A Mixed Integer Programming Model for Production Planning in Labor Intensive Manufacturing Systems

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Abstract

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The literature about labor intensive production planning addresses certain problems such as workforce sizing problems, workforce transfer problems and multi skill-level workforce utilization problems. Some of them considered quality related issues or workforce learning effects in modeling and solving these problems. In this thesis, a production planning model is developed for small to medium sized labor intensive production systems. It aims at deciding the optimal production plans for producing different types of products and assigning workers of different skill levels to production stations in the considered system. The main production planning model is formulated with the considerations of learning effect, quality issues, overtime work hours, and possible delays in product delivery. Numerical example problems based on practical cases are presented to illustrate the considered problems and the behavior of the developed model in solving these problems. The strength of the proposed model lies in the integration of some critical issues in a production system. A main advantage of using the approach developed in this thesis is to provide shop managers different options in deciding the number of production lines, overtime work time, on time or late product delivery, when the demand is seasonal and volatile.

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Chapter 1

Introduction

1.1 Introduction of Production Methods

Production planning is concerned with production and manufacturing processes in a company or industry, considering resource allocation such as human, material and machine capacity. For different production types or patterns, the method of production planning can be different. In general, mathematical models have been extensively used in production planning since it may give decision makers a chance to optimize his choices for obtaining better results.

In general, there are three methods of production systems: job production, batch production and mass production. These production methods and their characteristics are briefly discussed below.

Job production, or unit production, is a customer-oriented production method and can be classical craft production. It can be therefore labor intensive. Generally, compared with mass production, job production contains more actions at smaller scale. More specialized and skilled operators are typically used to perform these operations, because higher product flexibility is required in job production. Whether the jobs are small-scale and of low technology or complex requiring high technology, labor cost is the main part of the total production cost. Consequently, the key characteristics of job production are highly flexibility to produce customized products, requiring specialized labor skills and associated with high unit production cost.

Batch production is a process to manufacture products in batches of different sizes, typically from several units to several hundred units per batch. The subsequent operations can be undertaken after the previous operations finished. Batch production is probably the most common method for manufacturing. It is suitable for making seasonal products, products with unclear demand forecast and that may not be produced continuously. A typical example is bakery product in food processing industry. Some batch production systems may involve a single production line with several workers, each one having specialized jobs.

Mass production system is usually used for high-volume production. The plant is often equipped with specialized and fast tools designed for manufacturing a single type of products. According to Sule (1994), mass production systems can be classified into two ways: an assembly line for producing discrete products, and the flow line used for continuous production process. In general, mass production is based on two principles, specialization of human labor and utilization of automated tools or equipment. Many important issues related to mass production systems, such as reducing production cost, improving product quality, and increasing system flexibility, have been studied by many researchers. In mass production systems involving manual operations, Taylor and Gilbreth (Mize, 1992) focused on organizing labor, controlling flow of work and handling the details since early 20th century. In the following decades, Henry Ford and his colleagues contributed to mechanizing their factory processes, minimizing worker movements and organizing production with job specialization (Mize, 1992).

In a capital intensive production system, although the considered manufacturing processes may have high proportion of machinery comparing to the number of its employed workers, the majority of assembly operations may still be performed manually such as in typical apparel factories or in manufacturing facilities of certain aerospace products. In addition, since mass production systems have, in general, very limited flexibility, production lines combining automatic and manual operations are widely used (ElMaraghy and Manns, 2007).

1.2 Learning Process and Learning Curve Functions

In production lines with manual operations, both skilled and unskilled workers are often employed. Their jobs need to be coordinated so that they can work efficiently and effectively. In such an environment, "learning" is required for both type of workers, especially for unskilled ones. "Learning curve" functions can be used to describe productivity progress of workers according to the accumulation of skills through their activities.

1.2.1 Learning Curves

Learning curve occurs when a worker is employed for a new job, he or she repeats the task in a series of trials and his/her knowledge improves over time. The concept of learning curve was first introduced in 1885 by Ebbinghaus, Ruger and Bussenius (1913) and the most widely used mathematical function was first given in Wright (1936) describing the effect of learning on production costs in aircraft industry. For predicting the costs and time in constructing ships and airplanes during World War II, learning curve began to receive more attention (Alchian, 1963).

In addition to the log-linear model presented by Wright (1936), other versions of learning curves have been proposed for better description of learning processes in different production systems. For instance, Stanford-B model is considered as the best learning curve for certain manufacturing processes of Boeing 707 production. As discussed Plateau learning curves are widely used in machine-intensive industries while DeJong model and S-model are also popularly used in other industries (Yelle, 1979).

As an example, the log-linear learning curve function can be presented as $Y = aX^n$, where Y is the production time for the X^{th} unit, a is the production time for the first unit, and n is the learning index given by $n = \log b / \log 2$. b is the learning rate as discussed later.

A learning curve function, in general, may have two phases: the cognitive learning and the motor learning (Dar-el, Ayas and Gilad, 1995). According to these authors, at the beginning of the learning process, workers use both cognitive system and motor system to perform the tasks correctly, and motor learning becomes the dominant factor once they have enough experience on similar tasks. In general, the progress in the cognitive learning phase is faster than that in the motor learning phase (Jaber and Kher, 2002). If the task is simple, however, only the motor learning will dominate the learning process and the cognitive learning may not be recognizable.

1.2.2 Learning Rate

During the learning process, it is assumed that the production time and cost to manufacture each unit of products will decrease by a constant value. This is typically a percentage value or a "learning rate" (McDonald and Schrattenholzer, 2001), indicating the changes to occur comparing to those of producing the current products. In the log-linear learning curve function, b represents the learning rate.

According to Wright (1936), learning rate can be a constant number (i.e. 90%, 80%, 70%, etc.). The learning processes with different learning rates can be illustrated using the relationship between production time per unit and the cumulative number of units as shown in Figure 1.1. In this figure, all of the initial production time per unit is assumed to be one hour.

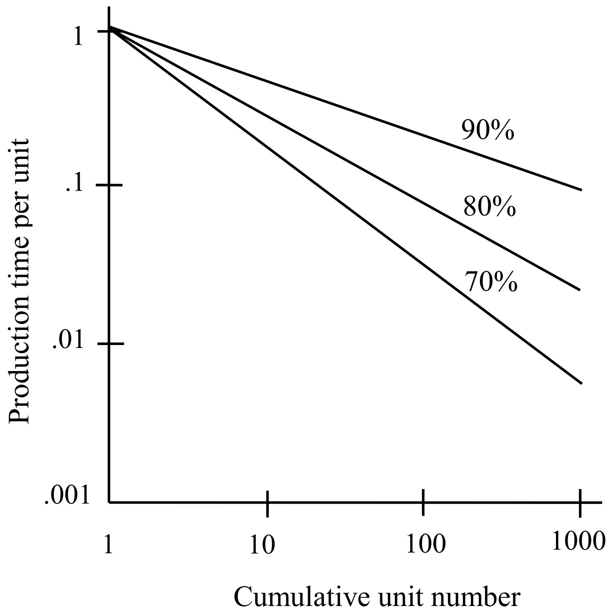

Figure 1.1 Typical learning curve with different rate (Yelle, 1979)

Generally, learning rate can be influenced by age, gender and experience of the operators as well by the type of operations and break period (Hancock, 1967). Learning rates for manual operators are usually from 80% to 90% (Yelle, 1979). Learning rate in machine intensive systems is generally smaller. Some historical data about learning rate of machine-paced labor is presented in Table 1.1.

Table 1.1 Learning Rate of System with Different Proportion of Labor (Yelle, 1979)

Machine-Paced Labor (as a percent of total labor)	Learning Rate	Progress Ratio
25%	80%	20%
50%	85%	15%
75%	90%	10%

1.2.3 Inaccuracies of Original Learning Curve

There are two factors that may cause inaccuracies in the learning curve function originally proposed by Wright (1936). The first factor is that learning rate may change during the learning process (Jaber and Kher, 2002) and the second one is that forgetting was not considered while it happens when there are interruptions in manufacturing (Jaber and Bonney, 1996).

1.3 Product Quality

Quality has been one of the most important issues in modern manufacturing systems. According to one of the widely accepted definitions, product quality is the ability that products should meet given requirements (Montgomery, 2007). In the 20^{th} century, manufacturing industry extended the use of statistical quality control methods to improve product quality and to eliminate product defects. In the past several decades, statistical process control (SPC), Total Quality Management (TQM) and Six Sigma were notable approaches to quality control and improvement. Nowadays, the goals of higher product quality have largely been driven by customer concerns and preferences.

The lack of quality affects a company in various ways. High proportion of defects will result in significant increase of direct cost because of waste of raw materials, labor and investment. In addition, defective products lead to low productivity. Most importantly, products with defects and defective products are not safe for customers to use. Costs for recalls and compensations are typically high. Companies always lose customers if the level of their product quality is too low or unacceptable.

1.4 Thesis Outline

In this thesis, we study a production planning problem to allocate manual labor force considering learning processes and concern of quality issues. For solving this problem, we developed a mixed integer programming model, and several linear sub-models for different cases based on a small confectionary production facility. The outline of this thesis is presented as follows.

This thesis has five chapters. The following Chapter Two presents a review of the literature mainly in manual production and some in automated production as well. Chapter Three provides the description of the considered problem and introduce the main model, as well as the revised models for different cases. Numerical examples are presented and solved in Chapter Four. Conclusions and future direction of research are introduced in the last Chapter Five.

Chapter 2

Literature Review

2.1 Introduction

This thesis studies production planning problems related to production systems with manual operations, automated production lines, and production systems with mixed manual and automated production lines. In this chapter, we review research literature on the following topics.

- Production system with manual operations
- Automated production systems
- Production system with manual and automated production lines

2.2 Manual Production Line Analysis

Production demand and availability of human resources play an important role in managing and optimizing a production system with manual production lines. Manual system production planning involves many aspects such as: human resource classification including temporary workers and permanent workers, transferring workers within and between production cells, employing skilled or unskilled workers, training unskilled workers, etc. In addition, a specific production system or process may require specially developed methods to solve these considered problems.

2.2.1 Workforce Size and Transfer

Ighravwe and Oke (2014) proposed a mathematical model for workforce planning in a manual production system. Considering the number of available technicians, fixed routine maintenance time and budget constraint, they formulated a multi-objective optimization model to minimize the number of workers and to maximize productivity. In solving their model, the authors used factorial design to determine optimal values of system parameters. Two factors in the experimental design were occupied time and ratio of full time workers to part time workers. For each experiment, they used branch-and-bound to obtain the minimum number of maintenance personnel within regular work time, as well as appropriate ratio of full and part time workers.

In addition to workforce sizing, issues related to manpower transfer should also be addressed. Süer and Dagli (2005) presented a simple optimization model for solving intra-cell manpower transfer problem. They developed a simple assignment model to obtain the best worker assignment based on the following considerations: producing one item requires certain operations in sequence, each operation requires different operation time, and several items can be on the waiting list. Depending on the results of worker assignment, the authors worked on rearranging items' producing sequence to minimize workforce transfer and production makespan. Final results provided optimal makespan and identified process bottleneck. Although it focuses on intra-cell transfer, the model can be extended to a mixed production system with intra-cell and inter-cell transfers.

Francas *et al* (2011) developed a mathematical model and used two parameters to present maximum amount of temporary workforce and the percentage of personnel transfer. By adjusting these two parameters, one may decide when to have only temporary workers or only personnel transfer, as well as the optimal capacity and cost in a two-plant, two-product manufacturing system. Results indicated that utilization of personnel transfer can be impacted by collaborations of the two plants. In addition, comparing with employing permanent staffs, either of the labor flexibility instruments - using personnel transfer and using temporary workers - had a positive influence on increasing profit. The authors also pointed out that using temporary workers requires less investment, while personnel transfer is more efficient. They showed the necessity to use both instruments in the factory. Moreover, considering inter-plants activities, personnel transfer is a better choice in larger systems.

Song and Huang (2008) considered workforce transfer issues in supply chain management. In developing a model to solve their problem, they assumed that employees may be transferred between departments at the beginning of each time period. In the considered problem, each department could be treated as a station in a production system, with some workers coming or leaving. The total number of employees in the system is a fixed number. Hiring and firing can only occur at the beginning and the end of the whole time horizon respectively. Incorporating constraints of turnover cost limit and capacity threshold, a multi-time period optimization model was developed for workforce management. To solve large size problems, they used successive convex approximation.

2.2.2 Workforce Cost

Stewart *et al* (1994) proposed several mathematical models to minimize training cost and maximize worker flexibility. These models were presented separately for minimum training cost, maximum worker flexibility, and minimum training time. The authors also proposed a multi-objective model for simultaneously minimizing cost and maximizing flexibility. Their standard constraint sets were human resource limit, machine quantity and capacity, as well as production requirement. In their research, staff training was divided into two levels, and persons completed both levels of training were considered as skilled workers. Time horizon was affected by worker's skill levels. Furthermore, each machine required the workers to have a specific skill level. These models were developed aiming at solving large size problems.

A more recent research on workforce assignment was proposed by Jennings and Shah (2014). They presented a non-linear programming model aiming at minimizing the sum of workforce cost, overtime cost and technology cost over multiple time periods. This model considers different supplier categories, workforce categories and technology types. The model constraints have those for capacity threshold, resource limit and learning rate. Model solutions are to decide workforce arrangement, production cost, processing time and wasting time. The cost function includes workforce cost, maintaining cost, service cost and customer cost. The authors also conducted uncertainty analysis considering that some of the model input was uncertain.

In a production planning problem involving human operators, learning effects are usually incorporated into formulating the models to reflect the fact that entry-level workers are typically not as productive as skilled workers. In addition, learning procedure usually takes considerable amount of time. It is also of common understanding that higher skill-level workers have better learning skills than lower skill-level workers. On the other hand, lower skill-level workers may require further training if the tasks are more complex. Training is always associated with cost.

Some researchers investigated work-task assignment issues and proposed linear or nonlinear models to solve such problems. Examples can be found in Nembhard (2001), Corominas, Pastor and Rodriguez (2006), and Corominas, Olivella and Pastor (2010). Nembhard and Bentefouet (2012) discussed a work-task assignment problem considering labor learning and forgetting in a system with parallel production lines. To reduce computational time in obtaining optimal work schedule, the authors assumed that the number of workers was equal to the number of tasks and revised existing nonlinear models to build a linear model with multiple time periods. Furthermore, based on the linear model, they introduced an extra parameter representing learning and forgetting.

In production planning and scheduling, earliness and tardiness penalty is another category of costs to be considered in modeling and problem solving. Especially in systems using just-in-time policy, earliness-tardiness cost can be expensive. Earliness may cause extra unprocessed parts and work-in-process storage, while tardiness leads to monetary penalties as well as customer complaints.

Some of the basic single-machine scheduling models with earliness and tardiness are widely discussed in literature. For instance, Baker and Trietsch (2009) presented a mathematical model for solving such problems.

Based on the basic scheduling model, Khoshjahan *et al* (2013) presented a similar model for production planning with limited resource capacity. They developed a solution method based on genetic algorithm (GA) and simulated annealing (SA) to solve the problem. They found that SA performs better than GA in obtaining optimal or near-optimal solutions, while

GA based search may reach a better near-optimal solution faster.

Stratman *et al* (2004) studied a quality improvement problem in a system with assembly operations and quality assurance programs to minimize quality cost and labor cost with learning. In their model, a simple learning and forgetting function was introduced to estimate processing time and to calculate product defective rate. In solving the problem, the authors used experimental design with two factors: 1) diverse lot size and category combination; 2) location and proportion of permanent and temporary workers. Then simulation and ANOVA analysis were utilized to identify the impact of lot sizing and workforce assignment on optimal solution in solving a real case problem.

Moore *et al* (2007) studied the problem on productivity of technical workers when they are reassigned to different tasks. Technicians with lower productivity tend to cause more defective products such as dropped-object products. The authors collected actual performance data of technicians and data from maintenance stations, analyzed their productivity to identify the values of the investigated key performance indices (KPI), then made recommendations for reducing quality related cost.

2.3 Analysis of Automated Systems

2.3.1 Machine Selection and Scheduling

Subramaniam *et al* (2000) studied machine assignment problems and proposed three machine selection rules, lowest average cost (LAC), least average process time (LAP) and least aggregate cost and process time (LACP) for assigning appropriate machines to several operations. The authors used experimental design to determine the best combinations of three factors: due date, the number of operating machine for one job, and machine breakdown, with each of them divided into two levels. Thus, eight experiments were conducted in total. Additionally, four dispatching rules – Random, FIFO, earliest due date, and shortest processing time – were applied to each experiment. The authors obtained improved results using numerical simulations for each experiment with selected rules.

Cao *et al* (2005) proposed a method for parallel machine selecting and scheduling. The problem is to allocate N jobs to K machines with the condition that one machine can process just one job once. The objective function is to minimize holding cost and tardiness penalty cost. The developed integer programming model can be solved directly for small size problems and while for large practical problem the authors developed a heuristic solution method using Tabu Search.

Yu *et al* (2014) studied a multi-machine scheduling problem to minimize total load on machines. The authors proposed four mathematical models. The first model is to solve the problem of unrelated parallel machines with non-decreasing processing time of jobs. The fourth model is also for unrelated machines with non-monotone processing time. These two models could be transformed to a regular assignment problem. The second model is to solve a problem with parallel identical machines with non-decreasing processing time based on machine balance principle. The third model is to solve a problem of parallel identical machine with non-decreasing processing time and non-decreasing job position. The problem was solved using longest processing time first rule.

2.3.2 Machine Cost

Gulledge and Khoshnevis (1987) presented a review paper on research related to production rate, learning curve and programming cost of automated manufacturing systems. The authors pointed out that learning curve does exist in manual production systems as well as in automated production systems. The authors also explained the concept of "experience curve" and its impact on productivity improvement. They proposed an economic planning model to maximize total output considering both production rate and learning curve cost in a made-to-order production system. Dynamic programming or calculus of variations could be used to solve the developed model.

Zhu and Heady (2000) investigated a multi-machine scheduling problem with earliness/tardiness. The problem to allocate N jobs to a single machine with minimum earliness and tardiness cost ($N/1/ET$) has been studied by many researchers, e.g., Coleman (1992) and Davis and Kanet (1993). Zhu and Heady presented an integer programming model for solving the same $N/M/ET$ problem. The integer programming model contained many binary integer variables and may take extensive computational time to solve large-size real problems. The authors suggested that practical constraints can be used to improve the original integer programming model as they may reduce the search space. The authors also suggested different solution approaches to solve the NP-hard problem such as Lagrangian relaxation, triangle inequality restriction, and approximate solution technique, etc.

