Examining the Types of Problem Solving Strategies Used by Children with Intellectual Disabilities During Modified Schema Based Instruction

Kim Desmarais

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By: Kim Desmarais

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Signed by the final examining committee:

Yasaman Jalali-Kushki ______ Chair

Dr. Hariett Hariclia Petrakos ______ Examiner

Dr. Diane Pesco ______ Examiner

Dr. Helena P. Osana ______ Supervisor

Approved by Dr. Richard Schmid ______
Chair of Department

André Roy ______
Dean of Faculty

Date June 2, 2015 ______

Abstract

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Kim Desmarais

The current study examined the problem solving strategies used by children with intellectual disabilities (ID group) before and after a modified Schema Based Instruction (MSBI) intervention, and compared the strategies to those of children who were (a) struggling with mathematics (SM group) and (b) of average mathematics ability (AM group). The potential impact of MSBI on children's ability to use appropriate strategies, to use more than one strategy to solve a given problem, and to identify the problem structure was also assessed. All three groups received three hours of MSBI on how to solve a specific type of addition and subtraction word problems. Results demonstrated marked differences in strategy types across groups, with children in the ID group favoring strategies unrelated to the mathematical action or context, and children in the AM and SM groups using standard algorithms during the pretest. Following the MSBI intervention, children in the ID group began to use Direct Modeling strategies. In both the SM and AM groups, most students continued to use the standard algorithm. In the SM group, however, there was more variability with some students using some Direct Modeling strategies. All children showed improvement on appropriateness of strategies and identification of word problem structure following instruction, with the biggest gains observed for children in the ID and SM groups, respectively. The results of this study show promise for the use of MSBI in inclusive classrooms to help students with different abilities learn to successfully solve word problems.

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"It always seems impossible until it's done." – Nelson Mandela

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Statement of the Problem

Human Resources and Skills Development Canada (HRSDC, 2011) states that there are 90, 590 students with disabilities who are integrated into mainstream schools with no special education classes, and an additional 55, 650 youth who are integrated into mainstream schools with some special education classes. The Quebec Education Act (Ministère de l'Éducation, du Loisirs et du Sport, MELS, 2012, Article 235) stipulates that every school board is responsible for creating a policy for the educational services of students with special needs to ensure their successful integration into regular classrooms and school activities. As such, two of the largest Anglophone school boards in Montreal, the Lester B. Pearson and English Montreal School Boards, have created policies for the integration of children with developmental disabilities. In their respective policies, both school boards emphasize their objectives to integrate, when possible, students with disabilities into regular classrooms (English Montreal School Board, EMSB, 2005; Lester B. Pearson School Board, LBPSB, 2003). The policy of the English Montreal School Board, for example, specifies that inclusion, "…involves melding special education and regular educational services, and instituting innovative instructional strategies and professional collaborative teaming approaches" (EMSB, 2005, p. 25).

On a North American scale, the No Child Left Behind Act (NCLB, 2002) in the United States necessitates that all children, including those with special needs, should have access to equal opportunities for high quality education that meets their educational needs. The NCLB Act further requires that all students demonstrate yearly progress on assessments in reading and mathematics. This includes progress and achievement in mathematics for students with disabilities (NCLB, 2002). The Individual with Disabilities Education Improvement Act (IDEA, 2004) states children with disabilities must have access to general education in regular classrooms to meet the developmental goals that have been established for all students. The

IDEA (2004) further stipulates that students can receive accommodation to help with their educational needs and eventual achievement.

With regards to mathematics education for the primary grades, the Quebec government and the MELS compiled a list of mathematics objectives and competencies for children with special needs (Gouvernement du Québec & MELS, 1996). Such children need to be able to solve addition and subtraction problems with meaning so that they may apply their learning to everyday situations. To achieve this goal, teachers should progress through three stages: concrete representations of the concepts, visual representations of the concepts, and finally, abstract representations, where children understand abstract forms of the concepts they are learning (Gouvernement du Québec & MELS, 1996).

Much of the research on children with mathematics difficulties has focused on children with learning disabilities, with little attention paid to children with intellectual disabilities. Research examining the mathematics abilities of children with learning disabilities has shown that these children can successfully solve word problems using a variety of strategies (Geary, Hoard, Byrd-Craven, & DeSoto, 2004). There is reason to believe that the same holds true for students with intellectual disabilities. Both groups of children present with similar profiles with regards to their executive functioning deficits. That is, both groups of children have deficits in working memory, shifting (shifting between mental sets), and processing speed (Baroody, 1999; Geary, Hoard, Byrd-Craven, & DeSoto, 2004).

Given the large numbers of students integrated into mainstream classrooms, it is of utmost importance that appropriate programs are put into place to help support their conceptual understanding of mathematics to make them more proficient problem solvers. Developing educational programming in mathematics for children with intellectual disabilities is difficult. The curricula in mainstream schools are not adapted to meet the needs of children with disabilities (Rose & Rose, 2007), despite integration of these children into mainstream classrooms.

Existing mathematics instruction for children with intellectual disabilities primarily focuses on procedural instruction, as opposed to conceptual understanding (Baroody, 1999). Traditional views of children with disabilities has been that they are passive learners -- that is, they are capable of learning rote skills, but are unable to devise new strategies for learning and are unable to the transfer skills they have learned (Baroody, 1999). An emerging view on the mathematical abilities of children with intellectual disabilities finds that they can transfer strategies for learning when the instruction is presented clearly and the children are developmentally ready for them (Baroody, 1999). To this effect, Baroody (1999) argued that mathematics instruction for children with intellectual disabilities should emphasize an understanding of mathematical concepts and procedures, as well as the development of inquiry skills, as opposed to the memorizing that is often prioritized in their instruction. Children (those with and without disabilities) who understand mathematics are more likely to transfer their knowledge to new situations (Baroody, 1999; Siegler, 2003). In addition, they will require less practice and review and will invent and monitor problem-solving strategies that are meaningful to them (Baroody, 1999).

Carpenter and Moser (1984) stated the importance of identifying the intuitive strategies used by children as a starting point for their formal instruction. Given the similarities between children with intellectual disabilities and learning disabilities, and the importance of identifying the strategies children use, it is essential to investigate the problem solving strategies used by children with intellectual disabilities. By identifying the types of strategies they use, teachers in inclusive settings can adapt their instruction to help further the conceptual and procedural mathematical understanding of these children. Little research has focused on the strategies used by children with intellectual disabilities. In fact, to my knowledge, there is no research examining the types of problem solving strategies used by children with intellectual disabilities. The present study will seek to address this gap in the research.

This study will also seek to provide insights on effective mathematics instruction on the performance of children with intellectual disabilities. Baroody (1999) argued that teachers should encourage children, including those with disabilities, to actively construct their own knowledge and create their own strategies for solving problems. He stated, "...teachers should consider being more of 'guides on the side' than a 'sage on the stage'" (Baroody, 1999, pg. 89). In keeping with this view, the children in the present study will be engaged in what has been called Schema Based Instruction (Fuchs, Fuchs, Finelli, Courey & Hamlett, 2004; Fuchs, Fuchs, Prentice, Hamlett, Finelli & Courey, 2004; Jitendra, DiPipi, & Perron-Jones, 2002; Jitendra & Star, 2011). During the instruction, children will be taught to look at the relationships between quantities in word problems and then they will be encouraged to generate and share their own problem solving strategies with their peers. Children's word problem performance, namely their ability to select an appropriate strategy to solve the problem, to use multiple strategies to solve problems, and to identify the underlying structure of the problem will be assessed.

The present study will have important implications. Because studies involving children with intellectual disabilities have often focused on identifying their deficits, this study will provide important information as to what these children are able to do with regards to mathematics. Furthermore, this study will provide insights as to the strategies these children use and how we can adapt mathematics instructions to teach concepts to these children. It is imperative that teachers make the connection between their intuitive problem solving strategies and the concepts being taught in schools (Ginsburg, 1997). Without conceptual understanding, children will not be able to use the skills they have learned flexibly or adaptively (Baroody, 1999). Children with intellectual disabilities need to learn adaptive mathematical skills (such as problem solving) to help them become as autonomous as possible in their daily lives.

Literature Review

Definitions

Intellectual disability. Many of the studies presented in the literature review focus on children with learning disabilities or mathematics difficulties. It is important to define what constitutes an intellectual disability, given that this is the population targeted in the present study. An intellectual disability involves cognitive deficits that impact adaptive functioning across all three areas: the conceptual domain, social domain, and practical domain (American Psychiatric Association, APA, 2013). The conceptual domain encompasses skills such as language, mathematical reasoning, and memory. Empathy, interpersonal communication skills, and social judgments are aspects of the social domain (APA, 2013). Lastly, the practical domain includes abilities related to personal care, money management, and organizing school tasks. For a child to receive a diagnosis of intellectual disability, he must have an IQ score below 70 (APA, 2013) and have been diagnosed during the developmental period (i.e., prior to the age of 18 in the DSM-IV) (APA, 2000). There must be a significant impact on adaptive functioning in all three domains to the extent that it affects their daily functioning.

Problems for addition and subtraction. Carpenter and colleagues (1999) have identified 11 different word problem types. In Schema Based Instruction (SBI) studies, the authors have combined these 11 problem types into three problem types: group, change, and compare problems (Jitendra, Griffin, McGoey, Gardill, Bhat, & Riley, 1998). Group word problems (also known as Part-Whole problems, Carpenter et al., 1999), describe two or more parts of a whole amount. The unknown -- that is, the element in the problem that has to be solved -- can be the part or the whole. Change problems (also known as Join and Separate problems, Carpenter et al, 1999), describe an action where a given set is either increased or decreased, resulting in a different final quantity. In these problems, the unknown can be the initial set, the change set, or the end set. Compare word problems involve two distinct sets that differ by a specific quantity (Carpenter et al, 1999; Jitendra, Griffin, McGoey, Gardill, Bhat, & Riley, 1998). In compare problems, the unknown can be the difference or one of the sets. In this proposal, word problems will be referred to as “Part/Whole” for group word problems, “Action” for change problems, and lastly “Seesaw” for compare problems.

Word Problem Solving Performance

De Corte, Verschaffel, and De Win (1985) proposed a theoretical model for mathematical word problem solving. This problem solving model consists of five stages. In the first stage, the child processes the verbal text and creates a mental representation of the word problem structure. In the second stage, the child selects the appropriate arithmetic operation to find the unknown. The operation is chosen based on the mental representation created in the first stage. In the third stage, the child executes the operation he has chosen. During the fourth stage, the child reactivates the mental representation, inserting the answer he has calculated. In the last stage, the child will verify if his answer is correct. If the child is unable to create the proper mental representation during the first stage of the model, it reduces the likelihood that he will be able to correctly solve the problem. It is imperative that children create accurate mental representations of word problem structures to accurately determine the relationships between the quantities in the problem. Understanding these relationships enhances children’s ability to appropriately identify strategies to solve problems (Lucangeli, Tressoldi, & Cendron, 1998).

Identification of word problem structure. Xin, Jitendra, and Deatline-Buchman (2002) stated that students with disabilities have difficulty creating representations of problems and identifying the relevant information for solving them. Some of the difficulty they have creating mental representations of mathematics problems can be explained by executive functioning deficits that have been identified in the literature for children with intellectual disabilities (Oznoff & Schetter, 2007). Children with disabilities are challenged when they solve problems that require visualization and working memory capacity (Stein & Krishnan, 2007).

Hutchinson (1993) studied 20 adolescents with learning disabilities to determine the effects of a cognitive strategy on problem solving. The cognitive strategy was created to help students learn how to represent and solve three different types of word problems. In-depth analyses were used to answer questions about children's ability to solve word problems. Hutchinson (1993) found evidence suggesting that students with learning disabilities failed to generate an equation and construct an appropriate representation of the word problem. Similarly, in their investigation of the problem solving skills of 90 adolescents with learning disabilities, Montague and Applegate (1993) found that the errors made by these children were caused by their inability to create an internal representation of the word problem structure. This led to a breakdown in applying appropriate strategies for solving the problems.

Judd and Bilsky (1989) studied the effects of memory aids, cue words, and problem context on children's ability to solve simple addition and subtraction word problems. They compared a group of children with intellectual disabilities and a group of typically developing children with respect to the effects of their intervention within each group (ID and TD). All students in the study received a single 30-minute session where they were asked to solve word problems flashed on a computer screen. Students in the treatment group received visual aids that remained on the screen until they solved the problem. The visual aids were dots that represented the quantities in the problem. The control group did not have any visual support. Their results demonstrated that students with an intellectual disability had difficulty solving these problems, especially without the memory aid. The authors argued that this is because the children were unable to create a representation of the problems in such a way that sets up the solution strategy.

Children's problem solving strategies.

Typically developing children. Research has shown that typically developing children use a multitude of strategies when solving word problems. The development of strategy use in the domain of whole numbers for children in kindergarten through third grade progresses from more primitive forms to more sophisticated ones (Carpenter, Fennema, Empson, Loef, & Levi, 1999; Carpenter & Moser, 1984). Consider the following word problem: *Sally has six stickers. Her friend, Janie, has seven stickers. How many stickers do they have in all?* Children use a number of different strategies to solve the problem. Direct Modeling is when children use their fingers or objects to directly "act out" the numbers and actions represented in the problem (Carpenter et al., 1999). In the case of the above example, a child using a Direct Modeling strategy could count six blocks and then count out an additional seven blocks. The child would then count all of the blocks to determine the answer. When using Counting Strategies, children do not need to represent both addends. If fingers or objects are used, they are used to keep track of counts (Carpenter et al., 1999). For example, a child using Counting Strategies to solve the above problem could count on from the number six using his fingers or other tools. That is, the child would count starting at six and would add seven more, using his fingers to represent the seven being added on. In this case, the answer would be the last number recited in the counting sequence. Children also use a Derived Fact strategy, the children use known mathematics facts to help them solve a problem (Carpenter et al, 1999). A child using this type of strategy to solve the above problem could say something like, "Well, I know 6 + 6 = 12, so one more is 13."

Generally, children begin by using Direct Modeling strategies (such as Joining All) and progress to Counting Strategies (such as Counting On From Larger) and Derived Number Facts

(Carpenter et al., 1999). Despite this progression, children do not always use the most efficient strategy. They often prefer to use more primitive strategies even if they are less efficient (Siegler, 2003).

Instruction is often not aligned with the intuitive strategies that children use. Carpenter and Moser (1984) stated that the curriculum fails to acknowledge the informal strategies that children bring with them to school, stating, “the analysis that children spontaneously use provides a much better basis for teaching problem solving” (p. 200). In their view, problem-solving instruction should build on children’s existing strategies to help them develop more sophisticated problem solving skills over time.

Children with disabilities. There has been some research on the differences in strategy use in children with learning disabilities compared to typically developing children. Geary (2004), for example, presented the cognitive profiles of children with mathematics learning disabilities (MLD). He stated that children with MLD presented with working memory deficits, difficulties attending to the task, inhibiting irrelevant information, as well as understanding abstract concepts. As such, Geary (2004) explained that children with MLD tend not to demonstrate a shift from immature problem solving strategies based on modeling to those based on memory that is seen in typically developing children, suggesting that children with MLD may have difficulty storing or accessing arithmetic facts in their long-term memories. With regards to strategy use, MLD students commit more counting errors and use immature problem solving strategies for longer (Geary, 2004).

There is some research, however, that supports the notion that children with intellectual disabilities can learn the same mathematical skills and concepts as their typically developing peers. Baroody (1999) pointed out that children with intellectual disabilities can learn oral counting, one-to-one correspondence, and cardinality. In addition, Bird and Buckley (2001) maintained that children with Down syndrome can learn number skills to the same level as their typically developing peers. Children with Down syndrome, however, require instruction to be explicit with many opportunities for practice. Moreover, Mackinnon (2005) stated that number skills for these students are linked to their general understanding and language development, as opposed to their disability. Thus, with better instruction, children can understand these concepts in much the same way as their typically developing peers.

Baroody (1996) was interested in examining whether children with intellectual disabilities (ID) could spontaneously invent more efficient addition strategies during simple addition problems without any additional metacognitive training. His study consisted of 30 children with mild (6 children) to moderate intellectual disabilities (24 children). All children received 51 20-minute individual sessions. The comparison group was tutored on the Individualized Education Plan (IEP) mathematics objectives, but any on addition was omitted. The students in the experimental group were given sessions that focused on accurate computation, which included being shown or helped enacting a Direct Modeling strategy for addition problems. Results demonstrated that children in the experimental group used substantially fewer inappropriate strategies than the control group. In addition, these children developed short cuts for the Counting All strategies they knew. Another interesting finding was that two of the participants, who had IQs situated in the moderate intellectual disabilities range (IQ less than 50, APA, 2000), invented advanced counting strategies that involved a keeping-track process that disregarded addend order. The results of this study also provided some evidence that specific-strategy training alone can be as effective as specific-strategy training coupled with self-management training, as participants in the study spontaneously modified their strategies without self-management training.

Effects of executive functioning deficits on strategy use. Research has shown that children's difficulties with regards to mathematical problem solving may be due to executive functioning deficits, namely deficits in working memory and cognitive flexibility (e.g., Geary, 2004; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007).

Working memory. Working memory is the ability to hold a mental representation of information in one's mind while simultaneously using other mental processes (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). In his review of children with mathematical learning disabilities (MLD), Geary (2004) found evidence for important deficits in working memory. Geary (2004) argued that the deficits appear to involve representation of information and manipulation of the language system. Children with MLD will use finger counting strategies to solve mathematics problems as this reduces the demands on working memory (Geary, 2004). Geary (2004) further argued that children with MLD's tendency to undercount or overcount may also be attributed to working memory deficits. In addition, poor conceptual understanding and lack of prior knowledge may contribute to delays in adopting more sophisticated problem solving strategies (Geary, 2004).

