Cooperative Diversity in CDMA Networks

Amr Mohamed Eid

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Abstract

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Spatial diversity is one of the well known diversity methods used in combating fading channels. Recently, cooperative diversity has been widely studied in literature as a spatial diversity technique. Different from multiple-input multiple-output (MIMO) systems, each user in the cooperative network is employed with a single transmit/receive antenna. In this thesis, we propose a cooperative diversity technique for asynchronous direct sequence code division multiple access (DS-CDMA) over frequency selective slow fading environment. First we assume the single cooperation relay case, where the bit-error-rate performance of the system is studied for both cases of perfect and imperfect inter-user channel (user-relay link). In order to mitigate the multi-access interference (MAI), decorrelator multiuser detectors are introduced at both relay and base station sides. Its effect on performance is studied and compared to the performance of the conventional matched filter receiver. Additionally, the performance of the system is studied and compared for different multi-path diversity scenarios in the inter-user and uplink channel. Furthermore, a coded multi-relay cooperation technique is proposed, where channel coding is introduced to minimize errors over the inter-user channel. All users are embedded with convolutional encoder and a Viterbi decoder. We study the performance of the coded system for different number of cooperating relays and over different multi-path diversity scenarios. Both simulation and analytical results are compared. Finally, we conclude that for a communication network to benefit from the cooperation diversity technique, a reliable communication link between active users and the cooperating relays should be secured (inter-user channel). We show that for an active user cooperating with V relays over a P-path frequency-selective fading channel, the expected diversity degree is $P(V+1)$.

Say: Lo! my worship and my sacrifice and my living and my dying are for Allah, Lord of the Worlds. (162) He hath no partner. This am I commanded, and I am first of those who surrender (unto Him). (163)

(Al-An'am 6: 162,163)

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Amr Mohamed Eid

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Acronyms

AAF Amplify-and-forward.

AWGN Additive white Gaussian noise.

BER Bit error rate.

BPSK Binary phase shift keying.

DAF Decode-and-forward.

DS-CDMA Direct sequence code division multiple access.

FDMA Frequency-division multiple access.

MAI Multi-access interference.

MIMO Multiple-input multiple-output.

MMSE Minimum mean square error.

MRC Maximum ratio combining.

OFDM Orthogonal frequency-division multiplexing.

RAD-MUD Relay-assisted decorrelating multiuser detector.

SIC Successive interference cancelation.

SINR Signal-to-interference plus noise ratio.

SNR Signal-to noise ratio.

TDMA Time-division multiple access.

List of Symbols

K No. of users in the network.

P No. of paths in the channel.

L Size of the transmitted frame.

V No. of cooperating relays.

S_k Data bit of user k.

C_k Spreading signature assigned to user k.

R_c Convolutional code rate.

T_b Bit period.

T_c Chip period.

τ_k User k's transmission delay.

$\tau_{k,p}$ The delay of the p^{th} path of user k.

E_{U_k} Energy received at the base station from user k.

$E_{I_{kv}}$ Energy received at the relay v from user k.

h_{kb}^{p} Fading coefficient for the uplink channel between user k and the base station b over the path p.

h_{kv}^{p} Fading coefficient for the inter-user channel between user k and the relay v over the path p.

$\mathbf{H}$ Fading coefficients matrix.

$\mathbf{R}$ Cross correlation matrix.

Chapter 1

Introduction

With the continuous growth of wireless communication services, the performance of communication systems has always been one of the major points studied in literature. Channel fading is considered the main challenge in wireless communications. Signals affected by fading can suffer from severe loss of signal-to noise ratio (SNR) in case of deep fade. Fading channels can be categorized into slow and fast fading, or flat and frequency-selective fading depending on the coherence time or coherence bandwidth of the channel, respectively.

Diversity is one of the most important methods used in combating different types of fading channels. Time diversity, frequency diversity and space diversity are the main forms of diversity. Time diversity is achieved by sending replicas of the same signal over independent fading channels during different time slots. The difference between the time slots should be equal or more than the coherence time of the channel [3]. Frequency Diversity is another diversity method where the same signal is sent over separate frequency bands. The different frequency bands should be larger than the coherence bandwidth of the channel. Both time and frequency diversity methods can be integrated with space diversity in order to gain higher diversity degrees and mitigate the fading effect on the system performance. Another common form of diversity is spatial, where it can be achieved by transmitting copies of the signal to the receiver over different spatial channels. Multiple-input multiple-output (MIMO) is one of the widely used schemes in space diversity, where multiple antennas are used to transmit and/or receive the desired signal [4], [5]. A MIMO system with n-transmit and n-receive antennas is shown in Fig. 1.1. It is known that full diversity gain can be achieved using MIMO techniques [6]; however, embedding multiple antennas at the transmitter or the receiver can sometimes be expensive.

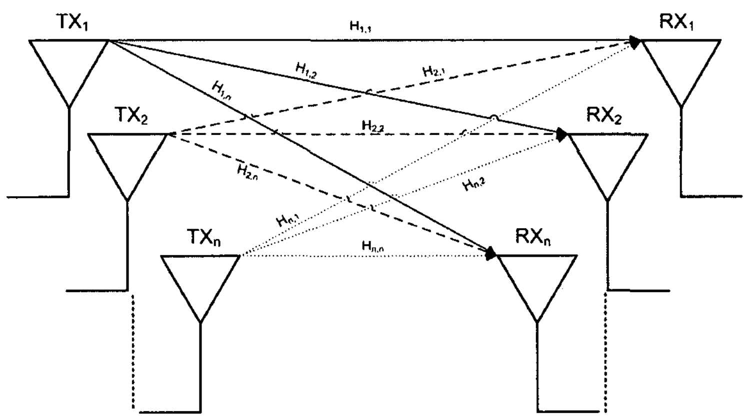

Figure 1.1: MIMO channel.

1.1 Cooperative Diversity

As an alternative to collocated antennas, cooperative diversity systems allow the receiver to see independent versions of information which yields to spatial diversity and/or coding gain compared to non-cooperative systems [7]. It was first introduced in [8], [9]. It has been shown that cooperative diversity can offer these gains by introducing both temporal and spatial correlation into the transmitted signals from different relays without increasing the total transmitted power. The cooperation among users can, in general, be categorized into two main methods; amplify-and-forward (AAF), and decode-and-forward (DAF).

1.1.1 Amplify-and-Forward

The AAF cooperating method is based mainly on the idea of forwarding an amplified version of the data over different spatial channels (Fig. 1.2). First, the source will start transmitting its own data to the destination and the relay, then the relay will amplify the received noisy version of the data and re-transmits it to the destination. Finally, the destination receives two independent noisy versions of the same data, and benefits from space diversity.

1.1.2 Decode-and-Forward

Different from AAF, the relays cooperating using DAF method decode the received signal from the source before re-transmitting it to the destination (Fig. 1.3). In other words, during the first transmission time the source transmits its own data to the destination and the relay, then in the second transmission time, the relay decodes the received signal and re-transmits the decoded bits to the destination.

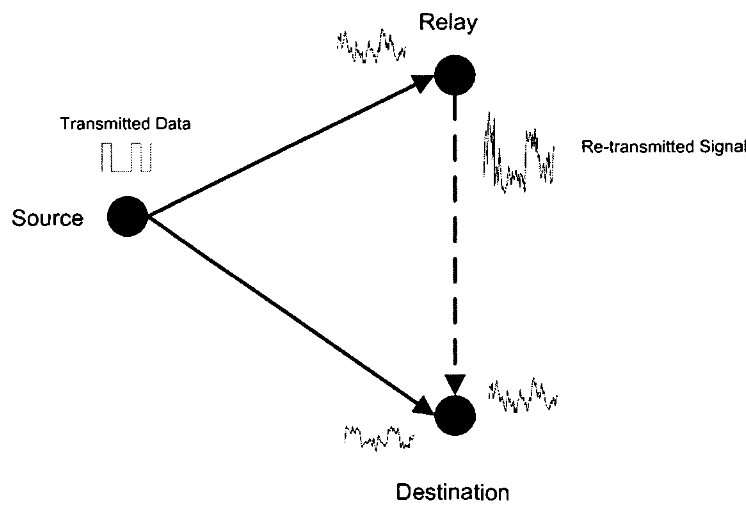

Figure 1.2: Amplify-and-forward cooperation method.

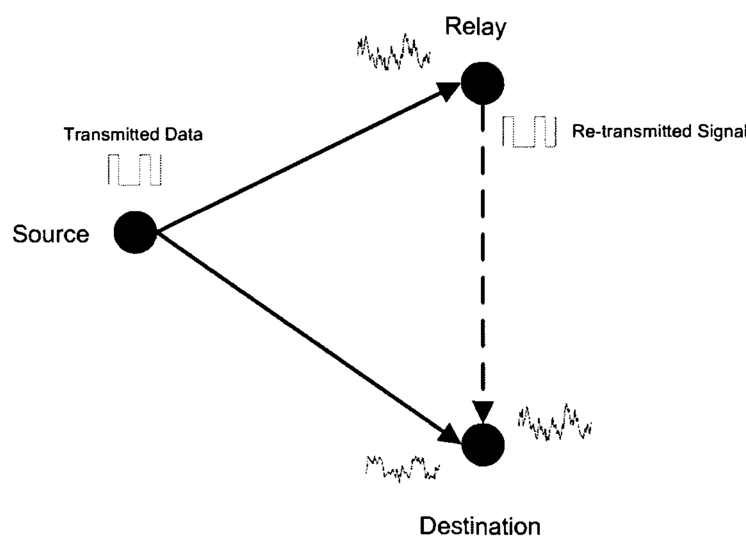

Figure 1.3: Decode-and-forward cooperation method.

1.2 Motivation

It was proved that the MIMO technique is an efficient technique used to gain transmit diversity benefits on cellular base stations. However, in some cases implementing this technique may not be practical. Cost, size and limitation of hardware can be considered as major obstacles in the way of implementing MIMO in wireless networks. Employing single transmit-receive antennas, cooperative diversity can be considered as the alternative technique that will be used in order to overcome the limitation of MIMO.

The performance of the cooperative networks was widely studied in literature. In [1], the authors combined the cooperative diversity with DS-CDMA using non-orthogonal codes for synchronous networks over flat fading channels, where each user is assigned a spreading signature which is used in modulating the transmitted data. Motivated by [1], we propose an asynchronous DS-CDMA network using cooperative diversity over the more realistic case of frequency-selective slow fading channels. Studying the impact of multiuser access interference (MAI) on the system performance, we introduce decorrelator multiuser detectors to the system, and we compare the performance of the decorrelator detectors to that of the conventional receiver. Moreover, we study the impact of the inter-user channel's condition on the system where we introduce channel coding in order to improve the inter-user channel reliability.

The main objective of this thesis is to design a cooperative diversity scheme for asynchronous DS-CDMA networks that can improve the system performance and provide full diversity gain.

1.3 Thesis Contributions

The main contributions of the thesis can be summarized as follow:

1. We propose a single-relay cooperating system for asynchronous DS-CDMA networks over frequency-selective slow fading channels using non-orthogonal codes.

2. Decorrelator multiuser detectors are introduced to the proposed system in order to mitigate the multiuser access interference (MAI). The performance of the system using decorrelator detectors is compared to the case of conventional matched filter showing the impact of MAI on the overall system performance.

3. An exact expression for probability of bit error for the perfect and imperfect inter-user channel cases for the uncoded system is evaluated and compared to numerical results for different multipath diversity scenarios.

4. A coded multi-relay cooperating system is introduced. In this system, we employ channel coding in the cooperating network in order to improve the performance over the uplink and inter-user channels. We evaluate the upper bound performance of the proposed system for the multi-relay case. We compare the evaluated formulas to the numerical results for different number of cooperating relays and multipath diversity scenarios. We show that the diversity gain of the system will depend mainly on the number of cooperating relays per active source.

1.4 Thesis Outline

The organization of the thesis is as follow: In chapter 2, different DS-CDMA MIMO, and cooperative networks are reviewed, showing the effect of MAI on the system performance, and employing different mitigation methods.

We propose a single-relay DS-CDMA cooperative network over frequency-selective slow fading channels in chapter 3. The performance of the DAF cooperative method is studied for asynchronous communication and using non-orthogonal spreading codes. We analyze mathematically the system's performance, providing an exact formula for the probability of bit error for both cases of perfect and imperfect inter-user channels. We compare the system performance with the numerical results for different number of channel paths.

In chapter 4, we introduce convolutional codes to a multi-relay cooperative network. We study the performance of the system over frequency-selective slow fading channels, and for asynchronous communication. We present an exact formula for the upper bound of the system performance, and compare it with numerical results for different number of cooperating relays and channel paths.

Finally, chapter 5 presents the thesis conclusions and the suggested future works.

Chapter 2

Background

Multiple-access systems have become very popular recently as a result of the rapid development in the area of wireless communications. Frequency-division multiple-access (FDMA), time-division multiple-access (TDMA) and code-division multiple-access (CDMA) are several ways of data multiplexing using orthogonal frequency bands, orthogonal time slots or orthogonal codes respectively [3]. Using DS-CDMA, each user's data is spread using a unique spreading code. Because of the non-orthogonality of the codes, MAI will be present, resulting in degradation on the overall system performance. Interference suppression methods proposed in [10] can be used to mitigate the effects of such MAI.

2.1 MIMO Networks

In this section we will focus on the implementation of MIMO systems using non-orthogonal CDMA codes. We will show the effect of multiple-access interference on the system and an overview on mitigation methods is presented.

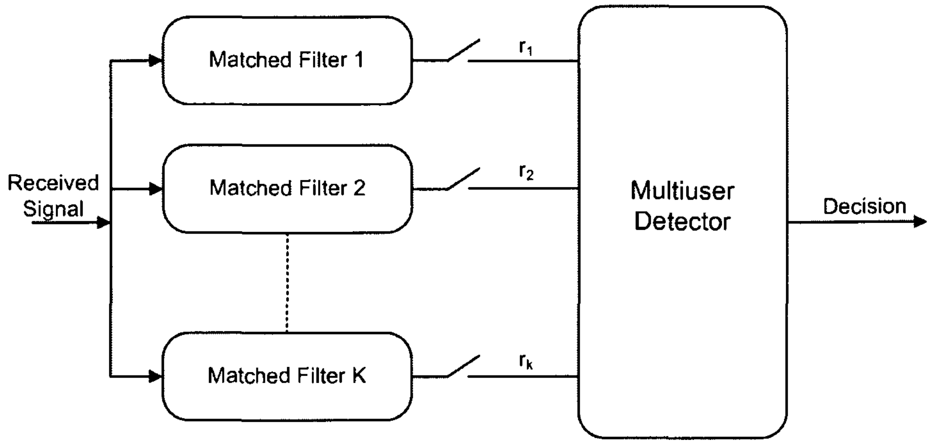

Figure 2.1: Multiple user detection.

2.1.1 Bell-Labs Layered Space-Time (BLAST)

The architecture of the Bell-labs layered space-time was first introduced in [11]. Using multiple antennas, BLAST takes advantage of spatial multiplexing in order to mitigate fading effects and achieve higher data rates. Vertical (V)-BLAST, horizontal (H)-BLAST, and diagonal (D)-BLAST are different types of BLAST.

For MIMO-CDMA networks, Huang *et. al.* [12] studied the down-link communication in a V-BLAST multiple transmit-receive antennas system. They proposed a K-user system with A_r receive antennas at each user side, and base station with A_t transmit antennas indicated by $m = 1, \cdots, A_t$. Each user's high data rate stream is split into G equal rate substreams, which is spread using spreading signatures of length N. In order to achieve transmit diversity, each substream will be transmitted from M_t transmit antennas where $M_t < A_t$. Two examples for the transmission strategies are shown in Fig. 2.2, where each row m represents a transmit antenna and columns represent different spreading code. In Fig. 2.2(a), it is shown that ten substreams G are spread using two different spreading codes, and transmit from five different transmission antennas with each substream sent only once with transmit diversity ($M_t = 1$). While in Fig. 2.2(b), it is shown that the transmit diversity $M_t = 2$, as each substream is sent twice using different spreading codes and from a different transmit antenna.

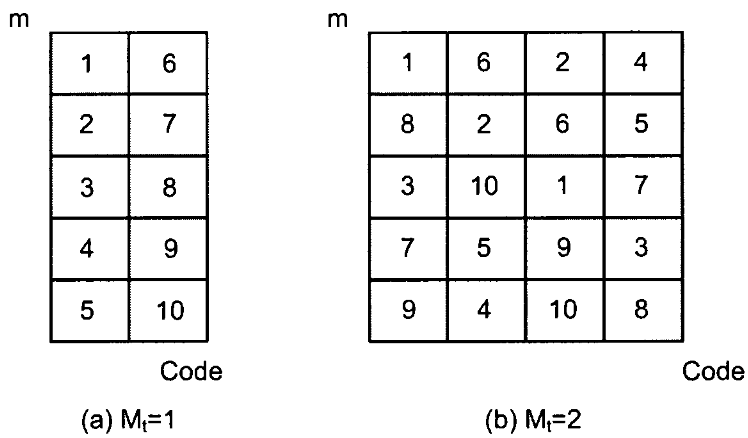

Figure 2.2: Transmit strategy in MIMO networks.

Assuming P multipath channel, the received signal model at antenna a_r in vector format can be expressed as

$$\mathbf{r}_{a_r} = \mathbf{C}\mathbf{H}_{a_r}\mathbf{S} + \mathbf{n}_{a_r}, \tag{2.1}$$

where $\mathbf{C}$ is the $N \times KGM_tP$ spreading code matrix, $\mathbf{H}_{a_r}$ is the channel coefficients matrix with dimensions $KGM_tP \times KG$, $\mathbf{S}$ is the KG data vector, and $\mathbf{n}_{a_r}$ is additive white Gaussian noise (AWGN) vector. The received signal passes through a bank of filters matched to the P multipath replicas of the KG spreading codes. Using the conjugate of the channel coefficients, the output of the matched filters is weighted and summed. Finally, the sufficient statistic vector $\mathbf{y}$ can be written as [12]

$$\begin{aligned}\mathbf{y} &= \sum_{a_r=1}^{A_r} \mathrm{Re}\left\{\mathbf{H}_{a_r}^H \mathbf{C}^T \mathbf{C} \mathbf{H}_{a_r}^H\right\} \mathbf{S} + \sum_{a_r=1}^{A_r} \mathrm{Re}\left\{\mathbf{H}_{a_r}^H \mathbf{C}^T \mathbf{n}_{a_r}\right\} \\ &= \mathbf{RS} + \mathbf{n}, \end{aligned} \tag{2.2}$$

where Re{.} donates the real part, $\mathbf{R}$ is the space-time code correlation matrix with dimensions of $KG \times KG$, and it can be shown as

$$\mathbf{R} = \sum_{a_r=1}^{A_r} \mathrm{Re}\left\{\mathbf{H}_{a_r}^H \mathbf{C}^T \mathbf{C} \mathbf{H}_{a_r}^H\right\}. \tag{2.3}$$

At the end of the detection process, vector $\mathbf{y}$ will pass through a V-BLAST detector to give [12]

$$\mathbf{z} = \left[\left(\mathbf{R}^{-1}\right)_{[1:A_t,1:A_t]}\right]^{-1} \left[\mathbf{R}^{-1}\mathbf{y}\right]_{[1:A_t]}, \tag{2.4}$$

where $[\mathbf{R}^{-1}\mathbf{y}]_{[1:A_t]}$ is the first A_t elements of the vector $\mathbf{R}^{-1}\mathbf{y}$, and $(\mathbf{R}^{-1})_{[1:A_t,1:A_t]}$ is the upper left $A_t \times A_t$ submatrix of $\mathbf{R}^{-1}$.

2.1.2 SINR Enhancement System

In [13], the authors studied the improvement of the signal-to-interference plus noise ratio (SINR) in the down-link of MIMO-CDMA networks over frequency-selective fading channels. As shown in Fig. 2.3, they proposed a cellular network with base station (BS) equipped with M_t transmit antennas, and a mobile station (MS) equipped with M_r receive antennas. At the base station, the transmitted signal passes through a L_t

tap transmit filter with complex response vector $\mathbf{V}_m^{(k)}$, while at the mobile station, the received signal passes through a L_r tap receiver filter with complex response vector $\mathbf{U}_n^{(k)}$.

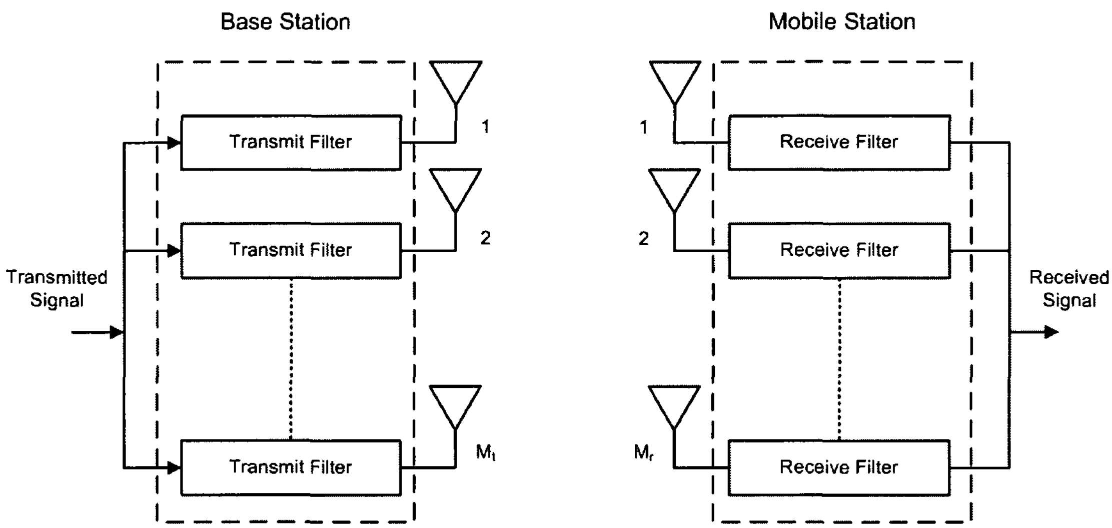

Figure 2.3: MIMO-CDMA antenna system.

Assuming that user k' is the desired user, it was shown in [13] that the decision statistic can be expressed as

$$\mathcal{X}^{(k')} = \mathbf{n}^{(k')} + \mathrm{Re}\left\{ \mathbf{U}^{(k')^H} \sum_{k=1}^{K} \mathbf{R}^{(k',k)} \mathbf{H}^{(k')} \mathbf{V}^{(k')} \right\}, \tag{2.5}$$

where $\mathbf{n}^{(k')}$ is the AWGN vector, $\mathbf{R}^{(k',k)}$ is the cross correlation matrix between user k' and user k, $\mathbf{H}^{(k')}$ is the channel coefficients matrix, $\mathbf{U}^{(k')} = [\mathbf{U}_1^{(k)^T}, \cdots, \mathbf{U}_{M_r}^{(k)^T}]^T$, and $\mathbf{V}^{(k')} = [\mathbf{V}_1^{(k)^T}, \cdots, \mathbf{V}_{M_t}^{(k)^T}]^T$. Equation (2.5) can be simplified and re-written as

$$\mathcal{X}^{(k')} = \mathcal{D}^{(k')} + \mathcal{S}^{(k')} + \mathcal{A}^{(k')} + \eta^{(k')}, \tag{2.6}$$

with $\eta^{(k')}$ representing the noise term, $\mathcal{D}^{(k')}$ the desired signal, $\mathcal{S}^{(k')}$ the self interference due to the multipath channel, and $\mathcal{A}^{(k')}$ is the multiple-access interference. Hence, the signal-to-interference plus noise ratio (SINR) of user k' can be written as

$$\text{SINR}^{(k')} = \frac{\mathcal{D}^{(k')}}{var(\mathcal{S}^{(k')}) + var(\mathcal{A}^{(k')}) + var(\eta^{(k')})}. \tag{2.7}$$

In order to maximize the SINR at the receiver, the transmit vector $\mathbf{V}^{(k)}$ and the receive vector $\mathbf{U}^{(k)}$ must be optimized as follow [13]

$$\left(\mathbf{V}^{(k)}, \mathbf{U}^{(k)}\right)_{opt} = arg_{||\mathbf{V}^{(k)}||=1,||\mathbf{U}^{(k)}||=1} max \left\{\text{SINR}^{(k)}\right\}. \tag{2.8}$$

Moreover, the authors in [13] compared the performance of the proposed system (SINR enhancement system) with the performance of the conventional rake receiver (system with one transmit antenna at the BS and one receive antenna at the MS with a rake receiver using maximum ratio combining (MRC) scheme), and the performance of the smart antenna rake system (system with one transmit antenna at the BS and multiple receive antennas at the MS with rake receiver). As concluded in [13], the SINR enhancement system showed a large improvement in the system performance when compared to the conventional rake system and the smart antenna rake system for different number of transmit/receive antennas.