Ko *et al* (2010) considered a manufacturing system with non-identical parallel machines and used a dispatching rule based method to solve the planning problem considering product quality. The goal is to minimize tardiness cost. The dispatching rule is to use the information including product deadline and quality data (such as C_{pk}) to calculate priority for the product and select the best combination of machine and job. The authors used simulation to study the system for different conditions such as high, low or mixed process capability.

2.4 Systems with Mixed Manual and Automated Manufacturing Processes

Literature on research concerning separated manual and automated production lines is abundant. On the other hand, research literature on manufacturing system optimization with mixed manual and automated operations or production lines is quite limited. We could not find relevant research in this area using quantitative methods or mathematical modeling. In this section, we discuss the research work using qualitative methods investigating such or similar systems involving manual and automated production operations.

Khan and Day (2002) introduced a knowledge based design methodology for manufacturing system design. It is a procedure to design assembly lines with both manual and automated operations. The method can also be used in designing systems with single production line, multi-production lines and mixed-production lines. The methodology starts from selecting the assembly line based on operation time and demand volume. The second step is to decide the parallelism of the system and to allocate jobs based on cycle time. Finally they obtained the rearranged stations layout with improved efficiency.

In addition to choosing proper lines, several researchers studied production flexibility including product mix, labor and machine flexibility. Some of them also presented research methods and results.

Karuppan and Ganster (2004) conducted an empirical study to investigate the interactive effects of labor and machine flexibility and product mix flexibility. The authors analyzed hundreds of samples from many production supervisors and operators. They proposed four indexes including range number (RN), range heterogeneity (RH), mobility (MOB) and uniformity (UN) to measure system flexibility.

Karuppan (2008) conducted similar study on labor flexibility and product mix flexibility. The author conducted a hierarchical regression analysis on labor flexibility and product mix flexibility based on survey results from large number of questionnaires.

2.5 Summary

The literatures in this chapter covers optimization problems in a production system with manual or automated production lines. Most of these reviewed articles have the objective of minimizing cost or/and maximizing productivity. Some of them focus more on the solution methods rather than formulating the model. While in this thesis, we present an optimization model with certain types of cost. Workers' learning curve and product defective rates are incorporated in our model formulation to make a better description of a manual production process. Although the mathematical models presented in several existing research articles may have some similarities to the optimization model presented in this thesis, our model is different in many different aspects. We aim at decreasing the total labor cost, overtime cost, delay penalty, fixed cost, and quality related cost. We can also obtain the improved workforce allocation and proper production schedule through this mathematic model.

In the following chapter, the optimization model will be presented in detail. Some modifications and simplified models will follows the main model, with the numerical examples for different production patterns presented in Chapter 4. Summary and conclusions are shown in the final chapter.

Chapter 3

Optimization Model

3.1 Introduction

As mentioned in the previous chapter, the research conducted in this thesis is to study production planning problem with workforce assignment to generate optimized production schedules in small to medium sized production systems. The production planning problem may have uncertainties related to initial production rate or time, customer demand and labor cost. The mathematical programming model was developed based on the manufacturing processes in a small sized manufacturing system producing various confectionary products.

In this chapter, we first present a mixed integer programming model to optimize the total cost in the production systems involving mainly manual workforce. The objective function of this mathematical model includes minimizing variable labor cost and fixed production line cost. Learning process of operators, workers efficiency functions, workforce capacity limits and product quality functions are considered as model constraints. The model was solved by CPLEX optimization software to obtain optimal solutions of the studied problems. The computational results allow us to compare different options such as overtime working, extra utilization of production lines and production delays, using skilled or unskilled workers leading to different product quantity and quality levels.

The developed model will be further studied under different production conditions. Different versions of the model will be presented as revised or sub models.

3.2 Problem Description and Modelling

3.2.1 Problem Description

In small to medium sized manufacturing systems (for example, a confectionary production factory producing consumer chocolate candies), automated production lines and manual production lines may be used simultaneously. Automated lines or machines are responsible for producing regular products of larger batches. These lines or machines can be used for product molding, forming, assembling, packaging, etc. In manual production lines, human operators can perform all the above mentioned operations with some of them (such as forming) requiring the use of powered machines.

In this study, we mainly focus on planning manual production lines. We have observed that productivity of a manual production line is affected, among other factors, significantly by worker effectiveness and efficiency. For example, in a chocolate production system, each chocolate can be produced very rapidly. However, if it is not handled well in various manual operations, a high percentage of formed chocolate products can be defective. They will be discarded or reworked with additional cost. Among other reasons, insufficient worker skill may cause poor product quality. Normally, skilled workers do a better job. Furthermore, workers with different skill levels may take different amount of time to complete the same job. The differences could be quite large up to 2 to 3 times. Undoubtedly, every operator needs to go through the process of learning, to certain extent.

Demands for confectionary products such as consumer chocolate products are highly seasonal. The manufacturing company must plan advance for having the optimal manpower in the system and preparing for high demand seasons every year, such as the Christmas-New Year and Valentine's Day celebrations.

3.2.2 Assumption

Based on the description of the confectionary production planning problem in a small factory, the following assumptions were considered in formulating an optimization model for production planning in a system with mixed manual and machine operations.

1. The model has multi time periods. Each time period can be a one week or a month, etc.
2. Operators working in the production line may be unskilled workers or have different levels of working skills.
3. Unskilled workers need to be trained and cause training cost. Training takes place while the workers are performing the jobs. The "on-the-job" training does not take extra resource and is a part of the learning process.
4. All workers will pass through a learning process. The first time period (a week, for example) is the learning period. After this period, a worker is considered to have acquired sufficient skills for the job and is no longer an unskilled labor.
5. Workers can be transferred from one work station to another in a production line. Since the types of jobs to perform at different stations are similar, the same skill level will be maintained if a worker is assigned to a different station.
6. The basic learning curves are used in describing the learning and training processes in formulating the model. Varying learning rate and forgetting factor in the learning process are not considered.

7. All workers in one station perform the same type of job to produce the same category of products.
8. The production line does not have work-in-process (WIP) inventory.

3.2.3 Notations

We present the following notations used in the model formulation to be presented and explained in the subsequent sections.

Index sets

$w = \{1,...,W\}$ The category of workforce

$s = \{1,...,S\}$ The category of product

$i = \{1,...,I\}$ The station in one production line

$j = \{1,...,J\}$ Number of production line

$t = \{1,...,T\}$ Number of time periods

Variables

X_{ijt}^{w} Total number of level w workforce at station i of production line j during time period t

XH_{ijt}^{w} Number of level w workforce hired for station i of production line j during time period t

XT_{ijt}^{w} Number of level w workforce need to be trained at station i of production line j during time period t

XF_{ijt}^{w} Number of level w workforce reduced from station i during time period t

WT_{jt}^{s} Working time hours for product s on production line j during time period t

OT_{jt}^{s} Overtime hours for product s on production line j during time period t

$$K_j = \begin{cases} 0, & \text{if the production line is not operating} \\ 1, & \text{otherwise} \end{cases}$$

C^{s} Real completion time for product s

P_{ijt}^{ws} Output during learning process of product s made by level w new employee at station i of production line j during time period t

D_{jt}^{s} Total output of product s on production line j during time period t

Parameters

α_t^{w} Regular weekly wage of a level w workers during time period t

β^{w} Hiring cost of a level w workforce per week

γ^{w} Training cost of a level w workforce per week

δ^{w} Firing cost of a level w workforce per week

τ^{w} Overtime cost of a level w workforce per hour

λ^{s} Delay penalty of product s workforce per week

d^{s} Due date of product s

ε^{s} Fixed cost of a production line based on different product s

g^{w} Learning index of level w workforce, $g^{w} = -\log b^{w} / \log 2$

b^{w} Learning rate of level w workforce

y_{0ijt}^{ws} Initial production time per unit of new employed level w workforce for one part of product s at station i of production line j in time period t

y_{ijt}^{ws} Stable production time per unit of level w workforce for one part of product s at station

i of production line j in time period t

ux_j Upper bound of workforce amount of production line j

ut_{jt} Upper bound of overtime hours on production line j during time period t

ub_t Upper bound of total working time hours during time period t

tr^w The ratio of output during learning process to output after learning process of level w workforce

η^s Customer demand for product s

3.2.4 The Main Model

The following model is a multi-period non-linear programming model consisting of learning curve functions and quality parameters to describe the small to medium scale production system. After we present the model, we will discuss certain other issues such as increasing the number of production lines, working overtime and late product deliveries during high seasons.

The objective function is shown in Eq. (3.1) below.

$$\begin{aligned} Min\ Z =& \sum_{w=1}^{W}\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T}(X_{ijt}^{w} \times \alpha_t^w) + \sum_{w=1}^{W}\sum_{t=1}^{T}\left(\max\left\{\sum_{i=1}^{I}\sum_{j=1}^{J}(XH_{ijt}^{w} - XF_{ijt}^{w}), 0\right\} \times \beta^w \right) + \\ & \sum_{w=1}^{W}\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T}(XT_{ijt}^{w} \times \gamma^w) - \sum_{w=1}^{W}\sum_{t=1}^{T}\left(\min\left\{\sum_{i=1}^{I}\sum_{j=1}^{J}(XH_{ijt}^{w} - XF_{ijt}^{w}), 0\right\} \times \delta^w \right) + \\ & \sum_{s=1}^{S}\sum_{j=1}^{J} K_j \times \varepsilon^s + \sum_{w=1}^{W}\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T}(OT_{jt}^{s} \times \tau^w) + \sum_{s=1}^{S}\left(\max\left\{C^s - d^s, 0\right\} \times \lambda^w\right) \end{aligned} \tag{3.1}$$

The seven items of the objective functions represent variable production cost, labor force hiring cost, training cost, firing cost, fixed production cost, overtime cost and delay penalty cost, respectively. In this model, the variable production cost can be different depending on the level of skills of the workers in different time period. Other costs are affected by worker skill levels. In the considered production system, workers are allowed to transfer among stations. Since transfer will not cause extra cost, we made subtractions of XH_{ijt}^{w} and XF_{ijt}^{w} in calculating the total cost. This feature will be elaborated by numerical examples in Section 4.3.

The optimization of the above cost function is subject to certain constraint conditions related to the production system. These constraint functions are presented and explained below. Eqs. (3.2) to (3.10) below are constraint functions related to workforce capacity and connections of workforces with different levels of work skill.

$$X_{ijt}^{w} = XH_{ijt}^{w}, t = 1, \forall w, i, j \tag{3.2}$$

$$XF_{ijt}^{w} = 0, t = 1, \forall w, i, j \tag{3.3}$$

$$m_{ijt}^{w} \times XH_{ijt}^{w} + \left(1 - m_{ijt}^{w}\right) \times XF_{ijt}^{w} = 0, \forall w, i, j, t \tag{3.4}$$

$$XH_{ijt}^{w} \geq 0, \forall w, i, j, t \tag{3.5}$$

$$XT_{ijt}^{w} = XH_{ijt}^{w}, w \geq 2, \forall i, j, t \tag{3.6}$$

$$XT_{ijt}^{w} = 0, w = 1, \forall i, j, t \tag{3.7}$$

$$X_{ijt}^{w} = X_{ij,t-1}^{w} + XH_{ijt}^{w} - XF_{ijt}^{w}, t \geq 2, \forall w, i, j \tag{3.8}$$

$$\sum_{w=1}^{W} \sum_{i=1}^{I} X_{ijt}^{w} \leq ux_{j}, \forall j, t \tag{3.9}$$

$$X_{ijt}^{w} \geq XF_{ijt}^{w} \geq 0, \forall w, i, j, t \tag{3.10}$$

Constraint Eqs. (3.2) and (3.3) show that the total number of workers is equal to the number of new employees in the first time period and no one can be fired. Eq. (3.4) means that hiring and firing should not occur in one station at the same time. Eqs. (3.6) and (3.7) indicate that unskilled workers require training. Eq. (3.8) shows the relationship between the number of workers at each station and the number of hired or fired workers at the same station. Eqs. (3.9) and (3.10) give the upper bounds of total number of workers and the number of workers who can be laid-off, respectively, during one time period.

In addition to the above functions, 5 fixed production cost related functions are presents below.

$$ux^s \times K_j \geq \sum_{i=1}^{I} X_{ijt}, \forall j,t \tag{3.11}$$

$$ux^s \times K_j \geq \sum_{i=1}^{I} XH_{ijt}, \forall j,t \tag{3.12}$$

$$ux^s \times K_j \geq \sum_{i=1}^{I} XF_{ijt}, \forall j,t \tag{3.13}$$

$$ux^s \times K_j \geq \sum_{i=1}^{I} XT_{ijt}, \forall j,t \tag{3.14}$$

$$\eta^s \times K_j \geq D_{jt}^s, \forall s,j,t \tag{3.15}$$

The above 5 constraints ensure that the number of workers and output are zero if a production line is not operating. Learning curve functions used in this model are shown next.

$$WT_{jt}^s + OT_{jt}^s = \int_0^{P_{ijt}^{ws}} y_{0ijt}^{ws} \times n^{-g^w} d(n), \forall w,s,i,j,t \tag{3.16}$$

$$y_{ijt}^{ws} = y_{0ijt}^{ws} \times P_{ijt}^{ws^{-g^w}}, \forall w,s,i,j,t \tag{3.17}$$

Eq. (3.16) calculates the output of the new employees during learning process. Eq. (3.17) is the learning curve function. In the following section, we explain these functions in detail. Finally, some restrictions for the total output and working time hours are presented below.

$$D_{jt}^{s} \div qr^{ws} \leq \sum_{w=1}^{W} XH_{ijt}^{w} \times P_{ijt}^{ws}, t = 1, \forall s, i, j \tag{3.18a}$$

$$\sum_{s=1}^{S} D_{jt}^{s} \times y_{ijt}^{ws} \div qr^{ws} \leq tr^{w} \times ub_{t} \times XH_{ijt}^{w}, t = 1, \forall w, s, i, j \tag{3.18b}$$

$$D_{jt}^{s} \div qr^{ws} \leq \sum_{w=1}^{W} XH_{ijt}^{w} \times P_{ijt}^{ws} + \sum_{w=1}^{W} (X_{ij,t-1}^{w} - XF_{ijt}^{w}) \times \frac{WT_{jt}^{s} + OT_{jt}^{s}}{y_{ijt}^{ws}}, t \geq 2, \forall s, i, j \tag{3.19a}$$

$$\sum_{s=1}^{S} D_{jt}^{s} \times y_{ijt}^{ws} \div qr^{ws} \leq ub_{t} \times (X_{ij,t-1}^{w} - XF_{ijt}^{w}) + tr^{w} \times ub_{t} \times XH_{ijt}^{w}, t \geq 2, \forall w, s, i, j \tag{3.19b}$$

$$\sum_{j=1}^{J} \sum_{t=1}^{C^{s}} D_{jt}^{s} \geq \eta_{s}, \forall s \tag{3.20}$$

$$\sum_{s=1}^{S} WT_{jt}^{s} \leq ub^{t}, \forall j, t \tag{3.21}$$

$$\sum_{s=1}^{S} OT_{jt}^{s} \leq ut_{jt}, \forall j, t \tag{3.22}$$

Eq. (3.18a) represents the total output produced by new employees during the first time period, and Eq. (3.19a) represents the total output produced by workers of all categories during the following time periods. Eqs. (3.18a) and (3.19a) are introduced to ensure that the final output is always less than the amount of items produced at any station during the same time period.

In addition to the the previous computing method, we have another method as Eqs. (3.18b) and (3.19b). They indicate the total operating time of all types of products is less than specified working time. These output constraint functions are linear without working time variables. Thus, all the restrictions about working time hours can be removed as well. It will be elaborated in the Secion 3.4.3.

These two calculating methods are suitable for two production modes. The difference between these two methods is working time at each station. Same operating time at all stations of each production line is guaranteed by the first method. For example, if the computed working time is 15 hours, the working time at all stations are 15 hours. In contrast, for the second method, total operating time for one product can be different with each other among all the stations. For example, the working time at three stations may be 15 hours, 16 hours and 14 hours individually.

3.3 Learning Curve and Output Functions

The basic learning curve used in our model is Eq. (3.17). Stable production time per unit is equal to the initial production time multiplying the total output during the learning process with the power of learning index $-g^{w}$. Eq. (3.16) calculates accumulative output of new employees and can be equal to

$$P_{ijt}^{ws} = \left[\frac{1-g^{w}}{y_{0ijt}^{ws}} \times (WT_{jt}^{s} + OT_{jt}^{s}) \right]^{\frac{1}{1-g^{w}}} \tag{3.23}$$

In our calculation, the first time period is typically the learning period for new employees. The shapes of the hourly production rate curve and learning curve are shown as Figure 3.1 below. And the shape of accumulative output calculated by Eq. (3.23) is shown in following Figure 3.2.

The curves presented in Figure 3.1 and 3.2 show 2 phases of production. The first phase is the one with learning and the second phase is the stable production. In the first phase, production time is relatively high. In the second phase, production time becomes stable. Therefore, the hourly output is increasing in first phase and becomes stable as well in the second phase.

