

SMART BETA PORTFOLIOS WITH MARKOV REGIME-SWITCHING MODELS

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## **ABSTRACT**

Smart Beta Portfolios with Markov Regime-Switching Models

Gabriel Barbe

While many researchers have studied the performance of style investing strategies such as value, growth or small caps, studies dealing with the performance of smart beta portfolios are limited. This study tests the performance of a dynamic asset allocation strategy based on various smart beta portfolios that rely on a Markov regime-switching model based on macroeconomic regimes. Results and backtests show that using Markov regimes increases the performance of a dynamic smart beta portfolio based on Markov regimes compared to a static benchmark in-sample, and that such performance begins to erode when utilized out-of-sample considering one friction (trade costs). Also, this study finds that the choice of the economic variable used to estimate the Markov regime switching model is important for the performance of smart beta portfolios using Markov regimes based on macroeconomic indicators.

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## 1. INTRODUCTION

One of the assumptions of the Capital Asset Pricing Model (CAPM) is that every investor holds the same efficient portfolio and that this portfolio is the capitalization weighted market portfolio. Based on this assumption, cap-weighted indices are common benchmarks for institutional investors. Many researchers show that cap-weighted indices are not mean-variance efficient ex post (Haugen and Baker, 1991; Grinold, 1992; Hsu, 2004). Hsu (2006) asserts that the sub-optimality of cap-weighted portfolios arise from the overweighting and underweighting of stocks that are overvalued and undervalued, respectively, based on their fundamentals. In contrast, Levy & Roll (2010) argue that while many of the common benchmarks are not mean-variance efficient, slight variations in parameters are enough to make them so, putting a grain of salt on a rationale for smart-beta indices.

Based on the findings on portfolio inefficiency, Arnott, Hsu and Moore (2005) create a set of fundamental indices that do not rely on market prices (Wood and Evans, 2003). The premise behind fundamental indices is that the fundamentals underlying a stock should determine its weight in the index rather than its market price. In constructing their fundamental indices, Arnott et al. (2005) use fundamentals such as book value, cash flow, revenue, sales, dividends and employment. Subsequently, many other indexing strategies have been created in parallel with risk factor models, since the risk factors used in those models have been shown to capture the variation in the performance of stocks.

The popularity of smart beta portfolios and the presence of regimes in financial time series and the forecasting ability of Markov regime-switching models has justified the combination of smart beta portfolios and Markov models. This thesis is situated in the common ground between smart beta indices and Markov regime-switching models. There are only a few papers that combine these areas (i.e. Gkatzilakis and Sivasubramanian, 2014; Boudt et al., 2015). Both articles show that such a combination can yield profitable asset allocation strategies. Therefore, the purpose of this thesis is to create a dynamic asset allocation strategy based on macroeconomic regimes that are estimated from a Markov-regime switching model.



The remainder of this thesis is organized as follows. In the next section, different risk factor models are reviewed, particularly in terms of their implications for the construction of smart beta indices. The literatures on Smart Beta portfolios and Markov regime-switching models in an asset allocation context are also examined. Then, the methodology of this study and its results are discussed. The last section provides some concluding comments.

## **2. LITERATURE REVIEW**

### **2.1 RISK FACTOR MODELS**

Fundamental indexing starts from the notion that different risk factors, such as firm size (market capitalization), relative valuation measured with the book to market ratio, or even momentum, capture the variance of the cross-section of stock returns. Many fundamental indexing strategies are built from the risk factors included in the asset pricing models. Among the 314 factors discovered, these are the most prominent ones (Harvey, Liu & Zhu, 2013).

The original factor model is the capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965), where the market is the only factor that explains the performance of a stock. The Intertemporal CAPM (Merton, 1973), the Consumption CAPM (Breedon, 1979), and the International CAPM (Stulz, 1981) are some of the extensions of the original CAPM.

Fama and French (1992, 1993) provide a three-factor version of the CAPM where the expected return of a stock is a linear combination of different factors and their sensitivities. In their three-factor model, they use the market proxy and two stock fundamentals: size (SMB) computed from the market capitalization of a firm, and value (HML) computed from the book-to-market ratio of a firm. These factors have their own covariances that are not captured by the covariance of the returns of a firm with that of the market (the market beta). Fama and French find that firms with smaller capitalizations and higher book-to-market ratios tend to outperform firms with higher capitalizations and firms with lower book-to-market ratios. Carhart (1997) adds a momentum factor (UMD) to this three-factor model to reflect the momentum effect anomaly found by Jegadeesh and

Titman (1993). The momentum effect states that a stock that has performed well in the twelve previous months tends to outperform in the following 3 to 12 months.

Acharya and Pederson (2005) link liquidity and stock performance in a liquidity-adjusted capital asset pricing model. They specify three different liquidity risks that each covary with the market. First, stock returns increase with the covariance between the asset's liquidity and the market's liquidity, because investors want to be compensated for the relative illiquidity of the stock compared to that of the market. Second, stock returns decrease with the covariance of the stock returns with market liquidity. In other words, low required returns are acceptable when the stock yields high returns in illiquid markets. Third, the stock returns decrease with the covariance between the stock's liquidity and the market return because their liquidity increases in down markets.

Ang, Hodrick, Xing & Zhang (2006) argue that stocks with an increased exposure to the volatility of the market have lower average excess returns and that stocks with high idiosyncratic volatility have low average returns. Ang et al. (2006) conclude that volatility should be an included factor based on their finding that the premium associated with size, value, momentum and liquidity cannot account for aggregate and idiosyncratic volatility.

Bali et al. (2011) use the maximum daily return over the past one month (MAX) as a factor based on the motivation that individual investors prefer lottery-like payoffs. The MAX factor is found to have a significant negative relationship with expected stock returns. Interestingly, the MAX factor reverses the relation of returns with idiosyncratic volatility of Ang et al. (2006) discussed in the previous paragraph. However, Aboulamer & Kryzanowski (2016) find that the negative relationship between the MAX factor and the expected stock returns is positive in Canadian markets.

Hou, Xue & Zhang (2014a) argue that the Fama-French three-factor model fails to explain many capital markets anomalies despite its influence on financial research. They replace the value factor by an investment factor and a return-on-equity factor calculated using the Fama-French approach. Their investment factor is the difference between the return of a low investment portfolio and the return of a high investment portfolio, and their return-on-equity factor is the difference between the return of a high return-on-equity (ROE) portfolio and the return of a low return-on-equity portfolio. Their investment factor

captures the valuation and growth opportunities of a firm by the investment-to-assets ratio, where investment includes the change in property, plant and equipment plus the change in inventories. The rationale is that the change in property, plant and equipment measures the long-term investment of a firm, while the change in inventories measures the short-term change in firm assets. The ROE, which is computed as income before extraordinary items divided by the book equity lagged one quarter, is a measure of how well the firm is investing its capital. More importantly, the significance of many documented anomalies that were previously found using the Fama-French three-factor model are reduced using the model of Hou et al. (2014a).

Most recently, Fama and French (2015) expand their three-factor model into a five-factor model that includes an investment and a profitability factor. Fama and French argue that these factors mimic the effects of the state variables without actually identifying them. Fama and French specify their investment and profitability factors in a slightly different manner from that of Hou et al. (2014a). Nichol and Dowling (2014) adapt the profitability and investment factors of both Hou et al. (2014) and Fama and French (2015) to the United Kingdom. They also argue that the Fama-French profitability factor offers the most potential for asset pricing models in the United Kingdom.

These factor models are the building blocks that lead to fundamental indexing. The way Fama and French created their portfolios and the intuition behind their models gave rise to fundamental indexed portfolios, commonly called smart beta portfolios. In the next section, smart beta indices, their performance and the theory behind them are presented.

## **2.2 SMART BETA INDICES**

Style investing, particularly value and momentum, are important ways to build portfolios. The methodology consists of sorting assets according to a given factor such as book to market, then using a capitalization weighting methodology to attribute weights. However, as noted above, Arnott et al. (2005) present a different way to obtain the weights for each stock in such portfolios. The set of methodologies used to create such portfolios are not only based on heuristics and factors but also optimization-based methodologies, such as minimum-variance or equal risk contribution methodologies. Smart beta, or

sometimes called exotic beta, refers to asset allocation techniques that use non-capitalization weights when forming portfolios.

Many researchers associate the outperformance of smart beta indices over capitalization-weighted indices to the increased exposure to value and size factors (see Amenc, Goltz and Lodh, 2012). Arnott, Hsu, Liu and Markowitz (2011) explain this phenomenon by the mean-reversion nature of stocks. Because low prices create a low book-to-market ratio and a low market capitalization, the mean reversion of prices creates outperformance for those value stocks and small caps. In contrast, a market cap-weighted index overweighs overvalued stocks and underweights undervalued stocks (Arnott, Hsu & Kalesnik, 2013).

Arnott, Hsu, Kalesnik and Tindall (2013) find that the upside down versions of the common smart beta portfolios, such as the maximum volatility portfolio or the inverse-ratio of maximum diversification, also beat the cap-weighted indices. Even a set of random weights, or in the case of Arnott et al. (2013), “Malkiel’s blindfolded monkeys throwing darts” outperform the cap-weighted indices, and also introduce a value tilt in most of their portfolios.

