Aggregate Production Planning with Uncertainty for Product Recovery and Remanufacturing

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A Thesis

in

The Department

of

Mechanical and Industrial Engineering

Presented in Partial Fulfillment of the Requirements

for the Degree of Master of Applied Science (Industrial Engineering) at

Concordia University

Montreal, Quebec, Canada

August 2010

Your file Votre référence
ISBN: 978-0-494-71001-2
Our file Notre référence
ISBN: 978-0-494-71001-2

Abstract

In this research, we study product recovery and remanufacturing systems in uncertain environments. A mixed integer mathematical model for aggregate production planning is proposed. The objective is to minimize the total costs of recovering and remanufacturing end-of-life (EOL) products. Parameters such as functional costs and demands are considered uncertain. The model can be taken as a decision support tool by remanufacturing managers and to develop formal production planning and product recovery systems. The solution methodology based on possibilistic programming in Liang (2007) is applied to solve the model. The main advantage of this solution method is that it allows the decision maker to modify the parameters until a satisfactory solution is obtained. A numerical example problem is carried out to test the mathematical model with analysis of the computational results.

Keywords: *Remanufacturing systems; Possibilistic linear programming; production planning.*

Acknowledgements

I would like to thank my supervisor Dr.Mingyuan Chen for all his guidance and support throughout this period as graduate student and preparation of this research. All his advises and patience were invaluable to achieve this personal goal.

I also would like to thank my parents, my wife, my brother and sister for all their support and encouragement. They became a great motivation for me since the beginning of this important project of my life.

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Chapter One

Introduction

1.1 Foreword

In recent years, worldwide firms have started to change significantly their way of manufacturing. Certain companies, most of them in electronic and auto parts industries, have started to recover their products and remanufacture them. Rubio (2008) pointed out some of the main reasons for companies to involve in product recovery systems.

1. Economic reasons; Less consumption of materials and reduction of disposal costs.

2. Legal reasons; Changes in the legislation, certain countries require companies to recover and dispose their manufactured products.

3. Social reasons; Pressure from environmental associations concerned about ecological issues, emissions and waste generation.

Some companies have modified their product designs and operational processes with the purpose to facilitate and improve recovery and remanufacturing activities. As a result, certain companies have turned remanufacturing into a profitable activity. Currently, many firms have added remanufacturing into their internal processes and some others are specialized only in remanufacturing activities.

According to Dowlatshahi (2005), remanufacturing business represents a $53 billion industry in the US. Examples are Xerox (toner cartridges), Kodak Fujifilm (cameras) and IBM (computers). They started to remanufacture damaged or end-of-life products recovered from the market and this activity has become more profitable than manufacturing new products.

1.2 Remanufacturing Systems

The process of remanufacturing involves getting back end-of-life products (EOL) or components to upgrade or turn them into their original specifications (Li, 2007). A formal definition of remanufacturing can be found in Sundin (2002): "The process of rebuilding a product, during which: the product is cleaned, inspected, and disassembled; defective components are replaced; and the product is reassembled, tested and inspected again to ensure it meets or exceeds newly manufactured product standards". Nowadays many everyday goods are remanufactured, from expensive jet engine fans, automobile engines and medical equipment to less valued goods such as cameras, auto parts, computer equipment and machine tools (Franke, 2006).

1.2.1 Characteristics of Remanufacturing Systems

Certain characteristics of remanufacturing systems make them more difficult to implement comparing to conventional manufacturing. They include:

- **Variability;** In some cases, the quality of the returned products may vary depending on different factors such as the type of final customers or the motive for their returning. For example, returned products sent back as warranties are often in better conditions than returned end-of-life products.

- **Operations Sequence;** Depending on the conditions of the returned products, the order of operations in remanufacturing processes may vary. For example, some returned products may need to be cleaned before passing to disassembly and some others do not. According to Sundin (2005), the operations and their sequences in remanufacturing processes may also vary depending on the type and volume of the returned products.

- **Procurement Methods;** In remanufacturing, raw materials are not obtained from selected suppliers as in conventional manufacturing business. Remanufacturers retrieve end-of-life products from end customers or purchase them from scrap yards and returned products brokers. It often makes difficult to predict the number and time of the returned products (Östlin, 2006). According to Javaraman (2005), there are three ways to obtain used products. Remanufacturers select the best alternative depending on their needs, the three procurement ways are:

    1. **Waste stream**; This means that manufacturing companies are responsible for taking back their products at the end of the product life. The remanufacturers do not have the option to select the products. They recover end-of-life products passively without considering their conditions.

    2. **The market driven**; In this approach, end-users are motivated by financial incentives to give back the end-of-life products to the original manufacturers or to companies specialized in recovering. Companies may have more control over the conditions of the returned products since they may reject some returned products depending on their conditions.

    3. **Third-party suppliers**; In this approach, third party suppliers collect the returned products. After gathering a certain amount, they offer the returned products to remanufacturers in batches. Suppliers may also carry out grading and

sorting of the returned products. This gives the remanufacturers more control over the quality of the returned products.

1.3 Product Acquisition Systems

In this research, we only consider the "Third-Party Suppliers" approach for remanufacturers to obtain the used products. According to Nikolaidis (2009), the effectiveness in the relationships between the remanufacturers and their suppliers may influence remanufacturing processes. He stated that as the product acquisition system (PAS) between remanufacturers and their suppliers evolves, the profitability increases as well. He also stated that if the PAS between remanufacturing companies and suppliers is not appropriate, it will lead to low quality raw materials used in remanufacturing systems. Figure 1.1 shows different states of a PAS. As can be seen, higher state of a PAS may help remanufacturers in better predicting returned product quality conditions. When the relationship is at the fourth state, remanufacturers are more capable of offering their clients good quality products for low prices.

1.4 Aggregate Production Planning in Uncertain Remanufacturing Systems

Aggregate production planning (APP) consists of determining the best way to meet future demands by the adjustment of production rates, inventory levels, labor levels and capacities, often from 6 to 12 months in advance. Different authors have analyzed uncertain APP problems. However, implementing aggregate production planning in remanufacturing systems often implies that parameters such as operational costs or quality conditions are not deterministic due to unavailability of information. For this reason, this research aims to developing APP in uncertain remanufacturing systems.

State 1

- Batches containing products in many quality levels
- Unstable quality mix form deal to deal
- Acceptance sampling is not carried out before the acquisition of any batch.
- Suppliers do not bring any added value to the batch they offer

↓

State 2

- Batches containing products in many quality levels
- Unstable quality mix form deal to deal
- Acceptance sampling is carried out before the acquisition of any batch.
- Suppliers do not bring any added value to the batch they offer

↓

State 3

- Batches containing products in many quality levels
- Almost stable quality mix form deal to deal
- Acceptance sampling is carried out before the acquisition of any batch.
- Suppliers manage their resources.

↓

State 4

- Batches containing products of a particular quality level
- Suppliers sort and grade products before selling them.

Figure 1.1.The States of PAS (Nikolaidis, 2009).

1.5 Possibilistic Linear Programming

As discussed in Liang (2007), possibilistic linear programming is a useful method to deal with uncertain APP problems. One of the advantages of this method is its computational efficiency, making its applications to solve practical problems easier than some other methods. In this research, we use possibilistic linear programming to solve the uncertain APP problems in remanufacturing systems.

1.6 Research Objective

The objective of this research is to develop a mixed integer linear programming (MILP) model for solving APP problems in uncertain recovery and remanufacturing systems. By applying the method of Liang (2007), a total cost in a set of solutions with a degree of satisfaction is obtained. The proposed possibilistic MILP model can be used as a decision support tool for remanufacturing system managers to derive satisfactory solutions of the complex problem with uncertain information.

1.7 Research Contributions

This research is an extension from that in Jayaraman (2006). He presented an approach to solve aggregate production planning and control problems for closed loop supply chains. This thesis also has the objective to solve an APP problem but for uncertain remanufacturing systems with product recovery. A new mathematical model is presented considering various practical issues and uncertain parameters and coefficients. The mathematical model also allows the selection of different pre-selected suppliers offering batches of returned products.

1.8 Organization of the Project

This thesis has five chapters. In Chapter Two, literature related to remanufacturing systems is reviewed and summarized. The mathematical model and methodology used to solve the considered problem are presented in Chapter Three. Chapter Four provides a numerical example to illustrate the mathematical model under different conditions. Conclusions and possible future research directions are given in Chapter Five.

Chapter Two

Literature Review

2.1 Introduction

Many researchers have extensively analyzed conventional forward supply chain systems. Since more companies are involved in recovery and remanufacturing activities, research opportunities in closed-loop supply chains have emerged in this area in recent years. In this chapter, we discuss articles on aggregate production planning (APP) for remanufacturing systems in both deterministic and uncertain environments. First, articles describing general characteristics of remanufacturing systems are presented. Second, we classify remanufacturing systems in systems with quality categorization of returned products and systems without it. Finally, articles on APP decision problems for uncertain systems solved using fuzzy linear programming (FLP) and possibilistic linear programming (PLP) are discussed.

2.2 General Characteristics of Aggregate Production Planning in Remanufacturing

Guide *et al* (2000) discussed production planning and control activities in remanufacturing environments. Seven complicating characteristics of remanufacturing systems were identified and described. They are: (1) uncertain timing and quantity of returned products, (2) balancing returned products with demands, (3) disassembly of returned products, (4) uncertainty in materials recovered from returned products, (5) requirement of reverse logistics networks, (6) material matching requirements, and (7) routing uncertainty and processing time uncertainty. They indicated that formal systems to deal with the complicating production planning remanufacturing characteristics need to be developed.

The new systems must be able to manage complex tasks broadly different from traditional manufacturing systems. Research opportunities for each complicating characteristic were described.

Guide and Jayaraman (2000) analyzed recovery and remanufacturing of end-of-life (EOL) products based on a survey conducted in North American remanufacturing companies. The survey consisted of 75 questions covering areas such as production planning and control, materials management, and demand management. Based on the survey findings, they concluded that there is a need of formal planning and control systems in remanufacturing activities. Therefore, a formal framework for Product Acquisition Management (PrAM) was proposed. PrAM is a decision support tool to coordinate and monitor product acquisition management. Some managerial guidelines for the organization of product acquisition management activities were also proposed.

Inderfurth and Teunter (2001) analyzed the seven complicating characteristics to planning and control in remanufacturing systems described in Guide *et al* (2000) and added a new one. The new characteristic applies to companies involved in simultaneously manufacturing of new products and remanufacturing of end-of-life products, commonly called hybrid systems. According to the authors, coordination of manufacturing and remanufacturing operations represents a real challenge due to the inherent uncertainty in them. They mentioned that if this coordination is not well managed, capacity shortages or excess in stocks may occur. They classified closed-loop supply chains with external returned products and those with internal returned products (rework) only.

Östlin *et al* (2008) identified and described seven types of closed-loop relationships in recovering of end-of-life products. The relationships are ownership-based, service-contract, direct order, deposit-based, credit-based, buy-back and voluntary-based relationships. Based on a case study, they concluded that the relationships may exist simultaneously to complement each other. In addition, the closeness of remanufacturing-supplier relationships depends on the importance of the products in the remanufacturing processes. Certain relationships such as ownership-based and service-contract allow remanufacturers more control over the timing and quantity of the returned products. The success of remanufacturing business often depends on the relationship between the remanufacturer and their customers.

Michaud and Llerena (2006) studied economic issues involved in current recovery and remanufacturing industry. The main factors affecting remanufacturing systems and their impact on profitability were analyzed. According to them, consumers play a key role in remanufacturing due to two reasons. First, their influence on the returning of used products to the original manufacturer. The second is that they are potential future buyers of the remanufactured products. A methodology to study the preferences of consumers to remanufacturing products was presented. They mentioned that economical feasibility of remanufacturing systems depends highly on coordination of all participants in closed loop supply chain systems.2.3 Remanufacturing Systems

2.3.1. Systems with Quality Consideration of Returned Products

Nikolaidis (2009) proposed a mixed integer linear programming model for aggregate production planning (APP) for product recovering and remanufacturing. To recover end-of-life products, remanufacturers select suppliers offering different quality of returned products in batches. He presented a scheme with four states of product acquisition systems (PAS). The model was applied in a telecommunication company dedicated to the purchasing, remanufacturing and reselling of used mobile phones.

Jayaraman (2006) presented a procedure for solving production planning and control problems of closed-loop supply chains. The procedure consists of a mathematical programming model called RAPP (Remanufacturing Aggregate Production Planning) to support decision-making in medium to long planning time horizons. The model is to minimize total remanufacturing cost. The results include decisions such as the number of disassembled, remanufactured, and disposed returned products. It assumed that the quality of the returned products is known. The mathematical model was tested using data of a company dedicated to remanufacturing of cell phones.

Zikopoulos and Tagaras (2007) studied the effects caused by variability in quality of returned products on profitability of recovery and remanufacturing systems. The problem consists of a reverse supply chain with two collection sites and one remanufacturing site. A model to maximize the total profit was formulated to solve the problem considering different situations as utilizing one or two collection sites. Demand for remanufactured components was considered as random variable

Galbreth and Blackburn (2006) studied the effects of sorting returned products on remanufacturing companies. They developed a mathematical model considering variable conditions of returned products. The effects on remanufacturing costs by taking different acquisition and sorting decisions were analyzed. An example problem of a remanufacturing company acquiring unsorted products from third-party suppliers to meet deterministic and uncertain demands was presented. The remanufacturer needs to decide the amount of returned products to acquire and the sorting acceptance level. Acquisition and sorting policies of returned products were studied under different conditions. Optimal acquisition and sorting policies were obtained for both deterministic and uncertain demands.

Aras *et al* (2003) presented an approach to evaluate the impact of returned products quality categorization on hybrid systems. Using a continuous-time Markov chain model, they showed that quality categorization of returned products in remanufacturing may lead to cost savings. The main characteristic of their model is the three types of inventories considered. The first is for remanufactured products, the second is for returned products requiring less remanufacturing and the third is for the rest of the returned products. They concluded that quality-based categorization is more effective when demand is low with more difference in quality among different types of returned products or demand is high with low quality of returned products.

Kiesmüller (2003) studied hybrid recovery systems for a single type of product considering lead times. A mathematical model was presented to determine optimal manufacturing/remanufacturing cost and disposal rates. In the model two types of inventories were analyzed. One is for returned products and the other is for remanufacturable returned products only. Demand and return rates were considered deterministic and dynamic. Two types of policies were studied, one allowing backlogs and the other forbidding them.

Ferguson *et al* (2009) studied production planning problems for hybrid systems considering different quality levels of the returned products. An optimization model was presented to maximize total profit considering deterministic and nondeterministic demands. The authors noticed that remanufacturing costs increase when quality of the returned products decrease. Product salvation instead of disposal for unused returned products was proposed. They mentioned that profit can be improved with grading returned products into different quality categories comparing to non-grading.

