Credit Risk, Liquidity Risk and Asset Dynamics: Theory and Empirical Evidence

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A Thesis

in

The John Molson School of Business

Department of Finance

Presented in Partial Fulfillment of the Requirements

for the Degree of

Doctor of Philosophy at

Concordia University

Montreal, Quebec, Canada

April, 2013

CONCORDIA UNIVERSITY

SCHOOL OF GRADUATE STUDIES

This is to certify that the thesis prepared

By: **Rui Zhong**

Entitled: **Credit Risk, Liquidity Risk and Asset Dynamics: Theory and Empirical Evidence**

and submitted in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY (Business Administration)

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ABSTRACT

Credit Risk, Liquidity Risk and Asset Dynamics: Theory and Empirical Evidence

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In this dissertation, we first generalize Leland (1994b)'s structural model from constant volatility to the state-dependent volatility with constant elasticity (CEV) and obtain the analytical solution for most variables of interest, including first-passage default probability, corporate debt and equity value. After incorporating jumps into asset dynamics, we develop an efficient algorithm to calculate the first passage default probability by adopting a restricted structure of default times and derive numerical solutions for the variables of interest. We find that the extra parameter in the CEV structural model has a significant impact on the optimal capital structure, the debt capacity, the term structure of credit spreads, the duration and convexity of risky debt, the equity volatility, the asset substitution impacts and the cumulative default probabilities.

Further, we incorporate the liquidity risk of the secondary bond market into the structural model with a constant elasticity of variance through the rollover channel and derive the analytical expressions for the variables of interest with an innovative method in Chapter 2. We find that state dependent volatility has noticeable impacts for all the interesting results, including the endogenous default boundary, the optimal leverage and the credit spreads, which depend on the value of the state dependence parameter.

In Chapter III, we compare the empirical performance of the two alternative volatility assumptions that we used in our study within the context of the Leland (1994b) model. Using time series data from both firm and risk level, We document that CEV structural model with the elasticity parameter around -0.67 on average exhibits a superior fitting in the CDS spreads across all the maturities. The relationship between the sign and value of β and the firm specific measures of default risk, such as leverage ratios, CDS spreads and current ratios indicates that there is a tendency for β to increase as the risk of the firm decreases, but that the tendency is weak and fluctuates. We also note that the CDPs generated by the CEV structural model can fit the Moody's observed data much better compared to these with constant asset volatility.

In the last Chapter, we study the market efficiency between the CDS and Loan CDS (LCDS) markets by constructing a CDS and LCDS parity relation under the no arbitrage assumption. We document persistent and significant violations of this relation with the cross sectional data from both markets. We identify time-varying and significant positive arbitrage profits from an artificial default risk-free portfolio that trades in both markets and simultaneously longs an undervalued contract and shorts the corresponding overvalued contract for exactly the same underlying firm, maturity, currency and restructure clauses. We show that the profits cannot be accounted for by trading costs or imperfect data about loan recovery rates in the event of default. Using panel regressions with macroeconomic and firm-level variables, we find that firm-level informational asymmetry and difficulty of loan recovery in case of default are much more important than macroeconomic factors in accounting for the arbitrage profits.

DEDICATION

To my dearest son, wife, father and mother

ACKNOWLEDGMENTS

This thesis would not have been possible without the support and help of many people. I would like to take this opportunity to truly and deeply thank them.

First and foremost, I am greatly indebted to my supervisor, Dr. Stylianos Perrakis, for his outstanding guidance and endless support throughout my doctoral career. His attitude of hard working and pursuing excellence had a direct influence on me. I am also extremely grateful to my current and former committee members, Dr. Lawrence Kryzanowski, Dr. Jan Ericsson, Dr. Zhiguo He, Dr. Nikolay Gospodinov, Dr. Sergey Isaenko and Dr. Peter Christoffersen for their insightful discussions, valuable comments and constructive suggestions. The generous support from other faculty members at the finance department at Concordia University, especially Dr. Harjeet S. Bhabra, Dr. Sandra Betton and Dr. Thomas Walker, are highly appreciated as well.

I also acknowledge the excellent research assistance from Ken Liu and help from supporting staff at the John Molson School of Business, Amr Addas, Karen Fada, Karen Fiddler, Edite Almeida and Catherine Sarrazin. In addition, I would like to thank my Ph.D. colleagues for the encouragement, help and especially friendship, which made my Ph.D. life colorful.

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1. Introduction

A very large number of studies, both theoretical and empirical, on corporate bond pricing and the risk structure of interest rates have appeared in the literature following the pioneering work of Merton (1974) and Black and Cox (1976), which in turn were inspired by the seminal Black and Scholes (1973) model of option pricing. These studies adopted the methodological approach of contingent claims valuation in continuous time, in which the value of a firm's assets played the role of the claim's underlying asset and allowed the valuation of the various components of the balance sheet under a variety of assumptions. This approach has been shown to be sufficiently flexible to tackle some of the most important problems in corporate finance, such as capital structure, bond valuation and default risk, under a variety of assumptions about the type of bonds included in the firm's liabilities. The resulting models came to be known as *structural models* of bond pricing, as distinct from another class of models known as reduced form models, in which there is no link between the default risk of bonds and the firm's capital structure.[1]

Under continuous coupon payment and first-passage default[2] assumptions, Leland (L, 1994a,b) and Leland and Toft (LT, 1996) first studied corporate debt valuation and optimal capital structure with endogenous default boundary for infinite maturity debt and

[1] For the reduced form models see Jarrow and Turnbull (1995), Duffie and Singleton (1999) and Duffie and Lando (2001). These models lie outside the topic of this paper.
[2] Under the first-passage default assumption, a firm will claim default when the asset value first crosses the pre-determined default boundary. This default boundary can be determined endogenously (Leland, 1994a,b, Leland and Toft, 1996) or exogenously (Longstaff and Schwartz, 1995).

finite maturity debt, respectively. Because of the computational complexity of the valuation expressions, a major emphasis in the structural models was placed on the derivation of closed form expressions, rather than numerical results based on approximations[3] or simulations.[4] Such a focus allowed relatively easy estimations of numerical values given the parameters of the model, but at the cost of maintaining simple formulations of the mathematical structure of the asset value dynamics, in which a univariate diffusion process still follows the original Black and Scholes (1973) and Merton (1974) assumption of a lognormal diffusion with constant volatility.[5] This is all the more surprising, in view of the fact that the option pricing literature has long recognized that such an assumption is no longer adequate to represent underlying assets in option markets, and has introduced factors such as rare events, stochastic volatility and transaction costs. Choi and Richardson (2009) studied the conditional volatility of the firm's assets by a weighted average of equity, bond and loan prices and found that asset volatility is time varying. Hilberink and Rogers (2002) and Chen and Kou (2009) extend the Leland (1994b) model by incorporating a Levy process with only upward jumps and with two-sided double exponential jumps[6], respectively. In their study of the term structure of credit default swaps (CDS), Huang and Zhou (2008) note that time varying asset volatility should potentially play a role in structural models in order to fit into the

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[3] Zhou (2001) and Collin-Dufresne and Goldstein (2001).

[4] Brennan and Schwartz (1978), and more recently Titman and Tsyplakov (2007) are examples of studies that rely on numerical simulations.

[5] Most structural models are univariate and assume a constant riskless rate of interest. Longstaff and Schwartz (1995), Briys and de Varenne (1997), and Collin-Dufresne and Goldstein (2001) use bivariate diffusion models, in which the term structure of interest rates follows the Vacisek (1977) model and the asset value is a constant volatility diffusion. As the empirical work in Chan *et al* (1992) shows, the Vacisek model does not fit actual term structure data. Further, Leland and Toft (1996) note that this bivariate diffusion refinement plays a very small role in the yield spreads of corporate bonds.

[6] Zhou (2001) was the first to introduce jumps into structural models under the first passage default assumption, but no analytical solution is presented and he did not study the impact on optimal capital structure with endogenous default boundary. Huang and Huang (2003) also incorporate double exponential jumps into a structural model, but they only focus on corporate debt valuation and credit spread.

empirical credit default spread data. Huang (2005), and Zhang, Zhou and Zhu (2008) incorporate stochastic volatility and jumps into the Merton (1974) model by assuming that default occurs only at maturity and find that incorporating jumps and stochastic volatility may help to improve the matching of the top quality credit spreads.

In this paper we incorporate similar generalizations into the dynamics of the asset value in the context of the L (1994b) and LT (1996) models, which allow default at times other than the maturity of the debt. We consider several alternative specifications that include a state-dependent volatility with and without jumps, and a stochastic volatility, again with and without jumps in both the asset value and the volatility processes. For the state dependent case we consider the Constant Elasticity of Variance (CEV) model, originally formulated by Cox (1975) in the context of option pricing,[7] which has an extra parameter and includes constant volatility as a special case. By borrowing heavily from the option pricing literature we manage to derive closed form expressions for almost all the variables of interest in the absence of jumps, including corporate debt value, total levered firm value, optimal leverage and equity value. Because of discontinuities, we only obtain quasi-analytical numerical solutions for several of these same variables when we include in the asset dynamics an independent jump component with multinomial amplitudes; for computational purposes default times are constrained to occur at fixed discrete intervals. As a special case, we also obtain quasi-analytical solutions for the variables of interest for the constant volatility model in the presence of jumps without restricting the type of distribution of the amplitude of the jump component.[8]

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[7] See also Emmanuel and MacBeth (1982), Cox and Rubinstein (1985), and Schroder (1989).

[8] Kou and Wang (2003) and Huang and Huang (2003) develop analytical solutions for this model, but at the price of restricting the jump amplitude distribution to a double exponential.

Similar results are also obtained from the stochastic volatility specification with and without jumps. In this more complex specification all results, with and without jumps, are derived quasi-analytically, with the important difference that without the discontinuities due to the jump components the default times are much more tightly spaced, approximating the continuous time default case. The general formulation and the nested nature of the various specifications allow us to gauge the impact of each additional feature of asset dynamics in approximating observed patterns in the variables of interest such as credit spreads and default probabilities. For this reason in the numerical applications we first compare the various cases of asset dynamics in the absence of jumps, and then we introduce jumps in each case.

Apart from the asset value dynamics, we follow the general assumptions initially formulated by Merton (1974), in which default occurs when the asset value hits a lower default-triggering threshold. While in Merton's model default could only take place at maturity, Black and Cox (1976), Leland (1994a,b,1998) and LT adopted debt assumptions that allowed default to take place before maturity. All these models can be included in our formulations, with the mixed jump-diffusion models that we present allowing default at discrete predetermined times. To our knowledge, this is the first paper to relax the constant volatility assumption of the earlier studies and still derive closed form solutions under continuous coupon payment and first passage default assumptions, and also the first model to incorporate jumps with distributions other than a double exponential at times other than debt maturity.[9]

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[9]Zhou (2001) is the first paper to introduce jumps into structural models by using simulation. Kou and Wang (2003) and Huang and Huang (2003) obtain the analytical solution of the first passage default time by restricting the jump amplitude distribution to a double exponential.

Since our extensions have implications for several strands of literature that have dealt with different problems in corporate finance, we review briefly the key issues examined by the class of models that we generalize. All these issues can be dealt with the same type of integrated models of the levered firm that we examine. The main such issue is the capital structure choice, which originates in the classic Modigliani and Miller (1963) analysis of the levered firm in the presence of taxes, according to which capital structure is chosen as a trade-off between the tax advantage of debt and the costs of possible bankruptcy. The pioneering work in this area that comes closest to our own approach is that of Leland (1994a,b,1998), and LT.[10] Since several authors have raised doubts on whether the trade-off approach can really be invoked to justify observed leverage ratios, several studies focused on agency problems between stockholders and debt holders, or stockholders and managers.[11] These and other related studies show clearly the importance of the structural models in linking the default probabilities and yield spreads to the capital structure decision, a linkage that is missing from the reduced form models.

As already noted, we use the asset value of the unlevered firm as the basic underlying process for the valuation of the various components of the balance sheet of the levered firm, following Leland (1994a, b) and LT. In a variant of the basic model, presented in Goldstein, Ju and Leland (2001), the firm value is estimated from the dynamics of the earnings before interest and taxes (EBIT), split between the claimholders and the government.[12] A direct modeling of the dynamics, division and valuation of the firm's cash flows would in principle also be possible in our models, but it will need to

[10] See also, Sarkar and Zapatero (2003), Ju *et al* (2005) and Titman and Tsyplakov (2007)
[11] See Mella-Barral and Perraudin (1997), Leland (1998), Morellec (2004), and Ju *et al* (2005).
[12] See also Sarkar and Zapatero (2003).

confront the troublesome issue of the valuation of non-traded assets, which is beyond the scope of this paper, and which in earlier studies is either avoided or carried out only under the most elementary assumptions.[13]

We close this literature review by noting a variant of the reduced form models, which is particularly popular in the financial mathematics literature. In that stream the primary asset dynamics, in the familiar forms of diffusion or jump diffusion, are applied not to the asset value but to the equity returns, as in the option pricing literature.[14] The advantage of this approach is that the equity returns, unlike firm value, are observable and available in high frequency data. Its disadvantage is, as with the reduced form models, that it does not allow the modeling of the firm's balance sheet and the linkage of the default process with the firm's capital structure, and for this reason will not be pursued in this paper. There exist statistical methods by which the parameters of the asset value dynamics can be estimated from the observed dynamics of the equity value for any given model.

In what follows we present in Section 2 the various cases of asset dynamics that have been used in the option pricing literature, as well as the fundamental building blocks of the L and LT structural models of the firm to which they will be applied. Section 3 presents the fundamental notions of the cumulative first passage to default probability (CFPD) and unit price (UP), on the basis of which all economic variables of interest can be computed, and develops the equations in closed form or semi analytical format for their estimation within each one of the asset dynamics presented in Section 2. Due to the

[13] Sarkar and Zapatero (2003, p. 38, footnote 1) avoid the issue, while Morellec (2004) assumes that agents are risk neutral and the approach of Goldstein *et al* (2001) is only suitable to constant parameter diffusion processes for the cash flows.
[14] See, in particular, Carr and Linetsky (2006) and Campi *et al* (2009).

numerical accuracy and convergence issues for the quasi analytical estimations, these key building blocks cannot be estimated to an acceptable degree of accuracy and time partition for the stochastic volatility models with and without jumps. For this reason we present numerical results for FPDP and UP only for the constant and state dependent volatilities and compare their performance with and without jumps. Section 4 compares all the variables of interest of the full L and LT models for the cases of constant and state dependent volatilities, the only cases for which a direct comparison is possible under the assumption that default is possible at any time point. Section 5 concludes.

2. Economic Setup

2.1 Unlevered asset dynamics

Following Leland (1994a, b), we consider a firm whose assets are financed by equity and debt with a tax-deductible coupon. As in all previous related literature, the values of the components of the firm's balance sheet are estimated as contingent claims of the state variable *V,* the value of the unlevered firm's assets representing its economics activities, which follows a mixture of a continuous diffusion process V^D with time-varying variance v_t together with an independent Poisson jump process (the physical or *P*- distribution):

$$\frac{dV}{V} = \left(\mu - q - \eta\mu_J\right)dt + \sqrt{v_t}\,dW + JdN \tag{2.1}$$

Where μ is the instantaneous expected rate of return of asset; q is the payout rate to the asset holders, including coupon payments to debt holders and dividends to equity holders; η is the jump arrival intensity and μ_J the mean of the logarithm of the amplitude distribution, $\ln(1+J)$; W is a standard Brownian motion; and N denotes the number of Poisson jumps. Three forms of time varying variance are considered: constant elasticity of variance (CEV), stochastic volatility (SV), and stochastic volatility with volatility jump (SVJ), which can be expressed by,

$$CEV : \sqrt{v_t} = \theta V_t^{\beta} \tag{2.2}$$

$$SV : dv_t = \kappa\left(\bar{v} - v_t\right)dt + \sigma\sqrt{v_t}\,dW^v \tag{2.3}$$

$$SVJ : dv_t = \kappa\left(\bar{v} - v_t\right)dt + \sigma\sqrt{v_t}\,dW^v + J^v dN^v \tag{2.4}$$

The constant risk free rate is denoted by r. Under the risk neutral measure (Q-distribution), Equation (2.1) becomes,

$$\frac{dV}{V} = \left(r - q - \eta^Q \mu_J^Q\right)dt + \sqrt{v_t}\,dW^Q + J^Q dN^Q \tag{2.5}$$

This mixed process continues until the asset value hits or falls below a threshold value, denoted by *K*, for the first time. In such a case, a default event will be triggered and liquidation takes place immediately. Assuming the absolute priority is respected, the bond holders will then receive $\left(1-\alpha\right)K$, while the equity holders receive nothing. The remaining asset value equal to αK is considered as a bankruptcy cost.

Unlike the case of the constant volatility or the pure CEV diffusion, the derivation of the *Q*-distribution for V_T given its *P*-distribution is not a trivial process in the presence of jumps, unless it is assumed that the jump component is non-systematic. Otherwise, the parameters of the jump component of the mixed process need also to be transformed in the transition from the *P*- to the *Q*-distribution, in addition to the replacement of μ by r. Similarly, the transition from the *P*- to the *Q*-distribution is not trivial for the stochastic volatility process, with or without jumps. In empirical applications in the option pricing literature the two distributions are extracted from the underlying and the derivative asset markets respectively. Nonetheless, the reconciliation between the two separate estimates has not generally been successful. We discuss this issue when we implement the model in subsequent sections.

We denote the bond maturity by *T*, and the first passage time when the asset value reaches the threshold value by τ. The risk neutral asset value dynamics then become,

$$\begin{cases} \dfrac{dV}{V} = \left(r - q - \eta^{Q}\mu_J^{Q}\right)dt + \sqrt{v_t}\,dW^{Q} + J^{Q}dN^{Q}, if\, 0 < t < \tau < T \\ V_t = \min\{V_\tau, K\}, \qquad if\, 0 < \tau \le t < T \end{cases} \tag{2.6}$$

In what follows we'll examine two kinds of first passage default time τ: continuous (or unrestricted) default, $\tau_c \in (0,T)$, and discrete (or restricted) default, τ_d. For discrete default, we discretize the maturity of debt, *T,* into *N* sub-intervals and define $\Delta t = T/N$; then, $\tau_d = i\Delta t$, $i = 1, 2, \cdots N$.

i Constant Volatility with jumps in asset value

We define $\sqrt{v_t} = \sigma^c$ which is constant and equation (2.6) becomes,

$$\begin{cases} \dfrac{dV}{V} = \left(r - q - \eta^{Q}\mu_J^{Q}\right)dt + \sigma^c dW^{Q} + J^{Q}dN^{Q}, if\, 0 < t < \tau < T \\ V_t = \min\{V_\tau, K\}, \qquad if\, 0 < \tau \le t < T \end{cases} \tag{2.7}$$

Without the jump component the asset dynamics follow a geometric Brownian motion with a constant volatility for which an analytical solution for the first passage default probability exists.[15] Incorporating jump components introduces discontinuities into the asset value dynamics, implying that an analytical solution for the first passage default probability exists only for particular forms of distribution of the jump amplitude such as the double exponential of Kou and Wang (2003), who use Laplace transform techniques to derive the first passage default probability.[16] Similarly, the numerical approximation of the first passage default probability by discretizing Fortet's equation, developed by Collin-Dufresne and Goldstein (2001), does not work after incorporating jumps. In the following section, we present a numerical algorithm to approximate the first passage default probability after incorporating the jump component without any restrictions on the distribution of jump amplitude given the characteristic function of the jump amplitude distribution, $\phi^J(\omega)$. The following Lemma, whose proof is obvious, applies to this case and is noted for future references.

Lemma 1: *When the asset dynamics follow* (2.7) *and* $\tau > T$ *, if the characteristic function of the jump amplitude distribution,* $\phi^J(\omega)$*, exists, then the characteristic function of the asset value* $\ln V_T$ *is:*

$$E\left[e^{i\omega \ln V_T}\right] = \varphi^{CV}(\omega) = \exp\left(i\omega(r-q)T - \frac{\omega^2(\sigma^c)^2 T}{2}\right)\exp\left(\eta^Q T(\varphi^J(\omega)-1)\right) \qquad (2.8)$$

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[15] See Black and Cox (1976), Leland (1994a, b) and Leland and Toft (1996).

[16] Huang and Huang (2003) also used this diffusion process to study the credit spread of corporate bonds.

The conditional distribution of V_T given the jump diffusion process (2.7) can be found by inverting (2.8).

ii CEV with jumps in asset value

By combining (2.2) and (2.6), the asset dynamics with CEV and jumps in asset value can be written as,

$$\begin{cases} \dfrac{dV}{V} = \left(r - q - \eta^Q \mu_J^Q\right) dt + \theta V_t^{\beta} dW^Q + J^Q dN^Q, if\, 0 < t < \tau < T \\ \qquad V_t = \min\{V_\tau, K\}, \qquad if\, 0 < \tau \le t < T \end{cases} \tag{2.9}$$

Without the jump components, the parameter β, the elasticity of the local volatility, is a key feature of the CEV model. For $\beta = 0$ the model becomes a geometric Brownian motion with constant volatility. For $\beta > 0$ ($\beta < 0$) (the state-dependent volatility is positively (negatively) correlated with the asset price.[17] In equity markets, the well-known "leverage effect" shows generally a negative relationship between volatility and equity price. There are also some suggestions that the economically appropriate range is $0 > \beta > -1$,[18] even though empirical evidence in the case of the implied risk neutral distribution of index options finds negative values significantly below this range. Jackwerth and Rubinstein (2001) find that the unrestricted CEV model when applied to the risk neutral distribution extracted from S&P 500 index options is able to generate as good out-of-sample option prices as the better known stochastic volatility model of

[17] As Emmanuel and Macbeth (1982, p. 536) were the first to point out, for $\beta > 0$ the local volatility becomes unbounded for very large values of V, and there are technical issues concerning the mean of the process under both the physical and the risk neutral distribution. This problem is solved by assuming that the volatility is bounded and becomes constant for V exceeding an upper bound; see Davydov and Linetsky (2001, p. 963), A similar lower bound when β is < 0 prevents the formation of an absorbing state at 0.
[18] See Cox (1996), and also Jackwerth and Rubinstein (1999), who term this model the *restricted* CEV. The arguments in favor of the restricted CEV model are mostly applicable to index options and will not affect our formulation.

Heston (1993). Note that all this empirical evidence only reveals the elasticity of volatility of equity value but not of asset value. The observed negative relationship between the equity value and equity volatility could be generated even with constant asset volatility or even with slightly positive elasticity of asset volatility. Hereafter we shall study positive, negative and zero β scenarios without any restrictions.

The CEV model yields a distribution of the asset value V_T conditional on the initial value V_t and, hence, initial volatility that has the form of a non-central chi-square $\chi^2(z,u,v)$, denoting the probability that a chi-square-distributed variable with *u* degrees of freedom and non-centrality parameter *v* would be less than *z*. The shape of this distribution is given analytically most often in terms of its complementary form 1-$\chi^2(z,u,v)$, denoting in our case the probability $V_T \geq v_T$. For $\beta < 0$ this probability is given analytically by,[19]

$$\Pr ob(V_T \geq v_T) = 1 - \chi^2(c,b,a) = \chi^2(a, 2-b, c) \tag{2.10}$$

Where

$$\begin{aligned} &a = \upsilon v_T^{-2\beta},\ c = \upsilon(V_t e^{(r-q)T})^{-2\beta},\ b = -\beta^{-1}, \\ &\upsilon = -\frac{2(r-q)}{\theta^2 \beta[e^{-2(r-q)\beta T} - 1]} \end{aligned} \tag{2.11}$$

This distribution is the equivalent of the lognormal when the volatility is constant. It has been tabulated and is easily available numerically. Several additional results hold about the $\chi^2(z,u,v)$ distribution when the parameter *u* is an even integer that can

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[19] See Schroder (1989, p. 213-214).

simplify the computations. Nonetheless, the main result necessary for the extension to the mixed jump diffusion process by using the chi-square distribution's characteristic function holds even for non-integral degrees of freedom.[20]

For the mixed jump-CEV diffusion process (2.9) when $\tau > T$, we derive a quasi-analytical form of the conditional distribution of the unlevered asset value V_T given the initial value V_t the equivalent of (2.1)-(2.2) for the CEV diffusion if the riskless rate r is replaced by the instantaneous drift μ. In order to obtain quasi-analytical solutions we shall also restrict the class of distributions of the amplitudes of the jump processes that we'll consider, to discrete multinomial jumps, presenting results only for the binomial case without loss of generality. The quasi-analytical form is derived by the inversion of the characteristic function of the distribution, for which efficient numerical procedures exist.

Let L_i, $i = 1,...,n$ denote the amplitude of the i^{th} jump given n jumps in the interval $\left[0, T\right]$, and let Y_T denote the jump component in that period, with Y_T^n, the conditional value of Y_T, where $Y_T^n = \prod_{i=1}^{n} L_i$. From (2.1) and the independence of the diffusion and the jump components, we have $V_T = V_T^D Y_T$. The following auxiliary result, proven in the appendix, will be necessary for the estimation of the distribution of V_T under the mixed process.

Lemma 2: *The characteristic function of the distribution of V_T is given by*

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[20] See Johnson *et al* (1995, p. 433).

$$E[e^{i\omega V_T}] = E\left[E\left[e^{i\omega Z_T} \mid Y_T\right]\right] = E\left[\phi_D(i\omega y_T)\right] \quad (2.12)$$

Where $y_T = Y_T^{-2\beta}$., $Z_T = \upsilon V_T^{-2\beta}$, *and*

$$\phi_D(i\omega) = E[e^{i\omega Z}] = \frac{\exp(\frac{i\omega c}{1-2i\omega})}{(1-2i\omega)^{\frac{b}{2}}} \quad (2.13)$$

and the parameters c, b are given by (2.11).

From this result we can now derive the distribution of V_T under the mixed process under a binomial distribution, by inverting the characteristic function given by Lemma 2. The following result is also proven in the appendix.

Proposition 1: *Let* $l_j = L_j^{-2\beta}\ j = u, d$ and *a given by* (2.11). *Then the probability distribution of* V_T *is given by*

$$\Pr ob(V_T \le v_T \left| V_t \right.) = \sum_{j=0}^{\infty} e^{-\eta T} \frac{(nT)^j}{j!} \sum_{i=0}^{j} \binom{j}{i} p_u^i (1-p_u)^{j-i} \Pr ob(Z_T \le a \mid N = j, y_T = l_u^{\,i} l_d^{\,j-1}) \quad (2.14)$$

where

$$\Pr ob(Z_T \le a \mid N = j, y_T = l_u^{\,i} l_d^{\,j-1}) = \frac{1}{2} - \frac{1}{\pi}\int_0^{\infty} \frac{\operatorname{Im}(e^{-i\omega Z_T}\phi_D(i\omega y_T))}{\omega}$$
$$= \frac{1}{2} - \frac{1}{\pi}\int_0^{\infty} \frac{\operatorname{Im}(e^{-i\omega Z_T}\phi_D(i\omega l_u^{\,i} l_d^{\,j-1}))}{\omega} \quad (2.15)$$

Although this distribution is not given in closed form, the expressions (2.14)-(2.15) yield semi-analytical expressions for it under the binomial jump amplitude assumption, which easily and obviously generalizes to a multinomial one.

iii SV with jump in asset value and jumps in asset volatility (SVJJ)

The diffusion process of asset value with stochastic volatility and jumps in both asset value and asset volatility can be written as follows by (2.1)-(2.4),

$$\begin{cases} \dfrac{dV}{V} = \left(r - q - \eta^{Q}\mu_J^{Q}\right)dt + \sqrt{v_t}\,dW^{Q} + J^{Q}dN^{Q}, if\, 0 < t < \tau < T \\ dv_t = \kappa\left(\bar{v} - v_t\right)dt + \sigma\sqrt{v_t}\,dW^{v} + J^{v}dN^{v} \\ V_t = \min\{V_\tau, K\}, if\, 0 < \tau \le t < T \end{cases} \tag{2.16}$$

This diffusion process was first used by Duffie, Pan and Singleton (DPS, 2000) to describe the dynamic process of equity value. As with the CEV process, the bivariate mixed jump diffusion of (2.16) was also used in option pricing, with the *P*- and *Q*-distributions extracted from the underlying asset and the option market respectively. As Eraker, Johannes and Polson (2003, p. 1294) note, such a joint estimation does not necessarily reduce the uncertainty in the estimates, unless the jump risk premia are arbitrarily restricted. Nonetheless, since in structural models equity and debt are derivative assets on the total asset value, SVJJ should also be a candidate process to be considered when we want to assess the effects of alternative asset dynamics specifications.

A major difficulty in empirical studies on structural models is that the unlevered asset value is a non-tradable asset and its *P*-distribution cannot be extracted from the financial markets. In what follows we study the effects of the SVJJ asset dynamics specification on the economic variables of interest and discuss empirical implementation in subsequent sections. Note that SVJJ contains stochastic volatility (SV) and stochastic volatility with jumps only in asset value (SVJ) as special cases.

There are three sources of risk in SVJJ: the stochastic volatility process, the jumps in asset value and the jumps in asset volatility. We follow DPS (2000) in the model specification, assuming that the jump components and the bivariate diffusion process are mutually independent, with $Cov(dW^Q, dW^v) = \rho_v$, and that the jump amplitude distributions in asset value and asset volatility are correlated as well with correlation ρ_J, and with common arrival intensity η. The distribution of the jump amplitude in asset variance is exponential with mean μ_J^v, while the jump amplitude in asset value is lognormally distributed with standard deviation, σ_J, and mean $\mu_J + \rho_J z^v$, where z^v is a realization of jump amplitude in asset volatility. The following result is extracted from DPS.

Lemma 3: *When the asset value dynamics follow* (2.16) *with* $\tau > T$, *the bivariate characteristic function of the distribution of lnV*$_T$ *and* v_T *at time* T *conditional on lnV*$_0$ *and* v_0 *at time 0 is,*

$$\begin{aligned} &E\left[e^{i(\phi_1 \ln V_T + \phi_2 v_T)}\right] = \varphi^{SVJJ}(\phi_1, \phi_2, \ln V_0, v_0) \\ &= \exp\left(A(t,\phi_1,\phi_2) + B_1(t,\phi_1)\ln V_0 + B_2(t,\phi_2)v_0\right) \end{aligned} \tag{2.17}$$

Where,

$$A = (r-q)t\phi_1 i - \eta t(1+\bar{\mu}_J\phi_1) + \frac{\kappa\bar{v}}{2a}\left[(D-b)t + 2\log\left(\frac{E+1}{E\exp(Dt)+1}\right)\right] + \eta\int_0^t h(B_1, B_2, \phi_1, \phi_2)$$

$$B_1(t,\phi_1) = i\phi_1$$

$$B_2 = \frac{\exp(Dt)}{-\frac{a}{D}\exp(Dt)+C} + \frac{-b+D}{2a}$$

Where a, b, c, D, E and $\int_0^t h(B_1, B_2, \phi_1, \phi_2)$ are defined in the proof in appendix B. The distribution of V_T conditional on the initial values V_0 and v_0 can now be found by inverting (2.17).

2.2 Stationary debt structure

We consider a claim such as a corporate bond on this underlying asset, denoted by $F(V, v, t)$ under the bivariate diffusion case, or $F(V, t)$ under the state-dependent volatility with and without jumps. This claim pays a continuous nonnegative coupon C per unit time as long as the firm is solvent,[21] and it must satisfy a partial differential equation (PDE) whose form depends on the asset dynamics. For instance, under a general state dependent volatility the following PDE has to be satisfied when the firm finances the net cost of the coupon by issuing additional equity, with the subscripts denoting partial derivatives and τ denoting the default time.

$$\begin{cases} \frac{1}{2}vV^2 F_{VV} + (r - q - \eta\mu_J)VF_V + F_t \\ +\eta E[F(V(1+J) - F] + C - rF = 0 \ \text{ if } \ 0 < t < \tau < T \\ \\ F(V,t) = (1-\alpha)\min\{V_\tau, K\} \ \text{ if } \ 0 < \tau \le t \end{cases} \tag{2.18}$$

A closed form solution for such an equation for debt claims that are generally time-dependent is not available even under constant volatility and without jumps. Nor is it

[21] The firm is solvent only when the asset value is above the threshold value for bankruptcy all the time and never below it, starting from the issue date of this corporate bond.

available for the more complex bivariate diffusion asset dynamics, in which the PDE also depends on the price of volatility risk. For the constant volatility case without jumps L and LT adopted particular debt maturity and repayment structures that allowed the solution of (2.18) as if the value of the debt claims were time-independent. In this paper we apply the same maturity and repayment structures and solve both the Leland (1994a,b) and the LT models for our asset dynamics cases, but we use L as our base case, since this model, with its exponential stationary debt structure, generates the most elegant results[22]. We assume that the debt has a total principal value P at time 0 when it is issued with coupon rate C. As time goes by, the firm retires this debt at a proportional rate *g*. Thus, the remaining principal value of this debt value at time *t* is $e^{-gt}P$, and the debt holders receive a cash flow $e^{-gt}(C+gP)$ at time *t*, provided the firm remains solvent. Hence, the average maturity of this debt will be, given that no default occurs,

$$T_a = \int_0^\infty gte^{-gt}dt = g^{-1} \tag{2.19}$$

Thus, the average maturity under the L model is the reciprocal of the proportional retirement rate. In order to get a stationary debt structure we assume that the firm replaces the retired debt with newly issued debt having the same principal and coupon so as to keep the total principal and total coupon payments independent of time. We denote the total value of all the outstanding debt by $D(V)$ or $D(V,v)$ for the bivariate diffusion. Because all outstanding debts are homogenous, the initial total principal *P*, the coupon

[22] Compared to LT, the L model yields a simpler analytical solution. The debt service rate is *C*+*gP* under L, while it is *C*+*P*/*T* under LT. The two models are fully consistent with each other in their results if L's retirement rate *g* is changed to match the average maturity of debt structure under LT. The LT model is discussed in the appendix.

rate *C,* and the retirement rate *g* (or equivalently, the average maturity T_a) define the debt characteristics and can be used at time *0* as control parameters to value all the outstanding debt. When the volatility of the unlevered asset value is constant L derived this value $D(V)$ analytically. If the volatility is time-varying then the solution of the corresponding PDE depends on the structure of time-varying volatility.

The key to the estimation of $D(V)$ or $D(V,v)$ lies in two basic concepts: the first passage default, probability and the unit price. Omitting for notational simplicity the arguments (V,v,K). The first passage probability is denoted by $f\left(\tau \mid t\right)$, where $t<\tau$ is the probability that the underlying asset hits or falls below the default boundary for the first time conditional on the initial status of the underlying asset at time t. Thus, the cumulative first passage density (CFPD)

$$F(T \mid t) = \int_t^T f(\tau \mid t) d\tau \tag{2.20}$$

Similarly, we define the unit price (UP) denoted by p_u as the price of a security which pays \$1 when the default event occurs before the maturity T of a claim. In a risk-neutral world with risk free rate, r, we have,

$$p_u(T \mid r,t) = \int_t^T e^{-r\tau} f(\tau \mid t) d\tau \tag{2.21}$$

We define also the value of a risky *perpetuity* $d(V)$, corporate debt of an infinite maturity such as the one examined in Leland (1994a), to be used as a building block is subsequent results

$$d(V) = [1 - p_d(r)]\frac{C}{r} + p_d(r)(1-\alpha)K, \text{ where } p_d(r) = p_u(\infty | r, 0) \qquad (2.22)$$

Under the Leland (1994b) debt structure the weighted-average maturity of the risky corporate debt is T_a, where $g = 1/T_a$ from equation (2.19). At time *0* the firm issues perpetual debt with principal *P* and coupon payment *C*. Since the debt payout rate is $e^{-gt}(C + gP)$ at time *t* and the debt holders' claim on the principal is $(1-\alpha)Ke^{-gt}$ in case of bankruptcy, the value $D(V,v)$ of this debt at time 0 is, for all asset dynamics,

$$D(V,v) = \int_0^\infty e^{-rt}[e^{-gt}(C + gP)](1 - F(t|0))dt + (1-\alpha)K\int_0^\infty e^{-(r+g)t} f(t|0)dt \qquad (2.23)$$

Which becomes after integration by parts,

$$D(V,v) = \frac{C + gP}{r+g}(1 - p_d(r+g)) + (1-\alpha)Kp_d(r+g) \qquad (2.24)$$

In the following section we estimate the CFPD and UP for all cases of asset dynamics examined in this paper. We derive an analytical solution for the CEV case and a quasi-analytical solution for the SV case of asset dynamics without jumps and develop an efficient numerical algorithm for all the cases with jumps: constant volatility, CEV and SVJJ, all under discrete time default.

3. First Passage Default (FPD) Probability and Unit Price (UP)

3.1 Unrestricted default: CEV and SV

For unrestricted (or continuous) default, we assume that the default events could occur at any time point before the maturity of the debt. In other words, the first passage default time is $\tau \in (0,T]$. This assumption has been used in most of the literature of structural models. The advantage of this assumption is that it is much easier to arrive at an analytical solution for the CFPD and UP under a diffusion process for the asset value[23]. Such an analytical solution is very convenient for the study of many important variables such as optimal capital structure, credit spreads, debt capacity, agency cost, etc. In this first subsection we focus on the continuous default assumption and derive an analytical solution for the CFPD and UP under the CEV diffusion process. We also study the CFPD and UP under the SV process by a numerical approximation algorithm proposed by Collin-Dufresne and Goldstein (2001).

i The CEV process

Without the jump components, the asset dynamic in (2.9) can be written as,

$$\begin{cases} \dfrac{dV}{V} = \left(r - q - \eta^{Q} \mu_J^{Q}\right) dt + \theta V_t^{\beta} dW^{Q}, if\, 0 < t < \tau < T \\ V_t = K, if\, 0 < \tau \le t < T \end{cases} \tag{3.1}$$

Since the value of a unit security is also that of a down and out barrier option with \$1 payment, we can use available results from option pricing to prove the following lemmas. Firstly, we study the UP for a claim with infinite maturity.

Lemma 4: *Under a general state dependent volatility $\sigma(V)$ the price of a unit security which pays one dollar when the asset value V hits the barrier K under the risk neutral distribution is given by*

[23] See Black and Cox (1976), Leland (1994a, b), Leland and Toft (1996), Kou and Wang (2003) and Sarkar and Zapatero (2003).

$$p_d(r) = \int_0^{\infty} e^{-r\tau} 1_{\tau<\infty} d\tau = \frac{\phi_r(V)}{\phi_r(K)} \tag{3.2}$$

where $\phi_r(V)$ is the decreasing fundamental solution of the following ordinary differential equation (ODE) for U(V,t),

$$\frac{1}{2}\sigma(V)^2 V^2 U(V,t) + (r-q) V U_V(V,t) - rU(V,t) = 0, V > 0 \tag{3.3}$$

Proof: See Proposition 1 of Davydov and Linetsky (2001).

When the volatility is constant, the above ODE has a solution $\phi_r(V) = V^{\gamma^*}$, where γ^* is the solution of a quadratic equation, yielding[24]:

$$\phi_r(V) = V^{\gamma^*} = V^{-\gamma - \sqrt{\gamma^2 + \frac{2r}{\sigma^2}}} \tag{3.4}$$

When the volatility is state-dependent, there is no analytical solution for the general form. We have, however, an analytical solution for the state-dependent variance under a CEV process given by

Lemma 5*: When the state dependent volatility is given by the CEV process $\sigma(V) = \theta V^{\beta}$, the solution of PDE* (3.3) *is given by*

$$\phi_r(V) = \begin{cases} V^{\beta+\frac{1}{2}} e^{\frac{\varepsilon}{2}x} W_{k,m}(x), \beta < 0, r \neq 0 \\ V^{\beta+\frac{1}{2}} e^{\frac{\varepsilon}{2}x} M_{k,m}(x), \beta > 0, r \neq 0 \end{cases} \tag{3.5}$$

Where,

[24] See Ingersoll (1987, p. 372) .

$$x = \frac{|r-q|}{\theta^2 |\beta|} V^{-2\beta}, \varepsilon = sign\left((r-q)\beta \right), m = \frac{1}{4|\beta|},$$
$$k = \varepsilon \left(\frac{1}{2} + \frac{1}{4\beta} \right) - \frac{r}{2\beta |r-q|}$$

$W_{k,m}(x)$ and $M_{k,m}(x)$ are the *Whittaker functions.*

Proof: See Proposition 5 of Davydov and Linetsky (2001).

The combination of equations (3.2)-(3.5) may now be used to provide the analytical solution for the UP while the CFPD is always equal to 1 because of the infinite maturity. The Whittaker functions $W_{k,m}(x)$ and $M_{k,m}(x)$ are the fundamental solutions for the Whittaker equation and are available in the *Matlab* (or *Mathematica)* software.[25] Since the sign and value of β affect the probability of default by increasing (decreasing) the volatility in "bad" states when $\beta < 0$ ($\beta > 0$), the shape of $\phi_r(V)$ is also strongly affected by that parameter. It is a monotonic decreasing (increasing) function with respect to asset value *V* when $\beta < 0$ ($\beta > 0$), In addition, the slope of the function increases with the absolute value of β.[26]

For the claim with finite maturity, *T,* the semi-analytical solution for UP and CFPD could be derived by the following lemma.

Lemma 6*: When the state dependent volatility is given by the CEV process* $\sigma(V) = \theta V^{\beta}$, *the cumulative first passage cumulative default probability,* $F_{CEV}(T\,|\,0)$, *and the price of a unit security,* $p_u(T\,|\,0)$ *equal*

[25] See Whittaker and Watson (1990, pp. 339-351).
[26] The relevant figures are available from the authors on request.

$$
\begin{aligned}
F_{CEV}\left(T \mid 0\right) &= L^{-1}\left[\frac{1}{\lambda}\frac{\phi_{\lambda}\left(V\right)}{\phi_{\lambda}\left(K\right)}\right] \\
p_{u}^{CEV2}\left(T \mid 0\right) &= L^{-1}\left[\frac{1}{\lambda}\frac{\phi_{r+\lambda}\left(V\right)}{\phi_{r+\lambda}\left(K\right)}\right]
\end{aligned}
\tag{3.6}
$$

Where L^{-1} *denote the inverse of the Laplace transform evaluated at the appropriate maturity T and* $\phi_{\lambda}\left(V\right)$ *is defined in equation (3.5).*

Proof: See appendix.

There are several numerical algorithms that can be used to invert the Laplace transform in order to get $F_{CEV}\left(T \mid 0\right)$ and $p_{u}^{CEV2}\left(T \mid 0\right)$ [27]. $F_{CEV}\left(T \mid 0\right)$ can also be estimated numerically with the one-dimensional Fortet equation.

ii The Stochastic Volatility process

After removing the jump components in the asset value and the asset volatility dynamics (2.16), the asset dynamics with a pure stochastic volatility process can be written as,

$$
\begin{cases}
\dfrac{dV}{V} = \left(r-q\right)dt + \sqrt{v_t}\,dW^{Q}, if\, 0 < t < \tau < T \\
dv_t = \kappa\left(\bar{v} - v_t\right)dt + \sigma\sqrt{v_t}\,dW^{v} \\
V_t = K, if\, 0 < \tau \le t < T
\end{cases}
\tag{3.7}
$$

[27] Davydov and Linetsky (2001) use an Euler numerical integration algorithm, with the details shown in their appendix D, p. 964. Kou and Wang (2003) use the Gaver-Stehfest algorithm, with the details showing in their section 5, p. 519. We tried both algorithms in this paper and arrived at the same results given our calibration.

The correlation between W^Q and W^v is ρ_v . To our knowledge, no analytical solutions for CFPD and UP exist for this general form of diffusion process. By restricting the drift of asset the diffusion process, $(r-q)=0$, and the correlation $\rho_v=0$ simultaneously, the analytical solution for CFPD and UP can be found by the method of images or the eigen-function expansion method[28]. Since the restrictions are unrealistic, we will not discuss this semi-analytical solution here.

As the continuous property of the asset diffusion process is maintained with the stochastic volatility, the Fortet equation algorithm could be used to compute the CFPD probability and the UP. Longstaff and Schwartz (1995) first introduced this algorithm into the finance literature to solve the default probability with a stochastic interest rate. Collion-Dufresne and Goldstein (2001) extend the Fortet algorithm from a one-dimensional to a two-dimensional Markov process.[29] Elkamhi, Ericsson, Jiang and Du (2012) applied this algorithm to the first passage default probability calculations with stochastic volatility.

Consider a two-factor Markov process $\{z_t, v_t\}$,where $z_t=\ln(V_t/K)$ as in (3.7) with a free transition density, denoted by $f_0(z_t, v_t, t \mid z_0, v_0, 0)$ with $z_0>0>z_t$ and a probability density that the first passage time through zero at time τ , given $0<\tau<t$, and the asset volatility takes the value v_τ at that time, denoted by $f_1(0, v_\tau, \tau \mid z_0, v_0, 0)$. Thus, the two-dimensional generalization of the Fortet's equation can be expressed as,

$$f_0(z_t, v_t, t \mid z_0, v_0, 0)=\int_0^t \int_0^{+\infty} f_1(0, v_\tau, \tau \mid z_0, v_0, 0) f_0(z_t, v_t, t \mid 0, v_\tau, \tau)\, dv\, d\tau \quad (3.8)$$

[28] See Lipton (2001)

[29] Collin-Dufresne and Goldstein (2001) also show that the Longstaff and Schwartz (1995) algorithm can only approximate the exact solution of their model.

The probability density functions $f_0(...)$ and $f_1(...)$ can be obtained by the discretization algorithm proposed by Collin-Dufresne and Goldstein (2001). Denote the maximum and minimum asset variance values as $\overline{v}$ and $\underline{v}$, respectively. We discretize the time T and asset variance v into N_T sub-periods and N_v sub-intervals, respectively. Define $t_j = j\Delta t$ with $\Delta t = T/N_T$, $j \in \{1,2,\cdots,N_T\}$, and $v_i = \underline{v} + i\Delta v$ with $\Delta v = (\overline{v} - \underline{v})/N_v$, $i = \{1,2,\cdots,N_v\}$. The discretized version of (3.8) is,

$$f_0\left(z_t, v_t, t \mid z_0, v_0, 0\right) = \sum_{t=0}^{T} \sum_{v=\underline{v}}^{\overline{v}} q_1\left(t_j, v_i\right) f_0\left(z_t, v_t, t \mid 0, v_\tau, \tau\right) \qquad (3.9)$$

Where

$$q_1\left(t_j, v_i\right) = \Delta t \Delta v f_1\left(0, v_i, t_j \mid z_0, v_0, 0\right)$$

The $q_1\left(t_j, v_i\right)$ can be calculated recursively as follows,

$$\begin{aligned} &q_1\left(t_1, v_i\right) = \Delta v f_0\left(z_{t_1}, v_i, t_1 \mid z_0, v_0, 0\right) \\ &q\left(t_j, v_i\right) = \Delta v\left[p\left(z_{t_j}, v_i, t_j \mid z_0, v_0, 0\right) - \sum_{k=1}^{j-1} \sum_{l=1}^{N_v} q\left(t_k, v_l\right) f_0\left(z_{t_j}, v_i, t_j \mid 0, v_l, t_k\right)\right], \\ &j \in \{2,3,\cdots,N_T\} \end{aligned} \qquad (3.10)$$

Given the joint probability density function $f_0\left(z_t, v_t, t \mid z_0, v_0, 0\right)$, the CFPD probability with stochastic volatility is given by,

$$F_{SV}(z_0, v_0, T) = \sum_{j=1}^{N_t} \sum_{i=1}^{N_v} q_1\left(t_j, v_i\right) \qquad (3.11)$$

3.2 Discrete default

Although the analytical (or semi-analytical) solution for CFPD and UP can be derived under a continuous default assumption, this assumption does not allow the derivation of CFPD and UP in the presence of jump components. Note also that in the empirical world default mostly occurs on certain critical discrete times such as the coupon payment date or the maturity date of the corporate bonds. On the other hand, incorporating jumps into the diffusion process of asset value seems to be very critical especially during financial crisis periods; such jumps cannot be included under continuous default assumption.[30] In order to be consistent with the discrete default data and also to incorporate jump components in the asset diffusion process, we present an efficient numerical algorithm for the approximation of CFPD and UP with the discrete default assumption under which the first passage default time, $\tau_d = i\Delta t$ where, $i = 1,2\cdots N$. The CFPD and UP of this algorithm converge to the results of continuous default when $\Delta t \to 0$.

The key to developing the equivalent of equation (3.1)-(3.2) after incorporating jump components is the default time density function $f(t,V,K)$, the first passage time probability distribution for this model for a given V_0. A closed form expression for $f(t,V,K)$ or for the value of a risky perpetuity does not exist for the mixed process. Instead, we develop an algorithm based on time discretization, as in the solution of

[30] Kou and Wang (2003) have to restrict the distribution of jump amplitude to be a double-exponential distribution in order to derive a semi-analytical solution for CFPD by Laplace transform since the double-exponential distribution is memoryless.

Fortet's equation,[31] using the distributions of V_T derived in previous section under each scenario. Given $F(v_T \left| v_t \right.) = \Pr ob(V_T \le v_T \left| V_t = v_t \right.)$, $v_t \ge K$, we set $T = t + \Delta t$, and evaluate from (2.10)-(2.11) the probability $F(v_{t+\Delta t} \left| t, v_t \right.)$ and the associated density $f(v_{t+\Delta t} \left| t, v_t \right.)$ for various values of v_t and $v_{t+\Delta t}$ in the two-dimensional array $[K,\infty)$. Define also, for $\Delta t = \dfrac{T}{n}$,

$$f_1(v_{\Delta t}) \equiv f(v_{\Delta t} \left| 0, v_0 \right.), \cdots, f_k\left(v_{t+\Delta t}\right) = \int_K^{\infty} f(v_{t+\Delta t} \left| t, v_s \right.) f_{k-1}(v_s)\, dv_s \qquad (3.12)$$

For $k \in [2,n]$. We then have the following result, whose proof is obvious and is omitted.

Proposition 2*: If* $\tau \in (0,T]$ *denotes the first passage time to default,* $G(T\left|0,V_0\right.) = \Pr ob\{\tau \le T \left| 0, V_0 \right.\}$ *denotes its distribution, and* Q_i *denotes the probability that the firm asset value will lie below its default value* K *at* $i\Delta t$ *given that it lies above it at* $j\Delta t, j = 1,2 \cdots i-1,$ *these probabilities are given by the relations*

$$\begin{aligned} Q_1 &= F(\Delta t, K \left| 0, V_0 \right.), Q_2 = \int_K^{\infty} F(2\Delta t, K \left| \Delta t, V_{\Delta t} \right.) f_1(\Delta t, V_{\Delta t}) dV_{\Delta t} \\ Q_i &= \int_K^{\infty} F(i\Delta t, K \left| (i-1)\Delta t, V_{(i-1)\Delta t} \right.) f_{i-1}((i-1)\Delta t, V_{(i-1)\Delta t}) dV_{(i-1)\Delta t}, \; i \in [2,n] \end{aligned} \qquad (3.13)$$

and $G(T\left|0,V_0\right.)$ *can be approximated by*

---

[31] See Collin-Dufresne and Goldstein (2001). Fortet's equation cannot be used either in its continuous or in its discrete time format, since the asset value path is discontinuous and at default the asset value may be less than K.

$$G(T|0,V_0)=\sum_{1}^{n}Q_i \tag{3.14}$$

Where $f_i(k\Delta t, V_{i\Delta t}),\ i=1,...,n$ *is given by* (3.12).

A key input in evaluating this algorithm is the probability density function $f\left(V_T,V_t\right)$, which has closed form expression for the constant volatility and CEV. After incorporating jumps an analytical expression exists only in the form of the characteristic function. The probability density function can be expressed as the inverse Fourier Transform of characteristic function, $\psi\left(\phi,V_t,\tau\right)$, which yields,

$$f\left(V_T,V_t\right)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}e^{-i\phi V_T}\psi\left(\phi,V_t,\tau\right)d\phi \tag{3.15}$$

The Fast Fourier Transform (FFT) technique[32] can be applied here to reduce the computational time significantly. Given $a<0, b>0$ are sufficiently large in absolute value, a valid approximation to $f\left(V_T,V_t\right)$ is,

$$f\left(V_T,V_t\right)=\frac{1}{2\pi}\int_{a}^{b}e^{-i\phi V_T}\psi\left(\phi,V_t,\tau\right)d\phi \tag{3.16}$$

Let $\Delta=\frac{b-a}{N}$, $\phi_j=a+j\Delta$ and $\psi_j=\psi\left(\phi_j,V_t,\tau\right)$ for $j=0,1,\cdots,N$. The integral can be approximated as,

$$f\left(V_T,V_t\right)=\frac{1}{2\pi}\Delta\sum_{j=0}^{N-1}e^{-i\phi V_T}\psi_j \tag{3.17}$$

[32] See Carr and Madan (1999).

Then, define $V_{T,k} = \dfrac{2\pi k}{N\Delta} \equiv \dfrac{2\pi k}{b-a}$ for k from the same grid as j. Thus, the approximation becomes,

$$f\left(V_T, V_t\right) = \frac{1}{2\pi}\Delta\sum_{j=0}^{N-1} e^{-i\phi V_T}\psi_j = \frac{1}{2\pi}\Delta e^{-iV_{T,k}a}\sum_{j=0}^{N-1} e^{-i(2\pi kj/N)}\psi_j \tag{3.18}$$

The summation term is a discrete Fourier transform, and the FFT technique is readily available in computation softwares, such as Matlab, R, etc. In addition, as the transition matrix is the same for each time step because of the Markov property and only needs to be calculated once, the computational time can be reduced further even for a complicated distribution. The key issue in evaluating the results of interest is the proper value of a and b. Although the discrete time methods to estimate the CFPD and UP measures are applicable to all asset dynamics presented in Section 2, their practical implementation in the SV and SVJ cases is limited for reasons of accuracy and computational time to a number of time partitions that is too small for most empirically interesting problems, For this reason we limit ourselves in what follows to the constant and state dependent volatility cases, and we examine the effects of our discretization for these cases. We will test the convergence of the first passage default probabilities produced by this algorithm to those under continuous default under the calibration of the constant volatility case.

3.3 Calibrations and numerical results

In this section, we will test the convergence of restricted default to un-restricted default under constant volatility asset dynamics and compare by calibrations the term structure of first passage default probabilities and unit prices generated by different asset dynamics, including constant volatility and CEV under un-restricted default. In our base case, we assume the risk free interest rate $r = 8\%$; firm's payout rate $q = 6\%$; initial asset volatility $\sigma_0 = 20\%$; the current asset value $V = \$100$; the exogenous default boundary $K = \$50$.[33] We consider $\beta = 1$ and $\beta = -1$, and set $\theta = \sigma_0 V^{-\beta}$ under CEV asset dynamic.

i Restricted default versus unrestricted default

In Section 3.2, we introduced the concept of restricted default under which the default events only occur at pre-specified discrete time points. It can be daily, weekly, monthly etc. The discrete default structure is more flexible but also more computationally intensive compared to the unrestricted default structure. The most important advantage of our algorithm is that it can apply to any asset dynamics including the jumps given the analytical expression of the characteristic function of the underlying asset. Theoretically, the CFPD probabilities under a restricted default structure should converge to those under the unrestricted default structure as the number of discrete default points goes to infinity. Numerically, Figure I-1 shows the clear convergence trend of the CFPD probabilities under restricted default to those under continuous default when the discrete default interval decreases from one quarter to one week.