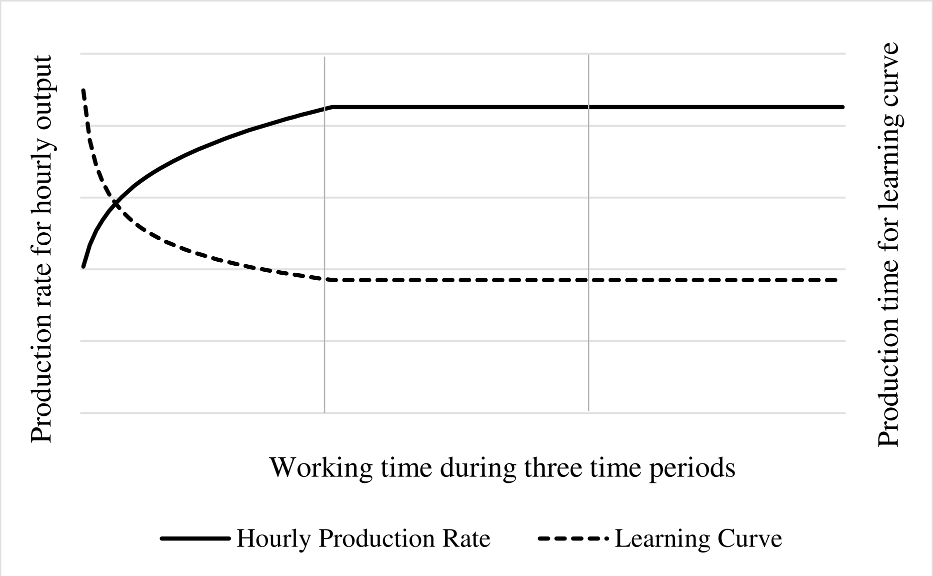

Figure 3.1 Hourly output curve and learning curve

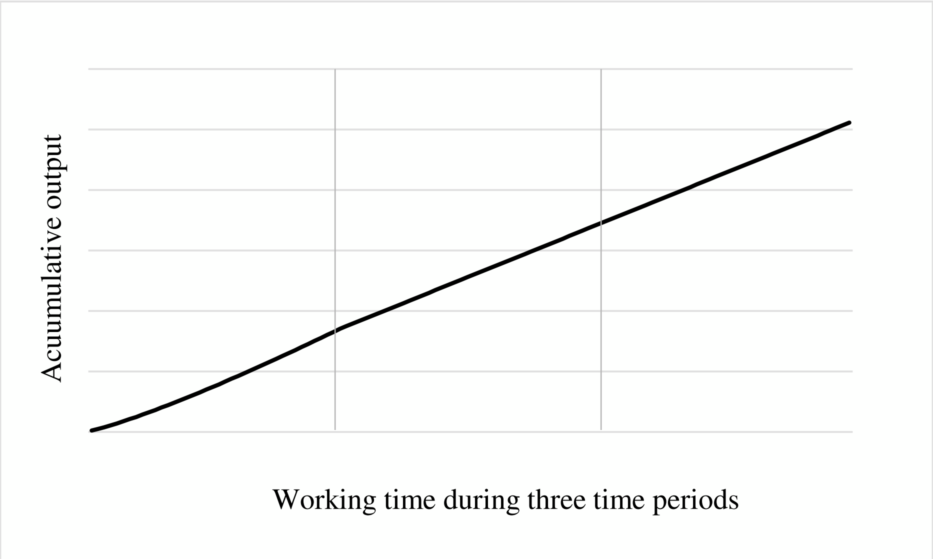

Figure 3.2 Accumulative output curve

3.4 Sub-Models for Different Situations

Based on the main model developed in the previous section, we present several different versions of the formulated model for solving more specific problems of different cases. These versions include those for single production line and for double production lines. They will be presented from simple to complex. As can be seen from the main model, some of the functions have nonlinear terms. We used an off-shelf software, CPLEX, to solve the developed model. The available version of the CPLEX software does not support nonlinear functions in the optimization model. Some of the simplified models presented below may be used without the original nonlinear terms, or with the original ones replaced by a simplified term. We believe that the reduced models still capture with close approximation the true features of the studied problem.

3.4.1 Single Production Line with One Station

We start from a production system of a single production line with one station. The production planning does not allow overtime or delay in product delivery. Both skilled and unskilled workers are employed to produce two types of products in a three-week time period.

In the proposed main model, there are three non-linear functions: learning curve function, new employee's accumulative output function during learning phase and the function to calculate the final product quantity. In solving this first sub-model, we removed the nonlinear learning curve function from main model. We calculate the production rate using Excel worksheet outside the model and used the Excel results as input data for the CPLEX program.

The nonlinear function to calculate new employ output P_{ijt}^{ws} can be approximated by a linear function as shown in Figure 3.2. The slope of the accumulated output line is regarded as a constant number. An additional parameter cn^{w} representing for the considered constant number.

The final quantity of products to be produced is the multiplication of the work time and the number of workers to be employed with both being decision variables. In the first stage of using the main model to determine the optimal production plan, we assume that the work time is a fixed value hence this nonlinear term is reduced to a linear term. We conducted a series of experiments combining several representative work time and manpower values. This will be discussed in the next chapter in detail.

Based on the above discussion, the simplified mathematical model is presented below.

$$\begin{aligned} Min\ Z = & \sum_{w=1}^{W}\sum_{t=1}^{T}(X_t^w \times \alpha_t^w) + \sum_{w=1}^{W}\sum_{t=1}^{T}\left(\max\left\{XH_t^w - XF_t^t,\ 0\right\}\times \beta^w\right) + \sum_{w=1}^{W}\sum_{t=1}^{T}\left(XT_t^w \times \gamma^w\right) \\ & - \sum_{w=1}^{W}\sum_{t=1}^{T}\left(\min\left\{XH_t^w - XF_t^t,\ 0\right\}\times \delta^w\right) \end{aligned} \tag{3.24}$$

Subject to:

$$X_t^w = XH_t^w, t = 1, \forall w \tag{3.25}$$

$$XF_t^w = 0,, t = 1, \forall w \tag{3.26}$$

$$m_t^w \times XH_t^w + \left(1 - m_t^w\right) \times XF_t^w = 0, \forall w, t \tag{3.27}$$

$$X_t^w = X_{t-1}^w + XH_t^w - XF_t^w, t \ge 2, \forall w \tag{3.28}$$

$$XT_t^w = XH_t^w, w \ge 2, \forall t \tag{3.29}$$

$$XT_t^w = 0, w = 1, \forall t \tag{3.30}$$

$$\sum_{w=1}^{W} X_t^w \le ux, \forall t \tag{3.31}$$

$$X_t^w \ge XF_t^w \ge 0, \forall w, t \tag{3.32}$$

$$XH_t^w \ge 0, \forall w, t \tag{3.33}$$

$$P_t^{ws} = cn^w \times WT_t^s, w = 1, \forall s, t \tag{3.34}$$

$$D_t^s \le \sum_{w=1}^{W} XH_t^w \times P_t^{ws}, t = 1, \forall s \tag{3.35}$$

$$D_t^s \le \sum_{w=1}^{W} XH_t^w \times P_t^{ws} + \sum_{w=1}^{W} (X_{t-1}^w - XF_t^w) \times \frac{WT_t^s}{y_t^{ws}}, t \ge 2, \forall s \tag{3.36}$$

$$\sum_{t=1}^{T} DC_t^s \ge \eta_s, \forall s \tag{3.37}$$

Eq. (3.34) is the approximate learning phase output function for skilled and unskilled workers. The value of cn^w will be explained in Chapter 4 with the presentation of numerical experiments.

3.4.2 Single Production Line with 2 Stations

The second sub-model is a production system of a single production line with two stations. The production planning does not allow overtime or delay in product delivery. Both skilled and unskilled workers are employed to produce two types of products in a three-week time period.

For this case, the mathematical programming model is similar to that discussed in the previous section. Station index *i* is added in this model, and amount of cn^w can be different in this model.

3.4.3 Single Production Line with 3 Stations

This sub-model is a production system of a single production line with three stations. The production planning does not allow overtime or delay in product delivery. Both skilled and unskilled workers are employed to produce two types of products in a three-week time period.

If we use the second method of simplification with Eqs. (3.18b) and (3.19b), it means that during learning procedure, the new employees have stable, but lower productivity than that of experienced operators. Thus we may keep the other constraints such as limits of human resource, while transform the output constraint functions into another form. Eqs. (3.34) to (3.36) are replaced by two functions Eqs. (3.38) and (3.39).

$$\sum_{s=1}^{S} D_t^{ws} \times y_{it}^{ws} \leq tr^{w} \times ub_t \times XH_{it}^{w}, t = 1, \forall w, i \tag{3.38}$$

$$\sum_{s=1}^{S} D_t^{ws} \times y_{it}^{ws} \leq ub_t \times (X_{i,t-1}^{w} - XF_{it}^{w}) + tr^{w} \times ub_t \times XH_{it}^{w}, t \geq 2, \forall w, i \tag{3.39}$$

3.4.4 Single Production Line Considering Product Quality Issues

In this section, we consider the same production system of a single production line with 3 stations as in the previous section. The production planning does not allow overtime or delay in product delivery. In addition, we consider that some of the products may not meet quality requirements due to various reasons such as unskilled workforce. Both skilled and unskilled workers are employed to produce two types of products in a three-week time period.

The products of the producing line will be inspected by quality inspectors. Nonconforming products (with problems in packaging, weight, etc.) will be rejected. Based on the consideration, we revise Eqs. (3.38) and (3.39) and incorporate a defective rate in these functions, seeing Eqs. (3.40) and (3.41).

$$\sum_{s=1}^{S} D_t^{ws} \times y_{it}^{ws} / qr^{sw} \leq tr^{w} \times ub_t \times XH_{it}^{w}, t = 1, \forall w, i \tag{3.40}$$

$$\sum_{s=1}^{S} D_t^{ws} \times y_{it}^{ws} / qr^{sw} \le ub_t \times (X_{i,t-1}^{w} - XF_{it}^{w}) + tr^{w} \times ub_t \times XH_{it}^{w}, t \ge 2, \forall w, i \tag{3.41}$$

3.4.5 Double Production Lines with Fixed Cost

More production lines may be required for a high season, so the sub-model in this section is built for calculating fixed cost. It focus on a production system of two production lines with three stations for each line. Only unskilled workers are employed to produce two types of products in a three-week time period.

Considering the computational complexity, we temporarily disregard the workforce category index *w*. And we assume all new workers are unskilled workers requiring training. Both objective and constraints change slightly, so we present the whole sub-model below. The sample data, optimal solution and discussion will be presented in Chapter 4.

$$\begin{aligned} Min\ Z = & \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T}(X_{ijt} \times \alpha) + \sum_{t=1}^{T}\left(\max\left\{\sum_{i=1}^{I}\sum_{j=1}^{J}(XH_{ijt} - XF_{ijt}), 0\right\} \times \beta\right) + \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T}(XT_{ijt} \times \gamma) \\ & - \sum_{t=1}^{T}\left(\min\left\{\sum_{i=1}^{I}\sum_{j=1}^{J}(XH_{ijt} - XF_{ijt}), 0\right\} \times \delta\right) + \sum_{j=1}^{J} K_j \times \varepsilon \end{aligned} \tag{3.42}$$

Subject to:

$$ux_j \times K_j \ge \sum_{i=1}^{I} X_{ijt}, \forall j, t \tag{3.43}$$

$$ux_j \times K_j \ge \sum_{i=1}^{I} XH_{ijt}, \forall j, t \tag{3.44}$$

$$ux_j \times K_j \ge \sum_{i=1}^{I} XF_{ijt}, \forall j, t \tag{3.45}$$

$$ux_j \times K_j \geq \sum_{i=1}^{I} XT_{ijt}, \forall j, t \tag{3.46}$$

$$\eta^s \times K_j \geq D_{jt}^s, \forall s, j, t \tag{3.47}$$

$$X_{ijt} = XH_{ijt}, t = 1, \forall i, j \tag{3.48}$$

$$XF_{ijt} = 0, t = 1, \forall i, j \tag{3.49}$$

$$m_{ijt} \times XH_{ijt} + (1 - m_{ijt}) \times XF_{ijt} = 0, \forall i, j \tag{3.50}$$

$$X_{ijt} = X_{ij,t-1} + XH_{ijt} - XF_{ijt}, t \geq 2, \forall i, j \tag{3.51}$$

$$XT_{ijt} = XH_{ijt}, \forall i, j, t \tag{3.52}$$

$$\sum_{i=1}^{I} X_{ijt} \leq ux_j, \forall j, t \tag{3.53}$$

$$X_{ijt} \geq XF_{ijt} \geq 0, \forall i, j, t \tag{3.54}$$

$$XH_{ijt} \geq 0, \forall i, j, t \tag{3.55}$$

$$\sum_{s=1}^{S} D_{jt}^s \times y_i^s \leq tr^w \times ub_t \times XH_{\mathrm{ijt}}, t = 1, \forall i, j \tag{3.56}$$

$$\sum_{s=1}^{S} D_{jt}^s \times y_i^s \leq ub_t \times (X_{\mathrm{ij,t-1}} - XF_{\mathrm{ijt}}) + tr^w \times ub_t \times XH_{\mathrm{ijt}}, t \geq 2, \forall i, j \tag{3.57}$$

$$\sum_{j=1}^{J} \sum_{t=1}^{T} DC_{jt}^s \geq \eta_s, \forall s \tag{3.58}$$

3.4.6 Double Production Lines with Overtime

The sixth sub-model mainly focus on working overtime. It discusses a problem in a production system of two production lines with three stations for each line. Only unskilled workers are employed to produce two types of products in a three-week time period. The linear functions representing for output are presents as Eqs. (3.59) and (3.60) replacing Eqs. (3.54) and (3.55). Other two constraint functions represent limits of overtime.

$$\sum_{s=1}^{S}(D_{jt}^{s} \times y_i^s - OT_{jt}^{s\,\prime}) \le tr^{w} \times ub_t \times XH_{\mathrm{ijt}}, t = 1, \forall i, j \tag{3.59}$$

$$\sum_{s=1}^{S}(D_{jt}^{s} \times y_i^s - OT_{jt}^{s\,\prime}) \le ub_t \times (X_{\mathrm{ij,t\text{-}1}} - XF_{\mathrm{ijt}}) + tr^{w} \times ub_t \times XH_{\mathrm{ijt}}, t \ge 2, \forall i, j \tag{3.60}$$

$$\sum_{s=1}^{S} OT_{jt}^{s\,\prime} \le ut_{jt} \times \sum_{i=1}^{I} X_{ijt}, j = 1, \forall i, t \tag{3.61}$$

$$\sum_{s=1}^{S} OT_{jt}^{s\,\prime} = 0, j = 2, \forall i, t \tag{3.62}$$

Since it is an experiment for overtime, the constraint functions of fixed cost – Eqs. (3.43) to (3.47) - are disregarded. In addition to overtime cost in objective functions, the constraint functions for calculating output change slightly, as Eqs. (3.59) and (3.60). And Eq. (3.61) indicates the upper limit of overtime, with the same meaning as Eq. (3.22) in main model. In this example, we assume that single production line supports for working overtime, thus Eq. (3.62) is required.

To ensure that linear functions are used in CPLEX, in the above functions, $OT_{jt}^{s\,\prime}$ replacing OT_{jt}^{s} are used in the sub-model. $OT_{jt}^{s\,\prime}$ represents for all operators' total overtime hours in a week. The overtime hours for each operator can be calculated based on optimal solution of overtime and workforce assignment, which will be elaborated in Chapter 4. Since $OT_{jt}^{s\,\prime}$ is the total overtime of all operators, the upper bound of overtime is the workforce amount in a whole production line multiplying original overtime hours a week OT_{jt}^{s}.

3.4.7 Double Production Lines with Overtime and Delay

The seventh sub-model discusses a problem in a production system of two production lines with three stations for each line. Only unskilled workers are employed to produce two types of products in a four-week time period. Both production lines support for overtime. Constraint functions are similar, while delay penalty of one week is λ^s shown as the objective function of main model. The comparison of delay and overtime will be discussed in Section 4.7.

3.5 Solution Method and Summary

A mixed integer programming model and seven simplified sub-models are detailed in this section. We use CPLEX for calculating the optimal solutions. Consequently, we have seven CPLEX sub-models to investigate the production system in detail with the numerical experiments in the next chapter.

Chapter 4

Numerical Example and Analysis

In this chapter we present numerical examples to illustrate different versions of the mathematical model described in the previous chapter. CPLEX 12.5.1 was used for solving these problems. The CPLEX codes are presented in Appendix D to Appendix J at the end of this document.

The data and information used in the examples are based on the production system in a confectionary production company in Montreal, Quebec. No real data are utilized directly due to confidentiality reasons. The computational results show the validity of our model and optimal solutions for potential practical applications.

4.1 Single Production Line with Single Station

4.1.1 Data for the Production System

The production system has a single production line (J=1) with one station (I=1) and two labor levels (W=2) to produce two types of products (S=2) in a three week time period (T=3). Labor costs are given in Table 4.1 below.

Table 4.1 Labor Cost [Example 1]

Workforce category	Hourly wage			Hiring cost	Training cost	Firing cost
	Time period					
	1	2	3			
1	12.5	13.75	13.75	50	10	60
2	10	11	11	40	8	48

The unit of the labor cost is dollars per person. The hourly wage can be different for each time period.

For the learning curve function $y_{ijt}^{ws} = y_{0ijt}^{ws} \times P_{ijt}^{ws^{-g^w}}$ shown in Eq. (3.17), other researchers (Smunt and Morton, 1985, Smunt, 1999) have used the constants 200 units/hour or 500 units/hour for P_{ijt}^{ws}. Since in the considered production system, the output during first time period is similar to 500 units, we use 500 for P_{ijt}^{ws} in the original equation to simplify the expression of the learning curve. Then the learning curve function will be $y_t^{ws} = y_{0t}^{ws} \times 500^{-g^w}$. And we have the stable production rate as shown in Table 4.2 below.

Table 4.2 Stable Production Rate and Demand [Example 1]

Product category	Workforce category		Demand
	1	2	
1	22	13	3000
2	24	15	4000

In addition, the single station can have a maximum of 5 workers working together. Since the tasks are not complex, the learning rates are set as 89% and 90% for the first and second level of workers, respectively.

4.1.2 Simplified Output Curve and Value of the Additional Parameter

After calculating with the data in the previous section, the new accumulative output curves are shown in Figure 4.1 and Figure 4.2 for skilled workers and unskilled workers, respectively. In these figures, the "original output curve" is based on the function in Eq. (3.23): $P_t^{ws} = \left[\frac{1-g^w}{y_{0t}^{ws}} * WT_t^s \right]^{\frac{1}{1-g^w}}$. The piecewise linear function presents similar shape of the original output curve. The data used to plot the curves in these two figures are shown in Appendix A.

