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On Exponential Stability of Linear Networked Control Systems

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SUMMARY

This paper addresses exponential stability of linear networked control systems (NCS). More specifically, the paper considers a continuous-time linear plant in feedback with a linear sampled-data controller with an unknown time varying sampling rate, the possibility of data packet dropout, and an uncertain time varying delay. The main contribution of this paper is the derivation of new sufficient stability conditions for linear NCS taking into account all of these factors. The stability conditions are based on a modified Lyapunov-Krasovskii functional (LKF). The stability results are also applied to the case where limited information on the delay bounds is available. The case of linear sampled-data systems is studied as a corollary of the networked control case. Furthermore, the paper also formulates the problem of finding a lower bound on the maximum network-induced delay that preserves exponential stability as a convex optimization program in terms of linear matrix inequalities (LMIs). This problem can be solved efficiently from both a practical and theoretical point of view. Finally, as a comparison, we show that the stability conditions proposed in this paper compare favorably with the ones available in the open literature for different benchmark problems.

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KEY WORDS: Networked control systems; Sampled-data systems; Lyapunov-Krasovskii functional; Linear systems; Linear matrix inequalities

1. INTRODUCTION

In an NCS, sensory information and feedback signals are exchanged among different components of the system (i.e. sensors, actuators, and controllers) through a communication network. In a modern long-range aircraft for instance, there exist about 170 (Km) of signal wiring which account for almost 700 (Kg) of the weight of the aircraft [1]. Other than weight, the main drawbacks of wired communication links include connector/pin failures, cracked insulation issues, arc faults, and maintenance/upgrade difficulties [2]. The inherent benefits of wireless communication systems and the recent advancements in this field have led to a growing interest in wireless flight control systems (i.e. fly-by-wireless) [3]. However, the effects of non-ideal communication networks on stability and performance of the system become more prominent in the case of wireless communication networks [4] and motivate a thorough study of NCSs. We refer the reader to [5, 6, 7] for applications of NCSs in document printing control systems, air vehicle systems and satellites, and an inverted pendulum, respectively.

In an NCS (as well as a sampled-data system and a time-delay system, as special cases of NCSs), the vector field is defined as a function of the current and the past values of the state vector. *Retarded functional differential equations* [8] are widely used as a framework for modeling, stability analysis, and controller synthesis of deterministic and stochastic NCSs (see [8, 9, 10] and the references

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therein). The main approaches for studying NCSs include the lifting approach [11, 12, 13, 14], the impulsive model approach [15, 5, 16, 17], and the input delay approach [18, 19, 20, 21, 22].

In the lifting approach, the retarded system is modeled as a finite dimensional discrete-time system. Lifting is used in studying sampled-data systems with constant or uncertain sampling rates [23]. However, the lifting approach is not applicable to systems with uncertain parameters. In the impulsive model approach, the retarded system is modeled as an impulsive system which exhibits continuous state evolutions (described by ordinary differential equations) and instantaneous state jumps. In the input delay approach, the retarded system is modeled as a continuous-time system with a delayed control input. Both the impulsive model and input delay approaches use Razumikhin-type [9] or Krasovskii-type [24] theorems to prove stability of the retarded system. While the Razumikhin-type theorems are based on classical Lyapunov *functions*, Krasovskii-type theorems use Lyapunov *functionals* and are known to be less conservative [9, 18, 25]. The evolution of LKFs over the past decade has yielded less conservative stability conditions. These conditions are usually cast in terms of LMIs which can efficiently be solved using software packages such as SeDuMi [26] and YALMIP [27].

In an NCS, a continuous-time plant is in feedback with a discrete-time emulation of a controller. The control signal is computed using state measurements that are sampled in intervals that are not necessarily uniform [16, 20, 7]. These signals go through a quantization process [28], and experience uncertain and time varying delays [29, 30], data packet dropouts, and congestion over the communication network. Most of the works in the literature focus on only one aspect of NCSs. There are papers, however, that study two or more features of an NCS at the same time. Reference [6] studies H_∞ control of a class of uncertain stochastic NCSs with both network-induced delays and packet dropouts. Sufficient conditions are proposed to ensure exponential stability in mean square of the closed-loop system subject to a performance measure. The robust filtering problem is addressed in [31] for a class of discrete-time uncertain nonlinear networked systems with both multiple stochastic time-varying communication delays and multiple packet dropouts. A method for designing linear full-order filter is proposed such that the estimation error converges to zero exponentially in the mean square while the disturbance rejection attenuation is constrained to a given level. Reference [32] studies the distributed finite- horizon filtering problem for a class of time-varying systems over lossy sensor networks with quantization errors and successive packet dropouts. Through available output measurements from a sensor and its neighbors (according to a given topology), a sufficient condition is established for the desired distributed finite-horizon filter to ensure that the prescribed average filtering performance constraint is satisfied.

The NCS considered in [33] comprises a linear sampled-data controller and an uncertain, time varying delay. Two drawbacks of that model are that the sampling intervals are assumed to be constant and the delay is assumed to be upper bounded by the sampling period. A more general model of NCSs is studied in [17, 22], where a linear sampled-data controller with uncertain sampling rates, the possibility of data packet dropouts, and an unknown, time varying delay are considered. While the stability theorems in [22] are less conservative than the corresponding theorems in [17], they are more computationally expensive as they involve solving two times as many LMIs. Moreover, due to the complexity of the LKF in [22], the number of LMIs increases even more if additional information on the time varying delay (e.g. a lower bound) is available.

Similar to [17, 22], in this paper we focus on linear NCSs. In particular, we study a continuous-time linear plant in feedback with a linear sampled-data controller with an unknown, time varying sampling rate, the possibility of data packet dropout, and an uncertain, time varying delay. In contrast to [22], our paper improves the stability conditions of [17] without increasing the computational cost of the resulting optimization program. We first consider the general case where information on the lower and upper bounds of the time-delay are available, and then study the case with limited information on the time-delay. In all those scenarios, our goal is to find a lower bound on the maximum network-induced delay that preserves exponential stability of the system.

The main contribution of this paper is the derivation of new sufficient stability conditions for linear NCS taking into account all of the factors mentioned before. The stability conditions are based on a modified LKF. The stability results are also applied to the case where limited information on

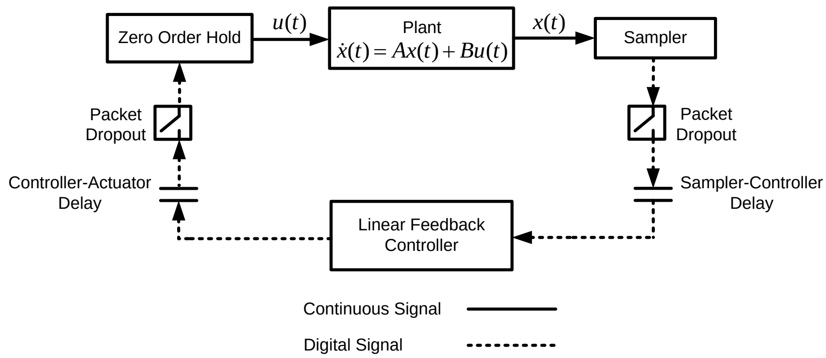

Figure 1. A linear networked control system

the delay bounds is available. The case of linear sampled-data systems is studied as a corollary of the networked control case. Furthermore, the paper also formulates the problem of finding a lower bound on the maximum network-induced delay that preserves exponential stability as a convex optimization program in terms of LMIs. This problem can be solved efficiently from both a practical and theoretical point of view. Finally, as a comparison, we show that the stability conditions proposed in this paper compare favorably with the ones available in the open literature for different benchmark problems.

The paper is organized as follows. Section 2 presents the linear NCS model. Section 3 starts by introducing a modified LKF. Next, we present theorems that provide sufficient conditions for exponential stability of linear NCSs. Furthermore, the problem of finding a lower bound on the maximum network-induced delay that preserves exponential stability as an optimization program is formulated in terms of LMIs. Finally, the new approach is applied to different examples in Section 4.

Notation. The zero matrix and the identity matrix of the appropriate size are represented by 0 and I, respectively. The notation $Z_1 > Z_2$ (or $Z_1 < Z_2$), where Z_1 and Z_2 are symmetric matrices, denotes that $Z_1 - Z_2$ is positive (or negative) definite. The transpose of a matrix Y is shown by Y^T. The scalar $\lambda_{\max}(.)$ represents the maximum eigenvalue of a matrix. A diagonal matrix with diagonal entries $d_1, \ldots, d_m$ is denoted by $diag\left(d_1, \ldots, d_m\right)$. Where there is no confusion, a vector $x(t)$ is written as x. The notation $|.|$ denotes the norm of a vector.

2. PRELIMINARIES

Consider the linear system

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$

where $x \in \mathbb{R}^{n_x}$ denotes the state vector, $A \in \mathbb{R}^{n_x \times n_x}$, $B \in \mathbb{R}^{n_x \times n_u}$, and $u \in \mathbb{R}^{n_u}$ is the control input. Let a continuous-time linear controller for (1) be defined by

$$u(t) = Kx(t), \tag{2}$$

where $K \in \mathbb{R}^{n_u \times n_x}$. In this paper, we study the stability of system (1) where controller (2) is implemented through a network. The network comprises a time driven sampler and an event driven zero order hold (see Figure 1). The possibility of data packet dropout and communication delays are also considered in the network's model. The networked controller is characterized through Assumptions 1-4.

Assumption 1
The state vector is measured at the sampling instants s_k, $k \in \mathbb{N}$. Each sampled state vector is sent over the network in one data packet.

