THE ESTIMATION OF EQUITY RISK PREMIUM FOR CANADA

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ABSTRACT

The Estimation of Equity Risk Premium for Canada

In this paper I assess the equity risk premium for Canada. In order to accomplish the task, I use three different procedures. The first procedure extends the seminal Mehra and Prescott (1985) article and examines whether the equity risk premium puzzle exists for Canada during the last fifty years. This approach incorporates the conventional parameters of risk aversion and the time discount factor generally accepted by the existing literature. Moreover, the estimates of those parameters are derived which correspond to the observed premium provided by Canadian equities over risk free debt.

The second procedure is based on Fama and French (2002). Different estimates of the risk premium are calculated using two *growth* models based on the growth rate of aggregate dividends and the growth rate of operating earnings. The third, and final, is the *decomposition* model based on the methodology employed by Dimson, Marsh and Staunton (2006).

The main results argue for the existence of an equity premium puzzle for Canada, as the estimated parameter of risk aversion of the average investor is unrealistically high. Additionally, the estimates of the expected risk premium in real values using the growth and decomposition models argue for a smaller magnitude the historical risk premium.

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1. INTRODUCTION

The historical gap between the long-term returns on stocks and the return on bonds in the major markets of the world has been consistently positive. There is substantial evidence that U.S. stocks in particular had considerably higher returns than T-bills over the last century. However, it is essential to make a distinction between the forward looking concept of the *expected* equity risk premium and the *ex-post* realized excess return that equity provides over risk-less assets. The forward looking expected risk premium is defined as the expected incremental return that induces risk-averse investors to hold more risky securities such as stocks rather than the risk free securities.

The main challenge of this study is to try to estimate what the sign and the size of the expected equity risk premium are for the Canadian financial market. In order to achieve this goal, the analysis has been done in three main directions. The first question to be answered is whether the equity premium puzzle, as documented by Mehra and Prescott (1985) for the United States, exists in Canada. The next issue discussed is the estimation of equity risk premium for Canada using growth models. The expected risk premium is estimated by using the growth rates of dividends and operating earnings. I argue that the equity risk premium is declining and the investors in Canadian equity shouldn't expect a repetition of the historical outperformance of equity. The final part of the study provides a decomposition of the equity risk premium to better understand of its drivers and their effects.

The main advantage of this combined methodology is that it provides more robust and precise information about the magnitude of the risk premium that investors in Canada could reasonably expect.

This study examines the historical risk premium for Canadian market by using S&P/TSX Composite index as a proxy for a well-diversified equity portfolio and the Government of Canada Treasury Bill for the risk free security.

The Canadian Financial Markets Research Centre (CFMRC) database includes a wide range of historical estimates for the Canadian equity risk premium. It includes historical estimates that are calculated by taking both the arithmetic and geometric means between annualized monthly returns on S&P/TSX Composite Total Return Index and several Government of Canada Treasury Bills and Bonds (short term – 91 day T-bills, medium term – 7.5 year average term and long term – 17 year average term). Estimates for Canadian equity premium available in the database as of December 2006 and the premium over short-term bills are 6.11% (arithmetic mean) and 4.96% (geometric mean) respectively. Switzer (2007) reports somewhat similar measures of 4.2% (3.0%) arithmetic (geometric) mean annualized premiums for Canadian equity.

Since the seminal articles by Shiller (1984) and Mehra and Prescott (1985) the magnitude of the historical realized premium for U.S. market has been questioned by researchers and practitioners. There have been numerous studies in finance and macroeconomics such as Benartzi and Thaler (1995), Goyal and Welch (2003), Arnott and Bernstein (2002), Fama and French (2002) and etc. that attempted to explain the puzzle, but so far the realized large magnitude of risk premium is not completely explained. Kocherlakota (1996) surveys the theoretical literature aimed to "solving the puzzle" by introducing market frictions such as trading costs into the model and/or incorporating higher risk aversion parameters. Nevertheless, the puzzle in terms of the size of risk premium remains an unsolved mystery for most financial economists.

The standard economic utility theory fails to provide a basis for the observed large excess returns to equity. In this paper, I develop a benchmark for the equity risk premium in Canada for the last fifty years by applying the Mehra (2003) procedure, which extends Mehra and Prescott (1985). In addition, similar to Mehra and Prescott (1985) I look at the question inversely: I attempt to identify and estimate the values of the parameters that are consistent with the historical risk premium.

Additionally, I employ the valuation methodology to estimate the expected equity returns used by Fama and French (2002) to assess the expected equity premium based on fundamentals such as dividend yields, operating earnings and growth rates of aggregate dividends and operating earnings.

Dimson, Marsh and Staunton (2006) argue that the historical estimated equity risk premia are not reliable predictors of the future expected risk premium. They suggest a decomposition of the equity risk premium using an alternative methodology that I also apply here using Canadian data. In their original paper, they demonstrate that the expected world equity risk premium does not exceed 3-5% (based on their calculation which appears to argue for 1-2% lower risk premium than the historical value).

This paper is organized as follows. Section 2 provides a review of the current literature on this topic, Section 3 describes the data used in the calculations, and Section 4 presents the methodology employed. The paper concludes with a summary section.

2. LITERATURE REVIEW

The risk-return tradeoff is one of the pillars of finance theory. It is strongly believed that the more risk the rational investor takes on the more reward he or she will demand. The equity risk premium provides a measure of the trade-off that is in the center of interest to both academics and practitioners. Welch (2000) calls the equity risk premium as “perhaps the single most important number in financial economics”.

The expected risk premium has great importance for institutional investors in the asset allocation decisions of their portfolios. Asset allocation is investor’s decision-making process of assigning specific weights to the broad asset classes, such as stocks, bonds and cash in the portfolio. The main target of asset allocation is the maximization of the expected risk to return ratio, given the investor’s level of risk aversion and other investment constraints such as time horizon, liquidity preference, etc. Depending on the strategy employed, three level of asset allocation could be identified. The first level of asset allocation is sometimes referred to as indexing, because it argues for replicating the weights of the benchmark portfolio. The strategic asset allocation focuses on the long-term investment horizon (usually 5 years) and is based on long-term forecasts of asset class performance. The tactical asset allocation is more short-term in nature and calls for active management of investment portfolios. An active investment strategy typically begins with an assessment of expected equity risk premium. The large expected equity premium dictates much greater tilt towards equity rather than debt bringing to larger holdings of common stocks in mutual funds and pension plans. According to the capital asset pricing theory, the expected return on common stock is strongly associated with equity premium; hence the determination of riskiness of each stock depends on the relative size of the equity premium, as the risky stocks that have large betas should provide returns that exceed the risk premium.

The equity risk premium is also important in the determination of the cost of capital for firms. Firm's cost of capital is the weighted average of the cost of equity and the cost of debt. The cost of debt generally is estimated relatively easily as the bond yields could be matched with similar bonds in the fixed income market, the equity risk premium contains valuable information for the correct estimation of the cost of equity and consequently the cost of capital. Thus, the equity risk premium is indirectly incorporated in the derivation of firm's cost of capital. Firm's weighted average cost of capital is the appropriate discount rate for future expected cash flows used in the firm valuation. Moreover, the cost of capital is crucial for managers of firms trying to determine the capital structure that maximizes its value.

The equity risk premium is also essential for business valuators for decision-making processes in terms of capital expenditures and M&As (Switzer 2007).

The issue of equity risk premium has critical meaning for public policy makers. There are many propositions to reform the social security system and to move the public pension funds towards equity investments rather than fixed income. The benefits could be obtained if the stock market continues to outperform the risk free assets in the future, as it did in the past.

The concept of equity risk premium is one of the most important issues in the finance and measuring its magnitude is of paramount importance. Using average excess returns on equity over risk free assets as a proxy for the equity risk premium is problematic for statistical reasons, particularly because the distribution of returns is time varying. The most common definition of the equity risk premium states that the equity risk premium is the incremental reward in terms of returns required by the investors for holding the assets that are relatively riskier than the assets providing risk free rates of returns. Historically, the standard deviation of stocks have been considerably higher than the standard deviation of bonds, hence, risk-averse investors require a positive premium in order to hold equity rather than bonds. The 20% per year standard deviation of the returns on equity has been five times the standard deviation of returns on bonds (about 4% per year) in U.S[1] over the last century.