[Insert Figure I-1 about Here]

[33] The calibration of risk free rate, payout ratio and asset volatility are similar to Leland and Toft (1996).

ii CFPD probabilities under CEV and Jumps

In this section, we will only do a static analysis of the jump impacts on the cumulative default probabilities and associated implied volatilities, in which the lowest value of β is limited to -1.

[Insert Figure I-2 about Here]

[Insert Figure I-3 about Here]

Figure I-2 and Figure I-3 report respectively the term structure of CFPD probabilities and term structure of IVs for diffusion-jump and CEV-jump processes under varying calibrations. In order to make all the scenarios comparable, we use the base calibration for asset dynamics without jumps and set the base case for jump at $\mu_J = -0.05, \sigma_J = 0.2, \eta_J = 1/10$. Both diffusion and CEV models with and without base case jumps are plotted by dashed and solid lines. For the CFPD probabilities, the presence of jump components shifts upwards sharply the default probabilities compared to that of the base case without jumps across all the models. The shift depends on the jump calibration. For instance, increasing jump intensity or the volatility of jump amplitude or decreasing the expected value of jump amplitude will increase the CFPD probabilities, even doubling the CFPD probabilities for the longest maturity of 20 years compared to the no jump case when the intensity is equal to $\eta_J = 1/2$. As for the term structure of IVs shown in Figure 14, we find that the jump components shift the term structure of IVs upward from the no jump base case under all scenarios and also twist the shape of term structure according to the model. For the constant volatility case($\beta = 0$), the jump component would increase the IV for short term debt, not enough for our parameter choices to account for the observed downward term structure of empirical IV, but raising the possibility that a higher or a systematic jump risk may indeed explain the observed structure. For the CEV-jump models the jump component has a more noticeable impact on the long term maturity, especially for positive β .

We conclude that jump components, for all the difficulties that they present in deriving analytical solutions, have significant impacts on default probabilities. The evidence that we present from our numerical algorithm can only be considered preliminary, and more research is needed, especially with respect to improving the accuracy of the derived solutions. Such improvements may allow a shorter discretization interval and, thus, bring the results of the algorithm closer to the unknown continuous time solutions.

4. The Structural Model Under CEV Diffusion Process

Since we derived an analytical solution for the value of the risky debt with stationary structure as in Section 2 under the CEV diffusion process by combining equation (3.2) and (3.5), the analytical solution for the equity and asset value will be derived as well in this section. Then we will study in this section the endogenous default boundaries, optimal capital structure, term structure of credit spreads, debt capacity, duration and convexity of corporate debt, equity volatility and asset substitution effect.

4.1 Equity value and asset value

The value of the equity can be derived by valuing the tax shield due to the deductibility of the coupon interest and the bankruptcy cost. The corporate tax rate for the firm is denoted by w . As the interest paid to the bondholder is tax-deductible, the firm's total value is increased by the tax shield due to debt financing. However, the bankruptcy costs increase as well if the firm issues more debt to finance its projects. According to the trade-off theory, the manager of this firm should balance the tax benefit and the bankruptcy cost by maximizing the total firm value. This value can be expressed by,

$$v(V,K) = V + TB(V,K) - BC(V,K) \tag{4.1}$$

Where $v(V,K)$ is the total firm value, *TB(V,K)* is the tax benefit due to debt financing and $BC(V,K)$ is the bankruptcy cost. For a risky debt with infinite maturity, the discount rate under the risk-neutral distribution to calculate the expected present value of one dollar when default occurs for the first time is the risk free rate. For a risky debt with finite maturity *T*, the discount rate will be the sum of the risk free rate plus the proportional retirement rate *g* that depends on the maturity of the debt. The tax benefit available to the firm equals the total tax benefit for a risk-free bond minus the tax benefit loss due to the default event[34], which yields,

$$TB(V,K) = \frac{wC}{r} - \frac{wC}{r}\frac{\phi_r(V)}{\phi_r(K)} \tag{4.2}$$

The bankruptcy cost is the present value of the loss due to the default event, equal to,

$$BC(V,K) = \alpha K \frac{\phi_r(V)}{\phi_r(K)} \tag{4.3}$$

Thus, we have,

[34] We assume that the firm always benefits fully from the tax deductibility of coupon payments when it is solvent, as in Leland (1994a, b).

$$v(V,K) = V + \frac{wC}{r} - \frac{wC}{r}\frac{\phi_r(V)}{\phi_r(K)} - \alpha K \frac{\phi_r(V)}{\phi_r(K)} \tag{4.4}$$

Since we assumed that the firm is financed by risky debt and equity, the value of the corresponding equity equals the total value of the firm minus the total value of the risky debt, which yields,

$$\begin{aligned} E(V,K) &= v(V,K) - D(V) \\ &= V + \frac{wC}{r}\left[1 - \frac{\phi_r(V)}{\phi_r(K)}\right] - \frac{\phi_r(V)}{\phi_r(K)}\alpha K \\ &- \frac{C+gP}{r+g}\left(1 - \frac{\phi_{r+g}(V)}{\phi_{r+g}(K)}\right) - (1-\alpha)K\frac{\phi_{r+g}(V)}{\phi_{r+g}(K)} \end{aligned} \tag{4.5}$$

4.2 The endogenous bankruptcy trigger

In the previous sections, we assumed that default happens when the state variable V drops below a default boundary, K. This default trigger value can be determined exogenously or endogenously. If a firm cannot choose its default boundary value, then this boundary can be determined by a zero-net worth trigger[35] or by a zero cash flow trigger[36]. Under the zero-net worth trigger assumption, the default occurs when the net worth of the firm becomes negative for the first time, which implies that the default trigger value equals the total face value of the outstanding debt, namely $K = P$. However, we often observe that firms are still alive even though their net worth is negative in the financial markets. Thus, in order to improve the simple zero net worth trigger, Moody's KMV defines as trigger value $K = P_{Short} + 0.5 * P_{Long}$. Under zero cash-flow trigger, a firm claims default when the current net cash flow to the security holders cannot meet the

[35] See Brennan & Schwartz (1978), and Longstaff & Schwartz (1995).
[36] See Kim, Ramaswamy, &Sundaresan (1993).

current coupon payments, which implies $K = C / \delta$, where δ is the net cash flow to the security holders. The problem for this trigger value is that sometimes the equity value is still positive even though the current net cash flow is zero. In this case, a firm will prefer to issue more equity so as to meet the current coupon payment, instead of announcing default. On the other hand, if a firm is capable to choose its default boundary value, this default boundary value will be set endogenously by maximizing the total firm value. Following Leland (1994a) and LT, we may find the optimal endogenous default boundary by the smooth-pasting condition,

$$\frac{\partial E(V,K)}{\partial V}\Big|_{V=K} = 0 \tag{4.6}$$

This default boundary value maximizes the value of the equity at any asset level[37]. Applying (4.6) to (4.5), we get the following results, proven in the appendix.

Proposition 3*: According to the smooth pasting condition* (4.6)*, the endogenous default value under the CEV diffusion process, denoted by* K_e*, can be obtained by solving following equation for given parameter values* θ*,* β *and with the auxiliary variables defined in* (3.5)

$$\begin{aligned} 1 - &\left[\frac{wC}{r} + \alpha K_e\right]\left[\frac{1}{\phi_r\left(K_e\right)}\frac{\partial \phi_r\left(K_e\right)}{\partial K_e}\right] \\ + &\left[\frac{C+gP}{r+g} - \left(1-\alpha\right)K_e\right]\frac{1}{\phi_{r+g}\left(K_e\right)}\frac{\partial \phi_{r+g}\left(K_e\right)}{\partial K_e} = 0 \end{aligned} \tag{4.7}$$

Where,

[37] See Leland (1994a)

$$\frac{1}{\phi_r(K)}\frac{\partial \phi_r(K)}{\partial K}=\begin{cases}\dfrac{\beta+0.5}{K}+\left[0.5\varepsilon+0.5-\dfrac{k}{x}-\dfrac{W_{k+1,m}(x)}{W_{k,m}(x)x}\right]x',if\,\beta<0\\ \dfrac{\beta+0.5}{K}+\left[0.5\varepsilon+\dfrac{M_{k+1,m}(x)(k+m+0.5)}{M_{k,m}(x)x}-\dfrac{k}{x}+\dfrac{1}{2}\right]x',if\,\beta>0\end{cases}$$

Proposition 3 yields an endogenous default boundary by solving equation (4.7). Although there is no explicit solution for the endogenous default value, it is straightforward to find it from equation (4.7) by using a root finding algorithm, provided a positive root exists. We examine the properties of the solution, as well as the other variables of interest of the model in numerical examples in the following sections.

We analyze the impact of state-dependent volatility on endogenous default triggers, debt values, optimal capital structure and term structure of credit spreads by considering their values in a base case with the following parameters: current asset value $V=100$, risk-free rate $r=0.08$, firm's payout rate $q=0.06$, tax rate $w=0.35$, proportional bankruptcy cost $\alpha=0.5$, and initial volatility of assets $\sigma_0=20\%$. Although some of these parameters may not reflect current conditions, they were chosen based on previous studies closely related to this paper, such as LT and Leland (2004), with which the results of this study need to be compared in order to assess the impact of the more general formulation. The remaining parameters will assume various values according to the studied topic.

[Insert Figure I-4 about Here]

Figure I-4 shows the values of endogenous default boundaries for the L model, which is a special case of the CEV structural model when β equals zero, and four CEV structural models with $\beta = -1, -0.5, 0.5, 1$. In all cases the endogenous default trigger declines as the average maturity of the bond increases. This is consistent with the findings in the L and LT models. The economic interpretation is that the equity holder would rather sell equity to finance the required cash flow for debt servicing than choose to go bankrupt, even though the net worth of the firm may be negative, provided the anticipated equity appreciation is greater than the contribution required from the equity holders to keep a firm alive. For long term debt structure, the endogenous default boundary is usually less than the face value of the debt, which implies that the expected appreciation of equity for such a debt structure should be relatively higher than for the short-term debt structure. After incorporating the CEV process, we find that the smaller β is, the faster the endogenous default boundary declines. If β is negative and large in absolute value, and the average maturity of the debt is long enough, the corresponding endogenous default boundary could be close to zero, which implies that this firm would never choose to go bankrupt endogenously even though the net worth of the firm may be negative. On the other hand, if β is positive and large in absolute value, the endogenous default trigger of the CEV model decreases slowly and is greater than that of the L model.

These changes of endogenous default triggers under the CEV structural model can be understood economically from the point of view of the relationship between anticipated equity value and volatility. When β is negative (positive), the volatility of the asset increases (decrease) when the asset value decreases. Recall that it is well known since Merton (1974) that the equity in a levered firm can be interpreted as a call option on the value of the assets. Similarly, Merton (1973) showed that in many cases of underlying asset dynamics, including those used in this paper, the value of the option is an increasing function of the volatility. It follows that *ceteris paribus* the anticipated increase in the value of the equity will be inversely proportional to the value of β, while the default boundary will also vary inversely with this anticipated equity appreciation. In other words, at low values of V where the probability of default is high, the increase in volatility when β is negative will counteract the fall in the value of equity because of the fall in V.

[Insert Figure I-5 about Here]

Next we examine the effect of the leverage ratio, $D(V,K,g)/v(V,K,g)$, on the default boundary for two different values of maturity, or of its inverse *g*. The debt value at time 0 is set at par, implying that the RHS of equations (2.24) is set equal to *P*. The L model shows a strictly increasing function of this endogenous default boundary with respect to the leverage ratio, depicted by the solid lines in Figure I-5. As expected, the monotone increasing property of the default boundary as a function of the leverage ratio is preserved, but the speed of increase depends on the value of β. For the 20-year average debt maturity the shape of the function is convex for all β. The positive relationship between the value of β and the default boundary is maintained for negative β's at all leverage ratios, but not for positive β's, where we see a reversal for low values of the leverage ratio. Again, this is consistent with the option interpretation of the equity, since a low leverage ratio corresponds to a deep in the money call option, for which the volatility effect is weak and may be swamped by other factors in solving equation (4.7).

Note also that for the firms with a low leverage ratio and a negative β there is a non-endogenous default zone in which a firm would never choose to go bankrupt endogenously, especially for the lowest value of $\beta = -1$. In Figure I-5, this non-endogenous default zone with $\beta = -1$ starts from zero leverage and ends at 29% leverage for 5-year bonds and at 43% leverage for 20-year bonds. This shows a positive relationship between the range of the non-endogenous default zone and the average maturity of corporate bonds. By comparing the endogenous default triggers for CEV structural models with $\beta = -1$ and $\beta = -0.5$ we see the non-endogenous default zone becomes wider if β decreases. Again, this lowering of the default boundary to about zero is consistent with the volatility effect that causes *ceteris paribus* an appreciation of the equity treated as an option whenever the underlying value V decreases. In the non-endogenous default zone, the anticipated equity value is high enough to dominate the required cash outflow for debt required to keep the firm alive. Thus, the equity holders will choose to retire equity in order to fund the coupon payment for the debt holders until the equity value goes to zero. This behavior leads to lower recovery value for the bond holders, which increase the risk of corporate bonds.

4.3 Optimal capital structure

Under an endogenous default boundary and a pre-determined debt structure, the optimal capital structure that maximizes the total firm value can be achieved by altering the leverage ratio, the ratio of the total outstanding debt value over the total firm value, D / v. Figure I-6 examines the relationship between total firm value and leverage ratio for bonds with average maturities from 1 to 20 years, and Table I-1 reports the optimal leverage ratios and the values of key endogenous variables at optimal leverage. As the average maturity increases, the optimal leverage ratio increases and the optimal total firm value increases as well. For instance, for 1-year debt, the optimal leverage ratios are

32.88%, 28.44% and 27.20% for $\beta = -0.5$, $\beta = 0$ and $\beta = 0.5$ respectively, while for 10-year debt, the corresponding optimal leverage ratios are 66.39%, 59.61% and 51.63%. This relationship was first reported by L and is still preserved under CEV volatility.

[Insert Figure I-6 about Here]

[Insert Table I-1 about Here]

Based on the results Figure I-4 and Figure I-5, we anticipate that the optimal leverage ratios are affected by the value of β for given debt characteristics. We also expect that negative values of β are more leverage-friendly. These do indeed turn out to be the case. For intermediate or long term debt structures, the optimal leverage ratios increase (decrease) with the absolute value of β when β is negative (positive). For instance, the optimal leverage ratio increases from 51.43% to 67.06% when β decreases from 0 to -1, and decreases to 40.79% when β increases to 1 for the 5-year average maturity debt. For short-term debt structures, less than or equal to 1-year, the optimal leverage ratios decrease first and then start to increase when β increases from 0 to 1.

Similarly, the total firm value and the total debt value are both increasing functions while the total equity value is a decreasing function of the average maturity of the debt under each scenario considered in Table I-1. This effect is consistent with the trade-off theory, which balances the tax benefits and bankruptcy costs of the firm in order to maximize the total firm value. Given that the anticipated bankruptcy costs are invariant to debt maturity, in long term debt the anticipated tax benefits that accumulate over time should dominate the anticipated bankruptcy costs, thus increasing both optimal leverage and firm value. Conversely, the anticipated bankruptcy costs should dominate the anticipated tax benefits for short-term debt.

Under the CEV model the strong effect of β on the optimal leverage also has predictable effects on total firm value. If β is negative, the optimal leverage ratio and total firm value will increase compared to the L model in which β is zero. The value effect is, however, small. For instance, for a 5-year average maturity the total firm value increases by about 5.5% when β changes from 0 to -1, a much smaller change than the respective changes in the debt and equity values. On the other hand, for positive values of β the effects of β on firm value do not have a consistent sign, with the effect dependent on debt maturity. Again, the effects on total value are small even though the shifts in the composition of capital structure are significant.

Table I-1 also examines the risk characteristics of the optimally levered firm with the equity risk measured by equity volatility and the debt risk measured by debt volatility and credit spread. For a given debt maturity, both equity volatility and debt volatility monotonically increase when β decreases; the same is true for the credit spread for all but the largest maturity. For all the debt maturities considered in Table I-1, the largest equity risk and debt risk measured by both risk metrics is always reached when β equals -1, the smallest β in our calibrations. On the other hand, for any given value of β, the volatility of debt increases monotonically with the average debt maturity. The volatility of equity and the credit spread, however, first increase and then decrease with maturity for β equal to -1,-0.5 and 1, while it increases with maturity for β equal to 0.5 and 0. Apparently, the impact of the size of β on optimal debt financing is major.

4.4 Credit spread and debt capacity

We calculate the term structure of credit spreads ($C/D-r$) of newly issued debt for alternative leverage ratios shown in Figure I-7. For a given maturity debt, a high leverage ratio implies a high credit spread. The humped term structure, which first increase and then decreases, can be observed clearly for moderate-to-high leverage ratios for all values of β. These patterns are consistent with the findings of the L and LT models. Under the CEV structural model credit spreads are higher for negative than for positive β for all maturities and under all the leverage ratios considered in Figure I-7. The humped shapes of term structure for moderate-to-high leverage ratios are still preserved with state-dependent volatility. This credit spread is inversely proportional to p_d, the present value of one dollar paid to debt holders when default occurs, which is given by Lemmas 1 and 2 for the risk neutral distribution. Since the default boundary is endogenously determined for given β, leverage ratio and debt maturity, these are also the variables that affect the credit spread.

[Insert Figure I-7 about Here]

Debt capacity is the maximum value of total debt under endogenous default boundary. L and LT found that debt capacity falls as the volatility of asset value increases under their constant volatility models. For the CEV model, debt capacity increases when β is negative and decreases when β is positive. Figure I-8 depicts the debt value as a function of the leverage ratio for debt with 5-year average maturity. For the three values of β in the figure, the maximal debt value tends to be reached at approximately equal leverage ratios, which lie between 80% and 90%. Since a firm with a negative β has a higher optimal leverage, such a firm would typically experience a high debt value and a high total firm value compared to a firm with a positive β with exactly the same leverage ratio.

[Insert Figure I-8 about Here]

4.5 Duration and convexity of corporate debt

For the debt portfolio, the Macaulay duration, which measures the percentage change of bond value with respect to the change of the risk free interest rate, is one of the most popular and simple ways to measure interest rate risk for bonds with no default risk. For coupon-paying corporate bonds with default risk, L and LT studied the relationship between effective duration, which measures the real change of bond value with respect to the change in the risk free interest rate, and Macaulay duration. They found that the Macaulay duration is much longer than effective duration as the leverage ratio (or credit spread) increases, which implies that the traditional duration-matching methods for immunization should be adjusted when using corporate bonds. Following L, the Macaulay duration is given by $1/(g+C/D)$, while the effective duration is equal to $\left(\partial D/\partial r\right)*(1/D)$. In Figure I-9, we fix the leverage ratio at 50% and show the relationship between Macaulay duration and effective duration for the CEV model under different values of β. For a constant volatility (solid line), the Macaulay duration is generally longer than the effective duration for all maturities, but under the CEV (dashed line for $\beta=-1$ and dotted line for $\beta=1$), the Macaulay duration is much closer to the real effective duration for any given effective duration. This implies that the traditional duration-matching method should be more effective under state-dependent than under constant volatilities.

[Insert Figure I-9 about Here]

For default-free debt, the debt value is a convex function of the interest rate, which is a critical property for the traditional duration matching method. However, L and LT found that this convexity does not necessarily hold any longer for corporate risky debt under the constant volatility assumption, a result confirmed in the scenarios considered in Figure I-10 (solid lines). Under the CEV model, the convexity relation may appear again depending on the value of β. In Panel A of Figure I-10, both the positive β ($\beta = 1$, dotted line) and the negative β ($\beta = -1$, dashed line) show a convex relationship for the debt with 5-year average maturity and 40% leverage ratio. When, however, the leverage ratio increases from 40% to 50% in Panel B, only the negative β preserves the convexity relationship. Thus, a dynamic duration-convexity hedge strategy for a bond portfolio should be implemented differently for different asset volatility assumptions, debt maturity and leverage ratio. For instance, for 20-year average maturity and 50% leverage ratio constant volatility yields a bond value that is a concave function of the interest rate, while for a CEV process with $\beta = -1$, the traditional duration-convexity hedging strategy still works because the convexity relationship still holds.

[Insert Figure I-10 about Here]

4.6 Equity volatility

Most structural models assume asset dynamics following a diffusion process for the unlevered firm value. Since this is a non-tradable asset and an unobservable variable, we need to estimate the drift and volatility of asset value from observable data of traded assets, such as the stock or bond price. For the model presented in this paper the parameters of the diffusion process may be estimated by a maximum likelihood method initially proposed by Duan (1994) that yields the asset value and volatility from observed equity value. By using three different models of asset dynamics, Ericsson and Reneby (2005) show that this method has superior properties compared to other estimators.

Once the parameters of the diffusion process have been estimated by the maximum likelihood method, it is straightforward to get the equity volatility and equity value. Since the analytical solution for the value of the equity has been derived under the CEV structural model, the volatility is given by applying Ito's Lemma to equation (4.5), equal to

$$\sigma_{Equity} = \frac{\partial E(V)}{\partial V} \frac{V}{E} \theta V^{\beta} \tag{4.8}$$

[Insert Figure I-11 about Here]

In Figure I-11, we assume that the firm is optimally levered under an endogenous default boundary and we examine the relationship between equity value and equity volatility for the L model and CEV structural models with varying β's. The L constant volatility model (solid line) indicates a negative correlation between equity value and equity volatility. As we use the whole US market's average data for our calibrations, this negative correlation is consistent with the findings for the market index data, which is popularly known as the "leverage effect". Under the CEV model, the correlation between equity value and equity volatility depends on the value of β. The smaller β is, the stronger the negative correlation between equity value and equity volatility. For instance, the dotted line ($\beta = -0.5$) is steeper than the solid line ($\beta = 0$) and the dashed line ($\beta = -1$) is even steeper compared to the case $\beta = -0.5$. On the other hand, when β is positive and large enough, it can also indicate a positive relationship between equity value and equity volatility, as in the case that $\beta = 1$ (plus-dashed line). While for the market index the leverage effect has been well documented, for an individual firm the correlation between equity value and equity volatility could be positive or negative, depending on the particular firm's characteristics. Thus, the CEV model allows more flexibility and generalization than the constant volatility model, which is only a special case of CEV structural models.

4.7 Agency effects: debt maturity and asset substitution

We noted in Table I-1 that the total firm value increases when the maturity of the debt becomes longer under optimal capital structure. Rationally then all firms should use long-term debt to finance their projects in order to maximize total firm value. Why are short-term debts still traded in the bond market? Leland and Toft (1996) answer this question by studying the asset substitution effect for different debt maturities. The asset substitution originated from Jensen and Meckling (1976) and refers to the effect that equity holders will try to transfer value from debt to equity by increasing the riskiness of the firm's activities. By analyzing the relationship between $\partial E / \partial \sigma$ and $\partial D / \partial \sigma$ for different levels of asset value, LT find that

"the existence of potential agency costs implies that firms with higher asset risk will shorten their optimal debt maturity as well as decrease their optimal amount of debt."

[Insert Figure I-12 about Here]

We re-examine the asset substitution effect in the context of the CEV model in Figure I-12. This figure shows the sensitivity of equity value and debt value to the total asset risk, $\sigma = \theta V^{\beta}$, respectively for the maturities of 1-year, 5-year, 10-year and perpetual. When the signs of $\partial E/\partial \sigma$ and $\partial D / \partial \sigma$ are the same, the interests of equity and debt holders are positively correlated, indicating a zero asset substitution effect, and *vice versa*. The constant volatility case ($\beta = 0$) is our benchmark for each scenario. As maturity increases from 1-year to perpetual, the asset substitution effect increases for all the scenarios being considered, which is consistent with LT's findings. For $\beta = -1$ the asset substitution effects are more severe, especially for intermediate- and long-term maturity, compared to those of the benchmark cases. On the other hand, for $\beta = 1$, most of the time the interests of equity holders are in line with those of the bond holders, provided the asset value is greater than the corresponding bankruptcy trigger for each maturity, indicating that increasing asset risk will decrease both equity value and debt value simultaneously. These observed stylized factors imply that firms would use short-term debt and suboptimal amounts when their asset value follows a CEV process with negative correlation between asset value and asset volatility, the most commonly assumed feature of asset dynamics.

5. Conclusion

In this Chapter, we have presented a new structural model of the firm that generalizes the asset dynamics assumptions of Leland (1994a,b) and Leland and Toft (1996), among others. The generalizations are twofold. First, we introduce a state dependent volatility that varies with the underlying asset, the value of the unlevered firm, under the constant elasticity of variance form. We derive closed form expressions for almost all the variables of interest on the balance sheet, including corporate debt values, total levered firm values and equity values. By comparing the term structure and unit price generated by CEV and stochastic volatility asset dynamics given an exogenous default boundary, we found that CEV is a simplified form of stochastic volatility and could mimic the term structure of CDP generated with stochastic volatility to a certain extent. Under the derived endogenous default trigger we study the impact of state dependent volatility on default probabilities, optimal leverage, credit spreads, debt capacity, duration and convexity of corporate debt, and the agency effects of debt. Most of the results are given implicitly, but efficient numerical methods allow us to reach the solutions easily.

Second, we introduce jump components into both the simple, the state dependent and stochastic volatility diffusion dynamics. We derive quasi-analytically the asset value distribution under log-normal jumps with constant volatility, multinomial jumps with state dependent volatility and double jumps with stochastic volatility and derive an efficient discrete time algorithm for the first passage time distribution under restricted default times. Although the lack of analytical expressions prevents several important derivations, we nonetheless establish that the presence of even unsystematic jump risk increases significantly default probabilities and the term structure of default volatilities.

An interesting question is whether the new structure model provides a better fitting for the cross section empirical data from equity, debt, option and CDS market. To answer this question, the empirical study will be conducted and results will be discussed in Chapter 3.

Appendix

A. The value of corporate debt under the CEV model and the LT debt assumptions

In LT's stationary debt structure the firm continuously sells a constant amount of new debt with maturity T and redeems the same amount of matured debt in order to keep the total outstanding principal and coupon payment rate constant and equal to P and C, respectively. Suppose $d(V,K,t)$ denotes the price of one unit of outstanding debt with finite maturity t, continuous coupon payment C, and principal P, which can be expressed by

$$d\left(V,K,t\right)=\begin{cases}\int_0^t e^{-rs}Cds+e^{-rt}P, 0<t<\tau \\ \int_0^\tau e^{-rs}Cds+e^{-r\tau}(1-\alpha)K, 0<\tau<t\end{cases} \quad \text{(A.1)}$$

Given the first passage probability density function $f(\tau,V,K)$ under the CEV process, we define, omitting arguments for notational simplicity

$$B\left(t\right)=\int_{\tau=0}^{t} e^{-r\tau}f(\tau,V,K)d\tau, \quad \text{(A.2)}$$

The expected price of one dollar payment when default occurs during the period (0,t). From equation (3) in LT we find the price of this bond equals

$$d\left(V,K,t\right)=\frac{C}{r}+e^{-rt}\left[P-\frac{C}{r}\right]\left[1-A(t)\right]+\left[\left(1-\alpha\right)K-\frac{C}{r}\right]B(t). \quad \text{(A.3)}$$

$A(t)$, the cumulative first passage default probability, was defined in Section 3.3. Under the CEV process analytical forms for $A(t)$ and $B(t)$ are given in the following Lemma.

Lemma A1: *When the state dependent volatility is given by the CEV process* $\sigma(V)=\theta V^{\beta}$, *the first passage cumulative default probability,)* $A(t)$ *and the expected price of one dollar payment when first passage default occurs during the period from time 0 to t equal*

$$\begin{aligned} A(t) &= \mathcal{L}^{-1}\left[\frac{1}{\lambda}\frac{\phi_{\lambda}(V)}{\phi_{\lambda}(K)}\right] \\ B(t) &= \mathcal{L}^{-1}\left[\frac{1}{\lambda}\frac{\phi_{r+\lambda}(V)}{\phi_{r+\lambda}(K)}\right] \end{aligned} \tag{A.4}$$

Where $\mathcal{L}^{-1}$ *denote the inverse of the Laplace transform evaluated at the appropriate debt maturity t and* $\phi_{\lambda}(V)$ *is defined in equation* (3.7)

Proof: It suffices to prove the second part of (A.4) since the first part follows immediately by setting *r=0*. By definition the Laplace transform $\Lambda(\lambda)$ of $B(t)$ is given by

$$\Lambda(\lambda)=\int_0^{\infty} e^{-\lambda t}\left[\int_0^{t} e^{-r\tau} f(\tau,V,K)d\tau\right]dt=\int_0^{\infty} e^{-\lambda t}E_{\tau}\left[1_{\tau\leq T}e^{-r\tau}\right]dt \tag{A.5}$$

By changing the order of integration (A.5) becomes

$$\Lambda(\lambda)=\int_0^{\infty}\left[\int_{\tau}^{\infty} e^{-\lambda t}dt\right]e^{-r\tau} f(\tau,V,K)d\tau=\int_0^{\infty}\frac{1}{\lambda}e^{-(r+\lambda)\tau} f(\tau,V,K)d\tau \tag{A.6}$$

Since the RHS of (A.6) is a constant times the value of a \$1 perpetual claim in the first passage time under the CEV distribution, (A.4) follows immediately from (A.6) by Lemmas 1 and 2, QED.

By inserting equation (A.4) into (A.3), we arrive at the solution for the price of risky corporate debt with finite maturity T under the CEV diffusion process. Thus, under the LT model's stationary debt structure, the value of all outstanding debt with maturity T, the equivalent of (3.8)-(3.9) for $g = T^{-1}$, is from (A.3),

$$D(V,K,T) = \int_{t=0}^{T} d(V,K,t)\,dt$$
$$= \frac{C}{r} + \left(P - \frac{C}{r}\right)\left(\frac{1-e^{-rT}}{rT} - I(T)\right) + \left((1-\alpha)K - \frac{C}{r}\right)J(T) \tag{A.7}$$

Where

$$I(T) = \frac{1}{T}\int_0^T e^{-rt} A(t)dt, \quad J(T) = \frac{1}{T}\int_0^T B(t)dt$$

An analytical expression for $I(T)$ is

$$I(T) = \frac{1}{T}\int_0^T e^{-rt} A(t)dt = \frac{1}{rT}\left(B(T) - e^{-rT} A(T)\right)$$

Unfortunately there is no analytical solution for $J(T)$, which must be evaluated numerically from the function *B(T)* estimated by(A.4).

Note that for debt with maturity t the first passage probability density function, $f(\tau, V, K)$ should be exactly the same for both L and LT stationary debt structures under the CEV process. Hence, we may define the equivalent retirement rate g' as the one that makes the expected price of a one dollar payment when default occurs equal under both L and LT debt structures, or $B_L(g') = B_{L\&T}(t)$, for *t* corresponding to T_a and *g=g'* in (3.2).

$$\int_{\tau=0}^{t} e^{-r\tau} f(\tau,V,K)d\tau = \int_{0}^{\infty} e^{-(r+g')\tau} f(\tau,V,K)d\tau \qquad \text{(A.8)}$$

Since the RHS of (A.8) is available analytically from the inversion of the Laplace transform in the second part of (A.4), the value of corporate debt under the LT stationary debt structure can be found for any *t* from (A.8) by applying the equivalent retirement rate g' and setting $t=g'^{-1}$. [38]

B. Proof of Lemmas and Propositions

<u>*Proof of Lemma 1*</u>

The characteristic function (2.8) of a non-central χ^2 variable is a well-known result.[39] (2.7) follows then immediately from (2.6) and the definitions of Z_T and y_T, QED.

<u>*Proof of Lemma 3*</u>

We have the bivariate diffusion Process (2.16) under risk neutral distribution. With correlated Brownian motions $d\left(W,W^v\right)_t = \rho dt, d\left(N,N^v\right)_t = \rho_J dt$ and the auxiliary variables are given by,

[38] Note, however, that the total debt (A.7) in the LT model is not equal to the total debt given by (3.9) for $g = T^{-1}$, but, given T, we can find the corresponding retirement rate g by setting the total debt value of (A.7) equal to that of (3.9).
[39] See, for instance, Walck (2007, p. 110).

$$K_0 = \begin{pmatrix} r - q - \eta\bar{\mu}_J \\ \kappa\bar{v} \end{pmatrix},\ K_1 = \begin{bmatrix} 0 & -\frac{1}{2} \\ 0 & -\kappa \end{bmatrix} \tag{B.1}$$

$$H_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ H_{(1,1)} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, H_{(1,2)} = \begin{bmatrix} 1 & \rho\sigma_v \\ \rho\sigma_v & \sigma_v^2 \end{bmatrix} \tag{B.2}$$

$$l_0 = \eta,\ l_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{B.3}$$

We conjecture the structure of the bivariate characteristic function as follows,

$$\varphi(V, v, \phi_1, \phi_2, t) = \exp\left[A + B_1 V + B_2 v\right] \tag{B.4}$$

Following *Proposition 1* of Duffie, Pan and Singleton (2000) , we have to solve the following system of ordinary differential equations,

$$\frac{\partial A}{\partial t} = (r - q - \eta\bar{\mu}_J)B_1(t,\phi_1) + \kappa\bar{v}B_2(t,\phi_2) + \eta(h(B_1, B_2, \phi_1, \phi_2) - 1) \tag{B.5}$$

$$\frac{\partial B_1(t,\phi_1)}{\partial t} = 0 \tag{B.6}$$

$$\frac{\partial B_2(t,\phi_2)}{\partial t} = aB_2(t,\phi_2)^2 + bB_2(t,\phi_2) + c \tag{B.7}$$

Subject to the boundary condition $A(0) = 0, B_1(0) = i\phi_1, B_2(0) = i\phi_2$. Define another set of auxiliary variables as,

$$a = \frac{1}{2}\sigma_v^2, b = \rho\sigma_v B_1 - \kappa, c = \frac{1}{2}(B_1^2 - B_1) \tag{B.8}$$

Apparently,

$$B_1(t,\phi_1) = i\phi_1 \tag{B.9}$$

Thus,

$$a = \frac{1}{2}\sigma_v^2, b = \rho\sigma_v\phi_1 i - \kappa, c = -\frac{1}{2}\phi_1(\phi_1 + i) \tag{B.10}$$

We know that (B.7) is a standard Riccati equation and $y_1 = \dfrac{-b+D}{2a}$ is one of the particular solution, where $D = \sqrt{b^2 - 4ac}$. Define $z = \dfrac{1}{B_2 - y_1}$. Thus,

$$B_2 = \frac{1}{z} + y_1, \ \frac{\partial B_2}{\partial t} = -\frac{1}{z^2}\frac{\partial z}{\partial t} \tag{B.11}$$

Insert (B.11) into (B.7) and rearrange the terms,

$$\frac{\partial z}{\partial t} = -Dz - a \tag{B.12}$$

(B.12) is a first order linear differential equation and the general solution is given by

$$z = \frac{-\frac{a}{D}\exp(Dt) + C}{\exp(Dt)} \tag{B.13}$$

where C is a constant and determined by the boundary condition. Therefore, we have

$$B_2 = \frac{\exp(Dt)}{-\frac{a}{D}\exp(Dt) + C} + \frac{-b+D}{2a} \tag{B.14}$$

As we know $B_2(0) = i\phi_2$, apply to (B.14) and have,

$$C = \frac{a}{D}\frac{b + D + 2a\phi_2 i}{b - D + 2a\phi_2 i} \tag{B.15}$$

We define,

$$E = -\frac{b - D + 2a\phi_2 i}{b + D + 2a\phi_2 i} \tag{B.16}$$

Thus,

$$B_2 = \frac{1}{2a}\left[-b - D\frac{E\exp(Dt) - 1}{E\exp(Dt) + 1}\right] \tag{B.17}$$

Plug (B.9) and (B.17) into (B.5) and solve this first order ordinary differential equation, we have

$$\begin{aligned} A = {} & (r - q)t\phi_1 i - \eta t(1 + \bar{\mu}_J\phi_1) + \frac{\kappa\bar{v}}{2a}\left[(D - b)t + 2\log(\frac{E + 1}{E\exp(Dt) + 1})\right] \\ & + \eta\int_0^t h(B_1, B_2, \phi_1, \phi_2) \end{aligned} \tag{B.18}$$

As we assume that jumps in asset value $\ln V$ and jumps in asset variance v are simultaneously correlated with common arrival intensity η^c. The marginal distribution of the jump amplitude in asset variance is exponential with mean μ_{cJ}^v conditional on a realization, z_J^v of the jump amplitude in asset variance, the jump amplitude in asset value is normally distributed with mean $\mu_{cJ}^V + \rho_J z_J^v$ and variance σ_{cJV}^2.

Thus, we have

$$h(x_1, x_2) = \frac{\exp\left(\mu_{cJ}^{V} x_1 + \frac{1}{2}\sigma_{cJV}^2 x_1^2\right)}{1 - \mu_{cJ}^{v} x_2 - \rho_J \mu_{cJ}^{v} x_1} \tag{B.19}$$

Therefore,

$$\int_0^t h(B_1(s,\phi_1), B_2(s,\phi_2), \phi_1, \phi_2) ds = \exp\left(\mu_{cJ}^{V}\phi_1 i - \frac{1}{2}\sigma_{cJV}^2 \phi_1^2\right) \frac{k_1 t + 2a \ln(\frac{k_1 E + k_2}{k_1 E e^{Dt} + k_2})}{k_1 k_2} \tag{B.20}$$

Where,

$$m_1 = \frac{\mu_J^v}{2a}(b + D) + 1, m_2 = \frac{\mu_J^v}{2a}(b - D) + 1 \tag{B.21}$$

$$k_1 = m_1 - \rho_J \mu_{cJ}^v \phi_1 i, k_2 = m_2 - \rho_J \mu_{cJ}^v \phi_1 i \tag{B.22}$$

Therefore, the bivariate characteristic function of asset value and asset volatility is given by *Lemma 3*.

Proof of Proposition 1

The characteristic function inversion (2.10) is well-known; see, for instance, Heston (1993, p. 331). (2.9) follows then immediately by noting that

$\Pr ob(V_T \le K) = \Pr ob(Z_T \le a)$ and that

$\Pr ob(V_T \le K) = E[\Pr ob(V_T \le K \mid N = j, y_T = l_u^{\ i} l_d^{\ j-1})]$, QED.

Proof of Proposition 3

Under CEV diffusion process, the equity value can be found from equations (4.5),

$$E(V,K)=v(V,K)-D(V)$$

$$=V+\frac{wC}{r}\left[1-\frac{\phi_r(V)}{\phi_r(K)}\right]-\frac{\phi_r(V)}{\phi_r(K)}\alpha K-\frac{C+gP}{r+g}\left(1-\frac{\phi_{r+g}(V)}{\phi_{r+g}(K)}\right)-(1-\alpha)K\frac{\phi_{r+g}(V)}{\phi_{r+g}(K)} \tag{B.23}$$

Where,

$$\phi_r(V)=\begin{cases} V^{\beta+\frac{1}{2}}e^{\frac{\epsilon}{2}x}W_{k,m}(x), \beta<0, r\neq 0 \\ V^{\beta+\frac{1}{2}}e^{\frac{\epsilon}{2}x}M_{k,m}(x), \beta>0, r\neq 0 \end{cases} \tag{B.24}$$

Where,

$$x=\frac{|r-q|}{\theta^2|\beta|}V^{-2\beta}, \varepsilon=sign((r-q)\beta), m=\frac{1}{4|\beta|}$$

$$k=\epsilon\left(\frac{1}{2}+\frac{1}{4\beta}\right)-\frac{r}{2|(r-q)\beta|}$$

$W_{k,m}(x)$ and $M_{k,m}(x)$ are the Whittaker function.

Following Leland (1994a), the smooth pasting condition implies,

$$\frac{\partial E(V,K)}{\partial V}|_{V=K}=0 \tag{B.25}$$

We define,

$$\frac{\partial W_{k,m}(x)}{\partial x}=W', \frac{\partial M_{k,m}(x)}{\partial x}=M', \frac{\partial x}{\partial V}=x'$$

From the Mupad notebook in Matlab software[40], we have,

$$
\begin{aligned}
W' &= -\left(\frac{k}{x}-\frac{1}{2}\right)W_{k,m}\left(x\right)-\frac{W_{k+1,m}\left(x\right)}{x} \\
M' &= \frac{M_{k+1,m}\left(x\right)(k+m+0.5)}{x}-(\frac{k}{x}-\frac{1}{2})M_{k,m}\left(x\right)
\end{aligned}
\tag{B.26}
$$

From (B.23) we have,

$$
\frac{\partial E(V,K)}{\partial V}=1-\left[\frac{wC}{r}+\alpha K\right]\left[\frac{1}{\phi_r\left(K\right)}\frac{\partial\phi_r\left(V\right)}{\partial V}\right]+\left[\frac{C+gP}{r+g}-\left(1-\alpha\right)K\right]\frac{1}{\phi_{r+g}\left(K\right)}\frac{\partial\phi_{r+g}\left(V\right)}{\partial V} \tag{B.27}
$$

Where,

$$
\frac{\partial\phi_r\left(V\right)}{\partial V}=\begin{cases}\left[\beta+0.5+0.5Vx'\varepsilon\right]V^{\beta-0.5}e^{0.5\varepsilon x}W_{k,m}\left(x\right)+V^{\beta+0.5}e^{0.5\varepsilon x}W'x', if\,\beta<0\\ \left[\beta+0.5+0.5Vx'\varepsilon\right]V^{\beta-0.5}e^{0.5\varepsilon x}M_{k,m}\left(x\right)+V^{\beta+0.5}e^{0.5\varepsilon x}M'x', if\,\beta>0\end{cases}
$$

Applying the smooth pasting condition that sets the RHS of (B.27) to 0, we have

$$
1-\left[\frac{wC}{r}+\alpha K\right]\left[\frac{1}{\phi_r\left(K\right)}\frac{\partial\phi_r\left(K\right)}{\partial K}\right]+\left[\frac{C+gP}{r+g}-\left(1-\alpha\right)K\right]\frac{1}{\phi_{r+g}\left(K\right)}\frac{\partial\phi_{r+g}\left(K\right)}{\partial K}=0 \tag{B.28}
$$

Where,

$$
\frac{1}{\phi_r\left(K\right)}\frac{\partial\phi_r\left(K\right)}{\partial K}=\begin{cases}\frac{\beta+0.5}{K}+\left[0.5\varepsilon+0.5-\frac{k}{x}-\frac{W_{k+1,m}\left(x\right)}{W_{k,m}\left(x\right)x}\right]x', if\,\beta<0\\ \frac{\beta+0.5}{K}+\left[0.5\varepsilon+\frac{M_{k+1,m}\left(x\right)(k+m+0.5)}{M_{k,m}\left(x\right)x}-\frac{k}{x}+\frac{1}{2}\right]x', if\,\beta>0\end{cases}
$$

which corresponds to equation (4.7), QED.

[40] The first derivative of the Whittaker function with respect to x can be found by command: diff(whittakerM(a,b,z),z) and diff(whittakerW(a,b,z),z)

Figure I-1: Convergence of Restricted Default

This figure depicts the term structure of cumulative default probabilities for both restricted default and unrestricted default with constant volatility asset dynamics. It is assumed that the risk free rate, payout rate, initial asset volatility, initial asset value and exogenous default boundary are the same as in the base case. Under restricted default, weekly (dash line), monthly (dash-dot line) and quarterly (dot line) default is considered. Under continuous default, the cumulative default probabilities are calculated by the analytical expression in Leland and Toft (1996) and shown by solid line.

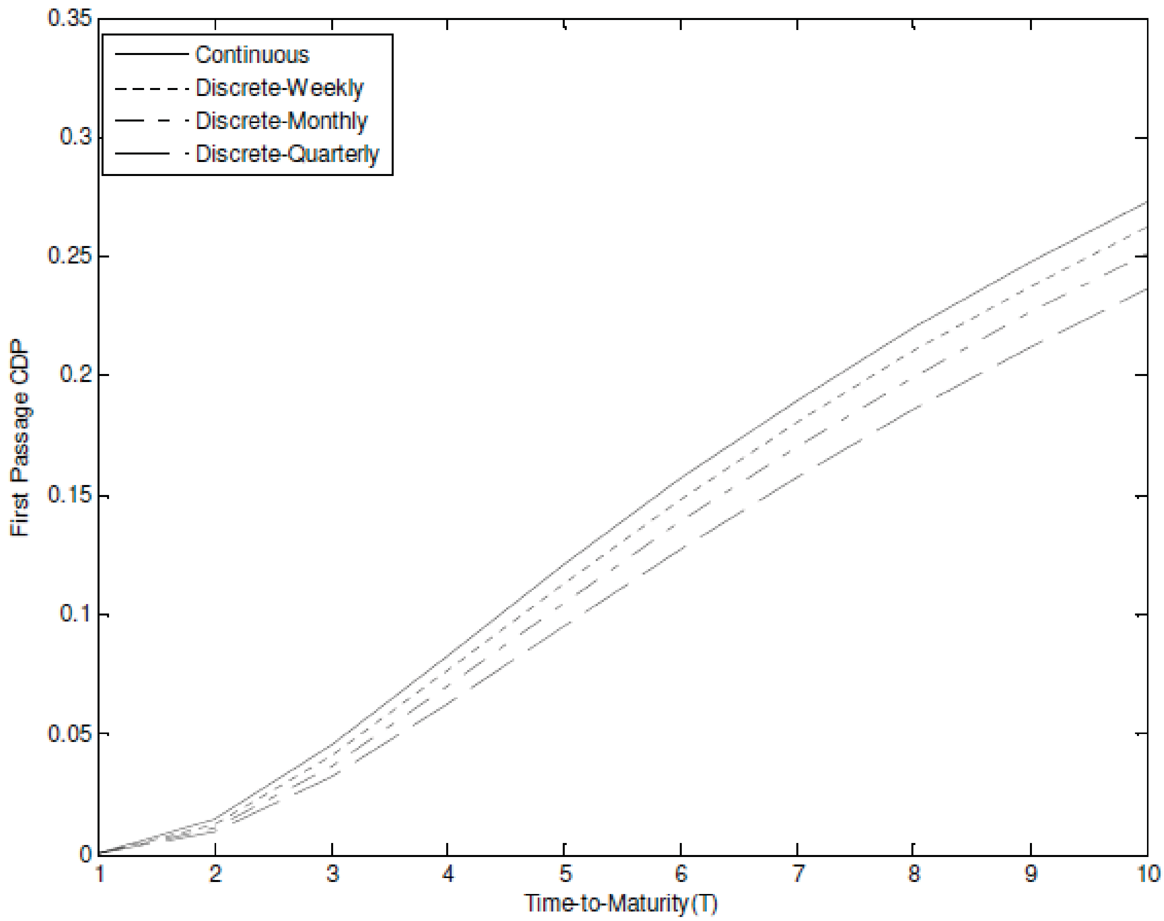

Figure I-2: FPCD probabilities for CEV with jumps

This figure depicts the CDPs for diffusion-jump model ($\beta = 0$) and CEV-jump models ($\beta = 1, \beta = -1$) under varying calibrations of jump parameters. The dashed lines show the diffusion model and CEV model without jumps. The solid lines show the base case for the diffusion-jump and CEV-jump models. For the base case of jump calibration, $\mu_J = -0.05, \sigma_J = 0.2$ and $\eta_J = 1/10$. The dashed-dot lines show the case of $\eta_J = 1/2$; the dot lines show the case of $\mu_J = -0.1$; the plus lines show the case of $\sigma_J = 0.4$ with the other parameters same as in the base case. The initial asset value, $V_0 = \$100$, risk free rate, $r = 0.08$, payout ratio, $q = 0.06$ and initial asset volatility $\sigma_0 = 20\%$. The exogenous default boundary equals $50 for all the scenarios.

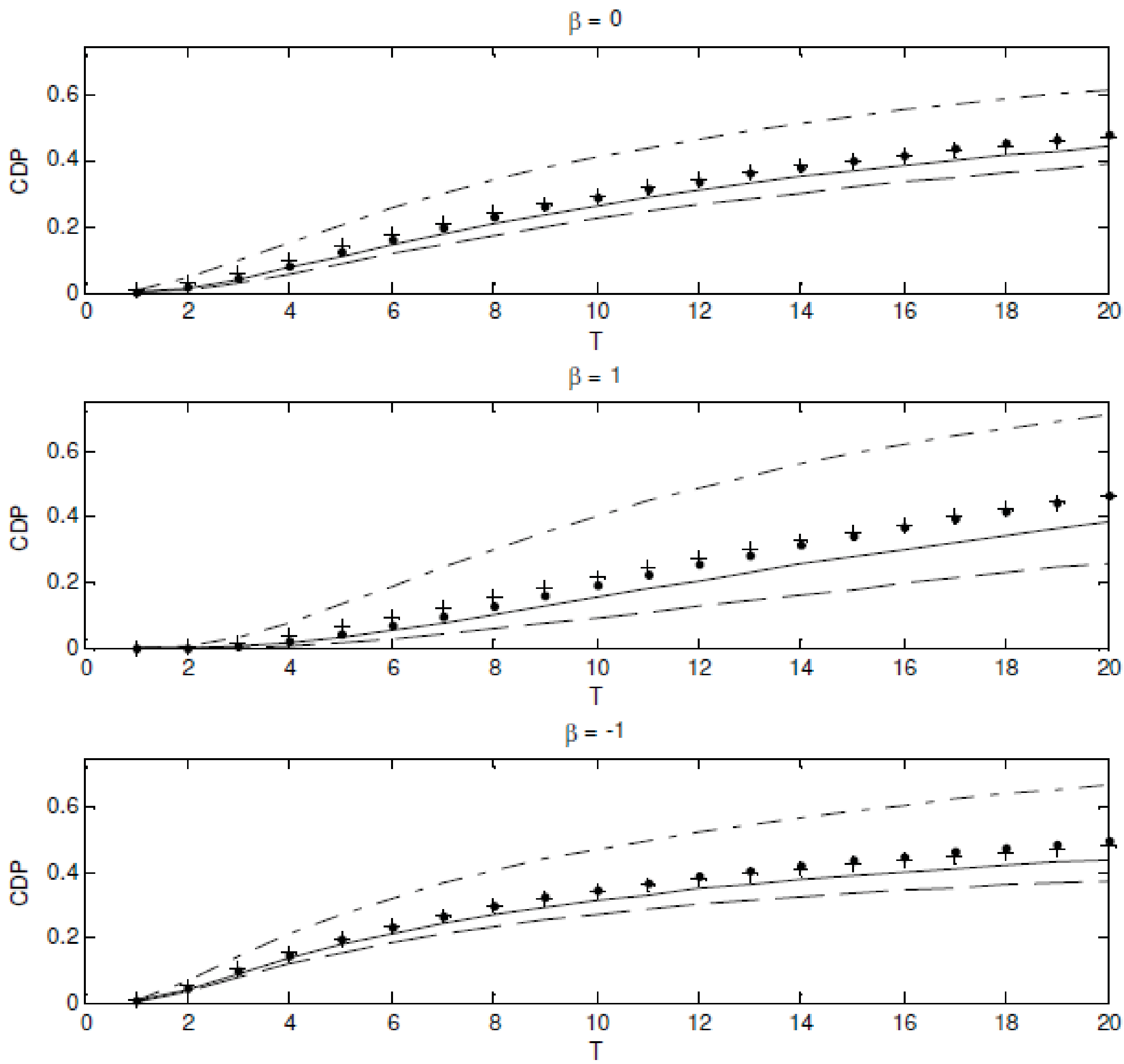

Figure I-3: Term structure of IVs for CEV with jumps

This figure depicts the term structure of IVs for diffusion-jump model ($\beta = 0$) and CEV-jump models ($\beta = 1, \beta = -1$) under varying calibrations of jump parameters. The dashed lines show the diffusion model and CEV model without jumps. The solid lines show the base case for diffusion-jump and CEV-jump models. For the base case of jump calibration, $\mu_J = -0.05, \sigma_J = 0.2$ and $\eta_J = 1/10$. The dashed-dot lines show the case of $\eta_J = 1/2$; the dot lines show the case of $\mu_J = -0.1$; the plus lines show the case of $\sigma_J = 0.4$ with the other parameters same as in the base case. The initial asset value, $V_0 = \$100$, risk free rate, $r = 0.08$, payout ratio, $q = 0.06$ and initial asset volatility $A(t) = \int_{\tau=0}^{\tau=t} f(\tau) d\tau$. The exogenous default boundary equals \$50 for all the scenarios.

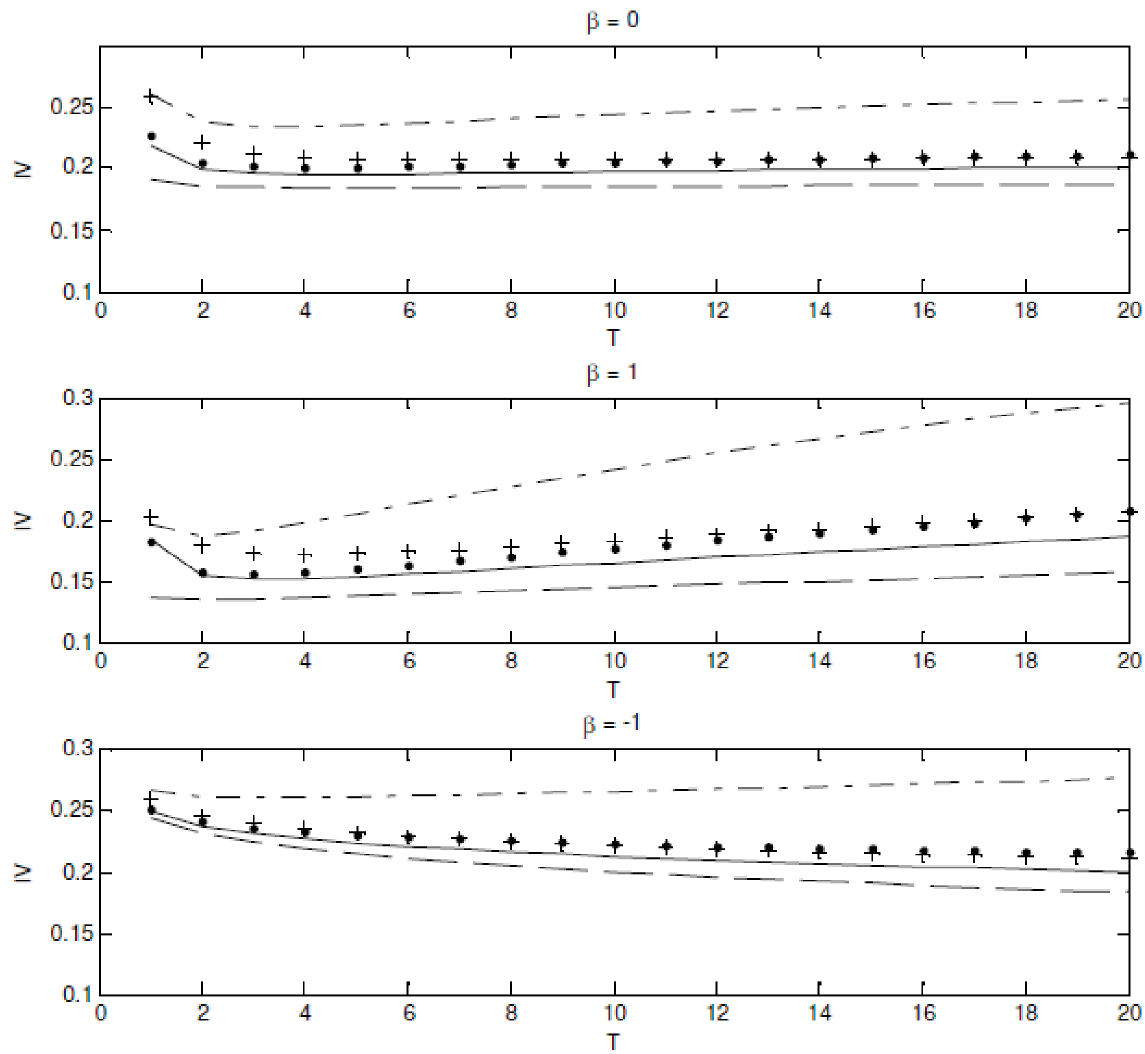

Figure I-4: Endogenous Default Trigger as a function of average maturity

This figure depicts the values of endogenous default triggers for the Leland (1994b) model (bold solid line) and CEV structural models with $\beta = -1$ (dashed line), $\beta = -0.5$ (dotted line), $\beta = 0.5$ (Bold dotted line) and $\beta = 1$ (bold dashed line). It is assumed that current asset value $V = 100$, current debt value $D = 50$, risk free rate $r = 0.08$, the firm's payout rate $q = 0.06$, tax rate $w = 0.35$, and proportional bankruptcy cost $\alpha = 0.5$. The coupon rate is determined by making the debt issued at par value under the endogenous default boundary. The initial volatilities are the same for all these scenarios, and $\sigma_0 = 20\%$. For each given β under the CEV diffusion process, $\theta = \sigma_0/V^{\beta}$.

