MODELING AND OPTIMIZING ROUTE CHOICE FOR MULTIMODAL TRANSPORTATION NETWORKS

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ABSTRACT

Modeling and Optimizing Route Choice for Multimodal Transportation Networks

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Traffic congestion has been one of the major issues that most urban areas are facing and thus, many solutions have been developed and deployed in order to mitigate its negative effects. Advanced Traveler Information Systems (ATIS) have been used over the past two decades to provide travelers with pre-trip or real-time traffic information. Most of the efforts have focused on providing timely traffic information at locations with regularly occurring congestion. ATIS can be used to provide travelers with pre-trip and on-route travel information necessary to improve trip decision making with respect to various criteria (e.g. minimizing delay, constraining travel to specific modes). Many jurisdictions within Canada and the United States have implemented the 511 travel information system that provides traffic information, road conditions and closures, traffic cameras, etc.

Several studies were conducted on vehicle routing optimization methods in ATIS. Most of them consider passenger vehicles as the only transportation mode in their routing algorithm. Others that include two transportation modes are mostly based on shortest path algorithms. However, a probabilistic based route optimization approach could better capture the stochastic characteristic of road traffic conditions. This research investigates an adaptive routing methodology for multi-modal transportation networks. A routing algorithm based on Markov decision processes is proposed to capture short-term traffic characteristics of transportation networks. Graph theory is used to model typical travel behavior within a multimodal network.

This thesis proposes to use special network modeling elements, e.g. super nodes, to allow the integration of public transportation schedule into the model via the publicly available predefined timetables. The proposed routing algorithm applies an iterative function to select the optimal transportation mode/route through the network junctions along a given path.

The proposed methodology is applied to several real-world networks of motorized and non-motorized modes located in the central business district in Toronto, Ontario, and Montreal and Longueuil in Quebec. The networks include train, bus, streetcar, subway and bicycle transportation facilities. Microsimulation models of the networks developed in VISSIM and AIMSUN are used to estimate travel times along major arterials, for all transportation modes and for different traffic demands and congestion levels. The simulation models were calibrated using volume and speed data. The developed routing algorithm is applied to several scenarios in order to estimate optimal routes for a hypothetical traveler moving between two arbitrarily selected nodes in the network. The results identify the most efficient combination of transportation modes that the travelers have to use given specific constraints pertaining to traffic and transit service conditions. It is also shown that by applying the proposed algorithm to bus lines, transit agencies can have significant cost savings by rerouting their fleet.

The results of the proposed research have the potential to be integrated into various Intelligent Transportation Systems applications by combining available traveler information services. It can assist travelers in making more informed decisions regarding their travel plans and provide transportation agencies with an overall assessment of the system and its performance. For example, it can be used to minimize the impact of congested traffic conditions on the overall travel time and/or cost incurred by travelers as well as the operating cost of transit agencies.

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To my lovely parents and brother, this thesis is dedicated.

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LIST OF SYMBOLS

Symbol	Description
s_i	Transit Station i
r_i	Transit Route number i
z_i	Transit mode i
$t_r^{z_i}$	Departure/Arrival time at route number r, for mode z_i
$w^r(\tau)$	Link weight (travel time) on route r, at time t
P^T	One-step transition probability matrix
O	Origin node
D	Destination node
N	Number of nodes
I	Number of states
$P^{(n)}$	State probability vector for the n^{th} node
$e^{(n)}$	Expected total penalties (travel time/cost) between node n and destination
$Q_{n+1}^{(n)}$	Matrix of penalties from node n to $n+1$, in each possible state
$\mathcal{R}$	Route list to include all the interconnected nodes that build a route between the origin and destination
$\mathcal{K}_n$	Set of directly connected (adjacent) nodes to node n
$\mathcal{M}_j$	Set of transportation modes related to each node in $\mathcal{R}$
R_{opt}	Optimal route
E_{min}	Expected minimum penalty for the optimal route
T_{tr}	Time-step threshold

LIST OF ABBREVIATIONS

Abbreviation	Description
AMT	Agence Métropolitaine de Transport
APTS	Advanced Public Transit Systems
ATIS	Advanced Traveler Information Systems
CBD	Central Business District
EB	East Bound
GTA	Greater Toronto Area
ITS	Intelligent Transportation Systems
MDP	Markov Decision Process
MTO	Ministry of Transportation Ontario
MTQ	Ministry of Transportation Quebec
NB	North Bound
RTL	Réseau de transport de Longueuil
SB	South Bound
STM	Société de Transport de Montréal
TTC	Toronto Transit Commission
VB	Visual Basic
WB	West Bound

CHAPTER 1. INTRODUCTION

Deployment of Intelligent Transportation Systems (ITS) improves the performance of the transportation system by using modern electronics and communications technologies. Advanced Traveler Information System (ATIS) and Advanced Public Transit Systems (APTS) are two types of ITS user services that aim to ameliorate traffic operations on urban transportation networks. The benefits of ATIS and APTS implementation are more evident under congested traffic conditions, either recurrent (due to morning and afternoon peak travel demand periods) or non-recurrent (e.g. due to incidents that hinder available road capacity).

1.1 Problem Statement

Several studies have shown the benefits of using various ATIS and/or APTS applications to reduce traffic congestion, economic productivity loss and greenhouse gas emissions. Currently, several Canadian agencies have deployed different ITS applications available to the travelling public. For example, up-to-date information about highways is accessible via phone or a dedicated website for several provinces, e.g. 511 Traveler Information Center in Nova Scotia (Road Conditions 511, 2012) and Quebec (Quebec 511, 2012). Moreover, many travel operators provide real time schedule of their services. However, corroborating road traffic conditions with public transportation services and other non-motorized transportation modes information is expected to further enhance the traveling experience by providing more efficient and reliable transportation services. Within large urban agglomerations, this is expected to be beneficial for both the traveling public and transportation operators.

ATIS/APTS can be used to provide travelers with pre-trip and/or on-route information concerning traffic conditions, travel options as well as real-time advice on navigating through the transportation network, where travel conditions may change rapidly several times during the course of a typical day. The major benefits of this type of ITS applications, as shown in several studies, are the expected reduction in travel time delay, typically obtained by providing optimal route information. Mostly, this is done based on the online data made available at any time to the travelers who want to plan or make necessary adjustments to minimize their trip travel times. Moreover, transit operators would benefit by managing their fleet more efficiently and by providing passengers with more reliable services. For example, in case of incidents that cause severe traffic congestion, a transit agency would be able to minimize the disruption to the original timetable and reduce the impact on the operating costs by rerouting buses at certain nodes using real-time information about traffic conditions. This research proposes a novel and versatile methodology to provide adaptive routing in multimodal transportation networks. The proposed methodology is applied to real-world test cases to validate its effectiveness.

1.2 Research Objectives

The main objectives of the research presented in this dissertation are:

[1] to advance a novel routing methodology based on graph theory;

[2] to integrate different transportation modes into the methodology;

[3] to demonstrate that the proposed methodology is able to realistically capture the stochastic effects of traffic conditions within a multimodal transportation network; and

[4] to identify the optimum route within a multi-modal transportation network. This can be achieved by targeting a specific optimization criterion. For example, one may use the proposed modeling approach in order to minimize the impact of congested traffic conditions on the overall travel time and/or cost for travelers and/or transit agencies.

1.3 Research Methodology

The proposed methodology uses a routing algorithm implemented based on a Markov Chain with Reward model. The optimization criterion used by the developed algorithm seeks to minimize the negative impact of congested traffic conditions on users' routing. Several real-world case studies are tested to demonstrate the feasibility of the proposed method. To achieve the proposed objectives, the following tasks have been conducted:

1. Defining a generic representation of each physical transportation network corresponding to different transportation modes (i.e. road, transit and rail). Since public transportation services operate based on a predefined schedule, in order to be able to realistically integrate this type of networks, the developed model accounts for the fixed schedule of public transportation services as well as stochastic variability of the observed travel times.
2. Collecting and processing transportation related data from several sources (e.g. provincial and municipal transportation authorities, etc.) to integrate in the transportation network model. In this task, several traffic data and information sources were considered depending on the location and the type of the transportation networks used. A set of stochastic properties pertaining to the study networks were investigated

- including short-term fluctuation of travel demand, transit schedule and non-recurrent congestions along major arterials. Traffic data (e.g. speed, travel time, etc.) were obtained from transit authorities/agencies.

We have acquired traffic data from the Ministry of Transportation of Quebec. The schedule of the transit services was collected from the *Agence Métropolitaine de Transport* (AMT), the *Société de Transport de Montréal* (STM) and Toronto Transit Commission (TTC). Additional information about the bicycle sharing services (i.e. BIXI in Montreal and Bike Share in Toronto) was obtained from the transportation departments of Montréal and Toronto. A database was developed in Excel to provide concurrent access to the collected data.

3. Developing and calibrating microscopic simulation models in Vissim and Aimsun to estimate travel times for major arterials and all transportation modes under several traffic demand and congestion scenarios. The results were used to emulate the lack of available historical traffic data of the real-world transportation networks used in this thesis to test the route optimization algorithm.
4. Developing an optimal route algorithm that can be used in user equilibrium and system optimal models. The proposed algorithm integrates the time and/or cost (of the trip) constraint into a single performance measure. The algorithm was validated with historical and real-time information about travel conditions in a stochastic and time dependent modeling approach.
5. Developing a traffic state prediction model to better capture the stochastic behavior of transportation networks. The applied methodology uses changes in traffic speed as the

traffic condition indicator to predict congestion level. The results were used to estimate the transition probabilities of Markov Chain model.

6. Applying the proposed methodology to several real-world transportation networks to identify the benefits of developed a routing algorithm. The studied transit network was chosen to investigate the application of the proposed method in on-demand transit re-routing for transportation agencies (e.g. bus line alignment changes with the network traffic). In addition, multimodal networks were built to study travelers' optimized routing in transit networks in case of congestion or unforeseen delays.

The main thesis contributions are as follows:

This thesis presents a new approach in developing an ITS methodology by combining available services and providing an integrated public and roadway traffic application. Previous route optimization studies consider passenger vehicles as the only transportation mode in their routing algorithm. In this research, a methodology is developed to use Markov process in route optimization algorithm for a multi-modal transportation network. The proposed approach applies probabilistic based methods to better estimate the parameters related to the stochastic nature of traffic parameters in a transportation network.

The proposed route optimization model can benefit various stakeholders, particularly transit operators and users, local transit agencies that provide feeder bus services to regional bus passengers to commute within the suburban communities, government agencies and industries related to the ITS. This methodology can be integrated into end-user products that would be beneficial for both travelers and transportation service providers.

For example, transit users would be able to modify their plans and choose other modes of transportation if the current mode experiences delays due to traffic conditions. In regards to transit operators, the proposed method enables them to share information on several transit services and stops/stations and reduce passengers' wait time and/or transit delay costs, assist planners in revising transit schedules periodically and provide real-time routing guidance to bus drivers.

1.4 Thesis Organization

Chapter 2 presents a literature review on the existing methods adopted by authorities and researched by academics for route optimization in transportation networks. The different approaches are discussed and investigated. Limitations and gaps in each approach are presented. Chapter 3 includes the development of the proposed methodology. It starts by developing the optimization methodology based on Markov Decision Process, followed by the traffic state prediction methodology. In addition, it includes a flowchart explaining the implementation of the proposed methodology. An example is presented to demonstrate the application of the proposed methodology in a small network. The chapter concludes with a discussion on improving the state transition probabilities used within a network. Chapter 4 presents three case studies developed to validate and implement the proposed methodology and prove its application in transportation networks. It also includes the data collection stage, which is an integral part of the research and model development. Finally, Chapter 5 summarizes the research and presents the thesis contributions and identifies future work directions.

CHAPTER 2. LITERATURE REVIEW

Given that traffic conditions on real-world transportation networks show stochastic and time dependent properties (e.g. occurrence and duration of non-recurrent congested conditions, fluctuations in demand for transit usage, etc.) transportation operators and passengers can benefit from deployment of ITS applications such as Advanced Traveler Information System (ATIS) and Advanced Public Transit Systems (APTS). By providing travelers with updated travel time/route information, they will be able to make more informed route choice decisions, mainly to minimize travel delay. ITS deployments provide benefits to the public transportation operators. By incorporating adequate online information, transit authorities will be able to adjust the operations of their fleet to respond more efficiently to various conditions hindering normal operation and, consequently, minimize the negative effects on passengers' travel time. An overview of recent studies on the vehicle routing is performed and categorized as described here after.

2.1 Passenger Vehicle Routing

In recent years, many studies investigated different network assignment and vehicle routing algorithms and related ATIS applications. ATIS is intended to improve traveler decision making by collecting, processing and disseminating information that helps travelers decide when to travel, the mode to choose and the route to take. For example, Huang and Li (2007) presented a traffic equilibrium model to evaluate the effect of ATIS as a travel information service on travel behavior. Their multi-criteria, logit-based model used a trade-off between travel time and travel cost to make route choices based on the value of time for different users. The authors assumed that all users select the routes with minimum perceived travel disutility, which is a linear bi-criteria combination of travel time and monetary travel cost. Two types of users were studied: equipped and non-equipped with ATIS. The authors found that their model had a better estimation of network benefits of ATIS compared to other single-criterion models (i.e. travel time-based or travel cost-based single-criterion models).

Other studies had investigated travel time prediction under ATIS. For example, Abdalla and Abdel-Aty (2006) studied the benefits of ATIS in route choice at microscopic level. The authors used a mixed linear modeling approach to study travel time under ATIS. They used a real world network with 40 links and 25 nodes, and vehicle flows in a travel simulator, where a traveler drives in a simulated environment, to generate dynamic route choice data. Travelers were provided with one of five different levels of information and/or advice, including no information, pre-trip/on-route information with/without advice. The authors analyzed travel time of total of 630 trial trips completed by the 63 travelers. The authors' study focused only on drivers using passenger cars and concluded that by increasing the level of information (i.e. adding on-route knowledge to pre-trip information) the average travel time decreased.

Bingfeng et al. (2008) presented a bi-level programming model to determine the optimal system performance of traffic network within an ATIS environment. In their model traffic authority is the decision maker in the upper-level problem, and drivers - with or without ATIS, are the decision makers in the lower-level problem. They used a numerical example to demonstrate the application of model and investigated the traffic behaviors under three cases of the ATIS environment: (i) ATIS provides drivers with parking and route information, (ii) ATIS provides drivers with route information only, and (iii) ATIS provides drivers with parking information only. Their findings showed that ATIS with parking information would be most effective when parking demand is reaching capacity, and the roads are not congested.

Yang and Luk (2008) studied the impact of ATIS on the performance of road network. The authors used traffic simulation module to represent the traffic and calculate network delay as the main performance measure. They considered four categories of drivers, based on the level of access to traffic information. The route choice model proposed by the authors categorized drivers into four groups, drivers with i) no traffic information; ii) pre-trip information; iii) real-time traffic information; and iv) displaying messages using variable message sign, respectively. The method was applied to a case study in Singapore consists of express ways and arterials. The authors also conducted an analysis of different percentage of market penetration (i.e. percentage of travelers that have access to driving information system). Their results showed that providing traffic information to drivers can reduce the total network delay by 7.5%. In addition, they found the optimal level of market penetration for each demand, which would result in the performance of a real-time information system to be better than or equal to that of the pre-trip system. Their results were mainly based on simulation and were not validated with real world data.

Another category of studies are those that evaluated different route choice techniques. The reviewed literature shows that most discrete choice models for route choice analysis are based on static and deterministic networks. Examples of such models are Path Size Logit (Ben-Akiva and Ramming, 1998; Ben-Akiva and Bierlaire, 1999) and C-Logit (Cascetta et al., 1996). These models are non-adaptive path choice models because travelers are not allowed to adjust their route choices on-route in response to the revealed traffic conditions. Several studies of path choice models with real-time information, both pre-trip and on-route, and a recent related literature review can be found in Abdel-Aty and Abdalla (2006).

Some models investigate drivers' behavior to predict the decision to switch from a previously chosen or experienced route and others are route choice models with explicit choice sets of paths. For example, Srinivasan and Mahmassani (2003) studied the effect of observed heterogeneity due to age and gender effects in user decisions under real-time information. Abdel-Aty and Abdalla (2004) investigated drivers' diversion from their normal routes under different scenarios of providing traffic information (i.e. no information, pre-trip information without and with advice, and on-route information without and with advice). Their study network was located in the area around the University of Central Florida and included 25 nodes and 40 links. In total 630 trial trips by 63 drivers were studied. Their results showed that the travel time of the normal and diverted routes are significant in encouraging drivers to divert from normal routes. Also it was shown that expressway users may divert from the expressway if they are guided to a route with a temporarily less travel time. Bogers et al. (2005) investigated learning, risk attitude under uncertainty, habit and the impacts of advanced travel information service on route choice behavior. The authors developed a conceptual framework to integrate these aspects and used the interactive travel simulator of Delft University of Technology (TSL) to investigate route choice among a given number of paths for travelers. They concluded that people perform best under the most elaborate information scenario and that habit with on-route information plays a major role in route choice behaviors.

Ukkusuri and Patil (2007) developed a methodology for traffic assignment by accounting for travelers' recourse actions (opportunity for the traveler to evaluate his or her remaining path when en-route information is available). They applied a methodology based on Logit model and Hyperpaths (i.e. subset of links connecting adjacent nodes with different probabilities). The authors' methodology includes a utility function based on minimizing the total cost of traveling between the origin and destination nodes. In their model the link cost is a function of traffic flow and has to be recalculated for all the links in each iteration. An iterative stochastic user equilibrium approach is utilized to find the most efficient Hyperpath (minimum cost). The authors applied the proposed method to a test network and achieved convergence in less than 100 iterations. They concluded that the methodology could be efficiently adopted for stochastic user equilibrium with recourse.

Some studies proposed various algorithms to solve different routing policy problems. For example, Gao and Chabini (2006) studied the optimal routing policy (ORP) problem in stochastic networks. The authors reviewed different variations of optimal routing policy problem in the literature. They implemented an ORP algorithm that accounts for stochastic dependency among link travel times and they investigated the role of information in routing decision making. Gao et al. (2008) presented a route choice model to capture travelers' behavior when adapting to the provided traffic conditions en-route, in a stochastic network. The authors proposed a routing policy to represent drivers' adaptive behavior. Their routing policy is defined as a decision rule that maps all possible traffic conditions to the next links at a decision node (e.g. Choosing the route between the two traveling points with minimum travel delay). A variable message sign was used to provide information about congestion status on the network links.

Their findings showed that between the routing policy model and the non-adaptive path model, where traveler cannot change their path while on-route, there is a significant difference in terms of expected travel time, when the network is more unpredictable (i.e. the probability of an incident is in the medium range).

Nikolova and Karger (2008) presented an optimal solution approach to find an optimal policy that minimized the expected cost of travel for the Canadian Traveler problem. The Canadian traveler problem is a stochastic shortest path problem in which travelers learn the cost of a link only when they arrive at its connecting junction. The authors applied a mix of techniques from algorithm analysis and the theory of Markov Decision Processes to develop algorithms for directed acyclic graphs. The proposed solution was not validated for other types of graphs.

Other studies investigated in-vehicle routing solutions by using real-time information. For example, Du et al. (2013) proposed a coordinated online in-vehicle routing mechanism for smart vehicles with real-time information exchange and portable computation capabilities. This study considered that at a given short time period, there was a group of smart vehicles which need to make route choice decisions among a number of candidate routes, according to the latest real-time traffic information. The authors proposed a coordinated online in-vehicle routing mechanism and modeled it as a mixed strategy routing game, in which the process that smart vehicles decided their own route choice priorities was treated as a negotiation and coordination process among other smart vehicles. In a routing coordination group, each smart vehicle was seeking to find the best online route choice priority, which leads to the probabilities of choosing the candidate paths with minimum expected travel time. The coordinated vehicles iteratively updated and proposed their routing choice priority in responding to their evaluation of near future traffic condition based on shared online traffic information. The negotiation process repeated several iterations until all travelers accepted and would not change their route choice priorities (i.e. an equilibrium route choice priority decision). The transportation network is represented by a directed graph. At each iteration individual vehicle predicted the expected traffic flow based on the latest traffic flow information and other vehicles' route choice proposals. When new traffic condition becomes available, each smart vehicle computes its new targeting route choice priority through a multinomial logit choice model. The utility function of the model was expected travel time on each path during current iteration. The process of updating traffic condition and proposing new route choice process was repeated until the targeted route distribution was the same as the current route choice priority for all the vehicles (equilibrium routing decision). Authors conducted numerical experiments to demonstrate the performance of their proposed routing mechanism by modeling a sample network of Sioux Falls City. The results showed that by increasing the percentage of smart vehicles, the ratio of average travel times between the proposed and traditional methods became smaller, which indicated shorter travel times under the coordinated routing method. In addition, the authors showed that their method outperformed the traditional routing method, in which each smart vehicle decides its route choice priority independently without coordination.

In another recent study Xiao and Lo (2014) proposed an in-vehicle navigation algorithm based on adaptive control. The proposed algorithm incorporates historical traffic information to minimize the expected on-route travel time. The transportation network is divided into a finite number of nodes or decision points (e.g. intersection) and links (e.g. arterials). The authors assumed a different traffic state at each node and formulated the travel time between two nodes as a function of estimated travel time on the link between two nodes and the uncertainty between actual and estimated travel time. The traffic states were defined as factors that will influence the uncertainty of travel time (e.g. traffic signal) and the travel time was calculated by applying the traffic state vector and the probability of occurrence of each traffic state to the estimated travel times. Ultimately, a cumulative expected travel time from origin to destination was defined and minimized to identify the optimal routing policy. The proposed optimization did not produce a predetermined route for the vehicle. Instead, the next direction to be taken was a function of the arrival state at a node. Therefore, the decision rule was adaptive to the most recent traffic states encountered. The author then compared their methodology with a deterministic algorithm that calculated an instantaneous shortest path and showed that the adaptive routing policy outperformed the instantaneous shortest path algorithm through an example network. Their results showed that, for most of the links, the average path travel times of the proposed routing policy were between 1% and 7% shorter than those of the time-dependent instantaneous shortest paths, particularly when the traffic volume was high. The main limitation of this study is the assumption of conditioning factors that influence traffic state and the variability in travel time. These factors need to be calibrated based on historical data or through several scenarios within simulation models. A more realistic approach to estimate the probability of traffic states should be used. In addition, the authors only considered one single mode (private cars) in their methodology.

The above studies reveal different vehicle routing optimization methods in ATIS. However, they only consider a single transportation mode (i.e. passenger vehicles) in their routing algorithm, while many commuters of large urban agglomerations often times used a combination of at least two transportation modes. This research work includes both private and transit modes in optimal travel route calculations. This approach would enable both travelers and transit agencies to benefit a multi-modal route optimization.

2.2 Transit Routing

There are a few studies about dynamic routing in transit networks. Jeremy and Mathew (2011) developed an optimization method for bus transit system design using intelligent agent architecture, which allows for more efficient evaluation of trade-offs between passenger cost and operator cost. The authors applied their method to transit networks in Switzerland and India, which were previously investigated. According to the authors, for both networks the agent optimization system improved on the best of the previous solutions, both in terms of operator cost and passenger utility. Wang et al. (2009) presented a simulation-based optimization method for campus bus routing. The objective of their study was to find the minimum-cost route, while minimizing each passenger's inconvenience by satisfying their request (fewer complains). The authors used numerical experiments to validate their proposed simulation model. The method was applied to a High-Tech zone in Dalian city in China. The authors conducted a numerical experiment based on the real data from a university campus bus service to evaluate the validity of their proposed model. Their new vehicle routing methodology was tested to divert a bus away from its fixed route in response to a new customer request and was found to be beneficial to high-tech zone campus bus managers by reducing their costs.

Fu and Lam (2014) presented an activity-based network equilibrium model for scheduling daily activity travel patterns in transit networks under uncertainty. The authors uses supernetwork to simultaneously consider individuals' activity and travel choices (i.e. time and space coordination, activity location, activity sequence and duration, and route/mode choices). They assumed the activity utilities to be time-dependent and stochastic in relation to the activity types and modeled activities with different durations or different start times as different activity links. The authors proposed a route searching algorithm based on method of successive average to solve for equilibrium. The objective was to maximize a daily activity utility function by considering value of time and link travel costs. The studied network consisted of one subway line and two bus lines. The results of their study showed that individuals' travel choices were affected by travel dis-utilities of different transit lines. For example, when the network was not congested (i.e. low population), the dis-utilities of different transit lines were all quite small and therefore, the percentages of people choosing different lines are almost equal. As the population increased, they found a significant difference between the demands for two bus lines until the network became extremely congested where the individuals had little preference towards two bus lines. This network only included transit (not road network). The authors used a very small network to apply their proposed methodology and their results were not calibrated with real data. Moreover, the effect of road congestion and variations in travel times was not considered in the analysis.

Crainic et al. (2008) used a demand adaptive model to capture the behavior of transit systems with mandatory and optional stops (i.e stops requested by passengers and that may lead to changes in the default bus route). The authors suggested that a master schedule has to be developed based on time windows associated with mandatory stops. The authors employed a particular sampling technique to solve a master schedule problem for a single bus line. The efficiency of the proposed methodology was tested using various lengths and scenarios of the hypothetical transit line.

In a different study, Panou (2012) investigated the optimization of public transport (PT) information services that are provided on mobile devices, for travelers of PT means, through their personalization. The author proposed an algorithm along with the necessary parameters (dynamic and semi-dynamic) that supported a holistic personalization, based on each user specific profile and the history of their previous selections. The dynamic parameters were calculated automatically by the system and included: Walking distance, preferred transit mode, Number of changes between transport modes, distance and cost of each route. Semi-dynamic parameter was chosen each time the traveler used the application and included the reason for travelling (Tourist, Commuter, Recreational or Emergency). During the learning process of user preferences, the selected parameters were monitored and the corresponding values were stored based on the selected route, each time the user makes use of the system. The history of user preferences was used for future route recommendations. The author tested the proposed model with 10 users and evaluated its performance by providing users with a questionnaire and asking for their feedback. By using an average scoring value for the questions, the author concluded that the users had a positive opinion for the application of personalized routes provided. This study showed the possibility of providing beneficial information about travelers routes/modes via mobile tools. However, only the scheduled travel time of transit modes are used for analysis.

Lin and Bertini (2004) proposed a Markov chain model for bus arrival time prediction that captures the behavior of bus operators in putting delayed or ahead of schedule buses back on their predefined schedule. They used a link-node representation of the bus network. The proposed methodology is demonstrated for a hypothetical case of equally spaced bus stops and a possible solution for more realistic bus lines is discussed. They suggested using three performance measures to evaluate the effectiveness of their prediction algorithm: overall performance (minimum total prediction error), robustness (minimize the occurrence of large deviations) and Stability (prediction of travel time does not fluctuate from time to time). The transition probability in the proposed model represented the conditional probability of a bus being delayed at downstream stop, given the delay at current stop. The authors suggested integrating the proposed model to a bus arrival prediction algorithm. There are several limitation in this study. The authors assumed that bus tops are uniformly spaced. The effectiveness of this algorithm was not tested with actual data from transit operators. Moreover, the authors did not use real data to calibrate their transition probability matrix.

In another study, Wong (2009) developed a dynamic mathematical model to estimate regional bus journey time using Artificial Neural Networks (ANN). The model updates bus arrival time using real-time Global Positioning System (GPS) information and also real-time highway loop detector data of volume, speed and occupancy. The ANN model was used for predicting arrival time of one bus route within Toronto and was compared to two other forecasting methods, historical average and linear regression, and outperformed them by an average of 35 and 26 seconds in travel time calculations respectively. The author reports overestimation of arrival time in their model, which could result in passengers missing the bus. This study only predicts the bus travel time travelling mainly along EB direction of Gardiner Expressway in Toronto and does not consider other bus lines travelling on parallel arterials. Also, due to limited available data, the time period used in this covers only morning period: 5:30 till noon and does not consider Toronto's rush hour traffic during PM peak.

Polyviou (2011) proposed a simulation-based model capable of modeling the details of bus and traffic incidents (SIBUFEM) in order to assess the impact of incidents on overall bus performance and suggest potential fleet management strategies for improved efficiency. The author considered three key performance measures to evaluate the effect of incidents: (i) average bus journey time, (ii) average bus speed and (iii) average excess waiting time. The author used data from the Portswood corridor bus route in Southampton, UK to calibrate and test the model. The results showed that the higher the severity (capacity reduction) of a traffic incident, the higher is the expected impact of the event on the overall bus performance. Their finding showed that 25% and 40% reduction of capacity caused 0.25 and 0.54 minutes average increase in travel time respectively. Also, the author concluded that a longer duration of a traffic incident causes more severe effects on the overall bus performance: i.e. similar incidents lasting for 20, 40 or 60 minutes, caused 0.1%, 1.3% and 2.8% increase in the average bus travel time. The authors suggested that the travel time results of SIBUFEM could be used for further evaluation of the impact of the incidents on bus patronage (ridership) levels. This study quantifies the expected delay for one bus line during limited number of scenarios however, it does not provide any re-routing solution via alternative route. Moreover, There are other limitations in the simulation model: the computer model was not calibrated for vehicular traffic and the traffic signals were not coded and their effect was replaced by an additional delay. In addition, the severity of incidents is coded as a reduction in the road capacity, while a microsimulation model that simulates different lane closures and interactions between vehicles trying to change lane, could provide better and more realistic results.

Wang and Cheng (2012) proposed an allocation method for increasing the Public Transit (PT) level of service in an urban network. Their proposed method is based on Hub-Spoke structure that integrates PT lines with transfer hubs. Their approach focused on planning bus rapid transport (BRT) and regular bus lines. The authors did not include other transit modes (e.g. subway) nor transfer hubs (i.e. terminals or transfer stations) in the optimization process. They used an objective function for BRT line to maximize the operating efficiency (ratio of passenger person kilometers to total kilometers traveled by all buses over a day) within the network. Similarly, for bus lines, their objective function was to maximize the density of nonstop passenger volume (the ratio of the nonstop passenger volume of the bus line divided by the length, i.e. distance, of the bus line). Nonstop passenger was referred to a traveler who would not get off the bus until the final stop. They applied the proposed method to a PT network with 16 Traffic Analysis Zone (TAZ) in the City of Fuzhou in China. Operating efficiencies of feasible bus lines between two transfer hubs were calculated and the path with maximum efficiency was selected as the optimal BRT/bus line for that path. The authors proposed their methodology could be used in PT line planning. This study has some limitations. The solution method and the constraints used for the analysis need validation. Any modification to the network size has a large effect on the solution time.

Other studies have addressed public transit schedule reliability and system efficiency issues. For example, Teklu et al. (2007) evaluated the transit assignment of systems characterized by small capacity buses (12-20 passengers) and stochastic headways (no timetables). Particularly, the authors proposed a composite frequency-based and schedule-based Markov process model for capacity constraint transit networks. Their model included bus and passengers' simulator and a random utility model for transit route choice. Their approach assumed passengers' routes were defined by a sequence of transfer stops connected by alternative route-sections to represent the attractive lines passengers could choose to travel on. Their generalized cost function consisted of in-vehicle travel cost, waiting cost and transit fares. A multinomial Probit route choice model that considers the cost correlations between alternative routes was used to model passengers route choice. The authors applied their model to a small network with 4 bus stops and 2 bus lines. Their results showed that passengers on relatively congested sections of the network experienced higher cost variability, due to the additional stochasticity associated with finding spare seats in the buses. The authors concluded that their model could be applied for network where transit vehicles are small and not operating to timetables to represent the variability in flows and costs and enable planners make more informed decisions.

The above literature shows applications of route choice models in evaluation of performance and schedule reliability and estimation of arrival time in public transit network. These studies mainly consider one transit mode models which can be further developed by including other transportation modes (i.e. private and other public transit modes). By including several transit modes, travelers would be able to modify their plans and choose other modes of transportation if their current mode experiences delays due to traffic conditions. Moreover, commuters using their own vehicles could have a better planning for their trips if the option of switching between car and transit is available to them.

2.3 Multimodal Transportation Network Models

Multimodal transportation involves the usage of at least two modes of transportation to complete a single trip. Common modes of urban transport today mainly include car, bus, rail, motorcycle, bicycle and walking. Multimodal networks inherently offer redundancy and flexibility by offering multiple choices and routes, while mitigating the negative effects of traffic congestion. On the other hand, transfers between different modes carry a certain overhead in terms of waiting time and convenience and sometimes are a key factor in the decision of making a specific multimodal trip. Modeling multimodal transport requires identifying the availability of various transportation modes at specific locations and the ability and reliability of transfers between these different modes.

A limited number of studies attempted to model multimodal transportation networks that include both private and public transportation vehicles. Nagurney and Smith (2003) proposed to represent this type of networks as a supernetwork. Supernetwork framework allows one to formalize the alternatives available to decision-makers, to model their individual behavior and to compute the flows on the supernetwork, which may consist of travelers between origins and destinations as well as the associated costs. The supernetwork has the advantage that it can model simultaneously multiple physical networks while accounting simultaneously for different trip features (e.g. route choice, transfer/waiting time, cost of transfers, etc.). The authors presented an overview of development and application of the supernetwork concept in transportation and decision-making concepts. The authors did not provide any specific case study or real world example of such modeling approach.

Zhang et al. (2011) presented a generic multimodal transportation network model for ATIS application to be used for large-scale transportation systems. The authors proposed using a supernetwork framework that integrates individual networks representing different transportation modes. Their model included dynamic travel times and timetable of public transportation services. The authors used the basic Dijkstra algorithm for routing purposes and they used the travel time as the performance measure for best route choice. Their results indicated that the model could be used to find optimal routes in short computation time for realistic networks. The main limitation of their model was long computation time to read and compile the integrated network, which depending on the size of network may take several hours.

Casey et al. (2013) presented an analysis of the computational performance of two shortest path algorithms for a multimodal multiobjective trip planner tool. The authors used Graph theory to create the road and public transport networks where a set of nodes and links that connect neighboring nodes together. They compared the performance of two shortest path algorithms: simple Dijkstra and A* heuristic, which improves upon Dijkstra's algorithm. Dijkstra's algorithm only considers the cost to travel from the origin node to the candidate nodes and sorts candidate nodes in the order of their cost from the origin node. However, the A* heuristic method estimates of the cost to travel from the candidate node to the destination node, plus the calculated cost from the origin node to the candidate node, and orders the nodes based on total origin-destination cost. They applied the proposed methodology to an area of suburbs (as origins) and major destinations (e.g. CBD and airport) in the South East Queensland region in Australia. A set of constraints were set for the analysis which included: maximum number of transfers and maximum walking/transfer distance. The travel time value was used as the performance measure. The authors concluded for road only network, A* outperformed Dijkstra's algorithm while for public transport and/or multimodal networks, using Dijkstra as the shortest-path algorithm produces adequate results, with the average search completing within 5-10 seconds compared to minimum 15 seconds in the A* method. This study did not include any real time information and cannot be implemented in time dependent networks.

Meng et al. (2014) presented a dynamic traffic assignment model for urban multi-modal transportation network by constructing a mesoscopic simulation model. The authors used MesoTS simulation laboratory previously developed by Yang (1997). The proposed model updates the path travel time at the beginning of each iterative phase, finds the shortest path with the k-shortest path algorithm, and finally assigns the traffic flow based on a C-Logit model. Travel utility function was used to calculate the updated link travel times at the beginning of each iteration. The k-shortest path algorithm is an extension of the typical Dijkstra algorithm with the possibility to calculate a set of shortest travel time paths, and determine the distinct set of the shortest path numbers according to different criteria. The authors applied their proposed model to an area in the Chaoyang district in Beijing. The network consists of 5 subway lines with 39 stations, 18 bus lines with 51 stations and 188 road links with 122 nodes. The authors conducted several experiments to study the effect of different factors (e.g. increase in demand, parking fees and car ownership) on the percentage of car and transit (bus/subway) travel trips. In one of the conducted experiments, they examined the effect of traffic information on the travel mode choice. The results showed that when the car transfer information was not provided, (drivers had no opportunity to transfer to other modes) the private car trip increased from 61.5% to 86.5%. Similarly, when the transit transfer information was not provided, the one transit line trip increased from 3.9% to 6.2% (lower number of mode changes due to lack of transfer information). In both cases the average travel time increased by 1-2%. Finally, when no transfer information was provided, most of the travelers chose the private car trip, and the average travel time increased as the traffic congestion aggravates. Authors reported slow computation process for the time-dependent shortest path algorithm. In addition, this methodology does not account for real-time traffic information and changes in traffic conditions.

Arentze and Timmermans (2004) developed a network representation of multimodal transportation systems that allows the modeling of multi-activity, multimode routes by means of a standard least-cost path finding method. A case study was tested along the Almere–Amsterdam corridor in the Netherlands. The purpose of their study was to assess the sensitivity of activity-travel choice behavior on the travel time and cost from Almere to Amesterdam. The authors implemented a two-activity program (working and shopping) under various specifications of cost functions (e.g. value of time, transit tickets, parking fees and penalty for inconvenience of transferring to another mode). Their results showed that key choices such as main mode, access station, activity location and making an intermediate trip home were strongly interrelated and fairly sensitive to prices, search times, activity–location preferences and the activity program. The authors found that a secondary activity during the trip might work in favor of transit use because of extra costs generated for cars related to parking. Their model identifies least cost path based on several assumed parameters that need to be calibrated with real data. The proposed methodology does not account for changes in travel speed/link cost during the trip.

Abdalla and Abdel-Aty (2006) studied travelers' mode/route choice behavior under different levels of Advanced Traveler Information System (ATIS). ATIS is a group of services that provide travelers with information that will facilitate their decisions concerning route choice, departure time, trip delay or elimination, and mode of transportation. The authors recruited 65 subjects and instructed them about the experiment, they combined a travel simulator with real network and traffic data in order to model five mode/route choices under ATIS: i) travelers' mode choice (Car vs. bus); ii) drivers' diversion from the normal route; iii) drivers' compliance with pre-trip advised route; iv) driver's compliance with on-route short-term traffic information choice; and v) drivers' long-term route choices. The authors showed that travel time and familiarity with devices that provide the information are the factors that have significant effects on drivers' behavior. They also found that qualitative information (e.g. showing congestion level by using different colors) is more beneficial than quantitative information (e.g. travel time of roads) to drivers in assisting their on-route short-term choices. In addition, a high number of traffic signals on a route increased the probability of diversion. The authors suggested that their findings could be used to enhance ATIS devices and incorporated in dynamic network assignment models.

Arentze (2013) proposed a Bayesian method to incorporate the learning of users' personal travel preferences in a multimodal routing system. The proposed method learns the preference profile of a user (as parameters of link costs functions) incrementally from observations of preferred travel options (routes) in choice situations. The author applied multinomial-logit framework to model the choice behavior as a function of preference parameters/route attributes (e.g. travel time, walking time, travel/parking cost). The data were obtained from a travel choice experiment where 438 individuals were presented travel alternatives and indicated their choice for a trip of approximately 20km. The choice alternatives presented to individuals consisted of using car for the entire trip, using public transport for the entire trip and using a combination of public transport and car. The application of logit model for route choice evaluation in a multi-modal transportation network in this study is based on predefined link travel time/costs.

Khani et al (2012) proposed an algorithm to find the optimal path in an intermodal urban transportation network with multiple modes (auto, bus, rail, walk, etc.). Their proposed method found the optimal path according to the generalized cost, including private-side (travelers), public-side (transit agencies) and transfer related travel costs. The authors applied a trip-based transit shortest path algorithm and a label-correcting algorithm using park-and-ride facilities to find the best transfer location (i.e., park-and-ride) from the origin, considering the cost for the transit part of the trip. Optimal path was chosen based on the time-dependent shortest path algorithm for cars and transit and mode change links (access and egress links between nodes in the auto network/transit stops). To reduce the number of iterations/complexity of transit network, the authors only considered transfer stops as the eligible alighting nodes to be scanned during the process. The results of their study showed that applying the proposed shortest path algorithm to both car and transit routes could improve the computational performance by 75%. Nassir et al. (2012) applied the proposed algorithm by Khani et al (2012) to find the intermodal optimal tour (origin to origin) in time-dependent transportation networks for a traveler with a sequence of destinations to visit. The authors proposed a methodology to identify the best combination of modes and park-and-ride (transfer) locations to allow traveler visiting a sequence of destinations, as well as the optimal path for each segment of trip. Their proposed mathematical approach minimizes objective function with the following decision variables: i) availability of link from the current node to adjacent node that serves destination, ii) waiting time before departure, and iii) time of arrival to current node. Authors applied the proposed method to the Rancho Cordova bimodal network in Sacramento with 447 nodes, 850 links in the auto network, 163 bus stops, 6 bus routes and Two park-and-ride facilities that connect the auto network to the transit network. The optimal path found for selected origin and destination was 62 min long and uses auto-only mode, vs 71 and 78 minutes for the two alternative routes that used one of the two available park-and-ride nodes. The above studies improved the computational time for evaluating optimal route, with minimum travel time, in multimodal network, however, they did not consider the stochastic attribute of traffic flow (variable traffic condition/travel time).

The above literature review reveals a limited number of studies on route optimization in multi-modal transportation networks. Among them, mainly a simple Dijkstra Shortest Path (DSP) algorithm or a routing policy based on DSP was used to identify optimal routing, while optimization constraints are very basic. The optimization used in these studies mainly included the shortest distance or travel time based on a constant speed/traffic condition.