However, smart beta has its critics. Amenc et al. (2015) question the theoretical underpinnings of the strategies, arguing that the selection of stocks based on fundamental data is not supported by any research. They argue that stock mispricing is inconsistent with the efficient market hypothesis, and that the performances of such indices are “highly sensitive to strategy specification choice”, which is not a desirable model attribute. They also argue that smart beta portfolios neither control for exposures to unrewarded strategy-specific risks nor have risk controls for systematic risk factors. Smart beta 2.0 methods that include risk management to the smart beta indices help to address this latter criticism (Amenc, Goltz & Martellini, 2013). The main remedy to these drawbacks are to establish a consistent framework, avoid data and model mining, control for unrewarded risk, diversify across risk factors and adopt full transparency regarding model specification.

Since the performance of each smart-beta index is attributable to certain market conditions, Amenc et al. (2012) create a combination of such strategies to diversify model

selection risk. Their combination leads to less volatile performance that may better suit investors who do not have forward-looking views about equity markets.

Based on the intuition of Amenc et al. (2012), it seems reasonable to believe that a regime-switching model may be able to capture part of that variation since the performance of a smart beta index relies on market conditions. Therefore, the next section reviews regime switching models as a predecessor to their use in tactical asset allocation strategies.

### **2.3 SWITCHING MODELS**

Hamilton (1989) applies an estimation framework for capturing the distribution of the returns in each regime and also the time distribution of those regimes using U.S. economic data. Kim and Nelson (1999) document the econometric applications of regime-switching models to many economic and financial data sets.

Ang and Bekaert (2002, 2004) test the presence of regimes in financial data and investigate their effects on portfolio management. Based on out-of-sample tests, Ang and Bekaert (2004) conclude that it is possible for two-state regime-switching strategies to outperform static strategies in country equity portfolios.

Ammann and Verhofen (2006) investigate the effects of market regimes on style allocation. They develop a dynamic tactical asset allocation strategy based on switching within an asset class instead of between countries as in Ang and Bekaert (2002). They demonstrate outperformance relative to a static strategy for a value investing strategy during a high-variance regime and a momentum investing strategy during a low-variance regime.

Guidolin and Ria (2010) apply Markov regime-switching to the mean-variance efficient frontier arguing that regimes of the Markowitz mean-variance efficient frontier exist and that it is possible to profit from them. Their back-testing shows that the switching mean-variance strategy can yield better risk-adjusted payoffs than a static strategy.

Bulla et al. (2011) show that a Markov regime-switching strategy using daily data reduces the market's exposure to volatility. Since the performance of regime-switching models rely heavily on the estimation of the regimes, using daily returns limits the impact

of estimating wrong regimes to a single day and not some lower-order frequency. By switching between a risk-free asset (cash) and a risky index (e.g., S&P 500), their model outperforms a buy-and-hold strategy. Other researches also report evidence that a regime-switching asset allocation strategy outperforms (e.g., Guidolin & Timmermann, 2007; Ang & Timmermann, 2012).

Kritzman, Page & Turkington (2012) use a Markov regime-switching methodology to create a dynamic asset allocation strategy. They argue that in a world where economic conditions are linked to the performance of assets, such an asset allocation strategy must outperform a static strategy. They use the Chow et al. (1999) market turbulence approach (a multivariate distance measure known as the Mahalanobis distance) to specify the regimes and their model switches between asset classes according to the performance of asset-risk premiums.

Angelidis and Tessaromatis (2014) show that using a regime-switching approach when applied to time global style allocation portfolios using international country-based factor portfolios outperforms benchmark portfolios. They also show that holding the benchmark portfolio with the style global portfolio increases the performance of the strategy. In addition, they show that holding the world market portfolio along with the country-based factor portfolio increases the risk-adjusted performance of the strategy because of the increased exposure to the value, size and momentum factors.

Asset allocation strategies using macroeconomic variables exist. Avramov and Chordia (2006) create an optimal mean-variance portfolio using stock returns estimates conditional on business cycle variables such as the dividend yield, the default spread, the term spread or the Treasury bill yield. The authors show that their approach outperforms static and dynamic investment strategies as well as the Fama-French plus momentum factors. The outperformance is attributed to the size, book-to-market and momentum effects. Their analysis shows that investors overweight small-cap stocks and underweight momentum stocks in NBER recession periods.

The research aims to fill the gap between smart betas and Markov-switching models that rely on macroeconomic conditions. This thesis is situated in the common ground between smart beta indices and Markov regime-switching models. To the extent of my

knowledge, the only two papers that combine these two areas are Gkatzilakis and Sivasubramanian (2014) and Boudt et al. (2015). Gkatzilakis and Sivasubramanian (2014) show that a passive asset allocation strategy is outperformed by a strategy where a Markov switching model based on market volatility (measured by the VIX) dictates the allocation of assets towards different smart beta indices according to the high or low volatility regime. Similarly, Boudt et al. (2015) find a better risk-adjusted performance, lower drawdown, and more adaptiveness to market conditions for a tactical asset allocation strategy that switches between equity investments and cash depending on an underlying regime based on macro-economic, macro-financial and smart beta momentum variables.

Finally, financial literature sometimes question the effectiveness of market timing strategies. However, Dichtl et al. (2016) argue that even if a forecast model has a low success ratio (hit ratio), certain investors desire market timing because of short-termism in performance evaluation.

### **3. METHODOLOGY**

In this section, the existing smart beta portfolios used in this thesis are explained before describing the smart beta indices assessed herein. In addition, the specification of the Markov regime-switching model and the variables used to determine the regimes are presented. Finally, the dynamic asset allocation strategy that uses the smart beta portfolios and the Markov regimes is discussed.

#### **3.1 SMART BETA INDEX STRATEGIES**

Strategies that are used subsequently are classified into two categories. The heuristics-based weighting methodologies are based on simple rules that are established beforehand. The optimization-based methodologies are often statistical methodologies that maximize a portfolio's *ex ante* measure of performance, such as the Sharpe ratio, while using constraints (see Chow et al., 2011; Clare, Motson & Thomas, 2013ab). While the selection of smart beta indices included in this paper is far from exhaustive, it offers a fairly representative sample of the smart beta portfolios used in practice.

For each of the following strategies, the vector of portfolio weights is  $w$  such that  $w = [w_1, w_2, \dots, w_N]$  and  $N$  is the number of stocks in the universe. The restrictions are that the portfolio should be fully invested at all times and that there are no short sales. The universe consists of U.S. equity stocks included in the CRSP and Compustat databases. Exchange-traded funds (ETFs) and American Depositary Receipts (ADRs) are excluded. In the subsequent paragraphs, each portfolio weighting strategy is described.

The first heuristics-based index strategy is equal weighting. This strategy is the simplest one to implement. The first step is to take the largest  $N$  stocks by market capitalization from the investor's universe. Each stock is attributed the same weight,  $w_i = \frac{1}{N}$ , where  $N$  represents the number of stocks in the universe. The naïve strategy does not rely on historical data to estimate its parameters, therefore it is not exposed to sampling errors (Duchin & Levy, 2009). Additionally, Pflug et al. (2012) argue that this strategy is not only rational but optimal when there is model uncertainty. DeMiguel et al. (2009a) comes to a similar conclusion.

The second heuristics-based strategy is diversity weighting. This strategy was created in order to counter the two shortcomings of the equal weighting strategy mentioned above. Diversity weighting lies somewhat between the capitalization weighted and the equally weighted strategy by tilting the capitalization weights towards a more equal balance.

In Fernholz (1995), the diversity weights are defined as:

$$x_{Diversity,i} = \frac{(x_{Market,i})^p}{[\sum_{i=1}^N (x_{Market,i})^p]}$$

where  $p$  is a constant between 0 and 1 ( $p \in (0,1)$ ). The power  $p$  also affects the level of a portfolio's tracking error. As a heuristic, the strategy redistributes the weights from the largest weights to the lowest weights. As  $p$  moves to 1, the portfolio moves closer to the capitalization weighted portfolio, and as  $p$  moves closer to 0, it moves closer to the equal weighted portfolio. The chosen  $p$  for the analysis is 0.76, which is the base value suggested by Fernholz, Garvy and Hannon (1998). The creators of the weighting scheme



justified the parameter  $p = 0.76$  as a way to obtain 40 to 60 basis point while retaining characteristics of a large stock index. The sensitivity of the parameter  $p$  is tested. Results are shown in **Table 1** of the appendix. Because the choice of parameter is a qualitative choice, the main parameter used to generate results will be 0.76.

The third heuristics-based strategy is the fundamental weighting strategy of Arnott et al. (2005). The fundamental weighted portfolio used herein is an equal weighted portfolio of our fundamentally weighted portfolios based on the averages over the last five years for gross sales, cash flows, and revenues and the previous year's book value. The weight of stock  $i$  in fundamental weighted portfolio based on metric  $j$  is given by:

$$W_{Fundamental\ Metric_{i,j}} = \frac{Fundamental\ Metric_{i,j}}{\sum_{i=1}^N Fundamental\ Metric_{i,j}}$$

The fourth and last heuristics-based strategy is the inverse volatility weighting strategy described in Clare, Motson and Thomas (2013a) whose aim is to create an index where stocks with lower volatility have a higher weight. Only stocks with a five year rolling standard deviation of the monthly returns (denoted  $\sigma_{5yr_i}$ ) are considered. The attributed weights are given as:

$$W_{inv.vol.,i} = \frac{\frac{1}{\sigma_{5yr_i}}}{\sum_{i=1}^N \frac{1}{\sigma_{5yr_i}}}$$

The optimization-based indices are more complex than the heuristics-based indices to compute since they use a statistical optimizer in order to maximize or minimize an objective function. The challenge lies in the estimation of the parameters used in the optimization. The covariance matrix used herein is created using a Bayesian shrinkage method described by Ledoit and Wolfe (2004). The method re-centers outliers in order to minimize estimation errors, thus reducing the perturbation of the optimizer caused by extreme values. Other parameters such as the expected returns are also estimated using different techniques according to the smart beta portfolio.

