Comparison of Deterministic and Stochastic Production Planning Approaches in Sawmills by Integrating Design of Experiments and Monte-Carlo simulation

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This is to certify that the Thesis prepared,

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Abstract

Forest industry is one of the key economic initiatives in Quebec (Canada). This industry has recently faced some obstacles, such as the scarcity of raw material, higher competitiveness in the market and new obligations applied in North America regarding sustainable development. These problems force lumber industries to improve their efficiency and become more service sensitive, in order to ensure the on time demand fulfillment. To achieve the goals aforesaid, one solution is to integrate the uncertainties more appropriately into production planning models. Traditional production planning approaches are based on deterministic models which in fact, ignore the uncertainties. A stochastic production planning approach is an alternative which models the uncertainties as different scenarios. Our goal is to compare the effectiveness of deterministic and stochastic approaches in sawing unit of sawmills on a rolling planning horizon. The comparison is performed under different circumstances in terms of length of planning horizon, re-planning frequency, and demand characteristics defined by its average and standard deviation. The design of experiments method is used as a basis for performing the comparison and the experiments are ran virtually through Monte-Carlo simulation. Several experiments are performed based on factorial design, and three types of robust parameter design (Taguchi, combined array, and a new protocol) which are integrated with stochastic simulation. Backorder and inventory costs are considered as key performance indicators. Finally a decision framework is presented, which guides managers to choose between deterministic and stochastic approaches under different combinations of length of planning horizon, re-planning frequency, and demand average and variation.

Key words: *sawmills, production planning, design of experiments, robust parameter design, uncertainty, Monte- Carlo simulation*

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I dedicate my thesis to my parents and my dear sister who always stood beside me in my life.

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1 Introduction

1.1 Foreword

The lumber industry is one of the most important business sectors in Quebec (Canada). In recent years this industry has faced serious difficulties. Cid Yáñez et al. [1] mentioned some of these obstacles. The scarcity of logs in both quantity and quality aspects in public forests, especially in eastern Canada, is one of the challenges in lumber industry, which highlights the need for using raw materials more efficiently. On the other side, the lumber industry market has become more competitive which justifies Quebec sawmills to have higher flexibility in demand fulfillment. New market obligations in the North America to consider sustainable development of value-added wood-based products are forcing lumber producers to replace the widely accepted traditional price-based push strategy with demand-driven approaches of production planning.

One of the most important issues influencing sawmill industry performance and profitability is the uncertainty. Santa-Eulalia et al. [2] mentioned three sources for uncertainty in sawmills: supply, manufacturing process, and demand uncertainty. According to Santa-Eulalia et al. [3] these uncertainties create a complex planning environment in which decision makers have to analyze different alternatives before implementing any of them. According to Kazemi Zanjani et al. [4-6], random process yields, due to non-homogeneity in the quality of raw materials (logs), and demand variation, caused due to difficulties in forecasting market condition, are two sources of uncertainty in sawmills.

As sawmill industry is categorized as a divergent production process, ignoring demand uncertainties may lead either to increased backorder levels (lost customers), or to huge inventory of products with low or zero demand in the market. In addition, ignoring the random yield in production planning models can lead to large inventory sizes of low quality and price products, and lack of high quality and price products. As a consequence, the industry will face with financial penalties and reduced capability of competition in the market. This research focuses on this issue in the sawing unit of sawmills to help this industry to deal with the impact of uncertainties and to increase the efficiency.

The focus of this study is on production planning in the sawing unit of sawmills. There are two approaches for production planning in sawmills. The production planning approach in Quebec sawmill industry is currently based on deterministic optimization models, by considering the average of random parameters. Kazemi Zanjani et al. [4-6] illustrated that ignoring the uncertainty in production planning models may lead to serious consequences in terms of failure to fulfill the demand at the right time. This can affect the company's reputation significantly regarding customer service level and may impose back order penalties to the company. Kazemi Zanjani et al. [4-6] proposed stochastic programming models, which considers different scenarios of random process yields and demand.

1.2 Goal of the study

Our goal is to study and compare the effectiveness of deterministic and stochastic production planning models in sawing units of sawmills. Since production planning in real sawmills is performed on a rolling planning horizon, the comparison between these models in this study is also performed on a rolling planning horizon. While in fix planning horizon the production plan is fixed, in rolling planning horizon the production plan can be updated in predetermined intervals based on the feedback data received from the implementation of plans in previous periods.

The final goal of the comparison among the two models in this study is to propose a decision framework, in order to identify which production planning approach is appropriate under different circumstances in the sawing units of sawmills, in terms of length of planning horizon, re-planning frequency, and demand average and standard deviation.

The influence of the above factors and their interactions on the performance of stochastic and deterministic models on a rolling planning horizon will be compared by conducting designed experiments. We considered backorder and inventory costs as our key performance indicators. As it is not possible and economically reasonable to interrupt the production line in real sawmills, the comparison is performed via Monte-Carlo simulation. The simulation experiments were implemented based on the design of experiments (DOE) and robust parameter design as the main methodologies. The simulation results are finally analyzed by means of statistical approaches.

1.3 Research contribution

Some studies have been previously done to compare the performance of production planning approaches in sawmill industries. Santa-Eulalia et al. [3] compared the performance of different planning and control policies by means of robust parameter designs for a lumber supply chain industry. Lemieux et al. [7] studied the performance of pull and push strategies by referring to a virtual lumber case. In another attempt Kazemi

Zanjani et al. [5] compared a two stage stochastic production planning model considering process random yields with the deterministic production planning model on a fix planning horizon. They proved the superiority of stochastic model regarding backorder amounts and the level of model precision.

Although all the attempts aforesaid aimed to compare the performance of different production planning approaches in sawmill industry, they included one or some of the following shortcomings: i) ignoring the rules of design of experiments, ii) implementing the experiments on a fixed planning horizon instead of rolling planning horizon, and iii) implementing only one type of robust parameter design methodologies. In an attempt to overcome these shortcomings we extend the existing contributions by:

- Performing the comparison based on designed experiments.
- Implementing the experiments on a rolling planning horizon.
- Integrating the robust parameter design with stochastic simulation to compare the performance of stochastic and deterministic production planning models in sawing unit of sawmills.
- Using three types of robust parameter design in addition to factorial design to propose a decision framework.

1.4 Summary and research overview

In this chapter, we provided an introduction regarding problem statement, goal of this study, the applied methodology to achieve the goal, and the contribution of this study by comparing the previous contributions in the literature on this topic. Chapter 2 includes a literature review regarding previous attempts to compare different production planning approaches in sawmills, a brief introduction to DOE/ Robust parameter design, and previous studies on integration of stochastic simulation and robust parameter design. Chapter 3 describes the methodology applied in this study. Chapter 4 provides the results and analysis of the sawmill case study. It also presents a decision framework based on the achieved results and analysis. Chapter 5 provides a conclusion and further steps.

2 Literature Review

In this chapter we review literature related to this thesis in the following topics:

2.1 Sawmill processes and characteristics

2.2 Production planning approaches in the sawing unit of sawmills

2.3 Previous attempts to compare different production planning approaches in sawmills

2.4 Design of experiments (DOE)

2.5 Factorial designs

2.6 Robust Parameter Design (RPD)

2.7 Comparison between Taguchi and combined array methodologies

2.8 Integrated robust parameter design and stochastic simulation

2.9 A new protocol to integrate robust parameter design and stochastic simulation

2.1 Sawmill processes and characteristics

Since our case study is focused on sawmills industry, in this section, we briefly describe sawmill processes and characteristics. Log sorting, sawing, drying, planing and grading (finishing) can be considered as main sawmill processes. Logs from different districts of forest are the raw material in sawmills industry. After bucking the felled trees, they are transported to sawmill. Logs are sawn, dried and finally graded in a finishing process to be transported to domestic and international markets. Figure 1 illustrates the sawmill main processes.

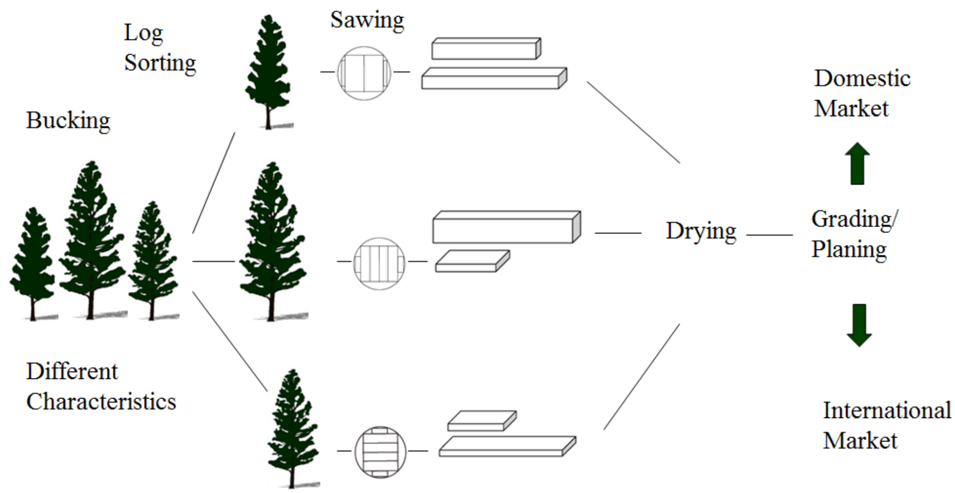

Figure 1- Illustration of sawmills main processes

Although our focus in this study is on sawing unit of sawmills, other processes are also explained briefly in this section. According to Gaudreault et al. [8] and Kazemi Zanjani et al. [4-6] different production units in sawmills operate as follows:

2.1.1 Sawing process

The bucked trees (logs) are classified based on common attributes such as diameter class, length, taper, etc. when they are shipped to sawmill. In sawing unit, the logs are cut into different dimensions of rough pieces of lumbers (e.g. 2(in) ×4(in) ×8(ft.), 2(in) ×4(in) ×10(ft.), 2(in) ×6(in) ×16(ft.)…) by means of different cutting patterns. Figure 2 illustrates three different sample sets of lumbers achieved by different cutting patterns:

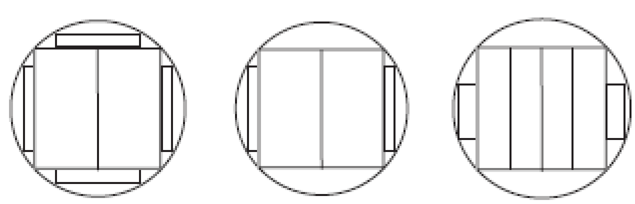

Figure 2- Different sample sets of lumbers sawn by different cutting patterns [6]

The lumbers' quality and quantity obtained from each cutting pattern depends on the quality and characteristics of the input logs. Although the logs are classified in sawmills, variable characteristics are observed in different logs in each class. In fact, natural variable conditions during the growth period of trees affect the characteristics of logs. The non-homogeneity in logs, as a consequence, makes it impossible to anticipate the exact yield of a log while deciding on the production plan.

2.1.2 Drying process

Drying operation decreases the moisture of lumbers. The required drying level is usually based on industrial standards, but in some cases it depends also on customer's required specification. Drying is a complicated process and can take several days. The lumbers should be loaded and dried in kiln dryers in batches. There are several constraints and each company has its own loading patterns. It is useful to mention that sometimes a combination of air drying and kiln drying is applied to increase the lumbers final quality.

2.1.3 Finishing process

During this final process, lumbers are planed and sorted according to their quality. More precisely, different quality levels are defined based on the degree of moisture and physical defects. The drying process used for each batch dictates the achievable products in the finishing unit. In other words, the output products of different drying processes are considered as different inputs for the finishing unit.

2.2 Production planning approaches in the sawing unit of sawmills

Since, the focus of current study is on sawing unit of sawmills, in this section, we are going to review the literature of production planning in the sawing unit.

According to Kazemi Zanjani et al. [4-6] and Guadreault et al. [8] the goal of production planning models in the sawing units of sawmills is to determine the optimal quantity of log consumption from different classes and the selection of corresponding cutting patterns in order to fulfill lumber demand. The objective is to minimize total cost including log consumption, inventory and backorder costs.

Ignoring the uncertainties aforesaid in sawmill production planning will lead to increased back order levels (lost customer), as well as huge inventory of products with low or zero demand in the market, and consequently, facing with financial penalties and reduced capability of competition in the market.

Different production planning approaches in sawmills are classified in the literature based on the way they deal with uncertainties:

2.2.1 Deterministic production planning models

Deterministic optimization models are the traditional production planning tools in many sawmills. In the deterministic approach, the objective function minimizes back order, inventory and production costs subject to production capacity constraints and product and material inventory balance. Deterministic models ignore uncertainty by considering the average of random process yields. This method usually results in extra inventory of products with lower quality and price and on the other hand backorder of products with higher quality and price. This will make sawmills to fail in demand fulfillment and consequently facing with decreased service level. It also results in poor utilization of raw materials and reduced competition ability in the market.

A common practice in many sawmills in order to take into account the uncertainties while adopting deterministic production planning approaches is the rolling horizon planning. In this approach, at the beginning of a planning horizon, the decision maker generates a plan based on the available forecasted data (e.g., demand). The production plan is then updated at predetermined intervals during the planning horizon as the information with regard to demand and actual inventory/backorder level of products and raw materials are updated. Nevertheless, as the rolling horizon planning is a reactive approach, it cannot address completely the issue of uncertainty.

2.2.2 Stochastic production planning models

In the stochastic programming approach, the random yields and demand are modeled as a set of scenarios and/or a scenario tree. The objective of such stochastic production planning models is to generate a plan with the minimum expected cost for all possible scenarios for random yield and demand. As the stochastic programming predicts different scenarios, it can be considered as a proactive approach to deal with uncertainties. The only drawback of such stochastic programming models is their complexity in terms of modeling and solving. Moreover, estimating realistic scenarios for random parameters is another important issue.

Kazemi Zanjani et al.[5] proposed a two stage stochastic model for sawmill production planning while considering random process yields. For considering both yield and demand uncertainties, K. Zanjani et al. [4, 6] proposed a multi stage stochastic model.

2.3 Previous attempts to compare different production planning approaches in sawmills

In this section we will review the literature related to comparing different production planning approaches in sawing units. Their shortcomings and relation to our study are presented at the end of this section.

Santa-Eulalia et al. [2] performed robust experiments to study the effect of different planning and control policies for lumber supply chain industry. They also tried to find the optimum combination of effective factor levels in order to minimize the impact of uncertainties of supply, manufacturing and demand. They identified three controllable factors which are the control levels, planning method and planning horizon. Back order and inventory averages were defined as key performance indicators (KPIs) for evaluating the system performance. They came to the conclusion that regarding the backorder cost, the control level has the most important role and planning horizon and planning method do not have great impact. In other words more frequent control provides better customer service. For inventory level, their result illustrated that none of the factors solely has great impact. Based on their results, in order to decrease the inventory level a suitable combination of all factors levels should be considered.

Lemieux et al. [7] compared the results of implementing two different production planning strategies (pull/ push) in sawmills, where they used a virtual lumber case derived from actual Canadian lumber industry. They performed two experiments, where in the first one only push and pull production planning models were compared. In the second experiment, another approach regarding a combination of pull and push production planning has been added to this comparison. The results indicated that the fill rate will not be affected considerably by the production planning method. On the other hand, the combined pull/push method will result in the least back order.

In another attempt, Kazemi Zanjani et al. [5] compared a two stage stochastic production planning model by considering random yields with the mean value deterministic production planning model on a fix planning horizon. Their results illustrated that the stochastic approach is superior to the deterministic model as it leads to smaller amounts for back order and besides provides higher precision of the production plan.

Although all the above literature attempted the comparison between different production planning models in sawmills, there are still some shortcomings and some areas for extending and improving the studies aforesaid. For example some of them do not follow the rules of design of experiments, or are implemented on a fixed planning horizon. As a consequence, in this research we plan to consider:

1. Performing the comparison based on designed experiments
2. Implementing the experiments on a rolling planning horizon
3. Using three types of robust parameter design in addition to factorial design to propose a decision framework
4. Integrating the robust parameter design with stochastic simulation

2.4 Design of Experiments (DOE)

According to Montgomery et al. [9] the purpose of running designed experiments is to define the relation between input and output variables of a system. By analyzing the experimental results, we aim to find the impact of input factors and their changes on the response variable(s). It is important to mention that in many cases the final goal is to propose the level of input factors which make the system or process robust against noise factors (sources of system variability).

Montgomery et al. [9] also indicated that designing the experiment is the first step of any experimental problem. The second important step is to analyze the achieved experimental results. These two steps are linked together, since the well-designed experiment is vital to guarantee trustable results. A statistical approach for designing the experiment and collecting the data is necessary to achieve appropriate data and to find meaningful conclusions based on statistical methods.

Montgomery et al. [9] mentioned 7 major steps for designing the experiments as follows: "i) recognition and statement of the problem, ii) selection of response variables, iii) choice of factors, levels and range, iv) choice of experimental design, v) performing the experiment, vi) statistical analysis of the data, and vii) conclusion and recommendation."

There are different types of DOE which can be applied in related steps. The initial step is usually to perform factorial experiments especially 2^k designs where "k" represents the number of factors. In the next steps, more detailed experiments such as robust parameter design are run. In the following, different types of DOE are provided.

2.5 Factorial designs

Montgomery et al. [9, 10] have explained the factorial designs. They mentioned that many experiments include two or more factors. The experimenter tries to find the influence of these factors on the output response of the system. The right approach to deal with cases including several factors is to conduct a factorial experiment. A complete factorial design analyses all possible combinations of all factor levels. "For example, if there are “a” levels of factor “A” and “b” levels of factor “B”, each replication contains all “a × b” level combinations.” [9, 10]

The main effect of a factor is the change in the value of response variable caused by changing the factor level. If the effect of a factor on response variable depends on the level of another factor(s), there is an interaction between them. It means for example, the effect of factor “A” depends on the level chosen for factor “B”. By the help of factorial design, we can study not only the main effects but also the interactions effects between factors. This ability to investigate the interaction effects is the main superiority of factorial designs comparing to classical method of exploring the factor effects one by one. Regression models prepared for response variables (KPIs) besides ANOVA tests are used to decide if a special factor or interaction is statistically significant.

Some special cases of the general factorial designs are important because they are widely used in many industries. The most important one is the experiment which considers “k” factors, and allocates only two levels to each of them. These levels may be quantitative or qualitative. “A complete replicate of such a design requires $2 \times 2 \times 2 \times \ldots \times 2 = 2^k$ observations which is also called a 2^k factorial design.” [9, 10]. This type of experiment is especially appropriate in the first steps of experimentation. The reason is that in early stages, there are many potential effective factors. By applying 2^k design we can study the impact of these factors and all their interactions with the minimum number of necessary experiments.

Although the 2^k design is a useful and trustable methodology, there are more detailed and coherent experiments which can be applied in cases including noise (uncontrollable) factors. The following section summarizes those types of experiments.

2.6 Robust Parameter Design (RPD)

There are two types of factors in each system or process: i) controllable factors, and ii) noise (uncontrollable) factors. The classical DOE which was explained in the previous section is only capable to include controllable factors in the experimental structure. There have been several attempts to deal with the noise factors in processes. The initial attempts were concentrating on removing the noise factors. This method is not only economically inefficient, but also in some cases it is not even applicable. Later, some attempts were performed in order to make the system robust against the noise impacts instead of removing them. To do so, it seemed to be vital to develop an experimenting approach which considers the noise factors in the experimental design.

Based on Montgomery et al. [9] robust parameter design (RPD) is an approach and a particular type of DOE that includes both controllable and noise factors in its structure. It emphasizes on choosing the levels of controllable factors in a process or a product to achieve robustness, which includes two objectives: i) to ensure that the average of the response variable remains close to a desired value, and ii) to ensure that the variability around this target value is as small as possible.

Although noise factors are not controllable in usual processing level, they may be controlled in research or developing level and during the RPD experimentation. By fixing the levels of noise factors artificially during the experiments, RPD aims to find the levels of controllable factors, which can minimize the variability caused by the noise (uncontrollable) factors. This setting will result in a robust process/system that is, a process/system affected minimally by variations. There are two main methodologies for robust parameter design which are reviewed in the following sections.

2.6.1 Taguchi crossed array methodology for robust design

The initial approach in robust parameter design was proposed by Taguchi [11] in 1980s. Taguchi's method is based on the fact that it is not always possible or economically reasonable to remove or reduce the sources of variation. This method intends to make the system insensitive to noise factors. Taguchi method is based on new quality control approaches. In these approaches, reducing the variability around the response is superior to simply achieving a desirable response value. The reason is that in traditional quality control approaches, the controllable factor levels were set in a way to achieve the best possible response variable. In this case, the variability caused by noise factors could easily make the chosen levels useless. In other words it made it necessary to run another set of experiments in order to find the new optimum levels of controllable factors under new circumstances caused by the impact of variability.

Taguchi [12] has proposed a special structure to include the noise factors in the experimental designs. This structure contains two separate arrays: i) inner array which is allocated to the controllable factors, and ii) outer array which is allocated to the noise factors. This type of experimental design is called a "crossed array design".

The following table illustrates a sample of crossed array design based on Montgomery et al. [9]. The experiment consists of four controllable factors *A, B, C* and *D* each at two levels, and one noise factor *E* which has two levels as well.

To run the experiments, these two arrays will cross each other. In other words, every combination (run) of the inner array will be performed for all combinations (low and high levels of E) of the outer array. This way, the noise factor is included in our experiments

Table 1- A sample of crossed array design [9]

Inner Array

Outer Array

				Low Level	High Level
-	-	-	-	-	+
+	-	-	+	-	+
-	+	-	+	-	+
+	+	-	-	-	+
-	-	+	+	-	+
+	-	+	-	-	+
-	+	+	-	-	+
+	+	+	+	-	+

and if there is an interaction between noise and controllable factors, it is possible to choose the appropriate levels for controllable factors to reduce the impact of the noise factor.

The designs of inner and outer arrays are independent. There are several design possibilities. Taguchi et al. [12] proposed orthogonal arrays in his “quality engineering handbook”. Montgomery et al. [9] proposed fractional factorial designs for inner arrays and full factorial designs for outer arrays. By using orthogonal arrays or fractional factorial designs, some interaction effects will be missed due to the aliased structure. The accuracy of results depends on the size and type chosen for orthogonal arrays or level of fractioning for factorial designs. It is also possible to have full factorial designs for both arrays which may lead to higher accuracy, but larger size of experimental designs.

Taguchi et al. [12] defines the signal to noise ratio (S/N ratio) as an indicator of variability. The S/N ratio formula depends on the desired response variable[13]. Indeed, it is always desired to have the highest S/N ratio for better robustness. Let us denote by y_i, $\overline{y}$ and s are the value, the average and the variance of response variable, respectively. Three types of response variables are illustrated in [13], as follows:

- when a larger response value is better:

$$\text{S/N ratio}= -10\log\left(\frac{\sum_{i=1}^{n} 1/{y_i}^2}{n}\right) \tag{1}$$

- when a smaller response value is better:

$$\text{S/N ratio}= -10\log\left(\frac{\sum_{i=1}^{n} {y_i}^2}{n}\right) \tag{2}$$

- when the nominal response value is the best:

$$\text{S/N ratio}= 10\log\left(\frac{\overline{y}^2}{s^2}\right) \tag{3}$$

Taguchi's method is suitable to find the optimum levels of controllable factors among the predefined values of factors levels. More precisely, it does not have the capability to explore other amounts for factors levels. Due to this shortcoming, always the combination of Taguchi method and regression model or response surface method has been considered. In this way, we have the ability to evaluate the impact of other values for controllable factors, which lay within the low and high levels defined for the experiment.

Montgomery et al. [9] proposed a revision to the Taguchi method to use the variance of the response variable directly, as the indicator of variability, instead of S/N ratio. They mentioned that S/N ratio has complicated calculations, and besides they criticized its accuracy to find the factor effects on variation. They suggested considering mean and variance of each treatment of inner array as response variables, and apply usual regression analysis to find important factors and interactions. Table 2 illustrates an example regarding mean and direct variance calculations. It is important to mention that as the sample variance follows a chi-square distribution, Montgomery et al. [9] suggested analyzing the natural logarithm of the variance, instead.

Table 2- A sample of Montgomery proposed method [9]

				Low Level	High Level	Mean	Variance
-	-	-	-	-	+	7.54	0.09
+	-	-	+	-	+	7.9	0.071
-	+	-	+	-	+	7.52	0.001
+	+	-	-	-	+	7.64	0.008
-	-	+	+	-	+	7.6	0.074
+	-	+	-	-	+	7.79	0.053
-	+	+	-	-	+	7.36	0.03
+	+	+	+	-	+	7.66	0.017

2.6.2 Combined array methodology for robust design

Although the crossed array design is a considerable step regarding the inclusion of noise factors in designing the experiments, it has some shortcomings. Firstly, although it provides information about interactions between controllable and noise factors, the resulted regression model does not include any direct terms regarding the main effect of noise factors or any term representing the interaction effect between noise and controllable factors. Also, this method can lead to a very large size of experiments because of its special structural design.

To solve the problems aforesaid, Welch et al. [14] proposed another method called combined array design. The crossed array design forces the experimenter to include high order interactions of noise and controllable factors. In this method there are no inner and outer arrays and the noise factors are included in the same design as the controllable factors. Combined array method results in empirical model considering some terms regarding interactions between controllable and noise factors. It can also provide the same level of information by smaller size of experiments.

Similar to the revised method Montgomery suggested for Taguchi approach, the combined array method tries to find the appropriate empirical models for the mean and variance of response variable. The difference is that in the revised Taguchi method, the mean and variance for each factor combination is calculated and behaves as a response value. Then the regression model is calculated for the mean and variance separately. In the combined array design, on contrary, there will be one regression model for the response value. Then the functions regarding mean and variance will be extracted based on this regression model. In other words, the regression model found for the response variable will be the basis for both functions of mean and variance.

The following example illustrates the sample steps for a case including only one noise factor based on Montgomery et al. [9]:

1. To find the appropriate regression model for the response value:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \gamma_1 z_1 + \delta_{11} x_1 z_1 + \delta_{21} x_2 z_1 + \varepsilon \tag{4}$$

where y refers to response variable, β_0 is an intercept representing the average of all achieved response values, x_1 and x_2 refer to controllable factors, z_1 is the noise factor and ε represents the error.

2. In order to find the model for the mean, it is enough to take the expected value of equation (4) over z_1. In the combined array design the coded variables are used for both noise and controllable factors. By coded we mean that the variables are centered on zero and have upper and lower limits as $\pm a$. As a result the expected value of noise factors and subsequently any term including noise factors are equal to zero. In addition, as the error is supposed to have random distribution, it is expected to have zero mean value as well. As a result the mean regression model will be a function of controllable factors and it will be defined as follows:

$$E_z(\mathrm{y}) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 \tag{5}$$

3. To find the model regarding the variance of response variable, Montgomery [9] proposes two steps. First they expand the response model in a first-order Taylor series around $z_1 = 0$. The result would be as follows:

$$\mathrm{y} \cong \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + (\gamma_1 + \sigma_{11} x_1 + \delta_{21} x_2) z_1 + \mathrm{R} + \varepsilon \tag{6}$$

where R is the remainder term in tailor series and can be ignored. Therefore, the final variance function over z_1 will be as follows:

$$V_z(\mathrm{y}) = \sigma_z^2\,(\gamma_1 + \sigma_{11} x_1 + \delta_{21} x_2)^2 + \sigma^2 \tag{7}$$

where σ_z refers to the variance of noise factor and σ refers to the variance of error. This function contains only controllable factors, but the impact of noise factor is considered by including the regression coefficient of interaction between noise and controllable factors.

The achieved regression models can be used for further analyzes as the Taguchi method. These two methods are the milestones of robust design. The next section reviews the literature regarding the comparison of Taguchi and combined array methodologies.

2.7 Comparison between Taguchi and Combined array methodologies

Taguchi and combined array designs are the basis of robust parameter design. Several attempts have been performed to compare the performance of these two methods.