2.2 Cooperating Networks

An overview on DS-CDMA cooperating networks is presented in this section. We will explore different cooperating network schemes that were introduced previously in literature. Also we will look at various MAI mitigation schemes employed in cooperating networks.

2.2.1 Conventional Detector

For K-user CDMA system, the conventional detector consists of a bank of K filters matched to the users' signatures. The filters are followed by samplers, which are used to sample the output of the filters at the end of every bit duration T_b. As shown in Fig. 2.4, the received signal first passes through the matched filters, the output is then sampled every bit time yielding to a soft estimate of the transmitted data. Finally, a hard decision is taken relying on the signal's sign. One of the main disadvantages of the conventional detector is due to MAI, which has a severe impact on the detector's performance in case of non-orthogonal users' signatures.

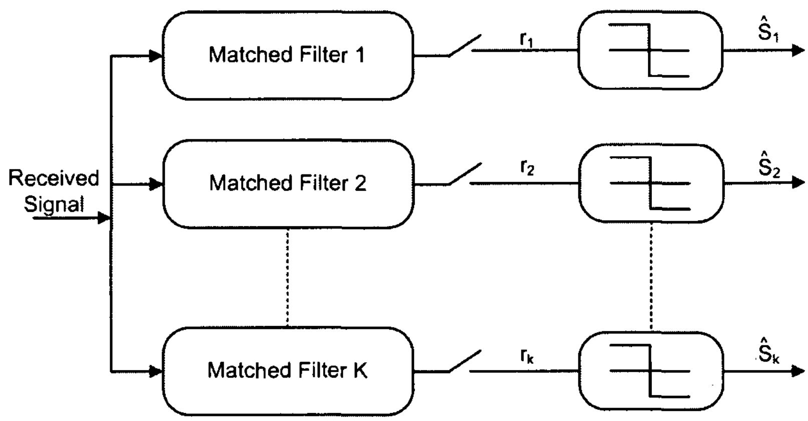

Figure 2.4: Conventional detector structure.

Considering DAF cooperative technique, Sendonaris *et. al.* [14], [15] studied the performance of an orthogonal CDMA implementation of a synchronous cooperating network. In their scheme, each mobile user receives a noisy version of its partner's signal, combines it with its own data and forwards it to the base station as shown in Fig.

2.5. The base station then receives the sum of the signals from both users. Assuming possible echo cancelation, the received signals can be illustrated mathematically as

$$Y_0(t) = h_{10}X_1(t) + h_{20}X_2(t) + n_0(t) \tag{2.9}$$

$$Y_1(t) = h_{21}X_2(t) + n_1(t) \tag{2.10}$$

$$Y_2(t) = h_{12}X_1(t) + n_2(t), \tag{2.11}$$

where $Y_0(t), Y_1(t), Y_2(t)$ represent the received signals at the base station, user 1 and user 2, respectively, X_i is the user's i transmitted data, h_{ij} is the fading channel coefficients (assuming that $h_{ij} = h_{ji}$) and n_i is the additive noise.

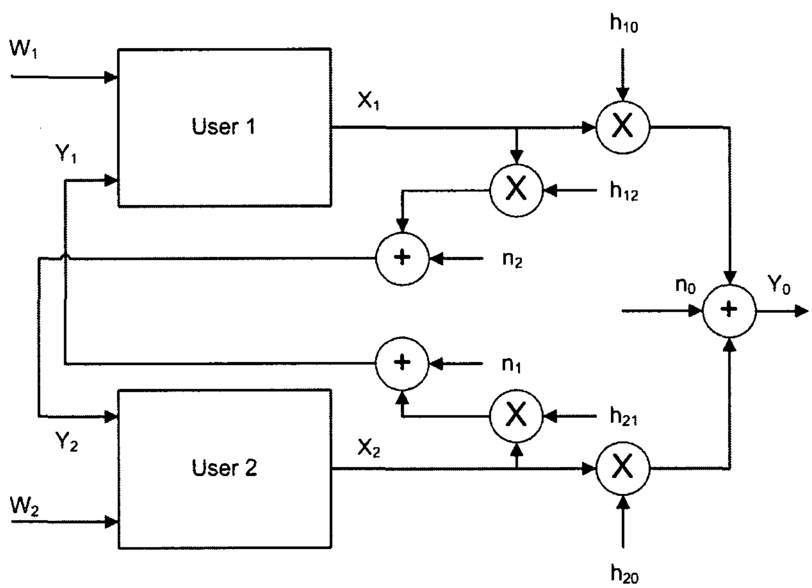

Figure 2.5: Cooperation model.

Using orthogonal spreading codes and assuming the coherence time $L = 3$, the authors in [14] showed that for a non-cooperative case and during three successive periods, the two cooperating users would transmit

$$\begin{aligned} X_1(t) &= a_1 S_1^{(1)} C_1(t), a_1 S_1^{(2)} C_1(t), a_1 S_1^{(3)} C_1(t) \\ X_2(t) &= a_2 S_2^{(1)} C_2(t), a_2 S_2^{(2)} C_2(t), a_2 S_2^{(3)} C_2(t), \end{aligned} \tag{2.12}$$

where $S_j^{(i)}$ is the i^{th} bit of user j, $C_j(t)$ is user's j spreading code, and a_i is the power allocated to each transmitted bit. While in the cooperation scheme, the transmitted signal during the three periods will be

$$\begin{aligned} X_1(t) = a_{11} S_1^{(1)} C_1(t), a_{12} S_1^{(2)} C_1(t), a_{13} S_1^{(2)} C_1(t) + a_{24} \hat{S}_2^{(2)} C_2(t) \\ X_2(t) = a_{21} S_2^{(1)} C_2(t), a_{22} S_2^{(2)} C_2(t), a_{23} \hat{S}_1^{(2)} C_1(t) + a_{24} S_2^{(2)} C_2(t), \end{aligned} \tag{2.13}$$

where $\hat{S}_j^i$ is the estimate of S_j^i, and a_{ji} is the factor that controls the power allocation for the transmitted bits. As one can realize from (2.13), the transmitting users use period one in order to transmit its own data to the base station, while on the second period users send its data to the base station and the cooperating partner. After receiving the partner's data, each user takes a hard decision on the received data and combines it with its own data during the third period to build a cooperative signal.

Comparing data rates of the non-cooperative and cooperative cases, users during the second and third periods transmit the same data bit. This means that each user will be able to transmit two data bits every three transmitting periods, while in the non-cooperative case users will be able to send three new data bits during the three periods. Taking into consideration the performance benefits of the cooperation scheme, the data rate loss can be disregarded.

The case shown by (2.13) can be generalized to L periods, where the two cooperating users will use $2L_c$ periods for cooperation, where the value of L_c varies in the range of $[0, \frac{L}{2}]$. The remaining $L_n = L - 2L_c$ will be used to send a non-cooperative data. This means that for the non-cooperative case $L_c = 0$, while for the fully cooperative case $L_c = \frac{L}{2}$. The $2L_c$ cooperation periods will be organized as follow: odd periods are used to send user's own data to the partner and base station while during the even periods, users send the cooperated signal.

Studying the performance of the cooperation scheme and focusing on user one, it is shown in [15] that during the $L - 2L_c$ non-cooperative periods, user one sends $\mathbf{X}_1 = a_{11}S_1C_1$, which is received at the base station as $\mathbf{Y}_0 = h_{10}\mathbf{X}_1 + h_{20}\mathbf{X}_2 + \mathbf{n}_0$. The estimate of user one's bit at base station is $\hat{S}_1 = \mathrm{sgn}\,(h_{10}a_{11}S_1 + n_0)$, and the probability of bit error can be expressed as [15]

$$P_{e_1} = Q\left(h_{10}a_{11}\frac{\sqrt{N_c}}{\sigma_0}\right), \tag{2.14}$$

where N_c is the spreading gain, and σ_0^2 is the noise variance. During the $2L_c$ cooperative periods, each user will send its own data to its partner and the base station. The signal sent by user one can be expressed as $\mathbf{X}_1 = a_{12}S_1C_1$, which is received at the partner as $\mathbf{Y}_1 = h_{12}\mathbf{X}_1 + \mathbf{n}_1$. By forming a hard decision on the received signal, the probability of bit error at the partner side can be shown as [15]

$$P_{e_{12}} = Q\left(h_{12}a_{12}\frac{\sqrt{N_c}}{\sigma_1}\right). \tag{2.15}$$

The signal received at the base station can be expressed as $\mathbf{Y}_0^{odd} = h_{10}\mathbf{X}_1 + h_{20}\mathbf{X}_2 + \mathbf{n}_0^{odd}$. In contrast, the base station forms a soft estimate of the desired bit which is then combined with the signal received during the even periods to form the final decision statistic. The signal transmitted by the users in the even periods can be shown as

$$\mathbf{X}_1 = a_{13}S_1C_1 + a_{14}\hat{S}_2C_2$$

$$\mathbf{X}_2 = a_{23}\hat{S}_1 C_1 + a_{24} S_2 C_2, \tag{2.16}$$

and the received signal at the base station $\mathbf{Y}_0^{even} = h_{10}\mathbf{X}_1 + h_{20}\mathbf{X}_2 + \mathbf{n}_0^{even}$. Now if we consider the following suboptimal detector,

$$\hat{b}_1 = \text{sgn}([h_{10}a_{12} \quad \lambda(h_{10}a_{13} + h_{20}a_{23})]) \tag{2.17}$$

where λ represents the base station confidence in the decision taken by the partner [15]. In case of perfect inter-user channel ($P_{e_{12}} = 0$), the value of $\lambda = 1$, while as the inter-user channel becomes more unreliable the value of λ will decrease to zero. Using this suboptimal receiver, the probability of bit error is given by [15]

$$P_{e_2} = (1 - P_{e_{12}})Q\left(\frac{v_\lambda^T v_1}{\sqrt{v_\lambda^T v_\lambda}}\right) + P_{e_{12}} Q\left(\frac{v_\lambda^T v_2}{\sqrt{v_\lambda^T v_\lambda}}\right), \tag{2.18}$$

where $v_\lambda = [h_{10}a_{12} \quad \lambda(h_{10}a_{13} + h_{20}a_{23})]^T$, $v_1 = [h_{10}a_{12} \quad (h_{10}a_{13} + h_{20}a_{23})]^T \left(\frac{\sqrt{N_c}}{\sigma_0}\right)$, and $v_2 = [h_{10}a_{12} \quad (h_{10}a_{13} - h_{20}a_{23})]^T \left(\frac{\sqrt{N_c}}{\sigma_0}\right)$.

2.2.2 Decorrelator Detector

In order to overcome the impact of MAI on the conventional detector performance, the inverse of cross correlation matrix $\mathbf{R}^{-1}$ is applied to the output of the matched filters defining the so called decorrelator multiuser detector. The main advantage of the decorrelator detector is the ability to suppress MAI, which provides a significant performance gain over the conventional detector.

In [16], the impact of non-orthogonality and asynchronous communications in flat-fading channels for a CDMA system that employs DAF cooperation technique was studied. In [16] the set $\mathcal{S} = K$ is the set of all users of the network, the set $\mathcal{C} = V$ represents the cooperating users $\mathcal{C} \subset \mathcal{S}$, and set $\mathcal{N} = K - V$ is the set of non-cooperating users. Each user is assigned a non-orthogonal spreading code. During the first time interval, each user k will send its information using its own code. In the second time interval, all the relays in set $\mathcal{C}$ that are capable to decode user k information, will decode and forward the information to the base station asynchronously.

The received signal at the base station from K users with V cooperating users and $V-1$ relays can be expressed as

$$\begin{aligned} r(t) &= \sum_{k=1}^{V} \sum_{l=1_{l\neq k}}^{V-1} \sum_{i=0}^{L-1} S_{l,k}[i] h_l C_k(t - iT_s - \tau_l) \\ &+ \sum_{k=V+1}^{K} \sum_{i=0}^{L-1} S_{k,k}[i] h_k C_k(t - iT_s - \tau_k) + n(t), \end{aligned} \tag{2.19}$$

where L is the block length, T_s is the symbol period, $n(t)$ is the additive Gaussian noise, C_k is the spreading code assigned to user k, h_l is the fading coefficient, and $S_{l,k}[i]$ is the information bit of user k transmitted by relay l. The received signal then passes through bank of filters that are matched to the delayed versions of the spreading codes, the output of the matched filters can be written in vector format as

$$\mathbf{r} = \mathbf{H}\mathbf{R}\mathbf{H}^H\mathbf{S} + \mathbf{n}, \tag{2.20}$$

with $\mathbf{H}$ represents the channel coefficients diagonal matrix, $\mathbf{R}$ is the cross correlation matrix, $\mathbf{S}$ is the data vector, and $\mathbf{n}$ is the noise vector. Applying the received signal $\mathbf{r}$ to a decorrelator detector, yields to

$$\mathbf{y} = (\mathbf{H}\mathbf{R})^{-1}\mathbf{r} + \tilde{\mathbf{n}}, \tag{2.21}$$

where $\tilde{\mathbf{n}}$ is the additive noise vector with zero mean and covariance $N_0\mathbf{R}^{-1}$.

In [16] the outage probability of the system was analyzed, where an exact expression for underloaded CDMA uplink, fully-loaded CDMA uplink, and overloaded

CDMA uplink are obtained. From the major weakness of the decorrelator detector is the noise enhancement caused by applying the inverse of cross correlation $\mathbf{R}^{-1}$ to the noise term as can be seen in (2.21).

2.2.3 Minimum Mean Square Error (MMSE) Detector

The MMSE multiuser detector applies a modified inverse of the cross correlation matrix to the output of the matched filters. Assuming that $\mathbf{Y}$ is the vector representing the output of the matched filters, then the output of the MMSE detector can be written as $\mathbf{Z} = W_{\text{MMSE}}\mathbf{Y}$, where W_{MMSE} is the matrix needed to minimize the MSE between the transmitted data and the output of the detector. It can be shown that [17] $W_{\text{MMSE}} = [\mathbf{R} + \sigma_n^2 \mathbf{A}^{-2}]^{-1}$, where $\mathbf{R}$ is the cross correlation matrix, σ_n^2 is the AWGN variance, and $\mathbf{A}$ is the signal amplitude matrix. In addition to the interference cancelation, the MMSE detector takes the background noise into consideration, which gives it the advantage to deliver a better performance compared to the decorrelator detector.

Employing non-orthogonal spreading codes, Venturino *et. al.* [1] investigated the performance of cooperative networks based on synchronous DS-CDMA, and using MMSE detector. In their work, they considered both DAF and AAF in cases of perfectly and partially known channel conditions. All channels are assumed to be frequency non-selective, also reciprocity of the channel is assumed. The authors assumed the cooperation takes place in two transmission phases. First, during the odd time intervals each user transmits its own data to its partner and to the base station. Then, each partner forms an estimate for the received signal and starts forwarding it to the base station in the even intervals.

Assume K-user network, where $\mathcal{F} = (1, 2, \cdots, V)$ is the subset of users that are welling to cooperate. Let $[k, f(k)] \epsilon \mathcal{F}$ be two cooperating partners, the received signal during the odd time interval at user $v \epsilon \mathcal{F}$ can be expressed as

$$\mathbf{r}_v(2i-1) = \sum_{k=1, k\neq v}^{V} \sqrt{(2-\rho_k)} h_{k,v} C_k S_k(i) + \sum_{k=V+1}^{K} h_{k,v} C_k x_k(2i-1) + \mathbf{n}_v(2i-1), \quad (2.22)$$

where C_k represents the signature of user k received at user v over the fading channel $h_{k,v}$ and with processing gain N, $S_k(i)$ is the symbol sent by cooperating user k, while $x_k(i)$ is the symbol of the non-cooperative users, and $\rho_k \epsilon [0, 2]$ is the factor that controls the power allocation during the two cooperating time intervals, $\mathbf{n}_v(2i-1)$ is the AWGN vector. The received signal at the base station during the even time intervals can be represented as

$$r_0(2i) = \sum_{k=1}^{V} \sqrt{\rho_{f(k)}} h_{f(k),0} C_{f(k)} \tilde{S}_k(i) + \sum_{k=V+1}^{K} h_{k,0} C_k x_k(2i) + \mathbf{n}_0(2i), \quad (2.23)$$

where $\mathbf{n}_0(2i)$ is the AWGN vector, $\tilde{S}_k(i)$ is the estimate of $S_k(i)$. For the DAF technique, the estimate taken by the partner is a hard estimate, on the contrary a soft estimate is taken when using AAF technique.

Studying the performance at the relay side, assuming that $f(q)$ is the terminal of interest and using DAF technique, the output of the MMSE receiver can be written as [1]

$$\begin{aligned} \mathbf{m}_{q,f(q)} &= \mathbf{C}_{f(q)}^{-1} \mathbf{s}_{q,f(q)} \\ &= \sum_{k=1, k\neq f(q)}^{K} \mathbf{s}_{k,f(q)} \mathbf{s}_{k,f(q)}^{H} + \mathbf{n}_{f(q)} \mathbf{I}_N \end{aligned} \quad (2.24)$$

with $\mathbf{C}_{f(q)} = E[\mathbf{r}_{f(q)}(2i-1)\mathbf{r}_{f(q)}(2i-1)^H]$, $\mathbf{I}_N$ is $N \times N$ identity matrix, $\mathbf{s}_{k,f(q)} =$

$h_{k,f(q)}\mathbf{C}_k$, and the decoded symbol to be relayed can be expressed as

$$\tilde{S}_k(i) = (\mathrm{sgn}[\mathrm{Re}\{\xi\}] + j\mathrm{sgn}[\mathrm{Im}\{\xi\}])/\sqrt{2}, \quad i = 1, \cdots, L \tag{2.25}$$

where $\xi = \mathbf{m}_{q,f(q)}^H \mathbf{r}_{f(q)}(2i-1)$. The probability of error conditioned on the fading channel, h, is equivalent to [1]

$$P_{q,f(q)}\left(h_{f(q)}\right) = Q\left(\sqrt{\mathrm{SINR}_{q,f(q)}(h_{f(q)})}\right), \tag{2.26}$$

where

$$\begin{aligned} \mathrm{SINR}_{q,f(q)}(h_{f(q)}) &= \frac{E[\mathrm{Re}\{\xi\}|\mathrm{Re}\{S_q(i)\}]^2}{\mathrm{Var}[\mathrm{Re}\{\xi\}|\mathrm{Re}\{S_q(i)\}]} \\ &= \frac{\frac{1}{2}\mathrm{Re}\left[\mathbf{m}_{q,f(q)}^H \mathbf{s}_{q,f(q)}\right]^2}{\frac{1}{2}\sum_{k=1,k\neq q,f(q)}^{K} \left|\mathbf{m}_{q,f(q)}^H \mathbf{s}_{k,f(q)}\right|^2 + \frac{\mathcal{N}_{f(q)}}{2}||\mathbf{m}_{q,f(q)}||^2}. \end{aligned} \tag{2.27}$$

For the AAF technique, the soft estimate formed by the partner and forwarded to the base station on the even time intervals can be expressed as

$$\begin{aligned} \tilde{S}_q(i) &= \frac{\mathbf{m}_{q,f(q)}^H \mathbf{r}_{f(q)}(2i-1)}{G_{f(q)}} = \underbrace{\psi_{q,f(q)} S_q(i)}_{Useful\ Signal} \\ &+ \underbrace{\sum_{k=1,k\neq q,f(q)}^{V} \psi_{k,f(q)} S_k(i)}_{Residual\ Interference} + \underbrace{\sum_{k=V+1}^{K} \psi_{k,f(q)} x_k(2i-1) + v_{f(q)}(2i-1)}_{Filtered\ Noise} \end{aligned} \tag{2.28}$$

where $\psi_{k,f(q)} = \frac{\mathbf{m}_{q,f(q)}^H \mathbf{s}_{k,f(q)}}{G_{f(q)}}$, $v_{f(q)}(2i-1) = \frac{\mathbf{m}_{q,f(q)}^H \mathbf{n}_{f(q)}(2i-1)}{G_{f(q)}}$, and $G_{f(q)}$ is a power normalization factor.

At the end of the partner transmission, the base station combines both signals received during the two transmission intervals. Using DAF technique, the combined received signal will be equivalent to

$$\mathbf{r}(i) = \sum_{k=1}^{V} \mathbf{H}_k \mathbf{b}_k(i) + \sum_{k=V+1}^{K} \mathbf{h}_k x_k(2i) + \mathbf{n}(i) \tag{2.29}$$

where $\mathbf{H}_k = [\mathbf{h}_k^{(1)}, \mathbf{h}_{f(k)}^{(2)}]$, $\mathbf{h}_k^{(1)} = [\mathbf{s}_{k,0}, \mathbf{0}_N]^T$, $\mathbf{h}_k^{(2)} = [\mathbf{0}_N, \mathbf{s}_{k,0}]^T$, $\mathbf{S}_k(i) = [S_k(i), \tilde{S}_k(i)]^T$, and assuming that $x_k(2i) = x_k(2i-1)$. It can be shown that the decision statistic is equivalent to [1]

$$\begin{aligned}\xi = \mathbf{m}_q^H \mathbf{r}(i) &= \mathbf{m}_q^H \left[\mathbf{h}_q^{(1)} + \delta_q(i)\mathbf{h}_{f(q)}^{(2)}\right] S_q(i) \\ &+ \sum_{k=1,k\neq q}^{V} \mathbf{m}_q^H \mathbf{H}_k \mathbf{S}_k(i) + \sum_{k=V+1}^{K} \mathbf{m}_q^H \mathbf{h}_k \mathbf{x}_k(2i) + \mathbf{m}_q^H \mathbf{n}(i), \quad (2.30)\end{aligned}$$

where $\delta_q(i)$ is a random variable. Assuming that the interference is Gaussian distributed, the conditional bit-error rate (BER) can be expressed as [1]

$$P_q(h) = \sum_{\lambda \epsilon \{\pm 1, \pm j\}} P_r\{\delta_q(i) = \lambda\} Q\left(\sqrt{\text{SINR}_q(h,\lambda)}\right) \quad (2.31)$$

with

$$\text{SINR}_q(h,\lambda) = \frac{\frac{1}{2}\text{Re}\left[\mathbf{m}_q^H\left(\mathbf{h}_q^{(1)} + \lambda \mathbf{h}_{f(q)}^{(2)}\right)\right]^2}{\frac{1}{2}\text{Im}\left[\mathbf{m}_q^H\left(\mathbf{h}_q^{(1)} + \lambda \mathbf{h}_{f(q)}^{(2)}\right)\right]^2 + \phi + ||\mathbf{m}_q||^2 \frac{\mathcal{N}}{2}}, \quad (2.32)$$

and

$$\begin{aligned} P_r\{\delta_q(i) = 1\} &= 1 - \left\{2P_{q,f(q)}\left(h_{f(q)}\right) - \left[P_{q,f(q)}\left(h_{f(q)}\right)\right]^2\right\} \\ P_r\{\delta_q(i) = -1\} &= \left[P_{q,f(q)}\left(h_{f(q)}\right)\right]^2 \\ P_r\{\delta_q(i) = \pm j\} &= P_{q,f(q)}\left(h_{f(q)}\right)\left[1 - P_{q,f(q)}\left(h_{f(q)}\right)\right], \quad (2.33)\end{aligned}$$

where $\phi = E[\text{Re}\{z_q(i)\}\text{Re}\{z_q(i)\}]$. Using Monte Carlo method, the probability of error can be evaluated by averaging (2.31) over realizations of the fading channel.

