# Developing Preservice Teachers’ Professional Noticing of Students’ Learning

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## ABSTRACT

**Developing preservice teachers' professional noticing of students' learning**


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This study examined the effects of an intervention included in a mathematics methods course on preservice teachers' ($N = 29$) ability to specify learning goals of a lesson (Skill 1), collect evidence of student learning (Skill 2), generate hypotheses about the effect of teaching on student learning (Skill 3), and use the analysis to propose alternative teaching strategies to improve the lesson (Skill 4; Hiebert et al., 2007). I examined the effect of direct instruction on Skill 1 on the development of the other three skills. Also, the study examined the nature of specifying learning goals (Skill 1) following instruction that did and did not address this skill. A two-group pretest-posttest experimental design was used to compare the effect of two conditions (Students Learning and Learning Goals) on skill development. Both conditions received classroom instruction on Skills 2, 3, and 4, but only the Learning Goals condition received instruction on Skill 1. The instruction included skill-based instruction and video analysis using an observation framework that was designed by the participants in the study. Four topics related to the development of children's algebraic reasoning were used for instruction, practice, and assessment of all four skills. A subsample of preservice teachers from both conditions ($n = 8$) were individually interviewed to examine the nature of Skill 1. The results revealed significant improvement on Skills 2, 3, and 4 following instruction, however the instruction provided to each groups had the same effect on the development of these three skills. The results demonstrated no difference on mean Skill 1 performance on the post-assessment. The interview data revealed qualitative differences in the nature of Skill 1. Specifically, compared to the preservice teachers in the Learning Goals group, those without Skill 1 training (Student Learning group) showed a greater tendency to focus on students' behaviors to identify learning goals, and this limited their ability to specify learning goals across different teaching contexts. Overall, the results indicated that Skills 2, 3, and 4 do not develop naturally and are learned. As such, the results lend support for teacher training programs to incorporate instruction on these three skills.

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## Contribution of Authors

The first author of this dissertation was Vanessa Rayner, who designed the study, created the instruction and measures, collected the data, created the scoring and coding rubrics, prepared and analyzed the data, and wrote this dissertation. The second author is Helena Osana, her research supervisor, who helped with the study's design and execution, and facilitated the production of this document. Emmanuelle Adrien is the third author as she was one of the instructors during the study. Nathalie Duponsel is the fourth author as she scored a portion the post-assessment data to help with inter-rater reliability. Finally, Anna Tomaszewski is the last author as she double coded the interview data.

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# CHAPTER 1: STATEMENT OF THE PROBLEM

For decades, educational leaders and policy makers have called for more complex and ambitious instruction to prepare students for the demands of the 21st century (Ball & Forzani, 2009). Reform in mathematics education, for instance, requires teachers to introduce new technologies, as well as to engage students in meaningful activities and productive discussions that elicit explanation, reflection, and evaluation (National Council of Teachers of Mathematics, 2000; NCTM).

Preparing teachers to meet these teaching demands is a challenging task. Evidence supporting the link between teachers' subject matter knowledge and student learning (e.g., Ball, Hill, & Bass, 2005) has centralized the role of mathematical knowledge for teaching in teacher education. That said, however, the role of instructional practice within this relationship cannot be ignored (Fennema & Franke, 1992). With this in mind, mathematics education researchers (e.g., Ball & Forzani, 2009; Ball, Sleep, Boerst, & Bass, 2009; Hiebert, Morris, Berk, & Jansen, 2007) have taken a broader perspective on teacher training, placing practice at the core of mathematics teacher preparation. Indeed, Ball and Forzani (2009) proposed that placing practice at the core of teacher education involves illustrating, in detail, the "work of teaching," or the core tasks that are necessary to effectively support student learning. Accordingly, Ball et al. (2009) have recently proposed a practice-based professional teaching curriculum that blends the learning of how to enact key pedagogical practices with knowledge of how mathematics content knowledge is uniquely used by teachers. In general, these learning objectives specify the professional role of teachers and the unique way in which teachers understand subject matter, making the knowledge and skills necessary for effective teaching more accessible to teachers in training.

While Ball et al.'s (2009) model of teacher education may develop several aspects of a preservice teacher's Mathematical Knowledge for Teaching (Ball, Thames, & Phelps, 2008), Hiebert, Morris, and Glass (2003) maintain it would be more effective for teacher education programs to focus on how to learn from teaching as opposed to learning about what experienced teachers know and do in the classroom. That is, understanding the nature and practice of teaching may support a teacher's ability to implement mathematics lessons however, without well developed skills to systematically inquire into the effectiveness of the instruction, how will he or she know (a) what students learned and (b) how the teaching helped students learn what was intended?

In line with this reasoning, Hiebert, Morris, Berk, and Jansen (2007) proposed an alternative framework for teacher education that underscores the importance of preparing preservice teachers to *learn from their teaching* when they enter the profession. Writ large, the Learning from Teaching model posits that in addition to emphasizing subject matter knowledge, teacher education should prepare preservice teachers for systematic lifelong learning by developing skills and knowledge that support a teacher's ability to evaluate, refine, and improve his or her practice. Hiebert et al.'s model for analyzing teaching is supported by four skills: (a) unpacking a lesson to specify the learning goals (Skill 1), (b) collecting evidence of student learning to determine whether and to what extent the learning goals are achieved (Skill 2), (c) generating hypotheses about the cause-effect relationship between teaching and student learning (Skill 3), and (d) using the analysis to propose improvements/alternatives in teaching (Skill 4).

The view that purposeful inquiry and reflection on teaching could support the ability to learn from teaching is not new and has theoretical (e.g., Mason, 2002) and empirical support (e.g., Ma, 1999; Sherin & van Es, 2009; Turner & Rowland, 2011). In addition, there is evidence linking teachers' practice of systematic inquiry in teaching to student learning (Carpenter, Fennema, Franke, Levi, & Empson, 1989). Unique to Hiebert et al.'s (2007) model is its focus on developing *lifelong learning* skills that go beyond a focus on student learning to the role of teaching in students' learning.

Research that aligns with this model of teacher education is thinly developed, and thus little is known regarding its integration and role in teacher education programs. Morris (2006), for instance, examined the degree to which entry-level preservice teachers possess three of the four skills outlined in Hiebert et al. (2007). Preservice teachers were randomly assigned to one of two conditions and asked to analyze the effects of an elementary mathematics lesson on students' learning. While preservice teachers in both conditions were given the same classroom clip to analyze, the preservice teachers in one condition were not informed on whether the lesson was or was not successful, whereas those in the second condition were informed that the lesson was not successful.

The results of her study indicated that narrowing preservice teachers' expectations regarding the outcome of the lesson prompted participants to focus their attention on student behaviors and responses, in addition to observing teacher actions. This tendency to broaden their analysis to both the teacher *and* the students supported more analyses on the cause-effect relationship between instructional events and students' learning. That said, additional analyses revealed limitations in the preservice teachers' ability to effectively reason about the cause-effect hypotheses they produced. That is, for both conditions, the majority of preservice teachers were unable to distinguish student responses that revealed students' learning from those did not, impairing their ability to effectively link specific teaching events with students' learning. Based on these results, then, under the right conditions, preservice teachers can produce hypotheses concerned with students' learning, but they still require professional development to understand what constitutes evidence of students' learning (Spitzer, Phelps, Beyers, Johnson, & Sieminski, 2011), how to use this evidence to generate hypotheses that address the impact of teaching on student learning (Yeh & Santagata, 2015), and how to propose alternative approaches to instruction (Santagata & Angelici, 2010).

Research that has examined preservice teachers' ability to specify learning goals has also showed that Skill 1 does not develop naturally (Morris, Hiebert, & Stigler, 2009). The results from Morris, Hiebert, and Stigler (2009) demonstrated that preservice teachers are capable of identifying the learning goals of a lesson, but this skill is limited without professional development. That is, preservice teachers successfully identified learning goals in contexts where they did not have to search for the mathematical concepts addressed during the lesson because they were clearly indicated by the students' responses (e.g., incorrect answers). These "supportive" contexts elicited a strategy of using student responses to discern learning goals rather than identifying learning goals based on elements of the lesson (e.g., tasks) and knowledge of the mathematical concepts they target. When the preservice teachers' Skill 1 abilities were assessed in contexts where the mathematical concepts could not be easily identified (i.e., "nonsupportive contexts"), they could not produce the subject matter knowledge needed to unpack the lesson and identify its learning goals. This suggests that in absence of certain cues (e.g., incorrect responses), preservice teachers are likely to struggle with unpacking a lesson to identify a main learning goal and its subcomponents (subgoals), a skill that is necessary when planning a lesson (Morris et al., 2009).

The assumption that preservice teachers can learn the skills in Hiebert et al. (2007) is tied to previous research on teacher noticing (e.g., Star and Strickland) and learning from teaching (Santagata & Angelici, 2010; Spitzer et al., 2011; Yeh & Santagata, 2015). *Teacher noticing* is a form of professional noticing that underscores the role of attention, reasoning, and action during instruction. Generally speaking, teacher noticing involves, (a) identifying what is important or noteworthy in a particular situation, (b) reasoning and interpreting about what is identified, and (c) making informed decisions on the basis of what was observed, which may involve making connections between the identified event and the broader principles of teaching and learning (van Es, 2011). Together, all three components form a model of teaching practice that incorporates "seeing" during teaching to make sense of important events and interactions that inform and shape what a teacher does in the classroom (van Es, 2011). The research has demonstrated that expertise in aspects of teacher noticing (Star & Strickland, 2008), and specialized forms of teacher noticing (Jacobs, Lam, & Philipp, 2010) is supported by professional development of this skill.

In addition, a few studies have showed some success using classroom interventions to develop Hiebert et al.'s (2007) learning from teaching skills with preservice teachers (Santagata & Angelici, 2010; Spitzer et al., 2011; Yeh & Santagata, 2015). However, one of the four skills, specifying learning goals (Skill 1), has received very little attention (Morris et al., 2009). It is hard to imagine that analyzing the impact of teaching on student learning can occur in absence of a clear understanding of what the teacher intended the students to learn (Morris et al., 2009). Although to my knowledge no studies have examined the development of Skill 1, there is reason to believe that specifying learning goals is the starting point for developing the other three skills (Morris et al., 2009).

Previous research has shown that preservice teachers tend to rely on students' correct responses as evidence of learning even when the answer provided is unrelated to the learning goals of the lesson (Morris, 2006). In these situations, for example, a preservice teacher would accept a student's correct use of the equal additions algorithm as evidence of successful learning of regrouping (Spitzer et al., 2011). Results from Spitzer, Phelps, Beyers, Johnson, and Sieminski (2011) showed that preservice teachers' difficulties understanding what constitutes evidence of student learning (Skill 2), to some extent, persisted following an intervention designed to develop this skill. It is possible that their improvement in Skill 2 may have been constrained by difficulties identifying learning goals (Hiebert et al., 2007). In line with this, I would argue that developing Skill 1 would help narrow preservice teachers' focus on evidence that is revealing of student learning *and* relevant to the learning goal (Skill 2). Developments in Skill 1 *and* Skill 2 are likely to have a strong impact on how a preservice teacher reasons about the effects of teaching on students' learning (Skill 3; Yeh & Santagata, 2015). Because Skill 3 is said to directly impact the ability to propose alternative teaching strategies (Skill 4; Santagata & Angelici, 2010; Yeh & Santagata, 2015), improvements in all three skills are likely to influence the development of Skill 4.

My study examined this line of reasoning in the context of a mathematics methods course. More specifically, two groups (Learning Goals group and the Students Learning group) received instruction on how to: (a) collect evidence about what students learned (Skill 2), (b) form hypotheses about how teaching helped students learn (Skill 3), and (c) revise the lesson with the intention to improve student learning (Skill 4; Hiebert et al., 2007), but only the Learning Goals group received instruction that specifies how to unpack a lesson to identify what students were intended to learn, or the learning goals (Skill 1).

Hiebert et al.'s (2007) model stands to equip preservice teachers with the ability to accumulate knowledge of his or her teaching over time, and thus is one of the ways in which ongoing improvements to teaching can be supported. Although the skills in Hiebert et al.'s framework support teaching practices central to educational reform, only a paucity of research has examined the development of these skills in teacher education (Santagata & Angelici, 2010; Spitzer et al., 2011; Yeh & Santagata, 2015). Research, then, that contributes to our understanding of how to help preservice teachers acquire the skills outlined in the framework is warranted.

# CHAPTER 2: REVIEW OF THE LITERATURE

## Unpacking the Learning from Teaching Framework

Hiebert, Anne, Berk and Jansen (2007)'s Learning from Teaching framework is based on four skills involved in effective teaching practices and reflecting on teaching. The first skill, specifying the learning goals, involves using subject matter knowledge of a topic to unpack the lesson into the main goal and subgoals. In the context of elementary mathematics teaching, the main learning goal of the lesson, and its subgoals, address elements of mathematics concepts and procedures, and ways of reasoning about mathematics. For this skill, *identifying a learning goal* is distinct from *specifying learning goals*, the former being more general and the latter more detailed. To illustrate, the goals to develop a conceptual understanding of place value can be identified as "the purpose of the lesson is to understand the concept of place value" *or* specified as understanding that: (a) the value of the digit is based on the place of the digit in the numeral, (b) values of places on either side of a digit are more or less than each other by a factor of 10, (c) the role of zero as a place value holder, and (d) in a numeral each place only has one digit. Compared to identifying learning goals, then, specifying learning goals outlines the pieces of place value knowledge that collectively support a conceptual understanding of the topic and as such, clearly indicates to the teacher (or observer) what students must grasp to claim that learning of place value occurred.

Collecting evidence of students' learning (Skill 2), involves (a) understanding that evidence of student learning can be used to evaluate the effectiveness of the lesson, (b) identifying relevant evidence of student learning and ignoring less informative evidence and, (c) anticipating moments during the lesson when evidence could be collected. Hiebert et al. (2007) proposed that identifying relevant evidence of student learning involves attending to evidence that is both *related* to what the students should learn and *revealing* of understanding. More specifically, evidence of student learning includes evidence of students' mathematical thinking (i.e., written work, verbal responses, or strategies) related to the learning goals but not responses (e.g., nodding of the head, or final answer) that lack goal-related information.

Forming hypotheses on how the teaching helped students learn is the third skill. The purpose of constructing hypotheses is to propose cause-effect statements that link teaching and learning. Hypotheses can be framed more generally such as those that refer to several tasks, or specific to one teaching activity such as one that refers to a particular teaching activity. The final skill, proposing teaching alternatives (Skill 4), is used by teachers when he or she revises and improves the lesson to better help the students learn. This last skill completes the learning from teaching cycle by connecting the information gathered about the lesson (Skills 2 and 3) with the planning of future lessons (Skill 1). Accordingly, it is important that alternatives are grounded in evidence observed during the lesson and justified using pedagogical knowledge of teaching and learning.

**Why are these Skills Beneficial for Mathematics Teaching?**

**Enhanced teacher knowledge.** Given the range of competencies needed for effective teaching, why are these particular skills emphasized in the Learning from Teaching framework? A first reason the four skills are emphasized in the framework is that they highlight the nature of mathematics knowledge for teaching and the "working" relationship between subject matter knowledge and pedagogical knowledge. Indeed, prior to teacher training, a preservice teacher's understanding of teaching is primarily framed by his or her experiences as a learner of mathematics (Hiebert, Morris, & Glass, 2003; Lortie, 1975). From the learner's perspective, mathematics knowledge in the classroom is limited to explaining content. Learning how to specify learning goals (Skill 1) and gather evidence of student learning (Skill 2) reframes this conception and underscores how mathematics content is linked with classroom teaching practices.

Moreover, while frameworks of teacher knowledge have identified independent types of mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008; Hill, Ball, & Schilling, 2008), there is also evidence that some mathematics teachers exhibit levels of "cognitive connectedness" between their content knowledge and pedagogical content knowledge (Krauss et al., 2008). Indeed, from a theoretical perspective although it makes sense to present different types of mathematical knowledge for teaching as distinct constructs, more practically, these different ways of knowing interact in important ways in the classroom (e.g., Speer & Wagner, 2009). The research on teacher knowledge used during inquiry-based mathematics instruction underscores the importance of bridging content knowledge to knowledge of students' mathematical thinking (Inoue & Buczynski, 2011; Johnson & Larsen, 2011; Speer & Wagner, 2009). That is, the strategies used during inquiry-based instruction rely heavily on listening to students to make sense of their thinking on the fly. Knowledge of the subject matter, then, is needed to make sense of students' mathematical reasoning and make decisions about which aspects of the content will likely mobilize student learning (Inoue & Buczynski, 2011). Research has shown that in addition to subject matter knowledge, skills in evaluating student understanding are required to effectively "listen" to students (Johnson & Larsen, 2011). If teacher education aims to promote instruction that builds on student thinking (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Kilpatrick, Swafford, & Findell, 2001), preservice teachers require opportunities to develop pedagogical knowledge and subject matter knowledge, *and* form connections between them. Because Hiebert et al.'s (2007) skills are tied to both subject matter (Skills 1 and 2) and pedagogical knowledge (Skills 3 and 4), activities aimed at developing all four skills stand to bridge these two forms of mathematical knowledge for teaching.

In addition, based on Turner and Rowland's (2011) study, skills involved in systematic analysis of teaching can positively impact changes in preservice teachers' subject matter knowledge. Turner and Rowland trained preservice teachers to use the Knowledge Quartet framework (Turner & Rowland, 2008) to guide analyses of mathematics lessons. The Knowledge Quartet framework is one that is based in mathematical knowledge for teaching (Ball et al., 2008) and is therefore designed to support preservice teachers' skills in identifying, describing, and analyzing a teacher's mathematics content knowledge in the classroom. Turner and Rowland found that using the Knowledge Quartet to analyze mathematics lessons was linked to preservice teachers' adoption of a problem solving and inquiry approach to teaching as well as improvements in their subject matter knowledge. The results suggest that developing skills in specifying mathematics topics (Skill 1) and analyzing the learning of mathematics content (Skill 2) focuses attention on the mathematics content in the lesson which, in turn, can influence changes in preservice teachers' own subject matter knowledge.

**Changes in teacher practice.** A second reason the four skills are emphasized in the Learning from Teaching framework is based on the research linking systematic analyses of teaching to changes in teacher practice (Ma, 1999; Sherin & van Es, 2009). For example, Sherin and van Es (2009) conducted a series of video-club meetings with a group of elementary teachers to study developments in teacher noticing. Changing the way a teacher notices classroom events involves the developing his or her *professional vision* (Goodwin, 1994), a perceptual framework used by teachers to see and understand classroom events in particular ways. During these meetings, a researcher prompted and guided the teachers' discussion of classroom videos. The prompts included questions centered on what the teachers noticed, whether the teachers attended to student thinking, and if so, how they would discuss the student thinking they noticed. Analyzing the video club discussions and the teachers' classroom instruction provided evidence that changes in teacher practice could be traced to the professional vision that emerged during the video club. For all of the participating teachers, the observations of teaching conducted toward the end of the school year (i.e., after participating in the video club) indicated that the teachers viewed student comments as objects of inquiry (e.g., "let's try to understand what Mark is saying"; Sherin & van Es, 2009, p. 30) as opposed to correct or incorrect answers, suggesting the focus on identifying students' mathematical ideas in the video club extended to the classroom. The results from Sherin and van Es lend support to the notion that guided systematic inquiry into teaching can impact positive changes in practice.

**Changes in student learning.** Prior research examining systematic inquiry into students' mathematical thinking and learning can be used to support the notion that developing the skills in the Learning from Teaching framework can have a positive impact on students' performance in mathematics (Hiebert et al., 2007). In Carpenter et al. (1989), for instance, first-grade students of teachers who received professional development in analyzing student thinking demonstrated greater gains compared to the control group on topics more frequently addressed by those teachers (i.e., problem solving) in addition to content that the teachers de-emphasized (i.e., number fact knowledge).

One reason systematic inquiry into teaching may impact student learning could be based on how the skills support a focus on classroom discourse (Sherin, 2002). In contrast with teaching by "telling", the *Professional Standards for Teaching Mathematics* (NCTM, 1991) recommend that teachers coordinate classroom discourse of mathematics in a way that draws out students' ideas so that students' mathematical thinking can be assessed. The skills in Hiebert et al. (2007) are based on the notion that students are the source from which student learning, and the effect of teaching, can be understood. In line with this reasoning, the framework centralizes the observation of students' thinking and behaviors, which requires a teacher's attention to and interpretation of classroom discourse.

Added to this, changes in teachers' own subject matter knowledge (Turner & Rowland, 2011) may also contribute to the link between systematic inquiry into teaching and student learning. Indeed, research examining the mathematics knowledge teachers use in the classroom demonstrated that a teacher's level of subject matter knowledge (both common content knowledge and specialized content knowledge; Ball et al., 2008) predicted students' achievement in mathematics; more specifically, teachers with high levels of subject matter knowledge predicted gains in student performance equivalent to 2 to 3 weeks of instruction (Ball, Hill, & Bass, 2005). Based on Turner and Rowland's (2011) findings, then, the skills in the learning from Teaching framework (Hiebert et al., 2007) stand to improve teachers' own subject matter knowledge and these improvements could, in turn, positively influence students' mathematical understanding (Ball et al., 2005).

**Linking learning during teacher education to learning in the classroom.** The final reason that the four skills are emphasized in the Learning from Teaching framework is that developing the four skills addresses the challenge of building professional knowledge outside the context in which it will be used. Said otherwise, developing these skills can address the gap between theory and practice (Santagata & Guarino, 2011). Indeed, preparing preservice teachers for the complexities of teaching while simultaneously encouraging them to develop new ways of thinking is a daunting task. Activities designed to foster the skills in the Learning from Teaching framework (e.g., Santagata, Zannoni, & Stigler, 2007; Yeh & Santagata, 2015) provide opportunities to link teacher education to classroom practices by emphasizing: (a) representations of practice, (b) decomposing of practice (unpacking practice for purposes of teaching and learning), and (c) approximations of practice (engaging in practice that closely resembles teaching practices; Grossman et al., 2009).

In sum, a close examination of Hiebert et al.'s (2007) model suggests that becoming adept at accumulating knowledge about the effectiveness of ones teaching and using that knowledge to inform practice is a noteworthy goal for prospective teachers. The importance of systematic inquiry and reflection is not new, but researchers have primarily focused on its development with inservice teachers (e.g., Elbaz, 1983, 1991; Fenstermacher, 1994; Mason, 2002; Schön, 1983) and less often with preservice teachers (e.g., Stockero, 2008). Based on the benefits to mathematics teaching and learning outlined above, one can argue that systematic inquiry and reflection should be something teachers know how to do when they enter the profession. The question remains, however, on how to foster such skills in preservice teachers.

## Developing the Skills in the Learning from Teaching Framework

Santagata initiated the Learning to Learn from Mathematics Teaching project (2007) to study the development of learning from teaching skills (Hiebert et al., 2007) during teacher training. The project so far has examined the short-term effects of the Learning from Mathematics Teaching course, a mathematics methods course that blends a traditional course curriculum with the Learning to Learn from Mathematics Teaching (LLMT) curriculum[1]. Specifically, the course provides instruction to develop preservice teachers' knowledge for teaching mathematics and uses video analysis activities to promote the skills from Hiebert et al.'s framework. Videos of mathematics lessons are analyzed using the *Visibility*software developed at LessonLab (www.lessonlab.com). The software links the lesson videos to other related materials (e.g., video index, transcripts, and worksheets given to the students). Prior to analyzing videos, questions from the Lesson Analysis Framework (Santagata et al., 2007) serve to guide the process of analysis. The Lesson Analysis Framework includes a series of questions that prompt preservice teachers to focus on key elements of the lesson (i.e., the mathematics concepts), student learning, and teaching strategies. The questions also prompt preservice teachers to reflect on the effectiveness of instruction, and how to modify and improve the lesson.

Santagata, Zannoni, and Stigler (2007), for example, used the Lesson Analysis Framework with a group of preservice teachers enrolled in a specialized two-year university program for secondary teaching (School for Specialization in Secondary Teaching). During the course, the preservice teachers used the Lesson Analysis Framework to analyze three videotaped mathematics lessons. Preservice teachers individually watched each video three times, each time focusing on one of three aspects of the lesson: (a) parts of the lesson and the learning goals, (b) students' thinking and learning, and (c) alternative teaching strategies. The questions in the Lesson Analysis Framework served to guide what aspects of the lesson the preservice teachers should direct their attention to each time they viewed the video. To guide their first video viewing, the preservice teachers were asked to identify the main mathematical concepts the teacher intended the students to learn during the lesson. This question was designed to help the preservice teachers use elements of the lesson to determine the mathematical concepts being addressed (Skill 1). Prior to viewing the same video a second time, five questions from the Lesson Analysis Framework were used to shift preservice teachers' analysis of learning goals to students' learning (Skill 2) and the impact of the teaching strategies on students' learning (Skill 3). Prior to the final video viewing, the question from the Lesson Analysis Framework prompted

[1] The project also involves studying the long-term effects of the LLMT curriculum.

preservice teachers to reflect on alternative teaching strategies that would improve the lesson (Skill 4). To be clear, the preservice teachers were explicitly told what to focus on while viewing the video but, to my understanding, no class time was devoted to learning *how* to attend to the specified aspects of the lesson. Following individual analysis and reflection on the lessons, the preservice teachers discussed their analyses of the video lessons with the whole class.

The preservice teachers' ability to analyze lessons was measured at the beginning and at the completion of the course. The assessment required them to watch a video of an eighth-grade mathematics lesson[2] to identify interesting events and explain why that particular moment was interesting. The preservice teachers' comments were categorized in one of the following five categories: (a) elaboration, (b) links to evidence, (c) mathematics content, (d) student learning, and (e) critical approach and subsequently coded in terms of quality (i.e., low or high).

Comments in the elaboration category were coded as high-quality when a reason(s) was provided to explain his or her interest in the event and as low-quality when the reason was omitted. Links to evidence was a category for comments that involved linking evidence to general aspects of the lesson (low-quality) or to specific teacher/student actions observed during the video (high-quality). For the mathematics content category, high-quality comments involved analyzing teacher and students' actions in relation to the mathematical content, while low-quality codes did not mention the mathematics presented in the lesson. Comments in the student learning category that were given a high-quality code referenced the students' behaviors or made inferences about student thinking and learning. Low-quality codes given to comments in this category focused on the teacher. Finally, for the critical approach category, all positive comments about the teaching were scored as low-quality and comments that critically analyzed the teacher's actions or those that proposed alternative teaching strategies were scored as high-quality.

Each participant received a score for each category that reflected the ratio of low quality to high-quality statements: (a) a score of 1 indicated that there were more low-quality comments relative to high-quality comments, (b) a score of 2 indicated that the number of low-quality and high-quality were balanced and, (c) a score of 3 indicated a relatively high frequency of high-

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[2] The video was not the same as the one used during the course but, the same video was used on the pre- and post-analysis.

quality comments.

For all five categories, the preservice teachers' quality score significantly improved from on the posttest compared to the pretest. That said, the mean quality scores reported on the posttest for the mathematics content ($M$ = 2.02) and for the student learning ($M$ = 2.08) categories suggest a balanced number of low- and high-quality comments. A high number of high-quality comments for the following categories were reported: elaboration ($M$ = 2.77), links to evidence ($M$ = 2.68), and critical approach ($M$ = 2.48). Based on these results, at the end of the video-based program the authors concluded that the preservice teachers' analysis of mathematics lessons were more descriptive, targeted specific events in the video related to the teacher and the students, and were more critical of teaching. The mean posttest score for the student learning category, however, showed that the preservice teachers continued to use observations of the teacher to analyze students' learning. For the mathematics content category, the preservice teachers' comments revealed an increase in using their mathematics knowledge to interpret their observations and evaluate the teacher's decisions, but comments that by-passed the mathematics content in the lesson were equally present.

Santagata et al. (2007) repeated the study with a larger sample of preservice teachers enrolled in the same program the following year. The categories used to code the comments were modified slightly to separate comments that criticized teaching (critical approach) from comments that proposed alternatives to teaching (alternatives). The links to evidence category was also removed. Similar to Study 1, significant differences from pretest to posttest were found for each category, but none of the mean scores on the posttest exceeded 2.30. In addition, contrary to what was found in Study 1, the preservice teachers' gains on the critical approach and alternatives categories were more modest.

Although Hiebert et al. (2007) specified criteria to measure each of the four skills in the framework (see Table 1), Santagata et al. (2007) did not use these criteria to frame the analysis. Rather, the categories used in Santagata et al. emerged from the data and as such, the categories do not align cleanly to each of the skills in Hiebert et al.'s framework. Thus, while the results describe how the participants' analyses of lessons changed, one cannot conclude that the preservice teachers in the Santagata et al. study improved on the four skills in the Learning from Teaching framework. That said, two categories in Santagata et al.'s study, student learning and critical approach, can be linked to two skills in the framework (Hiebert et al., 2007), collecting

Table 1

*Criteria for Assessing Components of Learning from Teaching Framework (Hiebert et al., 2007)*

| **(Skill #) Skill Name** | **Description** | **Criteria for Measurement** |
|---|---|---|
| (1) *Specifying the Learning Goal(s)* | Specifying and describing learning goals. Involves unpacking the main goal into sub goals. | 1. Level of specificity of goal description.<br>2. Accuracy and frequency of math-specific language. |
| (2) *Collecting Evidence of Student Learning* | Knowing *that* evidence about students' learning is essential for understanding the effect of teaching on learning. Knowing *what* counts as evidence (*related* and *revealing*). Knowing *how* to identify key moments in the lesson where students' learning should be apparent. Shift in focus of attention from the teacher to the students. | 1. The evidence should involve a detailed description of student behavior and responses that are relevant to interpreting students' understanding.<br>2. The evidence represents a range of student thinking rather than being focused on the most vocal students. |
| (3) *Constructing Hypotheses About Teaching and Student Learning* | Developing hypotheses that propose a cause-effect relationship between teaching and learning. Connections between teaching and learning can be framed more generally (series of teaching activities) or | 1. Hypotheses focus on how students' learning was influenced by teaching activities (Morris, 2006).<br>2. Hypotheses provide enough detail (i.e., reference students' observable behavior and responses) to allow teacher to test |

(continued)

| | | |
|---|---|---|
| | specific to one teaching activity. | their hypotheses in subsequent lessons (Morris, 2006).<br>3. Align with principles of the teaching and learning of the mathematics content.<br>4. Recognize the complexity of the teaching-learning relationship. |
| (4) *Propose* | Using evidence of students' learning to propose ways to improve the lesson and its impact on students' learning of learning goal(s). | 1. High-quality alternatives demonstrate reasoning based on (a) evidence of students' learning and (b) hypotheses of cause-effect relationship between teaching and students' learning (Morris, 2006).<br>2. Revisions provide greater insight into assessing the achievement of the learning goal(s) (i.e., greater access to student thinking).<br>3. Alternatives proposed align with principles of the teaching and learning of the main goal and sub goals. |

evidence of student learning (Skill 2) and proposing alternatives to teaching (Skill 4). Based on this notion, then, following Santagata et al.'s lesson analysis program, preservice teachers demonstrated inconsistent improvements on skill 4 and some improvement on Skill 2.

Overall, both studies reported in Santagata et al. (2007) contribute to the research on understanding how to help preservice teachers' develop lesson analysis skills. Specifically, instructing the preservice teachers to focus on a specific aspect of the lesson can positively influence certain skills in Hiebert et al.'s (2007) framework. A limitation of Santagata et al.'s intervention, perhaps, is that preservice teachers may require instruction on *how* to analyze learning goals and students' learning, and to propose alternatives to teaching.

Spitzer, Phelps, Beyers, Johnson, and Sieminski (2011) designed an intervention focused on developing one of the skills in Hiebert et al. (2007), collecting relevant evidence of student learning (Skill 2). According to Hiebert et al., the process of collecting evidence involves evaluating whether information is *related* to the learning goals and *reveals* students' understanding those goals. Using a researcher-designed transcript of a mathematics lesson, the intervention involved two instruction sessions addressing (a) how preservice teachers incorrectly interpret students' responses and make claims about student learning, and (b) how to collect relevant evidence of student learning. In the first session, the preservice teachers read a researcher-designed transcript of a mathematics lesson where the teacher provided a conceptual explanation of a place value concept. The students' responses in this transcript were intentionally designed so that no evidence of the students achieving the learning goal of the lesson was included. More specifically, the students responded to the teacher by (a) nodding in agreement, (b) demonstrating an understanding of mathematics unrelated to the lesson goal, and (c) demonstrating an understanding of procedural knowledge of place value. The preservice teachers were asked to rate the students' achievement and then their claims were discussed with the class to provide opportunities to identify whether a claim was correct or incorrect and, if incorrect, reflect on the source of the errors. For the second lesson, the intervention involved a card-sorting task to enhance preservice teachers' skill in evaluating evidence of students' learning that is both related to the learning goal and revealing of understanding that goal. For this task, the set of cards comprised *learning goal cards* and *student response cards*, the purpose, then, was to match a student's response with the corresponding learning goal.