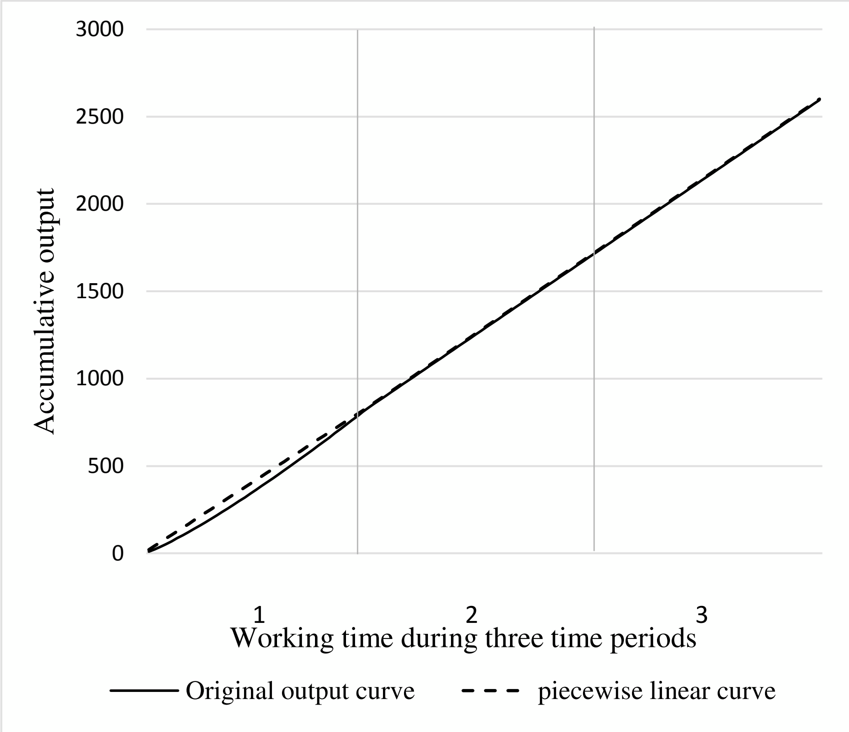

Figure 4.1 Accumulative output curve and approximate piecewise curve for skilled worker

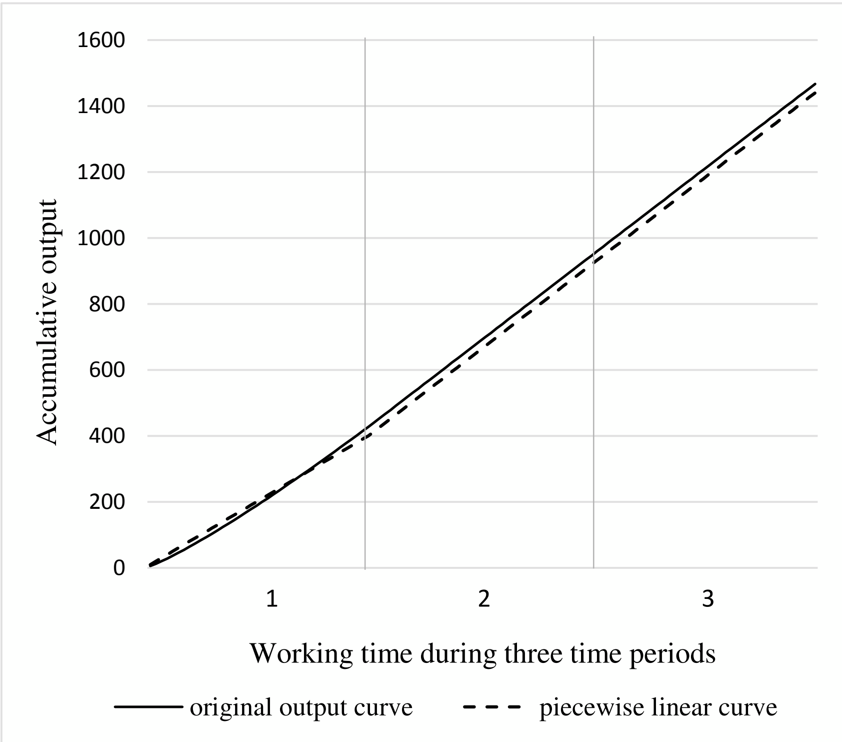

Figure 4.2 Accumulative output curve and approximate piecewise curve for unskilled worker

From the discussion in Section 3.4.1, the linearized output function needs a new parameter cn^{w} to describe the slope of the first segment of the “piecewise linear curve”. For this example problem, its value is set at 21.0 units and 10.0 units for skilled and unskilled workers, respectively.

4.1.3 Experiments and Optimal Solutions

In solving this example problem, we allocate different working hours to each type products from 0 to 40 hours. Each computational instance has a 10 hours increment for producing one type of products and 10 hours decrease for the other. Thus there are 5 levels of changes for each time period and a total of 125 experiments were conducted for the considered 3 time periods. Among these 125 instances, 74 instances have feasible solutions. Production hour allocations and optimal objective values of some of the feasible solutions are shown in Table 4.3.

In Table 4.3, each instance contains two rows. The first row is working time hours for type 1 products and the second row is working time hours for type 2 products. For example, the result of Instance 4 is to allocate all production hours to produce type 2 products in the first and second time periods. In the third time period, 30 hours will be used to produce type 1 products and 10 hours are used to produce the second type of products.

Table 4.3 Working Time Assigned for the Two Products [Example 1]

Instance	Product	t=1	t=2	t=3	Optimal objective function value
4	1	0	0	30	6250
	2	40	40	10	
5	1	0	0	40	5550
	2	40	40	0	
8	1	0	10	20	7250
	2	40	30	20	
9	1	0	10	30	5778
	2	40	30	10	
10	1	0	10	40	5778
	2	40	30	0	
12	1	0	20	10	7750
	2	40	20	30	
13	1	0	20	20	5600
	2	40	20	20	

Table 4.4 Optimal Working Time Allocation [Example 1]

Instance	Product	t=1	t=2	t=3	Optimal objective function value
5	1	0	0	40	5550
	2	40	40	0	
38	1	10	20	20	
	2	30	20	20	
41	1	10	30	10	
	2	30	10	30	
78	1	30	0	30	
	2	10	40	10	
82	1	30	10	20	
	2	10	30	20	
115	1	40	30	0	
	2	0	10	40	

Table 4.5 Optimal Workforce Assignment [Example 1]

Workforce category	t=1	t=2	t=3
1	3	3	4
2	0	0	0

Results of the optimal solutions are presented in Table 4.4 and Table 4.5. In Table 4.4, six optimal solutions provide different working time allocations with the same objective function value of 5550 dollars. These solutions have the same workforce assignment as shown in Table 4.5. Only first level of workers are required in the production because of high productivity of the skilled workers.

4.2 Single Production Line with Double Stations

4.2.1 Data of Production System

The second example is a production system of a single production line (J=1) with two stations (I=2) and two labor levels (W=2) to produce two types of products (S=2) in three weeks (T=3). Labor related costs are given in Table 4.6. Table 4.7 presents stable production rates and total demand from customers. The production line may have up to 10 workers in total. Other data are the same as those in the previous example.

Table 4.6 Labor Cost [Example 2]

<table>
<tr><th rowspan="3">Workforce category</th><th colspan="3">Hourly wage</th><th rowspan="3">Hiring cost</th><th rowspan="3">Training cost</th><th rowspan="3">Firing cost</th></tr>
<tr><th colspan="3">Time period</th></tr>
<tr><th>1</th><th>2</th><th>3</th></tr>
<tr><td>1</td><td>12.5</td><td>13.75</td><td>13.75</td><td>50</td><td>10</td><td>60</td></tr>
<tr><td>2</td><td>10</td><td>11</td><td>11</td><td>40</td><td>8</td><td>48</td></tr>
</table>

Table 4.7 Stable Production Rate and Demand [Example 2]

<table>
<tr><th rowspan="4">Product category</th><th colspan="4">Workforce category</th><th rowspan="4">Demand</th></tr>
<tr><th colspan="2">1</th><th colspan="2">2</th></tr>
<tr><th colspan="2">Station</th><th colspan="2">Station</th></tr>
<tr><th>1</th><th>2</th><th>1</th><th>2</th></tr>
<tr><td>1</td><td>29</td><td>19</td><td>15</td><td>13</td><td>2000</td></tr>
<tr><td>2</td><td>29</td><td>26</td><td>15</td><td>18</td><td>2500</td></tr>
</table>

The data of the output functions can be found in Appendix B. In this example, cn^w were set to 17.0 units and 11.0 units for skilled and unskilled workers, respectively. The optimal solutions computed by CPLEX will be presented and discussed next.

4.2.2 Experiments and Optimal Solution

In each time period, we changed the working hours allocated to each type products from 0 to 40 hours with 10 hours of increment for each experiment and kept the working time hours of all stations the same. Thus there are 5 levels for each time period, and 125 instances in total. Among these 125 instances, 104 instances are feasible. Some of the feasible examples are shown in Table 4.8 below.

Table 4.8 Working Time Assigned for the Two Products [Example 2]

Instance	Product	t=1		t=2		t=3		Optimal objective function value
		Station		Station		Station		
		1	2	1	2	1	2	
4	1	0	0	0	0	30	30	8488
	2	40	40	40	40	10	10	
5	1	0	0	0	0	40	40	8176
	2	40	40	40	40	0	0	
8	1	0	0	10	10	20	20	9300
	2	40	40	30	30	20	20	
9	1	0	0	10	10	30	30	8278
	2	40	40	30	30	10	10	
10	1	0	0	10	10	40	40	7750
	2	40	40	30	30	0	0	
12	1	0	0	20	20	10	10	10518
	2	40	40	20	20	30	30	
13	1	0	0	20	20	20	20	8588
	2	40	40	20	20	20	20	

Table 4.9 Optimal Working Time Allocation [Example 2]

Instance	Product	t=1		t=2		t=3		Optimal objective function value
		Station		Station		Station		
		1	2	1	2	1	2	
79	1	30	30	0	0	30	30	7200
	2	10	10	40	40	10	10	

Table 4.10 Optimal Workforce Assignment [Example 2]

Workforce category	t=1		t=2		t=3	
	Station		Station		Station	
	1	2	1	2	1	2
1	2	2	2	2	2	3
2	0	0	0	0	0	0

Table 4.9 shows the optimal working schedule. For example, it shows in the first time period, 30 hours are assigned to both stations to produce type 1 products and 10 hours are assigned to produce type 2 products. In addition, based on the first two experiments and their solutions, we notice that more unskilled labors are employed if the difference of the production rates between the two levels of labors is close.

4.3 Single Production Line with Three Stations

4.3.1 Data for the Production System

The third example is a single production line (J=1) with three stations (I=3) and two labor levels (W=2) to produce two types of products (S=2) in three weeks (T=3). Labor costs are given in Table 4.11. Table 4.12 presents stable production rates and total demand of customers.

Table 4.11 Labor Cost [Example 3]

<table>
<tr><td rowspan="3">Workforce category</td><td colspan="3">Hourly wage</td><td rowspan="3">Hiring cost</td><td rowspan="3">Training cost</td><td rowspan="3">Firing cost</td></tr>
<tr><td colspan="3">Time period</td></tr>
<tr><td>1</td><td>2</td><td>3</td></tr>
<tr><td>1</td><td>12.5</td><td>13.75</td><td>13.75</td><td>50</td><td>10</td><td>60</td></tr>
<tr><td>2</td><td>10</td><td>11</td><td>11</td><td>40</td><td>8</td><td>48</td></tr>
</table>

Table 4.12 Stable Production Rate and Demand [Example 3]

<table>
<tr><td rowspan="4">Product category</td><td colspan="6">Workforce category</td><td rowspan="4">Demand</td></tr>
<tr><td colspan="3">1</td><td colspan="3">2</td></tr>
<tr><td colspan="3">Station</td><td colspan="3">Station</td></tr>
<tr><td>1</td><td>2</td><td>3</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>1</td><td>29</td><td>17</td><td>23</td><td>13</td><td>11</td><td>17</td><td>4000</td></tr>
<tr><td>2</td><td>29</td><td>26</td><td>19</td><td>14</td><td>18</td><td>15</td><td>4500</td></tr>
</table>

This production line can have a maximum of 15 workers. Furthermore, in this example problem, we set the upper limit of regular working time ub_t to 40 hours.

Based on the data in Table 4.12 and before solving the optimization model, we used Microsoft Excel to calculate tr^w, the ratio of the output during learning to the output after learning. Figure 4.3 shows the accumulative output for both skilled and unskilled workers. And the output ratio tr^w is from 40% to 90% or higher depending on the cumulative working time hours during the learning phase. This is shown in Figure 4.4. The data to generate the curves in Figure 4.4 are presented in Appendix C.

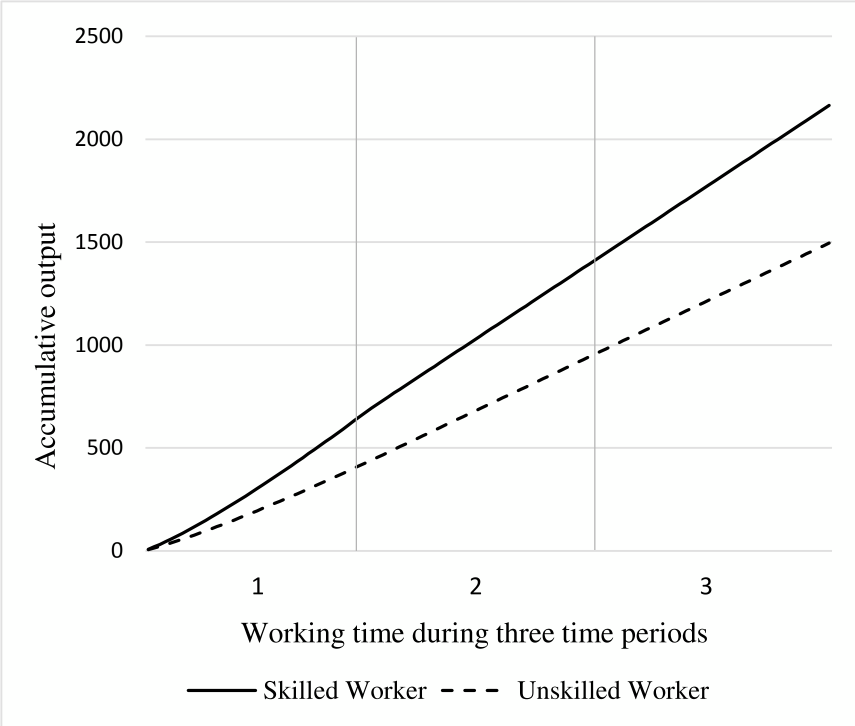

Figure 4.3 Accumulative output curve for both skilled and unskilled workers

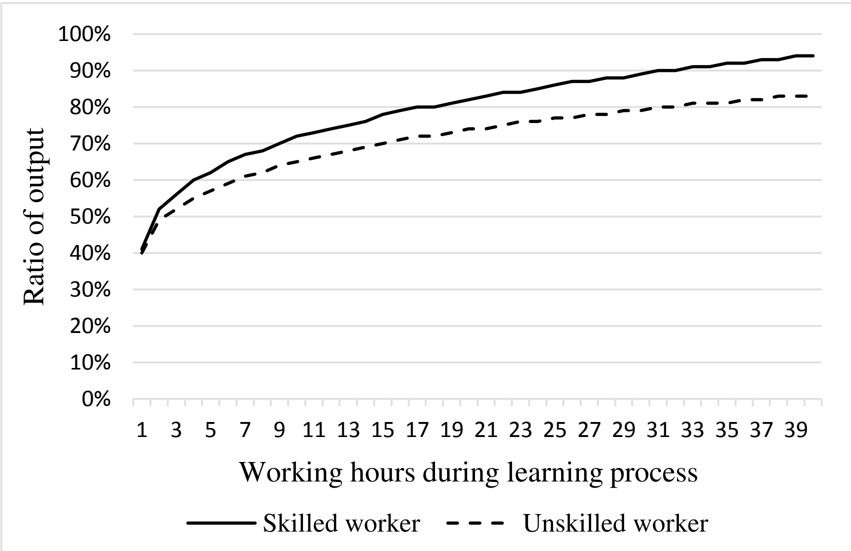

Figure 4.4 Ratio of output tr^{w} for both skilled and unskilled workers

When the working time is one hour in the learning phase, newly hired skilled workers can produce 8 units while unskilled workers produce 5 units as shown in Figure 4.3. After the learning phase, the experienced skilled workers can produce 18 units per hour while unskilled workers produce 13 units. The ratio of output during learning is 40% for both levels of workers. When the working time is 40 hours in the learning phase, the ratio of output will reach up to 94% and 83% for skilled and unskilled workers, respectively. Therefore, we may conduct two experiments using the lowest ratio and the highest ratio separately in determine the production planning solutions. This will be presented next in Section 4.3.2.

4.3.2 Optimal Solution

We first present the experiment with the lowest output ratio in the learning phase. The output ratio used in this first example is 40% of the regular output. After the problem is solved by CPLEX, the optimal objective function value (total cost) is 20400 dollars. The optimal solutions are presented in Tables 4.13 and 4.14.

Table 4.13 Optimal Workforce Assignment with Lowest Output Ratio [Example 3]

Workforce category	t=1			t=2			t=3		
	Station			Station			Station		
	1	2	3	1	2	3	1	2	3
1	3	5	4	3	4	5	4	4	5
2	0	0	0	0	0	0	0	0	0

Table 4.14 Output Results with Lowest Output Ratio [Example 3]

Workforce category	t=1		t=2		t=3	
	Product		Product		Product	
	1	2	1	2	1	2
1	1386	0	1462	2006	1152	2494
2	0	0	0	0	0	0

The output results indicate that operators used all production time to produce the first type of products in the first time period. In the second time period, 1462 units of first type of products and 2006 units of second type of products were produced, and one worker was transferred from the second station to the third station in this time period.

From the output results in Table 4.14 and the production rate data shown in Table 4.12, we notice that the total production time for the first type of products in the first time period is about 48 hours for the first station. And they are 80 hours and 60 hours for the second and third stations, respectively. From the data shown in Table 4.13, one can easily find that the average production time per operator are 16 hours, 16 hours and 15 hours, respectively, at these three stations. This indicates that not all the workers at the third station are fully occupied in this time period. Since the output of type 2 products in the first time period is 0 units, the total production time of type 2 products is 0 hours as well. Other results of average production time (hours) per operator are shown in Table 4.15 below.

Table 4.15 Optimal Working Time Allocation with Lowest Output Ratio [Example 3]

Workforce category	t=1			t=2			t=3		
	Station			Station			Station		
	1	2	3	1	2	3	1	2	3
1	16	16	15	40	40	34	31	40	36
2	0	0	0	0	0	0	0	0	0

We also used the same sub-model to generate a new production plan example with a highest output ratio of 94% for skilled workers and 83% for unskilled workers. Other data are kept the same. The resulting objective function value is 16600 dollars. Other information and data from the optimal solution are presented in Tables 4.15 and 4.16.