Following the same procedure, the combined received signal at the base station when using AAF cooperating technique will be equivalent to

$$\begin{aligned}\mathbf{r}(i) &= \sum_{k=1}^{V} \bar{\mathbf{h}}_k S_k(i) + \sum_{k=V+1}^{K} \bar{\mathbf{h}}_k x_k(2i-1) + \sum_{k=V+1}^{K} \mathbf{h}_k^{(2)} x_k(2i) + \mathbf{n}(i) \\ &= \sum_{k=1}^{V} \bar{\mathbf{h}}_k S_k(i) + \sum_{k=P+1}^{K} \bar{\mathbf{u}}_k x_k(2i) + \bar{\mathbf{n}}(i) \quad (2.34)\end{aligned}$$

where $\bar{\mathbf{h}}_k = [\mathbf{s}_{k,0}, \bar{\mathbf{s}}_{k,0}]^T$, $\bar{\mathbf{u}}_k = [\mathbf{s}_{k,0}, \bar{\mathbf{s}}_{k,0} + \mathbf{s}_{k,0}]^T$. The conditional probability of error can be expressed as [1]

$$P_q(h) = Q\left(\sqrt{\mathrm{SINR}_q(h)}\right) \tag{2.35}$$

where

$$\begin{aligned} \mathrm{SINR}_q(h) &= \frac{E[\mathrm{Re}\{\xi\}|\mathrm{Re}\{S_q(i)\}]^2}{\mathrm{Var}[\mathrm{Re}\{\xi\}|\mathrm{Re}\{S_q(i)\}]} \\ &= \frac{\frac{1}{2}\mathrm{Re}\{\mathbf{m}_q^H\bar{\mathbf{h}}_q\}^2}{\frac{1}{2}\mathrm{Im}\{\mathbf{m}_q^H\bar{\mathbf{h}}_q\}^2 + \frac{1}{2}\sum_{k=1,k\neq q}^{V}|\mathbf{m}_q^H\bar{\mathbf{h}}_k|^2 + \frac{1}{2}\sum_{k=V+1}^{K}|\mathbf{m}_q^H\bar{\mathbf{u}}_k|^2 + \frac{1}{2}\mathbf{m}_q^H\mathbf{R}_{\bar{n}}\mathbf{m}_q}, \end{aligned} \tag{2.36}$$

and $\mathbf{R}_{\bar{n}} = E\left[\bar{\mathbf{n}}(i)\bar{\mathbf{n}}(i)^H\right]$. Finally, the BER can be obtained by averaging (2.36) over realizations of the fading channel using Monte Carlo method.

In [1], the authors studied the impact of multiuser interference (MUI) on the system performance, where they compared the performance of the system using matched filter receivers at the relay side with the performance when the relays employ a linear MMSE receiver. Fig. 2.6 shows the results for both DAF and AAF cooperating techniques, comparing it with the non-cooperating network performance (NC network), and the ideal case of perfect inter-user channels (IC network).

Moreover, the authors in [1] compared the performance of the DAF cooperating technique for both optimum (the base station has the knowledge of the BER in the inter-user channels) and sub-optimum (the base station assumes that the relay's estimate is always correct) combining methods. Their results show that in case of optimum combining technique the system performance converges to the IC network performance.

In [18], a modified minimum mean-square-error (MMSE) detector was introduced for CDMA cooperative networks in synchronous flat-fading channels. A system with

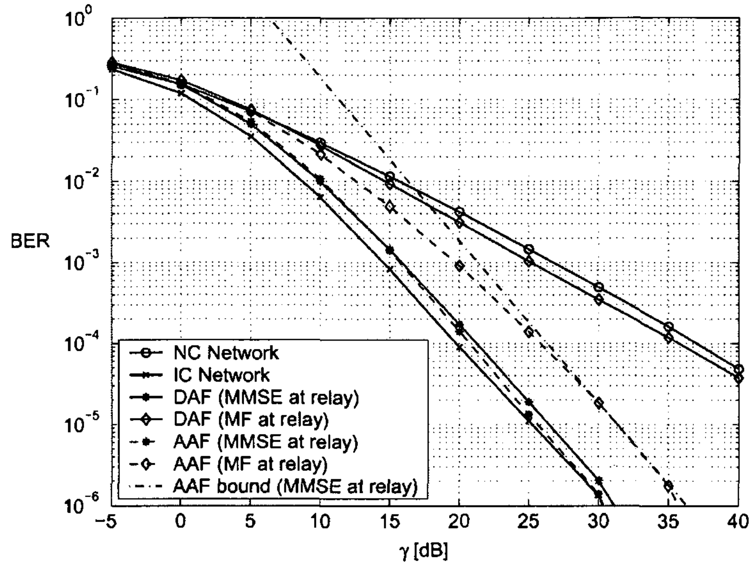

Figure 2.6: Impact of MUI on the performance of cooperating networks [1].

K-mobile nodes $(U_1, U_2, \cdots, U_K)$ and a base station U_d is proposed. The K users are split into two groups G_T and G_R, such that each node in the G_T group will be assigned a cooperating partner from the G_R group.

The received signals at the relay and the base station during the first and second transmission times are expressed in vector format as

$$\begin{aligned} \mathbf{Y}_2[1] &= \mathbf{R}\mathbf{A}_{d1}\mathbf{H}_2\mathbf{S}_1 + \mathbf{N}_2[1] \\ \mathbf{Y}_d[1] &= \mathbf{R}\mathbf{A}_{d1}\mathbf{H}_{d1}\mathbf{S}_1 + \mathbf{N}_d[1] \\ \mathbf{Y}_d[2] &= \mathbf{R}\mathbf{A}_{d2}\mathbf{H}_{d2}\mathbf{S}_2 + \mathbf{N}_d[2], \end{aligned} \tag{2.37}$$

where $\mathbf{R}$ is the cross correlation matrix, $[\mathbf{H_2}, \mathbf{H_{d1}}, \mathbf{H_{d2}}]$ are the $K \times K$ fading channel matrices, $[\mathbf{N}_2, \mathbf{N}_d]$ are the zero mean noise vectors, and $\mathbf{S}_1, \mathbf{S}_2$ represent the data vectors and are equivalent to

$$\begin{aligned} \mathbf{S}_1 &= \left[S_1[1], \hat{S}_2[0], S_3[1], \hat{S}_4[0], \cdots, S_{K-1}[1], \hat{S}_K[0]\right]^T \\ \mathbf{S}_2 &= \left[\hat{S}_1[1], S_2[0], \hat{S}_3[1], S_4[2], \cdots, \hat{S}_{K-1}[1], S_K[2]\right]^T, \end{aligned} \tag{2.38}$$

with S_k as the k^{th} user transmitted bit. The $[\mathbf{A}_{d1}, \mathbf{A}_{d2}]$ are $K \times K$ diagonal matrices and can be shown as

$$(\mathbf{A}_{d1})_{kk} = (\mathbf{A}_{d2})_{k+1,k+1} = \sqrt{\frac{P_{k,d}}{T_s}}, \tag{2.39}$$

where $P_{k,d}$ is the transmission power, and T_s is the symbol period. The performance of cooperating network using MMSE detector is shown in [18]. The results show that as the inter-user channel becomes more reliable, the system performance improves and more power should be allocated for cooperation.

2.2.4 Relay-Assisted Decorrelating Multiuser Detector (RAD-MUD)

In order to mitigate the effect of interference, a relay-assisted decorrelating multiuser detector for three relay cooperating methods was proposed in [2]:

- Transmit beamforming: It is based on beamforming coefficients that will be assigned for each transmitting source data. Those coefficients will arrange the power allocated to the cooperated signals. It can be implemented for the relay-destination (R-D) link or the source-relay-destination (S-R-D) link. This method can be used when full channel state information (CSI) is available.

- Selective relaying: When the relays have only the channel knowledge for its local (S-R) and (R-D) links, it can use threshold selection strategy. In this case, the received signal power is compared to a threshold and based on that the relay is selected. In case of global CSI availability, best selection strategy can be used.

- Distributed space-time coding (DSTC): In this method, space time codes can be generated at the relay side by multiplying the symbol period by a randomly generated unitary matrix. This method is used in the case of missing knowledge of the CSI at the relays.

A synchronous CDMA network with K sources $(s_1, \cdots, s_K)$ and V relays $(r_1, \cdots, r_V)$ is shown in Fig. 2.7. Each relay is considered to be able to cooperate with multi-sources simultaneously, where the relay decodes the received signals and re-transmits it to the destination.

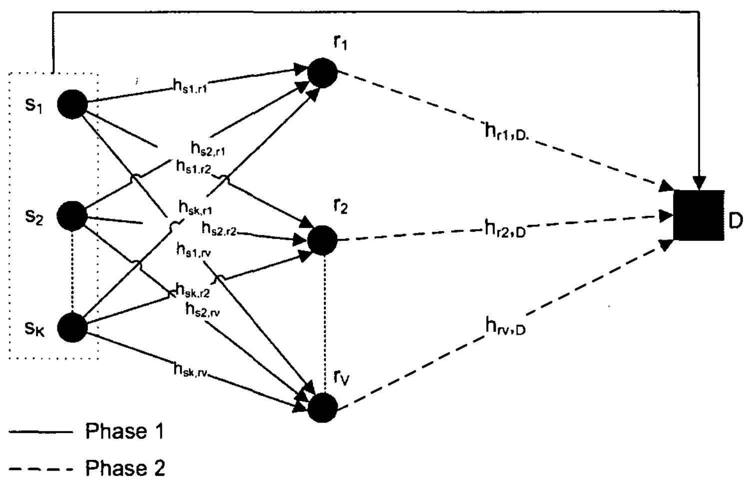

Figure 2.7: K users, V relays cooperative CDMA network.

For the proposed system, the received signal at the destination during the first transmission phase can be expressed as

$$y_1(t) = \sum_{l=1}^{L}\sum_{k=1}^{K} h_{s_kD}\sqrt{P_{s_k}}S_k[l]C_k(t - lT_s) + n_I(t), \tag{2.40}$$

where L is the data block length, T_s is the symbol period, P_{s_k} is the bit transmission power, h_{s_kD} is the fading channel coefficient between user s_k and destination D, $n_I(t)$ is the AWGN, S_k is user's k data, and C_k is user's k spreading signature. The output of the matched filter at the destination for the l^{th} period of user k during the first transmission phase can be expressed in vector format as

$$\mathbf{y}_I[l] = \mathbf{R}\mathbf{H}_{SD}\mathbf{S}[l] + \mathbf{n}_I[l], \tag{2.41}$$

where $\mathbf{H}_{SD}$ is the uplink channel matrix, $\mathbf{R}$ is the cross correlation matrix of the spreading signatures, and $\mathbf{S}$ representing the data vector. While the output of the MF at the relay v during the l period can be shown as

$$\mathbf{u}_v[l] = \mathbf{R}\mathbf{H}_{SR_v}\mathbf{S}[l] + \mathbf{n}_v[l], \tag{2.42}$$

with $\mathbf{H}_{SR_v}$ as the inter-user fading channel matrix, and $\mathbf{n}_v$ is the AWGN vector. During the second transmission phase, the V relays start transmitting the cooperated signals to the destination. In order to implement the proposed RAD-MUD scheme, and depending on the cooperating technique used, the relay r_v first starts to encode the received data matrix $\hat{\mathbf{S}}_v$ into another matrix $g_v(\hat{\mathbf{S}}_v)$, which will be multiplied by the matrix $\mathbf{L}^{-H}$, where $\mathbf{L}$ is the Cholesky decomposition of the cross correlation matrix $\mathbf{R} = \mathbf{L}\mathbf{L}^H$. The transmitted signal by relay r_v can be written as $\mathbf{T}_v = f(\hat{\mathbf{S}}_v) = \mathbf{L}^{-H}g_v(\hat{\mathbf{S}}_v)$, and the output of the matched filter at the destination during the second transmission phase can be expressed as

$$\mathbf{y}_{II} = \sum_{v=1}^{V} h_{R_v D} \mathbf{R} \mathbf{L}^{-H} g_v(\hat{\mathbf{S}}_v) + \mathbf{N}_{II}. \tag{2.43}$$

Multiplying (2.43) by $\mathbf{L}^{-1}$ yields to

$$\check{\mathbf{y}}_{II} = \sum_{v=1}^{V} h_{R_v D} g_v(\hat{\mathbf{S}}_v) + \check{\mathbf{N}}_{II}. \tag{2.44}$$

where $\check{\mathbf{N}}_{II} = \mathbf{L}^{-1}\mathbf{N}_{II}$ is the additive noise term, with covariance matrix equals to $\mathbf{E}[\check{\mathbf{N}}_{II}[k]\check{\mathbf{N}}_{II}[k]^H] = \sigma_n^2 \mathbf{I}_{K\times K}$. Thus $\mathbf{L}^{-1}$ can be considered as a whitening filter. Finally, the MMSE combining technique will be used to combine the received signals during the two transmission phases at the destination side.

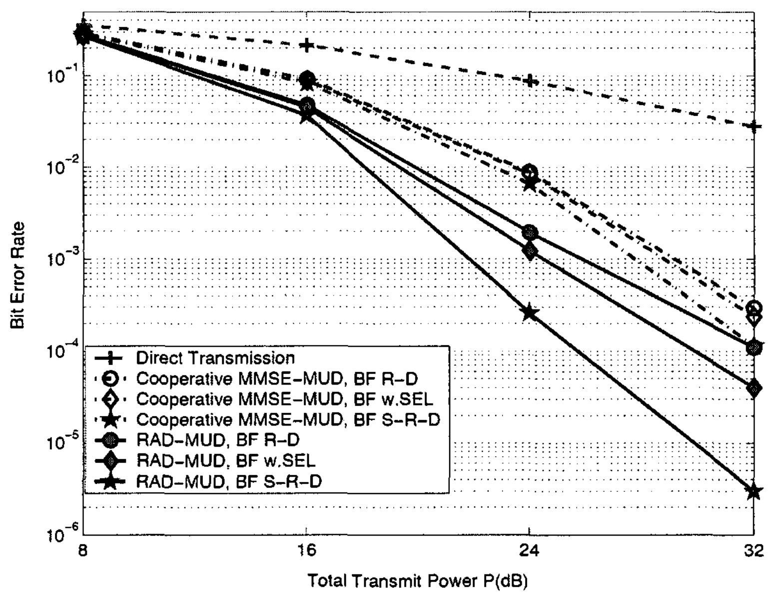

Figure 2.8: Performance of MMSE and RAD-MUD in cooperative networks [2].

Fig. 2.8 shows the performance of the RAD-MUD technique compared to the MMSE-MUD for different cooperation schemes. The results show the improvement in the system performance when the RAD-MUD technique is introduced.

2.2.5 Successive Interference Cancelation (SIC) Detector

The SIC technique is based on removing the MAI from the received signal by subtracting users' interference one at a time. The user's data with the strongest signal is demodulated and detected. The detected data is then subtracted from the original received signal, removing the interference caused by this user.

In [19] a successive interference cancelation for synchronous CDMA cooperative networks is proposed. It is assumed that users form a cooperating user pairs $(\{1,2\},\cdots,\{K-1,K\})$. In the proposed system, the authors show that the received signal at the relay v during the first time instant can be expressed as

$$r_v(t) = \sqrt{P_k}h_{kv}(t)S_k(t)C_k(t) + n_v(t), \tag{2.45}$$

where $h_{kv}(t)$ is the fading channel between the pair $\{v,k\}$, P_k is the signal power of user k, $S_k(t)$ is the transmitted user's k data, $C_k(t)$ is user k spreading code, and $n_v(t)$ is the additive noise. While the received signal at the base station during the first and second time intervals can be shown respectively as,

$$\begin{aligned} r_d(t) &= \sum_{k=1}^{K} \sqrt{P_k}h_{kd}(t)S_kC_k(t) + n(t) \\ r'_d(t) &= \sum_{k=1}^{K} \sqrt{P_v}h_{vd}(t)S'_k(t)C_k(t) + n(t), \end{aligned} \tag{2.46}$$

where $S'_k(t)$ is the estimate of bit $S_k(t)$. The proposed SIC technique is then applied to the received signal in the first time interval, where the user's data with the strongest signal is estimated and its MAI is canceled from the actual received signal. The output of the matched filter for the user with the maximum signal power can be written as

$$z_{\text{max}} = \max \left\{ \frac{1}{T_b} \int_0^{T_b} \mathbf{r}_d \mathbf{C}_k \right\}. \tag{2.47}$$

During the second time interval, the same user's estimate is extracted from the received signal $r'_d(t)$, and the output of the matched filter can be expressed as

$$z'_{\text{max}} = \frac{1}{T_b} \int_0^{T_b} \mathbf{r}'_d \mathbf{C}_{\text{max}}. \tag{2.48}$$

The two estimated data during both time intervals are then combined and a final hard decision is taken. Finally, the MAI for the detected user is then subtracted from the original received signal as follow

$$\begin{aligned} \mathbf{r}_d &= \mathbf{r}_d - z_{\text{max}} \mathbf{C}_k \\ \mathbf{r}'_d &= \mathbf{r}'_d - z'_{\text{max}} \mathbf{C}_k. \end{aligned} \tag{2.49}$$

In [19], the performance comparison for the SIC receiver and the conventional matched filter (MF) detector for different inter-user channels SNR was shown. It was shown that the performance gap between both receivers gets wider as the inter-user channel SNR increases.

2.3 Summary

In this chapter, we presented an overview of both CDMA-MIMO and CDMA cooperating networks. We investigated the effect of multi-access interference on the system performance, and we showed the extreme impact of the interference on the system.

On the other hand, we reviewed different MAI mitigation techniques presented previously in the literature. For MIMO networks, we explored the SINR enhancement system by studying the receive-transmit filters.

Moreover, we presented different MAI mitigation methods for cooperating networks. The performance of the decorrelator detector, MMSE, the relay-assisted decorrelating multiuser detection, and successive interference cancelation was explored and compared to the performance of the conventional detector.

Chapter 3

Uncoded Single Relay Cooperation

In the previous works, related to CDMA systems, the performance of the cooperative network was either accompanied with the assumption of orthogonal subchannels (e.g., [14], [20]) or considering synchronous communications (e.g., [1], [2]). The impact of the non-orthogonality and asynchronous communication was studied in [16] for CDMA cooperative network but over flat fading channels. In this chapter, and different from previous works, we study the impact of MAI on the performance of asynchronous DS-CDMA cooperative networks over the more realistic case of frequency-selective fading channels. We show that the full benefits of cooperative diversity cannot be achieved if no multiuser interference suppression is employed for the inter-user channel (source-relay). Our results show that in the presence of interference, the performance of the cooperative network is greatly affected resulting in a performance worse than the non-cooperative case. To this end, we analyze the performance of a system that employs interference suppression at both the cooperative user side and base station. In that, we consider two scenarios; (i) perfect inter-user channel (source-relay) where the relay (also referred as partner) correctly decodes the received user signal. This case is important since it serves as the optimal performance achieved using cooperation, where the distributed MIMO system reaches the performance of that of the centralized one (ii) imperfect inter-user channel where we assume that each relay has an error detection capability (e.g., cyclic redundancy check (CRC)) to decide weather cooperation can take place or not. In case of error, the cooperating user keeps silent (no cooperation). The receiver under consideration employs RAKE combining after interference suppression. The RAKE receiver exploits the path diversity inherent to multipath propagation where the decorrelator detector is used at both the relay side and base station to mitigate the effect of MAI and the known near-far problem [21]. The communication protocol considered is full-duplex where all users can transmit their data and act as relays for other users at the same time [1].

The rest of this chapter is organized as follows: The asynchronous DS-CDMA system model used in this chapter is described in section 3.2. In section 3.3, the system performance for a DAF scheme is examined for the case of a multiuser system where we obtain the probability density function (pdf) of the signal-to noise ratio at the receiver output and then derive the probability of bit error. The performance of DAF with imperfect inter-user channel is also analyzed in this section. In section 3.4, both analytical and simulation results are compared and discussed. Finally, conclusions are given in section 3.5.

3.1 Introduction

The fourth generation (4G) of wireless networks has been recently one of the most widely studied topic in literature [22], [23], [24]. Inter-network cooperation, cooperation at the application layer, cooperation at the IP layer and cooperation at the physical layer are all different cooperation techniques in the 4G world that was introduced by the authors in [25].

For a synchronous cooperative CDMA network, the authors in [2] considered a different cooperation technique where each relay can cooperate with multiple active users over flat fading channels. The opportunistic relaying (OR) and the selection cooperation (SC) are two relay selection methods studied in [26]. Closed form expressions for the outage probability and probability of error were provided for the uncoded single relay cooperation case. Furthermore, the impact of asynchronous communication on cooperative networks in frequency non-selective fading channels was studied in [27].

Other cooperative techniques including coded cooperation, where the cooperative diversity integrates with channel coding, have been considered in [28], [29]. A scenario where multiple transmitting sources and destinations using MMSE communicate simultaneously through number of relays was proposed in [30]. In [31], a cooperative wireless network using MIMO antenna selection algorithm was proposed, where transmitting users and relays employ multiple antennas. Using partial DAF strategy introduced in [32], the authors in [33] investigated the problem of relay selection in a multi-relay environment and studied the impact of the relay transmission power on the performance of the network.

3.2 System Model

3.2.1 Proposed Diversity Scheme

Consider an uplink transmission for an asynchronous K-user DS-CDMA system. The system employs one transmit antenna at the transmitter side and one receive antenna at the receiver side. In what follows, to simplify the notation, we refer to the base station with subscript b.

The transmission scheme considered is described in Fig. 3.1, 3.2, for a 2-user system [1]. As shown, during the first transmission period each user will send its own data to the base station and to its partner (Fig. 3.1), while during the second transmission period the cooperating user transmits the decoded version of its partner to the base station (Fig. 3.2).

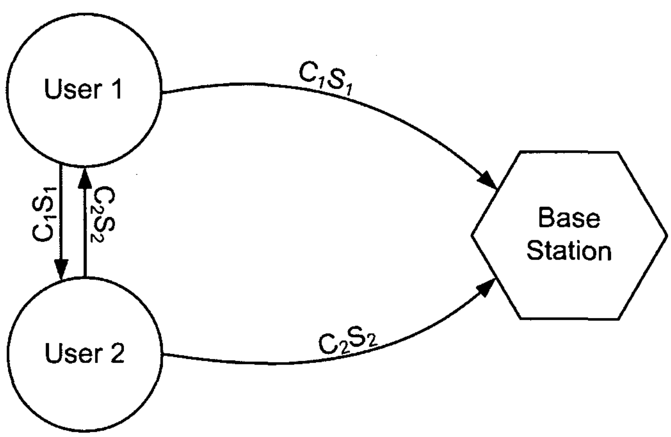

Figure 3.1: First transmission period (single relay).

Consider a multipath channel with P paths during each transmission period. For

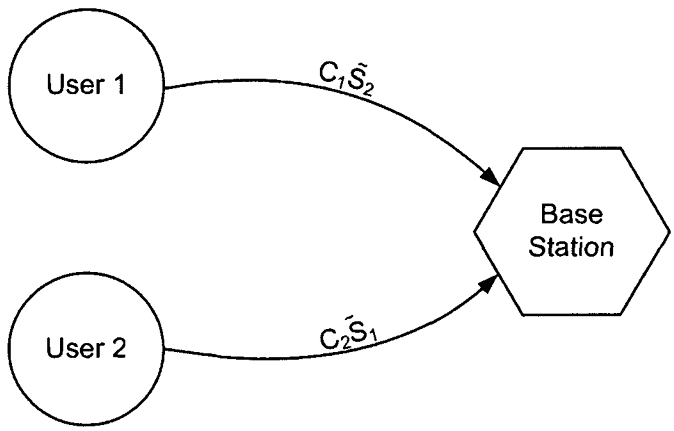

Figure 3.2: Second transmission period (single relay).

the DAF, the low pass equivalent of the received signal at the base station and relay (r) during the first transmission period can be expressed respectively as

$$r_{b1}(t) = \sum_{m=0}^{L-1}\sum_{k=1}^{K}\sum_{p=1}^{P}\sqrt{E_{U_k}}S_k(m)C_k(t-\tau_k-\tau_{k,p}-mT_b)h_{kb}^{p}(m)+n_{b1}(t) \tag{3.1}$$

$$r_r(t) = \sum_{m=0}^{L-1}\sum_{k=1_{k\neq r}}^{K}\sum_{p=1}^{P}\sqrt{E_{I_k}}S_k(m)C_k(t-\tau_k-\tau_{k,p}-mT_b)h_{kr}^{p}(m)+n_r(t) \tag{3.2}$$

where L is the frame size, E_{U_k} and E_{I_k} are the received signal energies per path of user k at the base station (uplink channel) and the relay (inter-user channel) respectively, $S_k(m)$ is the m^{th} data bit of user k, $C_k(t)$ is the spreading code assigned to the k^{th} user with processing gain (T_b/T_c), where T_b is the bit period and T_c is the chip period, and τ_k is the random transmit delay of the k^{th} user which is assumed uniformly distributed along the symbol period. The parameter $\tau_{k,p}$ represents the delay of the p^{th} path of user k during one transmission period. The channel coefficient h_{kr}^{p} models the fading of the inter-user channel between users k and r over the p^{th} path, while h_{kb}^{p} represents the fading coefficients for the uplink channel between user k and the base station

b over the path p. These fading coefficients are modeled as independent Gaussian random variables with zero mean and unit variance. The noise $n_{b1}(t)$ and $n_r(t)$ are complex Gaussian distributed each with zero mean and variance $\sigma_n^2 = N_o/2$.

