Preserving Privacy Against Side-Channel Leaks

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Abstract

Preserving Privacy Against Side-Channel Leaks

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The privacy preserving issues have received significant attentions in various domains. Various models and techniques have been proposed to achieve optimal privacy with minimal costs. However, side-channel leaks (such as, publicly-known algorithms of data publishing, observable traffic information in web application, fine-grained readings in smart metering) further complicate the process of privacy preservation. In this thesis, we make the first effort on investigating a general framework to model side-channel attacks across different domains and applying the framework to various categories of applications.

In privacy-preserving data publishing with publicly-known algorithms, we first theoretically study a generic strategy independent of data utility measures and syntactic privacy properties. We then propose an efficient approach to preserving diversity.

In privacy-preserving traffic padding in Web applications, we first propose a formal PPTP model to quantify the privacies and costs based on the key observation about the similarity between data publishing and traffic padding. We then introduce randomness into the previous solutions to provide background knowledge-resistant privacy guarantee.

In privacy-preserving smart metering, we propose a light-weight approach to simultaneously preserving privacy on both billing and consumption aggregation based on the key observation about the privacy issue beyond the fine-grained readings.

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Chapter 1

Introduction

1.1 Background and Motivation

The privacy preserving issue has attracted significant attentions in various domains, such as, data publishing and data mining, location-based service, mobile and wireless network, social network, web application, smart grid, and so on. However, side-channel leaks further complicate the privacy preservation. Various side-channel attacks have been discovered in many different domains, such as:

- data publishing (e.g. adversarial knowledge about a generalization algorithm itself may allow adversaries to infer sensitive information from the disclosed data);

- Web-based Application (e.g. user input can be inferred from the packet sizes of encrypted traffic between client side and server side);

- smart metering (e.g. the fine-grained meter readings may be used to track the appliance usage patterns and consequently the sensitive information of households, such as, the daily activities);

- cloud computing (e.g. the sharing of physical infrastructure among users allow adversaries to extract information about co-resident VMs);

- Android smartphone (e.g. per data-usage statistics and speaker status may allow an application without any permission to obtain the smartphone user's identity and geo-location as well as driving route);

- VoIP telephony (e.g. users' conversations can be partially reconstructed from encrypted VoIP packets due to the use of VBR codecs for compression and length-preserving stream ciphers for encryption in VoIP protocols);

- cryptography (e.g. information about the secret key may be retrieved from the physical characteristics of the cryptographic modules during the algorithm execution, such as, timing, power consumption, and so on).

In summary, side-channel attacks are prevalent in different applications. On the other hand, several approaches have been proposed to mitigate the threats of such attacks for each specific application, such as, traffic shaping [95], traffic morphing [103], side-buster [110] for web traffic, HTTPO [76] against encrypted HTTP leaks, DREAM [1], EPPA [75] for smart metering, and so on.

However, there are no existing works on studying the generic model under the same framework for different side-channel leaks. Such a study will establish a common understanding on the side-channel attacks, and enable us to apply similar solutions to different applications and new applications.

In this thesis, we make the first effort on investigating a general framework to model side-channel attacks across different domains and applying the framework to three emerging domains, namely, privacy-preserving data publishing (PPDP), privacy preserving traffic padding (PPTP), and privacy-preserving smart metering (PPSM). More specifically, the following questions are to be answered in each phase of the research.

- *Firstly, can we design a generic model for different side-channel attacks?*

While different domains may have different requirements for privacy, they similarly need to balance two seemingly conflicting goals: privacy preservation and cost minimization. Our main goal is to formulate a generic model for privacy preserving against side-channel leaks. The model encompasses privacy requirements (such as, indistinguishability, diversity, uncertainty), costs (such as, data utility, data accuracy, communication overhead, computation overhead), the corresponding methods to ensure privacy and minimize the costs [1]. We will address this question in Chapter 8. Nonetheless, to make our discussion more concrete and easy the understanding, we will first discuss three specific applications.

- *Secondly, can we apply the generic model to data publishing with publicly-known algorithms (PPDP)?*

Most of existing solutions for PPDP assume that the only knowledge the adversaries possess are the disclosed table and the required privacy properties. Actually, the adversaries may also know the disclosure algorithm. Such extra knowledge may assist the adversaries in further precisely predicting the possible original micro-data tables, and finally compromise the privacy properties.

For example, the generalization $g_2(t_0)$ shown in Table 1.1(c) satisfies 3-diversity (the highest ratio of any person being associated with any condition is no greater than $\frac{1}{3}$, see Chapter 2 for detail). However, when the adversary knows the generalization algorithm has considered $g_1(t_0)$ shown in Table 1.1(b) before it discloses $g_2(t_0)$, he can infer that both Charlie and David are definitely associated with cancer. We will detail the theoretical study and efficient solution later in Chapters 3 and 4 , respectively.

(a). A Micro-Data Table t_0

Name	**DoB**	**Condition**
Alice	1990	flu
Bob	1985	cold
Charlie	1974	cancer
David	1962	cancer
Eve	1953	headache
Fen	1941	toothache

(b). Generalization $g_1(t_0)$

DoB	**Condition**
1980∼ 1999	flu cold
1960∼ 1979	cancer cancer
1940∼ 1959	headache toothache

(c). Generalization $g_2(t_0)$

DoB	**Condition**
1970 ∼ 1999	flu cold cancer
1940 ∼ 1969	cancer headache toothache

Table 1.1: A Micro-Data Table and its Two Generalizations

[1]We shall explain the concepts and discuss the details in the following chapters.

- *Thirdly, can we apply the generic model to privacy-preserving traffic padding in Web-applications (PPTP)?*

Web-based applications essentially rely on the distrusted Internet as an internal component for carrying the continuous interaction between users and servers. While encryptions prevent adversaries from reading the data between these two components, some information is still observable, such as, packet sizes, directions, timings. By analyzing such encrypted traffic features, the adversaries can potentially identify an application's internal state transitions as well as users' inputs.

For example, Table 1.2 shows the identifiable packet sizes of each char as the first keystroke entered in a popular real-world search engine. We can observe that six characters (i, j, p, r, v, x) can be identified by a unique packet size. We will elaborate on the formal PPTP model and the enhancement of prior-knowledge resistance later in Chapters 5 and 6 , respectively.

Char	a	b	c	d	e	f	g
Size	509	504	502	516	499	504	502
Char	h	i	j	k	l	m	n
Size	509	492	517	499	501	503	488
Char	o	p	q		r	s	t
Size	509	525	494		498	488	494
Char	u	v	w		x	y	z
Size	503	522	516		491	502	501

Table 1.2: Packet Sizes for the First Character Input of a Search Engine

- *Fourthly, can we apply the generic model to privacy-preserving smart metering (PPSM)?*

Smart grid essentially relies on the fine-grained usage information to provide significant benefits for both utility and customers. However, the fine-grained meter readings could also be used to track the appliance usage pattern and then infer sensitive information of households, such as their daily activities.

For example, Table 1.3(b) shows when the reading is 300, the adversary can infer that Fan is definitely used at that read period in the case that all the appliances in a household are shown in Table 1.3(a) (To ease the understanding of main problem, we assume that each appliance consumes the labeled electricity). We will discuss the formal PPSM model in detail later in Chapter 7.

(a). Appliance Set

Item	**Labeled** (Watts)
Fan	200
Bulb	100
TV	100

(b). Possible Readings

Reading	**Use of Appliances**
400	$\{\{$Fan,Bulb,TV$\}\}$
300	$\{\{$Fan,Bulb$\}$, $\{$Fan,TV$\}\}$
200	$\{\{$Fan$\}$, $\{$Bulb,TV$\}\}$
100	$\{\{$Bulb$\}$, $\{$TV$\}\}$
0	$\{\emptyset\}$

Table 1.3: An Reading Example for Smart Metering

1.2 Summary of Contributions

As introduced in Section 1.1, the main purpose of this research is to understand and provide model and solution to the privacy threats in different applications due to various side-channel attacks. The rest of this Section overviews the five lines of the research in three categories of applications as well as the study on the generic model, and delay the details to the corresponding chapters.

Privacy-preserving data publishing with publicly-known algorithms

- Recent studies show that adversarial inferences using knowledge about a disclosure algorithm can usually render the algorithm unsafe. The first line of the research theoretically study a generic yet costly strategy which is independent of data utility measures and syntactic privacy property [73, 74].

  More specifically, we first show that a given unsafe generalization algorithm can be transformed into a large family of distinct algorithms under a novel strategy, called *k*-

jump strategy. Second, we discuss the computational complexity of such algorithms and prove that different algorithms under the k-jump strategy generally lead to incomparable data utility. We also confirm that the choice of algorithms must be made among safe algorithms (Chapter 3).

- While k-jump strategy is theoretically superior to existing ones due to its independence of utility measures and privacy models, it incurs a high complexity. To overcome this challenge, the second line of the research proposes an efficient *privacy streamliner* approach to preserving diversity [68].

  More specifically, we first observe that a high computational complexity is usually incurred when an algorithm conflates the processes of privacy preservation and utility optimization. Based on such observations, we then propose a novel privacy streamliner approach to decouple those two processes for improving algorithm efficiency. We also confirm our algorithms to be efficient through both complexity analysis and experimental results (Chapter 4).

Privacy-Preserving Traffic Padding in Web Applications

- Recent studies show that many popular Web applications actually leak out highly sensitive data from encrypted traffic due to side-channel attacks. The third line of the research proposes a formal model for privacy-preserving traffic padding (PPTP) which can quantify the effectiveness of mitigation techniques [69–71].

  More specifically, we first observe an interesting similarity between PPTP and PPDP issues. Based on such a similarity, we then establish a mapping between these two issues and propose a formal PPTP model, which encompasses the quantification of privacy requirements and padding costs. Such a model lays the foundation for further studies of this issue. We also design efficient heuristic algorithms and confirm their effectiveness and efficiency through experiments using real-world Web applications (Chapter 5).

- While the model in previous line of research is among the first efforts on formally addressing the PPTP issue, it relies on the assumption that adversaries do not possess prior background knowledge about possible user inputs. In the fourth line of the research, we propose a novel random ceiling padding approach whose results are resistant to such adversarial knowledge [72].

  More specifically, the approach injects randomness into the process of forming padding groups, such that an adversary armed with background knowledge would still face sufficient uncertainty in estimating user inputs. We formally present a generic scheme and discuss two concrete instantiations. We then confirm the correctness and performance of our approach through both theoretic analysis and experiments with two real world applications (Chapter 6).

Privacy-Preserving Smart Metering

- Recent studies show that fine-grained meter readings may allow adversaries to infer sensitive information about the households. In the fifth line of the research, we propose a novel light-weight technique for privacy-preserving smart metering (PPSM), which can achieve multiple objectives through a single set of data and procedures.

  More specifically, we first observe that satisfying certain privacy property for reading does not necessarily lead to preserving the household's privacy. Based on such observations, we propose a formal PPSM model, which encompasses the privacy properties and consumption accuracy. This model is among the first efforts on preserving the household's sensitive information (compared with preserving the readings). We also design efficient algorithms and analyze their privacy preservation (Chapter 7).

Generic Model for Privacy Preserving against Side-Channel Leaks

- We make the first step to extract a generic model under the same framework for various side-channel leaks in different categories of applications. Such a study will bridge the gap among different communities on study of side-channel attacks. To the best of our knowledge, there was no such an effort on the generic model in the literature (Chapter 8).

Implications of Our Study

Our research shows the possibility of a generic framework for privacy preserving against side-channel leaks and also leads to practical solutions with quantifiable privacy guarantee for different applications against side-channel attacks. The generic framework will facilitate the works on preventing such attacks with less effort.

Chapter 2

Related Work

2.1 Privacy Preservation

The privacy preserving issue has received significant attentions in various domains, such as, data publishing and data mining [25,46,90], mobile and wireless network [9] [11] [45], social network [31] [83] [44], outsourced data [18] [97], multiparty computation [82], web applications [9] [14] [22] [94], and so on.

In the context of privacy-preserving data publishing, various generalization techniques and models have been proposed to transform a micro-data table into a safe version that satisfies given privacy properties and retains enough data utility. In particular, data swapping [30, 35, 39] and cell suppression [29] both aim to protect micro-data released in census tables, but those earlier approaches cannot effectively quantify the degree of privacy. A measurement of information disclosed through tables based on the perfect secrecy notion by Shannon is given in [80]. The authors in [34] address the problem ascribed to the independence assumption made in [80]. The important notion of k-anonymity has been proposed as a model of privacy requirement [90]. The main goal of k-anonymity is to anonymize the data such that each record owner in the resultant data is guaranteed to be indistinguishable from at least $k - 1$ other record owner. That is, each quasi-identifier value in a micro-data should be at least shared by k tuples. Since the data owner modifies the data, some information is distorted. Therefore, it is desirable to find the modified table for k-anonymity with the minimum information loss. However, to achieve optimal k-anonymity with the most data utility is proved to be computationally infeasible [79].

Since the introduction of k-anonymity, privacy-preserving data publishing has received tremendous interest in recent years [27, 28, 37, 42, 46, 93, 100]. A model based on the intuition of *blending individuals in a crowd* is proposed in [21]. A personalized requirement for anonymity is studied in [104]. In [17], the authors approach the issue from a different perspective, that is, the privacy property is based on generalization of the protected data and could be customized by users. Much efforts have been made around developing efficient k-anonymity algorithms [3–5, 12, 38, 61, 90], whereas the safety of the algorithms is generally assumed.

Many more advanced models are proposed to address limitations of k-anonymity. Many of these focus on the deficiency of allowing insecure groups with a small number of sensitive values. For instance, l-diversity [77] requires that each equivalence class on the disclosed table should contain at least l well-represented sensitive values; t-closeness [63] requires that the distribution of a sensitive attribute in any equivalence class is close (roughly equal) to the distribution of the attribute in the whole table; (α, k)-anonymity [102] requires that the number of tuples in any equivalence class is at least k and the frequency (in fraction) of each sensitive value is at most α, where k and α are data publisher-specified thresholds. In addition, a generic model called GBP was proposed to unify the perspective of privacy guarantees in both generalization-based publishing and view-based publishing [33]. These privacy models in PPDP have been adjusting and applying to other domains.

In contrast to micro-data disclosure, aggregation queries are addressed in statistical databases [2,42,92]. The main challenge is to answer aggregation queries without allowing inferences of secret individual values. The auditing methods in [23, 36] solve this problem by checking whether each new query can be safely answered based on a history of previously answered queries. The authors of [23, 56, 58] considered the same problem in more specific settings of offline auditing and online auditing, respectively.

Compared with the aforementioned syntactic privacy models, recently, a semantic privacy notation to provide provable resistance to adversaries' background knowledge, differential privacy [40] has been widely accepted as a strong privacy model mostly for answering statistic queries. Differential privacy aims to achieve the goal that the probability distribution of any disclosed information should be similar enough regardless of whether that disclosed information is obtained using the real database, or using a database without any one of the existing records.

However, although differential privacy is extended to privacy preserving data publishing (PPDP) [64, 106], most existing approaches that ensure differential privacy are random noise-based and are suitable for specific types of statistical queries. Further, Kifer *et al.* [57] disproved some popularized claims about differential privacy and showed that differential privacy cannot always guarantee the privacy in some cases. differential privacy is also less applicable to our traffic padding due to the less predictable but larger sensitivity and the nature of budget share among executions of web applications(Chapters 5, 6). Moreover, while the qualitative significance of the privacy parameter ϵ is well understood in the literature, the exact quantitative link between this value and the degree of privacy guarantee has received less attention. Actually, more and more works have concluded that both differential privacy and syntactic privacy models have their place, and any one cannot replace the other [26, 65]. It is also shown that differential privacy cannot always guarantee the privacy in some cases [57]. Due to these reasons, we focus on syntactic privacy properties in this research and regard the differential privacy as future work.

2.2 Side-Channel Attack

Various side-channel leakages have been extensively studied in the literature. By measuring the amount of time taken to respond to the queries, an attacker may extract OpenSSL RSA privacy keys [16], and similar timing attacks are proved to be still practical recently [15]. By differentiating the sounds produced by keys, an attacker with the help of the large-length training samples may recognize the key pressed [7]; Zhuang *et al.* further present an alternative approach to achieving such attack which does not need the training samples [115]. By exploiting queuing side channel in routers by sending probes from a far-off vantage point, an attacker may fingerprint websites remotely against home broadband users [49, 50]. Ristenpart *et al.* discover cross-VM information leakage on Amazon EC2 based on the sharing of physical infrastructure among users [87]. Search histories may be reconstructed by session hijacking attack [19], while web-browsing histories may be compromised by cache-based timing attacks [43]. Saponas *et al.* show the transmission characteristics of encrypted video streaming may allow attackers to recognize the title of movies [91].

Meanwhile, much efforts have been made on developing techniques to mitigate the threats of such leakages. Countermeasures based on traffic-shaping mechanisms (such as, padding, mimicking, morphing, and so on) are suggested against the exposure of identification of encrypted web traffic in [95]. HTTPOS, a browser-side system, is proposed to prevent information leakages of encrypted HTTP traffic through configurable traffic transformation techniques in [76]. Timing mitigator is introduced to achieve any given bound on timing channel leakage by delaying output events to limit the amount of information in [6]. Zhang *et al.* present an approach to verifying the VMs' exclusive use of a physical machine. The approach exploits a side-channel in the L2 memory cache as a defensive detection tool rather than a vector of attack [114]. Provider-enforced deterministic execution by eliminating all the internal timing channels has been proposed to combat timing channel attack in cloud [8]. In the rest of this section, we review the work related to the side-channel attacks targeted in our lines of research.

2.2.1 The case that disclosure algorithms is publicly known in PPDP

While most existing work assume the disclosed generalization to be the only source of information available to an adversary, recent work [111] [101] shows the limitation of such an assumption. In addition to such information, the adversary may also know about the disclosure algorithm. With such extra knowledge, the adversary may deduce more information and eventually compromise the privacy property. In the work of [111] [101], the authors discover the above problem and correspondingly introduce models and algorithms to address the issue. However, the method in [101] is still vulnerable to algorithm-based disclosure [52, 53], whereas the one in [111] is more general, but it also incurs a high complexity.

In [111], Zhang *et al.* presented a theoretical study on how an algorithm should be designed to prevent the adversary from inferring private information when the adversaries know the algorithm itself. The authors proved that it is NP-hard to compute a generalization which ensure privacy while maximizing data utility under such assumptions of adversaries' knowledge. The authors then investigate three special cases of the problem by imposing constraints on the functions and the privacy properties, and propose a polynomial-time algorithm that ensures entropy l-diversity.

Wong *et al.* in [101] showed that a minimality attack can compromise most existing generalization techniques with the aim of only a small amount of knowledge about the generalization algorithm. The authors assume that the adversaries only have one piece of knowledge that the algorithm discloses a generalization with best data utility. Under this assumption, minimality attacks can be prevented by simply disclosing sub-optimal generalizations. Unfortunately, the adversaries, equipped with knowledge of the algorithm, can still devise other types of attacks to compromise sub-optimal generalizations.

Since the problem is discovered, some work have been developed to tackle the problem in the case that $l - diversity$ is the desired privacy property [53, 68, 105, 113].

To improve the efficiency, a so-called exclusive strategy is proposed in [112] to penalize the cases where a recursive process is required to compute the adversarial mental image about the micro-data table. To examine the general case, we have proposed a k-jump strategy [73](see Chapter 3 for the first line of our research) to penalize such cases where with more control in the sense that only k, instead of all, generalization functions will be skipped. Our proposed family of algorithms is general to handle different privacy properties and different measures of data utility. Despite the improved efficiency, most of those methods are still impractical due to the high complexity.

The concept of l-cover in [113] has been proposed for efficient diversity preservation. However, no concrete methods for building identifier partitions that can satisfy the l-cover property was reported in [113], which is the main focus of the second line of our research(see Chapter 4). The correctness and flexibility of our approach can be further confirmed by the following work in the literature. The authors of [105] introduce algorithms that share the same spirit with our algorithms, and can achieve similar performance (more precisely, their algorithms are slightly less efficient than ours since their time complexity is $O(n^2 logn)$). In fact, under slight modification, their algorithms, such as ACE algorithm which is originally intended to publish dynamic datasets [108], can be regarded as another instantiation of our model and approach.

2.2.2 The case that visible patterns in encrypted traffic are observed in Web applications

In the context of web applications, many side-channel leakages in encrypted web traffic have been identified in the literature which allow to profile the web applications themselves and their internal states [8,19,22,50]. Meanwhile, several approaches have been proposed to analyze and mitigate such leakages, such as [6, 76, 95, 103]. Recently, a black-box approach has been proposed to detect and quantify the side-channel vulnerabilities in web application by extensively crawling a targeted application [20].

Chen *et al.* in [22] demonstrated through case studies that side-channel problems are pervasive and exacerbated in web applications due to their fundamental features. Then the authors further study approaches to identifying such threats and quantifying the amount of information disclosed in [110]. They show that an application-agnostic approach generally suffers from high overhead and low level of privacy protection, and consequently effective solutions to such threats likely will rely on the in-depth understanding of the applications themselves. Finally, they design a complete development process as a fundamental solution to such side channel attacks.

Traffic morphing is proposed in [103] to mitigate the threats by traffic analyzing on packet sizes and sequences through network. Although their proposed system morph classes of traffic to be indistinguishable, traffic morphing pads or splits packets on the fly which may degrade application's performance.

The aforementioned works share an important limitation, that is, they are lack of privacy requirements, In such case, the degree of privacy, which the transformation of the traffic is able to achieve, cannot be evaluated during the process of padding. Consequently, it cannot ensure the privacy being satisfied. In contrast, our proposed algorithms following the proposed model in the third line of our research theoretically guarantee the desired privacy property. Our model and solutions provide finer control over the trade-off between privacy protection and cost, and those solutions can certainly be integrated into the development process.

Nonetheless, these solutions assume that adversaries do not possess prior background knowledge about possible user inputs. Our fourth line of research enhance previous works by mitigating the threats of background knowledge. Closest to this research, most recently, a formal framework is proposed to measure security in terms of the amount of information leaked from the observations without the assumption of any particular attacks [10]. However, the main deficiency of [10] regarding to the estimation of privacy renders it less applicable in practice.

2.2.3 The case that readings are used for inferences in smart metering

Electrical appliances, even small electric devices, generate detectable electric consumption signatures [51, 60]. Based on such signatures, electric consumption data (collected at a pre-configured granularity) of a household can be decomposed to identify the status of appliances. A domestic electricity demand model based on occupant time-use data has been presented and its example implementation is made for free download [86]. Worse still, even simple off-the-shelf statistical tools can be used to extract complex usage patterns from the consumption data without a priori knowledge of household activities [81], while Rouf *et al.* showed that real-world automatic meter reading (AMR) systems are vulnerable to spoofing attacks due to the unsecured wireless transmission and continuous broadcast of fine-grained energy data [88].

Many surveys have been conducted to review and discuss the security and privacy requirements and challenges(e.g. [99]). Some efforts have been made to preserve privacy for the load monitoring [41]. In-residence batteries, together with corresponding battery privacy algorithms such as Non-Intrusive Load Leveling (NILL) and stepping approach, used to mask load variance of a household to the grid and consequently avoided recovering of appliance profiles by grid [78, 109]. A distributed Laplacian Perturbation Algorithm (DLPA) has been proposed to achieve provably privacy and optimal utility without the need of a third trusted party [1]. An aggregation protocol is introduced to privately sum readings from many meters without the need of disclose those raw meter readings [59]. A scheme is designed to provide personal enquiry and regional statistics through anonymously-sent readings [24]. EPPA achieves privacy-preserving multi-dimensional data aggregation by using the homomorphic cryptosystem [75].

The other efforts were made to preserve privacy for billing. Rail *et al.* proposed a set of protocols which allow uers themselves to produce a correct provable final bill without disclosing fine-grained consumption data [84], and then the authors combined differentially private mechanisms with oblivious payments to eliminate leakages drawn from the final bill [32]. Recently, Lin *et al.* used trusted platform module (TPM) and cryptographic primitive to support privacy preserving billing and load monitoring simultaneously [67].

Chapter 3

PPDP: k-Jump Strategy for Privacy Preserving Data Publishing

In this chapter, we study the privacy issue for data publishing in the case that the adversaries utilize the knowledge about the algorithms themselves as side-channel to refine their guess about the original data, and then propose the strategy to transform an existing unsafe algorithm into a large family of safe algorithms.

3.1 Overview

The issue of preserving privacy in micro-data disclosure has attracted much attention lately [46]. Data owners, such as the Census Bureau, may need to disclose micro-data tables containing sensitive information to the public to facilitate useful analysis. There are two seemingly conflicting goals during such a disclosure. First, the utility of disclosed data should be maximized to facilitate useful analysis. Second, the sensitive information about individuals contained in the data must be to an acceptable level due to privacy concerns.

The upper left tabular of Table 3.1 shows a toy example of micro-data table t_0. Suppose each patient's name, DoB, and condition are regarded as *identifier attribute*, *quasi-identifier attribute* and *sensitive attribute*, respectively. Simply deleting the *identifier* Name is not sufficient because the *sensitive attribute* Condition may still potentially be linked to a unique person through the *quasi-identifier* Age (more realistically, a quasi-identifier is usually a combination of attributes, such as Age, Gender, and Zip Code). Nonetheless, we shall not include identifiers in the remainder of the chapter for simplicity.

A Micro-Data Table t_0

Name	**DoB**	**Condition**
Alice	1990	flu
Bob	1985	cold
Charlie	1974	cancer
David	1962	cancer
Eve	1953	headache
Fen	1941	toothache

Generalization $g_1(t_0)$

DoB	**Condition**
1980∼1999	flu cold
1960∼1979	cancer cancer
1940∼1959	headache toothache

Generalization $g_2(t_0)$

DoB	**Condition**
1970∼1999	flu cold cancer
1940∼1969	cancer headache toothache

Generalization $g_3(t_0)$

DoB	**Condition**
1960∼1999	flu cold cancer cancer
1940∼1959	headache toothache

Table 3.1: A Micro-Data Table and Three Generalizations

To prevent such a *linking attack*, the micro-data table can be partitioned into *anonymized group* and then *generalized* to satisfy *k-anonymity* [90, 96]. The upper right tabular in Table 3.1 shows a generalization $g_1(t_0)$ that satisfies 2-anonymity. That is, each generalized quasi-identifier value is now shared by at least two tuples. Therefore, a linking attack can no longer bind a person to a unique tuple through the quasi-identifier.

Nonetheless, k-anonymity by itself is not sufficient since linking a person to the second group in $g_1(t_0)$ already reveals his/her condition to be cancer. To avoid such a situation, the generalization must also ensure enough diversity inside each group of sensitive values, namely, to satisfy the l-diversity property [77]. For example, assume 2-diversity is desired. If the generalization $g_2(t_0)$ is disclosed, a person can at best be linked to a group with three different conditions among which each is equally likely to be that person's real condition. The desired privacy property is thus satisfied.

However, adversarial knowledge about a generalization algorithm itself may cause additional complications [101, 111]. First, without considering such knowledge, an adversary looking at $g_2(t_0)$ in Table 3.1 can guess that the three persons in each group may have the three conditions in any given order. Therefore, the adversary's mental image of t_0 is a set of totally $3! \times 3! = 36$ micro-data tables, each of which is equally likely to be t_0 (a common assumption is that the quasi-identifier attribute, such as Age in t_0, is public knowledge).We shall call this set of tables the *permutation set* with respect to the given generalization. The left-hand side of Table 3.2 shows two example tables in the permutation set (with the identifier Name deleted).

t_1

DoB	**Condition**
1990	cancer
1985	flu
1974	cold
1962	cancer
1953	headache
1941	toothache

$g_1(t_1)$

DoB	**Condition**
1980∼1999	cancer flu
1960∼1979	cold cancer
1940∼1959	headache toothache

t_2

DoB	**Condition**
1990	cold
1985	flu
1974	cancer
1962	cancer
1953	headache
1941	toothache

$g_1(t_2)$

DoB	**Condition**
1980∼1999	cold flu
1960∼1979	cancer cancer
1940∼1959	headache toothache

Table 3.2: Two Tables in the Permutation Set and Their Corresponding Generalizations under g_1

The permutation set would be the adversary's best guesses of the micro-data table, if the released generalization is his/her only knowledge. However, adversary may also know the generalization algorithm, and can simulate the algorithm to further exclude some invalid guesses from the permutation set. In other words, such knowledge may allow adversary to obtain a more accurate estimation of the private information than that can be obtained from the disclosed data alone. For example, assume that the adversary knows the generalization algorithm has considered $g_1(t_0)$ before it discloses $g_2(t_0)$. In Table 3.2, t_1 is not a valid guess, because $g_1(t_1)$ satisfies 2-diversity and should have been disclosed instead of $g_2(t_0)$. On the other hand, t_2 is a valid guess since $g_1(t_2)$ fails 2-diversity. Consequently, the adversary can refine his/her guess of t_0 to a smaller set of tables, namely, the *disclosure set*, as shown in Table 3.3. Since each table in the disclosure set is equally like to be t_0, the desired 2-diversity should be measured on each row of sensitive values (as a multiset). From this set of tables, the adversary can infer that both Charlie and David, whose DoB are 1974 and 1962 respectively, are definitely associated with cancer. Clearly, 2-diversity is violated.

DoB	**Condition**			
1990	flu	cold	flu	cold
1985	cold	flu	cold	flu
1974	cancer	cancer	cancer	cancer
1962	cancer	cancer	cancer	cancer
1953	headache	headache	toothache	toothache
1941	toothache	toothache	headache	headache

Table 3.3: The Disclosure Set of $g_2(t_0)$

A natural solution to the above problem is for generalization algorithms to evaluate the desired privacy property, such as l-diversity, on disclosure set in order to determine whether a generalization is safe to disclose. For example, consider how we can compute the disclosure set of next generalization, $g_3(t_0)$, in Table 3.1. We need to exclude every table t in the permutation set of $g_3(t_0)$, if either $g_1(t)$ or $g_2(t)$ satisfies 2-diversity. However, to determine whether $g_2(t)$ satisfies 2-diversity, we would have to compute the disclosure set of $g_2(t)$, which may be different from the disclosure set of $g_2(t_0)$ shown in Table 3.3. The left-hand side of Table 3.4 shows such an example table t_3 in permutation set of $g_3(t_0)$. The disclosure set of $g_2(t_3)$ as shown in right-hand side of Table 3.4 is different from the disclosure set of $g_2(t_0)$. Clearly, such a recursive process is bound to have a high cost.

The contribution of this research is three fold. First, we show that a given generalization algorithm can be transformed into a large family of distinct algorithms under

t_3

DoB	Condition
1990	cancer
1985	cancer
1974	flu
1962	cold
1953	headache
1941	toothache

Disclosure Set of $g_2(t_3)$

DoB	Condition					
1990	cancer	cancer	cancer	cancer	cancer	cancer
1985	cancer	cancer	cancer	cancer	cancer	cancer
1974	flu	flu	flu	flu	flu	flu
1962	cold	cold	headache	headache	toothache	toothache
1953	headache	toothache	cold	toothache	cold	headache
1941	toothache	headache	toothache	cold	headache	cold

Table 3.4: A Table t_3 in the Permutation Set of $g_3(t_0)$ and its Corresponding Disclosure Set Under g_2

a novel strategy, called *k-jump strategy*. Intuitively, the k-jump strategy penalizes cases where recursion is required to compute the disclosure set. Therefore, algorithms may be more efficient under the k-jump strategy in contrast to the above safe strategy. Second, we discuss the computational complexity of such algorithms and prove that different algorithms under the k-jump strategy generally lead to incomparable data utility (which is also incomparable to that of algorithms under the above safe strategy). This result is somehow surprising since the k-jump strategy adopts a more drastic approach than the above safe strategy. Third, the result on data utility also has a practical impact. Specifically, while all the k-jump algorithms are still publicly known, the choice among these algorithms can be randomly chosen and kept secret, analogous to choosing a cryptographic key. We also confirm that the choice of algorithms must be made among safe algorithms. Furthermore, the family of our algorithms is general and independent of the syntactic privacy property and the data utility measurement. Note that in this research we focus on the syntactic privacy properties which has been evidenced as complementary and indispensable to the semantic notion of privacy, such as differential privacy [26, 64].

The rest of the chapter is organized as follows. Section 3.2 gives our model of two existing algorithms. Section 3.3 then introduces the k-jump strategy and discusses its properties. Section 3.4 presents our results on the data utility of k-jump algorithms. We analyze the computational complexity of k-jump algorithms in Section 3.5, and confirm that the secret choice must be made among safe algorithms such as the family of k-jump algorithms in Section 3.6. Section 3.7 concludes the chapter.

3.2 The Model

We first introduce the basic model of micro-data table and generalization algorithm in Section 3.2.1. We then review two existing strategies and related concepts in Section 3.2.2. Table 3.5 lists our main notations which will be defined in this section.

t_0, t	Micro-data table
a, a_{naive}, a_{safe}	Generalization algorithm
$g_i(.)$, $g_i(t)$	Generalization (function)
$p(.)$	Privacy property
$per(.)$, $per(g_i(t))$, per_i, per_i^k	Permutation set
$ds(.)$, $ds(g_i(t))$, ds_i, ds_i^k	Disclosure set
$path(.)$	Evaluation path

Table 3.5: The Notation Table

3.2.1 The Basic Model

A secret *micro-data table* (or simply a table) is a relation $t_0(QID, S)$ where QID and S is the *quasi-identifier attribute* and *sensitive attribute*, respectively (note that each of these can also be a sequence of attributes). We make the worst case assumption that each tuple in t_0 can be linked to a unique identifier (which the identifier is not shown from t_0) through the QID value (if some tuples are to be deemed as not sensitive, they can be simply disregarded by the algorithm). Denote by T the set of all tables with the same schema, the same set of QID values, and the same multiset of sensitive values as those of t_0.

We are also given a *generalization algorithm* a that defines a *privacy property* $p(.) : 2^T \to \{true, false\}$ and a sequence of *generalization functions* $g_i(.) : T \to G$ $(1 \leq i \leq n)$ where G denotes the set of all possible *generalizations* over T. Note that the discussion about Table 3.3 in Section 3.1 has explained why $p(.)$ should be evaluated on a set of, instead of one, tables, and we follow the widely accepted notion of generalization [90]. Given t_0 as the input to the algorithm a, either a generalization $g_i(t_0)$ will be the output and then disclosed, or $\emptyset$ will be the output indicating that nothing is disclosed.

Note that in a real world generalization algorithm, a generalization function may take an implicit form, such as a cut of the taxonomy tree [101]. Moreover, the sequence of generalization functions to be applied to a given table is typically decided on the fly. Our simplified model is reasonable as long as such a decision is based on the quasi-identifier (which is true in, for example, the Incognito [61]), because an adversary who knows both the quasi-identifier and the generalization algorithm can simulate the latter's execution to determine the sequence of generalization functions for the disclosed generalization.

3.2.2 The Algorithms a_{naive} and a_{safe}

When adversarial knowledge about a generalization algorithm is not taken into account, the algorithm can take the following *naive strategy*. Given a table t_0 and the generalization functions $g_i(.)$ $(1 \leq i \leq n)$ already sorted in a non-increasing order of data utility, the algorithm will then evaluate the privacy property $p(.)$ on each of the n generalizations $g_i(t_0)$ $(1 \leq i \leq n)$ in the given order. The first generalization $g_i(t_0)$ satisfying $p(g_i(t_0)) = true$ will be disclosed, which also maximizes the data utility. Note that our discussion does not depend on specific utility measures as long as the measure is defined based on quasi-identifiers.

Before giving the detail of *naive strategy*, we first formalizes the set of all tables in T whose generalizations, under a given function, are identical with that of a given table in Definition 3.1.

Definition 3.1 (Permutation Set) *Given a micro-data table t_0, a generalization function $g_i(.)$, the* permutation set *of t_0 under $g_i(.)$ is a function $per(.) : G \to 2^T$, defined by:*

$$per(g_i(t_0)) = \{t : g_i(t) = g_i(t_0)\}$$

Note that $per(g_i(t_0))$ is also written as per_i when both g_i and t_0 are clear from context. It is easily seen that, in the naive strategy, evaluating the privacy property $p(.)$ on a generalization $g_i(t_0)$ is equivalent to evaluating $p(.)$ on the permutation set $per(g_i(t_0))$.

Next we introduce the *evaluation path* in Definition 3.2. Informally, evaluation path represents the sequence of evaluated generalization functions.

Definition 3.2 (Evaluation Path) *Given a micro-data table t_0, an algorithm composed of a sequence of generalization functions $g_i(.)(1 \leq i \leq n)$, the* evaluation path *of t_0 under the algorithm is a function $path(.) : T \rightarrow 2^{[1,n]}$, defined by:*

$$path(t_0) = \{i : (\textit{the algorithm will evaluate } t_0 \textit{ under } g_i) \wedge (1 \leq i \leq n)\}$$

Note that although $path(t_0)$ is defined as a set, the indices naturally form a sequence (we shall need this concept for later discussions). With these two concepts, we can describe the above algorithm as a_{naive} shown in Table 3.6.

Input: Table t_0;
Output: Generalization g or $\emptyset$;
Method:
Let $path(t_0) = \emptyset$;
For $i = 1$ to n
    Let $path(t_0) = path(t_0) \cup \{i\}$;
    If $p(per(g_i(t_0))) = true$ **then**
        Return $g_i(t_0)$;
Return $\emptyset$;

Table 3.6: The Algorithm a_{naive}

Unfortunately, the naive strategy leads to an unsafe algorithm as illustrated in Section 3.1 (that is, an algorithm that fails to satisfy the desired privacy property). Specifically, consider an adversary who knows the quasi-identifier $\Pi_{QID}(t_0)$, the above algorithm a_{naive}, and the disclosed generalization $g_i(t_0)$ for some $i \in [1, n]$. Given any table t, by simulating the algorithm's execution, the adversary also knows $path(t)$.

First, by only looking at the disclosed generalization $g_i(t_0)$, the adversary can deduce t_0 must be one of the tables in the *permutation set* $per(g_i(t_0))$. This inference itself does not violate the privacy property $p(.)$ since the algorithm a_{naive} does ensure

$p(per(g_i(t_0)) = true$ holds before it discloses $g_i(t_0)$. However, for any $t \in per(g_i(t_0))$, the adversary can decide whether $i \in path(t)$ by simulating the algorithm's execution with t as its input.

Clearly, any $t \in per(g_i(t_0))$ can be a valid guess of the unknown t_0, only if $i \in path(t)$ is true. By excluding all invalid guesses, the adversary can obtain a smaller subset of $per(g_i(t_0))$. We call such a subset of $per(g_i(t_0))$ the *disclosure set*, as formally stated in Definition 3.3.

Definition 3.3 (Disclosure Set) *Given a micro-data table t_0, an algorithm composed of a sequence of generalization functions $g_i(.)(1 \leq i \leq n)$, the* disclosure set *of t_0 under $g_i(.)$ is a function $ds(.) : G \to 2^T$, defined by:*

$$ds(g_i(t_0)) = per(g_i(t_0)) \setminus \{t : i \notin path(t)\}$$

A natural way to fix the unsafe a_{naive} is to replace the permutation set with the corresponding disclosure set in the evaluation of a privacy property. From above discussions, after $g_i(t_0)$ is disclosed, the adversary's mental image about t_0 is $ds(g_i(t_0))$. Therefore, we can simply modify the algorithm to ensure $p(ds(g_i(t_0))) = true$ before it discloses any $g_i(t_0)$. We call this the *safe strategy*, and formally describe it as algorithm a_{safe} in Table 3.7.

Input: Table t_0;
Output: Generalization g or $\emptyset$;
Method:
Let $path(t_0) = \emptyset$;
For $i = 1$ to n
    Let $path(t_0) = path(t_0) \cup \{i\}$;
    If $p(ds(g_i(t_0))) = true$ **then**
        Return $g_i(t_0)$;
Return $\emptyset$;

Table 3.7: The Algorithm a_{safe}

Taking the adversary's point of view again, when $g_i(t_0)$ is disclosed under a_{safe}, the adversary can repeat the aforementioned process to exclude invalid guesses from $per(g_i(t_0))$, except that now ds_j ($j < i$) will be used instead of per_j. As the result, he/she will conclude that t_0 must be within the set $per(g_i(t)) \setminus \{t' : i \notin path(t')\}$, which, not surprisingly, coincides with $ds(g_i(t_0))$ (that is, the result of the adversary's inference is $t_0 \in ds(g_i(t_0))$). Since a_{safe} has ensured $p(ds(g_i(t_0))) = true$, the adversary's inference will not violate the privacy property $p(.)$. That is, a_{safe} is indeed a safe algorithm.

A subtlety here is that the definition of disclosure set may seem to be a circular definition: $ds(.)$ is defined using $path(.)$, $path(.)$ using the algorithm a_{safe}, which in turn depends on $ds(.)$. However, this is not the case. In defining the disclosure set, $ds(g_i(t))$ depends on the truth value of the condition $i \notin path(t)$. In table 3.7, we can observe that this truth value can be decided in line 3, right before $ds(g_i(t))$ is needed (in line 4). Therefore, both concepts are well defined.

On the other hand, we can see that for computing $ds(g_i(t_0))$, we must compute the truth value of the condition $i \notin path(t)$ for every $t \in per(g_i(t_0))$. Moreover, to construct $path(t)$ requires us to simulate the execution of a_{safe} with t as the input. Therefore, to compute $ds(g_i(t_0))$, we will have to compute $ds(g_j(t))$ for all $t \in per(g_i(t_0))$ and $j = 1, 2, \ldots, i-1$. Clearly, this is an expensive process. In next section, we shall investigate a novel family of algorithms for reducing the cost.

3.3 k-jump Strategy

In this section, we first introduce the k-jump strategy in Section 3.3.1, and then discuss its properties in Section 3.3.2.

3.3.1 The Algorithm Family $a_{jump}(\vec{k})$

In the previous section, we have shown that the naive strategy is unsafe, and the safe strategy is safe but may incur a high cost due to the inherently recursive process. First, we more closely examine the limitation of these algorithms in order to build intuitions toward our new solution. In Figure 3.1, the upper and middle chart shows the decision process of the previous two algorithms, a_{naive} and a_{safe}, respectively. Each box represents the i^{th} iteration of the algorithm. Each diamond represents an evaluation of the privacy property $p(.)$ on the set inside the diamond, and the symbol Y and N denotes the result of such an evaluation to be $true$ and $false$, respectively.

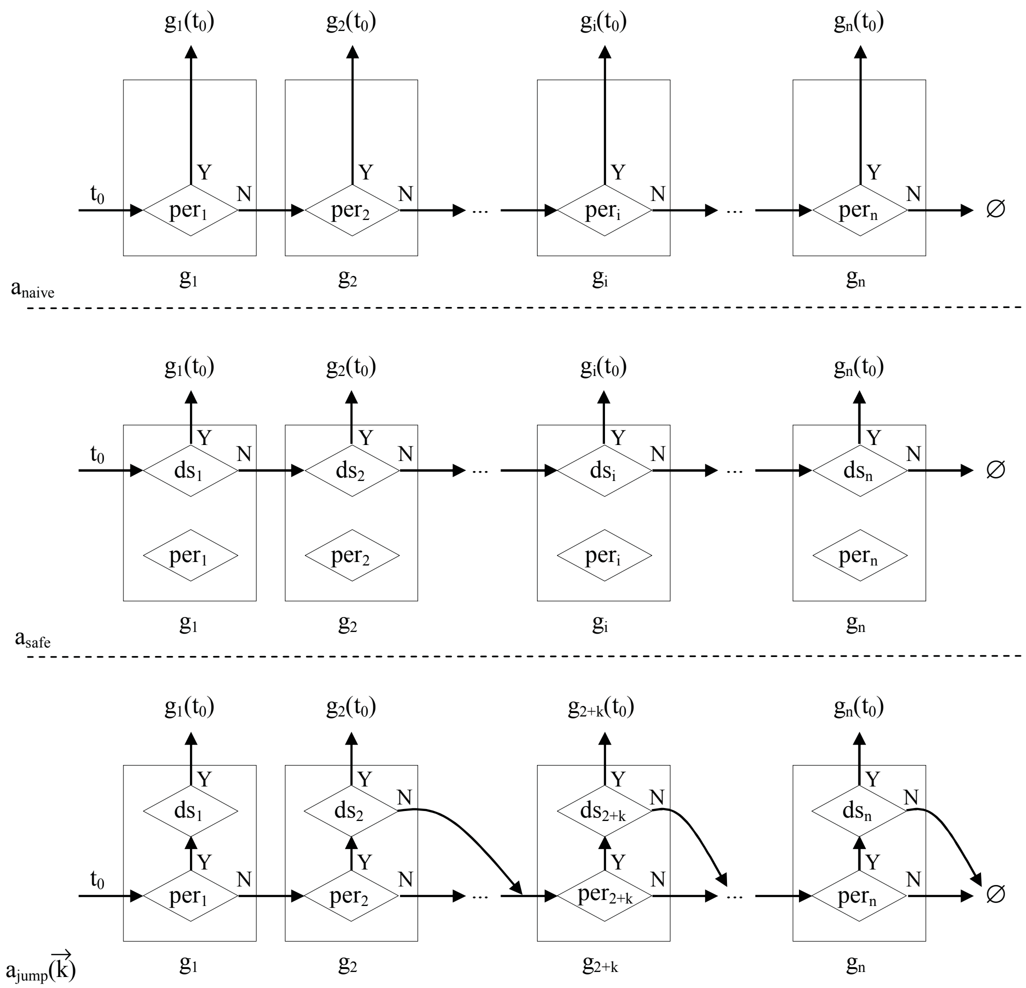


Figure 3.1: The Decision Process of Different Strategies

Comparing the two charts, we can have four different cases in each iteration of the algorithm (some iterations actually have less possibilities, as we shall show later):

1. If $p(per_i) = p(ds_i) = false$ (recall that per_i is an abbreviation of $per(g_i(t_0))$), then

clearly, both algorithms will immediately move to the next iteration.

2. If $p(per_i) = p(ds_i) = true$, both algorithms will disclose $g_i(t_0)$ and terminates.

3. We delay the discussion of the case of $p(per_i) = false \wedge p(ds_i) = true$ to later sections.

4. We can see the last case, $p(per_i) = true \wedge p(ds_i) = false$, is the main reason that a_{naive} is unsafe, and that a_{safe} must compute the disclosure set and consequently result in an expensive recursive process.

Therefore, informally, we penalize the last case, by *jumping* over the next $k-1$ iterations of the algorithm. As a result, we have the *k-jump strategy* as illustrated in the lower chart of Figure 3.1. More formally, the family of algorithms under the k-jump strategy is shown in Table 3.8.