St. Clair Thompson and Gathercole (2006) examined the role of a number of executive functions and their relationship to academic achievement. They studied the relationships between academic achievement and working memory (verbal and visuo-spatial), along with other aspects of executive functioning. Their findings revealed that working memory is closely related to achievement in mathematics, namely in mastering counting skills and mental arithmetic (St. Clair-Thompson & Gathercole, 2006).

Geary, Hoard, Byrd-Craven, Nugent, and Numtee (2007) examined the extent to which mathematics deficits were observed in children with MLD, low achieving (LA) children, and typically achieving (TA) children. Their study consisted of 115 children: 15 children with MLD, 44 LA children, and 46 TA children. The children were administered a battery of mathematical cognition tasks, working memory measures, and speed processing measures. Their results demonstrated that children with MLD had deficits across all mathematical cognition tasks, as compared to the other two groups. These deficits were mediated by working memory and processing speech deficits (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007), thereby contributing to these children's immature problem solving strategies.

Cognitive flexibility. Flexibility in strategy use is defined as adapting one's strategies to the characteristics of the task at hand (Van der Heijden, 1993, cited in Verschaffel, Torbeyns, De Smedt, Luvwel, & Van Dooren, 2007). There is some debate as to whether mathematics instructors should promote flexibility in students who are weak in mathematics or who have disabilities. Verschaffel, Torbeyns, De Smedt, Luvwel, and Van Dooren (2007) discussed this controversy and explored whether children with learning disabilities should be taught flexibility, or only a small handful of strategies for solving problems. There is an argument that children who have MLD or are low achieving in mathematics could benefit from learning a few strategies well, which would reduce cognitive load on working memory (Baxter, Woodward, & Olson, 2001, cited in Verschaffel, Torbeyns, De Smedt, Luvwel, &Van Dooren, 2007). On the other hand, others argue that flexibility should be taught from the beginning to enhance children's problem solving strategies (Verschaffel, Torbeyns, De Smedt, Luvwel, and Van Dooren, 2007). Verschaffel, Torbeyns, De Smedt, Luvwel, and Van Dooren (2007) argued that if the educational goal is for a deep understanding of mathematical concepts, procedures, and pattern recognition, all children (including those with MLD) should be taught flexibility.

Research has shown that children with disabilities tend to rely on immature counting strategies when solving problems. Ostad (1997) studied strategy use in 101 children with mathematical disabilities (MD) relative to a mathematically normal (MN) group of children. Specifically, he examined the use of "back-up strategies" versus the use of "retrieval strategies." Back-up strategies are overt strategies that are visible or audible, such as counting on one's fingers. Retrieval strategies are those in which the answers are retrieved from long-term memory and are generally preferred because they are faster and require fewer demands on working memory. Ostad's (1997) study demonstrated that MD students frequently used back up strategies over retrieval strategies throughout the primary grades, even in the advanced primary grades, reflecting the problem-solving patterns of younger children. In addition, Ostad (1997) found that MD children tended to use one strategy repeatedly, as opposed to their MN peers, who used several different strategies when solving problems. He defined MD children as having "strategy rigidity" because they repeatedly used a smaller number of primitive back up strategies when solving word problems.

Children with intellectual disabilities have difficulties with regards to flexibility. Children with Autism Spectrum Disorders, for example, have significant limitations with regards to planning and flexibility (Oznoff & Schetter, 2007), specifically at the conceptual level, making it difficult for these individuals to shift from one concept to the next. This is especially problematic during mathematics activities where children will often need to shift between various strategies or operations to solve problems.

Educational Interventions and Strategies for Students with Intellectual Disabilities

Traditional mathematics instruction for children with intellectual disabilities has emphasized procedures rather than concepts (Baroody, 1999). In the past, these children have been viewed as "passive learners" who are only capable of learning through rote repetition of basic procedures (Baroody, 1999). An example of a program that emphasizes procedures in their instruction for children with special needs is TouchMath (Innovative Learning Concepts Inc, 2013). This approach emphasizes procedural instruction using dot notation. Children are also taught to use a key word strategy to help identify the appropriate solution to use to solve the problem. Consider the following word problem taken from the TouchMath website: *At the dog* *park, there were 8 big dogs and 7 little ones. 3 of the dogs had spots. Find the sum of the dogs that were big and little*. Children are prompted to pick out the relevant information (number of big and little dogs) and to plug the numbers into an equation that has been provided for them. In the equation, blank boxes indicate where the children are expected to write the numbers. In addition, they are taught to look for key words to help identify a solution strategy. In the above example, the word "sum" indicates that the problem should be solved through addition.

There are several problems with using such an approach. In a key word strategy, children are taught "short cuts" to solving the problem. That is, they are not taught to look at the structure of the word problem (Parmar, Cawley, & Frazita, 1996). As a result, there are instances when children will choose inappropriate or faulty strategies to solve the problem. Furthermore, key word strategies inhibit the development of children's critical thinking skills (Molina, 2012). Parmar, Cawley, and Frazita (1996) studied the ability of children with mild disabilities to solve word problems of varying structures, including one and two-step problems and those containing irrelevant information. They observed that children with disabilities tend to overuse key word strategies and ignore the relationships between the quantities in the problem, leading them to error (Parmar, Cawley, & Frazita, 1996). As a result, the authors argue that there should be a shift from computation activities to those that encourage mathematical reasoning. Teachers should present word problems with a variety of structures to promote children's reasoning skills (Parmar, Cawley, & Frazita, 1996). In turn, learning a variety of different structures will help with transfer problems (Bransford, Brown, & Cocking, 1999).

Recent research has shown, however, that these children are capable of learning a number of mathematical skills and concepts with proper instruction (Baroody, 1999). Proper instruction builds on the child's strengths and informal knowledge and links the procedures to the mathematical concepts (Clements & Sarama, 2009; Ginsburg, 1997). Clements and Sarama

(2009) argued that all children learn from, “good mathematics instruction” (p. 245). In fact, children with special needs require more time and more mathematics instruction to bridge the gap between them and their typically developing peers (Clements & Sarama, 2009). The National Mathematics Advisory Panel (NMP, 2008) reported that mathematics instruction for children with intellectual disabilities should be systemic and should include the following components: concrete and visual representations; explicit explanations from instructors; discussions and collaborations among students; and explicit corrective feedback from instructors. Long sequences of information should be broken down into smaller units (Clements & Sarama, 2009). As motivation is often a factor for these children in learning mathematics, instruction should also be engaging (Bransford, Brown, & Cocking, 1999; Clements & Sarama, 2009).

Schema Based Instruction.

Theoretical framework. Schemata are knowledge structures that organize information in the learner’s long-term memory (Griffin & Jitendra, 2009). In problem solving, schemata assist the learner in categorizing information, identifying the relationships between the quantities in a problem, and determining the best strategy for solving the problem (Chen, 1999). Chen (1999) found that when students are able to internalize general schemata that represent the structure of word problems, they are better able to solve transfer problems. In addition, a general schema is one that is not linked to a specific procedure (Chen, 1999). When teachers provide students with multiple problems and diverse solution strategies, children can use the general schemata across a multitude of problems (Chen, 1999), offering more flexibility. The use of general schemata allows children to understand the semantic relations between the sets in the problem, which refers to the conceptual knowledge a child has about the increases, decreases, and combinations involving sets (Cummins, 1991).

Schema Based Instruction. The findings from several studies have shown that using Schema Based Instruction (SBI) helps teach youth with learning disabilities, children at-risk for mathematics failure, and typically developing youth how to solve different types of word problems (Fuchs, Fuchs, Finelli, Courey & Hamlett, 2004; Fuchs, Fuchs, Prentice, Hamlett, Finelli & Courey, 2004; Jitendra, DiPipi, & Perron-Jones, 2002; Jitendra & Star, 2011). SBI is an instructional approach that uses visual representations of word problem structures to teach students how to solve a variety of problems. In these studies, the authors refer to a "schema" as a schematic drawing of a word problem's structure. In this proposal, I will use "schema" to refer to the schematic drawings that represent the word problem structure that will be the focus of the instructional intervention. Traditionally, SBI is separated into two separate phases: Problem Schemata Phase and Problem Solution Phase. In the Problem Schemata Phase, children are shown the schemata and are taught to place the numbers of the word problem into a schema. This is to help children see the relationship between the quantities in the problem. In the Problem Solution Phase, children are taught strategies to solve specific problem types.

Xin and Jitendra (1999) argued that one of the reasons for the success of SBI is that it emphasizes conceptual understanding by creating representational links between the various aspects of word problems, thus enhancing students' ability to successfully solve them. SBI addresses the working memory and attention deficits of children with learning difficulties, and greatly differs from traditional mathematics instruction for children with intellectual disabilities, which tends to emphasize rote, procedural instruction (Cawley, Parmar, Yan, & Miller, 1998, as cited in Xin, Jitendra, & Deatline-Buchman, 2005). Another possible reason for the success of SBI, particularly for children with disabilities, is that the creation of visual representations of the problem structure helps children solve word problems by reducing the cognitive load on working memory. Jitendra and Star (2011) maintained that the explicit strategies taught in SBI, coupled with the visual representations of the word problem structure, accommodate the working memory deficits found in children with disabilities.

Evidence of conceptual understanding can be seen in a number of studies where participants were able to solve transfer problems following a Schema Based Instruction program (e.g., Fuchs, Fuchs, Finelli, Courey, & Hamlett, 2004; Jitendra, DiPipi & Perron-Jones, 2002). A study conducted by Fuchs, Fuchs, Prentice, Hamlett, Finelli, and Courey (2004) examined children's ability to solve immediate, near, and far transfer problems following SBI instruction. A total of 366 students participated in one of three conditions (control, SBI, and SBI plus sorting). The SBI and SBI plus sorting taught students how to solve word problems in 24 instructional sessions. Within these instructional sessions, students were given a variety of problems that had the same underlying structure, but differed in some superficial features (e.g., wording, irrelevant information). Following the instruction, students were given a posttest assessing their ability to solve near and far transfer problems. Students in both SBI groups outperformed the control group on all transfer measures (Fuchs, Fuchs, Prentice, Hamlett, Finelli, & Courey, 2004).

There is also some evidence for the maintenance effects of SBI. Rockwell, Griffin, and Jones (2011) studied the effects of Schema Based Instruction on the problem solving performance of a fourth grade student with autism. Using a single-case multiple probe design, a slightly modified version of the SBI program was implemented for 8 weeks, for a total of 540 instructional minutes. The program addressed three types of word problems: group, change, and compare. The unknowns in each problem type were placed at the end of the problem. Following SBI, a generalization session took place where the student was taught how to use SBI to solve for the unknown in the initial and middle positions in the word problems. The student demonstrated improvement in her ability to solve single-step addition and subtraction problems, and the effects of the instruction were maintained 6 weeks later.

Present Study

In the present study, I examined the impact of a Modified Schema Based Instruction (MSBI) program on three groups of children: children with intellectual disabilities, children who were struggling in mathematics and with no known disabilities, and children of average mathematics ability with no known disabilities. To my knowledge, no research has reported the types of problem solving strategies used by children with intellectual disabilities or compared their strategies to those of children without disabilities. In addition, most of the research on SBI has been conducted with individuals with various types of special education needs (e.g., learning disabilities, low achievers, and children at-risk of failing) (Fuchs, Fuchs, Finelli, Courey & Hamlett, 2004; Fuchs, Fuchs, Prentice, Hamlett, Finelli, & Courey, 2004; Jitendra, Hoff & Beck, 1999), but not children with intellectual disabilities, who have IQ scores under 70. Given that previous research has emphasized the importance of understanding children's strategies and that children with intellectual disabilities can invent their own problem solving strategies for simple addition and subtraction problems in much the same way as typically developing children (Baroody, 1996), I compared the three groups of children to determine to what extent they use similar strategies. Knowing the types of strategies that children with intellectual disabilities use will have important implications in terms of supporting their problem solving skills during instruction in school as well as outside of school.

For the present study, I examined the types of problem solving strategies used by children in the different groups before instruction. I also studied the potential effect of an MSBI intervention on the types of strategies that children, both with and without intellectual disabilities, used when solving one type of word problem, Action problems. Finally, I assessed if, following MSBI, there was a change in (a) the number of appropriate strategies used, (b) the use of multiple strategies to solve each problem, and (c) the ability to correctly identify word problem structures.

Individual assessments were administered to each participant before and after the instruction.

My specific research questions were as follows:

1) Prior to instruction:
   a. What strategies do children with intellectual disabilities use to solve word problems before instruction and how do they compare to strategies used by children struggling in mathematics and children with average mathematics ability?
   b. How appropriate are the strategies used by children with intellectual disabilities and how does this compare to the children in the SM and AM groups?
   c. How flexible in strategy use are the students with intellectual disabilities as opposed to the students in the SM and AM groups?
2) Following instruction:
   a. What was the problem solving performance of children with intellectual disabilities after instruction, and how do they compare to those of children in the SM and AM groups? In particular, are there differences with regards to the types of strategies chosen, the appropriateness of the strategy, their ability to use more than one strategy to solve a problem, and their ability to identify the underlying structure of the word problem?

For the purposes of this study, a strategy was considered as appropriate when it could lead to the right answer. That is, if a child made a computation mistake, but used a strategy that would have otherwise yielded a correct answer, his strategy was considered appropriate. Flexibility is defined as using a second problem solving strategy that falls into a different category than the first.

SBI helps children by providing concrete visual representations of the word problem structures thus allowing them to see the relationships between the sets within the problems, and it provides increased opportunities to apply and practice problem solving strategies. Previous research on SBI (e.g., Fuchs, Fuchs, Prentice, Hamlett, Finelli, & Courey, 2004; Xin, Jitendra, & Deatline-Buchman, 2005) thus allowed me to make the following predictions on three outcome measures: appropriateness of strategy type, flexibility, and identification of word problem structure. First, I predict that there will be an improvement between pre-and posttest on appropriateness scores for all three groups. In addition, because previous research on the use of general schemata has led to increases in children's ability to solve a variety of word problems (Chen, 1999), I predicted that I would see an improvement in the students' ability to use more than one strategy in all three groups after MSBI. Lastly, because children will have been repeatedly exposed to the structure of the problem, I believe that children from the three groups will be able to better recognize the correct structure following instruction.

Method

Participants

Nine children (*N* = 9) from the Montreal region were recruited to participate in the study. I recruited the students for the ID group (*n* = 3) through a government center in Montreal that provides services to children with intellectual disabilities (ID). I met with 15 educators from the center to identify individuals who met the study's eligibility criteria. Specifically, the participants in the ID group had a diagnosis of a mild intellectual disability (IQ score between 50 and 70, APA, 2013), two of which had a comorbid diagnosis of autism. In addition, none of the children presented with behavioral difficulties, and all three were able read and understand simple sentences as assessed by a subscale the language of the screening tool that was administered to the children by their educators prior to the intervention.

There were five children who were screened to participate in the study. Two students were excluded to due language difficulties. The remaining three students in the ID group consisted of two boys and one girl who were all 7 years old. The children in this group were given the Number Knowledge Test (NKT, Okamoto & Robbie, 1996) to determine eligibility to participate in the intervention. For this group, the educators at the center administered the NKT and I coded the results.

The remaining six children were recruited from a second grade classroom at a private school in a suburb outside Montreal. The second grade classroom consisted of nine students. All students were between the ages of 7 and 8. I asked their second grade classroom teacher to identify three children who were struggling in mathematics and three children who were of average ability in mathematics. Two final groups of typically developing students were thereby formed: the struggling in mathematics group (SM; $n = 3$, two girls, one boy) and the Average Mathematics ability group (AM; two boys, one girl). According to their teacher, the students in the SM group had substantial difficulty with the mathematical concepts that were being taught in class. The students in the AM group were not the highest achieving students in the class nor were they at risk for failing mathematics.

Table 1. *Number of correct answers at each level of the Number Knowledge Test*

Group	Level 1 Score	Level 2 Score	Level 3 Score
ID			
Student 1	6	3	N/A
Student 2	8	0	N/A
Student 3	9	4	N/A
SM			
Student 4	8	4	N/A
Student 5	8	4	N/A
Student 6	8	4	N/A
AM			
Student 7	8	8	2
Student 8	8	9	4
Student 9	9	9	4

Note. Each level had maximum scores of 9.

Table 1 presents the students' results on the NKT. To pass a level, children had to get at least 5 correct answers in each level. All children passed Level 1 of the NKT, and only the students in the AM group passed Level 2. No student received a passing score on Level 3. To participate in the MSBI intervention, the students had to have passed Level 1 of the NKT.

Design

There were phases to this study, as outlined in Figure 1.

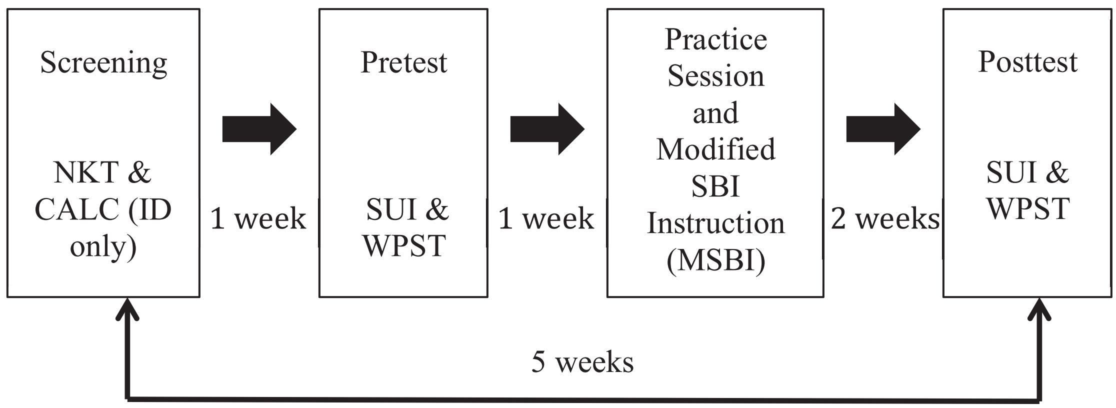

Figure 1. Study Design

Note. NKT = Number Knowledge Test; SUI = Strategy Use Interview; Word Problem Structure Test (WPST)

The study is a descriptive study that lasted five weeks between pretest and posttest. Students were administered the Number Knowledge Test (NKT) and the Clinical Assessment of Language Comprehension (CALC). The CALC was administered for the ID group only during a screening session conducted prior to the pretest. A trained research assistant or I administered the pretest individually to each student. For the SM and AM groups, the NKT portion of the screening test was administered in the same session as the pretest. A trained research assistant or I administered the Strategy Use Interview (SUI) and the Word Problem Structure Test (WPST) to all students.

