SHEAR STRESS ANALYSIS OF TUBULAR COMPOSITE BEAMS SUBJECTED TO BENDING BY SHEAR LOAD.

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ABSTRACT

Shear Stress Analysis of Tubular Composite Beams Subjected to Bending by Shear Load

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Tubular composite beams are of increasing interest due to their growing applications in the offshore and aerospace industries. Most analysis work done on tubular composite beams has been limited to pure bending, uniform axial loads or uniform torsion. These are also limited to the analysis of uniform section, uniform material and uniform thickness beams. In real applications, transverse shear loads are usually present and add complexity to the analyses. When a beam is under distributed or concentrated transverse loadings, regardless of the boundary conditions, the distributions of bending moments and internal transverse shear loads vary through the length of the beam. Analysis of such beams is very complicated. In this research, a systematic approach is presented to evaluate shear stress distribution across the cross section of thin walled tubular beams made of non homogeneous sections. Variation of shear stress through the thickness is ignored. Exact equations for the analysis of shear stresses in thin wall composite beams are derived in local coordinate systems. The results are projected in global coordinate system to facilitate evaluation and comparison of shear stress distribution in different beams. The method is applied to analyze beams with T, Triangular, Hexagonal, Octagonal and Decagonal sections. The pattern behaviour and shear stress variation in these beams is studied to predict the maximum shear stress in beams with circular cross section that has the same radius as the circumscribed circle of multi-gonal beams

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Finally, I would like to give my special thanks to my parents, Sukhwinder Singh and Daljeet kaur. Without their encouragement and understanding it would have been impossible for me to finish this work. To them, I dedicate this thesis.

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Chapter 1

Introduction, Literature Survey and Scope of the Thesis

1.1 Introduction

The present thesis has an objective to analyse the shear stress in composite tubular beams subjected to transverse loading. The three dimensional stress state at a point consists of 6 stresses, 3 normal stresses and 3 shear stresses. For a circular composite tube under bending (Fig.1.1a) the three shear stresses are $\tau_{r\theta}$ (interlaminar circumferential shear), τ_{rz} (interlaminar axial shear) and $\tau_{\theta z}$ (facial circumferential shear). For a tube made of homogeneous material, usually only $\tau_{\theta z}$ is considered because it comes from the application of shear load. For the case of composite tubes, other shears are also important. However in this thesis, only the $\tau_{\theta z}$ shear is considered.

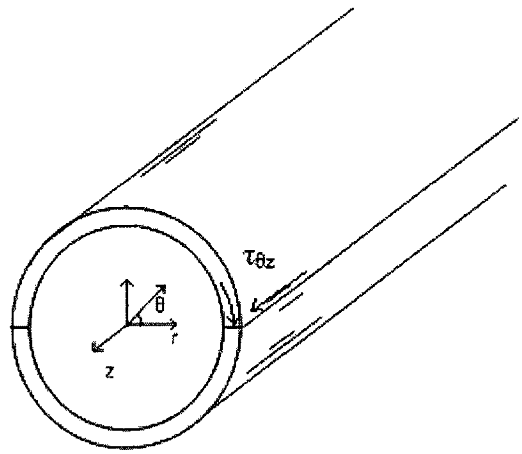

Fig. 1.1a: Circular Composite tube.

Fig. 1.1b shows different composite beams including multilayer circular composite beam, circular composite beam and hexagonal composite beam consisting of the same or different materials (segments 1 and 2) along the circumferential direction. In this thesis single layer composite beams consisting of the same or different materials along the circumferential direction are considered.

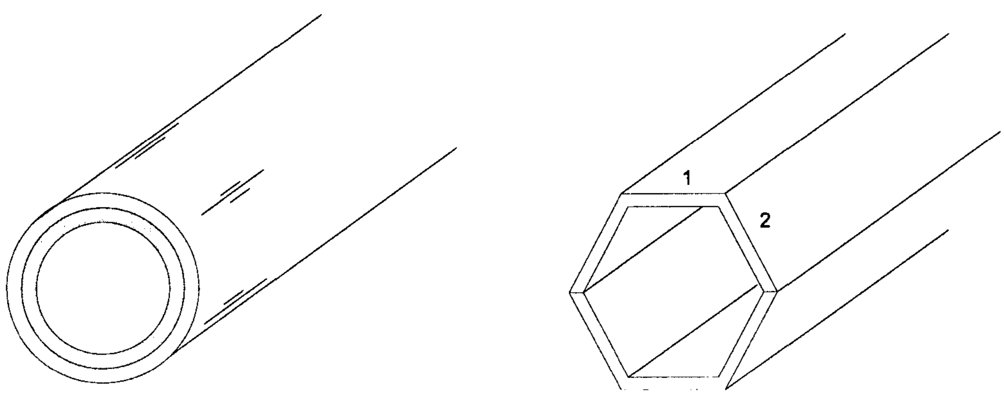

Fig. 1.1b: Multilayer Circular Composite beam and Hexagonal Composite beam

Currently composite beams are widely used in building structures and bridges. Due to their increasing applications, researchers have been motivated to develop simple approaches for stress analysis of complex composite beams. For example in the case of bridges and in other building structures, there is need to analyse the shear stress distribution in different sections of beams subjected to shear load. Also in the case when there is a need for connecting two parts to make a composite structure in which it is necessary to know the shear stress to prevent shear failure.

The present thesis was started with an attempt to perform stress analysis of multilayer circular composite beam which is widely used in many applications however it was modified to study beams with sections comprising of different material. The main concern in present thesis is shear stress $\tau_{\theta z}$.

1.2 Shear Stresses in beams

It took many years to understand and develop the concept of stresses. Galileo, Leonard Euler [1, 5, 6, and 13] and many others contribute towards this work. Shear plays an important role in designing of beams and needed to be considered to avoid failure. Shear stresses in beams are due to shear force generally denoted by 'V'. In the design of short beams, shear stress due to transverse loading plays an important role. Also shear stresses are taken into consideration when longitudinal shear strength of material is low as compared to longitudinal tensile/compressive strength (Example: Grain running along the length of wooden beam). If we consider that known amount of shear force is applied to the beam, then it is reasonable to assume that shear stress 'τ' is acting parallel to the shear force. When shear stresses develop on one side or on one face (let's say vertical face) of an element, they are accompanied by equal magnitude of shear stress on horizontal face of same element

Shear stresses do not exist in pure bending. This can be illustrated by giving an example that if a beam is made of separate flat pieces stacked on each other which are fixed together at one end and transverse load is applied on other end then pieces slide with respect to each other. But in case if there is only a couple applied then different pieces of such a composite beam will not slide with respect to each other but tends to bend in an arc of a circle.

Practically, in actual solid beams when the flat board/pieces are bonded together, the deformation results into distortion. Distortion in a bent beam due to shear is shown in fig.1.2:

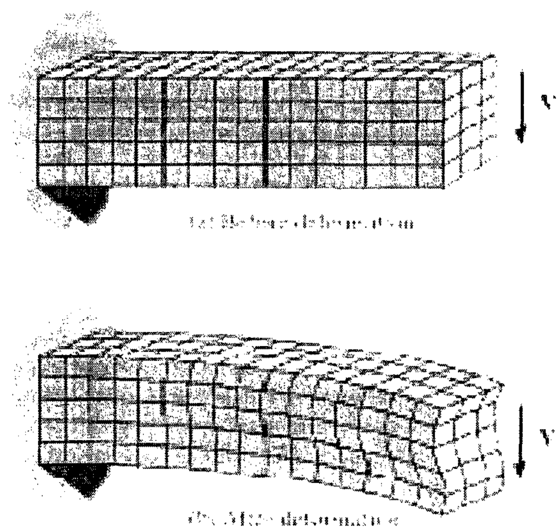

Fig. 1.2: Distortion due to Shear

Static equilibrium ($\sum F=0$) has to be taken into consideration in order to develop shear stress relations for beams. Now consider a prismatic beam[1] (Fig.1.3) having a vertical plane of symmetry under various types of loads (concentrated and distributed).

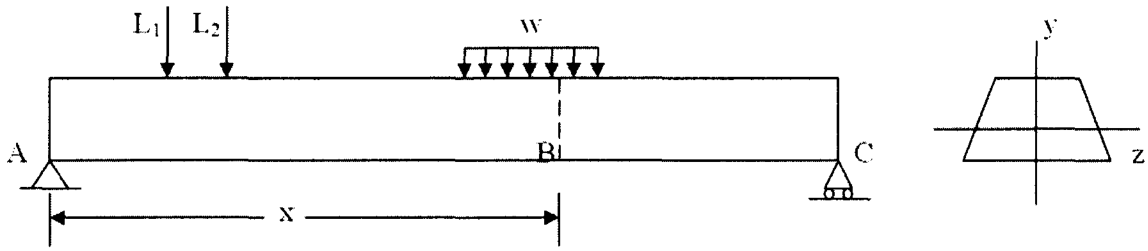

Fig. 1.3: Prismatic beam under concentrated and distributed loads

A beam element BDB'D' of length Δx has been detached at a distance x from end A and at a distance y_1 from neutral axis (Fig 1.4). The forces exerted on beam element are: Vertical shearing forces V_B and V_D, Horizontal shearing force ΔH (on the lower face of the element), Horizontal normal forces $\sigma_B dA$ and $\sigma_D dA$ and load $w\Delta x$.

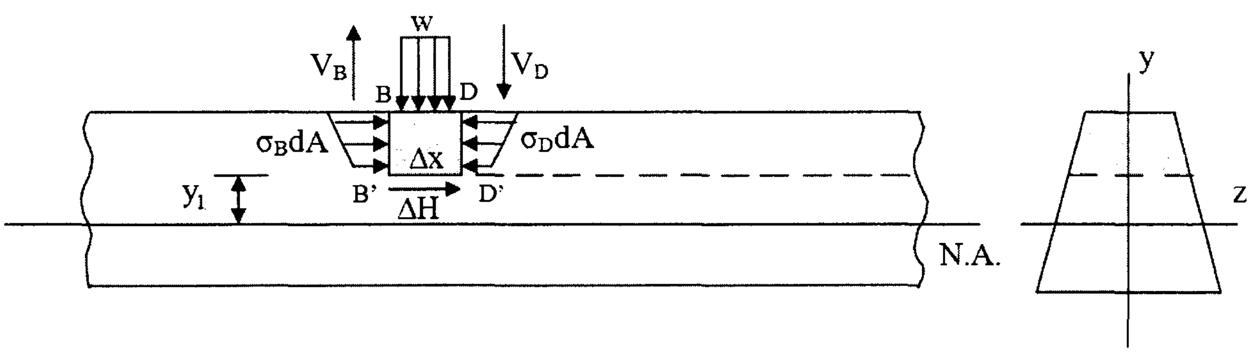

Fig. 1.4: Beam element

The equilibrium equation is $\sum F_x = 0$.

$$\Delta H + \int (\sigma_B - \sigma_D)\, dA = 0 \tag{1.1}$$

where the integral extends over the shaded area. Also the normal stresses can be expressed in terms of bending moments and equation 1.1 can be written as:

$$\Delta H = \left(\frac{M_D - M_B}{I}\right) \int y dA \qquad (1.2)$$

The integral represents first moment with respect to neutral axis of shaded portion and is denoted by Q. I represents centroidal moment of inertia of entire cross section. On the other hand the increment in bending moment is represented as

$$M_D - M_B = \Delta M = (dM/dx)\, \Delta x = V\Delta x$$

Therefore,

$$\Delta H = VQ\Delta x/I \qquad (1.3)$$

$$\text{Where } Q = \int y dA$$

Horizontal shear force per unit length also known as shear flow is given by

$$q = \Delta H/\Delta x = VQ/I \qquad (1.4)$$

The magnitude of shearing force exerted on the horizontal face of the element is given by ΔH. In order to obtain the average value of shearing stress on that face , shearing force is divided by area ΔA ($\Delta A = t\Delta x$; t = width of the element) of the face. Shear stress is given by

$$\tau = VQ/It \qquad (1.5)$$

Generally in case when width of the beam cross section is small as compared to its depth, there is no remarkable variation of shear stress along the width. Also variation of shear stress in narrow beam for which $t \leq h/4$ (t is width and h is depth of beam), across the width of the beam is less than 0.8% of average value of shear stress [1]. So generally we use the above written expression to calculate average shear stress at any point.

At upper and lower surfaces of the beam, horizontal shearing stress is equal to zero. So the maximum value of shear stress lies within the beam, where Q is maximum. In a beam with uniform cross section, shear and normal stress distribution is shown in fig. 1.5:

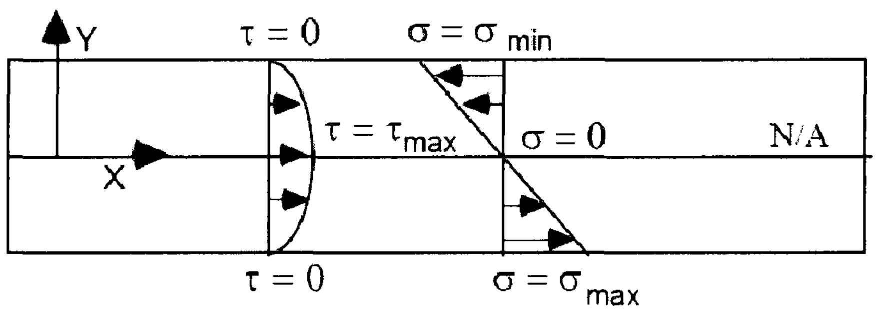

Fig. 1.5: Distribution of Shear and Normal stresses in beam with uniform cross section

Shear stress depends on distance from neutral axis. Shear deformation is the same in all elements having same distance from neutral axis. It is generally assumed that deformation caused by shearing stresses has no effect on distribution of normal stresses.

1.3 Literature review

In this section a literature survey is presented on the stress analysis of beams under different loading conditions. Significant work has been done on stress analysis of beams and other structural members, although the work done is limited to composite beam subjected to pure bending, uniform axial loads or uniform torsion. It is always a challenging task to obtain stress distribution expressions for composite structures and multilayer structures. The complexity of stress equations is much more in the case when the beam is made of different types of materials as compared to one made of isotropic materials.

1.3.1 Approaches presented in the literature

Many books and journals demonstrated the work done on the formulation of stress equations when beams are under the action of transverse loading. Most of the works are limited to the analysis of uniform material and uniform thickness beams. In reality however, the formulation and analysis is very complex as transverse shear loads are present. In most books equations have been derived to calculate the normal stresses and shearing stresses in beams. Beer, Johnston, Dewolf [1] and Haslach, Armstrong [5] and many others [2-4] presented the basic approach towards determining the stresses in different structural members. Most of the work is concentrated on homogeneous beams.

Gay and Hoa[7] presented an approach for the analysis of composite beams. Here composite beams are made of dissimilar materials or phases bonded together perfectly. Their approach defined the displacement equations analogous to stress and moment resultants for the applied loads. An approach has been developed for composite beams in flexure and thereafter torsion. Equilibrium and behaviour relations are used for the calculation of stress and displacement equations. . The isotropic phases of the beams are assumed to be perfectly bonded. Development of approach starts with flexure of composite beams with isotropic phases. This approach provides the basic guidelines for present thesis.

In the context of stress analysis of composite beams and tubes, the work is mainly concentrated on evaluation of displacements and stresses. Basic equations of anisotropic elasticity are available in Lekhnitskii [9] which provides a basis for almost all of the major studies in this field. He developed the governing equations for the analysis of a single-layer-anisotropic cylinder. A tube can be analysed by Lekhnitskii's stress function approach when it is in the state of generalised plane strain or generalized torsion. Jolicoeur and Cardou [8] and others [10-11] almost used the same approach to obtain the analytical solution for composite tubes. Jolicoeur and Cardou [8] extended the Lekhnitskii's method for layer-wise cylinders, and obtained the analytical solution for bending of coaxial orthotropic cylinders [8]. They studied two cases: no slip and no friction. In each case, perfect bonding conditions sustained. However it was found that for anisotropic materials, warping of the cross section develops even if just pure bending load is applied. Chouchaoui [10] also studied the layer wise anisotropy by assuming the perfect bonding between layers with no slip. Their method is based on analysis of each layer and satisfaction of interfacial continuity and boundary conditions. System of equations is complicated when many layers are involved.

Few others developed their work by other techniques. Tarn and Wang [13] used a state space approach to analyse a composite tube under extension, torsion, bending and shear loading. Their formulation suggests a symmetric way to determine the stress and deformation in a multilayered cylindrically anisotropic tube.

Budynas [14] in his book explained the concepts of stress analysis and topics from advanced mechanics of materials. This book provides the discussion for transverse shear stresses and shear flow in thin walled beams. Also it highlights radial and tangential stresses in curved beams. Other authors [17, 18, and 21] also provide the basis of stress analysis with explanation of stress transformation and other relations. Timoshenko [22] developed the beam theory after taking into account shear deformation and rotational inertia effects. Commonly this theory describes the behaviour of short beams and sandwich composite beams. The resulting equation is of 4^{th} order.

Sayman and Esendemir [19] presented the analytical elastic plastic stress analysis carried out on metal matrix composite beams under uniformly distributed transverse load. The composite layers are made of stainless steel fiber and aluminium matrix. Sayman [20] also extended the stress analysis of composite beams loaded by bending moments. An analytical solution is obtained by satisfying the governing equations and boundary conditions. Benachour [23] developed a closed form solution for interfacial stresses of simply supported beams with bonded prestressed FRP plate. A parametric study has been conducted to investigate the study of interface behaviour which further affects the magnitude of shear and normal stress in the composite member.

Furthermore, Sayed [24] presented an analytical expression to estimate the shear strength of joint cores in concrete beam-column connections. Chatterjee [25] performed the shear test analysis on short beams to predict the failure mode. It is shown that the failure loads can be predicted for small span-to-depth ratios based on the maximum shear stress at failure.

Alvarez-Dios and Viano [26] took into consideration beams of variable cross section, and derived a general model to evaluate the axial and shear stress distribution in the cross section. The three dimensional stress field at material discontinuities in composite beams is analysed analytically in agreement with finite element analysis.[27,28]. In the case of sandwich beams, structural analysis is carried out by transforming the web core sandwich beam into an equivalent homogeneous sandwich beam and stress components were calculated by re-evaluating the periodic structure of the beam.[29,30].

Ghugal and Shimpi [31] presented the review of shear deformation theories by taking into consideration the isotropic and anisotropic Beams. They discussed the merits and demerits of various theories like elementary theory of beam, first order, higher order and parabolic shear deformation theories etc. Cowper [32] and Murty [33] analyse the short beams and reviewed the shear correction factor and presented the discussion on shear coefficients in beam bending [34]. Donnell [35] came up with series solutions for deflections and stresses in continuously loaded beams. He also reviewed the effect of transverse shear strain and normal stress in the case of deflection of simply supported beam. Boley and Tolins [36] analzed the rectangular beams subjected to normal and shear force varying along the span and calculated the stresses and deflections by an iterative procedure.

Fatmi [37,38] and Mokos and Sapountzakis [39] presented a beam theory for homogeneous cross sections made of isotropic elastic material after considering the effects of torsion and shear forces. Theoretical development and analytical analysis has been presented starting with displacement model. They also conducted the comparison with classical beam theories. Some other researchers presented the shear stress analysis of beams using the finite element method [40] and presented the method to evaluate the warping properties of thin walled open and closed profiles. [41]

Wagner and Gruttmann [42] presented a nodal displacement method for the analysis of flexural shear stresses of prismatic beams by integration of equilibrium equations. Thin walled cross sections were assumed to have constant thickness. Furthermore for anisotropic composite beams with any ply orientation and stacking sequence, a solution is derived for shearing stresses. Also layer by layer analysis has been provided using classical beam theory.[43,44]. An approach has been presented to determine shear center for anisotropic thin walled beams [45]. Solution for orthotropic beams under normal and shear loading has been obtained [46]. Special case of box beam has been analyzed for the study of deflections and stresses [47]. Pagano [48] investigated the limitations of classical laminated plate theory and presented an analysis composite laminates under cylindrical bending.

As seen in the literature, most of the work is related with uniform material, fibre-matrix composite materials or beams having uniform thickness. Few researchers approached to predict the behaviour of shear stresses in composite beams. This thesis provides a shear stress analysis technique for composite beams subjected to shear loading.

1.4 Objective of the thesis

The primary objectives of the thesis are:

1. To develop a technique to analyze the shear stress ($\tau_{\theta z}$) distribution in composite beams where cross sections are made of different materials along the circumferential direction.
2. To compare the results with current shear stress determining technique for specific (homogeneous) cases.
3. To obtain shear stress distribution graphs for different cross sections and to predict the results for circular case.
4. To conduct a comparison between homogeneous and composite beams to show the importance of present approach.

This thesis presents the development of a new approach with which we can determine shear stress distribution in different types of composite beams under the action of applied shear force. The equations to determine the shear stress distribution are obtained for general cases and compared with the current 'VQ/It' technique for homogeneous cases.

1.5 Organisation of the thesis

The present chapter provides a brief introduction of shear stresses present in beams under different loading conditions. This chapter also gives an overview of background of mechanics of materials, literature survey of the work that has been done on the determination of stress in beams under different loading conditions, finally the primary objectives and scope of the present thesis.

Chapter 2 provides the basic idea for the development of this approach. The preliminary step for further formulations has been provided. The complete overview of the present thesis approach is presented. The approach is then applied to the determination of shear stresses in different sections of T beam.

Chapter 3 provides the detailed procedure to calculate the shear stresses in composite tubes subjected to shear loading. This chapter involves all the required steps-by-step derivations for getting the shear stress equations in four types of cross sections i.e. Triangular, Hexagonal, Octagonal and Decagonal.

In chapter 4, derivations are obtained in global coordinates on the basis of basic 'VQ/It' technique used for determining shear stress mainly in homogeneous cases. Detailed comparison has been made between the two approaches for very specific homogeneous cases. Comparison is made to validate the present thesis approach.

In chapter 5, graphs are plotted to show the flow of shear stress in the different sections of tubes. This chapter also highlights the generalisation of results for circular tubes after plotting the maximum value of shear stress based on the specific dimensions of different types of tubes. It also shows diagrammatically for clear understanding of the approach followed for approximating the results for circular case. In addition, graphs are obtained for normalized shear stress after defining the shape factor. Finally the comparison has been made for homogeneous case and composite case and this comparison is shown in different graphs. These graphs show the significance of the present thesis approach.

The thesis ends with chapter 6, which provides overall conclusion, contribution of this work and future recommendations for advanced formulations.

Chapter 2

Overview of the present approach and analysis of T beam

2.1 Introduction

In this chapter, the basic overview of the present approach is presented. Mono dimensional approach explaining the determination of stresses in composite beams is discussed. All derivations have been provided which form the bases of this approach. Furthermore analysis of T beam is conducted.

2.2 Overview of the Composite beams in flexure

Evaluation of stresses and displacements of beams under loading is always a challenging job. A number of mechanical members can be considered under the category of beams due to their slenderness. A mono dimensional approach has been purposed by Gay and Hoa [7]. This approach provides the basis for formulation of stresses of composite beams under flexure and torsion. Using the equilibrium and behaviour relations leads to derivation of stress expressions. First symmetric composite beams with homogeneous phases are considered (Fig. 2.1) [7] which provides a basis for extension to asymmetric section, transversely isotropic materials and orthotropic cases with more involvement of advanced formulation depending on geometry.

A symmetric beam is considered to be in bending in the plane of symmetry (x, z) under the action of external loads (also symmetric with respect to plane of symmetry). So the procedure started with finding displacement field equations and continued with perfect bonding conditions to determine stresses.