Franke (2006) studied a remanufacturing planning problem for mobile phones production. A linear programming model for the planning of remanufacturing capacities was presented. Quality conditions and quantities of mobile phones, processing times and capacities were considered uncertain. Based on the results of the linear optimization model, a simulation model was generated considering process capacities and resources for transport and storage. The required transport and storage capacities and the optimized performance of the remanufacturing system were obtained from the simulation analysis.

2.3.2 Systems without Quality Consideration of Returned Products

Xanthopoulos (2009) developed a two phase algorithm for optimal design of recovery systems for used electric appliances. In the first phase, the components to obtain from returned products are identified using goal programming. In the second phase, a mixed integer linear programming model is used for APP. The methodology can be taken as a decision support tool for medium level decision making. This approach combines optimal design for disassembly processes and aggregate production planning. The simplicity of the parameters used in the model makes its implementation easier for practical problems in recovery industry.

Taleb and Gupta (1997) studied the commonality of parts and materials in disassembly scheduling. They developed a material requirement planning algorithm for disassembly scheduling considering the issue that common parts and materials can be in the same product structure. The objective is to minimize the total number of root items to disassemble to fulfill demands of parts. The disassembly process was assumed to be perfect therefore the algorithm does not consider defective parts. Capacities for the disassembly processes were considered infinite.

Langella (2007) presented an extension of Taleb and Gupta (1997). He studied the recovering of end-of-life (EOL) products for remanufacturing. Returned products are disassembled to obtain their internal components and reassembled to get "as good as new" products. The components obtained from the driven-disassembly can be remanufactured either in external (third-party suppliers) or in internal operations. A heuristic to find an optimal solution was developed based on the one presented in Taleb and Gupta (1997). A main characteristic of the approach is that it deals with inventory costs and external procurement of used returned products. The efficiency of the model was tested solving a numerical example problem.

Kongar and Gupta (2006) presented a multi-criteria decision model disassembly-to-order remanufacturing systems under uncertainty. The model is to maximize total profit by determining the optimal amount of each product type to recover from collection sites or from end customers. The process consists of recovering returned products to obtain their internal components. Components that are not used may be recycled, disposed or kept in inventory. Cost parameters in the objective function were considered uncertain. The problem was solved using fuzzy goal programming. The number of products recovered and the number of components reused, disposed or recycled are included in the results. The model was tested using an example problem of remanufacturing computer equipment.

Depuy et al (2007) presented a methodology for production planning of remanufacturing activities. The quality of components obtained from retuned products disassembling was considered nondeterministic. They stated that if the rate of damaged components is high, problems to meet the demand for certain periods may occur. The probabilistic material requirement planning (MRP) approach determines the expected number of units to remanufacture at each time period. A procedure for generating a component purchase schedule to avoid shortages in periods with low probability of meeting demand was proposed. An example problem was presented with data obtained from an antenna remanufacturing process.

Clegg *et al* (1995) presented a linear programming model for aggregate production planning (APP) in manufacturing/remanufacturing production systems. The model is to determine the level of production to obtain optimal profit. The effects of disposal, disassembly, and remanufacture decision on system performance were studied. The mathematical model can be used as a decision support tool for hybrid system management. The model was tested using an example problem in a telephone remanufacturing facility.

Kim *et al* (2006) studied the remanufacturing processes of reusable parts in reverse logistics. A problem of a remanufacturing multi-facility company with two options for component procurement was considered. In the first option new components are purchased from external suppliers. The second option is to recover components from returned products and remanufacture them to become “as new“ products. A mixed integer LP model to maximize the whole cost saving was proposed. Numerical results include optimal number of components to remanufacture at each facility and the number of components to purchase from external suppliers. The model was tested using a numerical example with experimental data.

Jorjani *et al* (2004) presented a mathematical model for optimal disposal of components obtained from disassembling electronic equipment. The decisions are to classify and allocate the components with different disposal options. The model was tested using an example problem with 5 types of products. The results showed that the model can identify the products that have the best potential to recover for their components.

Chenglin (2008) studied supply planning decision problems in remanufacturing systems. A remanufacturing facility, a supplier facility and a collector agent were considered in the system. The collector agent recovers the used products from landfills to send them to the supplier facility. The supplier facility processes and sends the returned products to the remanufacturing facility. A mathematical model to minimize the total cost for the whole supply chain was proposed. A numerical example was presented to test the model.

2.4 Aggregate Production Planning Under Uncertainty

2.4.1 Fuzzy linear programming (FLP)

Mula *et al* (2006) presented a review on existing literature of production planning in uncertain environments. The review paper consists of 87 articles from 1983 to 2004. A detailed classification of production planning models under uncertainty was presented. Stochastic programming was found to be the most popular approach and dynamic programming the least. The authors indicated that fuzzy set theory is an important decision support tool for uncertain manufacturing production planning.

Tang *et al* (2000) studied multi-product aggregate production planning problems in uncertain environments considering fuzzy demands and capacities. The soft equation concept was used in the model to minimize total production and inventory costs. The problem was modeled using fuzzy quadratic programming with fuzzy objectives and constraints. Different alternatives can be provided to the decision maker to create aggregate production plans in uncertain systems.

Tang *et al* (2003) presented an extension of the work in Tang et al (2000). They made a simulation analysis for multi-product APP problems in uncertain environments with fuzzy demands and capacities. The fuzzy production-inventory balance equation was formulated as a soft equation. A simulation was run to check the performance and behavior of the system with uncertain parameters.

Wang and Liang (2004) presented a fuzzy multi-objective linear programming model to solve multi-product APP decision problems in fuzzy environments. The model was based on that presented in Hannan (1981). The first objective is to minimize total production, inventory and backlogging costs. The second objective is to minimize the rates of changes of labor forces. A set of compromise solutions and overall satisfaction are to be achieved. The main characteristics of the approach are the framework that facilitates the decision-making process and that interactively allows the adjustment of membership functions until a satisfactory solution is obtained. An example problem was presented to show the feasibility of the mathematical model.

Liang and Cheng (2009) applied fuzzy sets to solving integrated manufacturing and distribution planning decision problems. They presented a fuzzy multi-objective linear programming model to simultaneously minimize total costs and total delivery time. Inventory levels, available labor levels and machine capacities were considered. The fuzzy numbers were modeled using triangular distribution. Linear membership functions were used to represent the fuzzy objectives. The solving methodology enables the decision-maker to change the search direction during the process until a better set of solutions is obtained.

2.4.2 Possibilistic Linear Programming (PLP)

Inuiguchi (1993) presented some advantages of using possibilistic linear programming approach to solve production planning problems in uncertain environments. A comparison of solutions obtained using possibilistic linear programming and those using flexible and goal programming was made. Advantages of possibilistic programming include the simplicity to find optimal solutions and the possibility to adjust the solutions to satisfy the request of the decision-maker.

Lai and Wang (1992) presented a single-objective possibilistic linear programming model considering uncertain coefficients in the objective and constraint functions. The uncertain single LP model was converted to an auxiliary multi-objective LP model. The strategy was to maximize the most possible value of the fuzzy profit, minimize the risk of obtaining a low profit and maximize the possibility of obtaining higher profit. A numerical example problem was solved to illustrate the practicability of the proposed approach.

Hsu and Wang (2001) proposed a possibility linear programming model to solve assembly to order (ATO) decision problems in uncertain environments. The model integrates forecasting activities, material management and production planning. The fuzzy programming method of Zimmermann (1978) was used to obtain efficient satisfactory solutions. The strategy consists of substituting the uncertain objective function with three auxiliary objective functions. The results are to determine forecast demands, appropriate safety stocks and the number of machines to use in the process. An example problem to test the proposed model was presented.

Wang and Liang (2005) presented a single-objective possibilistic linear programming approach for solving aggregate production planning problems with uncertain parameters. The objective is to minimize total cost considering inventory levels, labor levels, overtime, subcontracting and backordering. The strategy is to simultaneously minimize the most possible value of the uncertain total costs, maximize the possibility of obtaining lower total costs, and minimize the risk of higher total costs. The model can determine efficient solutions with overall decision maker satisfaction.

Liang (2007) presented a multi-objective possibilistic linear programming model to interactively solve multi-product and multi-period APP problems. The objective functions and some parameters in the constraint functions are uncertain. The objectives are to minimize total production costs and changes in work-force level. Demand, cost coefficients, available resources and capacities are uncertain. The possibilistic linear programming model supports APP decision makers and enables them to interactively adjust uncertain parameters until a satisfactory solution is obtained.

Torabi and Hassini (2008) developed a supply chain master planning model consisting of multiple suppliers, multiple distribution centers and one manufacturer. They presented a multi-objective possibilistic mixed integer LP model that incorporates procurement, production and distribution planning activities. In the model, some parameters such as market demands, cost coefficients and capacity levels were considered uncertain. The problem was solved by converting the single-objective possibilistic model into an auxiliary uncertain multi-objective linear programming model.

2.5 Summary

Many researchers have studied aggregate production planning for deterministic remanufacturing systems. From the articles reviewed, we notice that only a few of them have investigated APP problems of remanufacturing systems in uncertain environments.

Possibilistic linear programming has been used effectively to solve APP problems for uncertain conventional manufacturing systems. In this thesis, we made attempt to use this approach to solve APP problems for recovering and remanufacturing systems in uncertain environments.

Chapter Three

Problem Statement and Modeling

3.1 Problem definition

In this chapter, the main characteristics of the studied problem and the methodology used to solve it are presented. We assume that a remanufacturing company needs to select suppliers offering returned products in batches. Returned products of different types are disassembled to obtain their internal components. The components obtained are remanufactured with the purpose to fulfill forecasted demands in different time periods. A possibilistic model is built for aggregate production planning to meet the forecasted demands with uncertain parameters such as operational costs, capacities and returned product conditions. Possibility distributions based on fuzzy set theory are used to model the uncertain parameters in the model. Some advantages of using fuzzy to represent the uncertain parameters are the computational efficiency and that we can keep the linearity of the original model in solving the considered problem.

The system begins from qualified suppliers offering returned products in batches to the remanufacturer. Once a supplier is selected and the batch is received by the remanufacturer, the returned products may go to different operations of the process. The first operation is to classify purchased products as remanufacturable or non-remanufacturable depending on their conditions. All the products classified as non-remanufacturable are salvaged to recover part of the purchasing cost. Returned products classified as remanufacturable may pass to disassembly, inventory or salvaging depending on the demands and further inspection. For products that cannot be salvaged the disposal option is available. The products sent to disassembly yield components that may go to three options: remanufacturing, inventory or disposal. After remanufacturing, components are sent to clients with "as new" conditions. If the demands for remanufactured components are not met in certain period, backlogging or purchasing of new components from third party suppliers may be necessary. Figure 3.1 shows the explained recovery and remanufacturing process.

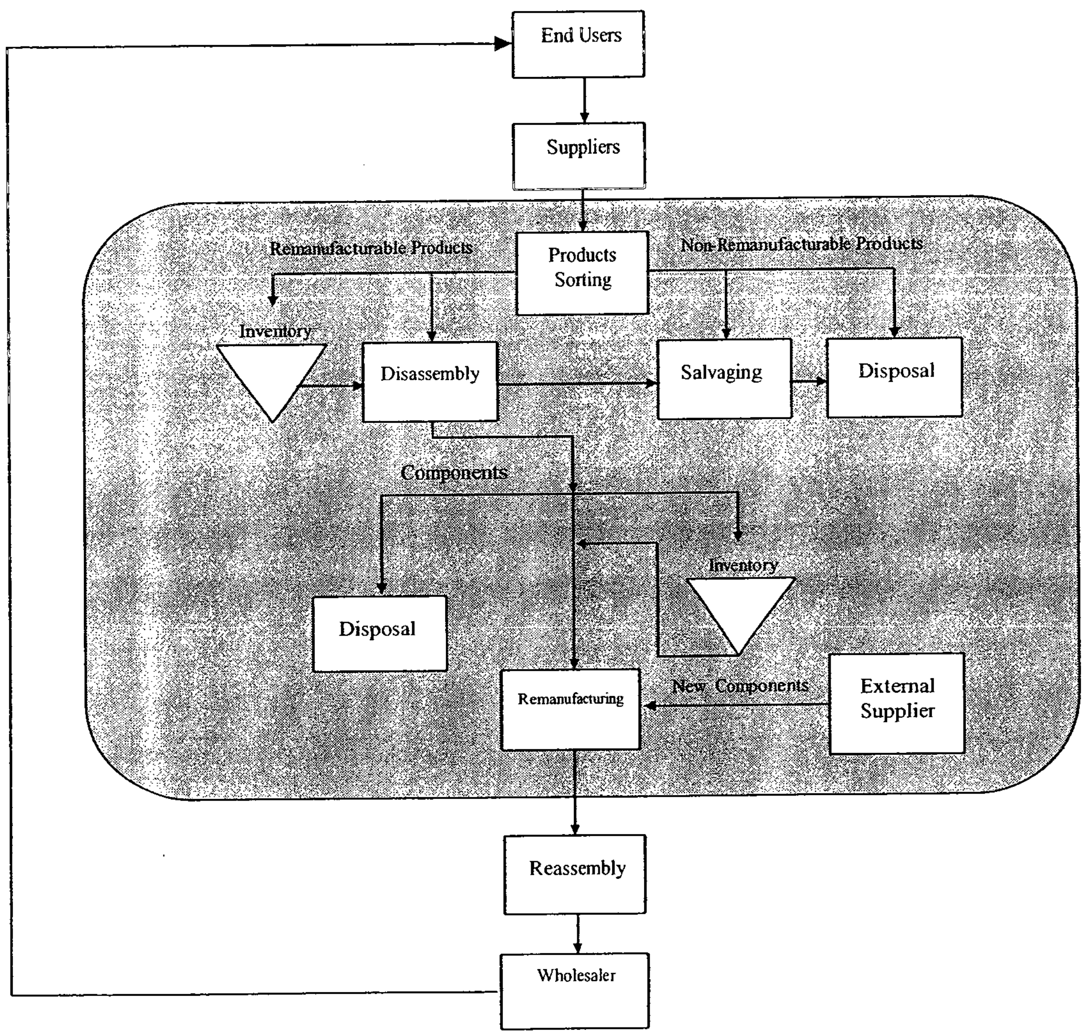

Figure.3.1.Remanufacturing Internal Process (shaded area).

3.2 Model Assumptions

The following assumptions are made in developing the mathematical model to minimize the cost of the considered remanufacturing system.

1. The remanufacturer-supplier relationship was described as the 4^{th} state in the PAS scheme as discussed in Nikolaidis (2009) and presented in Chapter 1.

2. The main components to be obtained from product disassembly are pre-identified.

3. There is a group of pre-selected suppliers offering batches of returned products at each time period.

4. The remanufacturer will receive the returned products at the beginning of each period.

5. Backlogs are allowed and the remanufactured components delivery may delayed with a penalty cost.

6. If there is no need to purchase returned products in a certain period, none of the suppliers will be selected.

7. If a supplier is selected, the whole batch of returned products will be purchased from the supplier.

3.3 Model Notation

The following notation is used in presenting the mathematical model. The symbol ˜ on some coefficients is used to indicate that those parameters are uncertain.