During the second transmission period, each two users k and $f(k)$ will cooperate together in which each cooperating user retransmits the received signal. Each cooperating user (relay) first decodes the partner's received signal, then using error checking techniques it decides whether or not to forward the estimated partner's data to the base station during the second transmission period. The low pass equivalent of the received signal at the base station during the second transmission period can then be expressed as

$$r_{b2}(t) = \sum_{m=0}^{L-1}\sum_{k=1}^{K}\sum_{p=1}^{P}\sqrt{E_{U_{f(k)}}}\tilde{S}_k(m)C_{f(k)}(t-\tau_{f(k)}-D_k-\tau_{f(k),p}-mT_b)h^p_{f(k)b}(m)+n_{b2}(t) \tag{3.3}$$

where $f(k)$ is the user cooperating with user k, $\tilde{S}_k(m)$ is the estimate of $S_k(m)$, the noise $n_{b2}(t)$ is real Gaussian with zero mean and variance $\sigma_n^2 = N_o/2$, D_k is the transmission delay during the second transmission period as shown in the timing diagram in Fig. 3.3, where user k will wait a time delay D_k for its partner $f(k)$ to finish transmission, and it is equivalent to

$$D_k = \begin{cases} |\tau_k - \tau_{f(k)}| & for\; user\; k \\ 0 & for\; user\; f(k) \end{cases}.$$

3.2.2 Multiuser Detector

In a K-user system, where each user signal is transmitted over a multipath channel with P-resolvable paths, a RAKE receiver is employed at the front-end of both relay

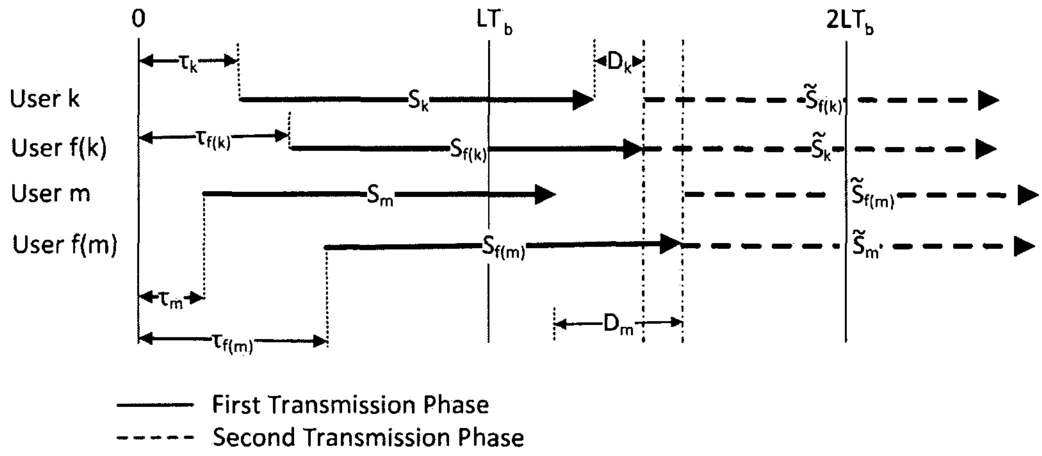

Figure 3.3: Cooperation timing diagram (single relay).

and base station sides. The outputs of these RAKE receivers represent the sufficient statistics for users' data detection where it consists of $(K-1)P$ filters at the relay side and KP filters at the base station. All filters are matched to the delayed versions of the normalized signature waveforms of each user. The output of the base station filter matched to the signature of user k, delayed over path p for the bit m, during the first transmission phase can be expressed in scalar form as

$$[Y_k^p(m)]_{b1} = \sum_{n=0}^{L-1}\sum_{w=1}^{K}\sum_{s=1}^{P}\sqrt{E_{U_w}}R_{k,w}^{(p,s)}(m,n)h_{wb}^{s}(n)S_w(n) + N_k^p(m) \qquad (3.4)$$

where $R_{k,w}^{(p,s)}(m,n)$ is the cross correlation value between bit m of user k transmitted over path p and the n bit of user w transmitted over path s, and it is equivalent to

$$R_{k,w}^{(p,s)}(m,n) = \int C_w(t-\tau_w-\tau_{w,s}-nT_b)C_k(t-\tau_k-\tau_{k,p}-mT_b)^*dt. \qquad (3.5)$$

The output of the bank of matched filters at both the base station and relay (r)

can be expressed in vector format, respectively, as

$$\mathbf{Y}_b = \mathbf{R}_b\mathbf{H}_b\mathbf{X}_b + \mathbf{N}_b \tag{3.6}$$

$$\mathbf{Y}_r = \mathbf{R}_r\mathbf{H}_r\mathbf{X}_r + \mathbf{N}_r \tag{3.7}$$

where $\mathbf{X}_b$ is the $(KLP \times 1)$ data vector of the K users and it can be expressed as

$$\mathbf{X}_b = \Big[\sqrt{E_{U_1}}S_1(1)\cdots\sqrt{E_{U_1}}S_1(1)\cdots\sqrt{E_{U_1}}S_1(L)\cdots\sqrt{E_{U_1}}S_1(L)\cdots\sqrt{E_{U_K}}S_K(1)\cdots$$
$$\cdots\sqrt{E_{U_K}}S_K(1)\cdots\sqrt{E_{U_K}}S_K(L)\cdots\sqrt{E_{U_K}}S_K(L)\Big]^T,$$

T represents transpose, $\mathbf{N}_b$ is $(KLP \times 1)$ noise vector with Gaussian elements each of zero mean and variance $N_o/2$, $\mathbf{H}_b$ is the $(KLP \times KLP)$ uplink channel matrix defined as

$$\mathbf{H}_b = \begin{bmatrix} h_{1b}^1(1) & 0 & \cdots & \cdots & 0 \\ \vdots & \cdots & \cdots & \cdots & \vdots \\ 0 & \cdots & h_{1b}^P(L) & \cdots & 0 \\ \vdots & \cdots & \cdots & \cdots & \vdots \\ 0 & \cdots & \cdots & \cdots & h_{Kb}^P(L) \end{bmatrix}. \tag{3.8}$$

Assuming the channels to be fixed over the whole frame L, the channel coefficients are equivalent $h_{kb}^p(1) = \cdots = h_{kb}^p(L)$. Similarly, at the relay side

$$\mathbf{X}_r = [\sqrt{E_{I_1}}S_1(1)\cdots\sqrt{E_{I_1}}S_1(L)\cdots\sqrt{E_{I_K}}S_K(1)\cdots\sqrt{E_{I_K}}S_K(L)]_{k\neq r}^T,$$

$\mathbf{N}_r$ is $((K-1)LP \times 1)$ Gaussian noise vector, and $\mathbf{H}_r$ is the $((K-1)LP \times (K-1)LP)$

inter-user channel matrix defined as

$$\mathbf{H}_r = \begin{bmatrix} h_{1r}^{1}(1) & 0 & \cdots & \cdots & 0 \\ \vdots & \cdots & \cdots & \cdots & \vdots \\ 0 & \cdots & h_{1r}^{P}(L) & \cdots & 0 \\ \vdots & \cdots & \cdots & \cdots & \vdots \\ 0 & \cdots & \cdots & \cdots & h_{Kr}^{P}(L) \end{bmatrix}_{k \neq r} . \quad (3.9)$$

The matrices $\mathbf{R}_b$ and $\mathbf{R}_r$ are the $(KLP \times KLP)$ base station and $((K-1)LP \times (K-1)LP)$ relay cross-correlation, given respectively by

$$\mathbf{R}_b = \begin{bmatrix} R_{1,1}^{(1,1)}(1,1) & \cdots & R_{1,1}^{(1,1)}(1,L) & \cdots & R_{1,K}^{(1,P)}(1,L) \\ \vdots & \cdots & \cdots & \cdots & \vdots \\ R_{1,1}^{(1,1)}(L,1) & \cdots & R_{1,1}^{(1,1)}(L,L) & \cdots & R_{1,K}^{(1,P)}(L,L) \\ \vdots & \cdots & \cdots & \cdots & \vdots \\ R_{K,1}^{(1,1)}(L,1) & \cdots & R_{K,1}^{(1,1)}(L,L) & \cdots & R_{K,K}^{(P,P)}(L,L) \end{bmatrix}, \quad (3.10)$$

$$\mathbf{R}_r = \begin{bmatrix} R_{1,1}^{(1,1)}(1,1) & \cdots & R_{1,1}^{(1,1)}(1,L) & \cdots & R_{1,K}^{(1,P)}(1,L) \\ \vdots & \cdots & \cdots & \cdots & \vdots \\ R_{1,1}^{(1,1)}(L,1) & \cdots & R_{1,1}^{(1,1)}(L,L) & \cdots & R_{1,K}^{(1,P)}(L,L) \\ \vdots & \cdots & \cdots & \cdots & \vdots \\ R_{K,1}^{(1,1)}(L,1) & \cdots & R_{K,1}^{(1,1)}(L,L) & \cdots & R_{K,K}^{(P,P)}(L,L) \end{bmatrix}_{k \neq r}, \quad (3.11)$$

where $R_{k,w}^{(p,s)}(m,n)$ is defined in (3.5). Note that the outputs of the matched filter bank in (3.6) and (3.7) suffer from MAI which can be eliminated using the one-shot decorrelator detector at both the relay and base station receivers [21]. In this case, the output of the matched filter bank $\mathbf{Y}_b$, $\mathbf{Y}_r$ are applied to linear mappers as follows

$$\mathbf{Z}_b = (\mathbf{R}_b)^{-1}\mathbf{Y}_b$$

$$\mathbf{Z}_r = (\mathbf{R}_r)^{-1}\mathbf{Y}_r$$

where $(\mathbf{R}_b)^{-1}$, $(\mathbf{R}_r)^{-1}$ are the inverses of the cross-correlation matrices. The $(KLP \times 1)$ vector $\mathbf{Z}_b$ represents the output of the one-shot decorrelator at the base station, defined by

$$\mathbf{Z}_b = \mathbf{H}_b\mathbf{X}_b + (\mathbf{R}_b)^{-1}\mathbf{N}_b \tag{3.12}$$

while the $((K-1)LP \times 1))$ vector $\mathbf{Z}_r$ represents the output of the decorrelator at the relay (r), given by

$$\mathbf{Z}_r = \mathbf{H}_r\mathbf{X}_r + (\mathbf{R}_r)^{-1}\mathbf{N}_r. \tag{3.13}$$

3.3 Performance Analysis

In this section, we analyze the average bit error rate (BER) at the base station decorrelator output using the DAF cooperative method and considering both cases of perfect inter-user channel, and the case of errors at the cooperating user side. For the sake of simplicity, we consider binary phase-shift-keying (BPSK) transmission.

3.3.1 Perfect Inter-user Channels

Here we consider perfect inter-user channels between the transmitting user and relays. This assumption is too optimistic, however, it serves as our reference for optimum performance (i.e., lower bound). Later we will consider the more realistic case where relays are subject to decoding errors.

Considering the m^{th} data bit of user 1 is the desired bit, the decorrelator output in the first and the second transmission periods at the base station can be expressed respectively as

$$[\mathbf{Z}_1]_{b1} = \mathbf{H}_b[\mathbf{X}_1]_{b1} + (\mathbf{R}_b)^{-1}[\mathbf{N}_1]_{b1}$$

$$[\mathbf{Z}_1]_{b2} = \mathbf{H}_b[\mathbf{X}_1]_{b2} + (\mathbf{R}_b)^{-1}[\mathbf{N}_1]_{b2}.$$

where $[\mathbf{X}_1]_{b1}, [\mathbf{X}_1]_{b2}$ represent the data vector transmitted during the first and the second transmission periods respectively

$$[\mathbf{X}_1]_{b1} = \Big[\sqrt{E_{U_1}}S_1(1)\cdots\sqrt{E_{U_1}}S_1(1)\cdots\sqrt{E_{U_1}}S_1(L)\cdots\sqrt{E_{U_1}}S_1(L)\cdots\sqrt{E_{U_K}}S_K(1)\cdots \cdots\sqrt{E_{U_K}}S_K(1)\cdots\sqrt{E_{U_K}}S_K(L)\Big]^T$$

$$[\mathbf{X}_1]_{b2} = \Big[\sqrt{E_{U_1}}\tilde{S}_{f(1)}(1)\cdots\sqrt{E_{U_1}}\tilde{S}_{f(1)}(1)\cdots\sqrt{E_{U_1}}\tilde{S}_{f(1)}(L)\cdots\sqrt{E_{U_1}}\tilde{S}_{f(1)}(L)\cdots \cdots\sqrt{E_{U_K}}\tilde{S}_{f(K)}(1)\cdots\sqrt{E_{U_K}}\tilde{S}_{f(K)}(1)\cdots\sqrt{E_{U_K}}\tilde{S}_{f(K)}(L)\Big]^T.$$

The P elements of $[\mathbf{Z}_1]_{b1}$ correspond to the decision statistics of bit m for user 1 before RAKE combining, during the first transmission

$$\begin{aligned}\left[Z_1^1(m)\right]_{b1} &= \sqrt{E_{U_1}}h_{1b}^1(m)S_1(m) + [(\mathbf{R}_b)^{-1}\mathbf{N}_{b1}]^1 \\ &\vdots \\ \left[Z_1^P(m)\right]_{b1} &= \sqrt{E_{U_1}}h_{1b}^P(m)S_1(m) + [(\mathbf{R}_b)^{-1}\mathbf{N}_{b1}]^P \end{aligned} \tag{3.14}$$

where $[Z_k^p(m)]_{b1}$ is the element that corresponds to the decision statistics of bit m for user k transmitted over path p before RAKE combining, and $[(\mathbf{R}_b)^{-1}\mathbf{N}_{b1}]^p$ is the p^{th} entry of the vector, while the P elements of $[\mathbf{Z}_1]_{b2}$ that represent the contribution of the cooperative phase (i.e., second frame), are given by

$$\begin{aligned}\left[Z_1^1(m)\right]_{b2} &= \sqrt{E_{U_{f(1)}}}h_{f(1)b}^1(m)\tilde{S}_1(m) + [(\mathbf{R}_b)^{-1}\mathbf{N}_{b2}]^1 \\ &\vdots \\ \left[Z_1^P(m)\right]_{b2} &= \sqrt{E_{U_{f(1)}}}h_{f(1)b}^P(m)\tilde{S}_1(m) + [(\mathbf{R}_b)^{-1}\mathbf{N}_{b2}]^P. \end{aligned} \tag{3.15}$$

Under the assumption of perfect inter-user channels, $\tilde{S}_1(m) = S_1(m)$, and by combining the $2P$ elements, the decision statistic of the desired user signal is

$$\begin{aligned}\grave{S}_1(m) &= h_{1b}^{1*}(m)\left[Z_1^1(m)\right]_{b1} + \cdots + h_{1b}^{P*}(m)\left[Z_1^P(m)\right]_{b1} + h_{f(1)b}^{1*}(m)\left[Z_1^1(m)\right]_{b2} + \cdots \\ &\quad + h_{f(1)b}^{P*}(m)\left[Z_1^P(m)\right]_{b2}. \qquad (3.16)\end{aligned}$$

Assuming that the two cooperating users are transmitting with the same energy $[E_{U_1} = E_{U_{f(1)}}]$, one can show that $\grave{S}_1(m)$ is equivalent to

$$\begin{aligned}\grave{S}_1(m) &= \sqrt{E_{U_1}}\left[|h_{1b}^1(m)|^2 + \cdots + |h_{1b}^P(m)|^2 + |h_{f(1)b}^1(m)|^2 + \cdots + |h_{f(1)b}^P(m)|^2\right] S_1(m) \\ &\quad + \mathrm{Re}\Big\{h_{1b}^{1^*}(m)\left[(\mathbf{R}_b)^{-1}\mathbf{N}_{b1}\right]^1 + \cdots + h_{1b}^{P^*}(m)\left[(\mathbf{R}_b)^{-1}\mathbf{N}_{b1}\right]^P + \\ &\quad h_{f(1)b}^{1^*}(m)\left[(\mathbf{R}_b)^{-1}\mathbf{N}_{b2}\right]^1 + \cdots + h_{f(1)b}^{P^*}(m)\left[(\mathbf{R}_b)^{-1}\mathbf{N}_{b2}\right]^P\Big\} \qquad (3.17)\end{aligned}$$

where $(*)$ denotes complex conjugate operation, and Re{.} donates the real part.

3.3.1.1 Conditional Probability of Error

From (3.17), the probability of error at the base station conditioned on the uplink channel coefficients can be expressed as

$$P_{b_P} = Q\left(\frac{\sum_{p=1}^{P}\sqrt{E_{U_1}}\left(|h_{1b}^p(m)|^2 + |h_{f(1)b}^p(m)|^2\right)}{\sqrt{\sigma_x^2}}\right) \qquad (3.18)$$

where $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}}\exp(-\frac{v^2}{2})dv$. In (3.18), σ_x^2 is the variance of the noise term in (3.17) which can be written as

$$\sigma_x^2 = \sigma_n^2 \sum_{p=1}^{P} |h_{1b}^p(m)|^2 (R_b)_1^{-2} + |h_{f(1)b}^p(m)|^2 (R_b)_{f(1)}^{-2}, \qquad (3.19)$$

where $(R_b)_k^{-2}$ is the square of the sum of the k^{th} row in the inverse of the cross-correlation matrix.

Let us define $\alpha_{1b}^p = |h_{1b}^p(m)|^2$ and $\alpha_{f(1)b}^p = |h_{f(1)b}^p(m)|^2$. Then α_{1b}^p , $\alpha_{f(1)b}^p$ are chi-square distributed of two degrees of freedom with variance σ^2 and characteristic function [34]

$$\phi(jw) = \frac{1}{1 - j2w\sigma^2}. \tag{3.20}$$

Also, by designating the two parameters $(R_b)_1^{-2}$, $(R_b)_{f(1)}^{-2}$, as C_1,$C_{f(1)}$, respectively, the noise variance σ_x^2 in (3.19) can be written as

$$\sigma_x^2 = \sigma_n^2 \sum_{p=1}^{P} \alpha_{1b}^p C_1 + \alpha_{f(1)b}^p C_{f(1)}. \tag{3.21}$$

Define the variable γ as

$$\gamma = \frac{A}{\sqrt{B}}, \tag{3.22}$$

where $A = \sum_{p=1}^{P} \alpha_{1b}^p + \alpha_{f(1)b}^p$ and $B = \sum_{p=1}^{P} \alpha_{1b}^p C_1 + \alpha_{f(1)b}^p C_{f(1)}$. Using the new conventions the conditional probability of error can be expressed as

$$P_{b_P} = Q\left(\sqrt{\frac{E_{U_1}\gamma^2}{\sigma_n^2}}\right). \tag{3.23}$$

3.3.1.2 Probability Density Function

In order to obtain the pdf of γ, we need to determine the joint characteristic function of A and B. As shown in Appendix A, the joint characteristic function can be expressed as

$$\phi_{A,B}(w_1, w_2) = \frac{\prod_{u=1}^{2P} \frac{j}{C_u}}{(2\sigma^2)^{2P}} \left(\sum_{U=1}^{2P} \frac{C_{Uk}}{\left(w_2 - \frac{y}{jC_U}\right)} \right), \tag{3.24}$$

where $C_{Uk} = \frac{K_U}{y^{2P-1}}$ is the residue term obtained from the partial fraction, and K_U is a function of the cross-correlations among users. The joint pdf, $f(A, B)$, is then given by

$$f(A, B) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi_{A,B}(w_1, w_2) \exp^{-jw_1 A} \exp^{-jw_2 B} dw_1 dw_2. \tag{3.25}$$

By solving the double integral as shown in Appendix A and letting $W = B$, we obtain

$$f(\gamma, W) = \frac{\prod_{u=1}^{2P} \frac{1}{C_u}}{4\pi^2(2\sigma^2)^{2P}} \sum_{U=1}^{2P} \Psi_U W^{\frac{1}{2}} \left(\gamma\sqrt{W} - \frac{W}{C_U}\right)^{2P-2} \exp^{\left(\frac{-\gamma\sqrt{W}}{2\sigma^2}\right)}, \tag{3.26}$$

where $\Psi_U = \frac{4\pi^2 K_U}{\Gamma(2P-1)}$. Hence, one can determine the pdf of the SNR as (see Appendix A)

$$f(\gamma) = \frac{\prod_{u=1}^{2P} \frac{1}{C_u}}{4\pi^2(2\sigma^2)^{2P}} \sum_{U=1}^{2P} \Psi_U R_U(\gamma), \tag{3.27}$$

where

$$R_U(\gamma) = \int_0^{C_U^2\gamma^2} W^{\frac{1}{2}} \left(\gamma\sqrt{W} - \frac{W}{C_U}\right)^{2P-2} \exp^{\left(\frac{-\gamma\sqrt{W}}{2\sigma^2}\right)} dW. \tag{3.28}$$

Using the binomial series expansion, setting $t = \sqrt{W}$ and expressing the equation in terms of confluent hypergeometric function, as shown in Appendix B we get

$$\begin{aligned} R_U(\gamma) &= 2\gamma^{4P-1} C_U^{2P+1} \exp^{\left(\frac{-C_U\gamma^2}{2\sigma^2}\right)} \sum_{m=0}^{2P-2} \begin{pmatrix} 2P-2 \\ m \end{pmatrix} \frac{(-1)^{2P-2-m}}{4P-m-1} \\ & {}_1F_1\left(1; 4P-m; \frac{C_U\gamma^2}{2\sigma^2}\right). \end{aligned} \tag{3.29}$$

Finally, the pdf of the SNR can be obtained by substituting (3.29) in (3.27).

3.3.1.3 Probability of Bit Error

The probability of bit error can be obtained by averaging (3.23) over the pdf given in (3.27)

$$P_{b_P} = \int_0^\infty Q(\sqrt{\delta\gamma^2}) f(\gamma) d\gamma, \tag{3.30}$$

where $\delta = \frac{E_{U_1}}{\sigma_n^2}$, and

$$Q(x) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp^{\frac{-x^2}{2\sin^2\theta}} d\theta$$

to give,

$$P_{b_P} = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\int_0^{\infty} \exp^{\frac{-\delta\gamma^2}{2\sin^2\theta}} f(\gamma)d\gamma d\theta$$
$$= \frac{\prod_{u=1}^{2P}\frac{1}{C_u}}{4\pi^3}\sum_{U=1}^{2P}\Psi_U F_U(\delta), \tag{3.31}$$

where

$$F_U(\delta) = \frac{1}{(2\sigma^2)^{2P}}\int_0^{\frac{\pi}{2}}\int_0^{\infty} \exp^{\frac{-\delta\gamma^2}{2\sin^2\theta}} R_U(\gamma)d\gamma d\theta$$
$$= 2C_U^{2P+1}\sum_{m=0}^{2P-2}\begin{pmatrix} 2P-2 \\ m \end{pmatrix}\left(\frac{(-1)^{2P-2-m}}{4P-1-m}\right)G_m(\delta), \tag{3.32}$$

and

$$G_m(\delta) = \frac{1}{(2\sigma^2)^{2P}}\int_0^{\frac{\pi}{2}}\int_0^{\infty}\gamma^{4P-1}\exp^{-\gamma^2(\frac{\delta}{2\sin^2\theta}+\frac{C_U}{2\sigma^2})}$$
$${}_1F_1\left(1;4P-m;\frac{C_U\gamma^2}{2\sigma^2}\right)d\gamma d\theta. \tag{3.33}$$

Letting $t=\gamma^2$ in (3.33), and using [35]

$$\int_0^{\infty}\exp^{-st}t^{b-1}{}_1F_1(a;c;kt)dt = \Gamma(b)(s-k)^{-b}{}_2F_1\left(c-a,b;c;\frac{k}{k-s}\right), \tag{3.34}$$

(3.33) yields (see Appendix C)

$$G_m(\overline{\delta}) = \frac{\Gamma(2P)}{2(P\overline{\delta})^{2P}}\int_0^{\frac{\pi}{2}}(\sin^2\theta)^{2P}{}_2F_1\left(4P-m-1,2P;4P-m;\frac{-C_U\sin^2\theta}{(P\overline{\delta})}\right)d\theta, \tag{3.35}$$

where $\overline{\delta} = \frac{E\left[|h_{kb}^p(m)|^2\right]E_{U_1}}{\sigma_n^2}$ is the average SNR of the channel. Substituting $V=\sin^2\theta$ in (3.35) and using [eq. (7.512.12), [35]],

$$\int_0^1(1-x)^{u-1}x^{v-1}{}_pF_q(a_1,...,a_p;b_1,...,b_q;ax)dx$$
$$= \frac{\Gamma(u)\Gamma(v)}{\Gamma(u+v)}{}_{p+1}F_{q+1}(v,a_1,...,a_p;u+v,b_1,...,b_q;a), \tag{3.36}$$

we have from Appendix C

$$G_m(\bar{\delta}) = \left(\frac{\Gamma(\frac{1}{2})\Gamma(\frac{4P+1}{2})}{8P(P\bar{\delta})^{2P}} \right) {}_3F_2 \left(\frac{4P+1}{2}, 4P-m-1, 2P; 2P+1, 4P-m; \frac{-C_U}{(P\bar{\delta})} \right). \tag{3.37}$$

Finally, using (3.37) and (3.32) the average probability of bit error in (3.31) can be evaluated as

$$\begin{aligned} P_{b_P} &= \left(\frac{\prod_{u=1}^{2P} \frac{1}{C_u}}{4\pi^3} \right) \sum_{U=1}^{2P} 2\Psi_U C_U^{2P+1} \\ & \sum_{m=0}^{2P-2} \begin{pmatrix} 2P-2 \\ m \end{pmatrix} \frac{(-1)^{2P-2-m}}{4P-1-m} \frac{\Gamma(\frac{1}{2})\Gamma(\frac{4P+1}{2})}{8P(P\bar{\delta})^{2P}} \\ & {}_3F_2 \left(\frac{4P+1}{2}, 4P-m-1, 2P; 2P+1, 4P-m; \frac{-C_U}{(P\bar{\delta})} \right). \end{aligned} \tag{3.38}$$

From (3.38), we can examine the asymptotic BER performance of the cooperative system as the SNR gets large. Noting that ${}_3F_2\left(\frac{4P+1}{2}, 4P-m-1, 2P; 2P+1, 4P-m; \frac{-C_U}{\bar{\delta}}\right) \rightarrow 1$ as $\bar{\delta} \rightarrow \infty$, one can see that $P_{b_P} \equiv \left(\frac{1}{\bar{\delta}^{2P}}\right)$, and the achieved diversity order $d = 2P$.