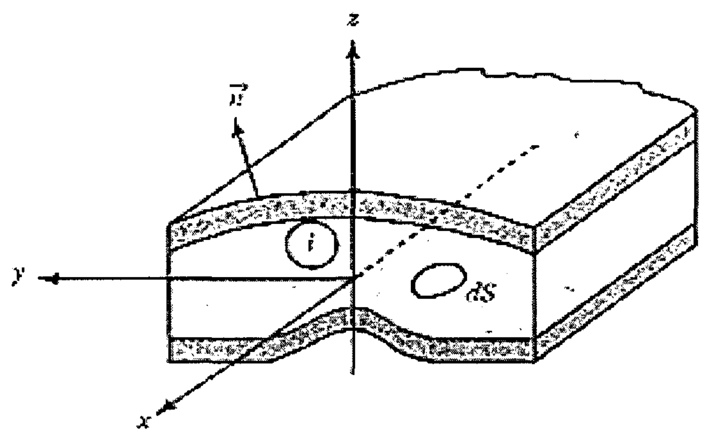

Fig. 2.1: Composite Beam [7]

Equivalent Stiffness

Equivalent stiffness in condensed form which includes the integrals of total cross section area for Extensional rigidity (ES), Bending rigidity (EI_y) and Shear rigidity (GS) are given as:

$$\langle ES \rangle = \int_D E_i dS = \sum_i E_i S_i \tag{2.1}$$

$$\langle EI_y \rangle = \int_D E_i z^2 dS = \sum_i E_i I_{yi} \quad (2.2)$$

$$\langle GS \rangle = \int_D G_i dS = \sum_i G_i S_i \quad (2.3)$$

where i represents the different phases.

Elastic center

The selection of elastic center geometry plays an important role. For symmetric sections the elastic is given by the following equation:

$$\int_D E_i z dS = 0 \quad (2.4)$$

Origin of the coordinate z can be chosen so that the above integral (2.4) is zero.

If geometric shape is different for example in case of asymmetric cross section, supplementary conditions for equivalent stiffness, displacements and elastic center appear which results into more complex mathematical calculations. But in that case also the same approach has to be followed.

Displacement field equations

Elastic displacement field equations which include longitudinal displacement along x axis (Fig. 2.1), rotation of the sections and transverse displacements along y and z direction can be derived by taking into consideration the effect of rotation and distortion. The displacement equations are given as:

$$u_x = u(x) - z\theta_y(x) + \eta_x(x,y,z) \quad (2.5)$$

$$u_y = \eta_y(x,y,z) \quad (2.6)$$

$$u_z = w(x) + \eta_z(x,y,z) \quad (2.7)$$

Where η represents the distortion of a cross section, η_x represents the longitudinal distortion of cross section and θ represents the rotation.

Perfect Bonding between the phases

Another important aspect to mention is the bonding between different phases is assumed to be perfect. Due to the perfect bonding, displacement field is continuous between the two bonded phases. So in perfect bonding conditions displacements between two phases i and j (Fig. 2.2) in contact are given as:

$$u_x(i) = u_x(j)$$
$$u_y(i) = u_y(j) \quad (2.8)$$
$$u_z(i) = u_z(j)$$

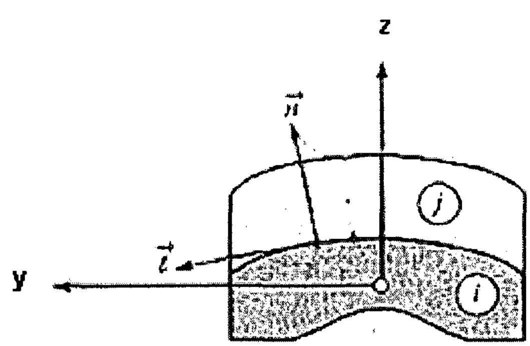

Fig. 2.2: Interface between two phases [7]

In the same way, for two bonded phases i and j, in the plane of an elemental interface with a normal vector n

$$\varepsilon_i^n = \varepsilon_j^n$$

Generally Strain transformation is given by:

$$\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xn} & \varepsilon_{xt} \\ \varepsilon_{xn} & \varepsilon_{nn} & \varepsilon_{nt} \\ \varepsilon_{xt} & \varepsilon_{nt} & \varepsilon_{tt} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & sin\theta \\ 0 & -sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & -sin\theta \\ 0 & sin\theta & cos\theta \end{bmatrix}$$

This simplifies to

$$\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xn} & \varepsilon_{xt} \\ \varepsilon_{xn} & \varepsilon_{nn} & \varepsilon_{nt} \\ \varepsilon_{xt} & \varepsilon_{nt} & \varepsilon_{tt} \end{bmatrix} =$$

$$\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy}cos\theta + \varepsilon_{xz}sin\theta & -\varepsilon_{xy}sin\theta + \varepsilon_{xz}cos\theta \\ \varepsilon_{xy}cos\theta + \varepsilon_{xz}sin\theta & \varepsilon_{yy}cos^2\theta + \varepsilon_{yz}sin2\theta + \varepsilon_{zz}sin^2\theta & (\varepsilon_{zz} - \varepsilon_{yy})sin\theta cos\theta - \varepsilon_{yz}(sin^2\theta - cos^2\theta) \\ -\varepsilon_{xy}sin\theta + \varepsilon_{xz}cos\theta & (\varepsilon_{zz} - \varepsilon_{yy})sin\theta cos\theta - \varepsilon_{yz}(sin^2\theta - cos^2\theta) & \varepsilon_{yy}sin^2\theta - 2\varepsilon_{yz}sin\theta cos\theta + \varepsilon_{zz}cos^2\theta \end{bmatrix}$$

In the plane of an elemental interface with a normal vector of n, the relation between strain tensors ε of phases i and j are:

$$\varepsilon_{xx}(i) = \varepsilon_{xx}(j)$$

$$\varepsilon_{xt}(i) = \varepsilon_{xt}(j)$$

$$\varepsilon_{tt}(i) = \varepsilon_{tt}(j)$$

It can also be written as:

$$\varepsilon_{xx}(i) = \varepsilon_{xx}(j)$$

$$-\varepsilon_{xz}n_y + \varepsilon_{xy}n_z \text{ (for i)} = \varepsilon_{xy}n_z - \varepsilon_{xz}n_y \text{ (for j)} \qquad (2.9)$$

$$\varepsilon_{zz}n_y^2 - 2\varepsilon_{yz}n_yn_z + \varepsilon_{yy}n_z^2 \text{ (for i)} = \varepsilon_{zz}n_y^2 - 2\varepsilon_{yz}n_yn_z + \varepsilon_{yy}n_z^2 \text{ (for j)}$$

On the interface having normal n, stress vector also remains continuous across an element of the interface.

$$\tau_{xy}n_y + \tau_{xz}n_z \text{ (for i)} = \tau_{xy}n_y + \tau_{xz}n_z \text{ (for j)}$$

$$\sigma_{yy}n_y + \tau_{yz}n_z \text{(for i)} = \sigma_{yy}n_y + \tau_{yz}n_z \text{(for j)} \quad (2.10)$$

$$\tau_{yz}n_y + \sigma_{zz}n_z \text{(for i)} = \tau_{yz}n_y + \sigma_{zz}n_z \text{ (for j)}$$

Equilibrium relations

In order to proceed further to get the expression of stresses, equilibrium relations need to be defined. In the absence of body forces, one can start with local equilibrium $\partial\sigma_{ij}/\partial x_j$ and integrate it over the cross section. So after integrating the two equations are:

$$\frac{d}{dx}\int_D \sigma_{xx} dS + \int_D (\partial\tau_{xy}/\partial y + \partial\tau_{xz}/\partial z)\, dS = 0$$

$$\frac{d}{dx}\int_D -z\sigma_{xx} dS + \int_D -z(\partial\tau_{xy}/\partial y + \partial\tau_{xz}/\partial z)\, dS = 0$$

$$\frac{d}{dx}\int_D \tau_{xz} dS + \int_D (\partial\sigma_{zz}/\partial z + \partial\tau_{yz}/\partial z)\, dS = 0 \quad (2.11)$$

Normal and Shear Stress resultants and moment resultant appears as:

$$\text{Normal stress resultant } N_x = \int_D \sigma_{xx} dS$$

$$\text{Shear stress resultant } T_z = \int_D \tau_{xz} dS \quad (2.12)$$

$$\text{Moment resultant } M_y = \int_D -z\sigma_{xx} dS$$

By plugging the values of stress resultants and moment resultants into equilibrium relations and taking into consideration the continuity of the expressions, one can obtain equations of equilibrium:

$$dN_x/dx = 0$$

$$dT_z/dx + p_z = 0 \quad (2.13)$$

$$dM_y/dx + T_z = 0$$

Where p_z is transverse density of loading on the lateral surface of the beam.

Constitutive Relations

The constitutive relation can be written by taking into account the homogeneous nature of different phases. It is given (for phase i) as:

$$\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz} \end{bmatrix} = \frac{1+v_i}{E_i} \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_{yy} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{bmatrix} - \frac{v_i}{E_i} [\sigma_{xx} + \sigma_{yy} + \sigma_{zz}] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2.14}$$

These equations are integrated over the domain occupied by cross-section area of the beam. The resulting equations are as follows:

$$\int_D \varepsilon_{xx} E_i \, dS = \int_D \sigma_{xx} dS - \int_D v_i (\sigma_{yy} + \sigma_{zz}) dS$$

and

$$\int_D -z\varepsilon_{xx} E_i \, dS = \int_D -z\, \sigma_{xx} dS + \int_D z v_i (\sigma_{yy} + \sigma_{zz}) dS = 0 \tag{2.15}$$

$$\int_D 2\varepsilon_{xz} G_i \, dS = \int_D \tau_{xz} dS$$

After simplifying the equations and taking into account the displacements, the results can be obtained for resultants as:

$$N_x = \langle ES \rangle \frac{du}{dx} + \int_D v_i(\sigma_{yy} + \sigma_{zz}) dS$$

$$M_y = \langle EI_y \rangle \frac{d\theta y}{dx} + \int_D v_i z(\sigma_{yy} + \sigma_{zz}) dS \tag{2.16}$$

$$T_z = \langle GS \rangle \left(\frac{dv}{dx} - \theta_y\right) + \int_D G_i \frac{\partial \eta_x}{\partial z} dS$$

Expressions for Flexure Stresses

Expression for normal stresses can be extracted by simplifying the constitutive relation. Normal stresses σ_{yy} and σ_{zz} are very small as compared to σ_{xx}.

The normal stress σ_{xx} is given as:

$$\sigma_{xx} = - E_i \frac{M_y}{\langle EI_y \rangle} z + E_i \frac{N_x}{\langle ES \rangle} \tag{2.17}$$

Bending Extension

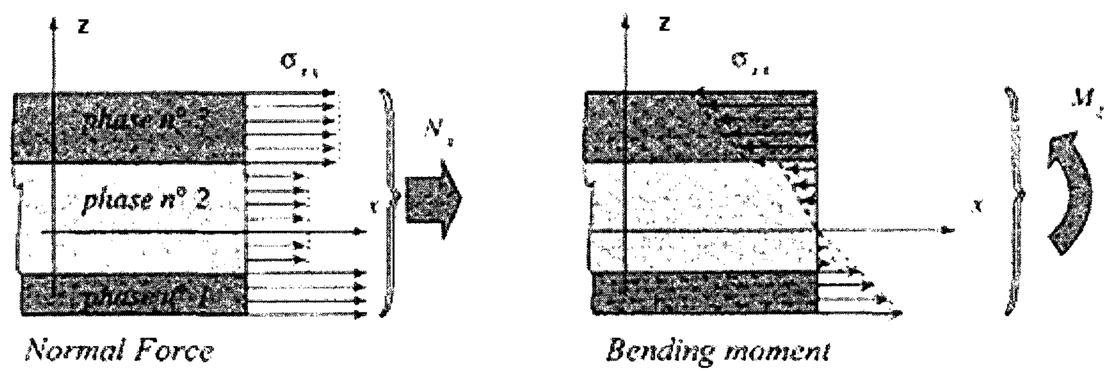

Fig 2.3: Normal and Bending stresses.

As shown in the Fig. 2.3, the normal stress is discontinuous due to the difference in longitudinal moduli.

Shear stress expressions are governed by warping function. The warping function has been introduced and simplified by using the above equilibrium equations, displacement field equations and normal stress expression. Starting equation was the equation of local equilibrium i.e. $\partial\sigma_{ij}/\partial x_j$. Taking into consideration equations 2.13 and 2.17 we can write:

$$\frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = -\frac{\partial \sigma_{xx}}{\partial x}$$

and

$$\frac{E_i}{\langle EI_y \rangle}\frac{dM_y}{dx}z - \frac{E_i}{\langle ES \rangle}\frac{dN_x}{dx} = -\frac{E_i}{\langle EI_y \rangle}T_z * z$$

Also with the displacement field in equations 2.5 to 2.7 and neglecting the variation of warping function between two neighbouring infinitely near sections one can write:

$$\frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = \frac{\partial}{\partial y}(G_i \gamma_{xy}) + \frac{\partial}{\partial z}(G_i \gamma_{xz})$$

$$\frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = G_i \left(\frac{\partial^2 \eta_x}{\partial y^2} + \frac{\partial^2 \eta_x}{\partial z^2} \right)$$

Thereafter

$$G_i \left(\frac{\partial^2 \eta_x}{\partial y^2} + \frac{\partial^2 \eta_x}{\partial z^2} \right) = -T_z \frac{E_i}{\langle EI_y \rangle} z$$

After mathematical formulation and putting

$$\eta_x = = \frac{T_z}{\langle GS \rangle} g(y, z)$$

yields to

$$\nabla^2 g = -\frac{E_i}{G_i}\frac{\langle GS \rangle}{\langle EI_y \rangle} z$$

Substituting the function g(y,z) with the function $g_o(y,z)$ such that

$$g_o(y,z) = g(y,z) + kz$$

Where coefficient k is analogous to shear coefficient for homogeneous beams.

One verifies that g_o is solution of the problem.

$$\nabla^2 g_o = -\frac{E_i}{G_i}\frac{\langle GS\rangle}{\langle EI_y\rangle} z \tag{2.18}$$

Where g_o (y,z) is longitudinal warping function for symmetric case and the following unique conditions and internal continuity has been taken into consideration

$$\frac{\partial g_o}{\partial n} = 0 \text{ on the boundary } \partial D.$$

$$\int_D E_i g_o dS = 0$$

$$g_{oi} = g_{oj}$$

After getting the expression for warping function, shear stress form can be determined as:

$$\tau_{xz} = G\gamma_{xz}$$

$$\tau_{xz} = G\left[\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right]$$

After putting the displacement values from equations 2.5, 2.6, 2.7 and then substituting the value of η_x and g_o (y,z), shear stress can be given by:

$$\tau_{xz} = G_i \frac{T_z}{\langle GS\rangle}\frac{\partial g_o}{\partial z} \tag{2.19 a}$$

Similarly we can obtain the expression for τ_{xy}.

$$\tau_{xy} = G_i \frac{T_z}{\langle GS\rangle}\frac{\partial g_o}{\partial y} \tag{2.19 b}$$

General shear stress form can be given as:

$$\vec{\tau} = \frac{G_i}{\langle GS\rangle} T_z \overrightarrow{grad}\, g_o \tag{2.19 c}$$

2.3 Shear Stress Analysis of composite T-Beam

2.3.1 Introduction

In this section, the approach presented above has been developed for T beam. The procedure basically starts with the calculation of elastic center for T beam. The warping function consists of constants and further advancement can be achieved by first determining the expressions for constants by using equilibrium conditions. Shear stress expressions can be obtained by plugging in the values of warping function. In the following derivation a few assumptions have been taken i.e. T beam is assumed to have uniform very thin cross section.

2.3.2 Step by step Procedure for shear stress expressions

The figure of T beam under the action of shear force T_z is shown below

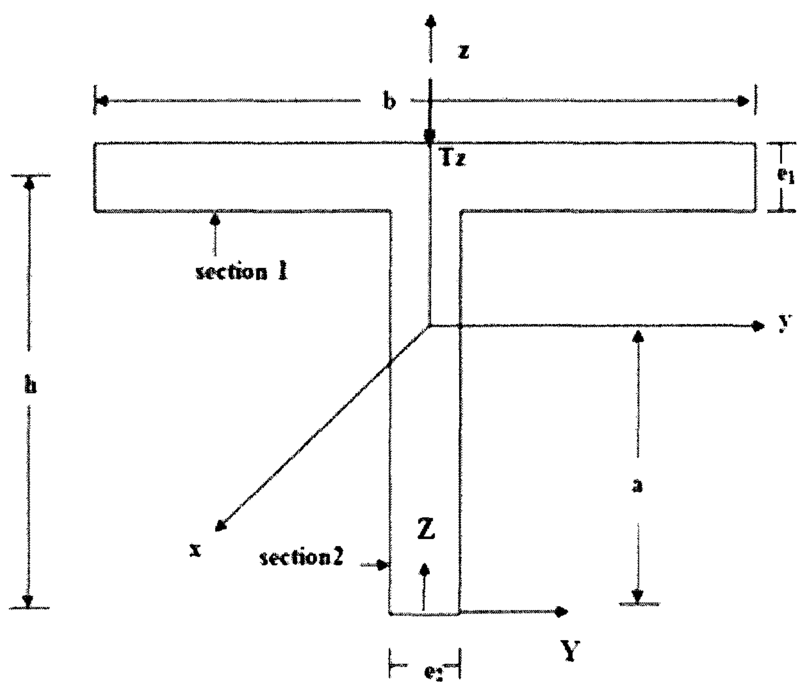

Fig. 2.4: T beam

In the figure T beam is shown to be made of two sections and two different materials (we can assume homogeneous also).The thickness of two sections is given by e_l and e_2. In this case of T beam we are considering height h up to the midpoint of section 1.

Elastic center Location

For elastic centre calculations we consider XYZ coordinate system starting from the bottom. The location of Elastic center can be calculated as

$$a = \frac{\int E_i Z ds}{\int E_i ds}$$

Where $\int E_i Z ds = \int_0^{h-\frac{e_1}{2}} E_2 Z e_2 dZ + \int_{h-\frac{e_1}{2}}^{h+\frac{e_1}{2}} E_1 Z b dZ$

$$= E_2 e_2 \left| \frac{Z^2}{2} \right|_0^{h-\frac{e_1}{2}} + E_1 b \left| \frac{Z^2}{2} \right|_{h-\frac{e_1}{2}}^{h+\frac{e_1}{2}}$$

$$= E_2 e_2 \left| \frac{(h-\frac{e_1}{2})^2}{2} \right| + E_1 b \left| \frac{(h+\frac{e_1}{2})^2}{2} - \frac{(h-\frac{e_1}{2})^2}{2} \right|$$

$$= \frac{E_2 e_2}{2}\left[h^2 + \frac{e_1^{\ 2}}{4} - he_1 \right] + \frac{E_1 b}{2}\left[h^2 + \frac{e_1^{\ 2}}{4} + he_1 - h^2 - \frac{e_1^{\ 2}}{4} + he_1 \right]$$

For $E_l = E_2 = E$ and $e_l = e_l = e$

$$\int E_i Z ds = \text{Ee}\left[\frac{h^2}{2} + \frac{e_1^{\ 2}}{8} - \frac{he_1}{2} + bh \right]$$

$$\int E_i ds = E_2 (h - \frac{e_1}{2}) e_2 + E_1 b e_1$$

$$\therefore \qquad a = \frac{Ee\left[\dfrac{h^2}{2} + \dfrac{e_1^2}{8} - \dfrac{he_1}{2} + bh\right]}{Ehe - E\dfrac{e^2}{2} + Ebe} \qquad (2.20)$$

For very thin cross section e_1 and e_2 is very small. For simple case assume b=h.

$$\therefore \qquad a = \frac{Ee\left[\dfrac{h^2}{2} + h^2\right]}{2Ehe}$$

$$a = 3h/4$$

Shear stress distribution

In this approach, shear stress depends on warping function which needs to be determined first. Shear stress expressions in different sections are governed by following equations. Due to the assumption of thin section, there is no variation along the thickness direction. That is why there is only one shear stress governing equation for each section.

In section 1 $\qquad \tau_{xy} = \dfrac{G_1}{\langle GS \rangle} T_z \dfrac{dh_{01}}{dy}$ (2.21)

In section 2 $\qquad \tau_{xz} = \dfrac{G_2}{\langle GS \rangle} T_z \dfrac{dh_{02}}{dz}$ (2.22)

Now from equation 2.18 $\qquad \dfrac{d^2 h_{01}}{dy^2} = -\dfrac{E_1}{G_1} \dfrac{\langle GS \rangle}{\langle EI_y \rangle} z$

After integrating

$$\frac{dh_{01}}{dy} = -\frac{E_1}{G_1}\frac{\langle GS\rangle}{\langle EI_y\rangle} zy + A \tag{2.23}$$

And for section 2

$$\frac{d^2 h_{02}}{dz^2} = -\frac{E_2}{G_2}\frac{\langle GS\rangle}{\langle EI_y\rangle} z$$

$$\frac{dh_{02}}{dz} = -\frac{E_2}{G_2}\frac{\langle GS\rangle}{\langle EI_y\rangle}\frac{z^2}{2} + B \tag{2.24}$$

There are two constants that need to be determined to get the expressions for shear stress.

Determination of constant B and Shear Stress τ_{xz}

Now at lower end, $\tau_{xz} = 0$ at z = - a

Therefore from equation 2.22 and 2.24,

$$\frac{dh_{02}}{dz} = 0 = -\frac{E_2}{G_2}\frac{\langle GS\rangle}{\langle EI_y\rangle}\frac{a^2}{2} + B$$

$$B = \frac{E_2}{G_2}\frac{\langle GS\rangle}{\langle EI_y\rangle}\left(\frac{a^2}{2}\right) \tag{2.25}$$

After plugging in the value of this constant in the expression of warping function (2.24) and further in shear stress expression (2.22), one can obtain shear stress in section 2 as:

$$\tau_{xz} = \frac{E_2}{\langle EI_y\rangle} T_z \left(\frac{a^2 - z^2}{2}\right) \tag{2.26}$$

Determination of constant A and Shear Stress τ_{xy}

Now At free ends, $\tau_{xy} = 0$ at y = b/2 and –b/2

$o < y \leq b/2$

$$0 = -\frac{E_1}{G_1}\frac{\langle GS\rangle}{\langle EI_y\rangle} z\frac{b}{2} + A$$

$$A = \frac{E_1}{G_1}\frac{\langle GS\rangle}{\langle EI_y\rangle}\frac{b(h-a+\frac{e_1}{2})}{2}$$

$$\tau_{xy} = \frac{E_1}{\langle EIy\rangle}T_z\left[\frac{b(h-a+\frac{e_1}{2})}{2} - (h-a+\frac{e_1}{2})y\right]$$

$$\tau_{xy} = \frac{E_1}{\langle EIy\rangle}T_z\left(h-a+\frac{e_1}{2}\right)\left[\frac{b}{2} - y\right] \qquad (2.27)$$

$- b/2 \leq y < 0$

$$A = -\frac{E_1}{G_1}\frac{\langle GS\rangle}{\langle EI_y\rangle}\frac{b(h-a+\frac{e_1}{2})}{2}$$

$$\tau_{xy} = -\frac{E_1}{\langle EIy\rangle}T_z\left(h-a+\frac{e_1}{2}\right)\left[\frac{b}{2} + y\right] \qquad (2.28)$$

Equilibrium Condition

Equilibrium condition is considered by taking into consideration the shear force equivalent flow. Here first equilibrium condition is considered at the junction i.e. summation of all the forces in x direction has to be zero for the body in equilibrium. Therefore

$$\sum F_x = 0$$

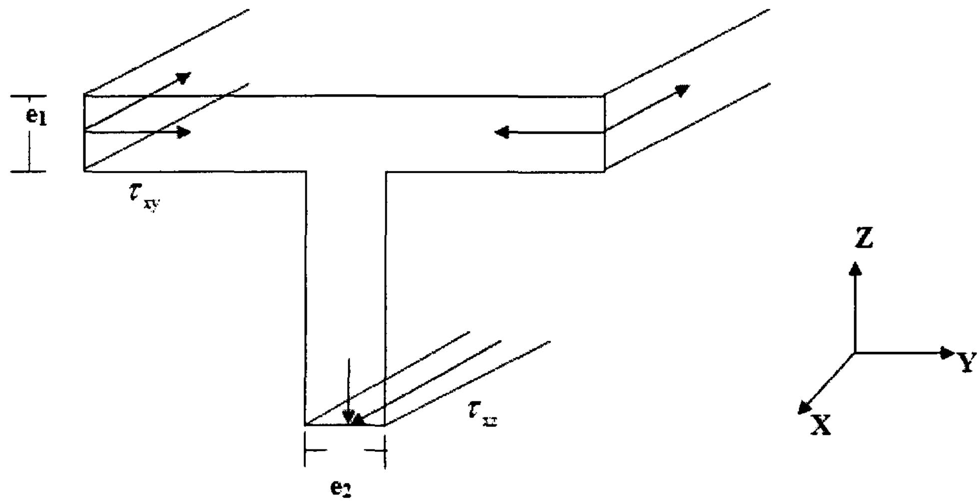

Fig. 2.5: Equilibrium Condition

$$-\tau_{xy}e_1 - \tau_{xy}e_1 + \tau_{xz}e_2 = 0$$

At y = 0 and z = h-a

$$2\tau_{xy}e_1 = \tau_{xz}e_2$$

$$2\left[\frac{E_1}{\langle EIy \rangle}T_z\left(h - a + \frac{e_1}{2}\right)\frac{b}{2}\right]e_1 = \frac{E_2}{\langle EI_y \rangle}T_z\left(\frac{a^2 - z^2}{2}\right)e_2$$

$$2E_1e_1\left(h - a + \frac{e_1}{2}\right)b = E_2e_2(2ha - h^2)$$

$$2\left(h - a + \frac{e_1}{2}\right)b = (2ha - h^2)$$

After putting b=h and ignoring small terms,

$$2h^2 - \frac{3h^2}{2} = \frac{3h^2}{2} - h^2$$

$$\frac{h^2}{2} = \frac{h^2}{2}$$

This satisfies the equilibrium equation.

