Reliability of Center-Notched and Edge-Notched Composite Laminates based on Experimental Investigation and Stochastic Finite Element Analysis

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ABSTRACT

Reliability of Center-Notched and Edge-Notched Composite Laminates based on Experimental Investigation and Stochastic Finite Element Analysis

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The reliability of notched composite laminates is determined to find its worthiness for the intended application and it is affected by various factors, such as, application of fatigue loading, notch location, notch shape and size, etc. In the present work, an attempt has been made to study the effects of fatigue loading and notch location on the reliability of notched composite laminates. The reliability values of notched laminates are calculated by using the following parameters: the stress developed in the laminate due to external loading and the strength of the corresponding un-notched laminates. Since composite laminates exhibit stochastic variations in material properties, Stochastic Finite Element Analysis (SFEA) is used to calculate the stresses developed in the laminate. The ultimate strengths of the un-notched laminates are found out experimentally. Tests are conducted on specimens made of NCT301 graphite/epoxy material to determine the material properties that are required for the SFEA. The material properties are modeled using Markov model based on the test data by two dimensional stochastic processes. Tests are also conducted on center-notched laminates subjected to fatigue loading conditions, Edge- notched laminates and un-notched laminates to determine their ultimate strength values. In practical applications, it is difficult to achieve a perfect circular profile during the drilling operation of notched composite laminates and there is a possibility that the drilled hole is offset from the desired location. These imperfections affect the reliability of the notched laminate. In the present work the perturbation in the circular profile of the hole is modeled using a hypotrochoid variation and the location of the hole center is modeled using a Gaussian random variable. The ultimate strength values of center-notched laminates subjected to fatigue loading conditions, Edge-notched laminates and un-notched laminates, obtained from experiments are used to determine the sets of characteristic length values for both the point stress criterion and average stress criterion. The distributions of the strength and characteristic length of laminates are determined. Stochastic simulations are performed on the laminates by subjecting them to tensile load. Probabilistic moments of the point stress and average stress parameters are found out for controlled hole and un-controlled hole laminates. The reliability values are calculated for the center-notched laminates subjected to fatigue loading conditions and Edge-notched laminates using point stress criterion and average stress criterion by combining the SFEA and experimental results.

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Nomenclature

ξ,η	Local co-ordinates for an element
N_i	Shape functions
$\{\varepsilon\}$	Strain vector
$\{\sigma\}$	Stress vector
[E]	Elasticity matrix
{d}	Displacement vector
$[K]^e$	Element stiffness matrix
$[B]^e$	Strain-nodal displacement matrix
[J]	Jacobian matrix
ν	Poisson's ratio
(ξ_p, η_q)	Sampling position
W_p, W_q	Weight factors
q	Uniformly distributed load
x,y,z	Global co-ordinates
R	Radius of the hole
d	Diameter of the hole
h	Thickness of the element; Thickness of the plate
t	Ply thickness
[A]	Extensional stiffness matrix
[B]	Axial-bending coupling stiffness matrix
[D]	Bending stiffness matrix

$[a]$	Extensional compliance matrix
$[\kappa]$	Curvature matrix
$E_x, E_y, G_{xy}, \nu_{xy}, m_{xy}$	Effective laminate normal moduli, Shear modulus, Poisson's ratio and tension-shear coupling coefficient in the x and y co-ordinate system
R_{aa}	Auto-correlation function
$[\zeta_x \ \ \zeta_y]^T$	Separation vector
a(X)	Fluctuating component of E_1
[L]	Lower triangular matrix
$[C_{aa}]$	Covariance matrix
c	Correlation length
θ	Ply orientation angle
δ	Small perturbation parameter
σ_o	Applied stress at infinity; Un-notched strength of the laminate
σ_N	Ultimate strength of the notched laminate
σ_y	Y component of normal stress for a finite width plate
μ_1, μ_2	Roots of the characteristic equation
K_T, K_T^{∞}	Stress concentration factors for finite and infinite plates respectively
σ_y^{∞}	Y component of normal stress for an infinite width plate
W	Width of the plate
M	Magnification factor

L	Gage length of the laminate
$E_1, E_2, \nu_{12}, G_{12}$	Young's modulus values of the NCT301 graphite/epoxy material in the fiber and transverse directions, major Poisson's ratio and shear modulus respectively
S_{max}, S_{min}	Maximum and Minimum stress values of the fatigue loading cycle
f	Frequency
N	Number of fatigue loading cycles
E_y	Young's modulus in the y-direction of the laminate
a_o , d_o	Characteristic lengths
D_a	hole diameter plus an allowance
s	Standard deviation
σ_p, σ_{avg}	Point and average stress
S	Entropy
$\lambda_0, \lambda_1, \lambda_2 \ldots \lambda_m$	Lagrangian multipliers
Z_R	Standardized variable
P_f	Probability of failure
R_s	Reliability of a system
R_p	Reliability value obtained using point stress criterion
R_{avg}	Reliability value obtained using average stress criterion

CHAPTER 1

INTRODUCTION

1.1 Composite materials and laminates

Conventional metallic materials are replaced by composite materials in many applications due to their less weight to strength ratio. Composite materials find wide applications in aerospace, automotive and construction industries. Especially the use of composite laminates, made up of Carbon or Graphite Fiber Reinforced Plastics (CFRP/GFRP), in military and commercial aircraft structures has progressed steadily over the past few decades. Composite laminates are used in both primary and secondary structures of aircraft. Primary structures include the wings, fuselage, horizontal and vertical tails, floor structure, etc. Secondary structure applications include landing gear doors, flight controls, stabilizers, etc.

Drilling holes and making cutouts in composite laminates are unavoidable for practical reasons. These holes (or) cutouts introduce stress concentrations near the hole (or) cutout edge and reduce the load-bearing capacity of the structure. Composite laminates used in aerospace applications are subjected to considerable fatigue loading due to turbulence and other service conditions. Automotive structures also are subjected to fatigue loadings. The stiffness and strength of the composite laminates are reduced due to the application of fatigue loading. Thus for an effective use of composite laminates in structural applications, the combined effect of notch and fatigue loading on the composite laminates has to be studied.

Apart from reducing the weight to stiffness ratio of the structure, composite laminates improves the fatigue properties, damage tolerance and acoustic performance of the structure. Improvement in aircraft performance, which is one of the important design criteria of aircraft structures, can be achieved by taking weight reduction measures coupled with strength, stiffness and stability improvement of the structure. The weight reduction in the aircraft structures can be put into effective use as increased passenger and cargo load, thereby increasing the revenue generation. Use of high percentage of composite materials in aircrafts reduces the direct operating cost of the aircraft industry.

Significant research work was done by researchers and investigators to find a material with less weight and sufficient strength and stiffness properties, which can withstand the aerodynamic loads experienced by the aircraft structures under different flight conditions. Composite materials and laminates are found to have very promising properties in this regard. Composites materials and laminates are also widely used in automotive, power generation and construction industries as advanced engineering materials for various structural applications. The increasing use of composite materials and laminates in the design of structural parts with high mechanical performance requires a better understanding and modeling of the behavior of these structures. The safety and reliability of these composite structures depend on the design of the constituent components.

The design of composite laminates involves optimization of the following four parameters [1]:

1. Ply or lamina properties

2. Ply orientation

3. Ply thickness

4. Lay-up sequence of the laminate

The optimization of composite laminate, considering the coupling effects of all the design parameters mentioned above is a mathematical challenge in structural optimization. To determine the stress distribution in composite laminates, simulations has to be carried out accompanied by experimental verification before using various laminate theories.

Composite laminates are sensitive to holes (or) cutouts, defects, fatigue loading and low-velocity impact damage that can significantly reduce their stiffness and strength properties. The stress distribution in notched composite laminates can vary according to the location of the notch in the laminate. For practical reasons, cutouts are made at the edges of composite laminates, which is capable of reducing the delamination effect in the laminates. In the present thesis, an attempt has been made to study the effect of notch location on the stress distribution of the laminate. The term, end-notched laminate, was widely used in the literature referring to composite laminates with cutouts on the edges. But in the present thesis work, the term edge-notched laminate was used.

Establishment of a strength criterion for composite materials and laminates is important to utilize these advanced materials to full potential. Considerable efforts have been taken to develop strength/failure criteria for anisotropic materials [1]. Some of the anisotropic strength criteria are an extension of isotropic yield criteria.

1.2 Stress concentration in composite laminates

Holes (or) cutouts in a structure cause stress concentration due to geometry discontinuity. Stress concentration is an important issue in both homogeneous isotropic and heterogeneous composite materials, as the point of maximum stress concentration is normally the location of initial failure. Composite laminates are more sensitive to stress concentrations due to its brittle behavior. It is necessary to analyze stress concentrations in the structure, to predict failure and to develop methods to reduce the effects of stress concentration. Some of the factors that create stress concentration in composite laminates are [2]:

1. Holes (or) cutouts
2. Material and geometric discontinuities
3. Joints in the structure.
4. Damage (or) voids formed during material fabrication

For composite laminates, there are no criteria, which is capable of predicting failure under a broad range of general stress states. Solutions to the stress concentration problems in composite laminates are still in the early stages of development. Finite element method is found to be a effective numerical tools for the analysis of composite laminates.

Composite laminates are generally used for structural application to withstand considerable mechanical loads. The design of these composite laminates requires a thorough knowledge of the mechanical behavior of material used to manufacture the laminate, which is obtained by testing laminate coupons. These laminate coupons are prone to damage during cutting operations (cutting laminate coupons from laminated composite plates, drilling operation, etc.) and it tends to reduce the strength of the laminate coupons. The standards tend to contain limited information in terms of cutting samples.

1.3 Randomness in the strength and stress parameters of composite laminates

Composite laminates exhibit significant randomness in their strength and stress parameters, which is attributed during the design and manufacturing stage of the laminate. In notched composite laminates, the uncertainly in the geometric profile of the notch also contributes to the randomness in the strength and stress parameters. During the design stage of the laminate, the test data obtained to calculate the strength of the notched composite laminate, engineering constants and material properties shows significant randomness. During the manufacturing stage, improper machining of the laminated composite plate leads to the variation in the geometric dimensions of the laminate coupons used for testing. In notched composite laminates, drilling operation causes damage near the hole region of the laminate and the degree of damage may vary from laminate to laminate, which leads to the randomness in the strength and stress parameters of the notched laminates.

Testing a single laminate coupon to determine the material parameters, such as, elastic constants, strength value, etc. may lead to a definite value pertaining to that laminate. But when a number of laminate coupons are tested, the material parameters fluctuate from laminate to laminate. The variations in the fiber size, fiber volume fraction, fiber orientation , matrix properties and thickness of the lamina results in the random variation of the engineering constant and strength values of the composite laminates.

1.4 Damage in composite laminates due to fatigue loading

Application of fatigue loading on composite laminates reduces the strength and stiffness of the laminate. In notched composite laminates, fatigue loading causes damage near the hole region of the laminate, which results in the pre-mature failure of the laminate. The three typical stages of fatigue damage in composite laminates under various cyclic loadings are described by Reifsnider [3] and Jie [4] and they are as follows:

(i) Initiation phase,

(ii) Stiffness and strength steady-degradation phase and

(iii) The phase of damage that leads to fracture and ultimate failure.

In the initiation phase, matrix cracking and fiber breaking are common from of damage in composite laminates. The initiation phase of the fatigue damage triggers crack coupling, interfacial de-bonding, delamination and its growth, which leads to the stiffness and strength steady-degradation phase of the fatigue damage. This cycle occupies most of the life time of service of the composite laminate. During the final phase of fatigue damage, rapid growth of delamination, concentrative fiber breaking, local fracture, etc. occurs, which leads to the fracture and ultimate failure of the composite laminate.

1.5 Literature Review

Holes (or) cutouts are drilled in composite laminates for many practical applications, such as, passage for hydraulic lines, avionic harnesses, access door in aircraft fuselage, etc. These holes (or) cutouts reduce the ultimate strength of the composite laminate by almost 50% [5]. Thus it is essential to analyze the stress concentration in notched laminates for the safe and effective use of laminate in structural applications. Greszczuk [6] derived the first analytical solution for composite laminates with circular hole. Greszczuk determined the failure strength and the point of failure based on Hencky-Von Mises theory using the equations given by Fischer [7]. Lekhnitskii [8] determined the stresses around holes of different shapes using series method. Jong [9] adopted a variant of Lekhnitskii [8] series method and the stress functions were determined using Cauchy intergrals. Daoust and Hoa [10] analyzed anisotropic plate with triangular hole considering different ratio of base length and height of the triangle. Ukandgaonker and Rao [11] extended the work of Daoust and Hoa [10] for multi-layered plates considering several in-plane loading conditions. In notched composite laminates, the presence of multiple layers and the anisotropy nature of material system make it difficult to optimize the hole (or) cutout geometry. Research work on circular and elliptical cutouts was also conducted by Shastri and Rao [12] and Lin and Ueng [13].

The ultimate strength of notched composite laminates is also affected by the application of fatigue loading and changing the notch location in the laminate. Reifsnider [3] has described the three typical stages of fatigue damage in composite laminates. Talreja [14] also studied the damage modes of composite laminates during fatigue loading and the results correlated with the works of Reifsnider [3]. The ultimate failure of composite laminate is caused by the large-scale fracture of fibers at different locations of delaminated layers. In notched composite laminates the large-scale fracture of fibers occur near the hole region, which leads to the ultimate failure of the laminate. The onset and propagation of delamination in composite laminates is considered to be the critical damage mode during fatigue loading condition.

The effect of fatigue loading on the strength of notched composite laminates has been analyzed by many researchers and investigators. Hwu and Hwang [15] analyzed the fatigue behavior of circular notched graphite/epoxy laminates and predicted the fatigue life of the laminate using various prediction equations. Point and Average Stress criteria proposed by Whitney and Nuismer [16] [17] were generalized to fatigue fracture criteria for notched laminates. Ramani and Williams [18] studied the notched and un-notched fatigue behavior of graphite/epoxy composites. The laminates were tested under both tension-tension and compression-compression loading conditions. Xiao and Bathias [19] analyzed the notched non-woven and woven composite laminates using point stress and average stress criteria. In the recent years, stochastic approach has been investigated for predicting the possibilities of fatigue damage or distribution of fatigue response parameter during fatigue testing. Ganesan [20] employed the data-driven stochastic approach to analyze the fatigue damage and response of composite laminates. Further developments in the fatigue analysis of notched composite laminates can be found in literature [21-27]. Hallett and Wisnow [28] developed a new approach to the finite element modeling of edge-notched laminates with triangular notch to analyze the progressive damage of the laminate.

Composite materials and laminates show variations in their material and strength properties. Generally, composite structures are analyzed under the assumption that the structure's material and strength parameters are deterministic quantities, not taking into account the random nature of external loading, boundary conditions, material and strength parameters, etc. in the composite structures. In many practical applications, this assumption is invalid and the probabilistic aspects of the composite structure have to be taken into account. Finite Element Analysis (FEA) of structures has developed in the past few decades to an extent that it has the ability to incorporate the random external loading conditions, boundary conditions, and material and strength parameters of the structures in the analysis. The FEA which incorporates these random effects is termed as Stochastic Finite Element Analysis (SFEA).

Rosen [29] and Zweben [30] have developed probabilistic theories for the tensile strength of unidirectional composites. But these probabilistic theories gives satisfactory results only when the composite structure failure is predominantly affected by the stochastic distribution of reinforcement fibers but are not suitable when there are other competing failure mechanisms. Further developments can be found in the works of Batdorf [31] and Yushanov and Jhosi [32].

In most works found in the literature related to the Stochastic Finite Element Analysis (SFEA) of structures, the stochastic variations of material and strength parameters of the structure were considered in the analysis and very rarely the stochastic variations of geometric properties were taken into account. Falsone and Impollonia [33] and Stefanou and Papadrakakis [34] performed the SFEA of structures based on the assumption that only one material or geometric properties exhibit stochastic variations and it was described by a stochastic field. But in practical situations, the physical mechanisms which leads to the random variations of one material or geometric parameter also creates random variations for other material or geometric variations. Ganesan [35] developed the finite element formulation to perform the vibration analysis of singular non-self-adjoint beam-column with two parameters (Young's modulus and mass density) behaving in a stochastic manner using virtual work method.

Shashank [2] performed the Stochastic Finite Element Analysis of notched composites laminates, considering the stochastic variations of material and geometric properties. $[0/90]_{4s}$ and $[0_2/\pm45]_{2s}$ laminates with hole diameter 5.1mm were considered for the analysis. Ibrahim [5]extended the previous work by performing the SFEA of $[0/90]_{4s}$ laminates with three different hole sizes, to study the effect of hole size on the stress distribution and reliability of the laminate. Ganesan and Hoa [36] performed the stress analysis of composite structures with stochastic parameters. Ganesan and Haque [37] developed the stochastic finite element methodology to analyze the probabilistic fracture behavior of composite laminates. Ganesan and Podugala [51] developed an effective finite element analysis methodology to evaluate the stochastic J-integral of laminated composites. Contreras [38], Vanmarcke and Grigoriu [39] and Yamazaki, Shinozuka and Dasgupta [40] reviewed the developments in the SFEA of composite structures.

1.6 Failure Criteria for Composite Laminates

In most failure criteria, the strength parameters of a structure were used to predict the failure of the structure. Once the stress distribution in the structure is determined, the point of interest is analyzed using the failure criterion to determine whether the structure will fail or not. In stress analysis, the strength of the structure is predicted by using the boundary conditions, displacements or both to solve for the unknowns. But there is no specific boundary condition that can be used for the failure criteria.

Considerable research work was done by researchers to formulate the failure criteria for composite laminates and to correlate with experimental data, but no criterion has been fully adequate. The analyses of Hoffman [41] and Fischer [42] are valid only for special cases and it limits their direct application to general materials. The ultimate strength of notched composite laminate depends on various factors, such as, laminate configuration, notch size, application of fatigue loading, notch location, etc. A laminate with different combination of these factors may lead to a different failure mechanism. Thus a failure criterion which can accept any kind of laminate configuration, notch size and location, and application of fatigue loading is essential and various failure criteria has been proposed by researchers in this regard.

In the failure criterion proposed by Waddoups, Eisenmann and Kaminski [43], linear elastic fracture mechanics was used to predict the failure of composite structure. The un-notched strength and length of intense energy region (characteristic length) of the composite structure were used in the failure criterion. These parameters were determined using experimental data.

Whitney and Nuismer [16] [17] developed a failure criterion for notched composite laminates. The failure criterion hypothesized that the strength of the notched laminate can be evaluated at a characteristic distance based on the point/average stress across the region from the notch edge. Based on the type of stress used in the failure criterion, it is classified into point stress criterion and average stress criterion. In point stress criterion, the failure is assumed to occur when the normal stress at a characteristic distance from the notch edge is equal to the un-notched strength of the laminate. In average stress criterion, it is assumed that failure occurs when the average of the normal stress over some characteristic distance from notch edge is equal to the un-notched strength of the laminate. The failure criteria suggested by Awerbuch and Madhukar [44] and El-zein and Reifsnider model [45] are extensions of the Whitney and Nuismer failure criteria.

Strictly speaking, the failure criteria proposed by Waddoups, Eisenmann and Kaminski [43] and Whitney and Nuismer [16] [17] are valid only for unidirectional composite laminates. These failure criteria have been widely used to predict the strength of the notched composite laminate with good results. But they should be considered as models rather than failure criteria, since they do not take into account the details of the complex failure mechanisms.

1.4 Scope and objective of the thesis

Composite laminate shows significant variation in material properties and geometric variations, such as, variations in hole shape and eccentricity of the hole from the laminate center. Thus the stress distribution in the laminates becomes stochastic in nature. It is more appropriate to analyze the notched laminates using a stochastic approach and to design the laminate based on a reliability-based design approach. The main objective of the present thesis work is to study the combined effect of stress concentration and fatigue on the reliability of composite laminates. Changing the notch location in composite laminates causes variation in the stress distribution. Thus an attempt is made in the present thesis to study the effect of changing the notch location in composite laminates on their stress distribution and reliability.

In the present thesis work,

(1) The stress distributions in notched laminates are determined using Stochastic Finite Element Analysis (SFEA), which incorporates the stochastic variations in material properties and the geometric variations in the laminate;

(2) A MATLAB® program is developed using the SFEA methodology, which will be capable of analyzing notch laminates with any laminate configuration and geometric dimensions;

(3) An extensive experimental investigation is conducted on center-notched and edge-notched laminates to find their ultimate strength and to determine the characteristic lengths of the laminate using the ultimate strength of the notched laminates and the ultimate strength of the corresponding un-notched laminates;

(4) The point/average stress parameters of notched laminate are calculated using Controlled Hole Laminate (CHL) and Un-Controlled Hole Laminate (UCHL) analyses. In CHL analysis; only the stochastic variation in the material properties was considered for the analysis; In UCHL analysis, both stochastic variation in material properties and geometric variations in the laminate were considered for the analysis;

(5) The reliability values of center-notched laminates subjected to fatigue loading conditions and edge-notched laminates are calculated using Gaussian distribution method; This requires the point/average stress parameters of notched laminates and the strength parameters of the corresponding un-notched laminates.

1.5 Organization of the thesis

Chapter 1 provides a brief introduction to composite materials and laminates and their practical applications. Literature survey is conducted to discuss the previous works done to analyze the effect of notch and fatigue loading on the strength, stiffness and reliability of composite laminates. Failure criteria of notched composite laminates proposed by number of researchers were discussed. The scope and objective of the thesis were also discussed in this chapter.

Chapter 2 describes the formulation for the Finite Element Analysis (FEA) of notched isotropic plate and notched composite laminates. An eight-node 2-D isoparametric element was used to model the notched laminates. The formulation for the Stochastic Finite Element Analysis (SFEA) of notched laminates, which considers the stochastic variations in material properties, is also discussed. MATLAB® programs were written to perform the FEA of notched isotropic plate and SFEA of notched composite laminates. Example problems are considered to validate the MATLAB® programs.

Chapter 3 explains the manufacturing and testing methods of center-notched and edge-notched laminates. Center-notched laminates were subjected to three different fatigue loading conditions and subsequently tested under static loading condition to find the reduction in the ultimate strength of laminate due to the application of fatigue loading. Three samples were tested for each fatigue loading condition to determine the average reduction in the ultimate strength of center-notched laminates. Fifteen samples of edge-notched laminates were tested under static loading condition to determine the average ultimate strength of edge-notched laminates. A comparison between the ultimate strengths of edge-notched laminate and center-notched laminate not subjected to fatigue loading was made to analyze the effect of changing the notch location on the ultimate strength of notched laminate. Point stress and average stress criteria, which calculate the characteristic lengths of notched laminates using the ultimate strengths of notched laminates and the ultimate strengths of the corresponding un-notched laminates, were discussed in detail. Micro structural analysis of center-notched and edge-notched laminates was conducted to analyze the damages in the laminates.

Chapter 4 discusses the stochastic simulations performed on center-notched and edge-notched laminates using Controlled Hole Laminate (CHL) and Un-Controlled Hole Laminate (UCHL) analysis. The number of simulations required to find the Mean and Standard Deviation (S.D.) values of point/average stress parameters of the notched laminates using CHL and UCHL analyses were found out. The point/average stress parameters of the laminates are required to calculate the reliability value of the laminates.

Chapter 5 explains the methodology to calculate the reliability value of center-notched and edge-notched laminates. Gaussian distribution method was used to calculate the reliability values of the laminates, which requires the point/average stress parameters of the notched laminates and the strength parameters of the corresponding un-notched laminates. Reliability values of notched laminates for different Factor of Safety (FOS) values on the applied load are calculated. Reliability graphs of the notched laminates, which show the variation in the reliability values with FOS values, are drawn. Finally, a comparison between the reliability values of center-notched and edge-notched laminates are made to analyze the effect of fatigue loading and notch location on the reliability values of notched laminates.

Chapter 6 provides the discussions and conclusions of the present thesis work with some recommendations for future work.

CHAPTER 2

STOCHASTIC FINITE ELEMENT ANALYSIS

2.1 Introduction

Composite laminates are being widely used in aerospace and automotive industries for structural applications. Holes or cutouts are being drilled in composite laminates for practical reasons, which create stress concentration near the hole or cutout and reduce the load-bearing capacity of the structure. The stress distribution of the notched laminate has to be determined to accurately predict the failure load. In the present chapter, the method and formulation used to analyze the stress distribution in notched laminates are discussed in detail.

The stress concentration effects in composite plates are complicated to analyze due to their directional anisotropy. Since closed form solutions exist only for very few cases, Finite Element Method (FEM) is used to find the stress distribution around the hole region of composite laminates. The minimum number of elements required in the finite element mesh to achieve a result close enough to the exact solution is studied. A MATLAB® code is developed using the finite element methodology and it has the capability to analyze any kind of laminate configuration and loading conditions. Both center-notched and edge-notched laminates were considered for the analysis to study the effect of hole location on the stress distribution in the notched laminate. The finite element formulation for isotropic plate is reviewed in Section 2.2.

2.2 Finite Element Formulation for Isotropic Plates

An eight-node two dimensional isoparametric element was used to analyze the stress concentration effect in the isotropic plates subjected to in-plane loadings. A rectangular element of this type is known as serendipity element and is shown in Figure 2.1.

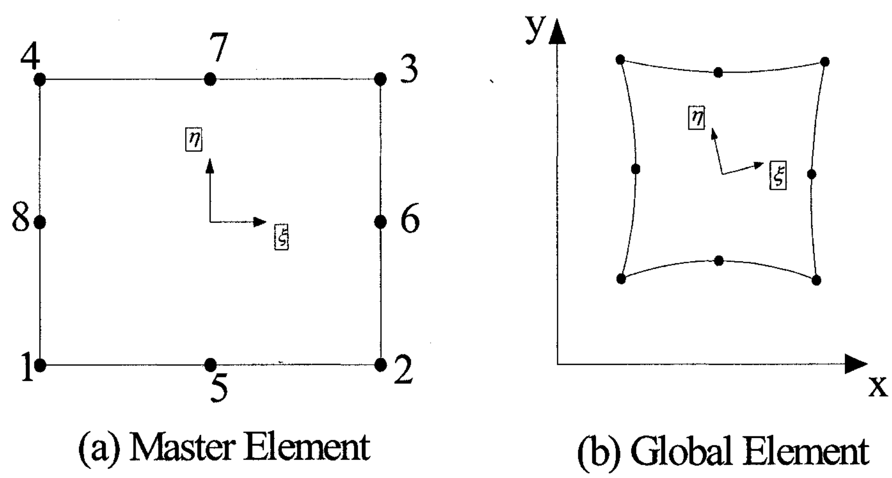


Figure 2.1 An eight-node 2-D isoparametric element (serendipity element) in both local and global coordinates

The interpolation or shape functions of the serendipity element [46] are given by

$$N_i = \left\{ \frac{1}{4}(1+\xi\xi_i)(1+\eta\eta_i)(\xi\xi_i+\eta\eta_i-1) \right. \qquad ;\ i = 1,2,3,4 \tag{2.1}$$

$$N_i = \left\{ \frac{\xi_i^2}{2}(1+\xi\xi_i)(1-\eta^2)+\frac{\eta_i^2}{2}(1+\eta\eta_i)(1-\xi^2) \right. \qquad ;\ i = 5,6,7,8 \tag{2.2}$$

where, ξ and η are the local co-ordinates of the element.

Since the loading of isotropic plate is in-plane and thickness of the plate is negligible compared to the in-plane dimensions, a 2-D plane stress case was considered for the analysis. In plane stress case, a two dimensional stress state exists in the x-y plane and the stresses σ_{zz}, τ_{xz} and τ_{yz} are equal to zero.

Considering only x and y directions, the matrices of displacements and strains can be written as

$$\{u\} = \begin{Bmatrix} u \\ v \end{Bmatrix} \tag{2.3}$$

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{2.4}$$

where,

$$\varepsilon_x = \frac{\partial u}{\partial x}; \quad \varepsilon_y = \frac{\partial v}{\partial y}; \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \tag{2.5}$$

The stress-strain relation equation is given by

$$\{\sigma\} = [E]\{\varepsilon\} \tag{2.6}$$

where,

$$\{\sigma\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \tag{2.7}$$

In linear elastic plane stress conditions, the elasticity matrix is given by

$$[E]=\frac{E}{1-\nu^2}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & \frac{(1-\nu)}{2}\end{bmatrix} \tag{2.8}$$

where, E and ν are the Young's modulus and the Poisson ratio respectively.

The element displacement matrix can be written as [47]

$$\{u\}^{(e)}=[N]^{(e)}\{d\}^{e} \tag{2.9}$$

where shape function matrix is,

$$[N]^{(e)}=\begin{bmatrix}N_1 & 0 & N_2 & 0 & \dots & \dots & N_8 & 0\\ 0 & N_1 & 0 & N_2 & \dots & \dots & 0 & N_8\end{bmatrix} \tag{2.10}$$

and displacement vector is,

$$\{d\}^{e}=\{u_1 \quad v_1 \quad u_2 \quad v_2 \quad \dots \quad \dots \quad u_8 \quad v_8\}^{T} \tag{2.11}$$

The element stiffness matrix can be written as

$$[K]^{(e)}=\int_{V_{(e)}}[B^{(e)}]^{T}[E][B^{(e)}]dV_{(e)} \tag{2.12}$$

where,

$$[B]^{(e)}=\begin{bmatrix}\frac{\partial}{\partial x} & 0\\ 0 & \frac{\partial}{\partial y}\\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x}\end{bmatrix}[N]^{(e)}, \tag{2.13a}$$

$$dV_{(e)} = h|J|d\xi d\eta \,, \tag{2.13b}$$

and h is the thickness of the element.

In equation 2.13b, $|J|$ is the determinant of Jacobian matrix.

The stiffness matrix coefficient linking nodes i and j in any element is given by

$$K_{ij}^{(e)} = \sum_{r=1}^{NGAUS} \sum_{s=1}^{NGAUS} \left[B_{ir}^{(e)}\right]^T \left[E_{rs}^{(e)}\right] \left[B_{sj}^{(e)}\right] h \left|J^{(e)}\right| d\xi d\eta \tag{2.14}$$

where, NGAUS is the order of Gauss quadrature used for numerical integration.

The elements of the stiffness matrix can be numerically evaluated as

$$K_{ij}^{(e)} = \sum_{r=1}^{NGAUS} \sum_{s=1}^{NGAUS} T\left(\xi_p, \eta_q\right)_{ij}^{(e)} W_p W_q \tag{2.15}$$

where,

$$T_{ij}^{(e)} = \sum_{r=1}^{NGAUS} \sum_{s=1}^{NGAUS} \left[B_{ir}^{(e)}\right]^T \left[E_{rs}^{(e)}\right] \left[B_{sj}^{(e)}\right] h \left|J^{(e)}\right| , \tag{2.16}$$

$\left(\xi_p, \eta_q\right)$ represent the sampling position, and

W_p and W_q are the weighting factors.

If q is the uniformly distributed load acting along the edge of an element (e) with length l, the nodal loads can be expressed as

$$\begin{bmatrix} Equivalent \;\; load \;\; at \;\; the \;\; left \;\; node \\ Equivalent \;\; load \;\; at \;\; the \;\; central \;\; node \\ Equivalent \;\; load \;\; at \;\; the \;\; right \;\; node \end{bmatrix} = \frac{ql}{6} \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix} \tag{2.17}$$

The flowchart for the computation of the element stiffness matrix, nodal loads and nodal displacements is given in Figure 2.2.

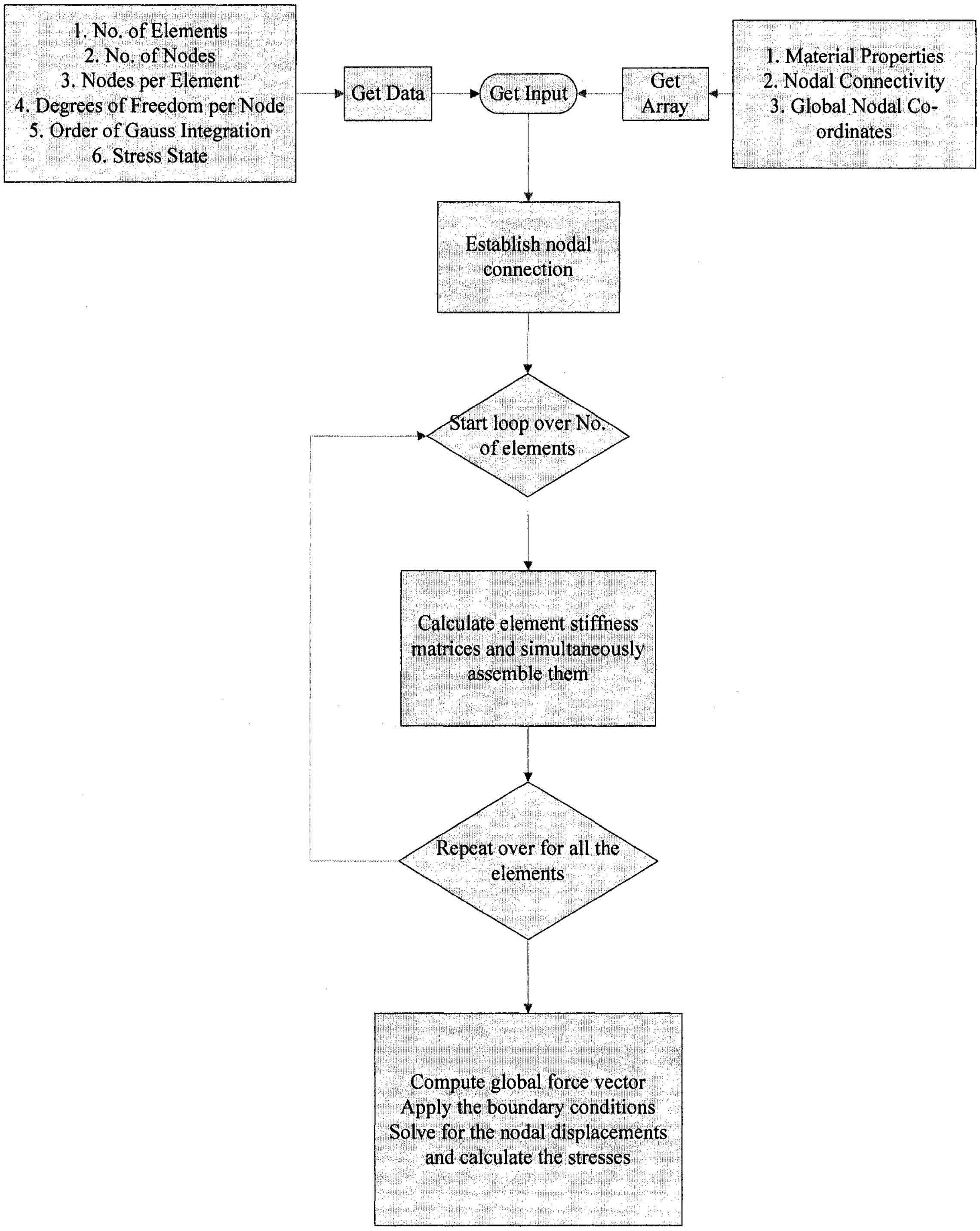


Figure 2.2 Flowchart representing the finite element analysis of isotropic plate

2.3 Finite Element Mesh

In reference [5], an attempt was made to find the convergence of the stress value at the hole edge with the increase in number of elements of the finite element mesh. Solutions obtained from MATLAB® program are compared with exact solution for an isotropic plate.

As per references [48] and [49], the exact solution for normal stress at any point at a distance 'x' from the center of the hole in an isotropic plate is given by

$$\sigma_y = \frac{\sigma}{2}\left(2 + \frac{R^2}{x^2} + 3\frac{R^4}{x^4}\right) \tag{2.18}$$

where σ is the remote applied stress,

x is the distance measured along x-axis and

R is the radius of the hole.

The maximum stress occurs at a distance 'x=R', which is at the edge of the hole boundary. Thus,

$$\sigma_y = 3\sigma \tag{2.19}$$

The finite element mesh with 200 elements along with the applied loading and boundary conditions is shown in Figure 2.3.

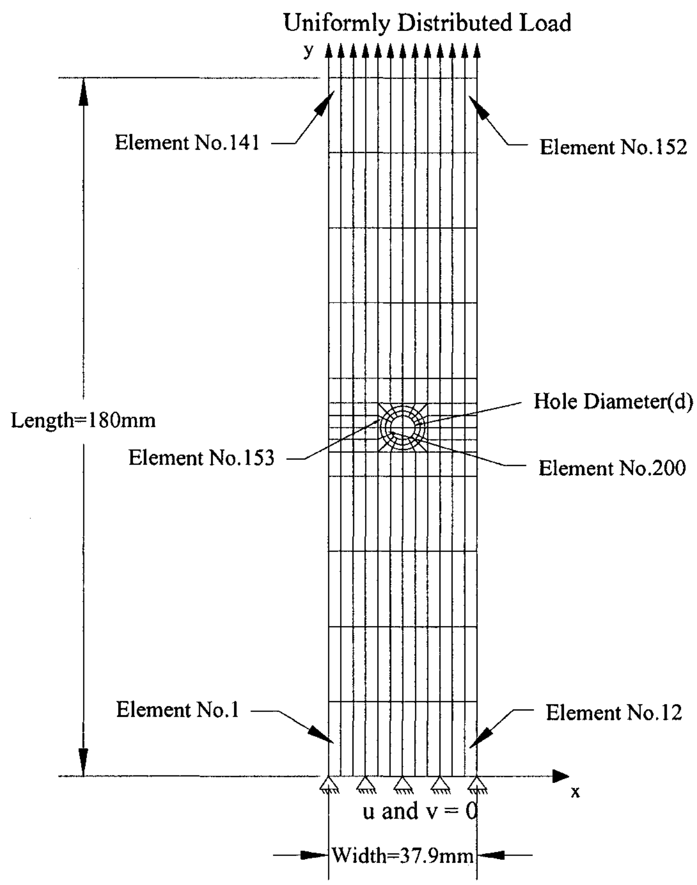


Figure 2.3 Finite element mesh with 200 elements along with the applied loading and boundary conditions

For validating the program and to see the convergence of stress value near the hole edge with increase in number of elements of the finite element mesh, an isotropic plate with hole diameter (d) = 6.35 mm, gage length (L) = 180mm, width (W) = 37.9mm and thickness (t) = 2mm was considered. Material properties assumed were as follows: Young's modulus (E) = 210 GPa and Poisson's ratio = 0.3. A uniformly distributed load of 1.5 x 10^6 N/m is applied on the top edge of the plate and appropriate boundary conditions were imposed which is shown in Figure 2.3.

Initially, 48 elements were used to mesh the isotropic plate. After which the number of elements used for meshing were increased to 80 and 200. The stress value at the hole edge, σ_y, with increase in number of elements of the finite element mesh is given in Table 2.1.

Total no. of Elements	FEA results	Exact solution	Percentage Difference
	σ_y (GPa)	σ_y (GPa)	%
48	1.991	2.250	11.51
80	2.008	2.250	10.75
200	2.132	2.250	5.24

Table 2.1 Comparison of stress value at hole edge (σ_y) in the isotropic plate, obtained by using different finite element meshes and exact solution

From Table 2.1, it can be seen that the stress value at hole edge, σ_y ,converge with exact solution with increase in the number of elements of the finite element mesh. Difference between MATLAB® program result and exact solution reduces to 5.24% from 11.51%, when the number of elements increases from 48 to 200. Stochastic analysis involves large scale computational work. Considering all these, the entire finite element analysis in the present work will be done by using 200 elements with total nodes of 668.

2.4 Finite Element Formulation for Composite Laminate

The constitutive equation of a laminated plate according to first order theory is given by [50]

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ & A_{22} & A_{26} & & B_{22} & B_{26} \\ symm & & A_{66} & symm & & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ & B_{22} & B_{26} & & D_{22} & D_{26} \\ symm & & B_{66} & symm & & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{2.20}$$

In a concise form it can be written as

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon \\ \kappa \end{Bmatrix} \tag{2.21}$$

For symmetric laminate, axial-bending coupling stiffness matrix [B] is a null matrix and for axial load there is no coupling between tension and bending. Thus for a symmetric composite plate loaded in tension, constitutive equation reduces to

$$\{N\} = [A]\{\varepsilon\} \tag{2.22}$$

Co-efficient of axial stiffness matrix [A] can be written as

$$A_{ij} = \sum_{k=1}^{n} Q_{ij}^{k} \left(h_k - h_{k-1}\right) \tag{2.23}$$

with i,j = 1,2,6 and

Q_{ij} denotes transformed reduced stiffness matrix coefficient.