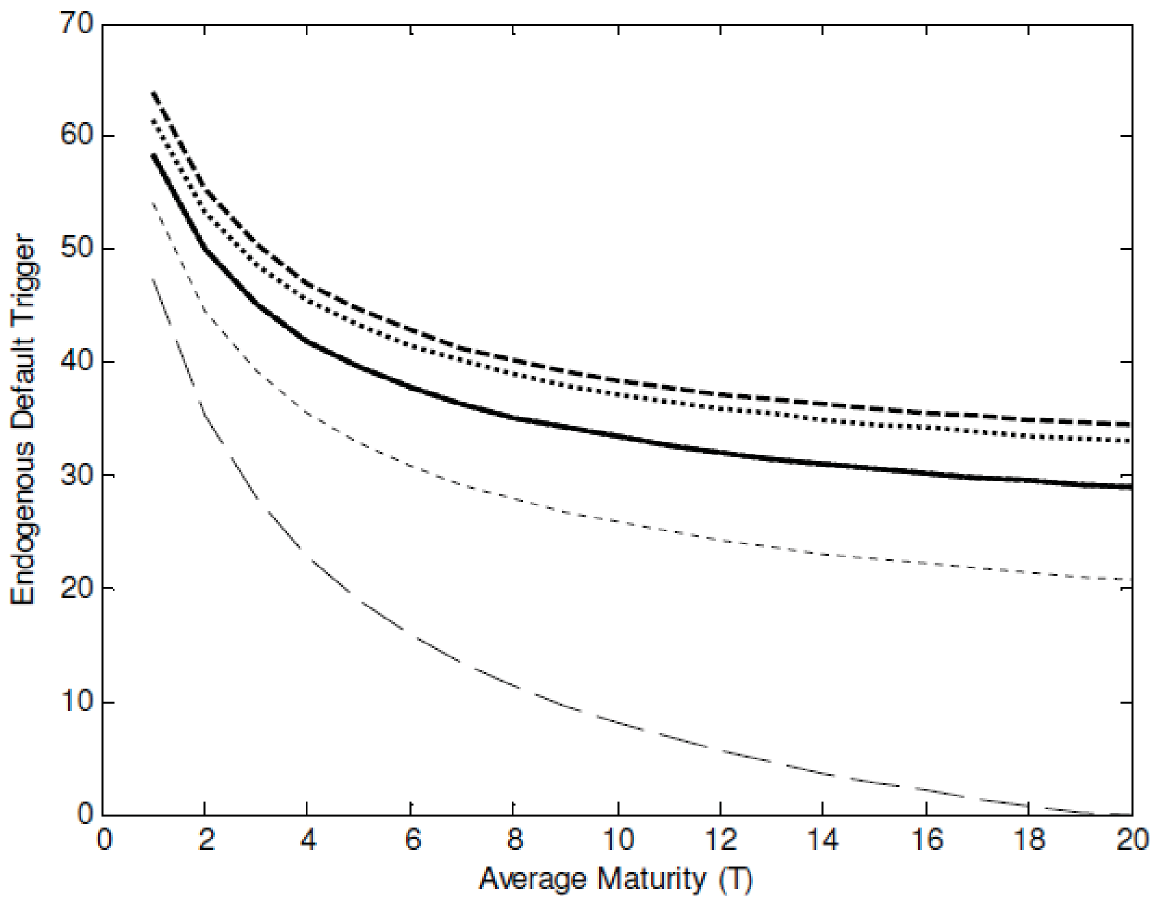

Figure I-5: Endogenous default trigger as a function of leverage ratio

This figure depicts the value of the endogenous default trigger with respect to the leverage ratio under the Leland (1994b) model (solid line) and CEV structural models with $\beta = -1$ (Bolded dashed lines), $\beta = -0.5$ (Bolded dotted lines), $\beta = 0.5$ (dotted lines) and $\beta = 1$(dash lines). It is assumed that current asset value $V = 100$, risk free rate $r = 0.08$, the firm's payout rate $q = 0.06$, tax rate $w = 0.35$, and proportional bankruptcy cost $\alpha = 0.5$. The coupon rate is determined by making the debt issued at par value under the endogenous default boundary. The initial volatilities are the same for these scenarios and $\sigma_0 = 20\%$. For each given β under the CEV diffusion process, $\theta = \sigma_0 / V^{\beta}$.

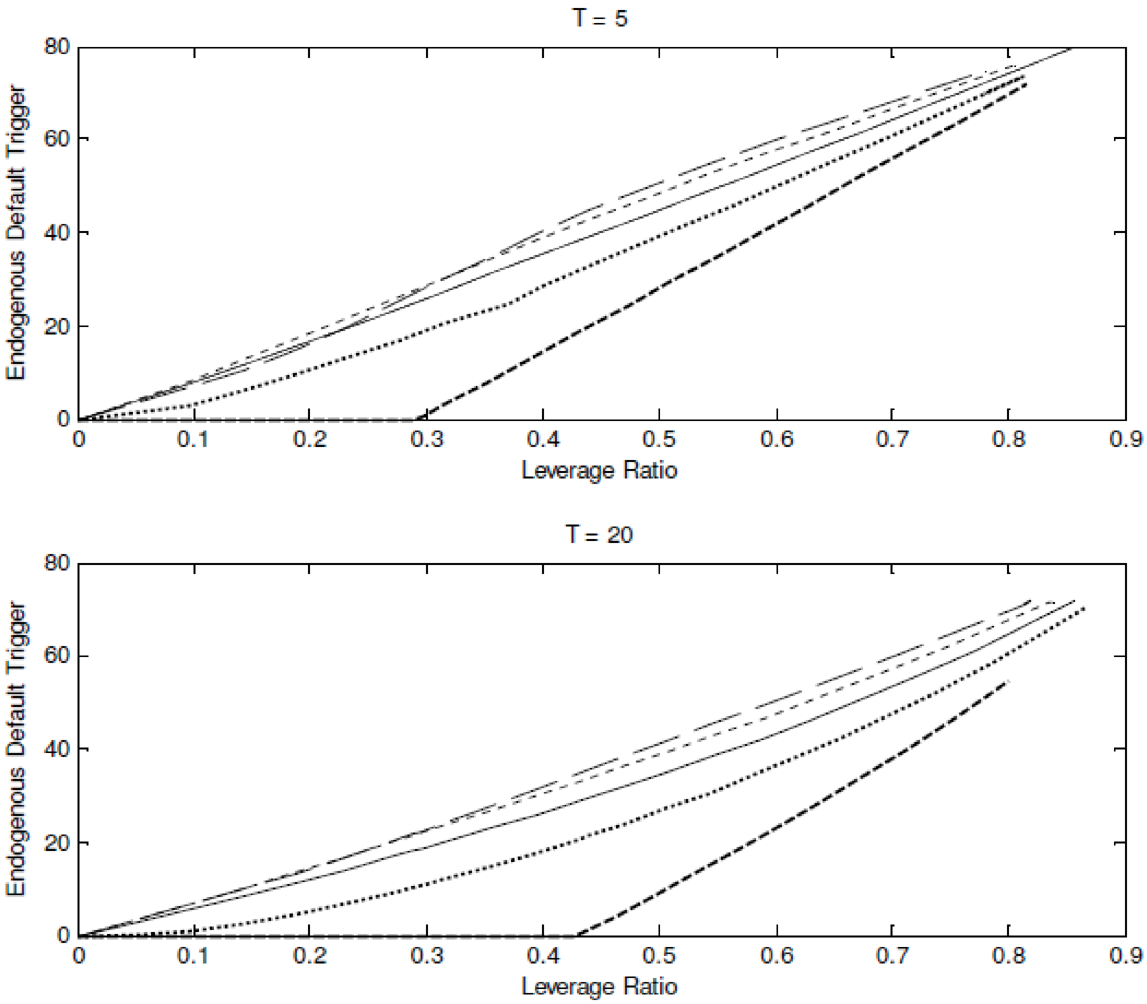

Figure I-6: Total firm value as a function of leverage ratio

This figure depicts the total firm value with respect to the leverage ratio under the Leland (1994b) model (solid line) and CEV structure models with $\beta = -0.5$(dashed lines) and $\beta = 0.5$ (dotted lines). Three scenarios of average debt maturity are considered, $T = 1$, $T = 10$ and $T = 20$. It is assumed that current asset value $V = 100$, risk free rate $r = 0.08$, the firm's payout rate $q = 0.06$, tax rate $w = 0.35$, and proportional bankruptcy cost $\alpha = 0.5$. The coupon rate is determined by making the debt issued at par value under the endogenous default boundary. The initial volatilities are the same for these scenarios and $\sigma_0 = 20\%$. For the given β under the CEV diffusion process, $\theta = \sigma_0 / V^{\beta}$.

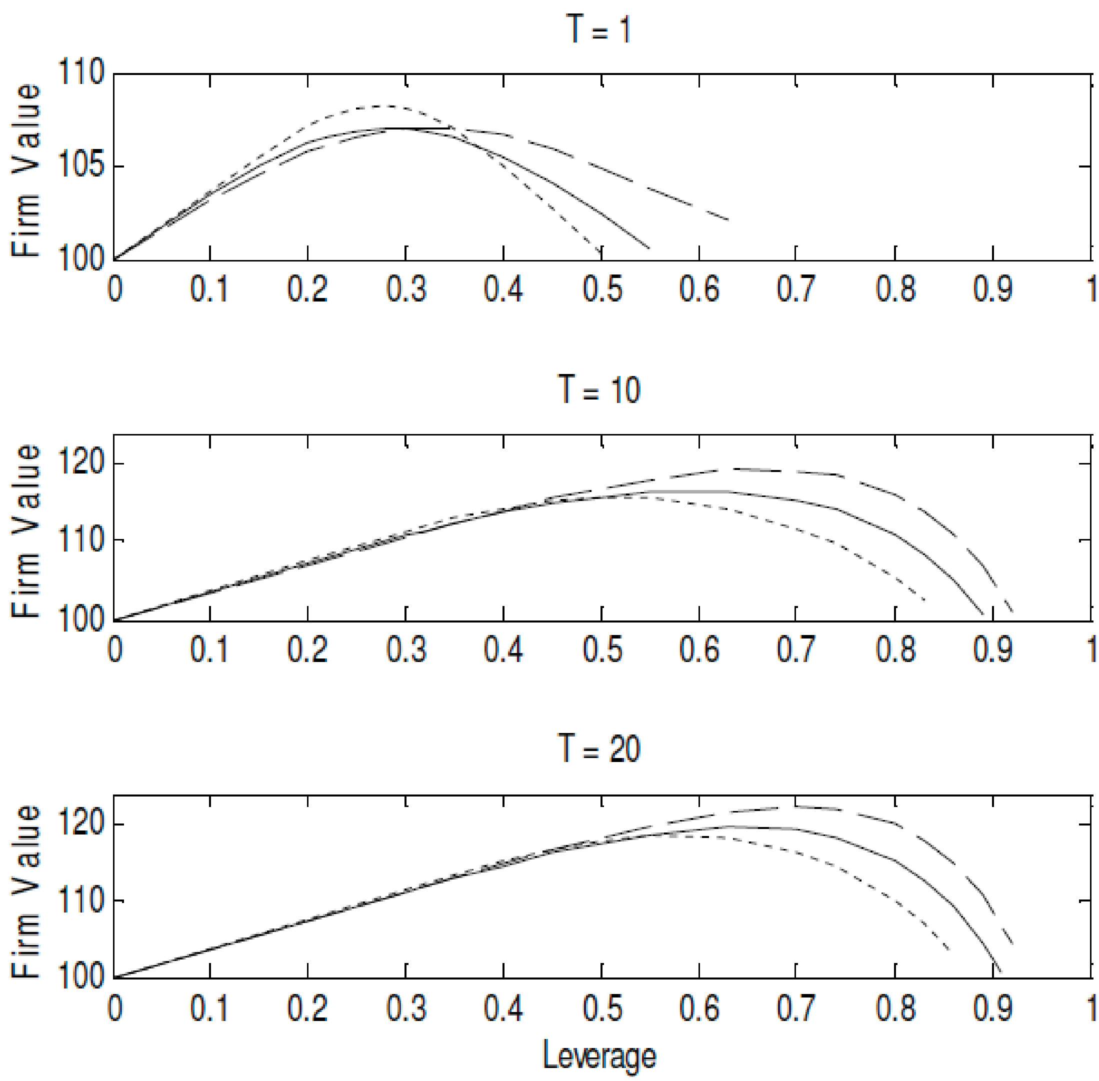

Table I-1: Characteristics of optimally levered firms under different models

This Table exhibits the characteristics of optimally levered firms under the Leland (1994b), Leland and Toft (1996) , and CEV structural models with 1-year's, 5-years', and 10-years' average maturity. It is assumed that current asset value $V = 100$ dollars, risk free rate $r = 0.08$, the firm's payout rate $q = 0.06$, tax rate $w = 0.35$, and proportional bankruptcy cost $\alpha = 0.5$. The leverage is chosen by maximizing total firm value and the coupon rate is determined by making the debt issued at par value under the endogenous default boundary. The initial volatilities are the same for these scenarios and $\sigma_0 = 20\%$. For each given β under the CEV diffusion process, $\theta = \sigma_0 / V^{\beta}$.

Models	Coupon (Dollars)	Bankruptcy Trigger (Dollars)	Optimal Leverage (Percent)	Firm Value (Dollars)	Equity Value (Dollars)	Total Debt Value (Dollars)	Equity Volatility (Percent)	Total Debt Volatility (Percent)	Credit Spread (Basis Points)
				1-year Average Maturity					
$\beta = -1$	3.59	37.52	39.05	108.59	66.18	42.41	35.63	0.71	48.31
$\beta = -0.5$	2.86	36.72	32.88	107.06	71.87	35.20	31.33	0.2	14.46
$\beta = 0.5$	2.35	36.25	27.20	108.19	78.76	29.42	26.26	3.04E-3	0.23
$\beta = 1$	2.54	38.76	28.90	110.00	78.21	31.80	25.94	3.24E-4	0.03
L($\beta = 0$)	2.44	35.67	28.44	107.06	76.61	30.45	27.99	3.12E-2	2.3
				5-year Average Maturity					
$\beta = -1$	8.70	51.83	67.06	119.44	39.34	80.10	58.40	8.90	286.30
$\beta = -0.5$	7.11	50.98	61.00	115.44	45.02	70.42	51.37	6.09	210.31
$\beta = 0.5$	3.99	41.72	42.75	112.74	64.54	48.20	32.85	0.67	27.51
$\beta = 1$	3.76	41.29	40.79	114.33	67.69	46.64	30.24	0.14	6.45
L($\beta = 0$)	5.23	46.36	51.43	112.99	54.88	58.12	40.82	2.69	100.51
				10-years Average Maturity					
$\beta = -1$	9.09	46.95	70.43	123.37	36.49	86.89	55.50	9.93	245.62
$\beta = -0.5$	8.07	49.51	66.39	119.38	40.12	79.26	52.70	8.06	218.09
$\beta = 0.5$	5.17	44.62	51.63	115.71	55.97	59.74	37.67	1.91	64.78
$\beta = 1$	4.58	43.27	47.74	116.83	61.05	55.78	33.60	0.53	20.98
L($\beta = 0$)	6.60	48.09	59.71	116.63	46.99	69.64	45.69	4.92	147.46
				Infinite Average Maturity					
$\beta = -1$	9.75	36.23	77.44	130.43	29.43	101.00	52.03	11.20	165.33
$\beta = -0.5$	9.20	42.97	74.65	126.85	32.16	94.69	52.06	10.13	171.30
$\beta = 0.5$	7.42	45.72	65.84	123.29	42.12	81.17	45.63	4.62	114.32
$\beta = 1$	4.98	36.19	50.73	120.95	59.60	61.35	33.52	0.39	12.36
L($\beta = 0$)	8.38	45.37	70.58	124.43	36.61	87.82	49.53	7.69	153.83

Figure I-7: The term structure of credit spreads under different leverage ratio

This figure depicts the term structure of credit spreads for the Leland (1994b) model (solid line) and CEV structural models with $\beta = -1$ (dashed lines), and $\beta = 1$ (dotted lines) under 40% (Top), 50% (Middle) and 60% (Bottom) leverage ratios. It is assumed that current asset value $V = 100$ dollars, risk free rate $r = 0.08$, the firm's payout rate $q = 0.06$, tax rate $w = 0.35$, and proportional bankruptcy cost $\alpha = 0.5$. The coupon rate is determined by making the debt issued at par value under the endogenous default boundary. The initial volatilities are the same for these scenarios and $\sigma_0 = 20\%$. For the given β under the CEV diffusion process, $\theta = \sigma_0 / V^{\beta}$.

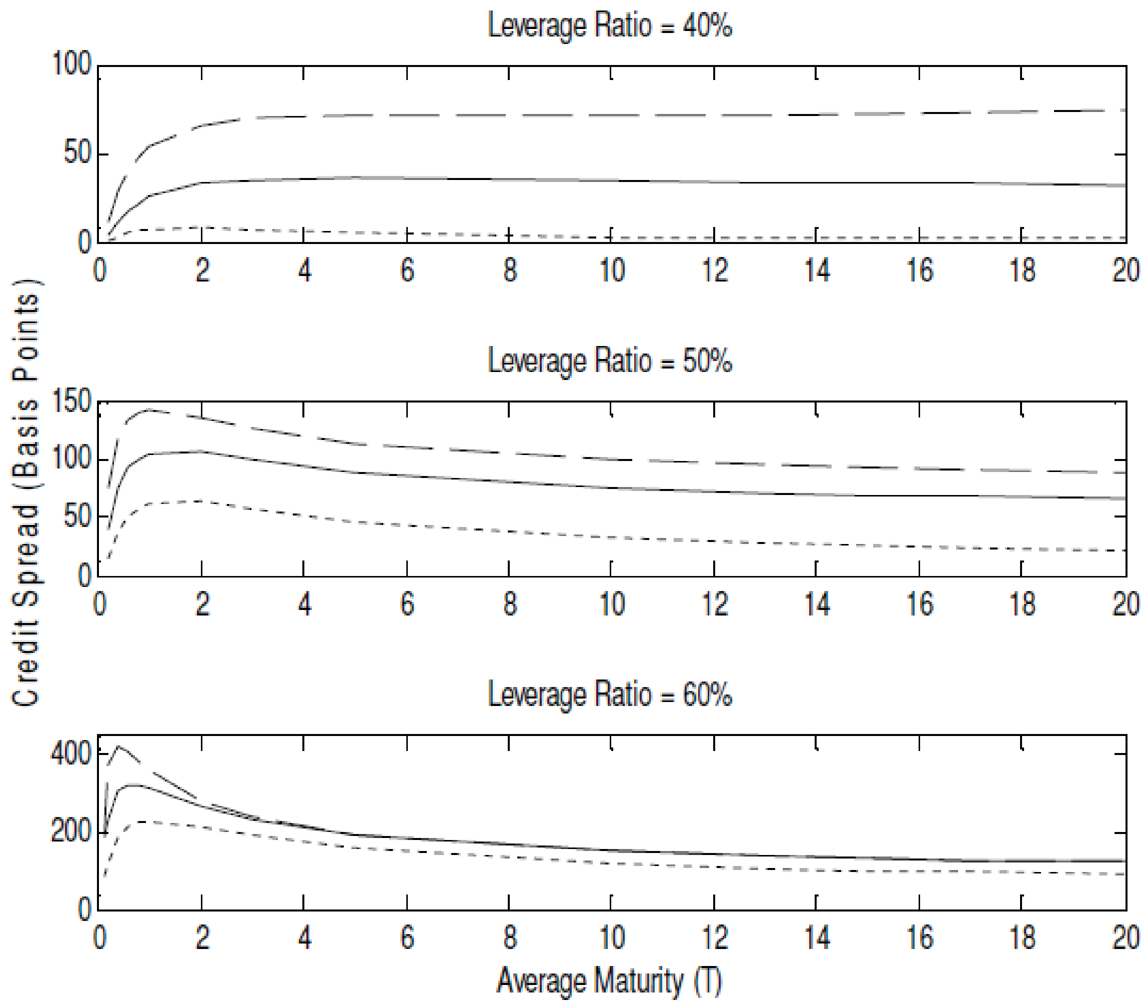

Figure I-8: Debt value as a function of leverage

This figure depicts the value of debt for different leverage ratios under the Leland(1994b) model(solid line), CEV structural model with $\beta = -1$ (dashed line), and $\beta = 1$ (dotted line). It is assumed that current asset value $V = 100$ dollars, risk free rate $r = 0.08$, the firm's payout rate $q = 0.06$, tax rate $w = 0.35$, and proportional bankruptcy cost $\alpha = 0.5$. The coupon rate is determined by making the debt issued at par value under the endogenous default boundary. The initial volatilities are the same for these scenarios and $\sigma_0 = 20\%$. For the given β under the CEV diffusion process, $\theta = \sigma_0 / V^{\beta}$. The average maturity of debt is assumed to be 5 years.

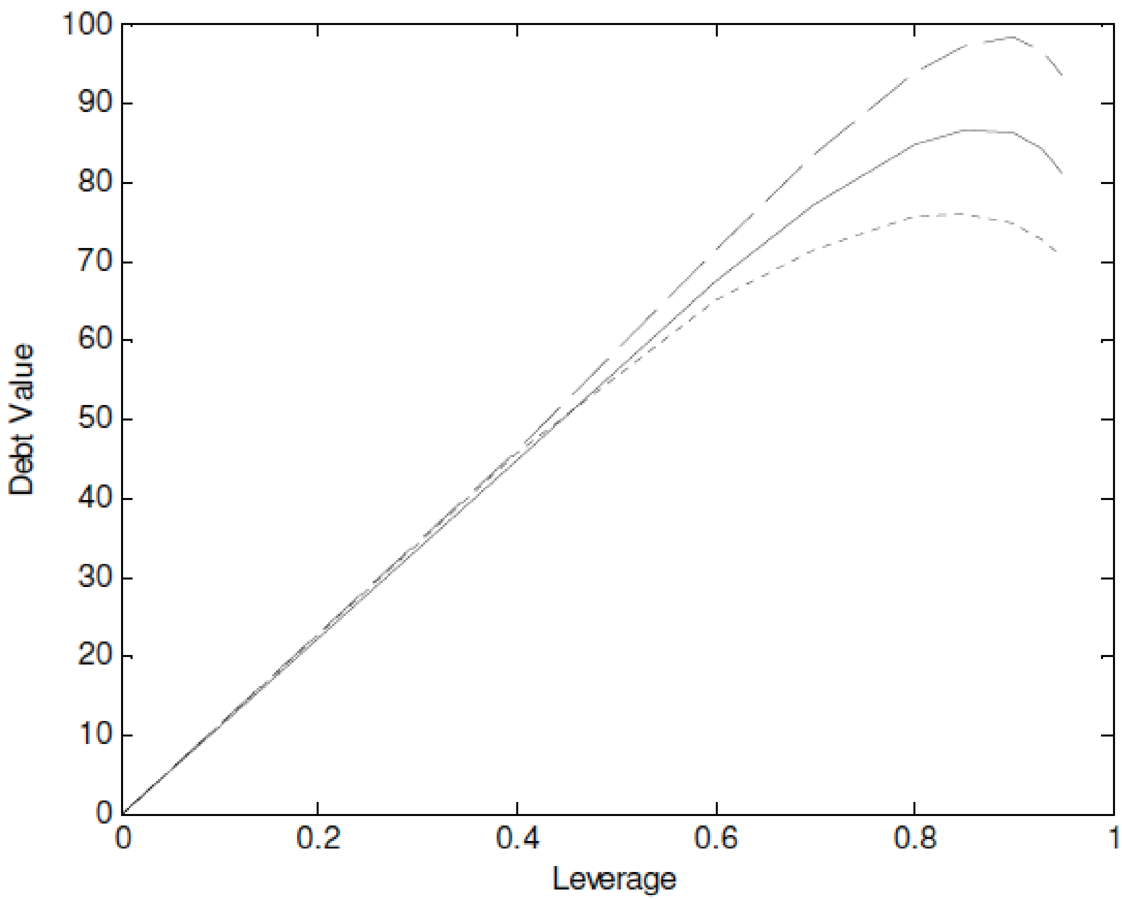

Figure I-9: Effective duration with respect to Macaulay duration

This figure depicts the change of effective duration of bonds with respect to their Macaulay Duration for the Leland (1994b) model (solid line)and CEV structural models with $\beta = -1$(dashed line) and $\beta = 1$(dotted line). It is assumed that current asset value $V = 100$ dollars, risk free rate $r = 8\%$, the firm's payout rate $q = 0.06$, tax rate $w = 0.35$, and proportional bankruptcy cost $\alpha = 0.5$. The coupon rate is determined by making the debt issued at par value under the endogenous default boundary. The leverage ratio is assumed to be 50%. The initial volatilities are the same for these scenarios and $\sigma_0 = 20\%$. For the given β under the CEV diffusion process, $\theta = \sigma_0 / V^{\beta}$.

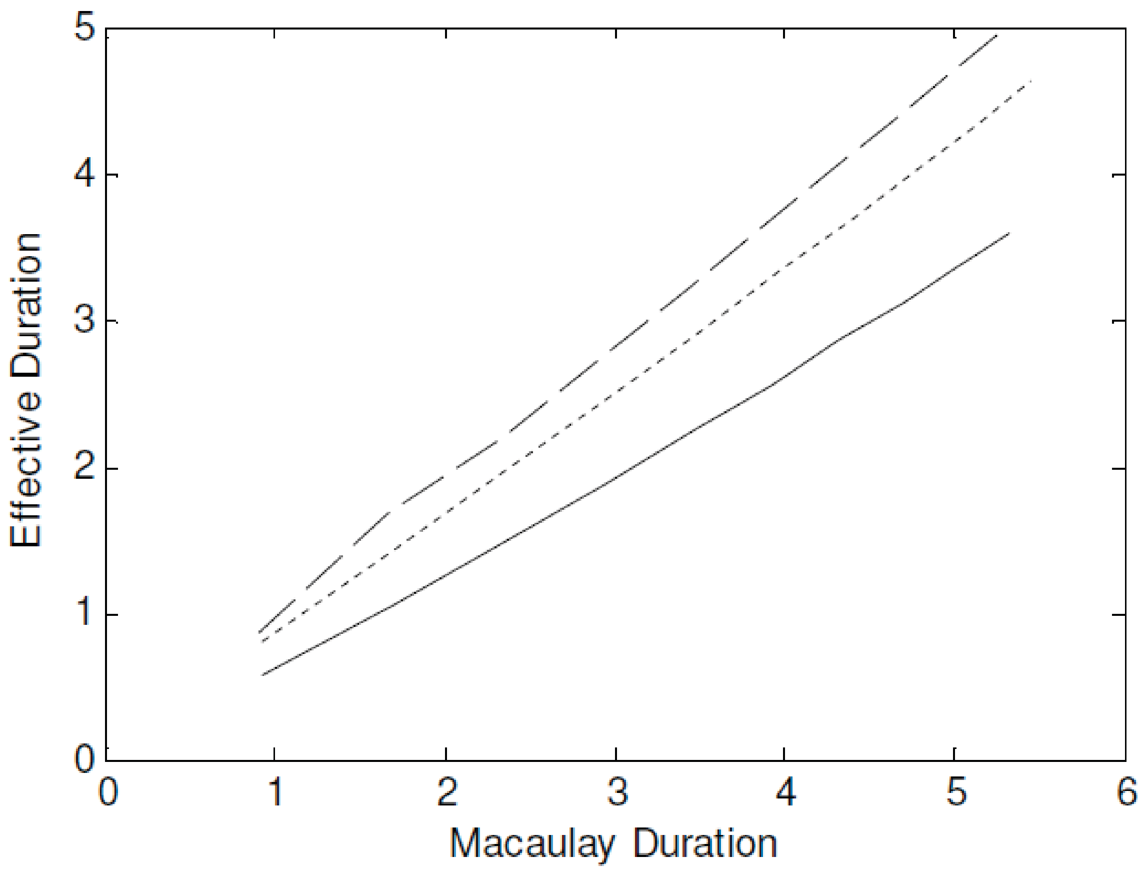

Figure I-10: Bond price as a function of the risk-free interest rate

The graphs depict the bond price per \$100 face value as a function of risk-free rate of interest for the Leland (1994b) structural model (solid lines) and CEV structural models with $\beta = -1$(dashed lines) and $\beta = 1$(dotted lines). Panel A shows the bonds with 5-year average maturity and 40% leverage ratio; Panel B shows the bonds with 5-year average maturity and 50% leverage ratio; Panel C shows the bonds with 20-year average maturity with 40% leverage ratio; and Panel D shows the bonds with 20-year average maturity with 50% leverage ratio. It is assumed that current asset value $V = 100$ dollars, the firm's payout rate $q = 0.06$, tax rate $w = 0.35$, and proportional bankruptcy cost $\alpha = 0.5$. The coupon rate is determined by making the debt issued at par value under the endogenous default boundary when the interest rate is 8%. The initial volatilities are the same for these scenarios and $\sigma_0 = 20\%$. For the given β under the CEV diffusion process $\theta = \sigma_0 / V^{\beta}$.

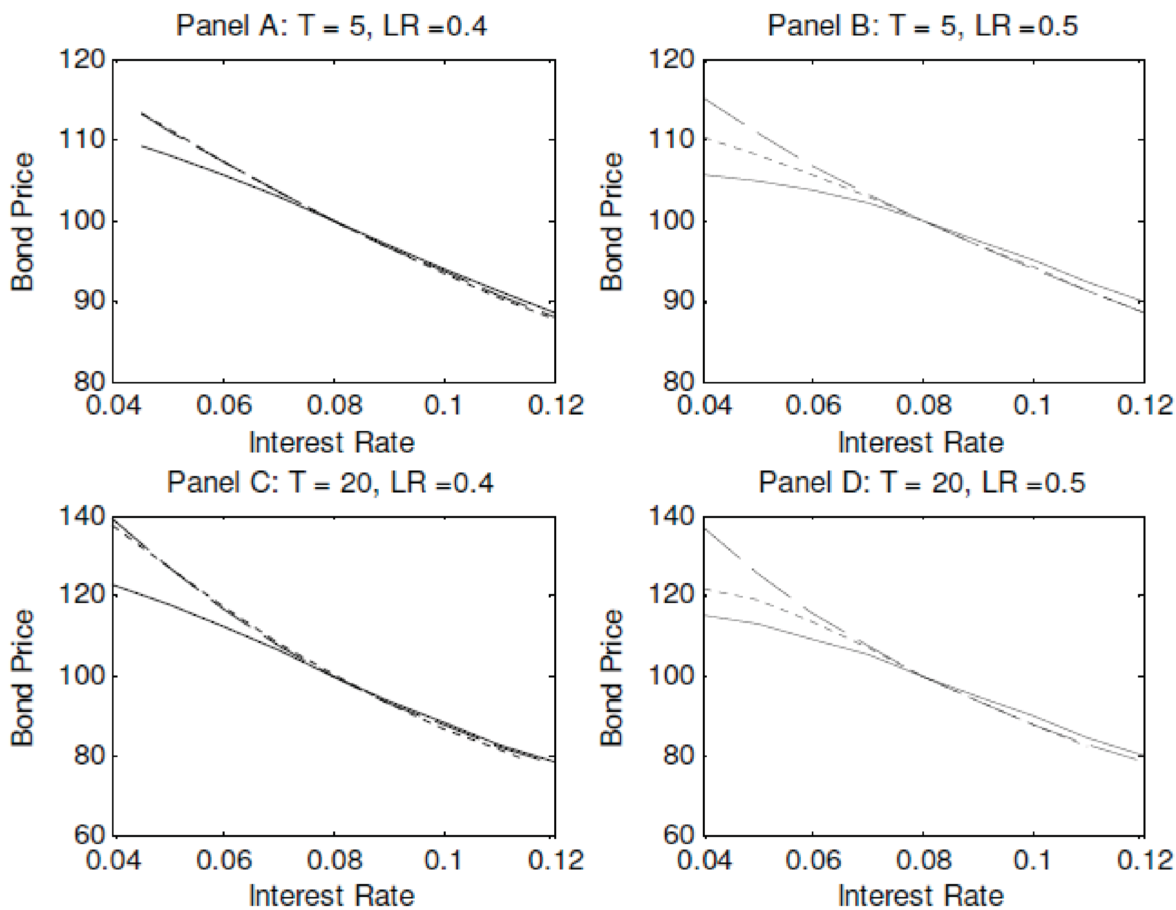

Figure I-11: Volatility of equity with respect to the level of equity value

This figure depicts the volatilities of equity under the Leland (1994b) model (solid line) and CEV structural models with $\beta = -1, \beta = -1$ (dashed line), $\beta = -0.5$ (dotted line), $\beta = 0.5$ (star-dashed line) and $\beta = 1$ (plus-dashed line). It is assumed that current asset value $V = 100$ dollars, risk free rate $r = 0.08$, the firm's payout rate $q = 0.06$, tax rate $w = 0.35$, and proportional bankruptcy cost $\alpha = 0.5$. The average maturity of debt is 5 years. The leverage ratio is the optimal leverage ratio computed under the endogenous default boundary. The coupon rate is determined by making the debt issued at par value under the ndogenous default boundary. The initial volatilities are the same for these scenarios and $\sigma_0 = 20\%$. For the given β under the CEV diffusion process $\theta = \sigma_0 / V^{\beta}$.

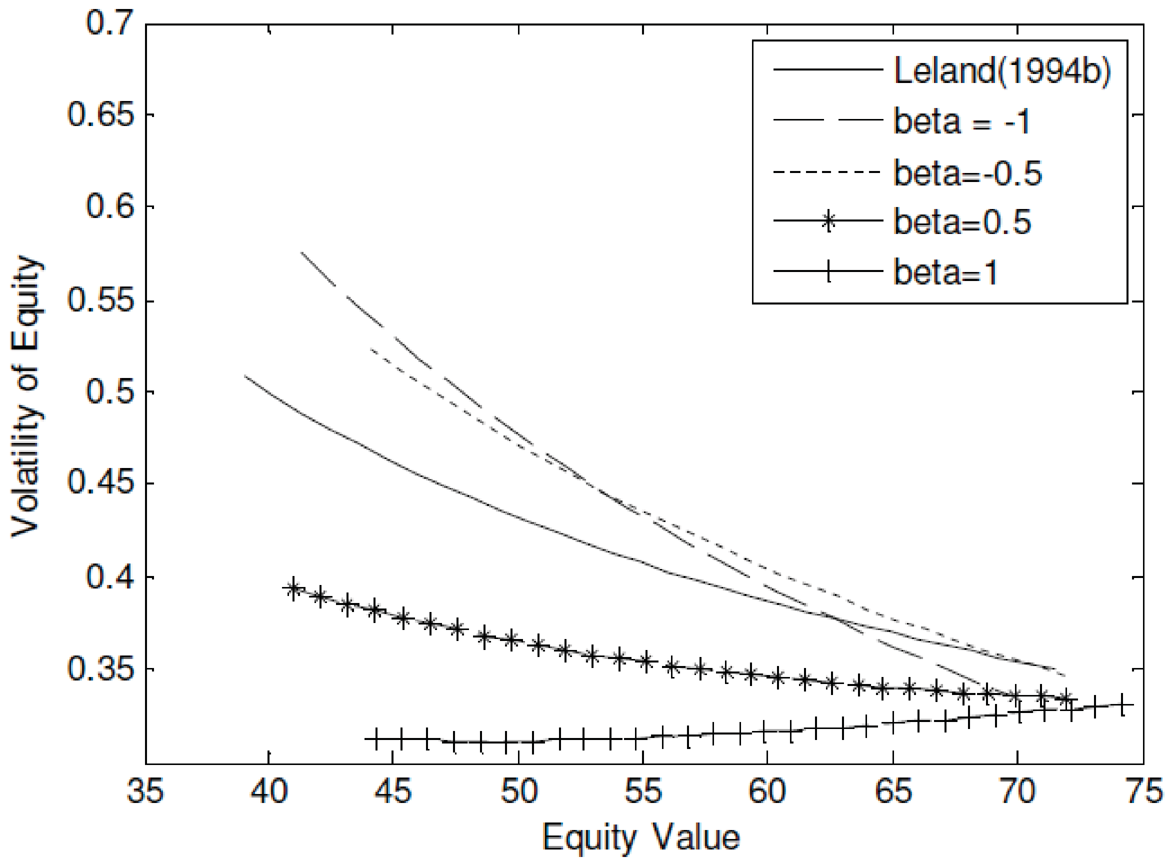

Figure I-12: Sensitivity of equity and debt values to total asset risk

This figure depicts the sensitivity of equity value and debt value to total asset risk measured by the volatility of assets, for 1-year, 5-year, 10-year and perpetual bonds, shown under the L model with solid (dashed) lines for the sensitivity of equity (debt), under the CEV structural model with $\beta = 1$ with plus-solid (plus-dashed) lines for the sensitivity of equity (debt), and the CEV model with $\beta = -1$ with star-solid (star-dashed) lines for the sensitivity of equity (debt). The total coupon payment and face value of debt are determined such that the capital structure is optimal for firm value $V = 100$. The particular values for each bond refer to Table I-1. It is assumed that the risk free rate $r = 0.08$, the firm's payout rate $q = 0.06$, tax rate $w = 0.35$, and proportional bankruptcy cost $\alpha = 0.5$.

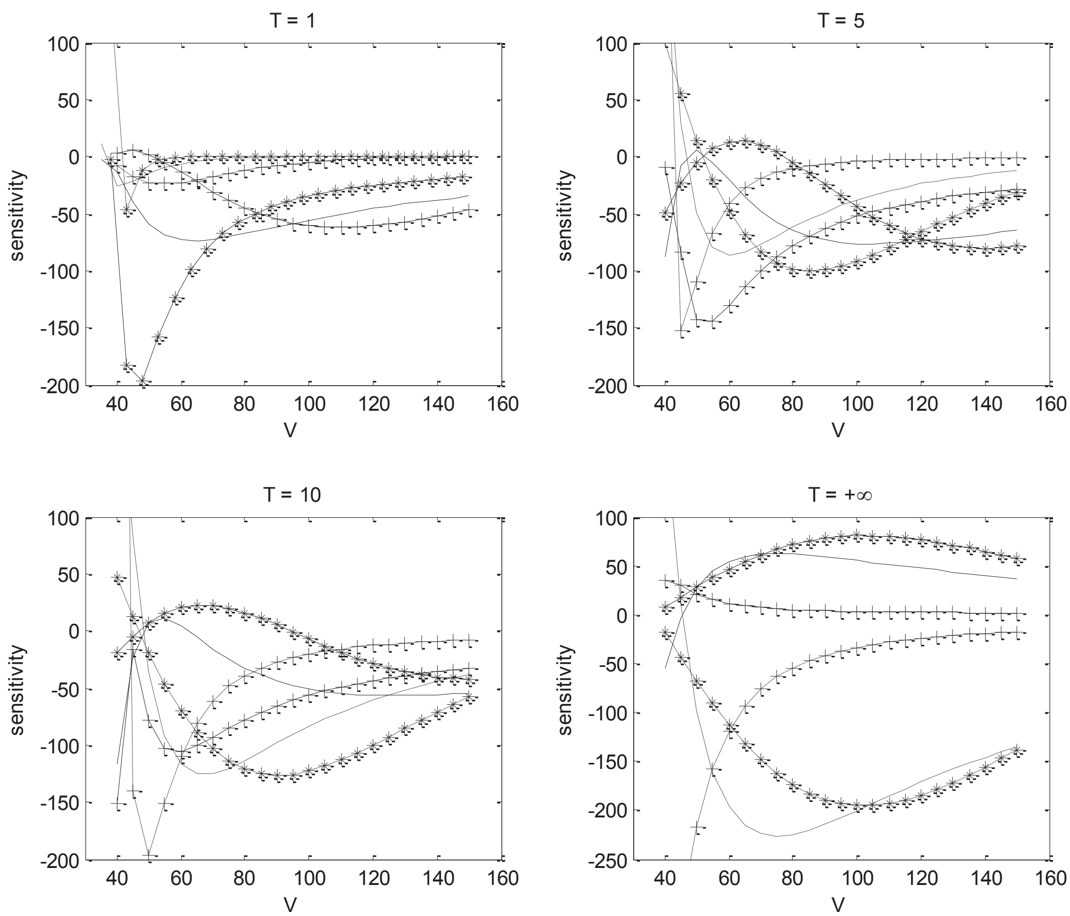

Table I-2: Characteristics of firms under different models with jumps

This table reports the characteristics of firms for Leland's model (L), Leland's model with jump (L-J), CEV structural model (CEV) and CEV structural model with jump (CEV-J). The bonds pays continuous coupon and the coupon payment equals the continuous coupon in Table I-1 for each scenario. The default events only occur on the semi-annual basis and the default boundary is equal to the endogenous default boundary of the optimally levered firm in Table 1 for each scenario. The intensity of the jump is $\eta_J = 1/10$. The jump amplitude for L-J follows a log-normal distribution and for CEV-J follows a binomial distribution with $\mu_J = -0.05, \sigma_J = 0.2$. It is assumed that current asset value $V = 100$ dollars, the firm's payout rate $q = 0.06$, tax rate $w = 0.35$, and proportional bankruptcy cost $\alpha = 0.5$. The initial volatilities are the same for these scenarios and $\sigma_0 = 20\%$. For each given β under the CEV diffusion process, $\theta = \sigma_0 / V^{\beta}$.

	Models	Leverage (Percent)	Firm Value (Dollars)	Equity Value (Dollars)	Total Debt Value (Dollars)	Credit Spread (Basis Points)
1-year maturity bond						
L($\beta = 0$)	L	0.2829	107.63	77.18	30.45	1.29
	L-J	0.2849	106.87	76.42	30.44	1.39
$\beta = -1$	CEV	0.3867	109.92	67.42	42.50	44.66
	CEV-J	0.3907	108.73	66.24	42.49	44.99
$\beta = -0.5$	CEV	0.3263	107.93	72.71	35.22	12.09
	CEV-J	0.3293	106.92	71.71	35.21	12.26
$\beta = 0.5$	CEV	0.2709	108.58	79.16	29.42	0
	CEV-J	0.2737	107.48	78.07	29.41	0
$\beta = 1$	CEV	0.2923	108.76	76.96	31.80	0
	CEV-J	0.2965	107.22	75.43	31.79	0
5-year maturity bond						
L($\beta = 0$)	L	0.5125	114.58	55.86	58.72	90
	L-J	0.5154	113.11	54.81	58.30	97
$\beta = -1$	CEV	0.6718	123.00	40.38	82.63	253
	CEV-J	0.6792	120.42	38.63	81.79	264
$\beta = -0.5$	CEV	0.6094	118.06	46.12	71.95	188
	CEV-J	0.6155	115.79	44.52	71.28	198
$\beta = 0.5$	CEV	0.4255	113.62	65.27	48.35	25
	CEV-J	0.4299	111.95	63.82	48.13	29
$\beta = 1$	CEV	0.4064	114.83	68.15	46.67	5.6
	CEV-J	0.4117	113.02	66.49	46.53	8
10-year maturity bond						
L($\beta = 0$)	L	0.5984	118.61	47.64	70.97	130
	L-J	0.5989	116.90	46.88	70.02	143
$\beta = -1$	CEV	0.7112	126.52	36.54	89.98	210
	CEV-J	0.7138	124.04	35.50	88.54	227
$\beta = -0.5$	CEV	0.6683	122.05	40.49	81.56	189
	CEV-J	0.6706	119.65	39.41	80.24	206
$\beta = 0.5$	CEV	0.5153	116.99	56.70	60.29	58
	CEV-J	0.5180	114.94	55.40	59.54	68
$\beta = 1$	CEV	0.4760	117.57	61.60	55.96	18
	CEV-J	0.4802	115.34	59.95	55.39	27

Table I-3: Characteristics of firms under different models with jumps

This table reports the characteristics of firms for Leland's model with jump (L-J) and CEV structural model with jump (CEV-J) under varying jump calibrations. The 5-year bond pays continuous coupon and the coupon payment equals the continuous coupon in Table I-1 for each scenario. The default events only occur on a semi-annual basis and the default boundary is equal to the endogenous default boundary of the optimally levered firm in Table I-1 for each scenario. For the base case, the intensity of the jump is $\eta_J = 1/10$. The jump amplitude for L-J follows a log-normal distribution, and for CEV-J follows a binomial distribution with $\mu_J = -0.05, \sigma_J = 0.2$. It is assumed that current asset value $V = 100$ dollars, the firm's payout rate $q = 0.06$, tax rate $w = 0.35$, and proportional bankruptcy cost $\alpha = 0.5$. The initial volatilities are the same for these scenarios and $\sigma_0 = 20\%$. For each given β under the CEV diffusion process, $\theta = \sigma_0 / V^{\beta}$.

Models	Leverage (Percent)	Firm Value (Dollars)	Equity Value (Dollars)	Total Debt Value (Dollars)	Credit Spread (Basis Points)
			L-J($\beta = 0$)		
Base Case	0.5154	113.11	54.81	58.30	97
$\eta_J = 1/5$	0.5182	111.71	53.82	57.89	103
$\eta_J = 1/20$	0.5140	113.84	55.33	58.51	94
$\sigma_J = 0.25$	0.5156	112.87	54.68	58.19	99
$\sigma_J = 0.15$	0.5153	113.33	54.93	58.40	95
$\mu_J = -0.1$	0.5178	112.13	54.07	58.07	101
$\mu_J = 0.05$	0.5109	114.84	56.17	58.67	91
			$\beta = -1$		
Base Case	0.6792	120.42	38.63	81.79	264
$\eta_J = 1/5$	0.6866	117.87	36.94	80.93	275
$\eta_J = 1/20$	0.6755	121.71	39.50	82.21	258
$\sigma_J = 0.25$	0.6799	120.09	38.44	81.64	266
$\sigma_J = 0.15$	0.6787	120.68	38.78	81.90	262
$\mu_J = -0.1$	0.6858	118.33	37.18	81.15	272
$\mu_J = 0.05$	0.6681	124.11	41.19	82.92	249
			$\beta = 1$		
Base Case	0.4117	113.02	66.49	46.53	8
$\eta_J = 1/5$	0.4165	111.34	64.97	46.37	11
$\eta_J = 1/20$	0.4093	113.82	67.23	46.59	7
$\sigma_J = 0.25$	0.4126	112.66	66.18	46.49	8.8
$\sigma_J = 0.15$	0.4110	113.29	66.73	46.56	7.5
$\mu_J = -0.1$	0.4152	111.87	65.43	46.45	9.5
$\mu_J = 0.05$	0.4065	114.72	68.09	46.64	6.2

Chapter II STATE DEPENDENT VOLATILITY, LIQUIDITY RISK AND CREDIT RISK

1. Introduction

The credit spread of a firm is defined as the yield increment of its bonds over the riskless rate. This spread is a key component of a firm's financing cost, and reflects primarily its default risk, but also other essential factors, such as the liquidity risk of its bond market and general macroeconomic conditions. The default risk has been modeled theoretically, and measured empirically, in a large number of studies on corporate bond pricing and the risk structure of interest rates, following the pioneering work of Merton (1974) and Black and Cox (1976), which in turn were inspired by the seminal Black and Scholes (1973) model of option pricing. The resulting models came to be known as *structural models* of bond pricing, as distinct from another class of models known as reduced form models, in which there is no link between the bonds of a given risk class and the firm's capital structure.[41] The liquidity of the firm's bond markets was till recently studied independently of its default risk. Ericsson and Renault (2006) develop a structural bond valuation model to simultaneously capture liquidity and credit risk directly and find positive evidence between the illiquidity and default components of yield spreads. An important recent study by He and Xiong (2012)[42] use structural model through the rollover channel to combine the effects of both default risk and liquidity risk. The framework followed the well-known Leland and Toft (1996) structural model of the firm, in which the basic stochastic process of the value of the unlevered firm's assets was

[41] For the reduced form models see Jarrow and Turnbull (1995), Duffie and Singleton (1999) and Duffie and Lando (2001). These models lie outside the topic of this paper.
[42] See also Ericsson and Renault (2006) , who found a positive correlation between bond market illiquidity and the default components of the yield spread.

constant volatility diffusion. They found that the market value of debt decreases and the endogenous default boundary increases in the presence of liquidity shocks in the bond market, which confirmed the positive relationship between liquidity risk and credit spread.

The adoption of Leland and Toft (1996) framework provides analytical solutions for debt and equity values in the presence of liquidity risk, but has to inherit the limitation of the constant volatility assumption. This is a common feature of structural models, since the complexity of the valuation expressions places a major emphasis on the derivation of closed form expressions, rather than numerical results based on approximations[43] or simulations.[44] Such a focus allowed relatively easy estimations of numerical values given the parameters of the model, but at the cost of maintaining simple formulations of the mathematical structure of the asset value dynamics, in which a univariate diffusion process still follows the original Black and Scholes (1973) and Merton (1974) assumption of a lognormal diffusion with constant volatility.[45] This is all the more surprising, in view of the fact that the option pricing literature has long recognized that such an assumption is no longer adequate to represent underlying assets in option markets, and has introduced factors such as rare events, stochastic volatility and transaction costs. Note also that empirical evidence shows that this assumption does not hold. Choi and Richardson (2009) studied the conditional volatility of the firm's assets by a weighted average of equity, bond and loan prices and found that asset volatility is time varying. In their study of the

[43] Zhou (2001) and Collin-Dufresne and Goldstein (2001).

[44] Brennan and Schwartz (1978), and more recently Titman and Tsyplakov (2007) are examples of studies that rely on numerical simulations.

[45] Most structural models are univariate and assume a constant riskless rate of interest. Longstaff and Schwartz (1995), Briys and de Varenne (1997), and Collin-Dufresne and Goldstein (2001) use bivariate diffusion models, in which the term structure of interest rates follows the Vacisek (1977) model and the asset value is a constant volatility diffusion. As the empirical work in Chan *et al* (1992) shows, the Vacisek model does not fit actual term structure data. Further, Leland and Toft (1996) note that this bivariate diffusion refinement plays a very small role in the yield spreads of corporate bonds.

term structure of credit default swaps (CDS), Huang and Zhou (2008) similarly note that time varying asset volatility should potentially play a role in structural models in order to fit into the empirical credit default spread data.

In this Chapter we revisit the combined effects of default risk and liquidity risk in a generalized model of the dynamics of the asset value by assuming that the diffusion volatility is state-dependent, varying with the asset value according to the constant elasticity of variance (CEV) model. Compared to constant volatility diffusion, the CEV model has an extra parameter and includes constant volatility as a special case. This model has had several applications in the realm of option pricing, and has even been mentioned in the context of structural models of the firm as early as 1976.[46] The fact that it has not attracted more attention is probably due to its analytical and computational complexity at a time when computational technology was relatively undeveloped.

In spite of the significant additional computational complexity of CEV we manage to derive closed form expressions for all the variables of interest, including corporate debt value and equity value. Other variables of interest such as the endogenous boundary and the optimal leverage under endogenous default boundary can be estimated numerically, as they are in several constant volatility models. As a result of the flexibility provided by the extra parameter, our structural model under the CEV process is able to produce numerical results that are considerably closer to the historical record of yield spreads, liquidity risk and default probabilities than the earlier constant volatility structural models.

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[46] See Cox and Ross (1976, pp. 163-165). This article examines the CEV model in two special cases, discussed further on in this paper.

The main economic justification for introducing state dependent asset volatility in credit default models of the firm is that volatility risk has increasingly been recognized as an important factor in the pricing of financial assets. Although the inverse variation of firm equity volatility with firm value can be justified theoretically because of the well-known leverage effect, the empirically documented index volatility fluctuations cannot be similarly justified, and it transmits itself to individual firms through capital market equilibrium.[47] An advantage of the CEV model is that it can approximate more complex stochastic volatility situations while remaining theoretically and empirically tractable.

Our main building block is the structural model of Leland (1994b), itself an extension of that author's seminal contribution published that same year. This model has some minor computational advantages in the derivation of closed form expressions, while retaining most of the attractive features of the subsequent Leland and Toft (1996). Leland (1994b) is similar to Leland (1994a) insofar as it uses infinite maturity debt financing, but it does achieve a debt structure with a finite average maturity by continuously retiring and refinancing the debt at a fixed rate with perpetual bonds. As a result, the debt maturity effect can be studied by focusing on the rollover rate, while the equivalence of this model's results with Leland and Toft (1996) can be easily demonstrated by an appropriate choice of parameters.[48] Further, the continuous debt rollover allows the study of the illiquidity effects of the bond markets on both equity and debt, while the default boundary is chosen endogenously by maximizing the value of the equity, as in Leland and Toft (1996).

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[47] See the empirical results for both market index and individual firms in Driessen *et al* (2009).
[48] This equivalence is demonstrated in the appendix to the Chapter 1 for both fixed volatility and CEV models.

Our approach has certain advantages vis-à-vis other attempts to generalize the asset dynamics of structural models. Hilberink and Rogers (2002) and Chen and Kou (2009) extend the Leland (1994b) model by incorporating a Levy process with only upward jumps and with two-sided double exponential jumps,[49] respectively. Apart from the highly stylized nature of the jump component model, which was chosen for its mathematical convenience, their diffusion component has constant volatility. Zhang, Zhou and Zhu (2008) incorporate stochastic volatility and jumps into the Merton (1974) model, but by necessity assume that default occurs only at maturity; they find that incorporating jumps and stochastic volatility may help to improve the matching of the top quality credit spreads. Elkamhi, Ericssion, Wang and Du (2012) introduce stochastic volatility into the asset value diffusion process and study the impact of volatility risk premium on the credit spread. Their results can be obtained only by numerically solving the two-dimensional Fortet equation approximation method for the first passage time distribution.[50]

In this Chapter we present generalized expressions for both equity and debt values in the presence of rollover risk under CEV that include constant volatility as a special case. We show theoretically and for all elasticity values that increases in the rollover risk parameters reduce the values of both debt and equity. In the numerical estimations with simulated data we find that the sign and magnitude of the elasticity parameter are major determinants of the model's results, on their own or in their interaction with other features of the model like leverage and debt maturity. As with constant volatility, the

[49] Zhou (2001) was the first to introduce jumps into structural models under the first passage default assumption, but no analytical solution is presented and he did not study optimal capital structure with endogenous default boundary. Huang and Huang (2003) also incorporate double exponential jumps into a structural model, but they only focus on corporate debt valuation and credit spread.
[50] See Colin-Dufresne and Goldstein (2001).

endogenous default boundary increases with the size of the rollover risk parameter, while a positive (negative) elasticity raises (lowers) the endogenous default boundary for all cases. Similarly, the elasticity parameter emerges as a major determinant of the optimal leverage in all cases, with the size of its effect depending on the maturity of the debt.