2.4 Conclusion and discussion.

In this chapter the method is explained to determine the shear stress of composite beams under bending. Approach is further developed for T beam to provide an idea of the procedure involved in the formulation and derivation. This chapter provides the basis for further analysis of complex shape composite beams like triangular, hexagonal, octagonal and decagonal beam.

Chapter 3

Analysis of Composite beams of different cross sections

3.1 Introduction

In this chapter, the approach is developed for the shear stress analysis of composite beams of different cross sections. Started with thin triangular beam, this chapter covers hexagonal beam, octagonal beam and decagonal beam. The step-by-step procedure which involves mathematical formulation and derivations is explained in each section.

3.2 Shear stress analysis of beams with Triangular section

In this section, beam under consideration is triangular composite beam. The beam is assumed to have a thin cross section. The Triangular beam of height 'h' is shown in Fig. 3.1.

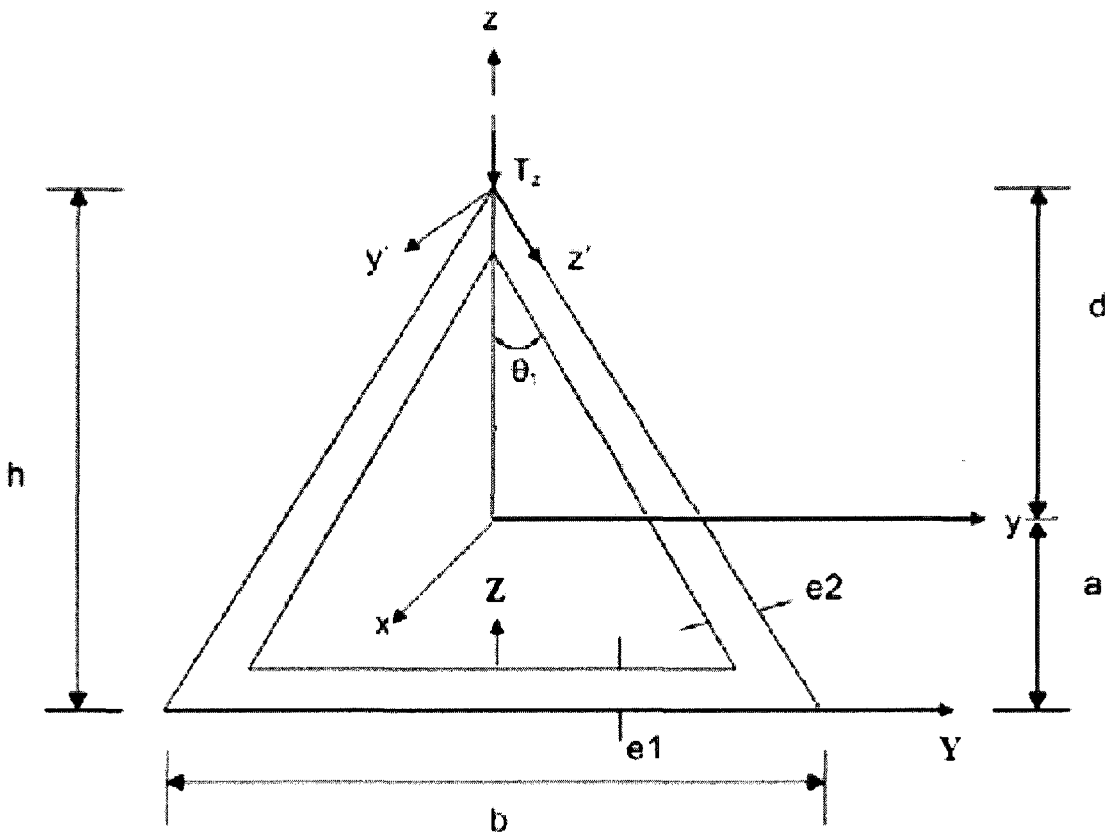

Fig. 3.1: Triangular Beam

3.2.1 Calculation of elastic center location

To determine the elastic center first a coordinate system X,Y,Z starting from the bottom is considered. And after that coordinates are switched to elastic center as origin for x,y,z system for further analysis. The location of elastic center can be calculated in the same way as explained in the previous chapter.

$$a = \frac{\int E_i Z ds}{\int E_i ds}$$

Elastic centre is given by above expression and denoted by 'a'. If we consider the homogeneous beam, then due to same material properties elastic center is at 1/3 of the total height of the beam.

3.2.2 Shear stress governing equations

In section 1, shear stress is governed by the expression:

$$\tau_{xy} = \tau_1 = G_1 \frac{T_z}{\langle GS \rangle} \frac{dh_{01}}{dy} \tag{3.1}$$

In section 2, due to the inclined cross section shear stress expression is obtained by stress transformation. In global coordinates shear stress expressions are:

$$\tau_{xy} = \frac{G_2}{\langle GS \rangle} T_z \frac{dh_{02}}{dy} \tag{3.2}$$

$$\tau_{xz} = \frac{G_2}{\langle GS \rangle} T_z \frac{dh_{02}}{dz} \tag{3.3}$$

3.2.2.1 Stress transformation and Coordinate transformations

In the case of triangular beam, the local and global coordinates do not coincide in the inclined sections. So to obtain the warping function and further shear stress expressions it is required to do the coordinate transformation and stress transformation. First the general transformation concept has been presented then it is modified for particular case.

General Concept

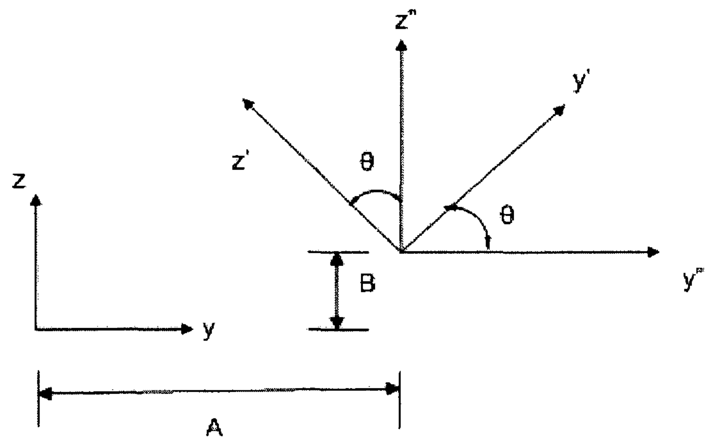

Fig. 3.2: Stress Transformation

The General transformation matrix is given by

$$\begin{Bmatrix} y' \\ z' \end{Bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} y'' \\ z'' \end{Bmatrix} \textit{where} \quad y = y''+A \text{ and } z = z''+B \ \ (\text{A and B are constant}) \tag{3.4}$$

$$\begin{Bmatrix} y' \\ z' \end{Bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} y-A \\ z-B \end{Bmatrix}$$

$$\begin{Bmatrix} y-A \\ z-B \end{Bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} y' \\ z' \end{Bmatrix}$$

Here angle θ is a general angle. It does not represent the actual sign and value of angle θ in the following figures. The stress transformation depends on angle rather than distance between coordinates. And for general case it is given by

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy'} & \tau_{xz'} \\ \tau_{xy'} & \sigma_{y'y'} & \tau_{y'z'} \\ \tau_{xz'} & \tau_{y'z'} & \tau_{z'z'} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & sin\theta \\ 0 & -sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_{yy} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & -sin\theta \\ 0 & sin\theta & cos\theta \end{bmatrix}$$

:

$$\tau_2 = \tau_{xz'} = -\tau_{xy} \sin\theta + \tau_{xz} \cos\theta \tag{3.5}$$

For inclined section of Triangular beam

In the inclined section of the triangular beam, the above transformation matrices are modified as:

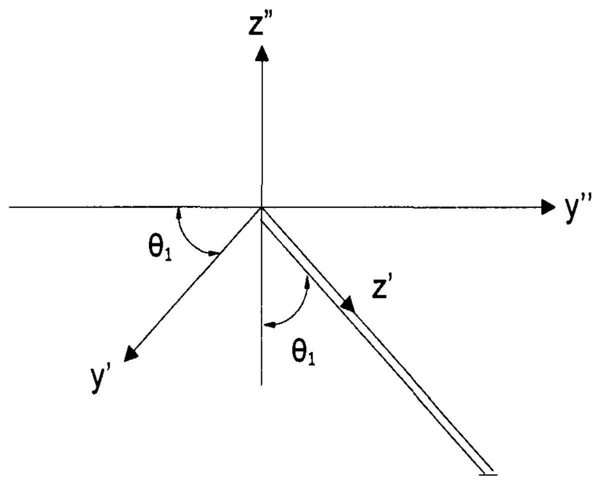

Fig 3.3: Inclination angle

Here angle is

$\theta = 180 + \theta_1$

$$\begin{Bmatrix} y' \\ z' \end{Bmatrix} = \begin{bmatrix} -\cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & -\cos\theta_1 \end{bmatrix} \begin{Bmatrix} y - A \\ z - B \end{Bmatrix}$$

Also A=0 , B=d

$$\begin{Bmatrix} y \\ z-d \end{Bmatrix} = \begin{bmatrix} -\cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & -\cos\theta_1 \end{bmatrix} \begin{Bmatrix} y' \\ z' \end{Bmatrix} \tag{3.6}$$

So stress expression in the case of triangular beam's inclined section is given by:

$$\tau_{xz'} = \tau_{xy}\sin\theta_1 - \tau_{xz}\cos\theta_1 \tag{3.7}$$

3.2.3 Derivation of Shear stress expressions for different sections of Triangular beam

Shear stress expressions are governed by warping function. As discussed in chapter 2, warping function is expressed as:

$$\nabla^2 h_0 = -\frac{E_i}{G_i}\frac{\langle GS\rangle}{\langle EI_y\rangle} z$$

Where $\nabla^2 h_0$ is the Laplacian of warping function and is be defined as:

$$\nabla^2 h_0 = \frac{\partial^2 h_0}{\partial y^2} + \frac{\partial^2 h_0}{\partial z^2}$$

In order to facilitate the analysis for inclined sections the warping function in global coordinate system might be transformed into a local coordinate system. Referring to Fig.3.3, the warping function gradient in local coordinate system is defined as:

$$\nabla^2 h_0 = \frac{\partial^2 h_{o2}}{\partial y'^2} + \frac{\partial^2 h_{o2}}{\partial z'^2}$$

The terms of second order differential equations of warping function with respect to y' and z' must be projected in the global coordinate system to be substituted in warping function equation with global coordinates. As y' and z' are function of y and z, according to the chain rule of derivation

$$\frac{\partial h_o}{\partial y'} = \frac{\partial h_o}{\partial y}\frac{\partial y}{\partial y'} + \frac{\partial h_o}{\partial z}\frac{\partial z}{\partial y'}$$

The second order differential shall be expressed as follows:

$$\frac{\partial^2 h_o}{\partial y'^2} = \frac{\partial}{\partial y}\left[\frac{\partial h_o}{\partial y}\frac{\partial y}{\partial y'} + \frac{\partial h_o}{\partial z}\frac{\partial z}{\partial y'}\right]\frac{\partial y}{\partial y'} + \frac{\partial}{\partial z}\left[\frac{\partial h_o}{\partial y}\frac{\partial y}{\partial y'} + \frac{\partial h_o}{\partial z}\frac{\partial z}{\partial y'}\right]\frac{\partial z}{\partial y'}$$

$$\frac{\partial^2 h_o}{\partial y'^2} = \frac{\partial^2 h_o}{\partial y^2}\left(\frac{\partial y}{\partial y'}\right)^2 + 2\frac{\partial^2 h_o}{\partial y \partial z}\frac{\partial z}{\partial y'}\frac{\partial y}{\partial y'} + \frac{\partial^2 h_o}{\partial z^2}\left(\frac{\partial z}{\partial y'}\right)^2$$

By substituting the values from equation 3.6, one can get,

$$\frac{\partial^2 h_o}{\partial y'^2} = \frac{\partial^2 h_o}{\partial y^2}\cos^2\theta_l + 2\frac{\partial^2 h_o}{\partial y \partial z}\cos\theta_l \sin\theta_l + \frac{\partial^2 h_o}{\partial z^2}\sin^2\theta_l$$

Similarly, for the other second order term of warping function gradient

$$\frac{\partial h_o}{\partial z'} = \frac{\partial h_o}{\partial z}\frac{\partial z}{\partial z'} + \frac{\partial h_o}{\partial y}\frac{\partial y}{\partial z'}$$

$$\frac{\partial^2 h_o}{\partial z'^2} = \frac{\partial}{\partial z}\left[\frac{\partial h_o}{\partial z}\frac{\partial z}{\partial z'} + \frac{\partial h_o}{\partial y}\frac{\partial y}{\partial z'}\right]\frac{\partial z}{\partial z'} + \frac{\partial}{\partial y}\left[\frac{\partial h_o}{\partial z}\frac{\partial z}{\partial z'} + \frac{\partial h_o}{\partial y}\frac{\partial y}{\partial z'}\right]\frac{\partial y}{\partial z'}$$

$$\frac{\partial^2 h_o}{\partial z'^2} = \frac{\partial^2 h_o}{\partial z^2}\left(\frac{\partial z}{\partial z'}\right)^2 + 2\frac{\partial^2 h_o}{\partial y \partial z}\frac{\partial y}{\partial z'}\frac{\partial z}{\partial z'} + \frac{\partial^2 h_o}{\partial y^2}\left(\frac{\partial y}{\partial z'}\right)^2$$

By substituting the values from equation 3.6, one can get,

$$\frac{\partial^2 h_o}{\partial z'^2} = \frac{\partial^2 h_o}{\partial z^2}\cos^2\theta_l - 2\frac{\partial^2 h_o}{\partial y \partial z}\cos\theta_l \sin\theta_l + \frac{\partial^2 h_o}{\partial y^2}\sin^2\theta_l$$

The final warping function equation can be achieved by adding the above two results:

$$\nabla^2 h_0 = \frac{\partial^2 h_o}{\partial y'^2} + \frac{\partial^2 h_o}{\partial z'^2} = \frac{\partial^2 h_o}{\partial z^2}\left(\cos^2\theta_1 + \sin^2\theta_1\right) + \frac{\partial^2 h_o}{\partial y^2}\left(\cos^2\theta_1 + \sin^2\theta_1\right)$$

$$\nabla^2 h_0 = \frac{\partial^2 h_o}{\partial y'^2} + \frac{\partial^2 h_o}{\partial z'^2} = \frac{\partial^2 h_o}{\partial z^2} + \frac{\partial^2 h_o}{\partial y^2}$$

In the case of triangular beam, due to thin section one can ignore the variation of warping function with respect to y'. So for the inclined section warping function is governed by the following equation:

$$\nabla^2 h_o = \frac{d^2 h_{02}}{dz'^2} = -\frac{E_2}{G_2}\frac{\langle GS\rangle}{\langle EI_y\rangle} z \tag{3.8}$$

In Horizontal Section 1

In section 1, warping equation is given as (refer to section 2.2):

$$\frac{dh_{01}}{dy} = -\frac{E_1}{G_1}\frac{\langle GS\rangle}{\langle EI_y\rangle} zy$$

In Section1, from above equation and equation 3.1, shear stress is given as

$$\tau_{xy} = \left[-\frac{E_1}{\langle EI_y\rangle} T_z\right] zy \tag{3.9}$$

In Inclined Section 2

As from equation 3.7, shear stress expression for inclined section depends on τ_{xy} and τ_{xz} which further depends on their respective warping functions. The global warping functions can be represented in local warping functions as:

$$\frac{dh_{02}}{dy} = \sin\theta_1 \frac{dh_{02}}{dz'}, \frac{dh_{02}}{dz} = -\cos\theta_1 \frac{dh_{02}}{dz'}$$

After putting these values in equations 3.2, 3.3 and 3.7, the governing equation of the shear stress can be written as:

$$\tau_{xz'} = \frac{G_2}{\langle GS \rangle} T_z \frac{dh_{02}}{dz'} \quad (3.9)$$

And the value of warping function (from equations 3.8 and 3.6) is given as:

$$\frac{d^2 h_{02}}{dz'^2} = -\frac{E_2 \langle GS \rangle}{G_2 \langle EI_y \rangle}(-y'\sin\theta_1 - z'\cos\theta_1 + d)$$

$$\frac{dh_{02}}{dz'} = -\frac{E_2 \langle GS \rangle}{G_2 \langle EI_y \rangle}(-y'z'\sin\theta_1 - \frac{z'^2}{2}\cos\theta_1 + dz') + C \quad (3.10)$$

Shear stress expression in inclined section is obtained by plugging equation 3.10 into equation 3.9

$$\tau_{xz'} = \frac{E_2 T_z}{\langle EI_y \rangle}\left[-(-y'z'\sin\theta - \frac{z'^2}{2}\cos\theta_1 + dz')\right] + C\frac{G_2}{\langle GS \rangle} T_z \quad (3.11)$$

At z'=0, the above expression presents the value of shear stress for a point on the axis of symmetry of the beam; as such, the value of shear stress shall be zero, resulting in zero value for constant C. Therefore, Equation 3.11 is simplified as:

$$\tau_{xz'} = \frac{E_2 T_z}{\langle EI_y \rangle}\left[-(-y'z'\sin\theta - \frac{z'^2}{2}\cos\theta_1 + dz')\right]$$

$$\tau_{xz'} = \frac{E_2 T_z}{\langle EI_y \rangle}\left[\frac{z'^2}{2}\cos\theta - dz')\right] \qquad \text{as } y'=0 \quad (3.12)$$

3.3 Shear Stress analysis of beams with Composite Hexagonal section

Now for hexagonal beam also the same procedure and same approach has been followed. The cross section is assumed to be thin.

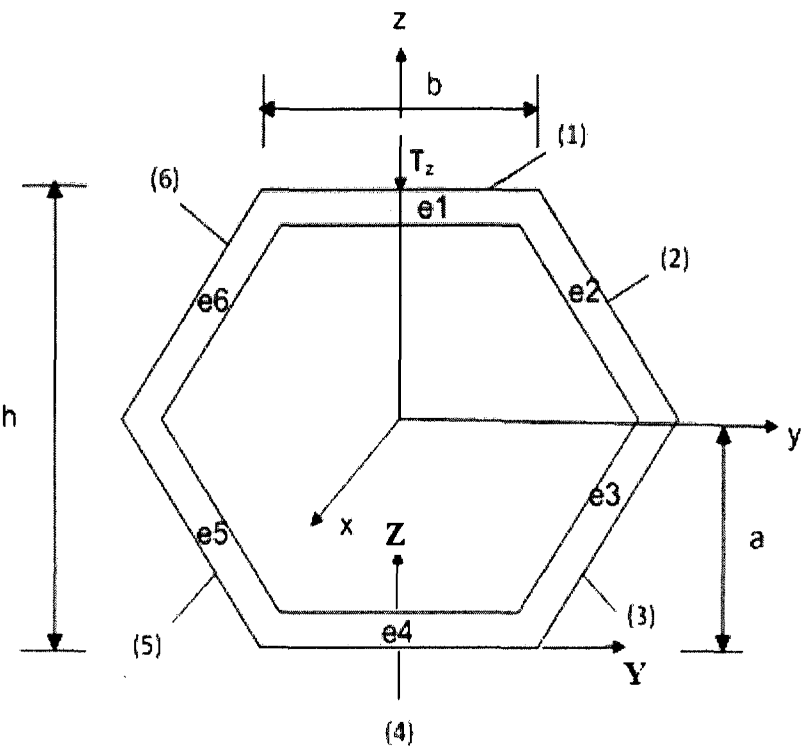

Fig. 3.4: Hexagonal Beam

In Fig. 3.4 a hexagonal beam with different cross sections is shown. The inclined sections are shown with inclination angles θ_1, θ_2 etc. And thickness of each cross section is represented by e_1, e_2 etc. It is assumed that elastic centre is exactly at the centre of the geometry for which $e_1 = e_4,\ e_2 = e_3 = e_5 = e_6$ and similarly $E_1 = E_4,\ E_2 = E_3 = E_5 = E_6$.

3.3.1 Elastic center location

To determine the elastic center first a coordinate system X,Y,Z starting from the bottom is considered. And after that coordinates are switched to elastic center as x,y,z system for further analysis. Elastic center location is given by

$$a = \frac{\int E_i Z ds}{\int E_i ds}$$

In the case of symmetric beam, 'a' can be taken exactly at the centre. Now in this composite beam elastic centre can be calculated in the same way as discussed in chapter 2 for T beam. Here if we consider this beam to be symmetric then 'a' lies on geometric centre of the figure.

3.3.2 Shear Stress Expressions

The governing expressions for shear stress are given below. The shear stress expressions depend on the warping function.