A practice session began one week after the pretest. Within their groups (ID, SM, and AM), students engaged in a 15-minute session on problem solving prior to the first MSBI session. The practice session was conducted by a trained research assistant or by me. The purpose of the session was to ask the students to discuss amongst themselves different ways they could solve division and multiplication word problems to get them thinking about different types of strategies. Immediately after the 15-minute practice session, the first Modified Schema Based Instruction (MSBI) session began. There were four 45-minute sessions that occurred twice a week over a two-week period. For the SM and AM groups, the sessions occurred over two consecutive days. Because of space constraints, the sessions for the ID group occurred on the same day, with a 30-minute break in between sessions. The posttests took place one week after the last instructional session. The posttest consisted of the SUI and WPST, and both measures were isomorphic versions of the tests administered at pretest.

Instrument and Measures

Screening. Children were administered a screening tool measure with two subscales: the Number Knowledge Test (NKT) and the Clinical Assessment of Language Comprehension (CALC). The NKT (Okatmoto & Robbie, 1996) assesses children's number knowledge and counting skills through four separate levels (0, 1, 2, and 3) corresponding to the ages of 4, 6, 8, and 10, respectively. Only the last three levels were used in the screening tool for the present study. There were a total of 33 items across the three levels. To pass a level, the children had to have at least 5 correct answers in each level. Children received one point for every correct answer. For questions with two parts, children had to get both answers correct to receive a point. The total points are summed for a total raw score. The results provide an approximate corresponding age. For example, children with a raw score between 15 and 19 correspond to the number knowledge of a 6 to 7 year old child. Children had to have passed Level 1 to participate in the study.

In addition, the ID group completed segments of the Clinical Assessment of Language Comprehension (CALC; Miller & Paul, 1995) to ensure that they had the necessary reading and comprehension prerequisites to participate in the intervention. The CALC was only administered to children in the ID group to ensure they had the necessary reading skills to participate in the program. I trained the educators on how to administer the measures and all data were returned to me.

Eight items from the CALC (Miller & Paul, 1995) were administered. The first subset of items assessed oral comprehension. The educator read a sentence (e.g., "The girl puts the blanket on the doll.") and showed the children a page with four different pictures on it. The child had to point to the picture that best corresponded to the meaning of the sentence that had been read to her. For the second part, she presented the child with a sentence written on a strip of paper. The child was asked to read the sentence out loud. The educator then presented a sheet with four different pictures on it and asked the child to point to the picture that corresponded to the meaning of the sentence she had read out loud. Children received a point for every correct response. There were four items in each section, for a total of eight items. Children who got more than two incorrect out of the eight possible answers were not eligible to participate in the intervention.

Assessment of strategy use. The Strategy Use Interview (SUI) is a task I designed to assess three outcomes measures: the types of strategies used, the proportion of appropriate strategies used, and the ability to use more than one strategy to solve a given problem. The complete SUI can be found in Appendix A. For the ID group, the SUI consisted of 6 single digit addition and subtraction problems. For the SM and AM groups, the problems were single- and double-digit addition and subtraction word problems, with and without regrouping. There were four Action problems, one Part/Whole problem, and one Seesaw problem. For the Action problems, the unknown was placed in different positions within the word problems. For the Part/Whole problem, the part was unknown, and in the Seesaw problem, the smaller set was unknown.

The SUI was administered to each student individually. The interviewer read each word problem out loud to the participant from an index card on which the problem was printed. She then placed the card on the top of the child's desk. The interviewer said, "I have a pencil, an eraser, some paper, and some tokens. I would like you solve the problems as best as you can in any way that makes sense to you. You can use the tokens if you wish, but you do not have to. You can also use paper to work out the problem. Please show your work and write your answers in the box at the bottom of the page in your workbooks." The student was permitted to solve the problem in whatever way that was most meaningful to him or her using any of the materials at hand and the problem was re-read to the student as often as necessary. The interviewer recorded the child's numerical answer at the bottom of pre-established coding scheme (refer to Figure 2 for the sheet used by the interviewer).

	Strategy Used	Appropriate (Y/N)	2nd Strategy Used	Appropriate (Y/N)
Melissa has 316 seashells. Her sister gives her some more. Now she has 113 seashells. How many seashells did her sister give her?	None ☐ DM ☐ CS ☐ DF ☐ RC ☐ Alg. ☐ Oth. ☐	/1	None ☐ DM ☐ CS ☐ DF ☐ RC ☐ Alg. ☐ Oth. ☐	/1
There are 270 pancakes on a plate. Timmy's family eats 97 of them. How many pancakes are left?	None ☐ DM ☐ CS ☐ DF ☐ RC ☐ Alg. ☐ Oth. ☐	/1	None ☐ DM ☐ CS ☐ DF ☐ RC ☐ Alg. ☐ Oth. ☐	/1
There are 406 French and English books in the classroom. 118 of them are in French and the rest are English. How many English books are in the classroom?	None ☐ DM ☐ CS ☐ DF ☐ RC ☐ Alg. ☐ Oth. ☐	/1	None ☐ DM ☐ CS ☐ DF ☐ RC ☐ Alg. ☐ Oth. ☐	/1

Figure 2. Part of the Strategy Use Interview coding sheet.

After the child arrived at an answer to the problem, the interviewer posed a series of questions designed to obtain more information about the strategy used. The interviewer asked, "Can you explain to me how you solved this problem?" Probes such as, "Can you tell me more about that?" were used to obtain as much information as possible. These questions provided information regarding the existing strategies the children used when solving word problems.

Six codes were used to classify children's strategies: Direct Modeling, Counting Strategies, Derived Facts/Invented Algorithm, Recall, Algorithm, or Other. A strategy was coded as Direct Modeling (DM) when the child physically represented the quantities and actions in the problem either by using fingers, manipulatives, or by drawing out the quantities (Carpenter & Moser, 1984). The Counting Strategy (CS) code was used when children were able to hold one quantity in their heads and count up to, or down to, a second quantity. When a child used a Derived Fact (DF), he used a previously known fact to help find the answer. That is, the child may have said something like, "5 + 5 = 10 so if I do 5 + 6, that is one more, so it is 11."

Children can also use an Invented Algorithm (IA) where they invent their own mental procedure to help them solve the problem (Carpenter & Moser, 1984). A code of Recall (R) was given when the child was able to recall the mathematical fact without calculating it. This code was only appropriate for children in the ID group because they were the only ones given single digit problems to solve. There was also a code for when children used a standard algorithm (ALG) to solve the problem. This code was only appropriate for students in the SM and AM groups because there were given multi-digit problems.

Lastly, the Other (OTH) code was used for strategies that were not interpretable or were unrelated to the mathematical action, structure, or quantities in the problem. For example, for the problem, "There are 17 pancakes on a plate. Timmy's family eats 7 of them. How many pancakes are left?," a student who states everything he knows about pancakes but could not explain the mathematical action happening in the problem (i.e., separating action) would be coded as Other. In addition, children were coded as using an Other strategy if they provided an incomplete answer, such as providing part of a strategy, but not following through to a solution. All strategies were coded immediately on the coding sheet by the interviewer during the interview.

Once the child had explained her strategy, the interviewer coded the child's strategy in terms of its appropriateness. That is, the child received 1 point if the strategy he had used was appropriate for the problem type, even if the child arrived at the wrong answer. The child received 0 points if the strategy that was selected was not appropriate for the problem type. I decided not to code for accuracy as doing so could underestimate the abilities of the students, especially those with intellectual disabilities and who were struggling in mathematics. For example, counting errors could lead such students to incorrect answers despite having used appropriate strategies. As such, points were given for appropriateness of strategy instead of accuracy of the answer. The appropriate score was calculated by summing up the total number of points. In addition, the proportion of appropriate strategies on the SUI was calculated and converted to percent.

Flexibility in strategy use was also assessed. After the child explained his first strategy, the interviewer then asked, "Is there another way you could solve that problem?" *If yes:* "Ok, please solve the problem again using this new way." *If no:* "Just try your best. I just want to know how you think about problems like this." These probes assessed if children had the ability to use more than one strategy to solve a given problem. Children received 1 point if their second strategy was in a different category than their first. That is, if a child used Direct Modeling with chips to solve the problem the first time, no points were given for using a second Direct Modeling with different tools. If the second strategy was counting on from a larger number, then the child received an additional point because Counting Strategies are in a different category than Direct Modeling. The flexibility score was the total number of distinct additional strategies used.

Word Problem Structure Test. In the same session, after the SUI was completed, the Word Problem Structure Test (WPST) was administered. The WPST is a measure I designed to assess the child's ability to identify the correct visual representation of a word problem's structure. The WPST can be found in Appendix B. The measure consists of 6 word problems: four Action problems, one Seesaw problem, and one Part/Whole problem. For each item on the WPST, the interviewer presented the child with a large cardboard sheet with three visual representations of the word problem structure and an option of "None of These" (refer to Figure 3 for the cardboard sheet used in the WPST). The positioning of the choices was different for each item, except for the option of None of These, which was always the fourth choice. The child was asked to point to the representation that best matched the word problem in each item.

An isomorphic version of the WPST was given at posttest. The posttest had 8 items on it. As with the pretest, there were four Action problems, one Seesaw, and one Part/Whole problem. There were two additional Action problems that were written such that the numbers presented in the problem were not in the typical start → action → end sequence. These problems, along with the Seesaw and Part/Whole problems, were unfamiliar problems for the students as these types were not taught during instruction.

The WPST was administered to each child individually. The interviewer read a word problem, and presented the problem typed on an index card to the child. The interviewer then placed the index card on the top left hand side of the desk. The interviewer said, "Now, I'm going to place a cardboard in front of you with different pictures. I want you to show me the picture that best matches the word problem I just read to you. I want you to try your best." The interviewer placed a cardboard with the three different visual representations and the fourth option of None of These on it in front of the child. The interviewer then asked the child to identify the word problem structure by pointing to the visual representation that best matched the word problem. The interviewer said, "Look at this cardboard. Which of these choices (circles to gesture the entire cardboard) best matches the problem I just read to you?" The interviewer wrote down the child's answers on a scoring sheet. A child received 1 point for each correct identification. The total number of correct answers, converted to a percent, was calculated for the WPST score.

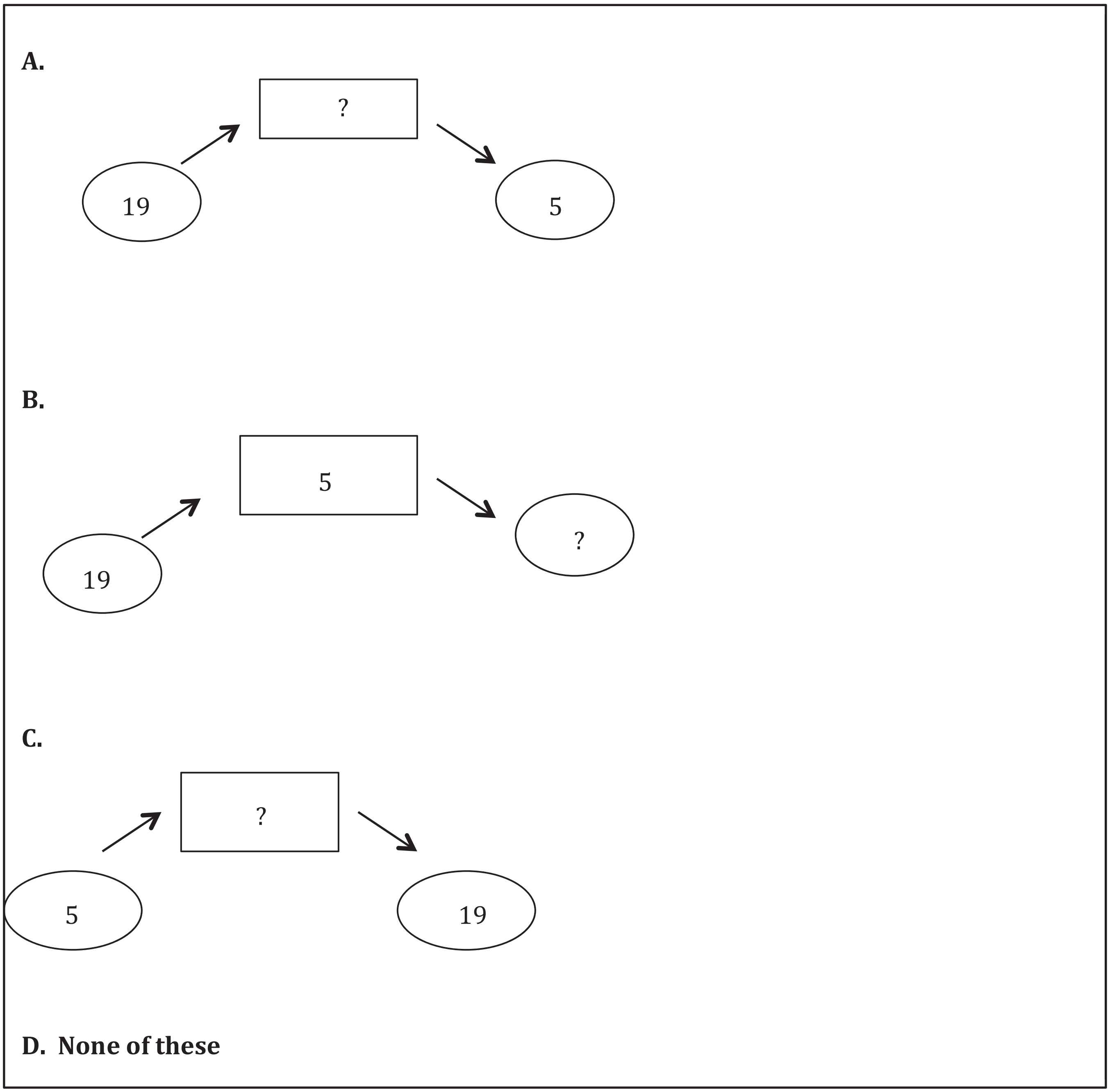

Figure 3. WPST word problem structure visual representations.

Description of the Intervention

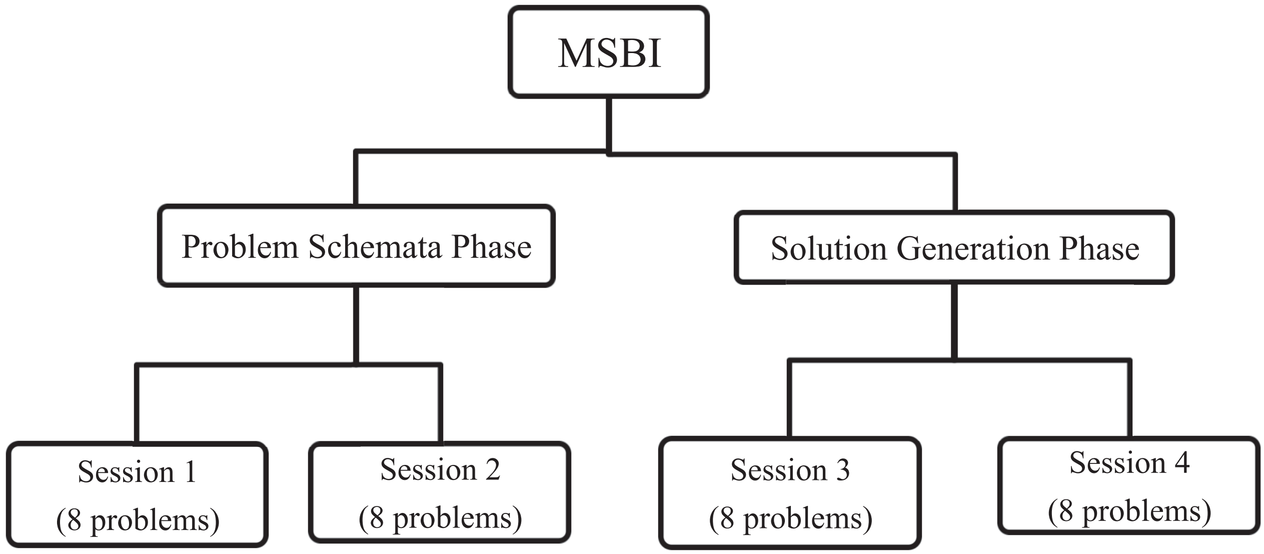

Figure 4. Outline of Modified Schema Based Instruction intervention.

Figure 4 depicts an outline of the MSBI intervention. The instruction was broken down into two phases: the Problem Schemata Phase and the Solution Generation Phase (Xin, Jitendra, & Deatline-Buchman, 2005; see also Jitendra, Griffin, McGoey, Gardill, Bhat, & Riley, 1998; Jitendra, Hoff, & Beck, 1999). In turn, each phase consisted of two sessions during each of which the children worked through eight different problems. The participants participated in 45-minute bi-weekly sessions during two consecutive weeks for a total of four sessions, which amounted to a total of three instructional hours.