Input: Table t_0, vector $\vec{k} \in [1, n]^n$;
Output: Generalization g or $\emptyset$;
Method:
Let $path(t_0) = \emptyset$;
Let $i = 1$;
While $(i \leq n)$
    Let $path(t_0) = path(t_0) \cup \{(i, 0)\}$; //the pair $(i, 0)$ represents per_i
    If $p(per(g_i(t_0))) = true$ **then**
        Let $path(t_0) = path(t_0) \cup \{(i, 1)\}$; //the pair $(i, 1)$ represents ds_i
        If $p(ds(g_i(t_0))) = true$ **then**
            Return $g_i(t_0)$;
        Else
            Let $i = i + \vec{k}[i]$; //$\vec{k}[i]$ is the i^{th} element of $\vec{k}$
    Else
        Let $i = i + 1$;
Return $\emptyset$;

Table 3.8: The Algorithm Family $a_{jump}(\vec{k})$

There are two main differences between $a_{jump}(\vec{k})$ and a_{safe}. First, since now in each iteration the algorithm may evaluate per_i and ds_i, or per_i only, we slightly change the definition of evaluation path to be $path(.) : T \rightarrow 2^{[1,n] \times \{0,1\}}$ so $(i, 0)$ stands for per_i and

$(i, 1)$ for ds_i. Consequently, the definition of a disclosure set also needs to be revised by replacing the condition $i \notin path(t)$ with $(i, 1) \notin path(t)$.

Second, the algorithm family $a_{jump}(\vec{k})$ takes an additional input, an n-dimensional vector $\vec{k} \in [1, n]^n$, namely, the *jump distance* vector. In the case of $p(per_i) = true \wedge p(ds_i) = false$, the algorithm will directly jump to the $(i + \vec{k}[i])^{th}$ iteration (note that jumping to the i^{th} iteration for any $i > n$ will simply lead to line 13 of the algorithm, that is, to disclose nothing). In the special case that $\forall i \in [1, n]\ \vec{k}[i] = k$ for some integer k, we shall abuse the notation to simply use k for $\vec{k}$.

Despite the difference between a_{safe} and $a_{jump}(\vec{k})$, the final condition for disclosing a generalization remains the same, that is, $p(ds_i) = true$. This simple fact suffices to show $a_{jump}(\vec{k})$ to be a safe family of algorithms.

3.3.2 Properties of $a_{jump}(\vec{k})$

We discuss several properties of the algorithms $a_{jump}(\vec{k})$ in the following.

3.3.2.1 Computation of the Disclosure Set

Again, the disclosure set is well defined under $a_{jump}(\vec{k})$, although it may seem to be a circular definition at first glance. First, $ds(g_i(t))$ depends on the truth value of the condition $(i, 1) \notin path(t)$. In table 3.8, we can then observe that this value can be decided in line 6, right before $ds(g_i(t))$ is needed (in line 7).

Although computing disclosure sets under $a_{jump}(\vec{k})$ is similar to that under a_{safe}, the former is generally more efficient. Specifically, recall that under a_{safe}, to compute $ds(g_i(t_0))$ we must first compute $ds(g_j(t))$ for all $t \in per(g_i(t_0))$ and $j = 1, 2, \ldots, i-1$. In contrast, this expensive recursive process is not always necessary under $a_{jump}(\vec{k})$.

Referring to the lower chart in Figure 3.1, to compute $ds(g_i(t_0))$ for any $2 < i < 2 + k$, we no longer need to always compute $ds(g_2(t))$ for every $t \in per_i$. By definition, $ds(g_i(t_0)) = per(g_i(t_0)) \setminus \{t : (i, 1) \notin path(t)\}$. From the chart, it is evident that $(i, 1) \notin$

$path(t)$ is true as long as $p(per(g_2(t))) = true$ (in which case $path(t)$ will either terminates at ds_2 or jump over the i^{th} iteration). Therefore, for any such table t, we do not need to compute $ds(g_2(t))$ in computing $ds(g_i(t_0))$.

As an extreme case, when the jump distance vector is $(n, n-1, \ldots, 1)$, all the jumps end at $\emptyset$ (disclosing noting). In this case, the computation of disclosure set is no longer a recursive process. To compute $ds(g_i(t_0))$, it suffices to only compute $per(g_j(t))$ for $t \in per(g_i(t_0))$ and $j = 1, 2, \ldots, i-1$. The complexity is thus significantly lower.

3.3.2.2 $ds(g_1(t_0))$ and $ds(g_2(t_0))$

The first two disclosure sets have some special properties. First of all, $ds(g_1(t_0) = per(g_1(t_0))$ is true. Intuitively, since any given table itself generally does not satisfy the privacy property, in computing ds_1, an adversary cannot exclude any table from per_1. More specifically, when $g_1(t_0)$ is disclosed, for all $t \in per(g_1(t_0))$, $path(t)$ must always end at ds_1, because $p(per(g_1(t))) = true$ follows from the fact that $per(g_1(t)) = per(g_1(t_0))$ (by the definition of permutation set) and $p(per(g_1(t_0))) = true$ (by the fact that $g_1(t_0)$ is disclosed). Therefore, $ds(g_1(t_0)) = per(g_1(t_0)) \setminus \{t : (1,1) \notin path(t)\}$ yields $ds(g_1(t_0) = per(g_1(t_0))$.

Second, we show that $ds(g_2(t_0))$ is independent of the distance vector $\vec{k}$. That is, all algorithms in $a_{jump}(\vec{k})$ share the same $ds(g_2(t_0))$. By definition, $ds(g_2(t_0)) = per(g_2(t_0)) \setminus \{t : (2,1) \notin path(t)\}$. As $ds(g_1(t_0) = per(g_1(t_0))$ is true, the case $p(per(g_1(t_0))) = true \wedge p(ds(g_1(t_0))) = false$ is impossible, and consequently the jump from ds_1 is never to happen (which explains the missing edge in the lower chart of Figure 3.1). Therefore, the condition $(2,1) \notin path(t)$ does not depend on the distance vector $\vec{k}$.

3.3.2.3 Size of the Family

First, with n generalization functions, we can have roughly $(n-1)!$ different jump distance vectors since the i^{th} $(2 \leq i \leq n)$ iteration may jump to $(n-i+1)$ different destinations , where the $(n+1)^{th}$ iteration means disclosing nothing. Clearly, $(n-1)!$ is a very large number even for a reasonably large n. Moreover, the space of jump distance vectors will be further increased when we *reuse* generalization functions in a meaningful way, as will be shown in later sections. Therefore, we can now transform any given unsafe algorithm a_{naive} into a large family of safe algorithms. This fact lays a foundation for making secret choices of k-jump algorithm to prevent adversarial inferences.

Note here the jump distance refers to possible ways an algorithm may jump at each iteration, which is not to be confused with the evaluation path of a specific table. For example, the vector $(n, n-1, \ldots, 1)$ yields a valid k-jump algorithm that always jumps to disclosing nothing, whereas any specific evaluation path can include at most one of such jumps. There is also another plausible but false perception related to this. That is, an algorithm with the jump distance k (note that here k denotes a vector whose elements are all equal to k) will only disclose a generalization under $g_i(.)$ where i is a multiplication of k. This perception may lead to false statements about data utility, for example, that the data utility for $k = 2$ is better than that for $k = 4$. In fact, regardless of the jump distance, an algorithm may potentially disclose a generalization under every $g_i(.)$. The reason is that each jump is only possible, but not mandatory for a specific table.

3.4 Data Utility Comparison

In this section, we compare the data utility of different algorithms. Section 3.4.1 considers the family of k-jump algorithms. Section 3.4.2 studies the case when some generalization functions are reused in an algorithm. Section 3.4.3 addresses a_{safe}.

3.4.1 Data Utility of k-Jump Algorithms

Our main result is that the data utility of two k-jump algorithms $a_{jump}(\vec{k})$ and $a_{jump}(\vec{k'})$ from the same family is generally incomparable. That is, the data utility cannot simply be ordered based on the jump distance of two algorithms. Note that, deterministically the data utility cannot be improved without the given table, and the data utility among algorithms is only comparable for the given table. In other words, here the comparison of data utility is independent of the given table, accordingly, the notation $a_{jump}(\vec{k})$ does not indicate the given table.

We do not rely on specific utility measures. Instead, the generalization functions are assumed to be sorted in a non-increasing order of their data utility. Consequently, an algorithm a_1 is considered to have better or equal data utility compared to another algorithm a_2 (both algorithms are from the same family), if we can construct a table t for which a_1 returns $g_i(t)$ and a_2 returns $g_j(t)$, with $i < j$.

Such a construction is possible with two methods. First, we let $path(t)$ under a_2 to jump over the iteration in which a_1 terminates. Second, when the first method is not an option, we let $path(t)$ under a_2 to include a disclosure set that does not satisfy the privacy property $p(.)$, whereas $path(t)$ under a_1 to include one that does. We first consider the following two special cases.

- $a_{jump}(1)$ *and* $a_{jump}(i)$ *(*$i > 1$*)* In this case, the evaluation path of $a_{jump}(1)$ can never jump over that of $a_{jump}(i)$ (in fact, a jump distance of 1 means no jump at all). Therefore, we apply the above second method, that is, to rely on different disclosure sets of the same disclosed generalization.
- $a_{jump}(i)$ *and* $a_{jump}(j)$ *(*$1 < i < j$*)* For this case, we apply the above first method, that is, by constructing an evaluation path that jumps over the other.

From now on, we shall add superscripts to existing notations to denote the distance vector of different algorithms. For example, ds_1^k means the disclosure set ds_1 under the algorithm $a_{jump}(k)$.

3.4.1.1 $a_{jump}(1)$ vs. $a_{jump}(i)$ ($i > 1$)

First, we need the following result.

Lemma 3.1 *For any $a_{jump}(1)$ and $a_{jump}(i)$ ($i > 1$) algorithms from the same family, we have $ds_3^i \subseteq ds_3^1$.*

Proof: By definition, $ds(g_3(t_0)) = per(g_3(t_0)) \setminus \{t : (3,1) \notin path(t)\}$. Obviously, for $a_{jump}(1)$, the disclosure set $ds_3^1(t_0)$ is derived from the permutation set of $g_3(t_0)$ by excluding those are disclosed under g_1 and g_2, while for $a_{jump}(i)$ ($i > 1$), the disclosure set $ds_3^i(t_0)$ is derived from the permutation set of $g_3(t_0)$ by excluding those permutation set are safe under g_1 or g_2. In other words, to remove a table t from $per(g_3(t_0))$, not only the permutation set but also disclosure set of $g_2(t)$ must satisfy the privacy property for $a_{jump}(1)$; while only permutation set of $g_2(t)$ must satisfy the privacy property for $a_{jump}(i)$ ($i > 1$), since in this case, no matter whether the disclosure set satisfies or not, $(3,1) \notin path(t)$. Formally,

$$\begin{aligned} ds_3^1(t_0) =& per_3(t_0)/\{t|(t \in per_3(t_0)) \wedge (p(per_1(t)) = true \\ & \vee (p(per_2(t)) = true \wedge p(ds_2^1(t)) = true))\} \end{aligned} \tag{3.1}$$

$$ds_3^i(t_0) = per_3(t_0)/\{t|(t \in per_3(t_0)) \wedge (p(per_1(t)) = true \vee p(per_2(t)) = true)\} \tag{3.2}$$

from which the result follows. ■

From Lemma 3.1, we can have the following straightforward result for the case that privacy property is set-monotonic (which $p(S) = true$ implies $\forall S' \supseteq S \; p(S') = true$). This result is needed for proving Theorem 3.1.

Lemma 3.2 *The data utility of $a_{jump}(1)$ is always better than or equal to that of $a_{jump}(i)$ ($i > 1$) when both algorithms are from the same family with a set-monotonic privacy property $p(.)$ and $n = 3$.*

Proof: As shown in Section 3.3.2.2, $per_1(t_0)$, $per_2(t_0)$, and $ds_2(t_0)$ are identical once the sequence of generalization functions are given. Therefore, either t_0 can be released by g_1 (or g_2) in both $a_{jump}(1)$ and $a_{jump}(i)$ ($i > 1$), or it cannot be in both of them.

For g_3, based on Lemma 3.1 and the definition of set-monotonic, $ds_3^i(t_0)$ satisfies privacy property only if $ds_3^1(t_0)$ satisfies. The proof is complete. ■

Theorem 3.1 *For any $i > 1$, there always exist cases in which the data utility of the algorithm $a_{jump}(i)$ is better than that of $a_{jump}(1)$, and vice versa.*

Proof: The key is to have different disclosure sets ds_3 under the two algorithms such that one satisfies $p(.)$ and the other fails. Figure 3.2 illustrates such evaluation paths.

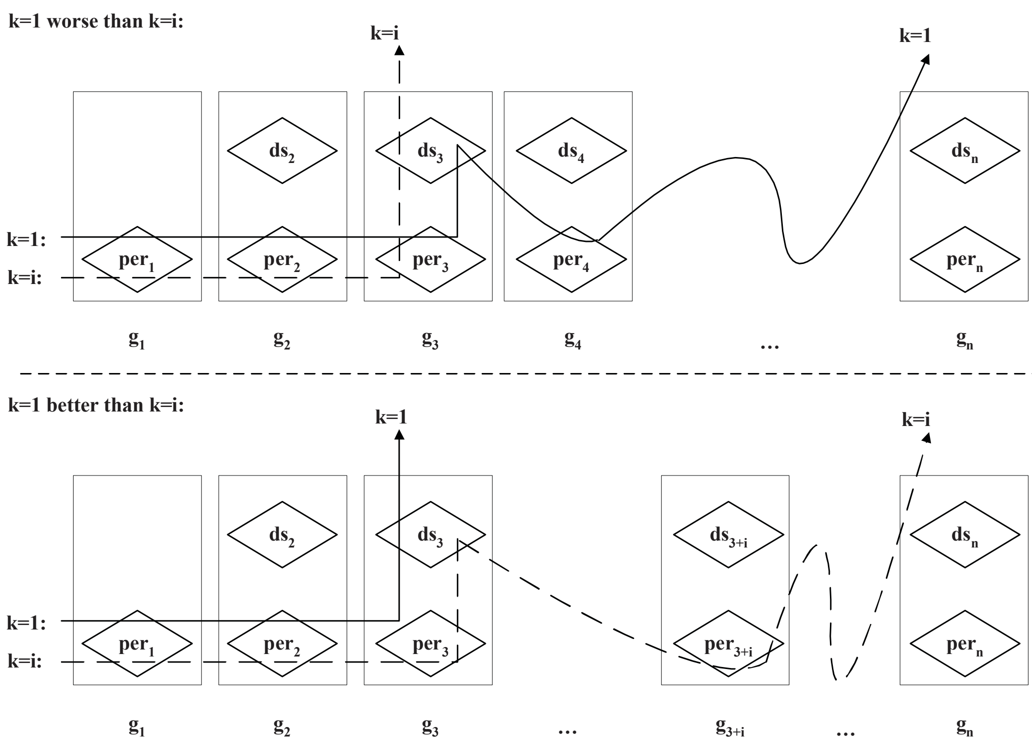


Figure 3.2: The Construction for $a_{jump}(1)$ and $a_{jump}(i)$ $(1 < i)$

By Lemma 3.2, the case where the data utility of $a_{jump}(1)$ is better than or equal to that of $a_{jump}(i)$ ($i > 1$) is trivial to construct and hence is omitted. We only show the other case where $a_{jump}(i)$ has better data utility. Basically, we need to design a table to satisfy the following. First, per_1 and per_2 do not satisfy $p(.)$ while per_3 does. Second, $p(ds_3^i) = true$ and $p(ds_3^1) = false$ are both true.

Table 3.9 shows our construction for the proof. The privacy property $p(.)$ is that the highest ratio of a sensitive value in a group must be no greater than $\frac{1}{2}$. Notice that here (and in the remainder of the paper) $p(.)$ is not necessarily set-monotonic. We show that $a_{jump}(i)$ can disclose using g_3, whereas $a_{jump}(1)$ cannot.

1. For this special case, $ds_3^k(t_0)$ can be computed by first excluding any table t for which $p(per_1(t)) = true$. The tables in $ds_3^i(t_0)$ must belong to one of the following four disjoint sets.

   In the first case, I has sensitive value C_6. The number of tables in this case is $\binom{2}{1} \times \binom{2}{1} \times (\binom{4}{1} \times \binom{3}{1}) \times (\binom{6}{2} \times \binom{4}{2}) = 48 \times 90 = 4320$. Denote this set by S_1. In the other three cases, I does not have C_6 and both N and O have C_7, C_8, or C_9, denoted respectively by S_2, S_3, and S_4. Each of these includes $\binom{2}{1} \times \binom{2}{1} \times (\binom{4}{1} \times \binom{3}{1}) \times \binom{2}{1} \times (\binom{4}{1} \times \binom{3}{1}) = 48 \times 24 = 1152$ tables.

   Now consider generalizing these tables using g_2. All tables in the last three sets cannot be disclosed under g_2 since each of their permutation sets under g_2 fails the privacy property. For the same reason, tables in the first set in which both N and O have C_7, C_8, or C_9, which is denoted as S_1', cannot be disclosed under g_2, either. The cardinality of S_1' is $\binom{2}{1} \times \binom{2}{1} \times (\binom{4}{1} \times \binom{3}{1}) \times \binom{4}{2} \times \binom{3}{1} = 48 \times 18 = 864$.

   For $a_{jump}(i)$, all the tables in $(S_1 \setminus S_1')$ will be excluded from $ds_3^i(t_0)$. The reason is the following. Each of their permutation sets under g_2 satisfies the privacy property, so $a_{jump}(i)$ will disclose them either under g_2 or after g_3. Therefore, $ds_3^i(t_0) = S_1' \cup S_2 \cup S_3 \cup S_4$. The highest ratio of sensitive value is that of A and B associated with C_0 or C_1, which is $\frac{1}{2}$. Since $ds_3^i(t_0)$ satisfies the privacy property, it can be disclosed using g_3 under $a_{jump}(i)$.

2. As to the case of $a_{jump}(1)$, the disclosure set of all the tables in $S_1 \setminus S_1'$ do not satisfy the privacy property and hence all of them cannot be removed from $ds_3^1(t_0)$. The reason is as follows. First, the permutation set of each such table under g_2 satisfies the privacy property. Next, consider their disclosure sets under g_2. The set $S_1 \setminus S_1'$ can be further divided into three disjoint subsets as follows.

- Either N or O has C_7 and the other has C_8. This subset has $\binom{2}{1} \times \binom{2}{1} \times (\binom{4}{1} \times \binom{3}{1}) \times \binom{1}{1} \times (\binom{4}{1} \times \binom{3}{1}) \times \binom{2}{1} = 48 \times 24 = 1152$ tables. Based on the sensitive value of H, this subset can be further divided into two disjoint subsets again.

  (a) H has C_6. This subset has $\binom{2}{1} \times \binom{2}{1} \times (\binom{3}{1} \times \binom{2}{1}) \times \binom{1}{1} \times (\binom{4}{1} \times \binom{3}{1}) \times \binom{2}{1} = 48 \times 12 = 576$ tables. For each table in this subset, to obtain its disclosure set, we must exclude the tables that can be disclosed under g_1 from its permutation set following the same rule as above. The tables in its disclosure set must satisfy that both H and I have C_6. The ratio of both H and I being associated with C_6 is $1.0 > 0.5$. This clearly violates the privacy property.

  (b) H does not have sensitive value C_6, but has either C_4 or C_5. This subset has $\binom{2}{1} \times \binom{2}{1} \times \binom{3}{1} \times \binom{2}{1} \times \binom{1}{1} \times (\binom{4}{1} \times \binom{3}{1}) \times \binom{2}{1} = 48 \times 12 = 576$ tables. Similarly, the tables in the disclosure set must satisfy that two from the set $\{E, F, G\}$ have C_6. Moreover, one and only one of H and I has C_6. Therefore, the ratio of both E, F, and G being associated with C_6 is $\frac{2}{3} > 0.5$. This also violates the privacy property.

  In summary, the disclosure set of every table in this subset under function g_2 will violate the privacy property, and consequently these tables cannot be disclosed under g_2. Therefore, the algorithm $a_{jump}(1)$ must continue to evaluate these tables under g_3 whose permutation set satisfies the privacy property.

- The other two cases are that N and O have C_7 and C_9, respectively, or C_8 and C_9, respectively. Similarly, each has 1152 tables, and for the same reason as above, the disclosure set of each table in each subset does not satisfy the privacy property, and hence cannot be disclosed under g_2.

Consequently, all the tables in $S_1 \setminus S_1'$ cannot be removed from $ds_3^1(t_0)$. Therefore, $ds_3^1(t_0) = S_1 \cup S_2 \cup S_3 \cup S_4$. The ratio of I being associated with C_6 is $\frac{48\times 90}{48\times(90+24\times 3)} =$

$0.556 > 0.5$. This violates the privacy property. Therefore, the given table cannot be disclosed using g_3 under $a_{jump}(1)$.

■

QID	g_1	g_2	g_3	...
A	C_0	C_0	C_0	...
B	C_1	C_1	C_1	...
C	C_2	C_2	C_2	...
D	C_3	C_3	C_3	...
E	C_4	C_4	C_4	...
F	C_5	C_5	C_5	...
G	C_6	C_6	C_6	...
H	C_6	C_6	C_6	...
I	C_6	C_6	C_6	...
J	C_7	C_7	C_7	...
K	C_7	C_7	C_7	...
L	C_8	C_8	C_8	...
M	C_8	C_8	C_8	...
N	C_9	C_9	C_9	...
O	C_9	C_9	C_9	...

Table 3.9: The Case Where $a_{jump}(i)$ Has Better Utility Than $a_{jump}(1)$

3.4.1.2 $a_{jump}(i)$ vs. $a_{jump}(j)$ $(1 < i < j)$

Next, we prove the data utility of $a_{jump}(i)$ and $a_{jump}(j)$ to be incomparable by constructing non-overlapping evaluation paths.

Theorem 3.2 *For any $j > i > 1$, there always exist cases where the data utility of the algorithm $a_{jump}(i)$ is better than that of $a_{jump}(j)$, and vice versa.*

Proof: Since both $a_{jump}(i)$ and $a_{jump}(j)$ can jump over iterations of the algorithm, we can easily construct evaluation paths for the proof. Figure 3.3 illustrates such constructed paths.

Firstly, the case where $a_{jump}(i)$ has better utility than $a_{jump}(j)$ $(1 < i < j)$ is relatively easier to construct. We basically need to construct a case satisfying the following

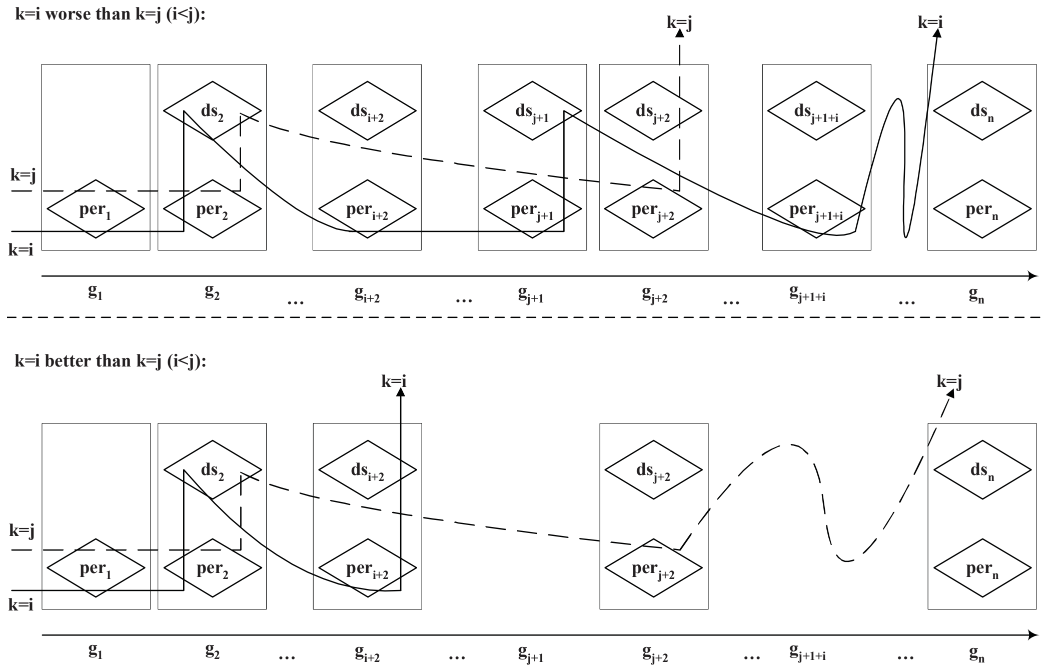


Figure 3.3: The Construction for $a_{jump}(i)$ and $a_{jump}(j)$ $(1 < i < j)$

conditions:

$$\begin{cases} (\text{if } \omega = 1), & p(per_\omega) = false; \\ (\text{if } \omega = 2), & p(per_\omega) = true \wedge p(ds_\omega) = false; \\ (\text{if } \omega = i+2), & p(per_\omega) = true \wedge p(ds^i_\omega) = true. \end{cases}$$

The above conditions imply that g_{i+2} will be used to disclose under $a_{jump}(i)$, while the algorithm $a_{jump}(j)$ will jump over the $(i+2)^{th}$ function to disclose under or after g_{j+2} since permutation set of g_2 satisfies privacy property while disclosure set of g_2 does not.

Secondly, we show the construction for the other case where $a_{jump}(i)$ has worse utility than $a_{jump}(j)$ $(1 < i < j)$. We basically need to construct a case satisfying the following conditions:

$$
\begin{cases}
(\text{if } \omega = 1), & p(per_\omega) = false; \\
(\text{if } \omega = 2), & p(per_\omega) = true \wedge p(ds_\omega^{i,j}) = false; \\
(\forall \omega \in [3, j]), & p(per_\omega) = false; \\
(\forall \omega \in [j+1, j+2]), & p(per_\omega) = true; \\
(\text{if } \omega = j+1), & p(ds_\omega^{i}) = false; \\
(\text{if } \omega = j+2), & p(ds_\omega^{j}) = true.
\end{cases}
$$

The above conditions imply that g_{j+2} will be used to disclose under $a_{jump}(j)$. On the other hand, when $a_{jump}(i)$ evaluates g_{i+2}, since its permutation set does not satisfy the privacy property, the algorithm will move to the next function, and repeat this until it reaches g_{j+1}. Since $ds_{j+1}^{i}(t_0)$ does not satisfy the privacy property, the algorithm will jump to g_{j+1+i} and will disclose using a function beyond g_{j+2}.

Table 3.10 shows our construction where the privacy property is again that the highest ratio of a sensitive value is no greater than $\frac{1}{2}$. We assume the table has many others tuples not shown (the purpose of these additional tuples is only to ensure the data utility of the generalizations is in a non-increasing order). The left (right) side of Table 3.10 shows the case where the data utility of $a_{jump}(i)$ is better (worse) than that of $a_{jump}(j)$ ($1 < i < j$). Without loss of generality, we discuss the first 12 tuples in these two tables.

Firstly, we discuss the left side of Table 3.10. The given table, denoted by t_0, cannot be disclosed under g_1 since $p(per_1) = false$. For g_2, we have $p(per_2) = true$. The tables in ds_2 (note that $ds_2^i \equiv ds_2^j$ as shown in Section 3.3.2.2) satisfy that E, F, G, and H have the sensitive value C_4, S, S, and C_5, respectively. Clearly, $p(ds_2) = false$, and $g_2(t_0)$ cannot be disclosed, either. Then $a_{jump}(i)$ and $a_{jump}(j)$ will jump to evaluate under g_{i+2} and g_{j+2}, respectively.

Now we show that $a_{jump}(i)$ can be disclosed using g_{i+2}. The ds^i_{i+2} can be computed first by excluding the tables $\{t : p(per_1(t) = true\}$. The tables in ds^i_{i+2} must belong to one of the following three disjoint sets.

1. Two of A, B, and C have S. This subset has $\binom{3}{1} \times \binom{5}{1} \times 4! \times 5! = 3 \times 5! \times 5!$ tables.

2. Both D and E have S. This subset has $5! \times 5!$ tables.

3. Both F and G have S. This subset also has $5! \times 5!$ tables.

Next, $a_{jump}(i)$ will evaluate these tables using g_2. Clearly, the permutation set of each of these tables satisfies privacy property. The $a_{jump}(i)$ will further evaluate their ds_2. As discussed above, all the tables in last set cannot be disclosed under g_2. Similarly, those in second set cannot either. For the first set, all the tables which D has S are safe under g_1. In other words, the ds_2 for each table in this set satisfies that two of A, B, and C have S, which violates the privacy property. Summarily, all these tables are in $ds^i_{i+2}(t_0)$. The ratio of A, B, and C being associated with S are $\frac{2}{5}$, which is the highest ratio. Thus, $g^i_{i+2}(t_0)$ can be safely released. Besides, $a_{jump}(j)$ must disclose table t_0 under or after g_{j+2}, therefore, in this case, $a_{jump}(i)$ has better data utility than $a_{jump}(j)$.

Secondly, we discuss the right side of Table 3.10. Similarly, $a_{jump}(i)$ will jump to evaluate g_{i+2} while $a_{jump}(j)$ will jump to g_{j+2}. For $a_{jump}(j)$, since $p(per_{j+2}) = true$ and $p(ds^j_{j+2}) = true$ (The ratio of A, B, C, J, K, and L being associates with S is $\frac{2}{9}$ which is highest ratio), therefore, $a_{jump}(j)$ will disclose g_{j+2}. For $a_{jump}(i)$, since $p(per_{i+2}) = false$, it will move to evaluate g_{i+3} and repeat until g_{j+1} due to the same reason. Obviously, the tables in ds^i_{j+1} satisfy that both F and G have sensitive value S, which violates the privacy property. Therefore, the algorithm $a_{jump}(i)$ jumps beyond g_{j+2} since $j + 2 < j + 1 + i$. Clearly, with these constructions, both $a_{jump}(i)$ and $a_{jump}(j)$ will follow the desired evaluation paths as shown in Figure 3.3. ■

(a). $a_{jump}(i)$ better than $a_{jump}(j)$

QID	g_1	g_2	$\ldots$	g_{i+2}	$\ldots$	g_{j+2}	$\ldots$
A	C_0	C_0	$\ldots$	C_0	$\ldots$	C_0	$\ldots$
B	C_1	C_1	$\ldots$	C_1	$\ldots$	C_1	$\ldots$
C	C_2	C_2	$\ldots$	C_2	$\ldots$	C_2	$\ldots$
D	C_3	C_3	$\ldots$	C_3	$\ldots$	C_3	$\ldots$
E	C_4	C_4	$\ldots$	C_4	$\ldots$	C_4	$\ldots$
F	S	S	$\ldots$	S	$\ldots$	S	$\ldots$
G	S	S	$\ldots$	S	$\ldots$	S	$\ldots$
H	C_5	C_5	$\ldots$	C_5	$\ldots$	C_5	$\ldots$
I	C_6	C_6	$\ldots$	C_6	$\ldots$	C_6	$\ldots$
J	C_7	C_7	$\ldots$	C_7	$\ldots$	C_7	$\ldots$
K	C_8	C_8	$\ldots$	C_8	$\ldots$	C_8	$\ldots$
L	C_9	C_9	$\ldots$	C_9	$\ldots$	C_9	$\ldots$
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$

(b). $a_{jump}(i)$ worse than $a_{jump}(j)$

QID	g_1	g_2	g_3	$\ldots$	g_j	g_{j+1}	g_{j+2}	$\ldots$
A	C_0	C_0	C_0	$\ldots$	C_0	C_0	C_0	$\ldots$
B	C_1	C_1	C_1	$\ldots$	C_1	C_1	C_1	$\ldots$
C	C_2	C_2	C_2	$\ldots$	C_2	C_2	C_2	$\ldots$
D	C_3	C_3	C_3	$\ldots$	C_3	C_3	C_3	$\ldots$
E	C_4	C_4	C_4	$\ldots$	C_4	C_4	C_4	$\ldots$
F	S	S	S	$\ldots$	S	S	S	$\ldots$
G	S	S	S	$\ldots$	S	S	S	$\ldots$
H	C_5	C_5	C_5	$\ldots$	C_5	C_5	C_5	$\ldots$
I	C_6	C_6	C_6	$\ldots$	C_6	C_6	C_6	$\ldots$
J	C_7	C_7	C_7	$\ldots$	C_7	C_7	C_7	$\ldots$
K	C_8	C_8	C_8	$\ldots$	C_8	C_8	C_8	$\ldots$
L	C_9	C_9	C_9	$\ldots$	C_9	C_9	C_9	$\ldots$
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$

Table 3.10: The Data Utility Comparison Between $a_{jump}(j)$ and $a_{jump}(i)$ ($1 < i < j$)

3.4.1.3 $a_{jump}(\vec{k_1})$ vs. $a_{jump}(\vec{k_2})$ ($\vec{k_1} \neq \vec{k_2}$)

Next, we extend the above results to the more general case in which the two algorithms $a_{jump}(\vec{k_1})$ and $a_{jump}(\vec{k_2})$ both have an n-dimensional vector as their jump distances.

Theorem 3.3 *For any $\vec{k_1}, \vec{k_2} \in [1, n]^n$, there always exist cases in which the data utility of the algorithm $a_{jump}(\vec{k_1})$ is better than that of $a_{jump}(\vec{k_2})$, and vice versa.*

Proof: Suppose the first element with different jump distance of $\vec{k_1}$ and $\vec{k_2}$ is the i^{th} element. Without the loss of generality, assume that $\vec{k_1}[i] < \vec{k_2}[i]$. Figure 3.4 illustrates such constructed paths. There are two cases as follows,

First, $\vec{k_1}[i] = 1$: Since $ds_l^{\vec{k_1}} = ds_l^{\vec{k_2}}$ for all $1 \leq l \leq i$, and $ds_{i+1}^{\vec{k_1}} \supseteq ds_{i+1}^{\vec{k_2}}$, we can construct in a similar way as in the proof of Theorem 3.1. Basically, we construct the following evaluation path: $per_1 \to per_2 \to \ldots \to per_i \to per_{i+1} \to ds_{i+1}$ so that in one case we have $p(ds_{i+1}^{\vec{k_1}}) = true \wedge p(ds_{i+1}^{\vec{k_2}}) = false$, whereas in the other case we have $p(ds_{i+1}^{\vec{k_1}}) = false \wedge p(ds_{i+1}^{\vec{k_2}}) = true$.

Second, $\vec{k_1}[i] > 1$: In this case, we consider two sub-cases.

1. $(\exists j)((i + \vec{k_1}[i] \leq j < i + \vec{k_2}[i]) \wedge (j + \vec{k_1}[j] > i + \vec{k_2}[i]))$:

   In this sub-case, we can construct the following two evaluation paths.

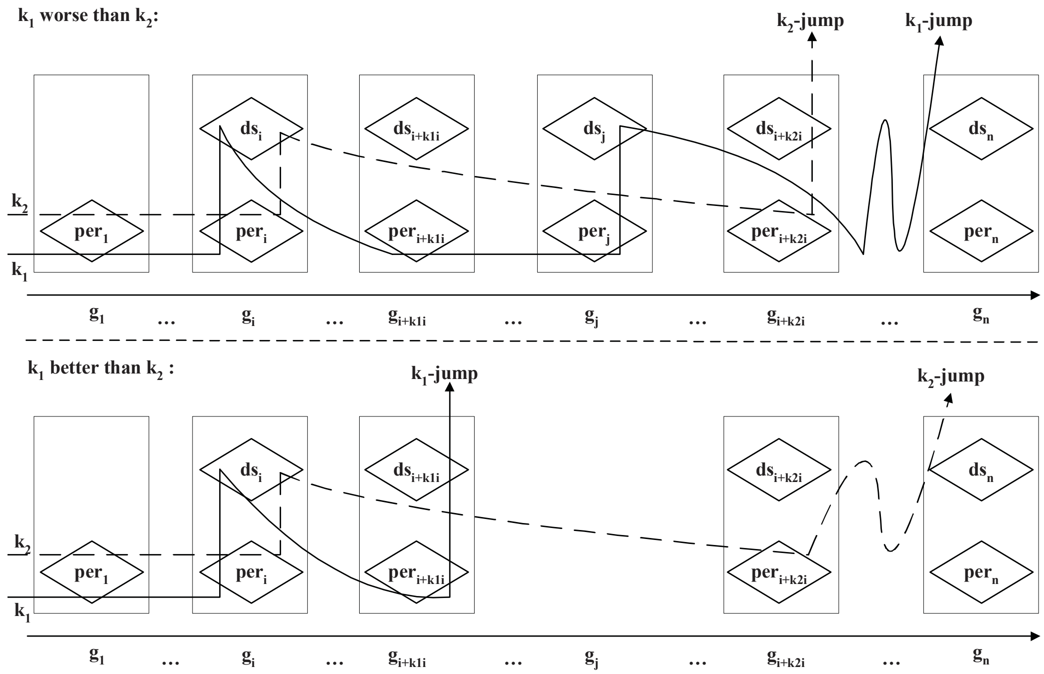


Figure 3.4: The Construction for $a_{jump}(\vec{k_1})$ and $a_{jump}(\vec{k_2})$ ($\vec{k_1} \neq \vec{k_2}$)

(a) $a_{jump}(\vec{k_1}) : per_1 \rightarrow per_2 \rightarrow \ldots \rightarrow per_i \rightarrow ds_i^{\vec{k_1}} \rightarrow per_{i+\vec{k_1}[i]} \rightarrow \ldots \rightarrow per_j \rightarrow ds_j^{\vec{k_1}} \rightarrow per_{j+\vec{k_1}[j]} \rightarrow \ldots$

$a_{jump}(\vec{k_2}) : per_1 \rightarrow per_2 \rightarrow \ldots \rightarrow per_i \rightarrow ds_i^{\vec{k_2}} \rightarrow per_{i+\vec{k_2}[i]} \rightarrow p(ds^{\vec{k_2}}_{i+\vec{k_2}[i]}) = true$

(b) $a_{jump}(\vec{k_1}) : per_1 \rightarrow per_2 \rightarrow \ldots \rightarrow per_i \rightarrow ds_i^{\vec{k_1}} \rightarrow per_{i+\vec{k_1}[i]} \rightarrow p(ds^{\vec{k_1}}_{i+\vec{k_1}[i]}) = true$

$a_{jump}(\vec{k_2}) : per_1 \rightarrow per_2 \rightarrow \ldots \rightarrow per_i \rightarrow ds_i^{\vec{k_2}} \rightarrow per_{i+\vec{k_2}[i]} \rightarrow \ldots$

Since $j + \vec{k_1}[j] > i + \vec{k_2}[i]$, the data utility of $a_{jump}(\vec{k_1})$ in the first case is worse than that of $a_{jump}(\vec{k_2})$. Meanwhile, since $i + \vec{k_1}[i] < i + \vec{k_2}[i]$, we have the converse result in the second case.

2. $\neg(\exists j)((i + \vec{k_1}[i] \leq j < i + \vec{k_2}[i]) \wedge (j + \vec{k_1}[j] > i + \vec{k_2}[i]))$:

   In this sub-case, $ds^{\vec{k_1}}_{i+\vec{k_2}[i]} \subseteq ds^{\vec{k_2}}_{i+\vec{k_2}[i]}$. We can reason as follows. The disclosure set of $g_{i+\vec{k_2}[i]}$ under $a_{jump}(\vec{k_2})$ is computed by excluding from its permutation set the tables which can be disclosed using g_1 and those which $p(per_2(t)) = true$; however, the disclosure set under $a_{jump}(\vec{k_1})$ needs to further exclude the tables which can be disclosed under some function g_j and $(j, 0)$ is in the evaluation path, where $(i + \vec{k_1}[i] \leq j \leq i + \vec{k_2}[i] - 1)$. Based on this result, we can construct the following evaluation paths.

(a) $a_{jump}(\vec{k_1}) : per_1 \to per_2 \to \ldots \to per_i \to ds_i^{\vec{k_1}} \to per_{i+\vec{k_1}[i]} \to \ldots \to per_{i+\vec{k_2}[i]} \to p(ds_{i+\vec{k_2}[i]}^{\vec{k_1}}) = false$
$a_{jump}(\vec{k_2}) : per_1 \to per_2 \to \ldots \to per_i \to ds_i^{\vec{k_2}} \to per_{i+\vec{k_2}[i]} \to p(ds_{i+\vec{k_2}[i]}^{\vec{k_2}}) = true$

(b) $a_{jump}(\vec{k_1}) : per_1 \to per_2 \to \ldots \to per_i \to ds_i^{\vec{k_1}} \to per_{i+\vec{k_1}[i]} \to \ldots \to per_{i+\vec{k_2}[i]} \to p(ds_{i+\vec{k_2}[i]}^{\vec{k_1}}) = true$
$a_{jump}(\vec{k_2}) : per_1 \to per_2 \to \ldots \to per_i \to ds_i^{\vec{k_2}} \to per_{i+\vec{k_2}[i]} \to p(ds_{i+\vec{k_2}[i]}^{\vec{k_2}}) = false$

Clearly, the data utility of $a_{jump}(\vec{k_1})$ in the first (second) case is worse (better) than that of $a_{jump}(\vec{k_2})$.

■

3.4.2 Reusing Generalization Functions

With the naive strategy, whether a generalization function satisfies the privacy property is independent of other functions. Therefore, it is meaningless to evaluate the same function more than once. However, we now show that with the k-jump strategy, it is meaningful to *reuse* a generalization function along the evaluation path. This will either increase the data utility of the original algorithm, or lead to new algorithms with incomparable data utility to enrich the the existing family of algorithms. That is, reusing generalization functions may benefit the optimization of data utility.

Theorem 3.4 *Given the set of generalization functions, there always exist cases in which the data utility of the algorithm with reusing generalization functions is better than that of the algorithm without reusing, and vice versa.*

Proof: Consider two algorithms a_1 and a_2 that define the functions $g_1, g_2, g_3, g_4, \ldots$ and $g_1, g_2, g_3, g_{2'}, g_4, \ldots$, respectively, where $g_{2'}(.)$ and $g_2()$ are identical. Suppose both algorithms has the same jump distance $k = 1$, and the privacy property is not set-monotonic. We can construct the following two evaluation paths.

1. $a_1(t_0) : per_1(t_0) \to per_2(t_0) \to ds_2^1(t_0) \to per_3(t_0) \to per_4(t_0) \ldots$
   $a_2(t_0) : per_1(t_0) \to per_2(t_0) \to ds_2^1(t_0) \to per_3(t_0) \to per_{2'}(t_0) \to ds_{2'}^1(t_0) \to p(ds_{2'}^1(t_0)) = true$

2. $a_1(t_0) : per_1(t_0) \to per_2(t_0) \to per_3(t_0) \to per_4(t_0) \to ds_4^1(t_0) \to p(ds_4^1(t_0)) = true$
   $a_2(t_0) : per_1(t_0) \to per_2(t_0) \to per_3(t_0) \to per_{2'}(t_0) \to per_4(t_0) \to ds_4^1(t_0) \to p(ds_4^1(t_0)) = false$

Clearly, the data utility of a_1 in the first case is worse than that of a_2, while in the second case it is better. ■

It is worth noting that although the same generalization function is repetitively evaluated, its disclosure set will depend on the functions that appear before it in the evaluation path. Take the identical functions g_2 and g_2' above as an example, the disclosure set of g_2 is computed by excluding from its permutation set the tables which can be disclosed under g_1; however, the disclosure set of g_2' needs to further exclude tables which can be disclosed under g_3. Therefore, $ds_{2'} \subseteq ds_2$. Generally, $ds_{i'} \subseteq ds_i$ when $g_i(.)$ is reused as $g_{i'}(.)$ in a later iteration. This leads to the following.

Proposition 3.1 *With a set-monotonic privacy property, reusing generalization functions in a k-jump algorithm does not affect the data utility under $a_{jump}(1)$.*

Proof: Suppose $g_i(.)$ is reused as $g_{i'}(.)$ in a later iteration of the algorithm. For any table t, since $ds_{i'}(t) \subseteq ds_i(t)$, $p(ds_{i'}(t)) = true$ implies $p(ds_i(t)) = true$ for any set-monotonic privacy property $p(.)$. Therefore, if $p(ds_{i'}(t)) = true$, the algorithm will disclose under $g_i(.)$; if $p(ds_{i'}(t)) = false$ then the algorithm will continue to the next iteration. In both cases, $g_{i'}(.)$ cannot exclude the tables from permutation set other than $g_i(.)$ can do, therefore, $g_{i'}(.)$ does not affect the data utility. ∎

On the other hand, when generalization functions are reused at the end of the original sequence of functions, some tables which will lead to disclosing nothing under the original sequence of functions may have a chance to be disclosed under the reused functions, which will improve the data utility.

Proposition 3.2 *Reusing a generalization function after the last iteration of an existing k-jump algorithm may improve the data utility when $p(.)$ is not set-monotonic.*

Proof: We construct a case in which reusing a function will improve the data utility. Consider two algorithms a_1 and a_2 that define the functions g_1, g_2, g_3 and $g_1, g_2, g_3, g_{2'}$, respectively, where $g_{2'}(.)$ and $g_2(.)$ are identical. Suppose both algorithms have the same jump distance $k = 1$, and the privacy property is not set-monotonic. We need to construct the following two evaluation paths by which a_1 will disclose nothing, while a_2 will disclose using $g_{2'}$.

1. $a_1(t_0) : per_1(t_0) \rightarrow per_2(t_0) \rightarrow ds_2^1(t_0) \rightarrow p(per_3(t_0)) = false$
2. $a_2(t_0) : per_1(t_0) \rightarrow per_2(t_0) \rightarrow ds_2^1(t_0) \rightarrow per_3(t_0) \rightarrow per_{2'}(t_0) \rightarrow ds_{2'}^1(t_0) \rightarrow p(ds_{2'}^1(t_0)) = true$

Table 3.11 shows our construction. The table will lead to disclosing nothing without reusing g_2, whereas reusing g_2 will lead to a successful disclosure. In this example, the jump distance is 1, and the privacy property is that the highest ratio of any sensitive value is no greater than $\frac{1}{2}$.

More specifically, the given table, denoted by t_0, cannot be disclosed under $g_1(.)$ or $g_3(.)$ since $p(per_1) = p(per_3) = false$. For g_2, we have $p(per_2) = true$. The tables in ds_2 must be in one of the following three disjoint sets.

1. C has the sensitive value C_3. The number of such tables is $\binom{2}{1} \times (\binom{4}{1} \times \binom{3}{1}) = 24$. Denote this set by S_1.

2. C does not have C_3, and both D and E have C_3. There are $\binom{2}{1} \times \binom{2}{1} \times \binom{2}{1} = 8$ such tables. Denote it by S_2.

3. C does not have C_3, and both F and G have C_3. Similarly, there are 8 such tables. Denote this set by S_3.

We then have $ds_2 = S_1 \cup S_2 \cup S_3$. The ratio of C being associated with C_3 is $\frac{24}{24+8+8} = 0.6 > 0.5$, so $g_2(t_0)$ cannot be disclosed, either.

