Issuing a Convertible Bond with Call-Spread Overlay: Incorporating the Effects of Convertible Arbitrage

Samira Shirgir

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By: **Samira Shirgir**

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Signed by the final examining committee:

____ Examiner

Dr. Patrice Gaillardetz

____ Examiner

Dr. J. Garrido

____ Thesis Supervisor

Dr. Cody Hyndman

Approved by ____

Chair of Department or Graduate Program Director

____

Dean of Faculty

Date ____

ABSTRACT

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In recent years companies issuing convertible bonds enter into some transactions simultaneously in order to mitigate some of the negative impacts of issuing convertible bonds such as the dilution of existing shares. One of the popular concurrent transactions is a call-spread overlay which is intended to reduce the dilution impact. This thesis explores the motivation for using these combined transactions from the perspective of the issuers, investors, and underwriters. We apply a binomial method to price the convertible bonds with call-spread which are subject to default risk. Based on previous empirical studies convertible bond issuers experience a drop in their stock price due to the activities of convertible bond arbitrageurs when the issuance of convertible bonds is announced. We propose a model to estimate the drop in the stock price due to convertible bond arbitrage activities, at the time of planning the issue and designing the security that will be offered. We examine the features of the model with simulated and real-world data.

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Chapter 1

Introduction

1.1 Introduction

Companies usually raise capital by issuing common stock or by borrowing money. They have three ways in order to borrow money: obtaining a loan from banks, issuing bonds, or issuing hybrid securities. Hybrid securities combine two or more financial instruments. They usually have both debt and equity characteristics. The most important subcategory of this asset class is the convertible bond.

A convertible bond is a combination of a fixed rate bond and an embedded call option. This security gives investors the opportunity to convert their bonds into a predefined number of ordinary shares during a prescribed conversion period. Convertible bonds typically pay lower interest than straight corporate debt because of the value of the call option that is embedded in this derivative security.

1.2 Advantages and Disadvantages of Convertible Bonds

Many researchers have studied convertible bonds and given reasons for issuing these bonds. We review the advantages and disadvantages from the point of view of issuers and investors.

1.2.1 From the Issuer's Perspective

One of the merits of issuing convertible bonds according to Hillier et al. (2010) is providing the company a way to issue its ordinary shares at a higher price than the currently prevailing stock price since the conversion price is always higher than the stock price on the issue date to prevent arbitrage activities. In fact, if the conversion price is lower than the stock price, arbitrageurs can make money from converting the convertible bonds into shares and immediately selling them at the higher price. Thus a company that thinks their ordinary shares are undervalued can defer equity financing to a time when their stock price performs well.

Another benefit is that issuing convertible bonds provides cheaper financing initially compared to straight bonds due to their lower interest rate. These decreased financing costs could be more significant for a young firm with potential growth and tight budgets in the first years after issuing. In fact, the value of the conversion option held by the convertible bondholders is reflected in the observed lower coupon rates of convertible bonds. Moreover, issuing convertible bonds can be preferable to equity issues for tax purposes, because dividend payments on stock are not tax deductible, while interest payments on debt are tax deductible, see De Spiegeleer and Schoutens (2011). Finally, Liu and Switzer (2013) argue that issuing convertible bonds is an optimal financial decision for firms that do not have a strong historical performance record but have promising new projects with uncertainty about when a new project will become fully operational. An alternative approach for the company to finance new projects is to issue straight bonds and then issue equity when the stock price goes up as a result of profitability of the project. However, this strategy could be more expensive given extra underwriting costs and issuing expenses.

Ross et al. (2009) argue that sometimes it is very costly to assess the risk of the company's projects and as a result a firm could not choose an appropriate instrument of financing. Issuing a convertible bond is a solution to this problem since a convertible bond has both debt and equity components. If the company's project turns out to be a low-risk after issuing the convertible bonds, the debt component of the convertible bond will be worth more than the equity component. On the contrary, if the company's project turns out to be a high-risk, the equity component will have a higher value than the debt component. Therefore, convertible bonds are a suitable financing instrument for companies that are not able to evaluate the risk of their projects.

Issuing convertible bonds provides an opportunity for the company to access a broader range of investors owing to the hybrid feature of convertible bonds. Consequently, the company can attract both fixed income and equity investors. For instance, some fund managers are restricted to investing only in fixed income instrument and not the stock market. Investing in convertible bonds allows them to relax this restriction and allocate some budget to the equity market, see De Spiegeleer and Schoutens (2011).

A firm with high stock price volatility is able to reduce its cost of debt capital by issuing convertible bonds owing to the fact that the conversion option in the convertible bond has more value when the underlying shares are highly volatile since there is a greater probability that the stock price will rise and conversion takes place. Therefore, firms with a volatile stock price are capable of lowering their interest rate charge by issuing convertible bonds compared to the issuing straight corporate debt, see De Spiegeleer and Schoutens (2011).

Convertible bonds can be combined with a variety of features, one of the most popular features is a call provision which gives the issuer the right to call back the bond and terminate the life of the bond by paying the early redemption amount. This is a perfect opportunity for the issuers who have a chance of refinancing at a lower interest rate which is discussed in Dong et al. (2013).

From the issuer perspective, issuing convertible bonds has some drawbacks. For instance, if the common stock price of the company falls during the life of the bond, the bondholders will not exercise their conversion option and the company will be forced to pay all the bond coupons and the face value on the maturity date when the company's cash flow may be under pressure, see Brown (2013).

According to De Spiegeleer and Schoutens (2011), by exercising the conversion option, convertible bond holders create new shares and dilute existing shareholders' stakes. This potential future dilution drags down the common stock price of the company on the announcement date of the convertible bonds which is a couple of days before the issue date. The size of this negative impact is a function of the conversion ratio. A high conversion ratio increases the possibility of potential dilution which leads to a higher negative effect.

Henderson and Zhao (2013) argue that convertible bonds are usually underpriced by an error made in the conversion factor, relative to their fair value. Some investors, usually hedge funds, try to benefit from this mispricing by purchasing the convertible bonds and simultaneously shorting an appropriate amount of shares to hedge their exposure to the company's stock price and default risk. This arbitrage activity affects the stock price negatively on the announcement date of a convertible bond since arbitrageurs' short sales absorb available liquidity. In Chapter 3 we will see that the more the convertible bond is equity-like, the larger the announcement effect will be since the arbitrageurs short sell more shares.

1.2.2 From an Investor's Perspective

Convertible bonds provide exposure to the upside potential of the common stock of the company through an embedded call option. Investors have an opportunity to convert their bonds to stock when the market value of the company's stock rises. Simultaneously, if the company's stock price falls, investors can benefit from the downside protection of receiving the bond's coupons and return of principal on the maturity date. Investing in convertible bonds can also be less volatile than holding the underlying shares but riskier than investing in the straight bonds, see De Spiegeleer and Schoutens (2011).

Liu and Switzer (2013) argue that investing in convertible bonds offer investors a combination of different financial instruments with the lower cost. In fact, if investors purchase the straight bond and an American call option to replicate the convertible bond's payoff, they are not able to replicate the exact payoff of the convertible bond since they miss some features of the convertible bond such as put rights, which are a common feature of convertible bonds that allow investors to sell back the bond to the issuer for a predetermined price and it costs them more.

The drawback of investing in convertible bonds is that if the stock price of the company performs poorly during the life of the bond investors will never convert their bonds and as a result they will earn a lower return compared to straight bonds due to the convertible bond's lower coupon payment compared to the straight bonds. However, this loss can be offset to some extent if investors short sell the common stock of the company in advance, see Brown (2013).

1.3 Convertible Bond Pricing Methods

There are three theoretical methods which have been used to price convertible bonds. The first pricing method is based on lattice models which restrict the result of a continuous stochastic process to some possible states over a finite number of time steps. Binomial and trinomial trees are examples of lattice models. Lattice models are popular based on their ability to incorporate several features of convertible bonds simultaneously. Hull (1988) and Hung and Wang (2002) used the standard tree method to price convertibles subject to default risk. Hung and Wang (2002) distinguish between the risky discount rate from the risk-free interest rate and combined the stochastic risk-free and risky discount rates into one tree.

The second method is finite-difference techniques which are used to solve partial differential equations, representing the value of the convertible bond, by replacing the continuous space for the share price and time with a two-dimensional discrete grid. Boundary conditions need to be imposed to find a solution. Lattice and finite-difference methods are effective when there is only one source of risk. However, when there is more than one source of risk, it is more difficult to implement these methods. Ayache et al. (2002) developed a valuation method for convertible bonds using finite-difference techniques.

Monte-Carlo methods are popular since they are easier to implement while working with multi-factor stochastic processes and can be used to price path-dependent securities with a complex payoff. Another advantage of this method is that it provides enough flexibility to specify the dynamics of the important financial variables such as the share price, interest rate, and credit risk models. In this method, a financial variable X in a set of k random values are generated for each of the n runs. The financial variable for path i is $X_{i,1}, ..., X_{i,k}$ with $i \in \{1, 2, ..., n\}$. Then the final payoff of the security is calculated for each path. The current price of the security is given by the average of the discounted payoffs. Longstaff and Schwartz (2001) introduced a Monte-Carlo method that can handle derivatives with early exercise features. They used this method to derive a lower bound for the price of an American option. American options and convertible bonds share a common feature which is their early exercise nature. Therefore, after the work of Longstaff and Schwartz, and through some recent work of Kind et al. (2008), Monte-Carlo methods have found wide acceptance in pricing convertible bonds.

1.4 Convertible Bond Credit Risk Models

Credit risk arises when there is a possibility that a financial institution or borrower fails to meet its contractual obligations such as repaying a loan. There are three methodologies to price the credit risk: structural models, credit spread models, and reduced form models.

1.4.1 Structural Models

The structural approach considers default as an endogenous event since it provides an explicit relationship between default risk and capital structure. According to Wang (2009), the structural models are commonly used by the practitioners in the area of credit portfolio and credit risk analysis. Indeed, these models require intensive computation that is why they are not used by the credit security trading practitioners who need fast computation tools to adjust themselves to quick market movements. Merton (1973) pioneered this approach. He uses the value of the firm as the underlying state variable and defines default when the value of the firm falls below the face value of its debt. The methods of Merton (1973) were modified by Longstaff and Schwartz (1995) who changed the default to a stopping time of a firm value to a certain boundary which was common among all the firm's debts.

The first pricing of a convertible bond based on the Black and Scholes (1973) model was done by Ingersoll (1977). He determined analytical solutions to price convertibles in specific cases under some restrictive assumptions such as no dividends, perfect markets, and constant conversion terms. The first model he considered was a non-callable convertible discount bond which was divided into a straight corporate bond and a European option. In his model the total asset value of the issuing company drives the price of the convertible. Ingersoll (1977) also established a method to determine of the optimal conversion strategy for the bondholders and the optimal call strategy for the issuer.

Brennan and Schwartz (1977) used the same framework as Ingersoll (1977) for valuing convertible bonds. The significant difference was that Ingersoll (1977) offered closed form solutions for the bond value while Brennan and Schwartz (1977) provided a general algorithm for pricing a convertible bond. Brennan and Schwartz (1977) applied numerical methods to solve the partial differential equation for pricing a convertible bond with call provisions, coupons and dividends. They also considered the probability that the firm defaults on the bond. Moreover, a risk-free interest rate is assumed to be known and constant.

Brennan and Schwartz (1980) extended their previous work with a new model that has an additional factor representing stochastic interest rates. The other difference from their early work was that they considered the possibility of senior debt in the firm's capital structure. Indeed, they considered the value of the firm as the sum of three components: outstanding senior debt, convertible bonds, and common stock.

Structural models can be difficult to use in practice because the value of the firm is not directly tradable or observable in the market. This fact complicates the estimation of model parameters. Moreover, defaultable assets are not equally ranked which adds complexity to the model. For instance, a convertible bond ranks before the shareholders while it may be subordinated to other firm's debt instruments.

1.4.2 Credit Spread Models

McConnell and Schwartz (1986) established a model to price a zero coupon convertible bond with call and put features where the cash flows from the convertible bonds were discounted at the risk-free rate plus a credit spread which is the extra yield that the investors demand when there is a possibility of default and the only source of uncertainty was the equity price. Their goal was to find a practical approach to price the contingent claim with numerical techniques. They used a finite-difference technique to solve the partial differential equation which is the function of the convertible bond price.

Bardhan et al. (1994)[1] constructed a credit spread model using a Cox, Ross and Rubenstein (CRR) stock price binomial tree where a weighted average of the risk-free rate and the risky rate was applied on all the cash flows in the binomial tree, based on the conversion probability, instead of applying the credit spread uniformly. The model assumes the underlying share price is the sole risk factor and other factors such as stock volatility, the issuer's credit spread and stock loan rate are known. The downside of this model is that investors will receive the stock even when default happens if they opt to convert. However, in this model the stock price does not drop to zero in the event of default. Finally, the model does not include any fraction of bond recovery in the case of default.

The Goldman Sachs model was improved by Tsiveriotis and Fernandes (1998) where the convertible was divided into a bond component and an equity component which were discounted at the risky rate and the risk-free rate respectively. The reason for choosing the risk-free rate as a discounting factor for the equity component is that this part can be hedged through shares so the company can always deliver its common stock but it might fail to pay promised cash payments. The model uses two partial differential equations to show the behaviour of the bond and equity component of the convertible bond. This model shares some of the same drawbacks of the Goldman Sachs model such as the stock price does not drop to zero in the case of default.

Ho and Pfeffer (1996) applied a two dimensional binomial tree for the stock price process and the interest rate risk to price the convertible bond with all the main features. The cash flow of the bond is discounted at the rate equal to the sum of a constant credit

[1] Goldman Sachs model

spread and the risk-free interest rate. They price the convertible bond through evaluating the building blocks of the convertible bond which are the corporate bond and an embedded warrant option.

1.4.3 Reduced Form Models

Reduced form models regard default as an exogenous event. These models are popular with practitioners and use the value of the firm's equity as the underlying state variable. The model parameters are estimated from trading securities of the same company such as corporate bonds.

Jarrow and Turnbull (1995) initiated the reduced form model where the default event is modelled as a Poisson process with an arrival rate λ. On default, some fraction of the face value of the bond, the recovery rate, will be paid back to the bondholders. This model was extended by Hung and Wang (2002) who combined the stock price process, the stochastic risk-free interest rate process, and the risky discount rate process into one single tree to value a convertible bond.