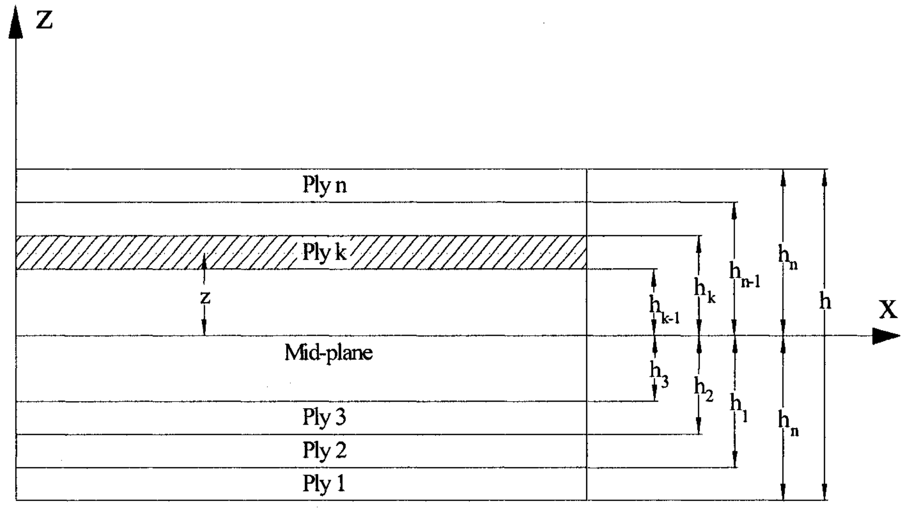


Figure 2.4 Orientation of layers in the composite laminate with respect to mid plane

In multilayer laminates, the total forces are obtained by summing the contributions from all layers. Thus for a laminate with n layers, as shown in Figure 2.4, the forces can be written as

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k dz \tag{2.24}$$

Equation (2.22) can be rewritten as follows

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} \tag{2.25}$$

Inversion of equation (2.25) gives

$$\begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{12} & a_{22} & a_{26} \\ a_{16} & a_{26} & a_{66} \end{bmatrix} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} \tag{2.26}$$

where, [a] is the extensional laminate compliance matrix, which is the inverse of the corresponding stiffness matrix, [A], as given below

$$[a] = [A]^{-1} \tag{2.27}$$

The average laminate stresses can be defined as

$$\overline{\sigma_x} = \frac{N_x}{h} \; ; \; \overline{\sigma_y} = \frac{N_y}{h} \; ; \; \overline{\tau_{xy}} = \frac{N_{xy}}{h} \tag{2.28}$$

where, h is the laminate thickness.

So, equation (2.26) can be rewritten in terms of average laminate stresses as

$$\begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} = \begin{bmatrix} ha_{11} & ha_{12} & ha_{16} \\ ha_{12} & ha_{22} & ha_{26} \\ ha_{16} & ha_{26} & ha_{66} \end{bmatrix} \begin{Bmatrix} \frac{N_x}{h} = \overline{\sigma_x} \\ \frac{N_y}{h} = \overline{\sigma_y} \\ \frac{N_{xy}}{h} = \overline{\tau_{xy}} \end{Bmatrix} \tag{2.29}$$

By superposition of the three loadings σ_x , σ_y and τ_{xy} the following stress-strain relation can be obtained in terms of engineering constants.

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{yx}}{E_y} & -\frac{1}{G_{xy}m_x} \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & -\frac{1}{G_{xy}m_y} \\ -\frac{m_x}{E_x} & -\frac{m_y}{E_y} & \frac{1}{G_{xy}} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \tag{2.30}$$

where, E_x, ν_{xy} and $m_x = -\frac{\gamma_{xy}}{\varepsilon_x}$ are the x directional modulus, Poisson's ratio and shear coupling coefficient respectively.

E_y , ν_{yx} and $m_y = -\frac{\gamma_{yx}}{\varepsilon_y}$ are the y directional modulus, Poisson's ratio and shear coupling coefficient respectively.

The equivalent elasticity matrix [E] for a composite laminate can be calculated by inverting equation (2.29) as

$$\begin{Bmatrix} \overline{\sigma_x} \\ \overline{\sigma_y} \\ \overline{\tau_{xy}} \end{Bmatrix} = \begin{bmatrix} ha_{11} & ha_{12} & ha_{16} \\ ha_{12} & ha_{22} & ha_{26} \\ ha_{16} & ha_{26} & ha_{66} \end{bmatrix}^{-1} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} \tag{2.31}$$

Now comparing equation (2.31) with equation (2.6) for calculating the elasticity matrix [E], one gets

$$[E] = \begin{bmatrix} ha_{11} & ha_{12} & ha_{16} \\ ha_{12} & ha_{22} & ha_{26} \\ ha_{16} & ha_{26} & ha_{66} \end{bmatrix}^{-1} \tag{2.32}$$

The elasticity matrix thus obtained will be incorporated in the finite element formulation through equation (2.12).

2.5 Stochastic Finite Element Analysis of Composite Plate

Composite laminates exhibit significant randomness in their material properties due to variations in fiber volume fraction, void content, fiber orientation of various layers, thickness of lamina, etc. As a result, test on one laminate coupon provides a specific value for each material parameter and mechanical property. But when a number of laminate coupons are tested, randomly distributed values are obtained for the same material property. Therefore the analysis of the laminate has to be performed based on a probabilistic approach so that a stochastic description of the material property can be provided.

2.5.1 Stochastic Field Modeling of Material Properties

The material properties are modeled in terms of two dimensional (statistically) homogeneous stochastic processes that have zero mean. Material properties such as Young's modulus, Poisson's ratio and shear modulus which have been obtained from experiments are considered for stochastic process. Sample realizations are obtained at each Gauss point in the finite element mesh. Using the sample realizations of material properties at each Gauss point, the stochastic elasticity matrix, [E], is calculated for each Gauss point. Stochastic elasticity matrix thus generated is incorporated in determining the element stiffness matrix. The flow chart for computing the stochastic fields of elastic constants is given in Figure 2.5.

Variations in material properties, such as, Young's modulus, Poisson's ratio and shear modulus are brought about using a fluctuating component a(X) associated with a material property, which has zero mean. For instance, the stochastic field of the Young's modulus in the fiber direction (E_1) is described below and a similar procedure is adopted for the other material properties such as E_2, G_{12}, ν_{12} and ν_{21} that are required for stochastic finite element analysis.

$$E_1 = \overline{E_1}[1 + a(X)] \quad ; \quad E\,[a(X)] = 0 \tag{2.33}$$

The auto-correlation function can be written as [37]

$$R_{aa}(\zeta) = E[a(X)a(X+\zeta)] \tag{2.34}$$

where,

$X = [x, y]^T$ is the position vector and

$\zeta = [\zeta_x, \zeta_y]^T$ indicates the separation vector between two points X and (X + ζ).

Material property is considered to vary at each and every Gauss Point. Thus, if n represents the number of finite elements present in the structure and m represents the order of Gauss quadrature, then there are N (=m x n) material property values associated with the structure. Only the fluctuating component of the homogeneous stochastic field is considered to model the material property variations around the expected value. These N values, $a_i = a(X_i)$, (i = 1,2,3…. N) are correlated random variables with zero mean. Also X_i corresponds to the location of each Gauss point. Their correlation characteristics can be specified in terms of the covariance matrix C_{aa} of order N x N, whose ij[th] component is given by

$$C_{ij} = Cov\lfloor a_i a_j \rfloor = E\lfloor a_i a_j \rfloor = R_{aa}(\zeta_{ij}); \quad i,j = 1,2,3 \ldots\ldots, N \tag{2.35}$$

where, $\zeta_{ij} = (X_j - X_i)$ is the separation distance between the Gauss points i and j.

Now the vector $\{a\} = [a_1 \quad a_2 \quad a_3 \quad \ldots. \quad a_N]^T$ can be generated by

$$\{a\} = [L]\,\{Z\} \tag{2.36}$$

where, $\{Z\} = [Z_1 \quad Z_2 \quad Z_3 \quad \ldots. \quad Z_N]^T$ is a vector consisting of N independent Gaussian random variables with zero mean and unit standard deviation and

L is a lower triangular matrix obtained by Cholesky decomposition of the covariance matrix $[C_{aa}]$.

Thus

$$[L][L]^T = [C_{aa}] \tag{2.37}$$

Once the Cholesky decomposition is accomplished, different sample vectors of {a} are easily obtained by generating different samples for the Gaussian random vectors {Z}. The correlation properties of the stochastic fields representing the fluctuating components of the material properties are expressed using the Markov correlation model, also known as the first-order autoregressive model. The choice of this model in this work is due to its wide use in the literature [52].

2.5.2 Markov Model

The first-order autoregressive correlation model or the Markov model is given by

$$R_{aa}(\zeta) = s_o^{\ 2} \exp\left[-\left(\frac{|\zeta|}{c}\right)\right] \tag{2.38}$$

where s_o is the standard deviation of the stochastic field a(X) and

c is a positive parameter called correlation length, which is defined such that correlation disappears more slowly when c is large.

The stochastic field a(X) represents the deviatoric components of the material property with auto correlation function as given in equation (2.38). The stochastic field value for each Gauss point is represented by the value a_g of a(X) at the Gauss point X_g of the structure i.e., $a_g = a(X_g)$.

The Young's modulus along the fiber direction can now be assumed to have a distribution as given by the vector {a} and can be represented by

$$E_{1g} = E_{1m}(1+a_g) \tag{2.39}$$

where, E_{1g} is the Young's modulus value in fiber direction at a Gauss point and

E_{1m} is the mean value of Young's modulus in fiber direction.

Young's modulus in the transverse direction,

$$E_{2g} = E_{2m}(1+b_g) \tag{2.40}$$

Major Poisson's ratio,

$$\nu_{12g} = \nu_{12m}(1+c_g) \tag{2.41}$$

Shear modulus,

$$G_{12g} = G_{12m}(1+d_g) \tag{2.42}$$

where E_{2m} is the mean value of Young's modulus in the transverse direction and

ν_{12m} and G_{12m} are the mean values of the major Poisson's ratio and shear modulus respectively.

It should be noted that the standard deviations of a_g, b_g, c_g and d_g represent the coefficients of variation of the material properties E_{1g}, E_{2g}, ν_{12g} and G_{12g}. Also the variation of ply orientation angle (θ_g) and ply thickness (t_g) are evaluated in a manner similar to equations (2.39 - 2.42) as

$$\theta_g=\theta_m(1+e_g) \tag{2.43}$$

$$t_g=t_m(1+f_g) \tag{2.44}$$

where θ_m and t_m are the mean values of the ply orientation angle and ply thickness respectively. The assumption of Gaussian distribution implies the possibility of generating negative values for the material properties. In order to avoid this difficulty, the values of the random variable, a_g in the case of Monte-Carlo simulation are confined to the range

$$-1+\delta \leq a_g \leq 1-\delta \tag{2.45}$$

where, δ is a very small perturbation parameter.

2.5.3 Flowchart Representing the Stochastic Finite Element Analysis

Sample realizations at each Gauss point in the finite element mesh are found out by applying stochastic processes using equations (2.38 - 2.42) for the material properties like Young's modulus, shear modulus, Poisson's ratio, etc. A similar procedure applies to θ and t. Using the generated sample realizations of material properties at each Gauss point the stochastic elasticity matrix, [E], is calculated for each Gauss point. The stochastic elasticity matrix thus generated is incorporated into the equation (2.14) for the element stiffness matrix. The flow chart for computing the stochastic fields of the elastic constants is given in Figure 2.5.

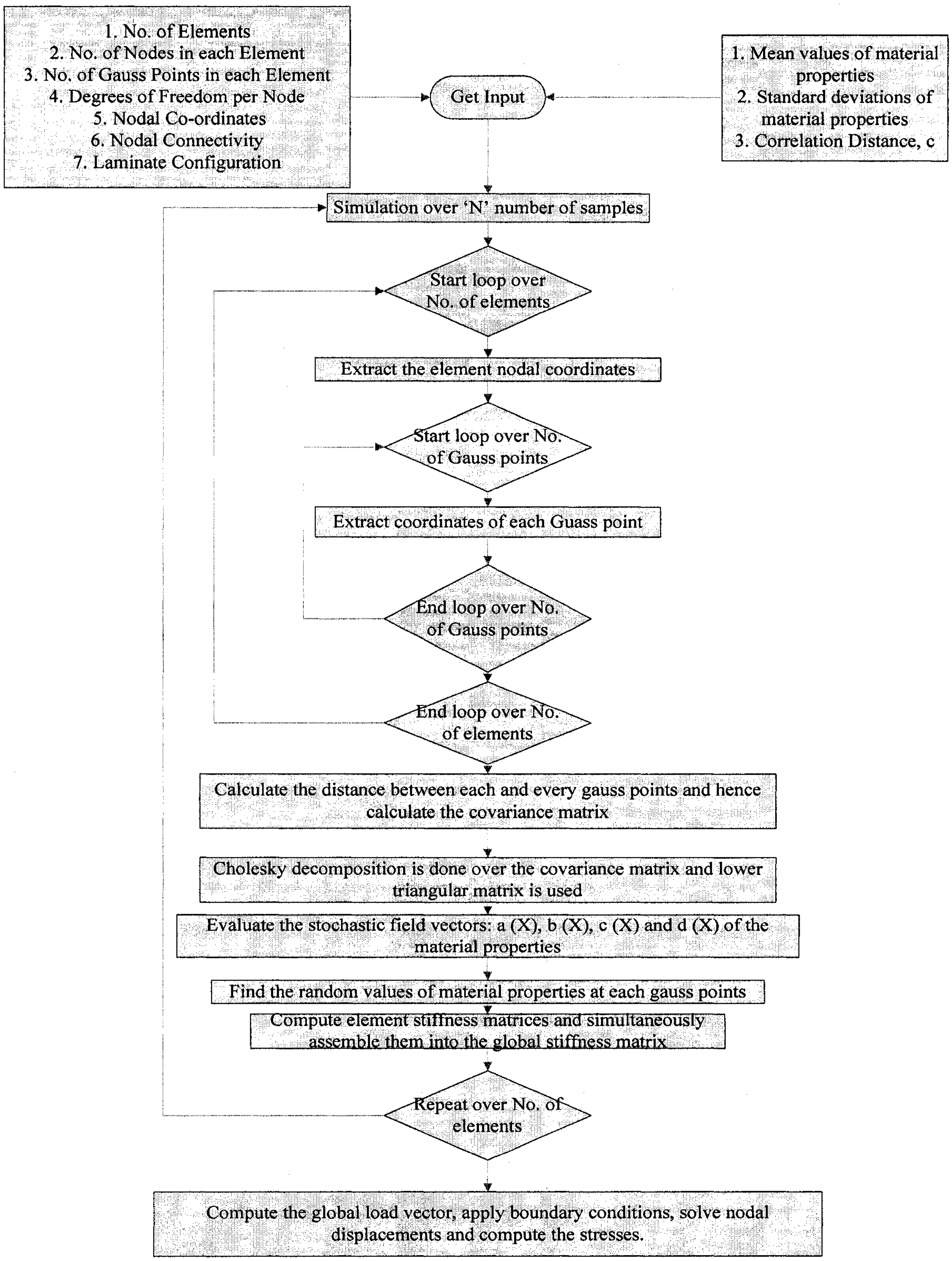


Figure 2.5 Flow chart used for the calculation of stochastic material properties, stiffness matrix, displacements and stresses for composite plate

2.6 Stress Concentration Effects in Composite Laminates

Stress Concentration Factor (SCF) can be defined as the local stress divided by the applied stress for uniform loading and can be written as

$$SCF = \frac{\sigma_y}{\sigma_o} \tag{2.46}$$

where,

σ_y is the local stress near the hole region and

σ_o is the remote applied stress

In metallic structures, the use of SCF as a design parameter is widespread and the effects of hole, plate, geometry and loading configuration on the value of this parameter have been extensively studied. But direct application of this methodology to notched composite laminates has produced some erroneous results. These errors occur due to the fact that the methodology is based on the conditions at a point on the hole boundary, while the strength of a notched composite laminate seems related to the in-plane elastic stresses within a region adjacent to the hole boundary [16]. Thus the SCF is not in itself an adequate measure of strength for a notched composite laminate; such a measure must be based on a more complete description of the stresses near the hole.

The transverse stress components $\sigma_y(x,0)$ along the x axis in an infinite orthotropic composite laminate containing a circular hole can be written as [53]

$$\sigma_y(x,0)=\sigma_o\left\{1+\operatorname{Re}\frac{1}{\mu_1-\mu_2}\left[\begin{array}{c}\dfrac{-\mu_2(1-i\mu_1)}{\sqrt{(\lambda^2-1-\mu_1^2)}\left(\lambda+\sqrt{\lambda^2-1-\mu_1^2}\right)}+\\ \dfrac{\mu_1(1-i\mu_2)}{\sqrt{(\lambda^2-1-\mu_2^2)}\left(\lambda-1-\mu_2^2\right)}\end{array}\right]\right\} \tag{2.47}$$

where,

σ_o is the applied stress at infinity,

$\lambda=\frac{x}{R}$ and R is the radius of the hole,

μ_1 and μ_2 are the roots of the characteristic equation given below

$$a_{22}\mu^4-2a_{26}\mu^3+\left(2a_{12}+a_{66}\right)\mu^2-2a_{16}\mu+a_{11}=0 \tag{2.48}$$

Coefficients a_{11}, a_{12}, a_{16}, a_{22}, a_{26} and a_{66} are all the components of extensional laminate compliance matrix obtained from equation (2.27).

Only principal roots should be chosen, i.e., two of the four roots that have a positive imaginary part.

2.6.1 Finite Width Correction Factor for Composite Laminates

The Finite-Width Correction Factor (FWCF) can be defined as a scale factor, which is multiplied with the notched infinite plate solution to obtain the solution for the notched finite plate. Based on the definition of FWCF stated above and an assumption that the normal stress profile for a finite plate is identical to that of an infinite plate except for the FWCF, the following relation is obtained:

$$\frac{K_T}{K_T^\infty}\sigma_y^\infty(x,0)=\sigma_y(x,0) \tag{2.49}$$

where, $\frac{K_T}{K_T^\infty}$ is the finite width correction factor, K_T represents the stress concentration factor at the hole edge of a finite width plate and K_T^∞ for an infinite width plate.

The parameters σ_y and σ_y^∞ are the normal stresses acting along y-axis for a finite width plate and an infinite width plate respectively.

For orthotropic laminate containing a central circular hole, the derivation of the FWCF is based on an approximate stress analysis. The solution for the inverse of the FWCF is given by [54]

$$\frac{K_T^\infty}{K_T} = \frac{3(1-\frac{2R}{w})}{2+(1-\frac{2R}{w})^3} + \frac{1}{2}\left(\frac{2R}{w}M\right)\left(K_T^\infty - 3\right)\left[1-\left(\frac{2R}{w}M\right)^2\right] \tag{2.50}$$

where, M is the magnification factor, defined as:

$$M^2 = \frac{\sqrt{1-8\left[\frac{3\left(1-\frac{2R}{w}\right)}{2+\left(1-\frac{2R}{w}\right)^3}-1\right]}-1}{2\left(\frac{2R}{w}\right)^2} \tag{2.51}$$

The SCF of an infinite orthotropic plate, K_T^∞, can be written as

$$K_T^\infty = 1 + \sqrt{\frac{2}{A_{66}}\left(\sqrt{A_{11}A_{22}} - A_{12} + \frac{A_{11}A_{22} - A_{12}^2}{2A_{66}}\right)} \tag{2.52}$$

where, A_{ij}, i,j=1,2,6 denote the effective laminate stiffness values. Axes 1 and 2 are parallel and transverse to the loading direction respectively.

2.7 Validation of stochastic finite element analysis for center-notched laminates

The MATLAB® program written for the Stochastic Finite Element Analysis (SFEA) of center-notched laminates is validated using an example application. Nodal displacements in y-direction, v, at the loading end of center-notched laminates subjected to Fatigue Loading Condition No.1 (F.L.C.1), F.L.C.2 and F.L.C.3 were determined using the MATLAB® program written for SFEA of center-notched laminates. The nodal displacement (v) values obtained from the MATLAB® program were compared with experimental results given in Table 3.3. In order to facilitate the comparison of SFEA and experimental results, geometric dimensions and laminate configuration of center-notched laminates used in the experimental investigation were considered for the SFEA. The geometric dimensions and laminate configuration of center-notched laminates are as follows: Gage Length (L) = 180mm, Width (W) = 37.9mm, Thickness (t) = 2mm, hole diameter (d) = 7.54mm and laminate configuration $[0/90]_{4s}$. NCT301 prepreg with thickness 0.125mm was used to prepare the center-notched laminates. The material properties of NCT301 prepreg, which is required for the SFEA, are given in Table 3.1.

The finite element mesh, boundary conditions, geometric dimensions, and applied loading of center-notched laminates are given in Figure 2.3. The Average Ultimate Strength values of Center-Notched Laminates (AUSCNL) subjected to Fatigue Loading Condition No.1 (F.L.C.1), F.L.C.2 and F.L.C.3, given in Table 3.3, were used as the applied load. The loading direction and node numbering at the loading end of center-notched laminates are given in Figure 2.6.

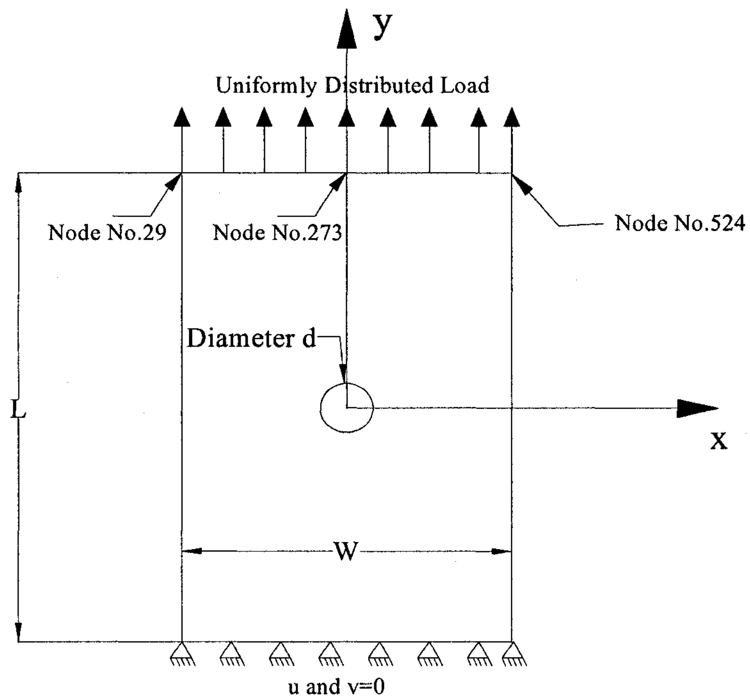


Figure 2.6 Loading direction and node numbering at loading end of center-notched laminates

The nodal displacements (v) at the loading end of center-notched laminates subjected to Fatigue Loading Condition No.1 (F.L.C.1), F.L.C.2 and F.L.C.3, calculated using Stochastic Finite Element Analysis (SFEA), were compared with experimental results and it is shown in Table 2.2. In reference [5], the nodal displacement (v) at the loading end of center-notched laminate not subjected to fatigue loading were obtained from Stochastic Finite Element Analysis (SFEA) and experiments, which is also given in Table 2.2. The nodal displacement (v) value of all the nodes at the loading end of center-notched laminate is the same, that is, $v_{node29} = v_{node273} = v_{node524}$.

S.No.	Center-notched laminates	Applied Load	SFEA Results	Experimental Results	Percentage difference
		N	mm	mm	%
1.	Not subjected to fatigue loading	34932	1.521	1.947	21.87
2.	Subjected to F.L.C.1	33819	1.492	1.804	17.29
3.	Subjected to F.L.C.2	33548	1.485	1.784	16.76
4.	Subjected to F.L.C.3	33418	1.453	1.696	14.32

Table 2.2 Comparison of displacement, v, at the loading end of center-notched laminates subjected to different fatigue loading conditions obtained from MATLAB® program and experimental investigation

The methodology used to obtain the experimental results of center-notched laminate subjected to fatigue loading conditions is explained in Chapter 3. From Table 2.2, it can be seen that the percentage difference in nodal displacement values (v) of center-notched laminates not subjected to fatigue loading, subjected to Fatigue Loading Condition No.1 (F.L.C.1), F.L.C.2 and F.L.C.3 are 21.87%, 17.29%, 16.76% and 14.32% respectively. The maximum percentage difference occurs for center-notched laminates not subjected to fatigue loading with 21.87%. The percentage difference is due to many practical problems that occurs during the experimental investigation of center-notched laminates, such as,

(i) Variation in geometric dimensions while cutting.

(ii) Damage near the hole edge during drilling operation.

(iii) Misalignment of MTS machine while testing.

(iv) Variation in the fiber angles of different layers during hand lay-up process.

2.8 Stochastic finite element analysis of edge-notched laminates

The methodology used in the Stochastic Finite Element Analysis (SFEA) of edge-notched laminates is similar to that of center-notched laminates, except for the finite element mesh. The notch location in edge-notched laminates is different from center-notched laminates, which leads to the variation in the finite element mesh. Center-notched laminates have a circular hole at the center of the laminate coupon. Stress concentration effect near the hole edge of center-notched laminates is analyzed by developing a fine mesh around the hole region and a coarse mesh towards the laminate boundary. Edge-notched laminates have two semi-circular holes on both ends of the laminate coupon. The stress concentration effect near the two semi-circular holes were analyzed by developing a fine mesh near the semi-circular hole regions and a coarse mesh towards the laminate boundary. A typical finite element mesh of an edge-notched laminate with 200 elements along the applied loading and boundary conditions is given in Figure 2.7.

In order to facilitate the comparison of results, geometric dimensions and laminate configuration of edge-notched laminates used in the experimental investigation were considered for the Stochastic Finite Element Analysis (SFEA). The geometric dimensions and laminate configuration of edge-notched laminates used in the SFEA are as follows: Gage Length (L) = 180mm, Width (W) = 37.9mm, Thickness (t) = 2mm, semi-circular hole radius (R) = 3.77mm and laminate configuration is $[0/90]_{4s}$. NCT301 prepreg with thickness 0.125mm was used to prepare the edge-notched laminates. The material properties of NCT301 prepreg, which is required for the SFEA, are given in Table 3.1.

The Average Ultimate Strength of Edge-Notched Laminates (AUSENL), which is given in Table 3.8, is considered to be the applied load in the Stochastic Finite Element Analysis.

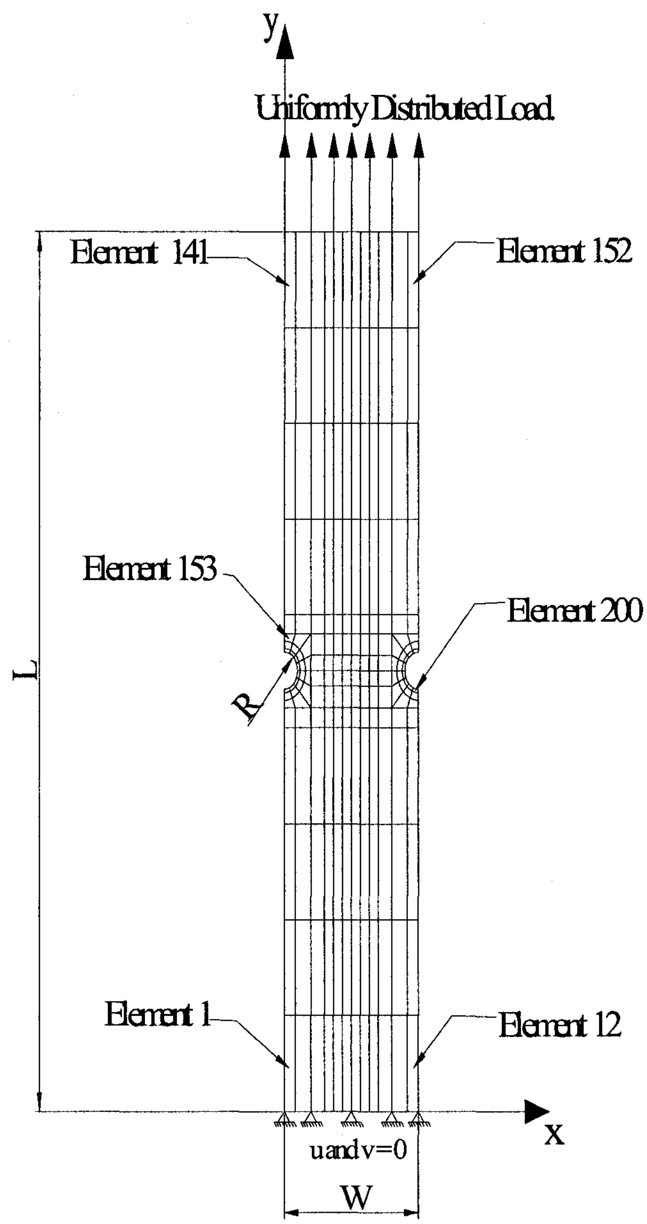


Figure 2.7 Finite element mesh of an edge-notched laminate with 200 elements along with the applied loading and boundary conditions

From Figures 2.3 and 2.7, it can be seen that the finite element mesh near the hole region of edge-notched laminates is different from that of center-notched laminates. The MATLAB® program written for the Stochastic Finite Element Analysis (SFEA) of center-notched laminates is modified, according to the finite element mesh of edge-notched laminates, to perform the SFEA of edge-notched laminates. The change in the global coordinate values of nodes near the hole region of edge-notched laminates, due to the change in notch location, are incorporated in the MATLAB® program.

The modified MATLAB® program, which is capable of performing the Stochastic Finite Element Analysis (SFEA) of edge-notched laminates, is used to calculate the maximum stress at hole edge (σ_y) of edge-notched laminates with 40, 80, 200 and 328 elements in the finite element mesh. An attempt has been made to see the convergence of the maximum stress at hole edge (σ_y) with the number of elements, used for finite element meshing, by calculating the percentage difference between the maximum stress at hole edge (σ_y) values of edge-notched laminates with 40, 80, 200 and 328 elements in the finite element mesh. Table 2.3 shows the maximum stress at hole edge (σ_y) value of edge-notched laminates with a finite element mesh of 40, 80, 200 and 328 elements.

S. No.	Number of Elements	Maximum Stress at hole edge (σ_y) (GPa)	Percentage difference (%)
1.	48	2.113	0
2.	80	2.361	10.50
3.	200	2.470	4.41
4.	328	2.561	3.55

Table 2.3 Comparison of the maximum stress at hole edge (σ_y) in edge-notched laminates obtained by using different meshes in MATLAB® program

From Table 2.3, it can be seen that the maximum stress at hole edge (σ_y) in edge-notched laminate converge as the number of elements in the finite element mesh is increased. The percentage difference between maximum stress at hole edge (σ_y) values of edge-notched laminates with 80 and 200 elements is 4.28% and it is 3.57% for edge-notched laminates with 200 and 328 elements. The error in the maximum stress at hole edge (σ_y) is less, when the number of elements in the finite element mesh of edge-notched laminate is increased from 200 to 328 elements. Stochastic analysis involves large scale computational work. Considering the computational work of edge-notched laminate with 328 elements, which is almost twice to that of edge-notched laminate with 200 elements, edge-notched laminate with 200 elements is used in the present analysis.

2.9 Validation of stochastic finite element analysis for edge-notched laminates

The MATLAB® program written for the Stochastic Finite Element Analysis (SFEA) of edge-notched laminates is validated using an example application. In the example application, the stress profile from hole edge A to laminate midpoint B of edge-notched laminate is determined using SFEA and the results were compared with ANSYS® results. Figure 2.8 shows the enlarged view of edge-notched laminate with nodes near the hole region and along the boundary.

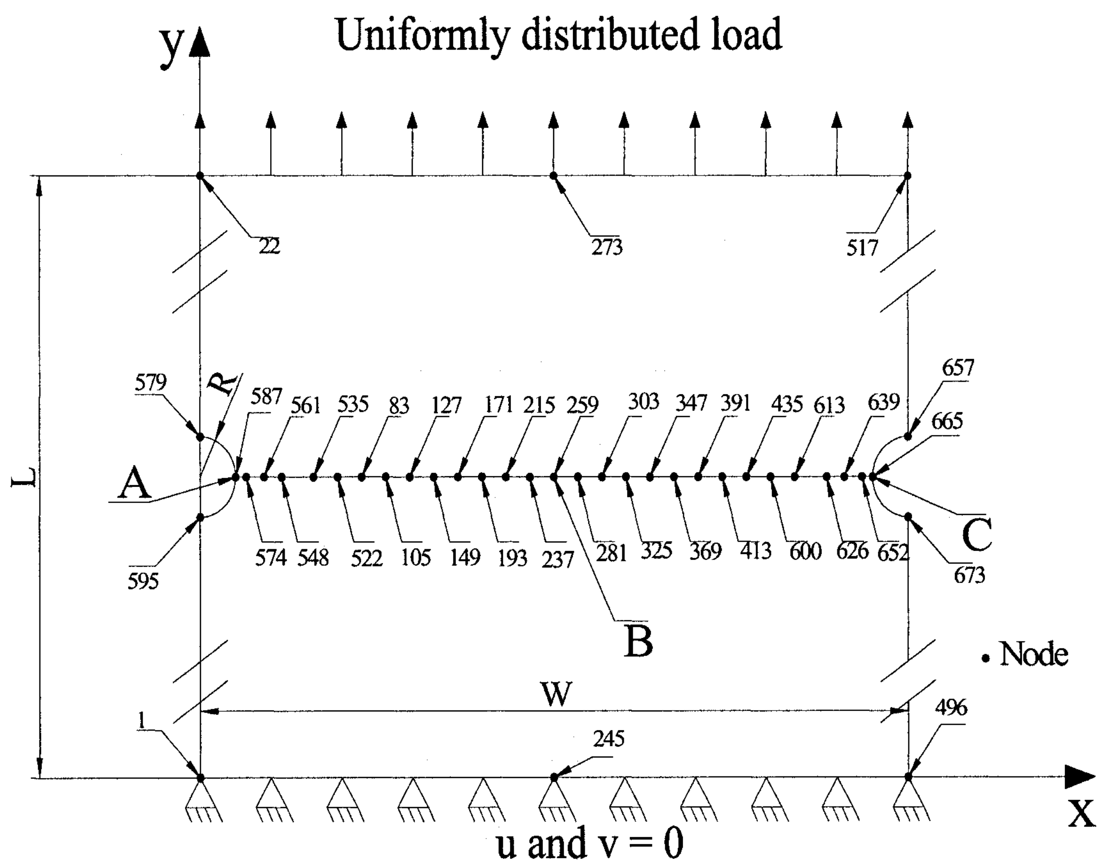


Figure 2.8 Enlarged view of edge-notched laminate with nodes near the hole region and along the boundary

The geometric dimensions and laminate configuration of the edge-notched laminate used in the Stochastic Finite Element Analysis (SFEA) are given in Section 2.8. The nodal stresses from hole edge A to the laminate midpoint B are determined using MATLAB® program. Standard commercial package ANSYS® is used for comparing the results obtained from MATLAB® program. In the ANSYS® package, a similar model with nearly the same number of elements in the finite element mesh is generated using SOLID46 element and analyzed. SOLID46 is a layered version of the 8 node structural solid designed to model layered thick shells or solids. The nodal stress values from hole edge A to the laminate midpoint B of edge-notched laminate obtained from Stochastic Finite Element Analysis (SFEA) and ANSYS® package are listed in Table 2.4.

From Table 2.4, it can see that the stress obtained in MATLAB® program are very close to the ANSYS® solutions. The percentage difference between the results obtained from MATLAB® program and ANSYS® package are in the range of 1 to 6%, indicating a correlation between the results.

Global Node	SFEA program results	ANSYS® results	Percentage Difference
	Stress(σ_y) (GPa)	Stress(σ_y) (GPa)	%
587	2.470	2.610	5.37
574	1.494	*	*
561	1.063	1.119	4.99
548	0.821	*	*
535	0.789	0.825	4.37
522	0.705	*	*
83	0.659	0.688	4.19
105	0.617	0.640	3.66
127	0.603	*	*
149	0.594	0.616	3.57
171	0.589	0.604	2.47
193	0.585	*	*
215	0.579	0.592	2.13
237	0.564	*	*
257	0.526	0.532	1.22

* There is no node in ANSYS® at the corresponding position.

Table 2.4 The nodal stress values from hole edge A to the laminate midpoint B of edge-notched laminate obtained from Stochastic Finite Element Analysis (SFEA) program and ANSYS® package

2.10 Conclusions and Discussions

The finite element formulation for an isotropic plate under plane stress condition is explained in Section 2.2. Using the finite element formulation, isotropic plate with center-notch is analyzed to find the maximum stress at hole edge (σ_y). Convergence of the maximum stress at hole edge (σ_y), with number of elements used to mesh the structure is shown in Table 2.1.

Finite element formulation for composite laminates is given in Section 2.4. Stochastic field modeling of material parameters is explained in Section 2.5. Stochastic variations in material properties over the laminate are established using Markov model and sample realizations of the material properties at each and every Gauss point are obtained. Necessary modifications are made in the MATLAB® program written for isotropic plate, to perform the stochastic analysis of composite laminates. The normal stress distribution near the hole boundary in an infinite orthotropic plate under the in-plane loading is explained in Section 2.6. The output includes the stress distribution along the axis perpendicular to the loading direction. Finite width correction factor is used to multiply the infinite plate solution to obtain the stress distribution of a finite plate.

Example application for center-notched laminate is given in Section 2.7. Nodal displacements at the boundary of the plate obtained using MATLAB® program are compared with experimental results and it is given in Table 2.2.

The Stochastic Finite Element Analysis (SFEA) of edge-notched laminates is explained in Section 2.7. Using the SFEA, edge-notched laminates were analyzed to find the maximum stress at hole edge (σ_y). Convergence of the maximum stress at hole edge (σ_y), with number of elements used to mesh the edge-notched laminates is shown in Table 2.3. Example application for edge-notched laminates is given in Section 2.9. Nodal stresses from hole edge A to laminate mid-point B obtained from MATLAB® program were compared with ANSYS solutions and it is given in Table 2.4.

CHAPTER 3

EXPERIMENTAL INVESTIGATION

3.1 Introduction

Composite materials have a comparatively less weight to strength ratio than metallic materials such as aluminum and steel, which results in their use in aerospace and automotive industries as a replacement for metallic materials. Composite materials exhibit variations in their material properties, which necessitates a detailed experimental investigation to understand their behavior. Composite laminates are made using fiber/matrix composite material. The variation in material properties and parameters of composite laminates are due to the stochastic spatial variations in fiber, matrix and their interface properties. The variation is also induced during the manufacturing process of composite laminates which produces number of defects in composite laminates such as void content, misalignment of the orientation of plies, variation in thickness of plies, etc. In order to determine the material property of composite laminate a number of samples have to be tested.

In the present work, experimental investigation is carried out to study the fatigue behavior of notched composite laminates. Fatigue loading induces fiber breakage, matrix cracking and delamination in the composite laminate structure, which lead to its premature failure. In notched composite laminate, fatigue loading induces more damage near the hole edge which leads to the redistribution of stress and reduction in the strength of composite laminate.

The experimental investigation of notched composite laminates consists of two phases, namely

(i) Manufacturing phase

(ii) Testing phase

In the manufacturing phase, laminated composite plates were manufactured using unidirectional preimpregnated NCT301 graphite/epoxy material. The material was supplied by Newport Adhesives and Composites Inc., U.S.A. In the testing phase, composite laminate coupons prepared from laminated composite plates were tested in both fatigue and static loading conditions using MTS 100KN testing machine to determine the effect of fatigue loading on the strength of laminate coupons.

3.2 Manufacturing of composite laminates

The manufacturing phase of composite laminates can be subdivided into three processes, namely

(i) Cutting process

(ii) Hand lay-up process

(iii) Autoclave cure process

3.2.1 Cutting process

The main purpose of this process is to cut plies from unidirectional prepreg with required dimensions and orientations according to the size and configuration of the laminated composite plate. Tools required for this process are: cutting board, sharp knife, steel ruler, triangles and protractor.

NCT 301 graphite/epoxy material is supplied in rolls of unidirectional impregnated prepreg of width 3 ft. According to the laminated composite plate size and configuration, plies are cut from the roll of unidirectional prepreg. Generally, laminated composite plates of size 12 in. x 11 in. (or) 10.5 in. x 11 in. are made which gives 8 (or) 9 laminate coupons respectively. An allowance of 0.5 in. x 0.5 in. is given to the dimension of plies while cutting because after the autoclave cure process the edges of the laminated composite plate will be rough and irregular. The edges of the laminated composite plate are made smooth by cutting it using a water-cooled rotary diamond cutter.

3.2.2 Hand lay-up process

The hand lay-up process is one of the oldest and common fabrication methods for composite laminates and structures. The tools and materials required for this process are an aluminum plate, sealant tape, release agent (safelease30), release film, bleeder ply, breather ply, vacuum bag, vacuum valve, sealant and a roller.