The first optimization-based strategy is the minimum-variance weighting strategy. This strategy only requires the covariance matrix to estimate the weight vector  $w$ . The

covariance matrix is estimated by using the previous 60 monthly excess returns. The weights are the optimized solution of the following problem:

$$\min_w [w' \hat{\Sigma} w] \text{ subject to } \begin{cases} \sum_{i=1}^N w_i = 1 \\ 0 \leq w_i \leq 0.05 \end{cases}$$

The constraints ensure full investment, limit short sales and avoid excess concentration in any single stock. While the weights are optimized from the complete universe of stocks, the resulting attribution of weights does not assure a non-zero weight for all stocks. The optimizer could attribute a 5% weight to 20 stocks or spread it more evenly across the full universe of stocks.

The second optimization-based strategy is the maximum Sharpe ratio weighting strategy.<sup>1</sup> This strategy requires the estimation of a covariance matrix and the expected returns. Following the intuition of Choueifaty and Coignard (2008), the estimated expected returns used in the optimization process is the vector of estimated return volatilities. The intuition behind this choice is the simple linear relationship between the expected premium and the return volatility of each stock given by:

$$E(R_i) - R_f = \gamma \cdot \sigma_i \quad \forall i$$

where  $\gamma > 0$ . The maximum Sharpe ratio objective function to optimize is:

$$\max_w \left[ \frac{w' \hat{\sigma}}{\sqrt{w' \hat{\Sigma} w}} \right] \text{ subject to } \begin{cases} \sum_{i=1}^N w_i = 1 \\ 0 \leq w_i \leq 0.1 \end{cases}$$

The rationale for the constraints is the same as for the minimum-variance strategy; namely, to ensure full investment, limit shorts sales and avoid excess concentration of investment in any specific stock.

The final optimization-based index is for the risk efficient strategy. This strategy also lies between the maximum Sharpe ratio and the equal weight strategy. The main assumption behind this strategy is that the expected returns of a stock have a linear

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<sup>1</sup> Note that Choueifaty and Coignard (2008) refer to this strategy as maximum diversification, but establish that it is equivalent to a maximum Sharpe ratio optimization.

relationship with its downside semi-volatility (or downside deviation). Therefore, this stock's risk premium should be directly related with that risk. The downside semi-volatility is defined as:

$$\delta_i = \sqrt{E \left[ \min(R_{i,t}, 0)^2 \right]}$$

where  $R_{i,t}$  is the return for stock  $i$  in period  $t$ . The maximum Sharpe ratio objective function to optimize then becomes:

$$\max_w \left[ \frac{w' \hat{\delta}}{\sqrt{w' \hat{\Sigma} w}} \right] \text{ subject to } \begin{cases} \sum_{i=1}^N w_i = 1 \\ \frac{1}{\lambda N} \leq w_i \leq \frac{\lambda}{N} \end{cases}$$

where  $N$  is the number of stocks in the universe and  $\lambda$  is a constant equal to or greater than 1. Note that the strict weight constraints are added in order to provide a tilt towards equal weighting, creating bounds for the weight of each stock. The parameter  $\lambda$  is set to 2 following Amenc et al. (2010).

These indices are benchmarked to the market-capitalization weighted index as proxied by the S&P 500. The constituents of the S&P 500 are generally the top 500 stocks by market capitalization.

### 3.2 MARKOV REGIME-SWITCHING MODEL

The distributions of the probabilities generated in Markov regime-switching models are dependent on the regimes of the underlying Markov process. Nystrup (2014) provides references for the mathematical framework associated with Markov-switching processes in discrete and continuous time.

In discrete time, the initial probability of being in regime  $i$  is given by the equation:

$$\Pr(X_i = i) = p_i$$

where  $X_i$  is the  $i$ -th state of the Markov regime-switching model. The transition probability matrix  $\Gamma$  represents the probability of a transition from state  $i$  to state  $j$ :

$$\Gamma = \begin{pmatrix} \gamma_{ii} & \gamma_{ij} \\ \gamma_{ji} & \gamma_{jj} \end{pmatrix}$$

where the transition probabilities are:

$$\gamma_{ij} = \Pr(X_t = j | X_{t-1} = i)$$

and  $t$  is the time variable.

The Markov regime-switching methodology allows for the estimation of the unobserved state processes. Each regime of the Markov chain generates observations from a given distribution. For example, a two-state, or two-regime, model that uses the Gaussian distribution is specified as:

$$X_t = \mu_{S_t} + \epsilon_{S_t} \text{ where } \epsilon_{S_t} \sim N(0, \sigma_{S_t}^2)$$

where  $\mu$  is the mean,  $S_t$  is the state (or regime) at time  $t$ , and  $\epsilon$  is the error term with mean 0 and a variance  $\sigma_{S_t}^2$  that is conditional on the regime  $S_t$ .

In this thesis, the number of regimes is limited to two for a matter of feasibility and efficiency. Therefore, the parameters of the underlying process for the two states specified by our model will be:

$$\mu_{S_t} = \begin{cases} \mu_1 & \text{if } S_t = 1 \\ \mu_2 & \text{if } S_t = 2 \end{cases}, \quad \sigma_{S_t}^2 = \begin{cases} \sigma_1^2 & \text{if } S_t = 1 \\ \sigma_2^2 & \text{if } S_t = 2 \end{cases}, \quad \Gamma = \begin{pmatrix} 1 - \gamma_{12} & \gamma_{12} \\ \gamma_{21} & 1 - \gamma_{21} \end{pmatrix}$$

The parameters of such a Markov regime-switching model are estimated through a maximum likelihood estimation method (MLE). The transition probabilities, also called filtered probabilities, resulting from this model estimation are then smoothed using the algorithm of Kim (1994)<sup>2</sup>. To delineate the regime at each point in time  $t$ , if the smoothed probability of being in an event regime is over 0.5, the regime is assumed to be an event. If the probability is below 0.5, the regime is assumed to be normal.

### 3.3 REGIME VARIABLES

The regimes are estimated according to the regime variables. The performance of each regime variable will be analyzed. First, the regimes are estimated based on a financial market turbulence index using the returns of the U.S. equities market, such a

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<sup>2</sup> The smoothed probabilities are a standard output for Markov switching models of software such as R and Matlab.

methodology is used in Kritzman et al. (2012). They measure financial turbulence using a squared Mahalanobis distance, which is a multivariate distance measure given by:

$$d_t = (y_t - \mu)\Sigma^{-1}(y_t - \mu)'$$

where  $y_t$  is the matrix of asset returns for period  $t$ ,  $\mu$  is the sample average vector of historical returns and  $\Sigma$  is the sample covariance matrix of historical returns over the full sample. The monthly time series are created from the mean daily turbulence scores within each calendar month from January 1<sup>st</sup>, 1960 to December 31<sup>st</sup>, 2014.

Second, as in Gkatzilakis and Sivasubramanian (2014), the regime is estimated based on a stock market volatility index. Since the VIX has a rather short data history, the market volatility index constructed is the past one month realized volatility of the S&P 500, computed as the standard deviation of the past month daily returns.

As in Kritzman et al. (2012), other variables are also tested such as economic growth (measured by the quarter on quarter real GNP growth) and inflation (measured by the percent change in CPI) because they are considered contrarian indicators. The excess equity returns, which is the S&P 500 total return minus the T-Bills yield, are also investigated as a regime variable following Ang & Timmermann (2011). The year over year change in the economic policy uncertainty index also is used as a regime variable.

### **3.4 DYNAMIC ASSET ALLOCATION STRATEGY**

Now that we have different portfolios to invest in, and we know that the financial markets are separated into normal regimes and event regimes, the question is: how can an investor profit from this situation? One would want to have the best performing portfolio in each type of regime. Other authors like Boudt et al. (2015) create similar strategies that switch investment across asset classes – for example from smart beta portfolio in normal regimes to cash in event regimes. However, this thesis investigates the viability of a different strategy where investors switch between smart beta portfolios – from an aggressive smart beta portfolio in normal regimes to a defensive smart beta portfolio in event regimes.

In order to do that, the performance of each smart beta methodology specified earlier is broken down and analyzed for each regime. The dynamic asset allocation strategy is explained as follows: in normal regimes, the investor invests his money in the best performing portfolio over the normal regimes, and in event regimes, the investors puts his money in the best performing portfolio over those event regimes. Based on the findings of Asness, Moskowitz and Pedersen (2013), a value-oriented portfolio and a momentum-oriented portfolio can potentially be two opposing strategies. Other articles such as de Boer & Norman (2014) praise the use of low volatility equity strategies in times of high volatility. Based on this insight, the minimum variance portfolio is expected to increase the performance of the strategy when used during event regimes.<sup>3</sup> Following that intuition, the performance of smart beta portfolios may be enhanced by investing in a defensive smart beta portfolio during event regimes. The performances of those mixed portfolios are examined using the regimes estimated using the full sample (in-sample) and the regimes estimated using expanding window regression forecasts (out of sample)<sup>4</sup>.

