T. Wang, W. F. Xie and Y. M. Zhang, “Sliding Mode Reconfigurable Fault Tolerant Control Without Explicit Knowledge of Faults and Failures,” *Proceedings of The Canadian Society for Mechanical Engineering Forum 2010*, Victoria, British Columbia, Canada, Jun. 7-9, 2010.

6. T. Wang, W. F. Xie and Y. M. Zhang, “Sliding Mode Reconfigurable Control with Application to Longitudinal Control of Boeing 747,” *Proceedings of 2010 AIAA Guidance, Navigation, and Control Conference*, Toronto, Ontario, Canada, Aug. 2-5, 2010.

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