INTEGRATED PRODUCTION-DISTRIBUTION PLANNING UNDER CONGESTION AND CARBON EMISSION CONSTRAINTS

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ABSTRACT

Integrated Production-Distribution Planning under Congestion and Carbon Emission Constraints

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The global warming, which is caused by increasing concentrations of carbon emissions, mainly results from human activities such as fossil fuel burning and deforestation. In order to alleviate global warming and its adverse effects, many countries including the United States and the European Union members have attempted to enact legislation or design market-based carbon trading mechanism for controlling carbon emission. Analyzing the impact of such governmental legislations on supply chain operations has particularly been noticed both in theory and practice. This implies that firms need to incorporate the governmental regulations into their decision making process. This thesis presents an integrated model of production-distribution planning in supply chains considering congestion and carbon emission capacity constraints. The objective of the model is to minimize the sum of production, inventory, and transportation cost subject to emission capacity constraints. Our model adopts a Carbon Cap regulation policy that requires the total carbon emission resulting from production and distribution of commodities from facilities to demand points to be constrained. Considering congestion at the production facilities for work in process (WIP) inventory, which may increase nonlinearly after a certain level of utilization (i.e. critical utilization), leads to a nonlinear multi-period mixed integer program. We then develop a robust approach that captures the uncertainty in estimating the emission of each of the logistic activities. We propose a Lagrangian relaxation approach and a heuristic to build feasible solutions which solves large instances. Finally, computational results on a set of instances are reported to assess the performance of the proposed MIP formulation and of our algorithmic approach.

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Dedicated to my Parents

Contents

List of Figures

List of Tables

List of Acronyms

CC	Carbon Cap
CF	Clearing Function
EU ETS	European Union Emission Trading System
FGI	Finished Goods Inventory
GHG	Green House Gas
GSCM	Green Supply Chain Management
MP	Master Problem
SP	Sub Problem
WIP	Work in Process

Chapter 1

Introduction

1.1. Foreword

It is widely reported that global warming, which has a direct relationship with the emission of carbon and other greenhouse gases (GHG), poses a grave threat to the world's ecological system and the human race. As global warming is expected to have fatal consequences at economic, ecologic, and social levels, it is necessary to reduce GHG emissions so as to prevent or at least reduce global warming. Public awareness toward the destructive impacts of GHGs has been growing significantly over the last few decades. For instance, in a study in European Union, 75% of respondents were willing to pay more for environmental friendly products and 17% had already done so (Eurobarometer 2008). This puts the governments under growing pressure in order to legitimate regulations to control the amount of these emissions. One of the first carbon emission control attempts was made in Kyoto Protocol, launched in 1997, with the aim of reducing GHG emissions caused by industrialized countries. The members of the Protocol has agreed to reduce their emission levels by five percent in the first commitment period, started from 2008 and ended in 2012, with further reduction of 18 percent from 2013 to 2020 compared to their emission level in 1990. The Protocol also obligates the members to report their annual emission inventory to UN Climate Change Secretariat (UNFCCC 1997).

The public awareness toward the destructive effects of GHGs along with the government regulations and the pressure from media force the manufacturers to take actions in pollution control, prevention and resource efficiently, and reduction of their carbon footprint (Carlson and Rafinejad 2011). These actions include investing in green manufacturing technology, reducing the supply chain waste, and increasing the efficiency of the green supply chain. For example, Walmart decreased its carbon emission by 400,000 tons with a small investment in reducing the fuel efficiency in its supply chain (Plambeck 2012). Walmart has also announced its goal to eliminate 20 million metric tons of GHG emissions from its global supply chain by 2015. Hewlett-Packard (HP) announced that it will decrease carbon content of its products by 40% in 2020 compared to its level in 2010 (Hewlett-Packard 2014). IBM has also decreased its emission by 59% from 1990 to 2013 (IBM 2014).

The new environmental friendly regulations may limit the total amount of carbon emitted by the industries. Specifically, it can be in the forms of (*i*) a strict cap, (*ii*) carbon tax, and (*iii*) cap-and-trade (Benjaafar *et al.* 2013). According to strict cap policy, the firms cannot produce more than a certain amount; this is also called *carbon cap*. In a carbon tax policy, firms pay for their emission in terms of a tax. In Norway, for example, the government has implemented carbon taxes based on the tons of carbon emission produced since 1991 (Bruvoll and Larsen 2004). In a cap-and-trade policy, although there is a cap on emission of firms, the firms are allowed to sell or buy the carbon allowances. Consequently, firms are subject to heavy fines if they do not fulfill the carbon allowances. If a firm emit less than its carbon allowance, it can either sell it through the carbon markets or save its allowance for future production. A cap-and-trade system called European Union Emissions Trading System (EU ETS) was initiated in 2005 which is known as the largest cap and trade system in the world. Around 11,000 power stations and manufacturing industry companies responsible for more than 45% of GHGs in Europe are now operating under the EU ETS. As a consequence of this action, emission produced by these firms will be reduced by 21% in 2020 compared to its level in 2005 (European Commission 2005).

Green Supply Chain Management (GSCM) deals with incorporating such regulations into the decision-making process of firms' managers and policy makers. In response to the carbon regulation firms usually choose one of the following options: (*i*) designing new products which need less emission for production, (*ii*) investing in energy efficient machinery and processes, or (*iii*) modifying the existent production processes. The first two options require strategic and long-term decisions as well as significant investment. With the uncertainty about future of environmental regulations, the first two options may seem less interesting, leading the firms to look for appropriate strategies to modify their operational decisions (Heindl and Löschel 2012). This explains why we can find a growing body of research focused in operational level of the green supply chain management (Arıkan and Jammernegg 2014, Battini *et al.* 2014, Cholette and Venkat 2009, Zhang and Xu 2013). The main focus is to explore the impact of government regulations, i.e., strict cap, carbon tax, and cap-and-trade, on firms' operational decisions, such as determining lot size, lead-time, and production planning.

1.2. Goal of the study

The goal of this study is to explore the impact of a strict cap policy on different operational decisions of a production-distribution system. Specifically, we study a production-distribution planning problem where we decide on the demand allocated to each facility and its level of production over a planning horizon. Regarding environmental concerns, we consider emissions produced by the supply chain as a result of manufacturing and distribution activities. Among the three types of environmental regulations that were mentioned earlier, we consider strict cap on total emission of the supply chain. Strict cap can also be a self-imposed emission target that the managers set to limit the firm's emission and decrease it over a period of time. This approach is a common one that already applied by many firms (Battini *et al.* 2014, Hoen *et al.* 2013). One of the interesting features of this research is to consider the congestion that may form in production facility. This enables us to study the effect of congestion levels on carbon emissions.

Another important aspect of this study is the way that we capture the uncertainty in the amount of emission. In reality, having an accurate estimation of the emission is a key factor for the production decision. The methods applied to measure the emission from different production-distribution activities may come with errors. It necessitates the decision makers to develop *robust* approaches to enable them to obtain solutions that work under different carbon-emission scenarios. Therefore, we consider uncertainty in measuring the emission associated with each activity and develop robust solutions that enable the managers to make their decisions with more confidence.

Including the aforementioned features in our model, there are a few issues that need to be addressed in terms of solution methodology. Considering congestion in our model results in a nonlinear mixed-integer programming. To deal with this, we use a linearization approach by approximating the function that relates work in process and the throughput through adding lines tangent to this function at different points. Furthermore, in order to minimize the error cause by this approximation, we employ an outer approximation algorithm that limits the error at the optimal solution.

In order to be able to solve large size instances, a Lagrangian relaxation approach is proposed. Since we are dealing with a minimization problem, the Lagrangian relaxation provides a lower bound on the optimal solutions. By relaxing two sets of constraints, we are able to decompose the problem into a number of single-facility production planning problems. The optimal solution from the Lagrangian relaxation is then used to build a feasible solution.

1.3. Research contributions

The key considerations in this study that contribute to the existing literature are summarized as follows:

- A strict cap on the total emission of the supply chain is considered in an integrated tactical planning model in the context of production-distribution planning while congestion at the production facilities is being considered.
- Uncertainty in estimating the emission of the supply chain is considered that enables us to account for the possible scenarios of uncertainty in emission estimation.
- A solution methodology based on Lagrangian relaxation approach is proposed to deal with large size instances.

1.4. Thesis outline

The remainder of this thesis is organized as follows. In the following chapter, we review the related literature. Chapter 3 provides the model formulation and solution methodology. We first explain the deterministic model and then discuss how we incorporate robustness into our stochastic version of our problem. The Lagrangian relaxation followed by a heuristic approach has been proposed to build a feasible solution. Numerical examples are provided in chapter 4. An illustrative numerical example is first developed to examine the effect of uncertainty on the operational decisions in our problem. We also solve instances with different sizes and parameters to examine the performance of our proposed solution algorithm. Finally, Chapter 5 provides concluding remarks and future research avenues.

Chapter 2

Literature Review

The literature in the green supply chain management include many different types of problems, from economic to operational and marketing perspectives (See Brandenburg *et al.* (2014), Dekker *et al.* (2012), Tang and Zhou (2012), and Wei *et al.* (2014) for an overview of articles in green supply chain management). The effect of environmental regulations on supply chain management can be discussed either from the firms' or policy makers' perspective.

2.1. Policy Maker's Perspective in GSCM

While the objective in most of studies is to minimize the total costs or maximize total profit of a firm under carbon regulations, there are studies which discuss the effect of environmental regulations on decision making of policy makers and governments. The objective function in these studies usually include maximizing social welfare, which can be measured by economic surplus, total carbon emission, or tax revenue (Brännlund and Nordström 2004, Eyland and Zaccour 2014, Huang *et al.* 2013, Krass *et al.* 2013). Brännlund and Nordström (2004) study the effect of environmental policies on the consumer response using a simulation method. They compare two scenarios where the revenues from doubling the carbon taxes is spent on either decreasing value added tax or subsidising the public transport. They show that the tax burden is distributed less even among households in the first scenario since household which live in a less urbanised area will have to pay the same amount as those who live in the urban areas while the first group take less advantage of subsidized transport. Krass *et al.* (2013) examine how the environmental taxes would motivate the choice of innovative and "green" manufacturing together. Moreover, they study the effect of subsidies and consumer rebates on this issue. To this end, they consider a problem where there is a leader-follower Stackelberg game between the firms and the regulator (government). They consider two settings in their analysis: (1) decentralized model, where the regulator and the firms act independently, and (2) a situation where the regulator has control over the prices and technology choice (centralized model). They show that the environmental taxes alone may be insufficient to coordinate the system. Instead, they explain that it would better to add other policy tools, such as fixed cost subsidies and consumer rebates, to increase its efficiency. Huang *et al.* (2013) examines subsidizing electric vehicles (EA) which have significantly less adverse impact on the environment compared to fuel vehicles (FA). EAs are currently being subsidized in many countries such United States, Canada, China, etc. in order to promote the use of such vehicles. Considering a duopoly where two automobile supply chains are competing, they show that such incentives are effective in promoting the EAs. Moreover, they compare this setting with one in which there is a centralized control with no subsidy and conclude that subsidizing EAs is more effective in decreasing the environmental impacts.

At the manufacturer level, the literature can be divided into two sub-categories: (i) strategic level and (ii) tactical and operational level.

2.2. Manufacturer's Perspective in GSCM

Manufacturers take environmental concerns into their decision making process with the aim of either regulation compliance or promoting their products through advertising on the greenness of them. Studies in this area can be divided into two categories: (1) Strategic Level, and (2) Operational Level.

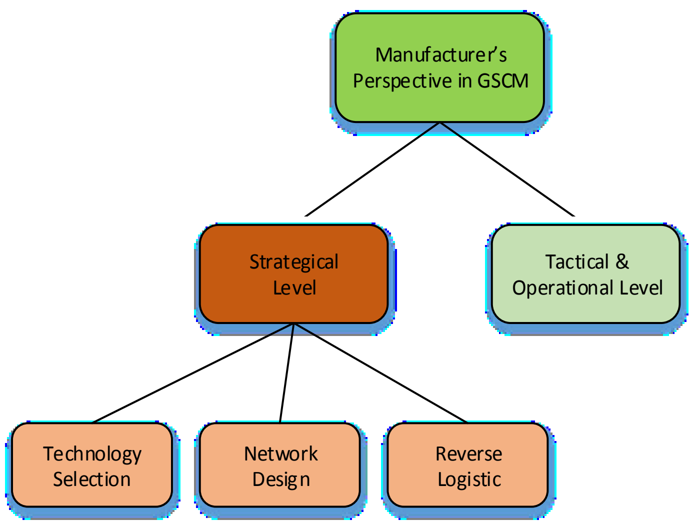

Figure 1. Overview of the literature from the manufacturer's perspective in GSCM

2.2.1. Strategic Level in GSCM

Decision making at the strategic level deals more with fundamental changes such as the choice of cleaner sources of energy, more sustainable production equipment, green machinery or raw materials, and greener transportation means (Debo *et al.* 2005, Drake 2012, Drake *et al.* 2012, Liu *et al.* 2012, Nouira *et al.* 2014, Walsh *et al.* 2014). These studies include problems that deal with technology selection, reverse logistic, and network design (Tang and Zhou 2012).

Nouira *et al.* (2014) study the technology (manufacturing process) and input material selection under the new environmental regulation. They assume that customers are environmental-sensitive, which means that customers are willing to pay more for green products. To this end, they introduce a factor to measure the greenness level of a product. The greenness level of a product is determined by the environmental impact of inputs and manufacturing processes. It increases as greener inputs and processes are chosen. They assume that demand has an inverse relationship with price and the price increases with greenness level of the product (Chen 2001). Using a numerical example in textile industry, they show that the price should be greater than a certain threshold, otherwise the firms would not invest in green products. They also discuss that how considering the relationship between environmental awareness of consumers and the demand can increase the total profit of a firm by offering green products to the customers. While most of the studies in the literature consider a linear relationship between amount technology investment and environmental improvement, (Liu *et al.* 2012) considers a nonlinear relationship. They argue that the environmental improvement should increase with declining rate as eco-friendly investment increases. Drake (2012) studies the eco-friendly investment decisions in the presence of foreign competitors under carbon tax policy. Unlike the previous studies, he assumes that the demand is exogenous and independent of product greenness. He considers domestic firm facing a decision on whether to invest on greener production or moving the production facilities to countries where there is no environmental regulations, e.g. China, where they may also benefit from lower production cost, although they also need to pay for the transportation. He also discuss the effect of putting carbon tariff on the carbon content of imported goods and see how this will affect firms' decisions. Carbon tariff are proposed to prevent carbon leakage which refers to the phenomenon of moving of production facilities to other countries because of asymmetric environmental regulations. He shows that putting carbon tariffs does not necessarily prevent firms from moving their firms to countries with no regulations and they may do so even in the presence of carbon tariff. In fact, they would just invest in cleaner products while they are producing outside the country to decrease the carbon tariff.

A number of studies discuss carbon abatement through reverse logistics (Beamon and Fernandes 2004, Diabat *et al.* 2013, Li *et al.* 2009, Lu and Bostel 2007, Shi *et al.* 2011). Reverse logistics can be done by either remanufacturing or recycling the products. Diabat *et al.* (2013) consider a facility location problem under cap-and-trade regulations and introduce numerical examples to draw some observations. For instance, they show that remanufacturing will become more interesting as the supply chain activities produces significant amount of carbon emission.

Firms can also achieve abatement in carbon emission by considering it into decisions such as plant location and network design problems (Altmann 2014, Chaabane *et al.* 2012, Ji *et al.* 2014, Ramudhin *et al.* 2010). A number of authors have developed multi-objective models that aim to minimize both cost and emission (Mallidis *et al.* 2012, Smith *et al.* 2014). Mallidis *et al.* (2012) study the effect of considering transportation emission on a supply chain network in a region, specifically in south-eastern Europe. The decisions include port of entry, transportation mode (truck, rail, or ship) and whether or not to use shared warehouses and transportation. They incorporate environmental concerns using a multi-objective modeling, where total cost and emission are being minimized. They show that using shared warehouse and transportation is efficient in terms of emission reduction, but it will increase the total cost. Wang *et al.* (2011) discuss a network design problem where environmental investment decisions are made through a multi-objective model. There is a trade-off between environmental investment and carbon emission such that as firms invest more in carbon efficient technologies, their associated carbon emission will decrease in the long-term. A multi-objective facility location problem is developed by Xifeng *et al.* (2013) with the aim of minimizing economic cost and transportation emissions while the minimum service reliability is being maximized. Transportation is the only source of emission that is affected by number of products being shipped and the distance between the facility and the customer. Service reliability is affected by the time needed to deliver goods. Therefore as the number of facilities increases, transportation emission decreases while the total cost and service reliability increases.

The benefits of the tactical level decisions can be observed after a relatively long period of time with the needs of significant initial investment. For recouping benefits within a short time period, it may be more effective to discuss changing operational decisions, such as determining lot size, lead-time, production planning. It has been shown that reducing carbon emission is also achievable through operational decisions as well (Benjaafar *et al.* 2013).

2.2.2. Operational Level in GSCM

A number of authors have incorporated environmental regulation in simple operational models and derive some important conclusions showing the effect of considering such regulations (Bonney and Jaber 2011, Bouchery *et al.* 2012, Hua *et al.* 2011, Wahab *et al.* 2011).

Using the EOQ model, Hua *et al.* (2011) develop a model based on a cap-and-trade system that determines the optimal ordering size. Regarding the sources of emission, they consider those emission caused by transportation and warehousing activities. They derive the optimal order quantity and compare it with the order quantity of the classical EOQ model. They conclude that total emission under a cap-and-trade system does not change as the cap changes and is only affected by the carbon price. Bouchery *et al.* (2012) also solve a multi-objective EOQ model called sustainable order quantity model where they show that environmental improvement are possible through relatively small changes in the total cost through operational adjustment. Benjaafar *et al.* (2013) model carbon emission in a supply chain in forms of strict cap, carbon tax, and cap-and-trade schemes. They model strict cap in form of a constraint and carbon price by adding a carbon cost term to the objective function. One insight from these models suggests that with a strict carbon cap, the amount of emission can be reduced significantly at a reasonable cost. In another observation, it is noticed that emission reduction by changing operational decisions could be reached at lower cost than those achieved by investing in more sustainable technologies. They also compare the benefits of collaboration in a supply chain under different regulations. Chen *et al.* (2013) implement environmental aspects by adding carbon cap as a constraint to a basic EOQ model. In order to calculate the total carbon emissions, they consider the emissions associated with ordering, holding, and production. A newsvendor model is discussed in Arikan and Jammernegg (2014), where there is a strict cap on carbon footprint of the product.

A number of studies model different types of regulations in the same model and compare their effect (Zakeri *et al.* 2014, Zhang and Xu 2013). Zhang and Xu (2013) incorporate cap and trade system into a multi-item production planning problem and derived an optimal policy for production planning. Comparing cap and trade policy with taxation policy, they conclude that if the carbon price and the carbon tax are equal, both policies have the same effect in terms of emission reduction. While one of the major components of operational decisions is transportation, it is not included in the aforementioned studies. Transportation is one of the important factors that needs to be incorporated along with production and warehousing as a factor that significantly affects carbon emission measurement of supply chain.

According to Inventory of U.S. GHG emissions and sinks: 1990-2012, transportation is the second largest source of GHG emissions after electricity (EPA 2014). Colman and Paster (2007) study the different sources of emission in winery industry in five regions of the world and show that the highest emission amount was due to shipping activities. Soysal *et al.* (2014) develop a multi-objective model that minimizes total costs of a food supply chain and its associated transportation emission. The model use the ε-constraint method to solve the model by keeping total cost as the objective function and the transportation emission objective is reformulated as a constraint. Based on the real data gathered from a beef supply chain where beef is being imported into European Union from Brazil, they show that carbon taxes can even lead to improvement in both economic and environmental aspects. Bauer *et al.* (2010) study an intermodal freight transport problem where they minimize carbon emission. Cachon (2014) model the layout of a supply chain and examine its effect on total emission caused by the supply chain and customer travel. He show that putting a carbon tax on emission does not result in significant emission reduction. Instead, increasing the fuel efficiency of customers' cars can save a significant amount of emission. Hoen *et al.* (2013) model a setting where a producer is deciding to reduce its transportation emission by putting a cap on total emission of outbound transportation. The objective function is to reduce total emission through using different transportation modes. They argue that significant emission abatement can be achieved with relatively small increases in the total costs. For instance, they show for a bulk liquid producer 10% reduction in emission can be obtained by only 0.7% increase in total cost.