3.3.2 Inter-User Channel Errors

As mentioned earlier, the performance of the cooperative system with perfect inter-user channel is not realistic in some cases and can only serve as a lower bound indicating optimal performance. Therefore, here we consider the effect of errors in the inter-user channels where cooperation among users can only take place if the relay detection is error-free. If errors exist, the corresponding relay (cooperating user) stays idle during the second transmission period.

3.3.2.1 Probability of Bit Error

Without loss of generality, consider the m^{th} data bit of user one as the desired bit, the decorrelator outputs at the cooperative user side $f(1)$ is given by

$$[\mathbf{Z}_1]_r = \mathbf{H}_r[\mathbf{X}_1]_r + (\mathbf{R}_r)^{-1}[\mathbf{N}_1]_r \tag{3.39}$$

where the P elements of Z_1 are

$$\begin{aligned} \left[Z_1^1(m)\right]_r &= \sqrt{E_{I_1}} h_{1f(1)}^1(m) S_1(m) + [(\mathbf{R}_r)^{-1}\mathbf{N}_1]^1 \\ &\vdots \\ \left[Z_1^P(m)\right]_r &= \sqrt{E_{I_1}} h_{1f(1)}^P(m) S_1(m) + [(\mathbf{R}_r)^{-1}\mathbf{N}_1]^P. \end{aligned} \tag{3.40}$$

At the output of the RAKE combiner, we have the user data decision statistic

$$\begin{aligned} \ddot{S}_1(m) &= \left[|h_{1f(1)}^1(m)|^2 + \cdots + |h_{1f(1)}^P(m)|^2\right] S_1(m) \\ &+ \operatorname{Re}\left\{h_{1f(1)}^{1*}(m)((\mathbf{R}_r)^{-1}\mathbf{N}_1)^1 + \cdots + h_{1f(1)}^{P*}(m)((\mathbf{R}_r)^{-1}\mathbf{N}_1)^P\right\}, \end{aligned} \tag{3.41}$$

and the probability of error of the inter-user channel

$$P_{b_E} = Q\left(\frac{\sum_{p=1}^{P} \sqrt{E_{I_1}}(|h_{1f(1)}^p(m)|^2)}{\sqrt{\sigma_{x_r}^2}}\right), \tag{3.42}$$

where

$$\sigma_{x_r}^2 = \sigma_n^2 \sum_{p=1}^{P} |h_{1f(1)}^p(m)|^2 (R_r)_1^{-2}, \tag{3.43}$$

and $(R_r)_k^{-2}$ is the square of the sum of the k^{th} row in the inverse of the cross-correlation matrix. Similar to the case of perfect inter-user channel, we define $\alpha_{1f(1)}^p = |h_{1f(1)}^p(m)|^2$with characteristic function given by (3.20). Also, letting $(R_r)_1^{-2} = C_1$, the noise variance $\sigma_{x_r}^2$ can be expressed as

$$\sigma_{x_r}^2 = \sigma_n^2 \sum_{p=1}^{P} \alpha_{1f(1)}^p C_1. \tag{3.44}$$

Following the same analysis as the perfect case, we define a variable γ_r as

$$\gamma_r = \frac{A_r}{\sqrt{B_r}}, \tag{3.45}$$

where $A_r = \sum_{p=1}^{P} \alpha_{1f(1)}^{p}$ and $B_r = \sum_{p=1}^{P} \alpha_{1f(1)}^{p} C_1$.Then, it is easy to show that

$$\phi_{A_r,B_r}(w_1, w_2) = \frac{\prod_{u=1}^{P} \frac{j}{C_u}}{(2\sigma^2)^P} \left(\sum_{U=1}^{P} \frac{C_{Uk_r}}{(w_2 - \frac{y}{jC_U})} \right), \tag{3.46}$$

where $C_{Uk_r} = \frac{K_{U_r}}{y^{P-1}}$ is the residue term obtained from the partial fraction and K_{U_r} can be obtained in terms of the cross-correlation among users. The pdf, $f(A_r, B_r)$, is then given by

$$f(A_r, B_r) = \frac{\prod_{u=1}^{P} \frac{1}{C_u}}{4\pi^2(2\sigma^2)^P} \sum_{U=1}^{P} \Psi_{U_r} \left(A_r - \frac{B_r}{C_U} \right)^{P-2} \exp^{\left(\frac{-A_r}{2\sigma^2}\right)}$$

with $\Psi_{U_r} = \frac{4\pi^2 K_{U_r}}{\Gamma(P)}$, and

$$f(\gamma_r) = \frac{\prod_{u=1}^{P} \frac{1}{C_u}}{4\pi^2(2\sigma^2)^P} \sum_{U=1}^{P} \Psi_{U_r} R_{U_r}(\gamma_r), \tag{3.47}$$

where

$$\begin{aligned} R_{U_r}(\gamma_r) &= 2\gamma_r^{2P-1} C_U^{P+1} \exp^{\left(\frac{-C_U\gamma_r^2}{2\sigma^2}\right)} \sum_{m=0}^{P-2} \begin{pmatrix} P-2 \\ m \end{pmatrix} \\ &\quad \left(\frac{(-1)^{P-2-m}}{2P-m-1} \right) {}_1F_1\left(1; 2P-m; \frac{C_U\gamma_r^2}{2\sigma^2}\right). \end{aligned} \tag{3.48}$$

The probability of bit error at the relay can then be obtained using (3.42) and (3.47),

$$\begin{aligned} P_{b_E} &= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \int_0^{\infty} \exp^{\frac{-\delta_r\gamma_r^2}{2\sin^2\theta}} f(\gamma_r) d\gamma_r d\theta \\ &= \frac{\prod_{u=1}^{P} \frac{1}{C_u}}{4\pi^3} \sum_{U=1}^{P} \Psi_{U_r} F_{U_r} \end{aligned} \tag{3.49}$$

with

$$F_{U_r}(\overline{\delta}_r) = 2C_U^{P+1}\sum_{m=0}^{P-2}\begin{pmatrix} P-2 \\ m \end{pmatrix}\left(\frac{(-1)^{P-2-m}}{2P-m-1}\right)G_{m_r}(\overline{\delta}_r),$$

where $\overline{\delta}_r = \frac{E\left[|h^p_{kf(k)}(m)|^2\right]E_{I_1}}{\sigma_n^2}$ is the average SNR of the inter-user channel, and

$$G_{m_r}(\overline{\delta}_r) = \frac{\Gamma(P)}{2(R_c dP\overline{\delta}_r)^P}\int_0^{\frac{\pi}{2}}(\sin^2\theta)^P {}_2F_1\left(2P-m-1, P; 2P-m; \frac{-C_U\sin^2\theta}{R_c dP\overline{\delta}_r}\right)d\theta. \tag{3.50}$$

Similar to the perfect case shown in Appendix C, the average probability of bit error for the inter-user channel can be expressed as

$$\begin{aligned} P_{b_E} \;=\; & \left(\frac{\prod_{u=1}^{P}\frac{1}{C_u}}{4\pi^3}\right)\sum_{U=1}^{P}2\Psi_{U_1}C_U^{P+1} \\ & \sum_{m=0}^{P-2}\begin{pmatrix} P-2 \\ m \end{pmatrix}\left(\frac{(-1)^{P-2-m}}{2P-m-1}\right)\left(\frac{\Gamma(\frac{1}{2})\Gamma(\frac{2P+1}{2})}{4(P)(R_c dP\overline{\delta}_r)^P}\right) \\ & {}_3F_2\left(\frac{2P+1}{2}, 2P-m-1, P; P+1, 2P-m; \frac{-C_U}{R_c dP\overline{\delta}_r}\right). \end{aligned} \tag{3.51}$$

Having obtained the probability of bit error at the relay, one can find the average BER at the base station as

$$P_b = P_{b_E}P_{b_{NC}} + (1-P_{b_E})P_{b_P}, \tag{3.52}$$

where P_{b_E} represents the probability of error at the relay, P_{b_P} represents the probability of error in the uplink channel (perfect inter-user channel case), and $P_{b_{NC}}$ is the average probability of error for the direct transmission between the active user and the base station. Note that if the inter-user channel is perfect ($P_{b_E} = 0$), the average probability of error P_b is equal to the full cooperation probability of error P_{b_P}, and the system gains full benefit from cooperation. While in case of very poor inter-user channels, the probability of error is dominated by the non-cooperative probability of error, $P_{b_{NC}}$. Similar to the discussion in the perfect inter-user channel, we can see that the maximum diversity order of $2P$ is achieved when the inter-user channel is perfect.

3.4 Simulation Results

In this section, we present simulation results to assess the performance of the cooperative system when considering different scenarios. Also, we examine the accuracy of our analytical results obtained in section 3.3. We consider the cooperative diversity spreading scheme in [1] with one transmit and one receive antennas. An 8-user asynchronous DS-CDMA system with BPSK transmission is assumed where every user data is spread using non-orthogonal Gold codes of length 31 chips. For the asynchronous channel, the transmitted frames are 100 bits each, the fading coefficients are fixed for a number of frames and the delay between users, τ_k, is uniformly distributed along the symbol period. We also assume perfect knowledge of the channel coefficients at the base station and the relay.

In Fig. 3.4, the performance of the cooperative system is simulated for a 2-path frequency-selective slow fading channels. The performance of the system will be compared to the non-cooperation case. In order to have a fair comparison, each transmitting user in the non-cooperation case will send its information bits twice to the base station. Considering errors in the inter-user channels, the performance of the network with conventional matched filter receivers at the relays is compared with the performance of the relays using decorrelator detectors. As shown, when the

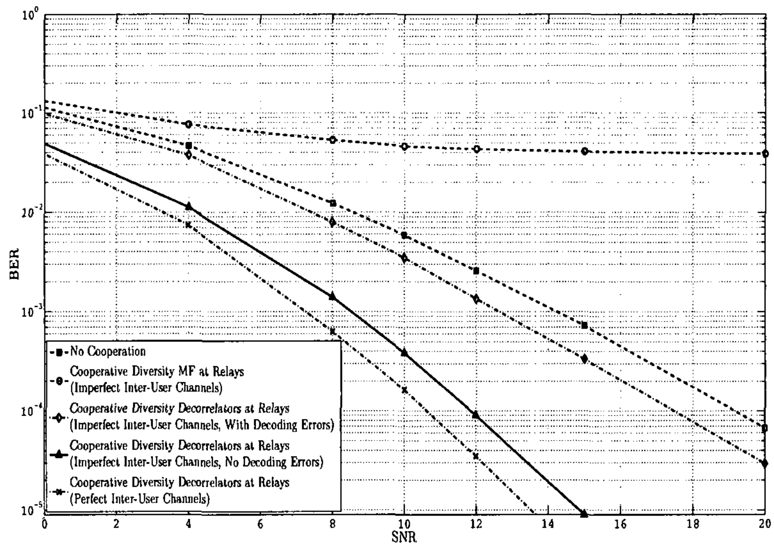

Figure 3.4: BER performance for an 8-user asynchronous DS-CDMA network over 2-path frequency-selective slow fading channels.

decorrelator receiver is used at the relay, the system is immune to multiple-access interference. However, the conventional receiver becomes almost ineffective resulting in an error floor due to the fact that the detected signals at both the relays and the base station suffer from high levels of interference. As a result, the performance of the cooperative network when employing matched filter receivers at the relays can in some cases be worse than the non-cooperative case. Comparing the two relay forwarding protocols, where the relay forwards the decoding errors with the case of silent transmission when errors exist at the relay, the results show that forwarding the decoding errors will cause degradation in the overall network performance shown in the loss of diversity gain. The results also show that the system can achieve the full diversity gain when considering perfect inter-user channels. These results suggest that, in the presence of MAI at the relay, the overall system performance degrades significantly and no benefit for user cooperation can take place.

Fig. 3.5 shows the impact of the inter-user channels over the overall performance of an 8 user cooperating network over 2-path frequency-selective slow fading channels. The figure compares the performance of the cooperating network as a function of the inter-user channel SNR (E_I) with the no cooperation case and the perfect inter-user channels case ($E_I = \infty$). The results show that as the inter-user channel SNR increases, the network gains from diversity and the performance gets closer to the perfect inter-user channel case.

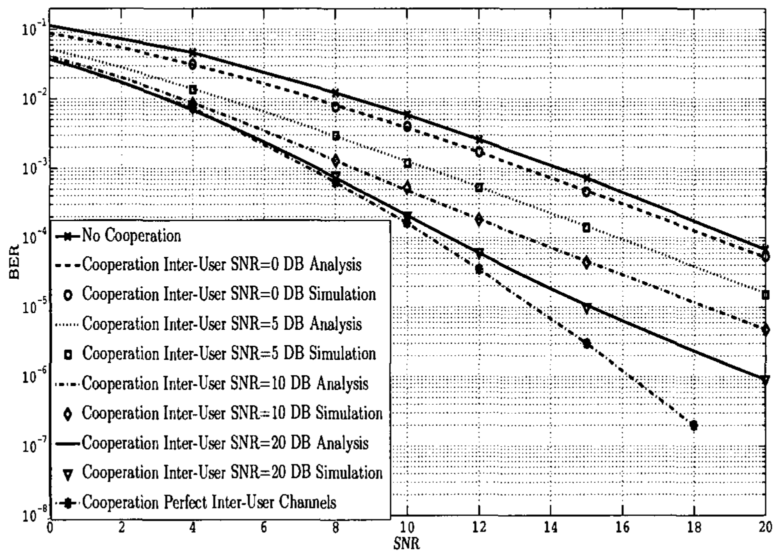

Figure 3.5: BER performance for an 8-user asynchronous DS-CDMA network over 2-path frequency-selective slow fading channels, for different inter-user channel SNR.

To show the full benefits of user cooperation, Fig. 3.6 shows both simulation and analytical results for the cooperative system with the same number of users under perfect inter-user channel and as a function of the number of resolvable paths. As can be seen from these results, the cooperative system always achieve higher diversity gains as the number of paths per channel increases. Note that the derived BER results are quite accurate when compared to simulated results. It is worthy to mention that the system, in this case, achieves the maximum diversity gain (cooperative and multipath diversities) of $2P$.

Finally, in Fig. 3.7, we consider the case of imperfect inter-user channel using both simulations and analytical results. By comparison with the perfect inter-user channel case, we can conclude that when the number of resolvable paths increases, the system can achieve high diversity gains, however, due to errors in the inter-user channel the system cannot gain the full cooperation diversity.

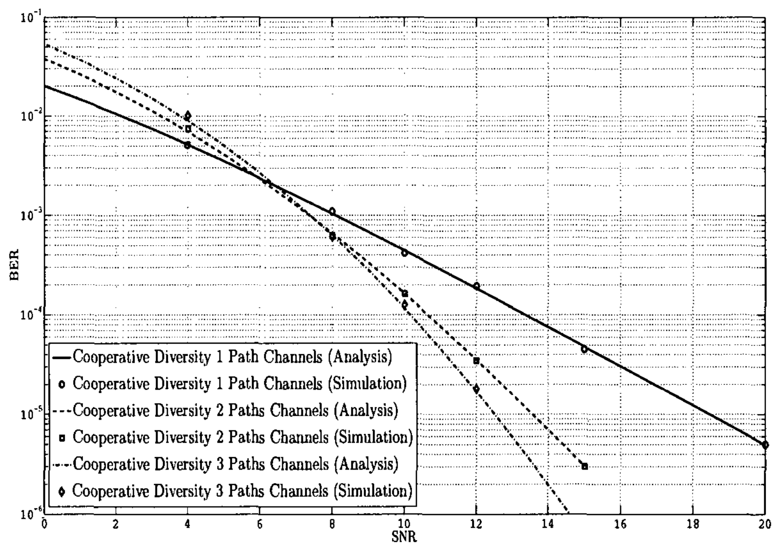

Figure 3.6: BER performance for an 8-user asynchronous DS-CDMA with DAF cooperation as a function of the number of paths over frequency-selective slow fading channels assuming perfect inter-user channels.

3.5 Conclusion

In this chapter, we examined the performance of cooperative diversity using DAF cooperation in asynchronous DS-CDMA systems over frequency-selective slow fading channels. Several issues have been studied, such as the impact of, multi-user interference, inter-user channel errors, and multipath diversity on the overall system performance. We have shown that if no multiple-access interference suppression is applied at the relay side, the system cannot achieve the diversity gains promised by the cooperative network. When considering multiuser interference suppression using the decorrelator detector at both the relay and base station sides, we showed that the system is capable of achieving large diversity gains. In that, we have analyzed the system performance for both the perfect and imperfect inter-user channel cases. We have provided mathematical forms for the probability of bit error for the cooperative DAF system for these two scenarios. The high accuracy of our analytical results was verified through simulation examples and under different system parameters.

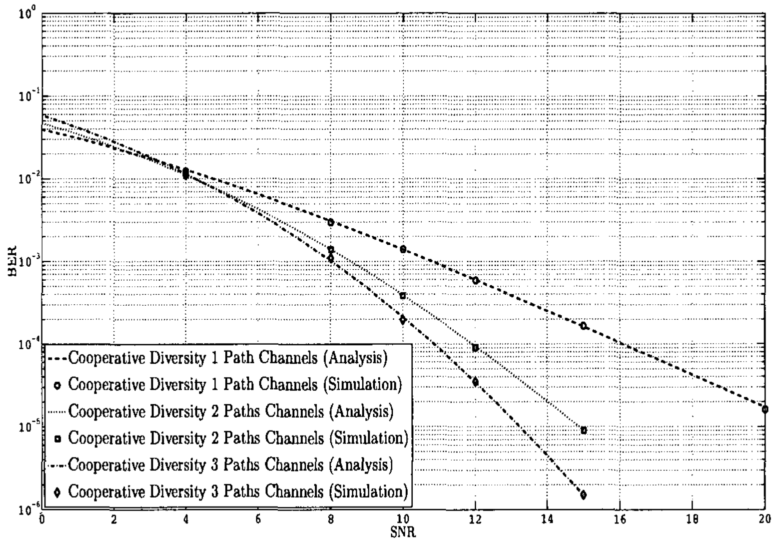

Figure 3.7: BER performance for an 8-user asynchronous DS-CDMA with DAF cooperation as a function of the number of paths over frequency-selective slow fading channels and considering errors in the inter-user channels.

Chapter 4

Coded Multi-relay Cooperation

In the previous chapter, we analyzed the performance of asynchronous DS-CDMA uncoded cooperative networks over frequency-selective fading channels. In this chapter, we study the impact of inter-user channel performance in asynchronous DS-CDMA multi-relay coded cooperative networks over the more realistic case of frequency-selective fading channels. In order to mitigate the effect of decoding errors in the inter-user channels, we integrate channel coding with the DAF cooperation technique for the more general case of multi-relay cooperative networks. We show the full benefits of cooperative diversity when channel coding is employed. Our results show that the performance of the cooperative network is greatly affected by the quality of inter-user channel. To this end, we analyze the performance of a system that employs convolutional coding at both the cooperative user side and base station. In that, we consider two scenarios; (i) perfect inter-user channel (source-relay) where the relay (also referred as partner) correctly decodes the received user signal. This case is important since it serves as the optimal performance achieved using cooperation, where the distributed MIMO system reaches the performance of the centralized one, (ii) imperfect inter-user channel where we assume that each relay has an error detection capability (e.g., cyclic redundancy check (CRC)) to decide weather cooperation can take place or not. In case of error, the cooperating user keeps silent (no cooperation).

The rest of this chapter is organized as follows: The following section describes the asynchronous DS-CDMA system model used in this chapter. In section 4.3, the system performance for a DAF scheme is examined for the case of a multiuser system where we obtain the probability density function of SNR at the receiver output. Then we derive the pairwise probability of error, which will be used to obtain an upper bound performance. The performance of DAF with imperfect inter-user channel is also analyzed in this section. In section 4.4, both analytical and simulation results are compared and discussed. Finally, conclusions are given in section 4.5.

4.1 Introduction

Coded cooperation integrates cooperative diversity with channel coding and it was introduced in [28] where the codewords for the active transmitting users are split and sent over a number of independent fading channels. In [36], serial concatenated convolutional coding was introduced to a half duplex TDMA single link cooperative scheme where the performance of the system was studied over flat fading channels for both cooperation schemes DAF and AAF. The performance of coded single relay cooperative network for OFDM systems was the point of research in [37]. Multi source cooperative TDMA network was studied in [38], where a distributed complex field coding (DCFC) was introduced and the performance was analyzed over Rayleigh fading channels. The performance of coded cooperation was also studied in [39], [40], [29]. In [41], adaptive coding protocol was introduced to cooperative networks, where active users and partners employ ACK/NACK concept in order to confirm the reception of correctly decoded data.

Multiple source-destination communication employing multi-relay cooperative network was introduced in [42], where the authors studied a multiple-relay case for CDMA network. More recently, the performance of multi-relay cooperative networks was studied using AAF in flat fading channels and employing orthogonal frequency division multiplexing (OFDM) [43].

The authors in [44] studied a coded multi-relay protocol for a half duplex cooperative system, also a distributed convolutional coding scheme was presented where the coded data is sent over several transmission periods. In [44] the performance of the system was investigated over slow fading channels. In [45], an antenna/relay selection protocol was presented, where each relay will be embedded with multiple antennas and an integration between relays and antennas selection was introduced. For a multi-relay cooperating network the authors in [46] introduced a parity forwarding DAF protocol, where each relay partially decodes and forward the source's messages. In [46], the authors proposed joint decoding strategy where the receiving node will be able to retrieve full information received from different relays. Recently, the authors in [47] studied a distributed coded cooperation method for a multi-relay convolutional coded cooperative network for M-ary phase-shift keying (M-PSK) modulation scheme, where the active users and their relays share their antennas for transmission.

4.2 Coded System

4.2.1 Proposed Diversity Scheme

Consider an uplink transmission for an asynchronous K-user DS-CDMA system. Each active transmitting user in the network will be cooperating with V relays. The system employs one transmit antenna at the transmitter side and one receive antenna at the receiver side. All users and relays are embedded with convolutional encoders of rate $R_c = m_c/n_c$ and a Viterbi decoder [48]. In what follows, to simplify the notation, we refer to the base station with subscript b and the v^{th} cooperating relay with subscript v.

The transmission scheme considered is described in Fig. 4.1, 4.2, for a V-relay system. As shown, during the first transmission phase each active transmitting user sends its own data to the base station and to its V partners (Fig. 4.1), while during the second transmission phase (relaying phase), each cooperating relay transmits the decoded version of its partner's data to the base station at different time slots (Fig. 4.2).