Takahashi et al. (2001) took advantage of the reduced form model to price a convertible bond with default risk based on the model of Duffie and Singleton (1999). They developed a consistent and practical model to price not only convertible bonds but also corporate bonds and equities.

Ayache et al. (2002) claimed that the approach of Tsiveriotis and Fernandes has some issues. In the case of default the stock price is not modelled as jumping to zero and the recovery on the bond is omitted. They considered the whole convertible bond as a contingent claim instead of splitting it into separate debt and equity components. They introduced a single-factor model where in the event of default a reduced form model was applied. In their model the stock price drops on default but it can be different from zero and the default intensity is a function of the stock price. Moreover, they considered a variety of recovery assumptions in the event of default.

1.5 Convertible Bond with Call-Spread Overlay

A convertible bond with call-spread overlay issue consists of three stages. In stage 1 a company issues convertible bonds through an underwriter such as a bank which is not related to the company and all the transactions between the company and the bank are at arms length. If conversion takes place the company can choose one of three kinds of settlements in order to meet its obligation upon conversion according to the terms of the offerings. The first type of settlement is a physical settlement where the company delivers the conversion shares to the bondholders. The second type is a cash settlement where the company delivers the value of the conversion shares in cash to the bondholders. The third type is a net share settlement where the company delivers the principle amount of the bonds in cash and the excess value of the conversion shares over the principle amount of the bonds in ordinary shares.

In stage 2 the company purchases a hedge from the bank on the convertible bonds issue date. Each hedge gives the company the right to buy from the bank a number of its ordinary shares, equal to the number of conversion shares, at a strike price equal to the conversion price of the convertible bonds. In other words, the hedge is a call option that the company purchases on its common stock that helps the company offset its position in the embedded call option in the convertible bonds. The hedge completely offsets the conversion feature and prevents the dilution to the company's common stock. The hedge is exercised automatically whenever the corresponding convertible bonds are converted by the bondholders. As a matter of fact, the company does not have the right to buy its ordinary shares under the hedge unless the bondholders convert their bonds. The hedge and convertible bonds have the same settlement manner and maturity date. The company pays the bank a premium for entering into the convertible bond hedge.

In stage 3 the company sells warrants to the bank on the convertible bonds issue date. Each warrant gives the bank the right to purchase the company's ordinary shares, equal to the number of conversion shares, at a strike price substantially higher than the strike price of the hedge. The warrants' settlement manner differs from the convertible bonds and the hedge. The warrants have a dilutive effect unless the company opts to settle the warrants in cash. The warrants are European style and their maturity date is a couple of months after the convertible bonds maturity date. The bank pays the company a premium for the warrants which is significantly lower than the premium paid for the hedge owing to the fact that the hedge has a lower strike price than the warrants.

If the conversion option is in-the-money the issuer will call the bonds and force the investors to convert their bonds so that the hedge will be exercisable. On the other hand, if the conversion option is deeply out-of-the-money and the bondholders believe that there is no possibility of conversion, they may decide to exercise their put option and sell the bonds back to the issuer.

The hedges and warrants should be treated as separate transactions for the following reasons. Exercising the hedges is triggered by exercising the conversion option while exercising the warrants does not relate to the bonds and the hedges. Moreover, the warrants are European style and expire several months after the hedges. The hedges and warrants are transferable, so the holder of the two instruments can sell one of them and retain its position in the other. The hedges and the warrants have different settlement mechanics. The hedges and the warrants do not contain a right of offset. That is, the two parities are not allowed to pledge their rights under the hedges (or the warrants) to secure their obligations under the warrants (or the hedges). Various formulas are applied in the hedges and warrants to determine the value of the company's common stock for the net settlement manner. The warrant is documented in a separate ISDA (International Swaps and Derivatives Association) Confirmation (which is part of the ISDA Master Agreement) from the hedge. The hedges and warrants are priced separately.

1.5.1 From the Issuer's Perspective

Companies that issue convertible bonds seek to reduce the dilution impact that occurs as a result of exercising the conversion option by the bondholders. Conducting the call-spread overlay concurrently with issuing the convertible bonds gives the company an opportunity to entirely remove this dilution impacts since the hedge will cancel out the conversion feature.

The announcement of the convertible bond issue will cause a potential future dilution which has a negative influence on the price of the company's ordinary shares on the announcement date. Issuing convertible bonds with call-spread overlay that eliminates the dilution effect enables company to reduce this negative announcement effect on the share price. However, it does not completely remove this effect because of convertible bond arbitrage activities that is another reason for the announcement effect. As mentioned earlier, arbitrageurs short the stock of the company to hedge their long position in convertible bonds. Companies try to reduce the impact of this short selling by issuing the convertible bonds quickly after the announcement date, using rule 144A, conducting a share repurchase program, or both, see Henderson and Zhao (2013), de Jong et al. (2011) and Duca et al. (2010).

As a consequence of integrating convertible bonds with hedges is that a synthetic fixed rate debt instrument is produced with an issue price equal to the convertible bond issue price minus the hedge premium. Therefore, the company will achieve a deductible original issue discount (OID)[2] equal to the hedge premium. This enables the company to use the cost of the hedge as a tax deduction. However, this integration can be prohibited under some situations. For instance, if the difference between the premium of the hedges and the warrants is not high enough [the net cost of the call-spread is typically 10%-15% of the bond's face value] the company will be under suspicion that entering into the hedge and warrant transactions has no purposes other than tax avoidance, see Memorandum (2007).

From the point of view of the issuer the ideal convertible bond would have low coupon payments to minimize the cost of financing and a high conversion price to minimize potential dilution. However, a high conversion price has an influence on the marketability of the bond and also means that the embedded call option is out of the money. Therefore, investors would demand a higher coupon as compensation. Companies that presume their convertible bonds deserve to have a greater conversion price may issue the convertible bonds with call spread overlay instead of issuing straight convertible bonds with a higher

[2]OID is a form of interest and is created when the redemption amount of the bond is greater than the issue price of the bond.

conversion price in order to lower their interest expense. Issuing convertible bonds with call-spread overlay allows the company to raise the effective conversion price, since the strike price of the warrants is greater than the conversion price, and reduce the coupon interest rate. In fact, issuing the convertible bonds with call-spread overlay enables the firm to report lower interest expenses for accounting purposes because the reported interest expense is not based on the effective conversion price but it is based on the stated conversion price and a higher effective interest rate for tax purposes, see Lewis and Verwijmeren (2011).

1.5.2 From the Underwriter's Perspective

When the underwriter enters into the hedge transactions, it brings an obligation to the underwriter to sell the company its common stock in a market equal to the number of conversion shares. Therefore, the underwriter has to enter into some transactions after pricing of the convertible bonds in order to hedge its positions. These transactions could vary from entering into cash-settled total returns swaps and over-the-counter derivative transactions to purchase the company's common stock in private or open market transactions. The impact of these activities could increase the market price of the company's common stock.

The underwriter can also make some money from the difference of the hedge premium and the warrant premium. If the company's stock price rises sufficiently above the strike price of the warrant at the warrant expiration, the underwriter would profit by exercising the warrant.

1.5.3 From an Investor's Perspective

Investing in the call-spread overlay increases the investor's ability to convert their convertible bonds sooner and receive an original stock of the company that is more valuable for two reasons, since it is not diluted upon conversion, compared to investing in the straight convertible bond. First, the stock price of the company may rise as a consequence of transactions that the underwriter will enter into after the pricing of the bonds. Second, investors are able to buy the convertible bonds with a lower conversion price since, as mentioned earlier, one of the effects of issuing convertible bonds with call-spread overlay is to raise the effective conversion price so the company sets the lower conversion price for its convertible bonds with call-spread in comparison to its straight convertible bonds. The hedge and the warrant transactions are separate transactions from the convertible bonds and do not affect the bondholder's rights.

1.6 Conclusion

In this chapter we explained a convertible bond and its advantages and disadvantages from both the issuer and the investor point of view. We pointed out the methods for pricing the convertible bond and different ways of incorporating credit risk in the valuation models. We gave an overview of the convertible bond with call-spread overlay and discussed the pros and cons of this combined financial instrument from the perspective of the issuer, underwriter and investor.

The remainder of this thesis is organized as follow: Chapter 2 introduces some definitions which are necessary to value the convertible bonds. The binomial model is applied to the convertible bond valuation problem. We give some evidence that the stock price of the company that plans to issue convertible bonds will drop on the announcement date of issuing convertible bonds due to the arbitrage activities and we apply the Almgren and Chriss (2000) model for this drop. In Chapter 3 we use a reduced form approach to incorporate the default in the valuation of the convertible bonds. We analyse the pricing of the hedge using optimal conversion and call policies. We conclude Chapter 3 by introducing the formula for pricing the convertible bond with call-spread. In Chapter 4 we provide numerical results for two sample convertible bonds with call-spread to demonstrate our approach. We also use our model to price the real convertible bonds traded in the market and compare the results. Chapter 4.2 concludes and discusses future research.

Chapter 2

Modelling a Non-defaultable Convertible Bond

2.1 Basic Terminology

In this section, we introduce some notations and concepts which are important for the formulation of the corporate bond pricing model.

Definition 2.1. The **face value**, denoted by F, is the notional amount of one single bond.

Definition 2.2. The **issue price** is the price that an investors should pay to purchase a convertible bond at issue. When the issue price is equal to the bond's face value, the bond is said to be **issued at par**. If a bond is issued at a price less than its face value, it is said to be **issued at discount** and if it is issued at a price higher than its face value, it is said to be **issued at a premium**.

Definition 2.3. The **redemption**, denoted by R, is the amount of money that will be paid to the bondholders at the maturity date which is the termination date of the bond if the issuer does not default during the life of the bond (and the bond is not converted into shares in the case of a convertible bond). It is often expressed as a certain percentage of the bond's face value.

Definition 2.4. The **maturity**, denoted by T, is the date at which the issuer has to redeem the bonds that are not converted to shares.

It is rational for the bondholders to convert their bonds prior or at the maturity date if the value of the shares received upon conversion exceeds the redemption amount (R) of the bond.

Definition 2.5. The **coupon**, denoted by c, is the interest payments that the issuer has to pay on the bond annually, semi-annually or quarterly. It is usually quoted as a percentage of the face value (fixed rate).

2.2 Bonds

Bonds are loans from one party to another. Governments and corporations issue bonds to finance their projects. Bonds are usually refereed to as fixed income securities since the issuer agrees to pay the bondholder a fixed amount of money at the maturity of the bond. Moreover, most bonds also pay annual or semi-annual coupons. Generally, the price of a bond is the sum of the present value of all the future cash flows.

2.2.1 Non-defaultable Bonds

According to Shreve (2005), let $r_0, ..., r_{N-1}$ be an interest rate process. Define the discount process by

$$D_n = \frac{1}{(1+r_0)...(1+r_{n-1})}, \tag{2.2.1}$$

for $n = 1, 2, ..., N$ and $D_0 = 1$. For $0 \leq n \leq m \leq N$, the price at time n, after the payments $c_0, ..., c_{n-1}$ have been made and the payment c_n has not been made, of a non-defaultable coupon paying bond maturing at time m, denoted by $B_{n,k}$, is defined by

$$\sum_{k=n}^{m} c_k B_{n,k} = E_n^Q \left[\sum_{k=n}^{m} \frac{c_k D_k}{D_n} \right], \tag{2.2.2}$$

where c_n, for $0 \leq n \leq m-1$, is the constant coupon payment at time n, c_m is the sum of the face value and the coupon payment at maturity, and E_n^Q denotes the conditional expectation under a risk-neutral measure based on the information at time n.

2.2.2 Defaultable Bonds

Duffie and Singleton (1999) showed that the fair value of a defaultable bond at time t, if it survives up to time t, with final payoff F at maturity T can be written as

$$B_t = E_t^Q \left[\sum_i e^{-\int_t^{T_i} R(u)du} c_i + e^{-\int_t^{T} R(u)du} F \right], \tag{2.2.3}$$

where $E_t^Q(.)$ denotes the conditional expectation under a risk-neutral measure Q given information available to investors at time t, c_i is the coupon payment at time T_i, and $R(t)$ is the default-adjusted discount rate.

$$R(t) :\, = r(t) + L(t)\lambda(t), \tag{2.2.4}$$

where $r(t)$ is the risk-free rate, $L(t)$ is the fractional loss rate of market value when the default happens, and $\lambda(t)$ is the default hazard rate. Note that the fractional loss rate is one minus the recovery rate.

2.3 Advanced Terminology and Analytic Ratios

Each kind of investors looks at a set of definitions to estimate the price of the financial instruments. The following definitions are used by the convertible bond's investors.

Definition 2.6. The **debt seniority** refers to the order of payment if a default takes place. Senior debt has a higher rank than subordinated debt and secured debt comes before the unsecured debt.

Definition 2.7. The **accrued interest**, denoted by Acc, is the amount of money that the investor should pay more than the price of the convertible bond in order to take the accrual of coupon into account if the settlement date of the convertible bond falls between two coupon dates. It would be calculated as follows:

$$\text{Accrued interest} = \{\text{Coupon payment/Number of days in a year}\}$$
$$\times \{\text{Number of days between the last coupon payment date and the settlement date}\}. \tag{2.3.1}$$

Definition 2.8. The **conversion ratio**, denoted by C_r, is the number of ordinary shares that a bondholder will receive if he converts one bond into shares.

Definition 2.9. The **conversion price**, denoted by C_p, is the price that the underlying shares are purchased when the conversion takes place. It is equal to:

$$C_p = \frac{F}{C_r}. \tag{2.3.2}$$

If the underlying share price is above the conversion price, the convertible bond is called **in-the-money**. If the underlying share price is below the conversion price, the convertible bond is called **out-of-the-money**. Conversion prices are fixed on the issue date while a convertible bond can be issued with flexible conversion prices which means that during the life of the convertible bond, the conversion price can be adjusted upwards or downwards.

Definition 2.10. The **call price**, denoted by K, is the price at which a convertible bond can be redeemed by the issuer before maturity. The investors will receive this amount if they accept the call and do not convert their bonds into shares. This price is determined on the issue date (often expressed as a percentage of the bond's face value).

Definition 2.11. The **call rights** are two types: **hard call and soft call**. During the hard call period, the convertible can be called by the issuer unconditionally while during a soft call period the convertible can only be called if the share price has reached some specific level, called **the trigger level**.