The children completed all sessions in their small groups (ID, SM, and AM). The instruction and the materials were the same for all groups. Each group had the same addition and subtraction word problems but the numbers in the problems differed by group. The ID group worked on single digit problems (refer to Appendix C); the SM group had double digit with and without regrouping problems; and lastly, the AM group had three digit with and without regrouping (refer to Appendix D for the problems for the SM and AM groups). All groups had the same problems in session 1 of the Problem Schemata Phase as the goal of the session was for children to learn where to place the numbers within the structure. The problems for both the MSBI phases depicted situations that the children could encounter in their everyday lives (e.g., purchasing items, sharing items with friends, items in a store).

Immediately before the first instructional session, children participated in a 15 minute practice session on how to solve multiplication and division word problems. The goal of this exercise was for the children to begin thinking about different ways of solving problems. Multiplication and division word problems were used to avoid a potential confound for the study. I did not want to use addition and subtraction word problems to avoid teaching them how to solve such problems prior to the instruction, where only addition and subtraction problems were used.

Problem Schemata phase. The Problem Schemata phase consisted of the first two instructional sessions. The visual representation for the structure of Action problems is shown in Figure 5. This visual tool was referred to as a “schemer” with the students. Children were taught to identify the components in the Action schemer for Action word problems. In addition, they learned where to place the numbers provided in the word problems into the correct spaces in the schemer.

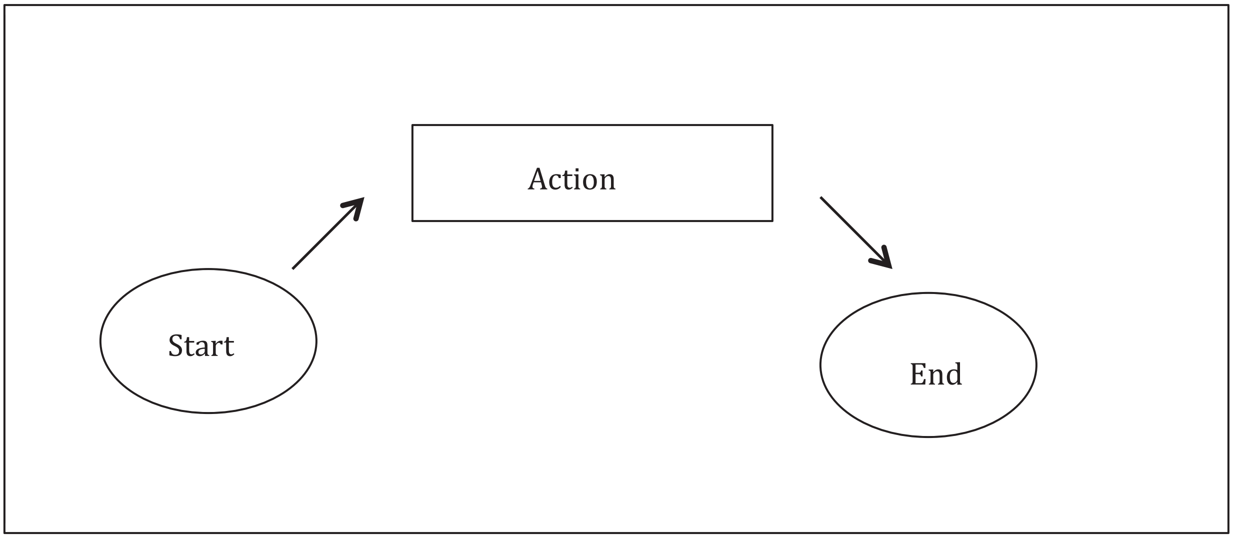

Figure 5. Schemer of an Action word problem (adapted from Jitendra et al, 1998).

The instructor drew the schemer on a whiteboard. The word problems that were used during the Problem Schemata phase were completed word problems known as “story scenarios” (Jitendra, Griffin, McGoey, Gardill, Bhat, & Riley, 1998). Story scenarios have no missing information for the children to solve. All of the story scenarios given during the sessions were in a workbook given to the participants at the beginning of each session. The workbooks consisted of eight story scenarios per session. Each story scenario was written at the top of a new page. Blank schemers were placed below the story scenario on the same page (refer to Figure 6 for a sample page of the workbook).

Kelly has 79 stickers in her sticker collection. She gave 27 stickers to her sister. Now Kelly has 52 stickers left in her collection.

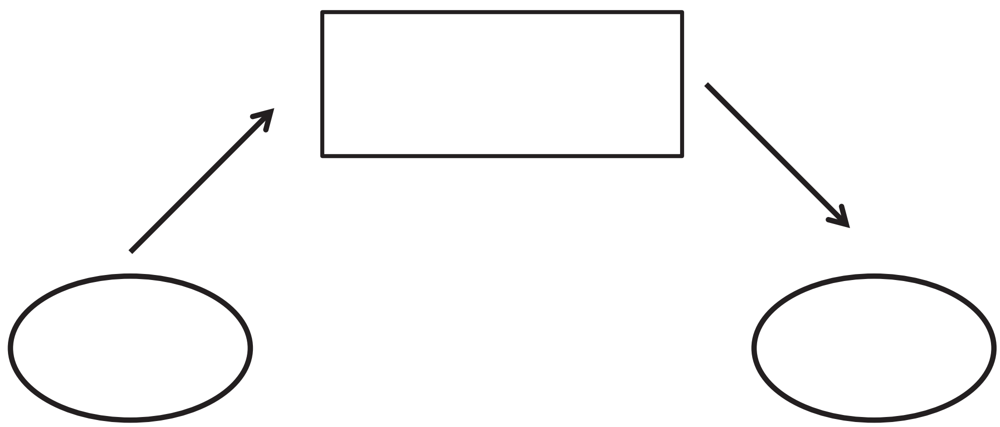

Figure 6. Sample workbook page used during the Problem Schemata Phase.

While referring to the schemer the instructor drew on the whiteboard, she named the schemer (Action schemer) and demonstrated its different parts: the start, the action, and the end. As she explained the different parts, she used a large checklist that was printed on a large sheet of paper placed next to the whiteboard. The checklist contained questions for the children to ask themselves to remind them of the different parts of the schemer and to place the numbers in the right parts (refer to Figure 7 for checklist). Once she had finished explaining the different components of the schemer, the instructor gave each student a paper with the same checklist she had posted earlier in the session. The instructor went through the different questions on the checklist that children need to ask themselves when trying to identify where the numbers go in the schemer. Following this explanation, she referred the children back to the checklist while they worked on each story scenario. The full script for the Problem Schemata sessions is presented Appendix E.

Action Checklist

Write it!

Write the numbers into the schemer.

- □ Does the problem have a start number?
- □ Does the problem have an action number?
- □ Does the problem have an end number?

Find your “x”.

Find it!

Plan your strategy to solve the problem.

- □ Do you want to add or subtract?
- □ How are you going to solve it?

Do it!

Solve the problem.

- □ Use your strategy to solve the problem.
- □ Check your work.

Share it!

Share your answer with your friends.

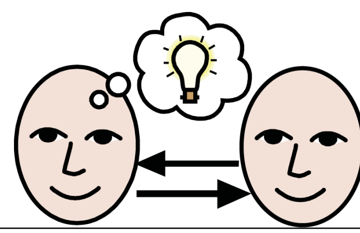

Figure 7. Checklist for Action word problems. Adapted from Jitendra, Griffin, McGoey, Gardill, Bhat, & Riley (1998)

With the checklist beside the picture of the schemer on the board, the instructor read a story scenario (corresponding to the first story scenario in the workbook) to the children and placed the numbers into the appropriate parts of the schemer. The first three problems in the children’s workbook were taught the same way. The instructor read the story scenario, repeated the parts of the schemer and their significance, and then used to checklist to place the numbers in the correct parts of the schemer. Figure 8 demonstrates an example of a story scenario with the numbers placed in the correct parts of the schemer for an Action problem used in the Problem Schemata Phase.

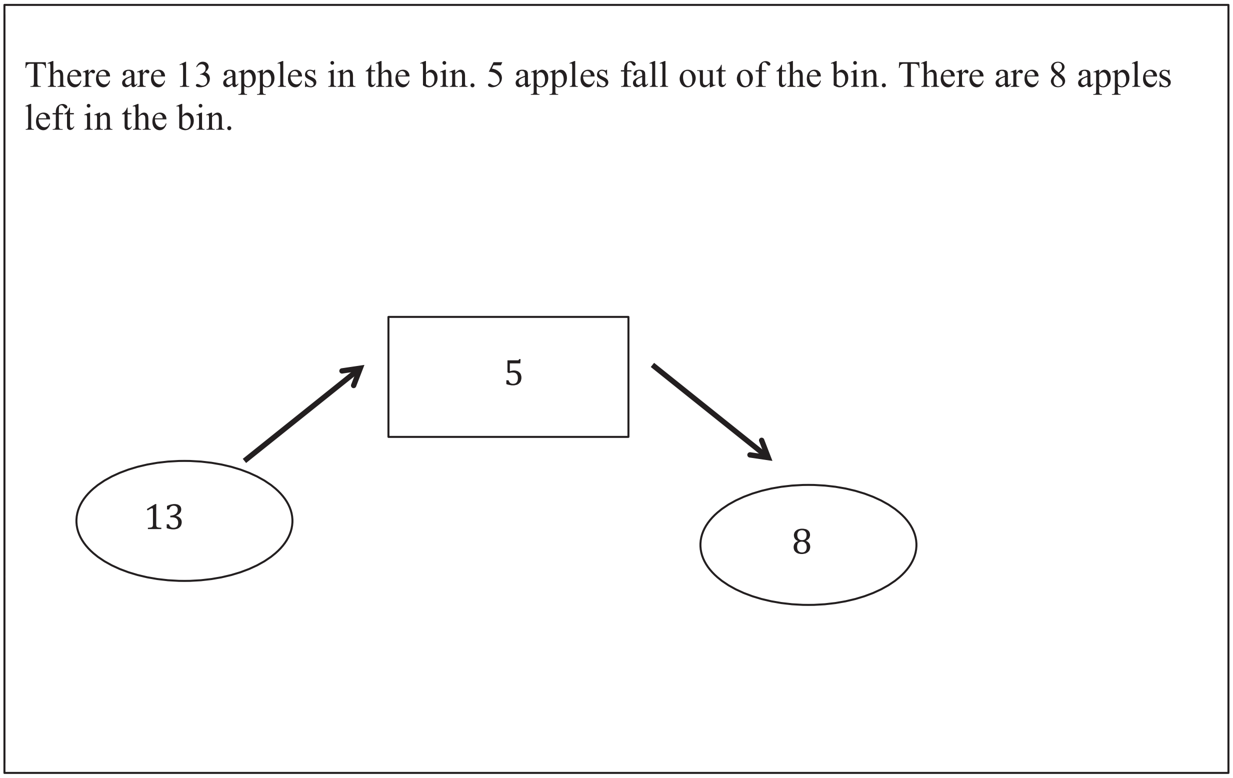

Figure 8. Completed Action schemer and Action scenario for the Problem Schemata Phase

Once the instructor had completed the first three scenarios in the workbook with the students, she encouraged them to tell her where the numbers would be placed in the schemer for the next two scenarios. The instructor read the story scenarios out loud to students and referred them to their checklist to help them determine where to place the numbers in the schemer. She waited for the children to tell her where the numbers should go in the schemer. As they made their suggestions, she wrote down the numbers in the appropriate parts of the schemer. When necessary, the instructor would prompt or guide children if they were having difficulty identifying where the numbers should be placed.

The children completed the three remaining story scenarios in the workbook under the supervision of the instructor. The children were permitted to work individually or with their peers. As the children were completing the problems in their workbook, the instructor observed their work to ensure that they were completing the scenarios correctly. If a child made a mistake, the instructor provided corrective feedback. When necessary, she reviewed the story scenario with the child to reinforce what was taught during the session. Both Problem Schemata sessions were delivered using the same procedure.

Solution Generation phase. In the Solution Generation phase (sessions 3 and 4), the participants solved word problems using the schemer. Each session in this phase began with a review of the problem type and the different parts of the schemer. Children were provided with a workbook (one for each session) and each page contained one word problem and a blank schemer. There was one problem per page. Each workbook consisted of eight word problems, each of which included an unknown. Figure 9 is a sample page from a workbook for the Solution Generation phase.

Brian gave some candies to his mother. His mother ate 114 candies. Now she has 225 candies left. How many candies did she have in the beginning?

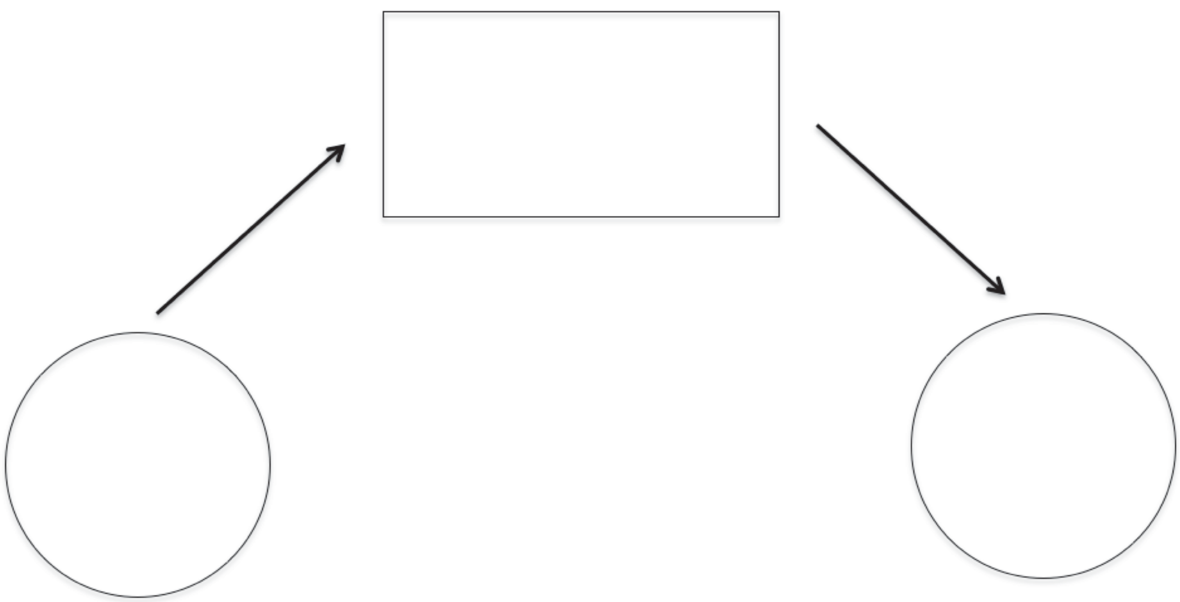

Figure 9. Sample worksheet from the Solution Generation Phase.

The instructor read each word problem aloud and asked the children to solve the problem. She reminded the children to use their checklists to guide them. The instructor encouraged the children to solve the problem on their own and then to share their problem solving strategies with their peers. She said, “There are many different and correct ways of solving this problem. I want you to solve the problem as best as you can, in any way you want. There are some blocks here or you can use the paper in your workbooks. You can use the blocks if you want, but you do not have to. When everyone is finished, we are going to share our answers.” The instructor permitted the children to solve the problems using the materials available to them.

After having given them some time to think about solving the problem, she then said, “Who can tell me one way to solve this problem? If you know, please raise your hand.” The instructor listened to the strategies given by the students and asked them questions about their strategies such as, “where did this number come from?” and “how did you know to subtract [or add] this number?” She then reviewed the strategy with group. If the strategy contained errors, the instructor asked the other children if they could correct it. She then said, “Great! Did anyone use a different strategy to solve the problem?” The instructor repeated the same procedure with each student. After three different strategies, she said, “That’s great! So we saw that there were many correct ways to solve this problem.” The instructor briefly reviewed the strategies before moving on to the next problem in the workbook. This procedure was repeated with the next four problems in the workbook.

For the last three problems, the instructor said, “So, we’ve learned many different ways of solving word problems. I would like you to finish the rest of the problems on your own using whatever strategy makes the most sense to you. You can use your own strategies or use some of the ones we talked about today. Remember, you can use the blocks or draw pictures if you wish.”

When the children finished their problems, the instructor collected their workbooks and checked the students' work with them to make sure it was correct.

The second session in the Solution Generation phase was conducted in the same way as the first. At the beginning of the session, the instructor began by asking the children to remind her of the strategies that were covered in the previous session. A full script for the Solution Generation session can be found in Appendix F.

Procedure

Pretest and posttest. Table 2 depicts the difference between the three groups in terms of which measures they received, who administered the tests, and when they were given. The educators administered the NKT and the CALC for the children in the ID group in a session prior to the pretest. In meeting 2, a trained research assistant or I administered the SUI and the WPST (pretest) to all groups. For the SM and AM groups, only one meeting took place during which the NKT, SUI and the WPST were administered by a trained research assistant or me. The NKT was videotaped for the SM and AM groups only, as it was administered at the same time as the pretest. The posttest consisted of isomorphic versions of the SUI and the WPST, and was administered by a trained research assistant or me. The NKT lasted 15 minutes and the SUI lasted approximately 30 minutes. In total, the pretest lasted approximately 50 minutes. A video camera recorded the students' hands and verbal answers to the interviews.