The plies are laid-up on a thick aluminum plate according to the laminate configuration. The surface of aluminum plate is cleaned to make it dust free followed by applying release agent (safelease30) onto the cure area of the plate. The purpose of release agent is to prevent the laminated composite plate from adhering to the aluminum plate during autoclave cure process. Once the release agent applied over the aluminum plate dries the lay-up process is started.

Each ply has a thin sheet of paper underneath it, which acts as a supporting layer to keep the graphite fiber and epoxy resin of ply intact and prevents it from bending. The supporting layer has to be removed from the ply during lay-up process. Now, according to the laminate configuration, plies are laid-up on the cure area of aluminum plate one over another until the last ply is laid-up. A roller is used to remove the entrapped air between plies, which is the primary cause for formation of void content in composite laminates. Then a release film, bleeder ply and breather ply are placed over the laminate in sequence. The release film prevents the bleeder ply from adhering to the laminate and bleeder ply is used to absorb the excess resin from the laminate during autoclave cure process.

In order to create vacuum around the laminate during cure process, a sealant tape, vacuum bag and vacuum valve are used. First the sealant tape is placed around the periphery of the cure area of aluminum plate and then the vacuum bag is placed over the sealant tape and pressed to avoid any air gap between them. The vacuum valve has two parts, a port base and a port. The port base is placed on top of the breather ply inside the vacuum bag and the port is installed through the vacuum bag. The breather ply helps to remove the air present inside the vacuum bag through the vacuum valve. A vacuum pump is used to create a vacuum of 28mm of Hg inside the vacuum bag. The hose (or) vacuum line of the vacuum pump is connected to the vacuum valve and checked for any leakage of air in the hand lay-up setup. If there is no leakage, then the laminate is ready for the autoclave cure process. The cross-sectional diagram of the hand lay-up setup is shown in Figure 3.1.

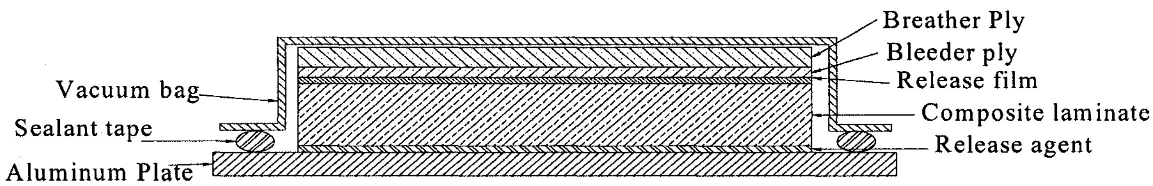


Figure 3.1 Cross-sectional diagram of the hand lay-up setup

3.2.3 Autoclave cure process

The curing process of composite laminate is carried out in an autoclave. The autoclave is a cylindrical pressure vessel with thermal insulation in which the pressure and temperature inside it can be controlled using a pressure valve and a temperature controller.

The aluminum plate with the hand lay-up setup is secured inside the autoclave. Then it is subjected to a two-stage cure cycle by varying the temperature of the autoclave. The pressure of the autoclave is maintained at 60 psi throughout the cure process. In the first stage, temperature of the autoclave is increased from room temperature to 106°C within

40 minutes and maintained at that temperature for 15 minutes (first dwell). During this stage the entrapped air and water vapor present in the matrix material escapes, allowing a free matrix flow which results in the compaction of the composite laminate. In the second-stage, temperature inside the autoclave is increased from 106°C to 145°C within 30 minutes and then maintained at that temperature for 45 minutes (second dwell). During this process cross-linking of the matrix material takes place which gives the material property to the laminate. The schematic diagram of the autoclave cure process is shown in Figure 3.2.

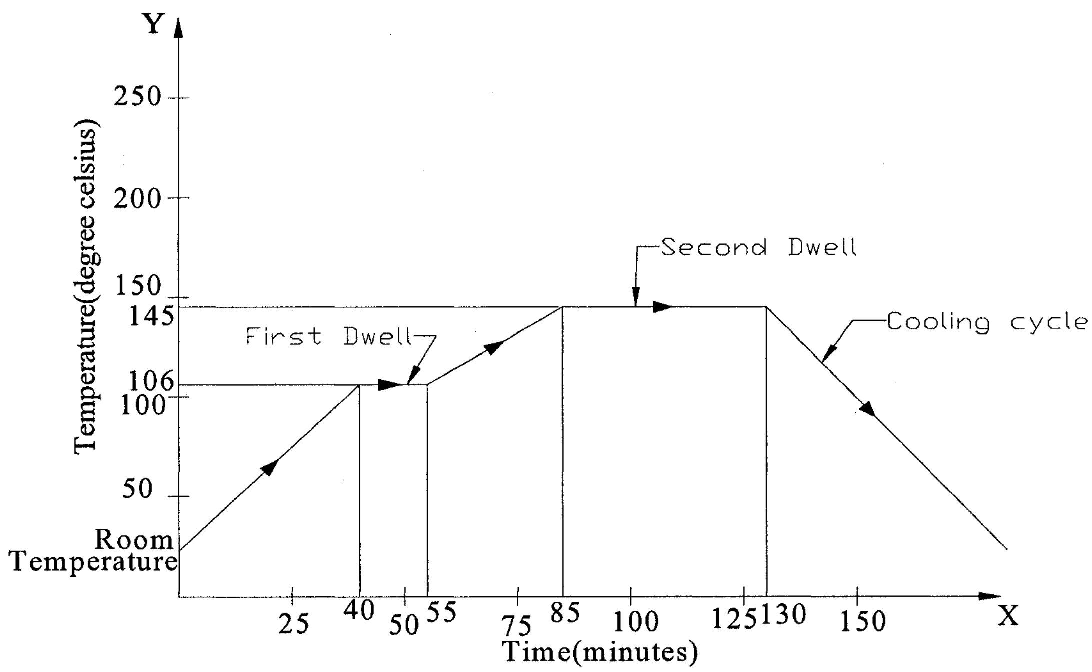


Figure 3.2 Graphical representation of the autoclave cure process

The composite laminate is allowed to cool inside the autoclave for at least 8 hours. After the cooling cycle, the entire accessory materials used during the hand lay-up process are removed and the composite laminate is inspected for any visible defects. The autoclave is shown in Figure 3.3. The aluminum plate with the hand lay-up set is shown in Figure 3.4.

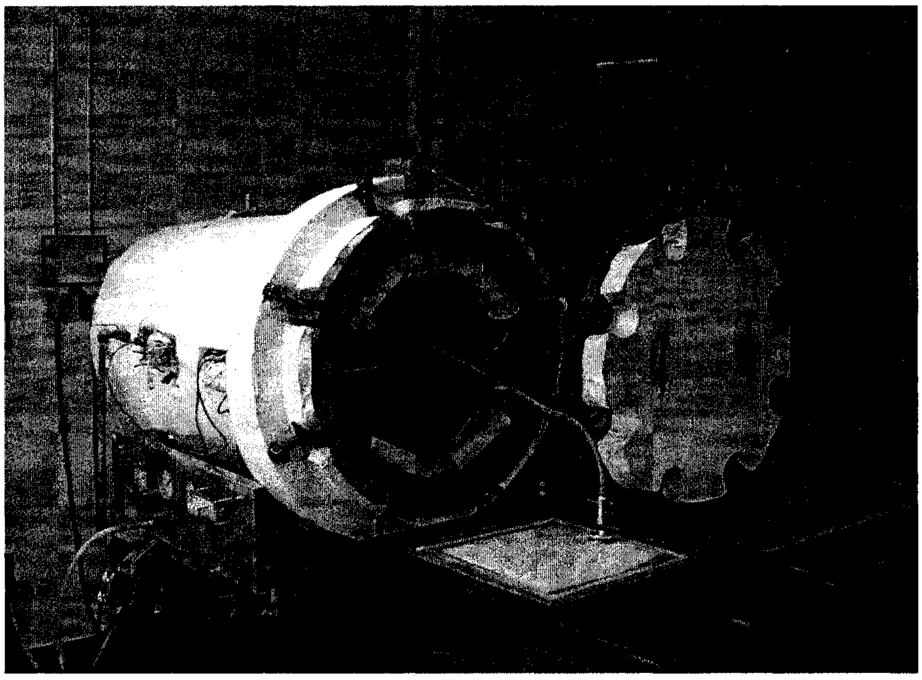


Figure 3.3 Autoclave

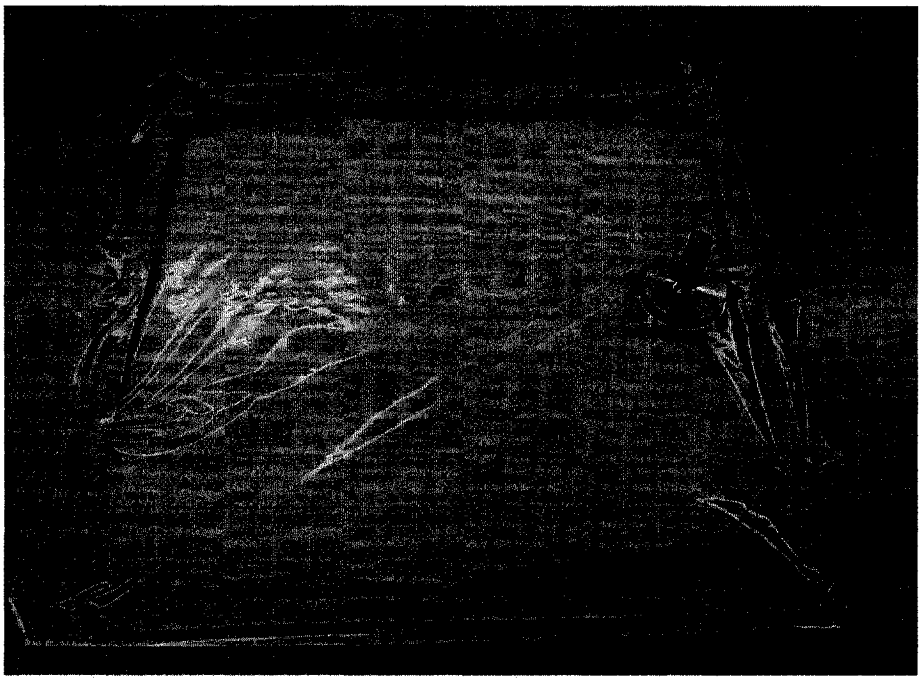


Figure 3.4 Aluminum plate with the hand lay-up setup

A water-cooled rotary diamond cutter is used to cut the laminated composite plate into smaller coupons of required dimensions, which can be used for testing. The water-cooled rotary diamond cutter is shown in Figure 3.5.

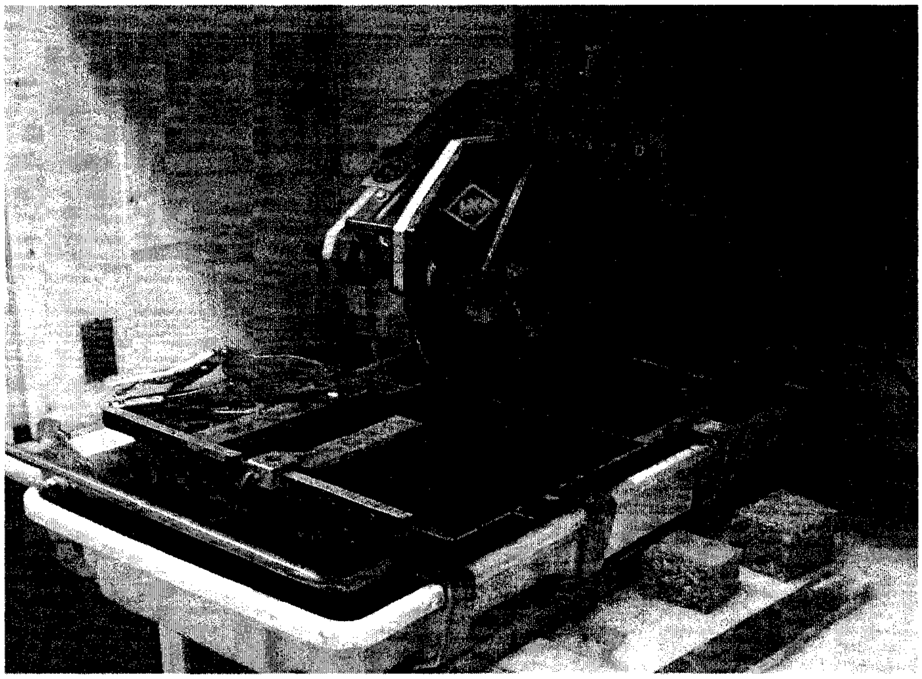


Figure 3.5 Water-cooled rotary diamond cutter

3.3 Drilling operation on the laminate coupons

The drilling operation on center-notched laminate coupons is performed using a titanium nitride coated HSS drill-bit. A small indent is made at the center of the laminate coupon using a centre punch for accurately placing the tip of the drill-bit at that point and a hole with required diameter is drilled. A wooden base is used as a support for the laminate coupon to avoid fiber breakage during drilling operation. Clamps are used to hold the laminate coupon rigidly in a position. The drilling operation is performed without a coolant. The drilling machine with the experimental setup is shown in Figure 3.6. The procedure for preparing and drilling edge-notched laminate coupons is different from that of center-notched laminates, which is described in section 3.10.

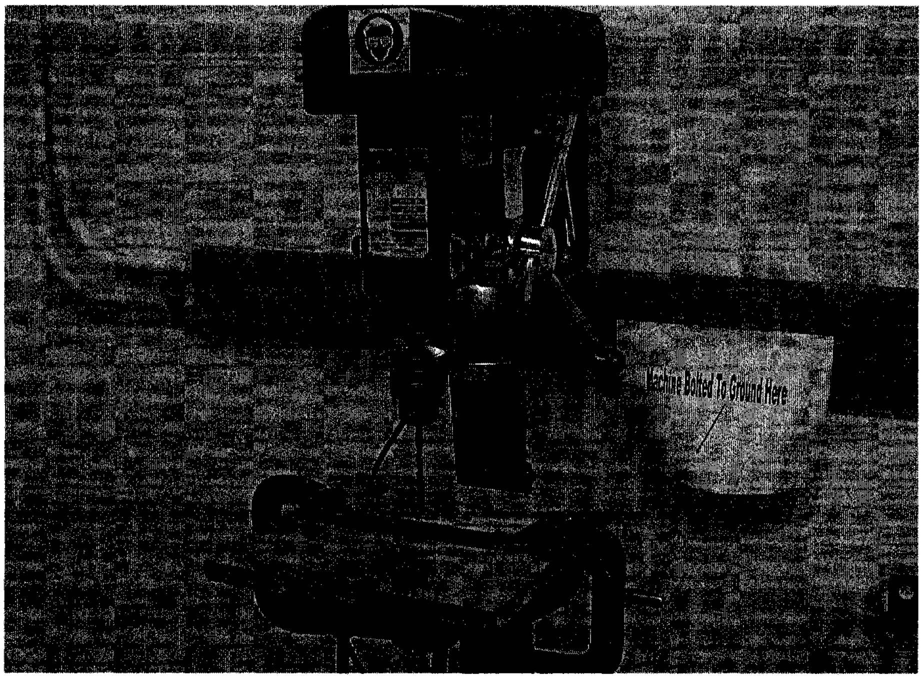


Figure 3.6 Drilling machine with the experimental setup

3.4 Laminate coupon configuration and dimension

Composite laminates are classified into unidirectional, cross-ply and off-axis laminates based on the orientation of plies and as symmetrical and asymmetrical laminates on the basis of stacking sequence. Unidirectional laminates contain only 0^{o} (or) 90^{o} plies and off-axis laminates contain only angle plies, especially $\pm 45^{o}$ plies. Both unidirectional and off-axis laminates are generally used to find the material property of the lamina, but for practical applications, composite laminates with a combination of 0^{o}, 90^{o}, $\pm 45^{o}$ plies are commonly used. Symmetrical laminates have the same stacking sequences above and below the laminate mid-plane, but asymmetrical laminates have different stacking sequences on both sides of the laminate mid-plane. In the present thesis, symmetrical cross-ply laminate with $[0/90]_{4S}$ laminate configuration is used due to its wide range of applications and the extensive study done on the tensile properties of the laminate in the previous work [5].

The dimensions and test methods for laminate coupons to be tested are decided by referring to the ASTM standards for composite materials. For example, the dimensions and test method of laminate coupons to find the tensile properties of polymer matrix composite material are referred from ASTM D 3039/D 3039M – 00 standard [55]. In the present work, the test methods of ASTM D 3039/D 3039M – 00 [55] and ASTM D 3479/D 3479M – 96 [56] standards have been used to conduct the static and fatigue testing of notched composite laminate respectively. But for the dimensions of laminate coupons, there are no specific ASTM standards for anisotropic materials with notch. In order to decide the dimensions for notched laminate coupons, the works of Tan [1] and Nuismer and Whitney [16] are referred to, who have carried out extensive studies on plates with circular openings for different laminate configurations. Based on their extensive work on notched laminates, they concluded that the ratio of the diameter of hole to the width of laminate coupon (2a/W) should lie in between 0 and 0.4. If the ratio is above 0.4, then the stress concentration factor tends to deviate from the exact values at regions away from the hole. This is true, as the theory is developed to find out only the maximum stress around the hole region. Based on the above considerations the dimensions of the laminate coupons are considered to have a width (W) of 37.9mm, gauge length (L) of 180mm and hole diameter (2a) of 7.54mm.

3.5 Material Properties of NCT301 graphite/epoxy material

The ply (or) lamina, which is cut from the unidirectional impregnated prepreg during the manufacturing phase, is the building block of a composite laminate. It is important to determine the material properties of the lamina for the design and analysis of composite laminate. The material properties of lamina, such as Young's modulus values in the fiber and transverse directions (E_1 and E_2), Poisson ratios (ν_{21} and ν_{12}) and Shear modulus (G_{12}) are determined by performing experiments according to the ASTM standards.

An extensive experimental work was done in reference [5] to determine the material properties of lamina. Since the same material is used in the present work, the material properties of lamina given in reference [5] are used in the present work. In reference [5], ASTM D 3039/3039M - 00 [55] was referred to determine the laminate configuration and dimension of laminate coupons, which were tested to find the Young's moduli of the lamina. Based on the ASTM recommendations, $[0]_{16}$ and $[90]_8$ laminate coupons were tested under uniaxial loading to find the Young's modulus in the fiber direction (E_1) and Young's modulus in the transverse direction (E_2) respectively. Similarly, referring to ASTM D 3518/3518M – 94 [57], $[\pm 45]_{4s}$ laminate coupons were chosen and tested under uniaxial loading to find the Poisson ratios (ν_{21} and ν_{12}) and Shear modulus (G_{12}). The dimensions of the $[\pm 45]_{4s}$ laminate coupons were also referred from the ASTM standards. Due to the randomness in the material properties, 25 laminate coupons in each laminate configuration were tested to determine the Mean, Standard Deviation (S.D.) and Coefficient of Variation (C.O.V.) values of the material properties of the lamina, which will be used in the stochastic analysis of composite laminate.

A hydraulic controlled 100 KN MTS machine was used to test all the laminate coupons in displacement control mode. A smooth grip was used to grip the composite laminate rigidly while testing. The load and displacement data of the testing were recorded in a computer attached to the machine. The data are retrieved and processed to get the material properties of the laminate.

The formula used to find the statistical parameters of the material properties are,

(i) Mean ($\overline{X}$) $= \frac{1}{n}\sum_{i=1}^{n} X_i$ (3.1)

where, X_i is the test data of ith laminate coupon and n is the number of laminate coupons tested.

(ii) Standard Deviation (S.D.) $= \sqrt{\frac{\sum_{i=1}^{n} X_i^2 - n\left(\overline{X}\right)^2}{n-1}}$ (3.2)

(iii) Coefficient of Variation (C.O.V.) $= 100 \times \frac{s}{\overline{X}}$ (3.3)

The Mean, S.D. and C.O.V. values of material properties of the lamina were calculated and they are shown in Table 3.1.

	E_1	E_2	ν_{21}	ν_{12}	G_{12}
	GPa	GPa			GPa
Mean	135.036	8.204	0.016	0.264	4.172
S.D.	5.43	0.465	0.002	0.025	0.1671
C.O.V.	4.02	5.67	10.54	9.6	4.006

Table 3.1 Mean, S.D. and C.O.V. values of the material properties of NCT301 graphite/epoxy composite material

3.6 Testing of notched laminate coupons

A hydraulic-controlled 100 KN MTS machine was used for testing the laminate coupons. It has a fixed upper jaw and a movable lower jaw. Each jaw have a pair of hydraulic controlled wedge-shaped smooth grips. The laminate coupon to be tested is fixed between the two jaws by means of the smooth grips. The loading parameters of the machine can be controlled by a computer attached to it. A program is created using the required loading conditions and it is used during testing. The output of the testing can also be stored in the computer. The MTS machine is capable of performing both fatigue and static testing of the laminate coupon. The MTS machine with the experimental setup is shown in Figure 3.7.

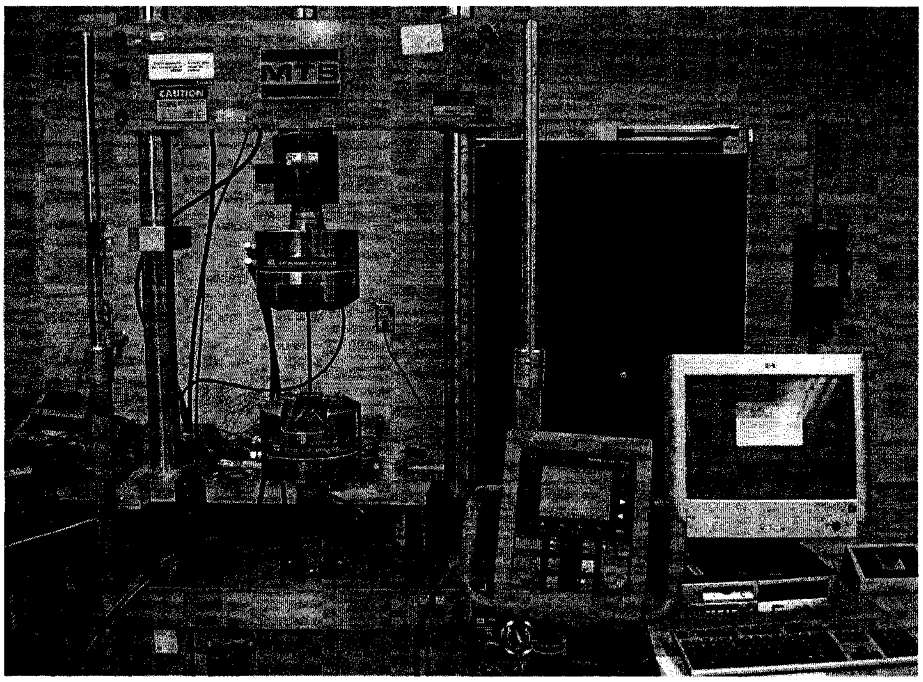


Figure 3.7 MTS machine with the experimental setup

The testing phase of the notched laminate can be categorized into two parts, namely

(i) Static testing of notched laminates

(ii) Fatigue testing of notched laminates

(iii) Static testing of fatigued notched laminates

Initially, the static testing of notched laminates are performed to determine the Average Ultimate Strength of Notched Laminate (AUSNL), which is used to calculate the loading parameters of the fatigue testing conditions. In reference [5], twenty-five samples of notched laminate with hole diameter 7.54mm was tested under static loading condition and the AUSNL value was determined. In the fatigue testing phase, the reduction in longitudinal Young's modulus (E_y) of notched laminate due to fatigue loading was studied and in the static testing of fatigue-tested laminates phase the reduction in the AUSNL value of the notched laminates was determined.

3.7 Fatigue testing of notched laminates

The following loading parameters were used to determine the fatigue loading conditions of notched laminates.

(i) Maximum stress of fatigue loading cycle (S_{max})

(ii) Minimum stress of the fatigue loading cycle (S_{min})

(iii) Stress ratio (R)

(iv) Frequency (f)

In reference [5], 25 samples of notched laminate that were not subjected to fatigue loading were tested under static loading conditions to determine the Average Ultimate Strength of Notched Laminate (AUSNL) value and it is given in Table 3.2.

Three different fatigue loading conditions were considered for fatigue testing of notched laminates. In all the three fatigue loading conditions, frequency (f) is kept constant with a value of 4 Hertz. The other three loading parameters for fatigue loading conditions are given below:

(i) Fatigue Loading Condition No.1 (F.L.C.1)

S_{max} = 50% AUSNL, S_{min}=0, R=0. (Tension-Tension fatigue)

(ii) Fatigue Loading Condition No.2 (F.L.C.2)

S_{max} = 70% AUSNL, S_{min}=0, R=0. (Tension-Tension fatigue)

(iii) Fatigue Loading Condition No.3 (F.L.C.3)

S_{max}= 50% AUSNL, S_{min}=-S_{max}\10, R= -0.1. (Tension-Compression fatigue)

Three samples were tested for each Fatigue Loading Condition to determine the Average Ultimate Strength of Notched Laminate (AUSNL) after fatigue loading. The fatigue testing of notched laminate was performed for 40 hours, which is approximately equivalent to ½ million cycles. For every 1 hour (or) 14400 cycles the displacement (x_i) and force (F) data obtained from the MTS machine were stored into the computer for 0.5 seconds, which gives the data for 2 fatigue cycles.

3.7.1 Calculation of Young's modulus (E_y) of notched laminate using the experimental data

The Young's modulus of the notched laminate is calculated for every 1 hour (or) 14400 cycles using the displacement and force data stored in the computer. The strain in the notched laminate is determined using the displacement data instead of a strain gage. The strain gage can be used to determine the strain in the notched laminate only once during the fatigue testing, whereas the Young's modulus of the notched laminate has to be calculated for every 1 hour (or) 14400 cycles. The Stress (σ) and Strain (ϵ) values of the notched laminate at any time t during the fatigue testing cycle can be calculated from force (F) and displacement (Δx) respectively at that time by using the formulae:

(i) $\sigma = F/A$ (3.4)

(ii) $\Delta x = x_i - x$ (3.5)

(iii) $\epsilon = \Delta x/L$ (3.6)

where,

A is the cross-sectional area of the notched laminate before loading,

x is the position of cross-head at zero loading,

x_i is the position of cross-head at any given time t and

L is the gage length of the notched laminate.

The pictorial representation of the notched laminate displacement at any given time in the MTS machine is shown in Figure 3.7 (a)

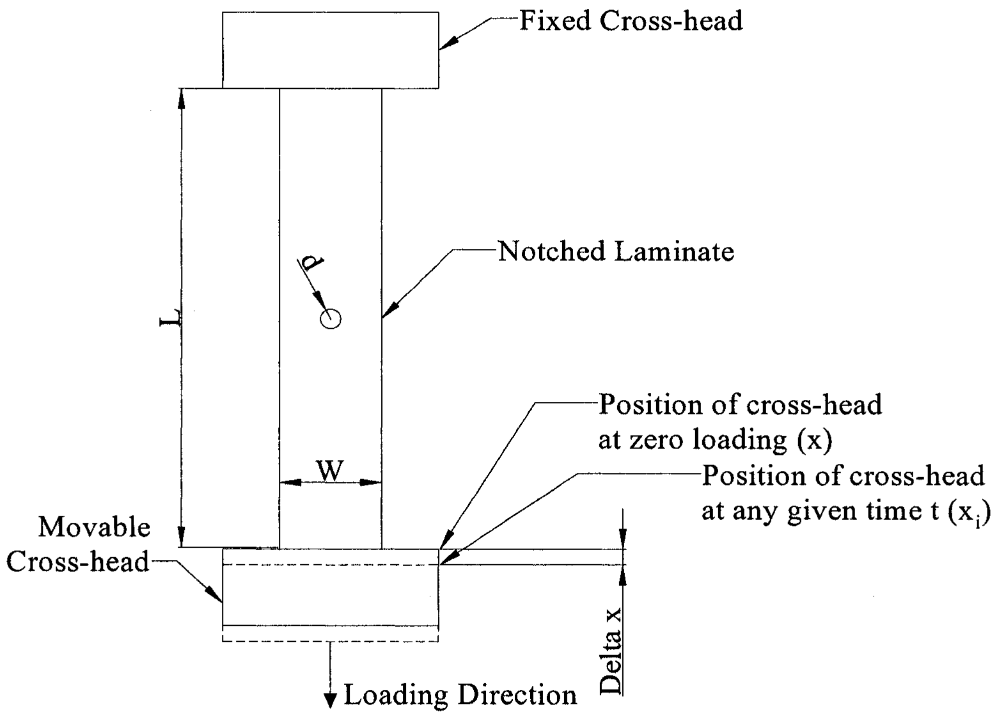


Figure 3.7(a) Pictorial representation of the notched laminate displacement at any given time in the MTS machine

In Figure 3.7(a), it can be clearly seen that the parameter, Δx, represents the change in length of the notched laminate at any given time, which is used to calculate the strain in the notched laminate. During fatigue testing, the deformation of the cross-head grip is very less when compared to the deformation of the notched laminate and hence it was not considered for calculating the strain in the notched laminate. Table 3.2 shows the sample test data of F and x_i obtained from the MTS machine during the fatigue testing of notched laminate subjected to Fatigue Loading Condition No.1 and the corresponding σ and ϵ values were calculated using equations 3.4-3.6.

Time	Axial Displacement	Axial Force	Stress	Displacement	Strain
t	x_i	F	$\sigma = F/A$	$\Delta x = x_i - x$	$\epsilon = \Delta x/L$
Seconds	mm	N	N/mm^2	mm	
1005.69	0.090	-46.12	-0.6068	-0.0014	-8.1E-06
1005.70	0.088	-68.78	-0.9051	-0.0031	-1.7E-05
1005.70	0.091	4.268	0.0561	0	0
1005.71	0.102	258.5	3.402	0.0113	6.3E-05
1005.71	0.122	642.3	8.451	0.0310	0.00017
1005.72	0.147	1146.2	15.08	0.0557	0.00030
1005.72	0.177	1763.9	23.20	0.0860	0.00047
1005.73	0.213	2485.0	32.69	0.1219	0.00067
1005.73	0.253	3297.1	43.38	0.1622	0.00090
1005.74	0.299	4194.7	55.19	0.2078	0.00115
1005.74	0.348	5160.7	67.90	0.2564	0.00142
1005.75	0.399	6177.3	81.28	0.3081	0.00171
1005.75	0.456	7236.1	95.21	0.3646	0.00202
1005.76	0.510	8315.9	109.4	0.4193	0.00232
1005.76	0.566	9405.5	123.7	0.4746	0.00263
1005.77	0.621	10484.4	137.9	0.5298	0.00294
1005.77	0.675	11538.5	151.8	0.5834	0.00324
1005.78	0.727	12550.7	165.1	0.6355	0.00353
1005.78	0.775	13501.7	177.6	0.6837	0.00379
1005.79	0.819	14384.7	189.2	0.7283	0.00404
1005.79	0.862	15186.6	199.8	0.7712	0.00428
1005.80	0.898	15889.5	209.0	0.8068	0.00448
1005.80	0.928	16486.0	216.9	0.8368	0.00464
1005.81	0.955	16959.0	223.1	0.8634	0.00479
1005.81	0.972	17312.2	227.7	0.8808	0.00489
1005.82	0.984	17536.9	230.7	0.8925	0.00495
1005.82	0.989	17633.6	232.0	0.8978	0.00498

Table 3.2 Sample test data of force (F) and axial displacement (x_i) obtained from the MTS machine during the fatigue testing of notched laminate subjected to Fatigue Loading Condition No.1 and the corresponding stress (σ) and strain (ϵ) values

In Table 3.2, the axial force (F) value close to zero loading is 4.268 N. Thus the axial displacement (x_i) value corresponding to 4.268 N is considered to be equivalent to x (position of cross-head at zero loading). A graph is drawn using the stress (σ) and strain (ϵ) values in Table 3.2 and it is shown in Figure 3.8.

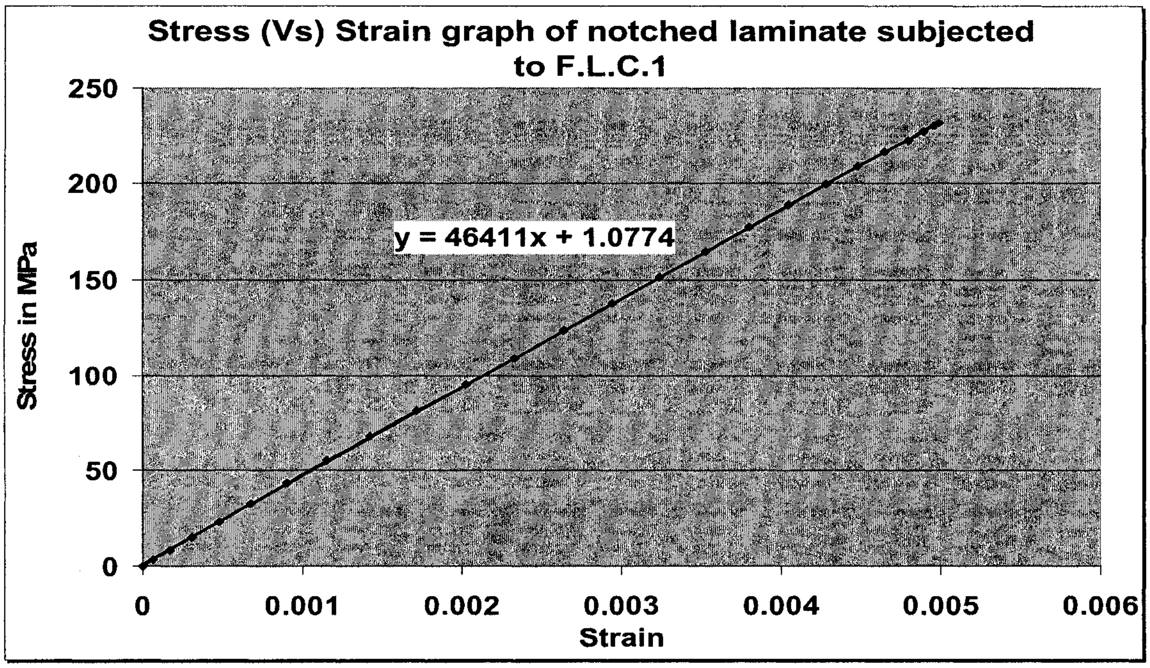


Figure 3.8 Stress (σ) *Vs* Strain (ϵ) graph of the notched laminate subjected to Fatigue Loading Condition No.1

In Figure 3.8, the slope of the line passing through all the data points gives the Young's modulus of the notched laminate subjected to Fatigue Loading Condition No.1 and this value is 46411 MPa (or) 46.411 GPa. By repeating the above procedure, Young's modulus values of the laminate for every 1 hour (or) 14400 cycles are calculated and the graph of Young's modulus (Ey) *Vs* No. of Cycles (N) is drawn and it is shown in Figure 3.9.

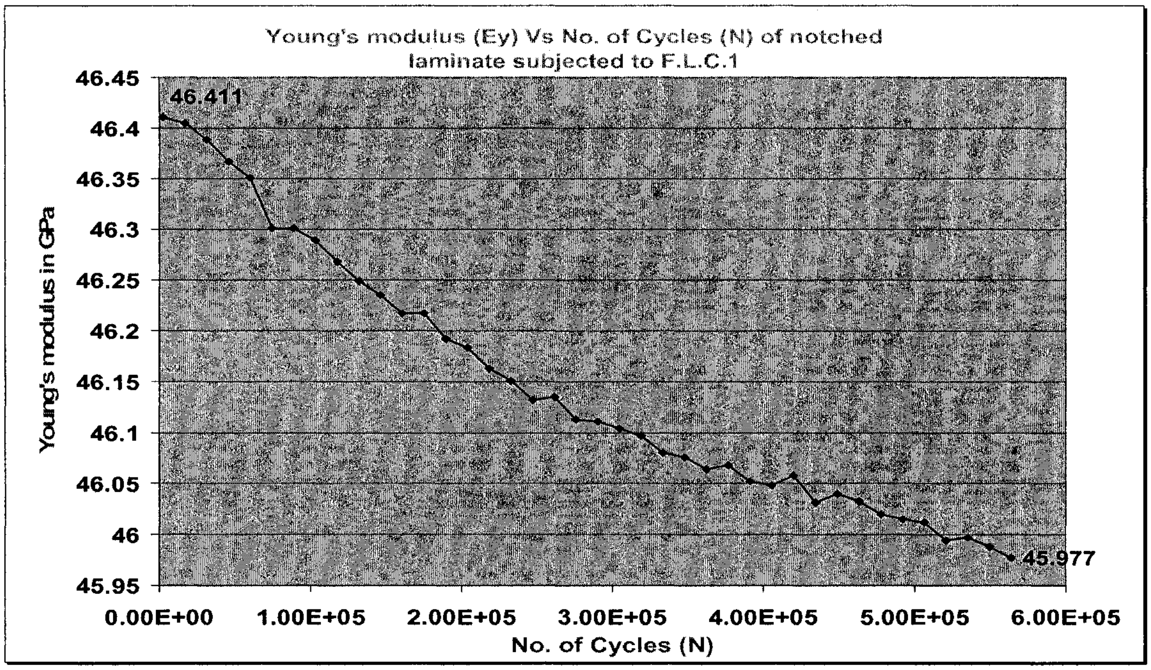


Figure 3.9 Young's modulus (E_y) *Vs* No. of cycles (N) graph of a notched laminate subjected to Fatigue Loading Condition No.1

3.7.2 Fatigue test results of notched laminates

The following results are obtained from the fatigue testing of notched laminates.

(i) Young's modulus (E_y) *Vs* No. of cycles (N) graph of notched laminates subjected to Fatigue Loading Conditions,

(i) Fatigue Loading Condition No.1 (F.L.C.1)

(ii) Fatigue Loading Condition No.2 (F.L.C.2)

(iii)Fatigue Loading Condition No.3 (F.L.C.3)

(ii) Strain (ϵ) *Vs* No. of cycles (N) graph of notched laminates subjected to Fatigue Loading Conditions,

(i) F.L.C.1, (ii) F.L.C.2, (iii) F.L.C.3.

The graphs are shown in Figures 3.10, 3.11, 3.12, 3.13, 3.14, 3.15 and 3.16.