Using a 200-year estimation period, Siegel (1999) shows that equity in U.S. and other major markets outperformed risk free securities, and the equity premia are positive for long horizons.

Most research on the equity risk premium focuses on the U.S. market. Fama and French (2002) note that the S&P 500 index which is the proxy for market portfolio during 1872-2000 provided 8.81% return per year and while the six-month commercial paper's annual return was 3.24% yielding a 5.57% equity risk premium.

However, one should differentiate between the realized excess returns (historical equity premium) and the expected equity risk premium. Grabowski (2005) asserts that the equity risk premium should be treated as a forward-looking concept. It is the investors' expectations as of the valuation date, which cannot be observed at the same date. The historical excess returns that investors have realized during the various holding periods are potentially unreliable predictors for the future expected risk premium.

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[1] See Mehra (2003)

Mehra and Prescott's (1985) seminal paper examines the historical (1889-1978) returns for U.S. market and concludes that the large average realized 6% excess return of S&P 500 index over the short term debt (T-bill) cannot be rationalized by standard expected utility maximization paradigm of neoclassical financial economics. This result is termed the "Equity Risk Premium Puzzle".

According to modern asset pricing theory, assets are priced in such a way that the loss in marginal utility of the market participant that occurs when foregoing current consumption in favor of future consumption by buying the assets with known prices, is balanced out by the gain in the marginal utility, which is due to expected gains in the future consumption, when the assets bought pay-off. Hence, it is important to differentiate the values of two assets that have the same pay-offs but provide them in different states of nature. The asset that provides the pay-off when the market is performing poorly and the consumption level is low, is more valuable than otherwise the same asset that pays-off when markets are doing well and the level of consumption is high. In other words, the asset highly correlated with the market (i.e. high beta asset) pays off more in bull market and less in bear. Such assets must provide a positive premium to persuade the investors to hold them. Mehra (2003) argues that when the markets are doing well and the consumption level is high, the high beta assets provide less incremental utility to investors than the assets that pay off when the consumption level is low. In other words, the asset that pays off more in bear markets, and less in bull help to smooth the consumption level over time, subsequently is more desirable by the investor. Based on these notions, it is shown that as both stocks and bonds pay off more or less in the same states of nature, these assets should command the rate of returns of approximately similar magnitude. However, the average realized equity premium has been much higher than the risk adjusted returns that Mehra and Prescott (1985) estimate using the standard utility theory.

Ait-Sahalia, Parker and Yogo (2004) attempt to partially solve the equity premium puzzle by introducing the nonhomothetic utility function of both luxury and basic goods. They argue that the correlation of the aggregate consumption with the stock market movements fails to justify such outperformance of stocks over short-term government debt. The authors suggest that the poor sector of the society generally does not participate in the stock market while the society members with relatively high net wealth have active participation in the market, hold the equity and, hence, are the main receivers of the equity premium. Therefore, since the poor are quite risk averse and the rich do not alter their consumption of basic goods (just the consumption of the luxury goods), when the equity risk premium shrinks, the authors show that by modifying the utility function, the marginal utility of the consumption of very rich significantly correlates with the return on equity and justifies the historical large premium. The suggested parameters of risk aversion are only slightly higher than the levels generally accepted by financial economists. However, the puzzle still remains for the average household that mainly consumes basic goods. Ait-Sahalia, Parker and Yogo (2004) assert that if it is believed that the average household participates in the stock market, then it has unrealistically high parameter of risk aversion.

There are various directions and models employed by researchers to assess the equity risk premium using conventional models[2]. The first direction of studies uses ex-post approach while the second direction includes the studies based on ex ante methods of inference.

Ibbotson and Chen (2003) present a summary of literature and categorize the equity risk premium derivations into four approaches, that are basically subcategories of main ex-post and ex-ante directions. The first type of studies implies that the past return observations are strong indicators of future market performance and strongly influence the expectations of market participants. Studies such as Ibbotson and Sinquefield (1976a, 1976b), Dimson, Marsh and Staunton (2003, 2006) and Switzer (2007) obtain the equity risk premium by looking at the difference between the realized return on the market portfolio of common stocks and the risk free interest rate. This direction is assuming that the historical risk premium is an unbiased estimate of the expected equity risk premium. Indeed, the arithmetic average realized return could be an unbiased estimate of the expected return if the return realizations are serially independent and stable over time. In general, the historical equity risk premium is measured using the U.S. T-bill yield (or sometimes long-term U.S. government bond yields) for the return on risk free securities and a broad stock market index such as S&P 500 for risky assets. Ibbotson Associates estimate a geometric premium of 7.3% for the period 1926-1999.

Siegel (2002) measures real stock, bond and bill returns and equity premia for 16 developed markets (including Canada) over century long holding period (1900-2002). He argues that it is important to conduct a study of historical market returns over a long

[2] Fernandez (2006) argues that since investors do not have homogenous expectations, there is no single expected equity risk premium or required equity risk premium, but several. Hence it is not possible to estimate the expected equity risk premium.

horizon and not just over the recent history in order to adjust for short term, non-persistent shocks where bad times are accompanied with good times to cancel each other's effect in the long run. For instance, after each major financial crisis such as the stock market crash of 1987, the bond market rout of 1994, Long Term Capital Management crisis of 1998, 9/11 attack or the recent developments in credit markets due to sub-prime mortgage industry blow-out, the average investor becomes extremely risk-averse, thus demanding higher than usual premium for holding equity considered to be the riskiest asset class. However, in longer term prospective, when the short-term turmoil in financial market is over, the investor's risk aversion normalizes, and risk is being taken again, the long-run expectations of equity risk premium should be more or less constant and not affected by short-term temporary shocks.

Furthermore, it is important to examine the historical performance of markets other than U.S. Bodie (2002), Dimson, March and Staunton (2006) and Arnott and Bernstein (2002) point that if the study looks only at the U.S. market, it most probably suffers from the survivorship bias, as many markets that have been active at the beginning of the observation period are no longer functioning. Dimson, March and Staunton (2006) refer to this concept as the country selection bias. They draw a parallel between the process of selecting Microsoft stock to estimate the risk premium and extrapolating it to other stocks and the process of selecting the U.S. market as a benchmark to make inferences about other markets. Their study includes estimates for 17 countries and in general, the results obtained for other countries are robust and consistent with studies conducted for U.S. The risky securities have outperformed riskless securities for a long holding period in all economies and the risk premium over both short-term government bills and long-term bonds are positive for all 17 countries. The 17 country average real dividend growth rate is -0.10 %, the average expansion in the price to dividends (P/D) ratio is 0.58% and they argue that those components of equity risk premium do not play significant role in the outperformance of the global equity. The largest contributor to the equity returns is the global geometric average dividend yield.

Dimson, March and Staunton (2006) use data for the 17 countries as of 2006 make up for 91% of world's market capitalization. The long run historical returns that they present include both arithmetic and geometric means for all these countries and for the world. The geometric mean of the annualized equity premia range from 2.87% for Denmark up to 7.08% for Australia, while based on the arithmetic mean the equity risk premia cross-sectionally vary from 4.51% (Denmark again) to 10.46% (Italy). The 4.54% Canadian geometric mean of the equity risk premium is the 9th in the world, while the 5.88% arithmetic mean annual equity premium is 12th.

Switzer (2007) provides the equity risk premium for both Canadian and U.S. markets that spans over the 50-year holding period (1956-2006). In addition, besides determining the equity premium of S&P 500 and S&P/TSX over short term T-bills, the alternative measures of risk premium are offered, such as Small Cap equity risk premium relative to 1 month T-Bills. All the measures based on both arithmetic and geometric means indicate that the equity risk premium for U.S. is higher. Canadian premium for the 50-year horizon is argued to be between 3- 4%.