Shear stress governing equations in Sections 1 & 2

In section 1

$$\tau_{xy} = G_1 \frac{T_z}{\langle GS \rangle} \frac{dh_{01}}{dy} \qquad \text{and} \qquad \frac{dh_{01}}{dy} = \frac{-E_1 \langle GS \rangle}{G_1 \langle EI_y \rangle} zy \tag{3.13}$$

In section 2

$$\tau_{xy} = G_1 \frac{T_z}{\langle GS \rangle} \frac{dh_{02}}{dy} \quad ; \quad \tau_{xz} = G_2 \frac{T_z}{\langle GS \rangle} \frac{dh_{02}}{dz} \tag{3.14}$$

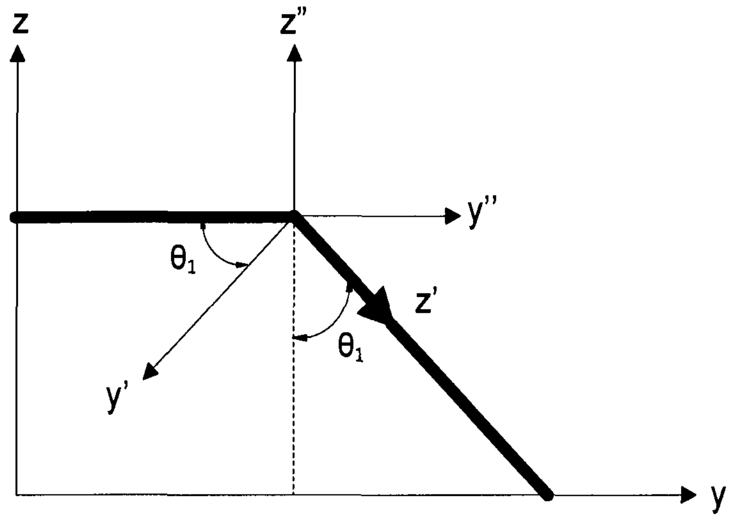

Fig.3.5: Sections 1 and 2

Coordinate Transformation

To proceed further coordinates needed to be transformed

Here angle $\theta = 180 + \theta_1$

$$\begin{Bmatrix} y' \\ z' \end{Bmatrix} = \begin{bmatrix} -\cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & -\cos\theta_1 \end{bmatrix} \begin{Bmatrix} y - A \\ z - B \end{Bmatrix}$$

$$\begin{Bmatrix} y - A \\ z - B \end{Bmatrix} = \begin{bmatrix} -\cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & -\cos\theta_1 \end{bmatrix} \begin{Bmatrix} y' \\ z' \end{Bmatrix}$$

And finally the basic shear stress equation for section 2 is given by:

$$\tau_2 = \tau_{xz'} = \tau_{xy} \sin\theta_1 - \tau_{xz} \cos\theta_1 \tag{3.15}$$

As from equation 3.15, shear stress expression for inclined section depends on τ_{xy} and τ_{xz} which further depends on their respective warping functions. The global warping functions can be represented in local warping functions as:

$$\frac{dh_{02}}{dy} = \sin\theta_1 \frac{dh_{02}}{dz'}, \frac{dh_{02}}{dz} = -\cos\theta_1 \frac{dh_{02}}{dz'}$$

And the final governing equation is given as:

$$\tau_{xz'} = G_2 \frac{T_z}{\langle GS \rangle} \frac{dh_{02}}{dz'} \tag{3.16}$$

So the above equation shows that the shear stress for inclined section depends on the local warping function. The warping function can be derived in the same way as we have in the case of triangular beam (equation 3.10) and is given as:

$$\frac{dh_{02}}{dz'} = -\frac{E_2\langle GS \rangle}{G_2\langle EI_y \rangle}(-y'z'\sin\theta_1 - \frac{z'^2}{2}\cos\theta_1 + Bz') + C_1$$

To determine the shear stress, the constant C_1 is needed to be calculated first. The constants can be determined by considering the equilibrium equations.

Equilibrium equations for calculation of constants

For the calculation of constant, here equilibrium conditions are considered for section 1 and section 2 of Fig. 3.6

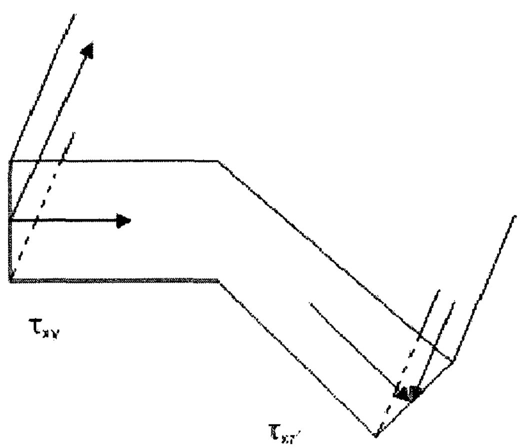

Fig 3.6: Shear Stress flow in Section 1 and Section 2

The static equilibrium condition is given by

$$\Sigma F_x = 0$$

$$e_1 \tau_{xy} = e_2 \tau_{xz'}$$

Where e_1 and e_2 are thicknesses of section 1 and section 2 respectively. The equilibrium condition is considered at the corner having coordinates:

$$y = b/2, \quad z = h/2$$

Here $y', z' = 0$,

Therefore by putting these values in equations 3.13 and 3.16, one can get

$$e_1 G_1 \left[-\frac{E_1}{G_1} \frac{\langle GS \rangle}{\langle EI_y \rangle} \frac{bh}{4} \right] = e_2 G_2 C_1$$

$$C_1 = -\frac{e_1 E_1}{e_2 G_2} \frac{\langle GS \rangle}{\langle EI_y \rangle} \frac{bh}{4} \tag{3.17}$$

So after plugging in the value of this constant in the warping function expression written above and then in the shear stress equation 3.16. the governing shear stress equations are given as:

In section 1 (from equation 3.13)

$$\tau_{xy} = \left[-\frac{E_1 T_z}{\langle EI_y \rangle} \right] \frac{h}{2} y \tag{3.18}$$

In section 2 (from equation 3.16)

$$\tau_{xz'} = \frac{E_2}{\langle EI_y \rangle} T_z \left[\frac{z'^2}{2} \cos\theta_1 - \frac{h}{2} z' - \frac{e_1 E_1}{e_2 E_2} \frac{bh}{4} \right] \tag{3.19}$$

Shear stress equations in Sections 3 & 4

Similar approach can be followed to determine the shear stress expressions for the section 3 and 4.

In section 4, shear stress is given by

$$\tau_{xy} = \frac{G_4}{\langle GS \rangle} T_z \frac{dh_{04}}{dy} \quad and \quad \left\{ \frac{dh_{04}}{dy} = -\frac{E_4 \langle GS \rangle .}{G_4 \langle EI_y \rangle} zy \right\} \tag{3.20}$$

In section 3

$$\tau_{xy} = \frac{G_3}{\langle GS \rangle} T_z \frac{dh_{03}}{dy}$$
$$\tau_{xz} = \frac{G_3}{\langle GS \rangle} T_z \frac{dh_{03}}{dz} \tag{3.21}$$

Sections 3 of hexagonal beam is shown in the figure given below:

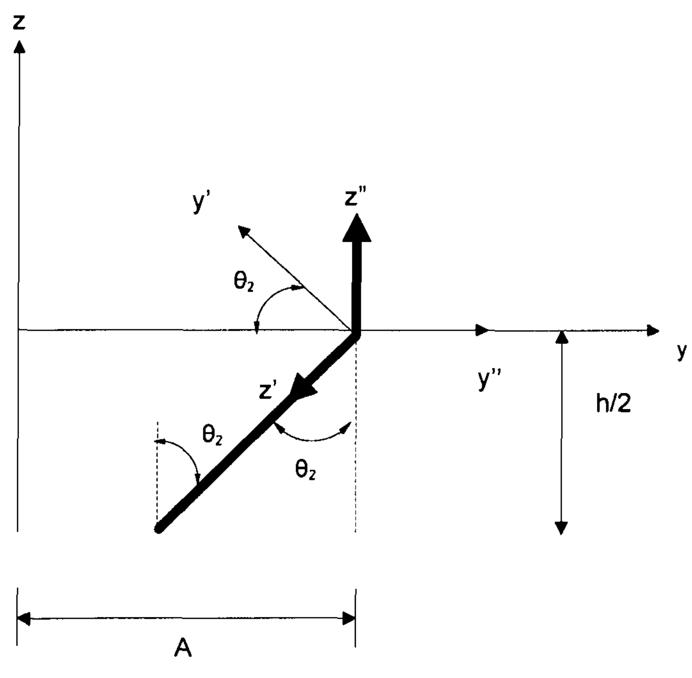

Fig. 3.7: Sections 3 with local coordinates

Coordinate Transformation

Here angle $\theta = 180 - \theta_2$

(Though θ_1 is equal to θ_2 in magnitude)

$$\begin{Bmatrix} y' \\ z' \end{Bmatrix} = \begin{bmatrix} -\cos\theta_2 & \sin\theta_2 \\ -\sin\theta_2 & -\cos\theta_2 \end{bmatrix} \begin{Bmatrix} y - A \\ z - B \end{Bmatrix}$$

$$\begin{Bmatrix} y - A \\ z \end{Bmatrix} = \begin{bmatrix} -\cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & -\cos\theta_2 \end{bmatrix} \begin{Bmatrix} y' \\ z' \end{Bmatrix}$$

Where the value of constant A is given as:

$$A = \frac{b}{2} + \frac{h}{2}\tan\theta_2, B = 0$$

Shear stress in section 3 depends on the warping function $\frac{dh_{03}}{dz'}$

and given as:

$$\tau_{xz'} = G_3 \frac{T_z}{\langle GS \rangle} \frac{dh_{03}}{dz'} \tag{3.22}$$

In the same way as discussed in previous section (3.2.3) the warping function is described as:

$$\frac{dh_{03}}{dz'} = -\frac{E_3\langle GS \rangle}{G_3\langle EI_y \rangle}\left(-y'z'\sin\theta_2 - \frac{z'^2}{2}\cos\theta_2\right) + C_2$$

Equilibrium equations for calculation of constants

For the calculation of constant C_2, equilibrium conditions are considered for section 3 and section 4

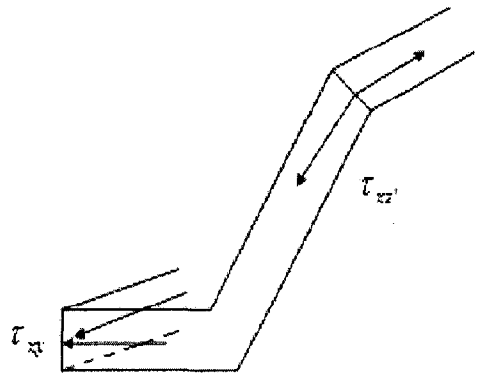

Fig. 3.8: Sections 3 and 4

$$\Sigma F_x = 0$$

$$e_4 \tau_{xy} = e_3 \tau_{xz'}$$

at $z = -h/2$ and $y = b/2$

$$y' = -\,(y-A)\cos\theta_2 + z\sin\theta_2$$

$$y' = -\left(\frac{b}{2} - \frac{b}{2} - \frac{h}{2}\tan\theta_2\right)\cos\theta_2 - \frac{h}{2}\sin\theta_2 = 0$$

$$z' = -\,(y-A)\sin\theta_2 - z\cos\theta_2$$

$$z' = \frac{h}{2}\tan\theta_2 \sin\theta_2 + \frac{h}{2}\cos\theta_2 = \frac{h}{2\cos\theta_2}$$

After plugging these values into above written equilibrium equation and equations 3.20 and 3.22, one can get

$$-\left\{e_4 G_4 \left[\frac{E_4}{G_4}\frac{\langle GS\rangle}{\langle EI_y\rangle}\frac{bh}{4}\right]\right\} = e_3 G_3\left[-\frac{E_3\langle GS\rangle}{G_3\langle EI_y\rangle}\left(\frac{-h^2}{8\cos\theta_2}\right) + C_2\right]$$

$$C_2 = \frac{-e_4 E_4}{e_3 G_3}\frac{\langle GS\rangle}{\langle EI_y\rangle}\frac{bh}{4} - \frac{E_3}{G_3}\frac{\langle GS\rangle}{\langle EI_y\rangle}\left(\frac{h^2}{8\cos\theta_2}\right) \tag{3.23}$$

After putting the value of this constant in warping function equation as discussed earlier

$$\frac{dh_{03}}{dz'} = -\frac{E_3\langle GS\rangle}{G_3\langle EI_y\rangle}(-y'z'\sin\theta_2 - \frac{z'^2}{2}\cos\theta_2) - \frac{e_4 E_4}{e_3 G_3}\frac{\langle GS\rangle}{\langle EI_y\rangle}\frac{bh}{4} - \frac{E_3}{G_3}\frac{\langle GS\rangle}{\langle EI_y\rangle}\left(\frac{h^2}{8\cos\theta_2}\right) \tag{3.24}$$

Shear stress governing equation are given after considering equations 3.20, 3.22 and 3.24

In section 4

$$\tau_{xy} = \frac{E_4}{\langle EI_y\rangle}T_z\frac{h}{2}y \tag{3.25}$$

In section 3

$$\tau_{xz'} = \frac{E_3}{\langle EI_y\rangle}T_z\left[\frac{z'^2}{2}\cos\theta_2 - \frac{e_4 E_4}{e_3 E_3}\frac{bh}{4} - \frac{h^2}{8\cos\theta_2}\right] \tag{3.26}$$

Equilibrium check (at the point M as shown in fig. 3.9 where Section 2 and Section 3 meet)

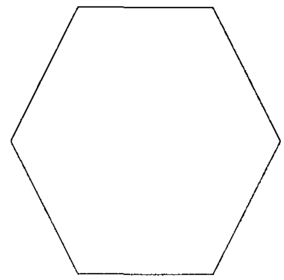

Fig. 3.9: Point M

In equation 3.19 for shear stress for section 2 put $z' = \dfrac{h}{2\cos\theta_1}$

$$\tau_{xz'} = \frac{E_2}{\langle EI_y \rangle} T_z \left[\frac{h^2}{8\cos\theta_1} - \frac{h^2}{4\cos\theta_1} - \frac{e_1 E_1}{e_2 E_2} \frac{bh}{4} \right]$$

$$\tau_{xz'} = \frac{E_2}{\langle EI_y \rangle} T_z \left[-\frac{h^2}{8\cos\theta_1} - \frac{e_1 E_1}{e_2 E_2} \frac{bh}{4} \right]$$

Also in equation 3.26 of shear stress for section 3 put $z' = 0$

$$\tau_{xz'} = \frac{E_3}{\langle EI_y \rangle} T_z \left[-\frac{h^2}{8\cos\theta_2} - \frac{e_4 E_4}{e_3 E_3} \frac{bh}{4} \right]$$

So for more specific case where we have $\theta_1 = \theta_2$, $e_1 = e_2 = e_3 = e_4 = e$ and same material (E), both equations are equal to each other.

3.4 Shear Stress analysis of beams with Composite Octagonal section.

The orthogonal beam with different sections is shown below:

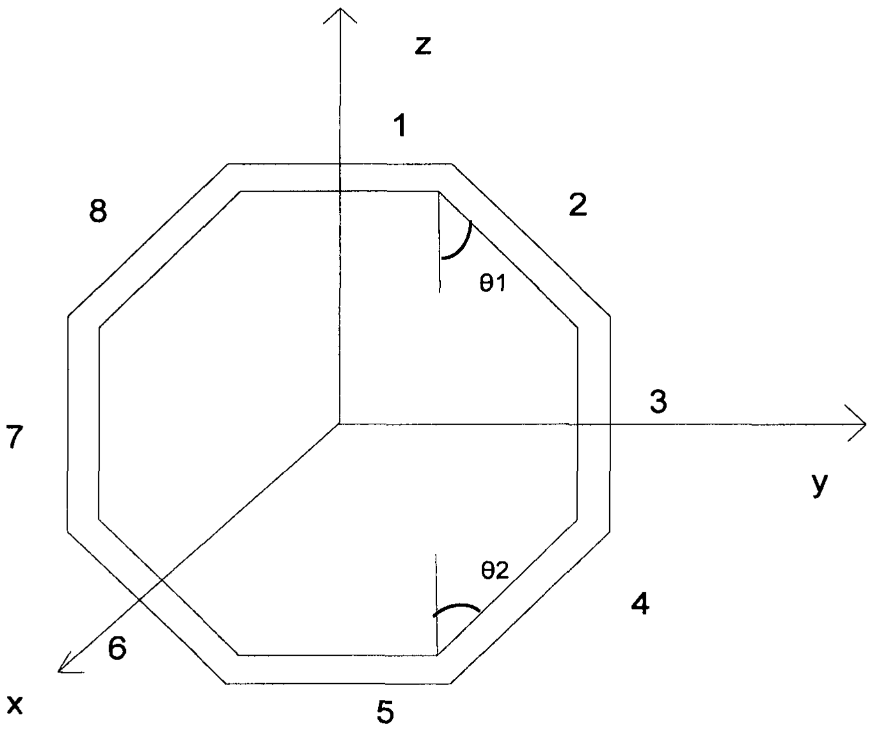

Fig. 3:10: Octagonal Beam

3.4.1 Elastic center location

To determine the elastic center first a coordinate system X, Y, Z starting from the bottom is considered. And after that coordinates are switched to elastic center as x,y,z system for further analysis. Elastic center can be obtained by following equation.

$$a = \frac{\int E_i Zds}{\int E_i ds}$$

To proceed further, It is assumed that elastic centre is exactly at the centre of the geometry having same thickness (e) and $E_1 = E_5, E_2 = E_4 = E_6 = E_8, E_3 = E_7$.

3.4.2 Shear stress expressions

The shear stress governing equations are derived using the same procedure as described in previous sections of triangular beam and hexagonal beam.

Shear stresses in Sections 1 and 2

In Section 1

$$\tau_{xy} = G_1 \frac{T_z}{\langle GS \rangle} \frac{dh_{01}}{dy}$$

In Section 2

$$\tau_{xy} = \frac{G_2}{\langle GS \rangle} \frac{T_z dh_{02}}{dy} \qquad \tau_{xz} = \frac{G_2}{\langle GS \rangle} \frac{T_z dh_{02}}{dz}$$

Shear stresses in sections 1 and 2 are same as in the sections 1 and 2 of hexagonal beam. So from equation 3.18 and 3.19, one can rewrite

In Section 1

$$\tau_{xy} = \left[\frac{-E_1 T_z}{\langle EI_y \rangle} \right] \frac{h}{2} y$$

In Section 2

$$\tau_{xz'} = \frac{E_2}{\langle EI_y \rangle} T_z \left[\frac{z'^2}{2} \cos\theta_1 - \frac{h}{2} z' - \frac{e_1 E_1}{e_2 E_2} \frac{bh}{4} \right]$$

Shear stresses in sections 4 and 5

Before proceeding to the derivation of shear stress, the coordinates are transformed for the case of inclined section.

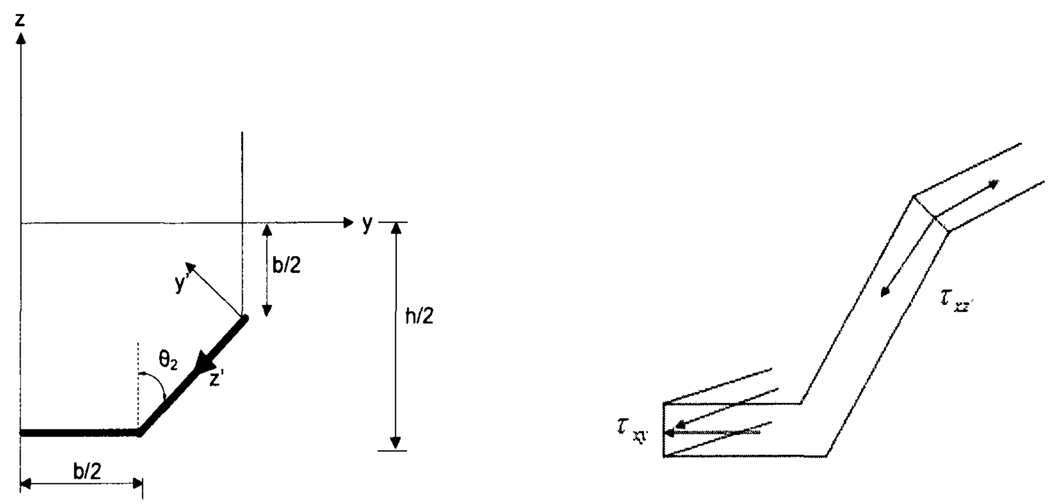

Fig. 3.11: Sections 4 and 5

Assume that all lengths are equal to b and for simplification in calculations. So the value of constants is given as:

$$A = \frac{b}{2} + \left(\frac{h}{2} - \frac{b}{2}\right)\tan\,\theta_2$$

$$\mathrm{B} = \frac{b}{2}$$

$$\begin{Bmatrix} \mathrm{y} - \mathrm{A} \\ \mathrm{z} + \mathrm{B} \end{Bmatrix} = \begin{bmatrix} -\cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & -\cos\theta_2 \end{bmatrix} \begin{Bmatrix} \mathrm{y'} \\ \mathrm{z'} \end{Bmatrix}$$

Shear stress expressions are given as:

For section 5

$$\tau_{xy} = G_5 \frac{T_z}{\langle GS \rangle} \frac{dh_{05}}{dy} \qquad \frac{dh_{05}}{dy} = \frac{-E_5 \langle GS \rangle}{G_5 \langle EI_y \rangle} zy$$

In Section 4, as described in the previous sections shear stress is given as:

$$\tau_{xz'} = G_4 \frac{T_z}{\langle GS \rangle} \frac{dh_{04}}{dz'}$$

And the warping function is given by the same equation as in the case of inclined section of hexagonal beam

$$\frac{dho_4}{dz'} = \frac{-E_4}{G_4} \frac{\langle GS \rangle}{\langle EI_y \rangle} (y'z'\sin\theta_2 - \frac{z'^2}{2}\cos\theta_2 - Bz') + C_2 \tag{3.27}$$

Equilibrium equations for calculation of constant

For the calculation of constant C_2, equilibrium conditions are considered for section 4 and section 5 (Fig 3.11)

$$\Sigma F_x = 0$$

$$e_5 \tau_{xy} = e_4 \tau_{xz'}$$

Equilibrium condition at $z = -\frac{h}{2}, y = \frac{b}{2}$

$$y' = 0$$

$$z' = -(y-A)\sin\theta_2 - (z+B)\cos\theta_2$$

$$= -\left[\frac{b}{2} - \frac{b}{2} - \frac{h}{2}\tan\theta_2 + \frac{b}{2}\tan\theta_2\right]\sin\theta_2 - \left[-\frac{h}{2} + \frac{b}{2}\right]\cos\theta_2$$

$$= \frac{h}{2}\tan\theta_2 \sin\theta_2 - \frac{b}{2}\tan\theta_2 \sin\theta_2 + \frac{h}{2}\cos\theta_2 - \frac{b}{2}\cos\theta_2$$

$$z' = \left(\frac{h}{2} - \frac{b}{2}\right)\frac{1}{\cos\theta_2}$$

After putting these values in shear stress expressions for section 4 and 5 and then in equilibrium equation, one can get the value of constant as:

$$-\left\{e_5 G_5\left[\frac{E_5}{G_5}\frac{\langle GS\rangle}{\langle EI_y\rangle}\frac{bh}{4}\right]=e_4 G_4\left[-\frac{E_4}{G_4}\frac{\langle GS\rangle}{\langle EI_y\rangle}\left(-\left(\frac{h}{2}-\frac{b}{2}\right)^2\frac{1}{2\cos\theta_2}-\frac{b}{2}\left(\frac{h}{2}-\frac{b}{2}\right)\frac{1}{\cos\theta_2}\right)\right]+C_2\right\}$$

$$C_2=-\frac{e_5 E_5}{e_4 G_4}\frac{\langle GS\rangle}{\langle EI_y\rangle}\frac{bh}{4}+\frac{E_4}{G_4}\frac{\langle GS\rangle}{\langle EI_y\rangle}\left(-\left(\frac{h}{2}-\frac{b}{2}\right)^2\frac{1}{2\cos\theta_2}-\frac{b}{2}\left(\frac{h}{2}-\frac{b}{2}\right)\frac{1}{\cos\theta_2}\right) \tag{3.28}$$

After putting the value of constant in into equation 3.27, one can obtain the value of warping function

$$\frac{dh_{04}}{dz'}=-\frac{E_4}{G_4}\frac{\langle GS\rangle}{\langle EI_y\rangle}\left(-\frac{z'^2}{2}\cos\theta_2-Bz'\right)-\frac{e_5 E_5}{e_4 G_4}\frac{\langle GS\rangle}{\langle EI_y\rangle}\frac{bh}{4}+\frac{E_4}{G_4}\frac{\langle GS\rangle}{\langle EI_y\rangle}*$$

$$\left(-\left(\frac{h}{2}-\frac{b}{2}\right)^2\frac{1}{2\cos\theta_2}-\frac{b}{2}\left(\frac{h}{2}-\frac{b}{2}\right)\frac{1}{\cos\theta_2}\right)$$

Therefore shear stresses in section 4 and 5 of octagonal beam are given by:

$$\tau_5=\frac{E_5}{\langle EI_y\rangle}T_z\frac{h}{2}y \tag{3.29}$$

$$\tau_4=\frac{E_4}{\langle EI_y\rangle}T_z\left[\frac{z'^2}{2}\cos\theta_2+Bz'-\frac{e_5 E_5}{e_4 E_4}\frac{bh}{4}-\left(\frac{h}{2}-\frac{b}{2}\right)^2\frac{1}{2\cos\theta_2}-\frac{b}{2}\left(\frac{h}{2}-\frac{b}{2}\right)\frac{1}{\cos\theta_2}\right] \tag{3.30}$$

Shear stresses in Section 3

As we have shear stress expressions for sections 1, 2, 4 and 5.