Now, consider the case that g_2 is reused as $g_{2'}$. To calculate the disclosure set of $g_{2'}$, the tables which can be disclosed under g_1, g_2, and g_3 must be excluded from $ds_{2'}$. After excluding the tables which can be disclosed under g_1, we have that the remaining tables in $ds_{2'}$ are the same as above, that is, $S_1 \cup S_2 \cup S_3$. These tables cannot be disclosed under g_2 as mentioned above. We further evaluate whether these tables can be disclosed using g_3. S_1 can be further divided into three disjoint subsets as follows.

1. One and only one of D and E has C_3, so does F and G. This subset has $\binom{2}{1} \times \binom{2}{1} \times \binom{2}{1} \times \binom{2}{1} = 16$ tables, and is denoted by S_{1_1}.

2. Both D and E have C_3. This subset has $\binom{2}{1} \times \binom{2}{1} = 4$ tables, and is denoted by S_{1_2}.

3. Both F and G have C_3. This subset also has 4 tables, and is denoted by S_{1_3}.

All the tables in S_{1_2}, S_{1_3}, S_2, and S_3 cannot be disclosed under g_3 since their permutation sets under g_3 do not satisfy the privacy property (the highest ratios of a sensitive value are respectively 0.6, 1.0, 0.6, and 1.0). On the other hand, the tables in S_{1_1} can be disclosed under g_3. We can reason as follows. Consider each table t in S_{1_1} under g_3. Since the tables which can be disclosed under g_1 must be excluded from $ds_3(t)$, the remaining tables in $ds_3(t)$ must be in one of the following two disjoint sets.

1. Both A and B have C_3. This subset has $3! \times 2! = 12$ tables, and is denoted by $S_{1_{1_1}}$.

2. Two of C, D and E have C_3. This subset has $(\binom{3}{1} \times \binom{2}{1}) \times \binom{3}{1} \times \binom{2}{1} = 36$ tables, and is denoted by $S_{1_{1_2}}$.

We must exclude from $ds_3(t)$ the tables which can be disclosed using g_2. The tables in $S_{1_{1_1}}$ cannot be disclosed under g_2 since their permutation sets under g_2 do not satisfy the privacy property. Furthermore, the tables in $S_{1_{1_2}}$ can be further divided into two disjoint subsets based on whether C has C_3. The tables in the case that C has C_3 cannot be disclosed using g_2 because of the same reason as those in $S_{1_{1_1}}$, while the tables in the case that C does not have C_3 cannot be disclosed using g_2 because of the similar reason as $g_2(t_0)$. In a word, all the tables in $S_{1_{1_1}}$ and $S_{1_{1_2}}$ cannot be disclosed using g_2, accordingly, these tables cannot be excluded from $ds_3(t)$. Thus, $ds_3(t) = S_{1_{1_1}} \cup S_{1_{1_2}}$. The ratio of C, D, E, F or G being associated with C_3 in $ds_3(t)$ is $\frac{1}{2}$ which is the highest ratio, accordingly, the tables in S_{1_1} can be disclosed under g_3.

Therefore, the disclosure set under the reused function $g_{2'}$ must exclude the tables in S_{1_1}, consequently, $ds_{2'} = S_{1_2} \cup S_{1_3} \cup S_2 \cup S_3$. The ratio of F and G being associated with C_3 are 0.5, which is the highest ratio. Therefore, $g_{2'}(t_0)$ can be safely disclosed. ■

QID	g_1	g_2	g_3	$g_{2'}$
A	C_1	C_1	C_1	C_1
B	C_2	C_2	C_2	C_2
C	C_3	C_3	C_3	C_3
D	C_4	C_4	C_4	C_4
E	C_5	C_5	C_5	C_5
F	C_3	C_3	C_3	C_3
G	C_3	C_3	C_3	C_3

Table 3.11: The Case Where Reusing Generalization Functions Improves Data Utility

3.4.3 a_{safe} and $a_{jump}(1)$

We show that the algorithm a_{safe} is equivalent to $a_{jump}(1)$ when the privacy property is either set-monotonic, or based on the highest ratio of sensitive values.

Given a group EC_i in the disclosed generalization, let nr_i be the number of tuples and ns_i be the number of unique sensitive values. Denote the sensitive values within EC_i by $\{s_{i.1}, s_{i.2}, \ldots, s_{i.ns_i}\}$. Denote by $n_{s_{i.j}}$ the number of tuples associated with $s_{i.j}$.

Lemma 3.3 *If the privacy property is either set-monotonic or based on the highest ratio of sensitive values, then a permutation set not satisfying the privacy property will imply that any of its subsets does not, either.*

Proof: The result is obvious if the privacy property is set-monotonic. Now consider a privacy property based on the highest ratio of sensitive values, which is supposed to be no greater than a given δ. Suppose that EC_i is a group that does not satisfy the privacy property, and in particular, $s_{i.j}$ is a sensitive value that leads to the violation. First, based on Lemma 3.5, we have that $\frac{n_{s_{i.j}}}{nr_i} > \delta$. Let nt be the cardinality of any subset of the permutation set. Since all tables in this subset have the same permutation set, each such table has totally $n_{s_{i.j}}$ appearances of $s_{i.j}$ in EC_i. Therefore, among these tables, the total number of appearances of $s_{i.j}$ in EC_i is $n_{s_{i.j}} \times nt$. On the other hand, assume that one subset of the permutation set with totally nt tables actually satisfies the privacy property. Then, the number of each sensitive value associated with a tuple should satisfy $|s_{i.j}| \leq \delta \times nt$. Therefore, the total number of sensitive values for all identities is:

$$nr_i \times |s_{i.j}| \leq nr_i \times (\delta \times nt) < nr_i \times \frac{n_{s_{i.j}}}{nr_i} \times nt = n_{s_{i.j}} \times nt. \tag{3.3}$$

Therefore, we have $n_{s_{i.j}} \times nt < n_{s_{i.j}} \times nt$, a contradiction. Consequently, the initial assumption that there exists a subset of the permutation set satisfying the privacy property must be false. ■

Since the disclosure set is computed by excluding tables from the corresponding permutation set, we immediately have the following.

Corollary 3.1 *When the privacy property is either set-monotonic or based on the highest ratio of sensitive values, the algorithm a_{safe} has the same data utility as $a_{jump}(1)$.*

For other kinds of privacy properties, we prove that the data utility is again incomparable between a_{safe} and $a_{jump}(1)$. First, we compare their disclosure set under the 3^{rd} generalization function.

Lemma 3.4 *The ds_3 under a_{safe} is a subset of that under $a_{jump}(1)$.*

Proof: By definition, we have the following (where the superscript 0 denotes a_{safe}).

$$ds_3^1(t_0) = per_3(t_0)/\{t|(t \in per_3(t_0)) \wedge (p(per_1(t)) \vee (p(per_2(t)) \wedge p(ds_2^1(t))))\} \tag{3.4}$$

$$\begin{aligned} ds_3^0(t_0) &= per_3(t_0)/\{t|(t \in per_3(t_0)) \wedge (p(ds_1^0(t)) \vee p(ds_2^0(t)))\} \\ &= per_3(t_0)/\{t|(t \in per_3(t_0)) \wedge (p(per_1(t)) \vee p(ds_2^1(t)))\} \end{aligned} \tag{3.5}$$

Therefore, we have $ds_3^1(t_0) \supseteq ds_3^0(t_0)$. ■

Theorem 3.5 *The data utility of a_{safe} and $a_{jump}(1)$ is generally incomparable.*

Proof: Based on Lemma 3.4, we can construct the following two evaluation paths.

1. $a_{jump}(1) : per_1 \rightarrow per_2 \rightarrow per_3 \rightarrow p(ds_3^1) = true$

   $a_{safe} : ds_1^0(per_1) \rightarrow ds_2^0 \rightarrow p(ds_3^0) = false$

2. $a_{jump}(1) : per_1 \rightarrow per_2 \rightarrow per_3 \ldots$

   $a_{safe} : ds_1^0 \rightarrow p(ds_2^0) = true$

Clearly, the data utility of $a_{jump}(1)$ in the first case is better than that of a_{safe}, while in the second case it is worse. ■

3.5 Computational Complexity of k-Jump Algorithms

In this section, we analyze the computational complexity of k-jump algorithms. Given a micro-data table t_0 and one of its k-jump algorithm a, let n_r be the cardinality of t_0, and n_p and n_d be the number of tables in its permutation set and disclosure set under function g_i, respectively. In the worst case, $n_p = n_r!$ and $n_d \approx n_p$, in which there is only one anonymized group and all the sensitive values are distinct.

Lemma 3.5 *Given a micro-data table t_0 under a generalization function, the distribution of sensitive values corresponding to each identity in the permutation set is coincident with the distribution of the multiset of sensitive values in the anonymized group the identity belongs to.*

The result of Lemma 3.5 is obvious due to the definition of permutation set. Besides, to evaluate a permutation set against k-anonymity privacy property, we only need to count the number of different sensitive values in each anonymized group. Based on these results, the running time of evaluating permutation set against privacy property reduces from $O(n_p \times n_r)$ to $O(n_r)$ for most existing privacy models, such as k-anonymity, l-diversity, and so on. Given a table t_0, let $e_p(t_0)$ and $e_d(t_0)$ be the running time of evaluating permutation set and disclosure set under a function g_i, respectively. Since generally the disclosure set does not satisfy Lemma 3.5, the running time of evaluating disclosure set is $O(n_d \times n_r)$. Nevertheless, for simplicity, we will consider that $O(e_p(t)) = O(1)$ and $O(e_d(t)) = O(1)$ in the following discussion. To facilitate the analysis, we elaborate the family of k-jump algorithms as shown in Table 3.12.

Basically, an $a_{jump}(\vec{k})$ algorithm checks the original table t_0 against privacy property $p(.)$ under each generalization function in the given order and discloses the first generalization g_i in the sequence whose permutation set per_i and disclosure set ds_i both satisfy the desired privacy property. To determine whether a table can be disclosed under certain generalization function g_i in the algorithm, its permutation set per_i is evaluated first. If

Algorithm $a_{jump}(t_0, s_g, \vec{k})$

Input: an original table t_0,
sequence of functions $s_g = (g_1, g_2, \ldots, g_n)$,
vector of jump distance $\vec{k}$,
and a privacy property $p(.)$;

Output: a generalization $g_i (1 \leq i \leq n)$ or $\emptyset$;

$i \leftarrow 1$;
while $(i \leq n)$ **do**
  if $(p(per(g_i(t_0))_= true)$ **then**
    if $(p(ds(t_0, i, s_g, \vec{k})) = true)$ **then**
      return $g_i(t_0)$;
    else
      $i \leftarrow i + \vec{k}[i]$;
    end if
  else
    $i \leftarrow i + 1$;
  end if
end while
return $\emptyset$;

Algorithm $ds(t_0, i, s_g, \vec{k})$

Input: a table t_0,
function i (to calculate t_0's disclosure set),
sequence of functions $s_g = (g_1, g_2, \ldots, g_n)$,
vector of jump distances $\vec{k}$,
and a privacy property $p(.)$;

Output: the disclosure set $ds_i(t_0)$;

$ds_i \leftarrow per(g_i(t_0))$;
for all $(t \in ds_i)$ **do**
  $j \leftarrow 1$;
  while $(j \leq i - 1)$ **do**
    if $(p(per(g_j(t))) = true)$ **then**
      if $(p(ds(t, j, s_g, \vec{k})) = true)$ **then**
        $ds_i \leftarrow ds_i / \{t\}$;
        break;
      else
        $j \leftarrow j + \vec{k}[i]$;
      end if
    else
      $j \leftarrow j + 1$;
    end if
  end while
  if $(j > i)$ **then**
    $ds_i \leftarrow ds_i / \{t\}$;
  end if
end for
return ds_i;

Table 3.12: Algorithms: $a_{jump}(\vec{k})$ and $ds_i^{\vec{k}}$ With Any Given Privacy Property $p(.)$

the permutation set does not satisfy the privacy property, the table will not be disclosed under this function and the algorithm moves to evaluate under next function, otherwise, its disclosure set ds_i is evaluated. If the disclosure set satisfies the privacy property, the table can be disclosed under this function g_i; otherwise, the algorithm will check the $(i + \vec{k}[i])^{th}$ generalization function in a similar way. This procedure will continue until the table is successfully disclosed under a function g_i $(1 \leq i \leq n)$ or fails to satisfy the privacy property for all functions and nothing is disclosed.

To compute the disclosure set ds_i of t_0 under generalization function g_i, we first enumerate all possible tables by permuting each group in the generalization $g_i(t_0)$. Then, by following the algorithm, for each table t in the permutation set $per_i(t_0)$, we first assume it is the original table t_0, check under the generalization functions in sequence following the paths of the generalization algorithm, then determine whether it will not be disclosed under generalization function g_i. Such tables may fall into two different cases. First, the table can be disclosured under certain generalization function $g_j(j < i)$ before g_i; Second, the table will not be checked by the generalization function g_i, even it cannot be disclosed before g_i, which has been discussed in Section 3.3.2.1.

Based on the above detailed analysis of the algorithm, it can be shown that the running time of evaluating whether a given disclosure set satisfies privacy property is different from the time of deriving that disclosure set. On one hand, we consider $O(e_d(t)) = O(1)$. On the other hand, to derive $ds_i(t_0)$, we must separately evaluate each table t in $per_i(t_0)$ to determine whether it is a valid guess.

With the aforementioned discussions, we can analyze the time complexity of k-jump algorithms as follows.

Theorem 3.6 *Given a micro-data table t_0, a generalization algorithm of k-jump strategy that considers the sequence of generalization functions $g_1, g_2, \ldots, g_n$ in the given order and the jump-distance k, the computational complexity of such k-jump strategy is $O((max_p)^{\frac{n}{k}})$ where max_p is the maximal cardinality of possible tables in the permutation set among the functions.*

Proof: Given a jump-vector, we prove the result by mathematical induction on n. For simplicity, we assume the jump-vector to be jump-distance k, where k is a constant.

The Inductive Hypothesis: To compute the disclosure set of micro-data table t_0 under generalization function i in k-jump strategy, its computational complexity is $O((max_p)^{\frac{i}{k}})$.

The Base Case: When $i = 1$, it is clear that we only need to evaluate whether the permutation set satisfies the privacy property, whose running time is $e_p(t)$.

For $i = 2, 3, \ldots, 1+k$, as mentioned before, the tables for which $p(per_j) = true$ for any $j < i$ will be removed from g_i's disclosure set. Therefore, the worst case is to evaluate all the permutation sets under each $j < i$ and evaluate both permutation set and disclosure set under function i. Thus, the running time is $O((i-1) \times max_p \times e_p(t) + e_p(t) + e_d(t)) = O((k \times max_p + 1) \times e_p(t))$, which is $O((max_p)^1)$.

The Inductive Assumption: Suppose the inductive hypothesis hold for any $j > 0$, the running time for $i \in [2 + j \times k, 1 + k + j \times k]$ is $O((max_p)^{j+1})$.

The Inductive Step: Now we show the hypothesis also holds for $j + 1$, and equivalently, for $i = 2+(j+1)\times k, 3+(j+1)\times k, \ldots, 1+k+(j+1)\times k$. Based on the assumption above, the most-time-consuming case is that for each table t in permutation set $per_i(t_0)$, there exists an evaluation of disclosure set $ps_m(t)$ where $m \in [2+j\times k, 1+k+j\times k]$. Therefore, the running time is $O(e_p(t) + \ldots + max_p \times O((max_p)^{j+1}) + e_d(t)) = O((max_p)^{j+2})$. Therefore, the assumption holds for any $j > 0$, and equivalently, for any $i \geq 2$. This concludes the proof. ■

Summarily, it is shown that the computational complexity of the family of algorithms is exponential in $\frac{n}{k}$. Although the worse case complexity is still exponential, this is, to the best of our knowledge, one of the first algorithms that allow users to ensure the privacy property and optimize the data utility given that the adversaries know the algorithms. Furthermore, unlike the safe algorithms discussed in [53, 105] which only work with l-diversity, the family of our algorithms $a_{jump}(\vec{k})$ is more general and independent of the privacy property and the measure of data utility.

3.6 Making Secret Choices of Algorithms

In this section, we discuss the feasibility of protecting privacy by making a secret choice among algorithms. Recall that we say an algorithm is safe if it can ensure the privacy property for any micro-data in the case that the adversary knows the algorithm itself, otherwise, we say it is unsafe.

3.6.1 Secret-Choice Strategy

From previous discussions, we know that the family of algorithms a_{jump} share two properties, namely, a large cardinality and incomparable data utility. The practical significance of this result is that we can now draw an analogy between a_{jump} and a cryptographic algorithm, with the jump distance $\vec{k}$ regarded as a cryptographic key. Instead of relying on the secrecy of an algorithm (which is security by obscurity), we can rely on the secret choice of $\vec{k}$ for protecting privacy.

On the other hand, as discussed in previous sections, a safe algorithm (e.g., a_{safe} or a_{jump}) usually incur a high computational complexity, therefore, one may suggest that we can make the secret choice among unsafe but more efficient algorithms instead of safe algorithms to reduce the computational complexity. We first formulate the secret-choice strategy.

The secret-choice strategy among a set of algorithms can take the following three stages. Given a table t_0 and the set of generalization functions $g_i(.)(1 \leq i \leq n)$, the strategy first defines a large set of generalization algorithms (either safe or unsafe) based on the set of functions, then randomly and secretly selects one of these algorithms, and finally executes the selected algorithm to disclose the micro-data. We can thus describe the above strategy as a_{secret} shown in Table 3.13.

There certainly exist many approaches to defining the sets of algorithms (the first stage of a_{secret}). We demonstrate the abundant possibilities through the following two

Input: Table t_0, a set of functions $g_i(.)(1 \leq i \leq n)$;
Output: Generalization g or $\emptyset$;
Method:
1. **Define** a large set of generalization algorithms $A = \{a_1, a_2, \ldots, a_m\}$ based on $g_i(i \in [1, n]])$;
2. **Select** an $j \in [1, m]$ randomly for representing one of the above algorithms a_j;
3. **Return** (**Call** a_j);

Table 3.13: The Secret-Choice Strategy a_{secret}

examples.

First, each generalization function is slightly revised to be a generalization algorithm. That is, instead of only evaluating whether the permutation set of a micro-data table under the function satisfies the desired privacy property, such generalization algorithm further discloses the generalization or nothing. To complete the random selection, the a_{secret} will randomly select one of such algorithms and then discloses its corresponding generalization if it satisfies privacy property or nothing otherwise. Intuitively, this approach may be safe as long as the cardinality of the set of functions is sufficient large. However, such randomness will generally lead to worse data utility since usually the number of functions under which the permutation sets of a given micro-data satisfy privacy property is relatively low compared to the total number of functions. Consequently, such algorithm will disclose nothing for the micro-data with considerably high probability. Therefore, in the following discussion, without loss of generality, the randomness refers to the selection of algorithms which is not to be confused with the selection of functions in an algorithm. In other words, we assume that the algorithms sort the functions in a predetermined non-decreasing order of the data utility.

The k-jump strategy is another possible approach to defining the set of algorithms based on a given set of generalization functions. In k-jump, k is the secret choice, while all the functions appear in each algorithm and are sorted based on data utility. Given the set of functions, the one and only difference among k-jump algorithms is the jump-distance (k). As discussed above, k-jump algorithms are safe and the adversaries can at most refine their mental image to the disclosure set no matter whether they know the k. In other words, it is not necessary to hide the k among the family of k-jump algorithms. Similarly, we do not need to make a secret choice among other categories of safe algorithms. Therefore, in the remainder of this section, we will restrict the discussions on the case of secret choice among the unsafe algorithms based on predetermined order of the generalization functions. We show that secret choice among such unsafe algorithms cannot guarantee the privacy through a family of unsafe algorithms.

3.6.2 Subset Approach

To facilitate our discussion, we design a straightforward *subset approach* to define the set of unsafe algorithms for the first stage of a_{secret}. Given a set of generalization functions $G = \{g_1, g_2, \ldots, g_n\}$, the *subset approach* first construct all the subsets S_G of G which includes at least 2 functions. Then the naive strategy discussed in Section 3.2.2 is adapted on each of such subsets to embody an algorithm. That is, the functions in a subset is sorted in the non-increasing order of the data utility, and then the first function under which the permutation set of given micro-data satisfies the privacy property is disclosed; otherwise, $\emptyset$ will be the output and nothing is disclosed as shown in Table 3.14. We assume that the adversaries know the set of functions G since they know the released micro-data and in most cases the generalization is based on the quasi-identifier. We also call the *secret-choice strategy* built upon subset approach *subset-choice strategy*.

Input: Set of function $G = \{g_1, g_2, \ldots, g_n\}$;
Output: Set of algorithms S_A
Method:
1. **Let** $S_A = \emptyset$;
2. **Let** $S_G = 2^G / \{\emptyset \cup \{g_i : 1 \leq i \leq n\}\}$;
3. **For** each element S_f in S_G
4.   **Create** in S_A an algorithm by applying naive strategy on S_f;
5. **Return** S_A;

Table 3.14: The Subset Approach For Designing the Set of Unsafe Algorithms

From the adversaries' point of view, when they know the disclosed data, the subset-choice strategy (that is, the secret-choice strategy with the subset approach as its first stage), the privacy property, and the set of functions G, they may be able to validate their guesses and refine their mental image about the original data. With the knowledge about G, the adversary can know there are $\binom{|G|}{2} + \binom{|G|}{3} + \ldots + \binom{|G|}{|G|} = 2^{|G|} - |G| - 1$ possible different secret choices; With the knowledge of the disclosed data, the adversary can further know the following two facts. First, the original micro-data is in the permutation set of the disclosed generalization. Second, the generalization function corresponding to the disclosed data should be a function in the selected algorithms, and consequently the number of possible secret choices in his/her mental image is reduced to be $2^{|G|-1} - 1$. Each secret choice corresponding to an algorithm is equally likely selected. For each of these refined secret choices, the adversary first assumes that it is the true secret choice, then deduces the disclosure set for given disclosed data and corresponding naive algorithm in a similar way discussed in Section 3.1. Finally, the adversary refines his/her mental image to be $(2^{|G|-1} - 1)$ disclosure sets.

Based on such a mental image, the adversary may refine his knowledge about an individual's sensitive information. For example, for entropy l-diversity, the adversary can calculate the ratio of an individual being associated with a sensitive value in each disclosure set, and then average the ratio among all disclosure sets. Whenever the average ratio among the disclosure sets of an individual being associated with a sensitive value is larger than $\frac{1}{l}$, the privacy of that individual is violated. Taking k-anonymity as another example, the adversary can simply count the number of sensitive values that an individual possibly being associated with among all disclosure sets. If the resultant number for any individual is less than k, the privacy of that individual is violated.

3.6.3 The Safety of Subset-Choice Strategy

In the following, we show that subset-choice strategy cannot ensure the privacy property by constructing a counter-example.

Theorem 3.7 *Given a subset-choice strategy, there exist cases that the strategy discloses an unsafe generalization.*

(a). The Table t_0

QID	**S**
A	C_0
B	C_0
C	C_0
D	C_1
E	C_2
F	C_3
G	C_4
H	C_5
I	C_6

(b). The set G of generalization functions for t_0

g_1		g_2		g_3		g_4		g_5	
QID	S	QID	S	QID	S	QID	S	QID	S
A	C_0	A	C_0	B	C_0	A	C_0	A	C_0
B	C_0	C	C_0	C	C_0	B	C_0	B	C_0
C	C_0	B	C_0	A	C_0	C	C_0	C	C_0
D	C_1	D	C_1	D	C_1	D	C_1	D	C_1
E	C_2	E	C_2	E	C_2	E	C_2	E	C_2
F	C_3	F	C_3	F	C_3	F	C_3	F	C_3
G	C_4	G	C_4	G	C_4	G	C_4	G	C_4
H	C_5	H	C_5	H	C_5	H	C_5	H	C_5
I	C_6	I	C_6	I	C_6	I	C_6	I	C_6

Table 3.15: The Counter Example for Secret Choice among Unsafe Algorithms

Possible S_G	**Probability**			**Possible S_G**	**Probability**		
	A	B	C		A	B	C
$\{g_1, g_5\}$	1	1	$\frac{1}{7}$	$\{g_1, g_2, g_5\}$	1	1	1
$\{g_2, g_5\}$	1	$\frac{1}{7}$	1	$\{g_1, g_3, g_5\}$	1	1	1
$\{g_3, g_5\}$	$\frac{1}{7}$	1	1	$\{g_1, g_4, g_5\}$	1	1	$\frac{1}{7}$
$\{g_4, g_5\}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\{g_2, g_3, g_5\}$	1	1	1
$\{g_1, g_2, g_3, g_5\}$	1	1	1	$\{g_2, g_4, g_5\}$	1	$\frac{1}{7}$	1
$\{g_1, g_2, g_4, g_5\}$	1	1	1	$\{g_3, g_4, g_5\}$	$\frac{1}{7}$	1	1
$\{g_1, g_3, g_4, g_5\}$	1	1	1	$\{g_1, g_2, g_3, g_4, g_5\}$	1	1	1
$\{g_2, g_3, g_4, g_5\}$	1	1	1				

Table 3.16: The Possible Subsets of Functions and the Corresponding Probability of A,B, and C Being Associated With C_0

Proof: One counter example, that an algorithm taking subset-choice strategy discloses a generalization while the privacy is actually violated, is sufficient to prove the theorem. Table 3.15 shows our construction for the proof. The left tabular shows the micro-data table t_0 whose identifiers are removed. The right tabular shows the five generalization functions in G. For clarification purposes, we intentionally keep the original value of QID. In other words, we only focus on the anonymized groups as illustrated by the horizontal lines while omitting the modification of quasi-identifiers. For example, by g_1, we partition t_0 into two anonymized groups: A and B form one anonymized group, while the others $(C - I)$ form another group. In this construction, the privacy property is 2-diversity and the data utility is measured by discernibility measure (DM).

Suppose that the algorithm select subset S_G of generalization functions to be $S_G = \{g_4, g_5\}$. Obviously, the permutation set of t_0 under function g_4 does not satisfy 2-diversity, while it does so under g_5. Therefore, based on the subset-choice strategy, the algorithm discloses $g_5(t_0)$.

Unfortunately, the knowledge of G and disclosed table will enable the adversary to refine his mental image about the original micro-data, and finally violate the privacy property since the adversary can infer that the ratio of A, B and C being associated with C_0 is $\frac{272}{315} > \frac{1}{2}$.

The adversary can reason as follows. There are totally $\binom{5}{2} + \binom{5}{3} + \binom{5}{4} + \binom{5}{5} = 26$ possible secret choices of S_G. By observing the disclosed data, the adversary knows that $g_5 \in S_G$ and then refines the number of possible choices to be $\binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4} = 15$. That is, one, two, three or all of g_1, g_2, g_3 and g_4 together with g_5 form S_G. Note that these 15 possible subsets are equally likely to be S_G. The possible subsets of functions are shown in the *possible* S_G column of Table 3.16.

By the data utility measurement DM, g_1, g_2, and g_3 have the same data utility which is better than that of g_4, and g_4 has better data utility than g_5. From the adversary's point of view, since g_5 is disclosed, the micro-data t_0 under any other functions in the selected S_G should violate the 2-diversity (otherwise, other generalization should be disclosed based on the subset-choice algorithm).

Based on the disclosed data g_5, the adversary knows that only three individuals can share the same sensitive value (C_0). Therefore, the anonymized group $\{C-I\}$ in g_1, whose cardinality is 7, cannot violate 2-diversity, neither do groups $\{B, D-I\}$ in g_2, $\{A, D-I\}$ in g_3, and $\{D-I\}$ in g_4. In other words, the reason that subset approach does not disclose t_0 using function g_1, g_2, g_3 or g_4 is that the group $\{A, B\}$, $\{A, C\}$, $\{B, C\}$ or $\{A, B, C\}$ respectively does not satisfy 2-diversity. For example, suppose that $S_G = \{g_1, g_5\}$ and g_5 is disclosed, then g_1 must violate 2-diversity, therefore, both A and B should be associated with C_0, while C can be associated with any sensitive value in set $\{C_i : i \in [0, 6]\}$. The similar analysis can be applied to other possible subsets S_G and the probability of A, B, and C being associated with C_0 are shown in Table 3.16 when corresponding subset S_G of G is selected. Since each S_G is equally likely selected, the ratio of A being associated with C_0 is $\frac{12\times1+2\times\frac{1}{7}+1\times\frac{2}{3}}{15} = \frac{272}{315} > \frac{1}{2}$, so do B and C. In other words, once the adversary knows G, the subset-choice algorithm, subset approach, and the disclosed data g_5, he/she can infer that A, B, and C is associated with C_0 with ratio higher than $\frac{1}{2}$ even in the case that she/he does not know the secret choice (the adversary does not know which subset of G is selected). This clearly violates the privacy property. Thus we have proved the theorem.

■

The counter example in the above proof is sufficient to demonstrate that secret choices made among unsafe algorithms does not always guarantee the privacy property. Therefore, safe algorithms are still necessary for preserving the privacy property.

3.7 Summary

In this chapter, we have proposed a novel k-jump strategy for preserving privacy in micro-data disclosure using public algorithms. We have shown how a given unsafe generalization algorithm can be transformed into a large number of safe algorithms. By constructing counter-examples, we have shown that the data utility of such algorithms is generally incomparable. The practical impact of this result is that we can make a secret choice from a large family of k-jump algorithms, which is analogous to choosing a cryptographic key from a large key space, to optimize data utility based on a given table while preventing adversarial inferences. It has been shown that the computational complexity of a k-jump algorithm with n generalization functions is exponential in $\frac{n}{k}$ which indicates a reduction in the complexity due to k (We shall discuss an efficient solution in next chapter). We have also shown that making a secret choice among unsafe algorithms cannot ensure the desired privacy property which embodies the need of safe algorithms from another standpoint.

Chapter 4

PPDP: An Efficient Strategy for Diversity Preservation With Publicly Known Algorithms

While the strategy in previous chapter is theoretically superior to existing ones due to its independence of utility measures and privacy models, and its privacy guarantee under publicly-known algorithms, it incurs a high computational complexity. In this chapter, we study an efficient strategy for diversity preserving data publishing with publicly known algorithms (algorithms as side-channel).

4.1 Overview

In many privacy-preserving applications ranging from micro-data release [46] to social networks [44, 83], a major challenge is to keep private information secret while optimizing the utility of disclosed or shared data. Recent studies further reveal that utility optimization may actually interfere with privacy preservation by leaking additional private information when algorithms are regarded as public knowledge [101, 111]. Specifically, an adversary can determine a guess of the private information to be invalid if it would have caused the disclosed data to take a different form with better utility. By eliminating such invalid guesses, the adversary can then obtain a more accurate estimation of the private information.

A natural solution to this problem is to simulate the aforementioned adversarial reasoning [73, 101, 111]. Specifically, since knowledge about utility optimization can assist an adversary in refining his/her mental images of the private information, we can first simulate such reasoning to obtain the refined mental images, and then enforce the privacy property on such images instead of the disclosed data. However, it has been shown that such approaches are inherently recursive and deemed to incur a high complexity [111].

In this chapter, we observe that the interference between privacy preservation and utility optimization actually arises from the fact that those two processes are usually mixed together in an algorithm. On the other hand, we also observe a simple fact that *to meet both goals does not necessarily mean to meet them at exactly the same time*. Based on such observations, we propose a novel *privacy streamliner* approach to decouple the process of privacy preservation from that of utility optimization in order to avoid the expensive recursive task of simulating the adversarial reasoning.

To make our approach more concrete, we study it in the context of micro-data release with publicly known generalization algorithms. Unlike traditional algorithms, which typically evaluate generalization functions in a predetermined order and then release data using the first function satisfying the privacy property, a generalization algorithm under our approach works in a completely different way: The algorithm starts with the set of generalization functions that can satisfy the privacy property for the given micro-data table; it then identifies a subset of such functions satisfying that knowledge about this subset itself will not assist an adversary in violating the privacy property (which is generally not true for the set of all functions, as we will show later); utility optimization within this subset then becomes simulatable by adversaries [56], and is thus guaranteed not to affect the privacy property. We believe that this general principle can be applied to other similar privacy preserving problems, although developing the actual solution may be application-specific and non-trivial.

The contribution of this chapter is twofold. First, our privacy streamliner approach is presented through a general framework that is independent of specific algorithmic constructions or utility metrics. This allows our approach to be easily adapted to a broad range of applications to yield efficient solutions. We demonstrate such possibilities by devising three generalization algorithms to suit different needs while following exactly the same approach. Second, our algorithms provide practical solutions for privacy-preserving micro-data release with public algorithms. As confirmed by both complexity analysis and experimental results, those algorithms are more efficient than existing algorithms.

The rest of this chapter is organized as follows. We first build intuitions through an example in the remainder of this section. We then present our main approach and supporting theoretical results in Section 4.2. Section 4.3 devises three generalization algorithms by following the approach. Section 4.4 experimentally evaluates the efficiency and utility of our algorithms. We discuss the possibilities for extending our approach and the practicality of the approach in Section 4.5. We finally conclude the chapter in Section 4.6.

A Micro-Data Table t_0

Name	**DOB**	**Condition**
Ada	1985	flu
Bob	1980	flu
Coy	1975	cold
Dan	1970	cold
Eve	1965	HIV

The Disclosure Sets

Name	**Condition**									
	t_{01}	t_{02}	t_{03}	t_{04}	t_{05}	t_{06}	t_{07}	t_{08}	t_{09}	t_{10}
Ada	flu	cold	flu	cold	flu	cold	flu	cold	HIV	HIV
Bob	flu	cold	flu	cold	HIV	HIV	flu	cold	flu	cold
Coy	cold	flu	cold	flu	cold	flu	HIV	HIV	cold	flu
Dan	cold	flu	HIV	HIV	cold	flu	cold	flu	cold	flu
Eve	HIV	HIV	cold	flu	flu	cold	cold	flu	flu	cold

Table 4.1: The Motivating Example

Motivating Example

The left table in Table 4.1 shows a micro-data table t_0 to be released. To protect individuals' privacy, the *identifier* Name will not be released. Also, the identifiers are partitioned into *anonymized groups*, with the *quasi-identifier* DOB inside each such group modified to be the same value [90] (in this chapter, we will only consider generalization and leave suppression [29] and bucketization [107] for the future work). For simplicity, we will focus on the partitioning of identifiers while omitting the modification of quasi-identifiers. For this particular example, we assume the desired privacy property to be that the highest ratio of a *sensitive value* Condition in any anonymized group must be no greater than $\frac{2}{3}$ [77].

By our *privacy streamliner* approach, we need to start with all partitions of the identifiers that can satisfy the privacy property. In this example, any partition that includes $\{Ada, Bob\}$ or $\{Coy, Dan\}$ will violate the privacy property, since the two persons inside each of those groups share the same condition. It can be shown that there are totally 9 partitions satisfying the privacy property, as shown below. We will refer to the set of such identifier partitions as the *locally safe set (LSS)*.

$$
\begin{aligned}
P_1 &= \{\{Ada, Coy\}, \{Bob, Dan, Eve\}\}, \\
P_2 &= \{\{Ada, Dan\}, \{Bob, Coy, Eve\}\}, \\
P_3 &= \{\{Ada, Eve\}, \{Bob, Coy, Dan\}\}, \\
P_4 &= \{\{Bob, Coy\}, \{Ada, Dan, Eve\}\}, \\
P_5 &= \{\{Bob, Dan\}, \{Ada, Coy, Eve\}\}, \\
P_6 &= \{\{Bob, Eve\}, \{Ada, Coy, Dan\}\}, \\
P_7 &= \{\{Coy, Eve\}, \{Ada, Bob, Dan\}\}, \\
P_8 &= \{\{Dan, Eve\}, \{Ada, Bob, Coy\}\}, \\
P_9 &= \{\{Ada, Bob, Coy, Dan, Eve\}\}
\end{aligned}
$$

It may seem to be a viable solution to start optimizing data utility inside the LSS, since every partition here can satisfy the privacy property. However, such an optimization may still violate the privacy property, because it is not simulatable by adversaries [56] unless if we assume the LSS to be public knowledge (that is, adversaries may know that each identifier partition in the LSS can satisfy the privacy property for the unknown table t_0). Unfortunately, this knowledge about LSS could help adversaries to violate the privacy property. In this case, it can be shown that adversaries' mental image about the micro-data table would only include t_{01} and t_{02} shown in the right table in Table 4.1. In other words, adversaries can determine that t_0 must be either t_{01} or t_{02}. Clearly, the privacy property is violated since *Eve* is associated with *HIV* in both cases.

Since the LSS may contain too much information to be assumed as public knowledge, we turn to its subsets. In this example, it can be shown that by removing P_7 from the *LSS*, the disclosure set becomes $\{t_{01}, t_{02}, t_{03}, t_{04}\}$. The privacy property is now satisfied since the highest ratio of a sensitive value for any identifier is $\frac{1}{2}$. We call such a subset of the LSS the *globally safe set (GSS)*. Optimizing data utility within the GSS will not violate privacy property, because the GSS can be safely assumed as public knowledge and the optimization is thus simulatable by adversaries.

However, there is another complication. At the end of utility optimization, one of the generalization functions in the GSS will be used to release data. The information disclosed by the GSS and that by the released data is different, and by intersecting the two, adversaries may further refine their mental image of the micro-data table. In this example, since the adversaries' mental image about the micro-data table in terms of the GSS is $\{t_{01}, t_{02}, t_{03}, t_{04}\}$, adversaries know both *Ada* and *Bob* must be associated with either *flu* or *cold*. Now suppose the utility optimization selects P_3, then from the released table, adversaries will further know that either *Ada* or *Eve* must have *flu* while the other has *HIV*. Therefore, adversaries can now infer that *Ada* must have *flu*, and *Eve* must then have *HIV*.

To address this issue, we will further confine the utility optimization to a subset of the GSS. In this example, if we further remove P_3, P_6, P_8 from the GSS, then the corresponding mental image of adversaries will contain all the 10 tables (from t_{01} to t_{10}). It can be shown that now the privacy property will always be satisfied regardless of which partition is selected during utility optimization. Taking P_1 as an example, from its corresponding generalized table, adversaries may further refine their mental image about t_0 as the first six tables (from t_{01} to t_{06}), but the highest ratio of a sensitive value is still $\frac{1}{2}$. We call such a subset of identifier partitions the *strongly globally safe set (SGSS)*. The SGSS allows us to optimize utility without worrying about violating the privacy property.

Therefore, the key problem in applying the privacy streamliner approach is to find the SGSS. The naive solution of directly following the above example to compute the LSS, GSS, and eventually SGSS is clearly impractical due to the large solution space. In the rest of this chapter, we will present more efficient ways to directly construct the SGSS without first generating the LSS or GSS.

4.2 The Model

We first give the basic model in Section 4.2.1. We then introduce the concept of *l-candidate* and *self-contained property* in Section 4.2.2. Finally, we prove that the SGSS can be efficiently constructed using those concepts in Section 4.2.3. Table 4.2 summarizes our notations.

t_0, t, $t(id, q, s)$	Micro-data table
$\mathcal{I}$, $\mathcal{Q}$, $\mathcal{S}$	Projection $\Pi_{id}(t)$, $\Pi_q(t)$, $\Pi_s(t)$
R_{iq}, R_{qs}, R_{is}	Projection $\Pi_{id,q}(t)$, $\Pi_{q,s}(t)$, $\Pi_{id,s}(t)$
$C(.\|t)$, $C_i(.\|t)$	A color of table t
$S^C(.\|t)$	The set of colors in t
$P(.\|t)$, $P_i(.\|t)$	A identifier partition of table t
$S^P(.\|t)$	A set of identifier partitions of t
$ss^l(.\|t)$	Locally safe set (LSS) of t
$ss^g(.\|t)$	Globally safe set (GSS) of t
$ss^s(.\|t)$	Strongly globally safe set (SGSS) of t

Table 4.2: The Notation Table

4.2.1 The Basic Model

We denote a micro-data table as $t_0(id, q, s)$ where id, q, and s denote the *identifier*, *quasi-identifier*, and *sensitive value*, respectively (each of which may represent multiple attributes). Denote by $\mathcal{I}$, $\mathcal{Q}$, $\mathcal{S}$ the set of identifier values $\Pi_{id}(t_0)$, quasi-identifier values $\Pi_q(t_0)$, and sensitive values $\Pi_s(t_0)$ (all projections preserve duplicates, unless explicitly stated otherwise). Also, denote by R_{iq}, R_{qs}, R_{is} the projections $\Pi_{id,q}(t_0)$, $\Pi_{q,s}(t_0)$, $\Pi_{id,s}(t_0)$, respectively.

As typically assumed, $\mathcal{I}$, $\mathcal{Q}$, and their relationship R_{iq} may be known through external knowledge, and $\mathcal{S}$ is also known once a generalization is released. Further, we make the worst case assumption that each tuple in t_0 can be linked to a unique identifier value through the corresponding quasi-identifier value. Therefore, both R_{is} and R_{qs} need to remain secret to protect privacy. Between them, R_{is} is considered as the private information and R_{qs} as the utility information.

We say a micro-data table t_0 is *l-eligible* if at most $\frac{|t_0|}{l}$ tuples in t_0 share the same sensitive value. We call the set of all identifier values associated with the same sensitive value s_i a *color*, denoted as $C(t_0, s_i)$ or simply C_i when t_0 and s_i are clear from the context. We use $S^C(t_0)$ or simply S^C to denote the collection of all colors in t_0.

Example 4.1 *The left-hand side of Table 4.3 (the right-hand side will be needed for later discussions) shows a micro-data table t_0 in which there are two colors: $C_1 = \{id_1, id_2\}$ and $C_2 = \{id_3, id_4\}$, so $S^C = \{C_1, C_2\}$.* $\boxdot$

R_{qs}

id	q	s
id_1	q_1	s_1
id_2	q_2	s_1
id_3	q_3	s_2
id_4	q_4	s_2
$\mathcal{I}$	$\mathcal{Q}$	$\mathcal{S}$

$$P_1 = \{\{id_1, id_3\}, \{id_2, id_4\}\}$$
$$P_2 = \{\{id_1, id_4\}, \{id_2, id_3\}\}$$
$$P_3 = \{\{id_1, id_2, id_3, id_4\}\}$$
$$P_4 = \{\{id_1, id_2\}, \{id_3, id_4\}\}$$

Table 4.3: An Example

We denote by $ss^l(t_0)$, $ss^g(t_0)$, and $ss^s(t_0)$ the *locally safe set (LSS)*, *globally safe set (GSS)*, and *strongly globally safe set (SGSS)* for a given t_0, respectively (those concepts have been illustrated in Section 4.1).

Example 4.2 *Continuing Example 4.1 and assuming the privacy property to be* 2*-diversity [77], it can be shown that* $ss^l(t_0) = \{P_1, P_2, P_3\}$ *and* $P_4 \notin ss^l$ *where* P_1, P_2, P_3, P_4 *are shown on the right-hand side of Table 4.3. Further,* $\{P_1, P_3\}$ *and* $\{P_2, P_3\}$ *are both GSS and SGSS.* $\boxdot$

We have previously given a sufficient condition for the SGSS, namely, the l*-cover* property [113]. In other words, a set of identifier partitions S^P is a SGSS with respect to l-diversity if it satisfies l-cover (however, no concrete method is given there to satisfy this property, which is the focus of this chapter). Intuitively, l-cover requires each color to be indistinguishable from at least $l-1$ other sets of identifiers in the identifier partition. If no ambiguity is possible, we also refer to a color C together with its $l-1$ covers as the l-cover of C. As these concepts are needed later in the proofs of our main results discussed in Section 4.2.3, we repeat them in Definition 4.1 and 4.2 (note the remaining content of this chapter can be understood without those definitions).

Definition 4.1 (Cover) *We say* $ids_1, ids_2 \subseteq \mathcal{I}$ *are* cover *for each other with respect to a set* $S^P \subseteq ss^l$*, if*

- $ids_1 \cap ids_2 = \emptyset$*, and*
- *there exist a bijection* $f : ids_1 \rightarrow ids_2$ *such that for any* $ids_x \in P_i$*,* $P_i \in S^P$*, there always exists* $P_j \in S^P$ *satisfying* $ids_x \setminus (ids_1 \cup ids_2) \cup f(ids_x \cap ids_1) \cup f^{-1}(ids_x \cap ids_2) \in P_j$ *[113].*

Definition 4.2 (l-Cover) *We say a set* $S^P \subseteq ss^l$ *satisfies the* l-cover *property, if every color* C *has at least* $l-1$ *covers* $ids_i (i \in [1, l-1])$ *with the bijections* f_i *satisfying that*

- *for any* $id \in C$*, each* $f_i(id)$ $(i \in [1, l-1])$ *is from a different color, and*

- *for any* $ids_x \in P$ *and* $P \in S^P$*, we have* $|ids_x \cap C| = |ids_x \cap ids_i| (i \in [1, l-1])$ *[113].*

Example 4.3 *Continuing Example 4.2 and considering* $S^P = \{P_1, P_3\}$*, the colors* $C_1 = \{id_1, id_2\}$ *and* $C_2 = \{id_3, id_4\}$ *provide cover for each other, since for* C_1 *we have* $f_1(id_1) = id_3$ *and* $f_1(id_2) = id_4$*, and for* C_2 *we have* $f_2(id_3) = id_1$ *and* $f_2(id_4) = id_2$*. Further,* S^P *satisfies the* l*-cover property where* $\{C_1, C_2\}$ *is the l-cover of both* C_1 *and* C_2*.*

Similarly, for $S^P = \{P_2, P_3\}$*,* C_1 *and* C_2 *provide cover for each other since for* C_1 *we have* $f_1(id_1) = id_4$ *and* $f_1(id_2) = id_3$*, and for* C_2 *we have* $f_2(id_3) = id_2$ *and* $f_2(id_4) = id_1$*. Further,* S^P *also satisfies the* l*-cover property.* $\boxdot$

4.2.2 l-Candidate and Self-Contained Property

We first give a necessary but not sufficient condition for l-cover, namely, l-*candidate*. As formally stated in Definition 4.3, subsets of identifiers can be candidates of each other, if there exists one-to-one mappings between those subsets that always map an identifier to another in a different color. We will prove later that any collection of subsets of identifiers can be l-cover for each other only if they form an l-candidate.

Definition 4.3 (l-Candidate) *Given an* l*-eligible micro-data table* t_0*, we say*

- $ids_1 \subseteq \mathcal{I}$ *and* $ids_2 \subseteq \mathcal{I}$ *are* candidate *for each other, if*
  - $ids_1 \cap ids_2 = \emptyset$ *and* $|ids_1| = |ids_2|$*, and*
  - *there exists a bijection* $f : ids_1 \rightarrow ids_2$*, such that every* $id \in ids_1$ *and* $f(id) \in ids_2$ *are from different colors.*
- $ids_1, ids_2, \ldots, ids_l \subseteq \mathcal{I}$ *form a* l-candidate*, if for all* $(1 \leq i \neq j \leq l)$*,* ids_i *and* ids_j *are candidates for each other.*
- *Denote by* $Can^l(.|t_0) = (can_1, can_2, \ldots, can_{|S^C|})$ *a sequence of* $|S^C|$ *l-candidates each* can_i *of which is the l-candidate for the color* C_i *in* t_0 *(note that there is exactly*

one l-candidate for each color in the sequence, and $Can^l(.|t_0)$ is not necessarily unique for t_0).