Due to the stochastic nature of traffic congestion and travel time/delay parameters in a transportation network, several authors proposed probabilistic modeling of different ATIS applications (e.g. estimation of expected freeway travel time, bus arrival time prediction, transit network assignment). The next section provides an overview of different route choice models used in transportation followed by a brief explanation of one of the frequently used probabilistic method, i.e. Markov Chain.

2.4 Route Choice Models for Transportation Networks

In this section an overview of the models used to generate routes for navigation within transportation network are presented. Traditionally traffic assignment models assumed very simple route choice that assumed drivers behave as if they have perfect knowledge of route cost/travel time. The most common route optimization method is the shortest path algorithm, such as the Dijkstra algorithm (Dijkstra 1959). Dijkstra's original variant found the shortest path between two nodes but a more common variant fixed a single node as the source node and finds shortest paths from the source to all other nodes in the graph, producing a shortest path tree. Attempts to improve the computational speed of the Dijkstra algorithm were reviewed by Wagner and Willhalm (2007) which can be broadly classified into two classical techniques, i.e., the bidirectional search and the goal-directed search, which is more commonly known as the A* algorithm proposed by Hart et al. (1968). Basically, these algorithms maintain two node lists, i.e., an open list from which a potential successor node is selected and a closed list of nodes that have been already selected.

The assumption of perfect knowledge of travel cost for drivers had been long considered inadequate for travel behavior. Consequently, probabilistic route choice models were developed in which drivers were assumed to minimize their perceived costs given a set of routes. An important extension of the simple shortest path approach is the generation of alternative paths, such as the k-shortest path approach by Eppstein (1999). The k-shortest paths algorithm lists the k paths connecting a given source-destination pair with minimum total length. Bell (2009) modified the classical A* shortest path algorithm to generate a set of attractive paths, which are called hyperpaths with a min-max exposure to delay strategy, leading to a link use probability inversely proportional to maximum link delay for attractive links leaving a given node. The advantage of hyperpaths is that multi-paths can be generated to accommodate drivers' preferences.

Several route choice models in the context of Stochastic User Equilibrium (SUE) were also developed. The 'stochastic' term is related to the probabilistic route choice model, instead of simply assuming the shortest path as in the deterministic user equilibrium model. SUE route choice models are generally derived from utility theory. The utility function cannot be measured directly. Therefore, to take into account the effects of unobserved attributes and characteristics, the utility of an individual is assumed to have a deterministic (or observable) component and a random component (error term). In these models, the stochasticity of perceived travel costs is accounted for by the random variable. Multinomial Logistics (MNL) followed by Multinomial Probit (MNP) regression models were among the early SUE route choice models.

The MNL model structure cannot capture similarities among alternatives and hence is not suitable to model route choice. This is because in typical networks, there is a fairly large amount of overlapping links among routes, which cause the violation of the basic assumption of the MNL model, i.e. the independence of irrelevant alternatives. Despite the theoretical problems, the MNL is still used in stochastic traffic assignment procedures. Later modifications of the MNL model, such as C-logit, were also adapted to route choice. These models overcome the overlapping problem while still retaining the MNL structure. In C-logit model, the similarity among routes is modelled by including a commonality specification in the deterministic component of the utility function. The commonality factor of path in such model is a measure of the degree of similarity between the subject path and other paths in an OD pair. The MNP was proposed by Daganzo and Sheffi (1977) to model route choice and is based on the assumption of a normal distribution for the random component. The calculation of the Probit choice probability when the number of alternatives is greater than two is not straightforward. While several methods were presented in the literature to evaluate the MNP choice probability by applying analytical methods, the analytical approximation methods cannot be applied for a large number of alternatives because the accuracy of the method deteriorates as the number of alternatives increases. Prashker and Bekhor (2004) reviewed different SUE route choice model and compared them with two numerical examples. They illustrated the importance of the specification of the similarity measures in the route choice model. Authors concluded that models that performed well in the two above networks may not perform as well in other examples.

Route choice models are incorporated as part of a bigger model system such as traffic assignment, route guidance and network design. For this reason, the choice models used in practice are simple, generally consisting of finding the minimum cost path. In state of the practice models, the MNL or modified versions like C-logit are implemented. The reviewed literature identifies also an alternative routing approach based on a dynamic discrete choice modeling. Travelers are modeled as taking on route decisions related to which link to use in order to better capture the short-term characteristics of transportation networks. This can be conducted by incorporating Markov property in the stochastic algorithm and is described in the following section.

2.5 Application of Markov Analysis in Transportation

Markov chain is a technique for statistical modeling of a random process in which the state of system changes through progression (i.e. system's evolution during time). A Markov chain is defined with the set of state definition, initial probabilities and transition probabilities. The transition probabilities are associated with the manner of state progression during the system evolution (the probability of system transitioning from one state to another). A system which has the Markov property satisfies the following: the conditional probability of the system being at the next state, S+1, given the current state, S, depends only on the current state and not on the previous states of the system.

Markov Process is a probabilistic model useful in analyzing traffic states and estimation of travel times. In Markov process the transition behavior is different from that in a Markov chain. In each state there are a number of possible events that can cause a transition. As a result, in this model transitions take place at random points in time. Markov analysis evaluates a given sequence of events in order to estimate the tendency of one event to be followed by another. Using this analysis, one can generate a new sequence of random but related events, which appear similar to the original sequence. The Markov model assumes that the future is independent of the past given known present conditions.

Markov Chain and Markov Process are two basic Markov analysis methods that have been widely used in transportation field, including traffic conditions analysis and transit schedule reliability. Most studies use the states of the Markov chain to represent different congestion levels along the transportation network links. Some studies investigated various problems related to traffic flow and control. For example, Yeon et al. (2008) investigated the application of Markov Chain for estimation of expected freeway travel time. They developed a model to estimate travel time on a freeway using discrete time Markov Chains where the states correspond to whether or not the link is congested. The transition probability matrix for Markov Chain was calculated by estimating link travel times for non-congested and congested conditions. They applied their methodology to an 8-mile freeway section along US 202, in Philadelphia, PA. Field measurements were used to validate the model. T-test was conducted to compare the expected travel time to measured travel time. The model developed was found to match field travel time estimates very well, with deviations of less than 3%. In this study only passenger vehicle routes along a short segment of highway are considered. The methodology lacks the inclusion of other modes within a large-scale transportation network.

Dong and Mahmassani (2009) proposed a methodology to predict travel time and its reliability in real time based on real-world measurements in light of the probabilistic nature of flow breakdown. They applied Markov chain approach for modeling traffic flow evolution that enabled prediction of flow breakdown probability, as well as the resulting flow rate. They defined each state of the Markov Chain by a flow–speed pair, namely a flow range and a speed level. They categorized flow rate range into several equal-width bins and as for speed it was categorized into two levels, high and low. To calibrate the transition matrix speed and flow rate, authors used data at 5-min intervals during a 1-year peak period collected at the Jeffrey section of I-405 northbound, where recurrent traffic breakdown had occurred during the morning peak period. The expected duration of breakdown was also derived from the probability transition matrix. Study suggested that to predict travel time and its variability along a path, the mean and variance of the constituent links could be summed up, assuming all the links are independent. Authors suggested that when the flow rate and speed are readily available from traffic sensors, their proposed methodology could be used to provide real-time traveler information. This study only considered traffic conditions along highways and for cars only. They did not provide any real case example for the proposed methodology.

Geroliminis and Skabardonis (2005) proposed an analytical model for platoon arrival profiles and queue length prediction considering platoon dispersion in arterial using Markov Process and loop detectors data. Traffic between successive traffic signals was modeled as a two-step

Markov decision process. Markov decision process is referred to Markov chain system where a decision can be made at each step. Platoon dispersion was modeled by using the kinematic wave theory and a functional relationship between the traffic flow and the traffic density and could be used to describe the speed at which change in traffic flow propagates either downstream or upstream from an origin point. Authors applied their proposed model to two real-life arterials in Washington, DC to estimate the queue length on each intersection approach. Their results indicated that the proposed model produced accurate estimates of queue lengths. In most of the cases, the difference between the model-predicted and the simulated queue lengths was less than four vehicles, with maximum 10% error. Although the main outcome of this study was analyzing queue propagation along signalized arterials, the methodology proved a successful implementation of MDP.

Other Markov chain applications in transportation attempt to solve transit network loading problems. For example, Kurauchi et al. (2003) investigated adaptive routing under uncertainty for passengers in a given transit network. They presented a capacity constrained transit assignment method that considered passenger strategies. Passengers could decide which transit lines to use based on minimizing an expected cost of travel. Expected travel cost included the cost of a risk of failing to board a train. To assign the traffic to the network, a Markovian loading process was applied. Their Markov chain used a transition matrix defining the probability of a traveler moving from one state to another. The states of the Markov chain in their model represented the origins, the intermediate vertices of the network and a destination. Graph model was used to represent the network with stop nodes and links connecting stops together. The cost of an arc path consisted of three elements: travel time, expected waiting time, and the implicit cost associated with the risk of failing to board. Their objective function minimized total cost to find the optimal path. This model considers the risk-aversion of passengers to overcrowded stations and combines the computation of common-line strategies with a probabilistic approach in which the boarding probability is determined by the residual capacity of the transit vehicles. Authors applied their proposed model to an example network consists of three stations and two transit lines and travel demand between OD pairs of 100 passengers. For each OD pair they identified the optimal routing strategy of combining one or both transit lines. They results showed that by increasing the risk awareness of passengers and when common lines are considered, passenger flow split between lines which is a realistic behavior.

Other applications of Markov processes in transport-related literature include indicatively pavement management and bridge maintenance management. Abaza et al. (2004) designed an effective decision-making tool for planning and scheduling of pavement maintenance and rehabilitation (M&R) work. Their system applied a Markovian model to predict pavement deterioration with the inclusion of pavement improvement resulting from M&R actions. Authors used a decision policy with two major options: i) optimizing a generalized objective function that is defined in terms of proportions of pavement sections in the five deployed condition states (i.e. excellent, good, fair, poor, bad), and ii) minimizing M&R cost subject to preset pavement condition requirements in terms of state proportions at the end of a selected study period. A Markovian model with discrete-time transition matrix was used to incorporate the five condition states and to predict process by applying the estimated transition probabilities. They tested the developed model to a total 20 lane-kilometers length of surveyed pavement sections.

Similar study by Ortiz-Garcia et al. (2006) showed that a Markov Chain process could be used in the determination of pavement deterioration. They proposed the development of three methods for the determination of transition probabilities and subsequently tested on six different sets of artificial data. In their first method, it was assumed that the original data, i.e. historical condition data for each of the sites, were available. The second method used regression equation obtained from the original data to estimate the transition probabilities. In this method, the raw data is used for estimation of transition probabilities after a regression equation has been obtained to describe the progression. Finally, in the third method the raw data were aggregated into bands of pavement condition and presented in the form of distributions. Their objective function aimed at minimizing the difference between the distributions of condition obtained from the raw data and the distributions obtained from the transition probabilities. Authors concluded that their third method yielded a distribution closer to the original distributions compared to the other methods.

In a more recent study Ramezani and Geroliminis (2012) applied Markov Chain procedure to estimate arterial route travel time distribution. They used a 2D diagram to graphically represent the joint distributions of successive link travel times. A Markov chain procedure was incorporated into the model and its initial and transition probabilities from identified from the observed data. Their raw measurements were experienced individual link travel times traversed by a set of probe vehicles, followed by finding trajectory of moving vehicles and link travel times. Afterwards, authors used travel times of all probe vehicles crossing two successive links during data collection period to construct a 2D diagram as a graphical representation of vehicles travel times joint distributions and defining the Markov chain states and determining the initial and transition probabilities. The data points in 2D diagrams represented travel times of a probe vehicle crossing two consecutive links. Authors tested their proposed model in two arterials, 650m and 1.1km long and each with 5 intersections. They estimated travel time distribution (TTD) for different demand and probe vehicle sample size. The Mean Absolute Error (MAE) between the observed and estimated TTD was calculated. The results showed a coherent performance capturing the fundamental characteristics of field measurements even under condition of low sample size for probe vehicles. They also compared results of 2- and 3-states Markov chain and noticed less good outcomes for 3-states compared to 2-states. The reason was that less demand made less congestion and thus introducing the third state regard to near capacity condition was not sensible. Authors suggested that the proposed methodology can be integrated in a real-time implementation and estimation of TTD. This study shows a successful implementation of Markov Chains in travel time estimation. However, the model does not take into account other transportation modes (e.g. transit) and was only tested in a short arterial section.

Markov process analysis has also been used for public transit schedule reliability. Schmocker et al. (2008) presented an approach to dynamic frequency-based transit assignment. To differentiate from static frequency-based approach and to deal with the line capacity constraints, they used “fail-to-board” probability for the circumstances where passengers are not able to board the first service arriving due to overcrowding. Authors modeled the network by using a series of links and nodes, representing transit lines/transfers and stations/stops. Their cost function minimized the travel time or generalized costs by considering transition probability for paths were defined to account for changes during time intervals. Authors applied their proposed methodology in a case study of inner London network, consists of 56 stations and 14 transit lines. They assumed a peaked demand distribution divided into twelve 15 min intervals and showed that the highest congestion occurs between 8.30 and 8.45 AM. The assignment was done with the assumption of different levels of passenger risk averseness with respect to delays and was shown that higher risk averseness led to fewer passengers failing to board. Authors suggested the higher risk averseness might also be enforced by transport operators through charging extra for the use of crowded stations. They believe that public transport congestion charging might lead to fewer passengers failing to board and might also be used in order to reduce crowding on platforms, which is a major safety concern. The authors concluded that the Markov assignment process could be efficiently used in dynamic assignment problems to remove the excess demand not being able to pass a bottleneck.

In a similar study, Bell (2002) proposed a transit assignment method based on Markov chains to solve the capacity constrained transit network loading problem within congested transit networks, where some passengers will not be able to board because of the absence of sufficient space. Their model also handled the common lines problem, where choice of route depends on frequency of arrivals. In their model passengers decided which transit lines to use based on minimizing an expected cost of travel. Expected travel cost included the cost of a risk of failing to board a transit vehicle. They tested their proposed method in a small network. The results showed that by increasing the risk of failing to board, passenger flow would begin to spread out to reduce the risk and to avoid nodes where they may fail to board until the point at which the cost of the risk of failing to board dominated those associated with travel time and waiting time. Authors recommended that their proposed model could be used to assess the capacity problems of a transit network and analyzing the effect of line capacity changes or changes in the infrastructure. In addition, their approach could calculated the number of passengers staying on the platform to represent platform congestion. Their method needs to be tested on a larger network. Proper calibration method for the risk of failing attribute should also be applied.

The literature review presented above shows that Markov process has been utilized in a variety of transportation problems. Based on the reviewed literature, Markov Process has not been applied for route optimization in a multi-modal transportation network. In this thesis a Markov-based method for route optimization in transit networks is developed. The Markov property allows the Markov Process application to better capture both probabilistic nature of travel time and the fundamental correlated feature of successive links travel times. In other words, traffic spatial progression in roadways can be captured through a methodology similar to a Markov Chain, where the current link travel time of a vehicle depends only on the travel time of immediate upstream link which is well-matched with physics of traffic. Therefore, Markov chain could be applied to transportation routing problems to better capture the short-term characteristics of transportation networks.

2.6 Traffic Conditions Prediction Models

Short-term traffic prediction is an important component in ATIS and APTS applications. Accurate prediction of traffic variables such as speed, travel time, headways, etc. is essential in traffic planning. There are several studies focused on development of mathematical or statistical models for traffic prediction. The mostly commonly used approaches for short-term traffic prediction are time series and Neural Network (NN) models. Time series analysis is usually used for data points taken over time that may have an internal structure, such as auto correlation, trend or seasonal variation. They became popular in short-term traffic prediction since the late 1990's. Hamed et al. (1995) developed a time-series model to predict future traffic volume values on urban arterials. The Box-Jenkins approach was employed in the analysis. A 1-minute data set representing traffic volume on five major urban arterials was available to construct the models. The most adequate model in reproducing all original time series was the Box-Jenkins autoregressive integrated moving average model of order (0, 1, 1). The model requires only the storage of the last forecasted error and current traffic observation.

In another study Ishak et al. (2003) performed an extensive experiment to evaluate the performance of the non-linear time series traffic prediction system implemented on the 40-mile corridor of I-4 in Orlando, Florida under various model parameters and traffic conditions. A generalized linear model was developed to tests the effects of the prevailing level of congestion, the prediction horizon, the rolling horizon, and their interaction on the model relative error of traffic speed prediction. The results show that the model performance deteriorates rapidly as congestion develops, and all the tested model parameters have significant influence on the relative error. The significant interactions between model parameter with congestion index indicate that shorter prediction and rolling horizon are more favorable during congested conditions. In addition, the performance of the system was evaluated in terms of the relative error of predicted travel time using the predicted speed information. It is found that the model has a slight tendency to underestimate the travel times.

An Artificial Neural Network (ANN) mimics the way the human brain works; it is a supervised learning tool, by which classification and prediction is made on the data through a learning process. ANN has been extensively used in short-term traffic prediction. Abdulhai (1999) developed a system based on Time Delay Neural Network model synthesized using Genetic Algorithm for short-term traffic prediction. The model predicts flow and density based on the contribution of temporal profiles as well the spatial contribution from neighboring sites. Both the simulated and real traffic data obtained from the California Test bed in Orange County were used to validate the model. In addition, the effects of the extent of prediction horizon, spatial contribution and the resolution of the data were investigated. The results indicate that the inclusion of three loop stations in both directions of the subject station is sufficient for practical purposes. Also, it is found that, for best accuracy, the resolution of available data (e.g. daily, weekly) should be comparable to the required resolution of predicted data.

Alecsandru et al. (2004) proposed a hybrid model-based and memory-based methodology to strengthen predictions under both recurring and nonrecurring conditions. The model-based approach relies on a combination of static and dynamic neural network architectures to achieve optimal prediction performance under various input and traffic condition settings. Concurrently, the memory-based component was derived from the data archival system that encodes the commuters' travel experience in the past. For each query case two prediction values are generated based on the two methods. The better of two values is identified via an error-based decision algorithm integrated into a prediction query manager.

Some of the studies applied the Kalman filter for short-term traffic prediction. Kalman filter is used to produce values that tend to be closer to the true values of the measurements observed over time that contain random variations and other inaccuracies. For example, Xia et al. (2009) developed a dynamic short-term corridor travel time prediction model using Kalman filter. This method involves a multi-step-ahead prediction of traffic condition with a seasonal autoregressive integrated moving average model. The authors embedded an adaptive Kalman filter to adjust the prediction error based on traffic flow data that becomes available in real time. The developed traffic prediction tool was applied to an on-line corridor. The results show that this method was able to capture the traffic dynamics and to provide accurate travel time prediction.

Most of the prediction models in the literature fall into the class of deterministic models, which assume some known specific properties of the traffic data and estimate values of the parameters of the model. Given the dynamic nature of traffic data, deterministic models are not appropriate for short-term traffic prediction. In long term traffic data shows a clear trend or seasonality however, in short-term the data is stochastic. Therefore, in order to simulate a specified dataset, the time series model has to include more previous data points and resort to more complex smoothing techniques. This could lead to poor generalization. NN models can fit complex dataset by adding more hidden layers and hidden neurons. However, NN models with large amount of coefficient have the same problem of poor generalization.

On the contrary, probabilistic models could characterize the traffic data as a random process, and therefore, are a good candidate for short-term traffic prediction to capture the stochastic properties of the data. For example, Qi and Ishak (2012) introduced the One-Step Stochastic Model for short-term traffic prediction. The authors considered the measured traffic speeds as a proxy for a generic traffic conditions indicator and employed speed transition matrices to describe the change of traffic conditions within various time horizons. Subsequently, using historical data, the cumulative probabilities and conditional expected values for negative and positive transitions were calculated based on the observed transition probabilities. The authors developed statistical models to fit the cumulative transition probability and expected value curves. The fitted models were used for short-term traffic speed prediction. The results of this study showed that the root mean square errors for most of the validation dataset were around 5 mph, implying a good performance of the models. The authors also introduced two probabilistic approaches, hidden Markov and one-step stochastic models for short-term traffic prediction. Authors used traffic conditions, as opposed to traffic parameters, as the target for short-term prediction. Traffic conditions in their method were defined using first and second order statistic of traffic parameters to encode the range and variation of traffic variables. The dynamic aspect of freeway traffic was addressed using transition probabilities. The traffic state at the end of the optimal states sequence was the predicted traffic condition in 5 minutes. The model performance was evaluated using prediction errors. Their model validation results showed that the prediction error and the variation of prediction error decreased as the transition window and prediction sequence length increased. In addition, authors concluded that model performance was not affected remarkably by peak period time (morning/afternoon), travel direction, and data gathering locations, with overall prediction errors less than 10%.

The purpose of short-term prediction for traffic management centers is to enable them to apply traffic control to prevent congestion and incident, while for road users it is served to aid them to make informed decision including departure time, travel route and so on. Therefore, the concern for traffic prediction is not what exactly the speed or volume would be in the next short period time interval, rather what the traffic condition would be shortly thereafter. Based on the literature review and to tackle the stochastic characteristic of traffic data, a probabilistic method is applied in this research work to predict traffic conditions along roadways within the network. The proposed method includes a stochastic model that accounts for randomness in traffic parameters and showed good performance for short-term traffic prediction applications.

2.7 Concluding Remarks

This chapter presented the review of available literature related to this research work. There are several studies on different vehicle routing optimization methods in ATIS. Most of them consider passenger vehicles as the only transportation mode in their routing algorithm. However, many commuters of large urban areas often use a combination of at least two transportation modes. According to GO Transit (2010), the Greater Toronto and Hamilton Area's interregional public transportation service agency, 80% of train riders and 60% of bus riders use GO transit park-and-ride to park their car and use transit for their trip. This research work includes both private and transit modes in optimal travel route calculations. This approach would enable both travelers and transit agencies to benefit from a multi-modal route optimization.

Among the few route optimization studies in multi-modal transportation networks, mainly the simple Dijkstra's Shortest Path (DSP) algorithm or a routing policy based on DSP was used to identify the best route. However, due to the stochastic nature of traffic parameters in a transportation network, probabilistic methods are able to provide better estimation of these parameters. A dynamic discrete choice modeling approach where travelers are seen as taking sequential decisions on which link to choose could better capture the short-term characteristics of transportation networks. This can be conducted by incorporating Markov property in the stochastic algorithm.

Markov process is a method that is widely used in transportation problems to analyze traffic states and estimate travel times. Literature review showed that Markov methods are widely used in transportation field, including traffic conditions analysis and transit schedule reliability. However, its application in route optimization within a multi-modal transportation network has not been investigated yet. Traffic spatial progression in roadways can be captured through a methodology similar to a Markov Chain, where the current link travel time of a vehicle depends only on the travel time of immediate upstream link, which is well matched with physics of traffic. Consequently, Markov models could be applied to transportation routing problems to better capture the short-term characteristics of transportation networks. In this research work, a Markov-based methodology for route optimization in transit networks is developed and applied to real world cases studies.

CHAPTER 3. METHODOLOGY

Multimodal networks can be used to model travel behavior more realistically within a complex urban transportation environment. Different transportation modes are typically represented simultaneously via several interconnected networks (e.g. roadway network, bicycle path network, transit system network, etc.) and using special nodes (i.e. station node and route node) and links (transfer link). Typically, these special nodes and links are used to model public transit stations, where travelers have the opportunity to transfer between different transportation modes. Therefore, a multimodal trip (i.e. a trip involving at least two transportation modes) can be represented by defining a path through a multi-layered network, as defined above, which includes at least one station/route node and one transfer link.

In this research, multimodal trips are modeled in two stages. The first stage encompasses building a generic representation of each physical network, corresponding to the different transportation modes used. In the second stage, a routing algorithm is developed to estimate optimal paths across the defined multimodal network. To better capture the stochastic behavior of transportation network a traffic condition prediction methodology is proposed that uses changes in traffic speed as the traffic condition indicator to predict congestion level. The procedure is then summarized in a flowchart and is followed by an example to demonstrate the application of the proposed methodology. This chapter includes a detailed description of these the above steps.

3.1 Network Modeling

Most surface transportation networks, namely road, transit and rail networks can be modeled using a set of nodes and links, which represent physical junctions/intersections and travel paths between adjacent junctions, respectively. Therefore, a graph can be used to represent these networks. A graph is formed by sets of nodes (vertices) and links (edges) connecting the nodes. Often the graph elements are labeled with letters (e.g. a,b,..) or numbers (e.g. 1,2,…). When a number is assigned to each link in graph, they are called weights of links. In a transportation network, such weights might represent, for example costs, lengths or capacities, depending on the nature of the modeled network. A directed graph or digraph is a graph in which links have orientations between the interconnected nodes (i.e. allowing for a specific direction of travel). A graph can be used as a set of interconnected nodes to represent a physical transportation network and its connectivity (West 2001). Within a road transportation network, vehicle routing junctions (i.e. intersections and interchanges) are represented as nodes within the graph. A physical road segment typically connects two junctions. Any two junctions represented by nodes u and v respectively, can be used to define a link $e = (u, v)$ established between the two nodes $(u, v) \in V,$ where V is the set of all the network nodes. In a road transportation network, every node of a graph can be defined with at least two attributes, x and y. These attributes represent the node's coordinates within the network, with respect to an arbitrarily selected origin. Similarly, the links of a graph can be characterized by a combination of any of the following three attributes: the physical length of the represented road segment, the expected average travel time, or the expected average travel speed on the road segment. In addition, to represent pedestrian flows in the network, dedicated pedestrian links, between any nodes, u and v, can be added for each road segment (u, v) accessible to pedestrians, (e.g. sidewalks, crosswalks at intersections, etc.).

Directed graphs with weights, or weighted digraphs, are used to represent networks that, depending on the applications in which they are used, include junctions and links characterized by specific quantitative properties. This thesis uses a weighted graph to model the impact of traffic conditions on travel time. In the proposed model the link weight, $w(l)$, represents the expected average travel time on the associated road segment. Link weights are used to calculate the optimal route in the network. The details of route optimization are described in section 3.2. The average vehicular travel time corresponding to a given link (i.e. road segment) is calculated based on the estimated average traffic speed and the physical length of the road segment. Similarly, the weights of the individual pedestrian links are calculated based on the length of the pedestrian link and an arbitrarily selected average walking speed of pedestrians.

The methodology proposed in this thesis integrates the transit network model introduced by Pajor (2009) into a multimodal transportation modeling and routing algorithm. Modeling public transportation requires additional steps to account for the predefined schedules available for each public transit stop in the network. A transit vehicle schedule is represented by a set of three elements (C, S, Z) where, C is a set of connections, S is the set of all stations and Z represents a set of transit vehicles. A connection $c_i \in C$ is defined by the set $(z_i, s_i, t_i, s_{i+1}, t_{i+1})$ where z_i is the transit vehicle, $z_i \in Z$, traveling between stations s_i and $s_{i+1} \in S$, which departs from station s_i at time t_i and arrives at station s_{i+1} at time t_{i+1}.

In the proposed multimodal network model a public transit connection represents one uninterrupted segment of the transit schedule, the travelled distance between two adjacent stops, s_i and s_{i+1}, along a path between two arbitrarily selected origin and destination nodes in the network. Consequently, a transit vehicle can be modeled traveling along a given path which consists of multiple connections.

Using the above notation, a public transit network can be represented via a set of connections and a set of nodes (representing transit links and stations, respectively). Additionally, a super-node is defined to enable modeling of different transit lines and their schedule. Consequently a super node is added to each station $s \in S$, which is referred to as a *station node*. However, station nodes are not directly interconnected. In order to build a connection between the station nodes that correspond to two adjacent stations, another type of nodes is introduced – the *route node,* r. Route nodes are associated with each station in the network and are implemented to allow tracking of transit vehicles by the routing algorithm. Route nodes represent the arrival and departure events at each station of the public transit network. Route nodes are characterized by three attributes: event type (arrival or departure), event timestamp, and additional service related factors (e.g. bus or train number).

A routing algorithm in a multimodal transportation network has to account for the transfer time encountered by travelers as they transfer between transportation modes and/or vehicles. Therefore, at each station, a *transfer link* is defined between each route node (r) and its corresponding station (s). This type of link is denoted by the pair (r, s). The weight of a transfer link represents the transfer time, and it can be calculated in various ways depending on the physical configuration of the station. For example, a traveler parking her bike in front of a subway station would encounter a transfer time calculated as the average expected time needed to reach from entrance of subway terminal to boarding platform (see Figure 1). The procedure to estimate the transfer time is explained next.

Let's denote by TR the set of available transit options between two adjacent transit stations. A transit option is defined by the transit mode and its corresponding departure time when moving between two adjacent stations. An element of the set TR, is denoted by tr, and it represents an arbitrary route. Each route $tr \in TR$ is used by transit vehicles that travel through a predetermined sequence of stations: $[s_1, s_2, \dots, s_k]$. The schedules of individual transit lines are used in the routing algorithm to identify the optimal route between a given pair of origin-destination nodes. Figure 1 shows a representation of section of a hypothetical transit network. Stations s_1 and s_2 are connected either by trains z_1 and z_2 that use the same route but depart from station s_1 at different times $(t_1^{z_1}, t_2^{z_2})$, or by train z_3 that uses a different route and schedule (t_3). The path of a traveler transferring between the trains serving different lines can be modeled via station node and route node. This is done through the transfer links between station node ($s_1 or\ s_2$) and route node ($r_1,\ r_2,\ r_3\ or\ r_4$). The model could then account for the additional time incurred due to the transfer time corresponding to applicable transfer links (i.e. links (r_1, s_1), (r_2, s_1), (r_3, s_2) and (r_4, s_2)).

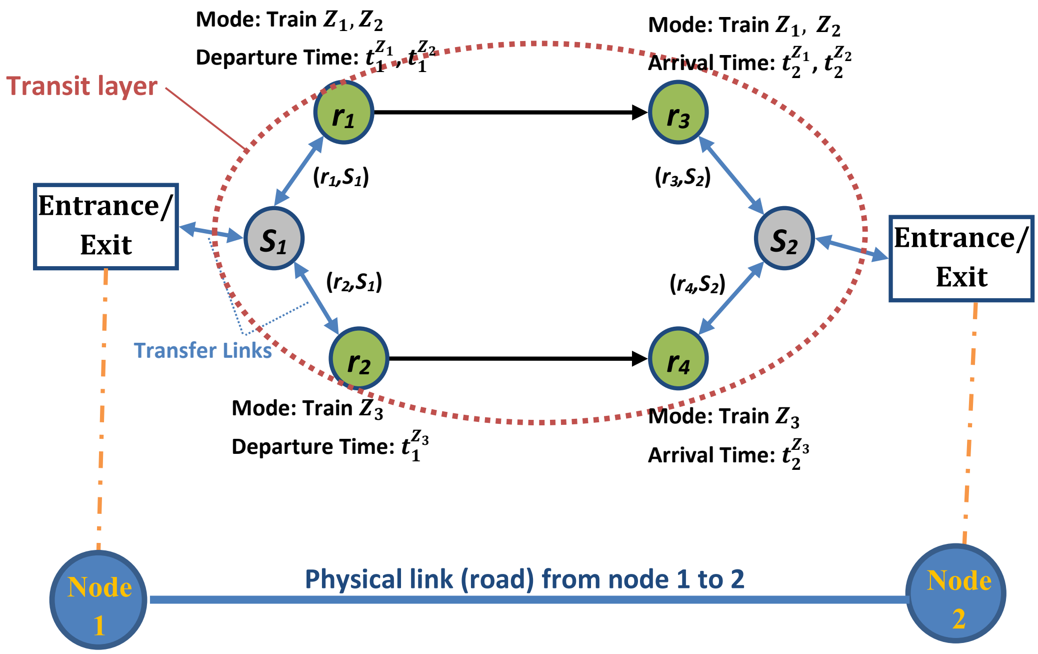

Figure 1: Station nodes (s_1, s_2) and their associated route nodes (r_1, r_2, r_3, r_4)

Using the notation introduced above, connections related to each route between two stations can be defined based on the available transit mode (z) and its arrival and departure time to/from the origin station respectively. For each connection $c_n = (z_n, s_i, s_{i+1}, t_i, t_{i+1})$ in the schedule of route r, the link weight at a given time τ: $w^r(\tau)$, where $\tau < t_i$, is sum of the travel time of the next available transit vehicle (n th train) on that route, $w^r(t_i)$, and the waiting time at station $(t_i - \tau)$:

$$w^r(\tau) = w^r(t_i) + (t_i - \tau) \qquad (1)$$

By following the above methodology, road, transit, cycling and walking networks can be modeled as different layers and then all individual networks can be integrated into a single transportation model able to represent a multimodal study network.

3.2 Routing Algorithm Using Markov Decision Processes (MDP)

After developing a model to represent the multimodal transportation network, the next step involves selecting an adequate routing algorithm to compute optimal paths for network users. The algorithm proposed in this thesis integrates the travel time and travel cost constraints into a single common performance measure. A stochastic and time dependent modeling approach can be applied to use historical and/or real-time information about travel conditions. The travel time (i.e. weight) of the next link in an evolving trip can then be determined during the search for an optimal path by keeping track of the time consumed up to the current node and retrieving the expected travel time depending on the arrival time to the current node.

In statistical analysis, random variables are treated as uncertain, numerical quantities. When random variables are indexed by time, they are referred to as stochastic processes. Stochastic processes usually model the evolution of a random system over time. In this study Markov Decision Processes (MDP) are used to develop an optimal routing methodology for stochastic time-dependent networks to minimize the overall travel cost based on current traffic conditions (i.e. level of congestion) in the network.

Suppose there is a physical or mathematical system that has I possible states and, at any one time, the system is in one and only one of its I states. If at a given observation period, e.g. t^{th} period, the probability of the system being in a particular state depends on its status at the previous, $(t-1)^{\text{th}}$ period, such a system is called Markov Chain. Markov processes are stochastic processes characterized by the Markov property (i.e. given the present state of the process, the past history does not affect conditional probabilities of events defined in the future, Markov and Nagorny 1988). Markov chains are discrete parameter Markov processes whose state space is finite or countable infinite. A set is countable infinite if its elements can be put in one-to-one correspondence with the set of natural numbers. A brief overview of Markov chains is presented below; additionally a detailed description can be found in the literature (Sheskin 2011):

A Markov chain is a sequence of states, $\{X_1, X_1, X_2, \dots, X_N\}$ observed at consecutive time instants $(t, t_1, \dots, t_N)$ at locations named nodes denoted by $n = (1,2, \dots, N)$. If at node n the system is at state $i = (1,2, \dots, I)$, then $X_n = i$ and its probability of occurrence is denoted by $\mathrm{P}(X_n = i)$. The conditional probability that the system will be in state j at the next observed period (i.e. node $n+1$), given it is currently (i.e. node n) in state i is called *transition probability* and is denoted by $p_{ij.}$ The Markov property states that a transition probability depends only on the present state of process (X_n) and the history of process prior to the present node can be ignored. Therefore:

$$p_{ij} = \mathrm{P}(X_{n+1} = j | X_n = i, X_{n-1} = k, \dots, X_1 = g) = \mathrm{P}(X_{n+1} = j | X_n = i) \tag{2}$$

The transition probabilities for a Markov chain with I states are recorder by an $I \times I$ matrix. This matrix is called a one-step transition probability matrix and denoted by P^T. The probability vector for the n^{th} node, denoted by $P^{(n)}$, is probabilities of observing possible states at that node and can be calculated by using equation (3):

$$P^{(n)} = P^{(n-1)} P^T \tag{3}$$

Where: $P^{(n-1)}$ is probability matrix for the $(n-1)^{th}$ node.

When a Markov process transitions from one state to another and it has certain quantifiable impact on a given parameter, the modeling system is called Markov Chain with Rewards (MCR), because the expected impact on the value of this parameter is typically positive it is referred to as "reward". Since in this study the affected parameter is travel cost, whose increase in value is associated with negative effects, we refer to this parameter as a "penalty". The reward is associated with the traffic condition in the transportation network and a value iteration approach can be used to evaluate its expected total value during a finite planning horizon. A MCR generates a sequence of penalties at each node, as it evolves over certain number of nodes from state i to state j, in accordance with one-step transition probability (P^T). The expected penalty observed from node n to node n+1 is denoted by $e_{(n+1)}^{(n)}$ and is calculated using equation (4):

$$e_{(n+1)}^{(n)} = P^{(n)}.Q_{n+1}^{(n)} \tag{4}$$

Where $P^{(n)}$ is the probability vector for node n and $Q_{n+1}^{(n)}$ is the vector of penalties from node n to node n+1 under different states $i = (1,2,\dots,I)$. The size of vector $Q_{n+1}^{(n)}$ is the same as the total number of states (I).

When decisions are added to a set of MCRs, the system is called Markov Decision Process (MDP). MDP generates a sequence of states and an associated sequence of penalties as it evolves over certain number of nodes from state to state, governed by both its transition probabilities and the series of decisions made (Sheskin 2011). A sample sequence of states, decisions, transitions and penalties for an MDP is shown in Figure 2. In this figure ($s_1, s_2, \dots, i, j$) represent states of the model at different times ($t_1, t_2, \dots, t_n, t_{n+1}$) and at nodes ($1, 2, \dots, n, n +$

1). At each node, the state probability vectors are $(P^{(1)}, P^{(2)}, \ldots, P^{(n)}, P^{(n+1)})$. In a transportation network, nodes represent available junctions and states represent different traffic conditions within the network. $d_m^{(n)}$ indicates decision m that is made at node n. In the figure, possible decisions that can be made at node n are shown as $(d_1^{(n)}, d_2^{(n)}, d_3^{(n)})$ that are made at respective state. To optimize an objective function (i.e. minimizing total travel time/cost), transportation routing algorithm decisions pertain to the selection of the next node in the route, as well as the transportation mode to use to move to the next node.

Decision m $(m \in \{1,2,3\})$ at node n identifies the actual mode and node that should be taken/followed after node n, which in this figure is node $n+1$. The incurred penalty at each node depends on the decision that is made given different network states. The penalties at node n are shown as ${q_i}^{{d_m}^{(n)}}$, which represents the penalty associated with decision ${d_m}^{(n)}$ given state i. In a transportation routing algorithm the penalty could be the travel time and/or the cost of travel. For example, if between node n and $n+1$ two possible routes/modes are available, one by car and one by bus, then ${q_i}^{{d_1}^{(n)}}$ could represent travel time under different traffic conditions when traveler drives between the two nodes. Similarly ${q_i}^{{d_2}^{(n)}}$ denotes travel time from n to $n+1$, under different traffic conditions, if traveler takes the bus.

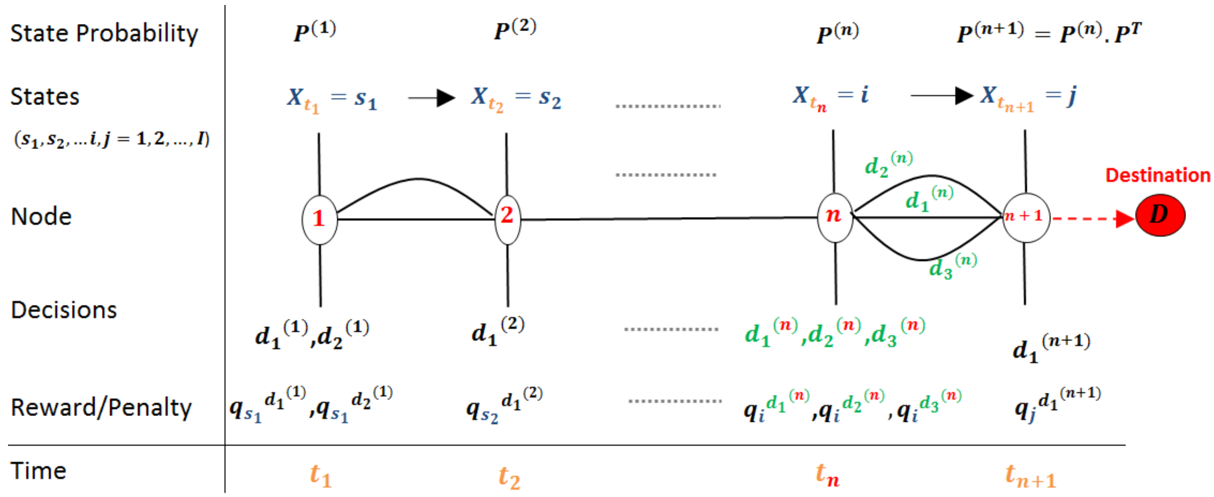

Figure 2: Sequence of states, decisions, transitions and penalties for an MDP.

Using this structure, one can define an optimal route over a finite planning horizon to minimize the expected total penalties received at the end of the given horizon when traveling between the origin (O) and the destination (D), as shown in equation (5):

$$e^{(n)} = \min_{Q}\left[P^{(n)}.\,Q_{n+1}^{(n)}\right] + e^{(n+1)} \tag{5}$$

$n = O, e^{(D)} = 0$

Where:

$e^{(n)}$: Expected total penalties (travel time/cost) between node n and destination (D)

$P^{(n)}$: Vector of the probabilities of the system to be in each state (traffic condition). The elements of $P^{(n)}$ are denoted as $p_i^{(n)}$, which is the probability of observing state i at node n.