Indices

$i \in \{1, \ldots, I\}$ Type of products

$j \in \{1, \ldots, J\}$ Type of components retrieved from products

$t \in \{1, \ldots, T\}$ Time periods

$k \in \{1, \ldots, K\}$ Products quality levels

$s \in \{1, \ldots, S\}$ Index of suppliers

$y \in \{1, \ldots, Y\}$ Components quality levels

Decision Variables

$x_{s,t}$ 1, if supplier s is selected in time period t, otherwise, 0

$g_{t,i,k}$ Number of type i products of quality level k disassembled in period t

$ie_{t,i,k}$ Number of type i products of quality level k disposed in period t

$se_{t,i,k}$ Number of type i products of quality level k salvaged in period t

$h_{t,i,k}$ Inventory level of remanufacturable type i products of quality level k in period t

$e_{t,j,y}$ Number of components j of quality y remanufactured in period t

$ke_{t,j,y}$ Number of components j of quality y disposed in period t

$je_{t,j,y}$ Inventory level of components j of quality y in period t

$bm_{t,j}$ Number of components j backlogged in period t

$f_{t,j}$ Number of components j purchased in period t

Cost Parameters

$\widetilde{CA}_{t,i,k}$ Acquisition cost of a type i product of quality level k in period t

$\tilde{o}_{i,k}$ Cost of disassembling a type i product of quality level k

$\tilde{R}_{i,k}$ Cost of disposing a type i product of quality level k

$\widetilde{CS}_{i,k}$ Cost of salvaging a type i product of quality level k

$\widetilde{CH}_{i,k}$ Cost of holding in inventory a type i product of quality level k

$\tilde{Q}_{j,y}$ Cost of remanufacturing a component j of quality y

$\tilde{V}_{j,y}$ Cost of disposing a component j of quality y

$\tilde{U}_{j,y}$ Cost of holding in inventory a component j of quality y

$\tilde{W}_j$ Cost of backlogging a component j

$\widetilde{TE}_{t,j}$ Cost of purchasing a new component j in period t

General Parameters

$M_{t,i,k}$ Revenue from salvaging a type i product of quality level k in period t

$RT_{s,t,i,k}$ Number of type i products of quality level k purchased from supplier s in period t

$\tilde{P}_{i,k}$ Fraction of remanufacturable type i products of quality level k

$NM_{i,k,j,y}$ Number of components j of quality y in a type i returned product of quality k

$\tilde{D}_{t,j}$ Demand for remanufactured components j in period t

$DC_{i,k}$ Expected fraction of damaged type i products of quality k

$PD_{j,y}$ Expected fraction of damaged components j of quality y

UB Components backlogging limit fraction.

VC_i Size of a type i product

SC Total products storage capacity

VM_j Size of a component j

SM Total components storage capacity

PC_t Components purchasing limit in period t

SA_t Products salvaging limit in period t

$\widetilde{TC}_{i,k}$ Time required to disassembly a type i product of quality k

$\widetilde{HD}_t$ Total hours available for disassembling in period t

$\widetilde{TM}_{j,y}$ Time required to remanufacture a component j of quality y

$\widetilde{HR}_t$ Total hours available for remanufacturing in period t

3.4 Mathematical Model Formulation

In this section, the uncertain mixed integer mathematical programming model to solve the APP problem is formulated. The parameters in the mathematical model considered as uncertain are:

In objective function: Acquisition cost of a returned product ($\widetilde{\boldsymbol{CA}}$), Cost of disassembling a returned product ($\widetilde{\boldsymbol{O}}$), cost of disposing a returned product ($\widetilde{\boldsymbol{R}}$), Cost of salvaging a returned product ($\widetilde{\boldsymbol{CS}}$), cost of holding in inventory a returned product ($\widetilde{\boldsymbol{H}}$), costs of remanufacturing a component ($\widetilde{\boldsymbol{Q}}$), cost of disposing a component ($\widetilde{\boldsymbol{V}}$), cost of holding in inventory a component ($\widetilde{\boldsymbol{U}}$), cost of backlogging a component ($\widetilde{\boldsymbol{W}}$), cost of purchasing a new component ($\widetilde{\boldsymbol{TE}}$),

In constraint functions: Demand of components ($\widetilde{\boldsymbol{D}}$), fraction of remanufacturable products ($\widetilde{\boldsymbol{P}}$), time required for disassembling a returned product ($\widetilde{\boldsymbol{TC}}$), total hours available for disassembling a returned product ($\widetilde{\boldsymbol{HD}}$), time required for remanufacturing a component ($\widetilde{\boldsymbol{TM}}$) and total hours available for remanufacturing a component ($\widetilde{\boldsymbol{HR}}$).

3.4.1 Objective function

$$\text{MIN } \tilde{z}_1 = \sum_s \sum_t \sum_i \sum_k (\widetilde{CA}_{s,i,k} \times RT_{s,t,i,k}) \times x_{s,t} + \sum_t \sum_i \sum_k \tilde{O}_{i,k} \times g_{t,i,k} +$$
$$\sum_t \sum_i \sum_k \widetilde{CS}_{i,k} \times se_{t,i,k} + \sum_t \sum_i \sum_k \widetilde{CH}_{i,k} \times h_{t,i,k} + \quad (3.1)$$
$$\sum_t \sum_i \sum_k \tilde{R}_{i,k} \times ie_{t,i,k} + \sum_t \sum_j \tilde{Q}_{j,y} \times e_{t,j,y} + \sum_t \sum_j \tilde{U}_{j,y} \times je_{t,j,y} +$$
$$\sum_t \sum_j \tilde{V}_{j,y} \times ke_{t,j,y} + \sum_t \sum_j \widetilde{TE}_{t,j} \times f_{t,j} + \sum_t \sum_j \tilde{W}_j \times bm_{t,j} -$$
$$\sum_s \sum_t \sum_i \sum_k M_{t,i,k} \times \left((se_{t,i,k} + \left((RT_{s,t,i,k} \times x_{s,t} \right) \times (1 - \tilde{P}_{i,k}) \right) \right)$$

The objective function (3.1) is to minimize the sum of the total remanufacturing costs less revenue obtained from salvaging the products. One part of the function is related to selection of suppliers and the other is to calculate the remanufacturing costs. The first term is the cost of supplier selection. The second term is the uncertain cost of disassembling a returned product. The third term is the cost of salvaging a returned product. The fourth term is the returned product holding cost. The fifth term corresponds to the cost of disposing a returned product. The sixth term is the cost of remanufacturing a component. The seventh term is the component holding cost. The eighth term corresponds to the cost of disposing a component. The ninth term is the cost of purchasing a new component from third-party suppliers. The tenth term is the component backlog cost. The last term is the revenue from salvaged products.

3.4.2 Constraint Functions

Suppliers Selection Constraint

$$\textstyle\sum_s \ x_{s,t} \leq 1, \ \forall t \quad (3.2)$$

Balance Constraints

$$\sum_s(RT_{s,t,i,k} \times x_{s,t}) \times \widetilde{\boldsymbol{P}}_{i,k} = g_{t,i,k} + ie_{t,i,k} + se_{t,i,k} + h_{t,i,k} - h_{t-1,i,k} \quad \forall t, \forall i, \forall k \qquad (3.3)$$

$$\sum_{i,k} NM_{j,y,i,k} \times g_{t,i,k} = e_{t,j,y} + ke_{t,j,y} + je_{t,j,y} - je_{t-1,j,y} \quad \forall t, \forall j, \forall y \qquad (3.4)$$

$$\sum_y e_{t,j,y} = \widetilde{\boldsymbol{D}}_{t,j} - f_{t,j} - bm_{t,j} + bm_{t-1,j}, \quad \forall t, \forall j \qquad (3.5)$$

$$\sum_s((RT_{s,t,i,k} \times x_{s,t}) \times (\widetilde{\boldsymbol{P}}_{i,k})) \times DC_{i,k} \le ie_{t,i,k}, \quad \forall t, \forall i, \forall k \qquad (3.6)$$

$$\sum_{i,k} (NM_{j,y,i,k} \times g_{t,i,k}) \times PD_{j,y} \le ke_{t,j,y}, \quad \forall t, \forall j, \forall y \qquad (3.7)$$

$$bm_{t,j} \le UB_t \times \widetilde{\boldsymbol{D}}_{t,j}, \quad \forall t, \forall j \qquad (3.8)$$

Capacity Constraints

$$\sum_{t,i,k} (VC_i \times h_{t,i,k}) \le SC, \qquad (3.9)$$

$$\sum_{t,j,y} (VM_j \times je_{t,j,y}) \le SM, \qquad (3.10)$$

$$\sum_j f_{t,j} \le PC_t, \quad \forall t \qquad (3.11)$$

$$\sum_{i,k} se_{t,i,k} \le SA_t, \quad \forall t \qquad (3.12)$$

$$\sum_{i,k} \widetilde{\boldsymbol{TC}}_{i,k} \times g_{t,i,k} \le \widetilde{\boldsymbol{HD}}_t, \quad \forall t \qquad (3.13)$$

$$\sum_{j,y} \widetilde{\boldsymbol{TM}}_{j,y} \times e_{t,j,y} \le \widetilde{\boldsymbol{HR}}_t, \quad \forall t \qquad (3.14)$$

Initial and Ending Constraints

$$bm_{0,j} = 0 \quad \forall j \qquad (3.15)$$

$$bm_{T,j} = 0 \quad \forall j \qquad (3.16)$$

$$h_{0,i,k} = 0 \quad \forall i, k \qquad (3.17)$$

$$je_{0,j,y} = 0 \quad \forall j, y \qquad (3.18)$$

$$g_{t,i,k}, se_{t,i,k}, ie_{t,i,k}, h_{t,i,k}, e_{t,j,y}, ke_{t,j,y}, je_{t,j,y}\, f_{t,j}, bm_{t,j} \ge 0 \in R_0^+ \qquad (3.19)$$

$$x_{s,t}, \in (0,1) \qquad (3.20)$$

Constraint (3.2) ensures that only one supplier will be selected at each period. Constraint (3.3) ensures that the number of disassembled, disposed, salvaged and held in inventory products equals the number of products available to use in the process less the products classified as non-remanufacturable. Constraint (3.4) ensures that the number of remanufactured, disposed, and held in inventory components equals the number of components yield from the disassembly of the products. Constraint (3.5) ensures that the number of remanufactured, purchased and backlog components equals the forecasted demands of remanufactured components. Constraint (3.6) ensures that the number of disposed products is equal or greater than the expected percentage of damaged products in the purchased batch size. Constraint (3.7) ensures that the number of disposed components is equal or greater than the expected percentage of damaged components in the purchased batch size. Constraint (3.8) ensures that the number of backlogs do not exceed the allowed limit fraction with respect of the demand. Constraint (3.9) ensures that the number of products held in inventory does not exceed the products storage capacity. Constraint (3.10) ensures that the number of components held in inventory does not exceed the components storage capacity. Constraint (3.11) ensures that the number of new purchased components does not exceed the upper allowed limit for that period Constraint (3.12) ensures that the number of salvaged products does not exceed the upper allowed limit for that period. Constraint (3.13) ensures that the process time used in products disassembling does not exceed the total time available for disassembling. Constraint (3.14) ensures that the process time used in components remanufacturing does not exceed the total time available for remanufacturing. Constraint (3.15) and (3.16) ensure that the number of backlogs for the beginning of the first and the end of last period equal zero. Constraints (3.17) and (3.18) ensure that the inventory of products and components at the beginning of the first period equals zero. Constraint (3.19) ensures that all decision variables except for the integers are real positive numbers. Constraint (3.20) ensures that variable x is an integer variable.

3.5 Solution Method

To solve the mathematical model presented in Section 3.4, the interactive approach proposed in Liang (2007) is used. This approach is an extension of the method in Lai and Wang (1992b). Some main advantages of this approach are its computational efficiency, flexibility of fuzzy arithmetical operations and its interactivity that allows the decision-maker to adjust the parameters until an efficient set of solutions is obtained. The strategy applied is to convert the uncertain linear programming model into an equivalent ordinary crisp LP model. To better understand the solving procedure, the concept of *possibility distribution* used in Liang (2007) is explained. This term was first presented in Zadeh (1978) based on fuzzy sets theory. He related the concept of *possibility distribution* to the membership grade of a fuzzy restriction acting as a constraint on the values assigned to a variable. A formal definition of the concept and detailed explanation can be found in Zadeh (1978).

The general procedure to solve the uncertain model in Liang (2007) is:

1. Convert the original uncertain objective function(s) $\tilde{z}_i$ into crisp function(s) z_i with respect to the possibility distribution pattern adopted to model its uncertain coefficients. In fuzzy sets, possibility distributions patterns such as triangular, trapezoid, bell-shaped, exponential and hyperbolic may be used. According to Liang (2007) the pattern of triangular distribution is used in many practical decision

problems because it is relatively easy to define the maximum and minimum limits of deviation from the central value of the fuzzy number.

2. The uncertain constraint functions are converted into crisp constraint functions. The weighted average method is used for constraint functions with uncertain coefficients in one side of the function. The fuzzy ranking concept is used for constraints with uncertain coefficients in both sides of the function.

   2.1 *Weighted average method*: In this method, weights $w_1 + w_2 \ldots + w_n = 1$ are associated to each uncertain coefficient. The decision maker may specify subjectively the weights according with his experience and knowledge. For fuzzy coefficients $\tilde{B} = (\tilde{b}_1 \ldots \tilde{b}_n)$ and their assigned weights $W = (w_1 \ldots w_n)$. A fuzzy constraint $a_i(x) \leq \tilde{b}_i$ can be converted into a crisp constraint with the form of: $a_i(x) \leq (b_i \times w_i \ldots b_n \times w_n)$. If the likelihood of some values is considered greater than others, the concept of most likely values is used to assign greater weights to those values.

   2.2 *Fuzzy ranking*: In this method, fuzzy coefficients are ranked according to their likelihood of occurrence. For fuzzy coefficients $\tilde{A} = (\tilde{a}_1 \ldots \tilde{a}_n)$ and $\tilde{B} = (\tilde{b}_1 \ldots \tilde{b}_n)$ ranked in ascendant order according to their likelihood, the fuzzy constraint $\tilde{a}_i(x) \leq \tilde{b}_i$ can be substituted by the following crisp constraints:

   $a_1(x) \leq b_{1,} \ \forall x$

   ...

   $a_n(x) \leq b_{n,} \ \forall x$

3. An ordinary auxiliary model is developed based on the max-min operator method of Zimmermann (1978). An effective method to solve multi-objective linear programming decision models by the application of fuzzy sets. If the fuzzy objectives are characterized by their membership functions, an efficient solution is defined as the intersection of the fuzzy sets defining the objectives. The solution with the highest degree of membership to the fuzzy decision-set is defined as the maximum decision.