The shear stress expression for section 2 is given as

$$\tau_{xz'}=\frac{E_2}{\langle EI_y\rangle}T_z\left[\frac{z'^2}{2}\cos\theta_1-\frac{h}{2}z'-\frac{e_1 E_1}{e_2 E_2}\frac{bh}{4}\right]$$

Shear stress expression for section 3 is given as:

$$\tau_{xz} = G_3 \frac{T_z}{\langle GS \rangle}\frac{dh_{03}}{dz} \qquad \frac{dh_{03}}{dz} = -\frac{E_3\langle GS \rangle}{G_3\langle EI_y \rangle}\frac{z^2}{2} + C_3 \tag{3.31}$$

To derive the expressions for shear stress for section 3, we need to consider the equilibrium condition at y'= 0 and z' = b (If we assume all lengths are equal to b)

$$e_2\tau_{xz'} = e_3\tau_{xz}$$

So after putting the values of y' and z' in to the above expressions for shear stress in section 2 and 3 and then in to the equilibrium equation, one can get

$$e_2\frac{E_2}{\langle EI_y \rangle}T_z\left[\frac{b^2}{2}\cos\theta_1 - \frac{bh}{2} - \frac{e_1}{e_2}\frac{E_1}{E_2}\frac{bh}{4}\right] = e_3\frac{G_3}{\langle GS \rangle}T_z\left[-\frac{E_3}{G_3}\frac{\langle GS \rangle}{\langle EI_y \rangle}\frac{b^2}{8} + C_3\right]$$

$$C_3 = \frac{e_2E_2\langle GS \rangle}{e_3G_3\langle EI_y \rangle}\left[\frac{b^2}{2}\cos\theta_1 - \frac{bh}{2} - \frac{e_1}{e_2}\frac{E_1}{E_2}\frac{bh}{4}\right] + \frac{E_3}{G_3}\frac{\langle GS \rangle}{\langle EI_y \rangle}\frac{b^2}{8}$$

After putting the value of constant in equation 3.31, we obtain the expression for shear stress for section 3

$$\therefore \tau_{xz} = \frac{E_3T_z}{\langle EI_y \rangle}\left[-\frac{z^2}{2} + \frac{e_2}{e_3}\frac{E_2}{E_3}\left(\frac{b^2}{2}\cos\theta_1 - \frac{bh}{2} - \frac{e_1E_1}{e_2E_2}\frac{bh}{4}\right) + \frac{b^2}{8}\right]$$

$$\tau_3 = \tau_{xz} = \frac{E_3T_z}{\langle EI_y \rangle}\left[-\frac{z^2}{2} + \frac{b^2}{8} - \frac{e_1}{e_3}\frac{E_1}{E_3}\frac{bh}{4} + \frac{e_2E_2}{e_3E_3}b\left(\frac{b\cos\theta_1}{2} - \frac{h}{2}\right)\right] \tag{3.32}$$

3.5 Shear stress analysis of beams with Composite Decagonal section

The decagonal composite beam is shown in the Fig. 3.12.

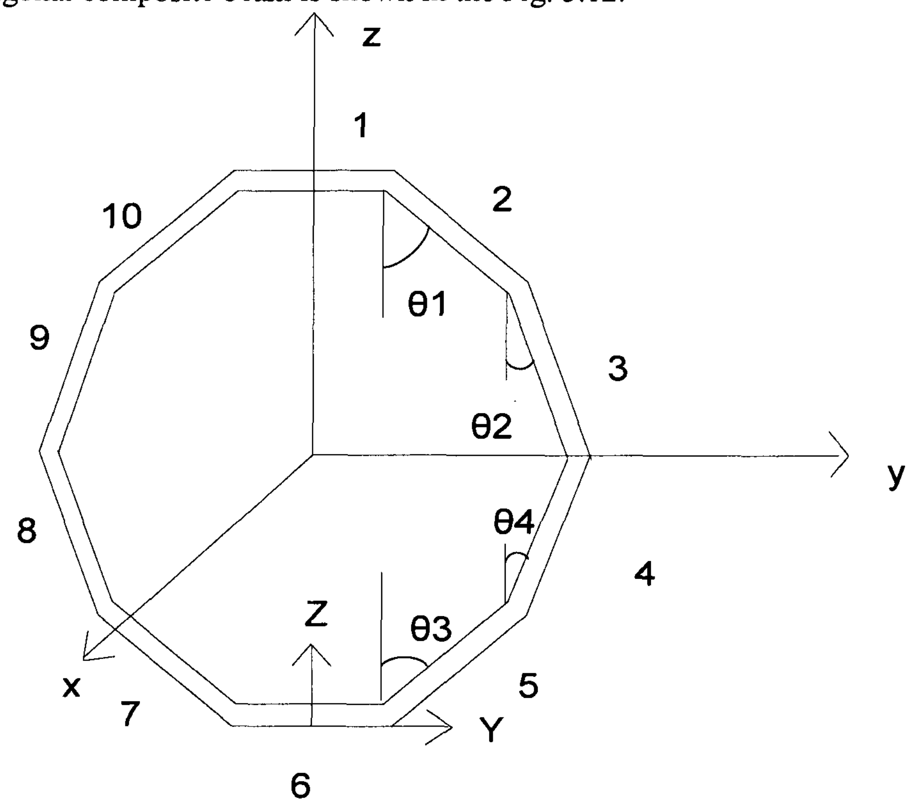

Fig. 3.12: Decagonal Composite beam

3.5.1 Elastic center.

To determine the elastic center first a coordinate system X, Y, Z starting from the bottom is considered. And after that coordinates are switched to elastic center as x,y,z system for further analysis. Elastic center can be obtained by following equation.

$$a = \frac{\int E_i Zds}{\int E_i ds}$$

To proceed further, It is assumed that elastic centre is exactly at the centre of the geometry having same thickness (e) and $E_1 = E_6$, $E_2 = E_{10} = E_5 = E_7$, $E_3 = E_9 = E_4 = E_8$.

3.5.2Shear stress expressions

Shear stress expressions for Sections 1 and 2

In Section 1

$$\tau_{xy} = G_1 \frac{T_z}{\langle GS \rangle} \frac{dh_{01}}{dy}$$

In Section 2

$$\tau_{xy} = \frac{G_2}{\langle GS \rangle} T_z \frac{dh_{02}}{dy}$$

$$\tau_{xz} = \frac{G_2}{\langle GS \rangle} T_z \frac{dh_{02}}{dz}$$

In section 1 and 2 shear stresses are same as in hexagonal beam. These shear stresses are given by the expressions:

In Section 1

$$\tau_{xy} = \left[-\frac{E_1 T_z}{\langle EI_y \rangle} \right] \frac{h}{2} y$$

In Section 2

$$\tau_{xz'} = \frac{E_2}{\langle EI_y \rangle} T_z \left[\frac{z'^2}{2} \cos\theta_1 - \frac{h}{2} z' - \frac{e_1 E_1}{e_2 E_2} \frac{bh}{4} \right]$$

Shear stresses in the sections 5 and 6

In section 6

$$\tau_{xy} = G_6 \frac{T_z}{\langle GS \rangle} \frac{dh_{06}}{dy}$$

In section 5

$$\tau_{xz'} = G_5 \frac{T_z}{\langle GS \rangle} \frac{dh_{05}}{dz'}$$

And the warping function is given as:

$$\frac{dh_{05}}{dz'} = -\frac{E_5}{G_5} \frac{\langle GS \rangle}{\langle EI_y \rangle} \left(y'z' \sin\theta_3 - \frac{z'^2}{2} \cos\theta_3 - Bz' \right) + C_2$$

$$\frac{dh_{05}}{dz'} = -\frac{E_5}{G_5} \frac{\langle GS \rangle}{\langle EI_y \rangle} \left(-\frac{z'^2}{2} \cos\theta_3 - Bz' \right) + C_2 \tag{3.33}$$

Coordinate transformations

Now at the point where local coordinates are shown in figure 3.13, the distance of local coordinates from global coordinates z and y is

$$A = \frac{b}{2} + \left(\frac{h}{2} - b\cos\theta_4 \right) \tan\theta_3$$

$$\mathrm{B} = \mathrm{b}\cos\theta_4$$

And the transformation matrix is given as:

$$\begin{Bmatrix} \mathrm{y - A} \\ \mathrm{z + B} \end{Bmatrix} = \begin{bmatrix} -\cos\theta_3 & -\sin\theta_3 \\ \sin\theta_3 & -\cos\theta_3 \end{bmatrix} \begin{Bmatrix} \mathrm{y'} \\ \mathrm{z'} \end{Bmatrix}$$

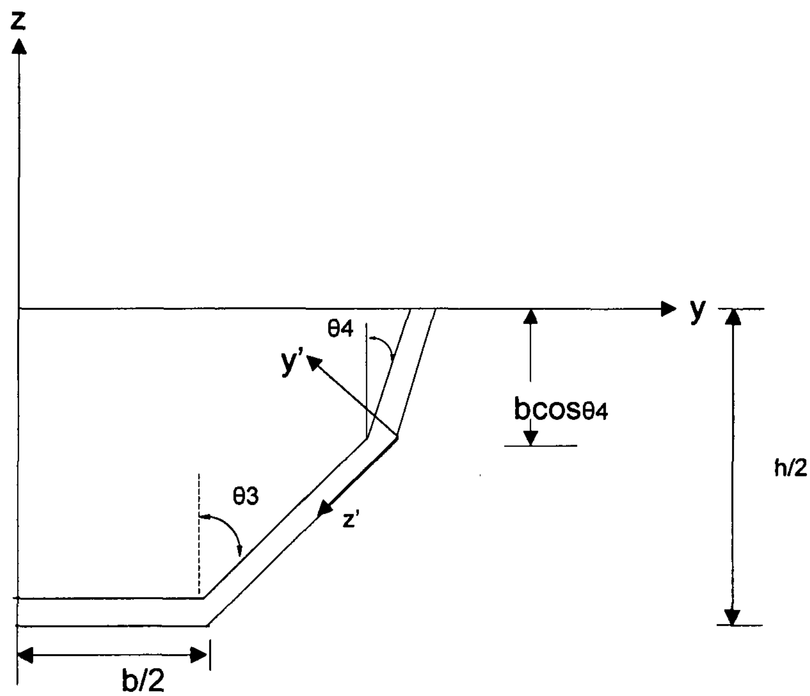

Fig. 3.13: Sections 4, 5 and 6

Now for the calculation of constant C_2, equilibrium condition is considered at

$$z=-\frac{h}{2}, y=\frac{b}{2}$$

Equilibrium equation is given as:

$$e_6\tau_{xy}=e_5\tau_{xz'}$$

$$y'=o$$

$$z'=-(y-A)\sin\theta_3-(z+B)\cos\theta_3$$

$$=-\left(\frac{b}{2}-\frac{b}{2}-\frac{h}{2}\tan\theta_3+b\cos\theta_4(\tan\theta_3)\right)\sin\theta_3-\left(-\frac{h}{2}+b\cos\theta_4\right)\cos\theta_3$$

$$=\frac{h}{2}\tan\theta_3\sin\theta_3-(b\cos\theta_4)\tan\theta_3\sin\theta_3+\frac{h}{2}\cos\theta_3-(b\cos\theta_4)\cos\theta_3$$

$$z' = \left(\frac{h}{2} - b\cos\theta_4\right)\frac{1}{\cos\theta_3}$$

Now after putting these values of y' and z' into equation 3.33 and then into shear stress and equilibrium equation, one can get:

$$-e_6 G_6\left[\frac{E_6}{G_6}\frac{\langle GS\rangle}{\langle EI_y\rangle}\frac{bh}{4}\right] = e_5 G_5\left[\frac{-E_5}{G_5}\frac{\langle GS\rangle}{\langle EI_y\rangle}\right]\left\{-\left(\frac{h}{2} - b\cos\theta_4\right)^2 *\right.$$

$$\left.\left(\frac{1}{2\cos\theta_3}\right) - \frac{b}{2}\left(\frac{h}{2} - b\cos\theta_4\right)\frac{1}{\cos\theta_3}\right\} + C_2$$

$$C_2 = \frac{-e_6 E_6}{e_5 G_5}\frac{\langle GS\rangle}{\langle EI_y\rangle}\frac{bh}{4} + \frac{E_5}{G_5}\frac{\langle GS\rangle}{\langle EI_y\rangle}\left\{-\left(\frac{h}{2} - b\cos\theta_4\right)^2\frac{1}{2\cos\theta_3} - \frac{b}{2}\left(\frac{h}{2} - b\cos\theta_4\right)\frac{1}{\cos\theta_3}\right\}$$

(3.34)

So the shear stresses for sections 5 and 6 are obtained by putting the value of constant in to warping function and then substituting into shear stress expressions.

In section 6

$$\tau_6 = \frac{E_6}{\langle EI_y\rangle}T_z\frac{h}{2}y \qquad (3.37)$$

In section 5

$$\tau_5 = \frac{E_5}{\langle EI_y\rangle}T_z\left[\frac{z'^2}{2}\cos\theta_3 + Bz' - \frac{e_6 E_6}{e_5 E_5}\frac{bh}{4} - \left(\frac{h}{2} - b\cos\theta_4\right)^2 *\right. \qquad (3.38)$$

$$\left.\frac{1}{2\cos\theta_3} - \frac{b}{2}\left(\frac{h}{2} - b\cos\theta_4\right)\frac{1}{\cos\theta_3}\right]$$

Shear stresses in Section 2 and 3

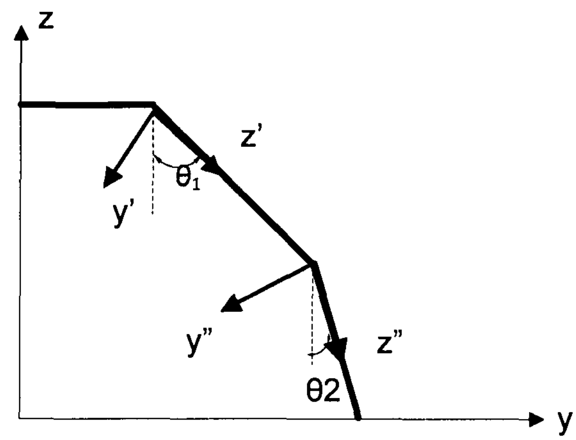

Fig. 3.14: Section 1, 2 and 3

Shear stresses in section 2 is

$$\tau_{xz'} = \frac{E_2}{\langle EI_y \rangle} T_z \left[\frac{z'^2}{2} \cos\theta_1 - \frac{h}{2} z' - \frac{e_1 E_1}{e_2 E_2} \frac{bh}{4} \right]$$

In section 3

$$\tau_{xz'} = G_3 \frac{T_z}{\langle GS \rangle} \frac{dh_{03}}{dz''}$$

And the warping function is given as:

$$\frac{dh_{03}}{dz''} = \frac{-E_3}{G_3} \frac{\langle GS \rangle}{\langle EI_y \rangle} \left(-y''z''\sin\theta_2 - \frac{z''^2}{2}\cos\theta_2 + Bz'' \right) + C_3$$

Transformation matrix is given as:

$$\begin{Bmatrix} y - A \\ z - B \end{Bmatrix} = \begin{bmatrix} -\cos\theta_2 & \sin\theta_2 \\ -\sin\theta_2 & -\cos\theta_2 \end{bmatrix} \begin{Bmatrix} y'' \\ z'' \end{Bmatrix}$$

$$A = \frac{b}{2} + b\sin\theta_1$$

$$B = \frac{h}{2} - b\cos\theta_1$$

Equilibrium condition (at y' = 0and z' = b) (y" and z"=0) is

$$e_2\tau_{xz'} = e_3\tau_{xz''}$$

So after putting in the values of coordinates into the shear stress expressions for section 2 and 3 and then into the equilibrium equation, one can get

$$e_2G_2\left[-\frac{E_2}{G_2}\frac{\langle GS\rangle}{\langle EI_y\rangle}\left(-\frac{b^2}{2}\cos\theta_1 + \frac{hb}{2} + \frac{e_1}{e_2}\frac{E_1}{E_2}\frac{bh}{4}\right)\right] = e_3G_3C_3$$

$$\therefore C_3 = \frac{e_2G_2}{e_3G_3}\left[-\frac{E_2}{G_2}\frac{\langle GS\rangle}{\langle EI_y\rangle}\left(-\frac{b^2}{2}\cos\theta_1 + \frac{hb}{2} + \frac{e_1}{e_2}\frac{E_1}{E_2}\frac{bh}{4}\right)\right] \quad (3.39)$$

$$\frac{dh_{03}}{dz''} = -\frac{E_3}{G_3}\frac{\langle GS\rangle}{\langle EI_y\rangle}\left(-y''z''\sin\theta_2 - \frac{z''^2}{2}\cos\theta_2 + Bz''\right)$$

$$+\frac{e_2G_2}{e_3G_3}\left[-\frac{E_2}{G_3}\frac{\langle GS\rangle}{\langle EI_y\rangle}\left(\frac{-b^2}{2}\cos\theta_1 + \frac{hb}{2} + \frac{e_1}{e_2}\frac{E_1}{E_2}\frac{bh}{4}\right)\right]$$

So shear stress in section 3 is given as

$$\tau_{xz''} = \frac{E_3}{\langle EI_y\rangle}T_z\left[\frac{z''^2}{2}\cos\theta_2 - Bz'' + \frac{e_2}{e_3}\frac{E_2}{E_3}\left(\frac{b^2}{2}\cos\theta_1 - \frac{bh}{2}\right) - \frac{e_1}{e_3}\frac{E_1}{E_3}\frac{bh}{4}\right]$$

$$\tau_{xz''} = \frac{E_3}{\langle EI_y\rangle}T_z\left[\frac{z''^2}{2}\cos\theta_2 - \left(\frac{h}{2} - b\cos\theta_1\right)z'' + \frac{e_2}{e_3}\frac{E_2}{E_3}\left(\frac{b^2}{2}\cos\theta_1 - \frac{bh}{2}\right) - \frac{e_1}{e_3}\frac{E_1}{E_3}\frac{bh}{4}\right]$$

$$= \frac{E_3}{\langle EI_y \rangle} T_z \left[\frac{z''^2}{2} \cos\theta_2 - (b\cos\theta_2) z'' + \frac{e_2}{e_3}\frac{E_2}{E_3}\left(\frac{b^2}{2}\cos\theta_1 - \frac{bh}{2} \right) - \frac{e_1}{e_3}\frac{E_1}{E_3}\frac{bh}{4} \right]$$

$$\tau_3 = \tau_{xz''} = \frac{E_3}{\langle EI_y \rangle} T_z \left[z''\left(\frac{z''}{2} - b \right)\cos\theta_2 + \frac{e_2}{e_3}\frac{E_2}{E_3}\left(\frac{b^2}{2}\cos\theta_1 - \frac{bh}{2} \right) - \frac{e_1}{e_3}\frac{E_1}{E_3}\frac{bh}{4} \right] \quad (3.40)$$

3.6 Conclusion

This chapter presented the step by step procedure to determine the shear stress equations for four different types of composite cross sections. The approach discussed in this chapter is valid for both homogeneous beams and for composite beams.

Chapter 4

Comparison and validation of shear stresses

4.1 Introduction

In this chapter the comparison is conducted between two approaches used to determine shear stresses. First approach is developed in previous chapters and the second approach used generally for homogeneous beams i.e. VQ/It technique. The main idea of this comparison is to validate the present approach for homogeneous cases. The comparison has been conducted by reducing problem to homogeneous cases and then comparing the governing stress equations of 'VQ/It' method.

4.2 Comparison of Shear stress in homogeneous triangular beam using two techniques

As the shear stress determining expression is given as [1]:

$$\tau_{ave} = \frac{VQ}{It}$$

Where V is vertical shear force, t is width of the section, Q is first moment of the area under consideration and I is moment of inertia of the entire cross section about neutral axis.

Calculation of Centroid for triangular cross section

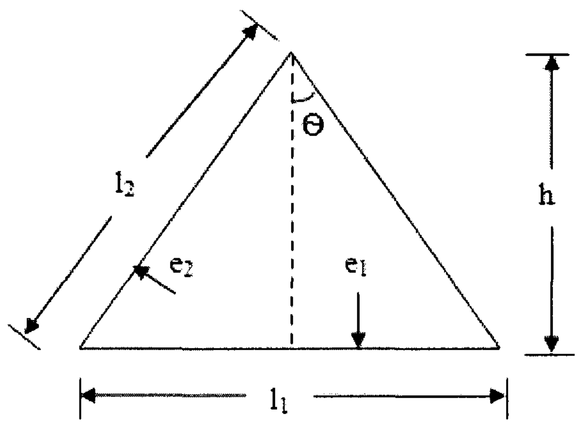

Fig. 4.1: Calculation of centroid

$$\bar{y} = \frac{2\ell_2 e_2 \times h/2}{2\ell_2 e_2 + \ell_1 e_1}$$

$$= \frac{2e_2\, h/2 \times \dfrac{h}{\cos\theta}}{\dfrac{2he_2}{\cos\theta} + \dfrac{2he_1 \sin\theta}{\cos\theta}} = \frac{e_2 h^2}{2he_2 + 2he_1 \sin\theta} = \frac{h/2}{1 + \dfrac{e_1 \sin\theta}{e_2}}$$

$$\bar{y} = \frac{h}{2(1 + \dfrac{e_1}{e_2}\sin\theta)}$$

Derivation of Shear stress expressions in local coordinates using VQ/It method

The general expression to determine shear stress is given above. For comparison with the approach given in the thesis, the general expression needs to be modified. Here Shear

Stress expressions are derived using the same general expression based on the local coordinates of the sections.