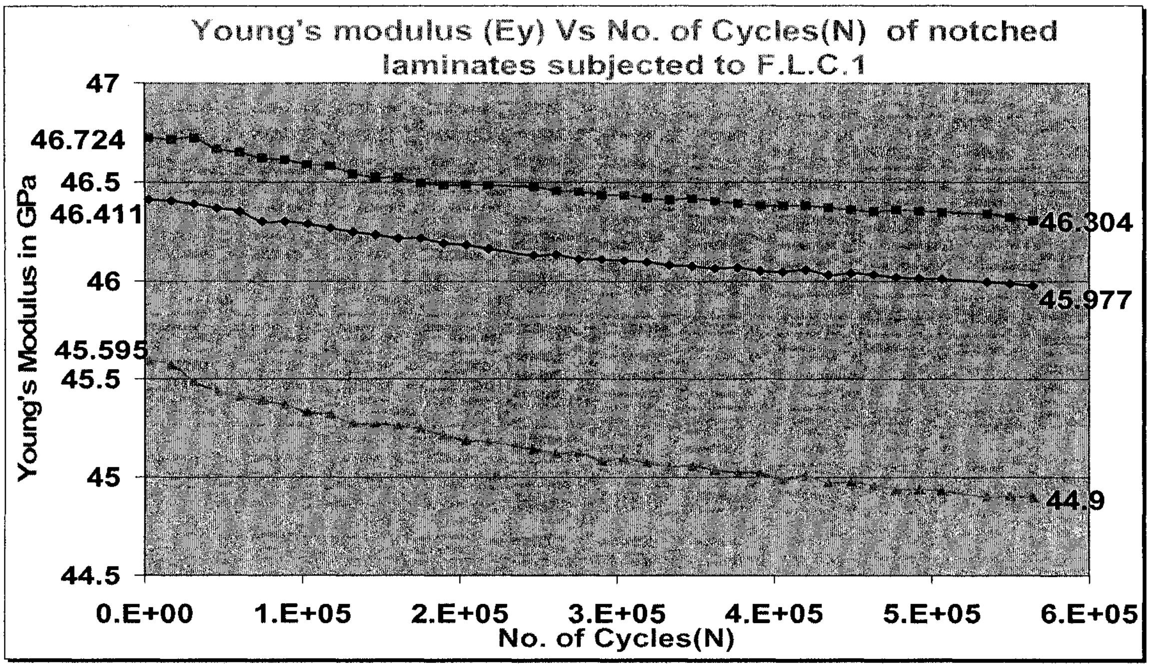


Figure 3.10 Young's modulus (E_y) *Vs* No. of cycles (N) graph of three notched laminate coupons subjected to Fatigue Loading Condition No.1

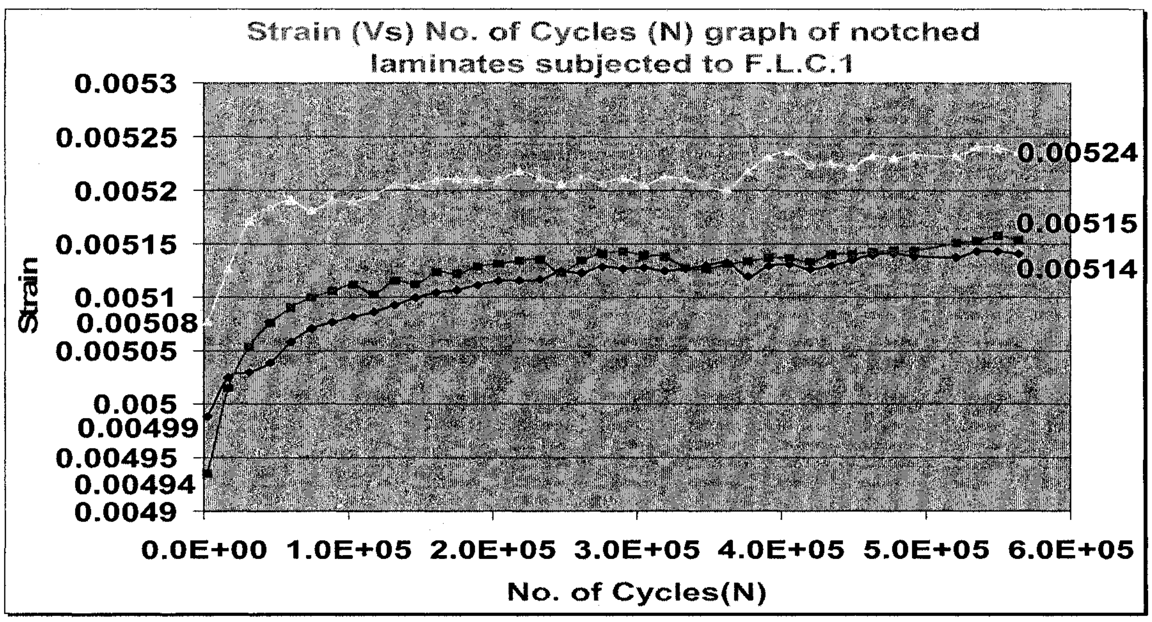


Figure 3.11 Strain (*Vs*) No. of cycles (N) graph of three notched laminate coupons subjected to Fatigue Loading Condition No.1

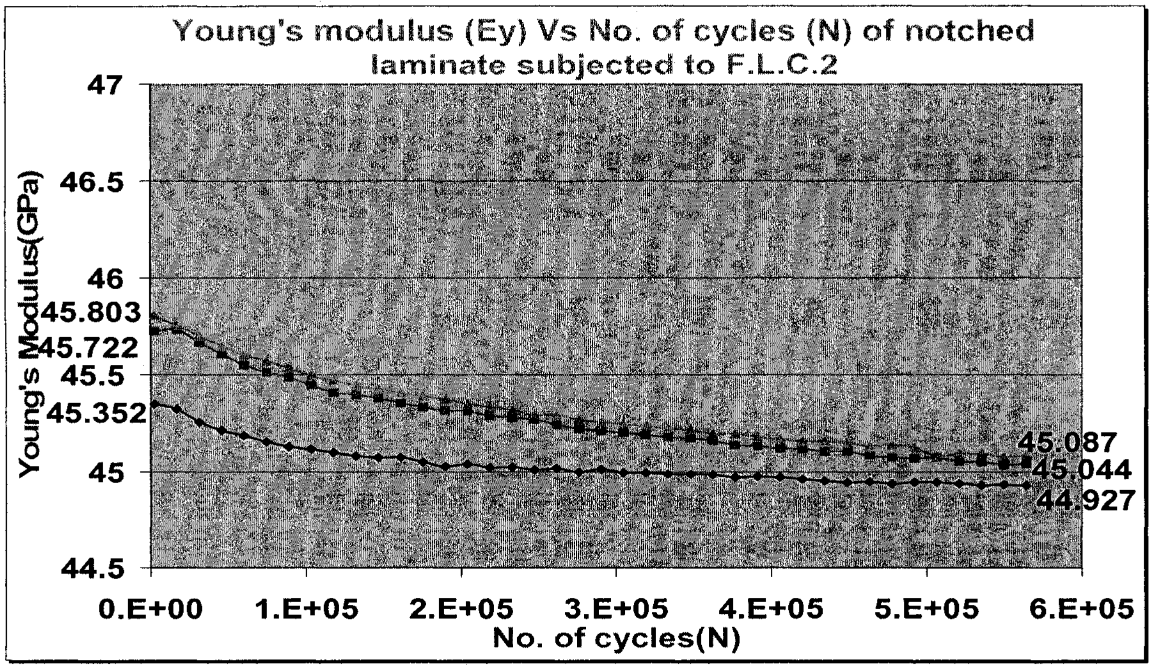


Figure 3.12 Young's modulus (E_y) *Vs* No. of cycles (N) graph of three notched laminate coupons subjected to Fatigue Loading Condition No.2

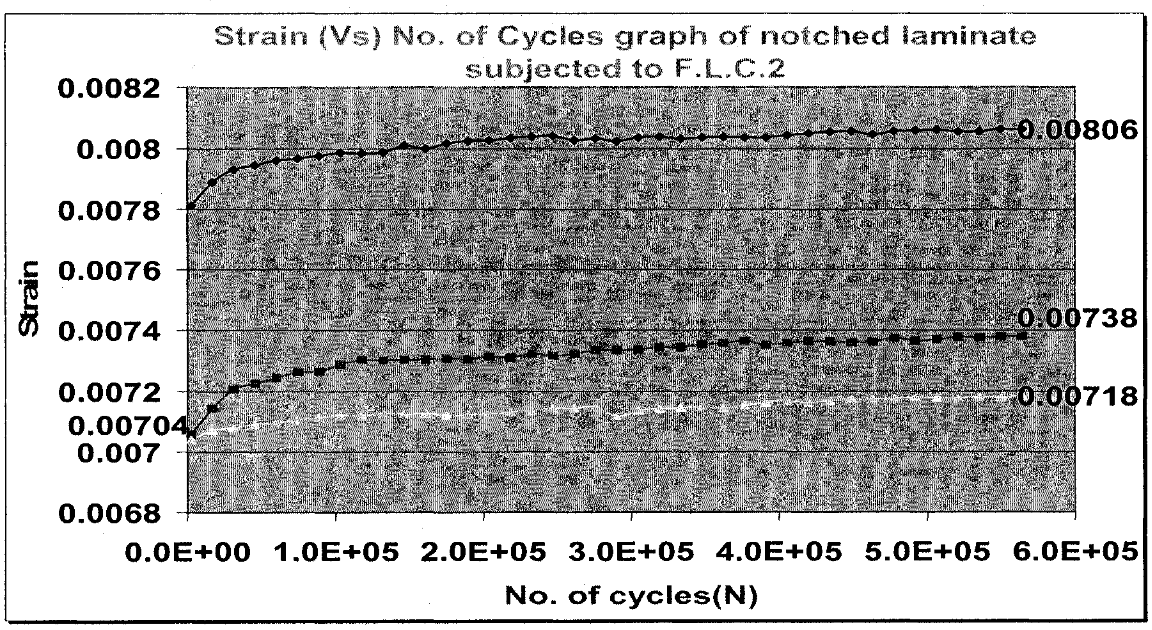


Figure 3.13 Strain (*Vs*) No. of cycles (N) graph of three notched laminate coupons subjected to Fatigue Loading Condition No.2

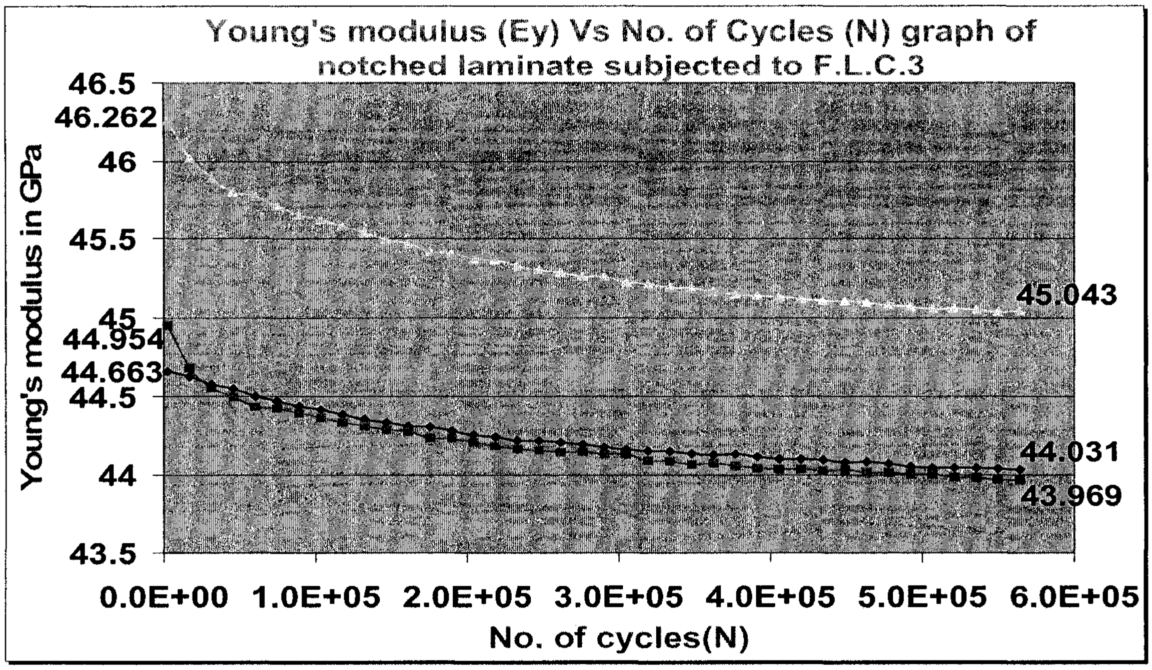


Figure 3.14 Young's modulus (E_y) *Vs* No. of cycles (N) graph of three notched laminate coupons subjected to Fatigue Loading Condition No.3

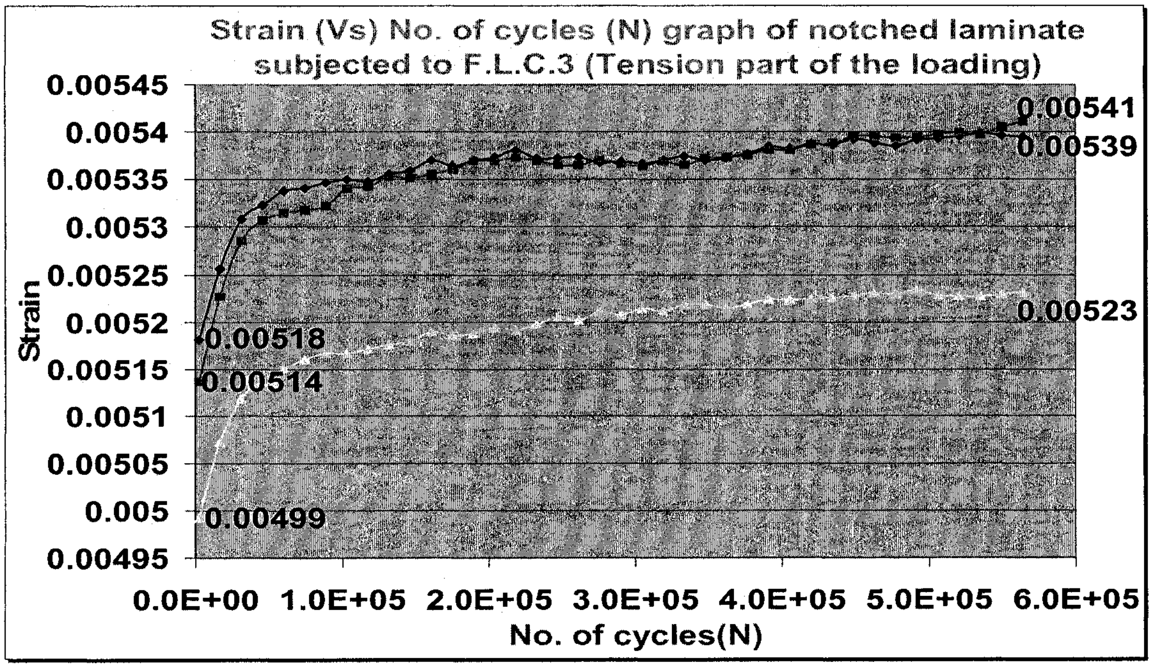


Figure 3.15 Strain (*Vs*) No. of cycles (N) graph of three notched laminate coupons subjected to Fatigue Loading Condition No.3 (Tension part of the loading)

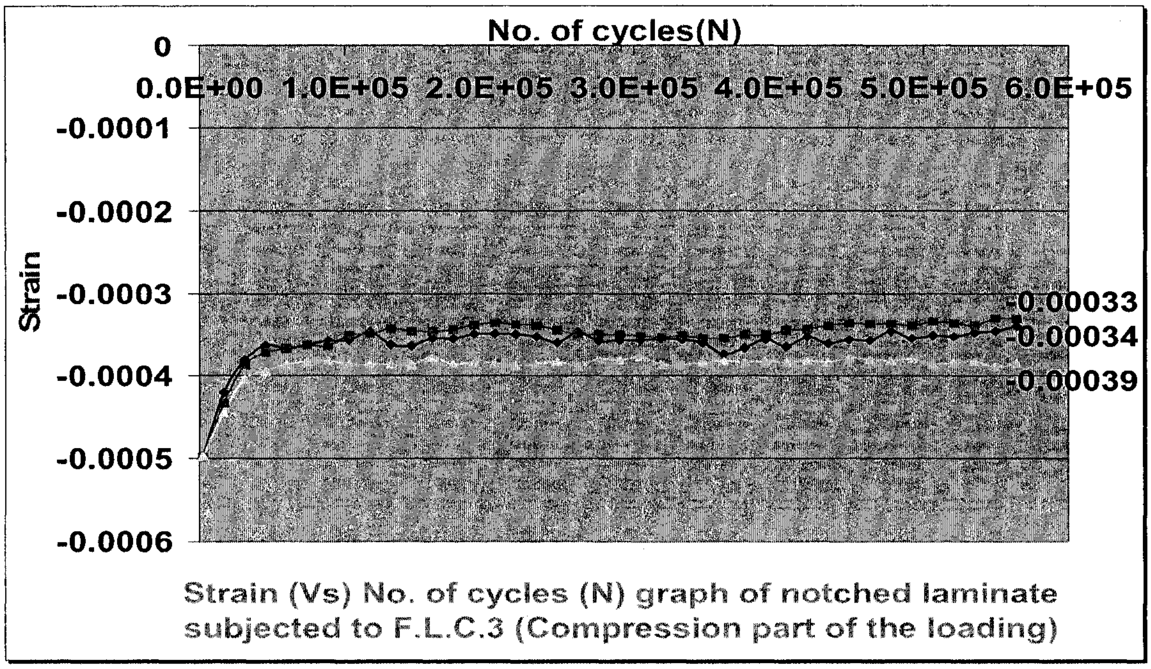


Figure 3.16 Strain (*Vs*) No. of cycles (N) graph of three notched laminate coupons subjected to Fatigue Loading Condition No.3 (Compression part of the loading)

Figures 3.10, 3.12 and 3.14 show the Young's modulus (E_y) *Vs* No. of cycles (N) graph of notched laminates subjected to Fatigue loading Condition No.1 (F.L.C.1), F.L.C.2 and F.L.C.3 respectively. From Figures 3.10, 3.12 and 3.14, the values of average reduction in Young's modulus (E_y) of notched laminates subjected to F.L.C.1, F.L.C.2 and F.L.C.3 are found to be 516.33 MPa (1.1%), 606.33 MPa (1.327%) and 945.33 MPa (2.086%) respectively. The difference between the average reduction in Young's modulus (E_y) of notched laminates subjected to F.L.C.2 and F.L.C.1 is 90 MPa, which is comparatively less indicating that the effect of changing the maximum stress value (S_{max}) of tension-tension fatigue loading cycles does not affect the reduction in Young's modulus (E_y) of notched laminate considerably.

But the difference between the average reduction in Young's modulus (E_y) of notched laminates subjected to Fatigue Loading Condition No.3 (F.L.C.3) and F.L.C.1 is 429 MPa, which is comparatively high indicating that the effect of changing the stress ratio (R) of tension-compression fatigue loading cycle affects the reduction in Young's modulus (Ey) of notched laminate considerably.

Figure 3.10 shows that the graph of Young's modulus (Ey) *Vs* No. of cycles (N) of notched laminate subjected to Fatigue Loading Condition No.1 (F.L.C.1) is relatively-linear, whereas it is non-linear for the notched laminates subjected to F.L.C.2 and F.L.C.3 as can be seen in Figures 3.12 and 3.14. The Young's modulus (Ey) reduces rapidly until about ¼ million cycles, after which the reduction is less rapid in the case of notched laminates subjected to F.L.C.2 and F.L.C.3. The variation in the reduction of Young's modulus (Ey) for each laminate coupon subjected to the same fatigue loading condition is due to the variation in the material property of the lamina.

Figures 3.11, 3.13 and 3.15 show the strain (ϵ) *Vs* No. of Cycles (N) graph of notched laminates subjected to Fatigue Loading Condition No.1(F.L.C.1), F.L.C.2 and F.L.C.3 (Tension part of the loading). The increase in the strain value of notched laminate is rapid until about 100 thousand fatigue cycles in all the three fatigue loading conditions, F.L.C.1, F.L.C.2 and F.L.C.3 (tension part of the loading), after which it is less rapid and relatively-linear. The main forms of damage in notched laminate during the initial stage of fatigue loading cycle are fiber breakage and matrix cracking [3], which cause severe deformation thus causing the rapid increase in strain value until 100 thousand cycles.

During the later stage of the fatigue loading cycle the main forms of damage in the notched laminate are delamination, crack coupling and interfacial debonding, which cause considerably less deformation than the initial stage thus causing the less and relatively-linear increase in strain after about 100 thousand cycles.

Figure 3.16 shows the strain (ϵ) *Vs* No. of Cycles (N) graph of notched laminate subjected to Fatigue Loading Condition No.3 (Compression part of the loading). The increase in strain value of notched laminate is rapid until about 50 thousand cycles, after which it is very less and almost a constant strain value is maintained. The rapid increase in strain value is due to fiber breakage and matrix cracking in the notched laminate during the initial stage, after which delamination is the main mode of damage in the notched laminate which contributes to very less deformation and thus a less increase in strain value.

3.8 Static testing of fatigued notched laminates

The strength and stiffness of notched laminates are reduced due to the fatigue loading and the laminate coupons are tested under static loading condition to determine the reduction in the strength value. The 100 KN MTS machine was used for the static testing of notched laminate. The loading parameters are determined by conducting trial tests on the notched laminates and the loading parameters are:

(i) Loading rate = 111 N/sec

(ii) Grip Pressure = 15 MPa

Using the loading parameters, a program is created in the computer attached to the MTS machine to control it during testing. A uniform axial load is applied to the laminate coupons until failure and the ultimate load is found out. During the testing, the load and displacement parameters are stored in the computer for every 0.5 seconds. All the three laminate coupons subjected to each fatigue loading condition were tested under static loading to find the Average Ultimate Strength of the Notched Laminate (AUSNL). The laminate coupons along with the experimental setup, before and after failure are shown in Figures 3.17 and 3.18. The laminate coupons subjected to Fatigue Loading Condition No.1 (F.L.C.1), F.L.C.2 and F.L.C.3 before and after failure are shown in Figures 3.19, 3.20 and 3.21 respectively.

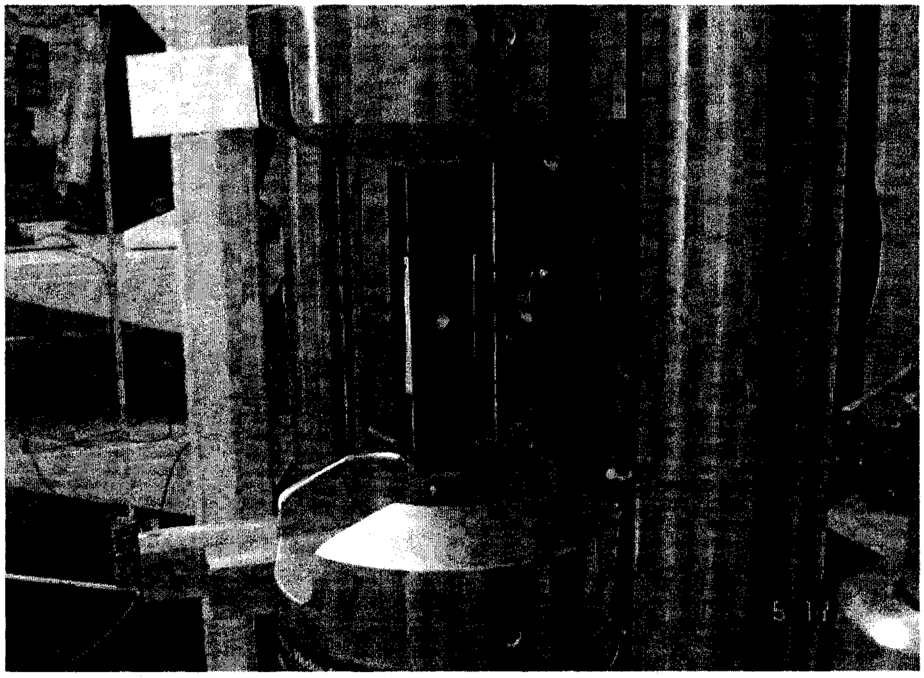


Figure 3.17 Notched laminate along with the experimental set-up before failure

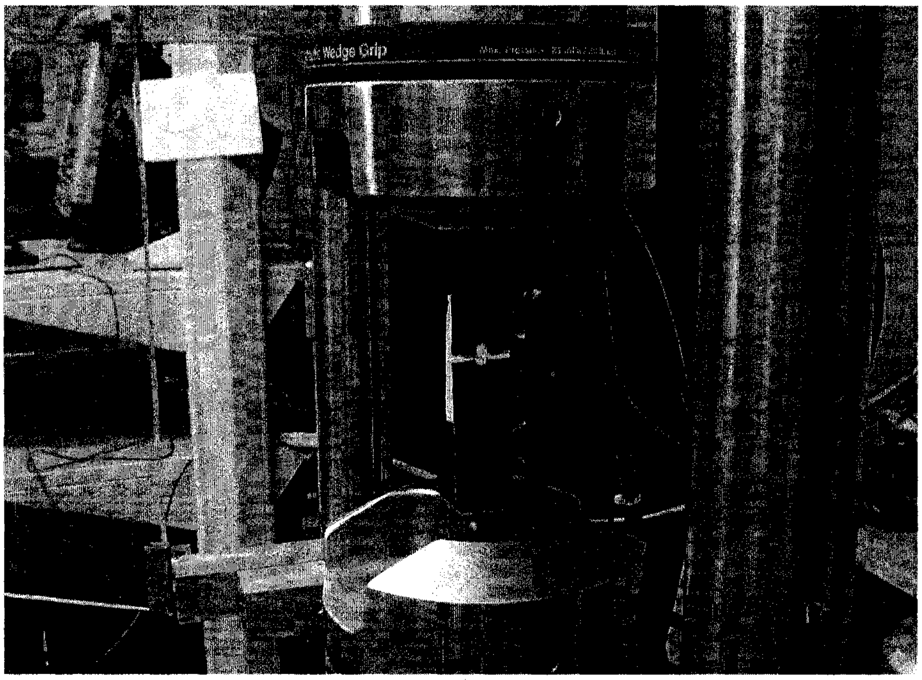


Figure 3.18 Notched laminate along with the experimental set-up after failure

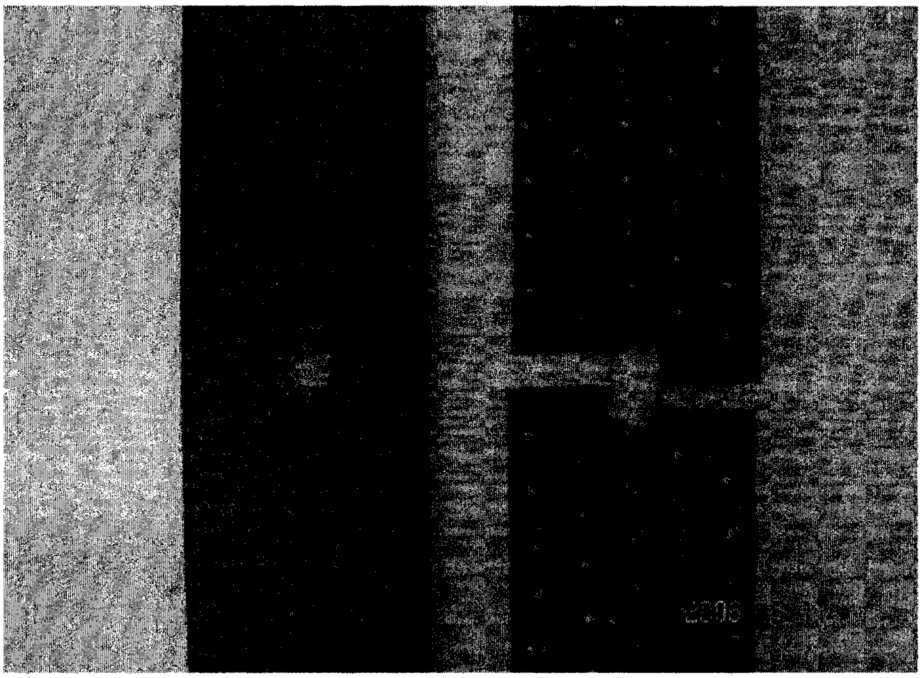


Figure 3.19 Notched laminate before and after failure subjected to F.L.C.1

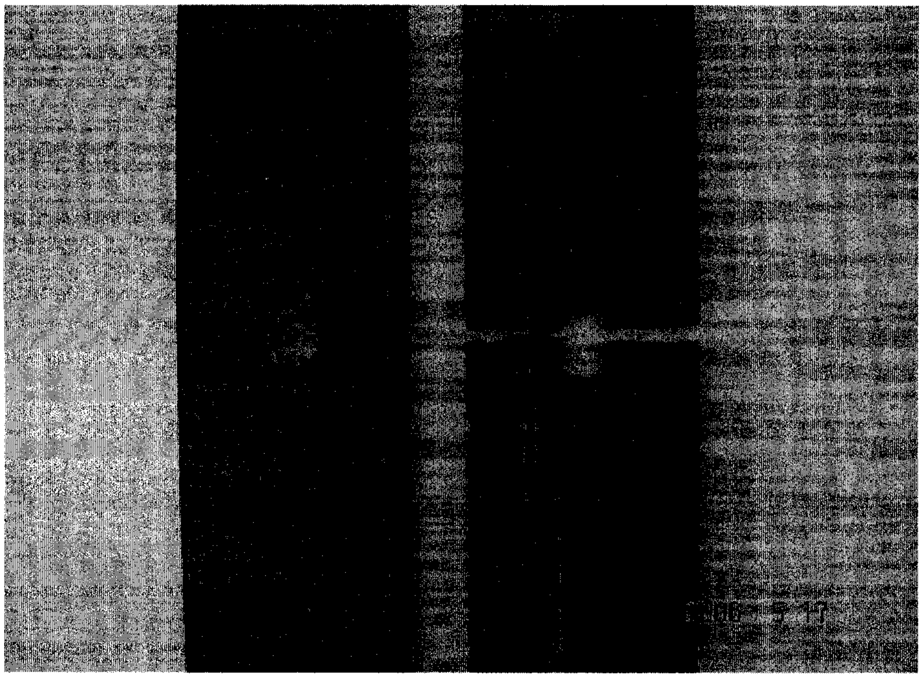


Figure 3.20 Notched laminate before and after failure subjected to F.L.C.2

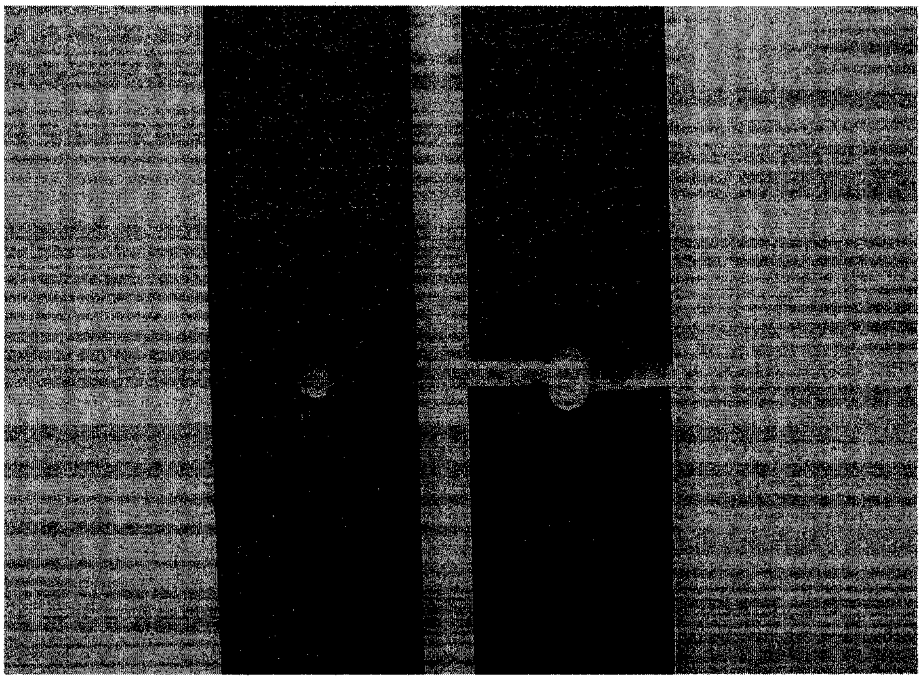


Figure 3.21 Notched laminate before and after failure subjected to F.L.C.3

The average ultimate load, average ultimate strength, S.D. and C.O.V. values of the notched laminate not subjected to fatigue loading and of those subjected to Fatigue Loading Condition No.1 (F.L.C.1), F.L.C.2 and F.L.C.3 are given in Table 3.3.

S.No.	Notched laminate	Average Ultimate Load (N)	Average Ultimate Strength (MPa)	S.D. value (MPa)	C.O.V. value (%)
1.	Not subjected to Fatigue Loading	34932	577.10	20.13	3.49
2.	Subjected to F.L.C.1	33819	553.76	15.57	2.81
3.	Subjected to F.L.C.2	33548	545.91	14.81	2.71
4.	Subjected to F.L.C.3	33418	536.49	11.023	2.05

Table 3.3 Comparison of average ultimate load, average ultimate strength, S.D. and C.O.V. values of notched laminate not subjected to fatigue loading and of those subjected to Fatigue Loading Conditions No.1 (F.L.C.1), F.L.C.2 and F.L.C.3

From Table 3.3, the reduction in Average Ultimate Strength of Notched Laminate (AUSNL) subjected to Fatigue Loading Condition No.1 (F.L.C.1), F.L.C.2 and F.L.C.3 is found to be 23.34 MPa (4.04%), 31.19 MPa (5.4%) and 48.46 MPa (8.4%) respectively. The difference between the AUSNL values of notched laminate not subjected to fatigue loading and that subjected to F.L.C.1 is 23.34 MPa (4.04%), though the difference between the Young's modulus (Ey) of notched laminate before and after subjecting to F.L.C.1 is 1.1%, which clearly indicates that the modes of damage during the later stage of fatigue loading such as delamination, crack coupling and interfacial debonding cause considerably less deformation thereby affecting the ultimate strength of the notched laminate. The difference between the AUSNL values of notched laminates subjected to

Fatigue Loading Condition No.2 (F.L.C.2) and F.L.C.1 is 7.85 MPa, which is comparatively less indicating that the effect of changing the maximum stress value (S_{max}) of tension-tension fatigue loading cycle does not affect the reduction in ultimate strength of notched laminate considerably.

But the difference between the Average Ultimate Strength of Notched Laminate (AUSNL) values of notched laminates subjected to Fatigue Loading Condition No.3 (F.L.C.3) and F.L.C.1 is 17.27 MPa, which is comparatively high indicating that the effect of changing the stress ratio (R) of tension-compression fatigue loading cycle affects the reduction in ultimate strength of notched laminate considerably. By comparing the S.D. and C.O.V. values of notched laminates not subjected to fatigue loading and that subjected to different types of fatigue loading conditions, it can be concluded that the notched laminates subjected to fatigue loading condition display lesser variability in their ultimate strength values than the notched laminates not subjected to fatigue loading. The fatigue loading confines the damage zone of notched laminates around the hole region, which leads to the lesser variability in ultimate strength values. The S.D. and C.O.V. values reduce when the fatigue loading condition of the notched laminate is changed from F.L.C.1 to F.L.C.2 and from F.L.C.2 to F.L.C.3.

3.9 Characteristic length of the notched laminates

In notched composite laminate, the stress concentration effect causes dispersed damage in a region around the hole and these are characterized by the characteristic length of notched laminate. Two different stress failure criteria of notched laminate are used in the present thesis to find the characteristic length, namely

(i) Point Stress Criterion (PSC)

(ii) Average Stress Criterion (ASC)

Both failure criteria require the following parameters to calculate the characteristic length of the notched laminate:

(i) Ultimate strength of the notched laminate (σ_N)

(ii) Ultimate strength of the unnotched laminate with the same dimension as that of the notched laminate (σ_o)

The methodology used in the point stress and average stress criteria to find the characteristic length is explained in Sections 3.9.1 and 3.9.2 respectively.

3.9.1 Point Stress Criterion

Point stress failure criterion [1] assumes that failure of the notched laminate occurs when the stress (σ_y), at some distance d_o away from the hole of the laminate is equal to or greater than the un-notched strength of the laminate (σ_o). This distance d_o represents the characteristic length of the notched laminate in point stress criterion.

Mathematically it can be written as,

$$\sigma_y (x,0) \,|_{\, x = R + do} = \sigma_o \qquad (3.7)$$

where, σ_y is the stress at any point in the laminate and R is the radius of the hole.

A graphical representation of the point stress criterion is shown in Figure 3.22.

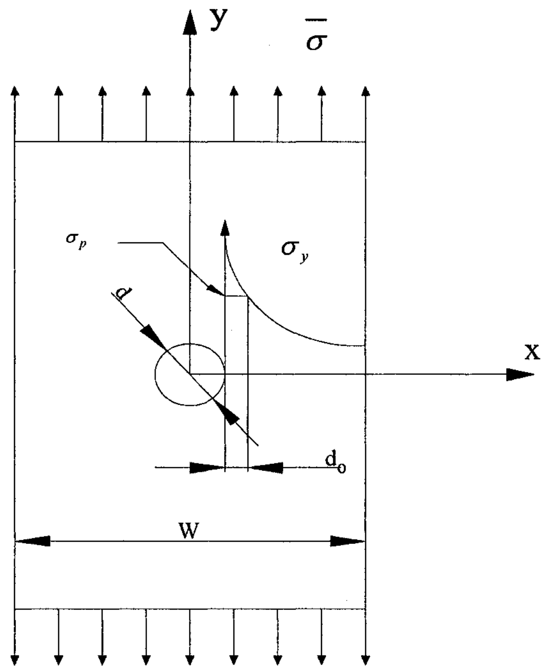


Figure 3.22 Graphical representation of point stress criterion

For infinite orthotropic plate containing a circular hole, the approximate solution of stress distribution [58] along the axis perpendicular to the loading direction is given by the formula

$$\sigma_y(x,0)=\frac{\bar{\sigma}}{2}\left\{2+\left(\frac{R}{x}\right)^2+3\left(\frac{R}{x}\right)^4-\left(K_T^\infty-3\right)\left[5\left(\frac{R}{x}\right)^6-7\left(\frac{R}{x}\right)^8\right]\right\} \text{ , for } x > R \tag{3.8}$$

where, $\bar{\sigma}$ is the stress applied at infinity and K_T^∞ denotes the stress concentration factor at the edge of the hole of the infinite plate and is given by the formula

$$K_T^\infty = 1+\sqrt{\frac{2}{A_{22}}\left[\sqrt{A_{11}A_{22}}-A_{12}+\frac{A_{11}A_{22}-A_{12}^2}{2A_{66}}\right]} \tag{3.9}$$

where, A_{ij}, i,j = 1,2,6 are the components of the in-plane stiffness matrix with 1 and 2 directions being parallel and transverse to the loading direction respectively.

Substituting equation (3.8) into equation (3.7) and replacing $\bar{\sigma}$ and σ_y (x,0) by σ_o and σ_N, we obtain

$$\frac{\sigma_N}{\sigma_o} = \frac{2}{2 + \xi_1^2 + 3\xi_1^4 - (K_T^\infty - 3)(5\xi_1^6 - 7\xi_1^8)} \tag{3.10}$$

where, $\xi_1 = \frac{R}{R + d_0}$.

3.9.2 Average Stress Criterion

The average stress criterion [1] considers the average stress over a characteristic length instead of considering the stress at a point as in the case of point stress criterion. In other words, the average stress criterion assumes that failure occurs when the average stress ($\bar{\sigma}_y$) over some distance (a_o) away from the hole edge is equal or greater than the ultimate strength of the un-notched laminate (σ_o). The distance a_o represents the characteristic length of the notched laminate in average stress criterion.

Mathematically it can be written as

$$\frac{1}{a_o} \int_R^{R+a_0} \sigma_y(x,o)dx = \sigma_o \tag{3.11}$$

where, R is the radius of the hole.

The graphical representation of the average stress criterion is shown in Figure 3.23.

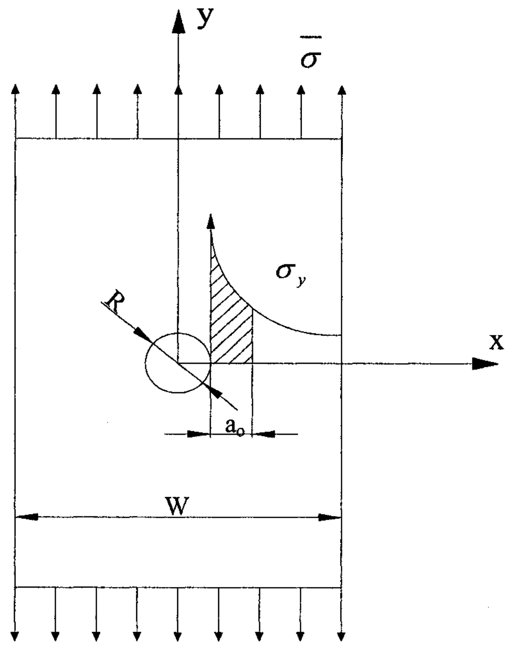


Figure 3.23 Graphical representation of average stress criterion

In the case of orthotropic plate with a hole, the solution is obtained by substituting equation (3.8) into equation (3.11) and it is given by

$$\frac{\sigma_N}{\sigma_o} = \frac{2(1-\xi_2)}{2-\xi_2^2-\xi_2^4+(K_T^\infty-3)(\xi_2^6-\xi_2^8)} \qquad (3.12)$$

where, $\xi_2 = \dfrac{R}{R+a_0}$ and K_T^∞ is same as in equation (3.9).

3.9.3 Calculation of characteristic lengths of notched laminates

The point stress and average stress criteria have three unknowns

(i) Ultimate strength of the notched laminate (σ_N)

(ii) Ultimate strength of the unnotched laminate (σ_o)

(iii) Characteristic length, d_o or a_o.

The procedure is to first obtain a set of un-notched and notched ultimate strengths of laminates from experiments. Then substitute these data in equations (3.10) and (3.12) to determine d_o and a_o respectively. In reference [5], twenty-five samples of un-notched laminates were tested to find the Average Ultimate Strength of Un-Notched Laminate (AUSUNL). The average ultimate load, average ultimate strength, S.D. and C.O.V. values of un-notched and notched laminates are given in Table 3.4.

S.No.	Laminate Type	Average Ultimate Load (N)	Average Ultimate Strength (MPa)	S.D. value (MPa)	C.O.V. value (%)
1.	Un-notched laminate	80496	1073.40	82.34	7.67
2.	Notched laminate	34932	577.10	20.13	3.49

Table 3.4 Comparison of average ultimate load, average ultimate strength, S.D. and C.O.V. values of un-notched and notched laminates

From Table 3.4, it can be seen that the Average Ultimate Strength of Un-Notched Laminate value (AUSUNL) is almost twice that of Average Ultimate Strength of Notched Laminate (AUSNL) value. This shows the effect of making notch in composite laminates on its ultimate strength. By comparing the S.D. and C.O.V. values of un-notched and notched laminates, it can be concluded that the notched laminates display lesser variability in their ultimate strength values than the un-notched laminates. The damage in notched laminates occurs in a small region around the notch, which leads to its ultimate failure. The failure region is small compared to the entire laminated composite plate size, which leads to less variation in their ultimate strength. But in the case of un-notched laminates, the damage is distributed throughout the laminated composite plate and leads to more variation in their ultimate strength.