The trigger level, denotes by (K_s), is usually stated as a percentage of the conversion price (e.g. $K_s = 130\%C_p$). This call trigger condition usually should be fulfilled for some days so that the issuer is allowed to call the convertible bond. The higher the level of call trigger, the higher the convertible bond price since the probability that the convertible bond is called by the issuer is lower. The intention behind the call right is to force conversion.

Definition 2.12. The **call protection period** is the period that the convertible bond cannot be called by the issuer.

Definition 2.13. The **call notice period** is the period that is given to the bondholder to decide whether to choose conversion after having received the call notice from the issuer or accept the call price. The issuer usually does not call the bond until the conversion value is well above the call price since there is a possibility that the conversion value falls during the call notice period as a result of decreasing stock prices in this period.

Definition 2.14. The **put right** gives the investor the right to sell the bond back to the issuer for the fixed put price (P_ν) on a predetermined date prior to maturity. The put price is paid to the convertible bond holder and terminates the life of the bond.

A high put price increases the price of the convertible bond since it protects the bond price against a drop in the share price while a low put price does not have any impact on the convertible bond price.

Definition 2.15. The **parity price**, denoted by P_a, is the market value of the equity part of the convertible bond. It shows the value of the investment if the investors convert their bonds into the underlying shares. It is also called the conversion value. If the convertible bond is quoted as a percentage of the face value, then the parity price is equal to:

$$P_a = \frac{S\ C_r}{F}, \tag{2.3.3}$$

where S is the stock price. If the convertible bond is quoted in units it is equal to:

$$P_a = S\ C_r. \tag{2.3.4}$$

Definition 2.16. The **premium to parity** is the amount that a convertible bond investor is willing to pay above the current market price of the share, for ownership of these shares in the future, through holding the convertible bonds. The coupon payment that is paid by the convertible bond is greater than the dividends paid by the shares; this difference rises the value of the conversion premium. It is also called conversion premium:

$$\frac{P_{CB} - P_a}{P_a}, \tag{2.3.5}$$

where P_{CB} is the price of the convertible bond. The convertible with low premium is more sensitive to the share price compared to the convertible with high premium.

Definition 2.17. The **bond floor**, denoted by B_F, is the present value of all the cash flows embedded in the convertible bond if the rights of conversion are ignored.

This present value B_F excludes any income coming from the convertible's equity option component; actually, it is the bond component of the convertible. This is often called investment value. It can be calculated as follows:

$$B_F = \sum_{i=1}^{N_c} c_{t_i} \exp(-r_b t_i) + F \exp(-r_b T) \tag{2.3.6}$$

where t_i is the time of the ith coupons, N_c is the number of coupons, c_{t_i} is the coupon paid out at time t_i, and F is the face value. The discount rate r_b is equal to the risk-free rate plus the credit spread. The interest rate and credit spread movements have an influence on the bond floor while the stock price does not affect it directly.

Definition 2.18. The **investment premium** is an indicator of the equity risk present in the convertible and increases if the share price performs well. It shows how much an investor is willing to pay for the option embedded in the convertible. If an investor buys a convertible at the price of the bond floor, he does not pay for the conversion right. It is also called the premium to bond floor or the risk premium:

$$\frac{P_{CB} - B_F}{B_F}. \tag{2.3.7}$$

Definition 2.19. The **dividend yield**, denoted by q, is an income generated by each share. It is calculated as:

$$\text{Dividend yield } = \frac{\text{Dividend per share}}{\text{Current share price}}, \tag{2.3.8}$$

$$\text{Dividend per share} = \frac{\text{Sum of dividends paid out over a year}}{\text{Number of outstanding ordinary shares over a year}}. \tag{2.3.9}$$

Definition 2.20. The **current yield**, denoted by CY, resembles the dividend yield on a stock. It is calculated as:

$$\text{Current yield} = \frac{\text{The value of the annualized coupon}}{\text{Current convertible price}}. \tag{2.3.10}$$

Definition 2.21. The **yield advantage** is the difference between the current yield on the convertible bond and the dividend yield on the stock $(CY - q)$.

Definition 2.22. The **yield to maturity**, denoted by YTM, is the discount rate that equates the present value of all the cash flows coming from the convertible (coupons (c) and final redemption) to the current market price of a bond. In other worlds, it is the rate of return that the investors will get if they hold the bond until maturity.

Definition 2.23. The **delta**, denoted by Δ, is a measure of the sensitivity of the convertible bond price to share price movements:

$$\Delta = \frac{\partial P_{CB}}{\partial S}. \tag{2.3.11}$$

A convertible bond with a 40% delta means that if the underlying share increases by 10%, the convertible bond price will increase by 4%.

2.4 The Convertible Bond Payoff

The convertible bond holder has the right to exchange the face value F of the bond for C_r shares with price S at maturity. The final pay off of the convertible bond can be written as

$$\max(F, C_r \times S). \tag{2.4.1}$$

By using equation (2.3.2) and omitting of the fact that the convertible pays a coupon lets us strip the convertible into a bond with face value F and C_r European call options. Equation (2.4.1) can be written as

$$F + C_r \times \max(0, S - C_p), \tag{2.4.2}$$

where the strike of the call options is equal to the conversion price of the convertible bond.

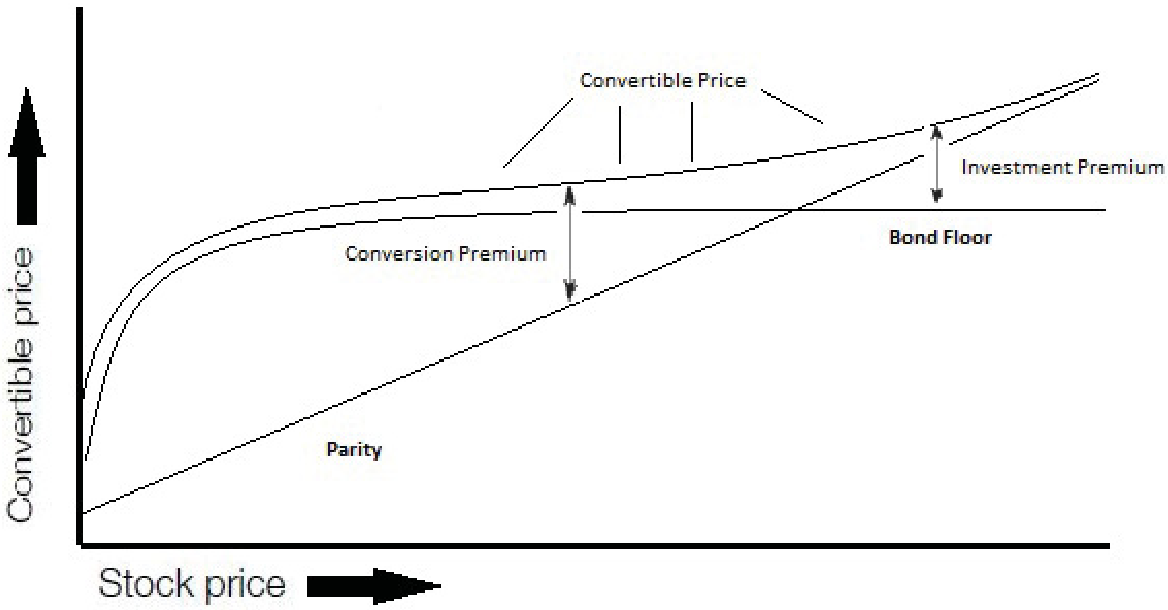
The above argument is true if the conversion right is limited to the maturity date. In this case the convertible bond is comprised of a corporate bond and a European call option. However, in reality most convertible bonds are American style and conversion can take place any time during a conversion period. The investors are entitled to receive coupon payments during the life of the bond and they can terminate the bond at or before maturity by receiving one the following payments: conversion value, put price, call price, bond's face value.

2.4.1 Convertible Bond Payoff Graph

When the share price falls, the convertible bond price will also fall but at a lower rate. At very low share prices the price of the convertible levels out to the bond floor. Furthermore, a dramatic fall in share prices may lead to a lower bond floor as a result of damaging the issuer's credit quality. However, according to De Spiegeleer and Schoutens (2011) we will make some simplifying assumptions so that the bond floor is flat. Figure 2.1 demonstrates the relationship between the stock price and the convertible bond price, see De Spiegeleer and Schoutens (2011) and RMF Investment Consultants (2002).

As it is shown in Figure 2.1 at high share prices the convertible price converges to the parity line and it acts like a share. Moreover, sometimes on default the convertible has an equity nature too and its price drops below the bond floor and approaches parity again. Therefore, parity is a real boundary condition and not the bond floor. While at high share prices the convertible has a low conversion premium, at low share prices it has a high conversion premium.

Figure 2.1: Convertible bond price



2.5 Non-callable Convertible Bonds

Early works in the field of pricing convertible bonds used the idea of pricing a convertible bond as a package of two instruments: a straight bond and an European call option. In fact, they priced the two components individually and added the result together. The issue with this method is that the two components are tied together inextricably and in many cases it gives us incorrect result. Ingersoll (1977) and Takahashi et al. (2001) proved that this technique is applicable under restrictive assumptions.

Takahashi et al. (2001) applied the default risk in their model based on the approach of Duffie and Singleton (1999). They showed that a convertible bond can be considered as a non-convertible corporate bond plus a call option on the underlying stock if conversion is allowed only at maturity:

$$CB_t = B_t + C_r E_t^Q \left[e^{-\int_t^T R(u)du} \max(S_T - C_p, 0) \right]. \tag{2.5.1}$$

In equation (2.5.1) B_t denotes the price of a non-convertible corporate bond (same as equation (2.2.3)) and the second term is the price of a call option with exercise price equal to the conversion price C_p. If the hazard rate is non-stochastic and the fractional loss of the market value is constant, $L(t) = L$, the Black-Scholes formula can be applied to price the call option.

Another drawback of using this method is that many convertible bonds have features like the issuer's call and the investor's put which cannot be included in equation (2.5.1). Finally, the embedded American style option of the convertible bond will be changed to European style when applying this method. However, if the convertible bonds are convertible into non-divided-paying stocks, it is not optimal for the bondholders to convert before maturity (which will be discussed in Chapter 3) so there is no difference between the American and European style convertible bond.

2.6 Callable Convertible Bonds

In this section, we would like to improve the pricing model of the non-callable convertible bond by allowing the conversion to take place before the maturity and adding the call and put provision in our pricing model.

Assume that we are in a binomial world where the stock price can either go up or down with two factors u and d respectively during a short time interval Δt. We also assume that the trades take place in an arbitrage free world with no transaction costs and bid-ask spreads. Although there are several sources of uncertainty that affect the price of a convertible bond we only take into account the stochastic characteristics of the stock price process and consider the other elements, like dividend yield, interest rate, and volatility as constant.

Consider a one step binomial model. We define $V_1(H)$ to be the amount that a derivative security pays if the stock price goes up to $S_1(H) = S_0u$ at time one and $V_1(T)$ if the stock price goes down to $S_1(T) = S_0d$ at time one.

To determine the price of the derivative security at time zero, we begin with short selling the derivative security at time zero for V_0, buying Δ_0 shares of stock at time zero, and investing $V_0 - \Delta_0 S_0$ in the risk-free market. We want to determine Δ_0 so that the price of the portfolio equals to the price of the derivative security at time one. Therefore, the value of our portfolio at time one if the stock pays a continuous dividend yield (q) for every time step Δt will be:

$$V_1(H) = \Delta_0 S_1(H) \exp(q\Delta t) + \exp(r\Delta t)(V_0 - \Delta_0 S_0), \tag{2.6.1}$$

$$V_1(T) = \Delta_0 S_1(T) \exp(q\Delta t) + \exp(r\Delta t)(V_0 - \Delta_0 S_0). \tag{2.6.2}$$

We solve for Δ_0 by subtracting (2.6.2) from (2.6.1)

$$\Delta_0 = \frac{V_1(H) - V_1(T)}{S_1(H) - S_1(T)} \exp(-q\Delta t). \tag{2.6.3}$$

We have constructed a risk-free portfolio since the value of our portfolio is known at time one regardless of whether the stock goes up or down. The portfolio of Δ_0 shares and a short position in the derivative security is hedged and risk-free.

By substituting (2.6.3) into either (2.6.1) or (2.6.2), we can solve for

$$V_0 = \exp(-r\Delta t) \left\{ V_1(H)\overline{p} + V_1(T)(1 - \overline{p}) \right\}, \tag{2.6.4}$$

where

$$\overline{p} = \frac{\exp((r-q)\Delta t) - d}{u - d}. \tag{2.6.5}$$

The probability that the stock price goes up is $\overline{p}$ and the probability of moving down is $(1-\overline{p})$. They are not the actual probabilities and are the result of the absent of arbitrage in our portfolio. These probabilities are called risk-neutral probabilities or sometimes called martingale probabilities. We can extend our binomial tree to multiple periods. Equation (2.6.4) expresses the fact that the value of a derivative security equals the present value of the expected pay off under the risk-neutral measure.

We shall assume that the stock price follows the Black and Scholes (1973) model in continuous time. The assumptions of this model are:

1. The distribution of future stock prices are lognormal and can be determined as follows:
$$S_t = S_0 \exp(\mu - \frac{\sigma^2}{2})t + \sigma W(t), \tag{2.6.6}$$
where μ denotes a drift, σ denotes volatility, and $W(t)$ denotes a Wiener process or Brownian motion.

2. In this frame work we assume that the interest rates are deterministic; in fact, we do not consider the interest rate as a stochastic process. The reason to do this is that interest rates have two opposing forces on the pricing of a convertible bond. As we know a convertible has both debt and equity characteristics; therefore, an increase in interest rates will decreases the debt component and increase the equity component. Brennan and Schwartz (1980) showed that if a reasonable range of interest rates is chosen, the errors of ignoring a stochastic interest rate process are insignificant.