Table 2. *Test administration, instructor, and testing meeting days for all groups.*

	Screening		Pretest		Posttest	
	NKT	CALC	SUI	WPST	SUI	WPST
ID	Meeting 1 (E)	Meeting 1(E)	Meeting 2 (I)	Meeting 2 (I)	Meeting 3 (I)	Meeting 3 (I)
SM	Meeting 1 (I)	N/A	Meeting 1 (I)	Meeting 1 (I)	Meeting 3 (I)	Meeting 3 (I)
AM	Meeting 1 (I)	N/A	Meeting 1 (I)	Meeting 1 (I)	Meeting 3 (I)	Meeting 3 (I)

Note. E = Educator; I = Instructor

At pretest, after the child had answered all of the questions on the SUI, the instructor collected the workbook and the WPST began. Once the WPST was completed, the instructor accompanied the child back to her classroom (for the AM and SM groups) or to her parents (ID group), who were in the waiting room at the Center. The posttest was administered one week after the final MSBI session and lasted approximately 45 minutes. The same procedures were used for the posttest as for the pretest.

Practice session and intervention. The intervention took place in a quiet room at the school or at the center. One of two trained instructors implemented the sessions. The instructors conducted the sessions according to the rotation schedule outlined in Table 3. To eliminate the possibility of instructor effects, each instructor taught all groups to ensure that all students received instruction from both instructors.

Table 3. *Schedule for the instructors during MSBI.*

Instructional Session	Group		
	ID	AM	SM
Schemata Phase – Session 1	A	B	A
Schemata Phase – Session 2	B	A	B
Solution Phase – Session 1	A	B	A
Solution Phase – Session 2	B	A	B

Note: A = Instructor 1; B = Instructor 2

For the practice session and the MSBI intervention, the children sat around a table with the instructor. The practice session lasted 15 minutes. For the intervention, which began immediately after the practice session, there was large whiteboard with markers and an eraser for the instructors to use during the sessions. Posters of the checklists were used alongside the schemer on the whiteboard. Each student was given a workbook, a pencil, an eraser, and colored tokens. The instructor brought the participants to the room from the classroom (for the AM and SM groups) or upon arrival to the center (for the ID group).

For the first 25 minutes of each session, the instructor engaged in instruction following a pre-prepared script (refer to Appendices E and F for scripts for all sessions). For the remaining 20 minutes, the participants were asked to complete questions in their workbooks. At the end of the sessions, the students returned back to their classrooms (for the AM and SM groups at the school) or to their parents (for the ID group at the center). The procedure used to deliver the Solution Generation instruction sessions (the last two sessions) were the same as those used for the Problem Schemata sessions.

Results

The present study examined the problem solving strategies used by children with intellectual disabilities following Modified Schema Based Instruction (MSBI). The goal was to document the problem solving strategies used by children with intellectual disabilities and compare them to other groups of children namely, a group of children struggling in mathematics (SM) and a group of children of average mathematics ability (AM). A total of nine children (each in their respective group) participated in four 45-minute instructional sessions on MSBI. Children were administered a pretest prior to instruction and completed an isomorphic posttest that documented children's strategy use and the identification of word problem structure. Specifically, my research questions were as follows:

Prior to instruction:

1. What strategies do children with intellectual disabilities use to solve word problems before instruction and how do they compare to strategies used by children struggling in mathematics and children with average mathematics ability?
2. How appropriate are the strategies used by children with intellectual disabilities and how does this compare to the children in the SM and AM groups?
3. How flexible in strategy use are the students with intellectual disabilities as opposed to the students in the SM and AM groups?

Following instruction:

4. What was the problem solving performance of children with intellectual disabilities after instruction, and how do they compare to those of children in the SM and AM groups? In particular, are there differences with regards to the types of strategies chosen, the appropriateness of their strategies, their ability to use

more than one strategy to solve a problem, and their ability to identify the underlying structure of the word problem?

My hypotheses were that there would be an improvement between pre-and posttest scores on use of appropriate strategies and identification of word problem structure for all groups. I also predicted that all groups would demonstrate greater flexibility in their strategy use following the MSBI intervention. That is, I hypothesized that participants in all three groups would improve in their ability to use more than one strategy to solve a given problem.

Children's Problem Solving Strategies

There were missing data for three of the children. Student 2 only answered 5 of 6 questions on the SUI at pretest. Because of time constraints during the posttest, Student 3 only answered 7 out of 8 questions on the SUI. Student 5 requested to stop the SUI posttest interview after the third question. She did, however, complete the full WPST at posttest.

Strategy type. Table 4 summarizes the types of problem solving strategies used before and after the MSBI intervention. Prior to instruction, the students in the ID group showed a strong preference for strategies coded as Other. Only one student in this group used Counting Strategies for two of the problems. In contrast, students in the SM and AM groups relied heavily on standard algorithms to solve the problems before instruction, especially those in the SM group. There was some variability in the SM group, however, with one student using a Direct Modeling strategy and another student using a strategy unrelated to the mathematical content in the problem (Other). One student (Student 7) in the AM group used Invented Algorithms. Although he could not articulate precisely what his strategy was, it was nevertheless coded an Invented Algorithm (IA).

Table 4. *Types of problem solving strategies used before and after MSBI*

Group	Pretest						Posttest					
ID	DM	CS	DF/IA	R	Alg.	Other	DM	CS	DF/IA	R	Alg.	Other
Student 1	0	0	0	0	--	6	7	1	0	0	--	0
Student 2	2	0	0	0	--	3	6*	0	0	0	--	2
Student 3	3	2	0	0	--	1	6*	0	0	1	--	0
SM												
Student 4	2	0	0	--	4	0	1	0	0	--	7*	0
Student 5	0	0	0	--	6	0	2	0	0	--	1	0
Student 6	0	0	0	--	5	1	8	0	0	--	8	0
AM												
Student 7	0	0	6	--	0	0	0	0	7*	--	1	0
Student 8	0	0	0	--	6	0	0	0	0	--	8*	0
Student 9	0	0	0	--	6	0	1	0	0	--	7*	0

Note. * denotes that a schemer was used with the strategy.

Note. Data missing for Student 2 (pretest), Student 3 (posttest), and Student 5 (posttest).

Following MSBI, there were clear differences between all groups with regards to strategy use before and after instruction. There was a marked change for the ID group. There was more consistent use of Direct Modeling strategies using tokens. Two of the students in the ID group also took to drawing the schemers prior to selecting a strategy. Furthermore, students in the SM group used more Direct Modeling strategies after instruction then they did before. Student 6 was the only student in the study to correctly identify two distinct solution strategies for the problems. This explains why he has a total of 16 answers on the posttest, whereas all other students have a total of 8 (with the exception of Student 5, who asked to stop the posttest during the SUI).

There was very little change in strategy use in the AM group. As seen on the both pre- and posttest, Student 7 used mostly Invented Algorithms whereas Students 8 and 9 used standard algorithms to solve most problems. All three students in the AM group, however, used the schemer to organize the information in the problem prior to solving it on the posttest.

Appropriateness of strategy use. Table 5 presents the appropriateness scores and the proportion of appropriate strategies used by each student on the SUI at both pretest and posttest. The results demonstrate that in general, most students were able to select more appropriate strategies following the intervention than at pretest. In fact, on average there were 16% more strategies that were appropriate at posttest than at pretest. Only Student 2 had a lower score at posttest.

Table 5. *Appropriateness scores and proportion of appropriate strategies on the SUI (first strategy used) at pretest and posttest*

Group	Pretest		Posttest	
	Score	Proportion (%)	Score	Proportion (%)
ID				
Student 1	0	0	3	37
Student 2	3	60	0	0
Student 3	5	83	6	85
SM				
Student 4	0	0	3	37
Student 5	3	60	3	100
Student 6	3	50	7	87
AM				
Student 7	6	100	8	100
Student 8	4	66	7	87
Student 9	3	50	6	85

There were several transfer items on the SUI. There were two items (one Part/Whole problem and one Seesaw problem) on the pretest and three on the posttest (one Part/Whole problem, one Seesaw problem, and one problem written out of sequence). Table 6 demonstrates the children's appropriateness scores on familiar items as well as the transfer items on the SUI at pre- and posttest. The results demonstrate that overall the children used a higher proportion of appropriate strategies on both familiar and transfer items at posttest. Only three students (Students 5, 7, and 9) were able to correctly answer problems that were written out of sequence (i.e., start → action → end). Otherwise said, the transfer problem that was the most difficult for children was the one that was written out of sequence. At posttest, most children (except Student 2) were able to use a higher proportion of appropriate strategies to solve the Seesaw problem and all except Student 4 answered the Part/Whole problem with an appropriate strategy.

Table 6. *Appropriateness scores and proportion of appropriate strategies for familiar (F) and transfer (T) items on the SUI at pre- and posttest.*

Groups	Pretest				Posttest			
	Score (F)	Score (T)	Prop in % (F)	Prop. in % (T)	Score (F)	Score (T)	Prop. In % (F)	Prop. in % (T)
ID								
Student 1	0	0	0	0	1	2	25	66
Student 2	2	1	50	50	0	0	0	0
Student 3	3	2	75	100	4	2	100	66
SM								
Student 4	0	0	0	0	2	1	40	33
Student 5	2	1	50	50	2	1	100	100
Student 6	1	2	25	100	4	3	60	100
AM								
Student 7	4	2	100	100	5	3	100	100
Student 8	3	1	75	50	5	2	100	66
Student 9	2	1	50	50	3	3	60	100

Note. Prop. = Proportion.

Flexibility

In terms of flexibility in strategy use, most students demonstrated little change in the ability to use more than one strategy from pretest to posttest. When asked for an alternative strategy during both pre- and posttest, most children would gave a response in the same category as their first strategy. This was seen for students across groups for all problems. For example, if a child used Direct Modeling with the tokens for his first strategy, then he might draw out the problem (directly modeling through a drawn picture) for his second strategy. A strategy that was noticed in three students (two from the AM group and one from the SM group) is that they would change the operation for the second strategy, both during the pre- and posttest. That is, they would also use the same algorithm for their second strategy but for a different operation. For example, Student 8 was asked to solve an Action problem where the action was the unknown on the pretest. For his first strategy, he performed the algorithm for 198 – 116 and correctly solved the problem. When asked for a second strategy, he used the standard algorithm for the opposite operation and solved 198 + 116. He initially seemed confused as to why he did not get the same answer, but nevertheless continued to do this throughout the pretest. At posttest, he did the same thing, except his second strategy was to use the multiplication algorithm (if he used addition in his first algorithm) or division (if he used subtraction in his first algorithm).

Student 6 in the SM group was the only child able to successfully use more than one strategy for all 8 problems on the posttest. For example, he used the algorithm as his first problem solving strategy for one of the end unknown problems. When asked for a different strategy, he explained what he would do to directly model the problem. He did not physically act it out by counting out tokens or drawing tallies, but clearly explained his direct modeling strategy. For example, Student 6 used the algorithm to solve 49 – 26 and for his second strategy, he said, "I would draw 49 dots and cross out 26 of them."

Identification of Problem Structure

The Word Problem Structure Test (WPST) assessed children's ability to correctly identify the underlying structure of word problems. Figure 10 demonstrates average percent scores on the WPST for the three groups. As shown, the AM group experienced virtually no change following the intervention (44% at pretest and 45% at posttest). The results show that the most marked changes were observed in the ID and SM groups. The scores increased 29 and 35 percentage points for the ID and SM groups, respectively.

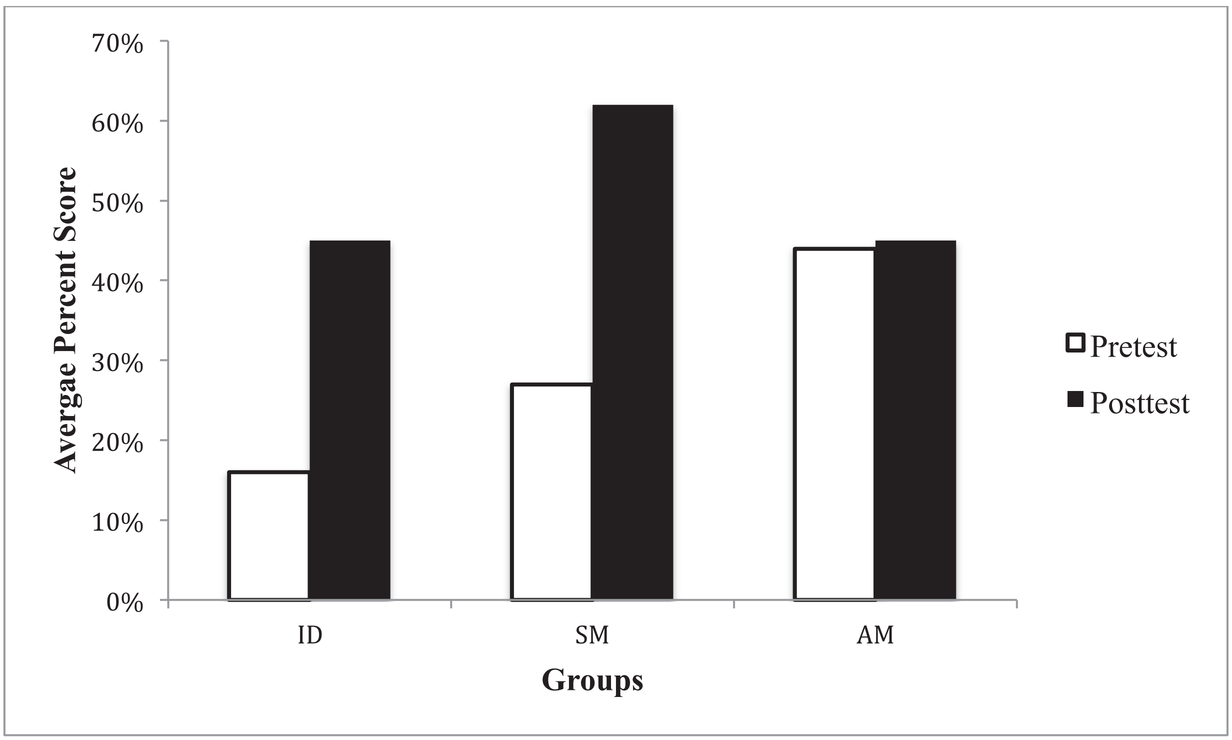

Figure 10. Mean percent scores on the WPST at pretest and posttest as a function of group.

Table 7 displays the mean percent scores on the WPST at posttest by group as well as the proportion of appropriate strategies used during the SUI.

Table 7. *WPST mean scores and proportion of appropriate strategies on SUI at pretest and posttest*

Group	Pretest		Posttest	
	WPST[a]	Prop. (%)	WPST[a]	Prop. (%)
ID	5.3	47	45	41
SM	27	36	62	74
AM	44	66	46	83
All groups	25.4	49.7	51	66

Note. "a" indicates scores in percent.

Note. Prop. = proportion

These results suggest that when the children were able to recognize the underlying structure of the word problem, they were more likely to select more appropriate problem solving strategies during the SUI. Specifically, except for the AM group, as the scores on the WPST increase, the proportion of appropriate strategies increases also. The AM group demonstrated almost no change on the WPST from pretest to posttest. Despite this, the proportion of appropriate strategies for the AM students on the posttest was 17% higher than at pretest (refer to Figure 11). Thus, it is possible that the for the AM group, the instruction helped reinforce the use of appropriate strategies even though no improvement was observed on the WPST.

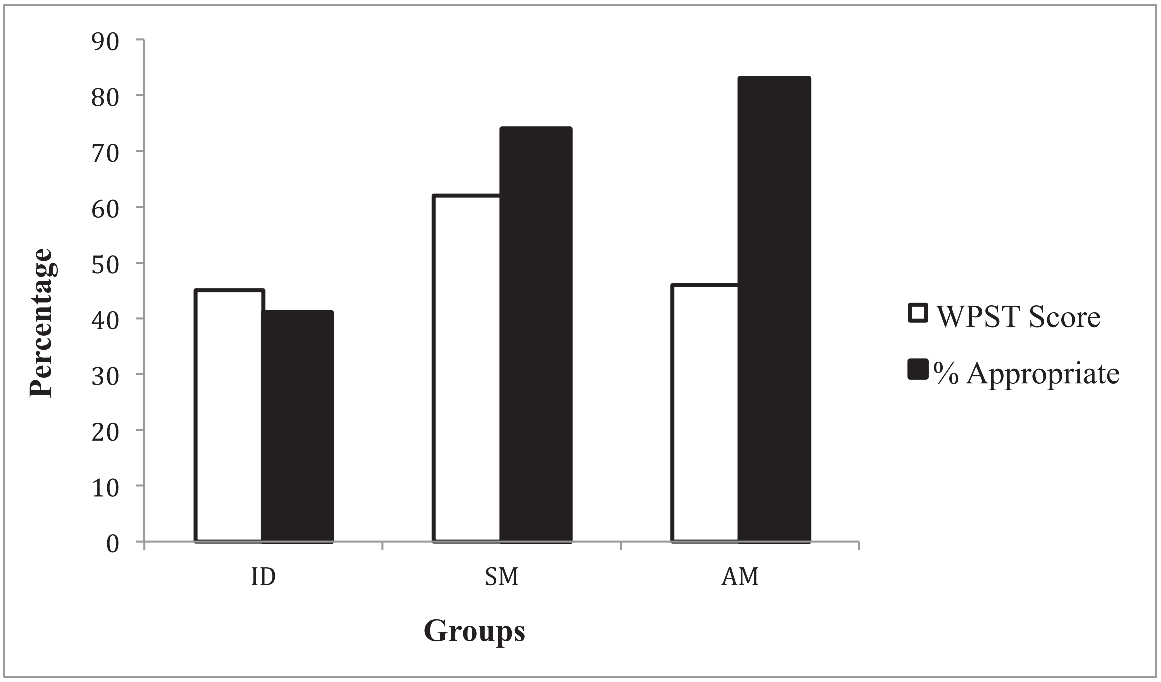

Figure 11. Mean percent scores on the WPST and proportion of appropriate strategies at posttest as a function of group.

Another interesting finding from the results of the WPST are children's responses to the transfer items on the posttest. The posttest contained four transfer items: two items in which the problem was not written in the standard start → action → end sequence; one Part/Whole problem; and one Seesaw problem. The results shown in Table 8 show the students' scores on the different types of transfer problems.