The preservice teachers completed a pre- and posttest designed to assess whether preservice teachers identify irrelevant evidence as relevant when analyzing students' learning. In particular, the preservice teachers were given a new researcher-designed transcript of a lesson that included six sections where students solved fraction comparison problems (e.g., "which is larger, $\frac{4}{5}$ or $\frac{3}{4}$?). For each section, the students' thinking and behaviors were irrelevant to evaluating students' learning of the lesson goals (or a conceptual understanding of the common denominator strategy for comparing fractions). The preservice teachers were asked to rate the students' achievement of learning and their responses were subsequently coded as: (a) "no claim" for phrases indicating insufficient evidence of student learning, (b) "weak claim" for phrases indicating uncertainty in his or her claim that student learning was evident, (c) "strong claim, positive" for phrases that claimed the students achieved the learning goals, and (d) "strong claim, negative" for phrases that claimed the students did not achieve the learning goal. The code for each section was scored as, 0 for "no claim", 1 for "weak claim", and 2 for either "strong claim" code. The total possible score for all six sections ranged from 0 to 12 whereby lower scores indicated higher quality analyses.

The mean pretest score ($M$ = 8.46) was significantly higher compared to the mean posttest score ($M$ = 7.17), implying that following the intervention, the preservice teachers were more likely to disregard irrelevant evidence when evaluating students' learning. Additional analyses revealed that changes from pretest to posttest were primarily attributed to improvements in understanding when evidence was unrelated to the learning goal. At the same time, however, the preservice teachers demonstrated less improvement in understanding whether evidence was or was not revealing students' understanding. Indeed, most preservice teachers on the posttest accepted procedural evidence as conceptual understanding despite the fact that the intervention heavily emphasized the distinction between evidence of procedural fluency and evidence of conceptual understanding.

In their discussion of this particular result, Spitzer et al. (2011) proposed that preservice teachers require a certain level of pedagogical content knowledge in mathematics prior to acquiring skills in evaluating student learning. Clearly, because knowledge of students' thinking about mathematics is a key component of a teacher's pedagogical content knowledge the notion that high levels of pedagogical content knowledge would lend itself to evaluating student learning is logically sound. I would add, however, that developing preservice teachers' skill in specifying the learning goal (Skill 1) would support improvements in collecting evidence of student learning (Skill 2). That is, perhaps knowledge of how to unpack the main learning goal to specify the subgoals may help clarify the types of mathematical thinking that support an understanding of the lesson goal (Hiebert et al., 2007).

Compared to the other skills in Hiebert et al.'s (2007) framework, the focus on preservice teachers' ability to analyze student thinking has received more attention (e.g., Spitzer et al., 2011). Building on this research, Yeh and Santagata (2015) examined preservice teachers' (*N* = 60) ability to generate hypotheses (Skill 3) as a function of two types mathematics methods courses. The Mathematics-Methods Course (MMC) focused on the development of content knowledge and pedagogical skills for teaching all elementary mathematics topics. Using a "learn-by-doing" approach, the course promoted knowledge of children's mathematics thinking, problem-based instruction, lesson planning and assessment. The MMC included some reflective activities but the process of reflection was not guided by a framework.

As mentioned previously, the Learning from Mathematics Teaching course (LMT) included analysis-of-teaching activities using the Lesson Analysis Framework (Santagata et al., 2007) in addition to developing content and pedagogy. In this study, the course involved individual and collaborative analysis of videotaped mathematics lessons using the Lesson Analysis Framework. The Lesson Analysis Framework was used to guide preservice teachers' analysis of videotaped lessons (other teachers as well as their own lessons) to support the development of habits of reflection that link student thinking and teaching.

Preservice teachers' ability to generate hypotheses was measured at two time-points (the beginning and end of the course). The assessment consisted of viewing four brief video clips of different mathematics lessons (using the *Visibility*software) and responding to the following prompt, "Discuss how the teacher and the student(s) interact around the mathematical content," (Yeh & Santagata, 2015, p. 25). The responses to the prompt were scored to indicate whether preservice teachers generated justified hypotheses for each video clip. Specifically, a justified hypothesis statement included all of these elements: (a) an analysis of the impact of teaching on students' learning, (b) accurate descriptions of the mathematics content, and (c) use of evidence of student learning from the video to justify the link between teaching and student learning.

The results indicated that the preservice teachers in the LMT course significantly outperformed those in the MMC condition at the end of the course. That is, at the beginning of the course, preservice teachers on average were able to generate a justified hypothesis statement for one of the four video clips (LMT mean performance at pretest was *M* = 1.26 and MMC mean performance was *M* = 1.17). The instruction provided in each method course developed the preservice teachers' ability to generate hypotheses statements in different ways. By the end of the course, preservice teachers in the LMT course significantly increased the number justified hypothesis statements (*M* = 3.14). The preservice teachers in the MMC course generated more hypothesis statements by the end of the course, but the number of justified hypotheses did not significantly improve at posttest (*M* = 1.48). A detailed analysis of the hypotheses generated by this group indicated that the preservice teachers were not adept in collecting evidence revealing of students' learning (Skill 2) and this impacted their ability to analyze the effect of teaching on students' learning (Skill 3).

Together, the research conducted by Santagata and colleagues (Santagata et al., 2007; Yeh & Santagata, 2015) and Spitzer et al. (2011) make several noteworthy contributions to our understanding of developing some of the skills in Hiebert et al.'s (2007) framework during teacher training. First, while preservice teachers' abilities in collecting evidence of student learning (Skill 2) and generating hypotheses (Skill 3) are not well developed, these skills can be learned. The results from Yeh and Santagata (2015) demonstrated that combining traditional methods for training preservice teachers with guided video analysis activities can develop learning from teaching skills. Although the studies I reviewed were not designed to investigate the relationship between the skills in the Learning from Teaching Framework, there is reason to speculate that certain skills (e.g., Skill 2) are a prerequisite for the development other skills in the model (e.g., Skill 3; Yeh & Santagata, 2015).

**Video Use and Teacher Education**

The research described to this point (Santagata et al., 2007; Spitzer et al., 2011; Yeh & Santagata, 2015), suggest two different approaches to incorporating Hiebert et al.'s (2007) framework in teacher education. The studies reported in Santagata et al. (2007), and more recent research (e.g., Yeh & Santagata, 2015) used hypermedia to represent practice and used a method of professional development similar to the video clubs reported in the research on teacher noticing (e.g., Sherin & Han, 2004; van Es, 2011). A video club is a context for professional development where teachers meet regularly to view and discuss videos of lessons from their own classroom or other classrooms. A researcher acts as the group facilitator and guides the discussion of events observed in the video. Ultimately, the goal of the meetings is to provide opportunities for teachers to reflect on practice and learn how to focus attention on students' thinking. In Santagata et al. for example, the preservice teachers viewed classroom videos of mathematics lessons independently during several lab sessions and subsequently discussed their analyses with the class during the discussion sessions. Rather than use video cases, Spitzer et al. (2011) used fictitious classroom transcripts and provided explicit instruction and activities to develop preservice teachers' ability to disregard irrelevant evidence of student learning.

The intervention I used is informed by the methodology used in Santagata and colleagues (Santagata & Angelici, 2010; Santagata et al., 2007; Yeh and Santagata, 2015), in addition to other research on teacher noticing (Jacobs, Lamb, & Philipp, 2010; Star & Strickland, 2008). Specifically, I used research-based video cases of mathematics lessons/interviews (Carpenter, Franke, & Levi, 2003) and asked preservice teachers to use an observation framework to guide what they notice. The observation framework I used at the beginning of the study was based on previous research (Star & Strickland, 2008) however, following explicit instruction on the skills outlined in Hiebert et al. (2007), the preservice teachers worked in small groups and revised the framework to incorporate the instruction content.

For decades, video use in teacher education has been an inexpensive way to support inservice and preservice teachers' reflection, interpretation, and fine-grained analysis of teaching (Sherin, 2004). Despite the mixed research results on video use (e.g., Kagan & Tippins, 1991) and its limitations (e.g., not all contextual features of the classroom are captured), video remains a central feature of teacher education (Sherin, 2004). Indeed, videos can be used to address a variety of learning goals of which include providing examples of: (a) generic problems experienced by teachers, (b) prototypes of teaching that link theoretical principles to principles of practice, (c) a specific pedagogical theory, and (d) students' thinking (e.g., Carpenter et al., 1999; Sherin, 2004).

Provided that videos are not used to prescribe what "good teachers" do, video use has been shown to be an effective approach to making expert teachers' tacit knowledge accessible to preservice teachers (e.g., van Es, 2011). Stockero (2008), for example, found that a video-based curriculum positively influenced preservice teachers' reflections of teaching *and* teacher practice during their practicum. The purpose of Stockero's study was to provide evidence on the effectiveness of video cases in teacher education. To address this goal, Stockero examined how the exclusive use of a video-case curriculum influences changes in preservice teachers' reflective stance. According to Stockero, a reflective stance is the ability to reflect on classroom events to identify students' mathematical understanding and link student understanding to the teacher's actions. The video-based program involved eight 3-hour modules, but included no instruction on how to observe and analyze the video cases provided. At the beginning of each module the preservice teachers worked individually on mathematical tasks related to linear growth and then participated in a classroom discussion the task. Following that, one or two video clips of students solving the same task were viewed. Video viewings were followed by individual reflection, guided by a facilitator who prompted the preservice teachers to identify interesting and relevant pedagogical and mathematical events in the video. After the individual reflection component of the module, the preservice teachers were asked to discuss their observations with the class and provide evidence (using video transcripts) to support any claims about students' mathematical understanding and the teaching practices that contributed to them. The results indicated that at the end of the course, preservice teachers demonstrated a greater tendency to (a) use evidence in their reflections on teaching, (b) analyze teaching in terms of its effect on student thinking, (c) consider multiple interpretations of student thinking, and (d) develop a more tentative stance of inquiry-based teaching.

**Frameworks to Support Video Viewing**

While the results in Stockero (2008) are promising, indicating that sustained observation and reflection on practice are key components to effective professional development, it is not always practical to offer a course for preservice teachers in 13 weeks entirely based on analyzing video cases. An alternative to extensive video analysis, perhaps, is to provide more support during the video viewings. Van Es and Sherin (2002), for instance, used a video-based program called Video Analysis Support Tool (VAST) to scaffold inservice teachers' analyses of the videos of their own mathematics lessons. The computer program included prompts that directed a teacher's attention to important classroom features. Following a "call out," or when the teacher identified an important event, the computer program provided a series of questions designed to scaffold interpretations of classroom interactions in a specific way. That is, the different types of questions would elicit teachers to (a) describe the events (call outs), (b) use evidence from the video, and (c) analyze or interpret students' thinking, the teacher's role, and classroom discourse. The results demonstrated that initially the teachers' analyses involved a chronological description of classroom events. As a result of using VAST, teachers provided detailed descriptions of the lesson that highlighted its key elements. These findings suggest that providing a framework to guide teachers' attention may be an effective approach to professional development on teacher noticing. This notion is supported by additional and more recent research (i.e., Santagata & Angelici, 2010; Santagata et al., 2007; Star & Strickland, 2008; Yeh & Santagata, 2015) in the context of teacher education.

More recently, Santagata and Angelici (2010) investigated the impact of the Lesson Analysis Framework (LAF) with a group of preservice teachers enrolled in the same teacher training program as those in Santagata et al. (2007; School for Specialization in Secondary Teaching). Thirty-eight preservice teachers were assigned to the Lesson Analysis Framework condition or the Teaching Rating Framework (TRF) condition. For both conditions, the preservice teachers were trained to view videos of mathematics lessons with a specific purpose. Those in the TRF condition were trained to rate separate elements of the instruction (e.g., learning goals, lesson structure, materials, and methods of student evaluation) using a 5-point scale and explain their ratings. The LAF, on the other hand, prompted preservice teachers to analyze the lesson in terms of teaching strategies and students' learning, and reflect on possible alternative teaching strategies that would improve student learning. In other words, the observation frameworks in each condition guided preservice teachers to focus on different aspects of the lesson (elements of a lesson versus teaching strategies and students' learning) and evaluate what they focused on in different ways (ratings versus reflection).

The intervention used with both groups involved learning to apply an observation framework (LAF or TRF) while analyzing videotaped mathematics lessons. The procedures used with the TRF group were slightly different to those used in the LAF condition however, both conditions provided preservice teachers with specific instructions on what to focus on when viewing the videotaped lesson.

In the LAF condition[3], preservice teachers were trained using two introductory tasks and a guided video analysis activity. The introductory tasks involved answering questions to become familiar with the lesson in the video (e.g., understanding the main learning goal) and analyzing the lesson plan. During the video analysis activity, the preservice teachers viewed three segments of a videotaped lesson, and after each segment they provided written responses to

[3] The process of applying the LAF in Santagata and Angelici (2010) slightly differed from the procedures used in Santagata et al. (2007).

specific prompts. The prompts required them to reflect on (a) the learning opportunities provided by the activities included in the lesson, (b) evidence of students' learning as they completed the activities, (c) evidence of students' difficulties, and (d) alternative teaching strategies that would improve students' learning.

The introductory task for the preservice teachers in the TRF group involved reviewing and rating the elements of the lesson plan using a 5-point scale (e.g., learning goals, materials, methods used to evaluate learning outcomes), and explaining their ratings. Following this, the preservice teachers watched the same video segments in the same order but were prompted to rate (using a 5-point scale) the (a) effectiveness of the lesson activities and teaching strategies, and (b) appropriateness of the students' responses.

The preservice teachers' ability to evaluate of the effectiveness of the lesson was assessed using a new video (the eighth-grade mathematics lesson used in Santagata et al., 2007) prior to and following the intervention. In particular, the preservice teachers used the *Visibility*software to analyze one videotaped lesson and were prompted to (a) identify the three most significant moments in the video and explain why they were chosen, (b) evaluate the activities and teaching strategies using a 5-point scale and justify the ratings, and (c) explain in detail which of the teaching strategies should be included future lessons and which should be changed (Skill 4).

The responses to these prompts at pretest and posttest were scored and used to compare the groups in terms of: (a) quality of comments when describing interesting moments in the lesson, (b) rating of the effectiveness of the teacher, (c) quality of explanations of the ratings, (d) the number of alternative teaching strategies proposed, and (d) the level of detail included in the alternatives.

The results indicated that compared to the preservice teachers in the TRF condition, those in the LAF included more elaborate comments in their descriptions of the lesson and provided higher quality explanations of their ratings of the teaching strategies following the intervention. In addition, the preservice teachers in the LAF group demonstrated significant improvements in proposing alternative teaching strategies (Skill 4) following the intervention, however they did not significantly outperform those in the TRF group on this skill. Nevertheless, the results suggest that preservice teachers' ability to evaluate the effectiveness of a lesson was better developed when the framework prompted reflection (versus rating) and guided preservice teachers to focus on analyzing the teaching strategies and students' learning.

In general, the research reviewed point to the importance of guided video analysis and the benefits of various observation frameworks. Rather than provide an observation framework, I speculated that it would be equally, or perhaps more, effective to allow preservice teachers to play a more active role in their own development. In one study, Jacobs, Lamb, and Philipp (2010) provided inservice teachers with professional development on noticing children's mathematical thinking and a completed framework was not provided. Rather in Jacobs et al.'s study, teachers worked together to create their own frameworks that reflected their reasoning about children's mathematical thinking. In particular, teachers worked together to solve math problems and then were asked to pose these problems to their students. In the sessions that would follow (5 sessions in total), the teachers analyzed the students' written work to make sense of their mathematical thinking and relate their observations to the frameworks of children's thinking that they developed. The results demonstrated that participating in Jacobs et al.'s professional development supported gains in interpreting children's understanding and using these interpretations to inform subsequent pedagogical decisions. In line with Jacob et al.'s professional development, the preservice teachers in my study were initially provided with an observation framework (see Star & Strickland, 2008) and then given the task to modify this framework so that it incorporates elements of the Learning from Teaching framework (Hiebert et al., 2007), which will be part of the course's curriculum.

**Mathematics Topics Addressed in Videos**

Similar to the research reviewed so far (e.g., Yeh & Santagata, 2015), the present study took place during a semester-long undergraduate mathematics methods course. In the present study, topics related to the development of children's algebraic reasoning were used during instruction (justifying conjectures, relational thinking, and eliciting conjectures) and during the pre- and post-measures (the meaning of the equal sign). Before the study began, both groups received a 30-minute review lesson in their methods class on children's thinking about the equal sign. Specifically, I reviewed the following conceptions of the equal sign: (a) "the answer comes next", (b) "use all numbers", (c) "extend the problem", and (d) "relational understanding" (Carpenter, Franke, & Levi, 2003). The "answer comes next" refers to the conception that the number that follows the equal sign is the answer to the expression preceding the equal sign. For example, if asked to determine what number goes in the box to make this number sentence true $8 + 4 = \square + 5$, a student with this view would put "12" in the box. Using the same problem, children with the “use all numbers” view would add all the numbers in the problem and the sum (i.e., 17) would be the number they put in the box. Children who think that the equal is used to “extend the problem” would also believe the answer is “17,” but they would put “12” in the box and extend the problem by performing a second operation (12 + 5) to get to 17. Finally, a student with a relational view of the equal sign would use the relationship between the numbers on either side of the equal sign to determine the number that goes in the box. In the example provided, a child using relational thinking would not operate on the numbers and would notice that 5 (on the right side of the equal sign) is 1 more than 4 (on the left side of the equal sign); thus, the number that goes in the box should be 1 less than 8. Because these different views may be elicited using number sentences in nonstandard form (i.e., non-canonical), the review included definitions of non-canonical and canonical number sentences.

## The Present Study

The research on developing the skills in Hiebert et al.’s (2007) framework has demonstrated that preservice teachers are not adept in specifying learning goals (Morris, Hiebert, & Spitzer, 2009), collecting evidence of students’ learning (Santagata et al., 2007; Spitzer et al., 2011), generating hypotheses (Yeh & Santagata, 2015), and proposing alternative teaching strategies (Santagata & Angelici, 2010). Although this area of research is thinly developed, we have some understanding of how teacher educators can integrate these skills in their courses (Santagata et al., 2007; Spitzer et al., 2011) and the short-term effects of such interventions (Santagata & Angelici, 2010; Yeh & Santagata, 2015). That said, each of these studies has focused on developing one of the four skills (e.g., Yeh & Santagata, 2015), and thus, we know very little about the collective development of these skills. Moreover, the assumption that skill development does not follow any particular sequence is questionable (Spitzer et al., 2011). According to Hiebert et al., each skill is theoretically linked to teaching activities that are deployed in a specific order: prior to, during, and following a lesson. That is, specifying learning goals (Skill 1) supports lesson planning, collecting evidence of student learning (Skill 2) and forming hypothesis (Skill 3) are used during the lesson (i.e., implementation), and proposing teaching alternatives (Skill 4) is used to reflect on the lesson. Because in practice Skill 1 is applied before Skills 2, 3, and 4, it may benefit preservice teachers to receive professional development that follows this sequence to approximate teaching practice (Grossman et al., 2009).

In line with this reasoning, this study examined the role of the first skill, specifying learning goals, when learning how to analyze the effect of teaching on students' learning. In particular, the present study was designed to explore the role of Skill 1 and to investigate whether its development would impact the other three skills in the model. Thus, the first goal of the current research was to understand the effects of explicit instruction on specifying learning goals (Skill 1) on preservice teachers' abilities to analyze what students learned (Skill 2), isolate the effects of teaching on students' learning (Skill 3), and revise a lesson accordingly (Skill 4). It was hypothesized that the Students Learning condition and the Learning Goals condition would develop the skills common to both conditions (Skills 2, 3, and 4) in different ways. More specifically, I predicted that following instruction those in the Learning Goals group would outperform those in the Students Learning group on collecting evidence of students' learning (Skill 2). Related to my predictions on the development of Skill 2, I also hypothesized that the instruction provided to both groups would not increase preservice teachers' attention to student behaviors that are not revealing of student learning (Spitzer et al., 2011). I also hypothesized that preservice teachers in the Learning Goals group would outperform those in the Students Learning group on generating hypotheses (Skill 3), and on proposing alternative methods of instruction (Skill 4) following instruction.

Furthermore, because the research has shown that specifying learning goals does not develop naturally (Morris et al., 2009), the second objective was to examine Skill 1 acquisition as a function of the presence or absence of instruction on that skill. In line with results from Morris, Hiebert, and Spitzer (2009), it was hypothesized that compared to the Students Learning group, the Learning Goals group would demonstrate a significantly higher Skill 1 performance on the post-assessment.

Finally, because the ability to specify learning goals has received less attention in the literature compared to Skills 2, 3, and 4 (Morris et al., 2009), my third objective was to qualitatively examine the nature of specifying learning goals (Skill 1) following instruction that did and did not address this skill.

# CHAPTER 3: DESIGN AND METHODOLOGY

## Method

### Design

A two-group pretest-posttest experimental design was used in the study. The study involved six phases: (a) an assessment of previously developed mathematical content knowledge for teaching and demographic information; (b) practice using an observation framework and review of key concepts related to teaching a lesson on the equal sign, (c) a pre-assessment of the skills addressed in both conditions (i.e., Skills 2, 3, and 4); (d) the experimental intervention; (e) a post-assessment of all four skills in Hiebert, Anne, Berk, and Jansen (2007)'s framework; and (f) the administration of a post-interview to a subsample of the participants to explore one of the skills (i.e., Skill 1) in Hiebert et al.'s framework in more depth. Figure 1 provides an overview of the study's design and measures.

The sample was stratified based on the number of completed mathematics methods courses[4]. Preservice teachers from each stratum were randomly assigned to one of two conditions, the Students Learning condition and the Learning Goals condition. Both conditions received five instruction sessions (Analysis of Learning sessions), which addressed skills in: (a) collecting evidence revealing of student learning (Skill 2), (b) forming hypotheses about how teaching helped students learn (Skill 3), and (c) proposing evidence-based teaching alternatives to improve student learning (Skill 4; Hiebert, Anne, Berk, & Jansen, 2007). Only the Learning Goals condition received instruction on how to identify and specify learning goals (Skill 1).

### Participants and Context

Twenty-nine (*N* = 29) preservice elementary teachers from a large university in a metropolitan area of Canada participated in this study on a voluntary basis. The sample included 26 female and 3 male preservice teachers. All participants were enrolled in a four-year undergraduate teacher education program specializing in early childhood and elementary education. Twenty-two participants were in their 3rd year of the program and 7 were in the 2nd year of the program. The program for certification in elementary education includes a balance of

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[4] This procedure was included to ensure group equivalence. Approximately half of the students enrolled in Teaching Mathematics III were concurrently enrolled in Teaching Mathematics II. The other half of the students enrolled completed Teaching Mathematics II during the previous academic year. As such, I stratified the sample to ensure a relatively equal number of students who had completed Teaching Mathematics II in each group.

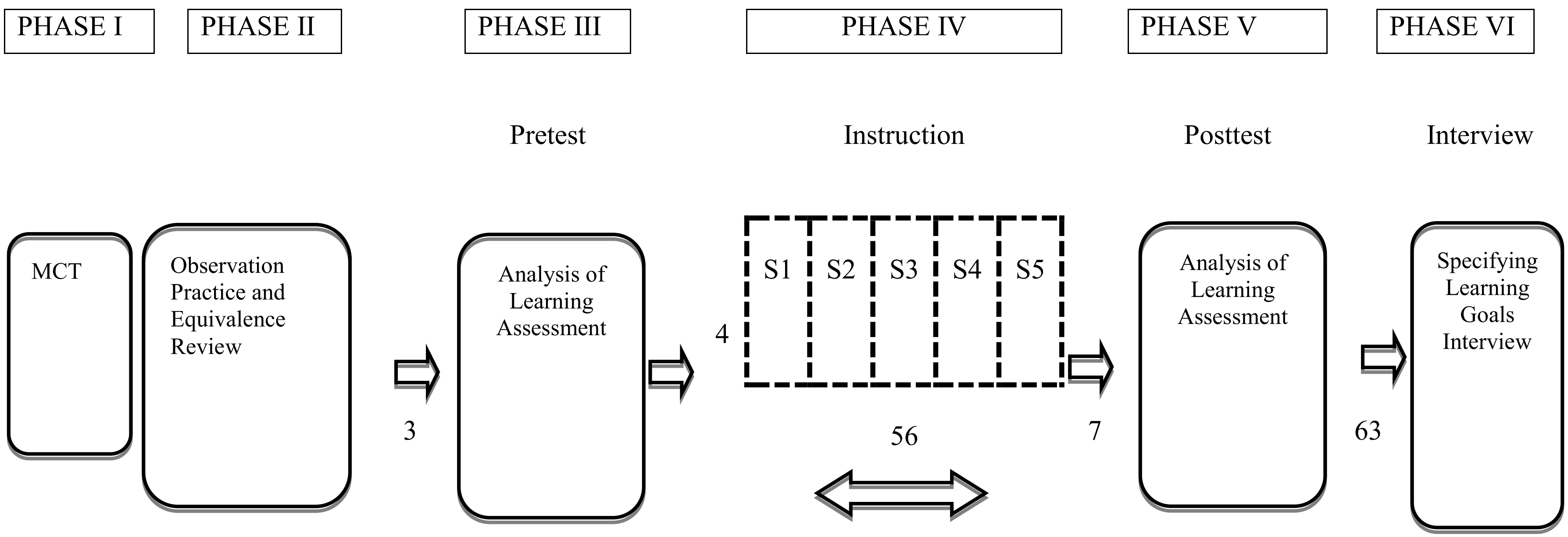


*Figure 1.* Overview of phases of the study.

theory courses (theories related to child development and teaching methods), method courses (language arts, mathematics, science, and social studies), and practica, undertaken in local preschool, kindergarten, primary, and elementary classrooms. Their preparation in mathematics consists of three required methods courses. In general, the sequence of the mathematics methods courses introduces a conceptual analysis of a wide range of elementary school mathematics topics (e.g., whole number operations, problem solving, algebraic reasoning, fractions, and geometry) and its application to the elementary classroom.

The preservice teachers in this study were enrolled in the third compulsory mathematics method course and I was the course instructor. The 13-week course involved a lecture and lab component. The lecture lasted 135 minutes and was scheduled once a week. Over the course of the semester, the topics addressed during the lecture included algebraic reasoning, fractions and operations with fractions, ratio, percent, and statistics. In addition to promoting subject-matter knowledge of these topics, the lecture component of the course aimed to enhance preservice teachers' knowledge of children's mathematical thinking related to these subject areas.

The lab component was also scheduled once a week, and each lab session lasted 75-minutes. Nine of the thirteen labs were dedicated to this study.

**Participation in the study.** Data for this study come from students who provided consent to give their work as data for my research. I remained blind to each preservice teacher's decision to participate in the study until after I submitted the final grades for the course. Two of the students who provided consent were not included in the study. One student had previously conducted research in mathematics education and was familiar with the details of the study. A second student received the majority of the instruction sessions on a one-on-one basis because he was absent for the majority of the labs. Because of his absence, he only participated once in activities that involved small group work and whole class discussions and therefore was not engaged in the instruction sessions in the same way as the other participants.

My research was reviewed and accepted by the University Human Research Ethics Committee. All preservice teachers who gave consent participated in a draw at the end of the study. Ten preservice teachers, five from each group, were randomly selected to receive a $25 gift card as compensation for participating the study. One randomly selected preservice teacher from both groups received an iPad mini.

**Participation in the interview.** After the final grades for the course were submitted, I began contacting preservice teachers via email to participate in a one-on-one interview. For both groups, I selected preservice teachers based on their post-assessment Skill 1 score. The final sample included 2 low-Skill 1 performers and 2 high-Skill 1 performers from each group ($n$ = 8). All preservice teachers who participated in the interview received $25.

**Measures**

**Demographic Survey.** An overview of all the measures is provided in Appendix A. A survey at the beginning of the course was administered to all the preservice teachers who gave consent to participate in the study (see Appendix B). The survey questions collected demographic information (i.e., age, gender, and year of entry in the program) and included additional questions pertaining to the preservice teachers' teaching experience and background in mathematics (i.e., completed university-level mathematics and mathematics methods courses).

**Mathematical Content Knowledge for Teaching (MCT) assessment.** I included the Mathematical Content Knowledge for Teaching (MCT) assessment in the study to ensure group equivalence on subject-matter knowledge prior to the instruction (see Appendix C). I constructed the paper and pencil assessment based on previous research measuring inservice teachers' mathematical knowledge for teaching (Learning for Mathematics Teaching Project, 2008) and subject-matter knowledge (Rayner, Osana, Lacroix, Halladjian, & Ing, 2010). Thirty-nine items were designed to measure preservice teachers' knowledge of mathematical content addressed in the first methods course. Specifically, the items in the MCT assessed the participants' knowledge of numeration and place value with whole numbers, properties of arithmetic, algorithms, children's counting, and the equal sign. Similar to previous research that has assessed components of teachers' mathematical knowledge for teaching (e.g., Hill et al., 2008), I used a multiple-choice format.

**Analysis of Learning assessment.** I designed a paper and pencil assessment that consisted of true/false and multiple choice items (or test items) and open-ended questions. The assessment was administered prior to and following instruction and was based on the criteria for each skill outlined in Hiebert et al. (Skills 1, 2, 3, and 4; 2007; see Table 1) as well as the method of assessment used in Morris (Skills 3 and 4; 2006) and Star and Strickland (Skills 1 and 2; 2008). The Analysis of Learning assessment at pretest was designed to assess all of the skills in

Hiebert et al. except for specifying learning goals (Skill 1) to avoid any pretest effects. At posttest, the assessment measured all four skills.

The Analysis of Learning assessment (Appendix D) was administered in a classroom with all the participants present at pretest and in two classrooms (one for each group) at posttest. The assessment was based on video featuring an elementary mathematics lesson on the equal sign that I presented to the students at the beginning of the testing. The same video was used at pretest and posttest. I transcribed this video to create a detailed list of the tasks used in the lesson, the teacher's questions and probes, and the students' responses (Star & Strickland, 2008) and the test questions were based on the transcript. The transcription is provided in Appendix E. After they viewed the video, the preservice teachers completed the paper-and-pencil test. The video viewing was approximately 15 minutes and 60 minutes were allocated to completing the test.

***Test items for Skills 1 and 2***. The items for assessing Skill 1 were included only on the posttest version of the test. I categorized the tasks in the video transcript based on whether they addressed the primary learning goal of the lesson (i.e., to understand the meaning of the equal sign) or one of the four subgoals outlined in Carpenter, Franke, and Levi (2003)[5]. I developed four true/false questions and seven multiple choice items to assess preservice teachers' ability to accurately identifying the primary and subgoals of the lesson in the video.

The teacher's questions and probes and students' responses from the transcript were used to create seven true/false and 12 multiple choice questions to measure Skill 2. Twelve of these 19 items included the student responses in the video that were revealing of their thinking about the equal sign (e.g., "it's backwards") and seven items addressed student responses that were not revealing (e.g., "No").

***Open-response items for Skills 1, 3, and 4.*** I also assessed Skill 1 on the post-assessment using two open-response questions: (a) "In as much detail as possible, can you identify the overall learning goal of the lesson?" and (b) "Now consider each activity/task presented during

[5] Carpenter et al. (2003) identifies four components, or sub-learning goals, for teaching and learning the meaning of the equal sign. Three of the four subgoals were addressed in the video: (a) eliciting students' conceptions (and misconceptions) about the equal sign, (b) guiding students' acceptance of non-canonical number sentences, and (c) developing students' understanding of a procedure used to solve equivalence problems.

the lesson. In as much detail as possible, describe what the teacher wanted the students to learn for each activity/task."

Open response items were also used to assess Skills 3 and 4 and were taken from previous research (Morris, 2006). Specifically, for the third skill, the question was: "Form a hypothesis (or more than one) about what the students learned and understood by the end of the lesson" (Hiebert et al., 2007). For the fourth skill, two questions were provided, (a) "If you were the teacher, what would you have done differently?" and (b) "Explain why you think the alternatives you propose would be more effective."