The second direction of the research in this area is based on fundamentals, when trying to measure the equity risk premium. The fundamental data used in modeling include price to earnings ratios, dividend yields, operational earnings and other supply-side factors. Grabowski (2005) states that the fundamental theoretical underpinning of this approach is that the investors price the assets by taking into consideration the current economic conditions of publicly traded firms and the overall health of the economy. In other words, the expected growth rates of dividends and earnings combined with the expected growth rates of Gross Domestic Product (GDP) and Consumption level play crucial roles in forming the investors' expectations on the equity risk premium. The key implication of studies based on fundamentals is that the investors could not have reasonably expected such outperformance by equity over risk free assets.

Ibbotson and Chen (2003) decompose the U.S. return on equity based on S&P 500 index into various drivers including corporate income, inflation, reinvestment return, and growth rate of price to earnings ratio, book value of equity, dividend payout ratio, GDP per capita, earnings per share, dividends, etc. Their main conclusion is that the largest part of the return on equity is attributable to dividend payments and nominal earnings growth, while the price to earnings ratio has a relatively small effect. Subsequently, the authors forecast the expected value of equity premium assuming current valuation levels, which are not anticipated to be changed in the near future. They suggest that the main drivers of the equity risk premium are the nominal earnings growth and dividend payments. Their estimate of the equity risk premium (3.97%) is slightly lower than the historical excess return to equity.

Additionally, they apply a forward-looking equity risk premium estimation technique based on the current dividend yield instead of its historical average. As the current level of dividend yield is significantly lower than the historical average the procedure results in much smaller risk premium. According to this method the risk premium is close to 0.24%! However, this approach violates the Modigliani and Miller propositions[3], the authors select the first estimate as a benchmark for the expected risk premium. They believe that the equity risk premium in the long run should be positive, which contrasts the studies suggesting negative or even close to zero risk premia such as Arnott - Bernstein (2002).

Another influential study done using supply side approach is Fama and French (2002). They use earnings and dividends as fundamentals to estimate the expected stock returns and subsequently the equity risk premium. They suggest that the information conveyed by fundamentals assists in evaluating the historical observed premium. The main suggested models are the dividend growth model and the earnings growth model. The authors decompose the unconditional expected return on stocks into the sum of average estimated dividend yield and/or average growth rate of earnings as an alternative in addition to the average growth rate of dividends. The long-term annual (1872-2000) estimate of the risk premium according to dividend growth model is 3.54%. One should mention that according to this fundamental approach the risk premium for the last 50 years is considerably smaller (2.55%) compared to the preceding period that included both the First and the Second World Wars. Fama and French (2002) suggest that this decrease is consistent with three observed phenomena. First, they indicate that the estimates from fundamentals are more precise as the standard error of the estimate from the dividend growth model has been smaller than the estimate from the average return. In

[3] Due to the current corporate tax laws, the firms are selecting the share buybacks as the most effective form to distribute the returns to their shareholders thus reducing their tax burden. This is considered the main cause for the current low levels of current dividend yields. In the meantime the M&M theory suggests that a company's dividend-payout ratio doesn't affect the total return that shareholders receive. It rather affects the form (capital gains vs. dividends) the total return is distributed. Subsequently, the current low dividend payout ratio should not affect the forecast of the long-term equity risk premium.

addition, the Sharpe ratio has been more or less constant during both 1872-1950 and 1950-2000. Finally, they argue that the standard valuation theory predicts much smaller equity risk premium in accordance with the lower estimate of risk premium from dividend growth model. The key conclusion of this paper is that the dividend growth model is preferable in estimating the expected real equity risk premium, and the realized large risk premium for the last fifty years is a result of the unexpected capital gains and the decline in the discount rate. The sources of these unexpected capital gains are the unanticipated growth rates of dividends and earnings. In other words, the expected risk premium for the last fifty years is much lower than the realized excess returns, because the market price to earnings (P/E) ratio has significantly increased.

The aggregate dividend ratio has been widely used in the literature to predict the equity risk premium. Dow (1920), Campbell and Shiller (1998) and Fama and French (2002) support the notion that the dividend yield is a statistically significant explanatory variable for forecasting the annual equity risk premium.

However, Goyal and Welch (2003) argue that the predictive ability of dividend yields is significant only for in-sample predictions, while it lacks any significance for out-of-sample forecasts. The authors provide an in-depth diagnosis of the predictive power of dividend yields through time. They conclude that the explanatory power of the aggregate dividend ratio of the one year ahead equity risk premium is time-varying and even though at the beginning of the period of their analysis the dividend yield has been a robust predictor, it gradually shifts towards predicting only itself due to high autocorrelations. Goyal and Welch (2003) show that the dividend ratios have explanatory power up until 1990. For yields to continue to be good predictors, they should have slowly increased until the mid-70s, followed by slowly decreasing values up until 1999, when the sharp increase should have been needed to follow. Another key implication is that the results are not robust with respect to the estimation period used.

The third direction of equity risk premium studies is based on surveys, in which experts such as financial economists and chief financial officers are pooled. Based on a survey of 226 experts, Welch (2000) obtains the risk premium expectations that are quite diverse, ranging from 1% to 15% with the mean of 7% over the next 30 years.

Welch (2001) surveys a larger number of finance and economics professors (more than 500). The survey reveals that finance and economics professors are more cautious and expect smaller premium in the future. The average estimate of the risk premium for 30-year horizon is revised and downgraded by 1.5% to 5.5%. Welch (2001) argues that because during these three years the markets were performing poorly, the subjects of the study were using short horizon experience to form their long-term expectations.

Graham and Harvey (2001) conduct a series of multi year (2000 Q1 – 2001 Q3) quarterly surveys of CFOs (about 200 responders per quarter with total of 1200 responses). They argue that CFOs use the Capital Asset Pricing Model (CAPM) in practice. Their results suggest that 10-year equity risk premium is about 4% and is demonstrably stable over time. However, the 1-year premium is quite time-variant and sensitive to the level of past returns.

Goldman Sachs[4] (2002) reports a survey conducted among its client base that includes 100 global senior portfolio managers and CIOs. The subjects have been asked about their estimates of the long-run equity risk premium for the markets they operate in.

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[4] "The Equity Risk Premium: It's lower than you think", CEO Confidential, Issue 2002/14, November 2002, Goldman Sachs & Co.

Most of the responses have been between 3.5% to 4.5% for major financial markets with 3.9% average expected equity risk premium for OECD countries. Even though these estimates are lower than the historical averages, they are higher than 2.4% expected equity risk premium estimate presented by the bank.

Grabowski (2005) mentions that in its "Quantitative profiles" monthly periodical, Merrill Lynch publishes the expected return estimates derived from its own developed multistage dividend discount model. The research covers a couple of hundred companies from S&P 500 broad market index. According to the periodical, the average risk premium estimate for the last fifteen years is believed to be around 4.6%.

3. DATA DESCRIPTION

The data used in this study are obtained from various sources. Fundamental data on personal expenditures on services and on non-durable goods as well as the total population for Canada are obtained from CANSIM (Statistics Canada) for the period 1961-2006.

The six-month average yields on Government of Canada Treasury bills and the annual Consumer Price Index (2001 basket content, base year of 1992= 100) for 1959-2006 are gathered from the Statistics Canada.

Dividend series are obtained from the Toronto Stock Exchange Monthly Reviews. Since February 2006 the periodical is available only electronically and has been renamed to "TSX E-Review". The growth rate of operating earnings is estimated by using the series kindly provided by Dr. Lorne N. Switzer.

The real growth rate of Canadian Gross Domestic Product and the real yields on long term (at least10 year) Government of Canada Treasury bonds are available through Organization for Economic Co-Operation and Development (OECD) database since 1971 till 2005.

The Canadian Financial Markets Research Centre (CFMRC) and CFMRC TSX database are used to obtain the historical estimates of various measures of Risk Premia published by Passport Financial Services Inc.

4.1 THE RISK PREMIUM PUZZLE IN CANADA

In this section I examine the equity risk premium puzzle in Canada using the same procedure as Mehra (2003) extending Mehra and Prescott (1985).