Shoemaker et al. [15] compared the performance of Taguchi method and combined array design. They compared these methods regarding several aspects and by means of few examples. Finally, they concluded that the most important benefit of combined array method is its smaller size of experiments comparing the Taguchi method. The reason is that Taguchi method forces the experimenter to evaluate all control by noise factor interactions, in case that so many of these interactions specially the high order ones are not necessary to be studied. This happens due to the special structure of Taguchi's method, which will also lead to miss some important data regarding interactions between controllable factors or the curvature effects. In other words, due to the special structure of Taguchi array, if the experimenter is interested to verify some interactions between controllable factors or curvature effects, a larger experiment should be run. This limitation does not exist in combined array methodology, and the experimenter can decide to which level the interaction between noise and controllable factors should be studied and the rest of degrees of freedom can be allocated to estimate desirable effects.

Borror and Montgomery [16] focused on the drawbacks of Taguchi method and compared it with a mixed resolution combined array approach. A mixed resolution design allows setting the experiment as to be able to study the impacts of factors, interactions, curvature and quadratic terms which are assumed and suspect to be important. Similar to Shoemaker et al. [15], they emphasized that the main advantage of combined array design is that it provides smaller necessary number of experiments comparing Taguchi method for evaluating the same parameters of interest.

They used the results of a sample experiment to compare the performance of these two methods. The comparison confirmed that combined array design has smaller size while covering more information. The proposed optimal settings for controllable factors were not the same for both models. Nevertheless, by comparing the two approaches, the settings of controllable factors provided by both models concluded in approximately the same “mean” of response variable. On the other hand, the setting provided by combined array design leads to less variance of response variable than Taguchi method. As achieving the least possible variance of response variable has the highest importance and priority in robust designs, it is another indicator of the superiority of combined array method.

In another attempt, Bates et al. [17] compared four different methodologies to perform robust designs by stochastic computer simulations. They reviewed four methodologies including Taguchi, combined array design, dual response surface analysis and the stochastic emulator strategy. The comparison was performed regarding an example including seven controllable factors and seven noise factors. The final comparison between the aforesaid methodologies indicated that the proposed factor settings were not always the same by all methods. More precisely, there were even some factors distinguished important by one method whereas the other method could not diagnose their effectiveness. It is important to note that although Taguchi method has been criticized due to its shortcomings in so many studies, it illustrated an acceptable performance in the mentioned example.

Kunert et al. [18] applied both Taguchi and combined array methods in an experiment and compared the results. They mentioned that based on the experiment's results, Taguchi method has been more capable and precise to distinguish some important factors (which the combined array was unable to find) regarding the variance of response variable. It is another indicator which supports the results by Bates et al. [17] confirming the validity of the Taguchi method.

Based on the literature aforesaid, we concluded that both methodologies are worthwhile to be implemented in our case. As we aim to implement our experiments by means of Monte-Carlo simulation, the next section covers the literature regarding the integration of robust design and stochastic simulation.

2.8 Integrating robust parameter design and stochastic simulation

Giovagnoli et al. [19] mentioned that the classical robust designs were initially introduced for physical experiments and hence, although the noise factors are varying in reality, they are fixed artificially during the experiments. In other words, in physical experiments just a few levels of the noise factors are included and investigated in the experiments. This limitation can be relaxed by means of capabilities of stochastic simulation to run less expensive and faster experiments which finally results in more number of trials and makes it possible to simulate the noise factors stochastically. As a result, the user is not forced to consider only few levels for noise factor. They mentioned that stochastic simulation is the best tool to perform robust design, as it can transmit the noise impact into the experimental outputs. Several attempts have been previously performed to combine robust design and stochastic simulation.

Antognini et al. [20] focused on the utilization and importance of computer simulation in performing quality improvement experiments and DOE. They mentioned that performing the experiments via computer simulation is a momentum in the design of experiments. Antognini et al. [20] covered some aspects regarding computer simulation experiments: i) sequential experimenting which follows a step by step experimenting with a stopping rule defined by economic constraints, and ii) the use of stochastic simulation including random factors to imitate the random behavior of the system under study. They mentioned that running the experiments by simulation has three major advantages comparing implementing them in physical systems: i) it becomes easier to study the relationship between input and output variables, ii) the system becomes more observable, and iii) running the experiments by the help of simulation is easier and cheaper than implementing them in real and physical circumstances.

Romano et al. [21] reviewed an integration of physical experimenting and computer simulation. They compared physical experiments versus computer ones through three features: feasibility, cost, and fidelity. They mentioned that in many cases it is not possible or it is expensive to run the experiments in physical circumstances. On the other hand, they indicated that computer experiments are not as trustable as physical ones. Consequently, they suggested a procedure for integrating computer and physical experimenting and implemented their procedure in two case studies. Their results illustrated that adding computer simulation to the experimenting procedure will result in better achievements comparing the cases using only physical experiments. The reason was that as computer experiments are cheaper comparing the physical ones, it is possible to find more details about the system by performing more experiments.

Santa-Eulalia et al. [2] performed robust experiments to compare different planning and control policies for lumber supply chain industry. They referred to a case study of sawmill supply chain. Three controllable factors including control level, planning method and length of planning horizon and three noise factors regarding demand, manufacturing and supply uncertainties were defined. They considered backorder and inventory sizes as the key performance indicators to compare the performance of different scenarios. Taguchi crossed array design and Monte-Carlo simulation were applied as the main methodology in their study. Through performing four replications for each experiment, they concluded that control level is the only main source of backorder variation. On the other hand, interaction effects are the source of change in the inventory size. They proposed optimal levels for controllable factors regarding the priority to decrease backorder.

Grubic et al. [22] argued that robust parameter design is an appropriate tool to improve the performance of a supply chain network. They tried to prove this argument by using a simple supply chain example and applying Taguchi method in a discrete event simulation. Delivery lead time and total cost of the supply chain were defined as KPIs. Control factors included number of system components, amount of products, buffer capacity and transporters capacity. The noise factors were the customer demand, distributer's inventory level and quality deviation. They performed 12 replications for each experiment and proposed the optimal factor levels considering the experimental result.

In another attempt Grubic et al. [23] used the robust design to analyse a supply chain network. Total cost of the system and product availability were considered as two KPIs in their study. They defined five controllable factors, including number of distributer's transporters, number of producer's transporters, the order of distributers, production volume and accumulation of orders. Five noise factors were also considered including order time deviation between producer and distributer and, between distributer and wholesalers, transporting time deviation between producer and distributer and, between distributer and wholesalers and finally end demand variation. They performed three replications for each experiment by means of discrete event simulation and proposed the optimal levels of controllable factors.

Shang et al. [24] proposed an integrated method which included simulation, Taguchi approach, response surface methodology and optimization to find the optimum levels of controllable factors so as to obtain the best possible performance of a supply chain. They considered six controllable factors: delayed differentiation, information sharing, capacity limit, reorder quantity, lead time and reliability. The defined noise factors were inventory holding cost and demand variability. They used a combination of methodologies aforesaid to find the optimum levels of controllable factors which minimize total cost and maximize service level as two desirable KPIs.

Veza et al. [25] mentioned that Taguchi crossed array design has the potential to be a suitable method to improve supply chain performance. It is applicable in all three planning levels (strategic, tactical and operational) and in all types of industries. They used discrete event simulation for modeling the system and supported their study with a practical example representing a simple supply chain network of a TV set assembly process. The controllable factors included three buffer capacities regarding assembly stations and one controllable factor for replenishment quantity. Three noise factors regarding the reworks and standard deviation of transportation time were also defined. Total cost of the system and the number of backorders were considered as KPIs. They finally proposed the optimum levels of controllable factors based on the results of the Taguchi and ANOVA methods.

Shukla et al. [26] proposed an integrated approach including simulation, Taguchi, regression analysis and optimization to minimize the total cost of supply chain network design. By applying the proposed approach the supply chain planners and decision makers can choose the level of operating factors such as appropriate plant capacity, reorder policy, lead time, etc. in order to achieve the optimal efficiency of the entire supply chain. By using simulation and referring to a sample supply chain, they suggested an approach for decision makers to understand the impact of controllable factors and make the system robust against demand uncertainty.

Deva et al. [27] also used Taguchi method as a tool to study the supply chain network behavior. They used a hypothetical supply chain network as a sample for their analysis and considered average inventory level as the KPI. They used discrete event simulation and through running five replications, they compared the results obtained from different factor levels under the noise impacts caused by demand and lead time variation.

Although all previous attempts have tried to integrate robust parameter design with stochastic simulation, they have used only the Taguchi method. Giovagnoli et al. [19] proposed a new methodology for integrating robust parameter design and stochastic simulation which is reviewed in following section. Their methodology has some similarities with Taguchi and combines array designs.

2.9 A new protocol to integrate robust parameter design and stochastic simulation

Giovagnoli et al. [19] categorized the noise factors into two groups: i) Z_1: Noise factors which are modeled and coded stochastically by simulation, and ii) Z_2: Noise factors which are fixed at different levels (similar to the traditional robust designs)

By considering "*Y*" as the response variable and "*X*" as the vector of controllable factors, they have defined two stages of the approach as follows:

Stage 1: In this stage "the computer experiment is performed by stochastically simulating the noise Z_1 for chosen level of (x,z_2), and the sample mean and variance of the observed responses are calculated." [19]. In other words, they perform several replications for each combination of controllable factors and type II noise factor(s). Each replication provides a different response value regarding the uncertainty caused by simulating Z_1. Next, the mean and variance of response values achieved for different replications are calculated with respect to Z_1. This process will be repeated for all pairs of (x,z_2). These values are then used to obtain the mean and variance regression models by means of ordinary least square methods. The following equations are provided by performing this stage. Equation 8 represents the expected value regression model. It is provided based on response value average amounts calculated for different replications of each pair of (x,z_2). Similarly, equation 9 represents the logarithm of variance regression model which is calculated based on observed variance values for different replication of each pair of (x,z_2).

$$E_{Z_1}(Y|\, Z_2 = \; z_2) = f\,(x)^T\beta + g(x, z_2)^T\gamma \tag{8}$$

$$Log\; Var_{Z_1}(Y|Z_2 = z_2) = h\,(x, z_2)^T\delta \tag{9}$$

Similar to Taguchi method, as direct variance follows a chi-squared distribution, the logarithm of variance which has the normal distribution is used for calculations. The unknown parameters β, ϒ and δ will be estimated by least square methods aforesaid.

Stage 2: This stage refers to the fact that the achieved regression models cover only the limited levels considered for Z_2 in the first stage. As the level of Z_2 is unknown and random in reality, they provided the models which are valid for all values of Z_2. Equation 10 represents the expected value function which is applicable for all levels of Z_2. To achieve this function, Giovagnoli et al. [19] proposed to find the expected value of average regression model in stage 1 (equation 8) with respect to Z_2. Similarly, equation 11 provides a valid variance function for all levels of Z_2. This model has two terms: i) the expected value of variance regression model in stage 1 (equation 9) with respect to Z_2 and, ii) variance of expected value regression model in stage 1 (equation 9) with respect to Z_2. These models are as follows:

$$E_Z(Y) = E_{Z_2}\left[E_{Z_1}(Y|Z_2)\right] \tag{10}$$

$$Var_Z(Y) = E_{Z_2}\left[Var_{Z_1}(Y|Z_2)\right] + Var_{Z_2}\left[E_{Z_1}(Y|Z_2)\right] \tag{11}$$

3 Methodology

This section presents the methodology applied in the current study. As mentioned in the introduction section, the goal of this study is to propose a decision framework, which suggests the appropriate production planning approach under different circumstances, in terms of length of planning horizon, re-planning frequency, and demand average and standard deviation. To compare the performance of these approaches, it is necessary to run designed experiments. As it is not possible and economically reasonable to interrupt the production line in real sawmills to perform the real experiments, they were performed via Monte-Carlo simulation.

In this section, we first explain the simulation process. Then, we explain the method for generating random components of simulation and implementing them in the simulation process. The last section describes the steps applied for running the designed experiments. The following 4 sections have been covered in this chapter:

3.1 Simulation process

3.2 Generating random components (yield and demand)

3.3 Implementing the random components in the simulation process

3.4 Applied steps for running the designed experiments

3.1 Simulation process

3.1.1 Monte-Carlo Simulation

As we mentioned earlier, all experiments are performed by the aide of Monte-Carlo simulation. The core idea of Monte-Carlo simulation is to use random samples of parameters or inputs to explore the behavior of a complex system or process which cannot be analyzed by an analytic approach. The simulation process designed for the implementation of different production planning models on a rolling planning horizon is explained and illustrated, as follows.

3.1.2 Different phases of the simulation process

The simulation process is divided into two phases: planning and implementing. In the "planning" phase, the production plans as well as the random demand profile and yield scenarios are prepared as the inputs to the simulator. Then the simulator implements the proposed plan for each re-planning interval. We call this stage as the "implementation" phase. The outputs of the simulator include the realized backorder cost of product and inventory costs of products and raw material (logs). The above process is then repeated for the whole simulation horizon which is considered as one year in our case. More details on the two phases of the simulation are provided in the following sub-sections.

3.1.3 Planning phase

In this phase, the deterministic and stochastic models are solved by $\text{CPLEX}^{(C)}$ optimization software, and the optimal production plan is obtained for each of them. These two models are presented in appendix I. The production plan illustrates how many times each process should be run in each period for the whole planning horizon. In addition, solving the plan by $\text{CPLEX}^{(C)}$ results in the expected amounts of backorder and inventory costs for each production planning model. The production plan is prepared for the whole planning horizon, which is considered as 15 and 30 days in this study.

3.1.4 Implementation phase

The second phase includes implementing the achieved production plan in planning phase on a rolling planning horizon. In the rolling planning horizon, the production plan is updated in predetermined intervals based on the feedback received from implementation of plans in previous periods. The re-planning frequency is considered to be 1 day and 7 days.

Starting from the beginning of the simulation horizon, the production plan for the first planning horizon (e.g., 30 days) is implemented through simulation for periods in the first interval (e.g., 10 days). Next, the data related to model parameters, such as the inventory level of raw material in addition to the inventory and backorder levels of products resulted, after implementing the plan, are calculated at the end of the first interval. These updated values, in addition to a newly generated random demand profile are then used as a feedback to update the plan for the next planning horizon. The updated plan will then be implemented via simulation for another re-planning interval. This process continues for all intervals till the end of the simulation horizon. Figure 3 illustrates the above process, where the planning horizon and re-planning frequency are considered as 30 and 10 days, respectively.

More detail of how to perform implementation phase are provided after describing the procedures of generating random components in the simulation process.

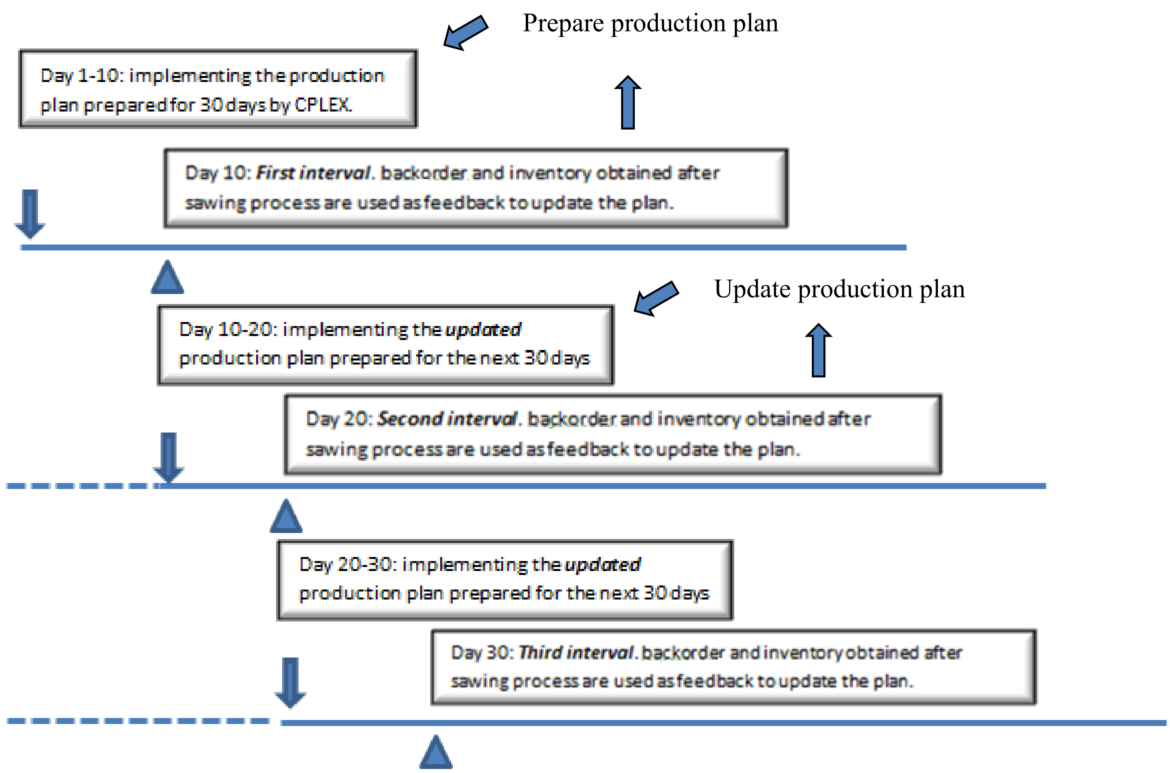

Figure 3- The simulation process

3.2 Random factors in the simulation

There are two random factors considered in the simulation process described earlier: random process yields and random demand. The method for generating random components should reflect the behavior of random yield and demand in real sawmills.

3.2.1 Random yield scenarios

Yield scenarios are generated based on a sample of yields prepared by a log sawing simulator (Optitek, FPI-innovation). Optitek is developed to simulate sawing process of Quebec sawmills industry. The class of raw material (logs), cutting pattern, and the number of logs to be sawn are the inputs for this simulator. Considering the input data and random characteristics of logs, Optitek provides the yield scenarios as the output. It is necessary to mention that although the logs are from the same class of raw material and they are sawn by the same cutting pattern, their yields are not the same. The reason is their nonhomogeneous characteristics caused during the growth period. Based on $\mathrm{Optitek}^{(C)}$ data, the probability distribution of process yields is estimated as follows [5]:

1 Take a sample of logs in each log class (e.g. 3000) and let them be processed by each cutting pattern. Compute the average yield of the sample.

2 Repeat step 1 for a number of replications (e.g., 30).

3 By the Central Limit Theorem (CLT) in statistics, the average yield has a normal distribution. Thus, based on the average yields computed for each replication in step 2, estimate the mean and variance of normal distribution corresponding to the average yields of each process (combination of cutting pattern and class of raw material).

Hence, for each process, a normal distribution is estimated. It is assumed that raw materials are supplied from the same section of forest during the planning horizon. Therefore, the yields distributions are supposed to be the same for all periods of the planning horizon. It is worth mentioning that these normal distributions are used to generate yield scenarios for both the stochastic production plan in the planning phase and the implementing phase in the simulation process.

3.2.2 Random demand profile

According to Lemieux et al. [7], two types of demand are observed in sawmills:

Spot market demand: the customer refers to the sales department and asks for required products. If the products are available at the desired time, the demand will become a formal order. Otherwise, it is assumed that the customer will choose another supplier. As a conclusion, the spot market demand does not affect the backorder cost.

Contract-based demand: In the contract-based demand, in contrast to the spot market demand, if a customer order cannot be fulfilled on time, the company will face a backorder cost. The procedure for generating the contract-based demand is as follow:

1 Consider the start date of simulation.

2 Generate a random interval for the next demand date.

3 If the calculated date for the next demand is prior to the end date of simulation, continue generating the next interval, else stop generating the demand dates.

4 Generate total demand quantity for each product, randomly, after generating all demand dates. This random quantity represents the total demand of each product during the simulation horizon.

5 Allocate to and divide the total demand of each product among demand dates.

The procedure to generate the total demand quantity for the simulation horizon (step 4 of above procedure) is as follows:

1 Calculate average demand for each product based on historical data.

2 Calculate maximum and minimum product demands based on the average amounts and by considering a standard deviation (e.g. 40%). We have considered 5% and 40% as standard deviation of demand for our case study, based on experts' point of view. Table 3 incorporates the average, and maximum and minimum amounts of demand. The uniform distribution for demand of each product can be calculated based on the values of these three parameters.

3 Generate total demand quantity for the simulation horizon for each product randomly based on the mentioned uniform distributions.

It is important to remind that this demand profile is updated at each re-planning interval by multiplying them by a noise factor (uniformly distributed on [0, 1]). The next section explains how these random components will be implemented in the simulation process.

3.3 Implementing the random components in the simulation process

In section 3.1 we briefly explained simulation process and its phases. In this section, the simulating procedure is described in more details regarding the information presented in section 3.2.

Table 3- Demand uniform distribution regarding 5% and 40% of demand standard deviation

Product Code	Average	Minimum (5% variation)	Maximum (5% variation)	Minimum (40% variation)	Maximum (40% variation)
P38-P41	102430	97308	107551	61474	143385
P42-P43	657199	624339	690059	394353	920045
P46-P49	947677	900293	995060	568606	1326747
P50-P53	1688576	1604147	1773005	1013154	2363998
P54-P56	2511002	2385452	2636552	1506617	3515387
P58-P61	15836936	15045089	16628783	9502187	22171684
P62-P63	17731	16844	18618	10664	24798
P66-P68	32524	30898	34150	19539	45509
P70-P71	464301	441086	487516	278597	650005
P74-P75	565455	537182	593728	339307	791603
P78-P80	3998081	3798177	4197985	2398865	5597296
P34-P37	1360316	1292300	1428332	816189	1904442
P30-P33	564412	536191	592632	338647	790176
P26-P29	751819	714228	789409	451125	1052512
P22-P24	453658	430975	476340	272211	635104
P18-P21	509961	484463	535459	305993	713929
P14-P17	108241	102828	113653	64944	151537
P13	1804907	1714662	1895152	1082952	2526861
P12	461129	438073	484185	276702	645556
P11	648196	615786	680605	388925	907466
P9	334007	317306	350707	200412	467601
P8	1590167	1510659	1669675	954125	2226209
P7	487759	463371	512147	292689	682829
P6	689513	655037	723988	413724	965301
P5	646301	613985	678616	387796	904805
P4	653411	620740	686082	392055	914767
P10	429796	408306	451285	257886	601705

To perform the simulation, first the optimal production plan identified by each production planning model indicates how many times each process should be run in each period. For example X_{11}= 20 is equivalent to repeat process 1 for 20 times in period 1. By "process" we mean a combination of a cutting pattern and a class of raw materials. If this production plan was implemented in real sawmills, the process yields would differ randomly for every replication of the process due to different characteristics of logs (as the raw materials). Kazemi Zanjani et al. [5] proposed the following process to simulate the implementation of production plans in sawmills by taking into account random process yields:

1 Get production plans proposed by both models in addition to random demand and yield scenarios as the inputs.

2 Simulate the production plan implementation as follows:

    2.1 Define the sample size equal to the number of times each process should be run in each period (production plan).

    2.2 Take randomly a sample of scenarios (with the defined size) for the yields of each process. Available scenarios for the yields of each process are based on the estimated normal distribution for each set of processes and products. Kazemi Zanjani et al. [5] explained that this step of simulation process is equivalent to selecting a sample of logs in each class of raw materials, randomly, and cut them by different cutting patterns.

3 Compute the total production size of each product at the end of each period, after simulating the plan implementation for that period.

4 Compute the backorder or inventory size of each product in each period based on the total production size of that product (computed in the previous step) and its demand for that period.

Figure 4 illustrates the above process.

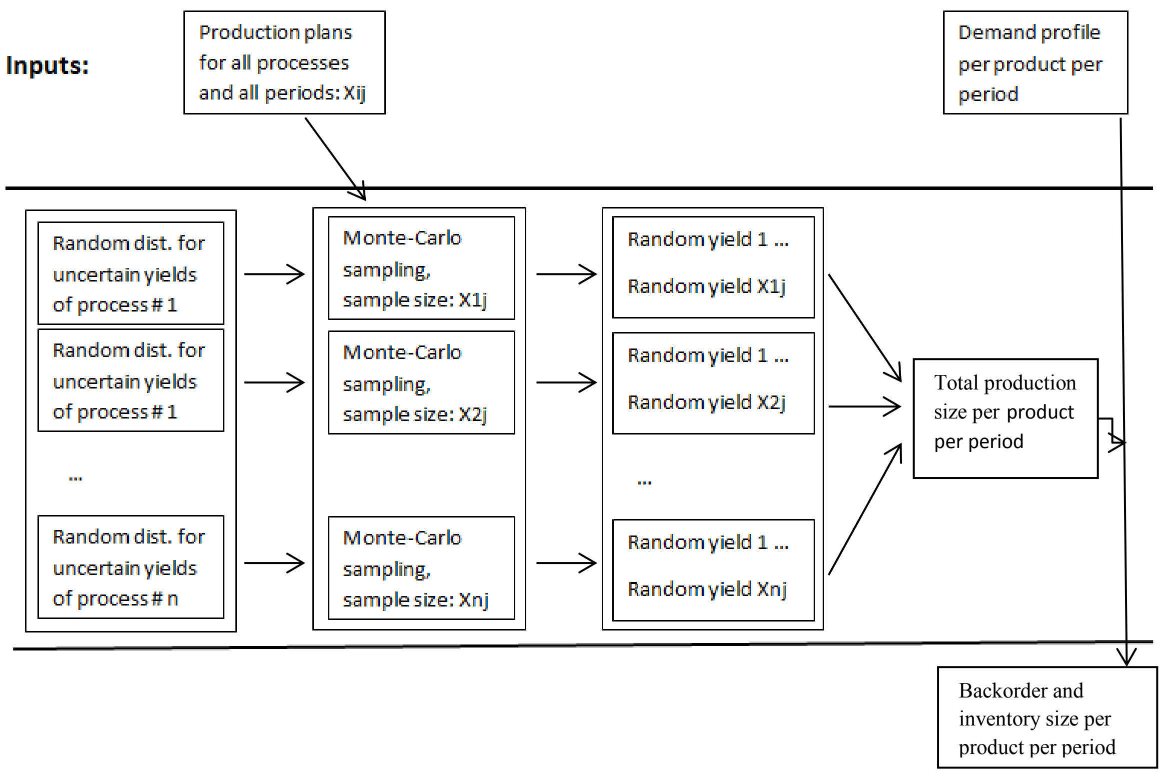

Figure 4- Simulation process of implementing the production plans in sawmills [5]

3.4 The steps applied for running the designed experiments

As we mentioned previously, the method to perform the experiments is based on rules of the design of experiments (DOE). We explained about different types of DOE, and summarized the major steps of applying these experiments. In this section, we explain how we have applied those steps in our case study.

- Recognition and statement of the problem: Our goal is to compare deterministic and stochastic production planning models for sawing units of sawmills. We also propose a decision framework that identifies under which circumstances it is suitable to use stochastic or deterministic production planning approaches.
- Selection of response variables: We need to introduce key performance indicators (KPIs) to evaluate and compare the performance of stochastic and deterministic approaches. Two KPIs were considered as follow:

  - Backorder cost: defined as the cost of unfulfilled demand at the end of the simulation horizon.
  - Inventory cost: defined as the cost of unsold products/ by-products at the end of the simulation horizon.

- Choice of factors, levels and their range: There are several factors which may influence the performance of production planning models. There are two types of factors in each system or process: controllable and noise (uncontrollable) factors. Four potential controllable factors and one noise factor are defined as follows:

  - Length of planning horizon: defined as the period of time on which the planning is applied.
  - Re-planning frequency: defined as re-planning intervals.
  - Demand level: it represents the average of demand and is defined as the percentage of total capacity of sawmill.
  - Planning approach: defined as stochastic and deterministic approaches.

- Demand variation (noise factor): defined as the standard deviation of demand probability distribution

We have considered two levels for each factor based on experts' point of view which will be explained in the experimental specification section in the next chapter.