Measuring the emission associated with logistic activities may not always be accurate (Monni *et al.* 2004). For example, in measuring the emission of production, the average time it takes to produce a unit and the energy consumption rate of the production machines may be considered in estimating the production emission. In reality, the production time may change based on different factors. Unexpected down time, failure of the machine, and parts failure are examples of events that may increase the production time and, therefore, energy consumption. Some of the transportation means produce more emission than others depending on the quality of the fuel, maintenance of the transportation means. Moreover, natural factors such as weather temperature may also play a role in the resulting emission level of the supply chain activities, for instance, heating emission will increase when the environment temperature is lower (Pulles and Meijer 2000). Hence, we need to consider uncertainty in measuring the emission of each source. We employ the robust optimization theory to deal with uncertainty in our problem. Robust optimization theory, presented by Mulvey *et al.* (1995), has been widely used in recent years to cope with problems under uncertainty (Gabrel *et al.* 2014). Uncertainty may appear in different coefficients, such as demand (Alem and Morabito 2012), cost coefficients (Wei *et al.* 2011), availability of raw material supply (Varas *et al.* 2014), etc. Uncertainty may affect the problem in two manners: (*i*) uncertainty on the feasibility of the solution (infeasibility), (*ii*) uncertainty on the optimality of the solution (sub-optimality). In the first case, we are looking for solutions which are feasible for any realization of the input data. In the latter case, the solution will be optimal for the worst-case-scenario. It is obvious that the solution obtained from robust optimization (robust solution) will be worse than the one obtained from a problem without uncertainty (nominal solution); however, the difference between the robust solution and the nominal solution depends on the risk aversion level of the manager. The more risk averse the decision maker is, the worse the robust solution will be with respect to the nominal solution.

Bertsimas and Sim (2003) propose a robust approach to deal with uncertainty when the distribution of uncertain coefficients is not known. In such problems, it is assumed that they change only within a certain range. The middle point of the range is called the nominal value. For an overview of other approaches in robust optimization the reader is referred to Ben-Tal and Nemirovski (2000), Ben-Tal *et al.* (2009), and Fischetti and Monaci (2009). This approach has been used in many production planning and network design problems (Alem and Morabito 2012, Bertsimas and Thiele 2006). Alem and Morabito (2012) implement this approach in a furniture setting where there is uncertainty on objective function parameters (cost parameters) and demand parameters separately. They first show that the uncertainty on cost parameters have no significant effect on the optimal solution while the demand uncertainty had more significant effects. They also show that choosing the budget of uncertainty is a very important factor in analyzing the effect of uncertainty. Therefore, choosing it correctly will become a matter of importance. They compare the robust optimization and worst-case deterministic approach and suggested using the robust approach with less conservative situations. Following Bertsimas and Thiele (2006), we consider uncertainty on estimated emission of each of the supply chain activities.

2.3 Conclusion

While previous studies on operational decisions with GHG emissions are considered as a transportation or production planning decision independently, we provide an integrated model to incorporate the effects of integrated production and transportation decisions. As mentioned earlier, transportation is very important in measuring the carbon footprint of the supply chain which needs to be considered along with the production activities. In our study, we consider a demand allocations and production planning problem with environmental concerns that appear in the form of a constraint on total periodic carbon emissions produced by the supply chain. Furthermore, uncertainty on the estimated emission of the supply chain activities are included in this study. To best our knowledge, no study in green supply chain management has considered uncertainty on estimating the emission of the supply chain to date.

In the following section, first, the problem is defined and then the solution methodology proposed to solve our problem is explained.

Chapter 3

Problem Statement and Methodology

In this study, we model a multi-period demand allocation and production planning problem for a multi-facility network. There are several demand regions and several facilities which are available to satisfy the demand of each region. There is no restriction on the number of facilities that can supply a region as well as number of regions that a facility can supply. The objective is to minimize the sum of costs associated with production, holding, transportation, and selecting a facility in all periods. Demand is assumed to be deterministic and might change from period to period. Backorder is not allowed and all demand should be satisfied in each period. There is also a limit on the total emission originated from production, holding inventory, transportation of goods, and selecting a facility in a period.

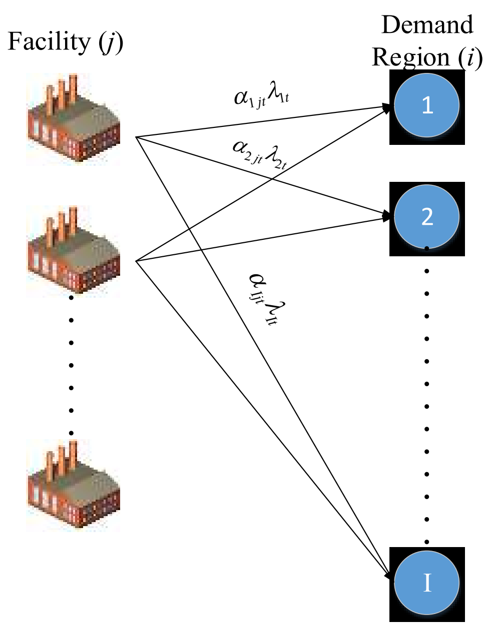

Figure 2. Overview of the problem

We use the following notation in formulating the problem:

Sets

i	Index for demand regions, $i = 1, 2, \ldots, i'$
j	Index for facilities, $j = 1, 2, \ldots, j'$
t	Index for time periods, $t = 1, 2, \ldots, t'$

Parameters

c_{jt}	Unit production cost of producing one product at facility *j* in period *t* (\$/unit)
h_{jt}	Cost of holding one raw material in facility *j* in period *t* (\$/unit)
τ_{jt}	Cost of holding one product at facility *j* in period *t* (\$/unit)
r_{jt}	Cost of raw material at facility *j* in period *t* (\$/unit)
κ	Cost of fuel (\$/litre)
s_{jt}	Setup cost of using facility *j* in period *t* (\$/facility)
c'_{jt}	Emission of producing one product at facility *j* in period *t* (kg CO_2/unit)
h'_{jt}	Emission of holding one raw material at facility *j* in period *t* (kg CO_2/unit)
τ'_{jt}	Emission of holding one product at facility *j* in period *t* (kg CO_2/unit)
r'_{jt}	Emission of raw material at facility *j* in period *t* (kg CO_2/unit)
c'_{f}	Emission of per liter of fuel consumption (kg CO_2/unit)
s'_{jt}	Fixed emission of selecting facility *j* in period *t* (kg CO_2/facility)
l	Fuel consumption for each unit of product (litre/km.unit)

The decision variables are as follows:

X_{jt}	Number of items produced at facility *j* in period *t*

W_{jt}	Number of raw materials at facility *j* at the end of period *t*
I_{jt}	Number of finished goods at facility *j* at the end of period *t*
R_{jt}	Number of raw material released to facility *j* at the beginning of period *t*
Z_{jt}	Binary variable that is equal to one if facility *j* at the beginning of period *t* is used and zero otherwise
α_{ijt}	Fraction of demand of region *i* allocated to facility *j* in period *t*

The fixed cost of allocating demand to a facility in a given period include the cost of setting up the production line, ordering cost, etc. A fixed emission is also considered for a facility selected for production. The fixed emission for selecting a facility can be equal to the emission due to the maintenance activities or other systems that are not used directly in the production process, such as cooling systems. Hence, we are dealing with two decisions for each facility. First decision would be whether to choose a facility for production in a period or not, which directly causes a fixed cost and emission. The second decision includes determining the fraction of demand from each district that the facility should satisfy (demand allocation). Based on the allocated demand in different periods, each facility's WIP level and production quantity is determined. In order to formulate the production planning and demand distribution (PD1) problem we use the following model:

$$[PD1]: \min \sum_{t\in T}\sum_{j\in J}\left(X_{jt}c_{jt} + I_{jt}\tau_{jt} + W_{jt}h_{jt} + R_{jt}r_{jt} + \left(\sum_{i\in I}\alpha_{ijt}\lambda_{it}D_{ij}l\right)\kappa + Z_{jt}s_{jt}\right) \quad (1)$$

Subject to

$$\sum_{t\in T}\sum_{j\in J}\left(X_{jt}c'_{jt} + I_{jt}\tau'_{jt} + W_{jt}h'_{jt} + R_{jt}r'_{jt} + \left(\sum_{i\in I}\alpha_{ijt}\lambda_{it}D_{ij}l\right)\kappa' + Z_{jt}s'_{jt}\right) \le CC \quad (2)$$

$$W_{jt} = W_{j,t-1} + R_{jt} - X_{jt}, \quad \textit{for all } j,t \quad (3)$$

$$I_{jt} = I_{j,t-1} + X_{jt} - \sum_{i\in I}\alpha_{ijt}\lambda_{it}, \quad \textit{for all } j,t \quad (4)$$

$$X_{jt} \le f\left(W_{j,t-1}, R_{jt}, X_{\max}\right), \quad \textit{for all } i,j,t \quad (5)$$

$$\sum_{j\in J}\alpha_{ijt} = 1, \quad \textit{for all } i,t \quad (6)$$

$$X_{jt} \le Z_{jt}M, \quad \textit{for all } j,t \quad (7)$$

$$X_{jt}, W_{jt}, I_{jt}, R_{jt}, \alpha_{ijt} \geq 0, \; Z_{jt} \in \{0,1\}, \quad \textit{for all } i, j, t \tag{8}$$

In the objective function (1), the first and the second terms represent production and finished good inventory (FGI) holding costs, respectively. The term $W_{jt}h_{jt}$ denotes the cost of holding WIP at period *t*, which may include warehousing costs, opportunity costs of capital, insurance expenses, etc. This is an important factor since it increases nonlinearly with the production rate increase. Here, we only consider the amount of WIP that is carried over to the next period identified as end of period WIP. The term $R_{jt}r_{jt}$ is the raw material release (RMR) cost. RMR cost might include cost of procurement, shipment of raw material from supplier to the production facility, and preparation of stored material to the shop floor, etc.

Another factor that affects the objective function is the shipping cost that results from shipping the products from each facility to the customers. This factor may play an important role when cost of transportation increases as a result of either transporting goods to a customer located farther from the facility, e.g. a foreign country, or using an expensive means of transportation. In the first case the cost of transportation increases because of increased distance. In the latter case, an expensive means of transportation can happen when using fast transportation (air freight) or when the final product requires special care during transportation, e.g. in the case of perishable items. The total fuel consumption for each facility and in each period is calculated and then multiplied by the fuel cost. Finally, $Z_{jt}s_{jt}$ is equal to the fixed cost of selecting facility *j*, where Z_{jt} is a binary variable and is equal to one when the facility *j* is in the production mode at period *t*.

Total amount of carbon emission produced should not exceed a fixed amount, called Carbon Cap (CC), and is shown in constraint (2). We consider the emission associated with production, holding of WIP and FGI, raw material releases, shipping, and fixed emission of a facility. The same sources were considered in Sundarakani *et al.* (2010) and Lee (2011), where they develop models to measure carbon footprint across supply chain.

Constraints (3) and (4) represent WIP and FGI balance equations. We assume that both throughput (TH) and WIP are measured the same unit. This means that each unit of WIP will be processed into one unit of product. The fraction of demand in region *i* that is assigned to facility *j* in period *t* is shown by α_{ijt}, varying between 0 and 1.

The fifth constraint define the maximum throughput (TH) as a function of the WIP carried over from the previous period and the amount of raw material released at the beginning of a period represented by the term $f\left(W_{j,t-1}, R_{jt}, X_{\max}\right)$. Note that the decision here is to determine WIP rather than TH. Once WIP level is decided, the TH is computed accordingly. In order to ensure that all demand is fulfilled, constraint (6) is added. Constraint (7) will set the binary variable, Z_{jt}, to one if the facility is in the production mode. Big M is large enough such that it does not limit the production quantity. Using a number equal to or larger than maximum capacity of the facility would serve this purpose.

In the following, we first explain how we implement the congestion effect in our model. We then illustrate how we deal with the nonlinearity in the model due to considering congestion using an outer approximation approach. We then use an exact algorithm to minimize the error of outer approximation approach. Using this exact algorithm, we also develop the uncertain model for this problem. Finally, we propose Lagrangian relaxation and a heuristic to build a feasible solution.

3.1. Modeling the congestion effect

The throughput function defined in (5) is a nonlinear function and should be expressed explicitly. In order to accomplish this, we incorporate the idea of clearing functions (CF). CF was first proposed by Graves (1986) where he considers a linear relationship between throughput and WIP. Further studies on the real data from industries revealed that there is a nonlinear relationship between throughput and WIP (Karmarkar 1989, Srinivasan *et al.* 1988). A significant number of studies have incorporated the idea of CF in the inventory and production planning literature. Missbauer (2011) model an order release planning problem using a new CF to define the clearing function and show that utilization increases nonlinearly as the WIP level increases. Benjaafar (1996) and Benjaafar and Gupta (1999) show how batch sizing will affect the clearing function. Selcuk *et al.* (2008) define four different CFs and compare them in order to find which one would best represent the capabilities of a shop. They concluded that CFs based on the short-term probabilistic behavior of a production model can better represent the relationship between WIP and TH of a shop than those CFs based on long-term average shop behavior. A number of authors have also incorporated congestion in production planning problems with multiple product (Asmundsson *et al.* 2006, Asmundsson *et al.* 2009). They first develop a multi-product single-period production planning problem and then extend it to a multi-period problem. They show that, if the CFs are estimated accurately, models with CFs reflect the production system performance better than those model that do not consider congestion effect. Another example of such CFs can be found in Albey *et al.* (2014) where they introduce multi-dimensional CF (MDCF) to be used in settings where a single machine is producing multiple products.

In this study, we use Equations (9) to express the CFs as proposed by Karmarkar (1989).

$$TH = C\frac{WIP}{WIP+K} \tag{9}$$

This approach uses the load to the system at the beginning of the period ($\overline{W}_{jt}$), which is equal to total number of items that are ready to be processed at the beginning of the period in the CF.

$$\overline{W}_{jt} = W_{j,t-1} + R_{jt} \tag{10}$$

$$f\left(\overline{W}_{jt}\right) = C\frac{\overline{W}_{jt}}{\overline{W}_{jt} + K'} \tag{11}$$

Karmarkar (1989) uses (11) to show the relationship between beginning WIP and maximum throughput. Parameter K' is the curvature of the CF and is estimated by $LC\left(1-\eta_c\right)$, where L is the average lead time, η_c is the critical utilization point, and C is the maximum throughput (Aouam and Brahimi 2013). It is assumed that the CF has no effect before a certain utilization, called critical utilization. In simple words, before reaching this level of utilization, the facility works in a low utilization mode and the congestion effect does not appear. Critical utilization point and lead time are assumed to be $\eta_c = 0.8$ and $L = 1$ period, respectively. This implies that for a utilization level below 80%, all the raw material released to the facility will be processed without congestion effects. We replace $f\left(W_{j,t-1}, R_{jt}, X_{\max}\right)$ with the right hand side of equation (11) in our model, which gives a nonlinear constraint. In the following, we explain how to deal with the nonlinear constraint using an outer linearization approach. This approximation will lead to error in computing WIP and TH level. We then propose an algorithm on how to minimize this error.

3.1.1. Linearization of the CF

In order to linearize the CF constraint, an outer linearization approach is used in the following form (Asmundsson *et al.* 2009, Kacar *et al.* 2012, Vidyarthi *et al.* 2009):

$$f\left(\overline{W}_{jt}\right) \le \min\left\{a^h \overline{W}_{jt} + b^h : \forall h \in H\right\} \tag{12}$$

Where a^h and b^h are the slope and intercept of the line, respectively, and H is the set of lines.

$$a^h = \frac{df\left(\overline{W}_{jt}\right)}{d\overline{W}_{jt}} \tag{13}$$

$$b^h = f\left(\overline{W}_{jt}\right) - a^h \overline{W}_{jt} \tag{14}$$

A set of points on the CF is chosen for approximating the CF. The closest approximate line h, which is the one that gives the minimum value $\left(h = \arg\min\left(h' \middle| \min\left(a^{h'} \overline{W}_{jt} + b^{h'}\right) : \forall h' \in H\right)\right)$, can be used to determine the TH (Figure 3).

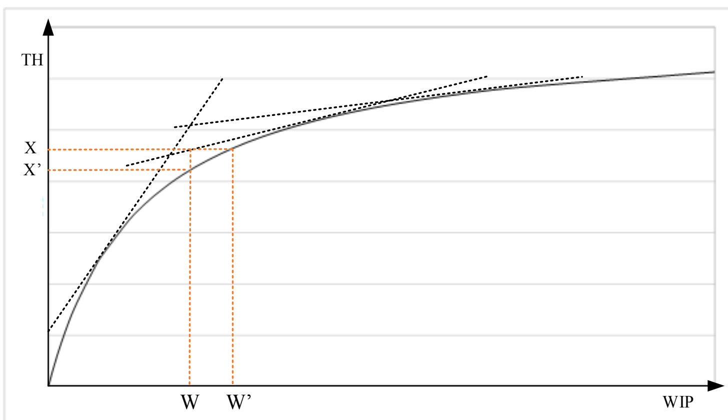

Figure 3. Piecewise Linearization of CF

Hence, the fourth constraint will be rewritten in the following form.

$$X_{jt} \le a^h \overline{W}_{jt} + b^h, \qquad \textit{for all } j, t, h \tag{15}$$

Due to the concavity of the CF, the slope parameter, a^h , decreases as h gets larger and is set to zero at maximum TH. Also, b^1 is set to zero in order to represent a zero TH when there is zero WIP.

Formulation [PD1] along with constraint (15) will give the following formulation:

$$[PD2]: \min \sum_{t \in T} \sum_{j \in J} (X_{jt} c_{jt} + W_{jt} h_{jt} + I_{jt} \tau_{jt} + R_{jt} r_{jt} + \left(\sum_{i \in I} \alpha_{ijt} \lambda_{it} D_{ij} l\right) \kappa + Z_{jt} s_{jt}) \tag{16}$$

s.t.

$$\sum_{t\in T}\sum_{j\in J}\left(X_{jt}c^{'}_{jt}+W_{jt}h^{'}_{jt}+I_{jt}\tau^{'}_{jt}+R_{jt}r^{'}_{jt}+\left(\sum_{i\in I}\alpha_{ijt}\lambda_{it}D_{ij}l\right)\kappa^{'}+Z_{jt}s^{'}_{jt}\right)\leq CC \tag{17}$$

$$W_{jt}=W_{j,t-1}+R_{jt}-X_{jt}, \quad for\ all\ j,t \tag{18}$$

$$I_{jt}=I_{j,t-1}+X_{jt}-\sum_{i\in I}\alpha_{ijt}\lambda_{it}, \quad for\ all\ j,t \tag{19}$$

$$X_{jt}\leq a^{h}\overline{W}_{jt}+b^{h}, \quad for\ all\ j,t,h \tag{20}$$

$$\sum_{j\in J}\alpha_{ijt}=1, \quad for\ all\ i,t \tag{21}$$

$$X_{jt}\leq Z_{jt}M, \quad for\ all\ j,t \tag{22}$$

$$X_{jt},W_{jt},I_{jt},R_{jt},\alpha_{ijt}\geq 0,\ Z_{jt}\in\{0,1\} \quad for\ all\ i,j,t \tag{23}$$

In order to have a zero error when using the outer approximation, an infinite number of lines tangent to the CF is needed. Since, having such a large set of lines is impossible, one needs to find a subset of tangent lines and dynamically update the constraints to make sure that nonlinear constraint is satisfied with a predetermined accuracy. In the following section, we provide an algorithm to minimize the error in finding an optimal solution to the problem.