Switzer (2007) provides the estimates of both short and long term Canadian equity risk premia. Over the post World War II holding period (1961-2006), the annualized equity risk premium over the short-term risk free security represented by the Scotia Capital 91-day T-bill has been 4.2%, while the spread over the long-term Government of Canada bond has averaged to 3%. Here, I examine whether the current economic theory justifies the magnitude of the above-mentioned outperformance (3-4%) of equity over risk free debt in Canada.

The following assumptions are required in order to illustrate the equity risk premium puzzle in Canada.

1. The economy is considered as frictionless.
2. The economy has a single representative described by the following preferences over the random consumption levels:

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\cdot U(C_t)\right], \text{ where } 0<\beta<1 \qquad (1)$$

Here the $E_0(\cdot)$ is the expectation operator conditional on the information available at time 0, β is the subjective time preference discount factor and C_t is the level of per capita consumption in Canada at time t.

The utility function employed is of the constant relative risk aversion (CRRA) class:

$$U(C,\alpha)=\frac{C^{1-\alpha}}{1-\alpha}, 0<\alpha<\infty \qquad (2)$$

The parameter α measures the curvature of the utility function and when α=1 the utility function in (2) is defined as logarithmic function which is the limit of the function as α approaches 1.

3. The economy consists of only one productive unit, which produces the perishable consumption output y_t at time t.

4. There is only one share with the price p_t at time t, which is competitively traded and since there is only one productive unit considered, the return on the share represents the return on the overall market and the claim on the stochastic process y_t.

An investor is facing the intertemporal choice, where in order not to be worse off, he or she should demand the loss in the utility associated with foregoing the current consumption in order to buy one additional share of the equity to be compensated with the increase in the utility at some future time via incremental consumption resulted from the selling of the additional equity share by the investor.

To buy one additional unit of equity, the investor at time t needs to spend p_t, thus the loss in the current utility is $p_t \cdot U'(C_t)$. Meanwhile at t+1 the investor sells the share at $p_{t+1}+y_{t+1}$ and the increase in the expected utility is $\beta \cdot E_t\left[(p_{t+1} + y_{t+1})U'(C_{t+1})\right]$.

The fundamental pricing relationship, which could be used both for stocks and bonds, is obtained by equating the foregone utility at time t with the incremental utility gained at time t+1.

$$p_t \cdot U'(C_t) = \beta \cdot E_t\left[(p_{t+1} + y_{t+1})U'(C_{t+1})\right] \qquad (3)$$

5. The growth rate of per capita consumption is assumed to be identically and independently distributed (i.i.d.). Let's denote it by: $x_{t+1} = C_{t+1} / C_t$

6. Let $\mu_x = E(\ln x)$ and $\sigma_x^2 = Var(\ln x)$, where $\ln x$ is the continuously compounded growth rate of consumption.

After some mathematical operations (see Appendix A) the expected returns of equity and risk free asset could be presented as:

$$\ln E_t(R_{e,t+1}) = -\ln \beta + \alpha \cdot \mu_x - \frac{1}{2} \cdot \alpha^2 \cdot \sigma_x^2 + \alpha \cdot \sigma_x^2 \tag{4}$$

$$\ln R_f = -\ln \beta + \alpha \cdot \mu_x - \frac{1}{2} \cdot \alpha^2 \cdot \sigma_x^2 \tag{5}$$

where $E_t(R_{e,t+1}) = \frac{p_{t+1} + y_{t+1}}{p_t}$ is return on equity and R_f is return on risk free asset.

The analysis here spans from 1961 till 2005 and covers several business cycles (see Figure 1). The average annual per capita consumption growth rate during that period has been about 3% with the variance of 0.02%. Mehra (2003) argues that the time preference discount factor should be 0.99 and the generally accepted coefficient of risk aversion must be around 10. Hence, these parameters become the starting point of the analysis. Consequently, by altering the input parameters the estimates are obtained so that they correspond to the observed 4.2% equity risk premium for Canada. *Table 1* illustrates the results.

The results for Canadian financial market strongly resemble the estimates of Mehra and Prescott (1985) for U.S market. Allowing the parameter of risk aversion to change from one to twenty, the Canadian equity risk premium does not break out from 1% barrier! The first row displays that the risk premium is 0.28% when employing the conventional parameters suggested by the existing literature. The results are very much similar to 0.35% equity risk premium benchmark for U.S. offered by Mehra and Prescott (1985). The second row displays the level of the equity risk premium when the time discount factor is held constant and the parameter of risk aversion varies. The results implicate that there is a positive relationship between the equity risk premium and the parameter of risk aversion. However the substantial increase (from 10 to 20) of the risk aversion parameter has only had marginal effect on the risk premium, bringing it up to 0.73% that is still less than 1%.

The third row displays the maximum value of the risk premium (0.56%), when the risk aversion parameter is being held constant at ten and the time discount factor is being gradually reduced. There is some evidence of negative relationship between these two variables.

Finally, the last row reports the estimates of the parameters of risk aversion and time preference that correspond to the historical risk premium. The results suggest that the average investor in 1961 should have had relatively large risk aversion (α=52) to anticipate the excess return that equity provided over the risk free asset class in the next fifty years. However, conventional wisdom argues against such level of risk aversion, which contradicts to the average investor's perceived risk aversion. The following example provided in Mankiw and Zeldes (1991) helps to demonstrate the point. An investor with the relative risk aversion parameter of 30 is indifferent between the gamble with a 50% chance of $100,000 consumption and 50% chance of $50,000 consumption and a certain consumption of $51,209. This level of risk aversion is not a typical characteristic of an average investor. In sum, this procedure provides evidence that the equity risk premium puzzle documented for U.S. by Mehra and Prescott (1985) exists for the Canadian financial market as well.

4.2 THE ESTIMATION OF EQUITY RISK PREMIUM BY GROWTH MODELS

In this part of the study, the expected equity risk premium for Canada is estimated using the fundamentals such as dividend yields, operating earnings. The procedure employed here is based on Fama and French (2002).

Here, the risk premium measure is a forward-looking concept estimated as the difference between the expected return on the market portfolio and the risk free rate. Fama and French (2002) argue that the expected forward-looking measure of the equity risk premium based on the fundamentals provides additional insights on whether the historical risk premium is close to its expected value when compared with the historical equity premium.

According to Gordon (1962) the current price of the equity equals the present value of the future expected dividend payments to investors discounted by the required rate of return:

$$P_0 = \frac{D_1}{1+r} + \frac{D_2}{(1+r)^2} + \cdots\cdots + \frac{D_n}{(1+r)^n} \qquad (6)$$

where r is the investor's required return in order to hold equity. Moreover, if dividends are assumed to grow at the constant rate (g) then the above mentioned equation becomes an annuity and could be presented as:

$$P_0 = \frac{D_1}{r - g} \tag{7}$$

Consequently, from (7) the required rate of return on equity is obtained as the ratio of expected dividends over the current price of equity plus the constant growth rate of dividends:

$$r = \frac{D_1}{P_0} + g \tag{8}$$

where

g = the expected growth rate of dividends in perpetuity

P_0 = the current price, at the end of year 0

D_1= the expected dividends at the end of year t = 1

r = the expected rate of return on marked based on the dividend discount model

The estimates of the expected rate of equity return are strongly impacted by dividends growth rate, hence the selection of the proper proxy for the growth rate is of great importance. Claus and Thomas (2001) argue that the growth rate of the dividends equals to the consensus of the forecasts by the sell-side analysts of 5-year earnings growth rate assuming that the dividend payout ratio is 50%. However, the authors mention that this estimate is systematically optimistic when compared to the realized earnings.

Meanwhile Easton, Taylor, Schroff and Sougiannis (2002) use the residual income model to estimate the growth rate of the dividends. Their methodology is based on two assumptions. First, the authors argue that accounting earnings could be summed over time. Also, they assume that the current price of equity could be expressed as a linear function of current book value and expected future earnings. By utilizing these concepts, the authors derive a simple linear relation between the ratio of the sum of the forecasted earnings to the current book value and the current price to book ratio. According to this model the expected rate of return on equity and the growth rate of dividends are estimated simultaneously.