Example 4.4 *In the table shown on the left-hand side of Table 4.3, the two colors $C_1 = \{id_1, id_2\}$ and $C_2 = \{id_3, id_4\}$ are candidates for each other, and they together form a 2-candidate $\{C_1, C_2\}$. Also, we have that $Can^l(.|t_0) = (\{\mathrm{C}_1, C_2\}, \{C_1, \mathrm{C}_2\})$ (note that $Can^l(.|t_0)$ denotes the sequence of l-candidates and we use the indices in the multiset to present the order in the remainder of this chapter, and if no ambiguity is possible, we shall not distinguish the notations between a collection and a sequence). In this special case, it has two identical elements, the first one for C_1 and the second one for C_2, since both colors have the same l-candidate.* $\boxdot$

Next we introduce the *self-contained* property in Definition 4.4. Informally, an identifier partition is self-contained, if the partition does not break the one-to-one mappings used in defining the l-candidates. Later we will show that the self-contained property is sufficient for an identifier partition to satisfy the l-cover property and thus form a SGSS.

Definition 4.4 (Self-Contained Property and Family Set) *Given a micro-data table t_0 and a collection of l-candidates Can^l, we say*

- *an anonymized group G in an identifier partition P is* self-contained *with respect to Can^l, if for every pair of identifiers $\{id_1, id_2\}$ that appears in any bijection used to define Can^l, either $G \cap \{id_1, id_2\} = \emptyset$ or $G \cap \{id_1, id_2\} = \{id_1, id_2\}$ is true.*
- *an identifier partition P is self-contained if for each $G \in P$, G is self-contained.*
- *a set S^P of identifier partitions is self-contained, if for each $P \in S^P$, P is self-contained; we also call such a set S^P a* family set *with respect to Can^l.*

Next we introduce the concept of *minimal self-contained identifier partition* in Definition 4.5 to depict those identifier partitions that not only satisfy the self-contained property but have anonymized groups of minimal sizes. Intuitively, for any given collection of l-candidates Can^l, a minimal self-contained identifier partition may yield optimal data utility under certain utility metrics (we will discuss this in more details later).

Definition 4.5 (Minimal Self-Contained Partition) *Given a micro-data table t_0 and a collection of l-candidates Can^l, an identifier partition P is called the* minimal self-contained partition *with respect to Can^l, if*

- *P satisfies the self-contained property with respect to Can^l, and*
- *for any anonymized group $G \in P$, no $G' \subset G$ can satisfy the self-contained property.*

Example 4.5 *In Example 4.4, assume the bijections used to define l-candidate for C_1 in Can^l are $f_1(id_1) = id_3$ and $f_1(id_2) = id_4$ while for C_2 are $f_2(id_3) = id_1$ and $f_2(id_4) = id_2$, then the identifier partitions P_1 and P_3 shown in the left-hand side of Table 4.3 satisfy the self-contained property, whereas P_2 does not. Also, P_1 is the minimal self-contained identifier partition, and $\{P_1\}$, $\{P_3\}$, $\{P_1, P_3\}$ are all family sets.* $\boxdot$

Similarly, assume the bijections used to define l-candidate for C_1 in Can^l are $f_1(id_1) = id_4$ and $f_1(id_2) = id_3$ while for C_2 are $f_2(id_3) = id_2$ and $f_2(id_4) = id_1$, then the identifier partitions P_2 and P_3 satisfy the self-contained property, whereas P_1 does not. Also, P_2 is the minimal self-contained identifier partition, and $\{P_2\}$, $\{P_3\}$, $\{P_2, P_3\}$ are all family sets. Finally, assume $f_1(id_1) = id_3$, $f_1(id_2) = id_4$ and $f_2(id_3) = id_2$, $f_2(id_4) = id_1$, then in this case only P_3 satisfies self-contained property, whereas P_1 and P_2 do not. It is clearly evidenced by this example that, given micro-data table, its minimal self-contained partition is determined not only by the Can^l, but also the corresponding bijections. In this chapter, we focus on deriving Can^l and constructing minimal self-contained partitions as well as family sets based on the bijections. Therefore, unless explicitly stated otherwise, Can^l is referred to itself together with the corresponding bijections in the remainder of this chapter.

4.2.3 Main Results

In this section, we first prove that the self-contained property and l-candidate provide a way for finding identifier partitions that satisfy the l-cover property, and then we prove results for constructing l-candidates. All the proofs can be found in the Appendix B due to space limitations.

First, in Lemma 4.1, we show that a minimal self-contained identifier partition always satisfies the l-cover property.

Lemma 4.1 *Given an* l-eligible *micro-data table* t_0*, every minimal self-contained partition satisfies the* l-cover *property. Moreover, for each color* C*, its corresponding* l*-candidate in* Can^l *is also an* l*-cover for* C *(that is,* C *together with its* $l-1$ *covers).*

Proof: To prove the lemma, we first show the procedure *l-candidate-to-*P^{lm} in Table 4.4 based on the self-contained property to construct its minimal self-contained partition.

Input: an l-eligible table t_0, a collection of l-candidates Can^l
Output: the minimal self-contained partition;
Method:
1. Create a set of anonymized groups $S^G = \emptyset$;
2. **For** each color C_i
3. **For** each $id_{i,a} \in C_i$
4. Create in S^G a anonymized group $G_{i,d} = \{id_{i,a}\} \bigcup_{u=1}^{l-1} \{f_{i,u}(id_{i,a})\}$;
5. Merge the anonymized groups which have common identifiers to build minimal self-contained partition (P^{lm});
6. **Return** P^{lm};

Table 4.4: procedure: l-candidate-to-P^{lm}

Then, we show that $P^{lm} \in ss^l$. As shown in Table 4.4, to satisfy the self-contained property, for each identifier $id_{i,a}$ in each color C_i, the identifiers to which $id_{i,a}$ is mapped in each of the l-1 candidates should be in the same final anonymized group. We call such set of identifiers ,$G_{i,a} = \{id_{i,a}\} \bigcup_{u=1}^{l-1} \{f_{i,u}(id_{i,a})\}$, for a^{th} identifier in color C_i is *transient group*. Obviously, each transient group itself satisfies entropy l-diversity. Furthermore, based on the Definition 4.4, for any color C_i in the micro-data table, if an identifier $id_{i,a}$ in C_i is in the final anonymized group, then its whole transient group $G_{i,a}$ will be in the final anonymized group. In other words, in any final anonymized group G, the ratio of any identifier in any C_i associated with the sensitive value S_i equals to $\frac{|n_{C_i}|}{|n_{C_i}| \times l + \delta}$ where $\delta \geq 0$ and $|n_{C_i}|$ is the number of identifiers from color C_i in the anonymized group. Therefore, it is less than or equal to $\frac{|n_{C_i}|}{|n_{C_i}| \times l} = \frac{1}{l}$. Thus, each anonymized group in minimal self-contained partition satisfies l-diversity, so does the minimal self-contained partition. We have thus proved that $P^{lm} \in ss^l$.

Next, consider the $l-1$ covers for each color $C_i \in S^C$. Without loss of generality, we rewrite its corresponding l-candidate as $can_i^l = \{C_i, ids_{i,1}, ids_{i,2}, \ldots, ids_{i,l-1}\}$ so that C_i is the first element, we show that for the set of identifier partition P^{lm} ($|P^{lm}| = 1$), $ids_{i,1}, ids_{i,2}, \ldots ids_{i,l-1}$ are $l-1$ covers of C_i. By Definition 4.1, C_i and $ids_{i,u} (u \in [1, l-1])$ should satisfy following two conditions:

- $C_i \cap ids_{i,u} = \emptyset$, and

- there exists a bijection $f_{i,u} : C_i \rightarrow ids_{i,u}$ satisfying that for any $ids_x \in P^{lm}$, $ids_{x'} = ids_x \setminus (C_i \cup ids_{i,u})) \cup f_{i,u}(ids_x \cap C_i) \cup f_{i,u}^{-1}(ids_x \cap ids_{i,u}) \in P^{lm}$.

The first condition is satisfied by the definition of l-candidate. For the second condition, let the bijection $f_{i,u}$ be the corresponding bijection for $ids_{i,u}$ in the l-candidate can_i^l. It is obvious that $ids_{x'} = ids_x$. Therefore, the second condition also holds.

Finally, we further show that the previous $l-1$ covers of C_i satisfy the following three conditions defined in the definition of l-cover.

- $\forall (u \neq w), ids_{i,u} \cap ids_{i,w} = \emptyset$, and

- $\forall (id \in C_i)$, each $f_{i,u}(id)$ $(u \in 1, l-1])$ is from different color.

- $\forall(ids \in P^{lm})$, $|ids \cap C_i| = |ids \cap ids_{i,u}|$ $(u \in [1, l-1])$.

The first two conditions follow directly from the definition of l-candidate. The last condition is satisfied by the property of self-contained. In other words, given such P^{lm}, all colors have their l-covers, therefore, P^{lm} satisfies l-cover property. Thus we have proved the lemma. ■

In Lemma 4.2, we prove that an anonymized group in any self-contained identifier partition must either also be a group in the minimal self-contained partition, or be a union of several such groups. This result will be needed in later proofs.

Lemma 4.2 *Given any* l-eligible t_0*, a collection of* l*-candidates* Can^l *and its corresponding minimal self-contained partition* $P^{lm} = \{ids_1, ids_2, \ldots, ids_k\}$*, any self-contained identifier partition* P *satisfies that* $\forall(G \in P)$*, either* $G \cap ids_i = \emptyset$ *or* $G \supseteq ids_i$ *(*$i \in [1, k]$*) is true.*

Proof: We prove by contradiction. First assume that there exist $G \in P$ and $ids_i \in P^{lm}$, such that $G \cap ids_i \neq \emptyset$ and $ids_i - G \neq \emptyset$. Then, due to $ids_i - G \neq \emptyset$, there must exist identifier $id_o \in ids_i$ such that $id_o \notin G$. Assume that $id_o \in G'$, where $(G' \in P) \wedge (G' \neq G)$. Moreover, due to $G \cap ids_i \neq \emptyset$, there also exists identifier $id_i \in ids_i$ such that $id_i \in G$. Thus there exist id_o and id_i which is a pair of identifiers for some bijection in Can^l, and $G \cap \{id_o, id_i\} = \{id_i\}$ and $G' \cap \{id_o, id_i\} = \{id_o\}$. However, By definition of *self-contained*, it has the following transitive property. That is, if $\{id_1, id_2\}$, $\{id_2, id_3\}$, ..., $\{id_{a-1}, id_a\}$ each pair satisfies that there exists bijections for the set of l-candidates such that $f_{i-1,i}(id_{i-1}) = id_i$ or $f_{i,i-1}(id_i) = id_{i-1}$. Then for any self-contained anonymized group G, either $G \cap \cup_{i=1}^{a}(id_i) = \emptyset$ or $G \supseteq \cup_{i=1}^{a}(id_i)$. Thus by definition, since $id_o \in ids_i$ and $id_i \in ids_i$, $\forall(G \in P)$, $G \cap \{id_o, id_i\} = \emptyset$ or $G \cap \{id_o, id_i\} = \{id_o, id_i\}$.

Therefore, neither G nor G' satisfies self-contained, so does P, leading to a contradiction. ■

Based on Lemma 4.1 and 4.2, we now show that similar results hold for any self-contained identifier partition and any family set, as formulated in Theorem 4.1.

Theorem 4.1 *Given an l-eligible t_0 and the l-candidates Can^l, we have that*

- *any self-contained identifier partition P satisfies the l-cover property. Moreover, for each color in t_0, the corresponding l-candidate in Can^l is also the l-cover for P.*
- *any family set S^{fs} satisfies the l-cover property. Moreover, for each color in t_0, the corresponding l-candidate in Can^l is also the l-cover for S^{fs}.*

Proof: *First, we prove that any self-contained identifier partition P satisfies l-cover property.*

We first show that $P \in ss^l$. Note that the privacy model l-diversity satisfies the monotonicity property. That is, for any two anonymized groups G_1 and G_2 satisfying l-diversity, the final anony-mized group derived by merging all tuples in G_1 and in G_2 satisfies l-diversity [100]. Based on Lemma 4.2, each anonymized group G in P satisfies $G = \cup_{X \subseteq \{1,2,\ldots,k\}} ids_X$. Therefore, each anonymized group G satisfies l-diversity, so does P.

Then we have proved that, given the l-candidate can_i^l of certain color C_i, the $can_i^l \backslash \{C_i\}$ are the $l-1$ covers of C_i for P, similar to the proof of Lemma 4.1.

Finally, the set of $l-1$ set of identifiers $can_i^l \backslash \{C_i\}$ are $l-1$ covers of color C_i which satisfy the three conditions of l-cover definition.

Second, we prove any family set satisfies the l-cover property.

We first show that $\forall (P \in S^{fs}), P \in ss^l$. Since the privacy model l-diversity satisfies the monotonicity property [100], based on the definition of family set, it is clear that the table generalization corresponding to each identifier partition in S^{fs} satisfies l-diversity.

Similar with previous proofs, for each color $C_i \in S^C$ and its corresponding l-candidate $can_i^l = \{C_i, ids_{i,1}, ids_{i,2}, \ldots, ids_{i,l-1}\}$, we have proved that for the family set S^{fs}, $ids_{i,1}, ids_{i,2}, \ldots, ids_{i,l-1}$ are the $l-1$ covers of C_i. Moreover, these $l-1$ covers of C_i satisfy the three conditions of l-cover. This completes the proof. ■

Based on the above results, once the collection of l-candidates is determined, we can easily construct sets of identifier partitions to satisfy the l-cover property. Therefore, we now turn to finding efficient methods for constructing l-candidates. First, Lemma 4.3 and 4.4 present conditions for subsets of identifiers to be candidates for each other.

Lemma 4.3 *Given an l-eligible t_0, any $ids \subseteq \mathcal{I}$ that satisfies $|ids| = |C|$ and $ids \cap C = \emptyset$ is a candidate for color C.*

Proof: By the definition 4.3, C and ids should satisfy the following two conditions:

- $C \cap ids = \emptyset$ and $|C| = |ids|$;
- there exists a bijection $f : C \rightarrow ids$, such that $\forall(id \in C)$, id and $f(id)$ are from different colors.

The first condition follows directly from the condition of the lemma. Since $|C| = |ids|$, there must exist bijection $f : C \rightarrow ids$. Moreover, since $C \cap ids = \emptyset$, $\forall(id_x \in ids)$, $id_x \notin C$, by the definition of color, id_x has sensitive value other than it of color C. In other words, id_x must belong to the other color C' other than C. Therefore, The second condition is also satisfied, which completes the proof. ■

Lemma 4.4 *Given an l-eligible t_0, any $ids_1, ids_2 \subseteq \mathcal{I}$ satisfying following conditions are candidates for each other:*

- *$|ids_1| = |ids_2|$ and $ids_1 \cap ids_2 = \emptyset$, and*
- *the number of all identifiers in $ids_1 \cup ids_2$ that belong to the same color is no greater than $|ids_1|$.*

Proof: The first constraint in the lemma respectively guarantees the first condition of definition 4.3. Consider the second condition. Since $|ids_1| = |ids_2|$, there must exist bijections between ids_1 and ids_2. Assume that the second condition of definition 4.3 does not hold. Then there must exist at least $|ids_1| + 1$ number of identifiers in $ids_1 \cup ids_2$ with identical sensitive value, which is in contradiction with the second constraint in lemma. Therefore, the second condition of definition 4.3 also satisfies. Since the two conditions both hold, the proof is complete. ■

Based on Lemma 4.3 and 4.4, we now present conditions for constructing l-candidates of each color in Theorem 4.2. We will apply those conditions in the next section to design practical algorithms for building the SGSS.

Theorem 4.2 *Given an l-eligible t_0, each color C together with any $(l-1)$ subsets of identifiers $\{ids_1, ids_2, \ldots, ids_{l-1}\}$ that satisfy following conditions form a valid l-candidate for C:*

- $\forall(x \in [1, l-1])$, $|ids_x| = |C|$ *and* $ids_x \cap C = \emptyset$;
- $\forall((x, y \in [1, l-1]) \wedge (x \neq y))$, $ids_x \cap ids_y = \emptyset$;
- *the number of all identifiers in* $\cup_{x=1}^{l-1} ids_x$ *that belong to the same color is no greater than* $|C|$.

Proof: To prove the theorem, we should show that any two sets of identifiers from the sets C and ids_x $(x \in [1, l-1])$ are candidate for each other. The fact that C and each ids_x $(x \in [1, l-1])$ are candidate follows the Lemma 4.3, while the fact any two ids_x, ids_y $((x, y \in [1, l-1]) \wedge (x \neq y))$ are candidate follows the Lemma 4.4. This completes the proof. ■

4.3 The Algorithms

In this section, we design three algorithms for constructing l-candidates for colors and analyze their complexities. It is important to note that there may exist many other ways for constructing l-candidates based on the conditions given in Theorem 4.2. This flexibility allows us to vary the design of algorithms to suit different needs of various applications, because different l-candidates will also result in different SGSSs and hence algorithms more suitable for different utility metrics. We demonstrate such a flexibility through designing three algorithms in the following.

To simplify our discussions, we say an identifier is *complete* (or *incomplete*) if it is (or is not) included in any l-candidate; similarly, we say a color is *complete* (or incomplete) if it only includes complete identifiers (or otherwise); we also say a set of identifiers is *compatible* (or *incompatible*) with an identifier id, if there does not exist (or exists) identifier in that set that is from the same color as id; finally, given any color, an identifier from other colors is said to be *unused* with respect to that color if it has not yet been selected as a candidate for any identifier in that color. Table 4.5 summarizes the notations used in the algorithms.

n	The number of (incomplete) tuples in t_0
C_i	The i^{th} color, or the set of (incomplete) identifiers in the i^{th} color
n_c	The number of (incomplete) colors in t_0
S^C	The sequence of (incomplete) colors in t_0
n_i	The number of (incomplete) tuples in color C_i
can_{ia}	The set of $(l-1)$ identifiers selected for identifier id_{ia} in color C_i
can_i	l-candidate for color i
Can^l	The collection of l-candidates

Table 4.5: Notations for Algorithms

4.3.1 The RIA Algorithm (Random and Independent)

The main intention in designing the RIA algorithm is to show that, based on our results in Theorem 4.2, l-candidate can actually be built in a very straightforward way, although its efficiency and utility is not necessarily optimal. In the RIA algorithm, to construct the l-candidates for each color C_i, $(l-1)$ identifiers can_{ia} are selected randomly and independently for each identifier id_{ia} in C_i. The only constraint in this selection process for any color is that the same identifier will not be selected more than once. Clearly, designing such an algorithm is very straightforward. Roughly speaking, for each identifier id_{ia} in any color C_i, RIA randomly selects $(l-1)$ identifiers from any other $(l-1)$ colors that are not selected by other identifiers in C_i, and then form l-candidate can_i for C_i from the can_{ia} of each identifier.

Input: an l-eligible Table t_0, the privacy property l;
Output: the set Can^l of l-candidates for each color;
Method:
1. **Let** $Can^l = \emptyset$;
2. **For** $i = 1$ to n_c
   // Iteratively construct l-candidate for each color C_i
3. **For** $a = 1$ to n_i
   // Iteratively select the $l - 1$ number of identifiers for
   //each identifier $id_{i,a}$ in color C_i
4. Randomly select $l - 1$ different colors S^C_{ia} from $S^C \backslash \{C_i\}$;
5. Randomly select one *unused* identifier from each color in S^C_{ia};
6. Form can_{ia} by collecting the previously selected $l - 1$ identifiers in any order;
7. **For** $i = 1$ to n_c
8. **For** $w = 1$ to $l - 1$
   // Create the l-candidate can_i for C_i based on the $can_{ia}(a \in [1, n_i])$
9. Create in can_i its w^{th} candidate: $\bigcup_{a=1}^{n_i}$ (the w^{th} identifier in can_{ia});
10. **Let** $Can^l = \{can_i : 1 \leq i \leq n_c\}$;
11. **Return** Can^l;

Table 4.6: The RIA Algorithm

The RIA algorithm is shown in Table 4.6. RIA first set $Can^l = \phi$ (line 1). Then, Given the l-eligible table t_0, RIA iteratively constructs l-candidate for all its colors (line 2-9). In each iteration, RIA first repeatedly selects $(l - 1)$ identifiers can_{ia} for each identifier $id_{i,a}$ in color C_i. These identifiers are from $(l - 1)$ different colors and not be used yet by the other identifier in current color. Then RIA builds the $(l - 1)$ candidates for current color. To construct the w^{th} candidate, RIA selects the w^{th} identifier from each can_{ia} for each identifier $id_{i,a}$ in color C_i. Consequently C_i, together with its $(l - 1)$ candidates, form the l-candidate, can_i, for color C_i. Finally, all the can_i for each color form the set Can^l of l-candidates, and RIA terminates and returns Can^l.

The computational complexity of RIA algorithm is $O(l \cdot n)$ since: since: first, for each color, each of its identifiers costs exactly $(l-1)$ many constant times (line 4-6) to select its $(l - 1)$ identifiers, and there are n_i identifiers in the color, so totally $(l - 1) \times n_i$. Then, based on these identifiers, it takes $(l - 1) \times n_i$ many times to create its l-candidate. There are totally n_c many colors in the micro-table. Finally it takes n_c many times to create the set of l-candidates. Therefore, in totally its computational complexity is $O(\sum_i^{n_c}(2 \times (l-1) \times n_i) + n_c) = O(l \times n)$, because the size of all colors adds up to be n, and $n_c \leq n$. Note that once an identifier select same identifier which was selected by the previously considered identifier in the color, RIA must reselect other identifier for that identifier. During the analysis of computational complexity, we ignore the time of solving such conflicts in colors and identifiers in line 4 and line 5 respectively. It is reasonable for most cases in the real life that $n_i \times (l-1) \ll n$, since in such case the probability of conflicts is very low. Note that the RIA algorithm only builds the l-candidates. In order to obtain the self-contained identifier partition and hence the SGSS (as shown in Theorem 4.1), we still need to merge the can_{ia}'s that share the common identifiers (which actually has a higher complexity than $O(l \times n)$, but we will not further discuss it since our intention of introducing the RIA algorithm is not due to its efficiency).

4.3.2 The RDA Algorithm (Random and Dependent)

The RDA algorithm aims at general-purpose data utility metrics that only depends on the size of each anonymized group in an identifier partition, such as the well known *Discernibility Metric (DM)* [13]. As we shall show through experiments, our RDA algorithm will produce solutions whose data utility by the DM metric is very close to that of the optimal solution, since the RDA algorithm can minimize the size of most anonymized groups in the chosen identifier partition.

Roughly speaking, for the color C_i that has the most incomplete identifiers, the algorithm randomly selects $(l-1)$ identifiers can_{ia} for each of its identifiers id_{ia}, one from each of the next $(l-1)$ colors with the most incomplete identifiers, until the number of incomplete colors is less than l. For the remaining identifiers, the algorithm simply selects any $l-1$ identifiers as their candidates from any compatible can_{ia}. The key difference from the RIA algorithm is that the RDA algorithm will not consider an identifier once it

Input: an l-eligible Table t_0, the privacy property l;
Output: the set Can^l of l-candidates for each color;
Method:
1. **Let** n_c be the number of colors in t_0;
2. **Let** S^C be the sequence of the colors in the non-increasing order of their cardinality;
3. **Let** C_i, n_i $(i \in [1, n_c])$ be the i^{th} color and its cardinality;
4. **While** $(n_c \geq l)$
   //Construct l-candidate for the color in which most number of incomplete identifiers
5. Determine the color C_i which has most number of incomplete identifiers;
6. **For** $a = 1$ to n_i
7. **If**($id_{i,a}$ is *complete*)
8. **Skip** to check the next identifier in current color;
   // Iteratively select $(l-1)$ identifiers for each identifier $id_{i,a}$ in color C_i
9. Randomly select $l-1$ *incomplete* identifiers from $l-1$ different colors in S^C with most *incomplete* identifiers;
10. Form can_{ia} by collecting the previously selected $l-1$ identifiers in any order;
11. Remove the *complete* colors from S^C, and recalculate n_c;
12. Reorder the colors in S^C based on their number of *incomplete* identifiers;
13. **If** $(n_c < l)$ **Break**;
14. **While** $(S^C \neq \emptyset)$
15. Select any *incomplete* identifier $id_{j,b}$ from the color $C_j \in S^C$ with the most number of *incomplete* identifiers;
16. Select any $l-1$ identifiers from the *compatible* can_{ia} with the minimal cardinality;
17. Form can_{jb} by collecting the previously selected $l-1$ identifiers in any order;
18. **If** (color C_i is *complete*) Remove it from S^C;
19. **For** $i = 1$ to n_c
20. **For** $w = 1$ to $l-1$
   // Create the l-candidate can_i for C_i based on the $can_{ia}(a \in [1, n_i])$
21. Create in can_i its w^{th} candidate: $\bigcup_{a=1}^{n_i}($ the w^{th} identifier in $can_{ia})$;
22. **Let** $Can^l = \{can_i : 1 \leq i \leq n_c\}$;
23. **Return** Can^l;

Table 4.7: The RDA Algorithm

has selected its candidates, or been selected as a candidate, in most cases. This difference not only improves the data utility by minimizing the size of anonymized groups in the identifier partition, but also ensures the sets of candidates selected for different identifiers to be disjoint, which eliminates the need for the expensive merging process required by the RIA algorithm.

The RDA algorithm is shown in Table 4.7. Compared to RIA algorithm, RDA simply skips and does not reselect the $l-1$ identifiers for the $l-1$ candidates if the identifiers have been selected (line 7-8), and ensures that each identifier is not selected as candidates (line 9). Specifically, RDA algorithm first sets n_c, C_i, n_i, and S^C to be the number of colors, the i^{th} color and its cardinality, and the sequence of colors in the non-increasing order of cardinality in t_0 respectively (line 1-3). Then, RDA iteratively selects $l-1$ identifiers $can_{i,a}$ for each identifier $id_{i,a}$ in color C_i until the number of incomplete colors is less than l (line 4-13) . Here C_i is the color which has the most number of incomplete identifiers in S^C. In each iteration, RDA first selects one incomplete color with most incomplete identifiers (line 5). Then for each of its incomplete identifiers, RDA forms can_{ia} by randomly selecting $(l-1)$ incomplete identifiers from $(l-1)$ different colors in S^C (line 9-10), and removes the completed colors from S^C, recounts n_c, and reorders the colors in S^C in the non-increasing order of the number of incomplete identifiers (line 11-12). Next, RDA forms can_{ia} for the remainder identifiers (line 14-18). In each iteration, RDA first selects any incomplete identifier $id_{j,b}$ from the color C_j with the most number of incomplete identifiers (line 15) , and then forms can_{jb} by collecting any $l-1$ identifiers from any compatible can_{ia} with smallest size (line 16-17). Finally, all the can_i for each color form the set Can^l of l-candidates, and RDA terminates and returns Can^l (line 19-23).

Note that, we can derive the minimal self-contained partition directly through the bijections in the l-candidates. In other words, each can_{ia} is a transient group (see proof of Lemma 4.1) for minimal self-contained partition, furthermore, it is the anonymized group in minimal self-contained partition when the intersection between any two can_{ia} is empty. Actually, the construction of the set of l-candidate based on can_{ia}s (Line 19-22 in RDA algorithm) is only used to prove its existence. Therefore, in order to ensure that can_{ia}s are disjoint, line 16-17 can be replaced by: Append $id_{j,b}$ to its compatible can_{ia} with the minimal cardinality. Since can_{ia}s are disjoint, the merge process in Table 4.4 can be bypassed. This will reduce the computational complexity and improve the data utility under certain type of utility measures based on the size of the QI-groups, such as DM.

Furthermore, we show that the computational complexity of Line 9-12 is linear in l. First, the remainder colors in S^C are incomplete, and we can also design certain additional data structure to store the incomplete identifiers in each incomplete color and record the cardinality. Therefore, Line 9-10 can be processed in time linear in l. Second, since after Line 9-10, only $l-1$ colors (besides color C_i) are affected and their cardinality is only reduced by 1, Line 11-12 also can be processed in time linear in l with the assistance of additional structure. Based on previous discussions, the computational complexity of RDA algorithm is $O(n)$. First, Line 1-3 runs in $O(n)$ time by applying bucket sort (Additionally, $n_c << n$ holds for general cases in real world). Second, from Line 4-17, each identifier in the micro-data table is considered once all through the process with the assistance of additional data structure. We will evaluate utility of the RDA algorithm through experiments in the next section.

4.3.3 The GDA Algorithm (Guided and Dependent)

For both the RIA and RDA algorithms, we have assumed that the utility metric is independent of the actual quasi-identifier values. Our intention of designing the GDA algorithm is to demonstrate how our approach also allows designing algorithms that optimize data utility based on actual quasi-identifier values. For this purpose, assuming the quasi-identifier is composed of attributes $q_1, q_2, \ldots, q_d$, we assign an integer *weight* $weight_i$ to each attribute $q_i (i \in [1, d])$, and a rank $rank \in [1, |q_i|]$ to each value of the attribute q_i. Given any tuple t_a in the micro-data table t_0 and its value of each quasi-identifier attribute $t_a[q_i]$, we define its *weighted-rank* as $wr_a = \sum_{i=1}^{d}(weight_i \times rank(t_a[q_i]))$. Given any two tuples t_a and t_b, we define their *QI-distance* as $d_{ab} = |wr_a - wr_b|$. Also, given a tuple t_a and a set of tuples t_B, we define the *average QI-distance* as $d_{aB} = \frac{\sum_{b \in t_B}(d_{ab})}{|t_B|}$. Intuitively, a smaller QI-distance indicates that placing the two tuples into the same anonymized group will produce better data utility (for example, patients from the same geographical region should be grouped together).

Roughly speaking, for each incomplete identifier $id_{i,a}$ in the color C_i with the most incomplete identifiers, the algorithm determines $l-1$ incomplete colors that can minimize the QI-distance between their first incomplete identifier with the largest weighted-rank and $id_{i,a}$, and then selects these $l-1$ identifiers to be the $l-1$ candidates for $id_{i,a}$, until the number of incomplete colors is less than l. For each remainder identifier $id_{j,b}$, GDA selects $(l-1)$ identifiers from its compatible can_{ia} which has the smallest average QI-distance from $id_{j,b}$.

Input: an l-eligible table t_0, the privacy property l;
Output: the set Can^l of l-candidates for each color;
Method:
1. **Let** n_c be the number of colors in t_0;
2. **Let** S^C be the sequence of the colors in non-increasing order of their cardinality;
3. **Let** C_i, n_i $(i \in [1, n_c])$ be the i^{th} color and its cardinality;
4. Compute the weighted-rank for each tuple in the table t_0;
5. Sort the tuples in each color in ascending order of their weighted-rank values;
6. **While** $(n_c \geq l)$
7.   Let C_i be the color with the most incomplete identifiers;
8.   **For** each incomplete identifier $id_{i,a}$ in C_i
9.   Create can_{ia} by selecting $l-1$ *incomplete* identifiers from the first $l-1$ colors that minimize the QI-distance;
        between their first and $id_{i,a}$;
10. **For** each incomplete identifier $id_{j,b}$
11.   Create can_{jb} by selecting $l-1$ identifiers with minimal QI-distance from compatible can_{ia} with the least average QI-distance;
12. **For** $i = 1$ to n_c
13.   **For** $w = 1$ to $l-1$
        // Create the l-candidate can_i for C_i based on the $can_{ia}(a \in [1, n_i])$
14.   Create in can_i its w^{th} candidate: $\bigcup_{a=1}^{n_i}$(the w^{th} identifier in can_{ia});
15. **Let** $Can^l = \{can_i : 1 \leq i \leq n_c\}$;
16. **Return** Can^l;

Table 4.8: The GDA Algorithm

The GDA algorithm is shown in Table 4.8. Given a micro-data table t_0 and an integer l, GDA first initialize the following: Set n_c, C_i, n_i, and S^C to be the number of colors, the i^{th} color and its cardinality, and the sequence of colors in the non-increasing order of cardinality in t_0 respectively (line 1-3); Compute the weighted-rank for each identifier (tuple) based on its quasi-identifier information (line 4); Sort the identifiers inside a color in ascending order of their weighted-rank values (line 5). After that, GDA iteratively constructs can_{ia} for each identifier in the micro-table t_0(line 6-11). In each iteration, GDA repeatedly selects $l-1$ identifiers can_{ia} for each identifier $id_{i,a}$ in color C_i. For each identifier $id_{i,a}$, we select the $l-1$ best colors among the whole set of colors other than C_i itself. To judge the best colors, we compare the QI-distance between the QI-attributes of $id_{i,a}$ and the first identifier in each color which is not yet mapped to any identifier in C_i. The less the QI-distance is, the better the identifier is. Finally, all the can_i for each color forms the set Can^l of l-candidates, and GDA terminates and returns Can^l (line 12-16). From the description above, the selection of l-candidate for each color is further decided by the selection of $l-1$ identifiers for each of its identifier, which in turn are selected based on the QI-distance, it is, the local optimization. Therefore, the transient groups are expected to be closer with regard to the QI-attributes, which may increase the data utility. However, this approach cannot assure the size of the anonymized group since there may exists many merges when construct the locally-minimal partition based on such set of l-candidates.

The computational complexity of GDA algorithm is $O(n \, log \, n)$ since after sorting each color based on the weighted-rank values, each identifier is processed only once throughout the process of building l-candidates. Since this algorithm aims at minimizing the average QI-distance inside each anonymized group, we will evaluate its data utility in the next section based on such a quasi-identifier value-dependent metric.

4.3.4 The Construction of SGSS

Remind that our ultimate objective is to construct strongly globally safe set (SGSS) in which the data utility is optimized later. Once Can^l has been constructed by RIA, RDA, or GDA algorithm, in this chapter we adopt the approach based on the corresponding bijections in Can^l to building the minimal self-contained partition and then the SGSS.

More specifically, for RDA and GDA algorithms, each can_{ia}, created in step 10 in Table 4.7 and in step 9 in Table 4.8 respectively, forms an anonymized group. Then we simply append the $id_{j,b}$, in step 15 in Table 4.7 and in step 11 in Table 4.8 respectively, to the selected can_{ia}. Similarly, for RIA algorithm, each can_{ia} created in step 6 in Table 4.6 forms an anonymized group, and we then merge the resultant anonymized groups which have common identifiers to be disjoint sets. The algorithms in the literature to achieve disjoint sets are applicable for our problem and the details are omitted here.

For the experiments in Section 4.4, we integrate the process in building the minimal self-contained partitions into the algorithms of constructing Can^l for RDA and GDA algorithms.

4.4 Experiments

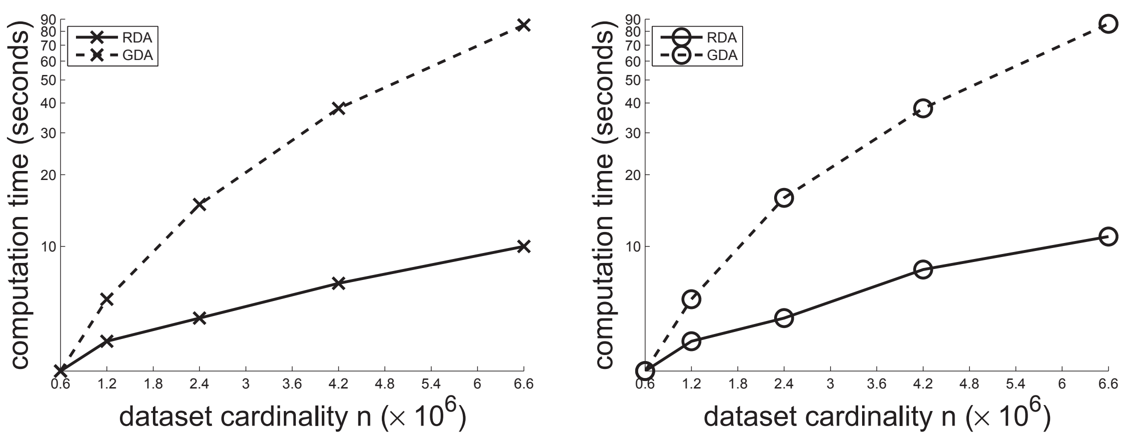
In this section, we evaluate the efficiency and utility of our proposed algorithms through experiments. To compare our results to that reported in [105], our experimental setting is similar to theirs. We adopt two real-world datasets, OCC and SAL, at the Integrated Public Use Micro-data Series [89]. Each dataset contains 600k tuples. The domain sizes of the six chosen attributes of both datasets are shown in Table 4.9. Among these, we select four attributes, *Age*, *Gender*, *Education*, and *Birthplace*, as the QI-attributes for both datasets, and we select *Occupation* and *Income* as the sensitive attribute for OCC and SAL, respectively. For our experiment, we adopt the *MBR* (*Minimum Bounding Rectangle*) function (similar to that in [105]) to generalize QI-values within the same anonymized group once we obtain an identifier partition using our algorithms. As mentioned before, the RIA algorithm is only introduced to demonstrate how simple an algorithm can be by following our approach, we will not evaluate its performance, but only focus on the RDA and GDA algorithm. In fact, in these two algorithms, each can_{ia} forms an anonymized group (transient group), and for the remainder identifiers shown in step 6 in Table 4.7 and step 7 in Table 4.8 are simply appended in the selected compatible anonymized groups (Step 19-22 in Table 4.7 and step 12-15 in Table 4.8 are used to represent the l-candidates). All experiments are conducted on a computer equipped with a 1.86GHz Core Duo CPU and 1GB memory.

Attribute	Age	Gender	Education	Birthplace	Occupation	Income
Domain Size	79	2	17	57	50	50

Table 4.9: Description of OCC and SAL Datasets

We evaluate computational complexity using execution time, and evaluate data utility of the released table using two measurements: Discernibility Metric (DM) [13] and Query Workload Error (QWE, which is a utility metric that depends on quasi-identifier values) [62].

4.4.1 Computation Overhead



(a) SAL Dataset (b) OCC Dataset

Figure 4.1: Execution Time vs. Dataset Cardinality n

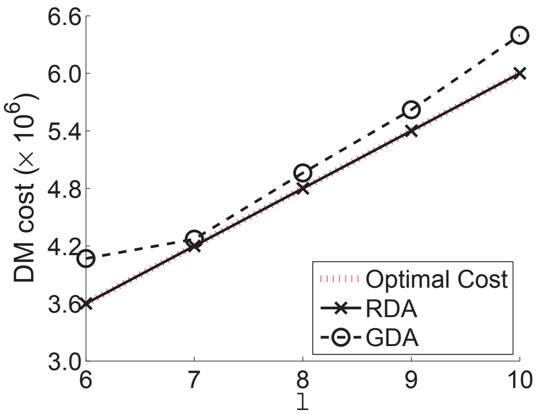
Figure 4.1 illustrates the computation time of both of our algorithms on both datasets against the dataset cardinality n. We generate n-tuple datasets by synthesizing $\frac{n}{600k}$ copies of OCC, SAL respectively (Reminder that both OCC and SAL contain 600k tuples). We set $l = 8$ for this set of experiments, and conduct the experiment 100 times and then take the average. From the results, it is clear that both of our algorithms are practically efficient, and the computation time increases slowly with n. The RDA algorithm is slightly more efficient than GDA. This is because, when selecting candidates for each identifier, RDA considers the $l - 1$ colors with the most incomplete identifiers while GDA considers the $l - 1$ colors whose incomplete identifiers have the least QI-distances. Therefore, the more complex computation required by the GDA algorithm results in slightly more overhead than RDA.

Comparing to Results in [105] In contrast to the results reported in [105], both of our algorithms are more efficient, while the RDA algorithm requires significantly less time than that in [105]. Although not reported here due to space limitations, we have also investigated the computation time against l as well as the number of QI-attributes. Both algorithms are insensitive to these two parameters. This is as expected since the computation complexity of both algorithms only depends on the cardinality of dataset n.

4.4.2 Data Utility

We first conduct a set of experiments on the original SAL and OCC dataset to evaluate the utility of released tables measured by the *DM* metric. Figure 4.2 shows the DM cost (the lower cost the better utility) of each algorithm against l. From the results, we can see that the DM cost of our RDA algorithm is very close to the optimal cost (calculated using a separate algorithm), while the DM cost of the GDA algorithm is only slightly higher than the optimal cost. This is as expected, because the RDA algorithm is specifically designed for a general-purpose utility metric that aims to minimize the size of each anonymized group regardless of actual quasi-identifier values, whereas the GDA algorithm will attempt to minimize the QI-distance (the assignment of weight and rank for the GDA algorithm is described below).

Following [105], we then evaluate the query workload error (QWE) by answering

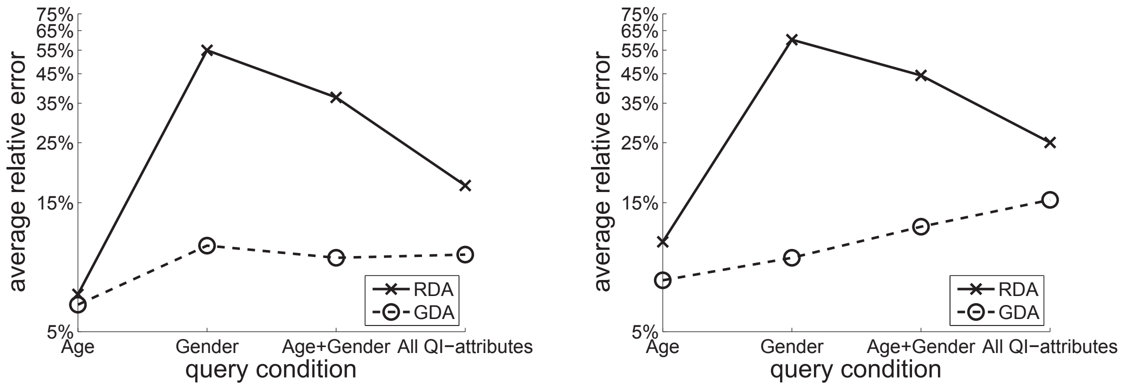


(a) SAL Dataset (b) OCC Dataset

Figure 4.2: Data Utility Comparison: DM Cost vs. l

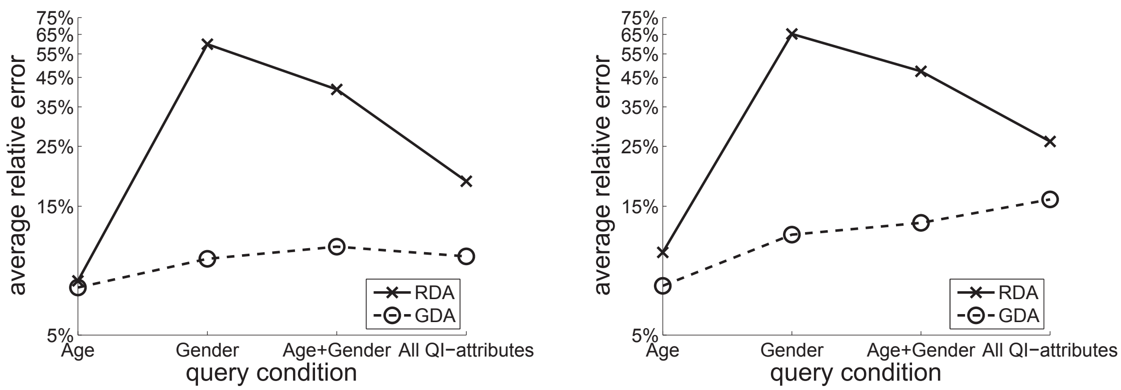
count queries. The intention is to compare our algorithms with a utility metric that depends on the actual quasi-identifier values. For this purpose, predicates on QI-attributes are constructed on *Age*, *Gender*, with an *and* operations between them, and with an *and* operations between all the QI-attributes, respectively. We set *weight* to be 1,10000,1, and 1 for *Age*, *Gender*, *Education*, and *Birthplace*, respectively. By processing 1000 randomly-generated queries for each type of predicates, we intend to investigate how well the released table preserves the R_{qs} relation. For each query, we first obtain its accurate answer *acc* from the original micro-data table, and then adopt the approximation technique in [62] to compute the approximate answer *app* from the released table output by our algorithms. The error of an approximate answer is formulated as $\frac{|acc-app|}{max\{acc,\delta\}}$ [105], where δ is set to 0.5% of the dataset cardinality. Then, the average error of all queries is taken as the QWE.