3.1.2. Exact algorithm to minimize the approximation error

Suppose that (X,W) denotes the optimal beginning WIP and TH for a facility using outer linearization described in previous section as depicted in Figure 3. This means that X units is planned to be produced during the period and in order to produce X units, W units are planned to be released at the beginning of the period. But according to the CF, only $X^{'}$ units will be produced during that period and as one can see in the graph $X^{'}\leq X$. Therefore, we won't be able to satisfy the demand as we have planned and the solution is infeasible. In fact, if we use the outer approximation, the given feasible solution will not be feasible unless (X,W) lies exactly on the clearing function, which happens only at points where the approximating line is tangent to the CF. Hence, as the number of lines in the outer linearization increases, the error decreases.

Any line that is tangent to the CF at any point can be added to the outer linearization. This indicates that infinite number of lines can be added to the problem since there are infinite number of points on the CF. Therefore, we need an algorithm that determines which points on the CF to be considered in the model in order to reduce the size of the problem. To this end, we first need to define the error between the actual TH and the linearization result. This error can affect the model in terms of (*i*) decision variables corresponding to primal solution, (e.g. X_{jt} and $\bar{W}_{jt}$), (*ii*) objective function value (Kefeli 2011). We choose the error in approximating the beginning WIP that is derived by fixing the TH and finding the true beginning WIP needed.

$$e_{jt} = \frac{f^{-1}\left(X_{jt}\right) - \overline{W}_{jt}}{f^{-1}\left(X_{jt}\right)} \tag{24}$$

In the following algorithm, we initially start with a set of lines and solve the problem to optimality. We then calculate the error in approximating each beginning WIP (e_{jt}). If e_{jt} is greater than a threshold value (ε), e.g. 10^{-3}, a line tangent to CF at point $\left(X_{jt}, \bar{W}_{jt}\right)$ in the optimal solution will be added to the set of lines (Kefeli 2011). We do this by calculating the slope and intercept of the tangent line and adding these values to the current set of slope and intercepts. After doing this procedure for every *j* and *t*, we solve the problem again with the new set of lines. Again we repeat the mentioned procedure. We repeat the whole procedure until the error for all optimal points on the CF would be less than the threshold value which means $e_{jt} \le \varepsilon$ for every *j* and *t*. To this end, we define φ_{jt}, which is equal to one if $e_{jt} \le \varepsilon$ and zero otherwise. This way we would be able to count the number of converged point in the optimal solution. Therefore, the algorithm will stop if the summation of all these φ_{jt} would be equal to the total number of points in the optimal solution. Assuming there are *m* potential facilities and *p* periods:

$$\sum\nolimits_{\forall j,t} \varphi_{jt} = (m)(p) \tag{25}$$

which indicates all $\overline{W}_{jt}$ are converged. The convergence algorithm can be presented as follows:

Convergence Algorithm

Start with an initial number of lines (H).

$\varphi_{jt} = 0$ for all j and t

While { $\sum_{j\in J, t\in T} \varphi_{jt} < (J)(T)$,

Solve [PD2] and find error for every $\overline{W}_{jt}$:

$$\mathrm{e}_{jt} = \frac{f^{-1}(\mathrm{X}_{jt}) - \overline{W}_{jt}}{f^{-1}(\mathrm{X}_{jt})}$$

for {every j and t,

if {$e_{jt} \leq \varepsilon$,

$\varphi_{jt} = 1$

else,

Add a new constraint (line) with the following properties:

$$a^{h'} = \frac{\partial f\left(\overline{W}_{jt}\right)}{\partial \overline{W}_{jt}}$$

$$b^{h'} = f\left(\overline{W}_{jt}\right) - a^{h'}\overline{W}_{jt}$$

Add h' to the current set of lines H.

$\varphi_{jt} = 0$

End if }

End for }

End while }

Hence, we presented a production planning and demand distribution problem and explained how we incorporate the congestion effect in our model. In the following, we first discuss how we deal with uncertainty in our model and then discuss our solution methodology.

3.2. Robust model of PD2

When facing uncertainty in a problem, there are two general approaches: (*i*) stochastic programming and (*ii*) robust optimization. The stochastic programming requires stochastic information of the uncertain factor in order to generate scenarios whereas the robust optimization needs no distributional information about it. Furthermore, in the stochastic programming, in order to accurately represent the uncertainty in the problem, large number of scenarios may be needed. On the other hand, in the robust optimization, the structure of the problem remains the same and is no harder than the deterministic problem to solve which makes this approach more appealing.

In this thesis, we assume that no information regarding the probability distribution of the emission of each activity is available. Hence, we develop a model based on the idea of robust optimization in order to incorporate uncertainty in our model. Following Bertsimas and Sim (2003), the model remains computationally tractable.

Before we explain how we adopt this approach, we present a background on robust optimization. Consider the following problem:

$$[RO]: Min \quad \sum_j c_j x_j \tag{26}$$

s.t.

$$\sum_{j=1}^{J} a_{ij} x_j \leq b_i, \qquad \forall i` \tag{27}$$

where we are uncertain about the exact value of a_{ij}.

For each a_{ij}, the nominal value and maximum deviation from the nominal value are represented by $\bar{a}_{ij}$, $\tilde{a}_{ij}$, respectively. Let ϱ be the ratio of $\bar{a}_{ij}$ to $\tilde{a}_{ij}$. In order to show the deviation of the input from the nominal value we use $u_{ij} = \frac{a_{ij}-\bar{a}_{ij}}{\hat{a}_{ij}}$ which belongs to $[-1,1]$. It is assumed that it is not realistic that all the parameters would take their worst value. Therefore, the budget of uncertainty was proposed by Bertsimas and Thiele (2006) which determines the risk aversion level of the decision maker and is shown by Γ (Γ_i for the above problem). Hence, Γ_i is equal to zero when there is no uncertainty and is equal to J at the worst case. Assuming all $\hat{a}_{ij} > 0$,

$$\max \sum_j \hat{a}_{ij} x_j u_{ij} \tag{28}$$

s.t.

$$\beta_i : \sum_{j \in J} \left| u_{ij} \right| \leq \Gamma_i, \qquad \forall i \tag{29}$$

$$\theta_{ij} : 0 \leq \left| u_{ij} \right| \leq 1, \qquad \forall i, j \tag{30}$$

here β_i and θ_{ij} are the dual values associated with each constraint in the optimal solution. Thus, we have a maximization problem within minimization problem. In order to overcome this problem we use the dual of above problem. In fact, since the above formulation is feasible and bounded, based on the strong duality, it can be replaced it with its dual problem which is also bounded and feasible. Hence, we will obtain a minimization problem. The above formulation can be replaced by its dual which gives:

$$\min\left(\Gamma_i \beta_i + \sum\nolimits_j \theta_{ij}\right) \tag{31}$$

s.t.

$$\beta_i + \theta_{ij} \geq \hat{a}_{ij} \left| x_j \right| \tag{32}$$

$$\beta_i, \theta_{ij} \geq 0 \tag{33}$$

Therefore, the robust counterpart of problem [RO] can be written as:

$$Min \ \sum\nolimits_j c_j x_j \tag{34}$$

s.t.

$$\sum\nolimits_{j=1}^{J} \mathrm{a}_{ij} x_j + \left(\Gamma_i \beta_i + \sum\nolimits_j \theta_{ij}\right) \leq b_i, \qquad \forall i \tag{35}$$

$$\beta_i + \theta_{ij} \geq \hat{a}_{ij} \left| x_j \right| \tag{36}$$

$$\beta_i, \theta_{ij} \geq 0 \tag{37}$$

In this study we follow the same procedure to obtain the counterpart of [PD2]. As mentioned earlier, we are interested in studying the uncertainty in emission parameter associated with each of the decision variables. Since the decision variables of our problem are all positive we can ignore the absolute value. In the following formulation, symbols with a bar (e.g. $\overline{h}'_{jt}$) and a hat (e.g. $\hat{h}'_{jt}$) represent the nominal and maximum deviations values, respectively.

$$\max \sum_t \left(W_{jt} \hat{h}'_{jt} u_{jt}^h\right) \tag{38}$$

s.t.

$$\beta_j^h : \sum_j u_{jt}^h \leq \Gamma_j^h \tag{39}$$

$$\theta_{jt}^h : 0 \leq u_{jt}^h \leq 1, \quad \forall j, t \tag{40}$$

Hence, from the strong duality, we have:

$$\min\left(\sum_{t}\left(\theta_{jt}^{h'}\right)+\beta_{t}^{h'}\Gamma_{j}^{h'}\right) \tag{41}$$

s.t.

$$\theta_{jt}^{h'}+\beta_{jt}^{h'}\geq W_{jt}\hat{h}'_{jt} \tag{42}$$

$$\theta_{jt}^{h'},\beta^{h'}\geq 0 \tag{43}$$

Doing the same procedure or all parameters, the robust counterpart of the problem would be obtained. The notations we use in developing the robust counterpart are summarized below:

$\bar{c}'_{jt}$	Nominal emission of producing one product at facility *j* in period *t* (kg CO2/unit)
$\bar{h}'_{jt}$	Nominal emission of holding one raw material at facility *j* in period *t* (kg CO2/unit)
$\bar{\tau}'_{jt}$	Nominal emission of holding one product at facility *j* in period *t* (kg CO2/unit)
$\bar{r}'_{jt}$	Nominal emission of raw material at facility *j* in period *t* (kg CO2/unit)
$\bar{\kappa}'$	Nominal emission of per liter of fuel consumption (kg CO2/unit)
$\bar{s}'_{jt}$	Nominal fixed emission of selecting facility *j* in period *t* (kg CO2/facility)
$\hat{c}'_{jt}$	Maximum deviation from the nominal emission of producing one product at facility j in period t (kg CO2/unit)
$\hat{h}'_{jt}$	Maximum deviation from the nominal emission of holding one raw material at facility j in period t (kg CO2/unit)
$\hat{\tau}'_{jt}$	Maximum deviation from the nominal emission of holding one product at facility j in period t (kg CO2/unit)
$\hat{r}'_{jt}$	Maximum deviation from the nominal emission of raw material at facility j in period t (kg CO2/unit)
$\hat{c}'_{f}$	Maximum deviation from the nominal emission of per liter of fuel consumption (kg CO2/unit)
$\hat{s}'_{jt}$	Maximum deviation from the nominal fixed emission of selecting facility j in period t (kg CO2/facility)
Γ	Budget of uncertainty
ϱ	Ratio of $\tilde{a}_{ij}$ to $\bar{a}_{ij}$

The robust counterpart is as follows:

$$[RPD2]: \min \sum_{t\in T}\sum_{j\in J}\left(X_{jt}c_{jt} + W_{jt}h_{jt} + I_{jt}\tau_{jt} + R_{jt}r_{jt} + \sum_{i\in I}\lambda_{it} l D_{ij}\kappa' \alpha_{ijt} + Z_{jt}s_{jt}\right) \quad (44)$$

s.t.

(18),(19),(20),(21),(22),(23)

$$\sum_{j\in J}\sum_{t\in T}\left(X_{jt}\bar{c}'_{jt} + W_{jt}\bar{h}'_{jt} + I_{jt}\bar{\tau}'_{jt} + R_{jt}\bar{r}'_{jt} + \left(\sum_i D_{ij} l \lambda_{it}\alpha_{ijt}\right)\bar{\kappa}' + Z_{jt}\bar{s}'_{jt}\right) +$$
$$\sum_{j\in J}\left(\beta_j^{c'}\Gamma_j^{c'} + \beta_j^{h'}\Gamma_j^{h'} + \beta_j^{\tau'}\Gamma_j^{\tau'} + \beta_j^{r'}\Gamma_j^{r'} + \beta_j^{\kappa'}\Gamma_j^{\kappa'} + \beta_j^{s'}\Gamma_j^{s'}\right) +$$
$$\sum_{j\in J}\sum_{t\in T}\left(\theta_{jt}^{c'} + \theta_{jt}^{h'} + \theta_{jt}^{\tau'} + \theta_{jt}^{r'} + \theta_{jt}^{c'_f} + \theta_{jt}^{s'}\right) \le CC \quad (45)$$

$$\theta_{jt}^{c'} + \beta_j^{c'} \ge X_{jt}\hat{c}'_{jt}, \quad for\ all\ j,t \quad (46)$$

$$\theta_{jt}^{h'} + \beta_j^{h'} \ge W_{jt}\hat{h}'_{jt}, \quad for\ all\ j,t \quad (47)$$

$$\theta_{jt}^{\tau'} + \beta_j^{\tau'} \ge I_{jt}\hat{\tau}'_{jt}, \quad for\ all\ j,t \quad (48)$$

$$\theta_{jt}^{r} + \beta_j^{r'} \ge R_{jt}\hat{r}'_{jt}, \quad for\ all\ j,t \quad (49)$$

$$\theta_{jt}^{c'_f} + \beta_j^{c'_f} \ge \sum_i D_{ij} l \lambda_{it}\alpha_{ijt}\hat{c}'_{f_{jt}}, \quad for\ all\ j,t \quad (50)$$

$$\theta_{jt}^{s'} + \beta_j^{s'} \ge Z_{jt}\hat{s}'_{jt}, \quad for\ all\ j,t \quad (51)$$

Solving the model for large instances, we noticed that CPLEX could not find a feasible solution in a reasonable amount of time. Thus, we developed a Lagrangian relaxation approach to solve large instances of the problem which we explain in the following section.

3.3. Lagrangian Relaxation

Considering the difficulty in solving [RPD2], we applied a Lagrangian relaxation approach for large instances. In Lagrangian relaxation approach, one or a set of constraints (complicating constraints) will be relaxed by taking them into the objective function using a penalty term. Complicating constraints are constraints that relaxing them would result in a problem that is easier to solve. Such constraints can be those which contain binary variables or those which link different sub-problems to each other. The reader is referred to Fisher (2004) for a comprehensive review of Lagrangian relaxation theory and its application.

Lagrangian relaxation has been used extensively in production planning problems (Jayaraman and Pirkul 2001, Kim and Kim 2000). Jayaraman and Pirkul (2001) study a locating production and distribution centers problem. Relaxing two linking constraints, the problem is decomposed into three sub-problems. They then propose heuristics to solve each of these sub-problems. Kim and Kim (2000) study a multi-period inventory/distribution problem. Similar to Jayaraman and Pirkul (2001), they employ Lagrangian relaxation by relaxing some constraints and decompose the problem into two sub-problem, where the first sub-problem is to determine the schedule of vehicles (scheduling problem) and the second problem is a demand allocation and production planning problem. In green supply chain management, Elhedhli and Merrick (2012) use this method to solve a network design problem. Resulted model is decomposed into two sub-problem where the second sub-problem was itself decomposed into *n* knapsack problems. Each of these knapsacks could easily be solved using a heuristic for knapsack problem. The Lagrangian relaxation approach has been used successfully in all these problems in decreasing the computation time of the solving problems and providing a decent bound. We apply the same Lagrangian relaxation by relaxing two sets of constraints and, hence, decompose our multiple facility production planning problem into several single-facility production planning problem.

Before we start explaining our solution methodology, the reader is provided with a brief review of Lagrangian relaxation based on Fisher (1985).

Consider the following integer program:

$$[P] \qquad Z^* = \min cx \tag{52}$$

$$subject\ to \quad Ax \geq b \quad \text{(complicating constraints)} \tag{53}$$

$$Dx \geq d \quad \text{(nice constraints)} \tag{54}$$

Let us relax the complicating constraints using Lagrangian multiplier. The resulting sub-problem would be:

$$[SP] \qquad Z^*_{sp} = \min \quad cx + u\left(b - Ax\right) \tag{55}$$

$$subject\ to \qquad Dx \geq d \quad \text{(nice constraints)} \tag{56}$$

Where $u \geq 0$. Since, some of the constraint in [P] are relaxed, the solution to [SP] will provide a lower bound to optimal solution of [P]. However, the quality of the lower bound (LB) depends heavily on the Lagrangian multiplier u. We need to solve the problem that finds the best Lagrangian multipliers by which the SP acquire its maximum value. Therefore, the best LB is

$$\max_{u>0} \left\{ ub + \min_{Dx>d} \left(cx - uAx \right) \right\} \tag{57}$$

In order to find the best LB, an iterative procedure is proposed in which the value of the Lagrangian multipliers are updated at each iteration. Assume that

$$x^k = \left\{ x \mid Dx \geq d, k = 1, 2, ..., K \right\} \tag{58}$$

In the above formula, k represents the iteration number.

We now present the Master Problem (MP).

$$[MP] \quad Z_{MP}^{*} = \max_{u>0} \quad ub + \min_{k=1,2,...,K} \left(cx^k - uAx^k \right) \tag{59}$$

Let us define

$$\eta = \min_{k=1,...,K} \left(cx^k - uAx^k \right) \tag{60}$$

Hence,

$$[MP] \quad Z_{MP}^{*} = \max_{u>0} \quad ub + \eta \tag{61}$$

$$s.t. \quad \eta < cx^k - uAx^k, \quad k = 1, 2, ..., K \tag{62}$$

The Lagrangian relaxation procedure is as follows:

The LB and the UB for the Lagrangian relaxation are initially set to $\left(LB, UB \right) = (-\infty, \infty)$. In the first iteration, an initial set of multipliers are put into the SP. Then, we solve the SP and obtain the variables which gives the Z_{sp}^{*} (x^1). Then, the LB is updated ($LB = \max(LB, Z_{sp}^{*})$). In the next step, the following problem will be solved:

$$[MP1] \quad Z_{MP}^{*} = \max_{u>0} \quad ub + \eta \tag{63}$$

$$s.t. \quad \eta < cx^1 - uAx^1 \tag{64}$$

The Lagrangian multiplier u which is obtained from solving MP is put as new Lagrangian multiplier in the SP in the next iteration. The new UB will be equal to $LB = \min(UB, Z^*_{MP})$. In the next iteration, SP is solved using the new Lagrangian multipliers.

In summary, at each iteration, first SP is solved and then, using the vector x obtained from optimal solution, a new constraint will be added to the MP. This procedure is continued iteratively until a desirable gap $(UB - LB)$ is obtained.

In order to able to verify the quality of the LB, an upper bound to the original problem is required. To this end, we need to develop a heuristic to build feasible solution which will be used as an upper bound. In the following, we first explain how we obtain the lower bound and then propose a heuristic to build a feasible solution based on the best lower bound solution.