- Choice of experimental design: It is necessary to define the suitable method and number of replications while designing the experiments. Moreover, the resolution of experiments should be decided as well. In our case, we started with 2^k experiments which is a special type of factorial design and completed the study with three methodologies of robust design, including Taguchi crossed array method, combined array method and a protocol for combining robust design and stochastic simulation. The number of replications is considered to be 10 and 20 in different experiments. In our study, as all levels of interactions are important to be investigated, we ran the full factorial design. In other words, the highest resolution was considered in all the designed experiments, mentioned above.

- Performing the experiment: As we mentioned earlier, it is not realistic to run the experiments in real sawmill. Hence, in this study we used Monte Carlo simulation to run each experiment.

- Statistical analysis of the data: statistical methods should by applied for analyzing in cases including experimental error. "Statistical methods help us to present the results of many experiments in terms of an empirical model, that is, an equation derived from the data that expresses the relationship between the response variable and the important design factor(s)" [9]. In current study we used regression models and analysis of variance as statistical (ANOVA) methods for analyzing the data.

- Conclusion and recommendation: The last step is to propose a conclusion. In our case, the decision framework which recommends the appropriate production planning approach represents the desired conclusion.

4 Experimental Results and Analysis

In this chapter, the results of applying the proposed methodology, presented in the previous section, for a realistic scale prototype sawmill is provided. It includes the step by step designed experiments performed in this study, and illustrates the procedure which led to final experimental setting and results.

We first present the explanation of the case study and specification of experiments. The results of designed experiments in addition to a decision framework are then provided. This chapter includes 6 sections:

4.1 Case study

4.2 Experimental specifications

4.3 Full Factorial design

4.4 Classical robust parameter designs

4.5 A protocol to combine robust parameter design and stochastic simulation

4.6 Decision framework

4.1 Case study

As we mentioned previously, the focus of this study is on sawing unit of sawmills. We considered a sawmill industry located in Quebec (Canada) as our case study. It includes 27 products/ by products (logs) with random demand which follow a uniform distribution as explained in the methodology chapter. The lumbers are sawn to produce the logs and they are classified based on their two end diameters into 3 classes. In addition, 5 different cutting patterns were defined for sawing the logs. In other words, there are totally 15 processes (the combination of each cutting pattern and each class of raw material) to produce the aforesaid 27 products. The process yields follow a normal distribution which is identified for each process as explained in the methodology chapter.

The next section explains about the experimental specifications such as the defined factors, KPIs and technical support of the experiments.

4.2 Experimental specifications

As we mentioned in our methodology, we initially defined four potential controllable factors including: length of planning horizon, re-planning frequency, demand level, and planning approach. In addition, we considered one noise (uncontrollable) factor in our study which is the demand variation.

Two levels have been considered for each factor which is illustrated in following tables:

Table 4- Description of controllable factor levels in experimental designs

Factor level / Factor	-	+
Planning Approach	Deterministic	Stochastic
Length of Planning Horizon	15 days	30 days
Re-planning Frequency	1 day	7 days
Demand Level (% of total capacity)	50%	100%

Table 5- Description of noise factor levels in experimental designs

Noise Factor		
Factor level / Factor	-	+
Demand Variation	5%	40%

Although we considered 100 percent as the high level for demand in the final sets of our experiments, we had tried other values in initial experimental revisions. It is also important to mention that it is tricky to define the exact sawing capacity of sawmill.

In order to evaluate and compare the experimental results, two KPIs have been defined: backorder and inventory costs.

As none of the interactions between controllable factors could be definitely considered to be unimportant, based on experts' point of view, it was decided to run the experiments for all possible factor interactions. In other words, the highest resolution was considered in our experiments.

We used the Microsoft C#© to code our Monte-Carlo stochastic simulation. In addition Cplex 12.1© was used for optimizing the production planning models to be implemented by simulation. The simulation horizon considered to be one year. Due to the fact that optimization and simulation were time consuming processes, especially for stochastic models, we used 10 to 12 parallel computers (CPU i7, with 16Gb RAM) to accelerate the procedure. Each set of experiments took approximately 1 to 2 weeks to be completed.

4.3 Full factorial design

In order to find the way that factors and their interactions are influencing the KPIs, a full factorial design was performed. As none of the factor interactions could be definitely considered to be negligible based on the experts' point of view, it was decided to run a full factorial design experiment.

Demand standard deviation was considered to be 5% in this set of experiments. The backorder and inventory costs were considered as KPIs. Each experiment was replicated for 10 times by considering four controllable factors, each at two levels. Hence, a total of 160 experiments were run. Table 6 illustrates the design of the experiment.

Table 6- Design of the full factorial experiment

2^4 Full Factorial Design				
Run	Planning Approach	Length of Planning Horizon	Re-planning Frequency	Demand Level (% of total capacity)
1	+	+	+	+
2	+	+	+	-
3	+	+	-	+
4	+	+	-	-
5	+	-	+	+
6	+	-	+	-
7	+	-	-	+
8	+	-	-	-
9	-	+	+	+
10	-	+	+	-
11	-	+	-	+
12	-	+	-	-
13	-	-	+	+
14	-	-	+	-
15	-	-	-	+
16	-	-	-	-

The achieved values for backorder and inventory costs are listed in appendix II. Analysis of variance (ANOVA) is the statistical tool that is applied to analyze the results of the designed experiments. By applying ANOVA, a regression model is achieved which illustrates the influence of controllable factors and their interactions on the output of the system, which can be measured by KPIs. R-squared (R-sq) is an indicator of the fitness of the regression model to the experimental data. Higher R-sq. is desired for fitting a regression model. As it is illustrated in table 7, the achieved R-sq. in the 2^4 factorial experiment is satisfactory to continue the analysis based on the resulted regression models for backorder and inventory costs.

Table 7- Adjusted R-sq. of the full factorial experiment

KPI	R-sq. (adj.)
Backorder cost	96.99%
Inventory cost	99.69%

In ANOVA, the most effective factors and their interactions are analyzed by reviewing P-values. Considering the experimental error (α-error) as 5%, any factor/ interaction with P-value less that 5% can be considered as factors/ interactions which highly affect the KPI. Table 8 incorporates factors, interactions and the associated P-values based on ANOVA results for the backorder cost.

Regarding the presented data in table 8, planning approach, demand level and their interaction are the most influencing factor/interactions on the backorder cost. If the interaction of some factors is important, only the interaction will be interpreted but not the main effect of each factor. Figure 5 illustrates the interaction plot of demand level and planning approach regarding the backorder cost.

It is important to mention that the coded units were applied in the experiments. This way "-1" presents the deterministic production planning approach and "+1" presents the

Table 8- Resulting P-value regarding simulated backorder cost of the full factorial experiment

Factor/ Interaction	P-value
Planning App	0.10%
Planning Hor.	58.00%
Re-planning	81.00%
Demand Level	0.00%
Planning App*Planning Hor.	67.20%
Planning App*Re-planning	86.60%
Planning App*Demand Level	0.10%
Planning Hor.*Re-planning	67.70%
Planning Hor.*Demand Level	58.00%
Re-planning *Demand Level	81.00%
Planning App*Planning Hor.*Re-planning	54.70%
Planning App*Planning Hor.*Demand Level	67.20%
Planning App*Re-planning *Demand Level	86.60%
Planning Hor.*Re-planning *Demand Level	67.70%
Planning App*Planning Hor.*Re-planning *Demand Level	54.70%

stochastic one. This figure can be interpreted as follows: in higher levels of demand, stochastic model performs better than the deterministic model, in terms of the backorder cost. In other words, the superiority of stochastic model versus deterministic one in higher demand levels is more considerable. It is again important to emphasize that the P-values of other factors and factor interactions are large, making it much less probable to be effective comparing the important factors/ interactions on the backorder cost.

Table 9 incorporates factors, interactions and the associated P-values based on ANOVA results for the inventory cost. As the table illustrates, the demand level is the most influencing factor impacting the inventory cost.

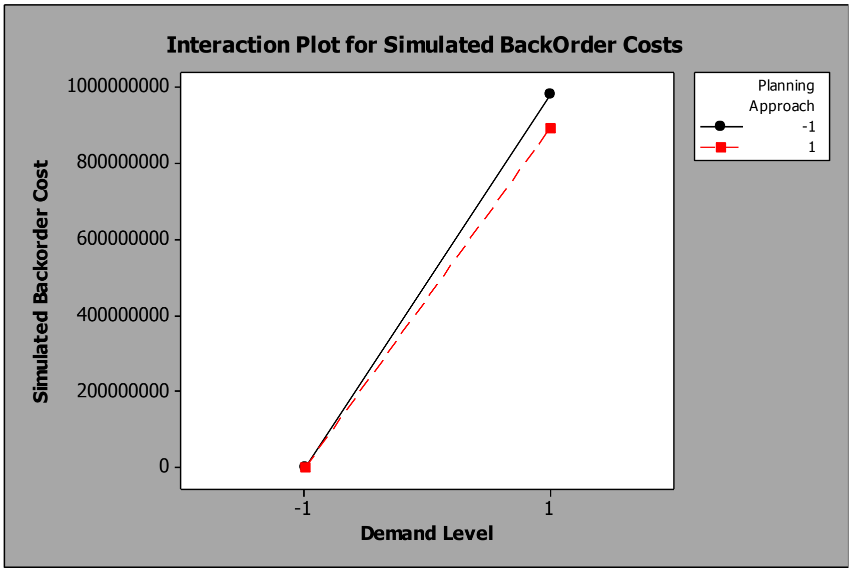

Figure 5- Demand level and planning approach interaction effect plot regarding the simulated backorder cost of the full factorial experiment

Table 9- Resulting P-value regarding the simulated inventory cost of the full factorial experiment

Factor/ Interaction	P-value
Planning App	92.60%
Planning Hor.	68.40%
Re-planning	38.00%
Demand Level	0.00%
Planning App*Planning Hor.	13.20%
Planning App*Re-planning	53.60%
Planning App*Demand Level	94.80%
Planning Hor.*Re-planning	50.10%
Planning Hor.*Demand Level	47.40%
Re-planning *Demand Level	43.40%
Planning App*Planning Hor.*Re-planning	67.70%
Planning App*Planning Hor.*Demand Level	9.50%
Planning App*Re-planning *Demand Level	58.90%
Planning Hor.*Re-planning *Demand Level	14.50%
Planning App*Planning Hor.*Re-planning *Demand Level	69.10%

Figure 6 illustrates the impact of the demand level on the inventory cost. It illustrates that by increasing the demand level the simulated inventory cost decreases, as it could be expected.

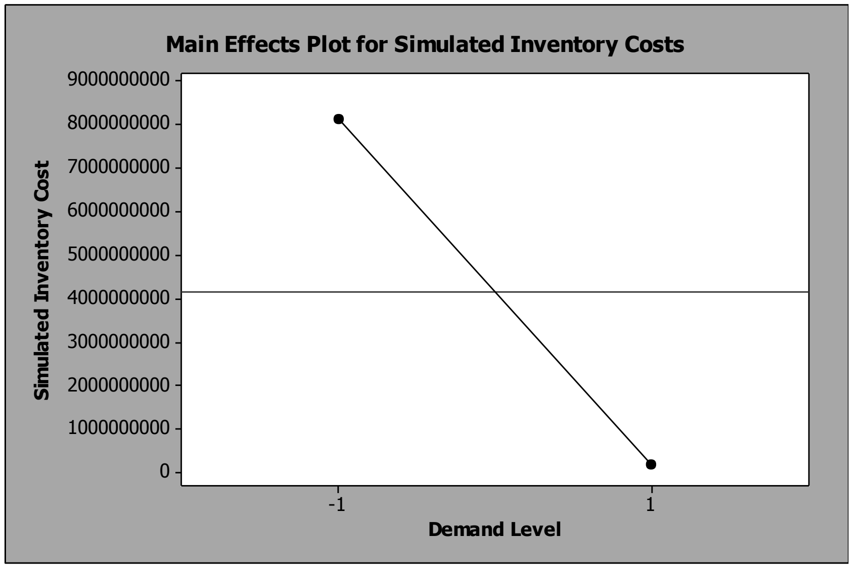

Figure 6- Demand level main effect plot regarding simulated inventory cost of the full factorial experiment

A decision framework thus can be proposed based on the results discussed in this section. The decision framework can differ regarding the KPI of interest:

- Considering the backorder cost: stochastic model has better performance especially in higher demand levels. In summary, it is better to apply stochastic approach when the demand gets close to the maximum capacity of sawmill.

- Considering the inventory cost: by increasing the demand level, the simulated inventory cost will decrease which is an expected result.

It is important to emphasize that the resulted decision framework is only valid considering the cases with 5% of demand variation. As the demand variation usually differs in reality and it is not necessarily a fixed amount, we continued our experiments considering other methods which can propose valid decision frameworks for uncertain demand variation levels.

4.4 Classical robust parameter designs

As we mentioned, the achieved results in the previous section are valid only for a fixed demand variation (e.g., 5%). As demand variation is a noise factor and is not usually under control in sawmills, we searched for a methodology which can support a decision framework while considering all levels of noise (uncontrollable) factors. Our goal is to present factor settings which are robust against variation.

In cases including the random noise factor, robust design is the appropriate option. This section covers the designs, results and analysis of two classical robust parameter methodologies. The methods include Taguchi (S/N ratio and direct variance) and combined array designs.

4.4.1 Taguchi robust parameter design

As we mentioned in section 2, the design structure in this approach is a combination of one inner and one outer array. The inner array includes controllable factors and the outer array includes noise factor(s). In our case, we have four controllable factors and one noise factor. Table 10 illustrates the experimental design for Taguchi method in current study. The factor levels definitions are as explained in tables 4 and 5. Considering two levels for all factors, totally 32 experiments were performed. Classical robust design was initially proposed for physical experiments. The core idea in such cases is to fix the noise factors artificially and perform the experiment. By fixing the noise factor in physical experiments, there will be no source of uncertainty and as a result, the initial version of Taguchi method does not consider replications in its procedure. However, this seems to be insufficient for our case study. We aim at imitating random behavior of demand by means of stochastic simulation. Despite fixing the levels of the demand variation, the demand profile in the simulation follows a uniform distribution. Hence, running the

Table 10- Design of Taguchi experiment

Inner Array					Outer Array	
Run	Planning Approach	Length of Planning Horizon	Re-planning Frequency	Demand Level (contract % of total capacity)	5% Demand variation	40% Demand variation
1	+	+		+	-	+
2	+	+	+	-	-	+
3	+	+	-	+	-	+
4	+	+	-	-	-	+
5	+	-	+	+	-	+
6	+	-	+	-	-	+
7	+	-	-	+	-	+
8	+	-	-	-	-	+
9	-	+	+	+	-	+
10	-	+	+	-	-	+
11	-	+	-	+	-	+
12	-	+	-	-	-	+
13	-	-	+	+	-	+
14	-	-	+	-	-	+
15	-	-	-	+	-	+
16	-	-	-	-	-	+

Taguchi design with one replication does not provide a realistic experimental condition. Through searching the literature more profoundly, we found some studies focused on combining Taguchi and stochastic simulation. More precisely, in these methods, several replications for each inner/outer array setting are considered. Consequently, the combination of Taguchi and stochastic simulation is applied in this study. The results are provided as follows:

4.4.1.1 Taguchi method by considering the S/N ratio

As we mentioned earlier, the initial proposed Taguchi method has two features for analyzing the experimental results: average and S/N ratio. The average is calculated regarding the achieved values for each combination of the inner array and several settings of the outer array. To clarify better, if the outer array includes for example two levels for the noise factor, there are two KPI values for each setting of inner array: one regarding the low level of noise factor and the other regarding the high level of noise factor. The average will then be calculated based on these two values. Table 11 illustrates the aforesaid combinations.

Table 11- Taguchi replications for inner/ outer array

Inner Array					Outer Array	
Run	Planning Approach	Length of Planning Horizon	Re-planning Frequency	Demand Level (% of total capacity)	5% Demand variation	40% Demand variation
1	+	+	+	+	-	+

Average

In the current study, each run has been repeated 20 times. Consequently, the average has been calculated regarding both outer array levels and different replications of each run.

We also calculated the S/N ratio as the indicator of KPI variance. Higher S/N ratio illustrates less transmitted variability from noise factors. As lower back order and inventory sizes are desired, the following formulation was applied to find S/N ratio:

$$\text{S/N ratio} = -\,10\log\left(\frac{\sum_{i=1}^{n} {y_i}^2}{n}\right) \tag{2}$$

where:

- "n" represents the number of replications (which is equal to 40 in our case) and
- "y_i^2" represents the squared value of KPI

Table 12 illustrates the calculated average and S/N ratio for backorder and inventory costs based on the simulation results.

Table 12- Calculated average and S/N ratio for backorder and inventory costs in the Taguchi method

Planning Approach	Length of Planning Horizon	Re-planning Frequency	Demand Level	Backorder Cost Average	Inventory Cost Average	Backorder Cost S/N Ratio	Inventory Cost S/N Ratio
Deterministic	15	1	0.5	95.4963405	8301619220	-54.70675386	-198.532039
Deterministic	15	1	1	1723320006	467548041	-186.189048	-176.555327
Deterministic	15	7	0.5	90459.1347	8355268316	-109.4223561	-198.582122
Deterministic	15	7	1	1868619665	431330457	-187.8290212	-175.835756
Deterministic	30	1	0.5	1983.76628	8695428971	-81.97040998	-198.946365
Deterministic	30	1	1	2197850234	545897422	-189.1314122	-177.933958
Deterministic	30	7	0.5	856563.929	8195497475	-133.5587681	-198.477399
Deterministic	30	7	1	1846494891	536156100	-187.4736256	-177.672318
Stochastic	15	1	0.5	332.338311	7703936229	-63.00094863	-197.905219
Stochastic	15	1	1	1704137894	492486673	-186.1397153	-177.235337
Stochastic	15	7	0.5	96156.0121	8018970168	-112.6421262	-198.193929
Stochastic	15	7	1	1864111003	554993374	-187.7629644	-177.772046
Stochastic	30	1	0.5	994.575487	8555485620	-71.60810796	-198.813369
Stochastic	30	1	1	1672545664	454809288	-185.7826523	-175.701194
Stochastic	30	7	0.5	152048.658	8216740001	-114.757237	-198.433089
Stochastic	30	7	1	2035337876	628663278	-188.449903	-178.56096

Detailed KPI values for each replication are presented in Appendix III. Considering the above results and 5% as experimental error, Figures 7 to 16 indicate and analyze the important factors/ interactions affecting average and S/N ratio based on ANOVA analysis. Significant factors/ interactions are highlighted by squared signs. Alphabetical codes are used to represent the factors, for simplicity, as follows:

- A: Planning approach
- B: Length of planning horizon
- C: Re-planning frequency
- D: Demand level

As it is illustrated in Figure 7, demand level is the only important factor influencing the backorder cost average; whereas re-planning frequency, demand level, their interaction

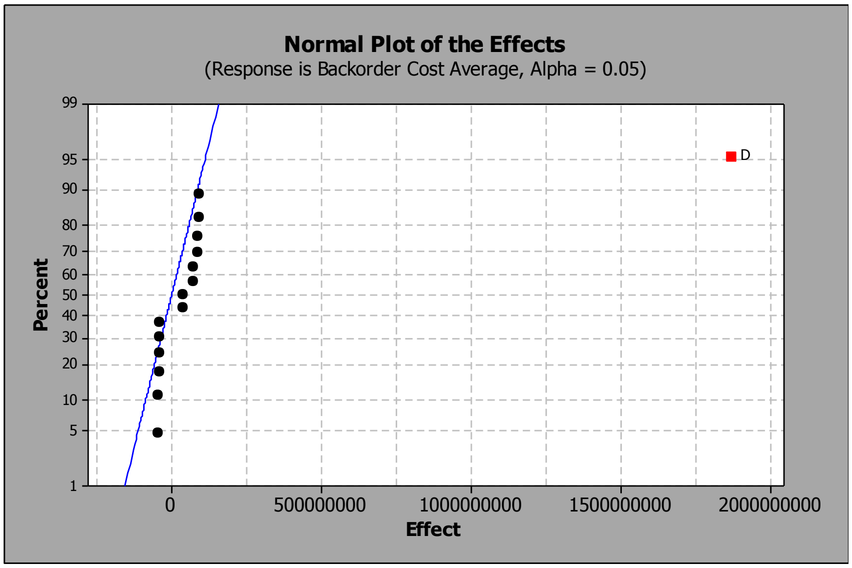

Figure 7- Normal plot of effects regarding backorder cost average of Taguchi method

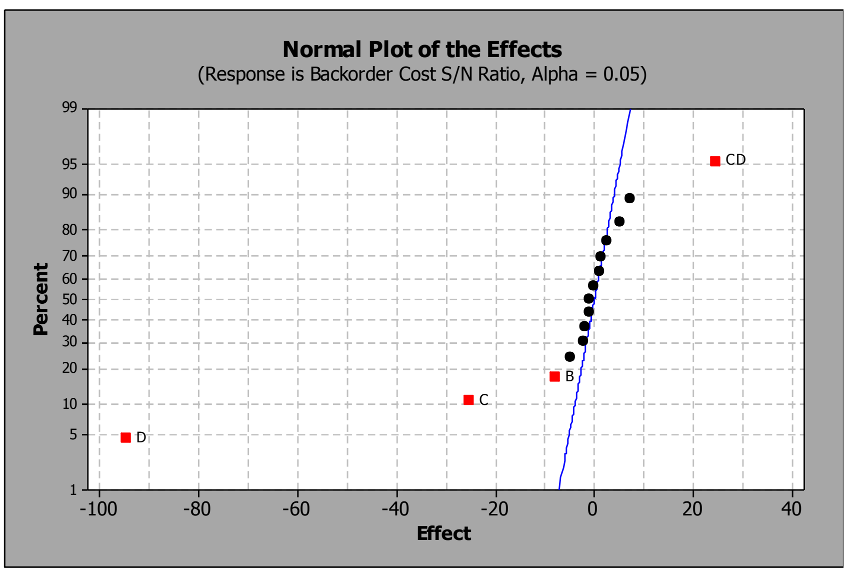

Figure 8- Normal plot of effects regarding backorder cost S/N ratio of Taguchi method

and length of planning horizon are important factors regarding the backorder cost S/N ratio. Reviewing the main and interaction effect plots is necessary to propose a decision framework.

By reviewing figures 9 to 11, it can be observed that both the backorder cost average and variation will be decreased at the low level of the demand. In other words, if the decision-maker prefers a low backorder cost he/ she should avoid contracts close to the maximum capacity of the sawing unit. On the other hand, a high S/N ratio (which is always desirable) can be achieved at the low level of re-planning frequency and planning horizon. Having short re-planning frequency is especially helpful while the demand is low. Prior to continue our analysis and decision framework, it is important to review the goal of robust design.

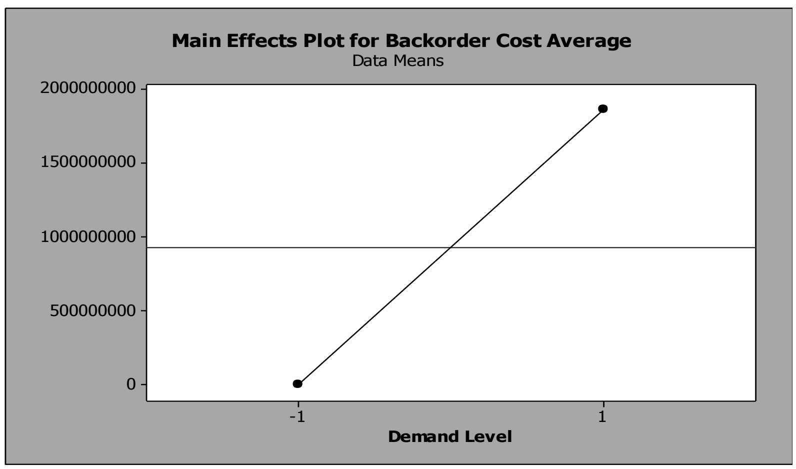

Figure 9- Main effect plot of demand level regarding backorder cost average of Taguchi method

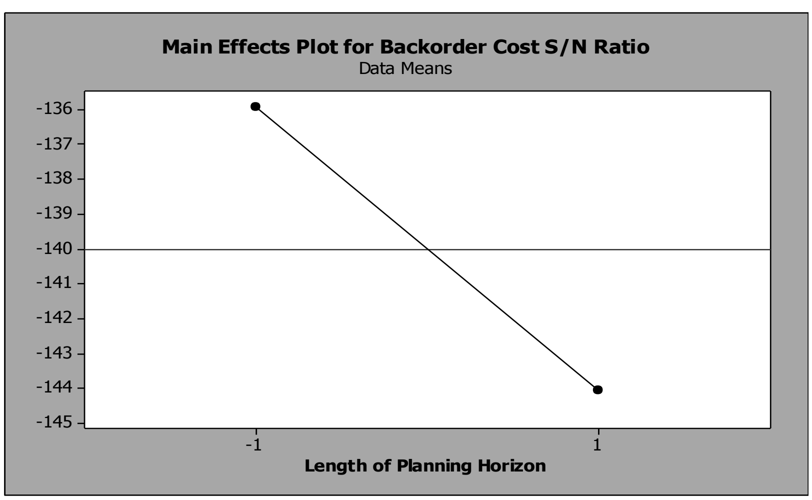

Figure 10- Main effect plot of planning horizon regarding backorder cost S/N ratio of Taguchi method

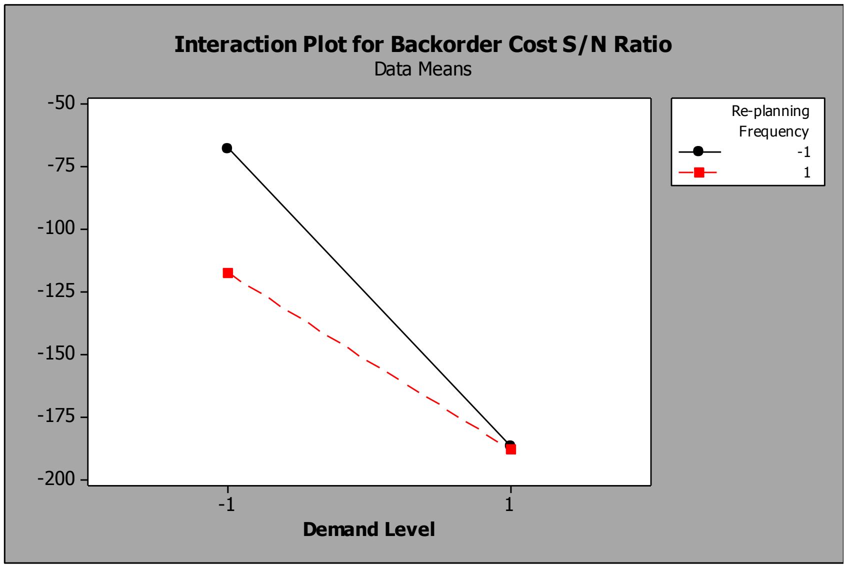

Figure 11- Interaction effect plot of demand level and re-planning frequency regarding backorder cost S/N ratio of Taguchi method

Robust design was initially proposed to find the optimum factor levels in a way that they can resist against variability as much as possible. By resistance, Taguchi meant two aspects: 1) to provide the maximum S/N ratio and 2) to optimize the average (maximum, minimum, or target). Hence, factors/ interactions with considerably higher effects (important factors) were considered for the optimization procedure. The initial Taguchi method proposed unique optimum important factor settings.

In the current study, as mentioned before the goal is to propose a decision framework; but not a unique optimal solution. The decision framework should guide the decision maker to choose between deterministic and stochastic production planning approaches under different circumstances. Different circumstances are caused by the other controllable factors levels (length of planning horizon, re-planning frequency and demand level) proposed by the decision maker. In fact, the decision maker defines the levels of these three factors as inputs and aims to find the appropriate planning approach based on the robust design results.