Figures 4.3, 4.4, 4.5, 4.6, and 4.7 show the average relative error against different types of predicates for $l = 6, 7, 8, 9$ and 10 respectively. Compared to RDA, GDA now has better utility, which is as expected since GDA does consider the actual quasi-identifier values in generating the identifier partition, as mentioned in Section 4.3. Particularly, the average relative error for querying on SAL and OCC with *Gender* as the only query condition for $l = 8$ is reduced from 64%, 69% (of RDA) to 10%, 18%, respectively. Finally, although not reported here due to space limitations, the utility result of our algorithms



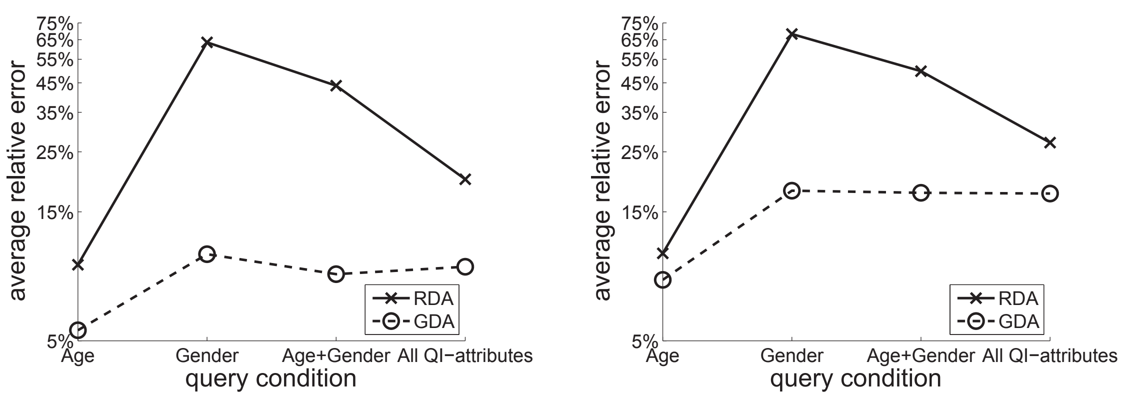
(a) SAL Dataset (b) OCC Dataset

Figure 4.3: Data Utility Comparison: Query Accuracy vs. Query Condition($l = 6$)



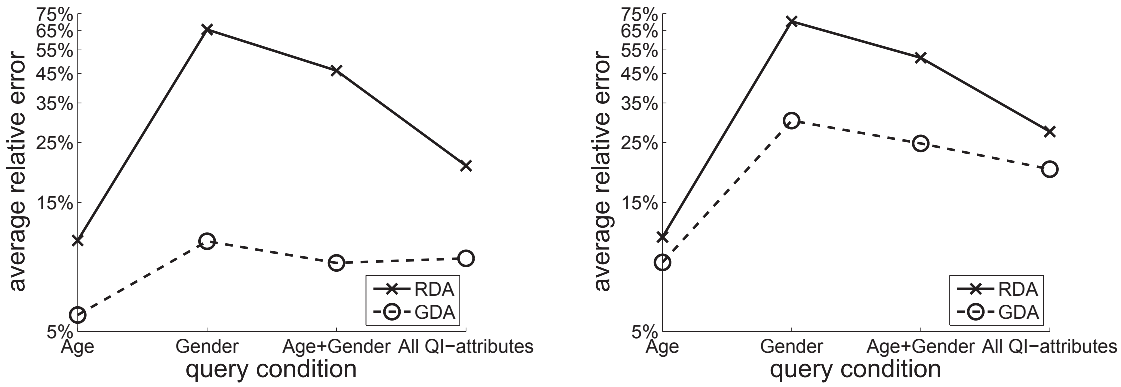
(a) SAL Dataset (b) OCC Dataset

Figure 4.4: Data Utility Comparison: Query Accuracy vs. Query Condition($l = 7$)



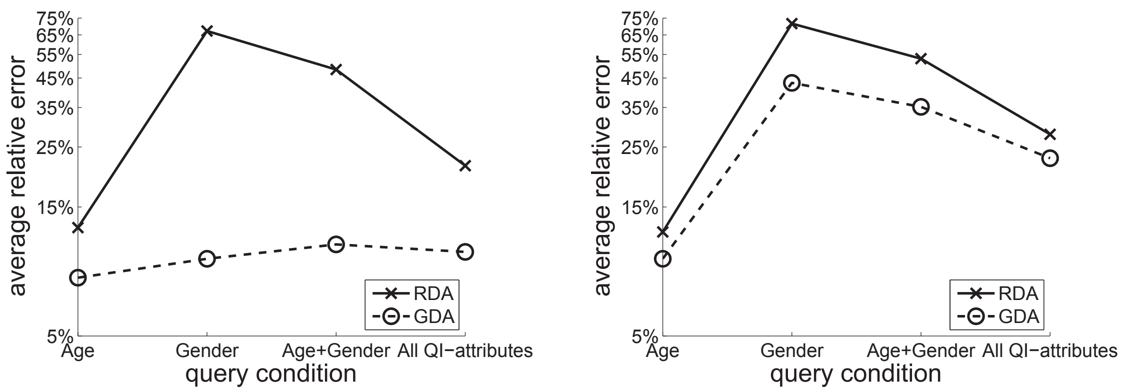
(a) SAL Dataset (b) OCC Dataset

Figure 4.5: Data Utility Comparison: Query Accuracy vs. Query Condition($l = 8$)



(a) SAL Dataset (b) OCC Dataset

Figure 4.6: Data Utility Comparison: Query Accuracy vs. Query Condition($l = 9$)



(a) SAL Dataset (b) OCC Dataset

Figure 4.7: Data Utility Comparison: Query Accuracy vs. Query Condition($l = 10$)

measured by QWE are close to the results reported in [105] (no result based on DM was reported there).

4.5 Discussion

Possible Extensions In this chapter, we have focused on applying the self-contained property on l-candidates to build sets of identifier partitions satisfying the l-cover property, and hence to construct the SGSS. However, there may in fact exist many other methods to construct the SGSS, which will lead to potential directions of future work. First, there are different ways for building the l-candidates for each color. As discussed above, theoretically any subset of $\mathcal{I}$ satisfying the constraints shown in Lemma 4.3 can be a valid candidate for a color, and $l-1$ such subsets together with that color will form a valid l-candidate for that color if they satisfy the constraints shown in Theorem 4.2. Second, once l-candidates are given, there still exist different ways, including applying the self-contained property, for constructing sets of identifiers to satisfy the l-cover property. Third, even the l-cover property is not necessarily the only valid way for directly building the SGSS. Finally, although we have focused on l-diversity and the utility measures DM and QWE, the principle of decoupling utility optimization from privacy preservation can potentially be applied to other privacy applications to yield efficient solutions.

Practicality of Our Approach We have demonstrated the practicality of our approach by showing through complexity analysis and experiments that our proposed algorithms are efficient enough to be applied to real world applications. It is important to note that it would be unfair to compare the performance of our algorithms to many existing algorithms that ignore the issue of privacy breaches caused by adversarial knowledge about algorithms [48, 98]. As to utility, as discussed earlier, our proposed algorithms produce results comparable to existing methods. We believe the flexibility of our approach may lead to other algorithms with further improved utility. For the QWE metric, note that our experiments only evaluate the QWE cost on the minimal self-contained partition. The utility may be increased by fine-tuning the weight information for each quasi-identifier, and by optimizing among the family set. We will conduct more experimental comparisons in terms of performance and utility between our algorithms and the traditional approaches in our future work.

The Focus on Syntactic Privacy Principles We have focused on syntactic privacy principles and methods, such as l-diversity and generalization, in this chapter. However, the general approach of decoupling utility optimization from privacy preservation is not necessarily limited to such a scope. In particular, one interesting issue is to consider its applicability to differential privacy [40], which is being accepted as one of the strongest privacy models and extended to privacy preserving data publishing [66]. On the other hand, since most existing approaches that ensure differential privacy are random noise-based and are suitable for specific types of statistical queries, we have regarded this direction as future work.

4.6 Summary

In this chapter, we have proposed a *privacy streamliner* approach for privacy-preserving applications. We reported theoretical results required for instantiating this approach in the context of privacy-preserving micro-data release using public algorithms. We have also designed three such algorithms by following the proposed approach, which not only yield practical solutions by themselves but also reveal the possibilities for a large number of algorithms that can be designed for specific utility metrics and applications. Our experiments with real datasets have proved our proposed algorithms to be practical in terms of both efficiency and data utility. Our future work will apply the proposed approach to other privacy-preserving applications and privacy properties in order to develop efficient algorithms.

Chapter 5

PPTP: k-Indistinguishable Traffic Padding in Web Applications

In this chapter, we present a formal PPTP model encompassing the privacy requirements, padding costs, and padding methods to prevent side-channel attacks due to unique patterns in packet sizes and directions of the encrypted traffic among components of the Web application.

5.1 Overview

Web-based applications are becoming increasingly popular. In contrast to their desktop counterparts, Web applications demand less client-side resources and are easier to deliver and maintain through using the Web browser as a thin client. On the other hand, Web applications also present new security and privacy challenges, partly because the untrusted Internet has essentially become an integral component of such applications for carrying the continuous interaction between users and servers. Recent study showed that the encrypted traffic of many popular Web applications may actually disclose highly sensitive data, and consequently lead to serious breaches of user privacy [22]. Specifically, by searching for unique patterns exhibited in packets' sizes and/or timing, an eavesdropper can potentially identify an application's internal state transitions and the corresponding users' inputs. Moreover, such side-channel attacks are shown to be pervasive and fundamental to most Web applications due to many intrinsic characteristics of such applications, such as low entropy inputs, diverse resource objects, and stateful communications.

Taking one popular real-world search engine as an example, Table 5.1 shows the sizes and directions of packets observed between users and the search engine. Observe that due to the user-friendly *auto-suggestion* feature, with each keystroke, the browser sends a b-byte packet to the server; the server then replies with two packets of 60 bytes and s bytes, respectively; finally, the browser sends a 60-byte packet to the server. In addition, in the same input string, the b value of the first keystroke is about 50 larger than that of the second one while each subsequent keystroke increases the b value by one byte from the third keystroke, and the s value depends both on the current keystroke and on all the preceding ones. Clearly, due to the fixed pattern in packet sizes (first, second, and last), the packets corresponding to each input string can be identified from observed traffic, even though the traffic has been encrypted.

User Input	Observed Directional Packet Sizes			
be**e**	$641 \rightarrow$,	$\leftarrow 60$,	$\leftarrow 544$,	$60 \rightarrow$,
	$585 \rightarrow$,	$\leftarrow 60$,	$\leftarrow 555$,	$60 \rightarrow$,
	$586 \rightarrow$,	$\leftarrow 60$,	$\leftarrow 547$,	$60 \rightarrow$
ca**b**	$641 \rightarrow$,	$\leftarrow 60$,	$\leftarrow 554$,	$60 \rightarrow$,
	$585 \rightarrow$,	$\leftarrow 60$,	$\leftarrow 560$,	$60 \rightarrow$,
	$586 \rightarrow$,	$\leftarrow 60$,	$\leftarrow 558$,	$60 \rightarrow$
	(b bytes)		(s bytes)	

Table 5.1: User Inputs and Corresponding Packet Sizes

Similar traffic patterns have also been observed in different categories of Web applications [22]. Therefore, we assume a worst case scenario in which an eavesdropper can pinpoint traffic related to a Web application (such as using de-anonymizing techniques [95]) and locate packets for user inputs using the above technique. We use search engines as examples in this chapter due to their distinct and representative patterns. In reality, the s value can be larger and more disparate as discussed in Section 5.6.

Moreover, the size of the third packet provides a good indicator of the input itself (which again can be found in many Web applications [22]). Specifically, Table 5.2 shows the s value for character (a, b, c and d) entered as the first (second column) and second (3-6 columns) keystroke for a different search engine. Observe that the s value for each character entered as second keystroke is different from that it is entered as the first, since the packet size now depends on both the current keystroke and the preceding one. Clearly, every input string can be uniquely identified by combining observations of packet sizes about the two consecutive keystrokes (for simplicity, we only consider $a-d$ combinations here, whereas in reality it may take more than two keystrokes to uniquely identify an input string).

		Second Keystroke			
First Keystroke		**a**	**b**	**c**	**d**
a	509	487	493	501	497
b	504	516	488	482	481
c	502	501	488	473	477
d	516	543	478	509	499

Table 5.2: s Value for Each Character Entered as the First (Second Column) and Second (3-6 Columns) Keystroke

A natural solution for preventing such a side channel attack is to pad packets such that each packet size will no longer map to a unique input (One extreme case is to pad all packets to the identical size, namely, *maximizing*). However, such a solution does not come free, since padding packets will result in additional communication and processing overhead. In fact, it has been shown that a straightforward solution, such as random padding (appending a random-length padding within a given interval to a packet) and rounding (rounding packet sizes to the nearest intervals), may incur a prohibitive overhead (e.g. 21074% for a well-known online tax system [22]). Thus, we face two seemingly conflicting goals. First, the difference in packet sizes needs to be sufficiently reduced to prevent eavesdroppers from distinguishing between different users inputs based on corresponding packet sizes. Second, the overhead for achieving such privacy protection should be minimized. Finally, a tradeoff naturally exists between these two objectives.

We now consider a different way for padding the packets as shown in Table 5.3. The first and last columns respectively show the s value and corresponding character with its prefix (e.g., $(c)d$ means the character d is entered as the second keystroke after its prefix c is entered for the same input string). The middle two columns give two options for padding packets (although not shown here, there certainly exist many other options). Specifically, each option first divides the six keystrokes into three (or two) *padding groups*, as illustrated by the (absence of) horizontal lines. Packets within the same padding group are then padded in such a way that the corresponding s values become identical to the maximum value in that group, and thus the characters inside the group will no longer be distinguishable from each other by the s values. The objective now is to find a padding option that can provide sufficient privacy protection and meanwhile minimize the padding cost. Note that gathering such packet information is practical for most Web applications, as we will discuss later in Section 5.6.1.

s **Value**	**Padding**		**(Prefix)Char**
	Option 1	**Option 2**	
473	477	478	$(c)c$
477	477	478	$(c)d$
478	499	478	$(d)b$
499	499	509	$(d)d$
501	509	509	$(c)a$
509	509	509	$(d)c$
Quasi-ID	**Generalization**		**Sensitive Value**

Table 5.3: Mapping PPTP to PPDP

Interestingly, this *privacy-preserving traffic padding (PPTP)* problem is naturally associated with another well studied problem, namely, *privacy-preserving data publishing (PPDP)* [47]. For example, in Table 5.3, if we regard the s value as a *quasi-identifier* (such as DoB), the input as a *sensitive value* (such as medical condition), and the padding options as different ways for generalizing the DoB into *anonymized groups* (for example, by removing the day from a DoB), then we immediately have a classic PPDP problem, that is, publishing DoBs and medical conditions while preventing adversaries from linking any published medical condition to a person through his/her DoB [47].

The above connection between the two issues implies that we may borrow many existing efforts in the PPDP domain to address the PPTP issue based on the similarity between these two problems. On the other hand, there also exist significant differences between the two. As an example, in Table 5.3, the second option will likely be considered as worse than the first in the PPDP domain in terms of typical data utility measures (intuitively, the second option leads to more utility loss due to its larger anonymized groups), whereas it is actually better in the PPTP domain with respect to padding cost (it can be shown that the second option incurs totally 24 bytes of overhead, in contrast to 33 by the first option). As another example, we will show later that the effect of combining two keystrokes will be equivalent to releasing multiple inter-dependent tables, which actually leads to a novel PPDP problem.

In this chapter, we first present a model of the PPTP issue based on the mapping to PPDP, which formally characterizes the interaction between users and Web applications, the observation made by eavesdroppers, the privacy requirement, and the overhead of padding. Based on the model, we then formulate several PPTP problems under different assumptions, and discuss the complexity. We show that minimizing padding cost under a given privacy requirement is generally intractable. Next, we design several heuristic algorithms for solving the PPTP problems in polynomial time with acceptable overhead. Finally, we demonstrate the effectiveness and efficiency of our algorithms by both analytical and experimental evaluations.

The contribution of this chapter is threefold. First, the identified similarity between PPTP and PPDP establishes a bridge between the two research areas, which will not only allow for reusing many existing models and methods in the well investigated PPDP domain, but serve to attract more interest to the important PPTP issue. Second, to the best of our knowledge, our formal model is among the first efforts on formally addressing the PPTP issue (refer to Chapter 2 for a detailed review of related work). Third, the proposed algorithms may provide direct and practical solutions to real world PPTP applications, as evidenced by our implementation and comparative experimental studies. Moreover, those algorithms demonstrate the feasibility of adapting existing PPDP methods to the PPTP domain, and the challenges in doing so.

The rest of the chapter is organized as follows. Section 5.2 defines our PPTP model. Section 5.3 formulates PPTP problems and analyzes the complexity. Section 5.4 devises heuristic algorithms for the formulated problems. Section 5.5 proposes an extended version of the PPTP solution to accommodate different likelihoods of possible inputs, including a re-defined privacy model, the new PPTP problems, and corresponding PPTP algorithms. Section 5.6 discusses the implementation of our solution, and experimentally evaluates the performance of our algorithms. Finally, Section 5.7 concludes the chapter.

5.2 The Model

Section 5.2.1 first presents the basic model of interaction and observation. Section 5.2.2 then maps PPTP to PPDP in order to quantify privacy protection and overhead. Finally, Section 5.2.3 extends the basic model to more realistic cases. We will also demonstrate its flexibility to adapt different privacy properties in Section 5.5. Table 5.4 lists main notations that will be used throughout the chapter.

a, $\vec{a}$, A_i or A	Action, action-sequence, action-set
s, v, $\vec{v}$, V_i or V	Flow, flow-vector, vector-sequence, vector-set
$\vec{a}[i], \vec{v}[i]$	The i^{th} element in $\vec{a}$ and $\vec{v}$
VA_i or VA	Vector-action set
$pre(a, i)$	i-Prefix
$dom(P)$	Dominant-vector
$vdis(v_1, v_2)$	Vector-distance

Table 5.4: The Notation Table

5.2.1 The Basic Model

We model the PPTP issue from two perspectives, the *interaction* between users and servers, and the *observation* made by eavesdroppers. First, Definition 5.1 formalizes the interaction. Our discussions about Table 5.2 demonstrated how one keystroke may affect another in terms of observations (packet sizes), and how an eavesdropper may combine such multiple observations for a refined inference. Such inter-dependent user *actions* are modeled as an *action-sequence* in Definition 5.1. The concept of *action-set* models a collection of actions whose corresponding observations may be padded together.

Definition 5.1 (Interaction) *Given a Web application, we define*

- *an* action a *as an atomic user input that triggers traffic, such as a keystroke or a mouse click.*

- *an* action-sequence $\vec{a}$ *as a sequence of actions with known relationships, such as consecutive keystrokes entered into real-time search engine or a series of mouse clicks on hierarchical menu items. We use* $\vec{a}[i]$ *to denote the* i^{th} *action in* $\vec{a}$.

- *an* action-set A_i *as the collection of all the* i^{th} *actions in a set of action-sequences. We will simply use* A *if all action-sequences are of length one.*

Example 5.1 *Assume "be**e**" and "ca**b**" in Table 5.1 to be the only possible inputs, we have six actions,* a_{11}, a_{12}, a_{13} *and* a_{21}, a_{22}, a_{23} *corresponding to* b, e *(as second keystroke),* ***e*** *(as third) in input "be**e**", and* c, a, ***b*** *(as third keystroke) in input "ca**b**". There are two action-sequences* $\vec{a}_1 = \langle a_{11}, a_{12}, a_{13} \rangle$ *and* $\vec{a}_2 = \langle a_{21}, a_{22}, a_{23} \rangle$*, and three action-sets* $A_1 = \{a_{11}, a_{21}\}$*,* $A_2 = \{a_{12}, a_{22}\}$*, and* $A_3 = \{a_{13}, a_{23}\}$. $\boxdot$

Definition 5.2 models concepts related to the observation made by an eavesdropper. Note that a *flow-vector* is intended to only model those packets that may contribute to identify an action (such as the s value in Table 5.1). Also, each action is not associated with a flow but a flow-vector, which is itself a sequence, since a single action may trigger more than one packet. Finally, unlike an action-set, a vector-set is defined as a multiset, since it may contain duplicates (that is, packets nay share the same size).

Definition 5.2 (Observations) *Given a Web application, we define*

- *a* flow-vector v *w.r.t. an action* a *as a sequence of* flows*, where each flow* s *represents the size of a directional packet triggered by* a*. Denoted the relation between* a *and* v *by* $f(a)=v$*.*

- *a* vector-sequence $\vec{v}$ *as a sequence of flow-vectors corresponding to an equal-length action-sequence* $\vec{a}$*, with each* $\vec{v}[i]$ *corresponding to* $\vec{a}[i]$ *(*$1 \leq i \leq|\ \vec{v}\ |$*).*

- *a* vector-set V_i *(or simply* V*) as the collection of all the* i^{th} *flow-vectors in a set of vector-sequences, which corresponds to an action-set in the straightforward way.*

Example 5.2 *Following Example 5.1, we have six flow-vectors,* $v_{11} = \langle 544\rangle$*,* $v_{12} = \langle 555\rangle$*,* $v_{13} = \langle 547\rangle$ *and* $v_{21} = \langle 554\rangle$*,* $v_{22} = \langle 560\rangle$*,* $v_{23} = \langle 558\rangle$ *(note that we only model those packets whose sizes can help to identify an action), corresponding to actions* a_{11}, a_{12}, a_{13} *and* a_{21}, a_{22}, a_{23}*, respectively. We have two vector-sequences* $\vec{v}_1 = \langle v_{11}, v_{12}, v_{13}\rangle$ *and* $\vec{v}_2 = \langle v_{21}, v_{22}, v_{23}\rangle$*, corresponding to action-sequences* $\vec{a}_1$ *and* $\vec{a}_2$*, respectively. We have three vector-sets* $V_1 = \{v_{11}, v_{21}\}$*,* $V_2 = \{v_{12}, v_{22}\}$ *and* $V_3 = \{v_{13}, v_{23}\}$ *corresponding to the three action-sets* A_1*,* A_2*, and* A_3 *in Example 5.1.* $\boxdot$

Finally, Definition 5.3 models the joint information about interaction and observation, which is the collection of the pairs of the action and its corresponding flow-vector.

Definition 5.3 (Vector-Action Set) *Given an action-set* A_i *and its corresponding vector-set* V_i*, a* vector-action set VA_i *is the set* $\{(v, a) : v \in V_i \wedge a \in A_i \wedge f_i(a) = v\}$.

Example 5.3 *Following above Examples, given the action-set A_1 and vector-set V_1, then the vector-action set is $VA_1 = \{(v_{11}, a_{11}), (v_{21}, a_{21})\}$. Similarly, $VA_2 = \{(v_{12}, a_{12}), (v_{22}, a_{22})\}$, $VA_3 = \{(v_{13}, a_{13}), (v_{23}, a_{23})\}$.*

5.2.2 Privacy and Cost Model

For simplicity, we first consider a simplified case where every action-sequence and flow-vector are of length one, namely, the Single-Vector Single-Dimension (SVSD) case. That is, all actions are independent, and each action triggers only a single packet that can be used to identify the action. In this case, we map a given vector-action set $VA = \{(v, a) : v \in V \wedge a \in A \wedge f(a) = v\}$ to a table $T(v, a)$ with two attributes, the flow-vector v (equivalent to a flow s here) as quasi-identifier and the action a as sensitive attribute. Note that we will interchangeably refer to a vector-action set and its tabular representation from now on.

Definition 5.4 quantifies the amount of privacy protection under a given vector-action set. This model follows the widely adopted approach of assuming a fixed privacy requirement while minimizing the cost.

Definition 5.4 (k-Indistinguishability) *Given a vector-action set VA, we define*

- *a* padding group *as any $S \subseteq VA$ satisfying that all the pairs in S have identical flow-vectors and no $S' \supset S$ can satisfy this property, and
- *we say VA satisfies* k-indistinguishability *(k is an integer) or VA is* k-indistinguishable *if the cardinality of every padding group is no less than k.*

Discussion One may argue that, in contrast to encryption, *k-indistinguishability* may not provide strong enough protection. However, as mentioned before, we are considering cases where encryption is already broken by side-channel attacks, so the strong confidentiality provided by encryption is already not an option. Second, in theory k could always be set to be sufficient large to provide enough confidentiality, although a reasonably large k

would usually satisfy users' privacy requirements for most practical applications. Finally, since most web applications are publicly accessible and consequently an eavesdropper can unavoidably learn about possible inputs, we believe focusing on protecting sensitive user input (by hiding it among other possible inputs) yields higher practical feasibility and significance than on perfect confidentiality (attempting to hide everything).

As demonstrated in Section 5.1, we can map the PPTP model to PPDP. Meanwhile, such mapped PPDP problems actually possess a unique characteristic. That is, the sensitive values (actions) are always unique. Thus, by satisfying k-indistinguishability, the vector-action set also satisfies *l-diversity* ($l = k$) in its simplest form [77]. Furthermore, a probabilistic approach based on *differential privacy* [40] is another possible extension to enhance our model such that the padding result will be immune to eavesdroppers' prior knowledge. Nonetheless, this simple model is sufficient to demonstrate the usefulness of mapping PPTP to PPDP. For simplicity, we will first focus on *k-indistinguishability* in Sections 5.2– 5.4, and delay the discussion about more general forms of l-diversity in Section 5.5 to address cases where not all actions should be treated equally in padding.

In addition to privacy requirement, we also need a quantitative measure for the cost of padding and processing. Across the whole vector-set, Definition 5.5 counts the number of additional bytes after padded, while Definition 5.6 counts the number of flows that are involved in padding. We focus on these simple models in this chapter while there certainly exist other ways for modeling such costs.

Definition 5.5 (Distance and Padding Cost) *Given a vector-set V, we define*

- *the* vector-distance *between two equal-length flow-vectors v_1 and v_2 as: $vdis(v_1, v_2) = \sum_{i=1}^{|v_1|}(|s_{1i} - s_{2i}|)$ where s_{1i} and s_{2i} are the i^{th} flow in v_1 and v_2, respectively.*

- *the* padding cost *of V as: $cost = \sum_{i=1}^{|V|}(vdis(v_i, v_i'))$ where v_i and v_i' denote a flow-vector in V and its counterpart after padding, respectively.*

Definition 5.6 (Processing Cost) *Given a vector-set V, we define the processing cost of V as the number of flows in V which corresponding packets should be padded.*

5.2.3 The SVMD and MVMD Cases

In the previous section, we have focused on the simplified SVSD case to facilitate a focused discussion on the privacy and cost model. We now look at the more realistic cases. First, we consider the Single-Vector Multi-Dimension (SVMD) case where each flow-vector may include more than one flows (that is, an action may trigger more than one packets that can be used to identify the action), whereas each action-sequence is still composed of a single action. In this case, the vector-action set needs to be mapped to a table $T(s_1, \ldots, s_{|v|}, a)$ with multiple quasi-identifier attributes (each flow corresponds to an attribute). Thus, based on Definition 5.4, flow-vectors can form a padding group only if they are identical with respect to every flow inside the vectors. Another subtlety is that the model of vector-action set requires all the flow-vectors to have the same number of flows, which is not always possible in practice. One solution is to insert dummy packets of size zero which will then be handled as usual in the process of padding.

Next, we consider the Multi-Vector Multi-Dimension (MVMD) case in which each action-sequence consists of more than one actions and each flow-vector includes multiple flows. Definition 5.7 expresses the relationship between actions in an action-sequence.

Definition 5.7 (i-prefix, adjacent-prefix, adjacent-suffix) *We define*

- *the* i-prefix *of an action-sequence $\vec{a} = \langle \vec{a}[1], \vec{a}[2], \ldots, \vec{a}[t] \rangle$ ($i \in [1, t]$), denoted as $pre(\vec{a}, i)$, as the sequence $\langle \vec{a}[1], \vec{a}[2], \ldots, \vec{a}[i] \rangle$, and we say $\vec{a}[i-1]$ is the* adjacent-prefix *(or simply prefix) of $\vec{a}[i]$, and $\vec{a}[i+1]$ is the* adjacent-suffix *(or simply suffix) of $\vec{a}[i]$,*

- *similarly, we define the* i-prefix *of vector-sequence $\vec{v}$, and the* prefix, suffix *of $\vec{v}[i]$.*

In the MVMD case, due to the prefix relationship, the flow-vector for an action may provide additional information about flow-vectors that correspond to the previous actions in the same action-sequence. Such knowledge may enable the eavesdropper to refine his guesses about an action. Such a scenario is illustrated in Figure 5.1. Also, we slightly change the definition of a vector-action set to accommodate the added prefix action information, as shown in Definition 5.8. We will delay the discussion about how a padding algorithm may satisfy k-indistinguishability in this case to the next section.

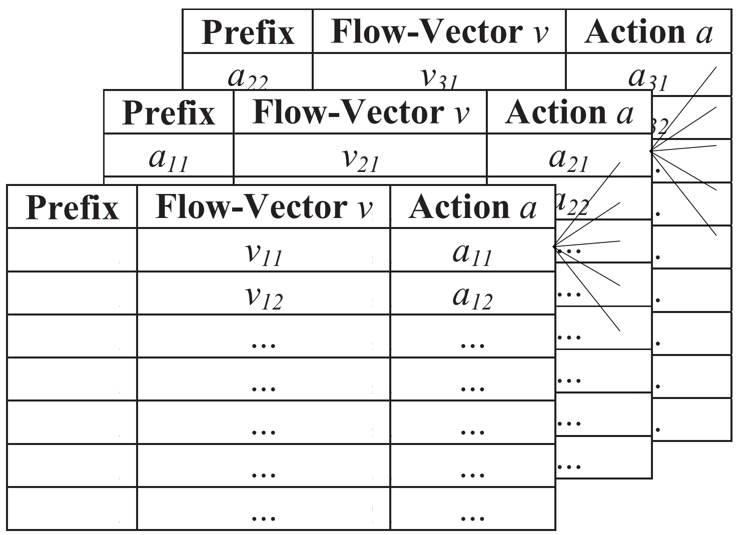


Figure 5.1: The Vector-Action Set in MVMD Case

Definition 5.8 (Vector-Action Set (MVMD Case)) *Given t action-sets $\{A_i : 1 \leq i \leq t\}$ and the corresponding vector-sets $\{V_i : 1 \leq i \leq t\}$, the* vector-action set *VA is the collection of sets $\{\{(v, a) : v \in V_i \wedge a \in A_i \wedge f_i(a) = v\} : 1 \leq i \leq t\}$.*

5.3 PPTP Problem Formulation

The formal model introduced in the previous section enables us to formulate a series of PPTP problems and study their complexity. We first discuss the choice of our ceiling padding approach among other possibilities in Section 5.3.1, and then address the SVSD and SVMD cases in Section 5.3.2 and the MVMD case in Section 5.3.3.

5.3.1 Ceiling Padding

In choosing a padding method, we need to address two aspects, privacy protection by satisfying the k-indistinguishability property, and minimizing padding cost. As previously mentioned, an application-agnostic approach, such as packet-size rounding and random padding, will usually incur high padding cost while not necessarily guaranteeing sufficient privacy protection [22]. We now revisit this argument by showing that a larger rounding size does not necessarily lead to more privacy. With our model, more privacy can now be clearly defined as satisfying k-indistinguishability for a larger k. Consider rounding the flows for the second keystrokes shown in Table 5.2 to a multiple of 64 (for example, 487 to $8 \times 64 = 512$). It can be shown that such rounding can achieve 2-indistinguishability (detailed calculations are omitted), while increasing the rounding size to 160 can achieve 3-indistinguishability. However, further increasing it to 256 can still only satisfy 2-indistinguishability.

From another point of view, as demonstrated in Section 5.1, we can now apply the PPDP technique of generalization to addressing the PPTP problem. A generalization technique will partition the vector-action set into groups, and then break the linkage among actions in the same group. One unique aspect in applying generalization to PPTP is that padding can only increase each packet size but cannot decrease or replace it with a range of values like in normal generalization. The above considerations lead to a new padding method given in Definition 5.9. Basically, after partitioning a vector-action set into groups, we pad each flow in a padding group to be the maximum size of that flow in the group.

Definition 5.9 (Dominance and Ceiling Padding) *Given a vector-set V, we define*

- *the* dominant-vector $dom(V)$ *as the flow-vector in which each flow is equal to the* *maximum of the corresponding flow among all the flow-vectors in V.*

- *a* ceiling-padded *group in V as a padding group in which every flow-vector is padded to the dominant-vector. We also say V is ceiling-padded if all the groups are ceiling-padded.*

Similar to the centroid in *k-means clustering* [55], dominant-vector is not necessary to be an actual vector in V. We will focus on the ceiling padding method in the rest of the chapter. When no ambiguity is possible, we will not distinguish between vector-set, vector-action set, flow-vector, and vector-sequence.

5.3.2 The SVSD and SVMD Cases

In the SVSD case, there is only a single flow in each flow-vector of the vector-set. Therefore, we only need to modify the vector-set by increasing the value of some flows to form padding groups. The padding problem can be formally defined as follows.

Problem 5.1 (SVSD Problem) *Given a vector-action set VA and the corresponding vector-set V and action-set A, the privacy property $k \leq |V|$, find a partition P^{VA} on VA such that the corresponding partition on V, denoted as $P^V = \{P_1, P_2, \ldots, P_m\}$, satisfies*

- $\forall(i \in [1, m]), |P_i| \geq k$;
- *The padding cost $\sum_{i=1}^{m}(dom(P_i) \times |P_i|)$ is minimal.* $\boxdot$

In the SVMD case, there are more than one flows in each flow-vector of the vector-set. The padding problem can be defined as follows:

Problem 5.2 (SVMD problem) *Given a vector-action set VA and the corresponding vector-set V (in which each flow-vector includes n_p flows) and action set A, the privacy property $k \leq\mid V\mid$, find a partition P^{VA} on VA such that the corresponding partition on V, denoted as $P^V = \{P_1, P_2, \ldots, P_m\}$, satisfies*

- $\forall(i \in [1,m]), |P_i| \geq k$;

- *The cost* $\sum_{i=1}^{m}(\sum_{j=1}^{n_p}((dom(P_i))[j]) \times |P_i|)$ *is minimal.* $\boxdot$

Theorem 5.1 shows that the above PPTP problem is intractable, and indicates that Problem 5.2 is NP-hard even when there are only two different flow values in the vector-set.

Theorem 5.1 *Problem 5.2 is NP-complete when* $k = 3$ *and the flow-vectors are from binary alphabet* $\sum$.

Proof: The proof follows the work in [4] for reduction from the well-known NP-hard problem, namely, the problem of Edge Partition Into Triangles (EPIT) [54], which is defined as follows:

Given a graph $G = (V, E)$ *with* $|E| = 3m$ *for some integer* m*, can the edges of* G *be partitioned into* m *triangles with disjoint edges? This problem is still NP-hard even* G *is simple.*

For an arbitrary instance of EPIT problem (without loss of generality, the graph is assumed to be simple), we construct a vector-action set VA with $3m$ pairs of (vector, action) where the vector has $|V|$ number of flows. Concretely, for each edge $u_iu_j \in E$, to which u_i and u_j are incident, we create a (v, a) pair in VA such that the two flows in v corresponding to u_i and u_j have z_b's and all the other flows have z_a's (each vertex is bijectively mapped to a flow in the flow-vector denoted by the subscript), where z_a and z_b are two positive integers and $z_a < z_b$. Obviously, this reduction works in polynomial time $O(3mn)$. Note that padding is represented here by modifying z_a's in some flows to be z_b's, and correspondingly, the cost is shown by the total number of z_a's which is changed to z_b's.

Now we show that the cost of optimal solution for Problem 5.2 is at most $3m$ if and only if E in G can be partitioned into a collection of m disjoint triangles.

Suppose that there exists a partition of edges in G for EPIT. Consider any triangle with vertexes u_i,u_j and u_l, and edges u_iu_j, u_ju_l and u_lu_i. Observe that, by modifying the l, i, and j flows from z_a to z_b respectively in the corresponding (v, a) pairs, we obtain a group in size 3 with identical flow-vectors. Consequently, we get a solution to Problem 5.2 with cost $3m$.

Conversely, suppose that there is a 3-indistinguishability solution for Problem 5.2 of cost at most $3m$. Since G is simple, any two pairs in VA are different in at least 2 flows (in the case that the corresponding edges in G share one common vertex). Consequently, to make two pairs have identical flow-vector values, we should at least modify one flow from z_a to z_b for each (v, a) pair. Therefore, the solution cost is exactly $3m$ and each pair is padded exactly one flow.

Since the solution satisfies 3-indistinguishability, any group in VA should be in size at least 3. Observe that, for any group with size larger than 3, any of its pairs will have at least two flows which are z_a and at least one other pair in the group has value z_b. In other words, in any such group, each pair should be at least padded two flows. Thus, each group has exactly three pairs. The only possibility is that the corresponding edges for the tuples in each group composes a triangle.

Besides, a given solution of Problem 5.2 can be verified in polynomial time whether it satisfies k-indistinguishability and the cost is less than a given value or not. ■

Note that, at first glance, the SVMD problem may resemble the problem of *k-means clustering* [55]. However, algorithms for *k-means clustering* cannot be directly applied to our problem due to following differences between these two problems. First, *k-means clustering* needs to partition a multiset into k groups, whereas in our problem, the minimal size of each group must be at least k. Second, *k-means clustering* is to minimize the within-cluster sum of squares, while our problem is to minimize the total distance between each of the flow-vectors and the dominant-vector.

5.3.3 MVMD Problem

As mentioned in Section 5.2.3, by correlating flow-vectors in the vector-sequence, an eavesdropper may refine his guesses of the actual action-sequence. We first discuss the challenges of traffic padding in such cases by observing the traffic for the sequence of two keystrokes as shown in Table 5.2.

Example 5.4 *To revisit Table 5.2, suppose an eavesdropper has observed the flow for the second keystroke. In order to preserve* 2*-indistinguishability with minimal padding overhead, one algorithm may partition the* 16 *cells into eight groups such that the size of each group is not less than* 2*, and assume that the queried strings* $(a)c$ *and* $(c)a$ *form one group. When the eavesdropper observes that the flow for the second keystroke is* 501*, she cannot determine whether the queried string is* $(a)c$ *or* $(c)a$*.*

However, suppose the eavesdropper also observes the flow corresponding to the first keystroke, she can determine that the first keystroke is either (a) *or* (c) *when the flow is* 509 *or* 502*, respectively. Consequentially, she can eventually infer the queried string by combining these observations.* $\boxdot$

One seemingly valid solution is padding the flow-vector for each keystroke so that 2-indistinguishability is satisfied separately for each keystroke. Unfortunately, this will fail to satisfy 2-indistinguishability. To pad traffic for the first keystroke, the optimal solution is to partition $\{509, 504, 502, 516\}$ into two padding groups, $\{502, 504\}$ and $\{509, 516\}$. However, when the eavesdropper observes the flow corresponding to the first keystroke, he/she can still determine it must be either (a) or (c) when the size is 516 or 504, respectively, because only when the first keystroke starts with (a) or (c) can the flow for second keystroke be padded to 501. Therefore, the eavesdropper will eliminate (b) and (d) from possible guesses, which violates 2-indistinguishability.

Another seemingly viable solution is to first collect all vector-sequences for the sequence of keystrokes and then pad them such that the current input string as a whole cannot be distinguished from at least $k-1$ others. Unfortunately, such an approach cannot guarantee the privacy property, either. First, the auto-suggestion feature requires the server to immediately respond to the client upon each single keystroke. Second, when receiving a single keystroke, the server cannot predict what would be the next input and hence cannot decide which padding option is suitable. For example, suppose the flow corresponding to (a) in $(a)c$ should be padded to 509, while in $(a)b$ to 516. When the server receives (a), it cannot determine whether to pad (a) to 509 or to 516.

The discussed challenges mainly arise due to the approach of padding each vector-set independently. We now propose a different approach. Intuitively, the partitioning of a vector-set corresponding to each action will *respect* the partitioning results of all the previous actions in the same action-sequence. More precisely, the padding of different vector-sets is correlated based on the following two conditions.

- Given two t-sized vector-sequences $\vec{v}_1$ and $\vec{v}_2$, any prefix $pre(\vec{v}_1, i)$ and $pre(\vec{v}_2, i)(i \in [2, t])$, can be padded together only if $\forall(j < i)$, $pre(\vec{v}_1, j)$ and $pre(\vec{v}_2, j)$ are padded together.

- For any two t-sized action-sequences $\vec{a}_1$ and $\vec{a}_2$ and corresponding vector-sequences $\vec{v}_1$ and $\vec{v}_2$, if $pre(\vec{a}_1, i) = pre(\vec{a}_2, i)(i \in [1, t])$, then $pre(\vec{v}_1, i)$ and $pre(\vec{v}_2, i)$ must be padded together.

Once a partition satisfies these conditions, no matter how an eavesdropper analyzes traffic information, either for an action alone or combining multiple observations of previous actions, the mental image about the actual action-sequence remains the same. Due to the similarity between the conditions and a related concept in graph theory, we call a partition satisfying such conditions the *oriented-forest partition*.

Problem 5.3 (MVMD problem) *Given a vector-action set* $VA = (VA_1, VA_2, \ldots, VA_t)$ *where* $VA_i = (V_i, A_i)$ *(*$i \in [1, t]$*), the privacy property* $k \leq |V_t|$*, find the partition* P^{VA_i} *on* VA_i *such that the corresponding partition* $P^{V_i} = \{P^i_1, P^i_2, \ldots, P^i_{m_i}\}$ *on* V_i *satisfies*

- $\forall((i \in [1, t-1]) \wedge (j \in [1, m_i]))$
$$\begin{cases} |P^i_j| \geq k, & \textit{if } (|V_i| \geq k), \\ |P^i_1| = |V_i|, & \textit{if } (|V_i| < k); \end{cases}$$

- $\forall(j \in [1, m_t]), |P_j^t| \geq k$;

- *The sequence of P^{V_i} is an oriented-forest partition;*

- *The total padding cost of P^{V_i} ($i \in [1, t]$) is minimal.* $\boxdot$

Obviously, Problem 5.3 is also NP-complete when $k \geq 3$ since Problem 5.2 is special case of Problem 5.3.

5.4 The Algorithms

In this section, we design three algorithms for partitioning the vector-action set into padding groups to satisfy a given privacy requirement. Our intention is not to design an exhaustive list of solutions but rather to demonstrate the existence of abundant possibilities in approaching this PPTP issue. Note that when the cardinality of vector-action set is less than the privacy property k, there is no solution to satisfy the privacy property. In such cases, our algorithms will simply exit, which will not be explicitly shown in each algorithm hereafter.

5.4.1 The svsdSimple Algorithm

The main intention of presenting the svsdSimple algorithm is to show that, when applying k-indistinguishability to PPTP problems, an algorithm may sometimes be devised in a very straightforward way, and yet achieve a dramatic reduction in costs when compared to existing approaches (as shown in the Section 5.6).

The svsdSimple algorithm shown in Table 5.5 basically attempts to minimize the cardinality of padding groups in the SVSD case. More specifically, svsdSimple first sorts the single flow in the flow-vector into a non-decreasing order of the flows, and then selects k pairs of (flow-vector, action) each time in that order to form a padding group. This is repeated until the number of pairs is less than k. The remainder of pairs is simply appended to the last padding group. The computational complexity is $O(nlogn)$ where $n = |VA|$, since step 2 costs $O(nlogn)$ time and each pair is considered once for the remaining steps.

Algorithm svsdSimple
Input: a vector-action set VA, the privacy property k;
Output: the partition P^{VA} of VA;
Method:
1. **Let** $P^{VA} = \emptyset$;
2. **Let** S^{VA} be the sequence of VA in a non-decreasing order of V;
3. **Let** $N = \frac{|S^{VA}|}{k}$;
4. **For** $i = 0$ to $N - 2$
5. **Let** $P_i = \bigcup_{j=i\times k+1}^{(i+1)\times k}(S^{VA}[j])$;
6. Create partition P_i on P^{VA};
7. Create $P_{N-1} = \bigcup_{j=(N-1)\times k+1}^{|S^{VA}|}(S^{VA}[j])$ on P^{VA};
8. **Return** P^{VA};

Table 5.5: The svsdSimple Algorithm for SVSD-Problem

5.4.2 The svmdGreedy Algorithm

The svmdGreedy algorithm, which aims at both SVSD and SVMD problems, is shown in Table 5.6. Roughly speaking, the svmdGreedy recursively divides the padding group P_i in P^{VA}, where $|P_i| \geq 2 \times k$, into two padding groups P_{i1} and P_{i2} until the cardinality of any padding group in P^{VA} is less than $2 \times k$. When svmdGreedy splits a padding group $P_i(VA_i)$ into two, these resultant padding groups, P_{i1} and P_{i2}, must satisfy that $(P_{i1} \cup P_{i2} = P_i) \wedge (P_{i1} \cap P_{i2} = \emptyset) \wedge (|P_{i1}| \geq k) \wedge (|P_{i2}| \geq k)$. Obviously, there must exist many solutions of P_{i1} and P_{i2}. svmdGreedy limits the optimizing process insides a subset of possible solutions as follows. For each flow, svmdGreedy first sorts the flow-vectors in non-decreasing order of that flow, then splits P_i into P_{i1} and P_{i2} at position pos in the sorted sequence where $(pos \in [k, |P_i| - k])$. There are totally $(n_p \times (|P_i| - 2 \times k))$ possible solutions for all flows in the flow-vector, where n_p is the number of flows in flow-vector. SvmdGreedy finally selects the one with minimal padding cost among this set of solutions. Clearly, this algorithm can solve SVSD-problem when n_p is set to be 1.

Algorithm svmdGreedy
Input: a vector-action set VA, the privacy property k;
Output: the partition P^{VA} of VA;
Method:
1. **If**($|VA| < 2 \times k$)
2.   Create in P^{VA} the VA;
3.   **Return**;
4. **Let** n_p be the number of flows in flow-vector;
5. **For** $p = 1$ to n_p
6.   **Let** S_p^{VA} be the sequence of VA in the non-decreasing order of the p^{th} flow in the flow-vector;
7.   **For** $i = k$ to $|S_p^{VA}| - k$
8.   **Let** $cost_{p,i}$ as the cost when S_p^V is split at position i;
9.   **Let** $cost_p$ be a pair (c, i) where c is the minimal in $(cost_{p,i})$ and i is the corresponding position;
10. **Let** $cost$ be a triple (c, p, i) where c is the minimal in c of $cost_p(p \in [1, n_p])$, and p and i are the corresponding p and i;
11. Split $S_{cost.p}^{VA}$ into VA_1 and VA_2 at position $cost.i$;
12. **Return** svmdGreedy(VA_1);
13. **Return** svmdGreedy(VA_2);

Table 5.6: The svmdGreedy Algorithm For SVMD-Problem

The svmdGreedy algorithm has an $O(n_p \times n^2)$ time complexity in the worst case (each time, the algorithm splits P_i into k-size P_{i1} and $(|P_i| - k)$-size P_{i2}), and $O(n_p \times n \times logn)$ in average cases (each time, the algorithm halves P_i), where $n = |VA|$.

5.4.3 The mvmdGreedy Algorithm

Both svsdSimple and svmdGreedy algorithms tackle cases where each action-sequence consists of a single action (correspondingly, each vector-sequence consists of a single flow-vector). Our intention now in devising the mvmdGreedy is to demonstrate how the two conditions mentioned in Section 5.3.3 facilitate the algorithm design. In this algorithm, we extend PPDP solutions to a sequence of inter-dependent vector-action sets. The only constraint in partitioning vector-action set VA_i is to ensure all flow-vectors in a padding group should have their prefix in an identical padding group of VA_{i-1}.

The mvmdGreedy algorithm for MVMD-Problem is shown in Table 5.7. Roughly speaking, mvmdGreedy partitions each vector-action set in the sequence in the given order, each for the flow-vector corresponding to an action in an action-sequence. More specifically, mvmdGreedy applies svmdGreedy to partition the first vector-action set in the sequence. For each remaining vector-action set VA_i, mvmdGreedy first partitions it into $|P^{VA_{i-1}}|$ number of padding groups based on the adjacent-prefix of the flow-vectors, and then applies svmdGreedy to further partition these padding groups.

Algorithm mvmdGreedy
Input: a t-size sequence D of vector-action sets, the privacy property k;
Output: the partition P^D of D;
Method:
1. **Let** $D = (VA_1, VA_2, \ldots, VA_t)$;
2. **Let** $P^1 = svmdGreedy(VA_1, k)$;
3. **For** each $(w \in [1, |P^1|])$, assign group $G^1_w \in P^1$ a unique $gid = w$;
4. **For** $i = 2$ to t
5. Create in P^i $|P^{i-1}|$ number of empty groups $G^i_w(w \in [1, |P^{i-1}|])$;
6. **For** each v_{ia} in VA_i
7. **Let** w be the gid of the group G^{i-1}_w in P^{i-1} that the prefix of v_{ia} in VA_{i-1} belongs to;
8. Insert v_{ia} into G^i_w;
9. **For** each $(w \in [1, |P^{i-1}|])$
10. $P^i = (P^i \setminus G^i_w) \cup svmdGreedy(G^i_w, k)$;
11. **For** each $(w \in [1, |P^i|])$, assign group $G^i_w \in P^i$ a unique $gid = w$;
12. **Return** $P^D = \{P^i : 1 \leq i \leq t\}$;

Table 5.7: The mvmdGreedy Algorithm For MVMD-Problem

Similarly, the mvmdGreedy also has an $O(n_p \times n^2)$ time complexity in the worst case (each time, the algorithm splits VA_i into k-size VA_{i1} and $(|VA_i| - k)$-size VA_{i2}), and $O(n_p \times n \times logn)$ in average cases (each time, the algorithm halves VA), where n is the total number of flow-vectors in those vector-sets.