We use the following notation in explaining the Lagrangian relaxation procedure:

v_{it}	Lagrangian multiplier associated with constraint (22)
ξ	Lagrangian multiplier associated with constraint (27)
ζ^k	Total emission at iteration k
χ^k	Total cost at iteration k

3.3.1. Lower bound for RPD2

In this thesis, we employ the Lagrangian relaxation approach proposed by Fisher (1985). We use Lagrangian multipliers v_{it} and ξ to relax constraints (22) and (27). These two constraints are the only constraints that link different facilities. Therefore, by relaxing them we will be able to decompose the problem into different facilities. This way, instead of solving a multi-facility multi-period demand allocation problem we will be solving several single-facility production planning and demand allocation problems.

$$
\begin{aligned}
&\min \sum\nolimits_{t\in T}\sum\nolimits_{j\in J}\left(X_{jt}c_{jt}+W_{jt}h_{jt}+I_{jt}\tau_{jt}+R_{jt}r_{jt}+\sum\nolimits_{i\in I}\lambda_{it}lD_{ij}\kappa\alpha_{ijt}+Z_{jt}s_{jt}\right)+\\
&\xi\left[\sum_{j\in J}\sum_{t\in T}\left(X_{jt}\bar{c}'_{jt}+W_{jt}\bar{h}'_{jt}+I_{jt}\bar{\tau}'_{jt}+R_{jt}\bar{r}'_{jt}+\left(\sum_{i}D_{ij}l\lambda_{it}\alpha_{ijt}\right)\bar{\kappa}'+Z_{jt}\bar{s}'_{jt}\right)+\right.\\
&\sum_{j\in J}\left(\beta_j^{c'}\Gamma_j^{c'}+\beta_j^{h'}\Gamma_j^{h'}+\beta_j^{\tau'}\Gamma_j^{\tau'}+\beta_j^{r'}\Gamma_j^{r'}+\beta_j^{c_f'}\Gamma_j^{c_f'}+\beta_j^{s'}\Gamma_j^{s'}\right)+\\
&\left.\sum_{j\in J}\sum_{t\in T}\left(\theta_{jt}^{c'}+\theta_{jt}^{h'}+\theta_{jt}^{\tau'}+\theta_{jt}^{r'}+\theta_{jt}^{c_f'}+\theta_{jt}^{s'}\right)-CC\right]+\sum_{i,t}\upsilon_{it}\left(\sum\nolimits_{j\in J}\alpha_{ijt}-1\right)
\end{aligned}
\tag{65}
$$

$s.t.$

(18),(19),(20),(21),(23), (45),(46),(47),(48),(49),(50),(51)

Let define ϕ_j^* as the optimal solution to the following sub problem (SP).

$$
\begin{aligned}
&[SP_j]:\phi_j=\min \sum\nolimits_{t\in T}\left(X_{jt}c_{jt}+W_{jt}h_{jt}+I_{jt}\tau_{jt}+R_{jt}r_{jt}+\sum\nolimits_{i\in I}l_0D_{ij}c_f n_{ijt}+\sum\nolimits_{i\in I}\lambda_{it}l_1D_{ij}\kappa\alpha_{ijt}+Z_{jt}s_{jt}\right)\\
&\xi\left[\sum_{t\in T}\left(X_{jt}\bar{c}'_{jt}+W_{jt}\bar{h}'_{jt}+I_{jt}\bar{\tau}'_{jt}+R_{jt}\bar{r}'_{jt}+\left(\sum_{i}D_{ij}l\lambda_{it}\alpha_{ijt}\right)\bar{\kappa}'+Z_{jt}\bar{s}'_{jt}\right)+\right.\\
&\left(\beta_j^{c'}\Gamma_j^{c'}+\beta_j^{h'}\Gamma_j^{h'}+\beta_j^{\tau'}\Gamma_j^{\tau'}+\beta_j^{r'}\Gamma_j^{r'}+\beta_j^{\kappa'}\Gamma_j^{\kappa'}+\beta_j^{s'}\Gamma_j^{s'}\right)+\\
&\left.\sum_{t\in T}\left(\theta_{jt}^{c'}+\theta_{jt}^{h'}+\theta_{jt}^{\tau'}+\theta_{jt}^{r'}+\theta_{jt}^{c_f'}+\theta_{jt}^{s'}\right)\right]+\sum_{i,t}\upsilon_{it}\alpha_{ijt}
\end{aligned}
\tag{66}
$$

$s.t.$

(18),(19),(20),(21),(23), (45),(46),(47),(48),(49),(50),(51)

Hence, the lower bound to [RPD2] would be:

$$
LB_{LR}=\sum\nolimits_{j}\phi_j^*-\sum\nolimits_{i,t}\nu_{it}-\xi(CC) \tag{67}
$$

The quality of the LB provided by the Lagrangian relaxation depends heavily on the Lagrangian multipliers. In order to improve the quality of the Lagrangian multipliers, we solve the master problem (MP), where the Lagrangian multipliers are the decision variables of the MP and the decision variables of optimal solution of SP are the used to build constraints in MP. Let ζ^k and χ^k be the total emission and total cost at iteration *k*.

$$\zeta^k = \Bigg[\sum_{j\in J}\sum_{t\in T}\Bigg(X^k_{jt}\bar{c}'_{jt} + W^k_{jt}\bar{h}'_{jt} + I^k_{jt}\bar{\tau}'_{jt} + R^k_{jt}\bar{r}'_{jt} + \Big(\sum_i D_{ij} l \lambda_{it}\alpha^k_{ijt}\Big)\bar{\kappa}' + Z^k_{jt}\bar{s}'_{jt}\Bigg) +$$

$$\sum_{j\in J}\Big(\beta^{c'}_{j,k}\Gamma^{c'}_{j,k} + \beta^{h'}_{j,k}\Gamma^{h'}_{j} + \beta^{\tau'}_{j,k}\Gamma^{\tau'}_{j} + \beta^{r'}_{j,k}\Gamma^{r'}_{j} + \beta^{\kappa'}_{j,k}\Gamma^{\kappa'}_{j} + \beta^{s'}_{j,k}\Gamma^{s'}_{j}\Big) +$$

$$\sum_{j\in J}\sum_{t\in T}\Big(\theta^{c'}_{jt,k} + \theta^{h'}_{jt,k} + \theta^{\tau'}_{jt,k} + \theta^{r'}_{jt,k} + \theta^{c'_f}_{jt,k} + \theta^{s'}_{jt,k}\Big)\Bigg] \tag{68}$$

And,

$$\chi^k = \sum\nolimits_{t\in T}\sum\nolimits_{j\in J}\Big(X^k_{jt}c_{jt} + W^k_{jt}h_{jt} + I^k_{jt}\tau_{jt} + R^k_{jt}r_{jt} + \sum_i D_{ij} l\lambda_{it}\alpha^k_{ijt}\kappa\alpha^k_{ijt} + Z^k_{jt}s_{jt}\Big) \tag{69}$$

Hence, the MP would be

$$[MP]: \max_{\xi\geq 0} \quad -(CC)\xi - \sum_{i,t}\upsilon_{it} + \min_{k=1,\ldots,K}\Big(\xi\zeta^k + \chi^k + \sum_{i,t}\upsilon_{it}\Big(\sum\nolimits_{j\in J}\alpha^k_{ijt}\Big)\Big) \tag{70}$$

Let us define

$$\eta = \min_{k=1,\ldots,K}\Big(\xi\zeta^k + \chi^k + \sum_{i,t}\upsilon_{it}\Big(\sum\nolimits_{j\in J}\alpha^k_{ijt}\Big)\Big) \tag{71}$$

Which leads us to

$$[MP]: \max_{\xi\geq 0} \quad -(CC)\xi - \sum_{i,t}\upsilon_{it} + \eta \tag{72}$$

s.t.

$$\eta \leq \xi\zeta^k + \chi^k + \sum_{i,t}\upsilon_{it}\Big(\sum\nolimits_{j\in J}\alpha^k_{ijt}\Big), \qquad \forall k = 1,\ldots,K \tag{73}$$

We start with an initial set of multipliers and solve the SP. Using the decision variables obtained from the optimal solution of SP, a new constraint $\left(\eta \leq \xi\zeta^k + \chi^k + \sum_{i,t}\upsilon_{it}\left(\sum_{j\in J}\alpha^k_{ijt}\right)\right)$ will be added to MP. Solving MP, new multiplier will be provided to SP. Using the new multipliers, we solve the SP again and add another constraint to the MP. SP provides a LB for the [RPD2] and MP gives an UB on the LB. We continue doing this loop until a desirable gap (ω) is reached; however, it is possible the LB would not improve after a certain point. In order to prevent getting stuck in the same loop, we define another stopping criterion which stops the procedure if the UB does not improve in the last *m* iterations. Here is summary of the Lagrangian procedure:

Lagrangian Relaxation Procedure

Start with an initial set of multipliers.

$StoppingCriterion = 0$

While $\{StoppingCriterion \neq 1,$

Solve SP and get an lower bound LB^k

Update the $LB = \max(LB, LB^k)$

add constraint *k* to the MP $\left(\eta \leq \xi\zeta^k + \chi^k + \sum_{i,t} \upsilon_{it} \sum_{j \in J} \alpha_{ijt}^k \right)$

Solve MP and get and a upper bound UB^k

Update the multipliers in the SP using solution to the MP

$UB = \max(UB, UB^k)$

if { $Gap < \omega$,

Stopping Criterion = 1

End if }

if { UB has not improved in the last n iterations,

Stopping Criterion = 1

End if }

End while }

The Lagrangian procedure will provide the LB to [RPD2] which may not be a feasible solution (unless it is the optimal solution). We need to develop a heuristic to find a decent feasible solution using the decision variables obtained from the LB.

3.3.2. Heuristic to build a feasible solution

We propose a two-step heuristic to build a feasible solution. In the first step, after the LB has been found, we check for used facilities and fix them in the heuristic problem. We do not fix the binary variables which are not used and let them be free. We then solve the [RPD2] which gives us a feasible solution. Based on the quality of the LB, the computation time will decrease due to the decrease in the number of binary variables. After doing so for a number of instances, we realized that some of the facilities that were set to be used in the first step are loaded very low (e.g. 10% or even lower). In the second step, we set those facilities that are loaded less than a certain value, ϖ, free and solve the problem again. Since we are relaxing some of the constraints in the first step, the solution in the second step be either equal or better than the one obtained from the first step.

Building a Feasible Solution Procedure
For {every *j* and *t*,
if {if the facility is used in the best LB,
$Z_{jt} = 1$
}
Solve RPD2
$UB = Optimal\ Value\ of\ RPD2$
$\Theta^* = UB$
For {every *j* and *t*,
if $\{\frac{X_{jt}}{Max\ Cap} < \varpi$,
relax $Z_{jt} = 1$
}
Solve RPD2
$UB = Optimal\ Value\ of\ RPD2$
$UB = \min(\Theta^*, UB)$

An overview of the whole solution methodology is brought here.

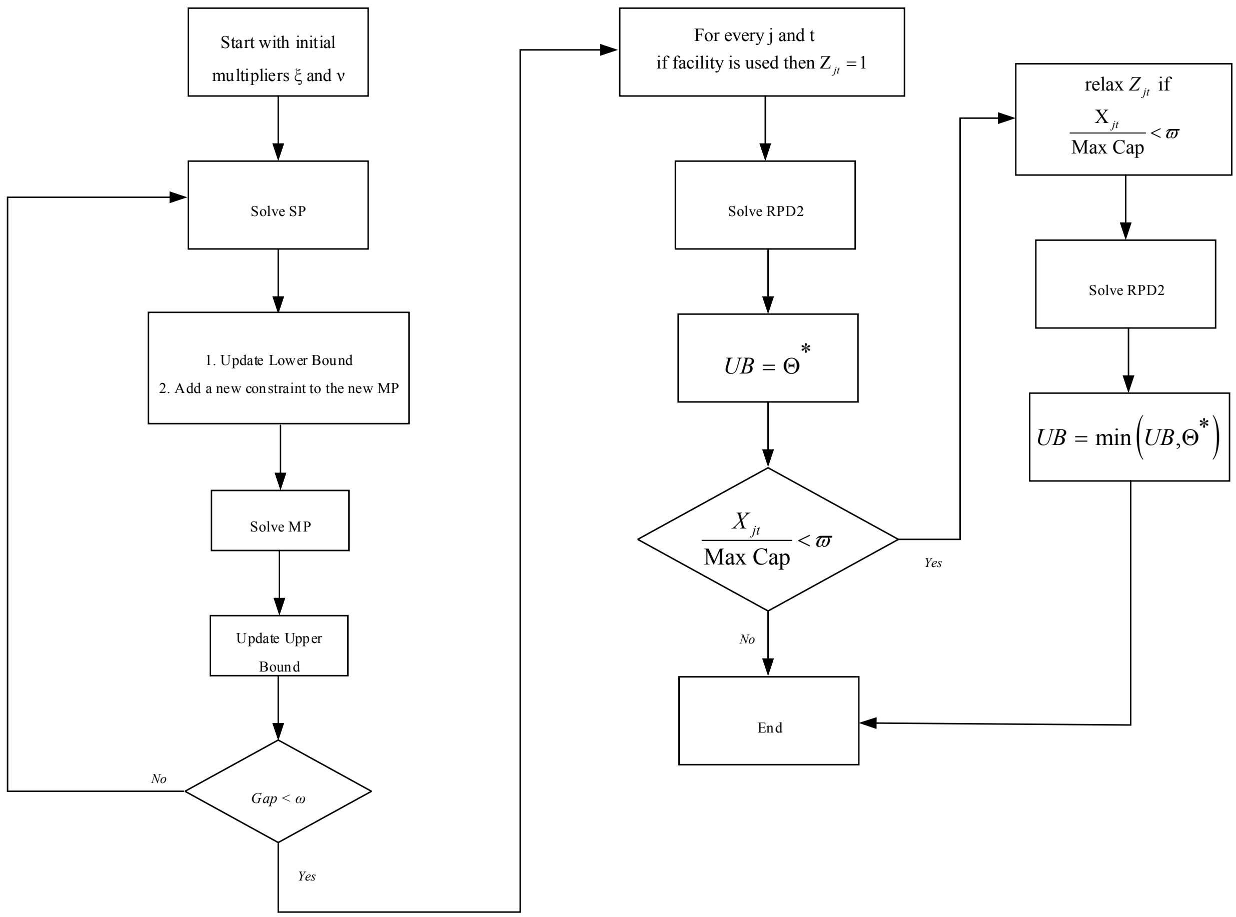

Figure 4. The Solution Methodology

In this chapter, we developed the deterministic model and, then, explained how we incorporate different aspects of the model, such as congestion. Due to the nonlinearity in the model, we employed an outer approximation approach. Using an exact algorithm we minimized the error of approximation. We then developed a robust model to consider uncertainty on emission of different activities. Finally, we used a Lagrangian relaxation approach to solve large instances. In the following chapter, we will develop numerical examples to analyze the effect uncertainty on operational decisions and the performance of our proposed methodology.

Chapter 4

Numerical Experiments

In this chapter, we first present an illustrative example and conduct experiments to show the impact of uncertainty in estimating the emission associated with each activity of the supply chain on the optimal solution. We then develop larger instances of the proposed model in order to analyze the performance of the proposed solution algorithms. All the experiments have been implemented in GAMS 22.5 software using CPLEX 12.2 solver and run on a Dell Vostro 3460 station with an Intel Core i5-3230M processor at 2.60 GHz and 6 GB of RAM running Windows 7 operating system.

4.1. Impact of uncertainty on operational decisions

In this section, we present an illustrative example in order to analyze the effects of considering carbon emissions in production planning and distribution decisions. We first set up the problem and solve it considering certain amount of emissions. We then extend our analysis by defining uncertainty in estimating each source of emission. Consider the following setting. There are five potential production facilities that we can be used in order to satisfy the demand distributed in four regions (see Figure 5). All the facilities are identical in terms of production capacity, cost, and emission parameters. The planning horizon is 10 periods.

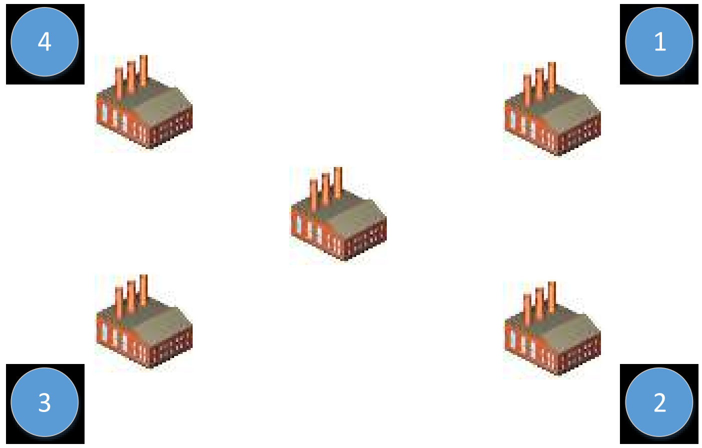

Figure 5. Overview of the numerical example

The demand and distances between demand regions and production facilities have been generated based on uniform distribution with a range [270,310] and [10,70], respectively. The maximum production capacity in each facility is 350; equal to 1.2 times the average demand. Unit production and holding costs are set to 0.3 kg CO_2 and 0.1 kg CO_2, respectively. Other cost parameters are set as $\tau = 1$, $r = 1.33(c)$, $c_f = 0.33(c)$, and $S = 120$. We set the emission associated with production, holding WIP, and holding FGI equal to one, whereas r' equal to $0.1(c')$ since procurement has no significant emission. The emission of establishing a facility in a period is 30 kg CO2. Finally, the coefficient of variation in the robust model (ϱ) is 0.2, and the critical utilization level is set to 80 percent of the maximum capacity of the facility. The latter assumption means that the congestion may be formed in production facility only if the utilization level of facility is at least 80 percent.

Table 1. Input Parameters

c	h	τ	r	κ	s	c'	h'	τ'	r'	κ'	s'	l	ϱ
0.3	0.1	1	0.4	0.1	120	1	1	1	0.1	0.1	30	0.1	0.2

To solve the model, we first provide the initial set of lines for piecewise linearization of the clearing function in Table 2.

Table 2. Clearing function approximation

Segment	slope	intercept
1	5.00	0.00

2	0.20	224.00
3	0.14	243.06
4	0.08	269.31
5	0.02	305.70

Figure 6 examines the effect of changing CC on the total cost and average utilization level. Note that the average utilization level is equal to the average utilization level of all used facilities in all periods.

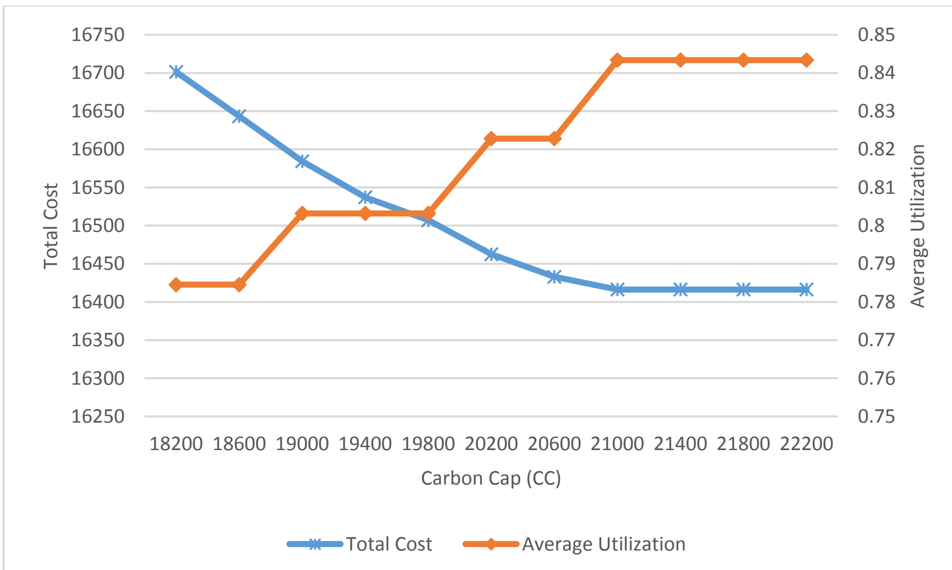

Figure 6. Effect of CC on Total Cost and Average Utilization

The first observation from Figure 6 is that decreasing the CC results in an increase in the total cost and a decrease in the average utilization level. The latter result indicates that when carbon cap decreases, the firm needs to use more facilities to serve the demand. This behavior is mainly due to the fact that using one more facility to avoid congestion would help us with reducing emission of holding WIP that has been produced because of highly loaded facilities. This claim is also supported by Table 3, where cost percentage of each activity is reported. As one can see in the table below, the percentage of WIP holding cost decreases monotonically as CC decreases. It can also be noticed that at some of CC values, e.g. from 20600 to 20200, the average utilization level does not change. Taking a look at the cost components percentage, one can see that although the average utilization level remains constant, the FGI holding cost percentage increases while fixed cost percentage does not change noticeably. This means the model chooses to produce in advance rather than to use more facilities. The same behavior happens when CC decreases from 19800 to 19000 which can be explained in the same manner.