## **4. DATA**

### **4.1 SECURITIES PRICES AND FUNDAMENTALS DATA**

One of the most important steps in setting up the smart beta indices is to choose the investment opportunity set (IOS), which encompasses all the securities that could be included in the portfolio. In this thesis, the IOS is the largest 500 stocks in the U.S. market based on their market capitalizations and its associated benchmark is the S&P 500 index. All securities prices and returns are from CRSP and annual fundamentals are from Compustat. For each index portfolio that includes a market capitalization screening variable, the market capitalization values are computed using the main share class and are not adjusted for float. While the universe and stock selection criteria do not exactly match Standard & Poor's rules, this discrepancy is not expected to systematically bias the results reported in this paper. All the smart beta portfolios created are for the period from January 1970 to December 2014. The 10 S&P 500 sector indices used as regime

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<sup>3</sup> We caution the reader that basing the choices on reported empirical findings may introduce some data snooping bias into our findings.

<sup>4</sup> More details on this matter in Section 5.5

generating variables are extracted from the Fama and French data library because it has the longest history. For our purpose, the data spans from January 1960 to December 2014.

## **4.2 ECONOMIC DATA**

Economic data such as real GNP and the CPI are gathered from the FRED St. Louis database. The 1-month Treasury bill rate is also extracted from the Fama and French data library as the risk-free rate series. The real GNP data spans from the first quarter of 1948 to the fourth quarter of 2014. The 1-month Treasury bill rate data spans from January 1970 to December 2014. The U.S. economic policy uncertainty index<sup>5</sup> was taken from the website: [www.policyuncertainty.com](http://www.policyuncertainty.com). It spans the period from 1900 to 2016. However, only a sample from 1950 is used in this thesis.

## **5. EMPIRICAL RESULTS**

### **5.1 PERFORMANCE CHARACTERISTICS AND SUMMARY STATISTICS OF SMART BETA PORTFOLIOS**

The time series characteristics of the monthly returns of each smart beta portfolio over the period from January 1970 to December 2014 are presented in **Table 2**. These results are similar to those obtained by the creators of each strategy, or with results reported in other published articles. The total returns stated are arithmetic and monthly. The equally weighted, and diversity weighted portfolios show very similar characteristics to the S&P 500. The highest return is the maximum Sharpe ratio portfolio, while the highest risk-adjusted return is for the minimum variance portfolio, which also has the lowest volatility of all portfolios. As expected, optimization-based portfolios have much higher turnover than heuristics-based portfolios since the optimizers used are usually very sensitive to the small changes of the returns and covariances estimates used in the process of creating them. Chow, Hsu and Kalesnik (2011) state that optimization-based portfolios often have higher tracking errors and lower volatilities, while heuristic-based strategies have lower tracking errors and higher volatilities, which is generally the case in our results. In addition, while the risk and return characteristics of the portfolios remain

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<sup>5</sup> Details on the construction of the index are presented on this website.

comparable through different rebalancing frequencies, the turnover rates of the monthly rebalanced portfolios are much higher than those of the annually rebalanced portfolios. Aside from the equally weighted portfolio, the risk characteristics of each portfolio show significantly lower downside risk, time under water and Cornish-Fisher 5% VaR, which are desirable traits for alternatively weighted portfolios.

**Figure 1** has line charts of the performance of each smart beta portfolio over different samples. In chart A, we can see that the heuristics-based portfolios, such as the equal weight, diversity weight and capitalization weight portfolios, tend to underperform compared to fundamental indexing portfolios and the optimization-based portfolios, namely the maximum Sharpe ratio, risk efficient and minimum-variance portfolios. However, an examination of different sub-periods shows that the minimum-variance portfolio performed very well from 1970 to 1990 but underperformed from 1990 to 2014. Consistent performers are the fundamental indexing portfolio, which showed resilience through the dot com bubble, and the maximum Sharpe ratio portfolio.

A breakdown of the portfolios' performance over each decade is presented in **Table 3**. Panel A and Panel B show very similar results as the frequency of rebalancing does not appear to affect gross returns. Examining portfolio performance through time, the periods from 1980 to 1999 have the best risk-adjusted performances, while the portfolios underperformed in the 1970s and the 2000s. While the minimum variance portfolio consistently has the lowest volatility over each decade, it has its best performance in the 1980s. The fundamental weighting index portfolio has the highest period of outperformance during the 1990s, a period where some smart beta portfolios underperformed the S&P 500. The 1990s are also the only subperiod where the benchmark outperformed many of the smart beta portfolios.

A more complete decomposition of performance is presented in **Table 4**. Each portfolio has positive alpha although only the minimum variance portfolio, fundamental weighting portfolio and the risk efficient portfolio are statistically significant at the 10% level; and only the fundamental weighted portfolio has a significant alpha at the 5% level. Heuristic-based portfolios show higher market beta than optimization-based portfolios since the optimization process often assigns a lower weight to higher beta stocks. The



five-factor risk decomposition shows that the higher returns of smart beta portfolios are attributed to a higher exposure to smaller caps and value stocks in most portfolios.

Up to this point, the presented returns are gross returns. In **Table 5**, the return and risk characteristics of each portfolios are presented, net of transaction costs created by the rebalancing of the portfolios. Anand et al. (2013) show empirical evidences that transaction costs vary according to the state of the market. They report that transaction costs vary from 17 basis points in a normal state of the market and can go up to 35 basis points on average during a turbulent state of the market. Therefore, in order to include this variation, the transaction costs are assumed to be 17 basis points per each 100% one-way turnover in normal regimes and 35 basis points per each 100% one-way turnover in event regimes. Thus, a higher rebalancing frequency is associated with higher transaction costs. For the monthly rebalanced portfolios shown in Panel A, the rebalancing costs reduce the total returns by, on average, 3 basis points monthly. For the annually rebalanced portfolios shown in Panel B, the rebalancing costs reduce the total returns by, on average, 2 basis points monthly. The rebalancing and transaction costs only slightly affect the risk characteristics of the portfolios. In addition, the transaction costs do not affect the main conclusions about the smart beta portfolios' performances relative to the benchmark. While each strategy has a different return and risk profile, they all beat the benchmark total return over the full time period. Since the transaction costs are spread out across the full sample, the performances during every subperiod of the smart beta portfolios relative to the benchmark are similar to their performances ex-transaction costs.

Before analyzing regimes through the Markov regime-switching methodology, a simple breakdown of the performances of the smart beta portfolios is done according to economic regimes such as the economic business cycle specified by the National Bureau of Economic Research (NBER). **Table 6** presents the returns characteristics of the annually rebalanced portfolios based on the NBER economic regime. As expected, all portfolios have higher returns during expansions than during recessions. The best performing portfolio is the risk efficient portfolio and the minimum variance portfolio,

respectively, during expansions and recessions. The presentation of this breakdown is done as a matter of comparison.

## 5.2 ESTIMATION OF MARKOV REGIMES

**Table 7** presents the in-sample regimes estimated by the Markov regime-switching models following the methodology presented in sections 3.2 and 3.3. For each regime variable time series, two regimes are estimated (a normal regime and an event regime). The normal regime can be seen as a calm, low-volatility regime while the event regime occurs in times of turbulence or uncertainty, characterized by a higher mean and higher volatility. By example, for the market turbulence index, the normal regime shows a low  $\mu$  ( $\mu$ ) and a low  $\sigma$  ( $\sigma^2$ ) meaning that the markets are in a low-volatility regime. On the other side, the event regime has a high market turbulence index mean and  $\sigma$ . Notice that as a rule of thumb, the normal regime has higher persistence and longer expected durations than event regimes. Similarly, higher volatility regimes tend to have lower persistence and lower expected duration. The results reveal the presence of two regimes for each variable. Those regimes are also statistically significant. For the market turbulence, market volatility and the inflation variables, all estimated parameters are shown to be statistically significant. The equity risk premium series is consistent with the low-volatility regimes as normal regimes and the high-volatility regimes as event regimes. The economic policy uncertainty falls on average during normal regimes and rises during event regimes. Note that in-sample regimes signify that the regimes are estimated using the full sample available for the regime variables. On the other hand, the out-of-sample regime estimations are done using an expanding window regression and using the  $t+1$  forecast to create a time series of probabilities. The resulting time series is assumed to be a reliable representation of how the model would behave at each point in time  $t$ .

In **Figure 2**, the historical probabilities of each regime variable are illustrated. Those probabilities are the smoothed probabilities used to create the delineation of the regimes. The smoothings are obtained from the algorithm of Kim (1994). There is some resemblance across different regime variables although each has its own particularities. Notice that each series identifies the 2009 crisis as a high-volatility event regime, while

the identification of the dot com bust as a high-volatility event is not consistent across the series.