The decision maker is supposed to be approximately aware of the demand average (demand level as a percentage of the sawing unit capacity) as an input of decision framework. On the other hand, the decision maker does not have exact forecast of the variance of demand. The latter makes it impossible to define the demand variation as a controllable (known) factor. Therefore, it was necessary to apply a method which can provide a function to link controllable factors with KPIs, while it is also valid for all levels of the noise factor. These functions can be used to find the appropriate planning approach for different levels of three other controllable factors.

In this study, Taguchi method was applied to find the valid regression model (function) for all demand variation levels between 5% and 40%. In summary, the regression models achieved by applying the Taguchi method are the decision frameworks for predicting the KPI behavior under different factor combinations and regarding different planning approaches. There are some points which should be considered while using the regression models:

- It is worth mentioning that although the planning approach is not identified as a significant factor, it does not mean that it has no effect on the backorder/ inventory costs. Being considered as an un-important factor is equal to the fact that effect is small "*comparing*" the impact of important factors. Figure 12 illustrates the main effect of the planning approach on the backorder cost average as an example. It is clear that applying stochastic model leads to less average backorder; however, the improvement is smaller comparing the impact caused by decreasing the demand level.

- The achieved regression models are based on coded units (± 1), so the actual factor levels should be converted to coded values in order to be used as inputs.
- The management may define not all but some of the input factors and try to find the optimum setting for all unknown factors, including the planning approach.
- As there are two regression models regarding the average and S/N ratio in the Taguchi method, usually the objective is to maximize the S/N ratio as the objective function and to define a target, maximum or minimum level for the average, as a constraint and solve the resulting optimization model. In the current study, the target or maximum levels for KPIs depend on management point of view and are flexible. Equations 12 and 13 represent the achieved regression models for backorder cost average and S/N ratio, respectively. The previously

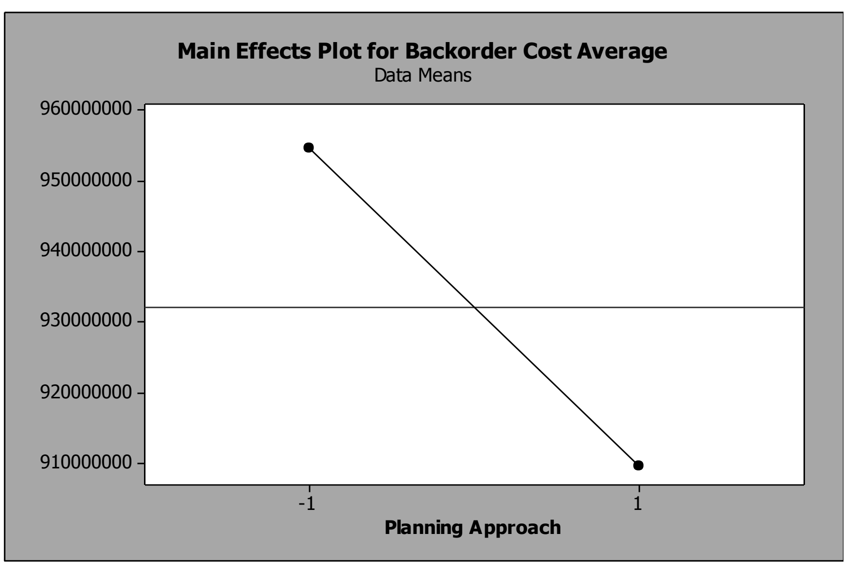

Figure 12- Main effect plot of planning approach regarding backorder cost average of Taguchi method

mentioned alphabetical codes are used for simplicity. Although the decision maker is the one who decides on the input factors, it is strongly recommended to have the lowest levels for planning horizon and re-planning frequency regarding the above figures and try to find the optimum planning approach regarding the demand level.

Backorder cost average ($millions) =

$932 - 23A + 37B + 20C + 932D - 20AB + 46AC - 22AD - 18BC + 37BD + 20CD + 44ABC - 20ABD + 46ACD - 18BCD + 44ABCD$ (12)

Backorder cost S/N ratio =

$-140.0 + 1.3\,A - 4.1\,B - 12.7\,C - 47.3\,D + 2.7\,AB + 0.6\,AC - 0.9\,AD + 0.7\,BC + 3.7\,BD + 12.2\,CD - 0.1\,ABC - 2.4\,ABD - 1.1\,ACD - 0.5\,BCD - 0.5\,ABCD$ (13)

The decision framework aforesaid is associated with the backorder cost and does not include the inventory cost, as another KPI. Figures 13 to 16 in addition to equations 14 and 15 represent the results for the inventory cost. The logic for interpreting these outputs is the same as the backorder cost.

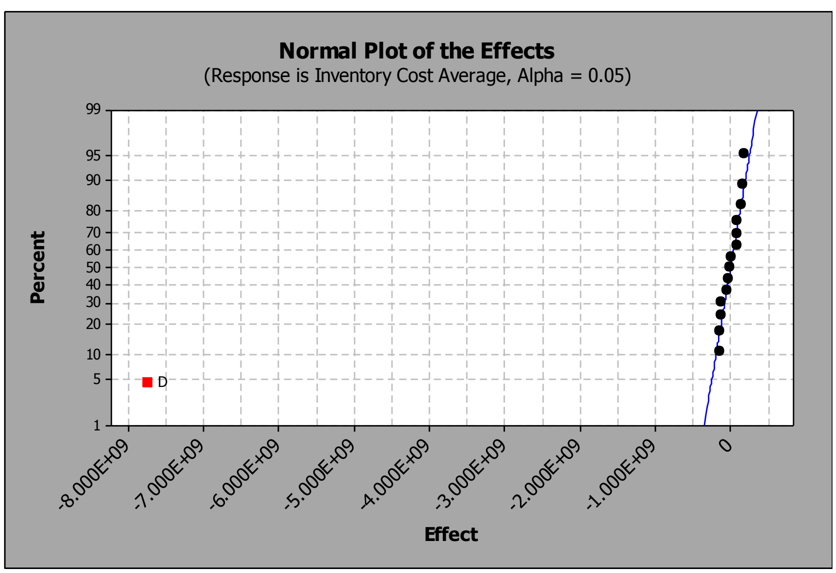

Figure 13- Normal plot of effects regarding inventory cost average of Taguchi method

Regarding the normal plots of effects, the only important factor is the demand level for both inventory cost average and S/N ratio. The main effect plots of demand for inventory cost average and S/N ratio are illustrated in figures 15 and 16.

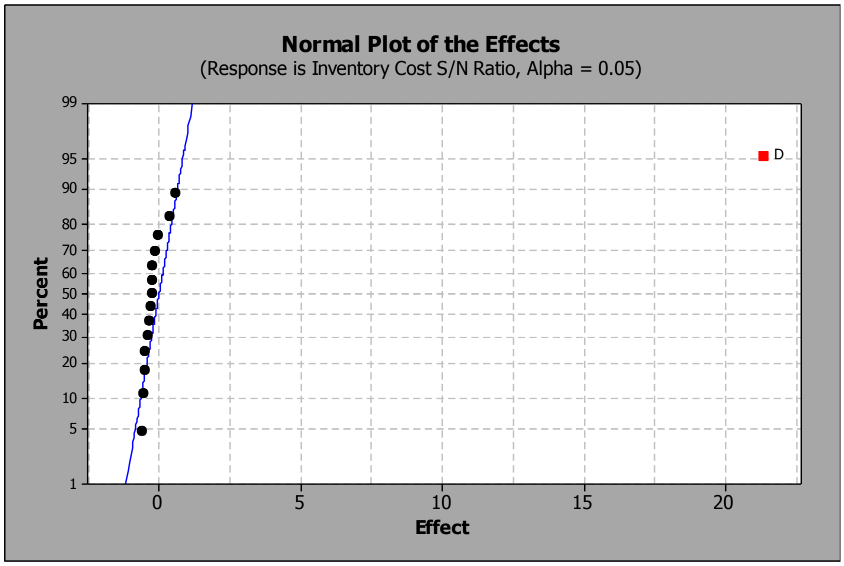

Figure 14- Normal plot of effects regarding inventory cost S/N ratio of Taguchi method

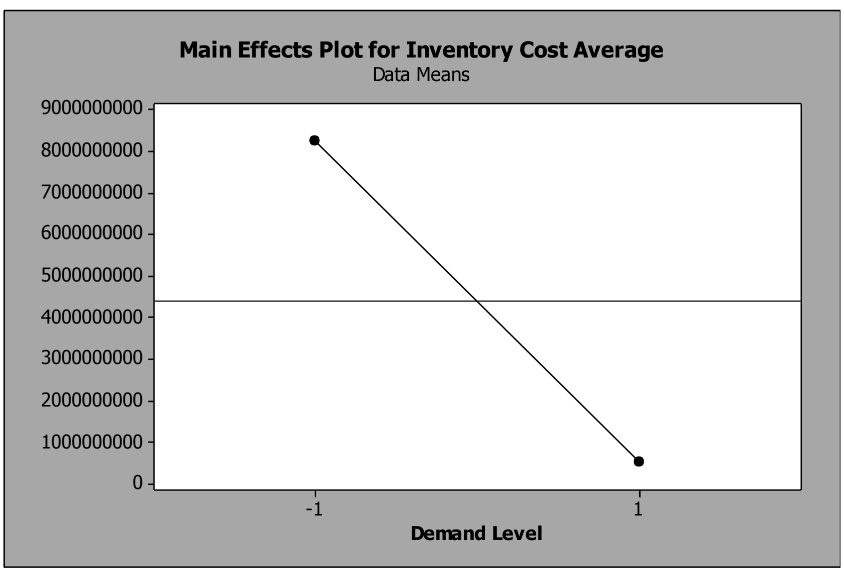

Figure 15- Main effect plot of demand level regarding inventory cost average of Taguchi method

According to the above figures, less inventory cost is expected for high demand levels. By reviewing Figure 16, we observe that high level demand will result in not only smaller average but also less variation for inventory cost for different levels of the noise factor. Equations 14 and 15 represent inventory cost average and S/N ratio regression models.

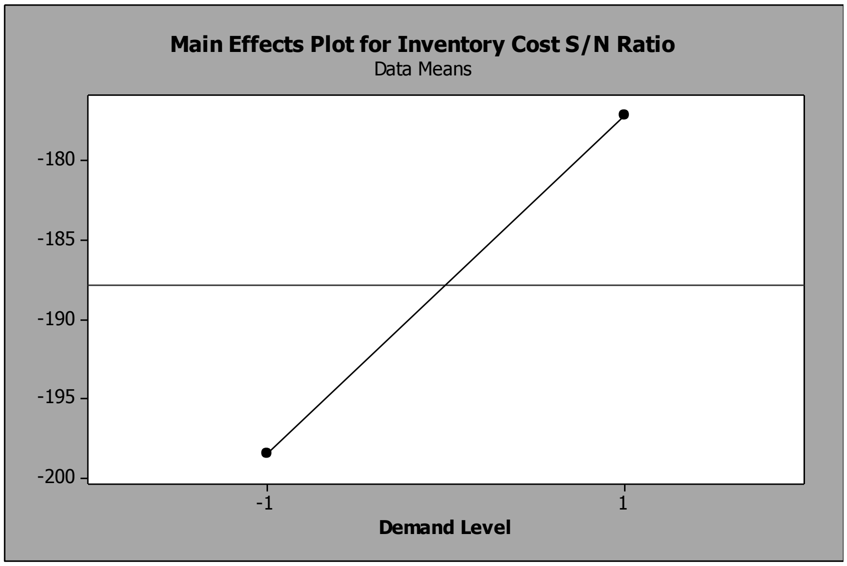

Figure 16- Main effect plot of demand level regarding inventory cost S/N ratio of Taguchi method

Inventory cost average ($ millions) =

$$4{,}385 - 56A + 94B - 17C - 3{,}871D + 42AB + 44AC + 75AD - 67BC - 67BD + 41CD - 0.9\,ABC - 60ABD - 9ACD + 84BCD + 12ABCD \quad (14)$$

Inventory cost S/N ratio =

$$-187.8 - 0.0\,A - 0.2\,B - 0.1\,C + 10.7\,D + 0.2\,AB - 0.3\,AC - 0.2\,AD - 0.1\,BC - 0.1\,BD - 0.2\,CD - 0.1\,ABC + 0.3\,ABD - 0.3\,ACD - 0.2\,BCD - 0.1\,ABCD \quad (15)$$

It is worth mentioning that the planning approach coefficient in the S/N ratio regression function was so small which was appeared as zero.

The proposed factor levels regarding different KPIs (backorder and inventory costs) may result in contradictory decision frameworks. In some cases a tradeoff must be considered regarding the priorities of KPIs. For example in the current study, the backorder cost has the higher priority.

4.4.1.2 Taguchi method by considering the direct variance

As we mentioned earlier, Taguchi proposed the S/N ratio as the indicator of variation. Montgomery [9] mentioned two shortcomings of S/N ratio: i) it needs complicated calculations, and ii) the accuracy and capability of the S/N ratio to find the appropriate solution for RPD problems can be argued. He consequently proposed to consider directly the variance of results, instead of calculating the S/N ratio. Since the variance follows chi-square distribution, the logarithm of variance which follows a normal distribution should be referred for further analysis. Table 13 illustrates the logarithm of variance for backorder and inventory costs. The detailed information for each replication is presented in appendix III.

Table 13- Backorder and Inventory cost logarithm of variance in the Taguchi experiment

Model	Planning Horizon	Rolling Horizon	Demand Volume Percentage	Backorder Cost Log. of Variance	Inventory Cost Log. of Variance
Deterministic	15	1	0.5	5.468060404	18.39154973
Deterministic	15	1	1	18.0859444	17.37985725
Deterministic	15	7	0.5	10.91061348	18.37937262
Deterministic	15	7	1	18.42164722	17.30609269
Deterministic	30	1	0.5	8.197040998	18.46544254
Deterministic	30	1	1	18.5369162	17.52077722
Deterministic	30	7	0.5	13.35259824	18.52433757
Deterministic	30	7	1	18.34941657	17.48468505
Stochastic	15	1	0.5	6.286364203	18.38812089
Stochastic	15	1	1	18.09275365	17.4681988
Stochastic	15	7	0.5	11.25278531	18.2345369
Stochastic	15	7	1	18.40885193	17.47440454
Stochastic	30	1	0.5	7.14107905	18.47268748
Stochastic	30	1	1	18.00633603	17.22791536
Stochastic	30	7	0.5	11.45177441	18.35290629
Stochastic	30	7	1	18.46670239	17.51984206

Based on the obtained results, the defined important factors and interactions are the same as S/N ratio results. The achieved regression models based on the direct variance of each KPI for different levels of the noise factor are provided as follows:

Backorder cost logarithm of variance =

$$13.777 - 0.138\,A + 0.411\,B + 1.300\,C + 4.519\,D - 0.283\,AB - 0.043\,AC + 0.086\,AD - 0.083\,BC - 0.367\,BD - 1.184\,CD + 0.019\,ABC + 0.232\,ABD + 0.122\,ACD + 0.035\,BCD + 0.065\,ABCD \quad (16)$$

Inventory cost logarithm of variance =

$$17.9119 - 0.0196\,A + 0.0342\,B - 0.0024\,C - 0.4892\,D - 0.0331\,AB + 0.0055\,AC + 0.0195\,AD + 0.0268\,BC - 0.0186\,BD + 0.0259\,CD + 0.0132\,ABC - 0.0312\,ABD + 0.0455\,ACD + 0.0137\,BCD + 0.0178\,ABCD \quad (17)$$

As it is illustrated, this model illustrates main effect of planning approach regarding inventory variation more precisely comparing to the S/N ratio approach.

As mentioned in the literature review, Taguchi method does not include any term regarding the main effect of noise factor(s) and interaction effects between the controllable and the noise factor(s) in its proposed regression models. Due to this shortcoming, we used the combined array design as an alternative approach.

4.4.2 Combined array design

The core idea of combined array design is to include the noise factor in the same array of controllable factors. This way, the achieved regression model contains the terms regarding main effect of noise factor and its interaction effects with controllable factors. Table 14 illustrates the designed combined array for the current case study. The experiment was performed regarding the highest resolution including all possible factor interactions. Considering two levels for each factor defined in tables 1 and 2, the total number of experiments is 32.

Table 14- Design of combined array experiment

Combined Array Design					
Run	Planning Approach	Length of Planning Horizon	Re-planning Frequency	Demand Level (% of total capacity)	Noise Factor (Demand variation)
1	+	+	+	+	+
2	+	+	+	+	-
3	+	+	+	-	+
4	+	+	+	-	-
5	+	+	-	+	+
6	+	+	-	+	-
7	+	+	-	-	+
8	+	+	-	-	-
9	+	-	+	+	+
10	+	-	+	+	-
11	+	-	+	-	+
12	+	-	+	-	-
13	+	-	-	+	+
14	+	-	-	+	-
15	+	-	-	-	+
16	+	-	-	-	-
17	-	+	+	+	+
18	-	+	+	+	-
19	-	+	+	-	+
20	-	+	+	-	-
21	-	+	-	+	+
22	-	+	-	+	-
23	-	+	-	-	+
24	-	+	-	-	-
25	-	-	+	+	+
26	-	-	+	+	-
27	-	-	+	-	+
28	-	-	+	-	-
29	-	-	-	+	+
30	-	-	-	+	-
31	-	-	-	-	+
32	-	-	-	-	-

The initial combined array methodology includes four steps:

1. To fix the noise factor levels.

2. To perform one replication of each experiment.

3. To use the achieved KPI to find a regression model (unlike the Taguchi method that uses the average and S/N ratio of KPIs to find regression models). The corresponding model is called the *response model* and includes the main and interaction effects of both controllable and noise factors.

4. To find the average and variance functions of the response model with respect to the noise factor.

Similar to Taguchi method, the initial version of combined array design does not include any replications. In our case, despite fixing the levels of the demand variation, the demand profile in the simulation follows a uniform distribution. Hence, running the combined array design with one replication does not provide a realistic experimental condition. Although we found no literature regarding the combination of combined array and stochastic simulation, we decided to apply it for 20 replications for each experiment. The average of the resulted KPIs was used to find the mentioned response model. Table 15 includes the resulted average backorder and inventory costs for each experiment regarding 20 replications.

The detailed results are provided in appendix III. Figures 17 and 18 present normal plots of effects for backorder and inventory costs, respectively, considering 5% as the experimental error and ANOVA as the analyzing tool. The normal plots indicate that the demand level is the only important controllable factor for both KPIs. Moreover, the noise factor (demand variation) is an important factor for both KPIs. As the achieved response model based on these results will be referred to find both average and variance functions, it is concluded that demand level is the only important controllable factor regarding average and variance of both KPIs, with respect to the noise factor.

Table 15- Resulted backorder and inventory costs averages in the combined array method

Planning Approach	Length of Planning Horizon	Re-planning Frequency	Demand Level	Demand variation	Backorder Cost Average	Inventory Cost Average
Determinist	15	1	0.5	5%	0	8158485246
Determinist	15	1	0.5	40%	190.9926809	8444753194
Determinist	15	1	1	5%	997808448.6	159134951.1
Determinist	15	1	1	40%	2448831563	775961130.7
Determinist	15	7	0.5	5%	0	8031525120
Determinist	15	7	0.5	40%	180918.2693	8679011511
Determinist	15	7	1	5%	963120734.8	149858650.1
Determinist	15	7	1	40%	2774118595	712802263.9
Determinist	30	1	0.5	5%	0	8214736217
Determinist	30	1	0.5	40%	3967.532555	9176121726
Determinist	30	1	1	5%	956052317.5	163958173.4
Determinist	30	1	1	40%	3439648151	927836670.9
Determinist	30	7	0.5	5%	0	8209101253
Determinist	30	7	0.5	40%	1713127.858	8181893697
Determinist	30	7	1	5%	941817804.6	152453451.2
Determinist	30	7	1	40%	2751171978	919858749.1
Stochastic	15	1	0.5	5%	0	8146474303
Stochastic	15	1	0.5	40%	664.6766224	7261398154
Stochastic	15	1	1	5%	918757069.1	155675384.6
Stochastic	15	1	1	40%	2489518719	829297960.5
Stochastic	15	7	0.5	5%	0	8100281225
Stochastic	15	7	0.5	40%	192312.0241	7937659110
Stochastic	15	7	1	5%	958949021.4	165953581.8
Stochastic	15	7	1	40%	2769272985	944033166.8
Stochastic	30	1	0.5	5%	0	8164664798
Stochastic	30	1	0.5	40%	1989.150974	8946306442
Stochastic	30	1	1	5%	932136876.6	168585772.2
Stochastic	30	1	1	40%	2412954451	741032804
Stochastic	30	7	0.5	5%	0	8178124329
Stochastic	30	7	0.5	40%	304097.3153	8255355672
Stochastic	30	7	1	5%	891410895.3	185825241.2
Stochastic	30	7	1	40%	3179264856	1071501316

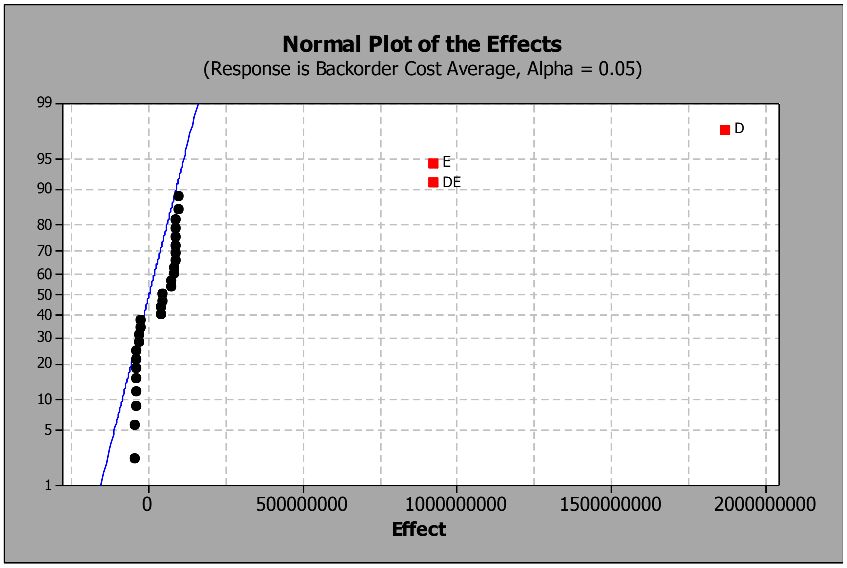

Figure 17- Normal plot of effects regarding backorder cost of combined array method

The backorder and inventory cost response models are presented as equations 18 and 19. "*E*" is the alphabetical code representing the demand variation.

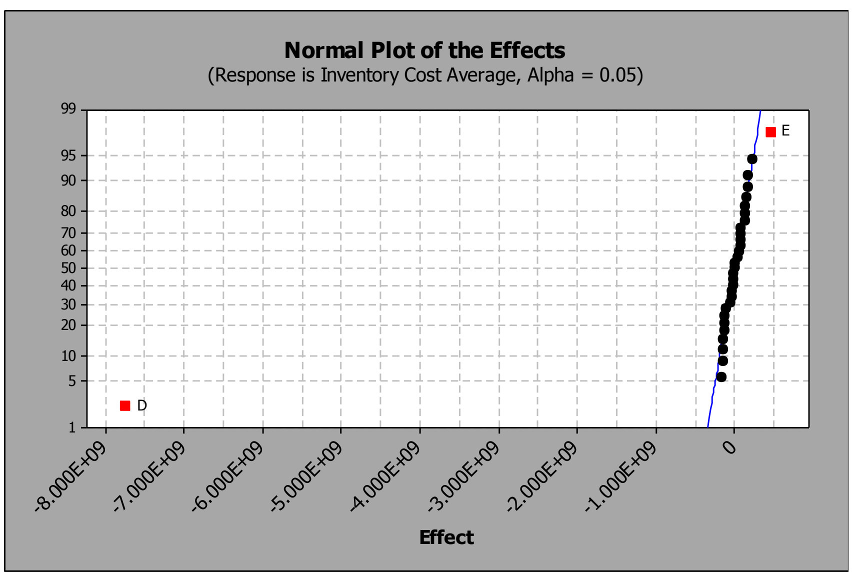

Figure 18- Normal plot of effects regarding inventory cost of combined array method

Backorder cost response model ($ millions) =

932- 23 A + 37B + 20 C + 931D + 459E - 19 AB + 45AC - 22AD - 12AE - 18BC + 36BD + 44BE + 19CD + 22CE + 459DE + 43ABC - 19ABD - 20ABE + 45ACD + 42E - 12ADE - 18BCD -

$14BCE + 44BDE + 22CDE + 43{,}761{,}443\ ABCD + 50ABCE - 20ABDE + 42ACDE - 14BCDE + 50ABCDE$ (18)

Inventory cost response model ($ millions) =

$4{,}384 - 56A + 93B - 17C - 3{,}870D + 228E + 41AB + 44AC + 75AD - 58AE - 66BC - 66BD + 70BE + 41CD - 7CE + 123DE - 0.9\ ABC - 60\ ABD + 48ABE - 8ACD + 34ACE + 70ADE + 84BCD - 78BCE - 48BDE + 30CDE + 11ABCD + 2ABCE - 70ABDE - 5ACDE + 95BCDE + 7ABCDE$ (19)

Considering the fact that the demand variation (noise) is modeled as coded values ($\pm\mathbf{1}$), its expected value and variance are zero and one, respectively. Consequently and based on calculations provided in the literature review, the average and variance functions with respect to the noise factor for backorder and inventory costs are as follows:

Backorder cost average ($ millions) = $Expected\ value_E$*(equation 18) =

$$932 - 22A + 37B + 19C + 931D - 19AB + 45AC - 22AD - 18BC + 36BD + 19CD + 43ABC - 19ABD + 45ACD - 18BCD + 43ABCD \quad (20)$$

Backorder cost variance ($ millions) = $Variance_E$* (equation 18) =

$$(459 - 12\,A + 44\,B + 22C + 459\,D - 20\,AB + 42\,AC - 12\,AD - 14\,BC + 44\,BD + 22\,CD + 50\,ABC - 20\,ABD + 42\,ACD - 14\,BCD + 50\,ABCD)^2 \quad (21)$$

Inventory cost average ($ millions) = $Expected\ value_E$* (equation 19) =

$4{,}384 - 56A + 93B - 17C - 3{,}870D + 41AB + 44AC + 75AD - 66BC - 66BD + 41CD - 0.9\ ABC - 60ABD - 8ACD + 84BCD + 11ABCD$ (22)

Inventory cost variance ($ millions) = $Variance_E$* (equation 19) =

$$(228 - 58A + 70B - 7C + 123D + 48AB + 34AC + 70AD - 78BC - 48BD + 30CD + 2ABC - 70ABD - 5ACD + 95BCD + 7ABCD)^2 \quad (23)$$

The only important factor regarding both backorder and inventory cost averages is demand level which is the same as the results of Taguchi method. Comparing to the

* Expected value and variance with respect to the noise factor (E)

Taguchi method, the combined array method could not find any important factor regarding the backorder cost variance except for the demand level. The reason is that the only important interaction between noise and controllable factors in response model is the interaction between demand level and demand variation. In addition, the combined array method could not find any important interaction between noise and controllable factors regarding inventory cost response model. Hence, there is no important factor regarding inventory variance. This might have happened due to the fact that the original combined array methodology does not include any procedure for replications (the initial version of Taguchi had the concept of replication which was extended by the users). Therefore the combined array design does not make profit of the possibility of having replications. We overcome the above shortcoming of the combined array design by applying another methodology in the following section.

4.5 A protocol to combine robust parameter design and stochastic simulation

This two stage approach methodology is proposed by Giovagnoli et al. [19] for combining robust parameter design and stochastic simulation. This method is considered as the general protocol which has similarities with combined array and Taguchi methods. According to [19], "Classical robust design relies on physical experiments whose factors are controllable factors and the noise factors. Noise factors, albeit they vary randomly in the process, are controlled in the experiment. Thus, only a few of them, and with few levels, are usually included in the design. This constraint can at times be relaxed in simulation." In order to relax this constraint, the noise factors are divided into two categories: i) the noises which are considered stochastically in experiments: Z_1 and, ii) the noises with fixed levels: Z_2.