Table 3. Cost Components under different CCs

Carbon Cap	Production	Fixed	Raw Material Release	WIP Holding	FGI Holding	Trans
18200	21.21%	30.90%	28.28%	0.43%	1.28%	17.91%
18600	21.28%	31.00%	28.38%	0.74%	0.75%	17.85%
19000	21.36%	30.39%	28.48%	0.99%	0.91%	17.88%
19400	21.42%	30.48%	28.56%	1.29%	0.44%	17.82%
19800	21.46%	30.53%	28.61%	1.58%	0.00%	17.82%
20200	21.52%	29.89%	28.69%	1.81%	0.34%	17.76%
20600	21.55%	29.94%	28.74%	2.05%	0.00%	17.72%
21000	21.58%	29.24%	28.93%	2.34%	0.25%	17.67%
21400	21.58%	29.24%	28.93%	2.34%	0.25%	17.67%
21800	21.58%	29.24%	28.93%	2.34%	0.25%	17.67%
22200	21.58%	29.24%	28.93%	2.34%	0.25%	17.67%

In what follows, we examine the effects of uncertainty in estimating each source of emission by changing the budget of uncertainty under different CC values.

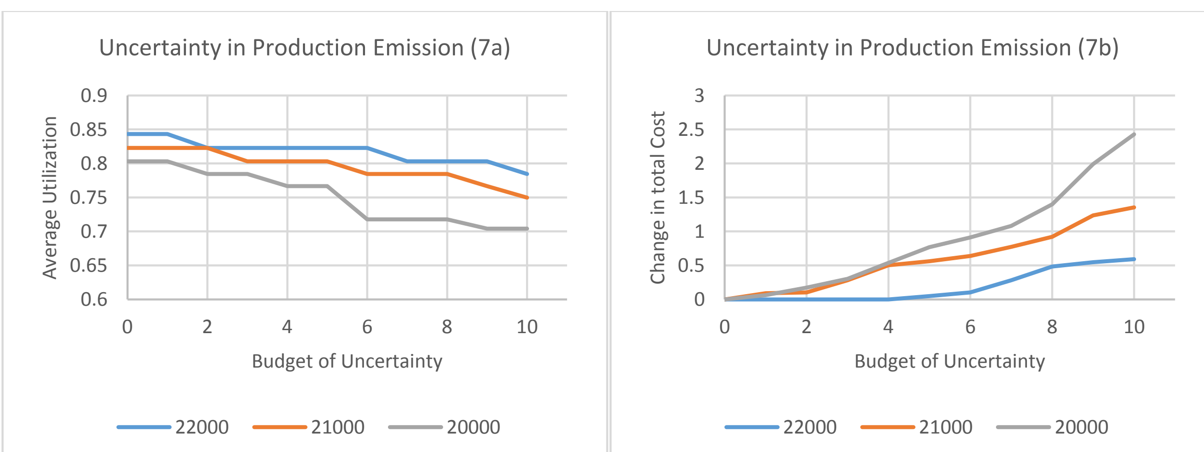

Figure 7. The effect of Uncertainty in Production Emission on Average Utilization (7a) and Total Cost (7b)

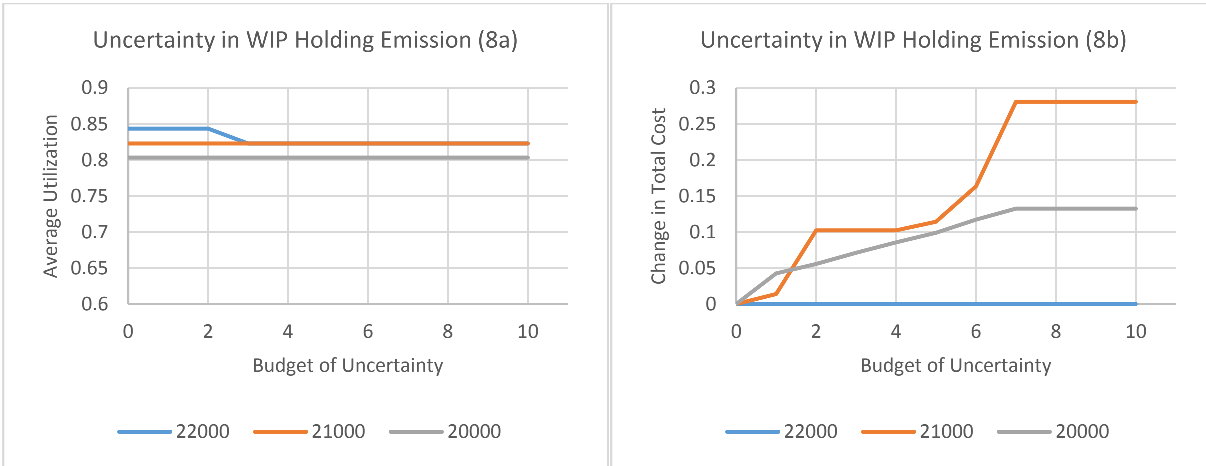

Figure 8. The effect of Uncertainty in WIP Holding Emission on
Average Utilization (Figure 8a) and Total Cost (Figure 8b)

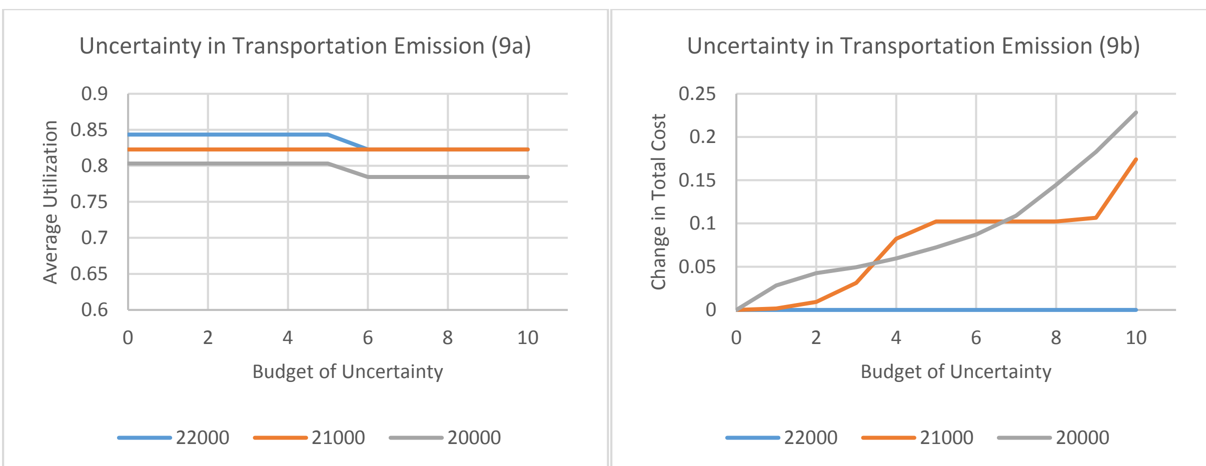

Figure 9. The effect of Uncertainty in Transportation Emission on
Average Utilization (Figure 9a) and Total Cost (Figure 9b)

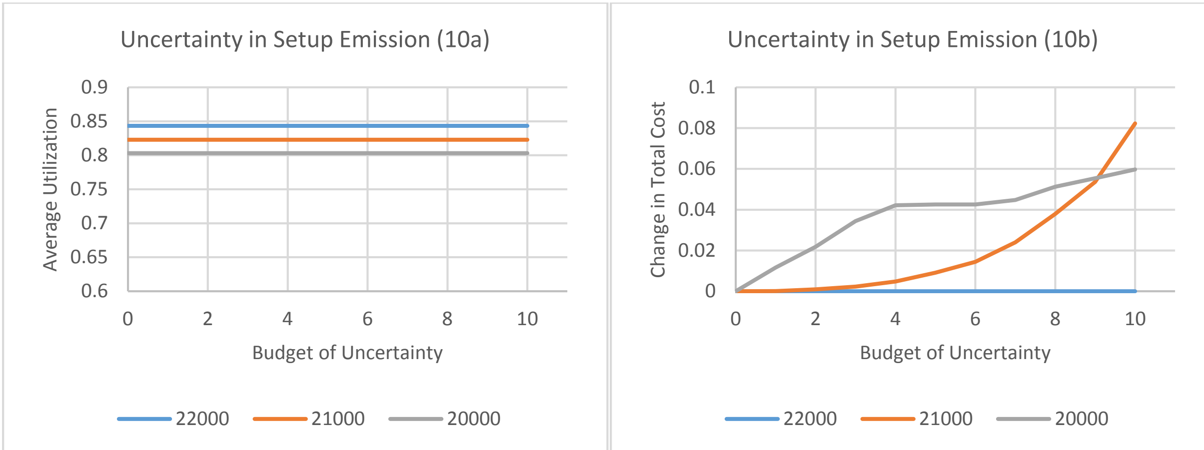

Figure 10. The effect of Uncertainty in Setup Emission on Average Utilization (Figure 10a) and Total Cost (Figure 10b)

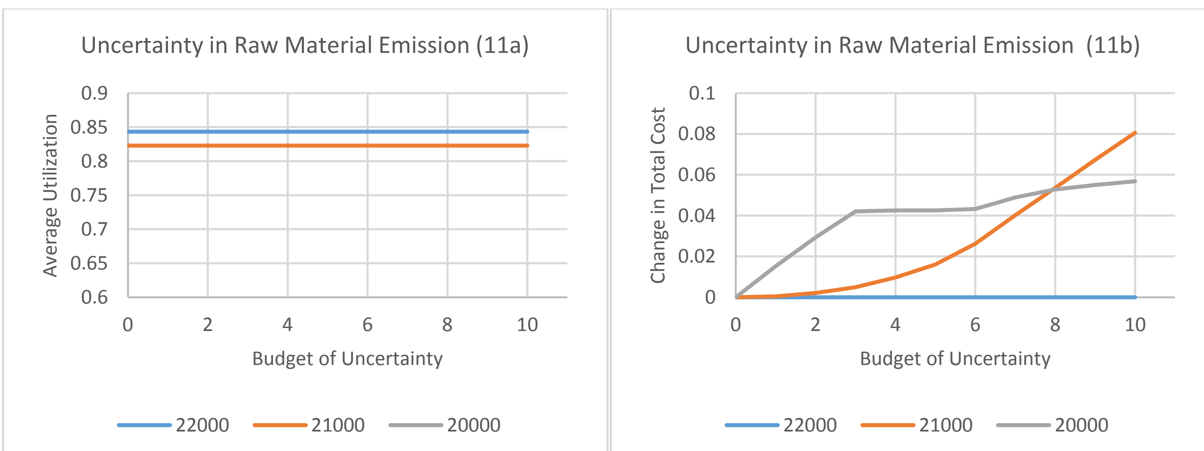

Figure 11. The effect of Uncertainty in Raw Material Procurement Emission on Average Utilization (Figure 11a) and Total Cost (Figure 11b)

The main takeaways from Figures 7-11 are summarized as follows:

- *Increasing uncertainty has a similar effect as that of decreasing the CC.* The rationale behind this observation is as follows. Note that when CC increases the production-distribution decisions would be taken with less sensitivity to the amount of emission, whereas with a tight CC, the decision maker has more concern not to exceed the CC, which results in more costly solutions.

The same rationale can explain the effect of uncertainty on the optimal decisions. Specifically, by increasing the budget of uncertainty we are capturing more uncertainty in the emissions of activities, and consequently, we have more conservative solutions. This means that we suspect the emission to be higher than the nominal emission and this situation gets worse as the risk aversion level of the decision maker (budget of uncertainty) increases. Therefore, the solution should work under any realization of the emission level based on a specific CC. Although having such conservative solutions guarantees that we will not exceed the CC, it comes at the cost of robustness; the more robust a solution is, the more the operational cost would be.

- *The effect of uncertainty on cost monotonically increases as CC decreases*. Note that, at one hand, the uncertainty is defined on the amount of emission resulted from different activities, and increasing the budget of uncertainty means that the emission may have higher perturbation than the nominal emission. On the other hand, this is not surprising that making production-distribution decision should be with utmost care in order to meet the limit on a tight CC. Therefore, having robust solutions, which requires a high level of budget of uncertainty, under a tight CC may lead to costly solutions. Moreover, the total cost *monotonically* increases in budget of uncertainty when CC decreases. This can be also observed from the above figures. Specifically, from Figures 6b, 7b, 8b, 9b, 10b, and 11b, having more robust solutions is more costly when the CC decreases either from 22,000 to 20,000 or from 21,000 to 20,000. That said, one can distinguish non-monotone increase in cost when the CC decreases from 21,000 to 20,000 as appeared in figures 8b and 9b. Observe that the total cost significantly increases when budget of uncertainty increases for CC equal to 21,000 comparing to a tighter CC, namely 20,000.
- *The effect of uncertainty is highly dependent on the level of emission.* By comparing Figure 7-Figure 11, it is straightforward to verify that the uncertainty in the amount of emission of production activities has higher impact on both average utilization and total cost compared to other activities, i.e., holding WIP, transportation, and setup emission. This comes from the difference in the level of emission resulted from different activities.

Table 4. Emission of different sources of emission (%)

Production	Transportation	WIP Hold.	Setup	Raw Material	FGI Hold.
56.2	18.3	13.8	5.7	5.6	0.2

Therefore, this observation can be explained based on the contribution of each kind of activity toward the total emission. As one can see in Table 4, production, which has the most significant effect among different activities, has also the biggest share in the total emission. On the other hand, FGI holding which has the smallest share in the total emission has the least effect on the total cost. Furthermore, the effect of uncertainty in setup emission and raw material emission, which both have very close emission levels, are very much similar to each other. This brings us to the conclusion that the effect of uncertainty increases with the share of emission of each activity.

4.2. Computational Results and Discussion

In this section, we present the results of computational experiments and analyze the performance of our proposed formulations and Lagrangian Relaxation approach using a wide variety of instances with different sizes and parameters.

4.2.1. Designing Test Problems

In order to analyze the performance of the solution algorithm, we develop four different scenarios; (*i*) base case scenario, (*ii*) dominant setup cost, (*iii*) dominant transportation cost, and (*iv*) very tight CC.

In developing the base case scenario, the model parameters are assumed to be equal to the parameters in Table 1. The budget of uncertainty is assumed to be equal to 5 for all parameters to maintain a certain level of uncertainty. The coefficient of variation in the robust model (ϱ) is set to 1%. The gaps for the lower bound derived from solving the LP model and the LR are equal to:

$$LP\ Gap = \frac{Best\ Feasible\ Solution - LP^*}{Best\ Feasible\ Solution}$$

$$LR\ Gap = \frac{Best\ Feasible\ Solution - LR_{LB}}{Best\ Feasible\ Solution}$$

The optimality tolerance for the Lagrangian problem was set to 10^{-2}. The LB improvement procedure will stop if the UB has not been improved in the last 50 iterations. In setting the initial values of Lagrangian multipliers, we use the dual values of the corresponding constraints in LP relaxation of the problem. If the feasible solution from the heuristic contains facilities which are loaded less than 50%, the algorithm sets those facilities free and solve the problem again. In that case, the feasible solution that gives the minimum total cost would be the best feasible solution.

In order to have a dominant setup cost, the value of setup cost in Scenario *ii* is doubled compared to its value in Scenario *i*. The cost of fuel is tripled to construct Scenario *iii*. We then multiply CC with 0.97 to generate a very tight CC for Scenario *iv*. We choose 0.97 to maintain the similar carbon reduction amount in all problem instances since multiplying CC by a factor less than 0.97 would be infeasible for some of the instances.

Each problem is denoted by (j,i,t) where j , i , and t are the number of potential facilities, demand regions, and periods, respectively. We set the number of facilities to 5, 10, 15, 25, and 50, the number of demand regions to 0.4, 0.6, and 0.8 times the number of potential facilities (to keep enough additional facilities to avoid congestion), and the number of periods to 5 and 10. Note that for experiments number 25-27, the Lagrangian relaxation approach could not find a decent optimal gap in a reasonable time for the 10 period instance. Therefore, we did not report the information on these instances in the following tables.

In Tables 5-8, we present the results regarding the comparisons of LP and LR solutions for theses 27 problem instances. All the numbers reported in the following tables are rounded to the nearest tenth. Regarding the Carbon Cap, we first run the model without a cap and measure the nominal emission for each of the instances. Since we did not consider uncertainty in measuring this value, putting CC equal to this value will give a relatively tight cap.

Table 5. Comparison of the bounds and heuristic performance: Base-Case Scenario

		Cost Component (%)						Gap			
No	J.I.T	Production	WIP Hold.	FGI Hold.	Procurement	Trans	Setup	LP	LR	FS Impv	CPU Time (sec)
1	5.2.5	21.3	0.2	0	28.4	17.2	32.9	9.9	2.7	3.7	15
2	5.2.10	21.2	1.5	0.7	28.4	17.2	30.8	7.9	2.4	5.0	70
3	5.3.5	21.5	1.1	0	28.7	14.8	34	12	2.9	6.7	44
4	5.3.10	22.6	1.3	0	30.1	13.6	32.3	7.1	1.3	1.9	135
5	5.4.5	21.6	2.7	0	28.8	17.8	29	7.0	0.7	0.0	80
6	5.4.10	21.6	2.3	0.3	28.9	17.7	29.2	6.8	1.1	0.6	222
7	10.4.5	22.9	0.8	0.2	30.5	12.3	33.2	9.1	1.8	0.1	116
8	10.4.10	22.8	1.4	0.3	30.7	12.4	33.3	5.9	1.7	2.8	447
9	10.6.5	23.3	1.4	0.2	31.4	12.6	32.2	9.8	0.9	0.0	367
10	10.6.10	22.4	0.9	0	29.9	15.2	31.6	7.9	1.1	0.9	658
11	10.8.5	22.6	0.6	0	30.1	14	32.7	7.3	1.3	2.7	302
12	10.8.10	22.5	1	0..3	30	14.7	31.5	7.1	1.3	0.9	1093
13	15.6.5	23	0.6	0	30.7	12.4	33.1	8.9	1.9	0.8	524
14	15.6.10	22.9	0.5	0.2	30.5	11.4	34.4	7.5	2.8	2.3	4525
15	15.9.5	23.1	0.8	0	30.8	12.1	33.2	8.3	1.6	3.4	540
16	15.9.10	22.9	0.7	0.2	30.5	12.4	33.3	9.2	2.3	3.1	2983
17	15.12.5	23.2	0.6	0.4	30.8	12.4	32.5	7.5	1.0	1.7	750
18	15.12.10	23.2	0.8	0.2	31	12.2	32.5	7.4	1.1	1.5	2838
19	25.5.5	23.5	1.3	0.4	31.4	10	33.4	9.0	2.7	3.2	646
20	25.5.10	23.4	1.9	0.2	31.2	10.7	32.6	9.5	3.4	1.7	2451
21	25.15.5	23.5	0.2	0	31.4	10.6	34.4	8.8	2.3	0.0	2131
22	25.15.10	22.9	0.6	0.2	30.4	10.3	35.5	10.1	5.4	4.2	6771
23	25.20.5	23.8	0.9	0	31.7	10.2	33.4	7.9	1.1	1.8	2355
24	25.20.10	23.5	0.8	0	31.4	11.3	33	7.5	3.9	2.0	6846
25	50.10.5	23.6	0.8	0.3	31.3	10.2	33.7	10.6	3.3	3.0	4400
26	50.30.5	23.7	0.6	0	31.6	8.8	35.2	8.9	4.8	7.6	10384
27	50.40.5	24	1.2	0	31.4	8.9	34.7	9.5	3.4	2.2	9039
	Min	21.2	0.5	0.0	28.4	8.8	29.0	5.9	0.3	0.0	15
	Max	24.0	2.7	0.7	31.7	17.8	35.2	11.7	4.8	9.8	10384
	Average	22.9	1.2	0.1	30.5	12.8	32.4	8.3	1.9	2.8	2249

Table 6. Comparison of the bounds and heuristic performance: Dominant Setup Cost Scenario