Goedhart, Koller and Wessels (2002) use the average growth rate of the Gross Domestic Product (GDP) as a proxy for the growth rate of earnings and dividends.

Here, the unconditional estimate of the expected stock return is calculated using two methods based on Gordon (1962). The first method is the dividend growth model expressed by the following equation:

$$Av.(RE_t) = Av.(\frac{D_t}{P_{t-1}}) + Av.(gd_t) \tag{9}$$

where the $Av.(\cdot)$ is the operator that provides the average value, the RE_t is the expected equity return, $\frac{D_t}{P_{t-1}}$ is the dividend yield at t and gd_t is the growth rate of the dividends and equals to $\frac{D_t - D_{t-1}}{D_{t-1}}$ where D_t is the total dividends for the year t on some broad market portfolio.

Similarly, the second method of estimating the expected equity return is called the earnings growth model:

$$Av.(RE_t) = Av.(\frac{D_t}{P_{t-1}}) + Av.(gy_t) \tag{10}$$

Here gy_t is defined as the growth rate of operational earnings and is set to be equal to $\frac{Y_t - Y_{t-1}}{Y_{t-1}}$, where Y_t is the aggregate dollar value of the reported operational earnings of the companies included in the broad market index for year t.

Fama and French (2002) argue that the stationarity of the dividend yield ratio is desired for the convergence of the growth rate of the dividends with the compound rate of capital gains. The existence of stationarity allows the growth models to be appropriate methods for obtaining the equity returns. Thus, the unit root tests for stationarity are performed here on the dividend yield. The unit root test used here is the Dickey-Fuller (1976) test. The descriptions of the hypothesis to be tested and the tests performed are presented in Appendix B.

Tables 2 and 3 indicate that the stationarity of the dividend yield is strongly supported. Table 2 reports the test statistics obtained when performing the Dickey-Fuller (1976) tests. The first and the second rows display the tests with a trend and no lags and one lag, while the third row reports the test without trend but with lags. When compared with the critical values for the tests reported in Table 3, all three tests indicate the dividend yield to be stationary.

In order to gain more insights, the estimation procedure is performed in both nominal and real terms. Moreover, both the geometric and arithmetic averaging procedures are reported.

The nominal valuation methods described below are not adjusted by the inflation and are applied both on the dividends and the operating earnings.

Let gd_{nom}^{t} be the growth rate of the dividends and its yearly values are obtained as follows:

$$gd_{nom}^{t} = \frac{d_t}{d_{t-1}} - 1 \tag{13}$$

Then the equity return for period t is:

$$RE_t = \frac{d_t}{P_{t-1}} + gd_{nom}^{t} \tag{14}$$

Consequently, the risk premium for the period could be expressed as:

$$ERP_t = RE_t - F_t \tag{15}$$

where F_t is the average yield on 6-month Government of Canada Treasury bill. The arithmetic and geometric means of the nominal ERP_t calculated based on the *dividends growth* methods are presented on the second and the fourth rows of the Table 4.

The real valuation methods are derived after adjusting the nominal values by the Consumer price index (CPI):

$$gd^t_{real} = \frac{d_t}{d_{t-1}} \times \frac{CPI_{t-1}}{CPI_t} - 1 \qquad (16)$$

where gd^t_{real} is the real growth rate of the dividends adjusted to S&P/TSX Composite index and CPI_t is the Consumer Price Index (1992=base) for Canada in year t. The return on equity is also expressed in real terms:

$$RE^r_t = \frac{d_t}{P_{t-1}} \times \frac{CPI_{t-1}}{CPI_t} + gd^t_{real} \qquad (17)$$

and accordingly the equity risk premium is obtained by subtracting the risk free rate from the return on equity:

$$ERP^r_t = RE^r_t - F_t \qquad (18)$$

Similarly, using the same logic both nominal and real measures of the equity risk premium could be obtained from the models that use the average growth rate of the operational earnings as alternative proxies for g (see Appendix C).

The results are presented in Table 4. From Table 4 we can see that the equity risk premia range from negative fifty basis points in real terms to 4-6% in nominal terms. The largest estimate of the equity risk premium in real terms is obtained from the operating earnings growth model and when the growth rate is averaged geometrically (about 2%). Three out of four nominal value estimates of the equity risk premium cluster around the 4% level. The largest estimate of the equity risk premium is 6.63% and it is derived from the operating earnings growth model using arithmetic averaging in nominal terms. This procedure provides evidence that the real expected equity risk premium is expected to be around 0-2% while the nominal expected equity risk premium between 4-6%. The results in real values are in line with the arguments of Arnott and Bernstein (2002) stating that the current level of the equity risk premium is close to 0 and is argued to be even below 0.

4.3 THE EXPECTED EQUITY RISK PREMIUM

In this section, the decomposition of the historical Canadian equity risk premium is performed using the procedures described in Dimson, Marsh and Staunton (2006). Fama and French (2002), Ibbotson and Chen (2003) and Arnott and Bernstein (2002) argue that the equity markets have outperformed the expectations of the investors and this unexpected rally by the equity markets is mostly due to the historical large equity risk premium. Moreover, it is not statistically proper to extrapolate the observed excess return that equity has provided over risk free debt in the past to forecast the expected equity risk premium. The decomposition of the equity risk premium into various components provides additional insights on the underlying drivers of the historical equity risk premium.

Therefore, the historical equity risk premium is decomposed into the following four components. The first component is the average dividend yield (Y_t) that could be presented as:

$Y_t = \frac{D_t}{P_t}$, where D_t is the dividend payment received at the period t on the broad market index and P_t is the closing price of the index at time t. The next component of the equity risk premium is the average growth rate of the real dividends (G_{dt}) adjusted for the inflation (I_t) :

$$G_{dt} = \frac{d_t}{d_{t-1}} - 1, \text{ where } d_t = \frac{D_t}{(1+I_t)^t} \text{ and } (1+I_t)^t = (1+I_1)\cdot(1+I_2)\cdots(1+I_t)$$

The next contributor is the average rate of change of the price to dividend ratio (G_{PDt}):

$$G_{PDt} = \frac{PD_t}{PD_{t-1}} - 1, \text{ where } PD_t = \frac{P_t}{D_t}$$

The last component of the risk premium is the average annualized real return on risk free security (RFR_t).

The total return on equity in real terms is the sum of capital appreciation and income from dividends both adjusted for inflation:

$$TR_t = CA_t + DI_t \quad (19)$$

where TR_t is the total return, CA_t is the capital gain and DI_t is the dividend income at time t. The capital gain and the dividend income could be presented as:

$$CA_t = \frac{P_t}{P_{t-1}} \cdot \frac{1}{1+I_t} \quad (20)$$

$$DI_t = \frac{D_t}{P_{t-1}} \cdot \frac{1}{1+I_t} \tag{21}$$

Furthermore, after some mathematical transformations presented below, the equity risk premium for the domestic Canadian investor is broken down into its components.

$$TR_t = CA_t + DI_t = \frac{P_t}{P_{t-1}} \cdot \frac{1}{1+I_t} + \frac{D_t}{P_{t-1}} \cdot \frac{1}{1+I_t} = \frac{1}{1+I_t} \cdot \left[\frac{P_t}{P_{t-1}} + \frac{D_t}{P_{t-1}} \cdot \frac{P_t}{P_t} \right] =$$

$$= \frac{1}{1+I_t} \cdot \frac{P_t}{P_{t-1}} \cdot \left[1 + \frac{D_t}{P_t} \right] = \frac{1}{1+I_t} \cdot \frac{P_t}{P_{t-1}} \cdot [1+Y_t] \tag{22}$$

$$(1+G_{dt}) \cdot (1+G_{PDt}) = \frac{d_t}{d_{t-1}} \cdot \frac{PD_t}{PD_{t-1}} = \frac{\dfrac{D_t}{(1+I_t)^t}}{\dfrac{D_{t-1}}{(1+I_{t-1})^{t-1}}} \cdot \frac{\dfrac{P_t}{D_t}}{\dfrac{P_{t-1}}{D_{t-1}}} = \frac{1}{1+I_t} \cdot \frac{P_t}{P_{t-1}} \tag{23}$$

The equation (23) is obtained through multiplication of the average growth rate of the real dividends plus 1 with the average rate of change of the price to dividend ratio plus 1.