**Specifying Learning Goals interview.** The Specifying Learning Goals interview was a semi-structured one-on-one interview used to assess Skill 1 for a subsample of the preservice teachers from each condition ($n$ = 8). Hiebert et al. (2007) claimed that measuring this skill involves examining two criteria: (a) the level of detail used to describe the learning goals, and (b) the degree to which mathematical language is used to describe the topic of the lesson. Both criteria were assessed during the interview using two teaching contexts, *planning* a lesson on the equal sign and *observing* a lesson on the equal sign. The interview protocol can be viewed in Appendix F. With respect to the planning context, I started each interview by asking the preservice teacher to propose a lesson on introducing elementary students to the meaning of the equal sign. I asked questions that focused the discussion on the learning goals and types of tasks she would include in the lesson so I could assess her skill in identifying *and* specifying learning goals. Linking learning goals and tasks (i.e., specifying learning goals) reflects an ability to situate the goals in the context of her own lesson.

For the observing context, I showed the same video used on the previously administered Analysis of Learning assessment. I then asked the preservice teacher to compare her lesson on the equal sign to the one that she observed in the video. Similar to the first series of questions used in the planning context, I included questions that guided the preservice teacher to focus her comparison on learning goals and tasks. Finally, the interview concluded with one question about how the preservice teacher reasoned about the learning goals she observed in the lesson.

**Procedures**

**Video selection.** I selected five videos in advance for the different phases in the study. The first video was used for practice conducted in the classroom prior to the instruction (Phase

II). This video was taken from University of Michigan's Deep Blue website (Mathematics Teaching and Learning to Teach, 2010).

The other four videos were used during the pre-assessment, intervention, post-assessment, and interview. The videos were taken from Carpenter et al. (2003). My selection of these four videos was based on the following dimensions: (a) topic, (b) context (i.e., classroom or one-on-one interview), and (c) elements of teaching and learning that were visible. The topics for the three videos used for instruction were: (a) justifying conjectures, (b) eliciting conjectures, and (c) developing relational thinking. The topic in the video at pretest, posttest, and the interview was the meaning of the equal sign.

I also selected videos for specific phases of the study based the level of complexity of the *context*. I determined the level of complexity based on the number of students and interactions that were represented in the video. A one-on-one interview was not considered complex because there is only one student to observe and the interactions occur between one teacher-student dyad. Compared to a one-on-one context, a classroom context is more complex because it involves observations of several students and contains several different interactions.

Finally, I paid attention to the *elements of teaching and learning* that were apparent in the videos. To analyze student learning and the effect of teaching, it was essential that the videos illustrated a situation where students are *learning* a concept or reasoning skill and where the teacher (or researcher) is engaging students in *activities to achieve a learning goal*. Observing student learning supports the (a) specification of the learning goals (Skill 1) and (b) collection of evidence pointing to the students' achieving the goals (Skill 2). Similarly, observing teaching strategies and activities, and how the students respond to them, promotes the (c) construction of hypotheses concerning the effects of teaching on student learning (Skill 3), and the (d) reflection on alternative teaching strategies and their potential influence on student learning (Skill 4).

**Phase I: Demographic and mathematical content knowledge assessment.** During the first scheduled lab of the semester, a trained research assistant administered the Demographic Survey to the preservice teachers who consented to participate. The survey collected demographic information pertaining to his or her teaching experience and the number of post-secondary mathematics courses completed. The preservice teachers were given 5 minutes to complete the questionnaire.

Once the Demographic Surveys were collected, I administered the MCT assessment. I explained that the purpose of the MCT was to get a sense of their knowledge of the topics addressed during their first mathematics method course, Teaching Mathematics I. The preservice teachers were given 30 minutes to complete the paper-pencil assessment.

**Phase II: Observation practice and review on the equal sign.** I used the last hour of the lab to prepare the preservice teachers for the Analysis of Learning assessment, which took place during the second lab. The preparation included practice using an observation framework when observing teaching and a 30-minute review on the equal sign. I began by introducing and explaining the details of Star and Strickland's (2008) observation framework (presented in Appendix H). Previous research indicates that preservice teachers' skills for observing teaching (Star & Strickland, 2008) and attending to relevant features of classrooms, such as noticing the mathematical details of students' strategies (e.g., Jacobs, Lamb, & Philipp, 2010), are generally not well-developed. Star and Strickland found that using an observation framework enhanced preservice teachers' ability to notice noteworthy classroom events. Their framework comprised five observation categories, namely Classroom Environment, Classroom Management, Tasks, Mathematical Content, and Communication. It should be noted that none of these categories directly focuses the viewer's attention on analyzing teaching in the way described in Hiebert et al.'s (2007) learning from teaching framework. Indeed, none of the categories in Star and Strickland's framework addressed the identification of learning goals nor the proposal of alternative teaching strategies. Moreover, in Star and Strickland's study, the Communication category focused on discourse in general and not on student thinking.

Following the introduction of the observation framework, I presented Video 1. This video is a classroom video clip taken from the University of Michigan's Deep Blue website (Mathematics Teaching and Learning to Teach, 2010). It includes a 10-minute video segment of a third-grade classroom discussing the concept of even and odd numbers. I showed the video without pauses and asked the preservice teachers to use the framework to record observations from video. The observation practice was followed by a guided discussion of what they noticed.

Following the observation practice, I provided a 30-minute review on the equal sign. The review on the equal sign addressed the definition of the equal sign and the different ways children interpret its meaning. Specifically, I reviewed the following conceptions of the equal sign: (a) “the answer comes next,” (b) “use all numbers,” (c) “extend the problem,” and (d) “relational understanding” (Carpenter et al., 2003).

**Phase III: Pre-assessment Skills 2, 3, and 4.** All the preservice teachers participated in Phase II as a class and were not yet assigned to their respective groups. At the beginning of the second lab, I distributed Star and Strickland’s (2008) observation framework and presented Video 2 (Carpenter et al., 2003). This video captures a 4th grade classroom discussion on the meaning of the equal sign. The teacher begins by presenting an open number sentence (i.e., 8 + 4 = ☐ + 5) to the students to evaluate their conceptions of the equal sign. Following the students’ responses to this problem, the teacher strategically follows up with a sequence of true/false number sentences to target the students’ misconceptions of the equal sign. It is 10 minutes and 19 seconds long and was paused every 2 to 3 minutes for 1 minute to facilitate note taking. I provided the following instructions to the preservice teachers prior to viewing the film:

> You will watch a video of a fourth-grade classroom’s mathematics lesson. While you watch the video, you may take notes using the table [framework] we used during the first class. After you watch this video, I will distribute a worksheet with some questions to see what you noticed. You are encouraged to use your notes if you want to, but I would like you to avoid discussing the video with other students while you complete the worksheet. Also, if there is a question you are not sure about, please avoid guessing and simply write “not sure.”

After the video viewing, which lasted 15 minutes, I distributed the Analysis of Learning assessment (Appendix D). The Analysis of Learning assessment was completed in approximately 30 minutes. Once Phase II was complete, I stratified the students according to whether they completed Teaching Mathematics II and randomly assigned all of the preservice teachers within each stratum to one of two conditions: the Students Learning group and the Learning Goals group.

**Phase IV: Instructional intervention.** I designed a series of instruction sessions, called the Analysis of Learning sessions, framed by the research on lesson analysis and teacher education (Santagata, Zannoni, & Stigler, 2007; see also Morris, 2006), by Hiebert et al.’s (2007) model for analyzing teaching, and by the research on teacher noticing (e.g., Sherin & van Es, 2005; Appendix I). A research assistant and I led the Analysis of Learning sessions. I approached this particular research assistant to co-instruct the sessions because (a) she was not directly involved in the course, and (b) she possessed the necessary knowledge for teaching these skills. Specifically, the assistant was a certified elementary teacher and was enrolled in the doctoral program in education at the same institution where the methods course was offered. Her research interests and experience centered on mathematics teaching and learning and children's cognitive development. She also had amassed relevant work experience as a teaching assistant for the same methods course (Teaching Mathematics I).

To avoid instructor effects, we provided instruction to both groups. Specifically, I was the instructor for three sessions with the Learning Goals group and two sessions with the Students Learning group. The research assistant was the instructor for three sessions with the Students Learning group and two sessions with the Learning Goals group. The research assistant and I each provided instruction on two of the four skills for each group. Prior to each instruction session, I trained the research assistant on the delivery of the session to ensure that the approach to teaching the skills was consistent across all sessions.

Two variations of the Analysis of Learning sessions were used in my study, one for the Students Learning condition and one for the Learning Goals condition. The instruction in both conditions was designed to develop preservice teachers' skills in noticing the effect of teaching on students' learning. That is, the instruction sessions were designed to develop a specialized type of noticing that involves a focus on students' learning. The Analysis of Learning sessions emphasized how to attend to relevant student responses and behaviors that reveal mathematical thinking so that conclusions could be drawn concerning (a) what students learned, (b) the effect of teaching on students' learning, and (c) alternative teaching strategies (Hiebert et al., 2007; Santagata et al., 2007).

Previous research suggests that there is a tendency for teachers to evaluate the effectiveness of teaching in terms of what teachers do (e.g., success of activity implementation and alignment with reform principles and standards) as opposed to students' responses to teaching activities (Hiebert et al., 2007). If the primary goal of teaching is to mobilize specific learning objectives, assessments of teaching that are limited to such elements cannot account for whether the learning goals were achieved; thus, evidence of students' learning is necessary and central to analyzing the effect of teaching on student learning. In line with this reasoning, the instruction provided to both conditions aimed to shift preservice teachers' focus of analysis from teacher practice to student learning to understand what students did, or did not, learn.

Both groups received instruction on how to: (a) collect evidence about what students learned (Skill 2), (b) form hypotheses about how teaching helped students learn (Skill 3), and (c) revise the lesson to improve student learning (Skill 4; Hiebert et al., 2007; Santagata et al., 2007). Only the Learning Goals condition received instruction on how to specify learning goals (Skill 1). An overview of each intervention session and the instruction PowerPoint slides are presented in Appendix I.

The Analysis of Learning sessions spanned over eight labs, but each preservice teacher only attended five of those eight labs. The lab schedule is presented in Appendix J. For some of the sessions, both instructors were available to deliver each group's instruction during the same lab, but for scheduling reasons, there were weeks where only one instructor was available to provide the instruction. For these weeks, one of the two groups would receive instruction and the second group would receive instruction the following week. In total, each group received approximately 6 hours of instruction. The Analysis of Learning sessions involved a combination of three main activities: (a) whole class video analysis, (b) skill-based instruction delivered by the instructor, and (c) participants' framework development that took place in small groups. The video analysis component had two purposes: (a) to reflect on the meaning of a skill using an observation framework (occurred before the skill-based instruction), and (b) to practice the skills learned using an observation framework (following the skill-based instruction). The skill-based instruction included (a) direct instruction on one or more of Hiebert et al.'s (2007) skills, or (b) a review of skills previously addressed. The direct instruction provided the preservice teachers with an explicit explanation of the meaning and use of the targeted learning from teaching skills.

The framework development activity involved working in small groups to modify the framework they had used while observing the video. At the end of each session, the instructor collected the students' frameworks along with their notes on how the framework should be modified. Prior to the following session, I modified the observation framework template accordingly. Each group's final observation framework (used for the post-assessment and interview) may be viewed in Appendix K.

***Analysis of Learning session 1.*** I delivered session 1 for the Learning Goals condition. The focus of session 1 for the Learning Goals group was to learn how to identify and unpack the primary learning goal of a lesson (Skill 1). Unpacking a lesson to understand its learning objectives involves specifying what the students need to understand (i.e., the subgoals) to achieve the primary learning goal of the lesson. I began the session by providing following instructions to the participants:

> We will watch a video and I would like you to take notes using the observation framework. I would like you to think about how you can identify the learning goals of the interview in detail because we will spend some time discussing your understanding of the learning goals afterward.

Video 3 (Carpenter et al., 2003) was presented to the group with periodic pauses (every 2 to 3 minutes for about 1 minute) to facilitate note taking. Video 3 was used in all sessions in which I provided instruction on Skills 1, 2, 3 and 4 (i.e., sessions 1, 3, and 4). In this 8-minute video, a second grader was asked during a one-on-one interview to justify $a + b - b = a$. The video was viewed and the total time spent watching the video was 15 minutes. Video viewing was followed by approximately 30 minutes of direct instruction on Skill 1 (see Appendix I). Following this, the preservice teachers were instructed to work in small groups to discuss how they would change the framework so that it includes a "learning goals" category. That is, the preservice teachers were asked to modify the structure and organization of the Star and Strickland (2008) observation framework to support the analysis of the primary and sub learning goals of a mathematics lesson. Also, I explained that any other aspects of Star and Strickland's framework may be revised and categories, if perceived as redundant, could be removed.

Following the framework development activity, the preservice teachers viewed the same video clip with one pause and were instructed to use the modified framework to record the primary and sub learning goals in the video. I then collected all the frameworks, which were subsequently used to create a new observation framework that incorporated the group's comments and suggestions on Skill 1. The revised framework was then used in session 2.

The research assistant delivered session 1 to the Students Learning group. She began the session by showing Video 3. The video was viewed with periodic pauses (every 2 to 3 minutes for about 1 minute) and the total time spent watching the video was 15 minutes. The following instructions were provided:

> We will watch a video and I would like you to take notes using the observation framework. I would like you to think about what you find interesting about this interview because we will spend some time discussing your observations afterward.

Following the video viewing (15 minutes), the research assistant did not provide any formal instruction. Rather, she prompted the preservice teachers to discuss the events they found to be interesting or noteworthy, avoiding any explicit discussion related to the learning goals of the lesson. The group discussion lasted approximately 15 minutes. Similar to the Learning Goals condition, the preservice teachers completed the framework development activity working in small groups for 20 minutes. The research assistant instructed them to discuss how they would change the organization and structure of Star and Strickland's (2008) framework based on the group's discussion of their observations. The research assistant explained that any of the categories in the framework could be revised and categories, if perceived redundant, could be removed. The research assistant presented Video 3 once more, with one pause, and instructed the preservice teachers to take notes using the modified framework. The research assistant collected all the frameworks and I created a new observation framework for use in the second session.

***Analysis of Learning session 2.*** I delivered session 2 to the Learning Goals group. The session began with a PowerPoint presentation that included feedback to the preservice teachers on their conceptions of primary and sub-learning goals and an explanation of the changes the group made to the observation framework. More specifically, the PowerPoint slides reviewed (a) the concept of a primary learning goal and subgoal, (b) the primary and subgoals observed in Video 3 during session 1, (c) the notes taken in small groups during the previous session on learning goals, and (d) the revised observation framework.

After the lecture, I presented Video 4 (Carpenter et al., 2003) to the preservice teachers. Video 4 is a one-on-one interview conducted with a second grade student. In this video, the interviewer gives a series of open number sentences (e.g., $67 + 83 = \square + 82$) to develop the student's relational thinking to solve the number sentences. I instructed the preservice teachers to practice Skill 1 using the revised framework while viewing Video 4. Video 4 was paused every 4 to 5 minutes to facilitate note taking. This video is approximately 17 minutes and 45 seconds and the total viewing time was 21 minutes. After viewing the video, 15 minutes of the session was dedicated to whole class discussion on the usefulness of the framework.

The research assistant delivered session 2 to the Students Learning Group. Session 2 began with a lecture that focused on reviewing the changes made to Star and Strickland's (2008) observation framework. Following the lecture, the research assistant presented Video 4 and instructed the preservice teachers to use the revised framework to guide their observations. Video 4 was paused every 4 to 5 minutes and the total viewing time was approximately 21 minutes. After the video, 12 minutes of the session was dedicated to whole class discussion on the usefulness of the framework.

***Analysis of Learning session 3.*** The research assistant led session 3 for the Learning Goals group. For both groups, the purpose of the third Analysis of Learning session was to learn how to collect evidence of student learning (Skill 2) and how to construct hypotheses about the effects of the teaching on learning (Skill 3). Skill 2 instruction addressed (a) strategies for identifying key moments in the lesson where evidence of student learning is likely to occur, and (b) how to distinguish informative from less informative student responses. Students' behaviors that inform teachers about their learning include those that are *revealing* of student thinking and *related* to the learning goals (Hiebert et al., 2007). Both components were used to define Skill 2 for the Learning Goals group. To avoid discussion of learning goals in the Students Learning condition, Skill 2 was defined as collecting evidence revealing of student thinking to evaluate learning. The instruction targeting hypotheses construction (Skill 3) was the same for both groups. Specifically, the instruction for Skill 3 defined the term hypothesis and discussed its role in analyzing the effect of teaching on student learning (see Appendix I).

At the beginning of the session, the research assistant provided the following instructions to the Learning Goals group:

> We will watch a video and I would like you to take notes using the framework from the last lab to record the learning goals you see in the video. I would also like you to think about how to collect evidence that pertains to what the student knows relative to the learning goal and form a hypothesis about what the student has learned by the end of the video. You can construct multiple hypotheses and you should be prepared to support your ideas.

Video 3 was presented to the group, with pauses to facilitate note taking, for approximately 15 minutes. After viewing the video, the research assistant dedicated 25 minutes to instruction on Skills 2 and 3. Following this, the preservice teachers were instructed to work in small groups for 15 minutes to discuss how they would change the framework so that it includes categories addressing Skills 2 and 3. Following the framework development activity, 10 minutes of the session were dedicated to whole group discussion on framework modifications. The research assistant collected all the frameworks, and I subsequently created a new observation framework based on the group's comments for use in the fourth session.

For the Students Learning group, the instructions I gave at the beginning of the session were as follows:

> We will watch a video and I would like you to take notes using the framework from your last lab to record your observations of the lesson. I would also like you to think about how to collect evidence that pertains to what the student knows and form a hypothesis about what the student understands by the end of the video. You can construct multiple hypotheses and you should be prepared to support your ideas.

The procedures that followed were virtually identical to the procedures used with the Learning Goals group. The direct instruction component on Skills 2 and 3, however, was approximately 20 minutes because I avoided discussion related to learning goals (Skill 1).

***Analysis of Learning session 4.*** I led session 4 for the Learning Goals group and the research assistant delivered this session to the Students Learning group. The purpose of the fourth session for both groups was to learn how to apply Skills 2 and 3 to make decisions on how to improve student learning (Skill 4). Instruction on Skill 4 was virtually identical for both groups. The only difference was the omission of Skill 1-related information for the Students Learning Group (see Appendix I).

At the beginning of session 4, I presented a lecture that reviewed the definitions of Skills 1, 2, and 3 with the Learning Goals group and the research assistant reviewed Skills 2 and 3 with the Students Learning group. Following this, each group received instructions for the video analysis activity. The instructions for the Learning Goals group was as follows:

> We will watch a video and I would like you to take notes using the framework from your last lab to (1) record the learning goals and evidence of student learning that is related to the learning goal, and (2) form hypotheses about the effect of the teaching in the video on student learning. I would also like you to propose alternative teaching strategies and speculate why these strategies might have an effect on students' learning. You will spend some time discussing your ideas in small groups.

The instructions provided to the Students Learning group was identical except the reference to learning goals was omitted.

The procedures for session 4 were the same for both groups. Video 3 was presented for approximately 15 minutes, including pauses. After viewing the video, approximately 10 minutes of the session were dedicated to instruction on Skill 4. Following this, the preservice teachers were instructed to work in small groups for 15 minutes to discuss how they would change the framework to include a category addressing Skill 4. At the end of the session, all the frameworks were collected and I created a final version of the framework based on the group's comments and suggestions for the final session.

***Analysis of Learning session 5.*** The research assistant delivered session 5 to the Learning Goals group and I delivered this session for the Students Learning group. The purpose of the fifth Analysis of Learning session was to review the instruction on all skills and allow for practice of these skills using the final version of the observation framework. For both groups, the session began with a presentation of PowerPoint slides that reviewed key skill concepts and specific examples from Video 3 to support these concepts. The slides also referenced examples from the preservice teachers' frameworks (see Appendix I). Twenty minutes of the Learning Goals group's session was dedicated to reviewing skill concepts, while the Students Learning group received 15 minutes of review. After the lecture, the following instructions were provided to the group:

> We will watch a video and I would like you to take notes using the framework to record everything we have talked about today and in the previous labs. You will be given some time after watching the video to write down your hypotheses and any alternatives that you would propose.

The instructor presented Video 5 and instructed the preservice teachers to practice all the skills addressed from sessions 1 through 4 while viewing the video. Similar to Video 2, the context of this video was complex and showed an edited clip of a classroom lesson on eliciting conjectures. The video was approximately 6 minutes in length. Video 5 was paused every 2 to 3 minutes and the total viewing time for both groups was approximately 10 minutes. Both groups used the final version of the observation framework to guide their analysis of the mathematics lesson observed in the video.

**Phase V: Post-assessment of Skills 1, 2, 3, and 4.** The Analysis of Learning assessment at posttest was administered at the same time to both conditions in separate locations. The final version of each group's framework was used during the assessment to analyze Video 2. The

Learning Goals group included sections in their framework related to Skill 1. For this reason, I conducted the post-assessment with both groups in separate rooms to avoid exposing participants to different frameworks.

The procedure for the post-assessment was similar to the pre-assessment. Video 2 was shown and the preservice teachers were asked to use their group's framework to guide their viewing of the mathematics lesson in the video. Following the video viewing, the Analysis of Learning test was administered in three phases (see Appendix D). Part I included the open-response questions measuring Skills 3 and 4. Once Part I was completed, the instructor collected Part I and distributed Part II. Part II included the open-response items measuring Skill 1. Once Part II was completed, Part III was distributed. Part III included the true/false and multiple choice questions measuring Skills 1 and 2. For all three parts of the assessment, the preservice teachers were encouraged to use their notes, but were asked not to share frameworks or discuss the video as they completed the tests. The entire lab was dedicated to the administration of the Analysis of Learning assessment, approximately 15 minutes to view the video and 60 minutes to complete the written test.

**Phase VI: Specifying Learning Goals interview**. I began contacting preservice teachers via email to request participation in a one-on-one interview after the final grades for the course were submitted. I contacted sixteen preservice teachers and the final sample included 8 participants. The first interview took place 9 weeks after the post-Analysis of Learning assessment was administered. All interviews were audio and video recorded.

I conducted all interviews using the interview protocol presented in Appendix F. The interview was semi-structured and began with a series of questions related to planning a lesson on the equal sign. Following that, I presented Video 2, which was the same video used during the pre- and post-assessment, and provided the preservice teacher with the final version of the framework of the group in which she participated during the intervention. Prior to the video, I read the following instructions:

> I will show you the last video we watched as a class in the lab. Just to remind you, the video is from a 4th grade math lesson. When you watch the video, I want you to focus on what the teacher's learning goals may have been for the students. You may use the framework your group designed to take any notes on this aspect of the lesson, but keep in mind that you do not need to focus on the other aspects of the lesson we discussed during

the labs unless you think it is necessary. After you've watched the video, I will ask you some questions and you can refer to your notes and the video itself at anytime when answering the questions. If you need me to, I can go back to a specific point in the video when you are answering your question.

The video was viewed without pauses. Following the video viewing, I asked a series of questions to compare the lesson the preservice teacher proposed in the first part of the interview to the lesson in the video. The final interview question asked the participant to elaborate on the strategies she used to identify learning goals when observing teaching. Each interview ranged from 30 to 50 minutes.

**Scoring Quantitative Measures**

A list of the scores for each of measure is provided in Table 2.

**Mathematical Content Knowledge for Teaching assessment.** Each item on the MCT was assigned 0 points for an incorrect answer or 1 point for a correct answer. The points were summed for the total MCT score. The maximum MCT score was 39.

**Test items: Pre- and Post-Analysis of Learning assessments.** On the pre- and post-Analysis of Learning assessment, I included two types of questions, closed test items (i.e., multiple choice and true/false items) and open-ended items. The test items were assigned 0 points for an incorrect answer and 1 point for a correct answer. The number of correct responses on the test items were summed to calculate three different scores: (a) a Skill 1 test score (score ranged from 0 to 11; post-assessment only); (b) a Skill 2 Evidence score for items focused on student evidence related to the learning goals and revealing of student thinking (score ranged from 0 to 12); and (c) a Skill 2 Not Evidence score for items focused on student evidence not revealing of student thinking (score ranged from 0 to 7). For the Skill 2 Not Evidence score, lower scores indicated that less attention was paid to student behaviors that were not revealing of their thinking.

**Open-ended items: Pre- and Post-Analysis of Learning assessments.** Prior to scoring the responses to the open-ended items, I incorporated measures to reduce experimenter effects during the scoring phase. First, I transcribed the responses into a word document. When I transcribed the responses, I only included the participant's identification number used during the study. Following that, I assigned a random numerical code to each participant's

Table 2

*List of Measures and Scoring*

| Variable Name | Range of Scores | Assessment |
|---|---|---|
| MCT | 0 to 39 | Mathematical Content Knowledge for Teaching |
| Skill 1 Test | 0 to 11 | Post-Analysis of Learning Assessment (closed-items) |
| Skill 1 Identification | 0 to 4 | Post-Analysis of Learning Assessment (open-ended items) |
| Skill 1 Specification | 1 to 5 | Post-Analysis of Learning Assessment (open-ended items) |
| Skill 1 Goals | 1 to 20 | Sum of three Skill 1 scores |
| Skill 2 Evidence | 0 to 12 | Pre- and Post-Analysis of Learning Assessment (closed-items) |
| Skill 2 Not Evidence | 0 to 7 | Pre- and Post-Analysis of Learning Assessment (closed-items) |
| Skill 3 Hypothesis | 0 to 4 | Pre- and Post-Analysis of Learning Assessment (open-ended items) |
| Skill 4 Alternative | 0 to 3 | Pre- and Post-Analysis of Learning Assessment (open-ended items) |

identification number. The numerical codes were used to replace the participant identification number in the document. This procedure allowed me to score the responses without the possibility of identifying the participants' identities or group membership.

Once the responses were ready to be scored, I designed a rubric to code and score responses to the open-ended questions assessing Skills 1 (post-assessment only), 3, and 4. My rubric used the criteria outlined in Hiebert et al. (2007) and Santagata et al.'s (2007) method for scoring response quality. The scoring rubric is presented in Appendix L.

***Skill 1.*** Two open-ended questions addressed Skill 1 on the post-assessment: (a) "In as much detail as possible, can you identify the overall learning goal of the lesson?" and (b) "Now consider each activity/task presented during the lesson. In as much detail as possible, describe what the teacher wanted the students to learn for each activity/task." Responses to each question were assigned an *identification* score and a *specification* score. The identification score indicated whether the response accurately identified the primary goal and the three subgoals. As previously mentioned, three possible subgoals were observed in the lesson: (a) eliciting students' conceptions (and misconceptions) about the equal sign, (b) guiding students' acceptance of non-canonical number sentences, and (c) helping students' develop a procedure to solve equivalence problems. The specification score was an indicator of response quality. For the primary goal, quality assessment was based on the details provided in the response and accuracy of information. Quality assessment for the subgoals was based on the description of each subgoal (i.e., whether the subgoal was correctly linked with a task from the video). The sum of the three Skill 1 scores (i.e., the test score, identification score, and specification score) was used for the analysis.

*Skill 1 identification.* For the first question that addressed the primary learning goal, I assigned 1 point for responses that accurately described the primary learning goal and 0 points if the response (a) contained inaccurate information or (b) included insufficient details about the learning goal topic. For the second question that addressed the subgoals of the lesson, I assigned 1 point for each accurately identified sub goal. The total identification score was calculated by summing the points, and it ranged from 0 to 4.

*Skill 1 specification.* For the first question about the primary learning goal, responses that described the primary goal with accuracy and sufficient detail were assigned 2 points. Responses that lacked details about the topic or contained inaccurate information were assigned

1 point. Responses that did not accurately specify the primary learning goal of the lesson were assigned 0 points.

The following is a response that would receive 2 points: "The overall goal of the lesson was to teach the students the meaning of the equal sign, that the amounts on both sides of the equal sign must be the same." This response accurately identifies that the lesson was about the equal sign and specifies what about the equal sign the teacher intended her students to learn. I would assign 1 point to the following response: "The overall goal of the lesson is for students to understand more about the equal sign and to address the misconception that the equal sign means the answer comes next." This response identifies the topic of the lesson, but does not accurately specify what about the equal sign the teacher wanted the students to understand by the end of the lesson. Further, this response suggests that the overall goal was to address a specific misconception about the equal sign observed in the video, but this was one of the subgoals of the lesson and not the primary learning goal. Finally, I would assign 0 points to the following response: "To learn an efficient way to make the number sentence true," because the topic identified is incorrect (i.e., no mention of the topic of equivalence).

For the second question, each subgoal in the response was coded as high quality or low quality. High quality responses accurately linked an activity/task to a learning goal and low quality responses either (a) did not link the activity/task to a learning goal or (b) incorrectly linked the activity/task to a learning goal. The following subgoal statement, "The teacher presented $7 = 3 + 4$ to provide an example where the answer does come next," would receive a high quality code. The teacher in the video provided this problem because the students revealed the misconception that the equal sign means the "answer comes next" when they answered that "12" should go in the box to make $8 + 4 = \square + 5$ true. Using a known number fact for fourth-grade students and presenting addends on the right side of the equal sign was a strategic way to address this misconception. In contrast, the following response would receive a low quality code, "the teacher used a 'reverse' number statement to explore their understanding of commutativity." This subgoal statement misunderstands the teacher's goal in using this task in her lesson. The teacher's goal was to address the misconception about non-canonical number sentences (i.e., "reverse") not students' understanding of the commutative property.

After assigning the high and low quality codes, I used a relative frequency score for the final specification score. Preservice teachers were not instructed to provide a specific number of sub goal statements. Because of this, I used a relative frequency score to indicate the number of high quality codes relative to the number of low quality codes (Santagata et al., 2007). Specifically, 3 points were awarded when the number of high quality statements exceeded the number of low quality statements. Two points were awarded when the number of high quality statements was equivalent to the number of low quality statements. One point was awarded when the number of high quality statements was less than the number of low quality statements. This specification score for the subgoals was added to the specification score for the primary learning goals for a total specification score, which range from 1 to 5.

***Skill 3 hypothesis.*** One open-ended question addressed Skill 3 on the pre- and post-assessment, "Form a hypothesis (or more than one) about what the students learned and understand by the end of the lesson." To assess the responses to this question on the pre- and post-assessment, I scored each hypothesis based on the criteria outlined in Hiebert et al. (2007). A hypothesis that only focused on teacher behaviors (i.e., cause) or student behaviors (i.e., effect) was assigned 0 points. For example, "By the end of the lesson, the students in the video understood that the addends can be placed on the right side of the equal sign provided that the amounts on both sides of the equal sign are the same," only mentions what the students understood without addressing how the teacher influenced this understanding. In this case, the effect is discussed in absence of the cause. Similarly, "By the end of the lesson the teacher returns to original problem or 8 + 4 = ☐ + 5 to see if the students will change their initial answer," addresses what the teacher did without referencing how this decision influenced student learning. This latter response would also be assigned 0 points.

When a hypothesis referenced a teacher behavior (i.e., cause) and explained its influence on student learning (i.e., effect), I assigned at least 1 point. The following hypothesis, for example, would receive 1 point, "The teacher exposed the students to non-canonical equations and as a result the students learned that equations do not always have to be written in a standard form."

Additional points could be assigned to hypothesis statements that linked teaching (i.e., cause) to student learning (i.e., effect) when the response included evidence from the video and reasoning about the teaching strategies. Specifically, there were three ways to elaborate on the cause-effect statement: (a) including specific details about student learning observed in the video, (b) including specific details about teaching observed in the video, and (c) reasoning about the purpose of the teaching and tasks observed in the video. For each of these three detail-oriented criteria, I would assign 1 additional point. In other words, each hypothesis that received 1 point for including a cause-effect statement could receive a maximum of 3 additional points depending on how many detail-oriented criteria were addressed. For example, I would assign 2 points for the following hypothesis,

> Because the teacher used non-canonical number sentences of varying levels of difficulty the students learned and understood the real meaning of the equal sign and modified their view that the equal sign means the answer comes next. We see this because the student's answer in the beginning of the lesson was 12, but by the end of the lesson the students said that 5 goes in the box.

This hypothesis would receive 2 points because in addition to stating how the teaching observed in the video influenced student learning (1 point), the statement also provides evidence of student thinking observed in the video and would thus receive 1 additional point.

The following statement would also receive 2 points because it includes a cause-effect statement (1 point) and 1 additional point would be assigned because the response includes details about the teacher's behavior: "By asking students 'is it the same on both sides' and using diagonals to illustrate that different addends on either side of the equal sign have the same sum, some students began demonstrating an understanding of the meaning of the equal sign."

