SWITCHING EQUIPMENT LOCATION/ALLOCATION IN HYBRID PONS

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Abstract

Switching Equipment Location/Allocation in hybrid PONs

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Our research goal is to investigate the FTTX (Fiber-to-the Home/Premises/Curb) passive optical network (PON) for the deployment of BISAN (Broadband Internet Subscriber Access Network) to exploit the opportunities of optical fiber enabled technologies as well as of passive switching equipment. Indeed, the deployment of FTTX PON is the most OPEX-friendly scenario, because it allows for completely passive access networks through minimizing the number of active components in the network. Previously, most FTTX PON architectures are designed based on the principle of either time division multiplexing (TDM) technology or wavelength division multiplexing (WDM) technology. We focus on designing the best possible architectures of FTTX PON, specifically hybrid PONs, which embraces both TDM and WDM technology. A hybrid PON architecture is very efficient as it is not limited to any specific PON technology, rather it is flexible enough to deploy TDM/WDM technology depending on the type (i.e unicast/multicast) and amount of traffic demand of the end-users. The advantages of a hybrid PON are of two folds: (i) it can offer increased data rate to each user by employing WDM technology, (ii) it can provide flexible bandwidth utilization by employing TDM technology.

In this thesis, we concentrate on determining the optimized covering of a geographical area by a set of cost-effective hybrid PONs. We also focus on the greenfield deployment of a single hybrid PON. It should be worthy to mention that while investigating the deployment of hybrid PONs, the research community around the world considers the specifications of either the physical layer or the optical layer. But an efficient planning for PON deployment

should take into account the constraints of the physical and optical layers in order that both layers can work together harmoniously. We concentrate our research on the network dimensioning and the selection as well as the placement of the switching equipment in hybrid PONs with the intention of considering the constraints of both physical and optical layers. We determine the layout of an optimized PON architecture while provisioning wavelengths in a hybrid PON. We also propose to select the switching equipment depending on the type (unicast/multicast) of traffic demand. Finally, we determine the best set of hybrid PONs along with their cascading architecture, type and location of their switching equipment while satisfying the network design constraints such as the number of output ports of the switching equipment and maximum allowed signal power loss experienced at each end user's premises.

In this thesis, we propose two novel schemes for the greenfield deployment of a single hybrid PON. The first scheme consists of two phases in which a heuristic algorithm and a novel column generation (CG) based integer linear programming (ILP) optimization model are proposed in the 1st and 2nd phase respectively. In the second scheme, a novel integrated CG based ILP cross layer optimization model is proposed for the designing of a single hybrid PON.

We also propose two novel schemes to deal with the greenfield deployment of multiple hybrid PONs in a given geographical area. These two schemes determine the best set of cost-effective hybrid PONs in order to serve all the end users in a given neighborhood. The first scheme executes in four phases in which two heuristic algorithms, a CG based ILP model and an ILP optimization model are proposed in the 1st, 2nd, 3rd and 4th phase respectively. In the second scheme, an ILP model as well as a CG based ILP model, another ILP model as well as another CG based ILP model, a CG based ILP model and an ILP optimization model are proposed during four consecutive phases.

Our proposed scheme can optimize the design of a set of hybrid PONs covering a given geographic area as well as the selection of the best cascading architecture (1/2/mixed-stage) for each selected PON. It minimizes the overall network deployment cost based on the location of the OLT and the ONUs while granting all traffic demands. The scheme emphasizes on the optimum placement of equipment in a hybrid PON infrastructure due to the critical dependency between the network performances and a proper deployment of its equipment, which, in turn depends on the locations of the users. It is a quite powerful scheme as it can handle data instances with up to several thousands ONUs. On the basis of the computational results, the proposed scheme leads to an efficient automated tool for network design, planning, and performance evaluation which can be beneficial for the network designers.

Acknowledgments

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Acronyms

ADSL	Asymmetric Digital Subscriber Line
APON	ATM Passive Optical Network
BSP	Best Set of PONs
BPON	Broadband Passive Optical Network
BISAN	Broadband Internet Subscriber Access Network
CPON	Composite Passive Optical Network
CDPD	Cellular Digital Packet Data
CDMA	Code Division Multiple Access
DSL	Digital Subscriber Line
EPON	Ethernet Passive Optical Network
FTTPC/FTTH/FTTB	Fiber-to-the-PC/Home/Building
FTTC	Fiber-to-the-Curb
GEOPONS	Geographical PON Set
GPON	Gigabit Passive Optical Network
GSM	Global System for Mobile Communications
HDSL	High-speed Digital Subscriber Line
ISP	Internet Service Provider
ISDN	Integrated Services Digital network
ITU	The International Telecommunication Union
LARNet	Local Access Router Network
L/A	Location/Allocation
OSPON	Optimized Set of PONs
OLT	Optical Line Terminal
ONU	Optical Network Unit
OCDMA	Optical Code Division Multiple Access

PSTN	Public Switched Telephone Network
PON	Passive Optical Network
RITENet	Remote Interrogation of Terminal Network
SLAPONS	Equipment Selection L/A in a set of PONs
SUCCESS-PON	Stanford University Access PON
SUCCESS-HPON	Stanford University Access Hybrid WDM/TDM PON
TDM PON	Time Division Multiplexed PON
UWB	Ultra-wide-band
VDSL	Very High-speed Digital Subscriber Line
WOBAN	Wireless Optical Broadband Access network
WiMAX	Worldwide Interoperability for Microwave Access
Wi-Fi	Wireless Fidelity
WWAN	Wireless wide area network
WMAN	Wireless metropolitan area network
WLAN	Wireless local area network
WPAN	Wireless personal area networks
WDM PON	Wavelength Division Multiplexed PON

Chapter 1

Introduction

1.1 General Background

Internet usage has been increased tremendously during the last decade. Consumers are using a number of broadband applications that are emerging everyday. To facilitate Internet access in a neighborhood, it is required to deploy Broadband Internet Subscriber Access Networks (BISANs). A BISAN consists of a digital communication link between a user and an Internet Service Provider (ISP) which is usually termed as last mile network from ISP's point of view or first mile network from subscriber's perspective.

BISANs can be broadly categorized into two groups: Wired access networks and Wireless access networks. Depending on the physical medium of data transmission, different types of wired access networks currently exist in the market with, either copper wire enabled technologies, or optical fiber enabled technologies, or hybrid technologies incorporating both optical fibers and copper wires.

The most widely deployed copper wire enabled technology is the digital subscriber line (DSL) which uses the higher frequency range on the traditional public switched telephone network (PSTN) line for higher speed data transmission. There are four basic variants of DSL: Integrated services digital network (ISDN) with a maximum speed of 144 Kbps in both upstream and downstream directions; asymmetric DSL

(ADSL) with a maximum speed of 800 Kbps and 8 Mbps in upstream and downstream directions respectively; high-speed DSL (HDSL) with a maximum speed of 1.44 Mbps in both directions; and very high-speed DSL (VDSL) with a maximum speed of 16 Mbps and 52 Mbps along the upstream and downstream direction respectively [Lam07]. The maximum transmission distance for ADSL, HDSL, and VDSL are 5500 meters, 3650 meters, and 1200 meters respectively [Lam07]. So, it is apparent that VDSL achieves much higher speed compared to HDSL and ADSL, but it compromises with the maximum allowable transmission distance. But the present traffic market asks for more speed to comply with high bandwidth-hungry applications such as high-definition television (HDTV), two-way video conferencing, video-on-demand (VoD), high definition multimedia interactive games, real-time transactions and Internet telephony without compromising the quality of service (QoS) and the maximum transmission distance.

Optical fiber enabled technologies can definitely be considered as an attractive solution for access networks to face the challenges of the new era. Optical fiber has already been deployed in the backbone and in the metropolitan networks. It is now penetrating into the access network domain mitigating the bandwidth bottleneck between the end users and the high capacity backbone network. Optical access networks, often termed as FTTX (Fiber-to-the Home/Premises/Curb), are considered as the last step for the future all-optic network revolution, which can be accomplished in either point-to-point (P2P) or point-to-multi-point (P2MP) fashion. In a P2P architecture, a dedicated fiber runs from ISP's central office (CO) to each customer in which high installation and maintenance cost of each individual fiber is a major economic barrier. P2MP architectures, offering an economically feasible solution compared to their P2P counterpart, may be either active or passive [Koo06]. An active architecture is usually established by deploying a remote curb switch close to the neighborhood, a single fiber from the CO to a switch, and a number of short branching fibers from the switch to each end user. But such an active star architecture does not attract ISPs as the curb switch requires electric power which is the most significant operational cost for the local ISP. On the other hand, passive architectures draw sensational attention not only from the ISPs but also from the researchers around the world as these are the most cost-effective solutions for optical access networks. Passive architectures are deployed in passive optical networks (PONs) which reduce the operational cost significantly by replacing the active switch by a passive optical power splitter/combiner.

PONs offer numerous advantages for local access networks as they allow longer distances between central offices and customer premises, minimize fiber deployment, provide higher bandwidth, allow downstream video broadcasting, eliminate the necessity of installing multiplexers and demultiplexers in the splitting locations, and allow easy upgrades to higher bit rates or additional wavelengths [KMP01]. As a result, PON based technologies are getting attention to a greater extent by the telecommunication industry nowadays. The maximum allowable distance from the CO to an end user can be extended up to 100 km by the deployment of long-reach PON (LPON). A typical LPON introduces an electro absorption modulator (EAM) as well as semiconductor optical amplifiers (SOAs) to extend its maximum reach.

Another flavor of wired access technology, adopted by the community antenna television (CATV) networks, is the hybrid wired technology, incorporating optical fibers and copper wires. Usually, a CATV is built as a hybrid fiber coax (HFC) network where an optical fiber runs up to the curb side optical nodes or CATV street cabinet, and from there, coaxial cables run to individual subscriber's home. Any CATV architecture encounters severe problems as it requires a number of active optical nodes at the curb site and allocates only about 36 Mbps of effective data throughput per optical node for upstream communication resulting in frustratingly low speed upstream capacity during peak hours [Kra05].

Wireless access networks are evolving as a promising network for the deployment of BISANs. While the wireline solutions for the access networks are dominating the mainstream, wireless solutions are fairly recent phenomenon representing divergent and challenging technology. Due to a number of advantageous features, wireless access networks are considered as a potential technology for the deployment of BISANs.

Wireless access networks can be broadly organized into four categories: Wireless wide area networks (WWANs), wireless metropolitan area networks (WMANs),wireless local area networks (WLANs), and wireless personal area networks (WPANs) [Par06]. WWANs, utilizing Global System for Mobile Communications (GSM), Cellular Digital Packet Data (CDPD), and Code Division Multiple Access (CDMA) technologies, have coverage over large geographic areas such as cities or countries. WMAN, facilitated through Worldwide Interoperability for Microwave Access (WiMAX) technology, can provide broadband wireless access within a metropolitan area.

The architecture of a WiMAX system consists of two parts: A number of WiMAX base stations (BSs) and hundreds of WiMAX receivers per base station which are referred to as subscriber station (SS) or customer premise equipment (CPE).WLAN exploits Wireless Fidelity (Wi-Fi) technology for establishing wireless connections within office buildings, restaurants, stores, homes, etc. WPAN technology allows very short range (up to 10 meters) wireless connectivity using Ultra-wide-band (UWB) technology. Different platforms promote different technologies, for example WiMAX works best for fixed wireless platforms, whereas 3G is more suitable for mobile wireless infrastructures.

In WiFi technology, any device containing the functionality of the 802.11 protocol is usually defined as a station; a group of stations that can communicate with one another under the direct control of a single coordination function (distributed coordinate function [DCF] or point coordinate function [PCF]) is termed as a basic service set (BSS); the geographic area covered by the BSS is known as the basic service area (BSA) [PR05].The fundamental building block of WiFi architecture supports the following two topologies [Std]: Independent basic service set (IBSS) and Extended service set (ESS) networks. IBSS is an ad hoc network in which self-managed stations are grouped under the umbrella of a single BSS without the aid of any administrator. IBSS is considered as a limited range network due to its single BSS constraint. On the other hand, ESS is an infrastructure network which requires a central authority known as Access Point (AP) to manage the network and to provide specific wireless services to the users. ESS is formed by integrating together multiple BSSs using a common distribution system (DS) in which APs function as the integration points required for network connectivity between multiple BSSs.

1.2 Scope of the Research Project

In this thesis, we concentrate on the FTTX passive optical network (PON) for the deployment of BISAN to exploit the opportunities of optical fiber enabled technologies as well as of passive switching equipment. Indeed, the deployment of FTTX PON is the most operational expenditure (OPEX)-friendly scenario, because it allows for completely passive access networks through minimizing the number of active components in the network. As considerable OPEX originates from central office operations, reducing costs means reducing the number of offices. However, office consolidation would result in enlarging the access network footprint and would demand enhanced capabilities from the access technologies. Consequently, new questions arise in the context of the access network evolution with respect to how the FTTX deployments can be supported in a cost-efficient manner when considering office consolidation strategies, and on what would be the impact on network architectures and related technologies. This motivated us to focus, in this thesis, on designing best possible architectures of FTTX PON, specifically hybrid PONs, built on the principle of time/wavelength division multiplexing (TDM/WDM) technology.

1.3 Thesis Contributions

In this thesis, we investigate the optimized covering of a geographical area by a set of cost-effective hybrid PONs. We also focus on the greenfield deployment of a single hybrid PON. It should be worthy to mention that while investigating the deployment of hybrid PONs, the research community around the world consider the specifications of either the physical layer or the optical layer. But an efficient planning for PON deployment should take into account the constraints of the physical and optical layers in order that both layers can work together harmoniously. We concentrate our research on the network dimensioning and the selection as well as the placement of the switching equipment in hybrid PONs with the intention of considering the constraints of both physical and optical layers. We determine the layout of an optimized PON architecture while provisioning wavelengths in a hybrid PON.

We also propose to select the switching equipment depending on the type (unicast/multicast) of traffic demand. In our research, we consider two types of switching equipment: (i) splitters, (ii) arrayed waveguide gratings (AWGs). It can be mentioned that splitters are best suited for multicast traffic whereas AWGs are appropriate for unicast traffic. Again, splitters are economically feasible switching equipment but they are badly susceptible to signal power loss with respect to the number of output ports. While selecting the switching equipment, unicast/multicast traffic together with the signal power loss experienced by the corresponding equipment plays a vital role. A splitter may be selected to satisfy multicast requests with the condition that the maximum allowable signal power loss is satisfied. On the contrary, an AWG can be chosen either to serve unicast requests or to satisfy the signal power loss constraint. In this thesis, we investigate the maximum signal power loss experienced at end users' premises. We scrutinize the selection of the switching equipment. We also study the impact of multicast traffic on the deployment cost of hybrid PONs.

Finally, we determine the best set of PON networks along with their cascading architecture, type and location of their switching equipment while satisfying the network design constraints such as the number of output ports of the switching equipment and maximum allowed signal power loss experienced at each end user's premises.

In this thesis, we propose two novel schemes for 'Switching Equipment Location/Allocation in a single hybrid PON'. The first scheme consists of two phases in which a heuristic algorithm and a novel column generation (CG) based integer linear programming (ILP) optimization model are proposed in the 1st and 2nd phase respectively. In the second scheme, a novel integrated CG based ILP cross layer optimization model is proposed for the designing of single PON.

We also propose two novel schemes to deal with 'Switching Equipment Location/Allocation in multiple hybrid PONs' in a given geographical area. These two schemes determine the best set of cost-effective PON networks in order to serve all the end users in a given neighbourhood. The first scheme executes in four phases in which two heuristic algorithms, a CG based ILP model and an ILP optimization model are proposed in the 1st, 2nd, 3rd and 4th phase respectively. In the second scheme, an ILP model as well as a CG based ILP model, another ILP model as well as another CG based ILP model, a CG based ILP model and an ILP optimization model are proposed during four consecutive phases.

We have generated a number of publications from this thesis.

The following research papers are published on the first proposed scheme for 'Switching Equipment Location/Allocation in a single hybrid PON'.

[1] Brigitte Jaumard and Rejaul Chowdhury, "Location and Allocation of Switching Equipment (Splitters/AWGs) in a WDM PON Network", The 20th International Conference on Computer Communications and Networks (ICCCN), pp. 1-8, Maui, 2011.

[2] Brigitte Jaumard and Rejaul Chowdhury,"Selection and Placement of Switching Equipment in a Broadband Access Network", International Conference on Computing, Networking and Communication (ICNC), pp 297-303 , Maui, 2012.

[3] Brigitte Jaumard and Rejaul Chowdhury, " An efficient optimization scheme for the designing of a WDM PON Network", Computer Communications, ELSEVIER, ACCEPTED.

The following research papers are published/submitted on the second proposed scheme for 'Switching Equipment Location/Allocation in a single hybrid PON'.

[1] Rejaul Chowdhury and Brigitte Jaumard,"A Cross Layer Optimization Scheme

for WDM PON Network Design and Dimensioning", IEEE International Conference on Communications (IEEE ICC), pp 3149-3154, Ottawa, 2012.

[2] Rejaul Chowdhury and Brigitte Jaumard,"An composite optimization scheme for the designing of a single Hybrid PON Network", Journal of Lightwave Technology, IEEE/OSA , SUBMITTED.

The following research paper is submitted on the first proposed scheme for 'Switching Equipment Location/Allocation in multiple hybrid PONs'.

[1] Rejaul Chowdhury and Brigitte Jaumard,"Optimized Covering of a Geographical Area by a Set of WDM PON Networks", Computer Networks, ELSEVIER, Minor Revision on-going.

The following research papers are published/submitted on the second proposed scheme for 'Switching Equipment Location/Allocation in multiple hybrid PONs'

[1] Rejaul Chowdhury and Brigitte Jaumard,"A p-center optimization scheme for the designing and dimensioning of a set of WDM PONs", IEEE GLOBECOM, 3001-3007, California, 2012.

[2] Rejaul Chowdhury and Brigitte Jaumard, "An efficient optimization scheme for 'greenfield' deployment of a set of Hybrid PONs", Journal of Optical Communications & Networking (JOCN), WILL BE SUBMITTED SHORTLY.

1.4 Plan of The Thesis

The thesis is organized as follows. In Chapter 2, we define and describe the technical background of PONs and classical location-allocation problem for the logistic systems planning as it bears significant resemblance with our investigated problem domain. In Chapter 3, we provide the previously published studies on PONs. Our proposed two optimization schemes for 'Switching Equipment Location/Allocation in a single hybrid PON' are presented in Chapter 4 and Chapter 5 respectively. In Chapter 6 and chapter 7, we present our other two proposed optimization schemes for 'Switching Equipment Location/Allocation in multiple hybrid PONs'. Conclusions and future directions are illustrated in Chapter 8.

Chapter 2

Technical Background

2.1 Evolution of PON Enabling Technologies

Passive Optical Network (PON) enabling technology is the latest broadband access network technology embraced by the Internet Service Providers (ISPs) to provide Internet access to the residential and business customers. In a typical PON architecture, there is an optical line terminal (OLT) at the central office (CO) of the ISP, a number of optical network units (ONUs), one or multiple passive switching equipment placed in a remote terminal (RT) between the OLT and the ONUs.

The ONUs are located either at end user premises resulting in FTTPC/FTTH/FTTB (Fiber-to-the-PC/Home/Building) solutions or at the curb site in case of a FTTC (Fiber-to-the-Curb) architecture, see Figure 1 for an illustration. In a typical PON, the presence of only passive elements from the OLT to the ONUs makes it relatively fault tolerant and decreases its operational and maintenance costs once the infrastructure has been laid down.

PONs are usually built following either time sharing principle known as time division multiplexed PON (TDM PON) or spectrum sharing principle recognized as wavelength division multiplexed PON (WDM PON) [UVlMD13]. In a TDM PON, the RT consists of passive optical power splitters. In a WDM PON, the RT consists of arrayed waveguide gratings (AWGs). The characteristic of a splitter is different

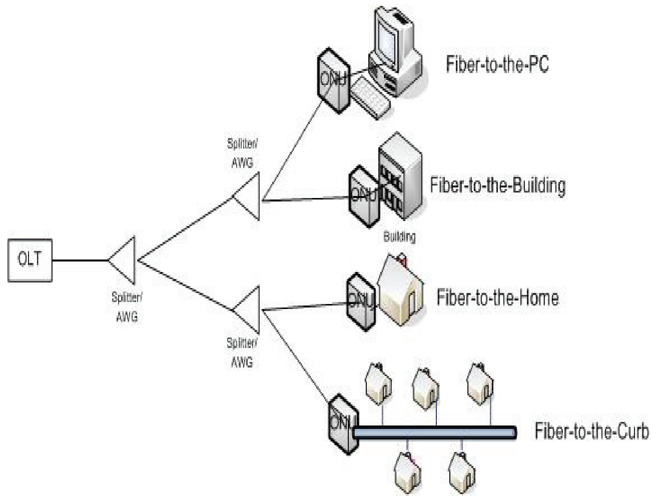


Figure 1: PON Architecture

than that of an AWG as the former equipment splits the optical power whereas latter equipment multiplexes/de-multiplexes optical wavelengths.

Recently, the research community proposes hybrid PONs which embraces both TDM PON and WDM PON technologies to avail the advantages of both technologies [MMCW13] [Rok12]. In such a PON, a splitter (in case of TDM PON) or an AWG (in case of WDM PON) can serve as the RT. Other categories of access mechanisms are sub-carrier multiple access (SCMA) in which an optical channel is shared among multiple users by allocating a dedicated electrical sub-carrier channel for each user [KH06] and optical code division multiple access (OCDMA) in which data is encoded/decoded into/from an optical pulse sequence by assigning orthogonal codes to all users [AP02].

2.1.1 Overview of TDM PON Technologies

The cost-effectiveness has led TDM PONs to emerge as the current PON generation. In a TDM-PON, a single wavelength channel is used along the downstream direction for broadcasting the same signal from the OLT to all ONUs by utilizing a passive optical power splitter or a cascade of passive splitters as the RT and another dedicated channel is used along the upstream direction for multiplexing signals from different ONUs in the time domain toward the OLT. TDM PONs can be implemented either by a space division duplex approach in which two separate fibers are used for upstream and downstream communications or by a coarse WDM (CWDM) approach in which the upstream and downstream wavelengths are multiplexed on the same fiber.

Several P2MP topologies are suitable for TDM-PON systems such as tree (single-stage and multi-stage), ring, and bus [Lam07]. Tree topologies are implemented using 1:N splitters, whereas bus and ring topologies are implemented utilizing 1:2 optical tap couplers. While designing a PON system, we have to select an appropriate topology considering that a PON is very cost sensitive and therefore, it should not include over-provisioning and should allow for incremental deployment.

The physical distance and splitting ratio in a TDM-PON are dependent on the power loss of the optical transmission medium. Power splitting allows the distribution of the cost of the OLT among ONUs and the reduction of the fiber mileage in the field, which is highly desirable in order to reduce the cost of an access network. Current commercial TDM PON specifications allow for 32 ONUs at a maximum transmission distance of 20 km from the OLT [SKM10], [LGW10]. Considering economic feasibility, a high splitting ratio is deserved as it eventually lowers the overall costs of the end users. But a high splitting ratio degrades the quality of service as well as the signal quality (higher attenuation) as the bandwidth of the OLT is shared among more ONUs allocating less bandwidth per user. So a trade off between the splitting ratio and the signal quality is required while designing a PON technology.

TDM PON technology faces several design challenges, regardless of the physical topology and splitting ratio. One of the major design issues is the selection of data-link technology (bearer protocol) upon which different blends of TDM-PONs have been standardized by several standard bodies which are ATM PON (APON)/ Broadband PON (BPON), Ethernet PON (EPON) and Gigabit PON (GPON). In all flavors of TDM PONs, the downstream traffic is broadcast to all ONUs; each ONU inspects the headers, extracts the packets addressed to it and discards the packets destined to other ONUs. During upstream transmission, each ONU is synchronized to a common time reference which puts one or more packets into a time slot that is allocated prior by the OLT. In case of not having any packet to send, an ONU fills the time slot with an idle signal. Although the broadcast transmission in the downstream and the time sharing transmissions in the upstream limit the bandwidth of each user, the resulting low transceiver cost plays a significant role to justify the trade off between the available bandwidth and economic feasibility [HRYK05] [Fin09].

APON/BPON

APON and BPON architectures are different aliases of TDM-PON architectures and refer to standards in the International Telecommunication Union-Telecommunication Standardization Sector (ITU-T) G.983 series. Those standards were promoted by the Full Service Access Network (FSAN) consortium, an international group formed by telecommunication operators and vendors around the world. While the name BPON reflects the system's support for broadband services and serves its marketing purposes, APON clearly specifies asynchronous transfer mode (ATM) technology as its layer-2 protocol. The basic APON/BPON standard comes with a symmetric data rate of 155.52 Mbps in both directions, and an asymmetric downstream bit rate of 622.08 Mbps and upstream bit rate of 155.52 Mbps [UOF+01]. The latest standard of this series, has an enhanced 2.5 Gbps downstream transmission rate and 311 Mbps in the upstream direction [SKM10]. The maximum splitting ratio is usually 32, and the maximum transmission distance between an OLT and ONU is 20 km [UOF+01]. In a basic APON/BPON system, the downstream and upstream traffic was positioned in the 1480-1580 nm and 1260-1360 nm wavelength band respectively; whereas in its latest standard, the band for downstream services is narrowed to 1480-1500 reserving one enhanced band (1539-1565 nm) for additional digital service and two more bands (1360-1480 nm, 1565 nm and beyond) for future use [Koo06] [Lam07]. As with ATM technology, APON/BPON transmits data encapsulating in a frame. Each frame consists of two types of ATM cells: (i) 54 payload cells (53 bytes each) of original data and 2 physical layer operation, administration, and maintenance (PLOAM) cells of control information. Although an APON/BPON provides built-in quality of service (QoS), it suffers from the ATM cell encapsulation overhead (5-byte header in a 53-byte cell). Moreover, ATM technology consumes network resources unnecessarily in case of a dropped or corrupted ATM cell which invalidates an entire IP datagram while the remaining cells carrying the portions of the same datagram propagate further in the network. As a result, this technology is not efficient for carrying the predominant component of Internet traffic, i.e., variable length Internet Protocol (IP) packets. The most vital shortcoming of APON/BPON is that it fails to become an inexpensive technology due to the excessive cost of ATM switches and network cards.

EPON

EPON, colloquially known as Ethernet in the First Mile (EFM) and standardized in the IEEE 802.3ah [DF06], carries data traffic encapsulated into the frame of IEEE 802.3 Ethernet technology. The Ethernet always tries to find application spaces where well-established high quality solutions already exist and eventually succeeds in displacing these solutions with their own paradigm of simplicity and low cost [Bec05]. At present, the Ethernet group is working for the development of EPONs after enjoying wide success in the networking world by delivering extremely successful standards such as 10BASE-T, Fast Ethernet (100BASE-T), Gigabit Ethernet (1000BASE-T), 10 Gigabit Ethernet and many others. Given the fact that 90 percent of the data traffic around the world originates and terminates in Ethernet frame [MMR04], Ethernet has become a universally accepted standard in the local area network (LAN) and consequently, it has appeared as a logical choice for an IP data optimized access network. Deployment of EPONs can overcome the need for the adaptation of data while communicating between LANs and an access network. EPON is capable of transporting variable length IP packets encapsulated in Ethernet frames at standard Gigabit Ethernet speed of 1 Gbps. It offers full duplex transmission by utilizing two separate wavelengths (1490 or 1510 nm for the downstream traffic and 1310 nm along the upstream direction). The maximum splitting ratio supported by EPON technology is 16 [KSG+07]. As the basic Ethernet technology only supports broadcast, EPON requires an efficient media access control (MAC) protocol, namely multi-point control protocol (MPCP) to facilitate bandwidth allocation along the upstream direction. Newly adopted IEEE 802.1p standard, a mechanism for implementing QoS at the MAC level, has facilitated EPON to support a class of services having different QoS requirements.

GPON

GPON, as recommended by the FSAN group and standardized in the ITU-T G.984 series, is capable of transporting both variable size IP datagrams and fixed size ATM cells. Along the downstream direction, it operates in the wavelength range of 1480-1500 nm and at a speed of 2.488 Gbps/1.244 Gbps; along the upstream direction, it operates in the wavelength range of 1260-1360 nm and at a speed of 155.52 Mbps/622.08 Mbps/1.244 Gbps/2.488 Gbps [Lam07]. GPON, employing either GPON encapsulation method (GEM) or ATM for framing, connects a maximum of 64 users per PON [KSG+07]. By incorporating the fast growing demand for network resources with the economic feasibility, GPON has advanced to become a key universal network protocol in communication technologies.

Long-Reach PON

A Long-Reach PON (LR-PON), proposed to provide more cost effective solution, extends the maximum transmission distance to 100 km. A LR-PON can accommodate 1024 ONUs with 10-Gbps transmission rate for upstream and downstream directions [SKM10]. In a standard LR-PON, there exists a 90 km feeder section between the OLT and local exchange as well as a 10 km drop section between the local exchange and end users. The necessity of signal power compensation is inevitable in a LR-PON due to large splitting and long distance transmission. As a result, optical amplifiers are installed at the OLT and at the local exchange. Although a LR-PON includes active components in its architecture, the overall cost is greatly reduced as it is distributed among large number of end users.

2.1.2 Overview of WDM PON Technologies

Although TDM PONs provide higher bandwidth compared to copper wire based access technologies, the demand for even higher data rates still remains strong. Moreover, TDM PON architectures are bandwidth limited as a single wavelength is shared among a number of users, resulting in a reduction of the average bandwidth per user to a few tens of megabits per second which can not fulfill the requirements of high capacity transmission [EAR12] [GKR+05]. The end users' demand for more bandwidth can be satisfactorily mitigated by employing WDM PON technology without drastically changing the fiber infrastructure. WDM PONs support multiple wavelengths in either or both upstream and downstream directions by using a passive WDM coupler/arrayed waveguide grating (AWG) router as the RT. In a traditional WDM PON, during downstream communication, multiple signals from the OLT are carried on different wavelengths using dense WDM (DWDM) technique and demultiplexed by the AWG router to the appropriate ONU; again during upstream communication, signals from all the ONUs are multiplexed at the AWG using DWDM technique and forwarded toward the OLT. Thus the WDM-PON technology provides virtual point-to-point connectivity between the OLT and each ONU.

Benefits of WDM technologies are manifold such as high performance, increased network capacity, flexibility with respect to network scalability, improved network security, isolation of service and service providers. Regardless of these aforementioned benefits, WDM PON has not been extensively commercialized due to the lack of an available market requiring high bandwidth, immature device technologies, and lack of suitable network protocols and software to support the architecture. DWDM PON has been first commercialized in the year of 2004 in Korea by Korea Telecom (KT) [PYP+07], [Hut08]. But it should be standardized internationally to be a competitive solution compared to TDM PONs [PK08]. Although WDM PONs define an ambitious access solution today, its commercial deployment will become more practical and viable in near future as the demand for dedicated per-subscriber bandwidth is increasing and the cost of optical components is slowly decreasing.

Variations of WDM PON Architecture

A simple WDM PON architecture requires expensive WDM components such as dedicated transceiver per user at the OLT and optical source at each ONU. To reduce the cost of WDM-PON technology, many architectures have been proposed and demonstrated by both academia and industry. Proposed architectures are mainly based on two approaches [KH06]. The first one is the remodulation method of the downlink signal at each ONU such as using saturated semiconductor optical amplifier (SOA) [TS03] [SKK+05], injection-locked Fabry-Perot laser diode (F-P LD) [CCT+02], mutually injected F-P LD [HCYK08]. The second approach is the controlling of the wavelength source in the CO rather than in the ONUs by using additional devices in CO or each ONU for the up-link wavelength source such as employing spectrum sliced light-emitting diode (LED) [RHZ+88], spectrum sliced LED with cyclic AWG [HSC+04], spectrum-sliced amplified spontaneous emission(ASE) of erbium-doped fiber amplifier(EDFA) [JSLC98], ASE injection locked F-P LD [KKL00], and the wavelength-seeded reflective SOA(RSOA) [HTF+01]. These two approaches introduce the concept of colorless ONUs which can be defined as ONUs having either no light source at all or only a broadband light source. In such an approach, the OLT generates and assigns wavelengths for each ONU in the PON. Several variations of WDM PON architectures are discussed below.

The Local Access Router Network (LARNet) architecture, proposed by Zirngibl *et al.* [ZJS+95], allows standard DWDM technique in the downstream direction in which a number of wavelengths at the OLT is generated by a multi-frequency laser, coupled onto a single fiber, demultiplexed by a AWG-based router, and sent to different ONUs. In the upstream direction, a commercially available LED is used as a signal source at each ONU whose broad range spectrum is sliced by the AWG-based router into discrete narrow optical bands for different ONUs. These optical bands are then combined/multiplexed on the same fiber and forwarded to a burst-mode receiver at the OLT. In this architecture, the cost of the ONU is reduced significantly as the expensive DFB LD is replaced by an inexpensive LED. LARNet limits the distance from the OLT to the ONU considerably [BPC+05].

The Remote Interrogation of Terminal Network (RITENet) architecture, proposed by Frigo *et al.* [FIM+94], uses a modulator at the ONU instead of an optical transmitter. In the downstream direction, standard DWDM technique is used, except that the downstream signal is split at each ONU with a portion of light detected by the receiver while the remainder is used for modulating the upstream data. In the upstream direction, a portion of downstream light signal is modulated with the upstream data and looped back toward the OLT. This architecture employs space division bi-directionality by dedicating two fibers to every subscriber as the same set of wavelengths are used for both upstream and downstream communications. Although RITENet architecture reduces the ONU cost and does not suffer from the spectral-slicing loss, it doubles the cost of fiber deployment and maintenance. The maximum transmission distance from the OLT to the ONU supported by this architecture is much less as the signal from the OLT has to propagate on a double distance.

LARNet and RITENet architectures suffer from two main difficulties: (1) scaling the number of ONUs once the network infrastructure is laid down, (2) encompassing new users beyond a certain fixed limit as the fabrication technology limits the AWG size. Aiming to surmount these difficulties, Multistage AWG-based WDM-PON architecture was proposed by Maier *et al.* [MMP00], which utilizes the periodic routing property of the AWG facilitating the re-usability of a given wavelength for more than one subscriber. This architecture is scalable with respect to both bandwidth and the number of users as it can either employ additional wavelengths at the CO or cascade multiple stages of AWGs allowing increased AWG coarseness at each stage [BPC+05].

DWDM SuperPON [TT05], a 100 km reach remotely-seeded system, employs an electro absorption modulator-semiconductor optical amplifier (EAM-SOA) as a colorless ONU to provide upstream customer data channels accommodating either 512 users at 2.5 Gbps or 128 users at 10Gbps. In this architecture, each 1 × N power splitter PON uses two DWDM bands, one for upstream and another one for downstream while low cost filters are used to separate these two bands. But this architecture is not passive as it incorporates active optical amplifiers and seed sources to facilitate long distance services which makes it compatible to compete with other metro solutions rather than PON solutions.

SUCCESS-DWA PON [HRS+04], a novel optical access network architecture under the SUCCESS (Stanford University Access) networking project, consists of tunable lasers (TL) and an AWG at the CO; a unique fixed-wavelength WDM filter and a burst mode receiver at each ONU. This architecture allows any TL at the OLT to address any ONU individually across all of the physical PONs at any given time. This eventually guarantees that a TL can communicate with any user on a separate PON by determining the wavelength passband of the user and the wavelength of the AWG going toward the particular PON. It initially deploys one TL and one AWG in the CO and then provides scalability either by adding more TLs to the AWG or by adding another AWG along with more TLs [BPC+05]. It utilizes an appropriate scheduling algorithm and employs dynamic wavelength allocation (DWA). This architecture provides excellent cost efficiency and high network scalability by sharing bandwidth across multiple physical PONs.

2.1.3 Overview of hybrid PON Technologies

A hybrid PON can be built by combining the architectures of both TDM PON and WDM PON networks. On the physical layer of a hybrid PON, both TDM and WDM transmission (downstream and upstream) channels are utilized in the same PON [Rok12]. A hybrid PON facilitates better bandwidth usage by adding a TDM layer on top of the WDM layer [MMCW13]. A hybrid PON architecture is very efficient as it is not limited to any specific PON technology, rather it is flexible enough to deploy

TDM/WDM technology depending on the type (i.e unicast/multicast) and amount of traffic demand of the end-users. The advantages of a hybrid PON are two fold: (i) it can offer increased data rate to each user by employing WDM technology, (ii) it can provide flexible bandwidth utilization by employing TDM technology.

There are several variations of hybrid PON Technologies. Composite PON (CPON), an earliest hybrid TDM/WDM-PON architecture, employs WDM technology in the 1550 nm band along the downstream direction and TDM in the 1330 nm band along the upstream direction [BPC+05]. Although CPON gets rid of the drawback of upstream WDM, it suffers from the cost of a single frequency laser, such as a distributed-feedback (DFB) laser diode (LD) which is required at each ONU.

Stanford University Access Hybrid WDM/TDM Passive Optical Network (SUCCESS-HPON) architecture [AGK+05], a next generation hybrid WDM/TDM optical access architecture facilitating smooth migration from TDM PON to WDM PON, is built using a single fiber collector ring with stars attached to it in which remote nodes (RN), being the center of stars, are connected to one another using the collector ring. In this architecture, TDM PONs and WDM PONs are combined. A TDM-PON RN has a CWDM band splitter and add/drop filters to add and drop wavelengths for upstream and downstream transmission; again a WDM-PON RN has one CWDM band splitter/filter for adding/dropping a group of DWDM wavelengths within a CWDM grid and an AWG for multiplexing/demultiplexing DWDM wavelengths [AGK+05].

2.2 Overview of Location-allocation (L/A) Problem

In this thesis, we investigate the network dimensioning and placement of equipment problem in PON. Such a problem resembles the classical location-allocation problem for the logistic systems planning. The location-allocation (L/A) problem is that of optimally locating a number of service facilities among a finite number of demand points and simultaneously assigning each demand point to be served by the closest service facility where closest may have different meanings according to the selected metric [Che83]. Usually this class of problems is characterized by four components: *(i)* demand points (customers) that are already located at specific points, *(ii)* service facilities to be located, *(iii)* a space either continuous or discrete in which customers and facilities are located, and *(iv)* a metric indicating distances between customers and facilities [RE05].

L/A problems are often solved exploiting clustering algorithms where the demand points are partitioned into a certain number of clusters (groups, subsets, or categories) such that all demand points within a cluster can be served by a service facility. Clustering algorithms have numerous classifications: Distance and similarity measurement based, hierarchical based, squared error based, graph theory based, combinatorial search techniques based, fuzzy measurement based, neural networks based, and kernel based etc [XI05]. A classical clustering approach for solving the L/A problem is the K-means clustering which is based on squared error measurement [JD88], [KR90]. A standard K-means algorithm yields a set of clusters by either iterative divisions or by partitioning of a set of objects into K clusters.

Another widely used clustering approach is hierarchical clustering which generates structured set of clusters consisting of demand points. Hierarchical clustering algorithms are implemented using either top-down or bottom-up strategy. The former strategy for hierarchical clustering proceeds by first considering all demand points in one single cluster and then splitting clusters recursively while moving down the hierarchy until individual demand points are obtained. The latter strategy for hierarchical clustering, also known as hierarchical agglomerative clustering (HAC), considers each demand point as a singleton cluster at the beginning and then merges pairs of clusters successively while moving up the hierarchy until all pairs of clusters are merged into a single cluster containing all demand points. There are mainly three different types of algorithm for HAC which are single-link algorithm (SLA), complete-link algorithm (CLA), average-link algorithm (ALA). These algorithms are implemented by determining the similarity between two clusters. In a L/A problem, usually, the geographical distance is considered as the similarity index. In SLA, the similarity of two clusters is the similarity of their most similar demand points with respect to geographical distance in which the distance between two clusters is equal to the shortest distance from any member of one cluster to any member of the other cluster[MRS08],[Bor94]. In CLA, the similarity of two clusters is the similarity of their most dissimilar demand points in which the distance between two clusters is equal to the longest distance from any member of one cluster to any member of the other cluster. In ALA, the distance between two clusters is equal to the average distance from any member of one cluster to any member of the other cluster.

A L/A problem can be described as follows: Given the location of a set of destinations in terms of their coordinates and a set of shipping costs (or distances) for the region of interest, determine the optimum location of a fixed number of sources and the allocation of the destinations to the sources that will minimize the overall cost (or distance) [Coo63].

Again, static and deterministic L/A problems can be categorized into three groups: (i) Median problems, (ii) Covering problems, and (iii) Center problems [OD98]. Median problems can be formulated as the minimization of the average distance between demand points and facility locations. The solution of this problem increases facility accessibility by decreasing mutual distance between demand points and facility locations. In some cases, when demands are not sensitive to the level of service, the efficiency of facility location is measured by demand-weighted distance in which each distance is weighted by the associated demand quantity. An extension of the Median problem is the p-median problem which can be defined as the determination of optimum locations of p facilities so that the total demand-weighted distance between demands and facilities is minimized.

Covering problems are intended to cover customers or demand nodes such that the distance between a customer and its closest facility is no greater than a pre-specified standard distance value. They are divided into two major sub-groups: location set covering problem and maximal covering problem. A set covering problem can be expressed as the minimization of the cost of facility locations guaranteeing a specified level of coverage. On the other hand, a maximal covering problem can be formulated as the maximization of the coverage of the number of customers (or the amount of demand) within the acceptable standard service distance by locating an economically feasible fixed number of facilities.

The goal of Center problems is to minimize the maximum distance between any customer (or demand) and its nearest facility. Again, p-center problems inquire about the location of a given number of facilities while minimizing the largest customer-facility distance. In a p-center problem[OD98],[SD96], p number of service facilities are allocated to a number of demand nodes such that the maximum distance between a demand node and its corresponding service facility is minimized. p-center problem and its many variations [SPS04],[MLH03],[ELP04] have been widely investigated for solving different kinds of L/A problem. A p-center problem can be solved either heuristically or exactly. Solving such a problem exactly is a very difficult one. That's why, in most of the cases, it is solved heuristically. In our research work, we adopt the concept of p-center problem and combine it with the concept of maximal covering problem.

The concept of classical L/A problem or one of its many variants for the logistic systems planning can be mapped onto the problem of network dimensioning and placement of equipment in access networks. For example, we can consider the scenario of a supply chain of a complex logistics system which consists of two parts: Production system and Distribution system as shown in Figure 2 [GLM04].