5.5 Extension to l-Diversity

In previous discussion, we implicitly assume that each action in an action-set is equally likely to occur. However, in real life, each action is not necessary to have equal probability to be performed. In this section, we discuss an extension to our model to further demonstrate that many existing PPDP concepts may be adapted to address PPTP issues. Specifically, we adapt the l-diversity [77] concept to address cases where not all actions should be treated equally in padding (for example, some statistical information regarding the likelihood of different inputs may be publicly known).

5.5.1 The Model

We first assign an integer *weight* to each action to catch the information about its *occurrence probability* among the action-set that it belongs to. The reason for assigning *weight* instead is due to the utilization of such as *access counter* for visit statistics in most applications.

Definition 5.10 (Weight-Set) *Given an action-set A_i, the weight-set W_i is defined as the collection of integer weights associated with the actions in that action-set.*

Definition 5.11 (Occurrence Probability) *Given an action-set A and corresponding weight-set W, the* occurrence probability *of an action a with weight w in A is defined as $pr(a, A) = \frac{w}{\sum_{i=1}^{|W|}(w_i)}$.*

Example 5.5 *To revisit Example 5.1, given the action-set $A_1 = \{a_{11}, a_{21}\}$, assume that the weight for $a_{11} = b$ and $a_{21} = c$ are 20 and 5 respectively (clearly, in practice the value of weight should be assigned based on the characteristics of applications). Then, the weight-set is $W_1 = \{20, 5\}$. Moreover, in action-set A_1, the occurrence probability of b and c is $\frac{20}{20+5} = 80\%$ and $\frac{5}{20+5} = 20\%$, respectively.* $\boxdot$

Then we slightly change the definition of vector-action set to accommodate the weight information. Since SVSD and SVMD are special cases of MVMD, w.l.o.g., we only redefine the concept for MVMD.

Definition 5.12 (Vector-Action-Weight Set) *Given t action-sets $\{A_i : 1 \le i \le t\}$, and the corresponding weight-sets $\{W_i : 1 \le i \le t\}$ and vector-sets $\{V_i : 1 \le i \le t\}$, the* vector-action-weight set *VAW is the collection of sets* $\{\{(v, a, w) : v \in V_i \wedge a \in A_i \wedge w \in W_i\} : 1 \le i \le t\}$.

Example 5.6 *Following Example 5.5, given the action-set $A_1 = \{b, c\}$, weight-set $W_1 = \{20, 5\}$ and vector-set $V_1 = \{544, 554\}$, the corresponding vector-action-weight set is* $VAW_1 = \{(544, b, 20), (554, c, 5)\}$. ⊡

We regard weight information as an additional information about the action, therefore, the mapping from PPTP to PPDP is consistent with what has been discussed before. Definition 5.13 applies *l-diversity* to quantify the amount of privacy protection under a given vector-action-weight set.

Definition 5.13 (l-Diversity) *Given a vector-action set VAW, we define*

- *a* padding group *as any $S \subseteq VAW$ satisfying that all the pairs in S have identical flow-vectors and no $S' \supset S$ can satisfy this property, and*
- *we say VAW satisfies* l-diversity *(l is an integer) or VAW is l-diverse if the* occurrence probability *of any action in any padding group is no greater than $\frac{1}{l}$.*

Example 5.7 *Following Example 5.6, the highest occurrence probability is b with $\frac{4}{5}$. Since $\frac{1}{1} > \frac{4}{5} > \frac{1}{2}$, VAW_1 does not satisfy 2-diversity.* ⊡

5.5.2 The Problem

With the revised definitions, we now formulate the diversity problems, namely, SVSD-Diversity, SVMD-Diversity, and MVMD-Diversity, for the SVSD case, SVMD case, and MVMD case, respectively. Clearly, the main difference between the l-diversity problems and aforementioned k-indistinguishability problems is the condition on the padding group. That is, for k-indistinguishability, the cardinality of each padding group should be at least k, whereas, for l-diversity, the maximal occurrence probability of each group should be at most $\frac{1}{l}$, as demonstrated by Problem 5.4 for MVMD case.

Problem 5.4 (MVMD-Diversity Problem) *Given a vector-action-weight set $VAW = (VAW_1, VAW_2, \ldots, VAW_t)$ where $VAW_i = (V_i, A_i, W_i)$ $(i \in [1,t])$, the privacy property $l \leq \frac{1}{\max\limits_{a \in A_t}(pr(a, A_t))}$, find the partition P^{VAW_i} on VAW_i such that the corresponding partitions $P^{A_i} = \{P_1^{A_i}, P_2^{A_i}, \ldots, P_{m_i}^{A_i}\}$ on A_i and $P^{V_i} = \{P_1^{V_i}, P_2^{V_i}, \ldots, P_{m_i}^{V_i}\}$ on V_i satisfy*

- $\forall((i \in [1, t-1]) \wedge (j \in [1, m_i]))$
$$\begin{cases} \max\limits_{a \in P_j^{A_i}}(pr(a, P_j^{A_i})) \leq \dfrac{1}{l}, & \textit{if } (\max\limits_{a \in A_i}(pr(a, A_i)) \leq \dfrac{1}{l}), \\ P_1^{A_i} = A_i, & \textit{if } (\max\limits_{a \in A_i}(pr(a, A_i)) > \dfrac{1}{l}); \end{cases}$$

- $\forall(j \in [1, m_t]), \max\limits_{a \in P_j^{A_t}}(pr(a, P_j^{A_t})) \leq \dfrac{1}{l}$;

- *The sequence of P^{V_i} is an oriented-forest partition;*

- *The total padding cost of P^{V_i} $(i \in [1,t])$ after applying ceiling padding is minimal.* $\boxdot$

Observe that when the weights of all actions in any VAW_i are set to be identical, and $l = k$, Problem 5.4 is simplified to Problem 5.3. Informally, Problem 5.4 is at least as hard as Problem 5.3.

Although l-diversity in PPTP shares the same spirit with that in PPDP, algorithms for l-diversity in PPDP cannot be directly applied to our PPTP problems due to the following main difference between these two problems. In PPDP, there are many tuples with same sensitive values in the micro-data table, while in our problem, the action in an action-set is not duplicated, and we assign a weight for each action to distinguish its possibility to be performed by a user from other actions.

5.5.3 The Algorithms

To facilitate the explanation, we first present the svsdDiversity algorithm for SVSD case to show the essence of design ideas to satisfy l-diversity as shown in Table 5.8. Roughly speaking, svsdDiversity algorithm first sorts the actions in non-increasing order of their *weight* values, and then among the actions with same *weight*, sorts them in a predefined order based on their flow-vectors. In this algorithm, we sort them in non-increasing order of the flows (note this step aims at reducing the padding cost in the resultant group and there must exist alternative solutions for ordering). Based on the sorted version S of vector-action-weight set, svsdDiversity iteratively removes actions from S to construct the padding group until S is empty. In each iteration, svsdDiversity splits the sequence S into two l-diverse sub-sequences, $P_{\alpha-}$ and $P_{\alpha+}$, such that the first sub-sequence $P_{\alpha-}$ has minimal possible cardinality.

Algorithm svsdDiversity
Input: a vector-action-weight set VAW, the privacy property l;
Output: the partition P^{VAW} of VAW;
Method:
1. **Let** $P^{VAW} = \emptyset$;
2. **Let** S be the sequence of VAW in a non-increasing order of its W;
3. **If**($pr(S[1], S) > \frac{1}{l}$)
4. **Return**;
5. **Sort** elements in S with same *weight* value
in non-increasing order of its V;
6. **While** ($S \neq \emptyset$)
7. **Let** $P_{\alpha-} = \{S[i] : i \in [1, \alpha]\}$, $P_{\alpha+} = \{S[i] : i \in [\alpha + 1, |S|]\}$;
8. **Let** $\alpha \in [l, |S|]$ be the smallest value such that:
$pr(S[1], P_{\alpha-}) \leq \frac{1}{l}$ and
($pr(S[\alpha + 1], P_{\alpha+}) \leq \frac{1}{l}$ or $P_{\alpha+} \equiv \emptyset$)
9. Create partition $P_{\alpha-}$ on P^{VAW};
10. $S = P_{\alpha+}$;
11. **Return** P^{VAW};

Table 5.8: The svsdDiversity Algorithm For SVSD-Diversity Case

Note that, in each iteration, the algorithm removes $P_{\alpha-}$ from S and further splits $P_{\alpha+}$. Before discussing the reasons, we first introduce the *undividable diverse group* concept to define the padding group which can not be further split without reordering the sequence.

Definition 5.14 (Undividable Diverse Group) *Given a vector-action-weight set VAW, and denote by $S = (S[1], S[2], \ldots, S[|A|])$ the sequence of VAW in the non-increasing order of its W, we say $P_{\alpha-} = (S[1], S[2], \ldots S[\alpha])$, a sub-sequence of S, is a* undividable diverse group, *if*

- *$pr(S[1], P_{\alpha-}) \leq \frac{1}{l}$, and*
- *there does not exist any integer $\beta \in [1, \alpha)$, such that both $pr(S[1], (S[1], \ldots, S[\beta])) \leq \frac{1}{l}$ and $pr(S[\beta+1], (S[\beta+1], \ldots, S[\alpha])) \leq \frac{1}{l}$ hold.*

The $P_{\alpha-}$ in step 9 is a undividable diverse group by reasoning as follows. If α is the smallest position that $P_{\alpha-}$ satisfies l-diversity, clearly, it cannot be further split. Otherwise, suppose that β is the smallest position such that $\beta < \alpha$ and $P_1 = (S[1], S[2], \ldots, S[\beta])$ satisfies l-diversity, then $P_2 = (S[\beta+1], \ldots, S[\alpha])$ is not l-diverse, since based on the condition in step 8, $pr(S[\beta+1], P_2) = \frac{w[\beta+1]}{\sum_{i=\beta+1}^{\alpha} w[i]} \geq \frac{w[\beta+1]}{\sum_{i=\beta+1}^{|S|} w[i]} > \frac{1}{l}$. Similarly, splitting $P_{\alpha-}$ at any position between β and α leads to same result, which confirms the statement.

Furthermore, svsdDiversity always terminates since appending action with smaller weight value to an l-diverse padding group will never produce a group violating l-diversity. Therefore, each iteration will result in either two l-diverse groups or one whole sequence together with an empty sequence.

The svsdDiversity algorithm has $O(nlogn)$ time complexity since step 2 and step 5 cost $(nlogn)$ time and each action is considered once for the remaining steps, where $n = |VAW|$.

Then, we design svmdDiversity and mvmdDiversity algorithms for SVMD-Diversity and MVMD-Diversity problems, respectively. Similar to svsdDiversity, svmdDiversity follows the conditions shown in step 8 in Table 5.8 to split S as shown in Table 5.9. In contrast to svsdDiversity, svmdDiversity first identifies all possible positions of given VAW which satisfy the conditions for each flow, and then selects the one with minimal cost among all possible positions in all the flows. The svmdDiversity has $O(n_p \times n^2)$ in the worst case and $O(n_p \times n \times logn)$ in average cases.

Algorithm svmdDiversity
Input: a l-diverse vector-action-weight set VAW, the privacy property l;
Output: the partition P^{VAW} of VAW;
Method:
1. **If**($|VAW| < 2 \times l$)
2. Create in P^{VAW} the VAW;
3. **Return**;
4. **Let** S be the sequence of VAW in a non-increasing order of its W;
5. **Let** n_p be the number of flows in flow-vector V;
6. **For** $p = 1$ to n_p
7. **Sort** elements in S with same *weight* value in non-increasing order of the p^{th} flow in its V;
8. **Let** $P_{\alpha_{p,i}-} = \{S[r] : r \in [1, \alpha_{p,i}]\}$, and $P_{\alpha_{p,i}+} = \{S[r] : r \in [\alpha_{p,i} + 1, |S|]\}$;
9. **Let** $Z_p \subseteq \{i : l \leq i \leq |S|\}$ be the set of values such that: $\forall(\alpha_{p,i} \in Z_p)$, $pr(S[1], P_{\alpha_{p,i}-}) \leq \frac{1}{l}$ and $(pr(S[\alpha_{p,i} + 1], P_{\alpha_{p,i}+}) \leq \frac{1}{l}$ or $P_{\alpha_{p,i}+} \equiv \emptyset)$
10. **Let** α_p be the value in Z_p such that the cost is minimal among all $\alpha_{p,i} \in Z_p$ when S is split at $\alpha_{p,i}$;
11. **Let** $\alpha \in \{\alpha_p : p \in [1, n_p]\}$ with the minimal cost;
12. **If** ($P_{\alpha+}$ is empty)
13. Create in P^{VAW} the $P_{\alpha-}$;
14. **Return**;
15. **Return** svmdDiversity($P_{\alpha-}$);
16. **Return** svmdDiversity($P_{\alpha+}$);

Table 5.9: The svmdDiversity Algorithm For SVMD-Diversity Case

Similar to mvmdGreedy, mvmdDiversity first ensures that the partitioning satisfies the conditions of an oriented-forest partition (refer to Section 5.4.3 for the main idea). There is still another complication. In mvmdGreedy, an initial padding group based on prefixes certainly satisfies k-indistinguishability only if the set of prefixes satisfies. However, this is not the case in mvmdDiversity since a vector-action set with size larger than l will not necessarily be a l-diverse set. To address this issue, we confine the Z_p in step 9 of svmdDiversity in Table 5.9 to further satisfy that both the set formed by all suffixes (refer to Definition 5.7) of $P_{\alpha_{p,i}-}$ and that of $P_{\alpha_{p,i}+}$ are l-diverse. To facilitate the evaluation of these two additional conditions, for each action, the algorithm can precompute the maximal value and the summation of *weight* values of all its suffixes. Clearly, such computation can be an integrated part of reading inputs, and does not increase the order of computational complexity. Thus, the mvmdDiversity algorithm has $O(n_p \times n^2)$ time complexity in the worst case and $O(n_p \times n \times logn)$ in average cases (detailed algorithm descriptions are omitted due to space limitations).

5.6 Evaluation

In this section, we evaluate the effectiveness of our solutions and efficiency through experiments with real world Web applications. First, Section 5.6.1 discusses the implementation of our techniques. Section 5.6.2 then elaborates on the experimental settings. Finally, Section 5.6.3, 5.6.4, and 5.6.5 present experimental results of the communication, computation, and processing overhead, respectively.

5.6.1 Implementation Overview

In previous sections, we have presented algorithms for determining the amount of padding for each flow given the vector-action set. To incorporate our techniques into an existing Web application requires following three steps. First, gather information about possible action-sequences and corresponding vector-sequences in the application. Second, feed the vector-action sets into our algorithms to calculate the desired amount of padding. Third, implement the padding according to the calculated sizes. The main difference between implementing an existing method (such as rounding) and ceiling padding lies in the second stage. Thus, we have focused on this stage in this chapter. Nonetheless, we will also briefly describe how to collect the vector-action sets in Section 5.6.2 and how to facilitate the third stage in Section 5.6.5.

One may question the practicality of gathering information about possible action-sequences since the number of such sequences can be very large. However, we believe it is practical for most Web applications due to following three facts. First, the aforementioned side-channel attack on web applications typically arises due to highly interactive features, such as auto-suggestion. The very existence of such features implies that the application designer has already profiled the domain of possible inputs (that is, action-sequences) for implementing the feature. Therefore, such information must already exist in certain forms and can be easily extracted at a low cost. Second, even though a Web application may take infinite number of inputs, this does not necessarily mean there would be infinite action-sequences. For example, a search engine will no longer provide auto-suggestion feature once the query string exceeds a certain length. Third, all the three steps mentioned above could be part of the off-line processing, and would only need to be repeated when the Web application undergoes a redesign.

Note that implementing an existing padding method, such as rounding, will also need to go through the above three steps if only the padding cost is to be optimized. For example, without collecting and analyzing the vector-action sets, a rounding method cannot effectively select the optimal rounding parameter.

5.6.2 Experimental Setting

We collect testing vector-action sets from four real-world web applications, two popular search engines $engine^B$ and $engine^C$ (where users' searching keyword needs to be protected) and two authoritative information systems, $drug^B$ for drugs and $patent^C$ for patents, from two national institutes (where users' health information and company's patent interest need to be protected, respectively). Such data can be collected by acting as a normal user of the applications without having to know internal details of the applications. For our experiment, these data are collected using separate programs whose efficiency is not our main concern in this chapter.

We observe that the flows of $drug^B$ and $patent^C$ are more diverse and larger than those of $engine^B$ and $engine^C$ evidenced by the standard deviations (σ) and the means (μ) of the flows, respectively. Besides, the flows of $drug^B$, $patent^C$ are much more disparate in values than those of $engine^B$, $engine^C$. Later we will show the effect of these different characteristics of flows on the costs.

All experiments are conducted on a PC with 2.20GHz Duo CPU and 4GB memory. We evaluate the overhead of computation, communication, and processing using execution time, padding cost ratio, and processing cost ratio, respectively. Specifically, for each application, we first obtain the total size of all flows ttl for all possible actions before padding, and then compute the padding cost $cost$ as shown in Definition 5.5 after padding. The padding cost ratio is formulated as $\frac{cost}{ttl}$. We also count the number of flows which need to be padded, and then formulate the processing cost ratio as the percent of flows to be padded among all flows. Clearly, given the interval Δ for random padding, theoretically the padding and processing cost ratio equal to $\frac{\Delta}{2\times ttl}$ and $1-\frac{1}{\Delta}$ respectively. Thus, we omit the comparison with it through the experiments.

To facilitate comparison, we use the $engine^B$ and $drug^B$ sets to compare the overheads for k-indistinguishability against an existing padding method, namely, packet-size rounding (simply rounding) [22], and the $engine^C$ and $patent^C$ sets to compare those for l-diversity against the other, namely, *maximizing* (a naive solution which pads each to be maximal size in the corresponding flow). For rounding, we set the rounding parameter $\Delta = 512$ and $\Delta = 5120$ for $engine^B$ and $drug^B$, respectively. Note that these Δ values just lead to results satisfying 5-indistinguishability in the padded data, and are adapted only for the comparison purpose. For l-diversity, we assign each action a uniformly random integer in a given range as its weight value (default $[1, 50]$). Note that our algorithms ensure the privacy for *l-diverse* vector-action sets and report exception for other sets regardless of the distribution and values of weights, and in real-life, the weight value could be assigned based on such as statistical results.

5.6.3 Communication Overhead

We first evaluate the communication overhead of our algorithms in the case of length-one action-sequences. In such cases, the svmdGreedy and svmdDiversity algorithms are equivalent to mvmdGreedy and mvmdDiversity, respectively. To apply the svsdSimple, svsdDiversity algorithms, we generate four vector-action sets by synthesizing the flow-vectors for the last action of the four collected sets.

For k-indistinguishability, figure 5.2(a) shows padding cost of each algorithm against k. Compared to rounding [22], our algorithms have less padding cost, while svmdGreedy incurs significantly less cost than that of rounding.

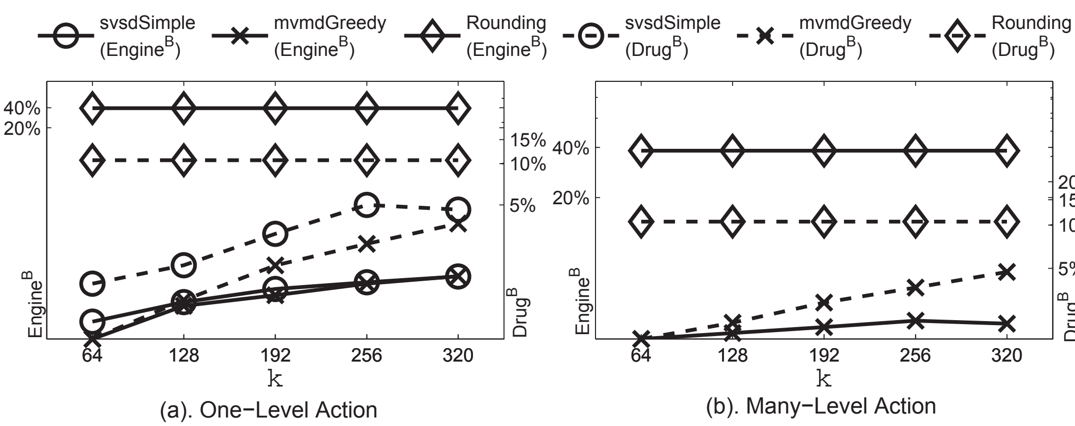


Figure 5.2: Padding Cost Overhead Ratio (k-Indistinguishability)

For l-diversity, figure 5.3(a) shows padding cost of each algorithm against l. From the results, the padding costs of our algorithms are significantly less than that of *maximizing*. We observe that our algorithms are superior specially when the number of flow-vectors in a vector-action set is larger since our algorithms have high possibility to partition the flow-vectors with close value into padding group.

We then compare our algorithms with existing methods in the case of action-sequences of lengths larger than one. Figure 5.2(b) and 5.3(b) show padding costs of our mvmdGreedy and the rounding algorithm against k, and our mvmdDiversity and the maximizing algorithm against l, respectively. Rounding and maximizing incur larger padding cost than

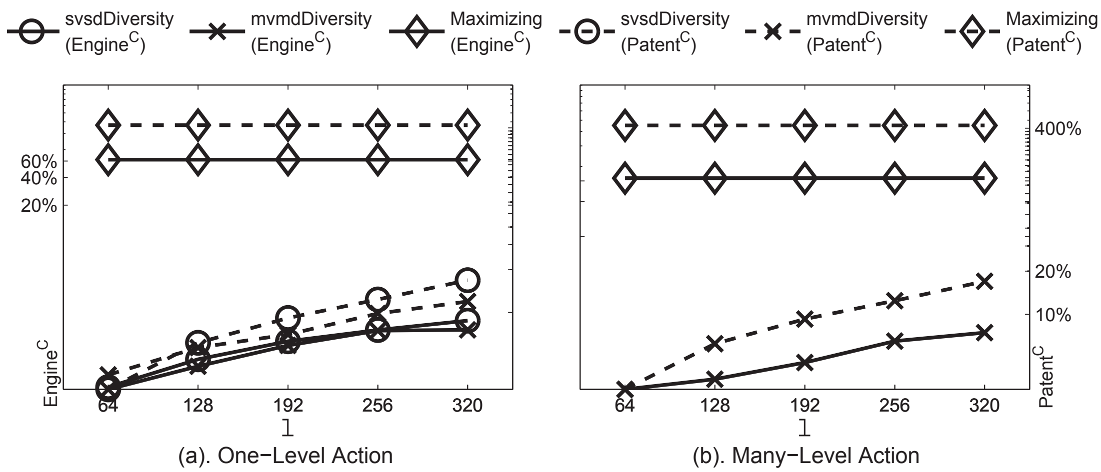


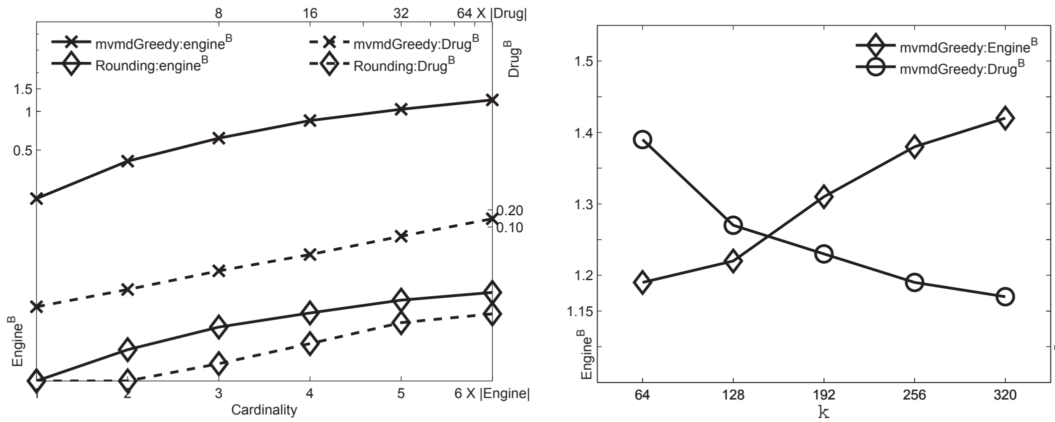
Figure 5.3: Padding Cost Overhead Ratio (l-Diversity)

mvmdGreedy and mvmdDiversity in all cases. For example, the padding cost ratio of maximizing for $patent^C$ is prohibitively high as 418%, which is 140 times higher than that of mvmdDiversity when $l = 64$. The reason for mvmdGreedy, mvmdDiversity algorithms have more padding cost in the case of many-level action than in one-level is as follows. In many-level action, these algorithms first partition each vector-action set (except VA_1) into padding groups based on the prefix of actions and regardless of the values of flow-vectors. Besides, the further ordering by the *weight* in mvmdDiversity results in slightly more overhead than mvmdGreedy when $l = k$.

5.6.4 Computational Overhead

We first study the computation time of our mvmdGreedy and mvmdDiversity against the flow data cardinality n as shown in Figure 5.4(a) and 5.5(a). We generate n-sized flow data by synthesizing $\frac{n}{\sum_i(|VA_i|)}$ copies of the four collected vector-action sets. We set $k(l) = 160$ for this set of experiments, and conduct each experiment 1000 times and then take the average.

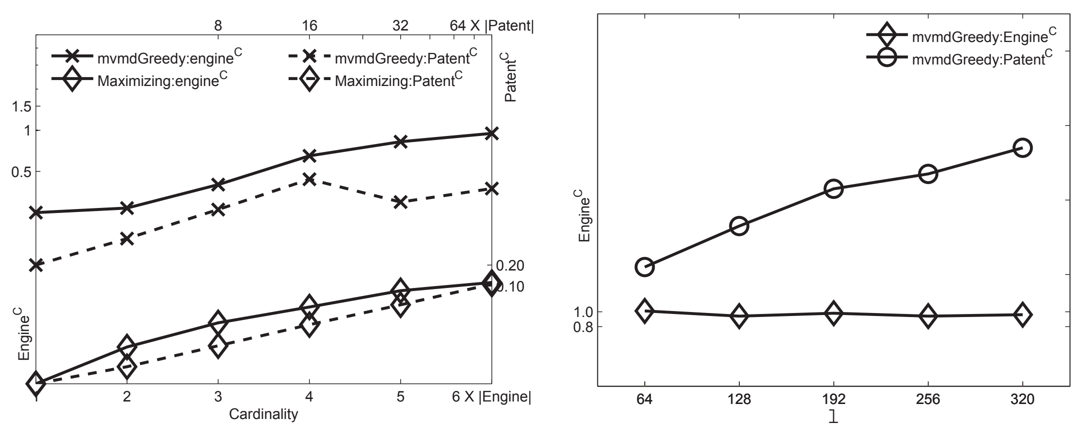
As the results show, our algorithms are practically efficient (1.2s and 0.98s for 2.7m flow-vectors for mvmdGreedy and mvmdDiversity, respectively) and the computa-



(a) Flow Data Cardinality (b) Privacy Property

Figure 5.4: Execution Time in Seconds (k-Indistinguishability)

tion time increases slowly with n, although our algorithms require slightly more overhead than rounding (when it is applied to a single Δ value) and maximizing. However, this is partly at the expense of worse performance in terms of padding cost. Note that the slight reduction of execution time observed in Figure 5.5(a) for $patent^C$ at $32 \times |patent^C|$ is reasonable since: first, the cardinality of each initial padding group based on the adjacent-prefixes may be smaller, which leads to less accumulated sorting time. Second, doubling the size of vector-action sets may result in less execution time based on the complexity in the average and worst cases ($2nlog2n < n^2$).



(a) Flow Data Cardinality (b) Privacy Property

Figure 5.5: Execution Time in Seconds (l-Diversity)

We then study computation time against privacy property k on the two synthesized vector-action sets ($6\times engine^B$ and $64\times drug^B$), and against l on the other two sets ($6\times engine^C$ and $64\times patent^C$). As expected, rounding and maximizing are insensitive to k and l since they do not have the concept of k and l, respectively. On the other hand, a tighter upper bound on the time required for mvmdGreedy is $O(n_p\times n\times 2k\times\lambda)$ in the worse case and $O(n_p\times n\times log(2k\times\lambda))$ in the average case, where λ is the maximal number of actions which has the same prefix in all action-sequences. Clearly, when λ is $O(n)$, the computational complexity here is equivalent to that in Section 5.4.3.

The reason for this tighter upper bound is that mvmdGreedy always feeds a vector-action set with maximal $2k\times\lambda$ cardinality to svmdGreedy (except VA_1 whose size is 26, a constant, for $search^B$), since: first, for each vector-action set VA_i, mvmdGreedy first partitions it into padding groups based on the prefix (which has $O(|VA_i|)$ solution). Second, there are at most $2k$ adjacent-prefixes in same padding group of VA_{i-1}. Therefore, when $2k\times\lambda \ll n$, the execution time of mvmdGreedy should be in the range of $[log(2k\times\lambda), 2k\times\lambda]$ times of $O(n_p\times n)$ which is the execution time of rounding algorithm. These two datasets in our experimental environment satisfy above condition, for example, $26(\lambda)\times 320(k)\ll 2.7m$ for $search^B$. Observe that, mvmdDiversity does not satisfy the tighter upper bound since a vector-action set with size larger than $2l$ probably cannot be split into two l-diverse subsets.

Figure 5.4(b) illustrates the computation time of mvmdGreedy against the privacy property k. Interestingly, the computation time increases slowly (from $1.19s$ to $1.42s$) with k for $engine^B$, and decreases slowly (from $0.147s$ to $0.136s$) for $drug^B$. Stress that the results are reasonable since both results fall within the expected range. Figure 5.5(b) shows that the computational time of mvmdDiversity increases slowly with l for $patent^C$, and is almost same for different l in the case of $engine^C$.

5.6.5 Processing Overhead

Our previous discussions have focused on reducing the communication overhead of padding while ensuring each flow-vector to satisfy the desired privacy property. To implement traffic padding in an existing Web application, if the HTTPS header or data is compressed, we can pad after compression, and pad to the header; if header and data are not compressed, we can pad to the data itself (e.g., spaces of required padding bytes can be appended to textual data). Clearly, the browser's TCP/IP stack is responsible for the header padding, while the original web applications regard the data padding as normal data. An application can choose to incorporate the padding at different stage of processing a request. First, an application can consult the outputs of our algorithms for each request and then pad the flow-vectors on the fly. Second, an application can modify the original data beforehand based on the outputs of our algorithms such that the privacy property is satisfied under the modifications. However, padding may incur a processing cost regardless of which approach to be taken. Therefore, we must aim to minimize the number of packets to be padded. For this purpose, we evaluate the processing cost ratio, which captures the proportion of flow-vectors to be padded among all such vectors.

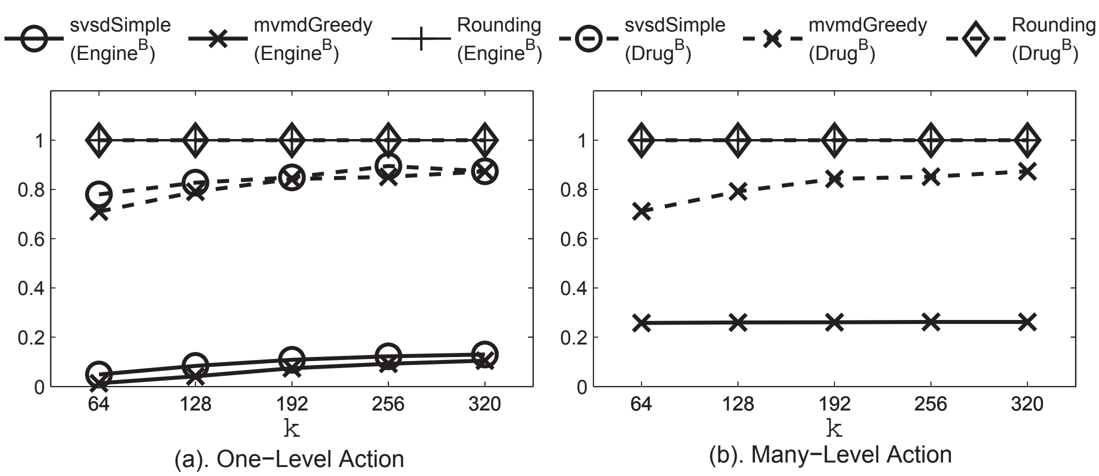


Figure 5.6: Processing Cost Overhead Ratio (k-Indistinguishability)

Figure 5.6 shows the processing cost of each algorithm against k. Rounding algorithm must pad each flow-vector regardless of the k's and the applications, while our algorithms have much less cost for $engine^B$ and slightly less for $drug^B$.

Similarly, from the results of the processing cost against l shown in Figure 5.7, we can see that maximizing algorithm almost pads each flow-vector regardless of the l's and the applications, while our algorithms have much less cost for $engine^C$ and slightly less for $patent^C$.

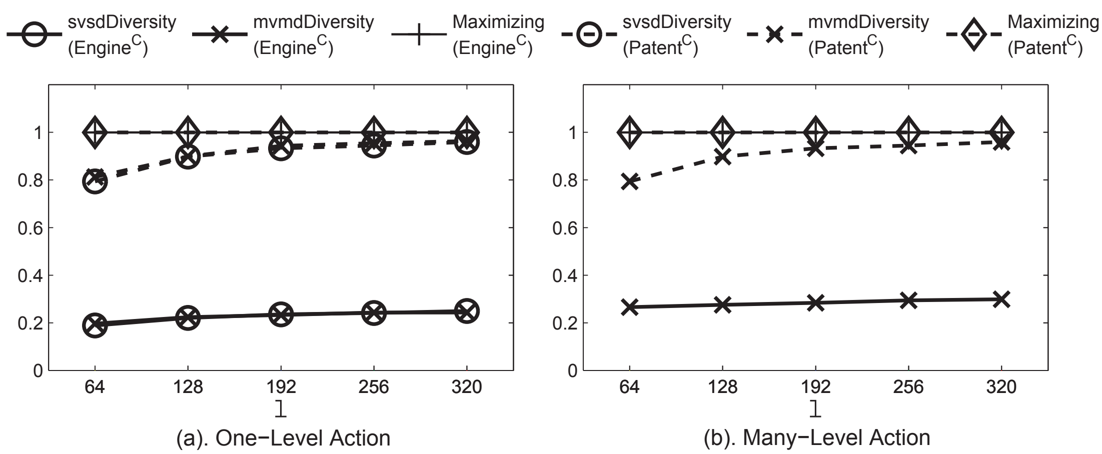


Figure 5.7: Processing Cost Overhead Ratio (l-Diversity)

5.7 Summary

As Web-based applications become more popular, their security issues will also attract more attention. In this chapter, we have demonstrated an interesting connection between the traffic padding issue of Web applications and the privacy-preserving data publishing. Based on this connection, we have proposed a formal model for quantifying the amount of privacy protection provided by traffic padding solutions. We have also designed algorithms by following the proposed model. Our experiments with real-world applications have confirmed the performance of our solutions to be superior to existing ones in terms of communication and computation overhead.

Chapter 6

PPTP: Background-Knowledge Resistant Traffic Padding for Privacy Preserving in Web Applications

The solutions in the previous chapter rely on the assumption that adversaries do not possess prior background knowledge about possible user inputs, which is a common limitation shared by most existing solutions. In this chapter, we propose a novel random ceiling padding approach whose results are resistant to such adversarial knowledge.

6.1 Overview

Today's Web applications allow users to enjoy the convenience of Software as a Service (SaaS) through their feature-rich and highly interactive user interfaces. However, recent studies show that such features may also render Web applications vulnerable to side channel attacks, which employ observable information, such as a sequence of directional packet sizes and timing, to recover sensitive user inputs from encrypted traffic [22]. Intrinsic characteristics of Web applications, including low entropy inputs, diverse resource objects, and stateful communications render such attacks a pervasive and fundamental threat in the age of cloud computing.

Existing countermeasures include *packet-size rounding* (increasing the size of each packet up to the closest multiple of given bytes) and *random padding* (increasing each packet size up to a random value). Those straightforward approaches have been shown to incur a high overhead and require application-specific implementation, while still not being able to provide sufficient privacy guarantee [22]. A more recent solution, *ceiling padding*, inspired by similar approaches in privacy-preserving data publication, partitions packets into *padding groups* and increases the size of every packet inside a group to the maximum size within that group in order to provide required privacy guarantee [71]. However, an important limitation shared by most existing solutions is that they assume adversaries do not possess any background knowledge about possible user inputs; the privacy guarantee may cease to exist when such knowledge allows adversaries to refine their guesses of the user inputs.

A natural way to address the above issue is to apply the well-known concept of differential privacy [40], which provides provable resistance to adversaries' background knowledge. Nonetheless, applying differential privacy to traffic padding will meet a few practical challenges. Specifically, introducing noises is more suitable for statistical aggregates (e.g., COUNT) or their variants, which have more predictable, and relatively small sensitivity; it is less applicable to traffic padding which has less predictable and often unbounded sensitivity (due to diverse resource objects), and individual packets' sizes, instead of their statistical aggregates, are directly observable. Moreover, while the qualitative significance of the privacy parameter ϵ is well understood in the literature, the exact quantitative link between this value and the degree of privacy guarantee is what an application provider would need to convince users about the level of privacy guarantee, which has received less attention. Therefore, the discussion of differential privacy is beyond the scope of this chapter and is regarded as a future direction.

In this chapter, we propose a novel *random ceiling padding* approach to providing background knowledge-resistant privacy guarantee to Web applications. We first adopt an information theoretic approach to modeling a padding algorithm's resistance to adversaries' prior knowledge about possible user inputs. Armed with this new *uncertainty* privacy metric, we then design a generic scheme for introducing randomness into the previously deterministic process of forming padding groups. Roughly speaking, the scheme makes a random choice among all the valid ways for forming padding groups to satisfy the privacy requirement. Consequently, an adversary would still face sufficient uncertainty even if s/he can exclude certain number of possible inputs to refine his/her guesses of the true input. We show that our proposed scheme may be instantiated in distinct ways to meet different applications' requirements by discussing two examples of such instantiation. Finally, we confirm the correctness (the algorithms provide sufficient privacy guarantee) and performance (the padding and processing cost), through both theoretic analysis and experiments with two real world Web applications.

The contribution of this chapter is twofold. First, the proposed random ceiling padding approach may lead to practical solutions for protecting user privacy in real-life Web applications. As evidenced by our experimental results, the two padding algorithms instantiated from the generic approach can provide required privacy guarantee with reasonable costs. Second, although we have focused on the traffic padding issue in this chapter, similar principles can be readily applied in other domains, such as privacy preserving data publication [47], in order to enhance syntactic privacy metrics [77, 90] with the capability of resisting adversarial background knowledge.

The rest of the chapter is organized as follows. The remainder of this section builds intuitions through a running example. Section 6.2 defines our models. Section 6.3 introduces a generic scheme and instantiates it into two concrete padding methods. Section 6.4 presents analysis on the privacy, costs, and complexity. Section 6.5 experimentally evaluates the performance of our algorithms. We conclude the chapter in Section 6.6.

Motivating Example

Consider a fictitious website which, upon the login of a user, displays information about the disease with which s/he is most recently associated. Table 6.1 shows a toy example of sizes and directions of encrypted packets for the diseases starting with the letter *C*. Clearly, the fixed patterns of directional sizes of the first, second, and last packets will allow an adversary to pinpoint packets corresponding to different diseases from the observed traffic. In this example, if an adversary observes a s-byte value to be 360 when a patient logins, s/he can infer that the patient was likely diagnosed $Cancer$ (note this example is simplified to facilitate discussions, and the traffic pattern may be more complicated in reality).

Diseases	**Observed Directional Packet Sizes**			
$Cancer$	801 →,	← 54,	← 360,	60 →
$Cervicitis$	801 →,	← 54,	← 290,	60 →
$Cold$	801 →,	← 54,	← 290,	60 →
$Cough$	801 →,	← 54,	← 290,	60 →
			(s bytes)	

Table 6.1: User Inputs and Corresponding Packet Sizes

We now examine two existing solutions, *rounding* [22] and *ceiling padding* [71], when applied to this example. Both solutions aim to pad packets such that each packet size will no longer map to a unique disease. In this example, we should pad s-byte such that each packet size maps to at least $k = 2$ different diseases, namely, 2-*indistinguishability*. In Table 6.2, the third column shows that a larger rounding size does not necessarily lead to more privacy, since rounding with $\Delta = 112$ and 176 cannot achieve privacy (the s-value of *Cancer* after padding is still unique), whereas $\Delta = 144$ does. Therefore, we may be forced to evaluate many Δ values before finding an optimal solution, which is clearly an impractical solution.

Next, the last column in Table 6.2 shows that the ceiling padding approach [71] achieves 2-indistinguishability. When an adversary observes a 360-byte packet, s/he can only infer that the patient has either *Cancer* or *Cervicitis*, but cannot be sure which is true.

Diseases	s Value	Rounding (Δ)			Ceiling Padding
		112	144	176	
$Cancer$	360	448	432	528	360
$Cervicitis$	290	336	432	352	360
$Cold$	290	336	432	352	290
$Cough$	290	336	432	352	290
Padding Overhead(%)		18.4%	40.5%	28.8%	5.7%

Table 6.2: Rounding and Ceiling Padding for Table 6.1

However, if the adversary happens to also possess some background knowledge through outbound channels that, say, this particular patient is a male, then it is obvious now that the patient must have *Cancer*.

In this chapter, we will adopt a different approach to traffic padding. Instead of deterministically forming padding groups, the server randomly (at uniform, in this example) selects one out of the three possible ways for forming a padding group. Therefore, we can see that a cancerous person will always receive a 360-byte packet, whereas the other patients have $\frac{2}{3}$ and $\frac{1}{3}$ probability to receive a 290-byte and 360-byte packet, respectively, as shown in Table 6.3.

Cancerous Person

Possible Padding Group	s Value (Padded)
{*Cancer, Cervicitis*}	360
{*Cancer, Cold*}	360
{*Cancer, Cough*}	360

Person Diagnosed with Cervicitis

Possible Padding Group	s Value (Padded)
{*Cervicitis, Cancer*}	360
{*Cervicitis, Cold*}	290
{*Cervicitis, Cough*}	290

Table 6.3: Proposed Solution for Table 6.1

To see why this approach provides better privacy guarantee, suppose an adversary observes a 360-byte packet and knows the patient to be a male. Under the above new approach, the adversary can no longer be sure that the patient has *Cancer*, because the following three cases will equally likely lead to a 360-byte packet to be observed. First, the patient has *Cancer* and the server selects either *Cervicitis*, *Cold*, or *Cough* to form the padding group. In the second and third case, the patient has either *Cold* or *Cough*, respectively, while the server selects *Cancer* to form the padding group. Consequently, the adversary now can only be 60%, instead of 100%, sure that the patient is associated with *Cancer*.

6.2 The Model

We first describe our traffic padding model in Section 6.2.1. We then introduce the concept of *uncertainty* in Section 6.2.2 and the *random ceiling padding* method in Section 6.2.3. Finally we define our cost metrics in Section 6.2.4. Table 6.4 lists our main notations.

a, $\vec{a}$, A_i or A	Action, action-sequence, action-set
s_i, v, $\vec{v}$, V_i or V	Flow, flow-vector, vector-sequence, vector-set
VA_i or VA	Vector-action set
$\vec{VA}$	Vector-action sequence
$dom(P)$	Dominant-Vector

Table 6.4: The Notation Table

6.2.1 Traffic Padding

We model the traffic padding issue from two perspectives, the *interaction* between users and servers, and the *observation* made by adversaries. For the *interaction*, we call an atomic input that triggers traffic an *action*, denoted as a, such as a keystroke or a mouse click. We call a sequence of actions that represents a user's complete input information an *action-sequence*, denoted as $\vec{a}$, such as a sequence of consecutive keystrokes entered into a search engine. We also call the collection of all the i^{th} actions in a set of action-sequences an *action-set*, denoted as A_i.

Correspondingly, for the *observation*, we denote a *flow-vector* as v to represent a sequence of *flows*, $\langle s_1, s_2, \ldots, s_{|v|} \rangle$, that is, the sizes of packets triggered by actions. We denote a *vector-sequence* as $\vec{v}$ to represent the sequence of flow-vectors triggered by an action-sequence, and a *vector-set* as V_i corresponding to the action-set A_i. Finally, given a set of action-sequences and corresponding vector-sequences, we define all the pairs of i^{th} actions and corresponding i^{th} flow-vectors as the *vector-action set*, denoted as VA_i or simply VA when no ambiguity is possible. For a given application, we call the collection of all the vector-action sets *vector-action sequence*, denoted as $\vec{VA}$.

6.2.2 Privacy Properties

We model the privacy requirement of a traffic padding scheme from two perspectives. First, when adversaries observe a flow-vector triggered by a single action, they should not be able to distinguish this action from at least $k-1$ other actions that could have also triggered that same flow-vector, which is formalized in the following.

Definition 6.1 (k-Indistinguishability) *Given a vector-action set VA, a padding algorithm $\mathcal{M}$ with output range $Range(\mathcal{M}, VA)$, we say $\mathcal{M}$ satisfies k-indistinguishability w.r.t. VA (k is an integer) if*

$$\forall(v \in Range(\mathcal{M}, VA)), |\{a : Pr(\mathcal{M}^{-1}(v) = a) > 0 \wedge a \in A\}| \geq k.$$

Example 6.1 *Assume that there are only four possible diseases in Table 6.2, then the ceiling padding solution as shown in the right column satisfies 2-indistinguishability.* $\boxdot$

In the previous section, we have illustrated how adversaries' background knowledge may help them to breach privacy even though the k-indistinguishability may already be satisfied. Therefore, our objective here is to first formally characterize the amount of *uncertainty* faced by an adversary about the real action performed by a user (we will then propose algorithms to increase such uncertainty in the next section). For this purpose, we apply the concept of entropy in information theory to quantify an adversary's uncertainty in the following.