$Q_{n+1}^{(n)}$: Matrix of penalties from node n to $n+1$, in each possible state (travel time/cost from n to $n+1$, under each traffic condition). The elements of $Q_{n+1}^{(n)}$ are denoted as ${q_i}^{{d_m}^{(n)}}$, which is the travel time under traffic condition i from n if decision m (i.e. next node and mode to follow) is made.

In the proposed methodology, similar to the Canadian Traveler problem (Nikolova and Karger 2008), it is assumed that travelers are able to make re-routing decisions based on the information available when they approach any given node in the network. The proposed routing algorithm identifies the next transportation mode and the associated link based on previously realized travel times and the availability of current traffic conditions on the links between the current and the destination nodes. In a transportation network, a system would be a (virtual) traveler and the state of the system would be the traffic condition at its location.

At each node in the network, the total time traveled in the network from beginning of the route is readily available. Therefore, at any node n within the network, the total travel time from the origin node (O) up to the next node, $n+1$, depends on the realized travel time up to current node n and on the estimated travel time from n to $n+1$. Using the notations presented in equations (2) through (5) one can represent the sequence of realized travel times through a Markov chain. Consequently, the nodes (i.e. junctions, stops, bike stations, etc.) in the transportation network can be modeled as nodes of a Markov chain and the traffic condition at a given node n is represented by the Markov chain's state X_{t_n}. Based on this notation a finite number of system states or traffic conditions, I, can be used to represent how travel times between adjacent nodes are affected by traffic conditions. The transition probabilities denote the probability that traffic conditions change while travelling from one node, corresponding to a specific state in the associated Markov chain, to another node. The proposed algorithm is used to determine which one of the available links between the current node and the following node should be used to receive minimum penalty. The penalty for this Markov chain is defined as travel time or the equivalent cost of travel time.

Equation (5) estimates the optimal route from origin (node O) to destination (node D), based on the available routing decisions. Total expected penalty (travel time) from node n to destination is the sum of the travel time from the current node (n) to the next node (term $P^{(n)}.Q_{n+1}^{(n)}$ in equation 5), identified based on selected route/mode, and the expected penalty (travel time) from the adjacent node to destination (term $e^{(n+1)}$ in equation 5). At each iteration, the probability of different traffic states occurrence at current node ($P^{(n)}$) is applied to the corresponding travel times on the link that connects nodes n and adjacent node ($n+1$). When the iterative calculation is completed until the destination node, the minimum expected penalty (travel time) from node n to destination among all available routes is reported as the optimal selection.

This methodology requires that at each node, the travel time of all the links, and all transportation modes from the current node to the destination node, to be recalculated considering the probability of having different traffic states. Travel time on each link depends on its corresponding traffic condition and their probabilities. The probabilities of travel times occurrences on each link and under different traffic conditions give the stochastic and time-dependent features of the network model (i.e. depending on the arrival time at the entry node of a link, traffic conditions may change). Therefore, travel time incurred by a traveler on any link may change with the arrival time at the entry node of the link. Next section describes a method that can be used to capture this change in traffic condition and travel times in the model.

3.3 Evaluation of Traffic Conditions and Transition Probability Matrix in MDP

In order to estimate the probability of observing different traffic conditions and to determine the associated transition probability matrix, average traffic speed can be used as a representative parameter to define traffic conditions at a specific location and time. To estimate changes in traffic condition based on average speeds, this thesis applies the method proposed by Qi and Ishak (2012) which is described here after. The speed data from the study area is divided into several intervals (bins), each indicating one traffic condition (from free-flow to congested traffic conditions). The transition probability for speed is calculated as the probability of speed change from one interval to another. It will be used as the probability of transitioning between related traffic conditions in the transition probability matrix of the proposed MDP algorithm.

Given an arbitrarily selected location within a transportation network, one can assess the traffic conditions via the observed range of speed of traffic at time t, denoted by X_t. Hence, X_t defines the current state of traffic at time t. Similarly, $X_{t+\delta}$ is used to define the future state of traffic some predefined δ time units later. Values of δ are chosen based on the frequency of anticipated changes in traffic conditions, typically measured by the speed of traffic stream. Different time horizons (e.g. 1, 2, 5 minutes) can be used to better capture the traffic conditions during the assessment period. The value of δ should also be correlated with the available time period of speed observations. The transition probability from current state X_t to a future state

$X_{t+\delta}$ given the current state X_t is denoted by $p(a,b,\delta)$, where a and b represent the current and future speed intervals, respectively. Equation (6) is used to estimate $p(a,b,\delta)$ from the speed observations at a specific location and for a specific time horizon δ:

$$p(a,b,\delta) = \frac{N\{X_{t+\delta} = b|X_t = a\}}{N\{X_t=a\}} \quad (6)$$

Where a and b are speed intervals and $N\{X_{t+\delta} = b|X_t = a\}$ is the number of instances when it was observed a change in the traffic speed from range a to range b, during period of δ time units. In equation (6) $N\{X_t = a\}$ is the number of instances when the speed range a was observed. Both $N\{X_{t+\delta} = b|X_t = a\}$and $N\{X_t = a\}$ are calculated directly from the available traffic speed data (i.e. observed real-world or simulation data).

The accuracy of estimated probability of changes in traffic conditions depends on road type, resolution of the collected data and the time period for which speed data is aggregated. For example, 5-minute speed data on arterials could capture the changes in traffic condition more accurately compared to highways. On higher speed roadways (i.e. highways), traffic data aggregated for shorter time periods (e.g. 2-minute instead of 5-minute) could result in better estimation of traffic condition.

3.4 MDP Flowchart to identify Optimal Route

After calculating the transition probability matrix by using the methodology described in the previous section, the associated cost values of individual links on the completed path will be used to calculate the cost of using that path, referred to as penalty in the MDP. This cost has to be evaluated for all the possible routes from the current node to the destination node, and, the route with the minimum cost will be selected as the optimal route. This iterative method uses the optimization criterion defined in equation (5) to identify the next link to be used by the traveler. Figure 3 presents a flowchart summarizing the procedure described above.

In the proposed procedure, the subject transportation network is modeled based on the graph theory and using the transportation network associated links and nodes. The origin and destination nodes are set, and possible traffic conditions (i.e. system states) are identified. State probability matrix at each node on any route between origin and destination can be calculated using the initial probability matrix at origin and the transition probability matrix. Finally, the vector of penalties to travel from each node to all its adjacent nodes under all possible traffic conditions and available modes (Q) is then estimated.

A route list ($\mathcal{R}$) is then created to register all the interconnected nodes that build a route between the origin and destination, in an ordered fashion. At the beginning of the procedure, the list is initialized with the origin node. As we progress through the network, at each node (n), a set of directly connected (adjacent) nodes are defined ($\mathcal{K}_n$). This set is used to keep track of visited nodes to avoid re-visiting them and eliminates the possibility of looping during the route computation process. The algorithm processes all adjacent nodes, and at each node the same process is applied. As new nodes are added to the route list, they are checked against the destination. When a route between the origin and destination is identified, the expected total penalty (travel time) between the two nodes will be estimated based on different state probabilities and by using equation (5). This computation is represented by the subroutine defined in the flowchart and is detailed in Figure 3.

The procedure required to distinguish between different transportation modes is implemented within the subroutine and is presented next. At each step the travel time between two consecutive nodes that belong to route ($\mathcal{R}$) are calculated considering the following rules:

1. Initial parameter initialization includes: Maximum number of mode transfers, waiting time for "Transit" mode, parking time for "Driving" mode;
2. A mode change when traveling from one node to another is acceptable only if total number of mode changes does not exceed a maximum prescribed value;
3. When switching from "Driving" mode to other modes, a fixed parking time is accounted for;
4. When switching to "Transit" mode, the waiting time is estimated based on the starting time at Origin, the time it takes to get from Origin to Transit node and the next available departure (according to transit schedule);

Calculated travel time for each route is compared with the previously identified optimum travel time. If a new route is found to have a smaller travel time it will be recorded as the updated optimal route.

The process continues with the remaining nodes within the network in order to determine all possible routes between origin and destination. The algorithm keeps track of the minimum penalty (expected travel time), E_{min}, for all calculated routes and the associated route list,

$\mathcal{R}_{opt}$. When all the nodes are visited, the algorithm reports the optimal route ($\mathcal{R}_{opt}$) and its expected minimum penalty (E_{min}). The time required to complete the iterative process depends on the size of network (i.e. number of nodes and links) and the computing performance of the machine used for the analysis.

The proposed algorithm is applied to an example network here after:

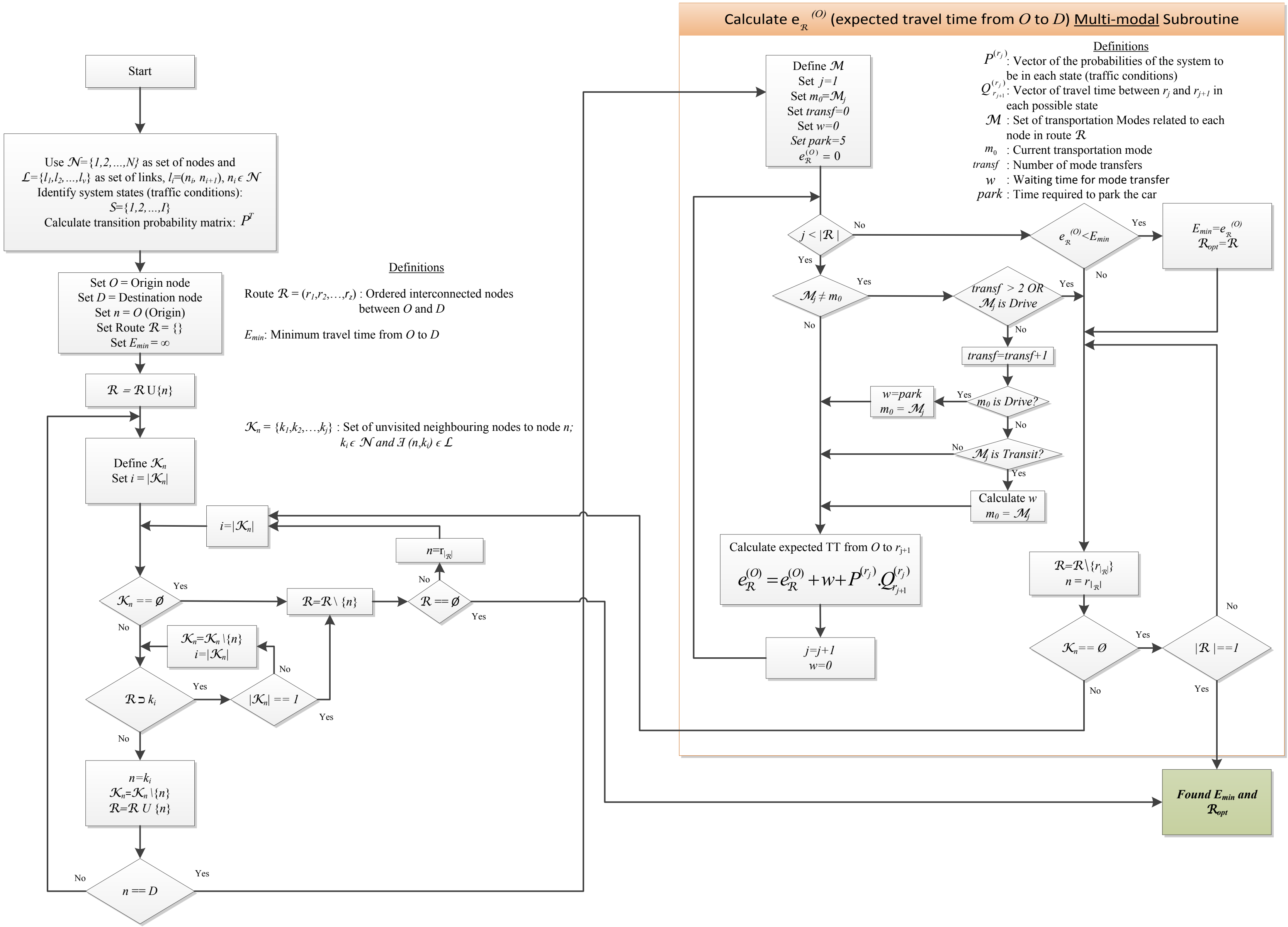

Figure 3: MDP Flowchart

This example calculated the optimal path between point O (origin) and point D (destination) in the network shown in Figure 4. The road network is modeled using five nodes (nodes "O, A, B, C, D") and seven links ($OA, OB, AB, AC, BC, BD, CD$), all bidirectional. These links are used for "Driving" mode. An alternative transportation mode, "Transit", is available between nodes $B\ and\ D$. The transit line is presented using the *route nodes* previously described in Section 3.1 (i.e. r_1, r_2). *Station nodes* are not used in network modeling, therefore are not shown in this figure. It is assumed that every 5 minutes a subway travels between nodes $B\ and\ D$. The time takes for traveler to get to the platform is shown on the *transfer links* in Figure 4. At the beginning the algorithm sets the route list $R = \emptyset$ and the minimum travel time from O to D: $E_{min} = \infty$.

It is assumed that two traffic states are available in the network: *i=1* (normal) and *i=2* (congested). The penalty associated with moving between only two adjacent nodes, as defined in equation 5, is considered to be the expected travel time between the two nodes. The vector of travel time values (penalties) under each traffic state (*i*) from each node to its adjacent node (denoted by Q) is shown on the corresponding link between the two nodes and is measured in minutes (see Figure 4). For example, the expected travel times under normal and under congested traffic states, between nodes "O" and "B", are 2 and 4 minutes, respectively. In this example, it is assumed that travel times on both directions of each link are equal (e.g. $Q_B^O = Q_O^B$).

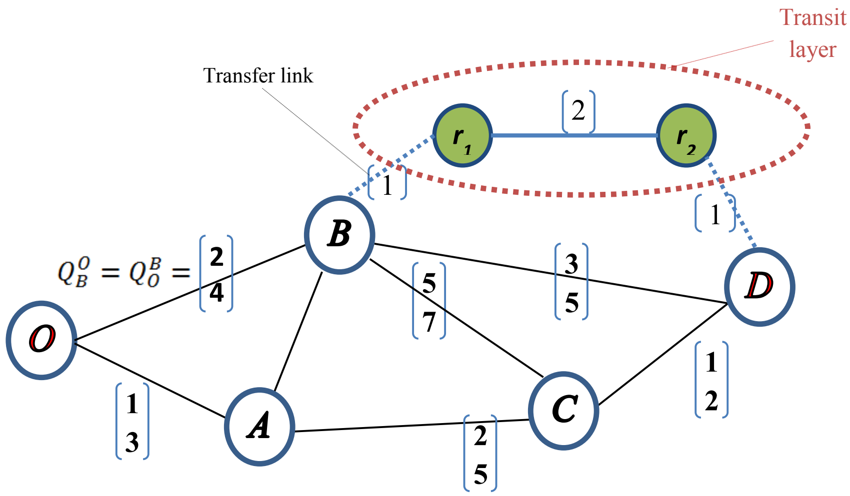

Figure 4: Example Network

At the beginning of process, after defining nodes and links and identifying the origin and destination nodes, the following steps are followed to identify the optimum path using the flowchart presented in Figure 3:

$n = O\ (Origin), \mathcal{R} = \emptyset, E_{min} = \infty,$

Add n to route list: $\mathcal{R} = \mathcal{R} \cup \{n\} = \{O\}$

Identify adjacent nodes to n: $\mathcal{K}_O = \{A, B\}, i = |\mathcal{K}_O| = 2$

$|\mathcal{K}_O| \neq 0$

$k_{i=2} = B \wedge k_i \notin \mathcal{R}$

$n = k_i = B\ (n \neq D), \mathcal{K}_O = \{A, \cancel{B}\}, \mathcal{R} = (O, B)$

Identify adjacent nodes to $n = B$: $\mathcal{K}_B = \{O, A, C, D, r_1\}, i = |\mathcal{K}_B| = 5$

$|\mathcal{K}_B| \neq 0$

$k_{i=5} = r_1 \wedge k_i \notin \mathcal{R}$

$n = k_i = r_1\ (n \neq D), \mathcal{K}_B = \{O, A, C, D, \cancel{r_1}\}, \mathcal{R} = (O, B, r_1)$

Identify adjacent nodes to $n = r_1$: $\mathcal{K}_{r_1} = \{B, r_2\}$, $i = |\mathcal{K}_{r_1}|$=2

$|\mathcal{K}_{r_1}| \neq 0$

$k_{i=2} = r_2 \wedge k_i \notin \mathcal{R}$

$n = k_i = r_2\ (n \neq D), \mathcal{K}_{r_1} = \{B, \cancel{r_2}\}, \mathcal{R} = (O, B, r_1, r_2)$

Identify adjacent nodes to $n = r_2$: $\mathcal{K}_{r_2} = \{r_1, D\}$, $i = |\mathcal{K}_{r_2}| = 2$

$|\mathcal{K}_{r_2}| \neq 0$

$k_{i=2} = D \wedge k_i \notin \mathcal{R}$

$n = k_i = D\ (n = D), \mathcal{K}_{r_2} = \{r_1, \cancel{D}\}, \mathcal{R} = (O, B, r_1, r_2, D)$

Calculate $e_R^{(O)}$ for $\mathcal{R} = (O, B, r_1, r_2, D)$

To calculate the optimal route using the proposed methodology, the initial probability vector, $P^{(O)}$, and the transition probability matrix, P^T, are defined as below:

$$P^{(O)} = [0.6 \quad 0.4],\ P^T = \begin{bmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{bmatrix}$$

The probability vector at each node can be calculated using equation 3. For node B:

$$P^{(B)} = P^{(0)}.P^{T} = [0.6 \quad 0.4].\begin{bmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{bmatrix} = [0.5 \quad 0.5]$$

There are no traffic conditions defined for "Transit" or "Walk" modes. Nevertheless, the steps within the travel-time calculation subroutine of the flow chart are as follows:

First, the transportation modes available at each node in route $\mathcal{R}$ are identified:

$\mathcal{M} = \{Drive, Walk, Transit, Walk\}$

Other initializations: $j = 1, m_0 = Drive, transf = 0, w = 0, e_R^{(O)} = 0$

$r_j = O, \mathcal{M}_j = Drive, r_{j+1} = B$

$\mathcal{M}_j = m_0$: No mode change happened from node j to $j + 1$.

$e_R^{(O)} = 0 + w + P^{(O)}.Q_B^{(O)} = [0.6 \quad 0.4].\begin{bmatrix} 2 \\ 4 \end{bmatrix} = 2.8\, min$

$j = 2, r_j = B, \mathcal{M}_j = Walk, r_{j+1} = r_1$

$\mathcal{M}_j \neq m_0$: mode change happened from node j to $j + 1$. $transf < 4$

$transf = 1, w = park = 5\, min$ (additional 5 minutes to park the car and get to the subway entrance), $m_0 = \mathcal{M}_j = Walk$

$e_R^{(O)} = 2.8 + w + Q_{r_1}^{(B)} = 2.8 + 5 + 1 = 8.8$ min

$j = 3, w = 0, r_j = r_1, \mathcal{M}_j = Transit, r_{j+1} = r_2$

$\mathcal{M}_j \neq m_0$: mode change happened from node j to $j + 1. transf < 4$

Assuming traveler started at 4:00 pm and the subway runs at 5 minute intervals between 4 and 5 pm:

$transf = 2, w = waiting\ time = 04{:}10{:}00 - (04{:}00{:}00 + 00{:}08{:}48) = 1.2\ min$

$m_0 = \mathcal{M}_j = Transit$

$e_R^{(O)} = 8.8 + 1.2 + Q_{r_2}^{(r_1)} = 8.8 + 1.2 + 2 = 12\ min$

$j = 4, w = 0, r_j = r_2, \mathcal{M}_j = Walk, r_{j+1} = D$

$\mathcal{M}_j \neq m_0$: mode change happened from node j to $j + 1. transf < 4$

$transf = 2,$

$e_R^{(O)} = 12 + 0 + Q_D^{(r_2)} = 12 + 1 = 13\ min$

$E_{min} = \infty > e_R^{(O)} \rightarrow E_{min} = e_R^{(O)} = 13 \wedge \mathcal{R}_{opt} = \mathcal{R} = (O, B, r_1, r_2, D)$

$\mathcal{R} = (O, B, r_1, r_2, D), |\mathcal{R}| = 4, n = r_{|\mathcal{R}|} = r_2$

$\mathcal{K}_{r_2} = \{r_1\}, \mathcal{K}_{r_2} \neq \emptyset \rightarrow i = |\mathcal{K}_{r_2}| = 1$

$(from\ above{:}\ \mathcal{K}_{r_2} = r_1)$

$$|\mathcal{K}_{r_2}| \neq 0 \rightarrow k_{i=1} = B \wedge k_i \in \mathcal{R}$$

$$|\mathcal{K}_{r_2}| = 1 \rightarrow \mathcal{R} = (O, B, r_1, \cancel{r_2}), \mathcal{R} \neq \emptyset \rightarrow n = r_{|\mathcal{R}|} = r_1, i = |\mathcal{K}_{r_1}| = 1$$

$$(from\ above: \mathcal{K}_{r_1} = B)$$

$$|\mathcal{K}_{r_1}| \neq 0 \rightarrow k_{i=1} = B \wedge k_i \in \mathcal{R}$$

$$|\mathcal{K}_{r_1}| = 1 \rightarrow \mathcal{R} = (O, B, \cancel{r_1}), \mathcal{R} \neq \emptyset \rightarrow n = r_{|\mathcal{R}|} = B, i = |\mathcal{K}_B| = 4$$

$$(from\ above: \mathcal{K}_B = \{O, A, C, D\})$$

$$|\mathcal{K}_B| \neq 0 \rightarrow k_{i=4} = D \wedge k_i \notin \mathcal{R}$$

$$n = k_i = D\ (n = D), \mathcal{K}_B = \{O, A, C, \cancel{D}\}, \mathcal{R} = (O, B, D)$$

Calculate $e_R^{(O)}$ for $\mathcal{R} = (O, B, D)$

To calculate the optimal route using the proposed methodology, the initial probability vector, $P^{(O)}$, and the transition probability matrix, P^T, are defined as below:

$$P^{(O)} = [0.6 \quad 0.4], P^T = \begin{bmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{bmatrix}$$

The probability vector at each node can be calculated using equation 3. For node B:

$$P^{(B)} = P^{(0)}.P^T = [0.6 \quad 0.4].\begin{bmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{bmatrix} = [0.5 \quad 0.5]$$

Now if one follows the steps in the travel time calculation subroutine of the flow chart:

$$\mathcal{M} = \{Drive\}$$

$$j = 1, e_R^{(O)} = 0$$

$$r_j = O, r_{j+1} = B$$

$$e_R^{(O)} = 0 + P^{(O)}.Q_B^{(O)} = [0.6 \quad 0.4].\begin{bmatrix} 2 \\ 4 \end{bmatrix} = 2.8\, min$$

$$j = 2, r_j = B, r_{j+1} = D$$

$$e_R^{(O)} = 2.8 + P^{(B)}.Q_D^{(B)} = 2.8 + [0.5 \quad 0.5].\begin{bmatrix} 3 \\ 5 \end{bmatrix} = 2.8 + 4 = 6.8\, min$$

$$E_{min} = \infty > e_R^{(O)} \rightarrow E_{min} = e_R^{(O)} = 6.8 \text{ and } \mathcal{R}_{opt} = \mathcal{R} = (O, B, D)$$

$$\mathcal{R} = (O, B, \cancel{D}), |\mathcal{R}| = 2,\ n = r_{|R|} = B$$

$$\mathcal{K}_B = \{O, A, C\}, \mathcal{K}_B \neq \emptyset \rightarrow i = |\mathcal{K}_B| = 3$$

$$k_{i=3} = C \wedge k_i \notin \mathcal{R}$$

$$n = k_i = C\ (n \neq D), \mathcal{K}_B = \{O, A, \cancel{C}\}, \mathcal{R} = (O, B, C)$$

Identify adjacent nodes to $n = C$: $\mathcal{K}_C = \{A, B, D\}$, $i = |\mathcal{K}_C| = 3$

$$|\mathcal{K}_C| \neq 0$$

$$k_{i=3} = D \wedge k_i \notin \mathcal{R}$$

$$n = k_i = D\ (n = D), \mathcal{K}_C = \{A, B, \cancel{D}\}, \mathcal{R} = (O, B, C, D)$$

Calculate $e_R^{(O)}$ for $\mathcal{R} = (O, B, C, D)$

$P^{(O)} = [0.6 \quad 0.4]$, $P^T = \begin{bmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{bmatrix}$

$P^{(B)} = P^{(0)}.P^T = [0.5 \quad 0.5]$

$P^{(C)} = P^{(B)}.P^T = [0.45 \quad 0.55]$

When following the steps in the travel time calculation subroutine of the flow chart, we have:

$\mathcal{M} = \{Drive\}$

$j = 1, e_R^{(O)} = 0\ , r_j = O, r_{j+1} = B\ , e_R^{(O)} = 0 + \ P^{(O)}.Q_B^{(O)} = [0.6 \quad 0.4].\begin{bmatrix} 2 \\ 4 \end{bmatrix} = 2.8\ min$

$j = 2, r_j = B, r_{j+1} = C, e_R^{(O)} = 2.8 + \ P^{(B)}.Q_C^{(B)} = 2.8 + [0.5 \quad 0.5].\begin{bmatrix} 5 \\ 7 \end{bmatrix} = 2.8 + 6 = 8.8\ min$

$j = 3, r_j = C, r_{j+1} = D, e_R^{(O)} = 8.8 + P^{(C)}.Q_D^{(C)} = 8.8 + [0.45 \quad 0.55].\begin{bmatrix} 1 \\ 2 \end{bmatrix} = 10.35\ min$

$e_R^{(O)} = 10.35, E_{min} = 6.8 < e_R^{(O)}$, Therefore E_{min} and $\mathcal{R}_{opt}$ will NOT change.

$\mathcal{R} = (O, B, C, \sout{D}), |\mathcal{R}| = 3,\ n = r_{|\mathcal{R}|} = C$

$\mathcal{K}_C = \{A, B\}, \mathcal{K}_C \neq \emptyset, i = |\mathcal{K}_C| = 2$

$|\mathcal{K}_C| \neq 0 \rightarrow k_{i=2} = B \wedge k_i \ \in \mathcal{R}$

$|\mathcal{K}_C| \neq 1 \rightarrow \mathcal{K}_C = \{A, \sout{B}\}, i = |\mathcal{K}_C| = 1$

$|\mathcal{K}_C| \neq 0 \rightarrow k_{i=1} = A \wedge k_i \notin \mathcal{R}$

$n = k_i = A\ (n \neq D), \mathcal{K}_C = \{\cancel{A}\}, \mathcal{R} = (O, B, C, A)$

There is no node from A to destination, which means this loop-processed node is a dead-end node. The algorithm is able to identify this situation and it returns to the previous node by following the steps below:

Identify adjacent nodes to $n = A$: $\mathcal{K}_A = \{O, B, C\}, i = |\mathcal{K}_A| = 3$

$|\mathcal{K}_A| \neq 0 \rightarrow k_{i=3} = C \wedge k_i \in \mathcal{R}$

$|\mathcal{K}_A| \neq 1 \rightarrow \mathcal{K}_A = \{O, B, \cancel{C}\}, i = |\mathcal{K}_A| = 2$

$|\mathcal{K}_A| \neq 0 \rightarrow k_{i=2} = B \wedge k_i \in \mathcal{R}$

$|\mathcal{K}_A| \neq 1 \rightarrow \mathcal{K}_A = \{O, \cancel{B}\}, i = |\mathcal{K}_A| = 1$

$|\mathcal{K}_A| \neq 0 \rightarrow k_{i=1} = O \wedge k_i \in \mathcal{R}$

$|\mathcal{K}_A| = 1 \rightarrow \mathcal{R} = (O, B, C, \cancel{A}), \mathcal{R} \neq \emptyset \rightarrow n = r_{|\mathcal{R}|} = C, i = |\mathcal{K}_C| = 0$

$(from\ above: \mathcal{K}_C = \emptyset)$

A was a dead-end node and was successfully removed from the route list. Now the algorithm processes the preceding node in the list, C.

$|\mathcal{K}_C| = 0 \rightarrow \mathcal{R} = (O, B, \cancel{C}), \mathcal{R} \neq \emptyset \rightarrow n = r_{|\mathcal{R}|} = B, i = |\mathcal{K}_B| = 2$

$(from\ above: \mathcal{K}_B = \{O, A\})$

Since there is no unvisited nodes available adjacent to node C, this is also be removed from the route list and the node before that (B) becomes the current node. By removing these nodes and identifying them as visited, the algorithm avoids looping infinitely. The process of identifying other routes continues from node B, where there is one more unvisited adjacent node available(i.e. A).

$|\mathcal{K}_B| \neq 0 \rightarrow k_{i=2} = A \wedge k_i \notin \mathcal{R}$

$n = k_i = A\ (n \neq D), \mathcal{K}_B = \{O, \cancel{A}\}, \mathcal{R} = (O, B, A)$

Re-identify adjacent nodes to $n = A$: $\mathcal{K}_A = \{O, B, C\}$, $i = |\mathcal{K}_A| = 3$

$|\mathcal{K}_A| \neq 0$

$k_{i=3} = C \wedge k_i \notin \mathcal{R}$

$n = k_i = C\ (n \neq D), \mathcal{K}_A = \{O, B, \cancel{C}\}, \mathcal{R} = (O, B, A, C)$

Re-identify adjacent nodes to $n = C$: $\mathcal{K}_C = \{A, B, D\}$, $i = |\mathcal{K}_C| = 3$

$|\mathcal{K}_C| \neq 0$

$k_{i=3} = D \wedge k_i \notin \mathcal{R}$

$n = k_i = D\ (n = D), \mathcal{K}_C = \{A, B, \cancel{D}\}, \mathcal{R} = (O, B, A, C, D)$

Calculate $e_R^{(O)}$ for $\mathcal{R} = (O, B, A, C, D)$

$P^{(O)} = [0.6 \quad 0.4]$, $P^T = \begin{bmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{bmatrix}$

$P^{(B)} = P^{(0)}.P^T = [0.5 \quad 0.5]$

$P^{(A)} = P^{(B)}.P^T = [0.45 \quad 0.55]$

$P^{(C)} = P^{(A)}.P^T = [0.425 \quad 0.575]$

Following the steps in the travel time calculation subroutine:

$\mathcal{M} = \{Drive\}$

$j = 1, e_R^{(O)} = 0\ , r_j = O, r_{j+1} = B\ , e_R^{(O)} = 0 + \ P^{(O)}.Q_B^{(O)} = [0.6 \quad 0.4].\begin{bmatrix} 2 \\ 4 \end{bmatrix} = 2.8\ min$

$j = 2, r_j = B, r_{j+1} = A, e_R^{(O)} = 2.8 + \ P^{(B)}.Q_A^{(B)} = 2.8 + [0.5 \quad 0.5].\begin{bmatrix} 2 \\ 3 \end{bmatrix} = 2.8 + 2.5 = 5.3\ min$

$j = 3, r_j = A, r_{j+1} = C, e_R^{(O)} = 5.3 + P^{(A)}.Q_C^{(A)} = 5.3 + [0.45 \quad 0.55].\begin{bmatrix} 2 \\ 5 \end{bmatrix} = 8.95\ min$

$j = 4, r_j = C, r_{j+1} = D, e_R^{(O)} = 8.95 + P^{(C)}.Q_D^{(C)} = 8.95 + [0.425 \quad 0.575].\begin{bmatrix} 1 \\ 2 \end{bmatrix} = 10.525\ min$

$e_R^{(O)} = 10.525, E_{min} = 6.8\ < e_R^{(O)}$, Therefore E_{min} and $\mathcal{R}_{opt}$ will NOT change.

$\mathcal{R} = (O, B, A, C, \not{D}), |\mathcal{R}| = 4,\ n = r_{|\mathcal{R}|} = C$

$\mathcal{K}_C = \{A, B\}, \mathcal{K}_C \neq \emptyset, i = |\mathcal{K}_C| = 2$

$|\mathcal{K}_C| \neq 0 \rightarrow k_{i=2} = B \wedge k_i \in \mathcal{R}$

$|\mathcal{K}_C| \neq 1 \rightarrow \mathcal{K}_C = \{A, \cancel{B}\}, i = |\mathcal{K}_C| = 1$

$|\mathcal{K}_C| \neq 0 \rightarrow k_{i=1} = A \wedge k_i \in \mathcal{R}$

$|\mathcal{K}_C| = 1 \rightarrow \mathcal{R} = (O, B, A, \cancel{C}), \mathcal{R} \neq \emptyset \rightarrow n = r_{|\mathcal{R}|} = A, i = |\mathcal{K}_A| = 2$

$(from\ above: \mathcal{K}_A = \{O, B\})$

$|\mathcal{K}_A| \neq 0 \rightarrow k_{i=2} = B \wedge k_i \in \mathcal{R}$

$|\mathcal{K}_A| \neq 1 \rightarrow \mathcal{K}_A = \{O, \cancel{B}\}, i = |\mathcal{K}_B| = 1$

$|\mathcal{K}_C| \neq 0 \rightarrow k_{i=1} = O \wedge k_i \in \mathcal{R}$

$|\mathcal{K}_C| = 1 \rightarrow \mathcal{R} = (O, B, \cancel{A}), \mathcal{R} \neq \emptyset \rightarrow n = r_{|\mathcal{R}|} = B, i = |\mathcal{K}_B| = 1$

$(from\ above: \mathcal{K}_B = \{O\})$

$|\mathcal{K}_B| \neq 0 \rightarrow k_{i=1} = O \wedge k_i \in \mathcal{R}$

$|\mathcal{K}_B| = 1 \rightarrow \mathcal{R} = (O, \cancel{B}), \mathcal{R} \neq \emptyset, n = r_{|\mathcal{R}|} = O, i = |\mathcal{K}_O| = 1 \qquad from\ above: \mathcal{K}_O = \{A\}$

$|\mathcal{K}_O| \neq 0 \rightarrow k_{i=1} = A \wedge k_i \notin \mathcal{R}$

At this point, all the possible paths from node B are processed. The algorithm was able to avoid the potential routing loops and is now going to check the possible routes from node A, which is the second (and last) adjacent node to the origin.

$n = k_i = A \ (n \neq D), \mathcal{K}_O = \{\cancel{A}\}, \mathcal{R} = (O, A)$

It should be noted that now $\mathcal{K}_O = \emptyset$, which means that there is no other node to process, after identifying all possible paths from A to destination.

Re-identify adjacent nodes to $n = A$: $\mathcal{K}_A = \{O, B, C\}, i = |\mathcal{K}_A| = 3$

$|\mathcal{K}_A| \neq 0$

$k_{i=3} = C \wedge k_i \ \notin \mathcal{R}$

$n = k_i = C \ (n \neq D), \mathcal{K}_A = \{O, B, \cancel{C}\}, \mathcal{R} = (O, A, C)$

Re-identify adjacent nodes to $n = C$: $\mathcal{K}_C = \{A, B, D\}, i = |\mathcal{K}_C| = 3$

$|\mathcal{K}_C| \neq 0$

$k_{i=3} = D \wedge k_i \ \notin \mathcal{R}$

$n = k_i = D \ (n = D), \mathcal{K}_C = \{A, B, \cancel{D}\}, \mathcal{R} = (O, A, C, D)$

Calculate $e_R^{(O)}$ for $\mathcal{R} = (O, A, C, D)$

$P^{(O)} = [0.6 \quad 0.4], P^T = \begin{bmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{bmatrix}$

$P^{(A)} = P^{(0)}.P^T = [0.5 \quad 0.5]$

$P^{(C)} = P^{(A)}.P^T = [0.45 \quad 0.55]$

The travel time calculation is performed using the subroutine of the flow chart (Figure 3):

$\mathcal{M} = \{Drive\}$

$j = 1, e_R^{(O)} = 0\ , r_j = O, r_{j+1} = A\ , e_R^{(O)} = 0 + \ P^{(O)}.Q_A^{(O)} = [0.6 \quad 0.4].\begin{bmatrix}1\\3\end{bmatrix} = 1.8\,min$

$j = 2, r_j = A, r_{j+1} = C, e_R^{(O)} = 1.8 + \ P^{(A)}.Q_C^{(A)} = 1.8 + [0.5 \quad 0.5].\begin{bmatrix}2\\5\end{bmatrix} = 1.8 + 3.5 = 5.3\,min$

$j = 3, r_j = C, r_{j+1} = D, e_R^{(O)} = 8.8 + P^{(C)}.Q_D^{(C)} = 5.3 + [0.45 \quad 0.55].\begin{bmatrix}1\\2\end{bmatrix} = 6.85\,min$

$e_R^{(O)} = 6.85, E_{min} = 6.8\ < e_R^{(O)}$, Therefore E_{min} and $\mathcal{R}_{opt}$ will NOT change.

$\mathcal{R} = (O, A, C, \cancel{D}), |\mathcal{R}| = 3,\ n = r_{|\mathcal{R}|} = C$

$\mathcal{K}_C = \{A, B\}, K_C \neq \emptyset, i = |\mathcal{K}_C| = 2$

$|\mathcal{K}_C| \neq 0 \rightarrow k_{i=2} = B \wedge k_i\ \notin \mathcal{R}$

$n = k_i = B\ (n \neq D), \mathcal{K}_C = \{A, \cancel{B}\}, \mathcal{R} = (O, A, C, B)$

<u>Re-identify</u> adjacent nodes to $n = B$: $\mathcal{K}_B = \{O, A, C, D, r_1\}, i = |\mathcal{K}_B| = 5$

$\mathcal{K}_B = \{O, A, C, D, r_1\}$, Set the size of $\mathcal{K}_B$: $i = 5$

$|\mathcal{K}_B| \neq 0$

$k_{i=5} = r_1 \wedge k_i\ \notin \mathcal{R}$

$n = k_i = r_1 \ (n \neq D), \mathcal{K}_B = \{O, A, C, D, \cancel{r_1}\}, \mathcal{R} = (O, A, C, B, r_1)$

Identify adjacent nodes to $n = r_1$: $\mathcal{K}_{r_1} = \{B, r_2\}$, $i = |\mathcal{K}_{r_1}| = 2$

$|\mathcal{K}_{r_1}| \neq 0$

$k_{i=2} = r_2 \wedge k_i \notin \mathcal{R}$

$n = k_i = r_2 \ (n \neq D), \mathcal{K}_{r_1} = \{B, \cancel{r_2}\}, \mathcal{R} = (O, A, C, B, r_1, r_2)$

Identify adjacent nodes to $n = r_2$: $\mathcal{K}_{r_2} = \{r_1, D\}$, $i = |\mathcal{K}_{r_2}| = 2$

$|\mathcal{K}_{r_2}| \neq 0$

$k_{i=2} = D \wedge k_i \notin \mathcal{R}$

$n = k_i = D \ (n = D), \mathcal{K}_{r_2} = \{B, \cancel{D}\}, \mathcal{R} = (O, A, C, B, r_1, r_2, D)$

Calculate $e_R^{(O)}$ for $\mathcal{R} = (O, A, C, B, r_1, r_2, D)$

$P^{(O)} = [0.6 \quad 0.4]$, $P^T = \begin{bmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{bmatrix}$

$P^{(A)} = P^{(0)}.P^T = [0.5 \quad 0.5]$

$P^{(C)} = P^{(A)}.P^T = [0.45 \quad 0.55]$

There is no traffic conditions defined for Transit or Walk modes. The travel time calculation is performed using the subroutine of the flow chart (Figure 3). First, the transportation modes associated with each node in route $\mathcal{R}$ is identified:

$\mathcal{M} = \{Drive, Drive, Drive, Walk, Transit, Walk\}$

Other initializations: $j = 1, m_0 = Drive, tranf = 0, w = 0, e_R^{(O)} = 0$

$j = 1, e_R^{(O)} = 0$, $r_j = O, \mathcal{M}_j = Drive, r_{j+1} = A$, $e_R^{(O)} = 0 + P^{(O)}. Q_A^{(O)} = [0.6 \quad 0.4]. \begin{bmatrix} 1 \\ 3 \end{bmatrix} = 1.8\, min$

$j = 2, r_j = A, \mathcal{M}_j = Drive, r_{j+1} = C,$ $e_R^{(O)} = 1.8 + P^{(A)}. Q_C^{(A)} = 1.8 + [0.5 \quad 0.5]. \begin{bmatrix} 2 \\ 5 \end{bmatrix} = 1.8 + 3.5 = 5.3\, min$

$j = 3, r_j = C, \mathcal{M}_j = Drive, r_{j+1} = B,$ $e_R^{(O)} = 5.3 + P^{(C)}. Q_B^{(C)} = 5.3 + [0.45 \quad 0.55]. \begin{bmatrix} 5 \\ 7 \end{bmatrix} = 11.4\, min$

$j = 4, r_j = B, \mathcal{M}_j = Walk, r_{j+1} = r_1,$

$\mathcal{M}_j \neq m_0$: mode change happened from node j to $j + 1. transf < 4$

$tranf = 1, w = park = 5\, min$ (additional 5 minutes to park the car and get to the subway entrance), $m_0 = \mathcal{M}_j = Walk$

$e_R^{(O)} = 11.4 + w + Q_{r_1}^{(B)} = 11.4 + 5 + 1 = 17.4$ min

$j = 5, w = 0, r_j = r_1, \mathcal{M}_j = Transit, r_{j+1} = r_2$

$\mathcal{M}_j \neq m_0$: mode change happened from node j to $j + 1. transf < 4$

Assuming traveler started at 4:00 pm and the subway runs at 5 minute intervals between 4 and 5 pm:

$tranf = 2, w = waiting\ time = 04{:}20{:}00 - (04{:}00{:}00 + 0{:}17{:}24) = 2.6\ min$

$m_0 = \mathcal{M}_j = Transit$

$e_R^{(O)} = 17.4 + 2.6 + Q_{r_2}^{(r_1)} = 17.4 + 2.6 + 2 = 22\ min$

$j = 6, w = 0, r_j = r_2, \mathcal{M}_j = Walk, r_{j+1} = D$

$\mathcal{M}_j \neq m_0$: mode change happened from node j to $j + 1. transf < 4$

$transf = 2,$

$e_R^{(O)} = 22 + 0 + Q_D^{(r_2)} = 22 + 1 = 23\ min$

$e_R^{(O)} = 23, E_{min} = 6.8 < e_R^{(O)}$, Therefore E_{min} and $\mathcal{R}_{opt}$ will NOT change.