Our theoretical results on the effects of liquidity risk in the CEV model parallel those of He and Xiong (2012) for the constant volatility case in the Leland and Toft (1996) model. Specifically, the liquidity premium reduces the value of both debt and equity and increases the level of the endogenously chosen default boundary. Comparing fixed and state dependent volatilities, the CEV model reduces the value of equity more than the constant volatility case, especially for the shorter maturities, while it reduces the value of debt for negative elasticity and leaves it approximately equal to the constant volatility case for positive elasticity. On the other hand, the effect on the endogenous boundary depends crucially on the sign of the elasticity, resulting in almost all cases in a lower boundary for negative and higher for positive elasticity than for constant volatility.

2. Economic Setup and Debt Valuation

2.1 The CEV diffusion model distribution for the unlevered asset

Following Leland (1994a,b), we consider a firm whose assets are financed by equity and infinite maturity debt with a tax-deductible coupon. As in all previous related literature, the values of the components of the firm's balance sheet are estimated as contingent claims of the state variable *V,* the value of the unlevered firm's assets representing its economics activities. If r denotes the riskless rate, q the payout rate to the stockholders, and $\theta V_t^{\beta+1} = \sigma(V)$ the state-dependent volatility, we have under the risk neutral distribution:

$$\begin{cases} \dfrac{dV_t}{V_t} = (r-q)dt + \theta V_t^{\beta} dW^Q, \text{ if } 0 < t < \tau < T \\ V_t = K \text{ if } 0 < \tau \le t < T \end{cases} \tag{2.1}$$

In (2.1) it is assumed that the diffusion process continues until the asset value hits or falls below a threshold value, denoted by *K*, for the first time τ. In such a case, a default event will be triggered and liquidation comes in immediately. Assuming the absolute priority is respected, the bond holders will then receive $(1-\alpha)K$, while the equity holders receive nothing. The remaining of asset value that equals to αK is considered a bankruptcy cost.

The parameter β, the elasticity of the local volatility, is a key feature of the CEV model. For $\beta = 0$ the model becomes a geometric Brownian motion with constant volatility. For $\beta > 0$ ($\beta < 0$) (the state-dependent volatility is positively (negatively) correlated with the asset price.[51] In equity markets, the well-known "leverage effect" shows generally a negative relationship between volatility and equity price. There are also some suggestions that the economically appropriate range is $0 > \beta > -1$,[52] even though empirical evidence in the case of the implied risk neutral distribution of index options finds negative values significantly below this range. Jackwerth and Rubinstein

[51] As Emmanuel and Macbeth (1982, p. 536) were the first to point out, for $\beta > 0$ the local volatility becomes unbounded for very large values of *V*, and there are technical issues concerning the mean of the process under both the physical and the risk neutral distribution. This problem is solved by assuming that the volatility is bounded and becomes constant for *V* exceeding an upper bound; see Davydov and Linetsky (2001, p. 963), A similar lower bound when β is < 0 prevents the formation of an absorbing state at 0.

[52] See Cox (1996), and also Jackwerth and Rubinstein (1999), who term this model the *restricted* CEV. The arguments in favor of the restricted CEV model are mostly applicable to index options and will not affect our formulation.

(1999) find that the unrestricted CEV model when applied to the risk neutral distribution extracted from S&P 500 index options is able to generate as good out-of-sample option prices as the better known stochastic volatility model of Heston (1993). Most of the option pricing literature has concentrated on two special cases, the square root model with $\beta = -\frac{1}{2}$ and the constant volatility model with $\beta = -1$, since these generate more tractable option pricing expressions.[53] Hereafter we will present our numerical results for both positive and negative values of the elasticity without any restrictions in our theoretical results.

The distribution of the asset value V_T conditional on the initial value V_0 has the form of a non-central chi-square $\chi^2(z,u,v)$, denoting the probability that a chi-square-distributed variable with u degrees of freedom and non-centrality parameter v would be less than z. This distribution is given analytically most often in terms of its complementary form 1- $\chi^2(z,u,v)$, denoting in our case the probability $V_T \geq v_T$. For $\beta < 0$ this probability is[54]

$$\Pr ob(V_T \geq v_T) = 1 - \chi^2(c,b,a) = \chi^2(a, 2-b, c), \tag{2.2}$$

Where

$$\begin{aligned} &a = \upsilon v_T^{-2\beta},\ c = \upsilon(V_t e^{(r-q)T})^{-2\beta},\ b = -\beta^{-1}, \\ &\upsilon = -\frac{2(r-q)}{\theta^2 \beta[e^{-2(r-q)\beta T} - 1]} \end{aligned} \tag{2.3}$$

This distribution becomes the lognormal when the volatility is constant. It has been tabulated and is easily available numerically. Several additional results hold about the

[53] See Beckers (1980) and Cox and Ross (1976).
[54] See Schroder (1989, p. 213-214).

$\chi^2(z,u,v)$ distribution when the parameter u is an even integer that can simplify the computations. Nonetheless, the main result necessary for the extension to the mixed jump diffusion process by using the chi-square distribution's characteristic function holds even for non-integral degrees of freedom.[55]

2.2 Stationary debt structure, rollover risk and debt value

The values of contingent claims on the value of a firm whose assets' dynamics follow an equation such as (2.1) are given by the solution of a partial differential equation (PDE), which expresses the notion that in the risk neutral world the instantaneous return on the value of the claim should be equal to the riskless rate. Merton (1974) and Black and Cox (1976) derived closed form solutions of the equation for the case of pure discount bonds. A time dependence term in this PDE prevents the derivation of a closed form solution for the pde when the claim is a conventional finite maturity coupon bond, even when the volatility is constant. Leland (1994a,b) and Leland and Toft (1996) adopted particular debt maturity, coupon and repayment structures, termed *stationary debt structures*, under which the firm continuously issues and retires debt simultaneously in order to keep the total value of outstanding debt time-independent and eliminate the corresponding term in the PDE.

In this Chapter we adopt the Leland (1994b) model as our base case, since this model, with its exponential stationary debt structure, preserves all the merits of Leland and Toft (1996)'s stationary debt structure, and also generates the most elegant analytical

[55] See Johnson *et al* (1995, p. 433).

results with both constant and state dependent volatility.[56] We assume that the infinite maturity debt has a total principal value P at time 0 when it is issued with coupon rate C. As time goes by, the firm retires this debt at a proportional rate g. Thus, the remaining principal value of this debt value at time t is $e^{-gt}P$, and the debt holders receive a cash flow $e^{-gt}(C+gP)$ at time t, provided the firm remains solvent. Hence, the average maturity of this debt will be, given that no default occurs,

$$T_a = \int_0^\infty gte^{-gt}dt = g^{-1} \tag{2.4}$$

Thus, the average maturity under this model is the reciprocal of the proportional retirement rate. In order to get a stationary debt structure we assume that the firm replaces the retired debt with newly issued debt having the same principal and coupon so as to keep the total principal and total coupon payments independent of time. We denote the total value of all outstanding debt by *D (V)*. Since all the outstanding debts are homogenous, the initial total principal *P*, the coupon rate *C,* and the retirement rate *g* (or equivalently, the average maturity T_a) define the debt characteristics and can be used at time *0* as control parameters to value all the outstanding debt.

The rollover risk follows the specification pioneered by He and Xiong (2012). In their model bond markets are occasionally subjected to Poisson-type liquidity shocks, during which a bondholder must sell her holdings at an exogenously given proportional cost k_1. In the presence of those shocks this proportional cost times the intensity ξ of the

[56] Compared to Leland and Toft (1996), the Leland (1994b) model yields a purely analytical solution in the CEV case. The debt service rate is *C+gP* under Leland (1994b), while it is *C+P/T* under Leland and Toft (1996). The two models can be made consistent with each other in their results, as shown in our online appendix.

shock induce a premium that must be paid by the firm when it refinances its debts through rollover. While He and Xiong (2012) focused on the effects of the bond market's liquidity parameters on credit risk, we concentrate in this paper on the interaction between the state-varying volatility of our CEV model with the liquidity risk in defining the firm's credit spreads and optimal capital structure.

Let $f(t,V,K)$ denote the first passage time to default density function and define

$$\begin{aligned} A(t) &= \int_{\tau=0}^{\tau=t} f(\tau,V.K)d\tau \\ B_r(V,K) &\equiv \int_0^{\infty} e^{-rt} f(t,V,K)dt \end{aligned} , \tag{2.5}$$

denoting respectively the cumulative default probability from time 0 to *t*, omitting for simplicity its arguments and the value of an instrument paying \$1 at the first passage time when the riskless rate is r. The functions $A(t)$ and $B_r(V,K)$ are the key building blocks of the closed form expressions for the values of the firm's financial instruments. Let also $D(V,g)$ denote the value of the debt at time 0 in the Leland (1994b) model. In the presence of liquidity risk as defined above and for a general state-dependent volatility $\sigma(V)$, the value $D(V,g)$ is given by the following equation, the counterpart of equation (5) in He and Xiong (2012) for the Leland and Toft (1996) model with a constant volatility.

$$\begin{aligned} &rD(V,g) = C + gP - gD(V,g) - \xi k_1 D(V,g) + (r-q)VD_V + \frac{1}{2}(\sigma(V))^2 V^2 D_{VV} \\ &\Rightarrow \left(r+g+\xi k_1\right) D(V,g) = C + gP + (r-q)VD_V + \frac{1}{2}(\sigma(V))^2 V^2 D_{VV} \end{aligned} \tag{2.6}$$

The solution of (2.6) is available in closed form if we know the functions $A(t)$ and $B_r(V,K)$. It is given by the following result, proven in the appendix.

Lemma 1: *The value* $D(V,g)$ *of the debt in the Leland (1994b) model is equal to*

$$D(V,g)=\frac{C+gP}{r+g+\xi k_1}+[(1-\alpha)K-\frac{C+gP}{r+g+\xi k_1}]B_{r+g+\xi k_1}(V,K) \quad (2.7)$$

(2.7) can also be written in the following more intuitive form

$$D\left(V,g\right)=\int_0^\infty e^{-(r+g+\xi k_1)t}\left(1-A(t)\right)\left[C+gP\right]dt+(1-\alpha)KB_{r+g+\xi k_1}(V,K)$$

The first part of above equation is the cash flow till default, while the last one is the payoff upon default. From these results we can now provide closed form solutions for the debt under both fixed volatility[57] and CEV.

Proposition 1: *Under constant asset volatility the debt is given by the following expression*

$$D(V,g)=\frac{C+gP}{r+g+\xi k_1}+[(1-\alpha)K-\frac{C+gP}{r+g+\xi k_1}]\left(\frac{V}{K}\right)^{-z}$$
$$z=\frac{\left(r-\delta-0.5\sigma^2\right)+\left[\left(r-\delta-0.5\sigma^2\right)^2+2\left(r+g+\xi k_1\right)\sigma^2\right]^{\frac{1}{2}}}{\sigma^2} \quad (2.8)$$

Under the CEV model the the debt is given by

$$D(V,g)=\frac{C+gP}{r+g+\xi k_1}+[(1-\alpha)K-\frac{C+gP}{r+g+\xi k_1}]\frac{\phi_{r+g+\xi k_1}(V)}{\phi_{r+g+\xi k_1}(K)} \quad (2.9)$$

With

[57] Hereafter we set $\sigma(V)=\sigma$ for all fixed volatility cases.

$$\phi_r(V) = \begin{cases} V^{\beta+\frac{1}{2}} e^{\frac{\epsilon}{2}x} W_{k,m}(x), \beta < 0, r \neq 0 \\ V^{\beta+\frac{1}{2}} e^{\frac{\epsilon}{2}x} M_{k,m}(x), \beta > 0, r \neq 0 \end{cases} \tag{2.10}$$

Where,

$$x = \frac{|r-q|}{\theta^2 |\beta|} V^{-2\beta}, \epsilon = sign\left((r-q)\beta\right), m = \frac{1}{4|\beta|}$$

$$k = \epsilon\left(\frac{1}{2} + \frac{1}{4\beta}\right) - \frac{r}{2\,|\,(r-q)\beta\,|}$$

$W_{k,m}(x)$ and $M_{k,m}(x)$ are the Whittaker functions.

Proof: (2.8) is a well-known result for the first passage time of a constant volatility diffusion process. See, for instance, Leland (1994b, p. 12) or Ingersoll (1987,p. 372). (2.9) is an auxiliary general result (*Proposition 1*) in the derivation of barrier options under state-dependent volatility underlying asset dynamics of Davydov and Linetsky (2001), while (2.10) is a similar result stated in *Proposition 5* of that same paper, QED.

The Whittaker functions $W_{k,m}(x)$ and $M_{k,m}(x)$ in (2.10) are the fundamental solutions for the Whittaker equation and are available in the *Matlab* (or *Mathematica)* software; they are described in more detail in the appendix.[58] Since the sign and value of β affect the probability of default by increasing (decreasing) the volatility in "bad" states when $\beta < 0$ ($\beta > 0$), the shape of $\phi_r(V)$ is also strongly affected by that parameter. It is a monotonic decreasing (increasing) function with respect to asset value V when

[58] See Whittaker and Watson (1990, pp. 339-351).

$\beta < 0$ $(\beta > 0)$. In addition, the slope of the function increases with the absolute value of β.[59]

3. Equity Valuation, Default Boundary and Leverage

3.1 Equity valuation under simple and CEV diffusion models

Let $E(V,g)$ denote the value of the equity and w denote the corporate tax rate. Omitting the arguments in the partial derivatives the value $E(V,g)$ in the presence of rollover risk and for a general state-dependent volatility $\sigma(V)$ satisfies the following equation, similar to equation (11) of He and Xiong (2012)

$$rE(V,g) = \left(r-q\right)VE_V + \frac{1}{2}\left(\sigma(V)\right)^2 V^2 E_{VV} + qV - \left(1-w\right)C + gD(V,g) - gP \tag{2.11}$$

In (2.11) the first two terms reflect the change in equity value because of the dynamic change in the unlevered firm's assets, the third and fourth terms are cash inflows from dividends and the after tax cost of the debt coupon, while the last two terms represent the change in equity value by the debt issuance cost absorption, with debt retired at face value but refinanced at market value. This absorption is affected by liquidity cost through the market value of debt. The boundary conditions are $E\left(K,g\right)=0,$ for all g, and $E(V,g)$ increases linearly with respect to V when $V \to \infty$.

The following general result, proven in the appendix, gives the value of $E(V,g)$ for both fixed and state-dependent volatility diffusions. This value is expressed in terms of the Unit Price (UP) function defined in Chapter 1 if this UP function is known analytically, as it is for both cases covered in this essay. In the appendix we also present

[59] The relevant figures are available from the authors on request.

the Laplace transform method proposed by He and Xiong (2012) to solve equation (2.11) when the volatility is constant and find exactly the same solution for the equity value (2.12). Unfortunately, this method cannot be used directly under CEV asset dynamics.

Proposition 2: *Under both constant and state-dependent asset volatility the equity value is given by the following expression*

$$E(V,g)=V+\frac{wC}{r}\left[1-B_r(V,K)\right]-\alpha K B_r(V,K)-\left[A_0+A_1B_{r+g+\xi k_1}(V,K)\right]$$
$$-\frac{\xi k_1}{g+\xi k_1}\left[\frac{C+gP}{r}+\left((1-\alpha)K-\frac{C+gP}{r}\right)B_r(V,K)-A_0-A_1B_{r+g+\xi k_1}(V,K)\right] \quad (2.12)$$

In (2.12), $B_{r+g+\xi k_1}(V,K)$ *is given by the same expressions as in* (2.8) *and* (2.9) *for the constant volatility and CEV cases respectively, while*

$$B_r(V,K)=\left(\frac{V}{K}\right)^{-y},\ y=\frac{\left(r-\delta-0.5\sigma^2\right)+\left[\left(r-\delta-0.5\sigma^2\right)^2+2r\sigma^2\right]^{\frac{1}{2}}}{\sigma^2} \quad (2.13)$$

for fixed volatility, and $B_r(V,K)=\frac{\phi_r(V)}{\phi_r(K)}$ *for CEV. The constant terms are as in (2.7):*

$$A_0=\frac{C+gP}{r+g+\xi k_1},\ A_1=[(1-\alpha)K-\frac{C+gP}{r+g+\xi k_1}]$$

It can be readily verified by using the terminal conditions $B_r(K,K)=B_{r+g+\xi k_1}(K,K)=1$ that $E\left(K,g\right)=0$. For $V\to\infty$ we have $B_r(V,K)$ and $B_{r+g+\xi k_1}(V,K)\to 0$, yielding

$$\lim_{V\to\infty}E(V,g)=V+\frac{wC}{r}-\frac{C+gP}{r+g+\xi k_1}\left(1+\frac{\xi k_1}{r}\right)$$

For $\xi k_1 = 0$ and constant volatility expression (2.12) coincides with equation (17) of Leland (1994b). A direct comparison of the limit of this last equation for $V \to \infty$ with the corresponding limit of (2.12) shows that at the limit the liquidity costs reduce the limiting value of the equity by an amount equal to $(C+gP)\frac{\xi k_1}{r+g+\xi k_1}\frac{g}{r+g}$, implying that the effect of liquidity costs varies with g, the rollover rate and the inverse of the average debt maturity.

A more intuitive understanding of the role of liquidity risk in the debt and equity values can be obtained by rewriting equation (2.12). Define three type of bonds, denoted by D_0, D_1, D_2, where D_0 is the value of a perpetual bond with coupon payment $C_0 = C + gP$, D_1 is the total value of perpetual bonds which are continuously issued and retired at a proportional rate g with coupon payment $C_1 = C$ and face value of total outstanding debt $P_1 = P$ in the absence of liquidity risk, and D_2 is the total value of bonds which are similar to D_1 but in the presence of liquidity risk. Then we have,

$$
\begin{aligned}
D_0 &= \frac{C+gP}{r} + \left((1-\alpha)K - \frac{C+gP}{r}\right)B_r(V,K) \\
D_1 &= \frac{C+gP}{r+g} + \left((1-\alpha)K - \frac{C+gP}{r+g}\right)B_{r+g}(V,K) \\
D_2 &= \frac{C+gP}{r+g+\xi k_1} + \left((1-\alpha)K - \frac{C+gP}{r+g+\xi k_1}\right)B_{r+g+\xi k_1}(V,K)
\end{aligned}
\tag{2.14}
$$

From (2.7) it can be easily seen that $D_0 > D_1 > D_2$; observe also that each $D_i, \forall\, i = 0,1,2$ can also be written under the form of (2.7). Substituting into equation (2.12), the equity value can be expressed by,

$$E(V,g)=V+\frac{wC}{r}\left[1-B_r(V,K)\right]-\alpha KB_r(V,K)-D_2-\frac{\xi k_1}{g+\xi k_1}\left[D_0-D_2\right]$$
$$=E_1-\left[D_2-D_1+\frac{\xi k_1}{g+\xi k_1}\left[D_0-D_2\right]\right] \qquad (2.15)$$

Where $E_1=V+\frac{wC}{r}\left[1-B_r(V,K)\right]-\alpha KB_r(V,K)-D_1$ is the equity value with D_1 in the absence of the liquidity risk given the default boundary K. Let

$$LC_E=\left[D_2-D_1+\frac{\xi k_1}{g+\xi k_1}\left[D_0-D_2\right]\right]$$

$$LC_D=D_1-D_2$$

If the default boundary K is exogenously determined and does not change in the presence of the liquidity risk then LC_E and LC_D represent the liquidity costs absorbed by the equity holders and debt holders, respectively. Observe that the losses of equity are always positive, since[60]

$$LC_E=\frac{\xi k_1}{g+\xi k_1}\left[D_0-D_2\right]-LC_D>0$$

provided the average maturity of total outstanding debts is finite ($g>0$). Specifically, for the debt with infinite maturity ($g=0$), the liquidity cost becomes zero since the rollover channel is closed.

Note that the total liquidity cost to both debt and equity holders can also be written as,

[60] This inequality can be proven very simply by noting from (2.7) that $D(V,g)$ is strictly convex with respect to the discount rate $r+g+\xi k_1$, which obviously implies $\frac{g}{g+\xi k_1}D_2+\frac{\xi k_1}{g+\xi k_1}D_0>D_1$.

$$LC_D + LC_E = \frac{\xi k_1}{g+\xi k_1}\underbrace{\left[D_1 - D_2\right]}_{LiquidityEffect} + \frac{\xi k_1}{g+\xi k_1}\underbrace{\left[D_0 - D_1\right]}_{RolloverEffect} \tag{2.16}$$

Where $\left[D_1 - D_2\right]$ represents the "Liquidity Effect", the decrease of debt value because of the presence of liquidity shocks, and $\left[D_0 - D_1\right]$ is the "Rollover Effect", the decrease of debt value because of the presence of the retirement. The total liquidity cost is the sum of the liquidity and rollover effects weighted by $\frac{\xi k_1}{g+\xi k_1}$. Further, as the liquidity risk of bond markets affects the value of equity through rollover channel, the liquidity cost to equity can also be decomposed into the weighted difference of rollover and liquidity effects as follows,

$$LC_E = \frac{\xi k_1}{g+\xi k_1}\underbrace{\left[D_0 - D_1\right]}_{RolloverEffect} - \frac{g}{g+\xi k_1}\underbrace{\left[D_1 - D_2\right]}_{LiquidityEffect} = \frac{\xi k_1}{g+\xi k_1} D_0 + \frac{g}{g+\xi k_1} D_2 - D_1 \tag{2.17}$$

In the second form of (2.17) the strict convexity of the function $D(V,g)$ implies that the equity losses increase with the rollover cost parameters ξk_1, just like the debt losses. Compared to the constant asset volatility, the state dependent volatility affects D_0, D_1 and D_2 through the probability density function of the first-passage default time. Thus, the total liquidity costs of equity are affected by the state dependent volatility through both the liquidity effect and rollover effect channels.

As already noted, the constant volatility case solution of (2.12) can also be derived by the Laplace transform technique used by He and Xiong (2012).[61] The derivation

[61] See appendix.

presented here, in addition to being computationally much simpler, is also more intuitive economically.

3.2 The endogenous default boundary

A firm's default trigger can be determined exogenously or endogenously. Generally, the exogenous default triggers such as the zero-net worth trigger[62] and the zero cash flow trigger[63] are not determined by the equity holders but rather by the creditors. Under the zero-net worth trigger assumption the default occurs when the net worth of the firm becomes negative for the first time, which implies that the default trigger value equals the total face value of the outstanding debt, namely $K = P$. However, we often observe that firms are still alive even though their net worth is negative in the financial markets. Thus, in order to improve the simple zero net worth trigger, Moody's KMV defines as trigger value $K = P_{Short} + 0.5 * P_{Long}$. Under zero cash-flow trigger, a firm claims default when the current net cash flow to the security holders cannot meet the current coupon payments, which implies $K = C / \delta$, where δ is the net cash flow to the security holders. The problem with this trigger value is that sometimes the equity value is still positive even though the current net cash flow is zero. In this case, a firm will prefer to issue more equity so as to meet the current coupon payment, instead of announcing default.

On the other hand, the endogenous default trigger determined by the equity holders is the optimal default boundary which can maximize the total asset value. The equity holders may receive the anticipated equity value and service the coupon payments and rolling costs even when the firm is insolvent. The equity holders will lower the default

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[62] See Brennan & Schwartz (1978), and Longstaff & Schwartz (1995).
[63] See Kim, Ramaswamy, &Sundaresan (1993).

boundary to keep the firm alive if the anticipated equity value is no less than the anticipated value of debt service. Even though the net worth of the firm may be negative, the equity holders may still be able to raise funds to service the debt cash flow by issuing new equity.

Following Leland and Toft (1996), and Chen and Kou (2009), we determine endogenously the default boundary K, from the smooth pasting condition[64] $\frac{\partial E}{\partial V_{V|V=K}} = 0$.

Differentiating (2.12) directly, we get

$$\begin{aligned} \frac{\partial E}{\partial V_{V|V=K}} &= 1 - \frac{wC}{r} B_r(V,g)_{V|V=K} - \alpha K B_r(V,g)_{V|V=K} - A_1 B_{r+g+\xi k_1}(V,g)_{V|V=K} \\ &- \frac{\xi k_1}{g+\xi k_1}\left[\left((1-\alpha)K - \frac{C+gP}{r}\right) B_r(V,g)_{V|V=K} - A_1 B_{r+g+\xi k_1}(V,g)_{V|V=K}\right] = 0 \end{aligned} \tag{2.18}$$

The solution of this equation depends on the asset dynamics. For both cases it is given by the following result, proven in the appendix.

Proposition 3: *Under constant asset volatility the default boundary is given by the following expression*

$$K = \frac{\frac{C+gP}{r+g+\xi k_1} z - \frac{wCy}{r} + \frac{(C+gP)\xi k_1}{g+\xi k_1}\left(\frac{y}{r} - \frac{z}{r+g+\xi k_1}\right)}{1 + \alpha y + (1-\alpha) z + \frac{\xi k_1 (1-\alpha)(y-z)}{g+\xi k_1}} \tag{2.19}$$

Under the CEV model there is no closed form expression for the default boundary. The solution of (2.18) is found numerically, by replacing the expressions for the derivatives of $B_r(V,K)$ and $B_{r+g+\xi k_1}(V,K)$ from the quantities given in the appendix.

[64] See Chen and Kou (2009).

From (2.19) it is clear that for $\xi k_1 = 0$ our boundary coincides with expression (19) of Leland (1994b). We also prove in the appendix the following result, the counterpart of the He and Xiong *Proposition 2*, with essentially the same proof as the one used in that paper.

***Proposition** 4: Under both constant and state dependent asset volatility an increase in the liquidity premium* ξk_1 *decreases the debt value and increases the default boundary* K.

The impact of the liquidity premium on the default boundary is illustrated in numerical results in the next section. In the CEV case it depends on the sign of the coefficient β, with the average debt maturity (the inverse of the parameter g by (2.4)), also playing a role in the relationship between fixed and state dependent volatilities.

From equation (2.19) it is also possible to derive the impact of g on the default boundary, which increases with g under some restrictions on the parameter values, as in He and Xiong (2012). Unfortunately no similar closed form results exist for the CEV case, due to the complexity of the Whittaker functions in (2.18). For this reason this average maturity effect is examined numerically in the next section.

3.3 Optimal leverage

At $t = 0$ the firm chooses its initial leverage by choosing the debt parameters, the coupon C and the principal P. Let V_0 denote the initial asset value and $v(C, P, V_0)$ the total value of the firm. To focus on the effects of liquidity risk on leverage we assume that debt is issued at par, in which case $P = D(V_0, C, P; K(C,P))$; this equality defines an implicit relation $P(C, V_0)$, which we substitute into the equation of the value of the firm

$$v(C,P,V_0) = E(V_0,C,P;K(C,P) + D(V_0,C,P;K(C,P)) \qquad (2.20)$$

(2.19) can now be maximized with respect to the optimal coupon C^* in order to find the optimal leverage. Setting $P^* = P(C^*, V_0)$, we measured the optimal leverage by the ratio $\dfrac{D(V_0, C^*, P^*; K(C^*, P^*))}{v(C^*, P^*, V_0)}$. The numerical results are shown in the next section.

4. Model Calibration and Numerical Results

In this section we present simulation results for the variables of interest, based on base case parameter values extracted from earlier empirical studies.[65] For the empirical work in the next Chapter we calibrate the model to individual firm data. Here, we set the riskless and payout rates $r = 8\%$ and $q = 2\%$, the tax and bankruptcy cost rates $w = 27\%$ and $\alpha = 40\%$, and the bond market illiquidity parameters at $\xi = 1$ and $k_1 = 1\%$. We also normalize the initial asset value at $V_0 = 100\$$. For the asset value volatility we set the initial volatility at $\sigma_0 = 23\%$ for both constant volatility and the CEV model, and we adjust the CEV parameter $\theta = \sigma_0 V_0^{-\beta}$ accordingly. We present results for $\beta = 1$, $\beta = 0$ (constant volatility) and $\beta = -1$, with intermediate values yielding similar results. Last, we use three values of the debt rollover parameter g, corresponding by (2.4) to average

[65] For instance Bao, Pan and Wang (2011), Chen *et al* (2007), He and Xiong (2012) and Zhang, Zhou and Zhu (2009).

maturities $T=1$, $T=5$, and $T=10$. The face value of total outstanding debt in the base case is $P=50$.

4.1 Debt and equity values and endogenous default boundary

Figure II-1 presents the endogenous default boundary for various values of the illiquidity parameter ξ ranging from 0 to 2, two benchmark maturities of $T=1$ and $T=10$, and two exogenously given leverages, with $P=50$ and $P=70$. The coupon rates have been chosen to make the debt issued at par for the benchmark case of constant volatility at 23% and $\xi=0$.

[Insert Figure II-1 about here]

The figure illustrates clearly the importance of the state dependence of the volatility, whose contribution to the rise or fall of the default boundary can be at least as large as that of the liquidity shock. The sign of the elasticity determines, in all but the lowest values of ξ, the relationship of the CEV boundary relative to the fixed volatility case, with a positive (negative) sign corresponding to a higher (lower) boundary at equal ξ 's. Not surprisingly, leverage increases the boundary at equal maturities in all cases, while the boundary increases with ξ and is uniformly higher for equal leverage for the shorter maturity, as in He and Xiong (2012, Figure 3).

These observed results can be understood from the relationship between the anticipated equity value appreciation and the anticipated debt costs to keep the firm alive, including both the rollover costs and the continuous coupon payment. For both constant and state dependent volatility cases, the presence of liquidity shocks in the debt market reduces the value of anticipated equity appreciation through the rollover channel while the coupon payments are the same. Hence, the default boundary is higher compared to that in the absence of liquidity shocks. In contrast to the constant volatility, the state dependent volatility, in this case the CEV, changes the first passage default probability density function, which has a direct impact on the market values of debt and equity. Specifically, a negative β, reflecting a negative relationship between asset value and asset volatility, increases the value of anticipated equity appreciation when the asset value decreases since the equity can be considered as an option on the firm's asset and, thus, rises in value with the volatility. The impact on the debt value is more complicated and depends on the debt structure, including maturity, coupon policy and firm leverage ratio. However, given the calibration in the base case, we note that the impact of state dependent volatility on the value of anticipated equity appreciation dominates that on the debt costs. Hence, we observe significantly lower endogenous default boundaries when β is negative compared to constant volatility. Note that we assume that the liquidity cost is proportional to the market value of debt. Hence, the magnitude of the endogenous default boundary increases as the intensity of liquidity shocks increases depending on the debt structure, including maturity, face value, coupon payment, etc., as well as the value of volatility elasticity.

Given the endogenous default boundaries under different volatility assumptions, Figure II-2 and Figure II-3 present the rollover losses for debt and equity respectively for $P = 50$ and $P = 70$ as functions of the parameter ξ ranging from 0 to 2, for two different average maturity values and for an endogenous default boundary. The coupon payment was chosen to make the debt issue at par in the absence of liquidity shocks when the volatility is constant. The results are presented as differences from the respective values

D_o and E_o for debt and equity in the absence of liquidity shocks, $D(\xi) - D_o$ and $E(\xi) - E_o$.

[Insert Figure II-2 about here]

[Insert Figure II-3 about here]

As the figures show, the effect of the state dependent volatility on the rollover losses is significant in many cases but its size and direction differ depending on the maturity of the debt and the sign of the elasticity parameter. In general, a negative (positive) relationship between the asset value and asset volatility results in relatively smaller (greater) losses of debt value compared to those of constant volatility. For debt the volatility effect is weak for low face value in all cases and moderately significant for high face value for both maturities. It is much more pronounced for equity in all cases, and it is especially strong for positive elasticity, high face value, and the shortest maturity of $T = 1$, where the loss of equity value from the volatility effect is approximately 25% lower in magnitude than the loss of value of the constant volatility when the liquidity parameter is equal to 2.

Of particular interest is the effect of compression or expansion of the optimal default boundary in the state dependent volatility cases. We study it by equalizing the default boundaries in all three elasticity parameter cases. From Figure II-1 it follows that the boundary is forced up (down) in most cases for negative (positive) elasticity, making default easier (harder). The consequences of this appear most clearly in the next set of figures, Figure II-4 and Figure II-5, showing the quantities $D(\xi) - D_o$ and $E(\xi) - E_o$

with an exogenous default boundary set equal to the endogenous boundary of the constant volatility case for each ξ.

[Insert Figure II-4 about here]

[Insert Figure II-5 about here]

A comparison of Figure II-2 with Figure II-4 shows clearly that equalizing the default boundary in all three elasticity cases also brings the corresponding debt values closer together than in the endogenous boundary cases. In equation (2.7) the raising (lowering) of the boundary K results in an increase (decrease) in the default probability $A(t)$ and in the term $B_{r+g+\xi k_1}(V,K)$, increasing (decreasing) the second term and decreasing (increasing) the first term. Hence, the optimal choice of the boundary may result in higher or lower rollover losses for debt, depending on maturity and face value. In the negative elasticity case the differences between Figure II-2 and Figure II-4 are very small in all cases, except for the case $T=1$ and $P=70$, where raising the boundary increases the rollover debt losses. In the positive elasticity case, on the other hand, lowering the boundary in Figure II-3 yields lower rollover losses for both maturities when $P=70$, and has an insignificant effect on the losses in the other two cases.

The situation is somewhat different in the comparison of the equity rollover losses in Figure II-3 and Figure II-5, since the losses are by definition lower in all Figure II-3 cases than the corresponding cases of Figure II-5. Most affected by the suboptimal choice of the boundary is again the positive elasticity case when $T=1$ and $P=70$, where the losses now exceed those of the constant volatility case, implying a rise in losses of more than 25% by the decrease in the default boundary. In all the other cases the boundary changes bring relatively small changes in equity losses. There is also some evidence of small, but measurable agency effects in the case $T = 10$ and $P = 70$, since the sum of the losses of equity and debt holders is higher for the optimally chosen rather than the fixed boundary, given that the former was chosen to represent the interests only of the equity holders.[66]

4.2 Optimal capital structure

Table II-1 presents various cases of the choice of the optimal initial leverage by maximizing the total firm value given in equation (2.20). As noted in Section 3.3, the coupon was first selected in order to make the debt issued at par, and then chosen optimally by maximizing (2.20). The table shows the optimal coupon, leverage, debt and equity values, as well as the endogenous default boundary for three values of ξ and three maturities, $T = 1$, $T = 5$ and $T = 10$.

[Insert Table II-1 about here]

The table illustrates several effects already expected from the earlier results of He and Xiong (2012). Specifically, the optimal leverage decreases as the rollover costs increase and increases with maturity in all cases. The strong negative volatility effect on leverage found in the earlier study becomes even more important when the volatility is state dependent, and its impact tends at times to swamp the maturity and even the rollover parameter effects. For instance, the leverage of the negative elasticity case with

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[66] Observe from (3.6) that the total value loss because of the liquidity and rollover effects is strictly dependent on the parameter set $(r, g, \xi k_1)$ and not on parameters such as the payout ratio δ. Hence, the endogenous choice of the default boundary does not necessarily maximize the value of the firm. See also the comments in Leland (1998, pp. 1224-1226) and Chen and Kou (2009, p. 353).

maximum rollover risk ($\xi = 2$) and $T = 5$ is higher than that of the positive elasticity case for $\xi = 0$ and $T = 10$.

In Table II-1 a negative elasticity always results in a higher leverage and a positive elasticity mostly in a lower leverage, with the exception of the low maturity and high rollover cost parameters. We also saw from Figure II-1 that at equal leverages the negative (positive) elasticity is also associated with a lower (higher) endogenous default boundary. In Table II-1, by contrast, the endogenous default boundary at optimal leverage varies inversely with the elasticity for all but the highest value of ξ, where the dependence reverses sign, or even results in a lowest boundary for the $\beta = 0$ case. We conclude that the volatility effects are predominant when it comes to the determination of the key endogenous variables of the firm, except for the extremes of the maturity and rollover cost ranges.

The strong volatility effect on leverage appears even more clearly in Figure II-6, which plots the optimal leverage as a function of the bond trading proportional cost parameter k_1 that varies from 0 to 150 basis points, corresponding to a change in the rollover cost parameter ξ from 1 to 1.5 if k_1 is kept equal to its base case value of 100 since the two appear always together. Here we vary the initial volatility and set it at two alternative values, $\sigma_0 = 20\%$ and $\sigma_0 = 25\%$, bracketing our base case of $\sigma_0 = 23\%$.

[Insert Figure II-6 about here]

This more detailed result confirms the conclusions derived from Table II-1. At low maturity and high trading costs the constant volatility case achieves the lowest optimal leverage, while at higher maturity it always lies between the two other cases. Further, a higher initial volatility results in a lower leverage in constant volatility and positive elasticity cases everywhere, while for negative elasticity leverage may perversely increase with initial volatility in the low maturity and low to moderate trading cost range. Note also that at the higher maturity the optimal leverage for both state dependent volatility cases is much less sensitive to trading costs than the constant volatility case.

4.3 Rollover cost and credit spreads

Figure II-7 plots the credit spreads $\frac{C+gP}{D}-(g+r)$ of the newly issued debt against the rollover risk parameter ξ for two different leverages and debt maturities. In all cases the spread increases almost linearly with ξ, but the slopes differ according to debt maturity, elasticity of the volatility and (especially) leverage. It is highest for all three elasticity cases in the high leverage, low maturity panel, in which its slope varies from approximately 150 to 250 basis points as ξ rises from 0 to 2. All three slopes are approximately equal to 100 basis points for both maturities when leverage is low.

[Insert Figure II-7 about here]

When the leverage is chosen optimally the change in ξ will affect the optimal leverage as well as the credit spread, in which case the change in spread will then include a default component, as well as a rollover cost component. The results in Table II-2 illustrate the contributions of these two components to the change in spread in a number of cases, covering two bond maturities, three different values of ξ, and two starting volatilities $\sigma_0 = 21\%$ and $\sigma_0 = 23\%$, coupled with corresponding bond trading proportional cost parameters $k_1 = 50$ and $k_1 = 100$ basis points respectively. The lower volatility and cost correspond to A-rated and the lower to B-rated bonds.

[Insert Table II-2 about here]

In the Table II-2 entries as ξ increases the default portion is defined as the amount $\left(\Delta Spread - \Delta\xi k_1\right)$. It is clear from the corresponding entries that the value and the share of the default portion in the total spread are increasing functions of the elasticity in all but one case. The default share also varies inversely with the maturity of the debt in all cases, even though the differences are proportionately smaller at the higher values of ξ. Last, we note the strong starting volatility effect on credit spread changes, which implies that for all maturities and elasticity values the low volatility A-rated bonds respond much less to a given change in ξ than the B-rated bonds. As He and Xiong (2012) note, this is a manifestation of the well-known flight to quality phenomenon associated with severe disruptions in the financial markets, in which the prices of lower rated bonds decrease much more than those of higher rated ones.

5. Conclusion

In this Chapter, we re-examine the impact of liquidity shocks of bond market on the credit risk through the rollover channel under the exponential stationary debt structure. We develop an innovative derivation methodology to obtain the analytical solution for the equity value in the presence of rollover risk. This approach provides a much intuitive expression for the equity value. Further, this methodology can be easily applied into CEV and stochastic volatility asset dynamic process provided the expression for the first passage CDP and unit price are available.

Taking advantage of the analytical expression of first passage CDP and unit prices derived in Chapter I, we analyze the impact of rollover risk on endogenous default boundary, credit risk and optimal capital structure under CEV asset dynamics. We show theoretically and for all elasticity values that increases in the rollover risk parameters reduce the values of both debt and equity. In the numerical estimations with simulated data we find that the sign and magnitude of the elasticity parameter are major determinants of the model's results, on their own or in their interaction with other features of the model like leverage and debt maturity. As with constant volatility, the endogenous default boundary increases with the size of the rollover risk parameter, while a positive (negative) elasticity raises (lowers) the endogenous default boundary for all cases. Similarly, the elasticity parameter emerges as a major determinant of the optimal leverage in all cases, with the size of its effect depending on the maturity of the debt.

Appendix

The following result, whose proof is obvious and will be omitted, will be used in several of the proofs of this appendix.

Lmma A.1

Let the risk neutral dynamic equation of an underlying asset with state-dependent volatility diffusion be

$$\frac{dV}{V} = \mu dt + \sigma(V) dW \tag{A.1}$$

Then the current value of a derivative asset that pays off \$1 when the underlying asset becomes equal to K *for the first time and 0 otherwise is given by* $B_r(V,K)$ *in* (2.5)*, with limiting values* $\lim_{V\to\infty} B_r(V,K) = 0$ *and* $B_r(K,K) = 1,$ *which satisfies the equation*

$$\frac{1}{2}(\sigma(V))^2 V^2 F_{VV} + \mu V F_V - rF = 0 \tag{A.2}$$

Proof of Lemma 1

Replacing from (2.7) into (2.6) and taking into account that $B_{r+g+\xi k_1}(V,K)$ satisfies (A.2) with $r+g+\xi k_1$ instead of r and $r-q$ instead of μ, and setting the constant term and coefficient in (2.7) A_0 and A_1 respectively, we get

$$
\begin{aligned}
&\left(r+g+\xi k_1\right)[A_0 + A_1 B_{r+g+\xi k_1}(V,K)] \\
&= C + gP + A_1[(r-q)VB_{r+g+\xi k_1}(V,K)_V + \frac{1}{2}(\sigma(V))^2 V^2 B_{r+g+\xi k_1}(V,K)_{VV}]
\end{aligned}
\tag{A.3}
$$

(A.3) obviously holds by the definitions of A_0 and A_1 and the terminal conditions of $B_{r+g+\xi k_1}(V,K)$ and the debt value, QED.

The Whittaker functions

The Whittaker functions are related to the confluent hypergeometric functions as follows

$$
\begin{aligned}
M_{k,m}\left(x\right) &= e^{-\frac{x}{2}} x^{m+\frac{1}{2}} {}_1F_1\left(\frac{1}{2}+m-k, 1+2m, x\right) \\
W_{k,m}\left(x\right) &= e^{-\frac{x}{2}} x^{m+\frac{1}{2}} U\left(\frac{1}{2}+m-k, 1+2m, x\right)
\end{aligned}
\tag{A.4}
$$

where ${}_1F_1\left(a,b,z\right)$ and $U\left(a,b,z\right)$ are, respectively, the first kind and second kind confluent hypergeometric functions. Mathematically, they can be expressed as follows, with $\Gamma(.)$ denoting the gamma function.

$$
{}_1F_1\left(a,b,z\right) = 1 + \frac{a}{b}z + \frac{a\left(a+1\right)}{b\left(b+1\right)}\frac{z^2}{2!} + \cdots = \sum_{k=0}^{\infty} \frac{\left(a\right)_k}{\left(b\right)_k}\frac{z^k}{k!}
\tag{A.5}
$$

Thus, the second kind confluent hypergeometric function can be expressed as,

$$U(a,b,z)=\frac{\Gamma(1-b)\,{}_1F_1(a,b,z)}{\Gamma(a-b+1)}+\frac{\Gamma(b-1)z^{1-b}\,{}_1F_1(a-b+1,2-b,z)}{\Gamma(a)}$$
$$=\frac{1}{\Gamma(a-b+1)}\left[\Gamma(1-b)\,{}_1F_1(a,b,z)+\frac{\Gamma(a-b+1)\Gamma(b-1)z^{1-b}\,{}_1F_1(a-b+1,2-b,z)}{\Gamma(a)}\right]$$
$$=\frac{1}{\Gamma(a-b+1)}\left[\Gamma(1-b)\,{}_1F_1(a,b,z)+beta(a-b+1,b-1)\,{}_1F_1(a-b+1,2-b,z)\right]$$

Replacing into the expression for $B_r(V,K)$, we get

$$B_r(V,K)=\frac{\phi_r(V)}{\phi_r(K)}=\begin{cases}\dfrac{VU\left(\frac{1}{2}+m-k,1+2m,x_1\right)}{KU\left(\frac{1}{2}+m-k,1+2m,x_2\right)}\exp[x_2-x_1],\beta<0,r-q\neq 0\\[2ex] \dfrac{{}_1F_1\left(\frac{1}{2}+m-k,1+2m,x_1\right)}{{}_1F_1\left(\frac{1}{2}+m-k,1+2m,x_2\right)},\beta>0,r-q\neq 0\end{cases}\tag{A.6}$$

Where,

$$x_1=\frac{|r-q|}{\theta^2|\beta|}V^{-2\beta},x_2=\frac{|r-q|}{\theta^2|\beta|}K^{-2\beta}$$

Proof of Proposition 2

From (2.12) we have, with subscripts denoting partial derivatives,

$$\begin{aligned}E_V&=1-\frac{wC}{r}B_r(V,g)_V-\alpha KB_r(V,g)_V-A_1B_{r+g+\xi k_1}(V,g)_V-\\ &\frac{\xi k_1}{g+\xi k_1}\left[\left((1-\alpha)K-\frac{C+gP}{r}\right)B_r(V,g)_V-A_1B_{r+g+\xi k_1}(V,g)_V\right]\end{aligned}\tag{A.7}$$

$$E_{VV} = -\frac{wC}{r} B_r(V,g)_{VV} - \alpha K B_r(V,g)_{VV} - A_1 B_{r+g+\xi k_1}(V,g)_{VV} - \frac{\xi k_1}{g+\xi k_1}\left[\left((1-\alpha)K - \frac{C+gP}{r}\right) B_r(V,g)_{VV} - A_1 B_{r+g+\xi k_1}(V,g)_{VV}\right] \tag{A.8}$$

The rest of the proof follows the same steps as in Lemma 1: replacing (2.12), (A.7) and (A.8) into (2.11) and collecting terms we find that the equation holds, since $B_r(V,K)$ satisfies (A.2) with $\mu = r - q$ and $B_{r+g+\xi k_1}(V,K)$ similarly satisfies (A.3) as in the proof of Lemma 1, QED.

Proof of Proposition 3

We use the definitions of $B_r(V,K)$ and $B_{r+g+\xi k_1}(V,K)$ from (2.12) and (2.7)-(2.8) respectively. (2.15) follows then directly from (2.14) by replacing and collecting terms, QED. Observe that in (2.15) the result is the same as expression (19) in Leland (1994b) if we set the rollover term ξk_1 equal to 0.

For the state dependent case, we concentrate on the case $\beta < 0$, with $\beta > 0$ treated as an extension. In such a case $\frac{1}{2} - k + m > 0$ and the Whittaker function in (2.10) is given by

$$W_{k,m}(x) = \frac{e^{-\frac{x}{2}} x^k}{\Gamma(\frac{1}{2} - k + m)} \int_0^\infty t^{-k-\frac{1}{2}+m} (1+\frac{t}{x})^{k-\frac{1}{2}+m} e^{-t} dt \tag{A.9}$$

We note from (2.10) that $\phi_r(V)$ and $\phi_{r+g+\xi k_1}(V)$ differ only as to their key parameter k, which has the respective values k_r as in (2.10) and $k_\xi = \epsilon\left(\frac{1}{2}+\frac{1}{4\beta}\right)-\frac{r+g+\xi k_1}{2|(r-q)\beta|}$.

Further, set from (2.10) the expression $x=\frac{|r-q|}{\theta^2|\beta|}V^{-2\beta}\equiv\psi V^{-2\beta}$, $\frac{dx}{dV}=-2\beta\psi V^{-(2\beta+1)}$.

Then we have:

$$B_r(V,g)_{V|V=K}=\frac{dB_r(V,g)}{dV_{V|V=K}}=\frac{1}{\varphi_r(K)}\frac{d\varphi_r(V)}{dV_{V|V=K}}=$$
$$\frac{\beta+\frac{1}{2}}{K}+2\beta\psi K^{-(2\beta+1)}-\frac{2\beta k_r}{K}-2\beta\psi K^{-(2\beta+1)}\Psi \tag{A.10}$$

Where we define $\Psi=-\dfrac{(k_r-\frac{1}{2}+m)\int_0^\infty t^{-k_r+\frac{1}{2}+m}(1+\frac{t}{x_K})^{k_r-\frac{3}{2}+m}e^{-t}dt}{x_K^2\int_0^\infty t^{-k_r-\frac{1}{2}+m}(1+\frac{t}{x_K})^{k_r-\frac{1}{2}+m}e^{-t}dt}$, $x_K=\psi K^{-2\beta}$.

$B_{r+g+\xi k_1}(V,K)_{V|V=K}$ is then found from the same expression, by replacing k_r by k_ξ. A similar computation takes place for the case $\beta>0$.

The terms $B_r(V,K)_V$ and $B_{r+g+\xi k_1}(V,K)_V$, instead of (A.9)-(A.10), can also be derived from (A.7)-(A.8), by using the following expressions

$$\frac{\partial\,{}_1F_1(a,b,z)}{\partial z}=\frac{a}{b}\,{}_1F_1(a+1,b+1,z)$$

$$\frac{\partial U(a,b,z)}{\partial z}=-aU(a+1,b+1,z).$$

These yield for $\beta>0$

$$B_r(V,K)_{V|V=K} = -\frac{\frac{1}{2}+m-k_r}{1+2m}\frac{{}_1F_1\left(\frac{3}{2}+m-k_r,2+2m,x_2\right)}{{}_1F_1\left(\frac{1}{2}+m-k_r,1+2m,x_2\right)}\frac{\partial x_2}{\partial K} =$$

$$-\frac{{}_1F_1\left(\frac{3}{2}+m-k_r,2+2m,x_2\right)}{{}_1F_1\left(\frac{1}{2}+m-k_r,1+2m,x_2\right)}\frac{2r}{\theta^2\left(2\beta+1\right)}K^{-2\beta-1}$$

For $\beta < 0$ we have

$$\begin{aligned} B_r(V,K)_{V|V=K} &= \frac{1}{K}+\frac{2\left(r-q\right)}{\theta^2}\exp\left[\frac{\left(r-q\right)K^{-2\beta-1}}{\theta^2\beta}\left(1-K\right)\right] \\ &+\frac{\frac{1}{2}+m-k_r}{1+2m}\frac{U\left(\frac{3}{2}+m-k_r,2+2m,x_2\right)}{U\left(\frac{1}{2}+m-k_r,1+2m,x_2\right)}\frac{\partial x_2}{\partial K} \\ &= \frac{1}{K}+\frac{2\left(r-q\right)}{\theta^2}\exp\left[\frac{\left(r-q\right)K^{-2\beta-1}}{\theta^2\beta}\left(1-K\right)\right] \\ &+\frac{U\left(\frac{3}{2}+m-k_r,2+2m,x_2\right)}{U\left(\frac{1}{2}+m-k_r,1+2m,x_2\right)}\frac{r-2\left(r-q\right)\beta}{\beta\theta^2}K^{-2\beta-1} \end{aligned} \tag{A.11}$$

Similar results also hold for $B_{r+g+\xi k_1}(V,K)_{V|V=K}$.

<u>*Proof of Proposition 4*</u>

For $D(V,g)$ the result follows immediately from (2.7), since both terms in the right-hand-side are decreasing functions of ξk_1, QED. For the equity, we note from (2.11) that we can write it as an expectation at time $0, \mathrm{E}_0$, of its discounted cash flows along the path of the value V till the default time t, writing explicitly the dependence of equity and debt on the important parameters.

$$E(V,K;\xi) = \mathrm{E}_0\left[\int_0^t e^{-rS}(qV_s - (1-w)C + g[D(V_s,K;\xi) - P])ds\right] \quad \text{(A.12)}$$

It is clear from (A.12) that for a fixed default boundary K the equity value decreases with ξk_1, given that the debt decreases.

Consider now two different values $\xi_1 < \xi_2$ and let K_1 and K_2 denote the corresponding endogenous default boundaries. Assume that $K_1 \geq K_2$. By definition, $E(K_1,K_1;\xi_1) = E(K_2,K_2;\xi_2) = 0$. Since the default boundaries were chosen optimally, we have $0 = E(K_1,K_1;\xi_1) > E(K_1,K_2;\xi_1)$. On the other hand, since the equity decreases as ξ increases, $E(K_1,K_2;\xi_1) > E(K_1,K_2;\xi_2) \Rightarrow E(K_1,K_2;\xi_2) < 0$. This is, however, not compatible with the definition of equity, which is non-negative whenever the starting asset value exceeds the default boundary, implying $E(K_1,K_2;\xi_2) \geq 0$ whenever $K_1 \geq K_2$. Thus, the latter assumption is false, and $K_1 < K_2$, QED.

Laplace Transform Method for Equity Value with Constant Volatility

The Laplace transform method can be used to solve equation (2.11) only when the volatility is constant. Under constant volatility, equation (2.11) can be written as,

$$rE = (r-q)VE_V + \frac{1}{2}\sigma^2 V^2 E_{VV} + qV - (1-w)C + gD(V) - gP, \quad \text{(A.13)}$$

with the boundary condition, $E\left(K\right)=0$ and $E_V\left(K\right)=l$. The equity value is linear in asset value when it approaches the default boundary *K*.

Define $m=\ln\left(\frac{V}{K}\right)$, and substitute into the above differential equation. We then have,

$$rE=\left(r-q-\frac{1}{2}\sigma^2\right)E_m+\frac{1}{2}\sigma^2E_{mm}+qKe^m-\left(1-w\right)C+gD(m)-gP, \quad \text{(A.14)}$$

with the boundary conditions, $E(0)=0$ and $E_M(0)=l$.

Define the Laplace transform of $E(M)$ as

$$F(s)\equiv L\left[E(m)\right]=\int_0^\infty e^{-sm}E\left(m\right)dm \quad \text{(A.15)}$$

Then, applying the Laplace transform to both sides of (A.14), we have

$$rF(s)=\left(r-q-\frac{1}{2}\sigma^2\right)L\left[E_m\right]+\frac{1}{2}\sigma^2L\left[E_{mm}\right]+\frac{qK}{s-1}-\frac{\left(1-w\right)C+gP}{s}+gL\left[D(m)\right] \quad \text{(A.16)}$$

Note that,

$$\begin{aligned} L\left[E_m\right]&=sF\left(s\right)-E(0)=sF\left(s\right)\\ L\left[E_{mm}\right]&=s^2F\left(s\right)-sE(0)-E_m(0)=s^2F\left(s\right)-l \end{aligned} \quad \text{(A.17)}$$

Thus, we have

$$\left[r-\left(r-q-\frac{1}{2}\sigma^2\right)s-\frac{1}{2}\sigma^2s^2\right]F(s)=\frac{qK}{s-1}-\frac{\left(1-w\right)C+gP}{s}+gL\left[D(m)\right]-\frac{1}{2}\sigma^2l \quad \text{(A.18)}$$

Define $\eta > 0$ and $-\gamma < 0$ to be the two roots of the following equation with respect to s,

$$r - \left(r - q - \frac{1}{2}\sigma^2\right)s - \frac{1}{2}\sigma^2 s^2 = 0 \tag{A.19}$$

Then we have, $-\frac{1}{2}\sigma^2(s-\eta)(s+\gamma) = 0$, $\eta = z - a > 1$ and $\gamma = a + z > 0$, where

$a = \dfrac{r - q - \frac{1}{2}\sigma^2}{\sigma^2}$ and $z \equiv \dfrac{\left(a^2\sigma^4 + 2r\sigma^2\right)^{1/2}}{\sigma^2}$.