Table 8. *Mean percent scores on WPST transfer items on the posttest as a function of group*

Group	Out of Sequence Score	Part/Whole Score	Seesaw Score
ID	0	100	0
SM	50	67	0
AM	0	66	0

Note. One of each problem type and two out of sequence transfer problems were included on the WPST at posttest.

All except two students were able to correctly identify the underlying structure for the Part/Whole problem. Only two students were able to identify the correct structure when the problems were written out of sequence. No student was able to correctly identify the structure for the Seesaw problem.

Discussion

The present study sought to document the problem solving strategies used by children with intellectual disabilities and to compare them to children who are struggling in mathematics and those of average mathematics ability. In addition, the study examined the potential effects of a modified version of Schema Based Instruction on children's ability to successfully solve Action word problems. Specifically, the study examined children's ability to use appropriate solution strategies, to use multiple strategies to solve a given problem, and to identify the underlying structure of word problems.

The results demonstrated marked differences in strategy choice amongst the different groups. Children in the three groups used different types of problem solving strategies prior to instruction, with children in the ID group using mostly strategies that were unrelated to mathematical content, and children in the SM and AM groups using standard algorithms. In addition, despite having only received three hours of Schema Based Instruction over two weeks, children's strategies became more appropriate following the intervention and they were better able to correctly identify the word problem structure. The largest gains on these latter two measures were seen in children in the ID and SM groups.

This study supports previous research about the effectiveness of SBI on children's ability to solve word problems. Previous SBI research (e.g., Fuchs, Fuchs, Finelli, Courey, & Hamlett, 2004) has shown that it is useful for different groups of children: children at risk for mathematics failure, typically developing youth, as well as children with learning disabilities. To my knowledge, this is the first study investigating whether or not a group of children with intellectual disabilities show improvement following SBI, thereby contributing to the research on SBI.

The present study contributes to the literature as it provides insight on the types of problem solving strategies used by children with intellectual disabilities. Some studies (e.g., Baroody, 1996) have investigated problem solving strategies, but this was conducted with older children. As far as I know, this is the first study investigating the problem solving strategies used by children with intellectual disabilities before and after Schema Based Instruction.

A particularly interesting finding from the study emerged from documenting the problem solving strategies that were used by children with intellectual disabilities. Prior to instruction, the children used a mix of Direct Modeling and inappropriate or incomplete strategies. Two of the children in the ID group in particular relied mostly on either incomplete strategies (e.g., using tokens to directly model one of the quantities in the problem and naming it as the answer) or ones unrelated to the mathematical content of the problem (e.g., talking about the non-mathematical context of the problem). By contrast, children in the SM and AM groups rarely (only once during the pretest) used an incomplete strategy, primarily relying on algorithms to solve the problems.

Following MSBI, children in the ID group used more Direct Modeling strategies. The students in this group, especially Students 1 and 3, began to use tokens to represent the numbers and the actions depicted in the problem. In contrast, there was less change in this regard in the SM and AM groups. There was some variability in the SM group, as students used Direct Modeling strategies and the standard algorithm. There was virtually no change in the AM group, however, with the children still preferring to use standard algorithms to solve problems.

The difference in strategy use prior to MSBI could be a result of the students' previous mathematics instruction. The children in the SM and AM groups all came from the same classroom, thus having received the same instruction prior to the intervention. In contrast, the children in the ID group all came from different schools. Thus, it is plausible that the students had received different kinds of prior exposure to mathematical concepts and problem solving. It could be that the previous instruction of the children in the SM and AM groups focused mainly on using algorithms to solve problems. Only one student (Student 4) used Direct Modeling at pretest. I suspect that her use of this strategy was only because there were tokens made available to the children during the practice session and she wanted to use them during the pretest.

In contrast, it is not surprising that children in the ID group used inappropriate strategies. Traditional instruction for children with disabilities tends to focus on applying procedures without meaning (Baroody, 1999; Jitendra, DiPipi, & Perron-Jones, 2002). That is, the link between the procedure and the concepts is not made explicit to the students. Indeed, these children are often viewed as being passive learners (Baroody, 1999), so this view could explain the limited strategies observed in this group prior to instruction. If these children are viewed as passive learners who can only learn rote procedures, they may not have been exposed to a wide range of solution strategies, or even given the opportunity to make sense of problems and ways to solve them prior to MSBI. This is consistent with the view that children with special needs should not be taught multiple strategies, but rather, be taught one or two strategies well to reduce working memory load (Baxter, Woodward, & Olson, 2001, cited in Verschaffel, Torbeyns, De Smedt, Luvwel, &Van Dooren, 2007).

The results also demonstrated that children from all groups chose more appropriate problem solving strategies following MSBI. Children were better able to select strategies that would lead to a correct answer. There are several possible reasons for this increase. It could be that, having been exposed to the underlying structure of the problems during MSBI, children learned to see the relationships between the numbers. This, in turn, may have made it easier for children to identify appropriate strategies for solving problems. Further, the children's success may have also been influenced by the teaching. The instructors provided all children with the same teaching with regards to the intervention. However, if a child was struggling, the instructor provided more individualized teaching to help the child learn the material. Another explanation is that the children were exposed to a number of appropriate strategies for addition and subtraction problems during the instruction. In the Solution Generation Phase, children presented their solution strategies to their peers. In doing so, the instructors reviewed the strategies and corrected any mistakes the children made. It is plausible, therefore, that this exposure and explicit feedback helped children identify which strategies would be suitable for a specific problem, thus helping them select their own appropriate ones. Research has shown that children, especially those with learning difficulties, benefit greatly from mathematics interventions which offer many opportunities for practice paired with explicit feedback (Butler, Miller, Lee, & Pierce, 2001).

Only one student (Student 2 in the ID group) did not show improvement on his ability to select an appropriate problem solving strategy, to use more than one strategy to solve a problem, or to identify the underlying problem structure. During the posttest, Student 2 was sick and attempted to rush through it. He did not take his time when attempting to solve the problems and the instructions had to repeated several times. In addition, the instructor had to stop the evaluation several times to manage problematic behaviors. This greatly affected his results on both the SUI and the WPST.

I hypothesized that there would be an improvement in children's ability to identify the appropriate underlying structure of word problems following instruction. The results support this hypothesis. Overall, children performed better on the WPST at posttest. Though the results were only at 50% at posttest (refer to Table 7), the percentage doubled from pretest, indicating an improvement for most students.

Some transfer effects were also observed. Most students were able to correctly identify the structure for the Part/Whole problem on the WPST posttest. It could be that there is something about the schemer for the Part/Whole problem that is intuitive for children. Another possibility could be that, because Action and Part/Whole problems are closely related (since most join/separate problems have a part/whole component to them), understanding one of the structures makes it easier to generalize to the other. Indeed, research on analogical reasoning has shown that when one grasps general schemata, transfer to other problems is enhanced (Chen, 1999; Gick & Holyoak, 1983). In contrast, however, none of the children were able to correctly identify the structure for the Seesaw problem. This could be because it is more difficult to identify the parts and the whole in Seesaw problems than in Action problems. Because this type of problem is more difficult for young children as compared to other types of addition and subtraction problems, children need a better understanding of the relationships between the numbers to solve Seesaw problems (Carpenter & Moser, 1984).

At posttest, six of the nine children used the schemers to help organize the numbers in the problem. The other three children did not use the schemers as part of their solution strategy, but they nevertheless improved at posttest. It is possible that these children had internalized the representation and thus did not need to draw it out.

During the WPST portion of the posttest, we were able to see that some children were using the schemers in a rote manner. That is, the children would select the schemer where the numbers were in the same order as those listed in the problem. When the numbers in the problem were written in start → action → end sequence, this led to a correct answer, but an incorrect response when the numbers in the problems were listed out of sequence (e.g., action → start → end). The WPST results helped demonstrate that some of the children did not understand the different parts of the schemer and the overall structure, but rather used it as a rote tool. These children would merely look at the order of the numbers and "match" it to the corresponding schemer. They did not pay attention to what the number was representing in the problem (e.g., the start number which is the number of things that are at the beginning of the problem prior to the action) or they did not decipher the parts in the problem. This is consistent with research on "psychological sets" (Luchins, 1949). Individuals can become used to solving things in a certain way, or as in this case, putting the numbers into the schemer in a certain order, so that it becomes rote. They do not stop to think about the problem, but rather engage in a behavior that has served them well in the past.

There were only two children (Students 5 and 6) who were able to see the structure despite the sequence. Student 6 answered both correctly, whereas Student 5 did not answer either correctly, but did not choose the answer that matched the sequence depicted in the problem. A reason for this could be that the children were not exposed to any problems that were written out of order except during the posttest. Future instruction should use problems written out of sequence throughout instruction. Perhaps this will help children to more fully understand the structure to the extent that they will be better able to apply it to novel problems. Of course, to assess conceptual learning of problem structure, other types of transfer problems would need to be incorporated on the posttest.

With regards to strategy flexibility, only one student demonstrated the ability to use more than one strategy to solve a given problem at posttest. Most children, both at pre- and posttest, would use one strategy to answer all problems, even if the strategy was not appropriate. This finding is consistent with the literature about strategy rigidity (e.g., Ostad, 1997). Even if the strategy would not result in the correct answer, children would continue to use the same strategy.

I speculate that one of the reasons that more of the children were not able to demonstrate flexibility in strategy use is because the types of strategies that were shared during the intervention may have all been of the same type. That is, as opposed to seeing a Direct Modeling strategy followed by a Counting Strategy, children may have seen two different ways to directly model the problem (e.g., using chips to represent the quantities and then drawing out the quantities). In fact, this is a pattern that emerged at posttest. Students used a different technique within the same strategy type. That is, children would stay within the same category of strategy type (e.g., Direct Modeling), but would use different materials. For example, Student 3 solved a problem by using Direct Modeling with tokens. In her second strategy, she drew out the quantities. Both of these strategies fall under Direct Modeling so I did not count them as being distinct. Had I chosen, however, to use a less conservative coding scheme, I may have seen more flexibility in strategy use for children. In addition, previous mathematics instruction could also have played a role in children's choice to use the same strategy. It is possible that during their regular instruction in their classrooms, they are not given the opportunity to explore different solution strategies.

The length of the intervention may also have impacted children's ability to demonstrate flexibility. As previously mentioned, all students received a total of three hours of instruction over a two week period. It is plausible that three hours is not enough time for children to become more flexible in strategy use. Increased opportunities for practice is especially important for children with special needs (Baroody, 1996). Perhaps with more practice, in addition to increased exposure to different types of strategies, children may have been more flexible following the intervention.

Implications and Future Directions

While promising, the results of this study should be interpreted with caution. The sample size was small and is not representative of the overall population in any of the groups. The children in the SM and AM groups, for example, all came from the same classroom (nine students in the class) in a small private school. Their instruction is not representative of what most children receive given the small class size. Their instruction may have been more individualized and they may have had access to more resources. The children were also not randomly selected, but rather were chosen to participate in the study by their teacher or by virtue of meeting the eligibility criteria set by the researcher. As such, I cannot talk about any causal effects of MSBI. Although the results provided some insight on the types of strategies used by different groups of children, I cannot generalize to specific populations nor conclude about the direct effects of MSBI on students' learning. Future experimental research is needed to yield causal conclusions.

Prior to the intervention, the participants were screened using the NKT and CACL to ensure that they had the necessary prerequisites to participate in the study (e.g., counting skills, reading comprehension). In a typical classroom setting, all children will not necessarily have the same level of skills, and those who have the prerequisites will not have the same level of proficiency. As such, future research is needed to see if children of different skill levels can benefit from MSBI or if additional teaching is needed prior to the intervention.

The present study has important implications for curriculum development in inclusive classrooms. As was demonstrated, MSBI showed promise across different groups of children. Though the gains made by each group differed, all groups learned to use more appropriate strategies and could identify the underlying structure of problems following the intervention. Given that increasing numbers of students are being integrated into mainstream classrooms (HRSDC, 2011), further research should investigate the effects of implementing a Schema Based Instruction program in inclusive settings, and in particular with larger, more representative samples in actual classroom settings. An important question to explore would be the effects for all students receiving less individualized instruction in larger classrooms.

As indicated earlier, I noticed that some students used the schemer in a rote fashion, presumably because the problems during instruction were always written in the start → action → end sequence. Future interventions should use problems written out of sequence from the beginning to encourage the children to pay closer attention to the components of the problems and what they mean. For children to be successful problem solvers, it is important that they understand the different components of the schemer. Furthermore, previous research has shown that children who learn concepts in a rote manner are not able to generalize to more novel problems (Cooper & Sweller, 1987). Future research could investigate whether students would show more flexibility and the ability to use appropriate strategies to solve a larger number of novel problems if they are able to correctly identify the correct structure for problems written out of sequence.

An area for future research is an examination of the schemers themselves. The schemers used for the Seesaw and Part/Whole problems in the pre- and posttests in the present study were different from the ones used in previous SBI studies (e.g., Jitendra, Hoff, & Beck, 1999). Despite these "untested" schemers, the children demonstrated success on identifying the structure of the Part/Whole problem following instruction. Further research should examine which schemers children find the most useful or intuitive. Having this information could change how SBI is taught so that the schemers become more meaningful to children, thus helping them better understand the underlying structure of word problems.

This study helped provide further evidence of transfer as demonstrated by the results of the SUI at posttest. Seven students were able to correctly solve the Part/Whole and Seesaw transfer problems on the SUI. It appears that learning the structure for Action problems helped children use strategies to successfully solve different types of addition and subtraction problems. This is supported by previous research with similar results (Fuchs, Fuchs, Prentice, Hamlett, Finelli, & Courey, 2004). When children understand the underlying structure of word problems, they are better able to solve transfer problems. It could be that helping children see the relationships between the numbers also contributed to identifying appropriate solution strategies for other problem types.

An important area for future research would examine how the schemers used in this study could be adapted or modified for multi-step addition and subtraction problems. It is currently unclear whether the schemers used in this study could be used one step at a time or if new schemers would need to be developed for instruction on more complex problems. With regards to the current mathematics curriculum in Quebec, a study examining the impact of SBI on children's ability to solve "situational problems" would be essential. Situational word problems are characterized by "real-life" elements and contexts, requiring multiple steps to a solution. In addition, situational problems are cross-curricular. Solving situational word problems is a central competency in the Quebec Education Plan, beginning in preschool and continuing through high school (Gouvernement du Quebec, Ministère de l'Éducation, 2001). A study examining the ways in which SBI could be used to help children solve these situational problems would help provide teachers with strategies for teaching children how to tackle complex word problems.

The length of the intervention was also short. Instructional time could be a factor as to why more children did not demonstrate the ability to use different strategies for solving a given problem. In total, children received three hours of instruction on how to solve Action word problems. Children, especially those with special needs, require more time and opportunities than their typically developing peers for practice to master mathematical concepts (Baroody, 1996; Chiak & Foust, 2008). Future research should look at impact of a longer intervention on children's ability to solve word problems. Specifically, it would be important to see if a longer intervention would help with children's ability to use more appropriate strategies, to be more flexible in strategy use, and to correctly identify the structure of word problems.

In addition to subtraction and addition problems, further research should investigate children's responsiveness to SBI with regards to different mathematics topics such as multiplication, division, fractions, and proportions. Some scholars have begun examining the effects of Schema Based Instruction on different topics, such as ratios and proportions (e.g., Jitendra, DiPipi, Perron-Jones, 2002) with children of either high, average, or low mathematics ability. Given the success observed in the present study for children in the ID group, future research should focus on the responsiveness of children with disabilities to SBI on different topics. This research would be of utmost importance to further inform mathematics instruction for children with special needs.

Conclusion

In sum, this study provided insight on how children with intellectual disabilities think when solving mathematical word problems, and it revealed the strategies that are meaningful to them. This research helped demonstrate that children with intellectual disabilities, similar to typically developing children, can make sense of mathematical word problems. They are able to reason through problems and identify appropriate ways to solve them. In fact, this study helped demonstrate that children with special needs are not “passive learners” (Baroody, 1999) but rather seek to actively solve problems presented to them using their own strategies. Despite important cognitive and executive functioning deficits (Rose & Rose, 2007; St. Clair-Thompson, 2006), the students in the ID group demonstrated important gains in strategy use, strategy appropriateness, and identification of word problem structure following MSBI.

In addition, this study shed light on effective instructional practices for teaching mathematics, specifically word problem solving, to children with intellectual disabilities. As the sessions were run in small groups similar in size to those in special education classes, the results were also more ecologically valid had the instruction been individualized. This differed from most of the research on children with intellectual disabilities, which was generally conducted with children in one-on-one settings (e.g., Rockwell, Griffin, & Jones, 2011).

To conclude, the students in the present study benefitted from an MSBI intervention. All students showed an improvement in their ability to select an appropriate problem solving strategy as well as identifying the correct underlying structure of addition and subtraction word problems. Further research is needed to examine the impact of MSBI on the word problem solving performance of children with special needs, but the present study shows promise for an intervention that could be used across a variety of different learners.

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Appendix A

Strategy Use Interview Protocol

Strategy Use Interview (SUI): Instructor Protocol (Pretest)
Materials
1) Pencils 2) Erasers 3) Blocks 4) Workbooks 5) Camera 6) Projector
Organization
1) The child will be seated at a desk or table in a quiet room, with the materials in front of him. 2) The interviewer will sit next to the child. 3) There will be a camera set up to film the child's hands/workbook on the table.