In our case, the demand distribution (uniform distribution) is considered to be Z_1 which is simulated randomly in the simulation process. The demand variation is allocated fixed levels and is behaved as a fixed level noise factor in the design structure (Z_2). The experimental structure is the same as combined array illustrated in table 14 in the previous section. The defined factor levels are the same as previous experiments, as well.

The methodology is based on two stages: in the first stage, the computer experiments are run based on stochastically simulated demand distribution (Z_1) for each experiment. We emphasize that each experiment includes a fixed level of demand variation as a noise factor type II (Z_2). The level can be either 5% or 40%. Average and variance of several replications of each experiment will be then calculated. Similar to the Taguchi method, as the calculated variance follows a chi-square distribution, the logarithm of variance should be referred for further analysis. The achieved results are illustrated in table 16.

Table 16- Resulted backorder and inventory cost averages and logarithm of variance of new protocol

Planning Approach	Length of Planning Horizon	Re-planning Frequency	Demand Level	Demand variation	Backorder Cost Average	Backorder Cost Log. of Variance	Inventory Cost Average	Inventory Cost Log. of Variance
Determinist	15	1	0.5	5%	0	0.00	8158485246	16.89
Determinist	15	1	0.5	40%	191	5.77	8444753194	18.69
Determinist	15	1	1	5%	997808449	16.92	159134951	15.54
Determinist	15	1	1	40%	2448831563	18.12	775961131	17.46
Determinist	15	7	0.5	5%	0	0.00	8031525120	16.81
Determinist	15	7	0.5	40%	180918	11.18	8679011511	18.67
Determinist	15	7	1	5%	963120735	15.90	149858650	15.46
Determinist	15	7	1	40%	2774118595	18.57	712802264	17.39
Determinist	30	1	0.5	5%	0	0.00	8214736217	16.94
Determinist	30	1	0.5	40%	3968	8.50	9176121726	18.73
Determinist	30	1	1	5%	956052318	15.95	163958173	15.42
Determinist	30	1	1	40%	3439648151	18.58	927836671	17.57
Determinist	30	7	0.5	5%	0	0.00	8209101253	16.71
Determinist	30	7	0.5	40%	1713128	13.65	8181893697	18.83
Determinist	30	7	1	5%	941817805	15.80	152453451	15.56
Determinist	30	7	1	40%	2751171978	18.46	919858749	17.50
Stochastic	15	1	0.5	5%	0	0.00	8146474303	16.83
Stochastic	15	1	0.5	40%	665	6.57	7261398154	18.66
Stochastic	15	1	1	5%	918757069	15.95	155675385	15.18
Stochastic	15	1	1	40%	2489518719	18.09	829297960	17.56
Stochastic	15	7	0.5	5%	0	0.00	8100281225	16.91
Stochastic	15	7	0.5	40%	192312	11.54	7937659110	18.53
Stochastic	15	7	1	5%	958949021	16.06	165953582	15.08
Stochastic	15	7	1	40%	2769272985	18.55	944033167	17.47
Stochastic	30	1	0.5	5%	0	0.00	8164664798	16.93
Stochastic	30	1	0.5	40%	1989	7.42	8946306442	18.75
Stochastic	30	1	1	5%	932136877	16.18	168585772	15.22
Stochastic	30	1	1	40%	2412954451	17.96	741032804	17.24
Stochastic	30	7	0.5	5%	0	0.00	8178124329	16.98
Stochastic	30	7	0.5	40%	304097	11.73	8255355672	18.66
Stochastic	30	7	1	5%	891410895	16.14	185825241	15.37
Stochastic	30	7	1	40%	3179264856	18.51	1071501316	17.42

Figures 19 and 20 present the normal plot of effects indicating important factors/ interactions for backorder cost average and logarithm of variance, respectively.

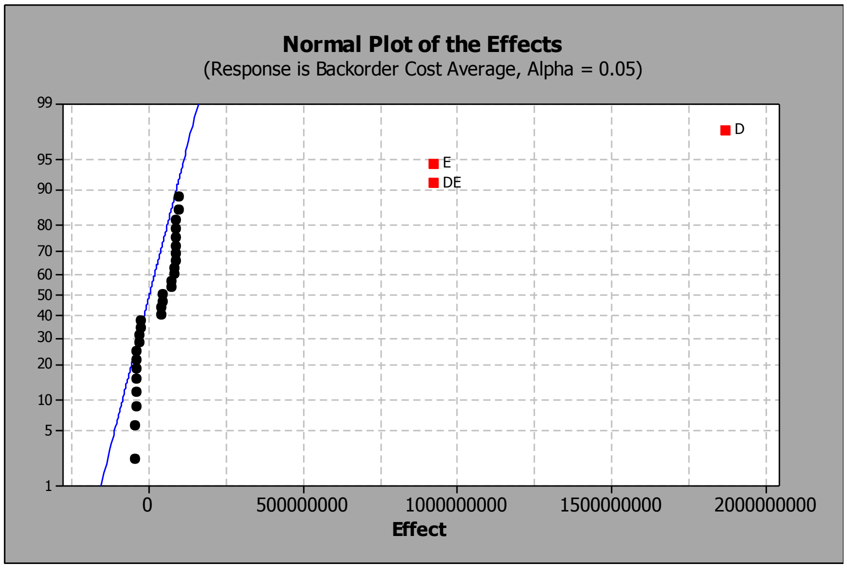

Figure 19- Normal plot of effects regarding backorder cost average of new protocol

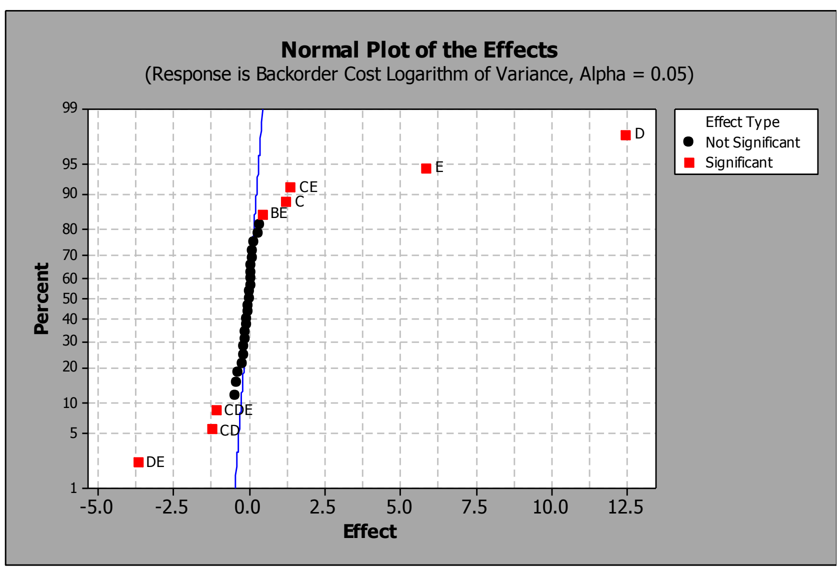

Figure 20- Normal plot of effects regarding backorder cost logarithm of variance of new protocol

As it is illustrated in figure 19, the interaction between demand level and demand variation are the only important factor considering backorder cost average. Figure 21 illustrates the interaction effect plot for the backorder cost average. Low level of demand has better performance in all levels of demand variation.

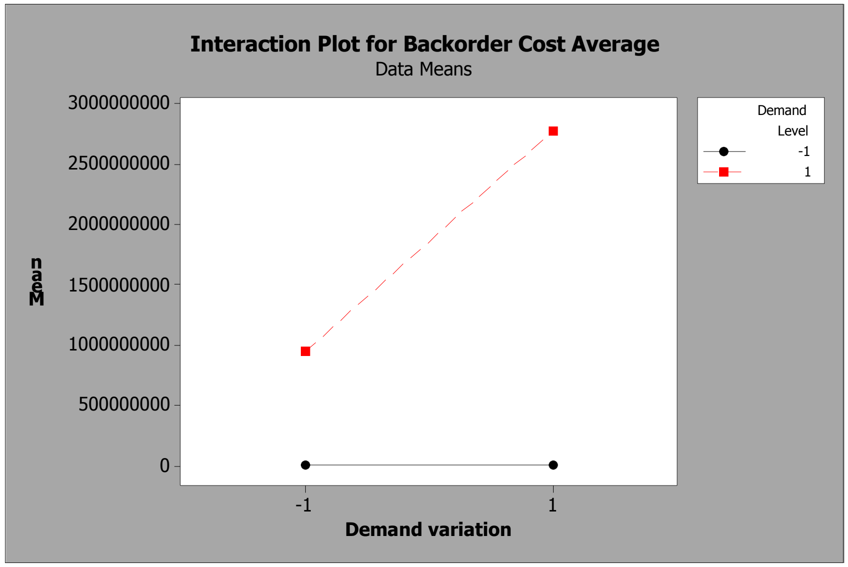

Figure 21- Interaction effect plot of demand level and demand variation regarding backorder cost average of new protocol

Figure 21 illustrates that i) the interaction of length of planning horizon and demand variation, and ii) the interaction between re-planning frequency, demand level and

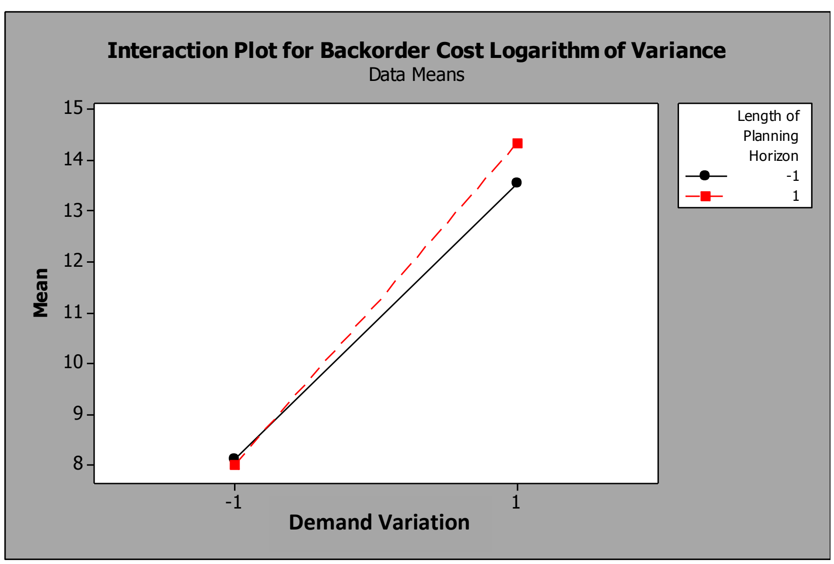

Figure 22- Interaction effect plot of demand level and demand variation regarding backorder cost logarithm of variance of new protocol

demand variation are significant regarding the backorder cost logarithm of variance. The figures 22 and 23 indicate the mentioned interactions. Based on figure 22, shorter planning horizon leads to less backorder variation especially in higher demand variation levels.

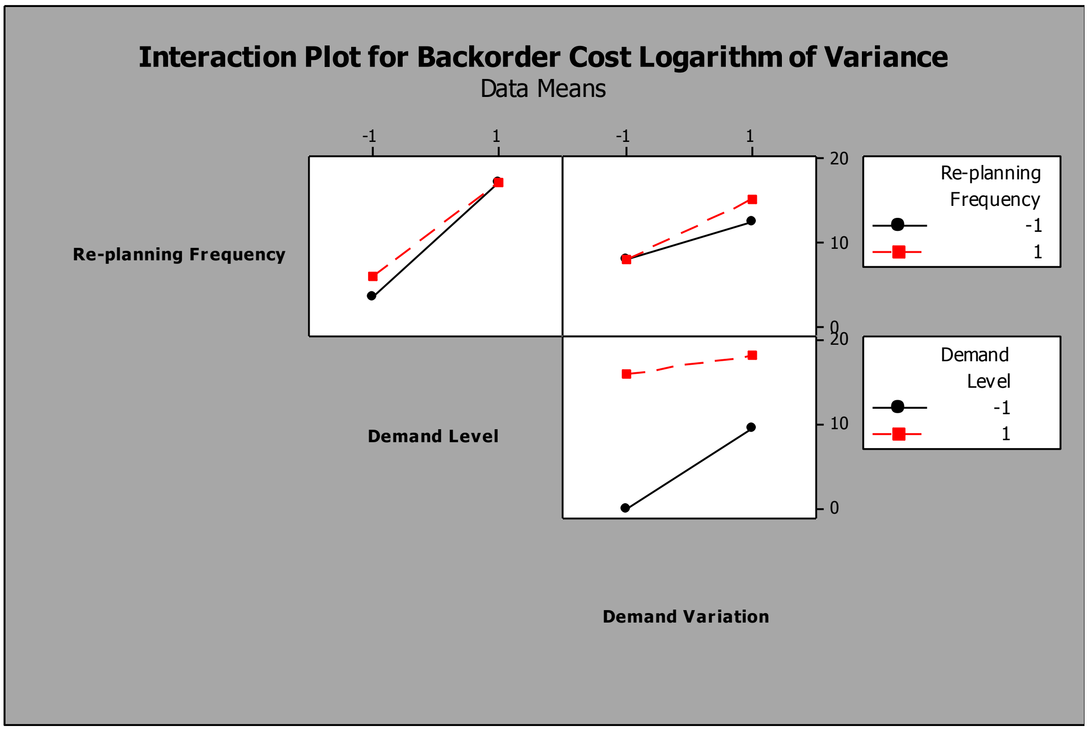

Figure 23- Interaction effect plot of re-planning frequency, demand level & demand variation on backorder cost logarithm of variance of new protocol

Figure 23 illustrates that lower demand level and shorter re-planning frequency lead to smaller backorder logarithm of variance.

Similar to the Taguchi method, in order to have a decision framework we need to consider the regression models. The achieved regression models are provided for average and logarithm of variance of backorder cost as follows:

Backorder cost average =

932,100,992 - 22,553,246 A + 37,054,040 B + 19,868,841 C + 931,951,162 D + 459,597,669 E - 19,592,641 AB + 45,507,684 AC - 22,465,799 AD - 12,706,655 AE - 18,313,528 BC + 36,950,972

BD + 44,380,127 BE + 19,719,863 CD + 22,959,857 CE + 459,447,839 DE + 43,672,820 ABC - 19,503,711 ABD - 20,148,937 ABE + 45,594,942 ACD + 42,483,421 ACE - 12,619,209 ADE - 18,415,959 BCD - 14,534,483 BCE + 44,277,058 BDE + 22,810,879 CDE + 43,761,443 $ABCD$ + 50,008,516 $ABCE$ - 20,060,007 $ABDE$ + 42,570,680 $ACDE$ - 14,636,913 $BCDE$ + 50,097,139 $ABCDE$ (24)

Backorder cost logarithm of variance =

11.003 - 0.084 A + 0.177 B + 0.627 C + 6.231 D + 2.946 E - 0.104 AB + 0.019 AC + 0.030 AD - 0.069 AE - 0.021 BC - 0.213 BD + 0.224 BE - 0.613 CD + 0.696 CE - 1.826 DE - 0.024 ABC + 0.157 ABD -0.190 ABE + 0.100 ACD - 0.057 ACE + 0.046 ADE + 0.036 BCD - 0.066 BCE - 0.166 BDE - 0.544 CDE + 0.002 $ABCD$ + 0.040 $ABCE$ + 0.071 $ABDE$ + 0.023 $ACDE$ -0.008 $BCDE$ + 0.065 $ABCDE$ (25)

The same interpretation is performed for the inventory cost, as the second KPI. Figures 24 and 25 illustrate the normal plots of effects regarding the inventory cost average and logarithm of variance, respectively.

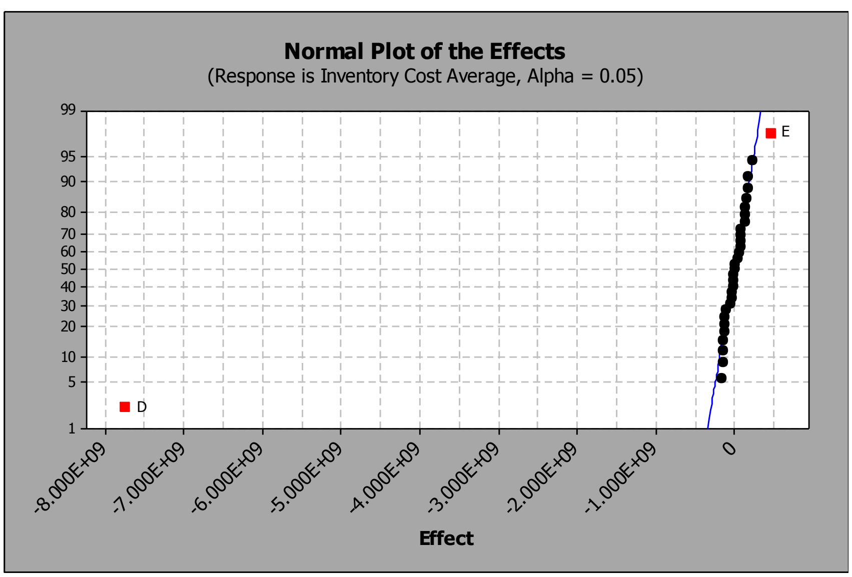

Figure 24- Normal plot of effects regarding inventory cost average of new protocol

Based on figure 24, only demand level and demand variation are significant regarding the inventory cost average. On the other hand, figure 25 indicates that the interaction between planning approach, demand level and demand variation is significant regarding

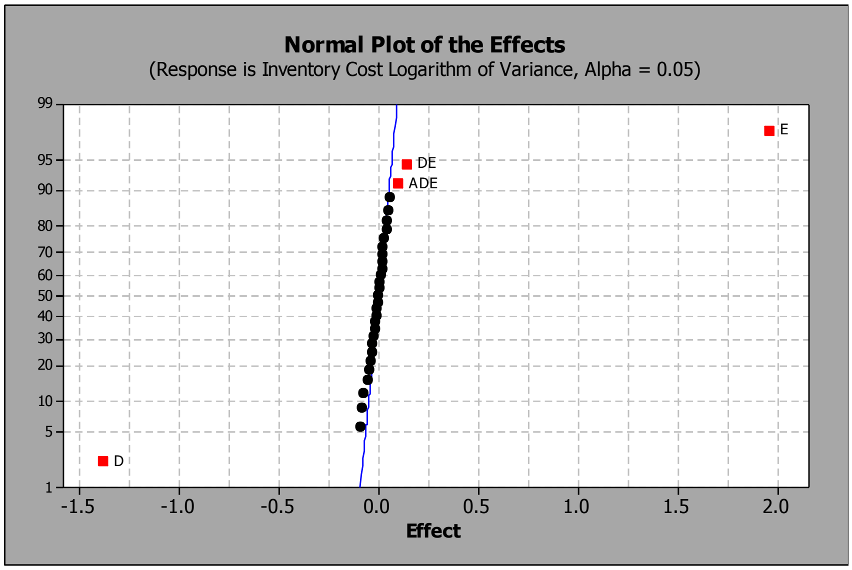

Figure 25- Normal plot of effects regarding inventory cost logarithm of variance of new protocol

the inventory cost logarithm of variance. Figures 26 and 27 present the main and interaction effect plots for important factors, mentioned above.

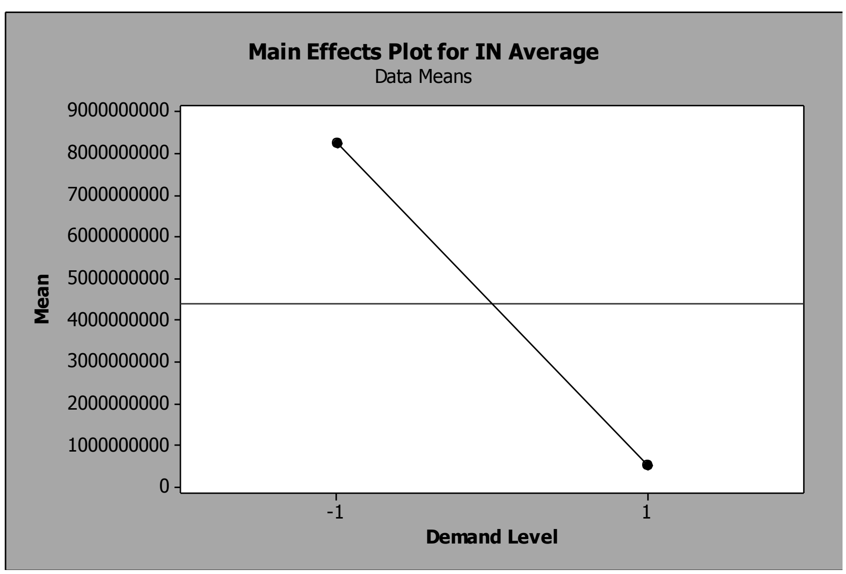

Figure 26- Main effect plot of demand level regarding inventory cost average of new protocol

Figure 26 illustrate the higher demand level will result in lower inventory cost average, as expected. On the other hand, figure 27 illustrates that higher demand level and stochastic planning approach lead in smaller amounts for inventory cost logarithm of variance. It is

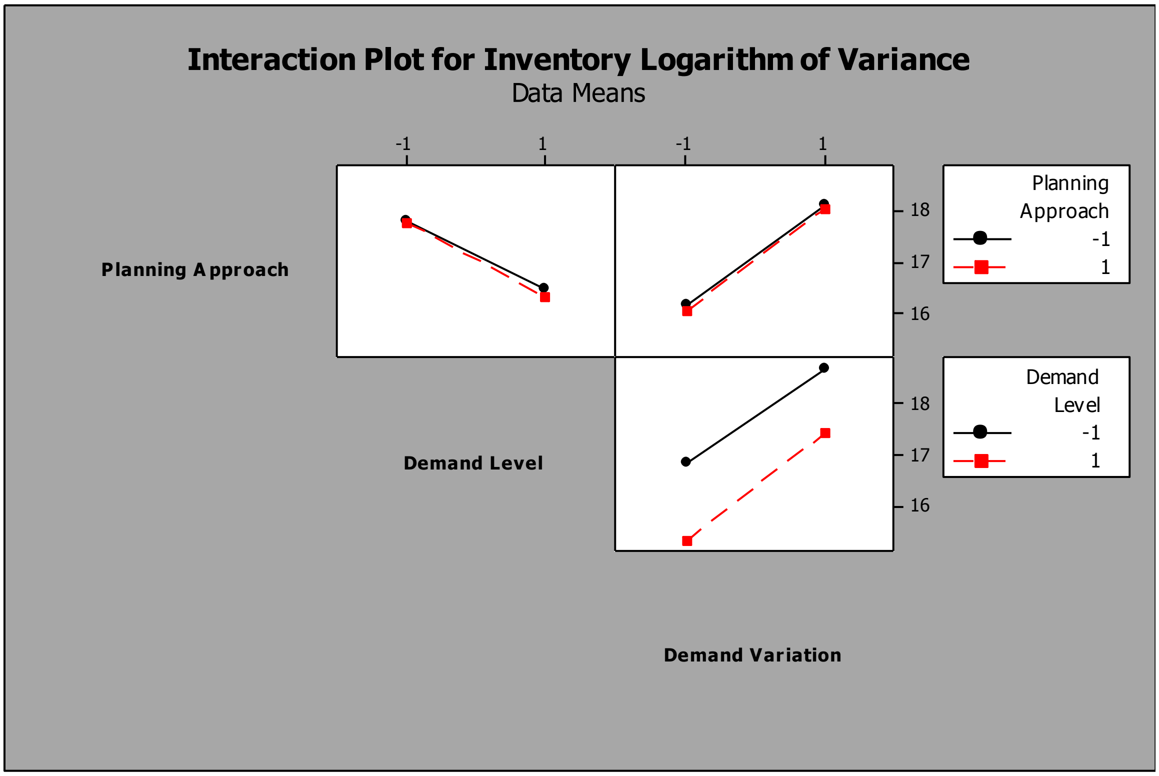

Figure 27- Interaction effect plot of planning approach, demand level and demand variation regarding inventory cost logarithm of variance of new protocol

important to highlight that larger demand will result in both lower inventory cost average and logarithm of variance. The associated equations for the inventory cost average and logarithm of variance are as follows:

Inventory cost average =

4,384,676,915 - 56,416,336 A + 93,907,855 B - 17,474,518 C - 3,870,691,335 D + 228,124,558 E + 41,756,113 AB + 44,055,645 AC + 75,168,910 AD - 58,062,059 AE - 6,684,6037 BC - 66,511,912 BD + 70,779,057 BE + 41,274,742 CD - 7,562,519 CE + 123,180,370 DE - 957,996 ABC - 60,153,926 ABD + 48,782,956 ABE -8,765,695 ACD + 34,795,630 ACE + 70,485,289 ADE + 84,073,981 BCD - 78,452,951 BCE - 48,408,122 BDE + 30,520,662 CDE + 11,566,875

ABCD + 2,322,183 *ABCE* - 70,351,272 *ABDE* - 5,543,017 *ACDE* + 95,089,289 *BCDE* + 7,137,984 *ABCDE* (26)

Inventory cost logarithm of variance =

17.0925 - 0.0427 *A* + 0.0221 *B* - 0.0085 *C* - 0.6903 *D* + 0.9781 *E* + 0.0006 *AB* + 0.0113 *AC* - 0.0415 *AD* + 0.0080 *AE* + 0.0223 *BC* - 0.0120 *BD* - 0.0048 *BE* + 0.0117 *CD* - 0.0042 *CE* + 0.0698 *DE* + 0.0099 *ABC* - 0.0144 *ABD* - 0.0362 *ABE* + 0.0023 *ACD* - 0.0149 *ACE* + 0.0473 *ADE* + 0.0252 *BCD* + 0.0042 *BCE* - 0.0242 *BDE* - 0.0057 *CDE* + 0.0097 *ABCD* + 0.0013 *ABCE* - 0.0226 *ABDE* + 0.0306 *ACDE* - 0.0174 *BCDE* + 0.0145 *ABCDE* (27)

The achieved functions for average and logarithm of variance in stage 1 are valid only for 5% and 40% of demand variation and considering random noise factor Z_1, based on Giovagnoli et al. [19]. To expand the above regression functions and get valid ones for all levels of demand variation (Z_2), they proposed the following calculations:

Average function = E_{Z_2}* (average function in stage 1)

Variance function= E_{Z_2}* (logarithm of variance function in stage 1) + Var_{Z_2} (average function in stage 1)

Considering coded variables (± 1) for Z_2, its expected value and variance are equal to zero and one, respectively. Consequently and with respect to explanation provided in the literature review, the final regression models for our case are calculated as follows:

Backorder cost average =

932,100,992 -22,553,246 *A*+ 37,054,040 *B*+ 19,868,841 *C*+ 931,951,162 *D* -19,592,641 *AB*+ 45,507,684 *AC* -22,465,799 *AD*- 18,313,528 *BC*+ 36,950,972*BD*+ 19,719,863 *CD*+ 43,672,820*ABC*-19,503,711 *ABD* + 45,594,942*ACD*-18,415,959 *BCD* +43,761,443*ABCD* (28)

* Expected value and variance with respect to Z_2

Backorder cost variance =

$$\underline{(11.003 - 0.084\,A + 0.177\,B + 0.627\,C + 6.231\,D - 0.104\,AB + 0.019\,AC + 0.030\,AD - 0.021\,BC - 0.213\,BD - 0.613\,CD - 0.024\,ABC + 0.157\,ABD + 0.100ACD + 0.036\,BCD + 0.002\,ABCD) +} (459{,}597{,}669 - 12{,}706{,}655\,A + 44{,}380{,}127\,B + 22{,}959{,}857\,C + 459{,}447{,}839\,D - 20{,}148{,}937\,AB + 42{,}483{,}421\,AC - 12{,}619{,}209\,AD - 14{,}534{,}483\,BC + 44{,}277{,}058\,BD + 22{,}810{,}879\,CD + 50{,}008{,}516\,ABC - 20{,}060{,}007\,ABD + 42{,}570{,}680\,ACD - 14{,}636{,}913\,BCD + 50{,}097{,}139\,ABCD)^2 \quad (29)$$

Inventory cost average =

$$4{,}384{,}676{,}915 - 56{,}416{,}336\,A + 93{,}907{,}855\,B - 17{,}474{,}518\,C - 3{,}870{,}691{,}335\,D + 41{,}756{,}113\,AB + 44{,}055{,}645\,AC + 75{,}168{,}910\,AD - 6{,}684{,}6037\,BC - 66{,}511{,}912\,BD + 41{,}274{,}742\,CD - 957{,}996\,ABC - 60{,}153{,}926\,ABD - 8{,}765{,}695\,ACD + 84{,}073{,}981\,BCD + 11{,}566{,}875\,ABCD \quad (30)$$

Inventory cost variance =

$$\underline{(17.0925 - 0.0427\,A + 0.0221\,B - 0.0085\,C - 0.6903\,D + 0.0006\,AB + 0.0113\,AC - 0.0415\,AD + 0.0223\,BC - 0.0120\,BD + 0.0117\,CD - + 0.0099\,ABC - 0.0144\,ABD + 0.0023\,ACD + 0.0252\,BCD + 0.0097\,ABCD) +} (228{,}124{,}558 - 58{,}062{,}059\,A + 70{,}779{,}057\,B - 7{,}562{,}519\,C + 123{,}180{,}370\,D + 48{,}782{,}956\,AB + 34{,}795{,}630\,AC + 70{,}485{,}289\,AD - 78{,}452{,}951\,BC - 48{,}408{,}122\,BD + 30{,}520{,}662\,CD + 2{,}322{,}183\,ABC - 70{,}351{,}272\,ABD - 5{,}543{,}017\,ACD + 95{,}089{,}289\,BCD + 7{,}137{,}984\,ABCD)^2 \quad (31)$$

The superiority of this model is highlighted by underlined terms, comparing to the combined array methodology. In this model, the expected value of logarithm of variance regression model has been included in final variance function in addition to the variance of average regression model. This way, the new protocol can benefit from the possibility of replications in the stochastic simulation and having the possibility of calculating the variance of those replications.