In order to apply our model for the convertible bond we must implement certain features in discrete time. Therefore, we use a binomial model which converges to the Black and Scholes (1973) model as the discretization time goes to zero. The required steps to use a binomial model to price a convertible bond are as follows:

1. Construct a binomial tree of stock prices from the current valuation date towards the maturity date. We use the Cox et al. (1979) model to build the tree of stock prices with up and down factors given by
$$u = \exp(\sigma\sqrt{\Delta t}), \tag{2.6.7}$$
$$d = \exp(-\sigma\sqrt{\Delta t}) = \frac{1}{u}, \tag{2.6.8}$$
where σ is the stock volatility and Δt is the time between discrete time points. The stock price at each node can be calculated as follows:
$$S_n(\omega_1, ..., \omega_n) = u^{\#H(\omega_1,...,\omega_n)} d^{\#T(\omega_1,...,\omega_n)} S_0, \tag{2.6.9}$$

where $\#H(\omega_1, ..., \omega_n)$ and $\#T(\omega_1, ..., \omega_n)$ are the number of up and down steps respectively. By substituting equations (2.6.7) and (2.6.8) into equation (2.6.5) the probability that the stock price goes up can be written as:

$$\overline{p} = \frac{\exp((r-q)\Delta t) - \exp(-\sigma\sqrt{\Delta t})}{\exp(\sigma\sqrt{\Delta t}) - \exp(-\sigma\sqrt{\Delta t})}. \tag{2.6.10}$$

After completing the tree of stock prices, we can construct a corresponding tree of convertible bond prices which should be built from the maturity of the convertible and rolling backwards in time to the valuation date.

2. Compute the price of the convertible bond at maturity nodes (N) which is the maximum of the redemption value plus the coupon and its conversion value plus the accrued interest or coupon (if the coupon payment falls on a conversion date). In fact, investors can choose to exercise their conversion option or let it expire and receive the redemption amount.

$$V_N = \max\{C_r S_N, R + c\} \tag{2.6.11}$$

3. Going backwards through the tree, the investors can hold the convertible bond or convert it to stock at each node. The price of the convertible bond is equal to the value $\overline{V_n}$ at time n if the investors want to wait for one further time period Δt without converting (continuation value):

$$\overline{V_n}(\omega_1, ..., \omega_n) = \exp(-r\Delta t)\left\{V_{n+1}(\omega_1, ..., \omega_n H)\overline{p} + V_{n+1}(\omega_1, ..., \omega_n T)(1-\overline{p})\right\} + c, \tag{2.6.12}$$

so the continuation value at time n depends on the first n coin tosses $\omega_1, ..., \omega_n$, where $n = N-1, \dots, 0$. On the nodes that the coupon is paid, c is the coupon payments and if the coupon is paid between the nodes c is the present value of any coupon payments (c_i) that are paid between the current node and the next node:

$$c = \sum_{t<t_i<t+\Delta t} c_i \ \exp(-r(t_i - t)). \tag{2.6.13}$$

Consequently, the price of the convertible bond at time n is the maximum of the continuation value and parity (conversion value):

$$V_n = \max\left\{C_r S_n, \overline{V_n}\right\}. \tag{2.6.14}$$

2.6.1 The Issuer's Call Option

A call provision gives the issuer the right to call back the bond at the call price (K) outside a period of call protection. The issuer will call the bond when the call price K is less than the continuation value $\overline{V}$. When the bondholder receives the call notice, he should decide whether to convert the bond into shares or to accept the issuer's call and take the early redemption amount. The price of the convertible bond can be written as:

$$V_n = \max\left\{C_r S_n, \min\left(K, \overline{V_n}\right)\right\}. \tag{2.6.15}$$

The call provision usually reduces the price of a convertible bond owing to the fact that it forces the bondholders to invoke their conversion option. Actually, rational investors prefer to convert the bond into shares if the call price K is lower than the parity value. The effects of the call provision is small when the share price is very low or very high. The reason is that at a very low share price the probability that the share price reaching the call trigger is low, so the issuer does not call the bond. At every high share price, the investors will exercise their conversion option since the conversion value is high enough so the call provision does not have any effects.

2.6.2 The Investor's Put Option

The bondholder can sell the convertible bond back to the issuer for cash equal to the put price P_ν during a predetermined period. The bondholders exercise their put option when the value from exercising the put option is greater than the continuation value. The investor will not receive coupon payments on top of the put price. In this case the value of the convertible bond at time n is

$$V_n = \max\left\{P_\nu, \overline{V_n}\right\}. \tag{2.6.16}$$

Generally, a convertible bond can be put, held, converted by the investor, and can be called by the issuer. The convertible bond price at time n will be

$$V_n = \max\left\{C_r S_n, P_\nu, \min\left(K, \overline{V_n}\right)\right\}. \tag{2.6.17}$$

As a matter of fact, the put option reduces the life of the instrument but in investor's favour. At low share prices, the put option is more valuable since there is a high probability that the put option is exercised by the bondholder so they will receive their money sooner.

The interest rates, the coupon level and the volatility of the underlying share have influence on the put option. High interest rates cause the put option to be more valuable, all else equal, due to the fact that put option shortens the life of the convertible bond and thus reduces the discounting factor. A put option is worth less when a convertible bond has high coupons, all else equal, since exercising the put will cancel the future coupons.

At each node that the accrued interest (Acc) is applied, the price of the convertible bond is:

$$V_n = \max\left\{C_r S_n + Acc, P_\nu, \min\left(K + Acc, \overline{V_n}\right)\right\}, \tag{2.6.18}$$

where Acc can be calculated by equation (2.3.1). At the nodes that a coupon c is paid then the accrued interest drops to zero, the price will be:

$$V_n = \max\left\{C_r S_n + c, P_\nu, \min\left(K + c, \overline{V_n}\right)\right\}. \tag{2.6.19}$$

Equations (2.6.18) and (2.6.19) are used to price the convertible bond with call and put features.

2.6.3 An Example

We illustrated how to use the binomial method to price a simplified convertible bond. In this section we calculate the price of the hypothetical convertible bond to demonstrate the binomial method. Table 2.1 shows the description of the convertible bond. We choose a convertible bond with two years call protection and an active put option in order to make it similar to a real world bond.

Table 2.1: Characteristics of the sample convertible bond

Face Value	\$100
Coupon	4.5% Annually
Maturity	5 years
Conversion ratio	0.8
Conversion	Life of the bond
Call	The bond can be called in the third year with the call price equal to 100
Put	The bond can be put in the second year with the put price equal to 108
Current stock price	\$100
Dividend yield (q)	2%
Volatility (σ)	18%
Interest rate	3%

Table 2.2 shows the stock prices tree with u equal to 1.1972 for each node and the time steps between two consecutive nodes is one year ($\Delta t = 1$). On an optional and forced conversion, the investors receive the accrued interest and they will get the coupon payment if it is paid on a call date or conversion date. Conversion values are calculated in Table 2.3.

Now we look at some specific nodes in the tree of convertible bond prices which is represented in Table 2.4.

Point A in Table 2.4 is an example of the maturity node where the investor chooses to convert their bonds into common stock because the conversion value plus coupon is greater than the redemption plus coupon.

In point B, the value of the convertible bond is equal to its continuation value since the continuation value ($\overline{V}_n = 123.3818$) is larger than the conversion value plus coupon

Table 2.2: 5-Step stock price binomial tree

t	1	2	3	4	5
					245.9603
				205.4433	
			171.6007		171.6007
		143.3329		143.3329	
	119.7217		119.7217		119.7217
100.0000		100.0000		100.0000	
	83.5270		83.5270		83.5270
		69.7676		69.7676	
			58.2748		58.2748
				48.6752	
					40.6570

Table 2.3: 5-Step conversion values binomial tree

t	1	2	3	4	5
					201.2682
				168.8547	
			141.7805		141.7805
		119.1664		119.1664	
	100.2774		100.2774		100.2774
80.0000		84.5000		84.5000	
	71.3216		71.3216		71.3216
		60.3141		60.3141	
			51.1199		51.1199
				43.4402	
					37.0256

Table 2.4: 5-Step convertible bond price binomial tree

t	1	2	3	4	5
					(A)201.2682
				169.9672	
			(C)141.7805		141.7805
		123.3818		(B)123.3818	
	116.5163		(D)104.5000		104.5000
(F)109.4554		(E)108.0000		105.9116	
	109.3081		104.5000		104.5000
		108.0000		105.9116	
			104.5000		104.5000
				105.9116	
					104.5000

($C_r S + c = 119.1664$) thus, the bondholders hold a convertible bond for one more step.

Point C in Table 2.4 is chosen to illustrate a forced conversion. The bondholders opt to covert the convertible bond into shares instead of accepting the call price offered by the issuer because the stock price at this node is 171.6007 which leads to a conversion value equal to 141.7805 ($= C_r S + c$) while the call price plus coupon is 104.5 and the continuation value is 146.066.

In point D the bond gets called by the issuer since the call price ($K = 104.5$) is less that the continuation value ($\overline{V}_n = 115.4683$). The rational investor accepts the call offer instead of converting to shares and receiving the conversion value of 100.2774 ($= C_r S + c$).

Point E in Table 2.4 illustrates that the investor puts back the bond to the issuer and gets the put price is equal to 108 since it is more economical to excise the put option rather than holding the bond for one more year ($\overline{V}_n = 105.9116$).

Finally, point F in Table 2.4 which is equal to 109.4554 represents the time zero price of the convertible bond.

2.7 The Announcement Effect

As mentioned earlier, the issuance of the convertible bonds is associated with the negative impact on the stock price, which in finance is measured by the concept of an "abnormal return".

2.7.1 Abnormal Return

An abnormal return is the difference between the actual return on a stock and the expected return from market movements (normal return). It is crucial measure to evaluate the impact of news that directly affect the stock price. The idea of this measure is to isolate the effect of the event from other general market movements. The abnormal return is measured by using the CRSP (Center for Research in Security Prices) equal-weighted index as a proxy for the market return. The daily abnormal return for each convertible bond issue i can be written as:

$$AR_{i,t}^{CRSPEW} = R_{i,t} - R_t^{CRSPEW}, \tag{2.7.1}$$

where $R_{i,t}$ is the stock return on day t obtained from CRSP, and R_t^{CRSPEW} is the total return to the CRSP equal-weighed index on day t [1].

2.7.2 The Effect of Convertible Bond Arbitrage on Abnormal Return

Duca et al. (2010) analysed the abnormal returns from the announcement of the convertible bond in three periods. In the first period (1984 to 1999), where the buyer of the convertible bonds were mostly long-only investors, the average abnormal stock return is -1.69% while in the second period (2000 to 2008) this abnormal return declines to -4.59%, because of investors shifting to convertible arbitrage funds. In the third period (2008 to 2009), when hedge funds played a minor role in the convertible bond market, they ob-

[1]The convertible bond issuers are usually small to medium size firms so the equal-weighted index is a suitable benchmark.

served a large decrease in the abnormal returns around the announcement date (-9.12%) which was attributed to the high underpricing of the convertible bonds in that period. Finally, their empirical research shows that there is no sign of decrease in equity and straight debt announcement returns during the past decade.

2.7.3 The Determinants of Concurrent Transactions

According to Henderson and Zhao (2013), since 2005, over 60% of the firms which issue convertible bonds have conducted at least one of the following transactions at the same time with their issuance of convertible bonds: a share repurchase program, call options, warrants, a seasoned equity, a share lending program. A share repurchase program is a plan announced by the issuer to buy back a specified number of its shares through the underwriter. The call options or the hedge are purchased by the issuer on their own stock. The warrants are sold to the underwriter by the issuer. Typically, firms combine a call option purchase with a sale of warrant which creates the call-spread overlay. A seasoned equity offering is an issuance of new equity by the company that its securities are already traded in the secondary market. A share lending program is the program which issuer lends a specified number of its shares to convertible bond arbitrageurs through underwriter to facilitate short selling in their own stock and enable them to hedge their position in convertible bonds.

An issuer that conducts share repurchase and call option purchase program needs to use on average 41.1% and 20.2% of the proceeds of the convertible bond respectively to pay the cost of these transactions,[2] see de Jong et al. (2011). Additionally, they reduce the dilutive effect on earnings per share while the other concurrent transactions raise funds and increase the impact of dilution.

Henderson and Zhao (2013) argue that the supply of capital by convertible bond arbitrageurs plays an important role in the convertible bond security design and the determinants of issuers' use of concurrent transactions owing to the fact that convertible bond arbitrageurs purchase the majority of newly issued convertible bonds. When firms en-

[2] Issuers usually reduce its cost to around 12% by combining the call option with the sale of warrant.

counter restricted capital supply they issue equity-like convertibles which is the preference of arbitrageurs and buy call options to mitigate the effects of dilution. Their empirical research established that the average abnormal returns for issuers conducting share repurchase and call option purchase are less negative (-1.9% and -2.37% respectively) compared to the issuers who do not use concurrent transactions (-4.78%). Moreover, the average abnormal returns are more negative for issuers conducting transactions which increase the effect of dilution.

2.7.4 The Convertible Arbitrage Hedging Technique

Convertible bond buyers can use three different hedging techniques to reduce the risk of purchasing the convertible bonds: a delta-neutral hedge, an under-hedge, or an over-hedge. In a delta-neutral hedge the theoretical delta of the convertible bond is used to calculate the number of shares of stock that the investors should short sell to hedge a long position in the convertible bonds. In other words, the position is arranged so that no profit or loss is produced from small stock price changes in the underlying asset of the company. In an under-hedge the convertible bond holders sell short fewer shares than implied by the theoretical delta. In fact, they believe there is a higher probability for the underlying asset to increase in value than to decrease. As such, this strategy is also called a bull hedge. In an over-hedge the holders of the convertible bond short more shares than calculated by the theoretical delta since they see more risk on the downside. Therefore, this strategy is also labelled a bear hedge.

According to de Jong et al. (2011) and Duca et al. (2010), convertible arbitrageurs use a delta-neutral hedging technique to calculate the number of shares that want to short sell after the announcement of convertible bonds. The expected number of shares shorted can be determined as follows:

$$\text{Expected number of shares short} = \frac{N_{CB} \times F \times \Delta}{Cp}, \tag{2.7.2}$$

where N_{CB} is the number of convertible bonds that will be issued, F is the face value of the convertible bond, Δ is the theoretical delta of the convertible bond and C_p is the conversion price. By using equation (2.3.2), we can rewrite the equation (2.7.2) as

$$\text{Expected number of shares short} = N_{CB} \times C_r \times \Delta, \tag{2.7.3}$$

where C_r is the conversion ratio. The number of convertible bond (N_{CB}) can be determined by dividing the offering proceeds by the face value of the convertible bond.

The convertible bond's delta measures the change in the price of convertible bond with respect to the change in the stock price:

$$\Delta = \frac{\partial P_{CB}}{\partial S}. \tag{2.7.4}$$

In order to determine the delta of the convertible bond, recall that the price of the convertible bond can be written as:

$$P_{CB} = B + Call, \tag{2.7.5}$$

where B denotes the price of the fixed rate bond and $Call$ denotes the price of the call option. By taking derivatives on both side of the equation (2.7.5) with respect to the stock price, we have:

$$\frac{\partial P_{CB}}{\partial S} = \frac{\partial Call}{\partial S}, \tag{2.7.6}$$

since the fixed rate bond price does not depend on the stock price.