Reasoning about the teacher's practice provides opportunities to make connections between the teaching observed and the broader principles for teaching the topic (Hiebert et al., 2007). Accordingly, 1 additional point was awarded to statements that explained the reasoning behind the teacher's decision to include certain tasks, pose questions, and probe student thinking. The following is an example of a hypothesis that includes this type of reasoning:

> Following the initial problem of $8 + 4 = \square + 5$, the teacher provided non-canonical equations with addends on one side of the equal sign, such as $6 = 6 + 0$ to correct students' misconception that the answer always follows the equal sign ($8 + 4 \neq 12 + 5$). The students began to understand that both sides of the equal sign need to have the same total and that it does not matter where the equal sign is placed in the equation.

This statement would receive 3 points: 1 point for referring both to teaching and student learning, a second point for providing details about the teaching observed in the video, and a third point for addressing why the equation $6 = 6 + 0$ was included in the lesson.

Similar to the open-response question pertaining to subgoals identification in Skill 1, the question associated with Skill 3 did not instruct the preservice teachers to construct a certain number of hypotheses. Because the number of hypotheses proposed on the pre- and post-assessment varied across the sample, a mean hypothesis score was calculated by dividing the total number of points for the response by the number of hypothesis statements provided. The score ranged from 0 to 4.

***Skill 4 alternatives.*** The responses to this question, "If you were the teacher, what would you have done differently?" were assessed to evaluate the preservice teachers' ability to propose teaching alternatives (Skill 4). Each response was scored based on criteria outlined in Hiebert et al. (2007) and Morris (2006). More specifically, my rubric included three types of alternatives, and I awarded 1 point for each distinct alternative addressed in the response. Responses that did not propose any of the three alternatives received 0 points. The total Skill 4 alternatives score ranged from 0 to 3.

One type of alternative involved using observations from the lesson to justify the proposed changes to the lesson. Specifically, when an alternative was justified using student evidence from the video (Skill 2) or was based on one of their hypotheses (Skill 3)[6], 1 point would be awarded. The following is an example of this type of alternative:

> Several students provided incorrect answers to the true/false equations and the teacher would move on to other number sentences without correcting them. For example, when the students said that 7 = 3 + 4 is false, she moved on to 6 = 6 + 0. I would have continued with 7 = 3 + 4 but use manipulatives to represent the equation. That way, the students would have a concrete representation of both amounts and could 'see' that the statement is actually true.

This response would receive 1 point because the proposal to use manipulatives to represent the equations was elicited in response to the answers the students provided in the video.

The second type of alternative involved using pedagogical knowledge about teaching the equal sign to justify the alternative proposed. One point was awarded if this type of alternative was included in the response. The following is an example:

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[6] I focused on the application of Skills 2 and 3 because both groups received instruction on these skills.

> There are many types of number sentences that can be used in a lesson about the equal sign. The teacher started with one of the more 'difficult' types and I would have started with a non-canonical true/false number sentence rather than a non-canonical open number sentence. Starting with 6 = 6 + 0, for example, would have made the meaning of the equal accessible at an earlier point in the lesson and then this understanding could be tested with non-canonical open number sentences.

This alternative would receive 1 point because it justifies changing the beginning of the lesson based on the knowledge of tasks that can be used in a lesson about the equal sign (Carpenter et al., 2003).

The third type of alternative involves changing the lesson to gain greater access to the students' thinking. One point was awarded if this type of alternative was included in the response. The following is an example:

> Before presenting the first number sentence 8 + 4 = □ + 5, I would ask the children to explain what they understand when they see the following symbol '='. The number sentence that I would choose to start the lesson would be based on the definitions the students provided me with.

This alternative would receive 1 point because the proposal to change the way the lesson begins is based on the view that a more explicit understanding of the students' thinking about the equal sign should inform the choice of tasks used in the lesson.

As previously mentioned, responses that did not propose any of the three types of alternatives received 0 points. Zero points were awarded to these responses because the statements did not allow me to assess the participants' ability to propose teaching alternatives. First, 0 points were awarded when the alternative focused on ways to manage the classroom or was not specifically linked to the learning goal of the lesson. For instance, "I may have had the students use manipulatives or chips," and "I would have given more time for the students to discuss the problems," would be awarded 0 points. In each of these examples, a suggestion to change the lesson is provided, but these proposals do not explain their alternative in the context of a lesson about the equal sign.

Responses that explicitly indicated that no changes should be made to the lesson were assigned 0 points and responses that did not explicitly indicate whether any modifications were necessary were also assigned 0 points. For example, the following statement "the teacher did do a great job at having the students understand why the answer was not 12, but instead 17" would receive 0 points because the response does not propose how to change the lesson observed in the video. Alternatively, "I like the way this teacher moved her lesson in a direction that her students needed to understand the concept, I would be alright not changing her method" would also receive 0 points because the statement explicitly states the view that changes to the lesson were not needed.

Inter-rater reliability was conducted with a second coder who coded 25% of the open-ended responses. Percentage of agreement was calculated for the coding of each skill and an overall kappa was calculated for all three skills combined. The second coder was a trained research assistant and had knowledge and expertise in the teaching and learning of elementary mathematics. The percentage of agreement for Skill 1 codes was 89.65%, Skill 3 codes was 87.9%, and Skill 4 codes was 84.2%. The kappa coefficient for all three skills was $\kappa = 88.33$. Inter-rater reliability for Skill 2 was not conducted because open-ended questions were not used to assess this skill.

**Coding for Qualitative Analysis**

I began by familiarizing myself with the data set, thoroughly reading and re-reading the interviews while taking note of interesting features in the data. I did not develop a coding rubric prior to coding the data because my goal was to provide a description of the participants' reflections and observations on teaching and learning, not compare them to a normative model from the literature.

First, I generated a list of codes from all the interviews. Following that, I grouped these codes into the following superordinate categories: (a) types of learning goals in the planning context, (b) types of tasks in the planning context, (c) types of learning goals in the observation context (observation of the video), (d) language used across both contexts, and (e) ways to reason about the learning goals. Once the codes were grouped, some codes were merged to reduce redundancy. The majority of the codes were renamed to make them more generalizable across the interviews (versus specific to a particular interview). I included these codes and their description in a codebook (see Appendix M). Using the codebook, I systematically re-coded the data set

A research assistant double coded the interview data. This second coder was not the same person who double-coded a portion of the pre- and post-assessment data. The second coder, who has some background knowledge on the teaching and learning of equivalence, was trained using this codebook and double-coded 7 of the 8 interviews (one interview was used for training purposes). The coding took place in three phases, and the second coder and I met after each coding phase to compare and discuss our coding. The first coding phase focused on codes for the types of tasks and types of learning goals. The second phase focused on the coding of language use. The third phase focused on the codes associated with reasoning about the learning goals viewed in the lesson. All discrepancies in coding were resolved through discussion.

**Learning goal codes.** For codes associated with the *types of learning goals*, each statement provided in the interview was reviewed and every learning goal mentioned received a code. Preservice teachers proposed (in the planning context) and observed (in the observing context) learning goals associated with teaching the equal sign and other elementary mathematics topics (e.g., the commutative property). The different types of learning goals were grouped into seven superordinate categories: (a) knowledge about the equal sign, (b) student misconceptions, (c) student thinking, (d) symbolic knowledge, (e) skill-based knowledge, (f) other mathematics topics, and (g) an other learning goals category.

**Task codes.** For codes associated with the *types of tasks*, each statement provided by the participant in the interview was examined and every task that was mentioned received a code. The majority of tasks were equations or number sentences. Three code names for number sentences were created and indicated the form of the number sentence and whether the number sentence was observed in the video. In particular, the task codes for number sentences included: (a) canonical (e.g., $a + b = c$) number sentence, (b) non-canonical number sentence observed in the video (e.g., $a + b = \square + d$), (c) non-canonical number sentence (e.g., $a + b = b + a$). Codes for tasks that did not involve number sentences also emerged from the data. Specifically, tasks that proposed using manipulatives, having students themselves create equivalence problems, and having students independently write definitions of the equal sign were also mentioned in the interview and were coded accordingly.

**Quality of language codes.** The *quality of language* codes were organized into six superordinate categories: (a) the meaning of the equal sign, (b) children's misconceptions, (c) terminology (technical terms related to mathematical principles and the equal sign), (d) lesson components (i.e., learning goals, tasks, and task sequence). Codes for describing the meaning of the equal sign referred to whether descriptions of the equal sign were correct, partially correct, or incorrect. Five codes focused on children's misconceptions about mathematics. Four of these codes were used for different types of misconceptions (e.g., the number that follows the equal sign is always the answer), and were assigned when a participant accurately described a specific misconception. One of the five misconception codes did not address a type of misconception but was used for statements describing sources of children's misconceptions.

Four terminology codes emerged from the data. Two codes were assigned when the participant without being prompted by the interviewer accurately used technical terms related to mathematical principles and the equal sign. Two codes were assigned when the participant accurately used technical terms related to mathematical principles and the equal sign only after the interviewer had used them.

Codes for statements describing tasks and learning goals in the planning and observing context also emerged from the data. These codes were used when the participant either elaborated on the task or learning goal, or justified the use of a task or learning goal. In addition to describing tasks and learning goals, some participants described the task sequence, discussing how a given task was linked to previous or subsequent tasks in the video lesson or could be associated in their lesson. Three description of task sequence codes were used for task sequences that were described, elaborated, or justified.

**Reasoning about the learning goal codes.** Four *reasoning about the learning goal* codes emerged from the data: (a) equations, (b) student responses, (c) task sequence, and (d) teacher behaviors. The equations code was used when the participant reported focusing on tasks or number sentence problems (e.g., is 7 = 3 +4 true or false?). The student response code was used when the participant reported focusing on the students' responses to the teacher's tasks and questions observed in the video. The task sequence code was used when the participant reported focusing on the order of the tasks presented in the lesson. The teacher behaviors code was used when the participant reported focusing various teacher behaviors, namely the teacher's responses to the students, the ways she represented concepts, and certain statements she repeated throughout the lesson (e.g., "the same on both sides"). It is important to note that all participants focused on more than one aspect of the lesson to reason about the learning goals, and as such participants received more that one type of reasoning code.

**Data preparation for analysis**. Once the data set was double-coded, I prepared the data for analysis in two ways. First, I calculated the frequency of each quality of language code.

Second, for each participant, I created a profile that consisted of a summary of her (a) lesson on the equal sign, (b) observations of learning goals and tasks included in the video lesson, and (c) reasoning about the learning goals in the video. To summarize each participant's lesson on the equal sign, I began by recording the unique collection of learning goal codes she identified in her lesson. Following that, I examined whether her learning goals were or were not linked with a task when they were discussed during the interview. In particular, when she identified a learning goal in her lesson and did not link it with a task I recorded that she *identified* the learning goal. When she identified a learning goal in her lesson and linked it with a task, I recorded that she *specified* the learning goal. The same procedure was used to summarize each participant's observation of the learning goals in the lesson in the video. In addition, each profile indicated the participant's unique collection of task codes to outline the tasks she included in her lesson. Finally, the participant's profile consisted of her unique collection of reasoning codes to describe what information from the video she used to discern the learning goals in the lesson.

Because my objective was to analyze group differences, I merged the participant profiles and created a profile for each group. The group profile was made up of the all the learning goal, task, and reasoning codes for each participant in the group. To summarize the lessons on the equal sign proposed by each participant in the group, the profile consisted of the group's collection of learning goal codes. For each learning goal, I indicated the number of participants in the group that linked the learning goal with a task (i.e., the number of participants that *specified* each learning goal included in the profile) and the number of participants that did not link the learning goal with a task (i.e., the number of participants in the group that *identified* each learning goal included in the profile). The same procedure was used to summarize the group's observation of the learning goals in the lesson from the video. Further, the group profile consisted of a collection of task codes unique to the participants in the group. For each task included in the group's profile, I indicated the number of participants in the group that included the task in their lesson. Also, the group profile consisted of the unique collection of reasoning codes to describe what information participants in the group used to discern the learning goals addressed in the lesson. In addition to this, I indicated the number of participants in the group that received each reasoning code included in the profile.

# CHAPTER 4: RESULTS

The current study was designed to address three research questions. The first research question examined the effects of explicit instruction on specifying learning goals (Skill 1) on preservice teachers' abilities to collect evidence on what students learned (Skill 2), isolate the effects of teaching on students' learning (Skill 3), and revise a lesson to improve student learning (Skill 4). To address this question, I used the data from the pre- and post-assessment and examined group differences on Skills 2, 3, and 4 prior to and following instruction. I will report results from four 2 x 2 ANOVAs using group (Students Learning and Learning Goals) as the between-group factor and time as the within-group factor.

The second research question focused on the effect of instruction on Skill 1 acquisition. I will report the results from one independent *t*-test using group (Students Learning and Learning Goals) as the between groups factor using Skill 1 Goals score at posttest as the dependent variable.

The third question concerned the nature of Skill 1 with and without direct instruction on that skill and how it compares to Skill 1 without instruction. To address this question, data from interviews conducted with a sub-sample of the preservice teachers from both conditions ($N = 8$) were analyzed qualitatively. I will describe the qualitative differences between both groups in terms of (a) specifying and identifying learning goals in two contexts (*planning* a lesson and *observing* a lesson), (b) the types of tasks proposed when planning a lesson on the equal sign, (c) the reasoning used to identify the learning goal(s) when observing teaching, and (d) the frequency of high quality language used when discussing learning goals and tasks. The means and standard deviations for each measure are reported in Table 3. All effects are reported as significant at $p < .05$.

**Mathematical Content Knowledge for Teaching**

To ensure group equivalence on mathematical content knowledge for teaching prior to the study, I conducted an independent *t*-test using group (Students Learning and Learning Goals) as the between-group factor and the Mathematical Content Knowledge for Teaching (MCT) score as the dependent measure (scores ranged from 0 to 39). On average, the preservice teachers in the Learning Goals group received a higher MCT score ($M = 31.20$, $SD = 4.33$) compared to the preservice teachers in the Students Learning group ($M = 30.71$, $SD = 6.19$), but this difference was not significant $t(27) = .25$, $p = .81$.

Table 3

*Means and Standard Deviations for Dependent Variables*

| | Learning Goals ($n$ = 15) | | Students Learning ($n$ = 14) | |
|---|---|---|---|---|
| Measure (possible score range) | Pre-Assessment *M (SD)* | Post-Assessment *M (SD)* | Pre-Assessment *M (SD)* | Post-Assessment *M (SD)* |
| MCT (0 to 39) | 31.20 (4.33) | - | 30.71 (6.19) | - |
| Skill 1 Test (0 to 11) | - | 8.07 (1.39) | - | 7.21 (1.85) |
| Skill 1 Identification (0 to 4) | - | 1.73 (0.59) | - | 1.71 (0.91) |
| Skill 1 Specification (1 to 5) | - | 3.67 (1.18) | - | 3.21 (1.31) |
| Skill 1 Goals (1 to 20) | - | 13.47 (2.61) | - | 12.14 (2.68) |
| Skill 2 Evidence (0 to 12) | 6.33 (2.13) | 7.00 (2.00) | 6.93 (1.90) | 8.21 (2.33) |
| Skill 2 Not Evidence (0 to 7) | 5.00 (1.20) | 5.33 (0.72) | 4.71 (1.33) | 5.14 (1.41) |
| Skill 3 Hypothesis (0 to 4) | 0.07 (.26) | 1.31 (1.10) | 0.07 (0.27) | 1.25 (0.97) |
| Skill 4 Alternative (0 to 3) | 0.53 (0.74) | 1.53 (1.06) | 1.14 (1.17) | 1.43 (1.09) |

**Research Question 1: The Effect of Skill 1 Instruction on Skills 2, 3, and 4**

I began by examining the effect of Skill 1 training on the development of Skills 2, 3, and 4. It was hypothesized that the instruction provided to the Learning Goals group who received instruction on all four skills, compared to the instruction provided to the Students Learning group who received the same instruction except on Skill 1 would differentially effect the development of Skills 2, 3, and 4. Specifically, preservice teachers who received Skill 1 training (Learning Goals condition) were expected to outperform those who did not receive Skill 1 instruction (Students Learning condition) on measures of Skills 2, 3, and 4 following instruction. To evaluate group differences on skill development, I conducted four 2 x 2 ANOVAs using skill scores as the dependent variable.

**Skill 2: Collecting evidence.** It was expected that Skill 1 instruction would differentially impact Skill 2 performance following instruction. I included two measures of Skill 2, one that assessed preservice teachers' ability to collect evidence revealing of student learning (Skill 2 Evidence score) and one that assessed the extend to which they attended to student behaviors not revealing of student learning (Skill 2 Not Evidence score). Higher Skill 2 Evidence scores at posttest indicated that the participants paid more attention to evidence of student learning following instruction. I hypothesized that the instruction would have a differential effect on the ability to collect evidence relevant to student learning. Specifically, I predicted that those in the Learning Goals group would outperform those in the Students Learning group on Skill 2 Evidence following instruction.

Lower Skill 2 Not Evidence scores at posttest indicated less attention to evidence not revealing of student learning. I predicted that preservice teachers' attention to evidence not revealing of student learning would remain the same, or decrease, following the instruction provided in both conditions.

***Collecting evidence revealing of student learning*.** Using the Skill 2 Evidence score (score ranged from 0 to 12) as my dependent variable, I conducted a 2 x 2 ANOVA using group (Students Learning and Learning Goals) as the between-group factor and time as the within-group factor. There was a significant main effect of time, $F(1,27) = 5.47$, $p = .03$, with a significant difference between the pre- assessment means ($M = 6.62$, $SD = 2.01$) and post-assessment means ($M = 7.59$, $SD = 2.21$), partial $\eta^2 = .17$. There was no significant main effect of group, $F(1,27) = 1.89$, $p = .18$. This indicates there was no difference between groups on the mean Skill 2 (Evidence) performance across both time points. Finally, the predicted interaction between group and time was not significant, $F(1,27) = .55$ , $p = .47$, indicating that the instruction provided to both groups had the same effect on the development of Skill 2.

***Collecting evidence not revealing of student learning.*** The Skill 2 Not Evidence score was used to examine change in the preservice teachers' attention to student behaviors that are not revealing of student learning. I predicted that for both groups, performance on Skill 2 Not Evidence would not significantly change following instruction.

Using the Skill 2 Not Evidence scores, which ranged from 0 to 7, as my dependent variable, I conducted a 2 x 2 ANOVA using group (Students Learning and Learning Goals) as the between-group factor and time as the within-group factor. The main effect of time was not significant, $F(1, 27) = 1.70$, $p = .20$, indicating there was no difference on mean Skill 2 Not Evidence performance on the post-assessment compared to the pre-assessment. The results also revealed no significant main effect of group, $F(1, 27) = .52$, $p = .48$. This indicates that there was no difference between the groups on Skill 2 Not Evidence performance across both time points (pretest and posttest). Finally, there was no significant interaction between group and time, $F(1, 27) = .03$, $p = .87$, indicating that the instruction used with both groups had the same effect on Skill 2 Not Evidence following instruction.

**Skill 3: Hypothesis construction.** To examine the effect of Skill 1 instruction on Skill 3 development, I ran the same 2 x 2 ANOVA using the Skill 3 Hypothesis score (scores ranged from 0 to 4) as the dependent variable. It was expected that presence of Skill 1 instruction would differentially impact the development of Skill 3 such that the Learning Goals group would significantly outperform the Students Learning group on Skill 3 following instruction.

The Skill 3 Hypothesis score indicated the average number of criteria addressed in the participants' hypothesis statements (Hiebert, Morris, Berk & Jansen, 2007). Scores close to 0 indicated that, on average, the hypothesis statements did not include the cause (teaching event) or the effect (student learning) in the cause-effect statement. Scores close to 1 indicated that, on average, the hypothesis statements included a cause (teaching event) and an effect (student learning) statement.

Hypothesis statements that included a cause and effect statement could receive additional points when statement included the following detailed-oriented criteria: (a) student-related details, (b) teacher-related details, or (c) pedagogically-related details. Specifically, scores close to 2 indicated that, on average, the hypothesis statements elaborated on the cause and effect statement using one of the three detail-oriented criteria. Scores close to 3 indicated that, on average, the hypothesis statements elaborated on the cause and effect statement using two of the three detail-oriented criteria. Scores close to 4 indicate that on average the hypothesis statements elaborated on the cause and effect statement using all 3 of the three detail-oriented criteria.

The number of participants mentioning each detail-oriented criterion in their responses on the pre- and post-assessment is reported in Table 4. As it can be seen in the table, the frequency for including each criterion increased from pretest to posttest. In particular, in the Learning Goals group no participants included student-related details and pedagogically-related details in the hypothesis statements at pretest. One participant in the Learning Goals group (i.e., 6.67% of the group) included teacher-related details in the hypotheses on the pre-assessment. Following instruction, 7 participants in the Learning Goals group (i.e., 46.67%) included student-related details, 8 participants included teacher-related details (i.e., 53.33%), and 4 participants (i.e., 26.67%) included pedagogically-related details in their hypotheses on the post-assessment.

In the Students Learning group, none of the participants included any of the detail-oriented criteria in their responses on the pre-assessment (see Table 4). On the post-assessment, 3 participants in the Students Learning group (i.e., 21.42%) included student-related details, 7 participants (i.e., 50.00%) included teacher-related details, and 2 participants (i.e., 14.29%) included pedagogically-related details in their hypotheses on the post-assessment.

To analyze group differences on the development of Skill 3, I conducted a 2 x 2 ANOVA using group (Students Learning and Learning Goals) as the between-group factor, time as the within-group factor, and the Skill 3 Hypothesis score as the dependent variable. There was a significant main effect of time, $F(1, 27) = 33.35$, $p < .001$, with a difference between the pre-assessment means ($M = 0.07$, $SD = 0.26$) and post-assessment means ($M = 1.28$, $SD = 1.02$), partial $\eta^2 = .55$. The results also revealed no significant main effect of group, $F(1, 27) = 0.03$, $p = .88$. This indicates that there was no difference between groups on Skill 3 performance across both time points (pretest and posttest). Finally, there was no significant interaction between group and time, $F(1, 27) = .03$, $p = .87$, indicating that the instruction provided to both groups had the same effect on the development of Skill 3.

**Skill 4: Proposing alternatives.** To examine the effect of Skill 1 instruction on the development of Skill 4, I ran the same 2 x 2 ANOVA using the Skill 4 Alternative score (scores

Table 4

*Number of Participants Providing Detailed-Oriented Criteria on the Pre- and Post-Assessment of Skill 3 Hypothesis*

| Criteria | Learning Goals ($n$ = 15) | | Students Learning ($n$ = 14) | |
|---|---|---|---|---|
| | Pre-Assessment | Post-Assessment | Pre-Assessment | Post-Assessment |
| Student-related Details | 0 (0.00%) | 7 (46.67%) | 0 (0.00%) | 3 (21.42%) |
| Teacher-related Details | 1 (6.67%) | 8 (53.33%) | 0 (0.00%) | 7 (50.00%) |
| Pedagogically-related Details | 0 (0.00%) | 4 (26.67%) | 0 (0.00%) | 2 (14.29%) |

ranged from 0 to 3) as the dependent variable. The Skill 4 Alternative score indicates how many types of alternatives preservice teachers proposed in their response. Based on Hiebert et al.'s (2007) criteria for this skill, three types of alternatives were each awarded 1 point. In particular, the three alternatives justified changes to the lesson based on (a) observations from the video lesson (i.e., the effect of teaching on student learning or evidence of student learning), (b) pedagogical knowledge of teaching the topic, and (c) the importance of gaining access to students' thinking.

Scores close to 0 indicated that none of the three alternatives were proposed. Scores close to 1 indicated that one of the three alternatives was addressed in the response. Scores close to 2 indicated that the response addressed two of the three alternatives. Scores close to 3 indicated that all three alternatives were addressed in the response.

The number of participants proposing each type of alternative at pretest and at posttest is reported in Table 5. As it can be seen in the table across and within groups, the number of participants proposing alternatives based on observations from the video increased from pretest to posttest, as did the number of participants proposing alternatives based on pedagogical knowledge for teaching the equal sign. In particular, on the pre-assessment 1 participant in the Learning Goals group (i.e., 6.67%) based their alternatives on observations from the video, 5 participants (i.e., 33.33%) based their alternatives on pedagogical knowledge of teaching a lesson on the equal sign, and 2 participants (i.e., 13.33%) based their alternatives on gaining access to student thinking. On the post-assessment, 9 participants in the Learning Goals group (i.e., 60.00%) based their alternatives on observations from the video, 11 participants (i.e., 73.33%) based their alternatives on pedagogical knowledge of teaching a lesson on the equal sign, and 3 participants (i.e., 20.00%) proposed alternatives to gain greater access to student thinking.

For the Students Learning group, 4 participants (i.e., 28.57%) based their alternatives on observations from the video, 7 participants (i.e., 50.00%) based their alternatives on pedagogical knowledge of teaching a lesson on the equal sign, and 5 participants (i.e., 35.71%) based their alternatives on gaining access to student thinking at pretest (see Table 5). Following instruction, 9 participants in the Students Learning group (i.e., 64.23%) based their alternatives on observations from the video, 9 participants (i.e., 64.23%) based their alternatives on pedagogical knowledge of teaching the topic, and 2 participants (i.e., 14.29%) proposed alternatives to gain greater access to student thinking.

Table 5

*Number of Participants Who Proposed Each Type of Alternative on the Pre- and Post-Assessment of Skill 4 Alternatives*

| Type of Alternative | Learning Goals ($n$ = 15) | | Students Learning ($n$ = 14) | |
|---|---|---|---|---|
| | Pre-Assessment | Post-Assessment | Pre-Assessment | Post-Assessment |
| Observations from the Video | 1 (6.67%) | 9 (60.00%) | 4 (28.57%) | 9 (64.23%) |
| Pedagogical Knowledge on Teaching and Learning | 5 (33.33%) | 11 (73.33%) | 7 (50.00%) | 9 (64.23%) |
| Access to Student Thinking | 2 (13.33%) | 3 (20.00%) | 5 (35.71%) | 2 (14.29%) |

I conducted a 2 x 2 ANOVA using group (Students Learning and Learning Goals) as the between-group factor and time as the within-group factor. There was a significant main effect of time, $F(1, 27) = 8.31$, $p = .01$, with an increase from the pre- assessment ($M = 0.83$, $SD = 1.00$) to post-assessment ($M = 1.48$, $SD = 1.07$), partial $\eta^2 = .24$. The results also revealed no significant main effect of group, $F(1, 27) = .67$, $p = 42$. This indicates that there was no difference between groups on mean Skill 4 performance across both time points (pretest and posttest). Finally, there was no significant interaction between group and time, $F(1, 27) = 2.57$, $p = .12$, indicating that the instruction provided to both groups had the same effect on the development of Skill 4.

**Research Question 2: The Effect of Skill 1 Instruction on Skill 1**

Following my examination of the effect of Skill 1 instruction on the development of Skills 2, 3, and 4, I conducted an independent *t*-test using group (Students Learning and Learning Goals) as the between-group factor and Skill 1 Goals score as the dependent variable (range was from 2 to 21). It was hypothesized that, compared to the Students Learning group, the Learning Goals group would demonstrate a significantly higher score on Skill 1. On average, the Learning Goals group received a higher Skill 1 Goals score ($M = 13.47$, $SD = 2.61$) compared to the Students Learning group ($M = 12.14$, $SD = 2.68$), but this difference was not significant $t(27) = 1.35$, $p = .19$.

**Research Question 3: The Nature of Skill 1 with and without Skill 1 Instruction**

The interview data were coded and subsequently analyzed to qualitatively describe each group in terms of: (a) specifying and identifying learning goals (for the planning and observing context), (b) the types of tasks proposed when planning a lesson on equivalence, (c) the reasoning used to discern learning goals when observing teaching, and (d) quality of language. Recall that identifying learning goals was defined as discerning the learning goal, but not linking that goal to any of the tasks observed or proposed. In contrast, specifying learning goals was defined as discerning the learning goal and linking it to tasks. Compared to identifying learning goals, specifying learning goals indicates a more advanced understanding of Skill 1 (Hiebert et al., 2007) because linking the goal to a task shows that the preservice teacher provided details about the learning goal (i.e., how it will be addressed during the lesson).

For learning goals, tasks, and reasoning codes, I report the number of participants in each group included in the group profiles to look for patterns grounded in the data. For the analysis of language quality, the frequencies for each code served to describe group differences.

**Analysis of specifying and identifying learning goals: Planning context.** I conceptually grouped the learning goals the preservice teachers proposed in their lessons into seven superordinate categories (see Table 6). Across all categories, the Learning Goals group either identified or specified 10 different learning goals and the Students Learning group identified or specified 8.

***Learning goals proposed by both groups.*** As it can be seen in Table 6, participants in both groups either identified or specified the same learning goals included in these three superordinate categories: (a) knowledge about the equal sign, (b) student thinking, and (c) other mathematics subjects.

In the promoting knowledge about the equal sign category, three learning goals were identified or specified by participants in both groups: (a) understanding the meaning of the equal sign, (b) learning a procedure, and (c) developing relational thinking. Relative to the other learning goals in this category, the first learning goal, understanding the meaning of the equal sign, was either identified or specified by a large number of participants in both groups. Evidence for this is provided in Table 6: three participants from the Learning Goals group and all four participants in the Students Learning group identified or specified this goal. Compared to the Students Learning group, however, more participants in the Learning Goals group specified this learning goal, meaning they linked the goal to a specific task. In sum, results for this category indicate that both groups emphasized the same goal, but more preservice teachers in the Learning Goals group discussed the tasks that would be linked with it during the lesson.

In the student thinking category, participants in both groups specified the same learning goal. Similar to what was found in the previous category (i.e., knowledge about the equal sign), more participants in the Learning Goals group linked the goal to elicit students' thinking about the equal sign with specific tasks during the interview (see Table 6).

In the other mathematics topics category, the preservice teachers in both groups identified or specified two learning goals about mathematics subjects not directly related to the equal sign: understanding the commutative property and part-whole knowledge. The evidence in Table 6 suggests that participants in the Students Learning group were more skilled in discussing these

Table 6

*Number of Participants in Each Group who Identified and Specified Learning Goals in the Context*

| | Learning Goals ($n$ = 4) | | Students Learning ($n$ = 4) | |
|---|---|---|---|---|
| | Identified | Specified | Identified | Specified |
| **Category I: Knowledge About the Equal Sign** | | | | |
| Understand the Meaning of the Equal Sign | 1 | 2 | 3 | 1 |
| Learn the Procedure for Solving Equivalence Problems | 0 | 1 | 0 | 1 |
| Develop Relational Thinking | 1 | 1 | 1 | 0 |
| **Category II: Student Misconceptions** | | | | |
| Address the Misconception that the Answer Comes Next | 1 | 2 | 0 | 0 |
| Address the Misconception that Non-canonical Number Sentences are Backwards | 1 | 3 | 2 | 0 |
| **Category III: Student Thinking** | | | | |
| Reveal Prior Knowledge on the Meaning of the Equal Sign | 0 | 2 | 0 | 1 |

(continued)

| | | | | |
|---|---|---|---|---|
| **Category IV: Symbolic Knowledge** | | | | |
| The Equal Sign Serves a Role/Purpose/Function | 0 | 0 | 2 | 0 |
| **Category V: Skill-based Knowledge** | | | | |
| Learn to use Manipulatives to Justify Thinking OR use Manipulatives to Solve Equivalence Problems | 0 | 1 | 0 | 0 |
| **Category VI: Other Mathematics Subjects** | | | | |
| Understand the Commutative Property | 0 | 1 | 0 | 2 |
| Part-whole Knowledge | 0 | 1 | 1 | 1 |
| **Category VII: Other Learning Goals** | | | | |
| Solidify Knowledge about the Equal Sign | 0 | 3 | 0 | 0 |
| Unclear | 0 | 2 | 1 | 2 |

learning goals (i.e., specified more of these learning goals) compared to those in the Learning Goals group.

***Differences between groups in learning goals.*** The participant frequencies in Table 6 also highlight group differences in identifying and specifying learning goals. Looking at the skill-based knowledge category in Table 6, only those in the Learning Goals group specified goals to use manipulatives to support learning about the equal sign. Further, the goal to solidify key concepts in their lesson (see Other Learning Goals category in Table 6) was specified by three participants in this group and not proposed by any participants in the Students Learning group. On the other hand, the data in the symbolic knowledge category indicate that two participants in the Students Learning group identified that goal and none of the participants in the Learning Goals proposed it.