In the production system, components and semi-furnished parts are produced in two manufacturing centers while finished goods are assembled at a different plant. The distribution system consists of an assembly plant which directly supplies goods to a number of *central distribution centers* (CDCs) from where goods are supplied to a number of *regional distribution centers* (RDCs) and finally each RDC supplies goods to several *retail outlets*. In analogy, the distribution system of the supply chain can be mapped to PON in which the assembly plant, CDCs/RDCs, and retail outlets

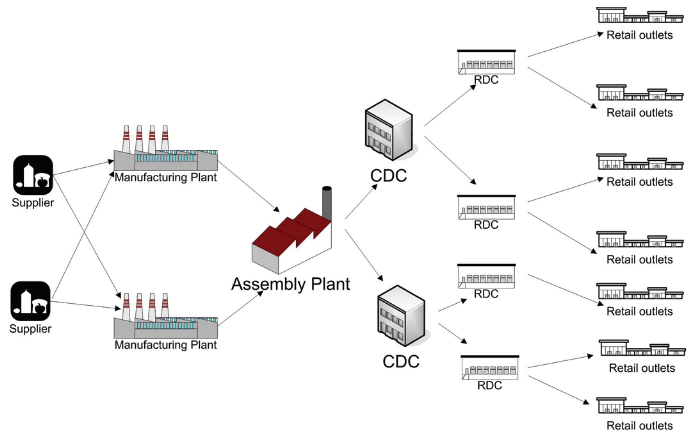


Figure 2: Logistics System Supply Chain [GLM04]

can be represented by the OLT, multi-level splitters/AWGs, and ONUs respectively. Thus the L/A problem of a logistics system can be a guideline for solving the L/A problem of network equipment in PON.

Chapter 3

Literature Review

In this chapter, we present a literature review on network dimensioning and placement of equipment in TDM/WDM PONs. Some of the studies on the placement of switching equipment in PONs have exploited the resemblance with the location/allocation (L/A) problem for the planning of logistic systems. For this reason, we describe the classical location allocation problem in Section 2.2 and explain how far the resemblance goes. While there are definitively some resemblance, there are also some differences such as the attenuation constraints which depend on the type of switching equipment and which limit the reach of the PON networks.

Another general comment is that most studies are conducted on the placement of splitters in TDM PON as well as placement of AWGs in WDM PONs. But, to the best of our knowledge, there is no published work on the selection and placement of splitters/AWGs in hybrid PONs. In this thesis, we do consider a mix of both switching equipment based on the characteristics of the traffic (e.g., mix of unicast and multicast requests) and on the location of the ONUs, as we do in the optimization process that is proposed in this thesis. We specify below, for each reference, the assumptions and limitations of the switching equipment selection.

In Section 3.1, we focus on the evolution of Location-Allocation Problem. In Section 3.2, we describe the previous research studies on the network planning and the placement of equipment in PONs.

3.1 Evolution of Location-Allocation Problem

Alfred Weber was the pioneer in the formal study of location theory while, in 1929, he formulated the problem of positioning a single warehouse at a location such that the total distance between the warehouse and several customers are minimized [web29], see [OD98] for a survey on location theory. Hakimi [Hak64] articulated location theory to find the optimum location of a 'switching center' in a communication network with the objective of minimizing the overall distance among the telecommunication users and the 'switching center'. His proposed location theory also locates the best place to build a 'police station' in a highway system with the objective of minimizing travel distance to reach the 'police station'. He shows that the optimum location of a switching center is always at a vertex of the communication network while the best location for the police station is not necessarily at an intersection.

Cooper [Coo63] proposed four heuristic algorithms for L/A problem which can be visualized in many ways such as locating factories, warehouses or supply points to serve customers at various locations. Their proposed heuristics are the basis of the most efficient heuristics of today.

The literature review on the L/A problem is replete with many variants and references [RE05], [GLM04], [HM03], [GGYX97], [HJK96]. We investigate which one of L/A problems, or which one of their many variants can be mapped onto the problem of network dimensioning and placement of equipment in passive access networks.

3.2 Network Planning and Placement of Equipment in PONs

The research on the placement of equipment in PONs can be motivated by the solutions of L/A problems of logistics systems. Li and Shen [LS08] investigate the problem of network planning for PON deployment. They decompose the problem into two subproblems: (1) Allocation subproblem in which clustering of ONUs is required to determine the groups of ONUs that will be connected to the same splitter, (2) Location subproblem to determine the optimal number and locations of the splitters. The objective function of both subproblems is to minimize the overall network deployment cost. They propose a scalable optimization approach for the solution of this problem. Their solution includes the total number, geographical locations, and varying splitting ratios (1:4, 1:8, 1:16, 1:32, 1:64) of required optical splitters along with the connection relationship between each ONU and its corresponding splitter assuming one level PON networks.

The authors remark that heuristics are the most practical solution to solve this optimization problem as both subproblems are NP-complete. Two heuristic algorithms are considered in their study. The first one is the extension of the benchmark sectoring algorithm in which the given parameters are a set of ONUs distributed in a full-circle (or an annulus fashion) and a maximum split ratio, S_r for the splitters to be deployed. In this algorithm, the circle is sliced into multiple sectors with each sector having S_r ONUs, except for the last one which may have less than S_r ONUs. It is a simple heuristic that does not consider the signal attenuation constraint between the OLT and the ONUs. The second heuristic is Recursive Allocation and Location Algorithm (RALA) which has been derived from Cooper's algorithm [Coo63]. RALA is designed to find a set of splitters so that each splitter should connect to a 'maximum number of ONUs such that the maximum split ratio, the maximal transmission distance, and the maximum differential distance of a standard PON network are satisfied. In RALA, during Step 1, a set of splitters is placed randomly on an Euclidean plane as an initial solution; in Step 2, the validity of the initial solution is checked to ensure that it meets all the PON system constraints; in Step 3, an efficient location of a set of splitters and the allocation of ONUs to each splitter are determined using a recursive process. The recursive process mainly proceeds in two sub-steps: (i) ONU allocation, and (ii) splitter relocation. During ONU allocation, any unconnected ONU is connected to its closest splitter first and this process continues until all the ONUs are linked to a splitter; any ONU making the system constraints unsatisfied will not be connected to that splitter. If the ONU allocation process fails to connect each ONU to one of the splitters for a given number of splitters, the splitters are relocated at random and the allocation process is repeated. If the allocation process is still unsuccessful to allocate the ONUs even after the relocation of splitters, the number of splitters is increased and the allocation process is repeated. Eventually, a solution will be obtained in which the number and location of the required splitters will be fixed, and all the ONUs will be arranged in several groups such that each and every ONU of a group will be connected to a common splitter allocated for that group. The authors also claim that they can further optimize the connection relationship between each ONU and the splitters by employing a Mixed Integer linear Programming (MILP) model for a small or medium-size design with up to several hundred ONUs. They carry out simulations to measure and compare the cost per user for three planning schemes namely, benchmark sectoring, RALA, and RALA incorporated with MILP. The results show that pure RALA scheme reduces the PON deployment cost 50%-70% compared to the sectoring scheme and for a medium-size design, RALA with MILP approach further reduces the corresponding cost about 10%. In this paper, both sectoring and RALA schemes can not determine the optimal location of splitters such that the distance between an ONU and its associated splitter is minimized. Even with MILP approach, the location of splitters is not optimal. Moreover, the authors do not investigate the compromise between one level networks with maximal signal splitting and two or more levels with reduced signal splitting.

Lee *et al.* [LKH06] examine design problem for the deployment of PONs by analyzing the location-allocation problem of splitters. They formulate the single splitting problem (SSP) and the distributed splitting problem (DSP) in which SSP includes single-level splitters and DSP multi-level splitters. The objective function minimizes the total expenditure of fiber and splitter cost which is subject to the following main constraints: (1) every demand should be assigned to splitters, (2) the sum of demands assigned to the splitters placed at a node should not exceed the total capacity of the splitters, (3) a single type of optical cable having enough capacity satisfying the required number of fibers should be installed at each link. For both problems, they use mixed integer programming (MIP) modeling to determine the optimal placement of splitters. In order to solve the MIP models, they provide a tighter representation by using the reformulation-linearization technique (RLT), and develop a column generation model taking advantage of polyhedral characteristics of the problems. Their proposed model assumes a tree-topology based PON having one access node at the root, several demand nodes at the leaf, and a number of intermediate nodes between the access node and each leaf node. Splitters can be placed at any node of the tree. For SSP, the column generation model was formulated by using tree configurations (i.e., columns) where each configuration is associated with one splitter and its set of incoming/outgoing links (so called tree generation by the authors, although it is a star centered at the node of concern). The tree generation formulation is executed for each node to find a star with the minimum reduced cost such that demands are assigned gratifying the splitter capacity constraint. For DSP, each configuration is defined as a tree having one primary splitter and several secondary splitters connected to the primary splitter; each configuration is further decomposed by defining a tree with a single secondary splitter. The authors present preliminary computational results for both SSP and DSP models where the lower bound is obtained by linear programming (LP) relaxation and disaggregation analysis. They compute upper bounds by using the CPLEX integer linear programming (ILP) solver (branch-and-bound method) on the restricted master problem defined by the set of columns. The optimality gaps (difference between lower and upper bounds) are quite large (up to 81%), so it is quite difficult to assess the quality of their solutions. In addition, there is no way to compare their results with other models, as there is no published model which measures the cost of splitter location-allocation problem.

Later, Kim *et al.* [KLH11] propose a relaxation of the objective function proposed in [LKH06] and, with the help of valid inequalities and a local search heuristic, they reduce the optimality gap between the solutions of their LP and ILP formulations, and therefore obtained a better estimation of the quality of their solutions.

Hajduczenia *et al.* [HLdSM07] investigate a multi-constrained optimization problem for automated PON deployment. The authors have stated this optimization problem as follows: Given a map containing a set of resources, a set of obstacles, and a set of access points, the objective function is to find the optimum path distribution in terms of optical power budget and deployment cost, and also to determine the optimum number of subscriber groups so that each group can be served with a separate PON or with a separate major network branch. They propose a model that takes into account several issues such as power budget, splitter location, existing network resources (trenches, aerial lines), and obstacles (both traversable: roads, greenfield areas and non-traversable: houses, industrial zones). They devise an optimization scheme to automate the selection of location of passive star couplers (PSC) and path deployment process. Genetic algorithms are adopted as the optimization technique for the optimum path deployment of PON. Clustering approach is applied to find optimum grouping of subscriber distribution. The authors have implemented a K-means clustering algorithm in which the input parameter representing an initial expected number of clusters is selected automatically by applying average silhouette width mechanism [DF02]. They execute the algorithm for different number of clusters, calculate an average silhouette width for each solution, select the cluster count which has the highest silhouette value, and finally constitute an independent PON branch for each of the identified clusters assuring sufficient splitter capacity. They apply their algorithm on artificial maps where the ONUs are scattered, although in reality ONUs are usually concentrated around building complexes. Their initial results show that the automated PON deployment tool can achieve lower network cost compared to the hand made cost computed by the experienced network planner. But the proposed technique does not specify how to find the optimal placement of ONUs and PSCs (splitters) in a given network environment.

Mitcsenkov *et al.* [APC09] propose a heuristic solution to address TDM PON topology planning minimizing deployment cost along with operational aspects. They also propose an ILP to serve as a reference for smaller cases so that the performance of their heuristic can be compared with the optimal solution obtained by the ILP. The ILP corresponds to a traffic flow problem where all customers are covered by a flow such that the splitters split an incoming flow to a set of outbound flows by the actual split ratio. The authors claim that the solutions obtained by their proposed heuristic are within the 10-20% of their computed ILP. Due to TDM technology, multiple splitters along with multiple feeder fibers are used which results in an overall increased of deployment cost. The topology supports only single stage splitting architecture. It neither supports a multi stage architecture nor it utilizes AWG in the PON topology. The ILP does not optimize the location of the splitting nodes, it only connects the customers with the given splitting nodes. In the formulation of the ILP, the distance between the CO and the customers are not taken into account which is required to take into account the attenuation of a splitter.

Zhang and Ansari [ZA09] present a heuristic scheme to minimize the cost of AWGs and of the optical cables in deploying WDM PON. While optimizing the trade-off between the AWG cost and optical fiber cable cost, they decompose the network planning problem into the following subproblems:(i) determine the subscribers connected to each AWG exploiting tree-partitioning algorithms, (ii) decide geometric locations of AWGs, (iii) determine the cascaded AWG architecture by proposing a recursive partition-combination based algorithm. No information is given on the performance and the efficiency of the proposed heuristic in terms of solution accuracies.

Li and Shen [LS09] formulate a mathematical optimization model to minimize the deployment cost of a single-stage architecture based PON. Their proposed optimization model is non-linear. Moreover, the authors assume that the cost factor of a splitter has a linear relationship with the number of output ports of the corresponding splitter (which is not true in practice). As their proposed model is not tractable in practice, experiments are conducted with the heuristic proposed in [LS08].

Khan and Ahmed [KA07] transform the PON layout design problem as a theoretical graph problem. They explore several graph techniques and propose an algorithm for designing a PON layout. They compare the results of their proposed approach with those of randomized layouts. But no information is provided on the effectiveness of the proposed approach with respect to realistic PONs.

Kokangul and Ari [KA11] develop optimization models for multi-hierarchy (two-stage) PON planning problem. First, they construct a large nonlinear mathematical model. Because of nonlinearity and NP-completeness, this model could not be solved. Then they propose a genetic algorithm (GA) based heuristic to solve the planning problem. Finally, they linearize the constructed nonlinear problem and obtain the optimal solution for a very small size problem instance. Exploiting GA and mathematical modeling, they optimize the positions of the primary and secondary nodes, the split levels of the nodes as well as assigning customers to secondary nodes and secondary nodes to primary nodes. Their proposed model has very limited capability as it considers only four possible primary node locations, twenty possible secondary node locations, and twenty-eight customers. In their proposed multi-hierarchy planning scheme, each secondary node can serve maximum eight customers and each primary node can serve maximum sixteen customers which implies that each PON can handle only sixteen customers. Moreover, the selection of split level of the primary and secondary nodes is also very much restricted.

Xiong *et al.* [XWW+11] propose a nonlinear ILP model for designing TDM PON networks. Their proposed model is formulated to determine the optimal number and locations of the OLT. Due to its nonlinearity, the proposed model can not be executed. Then they propose a partitioning algorithm with the same objective as the ILP model. But the objective function does not bear any significance for the designing of PON network. The authors consider single stage TDM PON in which different OLTs are situated in different locations and each OLT is connected to a single splitter that is connected to a number of subscribers in turn. But, in practice, the OLT is located at a single location, i.e., at the CO. Again, their proposed algorithm does not determine the location of the splitters. The authors do not take into account unicast/multicast traffic. They just consider total amount of traffic required by each ONU.

Roka [Rok12] investigates the designing of next generation PON (NG-PON)networks using the hybrid PON (HPON) network configuration. He builds a simulation tool to select the environment for the HPON configuration and its capabilities. The tool is created in Matlab 7.0 and Visual C++ 6.0 which includes graphical interface to insert the input parameters of the HPON. This tool provides heuristic solution for single stage PON and determines the number of required splitters, AWGs, ordinary lasers, tunable lasers, receivers based on the number of total subscribers and the capacity of the hybrid network. But the author does not describe the algorithm of the simulation tool. His created tool, at best, can serve as an approximation model as it considers that all ONUs are located at equal distance from the OLT which is very unrealistic. Moreover, the simulation tool neither takes into account the unicast/multicast traffic while selecting the splitters/AWGs nor determines the optimized location of these switching equipment for the HPON.

Recently, significant amount of research activities have been noticed to investigate different aspects of hybrid PONs. Mahloo *et al.* [MMCW13] investigate the design of multi-stage hybrid PONs. They compute the capital expenditure (CAPEX) for different architectures of hybrid PONs. Their investigated architectures consist of an AWG in the remote terminal 1 (RT1) and a number of splitters in the remote terminal 2 (RT2). They experiment with different number of output ports for the AWG as well as varying number of output ports for the corresponding splitters. But the authors do not propose any generalized optimization model or heuristic solution to calculate CAPEX of a hybrid PON.

We can summarize that in [LS09] the authors investigate the optimal grouping of ONUs to be served by a common splitter, in [LKH06] the authors develop mathematical models for single and double level splitting problems, in [HLdSM07] the authors implement clustering techniques to group the subscribers to be served with a separate PON network and then apply genetic algorithm to find the optimum path distribution, in [KA11] they authors propose a genetic algorithm (GA) based heuristic for multi-hierarchy (two-stage) PON planning problem to optimize the positions of the primary and secondary nodes, the split levels of the nodes as well as assigning customers to secondary nodes and secondary nodes to primary nodes, in [XWW+11] the authors propose a partitioning algorithm to determine the optimal number and locations of the OLT, in [Rok12] the author builds a simulation tool which provides heuristic solution for single stage PON and determines the number of required splitters, AWGs, ordinary lasers, tunable lasers, receivers based on the number of total subscribers and the capacity of the hybrid network.

As a summary of all the studies reviewed in this section, we note that none of the previously published heuristics and ILP formulations considers the traffic unicast/multicast flows of individual ONUs for the placement of equipment in PON. There is no study investigating the placement of both splitters and AWGs in a given hybrid PON network. In our research work, we plan to focus on the placement of splitters/AWGs based on the user density and required bandwidth of individual ONUs. In the solution process that will be proposed in the subsequent sections, we aim to find the optimum number and location of splitters/AWGs in a hybrid PON network according to the traffic demand (unicast/multicast) and the location of a set of ONUs, while taking care of the attenuation constraints. We will also propose a solution scheme to determine the optimal number of hybrid PONs to cover all ONUs (i.e., aggregated end users) in a neighborhood. Our proposed solution scheme will also determine the optimal coverage of each hybrid PON.

References	Equipment	Model H vs. ILP	Attenuation constraints	Traffic demand taken into account	# levels	Remarks
[LS08]	Splitters	H & MILP	No	No	1	MILP not presented.
[LKH06]	Splitters	ILP	No	Throughput # required optical fibers	1 & 2	
[KLH11]	Splitters	ILP	No	Throughout # required optical fibers	1 & 2	Traffic demand not justified,
[HLdSM07]	Splitters	H	No	No	1	Exploited Genetic algoritm, not an efficient tool to get an optimal solution.
[APC09]	Splitters	H & ILP	No	Yes Traffic flows	1	Traffic demand not realistic.
[ZA09]	AWGs	H	No	No	2	Assumptions are not realistic, multiple fibers are considered from OLT to AWG which should be multiple wavelengths in one fiber.
[LS09]	Splitters	H & MILP	No	No	1	MILP not simulated.
[KA07]	Splitters	H	No	No	1	Effectiveness of the heuristic is not verified.

Table 1: Literature Review Summary

Chapter 4

Switching Equipment Location/Allocation in a Single hybrid PON - Scheme 1

4.1 Introduction

In this chapter, we propose an efficient optimization scheme for network dimensioning and placement of switching equipment for a single hybrid PON network. Our proposed scheme executes in two phases which is elaborated in this chapter. The chapter is organized as follows. Our proposed optimization process as well as the problem statement is described in Section 4.2. Section 4.3 describes the first phase of the optimization process, i.e., the clustering algorithm. Phase II, a column generation (CG) based integer linear programming (ILP) model for selecting the type, location of the switching equipment and performing the network dimensioning, is presented in Section 4.4. The solution of the CG model is described in Section 4.5. Section 4.6 discusses the numerical experiments which have been conducted on various data sets. Summary of this chapter is given in the last section.

4.2 PON Deployment: Problem Statement and Optimization Process

4.2.1 Problem Statement

We propose to investigate the greenfield deployment of a hybrid PON network with the aim of minimizing the overall network deployment cost (i.e., infrastructure installation and maintenance cost). Installation cost comprises the price of the equipment (OLT, ONUs, splitters and AWGs), of the optical fiber cable and the cost for trenching and laying fibers. The cost of an equipment depends on the number of available outlet ports. Note that there is no maintenance cost for the switching equipment as it is a passive one. The optimization model excludes the installation and maintenance cost of the OLT and the ONUs as these are fixed and unavoidable costs. Our goal is to devise an efficient topology for a hybrid PON network. The topology of a PON network is defined by the grouping of ONUs, selection of a passive switching equipment for each group of ONUs and the placement of the selected switching equipment. We plan to propose a solution scheme to achieve our aforesaid goal.

The input parameters of our proposed scheme include the location of the OLT and of the ONUs, the set of potential/candidate equipment locations together with the unicast/multicast traffic demand matrix (normalized values with respect to the transport capacity of the wavelengths). The output parameters comprise of the grouping of ONUs as well as the locations and the type of the switching equipment for each group of the ONUs of the hybrid PON network while satisfying the network design constraints such as splitting ratio of splitters/AWGs and maximum allowed signal power loss at each ONU. As passive switching equipment, we are considering both splitters and AWGs. We assume that only one switching equipment can be assigned to serve a group of ONUs. We propose to select the switching equipment depending on the type (unicast/multicast) of traffic demand as splitters are best suited for multicast demand whereas AWGs are suitable for unicast traffic demand. Whenever a splitter is assigned to a group, all ONUs inside that group use one common upstream wavelength and another common downstream wavelength in which the corresponding splitter splits the downstream signal and combines the upstream signal of all ONUs. Whereas in case of an AWG, all ONUs inside any group use different upstream wavelengths and different downstream wavelengths. In both cases, each ONU uses a single wavelength for upstream and another wavelength for downstream transmission. This implies that each ONU requires single transmitter and single receiver. The upstream and downstream traffic demand of each ONU is normalized to the capacity of a single wavelength.

In this thesis, we have assumed the following: (i) each ONU accommodates aggregated traffic requests of a number of end users, (ii) the capacity of a single wavelength is sufficient enough to fulfill the traffic demand of each ONU, (iii) we have sufficient wavelengths to satisfy the total traffic demand requests of all ONUs. If an ONU's requested traffic demand exceeds the maximum capacity of a wavelength, another transmitter/receiver can be employed for that ONU. In this case, the ONU will be transmitting/receiving using two different wavelengths simultaneously. Here, we have considered static traffic demand requests for both unicast and multicast sessions. Our plan is to devise a network planning tool for L/A problem of a hybrid PON based on the projected future traffic demand of the ONUs over a period of years.

We also propose to investigate network dimensioning problem in a hybrid PON. Apparently, placing splitters/AWGs close to the OLT will increase the fiber cost significantly as separate fibers are required to connect each ONU to the splitter. On the contrary, locating the splitters/AWGs toward the proximity of ONUs will reduce the fiber cost but it will increase the number and the cascading of required switching equipment. We propose an optimization model which determines the optimal locations of splitters/AWGs such that the distance between an ONU and its corresponding splitter/AWG is minimized while satisfying the PON network design constraints, in particular the maximum allowed signal power loss (attenuation) at each ONU.

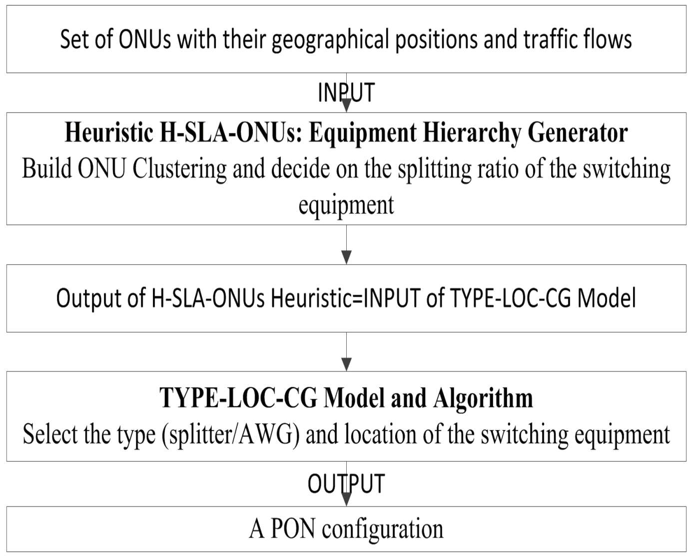


Figure 3: LAPON Solution Scheme

4.2.2 Optimization Process

The solution to the above mentioned problem statement consists of (i) first aggregating the ONUs into multiple clusters in order to determine the ONU-equipment association so that all ONUs inside a cluster can be served by a common switching element (ii) then determining the type and the location of the equipment for each cluster while provisioning the wavelengths based on traffic demand matrix. The first part as well as the second part of the solution can be proved to be NP-complete [LS09] [APC09] by reducing the problem into multiple knapsack problem [Jen09]. That's why, the problem, as a whole, is very difficult to solve using mathematical optimization scheme for a large planning scenario. This study about the problem has guided us to decompose the aforesaid problem into two sub-problems in which the first part (i.e., Sub-problem 1) of the problem is solved using a clustering heuristic and the second part (i.e., Sub-problem 2) is solved by formulating a mathematical linear optimization model.

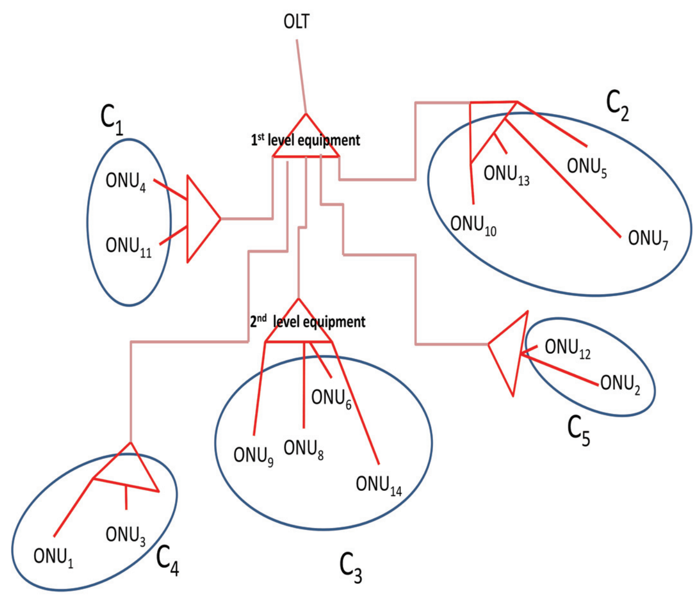


Figure 4: A two-stage Equipment Hierarchy

In order to solve these two subproblems, we propose the LAPON (Location/Allocation PON) algorithm which is a two-phase algorithm according to the process scheme depicted in Figure 3.

The first phase, detailed in Section 4.3, consists in generating several potential equipment hierarchies with the aid of a clustering heuristic algorithm. An equipment hierarchy can be defined as the physical cascading architecture of a PON network which includes the clustering of the ONUs and the number of levels/stages of switching equipment of a PON network. In this paper, we have considered two-stage equipment hierarchy in which all equipment are distributed on two levels such that all ONUs are connected to the 2nd level equipment and all 2nd level equipment are connected to the single 1st level equipment which is itself connected to the OLT, see Figure 4 for an example of such a hierarchy.

However, the type and geographical location of the passive equipment are not yet determined. The second phase, detailed in Section 4.4, consists in selecting for each potential hierarchy the best type and location of its passive equipment in terms of the minimum network deployment cost such that the distance between an ONU and its corresponding splitter/AWG is minimized. We assume that the location of the OLT and the ONUs along with the requested (unicast/multicast) traffic demand matrix of each ONU are given. The input data of the mathematical model also includes a potential set of equipment locations together with their distance matrices between any pair of potential locations. Note that those distances are not necessarily the shortest distance between the two locations, and do take into account the logistic obstacles for trenching and layering the optical fibers, as well as the available ducts to host the optical fibers.

At the end, the best hierarchy is selected.

4.3 Phase I: Equipment Hierarchies and Clustering Heuristic

In order to generate equipment hierarchies, we use a clustering algorithm, called H-SLA-ONUs. It relies on the classical single-link algorithm (SLA) [Har75],[Har81], [Pen95] for the clustering of the ONUs. SLA is an agglomerative hierarchical clustering method in which the geographical distance between clusters is defined as the distance between the closest pair of objects of the corresponding clusters. SLA is executed as follows: (i)Step 1: Each ONU is assigned to a cluster such that 'N' number of ONUs are confined to 'N' clusters, (ii)Step 2: Among all clusters, the closest pair of clusters is identified and merged them into a single cluster so that we have one cluster less than the previous step, (iii) Step 3: The distances between the new cluster and each of the old clusters are computed, (iv) Step 2 and Step 3 are repeated until all ONUs are grouped into a single cluster of size 'N'.

In each step of the H-SLA-ONUs algorithm, we get a new partition with a smaller number of clusters after the merging of the two closest clusters. Each partition leads to an equipment hierarchy by the process described below.

For each partition, the number of clusters defines the splitting ratio of the first level equipment, whereas the number of ONUs in a cluster defines the splitting ratio of the equipment of the corresponding cluster, i.e., the splitting ratio of the second level equipment. Some cluster re-organization is performed in order to reconcile the cardinality of the clusters with the standard splitting values, as described in Algorithm 1, see below. We illustrate in Figure 5 the cardinality adjustments of algorithm H-SLA-ONUs for a given clustering. Assume that clusters are ordered as follows: C_3, C_2, C_1, C_4, C_5. As the cardinality of C_3 is 5, it is rounded down to the closest available splitting ratio, i.e., 4. Consequently, we extract the ONU of C_3 which is the closest one to another cluster not yet considered, i.e., ONU_9 and we move it to C_4. Next, similarly, we move ONU_7 from C_2 to C_5. Finally, we round off the cardinalities of clusters C_5 and C_4 to 4. We are now done as all cardinalities matches standard splitting ratios.

Algorithm 1 H-SLA-ONUs

Apply the single-link algorithm (SLA) for a given number of clusters, say M, while forbidding the generation of clusters with more than the maximal allowed splitting ratio.
Order the clusters in the decreasing order of their cardinality
for all each cluster C in that order **do**
  Let card(C) be the cardinality of C
  Round off card(C) to the closest standard splitting ratio value
  if it corresponds to a rounding down **then**
    Extract from C the ONU which is the closest to another cluster which is, either smaller than C, or larger than C but with room for an additional ONU
    Repeat the operation until the number of ONUs in C is equal to the rounded down cardinality value
  end if
end for

4.4 Phase II: Optimization Model for Selecting the Location of the Passive Equipment

We propose the TYPE-LOC-CG-ILP algorithm that determines which switching equipment and where to locate it within a given hierarchy, as generated by the H-SLA-ONUs heuristic. Indeed, several potential equipment hierarchies will be generated by the H-SLA-ONUs heuristic. Once the best switching equipment and the best location have been found by the TYPE-LOC-CG-ILP algorithm for each potential hierarchy, the most economical equipment hierarchy will be selected. The TYPE-LOC-CG-ILP algorithm relies on a large scale optimization model that is described in Section 4.4.2 after setting the notations in Section 4.4.1. Its solution uses column generation techniques.

4.4.1 Notations

Hierarchy Parameters

For a given hierarchy, G is the set of ONU groups as well as 2nd level equipment in a given equipment hierarchy, i.e., g_0 the cluster of the 2nd level equipment associated with the single first level equipment and g any of the second level clusters, which is connecting a given subset of ONUs with the same switching equipment. We will denote by $|g|$ the splitting ratio of the switching equipment of cluster g. Let $G^\star$ be the set $G \setminus \{g_0\}$. In order to identify the membership of an ONU to a particular cluster, we use the parameter $\delta_{\text{ONU},g}$: It is equal to 1 if ONU belongs to cluster g in equipment hierarchy, and 0 otherwise.

A provisioned hierarchy is described by its switching equipment at each level by the following parameters: $a_{g_0,k} = 1$ if there is an equipment with $k \in K = \{2, 4, 8, 16, 32, 64\}$ output ports at the first level, below the OLT, leading cluster g_0, and 0 otherwise. Similarly, the equipment selected at the second level is described by the parameter $a_{g,k}$, for $g \in G^\star$.

Location Parameters

Let $V = \{\text{OLT}\} \cup V^{\text{ONU}}$ be the set of nodes where $V^{\text{ONU}} = \{\text{ONU}_1, \text{ONU}_2, \ldots, \text{ONU}_n\}$. We consider that all ONUs are capable of transmitting and receiving single or multiple wavelengths.

A set P of discrete locations, indexed by p, such that: $P = P_{\text{OLT}} \cup P_{\text{ONU}} \cup P_{\text{EQ}}$, where the following locations are assumed to be known: *(i)* $P_{\text{OLT}} = \{p_{\text{OLT}}\}$, the OLT location, *(ii)* $P_{\text{ONU}} = \{p_{\text{ONU}_1}, p_{\text{ONU}_2}, \ldots, p_{\text{ONU}_n}\}$, the ONU locations and *(iii)* P_{EQ} the set of potential locations for switching equipment. As all these locations are known, it is easy to determine their pairwise distances $d_{pp'}$.

Cost Parameters

We denote by $\text{COST}^k_{\text{S}}/\text{COST}^k_{\text{AWG}}$ the cost of a splitter/AWG with $k \in K$ output ports. Let COST_{FT} be the cost of the fiber and of the trenching per kilometre. Our optimization model excludes the cost of the OLT and the ONUs as we have assumed that the OLT and the ONUs have fixed costs, independent of the location of the switching equipment.

Traffic Parameters

Traffic matrix $T = (T_{s,d} \cup T_{s,D})$ such that $T_{s,d}$ is the amount of unicast bandwidth to be carried out from node v_s to node v_d where $v_s, v_d \in V$ and $T_{s,D}$ is the amount of multicast bandwidth to be carried out from node v_s to each node v_d where $v_s \in \{\text{OLT}\}$, $v_d \in D \subseteq V^{\text{ONU}}$ assuming D be the multicast destination sets and $\mathcal{D}$ be the overall set of multicast destination sets. We distinguish:

- Upstream traffic: it is made of unicast traffic flows, each flow from one ONU to the OLT, denoted by $T_{\text{ONU,OLT}}$,
- Downstream traffic: it is made of unicast or multicast traffic flows, each flow from the OLT to a single or subset of ONUs ($T_{\text{OLT},d}$ or $T_{\text{OLT},D}$ where $d \in P_{\text{ONU}}$,$D \in$

$\mathcal{D}$). The total number of wavelengths carried by the optical fiber is denoted by W.

4.4.2 Optimization Model

Location Configurations

A configuration c corresponds to the bandwidth demands that can be routed on a given wavelength, on a given equipment hierarchy where either an AWG or a splitter has been set at some of the intermediate nodes. A location configuration can be either upstream or downstream. As one configuration means one wavelength, it is not possible to have upstream as well as downstream traffic on a single wavelength. We denote the overall set of configurations by $\mathcal{C}$ such that $\mathcal{C} = \mathcal{C}^{\text{UL}} \cup \mathcal{C}^{\text{DL}}$, where $\mathcal{C}^{\text{UL}}$ (resp. $\mathcal{C}^{\text{DL}}$) is the set of uplink (resp. downlink) configurations. Let COST_c be the cost of configuration c. For a given equipment hierarchy, a configuration $c \in \mathcal{C}$ is characterized by:

- $t^c_{s,d} \in [0, 1]$ is the amount of unicast bandwidth carried out by configuration c for source node v_s and destination node v_d where $v_s, v_d \in V$. The parameter $t^c_{s,d}$ can be of two types: $t^c_{\text{OLT},d} \in [0, 1]$ for downstream where $d \in V^{\text{ONU}}$ and $t^c_{s,\text{OLT}} \in [0, 1]$ for upstream where $s \in V^{\text{ONU}}$.

- $t^c_{s,D} \in [0, 1]$ is the amount of multicast bandwidth carried out by configuration c for $(v_s, \{v_d : v_d \in D\})$. The parameter $t^c_{s,D}$ can be of single type: $t^c_{\text{OLT},D} \in [0, 1]$ for downstream.

- $a\_a^c_{p,g,k} = 1$ if an AWG with $k \in K$ output ports is set at location $p \in P_{\text{EQ}}$ serving the ONUs of cluster $g \in G$ in configuration c, 0 otherwise.

- $a\_s^c_{p,g,k} = 1$ if a splitter with $k \in K$ output ports is set at location $p \in P_{\text{EQ}}$ serving the ONUs of cluster $g \in G$ in configuration c, 0 otherwise.

- $\alpha^c_d = 1$ if destination d is served by configuration c and 0 otherwise where $d \in D$.

Variables

- $z_c \in \{0,1\}$ is a decision variable such that $z_c = 1$ if configuration c is selected, and 0 otherwise.

- $y\_s_{p,g,k} \in \{0,1\}$ is a decision variable such that $y\_s_{p,g,k} = 1$ if a splitter with $k \in K$ output ports is placed at location $p \in P_{\text{EQ}}$ serving either the ONUs of cluster $g \in G^\star$ or the 2nd level equipment $g \in g_0$ in the selected configurations (they must all concur for the switching equipment), and 0 otherwise.

- $y\_a_{p,g,k} \in \{0,1\}$ is a decision variable such that $y\_a_{p,g,k} = 1$ if an AWG with $k \in K$ output ports is placed at location $p \in P_{\text{EQ}}$ serving either the ONUs of cluster $g \in G^\star$ or the 2nd level equipment $g \in g_0$ in the selected configurations (they must all concur for the switching equipment), and 0 otherwise.

- $y_{p,p',g,k} \in \{0,1\}$ is a decision variable introduced for linearization purposes (see below), such that $y_{p,p'g} = 1$ if p (resp. p') are selected for the location of a switching equipment with $k \in K$ output ports in group $g \in G^\star$ (resp. g_0), where $p, p' \in P_{\text{EQ}}$ and 0 otherwise.

Objective

As mentioned before, the objective corresponds to the deployment cost of a given equipment hierarchy where the type and locations of its passive equipment are determined as to minimize the cost while satisfying the technological and traffic constraints. It is formally defined as follows:

$$\text{COST}(y) = \text{COST}^{\text{LINK}}(y) + \text{COST}^{\text{EQ}}(y) \tag{1}$$

where

$$\text{COST}^{\text{LINK}}(y) = \text{COST}_{\text{FT}} \sum_{i=1}^{3} \text{COST}_i^{\text{LINK}}(y) \tag{2}$$

$$\text{COST}_1^{\text{LINK}}(y) = \sum_{p \in P_{\text{EQ}}} \sum_{k \in K} d_{\text{OLT},p}(y\_s_{p,g_0,k} + y\_a_{p,g_0,k}) \tag{3}$$

$$\text{COST}_2^{\text{LINK}}(y) = \sum_{p \in P_{\text{EQ}}} \sum_{p' \in P_{\text{EQ}}} \sum_{g \in G^\star} \sum_{k \in K} d_{pp'}(y\_s_{p,g_0,k} + y\_a_{p,g_0,k})(y\_s_{p',g,k} + y\_a_{p',g,k}) \tag{4}$$

$$\text{COST}_3^{\text{LINK}}(y) = \sum_{g \in G^\star} \sum_{p \in P_{\text{EQ}}} \sum_{k \in K} \sum_{\text{ONU} \in P_{\text{ONU}}:\delta_{\text{ONU},g}=1} d_{p\text{ONU}}(y\_s_{p,g,k} + y\_a_{p,g,k}) \tag{5}$$

$$\text{COST}^{\text{EQ}} = \sum_{p \in P_{\text{EQ}}} \sum_{g \in G} \sum_{k \in K} (\text{COST}_{\text{S}}^k \, y\_s_{p,g,k} + \text{COST}_{\text{AWG}}^k \, y\_a_{p,g,k}) \tag{6}$$

where $\text{COST}_1^{\text{LINK}}(y)$ (resp. $\text{COST}_2^{\text{LINK}}(y)$, resp. $\text{COST}_3^{\text{LINK}}(y)$) are the fiber deployment costs associated with the first level, i.e., from the OLT to the first passive equipment of g_0 (resp. from the passive equipment of g_0 to the passive equipment of the clusters $g \in G^\star$, resp. from the passive equipment of the clusters $g \in G^\star$ to their ONUs), and COST^{EQ} the cost of the selected passive equipment.

In order to linearize the expression of (4), we introduce variables $y_{p,p',g,k}$ so that expression of $\text{COST}_2^{\text{LINK}}(y)$ becomes:

$$\text{COST}_2^{\text{LINK}}(y) = \sum_{p \in P_{\text{EQ}}} \sum_{p' \in P_{\text{EQ}}} \sum_{g \in G^\star} \sum_{k \in K} d_{pp'} \, y_{p,p',g,k} \tag{7}$$

with

$$y_{p,p',g,k} = (y\_s_{p,g_0,k} + y\_a_{p,g_0,k})(y\_s_{p',g,k} + y\_a_{p',g,k}),$$

together with the following additional constraints:

$$y\_s_{p,g_0,k} + y\_a_{p,g_0,k} + y\_s_{p',g,k} + y\_a_{p',g,k} - 1 \leq y_{p,p',g,k} \tag{8}$$

$$y\_s_{p,g_0,k} + y\_a_{p,g_0,k} \geq y_{p,p',g,k} \tag{9}$$

$$y\_s_{p',g,k} + y\_a_{p',g,k} \geq y_{p,p',g,k} \tag{10}$$

for all $p \in P_{EQ}, p' \in P_{EQ}, g \in G^\star, k \in K$.

The linearization is valid under the assumption that

$$y\_s_{p,g_0,k} + y\_a_{p,g_0,k} \leq 1 \quad g \in G, p \in P_{EQ}, k \in K$$

which is fulfilled due to constraints (16) (to be described in the sequel)

Constraints

There are three sets of constraints which decompose into the equipment hierarchy constraints, the equipment location constraints, and the demand constraints.