		Cost Component (%)						GAP (%)			
No	J.I.T	Production	WIP Hold	FGI Hold	Procurement	Trans.	Setup	LP	LR	FS Impv	CPU Time (sec)
1	5.2.5	16.7	0.6	0	22.3	14.5	45.9	9.5	0.4	8.1	6
2	5.2.10	16.3	0.9	0.4	21.9	13.4	47.2	10.8	3.3	2.2	92
3	5.3.5	17	0.9	0	22.8	11.7	47.5	9.3	0.8	2.7	52
4	5.3.10	17	0.8	0	22.8	10.4	48	10.6	1.6	1.5	147
5	5.4.5	16.8	2.1	0	22.3	13.8	45	9.4	0.2	0.0	61
6	5.4.10	16	1.7	0.2	21.5	13.2	47.2	8.8	0.8	0.0	284
7	10.4.5	17.3	0.1	0.1	23.4	9.4	48.5	8.2	0.8	2.1	169
8	10.4.10	17.5	1.2	0.2	23.4	9.5	48.1	7.9	1	1.2	642
9	10.6.5	17.5	1	0.2	23.4	9.4	48.4	9.3	0.6	0.0	262
10	10.6.10	17.1	0.8	0.1	22.8	11.6	47.4	8.9	1.1	0.5	1248
11	10.8.5	17.3	1.1	0.5	22.6	10.6	48	9.5	0.9	0.9	392
12	10.8.10	17.2	1.4	0.1	23	11.3	47	9.5	0.9	1.9	1182
13	15.6.5	17	0.9	0.3	22.7	9	50.1	13	4.2	10.2	368
14	15.6.10	17.6	0.9	0.1	23.5	9	48.9	8.8	1.9	0.8	1926
15	15.9.5	17.5	1.2	0.4	23.4	9.4	48.1	10.3	1.3	6.3	645
16	15.9.10	17.5	1	0.2	23.4	9.6	48.3	9.4	1.2	3.1	4111
17	15.12.5	17.5	1	0.2	23.5	9	48.5	9.7	1	2.4	1549
18	15.12.10	16.9	0.6	0	24.1	10.1	48.3	8.7	1.7	0.0	3678
19	25.5.5	17.2	1.4	0	23	7.5	51	13.8	4.1	1.9	801
20	25.5.10	17.7	1.7	0	23.6	8.6	48.3	9.5	2.3	3.7	3486
21	25.15.5	17.8	0.8	0.2	23.8	7.7	50	11.1	1	2.2	2242
22	25.15.10	16.8	0.1	0	22.4	7.5	53.2	15	4.2	9.9	7183
23	25.20.5	17.9	0.9	0	23.8	7.7	49.6	9.9	0.9	0.0	5298
24	25.20.10	17.7	1.1	0	23.6	8.5	49.1	10.2	4.1	4.3	6765
25	50.10.5	17.3	0.5	0	23	7	52	14.2	5	14.7	5030
26	50.30.5	17.7	0.3	0	23.6	6.5	51.8	11.9	3.7	8.1	8948
27	50.40.5	16.6	0.8	0.2	23.2	8.7	50.5	9.7	3.1	2.1	10263
	Min	16.0	0.1	0.0	21.5	6.5	48.0	7.9	0.2	0.0	6
	Max	17.9	2.1	0.5	24.1	14.5	52.2	15.0	5.0	14.7	10263
	Average	17.2	1.0	0.1	23.0	9.8	48.7	10.3	1.9	3.4	2475

Table 7. Comparison of the bounds and heuristic performance: Dominant Trans. Cost Scenario

		Cost Component (%)						Gap			
No	J.I.T	Production	WIP Hold.	FGI Hold	Procurement	Trans.	Setup	LP	LR	FS Impv	CPU Time (sec)
1	5.2.5	15.9	0.3	0	21.1	38.3	24.4	8.6	2	0	26
2	5.2.10	15.7	1.1	0.4	21.1	37.7	24	8.4	3.3	0	73
3	5.3.5	17.1	0.9	0.1	22.9	35.3	23.8	6.7	0.4	0	28
4	5.3.10	17.9	1.2	0	23.9	32.1	24.8	5.9	0.2	0	210
5	5.4.5	15.8	1.9	0	21.1	38.8	22.3	6.5	1.2	0	61
6	5.4.10	16	1.9	0.2	21.4	38.8	21.7	5.7	0.5	0	455
7	10.4.5	18.4	0.6	0.2	24.5	29.7	26.6	8.2	1.5	0	160
8	10.4.10	18.5	1.2	0.2	24.9	29.8	25.4	5.4	0.7	0	623
9	10.6.5	17.4	1	0	22.8	34.1	24.7	7.6	0.6	0.2	581
10	10.6.10	17.2	0.8	0	22.9	34.9	24.3	6.1	0.9	1.7	878
11	10.8.5	17.6	0.4	0.2	23.5	32.9	25.5	7.1	1.3	0.4	494
12	10.8.10	17.4	0.8	0.2	23.2	34	24.3	6.9	1.2	0.3	1513
13	15.6.5	18.8	0.9	0	25	28.6	26.7	8.4	1.4	3.1	478
14	15.6.10	19	0.4	0.4	25.3	26.8	28.1	7.7	2.9	1.4	1224
15	15.9.5	18.6	0.6	0	24.8	29.3	26.7	8.4	1.6	0.7	1070
16	15.9.10	18.3	0.9	0.2	25.2	29.8	25.6	9.3	0.7	0	623
17	15.12.5	18.7	0.3	0.6	24.8	29.3	26.2	7.4	0.9	1.2	1358
18	15.12.10	18.7	0.7	0	24.9	29.3	26.4	6.7	1	0.6	3954
19	25.5.5	20	1.4	0.1	26.6	23.5	28.3	9.6	2.4	4	622
20	25.5.10	20.1	1.7	0.1	26.9	23.8	27.4	11.0	1.6	4.5	2749
21	25.15.5	19.7	0.3	0	26.3	25.3	28.4	9.5	1.4	2.3	5173
22	25.15.10	19.3	0.2	0	25.7	25.2	29.6	10.0	1.3	4.8	7098
23	25.20.5	19.8	0.8	0	26.3	25.5	27.7	7.8	1.4	1.4	3586
24	25.20.10	18.9	0.5	0.1	24.9	28.1	27.5	6.2	2.1	1.5	10294
25	50.10.5	19.1	0.2	0	25.8	26.3	28.6	7.5	2.0	0.9	7412
26	50.30.5	19.3	0.3	0.1	26.1	26.2	27.8	11.2	1.9	1.8	7108
27	50.40.5	20.4	0.1	0	27.3	22.8	29.4	8.3	2.7	1.9	8702
	Min	15.7	0.1	0.0	3.9	22.8	2.3	5.4	0.2	0.0	26
	Max	20.4	1.9	0.6	27.3	38.8	29.6	11.2	3.3	4.8	10294
	Average	18.3	0.8	0.1	23.7	30.2	24.7	7.9	1.4	1.4	2464

Table 8. Comparison of the bounds and heuristic performance: Tight Carbon Cap Scenario

		Cost Component (%)						Gap			
No	J.I.T	Production	WIP Hold.	FGI Hold	Procurement	Trans.	Setup	LP	LR	FS Impv	CPU Time (sec)
1	5.2.5	20.7	0	0	27.6	16.7	34.9	12.7	4.8	0.0	16
2	5.2.10	17.7	0	1.7	29.5	15.8	35.1	8.8	5.9	1.6	55
3	5.3.5	21.6	0	0	28.9	15.2	34.2	9.6	4.1	1.9	27
4	5.3.10	22.2	0.4	0.5	29.6	13.6	33.7	8.9	1.3	2	122
5	5.4.5	21.2	1	0.4	28.3	17.8	31.2	8	2.7	1.5	52
6	5.4.10	21.3	0.7	0.9	28.4	17.7	31	7.9	2.4	0.0	162
7	10.4.5	22.9	0.7	0.3	30.5	12.3	33.2	9.9	1.8	0.0	122
8	10.4.10	22.5	0.2	1.6	30	12.2	33.3	7.2	1.4	0.6	338
9	10.6.5	22	0.2	0.5	29.4	15.3	32.4	8	2.3	0.2	176
10	10.6.10	22.3	0.6	0.2	29.7	15.2	31.9	7.3	1.5	0.9	813
11	10.8.5	22.2	0	0.2	29.7	14	33.8	9	2.6	0.6	240
12	10.8.10	22.5	0.9	0.4	30	14.7	31.5	8	1.4	1.0	1201
13	15.6.5	22.7	0.1	0.7	30.3	11.7	34.5	9.6	3.4	2.7	289
14	15.6.10	23.1	0.1	0.8	30.8	10.9	34.2	6.7	3.1	2.8	997
15	15.9.5	23	0.3	0	30.7	12.1	33.7	9.1	2	3.1	646
16	15.9.10	22.5	0.6	0.4	30	14.9	31.6	9.8	1.3	0.9	2422
17	15.12.5	23.2	1	0	30.8	12.3	32.6	7.7	1	0.0	2100
18	15.12.10	23.2	0.5	0.3	31	12.2	32.7	7	1.3	1.3	2482
19	25.5.5	23.3	0.6	0.9	31	10	34.2	11.3	4.1	9.6	495
20	25.5.10	23.4	1.2	0.2	21.2	10.8	33.1	9	2.4	4.3	7344
21	25.15.5	23.6	0.2	0.1	31.5	10.4	34.1	8.4	1.6	1.1	1778
22	25.15.10	23.4	0.5	0	30.7	12.1	33.3	10.1	1.5	0.4	4306
23	25.20.5	23.7	0.6	0	31.7	10.2	33.6	7.2	1.3	1.8	2677
24	25.20.10	23.8	0.9	0	27.2	11.9	36.2	11.1	3.1	2.5	5461
25	50.10.5	23.7	0.7	0.2	31.8	9.2	34.2	8.8	1.8	6.4	3855
26	50.30.5	23.4	0.3	0	31.9	10.2	34.2	9.2	2.3	6.1	8098
27	50.40.5	24	0	0.1	32.1	8.9	34.9	8	4.9	1.3	8851
	Min	17.7	0.0	0.0	21.2	8.9	31.0	7.2	1.0	0.0	16
	Max	24.0	1.2	1.7	32.1	17.8	36.2	12.7	5.9	9.6	8851
	Average	22.4	0.4	0.3	29.6	13.3	33.3	9.2	2.7	1.9	2042

Based on the results from solving different instances in Tables 5-8, we make the following observations:

- The results indicate that the heuristic algorithm finds good feasible solution in a reasonable amount of time for all instances. Specifically, it can solve any instance in less than 10384 seconds (2308 seconds on average for all 108 instances). The largest problem contains 40 demand regions, 50 facilities, and 5 periods, which was solved in 9039 seconds with a 3.4% gap. When solving the problem for larger instances, we observed larger gap for the Lagrangian relaxation method and the heuristic. For example, for a problem with 40 demand regions, 50 facilities, and 10 periods, the LR gap is equal to 22% which is obtained in 14292 seconds. We would like to note that a 22% gap on the feasible solution does not necessarily mean an inappropriate feasible solution, since such a large gap may be because of a worse lower bound obtained from the LR method. This implies that the real gap between the feasible and the optimal solution is less than 22%.
- The gap between the lower bound, obtained from the LR method, and the feasible solution obtained from the heuristic algorithm, varies between 0.2% and 5.1% with an average of 2.1% for all instances. In terms of the effectiveness of our proposed approach, the gap between the lower bound obtained from LP and the feasible solution obtained from our proposed heuristic can be up to 14.2% with an average of 10%. This confirms the efficiency of the Lagrangian relaxation method applied in our solution methodology.
- The time needed to obtain the feasible solution through heuristic is negligible; i.e., almost zero for any size of instance. It took nearly 1 to 2 seconds to solve the heuristic to build the feasible solution in most of the problem instances. The maximum computation time to obtain the feasible solution for a big problem is 6.2 seconds while the total CPU time is 10384 seconds (less than 0.06%). Such a low computation time along with the really small gap shows that proposed heuristic has been successful in finding a good feasible solution in a reasonable amount of time.
- The improvement achieved in the value of the objective function is reported in the "FS impv" column of the tables. It is obtained by dividing the difference between the first and second feasible solutions over the second heuristic. Note that the second solution usually dominates the first feasible solution since we may have relaxed some of the constraints in the second run. As one can verify from the results, the second-run solution tends to be a better than the

first-run solution in 88.9% of the instances with an average of 2.5% and the maximum of 14.7% improvement in the objective value.

In the tables above, we noticed that when the number of facilities are relatively large and the number of periods is 10, the gap increases significantly. For example, in experiment 27, when the number of periods was increased to 10, the LR gap and the heuristic bound are 19.6% and 22%. We would also be interested in exploring the performance of solution methodology on some problem instances of larger size. We develop these problem instances of interest by increasing the number of facilities in experiments 26 and 27 to 60 and 70 facilities. In Tables 9-12, we provide the results of same analysis on this new set of problem instances.

We also solve the original problem without applying our solution methodology using CPLEX which helps us to verify how efficient our proposed approach is compared to that if CPLEX solves the problem. The gap between the lower and upper bounds obtained from CPLEX is shown as "*CPLEX gap*". In order to have a fair comparison, we allow the CPLEX to run for a period of 14400 seconds (4 hours) and then compare the gap obtained from our methodology with the one obtained from CPLEX.

Table 9. Comparison of the bounds and heuristic performance: Base-Case Scenario for large instances

		Gap (%)				
No	J.I.T	LP	LR	CPLEX	FS Impv	CPU Time (sec)
1	25.15.5	8.8	2.3	0.7	1.5	2131
2	25.15.10	10.1	**5.4**	NA	4.2	6771
3	25.20.5	7.9	1.1	0.3	1.8	2355
4	25.20.10	7.5	**3.9**	NA	2.0	6846
5	50.10.5	10.6	3.3	1.5	3.0	4400
6	50.30.5	8.9	4.8	1.9	7.6	10384
7	50.40.5	9.5	**3.4**	NA	2.2	9039
8	60.30.5	11.9	**7**	NA	5.6	9143
9	60.40.5	9.8	**5.8**	NA	5.4	10278
10	70.30.5	13.3	**8.5**	NA	5.2	13541
11	70.40.5	14.7	**11.1**	NA	5.7	12648

Min		7.5	1.1	0.3	1.5	2131
Max		14.7	11.1	NA	7.6	13541
Average		10.3	5.1	NA	4.0	7958

Table 10. Comparison of the bounds and heuristic performance: Dominant Setup Cost Scenario for large instances

		Gap				
No	J.I.T	LP	LR	CPLEX	FS Impv	CPU Time (sec)
1	15.25.5	11.1	1	0.9	2.2	2242
2	15.25.10	15	4.2	1.7	9.9	7183
3	20.25.5	9.9	0.9	0.4	1.3	5298
4	20.25.10	10.2	**4.1**	NA	4.3	6765
5	10.50.5	14.2	5	0.8	14.7	5030
6	30.50.5	11.9	**3.7**	NA	8.1	8948
7	40.50.5	9.7	**3.1**	NA	2.1	10263
8	30.60.5	14.4	**5.9**	NA	10.4	11344
9	40.60.5	15.3	**5.6**	NA	3.7	10738
10	30.70.5	12.2	**7.4**	NA	2.9	12872
11	40.70.5	15.9	**11.9**	NA	4.1	13064
Min		9.7	0.9	0.4	1.3	2242
Max		15.9	11.9	NA	14.7	13064
Average		12.7	4.8	NA	5.8	8522

Table 11. Comparison of the bounds and heuristic performance: Dominant Trans. Cost Scenario for large instances

		Gap				
No	J.I.T	LP	LR	CPLEX	FS Impv	CPU Time (sec)
1	15.25.5	9.5	1.4	0.6	2.3	5173
2	15.25.10	10.0	1.3	0.3	4.8	7098
3	20.25.5	7.8	1.4	0.3	1.4	3586
4	20.25.10	6.2	2.1	0.7	1.5	10294
5	10.50.5	7.5	2.0	0.7	0.9	7412

6	30.50.5	13.6	**1.9**	NA	1.8	7108
7	40.50.5	8.1	**2.7**	NA	1.9	8702
8	30.60.5	15.8	**6.9**	NA	4.8	9433
9	40.60.5	17.2	**10.2**	NA	2.9	12169
10	30.70.5	10.6	**5.7**	NA	2.1	16054
11	40.70.5	13.2	**15.6**	NA	5.0	16581
Min		6.2	1.3	0.3	0.9	3586
Max		17.2	15.6	0.7	5.0	16581
Average		10.9	4.7	0.5	2.7	9419

Table 12. Comparison of the bounds and heuristic performance: Tight Carbon Cap Scenario for large instances

		Gap				
No	J.I.T	LP	LR	CPLEX	FS Impv	CPU Time (sec)
1	15.25.5	8.4	1.6	0.3	1.1	1778
2	15.25.10	10.1	**1.5**	NA	0.4	4306
3	20.25.5	7.2	1.3	0.2	1.8	2677
4	20.25.10	11.1	**3.1**	NA	2.5	5461
5	10.50.5	8.8	1.8	0.9	6.4	3855
6	30.50.5	9.2	**2.3**	NA	6.1	8098
7	40.50.5	8	**4.9**	NA	1.3	8851
8	30.60.5	12.9	**4.8**	NA	2.9	8694
9	40.60.5	13.9	**5.3**	NA	4.8	9281
10	30.70.5	14.6	**6.3**	NA	3.2	14052
11	40.70.5	15.9	**10.3**	NA	1.9	12932
Min		7.2	1.3	0.2	0.4	1778
Max		15.9	10.3	NA	6.4	14052
Average		10.9	3.9	NA	2.9	7271

Based on the results from solving larger problem instances in tables 9-12, we make the following observations:

- The results provided in the tables 9-12 indicate that the heuristic succeeds in finding good feasible solution in a reasonable amount of time for all instances. Specifically, it can solve any instances provided in less than 16581 seconds. The average LR gap and CPU time for all large instances was equal 4.6% and 8293 seconds, respectively. The largest gap is equal to 15.6 for experiment with 40 demand region, 70 facilities, and 5 periods in Scenario *iii*. When the number of facilities was increased to 80, it was noticed that the LR gap increased significantly and was equal to 18.3%. We also run the model for larger instances and the gaps were deteriorating as we increased the size of the problem.

- Similar to the experiments in tables 5-8, the second solution dominates the first feasible solution for large instances as well. In particular, as one can verify from the results, the second-run solution tends to be a better one than the first-run solution in all of the instances with an average of 3.9% and the maximum of 14.7% improvement in the objective value. This observation confirms the effectiveness of our proposed heuristic.
- Finally, we used CPLEX to solve the original problem without applying our solution methodology. Note that in some cases CPLEX could not find a feasible solution, which are indicated by "*NA*" (Not Available) in the tables above. In most cases where CPLEX could not find even a feasible solution, our solution methodology could successfully find reasonable gaps.

In summary, our solution methodology performs very well as it is able to obtain good solutions for all the instances of our numerical experiments. In order to explore the performance of our methodology, we compare the gap obtained from our approach to those obtained from the LP and CPLEX. The results suggest that our heuristic finds significantly better gaps than those has been found by the LP in a reasonable amount of time. We also show that our heuristic gives acceptable gaps for the large size instances of problem while the CPLEX could not obtain a feasible solution. The following chapter includes a summary of our study and provide some avenues for future work.

Chapter 5

Conclusion and Future Research Avenues

The objective of this thesis is to study a multi-period production distribution planning problem for a multiple facility network with GHG consideration. We modelled the GHG emissions generated by production, holding inventory, transportation, and establishing a facility by adding a constraint that puts an upper limit on the total emission produced. We also considered the impact of congestion on resource efficiency using non–linear CFs. To overcome the nonlinearity issue of CFs, we used a piecewise linearization approach, which, may generate some approximation errors. We then developed an algorithm to minimize the possible approximation error. To deal with the uncertainty that exists in estimating the real emission of supply chain activities, a robust optimization approach has been utilized that finds the best solution given all possible scenarios. We developed a Lagrangian relaxation approach to solve the large size problem instances. To illustrate the impact of including environmental concerns and uncertainty associated with the supply chain activities into our model, we conducted a numerical study. We further provided some examples to examine the performance of our proposed solution methodology.