Consequently, the delta of equation (2.7.4) in continuous time model can be written as:

$$\Delta = e^{-\delta T}\Phi(d_1) = e^{-\delta T}\Phi\left\{\frac{ln(\frac{S}{C_p}) + (r - \delta + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}\right\}, \tag{2.7.7}$$

where δ is the continuously compounded dividend yield, T is the time to maturity of the convertible bond, S is the stock price 5 days prior to the announcement date, C_p is the conversion price, r is the risk-free rate, σ is the annualized stock volatility, and $\Phi(.)$ is the cumulative standard normal probability distribution.

The delta of a convertible bond within the binomial framework can be written as:

$$\Delta = \frac{V_1(H) - V_1(T)}{S_1(H) - S_1(T)}, \tag{2.7.8}$$

where $V_1(H)$ and $V_1(T)$ are the prices of the call option at first step of the binomial tree when the stock price are $S_1(H)$ and $S_1(T)$ respectively. Equation (2.7.8) approximates equation (2.7.6) as the time discretization goes to zero.

The delta of the convertible bond takes value between 0 and 1. The closer the delta to 1, the more the convertible bond is equity-like because the bond price is more sensitive to the changes in the stock price, which implies a higher conversion probability. On the other hand, the convertible bond is more debt-like when its delta is closer to 0. Loncarski et al. (2006) found more downward pressure on the stock price between the announcement and issue date of convertible bond for equity-like convertible bonds compared to debt-like convertible bonds. This is explained by the fact that more stock needs to be shorted by convertible bond arbitrageurs for equity-like convertible bond issues since they have a higher delta.

2.7.5 Determining the Expected Stock Price Drop on the Announcement Date

In Section 2.7.4, we determined the total expected number of shares that convertible bond arbitrageurs will short sell between the announcement and the issue date, which is usually one day when the issue is structured as a Rule 144A offerings, and after issuing of the convertible bonds. This short selling activity absorbs market liquidity of shares and causes a drop in the stock price before and after the issue date.

In order to price the convertible bond more accurately we would like to incorporate in our model the expected stock price drop before the issue date that affects the time zero price of the stock on the binomial tree. The stock price at time zero on the binomial tree equals the stock price before the announcement minus the expected stock price drop on the announcement date.

In this section, we model this drop using the Almgren and Chriss (2000) model. However, there is no need to adjust our pricing model of convertible bonds to take into account the stock price drop after the issue date since it is absorbed in the stock price tree.

Almgren and Chriss (2000) consider an agent that wants to sell X units of a security in N steps before time T, with the goal of minimizing a combination of volatility risk and liquidation costs, and defines an optimal trading strategy which minimizes the expected cost. Define the discrete times $t_k = k\tau$, for $k = 0, ..., N$ and $\tau = \frac{T}{N}$. An optimal trading strategy specifies how many units of a security are sold between times t_{k-1} and t_k, denoted by n_k, with

$$X = \sum_{k=1}^{N} n_k. \tag{2.7.9}$$

We should consider two kinds of market impacts on the price of the security that is to be liquidated. First, permanent impact refers to the changes in the equilibrium price owing to our selling and it persists during the liquidation period. Second, temporary impact reflects short horizon imbalances in supply in demand as a result of our selling. When the number of units of the security that we want to sell at each period is large, the price of the security will fall constantly in this period because of using the supply of liquidity. However, liquidity will return in the next period so this effect is temporarily.

We express the permanent impact by introducing a permanent price impact function $g(\nu)$ which is a function of $\nu = \frac{n_k}{\tau}$ and represents the average rate of trading during the period of t_{k-1} and t_k. In this case the stock price is assumed to follow the following dynamics.

$$S_k = S_{k-1} + \sigma\tau^{1/2}\xi_k - \tau g(\frac{n_k}{\tau}), \tag{2.7.10}$$

for $k = 1, ..., N$, where S_k denotes the stock price after the sale of n_k shares of stock, S_{k-1} denotes the stock price before the sale of n_k shares of stock, σ denotes the stock volatility, and ξ_k are independent random variables with a standard normal distribution. For simplicity, we take the permanent impact function as a linear function of ν

$$g(\nu) = \gamma\nu. \tag{2.7.11}$$

By substituting equation (2.7.11) into equation (2.7.10) we have

$$S_k = S_{k-1} + \sigma\tau^{1/2}\xi_k - \gamma n_k, \tag{2.7.12}$$

where γ is a fixed cost and equals to:

$$\gamma = \frac{\text{Bid-ask spread}}{\alpha \times \text{Average daily trading volume}}, \tag{2.7.13}$$

and α takes values in $[0.1, 0.9]$, depending on the liquidity of the underlying stock. For highly liquid stock[3] α is close to 0.1.

Similarly, we have a temporary price impact function $h(\nu)$. The actual stock price after selling n_k shares of stock is:

$$S_k = S_{k-1} - h(\frac{n_k}{\tau}). \tag{2.7.14}$$

For linear temporary impact, $h(\frac{n_k}{\tau})$ takes the form

$$h(\frac{n_k}{\tau}) = \varepsilon\ sgn(n_k) + \frac{\eta}{\tau}n_k, \tag{2.7.15}$$

where sgn is the sign function, ε is a fixed part of the temporary cost, such as one half of the bid-ask spread, and

$$\eta = \frac{\text{Bid-ask spread}}{\beta \times \text{Average daily trading volume}}, \tag{2.7.16}$$

where β lies in the interval of $[0.01, 0.09]$, depending on the liquidity of the underlying stock. The more liquid the stock, the closer β is to 0.01.

Consequently, by applying both permanent and temporary impacts, the stock price after selling n_k shares of stock can written as:

$$S_k = S_{k-1} + \sigma\tau^{1/2}\xi_k - \tau g(\frac{n_k}{\tau}) - h(\frac{n_k}{\tau}). \tag{2.7.17}$$

[3]Highly liquid stocks have a high daily trading volume and narrow bid-ask spreads.

In our model the convertible arbitrageurs need to short sell X units of stock which can be calculated from equation (2.7.3) after the announcement of the convertible bonds. As we mentioned earlier, we only need to calculate the expected stock price drop on a daily basis (so $\tau = 1$), between the announcement and issue date. Therefore, in the Almgren and Chriss (2000) model, we should incorporate the drop on the first selling day if there is only one day between the announcement and issue date. The question is how many shares the arbitrageurs short sell between the announcement and issue date.

The largest fraction of short selling activities takes place after convertible arbitrageurs are actually able to buy the convertible bonds on the issue date, rather than the announcement date, since they do not know the parameters of the issue on the announcement date. Consequently, they cannot accurately calculate the number of shares that they want to short sell. Moreover, convertible arbitrageurs want to minimize the price impact of their short selling activities by not short selling a large fraction of shares in short period of time between the announcement and issue date, as the proceeds from the short sales are an important aspect of their hedging and investment strategy.

2.8 Cost of Capital

The cost of capital is the cost that the company must bear to raise capital. In this section, we discuss the cost of convertible bond issuance for the company and compare it with the cost of debt and equity financing.

The cost of debt is the interest rate that the company must pay for borrowing. There is an issue here that needs some attention. The interest payments are tax deductible, thus the cost of debt after tax can be calculated as:

$$R_B = \text{Yield to maturity of debt} \times (1 - \text{Tax rate}). \tag{2.8.1}$$

The cost of equity is more complicated since equity capital does not have an explicit cost. It is usually defined as the return stockholders expect from their investment in a company. If the stockholders do not receive a satisfactory return they will sell their shares and the stock price of the company will fall. Under the capital asset pricing model, the cost of equity capital can be estimated as

$$R_S = R_F + \beta \times (R_M - R_F), \tag{2.8.2}$$

where R_F is the risk-free rate, R_M is the expected return on the market portfolio, $R_M - R_F$ is the market risk premium, and β measures the expected return on the stock based on the stock's risk.

Many firms use a combination of debt and equity to finance their investments. The weighted average cost of capital (WACC) can be used to estimate their overall cost of capital as

$$R_{WACC} = \left(\frac{S}{S+B}\right) \times R_S + \left(\frac{B}{S+B}\right) \times R_B, \tag{2.8.3}$$

where S is the market value of the firm's equity, B is the market value of the firm's debt, R_S is the cost of equity, and R_B is the cost of debt.

According to Ross et al. (2009), there is a myth that firms can reduce the cost of financing by issuing convertible debt, compared to issuing straight debt, since convertible debt pays a lower coupon rate than the equivalent straight debt. In fact, this analysis does not take into account the call option embedded in the convertible bond.

If we consider a situation where the stock price of the company eventually rises above the conversion price and a conversion will take place, then the firm has to sell its stock to the convertible holders at a below-market price. This loss may not be offset by the lower coupon rate on a convertible. Thus, in this situation the convertible debt is a more expensive financing instrument compared to the straight debt. The opposite scenario happens when the stock price remains subsequently below the conversion price for the life of the convertible. In this case the conversion option will be worthless and the firm will benefit from issuing convertible debt due to its lower coupon rate instead of issuing straight debt.

Similarly, we may compare the convertible debt to equity. If the underlying stock of the company rises later (the stock price exceeds the conversion price), the firm will make a profit of issuing a convertible instead of equity, owing to the fact that the company issues common stock at the conversion price, which is higher than the current stock price when issuing convertible; therefore, this causes the lower cost of equity capital. If the company's underlying stock price drops subsequently below the conversion price, it would be better for the company to issue equity instead of a convertible, since the firm makes more money by issuing stock, which is worth more than the later stock price.

To summarize, convertible bonds are not a cheaper source of financing compared to debt or equity and we can not calculate the aggregate cost of convertible debt before the maturity date, because it depends on a future stock price. Moreover, we cannot predict the behaviour of stock prices in an efficient market, as a result we do not know when the bondholders decide to terminate the life of convertible bond and forfeit the future stream of coupon payments by converting the bond into shares.

Chapter 3

Modelling a Defaultable Convertible Bond with Call-Spread

3.1 Credit Risk Model

In this section, we apply the reduced form approach to incorporate the probability that the company will default on its convertible bonds. We also consider the recovery rate for the convertible bond holders in the case of default.

3.1.1 Survival and Default Probabilities

Definition 3.1. The **survival probability** is the probability that the convertible bond does not default before time t, it is denoted by $p_s(t)$.

Definition 3.2. The **default probability** is the probability that the convertible bond issuer goes bankrupt from time zero until time t, it is denoted by $1 - p_s(t)$.

Definition 3.3. The **conditional probability of default** is the probability that the issuer defaults on the convertible bond in the time interval between t and $t + \Delta t$, given its survival until time t. It can be written as:

$$\frac{p_s(t) - p_s(t + \Delta t)}{p_s(t)} \approx \log(p_s(t)) - \log(p_s(t + \Delta t)). \tag{3.1.1}$$

On the other hand, based on the intensity-based credit modelling or the reduced form model used by De Spiegeleer and Schoutens (2011), the conditional probability of default can be calculated in terms of default intensity $\lambda(t)$, if we suppose that the arrival time of default follows a Poisson process with a mean arrival rate $\lambda(t)$. The conditional probability that the company defaults in the time interval between t and $t+\Delta t$, conditional on survival until time t, is equal to $\lambda(t)\Delta t$. Therefore, in the limit and by using equation (3.1.1) we can write:

$$\lambda(t)dt = -d\log(p_s(t)) \tag{3.1.2}$$

or

$$-\lambda(t) = \frac{d\log(p_s(t))}{dt}. \tag{3.1.3}$$

By taking an integral and exponential of both sides we have:

$$p_s(t) = \exp(-\int_0^t \lambda(s)\, ds\ ). \tag{3.1.4}$$

If we assume that the default intensity is constant throughout the life of the convertible bond and independent of the level of the stock price[1], the probability of survival up to time t is

$$p_s(t) = \exp(-\lambda t). \tag{3.1.5}$$

According to De Spiegeleer et al. (2014), the value of λ can be estimated by two approaches:

1. Using the corporate bond yield. In fact, the liquid corporate bond which does not have any embedded option is a good source to deduce the default intensity. The average default intensity per year can be written as:

$$\lambda = \frac{cs}{1 - R_r}, \tag{3.1.6}$$

   where cs is a spread of the corporate bond yield over the risk-free rate (credit spread)

[1] In reality, the conditional default risk is used, which means that when the stock price collapses, the probability that the company goes bankrupt increases so the default intensity should increases too.

and R_r is the recovery rate. The equation (3.1.6) is called the credit triangle.

2. Using credit default swaps the default intensity can be calculated as

$$\lambda = \frac{cr}{1 - R_r}, \tag{3.1.7}$$

where cr is a credit default swap rate and R_r is the recovery rate. The credit default swap (CDS) rate is the coupon rate of the CDS contract which is a credit derivative contract between two counterparties. In fact, the default risk is traded by the credit default swap contract. One counterparty buys protection from the other one to hedge its exposure to the default risk. The buyer of the insurance pays coupon to the seller who promises to protect him in the case of default.

3.1.2 Incorporating the Default Risk in the Binomial Tree

According to De Spiegeleer and Schoutens (2011), in order to integrate a state of default into a binomial tree, we need to add an extra state in the binomial tree at each node like the Figure 3.1. We assume that the stock price falls to zero when the default takes place. Once the stock price reaches zero it can never go up again.

To calculate the price of a convertible bond with the probability of default we follow the same steps for pricing the callable convertible bond described in Section 2.6, but using different risk-neutral probabilities and the continuation value.