Let X be a finite set of all possible solutions to a decision problem. The fuzzy objective functions G and H are fuzzy subsets on X characterized by their membership functions.

$$\mu_G(x) \rightarrow [0, 1]$$
$$\mu_H(x) \rightarrow [0, 1] \qquad (3.21)$$

Then, G and H are combined to generate a fuzzy decision D on X. D is a fuzzy subset resulting from the intersection G and H, characterized by its membership function. Based on fuzzy sets logic, the intersection ($\cap$) of subsets G and H is formulated as follows:

$$\mu_{G \cap H}(x) = \mu_D(x) = \text{Min}\ (\mu_G(x), \mu_H(x)), \quad \forall x \in X \qquad (3.22)$$

The corresponding decision will be obtained by:

$$\text{Max}\ \mu_D(x) = \text{Max Min}\ (\mu_G(x), \mu_H(x)), \quad \forall x \in X \qquad (3.23)$$

As discussed in Liang (2007) it is equivalent to solve the following LP,

Max L

s.t.

$L \leq \mu_i(x) \quad \forall i$

$x \geq 0$

$$0 > L < 1 \tag{3.24}$$

where L is defined as the maximum degree of satisfaction for certain determined goals.

The steps of the max-min method of Zimmermann (1978) are:

1. Obtain the upper and lower bounds of the linear membership functions represented by the positive ideal solutions (PIS) and negative ideal solutions (NIS). They are obtained by solving the LP model for each auxiliary objective.
2. Specify the membership functions for each of the auxiliary objectives according to the possibility distribution pattern used to model the objective function
3. Develop an auxiliary ordinary model by using the minimum operator L based on equation (3.24).

4. The auxiliary model is solved by using linear programming to obtain a set of efficient solutions and the overall degree of satisfaction. If the initial set of solutions is not satisfactory, the PIS and NIS may be adjusted for several iterations until a satisfactory solution is obtained.

The general solving procedure of Liang (2007) is used to solve the MILP model presented in equations (3.1) – (3.20).

Step 1: Convert the uncertain objective function $\tilde{z}_i$ into crisp objective functions. In this thesis we assume that the uncertain coefficients are modeled with their triangular possibility distributions. Triangular distribution is often used in solving uncertain decision problems due to the simplicity in fuzzy arithmetical operations, the ease in data acquisition and definition of the maximum and minimum deviation limits from the central value. Taking as reference demand $(\widetilde{D}_{t,j})$, its triangular possibility distribution may be defined using three prominent points

$$(D^p_{t,j}, D^m_{t,j}, D^o_{t,j}), \quad \forall t, \forall j$$

where $D^p_{t,j}$ is the most pessimistic value with low possibility of belonging to the set of feasible solutions, $D^m_{t,j}$ is the most likely value with high possibility of belonging to the set of solutions and $D^o_{t,j}$ is the most optimistic value with low possibility of belonging to the set of solutions. An illustration of this distribution is shown in Figure 3.2.

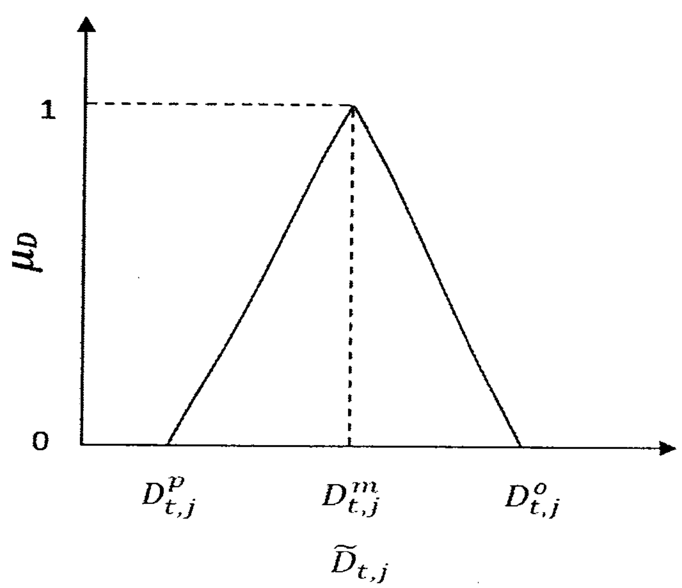

Figure.3.2.Triangular Distribution of Coefficient $\widetilde{D}_{t,j}$.

The rest of the uncertain coefficients are modeled in the same way. For example, the possibility distributions of the costs coefficient $(\tilde{O}_{i,k})$ will be:

$$(O_{i,k}^{o}, O_{i,k}^{m}, O_{i,k}^{p}), \quad \forall i, \forall k,$$

We assume that the objective function $\tilde{z}_1$ also has a triangular distribution and it is geometrically defined by 3 coordinate points: $(z_1^o, 0)$, $(z_1^m, 1)$ and $(z_1^p, 0)$. Minimization of the total cost requires the pushing of the 3 points on the horizontal axis towards the left side as shown in Figure 3.3

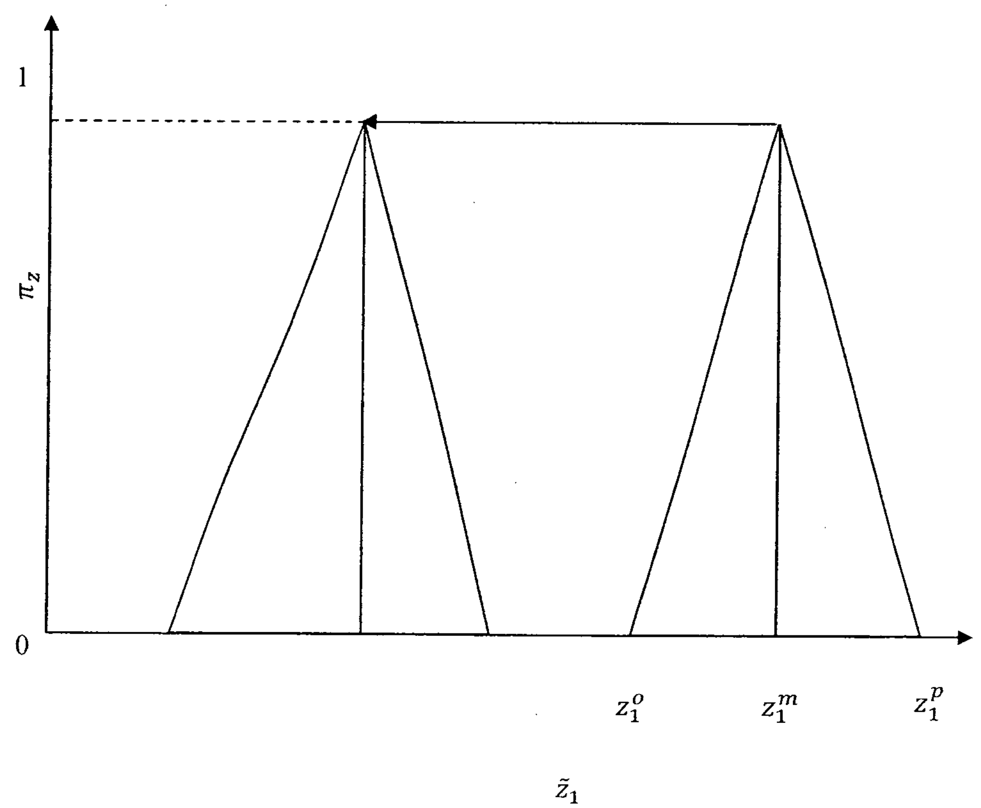

Figure.3.3.Strategy to Minimize the Total Cost

As can be seen from Figure 3.3, the strategy is to simultaneously minimize the most likely objective value z_1^m, maximize the possibility of obtaining a low total cost $(z_1^m - z_1^o)$ and minimize the risk of getting a high total cost $(z_1^p - z_1^m)$. The last 2 objectives are measures from the most likely value z_1^m. Using this strategy, the following 3 auxiliary objective functions will be used to replace equation (3.1) of the original model developed in this research to solve the remanufacturing APP problem:

$$\text{Min } z_2 = \sum_s \sum_t \sum_i \sum_k (CA^m_{s,i,k} \times RT_{s,t,i,k}) \times x_{s,t} + \sum_t \sum_i \sum_k O^m_{i,k} \times g_{t,i,k} +$$
$$\sum_t \sum_i \sum_k CS^m_{i,k} \times se_{t,i,k} + \sum_t \sum_i \sum_k CH^m_{i,k} \times h_{t,i,k} + \tag{3.25}$$
$$\sum_t \sum_i \sum_k R^m_{i,k} \times ie_{t,i,k} + \sum_t \sum_j Q^m_{j,y} \times e_{t,j,y} + \sum_t \sum_j U^m_{j,y} \times je_{t,j,y} +$$
$$\sum_t \sum_j V^m_{j,y} \times ke_{t,j,y} + \sum_t \sum_j W^m_j \times bm_{t,j} + \sum_t \sum_j TE^m_{t,j} \times f_{t,j} -$$
$$\sum_s \sum_t \sum_i \sum_k M_{t,i,k} \times \left((se_{t,i,k} + \left((RT_{s,t,i,k} \times x_{s,t}) \times (1 - \tilde{P}_{i,k}) \right) \right)$$

$$Max\ z_3 = (z_1{}^m - z_1{}^o) = \sum_s \sum_t \sum_i \sum_k ((CA^m_{s,i,k} - CA^o_{s,i,k}) \times RT_{s,t,i,k}) \times x_{s,t} +$$
$$\sum_t \sum_i \sum_k (O^m_{i,k} - O^o_{i,k}) \times g_{t,i,k} + \sum_t \sum_i \sum_k (CS^m_{i,k} - CS^o_{i,k}) \times se_{t,i,k} + \tag{3.26}$$
$$\sum_t \sum_i \sum_k (CH^m_{i,k} - CH^o_{i,k}) \times h_{t,i,k} + \sum_t \sum_i \sum_k (R^m_{i,k} - R^o_{i,k}) \times ie_{t,i,k} +$$
$$\sum_t \sum_j \sum_y (Q^m_{j,y} - Q^o_{j,y}) \times e_{t,j,y} + \sum_t \sum_j \sum_y (U^m_{j,y} - U^o_{j,y}) \times je_{t,j,y} +$$
$$\sum_t \sum_j \sum_y (V^m_{j,y} - V^o_{j,y}) \times ke_{t,j,y} + \sum_t \sum_j (W^m_j - W^o_j) \times bm_{t,j} + \sum_t \sum_j (TE^m_{t,j} - TE^o_{t,j}) \times f_{t,j} -$$

$$\sum_{s}\sum_{t}\sum_{i}\sum_{k} M_{t,i,k} \times \left((se_{t,i,k} + \left(\left(RT_{s,t,i,k} \times x_{s,t}\right) \times \left(1 - \widetilde{\boldsymbol{P}}_{i,k}\right)\right)\right)$$

$$Min\ z_4 = ({z_1}^p - {z_1}^m) = \sum_{s}\sum_{t}\sum_{i}\sum_{k}((CA^{p}_{s,i,k} - CA^{m}_{s,i,k}) \times RT_{s,t,i,k}) \times x_{s,t} +$$

$$\sum_{t}\sum_{i}\sum_{k}(O^{p}_{i,k} - O^{m}_{i,k}) \times g_{t,i,k} + \sum_{t}\sum_{i}\sum_{k}(CS^{p}_{i,k} - CS^{m}_{i,k}) \times se_{t,i,k} + \quad (3.27)$$

$$\sum_{t}\sum_{i}\sum_{k}(CH^{p}_{i,k} - CH^{m}_{i,k}) \times h_{t,i,k} + \sum_{t}\sum_{i}\sum_{k}(R^{p}_{i,k} - R^{m}_{i,k}) \times ie_{t,i,k} +$$

$$\sum_{t}\sum_{j}\sum_{y}(Q^{p}_{j,y} - Q^{m}_{j,y}) \times e_{t,j,y} + \sum_{t}\sum_{j}\sum_{y}(U^{p}_{j,y} - U^{m}_{j,y}) \times je_{t,j,y} +$$

$$\sum_{t}\sum_{j}\sum_{y}(V^{p}_{j,y} - V^{m}_{j,y}) \times ke_{t,j,y} + \sum_{t}\sum_{j}(W_j^{p} - W_j^{m}) \times bm_{t,j} + \sum_{t}\sum_{j}(TE^{p}_{t,j} - TE^{m}_{t,j}) \times f_{t,j} -$$

$$\sum_{s}\sum_{t}\sum_{i}\sum_{k} M_{t,i,k} \times \left((se_{t,i,k} + \left(\left(RT_{s,t,i,k} \times x_{s,t}\right) \times \left(1 - \widetilde{\boldsymbol{P}}_{i,k}\right)\right)\right)$$

Step 2: The uncertain constraint functions in the original model are also converted into crisp constraints. For constraints (3.3), (3.5), (3.6) and (3.8) with uncertain coefficients only on one side of the function, the weighted average method is used. 3 weight values are associated to the triangular possibility distribution of each uncertain coefficient. The weight $w_1 = 4/6$, is associated to the most likely value and weights $w_2 = w_3 = 1/6$ are associated to the most optimistic and most pessimistic values. With a minimum acceptable possibility β given by the decision maker, the uncertain constraints are converted to equations (3.28) - (3.31) below:

$$\textstyle\sum_s \left(RT_{s,t,i,k} \times x_{s,t}\right) \times (P^{m}_{i,k,\beta} \times w_1 + P^{o}_{i,k,\beta} \times w_2 + P^{p}_{i,k,\beta} \times w_3) = g_{t,i,k} + ie_{t,i,k} + se_{t,i,k} + h_{t,i,k} - h_{t-1,i,k}, \quad \forall t, \forall i, \forall k \quad (3.28)$$

$$\textstyle\sum_y e_{t,j,y} = (D^{m}_{t,j,\beta} \times w_1 + D^{o}_{t,j,\beta} \times w_2 + D^{p}_{t,j,\beta} \times w_3) - f_{t,j} - bm_{t,j} + bm_{t-1,j}, \quad \forall t, \forall j \quad (3.29)$$

$$\sum_s((RT_{s,t,i,k} \times x_{s,t}) \times (P^m_{i,k,\beta} \times w_1 + P^o_{i,k,\beta} \times w_2 + P^p_{i,k,\beta} \times w_3)) \times DC_{i,k} \leq ie_{t,i,k}, \quad \forall t, \forall i, \forall k \quad (3.30)$$

$$bm_{t,j} \leq UB_t \times (D^m_{t,j,\beta} \times w_1 + D^o_{t,j,\beta} \times w_2 + D^p_{t,j,\beta} \times w_3), \quad \forall t, \forall j \quad (3.31)$$