1) The interview gives the child a workbook, a pencil, an eraser, and some blocks.
2) The instructor says, "Today, I am going to read you some word problems. I would like you to solve the problems as best as you can in any way that makes sense to you. You can use the blocks if you want, but you do not have to. You can also use the space on the page in your workbook if you would like. Please solve the problem and write your answer in the box at the bottom of the page in your workbook."
3) The interviewer will read the word problem out to the child.
4) The interviewer will then say, "Please solve the problem. Just try your best."
5) When the child is done, the instructor will say, "Ok. Can you explain to me how you solved this problem?"
    a. Probes such as, "That is interesting, can you tell me more about that?" and "I think I understand it, but can you explain it to me a different way?" will be used.
6) The interviewer will then say, "Thank you for explaining it to me. Now, is there another way to solve this problem?"
    a) *If yes:* "Great. Can you solve it using that other way?"
    b) *If no:* "Just try your best. I just want to know about how you think about problems like this."
    c) Once the child has solved the problem, the instructor will repeat Step 5.
7) This procedure will be repeated for all 6 problems.
8) When the child is finished, the instructor will take the workbook and all materials from the child, clear the desk, and begin the WPST.

Action	Melissa has 316 seashells. Her sister gives her some more. Now she has 113 seashells. How many seashells did her Melissa's sister give her?
Action	There are 270 pancakes on a plate. Timmy's family eats 97 of them. How many pancakes are left?
Part/Whole	There are 406 French and English books in the classroom. 118 of them are in French and the rest are in English. How many English books are in the classroom?
Action	There are many birds sitting in a tree. There is a loud boom that scares 138 of them away. Now there are 114 birds left in the tree. How many birds were in the tree at the beginning?
Seesaw	During his summer vacation, Robert read 5 books. His friend, David, read 4 more books than Robert. How many books did David read during his vacation?
Action	Some butterflies are sitting on the grass. 78 butterflies come and sit on the grass. Now there are 313 butterflies sitting on the grass. How many butterflies were on the grass in the beginning?

Strategy Use Interview (SUI) - Posttest Word Problems	
Action	Lisa has 324 pennies. She finds some pennies on the sidewalk. Now she has 434 pennies. How many pennies did Lisa find on the sidewalk?
Action	Jenny ate a lot of cookies at school. She had 234 cookies in her lunchbox that mornings. Now she has 115 cookies left. How many cookies did she eat?
Action	There are 449 balloons at a party. 256 of the balloons pop. How many balloons are left?
Part/Whole	Sonia has 219 fish in her fishbowl. 166 of the fish are red. The rest are orange. How many orange fish does Sonia have in her fishbowl?
Action	Carl baked a lot of cupcakes. His father ate 289 cupcakes. Now there are only 179 left. How many cupcakes did Carl bake at the beginning?
Action	Mary has many stickers. She had 342 stickers before. Then, she got 165 for her birthday. How many stickers does Mary have?
Seesaw	Ricky has 107 Spiderman toys. He has 78 more than his friend Phillip. How many Spiderman toys does Phillip have?
Action	Robin had some toy cars. Her parents gave her more toys cars for her birthday. Then she had 312 toy cars. How many toy cars did Robin have before her birthday?

Appendix B

Word Problem Structure Test Protocol

Word Problem Structure Test (WPST): Instructor Protocol (Pretest)

Materials

1) Word problem structure visual representation strips
2) Coding sheet
3) Instructor word problem index cards

Organization

1) The child will be seated at a desk.
2) The interviewer will sit next to the child.
3) There will be a camera set up to film the child's hands/workbook on the table.

1) The instructor says, "Today, I am going to read you some word problems and I'm going to ask you a question about it."
2) The interviewer will read the word problem out then place the index card with the problem on it in front of the child.
3) The interview says, "Now, I'm going to place a cardboard in front of you. It has different pictures on it. I want you to point to the picture that best matches the word problem I just read to you."
4) The instructor will write the number of the visual representation the child has chosen on the scoring sheet.
5) The instructor will remove the strip.
6) The instructor will repeat steps 2-5 with all problems.
7) Once the child has finished all of the questions, the instructor will walk to the child back to class/to his parents.

Action	There are many ducks on Jon's farm. Jon's dad buys 7 more ducks and now there are 14 ducks. How many ducks did Jon have in the beginning?
Seesaw	Rusty has 9 toy soldiers. His friend, Timmy, has 6 more toy soldiers than Rusty. How many toy soldiers does Timmy have?
Action	Alice has 7 dolls. For her birthday, her friends give her more dolls. Now she has 16 dolls. How many dolls did she get on her birthday?
Action	Sally picked 20 flowers. She gave 11 flowers to her mom. How many flowers does Sally have left?

Part/Whole	Doug has a lot of Hot Wheels in his collection. He has 9 cars and 10 trucks. How many Hot Wheels does Doug have in his collection?
Action	Jesse's mom made cookies for the bake sale. She sold 19 cookies and she has 5 left over. How many cookies did Jesse's mom bake for the bake sale?

Word Problem Structure Test (WPST) - Posttest Word Problems	
Action	James has a lot of shirts. For his birthday, his mom gave him 3 new shirts. Now James has 11 shirts. How many shirts did James have at the beginning?
Action	Some of Jon’s crayons broke. He has 16 crayons and now he has 7 left. How many of Jon’s crayons broke?
Seesaw	Julie has 8 dolls. Her friend, Sarah, has 4 more dolls than Julie. How many dolls does Sarah have?
Action	Dan has 9 seashells. He finds some more at the beach. Now he has 13 seashells. How many seashells did Dan find at the beach?
Action	There are 16 crayons in a box. Sarah’s teacher open the box and 11 crayons fell on the floor. How many crayons stayed in the box?
Part/Whole	Connie has a lot of stickers in her collection. She has 7 butterfly stickers and 8 heart stickers. How many stickers does Connie have in her collection?
Action	Brian’s mom gave him 7 big marbles. Before that, he had only 6 marbles. How many marbles does Brian have now?
Action	A school had some prizes to give out for the school carnival. During the carnival, the school gave out 20 prizes. Now they have 6 prizes left. How many prizes did the school have at the beginning?

Appendix C

Word Problems for Instruction (ID Group)

Action – Sample Word Problems				
PHASE	**SESSION 1 (8 PROBLEMS)**		**SESSION 2 (8 PROBLEMS)**	
Problem Schemata	**Start Unknow n**	There are 13 apples in the bin. 5 apples fall out of the bin. There are 8 apples left in the bin. Mark saw 5 movies Saturday morning. He then saw 2 more movies on Saturday afternoon. He saw 7 movies in all on Saturday. Megan's mom made 18 bags of candy for Halloween. She gave out 10 bags and she has 8 left.	**Start Unknown**	There are many bananas in the bin. 5 bananas fall out of the bin. Now there are 8 bananas left in the bin. How many bananas were in the bin at the beginning? Mark saw many movies on Saturday morning. He saw on 2 more movies on Saturday afternoon. He saw 7 movies in all. How many movies did Mark see on Saturday morning? Megan's mom made many bags of candy for Halloween. She gave out 10 bags and she has 8 left. How many bags of candy did she have in the beginning?
	Change Unknow n	Richie has 9 Hot Wheels. He gives 3 Hot Wheels to his brother. Richie now has 6 Hot Wheels. Yesterday, Becky made 4 cupcakes for a party. Today, she made 8 more cupcakes. Now, Becky has 12 cupcakes. There are 5 trains at the station. 4 more trains pull into the station. Now there are 9 trains at the station.	**Change Unknown**	Richie has 9 Hot Wheels. He gives some Hot Wheels to his brother. Now Richie has 6 Hot Wheels. How many Hot Wheels did Richie give his brother? Yesterday, Becky made 4 cupcakes for a party. Today, she made some more cupcakes. Now, Becky has 12 cupcakes in all. How many cupcakes did Becky make today?

				There are 5 trains at the station. Some more trains pull into the station. Now there are 9 trains at the station. How many trains arrived at the station?
	Result Unknown	Vera painted 8 flowers in her picture. Vera paints 6 more flowers. Vera now has 14 flowers in her picture. Hugo has 7 books on trains. He gives 4 books to his brother. Hugo now has 3 books on trains.	**Result Unknown**	Vera painted 8 flowers in her picture. Later, she painted 6 more flowers. How many flowers did Vera paint in all? Hugo has 7 books on trains. He gives 4 books to his brother. How many books on trains does Hugo have now?
Sessions 3 and 4				
Solution Generation	**Start Unknown**	Brian gave some candies to his mother. His mother ate 4 candies. Now she has 5 candies left. How many candies did she have in the beginning? Jesse's mom made cookies for the bake sale. She sold 9 cookies and she has 4 left over. How many cookies did she bake for the bake sale? Sean has some books on snakes. His grandmother brings him 2 more. Now Sean has 6 books on snakes. How many books on snakes did Sean have in the beginning?	**Start Unknown**	Miriam has some Polly Pocket dolls. For her birthday, her family gave her 8 more dolls. Now she has 13. How many dolls did Miriam have at the beginning? Dena has some sweaters in her closet. 11 of her sweaters do not fit her anymore so she gives them away. Now she has 5 sweaters. How many sweaters did she have in the beginning? A school gave out 11 prizes during the school carnival. Now they have 3 prizes left. How many prizes did school have at the beginning?

	Change Unknown	Dena has a collection of 9 bottle caps. While walking to school one day, she dropped some. Now she has 2 bottle caps. How many did Dena lose? Kathy brought 3 bags of chips to a party. Her friend, Angela, brought some bags to the party, too. Now there are 5 bags at the party. How many bags did Angela bring? Rusty has 15 dollars. He wants to buy a game that costs 19 dollars. How much money does he need to earn to buy the game	**Change Unknown**	Cara had 15 grapes in her lunch box. During the day, she ate some of them. Now she has 4 grapes. How many grapes did Cara eat? In a tournament, Charles won 6 games of checkers. His sister, Carla, also won some games. Together, they won 13 games. How many games did Carla win? The bakery bakes 19 cookies every day. At the end of the day, there are only 8 left. How cookies were sold during the day?
	Result Unknown	Timmy's cat has 9 cat toys. He ate 5 of them. How many cat toys does Timmy's cat have left? Diana ate 2 popsicles. Her friend, Brian, ate 6 popsicles. How many popsicles did they eat altogether?	**Result Unknown**	Carrie baked 12 cookies. Her brother, Chris, ate 7 cookies. How many cookies are left? Allyson has 6 T-shirts. Her sister, Amy, has 9. How many T-shirts do they have altogether?

Appendix D

Problems for Instruction (SM & AM Groups)

Phase/Session	Problem Type	SM Group	AM Group
Problem Schemata Phase Session 2	Start unknown	25 + 42 = 59 – 26 = 31 + 81 =	25 + 142 = 402 – 216 = 437 + 181 =
	Action unknown	71 – 37 = 87 – 43 = 47 – 12 =	327 – 117 = 524 – 379 = 537 – 412 =
	End unknown	78 + 46 = 71 – 25 =	457 + 206 = 607 – 250 =
Solution Generation Phase Session 1	Start unknown	55 + 24 = 38 + 44 = 36 – 12 =	114 + 225 = 249 + 94 = 466 – 352 =
	Action unknown	69 – 32 = 75 – 53 = 39 – 15 =	419 – 282 = 545 – 363 = 279 – 125 =
	End unknown	29 – 15 = 22 + 26 =	309 – 125 = 298 + 346 =
Solution Generation Phase Session 2	Start unknown	34 – 18 = 32 + 9 = 33 + 14 =	34 – 18 = 32 + 9 = 33 + 14 =
	Action unknown	29 – 13 = 35 – 16 = 35 – 13 =	29 – 13 = 35 – 16 = 35 – 13 =
	End unknown	33 – 19 = 14 + 18 =	33 – 19 = 14 + 18 =

Appendix E

Problem Schemata Phase Script

Problem Schemata Phase – Action Problems, Session 1

Instructor Protocol

Materials (per child + instructor)

1. Workbooks
2. Pencil
3. Eraser
4. Posters of Schemer and Checklist
5. Story Scenario Cards

Organization

1. The children will be siting at a table with the instructor.
2. The materials will be placed near the instructor until it is time to use them (1 set per child).
3. The schema and checklist poster is placed on the board.

Introduction to the Materials

1. Today, I am going to teach you about word problems and their "schemer".
2. Do you have any questions before we start? *If yes, answer questions.*
3. Great! Let's get started.

Imitation

I: I want to tell you about the *schemers.*

I: Every question has a *schemer.* Do any of you know what a *schemer* is?

Instructor waits for children's answers.

I: A *schemer* is a picture of the question. It is a picture to help you think about the question.

I: The *schemer* tries to trick you!

I: But when you know the *schemer,* it makes it much easier to any questions.

I: So today, we are going to be talking about this *schemer.*

Instructor puts places a cardboard on the table.

I: In a question, we have some numbers. These numbers go in our *schemer*, in the circles and the box.

I: Before we talk about where to put the numbers, let me tell you about the different parts of the *schemer.*

I: The *schemer* has three parts.

I: We're going to look at them now.

I: At the beginning of the problem, you can have a bunch of things.

Instructor gestures to the start circle on the schemer.

I: This is called the *start.*

I: The *start* number can be a bunch of things. They can be apples, birds, toy cars, or anything!

I: In questions, something happens. There is an action.

I: And we can see this in this part called the *action.*

Instructor gestures to the action box on the schemer.

I: For the questions we will be working on, the *action* is what happens after the *start* number.

I: In the last part of the *schemer*, we have the *end* number.

Instructor gestures to the end circle.

I: The *end* number is the number that happens after the action number.

I: When you read a question, the *start* number can be missing.

Instructor gestures to the start circle.

I: Sometimes, it is the *action* number that is missing.

Instructor gestures to the action box.

I: Or sometimes, you don't know the *end* number.

Instructor gestures to the end circle.

I: Now that we understand the different parts of our *schemer* – the *start,* the *action,* and the *end*- and that the numbers in a question go in the *schemer,* questions become easier to answer!

I: I have a checklist that will help us remember. Let's look at it before we try to answer a question.

I: The first thing we have to do when answering a question is to write the numbers into the schemer. We have to ask ourselves three things.

I: The first thing is, does the problem have a start number? Remember, the start number is the number we have at the beginning and it goes here (*gestures to the start circle in the schemer)*.

I: The second thing we have to ask is, does the problem have an *action* number? Remember, the *action* is what happens after the *start* number and it goes here *(gestures to the action box in the schemer)*.

I: The last thing we have to ask is, does the problem have an *end* number? Remember, the *end* number is the number that we have at the end, after the *action* number, and it goes here *(gestures to the end circle in the schemer)*.

I: So when we are trying to answer a question, we put the numbers in the right place in the *schemer* to help us find the answer.

I: Let's look at an example of a story to see where the numbers would go in our *schemer*.

I: Remember, when we are trying to answer a question, there are three things we can ask ourselves to make sure we are putting the numbers in the right place in the *schemer*.

I: Let's take a look at a story and use our checklist to make sure we are putting the numbers in the right place.

Instructor puts an index card with the problem written on it to the left of the poster of the schemer.

I: There are 13 apples in the bin. 5 apples fall out of the bin. There are 8 apples left in the bin.

Instructor places checklist poster on the board, to the right of the schema.

I: Ok, now we know the numbers in our question. Let's see where they go in our schemer.

I: To help us, we can use our checklist.

I: So the first thing we ask is, "Does the story have the *start* number?"

I: The *start* number is the number that goes here *(points to the start circle on the schemer)*.

I It is the amount of things that are in the beginning.

I: Let's read the story again so we can think about it.

Instructor reads the problem again.

I: So, does our question have the start?

I: Yes it does! It says that there are 13 apples in the bin *(gestures to the 13 in the word problem)*.

I: So, I'm going to write the number 13 right here in the *start* circle. *Instructor writes the number 13 in the start circle.* I: These are the 13 apples that were in the bin at the beginning of the problem. I: Now, let's look at what the rest of the story says. I: It says that 5 apples fell out of the bin. I: That part is the *action*. Remember, the *action* number is what happens after the *start* number. I: So, I'm going to write 5 in the *action* box. *Instructor writes the number 5 in the action box.* I: These are the 5 apples that fell out of the bin. I: Let's see what is written in the last part of the story. I: It says, "There are 8 apples left in the bin." I: So the 8 apples that are left are the *end* number *(gestures to the end circle on the schemer).* I: So in the *end* circle, I am going to write 8 for the 8 apples that are left. *Instructor writes the number 8 in the end circle.* I: So in this story, the *start* number is 13 *(gestures to the start circle on the schemer)*, which was the number of apples in the bin at the beginning. The *action* number is 5 *(gestures to the action box on the schemer),* which was the number of apples that fell out of the bin. The *end* number is 8 *(gestures to the result on the schemer)*, which was the number of apples that were left in the bin at the end. I: So, now we know the different parts of the schemer and where the numbers go. This will make it easier for us to answer the question! I: For now, it is important to remember that we have to read the question carefully! We have to take the time to read the question and see where the numbers go in our *schemer*. I: Let's look at a second problem together. *Instructor takes down the first word problem and puts up the second one on the board, next to the schema.*
Process to be repeated for the next two word problems.
Structured Practice
I: Ok, let's do the next few problems together.

I: Please open your workbooks to the first page.

I: I'm going to read out the story and then ask you some questions.

I: Please raise your hands to answer the questions. You'll each get a turn.

I: I am going to keep our checklist here to help guide us through.

I: Let's look at this story.

I: You have the same story written in the first page of your workbook.

Instructor puts the problem up on the board next to the schemer.

I: Mark watched 5 movies on Saturday morning. He watched on 2 more movies on Saturday afternoon. He watched 7 movies in all. How many movies did Mark watch on Saturday morning?

I: So, the first thing we have to do it look at where the numbers in the story go in our *schemer*. What do we begin with?

Children say, "We have to find the start number!"

I: That's right. Let's look at our story again. Mark saw 5 movies on Saturday morning. What would this be?

Children say, "The start number."