Then,

$$\begin{aligned}\frac{1}{2}\sigma^2 F(s) &= -\frac{1}{(s-\eta)(s+\gamma)}\left\{\frac{qK}{s-1} - \frac{(1-w)C + gP}{s} + gL\left[D(m)\right] - \frac{1}{2}\sigma^2 l\right\} \\ &= -\frac{\frac{1}{s-\eta} - \frac{1}{s+\gamma}}{\eta+\gamma}\left\{\frac{qK}{s-1} - \frac{(1-w)C + gP}{s} + gL\left[D(m)\right] - \frac{1}{2}\sigma^2 l\right\}\end{aligned} \tag{A.20}$$

As we have seen, $D(V,g) = A_0 + A_1 B_{r+g+\xi k_1}(V,K)$, where

$$A_0 = \frac{C + gP - \xi k_0}{r + g + \xi k_1},\ A_1 = [(1-\alpha)K - \frac{C + gP - \xi k_0}{r + g + \xi k_1}],$$

$$B_{r+g+\xi k_1}(V,K) = \int_0^\infty e^{-(r+g+\xi k_1)t} f(t,V,K)dt$$

Under constant volatility assumption, we have,

$$B_{r+g+\xi k_1}(V,K) = \left(\frac{V}{K}\right)^{-y},\ y \equiv \frac{\left(r - q - 0.5\sigma^2\right) + \left[\left(r - q - 0.5\sigma^2\right)^2 + 2\left(r + g + \xi k_1\right)\sigma^2\right]^{\frac{1}{2}}}{\sigma^2}$$

Therefore,

$$\frac{1}{2}\sigma^2 F(s) = -\frac{\frac{1}{s-\eta}-\frac{1}{s+\gamma}}{\eta+\gamma}\left\{\frac{qK}{s-1}-\frac{(1-w)C+gP}{s}+\frac{gA_0}{s}+\frac{gA_1}{s+y}-\frac{1}{2}\sigma^2 l\right\} \tag{A.21}$$

The Laplace inverse can be derived as,

$$\begin{aligned}
E(m) &= \int e^{sm}F(s)ds \\
&= \frac{2}{\sigma^2}\left[\frac{\sigma^2}{2}V-\frac{qK}{\eta+\gamma}\left(\frac{1}{\eta-1}e^{\eta m}+\frac{1}{\gamma+1}e^{-\gamma m}\right)\right] \\
&+\frac{2}{\sigma^2}\left[\frac{(1-w)C+gP-gA_0}{\eta+\gamma}\left(\frac{1}{\eta}\left(e^{\eta m}-1\right)-\frac{1}{\gamma}\left(1-e^{-\gamma m}\right)\right)\right] \\
&+\frac{2}{\sigma^2}\left[\frac{\sigma^2 l}{2(\eta+\gamma)}\left(e^{\eta m}-e^{-\gamma m}\right)\right] \\
&-\frac{2}{\sigma^2}\left[\frac{gA_1}{\eta+\gamma}\left(\frac{1}{\eta+y}\left(e^{\eta m}-e^{-ym}\right)-\frac{1}{\gamma-y}\left(e^{-ym}-e^{-\gamma m}\right)\right)\right]
\end{aligned} \tag{A.22}$$

Note that, $\frac{gA_1}{\eta+\gamma}\left(\frac{-e^{-ym}}{\eta+y}-\frac{e^{-ym}}{\gamma-y}\right)=-\frac{gA_1e^{-ym}}{(\eta+y)(\gamma-y)}=\frac{\sigma^2 gA_1e^{-ym}}{2(g+\xi k_1)}$. Simplifying and rearranging, the equity value has the exactly same expression as (2.12), QED.

Figure II-1: Endogenous Default Boundaries

This figure depicts the endogenous default boundaries in the presence of liquidity shocks for the different asset volatility assumption, including constant volatility ($\beta = 0$) and CEV processes ($\beta = -1, \beta = 1$). T is the average maturity of total outstanding debts. P is the face value of total outstanding debts. It is assumed endogenous default and the coupon payment makes the debt issue at par in the absence of liquidity shocks with constant asset volatility. The rest of calibrations are same as the base case. To make the picture look more condense, we shift the endogenous default boundary of $\beta = -1$ up 20 units in Panel B and D, and 5 units in Panel A.

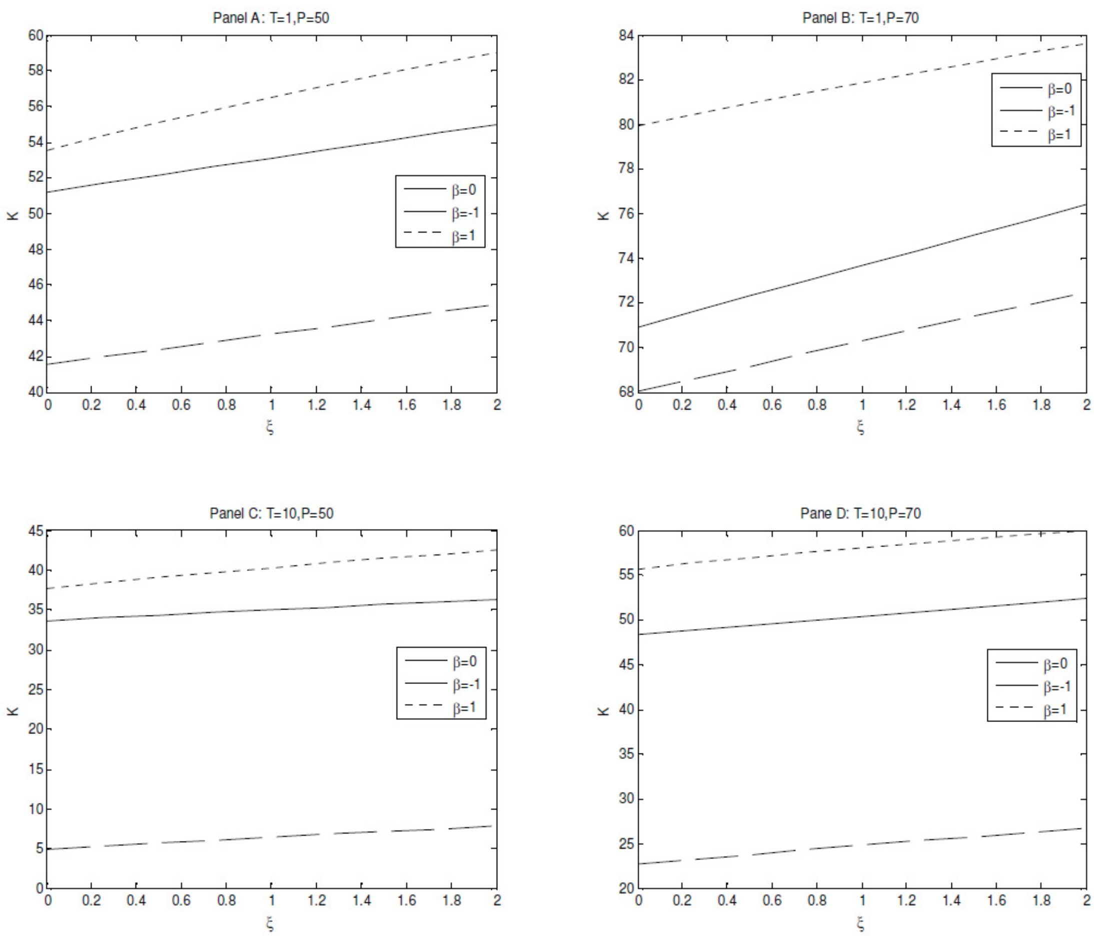

Figure II-2: Rollover Loss of Debt Holder with Endogenous Default Boundary

This figure depicts the rollover loss of debt with endogenous default boundary. The coupon payment is set to make the bond be issued at par in the absence of liquidity shocks with constant volatility. The rollover losses of debts holders equal $D(\xi)-P$, where $D(\xi)$ and P denote the value of total outstanding debt in the presence and in the absence of liquidity shocks, respectively.

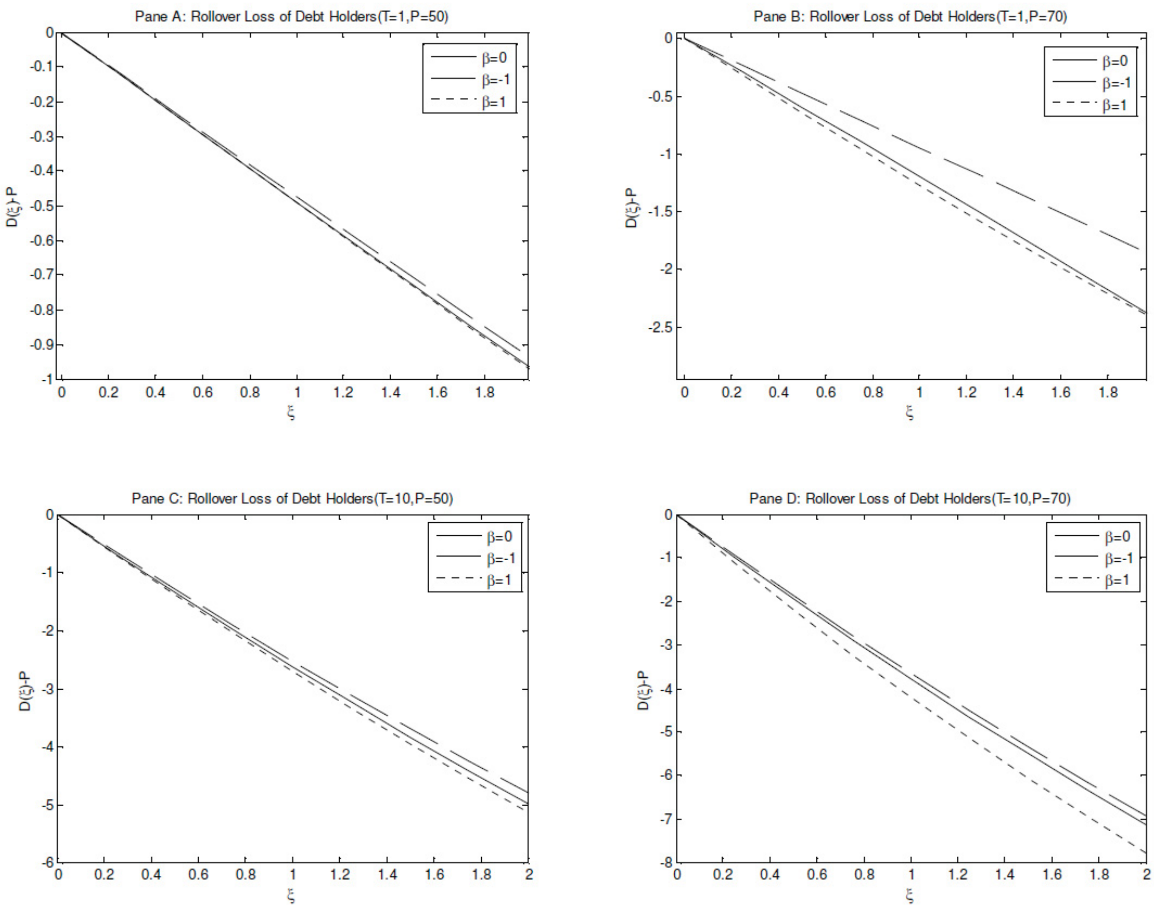

Figure II-3: Rollover Loss of Equity Holders with Endogenous Default Boundary

This figure depicts the rollover loss of equity with endogenous default boundary. The coupon payment is set to make the bond be issued at par in the absence of liquidity shocks with constant volatility. The rollover losses of equity holders equal $E(\xi)-E_o$, where $E(\xi)$ and E_o denote the value of total equity in the presence and in the absence of liquidity shocks, respectively.

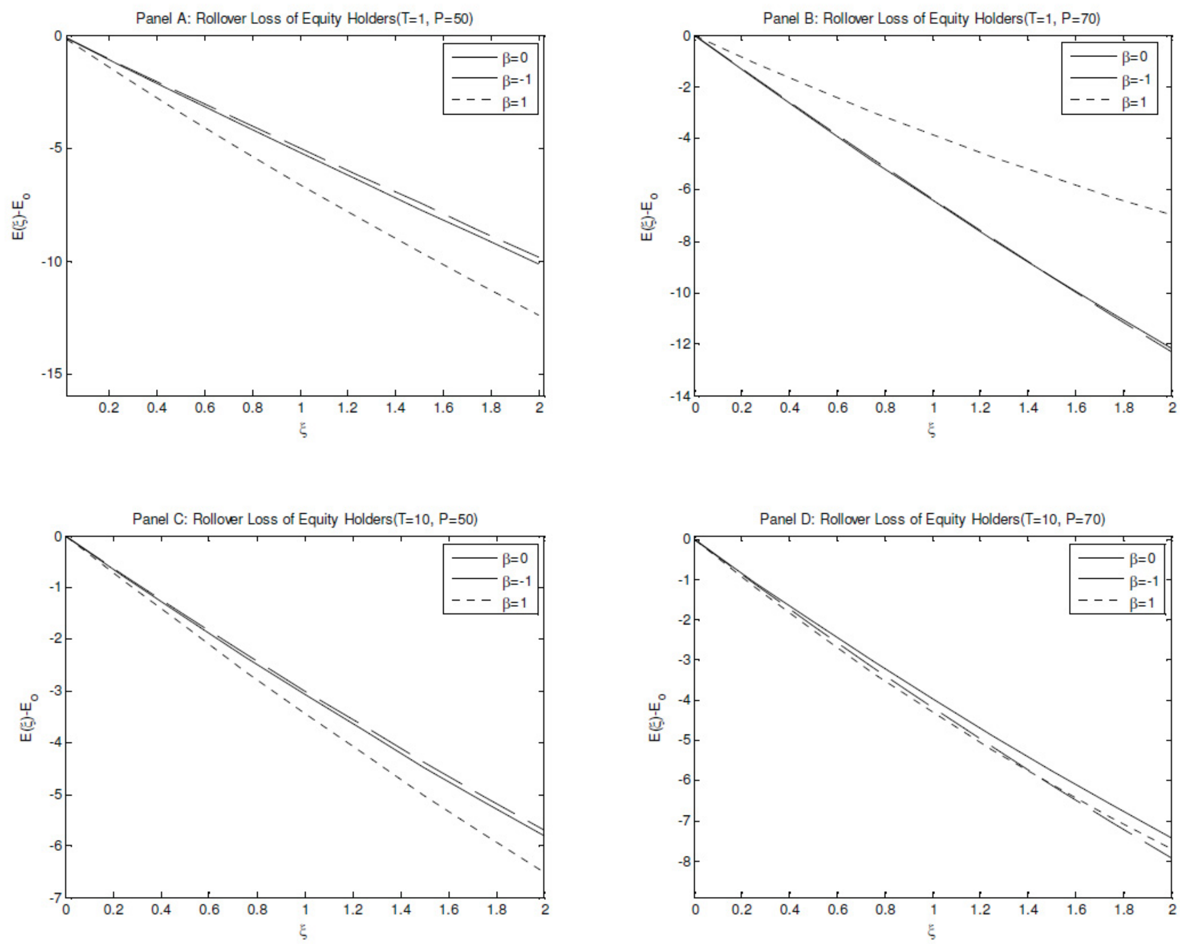

Figure II-4: Rollover Loss of Debts with Exogenous Default Boundary

This figure depicts the rollover loss of debt with exogenous default boundary. The coupon payment is set to make the bond be issued at par in the absence of liquidity shocks with constant volatility. The rollover losses of debts holders equal $D(\xi)-P$, where $D(\xi)$ and P denote the value of total outstanding debt in the presence and in the absence of liquidity shocks, respectively.

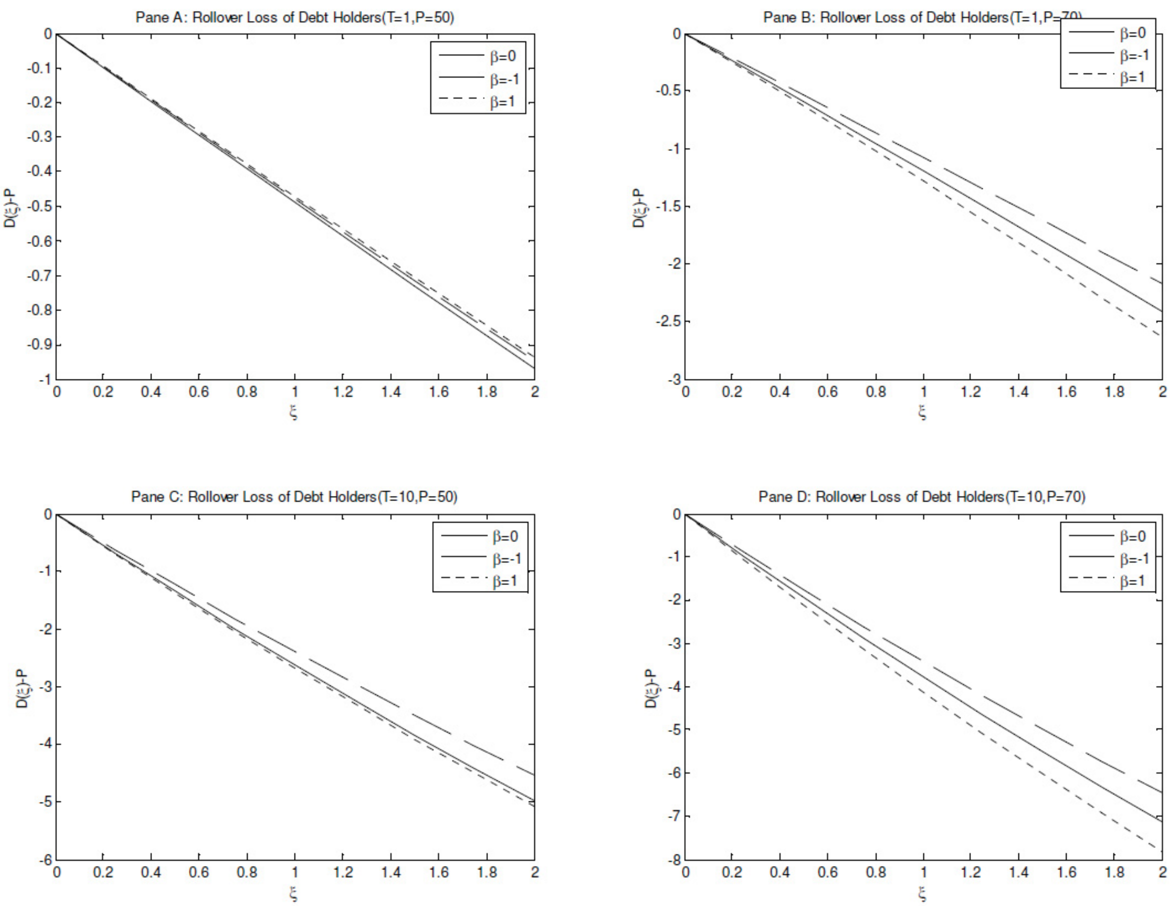

Figure II-5: Rollover Loss of Equity Holders with Exogenous Default Boundary

This figure depicts the rollover loss of equity with endogenous default boundary. The coupon payment is set to make the bond be issued at par in the absence of liquidity shocks with constant volatility. The rollover losses of equity holders equal $E(\xi)-E_o$, where $E(\xi)$ and E_o denote the value of total equity in the presence and in the absence of liquidity shocks, respectively.

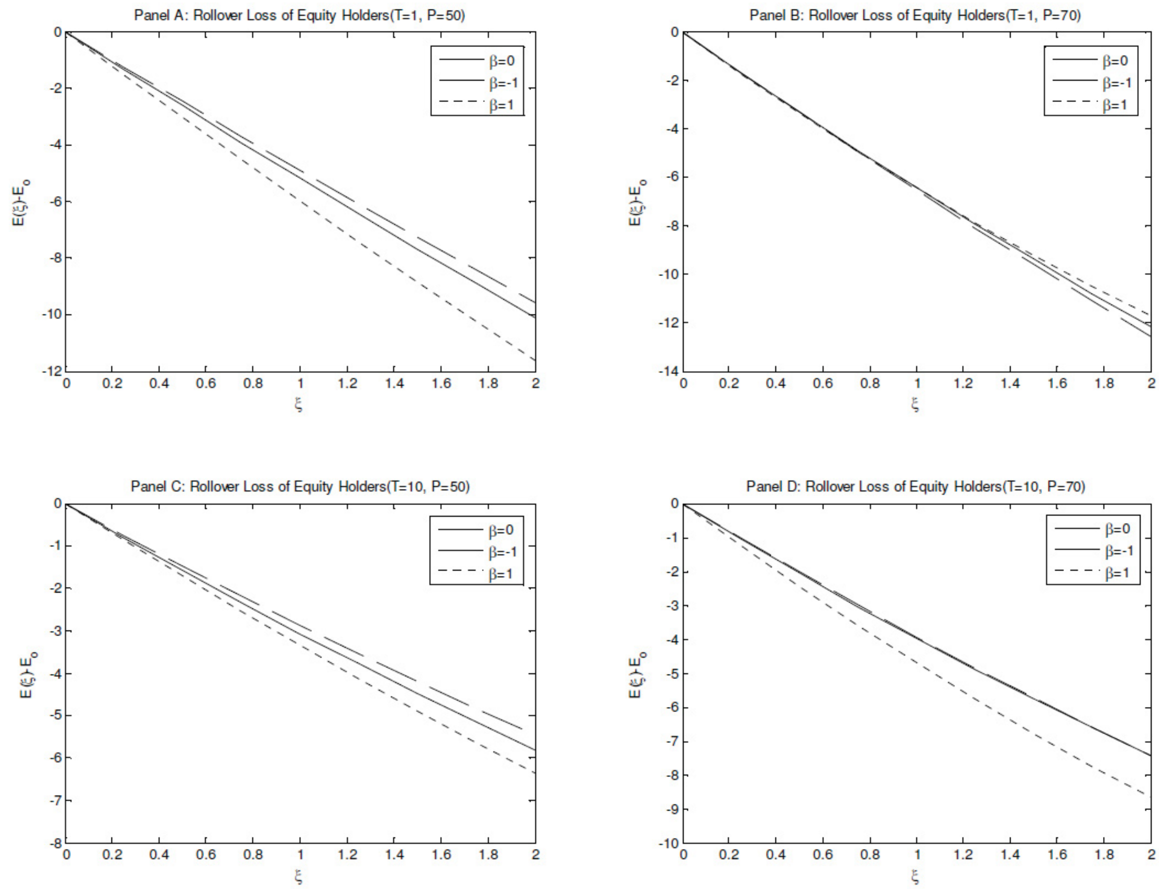

Table II-1: Characteristics of the Optimally Levered Firms

This table depicts the characteristics of the optimally levered firms. T is the average maturity of the total outstanding debts. C is the optimal coupon payment which made the debt issued at par. *Lev*, Δ *Lev*, *DV*, *EV* and *K* denote the optimal leverage, percentage of leverage compared to the case $\xi = 0$, debt value, equity value and endogenous default boundary, respectively. The rest of the calibrations are same as the base case.

Panel A: T=1

	$\xi = 0$					$\xi = 1$						$\xi = 2$					
	C	*Lev*	*DV*	*EV*	*K*	*C*	*Lev*	Δ *Lev*	*DV*	*EV*	*K*	*C*	*Lev*	Δ *Lev*	*DV*	*EV*	*K*
$\beta = -1$	6.04	55.42%	60.99	49.06	55.91	5.04	48.37%	-12.72%	51.08	54.53	44.68	3.72	35.47%	-35.99%	36.17	65.81	26.40
$\beta = 0$	3.43	39.17%	42.49	65.68	43.34	3.24	34.28%	-12.48%	35.83	68.68	38.00	2.72	26.71%	-31.81%	27.16	74.53	29.78
$\beta = 1$	3.41	38.45%	42.59	68.17	43.14	3.57	37.15%	-3.38%	39.61	67.01	41.99	3.60	34.99%	-8.99%	36.02	66.93	39.80

Panel B: T=5

	$\xi = 0$					$\xi = 1$						$\xi = 2$					
	C	*Lev*	*DV*	*EV*	*K*	*C*	*Lev*	Δ *Lev*	*DV*	*EV*	*K*	*C*	*Lev*	Δ *Lev*	*DV*	*EV*	*K*
$\beta = -1$	9.98	75.91%	90.74	28.80	57.11	9.51	74.25%	-2.18%	82.97	28.78	53.06	8.38	68.30%	-10.03%	71.75	33.30	44.18
$\beta = 0$	6.35	60.56%	68.35	44.52	52.50	5.60	53.66%	-11.39%	57.49	49.65	46.67	4.40	41.79%	-30.99%	42.88	59.73	36.68
$\beta = 1$	4.48	48.73%	55.42	58.32	45.24	4.51	46.13%	-5.3%	49.91	58.27	43.99	4.38	42.21%	-13.38%	43.70	59.84	41.40

Panel C: T=10

	$\xi = 0$					$\xi = 1$						$\xi = 2$					
	C	*Lev*	*DV*	*EV*	*K*	*C*	*Lev*	Δ *Lev*	*DV*	*EV*	*K*	*C*	*Lev*	Δ *Lev*	*DV*	*EV*	*K*
$\beta = -1$	10.57	82.62%	101.43	21.34	53.53	10.10	80.50%	-2.57%	91.43	22.14	49.38	9.03	74.11%	-10.3%	78.47	27.42	40.82
$\beta = 0$	6.89	64.50%	74.03	40.75	51.63	6.25	58.44%	-9.4%	63.25	44.99	47.01	5.11	47.63%	-26.16%	49.10	53.98	38.62
$\beta = 1$	4.98	53.05%	60.98	53.97	46.43	4.94	49.87%	-5.9%	54.26	54.54	45.12	4.71	45.16%	-14.87%	46.86	56.90	42.37

Figure II-6: Optimal Capital Structure

This figure depicts the relationship between firm's optimal capital structure and the bond proportional trading cost for the different scenarios. The calibrations, except the average debt maturity and initial volatilities, are the same as base case.

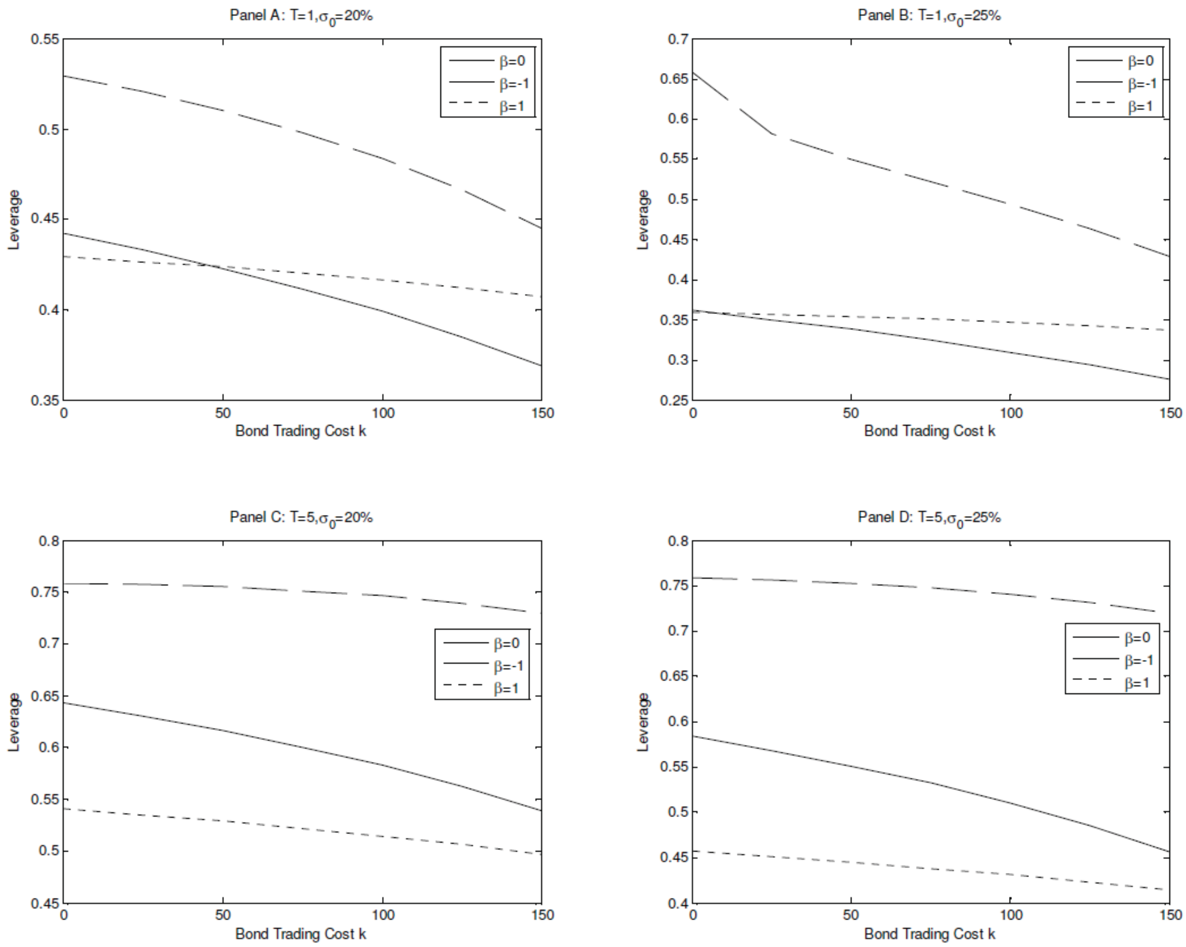

Figure II-7: Effect of liquidity demand intensity on credit spreads

This figure depicts the effect of liquidity demand intensity on the newly issued corporate debt's credit spreads. *T* is the average maturity of the total outstanding debts and *P* is the face value of total outstanding debts. It is assumed endogenous default and the coupon payment makes the debt issue at par in the absence of liquidity shocks with constant asset volatility. The rest of calibrations are same as the base case.

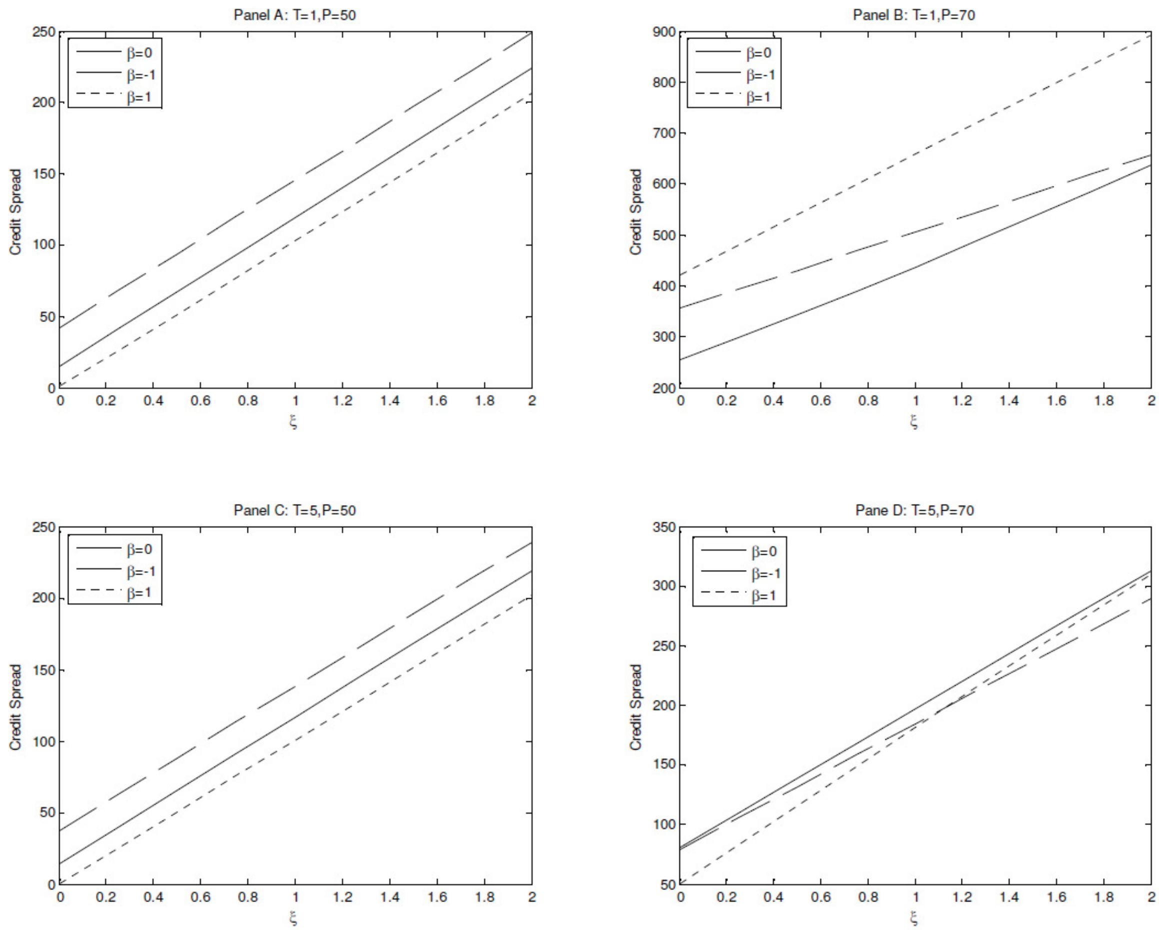

Table II-2: Response of Credit Spreads

This table reports the responses of different firm's credit spread to a liquidity shock and state dependent volatilities. It is assumed risk free interest rate $r = 8\%$, tax rate $w = 27\%$, proportional bankruptcy cost $\alpha = 40\%$, firm's payout rate $q = 2\%$, and the current asset value $V = \$100$. For A-rated firms, the initial volatility $\sigma_0 = 21\%$, proportional liquidity cost $k_1 = 50$ basis points (*bps*). For BB-rated firm, the initial volatility $\sigma_0 = 23\%$, proportional liquidity cost $k_1 = 100$ basis points. We find the coupon payment and face value of debt such that its newly issued par bonds with the specified maturity have an initial credit spread of 100 bps for A-rated firms and 330 bps for BB-rated firms with constant volatility.

Panel A (A-Rated, T=1)									
	$\xi = 1$	$\xi = 2$				$\xi = 4$			
Model	Spread (bps)	Spread (bps)	Δ Spread (bps)	Default part (bps)	Default part (Percent)	Spread (bps)	Δ Spread (bps)	Default part (bps)	Default part (Percent)
$\beta = -1$	136.07	189.53	53.46	3.46	6.47%	296.48	160.41	60.41	37.66%
$\beta = 0$	100.00	155.88	55.88	5.88	10.52%	268.70	168.7	68.7	40.72%
$\beta = 1$	69.86	126.86	57	7	12.28%	242.83	172.97	72.97	42.19%
Panel B (A-Rated, T=5)									
	$\xi = 1$	$\xi = 2$				$\xi = 4$			
Model	Spread (bps)	Spread (bps)	Δ Spread (bps)	Default part (bps)	Default part (Percent)	Spread (bps)	Δ Spread (bps)	Default part (bps)	Default part (Percent)
$\beta = -1$	114.85	166.17	51.32	1.32	2.57%	268.59	153.74	53.74	34.96%
$\beta = 0$	100.00	153.65	53.65	3.65	6.80%	260.60	160.6	60.6	37.73%
$\beta = 1$	63.52	118.99	55.47	5.47	9.86%	230.67	167.15	67.15	40.17%
Panel C (BB-Rated, T=1)									
	$\xi = 1$	$\xi = 2$				$\xi = 4$			
Model	Spread (bps)	Spread (bps)	Δ Spread (bps)	Default part (bps)	Default part (Percent)	Spread (bps)	Δ Spread (bps)	Default part (bps)	Default part (Percent)
$\beta = -1$	350.88	474.40	123.52	23.52	19.04%	722.73	371.85	171.85	46.21%
$\beta = 0$	330.00	475.05	145.05	45.05	31.06%	773.58	443.58	243.58	54.91%
$\beta = 1$	392.77	551.10	158.33	58.33	36.84%	850.20	457.43	257.43	56.28%
Panel D (BB-Rated, T=5)									
	$\xi = 1$	$\xi = 2$				$\xi = 4$			
Model	Spread (bps)	Spread (bps)	Δ Spread (bps)	Default part (bps)	Default part (Percent)	Spread (bps)	Δ Spread (bps)	Default part (bps)	Default part (Percent)
$\beta = -1$	261.80	372.58	110.78	10.78	9.73%	592.12	330.32	130.32	39.45%
$\beta = 0$	330.00	459.05	129.05	29.05	22.51%	709.44	379.44	179.44	47.29%
$\beta = 1$	410.10	544.79	134.69	34.69	25.76%	784.02	373.92	173.92	46.51%

Chapter III EMPIRICAL EVIDENCE: CEV AND CONSTANT VOLATILITY

In this chapter we compare the empirical performance of the two alternative volatility assumptions that we used in our study within the context of the Leland (1994b) model. Using time series data at both the macro and firm levels, respectively, we use two different estimation methods for the competing models' parameters for each firm (each rating class) in our sample. Given the estimates, we then compare the out-of-sample performance of the two models with respect to the endogenously generated variables of each model when compared to data that was not used for the parameter estimation.

In the first estimation we use the historical record of observed default probabilities for the bonds of each maturity and risk class in order to extract the volatility parameters of the two models. This is an area in which the constant volatility assumption does not perform very well, especially for short maturities in all risk classes, as shown in Leland (2004). This estimation is, however, flawed, insofar as it forces every firm in the sample to follow the observed default probabilities of the "average" firm of the corresponding rating class. Thus, although the CEV model outperforms the constant volatility in better approximating the historical record, the extracted values of the elasticity of variance are not considered reliable and are not consistent with the values extracted from the other two methods.

The second estimation is based on the observed leverage ratio, equity value and equity volatilities, extracted from either the intraday historical volatilities or from the implied volatilities as observed in the option market. Here the estimated parameters are extracted from firm-specific data and vary widely across the firms in our sample. We find that the constant volatility and CEV structural models have similar performances when using only these three variables for the parameter estimation.

Last, we estimate the parameters by fitting the two competing models to all the available historical data for our sample of firms. This data includes not only the equity volatilities but also the credit default swap (CDS) data for five different maturities. The parameters are extracted using the Generalized Method of Moments (GMM) method. We find that the CEV structural model, with its extra parameter, exhibits a much better in-sample fitting in the CDS spreads across all the maturities. In additional, we find a negative relationship between the value of β and the firm specific measures of default risk, such as leverage ratios, CDS spreads and current ratio etc. Last, we compare the estimates of the average cumulative default probabilities across all the bonds of a similar maturity and rating to the comparable historical record of bonds of the same maturity and risk class. In this out-of-sample comparison we find that the CEV model average probabilities are much closer to the historical record than the constant volatility model.

1. Empirical Evidence from Moody's Historical Cumulative Default Probabilities

1.1 Moody's historical default data

In this section we evaluate the capacity of the CEV structure to approximate available bond risk structure historical data. Such data can be under the form of corporate bond prices or yields, as in the empirical studies of Anderson and Sundaresan (2000) and

Eom, Helwege and Huang (2004), or default probabilities as in Leland (2004). The advantage of focusing on default probabilities rather than bond prices is that the cumulative default probabilities (CDP) are not affected by additional factors, such as illiquidity, state tax, etc. The physical asset dynamics of the underlying asset value are the main driver of the cumulative default probability. By contrast, the risk neutral process is the one that enters into corporate bond pricing. Although the two processes yield different results, the volatilities should be exactly the same under both risk-neutral and physical distributions for the diffusion and jump-diffusion models with diversifiable jump risk.[67]

[Insert Figure III-1 about Here]

The data for the cumulative default probability (CDP) of bonds of various risk classes and maturities is given in Moody's.[68] Figure III-1 exhibits the CDPs for Aa, A, Baa and Ba rated corporate bonds for the periods 1983-2008 and 1983-2010. Since the sub-prime financial crisis starts in 2008, these two data sets show the CDPs pre and post crisis. We only consider these four middle ratings because the default probabilities are rather low for Aaa bonds, while bond ratings lower than Ba are too risky. As the figure shows, the CDP term structure after the sub-prime financial crisis shifts upward and becomes much steeper relative to that before the crisis.

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[67] When the jump risk is systematic the market is incomplete, the physical and risk neutral jump process parameters are different, there are infinitely many possible transformations corresponding to the same physical distribution, and total volatility may be affected. For an analysis of this case see Oancea and Perrakis (2010).

[68] Moody's "Corporate Default and Recovery Rates 1920-2008", and "Corporate Default and Recovery Rates 1920-2010".

1.2 Term structure of implied volatilities of historical CDPs under the L and LT models

In order to examine the consistency of the L and LT models with the above data we examine the volatilities implied by the CDPs under the assumption that the CDP data represents an "average" firm for each rating class with characteristics corresponding to the averages reported by Moody's. Under both L and LT it is assumed that a firm's value follows a diffusion process with constant volatility under the risk-neutral distribution, as in equation (2.2) with a constant σ and jump intensity $\eta^Q = 0$. Then the cumulative default probability till time T is,[69]

$$F(T;\sigma) = N(h_1(T)) + \left(\frac{V}{K}\right)^{-2a} N(h_2(T))$$
$$a = \frac{r-q-0.5\sigma^2}{\sigma^2}, b = ln\left(\frac{V}{K}\right), h_1(T) = \frac{-b-a\sigma^2 T}{\sigma\sqrt{T}}, h_2(t) = \frac{-b+a\sigma^2 T}{\sigma\sqrt{T}} \quad (1.1)$$

The corresponding CDP under the physical distribution is found by replacing $r-q$ by the drift $\mu - q$ of the physical distribution. Thus given an observed CDP for a particular maturity, risk free rate, payout rate, current asset value and exogenous default boundary, *K,* the volatility σ_{im}^D implied by the simple diffusion is given by,

$$\sigma_{im}^D(T) = F^{-1}(T;b,\mu,q) \quad (1.2)$$

[Insert Table III-1 about Here]

We show in Table III-1 the information from Moody's used to find the implied volatility (IV) in (1.2). The risk premium, risk free rate, payout rate and average leverage for Aa, A, Baa and Ba rated corporate bonds are shown in Panel A, with the average

[69] See equation (4) in LT or equation (21) in L.

leverage ratio found from Moody's special comment in 2006.[70] The average leverage ratio increases when the corporate debt rating decreases. We use a constant payout rate 6%, risk free rate 8% and tax rate 35% for all the corporate bonds. In addition, we assume that the current asset values are the same and equal to 100 and that the risk premium is 4%[71] implying a 12% rate of return of underlying asset value for all the bonds. Leland (2004) shows that an exogenous or endogenous default boundary fits the observed default probabilities equally well provided default costs and recovery rates are matched. In this study exogenous default boundaries which equal the value of debt for the "average" firm of each rating are used to calculate the IVs. The debt value is set according to the historical average leverage ratio for each rating.

[Insert Figure III-2 about Here]

We compute the simple diffusion IVs for our samples and plot them in Figure III-2, while the average IVs for all the scenarios considered are shown in Panel B of Table I-2. As expected, the IVs after the sub-prime financial crisis are relatively higher than those before the crisis for all the ratings because of the higher CDPs after the crisis. The figure shows clearly that the term structure of IVs is not flat for all rating categories, which conflicts with the constant volatility assumption of both the L and LT diffusion models. Compared to the average IV for each rating category, the IVs are significantly higher for short-term corporate debt for all rating classes and occasionally higher for longer-term debt as well. As functions of maturity, the IVs are sharply decreasing initially and then become flat, with a slight increase when maturity is long enough. Under this asymmetric

[70] Moody's special comments: "The Distribution of Common Financial Ratios by Rating and Industry for North American Non-Financial Corporations: July 2006".
[71] A 4% asset risk premium is consistent with an asset beta of about 0.6, as used in Leland (2004).

"U" shape term structure of IV the IVs for medium-term debt, around 10 to 15 years, are the lowest for each rating category. This indicates that the risk of short-term and long-term debt issued by a firm is higher than the risk of medium-term debt. Comparing the average of IVs in Table III-1, we find that the sub-prime financial crisis does increase the volatility of asset value for all debt categories. As expected, the average IV increases with the average maturity of debt for all rating categories, since the risk of a firm is generally higher when it chooses to finance itself with longer term debt. On the other hand, the volatility does not always increase when the debt rating deteriorates, although the lowest rating has a much higher average volatility than the highest one; the slight drops in average volatility in intermediate ratings may be due to the composition of the sample. The U-shaped curve implies that if we use the average IV to predict the cumulative default probabilities of corporate debt for each rating, we will sharply under-estimate the default probability for short term and maybe slightly under-estimate the default probability for long-term. This is consistent with the Leland (2004) findings.[72] As most curvature of the IV appears in the short term CDPs, we only focus on the short term CDPs in the following analysis, ranging from 1-year to 10-year CDPs.

1.3 Term structure of implied volatilities under the CEV structural model

Compared to the L and LT models, the CEV structural model has one extra parameter β to capture the state-dependent volatility of asset value. Can the CEV structural model generate the downward sloping term structure of IV? In order to answer this question, we first try to fit the historical term structure of IVs by varying β and θ

[72] Note, however, that the evidence for the LT model from the sample of 182 firms used by Eom, Helwege and Huang (2004) to test the predictions of five structural models finds that LT *overpredicts* the yield spread for low maturity bonds.

and keeping the rest of the calibrations the same as in the LT model, which is used in most empirical work .[73]

[Insert Table III-2 about Here]

[Insert Figure III-3 about Here]

Table III-2 reports the values of β and θ which minimize the sum of absolute deviations between IVs from the CEV structural models and IVs of historical CDPs for Aa, A, Baa and Ba rated bonds during the periods of 1983-2008 and 1983-2010. To keep the optimization problem simple, we only consider integer values of β . Compared to the LT model with flat term structure of IVs, the CEV structural model can generate a downward sloping term structure of IV, which can be visualized in Figure III-3 for different rating categories during the periods we considered. At the same time, the sums of absolute differences between CEV IVs and Moody's historical IVs are small compared to those between LT IVs and Moody's historical IVs, especially for higher rated debt. For instance, the sum of the absolute errors of Aa bonds decreases by 85%, from 0.3146 to 0.0498 after incorporating the state-dependent volatility. Even for the Ba bond which has the highest β in our sample, the sum of the absolute errors still decreases by 25%, from 0.1208 to 0.0884. Therefore, the prediction of CDPs could be improved dramatically after introducing state-dependent volatilities to the asset value diffusion process, especially for higher rated bonds. Across different rating categories, β increases and the initial

[73] Since Leland (2004) shows that an exogenous or endogenous default boundary fits the observed default probabilities equally well provided default costs and recovery rates are matched, the exogenous default boundary is used here to keep the exercise simple.

volatility, $\sigma_0 = \theta S^{\beta}$, increases when the bond rating decreases.[74] This indicates a higher volatility increase of higher rated bonds per unit decrease of asset value compared to that of lower rated bonds. Although the absolute scale of CDPs of higher rated debt is small, especially for short term maturities, the change of the IVs is relatively greater across different maturities. This evidence is consistent with Coval, Jurek and Stafford (2008)'s findings that although default risk is less important in an absolute sense for senior CDO tranches, systematic risk is extremely important as a proportion of total spreads for these tranches. In additional, by comparing the β and σ_0 with the period of 1983-2010, we found that the sub-prime financial crisis makes the term structure of IVs of Aa and A bonds much steeper and has relatively little impact on the term structure of IVs of Baa and Ba bonds.

On the other hand, the elasticity estimates extracted from the CDP data for the CEV model are all negative and implausibly high in absolute value, varying from -2 to -5 or even to -6 for the period that includes the financial crisis. These values were estimated by ignoring individual firm information and are radically different from those extracted in the following two sections. Their only advantage lies in demonstrating the ability of the extra parameter to achieve a much better approximation to the historical record.

2. Data Description of Individual Firms

In the following sections, we recognize the characteristics of individual firms and conduct two exercises with the firm-level information from the financial statements,

[74] As we assume the exogenous default boundary equals the face value of the debt so as to keep the calculation simple, it makes the initial volatilities relatively small. The values of the implied initial volatilities increase and the values of β do not change when the exogenous default boundary decreases.

equity market, debt market, option market and Credit Default Swap (CDS) markets, both separately and simultaneously.

The credit spread data is obtained from the Markit database during the period from January, 2001 to December, 2011. We limited our sample only to United States firms for contracts denominated in US dollars. We select single name contracts with senior unsecured debts and modified restructure (MR) clause. We only keep the single name contracts which have at least 60 consecutive months' observations. As the reported frequency of CDS database is daily, the CDS spread on the last Wednesday in each month is extracted as the CDS spread in that month.

The accounting and equity information are extracted from the COMPUSTAT and CRSP data bases respectively. We calculate the total assets as the sum of book value of debt and market value of equity. The firms' payout ratio is represented by the sum of cash dividend and interest payment divided by the total asset. As the accounting information frequency is quarterly, we convert it into monthly by assuming that the values are constant within each quarter. The at the money call option implied volatilities are extracted from the *Optionmetrics* database. The risk free rates are interpolated from the observed 6month Libor rates and 1, 2, 3, 5, 7, 10 years interest rate swap rates.

[Insert Table III-3 about Here]

[Insert Table III-4 about Here]

The final sample consists of 103 firms whose detailed characteristics are reported in Table III-3. Table III-4 reports the distribution of individual firms in term of industries and ratings[75]. Approximately half of the firms belong to consumer goods and industrials. There are only four firms in technology and telecommunication services, while the rest of the firms are almost equally distributed in basic materials, energy, healthcare and consumer services. The average payout ratios are equal in all the industries, about 1%, which is much lower than the calibration values used in Leland and Toft (1996). The average leverage ratio across all the industries is around 38%. In term of the individual industries, healthcare has the lowest leverage, about 25%, while the other industries are more or less around 40%, with the highest leverage of 43% in industrials. The highest implied volatility from the option markets occurs in energy, around 31%, while the average implied volatility in the full sample is around 27%.[76] In term of credit default swap spreads, consumer services have the highest spreads and healthcare has the lowest spreads across all the maturities. We also note that the highest credit spreads do not coincide with the highest leverage ratios, indicating that industry is potentially an important factor in the determination of credit spreads, most probably because of the recovery rates in case of default, that enter into the CDS spread determination.[77]

Panel B reports the rating distribution of individual firms. Most of the sample firms are rated as *A* and *BBB*, approximately 82%, while the numbers of firms in the highest and lowest ratings are relatively small. Generally, as ratings decrease from *AAA* to *BB*, the leverage, implied volatility and credit spreads across all the maturities increase. As there is only 1 firm in the *B* and *CCC* ratings respectively, we attribute the abnormal

[75] Both industries and ratings classifications are extracted from the Markit database.
[76] The implied volatility of Telecommunication services is only 22%. As there is only one firm in this industry which is not enough to represent the whole industry, we leave this outside our discussion.
[77] Acharya, Bharath and Srinivasan (2007) studied the impact of the industry factor on credit spreads through the recovery channel and documented that creditors of defaulted firms recover significantly lower amounts in present-value terms when the industry of the defaulted firms is itself in distress.

behaviour of leverage, implied volatility and credit spreads in these ratings to firm specific characteristics.

In the subsequent two sections we will use the information of our sample firms to test whether the CEV model is a better structural model of the firm in terms of representing the observable characteristics of debt and equity instruments, as well as CDS spreads.

3. Empirical Evidence from Leverage, Equity value and Volatilities

3.1 Methodology

There are three types of equity volatilities: historical volatility, realized volatility and option implied volatilities, which contain the different information sets. The historical and realized volatility reflect the past and current information in the equity market, respectively, while the option implied volatility reflects the information in the option market. Cao, Yu and Zhong (2010) show that the option implied volatility is a more efficient forecast for future realized volatility compared to the historical volatility. By studying the co-movement among CDS, equity and option markets, Berndt and Ostrovnaya (2008) find that option prices reveal information about forthcoming adverse events at least as early as do credit spreads. In other words, the implied volatility from option markets contains certain future information compared to historical and realized volatilities. Since the current and future information sets are more interesting for our study purposes, we are going to use implied volatility from option markets for our base case results and realized volatility from the intraday dataset for robustness checks.

Denote the observed and model implied equity volatility by σ_E^{Obs} and σ_E^{Im}, equity value by E^{Obs} and E^{Im}, leverage ratio by Lev^{Obs} and Lev^{Im}, respectively. For the parameter estimation we use the Generalized Method of Moments (GMM) method. Denote the estimation parameter set as $\psi_1=(\sigma_0,\beta)$. This approach obviously nests the constant volatility case by setting $\beta=0$.

At each time point $t=1,\ldots,\mathrm{T}$ of our data base we have the following vector $f_1(\psi_1,t)$, a function of the parameter set

$$f_1(\psi,t)=\begin{Bmatrix} E^{Obs}(t)-E^{\mathrm{Im}}(t) \\ \sigma_E^{Obs}(t)-\sigma_E^{\mathrm{Im}}(t) \\ Lev^{Obs}(t)-Lev^{\mathrm{Im}}(t) \end{Bmatrix} \tag{3.1}$$

The model implied debt value $D^{\mathrm{Im}}(t)$, equity value $E^{\mathrm{Im}}(t)$ and volatility σ_E^{Im}, can be computed from equations (2.24), (4.5) and (4.8) in Chapter 1, respectively. The implied leverage can be computed by,

$$Lev^{\mathrm{Im}}(t)=\frac{D^{\mathrm{Im}}(t)}{D^{\mathrm{Im}}(t)+E^{\mathrm{Im}}(t)} \tag{3.2}$$

In order to determine the model implied moments numerically, we need both accounting and equity information. It is assumed that the time-varying exogenous default boundary equals the value of current debt plus one-half of the long term debt[78] for both candidate models. The asset values are the sum of book value of debt plus the market value of equity; the latter equals the product of stock price and outstanding shares. The firm's total payouts are the sum of cash dividends to shareholders and interest payments

[78] This exogenous default boundary was introduced by the KMV group and is widely known.

to debtholders. The corporate tax rate is assumed to be 35% and the recovery rate is equal to the estimated recovery rate in the Markit database. Under these assumptions, the Leland model has only one variable, asset volatility, to estimate, while the CEV structural model has one extra parameter, the elasticity of variance, besides the volatility level.

Since the GMM estimates ψ by minimizing $E[f_1(\psi_1, t)]$, we set

$$G_1(\psi_1, \mathrm{T}) = \frac{1}{\mathrm{T}} \sum_1^{\mathrm{T}} f_1(\psi_1, t) \tag{3.3}$$

and estimate ψ_1 by the relation,

$$\psi_1 = \arg\min G_1(\psi_1, T)' W_1 G_1(\psi_1, T) \tag{3.4}$$

In (3.4) W_1 is a matrix of weights that is computed by successive approximations.[79]

3.2 Results

[Insert Table III-5 about Here]

Table III-5 presents the average values of the parameters under the CEV structural model estimated as described in Section 3.1, with leverage ratios, equity values and implied equity volatilities. In Panel A, we observe that the value of β decreases from 1.19 to -1.08 as the rating class decreases from AAA to CCC, even though the β's in some rating classes are not significantly different from zero at conventional levels. We also note that for the whole sample, the average value of β is around 0.095 which is not significant from zero either, and the average value of initial volatility is around 15% and

[79] We choose W_1 by setting W_1^{-1} as the covariance matrix of moments. See pages 443-447 in Greene's *Econometric Analysis* (Sixth Edition).

significant different from zero. For the industry distribution of parameters reported in Panel B, the Industrials and Technology sectors have the relatively lowest β's , around -0.21 and -0.4, respectively, while Consumer Services and Healthcare have the highest β among all the sectors. However, none of these β's are significant different from zero conventional levels.