Equipment hierarchy constraints The number of selected configurations generated around one equipment hierarchy is limited by the number of available wavelengths:

$$\sum_{c \in C} z_c \leq W. \tag{11}$$

The next set of constraints imply that only configurations associated with the selected equipment hierarchy can be themselves selected in the optimal solution. For all $p \in P_{\text{EQ}}, g \in G, k \in K$, we have:

$$\sum_{c \in C} a\_s^c_{p,g,k}\, z_c \geq y\_s_{p,g,k} \tag{12}$$

$$\sum_{c \in C} a\_a^c_{p,g,k}\, z_c \geq y\_a_{p,g,k} \tag{13}$$

$$\sum_{c \in C} a\_s^c_{p,g,k}\, z_c \leq W y\_s_{p,g,k} \tag{14}$$

$$\sum_{c \in C} a\_a^c_{p,g,k}\, z_c \leq W y\_a_{p,g,k}. \tag{15}$$

Equipment location constraints All 2nd level equipment must connect to the same equipment of the 1st level (i.e., location of a 1st level equipment is same for all 2nd level equipment). The equipment of each group must be placed in a single location:

$$\sum_{p \in P_{\text{EQ}}} \sum_{k \in K} (y\_s_{p,g,k} + y\_a_{p,g,k}) = 1 \qquad g \in G. \tag{16}$$

A given location cannot be selected more than once in a given hierarchy:

$$\sum_{g \in G} \sum_{k \in K} (y\_s_{p,g,k} + y\_a_{p,g,k}) \leq 1 \qquad p \in P_{\text{EQ}}. \tag{17}$$

Demand constraints The upstream traffic will be granted if all its components are carried out.

$$\sum_{c \in \mathcal{C}^{\text{UL}}} t^c_{\text{ONU,OLT}}\, z_c \geq T_{\text{ONU,OLT}} \qquad \text{ONU} \in V^{\text{ONU}}. \tag{18}$$

The downstream traffic will be carried out only if every destination gets the signal and it is of two types:

$$\text{Unicast:} \sum_{c \in \mathcal{C}^{\text{DL}}} t^c_{\text{OLT},d}\, z_c \geq T_{\text{OLT},d} \qquad d \in P_{\text{ONU}} \tag{19}$$

$$\text{Multicast:} \sum_{c \in \mathcal{C}^{\text{DL}}} \alpha^c_d\, t^c_{\text{OLT},D}\, z_c \geq T_{\text{OLT},D} \quad d \in D, D \in \mathcal{D}. \tag{20}$$

4.5 Solution of the Model

4.5.1 Column Generation and ILP Solution

In order to solve the optimization model described in the previous section, we have two options: An off-line process in which all location configurations are pre-enumerated, or at least a subset of promising ones, or an on-line process in which location configurations are generated along with an iterative solution of the model. We choose the latter process relying on a column generation solution scheme, in which we start with a preliminary selection of a handful location configurations, and we add a new configuration only if it contributes to the improvement of the current solution of the linear relaxation of the model. For readers not familiar with column generation techniques, see, e.g., [Chv83] or [Las70].

A column generation solution scheme corresponds to a decomposition made of a so-called master problem (here the optimization model described in the previous section) and a so-called pricing problem (PP), to be viewed as a configuration generator. Note that in practice, one works with a so-called restricted master problem (RMP), as we only explicitly embed a subset of location configurations in the optimization model of Section 4.4.2. The PP guarantees the generation of an augmenting location configuration thanks to its particular objective, the so-called reduced cost, which has the following properties (again, readers not familiar with column generation techniques must refer to, e.g., [Chv83] or [Las70]): if there exists a location configuration with a negative reduced cost, its addition to the RMP will lead to a new solution with a reduced deployment cost, otherwise, we can claim that we have reached the optimal solution of the linear relaxation of the master problem.

Once the linear relaxation of the RMP has been solved optimally by the column generation algorithm, one needs to derive an integer solution. Here, rather than developing a costly branch-and-cut algorithm (see, e.g., [BJN+98]), we solve the ILP model made of the columns generated in order to obtain the optimal linear programming solution. It is well known that it usually does not provide the optimal ILP solution, but, as will be seen in the numerical results section, in practice, it was enough in order to obtain satisfactory optimized solutions. The execution flow of a CG based solution scheme is depicted in Figure 6.

We next describe the pricing problem, first its set of variables (Section 4.5.2), next its objective (Section 4.5.2), and then its set of constraints (Section 4.5.2).

4.5.2 Pricing Problem

In order to alleviate the notations, although each pricing problem is associated with a given equipment hierarchy, and a given equipment location configuration (c), we will omit the c index if there is no confusion.

Variables

The variables of the pricing are the coefficients of the z_c variables in the master problem, i.e., the generic coefficients of a column vector associated with a z_c variable (see their definitions in Section 4.4.2). Therefore, the variables of the pricing problem are:

- $t_{sD} \in [0,1]$ (values of the traffic are normalized using the transport capacity of a wavelength)
- $a\_a_{p,g,k} \in \{0,1\}$
- $a\_s_{p,g,k} \in \{0,1\}$
- $\alpha_d \in \{0,1\}$ where $\alpha_d = 1$ if any ONU $\in P_{\text{ONU}}$ is associated with a configuration.
- $\beta_g \in \{0,1\}$ where $\beta_g = 1$ if any ONU of group $g \in G^\star$ is associated with a configuration.

Objective

The objective of the pricing problem is defined by the minimization of the reduced cost (see [Chv83] if not familiar with linear programming concepts), which is expressed as follows for the upstream pricing problem:

$$\begin{aligned}\overline{\text{COST}}^{\text{UP}}(z) = &-\sum_{p \in P_{\text{EQ}}}\sum_{g \in G}\sum_{k \in K} u^{\text{S}}_{1_{p,g,k}} a\_s_{p,g,k} \\ &- \sum_{p \in P_{\text{EQ}}}\sum_{g \in G}\sum_{k \in K} u^{\text{AWG}}_{1_{p,g,k}} a\_a_{p,g,k} + \sum_{p \in P_{\text{EQ}}}\sum_{g \in G}\sum_{k \in K} u^{\text{S}}_{2_{p,g,k}} a\_s_{p,g,k} \\ &+ \sum_{p \in P_{\text{EQ}}}\sum_{g \in G}\sum_{k \in K} u^{\text{AWG}}_{2_{p,g,k}} a\_a_{p,g,k} - \sum_{\text{ONU} \in P_{\text{ONU}}} u^t_{\text{ONU}} t_{\text{ONU,OLT}} \qquad (21)\end{aligned}$$

where $u^{\text{S}}_{1_{p,g}}$ and $u^{\text{AWG}}_{1_{p,g}}$ are the dual values associated with constraints (12-p,g) and (13-p,g) respectively, $u^{\text{S}}_{2_{p,g}}$ and $u^{\text{AWG}}_{2_{p,g}}$ are the dual values associated with constraints (14-p,g) and (15-p,g) respectively, and u^t_{ONU} is the dual value associated with constraint (18-ONU).

The objective of the downstream pricing problem is expressed as follows:

$$\overline{\text{COST}}^{\text{DL}}(z) = -\sum_{p \in P_{\text{EQ}}} \sum_{g \in G} \sum_{k \in K} u^{\text{S}}_{1_{p,g,k}} a\_s_{p,g,k} - \sum_{p \in P_{\text{EQ}}} \sum_{g \in G} \sum_{k \in K} u^{\text{AWG}}_{1_{p,g,k}} a\_a_{p,g,k}$$
$$+ \sum_{p \in P_{\text{EQ}}} \sum_{g \in G} \sum_{k \in K} u^{\text{S}}_{2_{p,g,k}} a\_s_{p,g,k} + \sum_{p \in P_{\text{EQ}}} \sum_{g \in G} \sum_{k \in K} u^{\text{AWG}}_{2_{p,g,k}} a\_a_{p,g,k}$$
$$- \sum_{d \in P_{\text{ONU}}} u^t_{d,\text{UNI}}\, t_{\text{OLT},d} - \sum_{D \in \mathcal{D}} \alpha_d\, u^t_{d,\text{MULTI}}\, t_{\text{OLT},D} \quad (22)$$

where $u^{\text{S}}_{1_{p,g,k}}$ and $u^{\text{AWG}}_{1_{p,g,k}}$ are the dual values associated with constraints (12-p, g, k) and (13-p, g, k) respectively, $u^{\text{S}}_{2_{p,g,k}}$ and $u^{\text{AWG}}_{2_{p,g,k}}$ are the dual values associated with constraints (14-p, g, k) and (15-p, g, k) respectively, $u^t_{d,\text{UNI}}$ is the dual vector associated with constraint (19-d), and $u^t_{d,\text{MULTI}}$ is the dual vector associated with constraint (20-D).

The last term of the reduced cost is nonlinear, but we can easily linearized it: we can remove α_d in the above expression (22) of the reduced cost, and add the following constraint:

$$t_{\text{OLT},D} \leq \alpha_d \qquad d \in D, D \in \mathcal{D}, \quad (23)$$

as the values (i.e., the $t_{\text{OLT},D}$ values) of the traffic are normalized.

Constraints

Equipment Selection Constraints For each cluster g, at most one splitter/AWG with $k = |g|$ output ports can be placed in a potential location. In other words, for each $g \in G, k \in K : a_{g,k} = 1$, we have:

$$\sum_{p \in P_{\text{EQ}}} a\_a_{p,g,k} \leq 1 \quad (24)$$

$$\sum_{p \in P_{\text{EQ}}} a\_s_{p,g,k} \leq 1 \quad (25)$$

$$\sum_{p \in P_{\text{EQ}}} (a\_s_{p,g,k} + a\_a_{p,g,k}) = 1. \quad (26)$$

In the case where an AWG has been selected at the first level of the equipment hierarchy in the configuration under construction, we only need to select one equipment in the 2nd level as each configuration is associated with a single wavelength. However, if a splitter has been selected in the first level, we need to select $k_2 = |G^\star|$ equipment in the 2nd level. Those constraints are the purpose of the following constraint:

$$\sum_{g \in G^\star} \sum_{p \in P_{\text{EQ}}} \sum_{k_1 \in K} (a\_s_{p,g,k_1} + a\_a_{p,g,k_1}) = \sum_{p \in P_{\text{EQ}}} \sum_{k_2 \in K} (k_2 \times a\_s_{p,g_0,k_2} + a\_a_{p,g_0,k_2}). \qquad (27)$$

For each potential location, at most one equipment with a single splitting ratio can be placed.

$$\sum_{g \in G} \sum_{k \in K} (a\_s_{p,g,k} + a\_a_{p,g,k}) \leq 1 \qquad p \in P_{\text{EQ}}. \qquad (28)$$

For each cluster, at most one equipment with a single splitting ratio can be placed at a potential location.

$$\sum_{p \in P_{\text{EQ}}} \sum_{k \in K} (a\_s_{p,g,k} + a\_a_{p,g,k}) \leq 1 \qquad g \in G. \qquad (29)$$

Downstream Traffic Constraints If the optimization model selects a splitter in the first level, the summation of traffic requests of all clusters in the second level can be at most 1, in order not to exceed the transport capacity of a wavelength. If the selected switching equipment is an AWG in the first level, the individual traffic of each cluster can be at most 1. Similarly, if there is a splitter in the second level, the summation of traffic of all ONUs in the corresponding cluster can be at most 1 (and most likely even less than that in order to satisfy the capacity requirements at the upper level). Again, if there is an AWG in the second level, the individual traffic of each ONU can be at most 1.

For downstream traffic, we need to take into account both unicast and multicast traffic requests. We have considered the multicast traffic to facilitate the selection of the switching equipment. The optimization model will try to assign a splitter to a cluster if the ONUs of same multicast group resides in the same cluster so that a splitter can send the multicast traffic to all the ONUs within that cluster.

Constraints for unicast traffic are as follows:

$$\sum_{d\in P_{\text{ONU}}} t_{\text{OLT},d} \leq 1 \qquad t_{\text{OLT},d} \leq \alpha_d, \qquad\qquad d \in P_{\text{ONU}}. \tag{30}$$

Constraint 30 implies that the summation of traffic destined for all ONUs must not exceed 1 as the bandwidth of each wavelength is normalized to 1. It also ensures that an ONU can receive traffic only if the ONU is included in the configuration.

Constraints for multicast traffic are written as follows:

$$t_{\text{OLT},D} \leq \alpha_D \qquad \alpha_D \geq \alpha_d, \qquad\qquad d \in D, D \in \mathcal{D}. \tag{31}$$

Constraint 31 implies that a multicast destination set of ONUs can receive a multicast traffic demand only if all the ONUs in the destination set is included in the configuration.

Constraints for both unicast and multicast traffic are:

$$\sum_{d\in P_{\text{ONU}}:\delta_{d,g}=1} \alpha_d \leq \sum_{p\in P_{\text{EQ}}} \sum_{k\in K} (a\_a_{p,g,k} + |g| \times a\_s_{p,g,k}) \qquad\qquad g \in G^\star \tag{32}$$

$$\sum_{g\in G^\star} \beta_g \leq \sum_{p\in P_{\text{EQ}}} \sum_{k\in K} (a\_a_{p,g_0,k} + |g_0| \times a\_s_{p,g_0,k}) \tag{33}$$

$$\beta_g \geq \alpha_d \qquad\qquad g \in G^\star, d \in P_{\text{ONU}} : \delta_{d,g} = 1. \tag{34}$$

Constraints 32 states that if a splitter is assigned to a cluster g, at most $|g|$ ONUs confined to that cluster can receive traffic in each configuration. But in case of an AWG, only one ONU can receive traffic in each configuration.

Constraints 33 and 34 state that if the optimization model selects a splitter in the first level, all ONUs grouped into $|g_0|$ clusters can receive the data in each configuration. Again, if there is an AWG in the first level, at most one cluster of ONUs can receive the data in each configuration. A cluster is involved in a configuration only when any ONU confined to that group has downstream traffic demand to receive.

Upstream Traffic Constraints The upstream traffic only consists of unicast requests.

$$\sum_{\text{ONU} \in P_{\text{ONU}}} t_{\text{ONU,OLT}} \leq 1 \tag{35}$$

$$t_{\text{ONU,OLT}} \leq \alpha_{\text{ONU}} \qquad \text{ONU} \in P_{\text{ONU}} \tag{36}$$

$$\sum_{\text{ONU} \in P_{\text{ONU}}:\delta_{\text{ONU},g}=1} \alpha_{\text{ONU}} \leq \sum_{p \in P_{\text{EQ}}} \sum_{k \in K} (a\_a_{p,g,k} + |g| \times a\_s_{p,g,k}) \qquad g \in G^\star \tag{37}$$

$$\sum_{g \in G^\star} \beta_g \leq \sum_{p \in P_{\text{EQ}}} \sum_{k \in K} (a\_a_{p,g_0,k} + |g_0| \times a\_s_{p,g_0,k}) \tag{38}$$

$$\beta_g \geq \alpha_{\text{ONU}} \qquad g \in G^\star, \text{ONU} \in P_{\text{ONU}} : \delta_{\text{ONU},g} = 1 \tag{39}$$

Attenuation Constraints For each ONU, the power budget is limited to 20 dB. This implies that we have to make sure that the total signal loss from the OLT to each ONU must be less than 20 dB [LMKL07]. The total signal P_p is given by:

$$P_p = P_p^{\text{FIBER}} + P_p^{\text{THROUGH}} + P^{\text{INSERTION}} + P^{\text{MARGIN}} \tag{40}$$

where P_p^{FIBER} is the signal loss on the fiber to reach the ONU located at p, P_p^{THROUGH} is the loss provoked by going through the equipment towards the ONU located at p, $P^{\text{INSERTION}}$ is the overall insertion loss (i.e., the ratio of the power received at the end of a line to the power transmitted into the line) for all the lines in the PON topology, and P^{MARGIN} is a power margin to ensure that the calculation of the total loss is within the power budget range. The last two losses have a constant value.

We have experimented with two-stage cascaded architecture of switching equipment. A cascaded architecture of switching equipment consisting of more than two-stage will experience more signal attenuation caused by the additional switching equipment and may result in infeasible solution due to the limited power budget.

To calculate the first two losses, we introduce the variables $x\_\text{ATT}_p^g$ to evaluate the total attenuation in order to reach the ONU of cluster g located at p, $p \in P_{\text{ONU}}$. Let us assume a loss of 0.2dB/km caused by the optical fiber, and let ATT_k^{S} (resp. ATT^{AWG}) be the attenuation factor of the splitter S (resp. the AWG) heading cluster g, which depends on the number of outputs of S (resp. which is independent of the number of outputs of AWG). We get:

$$
\begin{aligned}
x\_\text{ATT}_p^g = & \sum_{k \in K} \sum_{p'' \in P_{\text{EQ}}} \big(\text{ATT}^{\text{AWG}}\, a\_a_{p'',g_0,k} + \text{ATT}_k^{\text{S}}\, a\_s_{p'',g_0,k} \\
& + \sum_{k \in K} \sum_{p' \in P_{\text{EQ}}} \big(\text{ATT}^{\text{AWG}}\, a\_a_{p',g,k} + \text{ATT}_k^{\text{S}}\, a\_s_{p',g,k}\big) \\
& + \sum_{k \in K} \sum_{p'' \in P_{\text{EQ}}} 0.2\, d_{\text{OLT}p''} (a\_a_{p'',g_0,k} + a\_s_{p'',g_0,k}) \\
& + \sum_{k \in K} \sum_{p'' \in P_{\text{EQ}}} \sum_{p' \in P_{\text{EQ}}} 0.2\, d_{p''p'} (a\_a_{p'',g_0,k} + a\_s_{p'',g_0,k})(a\_a_{p',g,k} + a\_s_{p',g,k}) \\
& + \sum_{k \in K} \sum_{p' \in P_{\text{EQ}}} 0.2\, d_{p'p} (a\_a_{p',g,k} + a\_s_{p',g,k}) \qquad p \in P_{\text{ONU}} : \delta_p^g = 1, g \in G^\star, \quad (41)
\end{aligned}
$$

where the first summation corresponds to the equipment attenuation at the first level (cluster g_0), the second summation corresponds to the equipment attenuation at the second level (cluster g), the third summation corresponds to the fiber attenuation between the OLT and the 1st level equipment at g_0, the fourth (resp, the fifth) corresponds to the fiber attenuation between the first level and the second level (resp. between the second level and the ONU located at p).

The fourth summation in (41) contains non linear terms. In order to linearize it, we add a new variable

$$a_{p',p'',g,k} = (a\_a_{p'',g_0,k} + a\_s_{p'',g_0,k})(a\_a_{p',g,k} + a\_s_{p',g,k}),$$

and the following constraints:

$$a\_a_{p'',g_0,k} + a\_s_{p'',g_0,k} + a\_a_{p',g,k} + a\_s_{p',g,k} - 1 \geq a_{p',p'',g,k} \tag{42}$$

$$a\_a_{p'',g_0,k} + a\_s_{p'',g_0,k} \leq a_{p',p'',g,k} \tag{43}$$

$$a\_a_{p',g,k} + a\_s_{p',g,k} \leq a_{p',p'',g,k} \tag{44}$$

for all $p'', p' \in P_{\text{EQ}}, g \in G^{\star}, k \in K$.

The last set of constraints expresses that the total loss for every ONU should not exceed 20 decibels:

$$x\_\text{ATT}_p^g + P^{\text{MARGIN}} + P^{\text{INSERTION}} \leq 20dB \qquad p \in P_{\text{ONU}}, g \in G^{\star}. \tag{45}$$

4.6 Numerical Results and Analysis

We implement the optimization model of Section 4.4.2 within the Optimization Programming Language (OPL) platform and solved the linear and integer linear programs using the CPLEX package [IBM11].

4.6.1 Data Instances

We conduct our experiments with four different scenarios (namely Scenario 1, Scenario 2, Scenario 3 and Scenario 4) consisting of randomly generated Manhattan pattern geographic locations of 16, 32, 64 and 128 ONUs respectively. Manhattan model is an ideal geometric model to represent a dense urban area in which ONUs are grouped in blocks and arranged in an array manner [MMCW13]. These ONUs are generated in a 40×20 km^2 rectangular grid such that the OLT is located at the middle of the corresponding grid, i.e., at location (20,10) as shown in Figure 7(a). ONUs are located along several vertical lines so that each value of x-coordinate can accommodate several ONU locations. There are 30 candidate/potential locations for the placement of the passive equipment which are randomly positioned inside the same rectangular grid as shown in Figure 7(b).

Table 2 contains the values taken for the cost of the equipment [CWMJ10], as well as the attenuation parameters, which depend on the number of output ports for the splitters, but not for the AWGs. For the costs related to optical fiber cables, we use the value of 7160$/km [CWMJ10], assuming it includes the cost of trenching and laying the optical fiber cables.

Table 2: Cost and Attenuation of Equipment

# output	Splitters		AWG	
ports	cost ($)	attenuation (dB)	cost ($)	attenuation (dB)
2	800	3	950	3
4	900	6	1,100	
8	1,100	9	1,400	
16	1,500	12	2,000	
32	2,300	15	3,200	
64	3,700	18	5,600	

We randomly generate the upstream unicast traffic flows within the range [0.05, 0.1] for each pair (ONU, OLT) (recall that our traffic parameters are normalized using the wavelength transport capacities, see Section 4.4.1). Towards downstream direction, we randomly generate both unicast and multicast traffic flows within the range [0.1, 0.4] for each pair (OLT, ONU), and a number of multicast requests. We assume 10GPON system for our experiment, referred to as XG-PON, which implies 10 Gbit/s transmission speed towards downstream and upstream direction [Rok12], [SKM10]. Our experimental unicast/multicast traffic flow can be translated according to the transmission speed of XG-PON. The number of generated multicast requests is 3,8,12 and 25 for different scenarios consisting of 16, 32, 64, 128 ONUs respectively. Each multicast request consists of 3 randomly generated destination ONUs. If the destination ONUs within a multicast group are geographically located in nearby regions, a regional multicast traffic is generated. There may also be generated a multicast group where the destination ONUs within that group are geographically located far apart from one another.

4.6.2 Accuracy of the Output Solutions

The accuracy of our proposed column generation based optimization scheme can be validated by computing the optimality gap between the LP and ILP solutions.

The optimality gap corresponds to:

$$\frac{\tilde{z}_{\text{ILP}} - z^{\star}_{\text{LP}}}{z^{\star}_{\text{LP}}},$$

where $z^{\star}_{\text{LP}}$ is a lower bound on the optimal value $z^{\star}_{\text{ILP}}$ (PON minimum cost) provided by the optimal value of the linear relaxation of the model (restricted master problem) described in Section 4.4, and $\tilde{z}_{\text{ILP}}$ is an upper bound on the optimal value $z^{\star}_{\text{ILP}}$ provided by the ILP solution of the ILP model associated with the last generated restricted master problem. As already observed by several authors for simpler ILP models, the optimality gaps are not very small, and vary from 0 to 7/10/11% in various case studies with 16/32/64 ONUs, therefore much smaller than those observed by, e.g., [LKH06].

4.6.3 Obtained Results and Analysis

The first step of our LAPON scheme is to run the H-SLA-ONUs heuristic in order to generate three equipment hierarchies for each of the experimental scenarios, using the number of clusters as a parameter:

(*i*) Hierarchy 1 with all the ONUs of a scenario grouped into 2 clusters, (*ii*) Hierarchy 2 with all the ONUs of a scenario grouped into 4 clusters,

(*iii*) Hierarchy 3 with all the ONUs of a scenario grouped into 8 clusters.

The next step of the LAPON scheme is to solve the column generation based optimization model in order to:

(i) Select the type (splitter or AWG) and location of the passive equipment,

(ii) Provision the traffic flows,

for each equipment hierarchy of each scenario. The last step of the LAPON scheme is then to select the best (minimum cost) equipment hierarchy. We now report on the numerical results, for various number of ONUs.

Table 3 shows a comparison of the PON 'greenfield' deployment costs for different hierarchies of Scenario 1 consisting of 16 ONUs. The type of the switching equipment, selected by the optimization model, is depicted in Figure 8 where the distribution of switching equipment is as follows:

Hierarchy 1. 1 AWG at the 1st level and 2 splitters at the 2nd level,

Hierarchy 2. 1 AWG at the 1st level and 3 splitters along with 1 AWG at the 2nd level,

Hierarchy 3. 1 AWG at the 1st level and 6 splitters along with 2 AWGs at the 2nd level.

For Scenario 1, the minimum cost hierarchy is the Hierarchy 1, consisting of 2 clusters, in which splitters are selected for the 2nd level and 1 AWG is selected for the 1st level. The selection of the switching equipment is made based on the best choice taking into account the cost, the traffic flows (some unicast, some multicast) and the attenuation constraints.

Table 4 shows a comparison of the PON deployment costs for different hierarchies of Scenario 2 consisting of 32 ONUs. The selection of switching equipment for all hierarchies is portrayed in Figure 9 which can be described as follows:

Table 3: Experimental Results for Scenario 1 (16 ONUs)

Hierarchy	Equip. Type	$z^{\star}_{\text{LP}}$	$\tilde{z}_{\text{ILP}}$	Optimality gap (%)
1	Mixed	1,296,120	1,296,640	0.04
2	Mixed	1,307,374	1,401,260	7.18
3	Mixed	1,301,220	1,301,220	0

Table 4: Experimental Results for Scenario 2 (32 ONUs)

Hierarchy	Equip. Type	$z^{\star}_{\text{LP}}$	$\tilde{z}_{\text{ILP}}$	Optimality gap (%)
1	Mixed	2,467,230	2,467,230	0
2	Mixed	2,254,340	2,254,340	0
3	Mixed	2,262,458	2,499,160	10.16

Hierarchy 1. 1 AWG at the 1st level and 2 splitters at the 2nd level,

Hierarchy 2. 1 AWG at the 1st level and 3 splitters along with 1 AWG at the 2nd level,

Hierarchy 3. 1 AWG at the 1st level and 5 splitters as well as 3 AWGs at the 2nd level.

We observe that Hierarchy 2 incurs minimum cost compared to other hierarchies in the case study of 32 ONUs.

In Table 5, we conduct experiments with Scenario 3 with three similar hierarchies. For Hierarchy 1 and Hierarchy 3, AWGs are selected for both 1st and 2nd level equipment; whereas for Hierarchy 2, the optimization model selects mixed-equipment PON architecture in which either a splitter or an AWG is assigned to each cluster, as displayed in Figure 10. The distribution of switching equipment is described below:

Hierarchy 1. 1 AWG at the 1st level and 2 AWGs at the 2nd level,

Table 5: Experimental Results for Scenario 3 (64 ONUs)

Hierarchy	Equip. Type	$z^{\star}_{\text{LP}}$	$\tilde{z}_{\text{ILP}}$	Optimality gap (%)
1	AWGs only	4,174,220	4,174,220	0
2	Mixed	3,957,240	3,957,240	0
3	AWGs only	3,295,240	3,685,280	11.83

Hierarchy 2. 1 AWG at the 1st level and 2 splitters along with 2 AWGs at the 2nd level,

Hierarchy 3. 1 AWG at the 1st level and 8 AWGs at the 2nd level.

We notice that Hierarchy 3 experiences minimum cost for the deployment of PON with the setting of 64 ONUs. It is obvious that if the optimization model could select splitters for all the clusters of a given hierarchy, the deployment cost would be the most economical one. However, there does not always exist a feasible passive equipment location/allocation with splitters only, due to the signal attenuation constraints. Indeed, in a splitter, the attenuation increases significantly with the increase of the number of output ports. However, the attenuation caused by an AWG is low and independent of the number of the output ports. While selecting the type of the equipment, the optimization model takes into account the attenuation constraint along with the bandwidth demand of each ONU and decides whether a splitter or an AWG will be assigned to a given cluster, according to the distance among the ONUs and the switching equipment.

Table 6 illustrates the deployment cost of a PON where all hierarchies of Scenario 4 are considered. The type of switching equipment selected in these hierarchies are shown in Figure 11 and described below:

Hierarchy 1. 1 AWG at the 1st level and 2 AWGs at the 2nd level,

Hierarchy 2. 1 AWG at the 1st level and 4 AWGs at the 2nd level,

Table 6: Experimental Results for Scenario 4 (128 ONUs)

Hierarchy	Equip. Type	$z^{\star}_{\text{LP}}$	$\tilde{z}_{\text{ILP}}$	Optimality gap (%)
1	AWGs only	7,865,230	7,865,230	0
2	AWGs only	6,352,560	6,352,560	0
3	AWGs only	6,000,173	6,060,140	0.99

Hierarchy 3. 1 AWG at the 1st level and 8 AWGs at the 2nd level.

Table 6 also reveals that the optimization model only assigns AWGs as the 1st and 2nd level equipment for all hierarchies. The reason behind it is that the selection of a splitter for a cluster can not generate any feasible solution due to the high power attenuation caused by the splitters. We perceive that Hierarchy 3 evolves as the best PON architecture for the Scenario 4 which consists of 128 ONUs.

In this thesis, we experiment with four different scenarios consisting of different number of ONUs. For each scenario, we consider three types of hierarchies which are generated to investigate the impact of the number of output ports (i.e., split/AWG ratio) of the 1st and the 2nd level switching equipment while optimizing the overall deployment cost of each scenario. For example, in Hierarchy 1 of Scenario 3, 64 ONUs are grouped into 2 clusters, the number of output ports is 2 for the AWG of the 1st level and is 32 for both AWGs of the 2nd level; again in Hierarchy 2 of the same Scenario, the number of output ports is 4 for the AWG of the 1st level and is 32, 32, 2,4 for two splitters and two AWGs of the 2nd level respectively; finally in Hierarchy 3 of the same Scenario, the number of output ports is 8 for the AWG of the 1st level and is 32,8,32,2,2,4,4,2 for eight AWGs of the 2nd level. Similarly, for all scenarios, different values of split/AWG ratio are taken into account. By using different hierarchies, we are getting insight of the number of output ports of the switching equipment and obtaining the optimal values of the corresponding number. Our optimization model considers the following factors while deciding on the minimal cost PON hierarchy for each scenario: (i) type of traffic demand (unicast/multicast), (ii) split/AWG ratio of the equipment, (iii) cost of the equipment and the fiber, (iv) signal power loss caused by equipment and fiber. There is a trade off between the splitting ratio and the maximum allowable distance from the OLT to the ONUs. Increasing the split ratio will accommodate more ONUs to be served by the single equipment, but it will decrease the maximum acceptable distance from the OLT to the ONUs as the attenuation of a splitter depends on its number of output ports. In such a situation, an AWG can be deployed in the network as its attenuation is much less compared to a splitter and does not increase with the increase of its number of output ports. But AWGs are much expensive than splitters. However, the cost of a splitter or an AWG depends on its number of output ports.

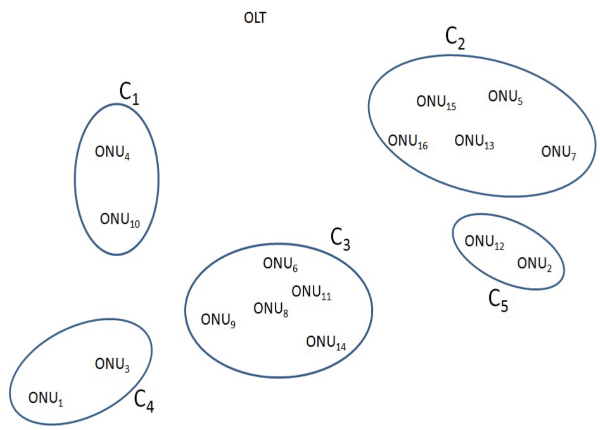
Moreover, the deployment cost increases with the number of ONU clusters: for the given ONU locations, more clusters mean not only a passive equipment to the ONUs but also more fiber cables in order to connect the passive equipment to the OLT. Again, there is also a trade off between the splitting ratio and fiber cable costs. For example, higher splitting ratio for the 2nd level equipment results in shorter sum of overall fiber cables. On the contrary, smaller splitting ratio for the 2nd level equipment results in longer sum of overall fiber cables, see Figure 12 for an illustration.

Our proposed optimization model takes into account all these aspects and selects different hierarchies for different scenarios as a economically feasible hybrid PON architecture.

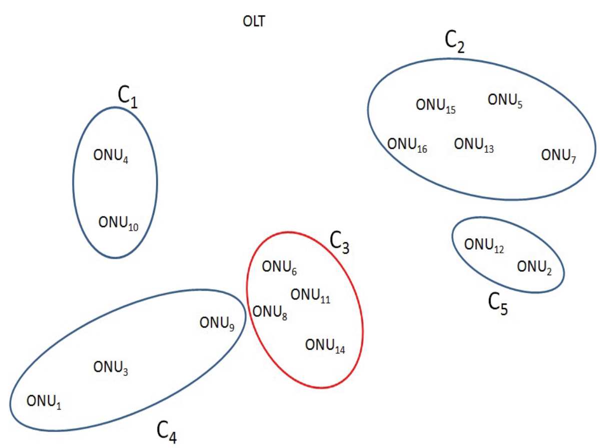
4.7 Summary

In this chapter, we present our first proposed scheme to solve the location allocation (L/A) problem of a single hybrid PON. We propose here an original optimization scheme for the deployment of greenfield PON networks where we minimize the overall deployment cost. The optimization scheme proceeds in two phases. In the first phase, we generate several potential equipment hierarchies, where each equipment hierarchy is associated with an ONU partition such that a switching equipment is associated with each cluster,each ONU belongs to a single cluster, and the splitting ratio of the equipment corresponds to the number of ONUs in the cluster. In the second phase, for each equipment hierarchy, we make use of a column generation (CG) mathematical model to select the type and location of the switching equipment that leads to the minimum cost multi-stage equipment topology which accommodates all the traffic demand. Finally, the best hierarchy among all the generated and dimensioned hierarchies is selected.

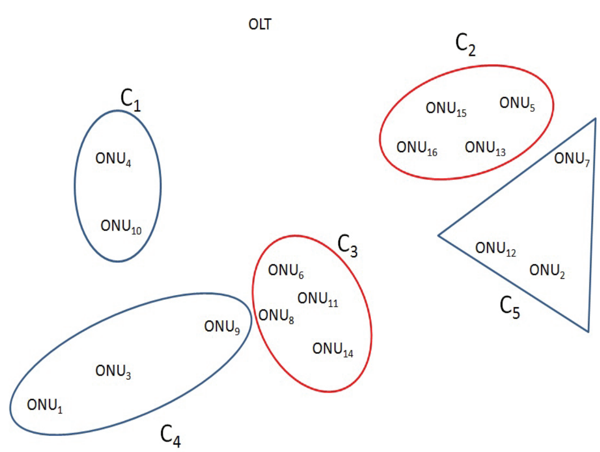
The optimization model encompasses the particular cases where all switching equipment are either splitters and AWGs, and outputs the location of the switching equipment together with the dimensioning of the PON network. We perform numerical experiments on various data sets in order to evaluate the performance of the optimization model, and to analyze the type of equipment hierarchies which are generated depending on the traffic and the location of the ONUs. As shown in the section on numerical results, the tool is quite powerful as data instances with up to 128 ONUs can be easily solved.



(a) Initial partition



(b) First modified partition



(c) Second modified partition

Figure 5: ONU partitioning

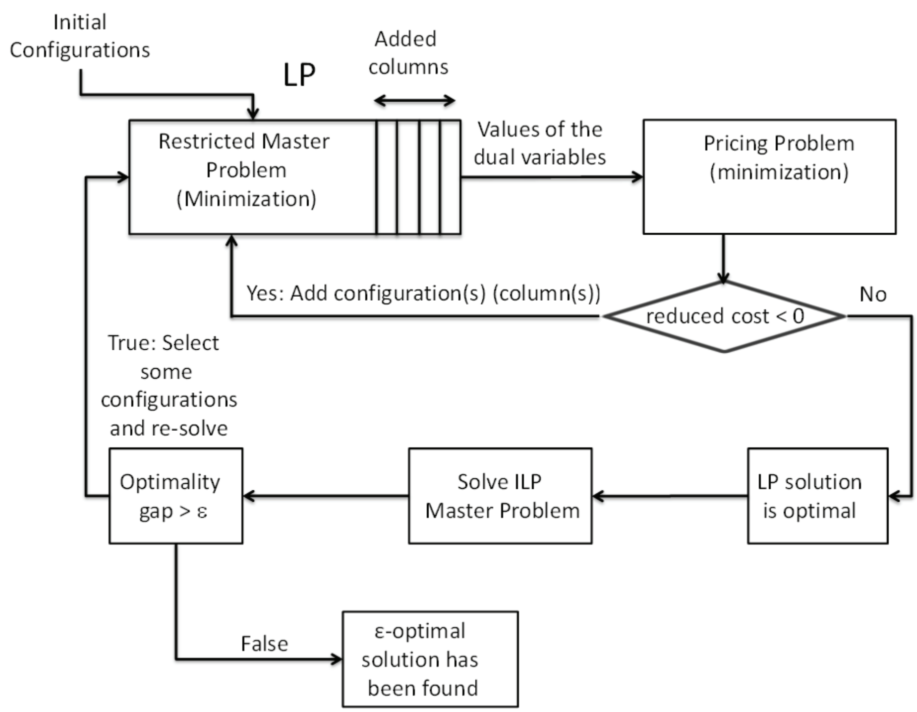
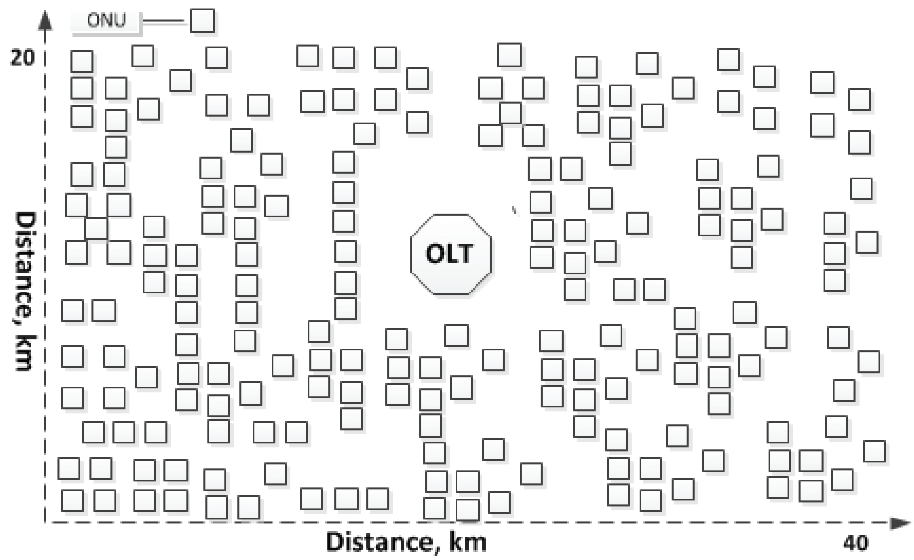
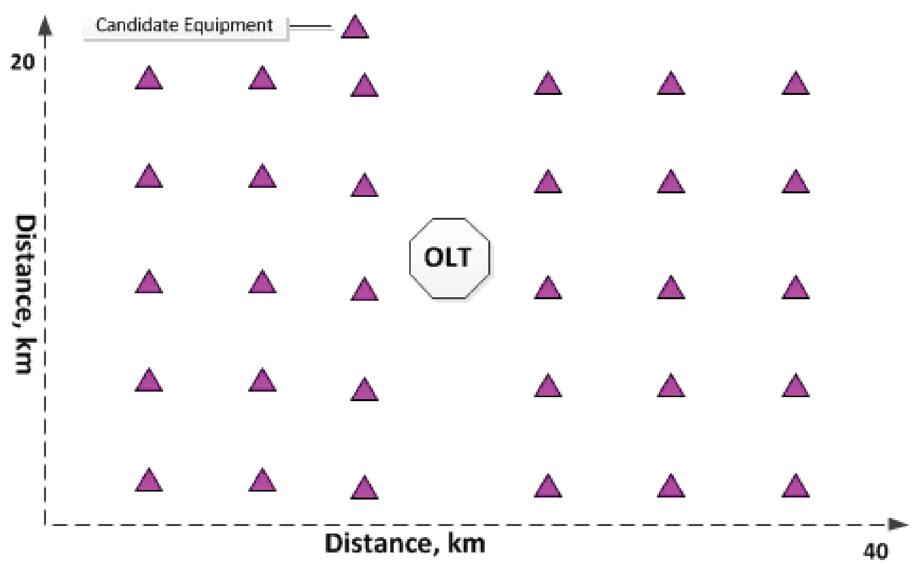