From these two equations, the historical total real return on equity is shown as:

$$TR_t = (1+G_{dt}) \cdot (1+G_{PDt}) \cdot (1+Y_t) \tag{24}$$

In order to obtain the equity risk premium the geometric difference of the risk free asset from the total return on equity is taken[5]:

$$ERP_t = (1 + G_{dt}) \cdot (1 + G_{PDt}) \cdot (1 + Y_t) \cdot \frac{1}{(1 + RFR_t)} \tag{25}$$

The results are presented in Table 5. The last column in Table 5 shows that the ex-post equity risk premium for the Canadian investor is 2.76%. The columns 1 and 3 demonstrate that the main contributors to the equity risk premium are the average dividend yield and the average growth rate of dividends. Meanwhile, the contribution of the expansion of the price to dividends ratio is somewhat limited. For the same holding period the real risk free rate averaged to 2.56%.

During the previous fifty years the institutional investors have benefited from the positive effects of portfolio diversification that reduced their portfolio risks. Moreover, the investors seeking international exposure to their portfolios have experienced additional rewards such as the benefits of international diversification. This process is considered to be the main driver of the average annual expansion of the price to dividend ratio (0.68%) according to Dimson, Marsh and Staunton (2006). In addition, Arnott and Bernstein (2002) argue that the historical expansion of the price to dividend ratio is non-recurring event and it is not expected to persist in the future. Thus, in order to make inferences about the future expected risk premium, it could be assumed that the

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[5] Dimson, Marsh and Staunton (2006) includes also the term for exchange rate since the study is designed to measure the realized equity risk premium for the U.S. investor. To be consistent with the previous estimates of the equity risk premium presented in the previous sections, the procedure is regarded from the point of the Canadian investor's view. Thus, Dimson, Marsh and Staunton (2006) is modified by leaving out the exchange rate factor.

expansion of the price to dividend ratio that have taken place in the past, will not recur in the next fifty years and its contribution could be taken out from the historical risk premium. Without the annual expansion of the price to dividend valuation multiple, the expected (forward looking) equity risk premium becomes closer to 2%.

4.4 THE ALTERNATIVE MEASURES OF THE EQUITY RISK PREMIUM

The Canadian Financial Markets Research Centre (CFMRC) Summary Information Database includes the special section for the risk premia. Both the short-term and the long-term Government of Canada bonds are employed to obtain the annual equity risk premia. The return on equity is proxied by the S&P/TSX Composite index. The estimation procedure incorporates both arithmetic and geometric averaging methods. The starting point reported in the database is 1988. Figures 2 and 3 demonstrate the variation through time of the various measures used to capture the annual equity risk premia. The evidence suggests that the levels of the annual risk premia have been contracting. Figure 2 displays the premia estimated using the arithmetic averaging while Figure 3 is for geometric averaging. Both figures include spreads over the short term as well as the long-term government bonds.

Both the arithmetic and geometric averaging techniques indicate that the equity risk premia over the long term government bonds have shrank by 2.07% and 1.96% respectively during 1988-2006 holding period. In the meantime, the decrease in the annual equity risk premium over 91-day T-bills has been only marginal (about 15 basis points on average).

Many researchers such as Goyal and Welch (2006) and Mehra (2003) contend that no other estimate can do better in forecasting the future expected risk premium than the historical average equity premium. Hence, by looking at the latest estimates of the arithmetic average of the annual equity risk premium over the long-term bonds for 2006, the inferences on the market expectations for the future expected risk premia could be done. The 2006 estimate indicates 4.5% premium somewhat larger than the previous estimates suggested in this study.

Moreover, the OLS regression analysis has been performed here where the significant positive relationship has been identified between the historical equity risk premium over 91-day T-bills (arithmetic averaging) and the growth rate of Canadian per capita consumption for the periods of 1988-2005. The correlation coefficient is 0.43 and statistically significant at α=10% level. The regression model argues that the equity risk premium is expected to be 5.7%, which is slightly lower than the observed premium of 6.11% for 2006 assuming the average growth rate consumption to be equal 2.7% (historical average for 1988-2005).

An alternative method of measuring the expected risk premium by deriving the forward-looking estimate is largely used among the practitioners such as the large investment banks.

The growth rate of the real GDP is used to approximate for the growth accompanied with the *current* levels of the dividend yield on a broad market index. The expected risk premium could be expressed as:

$$E(Equity\,\mathrm{Re}\,turn)_t = Growth(GDP_{\mathrm{Re}\,al})_t + DivYield_t \tag{26}$$

$$E(Risk\,\mathrm{Pr}\,emium)_t = E(Equity\,\mathrm{Re}\,turn)_t - RFR_t \tag{27}$$

Here the RFR represents the real return on the long-term bond provided by the Bank of Canada, which is a marketable bond that pays semi-annual interest based on a real interest rate. The rationale behind this procedure is that the estimate is forward looking and does not depend on the past information such as historical mean or standard deviation. In order to gauge the current level of equity risk premium one needs the current level of dividend yields and the current growth rate of the real economy[6]. Table 6 presents the expected values for the equity risk premia using the above-mentioned methodology for each year starting from 1991. The average value of the expected risk premium for the 1991-2005 period is slightly greater than 1%.

The most recent measure (3.49% for 2005) of the expected risk premium provides an assessment of the expectations on the magnitude of the equity risk premium.

The bottom line is that these alternative measures of the equity risk premium reveal that the investors should expect the equity risk premium lower than its historical average by 2-3%.

5. CONCLUSIONS

In the first section of this study, the existence of the equity risk premium "puzzle" is documented, and it is shown that the standard neoclassical theory of utility

[6] Source: OECD "In order to calculate the growth rate of GDP free of the direct effects of inflation, data at fixed, or constant, prices should be used. Price relativities change over time, and the *1993 System of National Accounts* recommends that the fixed prices used should be representative of the periods for which the growth rates are calculated, which means that new fixed prices should be introduced frequently, typically every year. The growth rates of GDP between successive periods are linked together to form chain volume indices."

maximization in modern financial economics is not able to rationalize the observed historical excess returns that the equity holders received over the risk free debt holders in the last fifty years in Canada. The generally accepted levels of the risk aversion and the time discount factors suggest that the premium should have been less than one third of 1%! This evidence is in line with the findings of the Mehra and Prescott (1985).

In addition, the following question has been raised: what the above-mentioned parameters should be in order to justify the large equity risk premium the Canadian equity provided versus the risk free bonds. Only a relatively large level (α=52) of the investor risk aversion has been able to explain the historical premium of 4.2%. However, such large measure of risk aversion is not reasonable, given the usual behavior and the investment choices that the average investor makes.

In the next section, the fundamentals such as the dividends and the operating earnings have been used to estimate the expected risk premium. The estimates of the real equity risk premium vary between 0 and 2%, while the nominal risk premium has been between 4% and 6%.

Furthermore, the historical equity risk premium has been decomposed to evaluate the forces that drive the expected risk premium. The obtained results suggest that the level of risk premium is 2.76% and investors should not expect the history to repeat itself in the future. Historically, the main contributor to the formation of the equity risk premium has been the level of dividend yields, while the expansion of the price to dividend ratio has only had a marginal impact.

The bottom line is that the three-step analysis performed in this study indicates that the outperformance of the risky assets is expected to be of smaller magnitude.