Definition 6.2 (Uncertainty) *Given a vector-action sequence $\vec{VA}$, a padding algorithm $\mathcal{M}$, we define*

- *the uncertainty of $v \in Range(\mathcal{M}, VA)$, where $VA \in \vec{VA}$, is defined as $\varphi(v, VA, \mathcal{M}) =$*

$$-\sum_{a \in A}(Pr(\mathcal{M}^{-1}(v) = a) log_2(Pr(\mathcal{M}^{-1}(v) = a));$$

- *the uncertainty of algorithm $\mathcal{M}$ w.r.t. VA is defined as*

$$\phi(VA, \mathcal{M}) = \sum_{v \in Range(\mathcal{M}, VA)} \varphi(v, VA, \mathcal{M}) \times Pr(\mathcal{M}(A) = v);$$

- *the uncertainty of algorithm $\mathcal{M}$ w.r.t. $\vec{VA}$ is defined as*

$$\Phi(\vec{VA}, \mathcal{M}) = \prod_{VA \in \vec{VA}} (\phi(VA, \mathcal{M}));$$

Example 6.2 *To illustrate the above notions, following Example 6.1, the uncertainty of the flow* 360*, denoted as* $\varphi(360, VA, \mathcal{M})$ *(or simply* $\varphi(360)$ *hereafter when no ambiguity is possible) can be calculated as* $\varphi(360) = -(\frac{1}{2} log_2(\frac{1}{2}) + \frac{1}{2} log_2(\frac{1}{2})) = 1$*. Similarly, we have* $\phi(VA) = \frac{\varphi(360)}{2} + \frac{\varphi(290)}{2} = 1$*. Further, since the vector-action sequence is composed of a single vector-action set,* $\Phi(\vec{VA}) = \phi(VA) = 1$*.* $\boxdot$

Finally, we model the privacy of a padding algorithm as its joint capabilities of satisfying k-indistinguishability and δ-uncertainty. Note that here the former serves as a basic privacy requirement (when no resistance to background knowledge is needed) while the latter can be regarded as an enhanced requirement. Both parameters may be adjusted according to different applications' unique requirements for privacy.

Definition 6.3 (δ-uncertain k-indistinguishability) *An algorithm $\mathcal{M}$ gives δ-uncertain k-indistinguishability for a vector-action sequence $\vec{VA}$ if*

- $\mathcal{M}$ *w.r.t. any* $VA \in \vec{VA}$ *satisfies* k*-indistinguishability, and*

- *the uncertainty of* $\mathcal{M}$ *w.r.t.* $\vec{VA}$ *is not less than* δ.

6.2.3 Padding Method

To be more self-contained, we first review the *ceiling padding* method [69, 71]. The method deterministically partitions a vector-action set into padding groups, each of which has a cardinality no less than k, and then breaks the linkage among actions in the same group by padding the flows to be identical, as described in the following.

Definition 6.4 (Dominance and Ceiling Padding [71]) *Given a vector-set* V*, we define*

- *the* dominant-vector $dom(V)$ *as the flow-vector in which each flow is equal to the maximum of the corresponding flow among all the flow-vectors in* V.

- *a* ceiling-padded *group in* V *as a padding group in which every flow-vector is padded to the dominant-vector.*

Clearly, the ceiling padding method is only designed to achieve the k-indistinguishability, and will not provide sufficient privacy guarantee if the adversary possesses prior background knowledge.

In this chapter, we propose to introduce randomness into the process of forming padding groups per each user request. Specifically, to response to an action, we first select at random, from certain distributions, $k-1$ other actions to form the padding group. Then, we apply ceiling padding on the resultant group. To differentiate from the aforementioned fixed padding group and the original ceiling padding method, we call the group formed on the fly with randomness the *transient group*, and our method the *random ceiling padding* in the following.

Definition 6.5 (Transient Group and Random Ceiling Padding) *We say a mechanism* $\mathcal{M}$ *is a* random ceiling padding *method if, when responding to an action* a*, it randomly selects*

$k{-}1$ other actions and pads the flow-vector of action a *to be the dominant-vector among the corresponding flow-vectors of selected $k-1$ actions together with the original flow-vector of action* a*. We also call those k actions a* transient group.

Example 6.3 *To achieve 2-indistinguishability, a mechanism $\mathcal{M}$ selects uniformly at random 1 other action to form the transient group (Table 6.2). Then, the following two cases could both lead to an observed $s = 360$ flow. First, the patient has $Cancer$ and $\mathcal{M}$ selects any one of the others to form the group (there are 3 possible transient groups in this case). Second, the patient does not have $Cancer$ but has one of the other threes, and $\mathcal{M}$ selects $Cancer$ to form the group. Each of them has only one possible transient group. Thus,* $\varphi(360) = -(\frac{1}{2}log_2(\frac{1}{2}) + 3 \times \frac{1}{6}log_2(\frac{1}{6})) \approx 1.79$.

Now, if the adversary knows that the patient can not have $Cervicitis$ and observes the s-byte value to be 360, s/he will no longer be able to infer which disease the patient has. Formally, in this case, $\varphi(360) = -(\frac{3}{5}log_2(\frac{3}{5}) + 2 \times \frac{1}{5}log_2(\frac{1}{5})) = 1.37$. $\boxdot$

6.2.4 Cost Metrics

In addition to privacy requirements, we also need metrics for the communication and processing costs. For the former, we measure the proportion of packet size increases compared to the original flow-vectors. For the latter, we measure how many flow-vectors need to be padded among all the vectors in a $\vec{VA}$, as formalized in Definition 6.6 and 6.7, respectively.

Definition 6.6 (Expected Padding Cost) *Given a vector-action sequence $\vec{VA}$, an algorithm $\mathcal{M}$,*

- *the* expected padding cost *of action a in $(a, v) \in VA$ where $VA \in \vec{VA}$ is defined as* $pcos(a, VA, \mathcal{M}) =$

$$||\sum_{v' \in Range(\mathcal{M}, VA)} (Pr(\mathcal{M}(a) = v') \times v') - v||_1;$$

- *the* expected padding cost *of a vector-action set* $VA \in \vec{VA}$ *under algorithm* $\mathcal{M}$ *is defined as* $pcos(VA, \mathcal{M}) = \sum_{(a,v) \in VA}(pcos(a, VA, \mathcal{M}))$ *and that of the vector-action sequence is defined as* $pcos(\vec{VA}, \mathcal{M}) = \sum_{VA \in \vec{VA}}(pcos(VA, \mathcal{M}))$.

Definition 6.7 (Expected Processing Cost) *The* expected processing cost *of a vector-action sequence* $\vec{VA}$ *under an algorithm* $\mathcal{M}$ *is defined as*

$$rcos(\vec{VA}, \mathcal{M}) = \frac{\sum_{VA \in \vec{VA}} \sum_{(a,v) \in VA}(Pr(\mathcal{M}(a) \neq v))}{\sum_{VA \in \vec{VA}} |\{(a,v) : (a,v) \in VA\}|};$$

Surprisingly, while introducing randomness into the process of forming padding groups improves the privacy, this improvement does not necessarily come at a higher cost, as shown in Example 6.4 (we will only compare the cost with the original ceiling padding method hereafter, since ceiling padding has much less overhead than other methods, such as rounding, as shown in our previous work [71]).

Example 6.4 *For ceiling padding shown in last column of Table 6.2, the expected padding cost can be calculated as* $pcos(VA, ceiling\ padding) = 70$*, and the expected processing cost as* $rcos(VA, ceiling\ padding) = 25\%$.

On the other hand, for the random ceiling padding $\mathcal{M}$ *shown in Example 6.3, we have* $pcos(VA, \mathcal{M}) = (360-360)+3\times((\frac{1}{3}\times 360+\frac{2}{3}\times 290)-290) = 70$ *and* $rcos(VA, \mathcal{M}) = \frac{0+3\times\frac{1}{3}}{4} = 25\%$.

That is, these two methods actually lead to exactly the same expected padding and processing costs, while the latter clearly achieves higher uncertainty (with the same k*-indistinguishability).* $\boxdot$

6.3 The Algorithms

We first introduce a generic random ceiling padding scheme in Section 6.3.1, and then discuss two example ways for instantiating the scheme into concrete algorithms in

Section 6.3.2. The main intention here is to show that the random ceiling padding method can potentially be instantiated in many different ways based on specific applications' needs. In the coming sections, we will further show that even those straightforward ways we describe here can still achieve good performance in terms of the privacy guarantee and the costs.

6.3.1 The Random Ceiling Padding Scheme

The main idea of our scheme is the following. In responding to a user input, the server will form a transient group on the fly by randomly selecting members of the group from certain candidates based on certain distributions (different choices of such candidates and distributions will lead to different algorithms, as demonstrated in Section 6.3.2).

Our goal is two-fold. First, the privacy properties, k-indistinguishability and δ-uncertainty, need to be ensured. Second, the costs of achieving such privacy protection should be minimized. Clearly, a trade-off naturally exists between these two objectives. We will demonstrate how to address this trade-off through two instantiations of the general scheme with different methods of forming transient groups.

The generic random ceiling padding scheme consists of two stages as shown in Tables 6.5 and 6.6. The first stage (Table 6.5), a one-time process, derives the randomness parameters and accordingly determines the probability of an action being selected as a member of a transient group. As exemplified later in Section 6.4, both δ and costs are related to k (which is considered as a given constant), the vector values and their cardinalities (which is determined for a given vector-action set), and the parameters of distribution from which the randomness is drawn. Clearly, to determine the randomness parameters such that δ is not less than a desired value while keeping the costs minimal is naturally an optimization problem. In this chapter, we simplify the process by setting the size of each transient group to k to ensure the indistinguishability (the proof is omitted due to space limitations).

Stage 1: One-Time Preprocessing
Input: the vector-action set VA, the privacy properties k_{min} and δ_{min}, the randomness generator G; **Output:** the parameters $\langle P \rangle$ of G **Method:** 1. **Let** V be the vector-set of VA, and A be the action-set of VA; 2. **If** $(\|VA\| \leq k_{min})$ **Return**; 3. Compute the distribution D_V of V; 4. Compute $\langle P \rangle$ based on its relation with δ, k, $pcos$, $rcos$, D_V when random ceiling padding is applied, such that (1). $k \geq k_{min}$ and $\delta \geq \delta_{min}$; (2). $pcos$ and $rcos$ are minimal; 5. **Return** $\langle P \rangle$;

Table 6.5: The Random Ceiling Padding Scheme: Stage One

Once the randomness parameters are set, then repeatedly, upon receiving an action a_0, the second stage (Table 6.6) selects, randomly following the results of stage one, $k - 1$ other actions from the corresponding action-set A of vector-action set VA to form the transient group, and then returns the dominant-vector of this transient group.

Stage 2: Real-Time Response
Input: the vector-action set VA, the randomness parameters $\langle P \rangle$ of G, the privacy properties k_{min} and δ_{min}, the action a_0 **Output:** the flow-vector $v_0^{'}$; **Method:** 1. **Let** V be the vector-set of VA, and A be the action-set of VA; 2. Create A_C by randomly selecting $k_{min} - 1$ actions from the subset of A based on $\langle P \rangle$ of G; 3. $A_C = A_C \cup \{a_0\}$; 4. **Let** V_C be the subset of vector-set V which corresponds to A_C; 5. **Return** the dominant-vector of V_C;

Table 6.6: The Random Ceiling Padding Scheme: Stage Two

6.3.2 Instantiations of the Scheme

In this section, we discuss two example instantiations of the proposed random ceiling padding scheme, and illustrate two different ways for reducing the padding and processing costs while satisfying the privacy requirements. Basically, the two instantiations differ in their ways of selecting candidates for members of the transient group, in order to reduce the cost. First, to facilitate discussions, we introduce two new notions.

In Definition 6.8, intuitively, function $f_v(.)$ sorts a vector-action set VA based on the padding cost, and we denote the resultant totally ordered set (chain) under the binary relation $\succcurlyeq_v$ by $\langle VA \rangle_v$. The main objective of this step is to adjust the probability of an action being selected as a member of the transient group, in order to reduce the expected costs. Besides, in the case that each flow-vector in V includes a single flow, the flows (integers) ordered by the standard larger-than-or-equal relation $\geq$ is also a chain that is naturally identical for each v. Therefore, although the chain $\langle VA \rangle_v$ for different $v \in V$ may be different, in the rest of this chapter, we will use a single chain (simplified as $\langle VA \rangle$) for analysis and experiment.

Definition 6.8 (Larger and Closer) *Given a vector-action set VA, a pair* $(a, v) \in VA$*, define a function* $f_v : V \to I$ *(I for integers) as* $f_v(v') = ||dom(\{v, v'\}) - v||_1$*. Then, we say, w.r.t.* (a, v)*,*

- $(a', v') \in VA$ *is larger than* $(a'', v'') \in VA$*, denoted by* $(a', v') \succcurlyeq_v (a'', v'')$*, if* $f_v(v') > f_v(v'') \vee ((f_v(v') = f_v(v'')) \wedge (a' \succ a''))$*, where* $\succ$ *is any predefined order on the action-sets;*

- $(a', v') \in VA$ *is closer to* (a, v) *than* $(a'', v'') \in VA$ *if* $|f_v(v')| < |f_v(v'')|$*.*

a) Option 1: Randomness from Bounded Uniform Distribution

The step 2 of stage 2 in Table 6.6 may be realized in many different ways by choosing group members from different subsets of candidates and based on different distributions.

Note that although choosing the members uniformly at random from all possible candidates certainly leads to highest possible uncertainty, this also will incur prohibitive processing cost. In fact, in Section 6.4, we will show through theoretical analysis that the uncertainty of an algorithm can be dramatically increased even by a slight increase in the cardinality of possible candidates for forming the transient group.

This first option draws candidates from a uniform distribution. It also allows users to constraint the cardinality of candidate actions to be considered (c_t) and the number of such actions that are *larger* than given action (c_l). More specifically, given a vector-action set VA, and a pair (a_i, v_i) being the i^{th} pair of its corresponding chain $\langle VA \rangle$, the transient group of (a_i, v_i) will be selected uniformly at random from the sub-chain of the chain in the range of $[max(0, min(i - c_l, |VA| - c_t)), min(max(0, min(i - c_l, |VA| - c_t)) + c_t, |VA|)]$ (complete algorithms will be omitted due to space limitations) .

The action in a transient group which is in the least position of the chain $\langle VA \rangle$ will determine the padding cost of (a, v) when a is performed. Thus, from this perspective, c_l should be as small as possible. However, c_l should also be sufficiently large. For example, if $c_l = 0$, each action should be deterministically padded. Moreover, the c_t determines the cardinality of possible transient groups. More possibilities of transient groups will complicate adversaries' tasks in attacking (collecting the data of directional packet sizes and analyzing the distribution of flow-vector information).

b) Option 2: Randomness from Normal Distribution

In this option, the action *closer* to a in the chain has higher probability to be selected as a member of a's transient group. To select a member, the distance between the selected action and the performed action a in the chain $\langle VA \rangle$ (that is, the difference of the positions) is drawn from normal distribution (rounded up to the nearest integer).

When the mean of normal distribution is set to zero, the two actions with equal distance in the both sides of the performed action a in the chain are equally likely selected. As mentioned before, the action in transient group with least position in the chain $\langle VA \rangle$

determines the padding cost. Thus, the mean can be adjusted to a positive integer, such that the actions in larger positions than a would have a higher chance to be selected, and consequently the expected cost will be reduced.

In addition, since increasing the standard deviation flattens the curve of the distribution and allows more chances to draw a value far from the mean, it yields a higher probability to select an action farther away from the performed one in the chain $\langle VA\rangle$. Thus, in practice, the standard deviation should be small enough to reduce the padding cost; it also should not be too small in order to prevent the adversary from collecting the data and analyzing the distribution of flow-vector values.

6.4 The Analysis

In this section, we evaluate the privacy degree, the costs, and the computational complexity of our solution. For simplicity, we analyze those parameters for scenarios in which each action-sequence and flow-vector are of length one, referred to as VA_s, and the randomness in our scheme (shown in Tables 6.5, 6.6) is drawn from a uniform distribution, denoted by $\mathcal{M}_u$.

To simplify the discussions, we use $\vec{s} = \left\langle s_1, s_2, \ldots, s_{|\vec{s}|}\right\rangle$ to denote the sequence of distinct flow values in decreasing order, and use $\vec{n} = \left\langle n_1, n_2, \ldots, n_{|\vec{s}|}\right\rangle$ to denote that there is n_i number of actions in VA_s whose flow value is s_i. We let $\vec{L} = \left\langle L_1, L_2, \ldots, L_{|\vec{s}|}\right\rangle$, where $L_i = \sum_{j=i}^{|\vec{s}|} n_j$ for ($i \in [1, |\vec{s}|]$). Also, we set $N = \sum_{i=1}^{|\vec{s}|} n_i$ ($L_1 = N$). We say an action a in VA_s is an s_i-type action if its flow value equals to s_i before padding, denoted by $a \in VA_{s_i}$.

6.4.1 Analysis of Privacy Preservation

For the purpose of analyses, we need to characterize the cardinality of transient groups. Given a vector-action set $VA_s = (V, A)$ and its action $a \in A$, the $\mathcal{M}_u$ algorithm selects $k-1$ other actions uniformly at random to form its transient group. For any action, the cardinality of sample space with respect to the set of all possible transient groups is equal to $\binom{N-1}{k-1}$ [85]. Given a s_i-type action, we partition its sample space Ω_i into i number of disjoint events $E_{i,j}$, where $j \in [1, i]$ and $E_{i,j}$ is the set of transient groups for which the maximal flow value is s_j, as shown in Table 6.7.

Clearly, the actions with the same flow-vector value have the sample space with similar events and corresponding cardinality. Note that there may exist some i such that $L_i < k$ (L_i as defined above). That is, the number of actions, whose flow values are less than or equal to s_i, is less than k. However, since our algorithms always select k different actions to form a transient group on-demand, without loss of generality, we assume that $n_{|\bar{s}|} \geq k$ for the purpose of the analysis whereas our algorithms does not need such assumption.

Sample Space	**Number of Possible Transient Groups**	**Events (Based on the Maximal Flow Value)**	
		Event	**Cardinality**
Ω_{i-1}	...	...	...
Ω_i	$\binom{N-1}{k-1}$	$E_{i,1}$	$\binom{L_1-1}{k-1} - \binom{L_2-1}{k-1}$
		$E_{i,2}$	$\binom{L_2-1}{k-1} - \binom{L_3-1}{k-1}$
		...	...
		$E_{i,i-2}$	$\binom{L_{i-2}-1}{k-1} - \binom{L_{i-1}-1}{k-1}$
		$E_{i,i-1}$	$\binom{L_{i-1}-1}{k-1} - \binom{L_i-1}{k-1}$
		$E_{i,i}$	$\binom{L_i-1}{k-1}$
Ω_{i+1}	...	...	...

Table 6.7: The Sample Space for Transient Groups by Random Ceiling Padding and Corresponding Events

For a s_i-type action a, if the actions selected in a transient group are from those whose flow value is less than or equal to s_i, the maximal flow value will be s_i in that group. There are totally $L_i - 1$ such actions (excluding action a itself). Therefore, the cardinality of $E_{i,i}$ is equal to $\binom{L_i-1}{k-1}$. Similarly, a transient group belongs to an event $E_{i,j}$ where $j < i$ only if, in that group, there is at least a s_j-type action and there is not any s_k-type action for all $k < j$. Therefore, the cardinality of $E_{i,j} (j < i)$ equals to $\binom{L_j - 1}{k-1} - \binom{L_{j+1}-1}{k-1}$. Clearly, for one given execution, if the resultant transient group is in event $E_{i,j}$, the flow value of action a is padded to s_j by the $\mathcal{M}_u$ algorithm. The cardinality of each event in the sample space for an action is shown in Table 6.7.

Note that the probability that the flow of an action is padded to a value is different from the probability that the traffic with a padded flow value is triggered by an action. For example, the flow of any s_1-type action is always padded to s_1. However, one observing a s_1-byte packet can only infer that the probability that this traffic is triggered by a s_1-type action is $\frac{n_1 \times \binom{N-1}{k-1}}{n_1 \times \binom{N-1}{k-1} + (N - n_1) \times (\binom{N-1}{k-1} - \binom{N-n_1-1}{k-1})} \approx \frac{n_1}{N}$.

Moreover, the adversary cannot collect the vector-action set even if s/he acts as a normal user of the application using random ceiling padding technique. The reason is as follows. First, the sample space is huge even for small-size vector-action set with reasonable k value. For example, when $|VA| = 100$ and $k = 20$, the cardinality of sample space for each action equals to $\binom{99}{19} \approx 2^{66}$. Second, since all users share one uniform random process in the scheme, the distribution of events cannot be sufficiently approximated by collecting flow-vector values for a special action just as many times as the cardinality of its sample space.

Lemma 6.1 *The $\mathcal{M}_u$ algorithm gives δ-uncertain k-indistinguishability for a VA_s, where*

$$\begin{aligned}\delta = & -\sum_{i=1}^{|\vec{s}|-1} \left(n_i \times \frac{\frac{\binom{L_i}{k}}{L_i}}{\binom{L_i}{k} - \binom{L_{i+1}}{k}} log_2 \frac{\frac{\binom{L_i}{k}}{L_i}}{\binom{L_i}{k} - \binom{L_{i+1}}{k}}\right) \\ & -\sum_{i=1}^{|\vec{s}|-1} \left(L_{i+1} \times \frac{\frac{\binom{L_i}{k}}{L_i} - \frac{\binom{L_{i+1}}{k}}{L_{i+1}}}{\binom{L_i}{k} - \binom{L_{i+1}}{k}} log_2 \frac{\frac{\binom{L_i}{k}}{L_i} - \frac{\binom{L_{i+1}}{k}}{L_{i+1}}}{\binom{L_i}{k} - \binom{L_{i+1}}{k}}\right) \\ & + log_2(n_{|\vec{s}|})\end{aligned}$$

Proof: First, for the s_i ($i \in [1, |\vec{s}| - 1)$, there are two cases for the action a that $\mathcal{M}_u(a) = s_i$ as follows.

- Action a is a s_i-type action. Denote the set of such actions by A_1. Clearly, $|A_1| = n_i$.

The $\mathcal{M}_u$ selects $k-1$ actions which flow value is no larger than s_i to form the transient group. The number of such transient groups for any s_i-type action is $\binom{L_i-1}{k-1}$. For all such n_i actions, there are $n_i \times \binom{L_i-1}{k-1}$ transient groups in total. Note that the transient group could be identical for different actions. For such cases, it should be counted once for each action since it is triggered by different actions.

- Action a is a s_j-type action ($j > i$, that is, $s_j < s_i$). Denote the set of such actions by A_2. Then, $|A_2| = \sum_{j=i+1}^{|\vec{s}|} = L_{i+1}$. The $\mathcal{M}_u$ selects at least one s_i-type actions and zero number of actions which flow value is larger than s_i to form the transient group. The number of such transient groups for any such action is $\binom{L_i-1}{k-1} - \binom{L_{i+1}-1}{k-1}$. For all L_{i+1} such actions, there are $L_{i+1} \times (\binom{L_i-1}{k-1} - \binom{L_{i+1}-1}{k-1})$ transient groups in total.

Since each transient group is formed equally likely, we then have $Pr((\mathcal{M}_u^{-1}(s_i)) = a) =$

$$\begin{cases} \frac{\binom{L_i-1}{k-1}}{n_i \times \binom{L_i-1}{k-1} + L_{i+1} \times (\binom{L_i-1}{k-1} - \binom{L_{i+1}-1}{k-1})} & \text{if } a \in A_1; \\ \frac{\binom{L_i-1}{k-1} - \binom{L_{i+1}-1}{k-1}}{n_i \times \binom{L_i-1}{k-1} + L_{i+1} \times (\binom{L_i-1}{k-1} - \binom{L_{i+1}-1}{k-1})} & \text{if } a \in A_2, \end{cases}$$

which leads to the first two lines of Equation 6.1.

Second, for the $s_{|\vec{s}|}$, the only case that $\mathcal{M}_u(a) = s_{|\vec{s}|}$ is as follows. Action a is a $s_{|\vec{s}|}$-type action and all the members of its transient group are also $s_{|\vec{s}|}$-type actions. The number of such transient groups for any $s_{|\vec{s}|}$-type action is $\binom{L_{|\vec{s}|}-1}{k-1}$. We then have $Pr((\mathcal{M}_u^{-1}(s_{|\vec{s}|})) = a) = \frac{1}{n_{|\vec{s}|}}$ for any $s_{|\vec{s}|}$-type action, which leads to the last line of Equation 6.1. Thus we have proved the lemma. ■

In summary, in random ceiling padding, an action cannot be distinguished from at least other $k-1$ different actions based on the traffic triggered, which satisfies k-indistinguishablility. Moreover, the adversary cannot deterministically infer the action only by the observation even s/he can further remove a limited number of actions based on prior knowledge.

6.4.2 Analysis of Costs

In this section, we first compare the padding cost between ceiling padding and random ceiling padding, then formulate the upper bound of padding cost for random ceiling padding.

First, we show that the padding cost and the processing cost of ceiling padding and random ceiling padding is deterministically incomparable. That is, these costs cannot simply be ordered based on the algorithms themselves and will depend on specific vector-action sets.

Lemma 6.2 *There exist cases in which the expected padding cost of random ceiling padding $\mathcal{M}_u$ is less than that of ceiling padding $\mathcal{M}_c$, and vice versa.*

Proof: For simplicity, we omit the action information and model the vector-action set as an integer vector, where each entry represents the single flow value corresponding to an action.

Firstly, we show the construction for the case where random ceiling padding has less expected padding cost than ceiling padding.

Equation 6.1 shows our construction for the proof, where $n = 2k - 1$ and $s_1 > s_2$. That is, $\vec{s} = \langle s_1, s_2 \rangle$ and $\vec{n} = \langle 1, 2k-2 \rangle$. Note that the equation presents a category of vector-action sets instead of one specific set.

$$VA_1 = \underbrace{\overbrace{s_1}^{1}, \overbrace{s_2, s_2, \ldots, s_2}^{2k-2}}_{2k-1} \tag{6.1}$$

To achieve k-indistinguishability, since the number of actions is less than $2k$, ceiling padding can only partition the sets into a padding group. Therefore, the flow corresponding to s_2-type actions must be padded to s_1. Consequently, its expected padding cost $pcos(VA_1, \mathcal{M}_c)$=$1 \times (100\% \times (s_1 - s_1)) + (2k-2) \times (100\% \times (s_1 - s_2)) = (2k-2) \times (s_1 - s_2)$.

On the other hand, in random ceiling padding, given any action, the algorithm can select any $k-1$ other actions from all the other $2k-2$ possible actions to form its transient group. Consequently, there are $\binom{2k-2}{k-1}$ number of different transient groups. For the s_1-type action, the dominant-vector of any combination is s_1 because $s_1 > s_2$. For the other $(2k-2)$ s_2-type actions, the corresponding flow will be padded to s_1 only if that s_1-type action is selected to form their transient groups, which has $\binom{1}{1} \times \binom{2k-3}{k-2} = \binom{2k-3}{k-2}$ possible combinations. Otherwise, it will be padded to s_2, which has $\binom{1}{0} \times \binom{2k-3}{k-1} = \binom{2k-3}{(2k-3)-(k-1)} = \binom{2k-3}{k-2}$ possible combinations. In other words, the flow of the s_2-type actions has a $\frac{\binom{2k-3}{k-2}}{\binom{2k-2}{k-1}} = 50\%$ chance of being padded to s_1, and a 50% chance of remaining the same (s_2). Therefore, the expected padding cost for a s_2-type action equals to $\frac{1}{2} \times (s_1 - s_2) + \frac{1}{2} \times (s_2 - s_2) = \frac{s_1 - s_2}{2}$. Thus, $pcos(VA_1, \mathcal{M}_u) = 1 \times (s_1 - s_1) + (2k-2) \times \frac{s_1 - s_2}{2} = (k-1) \times (s_1 - s_2)$.

In summary, for such category of vector-action sets, the expected padding cost of ceiling padding is as twice as that of random ceiling padding.

Secondly, we show the construction for the other case where ceiling padding has less expected padding cost that random ceiling padding.

Equation 6.2 shows our construction for the proof, where $n = 2k$ and $s_1 > s_2$. That is, $\vec{s} = \langle s_1, s_2 \rangle$ and $\vec{n} = \langle k, k \rangle$. Note again that the equation presents a category of vector-action sets.

$$VA_2 = \underbrace{\overbrace{s_1, s_1, \ldots, s_1}^{k}, \overbrace{s_2, s_2, \ldots, s_2}^{k}}_{2k} \tag{6.2}$$

To achieve k-indistinguishability, the padding cost of ceiling padding equals to 0, since ceiling padding can simply partition the actions into two groups. One group includes all the actions which flow value equals to s_1, and the other equals to s_2. That is, $pcos(VA_2, \mathcal{M}_c) = 0$.

On the other hand, in random ceiling padding, given an action, the algorithm will randomly select any $k-1$ actions from all the other $2k-1$ actions to form its group.

Consequently, there are $\binom{2k-1}{k-1}$ number of different combinations. Furthermore, for the actions with the flow is s_1, no matter which combination is selected, the dominant-vector is s_1 since $s_1 > s_2$. For those flow is s_2, if and only if the $k-1$ other actions are selected from the left $k-1$ s_2-type actions, the dominant-vector is s_2, otherwise the dominant-vector is s_1. Therefore, the expected padding cost for a s_2-type action equals to $\frac{\binom{k-1}{k-1}}{\binom{2k-1}{k-1}} \times (s_2 - s_2) + (1 - \frac{\binom{k-1}{k-1}}{\binom{2k-1}{k-1}}) \times (s_1 - s_2)$. Thus, $pcos(VA_2, \mathcal{M}_u) = k \times (s_1 - s_1) + k \times (1 - \frac{1}{\binom{2k-1}{k-1}}) \times (s_1 - s_2) \approx k \times (s_1 - s_2)$ for sufficiently large k.

In summary, for such category of vector-action sets, the expected padding cost of ceiling padding is zero while it of random ceiling padding is around $k \times (s_1 - s_2)$.

Finally, based on the two constructions above, we have proved the lemma. ■

Through similar constructions in the proof of Lemma 6.2, we have result that the processing cost between them is also incomparable (we omit the details in this paper due to space limitations). Next, more generally, we formulate the padding and processing cost of random ceiling padding as shown in Lemma 6.3.

Lemma 6.3 *The padding and processing cost of $\mathcal{M}_u$ for a VA_s are*

$$\begin{aligned} pcos(VA_s, \mathcal{M}_u) &= \sum_{i=1}^{|\vec{s}|} s_i \Big(\frac{N(\binom{L_i}{k} - \binom{L_{i+1}}{k})}{\binom{N}{k}} - n_i \Big), and \\ rcos(VA_s, \mathcal{M}_u) &= 1 - \frac{1}{\binom{N}{k}} \times \sum_{i=1}^{|\vec{s}|} \frac{n_i}{L_i} \binom{L_i}{k}, \textit{where } L_{|\vec{s}|+1} = 0. \end{aligned}$$

Proof: Based on Table 6.7, we can lead to the aforementioned results.

First, we prove the result for the expected padding cost.

For any s_1 type action, the expected padding cost equals 0. For an action which is any s_i-type action other than s_1-type,

$pcos(a \in VA_{s_i}, VA_s, \mathcal{M}_u) =$

$$\sum_{j=1}^{i-1} \frac{\binom{L_j - 1}{k-1} - \binom{L_{j+1} - 1}{k-1}}{\binom{N-1}{k-1}} s_j + \frac{\binom{L_i - 1}{k-1}}{\binom{N-1}{k-1}} s_i - s_i.$$

Thus, based on the definition of expected padding cost, we have $pcos(VA_s, \mathcal{M}_u)$

$$= \sum_{VA_{s_i} \in VA} |VA_{s_i}| \times pcos(a \in VA_{s_i}, VA_s, \mathcal{M}_u)$$

$$= \sum_{i=1}^{|\vec{s}|} s_i\left(\frac{n_i\binom{L_i-1}{k-1} + L_{i+1}(\binom{L_i-1}{k-1} - \binom{L_{i+1}-1}{k-1})}{\binom{N-1}{k-1}} - n_i\right)$$

$$= \sum_{i=1}^{|\vec{s}|} s_i\left(\frac{L_i\binom{L_i-1}{k-1} - L_{i+1}\binom{L_{i+1}-1}{k-1}}{\binom{N-1}{k-1}} - n_i\right)$$

which leads to the formula above.

Second, we prove the result for the expected processing cost.

For a s_i-type action (a, s_i),

$$Pr(\mathcal{M}_u(a \in VA_{s_i}) \neq s_i) = 1 - \frac{\binom{L_i-1}{k-1}}{\binom{N-1}{k-1}};$$

Thus, based on the definition of expected processing cost, we have $rcos(VA_s, \mathcal{M}_u)$

$$= \frac{\sum_{VA_{s_i} \in VA_s} (|VA_{s_i}| \times Pr(\mathcal{M}_u(a \in VA_{s_i}) \neq s_i))}{|VA_s|}$$

$$= \frac{1}{N} \times \sum_{i=1}^{|\vec{s}|} n_i\left(1 - \frac{\binom{L_i-1}{k-1}}{\binom{N-1}{k-1}}\right)$$

which leads to the above formula. Thus, we prove the lemma. ∎

6.4.3 Analysis of Computational Complexity

The computational complexity of random ceiling padding algorithm, in the case that randomness is drawn from a uniform random distribution, is $O(k)$ due to the following. First, the first stage of our scheme can be pre-calculated only once, when the vector-action set is given, and does not need to be repeatedly evaluated each time when the scheme is invoked to respond to an action. Therefore, although it runs in polynomial time of N (N as above defined), for continuous execution of the algorithm, the computational complexity for responding to each action is still $O(1)$. Second, to select k random actions without duplicates, Line 2 of second stage can be realized in $O(k)$ time with many standard algorithms. Finally, it takes $O(k)$ times to select the corresponding vector-set for the selected actions in Lines 3-4 and calculate their dominant-vector in Line 5.

6.5 Experiment

In this section, we evaluate the uncertainty and the cost under two implementation options of our scheme (see Section 6.3) through experiments with two real world Web applications. First, Section 6.5.1 describes the experimental settings, and then Section 6.5.2, 6.5.3, and 6.5.4 present experimental results for randomness drawn from bounded uniform distribution and normal distribution, respectively.

6.5.1 Experimental Setting

We collect testing vector-action sets from two real-world web applications, a popular search engine $Engine$ (where users' searching keyword needs to be protected) and an authoritative drug information system $Drug$ (where users' health information needs to be protected).

We compare our solutions with the svmdGreedy (short for SVMD) [71] on four-letter combinations in $Engine$ and the last-level data in $Drug$ due to the following. First, one vector-action set is sufficient to demonstrate the results. Thus, we use a single vector-action set instead of vector-action sequence. Second, as reported in [71], rounding and random padding [22] lead to even larger overheads while they cannot ensure the privacy. Thus, we compare our results with SVMD only.

In the first option (see Section 6.3.2), namely, *TUNI* option, we constraint the number of larger actions (c_l) and the minimal number of possible actions to be selected (c_t) when the probability of an action to be selected is drawn from uniform distribution. In the meantime, in the second option, namely, *NORM* option, we allow to adjust the mean (μ) and standard deviation (σ) when it is drawn from normal distribution.

All experiments are conducted on a PC with 2.20GHz Duo CPU and 4GB memory and we conduct each experiment 1000 times. To facilitate the comparisons, we use padding cost ratio, processing cost ratio to measure the relative overheads of the padding cost and processing cost, respectively.

6.5.2 Uncertainty and Cost v.s. k

The first set of experiments evaluates the uncertainty and cost of *TUNI* and *NORM* options against *SVMD*. Figure 6.1(a), (b), and (c) illustrate the padding cost, uncertainty, and processing cost against the privacy property k, respectively. In general, the padding and processing costs of all algorithms increase with k, while *TUNI* and *NORM* have less costs than those of *SVMD*. Meanwhile, our algorithms have much larger uncertainty for *Drug* and slightly larger for *Engine*.

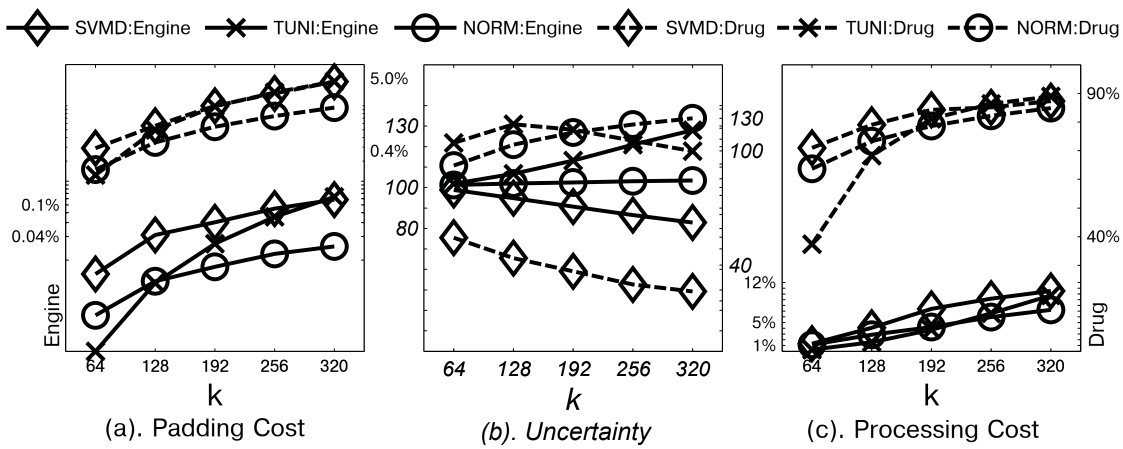


Figure 6.1: Uncertainty and Cost Against k

6.5.3 Randomness Drawn from Bounded Uniform Distribution

Figure 6.2 illustrates the uncertainty and cost of *TUNI* option on both vector-action sets against the top limit c_l. As expected, *SVMD* is insensitive to c_l since it does not have the concept of c_l. On the other hand, both costs increase slowly with c_l for *TUNI*. This is because, larger c_l allows the algorithm to have more chances to select *larger* actions for transient group. The largest action in the transient groups determines the padding cost in this case, and a single *larger* action leads to an increase of processing cost. From the results, *TUNI* performs worse on *Drug* than on *Engine* w.r.t. cost. This is because, the more diverse in the flow of *Drug* leads to more chances to select *larger* action even with a small increase of c_l. Despite the slight increase of cost with c_l, *TUNI* generally has less cost and larger uncertainty than *SVMD* for both vector-action sets.

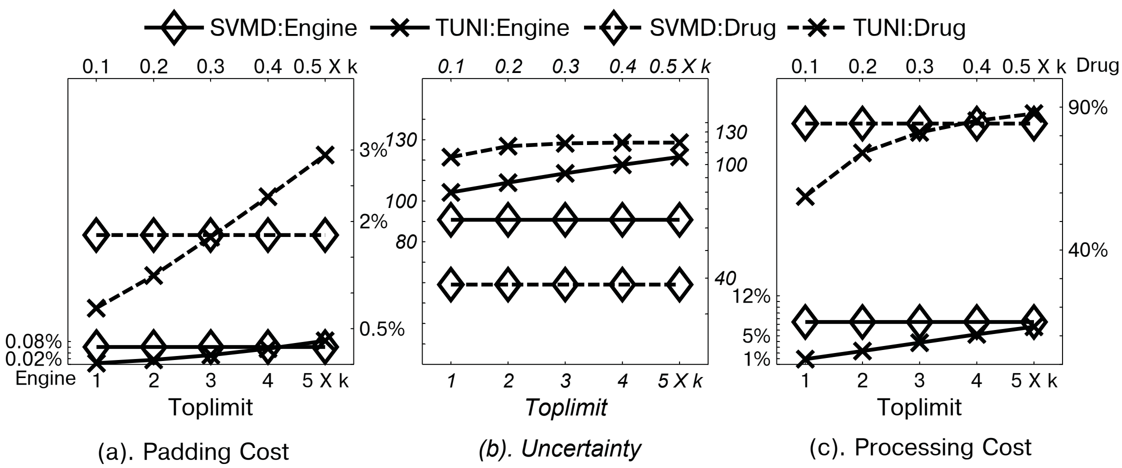


Figure 6.2: Uncertainty and Cost for Bounded Uniform Distribution Against Top Limit

Figure 6.3 shows the uncertainty and cost against the minimal cardinality c_t. Similarly, *SVMD* is insensitive to c_t due to the same reason. Also, *TUNI* demonstrates same results on *engine* in terms of both uncertainty and cost regardless of the value of c_t. This is because, the constraint of minimal cardinality works only when the cardinality of possible actions is less than c_t after applying c_l parameter. In *engine*, the number of actions which have the smallest flow value is extremely larger than the c_t values in the experiment.

In other words, c_t does not affect the results. For *drug*, the padding and processing costs increase slowly with c_t while the uncertainty decreases slowly.

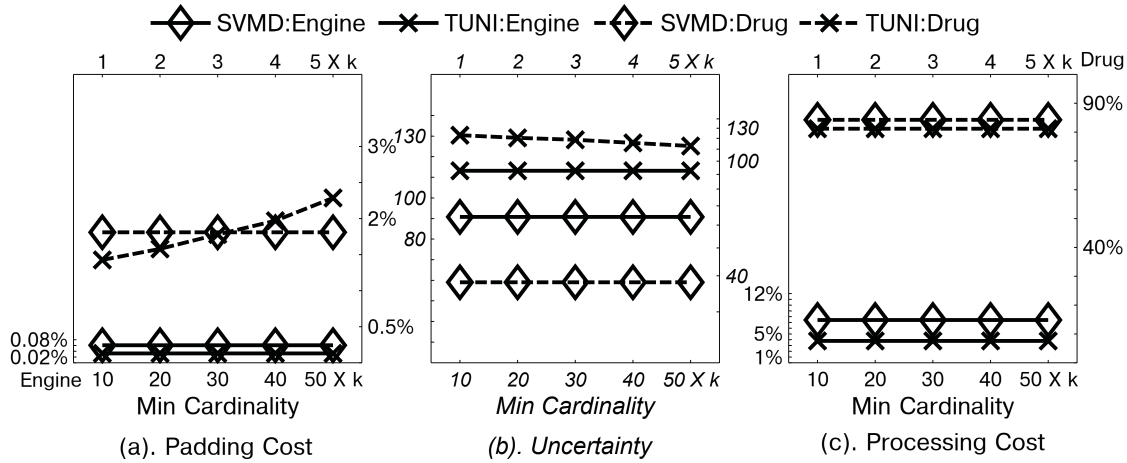


Figure 6.3: Uncertainty and Cost for Bounded Uniform Distribution Against Minimal Cardinality

6.5.4 Randomness Drawn from Normal Distribution

Figure 6.4 illustrates the uncertainty and cost of *NORM* option on both vector-action sets against the mean (μ) of normal random function. Compared with *SVMD*, *NORM* has less cost and yet higher uncertainty. The mean values do not affect the uncertainty and cost of *SVMD* since it does not take mean as a parameter. On the other hand, the cost of *NORM* decreases almost linearly with the increase of mean from 0 to 16, and rapidly as μ grows to 32 on both vector-action sets. In the meanwhile, the uncertainty of *NORM* slightly changes for the mean from 0 to 16, and decreases rapidly when $\mu = 32$. This is because, when $\mu = 32$, *NORM* has negligible chance to select a *larger* actions for the group. In other words, the vectors need not to be padded in most cases. Thus, in practice, we must tune the parameters (μ and σ) to avoid such situation.

Figure 6.5 shows the uncertainty and cost against the standard deviation σ of normal random function. Basically, all the three measurements decreases with the decrease of

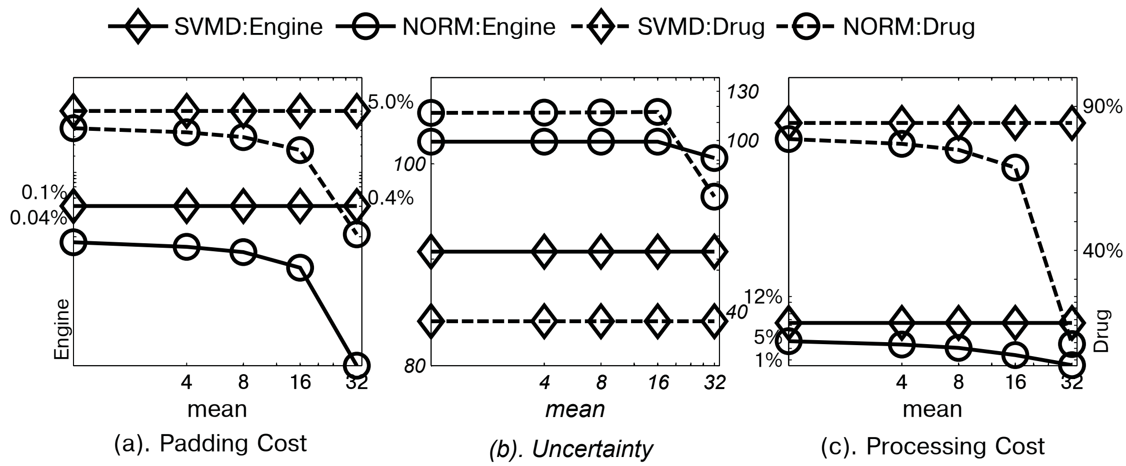


Figure 6.4: Uncertainty and Cost for Normal Distribution Against Mean

σ. Compared with *SVMD*, the less the σ, *NORM* exhibits better. This is as expected since the larger the standard deviation is, the flatter the curve of normal distribution is, and consequentially, the more chances to draw a value far from the mean, which is equal to select an action far from the performed one.

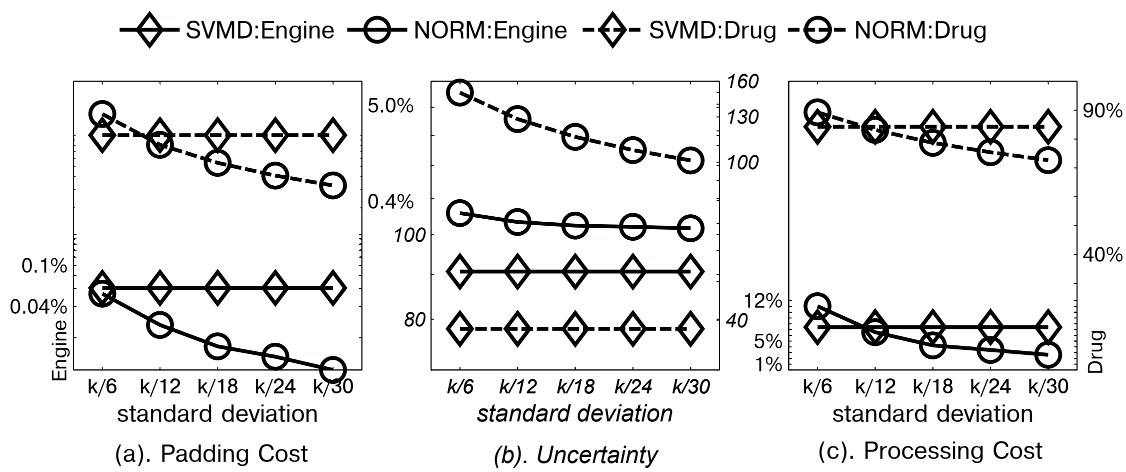


Figure 6.5: Uncertainty and Cost for Normal Distribution Against Standard Deviation

6.6 Summary

In this chapter, we have proposed a solution to reduce the impact of adversaries' background knowledge in privacy-preserving traffic padding. The approach can potentially be applied to other privacy-preserving issues, although we have focused on the traffic padding issues in this chapter. We have also instantiated two algorithms by following the proposed solution. Our experiments with real-world applications confirmed the performance of the proposed solution in terms of both privacy and overheads. Our future work will apply the proposed approach to privacy-preserving data publishing to achieve syntactic privacy while mitigating the threat of adversaries' prior-knowledge.