The ultimate strength values of notched laminate subjected to different fatigue loading conditions and un-notched laminate, given in Table 3.3 and Table 3.4 respectively, are substituted in equation (3.10) to find the characteristic length d_o of all the three laminate coupons for each fatigue loading condition. The Mean, Standard Deviation (S.D.) and Coefficient of Variation (C.O.V.) values of d_o are found out using equations (3.1), (3.2) and (3.3) respectively and they are shown in Table 3.5.

S.No.	Notched laminate	Mean Value of d_o	S.D. Value of d_o	C.O.V. Value of d_o
		mm	mm	%
1.	Not subjected to Fatigue Loading	0.8759	0.1096	12.51
2.	Subjected to F.L.C.1	0.9110	0.0481	5.957
3.	Subjected to F.L.C.2	0.9283	0.0445	5.660
4.	Subjected to F.L.C.3	0.9363	0.0324	4.243

Table 3.5 Mean, S.D. and C.O.V. values of characteristic length d_o for notched laminates not subjected to fatigue loading and that subjected to Fatigue Loading Condition No.1(F.L.C.1), F.L.C.2 and F.L.C.3

The ultimate strength values of notched laminates subjected to different fatigue loading conditions and un-notched laminate, given in Table 3.3 and Table 3.4 respectively, are substituted in equation (3.12) to find the characteristic length a_o of all the three laminate coupons for each fatigue loading condition. The Mean, Standard Deviation (S.D.) and Coefficient of Variation (C.O.V.) values of a_o are found out using equations (3.1), (3.2) and (3.3) respectively and they are presented in Table 3.6.

S.No.	Notched laminate	Mean Value of a_o	S.D. Value of a_o	C.O.V. Value of a_o
		mm	mm	%
1.	Not subjected to Fatigue Loading	3.0716	0.6144	20.00
2.	Subjected to F.L.C.1	2.6600	0.2536	9.533
3.	Subjected to F.L.C.2	2.5633	0.2281	8.899
4.	Subjected to F.L.C.3	2.4433	0.1617	6.616

Table 3.6 Mean, S.D, C.O.V. values of characteristic length a_o for notched laminates not subjected to fatigue loading and that subjected to Fatigue Loading Condition No.1 (F.L.C.1), F.L.C.2 and F.L.C.3

From Table 3.5 and Table 3.6, it can be seen that the characteristic lengths (a_o and d_o) of notched laminates subjected to Fatigue Loading Condition No.1 (F.L.C.1), F.L.C.2 and F.L.C.3, show less variability in their S.D. and C.O.V. values than the notched laminate not subjected to fatigue loading. The Mean value of a_o of notched laminate subjected to F.L.C.1, F.L.C.2 and F.L.C.3 is less when compared to notched laminate not subjected to fatigue loading, which clearly indicates that failure region around the notch reduces for the notched laminate subjected to fatigue loading conditions and leads to their premature failure. The characteristic length values d_o and a_o were used in calculating the point stress and average stress of the laminates for any loading, which is discussed in Chapter 4 in detail.

3.10 Static testing of edge-notched laminates

The effect of hole location on the ultimate strength of notched laminate was studied by testing laminate coupons with two semi-circular holes at both edges of the laminate, called as edge-notched laminates, under static loading condition. To facilitate comparison of results, laminate coupons with the same configuration and dimensions as that of center-notched laminate (Section 3.4) were used and the radius of the semi-circular hole is set equal to the radius of the center-notch. Considering the variations in the material properties, 15 samples were tested to get the Average Ultimate Strength of Edge-Notched Laminate (AUSENL). The manufacturing process of edge-notched laminate is similar to that of center-notched laminate (Section 3.2) until the preparation of laminated composite plate. However the method of preparation of laminate coupons using the laminated composite plate is different and it is explained in Section 3.10.1.

3.10.1 Preparation of edge-notched laminate coupons

In the case of center-notched laminate, laminate coupons of required dimensions were cut from the laminated composite plate using water-cooled rotary diamond cutter and it is followed by drilling a hole at the center of the laminate coupon. But for edge-notched laminate, due to the complexity in drilling two semi-circular holes at both edges of the laminate coupon, a series of holes were drilled in the laminated composite plate which is followed by cutting it into laminate coupons of required dimension. The step-by-step procedure of the preparation method of laminate coupons for edge-notched laminate is as follows.

(i) According to the number of laminate coupons required, laminated composite plate is prepared using the procedure mentioned in Section 3.2. Laminate plates of size 12 in. x 11 in. (or) 10.5 in x 11 in. were prepared to get 6 (or) 5 laminate coupons respectively.

(ii) A series of holes were drilled on the laminated composite plate with the required hole diameter. Figure 3.24 shows the laminated composite plate of size 12 in. x 11 in. with the series of drilled holes. The distance W is equivalent to the width of the laminate coupon and the distance D_a is equivalent to the hole diameter plus an allowance.

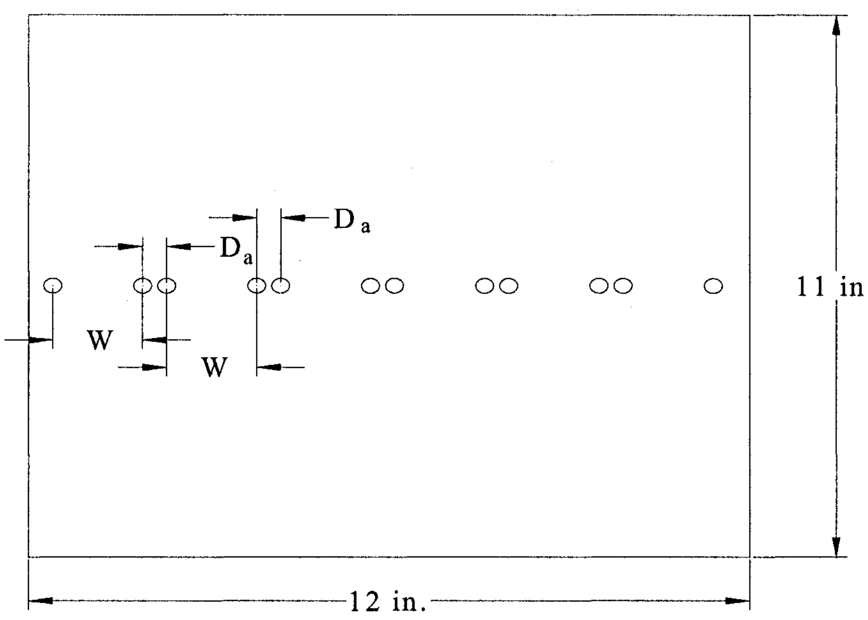


Figure 3.24 Laminated composite plate of size 12 in. x 11 in. with a series of drilled holes

(iii) Using water-cooled rotary diamond cutter, the laminated composite plate is cut along its length through the centers of each hole to get the edge-notched laminate coupons. Figure 3.25 shows the cutting lines along which the laminated composite plate is cut and the laminate coupon numbers from 1 to 6.

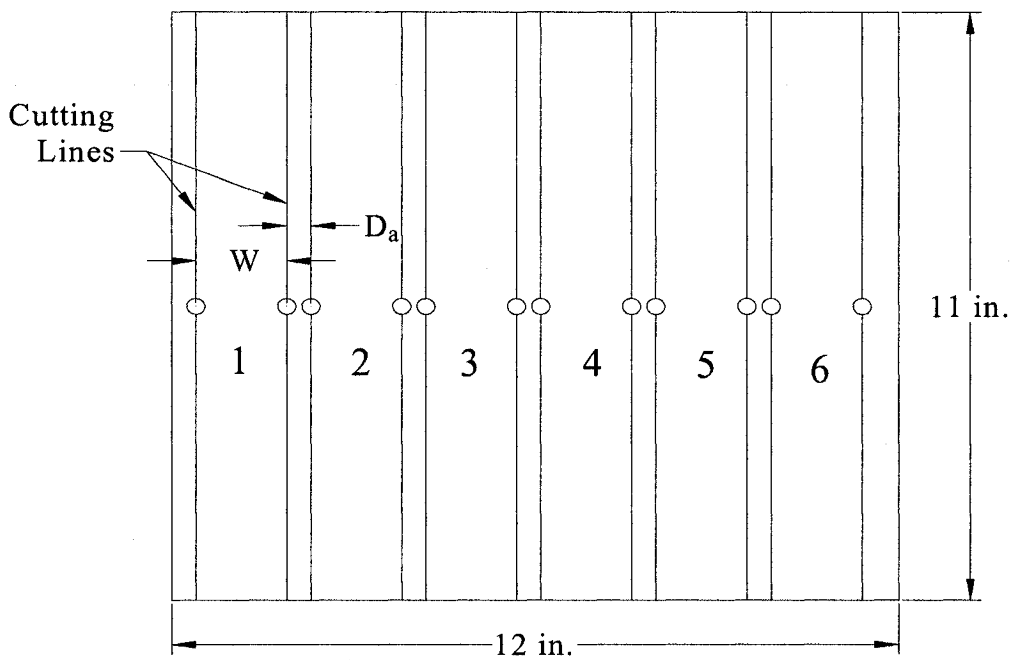


Figure 3.25 Laminated composite plate with the cutting lines and laminate coupon numbers

(iv) The six edge-notched laminate coupons were obtained from the 12 in. x 11 in. laminated composite plate. Figure 3.26 shows the laminate coupon No.1 obtained from the laminated composite plate.

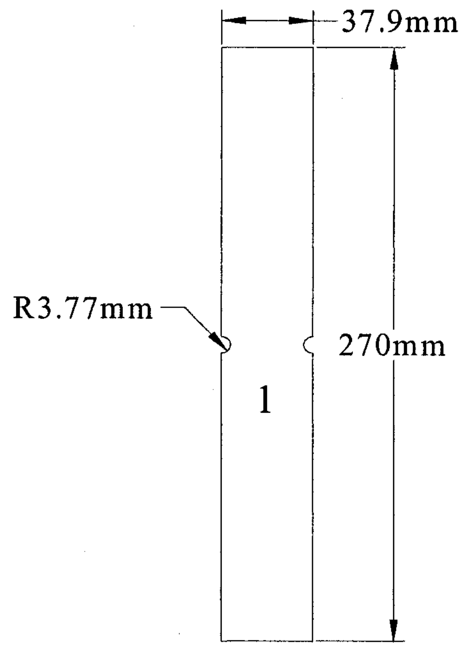


Figure 3.26 Laminate coupon No.1 obtained from the laminated composite plate

3.10.2 Static Test Results

The 100 KN MTS machine was used for the static testing of edge-notched laminates. The loading parameters were determined by conducting trial tests on the edge-notched laminates. The loading parameters are:

(i) Loading rate = 111 N/s

(ii) Grip Pressure = 15 MPa

A uniform axial load is applied to the edge-notched laminate coupons until failure and the ultimate load was determined. During the testing, the load and displacement parameters are stored in the computer for every 0.5 seconds. An edge-notched laminate coupon before and after failure is shown in Figure 3.27.

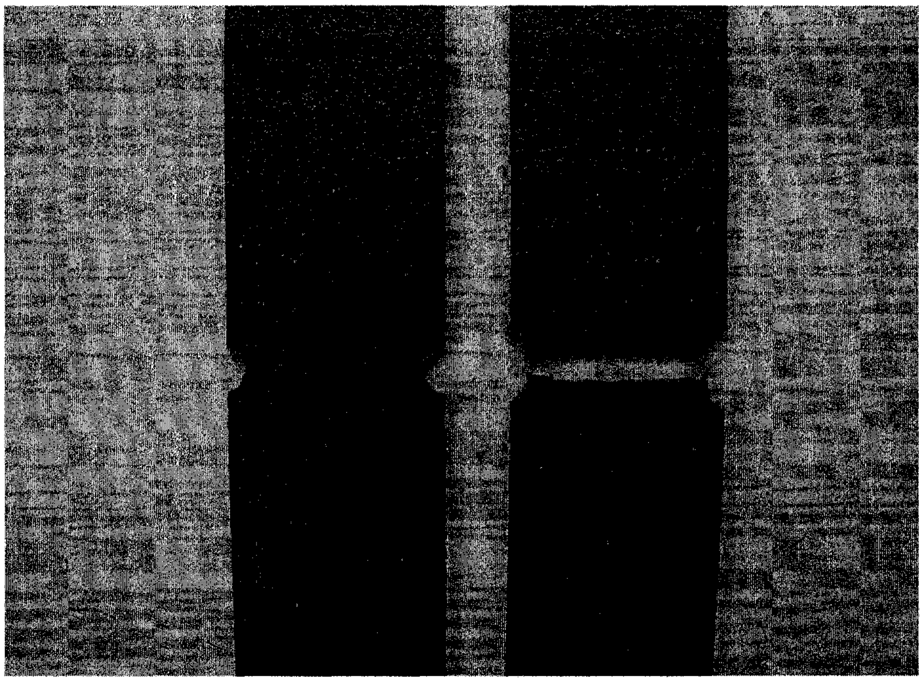


Figure 3.27 Edge-notched laminate coupon before and after failure

In Figure 3.27, it can be seen that the failure of edge-notched laminate occurs close to the center line connecting the two semi-circular hole edges. This indicates that the crack initiates near the two semi-circular hole edges and propagates along the center line until the ultimate failure of laminate coupon. Fifteen edge-notched laminate coupons were tested under static loading condition until failure. Table 3.7 shows the laminate coupon label, ultimate load, cross-sectional area and ultimate strength of the edge-notched laminates.

Laminate Coupon Label	Ultimate Load	Cross-sectional Area	Ultimate Strength
	N	mm^2	MPa
F4S1	33743	61.683	547.04
F4S2	32910	61.976	531.03
F4S3	33471	60.696	551.45
F4S4	35005	61.366	570.44
F4S6	34161	60.919	560.77
F5S1	33820	59.700	566.51
F5S2	33027	60.371	547.08
F5S3	34171	60.249	567.17
F5S4	32028	60.594	528.58
F5S5	32954	60.899	541.13
F6S1	34291	61.549	557.15
F6S2	34140	62.216	548.74
F6S3	31574	58.542	539.35
F6S4	33801	61.845	546.55
F6S5	34547	60.980	566.54

Table 3.7 Laminate coupon label, ultimate load, cross-sectional area and ultimate strength of edge-notched laminates

From Table 3.7, the average ultimate load and Average Ultimate Strength of Edge-Notched Laminate (AUSENL) can be calculated and it is compared with the average ultimate load and Average Ultimate Strength of Center-Notched Laminate (AUSCNL). Table 3.8 shows the average ultimate load, average ultimate strength, Standard Deviation (S.D.) value and Coefficient of Variation (C.O.V.) value of center-notched and edge-notched laminates.

S.No.	Laminate type	Average Ultimate Load	Average Ultimate Strength	S.D. value	C.O.V. value
		N	MPa	MPa	%
1.	Center-Notched Laminate	34932	577.10	20.13	3.49
2.	Edge-Notched Laminate	33576	551.30	12.765	2.32

Table 3.8 Average ultimate load, average ultimate strength, S.D. and C.O.V. values of center-notched and edge-notched laminates

The reduction between the Average Ultimate Strengths of Center-Notched and Edge-Notched laminates is 25.8 MPa, which shows the effect of changing the hole location in the laminate coupons on the ultimate strength. The damage near the hole edge of edge-notched laminate is more when compared to center-notched laminate, which leads to the early failure of edge-notched laminate. The edge-notched laminate also shows less variability in the ultimate load value of the laminate coupons than center-notched laminate, which is evident from the reduction in the S.D. and C.O.V. values of edge-notched laminate.

3.10.3 Characteristic lengths of edge-notched laminates

Point stress and average stress criteria were used to determine the characteristic lengths of edge-notched laminates. In both the stress criteria, ultimate strength of edge-notched laminate (σ_N) and ultimate strength of un-notched laminate with the same dimensions as that of edge-notched laminate (σ_o) are required to calculate the characteristic length. The methodologies used for the point stress and average stress criteria are explained in sections 3.10.1 and 3.10.2 respectively. A MATLAB® code was written using the methodology of point stress and average stress criteria to determine characteristic lengths d_o and a_o respectively. Table 3.9 shows the Mean, Standard Deviation (S.D.), Coefficient of Variation (C.O.V.), and Maximum and Minimum values of d_o for center-notched and edge-notched laminates.

S.No.	Laminate type	Mean value of d_o	S.D. value of d_o	C.O.V. value of d_o	Maximum Value of d_o	Minimum Value of d_o
		mm	mm	%	mm	mm
1.	Center-Notched Laminate	0.8759	0.1096	12.51	1.1180	0.6590
2.	Edge-Notched Laminate	0.8095	0.0933	11.52	0.9550	0.6540

Table 3.9 Mean, S.D., C.O.V. and Maximum and Minimum values of d_o for center-notched and edge-notched laminates

In Table 3.9, it can be seen that the characteristic length d_o of edge-notched laminate is less than that of center-notched laminate indicating that the point stress occurs nearer to the hole edge, which leads to the early failure of edge-notched laminate. There is also lesser variability in the d_o value of edge-notched laminate than center-notched laminate, which is evident from the S.D. and C.O.V values shown in Table 3.9.

Table 3.10 shows the Mean, Standard Deviation (S.D.), Coefficient of Variation (C.O.V.), and Maximum and Minimum values of a_o for center-notched and edge-notched laminates.

S.No.	Laminate type	Mean value of a_o	S.D. value of a_o	C.O.V. value of a_o	Maximum Value of a_o	Minimum Value of a_o
		mm	mm	%	mm	mm
1.	Center-Notched Laminate	3.0788	0.6160	20.01	4.56	1.96
2.	Edge-Notched Laminate	2.6980	0.4857	18.00	3.50	1.93

Table 3.10 Mean, S.D., C.O.V. and Maximum and Minimum values of a_o for center-notched and edge-notched laminates

In Table 3.10, it can be seen that the characteristic length a_o of edge-notched laminate is less than that of center-notched laminate indicating that the area of stress concentration near the hole edge for edge-notched laminate is less, which leads to the early failure of edge-notched laminate. There is also lesser variability in the a_o value of edge-notched laminate than center-notched laminate, which is evident from the S.D. and C.O.V. values shown in Table 3.10.

3.11 Micro structural study of edge-notched and center-notched laminate coupons

The micro structural study of laminate coupons is conducted to observe any form of defects in the laminate. At least one laminate coupon from each laminated composite plate manufactured is tested for defects. If any serious defects are found in the laminate coupon, the entire laminated composite plate is discarded due to the reason that testing laminate coupons with defects can give erroneous results. Some common defects that can occur in laminate coupons are: presence of void content, delamination, broken fibers, etc. The reasons for the formation of these defects are: uneven rolling of plies during hand lay-up process, poor vacuum bagging, improper pressurization of furnace during autoclave cure process, fiber breakage during drilling operation, etc.

The pictorial representation of the micro structural study of laminate coupons is shown in Figure 3.28. During drilling operation, the edges of the hole are prone to be damaged due to fiber breakage and it is considered as a major defect. So the region of interest for microscopic study is the cross-sectional area near the hole edge. First the laminate coupon is cut across the width at the center as shown in Figure 3.28(b). The cross-sectional area has to be polished in order to be seen clearly through the optical microscope. Since the specimens for micro structural study are quite small, it becomes difficult to polish the surface of the specimen. To overcome this problem, the region of interest is immersed in a bowl containing resin and heated to 80°C in a furnace for about 40 minutes. It is then allowed to cool down at room temperature. This in-turn creates a bigger surface and hence a better grip for polishing. The specimen with the resin bowl is shown in Figure 3.28(c).

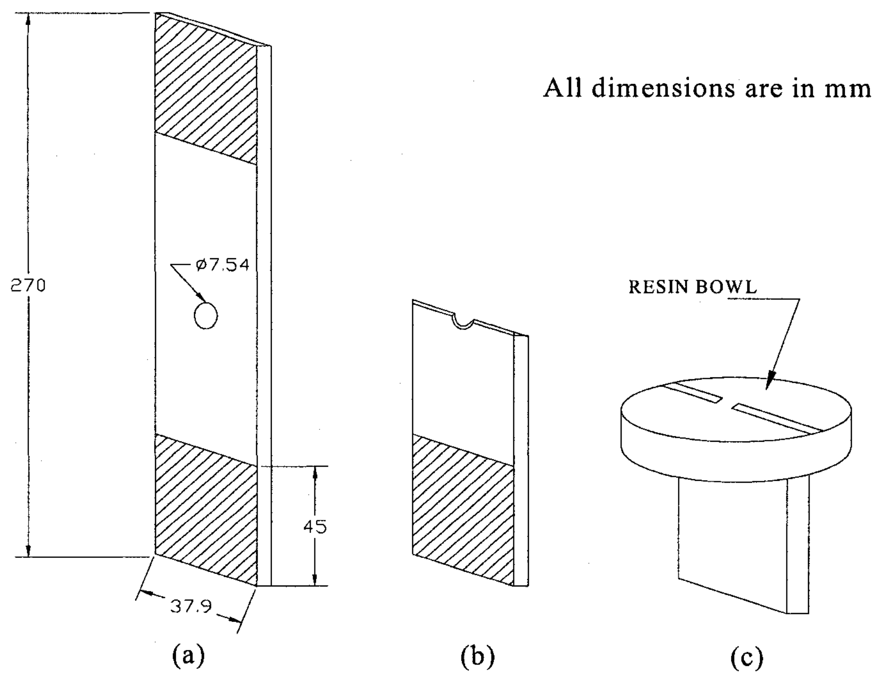


Figure 3.28 Pictorial representation of specimen preparation for microscopic study of center-notched laminate coupons

Now the surface of the resin bowl is polished by grinding operation. First the surface of the resin is treated with 200 grit SiC paper and subsequently with SiC papers that have 300, 400, 600, 800 and 1200 grit levels. While grinding the specimen, care should be taken to maintain a perfect horizontal surface. After the grinding operation, the specimens are put under an optical microscope fitted with a video camera. Microscopic images of the specimens are taken by using Clemex Vision software, which is connected to optical microscope.

A typical microscopic image of center-notched $[0/90]_{4s}$ specimen with 7.54 mm hole before failure is shown in Figure 3.29. A magnification value of 50X is used to see the image.

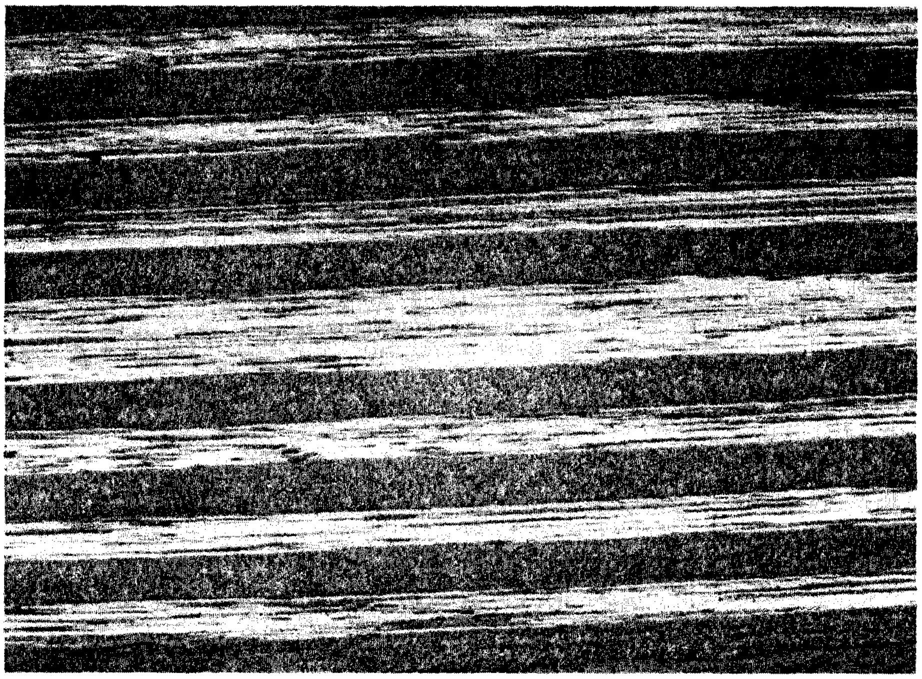


Figure 3.29 A typical image of $[0/90]_{4s}$ specimen with 7.54mm hole before failure observed under the optical microscope

In Figure 3.29, the white regions indicate the 90^0 layers and the dark regions indicate the 0^0 layers. There is no significant damage found in the image. The layers of 0^0 and 90^0 fibers are arranged in good shape and almost at equal distance from each other.

Damage occurs near the hole edge of center-notched laminates due to the application of fatigue loading. In order to study the damage caused by the application of fatigue loading, microscopic images of center-notched laminates subjected Fatigue Loading Condition No.1 (F.L.C.1), F.L.C.2 and F.L.C.3 are taken near the hole edge of the laminate coupon. Figures 3.30, 3.31 and 3.32 show the microscopic image of $[0/90]_{4s}$ center-notched laminates with 7.54mm hole diameter after failure subjected to Fatigue Loading Condition No.1, F.L.C.2 and F.L.C.3 respectively.

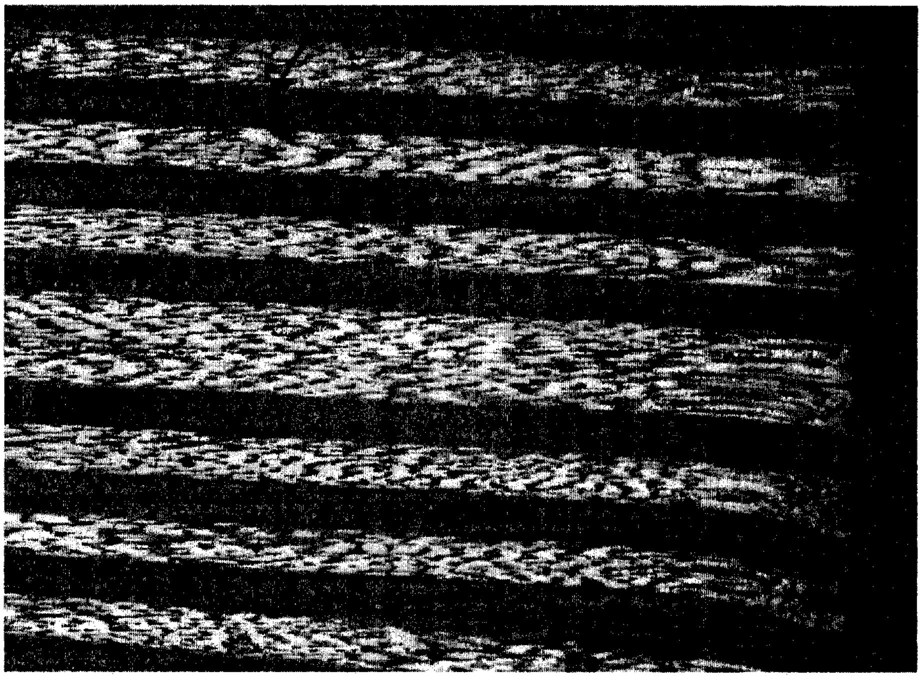


Figure 3.30 Microscopic image of center-notched laminate subjected to Fatigue Loading Condition No.1 near the hole region (after failure)

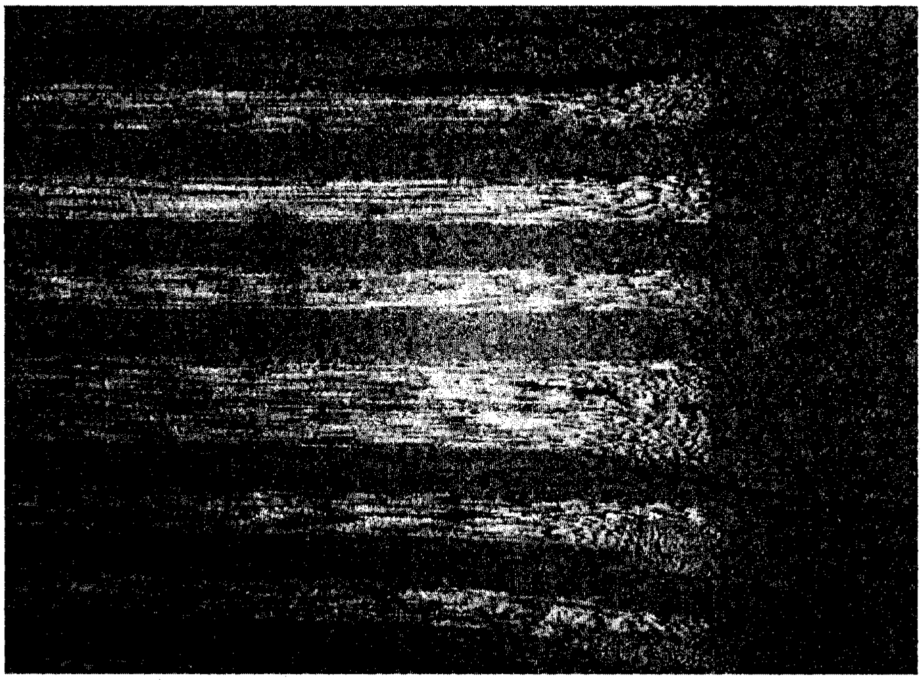


Figure 3.31 Microscopic image of center-notched laminate subjected to Fatigue Loading Condition No.2 near the hole region (after failure)

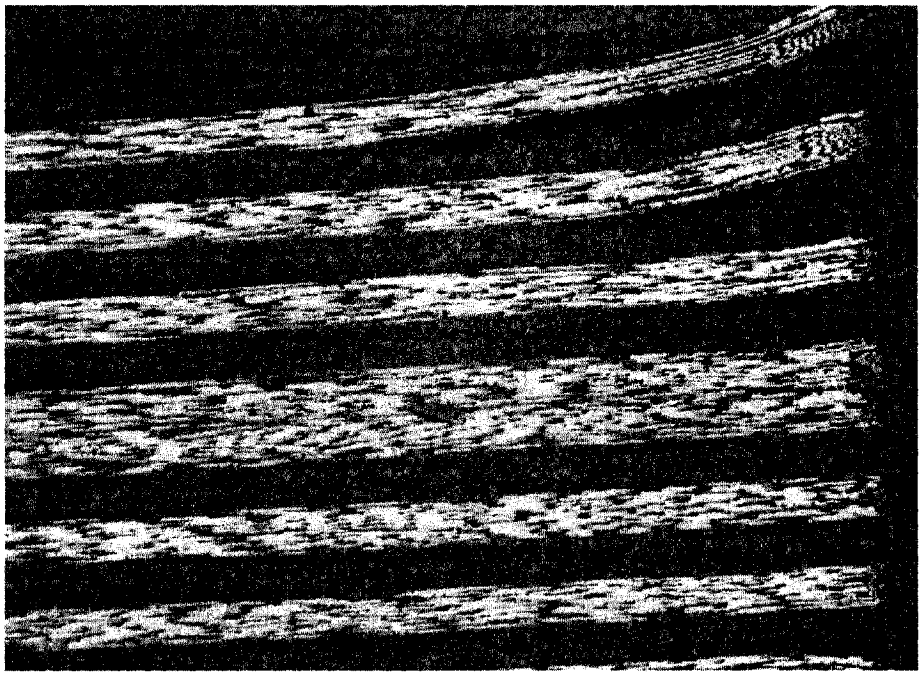


Figure 3.32 Microscopic image of center-notched laminate subjected to Fatigue Loading Condition No.3 near the hole region (after failure)

In Figures 3.30, 3.31 and 3.32, the damage near the hole region of center-notched laminates subjected to fatigue loading are clearly visible in the microscopic images. Damages such as matrix cracking, fiber breakage, fiber-matrix debonding, etc. occurs near the hole edge of center-notched laminates subjected to fatigue loading, which leads to their premature failure. From Figures 3.30 and 3.31, it can be seen that the damage in center-notched laminate subjected to Fatigue Loading Condition No.2 (F.L.C.2) is more when compared to center-notched laminate subjected to F.L.C.1. From Figures 3.31 and 3.32, one can see that the damage in center-notched laminate subjected to F.L.C.3 is even severe when compared to center-notched laminate subjected to F.L.C.2.

The damage near the hole region of edge-notched laminates are studied by taking the microscopic image of edge-notched laminate before and after failure. Figures 3.33 and 3.34 show the microscopic image of edge-notched laminate near the hole region before and after failure.

In Figure 3.34, it can be seen that the damage near the hole region of edge-notched laminate is more when compared to the center-notched laminates subjected to fatigue loading conditions. Thus the ultimate strength of edge-notched laminate is less when compared to the center-notched laminate subjected to fatigue loading conditions.

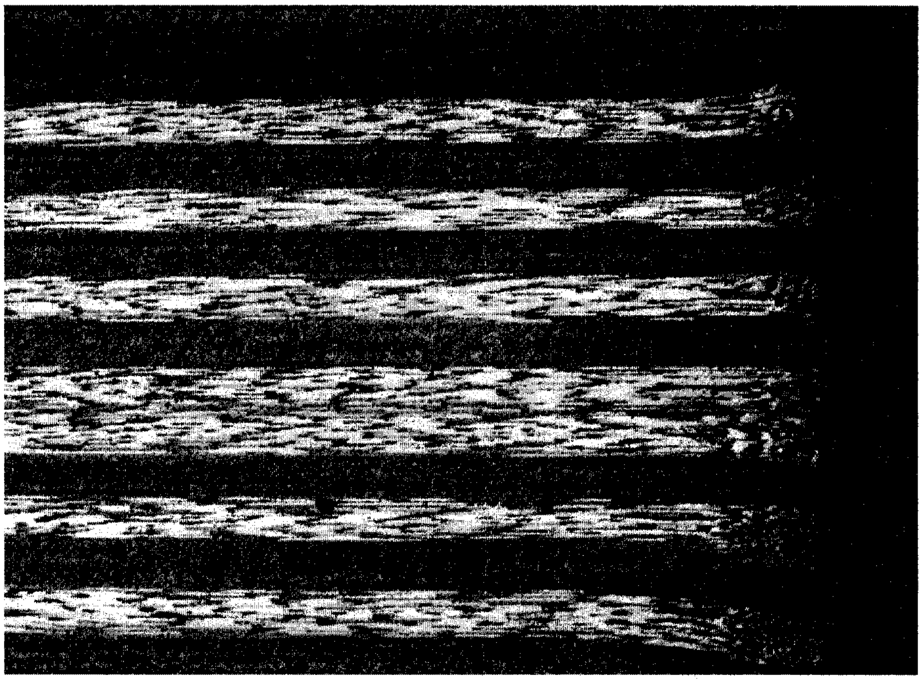


Figure 3.33 Microscopic image of edge-notched laminate near the hole region (before failure)

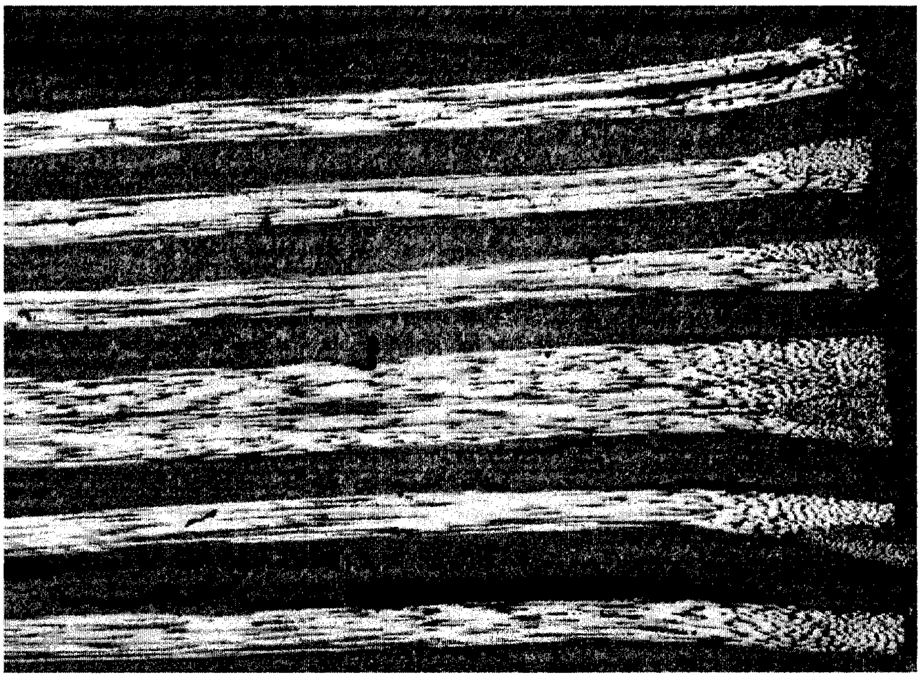


Figure 3.34 Microscopic image of edge-notched laminate near the hole region (after failure)

3.12 Conclusions and discussions

In Sections 3.2, 3.3, 3.4 and 3.10.1, the manufacturing methods and dimensions of center-notched and edge-notched laminate prepared for testing were discussed. In Section 3.5, tests conducted in reference [5] to get the material properties of the lamina were discussed. The experimental values of Mean, Standard Deviation (S.D.) and Coefficient of Variation (C.O.V) of Young's modulus, Poisson's ratio and shear modulus are listed in Table 3.1.

In Sections 3.7 and 3.8, the fatigue and static testing of center-notched laminates were discussed in detail. A linear reduction in Young's modulus was noticed for center-notched laminate subjected to Fatigue Loading Condition No.1 (F.L.C.1). For center-notched laminate subjected to F.L.C.2 and F.L.C.3, the reduction in Young's modulus was linear until ½ million cycles, after which it is non-linear. Table 3.3 clearly shows the effect of fatigue loading on the ultimate strength of center-notched laminates.

In Sections 3.8 and 3.10.2, photographs of center-notched laminate subjected to fatigue loading conditions and edge-notched laminate before and after failure are taken to explain the failure mode. In Section 3.11, micro structural studies on center-notched and edge-notched laminates are described to explain the micro structure of matrix and fiber, before and after failure.

The ultimate strength values of center-notched laminates subjected to fatigue loading, edge-notched laminates, un-notched laminates obtained from Tables 3.3, 3.4 and 3.7 were used to calculate the characteristic lengths, a_0 and d_0, by using average stress criterion and point stress criterion.

The value of characteristic length thus obtained will be used to conduct the stochastic finite element analysis for controlled hole laminates in Chapter 4, to obtain mean and standard deviation values of average and point stress values and finally to calculate the reliability.

CHAPTER 4

STOCHASTIC ANALYSIS

4.1 Introduction

Composite laminates show significant randomness in the stress distribution, due to variation in their material and geometric properties, which necessitates the implementation of stochastic simulation process in the present work. Variations in material properties are due to the stochastic spatial variations in fiber, matrix and their interface properties. The variations are also induced during the manufacturing process of composite laminates, which produces number of defects, such as, void content, misalignment of the orientation of plies, variation in the thickness of plies, etc. Geometric variations in composite laminates occur when they are machined. For example, delamination, a common form of damage in composite laminates occurs during drilling operation. Drilling holes in composite laminates create an irregularity around the circumference of the hole and leads to geometric variations. Due to the unpredictable variations in composite laminates, a study on stress distribution near the hole boundary of composite laminates is conducted.

The MATLAB® program developed for the Stochastic Finite Element Analysis (SFEA) of center-notched laminates in Chapter 2 is used in the present analysis with suitable modifications in the main program, which is given in Appendix-A, and with an addition of subroutine AVGSTR.m. The purpose of this subroutine is to calculate the point and average stress values for each and every laminate used in the analysis. The modified MATLAB® program is capable of handling the probabilistic distributions of material properties and geometric variations and can calculate the stresses over the composite laminate. The generalized program can also accept any laminate configuration, geometry and number of laminates to be analyzed.

The point stress and average stress are considered to be the prime design parameters of notched laminates. The methodology for the calculation of point and average stresses are explained in Sections 5.2 and 5.3. Subroutine AVGSTR.m is capable of calculating the point and average stress value for each and every laminate and can store them automatically in a MATLAB® file; simultaneously the maximum stress developed in the notched laminate due to the application of loading is also calculated and stored in the same file.

4.2 Controlled and Un-Controlled Hole Laminate Analysis

In this chapter, two different types of analysis were performed on notched laminates, namely

(i) Controlled Hole Laminate (CHL) Analysis

(ii) Un-controlled Hole Laminate (UCHL) Analysis

In CHL analysis, only the stochastic variation in material properties was considered in the analysis without taking into account the geometric variations. But in UCHL analysis, both stochastic variation in material properties and geometric variations, such as, variations in the hole shape and eccentricity of the hole from the center of the laminate were considered in the analysis.