Preservice teachers in each group identified or specified learning goals to address student misconceptions about the equal sign, but more participants in the Learning Goals group identified or specified such goals compared to the Students Learning group (see Table 6). Specifically, three participants in the Learning Goals group proposed to address the misconception the "answer comes next misconception," two of which specified this goal. Further, all four participants in the Learning Goals group proposed to address children's belief that non-canonical number sentences are "backwards," three of which specified this goal. While two preservice teachers in the Students Learning group also proposed to address this common misconception (i.e., non-canonical sentences being backwards), they only identified this learning goal with no connections to tasks.

Together, the interview data from the planning context revealed several noteworthy patterns about the nature of Skill 1 for those who did and did not receive direct instruction on this skill. When asked to plan a lesson on the equal sign, participants from the Learning Goals group proposed more learning goals compared to those in the Students Learning group. The data presented in Table 6 also revealed that participants from the Learning Goals group more often specified learning goals compared to participants in the Students Learning group. This means that they more often linked their learning goals to tasks, indicating that they included details about learning goals in their discussion of their lessons on the equal sign. Finally, more participants in the Learning Goals group provided details about learning goals relevant to teaching the equal sign (Carpenter, Franke, & Levi, 2003), namely eliciting students' thinking about the equal sign, understanding its meaning, solidifying knowledge about the equal sign, and addressing misconceptions related to the topic.

**Analysis of specifying and identifying learning goals: Observing context**

Similar to the planning context, I grouped the learning goals discussed in the observing context into superordinate categories. First, I conceptually grouped actual learning goals in the video (Carpenter et al., 2003) into four superordinate categories: (a) knowledge about the equal sign, (b) student misconceptions, (c) student thinking, and (d) other learning goals (see Table 7). Across these four categories, the Learning Goals group identified or specified seven of the teacher's learning goals and the Students Learning group identified or specified five.

Preservice teachers in both groups identified or specified learning goals that were not actual learning goals of the video. I grouped these perceived learning goals into four superordinate categories: (a) knowledge about the equal sign, (b) symbolic knowledge, (c) other mathematics subjects, and (d) other learning goals. Across the first three categories, preservice teachers in the Learning Goals group discussed three learning goals, and those in the Students Learning group discussed two.

***Video learning goals observed by both groups.*** In the knowledge about the equal sign category, the preservice teachers identified or specified two learning goals: (a) understanding the meaning of the equal sign, and (b) learning a procedure to solve equivalence problems. Looking at this category in Table 7, all participants from each group identified or specified this first goal, but more participants in the Students Learning group discussed their observations of this goal by linking it to tasks in the video lesson. In this context, then, more participants in the Students Learning group were skilled (i.e., specified this learning goal) in discussing their observations of the teacher's goal for students to understand the meaning of the equal sign.

Preservice teachers in both groups identified or specified two learning goals addressing student misconceptions about the equal sign. As it can be seen in Table 7, more participants in both groups identified or specified the teacher's goal to address misconceptions about non-canonical equations in her lesson. Another interesting finding in this category was that the same number of participants in both groups linked both learning goals with tasks in the video lesson. These findings suggest that participants in both groups were equally skilled in discussing the teacher's goals to address student misconceptions about the equal sign, even though more

Table 7

*Number of Participants in Each Group who Identified and Specified Learning Goals in the Observing Context*

| | Learning Goals ($n$ = 4) | | Students Learning ($n$ = 4) | |
|---|---|---|---|---|
| | Identified | Specified | Identified | Specified |
| **Video Learning Goals** | | | | |
| **Category I: Knowledge About the Equal Sign** | | | | |
| Understand the Meaning of the Equal Sign | 2 | 2 | 1 | 3 |
| Learn the Procedure for Solving Equivalence Problems | 1 | 1 | 1 | 2 |
| **Category II: Student Misconceptions** | | | | |
| Address the Misconception that the Answer Comes Next | 0 | 1 | 0 | 1 |
| Address the Misconception that Non-canonical Number Sentences are Backwards | 2 | 1 | 1 | 1 |
| **Category III: Student Thinking** | | | | |
| Reveal prior Knowledge on the Meaning of the Equal Sign | 1 | 0 | 1 | 1 |
| Reflect on the Meaning of the Equal Sign | 0 | 1 | 0 | 0 |
| **Category IV: Other Learning Goals** | | | | |

(continued)

| | | | | |
|---|---|---|---|---|
| Solidify Knowledge about the Equal Sign | 0 | 2 | 0 | 0 |
| **Perceived Learning Goals** | | | | |
| **Category I: Knowledge About the Equal Sign** | | | | |
| Develop Relational Thinking | 1 | 0 | 0 | 0 |
| **Category II: Symbolic Knowledge** | | | | |
| The Equal sign Serves a Role/Purpose/Function | 0 | 1 | 0 | 0 |
| **Category III: Other Mathematics Subjects** | | | | |
| Part-whole Knowledge | 0 | 1 | 0 | 0 |
| Understand the Commutative Property | 0 | 0 | 0 | 1 |
| Understand the Concept of 0 | 0 | 0 | 0 | 3 |
| **Category IV: Other Learning Goals** | | | | |
| Unclear | 0 | 1 | 0 | 0 |

participants in the Learning Goals group identified or specified the teacher's goal in addressing misconceptions about non-canonical equations.

Finally, participants in both groups identified or specified one of the learning goals in the student thinking category, eliciting students' thinking to reveal their understanding of the meaning of the equal sign. The data in Table 7 show more participants in the Students Learning attended to and specified this goal compared to those in the Learning Goals group.

***Differences between groups in video learning goals.*** Preservice teachers in the Learning Goals group identified or specified two learning goals not addressed by those in the Students Learning group. One goal, reflecting on the meaning of the equal sign, was in the student thinking category and the second, solidify knowledge about the meaning of the equal sign, was in the other learning goals category. As it can be seen in Table 7, participants discussed both learning goals with links to tasks from the video lesson.

***Differences between groups in perceived learning goals.*** Participants from both groups perceived different learning goals in the video. Only those in the Learning Goals group believed that the teacher included goals to develop (a) students' relational thinking, (b) part-whole knowledge, and (c) symbolic knowledge about the equal sign in her lesson (see Table 7). One participant in the group identified or specified these three learning goals, however. Preservice teachers in the Students Learning group believed that the teacher in the video included learning goals not directly related to the topic of the equal sign (i.e., the commutative property and the concept of 0). Compared to the Learning Goals group's discussion of developing part-whole knowledge, the data in Table 7 show that more participants in the Students Learning group linked learning goals that were not directly related to learning about the equal sign (learning the commutative property and the concept of 0) with tasks from the video lesson.

**Comparing the analysis of learning goals from both contexts.** Together, the results for identifying and specifying learning goals in the observing context differed compared to the planning context. Compared to the planning context, the Students Learning group specified more learning goals in their discussions of learning goals observed in the video lesson. Looking at Tables 6 and 7, for instance, the goal to understand the meaning of the equal sign was discussed by all participants in the Student Learning group in both contexts. In the planning context, however, only one participant in the group specified this goal, whereas three participants specified this goal in the observing context. It should be noted that this difference in specifying learning goals in the observing context addressed learning goals directly and not directly related to learning about the equal sign (e.g., concept of 0). Nevertheless, the greater number of specified learning goals in the observing context shows that the Students Learning group demonstrated greater Skill 1 abilities when learning goals were visible (i.e., observable in the video).

Preservice teachers in the Learning Goals group demonstrated similar Skill 1 abilities in both contexts. For example, for one of the learning goals, understand the meaning of the equal sign, the same number of participants in this group specified this goal in the planning and observing context. Also, the majority of participants in this group discussed goals to address misconceptions about the equal sign in both contexts. At the same time, however, these preservice teachers demonstrated greater Skill 1 abilities (i.e., higher frequency of participants specifying versus identifying goals in this category) when discussing goals related to student misconceptions in the planning context compared to the observing context (see Tables 6 and 7). Taken together, the results of my analysis across both contexts suggest that those who received instruction on Skill 1 used a similar strategy for specifying and identifying learning goals. On the contrary, the preservice teachers who did not receive Skill 1 instruction were skilled in specifying and identifying learning goals in a context where learning goals were more "visible" (i.e., observed in the video).

**Analysis of tasks proposed to teach a lesson on the equal sign.** In addition to learning goals, I asked the preservice teachers to propose tasks that would be included in their lesson on the equal sign. The majority of tasks that were proposed were number sentences. I analyzed the number of participants in each group who proposed: (a) canonical number sentences, (b) non-canonical number sentences included in the video, and (c) non-canonical number sentences that were not included in the video lesson. The results of my analysis of tasks proposed during the planning context are presented in Table 8.

Although canonical number sentences are not particularly effective in a lesson about the equal sign (Seo & Ginsburg, 2003), participants in both groups proposed to include these number sentences. Compared to those in the Learning Goals group, however, more preservice teachers in the Students Learning group proposed this task (see Table 8). Further, the task analysis indicated that preservice teachers in both groups integrated previous observations of the video in their lesson proposals. As it can be seen in Table 8, all four participants in each group proposed

Table 8

*Types of Tasks Proposed by Participants in Both Groups*

| | Learning Goals ($n$ = 4) | Students Learning ($n$ = 4) |
|---|---|---|
| Canonical Number Sentence | 1 | 2 |
| Non-canonical Number Sentence Observed in Video | 4 | 4 |
| Non-canonical Number Sentence | 2 | 3 |
| **Other Type of Tasks** | | |
| Using Manipulatives | 2 | 1 |
| Generate Definition | 1 | 0 |

non-canonical number sentences similar to those used in the video lesson. Table 8 also illustrates that non-canonical tasks not presented in the video lesson were also proposed by at least half of the preservice teachers from each group.

Preservice teachers in both groups also proposed tasks that were not focused on number sentences. The data in Table 8 show that a small number of participants in each group proposed these types of tasks. Nevertheless, the presence of these tasks in their lessons indicates that some preservice teachers in each group reflected on the teaching and learning about the equal sign in ways that were not observed in the video.

Taken together, these results suggest similar patterns for both groups for proposing tasks. Specifically, only a small number of preservice teachers in each group included tasks in their lessons that were not included in the video lesson. The majority of participants in both groups integrated the tasks from the video lesson.

**Analysis of how participants reasoned to identify learning goals.** Four reasoning about learning goal codes emerged from the data: (a) equations, (b) teacher behaviors, (c) student responses, and (d) task sequence (see Table 9). In both groups, all four preservice teachers reported that they used the equations and the teacher's behaviors to reason about the learning goals of the video lesson. As it can be seen in Table 9, the majority of preservice teachers in the Students Learning group also reporting relying on their observations of student responses in the video. Taken together, these results suggest that those who received Skill 1 instruction (i.e., the Learning Goals group) primarily relied on teacher behaviors to discern the learning goals of the lesson, whereas the preservice teachers who did not receive Skill 1 instruction attended to teacher and student behaviors. Because the Learning Goals group received instruction that emphasized unpacking elements of the lesson to identify learning goals, it follows that the preservice teachers in that group would focus on teacher behaviors and pay less attention to student responses.

**Analysis of the quality of language used during the interview.** To analyze the quality of language used during the interview, I began by conceptually grouping language codes into four superordinate categories: (a) description of the meaning of the equal sign, (b) description of children's misconceptions, (c) terminology, and (d) description of the lesson components. Following that, I calculated the frequencies for all codes across these categories. Table 10 reports the frequencies for the codes in every category.

Table 9

*Reasoning about Learning Goals Codes Observed During the Interview*

| | Learning Goals ($n = 4$) | Students Learning ($n = 4$) |
|---|---|---|
| Equations | 4 | 4 |
| Teacher Behaviors | 4 | 4 |
| Student Responses | 1 | 3 |
| Task Sequence | 1 | 1 |

Table 10

*Frequencies for Quality of Language Codes*

| | Learning Goals ($n = 4$) | Students Learning ($n = 4$) |
|---|---|---|
| Code | Frequency | Frequency |
| **Category I: Descriptions of the Meaning of the Equal Sign** | | |
| Accurate and Complete | 2 | 0 |
| Accurate and Incomplete | 9 | 7 |
| Inaccurate | 0 | 0 |
| Total | 11 | 7 |
| **Category II: Descriptions of Children's Misconceptions** | | |
| Misconception: Add all Numbers | 1 | 0 |
| Misconception: Non-canonical (Backwards) | 2 | 4 |
| Misconception: Related to "0" | 1 | 1 |
| Misconception: The Answer Comes Next | 7 | 1 |
| Sources for Misunderstanding the Equal Sign | 1 | 0 |
| Total | 12 | 6 |

(continued)

| | | |
|---|---|---|
| **Category III: Terminology** | | |
| Use of Equivalence Term | 7 | 24 |
| Prompted use of Equivalence Term | 1 | 1 |
| Use of Mathematical Term | 6 | 21 |
| Prompted use of Mathematical Term | 5 | 1 |
| Total | 19 | 47 |
| **Category IV: Description of Lesson Components** | | |
| Elaborates and Justifies Learning Goals | 14 | 18 |
| Elaborates on and Justifies Task | 40 | 31 |
| Describes, Elaborates, and Justifies Task Sequence | 10 | 26 |
| Total | 64 | 75 |

The data for the first category in Table 10 indicate that participants in the Learning Goals group more often described the meaning of the equal sign during the interview compared to those in the Students Learning group (frequency of 11 and 7, respectively). Added to this, only participants in the Learning Goals group accurately described its meaning (see Table 10). The most frequent description of the equal sign provided by participants in both groups was accurate, but incomplete.

Preservice teachers in both groups described common misconceptions students have about the equal sign and about numbers (i.e., the number 0) during the interview, but descriptions about these misconceptions were more often observed in the Learning Goals group compared to the Students Learning group (see Table 10). The most frequent misconception observed in each group was related to the equal sign. As it can be seen in Table 10, the most frequent misconception described by participants in the Learning Goals group was the misconception that the meaning of the equal sign is "the answer comes next." The Students Learning group most frequently described the misconception concerned with non-canonical number sentences.

Regarding the terminology category, the data in Table 10 also indicate relatively high frequencies of accurate use of technical terms related to the equal sign and mathematics principles, particularly for those in the Students Learning group (i.e., total frequency was 47). That is, compared to the Learning Goals group, those in the Students Learning group more frequently used technical terms related to the equal sign (i.e., frequency was 24 compared to 7) and mathematics principles (i.e., frequency was 21 compared to 6). Further, the Students Learning group's frequency of terminology prompted by the interview was very low, and at times less frequent (i.e., mathematical terms) compared to the Learning Goals group (see Table 10).

The descriptions of lesson components category included the highest frequencies compared to the other three categories (see Table 10). Evidence in Table 10 indicates that preservice teachers in both groups frequently described the tasks during the interview. Group differences, however, emerged on the other two lesson components, namely learning goals and task sequence. That is, the Students Learning group described learning goals as often as they described task sequences. On the other hand, the Learning Goals group described learning goals more often than task sequence (see Table 10).

# CHAPTER 5: DISCUSSION

The notion that it is important to attend to and interpret student thinking in the classroom is not new (Dewey, 1904; Erikson, 2011). These teaching practices, however, have a renewed value in the context of educational reform, which requires teachers to become experts in "listening" and adapting their teaching to students' learning. The nature of this approach to teaching is highly complex because it involves attending to, eliciting, and reasoning about student thinking *while* teaching. These teaching practices are supported by a set of pedagogical skills collectively referred to as *teacher noticing*. Van Es (2011) explained that teacher noticing involves (a) identifying what is important or noteworthy during a lesson, (b) reasoning and interpreting about what is identified, and (c) making informed decisions on the basis of what was observed. The development of these skills has been linked with improvements in analyzing student thinking, making connections between this analysis and the broader principles of teaching and learning, and implementing teaching practices that centralize student thinking (Sherin & Han, 2004; van Es, 2011).

While the development of teacher noticing skills involves reflecting on ones' teaching, its practice does not necessarily support a teacher's ability to learn from their teaching (Davis, 2006). Indeed, while reflection allows a teacher to reason about and respond to events that arise in the classroom, being skilled in learning from teaching provides a framework for improving practice (Mason, 2011). Some educational researchers argued that because reflection on teaching and learning from teaching are central to practices that align with educational reform, both should play a visible role in teacher education programs (Hiebert, Morris, Berk, & Jansen, 2007; Yeh & Santagata, 2015). What is less obvious is how to develop these necessary skills in teacher education.

To address this issue, Hiebert, Morris, Berk, and Jansen (2007) proposed a framework that integrates four skills that blend reflection on teaching and learning from teaching. The first skill, specifying the learning goals, involves using subject matter knowledge of a topic to decompose a lesson into learning goals (i.e., primary goal and sub-goals). This skill is used during the planning phase of a lesson and involves considering in detail what students need to know and understand to achieve the primary goal of the lesson. The second skill is applied during the lesson and involves collecting evidence of students' learning. More specifically, this skill requires an (a) understanding that evidence of student learning can be used to evaluate the effectiveness of the lesson, and an ability to (b) identify relevant evidence of student learning and ignoring irrelevant evidence and (c) anticipate moments during the lesson when evidence could be collected. Forming cause-effect hypothesis statements, the third skill, to link evidence of student learning with observations of teaching, is also carried out during the lesson. Pedagogical knowledge is used for this skill as well as for the fourth skill, proposing teaching alternatives (Spitzer, Phelps, Beyer, & Johnson, & Sieminski, 2011). This skill involves making decisions about future lessons based on knowledge about teaching the topic in addition to observations of the lesson. The development of proposing alternatives is somewhat challenging in teacher education. Developing expertise in this skill is contingent on opportunities to test the modifications in the classroom (Hiebert et al., 2007), and these opportunities are not always available for preservice teachers.

The research on developing these skills is thinly developed, and thus little is known regarding its role in teacher education programs (Santagata & Angelici, 2010; Santagata, Zannoni, & Stigler, 2007; Spitzer et al., 2011; Yeh & Santagata, 2015). What little research does exist has focused on the development of a select few of the four skills (e.g., Spitzer et al., 2011; Santagata & Angelici, 2010; Yeh & Santagata, 2015). The focus on developing certain skills in Hiebert et al.'s (2007) framework in absence of the others would only partially equip preservice teachers to learn from their teaching. Further, this also assumes that the development of all four skills does not follow any particular sequence (Spitzer et al., 2011). This assumption is questionable, however, because experts in the field have proposed that the specification of learning goals serves as a starting point for analyzing teaching (Morris, Hiebert, & Spitzer, 2009). In addition, according to Hiebert et al. (2007), each skill is theoretically linked to teaching activities that are deployed in a specific order: prior to, during, and following a lesson. That is, specifying learning goals (Skill 1) supports lesson planning, collecting evidence of student learning (Skill 2) and forming hypothesis (Skill 3) are used during the lesson (i.e., implementation), and proposing teaching alternatives (Skill 4) is used to reflect on the lesson. Because in practice Skill 1 is applied before Skills 2, 3, and 4, it may benefit preservice teachers to receive professional development that follows this sequence to approximate teaching practice (Grossman et al., 2009).

This study was designed to examine the development of learning from teaching skills in the context of an elementary mathematics methods course. I focused on understanding the effect of direct instruction on specifying learning goals (Skill 1) on the development of the other three skills because the role of this skill is not well understood. Compared to collecting evidence revealing of student learning (Skill 2; Morris, 2006; Spitzer et al., 2011), interpreting the effect teaching on student learning (Skill 3; Morris, 2006; Yeh & Santagata, 2015), and using this analysis to revise future instruction (Skill 4; Morris, 2006; Santagata & Angelici, 2010; Santagata & Guarino, 2011), specification of learning goals has received less attention in the literature (Morris et al., 2009). Because of this, I also examined the nature of specifying learning goals (Skill 1) following instruction that did and did not address this skill.

I used a two-group pretest-posttest experimental design to compare the effect of two conditions (Students Learning and Learning Goals) on the development of Skills 2, 3 and 4 in Hiebert et al.'s (2007) Learning from Teaching model. Both conditions received classroom instruction on Skills 2, 3, and 4, but only the Learning Goals condition received instruction on how to specify learning goals. Four topics related to the development of children's algebraic reasoning were used to teach (justifying conjectures), practice (relational thinking and eliciting conjectures), and assess the development (the meaning of the equal sign) of all four skills. In line with previous research on promoting teacher noticing (Jacobs et al., 2010; Sherin & Han, 2004; Sherin & van Es, 2009; Star & Strickland, 2008), reflection on teaching (Stockero, 2008), and learning from teaching (e.g., Santagata & Angelici, 2010; Santagata & Guarino, 201; Santagata et al., 2007; Yeh & Santagata, 2013), I supplemented the instruction with video analysis and framework development activities (Jacobs et al., 2010). Skill development was assessed prior to (Skills 2, 3, and 4) and following instruction (Skills 1, 2, 3, and 4). Following the post-assessment, a subsample of preservice teachers from each group participated in an interview designed to examine specifying learning goal abilities (Skill 1) in more depth. I will begin with a discussion on the development of Skills 2, 3 and 4.

Contrary to predictions, the results showed that learning how to identify and specify learning goals (Skill 1) did not support the development of Skills 2, 3, and 4. Thus, in line with Spitzer et al. (2011), the development of learning from teaching may not be contingent on Skill 1 instruction. Although Hiebert et al. (2007) proposed that Skill 1 is tied to teaching activities that precede Skills 2, 3, and 4, the results from this study suggest that the order in which to introduce the skills during teacher preparation may not follow the order in which they play out in the classroom. My observations on the nature of Skill 1 after instruction discussed later on may serve to explain the lack of group difference in the development of Skills 2, 3, and 4.

Although the results did not demonstrate group differences following instruction, the pre- and post-assessment results pertaining to Skills 2, 3, and 4 may be of interest for teacher educators. The pre-assessment of Skill 2 revealed two things about the preservice teachers' ability to collect evidence of student learning. First, prior to instruction, the preservice teachers in both groups attended to student behaviors that do not support the analysis of student learning. Second, the preservice teachers were not skilled in collecting evidence revealing of student learning. Together, the pretest results show a greater tendency to attribute final answers as evidence of learning than student justifications, explanations, and key words (e.g., "it's backwards). This finding is consistent with previous research demonstrating preservice teachers' difficulty understanding what constitutes evidence of student learning (e.g., Yeh & Santagata, 2015). Although the results of this study cannot confirm this, it is also possible that the preservice teachers focused more on the teaching strategies used during the lesson (Morris, 2006; Santagata et al., 2007), limiting their opportunities to focus on the students in the video and collect evidence revealing of their learning.

Following the instruction, the preservice teachers were more focused on student behaviors that support the analysis of student learning (i.e., reveal information about students' learning). At the same time, there was no significant change in their attention to student responses that are less informative of student learning. The preservice teachers' improvement in collecting evidence revealing of student learning (Skill 2) following instruction is consistent with previous research examining the use of video analysis in teacher education (Star & Strickland, 2008; Santagata & Angelici, 2010; Santagata et al., 2007). That is, although the results from Santagata and colleagues (2007; 2010) and Star and Strickland have not demonstrated that guided video analysis improves preservice teachers' focus on student learning (Santagata et al., 2007), they support claims that such activities improve preservice teachers' observational skills (Star & Strickland, 2008), reflections on teaching, and use of evidence to evaluate teaching (Santagata & Angelici, 2010; Santagata et al., 2007). The design of my study cannot confirm the link between skill development and the analysis of video cases but, it is nevertheless possible that these activities improved the preservice teachers' observation (Star & Strickland, 2008) and analytic skills (Santagata & Angelici, 2010; Santagata et al., 2007), which in turn helped focus preservice teachers' attention to behaviors revealing of students' learning.

Together, these results extend previous research examining the development of preservice teachers' skills in collecting evidence revealing of student learning. Spitzer et al.'s (2011) study on the development of Skill 2 demonstrated that the preservice teachers at the beginning of their teacher training were not adept in evaluating what constitutes evidence of student learning. My study shows that this skill was not well developed for preservice teachers nearing the end of their teacher training. In addition, the results from my study and from Spitzer et al. support the notion that collecting evidence of student learning (Skill 2) is a learned skill, and that a variety of instructional activities (e.g., card sort task, guided video analysis) may support its development.

Constructing hypotheses, the third skill, to link teaching with student learning involves attending to behaviors from both teachers and students, *and* making meaningful connections between them. Moreover, expertise in this skill involves elaborating on the cause effect statement using observations from the lesson and pedagogical knowledge related to teaching and student learning. The pre-assessment results for Skill 3 demonstrated that the preservice teachers' initial hypothesis statements offered no analysis of how the teaching observed in the video impacted student learning. Following the instruction, the preservice teachers' ability to construct hypotheses statements in line with some of Hiebert et al.'s (2007) criteria improved. That is, the responses included statements that reflected their ability to attend to noteworthy teacher and student behaviors and an understanding of how to make connections between them.

Consistent with recent research on this skill (Yeh & Santagata, 2015), the preservice teachers learned to generate hypotheses following instruction on learning from teaching. Results from Yeh and Santagata's (2015) study also showed that some improvement in hypothesis construction is possible in absence of direct instruction on learning from teaching skills. That is, in their study, a group of preservice teachers completed a mathematics methods course that did not include instruction and activities targeting learning from teaching skills. The authors observed that they generated more hypotheses following the course, but their gains in hypothesis generation were minimal and the quality of the hypotheses did not improve by the end of the course. Yeh and Santagata concluded that the lack of improvement in hypothesis quality in particular was influenced by their difficulty reasoning about the instruction they observed and recognizing what constitutes evidence of student learning. Based on this result, the ability to collect evidence revealing of student learning (Skill 2) may serve as a prerequisite for developing skills in hypothesis construction (Skill 3). Moreover, because of the positive effects of guided video analysis on preservice teachers' ability to reason and evaluate teaching found in the literature (e.g., Santagata & Guarino, 2011), it is possible that this type of activity plays an important role in the development of this skill. With respect to my study then, I speculate that the preservice teachers' improvement in collecting evidence of student learning (Skill 2) in combination with the video analysis I included during instruction contributed to the preservice teachers' abilities to form hypotheses following instruction.

My study also assessed preservice teachers' skills in revising the lesson observed in the video (Skill 4). This particular skill is said to complete the learning from teaching cycle by connecting the analysis of student learning (Skill 2) and interpretations of the effect of teaching on student learning (Skill 3) with future lessons (i.e., specifying learning goals, or Skill 1; Yeh & Santagata, 2015). Prior to instruction, preservice teachers in the Learning Goals group were not skilled in proposing alternatives to improve the lesson observed in the video. Specifically, the results on the pre-assessment of Skill 4 indicated that this group failed to propose alternatives (a) grounded in evidence from the video, (b) that used pedagogical knowledge of teaching the equal sign, and (c) that elicited student thinking (Hiebert et al., 2007). Those in the Students Learning group were more skilled at proposing alternatives on the pre-assessment and most often used pedagogical knowledge of teaching the equal sign to justify how they would improve the lesson. On the post-assessment of Skill 4, the preservice teachers' ability to propose alternatives improved substantially. In particular, the majority of preservice teachers in both groups (i.e., more than half) proposed alternatives using observations from the video and pedagogical knowledge of teaching this topic.

Improvements in proposing alternatives during a teacher's training is possible when preservice teachers learn to reason about teaching strategies while analyzing teaching (Santagata & Angelici, 2010). Santagata and Angelici (2010) explained that being prompted (i.e., asked to respond to specific questions during video analysis) to reason about teaching in terms of how it impacts students' learning focuses the observer's attention to the teaching strategies used in the lesson. Even in absence of actual teaching experience, this practice of evaluating teaching strategies leads to the generation of alternative teaching strategies (Santagata & Angelici, 2010).

In line with this, the video analysis and framework development activity I included in the instruction may have played a role in the development of this particular skill. Each group designed an observation framework to help them record relevant information as they analyzed videos. In particular, sections of the framework guided the preservice teachers to record important teaching behaviors (e.g., tasks, questions, key words), student responses, and form connections between them. These framework sections are similar to the prompts used in Santagata and Angelici (2010) in that they direct the preservice teachers' attention to key aspects of the lesson to reflect on the effectiveness of the teaching. It is possible that similar to the preservice teachers in Santagata and Angelici, the preservice teachers in my study improved their ability to propose alternatives following the instruction because the activities during instruction supported their reflection of the teaching strategies observed in the video.

The results also indicated that the preservice teachers who did not receive direct instruction on specifying learning goals (Students Learning group) demonstrated Skill 1 abilities similar to those that did receive instruction on this skill (Learning Goals group). Two reasons may explain this result. First, it is possible that the preservice teachers in the Learning Goals group required deeper subject matter knowledge about the equal sign to benefit from the instruction on Skill 1. Subject matter competence is likely to impact Skill 1 because unpacking learning goals requires a profound understanding of the subject (Hiebert et al., 2007; Ma, 1999). In the present study, Skill 1 was introduced with learning to efficiently justify conjectures about properties (i.e., $a + b - b = a$), and practiced with developing relational thinking and eliciting conjectures about fundamental properties (i.e., $a + b = a + b$). Because minimal amount of subject matter knowledge about the equal sign was provided to the preservice teachers in both groups, the Learning Goals group's Skill 1 abilities were possibly constrained on the post-assessment. In line with this reasoning, this result may imply that skills dependent on subject matter knowledge (i.e., specify learning goals) may be context dependent (Perkins & Salomon, 1989) and skills more closely tied to pedagogical content knowledge (i.e., Skills 3 and 4; Hiebert et al., 2007; Spitzer et al., 2011) may transfer more easily from one context to another.

An alternative interpretation of this result would be to assume that the preservice teachers in the Students Learning group were able to identify the learning goals of the lesson without applying Skill 1. This assumption is based on evidence that preservice teachers without Skill 1 training can discern learning goals in situations Morris et al. (2009) refer to as "supportive contexts." Morris et al. explained that in supportive contexts it is not necessary to examine elements of the lesson to identify learning goals, and thus the application of Skill 1 to identify learning goals can be avoided. That is, the key mathematics concepts of the lesson can be discerned from other sources of information, such as students' responses. Because of this, supportive contexts elicit a strategy whereby subject matter knowledge is used to interpret student responses; learning goals, then, are determined based on interpretations of students' responses, not the lesson itself. This strategy for identifying learning is problematic because it requires information (e.g., the students' responses to tasks) that is not available when a teacher plans a lesson. Moreover, Morris et al. showed that application of this strategy is limited to certain teaching situations. Situations referred to as "nonsupportive contexts" (Morris et al., 2009) are contexts where the mathematics concepts are not easily discerned from sources other than the elements of the lesson (e.g., tasks). Identifying learning goals in this context is more challenging for preservice teachers who have not received instruction on specifying learning goals (Morris et al., 2009).

Consistent with Morris et al. (2009), it would have been possible for the preservice teachers in the Students Learning group to identify learning goals of the video in absence of Skill 1 training had the video included "supportive" information. The video I used during the assessment phases could be considered a supportive context for identifying learning goals because it is possible to discern the mathematics concepts of the lesson based on the students' responses. Although this cannot be confirmed, group differences may have emerged with a different video (nonsupportive context) or if the assessment involved planning a lesson on the equal sign (i.e., no video analysis). The results from the interview that support this notion are discussed in more detail below.

The study investigated the nature of specifying learning goals based on the absence or presence of Skill 1 instruction. I interviewed a subsample of the participants to capture the level of detail used to discuss learning goals and the degree to which mathematics language is used to describe the subject matter of the learning goals (Hiebert et al., 2007). Both criteria were assessed during the interview using two teaching contexts, *planning* a lesson on the equal sign and *observing* a lesson the equal sign. The interview data revealed qualitative differences in the nature of this skill that were not captured by the quantitative analyses.

First, because the Learning Goals group received explicit training on this skill, they were able to identify and specify learning goals in both contexts: *planning* and *observing* a lesson on the equal sign. Across both contexts, participants in the Learning Goals group specified a larger variety of learning goals, and furthermore, the Learning Goals group was more skilled at including details about learning goals in the planning context compared to the observing context. My predictions of the qualitative differences in the nature of this skill was borne out, but only in the planning context. That is, the Students Learning group's abilities to specify learning goals resembled those demonstrated by the Learning Goals group when they observed the lesson in the video.