APPENDIX A

The utility function is assumed to be of the constant relative risk aversion (CRRA) class:

$$U(C,\alpha)=\frac{C^{1-\alpha}}{1-\alpha},0<\alpha<\infty \tag{1}$$

where α measures the curvature of the utility function and Ct is the level of per capita consumption in Canada at time t. By taking the first derivative of the utility function in respect to the consumption, we get:

$$U'(c_t) = c_t^{-\alpha} \tag{2}$$

Also the price of the equity share (p_t) is homogeneous of degree 1 in the dividend at time t (y_t), it could be represented as:

$$p_t = w \cdot y_t \tag{3}$$

By inserting the equations 1 and 2 into the following fundamental pricing relationship (4) obtained from equating the impact on the well being due to current foregone consumption and future benefits of holding the equity, we get the following:

$$p_t \cdot U'(C_t) = \beta \cdot E_t\left[(p_{t+1} + y_{t+1})U'(C_{t+1})\right] \tag{4}$$

$$w \cdot y_t \cdot c_t^{-\alpha} = \beta \cdot E_t\left[(w \cdot y_{t+1} + y_{t+1}) \cdot c_{t+1}^{-\alpha})\right] \tag{5}$$

$$w = \beta \cdot E_t\left[(w+1) \cdot (y_{t+1} / y_t) \cdot (c_{t+1}^{-\alpha} / c_t^{-\alpha})\right] =$$

$$= \beta \cdot w \cdot E_t\left[(y_{t+1} / y_t) \cdot (c_{t+1}^{-\alpha} / c_t^{-\alpha})\right] + \beta \cdot E_t\left[(y_{t+1} / y_t) \cdot (c_{t+1}^{-\alpha} / c_t^{-\alpha})\right] \quad (6)$$

$$w = \frac{\beta \cdot E_t\left[(y_{t+1} / y_t) \cdot (c_{t+1}^{-\alpha} / c_t^{-\alpha})\right]}{1 - \beta \cdot E_t\left[(y_{t+1} / y_t) \cdot (c_{t+1}^{-\alpha} / c_t^{-\alpha})\right]} \quad (7)$$

$$\frac{w+1}{w} = \frac{1}{\beta \cdot E_t\left[(y_{t+1} / y_t) \cdot (c_{t+1}^{-\alpha} / c_t^{-\alpha})\right]} \quad (8)$$

By definition:

$$E(R_{e,t+1}) = \frac{p_{t+1} + y_{t+1}}{p_t} = \frac{w \cdot y_{t+1} + y_{t+1}}{w \cdot y_t} = \frac{(w+1)}{w} \cdot \frac{y_{t+1}}{y_t} =$$

$$= \frac{1}{\beta \cdot E_t\left[(y_{t+1} / y_t) \cdot (c_{t+1}^{-\alpha} / c_t^{-\alpha})\right]} \cdot \frac{y_{t+1}}{y_t} \quad (9)$$

By using the logarithmic properties and by taking the natural logarithm on the both sides we can get:

$$E(R_{e,t+1}) = \frac{e^{\mu_z + 1/2 \cdot \sigma_z^2}}{\beta \cdot e^{\mu_z - \alpha \cdot \mu_x + 1/2 \cdot (\sigma_z^2 + \alpha^2 \cdot \sigma_x^2 - 2\alpha\sigma_{x,z})}} \quad (10)$$

$$\ln E_t(R_{e,t+1}) = -\ln \beta + \alpha \cdot \mu_x - \frac{1}{2} \cdot \alpha^2 \cdot \sigma_x^2 + \alpha \cdot \sigma_x^2 \quad (11)$$

APPENDIX B

Full model: $Y_t - Y_{t-1} = \alpha + \beta \cdot t + (\rho - 1) \cdot Y_{t-1} + \lambda_1 \cdot \Delta Y_{t-1}$

Restricted model: $Y_t - Y_{t-1} = \alpha + \lambda_1 \cdot \Delta Y_{t-1}$

The null and alternative hypothesis for the testing for stationarity could be represented:

Ho: Y_t has a unit root (random walk process) → β=0 and ρ=1

Ha: Y_t has not a unit root

Test statistic: Partial F statistic = $\dfrac{(N-K)\cdot(ESS_R - ESS_F)}{q \cdot ESS_F}$, where ESSR and ESSF are the sums of squared residuals in restricted and full models accordingly, N is the number of observations, K is the number of estimated parameters in full model and q is the number of parametric restrictions.

Decision rule: If calculated F statistic is large (> critical values from Table 3 for different level of significance), then we can reject the null hypothesis of random walk. It will mean that Y_t is stationary.

From the Table 2, we can see that all F statistics are significantly larger than the critical values even at 99% confidence level. Hence, the evidence strongly argues for the rejection of the null hypothesis

APPENDIX C

Nominal valuation method

In this model, the growth is proxied by the growth rate of operational earnings of the companies, included in the S&P/TSX Composite index:

$$gy_{nom}^{t} = \frac{OE_t}{OE_{t-1}} - 1 \tag{1}$$

where OE_t is the earnings level at time t. Hence, the return on equity at time t is presented as:

$$RE_t = \frac{d_t}{P_{t-1}} + gy_{nom}^{t} \tag{2}$$

Moreover, by subtracting the risk free rate at time t (here represented by F_t) from the return on equity, the equity risk premium is obtained:

$$ERP_t = RE_t - F_t \tag{3}$$

Real valuation method

The real valuation method is similar to the above mentioned model, except here the real values are employed by adjusting the nominal values for the effect of inflation.

$$gy_{real}^{t} = \frac{OE_t}{OE_{t-1}} \times \frac{CPI_{t-1}}{CPI_t} - 1 \tag{4}$$

$$RE_t^r = \frac{d_t}{P_{t-1}} \times \frac{CPI_{t-1}}{CPI_t} + gy_{real}^t \tag{5}$$

$$ERP_t^r = RE_t^r - F_t \tag{6}$$

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Table 1 – Equity Risk Premium Puzzle for Canada

This table reports the magnitude of risk premium corresponding to input variables of risk aversion, α and time discount factor, β. The sample includes growth rate of consumption which is annual rate of change of personal expenditures on services and non-durable goods from 1961-2006. μ_x is the continuously compounded average growth rate of personal expenditures on services and non durable goods. σ_x^2 is the variance of continuously compounded growth rate of personal expenditures on services and non durable goods. The second and third rows report the maximum level of risk premium when one of the input parameters is being held constant while the other varies between the indicated values. Last row reports the values of input parameters that correspond to observed risk premium for 1961-2006.

Risk aversion, α	**Time discount factor, β**	**Average growth rate of consumption, μ_x**	**Variance of the growth rate of consumption, σ_x^2**	**Risk Premium**
10	0.99	2.93%	0.0209%	**0.28%**
[10,20]	0.99	2.93%	0.0209%	max=0.73%
10	[0.5,0.99]	2.93%	0.0209%	max=0.56%
52	**0.9**	2.93%	0.0209%	**4.20%**

Table 2 – Unit Root Tests on the Dividend Yield Ratio

This table reports Dickey-Fuller (Dickey and Fuller, 1979) test for the absence of a unit root. The sample includes the monthly values of the dividend yield ratio of S&P/TSX Composite Index from 1956-2006 (total of 604 observations).

Test	Test statistic	Decision
Trend & 0 lag	-22.3584	Reject the null
Trend & 1 lag	-17.54617	Reject the null
No trend & 1 lag	-22.37556	Reject the null

Table 3 – Critical values for Dickey-Fuller Unit Root Test

This table displays the critical values for Dickey-Fuller test for the absence of a unit. The first column indicates that various specifications of Dickey-Fuller test are used.

Test	*1% level*	*5% level*	*10% level*
Trend & 0 lag	-3.978	-3.419	-3.132
Trend & 1 lag	-3.978	-3.419	-3.132
No trend & 1 lag	-3.444	-2.867	-2.569

Table 4 – Expected Equity Risk Premium Estimation by Growth Models

The sample includes 6-month Government of Canada Treasury bill average yield and the dividend yield ratio for S&P/TSX Composite Index from 1959 to 2006. g (1) and g(2) represent the growth rates of dividends adjusted to index (operating earnings) which equals to arithmetic average of $\frac{D_t - D_{t-1}}{D_{t-1}}$ (or $\frac{Y_t - Y_{t-1}}{Y_{t-1}}$) for g(1) and geometric average for g(2), where D_t (Y_t) is dividends adjusted to index (operating earnings) . Rows 1 and 3 report the results that are adjusted for inflation.