Figure 6: Execution Flow of CG Scheme



(a) ONU and OLT Distribution



(b) Potential Equipment Locations

Figure 7: Experimental Setup

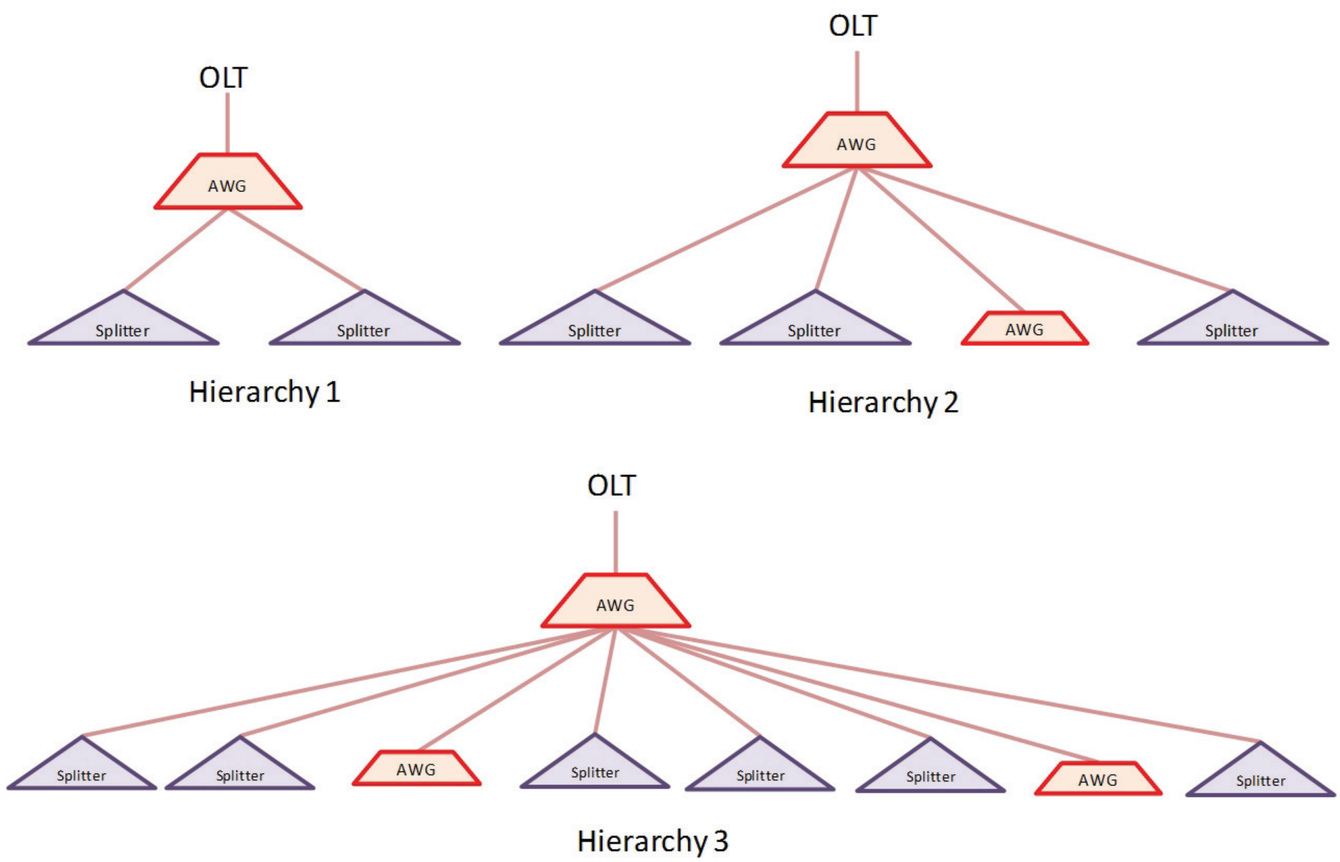


Figure 8: Type of equipment of Table 3

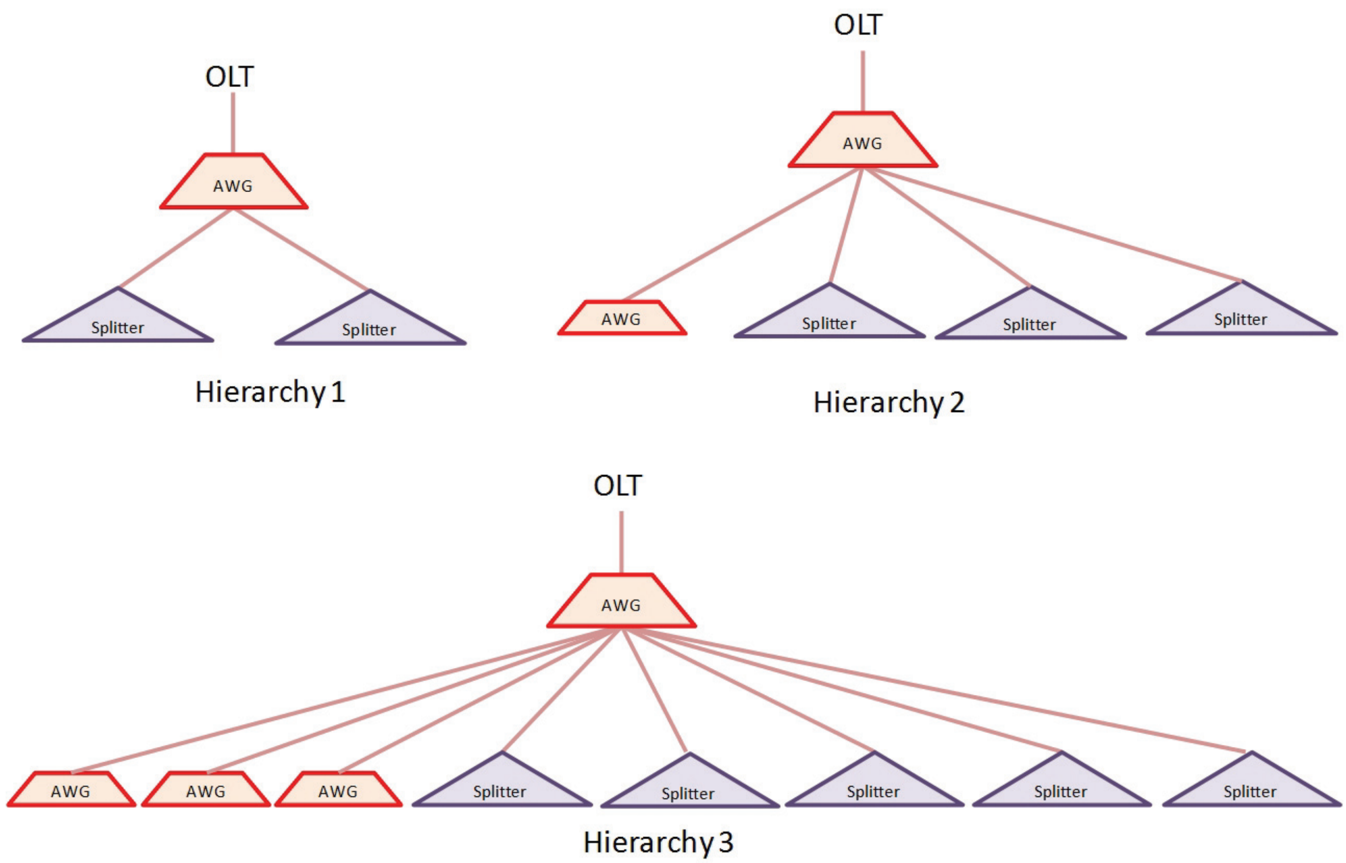


Figure 9: Type of equipment of Table 4

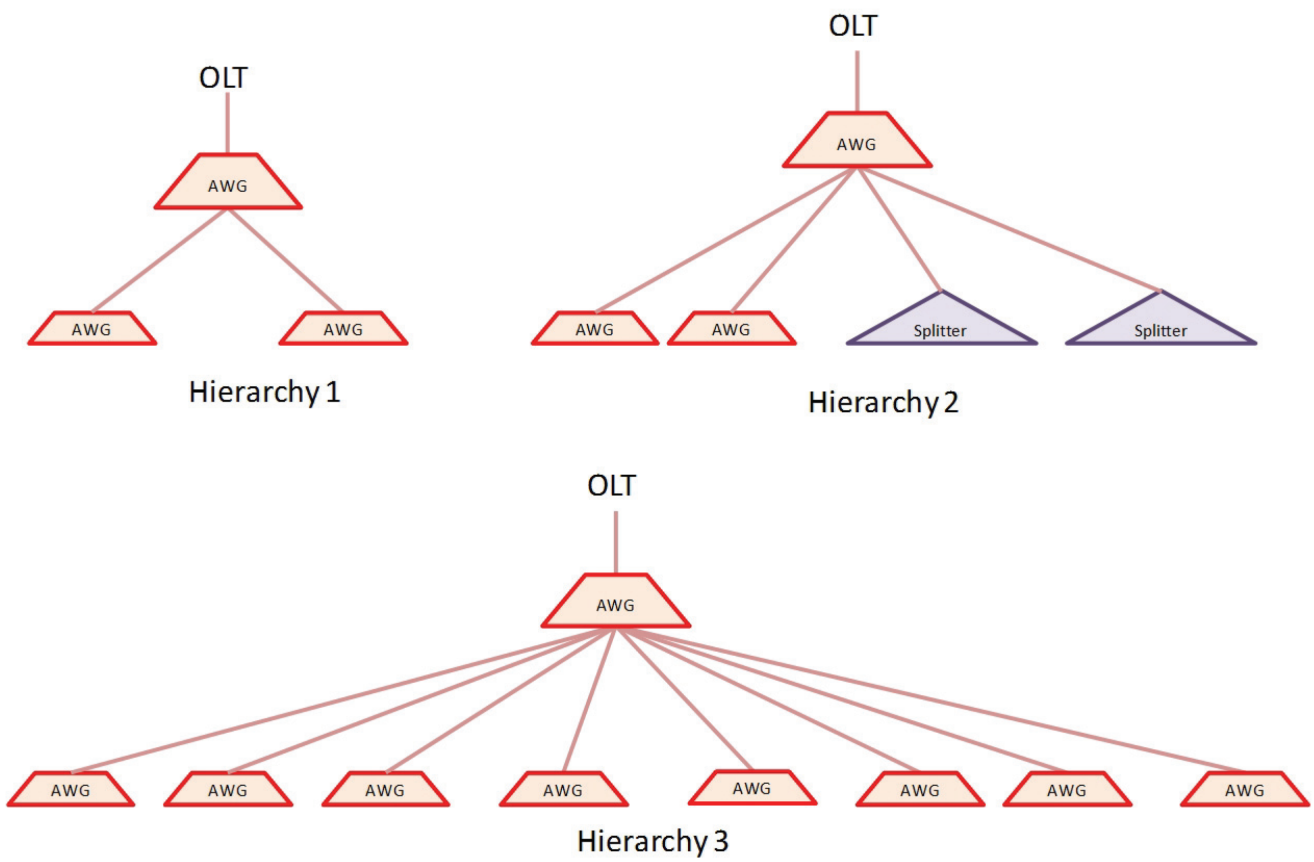


Figure 10: Type of equipment of Table 5

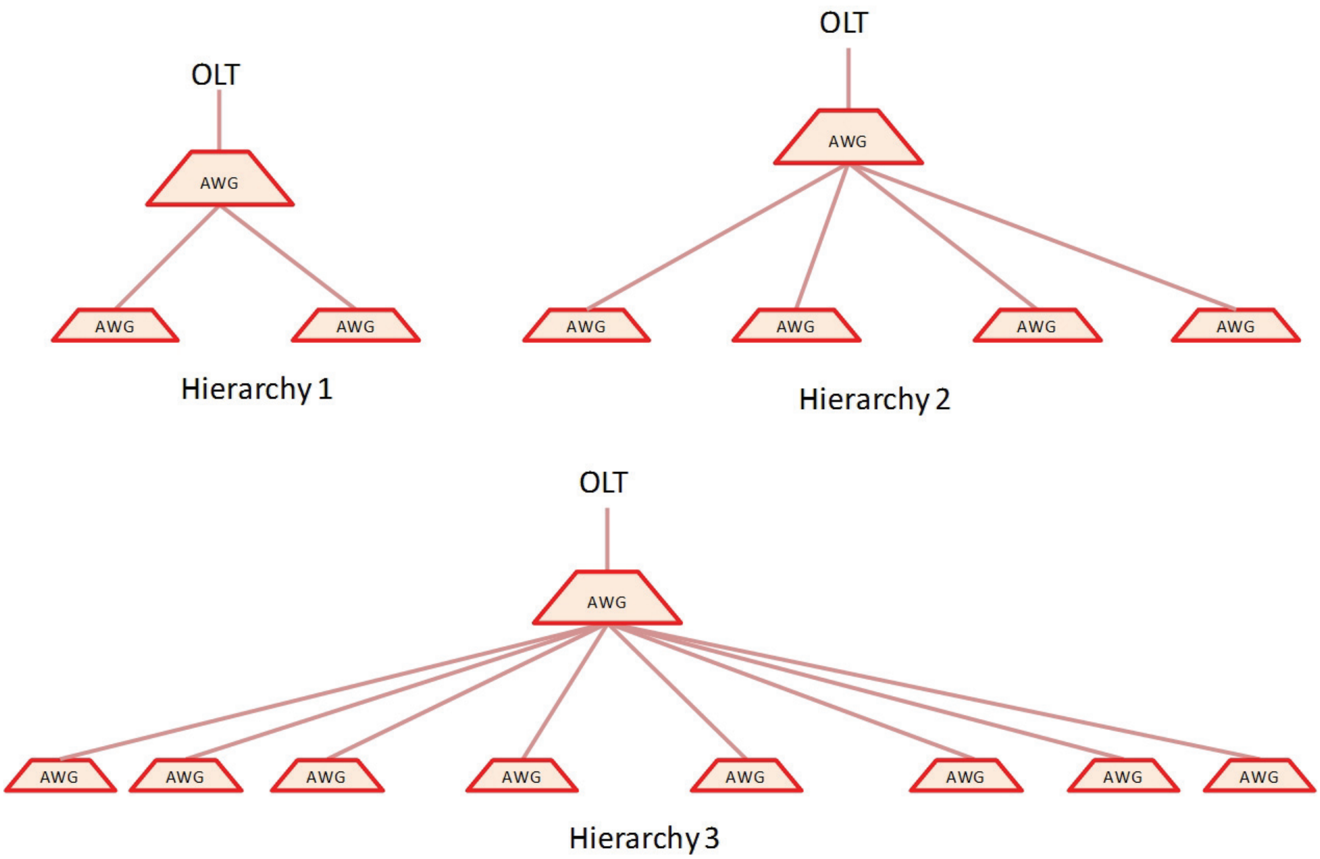


Figure 11: Type of equipment of Table 6

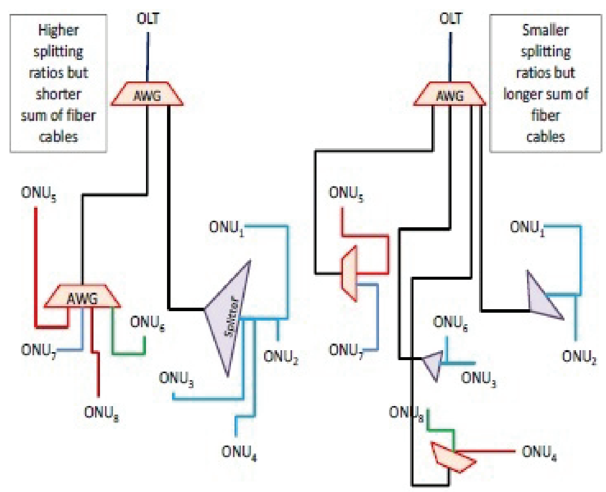


Figure 12: Minimum fiber cable costs vs splitting ratio

Chapter 5

Switching Equipment Location/Allocation in a Single hybrid PON - Scheme 2

5.1 Introduction

In this chapter, we investigate the layout of an optimal PON architecture and wavelength provisioning in a hybrid PON. We propose a novel cross layer optimization scheme by considering both physical and optical layer constraints, using a column generation model, to be solved using large scale optimization tools (i.e., decomposition techniques).

We have organized the chapter as follows.

In Section 5.2, a concise statement of the PON deployment problem is provided. We describe our proposed cross layer optimization model in Section 5.3 and computational results as well as analysis in Section 5.5. We have given the summary of this chapter in the last section.

5.2 PON Deployment: Problem Statement and Optimization Process

5.2.1 Problem Statement

Given the locations of the OLT, of a potential set of switching equipment locations and of the ONUs along with their incoming/outgoing traffic demands, we aim to determine the physical architecture of a hybrid PON network with the objective of minimizing the overall network deployment cost. The problem statement is described in detail in Section 4.2.1.

5.2.2 Optimization Process

We propose an integrated 'cross layer optimization scheme' by formulating a mathematical model which determines: *(i)* the optimum number of clusters the ONUs can be grouped into, *(ii)* ONU-cluster association information specifying which ONU belongs to which cluster, *(iii)* the type (splitter/AWG) and splitting ratio of the switching equipment of each cluster, *(iv)* locations of the selected equipment, *(v)* the provisioning of the traffic flows served by each wavelength. During optimization process, we not only consider the physical layer constraints of the PON (i.e., power attenuation and splitting ratio of the switching equipment as well as the maximum allowable signal power loss at each ONU) but also the optical layer constraints (i.e., number of wavelengths carried by optical fiber). Our proposed model assumes that each ONU accommodates aggregated traffic requests of a number of end users. For the deployment of a hybrid PON, we focus on two-stage cascading architectures of passive switching equipment (e.g. splitters/AWGs) such that all the ONUs are connected to the second level equipment by allocating each equipment to a group of ONUs, then all second level equipment are connected to the first level equipment, which is eventually connected to the OLT, as depicted in Figure 4.

5.3 Cross Layer Optimization Scheme

Our proposed optimization scheme is based on a large scale optimization model which is described in Section 5.3.2. The notations used in this model are illustrated in Section 5.3.1.

5.3.1 Notations

Location Parameters

Location parameters, used in the optimization model, are described in 4.4.1.

Cost Parameters

Cost parameters, used in the optimization model, are described in 4.4.1.

Traffic Parameters

Traffic parameters, used in the optimization model, are described in 4.4.1.

5.3.2 Linear Optimization Model

Location Configurations

First, we need to introduce the concept of location configurations. A configuration c corresponds to the bandwidth demands that can be routed on a selected wavelength, on a selected topology where either an AWG or a splitter has been set at some of the intermediate nodes. A configuration can be either upstream or downstream. As one configuration means one wavelength, it is not possible to have upstream as well as downstream traffic on a single wavelength. The overall set of configurations is denoted by $\mathcal{C}$ such that $\mathcal{C} = \mathcal{C}^{\text{UL}} \cup \mathcal{C}^{\text{DL}}$. A configuration $c \in \mathcal{C}$ is characterized by the following parameters:

- $t^c_{s,d} \in [0, 1]$ is the amount of unicast bandwidth carried out by configuration c for source node v_s and destination node v_d where $v_s, v_d \in V$. The parameter

$t^c_{s,d}$ can be of two types: $t^c_{\text{OLT},d} \in [0,1]$ for downstream where $d \in V^{\text{ONU}}$ and $t^c_{s,\text{OLT}} \in [0,1]$ for upstream where $s \in V^{\text{ONU}}$.

- $t^c_{s,D} \in [0,1]$ is the amount of multicast bandwidth carried out by configuration c for $(v_s, \{v_d : v_d \in D \in \mathcal{D}\})$. The parameter $t^c_{s,D}$ can be of single type: $t^c_{\text{OLT},D} \in [0,1]$ for downstream.

- $l^{\text{A},c}_{i,p}(l^{\text{S},c}_{i,p}) = 1$ if an AWG (a splitter) is set at location $p \in P_{\text{EQ}}$ serving either an ONU or any switching equipment or the OLT at $i \in P_{\text{ONU}} \cup P_{\text{EQ}} \cup P_{\text{OLT}}$ in configuration c and 0 otherwise.

Variables

- $z_c \in \{0,1\}$ is a decision variable such that $z_c = 1$ if configuration c is selected, and 0 otherwise.

- $y^{\text{A},\ell}_{p,k}(y^{\text{S},\ell}_{p,k}) \in \{0,1\}$ is a decision variable such that $y^{\text{A},\ell}_{p,k}(y^{\text{S},\ell}_{p,k}) = 1$ if an AWG (a splitter) with $k \in K$ output ports is placed at location $p \in P_{\text{EQ}}$ in at least one configuration and 0 otherwise where $\ell \in \aleph$, $\aleph \in \{\ell_1, \ell_2\}$ represents the level of the equipment.

- $L_{p,p'} \in \{0,1\}$ is a decision variable such that $L_{p,p'} = 1$ if there is a link between location p and p' where $p \in P_{\text{OLT}} \cup P_{\text{EQ}} \cup P_{\text{ONU}}$ and $p' \in P_{\text{EQ}}$

- $L^{\text{A}}_{i,p}(L^{\text{S}}_{i,p}) = 1$ if an AWG (a splitter) is set at location $p \in P_{\text{EQ}}$ serving either an ONU or any switching equipment or the OLT at $i \in P_{\text{ONU}} \cup P_{\text{EQ}} \cup P_{\text{OLT}}$ and 0 otherwise.

Objective

The objective corresponds to the deployment cost where the level (1st/2nd), type, number of output ports and locations of its passive equipment are determined as to minimize the cost while satisfying the technological and traffic constraints. It is formally defined as follows:

$$\text{COST}(y) = \text{COST}^{\text{LINK}}(y) + \text{COST}^{\text{EQ}}(y) \tag{46}$$

$$\text{COST}^{\text{LINK}}(y) = \sum_{p \in P_{\text{OLT}} \cup P_{\text{EQ}} \cup P_{\text{ONU}}} \sum_{p' \in P_{\text{EQ}}} L_{p,p'} * d_{p,p'} * \text{COST}_{\text{FT}} \tag{47}$$

$$\text{COST}^{\text{EQ}}(y) = \sum_{\ell \in \aleph} \sum_{p \in P_{\text{EQ}}} \sum_{k \in K} (\text{COST}_{\text{S}}^{k} \, y_{p,k}^{\text{S},\ell} + \text{COST}_{\text{A}}^{k} \, y_{p,k}^{\text{A},\ell}) \tag{48}$$

Constraints

Equipment selection constraints The number of selected configurations is limited by the number of available wavelengths.

$$\sum_{c \in C} z_c \leq W. \tag{49}$$

The following constraints are formulated to determine the type of the switching equipment (splitter/AWG) and the number of output ports of the corresponding equipment at the 2nd level:

$$\sum_{c \in C} l_{i,p}^{\text{S},c} \, z_c \leq M \, L_{i,p}^{\text{S}} \qquad i \in P_{\text{ONU}}, p \in P_{\text{EQ}} \tag{50}$$

$$\sum_{i \in P_{\text{ONU}}} L_{i,p}^{\text{S}} \leq \sum_{k \in K} k \, y_{p,k}^{\text{S},\ell_2} \qquad p \in P_{\text{EQ}} \tag{51}$$

$$\sum_{c \in C} l_{i,p}^{\text{A},c} \, z_c \leq M \, L_{i,p}^{\text{A}} \qquad i \in P_{\text{ONU}}, p \in P_{\text{EQ}} \tag{52}$$

$$\sum_{i \in P_{\text{ONU}}} L_{i,p}^{\text{A}} \leq \sum_{k \in K} k \, y_{p,k}^{\text{A},\ell_2} \qquad p \in P_{\text{EQ}} \tag{53}$$

The following constraints are formulated to determine the type of the switching equipment (splitter/AWG) and the number of output ports of the corresponding equipment at the 1st level:

$$\sum_{c \in C} l_{i,p}^{\text{S},c} z_c \leq M \, L_{i,p}^{\text{S}} \qquad i \in P_{\text{EQ}}, p \in P_{\text{EQ}} \tag{54}$$

$$\sum_{i \in P_{\text{EQ}}} L_{i,p}^{\text{S}} \leq \sum_{k \in K} k \, y_{p,k}^{\text{S},\ell_1} \qquad p \in P_{\text{EQ}} : l\_s_{\text{OLT},p} = 1 \tag{55}$$

$$\sum_{c \in C} l_{i,p}^{\text{A},c} z_c \leq M \, L_{i,p}^{\text{A}} \qquad i \in P_{\text{EQ}}, p \in P_{\text{EQ}} \tag{56}$$

$$\sum_{i \in P_{\text{EQ}}} L_{i,p}^{\text{A}} \leq \sum_{k \in K} k \, y_{p,k}^{\text{A},\ell_1} \qquad p \in P_{\text{EQ}} : l\_a_{\text{OLT},p} = 1 \tag{57}$$

The next set of constraints imply that the optimal configurations associated with the OLT and 1st level equipment will be selected.

$$\sum_{c \in C} l_{i,p}^{\text{S},c} z_c \leq M \, L_{i,p}^{\text{S}} \qquad i \in P_{\text{OLT}}, p \in P_{\text{EQ}} \tag{58}$$

$$\sum_{c \in C} l_{i,p}^{\text{A},c} z_c \leq M \, L_{i,p}^{\text{A}} \qquad i \in P_{\text{OLT}}, p \in P_{\text{EQ}} \tag{59}$$

Equipment location constraints Each location can contain at most one equipment of single type (splitter/AWG).

$$\sum_{k \in K} y_{p,k}^{\text{S},\ell} \leq 1 \qquad p \in P_{\text{EQ}}, \ell \in \aleph. \tag{60}$$

$$\sum_{k \in K} y_{p,k}^{\text{A},\ell} \leq 1 \qquad p \in P_{\text{EQ}}, \ell \in \aleph. \tag{61}$$

We can not select a given location more than once.

$$\sum_{\ell \in \aleph} \sum_{k \in K} (y_{p,k}^{\text{S},\ell} + y_{p,k}^{\text{A},\ell}) \leq 1 \qquad p \in P_{\text{EQ}}. \tag{62}$$

Topology constraints The global network for all configurations can be established by grouping common links in just one.

The following constraint establishes a link between the OLT and 1st level equipment, the 1st and 2nd level equipment as well as the 2nd level equipment and each ONU.

$$L^{\text{S}}_{i,p} + L^{\text{A}}_{i,p} = L_{i,p} \qquad i \in P_{\text{OLT}} \cup P_{\text{EQ}} \cup P_{\text{ONU}}, p \in P_{\text{EQ}}. \tag{63}$$

From an OLT, there must be only one outgoing link for the OLT which is to be towards an equipment.

$$\sum_{p \in P_{\text{EQ}}} L_{\text{OLT},p} = 1 \qquad \sum_{p \in P_{\text{ONU}}} L_{\text{OLT},p} = 0 \tag{64}$$

Each ONU must connect to single equipment.

$$\sum_{p \in P_{\text{EQ}}} L_{p',p} = 1 \qquad p' \in P_{\text{ONU}}. \tag{65}$$

All 2nd level equipment must connect to the same 1st level equipment such that the 2nd level equipment is connected to the ONUs and the 1st level equipment is connected to the OLT.

$$\sum_{p \in P_{\text{EQ}}: p \neq i} L_{\text{OLT},p}\, L_{i,p} \geq L_{p',i} \qquad i \in P_{\text{EQ}}, p' \in P_{\text{ONU}}. \tag{66}$$

Constraints (66) are nonlinear. In order to linearize them, we add new variables $L^{\text{OLT}}_{i,p} = L_{\text{OLT},p}\, L_{i,p}$, and the following constraints:

$$L_{\text{OLT},p} + L_{i,p} - 1 \leq L^{\text{OLT}}_{i,p} \qquad i \in P_{\text{EQ}}, p \in P_{\text{EQ}} : p \neq i \tag{67}$$

$$L_{\text{OLT},p} \geq L^{\text{OLT}}_{i,p} \qquad i \in P_{\text{EQ}}, p \in P_{\text{EQ}} : p \neq i \tag{68}$$

$$L_{i,p} \geq L^{\text{OLT}}_{i,p} \qquad i \in P_{\text{EQ}}, p \in P_{\text{EQ}} : p \neq i. \tag{69}$$

The number of output ports of an equipment must be greater than the number of outgoing links.

$$\sum_{p' \in P_{\text{EQ}}} L_{p',p} \leq \sum_{k \in K} k(y^{\text{S},\ell_1}_{p,k} + y^{\text{A},\ell_1}_{p,k}) \qquad p \in P_{\text{EQ}} \tag{70}$$

$$\sum_{p' \in P_{\text{ONU}}} L_{p',p} \leq \sum_{k \in K} k(y^{\text{S},\ell_2}_{p,k} + y^{\text{A},\ell_2}_{p,k}) \qquad p \in P_{\text{EQ}} \tag{71}$$

Demand constraints The upstream traffic will be granted if all its components are carried out.

$$\sum_{c \in \mathcal{C}^{\text{UL}}} t^c_{\text{ONU,OLT}}\, z_c \geq T_{\text{ONU,OLT}} \qquad \text{ONU} \in P_{\text{ONU}}. \tag{72}$$

The downstream traffic will be carried out only if every destination gets the signal and it is of two types:

$$\text{Unicast:} \sum_{c \in \mathcal{C}^{\text{DL}}} t^c_{\text{OLT},d}\, z_c \geq T_{\text{OLT},d} \qquad d \in P_{\text{ONU}} \tag{73}$$

$$\text{Multicast:} \sum_{c \in \mathcal{C}^{\text{DL}}} t^c_{\text{OLT},D}\, z_c \geq T_{\text{OLT},D} \qquad D \in \mathcal{D}. \tag{74}$$

Attenuation Constraints The total signal attenuation from the OLT to an ONU located at p, denoted by P_p, must not exceed 20 dB which is expressed by:

$$P_p = P^{\text{FIBER}}_p + P^{\text{THROUGH}}_p + P^{\text{INSERTION}} + P^{\text{MARGIN}} \tag{75}$$

where P^{FIBER}_p is the signal loss caused on the fiber to reach the ONU located at p, P^{THROUGH} is the loss provoked by going through the equipment towards the ONU located at p, $P^{\text{INSERTION}}$ is insertion loss caused by all the nodes on the link, P^{MARGIN} is a power margin. We just need to calculate the first two losses as other losses have constant values.

To calculate the first two losses, we introduce the variable x^{ATT}_p to evaluate the total attenuation to reach ONU located at p, $p \in P_{\text{ONU}}$. Let us assume a signal power loss of 0.2dB/km caused by optical fiber, and let ATT^{S}_k (resp. ATT^{A}) be the attenuation factor of splitter S, which depends on the number of output ports k of splitter S (resp. which is independent of the number of output ports of AWG A).

$$
\begin{aligned}
x_p^{\text{ATT}} = \sum_{p'' \in P_{\text{EQ}}} \sum_{k \in K} (\text{ATT}^{\text{A}}\, y^{\text{A},\ell_1}_{p'',k} + \text{ATT}^{\text{S}}_k\, y^{\text{S},\ell_1}_{p'',k}) \\
+ \sum_{p' \in P_{\text{EQ}}} \sum_{k \in K} (\text{ATT}^{\text{A}}\, y^{\text{A},\ell_2}_{p',k}\, L_{p,p'} + \text{ATT}^{\text{S}}_k\, y^{\text{S},\ell_2}_{p',k}\, L_{p,p'}) \\
+ \sum_{p'' \in P_{\text{EQ}}} L_{\text{OLT},p''}\, d_{\text{OLT},p''}\, 0.2 \\
+ \sum_{p'' \in P_{\text{EQ}}} \sum_{p' \in P_{\text{EQ}}} L_{p,p'}\, L_{p',p''}\, d_{p',p''}\, 0.2 \\
+ \sum_{p' \in P_{\text{EQ}}} L_{p,p'}\, d_{p,p'}\, 0.2 \qquad p \in P_{\text{ONU}} \qquad (76)
\end{aligned}
$$

The second element of the summation in (76) is nonlinear. In order to linearize it, we add two new variables

$$
y L\text{A}_{p,p',k} = y^{\text{A},\ell_2}_{p',k}\, L_{p,p'} \quad \text{and} \quad y L\text{S}_{p,p',k} = y^{\text{S},\ell_2}_{p',k}\, L_{p,p'},
$$

and the following constraints:

For all $p \in P_{\text{ONU}}, p' \in P_{\text{EQ}}, k \in K$,

$$
y^{\text{A},\ell_2}_{p',k} + L_{p,p'} - 1 \leq y L\text{A}_{p,p',k} \qquad (77)
$$

$$
y^{\text{A},\ell_2}_{p',k} \geq y L\text{A}_{p,p',k} \quad ; \quad L_{p,p'} \geq y L\text{A}_{p,p',k} \qquad (78)
$$

$$
y^{\text{S},\ell_2}_{p',k} + L_{p,p'} - 1 \leq y L\text{S}_{p,p',k} \qquad (79)
$$

$$
y^{\text{S},\ell_2}_{p',k} \geq y L\text{S}_{p,p',k} \quad ; \quad L_{p,p'} \geq y L\text{S}_{p,p',k}. \qquad (80)
$$

Again, the fourth element of the summation in (76) is nonlinear. In order to linearize it, we add a new variable

$$
LL_{p,p',p''} = L_{p,p'}\, L_{p',p''},
$$

and the following constraints:
For all $p \in P_{\text{ONU}}, p', p'' \in P_{\text{EQ}}$,

$$L_{p,p'} + L_{p',p''} - 1 \leq LL_{p,p',p''} \tag{81}$$

$$L_{p,p'} \geq LL_{p,p',p''} \tag{82}$$

$$L_{p',p''} \geq LL_{p,p',p''} \tag{83}$$

The total loss for every ONU should not exceed a given threshold (20 decibels in our experiments):

$$x_p^{\text{ATT}} + P^{\text{MARGIN}} + P^{\text{INSERTION}} \leq 20 \text{ dB} \qquad p \in P_{\text{ONU}}. \tag{84}$$

The above optimization model can be solved by first pre-enumerating all candidate configurations (i.e., traffic demand, grouping of ONUs, type and split ratio as well as the location of the switching equipment, connectivity between ONUs/equipment and equipment/OLT) and then selecting the promising configurations from the candidate set.

However, with the increase of network size, the number of candidate configurations increases exponentially which results in an inefficient optimization scheme. In this chapter, as an alternative solution to this linear mathematical model, we propose a large scale optimization method, namely column generation (CG) technique which is described in 4.5.

5.4 Solution Scheme

5.4.1 Column Generation and ILP Solution

Based on the CG technique, we only explicitly embed a very small subset of all location configurations in the optimization model of Section 5.3.2 which works as the restricted master problem (RMP), without hampering the reach of the optimal solution (linear programming relaxation). The pricing problem is described in Section 5.4.2.

5.4.2 Pricing Problem

The pricing problem is designed to generate meaningful configurations in which each configuration decides on the clustering of ONUs, type and location of the equipment (splitter/AWG) to serve the ONUs of the corresponding cluster, and provisioning of a wavelength. We next describe the pricing problem, first its set of variables (Section 5.4.2), next its objective (Section 5.4.2), and then its set of constraints (Section 5.4.2).

Variables

The variables of the pricing are the coefficients of the z_c variable in the master problem, i.e., the coefficients of a column vector associated to a z_c variable. Therefore, the variables of the pricing problem are:

- $t_{\text{OLT},d} \in [0, 1]$
- $t_{\text{OLT},D} \in [0, 1]$
- $t_{s,\text{OLT}} \in [0, 1]$
- $l^{\text{A}}_{i,p} \in \{0, 1\}$
- $l^{\text{S}}_{i,p} \in \{0, 1\}$
- $a\_s_p(a\_a_p) \in \{0, 1\}$ such that $a\_s_p = 1(a\_a_p = 1)$ if there is a splitter (an AWG) at $p \in P_{\text{EQ}}$,

Objective

The objective of the pricing problem is defined by the minimization of the reduced cost, which is expressed as follows for upstream pricing problem:

$$\begin{aligned}\overline{\text{COST}}^{\text{UP}}(z) = &\sum_{i \in P_{\text{ONU}}} \sum_{p \in P_{\text{EQ}}} u^{\text{S}}_{1_{i,p}} l^{\text{S}}_{i,p} \\ &+ \sum_{i \in P_{\text{ONU}}} \sum_{p \in P_{\text{EQ}}} u^{\text{A}}_{1_{i,p}} l^{\text{A}}_{i,p} + \sum_{i \in P_{\text{EQ}}} \sum_{p \in P_{\text{EQ}}} u^{\text{S}}_{2_{i,p}} l^{\text{S}}_{i,p} \\ &+ \sum_{i \in P_{\text{EQ}}} \sum_{p \in P_{\text{EQ}}} u^{\text{A}}_{2_{i,p}} l^{\text{A}}_{i,p} + \sum_{i \in P_{\text{OLT}}} \sum_{p \in P_{\text{EQ}}} u^{\text{S}}_{3_{i,p}} l^{\text{S}}_{i,p} \\ &+ \sum_{i \in P_{\text{OLT}}} \sum_{p \in P_{\text{EQ}}} u^{\text{A}}_{3_{i,p}} l^{\text{A}}_{i,p} - \sum_{\text{ONU} \in P_{\text{ONU}}} u^{t}_{\text{ONU}} t_{\text{ONU,OLT}} \end{aligned} \tag{85}$$

where $u^{\text{S}}_{1_{i,p}}$ and $u^{\text{A}}_{1_{i,p}}$ are the dual values associated with constraints (50-i,p) and (52-i,p) respectively, $u^{\text{S}}_{2_{i,p}}$ and $u^{\text{A}}_{2_{i,p}}$ are the dual values associated with constraints (54-i,p) and (56-i,p) respectively, $u^{\text{S}}_{3_{i,p}}$ and $u^{\text{A}}_{3_{i,p}}$ are the dual values associated with constraints (58-i,p) and (59-i,p) respectively,and u^{t}_{ONU} is the dual value associated with constraint (72-ONU).

The objective of the downstream pricing problem is expressed as follows:

$$\begin{aligned}\overline{\text{COST}}^{\text{DL}}(z) = &\sum_{i \in P_{\text{ONU}}} \sum_{p \in P_{\text{EQ}}} u^{\text{S}}_{1_{i,p}} l^{\text{S}}_{i,p} + \sum_{i \in P_{\text{ONU}}} \sum_{p \in P_{\text{EQ}}} u^{\text{A}}_{1_{i,p}} l^{\text{A}}_{i,p} \\ &+ \sum_{i \in P_{\text{EQ}}} \sum_{p \in P_{\text{EQ}}} u^{\text{S}}_{2_{i,p}} l^{\text{S}}_{i,p} + \sum_{i \in P_{\text{EQ}}} \sum_{p \in P_{\text{EQ}}} u^{\text{A}}_{2_{i,p}} l^{\text{A}}_{i,p} \\ &+ \sum_{i \in P_{\text{OLT}}} \sum_{p \in P_{\text{EQ}}} u^{\text{S}}_{3_{i,p}} l^{\text{S}}_{i,p} + \sum_{i \in P_{\text{OLT}}} \sum_{p \in P_{\text{EQ}}} u^{\text{A}}_{3_{i,p}} l^{\text{A}}_{i,p} \\ &- \sum_{d \in \mathcal{D}^{uni}} u^{t}_{d_{uni}} \, t_{\text{OLT},d} - \sum_{D \in \mathcal{D}^{mul}} u^{t}_{D_{mul}} \, t_{\text{OLT},D} \end{aligned} \tag{86}$$

where $u^{\text{S}}_{1_{i,p}}$ and $u^{\text{A}}_{1_{i,p}}$ are the dual values associated with constraints (50-i,p) and (52-i,p) respectively, $u^{\text{S}}_{2_{i,p}}$ and $u^{\text{A}}_{2_{i,p}}$ are the dual values associated with constraints (54-i,p) and (56-i,p) respectively, $u^{\text{S}}_{3_{i,p}}$ and $u^{\text{A}}_{3_{i,p}}$ are the dual values associated with constraints (58-i,p) and (59-i,p) respectively,and $u^t_{d_{uni}}$ is the dual vector associated with constraint (73-d), and $u^t_{D_{mul}}$ is the dual vector associated with constraint (74-D).

Constraints

Equipment Selection Constraints The following constraints ensure that there must be one type of equipment (an AWG /a splitter) in a potential location which can serve either an ONU or any switching equipment or the OLT.

$$l^{\text{S}}_{i,p} \leq 1 \qquad p \in P_{\text{EQ}}, i \in P_{\text{OLT}} \cup P_{\text{EQ}} \cup P_{\text{ONU}} \tag{87}$$

$$l^{\text{A}}_{i,p} \leq 1 \qquad p \in P_{\text{EQ}}, i \in P_{\text{OLT}} \cup P_{\text{EQ}} \cup P_{\text{ONU}} \tag{88}$$

$$l^{\text{S}}_{i,p} + l^{\text{A}}_{i,p} \leq 1 \qquad p \in P_{\text{EQ}}, i \in P_{\text{OLT}} \cup P_{\text{EQ}} \cup P_{\text{ONU}} \tag{89}$$

If an equipment (a splitter or an AWG) is selected in a potential location, then it can connect a number of ONUs/equipment.

$$l^{\text{S}}_{i,p} \leq a\_s_p \qquad p \in P_{\text{EQ}}, i \in P_{\text{OLT}} \cup P_{\text{EQ}} \cup P_{\text{ONU}} \tag{90}$$

$$l^{\text{A}}_{i,p} \leq a\_a_p \qquad p \in P_{\text{EQ}}, i \in P_{\text{OLT}} \cup P_{\text{EQ}} \cup P_{\text{ONU}} \tag{91}$$

$$a\_s_p + a\_a_p \leq 1 \qquad p \in P_{\text{EQ}} \tag{92}$$

Topology Constraints The topology is a tree which is routed at the OLT such that the leaves are a subset of ONUs.

Each ONU should have one predecessor which can be only an equipment.

$$\sum_{p \in P_{\text{EQ}}} (l^{\text{S}}_{i,p} + l^{\text{A}}_{i,p}) = 1 \qquad i \in P_{\text{ONU}} \tag{93}$$

As the architecture of a PON resembles a tree, there should be only one outgoing link from the OLT which is directed towards an equipment.

$$\sum_{p \in P_{\mathrm{EQ}}} (l^{\mathrm{S}}_{i,p} + l^{\mathrm{A}}_{i,p}) = 1 \qquad i \in P_{\mathrm{OLT}} \tag{94}$$

A second level equipment located at i must connect to the single first level equipment located at p

$$\sum_{p \in P_{\mathrm{EQ}}} (l^{\mathrm{S}}_{i,p} + l^{\mathrm{A}}_{i,p}) \leq 1 \qquad i \in P_{\mathrm{EQ}} \tag{95}$$

A second level equipment can be connected to a first level equipment only when the second level equipment connects an ONU and the first level equipment is connected to the OLT.

$$\sum_{p \in P_{\mathrm{EQ}}} (l^{\mathrm{S}}_{i,p} + l^{\mathrm{A}}_{i,p}) = (l^{\mathrm{S}}_{p',i} + l^{\mathrm{S}}_{p',i}) \qquad i \in P_{\mathrm{EQ}}, p' \in P_{\mathrm{ONU}} \tag{96}$$

$$(l^{\mathrm{S}}_{i,p} + l^{\mathrm{A}}_{i,p}) \leq (l^{\mathrm{S}}_{p',p} + l^{\mathrm{S}}_{p',p}) \qquad i \in P_{\mathrm{EQ}}, p \in P_{\mathrm{EQ}}, p' \in P_{\mathrm{OLT}} \tag{97}$$

In each configuration, the number of ONUs associated with each 2nd level splitter must be less than or equal to the splitting ratio of the corresponding splitter. Again, the number of 2nd level equipment associated the 1st level splitter must be less than or equal to the splitting ratio of the 1st level splitter.

$$\sum_{i \in P_{\mathrm{EQ}} \cup P_{\mathrm{ONU}}} l^{\mathrm{S}}_{i,p} \leq \max\_\mathrm{split} \times a\_s_p \qquad p \in P_{\mathrm{EQ}} \tag{98}$$

In case of an AWG at the 2nd (or 1st) level, only one ONU (or equipment) is associated with each configuration.

$$\sum_{i \in P_{\mathrm{EQ}} \cup P_{\mathrm{ONU}}} l^{\mathrm{A}}_{i,p} \leq a\_a_p \qquad p \in P_{\mathrm{EQ}} \tag{99}$$

All equipment can accommodate a number of ONU-equipment/equipment-equipment connections which is determined by the cumulative sum of the splitting ratio of all selected equipment.

$$\sum_{p \in P_{\text{EQ}}} \sum_{i \in P_{\text{EQ}} \cup P_{\text{ONU}}} (l^{\text{S}}_{i,p} + l^{\text{A}}_{i,p}) \leq \sum_{p \in P_{\text{EQ}}} (\text{max\_split} \times a\_s_p + a\_a_p) \quad (100)$$

For each cluster, the average distance from an equipment to all ONUs should be within a threshold (e.g.max_avg_val) value so that the the differential distance among the ONUs inside a cluster is minimized.

$$\sum_{i \in P_{\text{ONU}}} d_{i,p} \, (l^{\text{S}}_{i,p} + l^{\text{A}}_{i,p}) \leq \sum_{i \in P_{\text{ONU}}} (l^{\text{S}}_{i,p} + l^{\text{A}}_{i,p}) \times \text{max\_avg\_val} \qquad p \in P_{\text{EQ}}. \quad (101)$$

For each cluster, the distance between an ONU and its corresponding equipment should not exceed a threshold (max_dist_val) value, defined by the maximum attenuation that is allowed in order to get an acceptable signal level.

$$\sum_{p \in P_{\text{EQ}}} d_{i,p} \, (l^{\text{S}}_{i,p} + l^{\text{A}}_{i,p}) \leq \text{max\_dist\_val} \qquad i \in P_{\text{ONU}}. \quad (102)$$

Traffic Constraints Downstream Traffic.