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Since the controller is static and time-invariant, without loss of generality [5, 34, 35], the delay between the sensor (sampler) and the controller, the delay between the controller and the actuator, and the computation delay in the controller are modeled as one single delay.

Assumption 2
The state vector sampled at s_k, $k \in \mathbb{N}$, experiences an uncertain, time varying delay η_k as it is transmitted through the network. The delay η_k is bounded, i.e., $0 \leq \eta_{\text{min}} \leq \eta_k \leq \eta_{\text{max}}$.

Note that our model allows the delay η_k to grow larger than the sampling interval $[s_k, s_{k+1}]$ as opposed to the model in [33]. The possibility of data packet dropout is modeled via a switch in Figure 1. When the switch is closed, the data is transmitted through the network. When the switch is open, however, the data is assumed to be dropped. The actuator is updated with new control signals at the instants t_k,

$$t_k = s_k + \eta_k,\ k \in \mathbb{N}. \tag{3}$$

An event driven zero order hold keeps the control signal constant through the interval $[t_k, t_{k+1})$, i.e. until the arrival of new data at t_{k+1}.

Assumption 3
The control signals arrive at the actuator in the same order that their corresponding state vectors are sampled, i.e. $s_i < s_j \Longrightarrow t_i < t_j,\ \forall\ i, j \in \mathbb{N}$. If a sampled state vector arrives after a more recent sampled vector has arrived, the older sampled vector is dropped (cf. s_d and s_2 in Figure 2).

Without loss of generality, by the index k, $k \in \mathbb{N}$, we denote only the instants s_k and t_k for which a data packet is not dropped. In the interval between two actuator update instants t_k and t_{k+1}, the network-induced delay represented by ρ_s is defined as the time elapsed since the last available sampling instant s_k (see Figure 2), i.e.

$$\rho_s(t) = t - s_k = t - t_k + \eta_k,\ t \in [t_k, t_{k+1}), \tag{4}$$

where equation (3) is used in the second equality. Based on Assumption 2, the network-induced delay is greater than or equal to η_{min}. We denote the largest network-induced delay by τ, i.e.

$$\tau = \sup\left(\rho_s(t)\right) = \sup_{k \in \mathbb{N}}\left(t_{k+1} - s_k\right).$$

Therefore,

$$\eta_{\text{min}} \leq \rho_s(t) \leq \tau. \tag{5}$$

Furthermore, the time elapsed since the last actuator update instant t_k is denoted by ρ_t, i.e.

$$\rho_t(t) = t - t_k = t - s_k - \eta_k = \rho_s(t) - \eta_k,\ t \in [t_k, t_{k+1}). \tag{6}$$

Equation (5), equation (6), and Assumption 2 yield

$$0 \leq \rho_t(t) \leq \tau - \eta_{\text{min}}. \tag{7}$$

The following assumption models the fact that two actuator updates cannot occur simultaneously in practice. It is used in Section 3 to rule out the occurrence of the Zeno phenomenon and also plays an essential role in proving the convergence of the closed-loop vector field to the origin.

Assumption 4
There exists $\epsilon > 0$ such that $t_{k+1} - t_k > \epsilon$ for any $k \in \mathbb{N}$.

The control signal (2) is now redefined in the NCS framework as the piecewise constant function

$$u(t) = Kx(s_k),\ t \in [t_k, t_{k+1}), \tag{8}$$

with jumps at the actuator update instants t_k, $k \in \mathbb{N}$. Given a controller gain K that exponentially stabilizes the continuous-time system (1)-(2), our objective is to find a lower bound on the maximum network-induced delay that preserves exponential stability for the NCS defined by (1) and (8). To

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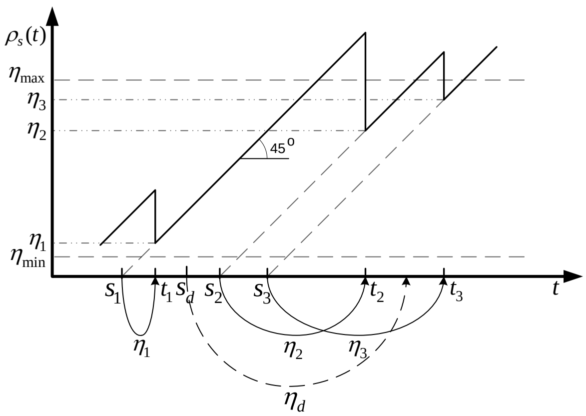

Figure 2. Network-induced delay

this end, we use the input delay approach to draw an analogy between NCSs and time-delay systems. Considering (4), we can rewrite (8) as

$$u(t) = Kx(t - \rho_s), \ t \in [t_k, t_{k+1}). \tag{9}$$

The linear NCS (1) with control input (9) can be viewed as a linear system with a discontinuous time varying input delay $d(t) = \rho_s$. In the literature of time-delay systems, LKFs are widely used to devise stability conditions (see [10, 29, 30, 36] and the references therein). Different LKFs are used for NCSs in [17, 22, 33] and sampled-data systems in [16, 18, 20, 21]. The subject of LKFs and stability of linear NCSs will be addressed in the next section where we present the main results of the paper.

3. MAIN RESULTS

We start this section by an observation that motivates our modified LKF. Next, we present the modified LKF and use it to provide new conditions for stability of linear NCSs. The case of linear sampled-data systems is also studied as a corollary. Finally, the problem of finding a lower bound on the maximum network-induced delay that preserves exponential stability is cast as an optimization program in terms of LMIs.

3.1. Motivation

The LKFs proposed in the literature for the analysis of linear time-delay systems contain integrals of quadratic functions of the state vector. For instance, when the derivative of the delay $d(t)$ is less than 1, i.e. $\dot{d}(t) < 1$, the following functional is widely used in the literature on linear time-delay systems as an LKF component [29, 30, 36]

$$V_{delay} = V_{delay_1} + V_{delay_2},$$

with

$$\begin{aligned} V_{delay_1} &= \int_{t-d(t)}^{t} x^T(s) W_1 x(s)\, \mathrm{d}s, \\ V_{delay_2} &= \int_{t-\eta_{\max}}^{t} x^T(s) W_2 x(s)\, \mathrm{d}s, \end{aligned} \tag{10}$$

where $W_1 > 0$ and $W_2 > 0$ are matrices in $\mathbb{R}^{n_x \times n_x}$. The two parts of V_{delay} contain information about the evolution of the states through the intervals $[t - d(t), t]$ and $[t - \eta_{\max}, t]$, respectively.

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Moreover, given $\dot{d}(t) < 1$, V_{delay_1} makes the LKF less conservative by using the available information about the time derivative of the delay. In NCSs however, the time derivative of the network-induced delay is equal to 1, i.e. $\dot{d}(t) = \dot{\rho}_s = 1$, for $t \in [t_k, t_{k+1})$. Evidently, $\dot{V}_{delay_1} = x^T W_1 x > 0$, for $t \in [t_k, t_{k+1})$. Therefore, adding the functional V_{delay_1} to the LKF is not beneficial for NCSs because it adds a positive term to its time derivative. To the best of our knowledge, no LKF in the literature of NCSs (or sampled-data systems) contains an integral similar to V_{delay_1}. As a result, the information about the time derivative of the network-induced delay is not fully exploited. In this paper, we add a new functional to the LKF to address that issue.

3.2. Lyapunov-Krasovskii functional

Let $\mathcal{C}([-\tau, 0], \mathbb{R}^{n_x})$ be the space of absolutely continuous functions with square integrable first-order derivatives, mapping the interval $[-\tau, 0]$ to $\mathbb{R}^{n_x}$. The function $x_t \in \mathcal{C}$ is defined as

$$x_t(\alpha) = x(t + \alpha), \; -\tau \le \alpha \le 0, \tag{11}$$

and similar to [20, 22], its norm is defined by

$$||x_t||_{\mathcal{C}} = \max_{\alpha \in [-\tau, 0]} |x_t(\alpha)| + \left[\int_{-\tau}^{0} |\dot{x}_t(\alpha)|^2 \, \mathrm{d}\alpha \right]^{\frac{1}{2}}. \tag{12}$$

We define our LKF as

$$V(t, x_t) = \sum_{j=0}^{8} V_j, \; t \in [t_k, t_{k+1}), \tag{13}$$

where

$$V_0 = x^T(t) P x(t), \tag{14}$$

$$V_1 = (\tau - \rho_s) \int_{t-\rho_t}^{t} [\dot{x}(r) - Bu(r)]^T R_1 [\dot{x}(r) - Bu(r)] \, \mathrm{d}r, \tag{15}$$

$$V_2 = (\tau - \rho_s) \int_{t-\rho_t}^{t} \dot{x}^T(r) R_2 \dot{x}(r) \, \mathrm{d}r, \tag{16}$$

$$V_3 = \int_{t-\eta_{\min}}^{t} (\eta_{\min} - t + r) \dot{x}^T(r) R_3 \dot{x}(r) \, \mathrm{d}r, \tag{17}$$

$$V_4 = \int_{t-\rho_s}^{t-\eta_{\min}} (\tau - t + r) \dot{x}^T(r) R_4 \dot{x}(r) \, \mathrm{d}r + (\tau - \eta_{\min}) \int_{t-\eta_{\min}}^{t} \dot{x}^T(r) R_4 \dot{x}(r) \, \mathrm{d}r, \tag{18}$$

$$V_5 = \int_{t-\rho_s}^{t} (\tau - t + r) \dot{x}^T(r) R_5 \dot{x}(r) \, \mathrm{d}r, \tag{19}$$

$$V_6 = \int_{t-\rho_s}^{t} (\tau - t + r) \dot{x}^T(r) R_6 \dot{x}(r) \, \mathrm{d}r, \tag{20}$$

$$V_7 = \int_{t-\eta_{\min}}^{t} x^T(r) Z x(r) \, \mathrm{d}r, \tag{21}$$

$$V_8 = (\tau - \rho_s) \begin{bmatrix} x^T(t) & x^T(t_k) \end{bmatrix} X \begin{bmatrix} x^T(t) & x^T(t_k) \end{bmatrix}^T, \tag{22}$$

ρ_s and ρ_t are defined in (4) and (6), respectively, and

$$X = \begin{bmatrix} X_1 & -X_2 \\ -X_2^T & X_2 + X_2^T - X_1 \end{bmatrix}, \tag{23}$$

where $P > 0$, $R_i > 0$, $i \in \{1, \ldots, 6\}$, $Z > 0$, $X_1 = X_1^T$, and X_2 are matrices in $\mathbb{R}^{n_x \times n_x}$. The reason for defining two similar functionals V_5 and V_6 becomes clear in the next subsection where we use V_5 to provide stability conditions that are independent of $\eta_{\max}$ and use V_6 to devise stability conditions for the case when $\eta_{\max}$ is known (see equations (49) and (50)).