Although this method results in a more precise variance function comparing to the combined array design, it is not very helpful in our case study. Although the new method distinguishes some important factors and interactions regarding backorder and inventory costs variances in the related logarithm regression models, their effects are trivialized in the final variance functions (stage two). This happens due to the fact that the coefficients derived from the regression models of logarithm of variance (the underlined sections) are very small comparing the coefficients derived from regression functions of the average. Therefore, when they are combined in the final variance functions, the effect of important factors in the logarithm of variance regression models are trivialized. It should be mentioned that the small coefficients in the aforementioned model illustrate the negligible impact of Z_1 (uniform distribution of demand) on the variation of response variable.

The next section focuses on choosing the most appropriate robust parameter design for our case study and proposes the decision framework, as the final goal of this study.

4.6 Decision framework

We applied three methodologies for robust parameter design in this study. The results indicated that the new protocol to combine robust parameter design and stochastic simulation provides the same results as the combined array methodology. Therefore, we considered Taguchi and combined array methodologies to propose the decision framework in our case study. Considering the results and discussions in the previous section, we chose the Taguchi method by using the direct variance, as it provided more accuracy regarding the main effect of planning approach comparing to the S/N ratio. Based on the experimental results provided in the previous section, it is not possible to propose a robust planning approach which results in smaller amounts for average and variance of backorder and inventory costs at all levels of demand variation. Hence, to propose the appropriate planning approach, we can follow two options:

4.6.1 The demand variation is unknown

In this case we use the regression models achieved by Taguchi or combined array methods to propose the decision framework. Taguchi [12] mentioned that in new quality control approaches the main goal is to minimize variance around the response value and then optimize the value of response variable. Based on this point of view and inspired from Chen [28], we suggest to follow the subsequent procedure to propose a decision framework:

$$Min\ Var_z[y(x)]$$

$$Subject\ to{:}\ Average_Z[y(x)]\ < m$$

$$-1 < x_i < +1\ where\ i = 2\ \cdots 4$$

$$x_1 = +1 \; or - 1$$

Or

$$Min\; Var_z[y(x)]$$

$$Subject\ to: Average_Z[y(x)] \; = m$$

$$-1 < x_i < +1 \; where\; i = 2 \cdots 4$$

$$x_1 = +1 \; or - 1$$

where, "m" is the target or maximum allowed value for the average of response variable (backorder or inventory costs) defined by management. x_2 to x_4 are representing planning horizon, re-planning frequency and demand level respectively whereas x_1 represents planning approach. The resulted regression models for average and variance of backorder and inventory costs are implemented separately in the above optimization model to propose the best planning approach under different levels of x_2 to x_4. There are two options:

Considering Taguchi method regression models based on direct variance values:

Equations 12 and 16 can be used as average and variance functions for backorder cost, whereas equations 14 and 17 are used for inventory cost optimization model.

Although the managers are supposed to make decision about the level of controllable factors as the inputs for the decision framework, the results of Taguchi method suggest the following level settings to decrease the average and variance of backorder and inventory costs:

- The demand level is the only important factor influencing the backorder cost average; whereas re-planning frequency, demand level, their interaction and length of planning horizon are important regarding the backorder cost logarithm of variance. It was observed that both the backorder cost average and variance will be decreased at the low level of the demand. In other words, if the decision maker prefers a low backorder cost he/ she should avoid contracts close to the maximum capacity of sawmills. On the other hand, lower variation of backorder cost can be achieved at the low level of re-planning frequency and planning horizon. Having short re-planning frequency is especially helpful while the demand level is low.

- Regarding the inventory cost, the demand level was the only distinguished important factor for both average and logarithm of variance. Higher demand level will result in not only less inventory cost average but also smaller variation around the average.

As it is observed here and we mentioned previously, in some cases the proposed factor levels are in contradiction for different KPIs. In such situation a tradeoff or prioritization is necessary. In our case, the system is service sensitive and the backorder cost has the first priority for decision making.

No matter what the levels of controllable factors would be, the following table compares the performance of two planning approaches based on decision framework under sample of different circumstances. By comparing the performances, it becomes possible to propose the appropriate production planning approach for each case.

Table 17- The performance of production planning approaches under sample of different circumstances based on Taguchi method

Planning Approach	Length of Planning Horizon	Re-planning Frequency	Demand Level	Backorder Cost Average	Inventory Cost Average	Backorder Cost Logarithm of Variance	Inventory Cost Logarithm of Variance
Deterministic	15	1	0.5	1	8300.9	5.468	18.3915
Stochastic	15	1	0.5	0	7703.1	6.288	18.3879
Deterministic	15	1	1	1723	466.9	18.084	17.3799
Stochastic	15	1	1	1705	493.1	18.092	17.4683
Deterministic	15	7	0.5	1	8355.1	10.91	18.3795
Stochastic	15	7	0.5	0	8020.9	11.254	18.2343
Deterministic	15	7	1	1867	433.1	18.422	17.3059
Stochastic	15	7	1	1865	554.9	18.41	17.4743
Deterministic	30	1	0.5	1	8695.1	8.198	18.4655
Stochastic	30	1	0.5	0	8556.9	7.142	18.4727
Deterministic	30	1	1	2199	545.1	18.538	17.5207
Stochastic	30	1	1	1669	454.9	18.006	17.2279
Deterministic	30	7	0.5	1	8196.9	13.352	18.5243
Stochastic	30	7	0.5	0	8219.1	11.452	18.3531
Deterministic	30	7	1	1847	534.9	18.348	17.4847
Stochastic	30	7	1	2037	629.1	18.468	17.5199

Considering combined array method average and variance functions

Equations 20 and 21 can be used as average and variance functions for backorder cost, whereas equations 22 and 23 are used for inventory cost optimization model.

As mentioned in the related section, combined array could distinguish demand level as the only important factor regarding backorder cost average and variance in addition to inventory cost average. It is also important to mention that no important factor was distinguished by this method regarding the inventory cost variance. Considering the fact aforesaid and regarding the combined array results, it is suggested to avoid contracts close to the maximum capacity of sawmills in order to achieve lower backorder cost average and variance. On the other hand, in order to have lower inventory average cost it is better to have higher demand levels, as expected.

Similar to Taguchi method, we propose the comparison between the performances of two planning approaches under a sample of different circumstances in table 18. The table is prepared based on the achieved results for combined array method.

Table 18- The performance of production planning approaches under sample of different circumstances based on combined array method

Planning Approach	Length of Planning Horizon	Re-planning Frequency	Demand Level	Backorder Cost Average	Inventory Cost Average	Backorder Cost Variance	Inventory Cost Variance
Deterministic	15	1	0.5	0	8,299	0	19,881
Stochastic	15	1	0.5	0	7,707	0	192,721
Deterministic	15	1	1	1,726	469	527,076	95,481
Stochastic	15	1	1	1,706	493	624,100	113,569
Deterministic	15	7	0.5	0	8,355	0	105,625
Stochastic	15	7	0.5	0	8,019	0	6,241
Deterministic	15	7	1	1,866	429	813,604	78,961
Stochastic	15	7	1	1,862	557	813,604	151,321
Deterministic	30	1	0.5	2	8,691	0	227,529
Stochastic	30	1	0.5	2	8,551	0	151,321
Deterministic	30	1	1	2,192	545	1,532,644	145,161
Stochastic	30	1	1	1,676	453	550,564	81,225
Deterministic	30	7	0.5	2	8,195	0	121
Stochastic	30	7	0.5	2	8,215	0	1,369
Deterministic	30	7	1	1,844	537	813,604	148,225
Stochastic	30	7	1	2,032	629	1,304,164	194,481

4.6.2 The demand variation is known

In this case, it is better to perform a factorial experiment for the specified variance level, similar to what was performed for 5% of demand variation in section 4.5. Hence, two cases might happen:

1. The planning approach is identified as an important factor or appears in important interactions: This makes it possible to definitely propose one of the planning approaches as the superior one. An example is the results of full factorial design for 5% of demand variation. In this experiment, the interaction of planning approach and demand level was considered to be important regarding backorder cost average. In the mentioned experiment, the results indicated that stochastic planning approach is superior to deterministic one especially for higher demand levels considering backorder average cost.

2. The planning approach is not identified as an important factor or it is not included in an important interaction: In this case it would be more appropriate to use the regression model, specifically estimated for the defined demand variation, as the decision framework for example. Based on the full factorial design for 40% of demand variation, the planning approach is not important regarding backorder or inventory costs averages and the demand level is the only important factor regarding these two KPIs. Therefore, the decision framework will be defined as follow:

$$\begin{gathered} Min\ Average\ (Backorder\ cost) \\ Subject\ to\colon -1 < x_i < +1;\ where\ i = 2 \cdots 4 \\ x_1 = +1\ or -1 \end{gathered}$$

And

$$\begin{gathered} Min\ Average\ (Inventory\ cost) \\ Subject\ to\colon -1 < x_i < +1;\ where\ i = 2 \cdots 4 \\ x_1 = +1\ or -1 \end{gathered}$$

where again x_2 to x_4 represent planning horizon, re-planning frequency and demand level respectively; whereas x_1 represents planning approach. Table 19 illustrates the results regarding backorder cost average for different combinations of other controllable factors for the 40% of demand variation as an example. The details of this experiment are presented in Appendix IV.

Table 19- The performance of production planning approaches under a sample of different circumstances based on 40% demand variation regarding backorder cost regression model

Planning Approach	Length of Planning Horizon	Re-planning Frequency	Demand Level	Backorder Cost Average ($millions)
Deterministic	15	1	0.5	1
Stochastic	15	1	0.5	0
Deterministic	15	1	1	2855
Stochastic	15	1	1	2525
Deterministic	15	7	0.5	1
Stochastic	15	7	0.5	0
Deterministic	15	7	1	2839
Stochastic	15	7	1	3085
Deterministic	30	1	0.5	1
Stochastic	30	1	0.5	0
Deterministic	30	1	1	3807
Stochastic	30	1	1	2669
Deterministic	30	7	0.5	1
Stochastic	30	7	0.5	0
Deterministic	30	7	1	2407
Stochastic	30	7	1	2469

5 Conclusions and future steps

This chapter provides a summary of research performed in this thesis and covers the most remarkable aspects and conclusions of this study. We also present the possible steps for future work.

5.1 Conclusions

The goal of this study was to compare the performance of deterministic and stochastic production planning models in sawing unit of sawmills. Regarding this objective, we conducted a related literature review which proved the necessity of performing a new study to compare these two models based on design of experiments and on a rolling planning horizon.

The experiments were performed virtually through Monte-Carlo simulation, as it was not possible and economically reasonable to run them in real sawmills. We started experimenting procedure by factorial design and continued it with three methods of robust parameter design (Taguchi crossed array design, combined array design and a new protocol to combine robust parameter design with stochastic simulation). The final goal of the comparison was to propose a decision framework in order to identify the appropriate production planning approach under different circumstances in terms of length of planning horizon, re-planning frequency and demand average and standard deviation. These applied steps covered the contributions of our study which were:

- Performing the comparison based on designed experiments.
- Implementing the experiments on a rolling planning horizon.

- Integrating the robust parameter design with stochastic simulation to compare the performance of stochastic and deterministic production planning models in sawing unit of sawmills.
- Using three types of robust parameter design in addition to factorial design to propose a decision framework.

The proposed decision framework depended on the available information regarding demand variation as the noise factor. Two cases might happen: i) demand variation was unknown, and ii) demand variation was known

In first case, it was suggested to refer to robust parameter design to propose the decision framework. The reason was that robust parameter design provided valid results for all levels of demand variation. Based on the achieved experimental results, the following points must be highlighted:

- As the new protocol provided the same results as combined array methodology, we decided to consider only Taguchi method and combined array design as two alternatives for proposing the decision framework.
- Inspired by Taguchi [12] and Chen [28], an optimization model was considered as the basis for the decision framework. In this optimization model, the variance equation resulted by the robust parameter design approach (Taguchi or combined array) was considered to be the objective function. The average function (with respect to the desired or the maximum level defined by management for each KPI), in addition to allowed levels for controllable factors, were considered as

constraints. The model seeks those factor combinations that minimize the variance around a target response (KPI) value.

- We proposed separate optimization models for each KPI. In some cases the proposed planning approaches might be in contradiction for different KPIs. In our case, the system was service sensitive and the backorder cost had the first priority for decision making.
- The management was supposed to identify the desired value for backorder and/ or inventory costs in addition to the length of planning horizon, re-planning frequency and demand level as the inputs of the optimization model. The appropriate planning approach would then be decided as the output of decision framework.
- The proposed decision framework was applicable to identify not only the appropriate planning approach but also the appropriate level for all other controllable factors.
- Considering Taguchi method, it was recommended to avoid contracts close to the maximum capacity of sawmills to have lower backorder cost average and variance. In addition, shorter re-planning frequency and planning horizon were helpful to achieve low backorder cost variance. On the other hand having higher demand level could result in lower levels of both inventory cost average and variance.
- Considering combined array method, it was recommended to avoid demand levels close to maximum sawmill capacity to have lower backorder cost average and variance. In addition, having higher demand level could result in lower inventory

cost average as expected. It was important to mention that combined array design did not distinguish any important factor regarding the inventory cost variance and had no suggestions in this regard.

- The achieved functions of Taguchi and combined array method provided almost the same results for KPI averages. This happened due to the fact that we considered the highest resolution for our experiments.
- Although it was not possible to choose the best method among Taguchi and combined array, we might suggest Taguchi method to propose a better decision framework in our case. The reason was that Taguchi method could distinguish more important factors regarding backorder and inventory cost variances. In addition, the sample decision frameworks illustrated in tables 17 and 18 indicated that combined array method could not distinguish any differences between the performance of deterministic and stochastic production planning approaches in some cases. It is also necessary to mention that these two methods may propose different suggestions in some circumstances which may happen due to their different approaches for calculating the variance functions.

As we mentioned, the proposed decision framework depended on the available information regarding demand variation as the noise factor. The above explanations are regarding the cases that demand variation is unknown. The other case happens when the demand variation is known. In this case we suggested performing factorial design which may lead to two situations:

- The planning approach was identified as an important factor or appears in important interactions. This made it possible to definitely propose one of the planning approaches as the superior one. An example was the results of full factorial design for 5% of demand variation which suggested stochastic production planning model in order to decrease the backorder cost average.

- The planning approach was not identified as an important factor or it was not included in an important interaction. In this case it would be more appropriate to use the regression model, specifically estimated for the defined demand variation, as the decision framework. This way, we had a new decision framework in which the achieved regression model for backorder or inventory cost averages were used as the objective function and the allowed levels for controllable factors were the constraints. We tried to find the best planning approach for each setting of controllable factor levels in order to minimize the KPI average. Table 19 compared the performance of deterministic and stochastic production planning approaches under a sample of different circumstances. The comparison was done based on the proposed decision framework for 40% demand variation.

Finally, we can conclude that the goals and objectives of this study are achieved, and it can be used as the basis for future works.

5.2 Future work

This case study compared the traditional deterministic production planning models in sawmills with stochastic production planning approach. The results of the current study and the guidelines it provided, especially regarding the appropriate methodology for such comparisons, can be used in similar works. The future steps can be defined as follows:

- The stochastic model in current study considers only random process yields. We suggest comparing the deterministic model with another stochastic model that considers both random yields and demand as uncertain parameters.

- To find more precise estimations for experimental environment. Experimental environment include the demand distribution and sawmill capacity. The data applied in this study can be improved in future work by using more updated historical data. The latter would result a more precise decision framework in terms of finding the superior planning approach.

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Appendix I

Deterministic and Stochastic Models for Sawmill Production Planning

Indices

p	product (lumber)
t	period
c	raw material (log) class
a	production process
r	resource (machine)

Parameters

h_{pt}	inventory holding cost per unit of product p in period t
b_{pt}	backorder cost per unit of product p in period t
m_{ct}	raw material cost per unit of class c in period t
I_{c0}	the inventory of raw material of class c at the beginning of planning horizon
I_{p0}	the inventory of product p at the beginning of planning horizon
S_{ct}	the quantity of material of class c supplied at the beginning of period t
d_{pt}	demand of product p by the end of period t
φ_{ac}	the units of class c raw material consumed by process a (consumption factor)
ρ_{ap}	the units of product p produced by process a (yield of process a)
δ_{ar}	the capacity consumption of resource r by process a
M_{rt}	the capacity of resource r in period t

Decision variables

X_{at}	the number of times each process a should be run in period t (production plan)
I_{ct}	inventory size of raw material of class c by the end of period t
I_{pt}	inventory size of product p by the end of period t
B_{pt}	backorder size of product p by the end of period t

Deterministic Model for Sawmill Production Planning

$$Minimize\ Z = \sum_{p\epsilon P}\sum_{t=1}^{T}(h_{pt}I_{pt} + b_{pt}B_{pt}) + \sum_{c\epsilon C}\sum_{t=1}^{T}\sum_{a\epsilon A} m_{ct}\,\varphi_{ac}\,X_{at}$$

Subject to

Material inventory constraint

$$I_{ct} = I_{ct-1} + S_{ct} - \sum_{a\epsilon A}\varphi_{ac}\,X_{at}$$
$$t = 1, \ldots, T, c\ \epsilon\ C,$$

Product inventory constraint

$$I_{p1} - B_{p1} = I_{p0} + \sum_{a \epsilon A} \rho_{ap} X_{a1} - d_{p1}$$

$$I_{pt} - B_{pt} = I_{pt-1} - B_{pt-1} + \sum_{a \epsilon A} \rho_{ap} X_{at} - d_{pt}$$

$$t = 2, \dots, T, p \in P$$

Product capacity constraint

$$\sum_{a \epsilon A} \delta_{ar} X_{at} \leq M_{rt}$$

$$t = 1, \dots, T, r \in R$$

non − negativity of all variables

$$X_{at} \geq 0, I_{ct} \geq 0, I_{pt} \geq 0, B_{pt} \geq 0,$$

$$t = 1, \dots, T, p \in P, c \in C, a \in A.$$

Stochastic Model for Sawmill Production Planning

$$Minimize\ Z = \sum_{c \epsilon C} \sum_{t=1}^{T} \sum_{a \epsilon A} m_{ct} \varphi_{ac} X_{at} + \sum_{i=1}^{N} \sum_{p \epsilon P} \sum_{t=1}^{T} p^i [h_{pt} I_{pt}^i + b_{pt} B_{pt}^i]$$

Subject to

$$I_{ct} = I_{ct-1} + S_{ct} - \sum_{a \epsilon A} \varphi_{ac} X_{at}$$

$$t = 1, \dots, T, c \in C$$

$$I_{p1}^i - B_{p1}^i = I_{p0} + \sum_{a \epsilon A} \rho_{ap}(\xi^i) X_{a1} - d_{p1}$$

$$I_{pt}^i - B_{pt}^i = I_{pt-1}^i - B_{pt-1}^i + \sum_{a \epsilon A} \rho_{ap}(\xi^i) X_{at} - d_{pt}$$

$$t = 2, \dots, T, p \in P, i = 1, \dots, N$$

$$\sum_{a \epsilon A} \delta_{ar} X_{at} \leq M_{rt}$$

$$t = 1,2, \dots, T, r \in R$$

$$X_{at} \geq 0, I_{ct} \geq 0, I_{pt}^i \geq 0, B_{pt}^i \geq 0$$

$$c \in C, p \in P, t = 1, \dots, T, a \in A$$

$$i = 1, \dots, N$$

Appendix II

The Backorder and Inventory Costs Values for Full Factorial Design

(5% Demand variation)

Planning Approach	Length of Planning Horizon	Re-planning Frequency	Demand Level	Simulated Inventory Cost	Simulated Backorder Cost
Deterministic	15	1	0.5	8,370,666,125	0
Deterministic	15	1	0.5	8,459,025,496	0
Deterministic	15	1	0.5	8,025,344,423	0
Deterministic	15	1	0.5	8,582,683,763	0
Deterministic	15	1	0.5	8,223,545,260	0
Deterministic	15	1	0.5	8,051,704,530	0
Deterministic	15	1	0.5	8,137,375,025	0
Deterministic	15	1	0.5	7,749,024,485	0
Deterministic	15	1	0.5	7,851,397,803	0
Deterministic	15	1	0.5	8,691,368,671	0
Deterministic	15	1	1	82,390,000	1,048,099,705
Deterministic	15	1	1	162,793,150	887,370,565
Deterministic	15	1	1	143,593,152	958,430,838
Deterministic	15	1	1	107,133,306	958,680,681
Deterministic	15	1	1	211,921,826	982,308,267
Deterministic	15	1	1	140,443,296	899,939,736
Deterministic	15	1	1	139,620,867	1,005,435,364
Deterministic	15	1	1	96,600,139	1,015,885,485
Deterministic	15	1	1	151,501,757	1,112,011,596
Deterministic	15	1	1	198,667,779	931,771,872
Deterministic	15	7	0.5	7,874,038,374	0
Deterministic	15	7	0.5	7,908,475,145	0
Deterministic	15	7	0.5	8,114,321,032	0
Deterministic	15	7	0.5	8,514,661,289	0
Deterministic	15	7	0.5	7,849,607,266	0
Deterministic	15	7	0.5	7,733,907,758	0

Deterministic	15	7	0.5	8,399,630,317	0
Deterministic	15	7	0.5	8,382,917,851	0
Deterministic	15	7	0.5	8,635,802,985	0
Deterministic	15	7	0.5	8,440,782,583	0
Deterministic	15	7	1	174,011,504	961,836,812
Deterministic	15	7	1	131,455,064	995,007,502
Deterministic	15	7	1	151,865,839	887,634,963
Deterministic	15	7	1	141,164,731	1,162,518,364
Deterministic	15	7	1	249,351,098	730,262,326
Deterministic	15	7	1	84,793,786	975,069,049
Deterministic	15	7	1	291,276,889	945,259,787
Deterministic	15	7	1	119,070,776	1,070,784,489
Deterministic	15	7	1	202,585,474	802,790,216
Deterministic	15	7	1	178,977,296	1,019,890,397
Deterministic	30	1	0.5	7,681,304,292	0
Deterministic	30	1	0.5	8,472,008,415	0
Deterministic	30	1	0.5	8,176,424,206	0
Deterministic	30	1	0.5	7,878,888,544	0
Deterministic	30	1	0.5	8,256,120,161	0
Deterministic	30	1	0.5	8,524,641,385	0
Deterministic	30	1	0.5	7,577,208,541	0
Deterministic	30	1	0.5	7,860,763,090	0
Deterministic	30	1	0.5	7,743,138,933	0
Deterministic	30	1	0.5	7,992,119,390	0
Deterministic	30	1	1	146,055,631	1,173,011,818
Deterministic	30	1	1	174,579,926	1,145,147,148
Deterministic	30	1	1	203,934,326	999,949,532
Deterministic	30	1	1	237,361,485	891,088,186
Deterministic	30	1	1	123,270,145	1,109,491,119
Deterministic	30	1	1	230,499,764	1,036,465,830
Deterministic	30	1	1	219,090,111	818,165,475
Deterministic	30	1	1	228,019,345	858,634,141

Deterministic	30	1	1	122,972,594	948,932,308
Deterministic	30	1	1	198,891,103	808,012,937
Deterministic	30	7	0.5	8,323,301,560	0
Deterministic	30	7	0.5	8,034,657,640	0
Deterministic	30	7	0.5	8,245,992,233	0
Deterministic	30	7	0.5	8,449,297,344	0
Deterministic	30	7	0.5	7,729,236,149	0
Deterministic	30	7	0.5	7,795,825,469	0
Deterministic	30	7	0.5	8,119,105,382	0
Deterministic	30	7	0.5	8,251,826,304	0
Deterministic	30	7	0.5	8,067,428,430	0
Deterministic	30	7	0.5	7,787,162,093	0
Deterministic	30	7	1	250,565,368	933,033,827
Deterministic	30	7	1	125,013,259	1,007,068,192
Deterministic	30	7	1	192,731,531	1,008,255,128
Deterministic	30	7	1	92,650,334	1,051,091,795
Deterministic	30	7	1	185,596,481	1,192,542,972
Deterministic	30	7	1	182,325,950	992,410,985
Deterministic	30	7	1	101,158,816	1,040,768,751
Deterministic	30	7	1	132,424,898	946,606,521
Deterministic	30	7	1	163,502,250	1,091,504,627
Deterministic	30	7	1	180,697,998	811,913,312
Stochastic	15	1	0.5	8,624,584,896	0
Stochastic	15	1	0.5	7,934,836,171	0
Stochastic	15	1	0.5	8,275,043,011	0
Stochastic	15	1	0.5	7,707,555,150	0
Stochastic	15	1	0.5	8,177,640,156	0
Stochastic	15	1	0.5	8,390,995,183	0
Stochastic	15	1	0.5	8,435,274,154	0
Stochastic	15	1	0.5	7,567,279,572	0
Stochastic	15	1	0.5	7,855,587,098	0
Stochastic	15	1	0.5	7,940,972,574	0

Stochastic	15	1	1	231,149,456	752,497,841
Stochastic	15	1	1	178,750,062	1,052,806,616
Stochastic	15	1	1	199,993,086	685,863,478
Stochastic	15	1	1	173,206,267	884,070,335
Stochastic	15	1	1	57,016,226	997,156,858
Stochastic	15	1	1	134,718,522	836,837,293
Stochastic	15	1	1	166,868,587	961,113,216
Stochastic	15	1	1	158,698,630	977,479,015
Stochastic	15	1	1	118,859,153	846,150,940
Stochastic	15	1	1	107,321,569	851,907,796
Stochastic	15	7	0.5	8,495,270,387	0
Stochastic	15	7	0.5	8,134,012,055	0
Stochastic	15	7	0.5	8,629,420,937	0
Stochastic	15	7	0.5	8,454,971,766	0
Stochastic	15	7	0.5	7,965,638,015	0
Stochastic	15	7	0.5	7,877,804,120	0
Stochastic	15	7	0.5	7,754,726,585	0
Stochastic	15	7	0.5	7,875,676,830	0
Stochastic	15	7	0.5	7,710,276,973	0
Stochastic	15	7	0.5	7,967,007,201	0
Stochastic	15	7	1	192,245,352	907,075,534
Stochastic	15	7	1	116,245,786	1,142,130,232
Stochastic	15	7	1	127,032,437	959,061,068
Stochastic	15	7	1	217,614,224	769,173,406
Stochastic	15	7	1	190,592,522	761,761,208
Stochastic	15	7	1	191,753,590	769,428,953
Stochastic	15	7	1	207,093,760	951,567,580
Stochastic	15	7	1	194,676,373	881,562,159
Stochastic	15	7	1	254,782,755	751,216,176
Stochastic	15	7	1	166,668,676	1,109,006,719
Stochastic	30	1	0.5	8,245,422,914	0
Stochastic	30	1	0.5	8,517,431,405	0