Un-Controlled Hole Laminate (UCHL) analysis of notched laminates is considered to be applied in practical situations. In reference [59], the imperfections around the hole boundary were expressed in the form of an equation and it is used in the UCHL analysis. Gaussian random value generating function is used to vary the coordinates of the hole within the notched laminate, thereby achieving the eccentricity of the hole. First order auto regressive model or Markov Model, explained in Section 2.5.2, is employed in the analysis to bring in the stochastic variation of material properties.

In this chapter, center-notched laminates subjected to Fatigue Loading Condition No.1, F.L.C.2, F.L.C.3 and edge-notched laminates were analyzed using Controlled Hole Laminate (CHL) and Un-Controlled Hole Laminate (UCHL) analysis. The laminate configuration and geometric dimensions of center-notched and edge-notched laminates used in the analysis are as follows: Gage Length (L) = 180mm, Width (W) =37.9mm, Thickness (t) = 2mm, Hole Diameter (d) = 7.54mm and laminate configuration $[0/90]_{4s}$.

Stochastic simulation is performed over a number of laminates, providing adequate provisions in the MATLAB® program to calculate the point and average stresses for each and every laminate. Simulation process is carried out until the fluctuations in the point and average stress parameter values culminate. Standard deviation and co-efficient of variation values was calculated.

4.3 Calculation of Point and Average stress Parameters

Using the modified MATLAB® program, stresses at Gauss points of each and every element in the finite element mesh can be calculated. The nodal stresses in the finite element mesh were found out using the Gauss point stresses, which contribute to the corresponding node. The point and average stress may not occur exactly at a nodal point. In order to calculate the point and average stresses, a stress distribution curve (stress profile) from hole edge A to the plate boundary B is developed and it is represented by a 10^{th} order polynomial, which is given by the equation.

$$\sigma_y(x) = b_0 + b_1 x + b_2 x^2 + b_3 x^3 + b_4 x^4 + b_5 x^5 + b_6 x^6 + b_7 x^7 + b_8 x^8 + b_9 x^9 + b_{10} x^{10} \tag{4.1}$$

where,

b_i, i = 0,1,...10 are the coefficients of the polynomial. A MATLAB® function, POLYFIT.m, is used to calculate the coefficients of the polynomial. The MATLAB® function requires the nodal stress values along line AB, nodal distance values along x-axis from hole edge and the order of the polynomial as its input to determine the coefficients of the polynomial of any degree by minimizing the sum of the squares of deviations of the data from the model (least-squares fit). The stress profile obtained from the polynomial function is then used to calculate the point and average stresses by using equations 3.7 and 3.8 respectively. The distribution of nodes from hole edge A to plate boundary B along x-axis is shown in Figure 4.1. Fifteen nodes are available along the line AB.

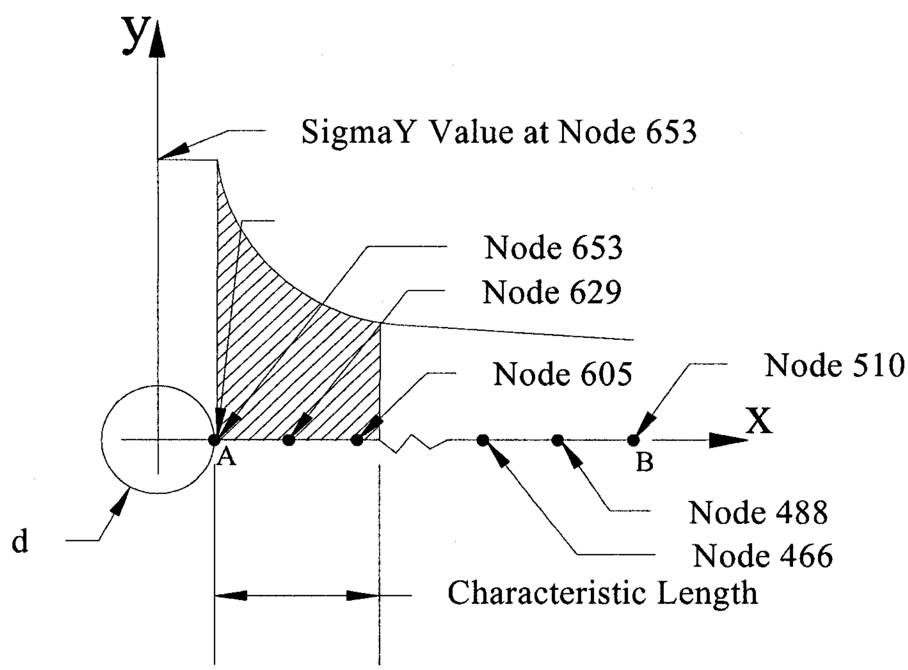


Figure 4.1 Node numbering from hole edge A to plate boundary B and stress profile near hole edge

The stress profile of center-notched laminate subjected to F.L.C.1 obtained by using 10th order polynomial and from the actual nodal stress data, which is determined by analyzing the notched laminate, is shown in Figure 4.1(a). In Figure 4.1(a), it can be seen that the stress profile curve obtained from 10th order polynomial and actual nodal stress data follow the same path, thus justifying the assumption of 10th order polynomial to describe the stress profile of the notched laminate.

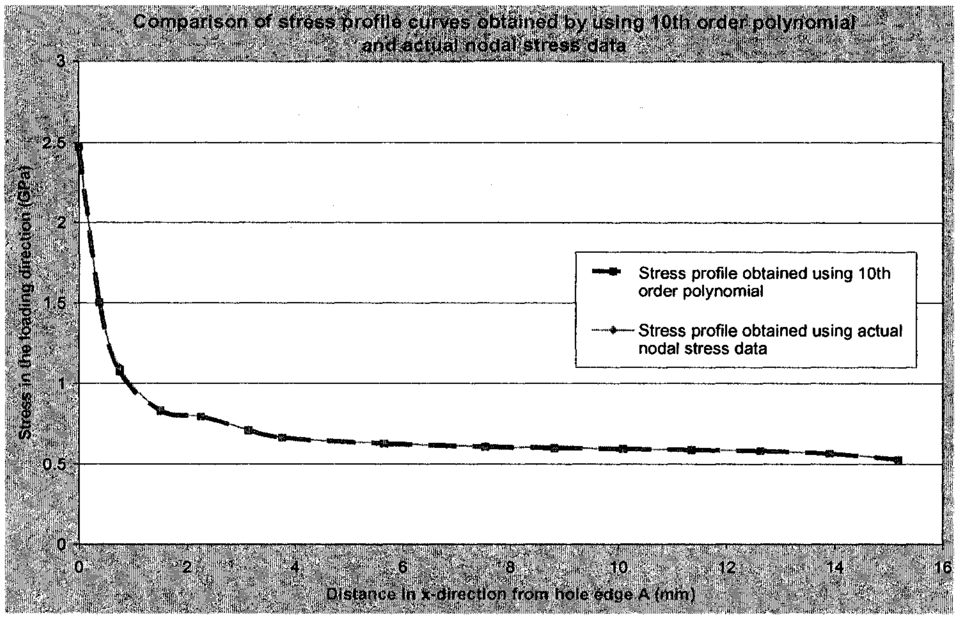
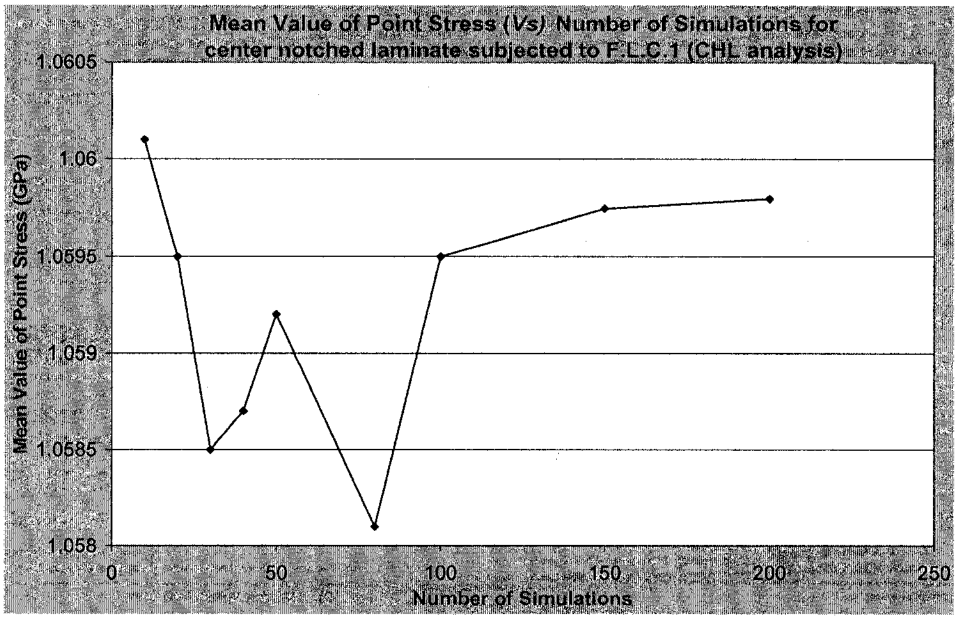


Figure 4.1(a) Comparison of stress profile curves obtained by using 10^{th} order polynomial and actual nodal stress data

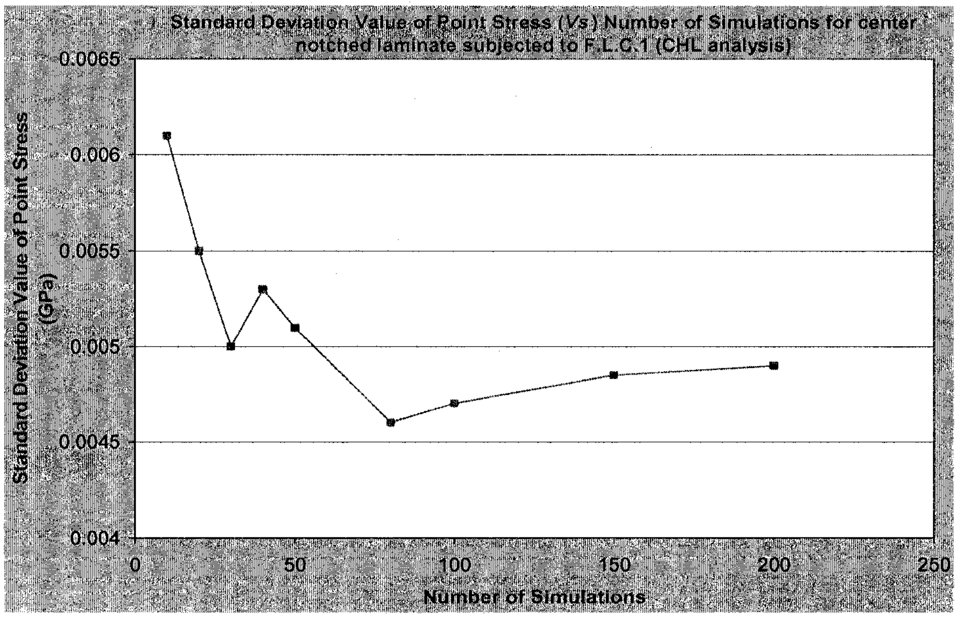
4.4 Stochastic simulation process for Controlled Hole Laminate (CHL)

In this section, Controlled Hole Laminate (CHL) analysis is performed on center-notched laminate subjected to Fatigue Loading Condition No.1 (F.L.C.1). The geometric dimensions and laminate configuration of the center-notched laminate used in the CHL analysis are explained in Section 4.2. The finite element mesh, boundary conditions and applied loading of the center-notched laminate is shown in Figure 2.3. The material properties of NCT301 prepreg, which is used to manufacture the center-notched laminate, are given in Table 3.1. The fluctuations in the material properties are expressed using stochastic processes as explained in Chapter 2. The Average Ultimate Strength of Center-Notched Laminate (AUSCNL) subjected to F.L.C.1 is used as the applied load in the present analysis. The influence of number of simulations, within the range of 1 to 200, on the mean value and Standard Deviation (S.D.) of the point stress (σ_p) and average stress

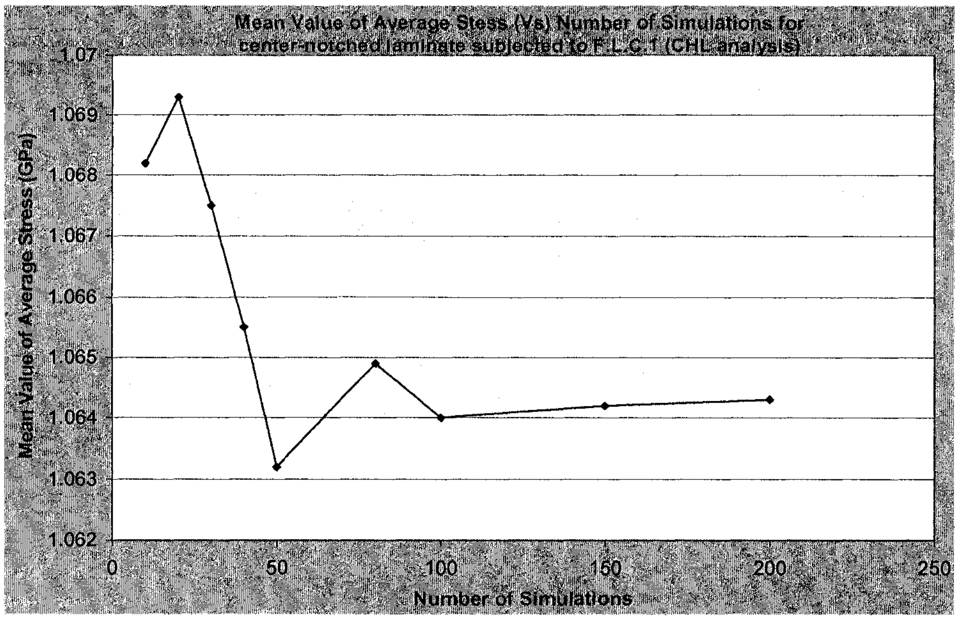
(σ_{avg}) parameters are studied. The variations in the mean values and S.D. values with the number of simulations for the two parameters are shown in Figures 4.2 (a) - 4.2 (d).



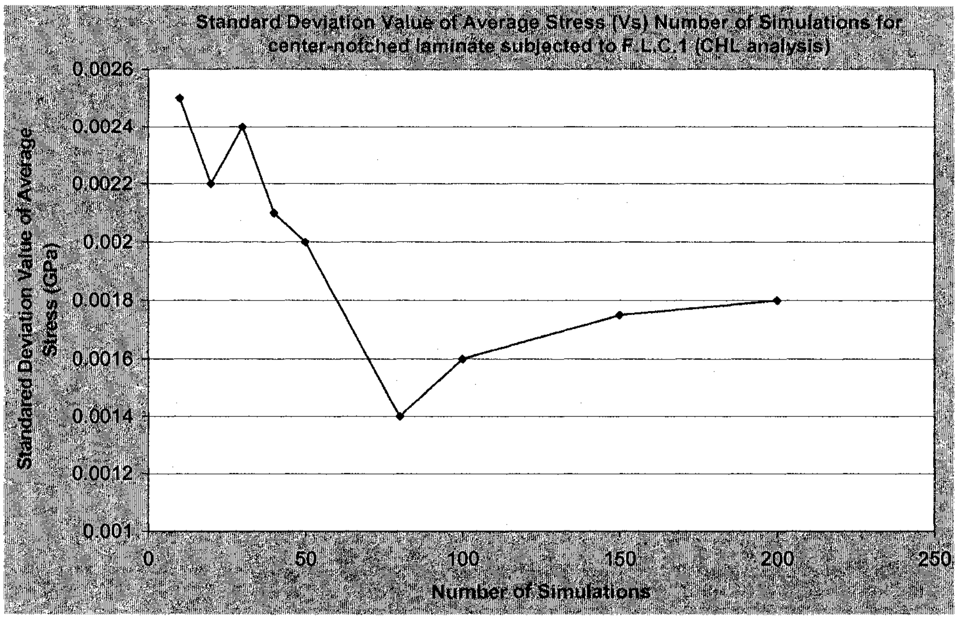
(a)



(b)



(c)



(d)

Figure 4.2 Stochastic simulation of center notched laminate subjected to F.L.C.1: (a) Mean values of point stress, (b) S.D. values of point stress, (c) Mean values of average stress and (d) S.D. values of average stress

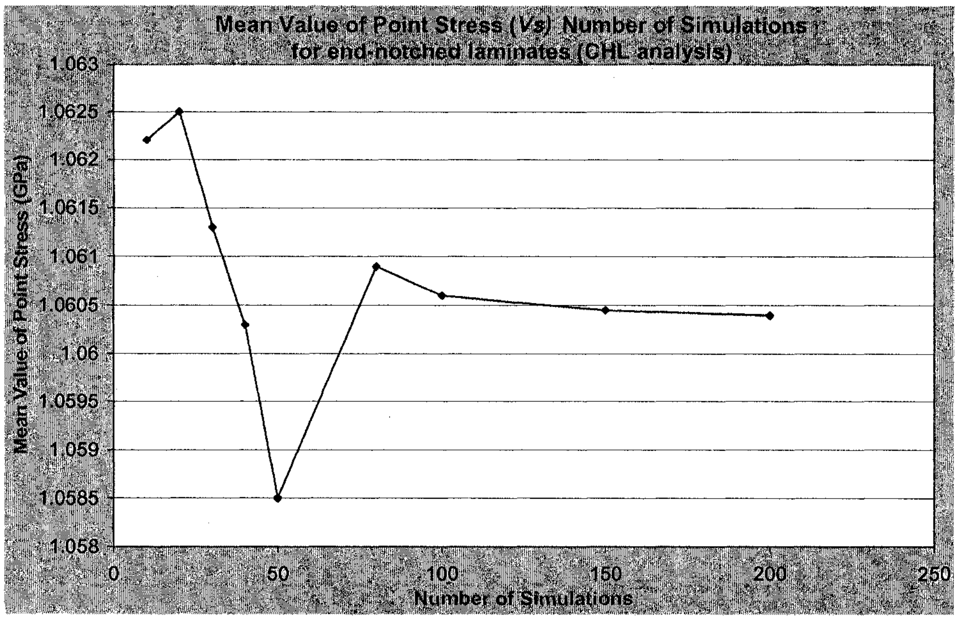
The stochastic simulation is carried out in the range of 1-200 simulations to check the convergence of the point and average stress values with number of simulations. From Figures 4.2 (a) and (b), it can be seen that the fluctuations in the mean and Standard Deviation (S.D.) values of point stress until 100 simulations are very high, but as the number of simulations is increased to 200, the mean and S.D. values of point value converges to an almost constant value. This implies that in Controlled Hole Laminate (CHL) analysis, a minimum of 200 laminates has to be simulated to achieve a mean point stress value. The mean and S.D. values of point stress after 200 simulations are 1.0598 GPa and 0.0049 GPa respectively.

The fluctuations in the mean and Standard Deviation (S.D.) values of average stress with number of simulations are shown in Figures 4.2 (c) and (d). The fluctuations are high in the first 100 simulations, but as the number of simulations is increased to 200 a steady state mean and S.D. values of average stress are achieved. The mean and S.D values of average stress after 200 simulations are 1.0643 GPa and 0.0018 GPa respectively. The initial variations in the mean and S.D values of point and average stress are attributed by the stochastic variations in the material properties. From Figures 4.2 (a) and (c), it can be seen that the mean value of point and average stress curves almost follows the same trajectory. A similar observation is made between the S.D. value of point and average stress as shown in Figures 4.2 (b) and (d).

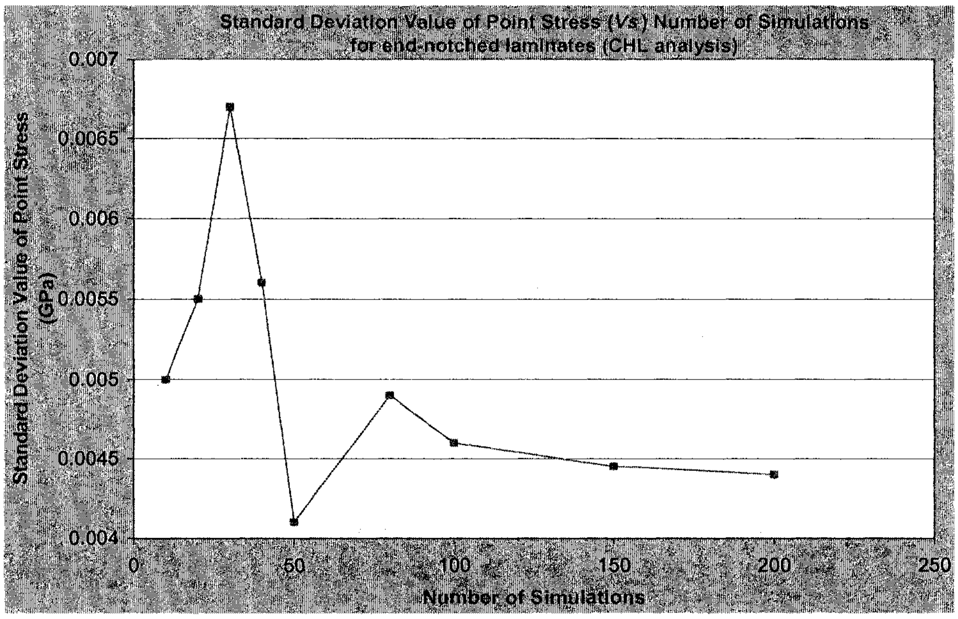
4.5 Controlled Hole Laminate (CHL) analysis of edge-notched laminates

Controlled Hole Laminate (CHL) analysis is performed on edge-notched laminates to analyze the fluctuations in the Mean and S.D. values of point and average stresses with number of simulations. The geometric dimensions and laminate configuration of center-notched laminate used in the CHL analysis are as follows: Gage Length (L) =180mm, Width (W) = 37.9mm, Thickness (t) = 2mm, Hole Radius (R) = 3.77mm and laminate configuration $[0/90]_{4s}$. The finite element mesh, boundary conditions and applied loading of the edge-notched laminate are shown in Figure 2.6. The material properties of NCT301 prepreg, which is used to manufacture the edge-notched laminates, are given in Table 3.1. The fluctuations in the material properties are expressed using stochastic processes as explained in Chapter 2. The geometric variations in the edge-notched laminates were not considered in the CHL analysis.

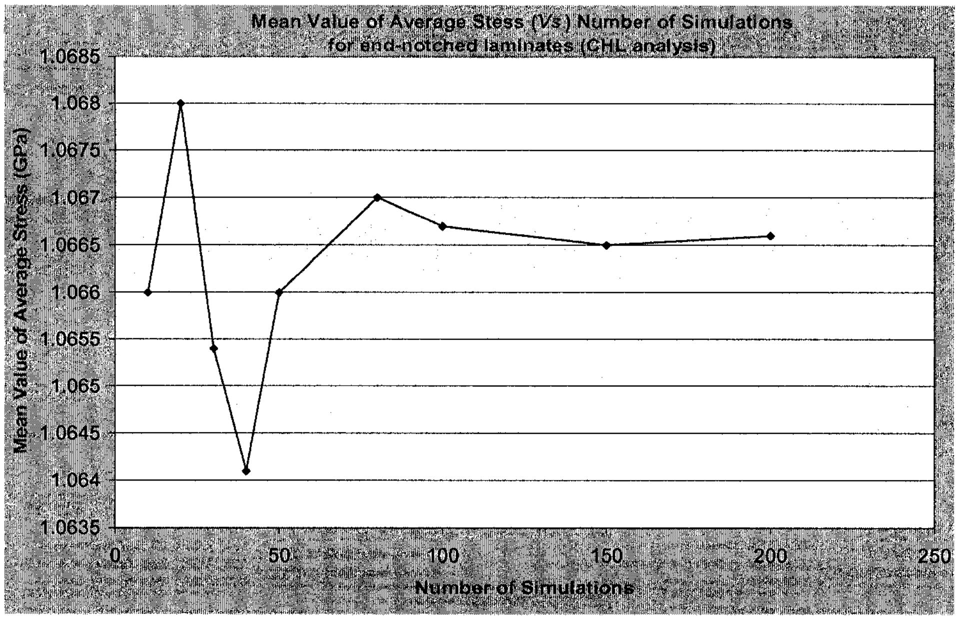
The Average Ultimate Strength of Edge-Notched Laminate (AUSENL), which is given in Table 3.8, is used as the applied load in the present analysis. The influence of number of simulations, within the range of 1 to 200, on the Mean and Standard Deviation (S.D.) values of point stress (σ_p) and average stress (σ_{avg}) parameters are studied. The variations in the Mean and S.D. values with increase in the number of simulations for the two parameters are shown in Figures 4.3 (a) - (d).


Mean Value of Point Stress (Vs) Number of Simulations
for end-notched laminates (CHL analysis)
Mean Value of Point Stress (GPa)
1.063
1.0625
1.062
1.0615
1.061
1.0605
1.06
1.0595
1.059
1.0585
1.058
0
50
100
150
200
250
Number of Simulations

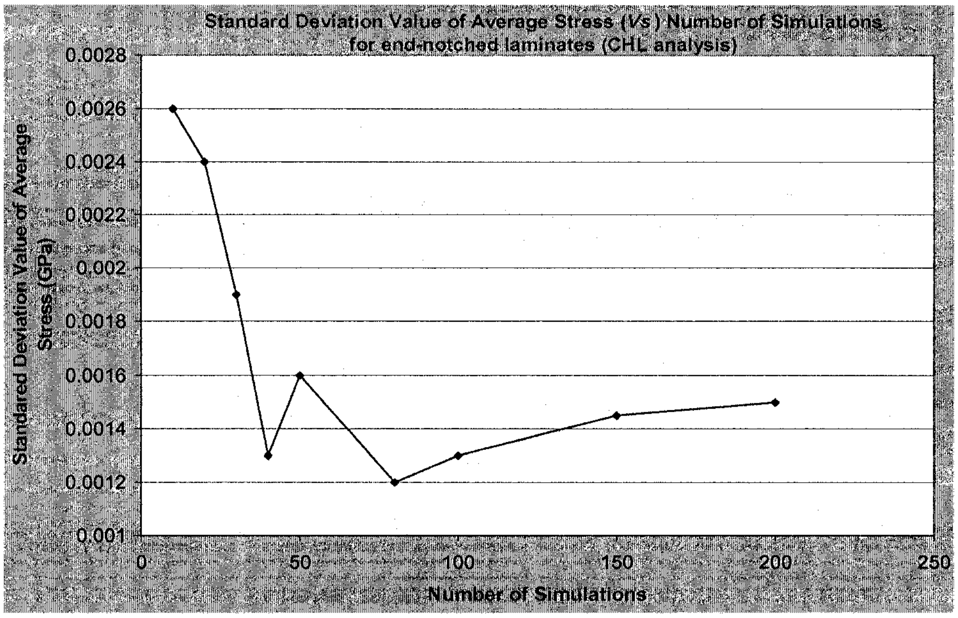
(a)


Standard Deviation Value of Point Stress (Vs) Number of Simulations
for end-notched laminates (CHL analysis)
Standard Deviation Value of Point Stress (GPa)
0.007
0.0065
0.006
0.0055
0.005
0.0045
0.004
0
50
100
150
200
250
Number of Simulations

(b)



(c)



(d)

Figure 4.3 Stochastic simulation of edge-notched laminates: (a) Mean values of point stress, (b) S.D. values of point stress, (c) Mean values of average stress and (d) S.D. values of average stress

From Figures 4.2 (a) and (b), it can be seen that the fluctuations in the mean and Standard Deviation (S.D.) values of point stress with number of simulations are more until 100 simulations. When the number of simulations is increased to 200, the mean and S.D. values of point value converges to a constant value in each case. Thus a minimum of 200 edge-notched laminates has to be simulated in Controlled Hole Laminate (CHL) analysis, to achieve a mean point stress value. The mean and S.D. values of point stress after 200 simulations are 1.0604 GPa and 0.0044 GPa respectively. The difference between the mean point stress value of center-notched laminates not subjected to fatigue loading and edge-notched laminates is 101 MPa, which indicates the effect of changing the hole location in composite laminates on their mean point stress value.

The fluctuations in the Mean and Standard Deviation (S.D.) values of average stress with number of simulations are shown in Figures 4.2 (c) and (d). As in the case of point stress, the fluctuations in the mean average stress value are high in the first 100 simulations, but as the number of simulations is increased to 200 a steady state mean and S.D. values of average stress are achieved. The mean and S.D values of average stress after 200 simulations are 1.0666 GPa and 0.0015 GPa respectively. The initial variations in the mean and S.D values of point and average stresses are attributed to the stochastic variations in the material properties. From Figures 4.3 (a) and (c), it can be seen that the trend of the mean values of point and average stress curves follows a similar path. A similar observation is made between the S.D. values of point and average stresses as it is shown in Figures 4.3 (b) and (d).

4.6 Formulation for Un-controlled Hole Laminate (UCHL) Analysis

In center-notched laminates, geometric variations occur when they are machined. The geometric variations, such as, changes in hole shape and eccentricity of the hole from the center of the laminate occur during the drilling operation of center-notched laminates. The causes for the geometric variations are as follows:

(i) Vibrations in the drilling machine during drilling operation

(ii) Improper contact of the drill-bit tip with the center of the center-notched laminate

(iii) Damage to the hole edge during drilling operation

The Un-Controlled Hole Laminate (UCHL) analysis of center-notched laminates, which includes stochastic variations in both material properties and geometric variations, is performed to find the Mean and Standard Deviation (S.D.) values of point and average stresses. Due to the eccentricity of the hole from the center of the laminate, characteristic lengths of center-notched laminates vary for each and every laminate used in the UCHL analysis and it is calculated using Gaussian distribution method.

The hypotrochoid variation in the hole shape of center-notched laminates is considered from reference [59] and the equations for variation in Cartesian co-ordinates are given by:

$$x = R(\cos\alpha + \psi\cos\rho\alpha),\ y = R(\sin\alpha - \psi\sin\rho\alpha) \tag{4.2}$$

where,

$\rho = 7$ (a non-negative integer), $\psi = 0.01$, $0 < (\rho\Psi) < 1$ and α is the angle at which a node is created on the circle while developing the finite element mesh.

From reference [2], a maximum tolerance of $\left(\frac{1}{20}\right)^{th}$ of an inch is considered for the eccentricity of the hole. Gaussian random variables were generated to control the movement of the hole on the center-notched laminates, depending on the number of the nodes associated with the hole. The MATLAB® program written for the Controlled Hole Laminate (CHL) analysis of center-notched laminates is modified to include the geometric variations. The modified MATLAB® program is used to perform the Un-Controlled Hole Laminate (UCHL) analysis of center-notched laminates. The change in the hole shape of center-notched laminates due to hypotrochoid variation is shown in Figure 4.4. Eccentricity of the hole from the laminate center is given in Figure 4.5.

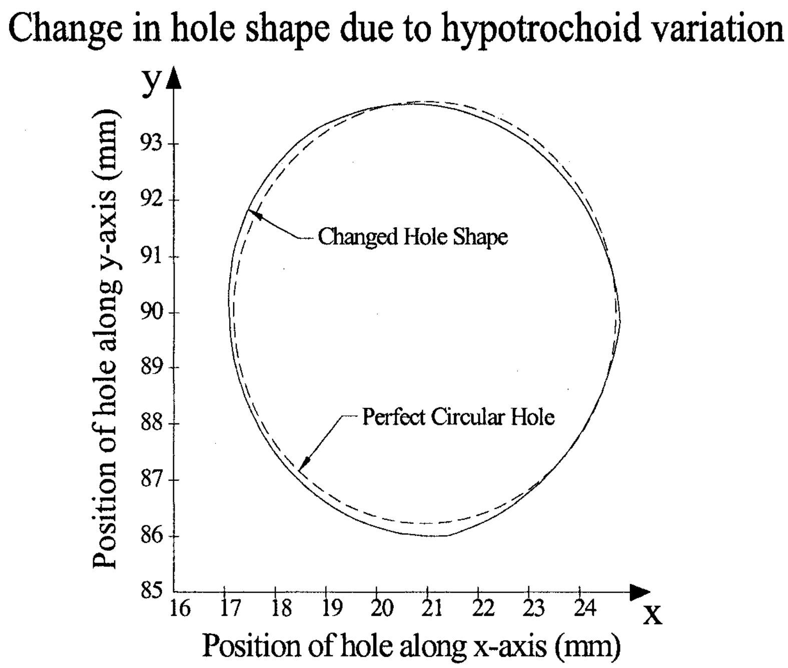


Figure 4.4 Change in hole shape due to hypotrochoid variation

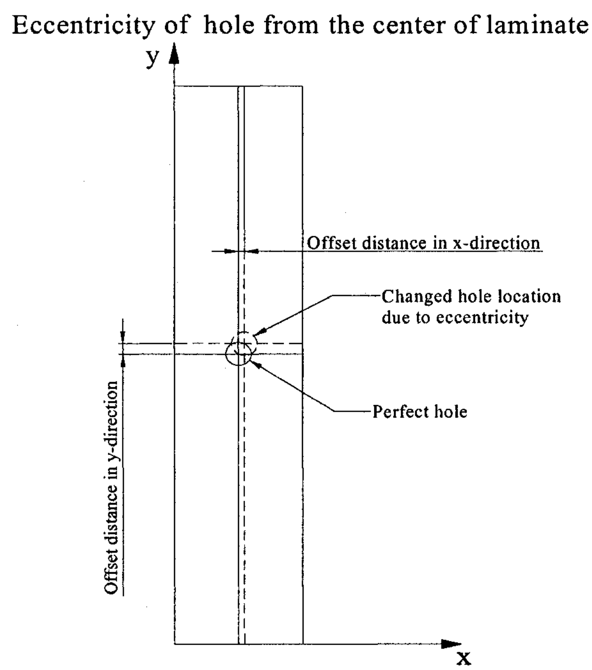


Figure 4.5 Eccentricity of the hole from plate center

In Un-Controlled Hole Laminate (UCHL) analysis, characteristic lengths of center-notched laminates vary for each and every laminate due to the eccentricity of hole from the plate center. A MATLAB® program is developed to generate a series of values for the characteristic lengths a_o and d_o by assuming that the variation in characteristic lengths follows a Gaussian distribution.

The MATLAB® program uses a sub-function:

$$[R] = a_2 + (a_1 - a_2) * rand(m,n) \tag{4.3}$$

where,

'rand' is the sub-function,

a_1 and a_2 are the maximum and minimum values of characteristic length,

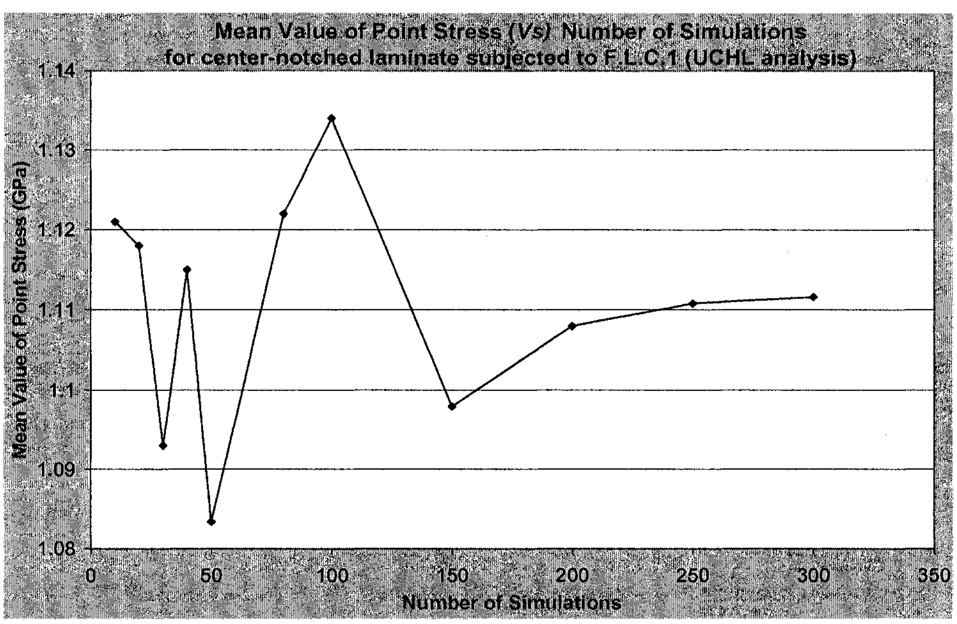
m is the number of rows of Gaussian random numbers to be generated and

n is the number of columns to be realized.

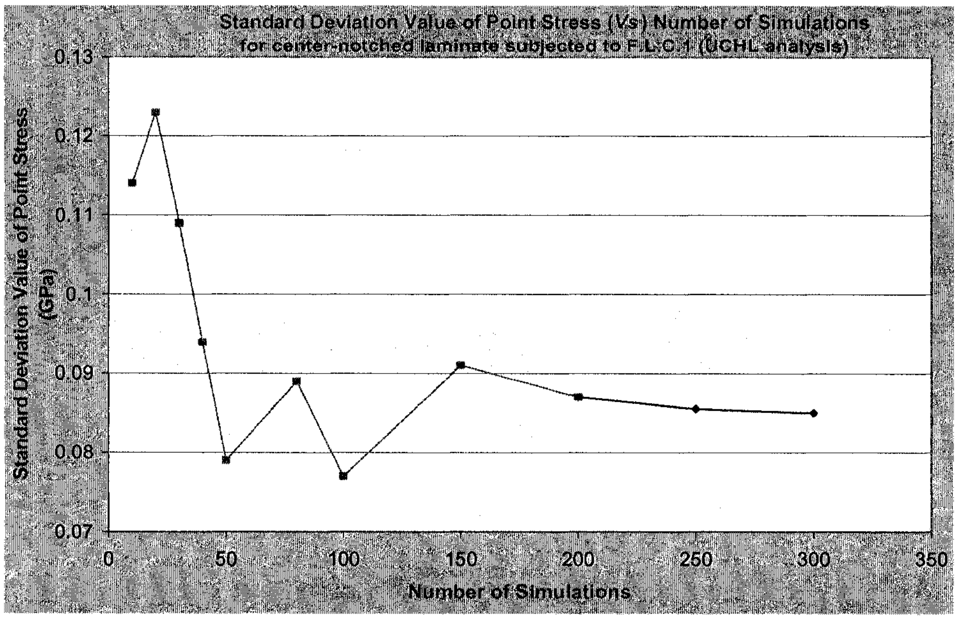
In the present analysis, m is equal to the number of laminates to be analyzed and n is equal to 1, giving matrix [R] a dimension of (m*n).

Un-Controlled Hole Laminate (UCHL) analysis of center-notched laminate subjected to Fatigue Loading Condition No.1 (F.L.C.1) is performed and the results obtained from the analysis were compared with the Controlled Hole Laminate (CHL) analysis results. In order to facilitate the comparison of results, geometric dimensions and laminate configuration of center-notched laminates used in the CHL analysis were considered for the UCHL analysis. The boundary conditions, finite element mesh and applied load of the center-notched laminate are shown in Figure 2.3. The finite element mesh used in the UCHL analysis assumes new co-ordinate values near the hole boundary based on the tolerance value set. The material properties of the NCT301 prepreg, which is considered for the UCHL analysis, are given in Table 3.1.

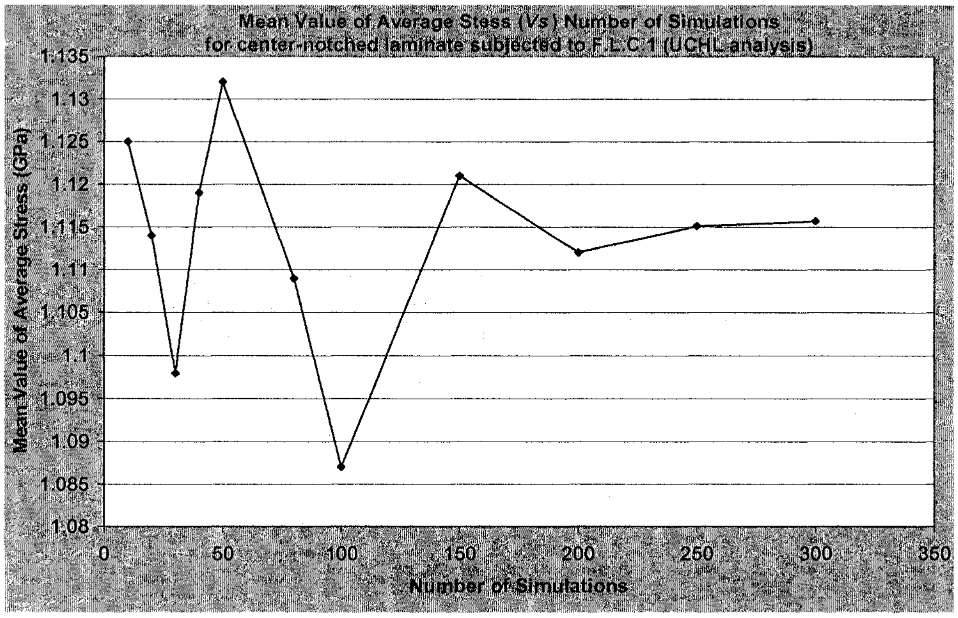
The fluctuations in the material properties are expressed using stochastic processes as explained in Chapter 2. The Average Ultimate Strength of Center-Notched Laminate (AUSCNL) subjected to F.L.C.1 is used as the applied load in the Un-Controlled Hole Laminate (UCHL) analysis. In UCHL analysis, the number of laminates analyzed to find the Mean and Standard Deviation (S.D.) values of point and average stresses were more than that used in the CHL analysis. The UCHL analysis considers the geometric variations in the center-notched laminate in addition to the variations in the material properties. The influence of number of simulations, within the range of 1 to 300, on the Mean and S.D. values of the point stress (σ_p) and average stress (σ_{avg}) parameters are studied. The variations in the Mean and S.D. values with the number of simulations for the two parameters are shown in Figures 4.6 (a) - 4.6 (d).