Consider a multipath channel with P paths during each transmission period. For the DAF, the low pass equivalent of the received signal at the base station and relay (v) during the first transmission phase can be expressed respectively as

$$\mathbf{r}_{b1}(t) = \sum_{m=0}^{L-1} \sum_{k=1}^{K} \sum_{p=1}^{P} \sqrt{R_c E_{U_k}} S_k(m) C_k(t - \tau_k - \tau_{k,p} - mT_b) h_{kb}^{p}(m) + n_{b1}(t) \quad (4.1)$$

$$\mathbf{r}_{v}(t) = \sum_{m=0}^{L-1} \sum_{k=1_{k\neq v}}^{K} \sum_{p=1}^{P} \sqrt{R_c E_{I_{kv}}} S_k(m) C_k(t - \tau_k - \tau_{k,p} - mT_b) h_{kv}^{p}(m) + n_{v}(t) \quad (4.2)$$

where L is the size of the transmitted frame (output of the modulator), $S_k(m)$ is the

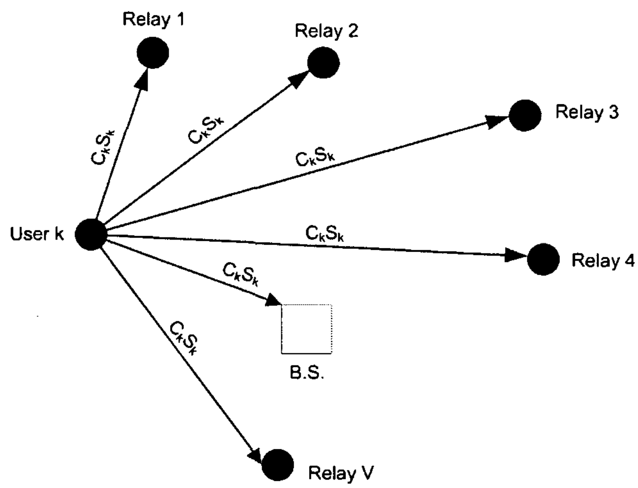

Figure 4.1: First transmission phase (multi-relay).

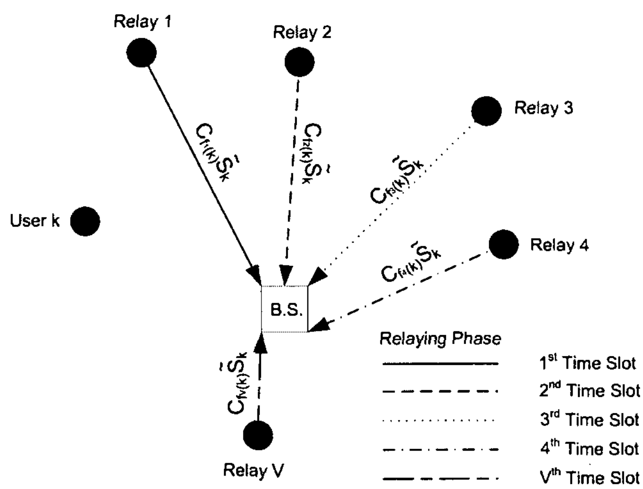

Figure 4.2: Second transmission phase (multi-relay).

m^{th} transmitted bit of user k, E_{U_k} and $E_{I_{kv}}$ are the received signal energies per path of user k at the base station (uplink channel) and the relay v (inter-user channel) respectively, $C_k(t)$ is the spreading code assigned to the k^{th} user with processing gain (T_b/T_c), where T_b is the bit period and T_c is the chip period, and τ_k is the random transmit delay of the k^{th} user which is assumed uniformly distributed along the symbol period. The parameter $\tau_{k,p}$ represents the delay of the p^{th} path of user k during one transmission period. The channel coefficient h^p_{kv} models the fading of the inter-user channel between users k and v over the p^{th} path, while h^p_{kb} represents the fading coefficient for the uplink channel between user k and the base station b over the path p. These fading coefficients are modeled as independent Gaussian random variables with zero mean and unit variance. The noise $n_{b1}(t)$ and $n_v(t)$ are complex Gaussian, each with zero mean and variance $\sigma_n^2 = N_o/2$.

During the second transmission phase (relaying phase), V set of relays $[f_1(k) \cdots f_V(k)]$ cooperate with user k in which each cooperating relay retransmits the received signal. Each cooperating relay first decodes the partner's received signal, then using error checking techniques it decides whether or not to forward the estimated partner's data to the base station in the relaying phase. The relaying phase is done in V time slots where each cooperating relay will start transmitting at a different time slot. Hence, the data transmission rate of the system will decline as the number of the cooperating relays per transmitting user increases as illustrated in Fig. 4.3. The low pass equivalent of the received signal at the base station during the relaying phase can then be expressed as

$$\mathbf{r}_{b2}(t) = \sum_{m=0}^{L-1}\sum_{k=1}^{K}\sum_{v=1}^{V}\sum_{p=1}^{P}\sqrt{R_c\frac{E_{U_{f_v(k)}}}{V}}\tilde{S}_k(m)C_{f_v(k)}(t-D_{k,v}-\tau_{v,p}-mvT_b)h^p_{f_v(k)b}(m)+n_{b2}(t) \quad (4.3)$$

where $f_v(k)$ is the relay v cooperating with user k, $\tilde{S}_k(m)$ is the estimate of the transmitted coded bit $S_k(m)$, the noise $n_{b2}(t)$ is Gaussian with zero mean and variance $\sigma_n^2 = N_o/2$, $D_{k,v}$ is the transmission delay during the relaying phase (see the timing diagram in Fig. 4.4), where each cooperating relay $[f_1(k)\cdots f_V(k)]$ will have to wait for a time delay of $D_{k,v}$ for the preceding relay to finish transmission, and it is equivalent to $D_{k,v} = \tau_k$

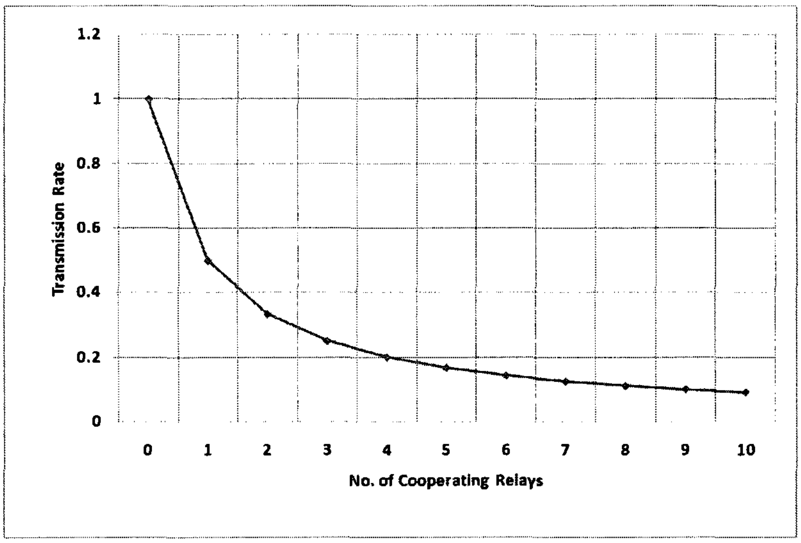

Figure 4.3: Data transmission rate for multi-relay cooperation.

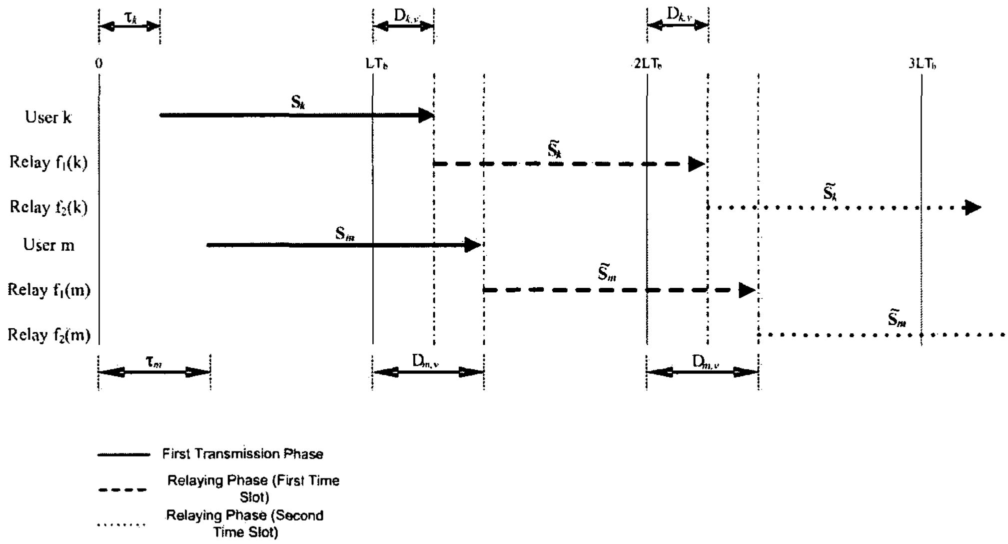

Figure 4.4: Cooperation timing diagram (multi-relay).

4.2.2 Multiuser Detector

In a K-user system, where each user signal is transmitted over a multipath channel with P-resolvable paths, a RAKE receiver is employed at the front-end of both sides of the relays and base station. The outputs of these RAKE receivers represent the sufficient statistics for users' data detection where it consists of $(K-1)P$ filters at the relay side and KP filters at the base station. All filters are matched to the delayed versions of the normalized signature waveforms of each user. The output of the base station filter matched to the signature of user k, delayed over path p, for the bit m during the first transmission phase can be expressed in scalar form as

$$[Y_k^p(m)]_{b1} = \sum_{n=0}^{L-1} \sum_{w=1}^{K} \sum_{s=1}^{P} \sqrt{R_c E_{U_w}} R_{k,w}^{(p,s)}(m,n) h_{wb}^s(n) S_w(n) + n_k^p(m) \tag{4.4}$$

where $R_{kw}^{(p,s)}(m,n)$ is the cross-correlation value between bit m of user k transmitted over path p and the n bit of user w transmitted over path s, and it is equivalent to

$$R_{k,w}^{(p,s)}(m,n) = \int C_w(t - \tau_w - \tau_{w,s} - nT_b)C_k(t - \tau_k - \tau_{k,p} - mT_b)^* dt. \tag{4.5}$$

The output of the bank of matched filters at both the base station and relay (v) can be expressed in vector format, respectively, as

$$\mathbf{Y}_b = \mathbf{R}_b\mathbf{H}_b\mathbf{X}_b + \mathbf{N}_b \tag{4.6}$$

$$\mathbf{Y}_v = \mathbf{R}_v\mathbf{H}_v\mathbf{X}_v + \mathbf{N}_v \tag{4.7}$$

where $\mathbf{X}_b$ is the $(KLP \times 1)$ data vector of the K users, expressed for the first transmission phase as

$$\mathbf{X}_b = [\sqrt{R_c E_{U_1}} S_1(1) \cdots \sqrt{R_c E_{U_1}} S_1(L) \cdots \sqrt{R_c E_{U_K}} S_K(1) \cdots \sqrt{R_c E_{U_K}} S_K(L)]^T, \tag{4.8}$$

$\mathbf{N}_b$ is $(KLP \times 1)$ noise vector with Gaussian elements each of zero mean and variance $N_o/2$, $\mathbf{H}_b$ is $(KLP \times KLP)$ uplink channel matrix defined as

$$\mathbf{H}_b = \begin{bmatrix} h_{1b}^1(1) & 0 & \cdots & \cdots & 0 \\ \vdots & \cdots & \cdots & \cdots & \vdots \\ 0 & \cdots & h_{1b}^P(L) & \cdots & 0 \\ \vdots & \cdots & \cdots & \cdots & \vdots \\ 0 & \cdots & \cdots & \cdots & h_{Kb}^P(L) \end{bmatrix}, \tag{4.9}$$

assuming the channels to be fixed over the whole frame L, the channel coefficients are equivalent $h_{kb}^p(1) = \cdots = h_{kb}^p(L)$. Similarly, at the relay side,

$$\mathbf{X}_v = [\sqrt{R_c E_{I_{1v}}} S_1(1) \cdots \sqrt{R_c E_{I_{1v}}} S_1(L) \cdots \sqrt{R_c E_{I_{Kv}}} S_K(1) \cdots \sqrt{R_c E_{I_{Kv}}} S_K(L)]^T, \tag{4.10}$$

$\mathbf{N}_v$ is $((K-1)LP \times 1)$ Gaussian noise vector, and $\mathbf{H}_v$ is $((K-1)LP \times (K-1)LP)$ inter-user channel matrix defined as

$$\mathbf{H}_v = \begin{bmatrix} h_{1v}^1(1) & 0 & \cdots & \cdots & 0 \\ \vdots & \cdots & \cdots & \cdots & \vdots \\ 0 & \cdots & h_{1v}^P(L) & \cdots & 0 \\ \vdots & \cdots & \cdots & \cdots & \vdots \\ 0 & \cdots & \cdots & \cdots & h_{Kv}^P(L) \end{bmatrix}. \tag{4.11}$$

The matrices $\mathbf{R}_b$ and $\mathbf{R}_v$ are the $(KLP \times KLP)$ base station and $((K-1)LP \times (K-1)LP)$ relay cross-correlation, given respectively by

$$\mathbf{R}_b = \begin{bmatrix} R_{1,1}^{(1,1)}(1,1) & \cdots & R_{1,1}^{(1,1)}(1,L) & \cdots & R_{1,K}^{(1,P)}(1,L) \\ \vdots & \cdots & \cdots & \cdots & \vdots \\ R_{1,1}^{(1,1)}(L,1) & \cdots & R_{1,1}^{(1,1)}(L,L) & \cdots & R_{1,K}^{(1,P)}(L,L) \\ \vdots & \cdots & \cdots & \cdots & \vdots \\ R_{K,1}^{(1,1)}(L,1) & \cdots & R_{K,1}^{(1,1)}(L,L) & \cdots & R_{K,K}^{(P,P)}(L,L) \end{bmatrix}, \tag{4.12}$$

$$\mathbf{R}_v = \begin{bmatrix} R_{1,1}^{(1,1)}(1,1) & \cdots & R_{1,1}^{(1,1)}(1,L) & \cdots & R_{1,K}^{(1,P)}(1,L) \\ \vdots & \cdots & \cdots & \cdots & \vdots \\ R_{1,1}^{(1,1)}(L,1) & \cdots & R_{1,1}^{(1,1)}(L,L) & \cdots & R_{1,K}^{(1,P)}(L,L) \\ \vdots & \cdots & \cdots & \cdots & \vdots \\ R_{K,1}^{(1,1)}(L,1) & \cdots & R_{K,1}^{(1,1)}(L,L) & \cdots & R_{K,K}^{(P,P)}(L,L) \end{bmatrix}_{k \neq v}, \tag{4.13}$$

where $R_{k,w}^{(p,s)}(m,n)$ is defined in (4.5). To overcome the effects of MAI, the output of the matched filter bank $\mathbf{Y}_b$, $\mathbf{Y}_v$ are applied to linear mappers as follows

$$\mathbf{Z}_b = (\mathbf{R}_b)^{-1}\mathbf{Y}_b \tag{4.14}$$

$$\mathbf{Z}_v = (\mathbf{R}_v)^{-1}\mathbf{Y}_v \tag{4.15}$$

where $(\mathbf{R}_b)^{-1}$, $(\mathbf{R}_v)^{-1}$ are the inverses of the cross-correlation matrices. The $(KLP \times 1)$ vector $\mathbf{Z}_b$ represents the output of the decorrelator at the base station, defined by

$$\mathbf{Z}_b = \mathbf{H}_b\mathbf{X}_b + (\mathbf{R}_b)^{-1}\mathbf{N}_b \tag{4.16}$$

while the $((K-1)LP \times 1))$ vector $\mathbf{Z}_v$ represents the output of the decorrelator at the relay (v), given by

$$\mathbf{Z}_v = \mathbf{H}_v\mathbf{X}_v + (\mathbf{R}_v)^{-1}\mathbf{N}_v. \tag{4.17}$$

4.3 Performance Analysis

In this section, an upper bound on the average bit error rate at the base station decorrelator output using DAF cooperative method and considering both cases of perfect and imperfect inter-user channels is derived. For the sake of simplicity, we consider binary phase-shift-keying transmission.

4.3.1 Perfect Inter-user Channels

The decorrelator output in the first transmission phase at the base station can be expressed as

$$[\mathbf{Z}_1]_{b1} \;=\; \mathbf{H}_b[\mathbf{X}_1]_{b1} + (\mathbf{R}_b)^{-1}[\mathbf{N}_1]_{b1}$$

where $[\mathbf{X}_1]_{b1}$ represents the data vector transmitted during the first transmission phase,

$$[\mathbf{X}_1]_{b1} = [\sqrt{R_cE_{U_1}}S_1(1)\cdots\sqrt{R_cE_{U_1}}S_1(L)\cdots\sqrt{R_cE_{U_K}}S_K(1)\cdots\sqrt{R_cE_{U_K}}S_K(L)]^T.$$

Also, the decorrelator output during the relaying phase at the base station for the time slot v can be expressed as

$$[\mathbf{Z}_1]_{b2,v} = \mathbf{H}_b[\mathbf{X}_1]_{b2,v} + (\mathbf{R}_b)^{-1}[\mathbf{N}_1]_{b2,v},$$

with $[\mathbf{X}_1]_{b2,v}$ being the data vector transmitted during the v^{th} time slot of the relaying phase,

$$[\mathbf{X}_1]_{b2,v} = [\sqrt{R_c\frac{E_{U_1}}{V}}\tilde{S}_1^{(v)}(1)\cdots\sqrt{R_c\frac{E_{U_1}}{V}}\tilde{S}_1^{(v)}(L)\cdots\sqrt{R_c\frac{E_{U_K}}{V}}\tilde{S}_K^{(v)}(1)\cdots\sqrt{R_c\frac{E_{U_K}}{V}}\tilde{S}_K^{(v)}(L)]^T,$$

where $\tilde{S}_k^{(v)}(l)$ is the estimate of the l^{th} bit of the active user cooperating with user k during the v^{th} time slot of the relaying phase. The P elements of $[\mathbf{Z}_1]_{b1}$ correspond to the decision statistics of the m^{th} bit for user one before RAKE combining, during the first transmission phase, are given by

$$\begin{aligned}
\left[\mathbf{Z}_1^1(m)\right]_{b1} &= \sqrt{R_cE_{U_1}}h_{1b}^1(m)S_1(m) + [(\mathbf{R}_b)^{-1}\mathbf{N}_{b1}]^1 \\
&\vdots \\
\left[\mathbf{Z}_1^P(m)\right]_{b1} &= \sqrt{R_cE_{U_1}}h_{1b}^P(m)S_1(m) + [(\mathbf{R}_b)^{-1}\mathbf{N}_{b1}]^P,
\end{aligned} \tag{4.18}$$

where $[(\mathbf{R}_b)^{-1}\mathbf{N}_{b1}]^p$ is the p^{th} entry of the vector, while the VP elements of $[\mathbf{Z}_1]_{b2}$ that represent the contribution of the relaying phase are given by

$$\begin{aligned}
\left[\mathbf{Z}_1^1(m)\right]_{b2,1} &= \sqrt{R_c\frac{E_{U_{f_1(1)}}}{V}}h_{f_1(1)b}^1(m)\tilde{S}_1(m) + [(\mathbf{R}_b)^{-1}\mathbf{N}_{b2,1}]^1 \\
&\vdots \\
\left[\mathbf{Z}_1^P(m)\right]_{b2,V} &= \sqrt{R_c\frac{E_{U_{f_V(1)}}}{V}}h_{f_V(1)b}^P(m)\tilde{S}_1(m) + [(\mathbf{R}_b)^{-1}\mathbf{N}_{b2,V}]^P.
\end{aligned} \tag{4.19}$$

Under the assumption of perfect inter-user channels, $\tilde{S}_1(m) = S_1(m)$, and by combining the $P(V+1)$ elements, the decision statistic of the desired user signal is

$$\grave{S}_1(m) = h_{1b}^{1*}(m)[\mathbf{Z}_1^1(m)]_{b1} + \cdots + h_{1b}^{P*}(m)[\mathbf{Z}_1^P(m)]_{b1} + h_{f_1(1)b}^{1*}(m)[\mathbf{Z}_1^1(m)]_{b2,1} + \cdots$$

$$+h^{P*}_{f_V(1)b}(m)[\mathbf{Z}^P_1(m)]_{b2,V}. \tag{4.20}$$

Assuming that all relays transmit with the same energy $E_{U_1} = E_{U_{f_1(1)}} = \cdots = E_{U_{f_V(1)}}$, one can show that $\grave{S}_1(m)$ is equivalent to

$$\begin{aligned}\grave{S}_1(m) \;=\; & \left[\sqrt{V}|h^1_{1b}(m)|^2 + \cdots + \sqrt{V}|h^P_{1b}(m)|^2 + |h^1_{f_1(1)b}(m)|^2 + \cdots + |h^P_{f_V(1)b}(m)|^2\right] \\ & \sqrt{R_c\frac{E_{U_1}}{V}}S_1(m) + \mathrm{Re}\left\{h^{1*}_{1b}(m)((\mathbf{R}_b)^{-1}\mathbf{N}_{b1})^1 + \cdots + h^{P*}_{1b}(m)((\mathbf{R}_b)^{-1}\mathbf{N}_{b1})^P + \right. \\ & \left. h^{1*}_{f_1(1)b}(m)((\mathbf{R}_b)^{-1}\mathbf{N}_{b2,1})^1 + \cdots + h^{P*}_{f_V(1)b}(m)((\mathbf{R}_b)^{-1}\mathbf{N}_{b2,V})^P\right\}. \end{aligned} \tag{4.21}$$

4.3.1.1 Conditional Pairwise Probability

The pairwise probability of error can be defined as the number of errors between the received codeword $\grave{\mathbf{S}}_1(m) = [\grave{S}^1_1(m), \grave{S}^2_1(m), \cdots, \grave{S}^{n_c}_1(m)]$ and the all zero transmitted codeword. From (4.21), the probability of pairwise error at the base station conditioned on the uplink channel coefficients can be expressed as

$$P_{b_P}(d) = Q\left(\frac{\sum_{p=1}^{P}\sqrt{dR_c\frac{E_{U_1}}{V}}\left(\sqrt{V}|h^p_{1b}(m)|^2 + \sum_{v=1}^{V}|h^p_{f_v(1)b}(m)|^2\right)}{\sqrt{\sigma_x^2}}\right), \tag{4.22}$$

where

$$\sigma_x^2 = \sigma_n^2\sum_{p=1}^{P}|h^p_{1b}(m)|^2(R_b)^{-2}_1 + \left[\sum_{v=1}^{V}|h^p_{f_v(1)b}(m)|^2(R_b)^{-2}_{f_v(1)}\right] \tag{4.23}$$

and $(R_b)^{-1}_k$ is the sum of the k^{th} row in the inverse of the cross-correlation matrix, and $(R_b)^{-2}_k$ is its squared value. Let us define $\alpha^p_{1b} = |h^p_{1b}(m)|^2$ and $\alpha^p_{f_v(1)b} = |h^p_{f_v(1)b}(m)|^2$. Then α^p_{1b} , $\alpha^p_{f_v(1)b}$ are chi-square distributed with two degrees of freedom with variance σ^2 and characteristic function (3.20). Also, by designating the two parameters $(R_b)^{-2}_1$, $(R_b)^{-2}_{f_v(1)}$, as C_1,$C_{f_v(1)}$, respectively, the noise variance σ_x^2 in (4.23) can be written as

$$\sigma_x^2 = \sigma_n^2\sum_{p=1}^{P}\alpha^p_{1b}C_1 + \left[\sum_{v=1}^{V}\alpha^p_{f_v(1)b}C_{f_v(1)}\right]. \tag{4.24}$$

Define the variable γ as

$$\gamma = \frac{A}{\sqrt{B}} \tag{4.25}$$

where $A = \sum_{p=1}^{P} \sqrt{V}\alpha_{1b}^{p} + \left\{\sum_{v=1}^{V} \alpha_{f_v(1)b}^{p}\right\}$ and $B = \sum_{p=1}^{P} \alpha_{1b}^{p} C_1 + \left\{\sum_{v=1}^{V} \alpha_{f_v(1)b}^{p} C_{f_v(1)}\right\}$. By generalizing (4.22), the pairwise conditional probability can be expressed as

$$P_{b_P}(d) = Q\left(\sqrt{\frac{dR_c E_{U_1}\gamma^2}{V\sigma_n^2}}\right). \tag{4.26}$$

4.3.1.2 Probability Density Function

In order to obtain the pdf of γ, the first step is to determine the joint characteristic function of A and B. Similar to the uncoded single-relay case discussed in chapter 3 (see Appendix A), the joint characteristic function can be expressed as

$$\phi_{A,B}(w_1, w_2) = \frac{\prod_{u=1}^{P(V+1)} \frac{j}{C_u}}{(2\sigma^2)^{P(V+1)}} \left(\sum_{U=1}^{P(V+1)} \frac{C_{Uk}}{\left(w_2 - \frac{y}{jC_U}\right)} \right), \tag{4.27}$$

where $C_{Uk} = \frac{K_U}{y^{P(V+1)-1}}$ is the residue term obtained from the partial fraction, and K_U is a function of the cross-correlation among users. The joint pdf, $f(A, B)$, is then given by

$$f(A,B) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi_{A,B}(w_1, w_2) \exp^{-jw_1 A} \exp^{-jw_2 B} dw_1 dw_2. \tag{4.28}$$

By solving the double integral and letting $W = B$, we obtain

$$f(\gamma, W) = \frac{\prod_{u=1}^{P(V+1)} \frac{1}{C_u}}{4\pi^2 (2\sigma^2)^{P(V+1)}} \sum_{U=1}^{P(V+1)} \Psi_U W^{\frac{1}{2}} \left(\gamma\sqrt{W} - \frac{W}{C_U}\right)^{P(V+1)-2} \exp^{\left(\frac{-\gamma\sqrt{W}}{2\sigma^2}\right)}, \tag{4.29}$$

where $\Psi_U = \frac{4\pi^2 K_U}{\Gamma(P(V+1)-1)}$. Hence, one can determine the pdf of the SNR as

$$f(\gamma) = \frac{\prod_{u=1}^{P(V+1)} \frac{1}{C_u}}{4\pi^2 (2\sigma^2)^{P(V+1)}} \sum_{U=1}^{P(V+1)} \Psi_U R_U(\gamma), \tag{4.30}$$

where

$$R_U(\gamma) = \int_0^{C_U^2\gamma^2} W^{\frac{1}{2}} \left(\gamma\sqrt{W} - \frac{W}{C_U}\right)^{P(V+1)-2} \exp^{\left(\frac{-\gamma\sqrt{W}}{2\sigma^2}\right)} dW. \qquad (4.31)$$

Using the binomial series expansion, setting $t = \sqrt{W}$ and expressing (4.31) in terms of confluent hypergeometric function, as shown in Appendix B for the uncoded case we get

$$\begin{aligned} R_U(\gamma) &= 2\gamma^{2P(V+1)-1} C_U^{P(V+1)+1} \exp^{\left(\frac{-C_U\gamma^2}{2\sigma^2}\right)} \sum_{m=0}^{P(V+1)-2} \left(\begin{array}{c} P(V+1)-2 \\ m \end{array}\right) \\ &\left(\frac{(-1)^{P(V+1)-2-m}}{2P(V+1)-m-1}\right) {}_1F_1\left(1; 2P(V+1)-m; \frac{C_U\gamma^2}{2\sigma^2}\right). \qquad (4.32) \end{aligned}$$

Finally, the pdf of the SNR can be obtained by substituting (4.32) in (4.30).