### **5.3 SMART BETA PORTFOLIO PERFORMANCE IN EACH REGIME**

After regimes are estimated, the next step is to examine how smart beta portfolios perform in each estimated regime. **Table 8** presents the performance of each smart beta portfolio in normal and event regimes. As a general rule, all portfolios performed better in the normal regime with lower volatility compared to lower returns with higher volatility during the event regimes. The market turbulence index regimes specified by Kritzman, Page & Turkington (2012) perform better for bull and bear markets than the market volatility methodology of Gkatzilakis & Sivasubramanian (2014) on an in-sample basis. The inflation, economic growth and equity risk premium regimes show the biggest difference between normal and event regimes. The minimum variance portfolio consistently shows the smallest variance in normal regimes and in event regimes. A *t*-test for the difference of means and a *F*-test for the difference of variances are conducted for the differences between normal and event regimes. While the differences of means are not significant for all regime variables used, the differences of variances are significant across most portfolios and regime variables.

### **5.4 DYNAMIC ASSET ALLOCATION STRATEGIES USING IN-SAMPLE REGIMES**

These results provide the basis for an examination of the performance of a tactical asset allocation strategy. Dynamic smart beta portfolios are created by using the better performing smart beta portfolio in each regime. While countless combinations of portfolios are possible for each regime-switching model, only the best performing combinations are presented in **Table 9**. The minimum-variance portfolio is a natural fit for high-volatility regimes as it has the lowest volatility, but is not always the best performing in those situations.

For the market turbulence index regimes, the normal event strategy is the maximum Sharpe ratio portfolio while the high-volatility regime portfolio is the fundamentally indexed portfolio. For the market volatility regimes, the combination is the same as the market turbulence index regimes. For the inflation regimes and the economic growth regimes,

the normal event strategy is the fundamentally indexed portfolio while the high-volatility regime portfolio is the minimum-variance portfolio. For the equity risk premium regimes, the normal event strategy is the risk efficient portfolio while the high-volatility regime portfolio is the minimum-variance portfolio. For the economic policy uncertainty regimes, the normal event strategy is the fundamentally indexed portfolio while the high-volatility regime portfolio is also the minimum-variance portfolio. Each dynamic strategy with the exception of the market volatility dynamic portfolio increases the Sharpe ratio compared to static smart beta portfolios. While the Sharpe ratio figures are comparable to the Sharpe ratio figure for the minimum-variance portfolio, the returns are higher with lower or comparable volatility, therefore showing better overall performance than the static portfolios.

**Table 9** also presents the risk values for the dynamic smart beta portfolios. All portfolios show a reduction of risk compared to the S&P 500 benchmark. The combinations allow us to control the risk, while attaining higher returns. The best performing portfolios are the ones conditional on inflation, economic growth, and equity risk premium regimes. They have the biggest reduction in risk based on various measures, while maintaining high total returns. They also show low tracking errors and high information ratios. In **Table 10**, the performances of the dynamic portfolios are assessed using a conditional performance evaluation model. The model is the conditional form of the Carhart (1997) model expressed as:

$$r_{i,t} = \alpha_{i0} + \alpha'_i x_{t-1} + b_{i01} r_{M,t} + b_{i02} SMB_t + b_{i03} HML_t + b_{i04} UMD_t + b'_{i1} (x_{t-1} r_{M,t}) + b'_{i2} (x_{t-1} SMB_t) + b'_{i3} (x_{t-1} HML_t) + b'_{i4} (x_{t-1} UMD_t) + u_{i,t}$$

where  $r_{M,t}$  is the excess return on the S&P 500;  $SMB_t$ ,  $HML_t$  and  $UMD_t$  are the factors specified in the Carhart model; and  $x_{t-1}$  is the conditioning variable. The conditioning variable is the demeaned dividend yield of the S&P 500 firms. The conditional risk-adjusted performance is the intercept ( $\alpha_{i0}$ ).  $u_{i,t}$  is the error term. Most dynamic portfolio created show a significant risk-adjusted alpha of 1 or 2 basis points per month, which means that they show superior performance. However, this may not cover costs other than trade costs in implementing these strategies.

## 5.5 SMART BETA PORTFOLIO PERFORMANCE IN OUT-OF-SAMPLE REGIMES

No in-sample regression analysis should be presented without its out-of-sample counterpart, especially for a tactical asset allocation strategy. Consistent results from in-sample to out-of-sample will be an effective robustness check for the viability of the strategy.

The first step of the out-of-sample robustness examination is to estimate the Markov regimes using an expanding window rolling regression methodology. For each regime variable specified in the methodology, the regression begins with a sample of 20 years' worth of observations. A forecast for the  $t+1$  probabilities is retained. The process is done iteratively adding one observation at a time (one month at a time) until it reaches the full sample of data available in order to complete the full set of out-of-sample probabilities. That being said, the estimation of the regimes becomes closer to the in-sample estimation as the sample size expands, especially for the market turbulence and market volatility indices. In general, the out-of-sample regime estimation is shown to be more volatile (or noisy) than the in-sample regimes. The inflation regimes, economic growth regimes, and the equity risk premium regimes perform particularly well out-of-sample. The regime probability graphs are shown in **Figure 4**. The prediction of the normal or event regime affects the choice of smart beta portfolio.

The second step is to analyze how each smart beta portfolio performs in the out-of-sample estimated regimes. The performances of the smart beta portfolios are presented in **Table 11**. While the out-of-sample regime consistently show higher returns and lower risk in normal regimes and lower returns and higher risk in event regimes, the in-sample performance is better illustrated by examining the disparity between the performances in normal and event regimes. For the out-of-sample regimes, the gap is smaller. The best performing regime indicators are the economic growth and equity risk premium variables based on the differences in means and differences in variances.

## 5.6 DYNAMIC ASSET ALLOCATION STRATEGIES USING EACH OUT-OF-SAMPLE REGIME

The third step is to test the performance of the dynamic smart beta portfolios created in section 5.4. The portfolio performance is not exactly the same as it was with the in-

sample regimes because of the disparity created by the estimation of the out-of-sample regimes. A comparison of the performance of the dynamic smart beta portfolios is done in order to measure the robustness of the tactical asset allocation strategies presented earlier. The returns and risk characteristics of each out-of-sample dynamic smart beta portfolios are presented in **Table 12**. The return and risk characteristics are presented for the gross returns in Panel A and for the returns net of transaction costs in Panel B. In Panel A, data shows that the out-of-sample returns and risk results of each portfolio remain consistent with the in-sample results. The risk measures are also reduced in the same way they did using the in-sample data. However, when we include the transaction costs, which are significant in such a dynamic strategy, some of the outperformance gets eroded. Nonetheless, the strategy using the inflation, economic growth and most significantly the equity risk premium regime variables still outperform the benchmark and the other smart beta portfolios<sup>6</sup>. Most importantly, the dynamic portfolios retain their risk reducing properties, which makes them desirable for the purpose of portfolio risk management. The resulting portfolios have relatively high returns, while maintaining lower volatilities than the benchmark. In **Table 13**, the performance of the dynamic portfolios is assessed using a conditional form of the Carhart (1997) model for performance evaluation. While the gross returns of the dynamic portfolios have significant risk-adjusted alpha, only the inflation, economic growth and equity risk premium portfolio show a significant and positive alpha of one or two basis points monthly.

In summary, the performance results for the tactical asset allocation strategies suggest that the previous in-sample results become less robust when considered out-of-sample, especially after incorporating the effects of trade costs. Thus, the performance of any smart-beta strategy is subject to an investor's execution prowess and the minimization of any other costs associated with the implementation of that strategy (e.g., taxes for a taxable investor, non-systematic risk exposure and management fees).

## 6. CONCLUSION

Markov regime-switching is able to dynamically estimate regimes with some accuracy. We find that Markov regime-switching models, even in their most basic forms,

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<sup>6</sup> Single strategy, non-dynamic smart beta portfolios.

can be useful to classify historical economic data into regimes that are consistent with financial or economic intuition. Few studies use Markov regimes to create strategies for a single asset class. This thesis provides some supporting evidence that using Markov regimes in a single asset class context can improve performance compared to a static benchmark. Using such a dynamic strategy can maintain returns while lowering the risk of the portfolio.

It is clear that not all regime variables create a breakdown of regimes that is suitable for portfolio management. Volatile variables tend to create higher turnover in the dynamic portfolios, thus inducing higher transaction costs, which erode the performance of a dynamic asset allocation strategy. Therefore, the choice of proper regime variables is a key element of how well strategies using Markov regime-switching models perform. According to the results, cyclical variables like inflation, economic growth and the equity risk premium work better for this purpose.

Dynamic strategies that results in higher turnover usually incur higher taxes unless the investor is not taxable, such as a pension fund. While this thesis does not directly analyze the impact of such additional costs, the possible impact of those costs will further erode the performance of any smart beta portfolio that has a higher turnover rate.

Future research may test additional variables for the estimation of the regimes and alternative smart beta portfolio strategies. Furthermore, with smart beta strategies expanding to other asset classes, it would be interesting to examine if the findings and methodology of this thesis would produce similar results for other asset classes.

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## 8. GLOSSARY

**Cornish-Fisher 5% Value at Risk:** 5% Value at risk metric that also includes the skewness and kurtosis of the returns.

**Diversity:** Extent to which the capital is spread amongst a number of stocks.

**Downside Deviation:** Measure of downside risk that focuses on returns that fall below a minimum threshold or minimum acceptable return. In this case, the minimum threshold is zero.