$\mathcal{R} = (O, A, C, B, r_1, r_2, \cancel{D}), |\mathcal{R}| = 6,\ n = r_{|\mathcal{R}|} = r_2$

$\mathcal{K}_{r_2} = \{B\}, \mathcal{K}_{r_2} \neq \emptyset \rightarrow i = |\mathcal{K}_{r_2}| = 1$

$|\mathcal{K}_{r_2}| \neq 0 \rightarrow k_{i=1} = B \wedge k_i \in \mathcal{R}$

$\left|\mathcal{K}_{r_2}\right| = 1 \rightarrow \mathcal{R} = (O, A, C, B, r_1, \cancel{r_2}), \mathcal{R} \neq \emptyset \rightarrow n = r_{|\mathcal{R}|} = r_1, i = \left|\mathcal{K}_{r_1}\right| = 0$

$\left|\mathcal{K}_{r_1}\right| \neq 0 \rightarrow k_{i=1} = B \wedge k_i \in \mathcal{R}$

$\left|\mathcal{K}_{r_1}\right| = 1 \rightarrow \mathcal{R} = (O, A, C, B, \cancel{r_1}), \mathcal{R} \neq \emptyset \rightarrow n = r_{|\mathcal{R}|} = B, i = |\mathcal{K}_B| = 4$

$(from\ above\ \mathcal{K}_B = \{O, A, C, D\})$

$k_{i=4} = D \wedge k_i \notin \mathcal{R}$

$n = k_i = D\ (n = D), \mathcal{K}_B = \{O, A, C, \cancel{D}\}, \mathcal{R} = (O, A, C, B, D)$

<u>Calculate $e_R^{(O)}$ for</u> $\mathcal{R} = (O, A, C, B, D)$

$P^{(O)} = [0.6 \quad 0.4], P^T = \begin{bmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{bmatrix}$

$P^{(A)} = P^{(0)}.P^T = [0.5 \quad 0.5]$

$P^{(C)} = P^{(A)}.P^T = [0.45 \quad 0.55]$

$P^{(B)} = P^{(C)}.P^T = [0.425 \quad 0.575]$

The travel time calculation is performed using the subroutine of the flow chart (Figure 3):

$j = 1, e_R^{(O)} = 0\ , r_j = O, r_{j+1} = A\ , e_R^{(O)} = 0 + P^{(O)}.Q_A^{(O)} = [0.6 \quad 0.4].\begin{bmatrix} 1 \\ 3 \end{bmatrix} = 1.8\ min$

$j = 2, r_j = A, r_{j+1} = C, e_R^{(O)} = 1.8 + P^{(A)}.Q_C^{(A)} = 1.8 + [0.5 \quad 0.5].\begin{bmatrix} 2 \\ 5 \end{bmatrix} = 1.8 + 3.5 = 5.3\ min$

$j = 3, r_j = C, r_{j+1} = B, e_R^{(O)} = 5.3 + P^{(C)}.Q_B^{(C)} = 5.3 + [0.45 \quad 0.55].\begin{bmatrix}5\\7\end{bmatrix} = 11.4\ min$

$j = 4, r_j = B, r_{j+1} = D, e_R^{(O)} = 11.4 + P^{(B)}.Q_D^{(B)} = 11.4 + [0.425 \quad 0.575].\begin{bmatrix}3\\5\end{bmatrix} = 15.5\ min$

$e_R^{(O)} = 15.5, E_{min} = 6.8 < e_R^{(O)}$, Therefore E_{min} and $\mathcal{R}_{opt}$ will NOT change.

$\mathcal{R} = (O, A, C, B, \cancel{D}), |\mathcal{R}| = 4,\ n = r_{|\mathcal{R}|} = B$

$\mathcal{K}_B = \{O, A, C\}, \mathcal{K}_B \neq \emptyset, i = |\mathcal{K}_B| = 3$

$|\mathcal{K}_B| \neq 0 \rightarrow k_{i=3} = C \wedge k_i \in \mathcal{R}$

$|\mathcal{K}_B| \neq 1 \rightarrow \mathcal{K}_B = \{O, A, \cancel{C}\}, i = |\mathcal{K}_B| = 2$

$|\mathcal{K}_B| \neq 0 \rightarrow k_{i=2} = A \wedge k_i \in \mathcal{R}$

$|\mathcal{K}_B| \neq 1 \rightarrow \mathcal{K}_B = \{O, \cancel{A}\}, i = |\mathcal{K}_B| = 1$

$|\mathcal{K}_B| \neq 0 \rightarrow k_{i=1} = O \wedge k_i \in \mathcal{R}$

$|\mathcal{K}_B| = 1 \rightarrow \mathcal{R} = (O, A, C, \cancel{B}), \mathcal{R} \neq \emptyset \rightarrow n = r_{|\mathcal{R}|} = C, i = |\mathcal{K}_C| = 1$

$(from\ above{:}\ \mathcal{K}_C = \{A\})$

$|\mathcal{K}_C| \neq 0 \rightarrow k_{i=1} = A \wedge k_i \in \mathcal{R}$

$|\mathcal{K}_C| = 1 \rightarrow \mathcal{R} = (O, A, \cancel{C}), \mathcal{R} \neq \emptyset \rightarrow n = r_{|\mathcal{R}|} = A, i = |\mathcal{K}_A| = 2$

$(from\ above\ \mathcal{K}_A = \{O, B\})$

$|\mathcal{K}_A| \neq 0 \rightarrow k_{i=2} = B \wedge k_i \notin \mathcal{R}$

$n = k_i = B \ (n \neq D), \mathcal{K}_A = \{O, \cancel{B}\}, \mathcal{R} = (O, A, B)$

Identify adjacent nodes to $n = B$: $\mathcal{K}_B = \{O, A, C, D, r_1\}, i = |\mathcal{K}_B| = 5$

$|\mathcal{K}_B| \neq 0$

$k_{i=5} = r_1 \wedge k_i \notin \mathcal{R}$

$n = k_i = r_1 \ (n \neq D), \mathcal{K}_B = \{O, A, C, D, \cancel{r_1}\}, \mathcal{R} = (O, A, B, r_1)$

Identify adjacent nodes to $n = r_1$: $\mathcal{K}_{r_1} = \{B, r_2\}, i = |\mathcal{K}_{r_1}| = 2$

$|\mathcal{K}_{r_1}| \neq 0$

$k_{i=2} = r_2 \wedge k_i \notin \mathcal{R}$

$n = k_i = r_2 \ (n \neq D), \mathcal{K}_{r_1} = \{B, \cancel{r_2}\}, \mathcal{R} = (O, A, B, r_1, r_2)$

<u>Re-identify</u> adjacent nodes to $n = r_2$: $\mathcal{K}_{r_2} = \{r_1, D\}, i = |\mathcal{K}_{r_2}| = 2$

$|\mathcal{K}_{r_2}| \neq 0$

$k_{i=2} = D \wedge k_i \notin \mathcal{R}$

$n = k_i = D \ (n = D), \mathcal{K}_{r_2} = \{B, \cancel{D}\}, \mathcal{R} = (O, A, B, r_1, r_2, D)$

<u>Calculate $e_R^{(O)}$ for</u> $\mathcal{R} = (O, A, B, r_1, r_2, D)$

$P^{(O)} = [0.6 \quad 0.4], P^T = \begin{bmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{bmatrix}$

$P^{(A)} = P^{(0)}.P^T = [0.5 \quad 0.5]$

There is no traffic conditions defined for Transit or Walk modes. The travel time calculation is performed using the subroutine of the flow chart (Figure 3). First, the transportation modes associated with each node in route $\mathcal{R}$ is identified:

$\mathcal{M} = \{Drive, Drive, Walk, Transit, Walk\}$

Other initializations: $j = 1, m_0 = Drive, transf = 0, w = 0, e_R^{(O)} = 0$

$j = 1, e_R^{(O)} = 0 \,, r_j = O, r_{j+1} = A \,, e_R^{(O)} = 0 + P^{(O)}.Q_A^{(O)} = [0.6 \quad 0.4].\begin{bmatrix} 1 \\ 3 \end{bmatrix} = 1.8\, min$

$j = 2, r_j = A, r_{j+1} = B, e_R^{(O)} = 1.8 + P^{(A)}.Q_B^{(A)} = 1.8 + [0.5 \quad 0.5].\begin{bmatrix} 2 \\ 3 \end{bmatrix} = 1.8 + 2.5 = 4.3\, min$

$j = 3, r_j = B, \mathcal{M}_j = Walk, r_{j+1} = r_1,$

$\mathcal{M}_j \neq m_0$: mode change happened from node j to $j + 1.\, transf < 4$

$tranf = 1, w = park = 5\, min$ (additional 5 minutes to park the car and get to the subway entrance), $m_0 = \mathcal{M}_j = Walk$

$e_R^{(O)} = 4.3 + w + Q_{r_1}^{(B)} = 4.3 + 5 + 1 = 10.3$ min

$j = 4, w = 0, r_j = r_1, \mathcal{M}_j = Transit, r_{j+1} = r_2$

$\mathcal{M}_j \neq m_0$: mode change happened from node j to $j + 1. transf < 4$

Assuming traveler started at 4:00 pm and the subway runs at 5 minute intervals between 4 and 5 pm:

$tranf = 2, w = waiting\ time = 04{:}15{:}00 - (04{:}00{:}00 + 00{:}10{:}18) = 4.7\ min$

$m_0 = \mathcal{M}_j = Transit$

$e_R^{(O)} = 10.3 + 4.7 + Q_{r_2}^{(r_1)} = 10.3 + 4.7 + 2 = 17\ min$

$j = 5, w = 0, r_j = r_2, \mathcal{M}_j = Walk, r_{j+1} = D$

$\mathcal{M}_j \neq m_0$: mode change happened from node j to $j + 1. transf < 4$

$transf = 2,$

$e_R^{(O)} = 17 + 0 + Q_D^{(r_2)} = 17 + 1 = 18\ min$

$e_R^{(O)} = 18, E_{min} = 6.8 < e_R^{(O)}$, Therefore E_{min} and $\mathcal{R}_{opt}$ will NOT change.

$\mathcal{R} = (O, A, B, r_1, r_2, \sout{D}), |\mathcal{R}| = 5,\ n = r_{|\mathcal{R}|} = r_2$

$\mathcal{K}_{r_2} = \{B\}, \mathcal{K}_{r_2} \neq \emptyset \rightarrow i = |\mathcal{K}_{r_2}| = 1$

$|\mathcal{K}_{r_2}| \neq 0 \rightarrow k_{i=1} = B \wedge k_i \in \mathcal{R}$

$|\mathcal{K}_{r_2}| = 1 \rightarrow \mathcal{R} = (O, A, B, r_1, \sout{r_2}), \mathcal{R} \neq \emptyset \rightarrow n = r_{|\mathcal{R}|} = r_1, i = |\mathcal{K}_{r_1}| = 0$

$|\mathcal{K}_{r_1}| \neq 0 \rightarrow k_{i=1} = B \wedge k_i \ \in \mathcal{R}$

$|\ \mathcal{K}_{r_1}| = 1 \rightarrow \mathcal{R} = (O, A, B, \sout{r_1}), \mathcal{R} \neq \ \emptyset \ \rightarrow \ n = r_{|\mathcal{R}|} = B, i = |\mathcal{K}_B| = 4$

$(from\ above\ \mathcal{K}_B = \{O, A, C, D\})$

$k_{i=4} = D \wedge k_i \ \notin \mathcal{R}$

$n = k_i = D\ (n = D), \mathcal{K}_B = \{O, A, C, \sout{D}\}, \mathcal{R} = (O, A, B, D)$

<u>Calculate $e_R^{(O)}$ for</u> $\mathcal{R} = (O, A, B, D)$

$P^{(O)} = [0.6 \quad 0.4],\ P^T = \begin{bmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{bmatrix}$

$P^{(A)} = P^{(0)}.\, P^T = [0.5 \quad 0.5]$

$P^{(B)} = P^{(A)}.\, P^T = [0.45 \quad 0.55]$

The travel time calculation is performed using the subroutine of the flow chart (Figure 3):

$\mathcal{M} = \{Drive\}$

$j = 1, e_R^{(O)} = 0\ , r_j = O, r_{j+1} = A\ , e_R^{(O)} = 0 + \ P^{(O)}.\, Q_A^{(O)} = [0.6 \quad 0.4].\begin{bmatrix} 1 \\ 3 \end{bmatrix} = 1.8\ min$

$j = 2, r_j = A, r_{j+1} = B, e_R^{(O)} = 1.8 + \ P^{(A)}.\, Q_B^{(A)} = 1.8 + [0.5 \quad 0.5].\begin{bmatrix} 2 \\ 3 \end{bmatrix} = 1.8 + 2.5 = 4.3\ min$

$j = 3, r_j = B, r_{j+1} = D, e_R^{(O)} = 4.3 + P^{(B)}.\, Q_D^{(B)} = 4.3 + [0.45 \quad 0.55].\begin{bmatrix} 3 \\ 5 \end{bmatrix} = 8.4\ min$

$e_R^{(O)} = 8.4, E_{min} = 6.8 < e_R^{(O)}$, Therefore E_{min} and $\mathcal{R}_{opt}$ will NOT change.

$\mathcal{R} = (O, A, B, \not{D}), |\mathcal{R}| = 3,\ n = r_{|\mathcal{R}|} = B$

$\mathcal{K}_B = \{O, A, C\}, \mathcal{K}_B \neq \emptyset, i = |\mathcal{K}_B| = 3$

$|\mathcal{K}_B| \neq 0 \rightarrow k_{i=3} = C \wedge k_i \notin \mathcal{R}$

$n = k_i = C\ (n \neq D), \mathcal{K}_B = \{O, A, \not{C}\}, \mathcal{R} = (O, A, B, C)$

Identify adjacent nodes to $n = C$: $\mathcal{K}_C = \{A, B, D\}, i = |\mathcal{K}_C| = 3$

$|\mathcal{K}_C| \neq 0$

$k_{i=3} = D \wedge k_i \notin \mathcal{R}$

$n = k_i = D\ (n = D), \mathcal{K}_C = \{A, B, \not{D}\}, \mathcal{R} = (O, A, B, C, D)$

Calculate $e_R^{(O)}$ for $\mathcal{R} = (O, A, B, C, D)$

$P^{(O)} = [0.6 \quad 0.4], P^T = \begin{bmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{bmatrix}$

$P^{(A)} = P^{(0)}.P^T = [0.5 \quad 0.5]$

$P^{(B)} = P^{(A)}.P^T = [0.45 \quad 0.55]$

$P^{(C)} = P^{(B)}.P^T = [0.425 \quad 0.575]$

The travel time calculation is performed using the subroutine of the flow chart (Figure 3):

$\mathcal{M} = \{Drive\}$

$j = 1, e_R^{(O)} = 0\ , r_j = O, r_{j+1} = A\ , e_R^{(O)} = 0 +\ P^{(O)}.\, Q_A^{(O)} = [0.6 \quad 0.4].\begin{bmatrix}1\\3\end{bmatrix} = 1.8\, min$

$j = 2, r_j = A, r_{j+1} = B, e_R^{(O)} = 1.8 +\ P^{(A)}.\, Q_B^{(A)} = 1.8 + [0.5 \quad 0.5].\begin{bmatrix}2\\3\end{bmatrix} = 1.8 + 2.5 = 4.3\, min$

$j = 3, r_j = B, r_{j+1} = C, e_R^{(O)} = 4.3 + P^{(B)}.\, Q_C^{(B)} = 4.3 + [0.45 \quad 0.55].\begin{bmatrix}5\\7\end{bmatrix} = 10.4\, min$

$j = 4, r_j = C, r_{j+1} = D, e_R^{(O)} = 10.4 + P^{(C)}.\, Q_D^{(C)} = 10.4 + [0.425 \quad 0.575].\begin{bmatrix}1\\2\end{bmatrix} = 11.97\, min$

$e_R^{(O)} = 11.97, E_{min} = 6.8\ < e_R^{(O)}$, Therefore E_{min} and $\mathcal{R}_{opt}$ will NOT change.

$\mathcal{R} = (O, A, B, C, \sout{D}), |\mathcal{R}| = 4,\ n = r_{|\mathcal{R}|} = C$

$\mathcal{K}_C = \{A, B\}, \mathcal{K}_B \neq \emptyset, i = |\mathcal{K}_B| = 2$

$|\mathcal{K}_C| \neq 0 \rightarrow k_{i=2} = B \wedge k_i\ \in \mathcal{R}$

$|\mathcal{K}_C| \neq 1 \rightarrow \mathcal{K}_C = \{A, \sout{B}\}, i = |\mathcal{K}_B| = 1$

$|\mathcal{K}_C| \neq 0 \rightarrow k_{i=1} = A \wedge k_i\ \in \mathcal{R}$

$|\mathcal{K}_C| = 1 \rightarrow \mathcal{R} = (O, A, B, \sout{C}), \mathcal{R} \neq\ \emptyset\ \rightarrow\ n = r_{|\mathcal{R}|} = B, i = |\mathcal{K}_B| = 2$

$(from\ above: \mathcal{K}_B = \{O, A\})$

$|\mathcal{K}_B| \neq 0 \rightarrow k_{i=2} = A \wedge k_i\ \in \mathcal{R}$

$$|\mathcal{K}_B| \neq 1 \rightarrow \mathcal{K}_B = \{O, \cancel{A}\}, i = |\mathcal{K}_B| = 1$$

$$|\mathcal{K}_B| \neq 0 \rightarrow k_{i=1} = O \wedge k_i \in \mathcal{R}$$

$$|\mathcal{K}_B| = 1 \rightarrow \mathcal{R} = (O, A, \cancel{B}), \mathcal{R} \neq \emptyset \rightarrow n = r_{|\mathcal{R}|} = A, i = |\mathcal{K}_A| = 1$$

$$(from\ above: \mathcal{K}_A = \{O\})$$

$$|\mathcal{K}_A| \neq 0 \rightarrow k_{i=1} = O \wedge k_i \in \mathcal{R}$$

$$|\mathcal{K}_A| = 1 \rightarrow \mathcal{R} = (O, \cancel{A}), \mathcal{R} \neq \emptyset \rightarrow n = r_{|\mathcal{R}|} = O, i = |\mathcal{K}_O| = 0$$

$$(from\ above: \mathcal{K}_O = \emptyset)$$

$\mathcal{K}_O = \emptyset$: which means that all adjacent nodes to the origin node have been visited.

$|\mathcal{K}_O| = 0 \rightarrow \mathcal{R} = (\cancel{O}), \mathcal{R} = \emptyset$ → Found the minimum path, Report:

$$E_{min} = 6.8\, min\, ,\ \mathcal{R}_{opt} = (O, B, D)$$

3.5 Discussion on Transition Probability in Transportation Network

When the proposed algorithm is applied for route optimization in transportation networks, the states of the system represent traffic conditions in the network. Similarly, transition probability matrix denotes the probability of changes in traffic condition when travelling from one node to another. In the previous sections, it was assumed that a unique transition probability matrix is applicable at any location in the network. However, transportation network consists of different road types (i.e. highway, arterial, etc.) which do not have the same traffic characteristics (e.g. average speed, traffic volume). Consequently, there will be different patterns of change in traffic conditions along each of these road types. Furthermore, traffic conditions depend on the peak period and are not necessarily the same at different locations within a large transportation network. Therefore, a more realistic representation of the network should consider distinct transition probability matrices associated with different road types and different peak directions within the network.

Using this approach, the estimation of transition probability matrix (P^T) used to calculate the state probability vector for the $(n+1)^{th}$ node, $P^{(n+1)} = P^{(n)}P^T$ as shown in Figure 2, will be specific to the link (road type) between nodes n and $n+1$. According to Figure 2 there are three different routes available between nodes n and $n+1$, $d_1^{(n)}, d_2^{(n)}$ and $d_3^{(n)}$. Depending on the type of the links related to each route, different transition probability matrices can be defined and applied to each type.

The methodology described in the previous sections, applies the one-step transition probability (P^T) during each step when moving from one node to its adjacent node. For example, according to Figure 2, when the traveler moves from node n at time step t_n to adjacent node ($n+1$), the time step at node $n+1$ is denoted by t_{n+1}. Expected penalty (travel time) from node n to $n+1$ is the difference between t_n and t_{n+1}, and is calculated using equation (3): $e^{(n)} = P^{(n)}.Q_{n+1}^{(n)}$. Then the state probability vector for node $n+1$ is updated using equation (2) and the one-step transition probability matrix: $P^{(n+1)} = P^{(n)}P^T$. Therefore, the probability vector for each node is updated each time traveler moves to that node, without considering the ($t_{n+1} - t_n$) time step and estimated travel time between the two nodes ($e^{(n)}$).

As proposed previously in Section 3.3 collected traffic speed data is used to estimate the transition probability matrix for the network. The resolution of the collected data (i.e. sampling frequency) and the intervals for which speed data is aggregated represents the minimum time period during which the changes in traffic conditions can be captured. For example, if aggregated speed data for 2-minute intervals were used for estimating the transition probability matrix, it can be assumed that if the travel time from one node to its adjacent node is less than 2 minutes, the probability vector of traffic conditions at the adjacent node does not need to be updated. Therefore, the resolution of traffic data can be used as an update threshold for applying the transition probability matrix and updating state probabilities at each node. Figure 5 presents a revised subroutine that can be used in the main flowchart to include the time step threshold in probability matrix calculation for each node. The new procedure temporarily stores the estimated travel time from node j to node $j+1$ into $t_{j,j+1}$, and checks it against the defined threshold (T_{ts}). If the estimated time between the two nodes is equal or greater than threshold, the traffic condition probability vector at node $j+1$ will be calculated by applying the one-step transition probability matrix, as described in the previous sections. However, if the expected travel time is less than the update threshold value, the state probability vector at $j+1$ will not change, as compared to node j.

This enhancement of the proposed methodology can capture more accurately the stochastic effects of traffic conditions along the network on travel time. The hypothetical example network used previously in this section is also use here to demonstrate its benefits. However, due to limitation of the available real-world data this enhancement has not been applied to any of the case studies in this thesis.

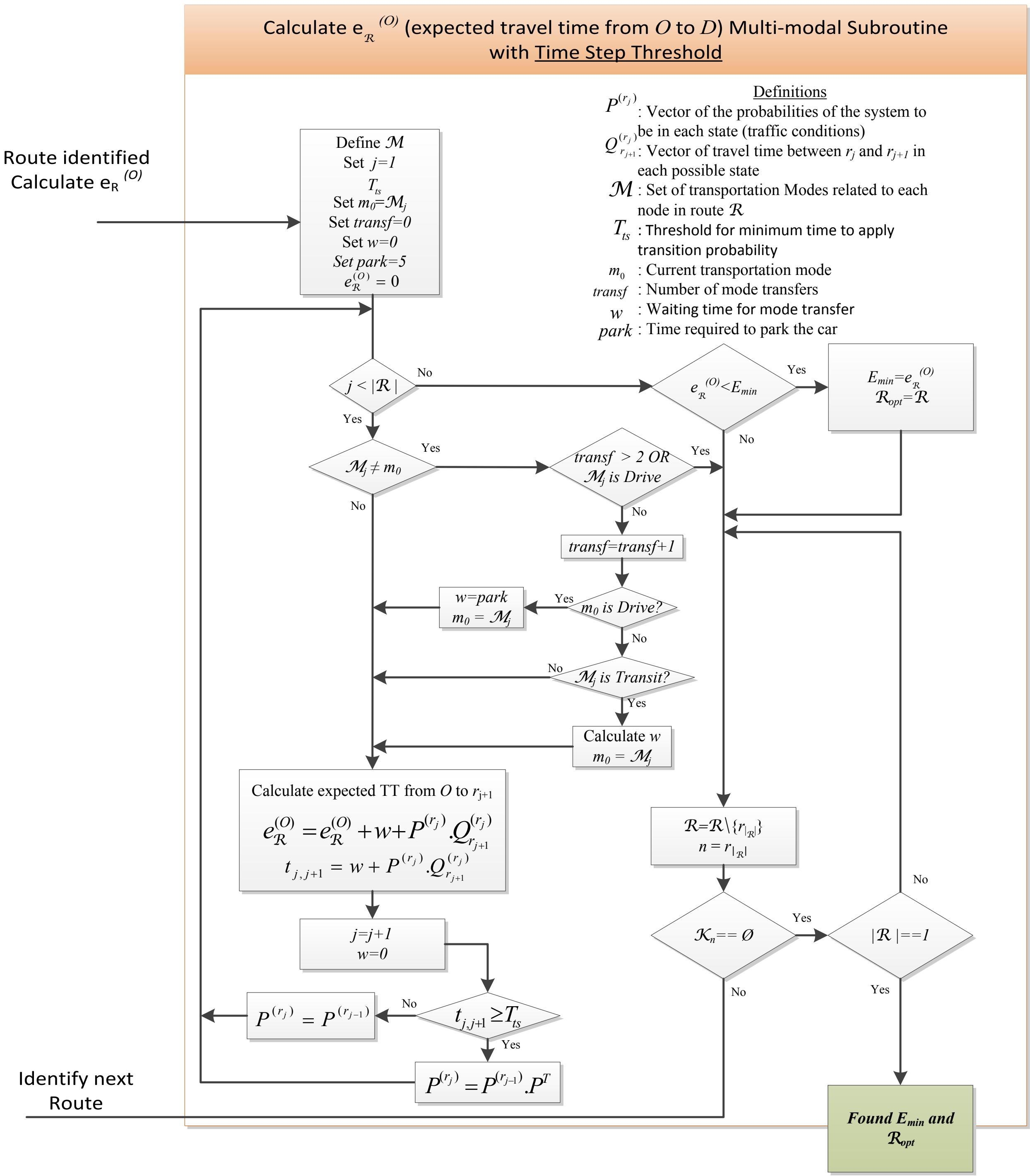

Figure 5: Time-Step Threshold Optimization Subroutine

The example provided in Section 3.4 applied the flowchart presented in Figure 3 to calculate the optimal path between point O (origin) and point D (destination) in a sample network (Figure 4). To demonstrate the application of the proposed time-step threshold, the revised subroutine shown in Figure 5 is used in conjunction with the flowchart. An arbitrary time-step threshold: $T_{tr} = 5\ minutes$ is assumed.

The optimal route calculation steps followed in the example to identify possible routes remain the same and are not repeated here. The only difference between the new time-step threshold subroutine and the one include in the flowchart is related to the calculation of $e_R^{(O)}$. Therefore the effect of introducing the time-step threshold in the example is shown by revising the $e_R^{(O)}$ calculations for each route identified in the previous example. Since the transition probability in this example was only applied to "$Driving$" mode, the changes are applied to the calculations related to this mode. These changes and the revised calculation for the optimal path calculations are listed in the following steps:

Calculate $e_R^{(O)}$ for $\mathcal{R} = (O, B, r_1, r_2, D)$

Mode change occurs at node B and continues until destination.

Calculate $e_R^{(O)}$ for $\mathcal{R} = (O, B, D)$

$P^{(O)} = [0.6 \quad 0.4]$, $P^T = \begin{bmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{bmatrix}$

To re-calculate the optimal route by applying the time-step threshold subroutine, the travel time between each pair of nodes is compared with $T_{tr} = 5$:

$j = 1, e_R^{(O)} = 0 + w + P^{(O)}.Q_B^{(O)} = 2.8\ min,\ t_{j,j+1} = P^{(O)}.Q_B^{(O)} = 2.8\ min$

$j = 2, t_{j,j+1} < T_{tr} \rightarrow P^{(B)} = P^{(O)} = [0.6\ \ 0.4]$

Since the expected travel time from O to B is less than threshold, the transition probability matrix is not considered for calculating the probability vector at node B.

$j = 2, e_R^{(O)} = 2.8 + P^{(B)}.Q_D^{(B)} = 2.8 + [0.6\ \ 0.4].\begin{bmatrix}3\\5\end{bmatrix} = 2.8 + 3.8 = 6.6\ min$

$E_{min} = e_R^{(O)} = 6.6$ and $\mathcal{R}_{opt} = \mathcal{R} = (O, B, D)$

The expected travel time calculated for route (O, B, D) by using the proposed time-step threshold is now less than the time estimated for the same in the previous example (i.e. 6.8 min).

Calculate $e_R^{(O)}$ for $\mathcal{R} = (O, B, C, D)$

Based on the previous route:

$P^{(O)} = [0.6\ \ 0.4], P^T = \begin{bmatrix}0.7 & 0.3\\0.2 & 0.8\end{bmatrix}$

$j = 1, e_R^{(O)} = 2.8\ min,\ tt = 2.8\ min$

$j = 2, t_{j,j+1} < T_{tr} \rightarrow P^{(B)} = P^{(O)} = [0.6\ \ 0.4]$

$j = 2, e_R^{(O)} = 2.8 + P^{(B)}.Q_C^{(B)} = 2.8 + [0.6\ \ 0.4].\begin{bmatrix}5\\7\end{bmatrix} = 2.8 + 5.8 = 8.6\ min, t_{j,j+1} = 5.8\ min$

$j = 3, t_{j,j+1} > T_{tr} \rightarrow P^{(C)} = P^{(B)}.P^{T} = [0.5 \quad 0.5]$

$e_R^{(O)} = 8.6 + P^{(C)}.Q_D^{(C)} = 8.6 + [0.5 \quad 0.5].\begin{bmatrix}1\\2\end{bmatrix} = 10.1\ min$

$e_R^{(O)} = 10.1, E_{min} = 6.6 < e_R^{(O)}$, Therefore E_{min} and $\mathcal{R}_{opt}$ will NOT change.

The calculations are conducted for all possible routes. The optimum route is $\mathcal{R}_{opt} = (O, B, D)$, similar to the previous example, and estimated travel time on that route is $6.6\ min$, which is different from the estimated travel time without considering the update time-step threshold.

CHAPTER 4. ANALYSIS AND RESULTS

The proposed methodology is applied to three different case studies. The first and second case studies investigate application of the proposed methodology to find the shortest route for travelers. Initially a small real-world network located in the central business district area of Montreal, Quebec is used to evaluate the potential benefits of proposed methodology to travelers using public transportation. In the second case study, a large area of Toronto CBD with several transportation modes (i.e. car, train, subway, bus, street car, bike and walk) is used to evaluate the benefits of proposed routing algorithm. In the third case study a bus line in Longueuil, Quebec is used to investigate benefits of alleviating the impact of traffic congestion on transit users and transit system's operating costs. Detailed description of each case study and achieved results are presented here below.

4.1 Montreal case study

The study area includes a small size road network in the city's central business district (CBD), delimited by three 1.5-km long arterials (i.e. a segment of a two-way road - Sherbrooke street, and two one-way road segments along de Maisonneuve Blvd. and Sainte-Catherine street. As it can be seen from Figure 6, the arterial roads are intersected by several smaller roads, between Guy street on the west side and University street on the east side. This study area is serviced by three subway stations and it includes pedestrian sidewalks along all the streets. Additional public transit services are provided by two bus lines, 24 and 15, both running eastbound, and are operated along Sherbrooke and Sainte-Catherine, respectively. In addition, a two-way bicycle lane has been integrated in recent years along de Maisonneuve, with a bicycle sharing service available nearly six months a year during the warm season. Figure 7 shows a sketch of all transportation modes available within the study area.

4.1.1 Analysis and Results

The proposed method is used to model the study area and find the optimal route for traveling between two arbitrarily selected nodes in the study area using any of the available modes, motorized or non-motorized. The intersections marked with A and B in Figure 6 correspond to the origin and the destination nodes, as shown in the associated public transportation network graph sketched in Figure 7. A traveler using the network shown in Figure 7, in order to reach point B from point A may choose among several alternative routes, four of which are identified as follows. First, the traveler starting at A could use the subway from the Guy-Concordia station to the McGill station and walk the last segment of the trip towards destination B. The second alternative could be to walk from origin A to one of the bus stops either at the intersection with Sherbrook St. or with Sainte-Catherine St. and travel by bus towards the destination B, where an additional distance to the destination B might have to be walked, depending which the bus line was used. Third, a traveler could decide to use the bicycle sharing system, which may include walking a certain distance to and/or from the closest bicycle station. Finally, travelers could use private passenger cars and drive from A to B via Sherbrooke St. or Sainte-Catherine St. To evaluate the model's ability to select the optimal route using the MDP algorithm described in Section 3.4, first traffic state conditions for each transportation mode within the network are identified. In this case study, two hypothetical conditions regarding the road traffic conditions and also the reliability of the transit network are considered.

Figure 6: Plan of study area (Source: Google Maps©)

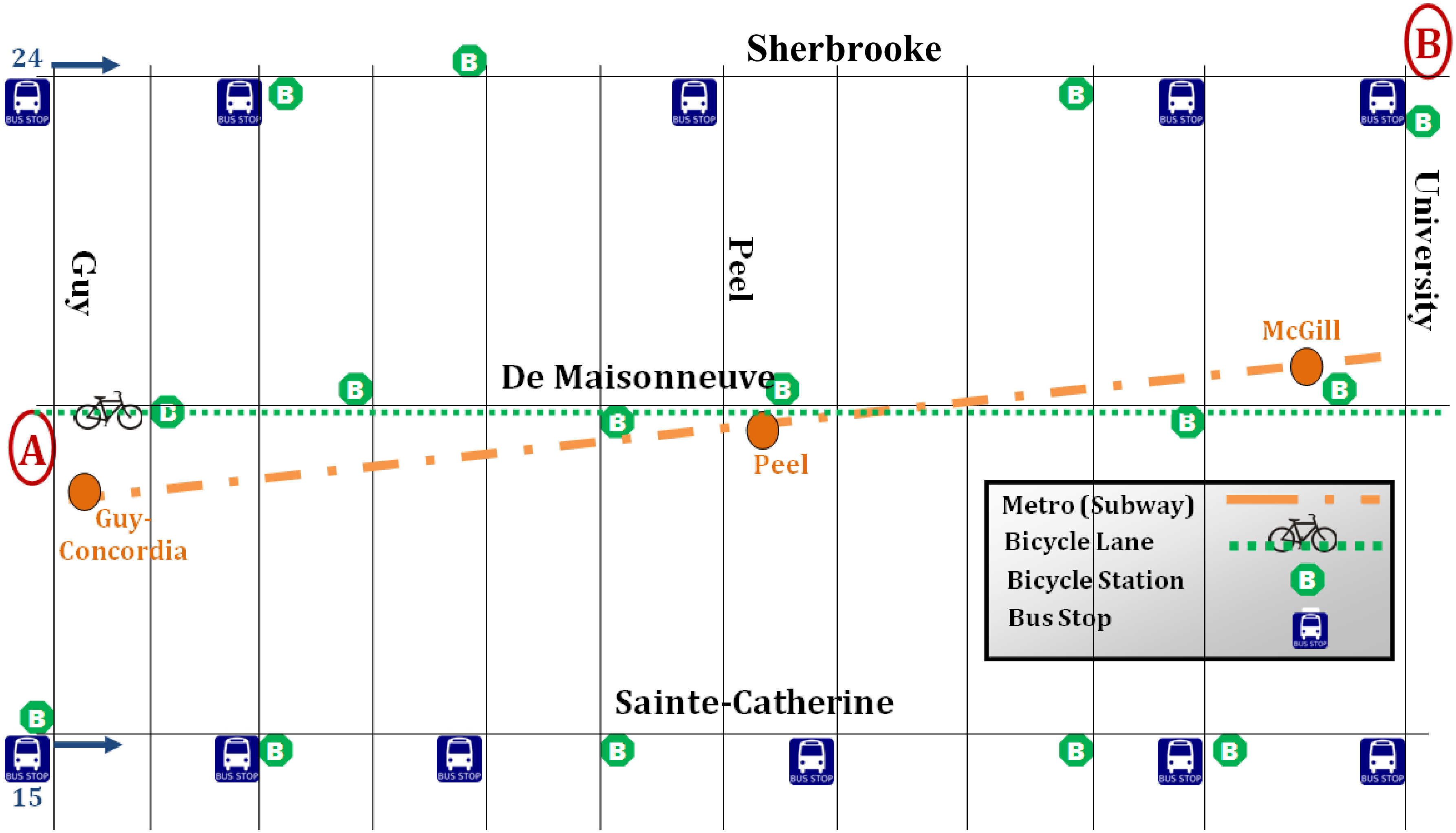

Figure 7: Public transportation network within the study area (*drawing not to scale)*

In order to estimate the probability of observing different traffic conditions and the related transition probability matrix, real-world traffic speed was used to identify different traffic conditions related to the study network. The following tasks have been performed to build the transition probability matrix for the MDP methodology:

1. Driving from point A to B, drivers would use either of two major arterials within the study area, Sherbrooke and Sainte-Catherine, respectively. Since traffic data for the subject streets was not available at the time of study, traffic speed data collected along a major arterial in Toronto (Yonge St, within downtown area) was collected and used in this example. This arterial was use due to its similarity with the study area (i.e. high pedestrian volumes and presence of bicycles mixed within vehicular traffic). The average speed data from GPS equipped vehicles was aggregated over 15-minute period intervals.
2. Speed graphs for 15-minute time intervals where produced and analyzed to better understand the range of changes in the average speed data along each direction. The analysis was done for the period between 14:00 and 19:00 and for all weekdays (during a calendar week in September 2012), the weekend data were excluded.
3. The processed speed values were divided into two arbitrary ranges to represent two types of traffic conditions: A) <u>off-peak</u>: speed > 40km/h; B) <u>peak</u>: speed <= 40 km/h.
4. A visual basic program was developed to calculate the elements of transition probability matrix by applying the methodology described in Chapter 3. The probability of observing a change in traffic conditions was estimated based on the number of changes in the

average traffic speed from one traffic condition range to another during each 15-minute interval. Final transition probability matrix is shown in Table 1.

Table 1: Road Network Transition Probability Matrix (Montreal Case Study)

Future Traffic State / Current Traffic State	A	B
A (off-peak)	0.86	0.14
B (peak)	0.08	0.92

5. Each row in Table 1 represents the probability of change in traffic conditions from initial state (Rows A and B) to another state (Columns A and B). Based on this table, the probability of the traffic conditions to improve from level B to level A is 0.08. There is also 14% chance that traffic condition would deteriorate from level A to level B. Moreover, the probabilities of traffic condition levels A and B to remain at the same level are 0.86 and 0.92, respectively.

It should be noted that in Table 1, the sum of probabilities for traffic change (each row) is equal to one. However, the values within a column of the table do not represent the same probability relationship between two states. In order to use the transition probability values, the initial traffic state should be chosen from the rows of matrix and the future traffic state should be selected from one of the columns.

Initially, it was assumed that there is no disruption in service provided by the subway. In order to estimate the probability of bicycle being available at BIXI® stations, their availability at five BIXI® stations within the study area were randomly checked during the afternoon peak hour. The bicycle readiness probability was estimated from the ratios between the number of times bicycles were available and the total number of observations. For the study period (between 16:00 and 17:00) the probability of bicycles not being available within the study area was 0.3. Based on the above assumptions, the transition probability matrices for the metro and BIXI® network were calculated and are shown in Table 2 and Table 3, respectively.

Table 2: Transition Probability Matrix for Metro Network

Current Metro State \ Future Metro State	A	B
A (normal service)	1	0
B (interruption)	1	0

Table 3: Transition Probability Matrix for Bixi Network

Current Bixi State \ Future Bixi State	A	B
A (bicycle available)	0.7	0.3
B (bicycle not available)	0.7	0.3

The network is modeled based on the methodology described in the previous section and is illustrated in Figure 8. In total the network includes 9 nodes and 14 links. The following constraints have been considered in this example:

1. Traveler is moving from point A (node 1) to point B (node 9);
2. It is assumed that a traveler starts the trip at 4:00 pm and has access to their automobile at point A;
3. The model accounts for the directional graph associated with the network. For example, Maisonneuve Blvd (connects nodes 1, 4 and 7) is a one way westbound arterial for vehicles. Therefore, the private car mode is not available on links 4 and 7 (See Figure 8);

4. The direction of traffic was considered when creating the links and separate links are created for each direction of traffic along arterials related to the study area. However, not all the bidirectional links are represented;
5. Since the scenarios are evaluated at the beginning of the afternoon peak period, the initial probability of traffic states was considered to be [0.4, 0.6], corresponding to higher chances of observing peak traffic conditions (60%).
6. Bus stops are located at nodes 2, 5 and 9 for line 24. Similarly, bus route 15 has stops at nodes 3,6 and 8;
7. Metro stations are located at nodes 1, 4 and 7;
8. A BIXI® station is available at all the nodes within the network;
9. A transfer/access time of 2 minutes between metro entrance and platform is considered;
10. Two-minute operating time for BIXI® bicycles at docking stations is assumed.