Table 4.16 Optimal Workforce Assignment with Highest Output Ratio [Example 3]

Workforce category	t=1			t=2			t=3		
	Station			Station			Station		
	1	2	3	1	2	3	1	2	3
1	2	4	3	3	3	4	3	4	4
2	0	0	0	0	0	0	0	0	0

Table 4.17 Output Results with Highest Output Ratio [Example 3]

Workforce category	t=1		t=2		t=3	
	Product		Product		Product	
	1	2	1	2	1	2
1	2195	0	248	2818	1557	1682
2	0	0	0	0	0	0

As can be seen from Table 4.16, a total of 9 workers were employed at the 3 stations. One additional worker is hired in the second time period with one more added in the third time period. Production amounts in the 3 time periods also show the similar trend as can be seen from Table 4.17. Production time is different as shown in Table 4.18.

Table 4.18 Optimal Working Time Allocation with Highest Output Ratio [Example 3]

Workforce category	t=1			t=2			t=3		
	Station			Station			Station		
	1	2	3	1	2	3	1	2	3
1	38	32	32	35	40	40	37	39	39
2	0	0	0	0	0	0	0	0	0

Since, in the second experiment, a higher output ratio during learning phase is used, less workforce is needed in this case. In both of the experiments in this section, no unskilled worker are employed.

4.4 Single Production Line Model with Quality Rate

4.4.1 Data of Production System

In this section, we consider an example problem with the same features and data used in Section 4.3. The main difference is that in this problem, we consider that the production process is subject to certain quality issues. We assume that some of the products will be rejected by inspection before they can be shipped to customer due to various quality related concerns such as packaging, sizes, appearances, etc. In the calculation, we consider used hypothetical quality conforming rates shown in Table 4.19.

Table 4.19 Quality Rate [Example 4]

Workforce category	Product category	
	1	2
1	85%	80%
2	70%	65%

From practical experiences, in general, quality rate is related to worker category and product category. Skilled workers correspond to better quality rates and less defective products.

4.4.2 Optimal Solution

When the output ratio is 40%, the objective function value of the optimal solution is 24250 dollars. This cost value is higher comparing to that in the previous example because some of the products do not meet quality requirement. Tables 4.20 to 4.22 present the main information of the optimal solutions.

Table 4.20 Optimal Workforce Assignment with Lowest Output Ratio [Example 4]

Workforce category	t=1			t=2			t=3		
	Station			Station			Station		
	1	2	3	1	2	3	1	2	3
1	4	5	5	4	5	6	4	6	5
2	0	0	0	0	0	0	0	0	0

Table 4.21 Output Results with Lowest Output Ratio [Example 4]

Workforce category	t=1		t=2		t=3	
	Product		Product		Product	
	1	2	1	2	1	2
1	715	671	985	2553	2300	1276
2	0	0	0	0	0	0

Table 4.22 Optimal Working Time Allocation with Lowest Output Ratio [Example 4]

Workforce category	t=1			t=2			t=3		
	Station			Station			Station		
	1	2	3	1	2	3	1	2	3
1	15	16	16	38	38	36	37	37	40
2	0	0	0	0	0	0	0	0	0

Comparing with the workforce assignment in the previous section as shown in Table 4.13, more operators are required in this example, because more products are produced. As shown in Table 4.20, 14 workers are employed in the first time period. In the second time period, one skilled worker is added to the third station. In the third time period, one skilled worker is transferred from the third station to the second station. The production line is operating at its full capacity.

When the same problem was solved with a higher output ratio of 94% for skilled workers and 83% for unskilled workers, the objective function value will becomes 23700 dollars. In addition, other solutions also changed. Some of the results of the new solutions are shown in Tables 4.23 to 4.25.

Table 4.23 Optimal Workforce Assignment with Highest Output Ratio [Example 4]

Workforce category	t=1			t=2			t=3		
	Station			Station			Station		
	1	2	3	1	2	3	1	2	3
1	4	5	5	4	5	5	4	5	6
2	0	0	0	0	0	0	0	0	0

Table 4.24 Output Results with Highest Output Ratio [Example 4]

Workforce category	t=1		t=2		t=3	
	Product		Product		Product	
	1	2	1	2	1	2
1	1397	1297	1470	1366	1133	1837
2	0	0	0	0	0	0

Table 4.25 Optimal Working Time Allocation with Highest Output Ratio [Example 4]

Workforce category	t=1			t=2			t=3		
	Station			Station			Station		
	1	2	3	1	2	3	1	2	3
1	28	32	31	30	34	33	31	33	30
2	0	0	0	0	0	0	0	0	0

4.5 Double Production Lines with Fixed Cost

4.5.1 Common Data of the Production System

In this example, we consider a production system with two production lines (J=2), three stations (I=3) for each line and one level (W=1) of workers to produce two types of products (S=2) in three periods (T=3). Labor costs are given in Table 4.26. Table 4.27 presents stable production rates.

Table 4.26 Labor Cost [Example 5]

Hourly wage	Hiring cost	Training cost	Firing cost
12.5	50	10	60

Table 4.27 Stable Production Rate [Example 5]

Product category	Station		
	1	2	3
1	29	17	23
2	29	26	19

To find the optimal solution of this problem, we allow a maximum of 15 workers for each production line. We did not specify further restriction on the number of workers at each work station. We assume that the demand for the second type of products is 80% of that for the first type of products. The lowest output ratio, 40% for both skilled and unskilled worker, is used in this example.

4.5.2 Optimal Solution without Fixed Cost

Several experiments have been conducted with the total demand from 3000 units to 11000 units. In the first set of experiments, we consider that the fixed cost is negligible as the equipment investment was made long time ago. The optimal solutions are presented in Table 4.28.

Table 4.28 Optimal Objective Function Value and Workforce Assignment [Example 5]

Demand for first type of product	Line	t=1			t=2			t=3			Optimal objective function value
		Station			Station			Station			
		1	2	3	1	2	3	1	2	3	
3000	1	0	0	0	0	0	0	0	0	0	12480
	2	2	3	3	2	3	3	2	3	3	
5000	1	2	3	3	3	4	4	3	4	4	20340
	2	1	1	1	1	1	1	1	1	1	
7000	1	2	3	3	2	3	3	2	3	3	28140
	2	2	3	3	3	4	4	3	4	4	
9000	1	3	4	4	3	5	4	3	5	4	35380
	2	3	4	4	3	4	4	3	4	4	
11000	1	3	5	4	4	5	5	4	5	6	43800
	2	4	4	5	4	6	5	4	6	5	

As can be seen from Table 4.28, when demand is 3000 units, the optimal solution is operating one production line with 8 workers. When demand is 5000 units or higher, both production lines are used. However, in some cases such as when the demand for the first type of products is 5000 units, neither of the two lines operates at their full capacities. This phenomenon is also related to that we did not include the fixed equipment cost in the objective function in solving this set of example problems.

4.5.3 Optimal Solution with Fixed Cost

Several more experiments with fixed cost have been conducted with the total demand from 5000 units to 9000 units. Normally, fixed cost may be approximately 10% of sales profit. We assume that the profit to produce one product is one dollar. Since each unit contains 8 products, the values of fixed costs are 80% of demand values. Different fixed cost values will be used in different experiment as shown in Table 4.29. Optimal objective function values are also presented in Table 4.29.

Table 4.29 Optimal Objective Function Value and Workforce Assignment [Example 5]

Demand for first type of product	Fixed cost	Line	t=1			t=2			t=3			Optimal objective function value
			Station			Station			Station			
			1	2	3	1	2	3	1	2	3	
5000	4000	1	3	4	4	4	5	5	4	5	5	23340
		2	0	0	0	0	0	0	0	0	0	
7000	5600	1	2	3	3	2	3	3	2	3	3	39340
		2	2	3	3	3	4	4	3	4	4	
9000	7200	1	3	4	4	3	5	4	3	5	4	49780
		2	3	4	4	3	4	4	3	4	4	

As can be seen from Table 4.29, single production line is used when the demand for the first type of product is 5000 units. The production line is operated at nearly its full capacity. When customer demand is 7000 or 9000 units, the workforce and production line arrangement does not change.

With further consideration, if fixed cost was included in the case with 5000 units demand in the previous Section 4.5.2, the optimal objective function value would reach up to 28340 dollars, because fixed cost of 4000 dollars would be double added to the original objective function value of 20340 dollars.

4.6 Double Production Lines with Overtime

As discussed in Chapter 3, the company may use overtime, instead of hiring new workers, to temporarily increase its production capacity. Hourly overcome wage is typically higher than regular time wage. Other features of this problem are similar to those of the problem discussed in Section 4.5.

In addition to the system features and basic data discussed in the precious section, we consider that an operator may be allowed to work for a maximum of 10 hours of overtime in each time period, or $ut_{jt} \le 10$. As discussed before, we use $12.5/hour as regular time wage. In this set of experiments, we consider that overtime wage can be at 1.5, 1.6 or 3.0 times of the regular wage and calculate optimal solutions using the same model accordingly.

These experiments were run with different values of customer demand from 3000 units to 15000 units in three time periods. Results of the optimal solutions corresponding to the above mentioned 1.5, 1.6 and 3.0 of overtime to regular time wage ratio are presented in Table 4.30, 4.31 and 4.32, respectively.

Table 4.30 Optimal Solutions with 1.5 of Overtime to Regular Wage Ratio [Example 6]

Demand for first type of product	Line	t=1					t=2					t=3					Optimal objective function value
		Station			Overtime Product category		Station			Overtime Product category		Station			Overtime Product category		
		1	2	3	1	2	1	2	3	1	2	1	2	3	1	2	
3000	1	1	1	1	0	7.3	1	2	2	0	6.0	2	2	2	10.0	0	9460
	2	0	0	0			0	0	0			0	0	0			
5000	1	2	3	2	8.5	0	2	3	3	0	7.0	2	3	3	7.5	0	15298.75
	2	0	0	0			0	0	0			0	0	0			
7000	1	2	4	4	6.0	0	3	4	4	0	7.7	3	4	4	7.7	0	20972.5
	2	0	0	0			0	0	0			0	0	0			
9000	1	3	5	5	6.8	0	3	6	5	0	6.4	4	5	5	8.2	0	26862.5
	2	0	0	0			0	0	0			0	0	0			
11000	1	4	5	5	7.0	0	4	6	5	0	6.7	4	6	5	6.7	0	34292.5
	2	1	1	1			1	1	1			1	1	1			
13000	1	5	5	5	6.8	0	4	5	5	0	5.5	4	5	5	5.3	0	41567.5
	2	2	3	2			2	3	3			2	3	3			
15000	1	4	5	5	4.6	0	5	5	5	0	5.5	5	5	5	5.5	0	48845
	2	3	4	4			3	5	4			3	5	4			

Table 4.31 Optimal Solutions with 1.6 of Overtime to Regular Wage Ratio [Example 6]

Demand for first type of product	Line	t=1					t=2					t=3					Optimal objective function value
		Station			Overtime Product category		Station			Overtime Product category		Station			Overtime Product category		
		1	2	3	1	2	1	2	3	1	2	1	2	3	1	2	
3000	1	1	1	1	0	10	1	2	2	0	6.0	2	2	2	8.7	0	9600
	2	0	0	0			0	0	0			0	0	0			
5000	1	2	3	3	6.4	0	2	3	3	0	6.3	2	3	3	6.3	0	15500
	2	0	0	0			0	0	0			0	0	0			
7000	1	2	4	4	5.8	0	3	4	4	0	7.5	3	4	4	8.2	0	21260
	2	0	0	0			0	0	0			0	0	0			
9000	1	3	5	5	6.5	0	3	6	5	0	6.4	4	5	5	8.6	0	27230
	2	0	0	0			0	0	0			0	0	0			
11000	1	4	5	5	7.0	0	4	6	5	0	6.7	4	6	5	6.6	0	34740
	2	1	1	1			1	1	1			1	1	1			
13000	1	4	6	5	5.4	0	4	6	5	0	5.4	4	6	5	5.2	0	42020
	2	2	2	3			2	2	3			2	3	3			
15000	1	4	5	5	4.5	0	5	5	5	0	5.5	5	5	5	5.5	0	49360
	2	3	4	4			3	5	4			3	5	4			

Table 4.32 Optimal Solutions with 3.0 of Overtime to Regular Wage Ratio [Example 6]

Demand for first type of product	Line	t=1					t=2					t=3					Optimal objective function value
		Station			Overtime Product category		Station			Overtime Product category		Station			Overtime Product category		
		1	2	3	1	2	1	2	3	1	2	1	2	3	1	2	
3000	1	1	2	2	5.6	0	1	2	2	0	6.0	2	2	2	3.5	0	11322.5
	2	0	0	0			0	0	0			0	0	0			
5000	1	2	3	3	6.4	0	2	3	3	0	6.3	2	3	3	6.3	0	18142.5
	2	0	0	0			0	0	0			0	0	0			
7000	1	2	5	4	5.5	0	3	4	4	0	6.6	3	4	4	0	7.4	25195
	2	0	0	0			0	0	0			0	0	0			
9000	1	3	5	5	6.8	0	4	5	5	0	8.3	3	6	5	6.4	0	32375
	2	0	0	0			0	0	0			0	0	0			
11000	1	3	5	5	1.8	0	3	5	5	0	4.3	3	5	5	4.3	0	40397.5
	2	2	3	2			2	3	3			2	3	3			
13000	1	3	5	5	2.7	0	3	5	5	0	3.6	3	5	5	3.6	0	47612.5
	2	3	4	4			3	5	4			3	5	4			
15000	1	2	4	4	2.4	0	4	5	6	0	4	4	5	6	4	0	55550
	2	4	6	5			4	6	5			4	6	5			

Tables 4.30 to 4.32 provide optimal solutions of workforce allocation, overtime hours and objective function values for this example problem. Overtime shown in these tables is overtime hours for each worker. For example, in Table 4.30, when demand is 3000 units, single production line is used, 3 workers are employed in the first time period and all of them should work overtime for 7.3 hours. It also show that 2 more workers are added in the second time period and all 5 workers should work overtime for 6.0 hours. The optimal objective function value is 9460 dollars.

In all experiments in this section, at least one production line operates at full capacity. Double production lines are used only if the demand is equal to or larger than 11000 units. We can also see from the optimal solutions that the overtime hours in all three time periods are similar. In each time period, only one type of products are produce during overtime. Comparing with the results in Section 4.5.2 (solutions without fixed cost), the objective function value of this model is lower, because less operators are used. It is more economical to extend working time hours than use extra production line or hire new employees.

Also from Tables 4.30 and 4.32, it is obvious that hourly overtime wage affects workforce assignment. When overtime wage is higher, less overtime hours are allocated and vice versa. Whereas, if one production lines operates at its full capacity, such as the case when demand is 9000 units, the overtime and workforce assignment are almost the same for different overtime wages. Its optimal solutions are the same for without using the other production lines.

4.7 Double Production Lines Allowing Delivery Delay

In the example problems in this section, we assume that each operator is allowed to work for a maximum of 5 hours of overtime on both production lines. That is, $ut_{jt} \le 5$. We assume that overtime wage is $20/hours, 1.6 times of regular wage. Only one type of products will be produced in this example problem. We present the following results and compare the optimal solutions for the two cases that product delivery delay is allowed and that it is not allowed.

Similar to the examples in the previous section, we consider that the customer demand is from 10000 units to 20000 units. Table 4.33 presents some results of the optimal solution when delay is not allowed. We re-calculated the same set of problems allowing product delivery to be delayed for one time period with delay penalty cost being $2000/period. Table 4.34 presents the results of the optimal solutions when such delay is allowed.

Table 4.33 Optimal Solutions without Delay [Example 7]

Demand	Line	t=1				t=2				t=3				Optimal objective function value
		Station			Overtime	Station			Overtime	Station			Overtime	
		1	2	3		1	2	3		1	2	3		
10000	1	0	0	0	0.0	0	0	0	0.0	0	0	0	0.0	20000
	2	2	5	3	3.0	3	5	4	3.8	3	5	4	3.3	
12000	1	0	0	0	0.0	0	0	0	0.0	0	0	0	0.0	23560
	2	3	6	4	2.4	3	6	4	3.5	4	6	4	4.3	
14000	1	2	4	3	2	2	4	3	3.3	2	4	3	3.3	28080
	2	2	3	2	2.6	2	3	2	4.3	2	3	2	4.3	
16000	1	2	4	3	3.1	3	5	3	4.0	3	5	4	3.8	31800
	2	2	3	2	4.3	2	3	2	4.3	2	3	2	4.3	
18000	1	2	3	2	2.4	2	3	2	4.3	2	3	2	4.3	35320
	2	3	6	4	3.0	3	6	4	3.5	3	6	4	3.5	
20000	1	2	4	3	1.9	2	4	3	3.3	3	4	3	4.3	39380
	2	3	6	4	3.5	3	6	4	3.5	3	6	4	3.5	

Table 4.34 Optimal Solution with One Time Period Delay [Example 7]

Demand	Line	t=1				t=2				t=3				t=4				Optimal objective function value
		Station			Over time	Station			Over time	Station			Over time	Station			Over time	
		1	2	3		1	2	3		1	2	3		1	2	3		
10000	1	2	3	2	2.3	2	4	3	1.6	2	4	3	3.3	2	4	3	3.3	21340
	2	0	0	0	0.0	0	0	0	0.0	0	0	0	0.0	0	0	0	0.0	
12000	1	2	3	2	2.7	2	4	3	3.2	3	5	3	3.8	3	5	4	3.8	24980
	2	0	0	0	0.0	0	0	0	0.0	0	0	0	0.0	0	0	0	0.0	
14000	1	2	4	3	2.4	3	5	3	4.1	3	5	4	3.8	3	6	4	3.5	28420
	2	0	0	0	0.0	0	0	0	0.0	0	0	0	0.0	0	0	0	0.0	
16000	1	3	5	4	3.0	3	6	4	3.5	3	6	4	3.5	3	6	4	3.5	31700
	2	0	0	0	0.0	0	0	0	0.0	0	0	0	0.0	0	0	0	0.0	
18000	1	0	0	0	0.0	0	0	0	0.0	1	1	1	5.0	1	1	1	5.0	36020
	2	3	6	4	2.9	3	6	4	3.5	3	6	4	3.5	3	6	4	3.5	
20000	1	3	6	4	3.5	3	6	4	3.5	3	6	4	3.5	3	6	4	3.5	39580
	2	0	0	0	0.0	1	1	1	5.0	1	2	1	3.8	1	2	2	3.0	

As can be seen from Table 4.33, multiple production lines are utilized when customer demand is equal to or larger than 14000 units. From Table 4.34, when one time period delay is allowed, the production lines are less utilized when customer demand is 14000 units or 16000 units. On the other hand, we can see that overtime is still used even when delay is allowed because overtime cost is less than delay penalty cost.