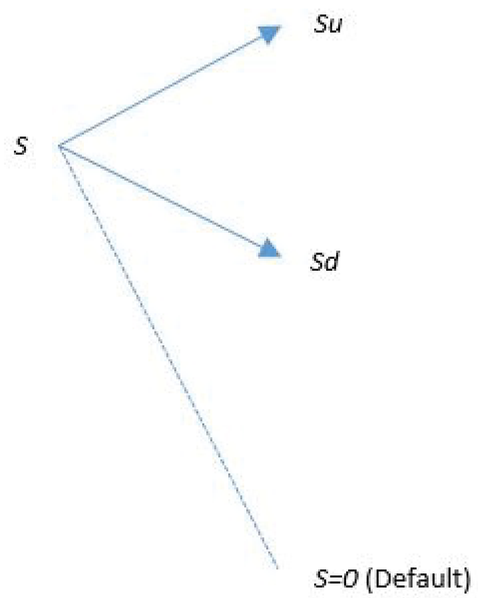
According to De Spiegeleer and Schoutens (2011), in the default-intensity model, the risk-neutral probabilities are conditional on the fact that the stock price does not jump to zero (default situation) and can be determined by imposing the following no-arbitrage condition:

$$S = \exp(-r\Delta t)\; p_s \left[\, \overline{p}\; Su \exp(q\Delta t) + (1 - \overline{p})\; Sd \exp(q\Delta t)\right], \tag{3.1.8}$$

where $p_s = \exp(-\lambda\Delta t)$. Then the probability of an up-move, $\overline{p}$, in the defaultable tree can be obtained from solving the above equation as

$$\overline{p} = \frac{\exp((r + \lambda - q)\Delta t) - d}{u - d}. \tag{3.1.9}$$

Figure 3.1: Stock price on default



The continuation value in every node is the sum of three components using the fact that the value of a derivative security equals the discounted value of the future cash flows:

$$
\begin{aligned}
\overline{V_n}(\omega_1, ..., \omega_n) = &\exp(-r\Delta t)\left\{p_s\left(V_{n+1}(\omega_1, ..., \omega_n H)\overline{p} + V_{n+1}(\omega_1, ..., \omega_n T)(1-\overline{p})\right)\right\} \\
&+ \exp(-r\Delta t)(1-p_s)P_{S=0} + c,
\end{aligned}
\tag{3.1.10}
$$

so $\overline{V_n}$ depends on the first n coin tosses $\omega_1, ..., \omega_n$, $p_s = \exp(-\lambda\Delta t)$ and where $P_{S=0}$ is the recovery value. The first term in equation (3.1.10) is the present value of the expected payoff when there is no default, the second term is the present value of the expected payoff in the case of default, and the third term is the present value of any coupon that is paid out between the nodes and can be calculated from equation (2.6.13).

Equity investors rank after the convertible bond holders so there will be some payoff to the convertible bond investors when the company goes bankrupt. This payoff is generated from selling all of the assets of the company and collecting the money from the owing clients and the cash accounts. Therefore, the convertible bond holders will receive a certain percentage of the risk-free bond floor of the convertible bond in case of default. This percentage is called the recovery rate (R_r) and it depends on the seniority of the convertible bond[2]. We can rewrite equation (3.1.10) as follows:

$$\overline{V_n}(\omega_1, ..., \omega_n) = \exp(-r\Delta t)\left\{p_s\left(V_{n+1}(\omega_1, ..., \omega_n H)\overline{p} + V_{n+1}(\omega_1, ..., \omega_n T)(1-\overline{p})\right)\right\} \\ + \exp(-r\Delta t)(1-p_s)(R_r B_{F,n+1}) + c. \tag{3.1.11}$$

In order to calculate the convertible bond price we must create a binomial tree for the risk-neutral value of the bond floor.

1. At maturity the risk-neutral value of the bond floor equals:

$$B_{F,N} = R + c, \tag{3.1.12}$$

   where R is the redemption value at maturity and c is the coupon payment.

2. Moving back through the tree, the value of the risk-free bond floor at each node from maturity to the valuation date can be written as:

$$B_{F,n} = \exp(-r\Delta t)\left[B_{F,n+1}(\omega_1, ..., \omega_n H)\overline{p} + B_{F,n+1}(\omega_1, ..., \omega_n T)(1-\overline{p})\right] + c, \tag{3.1.13}$$

for $n = N-1, ..., 0$.

3.1.3 Other Ways to Integrate Credit Risk

De Spiegeleer and Schoutens (2011) state that the advantage of the default intensity model, which was described in the previous section, is that it is linked to the Poisson-based default process by only one parameter λ.

De Spiegeleer and Schoutens (2011) argue that the credit spread is a fixed-income parameter and is created by the default intensity. Therefore, they do not agree to incorporate the credit spread as an input into the binomial model. The following discussion is about the structure of these models; the convertible bond price has two parts in such

[2] The recovery rate of a senior secured bond is higher than that for senior unsecured and subordinated bonds.

models:

$$V = V_{\text{No default}} + \alpha(\text{Spread impact}). \tag{3.1.14}$$

The first term in equation (3.1.14) is the price of a default free convertible bond where the risk-free rate is used as the discounting rate. The second part is a fraction of the credit spread impact on the bond part of the convertible bond. The spread impact is

$$\text{Spread Impact} = B_{\text{Risky rate}} - B_{\text{No default}}.$$

A risky rate $(r + CS)$ and a risk-free rate (r) are used as discount rates to calculate the risky and risk-free bond parts of the convertible bond, denoted by B, respectively. Various possible values of α are used. First, for the full impact $\alpha = 1$ is used. Second, the delta of the convertible is used ($\alpha = 1 - \Delta$), since as discussed in Section 2.7.4, the Δ of the convertible bond expresses the probability that the conversion option is exercised in the convertible bond. Therefore, it has influence on the discounting process. Third, the conversion probability (p_{conv}) is used. Bardhan et al. (1994) established a method where a hybrid discount rate is used in the binomial tree with a different probability of conversion at each node. Some nodes are more like a corporate bond, so there is no possibility of conversion. Hence the credit spread plus a risk-free rate is used to discount these nodes to the previous nodes. However, a risk-free rate is used as a discount rate at the node with an equity-like characteristic ($p_{conv} = 1$). Generally, the following discount rate is used at each node:

$$\text{Discount rate} = \text{Risk-free rate} \times (p_{conv}) + (\text{Risk-free rate} + \text{Credit spread}) \times (1 - p_{Conv}). \tag{3.1.15}$$

Finally, Tsiveriotis and Fernandes (1998) developed a method where a convertible bond has two parts: a cash only part, which uses the credit spread as a discount rate, and stock only part where the cash flows are discounted using the risk-free rate.

3.2 Pricing the Hedge

Purchasing the hedge entitles the issuer of the convertible bond to buy back its common shares upon conversion. In this section we formulate the price of the hedge when the issuer has the right to call back the convertible bonds after the call protection period. We make the following assumptions:

Assumption 1. *We assume that the following assumptions are satisfied in our financial market:*

(a) Capital markets are perfect and efficient which means that there are no transaction costs or taxes. Additionally, all investors have equal access to information.

(b) The conversion ratio is constant over the life of the convertible bond.

(c) Holders of the issuer's common stock do not receive any dividend payments.

(d) The term structure of the risk-free interest rate is not stochastic.

(e) When the issuer calls the convertible bond the bondholders do not have time to make a decision on converting their bonds or receiving the call price (no call notice period).

(f) The convertible bondholders and the issuer always try to maximize their own wealth which are the convertible bond price and underlying stock price respectively.

(g) The convertible bond holders and issuer act rationally and each party expects the other party to make an optimal decision.

(h) There are no arbitrage opportunities.

The optimal conversion strategy of the convertible bond investor is given by the following theorem.

Theorem 3.1. *If Assumptions 1. (a), (b), and (c) hold, it is optimal for the convertible bond holders not to convert their callable convertible bonds voluntarily, except at maturity or the call announcement.*

Theorem 3.1 was proved by Ingersoll (1977) and Feng et al. (2015). Ingersoll (1977) considered the callable convertible bond as a contingent claim on the value of the firm, while Feng et al. (2015) considered it as a derivative on the stock price.

It is optimal for the issuer to announce a call prior to maturity when the conversion value reaches the call price, $C_r S_t = K$ or, by using equation (2.3.2), where the stock price reaches the level

$$S_t = (\frac{K}{F})C_p. \tag{3.2.1}$$

If the stock price does not perform well, the issuer will not announce the call and the bondholders will hold the convertible bond until maturity. At maturity, if the conversion value $(\frac{F}{Cp})S_t$ is higher than the face value plus coupon $(F + c)$ the bondholders will convert their bonds, i.e. $S_T > (1 + \frac{c}{F})C_p$. We call $(1 + \frac{c}{F})C_p$ the adjusted conversion price.

Theorem 3.2. *If Assumptions 1. (a), (d)-(h) hold, the issuer of callable convertible bonds should call the convertible bonds back immediately after the underlying asset price reaches* $S_t = (\frac{K}{F})C_p$.

Theorem 3.2 was proved by Feng et al. (2015). Ingersoll (1977) also proved the theorem in a similar way to that used for the above theorem.

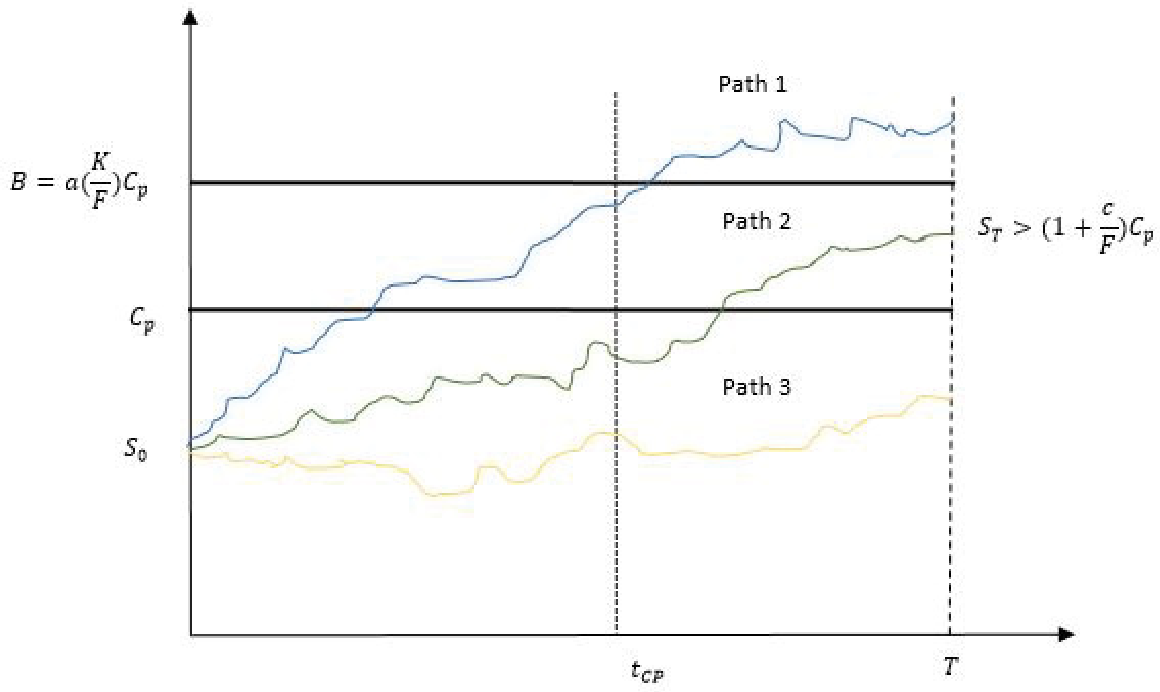
In practice, the issuer announces the call to force the bondholders to convert. For this reason the issuer always delays calling the convertible bond until the conversion value goes well above the call price since there is a chance that the stock price could decrease during the call notice period and below the call price which would deter the bondholders from converting. Therefore, the issuer will announce the call as soon as the stock price reaches

$$S_t = a(\frac{K}{F})C_p, \tag{3.2.2}$$

for some value $a > 1$.

Based on the optimal conversion strategies and the optimal call policies, given by Theorem 3.1 and Theorem 3.2 respectively, the underlying stock price can follow only three paths as shown in Figure 3.2. First, if the stock price reaches $S_t = a(\frac{K}{F})C_p$ prior to maturity, the issuer will call back the convertible bond and the bondholders will convert. Second, if the stock price does not reach $S_t = a(\frac{K}{F})C_p$ prior to maturity but it goes above the adjusted conversion price at maturity, the bondholders will convert voluntarily. Third, if the stock price does not reach $S_t = a(\frac{K}{F})C_p$ prior to maturity and does not exceed the adjusted conversion price at maturity then the bondholders will receive the face value plus coupon at maturity.

Figure 3.2: Paths of stock price



The exercise of the hedge is triggered by the conversion of the convertible bond. In fact, the hedge has positive payoff only if the convertible bonds are converted by the bondholders. As a result, the hedge is exercisable if the stock price follows the first and second paths in Figure 3.2.

If the stock price follows the first path the payoff to the hedge is similar to the payoff of an Up-and-In American call partial barrier option, since the convertible bond can be called only after call protection period. In this case the hedge has positive payoff when the stock price hits the barrier after call protection period. That is, if the stock price follows a path similar to path 1 in Figure 3.2 the payoff is

$$\max (S_t - C_p, 0) \quad \text{for} \qquad S_t \geq B, \quad t_{CP} < t \leq T, \tag{3.2.3}$$

where t_{CP} is the call protection period and $B = a(\frac{K}{F})C_p$ is a barrier.

If the stock price follows the second path, the payoff to the hedge is similar to the payoff of an Up-and-Out European call partial barrier option. In this case the payoff of the hedge is given by

$$\max (S_t - C_p, 0) \quad \text{for} \qquad S_t < B, \quad t_{CP} < t \leq T. \tag{3.2.4}$$

In all other cases the payoff of the hedge is equal to zero.

Consequently, the hedge price can be written as:

$$Call = Call^{Up-In} + Call^{Up-out}, \tag{3.2.5}$$

where $Call^{Up-In}$ is the price of an Up-and-In American call partial barrier option and $Call^{Up-out}$ is the price of an Up-and-Out European call partial barrier option.

3.2.1 Pricing UP and IN American Call Option with Monte-Carlo Simulation

In this section we use Monte-Carlo simulations to price the barrier options, and not the binomial method, since the binomial method is path dependent and when we increase the number of steps on the binomial tree the calculation time increases exponentially. Furthermore, according to Derman et al. (1995), the binomial method is not an accurate method to price the barrier option especially when the barrier level does not equal one of the stock prices on the nodes. Therefore, we use the method of Longstaff and Schwartz (2001) to price the Up-and-In American call option. The steps of the algorithm are as follows:

1. To generate the M paths of stock prices, each including $N + 1$ prices, we can use

the following equation:

$$S_{i,j+1} = S_{i,j} \exp\left[(r - \sigma^2/2)(t_{j+1} - t_j) + \sigma\sqrt{t_{j+1} - t_j} Z_{i,j}\right], \tag{3.2.6}$$

for $i = 1, ..., M$ and $j = 1, ..., N$. Here $Z_{i,j}$ denotes the standard normal random variable.