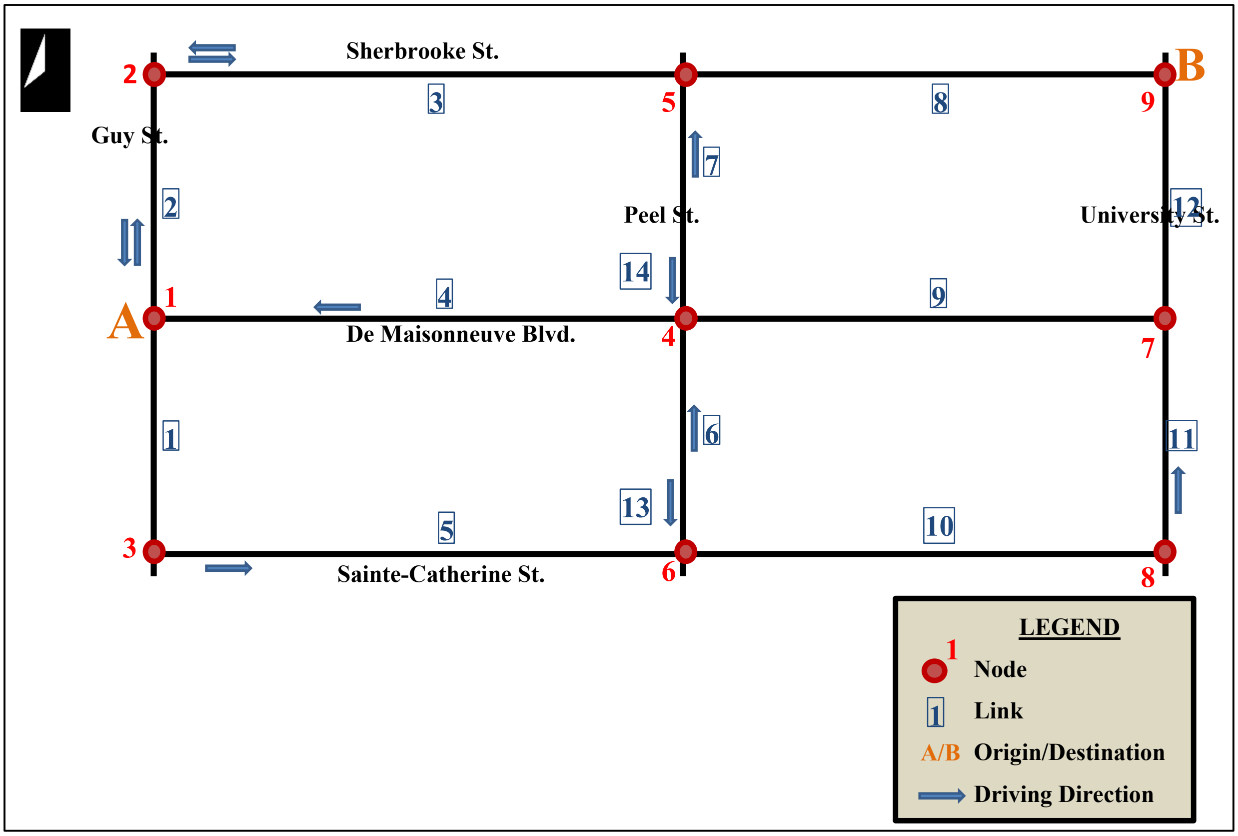

Figure 8: Modeled network (nodes/node), Montreal case study

The study used the regular weekday schedule of the city's subway system available through the local public transit agency, Société de transport de Montréal (STM). The transit schedule data was integrated into the route nodes parameters of transit stations, as was previously explained in Section 3.1 of the Methodology (Figure 1). Table 4 presents an example of data processing for the metro station Guy-Concordia and the line 24 bus stop at intersection of Peel and Sherbrooke.

Table 4: Sample route/station node parameters for metro and bus modes

	Metro Station Guy-Concordia	Bus Stop Sherbrooke/Peel
Route Node	1	2
Station Node	1	4
Mode	Metro	Bus
Transit Line	1	24
Transfer Time [min]	2	-
Route Event Type (Departure/Arrival)	Departure	Departure
Next Route Node	4	5
Departure Times	16:02, 16:06, 16:10, …	16:03, 16:11, 16:19, …

In order to estimate the driving time on the links, Google Maps® data is used. Travel time on each link was estimated by defining the beginning and end nodes of each link, as the starting and end point of an arbitrary trip in Google Maps® and the average estimated driving time during peak and off-peak hours during weekdays reported by Google Maps® was used for the calculations. For example, the westbound driving time on Sherbrooke St., between nodes 2 and 4 was estimated to be 3 and 5 minutes during off-peak and peak periods, respectively. Also, the departure time of bus/metro (Table 4) at each station is integrated into the transit model. It is assumed that a traveler's cost to use the public transportation or the bicycle sharing system is the same. Therefore, the performance measure used to determine the optimal route choice is given by the expected travel time between the origin and the destination. An average pedestrian walking and cycling speed of 3 and 15 km/h are used for calculating travel time on the links by each of these modes. The lengths of the E-W and N-S links are approximately 550 m and 175 m respectively. The expected travel time is evaluated using the flowchart previously shown in Figure 4 and summarized below in Table 5.

Table 5: Estimated travel times for select modes from A to B – No Service interruption at metro is expected (Departure at 4:00 pm)

Route No.	Travel Modes	Sequence of Nodes (See Figure 20)	Travel/Transfer Time [min]
1	Automobile	1(A), 2, 5, 9(B)	11.2+10*
2	Metro +walk	1(A), 4, 7, 9(B)	9**
3	Cycling	1(A), 2, 5, 9(B)	10.8
4	Walk + Bus (24)	1(A), 2, 5, 9(B)	17.4**

*10 minutes is allocated for parking the car and walking to destination

**Travel time includes the walking/access time to bus stop/metro platform, actual ride time on the bus/metro and walking time from bus stop/metro station to destination

It can be seen in Table 5 that, as expected, if a traveler begins the trip at 4:00 pm, the fastest route is provided by the subway and it amounts to about 9 minutes. Alternatively, a traveler can use BIXI® between A and B with a travel time of about 11 minutes. If traveler uses the bus line 24 on Sherbrooke St, total trip duration is 17 minutes. Driving an automobile between the A and B leads to the longest travel time, about 21 minutes including, an estimated 10 minutes time for parking the vehicle in the proximity of the destination and walking to point B. This additional parking/walking time is expected due to reduced availability of parking in the Montreal CBD, especially during the afternoon peak traffic.

The above estimations are valid assuming that the subway service with normal operations and no disruptions. However, if a 20% chance of an interruption in the Metro service is assumed (an arbitrarily selected value) and it continues during the whole peak hour, the expected travel time or route 2 increases to 21 minutes. Under this assumption, the metro mode does not provide the minimum travel time between A and B, and the route 3 is the optimal alternative (i.e. using the bicycle, as shown in Table 6).

Table 6: Estimated travel times for select modes from A to B – 20% chance of metro service interruption (Departure at 4:00 pm)

Route No.	Travel Modes	Sequence of Nodes	Travel/Transfer Time [min]
1	Automobile	1(A), 2, 5, 9(B)	11.2+10*
2	Metro +walk	1(A), 4, 7, 9(B)	20.8†
3	Cycling	1(A), 2, 5, 9(B)	10.8
4	Walk + Bus (24)	1(A), 2, 5, 9(B)	17.4**

*10 minutes is allocated for parking the car and towards destination

† Travel time includes the walking/access time to the platform, the trip time with the metro, the effect of service interruption and walking/egress time to the destination

**Travel time includes the walking time to the bus stop, the trip time with the bus, and the walking/egress time from bus stop to the destination

The above analysis was conducted based on a fixed departure time (at 4:00 pm). The time of trip in conjunction with transit schedule determines the waiting time for passengers. In addition to the departure time, several probabilistic parameters are used in the MDP algorithm which could affect the results and the optimal route the traveler should follow. The following section conducts a sensitivity analysis on these parameters.

4.1.2 Sensitivity Analysis

One of the parameters that could affect the traveler's route choice in a stochastic transportation network is their departure time. The first sensitivity analysis is conducted to evaluate the final route optimization results of the proposed algorithm based on different departure times. Three alternative departure times are considered: 4:15, 4:30 and 4:45 pm. For each departure time, two scenarios are considered for metro services: i) normal operations and ii) 20% chance of service disruption. The results are shown in Table 7 below.

Table 7: Estimated travel times (A to B) for different departure times

Metro Service	Travel Modes	Sequence of Nodes	Travel/Transfer Time [min]		
			Departure at 4:15 pm	Departure at 4:30 pm	Departure at 4:45 pm
Normal service	Driving	1(A), 2, 5, 9(B)	11.2+10	21.2	21.2
	Metro + Walk	1(A), 4, 7, 9(B)	9	10	7
	Cycling	1(A), 2, 5, 9(B)	10.8	10.8	10.8
	Walk + Bus (24)	1(A), 2, 5, 9(B)	17.4	18.4	21.4
Service interruption (0.2 probability)	Driving	1(A), 2, 5, 9(B)	21.2	21.2	21.2
	Metro + Walk	1(A), 4, 7, 9(B)	20.8	22	19
	Cycling	1(A), 2, 5, 9(B)	10.8	10.8	10.8
	Walk + Bus (24)	1(A), 2, 5, 9(B)	17.4	18.4	21.4

As it can be seen in Table 7, the departure time affects the estimated travel times of the trips made via the alternative routes with metro or the bus. As expected, when metro is operating normally, the optimal route from A to B is by using metro. However, when there is service interruption, using a BIXI® bicycle represents the fastest route to get to destination. If cycling is not an option for the traveler, depending on the departure time, the metro or bus routes become the alternative solution. For example, if the traveler starts from A at 4:15 or 4:30pm, the optimal route is given by the bus line on Sherbrooke. But, if the traveler departs after 4:45pm, the metro would provide the fastest route (assuming that cycling is not an option for the traveler).

Table 7 also provides the impact of metro service interruption in this hypothetical travel scenario. It can be seen that the metro service interruption could increase the travel time of this mode by 120%. Additional sensitivity analysis is conducted on testing different values of metro service interruption probability, when this mode would become the second alternative route. Figure 9 presents the changes in estimated travel time using metro under different probabilities of having a service interruption. It can be seen that, for departures at 4:00 pm and when the probability of metro service disruption is higher than 13%, the Bus alternative becomes the optimal route to move from A to B.

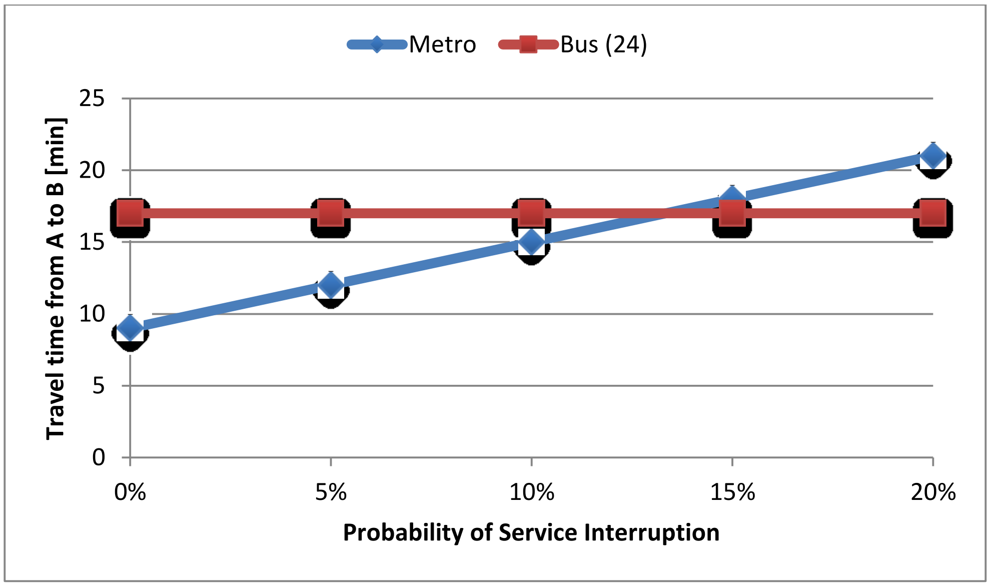

Figure 9: Travel time impact due to different probability values of service interruption at Metro (Departure 4:00 pm)

Similar sensitivity analysis can be conducted on different probabilities of bicycle availability at BIXI® stations and its effect on the optimal route. Figure 10 presents the estimated travel times for using the bicycle mode using different availability probability values. It can be seen that if there is a 50% chance of no bicycles available at the station, the expected travel time increases to nearly 15 minutes, which is comparable the expected travel time by bus. Depending on the convenience level of travelers and their value of time, using the bicycle may not be an option for them. Figure 10 also shows the expected travel time from A to B, under different probabilities of observing peak-period traffic congestion. Similarly, as explained above, these travel times include a flat 10-minute time for parking and walking to destination. On the other hand, assuming off-peak traffic conditions, the expected travel time from A to B, using the automobile, is comparable to the expected travel time using the bycicle, given that bicycles are available at all time, and this makes the bicycle mode less attractive compared to the convenience of driving, especially if the weather conditions deteriorate.

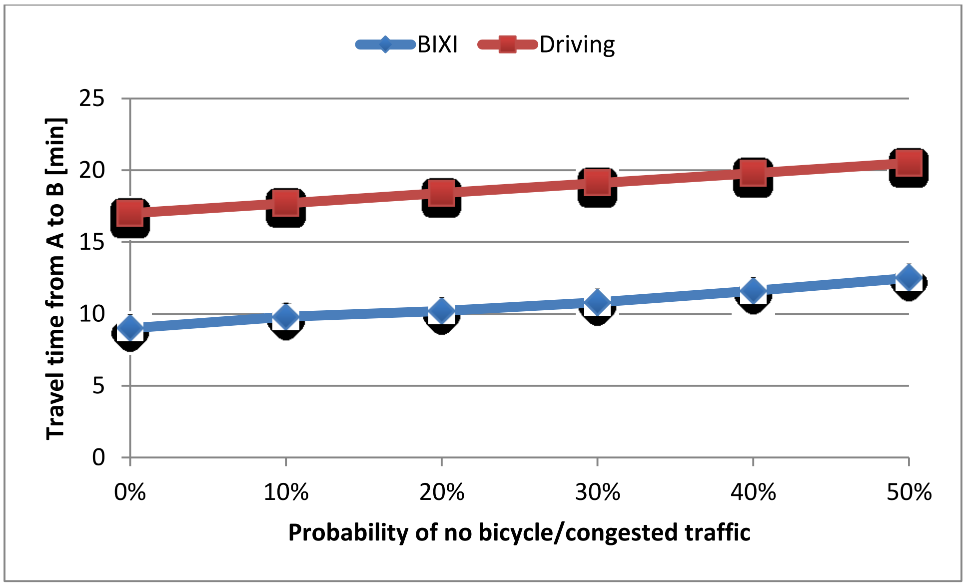

Figure 10: Travel time impact due to different probability values of bicycle unavailability and different probability values of traffic congestion occurrence

4.1.3 Discussion

In this case study the proposed Markov decision process-based routing algorithm was applied to model travelers' routing in multimodal transportation networks. The algorithm's objective function minimizes the travel time. The multi-modal network included four transportation modes: automobile, bus, metro and bicycle. In order to see the effect of changes of the traffic conditions, the transition probability matrix was estimated based on real-world speed data obtain from an arterial with similar traffic conditions. The concept of the super-node in a Markov chain was associated with transit station nodes to facilitate the integration of public transportation network into the multimodal network. The proposed methodology incorporated transit schedule into calculations and was applied to a real-world network located in Montreal, Quebec. The public transportation fixed schedule was used to compile the route node parameters for the modeled network. The case study demonstrated the calculation procedure of optimal routes for a traveler moving between two arbitrarily selected nodes in the network.

Several cases were tested by considering different operations conditions with a given probability of congested traffic and/or service disruption for two of the available transportation modes (Metro and Cycling). The results demonstrated the applicability of the proposed algorithm to identify the fastest route to destination. It was shown that automobile travelers can save up to 14 minutes of travel time by switching to another transportation mode (e.g. metro, bus, or bicycle).

The case study clearly shows the applicability of the proposed algorithm in evaluating the optimal route in a stochastic network and potential benefits as compared to shortest path algorithm. The results showed that by accounting for some stochastic parameters of different transportation networks, travelers could optimize their travel times when travel time delays occur for specific transportation modes. In this case study, the availability of traffic data for the study arterials was limited. Using wider range of speed data, for several days within the peak period, could improve the traffic condition predictability. In addition, the probability of the Bixi© bicycles being available could be more accurately estimated by using historical data related to availability of bicycles at bike stations during rush hour throughout the study area.

4.2 Toronto case study

The first case study investigated one application of the proposed methodology in a small network located within the downtown area of Montreal. In this section the MDP algorithm is applied to model general travelers' routing in a larger and more complex network of motorized and non-motorized modes. The 45 km^2 study area is located within the Greater Toronto Area (GTA). The network includes more than 80 km of major roadways, including QEW and Gardiner Expressway Highways (West of the CBD) and several major arterials within the Toronto CBD area and is shown in Figure 11.

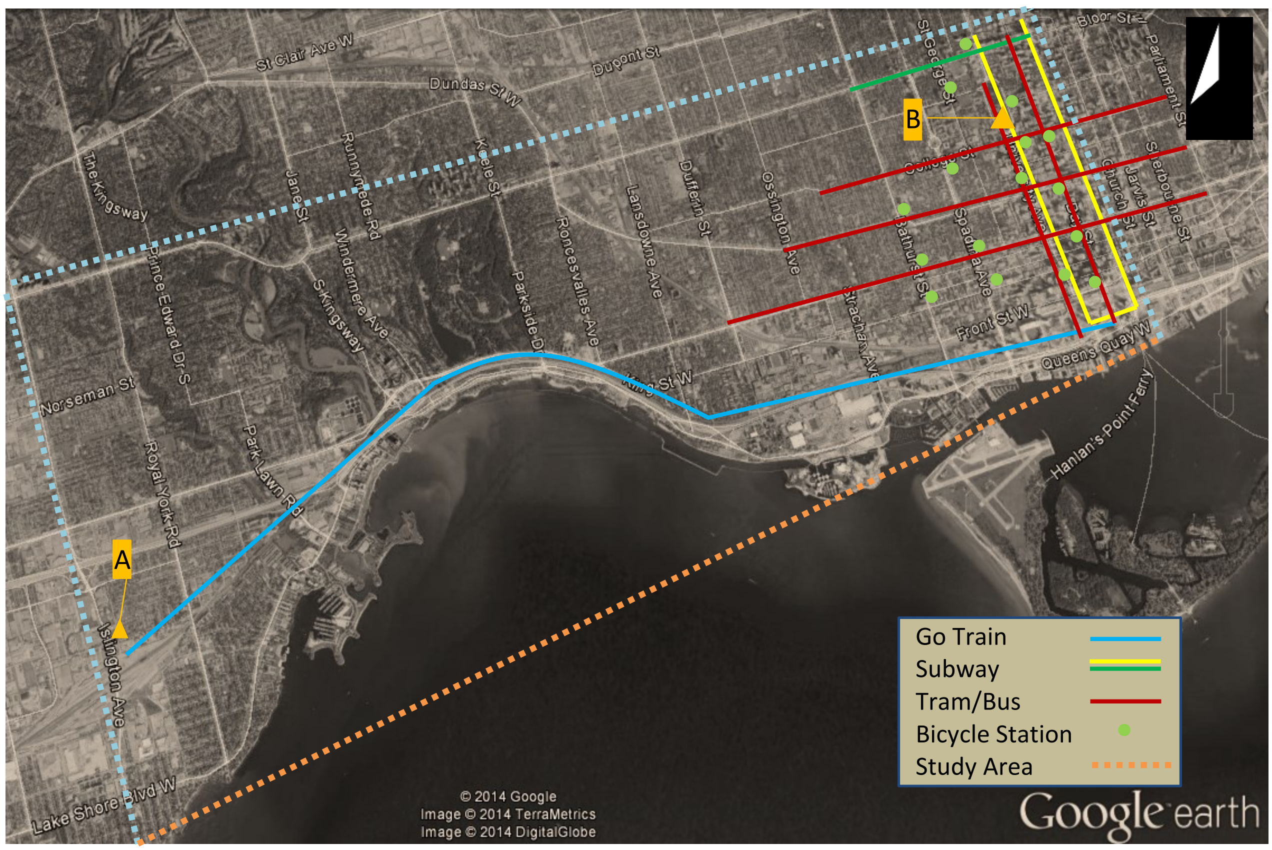

Figure 11: Layout of the study area, roadway network and select transportation modes (Source: Google Earth)

The boundaries of the study area are limited to Islington Avenue on the west, Bay St on the east, Lakeshore Blvd on the south and Bloor St on the north. The selected area with multiple transportation facilities (i.e. commuter trains, street cars, buses, subway and bicycle paths) enabled us to better model the stochastic behavior of a transportation networks and to evaluate the performance of proposed methodology in more realistic network.

The proposed method is used to find the optimal route for traveling between two arbitrarily selected node (marked as A and B in Figure 11) by using private and public transportation. This is a typical route for commuters living outside the City of Toronto. Travelers might combine carpooling parking lots and the commuter trains. The analysis includes the peak period and captures the heavy congestion along major highways (QEW and Gardiner Express) connecting the GTA suburbs to downtown Toronto. During the rush hour periods, heavy traffic on the EB highway/arterials towards downtown Toronto could be expected. The case study considers the stochastic traffic conditions along the roadways within the study network. Travel time along these roadways would change according to different traffic conditions.

4.2.1 Network Model

The network is modeled based on the methodology presented in Chapter 3. In total, 34 Nodes and 58 links, to represent intersections and arterials, are identified in the study network. Figure 12 and Figure 13 present schematic of the model with the created node and link numbers respectively. Nodes for the arterials represent the intersection of two streets. For Gardiner Exp. nodes represent intersection of the exit ramp with the crossing street. If there is no exit available (e.g. Gardiner Exp. at Bathurst St), no node is created (See Figure 12).

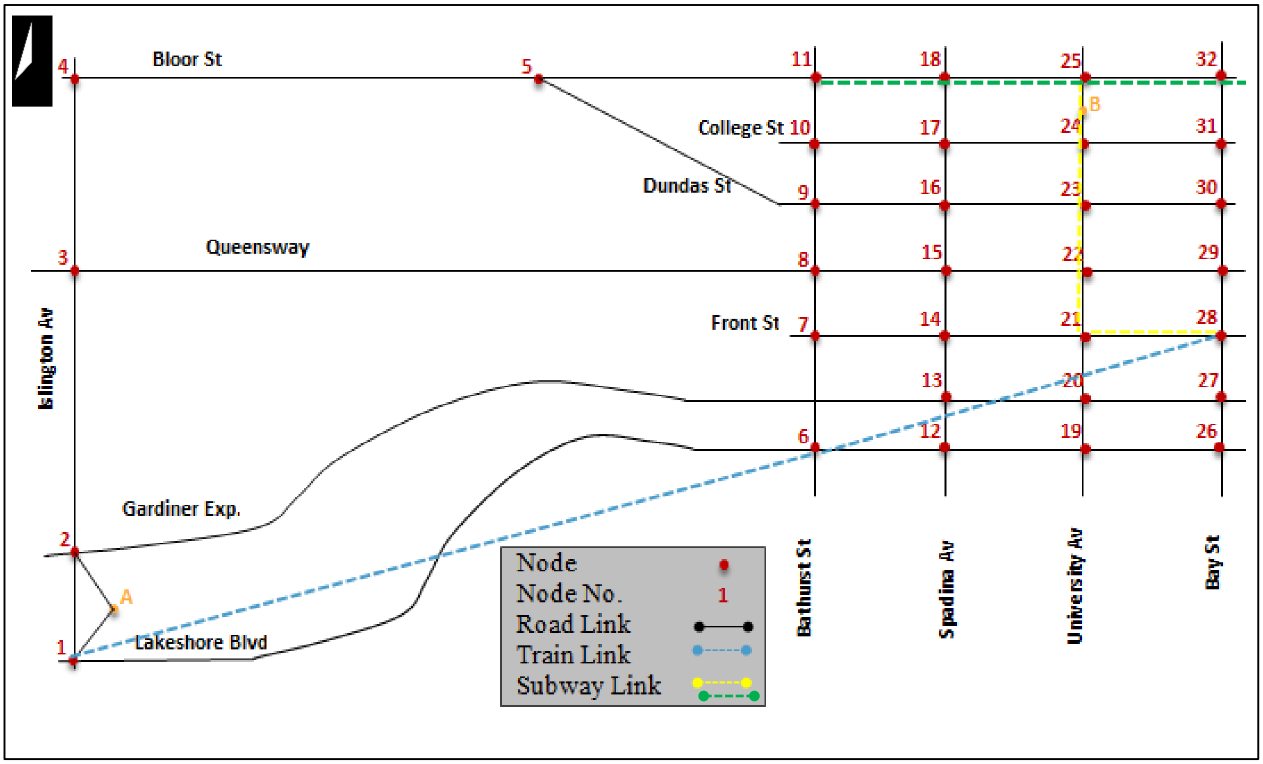

Figure 12: Schematic model of the nodes/node numbers within the network

The direction of traffic was considered when creating the links and separate links are created for each direction of traffic along arterials related to the study area. However, not all the bidirectional links are represented (See Figure 13).

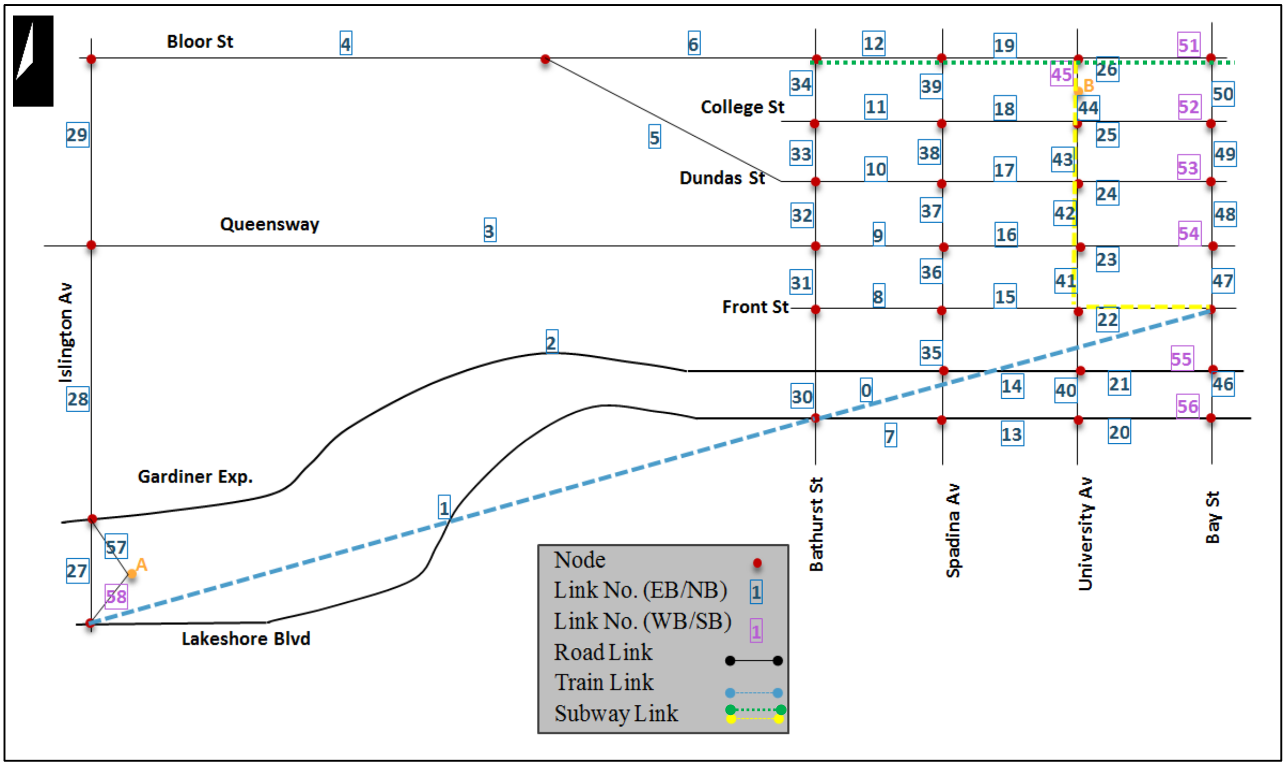

Figure 13: Schematic model of the links/link numbers within the network

In addition eight bus/tram lines, two subway lines and one commuter train route were coded in the model. The time-tables of the commuter train, tram/bus and subway at each station are integrated into the transit model. Travel time information for transit modes is evaluated using their regular daily schedule. The waiting time for next transit service is estimated based on original departure time and estimated arrival time of individual traveler at each station/top. Details regarding modeling the public transit lines are provided later in this section. Bicycle sharing facilities near different nodes are also integrated into the network. Travel time for bicycles/pedestrians on each link each are calculated based on typical average speeds used in similar studies conducted by FHWA 2006.

In order to estimate the travel time along the network links under different traffic conditions, a microscopic computer model of the study network was implemented using AIMSUN (Advanced

Interactive Microscopic Simulator for Urban and Non-Urban Networks). This software package is a proprietary microscopic simulator that can model traffic conditions of real-world traffic networks on a computer. The behavior of every single vehicle is continuously modeled throughout the simulation using several driver behavior models (e.g. car-following, lane-changing, gap-acceptance, etc.). AIMSUN is capable of performing microscopic and mesoscopic simulation as well as hybrid simulation, allowing modeling of large areas while focusing on the areas that require detailed traffic analysis. The reason for using AIMSUN in this case study is the ability to easily import and edit GIS files as the first step in developing the model, as explained here after.

In order to model the study area in AIMSUN, the following tasks were conducted:

1. The base geometry skeleton was developed by importing a GIS map of the study area in AIMSUN. The GIS maps from Land Information Ontario (LIO) (See Figure 14) was used which included key geometry attributes, i.e. number of lanes, road types, street names and speed limits in multiple datasets. These datasets were appropriately merged into one dataset based on a common identifier and joined with the road network map using ESRI ArcGIS software;

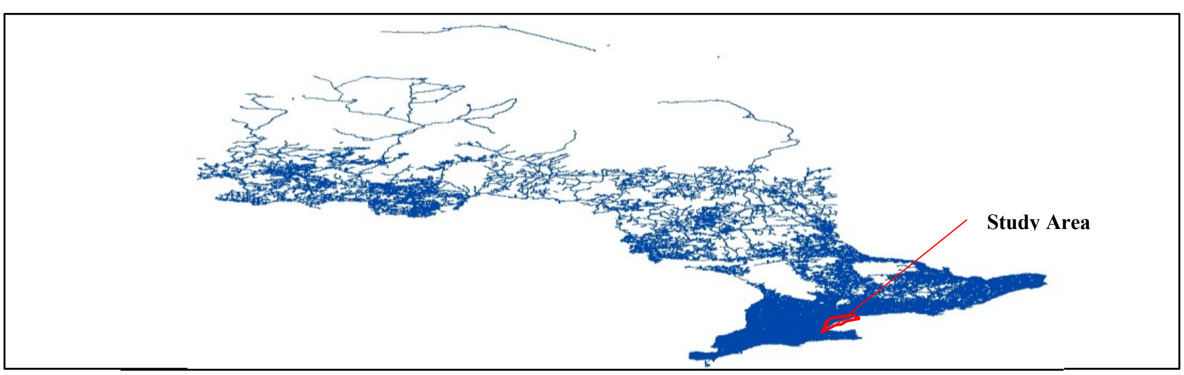

Figure 14: GIS network acquired from LIO and the study area in this project

2. The study area network with key attributes was imported in AIMSUN for development of the base geometry. The model was then refined by using Google Maps and Google Street View to make necessary changes regarding the geometry/number of lanes for all roadways within the model. Figure 15 shows the network model in AIMSUN;

Figure 15: Study area modeled in AIMSUN

3. The actual traffic demand as well as traffic signal timings was coded in the model. Separate traffic demand was available for a 4-hour period between 15:00 and 19:00. In order to create free-flow and congested scenarios, the simulation was conducted for a 4-hour period (15:00 – 19:00) with available hourly demands. A half hour warm-up period was used for the simulation. The microscopic model was calibrated using real data collected during PM peak period for major arterials and QEW highway (traffic counts, speed and travel time data). In order to account for the stochastic nature of

vehicle arrivals, five simulation runs with distinct random seeds were conducted. The average of five replications was used as the final result.

4. The simulation results were stored in a SQLITE database file by AIMSUN. The output was in the form of a table and included Section IDs, time intervals, and simulated travel time/speeds. Each section ID in AIMSUN corresponds to one link in the modeled network. SQL code was used to extract travel time and speed data for the required links within the network from the SQLITE database. The data were imported into Excel from SQLITE. A Visual Basic code was developed to import the results into a template created in an Excel worksheet. Link travel times were assigned to corresponding arterials and highway sections of the study area. (See Appendix A)

The simulation results were used to estimate the link travel times on Gardiner Expressway and major arterials in the network under different traffic congestion levels. Travel times for all the links were aggregated for 30 minute intervals within the 4-hour simulation period (3:00 – 7:00 pm). It was expected that during the first hour of simulation no congestion forms within the network and therefore the estimated travel time during the first hour was considered as the off-peak travel time. The simulation results for later hours within the simulation period were also analyzed to estimate the travel times corresponding to the peak and congested traffic conditions along each link.

4.2.2 Model Calibration

The control data used to calibrate the model was comprised of volume and speed data. The volume data was compiled from a mix of counts from different detectors. Speed data was collected for Gardiner Expressway, Lake Shore Boulevard and major arterials. For this study area GPS based speed information from proprietary data accessible via private data collection source was used. Signal timing plans were another important part of the model's calibration and was available from the municipalities with jurisdiction in the study area.

Driving behaviour

Driver behaviour is calibrated in four different categories within AIMSUN. The first category is the model's experiment properties. This is where the simulation step and reaction times are calibrated. These two parameters are tied together in that the reaction time must be a multiple of the simulation step. The simulation step and reaction time were adjusted to calibrate the overall speeds and level of congestion on the highway and major arterials.

The second category is the section properties. The section properties are used to calibrate the operation at on-ramps by adjusting how cooperative vehicles are with merging vehicles, the distance over which that they will cooperate, and the distance over which vehicles will merge into traffic. The section properties are also used to set speed limits.

The third category is the node properties. The node properties are used to calibrate how vehicles will react to upcoming turns in their path. Each turn has two variables that define the limits of three zones upstream. In the first zone, vehicles are unaware of the upcoming turn. In the second zone, vehicles recognize the upcoming turn but will still make lane choice decisions based on speed advantages. In the third zone, the vehicles will make whatever lane changes necessary; even if they must slow down to do so. These parameters are key to calibrating traffic operation at off-ramps and were adjusted in the model to reflect levels of congestion observed in the speed data collected.

The fourth category is the vehicle properties. This is where characteristics such as speed acceptance, maximum desired speed, and acceleration/deceleration curves are defined for each vehicle type (e.g. cars, light trucks, heavy trucks, etc.).

The visual observation of the model was completed under different simulation scenarios to ensure that the model overall is operating as expected and that driving behavior (merging, lane changing, speed reduction, queue formation, etc.) is reasonable.

Volume

Traffic volume calibration involves comparing average traffic volumes generated from the simulation model against the observed traffic volumes. The Geoffrey E. Havers (GEH) statistic formula is used for this comparison. The GEH statistic is an empirical formula similar to chi-square test that is used in traffic modelling to compare two sets of traffic volumes. GEH statistics can be used as an acceptance criterion for travel demand forecasting and model calibration as shown in Equation (7):

$$GEH = \sqrt{\frac{2(M-C)^2}{M+C}} \tag{7}$$

Where, M is the hourly traffic volume from the model and C is the hourly volume from the traffic count.

For calibrating traffic models in the "base case" scenario, according to the FHWA (2004) guidelines for microsimulation modelling, a GEH of less than 5.0 for 85% of the links is considered an acceptable match between the modelled and observed hourly volumes. In this model, volumes of all major arterials and Expressway within the network were considered for calibration. The calculated GEH for 90% of the volumes were less than 5.0 which shows that the model is well calibrated in terms of traffic volumes. Figure 18 shows the calculated GEH statistics for the model.

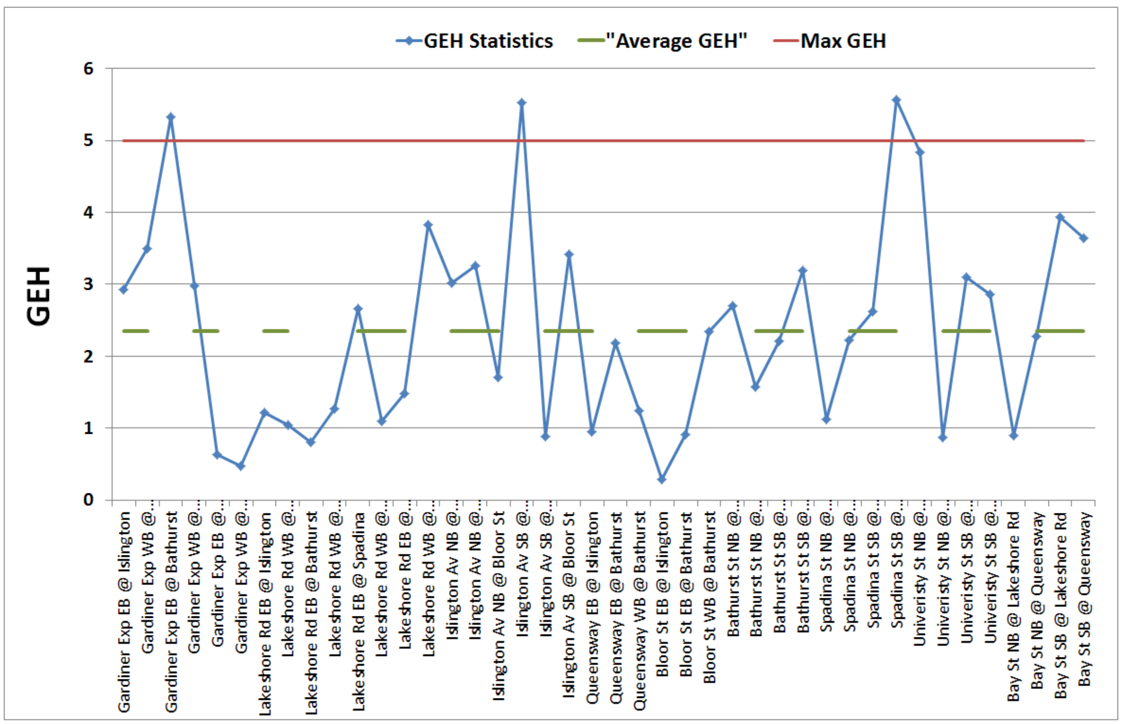

Figure 16: GEH Statistics Calculations for the modeled study area

Speed

Speed calibration was done for highway and major arterials. Speed profiles were constructed from the GPS data and compared with outputs from the model. The GPS data is aggregated based on changes in road cross-sections, resulting in a step-graph. The modeled speeds are plotted in two forms: (1) as a step-graph based on AIMSUN's section divisions and (2) a smooth line-graph based on detectors spaced every 100m. Figure 17 shows a sample speed profile for Gardiner Expressway.