**Dynamic Portfolios:** Portfolios that use the dynamic asset allocation strategy based on the normal or event regimes.

**Event Regime:** High-volatility regime. Opposite of a normal regime.

**Expected Duration:** Expected duration of being in a given regime in a number of periods.

**Information Ratio:** Measure of the risk-adjusted return of a financial security. It is computed as expected active return divided by tracking error, where active return is the difference between the return of the security and the return of a selected benchmark index.

**Maximum Time under Water:** Maximum time the asset or the trading strategy is under its high watermark

**Mu ( $\mu$ ):** Mean of the regime variable in the given regime.

**Normal Regime:** Low-volatility regime. Opposite of an event regime.

**One-Way Turnover:** Turnover measures the percentage change in the composition of an index at each index rebalancing. One-way would include the act of buying or selling, while roundtrip would include both buying and selling.

**Regime Variable:** Variable used in the process of estimating the Markov regime-switching model.

**Sharpe Ratio:** The ratio measures the excess return per unit of standard deviation in an investment asset or a trading strategy.

**Sigma ( $\sigma^2$ ):** Variance of the regime variable in the given regime.

**Smoothed Probabilities:** Probabilities of being in an event regime. The smoothing is done according to the algorithm of Kim (1994).

**Transition Probabilities:** Unsmoothed probabilities that are the direct output of the Markov regime-switching model.

**Tracking Error:** Measure of how closely a portfolio follows the index to which it is benchmarked. It is computed as the standard deviation of the difference between the portfolio and the index returns.

**x-%Value at Risk:** Statistical technique used to measure the level of financial risk over a specific time frame. It is an estimate of the x-% worst case scenario in normal market conditions.

## 9. APPENDIX

**Table 1 Sensitivity Testing for the Diversity Weighted Portfolio Parameter  $P$**

This table presents the return characteristics of the diversity weighting portfolio under different values for the parameter  $p$  where the portfolios are rebalanced monthly in Panel A and annually in Panel B. Total returns are the arithmetic mean of the monthly returns. Volatility is the standard deviation of the monthly returns. Tracking error is the standard deviation of the active returns, which are the portfolio returns minus the benchmark returns (S&P 500). Information ratio is the expected active return divided by the tracking error. C-F 5% VaR is the Cornish-Fisher 5% Value at Risk. The risk-free rate is the 1 month T-Bill rate provided in the Fama-French library. The time period examined is from January 1970 to December 2014. All rebalancing is at the beginning of each month in Panel A and the beginning of the year in Panel B.

	Total Return	Volatility	Sharpe Ratio	Return/Risk	Tracking Error	Information Ratio	One-way Turnover	Downside Deviation	Maximum Time Under Water	C-F 5% VaR (Monthly)
<b>Panel A: Portfolios Rebalanced Monthly</b>										
S&P 500	0.0092	0.0458	0.11	0.20	-	-	-	0.0299	66	0.0686
Capitalization weighted ( $p = 1$ )	0.0090	0.0448	0.11	0.20	0.0076	-0.0306	40.16%	0.0287	75	0.0641
Diversity weighted ( $p= 0.76$ )	0.0093	0.0459	0.11	0.20	0.0066	0.0059	41.10%	0.0293	67	0.0654
Diversity weighted ( $p= 0.50$ )	0.0095	0.0470	0.12	0.20	0.0070	0.0418	42.16%	0.0300	62	0.0668
Diversity weighted ( $p= 0.25$ )	0.0097	0.0480	0.12	0.20	0.0082	0.0605	43.41%	0.0306	51	0.0681
Equally weighted ( $p = 0$ )	0.0099	0.0488	0.12	0.20	0.0094	0.0698	45.72%	0.0312	50	0.0692
<b>Panel B: Portfolios Rebalanced Annually</b>										
S&P 500	0.0092	0.0458	0.11	0.20	-	-	-	0.0299	66	0.0686
Capitalization weighted ( $p = 1$ )	0.0091	0.0439	0.11	0.21	0.0065	-0.0294	5.57%	0.0281	75	0.0652
Diversity weighted ( $p= 0.76$ )	0.0093	0.0447	0.12	0.21	0.0049	0.0144	7.66%	0.0287	72	0.0667
Diversity weighted ( $p= 0.50$ )	0.0096	0.0457	0.12	0.21	0.0049	0.0655	11.96%	0.0292	62	0.0683
Diversity weighted ( $p= 0.25$ )	0.0098	0.0465	0.12	0.21	0.0059	0.0879	15.55%	0.0298	57	0.0698
Equally weighted ( $p = 0$ )	0.0099	0.0473	0.12	0.21	0.0071	0.0966	19.23%	0.0302	50	0.0710

**Table 2. Return-Risk Characteristics of Portfolios Rebalanced Monthly and Annually**

This table presents the return characteristics of each portfolio strategy where the portfolios are rebalanced monthly in Panel A and annually in Panel B. Total returns are the arithmetic mean of the monthly returns. Volatility is the standard deviation of the monthly returns. Tracking error is the standard deviation of the active returns, which are the portfolio returns minus the benchmark returns (S&P 500). Information ratio is the expected active return divided by the tracking error. C-F 5% VaR is the Cornish-Fisher 5% Value at Risk. The risk-free rate is the 1 month T-Bill rate provided in the Fama-French library. The time period examined is from January 1970 to December 2014. All rebalancing is at the beginning of each month in Panel A and the beginning of the year in Panel B.

	Total Return	Volatility	Sharpe Ratio	Return/Risk	Tracking Error	Information Ratio	One-way Turnover	Downside Deviation	Maximum Time Under Water	C-F 5% VaR (Monthly)
<b>Panel A: Portfolios Rebalanced Monthly</b>										
<b>S&amp;P 500</b>	0.0092	0.0458	0.11	0.20	-	-	-	0.0299	66	0.0686
<b>Capitalization Weighted</b>	0.0090	0.0448	0.11	0.20	0.76%	-0.03	40.16%	0.0287	75	0.0652
<b>Equally Weighted</b>	0.0099	0.0488	0.12	0.20	0.94%	0.07	45.72%	0.0312	50	0.0710
<b>Diversity Weighted</b>	0.0093	0.0459	0.11	0.20	0.66%	0.01	41.10%	0.0293	67	0.0667
<b>Fundamental Weighting</b>	0.0110	0.0436	0.16	0.25	1.71%	0.10	73.05%	0.0263	41	0.0581
<b>Inverse Volatility</b>	0.0104	0.0433	0.14	0.24	1.25%	0.09	41.95%	0.0269	44	0.0612
<b>Minimum Variance</b>	0.0099	0.0357	0.16	0.28	2.35%	0.03	65.32%	0.0217	44	0.0487
<b>Maximum Sharpe Ratio</b>	0.0108	0.0451	0.15	0.24	2.52%	0.06	90.38%	0.0285	65	0.0649
<b>Risk Efficient (<math>\lambda = 2</math>)</b>	0.0110	0.0464	0.15	0.24	1.70%	0.11	66.98%	0.0293	48	0.0670
<b>Panel B: Portfolios Rebalanced Annually</b>										
<b>S&amp;P 500</b>	0.0092	0.0458	0.11	0.20	-	-	-	0.0299	66	0.0686
<b>Capitalization Weighted</b>	0.0091	0.0439	0.11	0.21	0.65%	-0.03	5.57%	0.0281	75	0.0641
<b>Equally Weighted</b>	0.0099	0.0473	0.12	0.21	0.71%	0.10	19.23%	0.0302	50	0.0692
<b>Diversity Weighted</b>	0.0093	0.0447	0.12	0.21	0.49%	0.01	7.66%	0.0287	72	0.0654
<b>Fundamental Weighting</b>	0.0109	0.0428	0.16	0.26	1.68%	0.10	14.84%	0.0259	41	0.0573
<b>Inverse Volatility</b>	0.0104	0.0441	0.14	0.24	1.34%	0.09	17.30%	0.0273	44	0.0619
<b>Minimum Variance</b>	0.0098	0.0355	0.16	0.28	2.33%	0.02	42.24%	0.0218	45	0.0490
<b>Maximum Sharpe Ratio</b>	0.0111	0.0447	0.16	0.25	2.53%	0.07	60.20%	0.0283	65	0.0647
<b>Risk Efficient (<math>\lambda = 2</math>)</b>	0.0111	0.0456	0.15	0.24	1.66%	0.11	38.12%	0.0290	48	0.0665

**Figure 1. Comparison of the performance of smart beta portfolios rebalanced annually**

Chart A illustrates the performance of 1 dollar invested in January 1970 for each portfolio constructed, the S&P 500, and the risk free rate up to December 2015. Chart B illustrates the performance of 1 dollar invested in January 1970 for each portfolio constructed, the S&P 500, and the risk free rate up to December 1989. Chart C illustrates the performance of 1 dollar invested in January 1990 for each portfolio constructed, the S&P 500, and the risk free rate up to December 2014.