Chapter 7

PPSM: Privacy-Preserving Smart Metering

In this chapter, we present an efficient technique for privacy-preserving smart metering to simultaneously achieve multiple privacy objectives.

7.1 Overview

Smart meter with fine-grained consumption information benefits both utility (to better schedule electric production) and customers (to cut down the cost) . However, recent studies show that such features may also lead to serious breaches of customers' privacy. There typically exist two categories of approaches to prevent adversaries from violating the individual's privacy.

First, the smart meter, with the tariff information, accumulatively calculates the amount of billing and sends the billing information once to the service provider (utility) at each billing period. In such a way, the utility only knows the final billing and cannot compromise the customers' privacy. However, it may be challenging for such method to remain consistent of the tariff information between utility and meters. Furthermore, the utility cannot learn the trend of electrical consumptions for fine-grained period. Also, in the cases that arguments on the billing between users and providers, such information cannot be used as an evidence.

Second, at each time slot (e.g. 6 minutes), all the meters send the consumption information to the predetermined collector, which then sums up (through homomorphic encryption) and then sends the results to the utility. In such a way, the utility can learn the total consumptions for each reading period and consequently can dynamically adjust the producing of electric based on the consumptions. However, such approach does not provide individual information in terms of consumptions and billings, and consequently, the utility can only charge the collector totally and cannot charge individually.

Consequently, the aforementioned two solutions must both be applied to achieve these two objectives. In the sequel, the privacy of both objectives must be preserved. Recent solution in the literature integrates two objectives into one protocol with a single set of security primitives. However, the communication between the smart meters and corresponding counterparts remain separately which still incurs high overhead.

In this chapter, we observe that preserving the privacy with regard to the readings of a customer's electric consumption does not necessarily lead to preserving that customers' privacy. On the other hand, we also observe that to preserve the privacy of both aggregation of readings and the billing of consumptions can be concurrently achieved. Based on such observations, we propose a novel privacy model in smart meter to preserve semantic privacy, which allows the smart grid to have one unique set of consumption readings for each smart meter for the purpose of both aggregation analysis and billing.

The contribution of this chapter is two fold. First, we observe that the privacy issue is not due to the readings themselves but the sensitive information behind the readings. Second, to the best of our knowledge, our novel privacy model is the first effort on preserving the individual sensitive information (compared with preserving the readings information).

The rest of the chapter is organized as follows.We first build intuitions through a running example in the remainder of this section. Section 7.2 defines our model. Section 7.3 design the algorithm for the smart meters, and briefly introduce the algorithms for the other component of smart grid. We conclude the chapter in Section 7.4.

Motivating Example

The left table in Table 7.1 shows the electric appliances together with their corresponding labeled consumptions in watts for a fictitious household. For this particular example, to simplify our discussion, we make the following assumptions. The smart meter will send the consumption information once per 6 minutes (reading period), and the appliances will be on or off for a complete reading period. Further, the measured consumption of the appliances is consistent with the labeled one and the load type of the appliances are const load [60]. Later we will remove this assumption by allowing the appliances to consume in a given bounded range.

Appliance Set

Item	**Labeled** (Watts)
Fan	200
Bulb	100
TV	100

Possible Readings

Reading	**Use of Appliances**
40	{{Fan,Bulb,TV}}
30	{{Fan,Bulb}, {Fan,TV}}
20	{{Fan}, {Bulb,TV}}
10	{{Bulb}, {TV}}
0	$\{\emptyset\}$

Table 7.1: An Example

We first examine the privacy issue behind the readings. The right table in Table 7.1 shows all the possible readings and corresponding possible usage of appliances (note that the reading may be given by watts or kilowatt-hour. In this example, we use watt-hour for explanation). For example, when Fan is on, and either Bulb or TV is on, the reading is 30; when either Fan is on, or both Bulb and TV are on, the reading will be 20.

One existing solution is to add an amount of noise drawn from a geometric distribution with parameter $p = \frac{\epsilon}{\Delta}$ to the consumption in order to achieve ϵ-differential privacy for the readings, where Δ is the maximum difference of two readings of given household [32].

Unfortunately, achieving differential privacy in readings does not mean to preserve the privacy of the customers. For example, suppose that $\epsilon = 0.1$, a 20 reading implies that the probability that Fan is not used is as $3.90 \approx e^{1.36}$ times as that Fan is used.

The adversary can reason as follows. $\Delta = 40$ which is the reading difference between the case that all appliances are used and the case that no appliance is used, consequently, $p = \frac{0.1}{40} = \frac{1}{400}$. There are totally 5 possible cases which equally likely lead to a 20 noise reading as shown in Table 7.2. On one hand, the only case, that Fan is used, is that the original reading is 20 and the noise is 0. The probability of this case $Pr[\text{(Fan is on)} \wedge \text{(Reading is 20)}] = (1 - \frac{1}{400})^0 \times \frac{1}{400}$. On the other hand, there are four cases that Fan is not used. That is, none, one, or both of Bulb and TV is used and the noise is 20, 10, or 0, respectively. Therefore, when reading is 20, $\frac{Pr[\text{(Fan is off)}}{Pr[\text{(Fan is on)}}$=$\frac{\sum_{i=2}^{5} Pr[case_i]}{Pr[case_1]} \approx \frac{1+2\times 0.975+0.951}{1} = 3.90$.

	Use of Appliances			Original	Noise	Probability
	Fan	**Bulb**	**TV**	**Reading**	**Added**	$(1-p)^x \times p$
1	$\surd$			20	0	$(\frac{399}{400})^0 \times \frac{1}{400}$
2		$\surd$	$\surd$	20	0	$(\frac{399}{400})^0 \times \frac{1}{400}$
3		$\surd$		10	10	$(\frac{399}{400})^{10} \times \frac{1}{400}$
4			$\surd$	10	10	$(\frac{399}{400})^{10} \times \frac{1}{400}$
5				0	20	$(\frac{399}{400})^{20} \times \frac{1}{400}$

Table 7.2: The Possible Cases For a 200 Noise Reading

We have shown that, although the above mechanism achieves 0.1-differential privacy, the adversary still can learn that the Fan is more likely off in the case that the reading is 20. On the other hand, actually a special reading is not necessary to violate a given privacy in terms of the usages of the appliances.

We need to switch to the adversary's point of view. When an adversary observes a 20 reading, s/he knows that the customer either use the Fan or use both Bulb and TV, however, s/he is not sure that which option is. In other words, s/he can only know that these two options are equally likely true. However, when an adversary observes a 30 reading, s/he will be fully sure that Fan is used no matter whether Bulb or TV is used.

In this chapter, we will adopt a novel privacy property to quantify such privacy concerns. Intuitively, we must set the threshold of the maximal acceptable probability that any appliances in a given household are used in any given reading period. In this example, we assume that this probability is not less than $\frac{1}{2}$. Thus, the 20 reading is safe since all the three appliances have $\frac{1}{2}$ probability to be used. Moreover, the 30 reading is unsafe since Fan is used for sure. Similarly, reading 40 is unsafe while readings 10 and 0 are safe.

However, sometimes the cases that an appliance is not used will also release private information. Take an extreme case as an example. The reading 0 may infer that nobody is at home in that reading period. Therefore, we need to also set the threshold of the minimal acceptable probability that any appliances are used in any given reading period. In this example, we assume the minimal probability to be $\frac{1}{2}$ also. Considering both these two thresholds, the only safe reading is 20 (the probability that any of the three appliances is used is $\frac{1}{2}$).

Correspondingly, we will adopt a different approach to achieve the privacy. Basically, instead of adding noise to the reading such that the reading cannot be distinguished from others, we send the safe readings directly, while for the unsafe readings we send the most close safe readings (it could be larger or less than the original reading value) and leave the remainder (it could be positive or negative) to the next reading period. In such way, we will directly send 20 to the utility for a 20 reading while we will send 20 and leave the left 10 to a certain preassigned period when the consumption is 30. Therefore, for the reading sequence of consumption 30,20,10, we will send 20,20,20 to the utility. It is worthy noting that, when the adversary observes the third 20, s/he cannot infer that the original reading is 10 (either Bulb or TV is used) or 20 (either Fan is used or both Bulb and TV are used).

7.2 The Model

We first describe the adversary model and corresponding assumptions in Section 7.2.1. We then introduce the privacy property in Section 7.2.2. Finally, we define the cost metrics in Section 7.2.3. Table 7.3 lists our main notations which will be defined in this chapter.

A	Appliance Set
U	Utility, or Customer Set in Utility
G, G_s	Candidate Set, Safe Candidate Set
C_r	r-consumed Set (Consumption Set)
$\vec{R}$, $\vec{T}$	Reading Sequence, Tariff Sequence

Table 7.3: The Notation Table

7.2.1 Adversary Model

The adversary can eavesdrop the encrypted readings at any point in the traffic path between the smart meters and the utility. However, the adversary cannot break the security of a cipher unless the utility can decrypt it.

We have the following assumptions about the smart grid and the adversary's capability.

- The appliances will be on or off in the whole reading slot and will not switch between on and off during a reading slot.

- The households are aware of the appliances they will use upon the installations of the smart meters, and the maximal number of appliances in a household is not larger than 30.

- There exist some safe readings for the given set of appliances.

7.2.2 Privacy Property

For a given household h, we denote an *appliance set* as $A_h(id, l, d)$, where id, l, and d denote the identity of appliance unique to the household, its corresponding labeled electrical consumption in watts, and the possible bounded deviation from the labeled in percentage for the real consumption, respectively. We denote *reading frequency* as ϕ to represent the number of hours between two readings. Denote by ID and L the set of appliance identities $\prod_{id}(A_h)$, and labeled electrical consumption $\prod_{l}(A_h)$ (all projections preserve duplicates, unless explicitly stated otherwise). Note that different households may have different appliance set. When no ambiguity is possible, we will not specify the subscript for household, and will not distinguish between A and L.

First, we introduce the concept of *candidate set* and *r-consumed set* in Definition 7.1 to depict all the possible reading values for the given appliance set, and all the possible combination of appliances which can sum up to r watts.

Definition 7.1 (Candidate Set and Consumption Set) *Given an appliance set A, we define,*

- *the candidate set G as the set* $\{\cup\{sum : sum \in [\sum_{(id,l,d)\in S_A}(l\times(1-d)), \sum_{(id,l,d)\in S_A}(l\times(1+d))]\} : S_A \in 2^A\}$;
- *the r-consumed set C_r as the collection of sets* $\{\{id : (id,l) \in S_A\} : (S_A \in 2^A) \wedge (\sum_{(id,l,d)\in S_A}(l\times(1-d)) \leq r \leq \sum_{(id,l,d)\in S_A}(l\times(1+d)))\}$.

Example 7.1 *Given the appliance set A in the left tabular of Table 7.1, and assume that the deviation for each appliance is 0.0, then the candidate set $G = \{0, 100, 200, 300, 400\}$. Correspondingly, there are 5 consumption sets as shown in right tabular of Table 7.1, e.g., $C_{200} = \{\{Fan\}, \{Bulb, TV\}\}$. Note that reading 20 corresponds to C_{200} since* $20(\frac{watts}{6minutes}) \times 10(\frac{6minutes}{hour}) = 200(\frac{watts}{hour})$. ⊡

We then measure the probability that an appliance is used for a given reading r, as formalized in Definition 7.2.

Definition 7.2 (Occurrence Probability) *Given a r-consumed set C_r corresponding to the appliance set A, the* occurrence probability *of an appliance $id \in ID$ w.r.t. C_r is defined as*

$$pr(id, C_r) = \frac{|\{I : (id \in I) \land (I \in C_r)\}|}{|C_r|}$$

Example 7.2 *Following Example 7.1, $pr(Fan, C_{200}) = \frac{1}{2}$ since Fan appears in one of the two elements in C_{200}. Similarly, $pr(Bulb, C_{200}) = pr(TV, C_{200}) = \frac{1}{2}$.* $\boxdot$

Based on the *occurrence probability*, Definition 7.3 quantifies the amount of privacy protection under a given r-consumed set. Basically, a reading r satisfies (δ_1, δ_2)-bounded certainty, if the occurrence probability of each appliance in the corresponding r-consumed set falls in the range of $[\delta_1, \delta_2]$.

Definition 7.3 ((δ_1, δ_2)-Bounded Certainty) *Given an appliance set A and the corresponding candidate set G, we say a r-consumed set C_r ($r \in G$) satisfies (δ_1, δ_2)-bounded certainty ($0 \le \delta_1 \le \delta_2 \le 1$) or C_r is (δ_1, δ_2)-bounded w.r.t. A if*

$$\forall (id \in ID), \delta_1 \le pr(id, C_r) \le \delta_2.$$

Example 7.3 *Following Example 7.2, C_{200} w.r.t. A satisfies ($\frac{1}{2}, \frac{1}{2}$)-bounded certainty (or simply as 200 is ($\frac{1}{2}, \frac{1}{2}$)-bounded hereafter when no ambiguity is possible) since, for all appliances in A, the occurrence probability equals to $\frac{1}{2}$.* $\boxdot$

Finally, we model the privacy of an algorithm for a sequence of readings in Definition 7.4. Informally, the privacy model requires at least a pre-configured percentage of readings in the sequence is (δ_1, δ_2)-bounded.

Definition 7.4 (($\alpha, \delta_1, \delta_2$)-Undisclosed Privacy) *An algorithm $\mathcal{M}$ gives $(\alpha, \delta_1, \delta_2)$-undisclosed privacy for a reading sequence $\vec{R_{in}}$ (one record per ϕ hours) w.r.t. an appliance set A, if the output, another equal-length reading sequence, $\vec{R_{out}} = \mathcal{M}(\vec{R_{in}}, A)$ satisfies that*

$$1 - \frac{\left|\{r : r \in \vec{R_{out}} \wedge C_{\frac{r}{\phi}} \text{ is } (\delta_1, \delta_2) - bounded\}\right|}{\left|\vec{R_{out}}\right|} \leq \alpha$$

7.2.3 Cost Metrics

In addition to privacy requirements, we also need metrics for the billing accuracy for each user, the consumption accuracies for each user and for the utility. For the billing accuracy, we measure the billing difference in total for the given period, as formulated in Definition 7.5.

Definition 7.5 (Billing Error Rate) *Given an input reading sequence $\vec{R_{in}}$ for a customer u, the corresponding output reading sequence $\vec{R_{out}}$ and equal-length tariff sequence $\vec{T}$, the billing error rate of u is defined as*

$$err_b(u, \vec{R_{in}}, \vec{R_{out}}, \vec{T}) = \frac{|(\vec{R_{out}} - \vec{R_{in}}) \cdot \vec{T}|}{\vec{R_{in}} \cdot \vec{T}},$$

where $\cdot$ represents the dot product of two vectors.

Definition 7.6 measures the relative error rate of the readings for a customer in a given sequence.

Definition 7.6 (Customer Consumption Error Rate) *Given an input reading sequence $\vec{R_{in}}$ for a customer u, the corresponding output reading sequence $\vec{R_{out}}$, the customer consumption error rate of u is defined as*

$$err_c(u, \vec{R_{in}}, \vec{R_{out}}) = \frac{\sum_{i \in [1, |\vec{R_{in}}|]} |\vec{R_{out}}[i] - \vec{R_{in}}[i]|}{\sum_{i \in [1, |\vec{R_{in}}|]} \vec{R_{in}}[i]},$$

where $\vec{R_{in}}[i]$ and $\vec{R_{out}}[i]$ are the i^{th} reading in $\vec{R_{in}}$ and $\vec{R_{out}}$, respectively.

Example 7.4 *Following Example 7.3, given the input reading sequence $\vec{R_{in}} = \langle 30, 20, 10 \rangle$, the output reading sequence $\vec{R_{out}} = \langle 20, 20, 20 \rangle$ for the customer u in Table 7.1, suppose that the corresponding tariff sequence $\vec{T} = \langle 1, 2, 1 \rangle$, then the billing error rate $err_b(u, \vec{R_{in}}, \vec{R_{out}}, \vec{T})$ (or simply err_b hereafter when no ambiguity is possible) can be calculated as:*

$$err_b = \frac{(20-30)\times 1+(20-20)\times 2+(20-10)\times 1}{30\times 1+20\times 2+10\times 1} = 0.$$

Similarly, the customer consumption error rate is as follows,

$$err_c = \frac{|30-20|+|20-20|+|10-20|}{30+20+10} = \frac{20}{60} = \frac{1}{3}.$$ ⊡

Definition 7.7 (Utility Consumption Error Rate) *Given the utility $U = \{u_1, u_2, \ldots, u_{|U|}\}$, in which, each customer u_m has the corresponding input reading sequence $\vec{R^m_{in}}$ and output reading sequence $\vec{R^m_{out}}$, we define*

- *the utility consumption error rate of the i^{th} reading is defined as*

$$err(i, U) = \frac{|\sum_{u_m \in U}(\vec{R^m_{out}}[i] - \vec{R^m_{in}}[i])|}{\sum_{u_m \in U}(\vec{R^m_{in}}[i])},$$

- *the utility consumption error rate of the reading sequence is defined as*

$$err(U) = \frac{\sum_{i\in[1,|\vec{R^m_{in}}|]} err(i, U)}{|\vec{R^m_{in}}|}.$$

Example 7.5 *Following Example 7.4, assume that there is only another customer in the utility whose $\vec{R_{in}} = \langle 10, 30, 20 \rangle$, and $\vec{R_{out}} = \langle 20, 20, 20 \rangle$, then the error rate of first reading slot $err(1, U) = \frac{|(20-30)+(20-10)|}{30+30} = 0$. Similarly, $err(2, U) = \frac{1}{5}$ and $err(3, U) = \frac{1}{3}$. Thus, $err(U) = \frac{0+\frac{1}{5}+\frac{1}{3}}{3} = \frac{8}{45}$.* ⊡

7.3 The Algorithms

In this section, we design algorithms for revising the readings to satisfy given (δ_1, δ_2)-bounded certainty whereas minimize the aggregating error rate and billing error rate. Our intention is not to design an exhaustive list of solutions but rather to demonstrate the existence of abundant possibilities in ensuring such privacy property with negligible communication and computation overheads.

Remind that the smart grid makes use of the fine-grained readings. Without loss of generality, we split the consumption information into $K{\times}S$ readings as shown in Figure 7.1.

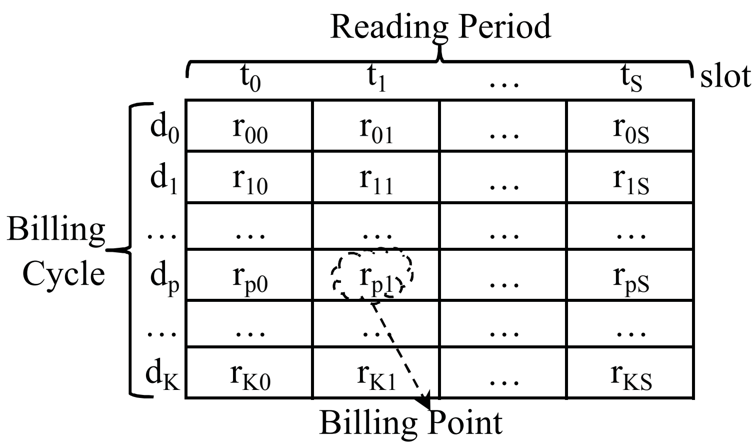


Figure 7.1: Sequence of Meter Readings For a Smart Meter

Basically, for a customer, a *reading period* (e.g. a day or a week) is split into S number of *reading slot*s such that any two neighbor reading slots have different tariff. A *billing cycle* (e.g. a month or a year) includes K number of days which is the period of time between billing statements as usual. In the *billing cycle* we preassign a special day, called *billing point*, for each reading slot. Correspondingly, we regard the reading sequence as a $K{\times}S$ reading matrix in our algorithms. Note that the subscripts in the matrix is just to show the position of a reading in the billing cycle for different tariff and aggregated consumption, and in our algorithm we handle each reading in real time.

The process in smart meter is divided into three steps. First, initialize the smart meter to determine the safe readings (one-time process). Second, modify the reading such that the resultant reading satisfies desired privacy property. Third, send the resultant reading to the utility in a secure way.

7.3.1 Smart Meter Initialization

Before modifying the readings, we should first tell apart the readings which satisfies given (δ_1, δ_2)-bounded certainty, called *Safe Candidate Set*, from all possible readings given the appliance set, called *Candidate Set*. In other words, we should identify all the possible (δ_1, δ_2)-bounded readings.

Note that building safe candidate set is part of the one-time off-line processing given the appliance set. In this section, we devise a very straightforward yet not necessarily efficient way to build it as shown in Table 7.4. Nonetheless, the discussion of optimal solution is regarded as future work.

Input: an appliance set A, the certainty property δ_1, δ_2,
    the reading frequency ϕ;
Output: the safe candidate set G_s;
Method:
1. **Let** each consumption set $C_r = \emptyset$;
2. **For** each S_A in 2^A
3.   **Let** $r = \sum_{(id,l) \in S_A}(l)$;
4.   Create S_A on C_r;
5. **For** each C_r
6.   **If** (C_r is (δ_1, δ_2)-bounded)
7.   Create r on G_s;
8. **Return** $G_s \times \phi$;

Table 7.4: The Safe Candidate Producer (SCP)

Roughly speaking, given the appliance set A, the SCP algorithm examines each possible appliance usage situations and corresponding electric power consumptions (lines 2-4), and then for each possible consumption value r, the algorithm justifies whether it is (δ_1, δ_2)-bounded by verifying the corresponding r-consumed set (lines 5-7 in Table 7.4).

The computational complexity of SCP algorithm is $O(|A| \times 2^{|A|})$ since: evaluating r

for each S_A costs $O(|A|)$ many times, and there are $2^{|A|}$ many S_As, so totally $O(|A| \times 2^{|A|})$. Then, evaluating each C_r also costs $O(|A|)$ many times and there are maximal $2^{|A|}$ many C_rs.

Besides the safe candidate set, we should also randomly select a special day for each reading slot, called *Building Point Set*. Note that, although for a single customer, the building points are randomly selected, we should select the building point of any reading slot for all the customers evenly distributes among the K days to reduce the errors of the aggregated consumption.

7.3.2 Reading Modifications

With the information of safe candidate set, we next modify the reading to satisfy (δ_1, δ_2)-bounded certainty. It is important to note that we may vary the design of algorithms to suit different needs of smart meter capabilities, and the privacy and accuracy requirements. We demonstrate such a flexibility through designing three light-weight algorithms in the following.

7.3.2.1 Option 1: the CRC algorithm

The cyclical reading converter (CRC) algorithm aims at maximal billing accuracy with reasonable utility consumption error as shown in Table 7.5. Roughly speaking, given a reading r_{ij} which is at the i^{th} reading period in the billing cycle and the j^{th} reading slot in that reading period, CRC first justifies whether the reading is on the billing point of j^{th} reading slot. If yes, CRC will sum up the current reading to the accumulated hold of j^{th} reading slot, reset the hold to be zero, and return the summation. Otherwise, the algorithm returns the closest value in G_s (the one has minimal absolute difference between it and the reading), and adds the difference to the hold of the j^{th} reading slot.

The CRC algorithm achieves 0% billing error rate (100% billing accuracy) without the knowledge of the detail tariff information while ensures $(\frac{1}{K}, \delta_1, \delta_2)$-undisclosed

Input: a reading r_{ij}, the safe candidate set G_s,
the billing point set $B[S]$, the hold set $H[S]$;
Output: the resultant reading r';
Method:
1. **If** (i=B[j]) //the billing point
2. **Let** $r' = H[j] + r_{ij}$;
3. **Let** $H[j] = 0$;
4. **Return** r';
5. **If** $(r_{ij} \in G_s)$
6. **Return** r_{ij};
7. **Let** $closest$ be $r \in G_s$ with the minimal $|r - r_{ij}|$ value;
8. $H[j] += (closest - r_{ij})$;
9. **Return** $closest$;

Table 7.5: The Cyclical Reading Converter (CRC)

privacy due to the following. First, the accumulated consumptions of any reading slots for any customer in any billing cycle is identical before and after modification. Formally, $\forall j, \sum_i (R_{out}[i][j] - R_{in}[i][j]) = 0$ Thus, the billing of R_{out} is identical with that of R_{in} . Second, there are at most S number of readings, which may be not (δ_1, δ_2)-bounded, among the total $S \times K$ readings.

7.3.2.2 Option 2: the PRC algorithm

The perpetual reading converter (PRC) algorithm aims at strict privacy preservation with reasonable billing error rate and utility consumption error rate as shown in Table 7.6. Compared with CRC algorithm, the hold of each reading slot in PRC algorithm will never be reset. Roughly speaking, if a reading r_{ij} is safe (in the safe candidate set), PRC returns that reading directly, otherwise, PRC returns the closest value in G_s (the one has minimal absolute difference between it and the reading), and adds the difference to the hold of the j^{th} reading slot.

The PRC algorithm ensures $(0, \delta_1, \delta_2)$-undisclosed privacy with reasonable billing error rate and utility consumption error rate. First, the algorithm ensures that each returned reading satisfies pre-determined (δ_1, δ_2)-bounded certainty. Second, the hold in each read-

Input: a reading r_{ij}, safe candidate set G_s, hold set $H[S]$;	
Output: the resultant reading r';	
Method:	
1.	**If** $(r_{ij} \in G_s)$
2.	**Return** r_{ij};
3.	**Let** $closest$ be $r \in G_s$ with the minimal $\lvert r - r_{ij} \rvert$ value;
4.	$H[j] += (closest - r_{ij})$;
5.	**Return** $closest$;

Table 7.6: The Perpetual Reading Converter (PRC)

ing slot is the only reason to cause the billing and consumption inaccurate, which can be deemed as negligible for negligible for a long-term period in most cases.

7.3.2.3 Option 3: the LRC algorithm

For both CRC and PRC algorithms, we assume that the smart meters are aware of the existence of different tariffs (note that such knowledge is different from knowing the different tariffs themselves and the smart meters using our algorithms do not need to know the tariffs). The parameters of those algorithms may need to be updated due to the change of the tariff structure. Our intention of designing LRC algorithm is to demonstrate the possibilities of totally tariff structure irrelevant solutions.

Input: a reading r, safe candidate set G_s;	
Output: the resultant reading r';	
Method:	
1.	**Let** $h = h + r$;
2.	**Let** $closest$ be $r_c \in G_s$ with the minimal $\lvert h - r_c \rvert$ value;
3.	**Let** $h = h - closest$;
4.	**Return** $closest$;

Table 7.7: The Light Reading Converter (LRC)

In contrast to previous two algorithms, all reading slots share a single hold to store the difference between original reading and returned reading in LRC algorithm as shown in Table 7.7. Roughly speaking, LRC first sums up current reading with the accumulated hold, then returns the closest value in G_s, and finally records the difference in the hold for the next readings.

7.3.3 Implementation Issues

In previous sections, we have presented a vital component of handling the information with regard to electric consumption exchanged between utility and customers. That is, how the smart meters can modify the reading information to ensure the privacy property while maximizing the billing accuracy and minimizing the consumption error.

To incorporate our techniques into the smart grid also requires ensure the integrity, non-reputation when sending the consumption information from smart meter to the utility. These objectives can be achieved by utilizing the existing solutions in the literature, such as, TPM, encryption, and so on. In this chapter, we specially focus on the process in smart meter to modify the information before communicating with the counterpart in the system.

7.4 Summary

In this chapter, we have proposed a novel light-weight approach to concurrently achieve two privacy objectives (billing and load monitoring) through a single set of data and fully under control of the smart meters. Based on the semantic explanation of privacies, we have presented a formal model for privacies in smart grid. We have also designed three efficient algorithms for reading modification and outlined implementation issue for our approach. We will continue this preliminary work with experiments using both real and synthetic data to confirm the effectiveness and efficiency of our solutions.

Chapter 8

Generic Model for Privacy Preserving against Side-Channel Leaks

In this chapter, we outline the preliminary work on generic model for privacy-preserving against side-channel leakages. We then demonstrate three example instantiations of the generic model through aforementioned three different scenarios.

As discussed in previous chapters, we apply the following similar idea for the aforementioned applications: we divide all the possible information into groups and then break the linkage inside each group by obfuscating the observable information, such as, encrypted packet sizes in web applications, smart reading for smart meters. Based on the similarity of the components and solutions, we can extract the common information from different domains and design a generic model over the models for specific applications.

8.1 Outline of Generic Model

8.1.1 Privacy-related Components of an Application

To prevent a side-channel attack, we need to first answer the following three questions: Who is the victim? What is the sensitive information? What is the side-channel information? Correspondingly, we need the following three concepts to model these three pieces of information. Note that, we preserve duplicates for all the sets in the remainder of this chapter, unless explicitly stated otherwise.

- *Identity Set:* Denote by I the set of identity of each victim which can be used to uniquely identify a victim. Note that the domain of identity set varies from application to application and side-channel to side-channel. For example, it could be the identifier of a record holder in a micro-data, such as, social insurance number, drive license number.

  Furthermore, the identities could be either permanent or temporary. For instance, a session ID between a client and the Web application may expire or be abandoned. However, it is typically assumed that this session information, together with additional information such as IP address and the access time of the client, enable an adversary to associate it with a victim.

- *Sensitive Set*: Denote by S the set of sensitive person-specific information about the victim for the given application. In contrast to confidentiality, for privacy, sensitive set itself alone can be not private. In other words, knowing the possible sensitive values does not mean to violate the privacy. However, the linkage between an identity and the sensitive values is regarded as private.

  For instance, the disease information in a medical record for a patient is considered as sensitive information. Note that the sensitive information may be legible in some scenarios and be illegible in others, specially in high-dimensional micro-data. Also, it varies from applications whether a piece of information with which an identity is associated is sensitive or not.

- *Observable Set*: Denote by O the set of observable information exposed due to the side-channel leakages. Such information is usually different from the sensitive information which need to be protected. However, it can allow the adversary to refine her knowledge about the possibility of an identity being associated with a sensitive value.

For example, the directional packet sizes in the encrypted traffic between victims and Web-based applications are visible to the adversaries. Different inputs or actions in a Web application may lead to different patterns of such observable information which allows adversaries to infer the victim's actions.

With these three concepts, we call the victims' information in a given scenario *Original Set*, denoted as a relation $t(i, o, s)$, where $i \in I$, $o \in O$, and $s \in S$ denote the *identity value*, *observable value*, and *sensitive value*, respectively. Denote by T the set of all relations with the same sets of I, O, and S as those of t.

We make the worst case assumption that each victim can be uniquely associated with an observable value. For example, An adversary knows the quasi-identifier of a victim (identity) in a micro-data through extra knowledge such as voter list. Note that such association could be time-sensitive. An adversary knows who triggers the traffic between client and web application through de-anonymizing techniques [95] and the methods discussed in Chapter 5 for a specially given time.

Therefore, we need to remain $\Pi_{i,s}(t)$ and $\Pi_{o,s}(t)$ secret to protect the privacy. There exist two seemingly conflicting goals. First, the sensitive information about an identity must be limited to a given acceptable level to preserve the privacy, such as k-anonymity, l-diversity, ϵ-differential privacy. Second, the costs to achieve the desired privacy should be minimized. For example, the data utility for analysis or information loss in privacy-preserving data publishing should be maximized or minimized; the padding overhead in privacy-preserving traffic padding in Web application should be minimized.

There exist many methods to protect the sensitive information as well as minimize the costs as discussed in Chapter 2. In the context of privacy preserving data publishing, *grouping-and-breaking* basically partitions the records into groups and then breaks the linkage between the quasi-identifier value and the sensitive value inside each group. In this generic model, we extend such operation in PPDP to other applications. However,the way to divide identities' records into groups and the way to break the linkage inside a group may be different. For example, for data publishing, we replace the quasi-identifier values by a less accurate one; for traffic padding in Web applications, we increase the packet sizes to a closer value; for privacy-preserving smart metering, we replace the readings by the closest safe one. We shall discuss the differences in more detail in the next section.

8.1.2 Privacy Properties

As mentioned above, privacy concerns that the degree about how likely an identity is associated with a sensitive value from the adversary's perspective. Therefore, we need to first formulate the adversary's mental image given the finally released information (see next subsection 8.1.3) and the side-channel information, such as, the algorithms themselves, the observable encrypted packet sizes. We slightly abuse the concept of fuzzy set to model the adversary's mental image.

The mental image of an adversary about the sensitive information of an identity in an application is denoted by a pair $(i, (S, f_{iS}))$, where i is the identity, (S, f_{iS}) is a corresponding fuzzy set denoting the probability that the given identity is associated with each sensitive value from the adversary's perspective. Obviously, the values of member function $f_{iS} : S \to [0, 1]$ could be different for different identity. We call the collection of all the $(i, (S, f_{iS}))$ pairs for the identities in an application *Inferred Set*, denoted as $r_I = (I, (S, f_{IS}))$. Different mechanisms may lead to different *Inferred Set*. Denote by $R_I(t)$ the set of all possible inferred sets for a given original set t, and short as R_I, if no ambiguity is possible.

The concept of *Inferred Set* is generic enough to model different syntactic privacy properties. Furthermore, *Inferred Set* simulates the view of an adversary instead of the released information itself. To illustrate, we model two main privacy properties mentioned in previous chapters in Definition 8.1 and 8.2.

Definition 8.1 (k-anonymity) *Given an inferred set $r_I \in R$, we say a r_I satisfies k-anonymity (k is an integer) if*

$$\forall((i,(S,f_{iS})) \in r_I), |\{s \in S : f_{iS}(s) > 0\}| \geq k.$$

Definition 8.2 (l-diversity) *Given an inferred set $r_I \in R$, we say a r_I satisfies l-diversity if*

$$\forall((i,(S,f_{iS})) \in r_I), \max_{s \in S}(f_{iS}(s)) \leq \frac{1}{l}.$$

8.1.3 Cost Metrics

In addition to privacy requirements, we also need metrics for the costs, such as information loss, padding cost, processing cost, reading error rate, billing error rate, and so on. The costs mainly incur due to the difference between the original information and released information. Similar with *inferred set*, we also model the released information based on the concept of fuzzy set.

The sensitive information of an identity of an application based on the released information is denoted by a pair $(o,(S,f'_{oS}))$, where o is the observable information, (S,f'_{oS}) is a fuzzy set denoting the probabilistic distribution of sensitive information corresponding to a given possible observable information. We call the collection of all the $(o,(S,f'_{oS}))$ pairs in an application *Released Set*, denoted as $r_D = (O,(S,f'_{OS}))$. Similarly, different mechanisms may lead to different *Released Sets*. Denote by $R_D(t)$ the set of all possible released sets for a given original set t, and short as R_D if no ambiguity is possible. Note that, the *Original Set* $t(i,o,s)$ can be also represented by a release set by removing identity information i.

These concepts, together with some necessary information such as fine-grained tariff for smart metering, allow us to model different costs. To illustrate, we exemplify the discernibility measure (DM) for data publishing in Definition 8.3 and customer consumption error rate for smart metering in Definition 8.4.

Definition 8.3 (Discernibility Measure (DM)) *Given a released set* $r_D = (O, (S, f'_{OS}))$ *for a privacy-preserving data publishing, the discernibility measure is formulated as follows:*

$$DM(r_D) = \sum_{o \in O} \left| \{o' : (o' \in O) \wedge (o' = o)\} \right| .$$

Definition 8.4 (Customer Consumption Error Rate) *Given a customer* i*, an original set* $t(i, o, s)$ *(a reading sequence of that customer) and its corresponding released set* $r_D(O, (S, f'_{OS}))$ *for a smart meter, the customer consumption error rate is formulated as follows:*

$$err_c(r_D, t) = \frac{\sum |t.o - r_D.o|}{\sum t.o}.$$

8.1.4 Obfuscating Mechanisms

Previous two sections model the privacy properties and the corresponding cost metrics introduced to satisfy the privacy. In this section, we formulate the effect of the mechanisms on the privacy degree and the overheads.

In addition to the released set, the adversary may also have some extra knowledge, denote as E, such as the generalization algorithms, the observable encrypted packet size, and so on. The effectiveness of mechanism $\mathcal{M}$ in terms of adversaries' mental image can now be formulated as $\mathcal{M} : T \times E \to R_D \times R_I$. A mechanism may have different methods to affect the adversary's inferred set and/or the released set, either by obfuscating the relation between I and O, or by obfuscating the relation between O and S, or by both. To facilitate the computation of the privacy guarantee and costs, we define an operation, called concatenation and denoted by $\odot$, for two fuzzy sets given the corresponding three sets as follows.

Definition 8.5 (Concatenation) *Given three sets* I*,* O *and* S*, the concatenation between* $(I \times O, f_{IO})$ *and* $(O \times S, f_{OS})$ *is defined as* $(I \times O, f_{IO}) \odot (O \times S, f_{OS}) = (I \times S, f_{IS})$, *where f_{IS} is calculated as follows.*

$$(\forall i \in I)(\forall s \in S) : f_{is} = \sum_{o \in O} (f_{io} \times f_{os})$$

Note that, $(I \times S, f_{IS})$ is a variant of *inferred set*, and the former facilitates the computation of the latter. The concrete mechanisms must be customized for applications.

8.2 Instantiations of Generic Model

Table 8.1 shows the mapping between the generic model and the models in previously chapters. We have customized the notions and notations in the three scenarios for the purpose of explanation in Chapters 3- 7. In this section, we briefly discuss the main challenges for the aforementioned three applications.

	PPDP	**PPTP**	**PPSM**
identity set	identifier	(session id)	(household)
observable set	quasi-identifier	vector-set	candidate set
sensitive set	sensitive attribute	action-set	consumption set

Table 8.1: Customized Notions in Three Scenarios

8.2.1 Privacy-Preserving Data Publishing

The main challenge for privacy-preserving data publishing is that the adversary may be able to further infer the sensitive information when she knows the generalization algorithm itself. The inferred set of the adversary is continuously refined during the running of the algorithm.

Table 8.2 shows a toy example of micro-data table $t(Name, gender, condition)$ (original set) to be released. The privacy objective is to ensure that the highest ratio of a *sensitive value* condition for each *identifier* Name must be no greater than $\frac{2}{3}$. When the adversary only knows the released set $g_1(t)$, the desired privacy property is satisfied.

A Micro-Data Table t

Identity (Name)	**Observable** (Gender)	**Sensitive** (Condition)
Ada	Female	flu
Bob	Male	HIV
Coy	Female	cold
Dan	Male	HIV
Eve	Female	cough

Generalization $g_1(t)$

Observable (Gender)	**Sensitive** (Condition)
person	flu
	HIV
	cold
	HIV
	cough

Table 8.2: A Micro-Data Table and its Generalization

However, assume that the adversary knows about the generalization algorithm works as follows: the algorithm releases $\prod(gender, condition)$, if it satisfies the privacy property; Otherwise, it further replaces the *gender* by person, then either releases it if it satisfies, or release nothing if it does not. Based on the released set, the adversary can reason that $\prod(gender, condition)$ will not be disclosed only if both males are associated with HIV, which violates the privacy. In such case, the inferred set is different from the released set.

Therefore, we need to evaluate the desired privacy property on the inferred set instead of the released set. However, the recursive nature of computing the inferred set is deemed to incur a high complexity. To avoid such recursion, we decouple the two processes, privacy preservation and utility optimization, to improve the efficiency.

8.2.2 Privacy-Preserving Traffic Padding

In privacy-preserving traffic padding against side-channel due to encrypted packet sizes, the side-channel information is modeled as observable set. We assume that the adversary can locate the traffic between a victim and the web server. A straightforward solution is to obfuscate the observable information by padding to the maximal in the group. However, the correlation among the observable information in the sequence of actions may cause additional complications.

The left tubular of Table 8.3 shows a toy example of the auto-suggestion feature in Web-based application (suppose that it shows all the possible inputs). The privacy objective in this example is to ensure that the adversary cannot distinguish from at least 2 different inputs when observing the encrypted packet sizes.

Original Set		Released Set	
Observable (s-byte)	**Sensitive** (user input)	**Observable** (s-byte)	**Sensitive** (user input)
$50 \rightarrow 75 \rightarrow 65$	$b \rightarrow u \rightarrow s$	$60 \rightarrow 75 \rightarrow 70$	$b \rightarrow u \rightarrow s$
$60 \rightarrow 55 \rightarrow 70$	$c \rightarrow a \rightarrow r$	$60 \rightarrow 75 \rightarrow 70$	$c \rightarrow a \rightarrow r$
$60 \rightarrow 55 \rightarrow 65$	$c \rightarrow a \rightarrow t$	$70 \rightarrow 55 \rightarrow 80$	$c \rightarrow a \rightarrow t$
$70 \rightarrow 55 \rightarrow 80$	$d \rightarrow o \rightarrow g$	$70 \rightarrow 55 \rightarrow 80$	$d \rightarrow o \rightarrow g$

Table 8.3: Original Set and Possible Released Set for an Action-Sequences

When an adversary only observes the packet size of second keystroke is 55, she can only infer that 'car', 'cat', and 'dog' are equally likely to be the real input. However, when she also observes the packet size of the first keystroke is 70, she can conclude that the input is 'dog', which violates the privacy requirement.

The aforementioned grouping and breaking techniques alone may lead to the released set as shown in right tabular of Table 8.3. That is, one algorithm may split cat and car into two different groups. Unfortunately, this released set cannot be used in real case. When the web server receives first keystroke 'c', it must immediately response due to auto-suggestion feature. However, since the server cannot predict the input is 'car' or 'cat', it cannot decide whether to remain 'c' to 60 or pad it to '70'. Therefore, we need to apply further constraints when splitting the inputs into groups as discussed in Chapter 5.

8.2.3 Privacy-Preserving Smart Metering

In privacy-preserving smart metering against side-channel due to fine-grained readings, the side-channel information is also modeled as observable set. A straightforward solution is to obfuscate the observable information by replacing the unsafe readings to the closest safe readings. However, such mechanism may not ensure the privacy requirements.

The left tubular of Table 8.4 shows a toy example of all the possible readings and the corresponding usage of appliances for a household. The privacy objective is to ensure

Original Set

Observable (reading)	**Sensitive** (usage of appliances)
400	$\{\{$Fan, Bulb, TV$\}\}$
300	$\{\{$Fan, Bulb$\}$,$\{$Fan,TV$\}\}$
200	$\{\{$Fan$\}$, $\{$Bulb, TV$\}\}$
100	$\{\{$Bulb$\}$,$\{$TV$\}\}$
0	$\{\emptyset\}$

Released Set

Observable (reading)	**Sensitive** (usage of appliances)
200	$\{\{$Fan, Bulb, TV$\}$, $\{$Fan, Bulb$\}$,$\{$Fan,TV$\}$, $\{$Fan$\}$, $\{$Bulb, TV$\}\}$
100	$\{\{$Bulb$\}$,$\{$TV$\}\}$
0	$\{\emptyset\}$

Table 8.4: Original Set and Possible Released Set for the Readings in a Household

that the probability that any appliance is used inferred by any reading is no greater than $\frac{1}{2}$. Obviously, the readings 200 is safe since all the three appliances have $\frac{1}{2}$ probability to be used. Similarly, readings 100 and 0 are safe, while 300 and 400 are not safe. The released set will be as shown in right tubular of Table 8.4 by replacing a unsafe reading to the closest one. Obviously, when an adversary observes a reading is 200, she can infer that Fan has $\frac{4}{5}$ probability to be used, which violates the privacy. Besides, it usually incurs a high computational complexity to enumerate all the possible usage of appliances. Therefore, it is a must to design efficient heuristic methods to ensure the privacy as well as minimize the billing and consuming error rate.

8.2.4 Others

Our generic model can also be applied to other categories of domains, such as, android applications, cryptography, and so on. For example,

- The data-usage statistical information can be modeled as the observable information in our model and apply grouping-and-breaking technique to make user's identity indistinguishable by observing the statistics for android system.
- The lengths of speech for voice guidance in Google Navigator for smart phone can be partitioned into different groups, and unified inside each group such that a route cannot be distinguished from sufficient other routes.

- We may consider the execution time of a cipher as the observable information and obfuscate it of one secret-key to be identical with sufficient large number of other keys of the cipher.

Chapter 9

Conclusion and Future Direction

9.1 Conclusion

As technology has advanced, new applications and products have emerged endlessly. Willingly or unwillingly, more and more information was spread out globally and rapidly. The privacy preserving issues are becoming increasingly severe and accordingly receiving significant attentions.

In this thesis, we studied privacy preservations against different types of side-channel leakages in different scenarios: publicly-known algorithms in data publishing (Chapter 3 and Chapter 4), observable encrypted traffic information in web applications (Chapter 5 and Chapter 6), and fine-grained reading in smart metering (Chapter 7). We then made the first effort on extracting a general framework to model side-channel attacks across different domains (Chapter 8). The main works throughout this thesis can be summarized as follows.

For data publishing, we have proposed a novel k-jump strategy for micro-data disclosure. This strategy ensures the data privacy even in the case that the adversaries know the disclosure algorithms. We have shown how to transform a given unsafe generalization algorithm into a large number of safe algorithms. By constructing counter-examples, we have shown that the data utility of such algorithms is generally incomparable.

To improve the efficiency, we have further proposed streamliner approach to preserving diversity for data publishing. Instead of sequentially evaluating generalization functions in a given order, and disclosing the first safe generalization, this strategy decouples the process of preserving the diversity from the process of optimizing the data utility, and consequently reduces the computation complexity.

For Web applications, we have established a mapping between the privacy-preserving traffic padding (PPTP) and privacy-preserving data publishing (PPDP) issues, which allows reusing many existing models and methods in PPDP as potential solutions for PPTP problems. We have also designed a formal model for the PPTP issue based on the mapping, which allows quantifying privacy properties and padding overheads.

To relax the assumption on the adversaries' prior knowledge about user input, we have further proposed random ceiling padding approach to providing background knowledge-resistant privacy guarantee to Web applications. Through our solution, the adversary would still face sufficient uncertainty even if s/he can exclude certain number of possible inputs to refine his/her guesses of the true input.

For smart metering, we have proposed a light-weight approach to simultaneously achieving the objectives of preserving privacy on both billing and consumption aggregation based on the key observation about the privacy issue beyond the fine-grained readings. Our solution precedes existing ones by efficiently realizing multiple privacy objectives.

Finally, we formulate a generic model for privacy preserving against side-channel leaks. The model encompasses privacy requirements, overheads, and methods to ensure privacy and minimize the overheads. Such a study will bridge the gap among different communities on study of side-channel attacks.

9.2 Future work

In the near future, I plan to focus on conducting the following studies. First, we will further study the side-channel attacks in different applications, extract their commonalities, and complete the generic model for privacy preserving against side-channel leakages. Second, we will propose a privacy-preserving querying system to allow users to request for desired micro-data through specially designed queries. Third, we will study data disclosure and its safety issue in different settings, such as, cloud computing, big data.

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