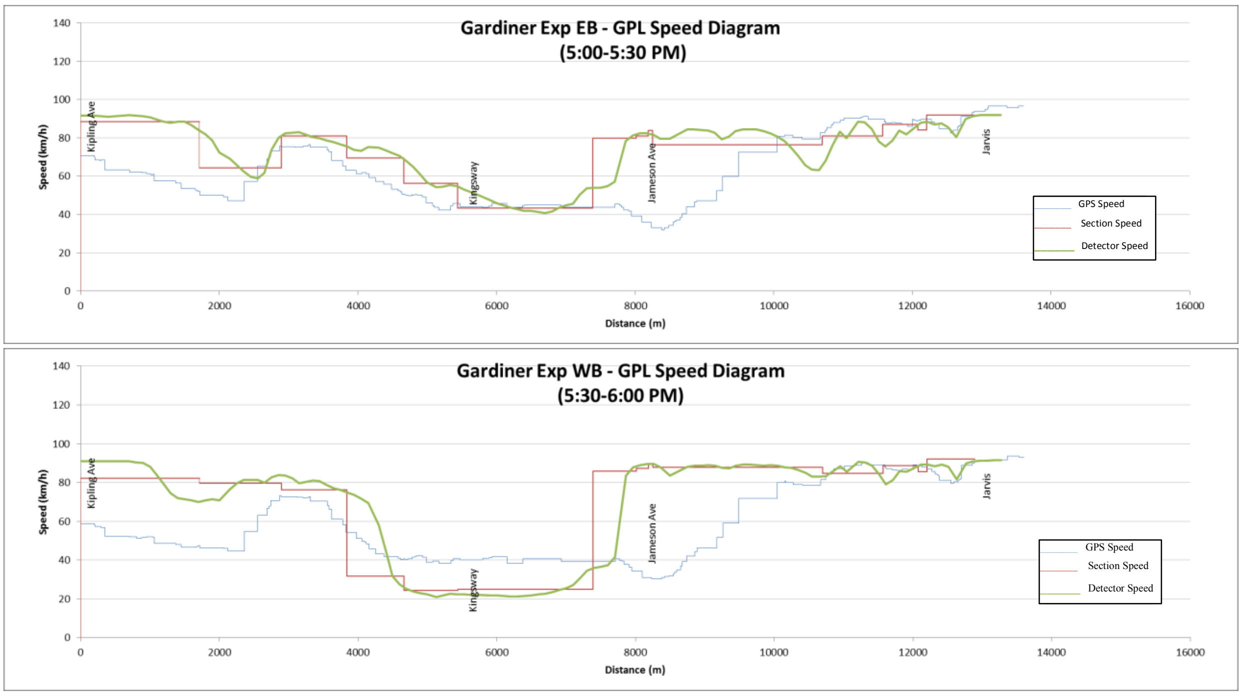

Figure 17: Speed profile used for calibration – Gardiner Expressway

4.2.3 Traffic Conditions and Transition Probabilities

The proposed MDP methodology incorporates transition probabilities which denote the probability that traffic conditions change from one node to another node within the network. In order to estimate the probability of observing different traffic conditions and the related transition probability matrix, real-world traffic speed was used to identify different traffic conditions related to the study network. The following tasks have been performed to build the transition probability matrix for the MDP methodology:

1. Traffic speed data collected for two different directions along four major arterials in Toronto during the 4th week of September 2012 was available. The average speed data from GPS equipped vehicles was aggregated over 15-minute period intervals. Speed data was available for about 50 kilometer length of major arterials was analyzed.
2. Speed graphs for 15-minute time intervals where created and analyzed to better understand the range of changes in the average speed data along each corridor/direction. The analysis was done for the period between 14:00 and 19:00 and for all weekdays, the weekend data were excluded. Figure 18 shows a sample of such graph for the northbound direction of one section along Dufferin St.

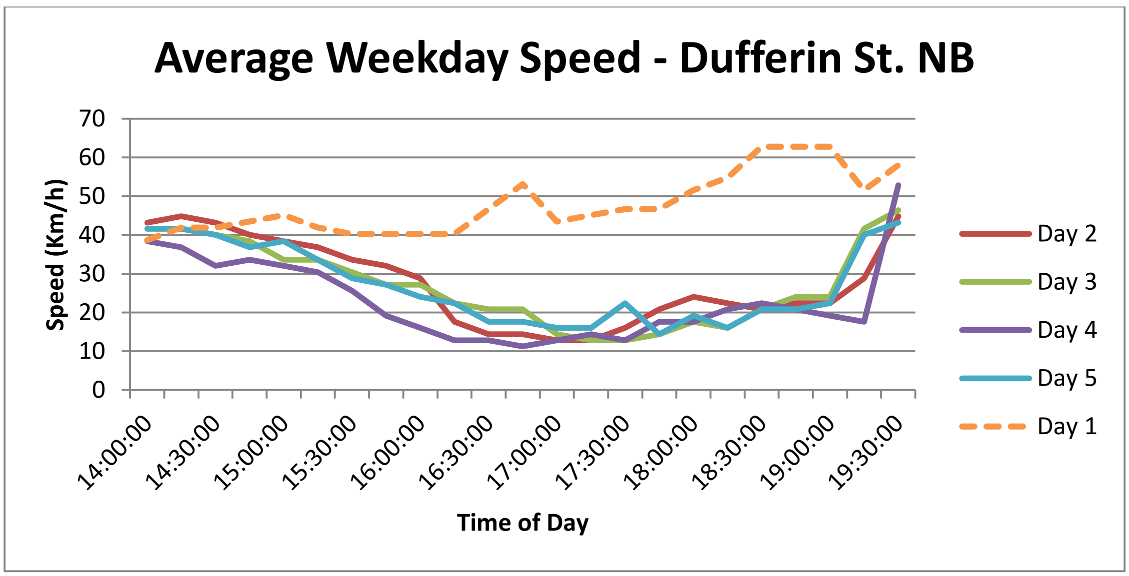

Figure 18: Sample of 15-minute interval average speed profile along Dufferin St. (NB)

3. Based on the observations from the speed graphs, outlier days/locations were removed from the analysis. For example, in case Dufferin St. speed graph shown in Figure 18, the average speed data related to “day 1” was identified as an outlier (with a different trend compared to the other days) and therefore was not considered for the

calculation. Day 1 was considered to be an outlier because more than 70% of the data points were outside the 95% confidence interval of the other four days.

4. The collected speed values were divided into three arbitrary ranges to represent three types of traffic conditions: A) off-peak: speed > 40km/h; B) peak: 20< speed <= 40 km/h; and C) congested: speed <= 20km/h.
5. A visual basic program was developed to calculate the elements of transition probability matrix by applying the methodology described in Chapter 3. The probability of observing a change in traffic conditions was estimated based on number of changes in the average traffic speed from one traffic condition range (defined above) to another during each 15-minute interval. This calculation was done based on weekday data (excluding weekend and outlier) and for each direction of travel along each arterial.
6. The calculated probability matrices of arterials were compared for the directions of traffic movement related to this study (northbound and eastbound). Final transition probability matrix was calculated by averaging the corresponding elements of each probability matrix of arterials and is shown in Table 8.

Table 8: Transition Probability Matrix (Arterials)

Future Traffic State / **Current** Traffic State	A	B	C
A (off-peak)	0.86	0.14	0.00
B (peak)	0.11	0.86	0.03
C (congested)	0.03	0.22	0.75

7. Each row in Table 8 represents the probability of the change in traffic condition from initial state (Rows: A, B and C) to another state (Columns: A, B and C). Based on this

table, the probability of the traffic conditions to improve from level C to level A and B is 0.03 and 0.22 respectively. Similarly, the probability of traffic condition level B to be improved to level A is 0.11. There is also 3% chance that traffic condition would deteriorate from level B to level C. Moreover, the probabilities of traffic condition levels A, B and C to remain at the same level are 0.86, 0.86 and 0.75 respectively.

As it can be seen in Table 8, the sum of probabilities for traffic change (each row) is equal to one. However, this relationship does not apply to the columns of this table. On the other hand, the numbers along columns of the table does not represent the same probability relationship between two states. The transition probability numbers are only applicable if the initial traffic state is chosen form rows and the future traffic state is selected from one of the columns.

8. The study area in this study includes one major east-west Highway (QEW) that connects Toronto to the west of the province. It should be noted that the changes in traffic flow parameters on highways (e.g. speed, density, shockwave speed) is different than those on arterials, therefore the probability of changes in traffic state is also different and should be considered in the analysis. For the purpose of this study, since the aggregated speed data was not available for QEW highway, a parallel highway (401) was used as reference for evaluating the transition probability matrix along highway section.

Spot speed data on two different sections of Highway 401 was available for one day (May 13 2011). Speed values were aggregated for 1-min periods between 15:00 and 19:00 on the count day. The same process as for the arterials was used to estimate the probability of change in the speed on vehicles on the EB direction of the highway, as per the study area. The collected speed values were divided into three arbitrary ranges to represent three types of highway traffic conditions: A) off-peak: speed > 70 km/h; B) peak: 50< speed <= 70 km/h; and C) congested: speed <= 50 km/h.

The final probability matrix was built by using the average probabilities for each change in traffic state on highway 401. Table 9 presents the results:

Table 9: Transition Probability Matrix (Highway)

Current Traffic State \ Future Traffic State	A	B	C
A (off-peak)	0.71	0.29	0.00
B (peak)	0.12	0.59	0.29
C (congested)	0.00	0.17	0.83

9. In order to have similar transition probabilities for the whole network, the average probabilities between the two matrices are used as the final transition probability matrix in the MDP algorithm (see Table 10)

Table 10: Transition Probability Matrix (Study Network)

Current Traffic State \ Future Traffic State	A	B	C
A (off-peak)	0.78	0.21	0.00
B (peak)	0.12	0.72	0.16
C (congested)	0.02	0.19	0.79

The initial transitions probability matrix can be created by using arbitrary probability values and based on the available departure time. Depending on the departure time and its observed traffic conditions, a higher probability should be assigned to related traffic state. For example, for the beginning of peak period, it is assumed that traffic state B (peak) has a higher probability compared to the other two states. Therefore, the arbitrary probability matrix presented in Table 11 can be used:

Table 11: Initial probability matrix when departure time is at the beginning of peak period

A	B	C
25%	50%	25%

4.2.4 Public Transit Network

The study area included the following transit modes/lines (as shown in Figure 19):

1. Go Transit Lakeshore east train, with 3 station within the study network (i.e. Mimico, Exhibition and Union);
2. Subway yellow line 1 (Yonge-University-Spadina), where 5 stations were included in the study area (i.e. Union, St Andrew, Osgoode, St Patrick and Queen's Park);
3. Subway green line 2 (Bloor-Danforth), where 15 stations starting from Islington on west to St George St were included in the study network;
4. Bus line 6, with 5 stops on Bay St (i.e. at Front, Queen, Dundas, College and Wellesley Streets);
5. Bus line 142 (express), with 4 stops on University St (i.e. at Queen, Dundas, College and Wellesley Streets);
6. Streetcar line 501/Bus line 301, with 3 stops along Queen St (at Bathurst, Spadina and University Streets);

7. Streetcar line 505, with 3 stops along Dundas St (at Bathurst, Spadina and University Streets);
8. Streetcar line 506/Bus line 306, with 3 stops along College St (at Bathurst, Spadina and University Streets); and
9. Streetcar line 511, with 3 stops along Bathurst St (at Queen, Dundas and College Streets).

The transit schedule for each line at each station/stop was derived from Toronto Transit Commission (TTC) website and is provided in Appendix B. Travel time on each link between each two steps within the network was then calculated based on the scheduled times at each station. In addition, any applicable waiting time was also calculated based on the difference between estimated traveler's arriving time at station and scheduled transit time and was added to travel time of related link.

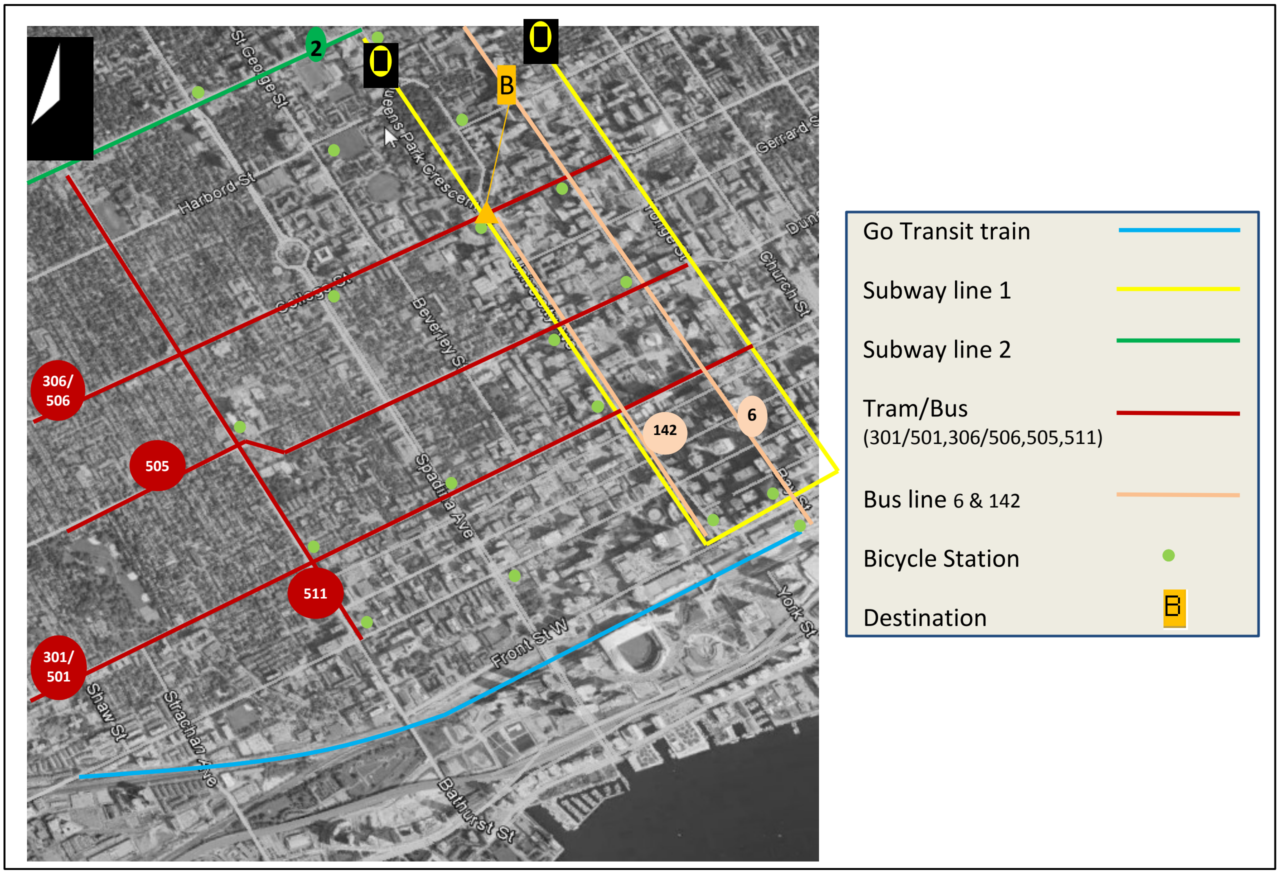

Figure 19: Transit network within the study area

The study area also includes 20 bike sharing stations of BikeShare Toronto. It was assumed that during the analysis, the bicycle docking stations have at least one available bike to rent or one available spot to return the bicycle. The travel time for the links by using bicycle mode was estimated based on the length of the link and by applying an average speed of 12 km/h for bicycles. An additional one minute was allocated for docking and undocking the bicycles at each station. The average walking speed of 5 km/h was used to calculate the walking time on each link.

4.2.5 Routing Constraints

In order to improve the efficiency of the routing algorithm and to better represent the actual behavior of passengers, the following constraints/assumptions were considered in developing the algorithm:

1. The maximum number of mode transfers (e.g. car to bus, bus to walking, etc.) is limited to two, to represent more realistically the expected behavior of commuters.
2. In regards to private mode (cars), mode change is only allowed from car to other transportation modes (and not vice versa). In this case, a transfer time of 5 to 15 minutes (variable based on the location) is considered for parking the vehicle and arriving at the station/stop. For example, in case of commuter train parking lots, which are outside the CBD, this transfer time is smaller than the expected transfer times for parking within the CBD area (i.e. roadside parking).
3. The expected average transfer time for accessing bicycle-sharing docks is two minutes.
4. All prohibited turn movements at various intersections are considered in the algorithm.
5. For transfers between the commuter train and other transit modes at the downtown central station (Union station) an average five-minute transfer time is assumed.

4.2.6 Analysis Results

The proposed algorithm was applied to find the optimal path between the arbitrarily selected points A and B. A starting time of 17:00 was used as reference time for travel time calculations. A Visual Basic (VB) program was developed to implement the routing model described in the previous chapter and to estimate the optimal route. The VB code used recursive function technique to process the MDP algorithm (See Appendix C).

Table 12 shows the results of estimating the optimal and select alternative routes of the hypothetical trip made by a commuter between A and B. It is assumed that traveler uses his own private vehicle to depart from A. Since the algorithm only identifies the optimal path, its implementation in the VB code was adjusted to obtain, for comparison purposes, two alternative paths presented in the following tables (Route No. 2 and 3).

Table 12: Estimated travel times for select routes from A to B

Route No.	Transit Modes	Sequence of Nodes (See Figure 20-Figure 22)	Travel Time [min]
1	Car, Subway, Bike/Walk	A,2,3,4,5,11,18,25,B	30
2	Car, Walk	A,1,13,20,21,22,23,24,B	61
3	Car, Go Train, Subway, Walk	A,1,6,28,21,22,23,24,B	34

According to Table 12 the following observations can be made regarding the results:

1. The optimal route for traveler (minimum travel time) is to drive north and then east along Bloor St, up to Bathurst St. and then take the subway to University St. Then, they can use bicycle or they can walk towards destination (See Figure 20). The total travel time of this path is 31 minutes, compared to 61 minute estimated time for using car only (driving from A to B).

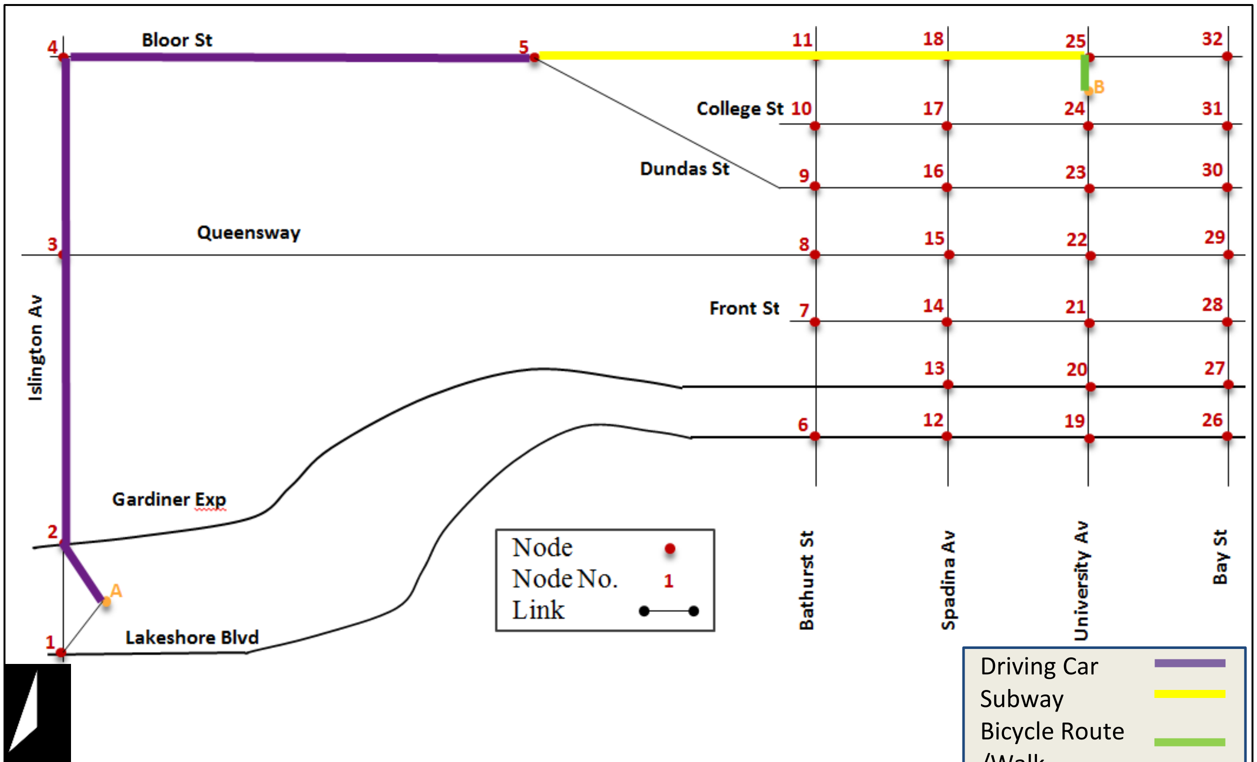

Figure 20: Traveler's optimal route

2. The second best option is to take the commuter train up to Union Station, and then switch to subway NB up to College St. Then, they can walk to destination (See Figure 21). Total estimated travel time is this scenario is 34 minutes.

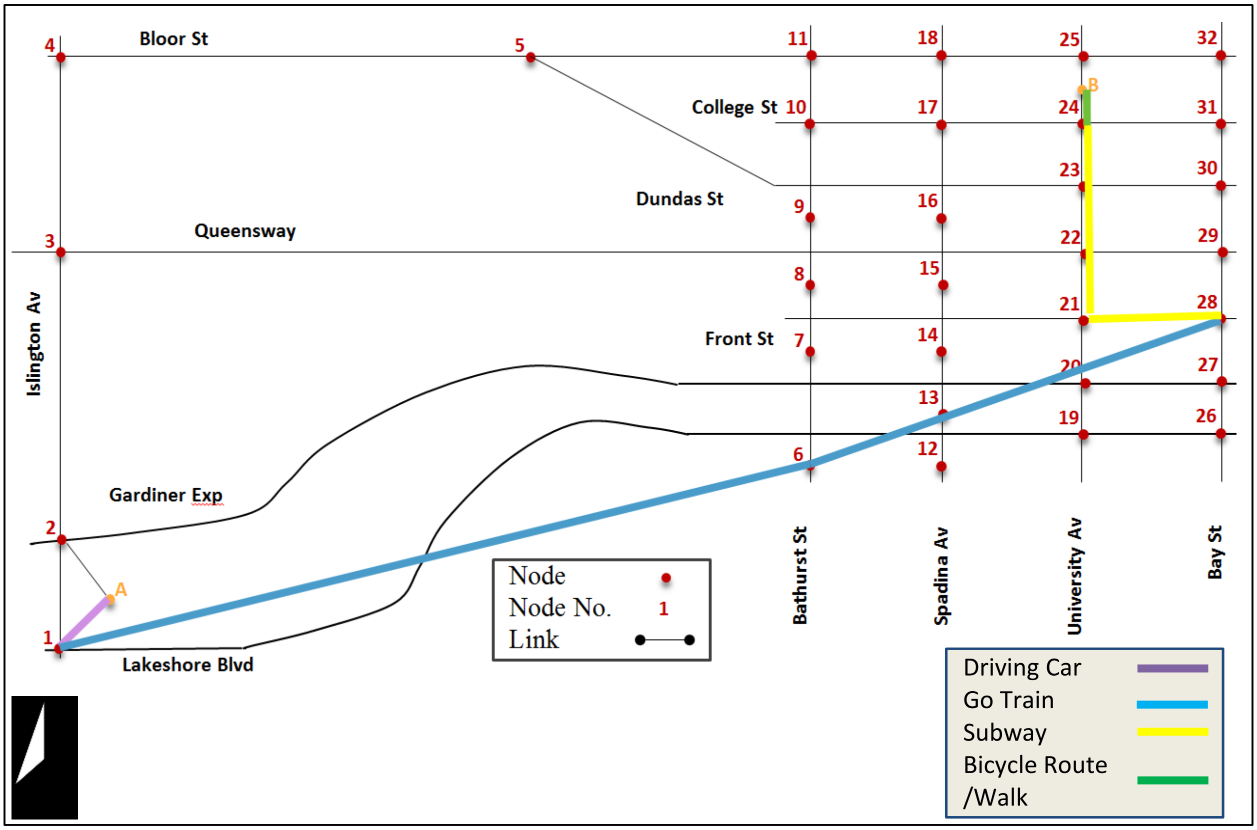

Figure 21: Second alternative route

3. Estimated travel time for driving along Gardiner highway EB and then University St NB (as shown in Figure 22) is about an hour, which is almost twice the time along the optimal route by using public transit. This is true during the congested traffic state. However, in case of normal traffic conditions (peak period) the estimated travel time for this route is 30 minutes.

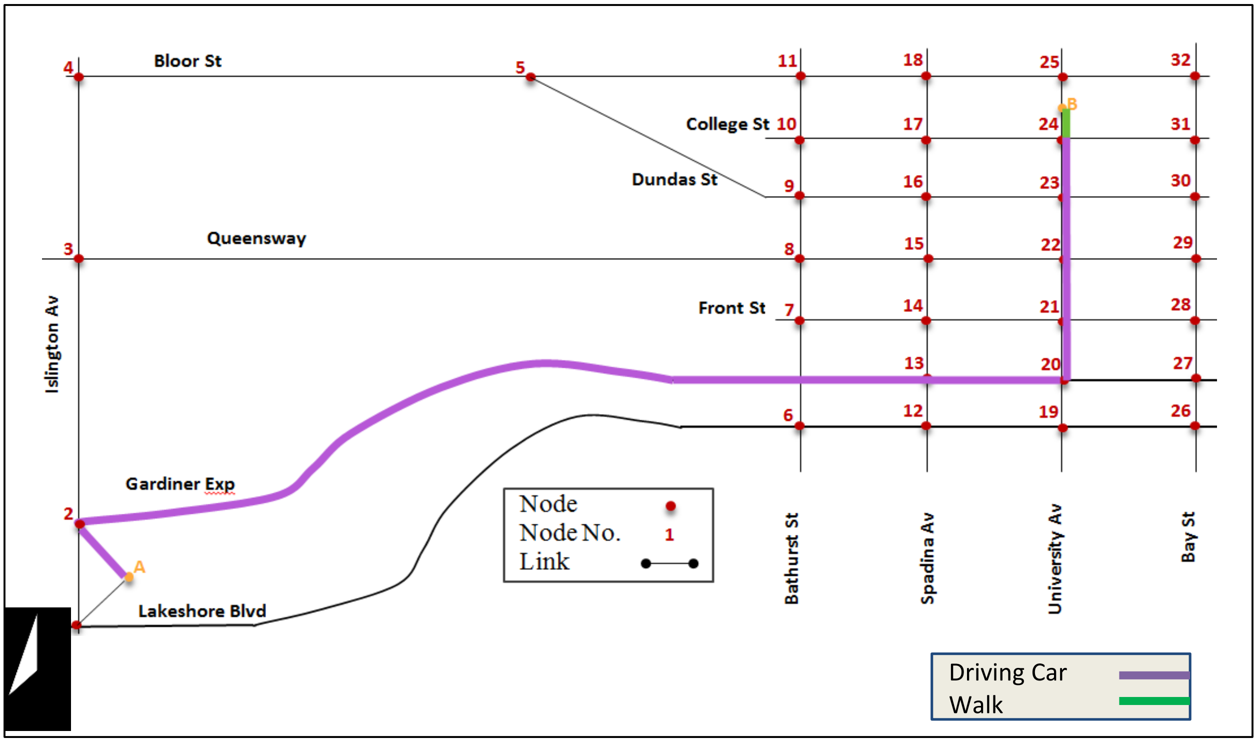

Figure 22: Preferred route under normal traffic condition

4.2.7 Sensitivity Analysis

In order to evaluate the effect of the stochastic parameters on the MDP algorithm results, a sensitivity analysis is conducted. The effect of arbitrary selected ranges of travelling speed, to represent different traffic conditions is evaluated. Initially, the following ranges of speed values are used to represent the following conditions: A) off-peak: speed > 45km/h; B) peak: 30< speed <= 45 km/h; and C) congested: speed <= 30km/h. The estimated travel times for previous routes are calculated using the proposed algorithm and results are shown in Table 13.

Table 13: Estimated travel times for select routes from A to B (revised transition probabilities)

Route No.	Transit Modes	Sequence of Nodes (See Figure 20-Figure 22)	Travel Time [min]
1	Car, Subway, Bike/Walk	A,2,3,4,5,11,18,25,B	31
2	Car, Walk	A,1,13,20,21,22,23,24,B	53
3	Car, Go Train, Subway, Walk	A,1,6,28,21,22,23,24,B	34

It can be seen that the expected travel time using the automobile mode could change by about 10% for different routes. However, the optimum route and the second preferred route do not change.

Additionally, the limits of speed ranges representing different traffic conditions are decreased to the following values: A) off-peak: speed > 30km/h; B) peak: 15< speed <= 30 km/h; and C) congested: speed <= 15km/h. Updated travel times for preferred routes are shown in Table 14.

Table 14: Estimated travel times for select routes from A to B (revised transition probabilities)

Route No.	Transit Modes	Sequence of Nodes (See Figure 20-Figure 22)	Travel Time [min]
1	Car, Subway, Bike/Walk	A,2,3,4,5,11,18,25,B	31
2	Car, Walk	A,1,13,20,21,22,23,24,B	54
3	Car, Go Train, Subway, Walk	A,1,6,28,21,22,23,24,B	34

The results show that there is minimal increase in travel time for route no. 2, by 1 minute. Nevertheless, the optimal and the second preferred route do not change. While under the tested case study the sensitivity analysis does not show significant impact on the optimal route, it can be seen that this model is able to capture such variations, and depending on the complexity of the network one can use the model to identify optimal routes under different traffic conditions.

The above analysis was conducted by applying similar transition probability matrices for arterials and highways. This was done using the average of two transition probability matrices (Table 10). As discussed in Section 3.5, the traffic pattern and changes in traffic speed may be different on arterials and highways and separate transition probability matrices may be used for different roadways within the network. To study the effect of this change, additional analysis was conducted to identify the optimum route by using the transition probability matrices for corresponding link types (i.e. Table 8 for arterials and Table 9 for highway) and the results are summarized in Table 15.

Table 15: Estimated travel times for select routes from A to B (Different transition probability Matrices for arterials and highway)

Route No.	Transit Modes	Sequence of Nodes (See Figure 20-Figure 22)	Travel Time [min]
1	Car, Subway, Bike/Walk	A,2,3,4,5,11,18,25,B	28
2	Car, Walk	A,1,13,20,21,22,23,24,B	55
3	Car, Go Train, Subway, Walk	A,1,6,28,21,22,23,24,B	34

The results show minor improvements in estimated travel time for the optimum (28 min vs. 31 min). This is due to the smaller chances of transitioning to congested traffic condition in the transition probability matrix for arterials as compared to the average transition probability matrix for the whole network.

4.2.8 Discussion

The proposed MDP algorithm was applied to a real world transportation network located within the Greater Toronto Area to find the optimal path for a traveler between to arbitrary points. The studied multi-modal network included Commuter train, Bus, Street car, Subway, Bicycle and Automobile modes. The transit network schedule was modeled using the Super node approach presented in the previous sections. In order to account for changes in traffic conditions, transition probability matrices were evaluated using real world speed data along several roadways in the Toronto City. The proposed routing algorithm successfully identified the optimal (shortest) route for a traveler moving between two arbitrarily selected locations in the network, using different travel modes. A sensitivity analysis was conducted to demonstrate the ability of the model to capture the effect of probabilistic parameters used for the route estimation. It was also shown that the proposed methodology can realistically account for the stochastic properties of traffic conditions along different types of transportation facilities. While there were some limitations of the data source (only one highway segment was used to collect highways-specific transition probability matrix) there was sufficient data to demonstrate how using different transition probability matrices for different type of links (i.e. arterials vs. highways) can lead to different outcome (i.e. optimal path). It is expected that if a more heterogeneous network is tested (i.e. different types of facilities and associated probability matrices) using specific matrices by road type would lead to more realistic modeling results when an aggregated transition probability matrix is used.

In this case study first a network wide transition probability was applied for route optimization. A sensitivity analysis was conducted to see the effect of applying separate transition probability matrices for arterial and highway corridors. Given that in a large transportation network there is a high degree of heterogeneity in terms of traffic conditions along different roadway types at any given time, the reliability of the methodology could be further improved by utilizing different transition probabilities for different sections of a large network.

4.3 Longueuil case study

The 2-km long study area is located on a corridor between the Champlain and Victoria bridges, in Longueuil, Quebec. The two-way, three lanes each direction arterial, Boulevard Taschereau, (henceforth referred to as the major street) crosses five local streets (minor approaches), see Figure 23. The major street is used on both directions by several bus lines from RTL (*Reseau de transport de Longueuil*). All bus stops are placed at the stop line of each intersection with the minor streets (i.e. near-side bus stops). In this study the west bound approach of the major street, starting from Churchill Blvd, is considered as the regular path for RTL bus line number 4. This bus line originates from the Longueuil terminal, heading west towards Cornwall and Maricourt and has five stops within the study area and cannot be skipped.

The reason for selecting this particular segment is that it is a straight section of a major roadway (134) that further downstream merges into highway 15. In addition, there are several arterials and minor streets (e.g. Rue Victoria, Rue de Mont Royal) available for rerouting the busses in case of congestion along the major road. The average spacing between bus stops is about 400 m. The extra distance that bus would travel in case of rerouting ranges between 200 and 700 meters.

In this case study all buses travel between nodes 1 and 5 (as shown in Figure 24), which represent the beginning and the destination nodes, respectively. If rerouting via minor roads is necessary, the stops can be relocated downstream of the intersection either on the major street or on the minor street, depending whether, in order to reach the next stop, the bus will get back on the predetermined route or will continue on an alternative route , respectively. For example, a bus may detour via Mont Royal Street in order to advance from node 2 to node 3. If the bus resumes its predetermined route on the major road then, in order to service the stop at node 3, it will stop on the major road downstream of the Charles Street intersection.

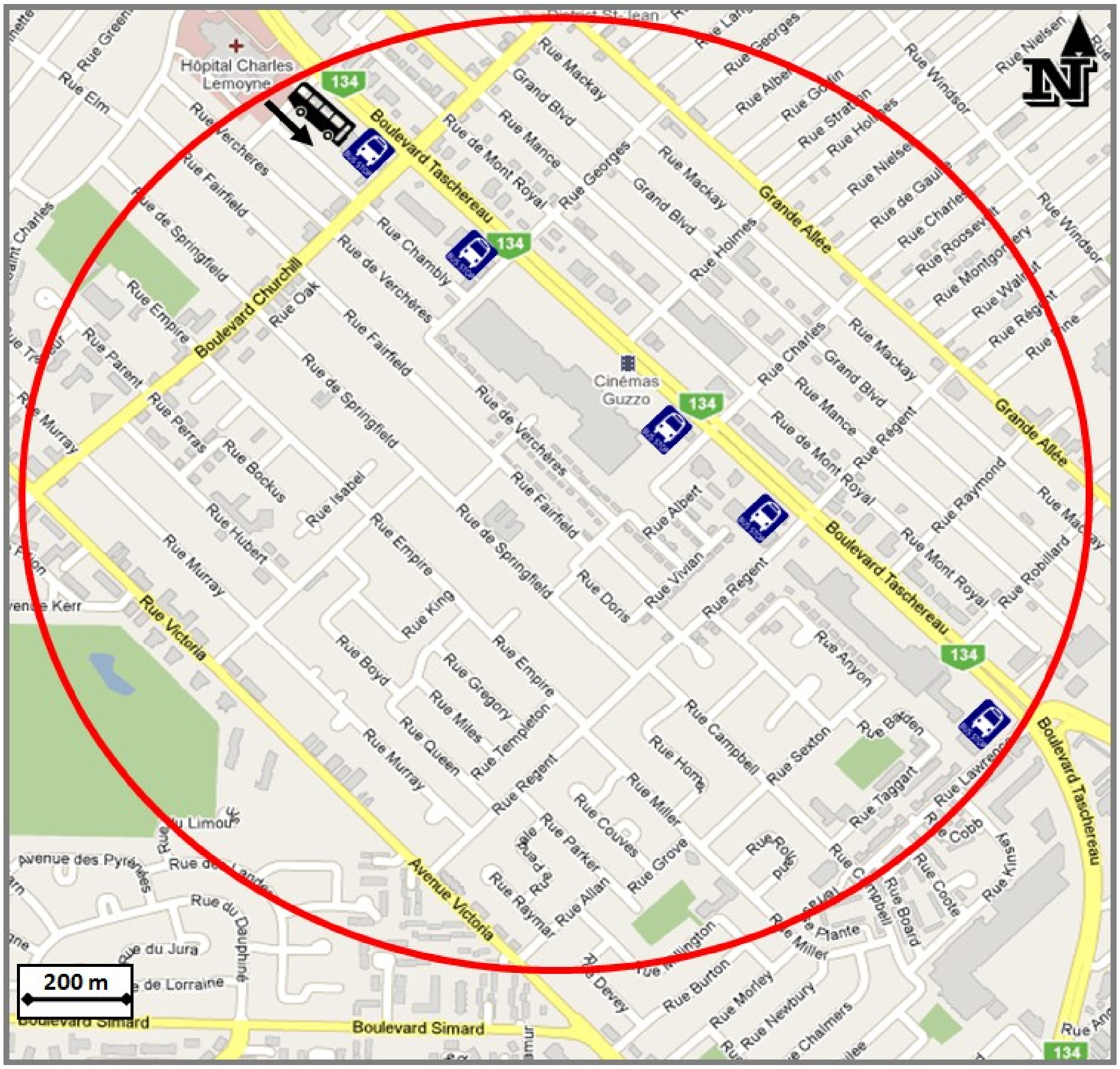

Figure 23: Layout of Study Area (Source: Google Maps)

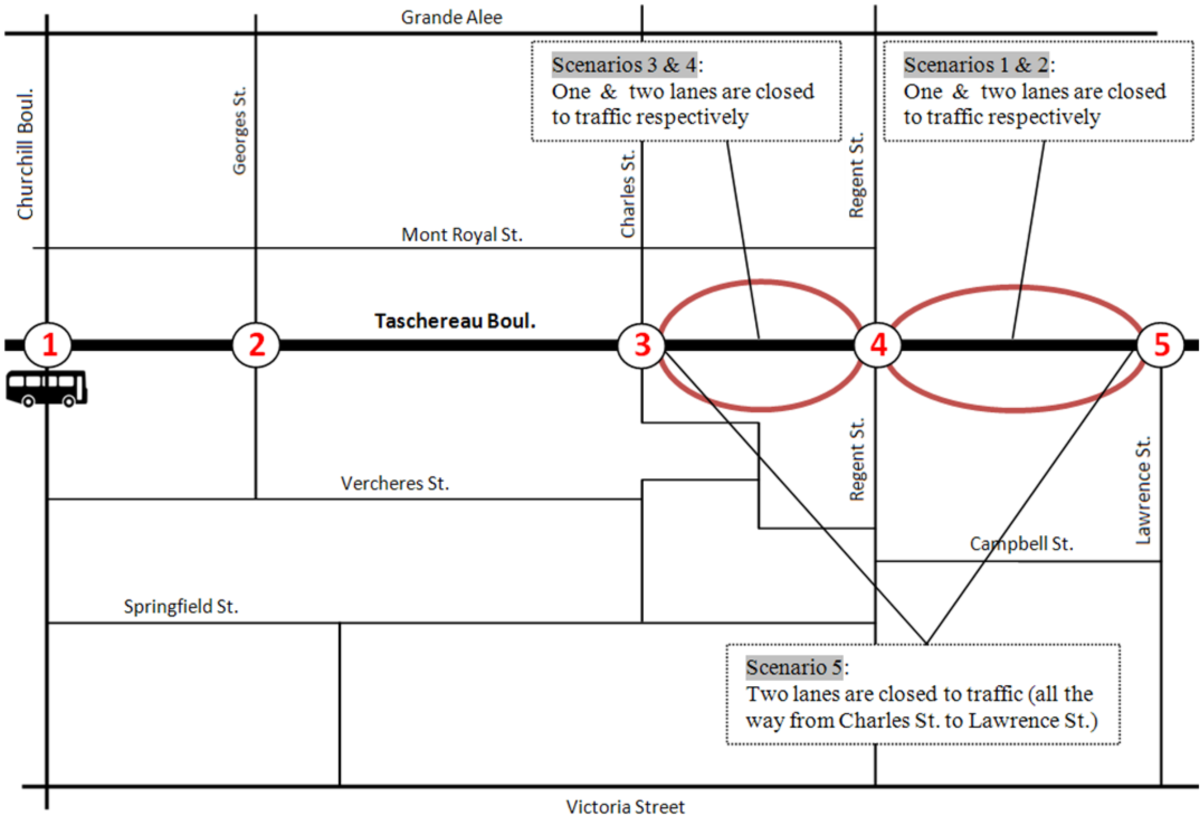

Figure 24: Alternative scenarios: lane closures in the network

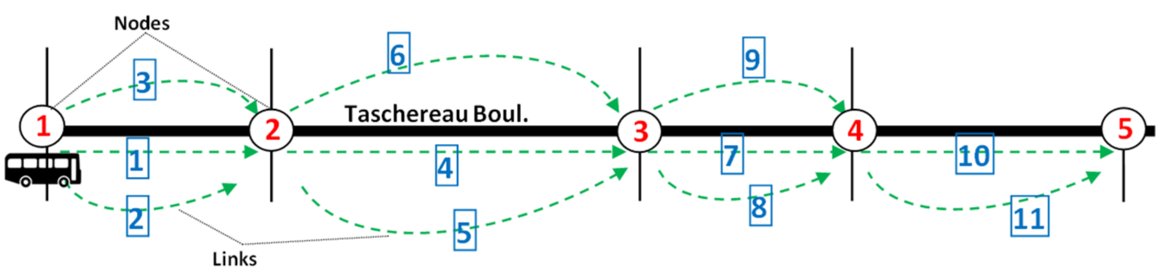

Figure 25: Bus routes: nodes and links

To evaluate the effectiveness of the proposed approach in the real world, traffic demand data and phasing times for signal controllers of intersections were collected from Quebec Transport Ministry and bus schedule and passenger count data was available through the local public transit agency, *Reseau de Transport du Longueil* (RTL). The bus schedule is used to generate bus fleet for microscopic simulations and also in model calibration to ensure buses arrive at consecutive bus stops according to their schedule. The passenger count data were used in the model as number of boarding/alighting for buses at each stop and to calculate their dwell time. Based on field observations, the afternoon peak hour demand on the major street varies between 800 and 2000 veh/h on each direction during the three peak hours between 15 and 18. Also, the volume of cars on minor approaches is estimated at 200 veh/h.