For downstream traffic, we need to take into account both unicast and multicast traffic requests. Constraints for unicast traffic are as follows:

The total traffic on the link going from the OLT towards an ONU(or a set of ONUs) can not be greater than the bandwidth of a wavelength (normalized at 1).

$$\sum_{d \in P_{\text{ONU}}} t_{\text{OLT},d} \leq 1 \quad (103)$$

The unicast traffic reaching ONU,d can be greater than zero only if the destination d is reached via an equipment in the configuration.

$$t_{\text{OLT},d} \leq \sum_{p \in P_{\text{EQ}}} (l^{\text{S}}_{d,p} + l^{\text{A}}_{i,p}) \qquad d \in P_{\text{ONU}} \tag{104}$$

Constraints for multicast traffic are as follows:

The sum of all multicast traffic granted in one configuration can not exceed the total traffic.

$$\sum_{D \in \mathcal{D}} t_{\text{OLT},D} \leq 1 \tag{105}$$

The multicast traffic destined for multiple ONUs (e.g.d_1,d_2,d_3) can be greater than zero only if all destination ONUs are reached via an equipment in the configuration.

$$\begin{aligned} 3 * t_{\text{OLT},D} \leq \sum_{p \in P_{\text{EQ}}} (l^{\text{S}}_{d_1,p} + l^{\text{A}}_{d_1,p} + l^{\text{S}}_{d_2,p} \\ + l^{\text{A}}_{d_2,p} + l^{\text{S}}_{d_3,p} + l^{\text{A}}_{d_3,p}) \qquad d_1, d_2, d_3 \in D, D \in \mathcal{D} \end{aligned} \tag{106}$$

The following constraint guarantees that the total amount of unicast and multicast traffic requests granted in one configuration must not exceed 1.

$$\sum_{d \in \mathcal{D}} t_{\text{OLT},d} + \sum_{D \in \mathcal{D}} t_{\text{OLT},D} \leq 1 \tag{107}$$

Upstream Traffic.

Upstream traffic consists of only unicast requests. The total traffic on the link incoming to the OLT from an ONU can not be greater than the bandwidth of a wavelength

$$t_{i,\text{OLT}} \leq 1 \qquad i \in P_{\text{ONU}} \tag{108}$$

The traffic originated from an ONU, i and destined to the OLT can be greater than zero only if the the ONU located at i is reached via an equipment residing at p

in the configuration.

$$t_{i,\mathrm{OLT}} \leq \sum_{p \in P_{\mathrm{EQ}}} (l^{\mathrm{S}}_{i,p} + l^{\mathrm{A}}_{i,p}) \qquad i \in P_{\mathrm{ONU}} \tag{109}$$

The sum of all upstream traffic granted in one configuration can not exceed the total traffic.

$$\sum_{i \in P_{\mathrm{ONU}}} t_{i,\mathrm{OLT}} \leq 1 \tag{110}$$

5.5 Computational Results and Analysis

We implement the optimization model of Section 4.4.2 within the Optimization Programming Language (OPL) platform and solve the linear and integer linear programs using the CPLEX package [IBM11].

We conduct our experiments with randomly generated Manhattan-pattern geographic locations of 16, 32, 64 and 128 ONUs in a 40×20 km^2 rectangular grid such that the OLT is located at the middle of the corresponding grid, i.e., at location (20,10) as shown in Figure 7(a). The locations of the ONUs are distributed along several vertical lines so that each value of x-coordinate can accommodate several "vertical" ONU locations. There are 30 candidate/potential locations for the placement of the passive equipment which are randomly positioned inside the same rectangular grid as shown in Figure 7(b). The cost and attenuation parameters of the passive switching equipment are presented in Table 2 (based on [CWMJ10]). For the costs related to optical fiber cables, we use the value of 7,160$/km [CWMJ10], assuming it includes the cost of trenching and laying the optical fiber cables.

We randomly generate the upstream unicast traffic flows within the range [0.05, 0.1] for each pair (ONU, OLT). Towards downstream direction, we randomly generate both unicast and multicast traffic flows within the range [0.1, 0.4] for each pair (OLT, ONU), and a number of multicast requests with 3 randomly generated destinations.

The number of generated multicast requests is 3,8,12 and 25 for different scenarios consisting of 16, 32, 64, 128 ONUs respectively.

Table 7: Deployment Cost of Different Scenarios

Scenario	No. of served ONUs	$z^{\star}_{\text{LP}}$	$\tilde{z}_{\text{ILP}}$	Optimality gap (%)
1	16	1,223,210	1,278,780	4.54
2	32	2,059,526	2,105,180	2.21
3	64	2,990,146	2,998,620	0.29
4	128	4,259,159	4,499,900	5.65

Our proposed column generation based cross layer optimization model determines the physical architecture of a hybrid PON and provisions the traffic flows. In Table 7, we report on the PON 'greenfield' deployment costs for four different scenarios (i.e., Scenario 1, Scenario 2, Scenario 3 and Scenario 4) consisting of 16, 32, 64 and 128 ONUs respectively. The deployment cost is obtained by determining an optimal physical architecture of a PON for each scenario. An optimal PON architecture is determined by: *(i)* deciding on the total number of clusters the ONUs can be grouped into, *(ii)* identifying the ONUs associated with each cluster *(iii)* assigning either a splitter or an AWG to each cluster, *(iv)* choosing the splitting ratio of the switching equipment (splitter/AWG) of each cluster, *(v)* allocating the locations of the selected switching equipment. While optimizing the physical architecture of a PON, our proposed model also provisions the traffic flows (some unicast, some multicast) destined to/from the ONUs and takes into account several constraints related to the topology, equipment location, traffic demand and signal power attenuation. The optimality gap (which measure the accuracy of our solutions) corresponds to: $\varepsilon = (\tilde{z}_{\text{ILP}} - z^{\star}_{\text{LP}})/z^{\star}_{\text{LP}}$, where $z^{\star}_{\text{LP}}$ is a lower bound on the optimal value $z^{\star}_{\text{ILP}}$ (PON minimum cost) provided by the optimal value of the linear relaxation of the model (restricted master problem) described in Section 5.3.2, and $\tilde{z}_{\text{ILP}}$ is an upper bound on the optimal value $z^{\star}_{\text{ILP}}$ provided by the ILP solution of the ILP model associated with the last generated restricted master problem.

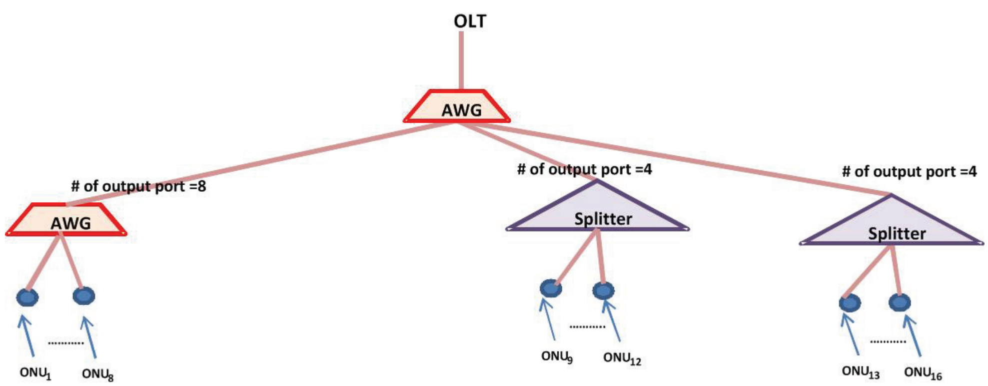
The proposed model is formulated to select an optimal two-stage hybrid PON architecture. Table 8 illustrates the optimal physical architecture of a PON by specifying the type and the number of switching equipment required for different scenarios. Table 9 represents the selected equipment and its corresponding selected number of required output ports for each of the generated clusters of Scenario 1. Here all 2nd level equipment form a single cluster with respect to the 1st level equipment that has been identified as "Cluster ID # 0". In Scenario 1, as shown in Figure 13, 16 ONUs are grouped into 3 clusters in which an AWG serves 8 ONUs of cluster 1 and one splitter serves 4 ONUs of cluster 2 and another splitter serves 4 ONUs of cluster 3. All these 2nd level equipment are connected to an AWG in the 1st level which is finally connected to the OLT. While selecting the type of equipment, the optimization model takes into account the attenuation constraint along with the bandwidth demand of each ONU and decides whether the best choice is to assign a splitter or an AWG to a given cluster.

Table 8: Physical Architecture of Different Scenario

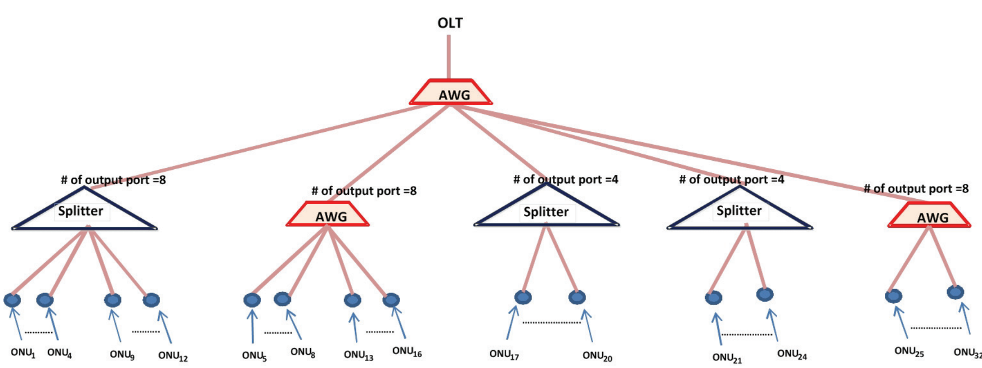
Scenario	No of generated clusters	No of selected AWGs	No of selected splitters
1	3	2	2
2	5	3	3
3	8	5	4
4	13	9	5

Figure 13 depicts the hybrid PON architecture of Scenario 2 in which 32 ONUs are grouped into 5 clusters, served by 3 splitters and 2 AWGs in the 2nd level as well as 1 AWG in the 1st level. The selected equipment and its corresponding selected number of required output ports for each of the generated clusters of Scenario 2 are presented in Table 10.

In Scenario 3, we carry on the experiment with 64 ONUs and obtain an optimal solution with 8 clusters where 4 AWGs and 4 splitters serve the ONUs confined to



(a) Scenario 1



(b) Scenario 2

Figure 13: PON Architecture

Table 9: Scenario 1 (Cluster vs. No. of Output ports)

Equipment Level	Cluster ID	Selected Equipment	No of selected output ports
1st	0	AWG	4
2nd	1	AWG	8
2nd	2	Splitter	4
2nd	3	Splitter	4

Table 10: Scenario 2 (Cluster vs. No. of Output ports)

Equipment Level	Cluster ID	Selected Equipment	No of selected output ports
1st	0	AWG	8
2nd	1	Splitter	8
2nd	2	AWG	8
2nd	3	Splitter	4
2nd	4	Splitter	4
2nd	5	AWG	8

these clusters. In this scenario, an AWG is selected as the 1st level equipment. Table 11 shows the selected equipment and its corresponding selected number of required output ports for each of the generated clusters of Scenario 3.

In Scenario 4, 128 ONUs are grouped into 13 clusters in which 8 AWGs and 5 splitters in the 2nd level together with 1 AWG in the 1st level are optimally selected. The selected equipment and its corresponding selected number of required output ports for each of the generated clusters of Scenario 4 are displayed in Table 12.

In all scenarios, the number of output ports of all equipment are optimally determined, with respect to the overall PON deployment cost.

Next, we extend our experiment with the intention of observing the impact of varying pattern of multicast traffic requests on the solution scheme. We considered multicast requests consisting of 3, 4 or 5 randomly generated destinations such that the multicast with 4 or 5 destinations are an extension of multicast with 3 destinations

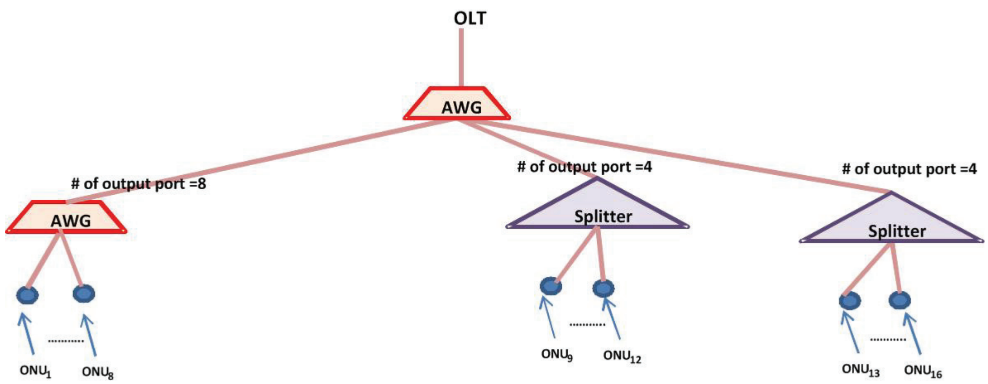
Table 11: Scenario 3 (Cluster vs. No. of Output ports)

Equipment Level	Cluster ID	Selected Equipment	No of selected output ports
1st	0	AWG	8
2nd	1	AWG	16
2nd	2	AWG	16
2nd	3	Splitter	4
2nd	4	Splitter	8
2nd	5	Splitter	8
2nd	6	Splitter	8
2nd	7	AWG	16
2nd	8	AWG	8

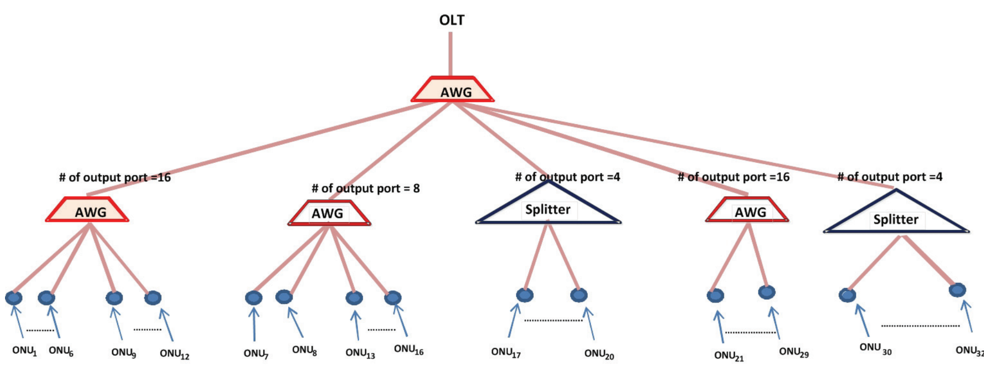
of previous scenarios (i.e., Scenario 1, Scenario 2, Scenario 3 and Scenario 4). In Table 13, we report on the PON 'greenfield' deployment costs for Scenario 5, Scenario 6, Scenario 7, Scenario 8 consisting of 16, 32, 64 and 128 ONUs respectively in which multicast traffic requests contain 3, 4, or 5 randomly generated destinations. The deployment cost is obtained by determining an optimal physical architecture of a PON for each scenario.

Table 14 illustrates the optimal physical architecture of a PON by specifying the type and the number of switching equipment required for different scenarios. Table 15 represents the selected equipment and its corresponding selected number of required output ports for each of the generated clusters of Scenario 5. In Scenario 5, as shown in Figure 14(a), 16 ONUs are grouped into 3 clusters in which an AWG serves 8 ONUs of cluster 1 and one splitter serves 4 ONUs of cluster 2 and another splitter serves 4 ONUs of cluster 3. Here, we observe same deployment cost for Scenario 5 with respect to Scenario 1.

The hybrid PON architecture of Scenario 6 is presented in Figure 14 in which 32 ONUs are grouped into 5 clusters, served by 2 splitters and 3 AWGs in the 2nd level as well as 1 AWG in the 1st level. In Table 16, the selected equipment and its corresponding number of selected output ports for each of the generated clusters of Scenario 6 are presented. We observe increased deployment cost with respect to



(a) Scenario 5



(b) Scenario 6

Figure 14: PON Architecture

Table 12: Scenario 4 (Cluster vs. No. of Output ports)

Equipment Level	Cluster ID	Selected Equipment	No of required output ports
1st	0	AWG	16
2nd	1	AWG	32
2nd	2	AWG	32
2nd	3	Splitter	4
2nd	4	AWG	16
2nd	5	AWG	16
2nd	6	AWG	16
2nd	7	AWG	8
2nd	8	AWG	8
2nd	9	Splitter	8
2nd	10	Splitter	4
2nd	11	Splitter	8
2nd	12	AWG	16
2nd	13	Splitter	8

Table 13: Deployment Cost of Different Scenarios

Scenario	No. of served ONUs	$z^{\star}_{\text{LP}}$	$\tilde{z}_{\text{ILP}}$	Optimality gap (%)
5	16	1,223,210	1,278,780	4.54
6	32	2,062,810	2,107,200	2.15
7	64	2,985,840	2,999,890	0.47
8	128	4,657,280	4,880,720	4.79

Scenario 2.

In Scenario 7, we carry on the experiment with 64 ONUs and obtain an optimal solution with 8 clusters where 5 AWGs and 3 splitters serve the ONUs confined to these clusters. In this scenario, an AWG is selected as the 1st level equipment. Table 17 shows the selected equipment and its corresponding selected number of required output ports for each of the generated clusters of Scenario 7. The deployment cost of Scenario 7 is higher compared to the Scenario 3.

In Scenario 8, experiments are carried on 128 ONUs. Here the solution of our

Table 14: Physical Architecture of Different Scenario

Scenario	No of generated clusters	No of selected AWGs	No of selected splitters
5	3	2	2
6	5	4	2
7	8	6	3
8	13	8	6

Table 15: Scenario 5 (Cluster vs. No. of Output ports)

Equipment Level	Cluster ID	Selected Equipment	No of selected output ports
1st	0	AWG	4
2nd	1	AWG	8
2nd	2	Splitter	4
2nd	3	Splitter	4

proposed scheme generates 13 optimal clusters in which 7 AWGs and 6 splitters are selected as the 2nd level together with 1 AWG as the 1st level equipment. The selected equipment and its corresponding number of selected output ports for each of the generated clusters of Scenario 8 are displayed in Table 18. We notice that the deployment cost of Scenario 8 is higher than that of Scenario 4.

From these experiments, we observe that varying the pattern of multicast request affects the clustering of ONUs as well the selection of type and number of output ports of each equipment which eventually affects the overall deployment cost. We observe increased deployment cost for the scenarios consisting of 3, 4 or 5 destinations in the multicast traffic set compared to the scenarios consisting of only 3 destinations in the multicast traffic set.

Finally, we intend to investigate the impact of increased number of multicast traffic requests on the deployment cost. We experiment with three different set of multicast traffic requests where the number of traffic requests is added incrementally.

Table 16: Scenario 6 (Cluster vs. No. of Output ports)

Equipment Level	Cluster ID	Selected Equipment	No of selected output ports
1st	0	AWG	8
2nd	1	AWG	16
2nd	2	AWG	8
2nd	3	Splitter	4
2nd	4	AWG	16
2nd	5	Splitter	4

Table 17: Scenario 7 (Cluster vs. No. of Output ports)

Equipment Level	Cluster ID	Selected Equipment	No of required output ports
1st	0	AWG	8
2nd	1	AWG	16
2nd	2	AWG	16
2nd	3	AWG	8
2nd	4	Splitter	4
2nd	5	AWG	16
2nd	6	Splitter	4
2nd	7	AWG	16
2nd	8	Splitter	4

We start with our Scenario 4 (consisting of 128 ONUs and the set of 3 destinations for multicast traffic requests) in which we consider 25 multicast requests. Later on, we experiment with 30 and 35 multicast requests. Impact of multicast traffic requests on the solution of our proposed scheme is shown in Table 19 as well as in Figure 15. We observe that deployment cost increases with the increase of number of multicast traffic requests.

Table 18: Scenario 8 (Cluster vs. No. of Output ports)

Equipment Level	Cluster ID	Selected Equipment	No of required output ports
1st	0	AWG	16
2nd	1	AWG	32
2nd	2	AWG	32
2nd	3	Splitter	4
2nd	4	AWG	16
2nd	5	AWG	16
2nd	6	AWG	32
2nd	7	Splitter	4
2nd	8	Splitter	4
2nd	9	AWG	32
2nd	10	AWG	32
2nd	11	Splitter	8
2nd	12	Splitter	8
2nd	13	Splitter	8

Table 19: Impact of multicast traffic requests on the Solution

#Multicast traffic	Deployment Cost
25	4,499,900
30	4,633,600
35	4,748,760

5.6 Summary

In this chapter, we present our second proposed scheme to solve the location allocation (L/A) problem of a single hybrid PON. We propose a novel cross layer optimization scheme for the design and dimensioning of greenfield hybrid PON networks. For a given geographical location of the OLT, the ONUs and their corresponding aggregated traffic demand, we propose a generic integer linear programming (ILP) model which optimally and simultaneously: *(i)* congregates the ONUs into clusters, *(ii)* determines

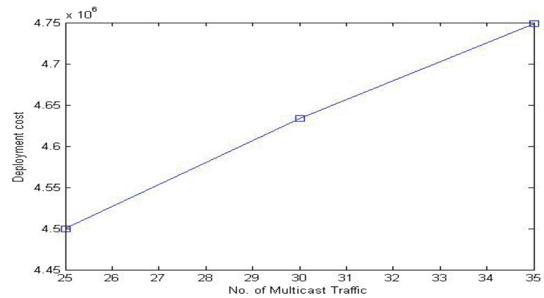


Figure 15: Impact of multicast traffic

the type (splitter/ AWG) and number of output ports of the passive switching equipment for all clusters so that all ONUs can be served, *(iii)* identifies the location of the switching equipment, *(iv)* determines the proper link dimensioning so as to allow the provisioning of the overall aggregated traffic demand destined to/from the ONUs.

The ILP model not only includes the physical layer constraints of PON (i.e., power attenuation and splitting ratio), but the optical layer constraints as well (i.e., number wavelengths carried by the optical fibers depending on the traffic and on the selected switching equipment). The resulting model is therefore the most general model proposed so far, and it guarantees the optimal solution in terms of minimum deployment cost for greenfield hybrid PON, with a two stage architecture. Computational results demonstrate the validation and effectiveness of the proposed solution scheme on various data sets with up to 128 ONUs.

Chapter 6

Switching Equipment Location/Allocation in multiple hybrid PONs - Scheme 1

6.1 Introduction

In this chapter, we investigate the selection and location/allocation of the switching equipment in a set of hybrid PONs covering a given geographical area. This chapter not only determines the cascading architecture and network dimension of a single hybrid PON but also decides on the total number of PON networks required to cover all ONUs in a given neighborhood. The aforesaid problem is an NP-complete one [LS09],[MPC11]. For this reason, we solve it using a multi-step solution scheme which consists of two heuristic algorithms for some pre-processing operations, an optimization model for generating promising PONs and then an integer linear programming (ILP) formulation for selecting the best PONs.

The chapter is organized as follows. In Section 6.2, we provide a concise statement of the multiple-PON deployment problem and an outline of our proposed 4-phase **SLAPONS** (Equipment Selection Location/Allocation in a set of PONs) scheme. The first phase, an ONU clustering algorithm is described in Section 6.3. The second phase, i.e., the generation of several potential PON hierarchies, is described in Section 6.4. In the third phase, described in Section 6.5 and 6.6, we propose an efficient and scalable optimization model, which establishes the selection, location/allocation of switching equipment and provisions traffic demand, with minimum PON network deployment cost for each potential PON hierarchy. In the fourth phase, detailed in Section 6.7, we propose an ILP model to select the best set of PON networks in order to cover the traffic demand in a given geographical area. Computational results and analysis are presented in Section 6.8 in order to validate the proposed **SLAPONS** scheme. Conclusions are drawn in the last section.

6.2 Multiple PON Deployment: Problem Statement and Optimization Process

6.2.1 The GEOPONS Problem Statement

We propose to investigate the **GEOPONS** (Geographical PON Set) problem, defined as follows. The goal is to determine the best set of PON networks, along with the topology of each PON network, which are required to serve all ONUs in a neighborhood. The topology of a PON network includes the selection, location and cascading (e.g., one/two/mixed-stage) architecture of passive switching equipment (e.g., splitters/AWGs) by allocating each equipment to a group of ONUs. We allow different cascading architectures for each PON network. In a one-stage architecture, there is only one switching equipment and all ONUs are connected/allocated to it as shown in Figure 16. In a two-stage architecture, all equipment are distributed on two levels such that all ONUs are connected to the 2nd level equipment and all 2nd level equipment are connected to the single 1st level equipment which is itself connected to the OLT which is shown in Figure 4. A mixed stage architecture, displayed in Figure 17, is an extension of the two-stage architecture in which ONUs can be either connected to the 1st level equipment or to the 2nd level equipment.

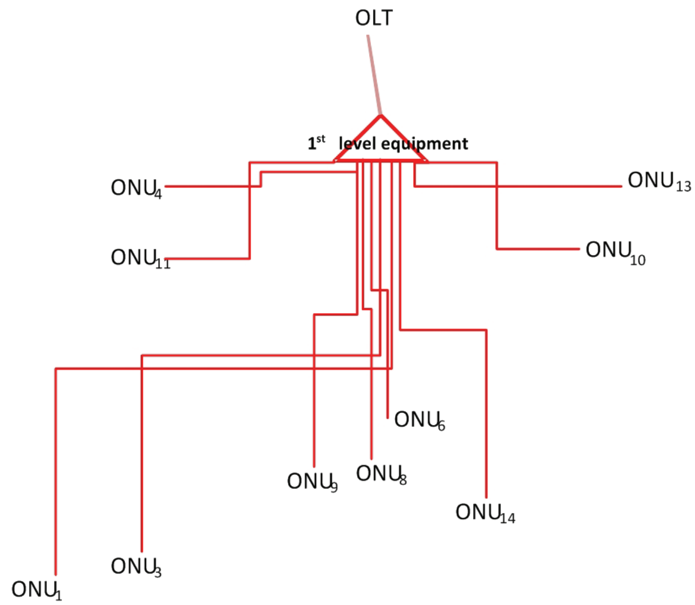


Figure 16: An one-stage Equipment Hierarchy

The **GEOPONS** problem is therefore to determine the best set of PON networks along with their cascading architecture, the type and location of their switching equipment while satisfying the network design constraints such as splitting ratio of splitters/AWGs and maximum allowed signal power loss at each ONU.

The input parameters of the **GEOPONS** problem include the location of the OLT and of the ONUs, the set of potential equipment locations and the unicast/multicast traffic. The output parameters consist of the number of required PON networks as well as the selected locations and type of the switching equipment (whether splitter or AWG) along with the cascading architecture of each PON network. The design is done with the objective of minimizing the network deployment cost, i.e., initial infrastructure installation and maintenance cost. Infrastructure installation cost is composed of the price of the switching equipment (splitter/AWG) and the optical fiber cables (including the cost of trenching and laying fibers). There is no maintenance cost for the switching equipment as it is a passive one. We assume that the OLT and the ONUs have already been installed, hence the installation and maintenance

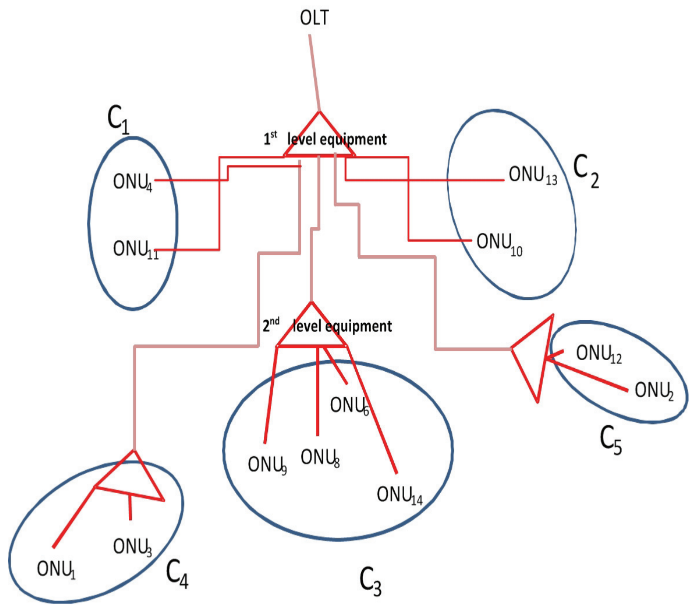


Figure 17: A mixed-stage Equipment Hierarchy

cost of these equipment are not taken into account. We also assume that each ONU accommodates aggregate traffic requests of a number of end users.

6.2.2 Optimization Process

We propose the 4-phase **SLAPONS** scheme, depicted in Figure 18, in order to solve the **GEOPONS** problem. The first phase consists in building various ONU clusterings. The second phase aims at generating several potential PON hierarchies. Each potential PON hierarchy relies on an ONU clustering, where each cluster confederates a set of ONUs connected to the same switching equipment. However, the type (splitter/AWG) and geographical location of the passive equipment are not determined at this stage. The third phase consists in selecting the best type and location/allocation of the passive equipment for each potential PON hierarchy, with the use of an ILP model. The fourth phase consists in selecting the best set of PON equipment hierarchies, with minimum deployment cost, covering all ONUs in a given neighbourhood.

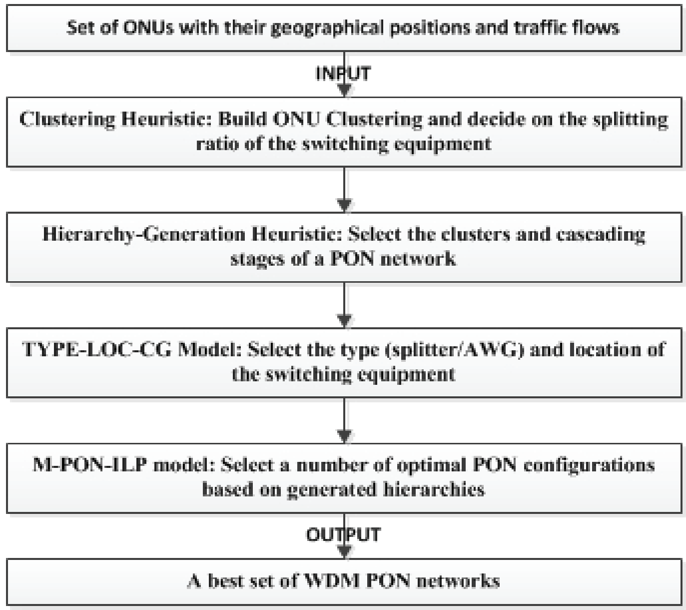


Figure 18: **SLAPONS** Scheme

6.3 Phase I: Clustering Heuristic

In the first phase, the ONUs are partitioned into geographically well separated clusters such that all ONUs within a cluster will be served by a single equipment. To build ONU clustering and decide on the splitting ratio of the switching equipment of the corresponding cluster, we use a clustering heuristic which relies on the classical single-link algorithm (SLA) [Har75],[Har81], [Pen95]. Note that the number of ONUs in a cluster defines the splitting ratio of the equipment confederating all the ONUS of the cluster.

The cardinality of the clusters (i.e., number of ONUs) will not necessarily be equal to one of the standard values of the split ratios (i.e., 1:2, 1:4, 1:8, 1:16, 1:32, 1:64) of a switching equipment. Some cluster re-organization is therefore performed in order to reconcile the cardinality of the clusters with the standard splitting values of the switching equipment. We round off the cardinality values to the closest ratio. In case of a rounding down, we expel from the cluster the extra ONUs which are the closest to another cluster, and append them to their closest cluster that can host them: either a cluster with a smaller cardinality, or a cluster with a larger cardinality if it has room. For example, the output of the heuristic of Section 6.3 is illustrated in Figure 19(b) where the ONUs are grouped into multiple clusters.

6.4 Phase II: PON Hierarchy Generation Heuristic

In this phase, we apply a simple heuristic to generate different PON hierarchies. A PON hierarchy corresponds to a subset of ONU clusters served by the same PON and the total set of ONU clusters results in a number of PON hierarchies. In order to generate a PON hierarchy, we exploit clustering/partitioning information of given ONUs obtained from the clustering heuristic of Section 6.3. Each potential PON hierarchy is made upon deciding on the scope/range (selection of clusters) of a PON, splitting ratio of each equipment and the number of levels/stages of the PON network. Throughout the selection of various PON hierarchies, we intend to allow one/two/mixed-stage architectures. We consider different combinations of feasible PON hierarchies so that we can obtain a near optimal solution with respect to the overall network deployment cost. For example, the input and the output of the heuristic of Section 6.4 are illustrated in Figure 19(b) and Figure 19(c) respectively, where a subset of ONU clusters constitutes a PON hierarchy.

6.5 Phase III: Generalized Optimization Model for Selecting the Type and Location of the Passive Equipment

We propose the TYPE-LOC-CG algorithm in order to determine the selection, location and allocation of the switching equipment in a given PON hierarchy, leading them to what we will call PON equipment hierarchy (or equipment hierarchy for short).

Remember that potential PON hierarchies, generated in Phase II, include one-stage, two-stage and mixed-stage architectures. The TYPE-LOC-CG algorithm relies on a large scale optimization model that is described in Section 6.5.2 after setting the notations in Section 6.5.1.

The TYPE-LOC-CG model that is described below is a generalized version of the model proposed in Chapter 4 of Section 4.4 which is limited to two-stage architectures only where the ONUs are not allowed to be connected to the first stage equipment (consequently, ONUs are possibly encountering more signal loss than needed, as packets need to go through the two switching equipment, even for the ONUs located nearby the switching equipment of the first stage). The TYPE-LOC-CG model is general enough in order to optimize the selection, location and allocation of the switching equipment, independently of the embedded cascading of the equipment hierarchy under construction.

6.5.1 Notations

Equipment Hierarchy Parameters

For a given hierarchy, G is the set of ONU groups as well as 2nd level equipment in a given equipment hierarchy, i.e., g_0 the cluster of ONUs and the 2nd level equipment associated with the single first level equipment and g any of the second level clusters, which is connecting a given subset of ONUs with the same switching equipment. We will denote by $|g|$ the splitting ratio of the switching equipment of cluster g. Note that first and second stages are merged for 1-stage cascading PON hierarchy. Let $G^\star$ be the set $G \setminus \{g_0\}$.For $g \in G$, cluster g is associated with the set of ONUs connected to the second level switching equipment except for the first level cluster, g_0 which is associated with the first level switching equipment. In order to identify the membership of an ONU to a particular cluster, we use the parameter $\delta_{\text{ONU},g}$: It is equal to 1 if ONU belongs to cluster g in equipment hierarchy, and 0 otherwise.

A provisioned hierarchy is described by its switching equipment at each level by the following parameters: $a_{g_0,k} = 1$ if there is an equipment with $k \in K = \{2, 4, 8, 16, 32, 64\}$ output ports at the first level, below the OLT, leading cluster g_0, and 0 otherwise. Similarly, the equipment selected at the second level is described by the parameter $a_{g,k}$, for $g \in G^{\star}$.

Location Parameters

Location parameters, used in the optimization model, are described in Section 4.4.1.

Cost Parameters

Cost parameters, used in the optimization model, are described in Section 4.4.1.

Traffic Parameters

Traffic parameters, used in the optimization model, are described in Section 4.4.1.

6.5.2 Linear Optimization Model

Location Configurations

Location configurations, used in the optimization model, are described in Section 4.4.2.

Variables

- $z_c \in \{0, 1\}$ is a decision variable such that $z_c = 1$ if configuration c is selected, and 0 otherwise.

- $y\_s_{p,g,k}(y\_a_{p,g,k}) \in \{0, 1\}$ is a decision variable such that $y\_s_{p,g,k}(y\_a_{p,g,k}) = 1$ if a splitter(respectively AWG) with $k \in K$ output ports is placed at location $p \in P_{\text{EQ}}$ serving the ONUs of cluster g in at least one configuration, and 0 otherwise.

- $y_{p,p',g,k} \in \{0,1\}$ is a decision variable such that $y_{p,p'g} = 1$ if p and p' are selected for the location of equipment having $k \in K$ output ports of cluster $g \in G^\star$ and g_0 respectively where $p, p' \in P_{\text{EQ}}$ and 0 otherwise.

Objective

As mentioned before, the objective corresponds to the deployment cost of a given equipment hierarchy where the locations of its passive equipment are determined as to minimize the cost while satisfying the technological and traffic constraints. It is formally defined as follows:

$$\textsc{cost}(y) = \textsc{cost}^{\text{LINK}}(y) + \textsc{cost}^{\text{EQ}}(y) \tag{111}$$

$$\textsc{cost}^{\text{LINK}}(y) = \textsc{cost}_{\text{FT}} \sum_{i=1}^{3} \textsc{cost}_i^{\text{LINK}}(y) \tag{112}$$

$$\textsc{cost}_1^{\text{LINK}}(y) = \sum_{p \in P_{\text{EQ}}} \sum_{k \in K} d_{\text{OLT},p}(y\_s_{p,g_0,k} + y\_a_{p,g_0,k}) \tag{113}$$

$$\textsc{cost}_2^{\text{LINK}}(y) = \sum_{p \in P_{\text{EQ}}} \sum_{p' \in P_{\text{EQ}}} \sum_{g \in G^\star} \sum_{k \in K} \bullet$$

$$d_{pp'}(y\_s_{p,g_0,k} + y\_a_{p,g_0,k})(y\_s_{p',g,k} + y\_a_{p',g,k}) +$$

$$\sum_{p \in P_{\text{EQ}}} \sum_{k \in K} \sum_{\text{ONU} \in P_{\text{ONU}} : \delta_{\text{ONU},g_0} = 1} d_{p,\text{ONU}}(y\_s_{p,g_0,k} + y\_a_{p,g_0,k}) \tag{114}$$

$$\textsc{cost}_3^{\text{LINK}}(y) = \sum_{g \in G^\star} \sum_{p \in P_{\text{EQ}}} \sum_{k \in K} \sum_{\text{ONU} \in P_{\text{ONU}} : \delta_{\text{ONU},g} = 1} d_{p,\text{ONU}}(y\_s_{p,g,k} + y\_a_{p,g,k}) \tag{115}$$

$$\textsc{cost}^{\text{EQ}} = \sum_{p \in P_{\text{EQ}}} \sum_{g \in G} \sum_{k \in K} (\textsc{cost}_{\text{S}}^k \, y\_s_{p,g,k} + \textsc{cost}_{\text{AWG}}^k \, y\_a_{p,g,k}) \tag{116}$$

where $\textsc{cost}_1^{\text{LINK}}(y)$ (resp. $\textsc{cost}_2^{\text{LINK}}(y)$, $\textsc{cost}_3^{\text{LINK}}(y)$) are the fiber deployment costs associated with the first level, from OLT to the first passive equipment of g_0 (resp. from the passive equipment of g_0 to the passive equipment of the groups $g \in G^\star$, from the passive equipment of the groups $g \in G^\star$ to their ONUs), and $\textsc{cost}^{\text{EQ}}$ the cost of the selected passive equipment.

In order to linearize the expression of (114), we introduce variables $y_{pp'g}$ so that expression of $\textsc{cost}_2^{\textsc{link}}(y)$ becomes:

$$\textsc{cost}_2^{\textsc{link}}(y) = \sum_{p \in P_{\textsc{eq}}} \sum_{p' \in P_{\textsc{eq}}} \sum_{g \in G^\star} \sum_{k \in K} d_{pp'}\; y_{p,p',g,k} + \sum_{p \in P_{\textsc{eq}}} \sum_{k \in K} \sum_{\textsc{onu} \in P_{\textsc{onu}} : \delta_{\textsc{onu},g_0} = 1} d_{p,\textsc{onu}} (y\_s_{p,g_0,k} + y\_a_{p,g_0,k}) \quad (117)$$

with

$$y_{p,p',g,k} = (y\_s_{p,g_0,k} + y\_a_{p,g_0,k})(y\_s_{p',g,k} + y\_a_{p',g,k}),$$

together with the following additional constraints:

$$y\_s_{p,g_0,k} + y\_a_{p,g_0,k} + y\_s_{p',g,k} + y\_a_{p',g,k} - 1 \leq y_{p,p',g,k} \qquad p \in P_{EQ}, p' \in P_{EQ}, g \in G^\star, k \in K \quad (118)$$

$$y\_s_{p,g_0,k} + y\_a_{p,g_0,k} \geq y_{p,p',g,k} \qquad p \in P_{EQ}, p' \in P_{EQ}, g \in G^\star, k \in K \quad (119)$$

$$y\_s_{p',g,k} + y\_a_{p',g,k} \geq y_{p,p',g,k} \qquad p \in P_{EQ}, p' \in P_{EQ}, g \in G^\star, k \in K \quad (120)$$

The linearization is valid under the assumption that

$$y\_s_{p,g_0,k} + y\_a_{p,g_0,k} \leq 1 \quad g \in G, p \in P_{EQ}, k \in K$$

which is fulfilled due to constraints (126) (to be described in the sequel)

Constraints

Equipment hierarchy constraints The number of selected configurations generated around one equipment hierarchy is limited by the number of available wavelengths.

$$\sum_{c \in C} z_c \leq W. \tag{121}$$

The next set of constraints imply that only configurations associated with the selected equipment hierarchy can be themselves selected. For all $p \in P_{\text{EQ}}, g \in G, k \in K$,

$$\sum_{c \in C} a\_s^c_{p,g,k} \, z_c \geq y\_s_{p,g,k} \tag{122}$$

$$\sum_{c \in C} a\_a^c_{p,g,k} \, z_c \geq y\_a_{p,g,k} \tag{123}$$

$$\sum_{c \in C} a\_s^c_{p,g,k} \, z_c \leq W y\_s_{p,g,k} \tag{124}$$

$$\sum_{c \in C} a\_a^c_{p,g,k} \, z_c \leq W y\_a_{p,g,k}. \tag{125}$$

Equipment location constraints All level 2 equipment must connect to the same equipment of level 1 (i.e., location of a 1st level equipment is same for all 2nd level equipment). The equipment of each group must be placed in a single location. In a single level hierarchy, all ONUs are connected with the 1st level equipment i.e., there is no second level equipment.

$$\sum_{p \in P_{\text{EQ}}} \sum_{k \in K} (y\_s_{p,g,k} + y\_a_{p,g,k}) = \sum_{k \in K} a_{g,k} \qquad g \in G. \tag{126}$$

A given location cannot be selected more than once in a given equipment hierarchy:

$$\sum_{g \in G} \sum_{k \in K} (y\_s_{p,g,k} + y\_a_{p,g,k}) \leq 1 \qquad p \in P_{\text{EQ}}. \tag{127}$$

Demand constraints The upstream traffic will be granted if all its components are carried out.

$$\sum_{c \in \mathcal{C}^{\text{UL}}} t^c_{s,\text{OLT}}\, z_c \geq T_{s,\text{OLT}} \qquad s \in P_{\text{ONU}}. \tag{128}$$

The downstream traffic will be carried out only if every destination gets the signal and it is of two types:

$$\text{Unicast:} \sum_{c \in \mathcal{C}^{\text{DL}}} t^c_{\text{OLT},d}\, z_c \geq T_{\text{OLT},d} \qquad d \in P_{\text{ONU}} \tag{129}$$

$$\text{Multicast:} \sum_{c \in \mathcal{C}^{\text{DL}}} \alpha^c_d\, t^c_{\text{OLT},D}\, z_c \geq T_{\text{OLT},D} \quad d \in D, D \in \mathcal{D}. \tag{130}$$

6.6 CG-BASED Solution of the Model

In order to solve the optimization model described in the previous section, we rely on a column generation solution scheme. The linear mathematical model, proposed in Section 6.5, works as the restricted master problem (RMP) of the column generation (CG) technique. We next describe the pricing problem, first its set of variables (Section 6.6.1), next its objective (Section 6.6.2), and then its set of constraints (Section 6.6.3). In order to alleviate the notations, although each pricing problem is associated with the definition of an equipment hierarchy, i.e., a given equipment location configuration (c), we will omit the c index if there is no confusion.