4.3.1.3 Pairwise Probability of Error

The pairwise probability of error can be obtained by averaging (4.26) over the pdf given in (4.30)

$$P_{b_P}(d) = \int_0^{\infty} Q(\sqrt{R_c d\delta\gamma^2}) f(\gamma) d\gamma, \qquad (4.33)$$

where $\delta = \frac{E_{U_1}}{V\sigma_n^2}$, it can be re-written as

$$\begin{aligned} P_{b_P}(d) &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\int_0^{\infty} \exp^{\frac{-R_c d\delta\gamma^2}{2\sin^2\theta}} f(\gamma) d\gamma d\theta \\ &= \frac{\prod_{u=1}^{P(V+1)} \frac{1}{C_u}}{4\pi^3} \sum_{U=1}^{P(V+1)} \Psi_U F_U(\delta) \qquad (4.34) \end{aligned}$$

with

$$\begin{aligned} F_U(\delta) &= \frac{1}{(2\sigma^2)^{P(V+1)}} \int_0^{\frac{\pi}{2}}\int_0^{\infty} \exp^{\frac{-R_c d\delta\gamma^2}{2\sin^2\theta}} R_U(\gamma) d\gamma d\theta \\ &= 2C_U^{P(V+1)+1} \sum_{m=0}^{P(V+1)-2} \left(\begin{array}{c} P(V+1)-2 \\ m \end{array}\right) \frac{(-1)^{P(V+1)-2-m}}{2P(V+1)-1-m} G_m(\delta), (4.35) \end{aligned}$$

and

$$G_m(\delta) = \frac{1}{(2\sigma^2)^{P(V+1)}} \int_0^{\frac{\pi}{2}} \int_0^{\infty} \gamma^{2P(V+1)-1} \exp^{-\gamma^2(\frac{R_c d\delta}{2\sin^2\theta}+\frac{C_U}{2\sigma^2})} {}_1F_1\left(1; 2P(V+1)-m; \frac{C_U\gamma^2}{2\sigma^2}\right) d\gamma d\theta. \quad (4.36)$$

Letting $t = \gamma^2$ in (4.36), and using (3.34), (4.36) yields

$$G_m(\overline{\delta}) = \frac{\Gamma(P(V+1))}{2(R_c dP\overline{\delta})^{P(V+1)}} \int_0^{\frac{\pi}{2}} (\sin^2\theta)^{P(V+1)} {}_2F_1\left(2P(V+1)-m-1, P(V+1); 2P(V+1)-m; \frac{-C_U\sin^2\theta}{R_c dP\overline{\delta}}\right) d\theta \quad (4.37)$$

where $\overline{\delta} = \frac{E\left[|h_{kb}^P(m)|^2\right]E_{U_1}}{V\sigma_n^2}$. Substituting $V_s = \sin^2\theta$ in (4.37) and using (3.36), we have

$$G_m(\overline{\delta}) = \frac{\Gamma(\frac{1}{2})\Gamma(\frac{2P(V+1)+1}{2})}{4(P(V+1))(R_c dP\overline{\delta}\ )^{P(V+1)}} {}_3F_2\left(\frac{2P(V+1)+1}{2}, 2P(V+1)-m-1, P(V+1); P(V+1)+1, 2P(V+1)-m; \frac{-C_U}{R_c dP\overline{\delta}}\right). \quad (4.38)$$

Finally, using (4.38) and (4.35), the average pairwise probability for V cooperating relays in (4.34) can be evaluated as

$$P_{b_P}(d) = \left(\frac{\prod_{u=1}^{P(V+1)} \frac{1}{C_u}}{4\pi^3}\right) \sum_{U=1}^{P(V+1)} 2\Psi_U C_U^{P(V+1)+1} \sum_{m=0}^{P(V+1)-2} \binom{P(V+1)-2}{m} \left(\frac{(-1)^{P(V+1)-2-m}}{2P(V+1)-1-m}\right) \left(\frac{\Gamma(\frac{1}{2})\Gamma(\frac{2P(V+1)+1}{2})}{4(P(V+1))(R_c dP\overline{\delta}\ )^{P(V+1)}}\right) {}_3F_2\left(\frac{2P(V+1)+1}{2}, 2P(V+1)-m-1, P(V+1); P(V+1)+1, 2P(V+1)-m; \frac{-C_U}{R_c dP\overline{\delta}}\right). \quad (4.39)$$

4.3.1.4 Upper Bound Probability of Bit Error

By obtaining the average pairwise error probability, the upper bound on the probability of bit error can be expressed as [3]

$$P_{e_P} < \frac{1}{m_c} \sum_{d=d_f}^{\infty} c(d) P_{b_P}(d), \quad (4.40)$$

where d_f is the code's minimum free distance, m_c is the number of information bits shifted to the encoder at the same time instant and $c(d)$ is the sum of errors for error events of distance d [49].

4.3.2 Inter-User Channel Errors

As mentioned earlier, the performance of the cooperative system with perfect inter-user channel is not realistic in some cases and can only serve as a lower bound indicating optimal performance. Therefore, here we consider the effect of errors in the inter-user channels where cooperation among users can only take place if the relay detection is error-free. If errors exist, the corresponding relay (cooperating user) stays idle during the second transmission period. First we derive the probability of pairwise error on the inter-user channels (source - relay channels), from which we can express the pairwise probability of error of the overall network.

Without loss of generality, consider the m^{th} bit of user one as the desired bit, the decorrelator outputs at the cooperative user side $f_v(1)$ is given by

$$[\mathbf{Z}_1]_v = \mathbf{H}_v[\mathbf{X}_1]_v + (\mathbf{R}_v)^{-1}[\mathbf{N}_1]_v, \tag{4.41}$$

where the P elements of Z_1 are

$$\begin{aligned} \left[\mathbf{Z}_1^1(m)\right]_v &= \sqrt{R_c E_{I_{1v}}} h^1_{1f_v(1)}(m) S_1(m) + [(\mathbf{R}_v)^{-1}\mathbf{N}_1]^1 \\ &\vdots \\ \left[\mathbf{Z}_1^P(m)\right]_v &= \sqrt{R_c E_{I_{1v}}} h^P_{1f_v(1)}(m) S_1(m) + [(\mathbf{R}_v)^{-1}\mathbf{N}_1]^P. \end{aligned} \tag{4.42}$$

At the output of the RAKE combiner, we have the user data decision statistic

$$\ddot{S}_1(m) \;=\; [|h^1_{1f_v(1)}(m)|^2 + \cdots + |h^P_{1f_v(1)}(m)|^2]\sqrt{R_c E_{I_{1v}}} S_1(m) +$$

$$\mathrm{Re}\left\{[h^{1*}_{1f_v(1)}(m)((\mathbf{R}_v)^{-1}\mathbf{N}_1)^1 + \cdots + \cdots + h^{P*}_{1f_v(1)}(m)((\mathbf{R}_v)^{-1}\mathbf{N}_1)^P\right\}, \quad (4.43)$$

and the pairwise probability of error over the inter-user channel

$$P_{b_E}(d) = Q\left(\frac{\sum_{p=1}^{P}\sqrt{dR_cE_{I_{1v}}}(|h^p_{1f_v(1)}(m)|^2)}{\sqrt{\sigma^2_{x_r}}}\right), \tag{4.44}$$

where

$$\sigma^2_{x_r} = \sigma^2_n \sum_{p=1}^{P} |h^p_{1f_v(1)}(m)|^2 (R_v)_1^{-2} \tag{4.45}$$

and $(R_v)_k^{-2}$ is the square of the sum of the k^{th} row in the inverse of the cross-correlation matrix. Similar to the case of perfect inter-user channel, we define $\alpha^p_{1f_v(1)} = |h^p_{1f_v(1)}(m)|^2$ with characteristic function given by (3.20). Also, letting $(R_v)_1^{-2} = C_1$, the noise variance $\sigma^2_{x_r}$ can be expressed as

$$\sigma^2_{x_r} = \sigma^2_n \sum_{p=1}^{P} \alpha^p_{1f_v(1)} C_1. \tag{4.46}$$

Following the same analysis as the perfect case, we define the variable γ_r as

$$\gamma_r = \frac{A_r}{\sqrt{B_r}} \tag{4.47}$$

where $A_r = \sum_{p=1}^{P} \alpha^p_{1f_v(1)}$ and $B_r = \sum_{p=1}^{P} \alpha^p_{1f_v(1)} C_1$. Then, it is easy to show that

$$\phi_{A_r,B_r}(w_1, w_2) = \frac{\prod_{u=1}^{P} \frac{j}{C_u}}{(2\sigma^2)^P}\left(\sum_{U=1}^{P} \frac{C_{Uk_r}}{(w_2 - \frac{y}{jC_U})}\right) \tag{4.48}$$

where $C_{Uk_r} = \frac{K_{U_r}}{y^{P-1}}$ is the residue term obtained from the partial fraction and K_{U_r} can be obtained in terms of the cross-correlation among users. The pdf, $f(A_r, B_r)$, is then given by

$$f(A_r, B_r) = \frac{\prod_{u=1}^{P} \frac{1}{C_u}}{4\pi^2(2\sigma^2)^P} \sum_{U=1}^{P} \Psi_{U_r}\left(A_r - \frac{B_r}{C_U}\right)^{P-2} \exp^{\left(\frac{-A_r}{2\sigma^2}\right)}$$

with $\Psi_{U_r} = \frac{4\pi^2 K_{U_r}}{\Gamma(P)}$, and

$$f(\gamma_r) = \frac{\prod_{u=1}^{P} \frac{1}{C_u}}{4\pi^2 (2\sigma^2)^P} \sum_{U=1}^{P} \Psi_{U_r} R_{U_r}(\gamma_r) \tag{4.49}$$

where

$$\begin{aligned} R_{U_r}(\gamma_r) &= 2\gamma_r^{2P-1} C_U^{P+1} \exp^{(\frac{-C_U \gamma_r^2}{2\sigma^2})} \sum_{m=0}^{P-2} \left(\begin{array}{c} P-2 \\ m \end{array} \right) \\ & \frac{(-1)^{P-2-m}}{2P-m-1} {}_1F_1 \left(1; 2P-m; \frac{C_U \gamma_r^2}{2\sigma^2} \right). \end{aligned} \tag{4.50}$$

The pairwise probability at the relay can then be obtained using (4.44) and (4.49),

$$\begin{aligned} P_{b_E}(d) &= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \int_0^{\infty} \exp^{\frac{-R_c d \delta_r \gamma_r^2}{2 \sin^2 \theta}} f(\gamma_r) d\gamma_r d\theta \\ &= \frac{\prod_{u=1}^{P} \frac{1}{C_u}}{4\pi^3} \sum_{U=1}^{P} \Psi_{U_r} F_{U_r}(\overline{\delta}_r) \end{aligned} \tag{4.51}$$

with

$$F_{U_r}(\overline{\delta}_r) = 2C_U^{P+1} \sum_{m=0}^{P-2} \left(\begin{array}{c} P-2 \\ m \end{array} \right) \frac{(-1)^{P-2-m}}{2P-m-1} G_{m_r}(\overline{\delta}_r)$$

where $\overline{\delta}_r = \frac{E\left[|h^p_{kf_v(k)}(m)|^2\right] E_{I_{1v}}}{\sigma_n^2}$ is the average SNR of the inter-user channel, and

$$G_{m_r}(\overline{\delta}_r) = \frac{\Gamma(P)}{2(R_c d P \overline{\delta}_r)^P} \int_0^{\frac{\pi}{2}} (\sin^2\theta)^P {}_2F_1 \left(2P-m-1, P; 2P-m; \frac{-C_U \sin^2\theta}{R_c d P \overline{\delta}_r} \right) d\theta. \tag{4.52}$$

Similar to the perfect case, the average pairwise probability for the inter-user channel can be expressed as

$$\begin{aligned} P_{b_E}(d) &= \frac{\prod_{u=1}^{P} \frac{1}{C_u}}{4\pi^3} \sum_{U=1}^{P} 2\Psi_{U_1} C_U^{P+1} \\ & \sum_{m=0}^{P-2} \left(\begin{array}{c} P-2 \\ m \end{array} \right) \left(\frac{(-1)^{P-2-m}}{2P-m-1} \right) \left(\frac{\Gamma(\frac{1}{2})\Gamma(\frac{2P+1}{2})}{4(P)(R_c d P \overline{\delta}_r)^P} \right) \\ & {}_3F_2 \left(\frac{2P+1}{2}, 2P-m-1, P; P+1, 2P-m; \frac{-C_U}{R_c d P \overline{\delta}_r} \right). \end{aligned} \tag{4.53}$$

4.3.3 Average Pairwise Probability

Having obtained the pairwise probability at the relay, one can find the average pairwise probability at the base station for the imperfect inter-user channels case as

$$P_b(d) = \underbrace{(1 - P_{b_E}(d))^V (P_{b_P}(d))_V}_{(1)} + \underbrace{\sum_{n=1}^{V-1} \binom{V}{n} (P_{b_E}(d))^n (1 - P_{b_E}(d))^{V-n} (P_{b_P}(d))_{V-n}}_{(2)} + \underbrace{(P_{b_E}(d))^V (P_{b_P}(d))_0}_{(3)}. \quad (4.54)$$

The first term represents the system pairwise probability in case of error-free inter-user channels (all V relays are cooperating) with $P_{b_E}(d)$ representing the pairwise probability per inter-user channel given by (4.53) and $(P_{b_P}(d))_V$ is the overall system's pairwise probability with (V) cooperating relays in (4.39). The second term expresses the combination of erroneous and error-free inter-user channels with $(P_{b_P}(d))_{V-n}$ as the overall system's pairwise probability with $(V-n)$ relays, while the third term is the pairwise probability in case of all inter-user channels are in error with $(P_{b_P}(d))_0$ as the pairwise probability for the case of zero cooperating relays (no cooperation).

4.4 Simulation Results

In this section, we present simulation results to assess the performance of a multi-relay coded cooperative system when considering different scenarios. Also, we examine the accuracy of our analytical results obtained in Sec. 4.3. We consider the cooperative diversity spreading scheme described before with one transmit and one receive antennas. A multi-user asynchronous DS-CDMA system with BPSK transmission is assumed where every user data is spread using non-orthogonal Gold codes of length

31 chips. Convolutional coding of rate $\frac{1}{2}$ with constraint length $[7, 5]$ is used at both active user and relay sides. For the asynchronous channel, the transmitted frames are 100 bits each, the fading coefficients are fixed for a number of frames and the delay between users, τ_k, is uniformly distributed along the symbol period. We also assume perfect knowledge of the channel coefficients at the base station and the relay. In order to maintain a constant data rate for all cases, the transmitting users in the non-cooperation case will transmit each data bit V times, so that the data rate is the same as the cooperating case.

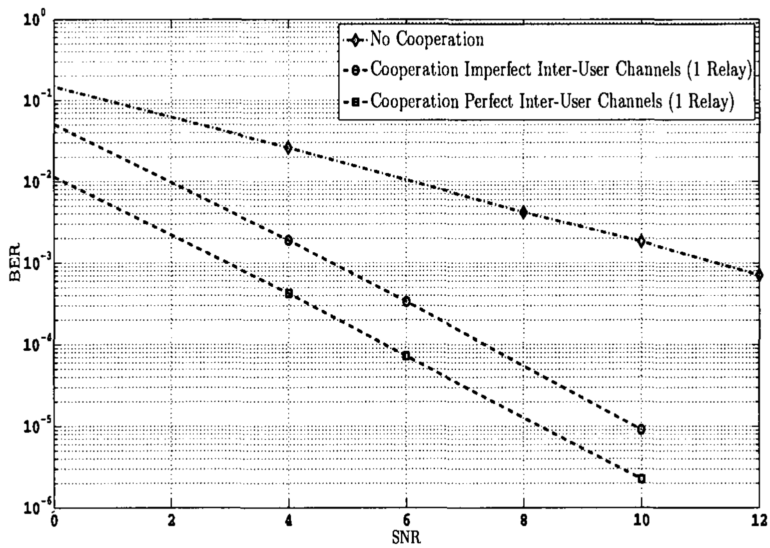

Figure 4.5: BER performance for a 8-user asynchronous DS-CDMA coded network over 2 paths frequency-selective slow fading channels (1 relay cooperation).

In Fig. 4.5, a wireless network using the DAF coded cooperating method is simulated over a 2-path frequency-selective fading channel. The SNR of the inter-user channel is assumed to be equal to the uplink channel. The performance of the non-cooperative case is also compared with the perfect and imperfect inter-user channels of a single-relay cooperation. Figure 4.6 shows the performance for the case of 2-relay cooperating network. We can conclude that by increasing the number of cooperating relays per active user, the overall system performance will be able to gain higher cooperative diversity degrees.

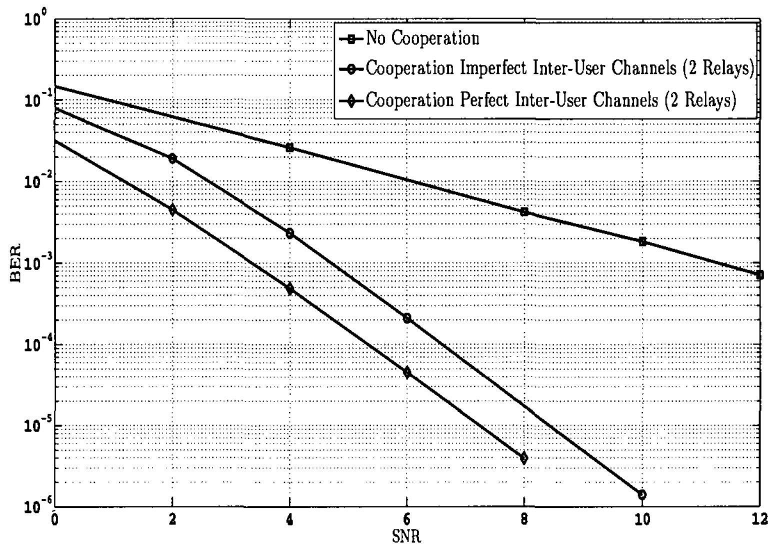

Figure 4.6: BER performance for a 8-user asynchronous DS-CDMA coded network over 2 paths frequency-selective slow fading channels (2 relays cooperation).

By fixing the inter-user channel SNR, Fig. 4.7 compares the performance of the coded cooperating network for eight users over 2-path slow frequency-selective fading channel. With the overall system performance for the non-cooperative case acting as the upper performance bound, and the perfect inter-user channel as the lower bound, it can be concluded that as the quality of the inter-user channel improves, a remarkable improvement in the system performance can be seen where large diversity gains achieved. It is shown that the system can achieve the full diversity gain when considering perfect inter-user channels. The results also suggest that, in the presence of MAI at the relay, the overall system performance degrades significantly and no benefit for user cooperation can take place.

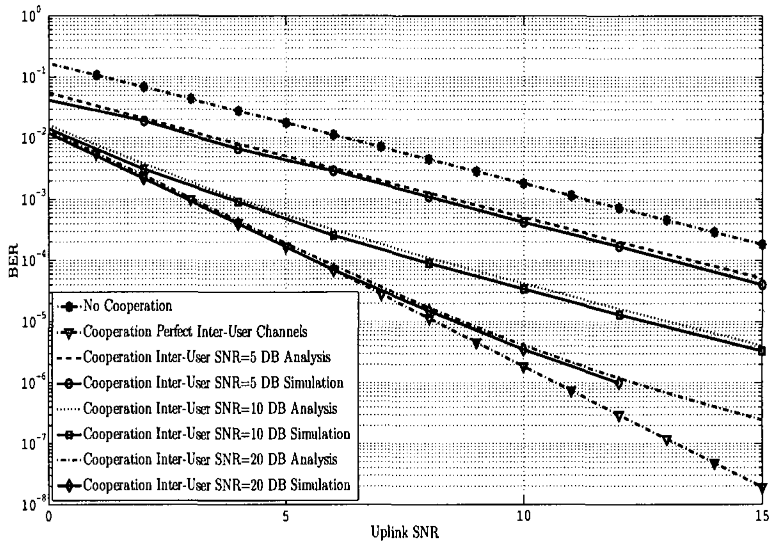

Figure 4.7: BER performance for a 8-user asynchronous DS-CDMA network over 2 paths frequency-selective slow fading channels (1 relay cooperation).

Figs. 4.8 and 4.9 show the performance of an eight-user DAF coded cooperating network simulated over a different number of resolvable paths for both the perfect and imperfect inter-user channel, respectively. In these results, the SNR of the inter-user channel is taken to be equal to the uplink channel (i.e., first transmission phase). For the single relay case, one can notice that as the number of paths per channel increases, the network benefits from higher diversity gains. Similarly, for the multi-relay case shown in Figs. 4.10 and 4.11, the overall diversity achieved is $(V + 1)P$ when the inter-user channels are error-free. That is, the overall diversity order of the system is a function of the number of cooperating relays V, and the number of resolvable paths P. In all the above scenarios, the accuracy of our analytical results is quite evident.

4.5 Conclusion

We examined the performance of a multi-relay coded cooperative diversity using DAF method in asynchronous DS-CDMA systems over frequency-selective slow fading channels. In order to mitigate the multi-access interference, decorrelator multiuser detectors have been employed at both the relays and the base station. The performance of the system has been studied, and an exact expression for the pairwise probability of error has been obtained for the case of imperfect and perfect inter-user channels. The impact of erroneous inter-user channels on the overall system performance was also studied, where we have shown that as the quality of the channel gets poorer, the system starts losing the diversity. As a result, we showed that DAF cooperating technique should be integrated with channel coding. Furthermore the impact of the number of cooperating relays have been studied in this work. We have presented and compared the performance of the system for different number of cooperating relays. A trade-off between the system performance and transmission rate was discussed, as the number of relays increases, the system gains higher diversity order, but on the other side the transmission rate starts to decline. Finally, our results show that the diversity gain of a multi-relay DAF cooperating network will mainly depend on the number of cooperating relays, inter-user channels conditions and the number of paths per channel.