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Table I. Comparison of the LKF in (13) with the LKFs proposed in [17] and [20]. The sign ✓ (respectively, ✗) denotes that a functional exists (respectively, does not exist) in the corresponding LKF.

LKF in (13)	V_0	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8
LKF in [17]	✓	✗	✗	✓	✓	✗	✓	✓	✗
LKF in [20]	✓	✗	✓	✗	✗	✗	✗	✗	✓

Remark 1
To the best of our knowledge, no other LKF in the literature of NCSs (or sampled-data systems) contains a term similar to V_1. The new functional V_1 is defined similar to V_2 (cf. equations (15) and (16)). However in V_1, the open-loop dynamics of the linear system are used as opposed to the closed-loop dynamics. Note that based on (1), V_1 can be rewritten as an integral in terms of a quadratic function of the state vector, i.e.

$$V_1 = (\tau - \rho_s)\int_{t-\rho_t}^{t} x^T(r)\overline{W}x(r)\,\mathrm{d}r, \tag{24}$$

where $\overline{W} = A^T R_1 A$. Observe that the definition of V_1 in the form of (24) is similar to the definition of V_{delay_1} in (10). The new term V_1 contains information about the evolution of the states through the interval $[t-\rho_t, t]$ and also exploits the available information about the time derivative of the network-induced delay, $\dot{\rho}_s = \dot{\rho}_t = 1$.

Table I compares the LKF in equation (13) with the LKFs in[17, 20]. Using the new functional V_1 and the proper use of the functional V_5, enables one to achieve less conservative stability criteria as will be shown in the next subsection.

3.3. Stability results

In this subsection, we present our stability theorems for linear NCSs. First, consider the definition of exponential stability in the context of retarded functional differential equations.

Definition [17] The linear NCS defined in (1) and (8) is said to be *globally uniformly exponentially stable*, if there exists a function $\beta(a,b) = ce^{-\lambda b}a$, for some $c > 0$ and $\lambda > 0$, such that for any initial condition $x_0 \in \mathcal{C}$ the solution is globally defined and satisfies $|x(t)| \leq \beta(||x_0||_{\mathcal{C}}, t)$, for all $t > 0$.

The following theorem provides a set of sufficient conditions for which the trajectories of the linear NCS are globally uniformly exponentially stable to the origin.

Theorem 1
Consider the linear NCS defined in (1) and (8) with Assumptions 1-4. Given the controller gain K and the scalars τ, $\eta_{\min}$, and $\eta_{\max}$, the NCS is globally uniformly exponentially stable if there exist symmetric positive definite matrices P, R_i, $i \in \{1,\ldots,6\}$, and Z, a symmetric matrix X_1, and matrices X_2, N_j, $j \in \{1,\ldots,5\}$, N_{6_a}, and N_{6_b}, with appropriate dimensions, satisfying

$$\begin{bmatrix} P & 0 \\ 0 & 0 \end{bmatrix} + (\tau - \eta_{\min})X > 0 \tag{25}$$

$$\begin{bmatrix} \Psi + \tau M_1 + (\tau-\eta_{\min})(M_2+M_4) + \eta_{\min}M_3 & \eta_{\min}N_3 & \eta_{\min}N_5 & \eta_{\max}N_{6_a} \\ \eta_{\min}N_3^T & -\eta_{\min}R_3 & 0 & 0 \\ \eta_{\min}N_5^T & 0 & -\eta_{\min}R_5 & 0 \\ \eta_{\max}N_{6_a}^T & 0 & 0 & -\eta_{\max}R_6 \end{bmatrix} < 0 \tag{26}$$

$$\begin{bmatrix} \Psi + \tau M_1 + (\tau-\eta_{\min})(M_4+M_5) + \eta_{\min}M_3 & \overline{N} \\ \overline{N}^T & D \end{bmatrix} < 0 \tag{27}$$

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where X is defined in (23) and

$$\Psi = \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix}^T \begin{bmatrix} P & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} P & 0 & 0 & 0 \end{bmatrix}^T \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix}$$
$$- \begin{bmatrix} I & -I & 0 & 0 \end{bmatrix}^T (N_1^T + N_2^T + N_{6_b}^T) - (N_1 + N_2 + N_{6_b}) \begin{bmatrix} I & -I & 0 & 0 \end{bmatrix}$$
$$- \begin{bmatrix} I & 0 & 0 & -I \end{bmatrix}^T N_3^T - N_3 \begin{bmatrix} I & 0 & 0 & -I \end{bmatrix} - \begin{bmatrix} 0 & 0 & -I & I \end{bmatrix}^T N_4^T - N_4 \begin{bmatrix} 0 & 0 & -I & I \end{bmatrix}$$
$$- \begin{bmatrix} I & 0 & -I & 0 \end{bmatrix}^T N_5^T - N_5 \begin{bmatrix} I & 0 & -I & 0 \end{bmatrix} - \begin{bmatrix} 0 & I & -I & 0 \end{bmatrix}^T N_{6_a}^T - N_{6_a} \begin{bmatrix} 0 & I & -I & 0 \end{bmatrix}$$
$$+ \begin{bmatrix} I & 0 & 0 & 0 \end{bmatrix}^T Z \begin{bmatrix} I & 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & I \end{bmatrix}^T Z \begin{bmatrix} 0 & 0 & 0 & I \end{bmatrix} - \begin{bmatrix} X & 0 \\ 0 & 0 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix}^T (R_5 + R_6) \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix},$$
$$M_2 = \begin{bmatrix} A & 0 & 0 & 0 \end{bmatrix}^T R_1 \begin{bmatrix} A & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix}^T R_2 \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix}$$
$$+ \begin{bmatrix} A & 0 & BK & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}^T \begin{bmatrix} X & 0 \end{bmatrix} + \begin{bmatrix} X \\ 0 \end{bmatrix} \begin{bmatrix} A & 0 & BK & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$
$$M_3 = \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix}^T R_3 \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix},$$
$$M_4 = \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix}^T R_4 \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix},$$
$$M_5 = \begin{bmatrix} 0 & 0 & BK & 0 \end{bmatrix}^T N_1^T + N_1 \begin{bmatrix} 0 & 0 & BK & 0 \end{bmatrix},$$
$$\overline{N} = \begin{bmatrix} (\tau - \eta_{\min})N_1 & (\tau - \eta_{\min})N_2 & \eta_{\min} N_3 & (\tau - \eta_{\min})N_4 & \tau N_5 & \eta_{\max} N_{6_a} & (\tau - \eta_{\min})N_{6_b} \end{bmatrix},$$
$$D = diag\left((\eta_{\min} - \tau)R_1,\ (\eta_{\min} - \tau)R_2,\ -\eta_{\min}R_3,\ (\eta_{\min} - \tau)R_4,\ -\tau R_5,\ -\eta_{\max}R_6,\ (\eta_{\min} - \tau)R_6\right).$$