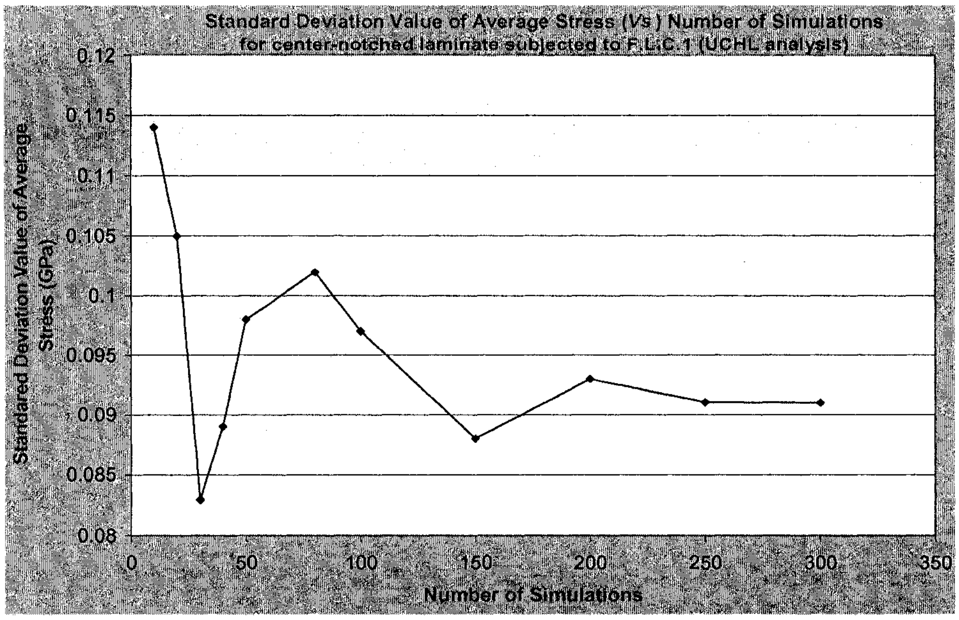
(a)


Standard Deviation Value of Point Stress (Vs) Number of Simulations
for center-notched laminate subjected to F.L.C 1 (UCHL analysis)
Standard Deviation Value of Point Stress (GPa)
Number of Simulations
0.13
0.12
0.11
0.1
0.09
0.08
0.07
0
50
100
150
200
250
300
350

(b)


Mean Value of Average Stess (Vs) Number of Simulations
for center-notched laminate subjected to F.L.C 1 (UCHL analysis)
Mean Value of Average Stress (GPa)
Number of Simulations
1.135
1.13
1.125
1.12
1.115
1.11
1.105
1.1
1.095
1.09
1.085
1.08
0
50
100
150
200
250
300
350

(c)



(d)

Figure 4.6 Stochastic simulation of center notched laminate subjected to F.L.C.1 (UCHL analysis): (a) Mean values of point stress, (b) S.D. values of point stress, (c) Mean values of average stress and (d) S.D. values of average stress

In the Un-Controlled Hole Laminate (UCHL) analysis of center-notched laminate subjected to F.L.C.1, stochastic simulations are carried out in the range of 1-300 simulations to check the convergence of the point and average stress values with number of simulations. From Figures 4.6 (a) and (b), it can be seen that the fluctuations in the Mean and Standard Deviation (S.D.) values of point stress are more until 150 simulations. When the number of simulations is increased to 300, the Mean and S.D. values of point stress converge to a constant value in each case. This implies that in UCHL analysis, a minimum of 300 laminates has to be simulated to achieve a mean point stress value. The Mean and S.D. values of point stress after 300 simulations are 1.1116 GPa and 0.085 GPa respectively.

The fluctuations in the Mean and Standard Deviation (S.D.) values of average stress with number of simulations are shown in Figures 4.2 (c) and (d) respectively. The fluctuations were high in the first 150 simulations. When the number of simulations is increased to 300 a steady state Mean and S.D. values of average stress are achieved. The Mean and S.D values of average stress after 300 simulations are 1.1157 GPa and 0.091 GPa respectively. The initial variations in the Mean and S.D values of point and average stresses are attributed to the stochastic variations in the material properties and geometric variations in the laminate. From Figures 4.6 (a) and (c), it can be seen that the mean values of point and average stress curves almost follow the same trajectory. A similar observation is made between the S.D. values of point and average stresses as shown in Figures 4.6 (b) and (d).

4.7 Comparison of stress parameters obtained from the CHL and UCHL analyses of center-notched laminate subjected to F.L.C.1

In order to analyze the effect of including the geometric variations of laminates in the stochastic analysis, the stress parameters obtained from the Controlled Hole Laminate (CHL) and Un-Controlled Hole Laminate(UCHL) analyses of center-notched laminate subjected to F.L.C.1 were compared. Tables 4.1 and 4.2 show the Mean and Standard

Deviation (S.D.) values of point and average stresses respectively, obtained from the CHL and UCHL analyses of center-notched laminate subjected to F.L.C.1.

Analysis type	Number of simulations	Applied Load	Mean Value of point stress	S.D. value of point stress
		MN/m	GPa	GPa
CHL	200	1.113	1.0598	0.0049
UCHL	300	1.113	1.1116	0.085

Table 4.1 Mean and S.D. values of point stress obtained from the CHL and UCHL analyses of center-notched laminate subjected to F.L.C.1

Analysis type	Number of simulations	Applied Load	Mean Value of average stress	S.D. value of average stress
		MN/m	GPa	GPa
CHL	200	1.113	1.0643	0.0018
UCHL	300	1.113	1.1157	0.091

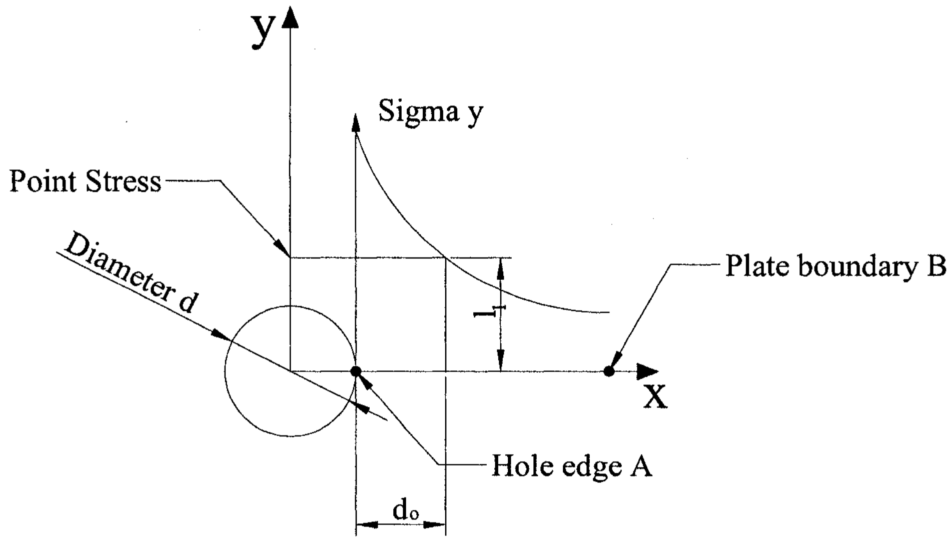
Table 4.2 Mean and S.D. values of average stress obtained from UCHL and CHL analyses for center-notched laminate subjected to F.L.C.1.

From Tables 4.1 and 4.2, it can be seen that the stress parameters obtained from the Controlled Hole Laminate (CHL) and Un-Controlled Hole Laminate (UCHL) analysis require 200 and 300 simulations respectively to converge to a stable value. In CHL analysis, only the stochastic variation of material properties is considered. Whereas

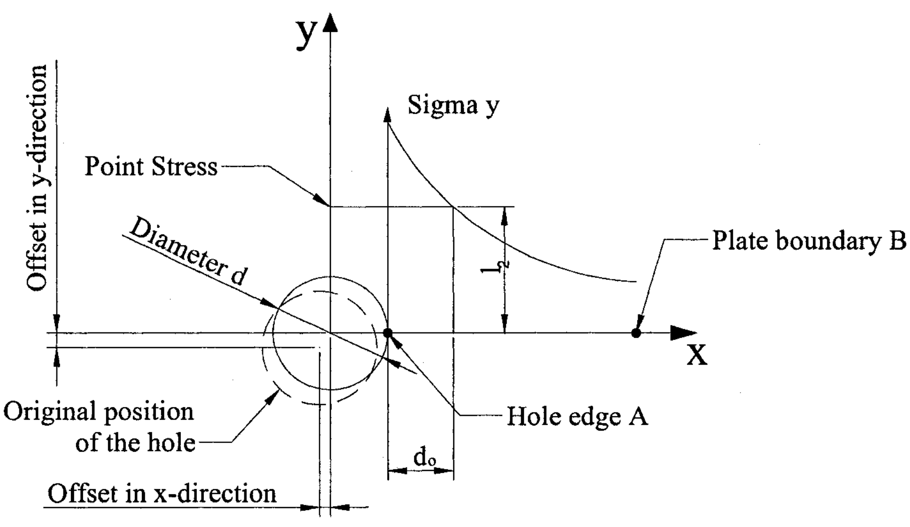
UCHL analysis takes into account the geometric variations in the laminate in addition to the stochastic variation of material properties, which contributes to the prolonged and non-uniform variations in pattern and hence leads to more number of simulations.

From Table 4.1, it can be seen that the difference between the Mean values of point stress obtained from CHL and UCHL analyses is 51.8 MPa and the difference between the Standard Deviation (S.D.) values of point stress obtained from CHL and UCHL analyses is 80.1 MPa. From Table 4.2, it can be noted that the difference between the Mean values of average stress obtained from CHL and UCHL analyses is 51.4 MPa and the difference between the S.D values of average stress obtained from CHL and UCHL analyses is 89.2 MPa. The increase in the Mean and S.D values of point and average stress in UCHL analysis are attributed to the irregularity in the hole shape and eccentricity of the hole from the laminate center. The cause for the increase in the stress parameters can be reasoned as follows; when the circular hole of the center-notched laminate moves closer to the laminate edge, a higher stress value is attained near the hole edge to maintain a uniform stress distribution along the axis perpendicular to the loading direction.

The pictorial representation of the increase in point and average stresses due to the hole eccentricity in the center-notched laminate is shown in Figures 4.7 and 4.8 respectively.

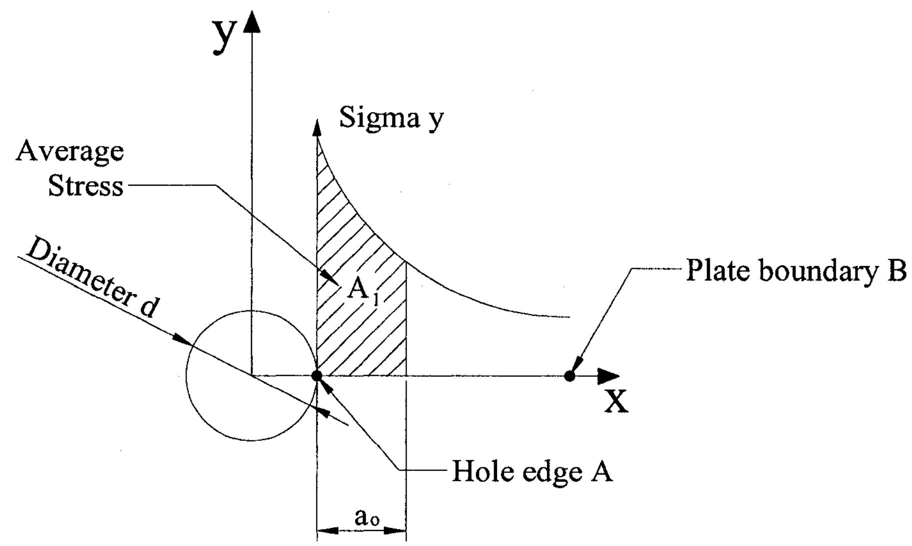


(a) Controlled Hole Laminate analysis

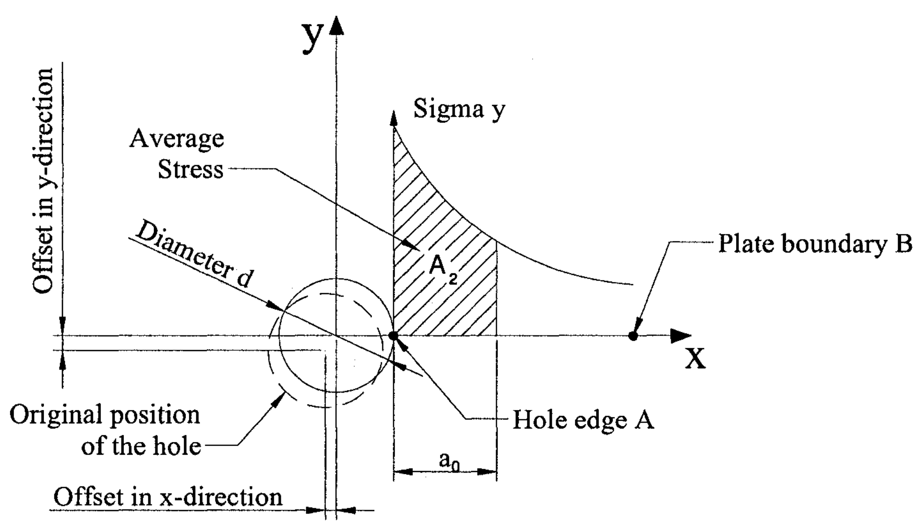


(b) Un-Controlled Hole Laminate analysis

Figure 4.7 Increase of point stress due to hole eccentricity



(a) Controlled Hole Laminate analysis



(b) Un-Controlled Hole Laminate analysis

Figure 4.8 Increase of average stress due to hole eccentricity

The values of characteristic lengths, a_o and d_o, of each laminate used in the Un-Controlled Hole Laminate (UCHL) analysis change as per the Gaussian distribution. But in Controlled Hole Laminate (CHL) analysis a fixed value for each of the characteristic lengths, a_o and d_o, were used for each and every laminate. The characteristic lengths used in the UCHL analysis might by higher, lower or equal to that used in the CHL analysis. Considering an equal value for each of the characteristic lengths, a_o and d_o, in the CHL and UCHL analyses, it can be seen in Figure 3.8 that, due to eccentricity of the hole, the area of integration A_2 in UCHL analysis is large when compared to the area of integration A_1 in CHL analysis, which explains the increase in the average stress for UCHL analysis. Similarly from Figure 3.7, it can be noticed that the point stress value l_1 in CHL analysis is less when compared to the point stress value l_2 in UCHL analysis, which explains the increase in the point stress value for UCHL analysis.

4.8 Comparison of stress parameters obtained from the CHL analysis of center-notched laminate not subjected to fatigue loading and edge-notched laminates

The stress parameters obtained from the Controlled Hole Laminate (CHL) analysis of center-notched laminate not subjected to fatigue loading and edge-notched laminates were compared to find the effect of changing the hole location in notched laminates on the stress parameters. Table 4.3 shows the Mean and Standard Deviation (S.D.) values of point and average stresses obtained from the CHL analysis of center-notched laminate not subjected fatigue loading and edge-notched laminates.

Laminate type	Number of simulation	Applied Load	Mean value point stress	S.D. value of point stress	Mean Value of average stress	S.D. value of average stress
		MN/m	GPa	GPa	GPa	GPa
Center-notched laminate not subjected to fatigue loading	200	1.154	1.05034	0.0051	1.0588	0.0019
Edge-notched laminate	200	1.111	1.0604	0.0044	1.0666	0.0015

Table 4.3 Mean and S.D. values of point and average stresses obtained from the CHL analysis of center-notched laminate not subjected to fatigue loading and edge-notched laminates.

From Table 4.3, it can be seen that the difference between the Mean values of point stress obtained from the CHL analysis of center notched laminate not subjected to Fatigue Loading Condition (F.L.C.) and edge-notched laminates is 10.06 MPa and the difference between the Standard Deviation (S.D.) values of point stress obtained from the CHL analysis of center notched laminate not subjected to F.L.C. and edge-notched laminates is 0.7 MPa . From Table 4.2, it can also be noted that the difference between the Mean values of average stress obtained from the CHL analysis of center notched laminate not subjected to F.L.C. and edge-notched laminates is 7.8 MPa and the difference between the S.D. values of average stress obtained from the CHL of center-notched laminate not subjected to F.L.C and edge-notched laminates is 0.4 MPa. The mean point and average stress values of edge-notched laminate is more when compared to that of center-notched laminate not subjected to fatigue loading, which clearly indicates the effect of changing the hole location on the stress parameters of notched laminates.

4.9 Maximum Entropy Method

Maximum Entropy Method (MEM) was used to determine the probability density function for the characteristic lengths of center-notched laminate not subjected to fatigue loading and edge-notched laminates. The MEM is based on Jayne's principle [60], which states that "the minimally prejudiced probability distribution is that which maximizes the entropy subject to constraints supplied by the given information".

In the MEM-based analysis of sample data, it is initially assumed that no information is available to convince that there are physical arguments that would suggest a particular distribution for the sample data. Therefore, maximizing the entropy of the variable's distribution should lead to the least biased estimate of its form. A convenient way to use the sample data is by evaluation of the sample moments. For a continuous random variable, the entropy (S) is defined as

$$S = -\int_R f(x)\ln[f(x)]dx$$

(4.4)

where,

f(x) is the probability density function and

$\int f(x)dx = 1$.

The central moments are given by the equation

$$\int x^i f(x)dx = m_i \quad , i = 1,2,\ldots,m \tag{4.5}$$

where, m is the number of moments to be used and

m_i is the i^{th} moment about the origin, which is determined numerically from the sample.

The analytical form for the maximum entropy density function is given by

$$f(x) = \exp\left(\lambda_0 + \sum_{i=1}^{m} \lambda_i x^i\right) \tag{4.6}$$

where,

$\lambda_0, \lambda_1, \lambda_2 \ldots \lambda_m$ are the Lagrangian multipliers. The procedure to evaluate the values of the λ's and to find the probability density function of the sample data is given in reference [60].

4.9.1 Application of MEM

The density functions of the characteristic lengths, a_o and d_o, of center-notched laminate not subjected to fatigue loading and edge-notched laminates are obtained using the Maximum Entropy Method (MEM), which is explained in Section 4.9. The number of moments used in the analysis is 4. Twenty-five and fifteen characteristic length values were used to obtain the density function of center-notched laminate not subjected to fatigue loading and edge-notched laminates respectively. The characteristic length values were determined using the ultimate strength of notched laminate and the ultimate strength of the corresponding un-notched laminates, which is explained in Chapter 3 in detail. The density functions of the characteristic lengths, a_o and d_o, of center-notched laminate not subjected to fatigue loading and edge-notched laminates, obtained using the Maximum Entropy Method (MEM), are given in Figures 4.9- 4.12. From Figures 4.9-4.12, it can be seen that the density function curves follow the same pattern in all the cases, with a single peak value and the tail extending on both ends. The values of the Lagrangian multipliers for the characteristic lengths of center-notched laminate not subjected to fatigue loading and edge-notched laminates are given in Table 4.4.

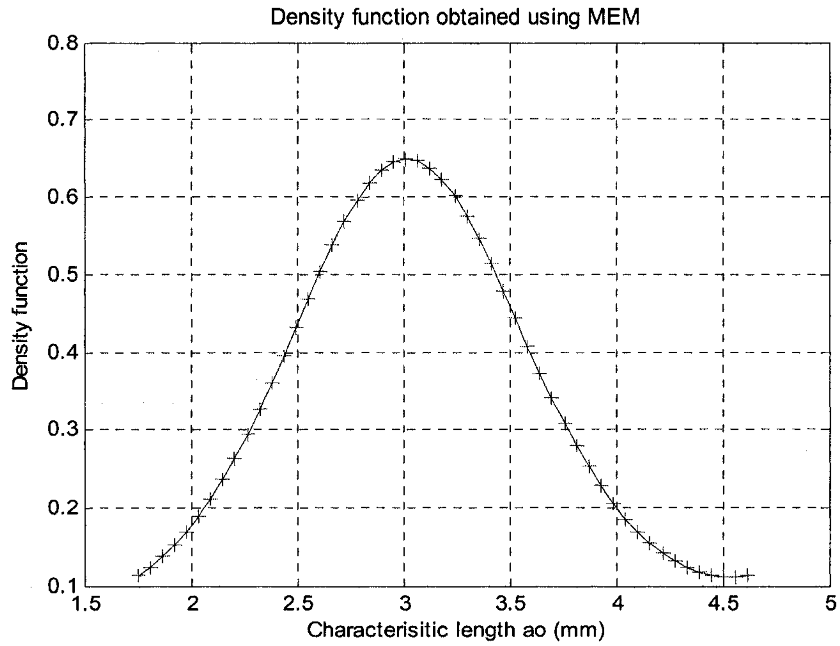


Figure 4.9 Density function of characteristic length, a_o, of center-notched laminate not subjected to fatigue loading obtained using MEM

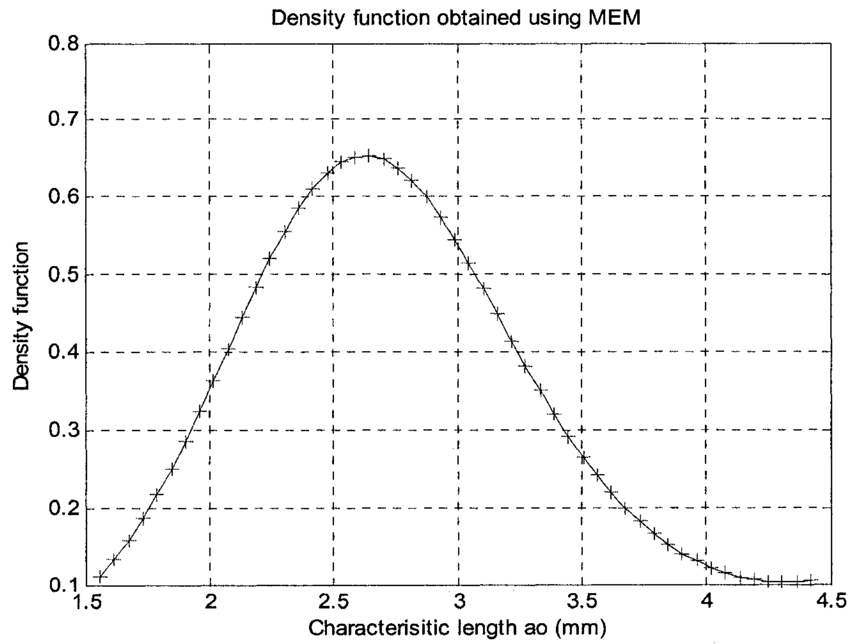


Figure 4.10 Density function of characteristic length, a_o, of edge-notched laminates obtained using MEM

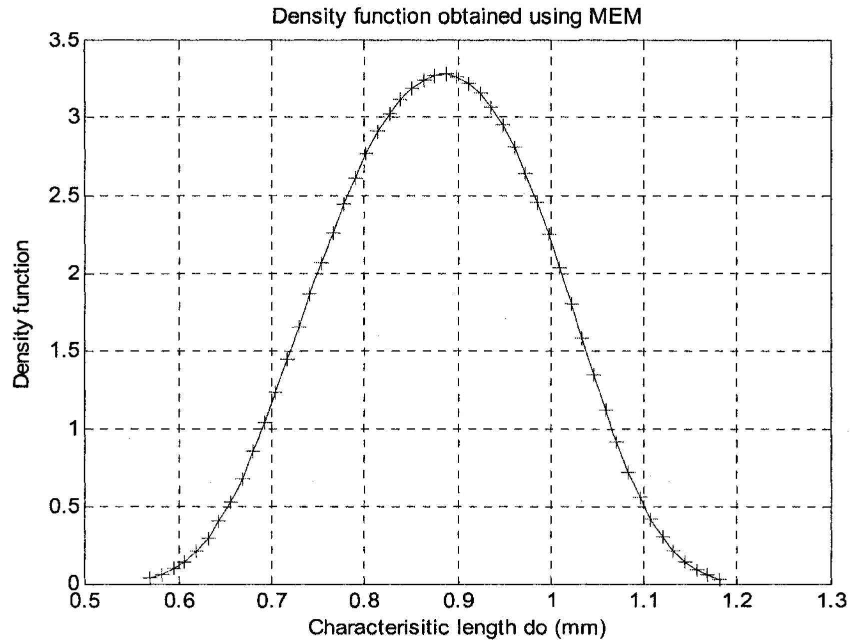


Figure 4.11 Density function of characteristic length, d_o, of center-notched laminate not subjected to fatigue loading obtained using MEM

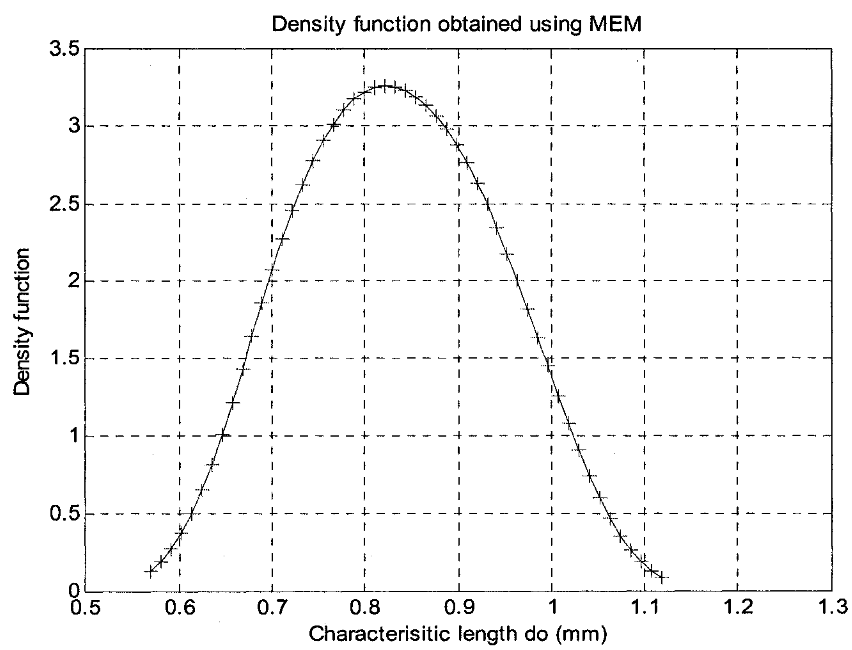


Figure 4.12 Density function of characteristic length, d_o, of edge-notched laminates obtained using MEM

Laminate Type	Characteristic lengths	λ_0	λ_1	λ_2	λ_3	λ_4
Center-notched laminate not subjected to fatigue loading	a_o	11.172	-25.088	15.731	-3.85	0.325
	d_o	-0.1758	0.7714	-1.2834	0.9668	-0.2782
Edge-notched laminates	a_o	14.532	-22.314	18.322	-1.45	4.235
	d_o	-0.1417	0.8323	-1.0361	1.1156	-0.1298

Table 4.4 Lagrangian multiplier values for the characteristic lengths of center-notched laminate not subjected to fatigue loading and edge-notched laminates

From Table 4.4, it can be seen that the Lagrangian multiplier values for the characteristic length, a_o, of center-notched laminate not subjected to fatigue loading and edge-notched laminates are close to each other. Likewise, a similar trend is noticed for the characteristic length, d_o, of center-notched laminate not subjected to fatigue loading and edge-notched laminates. Thus it can be concluded the there is not much variation in the density functions of the characteristic lengths of center-notched laminate not subjected to fatigue loading and edge-notched laminates and they follow the same trend.

4.9 Conclusions

In the present chapter, stochastic simulation process of center-notched laminates subjected to Fatigue Loading Condition No.1 (F.L.C.1), F.L.C.2, F.L.C.3 and edge-notched laminates were carried out to determine the probabilistic parameters of the stress parameters.

The center-notched laminates subjected to fatigue loading conditions were analyzed using two different types of analysis, namely, Controlled Hole Laminate (CHL) analysis and Un-Controlled Hole Laminate (UCHL) analysis. In CHL analysis, only the stochastic variations of material properties over the laminate were considered. In UCHL analysis, geometric variations, such as, change in the hole shape and eccentricity of the hole from the laminate center were considered in the analysis in addition to the stochastic variations of material properties. The value of characteristic lengths, a_o and d_o, varies for each and every laminate used in the UCHL analysis and it is calculated using Gaussian distribution method. The CHL and UCHL analyses of center-notched laminate subjected to fatigue loading conditions are explained in Sections 4.4 and 4.6 and the CHL analysis of edge-notched laminate is given in Section 4.7.

Stochastic simulation results show that, as the number of simulations is increased, a better convergence of the probabilistic parameters is achieved. It is shown that the Mean and Standard Deviation (S.D.) values of the point and average stresses converge to a constant value after 200 and 300 simulations for CHL and UCHL analyses respectively. The stochastic simulation results are shown in Figures 4.2, 4.3 and 4.6.

The Average Ultimate Strength of Center-Notched Laminate (AUSCNL) subjected to F.L.C.1, F.L.C.2 and F.L.C.3 and the Average Ultimate Strength of Edge-Notched Laminate (AUSENL) were used as the applied loads in the CHL and UCHL analysis. Point Stress and Average Stress criteria, explained in Chapter 2, were used to find the point stress and average stress in the stochastic analysis.

The point and average stress parameters obtained from the UCHL analysis are higher when compared to the CHL analysis. The increase in the stress parameters are attributed to the geometric variations, such as, change in hole shape and eccentricity of the hole from the laminate center. The reliability indices for the center-notched laminate subjected to fatigue loading conditions and edge-notched laminates are found out and presented in Chapter 5.

The density functions of the characteristic lengths of center-notched laminate not subjected to fatigue loading and edge-notched laminates were determined using Maximum Entropy Method (MEM) and it is explained in Sections 3.9 and 3.9.1. From Figures 4.9-4.12 and Table 4.4, it can be concluded that the density functions of the characteristic lengths of center-notched laminate not subjected to fatigue loading and edge-notched laminates show similar characteristics. In all the cases, the density function curves have a single peak value and the tails extend on both ends.

CHAPTER 5

RELIABILTY ANALYSIS

5.1 Introduction

The reliability analysis of notched composite laminates is performed to find the worthiness of the laminate for an intended application. Reliability is defined as the probability that a component, device or system will achieve a specified life without failure under a given loading [60]. In composite laminates, holes (or) cutouts are made for practical reasons, which induces stress concentration near the hole edge of the laminate and reduces the ultimate strength of the laminate considerably. There are other factors, such as, application of fatigue loading, hole location and hole shape and size, which can affect the ultimate strength of the notched composite laminate. Notched composite laminate can also exhibit local damages, such as, fiber breakage, matrix cracking, fiber-matrix debonding, delamination, etc. Considering all these factors, it becomes necessary to perform a reliable design of notched composite laminate based on a probabilistic approach.

The reliability of any structure can be calculated by using two parameters, namely

(i) Strength parameter of the structure

(ii) The stress developed in the structure due to external loading.

In the present analysis, point stress (σ_p) and average stress (σ_{avg}) parameters of center-notched laminates subjected to fatigue loading conditions and that of edge-notched laminates were used as the stress parameters of the laminate. The ultimate strength (σ_o) of the un-notched laminate is considered to be the strength parameter of the laminate.

5.2 Strength and stress distribution of notched composite laminates

The design and analysis of composite structures requires reliable experimental results. Experimental investigation on center-notched laminates subjected to fatigue loading conditions and edge-notched laminates was conducted and the results were presented in Chapter 3. The main objective of testing composite laminates is to determine the un-notched and notched laminate strength values, which are used to determine the distribution of characteristic length data of the laminates.

Probability distribution arises from experiments, where the outcome is subject to chance. Depending upon the nature of the experiment, a probability distribution which may be appropriate for modeling the resulting outcomes is chosen. In the present work, Gaussian distribution method is used to generate the probability density function (PDF) of the stress parameter. Probability density function is the basic tool for codifying and communicating uncertainty about the value of a continuously varying variable. This information along with the distribution of the point (σ_p) and average (σ_{avg}) stresses obtained using Stochastic Finite Element Analysis (SFEA) can be used to determine the reliability of the composite laminates.

The main purpose of SFEA is to determine the reliability considering the distributions of point/average stress ($\sigma_{p/avg}$) in notched laminate and strength of corresponding un-notched laminate (σ_o), at a critical location in the component. The distributions followed by each of these parameters might be in general different from each other and they can be represented as

$$F_1 = A\left(\mu_{\sigma_{p/avg}}, S_{\sigma_{p/avg}}\right) \tag{5.1}$$

$$F_2 = B\left(\mu_{\sigma_o}, S_{\sigma_o}\right) \tag{5.2}$$

where,

A and B represent the two different distributions followed by ($\sigma_{p/avg}$) and (σ_o) respectively.

5.3 Gaussian Distribution

The Gaussian distribution or normal distribution is the most commonly used two-parameter distribution. The distribution curve of the Gaussian distribution depends on the Mean and the Standard Deviation (S.D.) values of the distribution. The S.D. value is a measure of spread of the curve and indicates the amount of variation of the values from the mean. The Gaussian distribution is symmetric and have a bell shaped density curve with a peak value and the tail extends on both ends to infinity. The probability density function of the normal distribution is given by [61]

$$f(t)=\frac{1}{s\sqrt{2\pi}}\exp\left[-\frac{(t-\mu)^2}{2s^2}\right]; \quad -\infty < t < +\infty \tag{5.3}$$

where,

μ is the mean value and

s is the Standard Deviation (S.D.) value.

In the present analysis, there are two populations representing the stress and strength parameters. If the parameters are assumed to be normally distributed, then there is a possibility that the forward tail of the stress distribution might overlap with the backward tail of the strength distribution, which may result in some failures. The reliability of the notched composite laminate is determined by combining the two populations and computing the corresponding standardized variable Z_R, which is given by the equation 5.4. To determine the reliability, we combine the two populations and compute the corresponding standardized variable Z_R, which is given by [62]

$$Z_R=\frac{\mu_R}{s_R}=\frac{\mu_{\sigma_o}-\mu_{\sigma_{p/avg}}}{\sqrt{{s_{\sigma_o}}^2+s^2{}_{\sigma_{p/avg}}}} \tag{5.4}$$

where,

μ_{σ_o} is the Mean value of strength of the un-notched laminate,

$\mu_{p/avg}$ is the Mean value of point/average stress of the notched laminate,

s_{σ_o} is the Standard Deviation (S.D.) value of strength of the un-notched laminate and

$s_{\sigma_{p/avg}}$ is the S.D. value of point/average stress of the notched laminate.

Reliability of a system is defined as the probability of success and can be written as

$$R_s = 1 - P_f \tag{5.5}$$

where,

P_f is the probability of failure, which can be calculated by obtaining the value corresponding to Z_R from appendix K of reference [61]. The value corresponds to the area under the normal distribution curve of the combined population. Thus, we are able to quantify system reliability in terms of a number that lies between zero and one.

5.4 Reliability Calculation

In Chapter 4, center-notched laminates and edge-notched laminates subjected to uniaxial tensile load were analyzed using Controlled Hole Laminate (CHL) and Un-Controlled Hole Laminate (UCHL) analysis to get the probabilistic parameters, such as, Mean and Standard Deviation (S.D.) values of point and average stresses of the laminates. The reliability value of the laminates is obtained by using Gaussian distribution method and it is explained in Section 5.3. The Gaussian distribution method requires the probabilistic parameters of the point/average stress of the laminate and the strength of the corresponding un-notched laminate to calculate the reliability value. In order to have a better understanding of the load bearing capacity of the laminates, the laminates were loaded with different values of Factor of Safety (FOS) on the ultimate load and the stochastic simulation is performed to calculate the corresponding reliability value.

5.4.1 Reliability calculation using point stress parameters

The reliability values of center-notched laminate subjected to fatigue loading conditions calculated by using the probabilistic parameters of point stress and strength of the corresponding un-notched laminate in Gaussian distribution method is shown in Tables 5.1-5.3. Factor of Safety (FOS) values n the range of 1- 1.45 are considered for the applied load and the corresponding reliability values were calculated. In Tables 5.1-5.4, R_p, μ_p and S_p refer to the reliability value, Mean and Standard Deviation (S.D.) of point stress obtained from Point Stress Criterion. Both Controlled Hole Laminate (CHL) and Un-Controlled Hole Laminate (UCHL) analysis were considered in calculating the reliability values.

Factor of Safety (FOS)	Applied Load	CHL analysis			UCHL analysis		
	MN/m	μ_p	S_p	R_p	μ_p	S_p	R_p
		GPa	GPa	%	GPa	GPa	%
1.00	1.113	1.0598	0.0049	57.92	1.1116	0.085	37.36
1.05	1.060	1.0137	0.0047	76.53	1.0602	0.082	54.52
1.10	1.012	0.9740	0.0046	88.58	1.0134	0.077	70.28
1.15	0.968	0.9335	0.0044	95.52	0.9856	0.074	81.62
1.20	0.928	0.8854	0.0041	98.87	0.9321	0.069	90.58
1.45	0.768	0.7561	0.0036	≈100	0.7148	0.059	99.98

Table 5.1 Reliability values of center-notched laminate subjected to Fatigue Loading Condition No.1 (F.LC.1) obtained using point stress parameters

Factor of Safety (FOS)	Applied Load	CHL analysis			UCHL analysis		
	MN/m	μ_p	s_p	R_p	μ_p	s_p	R_p
		GPa	GPa	%	GPa	GPa	%
1.00	1.105	1.0619	0.0046	57.31	1.1144	0.081	36.12
1.05	1.052	1.0161	0.0045	75.64	1.0648	0.078	53.01
1.10	1.005	0.9807	0.0042	86.96	1.0193	0.076	68.55
1.15	0.961	0.9407	0.0041	94.63	0.9851	0.072	79.02
1.20	0.921	0.9018	0.0038	98.13	0.9427	0.068	88.95
1.45	0.762	0.7611	0.0033	≈100	0.7350	0.058	99.96

Table 5.2 Reliability values of center-notched laminate subjected to Fatigue Loading Condition No.2 (F.LC.2) obtained using point stress parameters

Factor of Safety (FOS)	Applied Load	CHL analysis			UCHL analysis		
	MN/m	μ_p	s_p	R_p	μ_p	s_p	R_p
		GPa	GPa	%	GPa	GPa	%
1.00	1.099	1.0626	0.0045	55.17	1.1213	0.080	33.83
1.05	1.047	1.0238	0.0044	72.63	1.0723	0.077	50.40
1.10	0.999	0.9939	0.0041	83.25	1.0298	0.075	65.21
1.15	0.956	0.9617	0.0039	91.23	0.9959	0.071	76.21
1.20	0.916	0.9290	0.0035	96.01	0.8723	0.065	85.24
1.45	0.758	0.7694	0.0031	≈100	0.7579	0.057	99.92

Table 5.3 Reliability values of center-notched laminate subjected to Fatigue Loading Condition No.3 (F.L.C.3) obtained using point stress parameters

From Tables 5.1-5.3, it can be seen that the reliability values of center-notched laminates subjected to fatigue loading conditions increase drastically when the Factor of Safety (FOS) is increased from 1 to 1.45 in both Controlled Hole Laminate (CHL) and Un-Controlled Hole Laminate (UCHL) analyses. The reliability value is close to 100% for all center-notched laminates, when a FOS value of 1.45 is used. But it should be noted that the increase in the reliability value is rapid for FOS value in the range of 1-1.20, after that the increase is very less. In Table 5.1, it can be seen that the reliability value of center-notched laminate subjected to F.L.C.1 increases drastically from 57.92% to 98.87% when the FOS value is increased from 1 to 1.2. But for FOS value in the range of 1.2-1.45, increase in the reliability value is very less. The selection of proper FOS value on the ultimate load of notched laminates optimizes the design cost for the intended reliability.