Chart A: Full sample

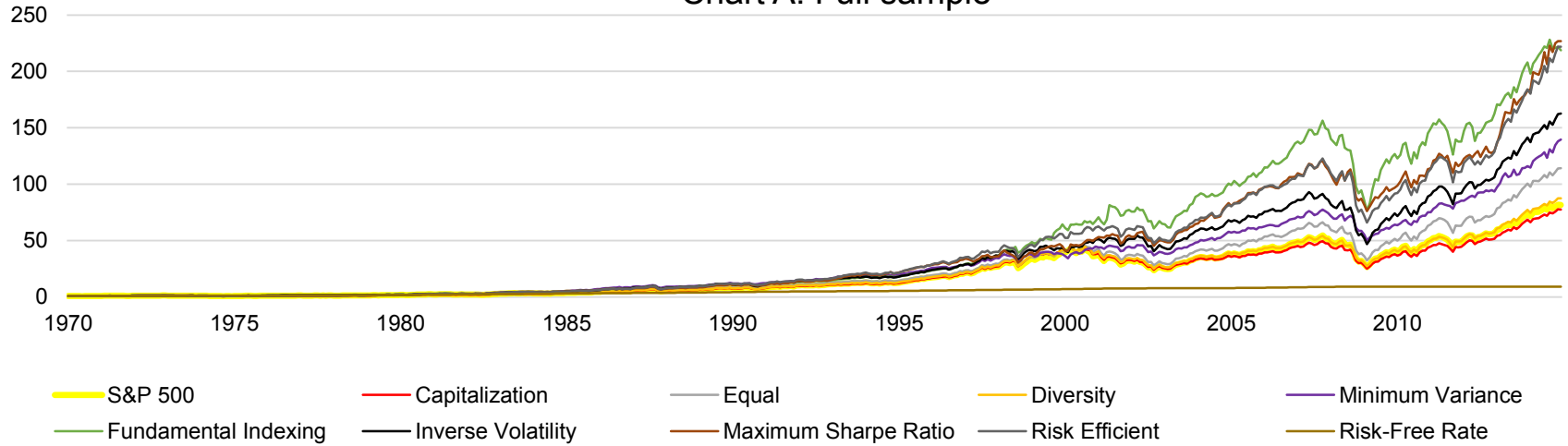


Chart B: 1970-1989

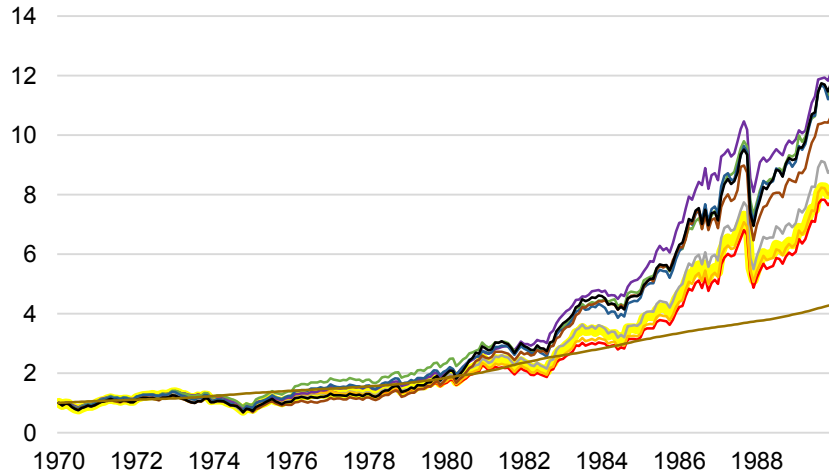
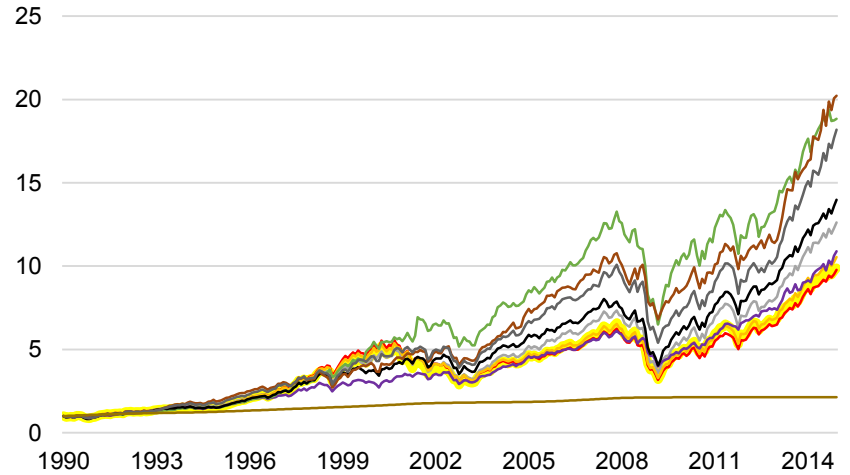


Chart C: 1990-2014



**Table 3. Return-Risk Characteristics of Portfolios Rebalanced Monthly and Annually by Decades**

This table presents the return characteristics of each portfolio strategy over each decade where the portfolios are rebalanced monthly in Panel A and annually in Panel B. Total returns are the arithmetic mean of the monthly returns. Volatility is the standard deviation of the monthly returns. The time period examined is from January 1970 to December 2014. All rebalancing is at the beginning of each month in Panel A and the beginning of the year in Panel B.

	1970-1979			1980-1989			1990-1999			2000-2009			2010-2014		
	Total Return	Volatility	Sharpe Ratio	Total Return	Volatility	Sharpe Ratio	Total Return	Volatility	Sharpe Ratio	Total Return	Volatility	Sharpe Ratio	Total Return	Volatility	Sharpe Ratio
<b>Panel A: Portfolios Rebalanced Monthly</b>															
Risk Free Rate	0.0051	0.0016	-	0.0071	0.0024	-	0.0040	0.0011	-	0.0023	0.0016	-	0.0000	0.0000	-
S&P 500	0.0062	0.0483	0.02	0.0139	0.0482	0.14	0.0142	0.0393	0.26	0.0014	0.0491	-0.02	0.0119	0.0389	0.30
Capitalization Weighted	0.0053	0.0478	0.00	0.0143	0.0459	0.16	0.0144	0.0381	0.27	0.0002	0.0490	-0.04	0.0127	0.0378	0.34
Equally Weighted	0.0065	0.0542	0.03	0.0146	0.0486	0.15	0.0129	0.0399	0.22	0.0035	0.0546	0.02	0.0141	0.0412	0.34
Diversity Weighted	0.0057	0.0497	0.01	0.0145	0.0468	0.16	0.0139	0.0383	0.26	0.0012	0.0502	-0.02	0.0131	0.0387	0.34
Fundamental Weighting	0.0081	0.0467	0.06	0.0145	0.0403	0.18	0.0150	0.0356	0.31	0.0071	0.0514	0.09	0.0097	0.0419	0.23
Inverse Volatility	0.0075	0.0502	0.05	0.0151	0.0457	0.18	0.0122	0.0366	0.22	0.0049	0.0430	0.06	0.0137	0.0356	0.39
Minimum Variance	0.0070	0.0426	0.04	0.0162	0.0352	0.26	0.0095	0.0307	0.18	0.0054	0.0364	0.09	0.0132	0.0271	0.49
Maximum Sharpe Ratio	0.0063	0.0553	0.02	0.0161	0.0427	0.21	0.0114	0.0386	0.19	0.0076	0.0442	0.12	0.0144	0.0406	0.35
Risk Efficient ( $\lambda = 2$ )	0.0072	0.0558	0.04	0.0163	0.0452	0.20	0.0127	0.0380	0.23	0.0059	0.0470	0.08	0.0149	0.0413	0.36
<b>Panel B: Portfolios Rebalanced Annually</b>															
Risk Free Rate	0.0051	0.0016	-	0.0071	0.0024	-	0.0040	0.0011	-	0.0023	0.0016	-	0.0000	0.0000	-
S&P 500	0.0062	0.0488	0.02	0.0139	0.0482	0.14	0.0142	0.0393	0.26	0.0014	0.0491	-0.02	0.0119	0.0389	0.30
Capitalization Weighted	0.0054	0.0469	0.01	0.0142	0.0464	0.15	0.0148	0.0384	0.28	0.0001	0.0454	-0.05	0.0127	0.0370	0.34
Equally Weighted	0.0066	0.0528	0.03	0.0145	0.0488	0.15	0.0137	0.0401	0.24	0.0030	0.0494	0.01	0.0141	0.0402	0.35
Diversity Weighted	0.0057	0.0486	0.01	0.0143	0.0472	0.15	0.0144	0.0386	0.27	0.0010	0.0462	-0.03	0.0131	0.0378	0.34
Fundamental Weighting	0.0081	0.0460	0.07	0.0144	0.0409	0.18	0.0149	0.0350	0.31	0.0068	0.0486	0.09	0.0100	0.0418	0.24
Inverse Volatility	0.0075	0.0510	0.05	0.0154	0.0453	0.18	0.0118	0.0367	0.21	0.0055	0.0456	0.07	0.0138	0.0364	0.38
Minimum Variance	0.0069	0.0427	0.04	0.0160	0.0362	0.24	0.0096	0.0305	0.18	0.0050	0.0346	0.08	0.0134	0.0269	0.50
Maximum Sharpe Ratio	0.0065	0.0546	0.03	0.0162	0.0432	0.21	0.0124	0.0391	0.22	0.0073	0.0424	0.12	0.0151	0.0397	0.38
Risk Efficient ( $\lambda = 2$ )	0.0073	0.0553	0.04	0.0163	0.0460	0.20	0.0135	0.0381	0.25	0.0051	0.0437	0.07	0.0156	0.0402	0.39



**Table 4. Five-Factor Decomposition of Annually Rebalanced Portfolios (Carhart + Liquidity)**

This table presents the five-factor model performance decomposition of the annually rebalanced portfolios' gross returns. The regression uses the full sample of monthly returns, which is from January 1970 to December 2014. The factors are gathered from the Fama-French library and the liquidity factor is the Pastor-Stambaugh liquidity factor taken from the Wharton Research Data Services library. The regression tests the hypothesis that the beta is statistically different from zero.