As we only fit the equity and account information in this exercise, it appears that the CEV structural model, which introduces skewness into the asset value distribution, does not outperform the Leland model with constant asset volatility.

4. Empirical Evidence From Equity, Debt and CDS Markets

4.1 Econometric methodology

In this section, we incorporate the observed information from the Credit Default Swap (CDS) market. For each firm in our data base we use $j = 1, 3, 5, 7, 10$ different maturities for the CDS spreads in our parameter estimation. Setting $g = T_j^{-1}$ everywhere the CDS spreads c_j are given by the following expression in our continuous time notation, with R denoting the estimated recovery rates, whose estimates are also available in the CDS data base.

$$c_j = \frac{(1-R)\int_t^{T_j} f(\tau, V.K)e^{-r\tau}d\tau}{\int_t^{T}[1-A(\tau)]e^{-r\tau}d\tau} \tag{4.1}$$

Discretizing this expression in terms of quarters τ_i, $i = 1,...,4T_j$, and setting $D(0,\tau_i)$ and $Q(0,\tau_i)$ for the discount factor and survival probabilities respectively in the time interval $\left[0,\tau_i\right]$, we have for $CDS\left(0,T_j\right)$, the total spread paid by the default protection buyers in $[0,T_j]$,

$$CDS\left(0,T_j\right)=\frac{\left(1\text{-}R\right)\sum_{i=1}^{4T_j}D\left(0,\tau_i\right)\left[Q\left(0,\tau_{i\text{-}1}\right)\text{-}Q\left(0,\tau_i\right)\right]}{\sum_{i=1}^{4T_j}D\left(0,\tau_i\right)Q\left(0,\tau_i\right)/4} \tag{4.2}$$

The first passage time to default probability distribution for the state dependent volatility processes is given in terms of its Laplace transform by the following expression[80]

$$A\left(T_j\right)=\left(\frac{1}{\lambda}\bullet\frac{\phi_\lambda\left(V\right)}{\phi_\lambda\left(K\right)}\right)^{-1} \tag{4.3}$$

with the expressions in braces given by (3.6) in Chapter I in the case of the CEV distribution.

For the parameter estimation we use the Generalized Method of Moments (GMM) method. At each time point $t=1,\ldots,\mathrm{T}$ of our data base we have the following vector $f_2\left(\psi_2,t\right)$, a function of the parameter set

$$f_2\left(\psi,t\right)=\left\{\begin{matrix} CDS^{Obs}\left(t,T_1\right)\text{-}CDS^{\mathrm{Im}}\left(t,T_1\right) \\ \vdots \\ CDS^{Obs}\left(t,T_{10}\right)\text{-}CDS^{\mathrm{Im}}\left(t,T_{10}\right) \\ \sigma_E^{Obs}\left(t\right)-\sigma_E^{\mathrm{Im}}\left(t\right) \end{matrix}\right\} \tag{4.4}$$

In (4.4) $CDS^{\mathrm{Im}}(t,T_j),\ j=1,2,3,5,7,10$ represent the CDS spreads estimated from (4.1)-(4.3) given the parameter set $\psi_2=\left(\sigma_0,\beta\right)$, while $CDS^{Obs}\left(t,T_j\right)$ is the corresponding

[80] See equations (3.6) in Chapter 1, based on Davydov and Linetsky (2001, Proposition 2).

observed CDS spread. Similarly, $\sigma_E^{\mathrm{Im}}(t)$ is the model-based equity volatility estimated from (4.8) in Chapter I given ψ_2, while $\sigma_E(t)$ is the observed equity volatility from the option market. Similar to section 3, we set

$$G_2(\psi_2, \mathrm{T}) = \frac{1}{\mathrm{T}} \sum_1^{\mathrm{T}} f_2(\psi_2, t) \tag{4.5}$$

and estimate ψ_2 by the relation

$$\psi_2 = \arg\min G_2(\psi_2, T)' W_2 G_2(\psi_2, T) \tag{4.6}$$

In (4.6) W_2 is a positive definite weighting matrix. We set this matrix equal to the variance matrix of the moment conditions[81] and compute by successive approximations.

4.2 Results

Following the procedure described in the previous section, we conduct the GMM estimation with seven moments including the information from financial statements, equity, option and CDS markets and report the average values of parameters in Table III-6. For the entire sample, the average values of β and σ_0 are around -0.67 and 21%, respectively, and both of them are significant different from zero at the highest conventional confidence level. Over 85% of the firms in the sample have negative β's, indicating a negative relationship between asset value and asset volatility. In other words, we document a significant skewness in the asset value distribution after incorporating the information from the CDS market.

[Insert Figure III-4 about Here]

[81] See "Econometric Analysis (6th Edition)" edited by William H. Green, on page 444.

[Insert Figure III-5 about Here]

Compared to the Leland structural model with constant volatility, the CEV structural model shows a clearly superior fitting of its estimates to the observed data, especially with respect to the CDS spreads. According to Figure III-4, the structural model with constant volatility consistently underestimates the CDS spreads with 1-year, 5-year and 10-year maturities, while the CEV model estimates lie much closer to the observed values. Since we fixed the default boundary to the one based on the KMV method, there is only one extra parameter, the elasticity of variance β, under the CEV structural model compared to the structural model with constant volatility. This extra parameter improves dramatically the fitting of the time series of CDS spreads across all the maturities while maintaining the fitting of the implied equity volatilities time series at comparable levels of accuracy to the constant volatility, as shown in Figure III-5.

[Insert Table III-6 about Here]

Table III-6 shows the distribution of the estimates of β across rating classes and industries. For the different rating classes, we observe a decrease of β as the rating decreases from AA to BB, while the value of the initial volatility σ_o first decreases and then increases. The observations in other rating classes are too few to arrive at a reliable conclusion. For the different industries, all the average values of β are negative and significantly different from zero, with the lowest values in Telecom Services, Technology and Consumer Goods. All the average value of σ_o are significant, with the highest value, around 28%, in the Energy sector. We also note that most of the positive β's fall into the Consumer Services, Industrials and Basic Materials sectors.

[Insert Table III-7 about Here]

As individual firms have their own specific characteristics which lead to different asset value distributions, we document a wide range of values of β from -2.34 to 1 for the whole sample. To assess the relationship between β and firm specific characteristics, we break down the whole sample into five sub-samples in ascending values of β. We pick the first and last 20 firms and put them into the first and last quantiles respectively, and split the rest into three quantiles evenly. Table III-7 reports the firm characteristics of the quantiles in term of both mean and median. Generally we observe that as the value of β increases, there is a tendency for both asset values and current ratios which are equal to current assets over current liabilities, to increase, while the leverage ratios and CDS spreads decrease. Nonetheless, the relationship is neither monotone nor very strong, and the ratios and spreads fluctuate in the intermediate quantiles, probably reflecting firm specific factors. It is well known that high leverage and CDS spreads are associated with high default risk while a high current ratio is associated with low default risk. Hence, the firms with higher default risk have a higher probability to have a negative β compared to those with a lower default risk. Since the sign of β shows the relationship between asset value and asset volatility, the firms with a high default risk have a higher probability to have a more skewed distribution of asset value compared to those with a relatively lower default risk. In addition, we did not see a clear relationship between the value of β and the firm's payout ratio and implied volatility.

4.3 Out of Sample Fitting of CDP

We observe from Table III-4 that we have fairly large sample firms in the A and BBB classes, with average values of leverages around 31% and 44%, respectively. Compared to the Moody's risk class information reported in Table III-1 in section 1, the leverage ratio of BBB firms is very similar to that of Baa firms, approximately 44%. [82] Thus, our samples of firms in the BBB class should be good proxies for Moody's Baa class. Since we have already calibrated the value of initial volatilities and elasticity parameters for each individual firm in the previous section with equity, option and CDS information, we may now verify the out of sample fitting for the historical term structure of cumulative default probabilities for the BBB class.

[Insert Figure III-6 about Here]

From Table III-6, we know that the average values of the initial volatility and the elasticity parameter β are 18.67% and -0.7037, respectively, under the CEV structural model, while the average asset volatility is around 15.29% under the Leland structural model with constant volatility. We assume 4% asset risk premium.[83] The exogenous default boundary, risk free rates and payout rates are the same as in the empirical data for all the firms in the BBB class of our sample. As we see in Figure III-6, the term structure of physical cumulative default probabilities generated by the CEV structural model can almost capture the level and trend of the observed information, while that generated by the structural model with constant volatility dramatically underestimates the CDP. Thus,

[82] We assume the S&P's BBB is equivalent to Moody's Baa class. The equivalence of the S&P A class with Moody's Aa is less clear, and for this reason we don't present the comparative CDP results for that class. In these results the observed CDP for the Aa class lies between the CEV and constant volatility cases.
[83] Leland (2004) assume 4% asset risk premium for the calibration. This number is consistent with 6% equity premium when the average firm has about 35% leverage.

the CEV structural model with its extra elasticity parameter shows significant flexibility to fit into the cross sectional information from major financial markets.

5. Conclusion

In this Chapter, we use real data from the equity, option and CDS markets for a sample of firms to compare the performance of structural models with constant volatility and CEV, using three alternative information sets.

First, using Moody's historical cumulative default probabilities, we show that the term structure of implied volatilities is not constant, especially for the short-term maturities. We show that the extra parameter of the CEV model achieves a much better approximation to the historical record than constant volatility. Further, we find that the elasticity estimates extracted from the CDP data for the CEV model are all negative and implausibly high in absolute value, varying from -2 to -5 or even to -6 for the period that includes the financial crisis.

Second, we use firm level information limited to the accounting data and equity and option market observations. With such an information set, we note that the CEV structural model has a performance similar to the structural model with constant volatility.

Third, we incorporate the information from CDS markets into the data set used in the second exercise and re-examine the performance of both models. We document that the CEV structural model exhibits a much better fitting to the CDS spreads across all maturities. In addition, we find that the estimated values of β are overwhelmingly and significantly negative for most of the firms in our sample across all industries and rating classes. The relationship between the sign and value of β and the firm specific measures of default risk, such as leverage ratios, CDS spreads and current ratios indicates that there is a tendency for β to increase as the risk of the firm decreases, but that the tendency is weak and fluctuates.

Last, we compared the estimated average cumulative default probabilities to the Moody's data for the BBB rating class in our sample for which there were sufficient numbers of firms for reliable inferences. We note that the CDP term structure generated by the CEV structural model can fit the observed data much better than the one estimated with constant asset volatility.

Overall, the elasticity parameter under the CEV structural model provides a significant degree of flexibility to fit the cross sectional information from financial statements, equity, option and CDS markets simultaneously compared to the competing structural model with constant volatility. More complex asset dynamics that include jump and/or stochastic volatility components may perhaps improve the fit compared to the CEV. Nonetheless, the theoretical and computational drawbacks of these dynamics, documented at length in Chapter I of this thesis, preclude their use in empirical research at this point in time.

Table III-1: Calibration of Model Parameters

Panel A				
	Aa	A	Baa	Ba
Average leverage(D/v)	31.6%	41.7%	44.8%	49.8%
Payout Rate	6%	6%	6%	6%
Risk Free Rate	8%	8%	8%	8%
Recovery Rate	50%	50%	50%	50%
Tax Rate	0.35	0.35	0.35	0.35
Risk Premium	4%	4%	4%	4%
Panel B				
V_0	100	100	100	100
K	31.6	41.7	44.8	49.8
Average Implied volatility(1983-2008)	18.97%	17.69%	19.11%	23.36%
Average Implied volatility(1983-2010)	19.33%	18.30%	19.50%	24.21%

Table III-2: CEV structural model parameter estimation by fitting Moody's historical CDPs

This table reports the estimation of β and σ_0 in the CEV structural model by minimizing the sum of absolute deviations from Moody's historical CDPs for terms from 1-year to 10-years.

$$\underset{\beta,\theta}{MIN} \sum \left| \sigma_{CEV}^{I} - \sigma_{M}^{I} \right|$$

The average leverage, payout rate, risk free rate, recovery rate, tax rate and risk premium are assumed to be the same as in Panel A in Table III-1 for different rating categories of debt. We assume the initial asset value is 100 and the exogenous default boundary equals the face value of the debt. $\overline{\sigma}_{M}^{I}$ is the average implied volatility by the LT model.

	Period of 1983-2008				Period of 1983-2010			
Rating	β	σ_0	$\sum \left| \sigma_{CEV}^{I} - \sigma_{M}^{I} \right|$	$\sum \left| \sigma_{M}^{I} - \overline{\sigma}_{M}^{I} \right|$	β	σ_0	$\sum \left| \sigma_{CEV}^{I} - \sigma_{M}^{I} \right|$	$\sum \left| \sigma_{M}^{I} - \overline{\sigma}_{M}^{I} \right|$
Aa	-5	6.5%	0.0498	0.3146	-6	6.3%	0.0530	0.3128
A	-4	8%	0.0621	0.1832	-5	7.6%	0.0778	0.2036
Baa	-4	9.5%	0.0747	0.1914	-4	9.8%	0.0836	0.1866
Ba	-2	16%	0.0884	0.1208	-2	16.5%	0.1010	0.1075

Figure III-1: Cumulative Default Probabilities of Aa, A, Baa and Ba rated corporate bonds

The dashed lines show the cumulative default probability (CDP) of corporate debt during 1983-2010. The solid lines show the cumulative default probabilities of corporate debt during 1983-2008.

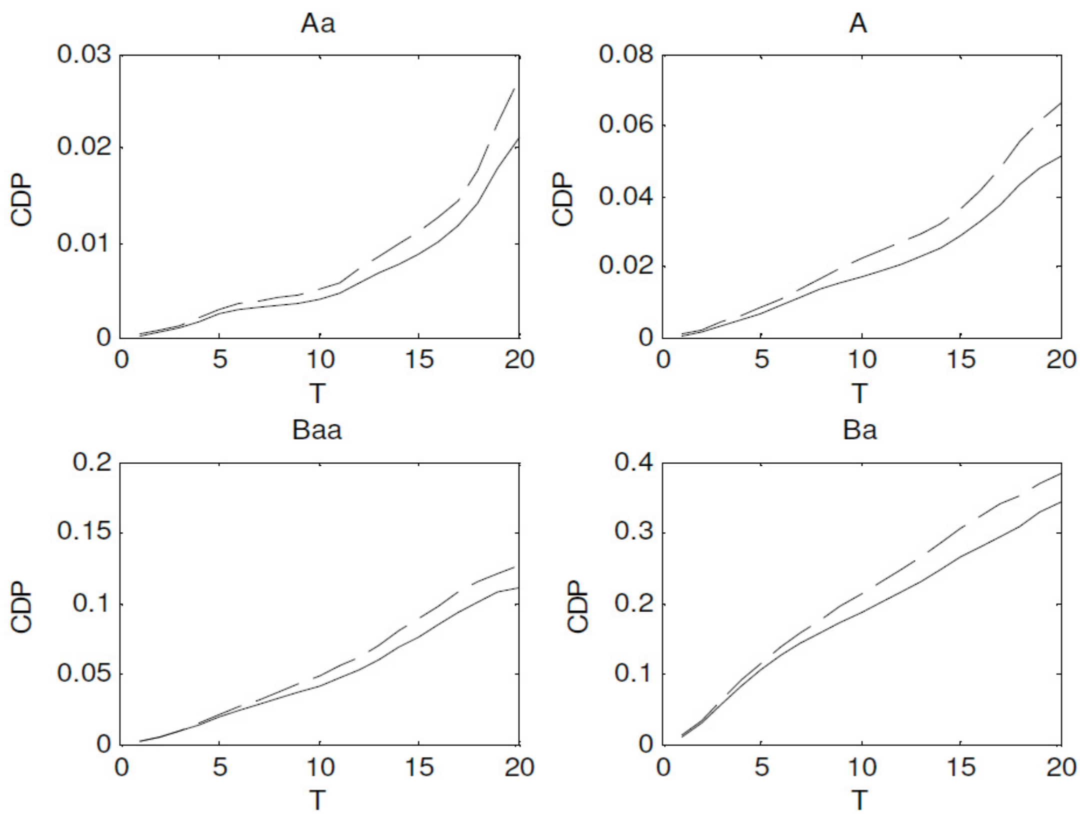


Figure III-2: Implied volatility of historical cumulative default probabilities

The dashed lines show the implied volatilities of the cumulative default probabilities during 1983-2010 by LT model, while the solid lines show the implied volatilities during 1983-2008. The initial asset value equals 100 and the debt value is chosen from the empirical leverage ratio for the different ratings. The coupon is calculated by making the debt issued at par value. Tax rate is 35% and recovery rate is 50% for all the debts. The average maturity of debt is 10 years. The exogenous default boundaries are equal to the value of debt for each scenario.

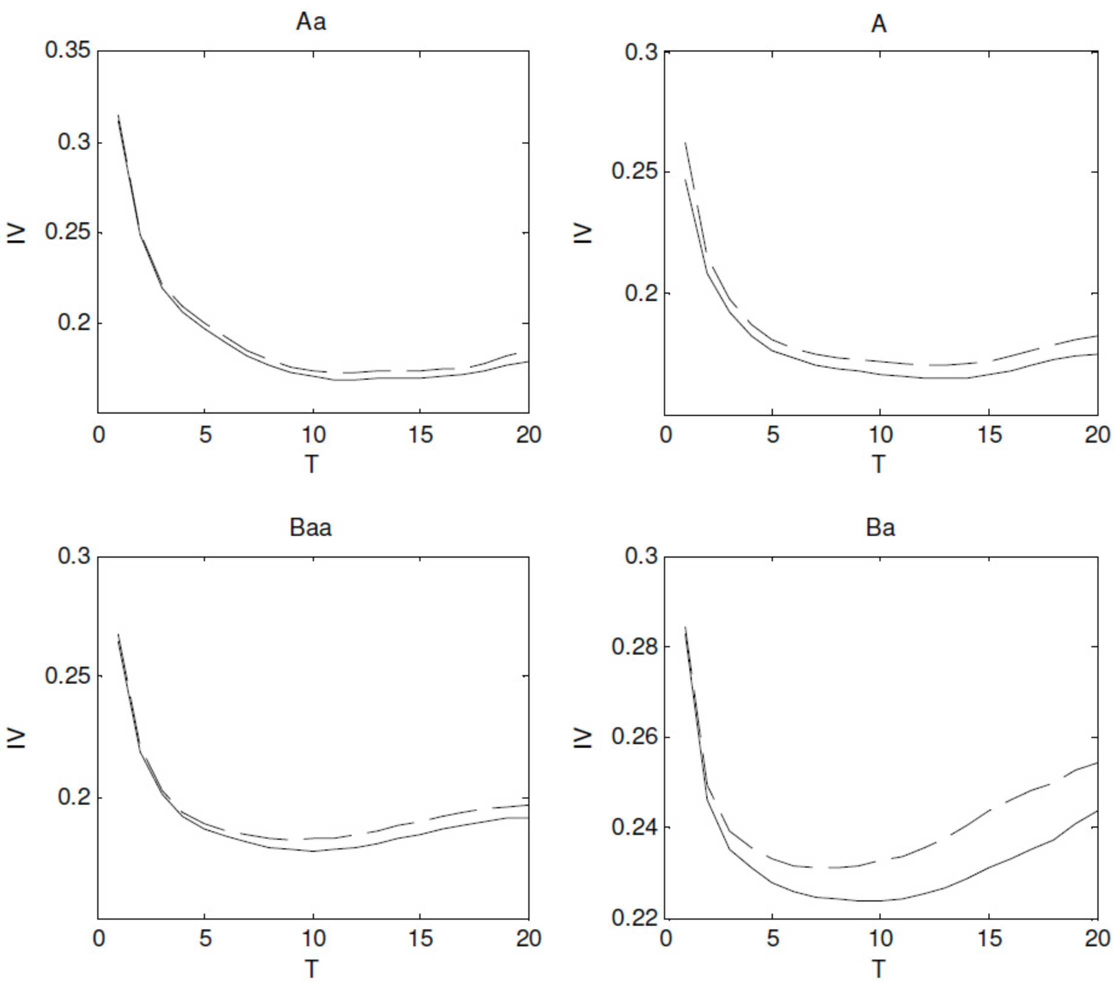


Figure III-3: Term structures of Implied Volatilities (IV) of CEV structure model

This figure depicts the term structure of IV for Moody's historical CDPs (solid lines) and CEV structural model (dashed lines). The LT model with exogenous default boundary is used to calculate the IVs. The average IVs for Moody's historical CDPs are shown by dashed-dot lines for Aa, A, Ba, B rated debt during the periods 1983-2008 and 1983-2010. The average leverage, payout rate, risk free rate, recovery rate, tax rate and risk premium are assumed to be the same as in Panel A in Table III-1 for different rating categories of debt. We assume the initial asset value is 100 and the exogenous default boundary equals the face value of the debt. The values of β and σ_0 in CEV structural model are calculated by minimizing the sum of absolute deviations from Moody's historical CDPs for terms from 1-year to 10-years and shown in Table III-2.

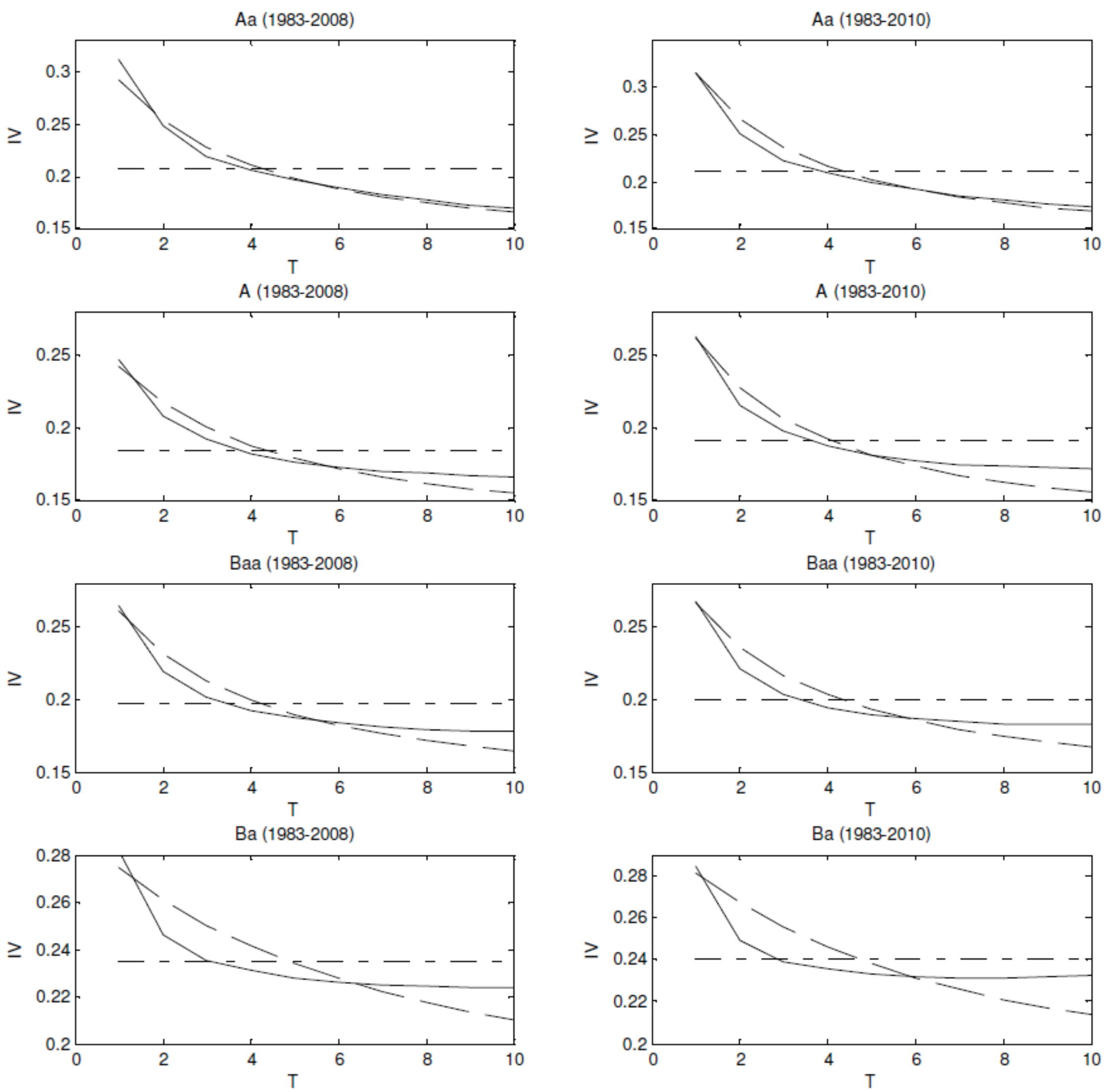


Table III-3: Summary Statistics of Individual Firms

This table reports the summary statistics of individual firms. Note that in sector column, *BM*, *CG*, *CS*, *EN*, *HC*, *IN*, *TE*, *TS* are abbreviations of *Basic Materials*, *Consumer Goods*, *Consumer Services*, *Energy*, *Healthcare*, *Industrials*, *Technology* and *Telecommunications Services*, respectively. The *payout ratio* is the sum of cash dividend and interest expense divided by the total asset. The *recovery rates* are the estimated recovery rates reported in Markit datasets. The *implied volatilities* are extracted from *Optionmetric* for the at the money call options.

Company Name	Sector	Rating	Begin date	End date	Total Asset (billion)	Payout Ratio	Leverage Ratio	Recovery Rate	Implied volatility
3M Co	IN	AA	04/2003	12/2011	69.56	0.01	0.18	0.40	0.22
Abbott Labs	HC	AA	10/2003	12/2011	98.95	0.01	0.23	0.40	0.21
Air Prods & Chems Inc	BM	A	04/2003	09/2008	20.78	0.01	0.30	0.40	0.22
Alcoa Inc.	BM	BBB	08/2001	09/2008	45.79	0.01	0.42	0.40	0.34
AmerisourceBergen Corp	CS	BBB	02/2004	12/2011	17.37	0.00	0.55	0.40	0.27
Anadarko Pete Corp	EN	BBB	01/2003	09/2008	40.52	0.01	0.48	0.40	0.31
Anheuser Busch Cos Inc	CG	A	06/2003	10/2008	51.64	0.01	0.25	0.40	0.18
APACHE CORP	EN	A	03/2003	09/2008	31.58	0.00	0.30	0.40	0.31
Archer Daniels Midland	CG	A	06/2003	09/2008	31.83	0.01	0.44	0.40	0.30
Arrow Electrs Inc	CG	BBB	11/2001	12/2011	7.43	0.00	0.57	0.40	0.38
Autozone Inc	CS	BBB	03/2003	07/2011	12.58	0.00	0.35	0.40	0.27
Avon Prods Inc	CG	BBB	01/2003	12/2011	19.04	0.01	0.25	0.40	0.31
Baker Hughes Inc	EN	A	11/2001	09/2008	20.82	0.01	0.17	0.40	0.34
Baxter Intl Inc	HC	A	02/2002	12/2011	36.91	0.01	0.26	0.40	0.26
Black & Decker Corp	CG	BBB	05/2002	01/2010	8.50	0.01	0.47	0.41	0.32
Boeing Co	IN	A	04/2001	09/2008	92.46	0.01	0.50	0.40	0.28
BorgWarner Inc	CG	BBB	11/2001	09/2008	5.21	0.01	0.43	0.40	0.31
Bristol Myers Squibb Co	HC	A	04/2003	12/2011	64.12	0.02	0.26	0.40	0.25
Campbell Soup Co	CG	A	06/2002	10/2011	17.43	0.01	0.32	0.40	0.21
Caterpillar Inc	IN	A	04/2001	09/2008	67.27	0.01	0.54	0.40	0.28
CenturyTel Inc	TS	BBB	03/2003	04/2008	8.99	0.01	0.50	0.40	0.22
Clorox Co	CG	BBB	07/2004	07/2009	13.16	0.01	0.32	0.40	0.21
Coca Cola Entpers Inc	CG	A	06/2003	09/2008	30.04	0.01	0.66	0.40	0.23
Colgate Palmolive Co	CG	AA	08/2003	12/2011	41.92	0.01	0.19	0.40	0.20
ConAgra Foods Inc	CG	BBB	08/2001	07/2011	20.08	0.03	0.42	0.40	0.22
ConocoPhillips	EN	A	01/2003	09/2008	152.40	0.01	0.45	0.39	0.25
Costco Whsl Corp	CS	A	07/2004	07/2011	35.93	0.00	0.29	0.40	0.25
CSX Corp	IN	BBB	01/2003	09/2008	29.17	0.01	0.58	0.40	0.29
Cytec Inds Inc	BM	BBB	02/2004	12/2011	4.30	0.00	0.49	0.40	0.36
Danaher Corp	IN	A	01/2004	12/2011	29.13	0.00	0.23	0.40	0.25
Diamond Offshore Drilling	EN	A	07/2003	09/2008	10.89	0.02	0.19	0.40	0.37
Dover Corp	IN	A	12/2004	12/2011	12.74	0.01	0.31	0.40	0.29
Dow Chem Co	BM	BBB	01/2002	09/2008	65.87	0.02	0.44	0.40	0.28
Eastman Chem Co	BM	BBB	01/2003	09/2008	8.57	0.01	0.52	0.40	0.25
FedEx Corp	IN	BBB	08/2002	07/2011	36.34	0.00	0.30	0.40	0.28
Gen Dynamics Corp	IN	A	11/2004	12/2011	41.54	0.01	0.36	0.40	0.24
Gen Mls Inc	CG	BBB	04/2002	07/2011	31.57	0.02	0.39	0.40	0.19
Goodrich Corp	IN	BBB	09/2001	09/2008	9.19	0.01	0.52	0.40	0.33

Halliburton Co	EN	A	02/2003	09/2008	34.94	0.01	0.30	0.40	0.33
H J HEINZ CO	CG	BBB	04/2001	10/2011	21.96	0.02	0.38	0.41	0.21
Home Depot Inc	CS	A	02/2002	09/2008	95.07	0.01	0.22	0.41	0.28
Honeywell Intl Inc	IN	A	11/2001	12/2011	54.86	0.01	0.41	0.40	0.30
Intl Business Machs Corp	TE	AA	04/2001	12/2011	233.09	0.01	0.34	0.40	0.25
Intl Paper Co	BM	BBB	04/2001	09/2008	39.13	0.01	0.56	0.40	0.27
Johnson & Johnson	HC	AAA	03/2003	12/2011	207.15	0.01	0.15	0.40	0.17
Kellogg Co	CG	BBB	03/2003	12/2011	27.45	0.01	0.33	0.40	0.18
Kimberly Clark Corp	CG	A	02/2004	12/2011	38.91	0.02	0.29	0.40	0.18
The Kroger Co.	CS	BBB	08/2006	10/2011	33.61	0.01	0.53	0.40	0.29
Eli Lilly & Co	HC	A	06/2003	12/2011	70.15	0.02	0.22	0.40	0.24
Ltd Brands Inc	CS	BB	03/2003	09/2008	12.76	0.01	0.29	0.40	0.32
Lockheed Martin Corp	IN	A	04/2001	12/2011	52.81	0.01	0.45	0.40	0.26
Lowes Cos Inc	CS	A	01/2003	09/2008	54.34	0.00	0.22	0.40	0.28
Marriott Intl Inc	CS	BBB	05/2002	09/2008	18.00	0.00	0.31	0.40	0.30
Masco Corp	CG	BB	07/2002	09/2008	18.45	0.01	0.39	0.41	0.31
Medtronic Inc	HC	A	09/2003	10/2011	62.35	0.01	0.16	0.40	0.24
Merck & Co Inc	HC	AA	03/2004	10/2009	105.48	0.02	0.23	0.40	0.27
Mohawk Inds Inc	CG	BBB	12/2004	12/2011	7.91	0.00	0.44	0.40	0.38
Molson Coors Brewing	CG	BBB	10/2005	12/2011	12.93	0.01	0.41	0.40	0.27
Monsanto Co	BM	A	04/2003	09/2008	31.34	0.01	0.22	0.40	0.32
Motorola Inc	TE	BBB	08/2002	09/2008	57.09	0.01	0.35	0.39	0.38
Newell Rubbermaid Inc	CG	BBB	05/2001	02/2009	11.75	0.02	0.43	0.41	0.30
Nordstrom Inc	CS	A	11/2001	09/2008	10.30	0.01	0.35	0.41	0.37
Norfolk Sthn Corp	IN	BBB	04/2001	09/2008	29.32	0.01	0.54	0.39	0.32
Northrop Grumman Corp	IN	BBB	04/2003	03/2011	36.98	0.01	0.46	0.40	0.22
OCCIDENTAL PETRO	EN	A	09/2002	09/2008	45.32	0.01	0.29	0.40	0.29
Omnicare Inc	CS	BB	11/2004	02/2011	7.65	0.01	0.50	0.26	0.40
Omnicom Gp Inc	CS	BBB	05/2002	12/2011	25.77	0.01	0.47	0.40	0.29
ONEOK Partners LP	EN	BBB	05/2006	12/2011	7.79	0.04	0.53	0.40	0.22
J C Penney Co Inc	CS	BB	06/2001	09/2008	20.37	0.01	0.51	0.38	0.39
Pepsico Inc	CG	A	06/2004	12/2011	124.03	0.01	0.19	0.40	0.18
Pfizer Inc	HC	AA	10/2003	12/2011	234.37	0.02	0.28	0.40	0.24
Pitney Bowes Inc	TE	BBB	11/2003	12/2011	15.76	0.02	0.53	0.40	0.24
PPG Inds Inc	BM	BBB	07/2001	12/2011	17.96	0.01	0.42	0.40	0.27
Praxair Inc	BM	A	10/2003	09/2008	25.16	0.01	0.26	0.40	0.23
Pride Intl Inc	EN	BBB	06/2003	09/2008	6.49	0.00	0.35	0.40	0.38
Procter & Gamble Co	CG	AA	04/2001	12/2011	213.09	0.01	0.25	0.40	0.19
Quest Diagnostics Inc	HC	BBB	09/2005	12/2011	14.15	0.01	0.30	0.40	0.24
Raytheon Co	IN	A	06/2003	12/2011	32.39	0.01	0.42	0.40	0.22
Rep Svcs Inc	IN	BBB	09/2004	12/2011	14.30	0.01	0.43	0.40	0.27
Reynolds Amern Inc	CG	BBB	11/2004	12/2011	26.53	0.02	0.39	0.40	0.23
Rohm & Haas Co	BM	BBB	05/2001	11/2008	15.81	0.01	0.39	0.41	0.27
Ryder Sys Inc	IN	BBB	01/2003	09/2008	7.26	0.01	0.62	0.39	0.29
Safeway Inc	CS	BBB	07/2005	12/2011	20.99	0.01	0.50	0.40	0.31
Schering Plough Corp	HC	A	04/2003	09/2008	40.65	0.01	0.23	0.40	0.28
Sealed Air Corp US	IN	B	02/2006	12/2011	7.05	0.01	0.47	0.40	0.31
Sherwin Williams Co	CG	A	06/2002	12/2011	9.60	0.01	0.31	0.40	0.29
Smithfield Foods Inc	CG	BB	07/2003	08/2008	7.53	0.01	0.57	0.39	0.29
Southwest Airls Co	IN	BBB	06/2003	12/2011	18.62	0.00	0.45	0.39	0.35

Sunoco Inc	EN	BB	07/2003	09/2008	14.55	0.01	0.51	0.40	0.34
SUPERVALU INC	CS	CCC	03/2003	09/2008	15.13	0.01	0.60	0.40	0.28
Sysco Corp	CS	A	03/2005	12/2011	24.48	0.01	0.26	0.40	0.23
Target Corp	CS	A	04/2002	09/2008	64.06	0.00	0.35	0.40	0.30
Textron Inc	IN	BBB	10/2002	09/2008	23.69	0.01	0.58	0.39	0.28
Un Pac Corp	IN	BBB	09/2003	09/2008	45.93	0.01	0.49	0.39	0.24
Utd Parcel Svc Inc	IN	AA	08/2004	12/2011	68.52	0.02	0.32	0.40	0.23
Utd Tech Corp	IN	A	06/2003	09/2008	85.24	0.01	0.33	0.40	0.20
Unvl Health Svcs Inc	HC	BB	03/2004	12/2011	5.10	0.01	0.42	0.40	0.31
UST Inc.	CG	BBB	04/2003	10/2008	8.99	0.03	0.17	0.40	0.22
V F Corp	CG	A	09/2004	12/2011	11.07	0.01	0.26	0.40	0.28
Wal Mart Stores Inc	CS	AA	01/2001	10/2011	296.93	0.01	0.28	0.40	0.23
Waste Mgmt Inc	IN	BBB	01/2004	08/2009	31.50	0.01	0.46	0.40	0.24
Whirlpool Corp	CG	BBB	04/2001	09/2008	12.73	0.01	0.59	0.40	0.33
Wyeth	HC	A	02/2003	07/2009	81.20	0.01	0.28	0.40	0.26

Table III-4: Distribution of Individuals Firms

This table reports the industry and rating distribution in Panel A and B, respectively. The average values of all the variables are reported. The 1, 3, 5, 7, 10 years credit default swap spreads are reported as basis points (*bps*).

Sector/Rating	Observations	leverage	Payout ratio	1 year Spread	3 year Spread	5 year Spread	7 year Spread	10 year Spread	Implied volatility
Panel A: Industry Distribution									
Basic Materials	10	0.40	0.01	26.85	39.11	51.92	59.30	68.51	0.28
Consumer Goods	27	0.37	0.01	33.53	48.82	63.90	71.57	79.79	0.26
Consumer Services	17	0.39	0.01	43.87	64.65	84.10	92.76	102.56	0.30
Energy	10	0.36	0.01	30.21	44.73	58.86	66.96	76.00	0.31
Healthcare	12	0.25	0.01	20.89	32.54	44.76	51.22	58.01	0.25
Industrials	23	0.43	0.01	26.79	39.48	52.73	59.99	67.94	0.27
Technology	3	0.41	0.01	37.66	54.12	68.55	76.46	85.50	0.29
Telecom Services	1	0.50	0.01	24.76	46.52	71.38	86.84	102.93	0.22
Panel B: Rating Distribution									
AAA	1	0.15	0.01	11.51	16.57	22.33	26.19	30.76	0.17
AA	9	0.26	0.01	15.68	23.18	30.89	35.72	41.42	0.23
A	38	0.31	0.01	17.32	26.50	35.91	41.95	49.11	0.26
BBB	46	0.44	0.01	37.10	53.84	71.12	79.88	89.44	0.28
BB	7	0.46	0.01	83.09	121.94	154.16	166.00	177.62	0.34
B	1	0.47	0.01	72.47	111.10	150.17	163.88	176.62	0.31
CCC	1	0.60	0.01	54.54	89.48	124.22	140.64	157.33	0.28
All Firms	103	0.38	0.01	31.32	46.24	61.07	68.76	77.29	0.27

Table III-5: Distribution of Parameters with Leverage and Equity

This table reports the average value of parameters by fitting leverage, equity value and equity implied volatility. The average p-values for each parameter are reported in the parentheses.

Panel A: Rating Distribution

	ALL			Negative Betas			Positive Betas		
	N	beta	sigma	N	beta	sigma	N	beta	sigma
AAA	1	1.1901	0.1700				1	1.1901	0.1700
		(0.3319)	(0.0008)					(0.3319)	(0.0008)
AA	9	0.5275	0.1505	4	-0.1005	0.1496	5	1.0299	0.1513
		(0.2657)	(0.0120)		(0.3290)	(<.0001)		(0.2150)	(0.0216)
A	38	0.1091	0.1703	20	-0.6523	0.1651	18	0.9550	0.1760
		(0.0992)	(0.0033)		(0.1052)	(0.0002)		(0.0925)	(0.0068)
BBB	46	0.0334	0.1393	27	-0.6962	0.1396	19	1.0701	0.1388
		(0.1303)	(0.0227)		(0.1137)	(0.0002)		(0.1538)	(0.0547)
BB	7	0.0463	0.1736	4	-0.5174	0.1704	3	0.7978	0.1778
		(0.1630)	(0.0216)		(0.1667)	(0.0010)		(0.1580)	(0.0490)
B	1	-1.0766	0.1213	1	-1.0766	0.1213			
		(<.0001)	(<.0001)		(<.0001)	(<.0001)			
CCC	1	-1.0801	0.1678	1	-1.0801	0.1678			
		(0.0556)	(0.0054)		(0.0556)	(0.0054)			
total	103	0.0950	0.1544	57	-0.6398	0.1516	46	1.0056	0.1580
		(0.1328)	(0.0139)		(0.1265)	(0.0003)		(0.1406)	(0.0308)

Panel B: Industry Distribution

	ALL			Negative Betas			Positive Betas		
	N	beta	sigma	N	beta	sigma	N	beta	sigma
Basic	10	0.2166	0.1545	4	-0.7157	0.1450	6	0.8382	0.1609
Materials		(0.0614)	(0.0322)		(0.0019)	(<.0001)		(0.1010)	(0.0536)
Consumer	27	0.0852	0.1425	16	-0.5375	0.1423	11	0.9910	0.1429
Goods		(0.1138)	(<.0001)		(0.1211)	(<.0001)		(0.1032)	(0.0003)
Consumer	17	0.5064	0.1869	8	-0.6178	0.1744	9	1.5056	0.1981
Services		(0.1208)	(0.0248)		(0.1498)	(0.0007)		(0.0949)	(0.0462)
Energy	10	-0.0236	0.1847	6	-0.3307	0.2074	4	0.4370	0.1506
		(0.1928)	(0.0405)		(0.1550)	(0.0013)		(0.2493)	(0.0994)
Healthcare	12	0.4163	0.1671	6	-0.3509	0.1613	6	1.1834	0.1729
		(0.2207)	(0.0211)		(0.2664)	(<.0001)		(0.1750)	(0.0422)
Industrials	23	-0.2159	0.1306	13	-0.9918	0.1306	10	0.7927	0.1306
		(0.1090)	(0.0013)		(0.0525)	(0.0005)		(0.1826)	(0.0024)
Technology	3	-0.4047	0.1097	3	-0.4047	0.1097			
		(0.2855)	(<.0001)		(0.2855)	(<.0001)			
Telecom	1	-1.8688	0.1487	1	-1.8688	0.1487			
Services		(<.0001)	(<.0001)		(<.0001)	(<.0001)			

Table III-6: Distribution of Parameters with Equity and CDS Spreads

This table reports the average value of parameters by fitting equity implied volatility and CDS spreads. The average p-values for each parameter are reported in the parentheses.

Panel A: Rating Distribution

	ALL			Negative Betas			Positive Betas		
	N	beta	sigma	N	beta	sigma	N	beta	sigma
AAA	1	-0.9971	0.1604	1	-0.9971	0.1604			
		(<.0001)	(<.0001)		(<.0001)	(0.0005)			
AA	9	-0.4186	0.2511	7	-0.5862	0.1812	2	0.1679	0.4958
		(0.0002)	(<.0001)		(<.0001)	(<.0001)		(0.0009)	(<.0001)
A	38	-0.6627	0.2307	33	-0.8087	0.2272	5	0.3006	0.2543
		(0.0002)	(<.0001)		(<.0001)	(<.0001)		(0.0012)	(<.0001)
BBB	46	-0.7037	0.1867	40	-0.9133	0.1908	6	0.6939	0.1590
		(0.0069)	(<.0001)		0.0009	(<.0001)		(0.0474)	(<.0001)
BB	7	-0.8919	0.2303	6	-1.1181	0.2349	1	0.4651	0.2028
		(<.0001)	(<.0001)		(<.0001)	(<.0001)		(<.0001)	(<.0001)
B	1	0.5274	0.1562				1	0.5274	0.1562
		(-0.0062)	(<.0001)					(0.0062)	(<.0001)
CCC	1	-1.0721	0.2736				1	-1.0721	0.2736
		(<.0001)	(<.0001)					(<.0001)	(<.0001)
total	103	-0.6709	0.2118	88	-0.8648	0.2073	15	0.4663	0.2384
		(0.0032)	(<.0001)		(0.0004)	(<.0001)		(0.0199)	(<.0001)

Panel B: Industry Distribution

	ALL			Negative Betas			Positive Betas		
	N	beta	sigma	N	beta	sigma	N	beta	sigma
Basic Materials	10	-0.4853	0.1822	7	-0.9851	0.1908	3	0.6809	0.1623
		(<0.0001)	(<0.0001)		(<0.0001)	(<0.0001)		(<0.0001)	(<0.0001)
Consumer Goods	27	-0.8375	0.1997	25	-0.9455	0.1888	2	0.5111	0.3358
		(<0.0001)	(<0.0001)		(<0.0001)	(<0.0001)		(<0.0001)	(<0.0001)
Consumer Services	17	-0.4826	0.2213	12	-0.8904	0.2392	5	0.4960	0.1782
		(0.0018)	(<0.0001)		(<0.0001)	(<0.0001)		(0.0061)	(<0.0001)
Energy	10	-0.5919	0.2871	9	-0.6950	0.2966	1	0.3357	0.2008
		(0.0035)	(<0.0001)		(0.0039)	(<0.0001)		(<0.0001)	(<0.0001)
Healthcare	12	-0.6231	0.1969	12	-0.6231	0.19692			
		(<0.0001)	(<0.0001)		(<0.0001)	(<0.0001)			
Industrials	23	-0.6847	0.2159	19	-0.8875	0.1916	4	0.2786	0.3315
		(0.0116)	(<0.0001)		(<0.0001)	(<0.0001)		(0.0669)	(<0.0001)
Technology	3	-1.0540	0.1686	3	-1.0540	0.1686			
		(<0.0001)	(<0.0001)		(<0.0001)	(<0.0001)			
Telecom Services	1	-1.1286	0.1382	1	-1.1286	0.1382			
		(<0.0001)	(<0.0001)		(<0.0001)	(<0.0001)			

Table III-7: Characteristics of Firms with Different Betas

This table reports the characteristics of firms within different beta quantiles. *Leverage* equals total debt over total assets. *Current ratio* equals current asset over current liability. *Payout ratio* equals the sum of cash dividend and interest expense divided by the total assets. *Implied volatilities* are the call option implied volatilities from option market.

Beta Quantiles	Total	0-20%	21%-40%	41%-60%	61%-80%	81%-100%
Beta values	-2.34 ~ 1	-2.34 ~ -1.04	-1.04 ~ -0.98	-0.98 ~-0.81	-0.81 ~ -0.28	-0.22 ~ 1
N	103	20	21	21	21	20
Panel A: Means						
Leverage	0.3786	0.4133	0.3896	0.4068	0.3573	0.3249
Total Assets	4.34E+10	2.88E+10	4.17E+10	4.17E+10	3.69E+10	6.83E+10
Current ratio	1.4533	1.2913	1.3540	1.4734	1.5876	1.5577
Payout ratio	0.0104	0.0108	0.0079	0.0081	0.0136	0.0120
1Y CDS Spreads	31.3216	35.9334	33.8354	38.5367	22.2912	25.9762
3Y CDS Spreads	46.2438	55.7609	48.3903	54.8282	33.5563	38.7813
5Y CDS Spreads	61.0658	75.2904	63.3211	69.8178	45.1666	51.9776
7Y CDS Spreads	68.7625	84.8468	71.0693	77.1313	52.0681	58.9980
10Y CDS Spreads	77.2913	94.6554	79.5323	85.4650	60.1437	66.9970
Implied Volatility	0.2733	0.2440	0.2814	0.2984	0.2675	0.2737
Panel B: Medians						
Leverage	0.3784	0.4187	0.3620	0.4301	0.3784	0.2946
Total Assets	2.74E+10	1.69E+10	2.08E+10	2.91E+10	2.45E+10	4.87E+10
Current ratio	1.3024	1.1594	1.1604	1.2889	1.3927	1.4940
Payout ratio	0.0095	0.0112	0.0073	0.0084	0.0114	0.0114
1Y CDS Spreads	22.7694	26.2025	18.5574	23.8004	18.1137	20.1085
3Y CDS Spreads	32.1538	45.7060	30.5065	32.9205	27.7378	29.1998
5Y CDS Spreads	43.2735	61.0262	40.0791	43.2819	37.6122	38.6436
7Y CDS Spreads	49.4273	68.6341	47.6000	51.0469	43.8566	44.6378
10Y CDS Spreads	57.8791	78.6446	57.0330	59.8140	52.7728	52.5273
Implied Volatility	0.2737	0.2438	0.2858	0.2848	0.2737	0.2795

Figure III-4: Time Series of CDS Spreads

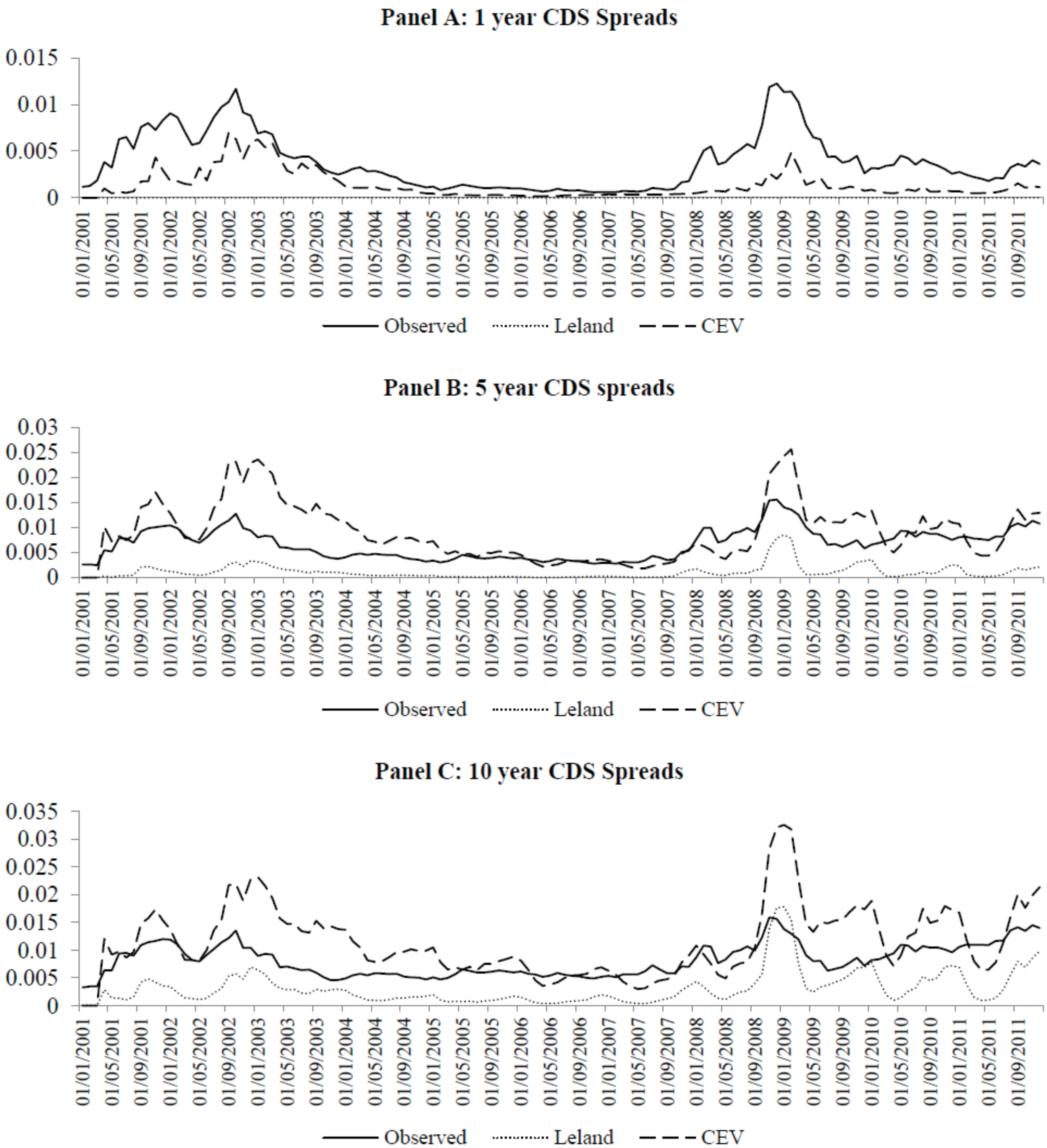


Figure III-5: Time Series of Equity Volatility

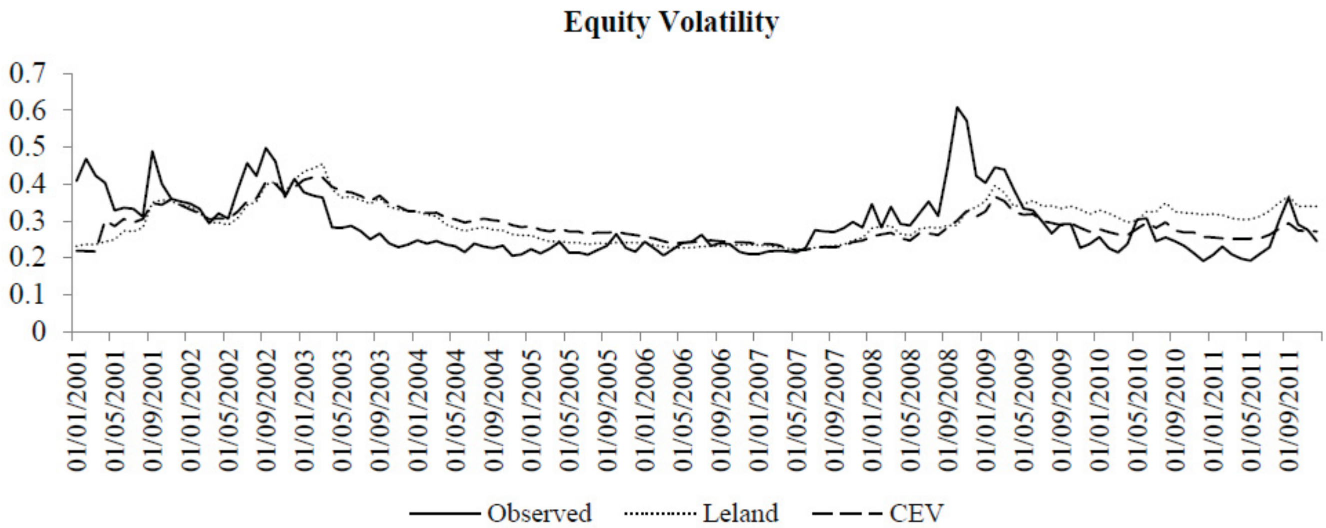
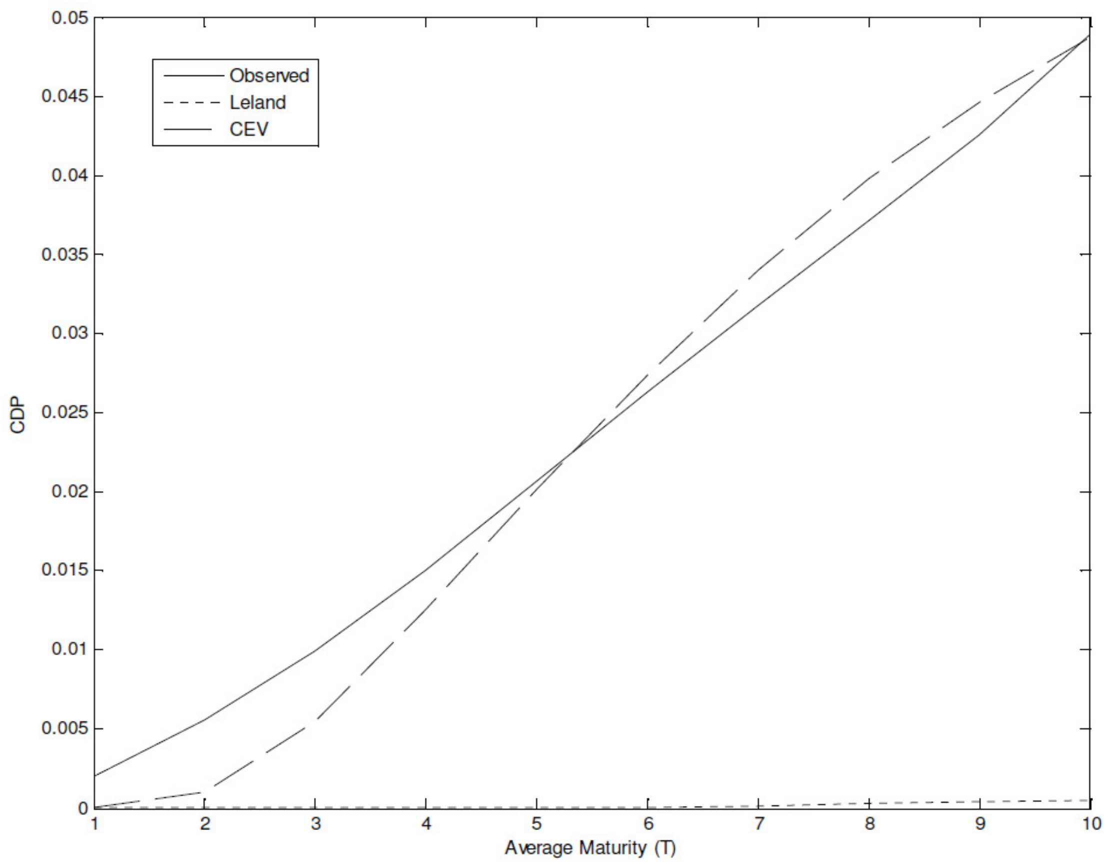


Figure III-6: Out of Sample Fitting for Historical Term Structure of CDP

This figure depicts the out of sample fitting for historical term structure of cumulative default probabilities (CDP) of Moody's Baa class. The value of σ and β come from the GMM estimation in Section 4.2. It is assumed that Moody's Baa equivalent to S&P's BBB. The average value of σ are equal to 15.29% and 18.67% for Leland and CEV structural models, respectively. The average value of β is -0.7037 for BBB class. The asset risk premium is assumed to be 4%. The exogenous default boundaries, risk free rates and payout rates are exactly same as the empirical data used in GMM estimation in Section 4.2 for BBB class.