This finding regarding the nature of Skill 1 for those who did not explicitly learn to specify learning goals (Students Learning group) is consistent with the research that has examined preservice teachers' Skill 1 abilities (Morris et al., 2009). As mentioned earlier, Morris et al. (2009) showed that preservice teachers without Skill 1 training are capable of identifying learning goals in supportive contexts. The observing context in the interview included the same video used on the pre- and post-assessment and so it was possible for the preservice teachers to identify the learning goals without analyzing the elements of the lesson. This strategy, however, would not support learning goal identification in the planning context. The planning context required participants to propose a primary learning goal and unpack the lesson to identify the subgoals; student responses could not be used to identify learning goals because they were not referencing an actual lesson. The Students Learning group's performance in the observing context (i.e., supportive context) and difficulty in the planning context (i.e., nonsupportive context) may indicate that they identified learning goals based on their interpretations of students' responses, rather than the application of Skill 1.

Evidence from the interview and post-assessment support this speculation. The preservice teachers across both groups demonstrated significant improvement on collecting evidence revealing of student learning (Skill 2) following instruction. Developments in this skill indicate that the preservice teachers (a) noticed more student behaviors and (b) analyzed each of those behaviors to determine whether they constituted evidence of learning. Improvements in Skill 2, then, may suggest that the preservice teachers were receptive to noticing the student behaviors in the video and capable of adequately interpreting these behaviors.

While participants across both groups developed this skill, evidence from the interview indicated that the majority of preservice teachers in the Students Learning group used observations of student learning to discern the learning goals in the lesson. Contrary to this, one participant in the Learning Goals group relied on student responses when identifying the learning goals in the video. Based on this result, it may be assumed that those in the Students Learning group showed a greater tendency to focus on student behaviors when reflecting on the lesson. Although attention to student behaviors is necessary for collecting evidence of student learning (Skill 2), a strategy that emphasizes the role of student responses in discerning learning goals is limiting and it therefore less effective (Morris et al., 2009).

The evidence discussed so far suggests that participants in the Students Learning group showed a tendency to attend to the student responses in the video and an ability to analyze them. Knowledge about students' thinking about the equal sign is also needed to analyze the student responses to identify learning goals. Although I did review this information prior to the instruction, it is possible that participants' subject matter knowledge may have been enhanced as a result of the systematic analysis of mathematics lessons that took place during instruction (Turner & Rowland, 2011), although the design of the study cannot confirm it. The data on language use during the interview suggested that the preservice teachers possessed adequate subject matter knowledge about the equal sign. For example, in their discussions about learning goals and tasks, the preservice teachers provided accurate descriptions of children's thinking about the topic (i.e., misconceptions), and accurately incorporated technical terms related to the topic. Also, the interview data on the types of tasks proposed in their lessons on the equal sign may support this assumption. That is, all participants in both groups integrated the tasks from the video lesson in their own lesson on the equal sign and very few proposed tasks that were not presented in the lesson. This may indicate that previous viewings of the video may have contributed to a schema on lessons about the equal sign, influencing their views *and* knowledge of teaching this topic.

Teacher educators rely on a number of pedagogical approaches to support preservice teachers' understanding of the complex nature of teaching (e.g., Lampert et al., 2013). In the context of teacher training, these pedagogical practices are designed to prepare preservice teachers to adopt approaches to teaching that are adaptive to student thinking (van Es & Sherin, 2002) and "intellectually ambitious" (Lampert et al. 2013). To do so, however, requires instructional activities that involve learning *and* enacting pedagogical practice (Grossman & McDonald, 2008). Similar to Lampert et al.'s (2013) cycle of enactment and investigation of pedagogical practice, the instruction sessions in my study went beyond providing skill-based instruction and encouraged preservice teachers to observe and analyze student learning and teaching in systematic ways. That is, the framework development activity and guided-video analysis served to provide a context in which the preservice teachers could enact their skill-based knowledge.

The results from my study demonstrated that preservice teachers nearing the end of their teacher training lacked the skills necessary to analyze student learning and reason about the role of teaching in students' learning. Following an intervention that provided the preservice teachers with representations of practice (i.e., research-based videos), the preservice teachers developed skills in decomposing student learning and teaching strategies and began to approximate the practice of learning from teaching (Grossman et al., 2009).

Given the study's design (i.e.. no control group), it is not possible to draw firm conclusions that the instruction itself resulted in the changes observed following instruction. Moreover, both instructors had knowledge of the theoretical underpinnings of the study and therefore the possibility of instructor effects cannot be ignored. Nevertheless, the results from my study indicate that three of the learning from teaching skills (Skills 2, 3, and 4) do not develop naturally and are learned. As such, the results have practical value for teacher educators. The results on the development of Skills 2, 3 and 4 lend support for Hiebert et al.'s (2007) contention that teacher training programs that incorporate instruction on these skills could enhance preservice teachers' analysis of teaching and the ability to reflect on the effectiveness of their own practice later on. Although the results indicated that Skills 2, 3, and 4 did not develop to the level of expert performance, the results nevertheless shed light on the development of learning from teaching skills during teacher training. In particular, developing more than one skill in the context of a methods course may be necessary for other skills, namely Skills 3 and 4. It is possible that the preservice teachers' developments in collecting evidence of student learning (Skill 2) contributed to their improvement in hypothesis construction (Skill 3) following instruction (Yeh & Santagata, 2015). I also speculate that the preservice teachers' skill in proposing alternatives (Skill 4) was impacted by their ability to reason and reflect deeply on teaching strategies (Santagata & Angelici, 2010), and that this reasoning was supported by the practice of constructing hypotheses (Skill 3). In addition, although the design of the study cannot confirm it, using video to develop these skills may only be effective when paired with practical activities that support the decomposition of teaching (Grossman et al., 2009). In my study, I used a framework development activity to guide the preservice teachers' analysis of the videos, however more recent research (Yeh & Santagata, 2015) has used video analysis in conjunction with preservice teachers' fieldwork experience (e.g., videotaping the preservice teachers' lessons and using them for analysis and discussion in the methods course).

Moreover, the data from the interview and pre- and post-assessments indicate that preservice teachers do not necessarily need to be trained on specifying learning goals (Skill 1) to identify learning goals in a lesson. Without Skill 1 training, however, the ability to identify learning goals may be limited to certain teaching situations. Providing training on this skill may support skills in identifying learning goals across a broader range of teaching contexts (i.e., both supportive and nonsupportive). These results, however, should be interpreted with caution. The results were observed with a small sample. In addition, the preservice teachers' Skill 1 abilities were not assessed prior to instruction, and therefore I cannot be certain that the Learning Goals group's Skill 1 abilities improved from pretest to posttest.

Nevertheless, these results are promising and future research building on these results should be considered. Findings would be more robust if I included a control group and used multiple Skill 1 measures with a larger sample (i.e., supportive and nonsupportive contexts). Further, similar to Yeh and Santagata (2015), it would be beneficial to develop learning from teaching skills with a wider variety of activities that promote the decomposition and approximation of teaching (Grossman et al., 2009; Yeh & Santagata, 2015). The ultimate goal, however, would be to examine the effect of these skills on student learning.

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# Appendix A

## Description of Measures

| | Demographic Survey | Mathematics Content Knowledge for Teaching (LMTP, 2008; Rayner et al., 2010) | Analysis of Learning Assessment | Specifying Learning Goals Interview |
|---|---|---|---|---|
| Purpose | Cohort Description | Baseline assessment | Pre- and Post-assessment | Post-assessment |
| Description of Measure | • Age<br>• Gender<br>• Teaching experience<br>• Number of mathematics methods courses completed<br>• Number of post-secondary mathematics courses completed<br>• Year of entry in the program | • Numeration<br>• Place value<br>• Properties of arithmetic<br>• Single-digit division<br>• Algorithms<br>• Equal sign<br>• Counting | • True/False and multiple choice items (Skills 1 and 2; Star & Strickland, 2008)<br>• Open-ended questions (Skills 1, 3 and 4; Hiebert et al., 2007; Morris, 2006) | • Specify primary learning goal of a lesson on the equal sign<br>• Specify the tasks to be used in the lesson and the learning goals of these tasks<br>• Discuss and compare learning goals of two lessons on the equal sign<br>• Describe strategies for identifying learning goals of a lesson |

Appendix B

Demographic Survey

Student Number: ________________________________________

Concordia University
Department of Education
EDUC 388/4: Teaching Mathematics III

Participant Demographics

**Instructions**: Please fill in all the information as accurately as possible. Your information will remain confidential and will only be used for research purposes.

a. Circle your gender: Male/ Female

b. Age: ______

c. When did you begin the ECEE Specialization Program?

    Semester: Fall or Winter

    Year: _____________

d. If applicable, please list the university-level mathematics courses you have completed.

   Please indicate the course name and number (e.g., Course Name: EDUC; Course Number 388).

   a. Course Name: _______ Course Number:_______

   b. Course Name: _______ Course Number:_______

   c. Course Name: _______ Course Number:_______

   d. Course Name: _______ Course Number:_______

e. When did you complete EDUC 386 Teaching Mathematics I?

   Year: ___________

f. Have you completed EDUC 387 Teaching Mathematics II?

Circle: **Yes** or **No**

g. If you circled yes, please indicate when you completed Teaching Mathematics II.

Year: ________

h. Do you have any individual or classroom-based teaching experience including substitute teaching, teaching stages, tutoring, working as a classroom aide, etc?

Circle: **Yes** or **No**

i. If you circled yes, please describe in detail your teaching experienced below.

| **Type of Teaching Experience** | **Details of Responsibilities/Tasks** | **Approximate Duration (in months)** |
|---|---|---|
| | | |
| | | |
| | | |

Please feel free to ask for another sheet if you need more space.

**Thank- you!**

Appendix C

Mathematical Content Knowledge for Teaching Assessment

Teaching Mathematics III EDUC 388/4

STUDENT NAME:________________________________________________

Concordia University
Department of Education

Vanessa Rayner
January 9th 2014

**NOTE: ANSWER ALL QUESTIONS ON THIS TEST SHEET.**

**<u>Instructions</u>. Please follow the directions for answering each question. The questions will involve marking <u>whether a statement is True (YES, NO, or I'M NOT SURE)</u>. NOTE THAT THERE CAN BE MORE THAN ONE TRUE STATEMENT FOR EACH QUESTION. Your responses will not be graded.**

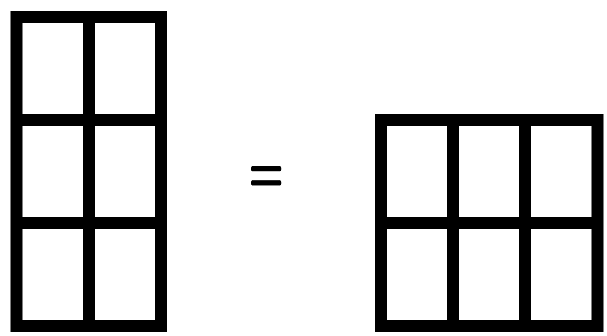
1. Below are five numbers. Indicate whether each of the five numbers is a <u>rational number</u>.

| **<u>Q1</u>** | | **Yes** | **No** | **I'm Not Sure** |
|---|---|---|---|---|
| **A)** | 32 | 1 | 2 | 3 |
| **B)** | $\sqrt{2}$ | 1 | 2 | 3 |
| **C)** | 1.45 | 1 | 2 | 3 |
| **D)** | $\frac{4}{2}$ | 1 | 2 | 3 |
| **E)** | π | 1 | 2 | 3 |

2. Below are 3 ways that $471_{10}$ has been renamed using Base-ten blocks. Indicate whether each of the following correctly represents $471_{10}$.

| **<u>Q2</u>** | **Yes** | **No** | **I'm Not Sure** |
|---|---|---|---|
| **A)** 4 *flats* + 3 *longs* + 41 *units* | 1 | 2 | 3 |
| **B)** 3 *flats* + 140 *longs* + 35 *units* | 1 | 2 | 3 |
| **C)** 3 *flats* + 150 *longs* + 10 *units* | 1 | 2 | 3 |

3. The rectangular array shown below is used to demonstrate a property of multiplication. Indicate whether each of the properties listed below matches the rectangular array shown.



| **Q3** | | **Yes** | **No** | **I'm Not Sure** |
|---|---|---|---|---|
| **A)** | The commutative property of multiplication | 1 | 2 | 3 |
| **B)** | The distributive property | 1 | 2 | 3 |
| **C)** | The associative property of multiplication | 1 | 2 | 3 |
| **D)** | The additive identity property | 1 | 2 | 3 |

4. Which of the following word problems matches the following question**:**

***"20 is how many groups of 5?"***

| **<u>Q4</u>** | | **Yes** | **No** | **I'm Not Sure** |
|---|---|---|---|---|
| **A)** | Sam has 20 candies. He wants to give 5 candies to his friends. How many friends can Sam give candy to? | 1 | 2 | 3 |
| **B)** | Sam has 20 candies. He wants to give all of his candy to 5 of his friends. How much candy will each friend get from Sam? | 1 | 2 | 3 |
| **C)** | Sam has 5 times as many candies as Jim. Jim has 20 candies. How many candies does Sam have? | 1 | 2 | 3 |
| **D)** | Sam has 20 candies. Jim has 5 candies. Sam has how many times more candies than Jim? | 1 | 2 | 3 |
| **E)** | Sam has 5 bags of candy. There are 20 candies in each bag. How many candies does Sam have altogether? | 1 | 2 | 3 |

5. You pose this problem to your 2nd Grade class,

*"What number would you put in the box to make this a true number sentence?"*

**5 + 9 = ☐ + 4**

The majority of the students say that the answer is 14. Based on this answer, which of the following statements is true?

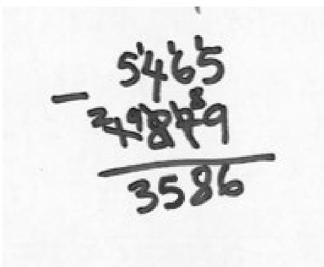
| **Q5** | | **Yes** | **No** | **I'm Not Sure** |
|---|---|---|---|---|
| **A)** | Students who answered 14 did not recognize the placement of the equal sign in the number sentence makes a difference. | 1 | 2 | 3 |
| **B)** | Students who answered 14 indicated an understanding that answers always follow the equal sign. | 1 | 2 | 3 |
| **C)** | Students who answered 14 indicated an understanding that the quantities on both sides of the equal sign must be the same. | 1 | 2 | 3 |

6. You ask your 3rd-Grade students to solve the following problem using any strategy they think makes sense:

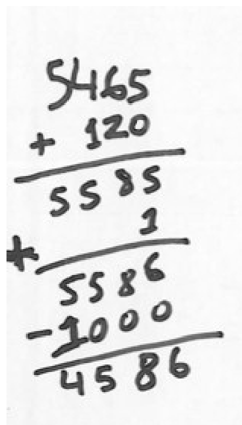
**5465 – 879 = ?**

Here are some of the answers that were given:

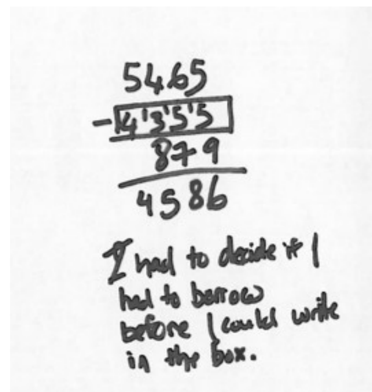
**Student A**




**Student B**




**Student C**




Which of the following statements about the students' strategies is true?

| Q6 | | Yes | No | I'm Not Sure |
|---|---|---|---|---|
| A) | Student A used an equal additions algorithm correctly. | 1 | 2 | 3 |
| B) | Student B used the fact that 5465 – 1000 + 121 = 5465 – 879. | 1 | 2 | 3 |
| C) | Student C's method would NOT work for all multidigit subtraction problems. | 1 | 2 | 3 |

7. Which of the following word problems matches the following question:

***"10 is 2 groups of what size?"***

| <u>**Q7**</u> | | **Yes** | **No** | **I'm Not Sure** |
|---|---|---|---|---|
| **A)** | Sam has 10 baseball cards. He wants to give 2 baseball cards to his friends. How many friends can Sam give baseball cards to? | 1 | 2 | 3 |
| **B)** | Sam has 2 times as many baseball cards as Jim. Jim has 10 baseball cards. How many baseball cards does Sam have? | 1 | 2 | 3 |
| **C)** | Sam has 2 boxes of baseball cards. There are 10 baseball cards in each box. How many baseball cards does Sam have altogether? | 1 | 2 | 3 |
| **D)** | Jim has 2 times as many baseball cards as Sam. Jim has 10 baseball cards. How many baseball cards does Sam have? | 1 | 2 | 3 |
| **E)** | Sam has 10 baseball cards. He wants to give all of his baseball cards to 2 of his friends. How many baseball cards will each friend get from Sam? | 1 | 2 | 3 |

8. When calculating 9270 – 581 using the standard subtraction algorithm, 9270 is regrouped. Which of the following numbers shows how 9270 is correctly regrouped?

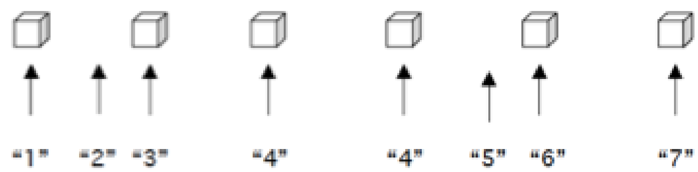
| **Q8** | | **Yes** | **No** | **I'm Not Sure** |
|---|---|---|---|---|
| **A)** | 8 thousands + 13 hundreds + 16 tens + 10 ones | 1 | 2 | 3 |
| **B)** | 8 thousands + 12 hundreds + 16 tens + 10 ones | 1 | 2 | 3 |
| **C)** | 8 hundreds + 11 tens + 26 ones | 1 | 2 | 3 |

9. For the following equation, which property(s) is demonstrated?

$$(76 + 34) + 13 = (13 + 76) + 34$$

| **Q9** | | **Yes** | **No** | **I'm Not Sure** |
|---|---|---|---|---|
| **A)** | The commutative property of addition | 1 | 2 | 3 |
| **B)** | The distributive property | 1 | 2 | 3 |
| **C)** | The associative property of addition | 1 | 2 | 3 |
| **D)** | The additive identity property | 1 | 2 | 3 |

10. A child is asked to count a set of cubes. Here is what he does:




Based on what this child did, which of the following counting principles can you say is violated?

| **Q10** | | **Yes** | **No** | **I'm Not Sure** |
|---|---|---|---|---|
| **A)** | Uniqueness principle | 1 | 2 | 3 |
| **B)** | One-to-one principle | 1 | 2 | 3 |
| **C)** | Cardinality | 1 | 2 | 3 |
| **D)** | Order irrelevance | 1 | 2 | 3 |

Appendix D

Pre- and Post-Analysis of Learning Assessment

STUDENT NAME:______________________________________________

Concordia University
Department of Education

Vanessa Rayner
January 13th 2014

**NOTE: ANSWER ALL QUESTIONS ON THIS TEST SHEET.**

**Part I: True/False and multiple choice**
**Instructions:** For each of the following questions select one answer. Circle your answer.

1) The majority of the students provided which of the following answers to the first problem?
   a. 12
   b. 7
   c. 17

2) The majority of the students' responses to the True/False sentence 7 = 7 suggests an understanding that:
   a. the equal sign means adding all the numbers
   b. the equal sign must be preceded by two numbers with a plus in between
   c. the equal sign means the answer comes next
   d. the equal sign represents a relation between two quantities

3) True or False: At the beginning of the lesson (when asked what goes in the box to make 8 + 4 = ☐ + 7 true), the students did not clearly articulate their conception of the equal sign.

4) True or False: When asked if 5 = 5 the students initially replied "Yes, it is true."

5) When asked which number goes in the box for 15 + 4 = ☐ + 11, many students told the teacher that:
   a. 8 goes in the box
   b. 19 goes in the box
   c. 30 goes in the box

6) True or False: When asked if 6 = 6 the students initially replied “Yes, it is true.”

7) The answer given for _______ indicated a rejection of non-canonical number sentences.
   a. 7 = 3 + 4
   b. 7 = 4 + 3
   c. 8 + 4 = ☐ + 7
   d. 15 + 4 = ☐ + 11

8) The majority of the students’ answers for 8 + 4 = ☐ + 7 (at the beginning of the lesson) suggests an understanding that:
   a. the equal sign means adding all the numbers
   b. the equal sign must be preceded by two numbers with a plus in between
   c. the equal sign means the answer comes next
   d. the equal sign represents a relation between two quantities

9) When the teacher put ________ on the whiteboard, a student said it was “backwards.”
   a. 6 = 0 + 6
   b. 6 = 6 + 0
   c. 7 = 3 + 4
   d. 7 = 4 + 3

10) A student used a canonical number sentence to justify why one of the non-canonical number sentences was true. Based on this, we can make claims about this student’s understanding of:
   a. the meaning of the equal sign
   b. non-canonical number sentences
   c. how to articulate his understanding of the equal sign

11) True or False: At the beginning of the lesson, one of the students said that “17” goes in the box.

12) Which of the following True or False number sentences was the first to elicit a student’s acceptance of non-canonical number sentences?
   a. 5 = 4 + 1
   b. 7 = 3 + 4
   c. 6 = 0 + 6
   d. 7 = 4 + 3
   e. 6 = 6 + 0

13) True or False: During the lesson (when asked what goes in the box to make 8 + 4 = ☐ + 7 true), the students did not clearly articulate their conception of the equal sign.

14) When asked why ________ is false, a student explained that it is not possible to count like that.
   a. 6 = 0 + 6
   b. 6 = 6 + 0
   c. 7 = 3 + 4
   d. 7 = 4 + 3

15) True or False: When asked if 7 = 7 the students initially replied "Yes, it is true."

16) True or False: At the end of lesson (when asked what goes in the box to make 15 + 4 = ☐ + 11 true), the students did not clearly articulate their conception of the equal sign.

17) For ________, the students articulated an understanding why non-canonical number sentences are acceptable.
   a. 5 = 1 + 4
   b. 5 = 4 + 1
   c. 8 + 4 = ☐ + 7
   d. 15 + 4 = ☐ + 11

18) The majority of the students' responses to the True/False sentence 6 = 6 suggests an understanding that:
   a. the equal sign means adding all the numbers
   b. the equal sign must be preceded by two numbers with a plus in between
   c. the equal sign means the answer comes next
   d. the equal sign represents a relation between two quantities

19) A student explained that 8 + 4 = 12 and 5 + 7 = 12. Based on this, we can make claims about this student's understanding of:
   a. the meaning of the equal sign
   b. non-canonical number sentences
   c. how to articulate his understanding of the equal sign

**Part II: Short answer**

**Instructions:** Answer the following questions using the space provided. Please be specific and provide details and/or examples from the video in your answer.

1) Explain what the students understand about the lesson and/or what the students have difficulty with at the beginning of the lesson.

2) Form a hypothesis (or more than one) about what the students learned and what they understood by the end of the lesson.

3) If you were the teacher, what would you have done differently when teaching this lesson?

4) Explain why you think the alternatives you propose (in question 3, above) would be more effective that what the teacher did.

STUDENT NAME:____________________________________________________

Concordia University
Department of Education

Vanessa Rayner
March 24th 2014

**NOTE: ANSWER ALL QUESTIONS ON THIS TEST SHEET**

**Part I: Short answer**

**Instructions:** Answer the following questions using the space provided. Please be specific and provide details and or examples from the video in your answer.

1) Form a hypothesis (or more than one) about what the students learned and understand by the end of the lesson.

2) If you were the teacher, what would you have done differently?

3) Explain why you think the alternatives you propose would be more effective.

STUDENT NAME:____________________________________________________

Concordia University
Department of Education

Vanessa Rayner
March 24th 2014

**NOTE: ANSWER ALL QUESTIONS ON THIS TEST SHEET**

**Part II: Short answer**

**Instructions:** Answer the following questions using the space provided. Please be specific and provide details and or examples from the video in your answer.

1) In as much detail as possible, can you identify the overall learning goal of the lesson?

2) Now consider each activity/task presented during the lesson. In as much detail as possible, describe what the teacher wanted the students to **learn for each activity/task**.

3) Explain what the students understand and/or what the students have difficulty with at the beginning of the lesson.

STUDENT NAME:________________________________________________

Concordia University
Department of Education

Vanessa Rayner
March 24th 2014

**NOTE: ANSWER ALL QUESTIONS ON THIS TEST SHEET.**

**Part III: True/False and multiple choice**

**Instructions:** For each of the following questions select **one** answer. Circle your answer.

1) The purpose of asking students <u>at the beginning of the lesson</u> “to think about what number would fit in the box” for 8 + 4 = ☐ + 5 was to:
   a. have students compare the numbers on either side of the equal sign without operating on the numbers
   b. have students recognize that the equal sign represents a relation between two equal amounts
   c. have students state what they think the equal sign means

2) The majority of the students provided which of the following answers to the first problem:
   d. 12
   e. 7
   f. 17

3) When the teacher put ________ on the whiteboard a student said it was backwards.
   e. 6 = 0 + 6
   f. 6 = 6 + 0
   g. 7 = 3 + 4
   h. 7 = 4 + 3

4) The purpose of giving the students 15 + 4 = ☐ + 11 was to:
   a. have students accept that number sentences do not always need to be written in the form $a + b = c$ (or non-canonical number sentences as true)
   b. have students compare the numbers on either side of the equal sign without operating on the numbers
   c. have students recognize that the equal sign represents a relation between two equal amounts

5) The majority of the students' answer for 8 + 4 = ☐ + 5 (at the beginning of the lesson) suggests an understanding that

e. the equal sign means adding all the numbers
f. the equal sign must be preceded by two numbers joined by a plus
g. the equal sign means the answer comes next
h. the equal sign represents a relation between two numbers

6) True or False: When asked if 5 = 5 the students initially replied "Yes."

7) The purpose of drawing diagonal lines under numbers and then writing their sum was to:

a. have students accept that number sentences do not always need to be written in the form *a* + *b* = *c* (or non-canonical number sentences as true)
b. have students state what they think the equal sign means
c. have students recognize that the equal sign represents a relation between two equal amounts

8) The majority of the students' response to the True/False sentence 7 = 7 (when presented at the beginning of the lesson), suggests an understanding that

e. the equal sign means adding all the numbers
f. the equal sign must be preceded by two numbers joined by a plus
g. the equal sign means the answer comes next
h. the equal sign represents a relation between two numbers

9) When asked why ________ is false a student explained that it is not possible to count like that.

e. 6 = 0 + 6
f. 6 = 6 + 0
g. 7 = 3 + 4
h. 7 = 4 + 3

10) True or False: The overall learning goal of the lesson was to have students compare the numbers on either side of the equal sign without operating on the numbers.

11) True or False: When asked if 7 = 7 the students initially replied "Yes."

12) For ________ the students articulated an understanding why non-canonical number sentences are acceptable.

e. 5 = 1 + 4
f. 5 = 4 + 1
g. 8 + 4 = ☐ + 5
h. 15 + 4 = ☐ + 11

13) The purpose of asking students whether 6 is equal to the sum of 0 and 6 (no order of addends implied) was to:

a. have students state what they think the equal sign means
b. have students accept that number sentences do not always need to be written in the form $a + b = c$ (i.e., non-canonical number sentences as true)
c. have students solve a problem using relational thinking (without operating on numbers)

14) The answer given for _______ indicated a rejection of non-canonical number sentences.

e. 7 = 3 + 4
f. 7 = 4 + 3
g. 8 + 4 = ☐ + 5
h. 15 + 4 = ☐ + 11

15) True or False: At the end of lesson (when asked what goes in the box to make 15 + 4 = ☐ + 11 true) the students <u>did not</u> articulate their conception of the equal sign.

16) The purpose of asking students <u>towards the end of the lesson</u> if they "still agree that 8 + 4 equals [or is] the same as 12 + 5" was to:

a. have students compare the numbers on either side of the equal sign without operating on the numbers
b. have students recognize that the equal sign represents a relation between two equal amounts
c. have students accept that number sentences do not always need to be written in the form $a + b = c$ (or non-canonical number sentences as true)

17) True or False: At the beginning of the lesson, one of the students said that "17" goes in the box.

18) Which of the True or False number sentences was the first to elicit a student's acceptance of non-canonical number sentences?

f. 5 = 4 + 1
g. 7 = 3 + 4
h. 6 = 0 + 6
i. 7 = 4 + 3
j. 6 = 6 + 0

19) True or False: The overall learning goal of the lesson was to change the students' understanding of what the equal sign means.

20) When asked which number goes in the box for 15 + 4 = ☐ + 11 a lot of the students told the teacher that,

d. 8 goes in the box
e. 19 goes in the box
f. 30 goes in the box

21) True or False: At the beginning of the lesson (when asked what goes in the box to make 8 + 4 = ☐ + 5 true) the students <u>did not</u> articulate their conception of the equal sign.

22) The purpose of asking students whether they think 6 = 6 is true was to:

a. have students compare the numbers on either side of the equal sign without operating on the numbers
b. have students state what they think the equal sign means
c. have students recognize that the equal sign represents a relation between two equal amounts

23) A student explained that 8 + 4 = 12 and 5 + 7 = 12. Based on this, we can make claims about this student's understanding of:

d. meaning of the equal sign
e. non-canonical number sentences
f. how to articulate his understanding of the equal sign

24) True or False: The overall learning goal of the lesson was to have students accept that number sentences do not always need to be written in the form *a* + *b* = *c* (or non-canonical number sentences as true).

25) The majority of the students' response to the True/False sentence 6 = 6, suggests an understanding that

e. the equal sign means adding all the numbers
f. the equal sign must be preceded by two numbers joined by a plus
g. the equal sign means the answer comes next
h. the equal sign represents a relation between two numbers

26) True or False: During the lesson (when asked what goes in the box to make 8 + 4 = ☐ + 5 true) the students <u>did not</u> articulate their conception of the equal sign.

27) A student used a canonical number sentence to justify why one of the non-canonical number sentences was true. Based on this, we can make claims about this student's understanding of:

d. meaning of the equal sign
e. non-canonical number sentences
f. how to articulate his understanding of the equal sign

28) True or False: When asked if 6 = 6 the students initially replied “Yes.”

29) True or False: The overall learning goal of the lesson was to have students state what they think the equal sign means.