To make a better comparison of optimal objective function values between Table 4.33 and Table 4.34, Figure 4.5 is presented as follows.

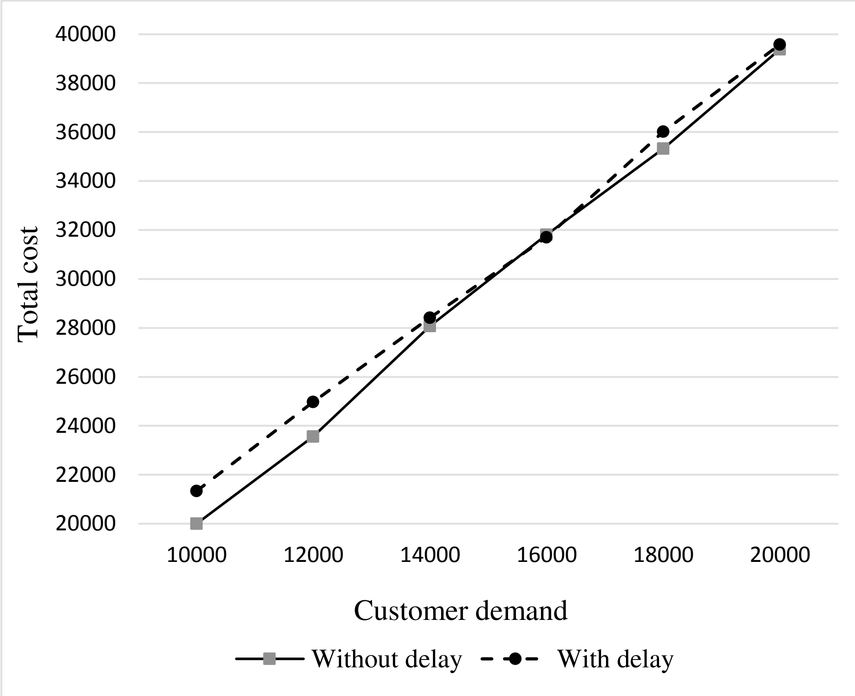

Figure 4.5 Total cost comparison when delay is allowed or not allowed

We can see from Figure 4.5, in most cases, the objective function values without delay penalty are smaller except that when demand is 16000 units. Further study on delay penalty is conducted and we have identified a critical value of the delay penalty cost as shown in Table 4.35. In each case, if the delay penalty cost is lower than the critical value shown in this table, the total cost of production planning with one time period delay is lower. For instance, as the customer demand is 10000 units, if the delay penalty cost is smaller than \$660/period, we may choose the planning with one time period delay in product delivery.

Table 4.35 Critical Value of Delay Penalty [Example 7]

Demand	10000	12000	14000	16000	18000	20000
Delay penalty critical value	660	580	1660	2100	1300	1800

4.8 Summary

Seven numerical example problems based on a real production system are presented in this chapter. The observations from the results are reasonable from practical point of view. By comparing different production plans and cases, some useful insights on the production system can be obtained from the optimal solutions of these example problems.

Chapter 5

Conclusion and Future Research

In this thesis, we introduced a mixed integer programming model for production planning in a labor intensive production system in a small confectionary factory. In studying the system and developing the mathematical model, we considered several practical issues such as different levels of productivities and product quality issues related to the level of operator skills. We also considered different options to handle the variations of customer demand which is often seasonal and uncertain. These options include using additional production lines, allowing overtime working hours and delaying product delivery with penalty costs incurred. The developed model can be revised for solving production planning problems of similar labor intensive production systems.

We further presented several revised models or sub-models based on the developed main model. They may have slightly different or revised constraint functions to reflect various practical considerations. Some of the sub-models can be solved to optimality with minimum computing effort for small size problems. Using these sub-models, we solved a number of numerical example problems with different cases. Some of the example problems were solved to find the optimal workforce assignment and working time arrangement. Other example problem solutions include multi production lines, overtime working and delay penalty. These examples are more representative for production planning in high demand seasons.

The original model has several nonlinear functions and containing certain specific nonlinear terms. One of the nonlinear functions can be replaced by a piece-wise linear function.

Most of the remaining non-linear terms can be linearized by adding additional variables and large number of linear constraint functions. In this research, we used revised and simplified sub-models to find optimal or near optimal solutions of some of the simpler problems obtained from our on-site study in the factory.

In summary, the main contributions made in this thesis research can be outlined below.

- A mixed integer mathematical programming model was developed for labor intensive production planning with several practical features which have not been considered by other researchers
- The model was solved for several practical cases and the solutions are satisfactory and can be adjusted for practical implementation.
- The considered practical problems can be solved without extensive computing effort.

In the future research in this direction, the further development of the main model should be conducted for efficient solution methods so that the entire model can be solved. In addition, modeling and solving the production planning problem considering the interactions between manual and automated production lines should be investigated. In addition, the feasibility, effectiveness and efficiencies of using such an integrated model to solving larger size problems should be studied. Finally, efficient heuristic solution methods should be developed for solving larger size problems.

References

1. Alchian, A. (1963), "Reliability of progress curves in airframe production", *Econometrica*, Vol. 31, pp. 679-693
2. Baker, K. R. and Trietsch, D. (2009), Principles of sequencing and scheduling, John Wiley & Sons
3. Cao, D., Chen, M. Y., and Wan, G. H (2005), "Parallel machine selection and job scheduling to minimize machine cost and job tardiness", *Computers & Operations Research*, Vol. 32, pp. 1995–2012
4. Coleman, B. J. (1992), "A simple model for optimizing the single machine early/tardy problem with sequence-dependent setups", *Production and Operations Management*, Vol. 1, pp. 225-228
5. Corominas, A., Olivella, J., and Pastor, R. (2010), "A model for the assignment of a set of tasks when work performance depends on experience of all tasks involved", *Int. J. Production Economics*, Vol. 126, pp. 335-340
6. Corominas, A., Pastor, R., and Rodriguez, E. (2006), "Rotational allocation of tasks to multifunctional workers in a service industry", *Int. J. Production Economics*, Vol. 103, pp. 3-9
7. Dar-el, E. M., Ayas, K., and Gilad, I. (1995), "A dual-phase model for the individual learning process in industrial tasks", *IIE Transactions*, Vol. 27, pp. 265-271
8. Davis, J. S. and Kanet, J. J. (1993), "Single-machine scheduling with early and tardy completion costs", *Naval Research Logistics*, Vol. 40, pp. 85-101
9. Ebbinghaus, H., Ruger, H. A., and Bussenius, C. E. (1913), Memory: A contribution to

experimental psychology, Teachers College Press

10. ElMaraghy, H. A. and Manns, M. (2007), "Transition of interarrival time patterns between automated and manual configurations of assembly systems", *Journal of Manufacturing Systems*, Vol. 26, pp. 1-11
11. Francas, D., Löhndorf, N., and Minner, S. (2011), "Machine and labor flexibility in manufacturing networks", *Int. J. Production Economics*, Vol. 131, pp. 165-174
12. Gulledge, T. R., Jr. and Khoshnevis, B. (1987), "Production rate, learning, and program costs: survey and bibliography", *Engineering Costs and Production Economics*, Vol. 11, pp. 223-236
13. Hancock, W. M. (1967), "The prediction of learning rates for manual operations", *The Journal of Industrial Engineering*, Vol. 18, pp. 42-47
14. Ighravwe, D. E. and Oke, S. A. (2014), "A non-zero integer non-linear programming model for maintenance workforce sizing", *Int. J. Production Economics*, Vol. 150, pp. 204-214
15. Jaber, M. Y., and Kher, H.V. (2002), "The dual-phase learning-forgetting model", *Int. J. Production Economics*, Vol. 76, pp. 229-242
16. Jaber, M. Y., and Bonney, M. (1996), "Production breaks and the learning curve: The forgetting phenomenon", *Appl. Math. Modelling*, Vol. 20, pp. 162-169
17. Jennings, M. G. and Shah, N. (2014), "Workforce planning and technology installation optimization for utilities", *Computers & Industrial Engineering*, Vol. 67, pp. 72-81,
18. Karuppan, C. M. (2008), "Labor flexibility: rethinking development", *International Journal of Business Strategy*, Vol. 8, pp. 108-113

19. Karuppan, C. M. and Ganster, D. C. (2004), "The labor-machine dyad and its Influence on mix flexibility", *Journal of Operations Management*, Vol. 22, pp. 533-556,

20. Khan, A. and Day, A. J. (2002), "A knowledge based design methodology for manufacturing assembly lines", *Computer & Industrial Engineering*, Vol. 41, pp. 441-467,

21. Khoshjahan, Y., Najafi, A. A., and Afshar-Nadjafi, B. (2013), "Resource constrained project scheduling problem with discounted earliness–tardiness penalties: mathematical modeling and solving procedure", *Computers & Industrial Engineering*, Vol. 66, pp. 293-300

22. Ko, H. H., Kim, J., Kim, S. S., and Baek, J. G. (2010), "Dispatching rule for non-identical parallel machines with sequence-dependent setups and quality restrictions", *Computers & Industrial Engineering*, Vol. 59, pp. 448-457,

23. McDonald, A. and Schrattenholzer, L. (2001), "Learning rates for energy technologies", *Energy Policy*, Vol. 29, pp. 255-261

24. Mize, J. H., Case, K. E., Turner, W. C., and Nazemtz, J. W. (1992), Introduction to Industrial and Systems Engineering, Prentice Hall

25. Montgomery, D. C. (2007), Introduction to Statistical Quality Control, John Wiley & Sons

26. Moore, T. D., Johnson, A. W., Rehg, M. T., and Hicks, M. J. (2007), "Quality assurance staffing impacts in military aircraft maintenance units", *Journal of Quality in Maintenance Engineering*, Vol. 13, pp. 33-48

27. Nembhard, D. A. (2001), "Heuristic approach for assigning workers to task based on

individual learning rates", *International Journal of Production Research*, Vol. 39, pp. 1968-1995

28. Nembhard, D. A. and Bentefouet, F. (2012), "Parallel system scheduling with general worker learning and forgetting", *Int. J. Production Economics*, Vol. 139, pp. 533–542
29. Smunt, T.L. and Morton, T.L. (1985), "The effects of learning on optimal lot sizes: Further developments on the single product case", *IIE Transaction*, Vol. 17, pp. 33-37
30. Smunt, T.L. (1999), "Log-linear and non-log-linear learning curve models for production research and cost estimation", *International Journal of Production Research*, Vol. 37, pp. 3901-3911
31. Song, H. Q. and Huang, H. C. (2008), "A successive convex approximation method for multistage workforce capacity planning problem with turnover", *European Journal of Operational Research*, Vol. 188, pp. 29-48
32. Stewart, B. D., Webster, D. B., Ahmad, S., and Matson, J. O. (1994), "Mathematical models for developing a flexible workforce", *Int. J. Production Economics*, Vol. 36, pp. 243-254
33. Stratman, J. K., Roth, A. V., and Gilland, W. G. (2004), "The deployment of temporary production workers in assembly operations: A case study of the hidden costs of learning and forgetting", *Journal of Operations Management*, Vol. 21, pp. 689–707
34. Subramaniam, V., Lee, G. K., Ramesh, T., Hong, G. S., and Wong, Y. S. (2000), "Machine selection rules in a dynamic job shop", *The International Journal of Advanced Manufacturing Technology*, Vol. 16, pp. 902–908
35. Süer, G. A. and Dagli, C. (2005), "Intra-cell manpower transfers and cell loading in

labor-intensive manufacturing cells", *Computers & Industrial Engineering*, Vol. 48, pp. 643-655,

36. Sule, D. R. (1994), Manufacturing facilities, location, planning, and design, PWS Publishing Company
37. Wright, T. P. (1936), "Factors Affecting the Cost of Airplanes", *Journal of the Aeronautical Sciences*, Vol. 3, pp. 122-128
38. Yelle, L. E. (1979), "The learning curve: Historical review and comprehensive survey", *Decision Sciences*, Vol. 10, pp.302-328
39. Yu, X. Y., Zhang, Y. L. and Huang, K. (2014), "Multi-machine scheduling with general position-based deterioration to minimize total load revisited", *Information Processing Letters*, Vol. 144, pp. 399–404
40. Zhu, Z. W. and Heady, R. B. (2000), "Minimizing the sum of earliness/tardiness in multi-machine scheduling: A mixed integer programming approach", *Computers & Industrials Engineering*, Vol. 38, pp. 297–305

Appendix A

Output of Learning Phase in the First Sub-model

Working Hours in Learning Phase	Original Output Curve		Piecewise Linear Curve	
	w=1 (y_0=0.123)	w=2 (y_0=0.200)	w=1	w=2
1	9.9772	5.4991	21	10
2	22.9216	12.4596	42	20
3	37.2868	20.1045	63	30
4	52.6602	28.2307	84	40
5	68.8295	36.7346	105	50
6	85.6627	45.5522	126	60
7	103.069	54.6395	147	70
8	120.9814	63.9643	168	80
9	139.3483	73.5017	189	90
10	158.1287	83.2322	210	100
11	177.2891	93.1397	231	110
12	196.8014	103.2109	252	120
13	216.6420	113.4344	273	130
14	236.7905	123.8006	294	140
15	257.2292	134.3010	315	150
16	277.9423	144.9283	336	160
17	298.9162	155.6759	357	170
18	320.1383	166.5380	378	180
19	341.5978	177.5092	399	190
20	363.2844	188.5850	420	200
21	385.1890	199.7609	441	210
22	407.3033	211.0330	462	220
23	429.6196	222.3978	483	230
24	452.1309	233.8519	504	240
25	474.8306	245.3923	525	250
26	497.7127	257.0161	546	260
27	520.7715	268.7206	567	270
28	544.0018	280.5034	588	280
29	567.3986	292.3623	609	290
30	590.9574	304.2950	630	300
31	614.6738	316.2995	651	310
32	638.5438	328.3739	672	320
33	662.5634	340.5165	693	330
34	686.7290	352.7255	714	340
35	711.0372	364.9993	735	350
36	735.4848	377.3364	756	360
37	760.0686	389.7353	777	370
38	784.7856	402.1947	798	380
39	809.6330	414.7133	819	390
40	834.6083	427.2898	840	400

Appendix B

Output of Learning Phase in the Second Sub-model

Working Time in Learning Phase	Original Output Curve		Piecewise Linear Curve	
	w=1 (y_0=0.140)	w=2 (y_0=0.195)	w=1	w=2
1	8.2922	5.6658	17	11
2	19.0506	12.8375	34	22
3	30.9898	20.7142	51	33
4	43.7669	29.0868	68	44
5	57.2056	37.8486	85	55
6	71.1960	46.9336	102	66
7	85.6627	56.2965	119	77
8	100.5501	65.9041	136	88
9	115.8152	75.7307	153	99
10	131.4240	85.7563	170	110
11	147.3485	95.9642	187	121
12	163.5656	106.3409	204	132
13	180.0556	116.8744	221	143
14	196.8014	127.5550	238	154
15	213.7884	138.3738	255	165
16	231.0035	149.3234	272	176
17	248.4353	160.3969	289	187
18	266.0735	171.5884	306	198
19	283.9088	182.8923	323	209
20	301.9330	194.3040	340	220
21	320.1383	205.8188	357	231
22	338.5180	217.4328	374	242
23	357.0655	229.1422	391	253
24	375.7751	240.9437	408	264
25	394.6413	252.8340	425	275
26	413.6591	264.8103	442	286
27	432.8237	276.8697	459	297
28	452.1309	289.0099	476	308
29	471.5765	301.2284	493	319
30	491.1567	313.5230	510	330
31	510.8679	325.8915	527	341
32	530.7066	338.3321	544	352
33	550.6698	350.8429	561	363
34	570.7544	363.4221	578	374
35	590.9574	376.0682	595	385
36	611.2763	388.7794	612	396
37	631.7083	401.5543	629	407
38	652.2512	414.3916	646	418
39	672.9024	427.2898	663	429
40	693.6598	440.2477	680	440

Appendix C

Output in Learning Phase

Working Time	Productivity of First Type of Workers			Productivity of Second Type of Workers		
	Learning Phase	Stable	Ratio	Learning Process	Stable	Ratio
1	8.2922	20.03	41%	5.6658	13.34	41%
2	19.0506	40.06	48%	12.8375	26.68	48%
3	30.9898	60.09	56%	20.7142	40.02	52%
4	43.7669	80.12	60%	29.0868	53.36	55%
5	57.2056	100.15	62%	37.8486	66.7	57%
6	71.1960	120.18	65%	46.9336	80.04	59%
7	85.6627	140.21	67%	56.2965	93.38	60%
8	100.5501	160.24	68%	65.9041	106.72	62%
9	115.8152	180.27	70%	75.7307	120.06	63%
10	131.4240	200.30	72%	85.7563	133.4	64%
11	147.3485	220.33	73%	95.9642	146.74	65%
12	163.5656	240.36	74%	106.3409	160.08	66%
13	180.0556	260.39	75%	116.8744	173.42	67%
14	196.8014	280.42	76%	127.5550	186.76	68%
15	213.7884	300.45	78%	138.3738	200.1	69%
16	231.0035	320.48	79%	149.3234	213.44	70%
17	248.4353	340.51	80%	160.3969	226.78	71%
18	266.0735	360.54	80%	171.5884	240.12	71%
19	283.9088	380.57	81%	182.8923	253.46	72%
20	301.9330	400.60	82%	194.3040	266.8	73%
21	320.1383	420.63	83%	205.8188	280.14	73%
22	338.5180	440.66	84%	217.4328	293.48	74%
23	357.0655	460.69	84%	229.1422	306.82	75%
24	375.7751	480.72	85%	240.9437	320.16	75%
25	394.6413	500.75	86%	252.8340	333.5	76%
26	413.6591	520.78	87%	264.8103	346.84	76%
27	432.8237	540.81	87%	276.8697	360.18	77%
28	452.1309	560.84	88%	289.0099	373.52	77%
29	471.5765	580.87	88%	301.2284	386.86	78%
30	491.1567	600.90	89%	313.5230	400.2	78%
31	510.8679	620.93	90%	325.8915	413.54	79%
32	530.7066	640.96	90%	338.3321	426.88	79%
33	550.6698	660.99	91%	350.8429	440.22	80%
34	570.7544	681.02	91%	363.4221	453.56	80%
35	590.9574	701.05	92%	376.0682	466.9	81%
36	611.2763	721.08	92%	388.7794	480.24	81%
37	631.7083	741.11	93%	401.5543	493.58	81%
38	652.2512	761.14	93%	414.3916	506.92	82%
39	672.9024	781.17	94%	427.2898	520.26	82%
40	693.6598	801.20	94%	440.2477	533.6	83%