Chapter IV MARKET EFFICIENCY AND DEFAULT RISK: EVIDENCE FROM THE CDS AND LOAN CDS

1. Introduction

The market for Loan Credit Default Swaps (LCDS), which are credit derivatives on syndicated secured loans, has grown rapidly since its 2006 launch until the recent Great Recession. Compared to traditional Credit Default Swap (CDS) contracts, LCDS contracts have higher recovery rates and cancellability options. Unlike non-cancellable LCDS (or US LCDS) that are generally used for trading purposes,[84] the protection buyer of a European LCDS stops paying a premium to the protection sellers once the loan is cancelled by a refinancing activity. Thus, the refinance rate and the LCDS spread are usually negatively correlated. Compared to selling the loan directly, banks can keep the loans on their balance sheets while transferring the credit risks of loans to third parties, usually large financial institutions. As the banks have no incentive to keep monitoring the loan actively after buying a LCDS contract, this causes what is referred to as the "Empty Creditor Problem".[85]

In this paper we take advantage of the fact that both CDS and LCDS contracts depend on the same credit event in order to examine the efficiency of their markets as reflected by the co-movement of their time series. Assuming market efficiency, we identify a model-free parity relation and we construct a zero-net-cost portfolio that trades simultaneously in both markets and uses observed CDS and LCDS spreads, as well as reported recovery rates in case of default from both types of contracts, We identify

[84] Merrill Lynch, *Credit derivative strategy*, Feb. 14, 2007.
[85] See Bolton and Oehmke (2011).

persistent and positive arbitrage profits from that portfolio over the data period, which imply that the observed CDS and LCDS data violates the parity relation. The arbitrage profits are large and exist for all rating classes of bonds in our data base.

We show that these profits cannot be justified by transaction costs or by imperfect information with respect to the recovery rates. Further, we identify a subset of firms that present persistent evidence of market failure in the form of extreme and persistent violations of the market efficiency condition. We analyze the determinants of both profits and market failure in terms of macroeconomic and firm-level variables. We find that firm-level effects, especially those related to informational asymmetry and difficulty of loan recovery in case of default, are much more important than macroeconomic factors in accounting for arbitrage profits and market failure.

Our paper contributes to the growing literature on violations of market efficiency in financial markets, which has been noted in several different venues. Since both CDS and LCDS contracts are derivatives that depend on the default risk of the borrowing firm, their spreads should be determined simultaneously with the valuation of the entire set of financial instruments that lay claim to the firm's cash flows. The theoretical models that value the firm's financial claims are known as *structural models* of the firm, pioneered by Merton (1974), with important contributions by Leland (1994) and Leland and Toft (1996). Nonetheless, the complexity of the structural models and the uncertainty of their parameters have limited their empirical applications to CDS valuation.[86] For this reason most violations of market efficiency have been documented in other derivatives markets, in equity option markets, in the pair of index and equity option markets, or in index

[86] Zhang, Zhou and Zu (2008) is one of the few studies that has adopted this approach.

futures options markets.[87] To our knowledge, this paper is the first one to document efficiency violations in credit default swap markets.

Most previous integrated studies of stock, bond, option and CDS markets have been purely empirical. Acharya and Johnson (2007) study the information flow between the CDS market and the stock market from the perspective of insider trading. They find that information revelation in the CDS market occurs only for negative credit news and for entities that subsequently experience adverse shocks, and increases with the number of a firm's relationship banks. Berndt and Ostrovnaya (2008) extend the work of Acharya and Johnson (2007) by incorporating the option market and find that prices of options reveal information about forthcoming adverse events at least as early as do credit spreads. Norden and Weber (2009) and Forte and Pena (2009) analyze the co-movements of credit default swap, bond and stock markets from real observed prices and implied CDS spreads, respectively. The main advantage of our study is that although it is consistent with structural models of the firm, it is completely model-free and depends only on market efficiency and the quality of the data.

If both CDS and LCDS contracts are written on the same firm, the claims are triggered by the same default events which are defined by the International Swap and Derivatives Association (ISDA). Thus, the default and survival probabilities of these credit derivatives should be exactly the same given the same maturities, restructuring clauses and denominated currencies. However, the claim sequences of CDS and LCDS are different, which leads to different recovery rates, for which estimates are provided in the respective data bases. Generally speaking, the syndicated secured loans which are the

[87] See for instance Goyal and Saretto (2009), Driessen *et al* (2009), and Constantinides *et al* (2011).

underlying assets of the LCDS have higher priority during the bankruptcy process compared to senior unsecured debts which are the underlying assets of CDS. Therefore, relatively higher recovery rate estimates of LCDS contracts are observed in the data bases (approximately 70% on average) compared to similar estimates for recovery rates on CDS contracts (around 40% on average). Further, the LCDS recovery rates are contingent on the values of the collateral assets, which are independent of the value of the borrowing firm, unlike the CDS rates.

Based on these observations, we construct CDS and LCDS parity under the no arbitrage assumption, which should hold in the absence of market frictions. With single name CDS and LCDS daily observations during the period from April 2008 to March 2012, we document a time-varying and significantly positive arbitrage profit generated by an artificial default risk-free portfolio that simultaneously longs the undervalued and shorts the overvalued contract based on CDS and LCDS parity. We also identify a subset of firms (hereafter termed "market failure set") for which the only recovery rate in the CDS market consistent with the parity relation is negative for continuous time intervals of at least ten days' length. In order to understand these observed parity deviations, we then address several follow-up research questions. First, do the observed arbitrage profits persist in the presence of transaction costs? Second, are the reported data on recovery rates in the data base reliable as estimates of "true" recovery rates upon default for both CDS and LCDS markets? Third, what is the impact on deviations in spreads from their parity relation of firm-specific and also macroeconomic variables, and the set of publications on North American LCDS released simultaneously by the ISDA? Last, to the extent that the deviations from parity cannot be explained from the above factors, what do these violations say about the efficiency of the CDS and LCDS markets?

To address the first two questions, we first compute the arbitrage portfolio by including transaction costs in the form of bid-ask spreads of the CDS market as reported in earlier studies and in the Bloomberg database. We find that the profits survive this inclusion, since the spreads are much lower than the estimated profits. To address the second question, we examine the real default data from a number of firms within our data base that failed as well as all the default events on the senior unsecured debts documented by Moody's default and recovery database during the study period. We find that the realized recovery rates vary widely between firms and the default types. We also compare the estimated recovery rates to other reported estimates from earlier studies for senior unsecured debt like the ones traded in the CDS market and find that the estimates in our data base are, if anything, rather conservative with respect to the existence of the arbitrage profits.

To address the third research question, we run panel regressions on arbitrage profits from violations of the parity relation on firm-specific and macroeconomic variables for the entire sample of firms, for several subsamples differentiated by rating class, and separately for the market failure set of firms. We include in the independent variables standard firm-specific variables like firm size, leverage, current asset ratio and tangible assets ratio. We also include the idiosyncratic volatilities calculated from the residuals of a Fama-French three-factor model. In the macroeconomic variables we include an important event during our sample period that would affect CDS and LCDS spreads and their deviations. The event consists of the positive news associated with the simultaneous release of a set of publications to regulate and standardize North American LCDS by ISDA on April 5, 2010. The results for the firm-specific variables are consistent with prior expectations on the basis of earlier studies about the determinants of the level and changes in CDS spreads[88] and their effects on information asymmetry, one of the most probable sources of the arbitrage profits. Specifically, we find that leverage turns out to be always positively correlated with profits, again an expected result since its effect on the default event is symmetric for both CDS and LCDS markets, while its effect on recovery rates is going to affect primarily the CDS market. The a priori effects of firm size and current asset ratio are less clear, since they impact both credit spreads and recovery rates. On the other hand, the idiosyncratic risk of firms turns out to be strongly positively associated with profits, an expected result given the fact that such risk is an indicator of information asymmetry. Equally interesting is the impact of the macroeconomic variables. The ISDA, by establishing global standards for LCDS contracts from the aspects of definition, qualification, settlements, continuity and documents, increased market efficiency in term of reducing the deviations between the two spreads in all the samples, but the effect is statistically significant at the conventional level only for the not-rated firms. On the other hand, variables associated with market downturns have a strongly positive effect on arbitrage profits for almost all samples. These findings provide some further indirect understanding of the dislocations caused in various markets by the collapse of Lehman Brothers[89] and adds some new insights to a

[88] Collin-Dufresne, Goldstein and Martin (2001), Ericsson, Jacobs and Oviedo (2009) and Cao, Yu and Zhong (2010).

[89] Some of the published studies include Baba and Packer (2009) who examine dislocations in the foreign exchange swap market; De Haas and Van Horen (2012) on the impact on cross-border bank participation in the syndicated loan market; and Aragon and Strahan (2012) on the impact on the provision of liquidity by hedge funds.

growing literature on the adverse (beneficial) effects of asset complexity (standardization) on price volatility and trade efficiency (e.g., Carlin, Kogan and Lowery, forthcoming).[90]

Last, we find that the failure firm subset has significant differences in the size of all firm-specific variables from the remaining firms in our sample, as well as in the levels of their respective CDS and LCDS spreads and recovery rates. On the other hand, the results of the panel regressions have much lower explanatory power for this subset, while not being noticeably different as to the size and significance of the coefficients. We conclude that the failure firm subset consists of small, heavily indebted firms with high idiosyncratic risk and, hence, likely to be subject to a significant degree of information asymmetry.

The rest of the paper is organized as follows. In Section 2 we present the CDS and LCDS parity and construct the trading strategy. In Section 3 we describe our sample datasets. In Section 4 we report and analyze the empirical evidence for the co-movement of CDS and LCDS markets for both the short and long runs and examine its robustness with respect to transaction costs and recovery rate estimates. In Section 5 we present the results of the panel regressions of the realized profits from our parity violations arbitrage strategy on the macroeconomic and firm-specific variables. Section 6 concludes.

2. EMPRICAL METHODOLOGY: THE TRADING STRATEGY

We construct a model-free trading strategy to detect whether an arbitrage[91] opportunity exists in the CDS and LCDS markets. According to the definition of a CDS

[90] Carlin, Kogan and Lowery (forthcoming) conclude that their experimental results imply that regulation requiring asset standardization should decrease price volatility and increase liquidity. Carlin and Manso (2011) explore the dynamic relationship between obfuscation and sophistication in retail financial markets accounting for the important role played by learning mechanisms within the investor population.

contract, the premium of a CDS contract (denoted by c) received by the protection seller (or paid by the protection buyer) has to make the present value of the expected premium leg equal to the present value of the expected default leg in order to rule out an arbitrage opportunity. This can be expressed mathematically as follows under continuous time,

$$c = \frac{(1-R)\int_t^T P^D(\tau \mid t)e^{-r\tau}d\tau}{\int_t^T P^S(\tau \mid t)e^{-r\tau}d\tau} \tag{2.1}$$

R denotes the recovery rate; $P^D(\tau \mid t)$ denotes the probability that a default event occurs at time τ for the first time conditional on the information at time t; and $P^S(\tau \mid t) = 1 - \int_t^{\tau} P^D(s|t)ds$ denotes the probability that a firm survives until time τ conditional on the information at time t. Letting

$$\int_t^T P^D(\tau \mid t)e^{-r\tau}d\tau \equiv G(T|t), \ \int_t^T P^D(s|t)ds \equiv F(T|t) \tag{2.2}$$

and integrating by parts, we find that the denominator of (2.1) is given by

$$\int_t^T P^S(\tau \mid t)e^{-r\tau}d\tau = \frac{e^{-rt}}{r} - \frac{e^{-rT}}{r}[1-F(T|t)] + \frac{G(T|t)}{r} \tag{2.3}$$

The expressions in (2.2) and (2.3) are given in particular structural models of the firm in terms of the parameters of the asset dynamics process.[92] The estimation of the parameters is done by calibrating the particular model to the observed spreads and to other observable variables of the model, as shown in the appendix. Nonetheless, the

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[91] In this chapter, "arbitrage" means the violation of parity which may or may not lead to profitable trading in the financial markets.
[92] See, for instance, Leland and Toft (1996, p. 990).

availability of the CDS and LCDS data sets allows the model-free examination of the two markets. Following the underlying logic of a structural model, the first passage default probability and the survival probability should only be driven by the distance between the firm's asset level and default boundary so that default risk and distance are inversely related. Thus, the US LCDS and traditional CDS issued on the same firm with the same default clause and maturity should share exactly the same first passage default probability and survival probability. If we denote the traditional CDS and US LCDS premiums by c_{CDS}, c_{LCDS} and recovery rates by R_{CDS}, R_{LCDS}, respectively, it follows that,

$$c_{CDS} = \frac{(1-R_{CDS})\int_t^T P^D(\tau \mid t)e^{-r\tau}d\tau}{\int_t^T P^S(\tau \mid t)e^{-r\tau}d\tau}, c_{LCDS} = \frac{(1-R_{LCDS})\int_t^T P^D(\tau \mid t)e^{-r\tau}d\tau}{\int_t^T P^S(\tau \mid t)e^{-r\tau}d\tau} \tag{2.4}$$

Thus, the following equality must be satisfied in order to rule out arbitrage opportunities given no market frictions and no errors in the recovery rate estimates,

$$c_{CDS} = c_{LCDS}\frac{1-R_{CDS}}{1-R_{LCDS}} \tag{2.5}$$

Thus, in the absence of transaction costs, the arbitrage payoffs of the portfolio constructed based on (2.5) can be computed as,

$$PR = \begin{cases} c_{CDS} - c_{LCDS}\dfrac{(1-R_{CDS})}{(1-R_{LCDS})} & if, c_{CDS} > c_{LCDS}\dfrac{(1-R_{CDS})}{(1-R_{LCDS})} \\ 0 & if, c_{CDS} = c_{LCDS}\dfrac{(1-R_{CDS})}{(1-R_{LCDS})} \\ c_{LCDS}\dfrac{(1-R_{CDS})}{(1-R_{LCDS})} - c_{CDS} & if, c_{CDS} < c_{LCDS}\dfrac{(1-R_{CDS})}{(1-R_{LCDS})} \end{cases} \tag{2.6}$$

In the presence of the one-way proportional transaction costs which are proportional to the nominal amount of contracts,[93] denoted by k, there is a non-trading zone $\mathbb{Z} = \left[\overline{c}_{CDS}, \underline{c}_{CDS}\right]$, where,

$$\overline{c}_{CDS} = \left(c_{LCDS} + k\right)\frac{1-R_{CDS}}{1-R_{LCDS}} + k, \underline{c}_{CDS} = \left(c_{LCDS} - k\right)\frac{1-R_{CDS}}{1-R_{LCDS}} - k \qquad (2.7)$$

If the observed CDS spreads fall in the non-trading zone $\mathbb{Z}$, there is no arbitrage opportunity. Otherwise, we are able to construct a trading strategy to generate arbitrage profits, which gives,

$$PR\_TC = \begin{cases} c_{CDS} - \left[\left(c_{LCDS} + k\right)\frac{1-R_{CDS}}{1-R_{LCDS}} + k\right] & if \quad c_{CDS} > \overline{c}_{CDS} \\ \left[\left(c_{LCDS} - k\right)\frac{1-R_{CDS}}{1-R_{LCDS}} - k\right] - c_{CDS} & if \quad c_{CDS} < \underline{c}_{CDS} \\ 0 & if \quad \overline{c}_{CDS} \le c_{CDS} \le \underline{c}_{CDS} \end{cases} \qquad (2.8)$$

Specifically, when the observed CDS spread $c_{CDS} < \underline{c}_{CDS}$, we buy one share CDS contract with \$1 notional amount and pay c_{CDS} premium continuously given that no default occurs and finance this transaction by shorting $\frac{1-R_{CDS}}{1-R_{LCDS}}$ shares of the US LCDS contract with \$1 notional amount per contract. If a default event occurs, we receive $\left(1-R_{CDS}\right)$ dollars from the CDS leg contract and pay $\frac{1-R_{CDS}}{1-R_{LCDS}} * \left(1-R_{LCDS}\right) = \left(1-R_{CDS}\right)$ dollars to the holder of the US LCDS leg. Given no estimation risk of recovery rates, this

[93] Given a CDS contract with 1\$ notional value and premium c, we have to pay $\left(c+k\right)$ when we buy and receive $\left(c-k\right)$ when we sell it.

portfolio can be considered as default risk free. It can, however, generate a positive profit at the initial time. When the observed CDS spread $c_{CDS} > \overline{c}_{CDS}$ positive arbitrage profits can be generated by selling CDS contracts and buying corresponding LCDS contracts. In the following sections, we will test the violation of this equality by the observed CDS and LCDS spread and estimated recovery data.

3. SAMPLE AND DATA

The CDS market has existed for a long time but the LCDS market was launched in 2006 in both US and Europe. We obtain our CDS and LCDS data from Markit who collects the quotes on LCDS spreads from large financial institutions and other high quality data sources and produces the LCDS spread database on a daily basis starting from April 11, 2008. Our sample is from April 11th, 2008 to March 30th, 2012, which encompasses the credit crisis and the Great Recession. Since the LCDS contracts can be divided into US LCDS and Euro LCDS based on the embedded cancellable feature, we only use US LCDS to construct the portfolio in order to keep our analysis model free.

In the CDS market, the contracts on senior unsecured debts are selected since this type of contract is the most liquid and is used frequently in the literature. In the LCDS market, the contracts on the first-lien syndicated loans are selected since the claims on collateral for the first-lien loans are senior to those of the second-lien loans, which indicate more reliable estimated recovery rates. In addition, the LCDS contracts on first-lien loans are the majority and more liquid compared to those on the second-lien loans. We restrict our CDS and LCDS contracts to those in the United States and denominated in US dollars. To ensure that the first-passage default and survival probabilities of the

CDS contracts are exactly the same as those of the corresponding LCDS, we match the daily LCDS and CDS data based on company name, denominated currency, restructure clauses and time to maturity. We only study the contracts with a 5-year maturity since they are the most liquid contracts and the most studied in the previous literature.[94] Markit also reports the estimated recovery rates obtained from their clients. These recovery rate expectations at time of issue may differ from subsequent recovery-rate expectations and actual recovery rates, especially during bad economic times.[95] Nevertheless, these data represent the only available proxy for the real recovery rates[96] (especially for LCDS contracts) and have been used in previous studies.[97] Table IV-1 reports the summary statistics for our full sample and sub-samples. We eliminate the observations whose CDS spreads (or LCDS spreads) are greater than 1 and the single name contracts which have less than 120 consecutive daily observations. In addition, we obtain the accounting variables from COMPUSTAT, economic macro variables from Federal Reserve H.15 database and equity information from Bloomberg. After merging all these datasets and removing the missing observations and private firms, the full sample contains 68,147 firm-clause-daily cross-sectional observations for 120 single names during the sample period from April 11, 2008 to March 30, 2012.

[Insert Table IV-1 about Here]

[94] See Jorion and Zhang (2007), Cao, Yu and Zhong (2010, 2011), Schweikhard and Tsesmelidakis (2011), Qiu and Yu (2012) and Zhang, Zhou and Zhu (2009).
[95] Jokivuolle and Peura (2003), Altman, Brady, Resti and Sironi (2005), Hu and Perraudin (2002) and Chava, Stefanescu and Turnbull (2006) report that the recovery and default rates are negatively correlated.
[96] The real recovery rates are collected from Moody's Default and Recovery Database and discussed in section 4.
[97] See Huang and Zhu (2008), Zhang, Zhou and Zhu (2008), and Elkamhi, Ericsson and Jiang (2012).

In the full sample, the mean LCDS and CDS spreads are around 3.7% and 4.6%, respectively. Both medians are smaller than their corresponding means which indicate asymmetric distributions and fat tails, especially on the right side. These style factors are also verified by positive skewness and high kurtosis for the CDS and LCDS spreads. The distributions of recovery rates for the LCDS and CDS contracts are close to a Gaussian distribution with slightly negative skewness. Both the mean and median of the LCDS recovery rates, around 65% and 70% respectively, are greater than the corresponding statistics for the CDS contracts, around 38% and 40% respectively. Intuitively, the syndicated secured loans which are the underlying asset of LCDS are usually backed up with collateral and have claim priority compared to the senior unsecured debts which are the underlying asset that backs the CDS once the default event occurs. The sub-sample of investment grades (includes firms rated greater than or equal to BBB), accounts for more than 60% of the total observations, while junk rated contracts and not rated contracts share almost equally the rest of the observations, approximately 20% each. As expected, both the mean and median of the CDS and LCDS spreads in the investment grade sub-sample are relatively lower, while the mean and median of the recovery rates are relatively higher compared to the junk subsample. In terms of the accounting variables, the average values in the full sample are around 25 million for total assets, 61% leverage ratio, 54% tangible asset ratio and 1.54 current ratio. Compared to the statistics of the investment grade firms, the means for the junk firms are higher for the leverage ratio and lower for total assets, tangible asset ratios and current ratios. The not rated firms are mostly relatively small firms in terms of their total assets. Their leverage ratios, tangible asset ratios and current ratios are diverse.

The daily idiosyncratic volatilities[98] of the full sample have a mean around 2.4% with positive skewness and extremely high kurtosis. As expected, both the mean and median of daily idiosyncratic volatilities of the investment grade firms are relatively lower than those of junk rated firms. For the not rated firms, the daily idiosyncratic volatilities are more volatile compared to the other sub-samples.

4. THE EFFICIENCY OF THE CDS AND LCDS MARKETS

In this section we examine the violation of CDS and LCDS parity constructed in Section 2 at the index and firm levels, respectively. The results are first presented in the absence of both transaction costs and uncertainty of recovery rates, and then subsequently extended by the inclusion of these two missing items.

4.1 Trading Strategies

Following the CDS and LCDS parity in the presence and in the absence of transaction costs discussed in Section 2, we first examine the relationship in (2.6) and (2.8) with the observed CDS and LCDS data. Figure IV-1 reports the distribution of trading strategies with and without transaction costs in Panel A and Panel B, respectively.

[Insert Figure IV-1 about Here]

Interestingly, without transaction costs the CDS and LCDS parity in (2.5) does not hold at all, which implies huge arbitrage profits in these markets in the absence of recovery rate risk. Generally, buying CDS contracts and selling corresponding LCDS contracts can generate positive arbitrage profits, which indicates that the LCDS spreads are overpriced compared to the corresponding CDS spreads, especially for the not-rated

[98] The calculation details are provided in Section 5.

single name contracts. But there is still around a 23% percent chance to make positive profits by selling the CDS contracts and buying corresponding LCDS contracts in the full sample.

In the presence of transaction costs, we observe that only approximately 19% of the cross-sectional observations in the full sample cannot generate positive arbitrage profits. There are still a large number of opportunities to make positive arbitrage profits, especially for the not-rated single name contracts. Similar to the case without transaction costs, buying CDS contracts and selling the corresponding LCDS contracts dominates the opposite trading strategy.

4.2 Portfolio Results without Transaction Costs and Uncertainty of Recovery Rates

In this subsection it is assumed that the CDS and LCDS markets are frictionless and that the recovery rates reported by the Markit database are the "real recovery rates" once the default events occur. As noted earlier, we construct the default-free portfolio by checking the equality of equation (2.5). There is arbitrage opportunity if the equality does not hold. Particularly, we simultaneously buy the CDS contract and sell the weighted LCDS contract provided $c_{CDS} < c_{LCDS} \frac{1-R_{CDS}}{1-R_{LCDS}}$ and *vice versa*.

Relying on the CDS and LCDS parity with the no-arbitrage assumption, we build the portfolio for each single name contract on a daily basis and analyze the payoff based on the cross sectional observations. The summary statistics are presented in Panel A of Table IV-2. The average trading profit across all observations is approximately a daily 3.75% indicating that the portfolios constructed by the contracts which violate the CDS and LCDS parity are able to generate a 3.75% arbitrage profit per day per single name contract on average over the whole sample period. This is incredibly large compared to the average daily returns in the traditional equity and bond markets. Note that the high standard deviation, 7.5%, and high kurtosis, 117.53, confirm that the mean may be driven by some outliers. Nevertheless, the 1.6% median return which is not affected by extreme values is still noticeable large and positive on a daily basis.

[Insert Table IV-2 about Here]

We now divide the full sample into three sub-samples based on rating status. The junk-rated contracts generate relatively lower returns in terms of both mean and median compared to the investment-grade contracts, while the not-rated contracts generate the highest profits among all the sub-samples. This can be interpreted intuitively from the perspective of asymmetric information. Compared to the rated firms, the not-rated firms likely have higher asymmetric information effects since they release less information to the markets. In turn, this can be expected to reduce the market efficiency between the CDS and LCDS markets and increase the deviations from the parity relation between these two markets, resulting in the large arbitrage profits for the arbitrage portfolio for these firms.

As the time span of the single name contracts varies, the cross-sectional average puts more weight on the firms with a longer life. In order to remove this bias, we first calculate the daily average profit for each single name across the life of the contract and then present the statistical properties of the sample reported as “Firm Daily Average Profits” in Panel A. The distribution is much closer to the Gaussian with 4.5% mean and

2.5% median daily return, which are even greater than those based on the cross-sectional observations.

In order to check the time trend of the arbitrage profits, we aggregate the trading profits per day across all the available paired single name contracts and then divide by the total number of single name contracts per day to construct a payoff index. Suppose there are N_t pairs of single name contracts on day t. The payoff of each pair i on day t is denoted by r_{it}. The payoff index return R_t on day t is then expressed by,

$$R_t = \frac{1}{N_t} \sum_{i}^{N_t} r_{it} \tag{3.1}$$

As expected, the distribution of index returns is almost Gaussian for all the samples. Similar to the cross-sectional results, the average profit for the not-rated sub-sample dominates in terms of the mean and median the rated sub-samples but also has the highest standard deviation, 3.3%. The investment grade single names generate lower profits than the junk-rated single names. The time trend of daily average profits of the different samples can be observed visually in Figure IV-2. In the full and Investment grade samples we note that the arbitrage profits are relatively higher during the great recession period from mid-2008 to late 2009, compared to the rest of the periods, and gradually decrease in recent years. The junk-rated and not-rated samples have significantly higher volatilities than the investment grade firms. As mentioned before, both junk-rated firms and not-rated firms are small firms in terms of total assets. They have relatively lower tangible ratios which make them more vulnerable, especially under turbulent financial market environments. This style factor turns into the higher volatilities for these two sub-samples compared to the investment grade firms.

4.3 Portfolio Profits Given Transaction Costs

The liquidity problem in the CDS market has been studied from different perspectives in the literature. Acharya and Johnson (2007) document that the median CDS bid-ask spread is around 20 basis points using CDS quotes for the most widely traded North American entities for the period from January 1, 2001 to October 20, 2004.[99] With the CDS data from 1997 to 2006, Tang and Yan (2007) report that the bid-ask spread is approximately 22 basis points on average.[100] Note that the time spans in these studies do not overlap with our sample period, which covers the Great Recession and afterwards. The Great Recession starting from 2008 affected the CDS market dramatically and may have adversely affected its liquidity. Since the Markit database only provides the composite quotes for the CDS and LCDS spreads, we match the single names in our sample with the Bloomberg database and find that 66 out of 120 firms are quoted in the Bloomberg historical CDS dataset.[101] The bid-ask quote information is retrieved during the period from January 2nd, 2008 to November 23rd, 2012, which covers the time span in our study. Table IV-3 reports the summary statistics of both firm average and daily average bid-ask spreads. The median of the daily average bid-ask spread at around 18 basis points is a little lower but close to the numbers documented by Acharya and Johnson (2007) and Tang and Yan (2007). The positive skewness and extremely high kurtosis imply fat tails, especially on the right. This turns into a relatively high mean of around 35 basis points in contrast to the median.

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[99] Cite from Table 1 on page 117 in Acharya and Johnson (2007).

[100] The bid-ask spread is computed by combining the summary statistics results in Table 1 and Table 3 in Tang and Yang (2007).

[101] As Bloomberg does not provide the information about restructuring clauses, we can only match with firm name and we need to assume that the restructure clauses are the same as the single name contracts in the Markit database.

[Insert Table IV-3 about Here]

Intuitively, the one way transaction cost represented by one-half of the quoted bid-ask spread in the LCDS market should be greater than its counterpart in the CDS market since the LCDS market is relatively smaller and less liquid. However, we are unable to identify the real bid-ask spread in the LCDS market.

Panel B of Table IV-2 reports the numerical results in the presence of time-varying bid-ask spreads for both the firm and index levels. Compared to the scenarios in the absence of transaction costs (Panel A in Table IV-2), both mean and median of the profits decrease for all samples but are still significantly positive. Specifically, the full cross-sectional sample generates 3.38% in average profits. In terms of time trend, the profits in the presence of transaction cost are very similar to those in the absence of transaction costs, which can be observed visually in Figure IV-2. Therefore, the significantly positive arbitrage profits survive the introduction of transaction costs.

[Insert Table IV-4 about Here]

If we now assume that all the arbitrage profits are caused by relatively less liquidity, we can calculate the value of transaction costs which make the arbitrage profits equal to zero. The computation is straightforward and can be expressed as,

$$\hat{k}_{it} = \begin{cases} \dfrac{c_{LCDS}\left(1-R_{CDS}\right)-c_{CDS}\left(1-R_{LCDS}\right)}{2-R_{CDS}-R_{LCDS}} & if, c_{CDS} < \dfrac{c_{LCDS}\left(1-R_{CDS}\right)}{\left(1-R_{LCDS}\right)} \\ \dfrac{c_{CDS}\left(1-R_{LCDS}\right)-c_{LCDS}\left(1-R_{CDS}\right)}{2-R_{CDS}-R_{LCDS}} & if, c_{CDS} > \dfrac{c_{LCDS}\left(1-R_{CDS}\right)}{\left(1-R_{LCDS}\right)} \end{cases} \qquad (3.2)$$

The summary statistics for the implied round-trip transaction cost $2\hat{k}_{it}$, are reported in Table IV-4. The average implied transaction cost for the cross-sectional observations of around 200 basis points is more than ten times the realized bid-ask spreads documented for the same sample period. These results show that transaction costs can only explain a small portion (approximately 10%) of the observed abnormal positive profits generated by the portfolio.

4.4 Uncertainty of Recovery Rates and Market Failure

After the introduction of transaction costs, there are still about 90% of the abnormal positive profits that remain unexplained. Note that we have assumed that the estimated recovery rates reported in the Markit datasets are reasonable proxies for the "real recovery rates" in the presence of default events. Generally, the real recovery rates depend on the type of default events and can only be observed once the default events occur. For instance, if a default event is triggered by missing an interest payment, the real recovery rate is usually higher than if the default event is triggered by the filing of Chapter 11 or Chapter 7. Hence, the uncertainty of the real recovery rates should be a source of risk that may explain the observed abnormal profits from an arbitrage portfolio between the CDS and LCDS markets.

i Real recovery rates versus estimated recovery rates

In the LCDS market, the underlying assets are the Line 1 syndicated secured loans with collateral. The value of the collateral is easier to estimate and incorporate into the estimation of the LCDS recovery rates. In addition, syndicated secured loans have claim priority compared to the senior unsecured debts which are the underlying assets of the

CDS markets in bankruptcy. Thus, the estimated recovery rates should be a good proxy for the "real recovery rates" of LCDS contracts.

In order to check the quality of the estimated recovery rates in the CDS markets obtained from Markit, we try to match the single names in our sample with those in the Moody's Default and Recovery Database which documents almost all the historical default events and their corresponding real recovery rates. We find that four firms have default events on senior unsecured debts during the period from April 11, 2008 to March 30, 2012. The detailed firm list and statistics of real recovery rates are reported in Table IV-5. The mean and median estimated recovery rates are calculated for all days prior to the default event, and the real recovery rate is calculated as the percentage of debt market value one month after the default event divided by the face value of the debt. The real recovery rates are much higher than the estimated recovery rates in term of both mean and median.

[Insert Table IV-5 about Here]

As the real recovery rate sample is too small to make conclusive conclusions, we collect all the observed default events on the senior unsecured debts (1535 observations) during the sample period from April 11, 2008 to March 30th, 2012 and report the summary statistics in Panel B. Apparently, these results verify that the real recovery rates depend on the type of default events. We note that around half of the default events are triggered by "Distressed Exchanges".[102] As distressed exchanges can trigger default

[102] Distressed exchange is the substitution of a bond by its issuer with a financial asset of lower value. A distressed exchange was possible but sometimes not likely to trigger a default event in the CDS market. As of April 2009, distressed exchanges are no longer considered a default event (see Altman and Karlin, 2009).

events for some CDS contracts with modified-restructuring (MR) clauses, Modified-modified restructuring (MM) clauses under certain circumstances and sometimes not,[103] we also report the average real recovery rates after excluding the distressed exchanges. Compared to the real recovery rates, the estimated recovery rates are much higher in term of both the mean and median. Given the LCDS recovery rates, buying CDS contracts and selling LCDS contracts generates much more profits when the real recovery rates are low, which is generally the case for our trading strategies.

Next, we benchmark our recovery rates to those reported in the literature. Acharya, Bharath and Srinivasan (ABS, 2007, pp. 797-798) provide recovery rate data from a sample of defaulted firms for the 18-year period ending in 1999. They report median rates of 91.55%, 61.99% and 54.63% for bank loans, senior secured debt and senior unsecured debt, respectively. The first two recovery rates correspond to our LCDS and the last one to our CDS. The average recovery rates for senior secured bank loans and senior unsecured bonds are 70.47% and 36.69%, respectively, during the period from 1982 to 2007, as reported in Moody's special comment.[104] The results from Moody's special comment (1982-2007) and observed defaults including distressed exchanges in Panel B of Table IV-5 (2008-2012) are much closer to the estimates given in our Markit data than the ABS results. They preserve in the entirety or even increase the estimated arbitrage profits that we report.

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[103] During the current 2012 negotiations regarding the restructuring of Greek sovereign debt, one important issue is whether the restructuring will trigger CDS payments. ECB and IMF negotiators are trying to avoid these triggers as they may jeopardize the stability of major European banks who have been protection writers. (Source: http://en.wikipedia.org/wiki/Credit_default_swap#Terms_of_a_typical_CDS_contract)

[104] Moody's special comment, "Corporate Default and Recovery Rates, 1920-2007", February 2008.

ii Implied recovery rates

Although we verified empirically that on average the real recovery rates are smaller but close to the estimated recovery rates, deviations between the real recovery rates and estimated recovery rates of the CDS contracts vary dramatically among the individual cross-sectional observations. To verify the cross sectional and time series sources of this variability, we now check the implied recovery rates in the presence of transaction costs by setting the profits of the arbitrage portfolio equal to zero. Mathematically, the implied recovery rates can be computed as,

$$\hat{R}_{CDS} = \begin{cases} 1-\dfrac{c_{CDS}-k}{c_{LCDS}+k}\left(1-R_{LCDS}\right) & if \quad c_{CDS} > \overline{c}_{CDS} \\ 1-\dfrac{c_{CDS}+k}{c_{LCDS}-k}\left(1-R_{LCDS}\right) & if \quad c_{CDS} < \underline{c}_{CDS} \\ R_{CDS} & if \quad \overline{c}_{CDS} \le c_{CDS} \le \underline{c}_{CDS} \end{cases} \tag{3.3}$$

Where $\hat{R}_{CDS}$ is the implied recovery rate of the CDS contract. Compared to the daily average of estimated recovery rates, the daily average implied recovery rates that are reported in Panel A of Figure IV-3 are greater and also more volatile.

[Insert Figure IV-3 and Table IV-6 about Here]

Theoretically, the recovery rates cannot be negative because of the limited liability of the debt holder. However, we observe some negative implied recovery rates and some of them even last for periods as long as a couple of months. According to (3.3), the observed CDS and LCDS spreads, LCDS recovery rates and the transaction costs affect the implied CDS recovery rates directly. In order to identify the most important of these factors in terms of the negative implied recovery rates, we collect all the turning days on which the implied CDS recovery rates become negative (First Day). Thus, each such identified implied CDS recovery rate is positive on the day before the turning day (1 Day Before). Table IV-6 reports the means and medians of all the variables which might affect or be affected by the implied recovery rates. Based on the two-sides Wilcoxon two sample test, we observe that the LCDS recovery rates decrease significantly on the turning days and their spreads also decrease on those days (albeit with less than conventional significance). Intuitively, when the LCDS recovery rates decrease, the corresponding LCDS spreads should increase because the loans have become more risky. Such counter-intuitive market behavior supports the failure of the LCDS market as reflected in the negative implied CDS recovery rates. Both CDS spreads and recovery rates increase but not significantly at conventional levels. Since the CDS spreads depend on both default probabilities and recovery rates (see (2.4)), we test for changes in the spread ratios defined as c_{CDS}/c_{LCDS} and recovery rates ratios defined as $(1-R_{CDS})/(1-R_{LCDS})$ in order to rule out any impacts on default probabilities. Not surprisingly, the spread ratios significantly increase but the recovery rates ratios decrease, which indicate market failure. In the presence of such market failure, the mean profit of our trading strategy on the first day of about 5.7% is significantly greater (almost double) the mean profit on the one day before of approximately 2.9%. We also observe a decrease in equity returns, CDS spreads and an increase of CDS recovery rates, but none of them are significant at conventional significance levels.

iii Market Failure

Given market frictions, negative implied recovery rates could appear occasionally but should disappear after a reasonable period of time because prices should adjust quickly in a well-functioning market. In this section, we define 19 single name contracts in our sample as market failure contracts because they have ten or more consecutive days (approximately two weeks) on which their respective implied CDS recovery rates are negative.

[Insert Table IV-7 about Here]

Comparison statistics of failure and non-failure firms are reported in Table IV-7. Compared to the non-failure firms, the failure firms are usually small firms in terms of total asset value with relatively higher leverage on average. The idiosyncratic volatilities of the failure firms are noticeably higher than those of non-failure firms. We also note dramatically higher CDS spreads and slightly higher LCDS spreads for the failure firms compared to the non-failure firms, although their CDS and LCDS recovery rates are very similar. Based on the results reported in Panel C of Table IV-7, all of the means and medians of the differences in these variables between failure and non-failure firms with the exception of median difference of the LCDS spreads is significant at the 1% level. Given these differences in firm-specific characteristics, the trading strategy of selling CDS contracts and buying the corresponding LCDS contracts of failure firms dominates all other strategies for non-failure and failure firms, as depicted in Figure IV-4. This indicates that the CDS spreads are generally over-priced for the failure firms compared to their corresponding LCDS spreads. We also note that the market failure behaviors reflected by the negative implied recovery rates are clustered and occur with greater frequency during the 2008 financial crisis, as depicted in Figure IV-5.

[Insert Figure IV-4 and Figure IV-5 about Here]

After removing all the market failure firms from the full sample, we find that the implied recovery rates become less volatile and are consistently above the estimated recovery rates with only one (inverted) spike during May 2011, as shown in Panel C of Figure IV-3. It appears that the uncertainty of the implied CDS recovery rates is mostly contributed by the failure firms.

5. IMPACT OF MACRO AND FIRM-SPECIFIC VARIABLES

The outcomes of the risk-free portfolio differ among the single names and are also time-varying. In this section, we study the impacts of different macro-economic and firm-specific variables on the outcomes of the arbitrage portfolios in the presence of transaction costs. As the outcomes of the arbitrage portfolios depend on the CDS and LCDS spreads and recovery rates, we list the most important factors explaining the levels and changes of credit spreads reported in the existing literature, including Collin-Dufresne, Goldstein and Martin (2001), Acharya, Bharath and Srinivasan (2007), Acharya and Johnson (2007), and Cao, Yu and Zhong (2010). Then we refine the list based on multicolinearity[105] and data availability to arrive at the subsequently discussed variables whose correlations are reported in Table IV-8.

[Insert Table IV-8 about Here]

5.1 Firm specific variables

Logarithm of total assets (LOGA): the logarithm of total asset value. The total asset value equals the sum of book value of total liabilities and the market value of total equity

[105] For instance, we use the yields on 5-year US treasury bonds since both CDS and LCDS contracts in our sample have five years to maturity. We use the spread between the yields on Aaa and Baa corporate bonds (CBS) and eliminate the VIX because we find that these two variables are highly correlated and the CBS has better explanatory power than VIX.

including traded equity and non-traded equity. The total asset value is used to control the size impacts on arbitrage profits. The relationship between total assets and arbitrage profits is ambiguous.

Current asset over current liability ratio (CAL): this ratio equals current assets divided by current liabilities. This ratio indicates the ability of a firm to pay short term debt with current assets (i.e., assets that should be easier to liquidate). The current ratio is expected to be inversely related with the default probability. Lower default probabilities are expected to decrease both the CDS and LCDS spreads. As the arbitrage profit measures the deviations between the CDS and LCDS markets, the impact of CAL on arbitrage profits is indeterminate.

Leverage ratio (LEV): this ratio equals total liabilities divided by total assets, which indicates the capital structure of a firm. Collin-Dufresne, Goldstein and Martin (2001) find that the change of the leverage ratio is positively correlated with the change of credit spreads for groups with leverages greater than 15%,[106] while Acharya and Johnson (2007) find that the leverage ratio has an insignificant impact on the level of the credit spread.[107] Economically, firms with high leverage ratios will have both higher CDS and LCDS spreads due to an increased probability of default. As the impact on the spread moves in the same direction for both markets, it may not change the deviation between these two markets. However, as highly levered firms issue more debt but with less equity backing, the recovery rates of the senior unsecured debts should decrease much more compared to the syndicated secured loans which usually have collateral backing and claim priority.

[106] Table 2 on page 2186 in Collin-Dufresne, Goldstein and Martin (2001).
[107] Table 10 on page 136 in Acharya and Johnson (2007).

Such a decrease of CDS recovery rates are expected to increase the profits of the portfolio. Overall, we expect the leverage ratio to be positively correlated with the arbitrage profits.

Tangible assets (TANG): this variable equals the total value of property, plant and equipment divided by total assets. Acharya and Johnson (2007) find that the tangible asset ratio increases the credit spread using Fama-MacBeth regressions but has an insignificant effect using panel regressions. As tangible assets can be considered as a collateral proxy, the recovery rates of CDS should be higher with higher tangible asset ratios. Furthermore, tangible assets are much easier to estimate and monitor compared to intangible assets, making the estimate of recovery rates much closer to real recovery rates. Higher expected recovery rates of CDS contracts are expected to decrease the expected profits of the arbitrage portfolio. Hence, we expect to have a negative coefficient for the *TANG* factor.

Idiosyncratic volatilities (IDIO): The idiosyncratic volatilities are the conditional volatility of equity return residuals which cannot be explained by the asset-pricing model. For the calculation, we collect the daily closing equity price, denoted by p_{it}, for firm i at day t and calculate the daily returns by $r_{it} = p_{it} / p_{it-1} - 1$. We run the following regression using the Fama-French three-factor model to get the residual ε_{it},

$$r_{it} - rf_t = \alpha_t + \beta_1 \left(R_t - rf_t \right) + \beta_2 SMB + \beta_3 HML + \varepsilon_{it} \tag{4.1}$$

The idiosyncratic volatilities, $\sqrt{h_{it}}$, which are the conditional volatilities of the residuals, are filtered by an EGARCH model, given as follows,

$$\varepsilon_{it} = \xi_{it}\sqrt{h}_{it}, \xi_{it} \sim N\left(0, \sqrt{h}_{it}\right)$$
$$\ln(h_{it}) = \omega + \beta\left[\theta\xi_{it-1} + \gamma\left(\left|\xi_{it-1}\right| - E\left|\xi_{it-1}\right|\right)\right] + \alpha\ln(h_{it-1}) \tag{4.2}$$

The idiosyncratic volatilities can be considered as measures of the firm-specific noise. Idiosyncratic volatility is used as a measure of pricing uncertainty or price informativeness. While some empirical researchers argue that greater idiosyncratic return volatility is an indicator of more informative stock prices (e.g., Brockman and Yan, 2009), most argue that it is an indicator of less informative stock prices due to more noise and pricing errors and greater asymmetric information (e.g., Chen, Huang and Jha, 2012; Krishnaswami and Subramaniam, 1999). Lee and Liu (2011) reconcile these two opposing views by empirically documenting that the relation between idiosyncratic volatility and price informativeness is either U-shaped or negative. We conjecture that higher idiosyncratic volatility is associated with lower market efficiency, since idiosyncratic noise generally reflects firm-specific factors which indicate increased information asymmetry. Thus, we expect that higher idiosyncratic volatilities are associated with increased arbitrage profits.

5.2 Macro variables

Publication of ISDA dummy (ISDA): As an administrator of the globally agreed standards of credit default swaps, the International Swaps and Derivative Association (ISDA) became more proactive after the sub-prime financial crisis and released a series of publications providing guidance and standards to try to protect investors and improve the efficiency of the CDS market. As our analysis focuses on the relative efficiency of the CDS and LCDS markets (especially the US market), we examine the impact of the release on April 5, 2010 of a series of documents published by the ISDA regarding the

North American Loan CDS market. The documents include the "Bullet Syndicated Secured Loan Credit Default Swap Standard Term Supplement", "Bullet Syndicated Secured Loan Polling Rules", "Bullet LCDS Auction Rules and LCDS Auction Settlement Terms" and "Bullet LCDS Continuity Procedures".[108] Further details on each publication are found in the appendix. Overall, these rules and supplements established the global standards for LCDS contracts from the aspect of definition, qualification, settlements, continuity and documentation. Thus, we expect that the LCDS market should become more efficient with standardization and that the deviation from efficiency should be narrowed after this event date. The *ISDA* dummy variable is equal to zero before the publication of ISDA to regularize and standardize the LCDS market, including the publication day and equals to one after this event date. Hence, we expect a negative coefficient for this dummy variable.

5-year US treasury bond yield (TB5Y): is the yield on US 5-year treasury bonds. This bond yield is usually considered as a risk-free rate and also as an indicator of the US economy. Lower interest rates usually coincide with a weakening economy as the government keeps interest rates at a low level to try to stimulate the economy. In a recession, recovery rates are generally low and the credit spread is generally high. From a different perspective, Longstaff and Schwartz (1995) point out that the static effect of a lower spot rate decreases the risk-neutral drift of the firm value process, which, in turn, increases the probability of default.[109] Since a high credit spread caused by an increase of default probabilities should affect the CDS and LCDS markets in the same direction, the net impact on the deviation of these two markets should be small. However, the decrease

[108] http://www.isda.org/publications/isdacredit-deri-def-sup-comm.aspx#ra
[109] This result is also verified by Duffee (1998).

of CDS recovery rates will increase the deviations, which should increase the arbitrage profits. Therefore, we expect a negative coefficient for this factor.

Slope of the term structure (SL): this variable is measured by the difference between the yields on 5- and 1-year US treasury bonds. The slope of the term structure is one of the most important factors documented by Litterman and Scheinkman (1991). A low slope indicates low forward interest rates, which means that the level of the current spot interest rate is high. For a higher interest rate level, the theory predicts a lower default probability and a higher recovery rate. Thus, higher CDS recovery rates indicate lower arbitrage profits and the expectation of a positive coefficient for this factor.

Yield spread between Aaa and Baa corporate bonds (CBS): this variable equals the difference between the yields of Aaa corporate bonds and Baa corporate bonds. The increase of the spread between these corporate bonds indicates an increase of credit spreads, especially the CDS spreads as CDS contracts usually have lower ratings compared to the same-firm LCDS. Thus, the yield spread between Aaa and Baa corporate bonds can be considered as the spread between CDS and LCDS. Intuitively, an increase in their yield spread will increase the probability of violations of CDS and LCDS parity and lead to higher arbitrage profits.

Return of the S&P 500 total return index (SP): this is the daily return on the S&P 500 total return index. This is an economic indicator from the perspective of the equity markets. Intuitively, positive S&P 500 returns imply a strong economy with a low default probability and high recovery rates. Thus, we would expect to have a negative coefficient for this factor.

The accounting variables, including total assets, book value of total liabilities, market value of equity, current assets, current liabilities and tangible assets, are obtained from the COMPUSTAT database via the WRDS platform. The data are updated quarterly. We convert the frequency from quarterly to daily by keeping the value constant within each quarter and then take a one quarter lag. The fixed income macro variables, including the yields on 1- and 5-year US treasury bonds, and Aaa and Baa corporate bond yields are obtained from the US Federal Reserve H15 database. The equity prices and S&P 500 total return index data are obtained from Bloomberg.

5.3 Regression results

The panel regression results are reported in Table IV-9. Overall, the combination of the firm specific variables and macro variables is able to explain 63.9% of the deviations between the CDS and LCDS markets in the presence of transaction costs on average. The lowest R-square of 39.20% is observed for the regression for the failure firm sub-sample followed by an R-square of 46.81% for the regression for the investment grade sub-sample. The R-squares for the junk-rated, not rated and non-failure subsets are all above 79%. With the exception of the S&P 500, whose coefficient is not significant for all subsets, the coefficient estimates of all the independent variables are significant for most subsets. The signs of the estimated coefficients that are significant are generally consistent across the subsets.

The leverage ratio consistently increases the profits of the arbitrage portfolio for all the samples apart from the not-rated firms and exhibits greater sensitivity for failure firms. A higher idiosyncratic volatility increases the arbitrage profits significantly across most of the samples except for the junk and failure firms. In particular, for the investment grade firms the arbitrage profits increase by 0.37% on average for every 1% increase in idiosyncratic volatility. We also note that the impact of idiosyncratic volatility is much greater for the non-failure firms compared to that for the failure firms. In addition, although the logarithm of asset value has an expected positive relationship with arbitrage profits, its impact is unclear for the different sub-samples as all the coefficients are not significant.

[Insert Table IV-9 about Here]

For the macro factors, only the yield on 5-year US treasury bonds (TB5Y), the slope of the yield curve for treasury bonds (SL) and the spreads between Aaa and Baa corporate bonds (CBS) are significant at the conventional level for the full sample. The coefficients of the ISDA publication dummy are not significantly different from zero expect for the not-rated sub-sample. Compared to the rated firms, the ISDA regulations seem to be more important for not-rated firms in terms of the magnitude of their reduction in arbitrage profits of around 0.65% after the ISDA publications. According to Table IV-2, we note that the not-rated sample has the highest volatility for the profits compared to others. Since the purpose of the ISDA publications is to standardize and regulate the LCDS markets, their effect should be much more important for the samples with the most volatile profits, as in our empirical findings. With the exception of the failure firms, the variables associated with the state of the economy generally have their expected impact on arbitrage profits, with economic downturns corresponding to higher profits. The impact of the yield of 5-year US treasury bonds on arbitrage profits is consistently negative for all sub-samples but not significant for not-rated and failure samples. Numerically, a 1% increase in the yield of 5-year US treasury bonds decreases arbitrage profits by 0.51% on average for the full sample. The slope of the yield curve for treasury bonds positively affects arbitrage profits (it is not significant only for the failure sub-sample), which is consistent with theoretical predictions. The spread between the yields on Aaa and Baa corporate bonds (CBS) significantly increases the profits on the arbitrage portfolio for the full sample. Since an increase in the yield spread is generally associated with an economic downturn, this is consistent with our predictions. Further, we note that the impact of CBS is more important for the not-rated firms compared to the rated firms. The sign of the S&P 500 total index return coefficient is negative and insignificant for the full sample and all the sub-samples.

5.4 Robustness test

The separate contributions of the macro and firm-specific factors are studied by conducting regressions for restricted models and the results are reported in Table IV-10. For the restricted model of fixed effects only, the goodness of fit of about 58.18% is slightly lower than that of the unrestricted model of about 63.90%. This implies that the cross sectional effect is much more important than the time series effect. Then, we restrict the model with only one firm-specific factor at a time in order to detect the maximum contribution of individual factors. As expected, *LOGA* and *LEV* are very important to the arbitrage profits. Interestingly, we find *IDIO* is also noticeably important, which emphasizes the role of information asymmetry in arbitrage profits. We also find that the coefficient of *LOGA* is now negative unlike its sign in the full sample due to its highly negative correlation with both *LEV* and *IDIO*. Hence, after removing the impact of *LEV* and *IDIO*, the relationship between *LOGA* and arbitrage profits becomes negative (consistent with expectations), indicating that arbitrage profits decrease with higher total assets.