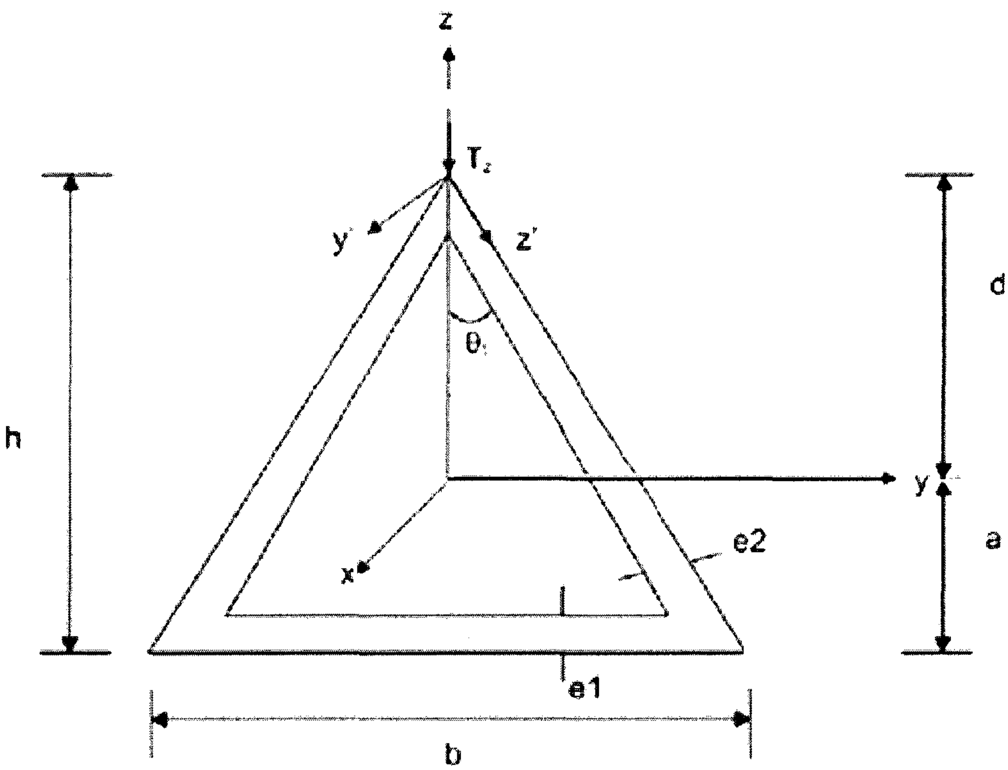

Fig. 4.2: Triangular homogeneous beam

From the expression of shear stress it is clear that shear stress depends on Q which further depends on area under consideration and the distance from neutral axis to the centroid of that area. Now

For section 1

$$Q = A\bar{z} \quad = ye_1a$$

$$\tau = \frac{VQ}{Ie_1} = \frac{V}{I}(y.a) \tag{4.1}$$

For section 2

$$Q = A(\bar{z}) = (z'e_2)(d - \frac{z'\cos\theta}{2})$$

$$\tau = \frac{VQ}{Ie_2} = \frac{V}{Ie_2}\left[z'e_2(d - \frac{z'\cos\theta}{2})\right]$$

$$\tau = \frac{V}{I}\left[dz' - \frac{z'^2}{2}\cos\theta\right] \qquad (4.2)$$

So these are the expressions of shear stress obtained using the VQ/It technique in local coordinates for homogeneous case. If we recall the expressions for shear stress in triangular beam (chapter 3), the expressions are:

For section 1,

$$\tau_{xy} = \left[-\frac{E_1}{\langle EI_y\rangle}.T_z\right]zy$$

For section 2

$$\tau_{xz'} = \frac{E_2T_z}{\langle EI_y\rangle}\left[\frac{z'^2}{2}\cos\theta - dz')\right]$$

As its clear that, if we consider the homogeneous case, these expressions are equal in magnitude with the expressions obtained in above section for triangular beam. This shows the validity of our approach.

4.3 Comparison of Shear stress in homogeneous hexagonal beam using two different techniques

Similar kind of comparison is conducted for hexagonal beam.

Derivation using $\frac{VQ}{It}$ expression in local coordinates

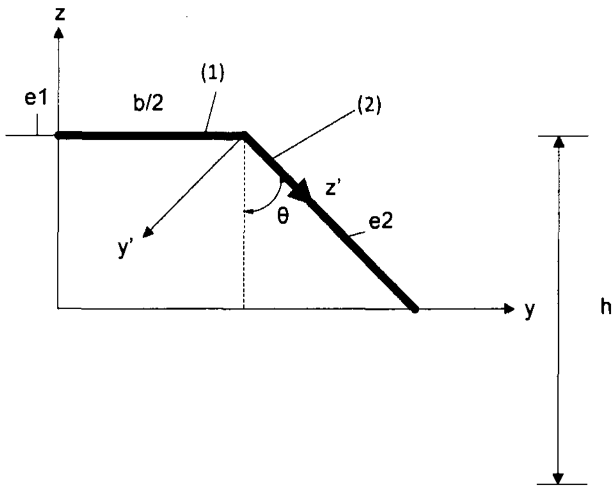

Fig. 4.3: Sections 1 and 2 of Hexagonal beam

In Section 1

$$\tau_1 = \frac{VQ}{It} = \frac{V}{Ie_1} e_1 y \frac{h}{2}$$

$$\tau_1 = \frac{V}{I} y \frac{h}{2} \quad (4.3)$$

In Section 2

$$Q = e_1 \frac{b}{2} \times \frac{h}{2} + e_2 z' \left(\frac{h}{2} - \frac{z' \cos\theta}{2} \right)$$

$$\tau_2 = \frac{VQ}{It} = \frac{V}{I} \left[\frac{e_1}{e_2} \frac{bh}{4} + \frac{z'h}{2} - \frac{z'^2}{2} \cos\theta \right] \quad (4.4)$$

Section 3 and 4

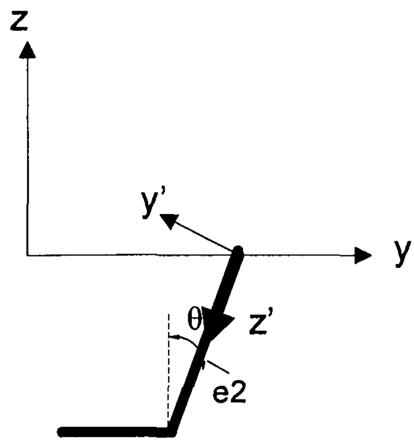

Fig. 4.4: Sections 3 and 4 of hexagonal beam

$$Q = e_1 \frac{bh}{4} + \frac{e_2 h}{2\cos\theta_1} \times \frac{h}{4} - e_2 z' \times \frac{z'\cos\theta}{2}$$

$$\tau_3 = \frac{VQ}{It} = \frac{V}{I}\left[\frac{e_1}{e_2}\frac{bh}{4} + \frac{h^2}{8\cos\theta} - \frac{z'^2}{2}\cos\theta\right] \quad (4.5)$$

And the expressions derived in chapter 3 are:

In section 1

$$\tau_{xy} = \left[\frac{-E_1 T_z}{\langle EI_y \rangle}\right]\frac{h}{2} y$$

In section 2

$$\tau_{xz'} = \frac{E_2}{\langle EI_y \rangle} T_z \left[\frac{z'^2}{2}\cos\theta_1 - \frac{h}{2} z' - \frac{e_1 E_1}{e_2 E_2}\frac{bh}{4}\right]$$

In section 3

$$\tau_{xz'} = \frac{E_3}{\langle EI_y \rangle} T_z \left[\frac{z'^2}{2}\cos\theta_2 - \frac{e_4 E_4}{e_3 E_3}\frac{bh}{4} - \frac{h^2}{8\cos\theta_2}\right]$$

So again the expressions derived here match with the Shear stress expressions derived in chapter 3 for the case of hexagonal beam, if we consider homogeneous beam only.

4.4 Comparison of Shear stress in homogeneous Octagonal beam using two different techniques

Derivation of shear stress expression from basic technique

For Section 1

$$Q = A\bar{z} = e_1.y.\frac{h}{2}$$

$$\tau_1 = \frac{VQ}{It} = \frac{V}{Ie_1}.e_1 y\frac{h}{2} = \frac{V}{I}\left(\frac{1}{2}hy\right) \qquad 0 < y < \frac{b}{2} \qquad (4.6)$$

For Section 2

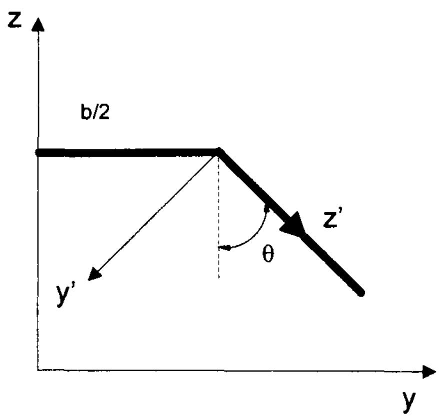

Fig. 4.5: Sections 1 and 2 of Octagonal homogeneous beam

$$Q = \frac{b}{2}\times\frac{h}{2}\times e_1 + z'e_2\left[\frac{h}{2} - \frac{z'}{2}\cos\theta\right]$$

$$\tau_2 = \frac{VQ}{It} = \frac{V}{Ie_2}\left[\frac{bhe_1}{4} \frac{+he_2 z'}{2} - \frac{z'^2 e_2 \cos\theta}{2}\right] \qquad o < z' < b \qquad (4.7)$$

For Section 3

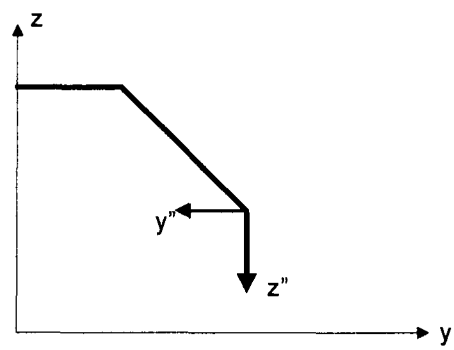

Fig. 4.6: Section 1, 2 and 3 of Octagonal Beam

$$Q = \frac{b}{2} \times \frac{h}{2} \times e_1 + be_2\left[\frac{h}{2} - \frac{-b\cos\theta}{2}\right] + z''e_3\left[\frac{h}{2} - \frac{b\cos\theta - z''}{2}\right]$$

$$\tau_3 = \frac{VQ}{It} = \frac{V}{Ie_3}\left[\frac{bhe_1}{4} + \frac{bhe_2}{2} - \frac{b^2 e_2 \cos\theta}{2} + z''e_3\frac{h}{2} - z''e_3 b\cos\theta - z''^2\frac{e_3}{2}\right]$$

Now $\frac{h}{2} - b\cos\theta = \frac{b}{2}$ $\qquad h = b(1 + 2\cos\theta)$

$$\Rightarrow z = \frac{b}{2} - z'' \Rightarrow \frac{z''}{2} = \frac{b}{4} - \frac{z}{2}$$

$$\Rightarrow \frac{b}{2} - \frac{b}{4} + \frac{z}{2} = \frac{b}{4} + \frac{z}{2}$$

$$\tau_3 = \frac{V}{Ie_3}\left[\frac{bhe_1}{4} + be_2\left(\frac{h}{2} - \frac{b}{2}\cos\theta\right) + \frac{z''e_3}{2}\left(b - z''\right)\right]$$

$$z'' = \frac{b}{2} - z$$

$$\tau_3 = \frac{V}{I}\left[\frac{bhe_1}{4e_3} + \frac{be_2}{e_3}\left(\frac{h}{2} - \frac{b}{2}\cos\theta\right) + \frac{1}{2}\left(\frac{b^2}{4} - z^2\right)\right] \quad (4.8)$$

The expressions obtained in chapter 3 for octagonal beam are:

In Section 1

$$\tau_{xy} = \left[\frac{-E_1 T_z}{\langle EI_y \rangle}\right]\frac{h}{2}y$$

In Section 2

$$\tau_{xz'} = \frac{E_2}{\langle EI_y \rangle}T_z\left[\frac{z'^2}{2}\cos\theta_1 - \frac{h}{2}z' - \frac{e_1 E_1}{e_2 E_2}\frac{bh}{4}\right]$$

For Section 3

$$\tau_3 = \tau_{xz} = \frac{E_3 T_z}{\langle EI_y \rangle}\left[-\frac{z^2}{2} + \frac{b^2}{8} - \frac{e_1}{e_3}\frac{E_1}{E_3}\frac{bh}{4} + \frac{e_2 E_2}{e_3 E_3}b\left(\frac{b\cos\theta_1}{2} - \frac{h}{2}\right)\right]$$

These expressions for different sections of octagonal beam (for homogeneous case) are same in magnitude as the expressions obtained in chapter 3

4.5 Comparison of Shear stress in homogeneous Decagonal beam using two different techniques

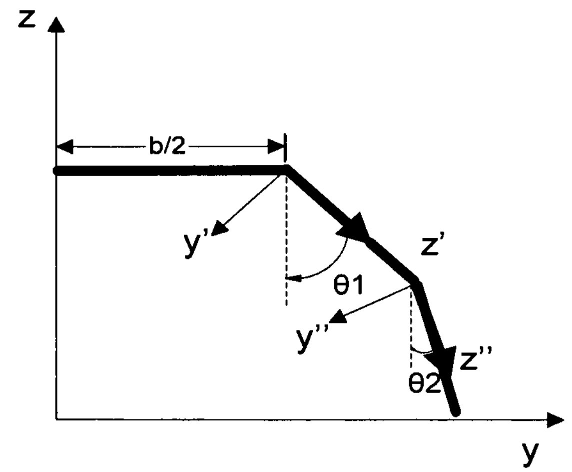

Fig: 4.7: Sections 1, 2 and 3 of Decagonal beam

For section 1

$$\tau_1 = \frac{V.y.e_1.\frac{h}{2}}{Ie_1} = \frac{V}{I}\left(\frac{1}{2}hy\right) \tag{4.9}$$

For section 2

$$Q = \frac{be_1}{2}\frac{h}{2} + z'.e_2\left(\frac{h}{2} - \frac{z'\cos\theta_1}{2}\right)$$

$$\tau_2 = \frac{V}{I}\left[\frac{bh}{4}\frac{e_1}{e_2} + \left(\frac{hz'}{2} - \frac{z'^2\cos\theta_1}{2}\right)\right] \tag{4.10}$$

For section 3

$$Q = \frac{be_1}{2}\frac{h}{2} + be_2\left(\frac{h}{2} - \frac{b\cos\theta_1}{2}\right) + z''.e_3\left(\frac{h}{2} - b\cos\theta_1 - \frac{z''\cos\theta_2}{2}\right)$$

$$\tau_3 = \frac{V}{I}\left(\frac{bh}{4}\frac{e_1}{e_3} + \frac{be_2}{e_3}\left(\frac{h}{2} - \frac{b\cos\theta_1}{2}\right) + z''\left(\frac{h}{2} - b\cos\theta_1 - \frac{z''\cos\theta_2}{2}\right)\right)$$

Now $\frac{h}{z} - b\cos\theta_1 = b\cos\theta_2$

$$\therefore \tau_3 = \frac{V}{I}\left[\frac{bh}{4}\frac{e_1}{e_3}\left(\frac{h}{2} - \frac{b\cos\theta_1}{2}\right) + z''\left(b - \frac{z''}{2}\right)\cos\theta_2\right]$$

$$\tau_3 = \frac{V}{I}\left[-\frac{z''^2}{2}\cos\theta_2 + b\, z''\cos\theta_2 + \frac{e_1}{e_3}\frac{bh}{4} + \frac{e_2}{e_3}\left(\frac{bh}{4} - \frac{b^2\cos\theta_2}{2}\right)\right] \tag{4.11}$$

And the expressions obtained in chapter 3 are:

In Section 1

$$\tau_{xy} = \left[-\frac{E_1 T_z}{\langle EI_y \rangle}\right]\frac{h}{2}y$$

In Section 2

$$\tau_{xz'} = \frac{E_2}{\langle EI_y \rangle}T_z\left[\frac{z'^2}{2}\cos\theta_1 - \frac{h}{2}z' - \frac{e_1 E_1}{e_2 E_2}\frac{bh}{4}\right]$$

For section 3

$$\tau_3 = \tau_{xz''} = \frac{E_3}{\langle EI_y \rangle}T_z\left[z''\left(\frac{z''}{2} - b\right)\cos\theta_2 + \frac{e_2}{e_3}\frac{E_2}{E_3}\left(\frac{b^2}{2}\cos\theta_1 - \frac{bh}{2}\right) - \frac{e_1}{e_3}\frac{E_1}{E_3}\frac{bh}{4}\right]$$

These expressions for decagonal beam are similar to the expressions obtained in above section for homogeneous beam.

4.6 Conclusion

This chapter shows the comparison between present thesis approach and the conventional VQ/It approach for homogeneous beams. The shear stress expressions obtained in this chapter match with the expressions obtained by present thesis approach in chapter 3 of thesis for homogeneous case. All the results match with each other and hence validate the present thesis approach.

Chapter 5

Results, Analysis and Discussion

5.1 Introduction

This chapter provides graphical results of shear stress distribution for composite beams having different cross sections. At the end, the results are obtained for maximum shear stress magnitudes in all beams by taking into consideration the weight and type of beams. Also comparison is presented between present thesis approach for composite beams and conventional approach for homogeneous beams.

5.2 Variation of shear stress across different Sections of beams

In this Section, variation of shear stress with respect to different cross sections is demonstrated. Starting with box beam graphical results are presented for more complex beams like Hexagonal beams, Octagonal beams and Decagonal beams. In these Sections a graph showing the distribution and variation of shear stress in each Section of beam are plotted for four type of beams stated above. It is noteworthy that here all beams are symmetric so graphs of shear stress distribution are plotted for half portion of the beams. It is also important to note that here a specific case is considered for simplification. It is assumed that beams are made of homogeneous materials and the thickness is the same in all the sections. This graphical presentation provides the basis for further advanced analysis.

5.2.1 Shear Stress distribution in Box Beam

The governing equations of shear stresses in different sections of box beam [7] are written below. fig. 5.1 shows the different sections of the box beam.

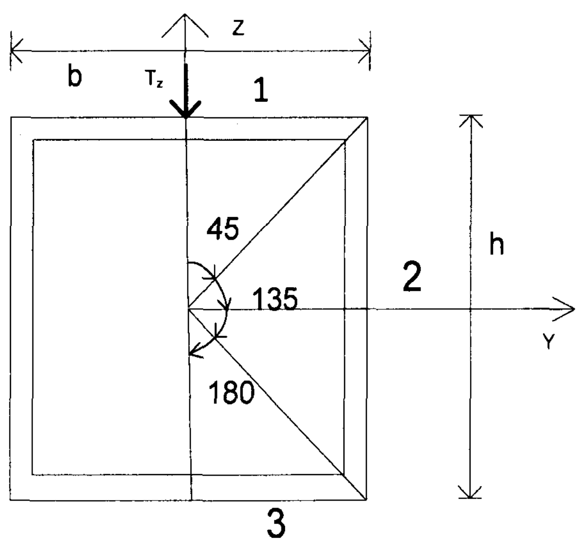

Fig. 5.1: Sections 1, 2 and 3 of box beam

For Section 1,

$$\tau_{xy} = \left[\frac{E_1 T_z}{\langle EI_y \rangle} \right] \frac{h}{2} y \tag{5.1}$$

For Section 2,

$$\tau_{xz'} = \frac{E_2}{\langle EI_y \rangle} T_z \left[-\frac{z^2}{2} + \frac{h^2}{8} + \frac{e_1 E_1}{e_2 E_2} \frac{bh}{4} \right] \quad (5.2)$$

For Section 3

$$\tau_{xy} = \left[-\frac{E_1 T_z}{\langle EI_y \rangle} \right] \frac{h}{2} y \quad (5.3)$$

In the case of box beam, fig. 5.2 shows the variation of shear stress in different sections from 0° to 180°. Here angle represents different cross sections. Shear stress in Section 1 and Section 3 is τ_{xy} and shear stress in Section 2 is τ_{xz}. So for Section 1 and Section 3 shear stress is varying with respect to y and variation is linear as shown in graph (from 0° to 45° and then from 135° to 180°). For Section 2 shear stress is varying with respect to *z* and variation is parabolic as shown in graph from 45° to 135°.

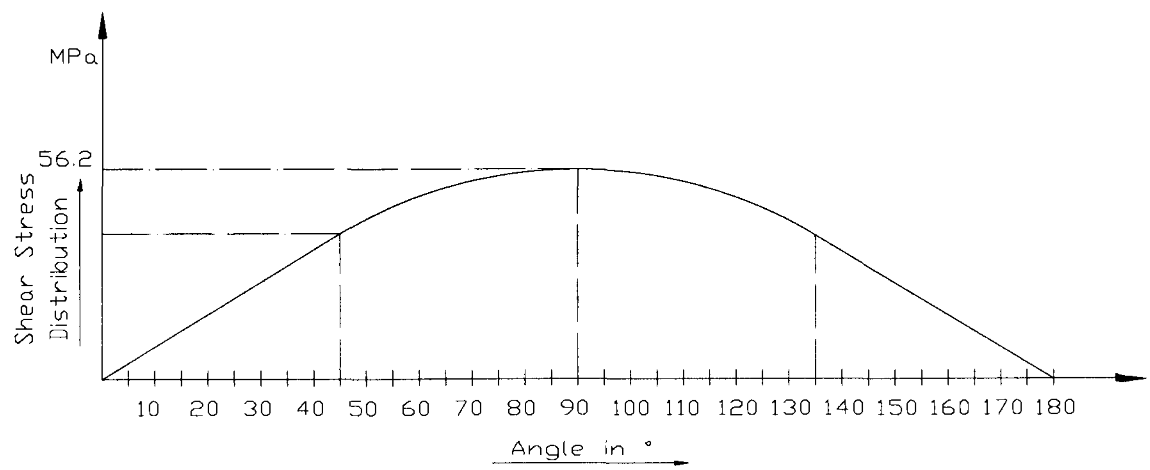

Fig. 5.2: Variation of Shear Stress with respect to Sections 1, 2 and 3 (homogeneous beam)

Maximum magnitude of shear stress occurs in section 2 is given as:

$$\tau_{\max.} = \frac{E}{\langle EI_y \rangle} T_z \left[\frac{h^2}{8} + \frac{bh}{4} \right] \quad (5.4)$$

5.2.2 Shear Stress distribution in Hexagonal Beam

Different sections of Hexagonal beam are shown in the fig. 5.3.

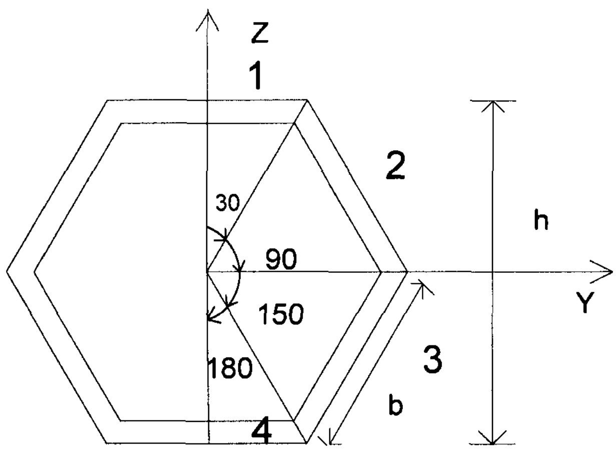

Fig. 5.3: Different Sections of Hexagonal beam

In the same way, shear stress equations for hexagonal beam are given as:

For Section 1,

$$\tau_{xy} = \left[\frac{E_1 T_z}{\langle EI_y \rangle} \right] \frac{h}{2} y \quad (5.5)$$

For Section 2,

$$\tau_{xz'} = \frac{E_2}{\langle EI_y \rangle} T_z \left[-\frac{z'^2}{2} \cos\theta_1 + \frac{h}{2} z' + \frac{e_1 E_1}{e_2 E_2} \frac{bh}{4} \right] \quad (5.6)$$

For Section 3,

$$\tau_{xz'} = \frac{E_3}{\langle EI_y \rangle}\frac{T_z}{}\left[-\frac{z'^2}{2}\cos\theta_2 + \frac{e_4 E_4}{e_3 E_3}\frac{bh}{4} + \frac{h^2}{8\cos\theta_2}\right] \tag{5.7}$$

For Section 4,

$$\tau_{xy} = -\frac{E_4}{\langle EI_y \rangle} T_z \frac{h}{2}(y) \tag{5.8}$$

In hexagonal beam, again the variation of shear stress is shown for different sections from 0° to 180°. Shear stress in Section 1 and Section 4 is τ_{xy} and shear stress in Sections 2 and 3 is $\tau_{xz'}$. Shear stress variation is linear for Section 1 and Section 3 as shown in graph from 0° to 30° and then from 150° to 180°. For Section 2 and Section 3 shear stress is varying with respect to local coordinates and variation is parabolic as shown in graph from 30° to 150°.