Proof
First we show that $P > 0$, $R_i > 0$, $i \in \{1, \dots, 6\}$, $Z > 0$, and LMI (25) are sufficient conditions for the LKF (13) to satisfy

$$c_1|x_t(0)|^2 \le V(t, x_t) \le c_2||x_t||_{\mathcal{C}}^2, \tag{28}$$

for some $c_1 > 0$ and $c_2 > 0$. Adding V_0 and V_8 yields

$$V_0 + V_8 = \begin{bmatrix} x(t) \\ x(t_k) \end{bmatrix}^T \left(\begin{bmatrix} P & 0 \\ 0 & 0 \end{bmatrix} + (\tau - \rho_s)X \right) \begin{bmatrix} x(t) \\ x(t_k) \end{bmatrix}, \tag{29}$$

for $t \in [t_k, t_{k+1})$. Based on (5), ρ_s varies between $\eta_{\min}$ and τ. Since (29) is affine in ρ_s, LMI (25) and $P > 0$ are sufficient conditions for the existence of a sufficiently small $c_1 > 0$ such that

$$\begin{bmatrix} c_1 I & 0 \\ 0 & 0 \end{bmatrix} \le \begin{bmatrix} P & 0 \\ 0 & 0 \end{bmatrix} + (\tau - \rho_s)X,$$

for any $\rho_s \in [\eta_{\min}, \tau)$. Therefore, based on (11) and (29) we can write

$$c_1|x(t)|^2 = c_1|x_t(0)|^2 \le V_0 + V_8.$$

Moreover, note that the constraints $R_i > 0$, $i \in \{1, \dots, 6\}$, and $Z > 0$ are sufficient conditions for V_j, $j \in \{1, \dots, 7\}$, to be non-negative at any time. Therefore, the lower bound on V in inequality (28) is computed as

$$c_1|x_t(0)|^2 \le V_0 + V_8 \le V.$$

Considering (5) and (12), observe that at any time t and for all $\alpha \in [-\rho_s, 0]$, $|x_t(\alpha)| \le ||x_t||_{\mathcal{C}}$. Equivalently,

$$|x(r)| \le ||x_t||_{\mathcal{C}},\ \forall\, r \in [t - \rho_s, t]. \tag{30}$$

Therefore, $\left| \begin{bmatrix} x^T(t) & x^T(t_k) \end{bmatrix}^T \right| < \sqrt{2}||x_t||_{\mathcal{C}}$. Based on (29),

$$V_0 + V_8 \le 2 \max_{\rho_s \in [\eta_{\min}, \tau]} \left\{ \lambda_{\max} \left(\begin{bmatrix} P & 0 \\ 0 & 0 \end{bmatrix} + (\tau - \rho_s)X \right) \right\} ||x_t||_{\mathcal{C}}^2. \tag{31}$$

Note that

$$[\dot{x}(r) - Bu(r)]^T R_1 [\dot{x}(r) - Bu(r)] \le \lambda_{\max}(R_1) |\dot{x}(r) - Bu(r)|^2.$$

Moreover, according to the parallelogram law [37], $|v_1 - v_2|^2 + |v_1 + v_2|^2 = 2|v_1|^2 + 2|v_2|^2$, where v_1 and v_2 are vectors in $\mathbb{R}^m$. Therefore, $|v_1 - v_2|^2 \le 2|v_1|^2 + 2|v_2|^2$. Thus, using (5),

$$V_1 \le (\tau - \eta_{\min}) \lambda_{\max}(R_1) \left(\int_{t-\rho_t}^{t} 2|\dot{x}(r)|^2 \, \mathrm{d}r + \int_{t-\rho_t}^{t} 2|Bu(r)|^2 \, \mathrm{d}r \right). \tag{32}$$

With a change of variables, considering (7), and using the definition of norm in (12), we can write

$$\int_{t-\rho_t}^{t} 2|\dot{x}(r)|^2 \, \mathrm{d}r = 2 \int_{-\rho_t}^{0} |\dot{x}(t+\alpha)|^2 \, \mathrm{d}\alpha = 2 \int_{-\rho_t}^{0} |\dot{x}_t(\alpha)|^2 \, \mathrm{d}\alpha \le 2||x_t||_{\mathcal{C}}^2. \tag{33}$$

Based on (8), note that $u(r) = Kx(s_k)$ is constant for $r \in [t - \rho_t, t] = [t_k, t]$, $t \in [t_k, t_{k+1})$. According to (30), $|x(s_k)| \le ||x_t||_{\mathcal{C}}$. Therefore, considering (7),

$$\int_{t-\rho_t}^{t} 2|Bu(r)|^2 \, \mathrm{d}r = 2 \int_{t-\rho_t}^{t} |BKx(s_k)|^2 \, \mathrm{d}r \le 2(\tau - \eta_{\min}) \lambda_{\max}(K^T B^T B K) ||x_t||_{\mathcal{C}}^2. \tag{34}$$

From (32)-(34),

$$V_1 \le 2(\tau - \eta_{\min}) \lambda_{\max}(R_1) \left(1 + (\tau - \eta_{\min}) \lambda_{\max}(K^T B^T B K)\right) ||x_t||_{\mathcal{C}}^2. \tag{35}$$

Similarly, it can be shown that

$$V_2 \le (\tau - \eta_{\min}) \lambda_{\max}(R_2) ||x_t||_{\mathcal{C}}^2, \tag{36}$$
$$V_3 \le \eta_{\min} \lambda_{\max}(R_3) ||x_t||_{\mathcal{C}}^2, \tag{37}$$
$$V_4 \le 2(\tau - \eta_{\min}) \lambda_{\max}(R_4) ||x_t||_{\mathcal{C}}^2, \tag{38}$$
$$V_5 \le \tau \lambda_{\max}(R_5) ||x_t||_{\mathcal{C}}^2, \tag{39}$$
$$V_6 \le \tau \lambda_{\max}(R_6) ||x_t||_{\mathcal{C}}^2. \tag{40}$$

Based on (5), $[-\eta_{\min}, 0] \subset [-\rho_s, 0]$, i.e. $[t - \eta_{\min}, t] \subset [t - \rho_s, t]$. Therefore, using (30),

$$V_7 \le \eta_{\min} \lambda_{\max}(Z) ||x_t||_{\mathcal{C}}^2. \tag{41}$$

Adding inequalities (31) and (35)-(41) leads to the upper bound on V in (28), i.e.

$$\begin{aligned} c_2 =& 2 \max_{\rho_s \in [\eta_{\min}, \tau]} \left\{ \lambda_{\max} \left(\begin{bmatrix} P & 0 \\ 0 & 0 \end{bmatrix} + (\tau - \rho_s) X \right) \right\} + (\tau - \eta_{\min})(\lambda_{\max}(R_2) + 2\lambda_{\max}(R_4)) \\ &+ 2(\tau - \eta_{\min}) \lambda_{\max}(R_1) \left(1 + (\tau - \eta_{\min}) \lambda_{\max}(K^T B^T B K)\right) + \tau(\lambda_{\max}(R_5) + \lambda_{\max}(R_6)) \\ &+ \eta_{\min}(\lambda_{\max}(R_3) + \lambda_{\max}(Z)). \end{aligned}$$

So far, it was shown that the LKF is positive definite and decrescent. Following Lyapunov theorem, to prove stability, it suffices to show that the LKF is decreasing. Since the LKF is discontinuous at actuator update instants t_k, we first show that the LKF is non-increasing at $t = t_k$, $k \in \mathbb{N}$. Next, computing the time derivative of V for $t \in (t_k, t_{k+1})$, $k \in \mathbb{N}$, it is proved that LMIs (26) and (27) are sufficient conditions for the LKF to be decreasing in the interval between two actuator update instants. To this end, note that V_j, $j \in \{1, \ldots, 7\}$, and $V_0 + V_8$ are always non-negative. Also observe that V_0, V_3, and V_7 are continuous functions. The functionals V_1 and V_2 vanish at the actuator update instants since $\rho_t = 0$ at $t = t_k$. The first integral in the functional V_4 is non-increasing at the actuator update instants $t = t_k$ because the integrand is non-negative and based on Assumption 3 the lower limit of the integral changes from s_{k-1} to s_k (see Figure 2). Note that the second part of V_4 is a continuous function. Using the same reasoning, the functionals V_5 and V_6 are non-increasing at the actuator update instants $t = t_k$ because the integrands are non-negative and the lower limit of the integrals change from s_{k-1} to s_k. The last component of the LKF, i.e. V_8, vanishes at the actuator update instants because $x(t) = x(t_k)$ at $t = t_k$ and the sum of the entries of X is equal to zero. Therefore, the LKF is non-increasing at instants t_k, $k \in \mathbb{N}$. The LKF is differentiable in the interval between two actuator update instants. For $t \in (t_k, t_{k+1})$, $\dot{V}$ is composed of nine terms computed as follows. The time derivative of V_0 is

$$\dot{V}_0 = \dot{x}^T P x + x^T P \dot{x}. \tag{42}$$

From (4) and (6), we have $\dot{\rho_s} = \dot{\rho_t} = 1$. Hence, applying the Leibniz integral rule to V_1 yields

$$\dot{V}_1 = -\int_{t-\rho_t}^{t} [\dot{x}(r) - Bu(r)]^T R_1 [\dot{x}(r) - Bu(r)]\,\mathrm{d}r + (\tau - \rho_s)\,[\dot{x} - Bu]^T R_1 [\dot{x} - Bu]\,. \tag{43}$$

Since R_1 is positive definite, for any arbitrary time varying vector $h_1(t) \in \mathbb{R}^{n_x}$ we can write

$$\begin{bmatrix} \dot{x}(r) - Bu(r) \\ h_1 \end{bmatrix}^T \begin{bmatrix} R_1 & -I \\ -I & R_1^{-1} \end{bmatrix} \begin{bmatrix} \dot{x}(r) - Bu(r) \\ h_1 \end{bmatrix} \geq 0.$$

Therefore,

$$-[\dot{x}(r) - Bu(r)]^T R_1 [\dot{x}(r) - Bu(r)] \leq h_1^T R_1^{-1} h_1 - [\dot{x}(r) - Bu(r)]^T h_1 - h_1^T [\dot{x}(r) - Bu(r)].$$

Note that $u(r) = Kx(s_k)$ is constant for $r \in (t_k, t_{k+1})$, and $x(r) = x_r(0) \in \mathcal{C}$ is absolutely continuous. Therefore, integrating both sides from $t - \rho_t$ to t, with respect to r, we have

$$\begin{aligned} -\int_{t-\rho_t}^{t} [\dot{x}(r) - Bu(r)]^T R_1 [\dot{x}(r) - Bu(r)]\,\mathrm{d}r \leq & \rho_t h_1^T R_1^{-1} h_1 - [x - x(t_k) - \rho_t Bu]^T h_1 \\ & - h_1^T [x - x(t_k) - \rho_t Bu]. \end{aligned} \tag{44}$$