Stochastic	30	1	0.5	7,845,399,520	0
Stochastic	30	1	0.5	8,347,987,310	0
Stochastic	30	1	0.5	7,852,092,622	0
Stochastic	30	1	0.5	8,028,436,564	0
Stochastic	30	1	0.5	7,764,696,183	0
Stochastic	30	1	0.5	7,659,280,394	0
Stochastic	30	1	0.5	8,559,542,906	0
Stochastic	30	1	0.5	7,780,179,715	0
Stochastic	30	1	1	208,162,086	965,714,857
Stochastic	30	1	1	188,227,291	817,031,275
Stochastic	30	1	1	141,470,990	965,417,452
Stochastic	30	1	1	239,821,078	715,616,320
Stochastic	30	1	1	185,392,322	933,057,681
Stochastic	30	1	1	185,983,162	1,068,420,064
Stochastic	30	1	1	195,198,809	761,037,727
Stochastic	30	1	1	167,421,692	771,950,998
Stochastic	30	1	1	144,645,089	974,105,881
Stochastic	30	1	1	191,976,540	956,497,481
Stochastic	30	7	0.5	8,002,038,546	0
Stochastic	30	7	0.5	8,058,435,633	0
Stochastic	30	7	0.5	8,245,001,802	0
Stochastic	30	7	0.5	8,227,522,815	0
Stochastic	30	7	0.5	7,881,879,094	0
Stochastic	30	7	0.5	8,453,315,508	0
Stochastic	30	7	0.5	8,458,539,860	0
Stochastic	30	7	0.5	8,662,959,133	0
Stochastic	30	7	0.5	8,500,040,601	0
Stochastic	30	7	0.5	8,137,174,939	0
Stochastic	30	7	1	159,759,198	1,069,464,264
Stochastic	30	7	1	96,822,310	1,006,835,694
Stochastic	30	7	1	262,227,924	834,691,344
Stochastic	30	7	1	180,475,571	861,679,597

Stochastic	30	7	1	110,937,290	913,677,536
Stochastic	30	7	1	144,363,157	791,601,678
Stochastic	30	7	1	237,025,551	732,152,803
Stochastic	30	7	1	157,010,954	787,592,028
Stochastic	30	7	1	113,969,894	1,063,510,542
Stochastic	30	7	1	176,754,257	926,085,507

Appendix III

The Backorder and Inventory Costs Values for RPD

Planning Approach	Length of Planning Horizon	Re-planning Frequency	Demand Level	Demand variation	Simulated Inventory Cost	Simulated Backorder Cost
Determinist	15	1	0.5	40%	10,645,728,930	0
Determinist	15	1	0.5	40%	8,121,944,967	0
Determinist	15	1	0.5	40%	9,510,674,265	0
Determinist	15	1	0.5	40%	8,408,518,672	3,420
Determinist	15	1	0.5	40%	4,468,786,635	0
Determinist	15	1	0.5	40%	10,001,079,198	0
Determinist	15	1	0.5	40%	8,443,056,345	0
Determinist	15	1	0.5	40%	7,063,046,211	0
Determinist	15	1	0.5	40%	11,132,868,137	0
Determinist	15	1	0.5	40%	6,409,117,126	0
Determinist	15	1	0.5	40%	8,984,952,054	0
Determinist	15	1	0.5	40%	11,615,647,892	0
Determinist	15	1	0.5	40%	6,095,052,744	352
Determinist	15	1	0.5	40%	3,538,024,025	0
Determinist	15	1	0.5	40%	10,473,862,061	0
Determinist	15	1	0.5	40%	9,303,501,805	48
Determinist	15	1	0.5	40%	11,533,791,633	0
Determinist	15	1	0.5	40%	8,091,265,906	0
Determinist	15	1	0.5	40%	7,670,960,526	0
Determinist	15	1	0.5	40%	7,383,184,754	0
Stochastic	15	1	0.5	40%	6,985,571,865	0
Stochastic	15	1	0.5	40%	7,147,148,484	0
Stochastic	15	1	0.5	40%	6,062,554,557	0
Stochastic	15	1	0.5	40%	11,822,908,195	0
Stochastic	15	1	0.5	40%	9,762,273,302	0
Stochastic	15	1	0.5	40%	8,713,069,270	0
Stochastic	15	1	0.5	40%	5,440,638,224	0
Stochastic	15	1	0.5	40%	7,285,331,480	0
Stochastic	15	1	0.5	40%	10,020,823,303	0
Stochastic	15	1	0.5	40%	5,521,205,291	0
Stochastic	15	1	0.5	40%	5,658,133,450	0
Stochastic	15	1	0.5	40%	4,228,229,405	704
Stochastic	15	1	0.5	40%	4,387,307,629	6,092
Stochastic	15	1	0.5	40%	5,604,058,249	0

Stochastic	15	1	0.5	40%	9,625,593,466	0
Stochastic	15	1	0.5	40%	10,293,339,946	0
Stochastic	15	1	0.5	40%	7,705,484,017	0
Stochastic	15	1	0.5	40%	6,039,421,508	6,498
Stochastic	15	1	0.5	40%	7,181,481,090	0
Stochastic	15	1	0.5	40%	5,743,390,355	0
Determinist	15	1	0.5	5%	8,462,951,759	0
Determinist	15	1	0.5	5%	7,938,073,327	0
Determinist	15	1	0.5	5%	7,921,064,356	0
Determinist	15	1	0.5	5%	8,631,357,553	0
Determinist	15	1	0.5	5%	7,999,403,979	0
Determinist	15	1	0.5	5%	7,536,016,453	0
Determinist	15	1	0.5	5%	8,470,454,574	0
Determinist	15	1	0.5	5%	7,818,592,967	0
Determinist	15	1	0.5	5%	8,127,010,456	0
Determinist	15	1	0.5	5%	7,773,807,596	0
Determinist	15	1	0.5	5%	8,294,522,228	0
Determinist	15	1	0.5	5%	8,303,618,633	0
Determinist	15	1	0.5	5%	8,330,744,068	0
Determinist	15	1	0.5	5%	8,481,411,181	0
Determinist	15	1	0.5	5%	8,200,457,671	0
Determinist	15	1	0.5	5%	8,108,496,060	0
Determinist	15	1	0.5	5%	8,061,155,969	0
Determinist	15	1	0.5	5%	8,028,231,718	0
Determinist	15	1	0.5	5%	8,301,118,528	0
Determinist	15	1	0.5	5%	8,381,215,838	0
Stochastic	15	1	0.5	5%	8,362,589,916	0
Stochastic	15	1	0.5	5%	7,892,567,945	0
Stochastic	15	1	0.5	5%	8,475,640,112	0
Stochastic	15	1	0.5	5%	8,236,514,387	0
Stochastic	15	1	0.5	5%	7,768,416,612	0
Stochastic	15	1	0.5	5%	8,350,077,337	0
Stochastic	15	1	0.5	5%	8,292,066,000	0
Stochastic	15	1	0.5	5%	8,053,277,624	0
Stochastic	15	1	0.5	5%	8,205,290,613	0
Stochastic	15	1	0.5	5%	7,857,490,666	0
Stochastic	15	1	0.5	5%	7,997,082,536	0
Stochastic	15	1	0.5	5%	8,203,373,644	0
Stochastic	15	1	0.5	5%	8,034,662,318	0
Stochastic	15	1	0.5	5%	8,325,642,187	0
Stochastic	15	1	0.5	5%	8,388,368,577	0

Stochastic	15	1	0.5	5%	8,369,140,524	0
Stochastic	15	1	0.5	5%	7,613,176,501	0
Stochastic	15	1	0.5	5%	8,591,240,129	0
Stochastic	15	1	0.5	5%	8,028,017,978	0
Stochastic	15	1	0.5	5%	7,884,850,452	0
Determinist	15	1	1	40%	255,744,280	1,644,647,589
Determinist	15	1	1	40%	476,300,444	2,391,447,154
Determinist	15	1	1	40%	450,963,825	1,282,363,843
Determinist	15	1	1	40%	381,166,280	2,820,448,415
Determinist	15	1	1	40%	944,175,955	2,208,347,817
Determinist	15	1	1	40%	190,651,857	2,143,938,213
Determinist	15	1	1	40%	1,209,518,594	2,552,887,349
Determinist	15	1	1	40%	1,006,169,397	2,184,014,631
Determinist	15	1	1	40%	980,465,172	2,008,177,436
Determinist	15	1	1	40%	785,731,596	1,173,741,181
Determinist	15	1	1	40%	1,007,762,979	4,922,797,654
Determinist	15	1	1	40%	726,920,314	1,733,316,503
Determinist	15	1	1	40%	192,299,930	2,669,766,994
Determinist	15	1	1	40%	2,303,659,573	3,387,869,247
Determinist	15	1	1	40%	753,808,452	904,209,461
Determinist	15	1	1	40%	356,801,462	1,725,361,778
Determinist	15	1	1	40%	1,314,788,957	5,390,732,757
Determinist	15	1	1	40%	1,445,683,767	1,672,892,985
Determinist	15	1	1	40%	30,831,647	2,775,868,465
Determinist	15	1	1	40%	705,778,133	3,383,801,781
Stochastic	15	1	1	40%	494,055,909	1,289,776,511
Stochastic	15	1	1	40%	224,700,924	2,468,463,070
Stochastic	15	1	1	40%	922,891,884	1,262,871,182
Stochastic	15	1	1	40%	1,789,829,264	2,404,644,533
Stochastic	15	1	1	40%	154,403,696	2,145,840,048
Stochastic	15	1	1	40%	1,349,709,321	4,177,582,689
Stochastic	15	1	1	40%	252,982,161	2,284,726,517
Stochastic	15	1	1	40%	511,282,010	3,310,521,721
Stochastic	15	1	1	40%	139,859,113	3,752,247,027
Stochastic	15	1	1	40%	692,736,855	1,446,823,892
Stochastic	15	1	1	40%	449,785,647	2,323,408,650
Stochastic	15	1	1	40%	1,864,704,729	3,434,527,370
Stochastic	15	1	1	40%	1,258,053,488	4,239,908,442
Stochastic	15	1	1	40%	1,026,386,235	1,285,376,871
Stochastic	15	1	1	40%	116,790,595	3,104,611,263
Stochastic	15	1	1	40%	991,230,866	4,633,514,684

Stochastic	15	1	1	40%	542,199,706	1,589,004,275
Stochastic	15	1	1	40%	476,056,099	1,212,309,193
Stochastic	15	1	1	40%	1,304,497,365	1,531,747,629
Stochastic	15	1	1	40%	2,023,803,341	1,892,468,811
Determinist	15	1	1	5%	287,638,830	747,494,025
Determinist	15	1	1	5%	133,271,958	1,165,340,307
Determinist	15	1	1	5%	178,680,093	676,734,864
Determinist	15	1	1	5%	110,013,129	912,178,247
Determinist	15	1	1	5%	115,121,282	1,057,105,696
Determinist	15	1	1	5%	127,694,690	950,587,155
Determinist	15	1	1	5%	189,299,818	890,153,152
Determinist	15	1	1	5%	155,626,095	985,703,116
Determinist	15	1	1	5%	205,404,796	957,315,020
Determinist	15	1	1	5%	242,106,749	2,105,980,789
Determinist	15	1	1	5%	288,374,465	718,933,987
Determinist	15	1	1	5%	133,127,488	1,027,304,411
Determinist	15	1	1	5%	113,814,082	945,569,639
Determinist	15	1	1	5%	64,442,799	965,694,838
Determinist	15	1	1	5%	165,046,963	1,060,026,142
Determinist	15	1	1	5%	153,872,107	1,009,960,403
Determinist	15	1	1	5%	145,214,367	838,646,720
Determinist	15	1	1	5%	140,991,074	911,740,394
Determinist	15	1	1	5%	134,140,065	1,036,267,763
Determinist	15	1	1	5%	98,818,172	993,432,302
Stochastic	15	1	1	5%	98,871,327	1,118,103,791
Stochastic	15	1	1	5%	189,856,850	869,126,149
Stochastic	15	1	1	5%	208,172,680	851,031,085
Stochastic	15	1	1	5%	77,382,127	852,866,588
Stochastic	15	1	1	5%	137,794,300	798,841,933
Stochastic	15	1	1	5%	150,009,786	942,071,242
Stochastic	15	1	1	5%	196,923,468	967,584,462
Stochastic	15	1	1	5%	227,809,337	792,499,866
Stochastic	15	1	1	5%	191,399,210	994,992,158
Stochastic	15	1	1	5%	139,974,560	1,015,588,687
Stochastic	15	1	1	5%	174,798,823	875,292,320
Stochastic	15	1	1	5%	133,145,073	771,870,543
Stochastic	15	1	1	5%	137,596,206	1,031,253,376
Stochastic	15	1	1	5%	137,957,739	884,478,504
Stochastic	15	1	1	5%	100,033,057	1,057,494,378
Stochastic	15	1	1	5%	199,831,979	946,938,922
Stochastic	15	1	1	5%	141,033,245	959,175,859

Stochastic	15	1	1	5%	146,143,748	940,483,535
Stochastic	15	1	1	5%	161,312,855	887,869,276
Stochastic	15	1	1	5%	163,461,319	817,578,706
Determinist	15	7	0.5	40%	9,203,776,524	0
Determinist	15	7	0.5	40%	9,783,178,151	0
Determinist	15	7	0.5	40%	10,386,563,580	4,400
Determinist	15	7	0.5	40%	9,117,513,851	121
Determinist	15	7	0.5	40%	11,408,869,397	0
Determinist	15	7	0.5	40%	10,710,398,444	0
Determinist	15	7	0.5	40%	11,712,311,547	0
Determinist	15	7	0.5	40%	10,440,918,019	0
Determinist	15	7	0.5	40%	6,495,694,337	0
Determinist	15	7	0.5	40%	7,304,916,065	44,855
Determinist	15	7	0.5	40%	6,750,337,652	248
Determinist	15	7	0.5	40%	6,044,340,097	1,279,572
Determinist	15	7	0.5	40%	9,256,209,908	69,875
Determinist	15	7	0.5	40%	8,782,179,428	425,425
Determinist	15	7	0.5	40%	7,711,559,850	0
Determinist	15	7	0.5	40%	5,919,415,745	713,020
Determinist	15	7	0.5	40%	9,725,806,033	0
Determinist	15	7	0.5	40%	7,251,300,516	0
Determinist	15	7	0.5	40%	4,007,744,623	1,080,848
Determinist	15	7	0.5	40%	11,567,196,460	0
Stochastic	15	7	0.5	40%	4,816,543,203	2,167,031
Stochastic	15	7	0.5	40%	9,895,662,690	8,325
Stochastic	15	7	0.5	40%	6,112,955,670	0
Stochastic	15	7	0.5	40%	8,408,792,584	0
Stochastic	15	7	0.5	40%	8,820,116,308	0
Stochastic	15	7	0.5	40%	6,816,526,931	1,628,789
Stochastic	15	7	0.5	40%	7,342,532,464	41
Stochastic	15	7	0.5	40%	11,113,198,970	0
Stochastic	15	7	0.5	40%	7,520,717,016	0
Stochastic	15	7	0.5	40%	5,601,193,134	0
Stochastic	15	7	0.5	40%	9,449,259,581	0
Stochastic	15	7	0.5	40%	7,708,813,036	0
Stochastic	15	7	0.5	40%	9,678,604,944	1,770
Stochastic	15	7	0.5	40%	11,728,880,023	17,048
Stochastic	15	7	0.5	40%	7,275,941,450	19,936
Stochastic	15	7	0.5	40%	6,148,028,169	3,299
Stochastic	15	7	0.5	40%	7,293,873,229	0
Stochastic	15	7	0.5	40%	5,758,011,732	0

Stochastic	15	7	0.5	40%	8,206,464,330	0
Stochastic	15	7	0.5	40%	9,057,066,734	0
Determinist	15	7	0.5	5%	8,257,429,519	0
Determinist	15	7	0.5	5%	8,375,185,774	0
Determinist	15	7	0.5	5%	7,687,721,583	0
Determinist	15	7	0.5	5%	8,452,318,103	0
Determinist	15	7	0.5	5%	8,281,725,863	0
Determinist	15	7	0.5	5%	7,806,004,484	0
Determinist	15	7	0.5	5%	7,891,460,812	0
Determinist	15	7	0.5	5%	8,513,756,608	0
Determinist	15	7	0.5	5%	7,871,786,454	0
Determinist	15	7	0.5	5%	8,067,247,095	0
Determinist	15	7	0.5	5%	7,874,097,292	0
Determinist	15	7	0.5	5%	7,684,387,571	0
Determinist	15	7	0.5	5%	8,075,513,443	0
Determinist	15	7	0.5	5%	7,914,151,303	0
Determinist	15	7	0.5	5%	7,736,721,705	0
Determinist	15	7	0.5	5%	8,157,491,456	0
Determinist	15	7	0.5	5%	7,770,604,123	0
Determinist	15	7	0.5	5%	7,919,432,401	0
Determinist	15	7	0.5	5%	8,196,194,246	0
Determinist	15	7	0.5	5%	8,097,272,555	0
Stochastic	15	7	0.5	5%	7,800,455,304	0
Stochastic	15	7	0.5	5%	7,745,428,396	0
Stochastic	15	7	0.5	5%	7,805,955,887	0
Stochastic	15	7	0.5	5%	8,325,796,868	0
Stochastic	15	7	0.5	5%	7,806,794,449	0
Stochastic	15	7	0.5	5%	8,186,030,217	0
Stochastic	15	7	0.5	5%	8,515,865,683	0
Stochastic	15	7	0.5	5%	7,600,440,426	0
Stochastic	15	7	0.5	5%	8,584,930,815	0
Stochastic	15	7	0.5	5%	8,276,460,383	0
Stochastic	15	7	0.5	5%	8,410,830,206	0
Stochastic	15	7	0.5	5%	7,823,598,951	0
Stochastic	15	7	0.5	5%	8,195,863,139	0
Stochastic	15	7	0.5	5%	8,147,600,537	0
Stochastic	15	7	0.5	5%	8,243,278,151	0
Stochastic	15	7	0.5	5%	8,059,163,714	0
Stochastic	15	7	0.5	5%	8,233,743,975	0
Stochastic	15	7	0.5	5%	7,777,893,688	0
Stochastic	15	7	0.5	5%	8,402,500,052	0

Stochastic	15	7	0.5	5%	8,062,993,667	0
Determinist	15	7	1	40%	311,895,113	1,890,560,837
Determinist	15	7	1	40%	1,472,508,085	1,967,805,831
Determinist	15	7	1	40%	55,674,250	2,822,978,732
Determinist	15	7	1	40%	234,283,409	1,491,116,944
Determinist	15	7	1	40%	198,357,331	2,891,437,832
Determinist	15	7	1	40%	709,658,569	2,332,514,526
Determinist	15	7	1	40%	1,796,687,167	1,584,033,823
Determinist	15	7	1	40%	942,070,506	7,826,475,826
Determinist	15	7	1	40%	751,667,463	1,932,713,596
Determinist	15	7	1	40%	510,902,151	2,332,412,074
Determinist	15	7	1	40%	1,079,293,118	1,801,177,171
Determinist	15	7	1	40%	567,066,373	1,819,418,000
Determinist	15	7	1	40%	360,587,042	2,318,730,505
Determinist	15	7	1	40%	145,363,374	4,176,169,249
Determinist	15	7	1	40%	1,195,831,813	1,347,042,401
Determinist	15	7	1	40%	451,332,713	1,149,445,647
Determinist	15	7	1	40%	538,345,245	1,543,244,972
Determinist	15	7	1	40%	1,018,422,242	5,340,094,699
Determinist	15	7	1	40%	416,068,747	1,592,808,088
Determinist	15	7	1	40%	1,500,030,569	7,322,191,138
Stochastic	15	7	1	40%	431,603,577	1,117,786,513
Stochastic	15	7	1	40%	924,779,588	3,141,015,338
Stochastic	15	7	1	40%	1,105,603,011	1,625,965,002
Stochastic	15	7	1	40%	347,612,832	1,595,257,661
Stochastic	15	7	1	40%	310,407,195	2,482,974,861
Stochastic	15	7	1	40%	603,091,136	1,041,209,282
Stochastic	15	7	1	40%	1,828,659,477	2,522,348,655
Stochastic	15	7	1	40%	534,117,300	2,038,664,569
Stochastic	15	7	1	40%	545,556,876	2,073,726,736
Stochastic	15	7	1	40%	2,153,400,352	6,889,132,882
Stochastic	15	7	1	40%	865,367,342	1,558,707,555
Stochastic	15	7	1	40%	1,016,043,243	6,671,696,767
Stochastic	15	7	1	40%	63,999,223	1,263,726,105
Stochastic	15	7	1	40%	914,194,715	4,858,549,017
Stochastic	15	7	1	40%	965,727,717	6,214,830,233
Stochastic	15	7	1	40%	866,132,213	1,785,614,540
Stochastic	15	7	1	40%	1,577,496,506	2,401,375,692
Stochastic	15	7	1	40%	838,190,939	1,413,805,811
Stochastic	15	7	1	40%	1,541,173,225	3,298,816,636
Stochastic	15	7	1	40%	1,447,506,869	1,390,255,850

Determinist	15	7	1	5%	232,411,422	976,229,463
Determinist	15	7	1	5%	188,001,140	876,662,440
Determinist	15	7	1	5%	149,537,680	832,968,801
Determinist	15	7	1	5%	119,272,702	880,984,284
Determinist	15	7	1	5%	154,438,179	1,083,291,378
Determinist	15	7	1	5%	220,870,473	929,068,574
Determinist	15	7	1	5%	192,311,632	958,095,946
Determinist	15	7	1	5%	91,250,227	998,945,264
Determinist	15	7	1	5%	66,962,438	1,001,556,977
Determinist	15	7	1	5%	240,734,207	890,322,554
Determinist	15	7	1	5%	235,561,159	1,032,729,019
Determinist	15	7	1	5%	109,977,285	938,354,811
Determinist	15	7	1	5%	93,105,967	907,257,385
Determinist	15	7	1	5%	136,751,089	1,102,603,391
Determinist	15	7	1	5%	149,357,984	977,943,774
Determinist	15	7	1	5%	131,812,353	941,213,804
Determinist	15	7	1	5%	161,464,134	996,238,445
Determinist	15	7	1	5%	142,048,932	1,185,369,737
Determinist	15	7	1	5%	94,033,116	837,503,747
Determinist	15	7	1	5%	87,270,883	915,074,903
Stochastic	15	7	1	5%	169,721,478	991,121,356
Stochastic	15	7	1	5%	135,700,508	989,694,323
Stochastic	15	7	1	5%	233,681,170	823,563,484
Stochastic	15	7	1	5%	149,948,488	1,133,818,511
Stochastic	15	7	1	5%	128,037,734	1,055,153,181
Stochastic	15	7	1	5%	146,179,578	946,857,884
Stochastic	15	7	1	5%	163,446,435	870,415,349
Stochastic	15	7	1	5%	197,818,904	966,529,359
Stochastic	15	7	1	5%	189,334,996	764,345,609
Stochastic	15	7	1	5%	121,609,367	956,672,576
Stochastic	15	7	1	5%	198,580,369	945,795,733
Stochastic	15	7	1	5%	186,869,583	978,073,227
Stochastic	15	7	1	5%	172,973,624	789,374,100
Stochastic	15	7	1	5%	121,092,688	917,342,533
Stochastic	15	7	1	5%	108,820,793	924,957,494
Stochastic	15	7	1	5%	211,559,460	997,198,155
Stochastic	15	7	1	5%	200,217,059	1,036,589,147
Stochastic	15	7	1	5%	162,575,173	1,179,559,747
Stochastic	15	7	1	5%	133,800,886	1,061,527,967
Stochastic	15	7	1	5%	187,103,343	850,390,693
Determinist	30	1	0.5	40%	11,407,423,983	0

Determinist	30	1	0.5	40%	12,185,739,400	0
Determinist	30	1	0.5	40%	8,088,920,668	0
Determinist	30	1	0.5	40%	11,383,928,866	0
Determinist	30	1	0.5	40%	9,607,849,383	0
Determinist	30	1	0.5	40%	9,111,644,332	0
Determinist	30	1	0.5	40%	5,030,399,902	0
Determinist	30	1	0.5	40%	10,362,479,697	0
Determinist	30	1	0.5	40%	10,084,399,752	0
Determinist	30	1	0.5	40%	10,747,574,923	0
Determinist	30	1	0.5	40%	13,019,277,808	0
Determinist	30	1	0.5	40%	4,776,940,661	79,351
Determinist	30	1	0.5	40%	8,726,802,015	0
Determinist	30	1	0.5	40%	8,617,668,562	0
Determinist	30	1	0.5	40%	8,568,405,288	0
Determinist	30	1	0.5	40%	7,438,558,186	0
Determinist	30	1	0.5	40%	9,451,939,719	0
Determinist	30	1	0.5	40%	7,678,225,803	0
Determinist	30	1	0.5	40%	5,465,136,829	0
Determinist	30	1	0.5	40%	11,769,118,736	0
Stochastic	30	1	0.5	40%	11,352,258,222	16,918
Stochastic	30	1	0.5	40%	10,425,618,843	0
Stochastic	30	1	0.5	40%	11,250,093,252	0
Stochastic	30	1	0.5	40%	9,597,480,565	0
Stochastic	30	1	0.5	40%	6,491,053,377	0
Stochastic	30	1	0.5	40%	11,096,382,182	0
Stochastic	30	1	0.5	40%	5,718,283,252	0
Stochastic	30	1	0.5	40%	7,314,550,796	2,624
Stochastic	30	1	0.5	40%	7,052,115,628	1,811
Stochastic	30	1	0.5	40%	7,910,282,034	16,733
Stochastic	30	1	0.5	40%	10,744,208,480	1,697
Stochastic	30	1	0.5	40%	10,779,330,718	0
Stochastic	30	1	0.5	40%	4,545,866,701	0
Stochastic	30	1	0.5	40%	12,734,678,742	0
Stochastic	30	1	0.5	40%	10,237,351,166	0
Stochastic	30	1	0.5	40%	11,025,101,930	0
Stochastic	30	1	0.5	40%	7,170,325,759	0
Stochastic	30	1	0.5	40%	9,801,134,760	0
Stochastic	30	1	0.5	40%	4,981,129,837	0
Stochastic	30	1	0.5	40%	8,698,882,594	0
Determinist	30	1	0.5	5%	7,924,918,198	0
Determinist	30	1	0.5	5%	7,459,329,325	0