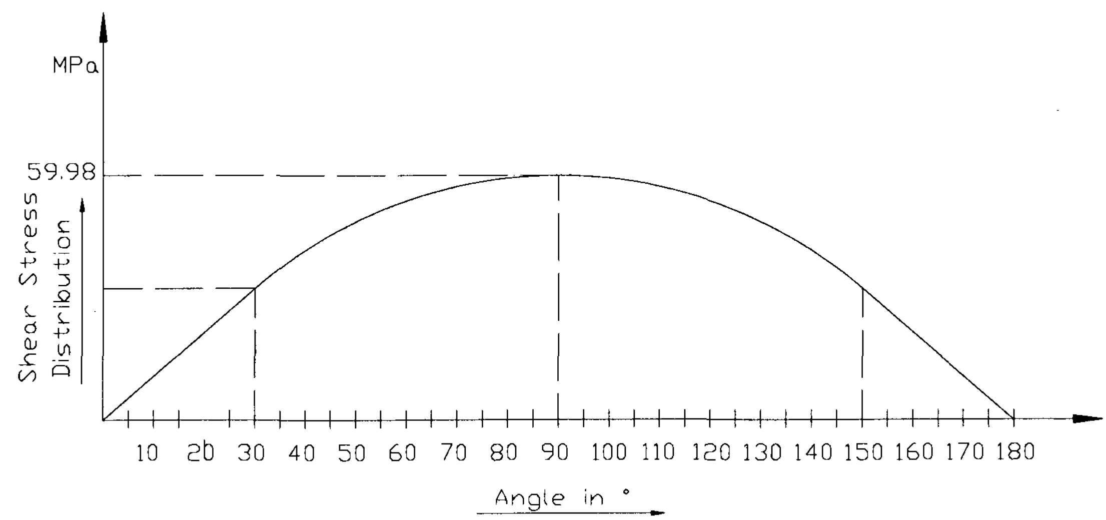

Fig. 5.4: Variation of Shear Stress with respect to Sections 1, 2, 3 and 4 (homogeneous beam)

5.2.3 Shear Stress distribution in Octagonal Beam

In the same way, the Octagonal beam (with height h and each side equal to b) is shown in the fig. 5.5

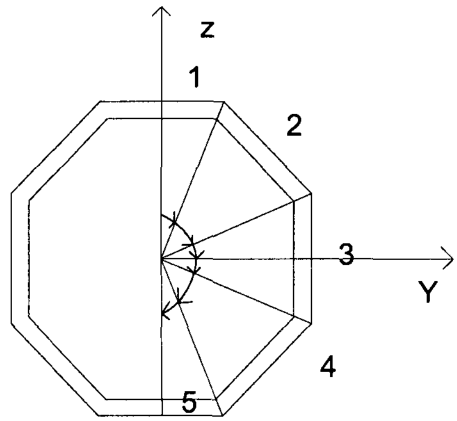

Fig. 5.5: Different Sections of Octagonal beam

The governing equations for shear stress in different sections of octagonal beam are given as:

For Section 1,

$$\tau_{xy} = \left[\frac{E_1 T_z}{\langle EI_y \rangle}\right]\frac{h}{2}y \tag{5.9}$$

For Section 2,

$$\tau_{xz'} = \frac{E_2}{\langle EI_y \rangle}T_z\left[-\frac{z'^2}{2}\cos\theta_1 + \frac{h}{2}z' + \frac{e_1 E_1}{e_2 E_2}\frac{bh}{4}\right] \tag{5.10}$$

For Section 3,

$$\tau_{xz} = \frac{E_3}{\langle EI_y \rangle}\frac{T_z}{}\left[-\frac{z^2}{2} + \frac{b^2}{8} + \frac{e_1}{e_3}\frac{E_1}{E_3}\frac{bh}{4} - \frac{e_2 E_2}{e_3 E_3}b\left(\frac{b\cos\theta_1}{2} - \frac{h}{2}\right)\right] \tag{5.11}$$

In Octagonal beam, variation is shown for different Sections (fig.5.6). Shear stress in Section 1 and Section 5 is τ_{xy} and shear stress in Section 2 and 4 is$\tau_{xz'}$ and shear stress in Section 3 is τ_{xz}. As shown in the figures shear stress variation is linear for Section 1 and Section 5 and variation is parabolic for Sections 2, 3 and 4.

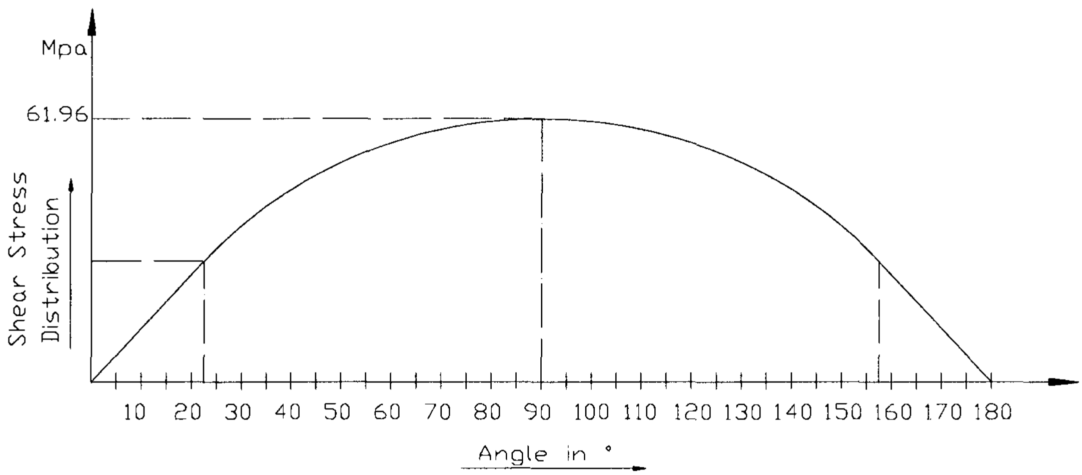

Fig. 5.6: Variation of Shear Stress with respect to Sections 1, 2 ,3 and 4

(Homogeneous beam)

5.2.4 Shear Stress distribution in Decagonal Beam

The same procedure is followed for the case of decagonal beam (with height h and each side equal to b).

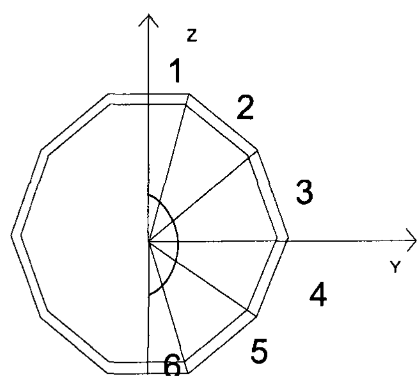

Fig. 5.7: Different Sections of Decagonal beam

The shear stress equations are given as:

For Section 1,

$$\tau_{xy} = \left[\frac{E_1 T_z}{\langle EI_y \rangle}\right]\frac{h}{2}y \tag{5.12}$$

For Section 2,

$$\tau_{xz'} = \frac{E_2}{\langle EI_y \rangle}T_z\left[-\frac{z'^2}{2}\cos\theta_1 + \frac{h}{2}z' + \frac{e_1 E_1}{e_2 E_2}\frac{bh}{4}\right] \tag{5.13}$$

For Section 3,

$$\tau_{xz''} = \frac{E_3}{\langle EI_y \rangle}\frac{Tz}{}\left[\frac{e_1}{e_3}\frac{E_1}{E_3}\frac{bh}{4} - \frac{e_2}{e_3}\frac{E_2}{E_3}\left(\frac{b\cos\theta_2}{2} - \frac{bh}{2}\right) - z''\left(\frac{z''}{2} - b\right)\cos\theta_2\right] \tag{5.14}$$

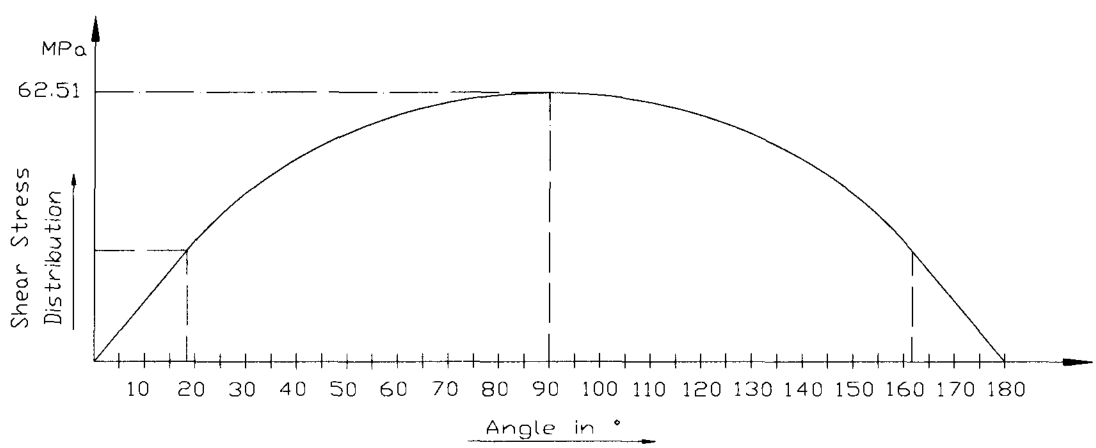

Fig. 5.8: Variation of Shear Stress with respect to Sections 1 to 6

(Homogeneous beam)

Shear stress in Section 1 and Section 6 is τ_{xy} and shear stress in Section 2 and 5 is $\tau_{xz'}$ and shear stress in Section 3 and 4 is $\tau_{xz''}$. As shown in the figure (fig. 5.8) Shear stress variation is linear or parabolic depending on the governing equations.

5.3 Maximum magnitude of shear stress in different type of beams.

In order to draw the graph of maximum magnitude of shear stress in different type of beams, moment of inertia ($I = I_c + Ad^2$) needs to be calculated.

Moment of Inertia

Now general expressions for moment of inertia in different cases are obtained as:

For Box beam:

$$I = \left[\frac{bt^3}{6} + \frac{bth^2}{2} + \frac{th^3}{6}\right] \tag{5.15}$$

Where h is height of the box beam and b is width of Section 1 and 3 and t is thickness (Fig.5.1).

For Hexagonal beam:

$$I = \left[\frac{bt^3}{6} + \frac{bth^2}{2} + \frac{t_2h^3}{8} + \frac{t_2h^3}{24}\right] \tag{5.16}$$

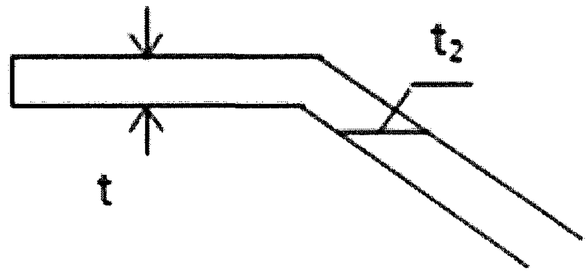

Fig.5.9 : t and t_2

$t_2 = t/\cos\theta$

Where t_2 is projection of thickness for Section 2 (Fig. 5.3) and it is measured parallel to horizontal axis. It is the same for all other inclined Sections. And t is the thickness of Section 1 and 4.

For Octagonal beam

$$I = \left[\frac{bt^3}{6} + \frac{bth^2}{2} + \frac{t_2}{3}\left(\frac{h}{2} - \frac{b}{2}\right)^3 + 2t_2(h-b)\left(\frac{h}{4} + \frac{b}{4}\right)^2 + \frac{t_3 b^3}{6}\right] \tag{5.17}$$

Where t_2 is projection of thickness of inclined Section on horizontal plane and t_3 is thickness of Section 3.

For Decagonal beam

$$I = \left[\begin{array}{l} \frac{bt^3}{6} + \frac{bth^2}{2} + \frac{t_2}{3}\left(\frac{h}{2} - b\cos\theta_2\right)^3 + 4t_2\left(\frac{h}{2} - b\cos\theta_2\right)\left(\frac{h}{4} + \frac{b}{2}\cos\theta_2\right)^2 + \frac{t_3}{6}(b\cos\theta_2)^3 \\ + 4t_3(b\cos\theta_2)\left(\frac{b}{2}\cos\theta_2\right)^2 \end{array} \right]$$

(5.18)

With the above expressions moment of inertia for each beam has been calculated. For the calculation of maximum shear stress, homogeneous case with same thickness of Section is taken into consideration.

In the calculations of maximum magnitude of shear stress, curve is shown to approach towards that of the circular beam. This plot shows that same approach can be followed to get the stress distribution equations for circular case. If the circular beam is made of different materials or, the calculation of shear stress with basic techniques is very complicated and cumbersome. But with this new approach one can consider different material properties side by side to obtain the distribution of shear stress.

Approach towards obtaining the results for circular beam

In this section it is clearly illustrated how the step by step procedure has been followed to calculate the shear stress of different beams including circular beam. Starting with box beam, each step proceeds further with increasing in the number of sides having the same height of beam. In fig 5.10, the geometric relation between different type of beams and circular beam is shown.

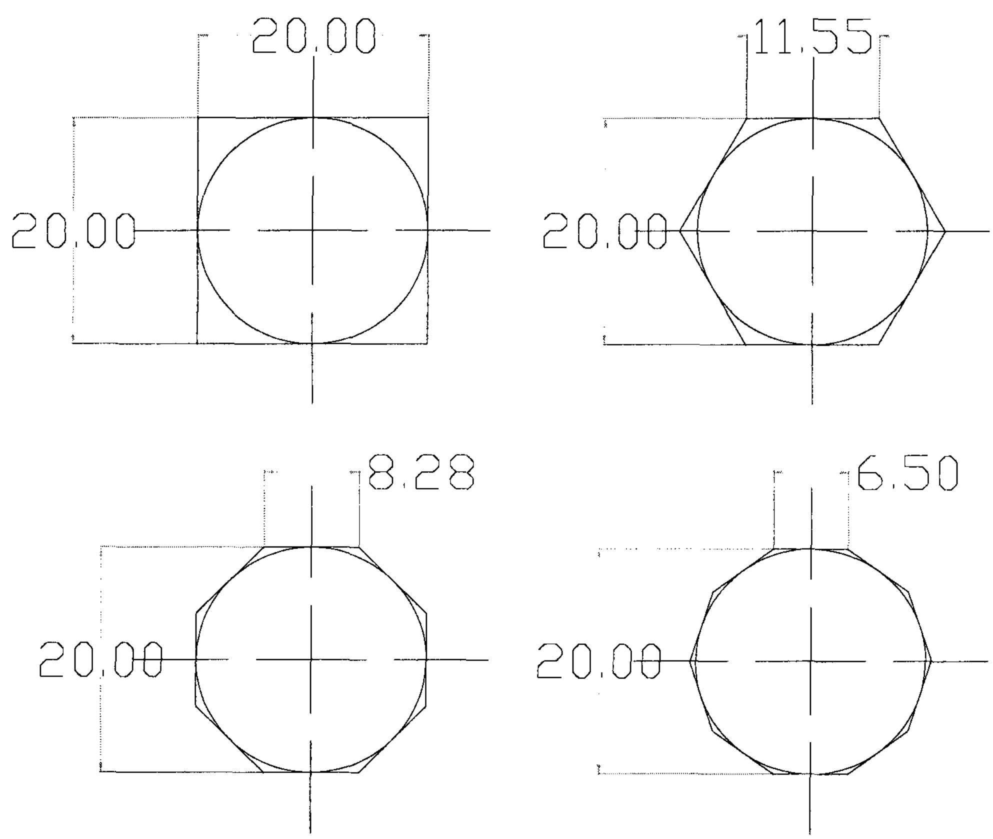

Fig. 5.10: Dimensional comparison of each beam with circular beam

In table 5.1, perimeter and width for the different type of beams are given.

Table 5.1: Perimeter and width of each side of different beams.

Number of sides	**Perimeter (mm)**	**Width of each side (b)**	**Height (h)**
Box beam (4)	80	20	20
Hexagonal beam (6)	69.28	11.52	20
Octagonal beam (8)	66.27	8.28	20
Decagonal beam (10)	64.98	6.50	20
Circle	62.83		20

Other parameters taken into consideration are:

Shear force (T_z) equals to 2 KN

Height of each beam 20 mm,

Thickness of each section equals to 1mm.

Calculation of shear stress

With the above given parameters and dimensions, maximum magnitude of shear stress is calculated in different type of beams and after that results are plotted on a graph.

Box beam

$$\tau_{\max.} = \frac{E}{\langle EI_y \rangle} T_z \left[\frac{h^2}{8} + \frac{bh}{4} \right] \tag{5.19}$$

$\tau_{\max.} = 56.2$ MPa

Hexagonal Beam

$$\tau_{\max} = \frac{E}{\langle EI_y \rangle} T_z \left[\frac{bh}{4} + \frac{h^2}{8\cos\theta_1} \right] \tag{5.20}$$

Where $\theta_1 = 30°$

$\tau_{\max.} = 59.98$ MPa

Octagonal Beam

$$\tau_{\max} = \frac{E}{\langle EI_y \rangle} T_z \left[\frac{b^2}{8} + \frac{3bh}{4} - \frac{b^2\cos\theta_1}{2} \right] \tag{5.21}$$

Where $\theta_1 = 45°$

$\tau_{\max.} = 61.96$ MPa

Decagonal Beam

$$\tau_{\max.} = \frac{E}{\langle EI_y \rangle} T_z \left[\frac{bh}{4} + b\left(\frac{h}{2} - \frac{b\cos\theta_1}{2} \right) - b\left(\frac{b}{2} \right)\cos\theta_2 \right] \tag{5.22}$$

Where $\theta_1 = 54°$ and $\theta_2 = 18°$

$\tau_{\max.} = 62.51$ MPa

For Circular beam

$I = \pi r^3 t$, $Q = r^2 t$

$\tau = 63.66$ MPa

After calculating the maximum magnitude of shear stress results are shown in the graph below.

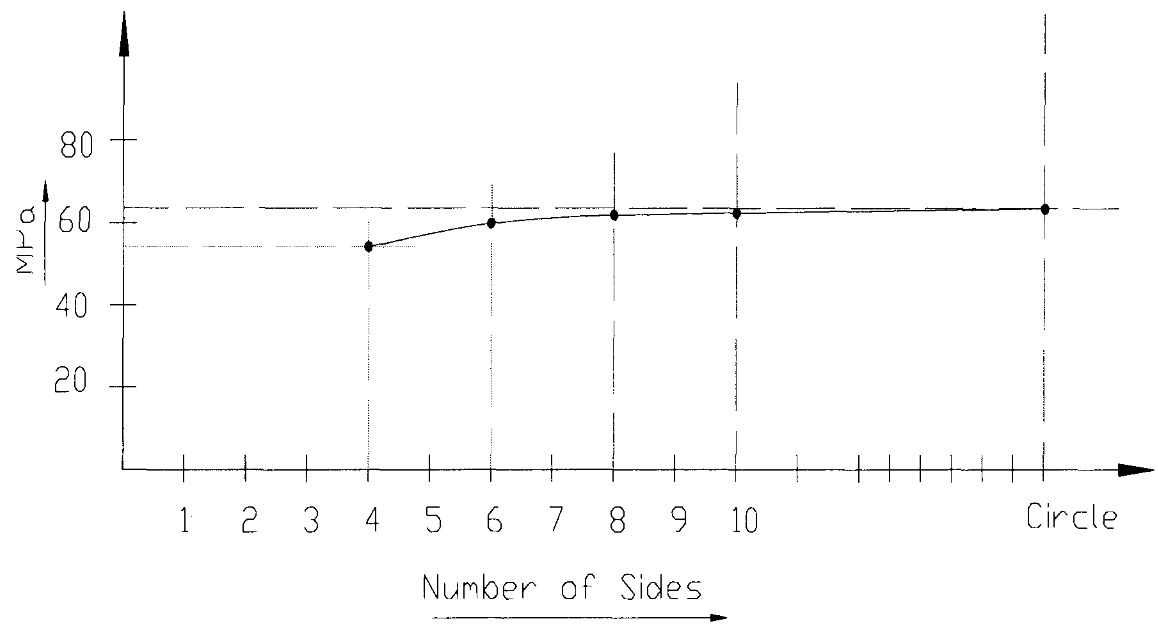

Fig. 5.11: Maximum magnitude of shear stress in different type of beams

The above graph shows the variation of maximum magnitude of shear stress with respect to the shape of the beam. As the number of sides increases from simple beam towards the circular, shear stress also varies. As shown above, same load is applied to all beams having same material and same thickness and same height. But in this case as shown in the table perimeter is decreasing as one goes from box beam towards circular beam. So to include the effect of less material used in the case of circular beam as compared to other beams, normalized stress is calculated as:

Normalized stress

In the calculation of normalized stress, weight is considered. Shape factor is defined as:

$$\eta = \frac{weight\ per\ unit\ length\ of\ the\ beam}{weight\ per\ unit\ length\ of\ square\ beam}$$

So normalized stress for different shape of beams is calculated as:

$$\tau_{normalized.} = \tau_{\max.} * \eta$$

Table 5.2: Normalized shear stress

Beam type	Normalized Shear Stress (MPa)
Box beam (4)	56.21
Hexagonal beam (6)	51.94
Octagonal beam (8)	51.32
Decagonal beam (10)	50.77
Circular beam	49.99

The calculated results are shown in the table 5.2. When number of sides increases and approaches towards the circle, shear stress decreases. The results of normalized shear stress are shown graphically in fig. 5.12. As we see that results are close but different from previous case. Also after considering dimensions and weight, if we go from box beam to circular beam, shear stress decreases.

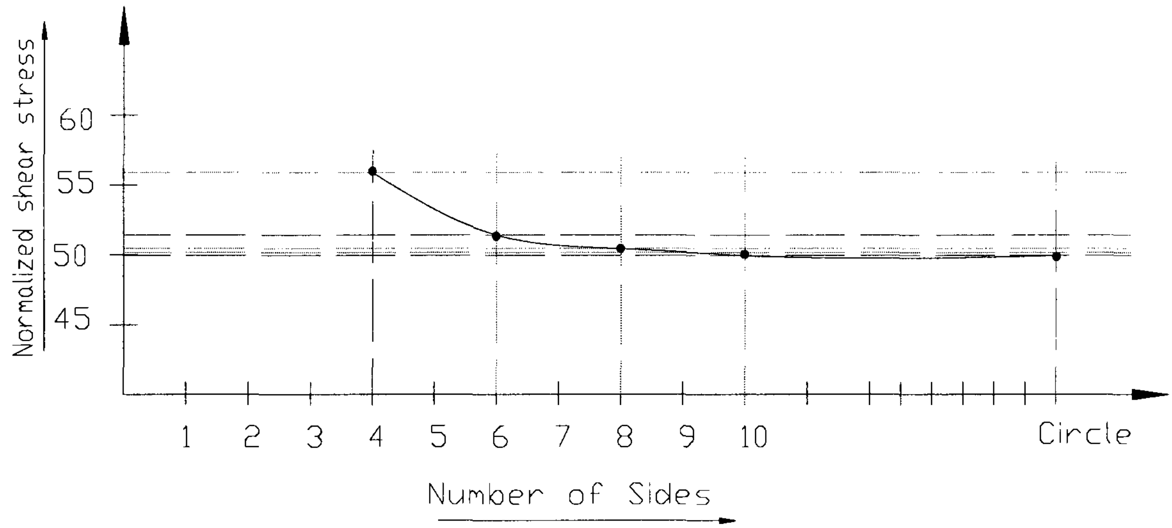

Fig. 5.12: Normalized shear stress in different type of beams

5.4 Comparison between Composite beam and homogeneous beam using present approach

In the case of homogeneous beams, determination of shear stress is easy as compared to composite beams. In open literature and in mechanics of materials books, methods are proposed to calculate the shear stress in homogeneous beams under transverse loading. But the approach presented in the present thesis is valid for both homogeneous and composite beams made of different materials. As shown above graphically in Section 5.1, shear stress varies in different cross section of different type of beams. So in this section basically a comparison has been presented between the shear stress distribution in homogeneous case and shear stress distribution when we are using different materials. Different materials and different layers can be used in the manufacturing of composite beams depends on the applications to increase the flexural strength of the structure.

For the comparison, as an example Hexagonal beam (Fig. 5.13) is taken. First the homogeneous case has been considered for the calculations. Then the calculations for shear stress distribution are extended to the case of composite beam with different materials.