For constraints (3.13) and (3.14) with uncertain coefficients on both sides, the fuzzy ranking method is used to convert them into crisp constraints. 3 equivalent constraints substitute the original uncertain constraint. With the minimum acceptable possibility β given by the decision maker, the new constraint functions are:

$$\sum_{i,k} TC^m_{i,k,\beta} \times g_{t,i,k} \leq HD^m_{t,\beta}, \quad \forall t \quad (3.32)$$

$$\sum_{i,k} TC^o_{i,k,\beta} \times g_{t,i,k} \leq HD^o_{t,\beta}; \quad \forall t \quad (3.33)$$

$$\sum_{i,k} TC^p_{i,k,\beta} \times g_{t,i,k} \leq HD^p_{t,\beta}; \quad \forall t \quad (3.34)$$

$$\sum_{j,y} TM^m_{j,y,\beta} \times e_{t,j,y} \leq HR^m_{t,\beta}, \quad \forall t \quad (3.35)$$

$$\sum_{j,y} TM^o_{j,y,\beta} \times e_{t,j,y} \leq HR^o_{t,\beta}, \quad \forall t \quad (3.36)$$

$$\sum_{j,y} TM^p_{j,y,\beta} \times e_{t,j,y} \leq HR^p_{t,\beta}, \quad \forall t \quad (3.37)$$

Step 3: The auxiliary multi-objective LP model obtained is converted into an auxiliary, equivalent LP model using the max-min operator method of Zimmermann (1978). This method associates the objective functions with fuzzy sets defined by their linear membership functions. First, the positive ideal solutions (PIS) and negative ideal solutions (NIS) that represents the maximum and minimum limits of the linear membership functions are obtained.

$$z_2^{PIS} = \text{Min } z_1{}^m, \quad z_2^{NIS} = \text{Max } z_1{}^m, \quad (3.38a)$$

$$z_3^{PIS} = \text{Max}\,(z_1{}^m - z_1{}^o), \quad z_3^{NIS} = \text{Min}(z_1{}^m - z_1{}^o), \quad (3.38b)$$

$$z_4^{PIS} = \mathrm{Min}(z_1{}^{p} - z_1{}^{m}), \quad z_4^{NIS} = \mathrm{Max}(z_1{}^{p} - z_1{}^{m}), \tag{3.38c}$$

Then, the linear membership functions for each of the three auxiliary objective functions are specified:

$$\mu_{z_2} = \begin{cases} 1, & if \quad z_2 < z_2^{PIS} \\ \frac{z_2^{NIS} - z_2}{z_2^{NIS} - z_2^{PIS}} & if \quad z_2^{PIS} \le z_2 \le z_2^{NIS} \\ 0, & if \quad z_2 > z_2^{NIS} \end{cases} \tag{3.39}$$

$$\mu_{z_3} = \begin{cases} 1, & if \quad z_3 > z_3^{PIS} \\ \frac{z_3 - z_3^{NIS}}{z_3^{PIS} - z_3^{NIS}} & if \quad z_3^{NIS} \le z_3 \le z_3^{PIS} \\ 0, & if \quad z_3 < z_3^{NIS} \end{cases} \tag{3.40}$$

$$\mu_{z_4} = \begin{cases} 1, & if \quad z_4 < z_4^{PIS} \\ \frac{z_4^{NIS} - z_4}{z_4^{NIS} - z_4^{PIS}} & if \quad z_4^{PIS} \le z_4 \le z_4^{NIS} \\ 0, & if \quad z_4 > z_4^{NIS} \end{cases} \tag{3.41}$$

The graphics of linear membership functions (3.39) - (3.41) are shown in Figure 3.4.

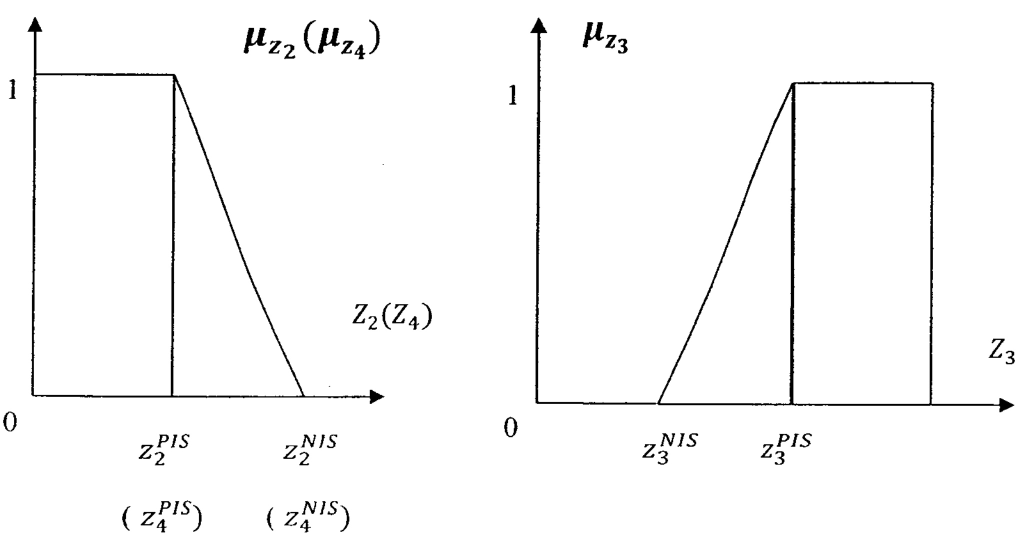

Figure 3.4.The linear Membership Functions of z_2, z_3 and z_4

Based on equation (3.24), the auxiliary ordinary LP model using the minimum operator L is formulated as below:

$MAX\ L$

$s.t.$

$L \leq \mu_{z_i}\ i = 2,3,4$

$\sum_s\ x_{s,t} \leq 1,\ \forall t$

$\sum_s(RT_{s,t,i,k} \times x_{s,t}) \times (P^m_{i,k,\beta} \times w_1 + P^o_{i,k,\beta} \times w_2 + P^p_{i,k,\beta} \times w_3) = g_{t,i,k} + ie_{t,i,k} + se_{t,i,k} + h_{t,i,k} - h_{t-1,i,k},$

$\forall t, \forall i, \forall k$

$\sum_{i,k}\ NM_{j,y,i,k} \times g_{t,i,k} = e_{t,j,y} + ke_{t,j,y} + je_{t,j,y} - je_{t-1,j,y},\quad \forall t, \forall j, \forall y$

$e_{t,j,y} = (D^m_{t,j,\beta} \times w_1 + D^o_{t,j,\beta} \times w_2 + D^p_{t,j,\beta} \times w_3) + f_{t,j} + bm_{t,j} - bm_{t-1,j},\quad \forall t, \forall j$

$\sum_s((RT_{s,t,i,k} \times x_{s,t}) \times (P^m_{i,k,\beta} \times w_1 + P^o_{i,k,\beta} \times w_2 + P^p_{i,k,\beta} \times w_3)) \times DC_{i,k} \leq ie_{t,i,k},\quad \forall t, \forall i, \forall k$

$\sum_{i,k}(NM_{j,y,i,k} \times g_{t,i,k}) \times PD_{j,y} \leq ke_{t,j,y},\quad \forall t, \forall j, \forall y$

$bm_{t,j} \leq UB_t \times (D^m_{t,j,\beta} \times w_1 + D^o_{t,j,\beta} \times w_2 + D^p_{t,j,\beta} \times w_3),\quad \forall t, \forall j$

$\sum_{t,i,k}\ (VC_i \times h_{t,i,k}) \leq SC,$

$\sum_{t,j}\ (VM_j \times je_{t,j}) \leq SM,$

$\sum_{i,k}\ se_{t,i,k} \leq SA_t,\quad \forall t$

$\sum_j f_{t,j} \leq PC_t,\quad \forall t$

$\sum_{i,k} TC^m_{i,k,\beta} \times g_{t,i,k} \leq HD^m_{t,\beta},\quad \forall t$

$\sum_{i,k} TC^o_{i,k,\beta} \times g_{t,i,k} \leq HD^o_{t,\beta},\quad \forall t$

$\sum_{i,k} TC^{p}_{i,k,\beta} \times g_{t,i,k} \leq HD^{p}_{t,\beta}, \qquad \forall t$

$\sum_{j,y} TM^{m}_{j,y,\beta} \times e_{t,j,y} \leq HR^{m}_{t,\beta}, \qquad \forall t$

$\sum_{j,y} TM^{o}_{j,y,\beta} \times e_{t,j,y} \leq HR^{o}_{t,\beta}, \qquad \forall t$

$\sum_{j,y} TM^{p}_{j,y,\beta} \times e_{t,j,y} \leq HR^{p}_{t,\beta}, \qquad \forall t$

$bm_{0,j} = 0 \qquad \forall j$

$bm_{T,j} = 0 \qquad \forall j$

$h_{0,i,k} = 0 \qquad \forall i, \forall k$

$je_{0,j,y} = 0 \qquad \forall j, \forall y$

$g_{t,i,k}, se_{t,i,k}, ie_{t,i,k}, h_{t,i,k}, e_{t,j,y}, ke_{t,j,y}, je_{t,j,y}, bm_{t,j}, f_{t,j}, \geq 0 \in R^{+}_{0}$

$x_{s,t}, \in (0,1)$

$0 \leq L \leq 1$

The auxiliary variable L may be considered as the overall degree of satisfaction for the goals established.

Step 4: The auxiliary model formulated in Step 3 is solved using linear programming. If the decision maker is not satisfied with the initial set of solutions, the uncertain parameters may be interactively adjusted for several iterations until a satisfactory solution is obtained.

The optimization software Lingo 10 is used to code and solve the auxiliary equivalent model on a PC of 2.5 GHZ processor and 1.5 GB RAM. Key decisions such as the number of disassembled, disposed or salvaged returned products and the number of remanufactured, disposed or held in inventory components are obtained in the final results.

Chapter Four

Numerical Examples and Analysis

4.1 Example Problem

An example problem is presented to test the validity and practicability of the proposed model and solution methodology presented in Chapter 3. This hypothetical example problem is based on the example given in Nikolaidis (2009). Certain adjustments were made in the data to fit in the considered problem in this research.

The original example problem was to develop optimal aggregate production plans in a medium-size company producing and remanufacturing electronic appliances. The goal of the mathematical model is to minimize the total remanufacturing cost considering some uncertain parameters such as demands, functional costs, capacities and rates.

In solving this example problem, we consider that 3 types of returned products can be acquired from 3 different suppliers with 3 types of components to be obtained from the disassembly of the returned products. Returned products and components are categorized in 2 different levels of quality. The number of time periods for the aggregate production planning is 5.

4.2 Example Problem Data

Tables 4.1 to 4.6 present the crisp values of the parameters for the example problem. Triangular possibility distributions used to model the uncertain parameters such as demands and operational costs are presented in Appendix A.

Table 4.1.Operational Costs

	Disassembling Cost (O)		Inventory Cost (H)		Salvaging Cost (CS)		Disposing Cost (R)	
	Quality 1	Quality 2	Quality 1	Quality 2	Quality 1	Quality 2	Quality 1	Quality 2
Product type 1	6.0	9.0	1.0	1.0	5.0	8.0	7.0	7.0
Product type 2	8.0	10.0	1.5	1.5	6.0	7.0	8.0	8.0
Product type 3	13.0	15.0	2.5	2.5	9.0	10.0	10.0	10.0
	Remanufacturing Cost (Q)		Inventory Cost (U)		Disposing Cost (V)		Backlogging Cost (W)	
	Quality 1	Quality 2	Quality 1	Quality 2	Quality 1	Quality 2		
Component type 1	11.0	15.0	1.6	1.6	4.0	4.0	9.0	
Component type 2	13.0	17.0	2.9	2.9	7.0	7.0	14.0	
Component type 3	15.0	19.0	3.3	3.3	8.0	8.0	19.0	

Cost of New Components Purchased from Third-Party Suppliers (TE)					
	Period 1	Period 2	Period 3	Period 4	Period 5
Component type 1	80.0	88.0	80.0	78.0	80.0
Component type 2	85.0	83.0	85.0	75.0	85.0
Component type 3	88.0	89.0	88.0	78.0	88.0

Table 4.2.Operational Times, Sizes and Damaged Fractions

	Time to Remanufacture a Component (hrs.)		Fraction of Damaged Components		Components Sizes (Cubic Feet)
	Quality 1	Quality 2	Quality 1	Quality 2	
Component type 1	0.15	0.18	0.05	0.10	0.5
Component type 2	0.21	0.23	0.05	0.10	0.8
Component type 3	0.25	0.28	0.05	0.10	1.0
	Time to Disassembly a Product (hrs.)		Fraction of Damaged Products		Products Sizes (Cubic Feet)
	Quality 1	Quality 2	Quality 1	Quality 2	
Product type 1	0.16	0.20	0.05	0.10	1.0
Product type 2	0.18	0.23	0.05	0.10	1.2
Product type 3	0.22	0.27	0.05	0.10	1.4

Table 4.3.Process Capacities

Products Salvaging (SA)	New Components Purchasing (PC)	Products Storage (SC)	Components Storage (SM)	Hours Available Remanufacturing in a Period (HR)	Hours Available Disassembling in a Period (HD)	Backlogs Limit Fraction (UB)
400.0	700.0	500.0	500.0	500.0	500.0	0.35

Table 4.4.Returned Products Offered from Suppliers

		Supplier 1		Supplier 2		Supplier 3	
		Quality 1	Quality 2	Quality 1	Quality 2	Quality 1	Quality 2
Period 1	Product type 1	800	300	700	200	800	300
	Product type 2	150	1300	900	150	150	1300
	Product type 3	100	600	100	900	100	600
Period 2	Product type 1	700	400	700	200	200	900
	Product type 2	1100	160	1000	100	400	700
	Product type 3	1000	100	300	400	400	600
Period 3	Product type 1	800	400	200	900	800	300
	Product type 2	600	200	400	700	150	1300
	Product type 3	800	300	400	600	100	600
Period 4	Product type 1	800	300	200	1000	200	1000
	Product type 2	150	1300	100	900	100	900
	Product type 3	100	600	400	700	400	700
Period 5	Product type 1	700	400	200	900	800	300
	Product type 2	1100	160	400	700	150	1300
	Product type 3	1000	100	400	600	100	600

Table.4.5.Number of Components Allocated in Products

		Component 1		Component 2		Component 3	
		Qua. 1	Qua. 2	Qua. 1	Qua. 2	Qua. 1	Qua. 2
Product 1	Quality 1	3	-	2	-	2	-
	Quality 2	-	3	-	2	-	2
Product 2	Quality 1	2	-	1	-	2	-
	Quality 2	-	2	-	1	-	2
Product 3	Quality 1	3	-	2	-	2	-
	Quality 2	-	3	-	2	-	2

Table 4.6.Other Parameters

Demand for Remanufactured Components (D)					
	Period 1	Period 2	Period 3	Period 4	Period 5
Component type 1	1,600	1,800	1,400	1,800	1,200
Component type 2	1,600	1,800	1,400	1,800	1,200
Component type 3	1,600	1,800	1,400	1,800	1,200

Revenue from Salvaged Products (M)										
	Period 1		Period 2		Period 3		Period 4		Period 5	
	Qua. 1	Qua. 2	Qua. 1	Qua. 2	Qua. 1	Qua. 2	Qua. 1	Qua. 2	Qua. 1	Qua. 2
Product type 1	37.0	33.0	37.0	34.0	37.0	33.0	37.0	33.0	37.0	33.0
Product type 2	39.0	34.0	35.0	35.0	36.0	36.0	34.0	33.0	37.0	34.0
Product type 3	39.0	33.0	37.0	35.0	37.0	37.0	35.0	34.0	37.0	33.0

	Returned Products Acquisition Cost (CA)				Fraction of Remanufacturable Products (P)
		Supplier 1	Supplier 2	Supplier 3	
Product type 1	Quality 1	78.0	77.0	68.0	0.90
	Quality 2	77.0	68.0	63.0	0.85
Product type 2	Quality 1	71.0	69.0	79.0	0.90
	Quality 2	67.0	67.0	69.0	0.85
Product type 3	Quality 1	63.0	73.0	74.0	0.90
	Quality 2	59.0	71.0	71.0	0.85

4.3 Solutions and Analysis

The methodology explained in Chapter 3 is applied to solve the example problem. To represent the limits of the linear membership functions, the positive ideal solutions (PIS) and negative ideal solutions (NIS) are specified. In practice they are obtained based on the single LP solutions of each auxiliary objective function. The linear membership functions for the three auxiliary functions are defined according to equations (3.39) – (3.41). Table 4.7 presents the initial positive ideal solutions and negative ideal solutions based on the LP solutions.