2. We need to start from maturity ($T = t_{N+1}$) since the algorithm is recursive backward in the time. The payoff of the call option at maturity ($\max(S_T - K, 0)$) has to be calculated for each generated stock price path.

3. To find the optimal exercise time, we move backwards in time and only consider the paths that are in the money, to better estimate the conditional expectation function and improve the efficiency of the algorithm. Let X denote the stock price at time $T - 1 = t_N$ and Y denote the corresponding discounted cash flows from time $T = t_{N+1}$, conditional on the fact that the option is not exercised at time $T - 1 = t_N$.

4. We find the conditional expectation function in order to calculate the expected cash flow from continuing the life of the option conditional on the stock price at time $T - 1 = t_N$. To calculate this function we regress Y on constants, X, X^2 and X^3.

5. To find the optimal exercise time, we should compare the value of immediate exercise at time $T - 1 = t_N$ (which is $\max(S_{T-1} - K, 0)$) with the value from continuation (which is calculated by substituting X into the conditional expectation function). It is optimal to exercise early if the value of immediate exercise is greater than the continuation value. If the option is exercised at time $T - 1 = t_N$, the subsequent cash flow at time $T = t_{N+1}$ will become zero owing to the fact that once the option is exercised there are no further cash flows and the option can only be exercised once.

6. Proceeding recursively, we need to examine whether the option should be exercised at time $T - 2$. To do this we repeat steps 3-5. The key to calculate Y is to discount

back the subsequent cash flows depending on their time, and since the option can only be exercised once, the future cash flows occur only at one of the subsequent times.

7. After identifying the cash flows generated by exercising the option at each date along each path, by working backwards from the maturity date to the first exercise date (t_2), the payoff for each path can be calculated by discounting each cash flow back to time zero (t_1).

8. To find the final payoff, we check the partial barrier condition for each path

$$\begin{cases} \text{if } S_{i,j}.1_{\{t_{CP}<j\leq N+1\}} \geq B & \text{then} \quad Call_i = \text{payoff} \\ \text{Otherwise} & \qquad\qquad Call_i = 0 \end{cases}$$

9. The price of the option can be estimated by taking an average over all paths

$$Call^{Up-In} = \frac{1}{M}\sum_{i=1}^{M} Call_i.$$

The method of Longstaff and Schwartz (2001) has some drawbacks. For instance, it is based on approximating the continuation values by regressing on the basis functions. Therefore, the result depends on the choice of different basis functions. Moreover, the method is also known to be biased.

3.3 Pricing the Warrant

The value of the warrant is equal to the value of a call option multiplied by the dilution factor:

$$W = \left(\frac{N_S}{N_S + N_W}\right) \times \text{Call}, \tag{3.3.1}$$

where N_S is the number of shares outstanding before the exercise of the warrants, N_W is the number of warrants, and Call is the price of a call option with the same strike and maturity as the warrants. Using the binomial tree can be calculated as follows

1. Build a CRR stock price tree with the following parameters

$$\text{CRR tree} = \begin{cases} u = \exp(\sigma\sqrt{\Delta t}) \\ d = \exp(-\sigma\sqrt{\Delta t}) \\ \overline{p} = \frac{\exp((r-q)\Delta t) - \exp(-\sigma\sqrt{\Delta t})}{\exp(\sigma\sqrt{\Delta t}) - \exp(-\sigma\sqrt{\Delta t})} \end{cases}$$

2. Moving backwards in time and computing the value of the call option at each node from the maturity date to the valuation date using the following pricing algorithm:

$$Call_N(\omega_1, ..., \omega_N) = \max\{S_T - K_w, 0\} \tag{3.3.2}$$

$$Call_n(\omega_1, ..., \omega_n) = \exp(-r\Delta t)\left[Call_{n+1}(\omega_1, ..., \omega_n H)\overline{p} + Call_{n+1}(\omega_1, ..., \omega_n T)(1 - \overline{p})\right], \tag{3.3.3}$$

for $n = N-1, ..., 0$ where K_w is a strike price of the warrant.

Now that we have methods for pricing the callable convertible bond, the hedge, and the warrant, we will estimate the proceeds of the company from issuing the convertible bond with call-spread overlay in the next section.

3.4 Pricing Convertible Bond with Call-Spread Overlay

There is a difference between the stock that convertible bondholders will receive upon conversion of the straight convertible bond and the convertible bond with call-spread overlay owing to dilution. In straight convertible bonds the bondholders are delivered diluted stocks of the company which are less valuable than the original stock before issuing new shares. Further, there is a reduction in the earnings per share after new shares are issued.

The value of the diluted stock can be calculated as follows. Assume a company has N_S shares outstanding before exercising the conversion option. If the company issues N_{CB} convertible bonds and each of them can be converted to C_r shares the aggregate number of new shares underling the convertible bonds will be $N_{CB}C_r$. The value of the firm before conversion is

$$V_{\text{Before}} = N_S S_{BC} + N_{CB} C_r C_p, \tag{3.4.1}$$

where S_{BC} is the value of stock before conversion. The value of the firm after conversion becomes:

$$V_{\text{After}} = N_S S_{AC} + N_{CB} C_r S_{AC}. \tag{3.4.2}$$

Therefore, the value of the diluted shares can be obtained by equating the value of the firm before conversion with the value of the firm after conversion, since the total value of the firm does not change on conversion. Setting equation (3.4.1) equal to equation (3.4.2) and solving for S_{AC} we find

$$S_{AC} = \frac{N_S S_{BC} + N_{CB} C_r C_p}{N_S + N_{CB} C_r} = (1-\gamma) S_{BC} + \gamma C_p, \tag{3.4.3}$$

where γ denotes a dilution factor given by

$$\gamma = \frac{N_{CB} C_r}{N_S + N_{CB} C_r}. \tag{3.4.4}$$

For the convertible bond with call-spread overlay we do not have a dilution impact (i.e $\gamma = 0$) because purchasing the hedge offsets the conversion feature entirely. Therefore, the number of shares outstanding does not change on conversion. By substituting $\gamma = 0$ in equation (3.4.3) we can see that the stock price after conversion is equal to the stock price before conversion:

$$S_{AC} = S_{BC}. \tag{3.4.5}$$

Consequently, an investor's gain from exercising the conversion option in the straight convertible bond can be written as

$$S_{AC} - C_p = \frac{N_S S_{BC} + N_{CB} C_r C_p}{N_S + N_{CB} C_r} - C_p = (1-\gamma)(S_{BC} - C_p), \tag{3.4.6}$$

while an investor's gain from exercising the conversion option in the convertible bond with call-spread is

$$S_{BC} - C_p. \tag{3.4.7}$$

Therefore, we can write the gain from straight convertible bonds in terms of the gain from convertible bonds with call-spread by using equation (3.4.6) as follows:

$$\text{Gain from straight CB} = (1-\gamma)\text{Gain from CB with CS}. \tag{3.4.8}$$

By rearranging the above equation, we obtain

$$\text{Gain from CB with CS} = \left(\frac{1}{1-\gamma}\right)\text{Gain from straight CB}. \tag{3.4.9}$$

According to equation (3.4.9), investors earn more from a convertible bond with call-spread than a straight convertible bond. Consequently, the value of the convertible bond that is concurrent with buying the hedges is more than the straight convertible bond.

$$V_0^{\text{CB with CS}} = \left(\frac{1}{1-\gamma}\right) V_0^{CB}, \tag{3.4.10}$$

where V_0^{CB} is the time zero price of the straight convertible bond defined in Section 3.1.2.

The sale of warrants affects the values of a convertible bond with call-spread by changing the parameters of the convertible bond, since it enables the company to issue the convertible bonds with a lower conversion price and coupon interest rate, compared to the straight convertible bonds.

Each component of the convertible bond with call-spread overlay is priced separately. Therefore, the proceeds of this combined product at the time of issuing can be written as:

$$P_{CBCS} = V_0^{\text{CB with CS}} + C_r(W_0 - Call_0), \tag{3.4.11}$$

where $Call_0$ and W_0 denote the hedge and the warrant price that can be calculated from equations (3.2.5) and (3.3.1) respectively.

3.5 Conclusion

In this chapter we incorporated credit risk in our binomial model. We used the optimal conversion strategy and optimal call policies theorems to price the hedge when the issuer has the right to call back the convertible bond after the call protection period; furthermore, we formulate the price of the convertible bond with call spread in terms of the straight convertible bond price. Finally, the proceeds of issuing the convertible bonds with call spread to the issuer is calculated by the equation (3.4.11).

We will illustrate in the next chapter the pricing model developed in this thesis by valuing two hypothetical convertible bonds with call-spread overlay and considering some real world examples.

Chapter 4

Sample Convertible Bonds with Call-Spread

4.1 Numerical Examples

In this chapter we consider hypothetical examples using simulated data to explore some of the features of the model. We also apply the model to some recent issues of convertible bonds with call-spread overlay to test the model with the real data.

We first consider two hypothetical companies with different stock liquidity that plan to issue convertible bonds with call-spread overlay. We examine how arbitrage activities on the announcement date may affect the security design and the decision to issue convertible bonds.

Suppose Company A's common stock has high liquidity which means that it has a large daily trading volume and narrow bid-ask spreads. However, Company B's common stock has low liquidity. A week prior to the announcement of issuing the convertible bonds, the companies would like to determine how much the underlying stock price will drop on the announcement date of issuing convertible bonds. Table 4.1 summarizes the convertible bonds and the characteristics of the companies.

According to the discussion in Section 2.7 the drop in the share price due to arbitrage activities is calculated in Table 4.2 for the two companies based on two extreme scenarios.

Table 4.1: Description of the sample convertible bonds

Issue Characteristics	**Company A**	**Company B**
Stock price (-5 days)	45	45
Face value	1000	1000
Coupon rate	1.8% Semi-annually	1.8% Semi-annually
Maturity	5	5
Conversion ratio	17	17
Conversion	0 to 5 years	0 to 5 years
Call	At year 3 at 1,000	At year 3 at 1000
Put	At year 4 at 1,000	At year 4 at 1000
Dividend yield	0%	0%
Volatility	25%	25%
Interest rate	3%	3%
Conversion price	58.8235	58.8235
Default intensity	6.25	6.25
Recovery rate	30%	30%
Strike of warrant	82	82
Number of CB	350,000	350,000
Issuer Characteristics		
Bid-ask spread	0.03	0.25
Average-daily-trading-vol	2,500,000	300,000
Shares outstanding at issue	92,000,000	92,000,000

In the first scenario arbitrageurs short sell a large fraction (50%) of the expected number of short shares on the announcement date. The second scenario is based on a smaller fraction (20%) of the expected number of shares shorted by the arbitrageurs on the announcement date. Note in Table 4.2 that the stock price of Company A is expected to fall slightly on the announcement date of the convertible bond with call-spread. However, Company B will experience a large stock price drop on the announcement date owing to its low liquid stock. Consequently, it is not optimal for Company B to issue convertible bonds without conducting a share repurchase program which, according to de Jong et al. (2011), mitigates the negative announcement effect of the convertible bonds due to arbitrage activities.

Table 4.2: The stock price drop on the announcement date

	Company A		Company B	
Fraction	0.2	0.5	0.2	0.5
Stock price drop	0.8457	2.0804	6.5112	16.1086
Stock price drop%	1.91	4.84	16.91	55.75
Stock price at issue	44.1543	42.9196	38.4888	28.8914

Table 4.3 shows the result of calculating the price of each combined product at the time of issue for Company A using equation (3.4.11). In fact, the proceeds to Company A after deducting the net cost of the call-spread will be \$329,440,767 and \$327,087,266 (in the best and worst case scenario respectively) from issuing 350,000 convertible bonds with call-spread overlay.

Table 4.3: Price of the combined product

	Company A	
Fraction	0.2	0.5
$V_0^{CB\ with\ CS}$	1,009.35	997.97
C_0	7.4802	6.8667
W_0	3.4748	3.1352
P_{CBCS}	329,440,767.28	327,087,266.95

Figure 4.1: The effect of volatility on the CB with call-spread

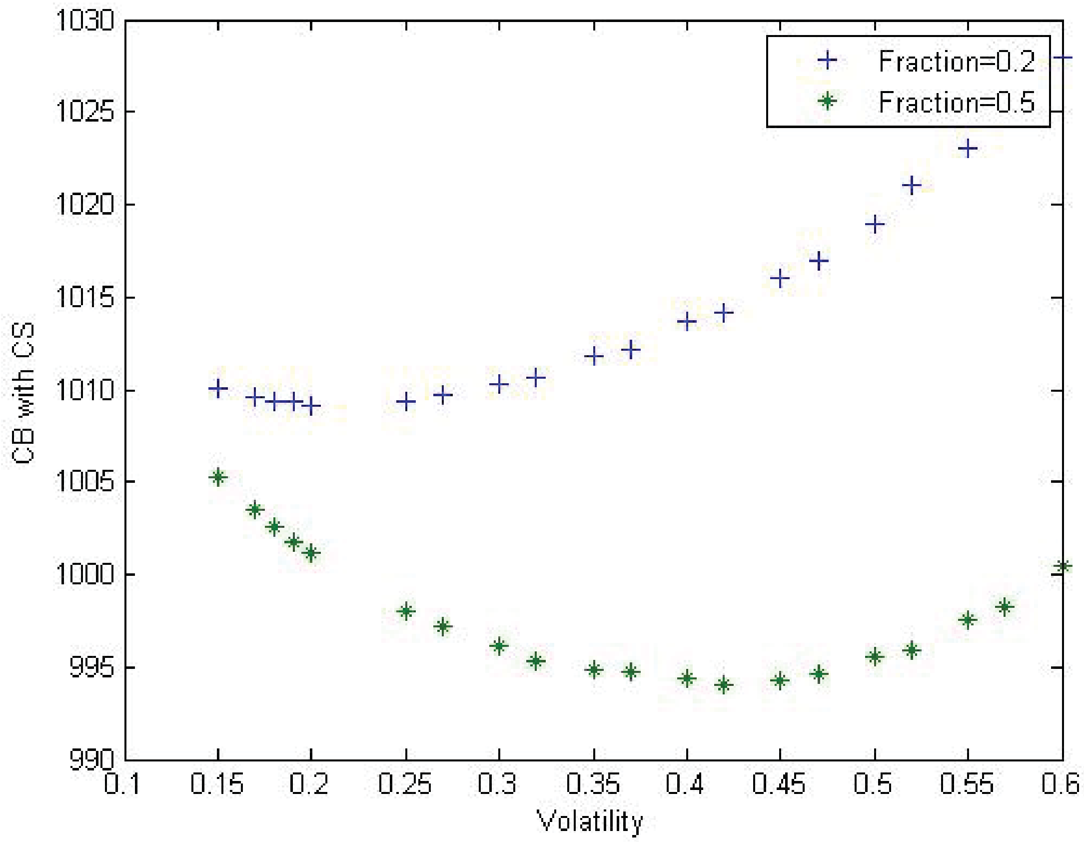


Figure 4.1 demonstrates the effect of the volatility on the theoretical price of the convertible bond with call-spread for the two extreme short selling scenarios. The value

of the convertible bond is calculated using equation (3.4.10). As mentioned in Chapter 1, for the volatile stock the embedded call option will be worth more, all else equal, and push up the price of the convertible bond. Note that in the lower short selling scenario we see the expected increasing relationship between the volatility and convertible bond price. However, in the high short selling scenario the relationship is not strictly increasing as expected since in this case the expected stock price drop on the announcement date is large which causes a lower convertible bond price.