Figures 5.1 and 5.2 show the change in reliability with Factor of Safety (FOS) value for center-notched laminate subjected to fatigue loading conditions. Both Controlled Hole Laminate (CHL) and Un-Controlled Hole Laminate (UCHL) analyses were considered. From Figures 5.1 and 5.2, it can be seen that the reliability curves of center-notched laminate subjected to Fatigue Loading Condition No.1 (F.L.C.1), F.L.C.2 and F.L.C.3, in both CHL and UCHL analyses, follow the same trend. The reliability curves of center-notched laminates subjected to F.L.C.1 and F.LC.2 are close to each other when compared to center-notched laminates subjected to F.L.C.2 and F.L.C.3. The reliability curves are non-linear in CHL analysis, whereas it is close to linear until a FOS value of 1.2 in UCHL analysis.

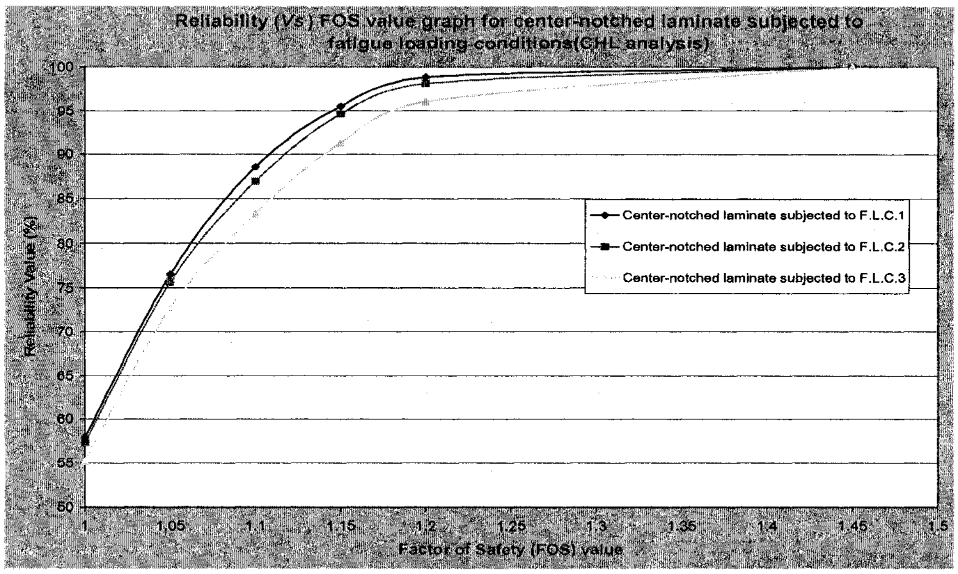


Figure 5.1 Reliability curves of center-notched laminate subjected to fatigue loading conditions and CHL analysis obtained using point stress parameters

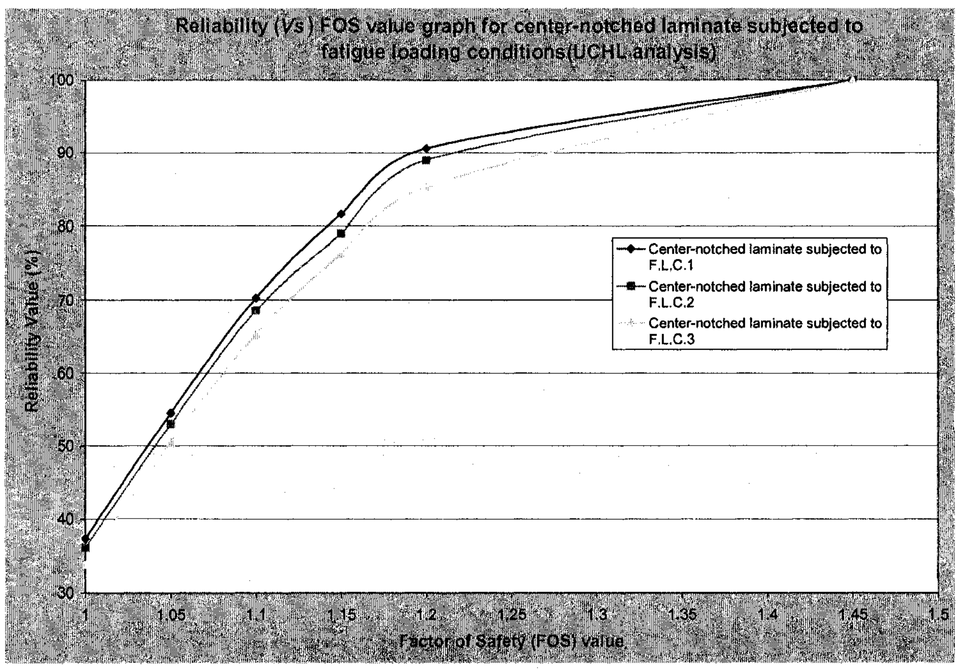


Figure 5.2 Reliability curves of center-notched laminate subjected to fatigue loading conditions and UCHL analysis obtained using point stress parameters

5.4.2 Reliability calculation using average stress parameters

The reliability values of center-notched laminate subjected to fatigue loading conditions determined by using the probabilistic parameters of average stress and strength of the corresponding un-notched laminate in Gaussian distribution method is shown in Tables 5.4-5.6. Average stress criterion was used to calculate the probabilistic parameters of average stress. Applying Factor of Safety (FOS) values in the range of 1- 1.45 on the ultimate load of center-notched laminates, the corresponding average parameters and reliability values were calculated. In Tables 5.4-5.6, R_{avg}, μ_{avg} and S_{avg} refer to the reliability value, Mean and Standard Deviation (S.D.) of average stress. Controlled Hole Laminate (CHL) and Un-Controlled Hole Laminate (UCHL) analysis were considered in calculating the reliability values.

Factor of Safety (FOS)	Applied Load	CHL analysis			UCHL analysis		
	MN/m	μ_{avg}	S_{avg}	R_{avg}	μ_{avg}	S_{avg}	R_{avg}
		GPa	GPa	%	GPa	GPa	%
1.00	1.113	1.0643	0.0018	54.38	1.1157	0.091	36.52
1.05	1.060	1.0173	0.0017	75.23	1.0584	0.088	54.95
1.10	1.012	0.9736	0.0015	88.72	1.0140	0.084	69.34
1.15	0.968	0.9394	0.0015	94.81	0.9763	0.081	79.97
1.20	0.928	0.9167	0.0013	97.15	0.9337	0.077	89.23
1.45	0.768	0.7580	0.0012	≈100	0.8049	0.063	99.52

Table 5.4 Reliability values of center-notched laminate subjected to Fatigue Loading Condition No.1 (F.L.C.1) obtained using average stress parameters

Factor of Safety (FOS)	Applied Load	CHL analysis			UCHL analysis		
	MN/m	μ_{avg}	s_{avg}	R_{avg}	μ_{avg}	s_{avg}	R_{avg}
		GPa	GPa	%	GPa	GPa	%
1.00	1.105	1.0659	0.0016	53.59	1.1205	0.090	34.97
1.05	1.052	1.0200	0.0014	74.19	1.0635	0.086	53.29
1.10	1.005	0.9809	0.0012	86.93	1.0210	0.083	67.31
1.15	0.961	0.9539	0.0011	92.67	0.9850	0.078	78.21
1.20	0.921	0.9314	0.0011	95.77	0.9414	0.074	88.34
1.45	0.762	0.7621	0.0010	≈100	0.8116	0.062	99.45

Table 5.5 Reliability values of center-notched laminate subjected to Fatigue Loading Condition No.1 (F.L.C.2) obtained using average stress parameters

Factor of Safety (FOS)	Applied Load	CHL analysis			UCHL analysis		
	MN/m	μ_{avg}	s_{avg}	R_{avg}	μ_{avg}	s_{avg}	R_{avg}
		GPa	GPa	%	GPa	GPa	%
1.00	1.099	1.0702	0.0015	51.60	1.1321	0.087	31.23
1.05	1.047	1.0270	0.0013	71.36	1.0720	0.084	50.46
1.10	0.999	0.9869	0.0012	84.25	1.0299	0.086	64.25
1.15	0.956	0.9736	0.0011	88.72	0.9984	0.078	74.58
1.20	0.916	0.9570	0.0009	92.11	0.9584	0.072	85.35
1.45	0.758	0.7654	0.0008	≈100	0.8288	0.061	99.15

Table 5.6 Reliability values of center-notched laminate subjected to Fatigue Loading Condition No.1 (F.L.C.3) obtained using average stress parameters

From Tables 5.4 to 5.6, it can be seen that the reliability value of center-notched laminate subjected to fatigue loading conditions obtained using average stress parameters increases drastically when the Factor of Safety (FOS) value is increased from 1.0 to 1.2, but after that the increase in the reliability value is less. For example, in Table 5.4, the reliability value of center-notched laminate subjected to Fatigue Loading Condition No.1 (F.L.C.1) increases from 36.52% to 89.23% for an increase in FOS value from 1 to 1.2 in UCHL analysis. But for an increase of FOS value from 1.2 to 1.45, the reliability value increases from 89.23% to 99.52%, which is comparatively less. The reliability value of the center-notched laminates is close to 100% when a FOS value of 1.45 is used in the CHL analysis. In UCHL analysis, the reliability value of center-notched laminates is in the range of 99.15 to 99.52% for a FOS value of 1.45.

Figures 5.3 and 5.4 show the change in reliability with Factor of Safety (FOS) value for center-notched laminate subjected to fatigue loading conditions. Both Controlled Hole Laminate (CHL) and Un-Controlled Hole Laminate (UCHL) analyses were considered. From Figures 5.3 and 5.4, it can be seen that the reliability curves of center-notched laminate subjected to Fatigue Loading Condition No.1 (F.L.C.1), F.L.C.2 and F.L.C.3 follow the same trend, in both CHL and UCHL analyses. The reliability curves of center-notched laminates subjected to F.L.C.1 and F.LC.2 are close to each other when compared to center-notched laminates subjected to F.L.C.2 and F.L.C.3. The reliability curves are non-linear in CHL analysis, whereas it is close to linear until a FOS value of 1.2 in UCHL analysis, after that it is non-linear.

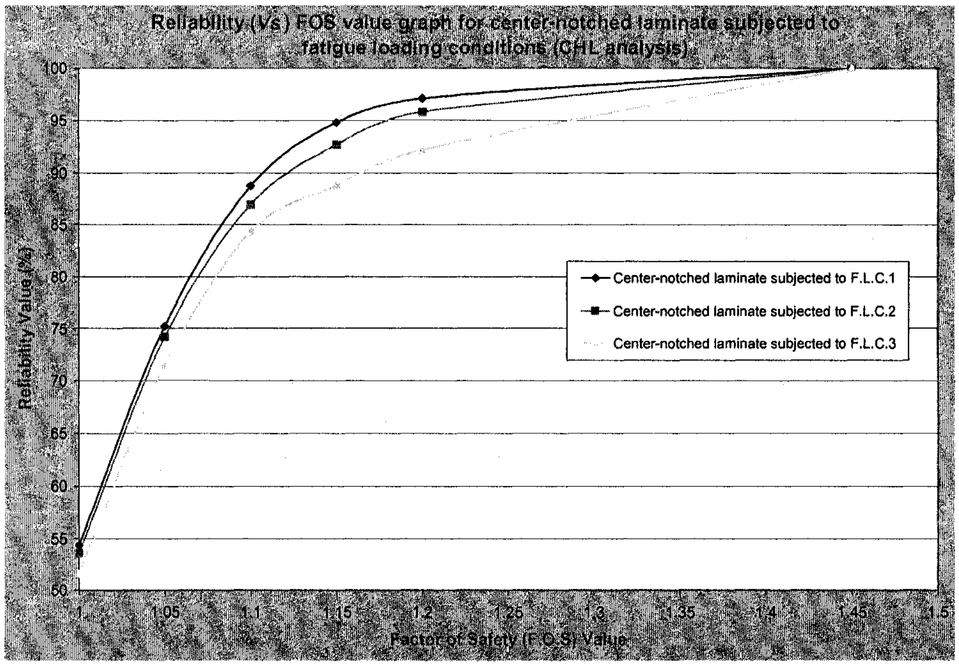


Figure 5.3 Reliability curves of center-notched laminate subjected to fatigue loading conditions and CHL analysis obtained using average stress parameters

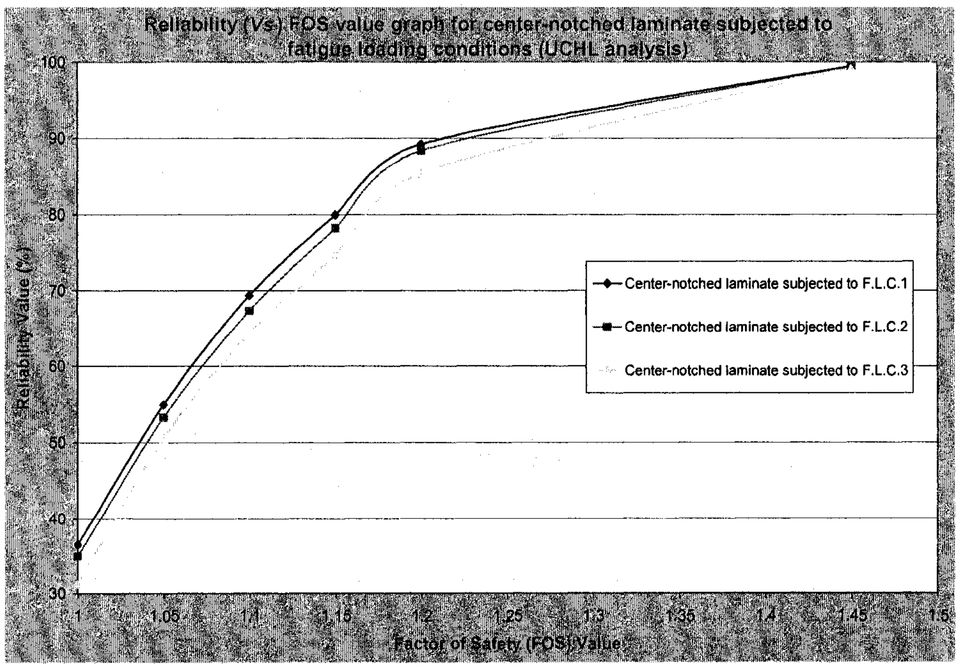


Figure 5.4 Reliability curves of center-notched laminate subjected to fatigue loading conditions and UCHL analysis obtained using average stress parameters

5.4.3 Reliability calculation of edge-notched laminates

The reliability values of edge-notched laminates are calculated by using the probabilistic parameters of point/average stress of edge-notch laminate and the strength of the corresponding un-notched laminate in Gaussian distribution method and they are shown in Table 5.7. Average stress criterion and point stress criterion were used to calculate the probabilistic parameters of average stress and point stress respectively. Factor of Safety (FOS) values in the range of 1- 1.45 on the ultimate load of edge-notched laminates were considered and the corresponding point/average parameters and reliability values were calculated. Controlled Hole Laminate (CHL) analysis is used in the calculation of reliability values of edge-notched laminates. The reliability curves of edge-notched laminates obtained using point and average stress criteria are shown in Figure 5.5.

Factor of Safety (FOS)	Applied Load	Point stress criterion			Average stress criterion		
	MN/m	μ_p	s_p	R_p	μ_{avg}	s_{avg}	R_{avg}
		GPa	GPa	%	GPa	GPa	%
1.00	1.111	1.0604	0.0044	57.51	1.0666	0.0015	53.28
1.05	1.058	1.0078	0.0043	78.67	1.0201	0.0014	74.12
1.10	1.01	0.9699	0.0040	89.54	0.9785	0.0014	87.53
1.15	0.9661	0.9415	0.0037	94.52	0.9565	0.0013	92.22
1.20	0.9258	0.9020	0.0035	98.12	0.9365	0.0012	95.17
1.45	0.7662	0.7734	0.0034	≈100	0.7753	0.0010	≈100

Table 5.7 Reliability values of edge-notched laminates obtained using point stress and average stress criteria

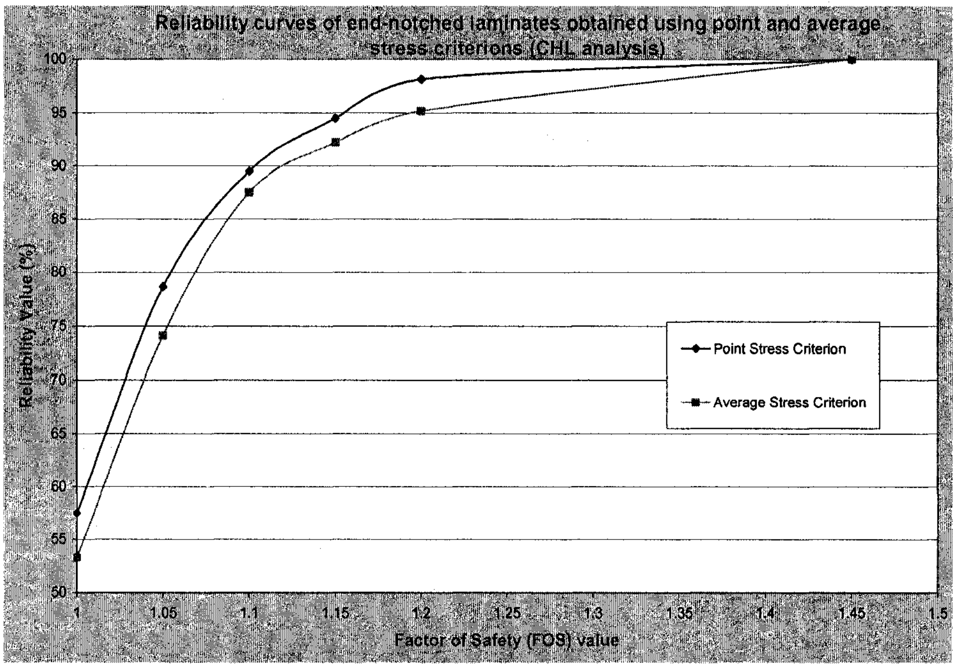


Figure 5.5 Reliability curves of edge-notched laminates obtained using point and average stress criterions

From Figure 5.5, it can be seen that the reliability curves of edge-notched laminates obtained using point and average stress criteria follow same pattern. The reliability value obtained from point stress criterion is more than that of average stress criterion for the same Factor of Safety (FOS) value. In Table 5.7, it can be noticed that for a FOS value 1, the reliability value of edge-notched laminate obtained from point and average stress criterion are 57.51% and 53.28% respectively. There is a difference of 4.23% in the reliability value, indicating that the reliability value obtained from average stress criterion gives conservative results and it provides a safer design of the laminate than the point stress criterion. Both the reliability curves are non-linear.

5.5 Comparison of reliability results

The reliability results of center-notched laminate subjected to fatigue loading conditions and of edge-notched laminates were compared to analyze the effect of fatigue loading and notch location on the reliability of notched laminates. In order to facilitate comparison of results, reliability values of laminates corresponding to the Factor of Safety (FOS) value 1, which is for the ultimate strength of the laminates, is used throughout this section. The effect of fatigue loading on the reliability of center-notched laminate is analyzed by comparing the reliability results of center-notched laminate subjected to fatigue loading conditions. Table 5.8 shows the reliability values of center-notched laminates obtained using point and average stress criteria. Both Controlled Hole Laminate (CHL) and Un-Controlled Hole Laminate (UCHL) analyses were considered. The reliability values of center-notched laminate not subjected to fatigue loading are considered from reference [5] and they are also given in Table 5.8.

Center-notched laminate	Applied Load	Reliability values obtained using point stress criterion (%)		Reliability values obtained using average stress criterion (%)	
	MN/m	CHL analysis	UCHL analysis	CHL analysis	UCHL analysis
Not subjected to fatigue loading	1.154	60.82	43.02	56.85	43.84
Subjected to F.L.C.1	1.113	57.92	37.36	54.38	36.52
Subjected to F.L.C.2	1.105	57.31	36.12	53.59	34.97
Subjected to F.L.C.3	1.111	55.17	33.83	51.60	31.23

Table 5.8 Comparison of reliability values of center-notched laminate not subjected to fatigue loading and of that subjected to fatigue loading conditions obtained using point and average stress criteria

In Table 5.7, it can be seen that the reliability values of all the center-notched laminates obtained using Un-Controlled Hole Laminate (UCHL) analysis are less than that of Controlled Hole Laminate (CHL) analysis for both point and average stress criteria. In UCHL analysis, both stochastic variations in material properties and geometric variations, such as, eccentricity of the hole the from the laminate center and change in hole shape were considered in the analysis. This leads to the increase in stress value around the hole region of the laminate, thereby increasing the probability of failure of the center-notched laminates. The reliability values of center-notched laminates calculated using UCHL analysis give conservative results, thus providing a safer design of the laminate than the CHL analysis. The difference between the reliability values of center-notched laminates obtained using CHL and UCHL analysis are in the range of 13.01% to 21.34%, which is considerably more.

The difference between the reliability values of center-notched laminates not subjected to fatigue loading and of that subjected to Fatigue Loading Condition No.1 (F.L.C.1) is in the range of 7.32% to 2.9%, which clearly indicates the effect of fatigue loading on the reliability value of center-notched laminate. The difference between the reliability values of center-notched laminates subjected to F.L.C.1 and F.L.C.2 are in the range of 1.55% to 0.61% and it is considerably less. Thus the effect of changing the maximum stress value (S_{max}) in the fatigue loading condition of center-notched laminate on the reliability value is less. The difference between the reliability values of center-notched laminates subjected to F.L.C.2 and F.L.C.3 are in the range of 3.74% to 1.99%. Thus the effect of changing the stress ratio (R) in the fatigue loading condition of center-notched laminate on the reliability value is more when compared to that of changing the maximum stress value (S_{max}).

The reliability results of center-notched laminates not subjected to fatigue loading and of edge-notched laminates obtained from Controlled Hole Laminate (CHL) analysis were compared to analyze the effect of changing the hole location on the reliability value of notched laminate and it is given in Table 5.8.

Laminate Type	Applied Load (MN/m)	Reliability value obtained using point stress criterion (%)	Reliability value obtained using average stress criterion (%)
Center-notched Laminate	1.154	60.82	56.85
Edge-notched laminate	1.111	57.51	53.28

Table 5.9 Comparison of reliability values of center-notched laminates not subjected to fatigue loading and of edge-notched laminates obtained using point and average stress criteria

In Table 5.8, it can be seen that the difference in the reliability values of center-notched and edge-notched laminates obtained from point and average stress criteria are 3.31% and 3.57% respectively, which clearly indicates the effect of changing the hole location on the reliability of notched laminate. The effect of changing the hole location in notched laminate is equivalent to that of subjecting the notched laminate with Fatigue Loading Condition No.1 (F.LC.1), as the differences in the reliability value of center-notched laminate not subjected to fatigue loading and of that subjected to F.L.C.1 are in the range of 3.74% to 1.99%.

5.7 Conclusions and Discussions

In the present chapter, reliability analysis is conducted on center-notched laminate subjected to fatigue loading conditions and on edge-notched laminates using point stress and average stress criteria. Both Controlled Hole Laminate (CHL) and Un-Controlled Hole Laminate (UCHL) analyses were considered in determining the reliability values. Gaussian distribution method is used to calculate the reliability values of the laminates, which requires the probabilistic parameters of point/average stress and the strength of the corresponding un-notched laminate.

In order to have a better understanding of the load bearing capacity of the laminates, the laminates were loaded with different values of Factor of Safety (FOS) on the ultimate load and the stochastic simulation is performed to calculate the corresponding reliability value. FOS values in the range of 1 -1.45 were considered in the analysis. A series of reliability values are obtained by varying the FOS value of center-notched and edge-notched laminates and it is given in Tables 5.1-5.7. The reliability results of the center-notched laminate subjected to fatigue loading conditions and of edge-notched laminates were compared to analyze the effects of fatigue loading and notch location on the reliability value of notched laminate and they are presented in Tables 5.8 and 5.9.

From Table 3.7, it can be concluded that the reliability values of the laminates obtained from UCHL analysis are always less than that of the CHL analysis, which is attributed by the geometric variations in notched laminates. The effect of changing the maximum stress (S_{max}) value of fatigue loading condition on the reliability of notched laminate is less. Whereas by changing the stress ratio (R) of the fatigue loading condition, the reliability value of the notched laminate increases considerably.

From Table 3.8, it can be concluded that the effect of changing the notch location has considerable effect on the reliability value of notched laminate. The reliability value obtained using average stress criterion is always less than that of point stress criterion. Thus it is recommended to design the notched laminates using average stress criterion considering Un-Controlled Hole Laminate (UCHL).

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

In the present thesis work, center-notched laminates and edge-notched laminates were analyzed experimentally and using Finite Element Analysis (FEA) to study the effect of fatigue loading and notch location on the reliability of notched laminates. The stress distributions in the notched isotropic plate and notched composite laminates were determined using Stochastic Finite Element Analysis (SFEA). A MATLAB© program has been developed using the SFEA methodology, which considers the stochastic variation of the material properties and geometric variations in the notched laminates. The program is capable of calculating the stress distributions in notched laminates with any laminate configuration and geometric dimensions.

Finite element formulation for isotropic plates and composite laminates and their corresponding flowcharts, which represent the systematic procedure of the analysis, were given in chapter 2. In Chapter 2, example problems were considered to validate the MATLAB© program written for the FEA of notched isotropic plates, and SFEA of center-notched and edge-notched laminates.

Experimental investigation on center-notched and edge-notched laminates was performed and it is discussed in Chapter 3. The geometric dimensions and laminate configurations of center-notched and edge-notched laminates used in the experiments were as follows:

Length (L) = 180mm, Width (W) = 37.9mm, Thickness (t) = 2mm, Hole Radius (R) = 7.54mm and laminate configuration $[0/90]_{4s}$. NCT301 graphite/epoxy material was used to manufacture the laminates. The center-notched laminates were subjected to three different fatigue loading conditions and subsequently tested under static loading condition to analyze the effect of fatigue loading on the ultimate strength of the laminates. Three samples are tested for each fatigue loading conditions. The loading parameters of the fatigue loading conditions were determined using the ultimate strength of center-notched laminate not subjected to fatigue loading condition. Fifteen samples of edge-notched laminate were tested under static loading condition to determine the average ultimate strength of the laminate. By analyzing the experimental results, it can be concluded that the effect of changing the stress ratio (R) value of fatigue loading condition on the ultimate strength of the laminates is more when compared to changing the maximum stress (S_{max}) value of fatigue loading condition. There is considerable difference between the ultimate strength of center-notched laminate not subjected to fatigue loading and edge-notched laminates, which indicates the effect of changing the notch location on the ultimate strength of the laminates. Microscopic study was performed on center-notched and edge-notched laminates to analyze the defects and damages in the laminates. The characteristic lengths of each notched laminate were calculated using point and average stress criteria, which requires the ultimate strength of the notched laminates and the ultimate strength of the corresponding unnotched laminates.

In chapter 4, stochastic simulations were performed on center-notched and edge-notched laminates using Controlled Hole Laminate (CHL) and Un-Controlled Hole Laminate (UCHL) analyses. In CHL analysis, only the stochastic variations in material properties were considered in the analysis. But in UCHL analysis, both stochastic variations in material properties and geometric variations in the laminate were considered. From the stochastic analysis, it was found that 200 and 300 simulations were required for CHL and UCHL analyses respectively to determine the mean and standard deviation values of point and average stress parameters of the laminates. By comparing the point and average stress parameters of the laminates obtained from CHL and UCHL analyses, it can be concluded that the stress values of the laminates obtained from UCHL analysis are always higher than that of CHL analysis. This is due to the consideration of geometric variations in the laminates for UCHL analysis. Stress values of the laminates obtained from average stress criterion is more than that of point stress criterion for both CHL and UCHL analyses.

In chapter 5, reliability analysis of the center-notched and edge-notched laminates was conducted to determine the reliability values of the laminates. The point/average stress parameters of the notched laminates and the strength parameters of the corresponding unnotched laminate were used in the Gaussian distribution method to find the reliability value of the laminates. Factor of Safety (F.O.S) values in the range of 1-1.45 were considered in the analysis for the applied load. The reliability graphs of the center-notched and edge-notched laminates, which show the variation in the reliability values with FOS values, were drawn.

By analyzing the reliability indices and graphs in Chapter 5, the following conclusions can be made:

(i) Changing the stress ratio (R) value in the fatigue loading condition of center-notched laminate has considerable effect on the reliability value of the laminate than by changing the maximum stress (S_{max}) value.

(ii) The reliability value of edge-notched laminate is less when compared with the reliability value of center-notched laminate not subjected to fatigue loading condition, which indicates the effect of changing the notch location on the reliability value of the laminate.

(iii) When the FOS value of the center-notched and edge-notched laminates is increased to 1.45, close to 100% reliability value is obtained. There is a sharp increase in the reliability value when the FOS value on the applied load is increased from 1 to 1.2. But for FOS values in the range of 1.2-1.45, the increase in the reliability value is relatively less.

(iv) The reliability values of both center-notched and edge-notched laminates, obtained from Un-Controlled Hole Laminate (UCHL) analysis are always less when compared to that of Controlled Hole Laminate (CHL) analysis.

(v) The reliability values of both center-notched and edge-notched laminates, obtained using average stress criterion is always less when compared to that of point stress criterion.

(vi) The notched laminates analyzed using average stress criterion and UCHL analysis will give conservative results, thereby providing a safer design for notched laminates.

The thesis can be further extended on the following topics, which will constitute the future research work:

- The number of fatigue loading cycles (N) used for the fatigue testing of notched laminate can be increased further, to analyze the effect of increasing the N value on the reliability of the center-notched and edge-notched laminates.
- The fatigue loading conditions can be used to test center-notched laminates with different hole sizes, to study the combine effect of notch size and fatigue on the reliability of center-notched and edge-notched laminates.
- The reliability analysis can be performed on notched laminates with different hole shapes, such as, elliptical, rectangular, triangular, etc.

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APPENDIX-A

MATLAB® Program for the Stochastic Finite Element Analysis (SFEA) of Notched Composite Laminates

```
clear all;
close all;

tic;
format long;

%%%%%%%%%%%%%%%%%%  Initializing all relevant variables  %%%%%%%%%%%%%%%%%%

dummy=0;          % Dummy variable %
nelem=0;          % Number of elements %
ndofn=0;          % Degrees of freedom per node %
nnode=0;          % Number of nodes per element %
ngaus=0;          % Order of Gaussian numerical integration %
ntype=0;          % Stress State %
nmats=0;          % Number of materials %
numnp=0;          % Total number of nodal points %
nstre=0; nstr1=0; % Number of stress components per element %
props = 0;        % Material properties %
lnods = 0;        % Node numbers %
coord = 0;        % Global co-ordinates %
kgaus = 0;        %  Keeps track over the Gauss points.


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

nlami = 300;      % Number of laminates %

[ao, do] = LENGTHAODO(nlami);           % Passes the characteristic length values obtained from point
and average stress criterion %
[xc_uchl, yc_uchl] = holeuncertainty(nlami); % Controls the eccentricity of hole from the laminate center %

for ilami = 1: nlami

fprintf(' \n SFEA of [0/90]4s Un-Controlled Hole Laminate (UCHL) simulation no:%d \n',ilami);


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


%%%%%%%%%%%%%%%%%%  GETDAT – Gets all relevant scalar variables  %%%%%%%%%%%

[nelem,ndofn,nnode,ngaus,ntype,nmats,numnp,nstre,nstr1,ntotg,iRuns,L,W,D]= GETDAT(dummy);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
%%%%%%%%%%%%%%% GETARR – Gets all the relevant datas %%%%%%% %%%%%%%

[props, tetag, t, matno, propg, lnods, coord, Plyorient]
=GETARR(nelem,ngaus,ndofn,nnode,ntotg,ilami,xc_uchl,yc_uchl,L,W,D);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


%%%%%%%%%%%%%%%%%%%% Establishes the nodal connectivity %% %%%%%%%%%%%%

nevab = ndofn*nnode;
globK = zeros(numnp*ndofn);        % Initializes the [K] matrix for each and every element %

[lm,id] = NCONNECT(ndofn,nnode,nelem,numnp,lnods);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


%%%%%%%%%%%%%%%%%%%% Computes the stiffness matrices %%%%%%%%%%%%%%%%%

for ielem = 1: nelem            % Loop over the total number of elements %

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


%%%%%%%%%%%%%%%%% Gets the coordinates of each node in the element %%%%%%%%%%

 for inode = 1: nnode
     lnode = round(abs(lnods(ielem,inode)));
         for idime = 1 : ndofn
             elcod(idime,inode) = coord(lnode,idime);
         end
 end

 shape = zeros(8,1);
 derivative = zeros(2,8);
 xjacm = zeros(2,2);
 cartd = zeros(2,8);
 estif = zeros(nevab,nevab);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


%%%%%%%%%%%%%%%%%%%% Starts Gaussian integration %%%%%%%%%%%%%%%%%%%%

 kgasp = 0;   % Keeps track over the Gauss points in each element %

 [posgp,weigp] = GAUSSIAN(ngaus);

 for igaus = 1 : ngaus                  % Loop over each Gauss points along the horizontal axis %

         for jgaus = 1 : ngaus          % Loop over each Gauss points along the vertical axis %

            kgasp = kgasp + 1;
            kgaus = kgaus + 1;
```

```
            lprop = matno(kgaus);
            Plyorient = tetag(kgaus,:);
            thick = sum(t(kgaus,:));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%% Evaluates the shape functions and their derivatives %%%%%%%%%%%%

            exisp = posgp(igaus);
            etasp = posgp(jgaus);

            [D] = MODPS(ntype,nstre,nmats,lprop,propg,Plyorient,kgaus);

            [shape,derivative] = SFR2(exisp,etasp);

            [xjacm,djacb,gpcod,cartd]= JACOBIAN(ielem,kgasp,ndofn,nnode,shape,derivative,elcod);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%% Evaluates the [B]  and [DB'] matrices %%%%%%%%%%%%%%%%%%

            [B] = BMATPS(nnode,cartd);

            [DB] = DBE(D,B);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%% Calculates the element stiffness matrices %%%%%%%%%%%%%%

             estif = estif + transpose(B)*DB*dvolu;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%% End of Gaussian integration %%%%%%%%%%%%%%%%%%%%%%

        end

 end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%% Assembles the element stiffness matrices %%%%%%%%%%%%%%%

  [globK] = ASSEMBLE(ielem,nevab,lm,estif,globK);

 end

%%%%%%%%%%%%%%%%%% End of Assembling %%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
%%%%%%%%%% Reads the nodal loads and assemble them into global force vector %%%%%%%%%

[eload] = LOAD(nelem,numnp,nnode,nevab,ndofn,ngaus,posgp,weigp,coord,lnods,matno,props,iRuns);

[forvec] = FORCE(nelem,nevab,lm,eload);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%% Solution for the nodal displacements %%%%%%%%%%%%%%%%

[Disp,eldis] = SOLVE(ndofn,coord,forvec,globK,lm,nelem,nevab,numnp,nnode,iRuns);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%% Solution for the Gauss point stresses %%%%%%%%%%%%%%%%

  strsp=zeros(nstre,ngaus*ngaus,nelem);
  sgtot=zeros(nstr1,ngaus*ngaus,nelem);

 [sgtot,strsp]= STREPS(nelem,matno,props,propg,tetag,ntype,nmats,nnode,ndofn,coord,ngaus,nstre,
 nevab,nstr1,eldis,lnods,iRuns);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%% Calculation of the stresses using point and average stress criterions %%%%%%%%%

[sigma_avg, stress_ab, sigma_p]= AVGSTR(sgtot,coord,ilami,ao,do,W,D,xc_uchl);

end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%% Displacements at loading end %%%%%%%%%%%%%%%%%%%%%%

Disp_v_29(ilami,1)=Disp(58);
Disp_v_273(ilami,1)=Disp(546);
Disp_v_524(ilami,1)=Disp(1048);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%% Stresses at nodes from point A to B %%%%%%%%%%%%%%%%
Sigma_y_653(ilami,1) = stress_ab(1,1);
Sigma_y_629(ilami,1) = stress_ab(1,2);
Sigma_y_605(ilami,1) = stress_ab(1,3);
Sigma_y_581(ilami,1) = stress_ab(1,4);
Sigma_y_557(ilami,1) = stress_ab(1,5);
Sigma_y_533(ilami,1) = stress_ab(1,6);
Sigma_y_334(ilami,1) = stress_ab(1,7);
Sigma_y_356(ilami,1) = stress_ab(1,8);
```

```
Sigma_y_378(ilami,1) = stress_ab(1,9);
Sigma_y_400(ilami,1) = stress_ab(1,10);
Sigma_y_422(ilami,1) = stress_ab(1,11);
Sigma_y_444(ilami,1) = stress_ab(1,12);
Sigma_y_466(ilami,1) = stress_ab(1,13);
Sigma_y_488(ilami,1) = stress_ab(1,14);
Sigma_y_510(ilami,1) = stress_ab(1,15);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


%%%%%%%%%%%%%%% Calculation of point and average stress parameters %%%%%%%%%%

Average_stress(ilami,1)= sigma_avg ;
Point_stress(ilami,1)=sigma_p;

end

SD_Sigma_y_653=std(Sigma_y_653);

Mean_average_stress = mean(Average_stress);
SD_average_stress =std(Average_stress);
COV_average_stress = (SD_average_stress/Mean_average_stress)*100;

Mean_point_stress= mean(Point_stress);
SD_point_stress =std(Point_stress );
COV_point_stress = (SD_point_stress/Mean_point_stress)*100;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


%%%%%%%%% Stores the average and point stress parameter results in a separate file %%%%%%%

fk = fopen('UCHL analysis results.m','w');
fprintf(fk,' \t Average stress parameters after simulation no.%d',nlami);

for ilami = 1 : nlami
        fprintf(fk,'\n-----------------------------------------------------------------------\n');
        fprintf(fk,'\nSimulation : %d\t   %13.10f  \n',ilami,Average_stress(ilami,1));
end

fprintf(fk,' \t   Mean average stress : %13.10f  \n\n',Mean_average_stress);
fprintf(fk,' \t   Standard Deviation of average stress : %13.10f  \n\n',SD_average_stress);
fprintf(fk,' \t   Coefficient of Variation of average stress : %13.10f  \n\n',COV_average_stress );

fprintf(fk,' \t Point stress parameters after simulation no.%d',nlami);
for ilami = 1 : nlami
        fprintf(fk,'\n--------------------------------------------------------------------\n');
        fprintf(fk,'\nSimulation : %d\t   %13.10f  \n',ilami,Point_stress(ilami,1));
end

fprintf(fk,' \t   Mean point stress: %13.10f  \n\n',Mean_point_stress);
fprintf(fk,' \t   Standard Deviation of point stress: %13.10f  \n\n',SD_point_stress);
fprintf(fk,' \t   Coefficient of Variation of point stress: %13.10f  \n\n',COV_point_stress );

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
disp(' Mean stress value at nodal points from point A to B')

Mean_Sigma_y_653 = mean(Sigma_y_653);
Mean_Sigma_y_629 = mean(Sigma_y_629);
Mean_Sigma_y_605 = mean(Sigma_y_605) ;
Mean_Sigma_y_581 = mean (Sigma_y_581);
Mean_Sigma_y_557 = mean(Sigma_y_557) ;
Mean_Sigma_y_533 = mean(Sigma_y_533) ;
Mean_Sigma_y_334 = mean(Sigma_y_334) ;
Mean_Sigma_y_356 = mean(Sigma_y_356) ;
Mean_Sigma_y_378 = mean(Sigma_y_378) ;
Mean_Sigma_y_400 = mean(Sigma_y_400) ;
Mean_Sigma_y_422 = mean(Sigma_y_422) ;
Mean_Sigma_y_444 = mean(Sigma_y_444) ;
Mean_Sigma_y_466 = mean(Sigma_y_466);
Mean_Sigma_y_488 = mean(Sigma_y_488) ;
Mean_Sigma_y_510 = mean(Sigma_y_510) ;

disp('Mean displacement of nodes at the loading end ')

Mean_Disp_v_29  = mean(Disp_v_29) ;
Mean_Disp_v_273 = mean(Disp_v_273);
Mean_Disp_v_524 = mean(Disp_v_524) ;

SD_Sigma_y_653 = std(Sigma_y_653);
COV_Sigma_y_653 =  (SD_Sigma_y_653/Mean_Sigma_y_653)*100;

Sigma_y_ref
Sigma_y_exact

fprintf(fk,' \t  Mean stress value at hole edge: %13.10f  \n\n',Mean_Sigma_y_653);
fprintf(fk,' \t  SD value of stress at hole edge: %13.10f  \n\n',SD_Sigma_y_653);
fprintf(fk,' \t  Displacement (v) at the loading end (node 29): %13.10f  \n\n',Mean_Disp_v_29);
fprintf(fk,' \t  Displacement (v) at the loading end (node 273): %13.10f  \n\n',Mean_Disp_v_273);
fprintf(fk,' \t  Displacement (v) at the loading end (node 524): %13.10f  \n\n',Mean_Disp_v_524);

save all;
fclose('all');
```