Replacing (44) in (43), yields

$$\begin{aligned} \dot{V}_1 \leq & \rho_t h_1^T R_1^{-1} h_1 - [x - x(t_k) - \rho_t Bu]^T h_1 - h_1^T [x - x(t_k) - \rho_t Bu] \\ & + (\tau - \rho_s)\,[\dot{x} - Bu]^T R_1 [\dot{x} - Bu]\,. \end{aligned} \tag{45}$$

Similarly, we can write the following equations

$$\dot{V}_2 \leq \rho_t h_2^T R_2^{-1} h_2 - [x - x(t_k)]^T h_2 - h_2^T [x - x(t_k)] + (\tau - \rho_s) \dot{x}^T R_2 \dot{x}, \tag{46}$$

$$\begin{aligned} \dot{V}_3 = & -\int_{t-\eta_{\text{min}}}^{t} \dot{x}^T(r) R_3 \dot{x}(r)\,\mathrm{d}r + \eta_{\text{min}} \dot{x}^T R_3 \dot{x} \\ \leq & \eta_{\text{min}} h_3^T R_3^{-1} h_3 - [x - x(t - \eta_{\text{min}})]^T h_3 - h_3^T [x - x(t - \eta_{\text{min}})] + \eta_{\text{min}} \dot{x}^T R_3 \dot{x}, \end{aligned} \tag{47}$$

$$\begin{aligned} \dot{V}_4 = & (\tau - \eta_{\text{min}}) \dot{x}^T(t - \eta_{\text{min}}) R_4 \dot{x}(t - \eta_{\text{min}}) - \int_{t-\rho_s}^{t-\eta_{\text{min}}} \dot{x}^T(r) R_4 \dot{x}(r)\,\mathrm{d}r + (\tau - \eta_{\text{min}}) \dot{x}^T R_4 \dot{x} \\ & - (\tau - \eta_{\text{min}}) \dot{x}^T(t - \eta_{\text{min}}) R_4 \dot{x}(t - \eta_{\text{min}}) \\ \leq & (\rho_s - \eta_{\text{min}}) h_4^T R_4^{-1} h_4 - [x(t - \eta_{\text{min}}) - x(s_k)]^T h_4 - h_4^T [x(t - \eta_{\text{min}}) - x(s_k)] \\ & + (\tau - \eta_{\text{min}}) \dot{x}^T R_4 \dot{x}, \end{aligned} \tag{48}$$

$$\begin{aligned} \dot{V}_5 = & -\int_{t-\rho_s}^{t} \dot{x}^T(r) R_5 \dot{x}(r)\,\mathrm{d}r + \tau \dot{x}^T R_5 \dot{x} \\ \leq & \rho_s h_5^T R_5^{-1} h_5 - [x - x(s_k)]^T h_5 - h_5^T [x - x(s_k)] + \tau \dot{x}^T R_5 \dot{x}, \end{aligned} \tag{49}$$

$$\dot{V}_6 = -\int_{t-\rho_s}^{t-\rho_t} \dot{x}^T(r) R_6 \dot{x}(r)\,\mathrm{d}r - \int_{t-\rho_t}^{t} \dot{x}^T(r) R_6 \dot{x}(r)\,\mathrm{d}r + \tau \dot{x}^T R_6 \dot{x}$$

$$\leq \eta_k h_{6_a}^T R_6^{-1} h_{6_a} - [x(t_k) - x(s_k)]^T h_{6_a} - h_{6_a}^T [x(t_k) - x(s_k)] + \rho_t h_{6_b}^T R_6^{-1} h_{6_b}$$
$$- [x - x(t_k)]^T h_{6_b} - h_{6_b}^T [x - x(t_k)] + \tau \dot{x}^T R_6 \dot{x}, \tag{50}$$
$$\dot{V}_7 = x^T Z x - x^T(t - \eta_{\min}) Z x(t - \eta_{\min}), \tag{51}$$
$$\dot{V}_8 = - \begin{bmatrix} x^T(t) & x^T(t_k) \end{bmatrix} X \begin{bmatrix} x^T(t) & x^T(t_k) \end{bmatrix}^T + (\tau - \rho_s) \begin{bmatrix} \dot{x}^T(t) & 0 \end{bmatrix} X \begin{bmatrix} x^T(t) & x^T(t_k) \end{bmatrix}^T$$
$$+ (\tau - \rho_s) \begin{bmatrix} x^T(t) & x^T(t_k) \end{bmatrix} X \begin{bmatrix} \dot{x}^T(t) & 0 \end{bmatrix}^T, \tag{52}$$

where $h_j(t)$, $j \in \{2, \ldots, 5\}$, $h_{6_a}(t)$, and $h_{6_b}(t)$ are arbitrary time varying vectors in $\mathbb{R}^{n_x}$. Although the functionals V_5 and V_6 were defined similarly in equations (19) and (20), their time derivatives were approximated differently in equations (49) and (50). $\dot{V}_6$ is approximated by a delay dependant functional and is used to devise stability conditions for the case when $\eta_{\max}$ is known. Since $\dot{V} = \sum_{i=0}^{8} \dot{V}_i$, adding (42) and (45)-(52) yields

$$\dot{V} \leq \dot{x}^T P x + x^T P \dot{x} + \rho_t h_1^T R_1^{-1} h_1 - [x - x(t_k) - \rho_t B u]^T h_1 - h_1^T [x - x(t_k) - \rho_t B u]$$
$$+ (\tau - \rho_s) [\dot{x} - Bu]^T R_1 [\dot{x} - Bu] + \rho_t h_2^T R_2^{-1} h_2 - [x - x(t_k)]^T h_2 - h_2^T [x - x(t_k)]$$
$$+ (\tau - \rho_s) \dot{x}^T R_2 \dot{x} + \eta_{\min} h_3^T R_3^{-1} h_3 - [x - x(t - \eta_{\min})]^T h_3 - h_3^T [x - x(t - \eta_{\min})]$$
$$+ \eta_{\min} \dot{x}^T R_3 \dot{x} + (\rho_s - \eta_{\min}) h_4^T R_4^{-1} h_4 - [x(t - \eta_{\min}) - x(s_k)]^T h_4 - h_4^T [x(t - \eta_{\min}) - x(s_k)]$$
$$+ (\tau - \eta_{\min}) \dot{x}^T R_4 \dot{x} + \rho_s h_5^T R_5^{-1} h_5 - [x - x(s_k)]^T h_5 - h_5^T [x - x(s_k)] + \tau \dot{x}^T R_5 \dot{x}$$
$$+ \eta_k h_{6_a}^T R_6^{-1} h_{6_a} - [x(t_k) - x(s_k)]^T h_{6_a} - h_{6_a}^T [x(t_k) - x(s_k)] + \rho_t h_{6_b}^T R_6^{-1} h_{6_b}$$
$$- [x - x(t_k)]^T h_{6_b} - h_{6_b}^T [x - x(t_k)] + \tau \dot{x}^T R_6 \dot{x} + x^T Z x - x^T(t - \eta_{\min}) Z x(t - \eta_{\min})$$
$$- \begin{bmatrix} x^T(t) & x^T(t_k) \end{bmatrix} X \begin{bmatrix} x^T(t) & x^T(t_k) \end{bmatrix}^T + (\tau - \rho_s) \begin{bmatrix} \dot{x}^T(t) & 0 \end{bmatrix} X \begin{bmatrix} x^T(t) & x^T(t_k) \end{bmatrix}^T$$
$$+ (\tau - \rho_s) \begin{bmatrix} x^T(t) & x^T(t_k) \end{bmatrix} X \begin{bmatrix} \dot{x}^T(t) & 0 \end{bmatrix}^T. \tag{53}$$

Recalling (1) and (8), we can write

$$\dot{x}(t) = \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix} \zeta(t), \text{ and } \dot{x}(t) - Bu(t) = \begin{bmatrix} A & 0 & 0 & 0 \end{bmatrix} \zeta(t), \tag{54}$$

where $\zeta(t) = \begin{bmatrix} x^T(t) & x^T(t_k) & x^T(s_k) & x^T(t - \eta_{\min}) \end{bmatrix}^T$, $t \in (t_k, t_{k+1})$. Replacing (8) and (54) in (53), setting $h_j(t) = N_j^T \zeta(t)$, $j \in \{1, \ldots, 5\}$, $h_{6_a}(t) = N_{6_a}^T \zeta(t)$, and $h_{6_b}(t) = N_{6_b}^T \zeta(t)$, where N_j, $j \in \{1, \ldots, 5\}$, N_{6_a}, and N_{6_b} are matrices in $\mathbb{R}^{4n_x \times n_x}$, and replacing ρ_t and η_k with $\rho_s - \eta_{\min}$ and $\eta_{\max}$, respectively, yields