6.6.1 Variables

The variables of the pricing are the coefficients of the z_c variable in the master problem, i.e., the coefficients of a column vector associated to a z_c variable. Therefore, the variables of the pricing problem are :

- $t_{s,d} \in [0, 1]$

- $t_{s,D} \in [0, 1]$

- $a\_a_{p,g,k} \in \{0, 1\}$

- $a\_s_{p,g,k} \in \{0,1\}$

- $\alpha_d \in \{0,1\}$ where $\alpha_d = 1$ if any ONU $\in P_{\text{ONU}}$ is associated with a configuration.

- $\beta_g \in \{0,1\}$ where $\beta_g = 1$ if any ONU of cluster $g \in G^\star$ is associated with a configuration.

6.6.2 Objective

The objective of the pricing problem is defined by the minimization of the reduced cost, which is expressed as follows for upstream pricing problem:

$$\begin{aligned}\overline{\text{COST}}^{\text{UP}}(z) = &- \sum_{p \in P_{\text{EQ}}} \sum_{g \in G} \sum_{k \in K} u^{\text{S}}_{1_{p,g,k}} a\_s_{p,g,k} \\ &- \sum_{p \in P_{\text{EQ}}} \sum_{g \in G} \sum_{k \in K} u^{\text{AWG}}_{1_{p,g,k}} a\_a_{p,g,k} + \sum_{p \in P_{\text{EQ}}} \sum_{g \in G} \sum_{k \in K} u^{\text{S}}_{2_{p,g,k}} a\_s_{p,g,k} \\ &+ \sum_{p \in P_{\text{EQ}}} \sum_{g \in G} \sum_{k \in K} u^{\text{AWG}}_{2_{p,g,k}} a\_a_{p,g,k} - \sum_{\text{ONU} \in P_{\text{ONU}}} u^t_{\text{ONU}} t_{\text{ONU,OLT}} \end{aligned} \quad (131)$$

where $u^{\text{S}}_{1_{p,g}}$ and $u^{\text{AWG}}_{1_{p,g}}$ are the dual values associated with constraints (122-p,g) and (123-p,g) respectively, $u^{\text{S}}_{2_{p,g}}$ and $u^{\text{AWG}}_{2_{p,g}}$ are the dual values associated with constraints (124-p,g) and (125-p,g) respectively, and u^t_{ONU} is the dual value associated with constraint (128-ONU).

The objective of the downstream pricing problem can be expressed similarly as upstream pricing problem.

6.6.3 Constraints

Equipment Selection Constraints

For each cluster g, at most one splitter/AWG with $k = |g|$ output ports can be placed in a potential location. In other words, for each $g \in G, k \in K : a_{g,k} = 1$, we have:

$$\sum_{p \in P_{\text{EQ}}} a\_a_{p,g,k} \leq 1 \tag{132}$$

$$\sum_{p \in P_{\text{EQ}}} a\_s_{p,g,k} \leq 1 \tag{133}$$

$$\sum_{p \in P_{\text{EQ}}} (a\_s_{p,g,k} + a\_a_{p,g,k}) = 1 \tag{134}$$

For each potential location, at most one equipment with a single splitting ratio can be placed.

$$\sum_{g \in G} \sum_{k \in K} (a\_s_{p,g,k} + a\_a_{p,g,k}) \leq 1 \qquad p \in P_{\text{EQ}}. \tag{135}$$

For each cluster, at most one equipment with a single splitting ratio can be placed at a potential location. In a single level hierarchy, all ONUs are connected with the 1st level equipment, i.e., the configuration should not include any 2nd level equipment.

$$\sum_{p \in P_{\text{EQ}}} \sum_{k \in K} (a\_s_{p,g,k} + a\_a_{p,g,k}) = \sum_{k \in K} a_{g,k} \qquad g \in G \tag{136}$$

Traffic Constraints

Downstream Traffic If the optimization model selects a splitter in the first level, the summation of traffic requests of all groups in the second level can be at most 1. Again, if there is an AWG in the first level, the individual traffic of each group can be at most 1. If there is a splitter in the second level, the summation of traffic of all ONUs in the corresponding group can be at most 1. Again, if there is an AWG in the second level, the individual traffic of each ONU can be at most 1.

For downstream traffic, we need to take into account both unicast and multicast traffic requests.

Constraints for unicast traffic are as follows:

$$\sum_{d \in P_{\text{ONU}}} t_{\text{OLT},d} \leq 1; \qquad t_{\text{OLT},d} \leq \alpha_d, \qquad\qquad d \in P_{\text{ONU}} \tag{137}$$

Constraints for multicast traffic are written as follows:

$$t_{\text{OLT},D} \leq \alpha_D; \qquad \alpha_D \geq \alpha_d, \qquad\qquad d \in D, D \in \mathcal{D} \tag{138}$$

The following constraints are for both unicast and multicast traffic. Constraint (139) states that if there is a splitter in a group g, at most $|g|$ ONUs confined to that group can receive traffic in each configuration. But in case of an AWG, only one ONU can receive traffic in each configuration.

$$\sum_{d \in P_{\text{ONU}}:\delta_{d,g}=1} \alpha_d \leq \sum_{p \in P_{\text{EQ}}} \sum_{k \in K} (a\_a_{p,g,k} + |g| \times a\_s_{p,g,k}) \qquad\qquad g \in G^{\star} \tag{139}$$

If a splitter is selected in the first level, $|g_0|$ is the maximum number for receiving the downstream transmission which includes the ONUs directly connected to the first level equipment along with the number of clusters such that any ONU from these clusters can receive the data. In case of an AWG in the first level, the maximum number for receiving downstream transmission is limited to at most 1 which implies that at most one ONU connected directly to the first level equipment or at most one group of ONUs can receive the data.

$$\sum_{g \in G^{\star}} \beta_g + \sum_{d \in P_{\text{ONU}}:\delta_{d,g_0}=1} \alpha_d \leq \sum_{p \in P_{\text{EQ}}} \sum_{k \in K} (a\_a_{p,g_0,k} + |g_0| \times a\_s_{p,g_0,k}) \tag{140}$$

A cluster is involved in a downstream transmission only when an ONU from the corresponding cluster receives any data from the OLT.

$$\beta_g \geq \alpha_d \qquad\qquad g \in G^{\star}, d \in P_{\text{ONU}} : \delta_{d,g} = 1 \tag{141}$$

Upstream Traffic The upstream traffic consists of only unicast requests.

$$\sum_{\text{ONU} \in P_{\text{ONU}}} t_{\text{ONU,OLT}} \leq 1 \tag{142}$$

$$t_{\text{ONU,OLT}} \leq \alpha_{\text{ONU}} \qquad \text{ONU} \in P_{\text{ONU}} \tag{143}$$

Constraint (144) states that if there is a splitter in a cluster g, at most $|g|$ ONUs confined to that group can send traffic in each configuration. But in case of an AWG, only one ONU can send traffic in each configuration.

$$\sum_{\text{ONU} \in P_{\text{ONU}} : \delta_{\text{ONU},g} = 1} \alpha_{\text{ONU}} \leq \sum_{p \in P_{\text{EQ}}} \sum_{k \in K} (a\_a_{p,g,k} + |g| \times a\_s_{p,g,k}) \qquad g \in G^{\star} \tag{144}$$

If a splitter is selected in the first level, the maximum number of transmitters is limited by the number $|g_0|$ which includes the ONUs directly connected to the first level equipment along with the number of clusters such that any ONU from these clusters can send the data. In case of an AWG in the first level, the maximum number of transmitters is at most 1 which implies that at most one ONU connected directly to the first level equipment or at most one group of ONUs having a splitter assigned to that cluster can transmit the data.

$$\sum_{g \in G^{\star}} \beta_g + \sum_{\text{ONU} \in P_{\text{ONU}} : \delta_{\text{ONU},g_0} = 1} \alpha_{\text{ONU}} \leq \sum_{p \in P_{\text{EQ}}} \sum_{k \in K} (a\_a_{p,g_0,k} + |g_0| \times a\_s_{p,g_0,k}) \tag{145}$$

A cluster is involved in a upstream transmission only when an ONU from the corresponding cluster transmits any data from the OLT.

$$\beta_g \geq \alpha_{\text{ONU}} \qquad g \in G^{\star}, \text{ONU} \in P_{\text{ONU}} : \delta_{\text{ONU},g} = 1 \tag{146}$$

Attenuation Constraints

To calculate the signal power loss caused by the optical fiber and passive equipment, we introduce the variable $x\_\text{ATT}_p^g$ to evaluate the total attenuation to reach ONU of cluster g located at p, $p \in P_{\text{ONU}}$. Let us assume a loss of 0.2dB/km caused by optical fiber, and let ATT_k^{S} (resp. ATT^{AWG}) be the attenuation factor of the splitter S heading group g, which depends on the number of outputs of S (resp. which is independent of the number of outputs of AWG).

$$
\begin{aligned}
x\_\text{ATT}_p^g = \sum_{k \in K} \sum_{p'' \in P_{\text{EQ}}} \big(\text{ATT}^{\text{AWG}}\, a\_a_{p'',g_0,k} + \text{ATT}_k^{\text{S}}\, a\_s_{p'',g_0,k} \\
+ \sum_{k \in K} \sum_{p' \in P_{\text{EQ}}} \big(\text{ATT}^{\text{AWG}}\, a\_a_{p',g,k} + \text{ATT}_k^{\text{S}}\, a\_s_{p',g,k}\big) \\
+ \sum_{k \in K} \sum_{p'' \in P_{\text{EQ}}} 0.2\, d_{\text{OLT}p''}(a\_a_{p'',g_0,k} + a\_s_{p'',g_0,k}) \\
+ \sum_{k \in K} \sum_{p'' \in P_{\text{EQ}}} \sum_{p' \in P_{\text{EQ}}} 0.2\, d_{p''p'}(a\_a_{p'',g_0,k} + a\_s_{p'',g_0,k})(a\_a_{p',g,k} + a\_s_{p',g,k}) \\
+ \sum_{k \in K} \sum_{p' \in P_{\text{EQ}}} 0.2\, d_{p'p}(a\_a_{p',g,k} + a\_s_{p',g,k}) \\
p \in P_{\text{ONU}} : \delta_{p,g} = 1, g \in G^{\star}. \quad (147)
\end{aligned}
$$

The fourth element of the summation in (41) is nonlinear, but can be easily linearized using classical technique.

We want the total loss for every ONU not to exceed 20 decibels:

$$
x\_\text{ATT}_p^g + P^{\text{MARGIN}} + P^{\text{INSERTION}} \leq 20 \text{ dB} \qquad p \in P_{\text{ONU}} : \delta_{p,g} = 1, g \in G^{\star}. \quad (148)
$$

Now we introduce another variable $y\_\text{ATT}_p^g$ to evaluate the total attenuation to reach ONU of group g_0 (i.e., 1st level equipment) located at p, $p \in P_{\text{ONU}}$. The following equations represent the attenuation constraint of the ONUs that are directly connected to the first level equipment.

$$y\_\text{ATT}_p^{g_0} = \sum_{k \in K} \sum_{p' \in P_{\text{EQ}}} (\text{ATT}^{\text{AWG}}\, a\_a_{p',g_0,k} + \text{ATT}_k^{\text{S}}\, a\_s_{p',g_0,k})$$
$$+ \sum_{k \in K} \sum_{p' \in P_{\text{EQ}}} 0.2\, d_{\text{OLT},p'} (a\_a_{p',g_0,k} + a\_s_{p',g_0,k})$$
$$+ \sum_{k \in K} \sum_{p' \in P_{\text{EQ}}} 0.2\, d_{p'p} (a\_a_{p',g_0,k} + a\_s_{p',g_0,k}) \qquad p \in P_{\text{ONU}} : \delta_{p,g_0} = 1 \quad (149)$$

$$y\_\text{ATT}_p^{g_0} + P^{\text{MARGIN}} + P^{\text{INSERTION}} \leq 20 \text{ dB} \quad p \in P_{\text{ONU}} : \delta_{p,g_0} = 1. \qquad (150)$$

6.7 Phase IV:Optimization Model for Selecting the Hierarchies

Once the best equipment type and location/allocation have been found by the TYPE-LOC-CG algorithm for each PON hierarchy, we need to decide on the selection of the best set of PONs for covering the traffic demands of a given geographical area. For this purpose, we next propose the M-PON-ILP optimization model.

6.7.1 Notations

Equipment Hierarchy Parameters

Let $\mathcal{H} = \{H_1, H_2, ..., H_n\}$ be a set of equipment hierarchies as output by the TYPE-LOC-CG algorithm. Parameter $\gamma_{H,\text{ONU}}$ indicates the association of an ONU, ONU $\in P_{\text{ONU}}$ with a given equipment hierarchy, $H \in \mathcal{H}$: $\gamma_{H,\text{ONU}} = 1$ if an ONU belongs to any cluster of an equipment hierarchy,H and 0 otherwise.

Cost Parameters

Let COST_H denote the cost of an equipment hierarchy H, with $H \in \mathcal{H}$.

Other Parameters

Let $n^{\max\_PON}$ denotes the maximum number of PONs supported by the CO.

6.7.2 Optimization Model

Variables

- $z_H \in \{0,1\}$ is a decision variable such that $z_H = 1$ if equipment hierarchy H is selected, and 0 otherwise.
- $x_{\text{ONU},H} \in \{0,1\}$ is a decision variable such that $x_{\text{ONU},H} = 1$ if an ONU $\in P_{\text{ONU}}$ of an equipment hierarchy, H is selected, 0 otherwise.

Objective

$$\min \text{COST} = \sum_{H \in \mathcal{H}} \text{COST}_H z_H \tag{151}$$

Constraints

Equipment Hierarchy Selection constraints Each ONU must be hosted in an equipment hierarchy.

$$\sum_{H \in \mathcal{H}} x_{\text{ONU},H} = 1, \qquad \text{ONU} \in P_{\text{ONU}} \tag{152}$$

An equipment hierarchy is selected only when any ONU of that hierarchy is not included in any other selected equipment hierarchies.

$$\sum_{H \in \mathcal{H}} \gamma_{H,\text{ONU}} z_H = 1, \qquad \text{ONU} \in P_{\text{ONU}} \tag{153}$$

The total number of equipment hierarchies is limited by the maximum number of

PONs supported by the central office (CO).

$$\sum_{H \in \mathcal{H}} z_H \leq n^{\max\_PON} \tag{154}$$

An ONU can be included in an equipment hierarchy only when the equipment hierarchy is selected by the optimization model:

$$x_{\text{ONU},H} = z_H \qquad H \in \mathcal{H}, \text{ONU} \in P_{\text{ONU}} : \gamma_{H,\text{ONU}} = 1 \tag{155}$$

Constraint (155) also guarantees that, whenever an equipment hierarchy is selected, all the ONUs (accordingly all clusters) of the corresponding equipment hierarchy must be selected.

6.8 Computational Results and Analysis

We implement the 4-phase **SLAPONS** scheme described in the previous sections, and in particular the optimization model of Section 6.5.2 using the Optimization Programming Language (OPL) platform. Linear and integer linear programs are solved using the CPLEX package [IBM11].

First, we conduct our experiments with randomly generated Manhattan-pattern geographic locations of 512 ONUs(1st set). Location of the ONUs are randomly generated in a 40×40 km^2 rectangular grid such that the OLT is located at the center of the grid, i.e., at location (20,20) as shown in Figure 20. ONU locations are distributed along several vertical lines so that each value of x-coordinate can accommodate several "vertical" ONU locations. Candidate/potential locations for the placement of the passive equipment are positioned inside the same rectangular grid. For the purpose of the experiments, they have been randomly generated, while in practice their locations are limited to some specific (easily accessible) locations. Table 2 contains the values taken for the cost of the equipment [CWMJ10], as well as the attenuation parameters, which depend on the number of output ports for the splitters. For the costs related to optical fiber cables, we use the value of 7,160$/km [CWMJ10], assuming it includes the cost of trenching and laying the optical fiber cables.

We randomly generate the upstream unicast traffic flows within the range [0.05, 0.1] (recall that our traffic parameters are normalized using the wavelength transport capacities, see Section 6.5.1). Towards downstream direction, we randomly generate both unicast and multicast traffic flows within the range [0.1, 0.4]. We consider a number of multicast traffic requests destined for different groups of ONUs.

The first phase of our **SLAPONS** scheme is to run the clustering heuristic which builds the ONU-cluster association for 512 ONUs and eventually decides on the splitting ratio of the switching equipment. We, first, start by considering 8 clusters such that each ONU must reside in one of these clusters. In the second phase, the hierarchy generation heuristic generates a number of different combinations of potential PON hierarchies based on the clustering information of the first phase. For example, in Figure 21(a), all ONUs are grouped into 8 clusters (i.e., $g_1, g_2, g_3, g_4, g_5, g_6, g_7, g_8$) such that each cluster of ONUs constitutes disjoint PON hierarchies (i.e., H1, H2, H3, H4, H5, H6, H7, H8). Therein, each hierarchy is considered as a separate one-stage PON network.

Next, as illustrated in Figures 21(b) and 21(c), additional PON hierarchies are defined by considering two neighboring clusters in a single PON where all the ONUs are connected to the 2nd level equipment resulting in a two-stage PON architecture (i.e., H9, H10, H11, H12, H13, H14, H15, H16).

We also consider mixed-stage PON hierarchies by connecting a group of ONUs to the 1st level equipment and another group of ONUs to the 2nd level equipment (i.e., H17, H18, H19, H20) as depicted in Figure 21(d).

Next, we consider 16 clusters, as illustrated in Figures 21(e), 21(f), where each of 512 ONUs resides in one of these clusters and all hierarchies constitute 2-stage PON architecture (i.e., H21, H22, H23, H24, H25, H26, H27, H28).

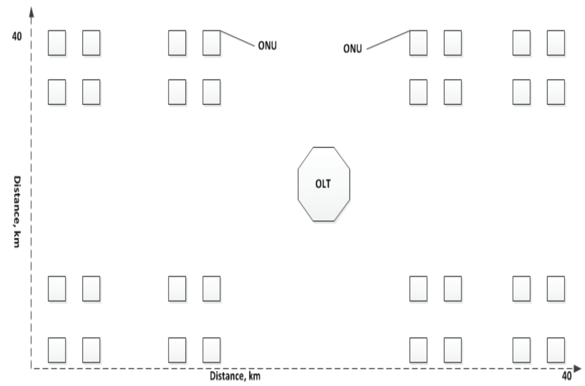
Finally, we consider 32 clusters, as illustrated in Figures 21(g), 21(h), where each of 512 ONUs resides in one of these clusters and all hierarchies constitute 2-stage PON architecture (i.e., H29, H30, H31, H32, H33, H34, H35, H36).

The next phase of the **SLAPONS** scheme is to solve the column generation based optimization TYPE-LOC-CG model in order to:

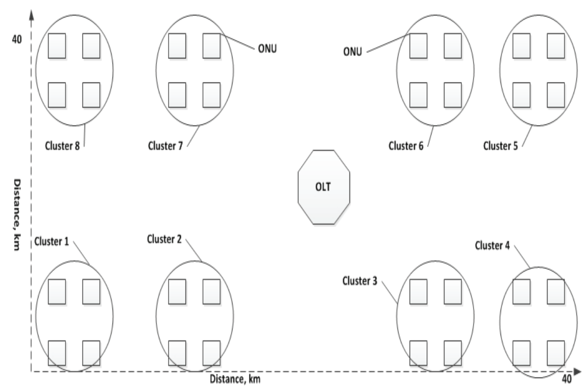
(i) select the type (splitter or AWG) and location of the passive equipment,

(ii) provision the traffic flows,

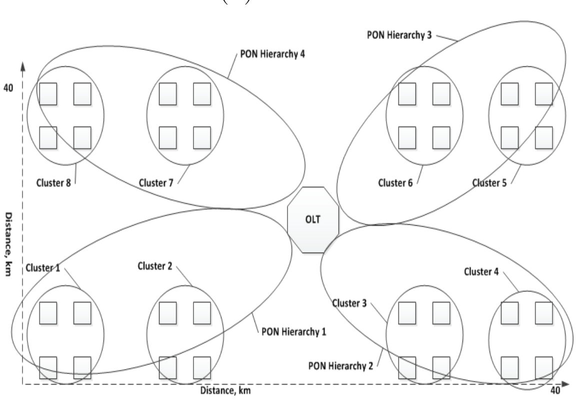
for each equipment hierarchy (i.e., PON network).



(a) Initial Phase



(b) Phase I



(c) Phase II

Figure 19: Illustration of Phases of **SLAPONS** Scheme

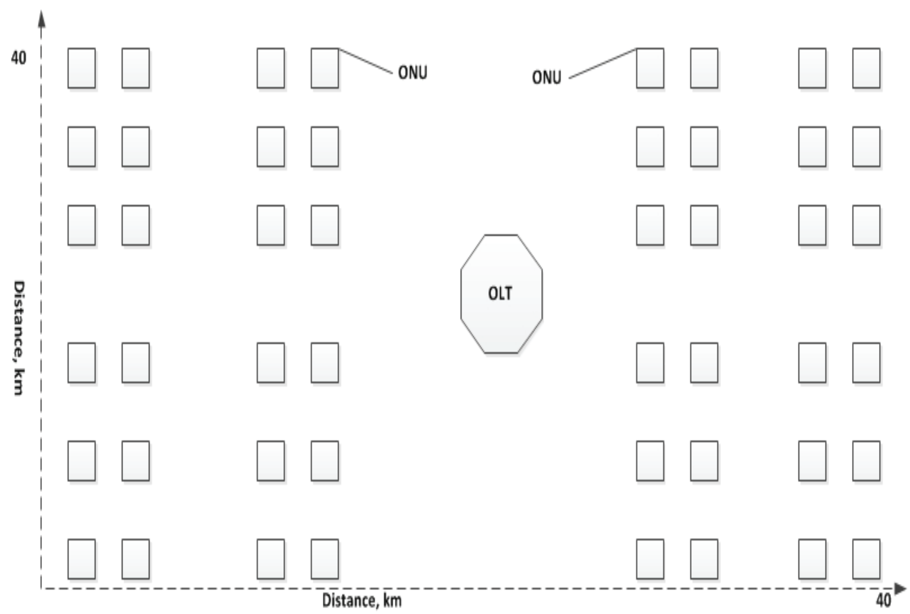


Figure 20: Experimental Scenario

Table 20, obtained by running the TYPE-LOC-CG algorithm, shows a comparison of the PON 'greenfield' deployment costs for different hierarchies where each hierarchy delineates a single PON network. For most of the investigated hierarchies, the optimality gap is equal to zero, as we manage to get integer solutions such that $z^{\star}_{\text{LP}} = \tilde{z}_{\text{ILP}}$.

We observe that different types of equipment are selected in different PON hierarchies. For example, only AWGs are selected as switching equipment in hierarchy H20, a single splitter in hierarchy H2, a mix of splitters and AWGs in hierarchy H16. In addition, cost varies depending on the number of served ONUs, with some hierarchies much more efficient than others.

The selection and the placement of the switching equipment are made based on the best choice taking into account the cost, the traffic flows (some unicast, some multicast) and the attenuation constraints.

The last phase of the **SLAPONS** scheme is to select the minimum cost PON hierarchies and thereby decide on the number of PON networks required to serve all the ONUs in a neighborhood. We now report on the computational results which are obtained through the simulation of M-PON-ILP model and are shown in Table 21.

M-PON-ILP model considers 36 PON equipment hierarchies as its input and finally selects 4 minimum cost hierarchies which corresponds to 4 pairwise disjoint PON networks. Table 21 shows that the following hierarchies, namely H29, H30, H31, H32 are identified as the most cost-effective combination of PON networks. It implies that we need four separate hybrid PON networks of two-stage cascading architecture in order to cover and serve the traffic requests of 512 ONUs.

Next, we extend our experiment with different distribution of the location of the ONUs (2nd set) in which the initial 40×40 km^2 rectangular grid is divided into 4 sub-squares. Each sub-square has different density of distribution of ONUs where sub-squares 1, 2, 3 and 4 have 227, 117, 70 and 98 ONUs respectively. But ONUs are uniformly distributed within each sub-square. We consider the same parameters and follow the same procedures as before while simulating our **SLAPONS** scheme with the new data-set (i.e., 2nd set of ONU locations).

Table 22, obtained by running the TYPE-LOC-CG algorithm for different distribution of ONUs, shows a comparison of the PON greenfield deployment cost for different hierarchies. It can be noticed that a mix of splitters and AWGs are selected for most of the PON hierarchies whereas some hierarchies consist of only splitters/AWGs.

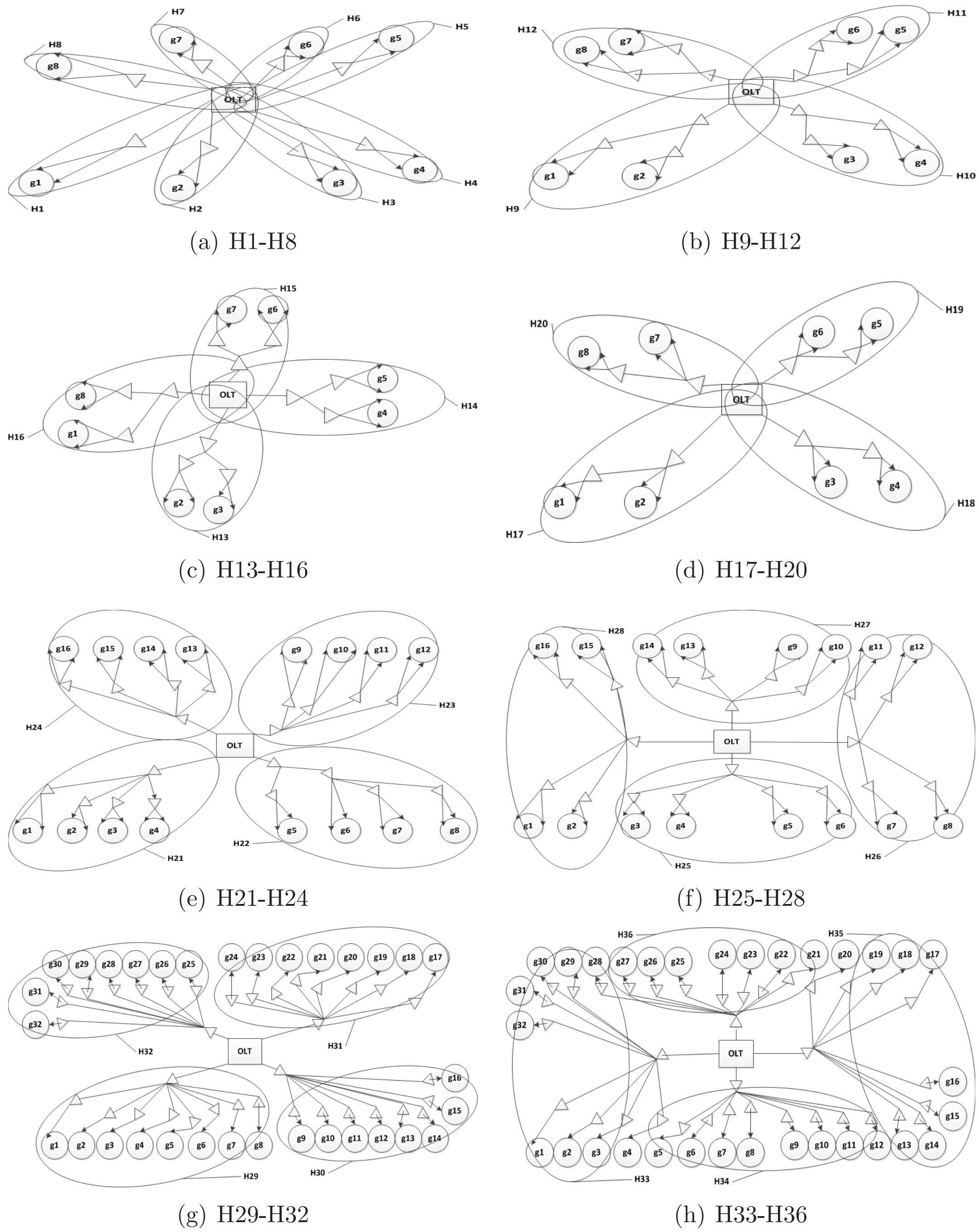
During the last phase of the **SLAPONS** scheme, we determine 4 minimum cost disjoint PON networks (i.e., H29, H30, H31, H32) from the Table 23 where 36 PON equipment hierarchies are taken into account. In all these selected PON networks, first the ONUs are partitioned into 32 clusters, then these 32 clusters constitute four separate hybrid PON networks of two-stage cascading architecture and finally the splitters/AWGs are selected as the 1st/2nd level switching equipment.

By simulating our **SLAPONS** scheme with two different sets of ONU distributions, we observe that the deployment cost of a set of hybrid PONs is dependent on the geographical location of the ONUs as well as on the type and the amount of traffic demand. We also observe that the best set of hybrid PONs is obtained when the 512 ONUs are grouped into 32 clusters and these clusters constitute 4 disjoint PON networks.

6.9 Summary

In this chapter, we present our first proposed scheme to solve the location allocation (L/A) problem of multiple hybrid PONs. We propose a novel network design optimization scheme, called **SLAPONS**, for greenfield deployment of hybrid PONs. The proposed **SLAPONS** scheme proceeds in four phases: The first phase consists in computing a set of potential ONU partitions using a hierarchical clustering heuristic. In the second phase, we generate several potential (equipment cascading) PON hierarchies, based on the various potential ONU partitions. In the third phase, we propose an efficient and scalable optimization (model and) algorithm, which selects the best passive switching equipment (AWG or splitter) and generate the cost effective (minimum deployment cost) location of the selected equipment for each potential PON hierarchy. In the fourth phase, we design an integer linear programming (ILP) model to select the best combination of PON networks (equipment hierarchies), among all the optimized single PON equipment hierarchies which have been output in the third phase.

The proposed scheme can optimize the design of a set of hybrid PONs covering a given geographic area as well as the selection of the best cascading architecture (1/2/mixed-stage) for each selected PON. Computational experiments have been conducted on a set of 512 ONUs in order to evaluate the performance of the **SLAPONS** scheme.



(a) H1-H8

(b) H9-H12

(c) H13-H16

(d) H17-H20

(e) H21-H24

(f) H25-H28

(g) H29-H32

(h) H33-H36

Figure 21: Set of Hierarchies

Table 20: Computational Results on Different Hierarchies (1st set)

Hierarchy	Equipment Type	$z^{\star}_{\text{LP}}$	$\tilde{z}_{\text{ILP}}$	Optimality gap (%)	# served ONUs
H1	AWG	9,239,640	9,239,640	0	126
H2	Splitter	768,420	768,420	0	21
H3	AWG	8,559,440	8,559,440	0	105
H4	AWG	1,062,880	1,062,880	0	20
H5	Splitter	689,660	689,660	0	20
H6	AWG	5,552,240	5,552,240	0	80
H7	Splitter	474,860	474,860	0	20
H8	AWG	8,602,400	8,602,400	0	120
H9	Mixed	9,794,960	9,794,960	0	147
H10	Mixed	7,833,120	7,833,120	0	125
H11	Mixed	5,176,760	5,176,760	0	100
H12	Mixed	8,899,960	8,899,960	0	140
H13	Mixed	9,157,720	9,157,720	0	126
H14	Mixed	1,503,640	1,503,640	0	40
H15	Mixed	7,109,960	7,109,960	0	100
H16	Mixed	25,883,520	25,883,520	0	246
H17	AWG	9,865,760	9,865,760	0	147
H18	AWG	7,975,520	7,975,520	0	125
H19	AWG	5,304,804	5,304,804	0	100
H20	AWG	8,934,960	8,934,960	0	140
H21	Mixed	7,546,060	7,546,060	0	147
H22	Mixed	6,331,454	6,338,380	1.09	125
H23	Mixed	4,032,860	4,032,860	0	100
H24	Mixed	6,759,176	6,901,660	2.10	140
H25	Mixed	6,252,460	6,252,460	0	126
H26	Mixed	3,271,500	3,271,500	0	81
H27	Mixed	4,190,380	4,190,380	0	100
H28	Mixed	11,691,740	11,691,740	0	205
H29	Mixed	6,476,040	6,476,040	0	147
H30	Mixed	4,954,120	4,954,120	0	125
H31	Mixed	3,005,640	3,005,640	0	100
H32	Mixed	5,065,860	5,065,860	0	140
H33	Mixed	5,235,420	5,235,420	0	126
H34	Mixed	2,567,960	2,567,960	0	81
H35	Mixed	3,276,460	3,276,460	0	100
H36	Mixed	10,001,640	10,001,640	0	205

Table 21: Computational Results on PON hierarchies (1st set)

List of Generated Hierarchies	List of Selected Hierarchies	Total Cost ($)
H1,H2, ... ,H36	H29, H30, H31,H32	19,501,660

Table 22: Computational Results on Different Hierarchies (2nd Set)

Hierarchy	Equipment Type	$z^{\star}_{\text{LP}}$	$\tilde{z}_{\text{ILP}}$	Optimality gap (%)	# served ONUs
H1	AWG	9,552,860	9,552,860	0	117
H2	AWG	8,761,460	8,761,460	0	110
H3	AWG	5,071,320	5,071,320	0	62
H4	splitter	4,962,840	4,962,840	0	55
H5	Splitter	4,603,240	4,603,240	0	40
H6	Splitter	4,453,280	4,453,280	0	30
H7	Splitter	3,982,620	3,982,620	0	54
H8	AWG	3,856,940	3,856,940	0	44
H9	Mixed	14,383,820	14,383,820	0	227
H10	Mixed	6,842,460	6,842,460	0	117
H11	Mixed	5,076,980	5,076,980	0	70
H12	Mixed	5,404,804	5,404,804	0	98
H13	Mixed	10,403,620	10,403,620	0	161
H14	Mixed	10,883,220	10,883,220	0	172
H15	Mixed	5,152,860	5,152,860	0	95
H16	Mixed	4,353,980	4,353,980	0	84
H17	Mixed	12,984,520	12,984,520	0	227
H18	Mixed	6,952,220	6,952,220	0	117
H19	Mixed	4,143,250	4,143,250	0	70
H20	Mixed	5,009,640	5,009,640	0	98
H21	Mixed	11,974,560	11,974,560	0	227
H22	Mixed	5,652,920	5,652,920	0	117
H23	Mixed	3,893,280	3,893,280	0	70
H24	Mixed	4,879,220	4,879,220	0	98
H25	Mixed	9,454,520	9,454,520	0	161
H26	Mixed	9,876,220	9,876,220	0	172
H27	Mixed	4,356,920	4,356,920	0	95
H28	Mixed	3,442,340	3,442,340	0	84
H29	Mixed	7,378,640	7,378,640	0	227
H30	Mixed	5,784,960	5,784,960	0	117
H31	Mixed	3,905,220	3,905,220	0	70
H32	Mixed	4,765,860	4,765,860	0	98
H33	Mixed	7,635,920	7,635,920	0	161
H34	Mixed	8,247,460	8,247,460	0	172
H35	Mixed	3,576,520	3,576,520	0	95
H36	Mixed	3,076,840	3,076,840	0	84

Table 23: Computational Results on PON hierarchies (2nd set)

List of Generated Hierarchies	List of Selected Hierarchies	Total Cost ($)
H1,H2, ... ,H36	H29, H30, H31,H32	21,834,680

Chapter 7

Switching Equipment Location/Allocation in multiple hybrid PONs - Scheme 2

7.1 Introduction

In this chapter, we investigate to determine the optimal number of hybrid PONs required to satisfy the traffic demands of all ONUs in a geographical area. We inspect the optimal covered region of each hybrid PON. We also focus on the greenfield deployment of a PON by selecting the optimal type (i.e., splitters/AWGs) and location of the switching equipment exploiting the mathematical formulation of the p-center problem oriented optimization scheme. To the best of our knowledge, no research activities are reported to solve the L/A problem of a hybrid PON network using the p-center problem. This motivated us for investigating a p-center based multi-phase solution scheme to solve the aforementioned problem, known to be an NP-complete one [LS09],[MPC11].

The chapter is organized as follows. In Section 7.2, we provide a concise statement of the multiple-PON deployment problem and an outline of our proposed 4-phase BSP (Best Set of PONs) scheme. The first phase, a p-center based integer linear programming (ILP) formulation, aiming at ONU clustering, is described in Section 7.3. An alternative scalable solution to the first phase, a p-center based column generation (CG) formulation is illustrated in Section 7.4. The second phase, another p-center based ILP is described in Section 7.5. An alternative scalable solution to the second phase, a p-center based CG formulation is illustrated in Section 7.6. The results obtained from these two phases are used to generate several potential PON hierarchies by fixing the location of the switching equipment. The third phase, in Section 7.7, takes care of the switching equipment selection with minimum network deployment cost for each potential PON hierarchy. In the fourth phase, detailed in Section 7.9, we propose an ILP model to select the best set of cost-effective PON networks in order to serve the traffic demand requests of all ONUs (consequently all end users) in a given geographical area. Computational results and analysis are presented in Section 7.10 in order to validate the proposed BSP scheme. Summary is drawn in the last section.

7.2 Multiple PON Deployment: Problem Statement and Optimization Process

7.2.1 Problem Statement

We propose to investigate the OSPON (Optimized Set of PONs) problem, defined as follows. It deals with the greenfield deployment of multiple hybrid PON networks in a given geographical area and consists of determining the best set of cost-effective PON networks in order to serve a number of ONUs covering all the end users in a given neighbourhood. It means partitioning a given geographical area into a number of sub-regions where each sub-region is covered by a single PON. The network topology of each PON has to be determined with the intention of minimizing the overall network deployment cost based on the location of the OLT and the ONUs while granting all traffic demands. The network topology is characterized by the selection, location and cascading architecture of splitters/AWGs, which allocates a switching equipment to each group of ONUs in a PON. We allow different two-stage cascading architectures for the various PON networks. In a two-stage architecture, as shown in Figure 4, we assume all equipment to be distributed on two levels such that all ONUs are connected to the 2nd level equipment and all 2nd level equipment are connected to the single 1st level equipment, which is itself connected to the OLT.

We also propose to select the switching equipment depending on the type (unicast/multicast) of traffic demand as splitters are best suited for multicast demand whereas AWGs are suitable for unicast traffic demand. Finally, we determine the best set of PON networks along with their cascading architecture, type and location of their switching equipment while satisfying the network design constraints such as splitting ratio of splitters/AWGs and maximum allowed signal power loss at each ONU.

The input parameters of our aforesaid problem contain the location of the OLT and of the ONUs, the set of potential/candidate equipment locations together with the unicast/multicast traffic demand matrix (normalized values with respect to the transport capacity of the wavelengths). The output parameters comprise the best set of selected PON networks as well as the locations and the type of the switching equipment (whether splitter or AWG) along with the cascading architecture of each PON network. The overall objective corresponds to the minimization of the network deployment cost (i.e., initial infrastructure installation and maintenance cost). Infrastructure installation cost is composed of the price of the switching equipment (splitter/AWG) and the optical fiber cables (including the cost of trenching and laying fibers). There is no maintenance cost for the switching equipment as it is a passive one. We have not taken into account the installation and maintenance cost of the OLT as well as of the ONUs assuming that these equipment are already in place. We also assume that each ONU accommodates aggregated traffic requests of a number of end users.

7.2.2 Optimization Process

We propose a 4-phase scheme, called BSP (Best Set of PONs), depicted in Figure 22, in order to solve the OSPON problem. In Phase I, a N_I-center based ILP model (namely, 1st-Phase-N_I-ILP) as well as an alternative CG model (namely, 1st-Phase-N_I-CG) is proposed to determine ONU clusters and the placement of the passive equipment (i.e., 2nd level equipment) for each cluster based on the geographical location and traffic demand of each ONU. In Phase II, a N_{II}-center based ILP model (namely, 2nd-Phase-N_{II}-ILP) as well as an alternative CG model (namely, 2nd-Phase-N_{II}-CG) is formulated to determine the clustering of the 2nd level equipment and the location of the 1st level passive equipment based on the locations of the 2nd level equipment selected during Phase I. Exploiting the results of Phases I and II, the covered region of each PON is determined and several potential PON hierarchies are generated. Each potential PON hierarchy, as shown in Figure 4, relies on an ONU clustering, where each cluster confederates a set of ONUs connected to the same passive equipment, and where the clustering of the 2nd level equipment corresponds to passive equipment all connected to a single 1st level passive equipment. However, the type (splitter/AWG) of the passive equipment is not yet determined at this stage. Phases I and II of the BSP Scheme are illustrated in Figure 23. Phase III consists in selecting the best type of the passive equipment in each potential PON hierarchy, with the use of a column generation (CG) ILP model. In Phase IV, the best set of PON equipment hierarchies is selected, with respect to minimum deployment cost, covering all ONUs in a given neighborhood. An illustration of the 4 phases is presented in Section 7.10 with the case study used in the numerical experiments.