If Company A would like to raise the same amount of money by issuing straight convertible bonds with a reasonable conversion price (i.e. conversion price that is high enough) instead of issuing convertible bonds with call-spread[1], it should increase the coupon rate to compensate the investors for the high conversion price. The coupon rates that need to be paid by the company were calculated in Table 4.4 for different conversion prices. As we can see in Table 4.4 increasing the conversion price should be compensated by increasing the coupon rate. Moreover, by increasing the conversion price the stock price drop on the announcement date is decreasing, since the straight convertible bond with high conversion price is more debt-like and arbitrageurs expect to short fewer shares on the announcement date based on equation (2.7.2).

Table 4.4: Straight convertible bond with different conversion price

Conversion price	82		72		62	
Fraction	0.2	0.5	0.2	0.5	0.2	0.5
Coupon rate	4.45 %	4.25 %	3.76 %	3.63 %	2.27 %	2.31 %
Stock price drop	0.3540	0.8616	0.5143	1.2519	0.7414	1.8438
Stock price at issue	44.6460	44.1384	44.4857	43.7481	44.2586	43.1562

To sum up, the companies with low liquid stock should not issue convertible bonds unless they also conduct a share repurchase program. Moreover, the proceeds from issuing the convertible bonds for the company with volatile stock price depends on the fraction of the expected number of short shares that the convertible arbitrageurs decide to short sell on the announcement date.

[1]The effective conversion price of the convertible bond with call-spread is the strike price of the warrants.

4.2 Real World Examples

In this section we apply our model to price five convertible bonds with call-spread that were issued in the U.S recently. Table 4.5 describes the features and parameters of these convertible bonds.

Table 4.5: Description of the convertible bonds

Company's name	HomeAway	Cornerstone	INVENSENSE	PROS Holdings	LinkedIn
Maturity	April 1,2109	July 1,2018	Nov 1,2018	Dec 1, 21019	Nov 1, 2019
Historical Volatility	0.380261	0.344672	0.481599	0.399505	0.439038
Aggregate principle	350,000,000	220,000,000	150,000,000	125,000,000	1,150,000,000
Over-allotment	52,500,000	33,000,000	25,000,000	18,750,000	172,500,000
CB rank	Sr Unsec.	Sr Unsec.	Sr Unsec.	Sr Unsec.	Sr Unsec.
Coupon rate	0.125% Semi-an	1.5% Semi-an	1.75% Semi-an	0.2% Semi-an	0.5% Semi-an
Face value	1,000	1,000	1,000	1,000	1,000
Conversion ratio	19.1703	18.5046	45.683	29.5972	3.3951
Announcement date	Mar 25,2014	Jun 11,2013	Nov 6,2013	Dec 4,2014	Nov 5,2014
Announcement stock price	39.54	41.54	16.52	30.28	238.43
Issue date	Mar 26,2014	Jun 12,2013	Nov 7,2013	Dec 5,2014	Nov 6,2014
Stock price at issue	38.64	40.03	15.92	25.99	218.18
Stock price drop	0.9	1.51	0.6	4.29	20.25
Conversion price	52.164	54.04	21.89	33.79	294.54
Strike of warrant	81.144	80.06	28.656	45.48	381.82
Call	No call	No call	No call	No call	No call
Put	No put	No put	No put	No put	No put
10-year U.S. treasury bond	0.0275	0.022	0.0267	0.0225	0.0279
Dividend yield	0	0	0	0	0
Default intensity	0.1445	0.1187	0.2319	0.1596	0.1927

Note in Table 4.5 that the stock price of the companies fell after the convertible bonds were announced because of the arbitrage activities. In order to test the arbitrage model that was discussed in Section 2.7, we use the parameters of the Table 4.6 to estimate the drop a week before the announcement of the convertible bonds. In this Table the fraction of shares that the arbitrageurs short sell between the announcement and the issue date is obtained statistically. The results given in Table 4.7 match the observed drops. Moreover, the confidence interval for the expected stock price drop is calculated in Table 4.7.

Table 4.6: Parameters of the arbitrage model

Company's name	HomeAway	Cornerstone OnDemand	INVENSENSE	PROS Holdings	LinkedIn
Stock price 5 days before announcement	43.07	39.7	18.62	27.94	205.35
Bid-ask spread	0.0339	0.0716	0.0148	0.085	0.2988
Average daily trading volume	1,555,367	366,205	2,562,500	113,500	1,956,336
Fraction	0.26	0.25	0.2	0.2	0.5
α	0.3	0.8	0.1	0.9	0.1
β	0.03	0.08	0.01	0.09	0.01

Table 4.7: The result of the arbitrage model

Company's name	HomeAway	Cornerstone OnDemand	INVENSENSE	PROS Holdings	LinkedIn
Expected drop	0.93	1.43	0.62	4.37	20.02
Confidence interval 95%	(0.9098 , 0.9502)	(1.4131 , 1.4469)	(0.6087 , 0.6313)	(4.3562 , 4.3838)	(19.9078, 20.1322)

As mentioned in Section 3.1.1, in order to price the convertible bond we need to deduct the default intensity from the corporate bond issued by the same company or from the credit default swaps data. However, this information is not available for any of these issuers. Therefore, we use the squared volatility (σ^2) as an estimator of the default intensity as Milanov et al. (2013) argue that in practice σ^2 and λ are usually very close or even identical. Moreover, we calculate the issue price of the convertible bonds with call-spread for three different recovery rates 26.5%, 34.4% and 39.3%. They are the first, second and third quartiles of the recovery rates that were reported for senior unsecured corporate bonds by Moody's Investors Service (2013) and Moody's Investors Service (2014),[2] since these five convertible bonds were issued in these years. The results are given in Table 4.8. However, all of these convertible bonds were issued at par so the recovery rates that make the issue price of these convertible bonds equal to \$1000 are presented in Table 4.9.

Table 4.8: Issue price of the convertible bonds

	Recovery rate		
Company's name	26.50	34.40	39.30
HomeAway	1,011.17	1,049.72	1,073.63
Cornerstone OnDemand	1,080.71	1,116.99	1,139.38
INVENSENSE	1,076.16	1,131.00	1,165.06
PROS Holdings	1,113.07	1,157.86	1,185.60
LinkedIn	987.23	1,032.00	1,059.76

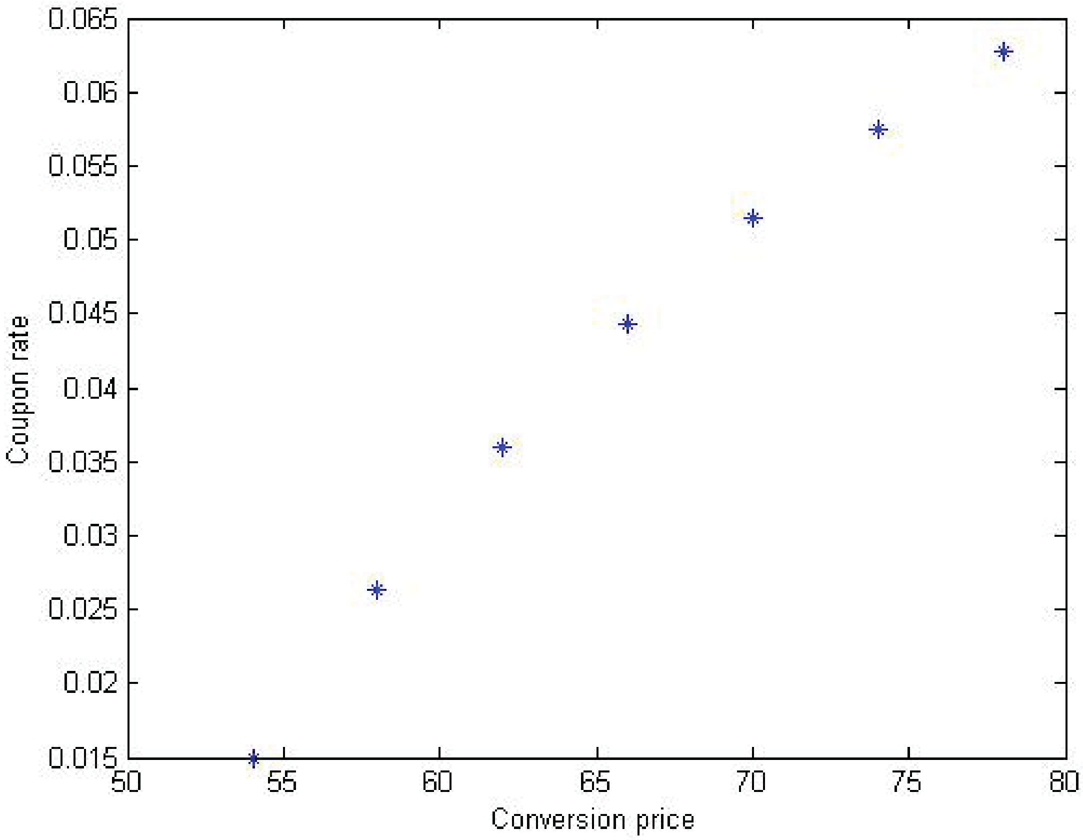
Table 4.9: Calibrated recovery rate

Company's name	HomeAway	Cornerstone OnDemand	INVENSENSE	PROS Holdings	LinkedIn
Recovery rate	24.85 %	8.9 %	15.54 %	6.555 %	28.75 %

Figure 4.2 demonstrates the relationship between the coupon rate and the conversion price for Cornerstone OnDemand company. As we expected, if the company wants to

[2] Moody's Investor Service is one of the most popular credit rating agencies that analyses the credit risks associated with fixed-income and hybrid securities.

Figure 4.2: The relationship between the coupon rate and the conversion price for Cornerstone OnDemand company



sell its convertible bond at par but at higher conversion price, it should compensate the convertible bond holders by increasing the coupon rate.

The price of the hedges and the warrants are calculated in Table 4.10 for each company using historical volatility of the stock price of that company. We use the binomial model to price the hedge which is an American call option since these convertible bonds do not have a call feature. The warrants are European style but cannot be exercised on one day. Warrants are exercisable on a contractually specified finite set of dates. The first expiration of the warrants are three months after the convertible bond's expiration. The number of warrants that can be exercised at each date are specified in the prospectus of each issue.

The calculated prices in Table 4.10 do not match the actual price of the hedges and the warrants that are mentioned in the prospectus of each deal. One possible explanation for

Table 4.10: Model calculated price of the call-spread

Company's name	HomeAway	Cornerstone OnDemand	INVENSENSE	PROS Holdings	LinkedIn
Hedge price	82,180,560	44,450,250	45,311,200	32,816,931	321,189,750
Warrant price	49,725,000	26,087,000	50,650,000	16,546,000	264,030,000

this discrepancy is that we have used the historical volatility and not the implied volatility or volatility surface obtained from option prices. Therefore, we calibrated the value of these options under our model to the reported values and record the implied volatilities in Table 4.11. The other possible reasons of this discrepancy are that we used a constant interest rate in our model, not stochastic interest rates and we did not consider additional costs other than option premiums. Moreover, it is possible that we did not incorporate some additional features in our model.

Table 4.11: Reported price of the call-spread

Company's name	HomeAway	Cornerstone OnDemand	INVENSENSE	PROS Holdings	LinkedIn
Imp volatility (hedge)	0.3947	0.3752	0.424	0.3639	0.3525
Hedge price	85,900,000	49,500,000	39,100,000	29,411,250	248,000,000
Imp volatility (warrant)	0.33667	0.3267	0.37696	0.32133	0.33203
Warrant price	38,300,000	23,200,000	25,600,000	17,106,250	167,300,000

This section provides some test of the models that developed in this thesis. As we saw the results for pricing the convertible bonds and modelling the arbitrage activities that are the focus of this thesis are reasonable, even with the simplifying assumptions.

Conclusion and Future Work

In this thesis we have reviewed different methods of valuing convertible bonds and discussed the reasons for issuing convertible bonds with call-spread from different points of view. We proposed a model to incorporate the expected stock price drop on the announcement of the convertible bond in order to estimate the convertible bond price at the time of designing the security that is going to be issued. In other words, at the planning time the company should choose an appropriate financial instrument based on its cost of capital to finance their new projects. Moreover, we discussed the valuation methods for each component of the convertible bond with call-spread overlay and then combined them to reflect the final price of this financial instrument for the company. We also considered some hypothetical and realistic examples to demonstrate our approach.

We have offered a mathematical model in order to incorporate the convertible bond arbitrage activities into a valuation model for the convertible bond which is different from previous studies, such as Duca et al. (2010) and de Jong et al. (2011) that only gave some empirical evidence of the effect of these arbitrage activities on the abnormal stock return. Moreover, this is the first study that analyses the convertible bond with call-spread overlay and introduces the model to value the hedge when the convertible bonds have a call provision.

This study can be improved by considering all the features of the convertible bond with call-spread which are included in the prospectuses of these deals. For example, we consider a constant conversion ratio for the life of the convertible bond while in the most recent prospectuses the conversion ratio will be adjusted on some specific dates based on the stock price on these dates. The other improvement can be modelling the effect of a share repurchase program on the expected stock price drop on the announcement date. On top of these, specifying the optimal fraction of shares that the arbitrageurs should short sell between the announcement and the issue date in order to hedge their long position on convertible bonds, improving the credit risk model by comparing it with other models, and incorporating the stochastic interest rates are interesting topics for future research.

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