Determinist	30	1	0.5	5%	8,488,799,874	0
Determinist	30	1	0.5	5%	8,082,724,087	0
Determinist	30	1	0.5	5%	8,521,726,367	0
Determinist	30	1	0.5	5%	8,550,426,599	0
Determinist	30	1	0.5	5%	7,893,774,258	0
Determinist	30	1	0.5	5%	8,302,060,253	0
Determinist	30	1	0.5	5%	8,124,981,499	0
Determinist	30	1	0.5	5%	8,412,894,890	0
Determinist	30	1	0.5	5%	8,098,444,514	0
Determinist	30	1	0.5	5%	8,236,467,733	0
Determinist	30	1	0.5	5%	8,074,864,607	0
Determinist	30	1	0.5	5%	8,288,336,071	0
Determinist	30	1	0.5	5%	7,975,963,925	0
Determinist	30	1	0.5	5%	8,151,733,793	0
Determinist	30	1	0.5	5%	8,075,243,055	0
Determinist	30	1	0.5	5%	8,598,469,200	0
Determinist	30	1	0.5	5%	8,716,548,756	0
Determinist	30	1	0.5	5%	8,317,017,334	0
Stochastic	30	1	0.5	5%	8,568,774,574	0
Stochastic	30	1	0.5	5%	8,448,685,190	0
Stochastic	30	1	0.5	5%	8,008,114,018	0
Stochastic	30	1	0.5	5%	8,389,919,046	0
Stochastic	30	1	0.5	5%	8,026,071,331	0
Stochastic	30	1	0.5	5%	7,643,606,451	0
Stochastic	30	1	0.5	5%	7,919,900,475	0
Stochastic	30	1	0.5	5%	8,569,767,375	0
Stochastic	30	1	0.5	5%	8,017,937,621	0
Stochastic	30	1	0.5	5%	7,743,132,402	0
Stochastic	30	1	0.5	5%	8,198,309,200	0
Stochastic	30	1	0.5	5%	8,365,795,422	0
Stochastic	30	1	0.5	5%	8,313,041,842	0
Stochastic	30	1	0.5	5%	8,407,868,420	0
Stochastic	30	1	0.5	5%	8,323,163,438	0
Stochastic	30	1	0.5	5%	7,693,137,768	0
Stochastic	30	1	0.5	5%	8,441,545,424	0
Stochastic	30	1	0.5	5%	7,780,431,641	0
Stochastic	30	1	0.5	5%	8,205,091,890	0
Stochastic	30	1	0.5	5%	8,229,002,443	0
Determinist	30	1	1	40%	861,405,234	7,498,216,904
Determinist	30	1	1	40%	527,321,111	2,432,352,028
Determinist	30	1	1	40%	846,248,302	1,335,486,384

Determinist	30	1	1	40%	1,154,366,293	1,265,856,681
Determinist	30	1	1	40%	244,450,359	1,903,453,924
Determinist	30	1	1	40%	334,065,384	2,368,267,319
Determinist	30	1	1	40%	69,591,945	3,569,545,088
Determinist	30	1	1	40%	524,108,793	1,951,278,571
Determinist	30	1	1	40%	1,117,826,852	3,397,193,598
Determinist	30	1	1	40%	869,307,785	4,908,616,867
Determinist	30	1	1	40%	2,101,530,773	4,036,001,622
Determinist	30	1	1	40%	1,689,767,353	875,082,885
Determinist	30	1	1	40%	1,363,319,960	5,825,476,953
Determinist	30	1	1	40%	532,492,494	3,126,698,752
Determinist	30	1	1	40%	1,437,618,981	5,917,024,736
Determinist	30	1	1	40%	1,183,995,906	6,023,372,184
Determinist	30	1	1	40%	751,400,792	1,938,607,106
Determinist	30	1	1	40%	2,234,858,747	6,027,968,714
Determinist	30	1	1	40%	199,374,933	1,809,690,057
Determinist	30	1	1	40%	513,681,421	2,582,772,638
Stochastic	30	1	1	40%	894,775,842	3,072,362,748
Stochastic	30	1	1	40%	473,375,605	1,907,692,539
Stochastic	30	1	1	40%	762,431,383	2,753,762,082
Stochastic	30	1	1	40%	759,705,872	1,145,892,498
Stochastic	30	1	1	40%	1,095,225,830	2,426,911,035
Stochastic	30	1	1	40%	323,938,058	1,997,524,793
Stochastic	30	1	1	40%	553,526,948	2,322,213,236
Stochastic	30	1	1	40%	818,653,175	1,533,705,731
Stochastic	30	1	1	40%	1,353,697,562	3,382,572,061
Stochastic	30	1	1	40%	464,686,869	1,026,387,384
Stochastic	30	1	1	40%	848,547,696	925,960,343
Stochastic	30	1	1	40%	397,070,807	4,490,096,481
Stochastic	30	1	1	40%	401,573,132	1,962,204,421
Stochastic	30	1	1	40%	371,553,277	2,383,663,470
Stochastic	30	1	1	40%	161,790,579	2,274,560,227
Stochastic	30	1	1	40%	880,586,417	4,033,341,655
Stochastic	30	1	1	40%	1,167,273,515	2,750,493,002
Stochastic	30	1	1	40%	1,423,961,795	1,844,265,147
Stochastic	30	1	1	40%	1,538,504,074	3,557,928,311
Stochastic	30	1	1	40%	129,777,643	2,467,551,856
Determinist	30	1	1	5%	205,527,725	799,989,950
Determinist	30	1	1	5%	150,388,939	906,285,039
Determinist	30	1	1	5%	179,995,933	930,597,363
Determinist	30	1	1	5%	156,417,700	949,134,577

Determinist	30	1	1	5%	142,245,567	948,196,179
Determinist	30	1	1	5%	240,237,656	753,507,016
Determinist	30	1	1	5%	147,793,732	1,076,913,895
Determinist	30	1	1	5%	144,309,391	1,088,719,210
Determinist	30	1	1	5%	231,072,206	945,230,612
Determinist	30	1	1	5%	160,516,851	1,002,808,816
Determinist	30	1	1	5%	167,352,348	1,036,888,515
Determinist	30	1	1	5%	74,701,256	1,064,300,615
Determinist	30	1	1	5%	197,471,508	939,945,544
Determinist	30	1	1	5%	139,189,180	864,115,942
Determinist	30	1	1	5%	131,995,500	910,137,171
Determinist	30	1	1	5%	137,941,245	965,480,363
Determinist	30	1	1	5%	197,296,793	951,729,367
Determinist	30	1	1	5%	279,750,542	1,102,192,338
Determinist	30	1	1	5%	120,473,410	853,792,916
Determinist	30	1	1	5%	74,485,987	1,031,080,921
Stochastic	30	1	1	5%	177,968,160	902,090,205
Stochastic	30	1	1	5%	249,327,266	846,266,375
Stochastic	30	1	1	5%	160,657,506	1,029,113,761
Stochastic	30	1	1	5%	168,879,342	995,411,018
Stochastic	30	1	1	5%	164,477,609	866,767,564
Stochastic	30	1	1	5%	142,190,266	849,302,311
Stochastic	30	1	1	5%	175,528,332	1,043,293,126
Stochastic	30	1	1	5%	153,113,540	944,856,209
Stochastic	30	1	1	5%	219,828,131	904,486,946
Stochastic	30	1	1	5%	119,934,842	1,084,499,653
Stochastic	30	1	1	5%	169,730,110	867,648,637
Stochastic	30	1	1	5%	227,664,970	711,607,042
Stochastic	30	1	1	5%	114,744,386	1,121,049,655
Stochastic	30	1	1	5%	192,484,799	1,086,008,350
Stochastic	30	1	1	5%	156,811,214	748,192,950
Stochastic	30	1	1	5%	158,363,016	858,460,938
Stochastic	30	1	1	5%	98,404,974	1,167,548,575
Stochastic	30	1	1	5%	161,584,047	868,826,479
Stochastic	30	1	1	5%	237,807,843	916,216,877
Stochastic	30	1	1	5%	122,215,090	831,090,863
Determinist	30	7	0.5	40%	4,992,489,205	844,144
Determinist	30	7	0.5	40%	9,363,826,797	0
Determinist	30	7	0.5	40%	8,319,694,894	442
Determinist	30	7	0.5	40%	11,927,229,570	0
Determinist	30	7	0.5	40%	8,085,763,510	165,883

Determinist	30	7	0.5	40%	9,465,589,871	0
Determinist	30	7	0.5	40%	9,840,527,833	0
Determinist	30	7	0.5	40%	3,061,156,336	30,036,114
Determinist	30	7	0.5	40%	7,119,410,640	1,880,280
Determinist	30	7	0.5	40%	11,629,759,343	0
Determinist	30	7	0.5	40%	6,051,827,193	24,900
Determinist	30	7	0.5	40%	8,695,367,404	33,657
Determinist	30	7	0.5	40%	4,570,961,730	0
Determinist	30	7	0.5	40%	8,509,950,110	0
Determinist	30	7	0.5	40%	4,635,316,742	1,102,010
Determinist	30	7	0.5	40%	10,160,646,612	0
Determinist	30	7	0.5	40%	11,803,057,467	1,895
Determinist	30	7	0.5	40%	9,814,724,190	0
Determinist	30	7	0.5	40%	9,995,056,471	4,333
Determinist	30	7	0.5	40%	5,595,518,034	168,899
Stochastic	30	7	0.5	40%	8,202,909,204	0
Stochastic	30	7	0.5	40%	10,137,724,327	0
Stochastic	30	7	0.5	40%	8,676,869,866	0
Stochastic	30	7	0.5	40%	5,314,798,309	1,945,376
Stochastic	30	7	0.5	40%	4,918,728,614	2,710,140
Stochastic	30	7	0.5	40%	10,264,221,757	3,954
Stochastic	30	7	0.5	40%	9,853,672,969	0
Stochastic	30	7	0.5	40%	11,407,973,452	0
Stochastic	30	7	0.5	40%	6,613,005,501	0
Stochastic	30	7	0.5	40%	6,242,253,255	805,743
Stochastic	30	7	0.5	40%	10,388,899,270	23
Stochastic	30	7	0.5	40%	7,924,531,320	0
Stochastic	30	7	0.5	40%	9,119,156,439	534
Stochastic	30	7	0.5	40%	6,884,569,394	18,214
Stochastic	30	7	0.5	40%	11,915,206,086	7,635
Stochastic	30	7	0.5	40%	4,589,545,618	384,925
Stochastic	30	7	0.5	40%	8,374,022,963	183,970
Stochastic	30	7	0.5	40%	7,070,622,925	21,432
Stochastic	30	7	0.5	40%	7,287,276,394	0
Stochastic	30	7	0.5	40%	9,921,125,772	0
Determinist	30	7	0.5	5%	8,237,218,683	0
Determinist	30	7	0.5	5%	8,001,451,988	0
Determinist	30	7	0.5	5%	8,245,187,359	0
Determinist	30	7	0.5	5%	8,373,191,748	0
Determinist	30	7	0.5	5%	8,459,123,515	0
Determinist	30	7	0.5	5%	8,306,267,969	0

Determinist	30	7	0.5	5%	8,141,419,357	0
Determinist	30	7	0.5	5%	8,600,187,232	0
Determinist	30	7	0.5	5%	8,288,389,408	0
Determinist	30	7	0.5	5%	8,241,725,947	0
Determinist	30	7	0.5	5%	7,858,528,795	0
Determinist	30	7	0.5	5%	7,960,361,523	0
Determinist	30	7	0.5	5%	8,213,917,430	0
Determinist	30	7	0.5	5%	8,000,166,622	0
Determinist	30	7	0.5	5%	8,247,794,734	0
Determinist	30	7	0.5	5%	8,408,944,842	0
Determinist	30	7	0.5	5%	8,196,640,144	0
Determinist	30	7	0.5	5%	7,715,709,307	0
Determinist	30	7	0.5	5%	8,105,874,083	0
Determinist	30	7	0.5	5%	8,579,924,368	0
Stochastic	30	7	0.5	5%	8,588,400,673	0
Stochastic	30	7	0.5	5%	7,850,846,183	0
Stochastic	30	7	0.5	5%	8,281,144,589	0
Stochastic	30	7	0.5	5%	8,324,604,533	0
Stochastic	30	7	0.5	5%	7,576,266,619	0
Stochastic	30	7	0.5	5%	8,124,072,600	0
Stochastic	30	7	0.5	5%	8,630,701,711	0
Stochastic	30	7	0.5	5%	8,438,381,486	0
Stochastic	30	7	0.5	5%	7,787,310,724	0
Stochastic	30	7	0.5	5%	7,892,466,023	0
Stochastic	30	7	0.5	5%	8,565,960,185	0
Stochastic	30	7	0.5	5%	7,869,960,000	0
Stochastic	30	7	0.5	5%	8,200,564,059	0
Stochastic	30	7	0.5	5%	8,243,284,867	0
Stochastic	30	7	0.5	5%	8,630,932,321	0
Stochastic	30	7	0.5	5%	7,893,475,083	0
Stochastic	30	7	0.5	5%	7,973,080,226	0
Stochastic	30	7	0.5	5%	8,094,476,251	0
Stochastic	30	7	0.5	5%	8,373,548,586	0
Stochastic	30	7	0.5	5%	8,223,009,870	0
Determinist	30	7	1	40%	741,147,020	1,325,273,112
Determinist	30	7	1	40%	390,565,953	2,842,404,156
Determinist	30	7	1	40%	424,097,530	3,015,938,076
Determinist	30	7	1	40%	1,333,317,694	1,818,893,970
Determinist	30	7	1	40%	870,907,124	3,466,989,603
Determinist	30	7	1	40%	1,484,221,727	2,463,139,544
Determinist	30	7	1	40%	1,041,328,752	1,578,110,943

Determinist	30	7	1	40%	1,977,077,807	5,646,928,370
Determinist	30	7	1	40%	329,147,652	2,549,335,961
Determinist	30	7	1	40%	1,783,422,391	6,292,745,966
Determinist	30	7	1	40%	1,624,893,212	4,837,171,542
Determinist	30	7	1	40%	347,504,520	990,525,681
Determinist	30	7	1	40%	483,398,418	2,285,908,752
Determinist	30	7	1	40%	1,344,610,412	2,601,964,574
Determinist	30	7	1	40%	1,336,845,258	6,008,410,036
Determinist	30	7	1	40%	378,094,012	1,191,100,810
Determinist	30	7	1	40%	477,071,007	1,196,252,211
Determinist	30	7	1	40%	1,266,000,440	777,847,610
Determinist	30	7	1	40%	619,110,092	1,602,108,668
Determinist	30	7	1	40%	144,413,961	2,532,389,966
Stochastic	30	7	1	40%	1,343,115,986	2,694,647,093
Stochastic	30	7	1	40%	1,652,964,468	5,774,782,187
Stochastic	30	7	1	40%	1,272,974,518	2,640,218,893
Stochastic	30	7	1	40%	1,033,699,133	2,753,378,608
Stochastic	30	7	1	40%	723,793,716	2,107,776,441
Stochastic	30	7	1	40%	1,548,604,381	628,480,383
Stochastic	30	7	1	40%	1,247,948,459	6,012,407,809
Stochastic	30	7	1	40%	2,445,659,880	6,835,724,022
Stochastic	30	7	1	40%	1,045,960,618	3,264,904,194
Stochastic	30	7	1	40%	730,153,970	6,186,594,749
Stochastic	30	7	1	40%	879,327,587	2,189,261,937
Stochastic	30	7	1	40%	393,525,184	4,095,141,745
Stochastic	30	7	1	40%	1,043,962,261	1,412,211,477
Stochastic	30	7	1	40%	1,083,698,326	2,550,247,046
Stochastic	30	7	1	40%	866,083,204	4,018,164,999
Stochastic	30	7	1	40%	223,434,699	2,179,331,431
Stochastic	30	7	1	40%	219,668,982	2,112,182,950
Stochastic	30	7	1	40%	935,529,852	3,651,230,281
Stochastic	30	7	1	40%	1,368,235,697	1,511,903,088
Stochastic	30	7	1	40%	1,371,685,395	966,707,781
Determinist	30	7	1	5%	240,784,947	889,433,907
Determinist	30	7	1	5%	134,200,881	961,973,119
Determinist	30	7	1	5%	93,287,044	904,895,011
Determinist	30	7	1	5%	72,487,583	879,609,820
Determinist	30	7	1	5%	71,297,140	1,100,186,879
Determinist	30	7	1	5%	142,574,535	1,011,651,266
Determinist	30	7	1	5%	128,043,524	868,724,337
Determinist	30	7	1	5%	157,945,101	929,191,803

Determinist	30	7	1	5%	261,526,870	822,876,259
Determinist	30	7	1	5%	148,013,498	1,101,863,071
Determinist	30	7	1	5%	184,929,160	926,145,725
Determinist	30	7	1	5%	199,496,506	961,342,873
Determinist	30	7	1	5%	262,826,026	844,944,689
Determinist	30	7	1	5%	74,807,838	968,933,807
Determinist	30	7	1	5%	185,866,581	874,995,941
Determinist	30	7	1	5%	149,760,841	917,890,257
Determinist	30	7	1	5%	107,783,822	1,077,978,737
Determinist	30	7	1	5%	211,908,295	937,240,489
Determinist	30	7	1	5%	103,256,284	963,407,741
Determinist	30	7	1	5%	118,272,547	893,070,362
Stochastic	30	7	1	5%	258,418,214	876,366,261
Stochastic	30	7	1	5%	192,933,334	706,805,421
Stochastic	30	7	1	5%	206,438,529	952,513,784
Stochastic	30	7	1	5%	194,431,026	1,033,798,687
Stochastic	30	7	1	5%	186,081,696	805,543,489
Stochastic	30	7	1	5%	141,558,553	907,914,999
Stochastic	30	7	1	5%	206,819,120	931,808,161
Stochastic	30	7	1	5%	216,931,975	882,724,379
Stochastic	30	7	1	5%	195,634,076	848,112,916
Stochastic	30	7	1	5%	216,491,870	651,200,648
Stochastic	30	7	1	5%	111,441,540	928,500,541
Stochastic	30	7	1	5%	164,916,312	1,059,220,734
Stochastic	30	7	1	5%	304,754,276	722,504,408
Stochastic	30	7	1	5%	207,195,079	970,541,099
Stochastic	30	7	1	5%	124,739,726	813,012,327
Stochastic	30	7	1	5%	138,597,619	836,127,554
Stochastic	30	7	1	5%	112,386,520	1,114,318,289
Stochastic	30	7	1	5%	210,666,228	908,122,728
Stochastic	30	7	1	5%	138,725,118	989,161,877
Stochastic	30	7	1	5%	187,344,015	889,919,605

Appendix IV

The Backorder and Inventory Costs Values for Full Factorial Design

(40% Demand variation)

Planning Approach	Length of Planning Horizon	Re-planning Frequency	Demand Level	Simulated Inventory Cost	Simulated Backorder Cost
Determinist	15	1	0.5	8,984,952,054	0
Determinist	15	1	0.5	11,615,647,892	0
Determinist	15	1	0.5	6,095,052,744	352
Determinist	15	1	0.5	3,538,024,025	0
Determinist	15	1	0.5	10,473,862,061	0
Determinist	15	1	0.5	9,303,501,805	48
Determinist	15	1	0.5	11,533,791,633	0
Determinist	15	1	0.5	8,091,265,906	0
Determinist	15	1	0.5	7,670,960,526	0
Determinist	15	1	0.5	7,383,184,754	0
Determinist	15	1	1	1,007,762,979	4,922,797,654
Determinist	15	1	1	726,920,314	1,733,316,503
Determinist	15	1	1	192,299,930	2,669,766,994
Determinist	15	1	1	2,303,659,573	3,387,869,247
Determinist	15	1	1	753,808,452	904,209,461
Determinist	15	1	1	356,801,462	1,725,361,778
Determinist	15	1	1	1,314,788,957	5,390,732,757
Determinist	15	1	1	1,445,683,767	1,672,892,985
Determinist	15	1	1	30,831,647	2,775,868,465
Determinist	15	1	1	705,778,133	3,383,801,781
Determinist	15	7	0.5	6,750,337,652	248
Determinist	15	7	0.5	6,044,340,097	1,279,572
Determinist	15	7	0.5	9,256,209,908	69,875
Determinist	15	7	0.5	8,782,179,428	425,425
Determinist	15	7	0.5	7,711,559,850	0
Determinist	15	7	0.5	5,919,415,745	713,020
Determinist	15	7	0.5	9,725,806,033	0
Determinist	15	7	0.5	7,251,300,516	0
Determinist	15	7	0.5	4,007,744,623	1,080,848
Determinist	15	7	0.5	11,567,196,460	0
Determinist	15	7	1	1,079,293,118	1,801,177,171
Determinist	15	7	1	567,066,373	1,819,418,000

Determinist	15	7	1	360,587,042	2,318,730,505
Determinist	15	7	1	145,363,374	4,176,169,249
Determinist	15	7	1	1,195,831,813	1,347,042,401
Determinist	15	7	1	451,332,713	1,149,445,647
Determinist	15	7	1	538,345,245	1,543,244,972
Determinist	15	7	1	1,018,422,242	5,340,094,699
Determinist	15	7	1	416,068,747	1,592,808,088
Determinist	15	7	1	1,500,030,569	7,322,191,138
Determinist	30	1	0.5	13,019,277,808	0
Determinist	30	1	0.5	4,776,940,661	79,351
Determinist	30	1	0.5	8,726,802,015	0
Determinist	30	1	0.5	8,617,668,562	0
Determinist	30	1	0.5	8,568,405,288	0
Determinist	30	1	0.5	7,438,558,186	0
Determinist	30	1	0.5	9,451,939,719	0
Determinist	30	1	0.5	7,678,225,803	0
Determinist	30	1	0.5	5,465,136,829	0
Determinist	30	1	0.5	11,769,118,736	0
Determinist	30	1	1	2,101,530,773	4,036,001,622
Determinist	30	1	1	1,689,767,353	875,082,885
Determinist	30	1	1	1,363,319,960	5,825,476,953
Determinist	30	1	1	532,492,494	3,126,698,752
Determinist	30	1	1	1,437,618,981	5,917,024,736
Determinist	30	1	1	1,183,995,906	6,023,372,184
Determinist	30	1	1	751,400,792	1,938,607,106
Determinist	30	1	1	2,234,858,747	6,027,968,714
Determinist	30	1	1	199,374,933	1,809,690,057
Determinist	30	1	1	513,681,421	2,582,772,638
Determinist	30	7	0.5	6,051,827,193	24,900
Determinist	30	7	0.5	8,695,367,404	33,657
Determinist	30	7	0.5	4,570,961,730	0
Determinist	30	7	0.5	8,509,950,110	0
Determinist	30	7	0.5	4,635,316,742	1,102,010
Determinist	30	7	0.5	10,160,646,612	0
Determinist	30	7	0.5	11,803,057,467	1,895
Determinist	30	7	0.5	9,814,724,190	0
Determinist	30	7	0.5	9,995,056,471	4,333
Determinist	30	7	0.5	5,595,518,034	168,899
Determinist	30	7	1	1,624,893,212	4,837,171,542
Determinist	30	7	1	347,504,520	990,525,681
Determinist	30	7	1	483,398,418	2,285,908,752

Determinist	30	7	1	1,344,610,412	2,601,964,574
Determinist	30	7	1	1,336,845,258	6,008,410,036
Determinist	30	7	1	378,094,012	1,191,100,810
Determinist	30	7	1	477,071,007	1,196,252,211
Determinist	30	7	1	1,266,000,440	777,847,610
Determinist	30	7	1	619,110,092	1,602,108,668
Determinist	30	7	1	144,413,961	2,532,389,966
Stochastic	15	1	0.5	5,658,133,450	0
Stochastic	15	1	0.5	4,228,229,405	704
Stochastic	15	1	0.5	4,387,307,629	6,092
Stochastic	15	1	0.5	5,604,058,249	0
Stochastic	15	1	0.5	9,625,593,466	0
Stochastic	15	1	0.5	10,293,339,946	0
Stochastic	15	1	0.5	7,705,484,017	0
Stochastic	15	1	0.5	6,039,421,508	6,498
Stochastic	15	1	0.5	7,181,481,090	0
Stochastic	15	1	0.5	5,743,390,355	0
Stochastic	15	1	1	449,785,647	2,323,408,650
Stochastic	15	1	1	1,864,704,729	3,434,527,370
Stochastic	15	1	1	1,258,053,488	4,239,908,442
Stochastic	15	1	1	1,026,386,235	1,285,376,871
Stochastic	15	1	1	116,790,595	3,104,611,263
Stochastic	15	1	1	991,230,866	4,633,514,684
Stochastic	15	1	1	542,199,706	1,589,004,275
Stochastic	15	1	1	476,056,099	1,212,309,193
Stochastic	15	1	1	1,304,497,365	1,531,747,629
Stochastic	15	1	1	2,023,803,341	1,892,468,811
Stochastic	15	7	0.5	9,449,259,581	0
Stochastic	15	7	0.5	7,708,813,036	0
Stochastic	15	7	0.5	9,678,604,944	1,770
Stochastic	15	7	0.5	11,728,880,023	17,048
Stochastic	15	7	0.5	7,275,941,450	19,936
Stochastic	15	7	0.5	6,148,028,169	3,299
Stochastic	15	7	0.5	7,293,873,229	0
Stochastic	15	7	0.5	5,758,011,732	0
Stochastic	15	7	0.5	8,206,464,330	0
Stochastic	15	7	0.5	9,057,066,734	0
Stochastic	15	7	1	865,367,342	1,558,707,555
Stochastic	15	7	1	1,016,043,243	6,671,696,767
Stochastic	15	7	1	63,999,223	1,263,726,105
Stochastic	15	7	1	914,194,715	4,858,549,017

Stochastic	15	7	1	965,727,717	6,214,830,233
Stochastic	15	7	1	866,132,213	1,785,614,540
Stochastic	15	7	1	1,577,496,506	2,401,375,692
Stochastic	15	7	1	838,190,939	1,413,805,811
Stochastic	15	7	1	1,541,173,225	3,298,816,636
Stochastic	15	7	1	1,447,506,869	1,390,255,850
Stochastic	30	1	0.5	10,744,208,480	1,697
Stochastic	30	1	0.5	10,779,330,718	0
Stochastic	30	1	0.5	4,545,866,701	0
Stochastic	30	1	0.5	12,734,678,742	0
Stochastic	30	1	0.5	10,237,351,166	0
Stochastic	30	1	0.5	11,025,101,930	0
Stochastic	30	1	0.5	7,170,325,759	0
Stochastic	30	1	0.5	9,801,134,760	0
Stochastic	30	1	0.5	4,981,129,837	0
Stochastic	30	1	0.5	8,698,882,594	0
Stochastic	30	1	1	848,547,696	925,960,343
Stochastic	30	1	1	397,070,807	4,490,096,481
Stochastic	30	1	1	401,573,132	1,962,204,421
Stochastic	30	1	1	371,553,277	2,383,663,470
Stochastic	30	1	1	161,790,579	2,274,560,227
Stochastic	30	1	1	880,586,417	4,033,341,655
Stochastic	30	1	1	1,167,273,515	2,750,493,002
Stochastic	30	1	1	1,423,961,795	1,844,265,147
Stochastic	30	1	1	1,538,504,074	3,557,928,311
Stochastic	30	1	1	129,777,643	2,467,551,856
Stochastic	30	7	0.5	10,388,899,270	23
Stochastic	30	7	0.5	7,924,531,320	0
Stochastic	30	7	0.5	9,119,156,439	534
Stochastic	30	7	0.5	6,884,569,394	18,214
Stochastic	30	7	0.5	11,915,206,086	7,635
Stochastic	30	7	0.5	4,589,545,618	384,925
Stochastic	30	7	0.5	8,374,022,963	183,970
Stochastic	30	7	0.5	7,070,622,925	21,432
Stochastic	30	7	0.5	7,287,276,394	0
Stochastic	30	7	0.5	9,921,125,772	0
Stochastic	30	7	1	879,327,587	2,189,261,937
Stochastic	30	7	1	393,525,184	4,095,141,745
Stochastic	30	7	1	1,043,962,261	1,412,211,477
Stochastic	30	7	1	1,083,698,326	2,550,247,046
Stochastic	30	7	1	866,083,204	4,018,164,999

Stochastic	30	7	1	223,434,699	2,179,331,431
Stochastic	30	7	1	219,668,982	2,112,182,950
Stochastic	30	7	1	935,529,852	3,651,230,281
Stochastic	30	7	1	1,368,235,697	1,511,903,088
Stochastic	30	7	1	1,371,685,395	966,707,781