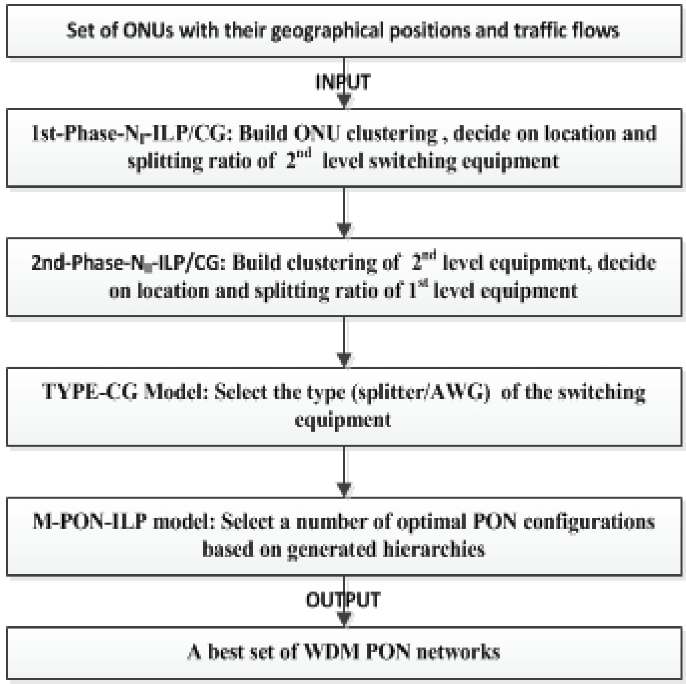
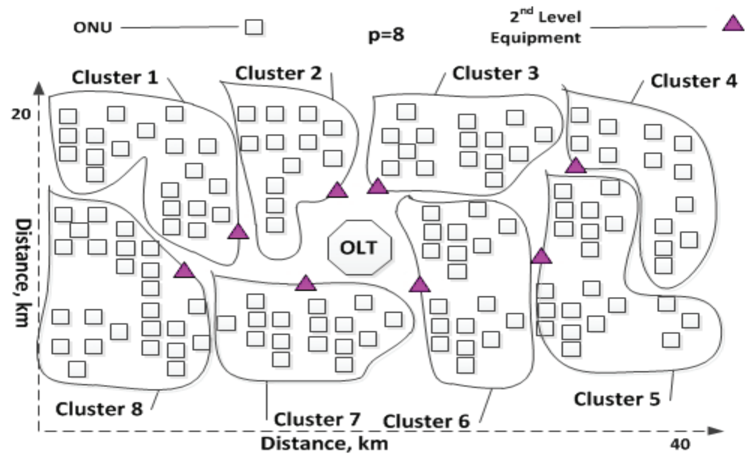


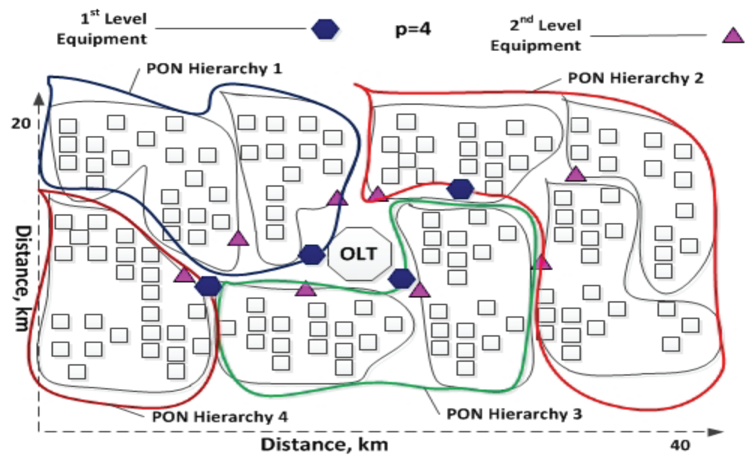
Figure 22: BSP Scheme

7.3 Phase I (1st-Phase-N_I-ILP): ILP Model for ONU Clustering and Determining the Location of 2nd Level Passive Equipment

During Phase I, in accordance with the standard p-center model, ONUs and 2nd level passive equipment potential locations are considered as demand nodes and service facilities respectively. Therein, the value of p corresponds to the number N_I of ONU clusters, i.e., the number of 2nd level passive equipment. In Phase I, the location and number of required output ports of the 2nd level equipment are determined. By varying the value of p, we can obtain different clustering of ONUs. Our proposed model extends the standard N_I-center based ILP formulation as it not only minimizes the largest distance of an ONU to the OLT, while going through the 2nd level passive equipment, but it also tries to aggregate the ONUs of the same multicast group into



(a) Phase I



(b) Phase II

Figure 23: Illustration of Phase I & Phase II of BSP Scheme

the same cluster in order to take advantage of the scattering characteristics of a splitter (usually cheaper than an AWG for a given number of ports).

7.3.1 Notations

Set of nodes $V = \{OLT\} \cup V^{ONU}$ where $V^{ONU} = \{ONU_1, ONU_2, \ldots, ONU_n\}$. We consider that all ONUs are capable of transmitting and receiving single or multiple wavelengths.

A discrete set P of locations, indexed by p, such that: $P = P_{OLT} \cup P_{ONU} \cup P_{EQ}$, where : *(i)* $P_{OLT} = \{p_{OLT}\}$, the OLT location, is known *(ii)* $P_{ONU} = \{p_{ONU_1}, p_{ONU_2}, \ldots, p_{ONU_n}\}$, the ONU locations, which are known as well, and *(iii)* P_{EQ} the set of potential locations for passive equipment.

The distance between ONU_i and potential equipment location p is denoted by d_{ip} and N_I (parameter of the p-center model) represents the total number of clusters to be formed. $d_{p,p'}$ denotes the distance between locations p and p'. Note that it does not necessarily corresponds to the geographical distance, but to the length of the fibers in order to connect p and p'.

We assume traffic values to be normalized with respect to the transport capacity of a wavelength, i.e., a value of 1 means a bandwidth requirement equal to the transport capacity of a wavelength.

7.3.2 Optimization Model

Variables

The variables are:

- z^I maximum (estimated) distance between an ONU and the OLT: it is the sum of the distance from the ONU to the second level passive equipment, and then the estimated distance from that equipment to the OLT.

- x^I_p decision variable such that $x^I_p = 1$ if a passive equipment is located in p, 0 otherwise, for $p \in P_{EQ}$.

- y_{ip} decision variable such that $y_{ip} = 1$ if an ONU_i is served by an equipment located in p, 0 otherwise, for $\text{ONU}_i \in V^{ONU}$, $p \in P_{EQ}$.

- v^D_p decision variable such that $v^D_p = 1$ if a downstream multicast request D has all its ONUs served by the same passive equipment located in p, 0 otherwise, for $p \in P_{EQ}$ and $D \in \mathcal{D} \subseteq V^{ONU}$ assuming D be the multicast destination sets and $\mathcal{D}$ be the overall set of multicast destination sets.

Objective

$$\min \quad z^I - \sum_{p \in P_{EQ}} \sum_{D \in \mathcal{D} \subseteq V^{ONU}} v_p^D. \tag{156}$$

The first part of the objective function minimizes the maximum distance between an switching equipment and an ONU, second part tries to aggregate the ONUs of same multicast group into the same cluster so that a splitter can send the multicast traffic request to all the ONUs within the same cluster.

Constraints

They correspond to a N_I-center model with side constraints, which are written as follows. The maximum distance between an ONU and a passive equipment within a cluster needs to be minimized, in order to ultimately minimize the distance from the OLT to each ONU in order to minimize the attenuation and keeps it acceptable:

$$z^I \geq d_{p,p_{OLT}} + d_{ip}\, y_{ip} \qquad ONU_i \in V^{ONU}, p \in P_{EQ}. \tag{157}$$

The ONUs will be grouped into exactly N_I clusters, i.e., N_I passive equipment will be selected and placed.

$$\sum_{p \in P_{EQ}} x_p^I = N_I. \tag{158}$$

An ONU must be assigned to exactly one potential equipment location, where some passive equipment has been set:

$$\sum_{p \in P_{EQ}} y_{ip} = 1 \qquad ONU_i \in V^{ONU}. \tag{159}$$

ONUs cannot be associated with locations where no passive equipment has been set.

$$y_{ip} \leq x_p^I \qquad ONU_i \in V^{ONU}, p \in P_{EQ}. \tag{160}$$

For each cluster, the average distance from an equipment to all ONUs should be within a threshold value (max_avg_val) in order to tentatively guarantee a similar quality of service to all users (ONUs) and the distance between an ONU and the OLT should not exceed a threshold value (max_rad_val), in order that the attenuation remains acceptable. It leads to the following constraints:

$$\sum_{ONU_i \in V^{ONU}} y_{ip}\, d_{ip} \leq \sum_{ONU_i \in P_{ONU}} y_{ip} \times \max\text{\_avg\_val}$$

$$p \in P_{EQ} \qquad (161)$$

$$d_{p,p_{OLT}} + \sum y_{ip}\, d_{ip} \leq \max\text{\_rad\_val} \qquad ONU_i \in V^{ONU}, p \in P_{EQ}. \qquad (162)$$

Constraints related to multicast traffic and cluster formation The following constraint ensures that $v_p^D = 0$ only when at least one of the ONU$_i$ belonging to the multicast destination set D is not linked to the passive equipment located in p.

$$v_p^D \leq y_{ip} \qquad ONU_i \in D, p \in P_{EQ}, D \in \mathcal{D}. \qquad (163)$$

7.4 Phase I (1st-Phase-N_I-CG): Column Generation Model for ONU Clustering and Determining the Location of 2nd Level Passive Equipment

We propose an alternative column generation (CG) model for Phase I which is more scalable compared to the ILP model proposed in Section 7.3. In our CG based solution scheme, the original problem is decomposed into two sub-problems: *(i)* so-called restricted master problem (RMP), *(ii)* so-called pricing problem (PP). In this approach, instead of pre-enumerating all candidate configurations, PP works as a configuration generator. Finally, we solve the ILP model made of the columns generated in order to obtain the optimal linear programming solution.

7.4.1 Master Model

Configuration

Before setting the optimization model, we need to introduce the concept of configurations. A configuration c corresponds to an active switching equipment, a number of ONUs in a cluster served by the switching equipment and the distance between the switching equipment and the farthest ONU assigned to it. We denote the overall set of configurations by $\mathcal{C}$ such that $\mathcal{C} = \bigcup_{p \in P_{EQ}} \mathcal{C}_p$ where $\mathcal{C}_p$ represents a configuration related to a potential switching equipment located at p in which $p \in P_{EQ}$. Let $COST_c$ be the cost of configuration c. A configuration $c \in \mathcal{C}$ is characterized by the following parameters:

- $y^c_{i,p} \in [0, 1]$ represents ONU-equipment association such that $y^c_{i,p} = 1$ if an ONU i is served by an equipment located at site p in configuration c and 0 otherwise where $ONU_i \in V^{ONU}$, $p \in P_{EQ}$.

- $COST_c$ indicates the distance between the switching equipment selected in configuration c and the farthest ONU assigned to it.

7.4.2 Notations

Notations of this CG model is described in Section 7.3.1.

Variables

- $z_c \in \{0, 1\}$ is a decision variable such that $z_c = 1$ if configuration c is selected, and 0 otherwise.

Objective

The objective corresponds to the minimization of the maximum distance between a 2nd level equipment and the farthest ONU assigned to it, expressed as follows:

$$min \sum_{c \in \mathcal{C}} COST_c \, z_c \tag{164}$$

Constraints

***p*-center related constraints** Each ONU must be served by one equipment in all configurations.

$$\sum_{p \in P_{EQ}} \sum_{c \in \mathcal{C}_p} y^c_{i,p} \, z_c \geq 1 \qquad ONU_i \in V^{ONU} \tag{165}$$

The ONUs will be grouped into N_I number of clusters, i.e., N_I number of 2nd level equipment will be placed.

$$\sum_{c \in \mathcal{C}} z_c \leq N_I \tag{166}$$

As each configuration is associated with a single equipment location, at most one configuration corresponding to each equipment will be selected.

$$\sum_{c \in \mathcal{C}_p} z_c \leq 1 \qquad p \in P_{EQ} \tag{167}$$

7.4.3 Pricing Model

Variables

- y_i is a decision variable such that $y_i = 1$ if an ONU i is served by an equipment selected in configuration c and 0 otherwise where $ONU_i \in V^{ONU}$, $c \in \mathcal{C}_p$, $p \in P_{EQ}$.

- z^I cost of each configuration c where $c \in \mathcal{C}_p$, $p \in P_{EQ}$.

- v^D is a decision variable such that $v^D = 1$ if a downstream multicast request D has all its ONUs served by an equipment selected in configuration c and 0 otherwise where $c \in \mathcal{C}_p$, $p \in P_{EQ}$ and $D \in \mathcal{D} \subseteq V^{ONU}$.

Objective

The reduced cost, i.e., the pricing problem objective can be written as follows:

$$\overline{COST_c} = z^I - \sum_{ONU_i \in V^{ONU}} y_i \; u_i^1 - u^2 - \sum_{p \in P_{EQ}} u_p^3 - \sum_{D \in \mathcal{D} \subseteq V^{ONU}} v^D \quad (168)$$

where u_i^1, u^2 and u_p^3 are the dual values associated with constraints (165) , (166) and (167) respectively.

Constraints

***p*-center related constraint** Maximum distance between the OLT and the farthest ONU assigned to it within a cluster needs to be minimized.

$$z^I \geq d_{p,p_{OLT}} + y_i \, d_{i,p} \qquad ONU_i \in V^{ONU} \quad (169)$$

For each cluster, the average distance from an equipment to all ONUs should be within a threshold (e.g.max_avg_val) value and the distance between an ONU and its corresponding equipment should not exceed a threshold (e.g.max_rad_val) value. The following two constraints work as an approximation for the attenuation of transmitted optical power signal.

$$\sum_{ONU_i \in V^{ONU}} y_i \, d_{i,p} \leq \sum_{ONU_i \in V^{ONU}} y_i \times \max\_\mathrm{avg\_val} \quad (170)$$

$$d_{p,p_{OLT}} + y_i \, d_{i,p} \leq \max\_\mathrm{rad\_val} \qquad ONU_i \in V^{ONU} \quad (171)$$

The following constraint ensures that the pricing problem generates efficient configurations such that each configuration consists of at least a minimum number of ONUs (i.e.,min_num_onu) in a cluster.

$$\sum_{ONU_i \in V^{ONU}} y_i \geq \min\_\mathrm{num\_onu} \quad (172)$$

Constraints related to multicast traffic and cluster formation The following constraint ensures that $v^D = 0$ only when at least one of the ONU$_i$ belonging to the multicast destination set D is not linked to the configuration with respect to a passive equipment. This constraint also relates the decision variables v^D and y_i.

$$v^D \leq y_i \qquad ONU_i \in V^{ONU}, D \in \mathcal{D} \subseteq V^{ONU} \tag{173}$$

7.5 Phase II (2nd-Phase-N_{II}-ILP): ILP Model for Clustering of 2nd level Equipment and Determining the Location of 1st Level Passive Equipment

During Phase II, in accordance with the standard N_{II}-center model, 2nd level and 1st level passive equipment are considered as demand nodes and service facilities respectively. All 2nd level equipment will be grouped into N_{II} clusters, each to be served by a 1st level equipment. In Phase II, the location and number of required output ports of the 1st level equipment are determined. Note that the value of N_{II} indicates the total number of 1st level equipment, i.e., the total number of PON hierarchies. By varying the value of N_{II}, we can obtain different sets of PON hierarchies. Our proposed model extends the standard N_{II}-center based ILP formulation as it minimizes the maximum distance between the OLT and one of the ONUs.

7.5.1 Notations

The parameters of the N_{II}-center model are the same as those of the N_I-center model of Phase I except that the set of potential locations for the passive equipment, P_{EQ} is divided into two sets such that $P_{EQ} = P_{EQ}^{L_1} \cup P_{EQ}^{L_2}$ where $P_{EQ}^{L_1}$ and $P_{EQ}^{L_2}$ represent the candidate location for the 1st level and the selected location for the 2nd level passive equipment respectively. Note that $P_{EQ}^{L_1}$ and $P_{EQ}^{L_2}$ does not necessarily define a partition of P_{EQ}.

7.5.2 Optimization Model

There are three sets of variables defined as follows.

- z^{II} maximum distance between an ONU and the OLT..
- x_p^{II} decision variable such that $x_p^{II} = 1$ if a 1st level equipment is located in p, 0 otherwise, for $p \in P_{EQ}^{L_1}$. Note that if a first level equipment is located in p, it is directly connected to the OLT.
- $y_{p,p'}$ decision variable such that $y_{pp'} = 1$ if a 2nd level equipment located in p is served by a 1st level equipment located in p', 0 otherwise, for $p \in P_{EQ}^{L_2}$ and $p' \in P_{EQ}^{L_1}$.

The objective, i.e., minimization of the deployment cost (which is proportional to the length of the deployed fibers), is written as follows:

$$\min \qquad z^{II}. \tag{174}$$

In order to minimize the largest distance between an ONU and the OLT, we need the following set of constraints:

$$z^{II} \geq d_{p',p_{OLT}} x_{p'}^{II} + d_{p,p'}\, y_{p,p'} + d_{ip}\tilde{y}_{ip} \qquad p' \in P_{EQ}^{L_1}, p \in P_{EQ}^{L_2}, ONU_i \in V^{ONU}, \tag{175}$$

where $\tilde{y}_{ip}$ is the output value of y_{ip} following the solution of the model of Phase I.

The 2nd level equipment will be grouped into N_{II} clusters, i.e., N_{II} 1st level equipment will be placed.

$$\sum_{p \in P_{EQ}^{L_1}} x_p^{II} = N_{II}. \tag{176}$$

Each 2nd level equipment will be assigned to only one 1st level equipment.

$$\sum_{p' \in P_{EQ}^{L_1}} y_{p,p'} = \tilde{x}_p^I \qquad p \in P_{EQ}^{L_2}, \tag{177}$$

where $\tilde{x}_p^I$ is the output value of x_p^I following the solution of the model of Phase I.

Each 2nd level equipment can be linked with a 1st level equipment, with respect to the locations where those equipment have been installed.

$$y_{p,p'} \leq x_{p'}^{II} \qquad p \in P_{EQ}^{L_2}, p' \in P_{EQ}^{L_1} \tag{178}$$

$$\sum_{p \in P_{EQ}^{L_2}} y_{p,p'} \geq x_{p'}^{II} \qquad p' \in P_{EQ}^{L_1}. \tag{179}$$

7.6 Phase II (2nd-Phase-N_{II}-CG): CG Model for Clustering of 2nd level Equipment and Determining the Location of 1st Level Passive Equipment

We propose an alternative column generation (CG) model for Phase II which is more scalable compared to the ILP model.

7.6.1 Master Model

Configuration

A configuration c corresponds to a 1st level switching equipment, a number of 2nd level switching equipment in a cluster served by the 1st level equipment and the distance between the 1st level switching equipment and the farthest 2nd level equipment assigned to it. We denote the overall set of configurations by $\mathcal{C}$ such that $\mathcal{C} = \bigcup_{P_{EQ}^{L_1}} \mathcal{C}_p$ where $\mathcal{C}_p$ represents a configuration related to a potential switching equipment located at p in which $p \in P_{EQ}^{L_1}$. Let $COST_c$ be the cost of configuration c. A configuration $c \in \mathcal{C}$ is characterized by the following parameters:

- $y_{p,p'}^c \in [0,1]$ represents 2nd level-1st level equipment association such that $y_{p,p'}^c = 1$ if a 2nd level equipment p is served by a 1st level equipment located at site p' in configuration c and 0 otherwise where $p \in P_{EQ}^{L_2}$, $p' \in P_{EQ}^{L_1}$.

- $COST_c$ indicates the distance between the 1st level switching equipment selected in configuration c and the farthest 2nd level equipment assigned to it.

Variables

- $z_c \in \{0,1\}$ is a decision variable such that $z_c = 1$ if configuration c is selected, and 0 otherwise.

Objective

The objective corresponds to the minimization of the maximum distance between a 1st lvel equipment and the farthest 2nd level equipment assigned to it, expressed as follows:

$$min \sum_{c \in \mathcal{C}} COST_c \, z_c \tag{180}$$

Constraints

***p*-center related constraints** Each 2nd level equipment must be served by one 1st level equipment in all configurations.

$$\sum_{p' \in P_{EQ}^{L_1}} \sum_{c \in \mathcal{C}_{p'}} y_{p,p'}^{c} \, z_c = 1 \qquad p \in P_{EQ}^{L_2} \tag{181}$$

All 2nd level equipment will be grouped into N_{II} number of clusters, i.e., N_{II} number of 1st level equipment will be placed.

$$\sum_{c \in \mathcal{C}} z_c \leq N_{II} \tag{182}$$

As each configuration is associated with a single 1st level equipment location, at most one configuration corresponding to each 1st level equipment will be selected.

$$\sum_{c \in \mathcal{C}_{p'}} z_c \leq 1 \qquad p' \in P_{EQ}^{L_1} \tag{183}$$

7.6.2 Pricing Model

Variables

- y_p is a decision variable such that $y_p = 1$ if 2nd level equipment p is served by a 1st level equipment equipment selected in configuration c and 0 otherwise where $p \in P_{EQ}^{L_2}$, $c \in \mathcal{C}_{p'}$, $p' \in P_{EQ}^{L_1}$.
- z^{II} cost of each configuration c where $c \in \mathcal{C}_{p'}$, $p' \in P_{EQ}^{L_1}$.

Objective

The reduced cost, i.e., the pricing problem objective can be written as follows:

$$\overline{COST_c} \quad = \quad z^{II} - \sum_{p \in P_{EQ}^{L_2}} y_p \quad u_p^1 - u^2 - \sum_{p' \in P_{EQ}^{L_1}} u_{p'}^3 \tag{184}$$

where u_p^1, u^2 and $u_{p'}^3$ are the dual values associated with constraints (181) , (182) and (183) respectively.

Constraints

***p*-center related constraint** In order to minimize the largest distance between an ONU and the OLT, we need the following set of constraints:

$$z^{II} \geq d_{p',p_{OLT}} + y_p \, d_{p,p'} + d_{ip}\tilde{y}_{ip} \qquad p' \in P_{EQ}^{L_1}, \qquad p \in P_{EQ}^{L_2}, \qquad ONU_i \in V^{ONU} \tag{185}$$

where $\tilde{y}_{ip}$ is the output value of y_{ip} following the solution of the model of Phase I.

For each cluster, the average distance from an equipment to all ONUs should be within a threshold (e.g.max_avg_val) value and the distance between an ONU and its corresponding equipment should not exceed a threshold (e.g.max_rad_val) value. The following two constraints work as an approximation for the attenuation of transmitted optical power signal.

$$\sum_{p \in P_{EQ}^{L_2}} y_p \, d_{p,p'} \leq \sum_{p \in P_{EQ}^{L_2}} y_p \times \max\_\text{avg\_val} \tag{186}$$

$$d_{p',p_{OLT}} + y_p \, d_{p,p'} \leq \max\_\text{rad\_val} \qquad p \in P_{EQ}^{L_2}, p' \in P_{EQ}^{L_1} \tag{187}$$

The following constraint ensures that the pricing problem generates efficient configurations such that each configuration consists of at least a minimum number of 2nd level equipment (i.e.,min _num_eqip) in a cluster.

$$\sum_{p \in P_{EQ}^{L_2}} y_p \geq \min\_\text{num\_eqip} \tag{188}$$

7.7 Phase III:TYPE-CG Model for selecting the type of Switching Equipment

In this phase, we have exploited simplified version of the model proposed in Chapter 6 of Section 6.5 in which the type and location of the switching equipment is determined using CG based solution scheme. We have exploited this model to select the type of the switching equipment as well as to provision the traffic flows for each PON hierarchy. While selecting the type of the switching equipment, this CG solution scheme takes into account the type (unicast/multicast) and amount of traffic demand together with signal power loss (attenuation) experienced at each ONU which is caused by the passive switching equipment and the optical fiber cable.

7.7.1 Notations

Equipment Hierarchy Parameters

For a given hierarchy, G is the set of ONU groups as well as 2nd level equipment in a given equipment hierarchy, i.e., g_0 the cluster of ONUs and the 2nd level equipment associated with the single first level equipment and g any of the second level clusters, which is connecting a given subset of ONUs with the same switching equipment. We will denote by $|g|$ the splitting ratio of the switching equipment of cluster g. Note that first and second stages are merged for 1-stage cascading PON hierarchy. Let $G^\star$ be the set $G \setminus \{g_0\}$.For $g \in G$, cluster g is associated with the set of ONUs connected to the second level switching equipment except for the first level cluster, g_0 which is associated with the first level switching equipment. In order to identify the membership of an ONU to a particular cluster, we use the parameter $\delta_{ONU,g}$: It is equal to 1 if ONU belongs to cluster g in equipment hierarchy, and 0 otherwise.

A provisioned hierarchy is described by its switching equipment at each level by the following parameters:

- $a_{g_0,k} = 1$ if there is an equipment with $k \in K = \{2, 4, 8, 16, 32, 64\}$ output ports at the first level, below the OLT, leading cluster g_0, and 0 otherwise.

- $a_{g,k} = 1$ if there is an equipment with $k \in K = \{2, 4, 8, 16, 32, 64\}$ output ports at the second level for cluster $g \in G^\star$, and 0 otherwise.

- $aa_{g_0,p} = 1$ if there is an equipment at location $p \in P_{EQ}$ at the first level, below the OLT, leading cluster g_0, and 0 otherwise.

- $aa_{g,p} = 1$ if there is an equipment at location $p \in P_{EQ}$ at the second level for cluster $g \in G^\star$, and 0 otherwise.

Location Parameters

Location parameters, used in the optimization model, are described in Section 4.4.1.

Cost Parameters

Cost parameters, used in the optimization model, are described in Section 4.4.1.

Traffic Parameters

Traffic parameters, used in the optimization model, are described in Section 4.4.1.

7.7.2 Linear Optimization Model

Location Configurations

Location configurations, used in the optimization model, are described in Section 4.4.2.

Variables

Variables, used in the optimization model, are described in Section 6.5.2.

Objective

Objective function of the optimization model is described in Section 6.5.2.

Constraints

Constraints of the optimization model is same as the constraints described in Section 6.5.2. In addition, we just need to add one constraint specifying that the equipment of each group with a already selected location must be deployed in the corresponding location.

$$\sum_{k \in K} (y\_s_{p,g,k} + y\_a_{p,g,k}) = 1 \qquad p \in P_{EQ}, g \in G : aa_{g,p} = 1. \tag{189}$$

7.8 CG-BASED Solution of the Model

Constraints of the pricing model is same as the constraints described in Section 6.6. We just need to add one additional constraint which specifies that for each cluster g associated with a selected location $p \in P_{EQ}$ at most one splitter/AWG with $k = |g|$ output ports can be placed in the corresponding location. In other words, for each $g \in G, p \in P_{EQ} : aa_{g,p} = 1$, we have:

$$\sum_{k \in K} (a\_s_{p,g,k} + a\_a_{p,g,k}) = 1 \tag{190}$$

7.9 Phase IV:Optimization Model for Selecting the Hierarchies

Once the best equipment type has been found by the TYPE-CG algorithm for each PON hierarchy, we need to decide on the selection of the best PON networks for covering the traffic demands of a given geographical area. For this purpose, we utilize our proposed M-PON-ILP optimization model which is described in 6.7

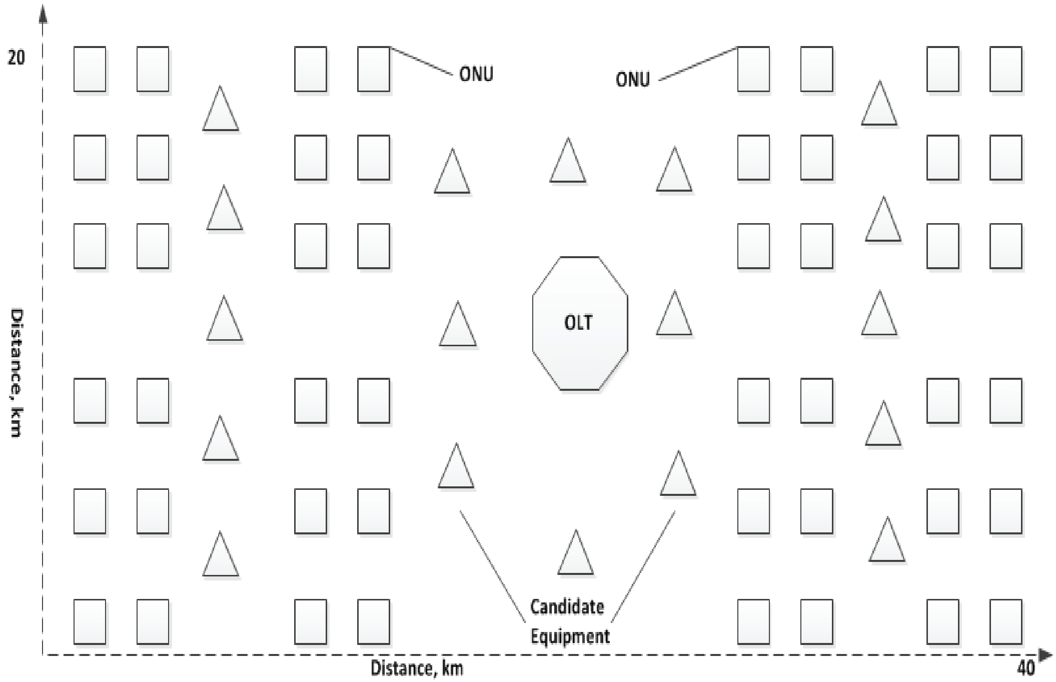


Figure 24: BSP Simulation Scenario for 128 ONUs

7.10 Computational Results and Analysis

We implement our 4-phase BSP scheme described in the previous sections using the Optimization Programming Language (OPL) platform. Linear and integer linear programs are solved using the CPLEX package.

We conduct our experiments with randomly generated Manhattan-pattern geographic locations of 128 ONUs and 30 candidate/potential locations for the placement of the passive equipment. Location of the ONUs and candidate equipment are randomly generated in a 40×20 km^2 rectangular grid such that the OLT is located at the center of the grid, i.e., at location (20,10) as shown in Figure 24. Table 2 contains the values taken for the cost of the equipment (taken from [CWMJ10]), as well as the attenuation parameters, which depend on the number of output ports for the splitters. For the costs related to optical fiber cables, we use the value of 7,160$/km [CWMJ10], assuming it includes the cost of trenching and laying the optical fiber cables.

We randomly generate the upstream unicast traffic flows within the range [0.05, 0.1]. Towards downstream direction, we randomly generate both unicast and multicast traffic flows within the range [0.1, 0.4].

During the simulation of our BSP scheme, at first, we solve the N_I-center ILP model of Phase I and obtain the grouping of ONUs to be served by a common 2nd level equipment. We experiment with two values of N_I (i.e., $N_I = 8$ and $N_I = 12$) so that the same set of ONUs can be grouped into different set of clusters in order to determine an efficient ONU clustering.

We next solve the N_{II}-center ILP model of Phase II to generate different PON hierarchies. We first consider 8 ONU clusterings (Phase I), which implies that there are eight 2nd level equipment. We run the N_{II}-center model by varying the values of N_{II} (i.e., $N_{II} = 2$ and 4). To illustrate the scenario, $N_{II} = 2$ indicates that there are two 1st level equipment resulting in two different PON hierarchies to serve all the ONUs through eight 2nd level equipment. On the other hand, $N_{II} = 4$ specifies that eight 2nd level equipment are grouped into four clusters, $\rightsquigarrow$ four 1st level equipment as well as four different PON hierarchies. Next, we consider 12 ONU clusterings obtained from the first phase which indicates that there are twelve 2nd level equipment. Then, we follow the same procedures as we do with 8 ONU clustering. Thus, with $N_{II} = 2$ and 4, we generate another 2 and 4 PON hierarchies respectively.

The next phase of the BSP scheme is Phase III in order to: (i) select the type (splitter or AWG) of the passive equipment, (ii) provision the traffic flows, for each PON hierarchy.

Table 24, obtained after solving Phase III, shows a comparison of the PON greenfield deployment cost for different hierarchies where each hierarchy delineates a single PON network.

The M-PON-ILP model of Phase IV considers 12 PON equipment hierarchies as its input and finally selects 4 minimum cost hierarchies which corresponds to 4 pairwise disjoint PON networks. Table 25 shows that hierarchies H9, H10, H11, H12 are the most cost-effective combination of PON networks. In all these selected PON

Table 24: Computational Results of P-center ILP on Different Hierarchies

Hierarchy	Equipment Type	p-val Phase I	p-val Phase II	Deployment cost (%)	# served ONUs
H1	Mixed	8	2	2,452,060	63
H2	Mixed	8	2	1,928,280	65
H3	Mixed	8	4	1,582,400	39
H4	Mixed	8	4	984,920	24
H5	Mixed	8	4	1,668,720	53
H6	Splitter	8	4	353,140	12
H7	Mixed	12	2	2,309,460	63
H8	Mixed	12	2	1,937,340	65
H9	Mixed	12	4	943,060	27
H10	Mixed	12	4	1,280,780	36
H11	Mixed	12	4	966,240	37
H12	Mixed	12	4	821,340	28

networks, first, the ONUs are partitioned into 12 clusters, then these 12 clusters constitute four separate hybrid PON networks and finally the splitters/AWGs are selected as the 1st/2nd level passive equipment. The construction of the best set of PON networks is elucidated in Figure 25 where we observe that the hierarchies H9, H10, H11, H12 are composed of 2, 3, 4 and 3 clusters of ONUs respectively such that each cluster is served by a 2nd level passive equipment (i.e., a splitter/an AWG) and all the 2nd level equipment of each hierarchy are connected to a single 1st level equipment (i.e., another splitter/AWG) to satisfy the traffic demand requests of 128 ONUs.

Table 25: Computational Results of P-center ILP on Different Hierarchies

List of Generated Hierarchies	List of Selected Hierarchies	Total Cost ($)
H1,H2, ... ,H12	H9, H10, H11,H12	4,011,420

Next, we implement our BSP scheme using N_I-center CG model of Phase I and N_{II}-center CG model of Phase II. The N_I and N_{II} values are chosen as before. For sub-sequent phases (i.e. Phase III and Phase IV), we follow the similar steps as specified before. Table 26, obtained after solving Phase III, shows a comparison of the PON greenfield deployment cost for different hierarchies.

Table 26: Computational Results of P-center CG on Different Hierarchies

Hierarchy	Equipment Type	p-val Phase I	p-val Phase II	Deployment cost (%)	# served ONUs
H1	Mixed	8	2	2,452,940	63
H2	Mixed	8	2	1,929,020	65
H3	Mixed	8	4	1,582,540	39
H4	Mixed	8	4	984,920	24
H5	Mixed	8	4	1,668,840	53
H6	Splitter	8	4	353,140	12
H7	Mixed	12	2	2,309,520	63
H8	Mixed	12	2	1,937,660	65
H9	Mixed	12	4	943,420	27
H10	Mixed	12	4	1,280,860	36
H11	Mixed	12	4	966,380	37
H12	Mixed	12	4	821,340	28

The computational results of Phase IV is summarized in Table 27 which shows that hierarchies H9, H10, H11, H12 are the most cost-effective combination of PON networks. We observe that the PON deployment cost for 128 ONUs are almost same for both p-center based ILP and CG models of our BSP scheme.

Table 27: Computational Results of P-center CG on PON Hierarchies

List of Generated Hierarchies	List of Selected Hierarchies	Total Cost ($)
H1,H2, ... ,H12	H9, H10, H11,H12	4,012,000

Next, we conduct our experiments with randomly generated Manhattan-pattern geographic locations of 512 ONUs (1st set) and 60 candidate/potential locations for the placement of the passive equipment. ONUs are uniformly distributed in a 40×40 km^2 rectangular grid such that the OLT is located at the center of the grid, i.e., at location (20,20) as shown in Figure 20.

During the simulation of the Phase I of our BSP scheme, we notice that our N_I-center ILP model is not scalable enough to run the experiment consisting of 512 ONUs. So we run our N_I-center CG model and obtain the grouping of ONUs to be served by a common 2nd level equipment. We experiment with two values of p (i.e.,$p = 16$ and $p = 32$) so that the same set of ONUs can be grouped into different set of clusters in order to determine an efficient ONU clustering.

In the second step, we simulate our N_{II}-center CG model to generate different PON hierarchies. We first consider 16 ONU clusterings (first phase), which implies that there are sixteen 2nd level equipment. We run this CG model by varying the values of p (i.e.,$p = 4, 6, 8$). To illustrate the scenario, $p = 4$ indicates that there are four 1st level equipment resulting in four different PON hierarchies to serve all the ONUs through sixteen 2nd level equipment. On the other hand, $p = 8$ specifies that sixteen 2nd level equipment are grouped into eight clusters, $\leadsto$ eight 1st level equipment as well as eight different PON hierarchies. Next, we consider 32 ONU clustering which indicates that there are thirty two 2nd level equipment. Then, we follow the same procedures as we do with 16 ONU clustering. Thus, with $p = 4$, 6, 8, we generate 4, 6 and 8 PON hierarchies respectively.

The next phase of the BSP scheme is to solve the CG based optimization TYPE-CG model in order to: (i) select the type (splitter or AWG) of the passive equipment, (ii) provision the traffic flows, for each PON hierarchy.

Table 28, obtained by running the TYPE-CG algorithm, shows a comparison of the PON greenfield deployment cost for different hierarchies where each hierarchy delineates a single PON network.

We observe that a mix of splitters and AWGs are selected for most of the PON hierarchies whereas some hierarchies consist of only AWGS. The selection of the switching equipment are made based on the best choice taking into account the cost, the traffic flows (some unicast, some multicast) and the attenuation constraints. While selecting the type of the equipment, the optimization model first takes into account the traffic flows (unicast/multicast) and tries to select splitters in the case of multicast traffic requests to minimize the number of used wavelengths as long as the constraint for power signal attenuation is satisfied. More over, a splitter is less expensive compared to an AWG. But there does not always exist a feasible solution with the selection of splitters due to the signal attenuation constraint. Indeed, in a splitter, the attenuation increases significantly with the increase of the number of output ports. However, the attenuation caused by an AWG is low and independent of the number of the output ports. As a result, only splitters are not always selected as a 1st/2nd level equipment for a PON hierarchy, although the deployment cost of a PON hierarchy consisting of only splitters would be the most economical one. During the last phase of the BSP scheme, we select the most cost effective set of PON hierarchies from the Table 29 which is obtained through the simulation of M-PON-ILP model.

M-PON-ILP model considers 36 PON equipment hierarchies as its input and finally selects 4 minimum cost hierarchies which corresponds to 4 pairwise disjoint PON networks. Table 29 shows that hierarchies H13, H14, H15, H16 are the most cost-effective combination of PON networks. In all these selected PON networks, first, the ONUs are partitioned into 36 clusters, then these 36 clusters constitute four separate hybrid PON networks and finally the splitters/AWGs are selected as the 1st/2nd level switching equipment.

The construction of the best set of PON networks are elucidated in Figure 26 where we observe that the hierarchies H13, H14, H15, H16 are composed of 9, 8, 8 and 7 clusters of ONUs respectively such that each cluster is served by a 2nd level switching equipment (i.e., a splitter/ an AWG) and all the 2nd level equipment of each hierarchy are connected to a single 1st level equipment (i.e., another splitter/AWG) to satisfy the traffic demand requests of 512 ONUs.

Now we analyze the transmission pattern of multicast traffic requests. In our experiment with the 1st set of 512 ONUs, we consider 25 multicast traffic requests in which each request consists of 3 randomly generated destination ONUs. We collect the statistics of these multicast requests and observe that only 15 requests are originally transmitted as multicast demand. It implies that 3 destination ONUs are served at a time. It is also noticed that 6 multicast requests are satisfied by transmitting 12 traffic streams in which each stream consists of either 2 or single destination. The remaining 5 multicast requests are served by 15 unicast traffic streams. We perceive that if the ONUs in a multicast group are within 45°, there is a high probability of being these ONUs inside the same cluster which will eventually facilitate the effective transmission of multicast traffic. But if the ONUs in a multicast group are located in significantly distant locations, the corresponding multicast traffic request will be served as 3 unicast traffic streams.

Next, we scrutinize the selection of the type of equipment by our proposed scheme. We know that an AWG can be selected due to two possible reasons: (i) to satisfy the attenuation constraint, (ii) to serve high bandwidth unicast traffic demand. On the contrary, a splitter can be selected to serve multicast traffic demand to a set of destination ONUS. In Figure 26(c), we detect that AWGs are selected as the 1st level equipment for all four (i.e., H13, H14, H15, H16) hierarchies. These 1st level AWGs are selected to reduce the signal power loss so that the attenuation constraint is satisfied. Basically, the AWGs of the 2nd level are picked by the optimization scheme in accordance with the high bandwidth traffic demand of individual ONUs.

Now we compare the results of our BSP scheme with those of the **SLAPONS** scheme proposed in Chapter 6. For both schemes, we experiment with the same set of 512 ONUs (1st set) and 60 candidate locations for the placement of passive switching equipment as well as the same set of unicast/multicast traffic demand. In **SLAPONS** scheme, 36 PON hierarchies consisting of one-stage, two-stage and mixed-stage architectures are considered as potential equipment hierarchies. Finally, this scheme selects four two-stage PON hierarchies with the total deployment cost of 19,501,660$ which is shown in Table 21. In all these selected PON networks, first the ONUs are partitioned into 32 clusters, then these 32 clusters constitute four separate hybrid PON networks of two-stage cascading architecture.

In our BSP scheme, four minimal cost two-stage PON hierarchies are selected with the total deployment cost of 13,739,040$ which is shown in Table 29. We notice that the most cost-effective PON hierarchies for this scheme are obtained when we consider the p values of Phase I and Phase II as 32 and 4 respectively (i.e. 512 ONUs are first grouped into 32 clusters which finally results in 4 disjoint PON hierarchies).

We observe that our proposed BSP scheme incurs less overall deployment cost compared to the cost obtained by applying **SLAPONS**scheme. The reasons why the BSP scheme outperforms **SLAPONS** scheme can be explained as follows. Both of the schemes execute in four phases. But first two phases (i.e., clustering of ONUs and generating equipment hierarchies) of the **SLAPONS** scheme are dependent on heuristic algorithmic solutions which can not guarantee optimal solution at all. On the contrary, all four phases of the BSP scheme are governed by the integer linear programming(ILP) formulation which are guaranteed to provide globally optimal solution. Thus we can claim that the BSP scheme results in a set of optimal PON equipment hierarchies. The total number of required PON networks as well as the covered region of each PON network is also optimal with respect to geographical location and unicast/multicast traffic demand.