$$\dot{V} \leq \zeta^T \Bigg(\begin{bmatrix} A & 0 & BK & 0 \end{bmatrix}^T P \begin{bmatrix} I & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} I & 0 & 0 & 0 \end{bmatrix}^T P \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix}$$
$$+ (\rho_s - \eta_{\min}) N_1 R_1^{-1} N_1^T - \begin{bmatrix} I & -I & -(\rho_s - \eta_{\min}) BK & 0 \end{bmatrix}^T N_1^T$$
$$- N_1 \begin{bmatrix} I & -I & -(\rho_s - \eta_{\min}) BK & 0 \end{bmatrix} + (\tau - \rho_s) \begin{bmatrix} A & 0 & 0 & 0 \end{bmatrix}^T R_1 \begin{bmatrix} A & 0 & 0 & 0 \end{bmatrix}$$
$$+ (\rho_s - \eta_{\min}) N_2 R_2^{-1} N_2^T - \begin{bmatrix} I & -I & 0 & 0 \end{bmatrix}^T N_2^T - N_2 \begin{bmatrix} I & -I & 0 & 0 \end{bmatrix}$$
$$+ (\tau - \rho_s) \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix}^T R_2 \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix} + \eta_{\min} N_3 R_3^{-1} N_3^T$$
$$- \begin{bmatrix} I & 0 & 0 & -I \end{bmatrix}^T N_3^T - N_3 \begin{bmatrix} I & 0 & 0 & -I \end{bmatrix}$$
$$+ \eta_{\min} \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix}^T R_3 \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix} + (\rho_s - \eta_{\min}) N_4 R_4^{-1} N_4^T$$
$$- \begin{bmatrix} 0 & 0 & -I & I \end{bmatrix}^T N_4^T - N_4 \begin{bmatrix} 0 & 0 & -I & I \end{bmatrix}$$
$$+ (\tau - \eta_{\min}) \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix}^T R_4 \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix}$$
$$+ \rho_s N_5 R_5^{-1} N_5^T - \begin{bmatrix} I & 0 & -I & 0 \end{bmatrix}^T N_5^T - N_5 \begin{bmatrix} I & 0 & -I & 0 \end{bmatrix}$$
$$+ \tau \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix}^T R_5 \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix} + \eta_{\max} N_{6_a} R_6^{-1} N_{6_a}^T - \begin{bmatrix} 0 & I & -I & 0 \end{bmatrix}^T N_{6_a}^T$$

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$$
\begin{aligned}
&- N_{6_a}\begin{bmatrix}0 & I & -I & 0\end{bmatrix} + (\rho_s - \eta_{\min})N_{6_b}R_6^{-1}N_{6_b}^T - \begin{bmatrix}I & -I & 0 & 0\end{bmatrix}^T N_{6_b}^T \\
&- N_{6_b}\begin{bmatrix}I & -I & 0 & 0\end{bmatrix} + \tau\begin{bmatrix}A & 0 & BK & 0\end{bmatrix}^T R_6\begin{bmatrix}A & 0 & BK & 0\end{bmatrix} \\
&+ \begin{bmatrix}I & 0 & 0 & 0\end{bmatrix}^T Z\begin{bmatrix}I & 0 & 0 & 0\end{bmatrix} - \begin{bmatrix}0 & 0 & 0 & I\end{bmatrix}^T Z\begin{bmatrix}0 & 0 & 0 & I\end{bmatrix} - \begin{bmatrix}X & 0\\ 0 & 0\end{bmatrix} \\
&+ (\tau - \rho_s)\begin{bmatrix}A & 0 & BK & 0\\ 0 & 0 & 0 & 0\end{bmatrix}^T \begin{bmatrix}X & 0\end{bmatrix} + (\tau - \rho_s)\begin{bmatrix}X\\ 0\end{bmatrix}\begin{bmatrix}A & 0 & BK & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\Bigg)\zeta. \qquad (55)
\end{aligned}
$$

Based on (5), ρ_s varies between $\eta_{\min}$ and τ. Considering (55) and using Schur complement [38], for $\rho_s = \eta_{\min}$, LMI (26) implies $\dot{V} < 0$. Similarly, LMI (27) implies $\dot{V} < 0$ for $\rho_s = \tau$. Since (55) is affine in ρ_s, LMIs (26) and (27) are sufficient conditions for $\dot{V} < 0$ to hold for any $\rho_s \in [\eta_{\min}, \tau]$, i.e. $\forall (t_k, t_{k+1})$, $k \in \mathbb{N}$. Note that there exists a sufficiently small scalar $c_3 > 0$ such that $\dot{V}(t, x_t) < -c_3||x_t||_{\mathcal{C}}^2$, for all $t \neq t_k$, $k \in \mathbb{N}$. Hence, inequality (28) yields

$$
\dot{V}(t, x_t) < -\frac{c_3}{c_2}V(t, x_t),\ \forall t \neq t_k,\ k \in \mathbb{N}. \qquad (56)
$$

Therefore, for any $k \in \mathbb{N}$,

$$
V(t_k^-, x_{t_k^-}) \leq e^{-\frac{c_3}{c_2}(t_k - t_{k-1})}V(t_{k-1}, x_{t_{k-1}}) \leq V(t_{k-1}, x_{t_{k-1}}),
$$

where $V(t_k^-, x_{t_k^-}) = \lim_{t \nearrow t_k} V(t, x_t)$. The second inequality is strict when the length of the interval (t_{k-1}, t_k) is nonzero. Note that according to Assumption 4, any interval (t_{k-1}, t_k), $k \in \mathbb{N}$, has a length greater than or equal to $\epsilon > 0$. Furthermore, it was shown at the beginning of the proof that V is non-increasing at the actuator update instants, i.e.

$$
V(t_k, x_{t_k}) \leq V(t_k^-, x_{t_k^-}),\ k \in \mathbb{N}.
$$

Therefore, for any $t \in [t_k, t_{k+1})$, $k \in \mathbb{N}$,

$$
\begin{aligned}
V(t, x_t) &\leq e^{-\frac{c_3}{c_2}(t - t_k)}V(t_k, x_{t_k}) \leq e^{-\frac{c_3}{c_2}(t - t_k)}V(t_k^-, x_{t_k^-}) \\
&\leq e^{-\frac{c_3}{c_2}(t - t_{k-1})}V(t_{k-1}, x_{t_{k-1}}) \leq e^{-\frac{c_3}{c_2}(t - t_{k-1})}V(t_{k-1}^-, x_{t_{k-1}^-}) \\
&\ \ \vdots \\
&\leq e^{-\frac{c_3}{c_2}t}V(0, x_0). \qquad (57)
\end{aligned}
$$

A similar conclusion could be drawn from Comparison Lemma [39]. Now, inequalities (28) and (57) yield

$$
|x(t)| \leq \left(\frac{V(t, x_t)}{c_1}\right)^{\frac{1}{2}} \leq \left(\frac{e^{-\frac{c_3}{c_2}t}V(0, x_0)}{c_1}\right)^{\frac{1}{2}} \leq \left(\frac{c_2}{c_1}\right)^{\frac{1}{2}} e^{-\frac{c_3}{2c_2}t}||x_0||_{\mathcal{C}}.
$$

Hence, the NCS is globally uniformly exponentially stable. Note that the Zeno phenomenon does not occur since, by Assumption 4, there exists $\epsilon > 0$ such that $t_{k+1} - t_k > \epsilon$. This finishes the proof. □

The following proposition compares the conservatism of the results of Theorem 2 in [17] and Theorem 1 in this paper. For the sake of completeness, the stability conditions in [17] are summarized in the Appendix.

Proposition 1
If there is a solution to the stability conditions in [17] then there is also a solution to the stability conditions of Theorem 1.

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Proof
Suppose that there exist matrices P, R_i, $i \in \{2,3,4,6\}$, Z, and $\overline{X}$, N_j, $j \in \{2,3,4\}$, and N_{6_a} satisfying the stability conditions in [17] (please refer to the appendix). Let $N_1 = N_5 = N_{6_b} = 0$ and $X_1 = X_2 = \overline{X}$. Then there exist matrices $R_1 = R_5 = \sigma I$, with a sufficiently small $\sigma > 0$, that satisfy the LMIs (25)-(27). The proof is complete since for any set of matrix variables satisfying the conditions of [17], there exist a set of matrix variables satisfying the conditions of Theorem 1. □

In Theorem 1, given the value of the network-induced delay τ and the lower and upper bounds on the delay, i.e. $\eta_{\min}$ and $\eta_{\max}$, we presented sufficient conditions for exponential stability of linear NCSs. In some practical problems, however, such information about the delay might not be available. Here, we present sufficient conditions for exponential stability of linear NCSs under limited information about the delay. The following corollary addresses the case where the upper bound on the delay $\eta_{\max}$ is unknown. To the best of our knowledge, this scenario was not studied in the literature before.

Corollary 1
Consider the linear NCS defined in (1) and (8) with Assumptions 1-4. Given the controller gain K and the scalars τ and $\eta_{\min}$, the NCS is globally uniformly exponentially stable if there exist symmetric positive definite matrices P, R_i, $i \in \{1,\ldots,5\}$, and Z, a symmetric matrix X_1, and matrices X_2, N_j, $j \in \{1,\ldots,5\}$, with appropriate dimensions, satisfying

$$\begin{bmatrix} P & 0 \\ 0 & 0 \end{bmatrix} + (\tau - \eta_{\min})X > 0$$

$$\begin{bmatrix} \Psi + \tau M_1 + (\tau-\eta_{\min})(M_2+M_4) + \eta_{\min} M_3 & \eta_{\min} N_3 & \eta_{\min} N_5 \\ \eta_{\min} N_3^T & -\eta_{\min} R_3 & 0 \\ \eta_{\min} N_5^T & 0 & -\eta_{\min} R_5 \end{bmatrix} < 0$$

$$\begin{bmatrix} \begin{pmatrix} \Psi + \tau M_1 + \eta_{\min} M_3 + \\ (\tau-\eta_{\min})(M_4+M_5) \end{pmatrix} & (\tau-\eta_{\min})N_1 & (\tau-\eta_{\min})N_2 & \eta_{\min} N_3 & (\tau-\eta_{\min})N_4 & \tau N_5 \\ (\tau-\eta_{\min})N_1^T & (\eta_{\min}-\tau)R_1 & 0 & 0 & 0 & 0 \\ (\tau-\eta_{\min})N_2^T & 0 & (\eta_{\min}-\tau)R_2 & 0 & 0 & 0 \\ \eta_{\min} N_3^T & 0 & 0 & -\eta_{\min} R_3 & 0 & 0 \\ (\tau-\eta_{\min})N_4^T & 0 & 0 & 0 & (\eta_{\min}-\tau)R_4 & 0 \\ \tau N_5^T & 0 & 0 & 0 & 0 & -\tau R_5 \end{bmatrix} < 0$$

where Ψ, M_j, $j \in \{1,\ldots,5\}$, are defined in Theorem 1 with $R_6 = 0$ and $N_{6_a} = N_{6_b} = 0$.