The network, i.e. arterials and minor streets presented in Figure 24, was modeled and calibrated in a microscopic simulator, VISSIM (PTV America 2011). VISSIM is a microscopic, time-step and behavior-based simulation model capable to simulate mixed vehicular traffic including public transportation operations. It is being used for more than two decades by researchers and practitioners in various transportation applications. In this study the latest available version was used, VISSIM 5.2. The reason for using VISSIM in this case study is the ability to easily define and work with transit lines/routes and bus stops within the model.

4.3.1 Model Calibration

The model was calibrated based on the available traffic demand from MTQ at several intersections along Taschereau Blvd. The simulated traffic volumes were compared with vehicle counts to make sure the FHWA (2004) guidelines for micro-simulation modelling (Equation 7), a

GEH value less than 5.0 for 85% of the links is met. A summary of calculated GEH statistics for different locations along the major road is presented in Figure 26. Speed data at the study corridor were not available to be used for the calibration.

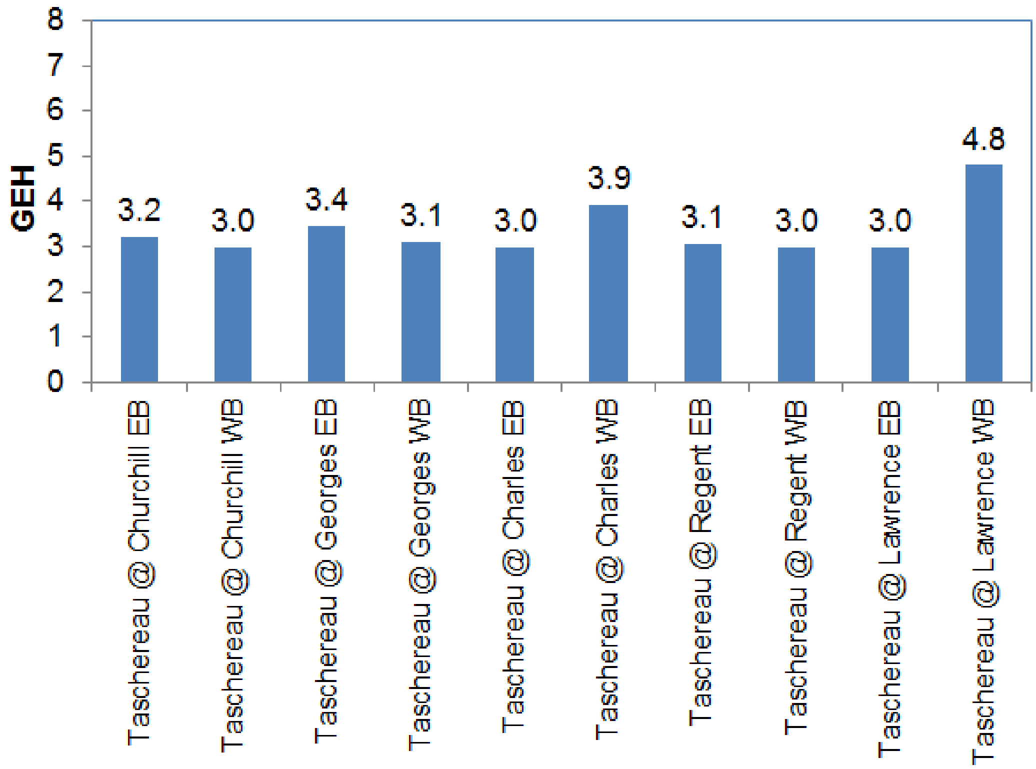

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Figure 26: GEH Statistics Data

4.3.2 Experimental Analysis

For illustrative purposes, in this study we assume that major street is characterized by one of the five different traffic conditions. Each traffic condition corresponds to a different state in the associated Markov system. One of the states represents free-flow operating traffic conditions under which bus operations on the pre-determined bus routes are not affected. The other five states correspond to five different traffic congestion levels based on possible lane closures scenarios as shown in Figure 24 and defined below:

- Scenario 1: One EB traffic lane is closed on Taschereau Boulevard, west bound at the end of the study corridor between Regent and Lawrence Streets (nodes 4 and 5 in Figure 25).
- Scenario 2: Two EB traffic lanes are closed to traffic on Taschereau Boulevard, west bound between Regent and Lawrence Streets.
- Scenario 3: One EB traffic lane on Taschereau Boulevard, between Margaret and Regent Streets is closed to vehicles (nodes 3 and 4 in Figure 25).
- Scenario 4: Two EB traffic lanes are closed to traffic on Taschereau Boulevard, west bound between Margaret and Regent Streets.
- Scenario 5: Two EB traffic lanes on Taschereau Boulevard, between Charles and Lawrence Streets are closed to traffic.

Each scenario was simulated in VISSIM for three hours using the same traffic demand input (See Table 16). Each scenario was run with the same ten distinct random seeds, to account for stochastic variations in the model and to allow for consistent comparison. Standard deviation and standard error of ten travel times based on ten different random seeds were calculated at different time stamps for each scenario. It was observed that the confidence interval for most of the travel times of buses were below 10% except for a few cases. This can be explained by the fact that due to inherent randomness of each simulation run, in certain instances buses may be affected by additional delay due to random vehicle arrival at the traffic signal. Since the simulated corridor is short, 2-km long, the impact of waiting an additional cycle length at one of the intersections along the corridor may have a substantial effect on the accumulated travel time on the links.

Table 16: Vehicle input volumes

Approach/Simulation Time	Total Vehicle Flow (Veh/h) First hour	Second hour	Third Hour	Movement	Ratio (% of total Flow)
Taschereau West bound	800	1600	2000	Through	80%
				Right/Left Turn	20%
Taschereau West bound	800	1600	2000	Through	60%
				Right/Left Turn	40%
Churchill Boul./Grande Allée/Victoria St.	200	200	200	Through	60%
				Right/Left Turn	40%

A total of 50 simulation runs were conducted. Average bus travel times were calculated from each simulated scenario. A t-test analysis was conducted to determine if the difference between average travel times estimated in alternative scenarios are statistically significant for similar links. Table 17 shows the results of the tests that compare average travel time, in seconds, of the base scenario – the scenario representing free-flow traffic conditions, μ_0 and the average travel time of buses in each of five traffic congestion alternative cases (μ_i , where $i = 1 \ldots 5$). The null hypothesis tested is that the two average values are not significantly different (H_0: $\mu_0 - \mu_i = 0$) at 95% confidence.

It can be seen in Table 17 that under certain traffic conditions, some links shows statistically significant different average travel times at 95% confidence. For example, it was found that there is a statistically significant difference between average travel times on link 7 when the base scenario and scenario 4 are compared. Moreover, link no. 4 shows a statistically significant difference for all simulated traffic conditions. This means that if buses are rerouted to travel through links 4 and 7, there will be a significant saving in travel times under certain traffic conditions. Furthermore, for other links, the t-test results show that statistically there is no difference between average travel times estimated for alternative scenarios. This indicates that these links may not have significant effect on the savings in travel time/costs, if the buses are rerouted to travel through them to get to the destination. The optimal routing calculations provided in the next section yield to the same conclusion.

It can be seen in Table 17 that link number 4 has the most significant results compared to the base scenario. This can be explained that by creating traffic congestion on the links downstream of link 4, the queuing condition occurs at upstream since the vehicles are not able to clear the intersection and travel via the available downstream lanes. Therefore, the travel time on link 4 increases compared to alternative routes. Due to the relatively low congestion level created in the network in scenarios 1-4 the other links did not show a statistically significant travel time difference. By modeling more severe congestion conditions, scenario 5, much longer queuing occurred upstream on link 4 and additional optimized paths were identified which are presented in the following section.

Table 17: Results of the t-test analysis for the travel time difference

	t-values for t-test ($H_0: \mu_0 - \mu_i = 0; where\ i = 1..5$)				
Link no.	Scen. 1	Scen. 2	Scen. 3	Scen. 4	Scen. 5
2	0.72	0.12	0.2	-1.5	-27*
4	-2.9*	-2.8*	-3.0*	-6.6*	-7.3*
5	-0.33	-0.83	-0.71	-4.3*	-130*
7	-1.1	-0.78	-1	-8.1*	-8.2*

* denotes statistically different means at 95% confidence (t-critical = 2.03)

4.3.3 Computation Results

The impact of the hypothetical traffic congestion scenarios defined above was evaluated through the proposed adaptive routing methodology. In the proposed methodology, a penalty (cost) is assigned in each step while moving from one node to another. Equation (5) is used to estimate total penalties earned at the end node and decision is made based on minimized penalties. In this study, the objective is to find the optimal route for which the total cost is minimized. The reward for traveling from one stop to the adjacent one is calculated considering both travel time and operating costs for buses. Based on a report released by Transport Canada (2006), the value of time for passengers is estimated as $29.7 per hour (2003 $). Using the latest annual report of American Public Transportation Association (2010), operating cost for buses is calculated as $5 per kilometer (2008 $) which includes Vehicle Operations, Maintenance, General Administration (Salaries and Wages, Materials and Supplies, Services) and Purchased Transportation.

Using the study area depicted in Figure 23 to Figure 25, buses travelling westbound between nodes 0, 1 or 2 and the next node may use one of the following three routes: i) the regular path along Taschereau, ii) north detour, via Mont Royal and minor roads back towards Taschereau, iii) or south detour, via Vercheres/Campbell St and back on Taschereau. Travel times between every two adjacent nodes were generated from VISSIM simulations of all possible routes. A sample of estimated travel times of links for different network conditions is shown in Table 18. The first row presents travel time on link no. 2 for all the scenarios. Rows 2 to 4 compares travel time on three available links between nodes 2 and 3 under different traffic conditions (i.e. Base alternative and scenarios).

Table 18: Average travel time simulation results for select links (see Figure 25)

Row No.	Link			Avg. Travel Time [sec] under different traffic conditions (Scenarios, Figure 7)					
	Link No.	From Node	To Node	Base Scenario	Scen. 1	Scen. 2	Scen. 3	Scen. 4	Scen. 5
1	2	1	2	92	92	92	92	92	108
2	4	2	3	37	38	38	38	219	513
3	5	2	3	147	147	147	147	149	237
4	6	2	3	106	106	104	101	105	87

The simulation results were used as realized travel time data for the links within the network. The average link travel times of the ten simulation results was calculated for each scenario. Thus, when the bus approaches a given node, the travel time on the adjacent links depends on the current simulation time (depending on the probability of observing specific traffic conditions on those links). Transition probability matrix is formed based on the assumption that the five different traffic conditions (i.e. free-flow, congestion scenario 1, 2, 3, 4 or 5) may occur with the same probability through the network (p_{ij}=0.2). A Visual Basic code was written to manipulate data and estimate the optimal route based on the proposed methodology (Figure 3). A recursive function is used in the code that accepts the beginning and destination nodes and arriving time at beginning node as parameters, calculates the links travel times based on the arrival time at nodes and calculates total cost by using Equation (5) for each chain of links between beginning and destination nodes. The total cost is evaluated by summing up the estimated travel time converted to cost and operating cost for the total distance traveled by bus. The combinations of links that yield the minimum total cost are selected as optimal route to be taken by the bus.

Table 19 shows select analysis results for the studied area. For example, under congested traffic conditions caused by closure of two traffic lanes between nodes 3 and 4 (Scenario 4), if buses arrives at $node\ 1$ at $t = 25min$, the regular path (links 1,4,7 and 10) remains the optimal route. However, if the bus arrives at $node\ 1$ at a later time (i.e. $t = 80\ min$) then the optimal route is determined by links 1,6,7 and 10. The estimated cost savings are per passenger on the bus.

Table 19: Sample optimal routing results for the study network

Departure Time [min]	Traffic condition	Beginning node	Routed links to reach *node* 5 (Figure 25)	Estimated cost for regular path [$]	Estimated cost by re-routing [$]	Travel time saved by re-routing [sec]
20	Scen. 4	0	1,4,7,10	8.9	-	-
80	Scen. 4	0	1,**6***,7,10	11.2	10.9	52
110	Scen. 4	0	1,**6***,7,10	11.9	11.7	114
125	Scen. 4	0	**3***,**6***,7,10	12.8	12.3	282
155	Scen. 4	0	**3***,**6***,7,10	13	12.5	292
95	Scen. 3	0	1,4,7,10	9.3	-	-
155	Scen. 3	0	1,4,7,10	10	-	-
155	Scen. 2	0	1,4,7,10	10	-	-
80	Scen. 5	0	**1***,**6***,1,1	10.3	7.5	325
85	Scen. 5	0	**1***,**6***,1,1	11.5	8.1	300
145	Scen. 5	0	**3***,**6***,1,1	13.8	8.8	740
*** denotes re-routes from regular path along the main corridor (links 1,4,7 and 10)**						

Table 19 shows that in some cases buses will run along links number 3 and 6 along Mont Royal St. without using Taschereau Boulevard, which is the main bus corridor, to pick up the passengers. In these cases, the regular bus stops should be temporary relocated to new location.

Although moving bus stops can be confusing for the riders, however, temporary relocation of bus stops is being practiced in the city of Montreal by local transportation agencies. For example, during special events, STM regularly cancels or relocates bus stops in the network. This happens for example in cases of accidents along the urban routes, or due to heavy snow falls during winter season. The STM mobile vehicles are responsible for posting necessary signs and guidelines. These changes will also be reflected in the online schedule and all the drivers will be informed by radio. The passengers will have to wait at new location to board the bus and buses will not wait for potential passengers to arrive so there will be no significant changes in their dwell time.

It can be seen in Table 19 that for some traffic conditions re-routing based on minimum cost method is not necessary. Nevertheless, the proposed methodology can help transportation operators to use flexible cost policies to manage their fleet and provide passengers with better service when specific traffic congestion conditions occur in the network. Since the case study is based on a small network with short distances, the savings in travel times are not considerable. If a larger area of transit network is considered and modeled accordingly by applying the proposed method to all bus lines, it is expected that total savings in the network would provide considerably higher benefits to both, the travelers and the transit authority.

4.3.4 Sensitivity Analysis

The bus optimal path identified by the proposed algorithm given the study network depends on several parameters. Due to changes in traffic conditions during the study period, one parameter is the departure time of the bus and the initial traffic conditions. The results presented Table 19 show effects of assuming different departure times on the trip optimal cost, under different traffic conditions. Since the results shown in Table 19 were based on total cost saving per individual passenger, additional analysis is conducted to estimate the cost savings based on the passenger occupancy of the bus. The estimations are done by assuming the average number of passenger in the bus to be 10 and 20 persons, respectively and the results are summarized in Table 20. It can be seen that when the average number of passengers in the subject bus increases, total cost savings by rerouting the bus during congested traffic conditions would also increase. This applies to only one bus line. The same estimation can be done for the whole fleet and total estimated cost savings can be used for justifying the rerouting exercise.

Table 20: The effects of re-routing on trip **cost-savings** under different scenarios

Departure Time [min]	Traffic condition	Beginning node	Routed links to reach $node\ 5$ (Figure 25)	Cost saving by rerouting [$] (**10 passengers** onboard)	Cost saving by rerouting [$] (**20 passengers** onboard)
20	Scen. 4	0	1,4,7,10	-	-
80	Scen. 4	0	1,**6***,7,10	3	6
110	Scen. 4	0	1,**6***,7,10	2	4
125	Scen. 4	0	**3***,**6***,7,10	5	10
155	Scen. 4	0	**3***,**6***,7,10	5	10
80	Scen. 5	0	**1***,**6***,1,1	28	56
85	Scen. 5	0	**1***,**6***,1,1	34	68
145	Scen. 5	0	**3***,**6***,1,1	50	100
*** denotes re-routes from regular path along the main corridor (links 1,4,7 and 10)**					

4.3.5 Discussion

The Longueuil bus line case study investigated one application of the proposed methodology by providing transportation operators with an overall situation of the system and its performance and by assisting decision makers in their assessment to improve the efficiency of transportation system. This case study applied the proposed MDP algorithm to estimate the optimal path a bus should follow while maintaining the scheduled bus stops. An objective function was defined to minimize the bus travel time by considering value of time and operating costs of the fleet. The proposed methodology was applied to a 2-km long section of a bus line operating along a 3-lane arterial, for which five different traffic congestion scenarios were tested. The assumed scenarios represent one or two lanes closed to all traffic at two different locations along the regular bus line. The simulation results for congestion conditions of scenario 4 showed that the travel time saved in several time intervals was in the range of 50 to 740 seconds. The sensitivity analysis showed that, depending on traffic conditions, the total cost savings per bus (i.e. value of passenger lost time due congested conditions) can be as high as $100, when an average of bus occupancy of 20 passengers is assumed.

This case study only considered one bus line and for a short section of their regular path. The potential benefits of cost savings for transit agencies could be better evaluated by considering several transit lines within their fleets and for a larger transit network. In addition, the traffic conditions prediction could be significantly improved by using real-world traffic data collected for the study arterials.

CHAPTER 5. CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

The research beforehand developed a novel routing procedure for multi-modal networks based on graph theory and Markov Decision Process. The most important benefit of this routing methodology stems from its application in a system optimization context. This is due to the flexibility of the objective defined by the proposed procedure. Mainly, the methodology seeks to minimize the impact of congested traffic conditions on the overall travel time and/or cost incurred by travelers as well as the operating cost of transit agencies. It was shown in this thesis that these different objectives are achieved by means of modeling the stochastic effects of traffic conditions as well as the ability of travelers to use different transportation modes. To demonstrate the benefits of the proposed system optimization methodology, three case studies involving real world networks were tested.

In the first and second case studies, the proposed routing algorithm was applied to model travelers' routing in a multi-modal transportation network. The algorithm's objective function minimizes the travel time for travelers. The proposed methodology has the capability of incorporating public transit schedules into the algorithm and was applied to two real-world multi-modal networks located in Montreal, QC and Toronto, ON. These multi-modal networks include passenger cars, public transit (i.e. commuter trains, bus, streetcar, subway) and bicycle facilities. The concept of super-node in a Markov chain was associated with transit station nodes to facilitate the integration of public transportation into the multimodal network. The public transportation fixed schedule was used to compile the route node parameters for the modeled network.

The Montreal case study demonstrated the calculation procedure of optimal routes for a traveler moving between two arbitrarily selected nodes in a multi modal network. In this case study, two scenarios were tested in a transit network. The first scenario assumed normal operation conditions, with a given probability of service disruption for two of the available transportation modes. The second scenario assumed the occurrence of an interruption in the metro service during the execution of a trip. A sensitivity analysis was conducted on several parameters used for estimating the optimal route. The effect of different probabilities of service interruption and departure times on the results were studied. The results demonstrated the applicability of the proposed algorithm to identify the fastest route to destination. It was shown that automobile travelers can save up to 14 minutes of travel time by switching to another transportation mode (e.g. metro, bus, or bicycle). It was also shown that in a stochastic network potential benefits could be achieved by using the proposed methodology compared to the shortest path algorithm.

In the Toronto case study, the proposed methodology was applied to a transportation network that consists of more than 80 km of major roadways, including the Gardiner Expressway and several major arterials within the Toronto CBD. The developed algorithm was used to find the optimal route for a typical commuter travelling from the suburb to downtown Toronto. Travelers had the option to travel with their car and/or one of the available transit modes (i.e. commuter train, two subway lines and 8 bus/street car lines). The study network was developed by using the proposed graph model. In addition, a microscopic simulation was modeled in AIMSUN to estimate travel times within the network under different traffic conditions. Real-world travel time and speed data was used to calibrate the model. Three different traffic conditions were evaluated: A) Off-peak; B) Peak and C) Congested. Furthermore, aggregated speed data for 15-minute intervals along several major arterials in the city of Toronto was used to calculate the required transition probability matrix of the MDP algorithm. Transit schedules were publicly available and processed from the Go Transit and Toronto Transit Commission web sites. A database was created to store the travel time data along all the links of the network and for all possible modes. The developed algorithm was implemented in a VB program and was used to find the optimal travel path. Results showed that while a typical driving route between two arbitrary points could take about 30 minutes; during congestion, the same route could take twice the time (61 minutes). However, during the congested period, the traveler could save about half an hour by taking an alternative path and switching to transit mode. The proposed algorithm was able to identify the optimal path for travelers considering the stochastic properties of traffic conditions and the benefits when compared to a shortest path methodology. A sensitivity analysis was performed to evaluate the effect of using different speed values to identify congestion conditions on expected travel times along the available routes.

The third case study used an arterial in Longueuil, QC to apply the Markovian optimal routing methodology to a fixed route public transit system. A Markov chain process with penalty was used to estimate the optimal path that a bus should follow while maintaining the exiting bus stops. An objective function was defined to minimize the bus travel time by considering the value of time and operating costs of the fleet. The proposed methodology was applied to a 2-km long section of a bus line operating along a 3-lane arterial. To evaluate the effectiveness of the routing algorithm, five different traffic congestion scenarios were tested. The assumed scenarios represent one or two lanes closed to all traffic at two different locations along the regular bus line. The simulation results showed that the average travel time savings per vehicle was in the range of 50 to 740 seconds. The case study presented an application of the proposed algorithm for transit agencies and its potential benefits by reducing total travel cost for bus line operator. A sensitivity analysis was conducted to show the magnitude of the total cost savings when the number of passengers increases.

5.2 Major Contributions

This research presents a novel integrated public and roadway traffic application in ITS which incorporates the stochastic behaviour of traffic flow into a route optimization methodology. Previous route optimization studies consider passenger vehicles as the only transportation mode in their routing algorithm. Available studies in multi-modal transportation networks mainly use the simple Dijkstra's Shortest Path algorithm or a routing policy based on DSP to identify the best route. This research developed a methodology to use Markov process in a route optimization algorithm for a multi-modal transportation network. The proposed approach applies probabilistic based methods to better estimate the parameters related to the stochastic nature of traffic parameters in a transportation network.

The proposed methodology can be integrated within an ATIS/APTS application to help alleviate the impact of traffic congestion on transit users and on the operating costs of the transit system. By providing an optimal route based on the online information available at any time, the bus operators can re-route their fleet so that passengers minimize their journey delay, while at the same time reducing the overall operating cost. Additionally, the transit agencies would benefit from having a more efficient fleet management and providing passengers with a more reliable service. These benefits are expected to have a positive effect on increasing the ridership. For example, in case of incidents that cause severe traffic congestion, a transit agency would be able to minimize the disruption to the original timetable and reduce the impact on the operating costs by rerouting buses at certain nodes based on real-time information about traffic conditions.

In order to calculate the optimal routes, traffic information should be available for all the links in the network. This type of information, compiled from various sources, is more and more readily available on major arterials of large urban agglomerations. Many agencies use transit vehicles equipped with on-board GPS units which can provide real-time location information that can be used to estimate real-time travel time and traffic conditions. The proposed methodology can be used in conjunction with this type of data to identify optimal re-routing scenarios under special circumstances (e.g. incident in the transit network or during especial events). The on-route information could be provided to travelers by using various transit information dissemination tools (i.e. many modern transit stations and subway cars are equipped with real-time display panels, or via specialized mobile applications).

Finally, in contrast to existing proprietary routing optimization tools – available online to the general public, this work can be made available to practitioners and researchers for deployment and future improvements as an open source multi-modal trip planner methodology. While individual users of a transportation network can benefit from different implementations, in general, transportation agencies are mostly interested in the overall system optimization criterion – feature not available through the existing online tools. However, while this thesis demonstrated the applicability of both optimization features, more work is envisioned toward enhancing the developed methodology and its applications.

5.3 Research Limitations

The main limitations of the proposed methodology are as follows:

- Currently the algorithm uses a system wide transition probability for each mode of transport in order to estimate the optimal path. This limitation would consider the same probabilities of change from one traffic state to another for different sections of a large network. The effect of this limitation on the results is much less when the system is in congestion mode given that there is less variability in the changes of traffic condition during congestion.
- The threshold introduced for applying transition probability matrix and updating state probabilities at each node is not incorporated in the analysis conducted through the real-world case studies. Nevertheless, it was shown that the proposed methodology is able to capture this enhancement of the routing algorithm using a hypothetical network.
- The methodology requires the subject network to be modeled using the graph theory and the Supernode concept for public transit. This process can be a considerable

amount of work. However, once the network is thoroughly modeled, it may be used for different applications that were tested in this research work (i.e. optimal path for traveling public or cost effective routes for transit agencies).

- The algorithm required initial travel time information for all the links in the model. If historical travel time data is not available, microscopic simulation can be used, with an additional computational and resource cost. Nevertheless, the travel times can be subsequently updated, when more recent traffic data becomes available (e.g. travel time studies or GPS data). It is expected that in the near future, travel times of the transit fleet would be easily estimated in an automated and real-time fashion by using on-board GPS units of transit vehicles, equipment that is more and more frequently adopted by various transit agencies.

5.4 Recommendation and Future Work

Although this thesis provides important contributions to the optimal route evaluation in a stochastic network, there is still considerable room for further research, mainly by addressing the limitations identified above. Several recommendations and potential future work is presented in the following section to better enhance the model and increase its reliability. This is presented in two parts: current study enhancement area and current study extension area.

5.4.1 Current study enhancement area

- The thesis work can be expanded to use different transition probability matrices within large networks by breaking them down into several sub networks. This can be done by considering several factors related to traffic and/or geometric conditions within the network, including but not limited to the peak period and peak direction of traffic and corridor types (i.e. Freeway, highway or arterial corridors).

- The case studies presented in this thesis apply the transition probability matrix at each step to evaluate probability vectors of traffic conditions at each node. A methodology is developed and suggested to use a minimum time as a threshold for applying transition probabilities at each step which can be used in future works. By using the resolution of the collected traffic speed data as the threshold, a more realistic change in traffic condition may be estimated.
- The resolution of traffic speed data being used as a representative parameter to define traffic conditions may not be available for all major corridors and highways within the network. In such cases, to increase the accuracy of capturing the changes in traffic conditions along different corridors, script based queries may be developed to use Google traffic information as an additional source.

5.4.2 Current study extension area

- The algorithm can be improved to learn from the history of unsuccessful routes used for identifying the optimal path in the previous steps. This could potentially improve the performance of the process by eliminating unnecessary calculations. A database platform could be implemented for the tool, enabling it to access the most recent travel time data on roadways as well as the most recent transit schedule.

- The algorithm could certainly be improved by integrating online (open source) transit schedule, real time arrival data regarding transit lines and bike sharing availability information from corresponding agencies.
- A user interface can be designed to use the proposed route optimization algorithm in providing the general traveling public with the necessary information to help them make informed decisions about the mode/route of their trips.
- Eventually, the proposed methodology can be extended for the whole metropolitan area of a city and be used as a model for other cities in Canada. The proposed approach can be applied to create a complete transportation package to provide transportation operators with an overall status of the system and its service performance and to assist decision makers in their assessment to improve the efficiency of the transportation system.

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APPENDIX A: AIMSUN Simulation Results

VBA Code for extracting data from Aimsun SQLite output file (CSV):

```
Sub ExtractAimsunResults()

    'Template sheet conainting secions
    inputSheet = "TrafficVolumes"
    'AIMSUN Result file (Excel)
    resultFile="C:\Users\behzad.rouhieh\Desktop\Behzad\Concordia\PhDProposal\PhD
Thesis\outputAVG.xlsx"
    ' Number of intervals for results (ent)
    interval = 8

    'Opens AIMSUN result file (Excel)
    Workbooks.Open resultFile

    'Number of rows
    N1 = Workbooks(1).Sheets(inputSheet).UsedRange.Rows.Count
    N2 = Workbooks(2).Sheets(1).UsedRange.Rows.Count

    j = 2
    For i = 4 To N1

        ID = Workbooks(1).Sheets(inputSheet).Cells(i, 5).Value
        pLane = Workbooks(1).Sheets(inputSheet).Cells(i, 14).Value
        auxL = Workbooks(1).Sheets(inputSheet).Cells(i, 16).Value
        lakeShore = Workbooks(1).Sheets(inputSheet).Cells(i, 17).Value
        nLanes = Workbooks(1).Sheets(inputSheet).Cells(i, 10).Value + auxL
        totLanes = Workbooks(1).Sheets(inputSheet).Cells(i, 9).Value

        ttLanes = nLanes
        If (nLanes >= 3 And lakeShore <> 1) Then
            ttLanes = ttLanes - 1
        End If

        Do While (j <= N2)

            If (Workbooks(2).Sheets(1).Cells(j, 2).Value = ID) Then

                'starting column for data input
                col = 18
```

```
For k = j To (j + (interval - 1) * totLanes) Step totLanes

    ' Initiate All Lanes parametrs: Volume, Travel Time, Speed
    volAll = 0
    ttAll = 0
    spAll = 0
    zsp = 0
    ztt = 0

    ' Calculate Travel Time and Speed
    For l = 1 + auxL To ttLanes
        'v = Workbooks(2).Sheets(1).Cells(k + l - 1, 7).Value
        sp = Workbooks(2).Sheets(1).Cells(k + l - 1, 25).Value
        tt = Workbooks(2).Sheets(1).Cells(k + l - 1, 29).Value

        ' Sum of parameters for all lanes
        'volAll = volAll + v

        If (sp <= 0) Then
            Workbooks(1).Sheets(inputSheet).Cells(i, 5).Interior.ColorIndex = 7
            zsp = zsp + 1 ' Number of zero/negative results
        Else
            If l <= (ttLanes) Then
                spAll = spAll + sp
            End If
        End If

        If tt <= 0 Then
            Workbooks(1).Sheets(inputSheet).Cells(i, 5).Interior.ColorIndex = 7
            ztt = ztt + 1 ' Number of zero/negative results
        Else
            If l <= (ttLanes) Then
                ttAll = ttAll + tt
            End If
        End If
    Next l

    'Calculate Volumes
    For l = 1 To totLanes
        v = Workbooks(2).Sheets(1).Cells(k + l - 1, 7).Value
        'sp = Workbooks(2).Sheets(1).Cells(k + l - 1, 25).Value
        'tt = Workbooks(2).Sheets(1).Cells(k + l - 1, 29).Value
        'zsp = 0
        'ztt = 0
```

```
   ' Sum of parameters for all lanes
   volAll = volAll + v
Next l

' If priority lane exists, GP Lanes and Priority Lane parameters will be calculated
If pLane = 1 Then
   'If highway (note Lake Shore)-> PL on the left
   If lakeShore <> 1 Then
      volPl = Workbooks(2).Sheets(1).Cells(k + auxL, 7).Value

      spPl = Workbooks(2).Sheets(1).Cells(k + auxL, 25).Value
      If spPl < 0 Then
         spPl = 0
      End If

      ttPl = Workbooks(2).Sheets(1).Cells(k + auxL, 29).Value
      If ttPl < 0 Then
         ttPl = 0
      End If

      volGpl = volAll - volPl
      If volGpl < 0 Then
         volGpl = 0
      End If
      spGpl = spAll - spPl
      If spGpl < 0 Then
         spGpl = 0
      End If
      ttGpl = ttAll - ttPl
      If ttGpl < 0 Then
         ttGpl = 0
      End If

      'Average for ttime and speed for GP lanes
      If (ttLanes - zsp - 1 - auxL = 0) Then
         spGpl = 0
      Else
         spGpl = spGpl / (ttLanes - zsp - 1 - auxL)
      End If

      If (ttLanes - ztt - 1 - auxL = 0) Then
         ttGpl = 0
      Else
```

```
      ttGpl = ttGpl / (ttLanes - ztt - 1 - auxL)
    End If

Else
    'If on Lake Shore -> Pl on the right
    volPl = Workbooks(2).Sheets(1).Cells(k + nLanes - 1, 7).Value

    spPl = Workbooks(2).Sheets(1).Cells(k + nLanes - 1, 25).Value
    If spPl < 0 Then
      spPl = 0
    End If

    ttPl = Workbooks(2).Sheets(1).Cells(k + nLanes - 1, 29).Value
    If ttPl < 0 Then
      ttPl = 0
    End If

    volGpl = volAll - volPl
    If volGpl < 0 Then
      volGpl = 0
    End If
    spGpl = spAll - spPl
    If spGpl < 0 Then
      spGpl = 0
    End If
    ttGpl = ttAll - ttPl
    If ttGpl < 0 Then
      ttGpl = 0
    End If

    'Average for ttime and speed for GP lanes
    If (ttLanes - zsp - 1 - auxL = 0) Then
      spGpl = 0
    Else
      spGpl = spGpl / (ttLanes - zsp - 1 - auxL)
    End If


    If (ttLanes - ztt - 1 - auxL = 0) Then
      ttGpl = 0
    Else
      ttGpl = ttGpl / (ttLanes - ztt - 1 - auxL)
    End If
End If
```

```
Else
   volPl = 0
   spPl = 0
   ttPl = 0

   volGpl = volAll
   If (ttLanes - zsp - auxL = 0) Then
      spGpl = 0
   Else
      spGpl = spAll / (ttLanes - zsp - auxL)
   End If

   If (ttLanes - ztt - auxL = 0) Then
      ttGpl = 0
   Else
      ttGpl = ttAll / (ttLanes - ztt - auxL)
   End If
End If

'Average for ttime and speed for ALL lanes
If (ttLanes - zsp - auxL = 0) Then
   spAll = 0
Else
   spAll = spAll / (ttLanes - zsp - auxL)
End If
If (ttLanes - ztt - auxL = 0) Then
   ttAll = 0
Else
   ttAll = ttAll / (ttLanes - ztt - auxL)
End If

' Write data into the tamplate sheet
Workbooks(1).Sheets(inputSheet).Cells(i, col).Value = ttAll
Workbooks(1).Sheets(inputSheet).Cells(i, col + interval).Value = ttGpl
Workbooks(1).Sheets(inputSheet).Cells(i, col + (2 * interval)).Value = ttPl

Workbooks(1).Sheets(inputSheet).Cells(i, col + (3 * interval)).Value = spAll
Workbooks(1).Sheets(inputSheet).Cells(i, col + (4 * interval)).Value = spGpl
Workbooks(1).Sheets(inputSheet).Cells(i, col + (5 * interval)).Value = spPl

Workbooks(1).Sheets(inputSheet).Cells(i, col + (6 * interval)).Value = volAll
Workbooks(1).Sheets(inputSheet).Cells(i, col + (7 * interval)).Value = volGpl
Workbooks(1).Sheets(inputSheet).Cells(i, col + (8 * interval)).Value = volPl
```

```
          col = col + 1

        Next k

        j = k
        If Workbooks(1).Sheets(inputSheet).Cells(i + 1, 5).Value = ID Then
          j = j - (interval * totLanes)
        End If

        Exit Do
      Else
        j = j + 1
      End If

    Loop

  Next i
End Sub
```

Sample of Aimsun output results (SQLite file):

did	oid	eid	sid	ent	lane	count	count_D	flow	flow_D	input_c...	input_c...	input_fl...	input_fl...	density	density_D	qmean	qmean_D	qmax	qmax_D	dtime	dtime_D	dtimeVQ	dtimeV...	speed	speed_D	hspeed
484433	340062	1500686...	0	0	1	382.6	17.27136...	191.3	8.635681...	0	0	0	0	16.06309...	0.937104...	0.956982...	0.063490...	10.2	2.167948...	18.93070...	0.617299...	18.93070...	0.617299...	33.30458...	0.603873...	25.79303...
484433	340062	1500686...	0	0	2	787.2	37.40588...	393.6	18.70294...	1040	32.58834...	520	16.29417...	17.92008...	0.911227...	2.115734...	0.125413...	12.8	1.923538...	19.06753...	0.384977...	19.06753...	0.384977...	32.74999...	0.302219...	25.61764...
484433	340062	1500686...	0	0	3	899.2	29.95329...	449.6	14.97664...	1022.4	16.54690...	511.2	8.273451...	19.44229...	0.612098...	2.344015...	0.114426...	13.2	1.303840...	18.72386...	0.548417...	18.72386...	0.548417...	33.29683...	0.684877...	25.93717...
484433	340062	1500686...	0	1	1	94.8	9.576011...	189.6	19.15202...	0	0	0	0	13.77360...	2.849214...	0.796739...	0.201282...	6.4	1.816590...	17.02349...	3.062481...	17.02349...	3.062481...	34.76334...	3.064743...	27.39851...
484433	340062	1500686...	0	1	2	197.2	14.53272...	394.4	29.06544...	249.8	19.97998...	499.6	39.95997...	16.05424...	1.422539...	1.949719...	0.180709...	11	0.707106...	17.87748...	0.622654...	17.87748...	0.622654...	33.99306...	0.714644...	26.47021...
484433	340062	1500686...	0	1	3	214	16.74813...	428	33.49626...	235.4	8.792041...	470.8	17.58408...	16.76777...	0.768915...	2.138064...	0.149481...	11.6	0.894427...	18.28899...	1.222430...	18.28899...	1.222430...	33.95873...	1.350619...	26.33177...
484433	340062	1500686...	0	2	1	96.6	14.74109...	193.2	29.48219...	0	0	0	0	16.38712...	3.008241...	0.970873...	0.190464...	6.8	1.643167...	18.03721...	1.115257...	18.03721...	1.115257...	33.98766...	1.603680...	26.44241...
484433	340062	1500686...	0	2	2	157.8	14.75466...	315.6	29.50932...	230.2	15.02331...	460.4	30.04663...	14.31922...	1.457122...	1.720749...	0.245937...	10.4	1.516575...	18.60610...	1.334691...	18.60610...	1.334691...	33.26918...	1.287582...	25.95394...
484433	340062	1500686...	0	2	3	180.2	15.53061...	360.4	31.06122...	222.4	13.22119...	444.8	26.44239...	14.92908...	1.444773...	1.815153...	0.267299...	10.8	1.788854...	17.51973...	1.254490...	17.51973...	1.254490...	34.56993...	1.620846...	26.88057...
484433	340062	1500686...	0	3	1	76.8	6.220932...	153.6	12.44186...	0	0	0	0	13.15243...	2.091549...	0.795083...	0.144897...	6.6	1.341640...	20.04251...	2.003608...	20.04251...	2.003608...	32.23309...	1.336945...	25.11392...
484433	340062	1500686...	0	3	2	220	21.27204...	440	42.54409...	260.6	19.16507...	521.2	38.33014...	18.21111...	1.785673...	2.406867...	0.269531...	12	1	20.02843...	0.608483...	20.02843...	0.608483...	31.86379...	0.805950...	24.96656...
484433	340062	1500686...	0	3	3	249.4	18.72965...	498.8	37.45931...	268	15.71623...	536	31.43246...	20.30353...	1.530343...	2.718078...	0.252603...	12	0.707106...	20.03272...	0.471111...	20.03272...	0.471111...	31.97555...	0.942236...	25.01774...
484433	340062	1500686...	0	4	1	114.4	6.228964...	228.8	12.45792...	0	0	0	0	20.65427...	1.114432...	1.253061...	0.078764...	9.6	2.792848...	20.51922...	1.408027...	20.51922...	1.408027...	32.23829...	1.368498...	24.76655...
484433	340062	1500686...	0	4	2	212.2	11.21160...	424.4	22.42320...	299.4	14.38054...	598.8	28.76108...	19.14031...	1.064457...	2.382040...	0.181500...	12.4	2.073644...	19.52039...	0.667016...	19.52039...	0.667016...	32.12746...	0.649539...	25.34010...
484433	340062	1500686...	0	4	3	255.6	7.602631...	511.2	15.20526...	296.6	10.16366...	593.2	20.32732...	21.25276...	1.318692...	2.702774...	0.236704...	13	1.581138...	18.65977...	1.032398...	18.65977...	1.032398...	33.13433...	1.101875...	25.95625...
484433	340062	1500686...	1	0	1	277	16	138.5	8	0	0	0	0	11.53168...	1.073863...	0.685421...	0.073440...	8	1.224744...	18.76538...	0.965576...	18.76538...	0.965576...	33.35751...	1.023023...	25.93021...
484433	340062	1500686...	1	0	2	601.6	25.82247...	300.8	12.91123...	778.2	23.76341...	389.1	11.88170...	13.53703...	0.641919...	1.606631...	0.105877...	11.2	0.836660...	18.95460...	0.569971...	18.95460...	0.569971...	32.96392...	0.473512...	25.73213...
484433	340062	1500686...	1	0	3	669.8	18.96575...	334.9	9.482879...	764.8	18.56609...	382.4	9.283049...	14.48026...	0.495844...	1.748995...	0.092281...	11.4	0.894427...	18.73709...	0.644821...	18.73709...	0.644821...	33.39894...	0.804886...	25.97758...
484433	340062	1500686...	1	1	1	70	7.582875...	140	15.16575...	0	0	0	0	10.07820...	2.533325...	0.582170...	0.177621...	5.6	1.516575...	16.86358...	3.278479...	16.86358...	3.278479...	34.92056...	3.350685...	27.57972...
484433	340062	1500686...	1	1	2	153.8	10.98635...	307.6	21.97271...	188.8	12.31665...	377.6	24.63331...	12.21343...	1.005407...	1.484375...	0.154104...	9.2	1.095445...	17.54214...	0.843997...	17.54214...	0.843997...	34.36976...	0.596654...	26.75667...
484433	340062	1500686...	1	1	3	160.4	10.50238...	320.8	21.00476...	178.6	6.107372...	357.2	12.21474...	12.66465...	0.610179...	1.622545...	0.138262...	9.4	1.516575...	18.51021...	1.413880...	18.51021...	1.413880...	33.92728...	1.551253...	26.23376...
484433	340062	1500686...	1	2	1	69.2	9.959919...	138.4	19.91983...	0	0	0	0	11.70518...	2.419591...	0.692351...	0.155603...	5.8	1.303840...	17.89807...	1.094338...	17.89807...	1.094338...	34.03664...	1.470958...	26.53316...
484433	340062	1500686...	1	2	2	116.8	7.949842...	233.6	15.89968...	168	12.08304...	336	24.16609...	10.49092...	0.713150...	1.267337...	0.099155...	8	1	18.64331...	1.394408...	18.64331...	1.394408...	33.36989...	1.259082...	25.96878...
484433	340062	1500686...	1	2	3	134.2	13.51665...	268.4	27.03331...	166	11.40175...	332	22.80350...	11.05396...	1.169741...	1.337068...	0.200622...	9	0.707106...	17.29906...	1.171527...	17.29906...	1.171527...	34.95036...	1.479882...	27.11034...
484433	340062	1500686...	1	3	1	54	3.535533...	108	7.071067...	0	0	0	0	8.979613...	1.456469...	0.539777...	0.105521...	4.8	1.643167...	19.52230...	2.776314...	19.52230...	2.776314...	32.53788...	1.878035...	25.56767...
484433	340062	1500686...	1	3	2	163.2	14.68672...	326.4	29.37345...	193.2	12.47798...	386.4	24.95596...	13.44654...	1.431130...	1.768477...	0.243313...	10	1.224744...	19.74589...	1.269424...	19.74589...	1.269424...	32.25944...	1.389399...	25.21289...
484433	340062	1500686...	1	3	3	184.8	16.70927...	369.6	33.41855...	195.2	14.54991...	390.4	29.09982...	14.80951...	1.268502...	1.984523...	0.208694...	10.4	1.140175...	19.77037...	0.583907...	19.77037...	0.583907...	32.34546...	0.972160...	25.24080...
484433	340062	1500686...	1	4	1	83.8	7.918333...	167.6	15.83666...	0	0	0	0	15.13193...	1.252037...	0.917667...	0.077193...	7.6	1.516575...	20.58244...	1.488061...	20.58244...	1.488061...	32.01921...	1.492800...	24.73942...
484433	340062	1500686...	1	4	2	167.8	13.47961...	335.6	26.95922...	228.2	12.91123...	456.4	25.82247...	14.95655...	1.161677...	1.902221...	0.189010...	10.6	1.140175...	19.69630...	0.696430...	19.69630...	0.696430...	32.07798...	0.867010...	25.24714...
484433	340062	1500686...	1	4	3	190.4	9.262828...	380.8	18.52565...	225	12.32882...	450	24.65765...	16.07491...	1.353496...	2.049327...	0.239935...	11	1	18.93891...	1.571580...	18.93891...	1.571580...	32.88286...	1.666143...	25.81434...
484433	340062	1500686...	10	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