Table 4.7.The initial PIS and NIS for Auxiliary Objective Functions

	LP Solutions		
Objective Function	**Min**	**Max**	**Initial (PIS, NIS)**
z_2 ($)	841,270	1,853,946	(841,270, 1,853,946)
z_3 ($)	7,834	406,607	(406,607, 7,834)
z_4 ($)	-30,776	402,597	(-30,776, 402,597)

The complete equivalent ordinary LP model is obtained using the Zimmermann (1978) max-min operator method according to equation (3.24).

4.4 Numerical Results

Solving the auxiliary model by using Lingo 10.0, an initial set of solutions $\tilde{z}_1$ = (1,256,350, 243,156, 146,856) with overall satisfaction degree L = 0.5901 is obtained. We assume that this solution is not accepted by the decision maker. The solution method requires the interactive adjustment of the positive ideal solutions (PIS) and negative ideal solutions (NIS) to find a better set of solutions. Table 4.8 shows the results of 12 iterations run with different values assigned to the PIS and NIS.

Table 4.8.Solutions of Iterations

Iteration	z_2		z_3		z_4		*L* Value
	PIS	NIS	PIS	NIS	PIS	NIS	(In 000 units)
0	1,256,350		243,156		146,856		0.5901
	841.2	1,853.9	7.8	406.6	-30.7	402.5	

Table 4.8.Solutions of Iterations (Continued)

Iteration	z_2		z_3		z_4		*L* Value
	PIS	NIS	PIS	NIS	PIS	NIS	(In 000 units)
1	1,045,175		231,930		180,844		0.5548
	600	1,600	410	10	5	400	
2	1,078,805		235,851		174,148		0.5791
	70	1,600	400	10	10	400	
3	1,063,307		225,158		164,405		0.5468
	700	1,500	400	15	10	350	
4	1,004,938		203,123		151,975		0.6600
	750	1,500	300	15	50	350	
5	975,552		186,174		153,332		0.7284
	780	1,500	250	15	80	350	
6	944,948		161,811		149,775		0.7935
	800	1,500	200	15	100	350	
7	932,812		150,238		138,317		0.8196
	830	1,400	180	15	110	350	
8	912,787		130,392		118,801		0.8547
	830	1,400	150	15	150	350	
9	900,738		118,450		107,072		0.8995
	845	1,400	130	15	80	350	
10	942,363		124,737		64,076		0. 8552
	848	1,500	145	5	30	300	
11	876,424		72,154		38,212		0.9593
	850	1,500	80	10	25	350	
12	861,284		63,958		34,687		0.9826
	850	1,500	65	5	30	300	

As can be seen in Table 4.8, a more efficient solution with better overall satisfaction degree is achieved. Therefore, the initial positive ideal solutions and negative ideal solutions are changed from (841,270, 1,853,946), (406,607, 7,834) (-30,776, 402,597) to (850,000, 1,500,000), (65,000, 5,000), (30,000, 300,000) respectively. The improved results are presented in Tables 4.9 to 4.16.

Table 4.9.Improved Set of Solutions

Improved Solutions	z_2	z_3	z_4	L value
	861,284	63,958	34,687	0.9826
$\tilde{z}_1$ = 861,284, 797,326, 895,971				

Note: *$\tilde{z}_1 = (z_2, z_2 - z_3, z_2 + z_4)$

Table 4.10.Disassembled Products

$g_{1,1,1}$	$g_{1,2,1}$	$g_{2,1,1}$	$g_{2,2,1}$	$g_{2,3,1}$	$g_{3,1,1}$	$g_{4,1,1}$	$g_{4,1,2}$	$g_{4,2,1}$	$g_{4,2,2}$	$g_{4,3,1}$	$g_{5,1,1}$	$g_{5,1,2}$
222.0	27.5	96.7	304.5	119.0	202.5	60.0	110.5	42.7	21.0	171.0	25.4	151.2

Table 4.11.Disposed Products

$ie_{1,1,1}$	$ie_{1,1,2}$	$ie_{1,2,1}$	$ie_{1,2,2}$	$ie_{1,3,1}$	$ie_{1,3,2}$	$ie_{2,1,1}$	$ie_{2,1,2}$	$ie_{2,2,1}$
92.9	85.0	20.2	63.7	2.2	382.5	15.7	8.5	22.5
$ie_{2,2,2}$	$ie_{2,3,1}$	$ie_{2,3,2}$	$ie_{4,1,1}$	$ie_{4,1,2}$	$ie_{4,2,1}$	$ie_{4,2,2}$	$ie_{4,3,1}$	$ie_{4,3,2}$
4.2	6.7	17.0	4.5	42.5	2.2	229.2	9.0	29.7

Table 4.12.Salvaged and Held in Inventory Products

$se_{1,2,1}$	$se_{1,3,1}$	$se_{2,1,2}$	$se_{2,2,2}$	$se_{2,3,1}$	$se_{2,3,2}$	$se_{4,2,2}$
357.2	42.7	76.5	38.2	9.2	153.0	132.5
$se_{4,3,2}$	$se_{5,1,2}$			$h_{2,1,1}$	$h_{4,1,1}$	$h_{4,1,2}$
267.7	120.8			202.5	25.4	271.9

Table 4.13.Remanufactured Components

$e_{1,1,1}$	$e_{1,2,1}$	$e_{1,3,1}$	$e_{2,1,1}$	$e_{2,2,1}$	$e_{2,3,1}$	$e_{3,1,1}$	$e_{3,2,1}$	$e_{3,3,1}$	$e_{4,1,1}$
685.1	448.0	685.0	916.6	699.1	334.6	716.7	384.8	716.6	601.4
$e_{4,1,2}$	$e_{4,2,1}$	$e_{4,2,2}$	$e_{4,3,1}$	$e_{5,1,1}$	$e_{5,1,2}$	$e_{5,2,1}$	$e_{5,2,2}$	$e_{5,3,1}$	$e_{5,3,2}$
143.2	479.6	195.7	424.6	72.5	601.1	48.3	294.2	225.1	408.2

Table 4.14.Disposed Components

$ke_{1,1,1}$	$ke_{1,2,1}$	$ke_{1,3,1}$	$ke_{2,1,1}$	$ke_{2,2,1}$	$ke_{2,3,1}$	$ke_{3,1,1}$	$ke_{3,2,1}$	$ke_{3,3,1}$	$ke_{4,1,1}$	$ke_{4,1,2}$
36.0	23.6	36.0	56.8	36.8	663.0	30.4	20.2	30.4	30.4	37.3
$ke_{4,2,1}$	$ke_{4,2,2}$	$ke_{4,3,1}$	$ke_{4,3,2}$	$ke_{5,1,1}$	$ke_{5,1,2}$	$ke_{5,2,1}$	$ke_{5,2,2}$	$ke_{5,3,1}$	$ke_{5,3,2}$	
25.2	24.2	30.4	373.6	3.8	45.3	2.5	30.2	3.8	45.3	

Table 4.15.Held in Inventory and Backlogged Components

$je_{2,1,1}$	$je_{2,3,1}$	$je_{3,1,1}$	$je_{4,1,2}$	$je_{4,2,2}$	$je_{4,3,1}$		$bm_{2,2}$	$bm_{4,1}$
163.5	139.4	24.2	192.9	22.0	152.6		99.5	155.3

Table 4.16.New Purchased Components

$f_{1,1}$	$f_{1,2}$	$f_{1,3}$	$f_{2,2}$	$f_{2,3}$	$f_{3,2}$	$f_{4,2}$	$f_{4,3}$	$f_{5,1}$	$f_{5,2}$
131.6	368.6	131.6	118.0	582.0	431.4	224.6	475.3	114.9	290.7

The comparison between the initial and improved solutions is presented in Figure 4.1.

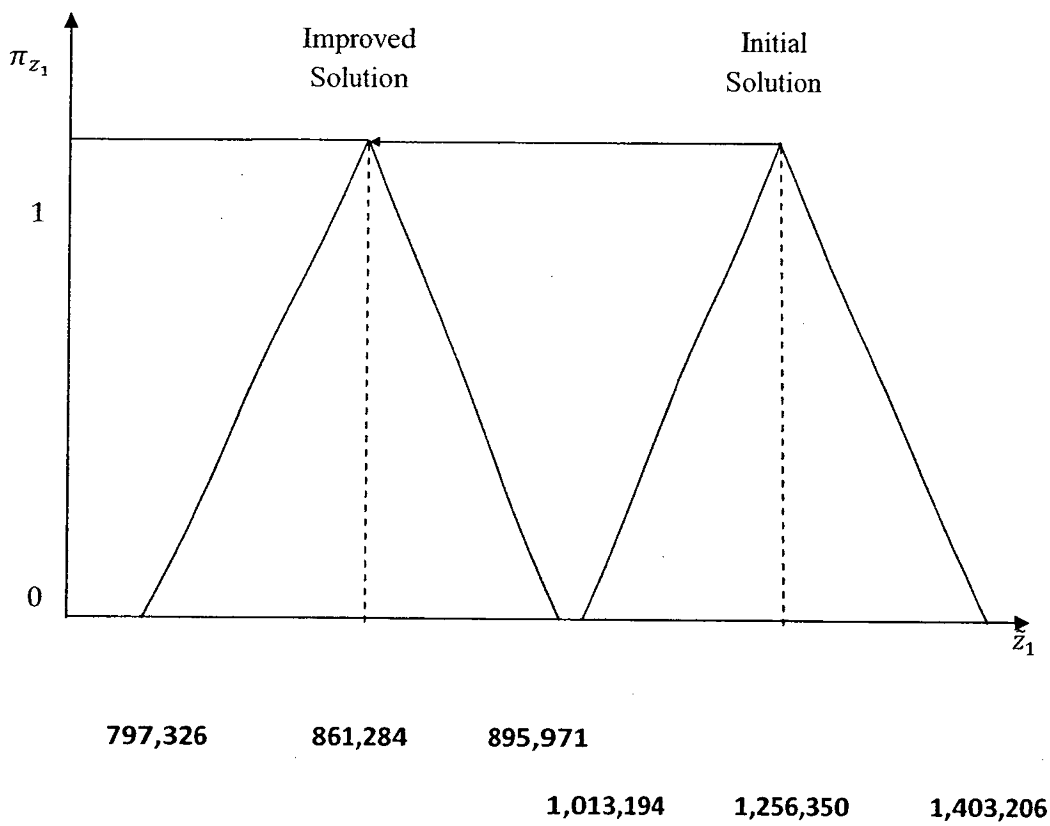

Figure 4.1.Comparison of Initial and Final Set of Solutions

The effects of different batch sizes of returned products and different remanufacturable fractions of products on the total production cost are analyzed. Some of the computational results are presented in Figure 4.2

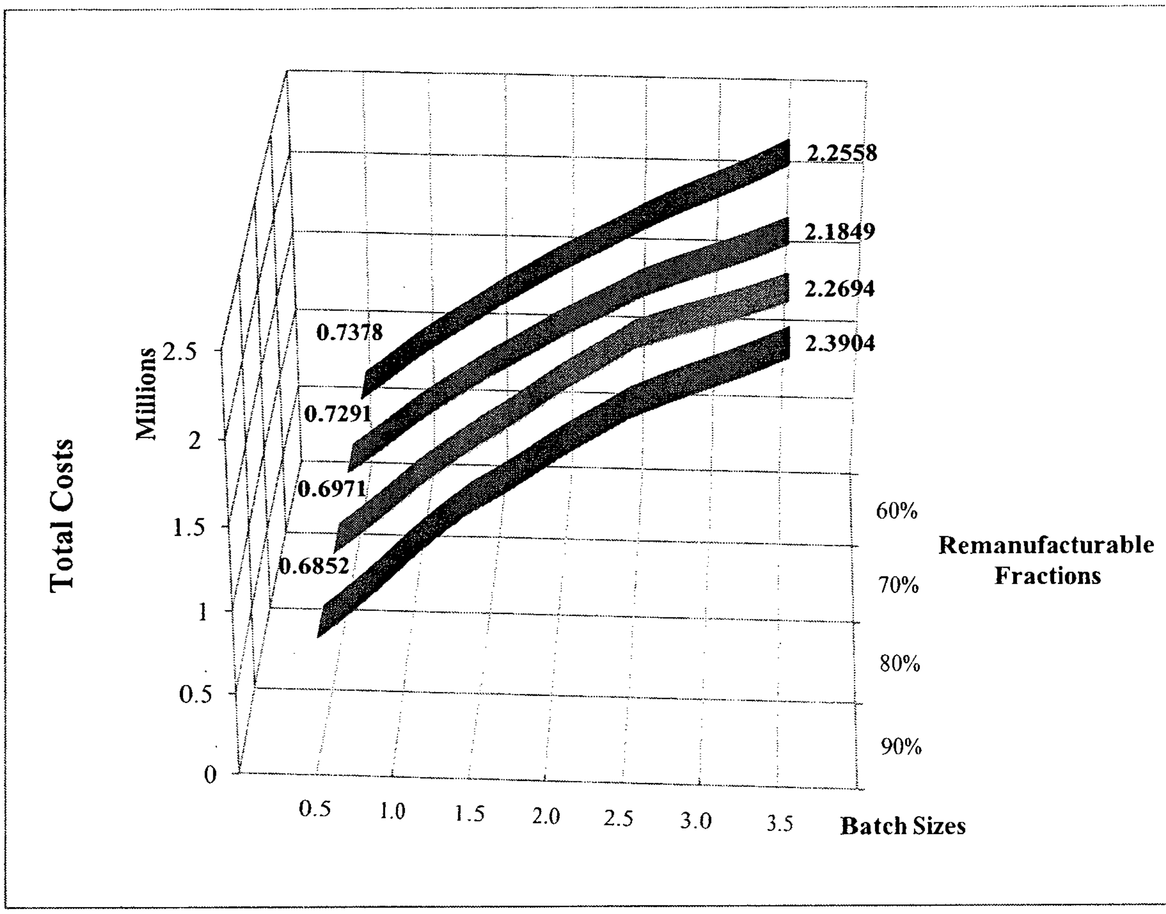

Figure 4.2.Different Batch Sizes and Remanufacturable Fractions

In Figure 4.2 the horizontal axes correspond to the combination of the different batch sizes of products and remanufacturable fractions. We can observe that when the batch size is 0.5, the remanufacturable fraction 0.90 corresponds to the least expensive option with a cost of 0.7378 millions. As the batch size increase to 3.5 times bigger, the remanufacturable fraction 0.70 corresponds to the least expensive option with a cost of 2.1849 millions.