In Table 30, we focus on the impact of p values on the solution of our proposed p-center based optimization scheme. We perceive that the overall deployment cost is lessened when the value of p of the Phase I CG model is increased from 16 to 32. The reason behind it is that when we select a small value of p during the 1st phase of our scheme, each cluster consists of large number of ONUs which requires a switching equipment (2nd level) of higher number of output ports for that cluster. As the attenuation of a splitter with high splitting ratio is much higher compared to an AWG, the optimization scheme selects AWGs for most of clusters resulting in increased deployment cost. On the contrary, when we select higher value of p during the 1st phase of our scheme, less number of ONUs are aggregated in each cluster, thereby a switching equipment of the corresponding cluster requires small number of output ports to connect the ONUs which results in minimized deployment cost due to the selection of the splitters.

In Figure 27, the graphical representation of Table 30, we observe that for each p value of the 1st-Phase-N_I-CG model (Phase I), increasing the value of p for the 2nd-Phase-N_{II}-CG model (Phase II) increases the overall deployment cost of all hierarchies which are constituted based on the aforesaid p values. This happens due to the increase of the number of switching equipment of the 1st level.

We also investigate the amount of signal power loss (attenuation) experienced at each ONU. The total signal attenuation from the OLT to an ONU located at p, denoted by P_p, must not exceed 20 dB which is expressed by:

$$P_p = P_p^{FIBER} + P_p^{THROUGH} + P^{INSERTION} + P^{MARGIN} \quad (191)$$

where P_p^{FIBER} is the signal loss caused on the fiber to reach the ONU located at p which is considered as 0.2dB/km, $P^{THROUGH}$ is the loss provoked by going through the equipment towards the ONU located at p which is shown in Table 2, $P^{INSERTION} = 0.1$ dB is the insertion loss caused by all the nodes on the link, $P^{MARGIN} = 1$ dB is the power margin.

Table 31 presents the amount of maximum signal attenuation (Max. Loss) and average attenuation (Avg. Loss) experienced at different hierarchies by the ONUs of the corresponding hierarchies. We observe that the maximum and the average signal attenuation vary significantly with the p values of the Phase I and Phase II of our proposed BSP scheme. Impact of p values on signal attenuation is summarized in Table 32 which shows that increasing the p value of Phase I degrades the received signal power of the ONUs transmitted from the OLT. The reason behind it is that the more the value of p is , the more the number of output ports of the 1st level equipment will be required. This results in more signal power loss in case of a splitter. The observation about the signal power attenuation also helps us to decide on the selection of p values of Phase I and II. We notice that the maximum attenuation is almost 20dB when $p = 32$ for Phase I. It suggests that if we increase the value of p beyond 32, it will exceed the maximum acceptable signal power loss (i.e., 20dB).

Next, we extend our experiment with different distribution of the location of the ONUs (2nd set) in which the initial 40×40 km^2 rectangular grid is divided into 4 sub-squares. Each sub-square has different density of distribution of ONUs where sub-squares 1, 2, 3 and 4 have 227, 117, 70 and 98 ONUs respectively. But ONUs are uniformly distributed within each sub-square. We consider the same parameters and follow the same procedures as before while simulating our BSP scheme with the new data-set (i.e., 2nd set of ONU locations).

Table 33, obtained by running the TYPE-CG algorithm for different distribution of ONUs, shows a comparison of the PON greenfield deployment cost for different hierarchies. It can be noticed that a mix of splitters and AWGs are selected for most of the PON hierarchies whereas some hierarchies consist of only splitters.

During the last phase of the BSP scheme, we select 4 minimum cost disjoint PON networks (i.e., H13, H14, H15, H16) from the Table 34 where 36 PON equipment hierarchies are taken into account. In all these selected PON networks, first the ONUs are partitioned into 32 clusters, then these 32 clusters constitute four separate hybrid PON networks and finally the splitters/AWGs are selected as the 1st/2nd level switching equipment.

Again, we compare the results of our BSP scheme with those of the **SLAPONS** scheme proposed in Chapter 6. For both schemes, we experiment with the same set of 512 ONUs (2nd set) and 60 candidate locations for the placement of passive switching equipment as well as the same set of unicast/multicast traffic demand. Finally, this scheme selects four two-stage PON hierarchies with the total deployment cost of 21,834,680$ which is shown in Table 23. In all these selected PON networks, first the ONUs are partitioned into 32 clusters, then these 32 clusters constitute four separate hybrid PON networks of two-stage cascading architecture.

In our BSP scheme, four minimal cost two-stage PON hierarchies are selected with the total deployment cost of 13,852,340$ which is shown in Table 34. We notice that the most cost-effective PON hierarchies for this scheme are obtained when we consider the p values of Phase I and Phase II as 32 and 4 respectively (i.e. 512 ONUs are first grouped into 32 clusters which finally results in 4 disjoint PON hierarchies). We observe that the BSP scheme also outperforms the **SLAPONS** scheme with the 2nd set of data.

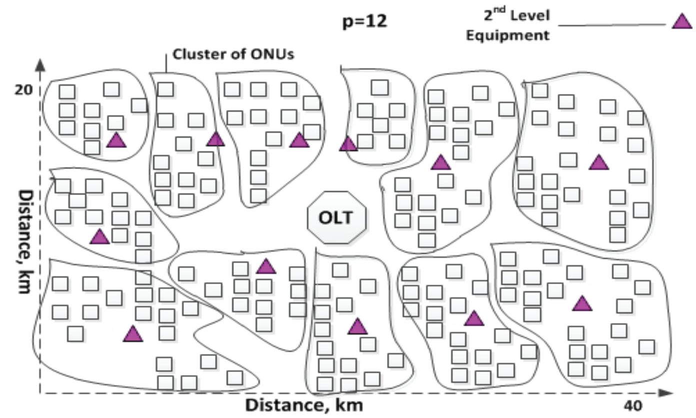
At last, we intend to investigate the impact of increased number of multicast traffic requests on the deployment cost. We experiment with three different set of multicast traffic requests where the number of traffic requests is added incrementally. We start with our 1st set of 512 ONUs in which we consider 25 multicast requests. Later on, we experiment with 50 and 75 multicast requests. During simulation, we consider $p = 32$ and $p = 4$ for our Phase I and Phase II CG models respectively. The selection of p values is done based on the optimal values obtained from the 1st set of locations of the previously experimented ONUs. During each experiment with incrementally added number of multicast requests, all ONUs are grouped into 32 clusters during the 1st phase of the BSP scheme which eventually formulate 4 separate PON hierarchies. Impact of multicast traffic requests on the solution of the BSP scheme is shown in Table 35 as well as in Figure 28. We observe that deployment cost increases with the increase of number of multicast traffic requests.

7.11 Summary

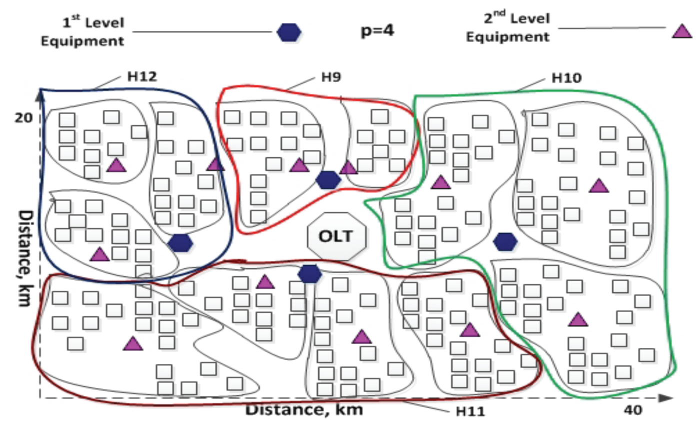
In this chapter, we present our second proposed scheme to solve the location allocation (L/A) problem of multiple hybrid PONs. Here, we propose a novel network design optimization scheme for greenfield deployment of a set of hybrid PONs.

The proposed BSP scheme proceeds in four phases: In the first phase, a p-center model is proposed to determine the best ONU clusterings and the placement of the switching equipment (i.e., 2nd level equipment). In the second phase, another p-center model is formulated to determine the clustering of 2nd level equipment and the location of the 1st level switching equipment. Exploiting the output results of the first two phases, the coverage of each PON is determined and several potential PON hierarchies are generated. The third phase consists in selecting the best type of passive equipment for each potential PON hierarchy. In the fourth phase, the best set of PON equipment hierarchies is selected in order to ensure a proper coverage of the initial set of ONUs.

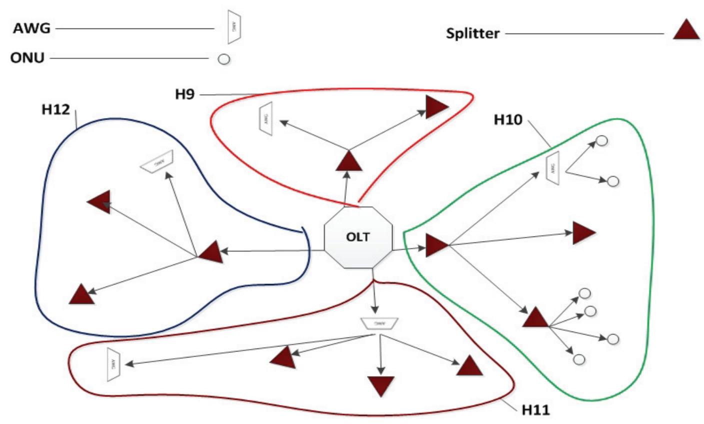
Computational experiments have been conducted on a set of 128 ONUs as well as different sets of 512 ONUs in order to evaluate the performance of the BSP scheme which outperforms our first scheme, namely **SLAPONS** proposed in Chapter 6 with the intention of solving the same problem.



(a) Phase I



(b) Phase II



(c) Phase III

Figure 25: Construction of Best Set (i.e, H9,H10,H11,H12) of PON Networks

Table 28: Computational Results on Different Hierarchies (1st Set)

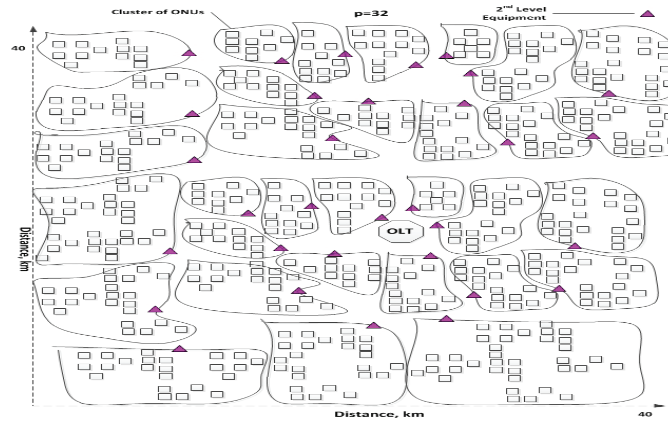
Hierarchy	Equipment Type	p-val Phase I	p-val Phase II	Deployment cost	# served ONUs
H1	AWG	16	4	6,146,060	160
H2	Mixed	16	4	4,789,420	128
H3	Mixed	16	4	4,832,380	128
H4	Mixed	16	4	3,411,500	96
H5	Mixed	16	8	3,726,540	96
H6	AWG	16	8	3,819,820	96
H7	Mixed	16	8	1,164,020	32
H8	Mixed	16	8	2,427,280	64
H9	Mixed	16	8	2,420,120	64
H10	Mixed	16	8	1,221,200	32
H11	Mixed	16	8	2,248,280	64
H12	Mixed	16	8	2,470,240	64
H13	Mixed	32	4	4,783,100	164
H14	Mixed	32	4	3,596,800	123
H15	Mixed	32	4	3,378,240	134
H16	Mixed	32	4	1,980,900	91
H17	Mixed	32	8	2,545,940	76
H18	Mixed	32	8	3,042,380	90
H19	AWG	32	8	1,814,620	56
H20	Mixed	32	8	857,640	40
H21	Mixed	32	8	1,584,000	62
H22	Mixed	32	8	836,160	39
H23	Mixed	32	8	1,197,460	50
H24	Mixed	32	8	2,341,500	89
H25	AWG	16	6	3,891,820	96
H26	Mixed	16	6	3,654,940	96
H27	Mixed	16	6	1,163,920	32
H28	Mixed	16	6	2,427,280	64
H29	Mixed	16	6	4,553,140	128
H30	Mixed	16	6	3,604,820	96
H31	Mixed	32	6	3,561,200	124
H32	AWG	32	6	2,360,780	73
H33	Mixed	32	6	2,718,480	94
H34	Mixed	32	6	1,066,780	51
H35	Mixed	32	6	2,758,580	107
H36	Mixed	32	6	1,464,380	63

Table 29: Computational Results on PON hierarchies (1st Set)

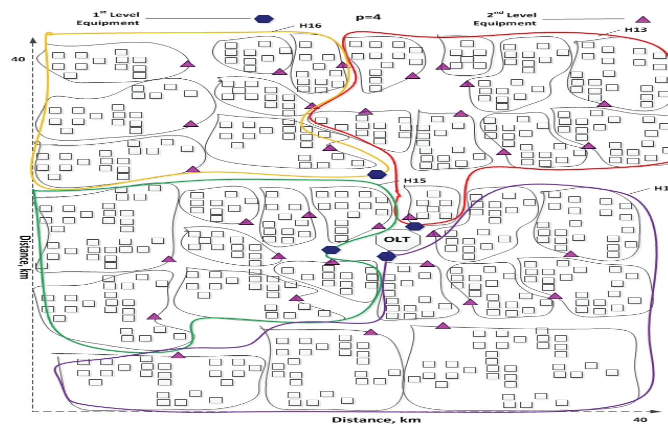
List of Generated Hierarchies	List of Selected Hierarchies	Total Cost ($)
H1,H2, ... ,H36	H13, H14, H15,H16	13,739,040

Table 30: Impact of p values on the Solution (1st Set)

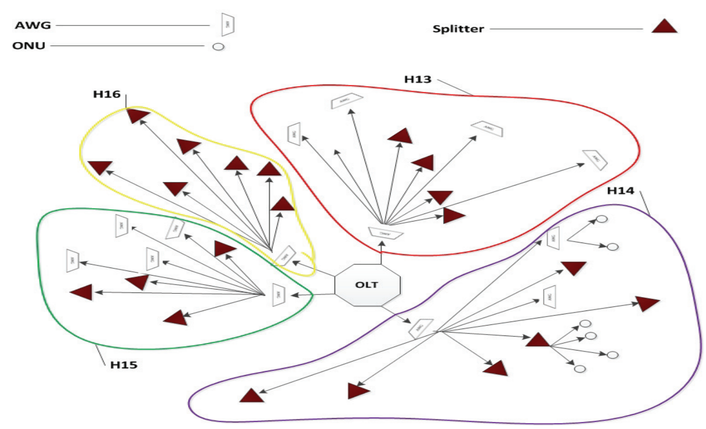
p-val (Phase I)	p-val (Phase II)	Total Deployment Cost	No. of Splitters	No. of AWGs
16	4	19,179,360	3	17
16	6	19,223,920	5	17
16	8	19,497,500	7	17
32	4	13,739,040	22	14
32	6	13,930,200	11	27
32	8	14,219,700	14	26



(a) Phase I

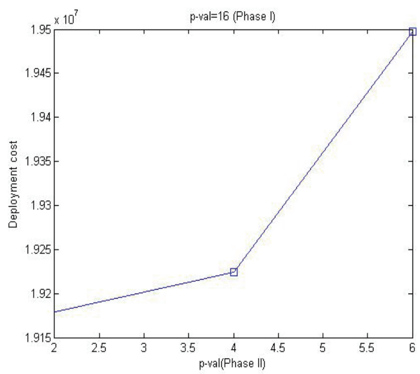


(b) Phase II

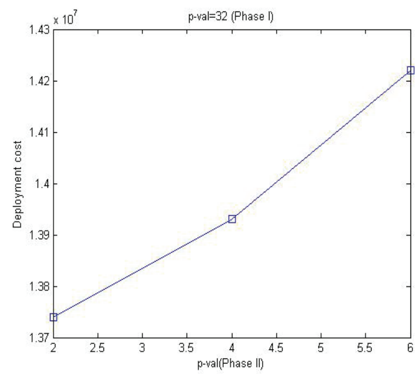


(c) Phase III

Figure 26: Construction of Best Set (i.e, H13,H14,H15,H16) of PON Networks



(a) p=16 for Phase I



(b) p=32 for Phase I

Figure 27: Impact of p values on the Solution

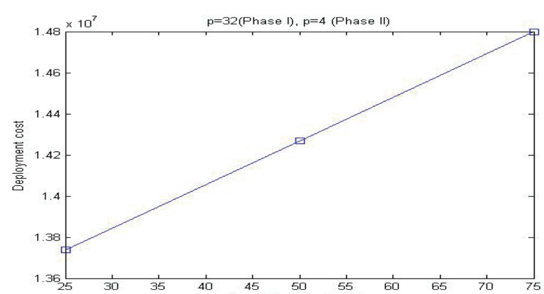


Figure 28: Impact of multicast traffic

Table 31: Observations on Signal Power Loss (1st Set)

Hierarchy	p-val Phase I	p-val Phase II	Max. Loss (dB)	Avg. Loss (dB)
H1	16	4	17.23	10.86
H2	16	4	11.24	10.39
H3	16	4	10.68	9.74
H4	16	4	12.47	11.46
H5	16	8	12.49	11.62
H6	16	8	17.23	10.88
H7	16	8	11.90	11.37
H8	16	8	10.19	9.61
H9	16	8	11.07	10.45
H10	16	8	9.69	8.82
H11	16	8	11.05	10.05
H12	16	8	11.66	11.03
H13	32	4	16.96	16.03
H14	32	4	19.70	15.82
H15	32	4	19.66	13.66
H16	32	4	19.46	19.16
H17	32	8	11.45	10.74
H18	32	8	19.16	12.30
H19	32	8	17.90	11.38
H20	32	8	11.79	11.49
H21	32	8	19.62	16.31
H22	32	8	12.03	11.72
H23	32	8	11.67	11.37
H24	32	8	11.39	10.96
H25	16	6	17.23	10.88
H26	16	6	11.07	10.19
H27	16	6	8.90	8.37
H28	16	6	10.19	9.61
H29	16	6	11.05	10.06
H30	16	6	11.05	10.10
H31	32	6	18.91	13.27
H32	32	6	17.90	11.39
H33	32	6	19.53	14.30
H34	32	6	12.03	11.72
H35	32	6	19.07	16.83
H36	32	6	11.67	11.37

Table 32: Impact of p values on Signal Power Loss (1st Set)

p-val (Phase I)	p-val (Phase II)	Max. Loss (dB)	Avg. Loss (dB)
16	4	17.23	10.57
16	6	17.23	10.08
16	8	17.23	10.62
32	4	19.70	15.91
32	6	19.53	13.54
32	8	19.62	12.02

Table 33: Computational Results on Different Hierarchies (2nd Set)

Hierarchy	Equipment Type	p-val Phase I	p-val Phase II	Deployment cost	# served ONUs
H1	Mixed	16	4	5,923,940	177
H2	Mixed	16	4	6,102,940	173
H3	Mixed	16	4	2,090,760	51
H4	Mixed	16	4	4,297,780	111
H5	Mixed	16	8	4,678,760	132
H6	Mixed	16	8	2,702,560	77
H7	Mixed	16	8	3,299,240	92
H8	Mixed	16	8	1,223,600	36
H9	Mixed	16	8	1,202,120	40
H10	Mixed	16	8	827,400	24
H11	Mixed	16	8	2,191,000	48
H12	Mixed	16	8	2,508,440	63
H13	Mixed	32	4	5,010,160	187
H14	Mixed	32	4	3,664,640	140
H15	Mixed	32	4	2,796,080	108
H16	Mixed	32	4	2,381,460	76
H17	Mixed	32	8	2,894,020	100
H18	Mixed	32	8	4,058,640	147
H19	Splitter	32	8	1,036,340	40
H20	Mixed	32	8	828,900	30
H21	Splitter	32	8	878,820	37
H22	Splitter	32	8	921,380	29
H23	Mixed	32	8	1,871,500	67
H24	Mixed	32	8	1,619,900	52
H25	Mixed	16	6	4,729,740	137
H26	Mixed	16	6	4,915,900	137
H27	Mixed	16	6	1,223,600	36
H28	Mixed	16	6	1,971,440	64
H29	Mixed	16	6	3,110,680	75
H30	Mixed	16	6	2,508,440	63
H31	Mixed	32	6	4,548,720	167
H32	Mixed	32	6	2,441,040	90
H33	Mixed	32	6	1,974,040	83
H34	Splitter	32	6	921,380	29
H35	Mixed	32	6	2,447,600	91
H36	Mixed	32	6	1,619,900	52

Table 34: Computational Results on PON hierarchies (2nd Set)

List of Generated Hierarchies	List of Selected Hierarchies	Total Cost ($)
H1,H2, ... ,H36	H13, H14, H15,H16	13,852,340

Table 35: Impact of multicast traffic requests on the Solution (1st Set)

#Multicast traffic	Cost (H1)	Cost (H2)	Cost (H3)	Cost (H4)	Total cost
25	4,783,100	3,596,800	3,378,240	1,980,900	13,739,040
50	5,033,700	3,890,160	3,341,840	2,003,680	14,269,380
75	5,348,740	4,148,220	3,384,800	1,918,160	14,799,920

Chapter 8

Conclusion and Future Directions

8.1 Conclusion

Deployment of a hybrid PON incorporates a highly inter-disciplinary diverse research area: from optics to electronics, from network architectures to network protocols, from computer algorithms to operations research. Usually, PON networking research community puts emphasis either on the physical layer or on the optical layer where one layer operates in isolation with another layer. An efficient planning for PON deployment should take into account the constraints of the physical and optical layers in order that both layers can work together harmoniously.

In this thesis, we have incorporated both physical and optical layer constraints while devising our proposed large scale optimization scheme for hybrid PONs.

First, we have investigated the greenfield deployment of a single hybrid PON. We have proposed a novel cross layer planning scheme for optimally devising a hybrid PON architecture and provisioning traffic flows depending on the geographic location of ONUs and their corresponding traffic demand. Our proposed scheme generates optimized clusters of ONUs, selects switching equipment for each cluster depending on the type (unicast/multicast) of traffic demand as splitters are best suited for multicast demand whereas AWGs are suitable for unicast traffic demand, determines optimal location for the corresponding switching equipment and provisions unicast/multicast traffic demand. It relies on an ILP with a CG formulation, a very generic and scalable mathematical model. It is the most comprehensive model proposed so far, and it guarantees an ε-solution (with ε as small as requested) in terms of minimum deployment cost for the greenfield hybrid PON.

Next, we extend our investigation for the greenfield deployment of multiple hybrid PONs. We propose a novel planning scheme for the deployment of a set of hybrid PONs which optimizes the selection, location and allocation of the passive switching equipment of each PON while provisioning the unicast/multicast traffic demand of individual ONUs in a given geographical area. Our proposed scheme also determines the covered region of each PON optimally. It relies on a simple ILP mathematical model as well as an ILP with a CG model in which both of the models are formulated exploiting the principle of p-center based optimization scheme. The proposed scheme can optimize the design of a set of hybrid PONs covering a given geographic area as well as the selection of the best cascading architecture (1/2/mixed-stage) for each selected PON. It minimizes the overall network deployment cost based on the location of the OLT and the ONUs while granting all traffic demands. The scheme emphasizes on the optimum placement of equipment in a hybrid PON infrastructure due to the critical dependency between the network performances and a proper deployment of its equipment, which, in turn depends on the locations of the users.It is a quite powerful scheme as it can handle data instances with up to several thousands ONUs. On the basis of the computational results, the proposed scheme leads to an efficient automated tool for network design, planning, and performance evaluation which can be beneficial for the network designers.

8.2 Future Directions

Our PhD research work can be an ideal guideline for network dimensioning and placement of equipment in hybrid Optical-Wireless Access Networks. An integrated optical/wireless architecture can be investigated for the greenfield deployment of future

BISAN. To implement a hybrid optical wireless access network, a hybrid network infrastructure can be proposed where fiber will be deployed as deeply as affordable/practical and then, wireless systems will be used to extend this connectivity to a large number of locations and ultimately connect the wireless end users.

The following key points related to 'Network Dimensioning and Placement of Equipment in hybrid Optical-Wireless Access Networks' can be addressed in future.

1. Propose efficient solution schemes for the dimensioning of optical and wireless links along with the placement of equipment in hybrid optical-wireless access networks that optimize the best of both worlds with respect to technical, economical, and deployment concerns.

2. Investigate the design of hybrid access networks with wireless access technology at the front end and PON technology at the back end.

3. Design an efficient algorithm in order to identify the best possible placement of equipment in hybrid access networks and evaluate the accuracy of the solutions.

4. Formulate mathematical models for the optimum placement of equipment in hybrid optical and wireless access networks in order to combine the capacity and reliability of optical fiber with the flexibility of wireless networks..

5. Compare the solution of these analytical models with that of the heuristic algorithms.

6. Investigate the failure-tolerant properties of hybrid access networks.

8.2.1 Literature Review on Future Directions

Sarkar *et al.* [SMD06a] investigate the problem of efficient placement of multiple ONUs in a hybrid wireless optical broadband access network (WOBAN). In this paper, the authors assume that the ONUs will also serve as BSs for the wireless portion of the hybrid network. But they do not consider at all the design aspects of the front-end wireless access networks such as transmission power, coverage region, signal quality and interference of wireless BSs. Moreover, the solution of greedy algorithm may get stuck in the local minimum of the problem domain.

Sarkar *et al.* [SMD06b] further elaborate the problem of placement of ONUs in a WOBAN infrastructure with minimum network cost (minimum distance). They apply two combinatorial techniques namely, simulated annealing (SA) and hill-climbing (HC) to obtain the globally optimum locations of multiple ONUs. But the authors neither take into account the design aspects of the wireless front end nor focus on the convergence of ONU and BS. They did not mention about how the number of required ONUs can be determined. Moreover, it is not guaranteed that SA and HC approach will produce globally optimum solution.

Sarkar *et al.* [SYDM07a] investigate the problem of the placement of BSs and ONUs in a WOBAN environment. They formulate the problem as a "Mixed Integer Programming (MIP)" model. But the authors do not describe the strategy of identifying the groups of BSs in which all BSs of a group should be supported by a single ONU. Moreover, the proposed algorithm does not have any scheme to determine the optimum locations of the ONUs required to satisfy the traffic demands from BSs.

Sarkar *et al.* [SYDM07b] [SYDM08b] propose and investigate the characteristics of the Delay-Aware Routing Algorithm(DARA) in order to handle packet delay in the wireless front end of the WOBAN. The authors claim that DARA minimizes average packet delay, generates less congestion, and improves load balancing in comparison with traditional routing algorithms.

Sarkar *et al.* [SYDM07b] proposed Risk-and-Delay-Aware Routing algorithm (RADAR) for the wireless front end of the WOBAN. RADAR can tackle not only the packet delay but also the packet loss due to multiple failure scenarios. It can be concluded that RADAR can provide protection for both the front end wireless mesh and back end the passive optical network (PON) of the hybrid wireless optical access network.

Sarkar *et al.* [SDM07] discuss the challenging factors for designing the hybrid wireless-optical broadband access network. First, they reviewed the algorithms proposed in [SMD06a], [SMD06b], [SYDM07a] for the optimum placement of ONUs in the hybrid access network. Later, they investigate and compare the performances of several routing algorithms, namely Minimum-Hop and Shortest Path Routing Algorithms (MHRA and SPRA), Predictive-Throughput Routing Algorithm (PTRA), Delay-Aware Routing Algorithm(DARA), and Risk-and-Delay-Aware Routing Algorithm for the wireless front end of the proposed hybrid network.

Sarkar *et al.* [SYDM08a] summarize their research activities related to the placement of equipment in WOBAN. They review greedy, Simulated Annealing, and Hill Climbing algorithms for the optimum placement of ONUs. The authors calculate the network deployment cost for PONs, WOBAN with WiMAX at the front end, and also WOBAN with WiFi. But the authors do not clearly describe the architecture of the front end of WOBAN, for example the required number and locations of deployed WiFi APs or WiMAX APs/BSs are not mentioned in the scenario of a given neighborhood. They do not devise any technique to find the optimum number for the ONUs and APs/BSs required to satisfy the bandwidth requests from all the users. They do not apply any clustering technique to divide the users into several groups such that each group of users can be served by an AP efficiently. They connect each ONU with only one BS resulting in wastage of huge bandwidth of an ONU, as each ONU has significantly higher bandwidth capacity compared to an AP/BS.

Finally, the authors propose a Combined Heuristic (CH) for joint optimization in a "greenfield" deployment of WOBAN that focuses on the placement of APs (on the basis of interference) in the front end, placement of ONUs (as returned by the greedy algorithm), and the minimum-cost fiber layout from OLT to the ONUs in the back end simultaneously. The authors do not specify how minimum spanning tree (MST) is constructed from the OLT to all the ONUs. Moreover, it should be mentioned that the topology of PON technology is implemented by Steiner tree not by MST, this fact creates ambiguity about the effectiveness of this heuristic.

Lin *et al.* [LTH11] investigate the dimensioning and site planning (DSP) of integrated PON and wireless cooperative networks (WCN) for fixed mobile coverage (FMC). They propose a mathematical formulation of the DSP problem with the objective of minimizing the overall infrastructure cost for integrated PON-WCN architecture and determining the location of network entities in such a network architecture. The proposed formulation of the DSP aims to provide better performance, including ONU-BS placement, splitter placement, fiber deployment and BS-user association while incorporating inter-cell cooperative transmission. Due to computational complexity, the authors decompose the DSP problem into two sub problems: (i) Subproblem 1 to minimize the total infrastructure cost for ONU-BS deployment, (ii) Subproblem 2 (a MILP) to minimize the total cost for PON deployment. As the Subproblem 1 is a mixed integer non linear program (MINLP), the authors reformulate it into a solvable MILP. Simulation results show that the proposed optimization frame work reduces the infrastructure cost significantly while improving spectral efficiency and scalability in capacity enhancement under cooperative service provisioning.

In the literature, the convergence challenges of optical and wireless access technology in a hybrid optical/wireless access network are not clearly described. Moreover, previous research activities do not take into account the constraints of specific technologies, e.g. WiMAX or WiFi, while developing the mathematical models or implementing the heuristic algorithms for a hybrid access network. It is obvious that placement of equipment in a network environment is dependent on the constraints of the equipment of a specific technology. The literature also lacks the research activities on how to determine the optimum number of equipment in a hybrid access network. Again, previous research activities do not consider any clustering techniques either for grouping the users to be served by one SS, or for grouping the SSs to be served by one RS, or for grouping the RSs to be served by one BS, or for grouping the BSs to be served by one ONU, or for grouping the ONUs to be served by one splitter/AWG, or for grouping the splitters/AWGs to be served by one OLT. Moreover, there is no investigation on the Hybrid PON technology and the level of splitting of PON in the back end of WOBAN to optimize the hybrid optical-wireless access network.

8.2.2 Detailed Guideline on Future Directions

Network planning tools can be developed in order to optimize the dimensioning of the optical and wireless parts of an hybrid access network so that we can decide efficiently about where to take over from the optical back end and to start the wireless front end.

Six hybrid access network architectures can be investigated: (i) TDM PON integrated with WiFi technology [Figure 29], (ii) WDM PON integrated with WiFi technology [Figure 29], (iii) TDM PON integrated with WiMAX technology [Figure 30], (iv) WDM PON integrated with WiMAX technology [Figure 30],(v) TDM PON integrated with both WiMAX and WiFi technologies [Figure 31](vi) WDM PON integrated with both WiMAX and WiFi technologies [Figure 31]. The reason of exploring both TDM and WDM PON architectures in the back end is in order to identify under which traffic assumptions both TDM and WDM PONs are able to converge with the wireless front end. Again, the placement of equipment in these hybrid access network architectures can be investigated. Each network architecture can be described as below.

In the first/second proposed architectures, the wireless stations are organized in a number of BSS where each station within the BSS is managed by an AP as specified by the IEEE 802.11 infrastructure-mode network, the APs are connected to the ONUs, the splitter/AWG, and the OLT in sequence. Based on such an architecture, the number of BSSs and the locations of corresponding APs satisfying the constraints such as the bandwidth requests from the users, transmission range of APs can be determined. As each ONU connects a number of APs, the optimum number and locations of ONUs can be determined so that all the APs are covered. Finally, based on the locations of ONUs, the optimum number and locations of the required splitters/AWGs are determined.

In the third/fourth architecture, the wireless (both fixed and mobile) users in a neighborhood are connected to the BS through the SSs and relay stations (RSs) in sequence using WiMAX technology (IEEE 802.16j standard), a number of BSs are linked to an ONU, a number of ONUs are connected to the OLT through a splitter/AWG. The optimum number and placement of RSs based on the locations of a given set of SSs can be determined. Then the optimum number and placement of required BSs and ONUs can be determined to cover all the RSs. Finally, the optimum number and locations of the required splitters/AWGs will be determined in order to accommodate all the ONUs.

In the fifth/sixth architecture, the wireless stations are organized according to the IEEE 802.11 infrastructure-mode network, a number of APs of WiFi technology are connected to a SS of WiMAX technology, each SS communicates with the BS through the RSs, each BS is connected to the ONU, splitter/AWG, and the OLT in sequence. Based on this architecture, the optimum number and locations of APs, SSs, RSs, BSs, ONUs, splitters/AWGs satisfying the constraints of each equipment can be obtained.

Optimization mathematical models can be proposed to determine the optimum placement of equipment in these above-mentioned hybrid access networks. The models will be a joint optimization one that will be capable of considering the design aspects of both the wireless front end, such as avoiding interferences among neighboring BSs/APs/RSs/SSs and the optical back end, such as minimizing the fiber layout.

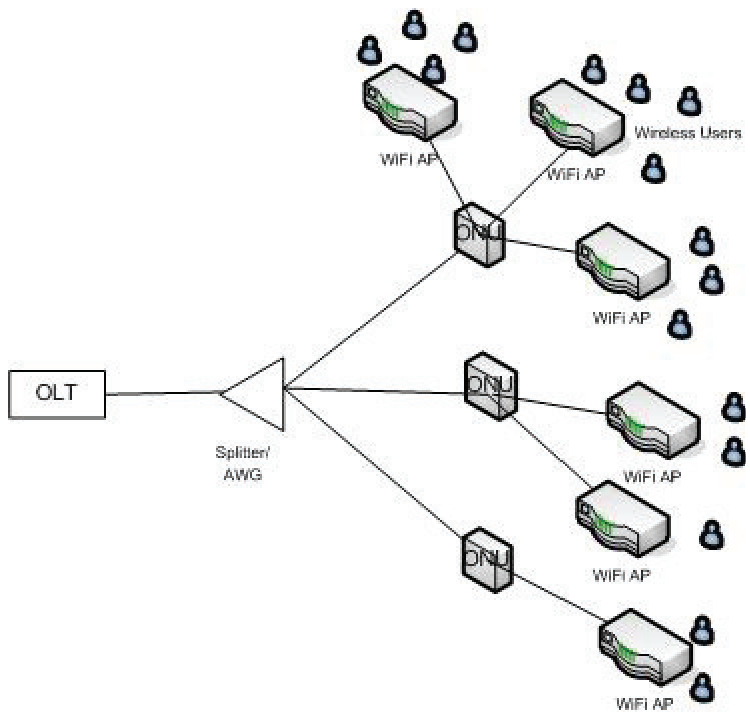


Figure 29: PON-WiFi Architecture

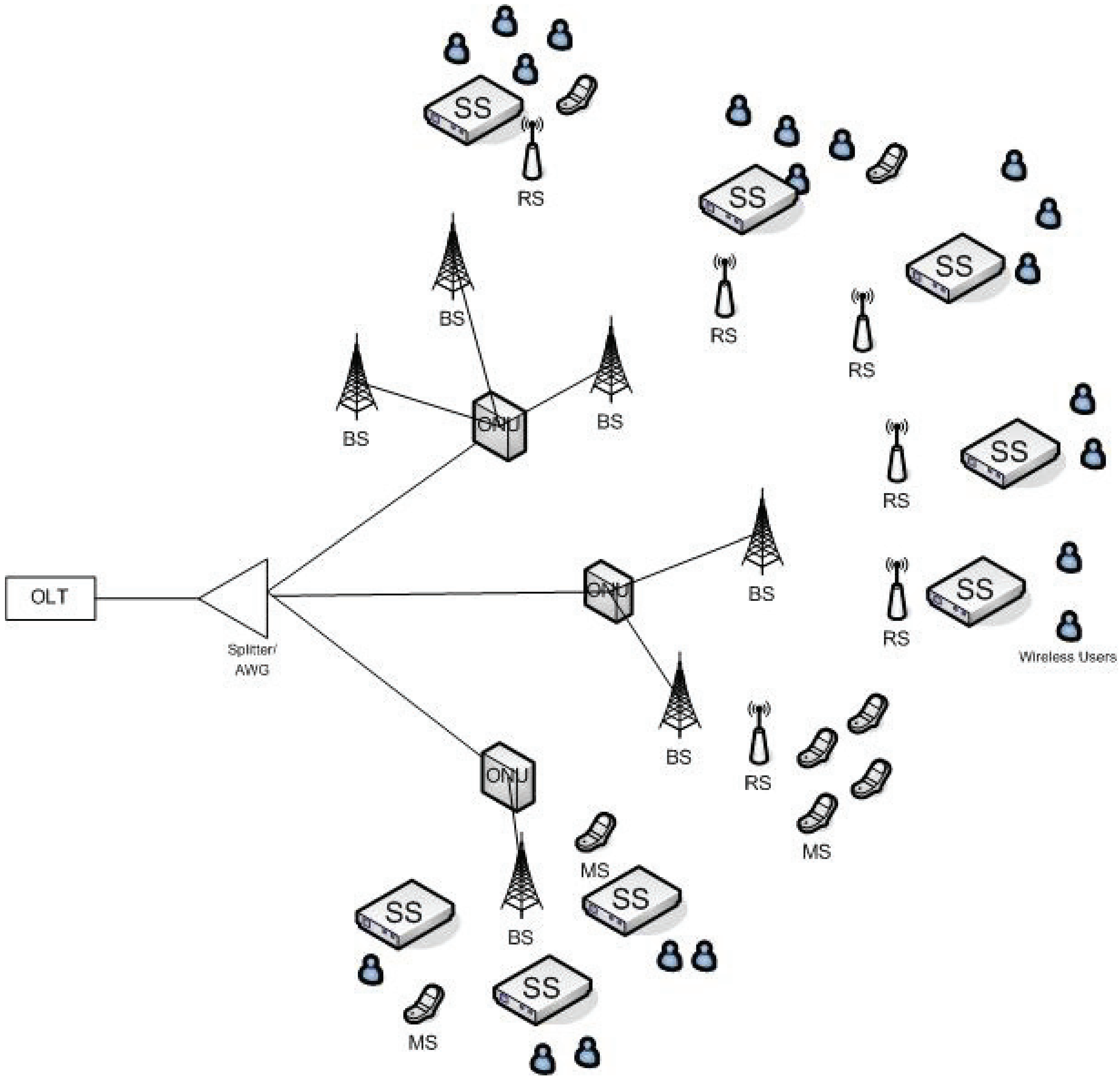


Figure 30: PON-WiMAX Architecture

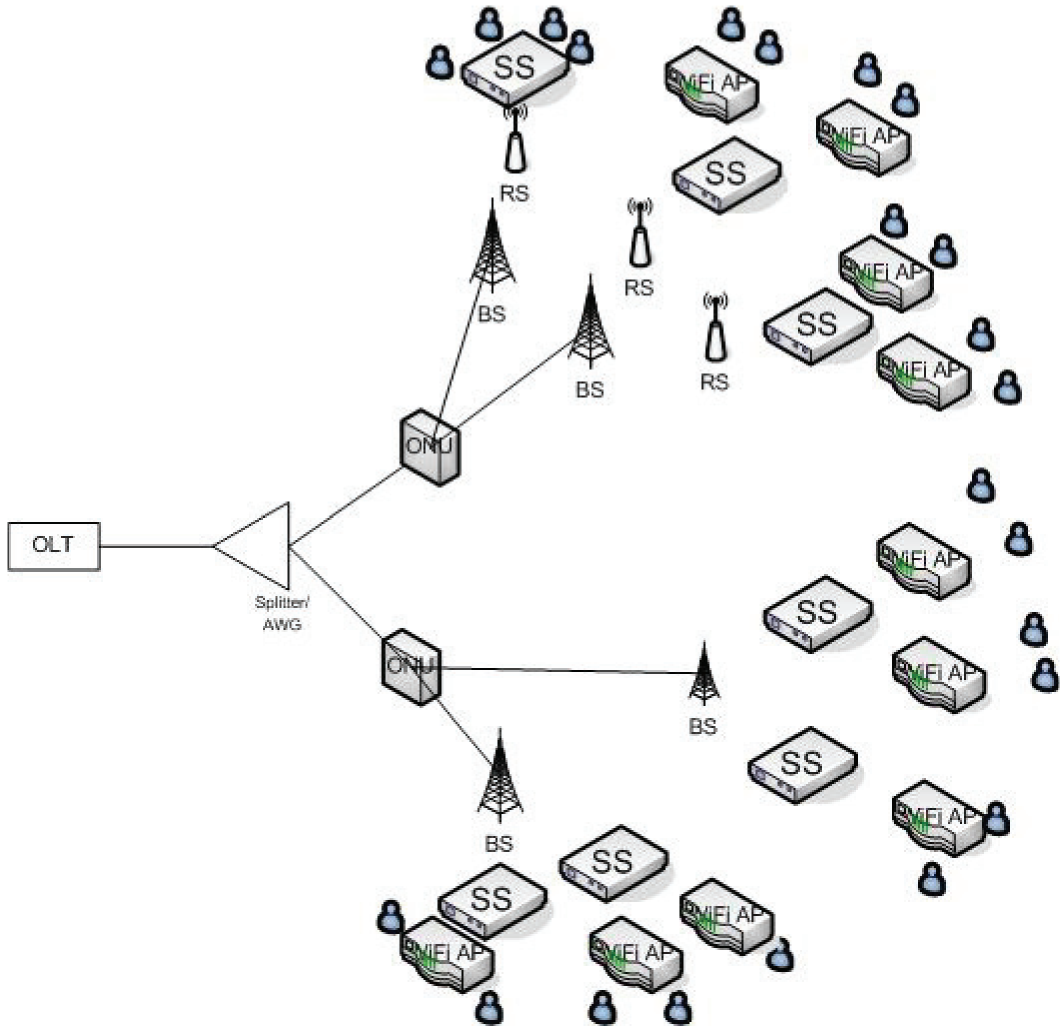


Figure 31: PON-WiMAX-WiFi Architecture

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