APPENDIX B: Public Transit Schedule

Transit Line	Link No.	Departures	Arrivals	Transit Line	Link No.	Departures	Arrivals
501/301	16	16:18	16:20	505	17	16:23	16:27
501/301	16	16:23	16:25	505	17	16:28	16:32
501/301	16	16:28	16:30	505	17	16:33	16:37
501/301	16	16:33	16:35	505	17	16:39	16:43
501/301	16	16:38	16:40	505	17	16:44	16:48
501/301	16	16:43	16:45	505	17	16:49	16:53
501/301	16	16:48	16:50	505	17	16:55	16:59
501/301	16	16:53	16:55	505	17	17:00	17:04
501/301	16	16:58	17:00	505	17	17:05	17:09
501/301	16	17:03	17:05	505	17	17:11	17:15
501/301	16	17:08	17:10	505	17	17:16	17:20
501/301	16	17:13	17:15	505	17	17:21	17:25
501/301	16	17:18	17:20	505	17	17:27	17:31
501/301	16	17:23	17:25	505	17	17:32	17:36
501/301	16	17:28	17:30	505	17	17:37	17:41
501/301	16	17:33	17:35	505	17	17:43	17:47
501/301	16	17:38	17:40	505	17	17:48	17:52
501/301	16	17:43	17:45	505	17	17:53	17:57
501/301	16	17:48	17:50	505	17	17:59	18:03
501/301	16	17:53	17:55	506/306	11	16:04	16:07
501/301	16	17:58	18:00	506/306	11	16:09	16:12
501/301	16	18:03	18:05	506/306	11	16:14	16:17
505	10	16:03	16:07	506/306	11	16:19	16:22
505	10	16:08	16:12	506/306	11	16:24	16:27
505	10	16:13	16:17	506/306	11	16:29	16:32
505	10	16:19	16:23	506/306	11	16:34	16:37
505	10	16:24	16:28	506/306	11	16:39	16:42
505	10	16:29	16:33	506/306	11	16:44	16:47
505	10	16:35	16:39	506/306	11	16:49	16:52
505	10	16:40	16:44	506/306	11	16:54	16:57
505	10	16:45	16:49	506/306	11	16:59	17:02
505	10	16:51	16:55	506/306	11	17:04	17:07
505	10	16:56	17:00	506/306	11	17:09	17:12
505	10	17:01	17:05	506/306	11	17:14	17:17
505	10	17:07	17:11	506/306	11	17:19	17:22
505	10	17:12	17:16	506/306	11	17:24	17:27
505	10	17:17	17:21	506/306	11	17:29	17:32
505	10	17:23	17:27	506/306	11	17:34	17:37
505	10	17:28	17:32	506/306	11	17:39	17:42
505	10	17:33	17:37	506/306	11	17:44	17:47
505	10	17:39	17:43	506/306	11	17:49	17:52
505	10	17:44	17:48	506/306	11	17:54	17:57
505	10	17:49	17:53	506/306	11	17:59	18:02
505	10	17:55	17:59	506/306	18	16:02	16:07
505	10	18:00	18:04	506/306	18	16:07	16:12
505	17	16:01	16:05	506/306	18	16:12	16:17
505	17	16:07	16:11	506/306	18	16:17	16:22
505	17	16:12	16:16	506/306	18	16:22	16:27
505	17	16:17	16:21	506/306	18	16:27	16:32

Transit Line	Link No.	Departures	Arrivals	Transit Line	Link No.	Departures	Arrivals
506/306	18	16:32	16:37	Subway 2: Green	4	17:33	17:44
506/306	18	16:37	16:42	Subway 2: Green	4	17:36	17:47
506/306	18	16:42	16:47	Subway 2: Green	4	17:39	17:50
506/306	18	16:47	16:52	Subway 2: Green	4	17:42	17:53
506/306	18	16:52	16:57	Subway 2: Green	4	17:45	17:56
506/306	18	16:57	17:02	Subway 2: Green	4	17:48	17:59
506/306	18	17:02	17:07	Subway 2: Green	4	17:51	18:02
506/306	18	17:07	17:12	Subway 2: Green	4	17:54	18:05
506/306	18	17:12	17:17	Subway 2: Green	4	17:57	18:08
506/306	18	17:17	17:22	Subway 2: Green	4	18:00	18:11
506/306	18	17:22	17:27	Subway 2: Green	6	16:00	16:07
506/306	18	17:27	17:32	Subway 2: Green	6	16:03	16:10
506/306	18	17:32	17:37	Subway 2: Green	6	16:06	16:13
506/306	18	17:37	17:42	Subway 2: Green	6	16:09	16:16
506/306	18	17:42	17:47	Subway 2: Green	6	16:12	16:19
506/306	18	17:47	17:52	Subway 2: Green	6	16:15	16:22
506/306	18	17:52	17:57	Subway 2: Green	6	16:18	16:25
506/306	18	17:57	18:02	Subway 2: Green	6	16:21	16:28
Subway 2: Green	4	16:00	16:11	Subway 2: Green	6	16:24	16:31
Subway 2: Green	4	16:03	16:14	Subway 2: Green	6	16:27	16:34
Subway 2: Green	4	16:06	16:17	Subway 2: Green	6	16:30	16:37
Subway 2: Green	4	16:09	16:20	Subway 2: Green	6	16:33	16:40
Subway 2: Green	4	16:12	16:23	Subway 2: Green	6	16:36	16:43
Subway 2: Green	4	16:15	16:26	Subway 2: Green	6	16:39	16:46
Subway 2: Green	4	16:18	16:29	Subway 2: Green	6	16:42	16:49
Subway 2: Green	4	16:21	16:32	Subway 2: Green	6	16:45	16:52
Subway 2: Green	4	16:24	16:35	Subway 2: Green	6	16:48	16:55
Subway 2: Green	4	16:27	16:38	Subway 2: Green	6	16:51	16:58
Subway 2: Green	4	16:30	16:41	Subway 2: Green	6	16:54	17:01
Subway 2: Green	4	16:33	16:44	Subway 2: Green	6	16:57	17:04
Subway 2: Green	4	16:36	16:47	Subway 2: Green	6	17:00	17:07
Subway 2: Green	4	16:39	16:50	Subway 2: Green	6	17:03	17:10
Subway 2: Green	4	16:42	16:53	Subway 2: Green	6	17:06	17:13
Subway 2: Green	4	16:45	16:56	Subway 2: Green	6	17:09	17:16
Subway 2: Green	4	16:48	16:59	Subway 2: Green	6	17:12	17:19
Subway 2: Green	4	16:51	17:02	Subway 2: Green	6	17:15	17:22
Subway 2: Green	4	16:54	17:05	Subway 2: Green	6	17:18	17:25
Subway 2: Green	4	16:57	17:08	Subway 2: Green	6	17:21	17:28
Subway 2: Green	4	17:00	17:11	Subway 2: Green	6	17:24	17:31
Subway 2: Green	4	17:03	17:14	Subway 2: Green	6	17:27	17:34
Subway 2: Green	4	17:06	17:17	Subway 2: Green	6	17:30	17:37
Subway 2: Green	4	17:09	17:20	Subway 2: Green	6	17:33	17:40
Subway 2: Green	4	17:12	17:23	Subway 2: Green	6	17:36	17:43
Subway 2: Green	4	17:15	17:26	Subway 2: Green	6	17:39	17:46
Subway 2: Green	4	17:18	17:29	Subway 2: Green	6	17:42	17:49
Subway 2: Green	4	17:21	17:32	Subway 2: Green	6	17:45	17:52
Subway 2: Green	4	17:24	17:35	Subway 2: Green	6	17:48	17:55
Subway 2: Green	4	17:27	17:38	Subway 2: Green	6	17:51	17:58
Subway 2: Green	4	17:30	17:41	Subway 2: Green	6	17:54	18:01

Transit Line	Link No.	Departures	Arrivals
6	47	16:02	16:09
6	47	16:07	16:14
6	47	16:13	16:20
6	47	16:18	16:25
6	47	16:24	16:31
6	47	16:29	16:36
6	47	16:35	16:42
6	47	16:40	16:47
6	47	16:46	16:53
6	47	16:51	16:58
6	47	16:57	17:04
6	47	17:02	17:09
6	47	17:08	17:15
6	47	17:13	17:20
6	47	17:19	17:26
6	47	17:24	17:31
6	47	17:30	17:37
6	47	17:35	17:42
6	47	17:41	17:48
6	47	17:46	17:53
6	47	17:52	17:59
6	47	17:57	18:04
6	48	16:03	16:05
6	48	16:09	16:11
6	48	16:14	16:16
6	48	16:20	16:22
6	48	16:25	16:27
6	48	16:31	16:33
6	48	16:36	16:38
6	48	16:42	16:44
6	48	16:47	16:49
6	48	16:53	16:55
6	48	16:58	17:00
6	48	17:04	17:06
6	48	17:09	17:11
6	48	17:15	17:17
6	48	17:20	17:22
6	48	17:26	17:28
6	48	17:31	17:33
6	48	17:37	17:39
6	48	17:42	17:44
6	48	17:48	17:50
6	48	17:53	17:55
6	48	17:59	18:01
6	48	18:04	18:06
6	49	16:00	16:03
6	49	16:05	16:08
6	49	16:11	16:14
6	49	16:16	16:19

Transit Line	Link No.	Departures	Arrivals
6	49	16:22	16:25
6	49	16:27	16:30
6	49	16:33	16:36
6	49	16:38	16:41
6	49	16:44	16:47
6	49	16:49	16:52
6	49	16:55	16:58
6	49	17:00	17:03
6	49	17:06	17:09
6	49	17:11	17:14
6	49	17:17	17:20
6	49	17:22	17:25
6	49	17:28	17:31
6	49	17:33	17:36
6	49	17:39	17:42
6	49	17:44	17:47
6	49	17:50	17:53
6	49	17:55	17:58
142	11	16:05	16:08
142	11	16:35	16:38
142	11	16:08	16:09
142	11	16:38	16:39
501/301	9	16:04	16:08
501/301	9	16:09	16:13
501/301	9	16:14	16:18
501/301	9	16:19	16:23
501/301	9	16:24	16:28
501/301	9	16:29	16:33
501/301	9	16:34	16:38
501/301	9	16:39	16:43
501/301	9	16:44	16:48
501/301	9	16:49	16:53
501/301	9	16:54	16:58
501/301	9	16:59	17:03
501/301	9	17:04	17:08
501/301	9	17:09	17:13
501/301	9	17:14	17:18
501/301	9	17:19	17:23
501/301	9	17:24	17:28
501/301	9	17:29	17:33
501/301	9	17:34	17:38
501/301	9	17:39	17:43
501/301	9	17:44	17:48
501/301	9	17:49	17:53
501/301	9	17:54	17:58
501/301	9	17:59	18:03
501/301	16	16:03	16:05
501/301	16	16:08	16:10
501/301	16	16:13	16:15

Transit Line	Link No.	Departures	Arrivals	Transit Line	Link No.	Departures	Arrivals
Subway 2: Green	6	17:57	18:04	Subway 2: Green	19	16:18	16:19
Subway 2: Green	6	18:00	18:07	Subway 2: Green	19	16:21	16:22
Subway 2: Green	12	16:00	16:01	Subway 2: Green	19	16:24	16:25
Subway 2: Green	12	16:03	16:04	Subway 2: Green	19	16:27	16:28
Subway 2: Green	12	16:06	16:07	Subway 2: Green	19	16:30	16:31
Subway 2: Green	12	16:09	16:10	Subway 2: Green	19	16:33	16:34
Subway 2: Green	12	16:12	16:13	Subway 2: Green	19	16:36	16:37
Subway 2: Green	12	16:15	16:16	Subway 2: Green	19	16:39	16:40
Subway 2: Green	12	16:18	16:19	Subway 2: Green	19	16:42	16:43
Subway 2: Green	12	16:21	16:22	Subway 2: Green	19	16:45	16:46
Subway 2: Green	12	16:24	16:25	Subway 2: Green	19	16:48	16:49
Subway 2: Green	12	16:27	16:28	Subway 2: Green	19	16:51	16:52
Subway 2: Green	12	16:30	16:31	Subway 2: Green	19	16:54	16:55
Subway 2: Green	12	16:33	16:34	Subway 2: Green	19	16:57	16:58
Subway 2: Green	12	16:36	16:37	Subway 2: Green	19	17:00	17:01
Subway 2: Green	12	16:39	16:40	Subway 2: Green	19	17:03	17:04
Subway 2: Green	12	16:42	16:43	Subway 2: Green	19	17:06	17:07
Subway 2: Green	12	16:45	16:46	Subway 2: Green	19	17:09	17:10
Subway 2: Green	12	16:48	16:49	Subway 2: Green	19	17:12	17:13
Subway 2: Green	12	16:51	16:52	Subway 2: Green	19	17:15	17:16
Subway 2: Green	12	16:54	16:55	Subway 2: Green	19	17:18	17:19
Subway 2: Green	12	16:57	16:58	Subway 2: Green	19	17:21	17:22
Subway 2: Green	12	17:00	17:01	Subway 2: Green	19	17:24	17:25
Subway 2: Green	12	17:03	17:04	Subway 2: Green	19	17:27	17:28
Subway 2: Green	12	17:06	17:07	Subway 2: Green	19	17:30	17:31
Subway 2: Green	12	17:09	17:10	Subway 2: Green	19	17:33	17:34
Subway 2: Green	12	17:12	17:13	Subway 2: Green	19	17:36	17:37
Subway 2: Green	12	17:15	17:16	Subway 2: Green	19	17:39	17:40
Subway 2: Green	12	17:18	17:19	Subway 2: Green	19	17:42	17:43
Subway 2: Green	12	17:21	17:22	Subway 2: Green	19	17:45	17:46
Subway 2: Green	12	17:24	17:25	Subway 2: Green	19	17:48	17:49
Subway 2: Green	12	17:27	17:28	Subway 2: Green	19	17:51	17:52
Subway 2: Green	12	17:30	17:31	Subway 2: Green	19	17:54	17:55
Subway 2: Green	12	17:33	17:34	Subway 2: Green	19	17:57	17:58
Subway 2: Green	12	17:36	17:37	Subway 2: Green	19	18:00	18:01
Subway 2: Green	12	17:39	17:40	Subway 1: Yellow	55	16:00	16:03
Subway 2: Green	12	17:42	17:43	Subway 1: Yellow	55	16:03	16:06
Subway 2: Green	12	17:45	17:46	Subway 1: Yellow	55	16:06	16:09
Subway 2: Green	12	17:48	17:49	Subway 1: Yellow	55	16:09	16:12
Subway 2: Green	12	17:51	17:52	Subway 1: Yellow	55	16:12	16:15
Subway 2: Green	12	17:54	17:55	Subway 1: Yellow	55	16:15	16:18
Subway 2: Green	12	17:57	17:58	Subway 1: Yellow	55	16:18	16:21
Subway 2: Green	12	18:00	18:01	Subway 1: Yellow	55	16:21	16:24
Subway 2: Green	19	16:00	16:01	Subway 1: Yellow	55	16:24	16:27
Subway 2: Green	19	16:03	16:04	Subway 1: Yellow	55	16:27	16:30
Subway 2: Green	19	16:06	16:07	Subway 1: Yellow	55	16:30	16:33
Subway 2: Green	19	16:09	16:10	Subway 1: Yellow	55	16:33	16:36
Subway 2: Green	19	16:12	16:13	Subway 1: Yellow	55	16:36	16:39
Subway 2: Green	19	16:15	16:16	Subway 1: Yellow	55	16:39	16:42

Transit Line	Link No.	Departures	Arrivals
Subway 1: Yellow	55	16:42	16:45
Subway 1: Yellow	55	16:45	16:48
Subway 1: Yellow	55	16:48	16:51
Subway 1: Yellow	55	16:51	16:54
Subway 1: Yellow	55	16:54	16:57
Subway 1: Yellow	55	16:57	17:00
Subway 1: Yellow	55	17:00	17:03
Subway 1: Yellow	55	17:03	17:06
Subway 1: Yellow	55	17:06	17:09
Subway 1: Yellow	55	17:09	17:12
Subway 1: Yellow	55	17:12	17:15
Subway 1: Yellow	55	17:15	17:18
Subway 1: Yellow	55	17:18	17:21
Subway 1: Yellow	55	17:21	17:24
Subway 1: Yellow	55	17:24	17:27
Subway 1: Yellow	55	17:27	17:30
Subway 1: Yellow	55	17:30	17:33
Subway 1: Yellow	55	17:33	17:36
Subway 1: Yellow	55	17:36	17:39
Subway 1: Yellow	55	17:39	17:42
Subway 1: Yellow	55	17:42	17:45
Subway 1: Yellow	55	17:45	17:48
Subway 1: Yellow	55	17:48	17:51
Subway 1: Yellow	55	17:51	17:54
Subway 1: Yellow	55	17:54	17:57
Subway 1: Yellow	55	17:57	18:00
Subway 1: Yellow	55	18:00	18:03
Subway 1: Yellow	41	16:00	16:01
Subway 1: Yellow	41	16:03	16:04
Subway 1: Yellow	41	16:06	16:07
Subway 1: Yellow	41	16:09	16:10
Subway 1: Yellow	41	16:12	16:13
Subway 1: Yellow	41	16:15	16:16
Subway 1: Yellow	41	16:18	16:19
Subway 1: Yellow	41	16:21	16:22
Subway 1: Yellow	41	16:24	16:25
Subway 1: Yellow	41	16:27	16:28
Subway 1: Yellow	41	16:30	16:31
Subway 1: Yellow	41	16:33	16:34
Subway 1: Yellow	41	16:36	16:37
Subway 1: Yellow	41	16:39	16:40
Subway 1: Yellow	41	16:42	16:43
Subway 1: Yellow	41	16:45	16:46
Subway 1: Yellow	41	16:48	16:49
Subway 1: Yellow	41	16:51	16:52
Subway 1: Yellow	41	16:54	16:55
Subway 1: Yellow	41	16:57	16:58
Subway 1: Yellow	41	17:00	17:01
Subway 1: Yellow	41	17:03	17:04
Subway 1: Yellow	41	17:06	17:07
Subway 1: Yellow	41	17:09	17:10
Subway 1: Yellow	41	17:12	17:13
Subway 1: Yellow	41	17:15	17:16
Subway 1: Yellow	41	17:18	17:19
Subway 1: Yellow	41	17:21	17:22
Subway 1: Yellow	41	17:24	17:25
Subway 1: Yellow	41	17:27	17:28
Subway 1: Yellow	41	17:30	17:31
Subway 1: Yellow	41	17:33	17:34
Subway 1: Yellow	41	17:36	17:37
Subway 1: Yellow	41	17:39	17:40
Subway 1: Yellow	41	17:42	17:43
Subway 1: Yellow	41	17:45	17:46
Subway 1: Yellow	41	17:48	17:49
Subway 1: Yellow	41	17:51	17:52
Subway 1: Yellow	41	17:54	17:55
Subway 1: Yellow	41	17:57	17:58
Subway 1: Yellow	41	18:00	18:01
Subway 1: Yellow	42	16:00	16:01
Subway 1: Yellow	42	16:03	16:04
Subway 1: Yellow	42	16:06	16:07
Subway 1: Yellow	42	16:09	16:10
Subway 1: Yellow	42	16:12	16:13
Subway 1: Yellow	42	16:15	16:16
Subway 1: Yellow	42	16:18	16:19
Subway 1: Yellow	42	16:21	16:22
Subway 1: Yellow	42	16:24	16:25
Subway 1: Yellow	42	16:27	16:28
Subway 1: Yellow	42	16:30	16:31
Subway 1: Yellow	42	16:33	16:34
Subway 1: Yellow	42	16:36	16:37
Subway 1: Yellow	42	16:39	16:40
Subway 1: Yellow	42	16:42	16:43
Subway 1: Yellow	42	16:45	16:46
Subway 1: Yellow	42	16:48	16:49
Subway 1: Yellow	42	16:51	16:52
Subway 1: Yellow	42	16:54	16:55
Subway 1: Yellow	42	16:57	16:58
Subway 1: Yellow	42	17:00	17:01
Subway 1: Yellow	42	17:03	17:04
Subway 1: Yellow	42	17:06	17:07
Subway 1: Yellow	42	17:09	17:10
Subway 1: Yellow	42	17:12	17:13
Subway 1: Yellow	42	17:15	17:16
Subway 1: Yellow	42	17:18	17:19
Subway 1: Yellow	42	17:21	17:22
Subway 1: Yellow	42	17:24	17:25
Subway 1: Yellow	42	17:27	17:28

Transit Line	Link No.	Departures	Arrivals	Transit Line	Link No.	Departures	Arrivals
Subway 1: Yellow	42	17:30	17:31	Subway 1: Yellow	43	17:54	17:55
Subway 1: Yellow	42	17:33	17:34	Subway 1: Yellow	43	17:57	17:58
Subway 1: Yellow	42	17:36	17:37	Subway 1: Yellow	43	18:00	18:01
Subway 1: Yellow	42	17:39	17:40	GoTrain	1	16:03	16:09
Subway 1: Yellow	42	17:42	17:43	GoTrain	1	16:24	16:30
Subway 1: Yellow	42	17:45	17:46	GoTrain	1	16:54	17:00
Subway 1: Yellow	42	17:48	17:49	GoTrain	1	17:24	17:30
Subway 1: Yellow	42	17:51	17:52	GoTrain	1	18:00	18:06
Subway 1: Yellow	42	17:54	17:55	GoTrain	0	16:09	16:20
Subway 1: Yellow	42	17:57	17:58	GoTrain	0	16:30	16:41
Subway 1: Yellow	42	18:00	18:01	GoTrain	0	17:00	17:11
Subway 1: Yellow	43	16:00	16:01	GoTrain	0	17:30	17:41
Subway 1: Yellow	43	16:03	16:04	GoTrain	0	18:06	18:17
Subway 1: Yellow	43	16:06	16:07	511	30	16:01	16:10
Subway 1: Yellow	43	16:09	16:10	511	30	16:04	16:13
Subway 1: Yellow	43	16:12	16:13	511	30	16:07	16:16
Subway 1: Yellow	43	16:15	16:16	511	30	16:10	16:19
Subway 1: Yellow	43	16:18	16:19	511	30	16:13	16:22
Subway 1: Yellow	43	16:21	16:22	511	30	16:16	16:25
Subway 1: Yellow	43	16:24	16:25	511	30	16:19	16:28
Subway 1: Yellow	43	16:27	16:28	511	30	16:22	16:31
Subway 1: Yellow	43	16:30	16:31	511	30	16:25	16:34
Subway 1: Yellow	43	16:33	16:34	511	30	16:28	16:37
Subway 1: Yellow	43	16:36	16:37	511	30	16:31	16:40
Subway 1: Yellow	43	16:39	16:40	511	30	16:34	16:43
Subway 1: Yellow	43	16:42	16:43	511	30	16:37	16:46
Subway 1: Yellow	43	16:45	16:46	511	30	16:40	16:49
Subway 1: Yellow	43	16:48	16:49	511	30	16:43	16:52
Subway 1: Yellow	43	16:51	16:52	511	30	16:46	16:55
Subway 1: Yellow	43	16:54	16:55	511	30	16:49	16:58
Subway 1: Yellow	43	16:57	16:58	511	30	16:52	17:01
Subway 1: Yellow	43	17:00	17:01	511	30	16:55	17:04
Subway 1: Yellow	43	17:03	17:04	511	30	16:58	17:07
Subway 1: Yellow	43	17:06	17:07	511	30	17:01	17:10
Subway 1: Yellow	43	17:09	17:10	511	30	17:04	17:13
Subway 1: Yellow	43	17:12	17:13	511	30	17:07	17:16
Subway 1: Yellow	43	17:15	17:16	511	30	17:10	17:19
Subway 1: Yellow	43	17:18	17:19	511	30	17:13	17:22
Subway 1: Yellow	43	17:21	17:22	511	30	17:16	17:25
Subway 1: Yellow	43	17:24	17:25	511	30	17:19	17:28
Subway 1: Yellow	43	17:27	17:28	511	30	17:22	17:31
Subway 1: Yellow	43	17:30	17:31	511	30	17:25	17:34
Subway 1: Yellow	43	17:33	17:34	511	30	17:28	17:37
Subway 1: Yellow	43	17:36	17:37	511	30	17:31	17:40
Subway 1: Yellow	43	17:39	17:40	511	30	17:34	17:43
Subway 1: Yellow	43	17:42	17:43	511	30	17:37	17:46
Subway 1: Yellow	43	17:45	17:46	511	30	17:40	17:49
Subway 1: Yellow	43	17:48	17:49	511	30	17:43	17:52
Subway 1: Yellow	43	17:51	17:52	511	30	17:46	17:55

Transit Line	Link No.	Departures	Arrivals	Transit Line	Link No.	Departures	Arrivals
511	30	17:49	17:58				
511	30	17:52	18:01				
511	30	17:55	18:04				
511	30	17:58	18:07				
511	30	18:01	18:10				

APPENDIX C: Visual Basic Code for the MDP Algorithm

```
'Nodes array includes "to nodes" from each node (0-33) and related links. Maximum 3 nodes possbible from each node
Public Nodes(33, 6) As Integer

'Transit schedule
Public transit(60, 60) As Date

'Traveltime (tt) array includes travel time on that link for each mode. 1-3. Cars: three travel times for each traffic state (congestion)- 4. transit: will be extracted based on the schedule- 5. Bike - 6. Walk
Public tt(60, 6) As Double

'Keep record of nodes/modes in each travel path
Public tPath(1000, 3) As Integer
'Keep record of "min travel time" and "best mode"
Public optPath(1000, 2) As Integer

'Transition Probabiity Matrix for cars: probability of change in traffic conditions: A, B and C:
'     A     B     C
'A  (1,1)  (1,2)  (1,3)
'B  (2,1)  (2,2)  (2,3)
'C  (3,1)  (3,2)  (3,3)
Public prob(3, 3) As Double

'Current traffic state. The current traffic state is the state with highiest probability at each time
Public trafficState(1, 3) As Double

'number of steps (links) to take and record in tPath
Global step As Integer

'Number of mode changes in each step: Maximum TWO is allowed. Transit-> Car is not allowed. Walking can be the third mode (transfer times is not considered in walking. they are included in the access time)
Global mChange(1000) As Integer

'Starting/End Nodes and Time -> to be set in main()
Global startTime As Date
Global startNode As Integer
Global destNode As Integer
Global lastNode As Integer
Global flag As Integer 'Check if next node is more than destination node: do not consider it
```

```
Global minTT As Double 'Keeps minimum travel time of all routes
Global atDestination As Boolean
Global parkTime As Double
Global Row As Integer

'DEBUG ONLY: MUST USE ONLY THIS MODE
Global mustMODE As Integer

Sub Initialization()

  'Populate Nodes Array based on "Network" sheet
  inpSheet = "Network"
  n = Worksheets(inpSheet).UsedRange.Rows.count

  For i = 0 To 33
    For j = 1 To 6
      Nodes(i, j) = -1
    Next j
  Next i

  For i = 2 To n

    Node = Worksheets(inpSheet).Cells(i, 1).Value
    toNode = Worksheets(inpSheet).Cells(i, 2).Value
    link = Worksheets(inpSheet).Cells(i, 3).Value

    If Nodes(Node, 1) = -1 Then
      Nodes(Node, 1) = toNode
      Nodes(Node, 4) = link
    Else
      If Nodes(Node, 2) = -1 Then
        Nodes(Node, 2) = toNode
        Nodes(Node, 5) = link
      Else
        Nodes(Node, 3) = toNode
        Nodes(Node, 6) = link
      End If
    End If
  Next i


  'Populate travel time (tt) array based on "Network" sheet. travel time will include access+wait times
  'Transit tt will be added later based on schedule/arriving time
```

```
inpSheet = "Network"
n = Worksheets(inpSheet).UsedRange.Rows.count
col = 4 'First column that includes travel times - Now is Column D

For i = 2 To n

   link = Worksheets(inpSheet).Cells(i, 3).Value

   'Cars: three travel times
   'State A
   tt(link, 1) = Worksheets(inpSheet).Cells(i, col).Value + Worksheets(inpSheet).Cells(i, col + 3).Value + Worksheets(inpSheet).Cells(i, col + 4).Value
   'State B
   tt(link, 2) = Worksheets(inpSheet).Cells(i, col + 1).Value + Worksheets(inpSheet).Cells(i, col + 3).Value + Worksheets(inpSheet).Cells(i, col + 4).Value
   'State C
   tt(link, 3) = Worksheets(inpSheet).Cells(i, col + 2).Value + Worksheets(inpSheet).Cells(i, col + 3).Value + Worksheets(inpSheet).Cells(i, col + 4).Value

   'Transit travel times, without considering schedule/arriving time
   tt(link, 4) = Worksheets(inpSheet).Cells(i, col + 5).Value + Worksheets(inpSheet).Cells(i, col + 6).Value + Worksheets(inpSheet).Cells(i, col + 7).Value

   'Bike
   tt(link, 5) = Worksheets(inpSheet).Cells(i, col + 9).Value + Worksheets(inpSheet).Cells(i, col + 10).Value + Worksheets(inpSheet).Cells(i, col + 11).Value

   'Walk
   tt(link, 6) = Worksheets(inpSheet).Cells(i, col + 13).Value + Worksheets(inpSheet).Cells(i, col + 14).Value + Worksheets(inpSheet).Cells(i, col + 15).Value

Next i

'Populate transit schedule based on "Bus-Train Schedule" sheet
inpSheet = "Bus-Train Schedule"
n = Worksheets(inpSheet).UsedRange.Rows.count
col = 4 'First column that includes travel times - Now is Column D
freq = 0 'Each schedule delarture

For i = 0 To 60
   For j = 1 To 60
      transit(i, j) = -1
   Next j
Next i
```

```
   link = Worksheets(inpSheet).Cells(2, 1).Value
   For i = 2 To n

      If Worksheets(inpSheet).Cells(i, 1).Value <> link Then
         link = Worksheets(inpSheet).Cells(i, 1).Value
         freq = 0
      End If

      freq = freq + 1
      transit(link, freq) = Worksheets(inpSheet).Cells(i, 2).Value

   Next i

   'Initialized mode change and travel path record
   For i = 1 To 1000
      mChange(i) = 0
      tPath(i, 1) = -1
      tPath(i, 2) = -1
      tPath(i, 3) = -1
      optPath(i, 1) = -1
      optPath(i, 2) = -1
   Next i

   tPath(0, 1) = startNode
   tPath(0, 2) = 1

   'Initialize transition probability matrix
   inpSheet = "Probability"

   For i = 1 To 3
      For j = 1 To 3
         prob(i, j) = Worksheets(inpSheet).Cells(i + 17, j + 8).Value
      Next j
   Next i

End Sub
Function getTraveltime(link As Integer, mode As Integer, traveltime As Double, pMode As Integer) As Double

' Return travel time on link for mode, to consider arriving time at startTime+traveltime

   'Temporaru stores next traffic state
   Dim nextState(1, 3) As Double
```

```
    'Temporary variable for returned travel time
    '-1 is to check if mode is available for that arrival time/link
    temptt = -1
    waitTime = -1


    'If bike or walk mode, then get travel times from the previously populated tt array
    If mode = 3 Or mode = 4 Then
      temptt = tt(link, mode + 2)
    Else

      'For Car mode, travel time should be calcualted considering probability of changes in traffic
  state
      If mode = 1 Then

        'Based on current traffic state
        temptt = trafficState(1, 1) * tt(link, 1) + trafficState(1, 1) * tt(link, 2) + trafficState(1, 1) *
  tt(link, 3)

        'Calculating next traffic state
        For i = 1 To 3
          nextState(1, i) = trafficState(1, i) * prob(i, 1) + trafficState(1, i) * prob(i, 2) +
  trafficState(1, i) * prob(i, 3)
        Next i

        'Set current traffic state equals to next traffic state
        For i = 1 To 3
          trafficState(1, i) = nextState(1, i)
        Next i

      Else
        'For Transit, travel time is calculated considering arrival time/schedule
        If mode = 2 Then

          'Calculates arrival time based on start/traveltime so far
          arrivalTime = DateAdd("n", traveltime, startTime)

          'Waiting time for arriving time. if arrives with transit, skip this
          If pMode = 2 Then
            waitTime = 0
          Else
            For j = 1 To 60
              If transit(link, j) >= arrivalTime Then
```

```
                    waitTime = Minute(transit(link, j) - arrivalTime)
                    Exit For
                  End If
                Next j
              End If

            'If one scheduled departure is available
            If waitTime > -1 Then
              temptt = tt(link, mode + 2) + waitTime
            End If

          End If

        End If


    End If

    'Returns estimated travel time for requested link/mode
    getTraveltime = temptt

End Function
Function modeChangeIsValid(link As Integer, pMode As Integer, mode As Integer) As Boolean

    Dim validity As Boolean

    validity = False

    If mustMODE > -1 Then
      If mode = mustMODE Then
        validity = True
      End If
    Else
      If (mode = 1 And tt(link, mode) > 0) Or (mode > 1 And tt(link, mode + 2) > 0) Then

        'Checking for mode change criteria. Maximum 2 is allowed. Walking(mode 4) is exception
at the end. NO mode change from others to car!
        If mode = 1 And pMode <> 1 Then
          validity = False
        Else
          If pMode <> mode Then
            mChange(step) = mChange(step - 1) + 1
          End If
```

```
          If mChange(step) <= 2 Then
            validity = True

            If pMode = 1 And mode > 1 Then
              Select Case link
                Case 1, 2, 3, 4, 5, 6, 27, 28, 29, 57, 58
                parkTime = 5
              Case Else
                parkTime = 15
              End Select
            End If
          Else
            mChange(step) = mChange(step) - 1
          End If

        End If

      End If
    End If

    modeChangeIsValid = validity

End Function

Function shortestPath(n As Integer, caltt As Double) As Double
    ' Finds the optimal route. gets "n" as the beginning node and "caltt" as the calculated travel time so far

    Dim nextNode, nextMode As Integer
    Dim i, j As Integer
    Dim traveltime As Double
    Dim returnTT As Double
    Dim modeChangeAllowed As Boolean


    'Initial large number for optimal traveltime
    ' No. of steps from origin to destination. Increases each time function is called (one route to be calculated)
    step = step + 1
    'Carry forward previous number of mode changes
    mChange(step) = mChange(step - 1)


    If n = destNode Then
```

```
      atDestination = True
      returnTT = caltt          ' reached destination node

   Else
      If (n = lastNode) Then
         flag = -1
         returnTT = 10000000          ' reached destination node

      Else
         ' Two inner loops:
         ' One for available links(nodes), maximum 3, from current node and the other for available modes, maximum 4, for each link
         For i = 1 To 3

            'Checking to see if next node (maximum 3) is actually available. in most cases there is only 2 available nodes from the current node!
            If Nodes(n, i) <> -1 Then

               flag = 1
               For j = 1 To 4
                  parkTime = 0
                  If (flag <> -1) Then

                     modeChangeAllowed = modeChangeIsValid(Nodes(n, i + 3), tPath(step - 1, 0), j)
                     If ((step = 1) Or (modeChangeAllowed)) Then

                        'Get travel time of link n->Nodes(n,i)=Nodes(n,i+3) when using mode j, pass current traveltime (caltt) for transit schedule check
                        'CHECK for -1 returned!!! No Link/Mode avaialble
                        temptt = getTraveltime(Nodes(n, i + 3), j, caltt, tPath(step - 1, 0))
                        'To temporary show what will be the next mode!
                        tPath(step, 0) = j
                        tPath(step, 3) = n

                        If (temptt > 0) Then

                           tPath(step, 1) = Nodes(n, i)
                           tPath(step, 2) = j

                           traveltime = caltt + temptt + parkTime
                           If traveltime <= minTT Then
                              traveltime = shortestPath(Nodes(n, i), traveltime)

                              If (atDestination) Then
```

```
                    'If reached destination, add time for park to car
                    If j = 1 Then
                       traveltime = traveltime + 15
                    End If

                    If (flag <> -1 And traveltime <= minTT) Then
                          minTT = traveltime
                          returnTT = traveltime
                          nextMode = j
                          ' Store travel path record: nodes and modes from destination to origin!

                          m = 0
                          Do
                             m = m + 1
                             optPath(m, 0) = minTT
                             optPath(m, 1) = tPath(m, 1)
                             optPath(m, 2) = tPath(m, 2)

                             'Excel
                             Sheets("Result").Cells(1 + Row, m) = optPath(m, 1)
                             Sheets("Result").Cells(2 + Row, m) = optPath(m, 2)


                          Loop While tPath(m, 1) <> destNode
                          Sheets("Result").Cells(2 + Row, m + 1) = minTT
                          Row = Row + 2
                          atDestination = False
                    End If

                 End If
              Else
                 traveltime = 10000
              End If 'for <=minTT

              returnTT = traveltime
           End If 'for temptt>0

           'If mode change occurred, deduct the number of lane changes by 1, for next mode to be checked!
           If tPath(step - 1, 0) <> j Then
              mChange(step) = mChange(step) - 1
           End If
```

```
                    End If ' For mode change allowed

                  End If 'for flag<>-1

                Next j

              End If

            Next i

            'step = step - 1

        End If
    End If

    'Returns the estimated minimum travel time so far
    mChange(step) = 0
    step = step - 1

    shortestPath = returnTT

End Function

Sub main()

    Dim myTravelTime As Double

    'Get start time from origin
    startTime = "17:00:00" 'TimeValue(InputBox("Enter start time (hh:mm):", "Start time at
Origin (A)", "17:00:00"))
    startNode = 0 'InputBox("Enter the beginning node:", "Origin", 0)
    destNode = 33 '33 'Destination Node = 33(B)
    atDestination = False
    lastNode = 33
    minTT = 1000000
    Row = 0 'PRINT IN EXCEL

    'Starting traffic state: 1:A, 2:B, 3:C
    trafficState(1, 1) = 0.25
    trafficState(1, 2) = 0.5
    trafficState(1, 3) = 0.25

    step = 0
```

```
    Sheets("Result").Cells.ClearContents

    'Populate Nodes and TravelTimes arrays
    Initialization

    'Initiates shortest path calculation from startNode to destNode
    myTravelTime = shortestPath(startNode, 0)

End Sub
```