Proof
Let an LKF be defined as $\sum_m V_m$, $m \in \{0,\ldots,5,7,8\}$. Here, we omit the functional V_6 because its derivative is approximated by a functional that depends on η_k (see inequality (50)). In turn, η_k is replaced in (55) by the upper bound $\eta_{\max}$. In this corollary, however, $\eta_{\max}$ is assumed to be unknown. Using the modified LKF, the rest of the proof is similar to the proof of Theorem 1. □

If the lower bound on the delay $\eta_{\min}$ is unknown, based on Assumption 2, we set $\eta_{\min} = 0$. The next corollary provides sufficient conditions for exponential stability of linear NCSs where $\eta_{\min}$ is unknown or similarly where $\eta_{\min} = 0$.

Corollary 2
Consider the linear NCS defined in (1) and (8) with Assumptions 1-4. Given the controller gain K and the scalars τ and $\eta_{\max}$, the NCS is globally uniformly exponentially stable if there exist symmetric positive definite matrices P, R_1, R_2, R_5, and R_6, a symmetric matrix X_1, and matrices X_2, N_1, N_2, N_5, N_{6_a}, and N_{6_b}, with appropriate dimensions, satisfying

$$\begin{bmatrix} P & 0 \\ 0 & 0 \end{bmatrix} + \tau X > 0$$

$$\begin{bmatrix} \Psi + \tau(M_1 + M_2) & \eta_{\max} N_{6_a} \\ \eta_{\max} N_{6_a}^T & -\eta_{\max} R_6 \end{bmatrix} < 0$$

$$\begin{bmatrix} \Psi + \tau(M_1 + M_5) & \tau N_1 & \tau N_2 & \tau N_5 & \eta_{\max} N_{6_a} & \tau N_{6_b} \\ \tau N_1^T & -\tau R_1 & 0 & 0 & 0 & 0 \\ \tau N_2^T & 0 & -\tau R_2 & 0 & 0 & 0 \\ \tau N_5^T & 0 & 0 & -\tau R_5 & 0 & \\ \eta_{\max} N_{6_a}^T & 0 & 0 & 0 & -\eta_{\max} R_6 & 0 \\ \tau N_{6_b}^T & 0 & 0 & 0 & 0 & -\tau R_6 \end{bmatrix} < 0$$

where Ψ, M_1, M_2, and M_5 are defined in Theorem 1 with $R_3 = R_4 = Z = 0$ and $N_3 = N_4 = 0$, and all the zero rows and columns (corresponding to $x(t - \eta_{\min})$) are omitted.

Proof
Let an LKF be defined as $\sum_m V_m$, $m \in \{0, 1, 2, 5, 6, 8\}$. Here, the functionals V_3, V_7, and the second term in V_4 are omitted because they vanish when $\eta_{\min} = 0$. Also note that when $\eta_{\min} = 0$, the first part of V_4 becomes identical to the functionals V_5 and V_6. Therefore, the first part of V_4 is dispensable in this case. Using the modified LKF, the rest of the proof is similar to the proof of Theorem 1. □

If $\eta_k = 0$, for all $k \in \mathbb{N}$, i.e. the case where the transmission and computation delays are negligible, the NCS is classified as a sampled-data system. The following corollary presents sufficient conditions for exponential stability of linear sampled-data systems.

Corollary 3
Consider the system defined in (1) and (8) with Assumptions 1-4 and assume that the delay is negligible. Given the controller gain K and the scalar τ, the resulting linear sampled-data system is globally uniformly exponentially stable if there exist symmetric positive definite matrices P, R_1, and R_2, a symmetric matrix X_1, and matrices X_2, N_1, and N_2, with appropriate dimensions, satisfying

$$\begin{bmatrix} P & 0 \\ 0 & 0 \end{bmatrix} + \tau X > 0$$

$$\overline{\Psi} + \tau \overline{M}_1 < 0$$

$$\begin{bmatrix} \overline{\Psi} + \tau \overline{M}_2 & \tau N_1 & \tau N_2 \\ \tau N_1^T & -\tau R_1 & 0 \\ \tau N_2^T & 0 & -\tau R_2 \end{bmatrix} < 0$$

where

$$\overline{\Psi} = \begin{bmatrix} A^T \\ K^T B^T \end{bmatrix} \begin{bmatrix} P & 0 \end{bmatrix} + \begin{bmatrix} P \\ 0 \end{bmatrix} \begin{bmatrix} A & BK \end{bmatrix} - \begin{bmatrix} I \\ -I \end{bmatrix} (N_1^T + N_2^T) - (N_1 + N_2) \begin{bmatrix} I & -I \end{bmatrix} - X,$$

$$\overline{M}_1 = \begin{bmatrix} A^T \\ 0 \end{bmatrix} R_1 \begin{bmatrix} A & 0 \end{bmatrix} + \begin{bmatrix} A^T \\ K^T B^T \end{bmatrix} R_2 \begin{bmatrix} A & BK \end{bmatrix} + \begin{bmatrix} A^T & 0 \\ K^T B^T & 0 \end{bmatrix} X + X \begin{bmatrix} A & BK \\ 0 & 0 \end{bmatrix},$$

$$\overline{M}_2 = \begin{bmatrix} 0 \\ K^T B^T \end{bmatrix} N_1^T + N_1 \begin{bmatrix} 0 & BK \end{bmatrix}.$$

Proof
When the transmission and computation delays are negligible, i.e. $\eta_k = 0$, $k \in \mathbb{N}$, we have $\eta_{\min} = 0$ and, according to (6), $\rho_s = \rho_t$. Let an LKF be defined as $V_{sd} = V_0 + V_1 + V_2 + V_8$. Here, with $\eta_{\min} = 0$, the functionals V_3, V_4, and V_7 are omitted for the same reasons as explained in Corollary 2. Note that with $\rho_s = \rho_t$, the functionals V_5 and V_6 become similar to the functional V_2. It is known in the literature [20, 21], however, that the functionals V_5 and V_6 lead to more conservative results compared to the functional V_2. Using the modified LKF V_{sd}, the rest of the proof is similar to the proof of Theorem 1. □

The following proposition presents sufficient conditions for exponential stability of linear NCSs with uncertain parameters.

Proposition 2
Suppose that the pair of system matrices $\Omega = \begin{bmatrix} A & B \end{bmatrix}$ in (1) is unknown but satisfies the following condition

$$\Omega \in \left\{ \sum_{j=1}^{p} \alpha_j \Omega_j,\ 0 \le \alpha_j \le 1,\ \sum_{j=1}^{p} \alpha_j = 1 \right\},$$

where $\Omega_j = \begin{bmatrix} A_j & B_j \end{bmatrix}$, $j \in \{1, ..., p\}$, denote the vertices of a convex polytope. If the LMIs in Theorem 1 (or Corollary 1-3) hold for each Ω_j, $j \in \{1, ..., p\}$, with the same matrix variables P, R_i, $i \in \{1, \ldots, 6\}$, Z, X_1, and X_2, then the uncertain linear NCS is globally uniformly exponentially stable.

Proof
Given that the LMIs in Theorem 1 (or Corollary 1-3) hold for each Ω_j, $j \in \{1, ..., p\}$, with the same matrix variables P, R_i, $i \in \{1, \ldots, 6\}$, Z, X_1, and X_2, it is guaranteed that the LMIs in Theorem 1 (or Corollary 1-3) also hold for any matrix parameter lying in the convex hull of Ω_j, $j \in \{1, ..., p\}$. Therefore, the uncertain linear NCS is globally uniformly exponentially stable. □

The LMIs in Theorem 1 are affine in τ, η_{min}, and η_{max}. Therefore, keeping two of these variables constant, we can use a line search approach to optimize for the other variable. For instance, given the lower and upper bounds on the delay, the problem of finding a lower bound on the maximum network-induced delay that preserves exponential stability is formulated as

Problem 1

$$\begin{aligned} &\text{maximize } \ \tau \\ &\text{subject to } \ P > 0,\ R_i > 0,\ i \in \{1, \ldots, 6\},\ Z > 0,\ X_1 = X_1^T,\ (25) - (27). \end{aligned}$$

We denote the computed lower bound on the maximum network-induced delay that preserves exponential stability by τ_{max}. Similarly, the LMIs in Corollary 1-3 can be used to write suitable optimization programs.

4. NUMERICAL EXAMPLES

In this section, we apply our stability theorems to three benchmark problems in the literature.