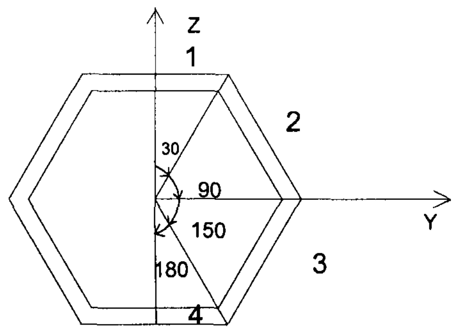

Fig 5.13: Hexagonal beam

The following data has been taken for the calculations:

Shear force (T_z) equals to 2 KN.

Height of the beam equals to 20 mm.

Thickness of each section equals to 1mm.

$t_2 = t/\cos\theta$ where $\theta = 30°$

Width equals to 11.52 mm.

The governing equations of shear stresses are given as

For Horizontal Section 1,

$$\tau_{xy} = \left[\frac{E_1 T_z}{\langle EI_y \rangle} \right] \frac{h}{2} y \tag{5.23}$$

For Inclined Section 2,

$$\tau_{xz'} = \frac{E_2}{\langle EI_y \rangle} T_z \left[-\frac{z'^2}{2} \cos\theta_1 + \frac{h}{2} z' + \frac{e_1 E_1}{e_2 E_2} \frac{bh}{4} \right] \tag{5.24}$$

Case I: Homogeneous Hexagonal beam

In homogeneous beam all sections are made of same material assuming to have same thickness. For homogeneous case as the material is same so E_1 equals to E_2.

Moment of inertia around y axis for this beam is given as:

$$I = \left[\frac{bt^3}{6} + \frac{bth^2}{2} + \frac{t_2 h^3}{8} + \frac{t_2 h^3}{24} \right] \tag{5.25}$$

After putting the values from above given data:

I= 3845.52 mm^4

Now after plugging in the values of 'I' and above given data in shear stress governing equations one can obtain:

For Section 1

τ_{xy} = 29.96 MPa for y = 5.76 (i.e. half of total width)

For Section 2

$\tau_{xz'}$= 59.98 MPa (maximum magnitude)

These are the shear stress values for homogeneous hexagonal beam.

Case II: Composite Hexagonal beam

Using analytical approach presented in Chapter 3, calculation of shear stresses in any arbitrary section of homogeneous and composite beams is feasible. To demonstrate the potentials of this approach through examples, shear stress distribution in a composite hexagonal beam, shown in Figure 5.14, is calculated. It can be observed that index 1 refers to horizontal sections that are made of material number 1. Index 2, however, refers to four inclined sections. The material for Section 2 varies according to Table 5.3. Even though for the sake of simplicity materials for Section 2 are introduced by E_2, it is obvious from Table 5.3 that 10 different materials are considered.

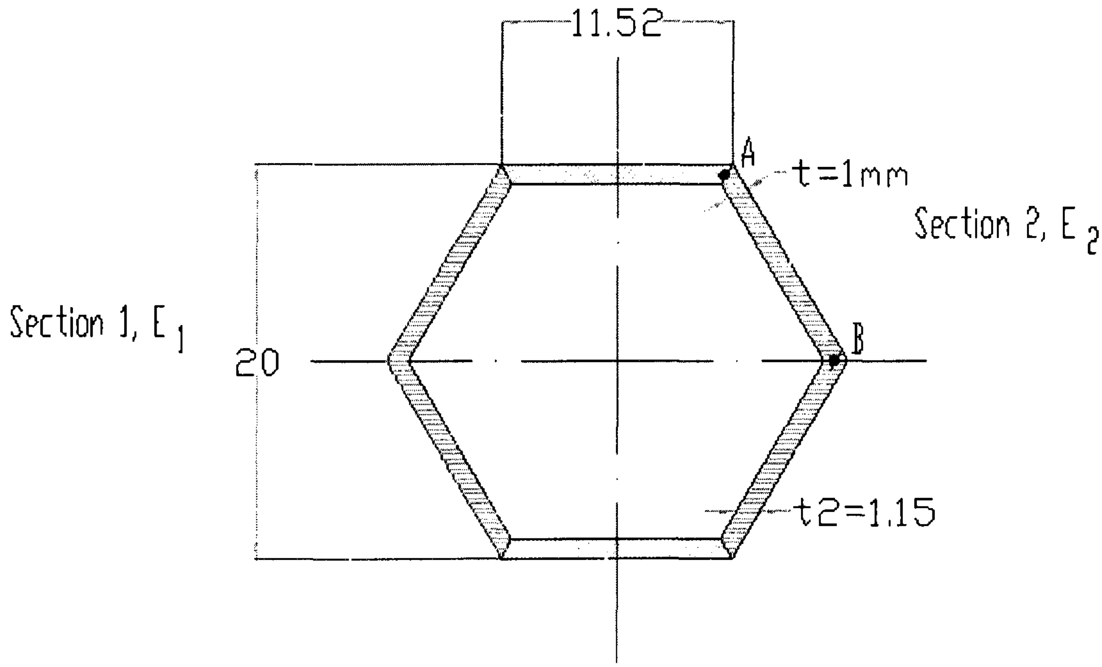

Fig 5.14: Different sections of Hexagonal beam

The equivalent flexural stiffness of the beam cross section is calculated according to the following equation:

$$\langle EI_y \rangle = 2\, E_1 I_1 + 4 E_2 I_2 = = E_1\left[\frac{bt^3}{6} + \frac{bth^2}{2}\right] + E_2\left[\frac{t_2 h^3}{8} + \frac{t_2 h^3}{24}\right] \tag{5.26}$$

By substitution of beam section geometry variables, b, t, h and t_2 from Fig.5.14 and material properties, E_1 and E_2, into this equation, equivalent stiffness of the beam cross section is calculated.

Equations 5.23 and 5.24 are used to calculate shear stresses distribution in section 1 and 2, respectively. For section 1, Equation 5.23 demonstrates a linear variation in shear stress, τ_{xy}, with respect to y coordinate. Therefore, shear stress value varies linearly from zero on symmetry axis to its maximum value at point *A* in Fig. 5.14. For Section 2, Equation 5.24 represents a second order equation and a non-linear variation in shear stress, $\tau_{xz'}$, with respect to z' coordinate. Therefore, shear stress value varies from its minimum value at z'=0 to its maximum value at point B in Fig5.14, which is located on neutral axis of the beam cross section. Table 5.3 demonstrates the maximum shear stress in horizontal section at point A and maximum shear stress in the whole cross section corresponding to point B in Fig 5.14.

Table 5.3: Shear Stresses in Composite beam made of different materials

E1	E2	E2/E1	τ_{xy} (MPa)	τ_{xz} (MPa)
210	210	1.000	29.96	59.98
210	190	0.905	31.14	59.39
210	170	0.810	32.43	58.74
210	150	0.714	33.83	58.04
210	130	0.619	35.35	57.28
210	110	0.524	37.01	56.45
210	90	0.429	38.84	55.53
210	70	0.333	40.86	54.52
210	50	0.238	43.10	53.39
210	30	0.143	45.61	52.14

The variation of shear stress in the horizontal section of the hexagonal beam with respect to Young's modulus ratio is shown in the Fig. 5.14. E_1 is the same in all cases and the ratio of E2/E1 is varying between 0 and 1.

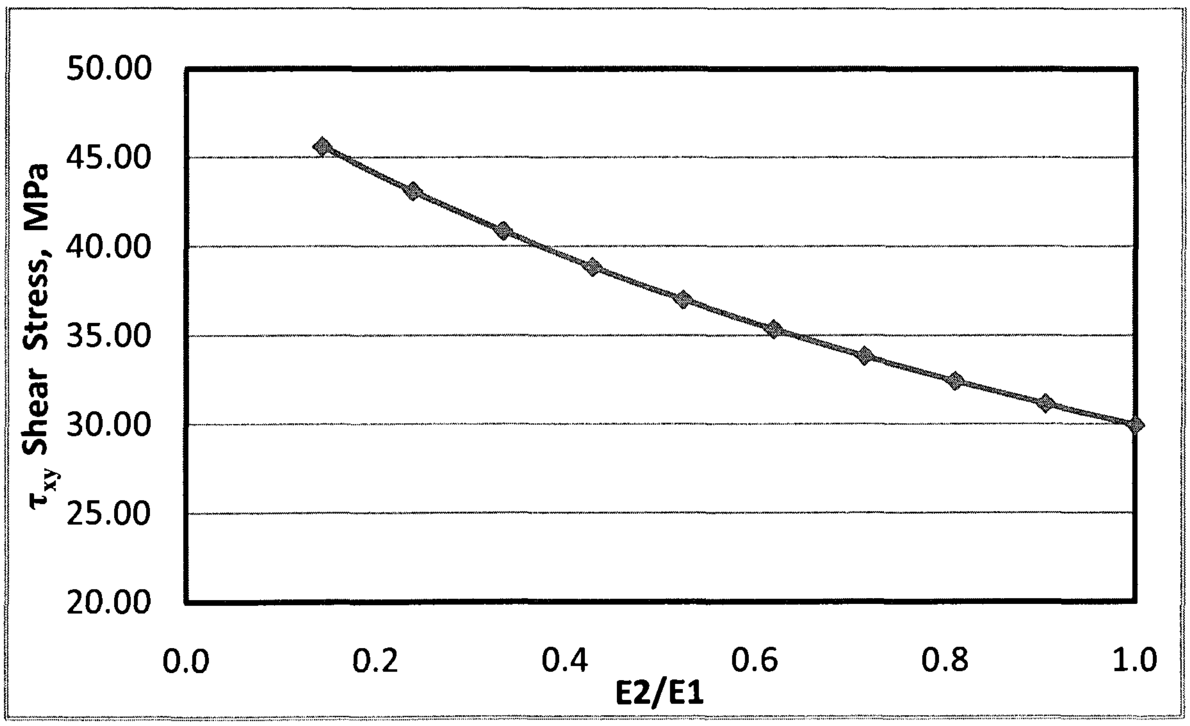

Fig 5.15: Shear Stress in horizontal section versus E_2/E_1 ratio (Hexagonal Beam)

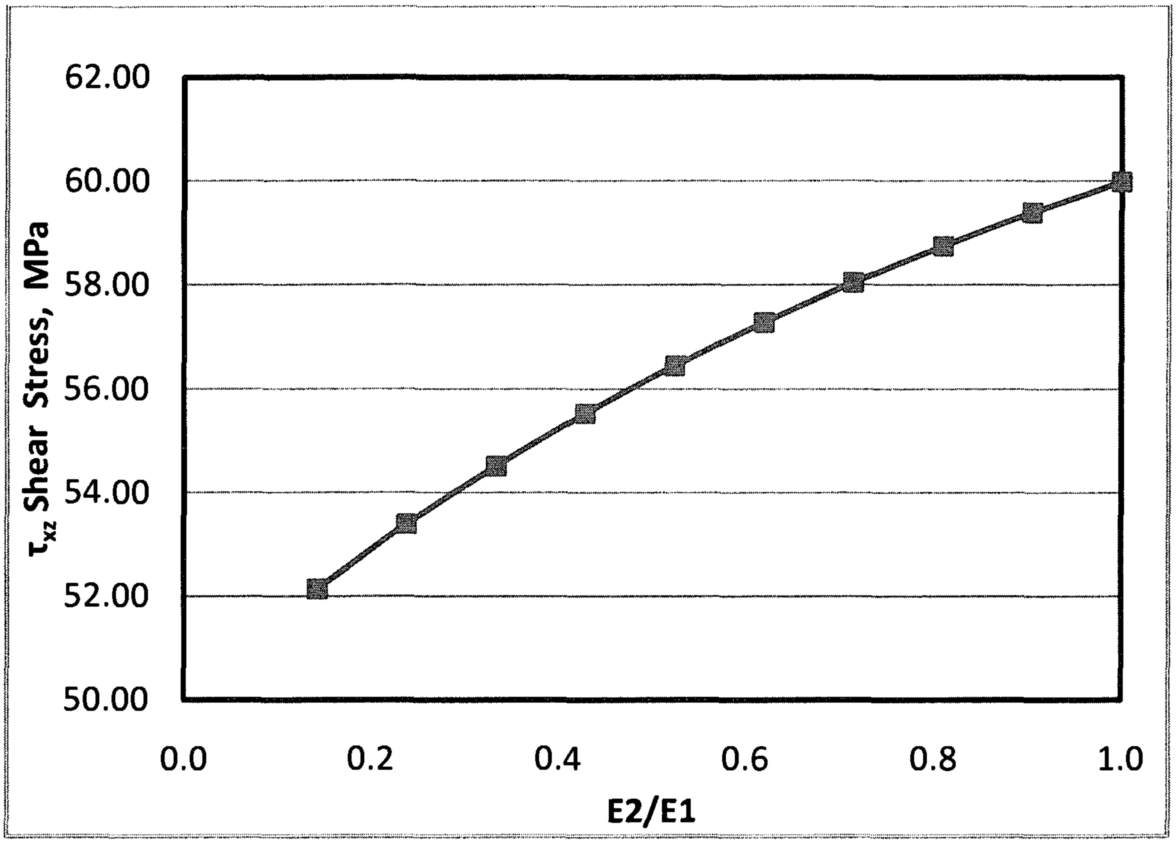

Fig 5.16: Shear Stress in inclined section versus E_2/E_1 ratio (Hexagonal Beam)

The same kind of shear stress distribution is shown for the inclined section of hexagonal beam in Fig 5.16. The combined variation of shear stresses in both horizontal and inclined sections is shown in the Fig. 5.17. Both the curves are made by using different materials. As it is clear from equation 5.26, when E_2 is increasing the equivalent stiffness is more. For shear stress in section 1, equivalent stiffness is in denominator (eq. 5.23).This is represented in graph 5.17, when E_2 is increasing the shear stress in section 1 is decreasing. For section 2, as the effect of section AB (Fig. 5.14) is more dominant, so in this case when E_2 is increasing shear stress is also increasing. When we are using material with smaller value of E_2 the effect of section AB is not significant in that case. For smaller value of E_2 shear stress in section 2 approaches to maximum shear stress value in section 1 as shown in the graph 5.17.

The shear stress variation is represented in these graphs using the present approach which shows the difference of this approach from conventional approaches.

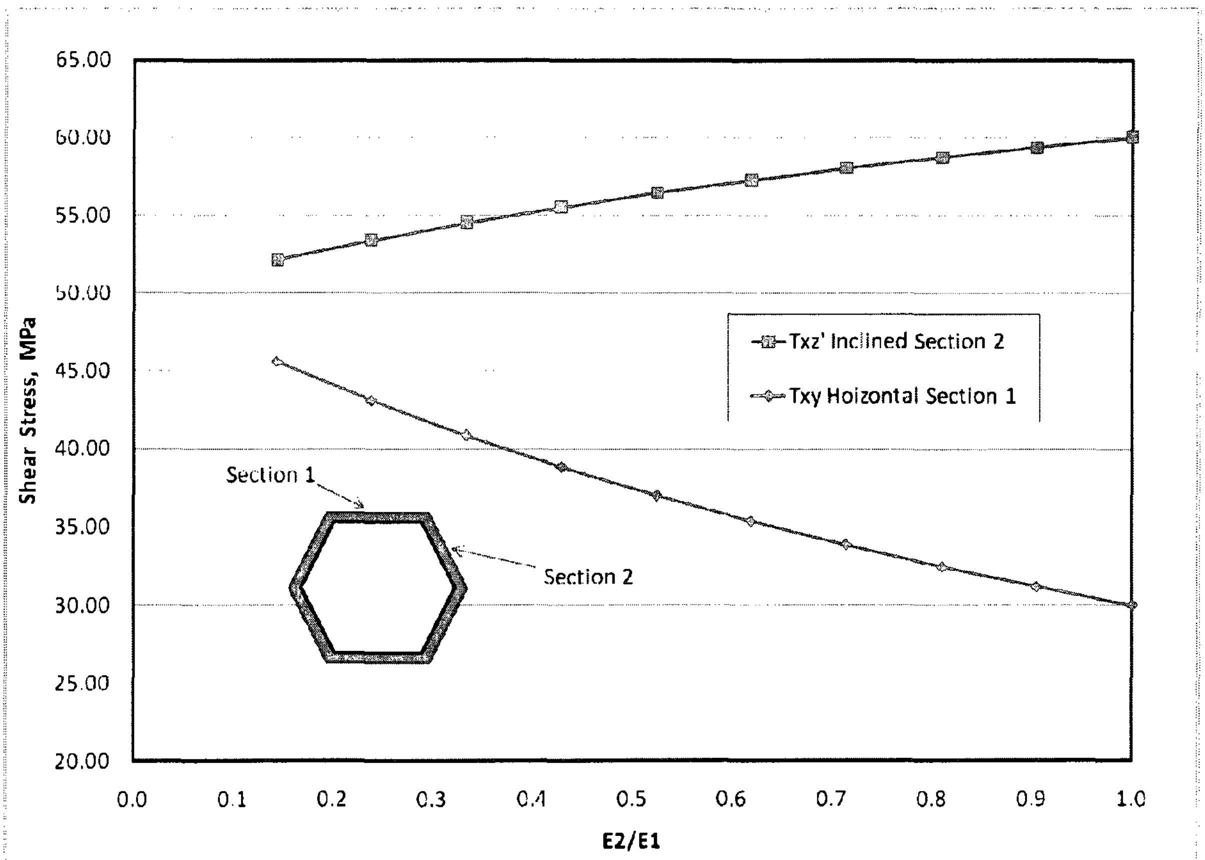

Fig 5.17: Shear Stress in inclined and horizontal section versus E_2/E_1 ratio

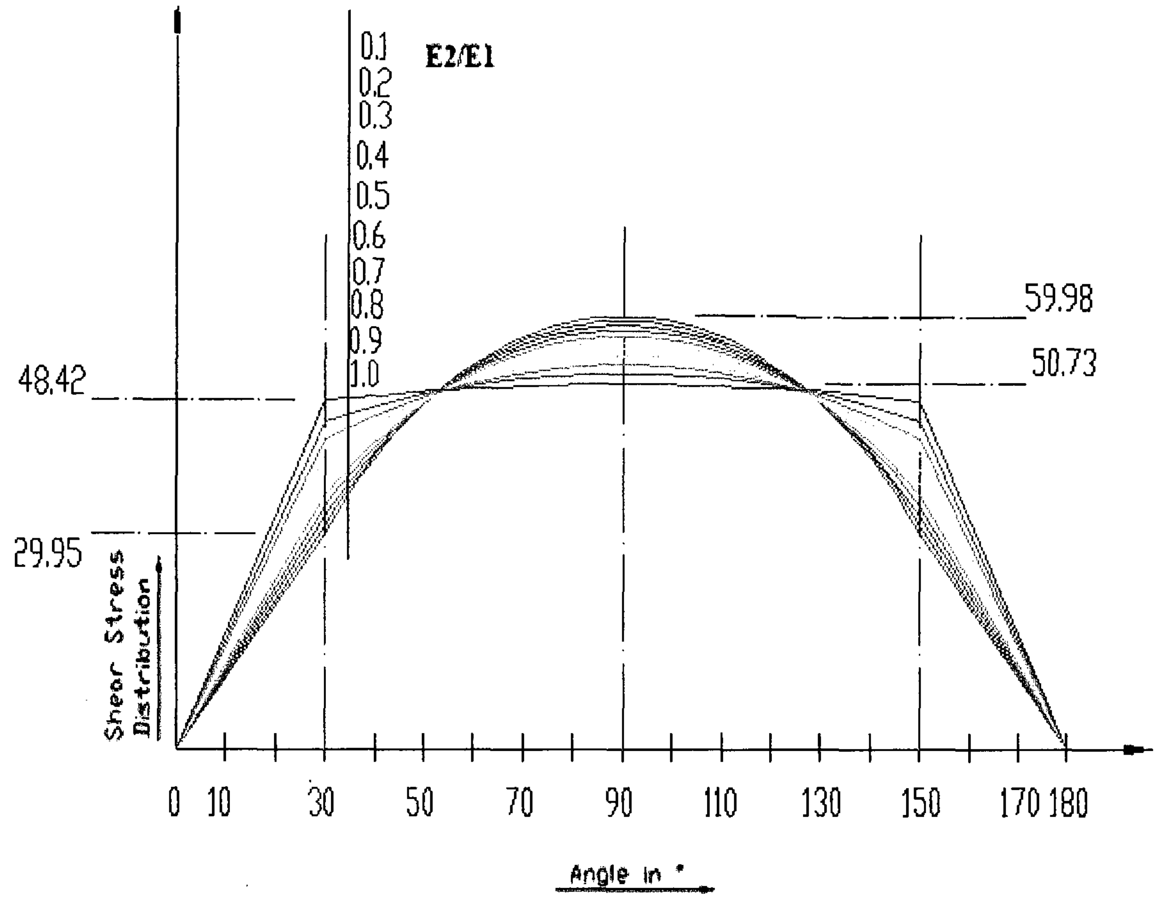

Fig. 5.18: Variation of shear stress in homogeneous and composite hexagonal beam

As shown in the Fig. 5.18, the shear stress distribution is different for Hexagonal composite beam and for Hexagonal homogeneous beam for three sections from 0° to 180°. When we are using different materials bonded together to form a beam the shear stress distribution curves are different. As we move from homogeneous case to composite case, with decreasing stiffness for inclined sections, the shear stress for horizontal section is increasing and shear stress for inclined section is decreasing. The variation of shear stress in horizontal sections of hexagonal beam is shown from 0° to 30° and from 150° to 180°. The variation is linear for horizontal section. On the other side the variation of shear stress in inclined sections is governed by second order equation as shown in the graph between 30° to 150°. Different curves represent different cases of composite beam made of different materials. The ratio of E_2/E_1 varies from 1.0 to 0.1 as shown in the fig.5.18. When we are using stiff material for both sections (horizontal and inclined $E_2=E_1$) the difference in shear stresses for both sections is more as shown in the Fig. 5.18. Shear stresses for that case are 29.95 MPa and 59.98 MPa for horizontal and inclined sections respectively. When the value of E_2 is decreasing the difference between the values of shear stresses in inclined and horizontal sections is also decreasing. It is 48.42MPa for horizontal section and 50.73 MPa for inclined section.

This section shows that the present thesis approach has advantages over conventional methods of finding the shear stress distributions. With the present approach composite beams made of two or more dissimilar materials can be analyzed easily which is very difficult and complicated with conventional methods.

5.5 Conclusion

This chapter presented the results in graphical form for different cross sections. Also maximum shear stress results and normalized shear stress results are highlighted. Finally, variation of shear stress in composite beam and in homogeneous beam has been illustrated on different graphs which show the significance of the present approach.

Chapter 6

Conclusions and Future Work

6.1 Conclusion and Contribution

Beam elements are widely used in building structures. So to determine the stresses in composite beams is always a challenging task. An approach is designed to cover the wide range of beams including homogeneous and composite beams.

In the thesis, a new approach has been developed to perform the stress analysis of composite beams in flexure. Different shapes of beams including T beams, triangular beam, hexagonal beam, octagonal beam, decagonal beam have been analysed. The pattern behaviour and shear stress variation in these beams is studied to predict the maximum shear stress in a circular beam that has the same radius as the circumscribed circle of multi-gonal beams. The approach is valid for both type of beams i.e. homogeneous beams and composite beams. After developing the approach for complex shapes of beams, a comparison has been conducted for homogeneous cases with basic technique to validate the approach.

Finally the shear stress distribution is shown graphically for different type of beams. The results showing the maximum magnitude of shear stress in different beams are presented. Difference of present approach from conventional approaches has been shown in the last section which shows the importance of the present approach.

There are constraints on this approach also. Mathematical constraints make it complex depending on the geometry of the beam. For complex geometric asymmetric shapes, the governing equations are complex. If elastic center does not fall on the center of the geometry then equations are more complex and then to determine the stress expressions is complicated and involve advanced mathematical derivations and calculations.

6.2 Future work

The approach can still be continued in the future based on the following recommendations:

- The approach presented in the thesis to determine the stresses of composite beams under shear load may be extended for the analysis of composite beams having more than one layer

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