30) The purpose of asking students whether 7 = 3 + 4 is True or False was to:

a. have students state what they think the equal sign means
b. have students accept that number sentences do not always need to be written in the form $a + b = c$ (or non-canonical number sentences as true)
c. have students solve a problem using relational thinking (without operating on numbers)

# Appendix E

## Video Transcript

T: I want you to think about what number would fit in the box, ok? So I am going to give you this number $8 + 4 = \square + 5$ think about it for a second and think about what number goes in the box. I see some people really looking thinking of some strategies. Got 4 people who know it (shown by putting hands on head) 5, 6…I think everybody knows it. Ok, let's see what you got, let's see what you got, what do you think?
S1 (boy with dark hair and long sleeve dark shirt with a t-shirt): 12
T: Ok we think 12, what's another answer? Anybody have another answer like another idea? How many people think it's 12 (she raises her hand)? Ok, so we are going to put 12 in the box. Let me ask you this, we are going to come back to that one in a second and we are going to double check it. Alright? Let's go to another one. Ready? Was that pretty easy?
Ss: yes
T: alright here we go. 7 = 3 + 4. Is that True or false? (says in Spanish). Yah, is it true that 7 = 3 + 4? Samuel what do you think?
S2: It's backwards
T: It's backwards! What's wrong with it?
S3 (can't see): 7 equals, it has to be 3 + 4 = 7.
T: It should be 3 + 4 = 7 (writes on the whiteboard), would that be right?
Ss: yah
T: so this is not right?
Ss: No
T: Why not?
S4: It is right but
S5: but you can't understand it like that
T: oh, you just can't understand it like this. And, so is 7 = 7 (does not write this just points to the 7s from both number sentences)? Does 7 = 7?
S6: No
T: No? 7 does not equal 7? Ok let me ask you this one. Let me take you to another one. True or False, 6 = 6 + 0? Is that true? Is it true that 6 is = 6 plus 0? Talk to each other and see. You guys can talk to each other if your not sure. No es verdad (talking to a student)
S7: es falsa
T: 6 = 6 + 0? o no es verdad. Falso o verdad. Speak you three (in Spanish). Ooh I see a lot of people with their hands up. A couple of people are still talking over here let's give them a second. Are you ready (in Spanish)? Here we go. What do you think, Eric what do you think?
S8(Eric): Falso
T: Falso, why?
S8: Why, why not? (in Spanish with subtitles). [Pause] It is not possible to count like that.
T: you can't count like that? [he shakes his head] Why not?
S8: How could it be 6 plus 0 (in Spanish with subtitles)
T: Tell me again

S8: How could it be 6 plus 0 (in Spanish with subtitles)
T: So you are saying how can that be? That it can't be [shakes his head]. Ok what else do you guys think?
S9 (Spanish boy with dark hair and white polo shirt): True
T: you think it's true, why?
S9: Because 6 = 6 is 6, 6 plus 0 is 6
T: 6 plus 0 is (draws diagonal lines under the 6 and 0 so that they meet and writes 6)
Ss: 6
T: and 6 equals (writes 6 = 6 underneath 6 = 6 + 0)
Ss: 6
T: 6? Do you think it is true that 6 = 6?
Ss: mix of yes and no
T: let's do it up here, "6 = 6" (writes on a different space on whiteboards) is that true?
Ss: mix of yes and no but the yes's are insisting that it is true
T: does 6 = 6?
Ss: all you hear is yes
T: oh, what about this? Does 5 = 5? (writes this)
Ss: most say yes maybe a few still think no
T: how many people think that 5 = 5? How many people think 6 = 6? (most of the students raise their hand but not all; see at least one who does not raise his hand) how about 10, does 10 = 10? How about this, does, if 5 = 5, does 5 = 4 + 1?
SS: large number of student say yes
T: Why?
Ss: because
T: Manuel
S10: just because 1 + 4 = 5
T: because this is still the
Ss: 5
T: it's still the 5, so is it the same on both sides?
Ss: yes
T: ok so then would 6 = 3 + 3?
Ss: Yes
T: why?
S11(Wayne): because 3 + 3 is 6
T: and then what happens to both sides
S11: they make 6
T: Are they the same?
Ss: yes
T: they are both the same. So 3 + 3 is the same as 6 (circles the 3 + 3 and then 6) because what does this equal (points to 3 + 3)
Ss: 6
T: So let's go back up here then. So you told me that 6 = 6 and 5 = 5, 5 = 4 + 1. Does it matter if I write like this 4 + 1 = 5
S12: no
T: or like this (points 5 = 4 + 1)
S12: no it's still the same

T: it's still the same right. Because what is this (draws diagonal lines under 4 and 1)
Ss: 5
T: so this is still 5 = 5 right
Ss: yes
T: so it's still coming out the same. So let me take you to this one again. I want you to talk to your partners. A little while ago you told me 8 plus 4 equals (some students say 12) the same as 12 + 5 [pause]. Talk to your partners and see if you still agree with that. Talk to each other. Think see what do you think if you agree, why do you agree if you do not agree, what would you put in that box to make it true. [mumbling of some answers]. What would you put in the box to make it true, talk to each other.
Teacher speaks with one student to discuss his answers. Asks if they are ready to share, the students are not quite ready so she goes around to the see what the students are thinking. Period of student discussion.
T: how many people think we should leave the 12 there? Do you think we should leave the 12 there?
S13: no
T: why not? Why can't we leave the, what's let's start with that, why can't we leave the 12 there?
S14: because 12 times 12 plus 5 doesn't equal 12
T: ok 12 + 5 equals how much (draws diagonal lines for both numbers and writes 17)
Ss: 17
T: and that does not equal [some say 12] what?
Ss: 12
T: where did you get the 12 from?
S14: from the 8 + 4
T: (writes 12) so that does not equal (draws diagonal lines to connect to 12) 12. So what do we need to do, Yvette? What do we need to do? (in Spanish with sub)
S15 (Yvette): 8 plus 4 is 12 (in Spanish with sub) and 5 plus 7 is 12 (teacher points to the 12 and erases it to put 7 in the box and erases the 17) How much is it?
S15: 12
T: So now are they the same?
Ss: yes
T: is that true?
Ss: yes
T: so now are both sides the same?
Ss: yes
T: ya now they are equal ok, good. Are you ready?
Ss: yes
T: one more
Writes on the whiteboard, "15 + 4 = □ + 11".
T: Go
Teacher walks around to discuss their solutions
T: what did you guys get?
S16: uh 19
T: you got 19, 19 is going in the box?
S16: yeah no no 8, 8

T: oh a lot of people told me 19 but is 19 the number that goes in the box?
Ss: no
T: you have to be super duper duper careful about what you are saying. You are going to put what in the box (points to S16)
S16: 8
T: 8 now, why are you going to put 8 in the box? Why did you guys decide
S17 (at the same table as S16, wearing green t-shirt): because 11 + 8 is (draws the diagonal lines)
Ss: 19
T: why were you trying to make 19 over here? Why did you have to put 19 over there? Why?
S15: because 19 is like the same one as 15 plus 4
T: 15 plus 4 is also (draws diagonal lines under the 15 and 4)
Ss: 19
T: 19, is 19 = to 19?
Ss: yes
T: Nice job.

Appendix F

Interview Protocol

# SPECIFYING LEARNING GOALS INTERVIEW PROTOCOL

*Interview questions prior to watching the video*

1. You are a 4th grade elementary teacher and you want to plan a lesson on introducing the meaning of the equal sign to elementary students, what would be the main goal of in terms of the learning of your students for that lesson? What would be your main learning goal?

2. In your opinion, what tasks would you need to include in your lesson to help students achieve the main learning goal?

3. What would you want the students to learn from each of these tasks?

*Instructions for video viewing:*

"I will show you the last video we watched as a class in the lab. Just to remind you, the video is from a 4th Grade math lesson. When you watch the video, I want you to focus on what the teacher's learning goals may have been for the students. You may use the framework your group designed to take any notes on this aspect of the lesson but keep in mind that you do not need to focus on the other aspects of the lesson we discussed during the labs unless you think it is necessary. After you've watched the video, I will ask you some questions and you can refer to your notes and the video itself at anytime when answering the questions. If you need me to, I can go back to a specific point in the video when you are answering your question."

*Show video. The participant is allowed to use his or her framework to take notes while watch the interview.*

1. Do you think the teacher's goals for the students in terms of their learning about the equal sign were similar or different to what you discussed before the video?
    a. Do you think the teacher in the video designed her lesson with the same main learning goal in mind? Why or why not? (*Make sure that the participant does identify the main learning of the lesson*)
        i. If the participant does not identify the main learning goal of the lesson probe further:
        "Before we watched the video you explained to me what the main goal in terms of the learning of your students would be. What do you think was the main goal in terms of learning for the students in the video? In other words, what do you think was the main learning goal of that lesson?"
    b. Did the teacher in the video use a similar sequence of tasks to what you proposed? Please explain.
    c. *Make sure that the participant does identify the sub goals of the lesson*
        i. If the participant does not identify the learning goals of the tasks used in the lesson probe further:
        "Before we watched the video, you proposed some tasks to include in your lesson to help students achieve the main learning goal. What do you think the teacher in the video wanted the students to learn from each of the tasks presented in her lesson?
    d. For the tasks that were similar to the ones you proposed, do you think the learning goals were also similar?

2. How did you identify the main learning goal and these other learning goals?

Can you describe what parts of the video informed your decision?

Appendix G

Consent Form

Student Consent to participate in Research (EDUC 388/4: Teaching Mathematics III).

**CONSENT TO PARTICIPATE IN THE TEACHER INSTRUCTION AND PROBLEM SOLVING RESEARCH PROJECT**

I understand that I have been asked to participate in a research project being conducted by Vanessa Rayner (v_rayner@education.concordia.ca). I understand that Vanessa Rayner is supervised by Dr. Helena Osana of the Department of Education at Concordia University (514-848- 2424 ext. 2543; osana@education.concordia.ca).

## A. PURPOSE

I have been informed that the purpose of the research is to study different methods to help teachers in training learn new techniques and strategies for analyzing student learning to assess the effectiveness of teaching.

## B. PROCEDURES

- I understand that I will be asked to complete three measures, which will each take approximately 30 minutes to an hour to complete (as part of the course requirements.)
- I understand that I will be asked to participate in four instructional sessions about learning to analyze student learning. These sessions will be conducted by Vanessa Rayner, the instructor of the course.
- I understand that I will be asked to watch video clips of examples of teaching and discuss what I observe.
- I understand that I will be asked to modify an observational framework, which will be collected at the end of each instructional session.
- I understand that my exposure to instruction on analyzing student learning can assist me in better understanding how to assess the effectiveness of my teaching in the future.
- I understand that Vanessa Rayner may or may contact me to request an interview outside of class time but that I am not obligated to accept her request (i.e., it will not be a course requirement to participate in the interview).
- I understand that my name will be kept confidential and will not be used for any other purposes other than this research.
- I understand that all activities are a required part of the course. My consent gives permission to Vanessa Rayner to use my written work as data for her research.
- I understand that none of the work that Vanessa Rayner will use as data will be formally evaluated in this course.
- I understand that my instructor, Vanessa Rayner, will not know whether I give consent or not until after the final grades have been submitted.

## C. RISKS AND BENEFITS

- I understand that my participation poses no known risks.
- I understand that my participation may result in a better ability to notice student learning and reason about (a) what students learned, (b) the effect of teaching on students' learning, and (c) ways to improve my mathematics teaching to elementary students.

## D. CONDITIONS OF PARTICIPATION

- I understand that I am free to withdraw my consent and discontinue my participation at anytime without negative consequences. (please contact Emmanuelle Adrien to withdraw your consent emmanuelle.adrien@education.concordia.ca)

- I understand that if I choose to withdraw my consent I am still required to participate in all the study's activities, but the results from my participation in the activities will not be used as data.

- I understand that my participation in this study is CONFIDENTIAL (i.e., Vanessa Rayner will know after my completion of this course, but will not disclose my identity)

- I understand that the data from this study may be published, but that no information will be reported that will expose my identity.

I HAVE CAREFULLY STUDIED THE ABOVE AND UNDERSTAND THIS AGREEMENT. I FREELY CONSENT AND VOLUNTARILY AGREE TO PARTICIPATE IN THIS STUDY.

NAME (please print) ____________________________________________

SIGNATURE ____________________________________________

If at any time you have questions about the proposed research, please contact the study's Principal Investigator:

*Vanessa Rayner, PhD Candidate*
*Department of Education, Concordia University*
*(514) 240-7134*
*v_rayner@education.concordia.ca*

If at any time you have questions about your rights as a research participant, please contact the Research Ethics and Compliance Advisor, Concordia University, 514.848.2424 ex. 7481
ethics@alcor.concordia.ca

Appendix H

Star and Strickland's (2008) Observational Framework (p. 113)

| Category | Description |
|---|---|
| Classroom environment | "Includes physical setting such as desk arrangements, materials and equipment available and utilized, demographics of students and teacher, class size, grade level, and course title" |
| Classroom management | "Includes the ways the teacher deals with disruptive events, pace changes, procedures for calling on students or handling homework, and the teacher's physical presence (e.g., patterns of moving around the classroom, strategies for maintaining visibility, tone and volume of voice)" |
| Tasks | "Refers more generally to activities students do in the class period (e.g., warm-ups, worksheets, taking notes, presentations, passing out papers) or future activities such as homework or upcoming quizzes" |
| Mathematical Content | "Includes representation of the mathematics(graphs, equations, tables, models), examples used, and problems posed" |
| Communication | "Refers to student-to-student as well as teacher-to-student talk and includes questions posed, answers or suggestions offered, and word choice" |

Appendix I

Overview of Instruction and Session PowerPoint Slides

<table>
<tr><th></th><th>Session 1</th><th>Session 2</th><th>Session 3</th><th>Session 4</th><th>Session 5</th></tr>
<tr><td>Learning Goals</td><td>• Video 3 viewed prior skill-based instruction<br>• Instruction on Skill 1<br>• Framework development (small group work)<br>• Video 3 viewed to apply skill-based instruction (Skill 1) to video analysis</td><td>• Review of skill-based instruction (Skill 1) and feedback on observation framework modifications<br>• Revised observation Framework distributed (addressing Skill 1)<br>• Video 4 viewed to practice Skill 1<br>• Whole class discussion on framework efficacy</td><td rowspan="2">• Video 3 viewed prior to skill-based instruction (Skills 2 and 3)<br>• Instruction on Skills 2 and 3<br>• Framework development (small group work)<br>• Whole class discussion on framework modifications</td><td rowspan="2">• Summarize skill-based instruction (Skills 2 and 3. Skill 1 if applicable).<br>• Revised observation Framework distributed (addressing Skills 2 and 3. Skill 1if applicable).<br>• Video 3 viewed prior to skill-based instruction (Skill 4)<br>• Instruction on Skill 4<br>• Framework development (small group work)</td><td rowspan="2">• Summarize skill-based instruction (Skills 2, 3, and 4. Skill 1 if applicable).<br>• Final version of framework distributed (addressing Skills 2, 3 and 4. Skill 1 if applicable).<br>• Video 5 viewed to practice all skill-based instruction</td></tr>
<tr><td>Students Learning</td><td>• Video 3 viewed<br>• Free discussion<br>• Framework development (small group work)<br>• Video 3 viewed to apply discussion to video analysis</td><td>• Feedback on framework modifications<br>• Revised observation Framework distributed (addressing discussion)<br>• Video 4 viewed to practice using new framework<br>• Whole class discussion on framework efficacy</td></tr>
</table>

## Learning to Specify Learning Goals

## WHAT LEARNING GOALS DID YOU IDENTIFY?

## Specify the Learning Goal(s) for the Instructional Episode

- What are students suppose to learn?
- Define learning goals precisely and explicitly
- But why?
  - Without explicitly specifying learning goals DIFFICULT TO ANALYZE WHAT STUDENTS' LEARNED
    - What evidence of student learning should be collected?
    - How student learning can be linked to **particular** instructional activities
    - Revise instruction

## What do I Need to Know?

- Unpack the main goal into sub-goals
  1. **Be specific**
  2. Use **language** of the subject to communicate the goals and sub-goals
- **Note: You are observing an episode of another teacher's lesson** you will not have the same insight/details of the lesson as a teacher teaching the lesson
- **Consider the general topic (e.g., the concept of place value) and specific knowledge pieces that support that topic**

## FOR EXAMPLE: PLACE VALUE

## Place Value Sub Goals

1. The value of the digit is based on the place of the digit in the numeral
2. Values of places on either side of a digit are more or less than each other by a factor of 10
3. The role of zero as a place value holder
4. In a numeral each place only has one digit

## CONJECTURES

- WHAT IS A CONJECTURE?:
  - Statement (e.g., property or principle) that is **believed to be true but not yet proved or disproved**
  - based on implicit knowledge of properties of mathematics
- PURPOSE:
  - Articulating conjectures to make informal knowledge explicit
  - Is it true for all numbers
  - Become fluent in symbolic manipulation
  - Symbolic manipulation meaningful

## JUSTIFYING CONJECTURES

## The Foundations Of Learning to Justify Conjectures

✓ Justification is an argument used to show a conjecture is true
  - Is the sentence always true?
  - How do you know the sentence true for all numbers?

✓ Replacing words with symbols when writing conjectures

✓ Writing sentences that are always true

✓ **Problems with variables**

✓ **Problems with multiple variables**

✓ Extend ability to use relational thinking

*Ricardo has 7 pet mice. He keeps them in two cages that are connected so that the mice can go back and forth between the cages. One of the cages is big and the other is small. Show all the ways that 7 mice can be in two cages.*

## I. Solving Problems with Multiple Variables

1. **Generate solutions** (combinations of mice in the small and big cage) without explicit thought of mathematical properties
2. More insightful solutions illustrates how **properties of mathematics** may be applied (i.e., the commutative property)
3. **Justify** that all possible solutions have been found
4. **Express conjecture about the number of solutions for any number** (of mice) **using algebraic notation**

Rico's list

| Big Cage | Small Cage |
|---|---|
| 5 | 2 |
| 4 | 3 |
| 1 | 6 |
| 0 | 7 |
| 7 | 0 |
| 2 | 5 |
| 6 | 1 |

Sandra's list

| Big Cage | Small Cage |
|---|---|
| 5 | 2 |
| 2 | 5 |
| 4 | 3 |
| 3 | 4 |
| 6 | 1 |
| 1 | 6 |

**Ms. S.:** How about 324 mice?
**Jamie:** It would be 325.
**Anna:** Yeah, it's always just one more than the number of mice. These are easy.
**Ms. S.:** Oh, how about if there are *n* mice?
**Anna:** It would be just one more than that.
**Ms. S.:** How would we write that? [*The children struggle with this question for a while. Finally Carla responds.*]
**Carla:** Would it be *n* + 1?

## II. Solving Number Sentences with Multiple Variables

$$q + p = 7$$

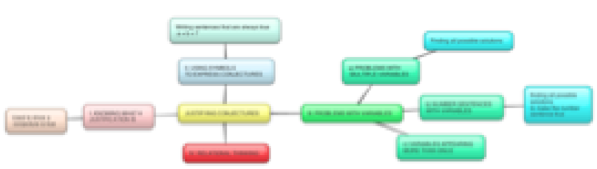
- Understanding that the problem involves finding all the possible numbers that can be substituted to make the number sentence true
  - Note the difference between these number sentence problems and the open number sentence problems used when teaching equivalence

## II. Solving Number Sentences with Multiple Variables

- The number chosen for one variable (several possibilities) *can* narrow/influence the number that can be used to replace the second variable
  - For example: $2 \times v + y = 14$; $v$ can be even or odd but when $v$ is a whole number, $y$ has to be an even number
  - For example: $s + t = 8$; $s$ has many possibilities but once the number for $s$ is picked, there can only be one possibility for $t$ (and vice versa)

## III. Variables Appearing more than Once

- *The mathematicians rule*, "when the same variable appears more that one time in a number sentence, the variable must be replaced by the same number throughout the number sentence for each substitution."



## Developing a Framework (Part I) Learning Goals and Sub-Goals

## Defining Learning Goals and Sub Goals

- What is intended for the students to learn as a result of (a) performing an activity and (b) answering questions about the activity
- The learning goal involves learning about mathematics concepts, procedures or how to reason about mathematics in new ways
- To achieve the learning goal, the lesson may involve sub-goals, or components of the learning goal that need to be understood/made explicit to support achievement of the learning goal

## Learning Goals Specific to Teaching Episode

- Learn how to efficiently justify a conjecture
  - Can the student justify whether the conjecture is true
- Explain reasoning (about what specifically)
- Extend thinking about relationships between numbers represented on either side of the equal sign

## Notes on Learning Goals

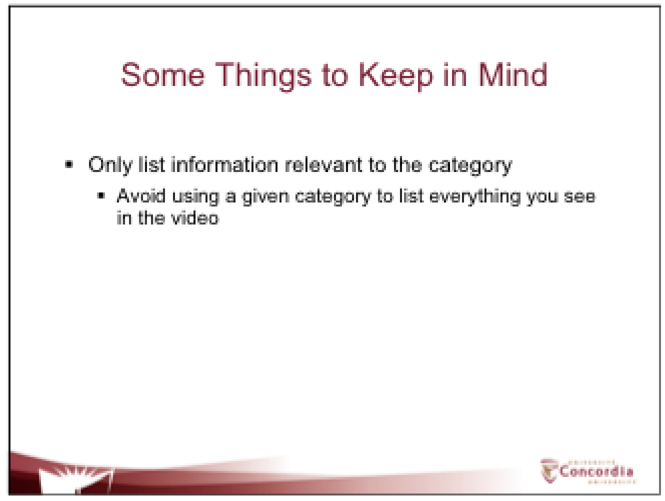
- Activities requires the student to (a) explain (b) demonstrate with examples (c) justify
- Teacher questions/guides
- Definitions of what learning goals and sub-goals are
- Examples of what students need to know
- The steps that the student went through to learn what was intended
  - Think about how to justify conjecture
  - Try out her approach
  - Reflect on whether the approach would work (would it prove that the conjecture is true for all numbers)
  - Consider any patterns/generalization
  - Can use conjectures known to be true to justify other conjecture

| Category | Description | NOTES | | |
|---|---|---|---|---|
| TOPIC | Identify the topic of the teaching episode | ☐ Equivalence | ☐ Conjectures | ☐ Conjectures about Classes of Numbers |
| PRIMARY LEARNING GOALS | Includes a specific description of what the student(s) are intended to learn during the teaching episode (e.g., learn the most efficient way to justify that a conjecture is true)<br><br>The learning goals are linked to the type of activity and can be guided by the teacher (e.g., questions used to probe student thinking) | A. Learning Goal of the Activity<br><br>B. Teacher cues related to the Learning Goal | | |

STUDENT NAME:

| | | |
|---|---|---|
| SUB-GOALS The components of the learning goal **(what do students need to understand/do to reach the primary learning goal?)** | A. What do the activities involve? | ☐ Justify/reason/explain (with or without demonstration) BE SPECIFIC. JUSTIFY WHAT, REASON ABOUT WHAT...<br><br>☐ Communicate what he or she understands about the topic (e.g., how do you know the conjecture is true)<br><br>☐ Describe strategy to solve a problem (e.g., what is your strategy to justify that $a + b - b = a$ is true?)<br><br>☐ Reflect on their strategies and/or understanding of the topic<br><br>☐ Communicate using mathematical language |


STUDENT NAME:
B. To achieve the learning goal, specify the mathematical knowledge student(s) need to make explicit or need to understand.
For example: To learn how to efficiently justify a conjecture a student has to start by understanding at least one way (try different types of numbers) to justify conjecture. Also, have to understand more generally what it means to justify a conjecture and know how to demonstrate/test their strategy.
☐ Mathematical knowledge needed to learn what is intended
(e.g., rules of mathematics, properties of mathematics, variables, solve problems with variables; relational thinking)


Some Things to Keep in Mind
Only list information relevant to the category
Avoid using a given category to list everything you see in the video

## Collecting Evidence of Student Learning *and* Hypotheses on the Effect of Teaching

## I. On Collecting Evidence

- **COLLECT EVIDENCE THAT PERTAINS TO WHAT STUDENTS KNOW <u>IN RELATION TO THE LEARNING GOAL</u>**
  1. **Related:** Evidence that indicates whether student(s) did and did not learn what was intended
  2. **Relevant:** Evidence of student understanding (verbal and nonverbal behaviors)
- **Evidence will allow support your claims that student(s) learned what was intended**

## What Evidence does NOT Count?

✗ **This is NOT <u>relevant</u>:** Teacher behaviors and explanations do not count as evidence of student learning
- Even if students NOD "yes" in response
- FOCUS SHOULD BE ON THE STUDENTS' THINKING AND BEHAVIORS

✗ **This is NOT always <u>related</u> to the learning goal:** If the answer is correct then it counts!
- If the answer is **not** related to the learning goal then it cannot be used to explain whether or not student learned what was intended

## For Example: 8 + 4 = ☐ + 5

- ***Sub-learning goal: Getting Children to Articulate the Meaning of the Equal Sign***

**Ricardo:** *[After a short pause]* It's 7.
**Ms. L.:** How do you know that it's 7?
**Ricardo:** Well, 8 and 4 is 12. So I had to figure out what to go with 5 to make 12, and I figured out that had to be 7.

✗ OVER-ESTIMATES WHAT STUDENTS UNDERSTAND

## For Example: 8 + 4 = ☐ + 5

- ***Sub-learning goal: Getting Children to Articulate the Meaning of the Equal Sign***

**Ms. L.:** Some kids have told me that 12 should go in the box. What do you think about that?
**Ricardo:** That's not right. Twelve and 5—that would be 17, and that's not equal to 12. These two sides have got to be the same.

## What Evidence Counts?

☑ <u>Verbal Evidence **related and relevant**</u>: Student justifications, explanations
- Articulating the meaning of the equal sign is both **relevant and related to the sub goal of getting students to communicate**

## What Evidence Counts?

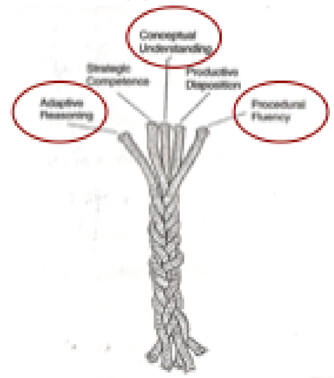
☑ <u>Nonverbal Evidence **related and relevant**</u>: Demonstrations of thinking (e.g., strategies)

- Students using showing in written form that the amounts on both sides of the equal sign need to be the same

$$\underbrace{8+4}_{12} = \underbrace{7+5}_{12} \quad or \quad 8+4=7+5,\ 12 = 12$$



## What do we Want Students to Learn?






## What Evidence Counts?

**3. Focus on the learning goal**

- ✓ **Is the intent to promote <u>conceptual understanding</u>?**
  - **Use evidence where students demonstrate (and/or do not demonstrate) a conceptual understanding**
- ✓ **Is the intent to learn <u>procedural fluency</u>?**
  - **Use evidence where students successfully execute (and/or do not) a procedure (e.g., the standard algorithm for multidigit addition)**
- ✓ **Is the intent to learn to use <u>adaptive reasoning</u>?**
  - **Use evidence where students demonstrate (and/or do not demonstrate) an understanding of how to reason in the way that is intended**
    - **Capacity for logical thought, reflection, explanation and justification**



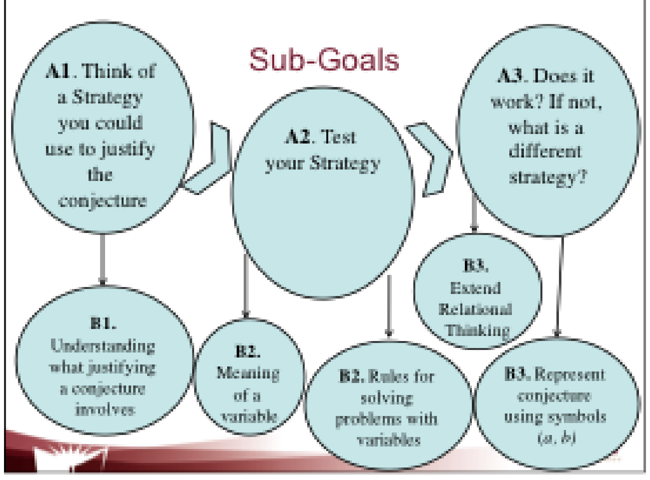
## Example for Learning Goals to Promote Adaptive Reasoning

- **PRIMARY GOAL**

✓ **How to efficiently justify a conjecture**



## Sub-Goals




## II. On Forming Hypotheses

- Using what you have observed
  1. Learning goals ➔ observations of what teacher did to promote the learning goal
     - Activities and questions related to the primary learning goal and sub-goals
  2. Evidence of student learning
- Cause effect relationship on the ways the teaching events (**cause**) could be linked to evidence of what students did or did not learn (**effect**)

## Why are these Good Examples of Hypotheses?

The teacher's development of the concept of square units had negative effects on student learning. *Examples:* The teacher's failure to develop the concept of square units led to misconceptions (specific misconceptions were described), lack of understanding, forgetting to write "square units" as part of the answer, and/or inability to solve problems.

The teacher did not develop the connections among the different approaches for finding area so the students did not understand the connections among the approaches.

The students did not have enough practice identifying $l$ and $w$, or $h$ and $b$ on figures, and plugging these numerical values into the relevant formula. Thus they had trouble identifying the dimensions of figures, plugging values into the formula, and/or selecting the correct formula.

## Why are these Good Examples of Hypotheses?

The teacher's development of the meaning of the formulas had negative effects on students' learning. *Examples:* The teacher just relied on examples of the formulas and the students did not learn the underlying concepts. This may lead to forgetting the formulas, inability to reconstruct the ideas, or lack of understanding of when to use the formulas and why to use the formulas.

By breaking down the formula for the area of a rectangle into parts, it made it easier for the students to identify what numbers went where and why they were used. Also by taking it slow and working with the students to develop the formula, the teacher gave the children time to connect the idea of counting square units to the idea of multiplying length times width.

## The Answer

- Hypotheses about the effects of specific instructional features on students' learning

The teacher assumed that everyone remembered concepts from previous lessons, such as "perpendicular," and did not refresh their memories. This interfered with learning new material that depended on these concepts.

The children experienced difficulty because they did not have any manipulatives or concrete materials to help them learn or understand. If children worked with concrete materials, they could cut up a triangle without a right angle in order to figure out the area, could invent ways for finding the area of rectangles and triangles themselves, and/or could use the concrete materials to develop an understanding of the formulas or to solve the problems.

## Why are these Bad Examples of Hypotheses?

The students understand what area is.

The students understand the concepts of length and width and how to identify them on a figure.

I don't know if the students understand the concept of area. Because the area of rectangles and triangles are formulas, it is easy for children to memorize the formula without fully understanding the concept.

Some of the students' basic mathematics skills were not very good. If students lack basic skills, then it does not matter if they know the formulas because their answers will be incorrect.

## That Answer

- Hypotheses about students' learning that made no reference to instructional features

One child asked whether she should divide by 2 to get the area of a rectangle. This tells me that maybe that child didn't understand it completely but when the teacher explained to do that only with triangles, the child seemed to understand better.

## I. Learning Goal

- Specific description of what is intended for the students to learn as a result of (a) performing an activity and (b) answering questions about the activity?
  - ✓ **Learn the most efficient way to justify that a conjecture is true**

## I. Sub Goals (Based on *Activities*)

- **What do students need to do** to achieve the primary learning goal?
  1. **Think of a Strategy you could use to justify the conjecture**
  2. **Test your Strategy**
  3. **Does it work? If not, what is a different strategy?**

## I. Sub Goals (Based on *Mathematical Knowledge* Involved)

- **What do students need to understand** to achieve the primary learning goal?
  1. Think of a Strategy you could use to justify the conjecture ➔ **How to show that a conjecture is true for all numbers?**
  2. Test your Strategy ➔ **Knowing what variables are** And **How to solve problems with variables**? (e.g., "mathematicians rule")
  3. Does it work? If not, what is a different strategy? ➔ **Extend Relational Thinking** and **Represent conjecture using symbols** ($a$, $b$)

## II. On Collecting Evidence

- **COLLECT EVIDENCE THAT PERTAINS TO WHAT STUDENTS KNOW <u>IN RELATION TO THE LEARNING GOAL</u>**
  1. **Related:** Evidence that indicates whether student(s) did and did not learn what was intended
  2. **Relevant:** Evidence of student understanding (verbal and nonverbal behaviors)
- **Evidence will allow support your claims that student(s) learned what was intended**

## III. On Forming Hypotheses

- Using what you have observed
  1. Learning goals ➔ observations of what teacher did to promote the learning goal
     - Activities and questions related to the primary learning goal and sub-goals
  2. Evidence of student learning
- Cause effect relationship on the ways the teaching events (**cause**) could be linked to evidence of what students did or did not learn (**effect**)

## Some Examples from the Video

- **<u>Sub Goal:</u>** *What Suzie is asked to do* ➔ **How to justify conjectures**
- **<u>Sub Goal:</u>** *What mathematical knowledge Suzie needs to know* ➔ **A strategy to show that $a + b - b = a$ is true for all numbers**
- **<u>Evidence about what Suzie knows related to this goal:</u>**
- *Suzie:* "Well if you used all the numbers it would take forever but if you got a lot of ways, one of each type of thing it might prove it. Fractions, regular numbers, negative numbers, really high numbers really low numbers."

## Examining the Effect of Getting Suzie to Think about how to Prove a Conjecture is True

- **Hypothesis:**
- By asking Suzie whether she thinks $a + b - b = a$ is true and then asking her if she thinks it is *always* true appears to get Suzie to realize that she "knows" the conjecture is true but she can't really explain that it is true for all numbers at the beginning of the interview.

## Some Examples from the Video

- **Sub Goal:** *What Suzie is asked to do* → **Testing the strategy**
- **Sub Goal:** *What mathematical knowledge Suzie needs to know* → **What variables are**
- **Evidence about what Suzie knows related to this goal:**
- Suzie knows for $a + b - b = a$ → choose numbers to replace variables to make the sentence true
  - ½ + 11-11 = ½
- *Suzie* : "Well if you have 11, and you minus that same 11 it will get you to zero. This 11 minus this 11 cancel each other out so it's more like ½ = ½."

## Examining the Effect of Getting Suzie to Test her Strategy

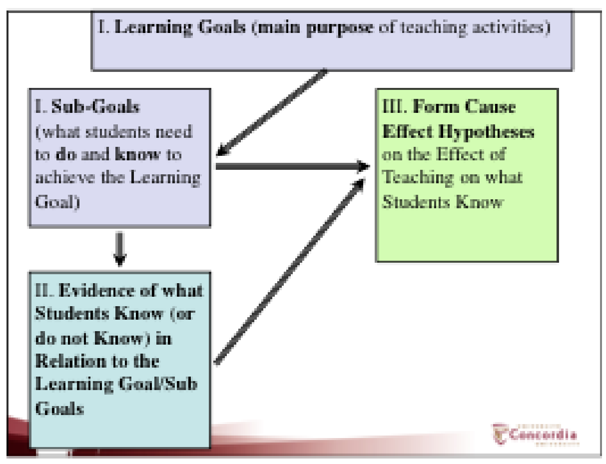
- **Hypothesis:**
- By getting Suzie to test her strategy of trying different types of numbers to see whether the conjecture is true for all numbers Suzie relies on her knowledge of the meaning of the equal sign to explain why the number sentence is true.