[Insert Table IV-10 and Table IV-11 about Here]

Although more than half of the coefficients of the macro factors are significantly different from zero, their contribution to arbitrage profits is much smaller compared to that of the firm-specific factors. Numerically, the maximum contribution of all macro factors is only approximately 1.87% in term of R-square. In other words, the marginal contribution of macro variables to the explanation of abnormal profits is very small compared to that of firm specific factors.

In addition, we also check the robustness of our findings when we reduce the frequency of our time series data. In these exercises, the daily profits are aggregated into weekly, monthly and quarterly time intervals and the panel regressions are repeated for the full sample in each case. The results are reported in Table IV-11. As the table shows, both the sign and (most of the times) the significance levels of the coefficients are very robust with respect to the level of aggregation, but their magnitude depends on the frequency of the data.

In summary, the contribution of the cross sectional effect on arbitrage profits dominates that of the time series effect. The firm-specific factors, especially firm size, leverage ratio and idiosyncratic volatility, are much more important than the macro factors in explaining the observed arbitrage profits. Our findings are very robust even with lower frequency data.

6. CONCLUSION

We identify a model-free parity relation between CDS and LCDS contract spreads under the no arbitrage assumption in the absence of market frictions, as well as a non-trading zone in the presence of market frictions. The parity relation uses only the observed CDS and LCDS spreads and the recovery rates of the underlying contracts in the event of default. We then examine whether these relations hold in a sample of paired CDS and LCDS contracts for exactly the same underlying firm, maturity, currency and restructure clauses. We document extensive violations of this parity relation, implying a time-varying and significant positive arbitrage profit from artificial default risk-free portfolios that simultaneously long the CDS contract and short the corresponding LCDS contract for each pair or vice-versa, depending on the direction of the violation of the parity.

We verify whether the arbitrage profits are robust with respect to the inclusion of proportional trading costs. We find that such costs can explain at most 10% of the profits. We then examine the reliability of the recovery rates reported in the data base and used in the parity relation. Using reported recovery rates from defaulted firms, including some in our data base, we find that the reported recovery rates are, if anything, overestimates of the true ones and understate the assessed arbitrage profits.

Given the inability of trading costs or recovery rate data to explain the observed arbitrage profits, we examine the possibility of inefficiency or market failure in the CDS-LCDS market pair. We construct artificial implied recovery rates for CDS under the assumption that parity holds in the presence of trading costs, and we identify a large number of data points for which these implied rates are negative. We also find a subset of firms for which CDS implied recovery rates are negative for at least ten consecutive days (the failure firm subset). We find that the failure firm subset differs significantly from the other firms in our data set in their levels of CDS and LCDS spreads, in their recovery rates, and in the size of every firm-specific variable used in our tests.

We use panel regressions of the arbitrage profits in the presence of trading costs against macroeconomic and firm-specific variables. We disaggregate the sample between bonds of various rating classes, as well as between failure and non-failure firms. We find that there are some significant differences in the sign and significance of the coefficients and the explanatory power of the regressions between failure and non-failure firms, and also between bonds of different rating classes.

We find that the contribution of the cross sectional effect to arbitrage profits dominates that of the time series effect. We find that firm-specific factors are much more important than various macro variables in the explanation of the observed arbitrage profits identified herein. These firm-specific factors measure a high level of indebtedness and difficulties of recovery in case of default (leverage ratio) and large firm size. Other important explanatory factors associated with arbitrage profits are those related to measures of pricing uncertainty or informational asymmetry. We find that the standardization of contracts and clarifications of the contract rules reduce profits (measure of informational asymmetry), while a firm's idiosyncratic volatility (measure of pricing uncertainty) from a Fama-French three-factor model is associated with positive profits.

Overall, we conclude that there is evidence of market inefficiency and failure of arbitrage to equalize the spreads in the CDS and LCDS markets. The failure is more prevalent in times of financial crisis, but is also present under more normal circumstances. Further, there is significant evidence that information asymmetry is an important contributing factor to this market failure. Last but not least, on the methodological side there is evidence that firm-specific variables play an important role in the determination of CDS and LCDS spreads, implying that structural models of the firm are more appropriate than reduced models to rationally value these spreads.

Appendix

A: Some Details about the North American Loan CDS Documentation Published on April 5, 2010 by the ISDA[110]

Document Name	Abstract
Bullet Syndicated Secured Loan Credit Default Swap Standard Terms Supplement	*"This template is designed to document credit default swap transactions where the Deliverable Obligations are limited to Syndicated Secured Loans of the Reference Entity. This form is primarily intended for use in the North American market. The contract: (a) has a "bullet" maturity, i.e. not subject to acceleration in the case where the Reference Entity's loans are repaid; (b) is subject to a credit event determination by a Determinations Committee; (c) provides for auction settlement if the Participating Dealers vote to hold an auction under the Bullet LCDS Auction Rules in relation to a Reference Entity and Designated Priority; and (d) contains specific rules and procedures for determining Successors to the Reference Entity (the procedures are contained in the Bullet LCDS Continuity Procedures). If no auction is held or the auction fails or is abandoned, Physical Settlement will apply to LCDS transactions under the most recently-published form of LSTA Physical Settlement Rider, which is available from the LSTA's website."*
Bullet Syndicated Secured Loan Polling Rules	*"This document contains the rules and procedures that apply to determine whether a loan qualifies as a "syndicated secured" loan of the Reference Entity, for purposes of the syndicated secured list."*
Bullet LCDS Auction Rules and LCDS Auction Settlement Terms	*"The Bullet LCDS Auction Rules and LCDS Auction Settlement Terms are designed to facilitate the settlement of Bullet Syndicated Secured Loan Credit Default Swap transactions."*
Bullet LCDS Continuity Procedures	*"The Bullet LCDS Continuity Procedures contain the procedural rules for determination of a Successor under the Bullet LCDS documentation*."

[110] The abstracts are quoted from ISDA website:http://www.isda.org/publications/isdacredit-deri-def-sup-comm.aspx#ra

B: Restructuring Clause[111]

Restructuring Clause	Details
Cum Restructuring (CR) or old restrucuring	Any Restructuring event is qualified as a credit event and any bond of maturity up to 30 years is deliverable. (1999 ISDA credit derivative definition)
Modified Restructuring (MR)	Restructuring events are considered as a credit event and the bonds with maturity of 30 months or less after the termination date of the CDS contract are deliverable. (2001, ISDA credit derivative definition)
Modified-Modified Restructuring (MM)	Restructuring events are considered as a credit event and the bonds with maturity of 60 months or less for the restructured obligations and 30 months for all the other obligations after the termination date of the CDS contract are deliverable. (2003, ISDA credit derivative definition)
Ex-Restructuring (XR) or without restructuring	All the restructuring events are not considered as a credit event.

[111] See Packer and Zhu (2005) and Berndt, Jarrow and Kang (2006).

Table IV-1: Summary Statistics

This table reports the summary statistics for the full sample and sub-samples during the period from April 11th, 2008 to March 30th, 2012. The idiosyncratic volatilities are the conditional daily volatilities of individual equity return residuals by fitting the Fama-French three-factor model. The total asset equals the sum of book value of total liabilities and market value of total equities. Leverage equals book value of total liabilities divided by the total asset value. Tangible ratio equals the book value of tangible assets divided by the total asset value. The current ratio equals the current asset divided by the current liabilities.

	CDS Spreads	CDS Recovery Rates	LCDS Spreads	LCDS Recovery Rates	Idiosyncratic Volatility	Total Asset (Thousands)	Leverage	Current Ratio
Full Sample (No. of Observations: 68147)								
minimum	0.0027	0.0125	0.0001	0.0750	0.0053	446.51	0.0834	0.2398
maximum	0.9651	0.7050	0.8984	0.9775	0.8429	295142.56	0.9857	5.8299
mean	0.0461	0.3817	0.0367	0.6523	0.0237	25193.94	0.6045	1.5413
median	0.0311	0.4000	0.0243	0.7000	0.0203	13565.72	0.6126	1.4112
standard deviation	0.0684	0.0567	0.0482	0.1128	0.0172	37329.28	0.1842	0.6878
skewness	6.7356	-1.8116	6.4265	-0.7765	10.8217	4.17	-0.2464	1.3736
1st Order Autocorrelation	0.9783	0.7730	0.9690	0.9273	0.8162	0.9904	0.9890	0.9874
Investment Grades (Above and include BBB, No. of Observations: 41327)								
minimum	0.0027	0.0722	0.0001	0.0750	0.0053	446.5	0.0834	0.2398
maximum	0.9453	0.6750	0.8671	0.8500	0.6991	295142.6	0.9746	5.2277
mean	0.0418	0.3809	0.0331	0.6386	0.0229	31707.0	0.5912	1.4944
median	0.0252	0.4000	0.0210	0.6750	0.0194	17942.9	0.5964	1.3890
standard deviation	0.0614	0.0506	0.0497	0.1152	0.0161	40757.4	0.1834	0.6286
skewness	5.4439	-3.2517	6.7415	-0.6919	8.5820	3.7	-0.2199	1.1164
1st Order Autocorrelation	0.9659	0.7369	0.9346	0.8946	0.7962	0.9594	0.9709	0.9748
Junk (Below BBB, No. of Observations: 11665)								
minimum	0.0028	0.0125	0.0001	0.3250	0.0054	446.51	0.0834	0.2398
maximum	0.9651	0.7050	0.6253	0.8500	0.5781	257135.61	0.9794	5.2277
mean	0.0543	0.3750	0.0384	0.6349	0.0240	23330.80	0.6341	1.4500
median	0.0391	0.4000	0.0257	0.6500	0.0215	13154.38	0.6582	1.3259
standard deviation	0.0765	0.0556	0.0416	0.1136	0.0155	38118.25	0.1939	0.6880
skewness	7.5180	-2.3062	3.0163	-0.6069	8.1152	4.50	-0.4206	1.8634
1st Order Autocorrelation	0.8493	0.6214	0.8495	0.8364	0.6903	0.8697	0.8719	0.8730
Not Rated (No. of Observations: 15155)								
minimum	0.0037	0.0188	0.0001	0.1000	0.0056	450.51	0.1008	0.2398
maximum	0.9463	0.7050	0.8984	0.9775	0.8429	249734.43	0.9857	5.8299
mean	0.0515	0.3889	0.0449	0.7030	0.0255	8867.08	0.6178	1.7393
median	0.0363	0.4000	0.0349	0.7250	0.0218	5739.02	0.6364	1.6201
standard deviation	0.0781	0.0707	0.0476	0.0882	0.0209	15260.94	0.1746	0.7961
skewness	7.4978	-0.2131	7.5601	-1.0831	13.8551	11.73	-0.1909	1.2968
1st Order Autocorrelation	0.7864	0.7350	0.8066	0.8261	0.6336	0.8855	0.8320	0.8339

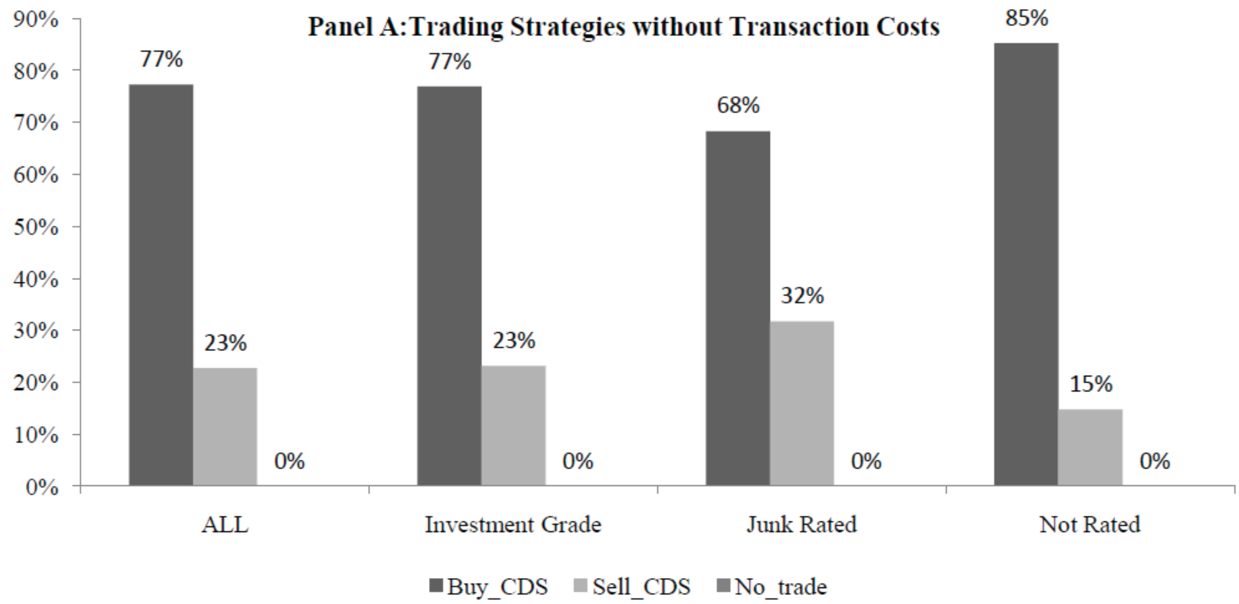
Table IV-2: Summary Statistics of Trading Profits

This table reports the summary statistics of the trading profits generated by the risk-free portfolio when the CDS and LCDS parity is violated for the cross-sectional daily observations, firm daily average across the time span and index daily across all the available firms during the sample period from April 11th, 2008 to March 30th, 2012. Panel A reports the results in the absence of transaction cost and Panel B reports the results in the presence of transaction costs. It is assumed that the transaction costs are same under CDS and LCDS market. The daily transaction costs come from the daily average bid-ask spread observed in Bloomberg database with the sample firms in Table IV-3.

Panel A: Profits in the absence of Transaction Costs							
	Minimum	maximum	Mean	Median	Standard Deviation	Skewness	Kurtosis
Cross-Sectional Daily Observations (68147 Observations)							
Full Sample	0.0000	1.6514	0.0375	0.0160	0.0754	8.4029	118.5388
Investment	0	1.6514	0.0301	0.0118	0.0656	11.3542	229.9689
Junk	3E-07	0.5186	0.0326	0.0142	0.0532	3.7779	18.3441
Not Rated	8.3E-06	1.3179	0.0615	0.0389	0.1043	5.7209	42.1163
Firm Daily Average Profits (120 Firm-Clause Contracts)							
Full Sample	0.0006	0.6853	0.0452	0.0250	0.0830	5.5332	36.6491
Index Daily Profits (959 Daily Observations)							
Full Sample	0.0214	0.0950	0.0369	0.0319	0.0134	1.1235	0.5672
Investment	0.0082	0.0823	0.0291	0.0253	0.0134	1.0790	0.7642
Junk	0.0036	0.1495	0.0350	0.0298	0.0213	1.4305	2.4680
Not Rated	0.0210	0.1499	0.0608	0.0500	0.0333	0.8264	-0.5001
Panel B: Profits in the presence of Transaction Costs							
	Minimum	maximum	Mean	Median	Standard Deviation	Skewness	Kurtosis
Cross-Sectional Daily Observations (68147 Observations)							
Full Sample	0.0000	1.6471	0.0338	0.0124	0.0740	8.5618	123.3978
Investment	0	1.6470	0.0266	0.0079	0.0649	11.6478	238.8615
Junk	0	0.5149	0.0292	0.0106	0.0527	3.8685	19.0626
Not Rated	0	1.2905	0.0568	0.0344	0.1015	5.7636	42.8601
Firm Daily Average Profits (120 Firm-Clause Contracts)							
Full Sample	0.0000	0.6572	0.0413	0.0210	0.0811	5.4707	35.5509
Index Daily Profits (959 Daily Observations)							
Full Sample	0.0166	0.0855	0.0332	0.0288	0.0120	1.0370	0.3901
Investment	0.0057	0.0781	0.0257	0.0225	0.0122	1.0986	1.1091
Junk	0.0012	0.1466	0.0315	0.0264	0.0207	1.4661	2.7208
Not Rated	0.0135	0.1386	0.0562	0.0462	0.0316	0.8427	-0.4417

Figure IV-1: Distribution of Trading Strategies

This figure depicts the distribution of trading strategies with and without transaction costs for the cross-sectional daily observations of the full sample during the sample period from April 11th, 2008 to March 30th, 2012.



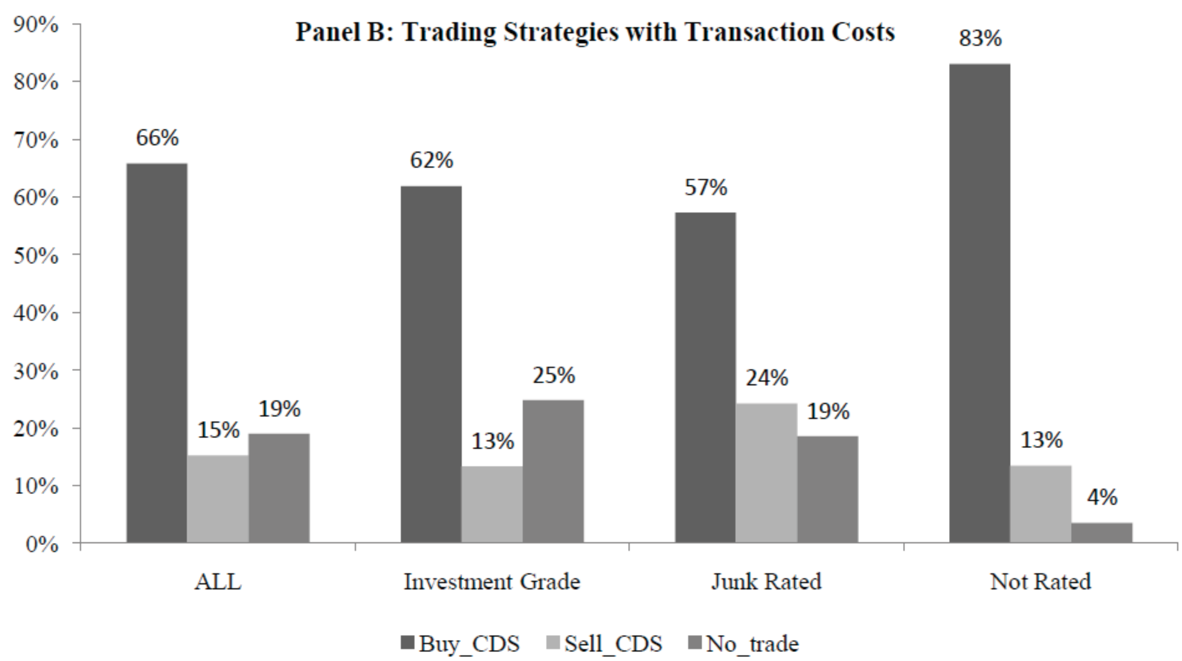
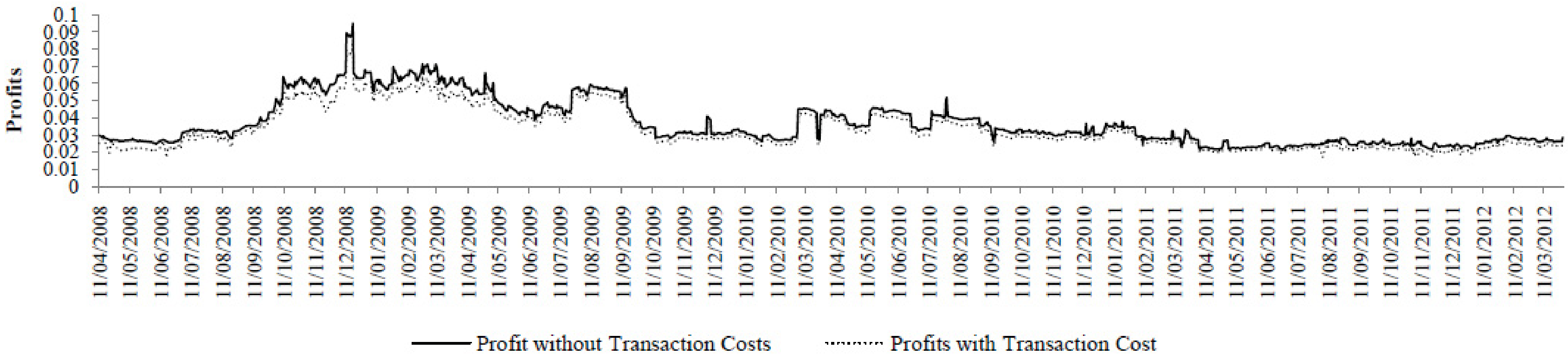
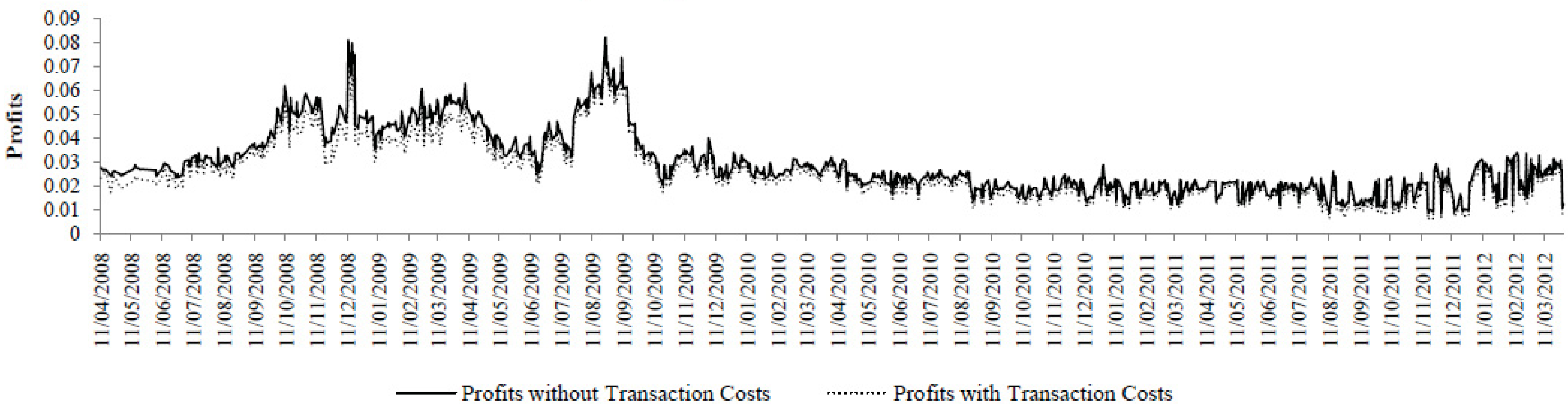


Figure IV-2: Daily Average Profits

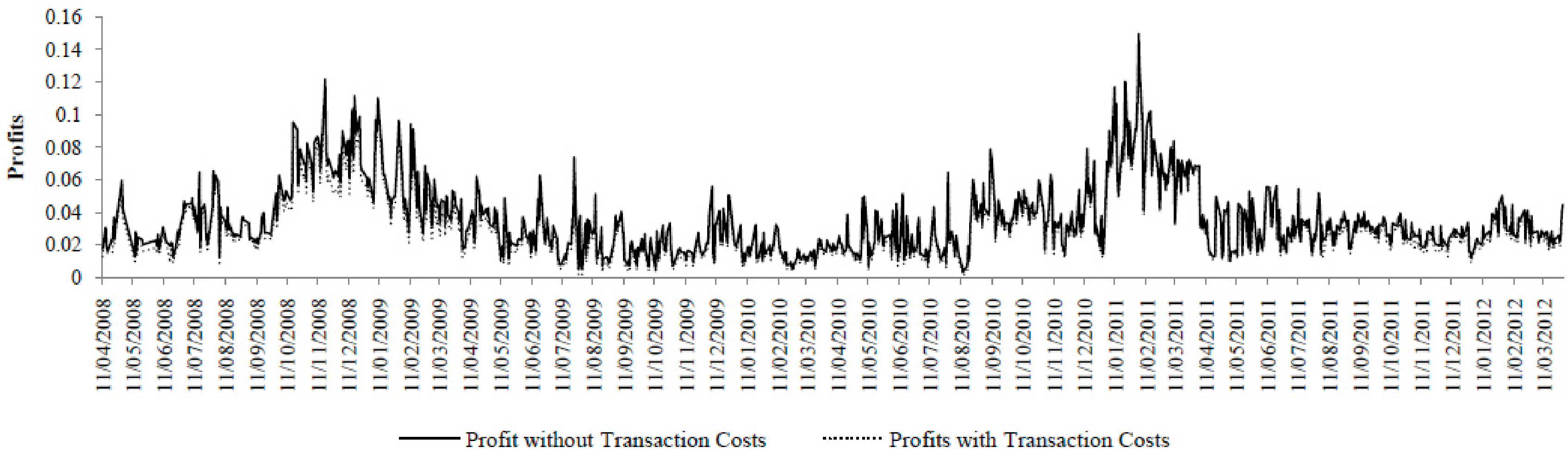
Panel A: Daily Average Profits of Full Sample



Panel B: Daily Average Profits of Investment Grades Contracts



Panel C: Daily Average Profits of Junk Rated Contracts



Panel D: Daily Average Profits of Not Rated Contracts

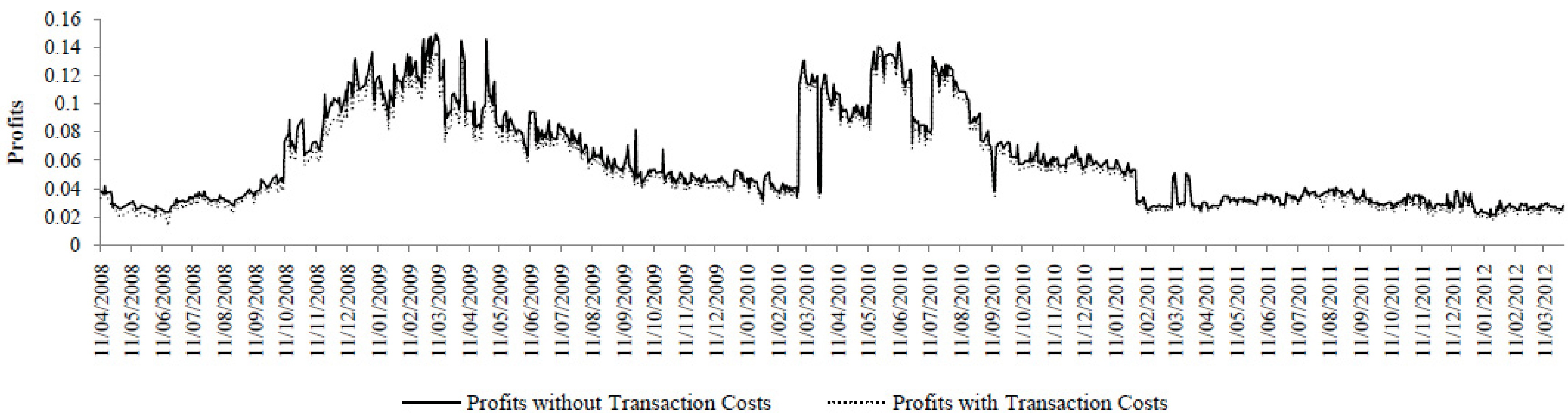


Table IV-3: Summary statistics of bid-ask spreads (Unit: basis points)

This table reports the summary statistics of bid-ask spreads. The *Firm Averages* shows the average bid-ask spread for each firm during the period from January, 2nd, 2008 to November 23rd, 2012 upon the data availability. The *Daily Average* shows the average bid-ask spread for each day across all the available firms. The unit is basis points.

	Firm Average	**Daily Average (Cross Firms)**
Minimum	3.76	4.50
Maximum	283.24	93.23
Mean	35.15	26.13
Median	17.68	21.62
Standard Deviation	47.35	14.83
Skewness	3.28	1.89
Kurtosis	13.27	3.31
No. of Observations	61 Firms	1219 Days

Table IV-4: Summary Statistics of Implied Transaction Costs in the absence of Profits

This table reports the summary statistics of the implied transaction costs under which the CDS and LCDS parity is not violated for the cross-sectional daily observations (Panel A), firm daily average across the time span (Panel B) and index daily across all the available firms (Panel C) during the sample period from April 11th, 2008 to March 30th, 2012. It is assumed that the transaction costs are same under CDS and LCDS market. The transaction costs showed in the table are round trip transaction cost (Bid-Ask spread) in basis points.

Panel A: Cross-Sectional Daily Observations (69805 Observations)							
	maximum	Minimum	Mean	Median	Standard Deviation	Skewness	Kurtosis
Full Sample	10254.70	0.00	199.89	73.49	404.40	8.97	159.88
Investment	10254.70	0.00	175.04	55.08	429.77	10.58	194.06
Junk	2547.22	0.00	172.70	44.46	350.95	3.61	14.41
Not Rated	6401.14	0.00	288.59	227.50	356.07	5.91	71.13
Panel B: Firm Daily Average Profits (141 Firm-Clause Contracts)							
	maximum	Minimum	Mean	Median	Standard Deviation	Skewness	Kurtosis
Full Sample	2032.92	0.00	202.52	116.02	288.07	3.46	16.24
Panel C: Index Daily Profits (1036 Daily Observations)							
	maximum	Minimum	Mean	Median	Standard Deviation	Skewness	Kurtosis
Full Sample	509.57	101.39	196.01	162.59	74.94	1.24	0.49
Investment	513.04	17.46	168.73	156.59	69.85	1.21	2.54
Junk	826.96	19.63	191.45	159.44	119.70	1.25	1.75
Not Rated	1084.12	111.29	283.12	216.36	168.70	1.78	2.53

Table IV-5: Real Recovery Rates versus Estimated Recovery Rates

This table reports the means and medians of both real recovery rates and estimated recovery rates of the firms which have default events documented by Moody's Default and Recovery Database during the sample period from April 11th, 2008 to March 30th, 2012. The real recovery rates (Real RR) are the bonds market value one month after default events divided by the face value of the bonds retrieved from Moody's Default and Recovery Database. The estimated recovery rates are the CDS recovery rates estimated by the data provided and reported in Markit datasets. Both real recovery rates and estimated recovery rates are all on senior unsecured bonds.

Panel A: Observed Defaults in the Sample

Ticker	Default Type	Observations Before Default	Real RR (mean)	Estimated RR (mean)	Real RR (median)	Estimated RR (median)
CCU	Distressed exchange	284	0.32	0.25	0.32	0.11
GGC	Missed interest payment	207	0.35	0.18	0.35	0.40
LVLT	Distressed exchange	123	0.96	0.33	0.96	0.36
UIS	Distressed exchange	185	0.99	0.21	0.99	0.16
Total		799	0.58	0.25	0.35	0.24

Panel B: All Moody's Observed Defaults of Senior Unsecured Debts

Default Types	No. of Observed Default Issues	Real RR (Mean)	Real RR (Median)	Real RR (Standard Deviation)
Bankruptcy	7	0.3464	0.3650	0.0748
Chapter 11	436	0.1377	0.1000	0.1185
Chapter 7	1	0.0053	0.0053	
Distressed exchange	799	0.5439	0.6500	0.2284
Missed interest payment	91	0.2794	0.2775	0.2034
Missed principal and interest payments	40	0.1905	0.0850	0.1707
Missed principal payment	8	0.3685	0.2500	0.2357
Payment moratorium	8	0.2036	0.2100	0.0215
Placed under administration	1	0.0700	0.0700	
Prepackaged Chapter 11	119	0.2404	0.1100	0.2818
Receivership	5	0.2650	0.2650	0.0000
Seized by regulators	16	0.0319	0.0300	0.0040
Suspension of payments	4	0.2040	0.2075	0.0540
Total	1535	0.3687	0.2975	0.2756
Total(without Distressed Exchange)	736	0.1785	0.1000	0.1800

Figure IV-3: Index of Implied Recovery Rates

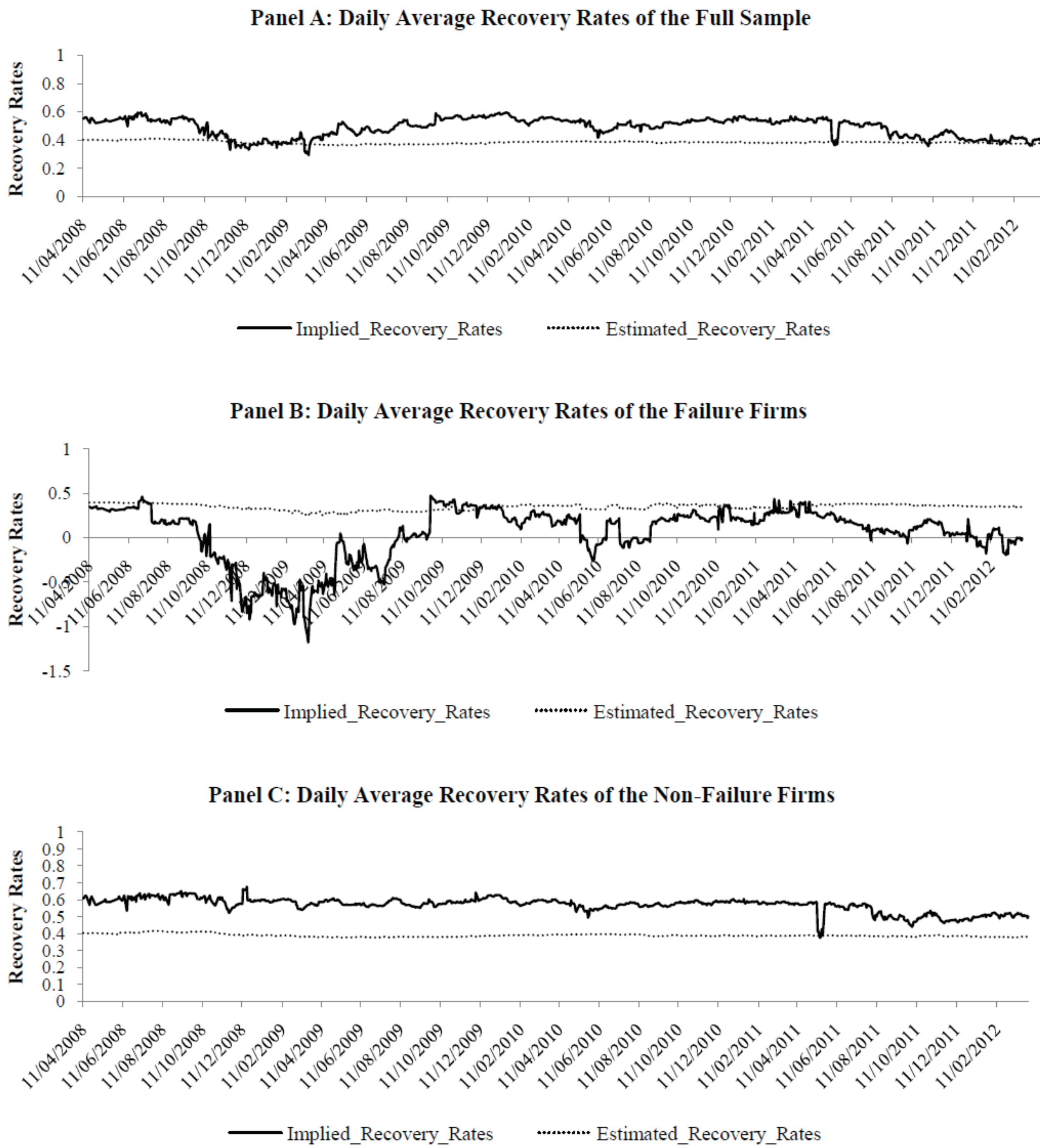


Figure IV-4: Trading Strategies of Failure and Non-Failure Firms

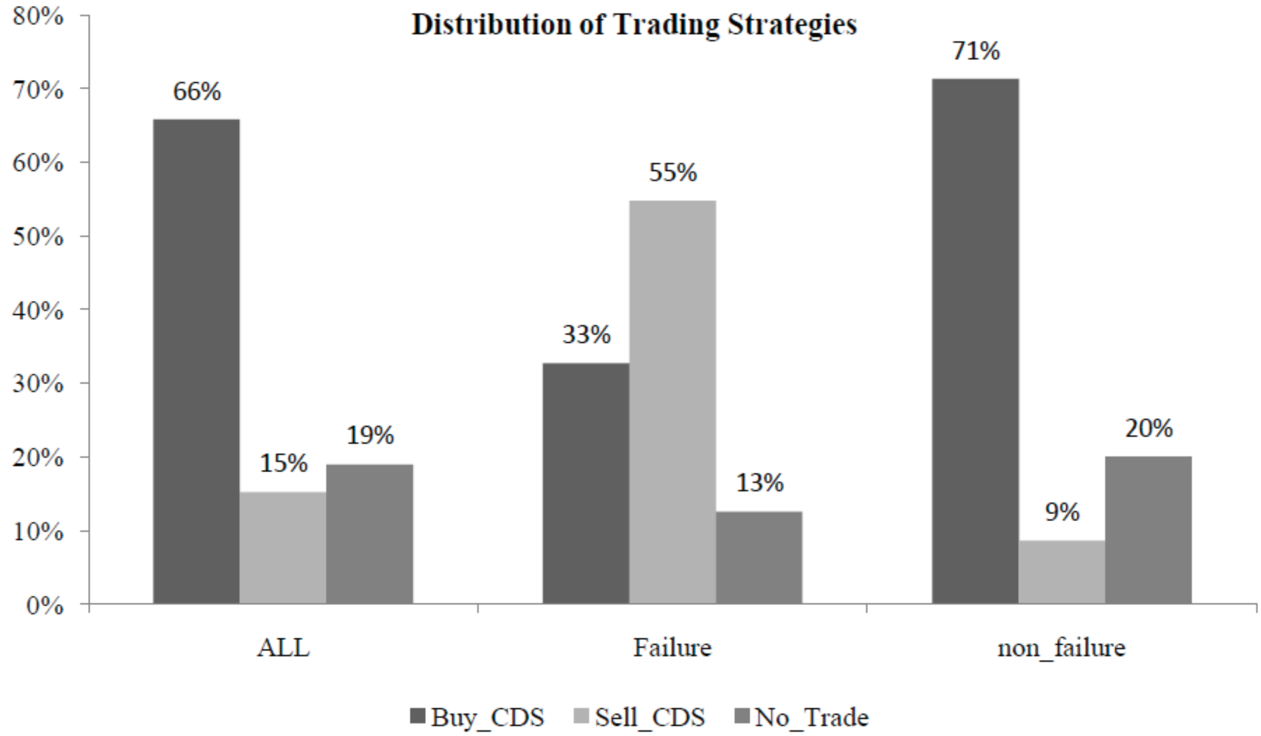


Figure IV-5: Time Distribution of the Negative Implied Recovery Rates

This figure depicts the percentage of the negative implied recovery rates over the total availabel observations for the full sample during the sample period from April, 11th . 2008 to March 30th, 2012.

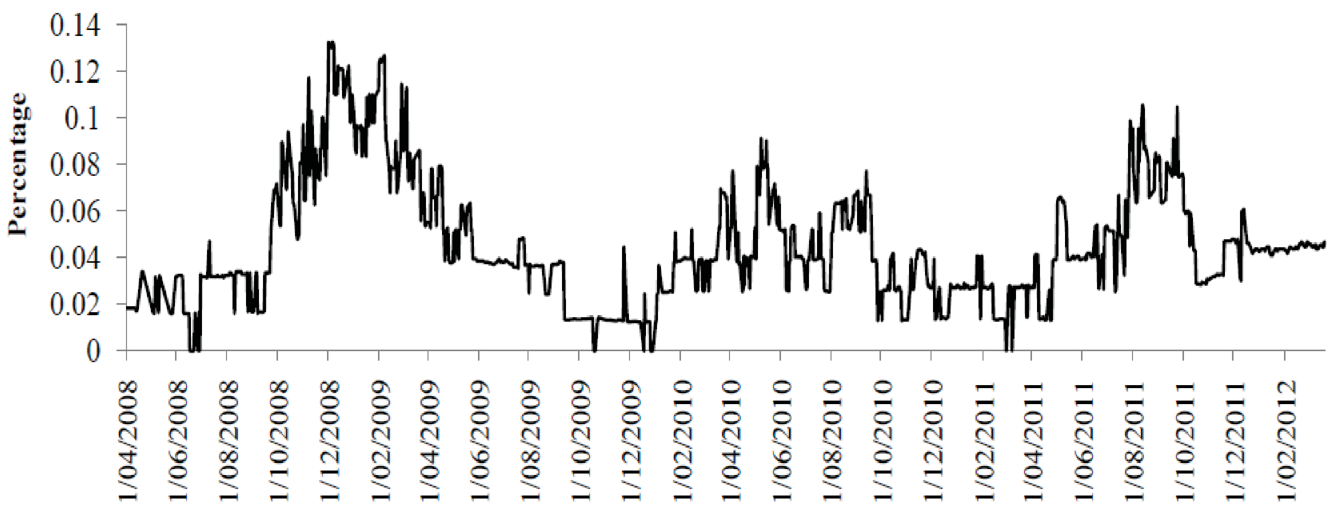


Table IV-6: Event study of Negative Implied Recovery Rates

This table reports the means and medians of the interesting variables on one day before (*1 Day Before*) and the first day (*First Day*) of the negative implied recovery rates for the full sample. *Spread ratios* are equal to the *CDS spreads*/*LCDS spreads*. *Recovery Rates Ratios* are equal to (1-*CDS Recovery Rates*)/(1-*LCDS Recovery Rates*). For the means, the two-sides two-sample tests with normal and t approximation are conducted and the corresponding *p*-values are reported as difference test. For the medians, the Wilcoxon median two-sample tests are conducted and the corresponding *p*-values are reported. ***, ** and * indicate the 1%, 5% and 10% significance level.

	Mean				Median		
	1 day Before	First Day	Difference Test Student t (p-value)	Difference Test Normal (p-value)	1 day Before	First Day	Wilcoxon Test (p-value)
CDS Spreads	0.1424	0.1582	0.2163	0.2160	0.0918	0.0935	0.2446
LCDS Spreads	0.0728	0.0639	0.1174	0.1169	0.0323	0.0309	0.1245
Spreads Ratios	6.0397	7.9953	<.0001***	<.0001***	2.8469	3.3583	0.019**
CDS Recovery Rates	0.3379	0.3405	0.4852	0.4852	0.3667	0.3667	0.5
LCDS Recovery Rates	0.6277	0.5977	0.0270**	0.0265**	0.7000	0.6708	0.0799*
Recovery Rates Ratios	1.9249	1.7988	0.0191**	0.0187**	2.0000	1.9355	0.0516*
Profits	0.0293	0.0567	<.0001***	<.0001***	0.0222	0.0347	<.0001***
Equity Returns	-0.0055	-0.0160	0.3304	0.3303	-0.0054	-0.0071	0.3224
Idiosyncratic Volatility	0.0469	0.0405	0.3360	0.3358	0.0305	0.0311	0.2446
Bid-Ask Spreads	0.0035	0.0034	0.3534	0.3532	0.0024	0.0024	0.3224
No. Observations	150	150			150	150	

Table IV-7: Summary Statistics of Failure Firms and Non-failure Firms

This table reports the summary statistics of implied CDS recovery rates for the failure firms and non-failure firms during the sample period from April 11th, 2008 to March 30th, 2012. The summary statistics of CDS and LCDS spreads and recovery rates, arbitrage profits and firms individual characteristics are reported as well. We define the market failure as that a firm has at least 10 consecutive negative implied recovery rates on all trading days. According to such criteria, we find 19 out of 102 firms experience market failures.

	CDS Spreads	LCDS Spreads	CDS Recovery Rates	LCDS Recovery Rates	Implied CDS Recovery Rates	Idiosyncratic Volatility	Total Asset (Thousands)	Leverage	Current Ratio	Profit With TC
Panel A: Failure Firms (19 Firms with 9745 observations)										
minimum	0.0060	0.0001	0.0125	0.0750	-6.1276	0.0078	1627.32	0.2795	0.2398	0.0000
maximum	0.9651	0.8984	0.6750	0.8100	0.9001	0.8429	70984.20	0.9854	3.1077	1.6471
mean	0.1059	0.0478	0.3443	0.6545	0.0255	0.0329	16019.43	0.6678	1.5216	0.0480
median	0.0653	0.0224	0.4000	0.7000	0.2837	0.0280	9774.30	0.7012	1.5002	0.0168
standard deviation	0.1421	0.0850	0.0905	0.1008	0.8329	0.0271	16562.91	0.1940	0.6384	0.1180
1st order autocorrelation	0.9706	0.9648	0.8623	0.9386	0.9504	0.7896	0.9914	0.9913	0.9863	0.9360
Panel B: Non-Failure Firms (83 Firms with 58402 observations)										
minimum	0.0027	0.0001	0.1500	0.3500	-12.3582	0.0053	446.51	0.0834	0.4169	0.0000
maximum	0.7014	0.6200	0.7050	0.9775	0.9918	0.6991	295142.56	0.9857	5.8299	1.2905
mean	0.0362	0.0348	0.3879	0.6519	0.5691	0.0221	26724.81	0.5939	1.5446	0.0314
median	0.0291	0.0243	0.4000	0.7000	0.5582	0.0194	14131.86	0.6048	1.4050	0.0115
standard deviation	0.0373	0.0385	0.0459	0.1147	0.2310	0.0144	39545.36	0.1803	0.6956	0.0635
1st order autocorrelation	0.9797	0.9698	0.7551	0.9252	0.9234	0.8212	0.9903	0.9885	0.9876	0.9177
Panel C: Difference Test (*p-value*) between Failure Firms and Non-Failure Firms										
Mean (Wilcoxon Paired *t* Approximation)	<.0001	0.3461	<.0001	0.0002	<.0001	<.0001	<.0001	<.0001	0.0005	<.0001
Median (Wilcoxon Median Test)	<.0001	<.0001	<.0001	<.0001	<.0001	<.0001	<.0001	<.0001	<.0001	<.0001

Table IV-8: Correlation Matrix of Variables

This table reports the correlation matrix of firm specific and macro variables, including the profits in the presence of transaction costs (*PR*), publication of ISDA dummy (*ISDA*), total asset (*LOGA*), current asset over current liability ratio (*CAL*), leverage ratio (*LEV*), tangible asset ratio (*TANG*), idiosyncratic volatility (*IDIO*), 5-year US treasury bond yields (*TB5Y*), slope of the yield term structure (*SL*), the spread between Aaa corporate bonds' yield and Baa corporate bonds' yield (*CBS*) and S&P 500 index return (*SP*). The numbers in the parentheses are the *p-values* of Pearson correlation coefficients.

	PR	**ISDA**	**LOGA**	**CAL**	**LEV**	**TANG**	**IDIO**	**TB5Y**	**SL**	**CBS**	**SP**
PR	1.00	-0.09	-0.23	0.00	0.26	-0.02	0.24	0.03	0.02	0.12	0.00
		(<.0001)	(<.0001)	(0.47)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(0.47)
ISDA	-0.09	1.00	0.05	0.03	-0.15	-0.05	-0.30	-0.62	-0.27	-0.55	0.01
	(<.0001)		(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(0.00)
LOGA	-0.23	0.05	1.00	-0.13	-0.42	-0.14	-0.32	-0.01	0.08	-0.06	0.01
	(<.0001)	(<.0001)		(<.0001)	(<.0001)	(<.0001)	(<.0001)	(0.02)	(<.0001)	(<.0001)	(0.11)
CAL	0.00	0.03	-0.13	1.00	-0.26	-0.16	0.02	-0.01	0.03	-0.04	0.00
	(0.47)	(<.0001)	(<.0001)		(<.0001)	(<.0001)	(<.0001)	(0.02)	(<.0001)	(<.0001)	(0.50)
LEV	0.26	-0.15	-0.42	-0.26	1.00	0.41	0.39	-0.01	-0.05	0.19	0.00
	(<.0001)	(<.0001)	(<.0001)	(<.0001)		(<.0001)	(<.0001)	(0.11)	(<.0001)	(<.0001)	(0.93)
TANG	-0.02	-0.05	-0.14	-0.16	0.41	1.00	0.11	0.01	0.01	0.07	0.00
	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)		(<.0001)	(0.04)	(0.00)	(<.0001)	(0.93)
IDIO	0.24	-0.30	-0.32	0.02	0.39	0.11	1.00	0.13	-0.14	0.39	-0.03
	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)		(<.0001)	(<.0001)	(<.0001)	(<.0001)
TB5Y	0.03	-0.62	-0.01	-0.01	-0.01	0.01	0.13	1.00	0.58	0.03	0.01
	(<.0001)	(<.0001)	(0.02)	(0.02)	(0.11)	(0.04)	(<.0001)		(<.0001)	(<.0001)	(0.03)
SL	0.02	-0.27	0.08	0.03	-0.05	0.01	-0.14	0.58	1.00	-0.26	0.04
	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(0.00)	(<.0001)	(<.0001)		(<.0001)	(<.0001)
CBS	0.12	-0.55	-0.06	-0.04	0.19	0.07	0.39	0.03	-0.26	1.00	-0.03
	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)		(<.0001)
SP	0.00	0.01	0.01	0.00	0.00	0.00	-0.03	0.01	0.04	-0.03	1.00
	(0.47)	(0.00)	(0.11)	(0.50)	(0.93)	(0.93)	(<.0001)	(0.03)	(<.0001)	(<.0001)	

Table IV-9: Panel Regression with Important Events and Macro Economic Factors

This table reports panel regression results with single name fixed effects during the sample period from April 11th, 2008 to March 30th, 2012. The variables are the intercept (*INT*), publication of ISDA dummy (*ISDA*), total asset (*LOGA*), current asset over current liability ratio (*CAL*), leverage ratio (*LEV*), tangible assets ratio (*TANG*), idiosyncratic volatility (*IDIO*), 5-year US treasury bond yields (*TB5Y*), slope of the yield term structure (*SL*), the spread between Aaa corporate bonds' yield and Baa corporate bonds' yield (*CBS*) and S&P 500 index returns (*SP*). The statistical significant coefficients are marked with ***, ** and * for significance at the 1%, 5% and 10% significance levels, respectively. The standard errors are calculated by the clustering standard error approach in order to remove the serial correlation effect.

Variables	Full Sample	Investment Grades	Junk	Not Rated	Failure Firms	Non-Failure Firms
INT	-0.1465 (0.5260)	-0.2019 (0.4530)	0.0618 (0.6767)	-0.0836 (0.8701)	-0.2841 (0.6363)	0.0487 (0.7067)
ISDA	-0.0018 (0.4162)	-0.0013 (0.6660)	-0.0027 (0.3599)	-0.0065* (0.0825)	-0.0101 (0.4049)	-0.0014 (0.5246)
LOGA	0.0137 (0.5266)	0.0197 (0.4374)	-0.0041 (0.7831)	0.0015 (0.9738)	0.0172 (0.7487)	-0.0019 (0.8745)
CAL	-0.0049 (0.4120)	-0.0045 (0.4850)	0.0002 (0.9571)	-0.0079 (0.3768)	-0.0324 (0.1863)	-0.0008 (0.8244)
LEV	0.1018*** (0.0064)	0.1314* (0.0563)	0.0811** (0.0190)	0.0631 (0.3233)	0.2328* (0.0697)	0.0582** (0.0248)
TANG	-0.0080 (0.7717)	-0.0124 (0.7453)	-0.0544** (0.0408)	0.0319 (0.6668)	-0.0092 (0.8933)	-0.0300* (0.0576)
IDIO	0.3187*** (0.0097)	0.3669** (0.0291)	0.2805 (0.1047)	0.1396* (0.0663)	0.1745 (0.2708)	0.3882*** (0.0069)
TB5Y	-0.5131** (0.0351)	-0.5193* (0.0984)	-0.9028* (0.0568)	-0.7315 (0.1861)	-0.5872 (0.4409)	-0.5936*** (0.0083)
SL	1.3424*** (<.0001)	0.8423** (0.0109)	1.4500*** (0.0047)	2.0243*** (0.0001)	1.9844 (0.1087)	1.2105*** (<.0001)
CBS	1.0319*** (0.0024)	0.3169 (0.4926)	0.9649*** (0.0012)	2.7882*** (<.0001)	1.2291 (0.5153)	0.9769*** (<.0001)
SP	-0.0022 (0.7171)	0.0110 (0.1467)	-0.0138 (0.3965)	-0.0112 (0.5405)	0.0067 (0.7537)	-0.0016 (0.7912)
No. of Observations	68147	41327	11665	15155	9745	58402
Adjusted R^2	63.90%	46.81%	86.16%	81.72%	39.20%	79.35%
MSE	0.0445	0.0474	0.0197	0.0435	0.0921	0.02887

Table IV-10: Regression Results of Restricted Models

This table reports the regression results for restricted models. The standard errors are calculated by the clustering standard error approach in order to remove the serial correlation effect.

Variables	Macro Factors Only	Firm Specific Factors with Fixed Effects Only	Fixed Effects Only	LOGA Only	CAL Only	LEV Only	TANG Only	IDIO Only
INT	-0.0013 (0.9234)	0.0700 (0.1816)	0.0299*** (<.0001)	0.1855*** (0.0003)	0.0333*** (<.0001)	-0.0283*** (0.0098)	0.0361*** (<.0001)	0.0098 (0.1105)
ISDA	0.0004 (0.9602)							
LOGA		-0.0107** (0.0134)		-0.0159*** (0.0019)				
CAL		-0.0002 (0.9735)			0.0003 (0.9424)			
LEV		0.0917*** (0.0028)				0.1026*** (<.0001)		
TANG		-0.0095 (0.4759)					-0.0043 (0.6282)	
IDIO		0.4087*** (0.0072)						1.0154*** (<.0001)
TB5Y	-0.0803 (0.8027)							
SL	0.9902** (0.0194)							
CBS	1.4874*** (<.0001)							
SP	-0.0051 (0.4601)							
No. of Observations	68147	68147	68147	68147	68147	68147	68147	68147
Adjusted R^2	1.87%	61.67%	58.18%	5.44%	.007%	6.52%	0.05%	5.58%
MSE	0.0733	0.0459	0.0479	0.720	0.074	0.0715	0.0740	0.0719

Table IV-11: Regression Results for Low Frequency Data

This table reports the regression results for lower frequency data. We aggregated the daily current payoffs for one week, one month and one quarter, respectively. The standard errors are calculated by the clustering standard error approach in order to remove the serial correlation effect.

Variables	Weekly	Monthly	Quarterly
INT	-0.1347 (0.5570)	-2.5571 (0.5262)	-4.7568 (0.7071)
ISDA	-0.0014 (0.5435)	-0.0580 (0.2988)	-0.3634 (0.2175)
LOGA	0.0124 (0.5614)	0.2361 (0.5244)	0.4258 (0.7036)
CAL	-0.0053 (0.3549)	-0.0998 (0.3380)	-0.0500 (0.9141)
LEV	0.1023*** (0.0042)	2.1274*** (0.0045)	6.5953*** (0.0037)
TANG	-0.0089 (0.7458)	-0.1265 (0.8007)	-0.2729 (0.8632)
IDIO	0.2727*** (0.0089)	7.0576** (0.0104)	21.2753*** (0.0026)
TB5Y	-0.4630* (0.0718)	-15.5054*** (0.0018)	-52.6298*** (0.0089)
SL	1.3360*** (<.0001)	29.6682*** (<.0001)	63.8917*** (0.0008)
CBS	1.0713*** (0.0019)	11.5147* (0.0572)	5.0155 (0.8387)
SP	-0.0153 (0.5129)	-0.0034 (0.9865)	-0.9076 (0.1086)
No. of Observations	15077	3663	1303
Adjusted R^2	64.02%	62.30%	61.26%
MSE	0.0446	0.8575	2.4245

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