Example 1
[16, 17, 20, 22] Consider the linear NCS defined in (1) and (8) with the following parameters

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix},\ K = -\begin{bmatrix} 3.75 & 11.5 \end{bmatrix}.$$

Here, we assume that $\eta_{\text{max}} = 0.8$ (s) and solve Problem 1 to find a lower bound on the maximum network-induced delay that preserves exponential stability for different values of η_{min}. Table II shows the computed τ_{max} by Theorem 1 and the Theorems in [17, 22, 40]. According to Table II, the stability criteria of Theorem 1 are less conservative (i.e. provide larger lower bounds on the maximum network-induced delay) for this benchmark problem than the previously existing results.

Now, consider the linear system with the same parameters in a sampled-data scenario. In other words, we let the transmission and computation delays to be negligible, i.e. $\eta_k = 0,\ \forall k \in \mathbb{N}$. However, the sampling intervals are assumed to be unknown and non-uniform. Our goal is to find a lower bound on the largest sampling interval that preserves exponential stability. The second row of Table III compares the results of Corollary 3 and the Theorems in [16, 20, 23, 40]. Based on Table III, the stability conditions of Corollary 3 are less conservative (i.e. provide larger lower bounds on the largest sampling interval) for this benchmark problem than the other results in the literature.

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Table II. Comparison of the computed lower bound on the maximum network-induced delay $\tau_{\max}$ (s) for $\eta_{\max} = 0.8$ (s) and different values of $\eta_{\min}$ in Example 1.

$\eta_{\min}$ (s)	0	0.2	0.4	0.6	0.75
[40]	1.04	-	-	-	-
[17]	0.87	0.89	0.92	0.97	1.02
([17] plus V_5) $\equiv$ (Theorem 1 with $V_1 = 0$)	0.87	0.89	0.93	0.98	1.03
([17] plus V_1) $\equiv$ (Theorem 1 with $V_5 = 0$)	1.06	1.02	1.00	1.01	1.03
[22]	1.10	-	-	-	-
Theorem 1	1.14	1.09	1.06	1.05	1.07

Table III. Comparison of the computed lower bound on the largest sampling interval (s) that preserves exponential stability in Example 1 and Example 2 with $\eta_k = 0,\ \forall k \in \mathbb{N}$.

	[40]	[23]	[16]	[20] $\equiv$ (Corollary 3 with $V_1 = 0$)	Corollary 3
Example 1	1.04	1.36	1.113	1.698	1.717
Example 2	1.06	0.87	0.732	1.641	2.015

Example 2
[20] Consider a linear sampled-data system with the following parameters

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ K = \begin{bmatrix} -1 & 1 \end{bmatrix}.$$

In the sampled-data scenario we assume that the transmission and computation delays are negligible, i.e. $\eta_k = 0,\ \forall k \in \mathbb{N}$. However, the sampling intervals are considered as unknown and non-uniform. Our goal is to find a lower bound on the largest sampling interval that preserves exponential stability. The third row of Table III compares the results of Corollary 3 and the Theorems in [16, 20, 23, 40]. Based on Table III, the stability conditions of Corollary 3 are less conservative (i.e. provide larger lower bounds on the largest sampling interval) for this benchmark problem than the other results in the literature.

Example 3
[16, 18] Consider the following linear sampled-data system (i.e. $\eta_k = 0,\ \forall k \in \mathbb{N}$) with polytopic uncertainty in matrix parameters

$$A = \begin{bmatrix} 1 & 0.5 \\ g_1 & -1 \end{bmatrix},\ B = \begin{bmatrix} 1 + g_2 \\ -1 \end{bmatrix},\ K = -\begin{bmatrix} 2.6884 & 0.6649 \end{bmatrix},$$

where $|g_1| \le 0.1$ and $|g_2| \le 0.3$. Our objective is to find a lower bound on the largest sampling interval that preserves exponential stability. Here, based on Proposition 2, we simultaneously check the stability criteria in Corollary 3 for each combination of A_i and B_j, $i, j \in \{1, 2\}$, defined by

$$A_1 = \begin{bmatrix} 1 & 0.5 \\ -0.1 & -1 \end{bmatrix},\ A_2 = \begin{bmatrix} 1 & 0.5 \\ 0.1 & -1 \end{bmatrix},\ B_1 = \begin{bmatrix} 0.7 \\ -1 \end{bmatrix},\ B_2 = \begin{bmatrix} 1.3 \\ -1 \end{bmatrix}.$$

Table IV compares the computed lower bound on the largest sampling interval that preserves exponential stability using Corollary 3 and the theorems in [18, 16, 20]. According to Table IV, the results of this paper compare favorably with the previously existing results for this benchmark problem.

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Table IV. Comparison of the computed lower bound on the largest sampling interval (s) that preserves exponential stability in Example 3 with $\eta_k = 0,\ \forall k \in \mathbb{N}$.

[18]	[16]	[20] $\equiv$ (Corollary 3 with $V_1 = 0$)	Corollary 3
0.35	0.447	0.591	0.699

5. CONCLUSION

In this paper, we addressed exponential stability of linear NCSs. We introduced a modified LKF that contains a functional in terms of the open-loop vector field of the linear system. Next, based on the modified LKF, new sufficient stability conditions were derived for linear NCSs. The paper studied the case of linear sampled-data systems as a corollary. Furthermore, the problem of finding a lower bound on the maximum network-induced delay that preserves exponential stability was formulated as a convex optimization program in terms of LMIs. The stability conditions of this paper were shown to be less conservative than previously existing results when applied to different benchmark problems.

APPENDIX

Here, the stability conditions in [17] are summarized for the sake of completeness. Note that the matrix variables in [17] are renamed in accordance with the notation of this paper.

Theorem [17] Consider the linear NCS defined in (1) and (8) with Assumptions 1-4. Given the controller gain K and the scalars τ, $\eta_{\min}$, and $\eta_{\max}$, the NCS is globally uniformly exponentially stable if there exist symmetric positive definite matrices P, R_i, $i \in \{2,3,4,6\}$, Z, and $\overline{X}$, and matrices N_j, $j \in \{2,3,4\}$, and N_{6_a}, with appropriate dimensions, satisfying

$$\begin{bmatrix} \overline{\Psi} + \tau\overline{M}_1 + (\tau-\eta_{\min})(\overline{M}_2+\overline{M}_4) + \eta_{\min}\overline{M}_3 & \eta_{\min}N_3 & \eta_{\max}N_{6a} \\ \eta_{\min}N_3^T & -\eta_{\min}R_3 & 0 \\ \eta_{\max}N_{6a}^T & 0 & -\eta_{\max}R_6 \end{bmatrix} < 0$$

$$\begin{bmatrix} \begin{pmatrix} \overline{\Psi} + \tau\overline{M}_1 + \eta_{\min}\overline{M}_3 \\ +(\tau-\eta_{\min})\overline{M}_4 \end{pmatrix} & (\tau-\eta_{\min})N_2 & \eta_{\min}N_3 & (\tau-\eta_{\min})N_4 & \eta_{\max}N_{6a} \\ (\tau-\eta_{\min})N_2^T & -(\tau-\eta_{\min})R_2 & 0 & 0 & 0 \\ \eta_{\min}N_3^T & 0 & -\eta_{\min}R_3 & 0 & 0 \\ (\tau-\eta_{\min})N_4^T & 0 & 0 & -(\tau-\eta_{\min})R_4 & 0 \\ \eta_{\max}N_{6a}^T & 0 & 0 & 0 & -\eta_{\max}R_6 \end{bmatrix} < 0$$

where

$$\begin{aligned} \overline{\Psi} &= \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix}^T \begin{bmatrix} P & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} P & 0 & 0 & 0 \end{bmatrix}^T \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix} \\ &\quad - \begin{bmatrix} I & -I & 0 & 0 \end{bmatrix}^T N_2^T - N_2 \begin{bmatrix} I & -I & 0 & 0 \end{bmatrix} - \begin{bmatrix} I & 0 & 0 & -I \end{bmatrix}^T N_3^T \\ &\quad - N_3 \begin{bmatrix} I & 0 & 0 & -I \end{bmatrix} - \begin{bmatrix} 0 & 0 & -I & I \end{bmatrix}^T N_4^T - N_4 \begin{bmatrix} 0 & 0 & -I & I \end{bmatrix} \\ &\quad - \begin{bmatrix} 0 & I & -I & 0 \end{bmatrix}^T N_{6a}^T - N_{6a} \begin{bmatrix} 0 & I & -I & 0 \end{bmatrix} + \begin{bmatrix} I & 0 & 0 & 0 \end{bmatrix}^T Z \begin{bmatrix} I & 0 & 0 & 0 \end{bmatrix} \\ &\quad - \begin{bmatrix} 0 & 0 & 0 & I \end{bmatrix}^T Z \begin{bmatrix} 0 & 0 & 0 & I \end{bmatrix} - \begin{bmatrix} I & -I & 0 & 0 \end{bmatrix}^T \overline{X} \begin{bmatrix} I & -I & 0 & 0 \end{bmatrix}, \\ \overline{M}_1 &= \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix}^T R_6 \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix}, \\ \overline{M}_2 &= \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix}^T R_2 \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix} + \begin{bmatrix} I & -I & 0 & 0 \end{bmatrix}^T \overline{X} \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix} \\ &\quad + \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix}^T \overline{X} \begin{bmatrix} I & -I & 0 & 0 \end{bmatrix}, \end{aligned}$$

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$$\overline{M}_3 = \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix}^T R_3 \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix},$$
$$\overline{M}_4 = \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix}^T R_4 \begin{bmatrix} A & 0 & BK & 0 \end{bmatrix}.$$

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