To test the proposed mathematical model in different situations, we run the model for 3 different scenarios regarding the quality of the retuned products with the same data set.

Scenario 1. Quality level of the returned products offered by the suppliers is mixed with about 50% high quality products and the rest low quality returned products

Scenario 2. The suppliers offer a better quality mix with about 80% high quality returned products.

Scenario 3. The suppliers offer a poor quality mix with about 20% high quality returned products.

The model was run for several iterations for each of the scenarios.

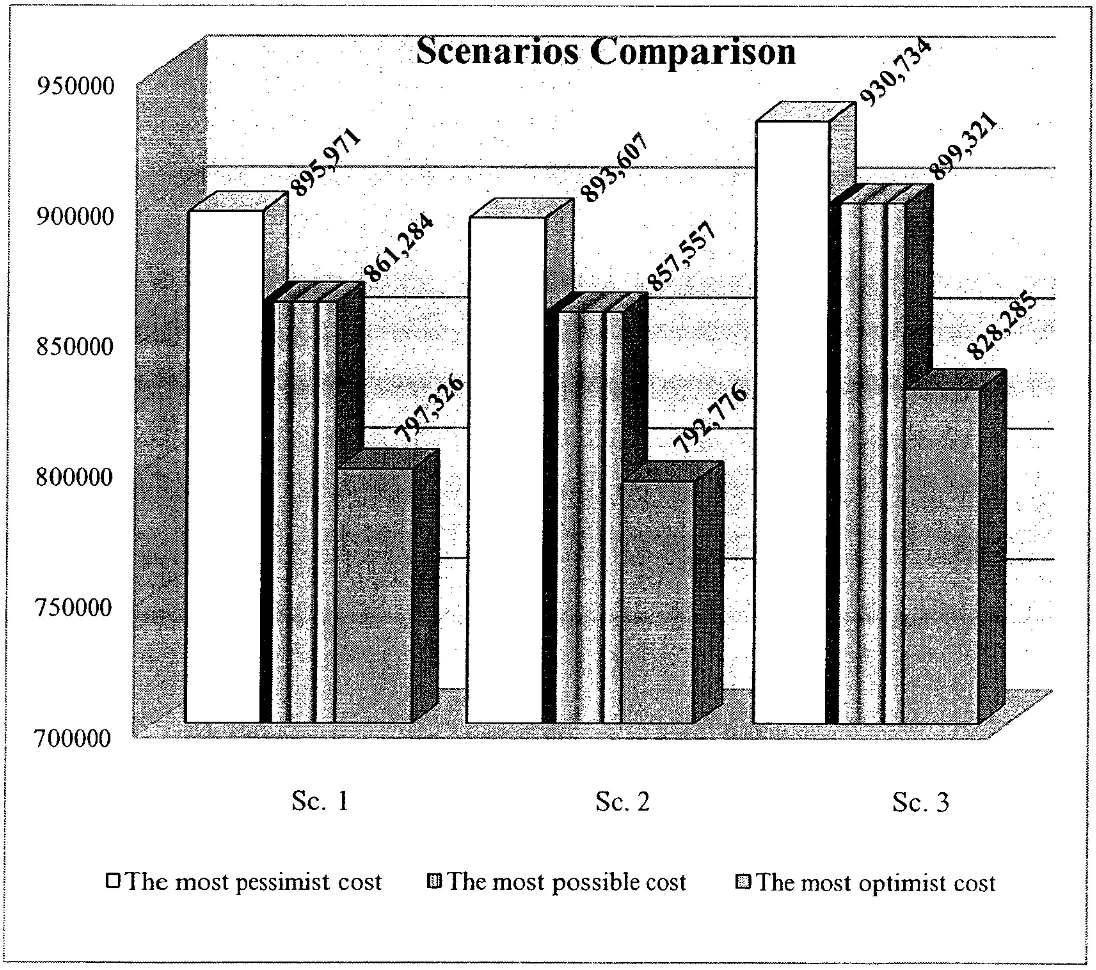

Figure 4.3.Comparison Between the 3 Scenarios.

The solutions corresponding to the best L values (0.9826, 0.9688, and 0.9960) in each scenario are shown in figure 4.2. It is observed that the second scenario yields the set of solutions representing the lowest total cost. Therefore, selecting batches containing a big rate of high quality returned products may represent a better alternative over the other 2. Even thought high quality returned products are more expensive they represent less operational costs compared to low quality returned products.

4.5 Summary

1. Using the presented solution methodology can yield efficient solutions. The main advantage of this method is that the decision maker may interactively adjust the search area until a satisfactory efficient solution is obtained.

2. To find a better solution, the search area is changed by adjusting the values of the positive and negative ideal solutions. As observed in the example, changes in PIS and NIS from z_1 (841,270, 1,853,946), z_2 (406,607, 7,834) and z_3 (-30,776, 402,597) to z_1 (850,000, 1,500,000), z_2 (65,000, 5,000), z_3 (30,000, 300,000) generate a better solution with higher L value. In practice, the ideal solutions may be estimated based on the experience and knowledge of the decision maker.

3. The comparison of the most possible final cost value (861,284) to the optimal value obtained solving the crisp single-objective LP (841,270), shows the efficiency of the solving method. To use LP model we considered the most possible parameters as crisp parameters.

4. The selection of the offered batches depends on different factors such as demands or quality of the returned products. At the fourth time period in the example problem solutions, results of Lingo 10 indicate that the first supplier is not selected even when it offers the less expensive batch with a cost of $ 224,950. Instead of that the third supplier was selected offering a batch with higher cost of $ 225,900.

Chapter Five

Conclusion

This research presents further development from the work in Jayaraman (2006) for production planning and control of closed-loop supply chains with product recovery and reuse. A mathematical programming model for remanufacturing aggregate production planning (RAPP) was developed in Jayaraman (2006). In this thesis, we present a possibilistic mixed integer linear programming model for production planning and control in remanufacturing and recovery systems. Certain parameters in the objective and constraint functions are considered uncertain as often they are in practical remanufacturing systems. The proposed model is flexible and can be easily modified for different recovery and remanufacturing applications.

The solution methodology presented in Liang (2007) is used to solve the developed model. It converts the uncertain objective function into 3 auxiliary functions. Also, the uncertain constraints are transformed into crisp constraints using weighted average method and fuzzy ranking method. The strategy consists of minimizing the most possible total cost, maximizing the possibility of obtaining a low total cost and minimizing the risk of getting a high total cost. If the obtained solution is not satisfactory, the solving method allows the interactive adjustments of the parameters for several iterations until a satisfactory solution is obtained.

A numerical example from the literature was used with different scenarios to test the model and solution methodology extensively. The results show that the developed model is valid and the solution method is effective.

The main contributions of this research are: the consideration of uncertain parameters in the model and the solution method based on possibility linear programming. The model is designed to support remanufacturing systems managers in medium to long aggregate production planning. The model may be used as a framework in development more formal systems for production planning, inventory control and end-of-life product recovery. The model is made for general recovery and remanufacturing systems, therefore its use is not limited to a specific area in the remanufacturing industry.

The further research of this thesis includes:

- Other objective for the single-objective linear programming model may be considered.
- A more detailed quality categorization for the returned products and components can be developed.
- The uncertain parameters can be modeled with other possibility distributions such as trapezoid or exponential.
- More attributes may be considered for the selection of the returned product suppliers.
- Other methodologies such as goal programming or utility theory may be used to solve the crisp multi-objective auxiliary LP model.

References

1. N. Aras, T. Boyaci and V. Verter, 2004. The effect of categorizing returned products in remanufacturing. IIE Transactions, **36**, 319-331.
2. A. Clegg and D. Williams, 1995, Production planning for companies with remanufacturing Capability. Proceedings of the IEEE Symposium on Electronics and the Environment. Orlando, Florida, 186-191.
3. V. Daniel and R. Guide 2000. Production planning and control for remanufacturing: industry practice and research needs. International Journal of Operations Management, **18**, 467-483.
4. V. Daniel, R. Guide and V. Jayaraman, 2000. Product acquisition management: current industry practice and a propose framework. International Journal of Production Research, **38**, 3779-3800.
5. G. DePuy, J. Usher, R. Walker and G. Taylor, 2007. Production planning for remanufactured products. Production Planning and Control, **18**, 573-583.
6. M. Ferguson, V. Guide, E. Koca and G. Souza, 2009. The value of quality grading in remanufacturing. Production Operations Management, **18**, 300-314.
7. C. Franke, B. Basdere, M. Ciupek and S. Seliger, 2006. Remanufacturing of mobile phones-capacity, program and facility adaptation planning, Omega, **34**, 562 – 570.
8. M. Galbreth and J. Blackburn, 2006. Optimal acquisition and sorting policies for remanufacturing. Production and Operations Management, **15**, 384-392.
9. H. Hsu and W. Wang, 2001. Possibilistic programming in production planning of assemble-to-order environments. Fuzzy Sets and systems, **119**, 59-70.

10. K. Inderfurth and H. Teunter, 2001. Production Planning and control of closed-loop supply chains. Econometric Institute Report EI, **39**, 1-21.

11. M. Inuiguchi, M. Sakawaa and Y. Kumeb, 1994.The usefulness of possibilistic programming in production planning problems. International Journal of Production Economics, **33**, 45-52.

12. V. Jayaraman, 2006. Production planning for closed-loop supply chains with product recovery and reuse: an analytical approach. International Journal of Production Research, **44**, 981-998.

13. S. Jorjani, J. Leu and C. Scott, 2004. Model for the allocation of electronic components to reuse options. International Journal of Production Research, **42**, 1131-1145.

14. G. Kiesmuller, 2003. Optimal control of a one product recovery system with leadtimes. International Journal of Production Economics, **81-82**, 333-340.

15. K. Kim, I. Song, J. Kim and B. Jeong, 2006. Supply planning model for remanufacturing system in reverse logistics environment. Computers and Industrial Engineering, **51**, 279-287.

16. E. Kongara and M. Gupta, 2006. Disassembly to order system under uncertainty. Omega, **34**, 550-561.

17. Y. Lai and C. Wang, 1992. A new approach to some possibilistic linear programming problems. Fuzzy Sets and Systems, **49**, 121-133.

18. I. Langella, 2007. Heuristics for demand-driven disassembly planning. Computers and Operations Research, **34**, 552-577.

19. Y. Li, J. Chen and X. Cai, 2005. Uncapacitated production planning with multiple product types, returned product remanufacturing and demand substitution. OR Spectrum, **28**, 101-125.

20. T. Liang, 2007. Application of possibilistic linear programming to multi-objective distribution planning decisions. Journal of the Chinese Institute of Industrial Engineers, **24**, 97-109.

21. T. Liang and H. Cheng, 2009. On application of fuzzy sets to manufacturing/distribution planning decisions with multi-product and multi-time period in supply chains. Expert Systems Applications, **36**, 3367-3377.

22. C. Michaud and D. LLerena, 2006. An economic perspective on remanufactured products: industrial and consumption challenges for life cycle engineering. CIRP International Conference on Life Cycle Engineering, **13**, 543-548

23. J. Mula, R. Poler, J. Garcia and F. Lario, 2006. Models for production planning under uncertainty: A review. International Journal of Production Economics, **103**, 271-285.

24. Y. Nikolaidis, 2009. A modeling framework for the acquisition and remanufacturing of used products. International Journal of Sustainable Engineering, **1**, 1-17.

25. J. Ostlin, E. sundin and M. Bjorkman, 2008. Importance of closed-loop supply chain relationships for product remanufacturing. International Journal of Production Economics, **115**, 336-348.

26. K. Taleb and M. Gupta, 1997. Disassembly of complex product structures with parts and materials commonality. Production Planning and control, **8**, 255 – 269.

27. J. Tang, R. Fung and K. Yung, 2003. Fuzzy modeling and simulation for aggregate production planning. International Journal of Systems Science, **34**, 10-20.

28. J. Tang, D. Wang and R. Fung, 2000. Fuzzy formulation for multi-product aggregate production planning. Production Planning and Control, **11**, 670-676.

29. S. Torabi and E. Hassini, 2008. An interactive possibilistic programming approach for multiple objective supply chain master planning. Fuzzy Sets and Systems, **159**, 193-214.

30. R. Wang and T. Liang, 2004. Application of fuzzy multi-objective linear programming to aggregate production planning. Computers & Industrial Engineering, **46**, 17-41.

31. R. Wang and T. Liang, 2005. Applying possibilistic linear programming to aggregate production planning. International Journal of Production Economics, **98**, 328-341.

32. A. Xanthopoulos and E. Iakovou, 2009. On the optimal design of the disassembly and recovery processes. Waste Management, **29**, 1702 -1711.

33. C. Zikopoulos and G. Tagaras, 2007. Impact of uncertainty in the quality of returns on the profitability of a single-period refurbishing operation. European Journal of Operational Research, **182**, 205 – 225.