I: Ok. Good! That's right! The 5 movies are the *start* number. Where is the *start* in this *schemer*?

Children point to the first circle on the left hand side of the schemer.

I: Excellent! So, I'm going to write the number 5 right here in the *start* circle. The 5 is for the 5 movies that Mark saw on Saturday morning. Please do the same in your workbooks.

Children write the number in the start in their workbooks.

I: Ok. What is next?

Children say, "We have to find the action number."

I: Correct! Let's see what our problem says now. Mark saw 2 more movies in the afternoon. Is this the *action number?*

Children say, "yes."

I: That's right! The *action* is what happens after the *start* number. And where do we find the *action* in our *schemer*?

Children point to the middle box on the schemer.

I: Terrific! That's right. So I am going to write 2 in the *action* box. It is the 2 more movies that Mark saw in the afternoon. Please write it down in your workbooks.

Children write the number down in their books.

I: Ok, let's see what the last part of the problem says. Mark saw 7 movies in all on Saturday. Where would we put this 7?

Children say, "In the end circle."

I: Excellent! You are right. This 7 is the *end* number. It is the number of movies that Mark saw in all on Saturday. Now, we will write 7 where in our schemer?

Children say, "In the end circle" (point to the proper circle).

I: Right. And now, we have written all the numbers from our story into our *schemer.* In our *start circle,* we have the number 5, for the 5 movies that Mark saw on Saturday morning. In our *action* box, we have the number 2, for the 2 movies more Mark saw on Saturday afternoon. And in our *end* number, we have 7. That's the total number of movies that Mark saw on Saturday.

I: You are all working so hard!

I: Let's do another one like this.

Instructor takes down the first word problem and puts up the second one on the board, next to the schema.

Process is repeated for the next two word problems.

Corrective Feedback

I: Now that we have practiced this, I'm going to let you do some problems on your own.

I: I'm going to leave the question checklist on the board for you, along with the schemer.

I: If you have any trouble, ask me and I'll help you.

Instructor observes the children doing the word problems and provides individual feedback.

When the instructor notices errors:

1. "Remember, when you look at the problem, you have to …"
2. Instructor will refer the children back to the checklist and the *schemer.*
3. The instructor will continually praise the children for their hard work.

End of Session

I: We are done for today.

I: I want to thank you all for your hard work.

Problem Schemata Phase – Action Problems, Session 2

Instructor Protocol

Materials (per child + instructor)

1. Workbooks
2. Pencil
3. Eraser
4. Poster of Schemer and checklist
5. Question Cards

Organization

1. The children will be siting at a table with the instructor.
2. The materials will be near the instructor until it is time to use them (1 set per child).
3. The schema and checklist poster will be near the instructor, posted on the wall.

Introduction to the Materials

1. We are going to keep working on putting the numbers into schemers. Earlier, you used stories to learn where to put the numbers in the *schemers.* Now, we are going to use questions.
2. I would like you to watch me and listen to what I am going to show you.
3. Do you have any questions before we start? *If yes, answer questions.*
4. Great! Let's get started.

Imitation

I: So, do you remember what *schemers* are?

Instructor waits for children's answers.

I: A *schemer* is a picture of the question. It is a picture to help you think about the question.

I: The *schemer* tries to trick you!

I: But when you know the *schemer,* it makes it much easier to any questions.

I: So now, we are going to use questions. We will put the numbers from the questions into our *schemer.*

Instructor puts places a cardboard on the table.

I: Now, remember, the *schemer* has three parts. Do you remember what they are?

Instructor waits for children to answer.

I: That's right! The *schemer* has three parts. There is the *start* number, which is the number at the beginning of the question. Then, there is the *action* number. The *action* is what happens after the *start* number. And the last part is the *end* number. This is the number that comes after the *action.*

I: When you read a question, the *start* number can be missing.

Instructor gestures to the start circle.

I: Sometimes, it is the *action* number that is missing.

Instructor gestures to the action box.

I: Or sometimes, you don't know the *end* number.

Instructor gestures to the end circle.

I: Now that we understand the different parts of our *schemer* – the *start,* the *action,* and the *end*- and that the numbers in a question go in the *schemer,* questions become easier to answer!

I: Remember, we have a checklist that will remind us of the things we have to ask ourselves when we are trying to answer a question.

I: Remember, we have to ask ourselves three things. Does anyone remember what those things are?

Instructor waits for children to answer.

I: That's right. The first thing is, does the problem have a start number? Remember, the start number is the number we have at the beginning and it goes here (*gestures to the start circle in the schemer)*.

I: The second thing we have to ask is, does the problem have an *action* number? Remember, the *action* is what happens after the *start* number and it goes here *(gestures to the action box in the schemer).*

I: The last thing we have to ask is, does the problem have an *end* number? Remember, the *end* number is the number that happens at the end, after the *action* number, and it goes here *(gestures to the end circle in the schemer).*

I: So when we are trying to answer a question, we put the numbers in the right place in the *schemer* to help us find the answer.

I: Now, we are going to read some questions and put the numbers in the right places in our *schemer.*

I: I am going to leave our checklist here, in case we need it.

Instructor places checklist poster on the board, to the right of the schemer.

I: You will see that it is a little different when we ask a question. In the stories you did with Kim, there were no missing numbers.

I: But in a question, there is a number that is missing.

I: Let's do a few questions together.

Instructor puts an index card with the problem written on it to the left of the poster of the schemer.

I: There are many bananas in the bin. 5 bananas fall out of the bin. Now there are 8 bananas left in the bin. How many bananas were in the bin at the beginning?

I: Ok, so now we know the numbers in the question. Let's see where they go in our schemer.

I: To help us, we can use our checklist.

I: So the first thing we ask is, "Does the story have the *start* number?"

I: The *start* number is the number that goes here *(points to the start circle on the schemer).*

I It is the amount of items that are in the beginning.

I: Let's read the story again so we can think about it.

Instructor reads the problem again.

I: So, does our question have the start?

I: The problem says that there were many bananas in the bin, but we don't know the number. So for now, I am going to write a ? in the *start* circle. I cannot write a number because the question does not give us the number of bananas that are in the bin.

Instructor puts a ? in the start circle.

I: Ok. Let's keep reading our question.

I: The question says that 5 bananas fell out of the bin. That's our *action* number! Remember, the *action* is what happens after the *start* number. So, I am going to write 5 in the *action* box.

Instructor puts the number 5 in the action box.

I: Ok. Let's go back to our question.

I: It says now there are 8 bananas left in the bin. That's our *end* number! Remember, the *end* number is the number that comes after the *action* number. So, I am going to write 5 in the *action* box.

Instructor writes 5 in the action box.

I: Now the rest of the question asks How many bananas were in the bin at the beginning?

I: So, let's look at our schemer. We have a question mark in our *start* circle, because we do not know the number of bananas that were in the bin. We have the number 5 in our *action* box. That is the number of bananas that fell out of the bin. And here, we have 8 in the *end.* Those are the number of bananas that were left in the bin after the other ones fell out. That means for our problem, we would have to find out the number for the *start.* We will talk about how to answer the question next week.

I: For now, we will keep practicing putting the numbers from a question into our *schemer.*

I: It is important to remember that we have to read the question carefully when we have to solve the problem. We have to take the time to look at the question and see where the numbers go in our schema.

I: Let's look at a second problem together.

Instructor takes down the first word problem and puts up the second one on the board, next to the schemer.

Process to be repeated for the next two word problems.

Structured Practice

I: Ok, let's do the next few problems together.

I: Please open your workbooks to the first page.

Instructor gives each of the children a workbook, pencil, and eraser.

I: I'm going to read out the question.

I: I am going to keep our checklist here to help guide us through.

I: Let's look at this question.

I: You have the same question written in the first page of your workbook.

Instructor puts the problem up on the board next to the schemer.

I: Mark saw many movies on Saturday morning. He saw on 2 more movies on Saturday afternoon. He saw 7 movies in all. How many movies did Mark see on Saturday morning?

I: So, the first thing we have to do it look at where the numbers from the question go in our *schemer*. What do we begin with?

Children say, "We have to find the start number!"

I: That's right. Let's look at our question again. Mark saw many movies on Saturday morning. What would this be?

Children say, "The start number."

I: Ok. Good! That's right! It would be our *start* number. But, does the question give us a number?

Children answer no.

I: That's right, we don't know how many movies Mark saw on Saturday morning. So in our *start* circle, I am going to write a ?.

Instructor writes ? in the start circle.

I: Please do the same in your workbooks.

Children write ? in the start circle in their workbooks.

I: Ok. What's next?

Children say, "We have to find the action number."

I: Correct! Let's see what our question says now. He saw 2 more movies in the afternoon. Is this the *action* number?

Children say, "yes."

I: That's right! The *action* is what happens after the *start* number. And where do we find the *action* in our *schemer*?

Children point to the middle box on the schemer.

I: Terrific! That's right. So I am going to write 2 in the *action* box. It is the 2 more movies that Mark saw on Saturday afternoon. Please write it down in your workbooks.

Children write the number down in their books.

I: Ok, let's see what the last part of the problem says. Mark saw 7 movies in all on Saturday. Where would we put this 7?

Children say, "In the end circle."

I: Excellent! You are right. This 7 is the *end* number. It is the number of movies that Mark saw on Saturday. Now, we will write 7 where in our schemer?

Children say, "In the end circle" (point to the proper circle).

I: The last part of our question asks how many movies did Mark see on Saturday morning? So this is what we would have to find out.

I: Let's look at the numbers in our *schemer.* In our *start circle,* we have the ? for the movies that Mark saw on Saturday morning. There was no number in the question. In our *action* box, we have the number 2, for the 2 more movies Mark saw on Saturday afternoon. And in our *end* number, we have 7. That's the total number of movies Mark saw on Saturday.

I: So for this question, we would have to find out the *start* number, which is the number of movies Mark saw on Saturday morning.

I: We will learn to do this next week.

I: You are all working so hard!

I: Let's do another one like this.

Instructor takes down the first word problem and puts up the second one on the board, next to the schema.

Process is repeated for the next two word problems.

Corrective Feedback

I: Now that we have practiced this, I'm going to let you do some problems on your own.

I: I'm going to leave question checklist on the board for you, along with the schemer of an Action problem.

I: If you have any trouble, ask me and I'll help you.

Instructor observes the children doing the word problems and provides individual feedback.

When the instructor notices errors:

4. "Remember, when you look at the problem, you have to …"
5. Instructor will refer the children back to the checklist and the *schemer.*
6. The instructor will continually praise the children for their hard work.

End of Session

I: We are done for today.

I: I want to thank you all for your hard work.

I: I will see you guys next week!

Appendix F

Solution Generation Phase Script

Solution Generation Phase – Sessions 3 & 4 Instructor Protocol
Materials (per child + instructor)
1. Workbook 2. Pencil 3. Paper 4. Eraser 5. Chips 6. Poster of Schemer and Checklist 7. Word Problem Cards 8. Stickers 9. Reinforcer chart
Organization
1. The children will all be sitting at the desk. 2. The materials will be off the table until it is time for the children to use them. 3. The schema and checklist posters will be placed on the board.
Introduction to the Materials
1. Last week, we talked about *schemers* and where to place the numbers from a question into the *schemer.* Today, we are going to talk about how we can answer these questions. I am going to ask you about how you answer questions and ask you to share your answers with your friends so we can learn many ways of answering questions. 2. Then, I am going to show you different ways of answering questions. 3. Do you have any questions before we start? *If yes, answer questions.* 4. Great! Let's get started.
Checklist Review and Problem Solving
I: So do you remember the *schemer* we learned last week?

I: We showed you where to put the numbers into the *schemer* and how knowing where the numbers go can help you figure out which part of the *schemer* is missing.

I: Do you remember the different parts of the *schemer?*

I: Let's see what you can remember. What is this part called?

Instructor points to the start circle on the schemer.

Children say, "The start!"

I: That's right. This is the *start* number. It is the number of things that we have at the beginning of a question.

I: What is this part called?

Instructor points to the action box on the schemer.

Children answer, "The action!"

I: Excellent! This is the *action* number. It is what happens after the *start* number in the question.

I: And what is this part called?

Instructor points to the end circle.

Children answer, "The end!"

I: Correct! That's the *end* number. This is the number of things we have at the end of the problem.

I: Now remember, in a question, any one of these parts can be missing.

I: Sometimes, we will not know the *start* number.

I: Sometimes, the *action* number will be missing.

I: And sometimes, we will not know the *end* number.

I: When we answer a question, we have to find the missing number.

I: Last week, you learned that when the number in a question is missing, you write a ? in that part of the *schemer.*

I: Today, we are going to talk about the different ways of answering a question.

I: There are many different ways of answering questions. Some ways may make more sense to you than others, but there are many ways of solving them.

I: We are going to answer some questions today and find out about the different ways you use when trying to figure it out.

I: I am going to give you a workbook. Please open your workbooks to the first page. That's the first problem we are going to solve.

I: You will see that there is an empty schemer on each page in your workbook. Sometimes, you won't have a schemer (when you are not here with us), but you can always draw it and use it to help you answer the question.

Instructor reads out the question and asks the children to solve the problem in whatever way they want.

Instructor then places a workbook, pencils, chips, and erasers in front of each child.

I: Ok, now. I want you to figure out how to answer the question in any way you can. You can use any of these materials (*instructor gestures to the materials in front of the children)* if you want, but you don't have to.

I: I am going to give you some time to think about the question and try to figure out.

I: After, I am going to ask you to share how you solved it with all of us.

I: Remember, we are here to learn from each other. It's ok if you do not know something. We will work through it together.

I: So, look at the question we just read. I want you to try to answer the question. It's the first question in your workbooks.

I: Remember to use your checklist and your *schemer* to figure out which number – *start, action, or end* – you have to find.

Corrective Feedback

Instructor observes the children solving the word problems and provides individual feedback if the child did not place the numbers into the appropriate spaces within the schemer.

When the instructor notices errors:

1. "Remember, when you look at the question, you have to ask…"
2. "Remember, this part of the schemer is the…"
3. Instructor will refer the children back to the question checklist and the schema or will repeat the information as many times as necessary.
4. Instructor will continually praise the children for their hard work.

Strategy Sharing Guidelines

I: Great job, guys! You are all working so hard.

I: Now, who would like to share how they figured out the answer?

Instructor chooses a child to share his/her strategy. Child explains his/her strategy to the group. If the child used their fingers, chips, or paper/pencil to solve the problem, the instructor will ask the child to show what he did using their fingers, blocks, or paper/pencil.

Instructor will probe the child to get as much information as possible as to what the child did to attempt to answer the question.

If the instructor notices errors:

1. In the *schemer* – Redirect the child to their checklist. Have them look at the numbers in their *schemer.*
2. In counting - Ask the child to count again so they have the correct number
3. If any other errors are noticed, the instructor will guide the child in the right direction. They will ask the appropriate questions and engage in re-explaining if necessary.

If the child says he or she does not know how to answer the question:

1. Instructor will encourage the child to try to figure it out.
2. Instructor will reassure the child to try their best.
3. If the child still states that they do not know what to do, the instructor will use scaffolding techniques to try to help the child.
    - For example, the instructor will use chips to count out the first part of the problem. She will say, "Well, I think I would try do something like this when trying to answer the question. I am going to count out 8 chips for the 8 cupcakes Molly baked. Now, what do you think I should do next?"

4. Instructor will go though parts of the lesson as necessary and will engage in direct instruction if the child does not know how to answer the question.

Once the child has explained their strategy, the instructor will repeat it for the group and the children will model the strategy together.

The instructor will then ask the same child if they can think of another way of solving the problem.

I: That was a great way of answering the question! Now, can you show me a different way of answering the question?

The instructor will prompt the child to solve the problem a different way.

If the instructor notices errors:

4. In the *schemer* – Redirect the child to their checklist. Have them look at the numbers in their *schemer.*
5. In counting - Ask the child to count again so they have the correct number
6. If any other errors are noticed, the instructor will guide the child in the right direction. They will ask the appropriate questions and engage in re-explaining if necessary.

If the child says they do not know how to answer the question:

1. Instructor will encourage the child to try to figure it out.
2. Instructor will reassure the child to try their best.
3. The instructor can also repeat parts of the lesson (where to place the numbers in the schemer, what the names of the parts are, etc.) if necessary.
4. If the child still states that they do not know what to do, the instructor will use scaffolding techniques to try to help the child.
    - For example, the instructor will use chips to count out the first part of the problem. She will say, "Well, I think I would try do something like this when trying to answer the question. I am going to count out 8 chips for the 8 cupcakes Molly baked. Now, what do you think I should do next?"
5. ****It is important to scaffold strategies that they child is able to do. That is, that matches the child's current level of problem solving.**
    - If the child is using their fingers to solve the problem, use scaffolding techniques (mentioned above) to bring the child to direct model using objects.
    - If the child has begun using counting strategies, but is not doing so consistently (that is, he/she reverts back to direct modeling strategies), use scaffolding techniques to reinforce counting strategies.
        - For example, the instructor could suggest that the child use chips to represent the first number in the problem. Once the number of chips has been confirmed ("Ok, so you counted out 8 chips for the 8 cupcakes"), the instructor could cover the 8 chips with her hands, and say, now can you count out the rest of the question (e.g., 7 more cupcakes) on your fingers?"

6. If the child still does not know, the instructor will use direct instruction to teach the child another strategy (a strategy that is as the same level as the child) so that there are two different strategies per problem.

Once the child has explained their second strategy, the instructor will repeat it for the group and the children will model the strategy together.

Then, the instructor will select another child to share their strategy. The process will be repeated for every child. The instructor will adapt scaffolding techniques to be appropriate for the level of the child.
Process is repeated for the four word problems.
End of Session
I: We are done for today. I: Thanks for all of your hard work. I: You all came up with great strategies! I: I will see you guys next week!