Appendix D

Frist Sub-model in CPLEX

```
/*define*/
 int w=2;
 int s=2;
 int t=3;
 range W=1..w;
 range W2=2..w;
 range S=1..s;
 range T=1..t;
 range T2=2..t;

/*parameters*/
 int alpha[W][T] = ...;
 int beta[W] = ...;
 int gamma[W] = ...;
 int delta[W] = ...;
 int eta[S] = ...;
 float y[W][S]= ...;
 int WT[S][T]=...;

/*variables*/
 dvar int+ X[W][T];
 dvar int+ XT[W][T];
 dvar int+ XH[W][T];
 dvar int+ XF[W][T];
 dvar boolean m[W][T];
 dvar int+ Z1[W][T];
 dvar int Z2[W][T];
 dvar int M[W][T];
 dvar boolean a[W][T];
 dvar int+ P[W][S][T];
 dvar int+ D[S][T];


/*objective*/
 minimize
           sum (w in W, t in T) X[w][t]*alpha[w][t]
         + sum (w in W, t in T) Z1[w][t]*beta[w]
         + sum (w in W, t in T) XT[w][t]*gamma[w]
         - sum (w in W, t in T) Z2[w][t]*delta[w];

/*constraints*/
```

```
subject to
{
 set1:
   forall (w in W, t in T)
     M[w][t] == XH[w][t]-XF[w][t];
 set2:
   forall (w in W, t in T)
     M[w][t]/1000 <= a[w][t];
   forall (w in W, t in T)
     (M[w][t]/1000)+1 >= a[w][t];
 set3:
   forall (w in W, t in T)
     M[w][t]-15*(1-a[w][t]) <= Z1[w][t];
   forall (w in W, t in T)
     Z1[w][t] <= M[w][t]+15*(1-a[w][t]);
 set4:
   forall (w in W, t in T)
     0 <= Z1[w][t];
   forall (w in W, t in T)
     Z1[w][t] <= 15*a[w][t];
 set5:
   forall (w in W, t in T)
     M[w][t]-15*a[w][t] <= Z2[w][t];
   forall (w in W, t in T)
     Z2[w][t] <= M[w][t]+15*a[w][t];
 set6:
   forall (w in W, t in T)
     15*a[w][t]-15 <= Z2[w][t];
   forall (w in W, t in T)
     Z2[w][t] <= 0;
 set7:
   forall (w in W)
     X[w][1] == XH[w][1];
 set8:
   forall (w in W)
     XF[w][1] == 0;
 set9:
   forall (w in W,  t in T)
     XH[w][t] <= 5* m[w][t];
 set10:
   forall (w in W, t in T)
     XF[w][t] <= 5* (1-m[w][t]);
 set11:
   forall (w in W, t in T2)
     X[w][t] == X[w][t-1]+XH[w][t]-XF[w][t];
```

```
  set12:
    forall (w in W2, t in T)
      XT[w][t] == XH[w][t];
  set13:
    forall (t in T)
      XT[1][t] == 0;
  set14:
    forall (t in T)
      1 <= sum(w in W) X[w][t];
  set15:
    forall (t in T)
      sum(w in W) X[w][t] <= 5;
  set16:
    forall (w in W, t in T)
      0 <= XF[w][t];
  set17:
    forall (w in W, t in T)
      XF[w][t] <= X[w][t];
  set18:
    forall (w in W, t in T)
      0<= XH[w][t];
  set19:
    forall (s in S, t in T)
      P[1][s][t] == 22*WT[s][t];
  set20:
    forall (w in W2, s in S, t in T)
      P[2][s][t] == 10*WT[s][t];
  set21:
    forall (s in S)
      D[s][1] <= sum(w in W)XH[w][1]*P[w][s][1];
  set22:
    forall (s in S, t in T2)
      D[s][t] <= sum(w in W)XH[w][t]*P[w][s][t]+sum(w in W)(X[w][t-1]
                 XF[w][t])*WT[s][t]/y[w][s];
  Set23:
    forall (s in S)
      sum(t in T) D[s][t] >= eta[s];
}

/***************************************************************************/
```

DATA:

```
SheetConnection sheet("mastermodel.xlsx");

alpha from SheetRead(sheet,"Sheet1!B2:D3");
```

```
beta from SheetRead(sheet,"Sheet1!B6:B7");
gamma from SheetRead(sheet,"Sheet1!B10:B11");
delta from SheetRead(sheet,"Sheet1!B14:B15");
eta from SheetRead(sheet,"Sheet1!B18:B19");
y from SheetRead(sheet,"Sheet1!H2:I3");
WT from SheetRead(sheet,"Sheet1!H6:J7");
```

Appendix E

Second Sub-model in CPLEX

```
/*define*/
 int w=2;
 int s=2;
 int i=2;
 int t=3;
 range W=1..w;
 range W2=2..w;
 range S=1..s;
 range I=1..i;
 range T=1..t;
 range T2=2..t;

/*parameters*/
 int alpha[W][T] = ...;
 int beta[W] = ...;
 int gamma[W] = ...;
 int delta[W] = ...;
 int eta[S] = ...;

{int} worker =...;
{int} order =...;
{int} station =...;
{int} time =...;

/*variables*/
 dvar int+ X[W][I][T];
 dvar int+ XT[W][I][T];
 dvar int+ XH[W][I][T];
 dvar int+ XF[W][I][T];
 dvar boolean m[W][I][T];
 dvar int+ Z1[W][T];
 dvar int Z2[W][T];
 dvar int M[W][T];
 dvar boolean a[W][T];
 dvar int+ P[W][S][I][T];
 dvar int+ D[S][T];

float ywsi[worker][order][station];
tuple ywsiStruct { int worker; int order;int station; float ywsi;};
{ywsiStruct} ywsiData = ...;
```

```
execute
{
   for (var y in ywsiData)
      {
         ywsi [y.worker][y.order][y.station] = y.ywsi;
      }
}

int WTsit[order][station][time];
tuple WTsitStruct { int order; int station;int time; float WTsit;};
{WTsitStruct} WTsitData = ...;

execute
{
   for (var x in WTsitData)
      {
         WTsit[x.order][x.station][x.time] = x.WTsit;
      }
}

/*objective*/
 minimize
            sum (w in W, i in I, t in T) X[w][i][t]*alpha[w][t]
          + sum (w in W, t in T) Z1[w][t]*beta[w]
          + sum (w in W, i in I, t in T) XT[w][i][t]*gamma[w]
          - sum (w in W, t in T) Z2[w][t]*delta[w];

/*constraints*/
 subject to
 {
  set1:
    forall (w in W, t in T)
      M[w][t] == sum (i in I) (XH[w][i][t]-XF[w][i][t]);
  set2:
    forall (w in W, t in T)
      M[w][t]/1000 <= a[w][t];
  set3:
    forall (w in W, t in T)
      (M[w][t]/1000)+1 >= a[w][t];
  set4:
    forall (w in W, t in T)
      M[w][t]-15*(1-a[w][t]) <= Z1[w][t];
  set5:
    forall (w in W, t in T)
      Z1[w][t] <= M[w][t]+15*(1-a[w][t]);
```

```
set6:
  forall (w in W, t in T)
    0 <= Z1[w][t];
set7:
  forall (w in W, t in T)
    Z1[w][t] <= 15*a[w][t];
set8:
  forall (w in W, t in T)
    M[w][t]-15*a[w][t] <= Z2[w][t];
set9:
  forall (w in W, t in T)
    Z2[w][t] <= M[w][t]+15*a[w][t];
set10:
  forall (w in W, t in T)
    15*a[w][t]-15 <= Z2[w][t];
set11:
  forall (w in W, t in T)
    Z2[w][t] <= 0;
set12:
  forall (w in W, i in I)
    X[w][i][1] == XH[w][i][1];
set13:
  forall (w in W, i in I)
    XF[w][i][1] == 0;
set14:
  forall (w in W, i in I, t in T)
    XH[w][i][t] <= 10* m[w][i][t];
set15:
  forall (w in W, i in I, t in T)
    XF[w][i][t] <= 10* (1-m[w][i][t]);
set16:
  forall (w in W, i in I, t in T2)
    X[w][i][t] == X[w][i][t-1]+XH[w][i][t]-XF[w][i][t];
set17:
  forall (w in W2, i in I, t in T)
    XT[w][i][t] == XH[w][i][t];
set18:
  forall (i in I, t in T)
    XT[1][i][t] == 0;
Set19:
  forall (i in I, t in T)
    1 <= sum(w in W) X[w][i][t];
set20:
  forall (t in T)
    sum(w in W, i in I) X[w][i][t] <= 10;
```

```
  set21:
    forall (w in W, i in I, t in T)
      0 <= XF[w][i][t];
  set22:
    forall (w in W, i in I, t in T)
      XF[w][i][t] <= X[w][i][t];
  set23:
    forall (w in W, i in I, t in T)
      XH[w][i][t] >=0;
  Set24:
    forall (s in S,  i in I, t in T)
      P[1][s][i][t] == 17*WTsit[s][i][t];
  Set25:
    forall (s in S,  i in I, t in T)
      P[2][s][i][t] == 10*WTsit[s][i][t];
  set26:
    forall (s in S, i in I)
      D[s][1] <= sum(w in W)XH[w][i][1]*P[w][s][i][1];
  set27:
    forall (s in S, i in I, t in T2)
      D[s][t] <= sum(w in W)XH[w][i][t]*P[w][s][i][t]+sum(w in W)(X[w][i][t-
                 1]-XF[w][i][t])*WTsit[s][i][t]/ywsi[w][s][i];
  set28:
    forall (s in S)
      sum(t in T) D[s][t] >= eta[s];
}

/**********************************************************************/
```

DATA:

```
SheetConnection sheet("mastermodel.xlsx");

alpha from SheetRead(sheet,"Sheet1!B2:D3");
beta from SheetRead(sheet,"Sheet1!B6:B7");
gamma from SheetRead(sheet,"Sheet1!B10:B11");
delta from SheetRead(sheet,"Sheet1!B14:B15");
eta from SheetRead(sheet,"Sheet1!B22:B23");

worker from SheetRead(sheet,"Sheet1!F27:F28");
order from SheetRead(sheet,"Sheet1!F27:F28");
station from SheetRead(sheet,"Sheet1!F27:F28");
time from SheetRead(sheet,"Sheet1!F27:F29");
ywsiData from SheetRead(sheet,"Sheet1!A27:D34");
WTsitData from SheetRead(sheet,"Sheet1!H27:K38";
```

Appendix F

Third Sub-model in CPLEX

```
/*define*/
 int s=2;
 int i=3;
 int w=2;
 int t=3;

 range S=1..s;
 range I=1..i;
 range T=1..t;
 range T2=2..t;
 range W=1..w;
 range W2=2..w;

/*parameters*/
 int alpha[W][T] = ...;
 int beta[W] = ...;
 int gamma[W] = ...;
 int delta[W] = ...;
 int eta[S] = ...;

{int} worker =...;
{int} order =...;
{int} station =...;

/*variables*/
 dvar int+ X[W][I][T];
 dvar int+ XT[W][I][T];
 dvar int+ XH[W][I][T];
 dvar int+ XF[W][I][T];
 dvar boolean m[W][I][T];
 dvar int+ Z1[W][T];
 dvar int Z2[W][T];
 dvar int M[W][T];
 dvar boolean a[W][T];
 dvar int+ D[W][S][T];

float ywsi[worker][order][station];
tuple ywsiStruct { int worker; int order;int station; float ywsi;};
{ywsiStruct} ywsiData = ...;

execute
```

```
{
   for (var y in ywsiData)
      {
         ywsi [y.worker][y.order][y.station] = y.ywsi;
      }
}

/*objective*/
 minimize
           sum (w in W, i in I, t in T) X[w][i][t]*alpha[w][t]
         + sum (w in W, t in T) Z1[w][t]*beta[w]
         + sum (w in W, i in I, t in T) XT[w][i][t]*gamma[w]
         - sum (w in W, t in T) Z2[w][t]*delta[w];

/*constraints*/
 subject to
 {
  set1:
    forall (w in W, t in T)
      M[w][t] == sum (i in I) (XH[w][i][t]-XF[w][i][t]);
  set2:
    forall (w in W, t in T)
      M[w][t]/1000 <= a[w][t];
  set3:
    forall (w in W, t in T)
      (M[w][t]/1000)+1 >= a[w][t];
  set4:
    forall (w in W, t in T)
      M[w][t]-15*(1-a[w][t]) <= Z1[w][t];
  set5:
    forall (w in W, t in T)
      Z1[w][t] <= M[w][t]+15*(1-a[w][t]);
  set6:
    forall (w in W, t in T)
      0 <= Z1[w][t];
  set7:
    forall (w in W, t in T)
      Z1[w][t] <= 15*a[w][t];
  set8:
    forall (w in W, t in T)
      M[w][t]-15*a[w][t] <= Z2[w][t];
  set9:
    forall (w in W, t in T)
      Z2[w][t] <= M[w][t]+15*a[w][t];
  set10:
```

```
  forall (w in W, t in T)
    15*a[w][t]-15 <= Z2[w][t];
set11:
  forall (w in W, t in T)
    Z2[w][t] <= 0;
set12:
  forall (w in W, i in I)
    X[w][i][1] == XH[w][i][1];
set13:
  forall (w in W, i in I)
    XF[w][i][1] == 0;
set14:
  forall (w in W, i in I, t in T)
    XH[w][i][t] <= 15* m[w][i][t];
set15:
  forall (w in W, i in I, t in T)
    XF[w][i][t] <= 15* (1-m[w][i][t]);
set16:
  forall (w in W, i in I, t in T2)
    X[w][i][t] == X[w][i][t-1]+XH[w][i][t]-XF[w][i][t];
set17:
  forall (w in W2, i in I, t in T)
    XT[w][i][t] == XH[w][i][t];
set18:
  forall (i in I, t in T)
    XT[1][i][t] == 0;
Set19:
  forall (t in T)
    sum(w in W, i in I) X[w][i][t] <= 15;
set20:
  forall (w in W, i in I, t in T)
    0 <= XF[w][i][t];
set21:
  forall (w in W, i in I, t in T)
    XF[w][i][t] <= X[w][i][t];
set22:
  forall (w in W, i in I, t in T)
    XH[w][i][t] >=0;
set23a:
  forall (w in W, i in I)
    sum(s in S)D[w][s][1]*ywsi[w][s][i] <= 16*XH[w][i][1];
set23b:
  forall (i in I)
    sum(s in S)D[1][s][1]*ywsi[1][s][i] <= 38*XH[1][i][1];
  forall (i in I)
```

```
      sum(s in S)D[2][s][1]*ywsi[2][s][i] <= 32*XH[2][i][1];
  set24a:
    forall (w in W, i in I, t in T2)
      sum(s in S)D[w][s][t]*ywsi[w][s][i] <= 40*(X[w][i][t-1]-XF[w][i][t])+
                                              16*XH[w][i][t];
  set24b:
    forall (w in W, i in I, t in T2)
      sum(s in S)D[1][s][t]*ywsi[1][s][i] <= 40*(X[1][i][t-1]-XF[1][i][t])+
                                              38*XH[1][i][t];
    forall (w in W, i in I, t in T2)
      sum(s in S)D[2][s][t]*ywsi[2][s][i] <= 40*(X[2][i][t-1]-XF[2][i][t])+
                                              32*XH[2][i][t];
  set25:
    forall (s in S)
      sum(w in W, t in T) D[w][s][t] >= eta[s];
}

/*************************************************************************/
```

DATA:

```
SheetConnection sheet("single line without learning curve.xlsx");

alpha from SheetRead(sheet,"Sheet1!B2:D3");
beta from SheetRead(sheet,"Sheet1!B6:B7");
gamma from SheetRead(sheet,"Sheet1!B10:B11");
delta from SheetRead(sheet,"Sheet1!B14:B15");
eta from SheetRead(sheet,"Sheet1!B22:B23");

worker from SheetRead(sheet,"Sheet1!F27:F28");
order from SheetRead(sheet,"Sheet1!F27:F28");
station from SheetRead(sheet,"Sheet1!F27:F29");
ywsiData from SheetRead(sheet,"Sheet1!A27:D38");
```