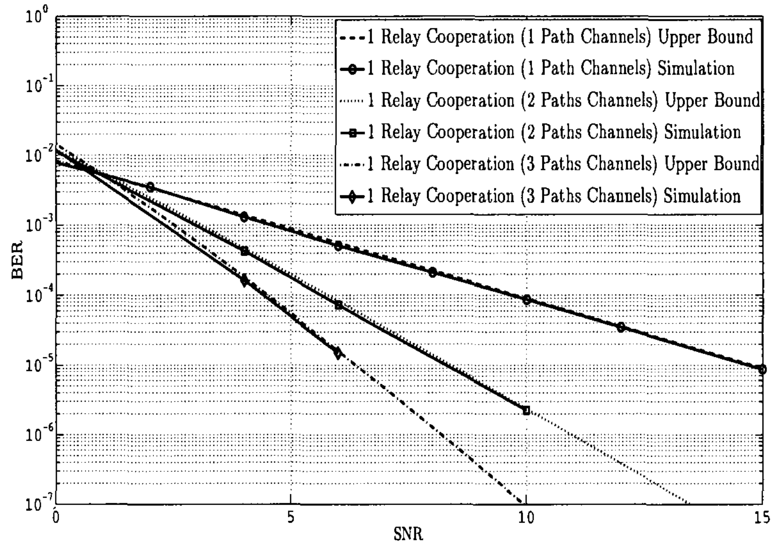

Figure 4.8: Performance for an 8-user network with DAF cooperation as a function of the number of paths for perfect inter-user channels (1 relay cooperation).

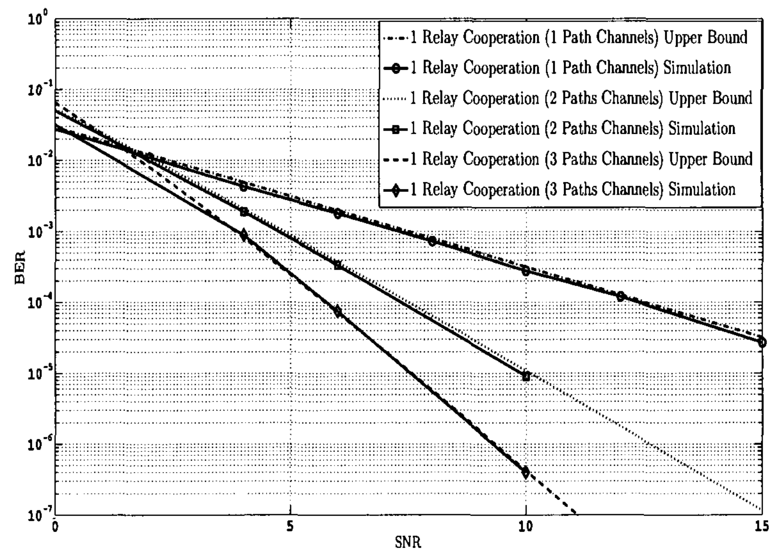

Figure 4.9: Performance for an 8-user network with DAF cooperation as a function of the number of paths for imperfect inter-user channels case (1 relay cooperation).

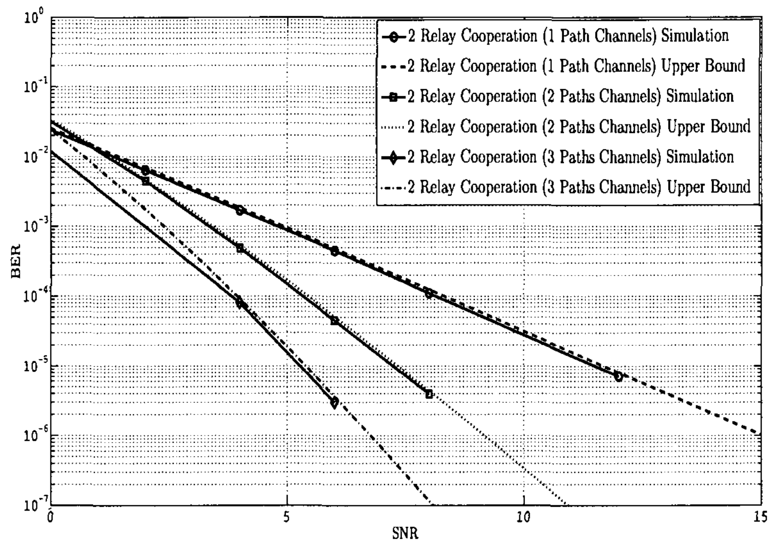

Figure 4.10: Performance for an 8-user network with DAF cooperation as a function of the number of paths for perfect inter-user channels case (2 relays cooperation).

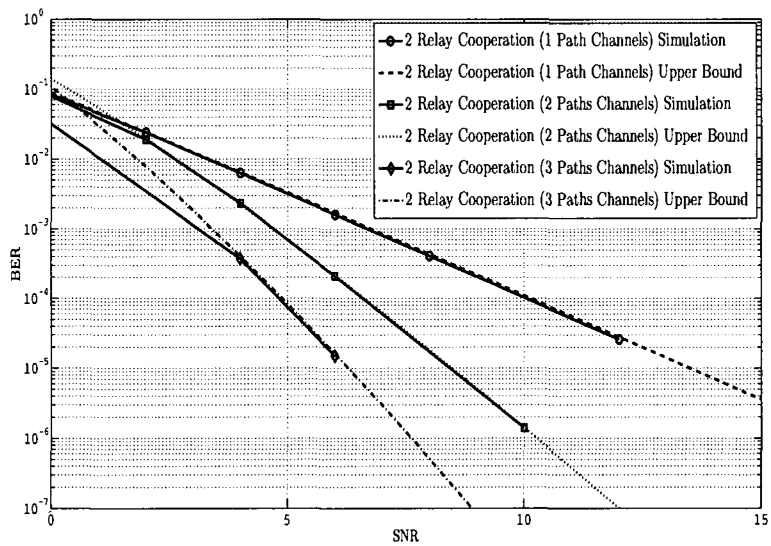

Figure 4.11: Performance for an 8-user network with DAF cooperation as a function of the number of paths for imperfect inter-user channels case (2 relays cooperation).

Chapter 5

Conclusions and Future Work

5.1 Conclusions

In this thesis we focused on studying the performance of cooperative diversity techniques over different channel conditions. First, we introduced DAF cooperative diversity to DS-CDMA networks over frequency-selective slow fading channels. Through the thesis, we proved that cooperative networks will start benefiting from the cooperative diversity gains as the inter-user channels condition improves. We studied both extreme cases assuming perfect inter-user channel cooperation and no cooperation case.

In chapter 3, the performance of DAF cooperative networks was analyzed for asynchronous DS-CDMA systems in frequency-selective slow fading environment. We studied the multiple-access interference impact on the performance of the cooperative networks. Both conventional matched filter receivers and decorrelator multiuser detectors were employed at the cooperating relay side and base station. When decorrelator detectors are employed at the relay, the system is immune to the multiple-access interference. It was shown that the conventional receiver becomes almost ineffective due to the fact that the detected signals at both relays and base station will be suffering of high levels of interference. As a result, the performance of the cooperative network employing matched filter receivers at the relays can in some cases be worse than the non-cooperative network. We showed that the full benefits of cooperative diversity cannot be achieved if no multiuser interference suppression is employed at the cooperative end. We considered two scenarios; perfect and imperfect inter-user channel. The bit-error-rate performance of the cooperative system was investigated for an uplink transmission where a decorrelator detector is used at both the relay and base station receivers. Both simulation and analytical results were presented to demonstrate the diversity gains of cooperative networks.

Through chapter 4, multi-relay decode-and forward cooperative transmission employing convolutional coding was studied for the case of asynchronous DS-CDMA systems over frequency-selective slow fading channels. The performance of the network was analyzed for both cases of perfect and imperfect inter-user channels. Decorrelator multiuser detectors were used at both the relay and base station receivers with the advantage of mitigating the multipath fading and multiple-access interference effects. We obtained an expression for the pairwise probability of error, where it was used to evaluate an upper bound on the BER. Moreover, simulation results were presented and compared to the performance analysis, showing that the network will be able to reach the full benefit of cooperation only by accompanying it with some type of channel coding. Also the system's performance was shown to gain more diversity degrees with the increase of the number of cooperating relays per active user. We found that for an active user transmitting to the base station over P paths and cooperating with V relays, the expected diversity degree is $P(V + 1)$.

5.2 Future Work

The following could be an interesting topics for future studies:

1. We studied the performance of cooperative networks over frequency-selective slow fading channels. This work could be extended to different channel conditions as fast fading channels, in which the channel state information perfect knowledge assumption is unrealistic.

2. We have assumed perfect channel and spreading codes knowledge at the relay side which is not fully practical. Channel estimation is an important aspect of our results. Therefore, a study of the effects of imperfect channel state information on the uncoded and coded systems is of great interest.

3. Through the thesis, we assumed random relay selection criteria, but future work can include more advanced criteria in relay selection, depending on the reliability of the (user-relay) and (relay-base station) links.

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Appendices

Appendix A

In order to obtain the pdf of γ, the first step is to determine the joint characteristic function of A and B as follows

$$\phi_{A,B}(w_1, w_2) = E\left[\exp^{j(w_1 A + w_2 B)}\right], \tag{A-1}$$

where $E[.]$ is the expected value of the enclosed argument. Equation (A-1) can be rewritten with respect to α and C as

$$\phi_{A,B}(w_1, w_2) = E\left[\exp^{j\sum_{p=1}^{P}\alpha_{1b}^{p}(w_1 + w_2 C_1)} . \exp^{j\sum_{p=1}^{P}\alpha_{f(1)b}^{p}(w_1 + w_2 C_{f(1)})}\right]. \tag{A-2}$$

Letting $y = \frac{1}{2\sigma^2} - jw_1$, and assuming independent fading channels, we can show that

$$\phi_{A,B}(w_1, w_2) = \prod_{p=1}^{P} E\left[\exp^{j\alpha_{1b}^{p}(w_1 + w_2 C_1)}\right] E\left[\exp^{j\alpha_{f(1)b}^{p}(w_1 + w_2 C_{f(1)})}\right]. \tag{A-3}$$

As α_{1b}^{p} and $\alpha_{f(1)b}^{p}$ are chi-square distributed each with two degrees of freedom, and using the characteristic function in (3.20), the joint characteristic function of A and B can be expressed as

$$\phi_{A,B}(w_1, w_2) = \frac{1}{(2\sigma^2)^{2P}} \prod_{p=1}^{P} \frac{1}{(y - jC_1 w_2)(y - jC_{f(1)} w_2)}, \tag{A-4}$$

or equivalently as

$$\phi_{A,B}(w_1,w_2) = \frac{1}{(2\sigma^2)^{2P}} \prod_{U=1}^{2P} \frac{1}{(y - jC_U w_2)}. \quad \text{(A-5)}$$

In order to simplify the analysis, we will use a partial fraction expansion of a rational function with high order poles method [50]. Equation (A-5) can be simplified to

$$\phi_{A,B}(w_1,w_2) = \frac{\prod_{u=1}^{2P} \frac{j}{C_u}}{(2\sigma^2)^{2P}} \prod_{U=1}^{2P} \frac{1}{(w_2 - \frac{y}{jC_U})}. \quad \text{(A-6)}$$

Consider Z a proper rational function having no zeros and $2P$ poles $\frac{y}{jC_U}$

$$Z(w_1,w_2) = \prod_{U=1}^{2P} \frac{1}{\left(w_2 - \frac{y}{jC_U}\right)}. \quad \text{(A-7)}$$

Using [eq.(1) in [50]],

$$Z(w_1,w_2) = \sum_{U=1}^{2P} \frac{C_{Uk}}{\left(w_2 - \frac{y}{jC_U}\right)}, \quad \text{(A-8)}$$

where C_{Uk} represents the expansion coefficients. The characteristic function in (A-5) can be simplified to

$$\phi_{A,B}(w_1,w_2) = \frac{\prod_{u=1}^{2P} \frac{j}{C_u}}{(2\sigma^2)^{2P}} \left(\sum_{U=1}^{2P} \frac{C_{Uk}}{\left(w_2 - \frac{y}{jC_U}\right)} \right), \quad \text{(A-9)}$$

where $C_{Uk} = \frac{K_U}{y^{2P-1}}$ is the residue term obtained from the partial fraction, and K_U is a function of the cross-correlations among users. The pdf, $f(A,B)$, is then given by

$$\begin{aligned} f(A,B) &= \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi_{A,B}(w_1,w_2) \exp^{-jw_1 A} \exp^{-jw_2 B} dw_1 dw_2 \\ &= \frac{\prod_{u=1}^{2P} \frac{j}{C_u}}{4\pi^2 (2\sigma^2)^{2P}} \sum_{U=1}^{2P} K_U I_U, \end{aligned} \quad \text{(A-10)}$$

where

$$I_U(A,B) = \int_{-\infty}^{\infty} \frac{\exp^{-jw_1 A}}{y^{2P}} \int_{-\infty}^{\infty} \frac{\exp^{-jw_2 B}}{\left(w_2 - \frac{y}{jC_U}\right)} dw_2 \, dw_1. \quad \text{(A-11)}$$

By solving the double integral, we can find that

$$I_U(A,B) = \frac{4\pi^2}{j\Gamma(2P-1)}\left(A - \frac{B}{C_U}\right)^{2P-2} \exp^{\left(\frac{-A}{2\sigma^2}\right)}, \tag{A-12}$$

where $\Gamma(.)$ represents gamma function. Hence, $f(A,B)$ is given by

$$f(A,B) = \frac{\prod_{u=1}^{2P} \frac{1}{C_u}}{4\pi^2(2\sigma^2)^{2P}} \sum_{U=1}^{2P} \Psi_U \left(A - \frac{B}{C_U}\right)^{2P-2} \exp^{\left(\frac{-A}{2\sigma^2}\right)}, \tag{A-13}$$

where $\Psi_U = \frac{4\pi^2 K_U}{\Gamma(2P-1)}$. Letting $W = B$, the pdf of γ and W as defined in (3.22) can be obtained from the relation $f(\gamma, W) = f(A,B)|\Omega(\gamma, W)|$ where $\Omega(\gamma, W) = \sqrt{W}$ is the Jacobian of the transformation. By substitution of (A-13) in this relation, we obtain

$$f(\gamma, W) = \frac{\prod_{u=1}^{2P} \frac{1}{C_u}}{4\pi^2(2\sigma^2)^{2P}} \sum_{U=1}^{2P} \Psi_U W^{\frac{1}{2}} \left(\gamma\sqrt{W} - \frac{W}{C_U}\right)^{2P-2} \exp^{\left(\frac{-\gamma\sqrt{W}}{2\sigma^2}\right)}. \tag{A-14}$$

Appendix B

We can determine the pdf of the SNR from equation (A-14) as follow

$$f(\gamma) = \frac{\prod_{u=1}^{2P} \frac{1}{C_u}}{4\pi^2(2\sigma^2)^{2P}} \sum_{U=1}^{2P} \Psi_U R_U(\gamma), \tag{B-1}$$

where

$$R_U(\gamma) = \int_0^{C_U^2\gamma^2} W^{\frac{1}{2}} \left(\gamma\sqrt{W} - \frac{W}{C_U}\right)^{2P-2} \exp^{\left(\frac{-\gamma\sqrt{W}}{2\sigma^2}\right)} dW. \tag{B-2}$$

Assume $X_1 = \left(\gamma\sqrt{W} - \frac{W}{C_U}\right)^{2P-2}$, then

$$X_1 = (-1)^{2P-2}\left(\frac{W}{C_U}\right)^{2P-2}\left(1 - \frac{\gamma\ C_U}{\sqrt{W}}\right)^{2P-2}. \tag{B-3}$$

Using binomial series, we can show that

$$\begin{aligned} X_1 \;=\; & (-1)^{2P-2}\left(\frac{W}{C_U}\right)^{2P-2} [1 + (2P-2)\left(-\gamma C_U \frac{1}{\sqrt{W}}\right) \\ & + \frac{(2P-2)(2P-3)}{2!}\left(-\gamma C_U \frac{1}{\sqrt{W}}\right)^2 + \cdots\cdots, \end{aligned} \tag{B-4}$$

or equivalently

$$X_1 = (-1)^{2P-2}(\frac{W}{C_U})^{2P-2} \sum_{m=0}^{2P-2} \begin{pmatrix} 2P-2 \\ m \end{pmatrix} (-1)^{-m}\gamma^m C_U^m \left(\sqrt{W}\right)^{-m}. \tag{B-5}$$

Then $R_U(\gamma)$ can be expressed as

$$\begin{aligned} R_U(\gamma) \;=\; & \sum_{m=0}^{2P-2} \begin{pmatrix} 2P-2 \\ m \end{pmatrix} (-1)^{2P-2-m} \frac{\gamma^m}{C_U^{2P-2-m}} \\ & \int_0^{C_U^2\gamma^2} \left(\sqrt{W}\right)^{4P-3-m} \exp^{\left(\frac{-\gamma\sqrt{W}}{2\sigma^2}\right)} dW. \end{aligned} \tag{B-6}$$

In what follows, we let

$$II_U(\gamma) = \int_0^{C_U^2\gamma^2} \left(\sqrt{W}\right)^{4P-3-m} \exp^{\left(\frac{-\gamma\sqrt{W}}{2\sigma^2}\right)} dW, \tag{B-7}$$

and use [35]

$$\begin{aligned}\int_{C_1}^{C_2} x^n \exp^{-ax} dx &= \frac{1}{a(n+1)} C_2^n (aC_2)^{\frac{-n}{2}} \\ &\exp^{\left(\frac{-aC_2}{2}\right)} M\left(\frac{n}{2}, \frac{n+1}{2}, aC_2\right) - C_1^n (aC_1)^{\left(\frac{-n}{2}\right)} \exp^{\left(\frac{-aC_1}{2}\right)} \\ &M\left(\frac{n}{2}, \frac{n+1}{2}, aC_1\right)],\end{aligned} \tag{B-8}$$

where $M(k, m, z)$ represents the WhittakerM function [51]

$$\begin{aligned}M(k, m, z) &= z^{\frac{1}{2}+m} \exp^{-\frac{z}{2}}[1 + \frac{\frac{1}{2}+m-k}{1!(2m+1)} z + \ldots. \\ &+ \frac{(\frac{1}{2}+m-k)(\frac{3}{2}+m-k)}{2!(2m+1)(2m+2)} z^2 + \ldots.].\end{aligned} \tag{B-9}$$

Now, by setting $t = \sqrt{W}$ in (B-7), we get

$$\begin{aligned}II_U(\gamma) &= 2\int_0^{C_U\gamma} t^{4P-2-m} \exp^{\left(\frac{-\gamma t}{2\sigma^2}\right)} dt \\ &= \frac{2^{2P-\frac{m}{2}+1}\sigma^{4P-m} C_U^{2P-1-\frac{m}{2}}}{\gamma(4P-1-m)} \exp^{\left(\frac{-C_U\gamma^2}{4\sigma^2}\right)} M\left(\frac{4P-2-m}{2}, \frac{4P-1-m}{2}, \frac{C_U\gamma^2}{2\sigma^2}\right).\end{aligned} \tag{B-10}$$

One can then express (B-10) in terms of confluent hypergeometric function using [35]

$$M(k, m, z) = z^{\frac{1}{2}+m} \exp^{\left(\frac{-z}{2}\right)} {}_1F_1\left(m - k + \frac{1}{2}; 1+2m; z\right), \tag{B-11}$$

where

$${}_1F_1(a; b; z) = \sum_{n=0}^{\infty} \frac{(a)_n z^n}{(b)_n n!}$$

is the confluent hypergeometric function and $(a)_n$,$(b)_n$ are pochhammer symbols.

Hence

$$II_U(\gamma) = \frac{2(\gamma C_U)^{4P-m-1}}{4P-m-1} \exp^{\left(\frac{-C_U\gamma^2}{2\sigma^2}\right)} {}_1F_1\left(1; 4P-m; \frac{C_U\gamma^2}{2\sigma^2}\right), \tag{B-12}$$

and by substitution in (B-6),

$$\begin{aligned} R_U(\gamma) &= 2\gamma^{4P-1} C_U^{2P+1} \exp^{\left(\frac{-C_U\gamma^2}{2\sigma^2}\right)} \sum_{m=0}^{2P-2} \left(\begin{array}{c} 2P-2 \\ m \end{array} \right) \left(\frac{(-1)^{2P-2-m}}{4P-m-1} \right) \\ & {}_1F_1\left(1; 4P-m; \frac{C_U\gamma^2}{2\sigma^2}\right). \end{aligned} \tag{B-13}$$

Finally, the pdf of the SNR can be obtained by substituting (B-13) in (B-1).

Appendix C

Using [35]

$$\int_0^\infty \exp^{-st} t^{b-1} {}_1F_1(a;c;kt)dt = \Gamma(b)(s-k)^{-b} {}_2F_1\left(c-a,b;c;\frac{k}{k-s}\right), \qquad \text{(C-1)}$$

we can solve the integral

$$\begin{aligned} G_m(\delta) &= \frac{1}{(2\sigma^2)^{2P}} \int_0^{\frac{\pi}{2}} \int_0^\infty \gamma^{4P-1} \exp^{-\gamma^2(\frac{\delta}{2\sin^2\theta}+\frac{C_U}{2\sigma^2})} \\ & {}_1F_1\left(1;4P-m;\frac{C_U\gamma^2}{2\sigma^2}\right) d\gamma d\theta. \end{aligned} \qquad \text{(C-2)}$$

Let $t=\gamma^2$, $s=\frac{\delta}{2\sin^2\theta}+\frac{C_U}{2\sigma^2}$, $a=1$, $c=4P-m$, $k=\frac{C_U}{2\sigma^2}$, and $b=2P$, then

$$G_m(\delta) = \frac{1}{2(2\sigma^2)^{2P}} \int_0^{\frac{\pi}{2}} \int_0^\infty t^{b-1} \exp^{(-ts)} {}_1F_1(a;c;kt)dtd\theta. \qquad \text{(C-3)}$$

Using (C-1), we can integrate (C-3) with respect to t

$$G_m(\overline{\delta}) = \frac{\Gamma(2P)}{2(P\overline{\delta})^{2P}} \int_0^{\frac{\pi}{2}} (\sin^2\theta)^{2P} {}_2F_1\left(4P-m-1,2P;4P-m;\frac{-C_U\sin^2\theta}{(P\overline{\delta})}\right) d\theta, \qquad \text{(C-4)}$$

where $\overline{\delta}$ is the average SNR of the channel. Define $x=\sin^2\theta$

$$\begin{aligned} G_m(\overline{\delta}) &= \frac{\Gamma(2P)}{4(P\overline{\delta})^{2P}} \int_0^1 (1-x)^{-\frac{1}{2}} x^{2P-\frac{1}{2}} \\ & {}_2F_1\left(4P-m-1,2P;4P-m;\frac{-C_U x}{(P\overline{\delta})}\right) dx. \end{aligned} \qquad \text{(C-5)}$$

From [eq. (7.512.12) [35]]

$$\begin{aligned} & \int_0^1 (1-x)^{u-1} x^{v-1} {}_pF_q(a_1,...,a_p;b_1,...,b_q;ax)dx \\ = & \frac{\Gamma(u)\Gamma(v)}{\Gamma(u+v)} {}_{p+1}F_{q+1}(v,a_1,...,a_p;u+v,b_1,...,b_q;a) \end{aligned} \qquad \text{(C-6)}$$

the integral (C-5) can be solved as

$$G_m(\bar{\delta}) = \left(\frac{\Gamma(2P)}{4(P\bar{\delta})^{2P}}\right)\left(\frac{\Gamma(\frac{1}{2})\Gamma(\frac{4P+1}{2})}{\Gamma(2P+1)}\right)$$
$$_3F_2\left(\frac{4P+1}{2}, 4P-m-1, 2P; 2P+1, 4P-m; \frac{-C_U}{(P\bar{\delta})}\right). \quad \text{(C-7)}$$

From $\Gamma(.)$ properties, $\Gamma(2P+1) = 2P\ \Gamma(2P)$, and (C-7) can be rewritten as

$$G_m(\bar{\delta}) = \left(\frac{\Gamma(\frac{1}{2})\Gamma(\frac{4P+1}{2})}{8P(P\bar{\delta})^{2P}}\right)$$
$$_3F_2\left(\frac{4P+1}{2}, 4P-m-1, 2P; 2P+1, 4P-m; \frac{-C_U}{(P\bar{\delta})}\right). \quad \text{(C-8)}$$