Model	Dividend yield	g(1) arithmetic average	g(2) geometric average	6-month Real T-bill rate	Risk premium (arithmetic)	Risk premium (geometric)
Dividend growth model - Real values	2.93%	2.50%	2.00%	5.41%	0.01%	-0.48%
Dividend growth model - Nominal values	3.07%	6.83%	6.35%	5.41%	4.50%	4.01%
Operating Earnings growth model* - Real values	2.95%	4.63%	1.95%	5.44%	2.15%	-0.53%
Operating Earnings growth model* - Nominal values	3.10%	8.97%	6.29%	5.44%	6.63%	3.95%

*- For operating earnings model the period covered is from 1959-2005

Table 5 – Decomposition of the Historical Equity Risk Premium

This table reports the decomposition of historical equity risk premium for 1957-2006 period. The average growth rate of real dividends is the geometric average growth rate of year-end dividends adjusted to S&P/TSX Composite return adjusted for inflation. Price-to-dividends ratio is obtained by dividing year end (December) closing price by the dividends adjusted to index. The geometric average dividend yield is calculated by adjusting the dividend yield for inflation. The proxy for risk-free rate is Government of Canada T-Bill rate.

	The average growth rate of real dividends	The average change rate of P/D ratio	The average real dividend yield	The average real risk free rate	Equity risk premium
Canada	1.54%	0.68%	3.10%	2.56%	2.76%

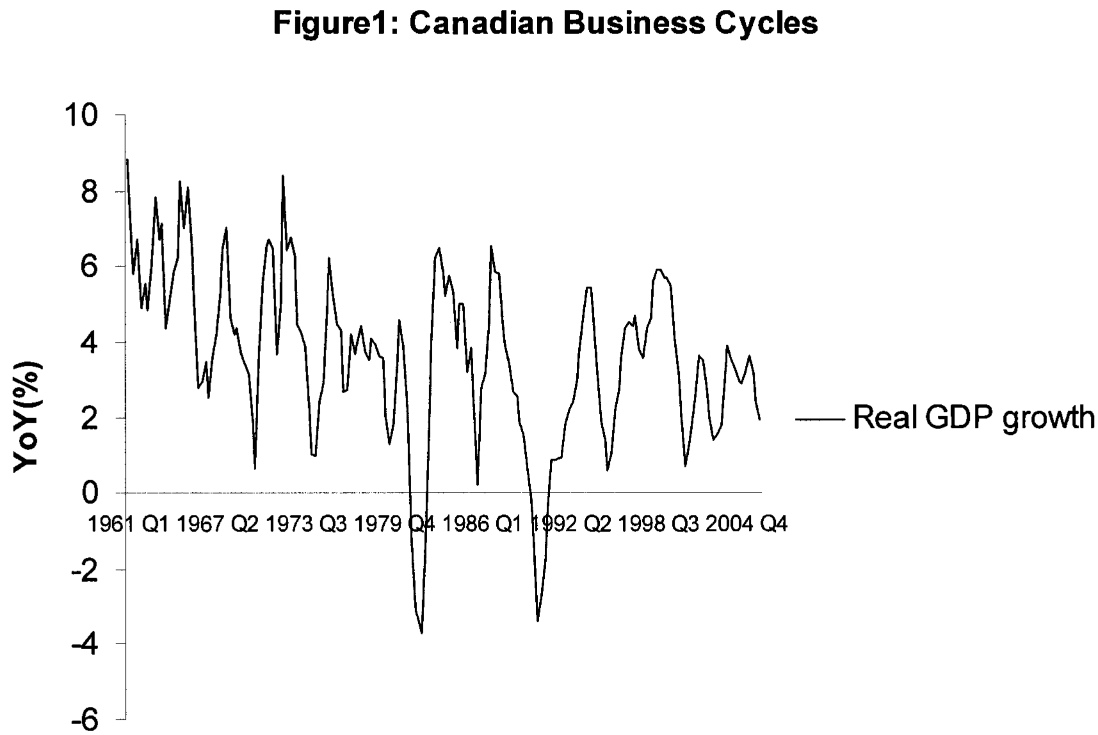
Table 6 – The Annual Expected Equity Risk Premium

This table reports the decomposition of historical equity risk premium for 1957-2006. Real GDP Growth Real Return Bond Yield is from OECD database. Real Return Bond Yield is from Bank of Canada. Dividend yield is on S&P/TSX Composite Index.

Year	Real GDP Growth* (%)	Dividend Yield** (%)	Real Return Bond Yield*** (%)	Expected Risk Premium (%)
1991	-2.10	3.18	4.45	-3.37
1992	0.90	3.05	4.62	-0.67
1993	2.30	2.26	3.78	0.78
1994	4.80	2.39	4.92	2.27
1995	2.80	2.27	4.42	0.65
1996	1.60	1.83	4.09	-0.66
1997	4.20	1.64	4.14	1.70
1998	4.10	1.68	4.11	1.67
1999	5.50	1.31	4.01	2.80
2000	5.20	1.26	3.42	3.04
2001	1.80	1.54	3.76	-0.42
2002	2.90	1.91	3.33	1.48
2003	1.80	1.64	2.79	0.65
2004	3.30	1.67	2.11	2.86
2005	2.90	2.03	1.44	**3.49**
Arithmetic Average	2.80	1.98	3.69	**1.08**
Geometric Average	2.78	1.98	3.69	**1.07**

* Source: OECD data
** Source: TSE review
*** Source: Bank of Canada

Figure 1 – Canadian Business cycles



Source: OECD data.

Figure 2 – Time Variance of the Expected Risk Premium (Arithmetical)

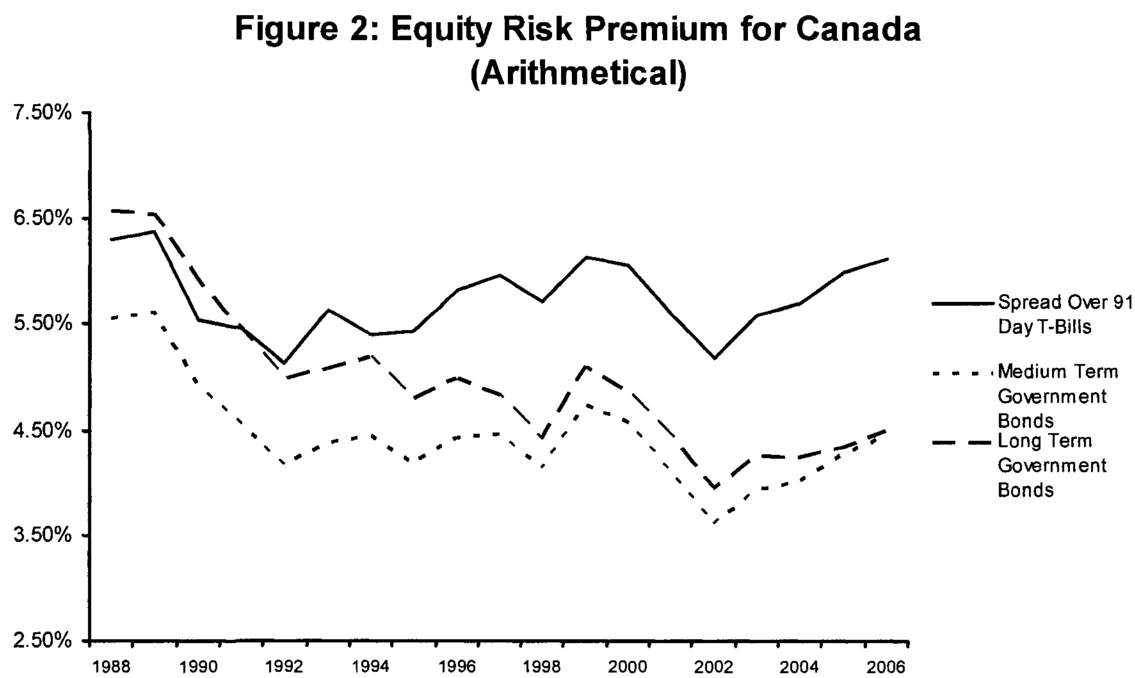


Figure 3 – Time Variance of the Expected Risk Premium (Geometrical)