Appendix G

Fourth Sub-model in CPLEX

```
/*define*/
 int s=2;
 int i=3;
 int w=2;
 int t=3;

 range S=1..s;
 range I=1..i;
 range T=1..t;
 range T2=2..t;
 range W=1..w;
 range W2=2..w;

/*parameters*/
 int alpha[W][T] = ...;
 int beta[W] = ...;
 int gamma[W] = ...;
 int delta[W] = ...;
 int eta[S] = ...;

{int} worker =...;
{int} order =...;
{int} station =...;

/*variables*/
 dvar int+ X[W][I][T];
 dvar int+ XT[W][I][T];
 dvar int+ XH[W][I][T];
 dvar int+ XF[W][I][T];
 dvar boolean m[W][I][T];
 dvar int+ Z1[W][T];
 dvar int Z2[W][T];
 dvar int M[W][T];
 dvar boolean a[W][T];
 dvar int+ D[W][S][T];

float ywsi[worker][order][station];
tuple ywsiStruct { int worker; int order;int station; float ywsi;};
{ywsiStruct} ywsiData = ...;

execute
```

```
{
   for (var y in ywsiData)
      {
         ywsi [y.worker][y.order][y.station] = y.ywsi;
      }
}

/*objective*/
 minimize
           sum (w in W, i in I, t in T) X[w][i][t]*alpha[w][t]
         + sum (w in W, t in T) Z1[w][t]*beta[w]
         + sum (w in W, i in I, t in T) XT[w][i][t]*gamma[w]
         - sum (w in W, t in T) Z2[w][t]*delta[w];

/*constraints*/
 subject to
 {
  set1:
    forall (w in W, t in T)
      M[w][t] == sum (i in I) (XH[w][i][t]-XF[w][i][t]);
  set2:
    forall (w in W, t in T)
      M[w][t]/1000 <= a[w][t];
  set3:
    forall (w in W, t in T)
      (M[w][t]/1000)+1 >= a[w][t];
  set4:
    forall (w in W, t in T)
      M[w][t]-15*(1-a[w][t]) <= Z1[w][t];
  set5:
    forall (w in W, t in T)
      Z1[w][t] <= M[w][t]+15*(1-a[w][t]);
  set6:
    forall (w in W, t in T)
      0 <= Z1[w][t];
  set7:
    forall (w in W, t in T)
      Z1[w][t] <= 15*a[w][t];
  set8:
    forall (w in W, t in T)
      M[w][t]-15*a[w][t] <= Z2[w][t];
  set9:
    forall (w in W, t in T)
      Z2[w][t] <= M[w][t]+15*a[w][t];
  set10:
```

```
  forall (w in W, t in T)
    15*a[w][t]-15 <= Z2[w][t];
set11:
  forall (w in W, t in T)
    Z2[w][t] <= 0;
set12:
  forall (w in W, i in I)
    X[w][i][1] == XH[w][i][1];
set13:
  forall (w in W, i in I)
    XF[w][i][1] == 0;
set14:
  forall (w in W, i in I, t in T)
    XH[w][i][t] <= 15* m[w][i][t];
set15:
  forall (w in W, i in I, t in T)
    XF[w][i][t] <= 15* (1-m[w][i][t]);
set16:
  forall (w in W, i in I, t in T2)
    X[w][i][t] == X[w][i][t-1]+XH[w][i][t]-XF[w][i][t];
set17:
  forall (w in W2, i in I, t in T)
    XT[w][i][t] == XH[w][i][t];
set18:
  forall (i in I, t in T)
    XT[1][i][t] == 0;
Set19:
  forall (t in T)
    sum(w in W, i in I) X[w][i][t] <= 15;
set20:
  forall (w in W, i in I, t in T)
    0 <= XF[w][i][t];
set21:
  forall (w in W, i in I, t in T)
    XF[w][i][t] <= X[w][i][t];
set22:
  forall (w in W, i in I, t in T)
    XH[w][i][t] >=0;
set23a:
  forall (w in W, i in I)
    sum(s in S)D[w][s][1]*ywsi[w][s][i]/qr[s][w] <= 16*XH[w][i][1];
set23b:
  forall (i in I)
    sum(s in S)D[1][s][1]*ywsi[1][s][i]/qr[s][w] <= 38*XH[1][i][1];
  forall (i in I)
```

```
      sum(s in S)D[2][s][1]*ywsi[2][s][i]/qr[s][w] <= 32*XH[2][i][1];
  set24a:
    forall (w in W, i in I, t in T2)
      sum(s in S)D[w][s][t]*ywsi[w][s][i]/qr[s][w] <= 40*(X[w][i][t-1]-
                 XF[w][i][t])+16*XH[w][i][t];
  set24b:
    forall (w in W, i in I, t in T2)
      sum(s in S)D[1][s][t]*ywsi[1][s][i]/qr[s][w] <= 40*(X[1][i][t-1]-
                 XF[1][i][t])+ 38*XH[1][i][t];
    forall (w in W, i in I, t in T2)
      sum(s in S)D[2][s][t]*ywsi[2][s][i]/qr[s][w] <= 40*(X[2][i][t-1]-
                 XF[2][i][t])+ 32*XH[2][i][t];
  set25:
    forall (s in S)
      sum(w in W, t in T) D[w][s][t] >= eta[s];
}

/**************************************************************************/
```

DATA:

```
SheetConnection sheet("single line without learning curve.xlsx");

alpha from SheetRead(sheet,"Sheet1!B2:D3");
beta from SheetRead(sheet,"Sheet1!B6:B7");
gamma from SheetRead(sheet,"Sheet1!B10:B11");
delta from SheetRead(sheet,"Sheet1!B14:B15");
eta from SheetRead(sheet,"Sheet1!B22:B23");
qr from SheetRead(sheet,"Sheet1!B18:C19");

worker from SheetRead(sheet,"Sheet1!F27:F28");
order from SheetRead(sheet,"Sheet1!F27:F28");
station from SheetRead(sheet,"Sheet1!F27:F29");
ywsiData from SheetRead(sheet,"Sheet1!A27:D38");
```

Appendix H

Fifth Sub-model in CPLEX

```
/*define*/
 int s=2;
 int i=3;
 int j=2;
 int t=3;

 range S=1..s;
 range I=1..i;
 range T=1..t;
 range T2=2..t;
 range J=1..j;

/*parameters*/
 int alpha = ...;
 int beta = ...;
 int gamma = ...;
 int delta = ...;
 int eta[S] = ...;
 float y[S][I] = ...;
 int epsilon = ...;

/*variables*/
 dvar int+ X[I][J][T];
 dvar int+ XT[I][J][T];
 dvar int+ XH[I][J][T];
 dvar int+ XF[I][J][T];
 dvar int+ Z1[T];
 dvar int Z2[T];
 dvar int M[T];
 dvar boolean a[T];
 dvar int+ D[S][J][T];
 dvar boolean k[J];

/*objective*/
 minimize
           sum ( i in I, j in J, t in T) X[i][j][t]*alpha
         + sum ( t in T) Z1[t]*beta
         + sum ( i in I, j in J, t in T) XT[i][j][t]*gamma
         - sum ( t in T) Z2[t]*delta
         + sum (j in J) k[j]* epsilon;
```

```
/*constraints*/
 subject to
 {
  set1:
    forall (t in T)
      M[t] == sum (i in I, j in J) (XH[i][j][t]-XF[i][j][t]);
  set2:
    forall (t in T)
      M[t]/100 <= a[t];
  set3:
    forall (t in T)
      (M[t]/100)+1 >= a[t];
  set4:
    forall ( t in T)
      M[t]-25*(1-a[t]) <= Z1[t];
  set5:
    forall (t in T)
      Z1[t] <= M[t]+25*(1-a[t]);
  set6:
    forall ( t in T)
      0 <= Z1[t];
  set7:
    forall ( t in T)
      Z1[t] <= 25*a[t];
  set8:
    forall ( t in T)
      M[t]-25*a[t] <= Z2[t];
  set9:
    forall (t in T)
      Z2[t] <= M[t]+25*a[t];
  set10:
    forall ( t in T)
      25*a[t]-25 <= Z2[t];
  set11:
    forall ( t in T)
      Z2[t] <= 0;
  set12:
    forall (j in J, t in T)
      15*k[j] >= sum (i in I) X[i][j][t];
    forall (j in J, t in T)
      15*k[j] >= sum (i in I) XH[i][j][t];
    forall (j in J, t in T)
      15*k[j] >= sum (i in I) XF[i][j][t];
    forall (j in J, t in T)
      15*k[j] >= sum (i in I) XT[i][j][t];
```

```
    forall (s in S, j in J, t in T)
      eta[1]*k[j] >= D[s][j][t];
  set13:
    forall (i in I, j in J)
      X[i][j][1] == XH[i][j][1];
  set14:
    forall ( i in I, j in J)
      XF[i][j][1] == 0;
  set15:
    forall (i in I, j in J, t in T2)
      X[i][j][t] == X[i][j][t-1]+XH[i][j][t]-XF[i][j][t];
  set16:
    forall (i in I, j in J, t in T)
      XT[i][j][t] == XH[i][j][t];
  set17:
    forall (t in T, j in J)
      sum( i in I) X[i][j][t] <= 15;
  set18:
    forall (i in I, j in J, t in T)
      0 <= XF[i][j][t];
  set19:
    forall (i in I, j in J, t in T)
      XF[i][j][t] <= X[i][j][t];
  set20:
    forall ( i in I, j in J, t in T)
      XH[i][j][t] >=0;
  set21:
    forall (i in I, j in J)
      sum(s in S)(D[s][j][1]*y[s][i]) <= 16*XH[i][j][1];
  set22:
    forall (i in I, j in J, t in T2)
      sum(s in S)(D[s][j][t]*y[s][i]) <= 40*(X[i][j][t-1]-XF[i][j][t])+
                                         16*XH[i][j][t];
  set23:
    forall (s in S)
      sum(j in J, t in T) D[s][j][t] >= eta[s];
}

/***************************************************************************/
```

DATA:

```
SheetConnection sheet("production line model2.xlsx");

alpha from SheetRead(sheet,"Sheet1!B2");
beta from SheetRead(sheet,"Sheet1!B6");
```

```
gamma from SheetRead(sheet,"Sheet1!B10");
delta from SheetRead(sheet,"Sheet1!B14");
eta from SheetRead(sheet,"Sheet1!B22:B23");
y from SheetRead(sheet,"Sheet1!F27:H28");
epsilon from SheetRead(sheet,"Sheet1!E18");
```

Appendix I

Sixth Sub-model in CPLEX

```
/*define*/
 int s=2;
 int i=3;
 int j=2;
 int t=3;

 range S=1..s;
 range I=1..i;
 range T=1..t;
 range T2=2..t;
 range J=1..j;

/*parameters*/
 int alpha = ...;
 int beta = ...;
 int gamma = ...;
 int delta = ...;
 int eta[S] = ...;
 float tau =...;
 int d[S] = ...;
 int lambda[S] = ...;
 int C[S] = ...;
 float y[S][I] = ...;

/*variables*/
 dvar int+ X[I][J][T];
 dvar int+ XT[I][J][T];
 dvar int+ XH[I][J][T];
 dvar int+ XF[I][J][T];
 dvar int+ Z1[T];
 dvar int Z2[T];
 dvar int M[T];
 dvar boolean a[T];
 dvar int A[S];
 dvar int Y[S];
 dvar boolean e[S];

 dvar int+ D[S][J][T];
 dvar int+ OT[S][J][T];

/*objective*/
```

```
 minimize
           sum ( i in I, j in J, t in T) X[i][j][t]*alpha
         + sum ( t in T) Z1[t]*beta
         + sum ( i in I, j in J, t in T) XT[i][j][t]*gamma
         - sum ( t in T) Z2[t]*delta
         + sum ( s in S, j in J, t in T) OT[s][j][t]*tau
         + sum (s in S) Y[s]* lambda[s];

/*constraints*/
 subject to
 {
  set1:
    forall (t in T)
      M[t] == sum (i in I, j in J) (XH[i][j][t]-XF[i][j][t]);
  set2:
    forall (t in T)
      M[t]/100 <= a[t];
  set3:
    forall (t in T)
      (M[t]/100)+1 >= a[t];
  set4:
    forall ( t in T)
      M[t]-25*(1-a[t]) <= Z1[t];
  set5:
    forall (t in T)
      Z1[t] <= M[t]+25*(1-a[t]);
  set6:
    forall ( t in T)
      0 <= Z1[t];
    forall ( t in T)
      Z1[t] <= 25*a[t];
  set7:
    forall ( t in T)
      M[t]-25*a[t] <= Z2[t];
    forall (t in T)
      Z2[t] <= M[t]+25*a[t];
  set8:
    forall ( t in T)
      25*a[t]-25 <= Z2[t];
    forall ( t in T)
      Z2[t] <= 0;
  set9:
    forall (s in S)
      A[s] == C[s]-d[s];
  set10:
```

```
  forall (s in S)
    A[s]/1000 <= e[s];
set11:
  forall (s in S)
    (A[s]/1000)+1 >= e[s];
set12:
  forall (s in S)
    A[s]-10*(1-e[s]) <= Y[s];
set13:
  forall (t in T)
    Y[s] <= A[s]+10*(1-e[s]);
set14:
  forall (s in S)
    0 <= Y[s];
set15:
  forall (t in T)
    Y[s] <= 10*e[s];
set16:
  forall (i in I, j in J)
    X[i][j][1] == XH[i][j][1];
set17:
  forall ( i in I, j in J)
    XF[i][j][1] == 0;
set18:
  forall (i in I, j in J, t in T2)
    X[i][j][t] == X[i][j][t-1]+XH[i][j][t]-XF[i][j][t];
set19:
  forall (i in I, j in J, t in T)
    XT[i][j][t] == XH[i][j][t];
set20:
  forall (t in T, j in J)
    sum( i in I) X[i][j][t] <= 15;
set21:
  forall (i in I, j in J, t in T)
    0 <= XF[i][j][t];
set22:
  forall (i in I, j in J, t in T)
    XF[i][j][t] <= X[i][j][t];
set23:
  forall ( i in I, j in J, t in T)
    XH[i][j][t] >=0;
set24:
  forall (i in I, j in J)
    sum(s in S)(D[s][j][1]*y[s][i]-OT[s][j][1]) <= 16*XH[i][j][1];
set25:
```

```
    forall (i in I, j in J, t in T2)
      sum(s in S)(D[s][j][t]*y[s][i]-OT[s][j][t]) <= 40*(X[i][j][t-1]-
                 XF[i][j][t])+16*XH[i][j][t];
  set26:
    forall (s in S)
      sum(j in J, t in T) D[s][j][t] >= eta[s];
  set27:
    forall (t in T)
      sum (s in S) OT[s][2][t] == 0;
  set28:
    forall (i in I, t in T)
      sum (s in S) OT[s][1][t] <=sum(i in I) X[i][1][t]*10;
  }

/**************************************************************************/
```

DATA:

```
SheetConnection sheet("production line model2.xlsx");

alpha from SheetRead(sheet,"Sheet1!B2");
beta from SheetRead(sheet,"Sheet1!B6");
gamma from SheetRead(sheet,"Sheet1!B10");
delta from SheetRead(sheet,"Sheet1!B14");
eta from SheetRead(sheet,"Sheet1!B22:B23");
tau from SheetRead(sheet,"Sheet1!E2");
d from SheetRead(sheet,"Sheet1!E6:E7");
lambda from SheetRead(sheet,"Sheet1!E10:E11");
C from SheetRead(sheet,"Sheet1!E14:E15");
y from SheetRead(sheet,"Sheet1!F27:H28");
```

Appendix J

Seventh Sub-model in CPLEX

```
/*define*/
 int i=3;
 int j=2;
 int t=4;
 range I=1..i;
 range T=1..t;
 range T2=2..t;
 range J=1..j;

/*parameter*/
 int alpha = ...;
 int beta = ...;
 int gamma = ...;
 int delta = ...;
 int eta = ...;
 float tau =...;
 int d = ...;
 int lambda = ...;
 int C = ...;
 float y[I] = ...;

/*variable*/
 dvar int+ X[I][J][T];
 dvar int+ XT[I][J][T];
 dvar int+ XH[I][J][T];
 dvar int+ XF[I][J][T];
 dvar int+ Z1[T];
 dvar int Z2[T];
 dvar int M[T];
 dvar boolean a[T];
 dvar int A;
 dvar int Y;
 dvar boolean e;
 dvar int+ D[J][T];
 dvar int+ OT[J][T];

/*objective*/
 minimize
           sum ( i in I, j in J, t in T) X[i][j][t]*alpha
         + sum ( t in T) Z1[t]*beta
         + sum ( i in I, j in J, t in T) XT[i][j][t]*gamma
```

```
        - sum ( t in T) Z2[t]*delta
        + sum ( j in J, t in T) OT[j][t]*tau
        + Y*lambda;

/*constraints*/
 subject to
 {
  set1:
    forall (t in T)
      M[t] == sum (i in I, j in J) (XH[i][j][t]-XF[i][j][t]);
  set2:
    forall (t in T)
      M[t]/100 <= a[t];
  set3:
    forall (t in T)
      (M[t]/100)+1 >= a[t];
  set4:
    forall ( t in T)
      M[t]-25*(1-a[t]) <= Z1[t];
    forall (t in T)
      Z1[t] <= M[t]+25*(1-a[t]);
  set5:
    forall ( t in T)
      0 <= Z1[t];
  set6:
    forall ( t in T)
      Z1[t] <= 25*a[t];
  set7:
    forall ( t in T)
      M[t]-25*a[t] <= Z2[t];
  set8:
    forall (t in T)
      Z2[t] <= M[t]+25*a[t];
  set9:
    forall ( t in T)
      25*a[t]-25 <= Z2[t];
  set10:
    forall ( t in T)
      Z2[t] <= 0;
  set11:
      A == C-d;
  set12:
      A/1000 <= e;
  set13:
      (A/1000)+1 >= e;
```

```
  set14:
      A-10*(1-e) <= Y;
  set15:
      Y <= A+10*(1-e);
  set16:
      0 <= Y;
  set17:
      Y <= 10*e;
  set18:
    forall (i in I, j in J)
      X[i][j][1] == XH[i][j][1];
  set19:
    forall ( i in I, j in J)
      XF[i][j][1] == 0;
  set20:
    forall (i in I, j in J, t in T2)
      X[i][j][t] == X[i][j][t-1]+XH[i][j][t]-XF[i][j][t];
  set21:
    forall (i in I, j in J, t in T)
      XT[i][j][t] == XH[i][j][t];
  set22:
    forall (t in T, j in J)
      sum( i in I) X[i][j][t] <= 15;
  set23:
    forall (i in I, j in J, t in T)
      0 <= XF[i][j][t];
  set24:
    forall (i in I, j in J, t in T)
      XF[i][j][t] <= X[i][j][t];
  set25:
    forall ( i in I, j in J, t in T)
      XH[i][j][t] >=0;
  set26:
    forall (i in I, j in J)
      D[j][1]*y[i]-OT[j][1] <= 16*XH[i][j][1];
  set27:
    forall (i in I, j in J, t in T2)
      D[j][t]*y[i]-OT[j][t] <= 40*(X[i][j][t-1]-XF[i][j][t])+16*XH[i][j][t];
  set28:
      sum(j in J, t in T) D[j][t] >= eta;
  set29:
    forall(i in I, j in J, t in T)
      OT[j][t] <= sum (i in I) X[i][j][t]*5;
}
```

```
/**********************************************************************/
```

DATA:

```
SheetConnection sheet("production line model3.xlsx");

alpha from SheetRead(sheet,"Sheet1!B2");
beta from SheetRead(sheet,"Sheet1!B6");
gamma from SheetRead(sheet,"Sheet1!B10");
delta from SheetRead(sheet,"Sheet1!B14");
tau from SheetRead(sheet,"Sheet1!E2");
eta from SheetRead(sheet,"Sheet1!B22");
d from SheetRead(sheet,"Sheet1!E6");
lambda from SheetRead(sheet,"Sheet1!E10");
C from SheetRead(sheet,"Sheet1!E14");
y from SheetRead(sheet,"Sheet1!F27:H27");
```