Appendix A

A.1 Uncertain Parameters Triangular Distributions

Table A.1.1 Operational Costs

	Products Disassembling Cost ($\tilde{O}$)						Products Inventory Cost ($\tilde{H}$)					
	Quality 1			Quality 2			Quality 1			Quality 2		
Product type 1	7.0	6.0	10.0	8.0	9.0	13.0	0.5	1.0	1.8	0.5	1.0	1.8
Product type 2	6.0	8.0	12.0	9.0	10.0	14.0	0.8	1.5	2.0	0.8	1.5	2.0
Product type 3	10.0	13.0	16.0	12.0	15.0	18.0	2.2	2.5	3.0	2.2	2.5	3.0
	Products Salvaging Cost ($\widetilde{CS}$)						Products Disposing Cost ($\tilde{R}$)					
	Quality 1			Quality 2			Quality 1			Quality 2		
Product type 1	3.0	5.0	8.0	6.0	8.0	9.0	4.0	7.0	9.0	4.0	7.0	9.0
Product type 2	5.0	6.0	10.0	5.0	7.0	11.0	6.0	8.0	11.0	6.0	8.0	11.0
Product type 3	7.0	9.0	11.0	9.0	10.0	12.0	8.0	10.0	13.0	8.0	10.0	13.0
	Components Remanufacturing Cost ($\tilde{Q}$)						Components Inventory Cost ($\tilde{U}$)					
	Quality 1			Quality 2			Quality 1			Quality 2		
Component type 1	8.0	11.0	19.0	12.0	15.0	21.0	1.2	1.6	2.0	1.2	1.6	2.0
Component type 2	11.0	13.0	23.0	14.0	17.0	24.0	2.3	2.9	3.1	2.3	2.9	3.1
Component type 3	12.0	15.0	24.0	16.0	19.0	27.0	2.6	3.3	3.6	2.6	3.3	3.6
	Components Disposing Cost ($\tilde{V}$)						Components Backlogging Cost ($\tilde{W}$)					
	Quality 1			Quality 2								
Component type 1	2.0	4.0	7.0	2.0	4.0	7.0	8.0		9.0		13.0	
Component type 2	5.0	7.0	9.0	5.0	7.0	9.0	10.0		14.0		16.0	
Component type 3	6.0	8.0	12.0	6.0	8.0	12.0	17.0		19.0		21.0	

Table A.1.2 Demand of Remanufactured Components ($\tilde{D}$)

	Component 1			Component 2			Component 3		
Period 1	2000	1600	1400	2000	1600	1400	2000	1600	1400
Period 2	2200	1800	1600	2200	1800	1600	2200	1800	1600
Period 3	1800	1400	1200	1800	700	1200	1800	1400	1200
Period 4	2000	1800	1600	2000	1800	1600	2000	1800	1600
Period 5	1800	1200	1000	1800	1200	1000	1800	1200	1000

Table A.1.3 New Components Cost ($\widetilde{TE}$)

Period 1	Component type 1	65.0	80.0	90.0
	Component type 2	70.0	85.0	95.0
	Component type 3	83.0	88.0	97.0
Period 2	Component type 1	65.0	88.0	90.0
	Component type 2	77.0	83.0	95.0
	Component type 3	81.0	89.0	92.0
Period 3	Component type 1	65.0	80.0	90.0
	Component type 2	70.0	85.0	95.0
	Component type 3	83.0	88.0	97.0
Period 4	Component type 1	70.0	78.0	90.0
	Component type 2	70.0	75.0	90.0
	Component type 3	72.0	78.0	93.0
Period 5	Component type 1	65.0	80.0	90.0
	Component type 2	70,0	85.0	95.0
	Component type 3	83.0	88.0	97.0

Table A.1.4 Products Acquisition Cost ($\widetilde{CA}$)

		Quality 1			Quality 2		
Supplier 1	Product type 1	58.0	78.0	88.0	57.0	77.0	87.0
	Product type 2	51.0	71.0	81.0	47.0	67.0	77.0
	Product type 3	53.0	63.0	83.0	49.0	59.0	79.0
Supplier 2	Product type 1	57.0	77.0	87.0	48.0	68.0	78.0
	Product type 2	49.0	69.0	79.0	47.0	67.0	77.0
	Product type 3	53.0	73.0	83.0	51.0	71.0	81.0
Supplier 3	Product type 1	48.0	68.0	78.0	43.0	63.0	73.0
	Product type 2	59.0	79.0	89.0	49.0	69.0	79.0
	Product type 3	54.0	74.0	84.0	51.0	71.0	81.0

Table A.1.5 Fraction of Remanufacturable Products ($\widetilde{P}$)

	Quality 1			Quality 2		
Product type 1	0.95	0.90	0.85	0.90	0.85	0.80
Product type 2	0.95	0.90	0.85	0.90	0.85	0.80
Product type 3	0.95	0.90	0.85	0.90	0.85	0.80

Table A.1.6 Operational Times and Time Capacities

Total Hours Available for Remanufacturing in a Period ($\widetilde{HR}$)			Time to Remanufacture a Component ($\widetilde{TM}$)					
400.0	500.0	600.0	Quality 1			Quality 2		
Component type 1			0.13	0.15	0.17	0.16	0.18	0.20
Component type 2			0.19	0.21	0.22	0.20	0.23	0.25
Component type 3			0.22	0.25	0.27	0.25	0.28	0.30
Total Hours Available for Disassembling in a Period ($\widetilde{HD}$)			Time to Disassembly a Returned Product ($\widetilde{TD}$)					
400.0	500.0	600.0	Quality 1			Quality 2		
Product type 1			0.14	0.16	0.18	0.18	0.20	0.22
Product type 2			0.15	0.18	0.21	0.20	0.23	0.26
Product type 3			0.20	0.22	0.24	0.24	0.27	0.29

Appendix B

Lingo Code of The Uncertain Remanufacturing Process

```
SETS:
PRODUCT/P1..P3/:VC;                                         !I;
COMPONENT/C1..C3/:VM,W,WO,WP;                               !J;
SUPPLIER/S1..S3/:;                                          !S;
QUALITYPROD/K1..K2/:;                                       !K;
QUALITYCOMP/Y1..Y2/ :;                                      !Y;
PERIOD/T1..T5/:;SA,UB,PC,HD,HDO,HDP,HR,HRO,HRP;             !T;

LINK1(PERIOD,COMPONENT):f,bm,D,DO,DP,TE,TEO,TEP;
LINK2(PRODUCT,QUALITYPROD,COMPONENT,QUALITYCOMP):NM;
LINK3(SUPPLIER,PERIOD):x;
LINK4(SUPPLIER,PERIOD,PRODUCT,QUALITYPROD):RT;
LINK5(PRODUCT,QUALITYPROD):P,PO,PP,TC,TCO,TCP,O,OO,OP,R,RO,RP,CH,CHO,
                            CHP,CS,CSO,CSP,DC;
LINK6(SUPPLIER,PRODUCT,QUALITYPROD):CA,CAO,CAP;
LINK7(PERIOD,PRODUCT,QUALITYPROD):g,h,ie,se,M,MO,MP;
Link8(PERIOD,COMPONENT,QUALITYCOMP):je,ke,e;
Link9(COMPONENT,QUALITYCOMP):Q,QO,QP,U,UO,UP,V,VO,VP,TM,TMO,TMP,PD;

ENDSETS

!**************************************DATA***********************;

DATA:
SC=500;
SM=500;
w1=4/6;
w2=1/6;
w3=1/6;
B=0.5
ENDDATA

!***************OBJECTIVE FUNCTION - COST MINIMIZATION***************;

MAX = L ;

L<=(NIS-z2)/(NIS-PIS);

L<=(z3-(NIS))/(PIS-NIS);

L<=(NIS-z4)/(NIS-PIS);


Z2=(@SUM(LINK4(S,T,I,K):x(S,T)*(CA(S,I,K)*RT(S,T,I,K)))+

    @SUM(LINK7(T,I,K):O(I,K)*g(T,I,K))+

    @SUM(LINK7(T,I,K):CS(I,K)*se(T,I,K))+

    @SUM(LINK7(T,I,K):CH(I,K)*h(T,I,K))+
```

```
     @SUM(LINK7(T,I,K):R(I,K)*ie(T,I,K))+

     @SUM(LINK8(T,J,Y):Q(J,Y)*e(T,J,Y))+

     @SUM(LINK8(T,J,Y):U(J,Y)*je(T,J,Y))+

     @SUM(LINK8(T,J,Y):V(J,Y)*ke(T,J,Y))+

     @SUM(LINK1(T,J):TE(T,J)*f(T,J))+

     @SUM(LINK1(T,J):W(J)*bm( T,J))-

     @SUM(LINK4(S,T,I,K):M(T,I,K)*(se(T,I,K)+((RT(S,T,I,K)*x(S,T))*
                        (1-P(I,K))))));

Z3=(@SUM(LINK4(S,T,I,K):x(S,T)*((CA(S,I,K)-CAO(S,I,K))*RT(S,T,I,K)))+

     @SUM(LINK7(T,I,K):(O(I,K)-OO(I,K))*g(T,I,K))+

     @SUM(LINK7(T,I,K):(CS(I,K)-CSO(I,K))*se(T,I,K))+

     @SUM(LINK7(T,I,K):(CH(T,I,K)-CHO(I,K))*h(T,I,K))+

     @SUM(LINK7(T,I,K):(R(T,I,K)-RO(I,K))*ie(T,I,K))+

     @SUM(LINK8(T,J,Y):(Q(J,Y)-QO(J,Y))*e(T,J,Y))+

     @SUM(LINK8(T,J,Y):(U(J,Y)-UO(J,Y))*je(T,J,Y))+

     @SUM(LINK8(T,J,Y):(V(J,Y)-VO(J,Y))*ke(T,J,Y))+

     @SUM(LINK1(T,J):(TE(T,J)-TEO(T,J))*f(T,J))+

     @SUM(LINK1(T,J):(W(J)-WO(J))*bm(T,J))-

     @SUM(LINK4(S,T,I,K):M(T,I,K)*(se(T,I,K)+((RT(S,T,I,K)*x(S,T))*
                        (1-P(I,K))))));

Z4=(@SUM(LINK4(S,T,I,K):x(S,T)*((CAP(S,I,K)-CA(S,I,K))*RT(S,T,I,K)))+

     @SUM(LINK7(T,I,K):(OP(I,K)-O(I,K))*g(T,I,K))+

     @SUM(LINK7(T,I,K):(CSP(I,K)-CS(I,K))*se(T,I,K))+

     @SUM(LINK7(T,I,K):(CHP(T,I,K)-CH(I,K))*h(T,I,K))+

     @SUM(LINK7(T,I,K):(RP(T,I,K)-R(I,K))*ie(T,I,K))+

     @SUM(LINK8(T,J,Y):(QP(J,Y)-Q(J,Y))*e(T,J,Y))+

     @SUM(LINK8(T,J,Y):(UP(J,Y)-U(J,Y))*je(T,J,Y))+

     @SUM(LINK8(T,J,Y):(VP(J,Y)-V(J,Y))*ke(T,J,Y))+
```

```
@SUM(LINK1(T,J):(TEP(T,J)-TE(T,J))*f(T,J))+

@SUM(LINK1(T,J):(WP(J)-W(J))*bm(T,J))-

@SUM(LINK4(S,T,I,K):M(T,I,K)*(se(T,I,K)+((RT(S,T,I,K)*x(S,T))*
                     (1-P(I,K))))));


!********************************CONSTRAINTS************************;

! SUBJECT TO;

!CONSTRAINT 1;
@FOR(PERIOD(T):
@SUM(SUPPLIER(S):x(S,T))<=1);

!CONSTRAINT 2;
@FOR(LINK7(T,I,K):
@SUM(SUPPLIER(S):(RT(S,T,I,K)* X(S,T))*((P(I,K)*(w1)*(B))+
(PO(I,K)*(w2)*(B))+(PP(I,K)*(w3)*(B))))=g(T,I,K)+se(T,I,K)+
h(T,I,K)+ie(T,I,K)-h(@WRAP(T-1,5),I,K));

!CONSTRAINT 3;
@FOR(LINK8(T,J,Y):
@SUM(LINK5(I,K):g(T,I,K)*NM(J,Y,I,K))=ke(T,J,Y)+e(T,J,Y)+je(T,J,Y)-
je(@WRAP(T-1,3),J,Y));

!CONSTRAINT 4;
@FOR(LINK1(T,J):
@SUM(QUALITYCOMP(Y):e(T,J,Y)=((D(T,J)*(w1)*(B))+
(DO(T,J)*(w2)*(B))+(DP(T,J)*(w3)*(B)))-f(T,J)-BM(T,J)+BM(@WRAP(T-1,5),J));

!CONSTRAINT 5;
@FOR(LINK7(T,I,K):
@SUM(SUPPLIER(S):DC(I,K)*((RT(S,T,I,K)*X(S,T))*((P(I,K)*(w1)*(B))+
(PO(I,K)*(w2)*(B))+(PP(I,K)*(w3)*(B)))))<=ie(T,I,K));

!CONSTRAINT 6;
@FOR(LINK8(T,J,Y):
@SUM(LINK5(I,K):PD(J,Y)*(g(T,I,K)*NM(J,Y,I,K)))<=ke(T,J,Y));

!CONSTRAINT 7;
@FOR(LINK1(T,J):BM(T,J)<= UB(T)*((D(T,J)*(w1)*(B))+
(DO(T,J)*(w2)*(B))+(DP(T,J)*(w3)*(B))));

!CONSTRAINT 8;
@SUM(LINK7(T,I,K):VC(I)*h(T,I,K))<=SC;

!CONSTRAINT 9;
@SUM(LINK8(T,J,Y):VM(J)*je(T,J,Y))<=SM;

!CONSTRAINT 10;
@FOR(PERIOD(T):
@SUM(LINK5(I,K):SE(T,I,K))<= SA(T));
```

```
!CONSTRAINT 11;
@FOR(PERIOD(T):
@SUM(MODULE(J):f(T,J))<= PC(T))  ;

!CONSTRAINT 12;
@FOR(PERIOD(T):
@SUM(LINK5(I,K):((TC(I,K))*(B))*g(T,I,K))<=HD(T)*(B));

!CONSTRAINT 13;
@FOR(PERIOD(T):
@SUM(LINK5(I,K):((TCO(I,K))*(B))*g(T,I,K))<=HDO(T)*(B));

!CONSTRAINT 14;
@FOR(PERIOD(T):
@SUM(LINK5(I,K):((TCP(I,K))*(B))*g(T,I,K))<=HDP(T)*(B));

!CONSTRAINT 15;
@FOR(PERIOD(T):
@SUM(LINK9(J,Y):((TM(J,Y))*(B))*e(T,J,Y))<=HR(T)*(B));

!CONSTRAINT 16;
@FOR(PERIOD(T):
@SUM(LINK9(J,Y):((TMO(J,Y))*(B))*e(T,J,Y))<=HRO(T)*(B));

!CONSTRAINT 17;
@FOR(PERIOD(T):
@SUM(LINK9(J,Y):((TMP(J,Y))*(B))*e(T,J,Y))<=HRP(T)*(B));

!CONSTRAINT 18;
@FOR(LINK1(T,J)|T#EQ#1:BM(@WRAP(T-1,5),J)=0);

!CONSTRAINT 19;
@FOR(LINK1(T,J)|T#EQ#5:BM(T,J)=0);

!CONSTRAINT 20;
@FOR(LINK8(T,J,Y)|T#EQ#1:je(@WRAP(T-1,5),J,Y)=0);

!CONSTRAINT 21;
@FOR(LINK7(T,I,K)|T#EQ#1:H(@WRAP(T-1,5),I,K)=0);

!CONSTRAINT 22;
@FOR(LINK3(S,T):@BIN(X(S,T)));

!CONSTRAINT 23;
@BND (0,L,1);


END
```