The results indicate that decreasing the CC would result in making decisions that contain producing less emission. Particularly, in our experiments, the optimal solutions suggest to use more facilities when the CC is decreased. Hence, there will be less congested facilities, and consequently, less WIP levels in the facilities. We then assessed the impact of uncertainty on the operational decisions by changing the budget of uncertainty. The main insights from these experiments are summarized below:

(i) Increasing uncertainty has a similar effect as decreasing the CC does.

(ii) The effect of uncertainty on cost monotonically increases as CC decreases.

(iii) The effect of uncertainty is highly dependent on the level of emission.

Comparing the effect of changing CC and the budget of uncertainty on the operational decisions, we concluded that increasing the uncertainty in estimating the emission associated with each activity of the supply chain has a similar effect on the operational decisions. Moreover, our findings suggest that the effect of uncertainty increases when CC decreases. Note that this observation is completely in line with the first observation. In the final observation, we noticed that the effect of uncertainty increases as the emission level of an emission source increases. We noticed that the effect of uncertainty diminishes when the emission level of that source is relatively small compared to other sources.

We compared the performance of our proposed solution methodology in terms of optimality gap and computation time with the one obtained from LP model and running CPLEX without using our solution algorithm. We found that our solution methodology performs very well as it is able to obtain good solutions for all the instances reported in the numerical experiment. Comparing the gap obtained from our approach to those obtained from the LP and CPLEX, we showed that the optimality gaps of our solutions are better than the gaps of those created by the LP in a reasonable amount of time. We showed that our heuristic yields good gaps for the big size instances while the CPLEX could not even obtain a feasible solution.

This research can be extended in a number of directions. One direction for the future work can be related to the carbon regulations. Considering other types of environmental regulations, such as cap-and-trade and carbon tax, and examining how these regulations will affect the optimal solution is subject to further investigation. Considering cap-and-trade system, the uncertainty in price of carbon allowances, which has been studied in the Economics literature, can be an interesting extension of this study.

Future studies might consider different features in the model such as allowing backorder or uncertainty in demand. Considering different setting in which different means of transportation (*green* and *not green*) or different raw materials with different impacts on the environment and different prices can be selected. In our model, we consider only one transportation mode. It is worthwhile to explore the role of transportation in cost structure and emissions abatement. For example, considering different type of transportation modes with different capacities and even different emission parameters could be another extension of this work. A wide variety of choices for the transportation means can complicate the problem even further. The decision maker may have multiple choices for raw material. The raw materials could differ in terms of price and their environmental impacts, i.e. the greener the raw material, the more expensive it will get.

Studying different heuristic methods to solve the problem would also be an extension of this thesis. The performance of other solution methodology such as, Subgradient optimization algorithm which is another approach to find the best LB, deserves further investigation.

In closing, we provided some insights on considering congestion and uncertainty in emission of supply chain activities in a production planning and demand distribution problem subject to environmental regulations. We examined the impact of considering congestion and environmental constraints, which has not been simultaneously studied in the literature before, on the solution of this problem. We proposed a solution methodology based on Lagrangian relaxation approach that provided feasible solution in a reasonable amount of time.

Chapter 6

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Appendix

GAMS Code:

```
OPTION MIP = Cplex;

OPTION LP = Cplex;

OPTION optcr=0;

OPTION ResLim = 120;

option limrow=1;

Set iter/1*3000/;


sets

i      regions / 1*I /

j      Facilities /1*J/

t      periods /1*T/

p      Possible points /1*20000/;


set

DP(p)    Dynamic subset

FP(p)    Future points  ;


Parameters

a(p)    slope of lines

b(p)    intersection   ;
```

```
Set y/1*2000/;

Set Dh(y);

Set Fh(y);



PARAMETER Sigma(i,j,t);

PARAMETER Nu(j,t);




**********SUB PROBLEM *****************

****************************************


a(p) = /Set of Initial Slopes/

;

b(p)=   /Set of initial Intercepts/

;


set  DP(p)   /1*19/;



Table d(i,j)     Distance
```

```
;

Table lambda(t,i)        Demand at period t in region i

;

*** Scalars declaration ***

scalar c    cost of production // ;

scalar h    holding cost of WIP // ;

scalar tau  holding cost of FGI // ;

scalar r    raw material cost // ;

scalar cf   cost of fuel //;

scalar co   cost of Selecting //;



scalar cp   emission  of production "c prime" // ;

scalar hp   holding emission of WIP //;

scalar taup holding emission of FGI //;

scalar rp   raw material emission //;

scalar cfp  Emission of fuel //;

scalar cop  Emission of Selecting //;


scalar fcr  fuel consumption rate per product//;

scalar GammaC    BO C //;

scalar GammaH    BO C //;
```

```
scalar GammaTAU  BO C //;

scalar GammaR    BO C// ;

scalar GammaCF   BO C // ;

scalar GammaFixed BO C //;


scalar SmallGamma    BO  //;




scalar CC   Carbon Cap //;


Parameter EpsilonMP//;

Table vMPNeg(i,t)

/initial multipliers/;


Table vMPPlu(i,t)

/initial multipliers/;


Parameter Dist(i);

scalar MaxiPro Max Production rate //;

scalar DistancePar //;


Variables

zobj            Objective function
```

W(t) WIP at the end of period t

BegWIP(t) Beginning WIP in period t

F(t) FGI at the end of period t

M(t) Raw material release

X(t) TH during period t

alpha(i,t) fraction of demand of i allocated to facility j at period t

BinServ(t) If used or not

thetaC(t) Dual for RO

BetaC Dual for RO

thetaH(t) Dual for RO

BetaH Dual for RO

thetaTAU(t) Dual for RO

BetaTAU Dual for RO

thetaR(t) Dual for RO

BetaR Dual for RO

thetaCF(t) Dual for RO

BetaCF Dual for RO

thetaFixed(t) Dual for RO

BetaFixed Dual for RO;

Binary variable BinServ(t) ;

positive variables BegWIP(t),n(i,t),W(t),F(t),M(t),X(t),alpha(i,t),TotalEmission(t),thetaC(t),BetaC,thetaH(t),BetaH,thetaTAU(t),BetaTAU,thetaR(t),BetaR,thetaCF(t),BetaCF,thetaFixed(t),BetaFixed;

alpha.up(i,t)=1;

x.up(t) = 350 ;

Equations

cost cost

BWIP(t) Balance Equation for WIP

BFGI(t) Balance Equation for FGI

CPC(p,t) Clearing Function

BinCons(t) For Binary Variable

BegWIPCons(t) Begining WIP

CRO(t) Dual Constraint

HRO(t) Dual Constraint

TAURO(t) Dual Constraint

RRO(t) Dual Constraint

CFRO(t) Dual Constraint

FixedRO(t) Dual Constraint

```
;

cost                  .. zobj =e= sum[(t), ( c*(x(t)) + W(t)*h + F(t)*tau + M(t)*r +
sum(i,[fcr*alpha(i,t)*lambda(t,i)] *DistancePar*Dist(i)*cf)+ co*BinServ(t) )]+
EpsilonMP*( sum[ (t), [ cp*x(t) + W(t)*hp + F(t)*taup + M(t)*rp +
sum(i,  [ fcr*alpha(i,t)*lambda(t,i)] *DistancePar *Dist(i)*cfp)+ cop*BinServ(t) ]  ] +
sum[(t),thetaC(t)+thetaH(t)+thetaTau(t)+thetaR(t)+thetaCF(t)+thetaFixed(t)]+
[BetaC*GammaC+BetaH*GammaH+                          BetaTAU*GammaTAU+BetaR*GammaR+
BetaCF*GammaCF+BetaFixed*GammaFixed  ] )
+ sum((i,t),vMPNeg(i,t)*((alpha(i,t)))) -sum((i,t),vMPPlu(i,t)*(alpha(i,t)))
;

BWIP(t)              .. W(t) - W(t-1) - M(t) + x(t) =e= 0  ;
BFGI(t)              .. F(t)- F(t-1) - x(t) + sum(i,alpha(i,t)*lambda(t,i)) =e= 0 ;
BegWIPCons(t)        .. BegWIP(t) =e=  W(t-1)+M(t);

CPC(DP,t)            .. x(t)-a(DP)*(BegWIP(t)) =l= b(DP);

BinCons(t)           .. x(t) =l= MaxiPro*BinServ(t) ;
```

```
CRO(t)          .. BetaC+      thetaC(t)     =g= cp*x(t)*SmallGamma;

HRO(t)          .. BetaH+      thetaH(t)     =g= W(t)*hp*SmallGamma;

TAURO(t)        .. BetaTAU+    thetaTAU(t)   =g= F(t)*taup*SmallGamma;

RRO(t)          .. BetaR+      thetaR(t)     =g= M(t)*rp*SmallGamma;

CFRO(t)         .. BetaCF+     thetaCF(t)    =g= sum(i,  [ fcr*alpha(i,t)*lambda(t,i) ] *DistancePar
*Dist(i)*cfp)*SmallGamma;

FixedRO(t)      .. BetaFixed+  thetaFixed(t) =g= cop*BinServ(t)*SmallGamma;



Model DemandAllocation /cost,
BWIP,BFGI,BegWIPcons,CPC,BinCons,CRO,HRo,TAURO,RRO,CFRO,FixedRO/;

DemandAllocation.OptFile=1;



****File opt Cpelx option file /cplex.opt/;

**put opt;

**put 'rhsrng Carboncap(t)'/;

**putclose opt;


*************************END OF SUB PROBLEM ************************

**********************Beginning OF Master PROBLEM **********************
```

Parameter WMP(y,t,j) WIP at the end of period t;

Parameter BegWIPMP(y,t,j) Begining WIP in period t

Parameter FMP(y,t,j) FGI at the end of period t

Parameter MMP(y,t,j) Raw material release

Parameter XMP(y,t,j) TH during period t

Parameter alphaMP(y,i,j,t) fraction of demand of i allocated to facility j at period t

Parameter BinServMP(y,j,t) If Servicing or not

Parameter thetaCMP(y,j,t) Dual for RO;

Parameter BetaCMP(y,j) Dual for RO ;

Parameter thetaHMP(y,j,t) Dual for RO ;

Parameter BetaHMP(y,j) Dual for RO ;

Parameter thetaTAUMP(y,j,t) Dual for RO ;

Parameter BetaTAUMP(y,j) Dual for RO ;

Parameter thetaRMP(y,j,t) Dual for RO ;

Parameter BetaRMP(y,j) Dual for RO ;

Parameter thetaCFMP(y,j,t) Dual for RO ;

Parameter BetaCFMP(y,j) Dual for RO ;

Parameter thetaFixedMP(y,j,t) Dual for RO ;

Parameter BetaFixedMP(y,j) Dual for RO ;

parameter thetaFactor(y);

```
Set Dh(y) /1*1/;


Variables

ObjMP

Epsilon

Vneg(i,t)

Vplu(i,t)

thetaa;


thetaa.up =   10000000;


positive variables

Epsilon,vplu(i,t),vneg(i,t);


Vneg.up(i,t)= 10000000;

Vplu.up(i,t)= 10000000;


equation

CostMP

Constraint(y);



CostMP    ..  ObjMP =e= -CC*Epsilon - sum((i,t), vneg(i,t)) +sum((i,t), vplu(i,t))+ thetaa;
```

```
Constraint(Dh)..  thetaa*thetaFactor(Dh) =l= thetaFactor(Dh)*( sum[(t,j), ( c*(xMP(Dh,t,j))

+ WMP(Dh,t,j)*h + FMP(Dh,t,j)*tau + MMP(Dh,t,j)*r + sum(i,[fcr*alphaMP(Dh,i,j,t)*lambda(t,i)

] *DistancePar*D(i,j)*cf)+co*BinServMP(Dh,j,t) )]

+Epsilon*( sum[ (t,j), [ cp*xMP(Dh,t,j) + WMP(Dh,t,j)*hp + FMP(Dh,t,j)*taup + MMP(Dh,t,j)*rp
+sum(i,  [ fcr*alphaMP(Dh,i,j,t)*lambda(t,i) ] *DistancePar *D(i,j)*cfp)+ cop*BinServMP(Dh,j,t) ] ]+
sum[(j,t),thetaCMP(Dh,j,t)+thetaHMP(Dh,j,t)+thetaTauMP(Dh,j,t)+thetaRMP(Dh,j,t)+thetaCFMP(
Dh,j,t)+thetaFixedMP(Dh,j,t)]+        sum[j,BetaCMP(Dh,j)*GammaC+BetaHMP(Dh,j)*GammaH+
BetaTAUMP(Dh,j)*GammaTAU+BetaRMP(Dh,j)*GammaR+         BetaCFMP(Dh,j)*GammaCF+
BetaFixedMP(Dh,j)*GammaFixed  ] )


+ sum((i,t),vneg(i,t)*(sum(j,alphaMP(Dh,i,j,t))))-sum((i,t),vplu(i,t)*(sum(j,alphaMP(Dh,i,j,t))))    );


Model MP /CostMP,Constraint/;



***************End of MP*************************

****** Parameter For COnvergence of LB *********


Set Iteration /1*200/;

scalar Converged;

Converged = 0;

parameter Convrg;

Convrg(t,j) = 0;
```

```
Parameter XLevel;

Parameter BegWIPLevel;

Parameter RealBegWIP;

Parameter IterObjValue;

Parameter error;

Parameter best;

Parameter NoOpenFacilty;

Parameter UtilLevel;


Parameter PeriodicAveUtil;


Parameter TOtalWorkInProcess;

Parameter Transportation;

Parameter TOtalEmissionBudget;

Parameter Rawtotal;

Parameter Prototal;

Parameter Fixtotal;


*************Parameter for Convergence of LB***************

Scalar ConvergedLB;

ConvergedLB = 0;

Scalar LB/-10/;

Parameter TempLB(iter);
```

```
Parameter  iterationLB(iter);


Parameter TempUB(iter);

Scalar UB/inf/;

Parameter IterationUB(iter);

Parameter  IterEpsilonMP(iter);

Parameter  iterVMPPlu(iter,i,t);

Parameter  iterVMPNeg(iter,i,t);

Parameter thetapar;

Parameter thetaIter;

Parameter KSUB;

Parameter EpsilonIter(iter);

Parameter CX;




Parameter          TempW(t,j)    ;

Parameter          TempBegWIPMP(t,j)    ;

Parameter          TempFMP(t,j) ;

Parameter          TempMMP(t,j)   ;

Parameter          TempXMP(t,j)   ;

Parameter          TempalphaMP(i,j,t);

Parameter          TempnMP(i,j,t)      ;
```

```
Parameter        TempBinServMP(j,t)  ;


Parameter        TempthetaCMP(j,t)     ;
Parameter        TempBetaCMP(j)        ;
Parameter        TempthetaHMP(j,t)     ;
Parameter        TempBetaHMP(j)     ;
Parameter        TempthetaTAUMP(j,t)       ;
Parameter        TempBetaTAUMP(j)         ;
Parameter        TempthetaRMP(j,t)          ;
Parameter        TempBetaRMP(j)          ;
Parameter        TempthetaCFMP(j,t)         ;
Parameter        TempBetaCFMP(j)          ;
Parameter        TempthetaFixedMP(j,t)          ;
Parameter        TempBetaFixedMP(j)              ;
Parameter        GAP(iter);


Parameter Zobject(j);


Parameter CheckLoop(iter);
CheckLoop(iter)=0;
Parameter NoImprovement(iter);
Parameter FinalGap;
 Parameter CPUTime;
```

```
CPUTime=0;

Parameter CPUTimeMP;

CPUTimeMP=0;

Parameter TotalCPUtime;

Parameter TotalCPUtimeforFUB;

Parameter TotalCPUtimeforSUB;

Parameter CPUT(iter,j);



Loop (iter$(not convergedLB),

                              **************Solve SP *****************

    loop (   iteration$(not converged),

        Loop(j,


         Dist(i)=D(i,j);



         solve DemandAllocation using mip minimizing zobj;

         abort$(DemandAllocation.modelstat=4) "SP Problem is infeasible";

         abort$(DemandAllocation.modelstat=10) "SP Problem is integer infeasible";

         abort$(DemandAllocation.modelstat=3) "SP Problem is unbounded";
```

```
CPUTime=CPUTime+ DemandAllocation.resusd ;

CPUT(iter,j)=DemandAllocation.resusd;


XLevel(iteration,t,j) = X.l(t);

BegWIPLevel(iteration,t,j) = BegWIP.l(t);

RealBegWIP(iteration,t,j) = (70*XLevel(iteration,t,j))/(350-XLevel(iteration,t,j));

IterObjValue(iteration) =  zobj.l;

error(iteration,t,j) = (RealBegWIP(iteration,t,j) - BegWIPLevel(iteration,t,j))/
(RealBegWIP(iteration,t,j)) ;



 loop(t,

    if(  (error(iteration,t,j) >= 10E-3) ,


      FP(p) = DP(p-1)-DP(p);

      a(FP) = (MaxiPro*70) / [(70 + RealBegWIP(iteration,t,j))* (70 +
RealBegWIP(iteration,t,j))];

      b(FP) =  XLevel(iteration,t,j) - a(FP) * RealBegWIP(iteration,t,j);

      DP(p) = DP(p)+FP(p);

      Convrg(t,j) = 0;

    else

      Convrg(t,j)= 1;

      );
```

```
            );



 TempW(t,j)=W.l(t)   ;
 TempBegWIPMP(t,j)= BegWIP.l(t)     ;
 TempFMP(t,j)=F.l(t) ;
 TempMMP(t,j)=M.l(t)  ;
 TempXMP(t,j)= X.l(t)  ;
 TempalphaMP(i,j,t)=alpha.l(i,t);
TempBinServMP(j,t)  =  BinServ.l(t);


 TempthetaCMP(j,t) = thetaC.l(t)    ;
 TempBetaCMP(j)    = BetaC.l     ;
 TempthetaHMP(j,t) = thetaH.l(t)      ;
 TempBetaHMP(j)     =  BetaH.l  ;
 TempthetaTAUMP(j,t) = thetaTAU.l(t)      ;
 TempBetaTAUMP(j)     = BetaTAU.l     ;
 TempthetaRMP(j,t)     = thetaR.l(t)      ;
 TempBetaRMP(j)         = BetaR.l   ;
 TempthetaCFMP(j,t)      = thetaCF.l(t)    ;
 TempBetaCFMP(j)         =  BetaCF.l    ;
 TempthetaFixedMP(j,t)    =  thetaFixed.l(t)        ;
```

```
        TempBetaFixedMP(j)        = BetaFixed.l        ;

        Zobject(j) =zobj.l;




);

***End of J****




converged$(sum[(t,j),Convrg(t,j)]>=10)=1;




            );

****End of Small Loop & SP solved***

            converged=0;
```

```
******Assiging Xh for the master problem
```

```
**                CX(iter) = sum[(t,j), ( c*(x.l(t,j)) + W.l(t,j)*h + F.l(t,j)*tau + M.l(t,j)*r +
sum(i,[fcr*alpha.l(i,j,t)*lambda(t,i)] *DistancePar*D(i,j)*cf)+ co*BinServ.l(j,t) )];

      Fh(y)=Dh(y-1)-Dh(y);

      WMP(Fh,t,j)=TempW(t,j)   ;

      BegWIPMP(Fh,t,j)= TempBegWIPMP(t,j)     ;

      FMP(Fh,t,j)= TempFMP(t,j) ;

      MMP(Fh,t,j)=TempMMP(t,j)  ;

      XMP(Fh,t,j)= TempXMP(t,j)  ;

      alphaMP(Fh,i,j,t)=TempalphaMP(i,j,t);

      BinServMP(Fh,j,t)  =  TempBinServMP(j,t);




      thetaCMP(Fh,j,t) = TempthetaCMP(j,t)    ;

      BetaCMP(Fh,j)    =  TempBetaCMP(j)     ;

      thetaHMP(Fh,j,t) = TempthetaHMP(j,t)    ;

      BetaHMP(Fh,j)     =   TempBetaHMP(j)  ;

      thetaTAUMP(Fh,j,t) =  TempthetaTAUMP(j,t)     ;

      BetaTAUMP(Fh,j)     =  TempBetaTAUMP(j)    ;

      thetaRMP(Fh,j,t)     = TempthetaRMP(j,t)      ;
```

```
BetaRMP(Fh,j)         = TempBetaRMP(j)   ;

thetaCFMP(Fh,j,t)      = TempthetaCFMP(j,t)    ;

BetaCFMP(Fh,j)         =  TempBetaCFMP(j)    ;

thetaFixedMP(Fh,j,t)    =   TempthetaFixedMP(j,t)        ;

BetaFixedMP(Fh,j)       = TempBetaFixedMP(j)       ;

Dh(y)=Dh(y)+Fh(y);


*********End of Assigning Xh********


TempLB(iter)=        sum(j,zobject(j))-        sum((i,t),vMPNeg(i,t))        +sum((i,t),vMPPlu(i,t))-
CC*EpsilonMP;


if (  TempLB(iter)>= LB,

Sigma(i,j,t)=TempalphaMP(i,j,t);

Nu(j,t)=TempBinServMP(j,t);

);


LB=max(LB,TempLB(iter));

iterationLB(iter)=LB;


*********Solving Master*******
```

```
Solve MP using mip maximizing ObjMP;


abort$(MP.modelstat=4) "MP Problem is infeasible";

abort$(MP.modelstat=10) "MP Problem is integer infeasible";

abort$(MP.modelstat=3) "MP Problem is unbounded";

*****Assigning U********

EpsilonMP=Epsilon.l;

VMPPlu(i,t)=vPlu.l(i,t);

VMPNeg(i,t)=vneg.l(i,t);


IterEpsilonMP(iter)=Epsilon.l;

iterVMPPlu(iter,i,t)=vPlu.l(i,t);

iterVMPNeg(iter,i,t)=vneg.l(i,t);



*   thetaPar =  sum(Dh,  [sum[(t,j), ( c*(xMP(Dh,t,j)) + WMP(Dh,t,j)*h +FMP(Dh,t,j)*tau + MMP(Dh,t,j)*r+   sum(i,[fcr*alphaMP(Dh,i,j,t)*lambda(t,i)]   *DistancePar*D(i,j)*cf)+ co*BinServMP(Dh,j,t)  )]  +  Epsilon.l*(  sum[  (t,j),  [  cp*xMP(Dh,t,j)  +  WMP(Dh,t,j)*hp  + FMP(Dh,t,j)*taup  +  MMP(Dh,t,j)*rp  +sum(i,   [  fcr*alphaMP(Dh,i,j,t)*lambda(t,i)  ]  *DistancePar *D(i,j)*cfp)+   cop*BinServMP(Dh,j,t)   ]   ]+ sum[(j,t),thetaCMP(Dh,j,t)+thetaHMP(Dh,j,t)+thetaTauMP(Dh,j,t)+thetaRMP(Dh,j,t)+thetaCFMP(Dh,j,t)+thetaFixedMP(Dh,j,t)]+   sum[j,BetaCMP(Dh,j)*GammaC+BetaHMP(Dh,j)*GammaH+ BetaTAUMP(Dh,j)*GammaTAU+BetaRMP(Dh,j)*GammaR+   BetaCFMP(Dh,j)*GammaCF+ BetaFixedMP(Dh,j)*GammaFixed   ]   )+   sum((i,t),vneg.l(t,i)*((sum(j,alphaMP(Dh,i,j,t)))))- sum((i,t),vplu.l(t,i)*(sum(j,alphaMP(Dh,i,j,t))))]);
```

```
thetaIter(iter) = thetaa.l;

EpsilonIter(iter) = Epsilon.l;


***********End of Assigning***********


TempUB(iter) = ObjMP.l;

UB=min(UB,TempUB(iter));

iterationUB(iter)=UB;


CPUTimeMP=CPUTimeMP+MP.resusd ;

**    KSUB=   Sum(Dh,  [    sum[(t,j), ( c*(xMP(Dh,t,j)) + WMP(Dh,t,j)*h + FMP(Dh,t,j)*tau + MMP(Dh,t,j)*r + sum(i,[fcr*alphaMP(Dh,i,j,t)*lambda(t,i)] *DistancePar*D(i,j)*cf)+ co*BinServMP(Dh,j,t) )]+Epsilon.l*( sum[ (t,j), [ cp*xMP(Dh,t,j) + WMP(Dh,t,j)*hp + FMP(Dh,t,j)*taup + MMP(Dh,t,j)*rp + sum(i, [ fcr*alphaMP(Dh,i,j,t)*lambda(t,i) ] *DistancePar *D(i,j)*cfp)+ cop*BinServMP(Dh,j,t) ] ] + sum[(j,t),thetaCMP(Dh,j,t)+thetaHMP(Dh,j,t)+thetaTauMP(Dh,j,t)+thetaRMP(Dh,j,t)+thetaCFMP(Dh,j,t)+thetaFixedMP(Dh,j,t)]+ sum[j,BetaCMP(Dh,j)*GammaC+BetaHMP(Dh,j)*GammaH+ BetaTAUMP(Dh,j)*GammaTAU+BetaRMP(Dh,j)*GammaR+ BetaCFMP(Dh,j)*GammaCF+ BetaFixedMP(Dh,j)*GammaFixed ] ) + sum((i,t),vneg.l(t,i)*(sum(j,alphaMP(Dh,i,j,t))))- sum((i,t),vplu.l(t,i)*(sum(j,alphaMP(Dh,i,j,t)))) ] );


*********End of SOlving Master Problem************

*************************************************

GAP(iter) = (UB-LB)/UB;
```

```
if (( (UB-LB)/UB )< 0.001,

convergedLB=1;

else

ConvergedLB=0;

);

****** end if

if (     iterationUB(iter-1)-iterationUB(iter)     <1,

NoImprovement(iter)=1;

else

NoImprovement(iter)=0;

);

CheckLoop(iter) =CheckLoop(iter-1)+NoImprovement(iter);

if (Checkloop(iter)=Checkloop(iter-1),
```

```
            Checkloop(iter)=0;
        else

                if ( Checkloop(iter)>50,

                ConvergedLB=1;
                );

        );

);

*end of BIG LOOOP*******
***Assigning Fixed Varialbes**

execute_unload "TempHeuristic2_5_Sec.gdx";
```

```
Parameter ConsforAlpha(i,t);
Parameter ConsforBin(j,t);

      Loop((i,t),
         if ( sum(j,Sigma(i,j,t))=1,

          ConsforAlpha(i,t)=1 ;

          else

          ConsforAlpha(i,t)=0;

             );
          );

           Loop((t,j),

           if (    TempXMP(t,j)>250  ,
           ConsforBin(j,t)=1 ;
```

```
        else
        ConsforBin(j,t)=0;


          );
         );

************MODEL FOR HEURISTIC******************



set
DPH(p)    Dynamic subset
FPH(p)    Future points  ;


Parameters
aH(p)    slope of lines
bH(p)     intersection   ;
```

```
**********SUB PROBLEM *****************
***************************************

aH(p) =
/initial slopes/;


bH(p)=
    /initial intercepts/
 ;


set  DPH(p)   /1*19/;
Variables
zobjHeuristic              OBJ
WH(t,j)           WIP at the end of period t
BegWIPH(t,j)      Begining WIP in period t
FHH(t,j)           FGI at the end of period t
MH(t,j)           Raw material release
XH(t,j)           TH during period t
alphaH(i,j,t)     fraction of demand of i allocated to facility j at period t
nH(i,j,t)         Number of truck from i to j
BinServH(j,t)     If Servicing or not
NoServH(t)        Total Number of Selected Fac.
```

```
thetaCH(j,t)      Dual for RO
BetaCH(j)         Dual for RO
thetaHH(j,t)      Dual for RO
BetaHH(j)         Dual for RO
thetaTAUH(j,t)    Dual for RO
BetaTAUH(j)       Dual for RO
thetaRH(j,t)      Dual for RO
BetaRH(j)         Dual for RO
thetaCFH(j,t)     Dual for RO
BetaCFH(j)        Dual for RO
thetaFixedH(j,t)  Dual for RO
BetaFixedH(j)     Dual for RO
CEmission

;

Binary variable    BinServH(j,t) ;
positive variables BegWIPH(t,j),NoServH(t),WH(t,j),FHH(t,j),MH(t,j),XH(t,j),alphaH(i,j,t),thetaCH(j,t),BetaCH(j),thetaHH(j,t),BetaHH(j),thetaTAUH(j,t),BetaTAUH(j),thetaRH(j,t),BetaRH(j),thetaCFH(j,t),BetaCFH(j),thetaFixedH(j,t),BetaFixedH(j);

alphaH.up(i,j,t)=1;
xH.up(t,j) = 350 ;
```