## Some Examples from the Video

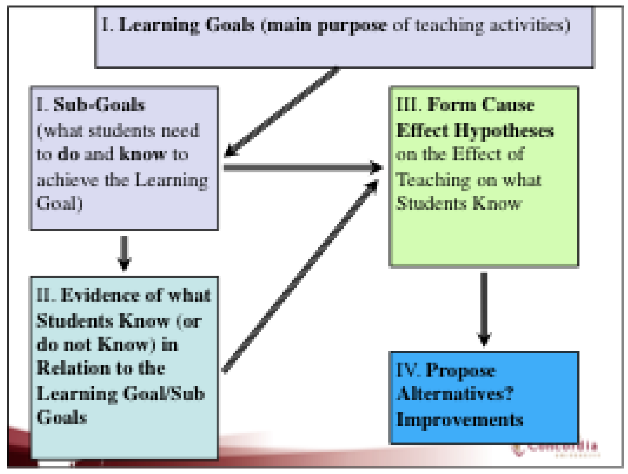
- **Sub Goal:** *What Suzie is asked to do* → **Is there a more efficient way?**
- **Sub Goal:** *What mathematical knowledge Suzie needs to know* → **Represent the conjecture using symbols (*a*, *b*)**
- **Evidence about what Suzie knows related to this goal:**
- Suzie identifies the two conjectures in $a + b - b = a$
- *Suzie* : "So its two conjectures makes up this conjecture (points at for $a + b - b = a$) so we've got two conjectures put together."

## Examining the Effect of Getting Suzie to Think of a Way that would Prove it is True for All Numbers

- **Hypothesis:**
- The probes the interviewer uses elicits Suzie to state conjectures such as "any number minus the same number gets you to zero."



## Proposing Alternatives/ Improvements




## IV. Proposing Alternatives/ Improvements

- What is involved?

1) **What would you do?**
   - Based on hypotheses

2) **Why would you do it that way?**
   - Based on evidence collected about what student(s) know in relation to the learning goal

## For Example

- Hypothesis:

> The teacher's development of the concept of square units had negative effects on student learning. *Examples:* The teacher's failure to develop the concept of square units led to misconceptions (specific misconceptions were described), lack of understanding, forgetting to write "square units" as part of the answer, and/or inability to solve problems.

- Proposal: Based on this what would you propose *and* why?

## *What* have we been doing and *Why?*

- Focus of teacher education
  - What good teachers do
  - What good teachers know
- Is this everything I need to learn how to teach?
- There are certain skills that focus on how you can learn from *your teaching*
  - Teaching involve lifelong learning
- Should these skills be taught in teacher education and how

## I. Learning Goal: The purpose of the **Lesson**

- We can easily discern what is intended for students to learn
- Your observations and work with frameworks made explicit *how you reason about learning goals*
- What do you focus on?
  1. The activities in the lesson
  2. The teacher questions/probes pertaining to the activity
  - ✓ **Learn the most efficient way to justify that a conjecture is true**

## I. Sub Goals: The Goals of the **Activities** throughout the Lesson

- **Not as intuitive**
- **Observe the steps involved getting students to reach overall goal**
- **Learning something new is a process that involves:**
  1. **Performing certain activities**
  2. **Eliciting and using certain knowledge about mathematics**

## I. Sub Goals: The Goals of the **Activities** throughout the Lesson

- **You observed that to get Suzie to learn what was intended she FIRST had to do and know the following:**
  1. **To do this: Think of a Strategy you could use to justify the conjecture → She has to know this: a conjecture is true for all numbers?**
  2. **To do this: Test your Strategy → She has to know this: Knowing what variables are AND how to solve problems with variables? (e.g., "mathematicians rule")**
  3. **To do this: Consider whether her strategy worked → She has to know this: How to think relationally AND represent conjecture using symbols ($a$, $b$)**

## II. On Collecting Evidence

- **USE OBSERVATIONS ABOUT WHAT SUZIE HAS TO KNOW (SUB GOALS AND PRIMARY LEARNING GOALS) TO COLLECT EVIDENCE THAT IS BOTH**
  1. **Related:** Evidence that indicates whether student(s) did and did not learn what was intended
  2. **Relevant:** Evidence of student understanding (verbal and nonverbal behaviors)
- **Evidence will allow support your claims that student(s) learned what was intended**

## III. On Forming Hypotheses

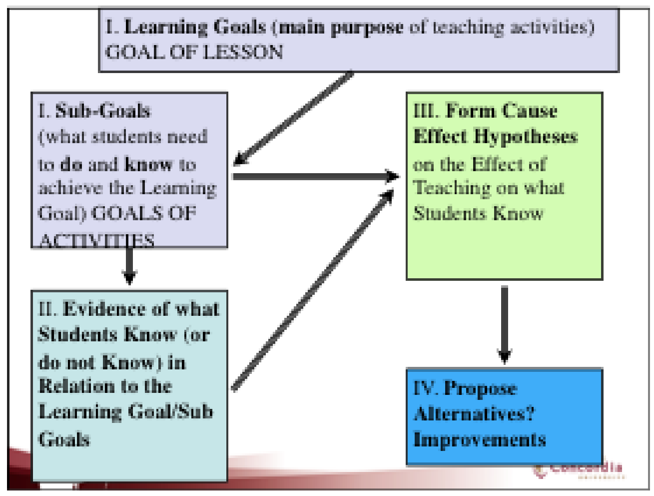
- SEQUENCE OF EVENTS: GOAL OF LESSON AND ACTIVITIES → EVIDENCE OF WHAT STUDENTS KNOW/ DO NOT KNOW → HYPOTHESES
  - YOU HAVE OBSERVED AND RECORDED
    - WHAT THE TEACHER WANTS SUZIE **TO DO** AND **TO LEARN/KNOW** (SUB GOALS AND PRIMARY GOAL)
    - EVIDENCE ON WHETHER LEARNING OCCURRED
- **Make sure your hypothesis is not just a statement about what students know**
- Cause effect relationship on the ways the teaching events (**cause**) could be linked to evidence of what students did or did not learn (**effect**)
- SHOULD ADDRESS BOTH **LEARNING GOAL** AND **SUB GOAL**

## IV. Proposing Alternatives/ Improvements

- BASED ON YOUR EVALUATION OF TEACHING (I.E., YOUR HYPOTHESES)

1) **What would you do?**
   - Based on hypotheses

2) **Why would you do it that way?**
   - Based on evidence collected about what student(s) know in relation to the learning goal




## YOUR OBSERVATIONS

## Examples of Sub Goals

- How to demonstrate conjecture is always true ➔ use of variable
- How to find alternative strategies to solving number sentences ➔ needs to know that a number subtracted from itself is always 0
- Using conjecture to lead to abstraction ➔ capable of identifying conjectures written in broaden context of number sentence

## Examples of Evidence Collected

**What student knows**

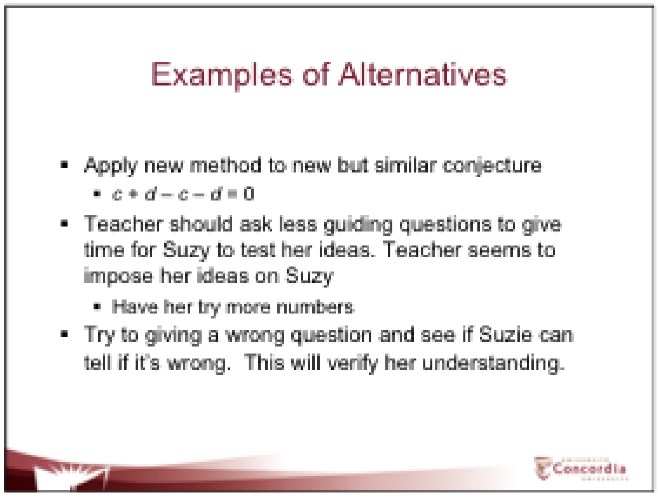
- Student suggests initial conjecture is probably true
- Uses different types of numbers
- Admits that not all numbers can be tried
  - It would take forever
- Student demonstrate knowledge of mathematician's rule
- Identifies $b - b = 0$ as a conjecture
- When using conjectures to prove others: Student identifies $b - b = 0$ and $a + 0 = a$

**What student does not know**

- Student cannot think of alternatives to trying different numbers in number sentence
  - Cannot prove a conjecture without trying different types of numbers

## Examples of Hypotheses

- *Teacher asked how to prove it was always true and the student was able to identify strategies (testing different kinds of numbers)*
- The student was able to identify two distinct conjectures in the variables sentence and relate them to the initial conjecture after the teacher had her consider if b-b was always 0.
- The teacher probes the student to think generally about the conjecture in order to have student think about conjecture as a general statement for all numbers
- The teacher provides clear guiding questions and the student provides clear responses to answer questions and further justify her reasoning/ point-of-view (which ultimately influence Suzie's approach to solve the conjecture)
- By restating the child's explanation and asking further questions, the student can better explain how they can use their current knowledge to explain a bigger task.


Examples of Alternatives
Apply new method to new but similar conjecture
$c + d - c - d = 0$
Teacher should ask less guiding questions to give time for Suzy to test her ideas. Teacher seems to impose her ideas on Suzy
Have her try more numbers
Try to giving a wrong question and see if Suzie can tell if it's wrong. This will verify her understanding.
Concordia

Appendix J

Lab Schedule

| LAB | TOPIC | GROUP |
|---|---|---|
| | | |
| | PHASE I AND II | LG/SL |
| | PHASE III | LG/SL |
| 1 | SESSION 1 | LG/SL |
| 2 | SESSION 2 | LG |
| 3 | SESSION 2 | SL |
| 4 | SESSION 3 | LG |
| 5 | SESSION 3 | SL |
| 6 | SESSION 4 | LG |
| 7 | SESSION 4 | SL |
| 8 | SESSION 5 | LG/SL |
| | PHASE V | LG/SL |

# Appendix K

## Final Observation Frameworks for Each Group

OBSERVATION FRAMEWORK # 1: **Session 5 Group A** 1
EDUC 388/4 TEACHING MATHEMATICS III

STUDENT NAME: ______________________

| **Topic**: | ☐ **Conceptual Understanding** ☐ **Procedural Fluency** ☐ **Adaptive Reasoning** | | | |
|---|---|---|---|---|
| **I. Primary Learning Goal**/What Do We Want Students to Learn | **I. Activity (Example Tasks)** | **I and III. Teacher Cues (Evidence of Learning Goals & Hypotheses)** | **II. Evidence of Student Learning (or Lack of Learning) Verbal and Nonverbal**<br><br>**Related** and **Relevant**<br>☐ **WHAT STUDENTS UNDERSTAND ASBOUT THE TOPIC AND/OR**<br>☐**WHAT STUDENT ARE LACKING IN UNDERSTANDING** | **III. Hypotheses on the Effect of Teaching (Cause) on Student Learning (Effect)** |
| **I. Sub Goal (GOAL OF ACTIVITY):** | | | | |
| **I. Sub Goal (GOAL OF ACTIVITY):** | | | | |
| **I. Sub Goal (GOAL OF ACTIVITY):** | | | | |
| **I. Sub Goal (GOAL OF ACTIVITY):** | | | | |
| | | | | |

OBSERVATION FRAMEWORK # 1: **Session 5 Group B** 1
EDUC 388/4 TEACHING MATHEMATICS III

STUDENT NAME: ______________________

| **Context** | | **Teacher Questions** | **Relevant Student Responses/Strategies** |
|---|---|---|---|
| ☐ **Interview**<br><br>☐ **Level of Student**<br>a) below grade-level b) at grade level c) above grade level | ☐ **Classroom (rate how formal the class is organized scale of 1 to 3)**<br>**1** Typical Classroom set up<br>**2** Less traditional but still used often<br>**3** Not traditional and rarely used | ☐ Probing Questions | |
| **Math Activities**<br>☐ Description of Math Tasks<br><br>☐ Math Tools<br>☐ Manipulatives ☐ Numbers ☐Pictures ☐Paper and pencil | | ☐ Clarifying Questions | ☐Level of Clarity (in expressing mathematical ideas)<br>a) not clear b) somewhat clear c) clear<br><br>☐ Math Terms |
| **Hypotheses** | | | |

Appendix L

Scoring Rubric for Analysis of Learning Assessment

**Skill 1 Assessment Items:**

- **i.** In as much detail as possible, can you identify the overall learning goal of the lesson?
- **ii.** Now consider each activity/task presented during the lesson. In as much detail as possible, describe what the teacher wanted the students to learn for each activity/task.

| **Criteria Description** | **Rubric for Identification Score (Score range from 0 to 4)** |
|---|---|
| **Primary Learning Goal (PLG)**: The identification of a primary learning goal that is both accurate and detailed.<br><br>**Sub Goal 1 (SG1):** Elicit students' understanding of the meaning of the equal sign. Allow students to reflect on their conceptions/misconceptions of the equal sign.<br><br>**Sub Goal 2 (SG2)**: To accept non-canonical sentences as true.<br><br>**Sub Goal 3 (SG3)**: To understand and apply a procedure for determining whether the amounts on either side of the equal sign are the same. | • 1 point if the Primary Learning Goal was accurately identified<br>• 1 point for each correctly identified Sub Goal (Carpenter et al., 2003) |

| Criteria Description | Rubric for Specification of the Primary Learning Goal (PLG) Score (Score range from 0 to 2) |
|---|---|
| 3. Accurately identifies the primary learning goal of the lesson<br><br>4. Describes **what about the equal sign** the teacher wants the students to understand.<br>*Examples*: Conceptual understanding of equal sign, the idea that both sides of the equal sign must have the same value. | **2** ➔ High quality score assigned to the identification of a primary learning goal that is both accurate and detailed.<br><br>**1** ➔Moderate quality score assigned to the identification of a primary learning goal that is accurate but lacks sufficient detail. Or, the primary learning goal is detailed but the description contains some inaccurate information regarding the overall goal of the lesson.<br><br>**0** ➔Low quality score assigned to the identification of a primary learning goal that is inaccurate and lacks detail. |

| Criteria Description | Rubric for Specification of Sub Goals (SG) Score (Score range from 1 to 3) |
|---|---|
| 1. The task/activity cited is accurately matched with its learning goal. Sub goals may be conceptualized in terms of identifying what the task/activity elicits the students to do or what mathematical knowledge is needed to complete the task. *Examples*:<br><br>a. 8 + 4 = ☐ + 5 presented at the beginning of the lesson was used to assess student understanding of the equal sign by eliciting students to share their conceptions (or misconceptions) of the meaning of the equal sign. The students are elicited to reflect on his or her understanding of what the equal sign means.<br>b. At the end of the lesson, 8 + 4 = ☐ + 5 and 15 + 4 = ☐+ 11 was used to assess students' understanding of the meaning of the equal sign by having students apply the procedure used during the lesson to determine what number goes in the box to make the number sentence true. Students must understand how to determine what number would make the amounts on both sides of the equal sign the same. Note that simply stating to "test understanding" is not acceptable as this can be said for any of the activities or tasks presented during the lesson. The activity/task and link to its learning goal must be specific.<br><br>c. By presenting 7 = 3 + 4, the teacher aims to guide students in understanding that the answer does not always immediately follow the equal sign. Understanding that this number sentence is true requires students to accept non-canonical sentences. | 1. Assign a **quality score** accordingly:<br><br>**2** ➔ High quality score assigned to sub goal statements that accurately link task/activity with purpose.<br><br>**1** ➔ Low quality code assigned to sub goal statements that inaccurately link task/activity with its purpose. No link between an activity/task and intended learning.<br><br>2. Assign a **final Specification of SG score** accordingly:<br><br>**3:** The number of high quality scores is greater than the number of low quality scores.<br><br>**2:** The number of high quality scores is equal to the number of low quality scores.<br><br>**1:** The number of low quality scores is greater than the number of high quality scores. |

d. By presenting 6 = 6 + 0, the teacher aims to guide students to accept non-canonical sentences. Students must understand that the sum on the right side of the equal sign has the same value as the amount represented on the left side of the equal sign.

e. By presenting 5 = 4 + 1, the teacher aims to guide students to understand that by finding the sum of the numbers on the left side of the equal sign, the equivalence of the amounts on both sides of the equal side can be verified.

| **Skill 3 Assessment Item**: Form a hypothesis (or more than one) about what the students learned and understand by the end of the lesson. | |
|---|---|
| **Criteria Description** | **Rubric for Criteria Score (Mean score range from 0 to 4)** |
| Developing hypotheses that propose a cause-effect relationship between teaching and learning.<br><br>Connections between teaching and learning can be framed more generally (series of teaching activities) or specific to one teaching activity. | For each hypothesis statement provided, list all criteria (e.g., C1-3, C2-3) that apply. Note that a 0 is assigned when none of the criteria are applicable.<br><br>0-3 ➔ No cause and effect stated. |
| **Criteria 1 (C1-3):** The hypothesis focuses on how students' learning was influenced by teaching activities (Morris, 2006). | C1-3 ➔ Cause and effect stated. |
| **Criteria 2 (C2-3):** The hypothesis provides enough detail (i.e., reference students' observable behavior and responses) to allow the teacher to test his or her hypothesis in subsequent lessons (Morris, 2006). | C2-3 ➔ Details from the video provided about the **effect** aspect of the hypothesis by referencing students' observable behavior. |
| **Criteria 3 (C3-3):** The hypothesis aligns with the principles of the teaching the mathematics content. The hypothesis reflects an understanding of what the students need to know and do in response to the teaching event. | C3-3 ➔ Details from the video provided about the **cause** aspect of the hypothesis. Details must refer to the topic of the equal sign. |
| **Criteria 4 (C4-3):** The hypothesis recognizes the complexity of the teaching-learning relationship. | C4-3 ➔ Some understanding of the complexity of teaching-learning relationship is evident. The information included in the hypothesis reflects an understanding of the principles (goals) of teaching and learning about the equal sign. Reflect an understanding of what the students need to know and to in response to the teaching event in question. |

| **Skill 4 Assessment Item**: If you were the teacher, what would you have done differently? | |
|---|---|
| **Criteria Description** | **Rubric for Criteria Score (range from 0 to 3)** |
| Proposing alternatives to improve the lesson and its impact on students' learning of learning goals based on evidence and reflection. | List all criteria (e.g., C1-4, C2-4) that apply for each alternative statement. Final criteria score is based on the sum of points received. 1 point assigned to C1-4, C2-4, and C3-4. 0 points assigned to C4-4, C5-4, and C6-4. |
| **Criteria 1 (C1-4):** High-quality alternative statements that demonstrate reasoning based on (a) evidence of students' thinking and (b) hypotheses constructed on the cause-effect relationship between teaching and students' learning (Hiebert et al., 2007; Morris, 2006). | **C1-4 (HyP):** The alternative incorporates ideas presented in one of the hypotheses. Note that only hypotheses that received a criteria score C1-3, C2-3, C3-3, or C4-3 are applicable.<br>**C1-4 (Rel):** The alternative incorporates revealing/related student evidence (specific **evidence** response to problem/teacher question in the video) |
| **Criteria 2 (C2-4):** High-quality alternative statements proposed to provide greater insight into assessing the achievement of the learning goal(s) (i.e., greater access to student thinking; Hiebert et al., 2007). | **C2-4:** Alternatives proposed intended to provide greater access to student thinking/reasoning about equal sign. |
| **Criteria 3 (C3-4):** High-quality alternative statements proposed align with principles of the teaching and learning of the main goal and sub goals. | **C3-4:** Alternatives proposed in line with teaching and learning of the equal sign. Changes are designed to alter the tasks used in the video and are explained in the context of teaching equivalence (not teaching strategies in general). Examples include, different types of number sentence, different way to represent the equation, different types of numbers; manipulatives (however the use of manipulatives has to explicitly be explained in the context of equivalence) |
| **Criteria 4 to 5 (C4-4; C5-4; C6-4):** Low-quality alternative statements. | **C4-4:** Other. Changes in managing the classroom (e.g., more group discussion). Changes not specific to teaching equivalence.<br>**C5-4:** States that no changes should be made.<br>**C6-4:** No Code. The statement provided does not indicate whether the preservice teacher would or would not change anything about the lesson. |

Appendix M

Codebook for Interview Data

| Types of Learning Goals | Description |
|---|---|
| **A. Promoting Conceptual and Procedural Knowledge Related to the Equal Sign** | |
| **1.** Understand the meaning of the equal sign | ○ Learn about what the equal sign represents. In a number sentence, indicates that the amounts to the left and right of the equal sign are the same.<br>○ When an equal sign is used in a number sentence, what the equal sign means.<br>○ Understanding the meaning of the equal sign. |
| **2.** Learn the procedure for solving equivalence problems | ○ Understand how to solve equivalence problems using a procedure.<br>○ For example, find the sum on one side of the equal sign. Solving the unknown involves finding the difference between the addend represented with the unknown and the sum represented on the other side of the equal sign.<br>○ Understand how to use a procedure to find the unknown.<br>○ Understanding how to find the number that goes in the box to make the number sentence true.<br>○ Understand how to determine whether both sides "match up". |
| **3.** Develop relational thinking | ○ Understand the relationship between the numbers on either side of the equal sign.<br>○ Use numbers on both sides of the equal without operating on these numbers to solve the equivalence problem (i.e., determine whether true or false, and find the number that goes in the box to make the number sentence true). |
| **B. Addressing Student Misconceptions** | |
| **4.** Address the Misconception that the Answer Comes Next | To have students understand that the equal sign is not a symbol that represents "the answer comes next". In addition to correcting this misconception, this learning goal may also involve having students recognize that they hold this misconception. |
| **5.** Address the Misconception about Non-canonical number sentences | ○ Learn about non-standard forms (e.g., a = b + c; a = a) of number sentences.<br>○ Understand that non-standard are accurate ways to represent equations. |
| **6.** Address Misconception | The specific type of misconception was not specified |

| | |
|---|---|
| not specified | |
| **C. Focus on Student Thinking Associated with the Equal Sign** | |
| **7.** Think about the meaning of the equal sign | ○ To have students think about how he or she defines/understands the meaning of the equal sign. |
| **8.** Reveal prior knowledge on the meaning of the equal sign | ○ To have students reveal to the teacher/class how he or she defines/understands the meaning of the equal sign so that the teacher can understand how students in the class perceive the meaning of the equal sign.<br>○ Teachers can infer students' misconceptions about the equal sign based on their answer |

| | |
|---|---|
| **D. Developing Symbolic Knowledge about the Equal Sign** | |
| **9.** The equal sign serves a role/purpose/function | ○ This code is used when the participant does not mention that the goal is for students to understand the meaning of the equal sign. Rather, the participant mentions that it serves a role, has a purpose. The terms used suggest that they want the students to understand that the symbol is there for a reason. The statement "serves a role" does not indicate that the goal is to understand *what* the symbol represents, but rather, that there is a reason why it is part of the number sentence.<br>○ Use of the term "purpose", "role", "function" not synonymous with "represents". |

| | |
|---|---|
| **E. Developing Skill-based Knowledge** | |
| **10.** Learn to use manipulatives to justify thinking OR use manipulatives to solve equivalence problems | ○ To have students justify his or her thinking about the equal sign using manipulatives.<br>○ To have students use manipulatives as a tool to help solve the problem. |
| **11.** Justify thinking about the equal sign. | ○ To have students justify his or her answer. |

| | |
|---|---|
| **F. Developing Knowledge in Other Mathematics Subject Areas** | |
| **12.** Understand the commutative property | Understand the commutative property. |

| | |
|---|---|
| **13.** Understand the concept of 0 | The goal of the task is to help students understand the concept of 0. |
| **14.** Part-whole knowledge | ○ To develop part-whole knowledge to understand how different parts are related to the same sum.<br>○ Understanding different ways to represent the same amount on either side of the equal sign. |
| **G. Other Learning Goals** | |
| **15.** Solidify knowledge about the equal sign | The goal is to verify whether the intended learning outcomes were achieved. |
| **16.** Recall math facts | To see whether students can recall math facts. |
| **17.** Unclear | A learning goal was referenced but is not clearly related to learning about the equal sign (e.g., the task was about the numbers). |

| a. Canonical Tasks | Description |
|---|---|
| 1. a + b = c | A canonical number sentence where without any unknowns. |
| 2. a + b = ☐ | A canonical number sentence with an unknown result. |
| 3. a + ☐ = c; ☐ + b = c | A canonical number sentence with an unknown addend. |

| b. Non-canonical Tasks | Description |
|---|---|
| 4. a = a | A non-canonical number sentence where the same values are represented on either side of the equal sign. |
| 5. a = ☐; ☐ = a | A non-canonical number sentence where the same values are represented on either side of the equal sign. One of the values is unknown. |
| 6. a = b + c | A non-canonical number sentence without any unknowns. The operation is on the right side of the equal sign only. |
| 7. a = ☐ + c; a = b + ☐ | A non-canonical number sentence where an addend is unknown. The operation is on the right side of the equal sign only. |
| 8. a + b = ☐ + d; a + b = c + ☐ | A non-canonical number sentence where an addend is unknown. The operation is on the both sides of the equal sign. The unknown is on the right side of the equal sign. |
| 9. a + b = b + a | A non-canonical number sentence where the numbers on both sides of the equal sign are the same. |
| 10. a + ☐ = b + a | A non-canonical number sentence where the numbers on both sides of the equal sign are the same. One of the addends is an unknown value. |
| 11. a + ☐ = c + d | A non-canonical number sentence where an addend is unknown. The operation is on the both sides of the equal sign. The unknown is on the left side of the equal sign. |
| 12. a = a + 0 | A non-canonical number sentence using 0 as an addend. |
| 13. a + b = c + d | A non-canonical number sentence where the numbers on both sides of the equal sign are not the same. |

| c. Other Number Sentences | Description |
|---|---|
| 14. a + b ☐ ☐ | A canonical number sentence where the result is unknown and the equal sign is missing. |
| **15.** ☐ = a + b = ☐ | A non-canonical number sentence with more than one unknown. |
| 16. a = b; a + b = a + c | A non-canonical number sentence that is false. A single value is represented on either side of the equal sign. Or, addends are represented on either side. The purpose of the task is to determine how much to add/subtract to make the number sentence true. |
| 17. Students **create** equivalence problems | |
| **18. Tasks Using Manipulatives** | ○ The task itself is to use manipulatives to determine/represent equivalent amounts.<br>○ Comparing amounts |
| **19. Submit definition of the equal sign** | Task requires students to directly state what they know about the equal sign. For example, write out the definition. |

| Reasoning about Learning Goals Code | Description |
|---|---|
| **Equations** | ○ This refers to the types of tasks/problems that were provided to the students by the teacher.<br>○ In some cases they may be referred to as “questions” but the term questions is used to refer to a symbolic expression as opposed to a worded question.<br>○ The strategic use of certain tasks at different points in the lesson |
| **Student Responses** | ○ How the students responded to the tasks/questions posed by the teacher |
| **Task sequence** | ○ The strategic ordering of tasks<br>○ The reasoning used to determine the learning goal is based on rationalization of the order of the tasks |
| **Teacher Behaviors** | ○ How the teacher demonstrated/represented procedures and concepts to the students<br>○ The questions the teacher asked the students<br>○ How the teacher spoke/responded to the students<br>○ Specific words used by the teacher (e.g., “the same on both sides”) to emphasize the meaning of a concept |

| Types of Quality of Language Code | Description |
|---|---|
| **A. Description of the Meaning of the Equal Sign** | |
| **1.** Accurate and Complete | The participant provides a description of the meaning of the equal sign that is both accurate and complete. The description used must be true for all types of examples of number sentences. The description states that the equal sign indicates that the amounts represented on both sides of the equal sign are the same, equivalent etc. Note that when the participant states that the *numbers* on both sides of the equal sign are the *same* does not receive this codes because this does not account for all types of number sentences (that are true). When the participant states that the *numbers* on both sides of the equal sign are *equivalent*, that receives this code. |
| **2.** Accurate and Incomplete | The participant provides a description of the meaning of the equal sign that is both accurate and incomplete. The description used is true for some types of examples of number sentences (e.g., a + b = b + a; a =a). |
| **3.** Inaccurate | The participant provides a description of the meaning of the equal sign that comprises inaccurate information; and thus, it cannot be used to accurately describe the meaning of the equal sign any types of examples of number sentences. |
| **B. Descriptions of Children's Misconceptions** | |
| **4.** Misconception: Add all Numbers | The participant accurately describes the misconception that some children add all numbers to solve equivalence problems. |
| **5.** Misconception: Non-canonical | The participant accurately describes the misconception that some children do not accept non-canonical number sentences as valid (true) equations. |
| **6.** Misconception: Related to "0" | The participant accurately describes the misconception some children have about the number 0. |
| **7.** Misconception: The answer comes next | The participant accurately describes the misconception that some children provide a solution to an equivalence problem based on the notion that the meaning of the equal sign is the answer comes next. |

| | |
|---|---|
| **8.** Sources for misunderstanding the equal sign | The participant provides an example of an **accurate** source of children's misconceptions of the equal sign (e.g., only experience canonical number sentences). |
| **C. Terminology** | |
| **9.** Use of equivalence term | The participant states a term typically (but not exclusively) used in the context of the topic of equivalence. When the participant states these types of mathematical terms *without being prompted to do so by the interviewer*, this code is assigned. Some examples of equivalence terms include, **non-canonical** (or its equivalent, non-standard), **open-number sentences, equivalent, true-false number sentences).** The terms "equals", 'equal sign" are **not** coded because it is to be expected that these terms would be used in the context of the interview. |
| **10.** Prompted use of equivalence term | The participant states a term typically (but not exclusively) used in the context of the topic of equivalence. When the participant states these types of mathematical terms *after the term was stated by the interviewer*, this code is assigned. Some examples of equivalence terms include, **non-canonical** (or its equivalent, non-standard), **open-number sentences, equivalent, true-false number sentences).** The terms "equals", 'equal sign" are **not** coded because it is to be expected that these terms would be used in the context of the interview. |
| **11.** Use of mathematical term | The participant states mathematical terms that reflect a **precise and accurate** understanding of mathematics used for a broad range of mathematics topics. When the participant states these types of mathematical terms *without being prompted to do so by the interviewer*, this code is assigned. Some examples of mathematical terms include **addends, descriptions of properties (the order of the numbers is reversed; 0 added to any number is that number) descriptions of types of numbers (e.g., two-digit numbers, single-digit, fraction) operation (only when referencing situations where numbers are being operated), sum.** Terms that lack precision are not coded such as, **result** (sum, difference, product would be coded as they reflect what the participant is referring to more precisely); **addition; multiplication.** |

| | |
|---|---|
| **12.** Prompted use of mathematical term | The participant states mathematical terms that reflect a **precise and accurate** understanding of mathematics used for a broad range of mathematics topics. When the participant states these types of mathematical terms *after the term was stated by the interviewer*, this code is assigned. Some examples of mathematical terms include **addends, descriptions of properties (the order of the numbers is reversed; 0 added to any number is that number) descriptions of types of numbers (e.g., two-digit numbers, single-digit, fraction), sum.** Terms that lack precision are not coded such as, **result** (sum, difference, product would be coded as they reflect what the participant is referring to more precisely); **addition; multiplication.** |
| **D. Description of the Learning Goals, of the Task, and of the Task Sequence** | |
| **13.** Elaborates on Learning Goals | This code is used when the participant provides more details about the learning goals. The participant explains the learning goal. For example, the statement reviews overall/in general *how* the learning is achieved. These statements are different from stating a specific task. |
| **14.** Justifies Learning Goals | This code is used when the participant explains *why* the learning goal is addressed in the lesson. |
| **15.** Elaborates on Task | This code is used when the participant provides more details about the task. *How* the task will be presented. |
| **16.** Justifies Task | This code is used when the participant explains why the task is included in the lesson (e.g., *in order to, so that)*. |
| **17.** Describes Task Sequence | This code is used when the participant lists the sequence of tasks, indicating the relative placement of each task within the lesson. |
| **18.** Elaborates on Task Sequence | This code is used when the participant provides more details about the sequence of two or more tasks, such as how the teacher would go from one task to the next task. |
| **19.** Justifies Task Sequence | This code is used when the participant explains the rationale behind the sequencing of the two or more tasks included in the lesson (e.g., *in order to, so that)*. |