Equations

costH cost

BWIPH(t,j) Balance Equation for WIP

BFGIH(t,j) Balance Equation for FGI

CPCH(p,t,j) Clearing Function

CarbonCapH Carbon Cap Calc

DemSatH(i,t) Demand satisfaction of i at period t

BinConsH(t,j) For Binary Variable

BegWIPConsH(j,t) Begining WIP

CROH(j,t) Dual Constraint

HROH(j,t) Dual Constraint

TAUROH(j,t) Dual Constraint

RROH(j,t) Dual Constraint

CFROH(j,t) Dual Constraint

FixedROH(j,t) Dual Constraint

CarbonEmission

*FixVarAlpha(i,j,t)

FixVarBin(j,t)

;

```
costH                 .. zobjHeuristic =e= sum[(t,j), ( c*(xH(t,j)) + WH(t,j)*h + FHH(t,j)*tau + MH(t,j)*r
+ sum(i,[fcr*alphaH(i,j,t)*lambda(t,i)] *DistancePar*D(i,j)*cf)+ co*BinServH(j,t) )];


*FixVarAlpha(i,j,t)$(ConsforAlpha(i,t)=1)      .. alphaH(i,j,t)=e=Sigma(i,j,t)    ;

FixVarBin(j,t)$(ConsforBin(j,t)=1)           .. BinServH(j,t) =e=1         ;


BWIPH(t,j)              .. WH(t,j) - WH(t-1,j) - MH(t,j) + xH(t,j) =e= 0  ;

BFGIH(t,j)              .. FHH(t,j)- FHH(t-1,j) - xH(t,j) + sum(i,alphaH(i,j,t)*lambda(t,i)) =e= 0 ;

BegWIPConsH(j,t)        .. BegWIPH(t,j) =e=  WH(t-1,j)+MH(t,j);


CPCH(DPH,t,j)            .. xH(t,j)-a(DPH)*(BegWIPH(t,j)) =l= b(DPH);


DemSatH(i,t)            .. sum(j,alphaH(i,j,t))=e= 1;


CarbonCapH              .. sum[ (t,j), [ cp*xH(t,j) + WH(t,j)*hp + FHH(t,j)*taup + MH(t,j)*rp + sum(i,
[    fcr*alphaH(i,j,t)*lambda(t,i)]    *DistancePar    *D(i,j)*cfp)+    cop*BinServH(j,t)    ]        ]    +
sum[(j,t),thetaCH(j,t)+thetaHH(j,t)+thetaTauH(j,t)+thetaRH(j,t)+thetaCFH(j,t)+thetaFixedH(j,t)]+
sum[j,BetaCH(j)*GammaC+BetaHH(j)*GammaH+
BetaTAUH(j)*GammaTAU+BetaRH(j)*GammaR+
BetaCFH(j)*GammaCF+BetaFixedH(j)*GammaFixed  ] =l= CC;

CarbonEmission            .. CEmission=e= sum[ (t,j), [ cp*xH(t,j) + WH(t,j)*hp + FHH(t,j)*taup +
MH(t,j)*rp + sum(i,  [ fcr*alphaH(i,j,t)*lambda(t,i)] *DistancePar *D(i,j)*cfp)+ cop*BinServH(j,t) ]  ]
;
```

```
BinConsH(t,j)          .. xH(t,j) =l= MaxiPro*BinServH(j,t) ;

CROH(j,t)              .. BetaCH(j)+      thetaCH(j,t)      =g= cp*xH(t,j)*SmallGamma;

HROH(j,t)              .. BetaHH(j)+      thetaHH(j,t)      =g= WH(t,j)*hp*SmallGamma;

TAUROH(j,t)            .. BetaTAUH(j)+    thetaTAUH(j,t)    =g= FHH(t,j)*taup*SmallGamma;

RROH(j,t)              .. BetaRH(j)+      thetaRH(j,t)      =g= MH(t,j)*rp*SmallGamma;

CFROH(j,t)             .. BetaCFH(j)+     thetaCFH(j,t)     =g= sum(i,  [ fcr*alphaH(i,j,t)*lambda(t,i) ]
*DistancePar *D(i,j)*cfp)*SmallGamma;

FixedROH(j,t)          .. BetaFixedH(j+   thetaFixedH(j,t)  =g= cop*BinServH(j,t)*SmallGamma;


Model                                                                    HEURISTICFS
/CostH,BWIPH,BFGIH,BegWIPConsH,CPCH,DemSatH,CarbonCapH,BinConsH,CROH,HROH
,TAUROH,RROH,CFROH,FIXEDROH,FixVarBin/;


Set IterationH /1*200/;

scalar ConvergedH;

ConvergedH = 0;

parameter ConvrgH;

ConvrgH(t,j) = 0;


Parameter XLevelH;

Parameter BegWIPLevelH;
```

```
Parameter RealBegWIPH;

Parameter IterObjValueH;

Parameter errorH;

Parameter bestH;

Parameter NoOpenFaciltyH;

Parameter UtilLevelH;


        loop (   iterationH$(not convergedH),

         solve HEURISTICFS using mip minimizing zobjHeuristic ;

         abort$(HEURISTICFS.modelstat=4) "HEURISTIC is infeasible";

         abort$(HEURISTICFS.modelstat=10) "HEURISTIC is integer infeasible";

         abort$(HEURISTICFS.modelstat=3) "HEURISTIC is unbounded";


         TotalCPUtimeforFUB= HEURISTICFS.resusd;

         XLevelH(iterationH,t,j) = XH.l(t,j);

         BegWIPLevelH(iterationH,t,j) = BegWIPH.l(t,j);

         RealBegWIPH(iterationH,t,j)          =          (70*XLevelH(iterationH,t,j))/(350-
XLevelH(iterationH,t,j));

         IterObjValueH(iterationH) =  zobjHeuristic.l;
```

```
errorH(iterationH,t,j) =  (RealBegWIPH(iterationH,t,j) - BegWIPLevelH(iterationH,t,j))/
(RealBegWIPH(iterationH,t,j)) ;


loop( j,
  loop(t,
        if(  (errorH(iterationH,t,j) >= 10E-3) ,


          FPH(p) = DPH(p-1)-DPH(p);
          aH(FP) = (MaxiPro*70) / [(70 + RealBegWIPH(iterationH,t,j))* (70 +
RealBegWIPH(iterationH,t,j))];
          bH(FP) =  XLevelH(iterationH,t,j) - a(FP) * RealBegWIPH(iterationH,t,j);
          DPH(p) = DPH(p)+FPH(p);
          ConvrgH(t,j) = 0;
        else
          ConvrgH(t,j)= 1;
          );



       );
    );



convergedH$(sum[(t,j),Convrgh(t,j)]>=10)=1;
```

```
        );
    convergedH=0;
Parameter BinForFeasSol(j,t);
BinForFeasSol(j,t)=BinServH.l(j,t);
Parameter FUB;
FUB=   zobjHeuristic.l;


execute_unload "TempHeuristic2_5_third.gdx";
**********************************************


Parameter XHH(t,j);
XHH(t,j)=Xh.l(t,j);
Parameter LowLoad(t,j);
Parameter SecondFUB;
Parameter BestFeasibleUB;


Parameter ProdCost;
Parameter WIPCost;
Parameter FGICost;
Parameter RawMCost;
Parameter TransportationCost;
Parameter SetupCost;
```

```
ProdCost=sum((j,t),XH.l(t,j))*c;

WIPCost=sum((j,t),WH.l(t,j))*h;

FGICost=sum((j,t),FHH.l(t,j))*tau;

RawMCost=sum((j,t),MH.l(t,j))*r;


SetupCost=sum((j,t),BinServH.l(j,t))*co;

TransportationCost = zobjHeuristic.l-(ProdCost+WIPCost+FGICost+RawMCost+SetupCost);


****Check for Low Loaded***

Loop((t,j),


     if ( XHH(t,j)>0,

              if (  (XHH(t,j)/MaxiPro)<0.3,

                             LowLoad(t,j)=1;

                             ConsforBin(j,t)=0;


                );


       );

);

****End of Checking*****
```

```
If ( sum(  (t,j),LowLoad(t,j)  )>0    ,

  loop (   iterationH$(not convergedH),

                 solve HEURISTICFS using mip minimizing zobjHeuristic ;
                 abort$(HEURISTICFS.modelstat=4) "HEURISTIC is infeasible";
                 abort$(HEURISTICFS.modelstat=10) "HEURISTIC is integer infeasible";
                 abort$(HEURISTICFS.modelstat=3) "HEURISTIC is unbounded";

                 TotalCPUtimeforSUB= HEURISTICFS.resusd;
                 XLevelH(iterationH,t,j) = XH.l(t,j);
                 BegWIPLevelH(iterationH,t,j) = BegWIPH.l(t,j);
                 RealBegWIPH(iterationH,t,j)          =          (70*XLevelH(iterationH,t,j))/(350-
XLevelH(iterationH,t,j));
                 IterObjValueH(iterationH) =  zobjHeuristic.l;
                 errorH(iterationH,t,j) =  (RealBegWIPH(iterationH,t,j) - BegWIPLevelH(iterationH,t,j))/
(RealBegWIPH(iterationH,t,j)) ;

                 loop( j,
                   loop(t,
                         if(  (errorH(iterationH,t,j) >= 10E-1) ,
```

```
                    FPH(p) = DPH(p-1)-DPH(p);

                    aH(FP) = (MaxiPro*70) / [(70 + RealBegWIPH(iterationH,t,j))* (70 +
RealBegWIPH(iterationH,t,j))];

                    bH(FP) =  XLevelH(iterationH,t,j) - a(FP) * RealBegWIPH(iterationH,t,j);

                    DPH(p) = DPH(p)+FPH(p);

                    ConvrgH(t,j) = 0;

                  else

                    ConvrgH(t,j)= 1;

                    );




                );

              );




            convergedH$(sum[(t,j),Convrgh(t,j)]>=10)=1;




        );

    convergedH=0;
```

```
    SecondFUB= zobjHeuristic.l;


 );

*****end of if***

Parameter EMision;

Emision = sum[ (t,j), [ cp*xH.l(t,j) + WH.l(t,j)*hp + FHH.l(t,j)*taup + MH.l(t,j)*rp + sum(i, [
fcr*alphaH.l(i,j,t)*lambda(t,i)] *DistancePar *D(i,j)*cfp)+ cop*BinServH.l(j,t) ] ] ;


BestFeasibleUB= min(FUB,SecondFUB);

*FinalGap=  (BestFeasibleUB-LB)/BestFeasibleUB;


Parameter ProdCost2;

Parameter WIPCost2;

Parameter FGICost2;

Parameter RawMCost2;

Parameter TransportationCost2;

Parameter SetupCost2;
```

```
ProdCost2=sum((j,t),XH.l(t,j))*c/zobjHeuristic.l;

WIPCost2=sum((j,t),WH.l(t,j))*h/zobjHeuristic.l;

FGICost2=sum((j,t),FHH.l(t,j))*tau/zobjHeuristic.l;

RawMCost2=sum((j,t),MH.l(t,j))*r/zobjHeuristic.l;


SetupCost2=sum((j,t),BinServH.l(j,t))*co/zobjHeuristic.l;

TransportationCost2 = 1-(ProdCost2+WIPCost2+FGICost2+RawMCost2+SetupCost2);

TotalCPUtime=CPUTime+CPUtimeMP+TotalCPUtimeforSUB+TotalCPUtimeforFUB;


execute_unload "TempHeuristic2_5_5.gdx";
```