	Alpha	Alpha p-value	Market (Mkt-Rf)	Size (SMB)	Value (HML)	Momentum (UMD)	Liquidity (PS)	R <sup>2</sup>
<b>S&amp;P 500</b>	-	-	1.000	0.000	0.000	0.000	0.000	1.00
<b>Capitalization Weighted</b>	0.01%	0.232	0.977	-0.157	-0.021	-0.001	0.003	1.00
<b>Equally Weighted</b>	0.01%	0.738	1.020***	0.064***	0.074***	-0.022***	0.023***	0.98
<b>Diversity Weighted</b>	0.01%	0.356	0.992***	-0.098***	0.009**	-0.006**	0.008***	1.00
<b>Fundamental Weighting</b>	0.13%	0.036	0.920***	-0.039*	0.265***	-0.051***	0.012	0.90
<b>Inverse Volatility</b>	0.07%	0.101	0.967***	-0.027**	0.244***	-0.074***	0.028**	0.95
<b>Minimum Variance</b>	0.07%	0.339	0.750***	-0.104***	0.265***	0.014	0.034*	0.81
<b>Maximum Sharpe Ratio</b>	0.17%	0.115	0.849***	0.049	0.181***	-0.022	0.047*	0.73
<b>Risk Efficient (<math>\lambda = 2</math>)</b>	0.12%	0.087	0.949***	0.065***	0.165***	-0.019	0.039**	0.89

\*\*\* Significant at the 1% level

\*\* Significant at the 5% level

\* Significant at the 10% level

**Table 5. Return-Risk Characteristics of Portfolios Rebalanced Monthly and Annually, Net of Transaction Costs**

This table presents the return and risk characteristics of each portfolio strategy where the portfolios are rebalanced monthly in Panel A and annually in Panel B. All returns displayed in this table are net of transaction costs. Total returns are the arithmetic mean of the monthly returns. Volatility is the standard deviation of the monthly returns. Tracking error is the standard deviation of the active returns, which are the portfolio returns minus the benchmark returns (S&P 500). Information ratio is the expected active return divided by the tracking error. C-F 5% VaR is the Cornish-Fisher 5% Value at Risk. The risk-free rate is the 1 month T-Bill rate provided in the Fama-French library. The time period examined is from January 1970 to December 2014. All rebalancing is at the beginning of each month in Panel A and the beginning of the year in Panel B.

	Total Return	Volatility	Sharpe Ratio	Return/Risk	Tracking Error	Information Ratio	One-way Turnover	Downside Deviation	Maximum Time Under Water	C-F 5% VaR (Monthly)
<b>Panel A: Portfolios Rebalanced Monthly, Net of Transaction Costs</b>										
S&P 500	0.0092	0.0458	0.11	0.20	-	-	-	0.0299	66	0.0686
Capitalization Weighted	0.0089	0.0449	0.11	0.20	0.76%	-0.05	40.16%	0.0288	76	0.0655
Equally Weighted	0.0097	0.0489	0.11	0.20	0.94%	0.05	45.72%	0.0313	51	0.0713
Diversity Weighted	0.0091	0.0459	0.11	0.20	0.66%	-0.02	41.10%	0.0294	72	0.0669
Fundamental Weighting	0.0107	0.0436	0.15	0.25	1.71%	0.09	73.05%	0.0265	61	0.0586
Inverse Volatility	0.0102	0.0433	0.14	0.24	1.25%	0.08	41.95%	0.0270	44	0.0614
Minimum Variance	0.0097	0.0357	0.16	0.27	2.35%	0.02	65.32%	0.0218	45	0.0492
Maximum Sharpe Ratio	0.0105	0.0451	0.14	0.23	2.52%	0.05	90.38%	0.0287	67	0.0653
Risk Efficient ( $\lambda = 2$ )	0.0108	0.0464	0.14	0.23	1.70%	0.09	66.98%	0.0295	48	0.0674
<b>Panel B: Portfolios Rebalanced Annually, Net of Transaction Costs</b>										
S&P 500	0.0092	0.0458	0.11	0.20	-	-	-	0.0299	66	0.0686
Capitalization Weighted	0.0090	0.0439	0.11	0.21	0.65%	-0.03	5.57%	0.0282	75	0.0642
Equally Weighted	0.0099	0.0473	0.12	0.21	0.71%	0.09	19.23%	0.0303	51	0.0693
Diversity Weighted	0.0093	0.0447	0.12	0.21	0.49%	0.01	7.66%	0.0287	72	0.0654
Fundamental Weighting	0.0107	0.0428	0.15	0.25	1.68%	0.09	14.84%	0.0260	61	0.0576
Inverse Volatility	0.0104	0.0441	0.14	0.24	1.34%	0.08	17.30%	0.0273	44	0.0620
Minimum Variance	0.0097	0.0355	0.16	0.27	2.34%	0.02	42.24%	0.0219	45	0.0493
Maximum Sharpe Ratio	0.0109	0.0446	0.15	0.24	2.53%	0.06	60.20%	0.0284	65	0.0649
Risk Efficient ( $\lambda = 2$ )	0.0110	0.0456	0.15	0.24	1.66%	0.10	38.12%	0.0291	58	0.0667

**Table 6. Return Characteristics of Annually Rebalanced Portfolios Based on the Economic Regime**

This table presents the return characteristics of each portfolio broken down into expansion and recession regimes. The analysis uses the full sample of monthly returns, which is from January 1970 to December 2014. The economic regimes are taken from the National Bureau of Economic Research (NBER) library.

	<u>Expansion</u>			<u>Recession</u>		
	Total Return	Volatility	Sharpe Ratio	Total Return	Volatility	Sharpe Ratio
<b>SP500</b>	0.1376	0.1399	0.65	-0.0539	0.2314	-0.54
<b>Capitalization Weighted</b>	0.1352	0.1350	0.65	-0.0496	0.2258	-0.53
<b>Equally Weighted</b>	0.1363	0.1434	0.62	-0.0509	0.2526	-0.48
<b>Diversity Weighted</b>	0.1293	0.1368	0.60	-0.0506	0.2338	-0.52
<b>Minimum Variance</b>	0.1311	0.1074	0.78	0.0033	0.1865	-0.36
<b>Fundamental Weighting</b>	0.1451	0.1273	0.77	-0.0227	0.2281	-0.41
<b>Inverse Volatility</b>	0.1400	0.1319	0.70	-0.0252	0.2272	-0.42
<b>Maximum Sharpe Ratio</b>	0.1511	0.1374	0.76	-0.0266	0.2258	-0.43
<b>Risk Efficient (<math>\lambda = 2</math>)</b>	0.1522	0.1371	0.77	-0.0324	0.2444	-0.42

**Table 7. Markov-Switching Model: Estimation Results**

This table presents the results for the estimation of the Markov regime switching models for each of the regime variables. The persistence represents the estimated transition probability of staying in the current regime. The expected duration of staying in the regime is presented in number of periods. All expected durations are in months unless specified otherwise. The tested hypothesis is that parameters are significantly different from zero.

	Regime 1 (Normal)				Regime 2 (Event)			
	Persistence	Mu ( $\mu$ )	Sigma ( $\sigma^2$ )	Expected Duration	Persistence	Mu ( $\mu$ )	Sigma ( $\sigma^2$ )	Expected Duration
<b>Market Turbulence</b>	98.10%	6.21	0.965	51.97	90.85%	27.868	2.950	10.930
Standard Error	0.380***	0.163***	0.060***		0.346***	2.294***	0.067***	
<b>Market Volatility</b>	97.20%	2.982	0.996	33.480	89.98%	6.704	10.132	9.980
Standard Error	0.159***	0.048***	0.069***		0.122***	0.290***	1.127***	
<b>Inflation</b>	98.48%	0.206	0.000	73.760	96.37%	0.522	0.277	24.230
Standard Error	0.188***	0.010***	0.039***		0.202***	0.043***	0.033***	
<b>Economic Growth (Quarterly)</b>	94.03%	1.020	0.635	16.740	73.99%	-0.220	1.015	3.840
Standard Error	0.227***	0.0779***	0.0749***		0.354*	0.374	0.3858***	
<b>Equity Risk Premium</b>	96.84%	0.011	0.001	31.63	93.51%	-0.004	0.003	15.42
Standard Error	0.199***	0.002***	0.0001***		0.289***	0.005	0.0004***	
<b>Economic Policy Uncertainty</b>	97.10%	-0.8073	471.66	34.45	92.30%	6.1365	3926.35	12.99
Standard Error	0.192***	1.2391	41.398***		0.242***	4.7513	489.29***	

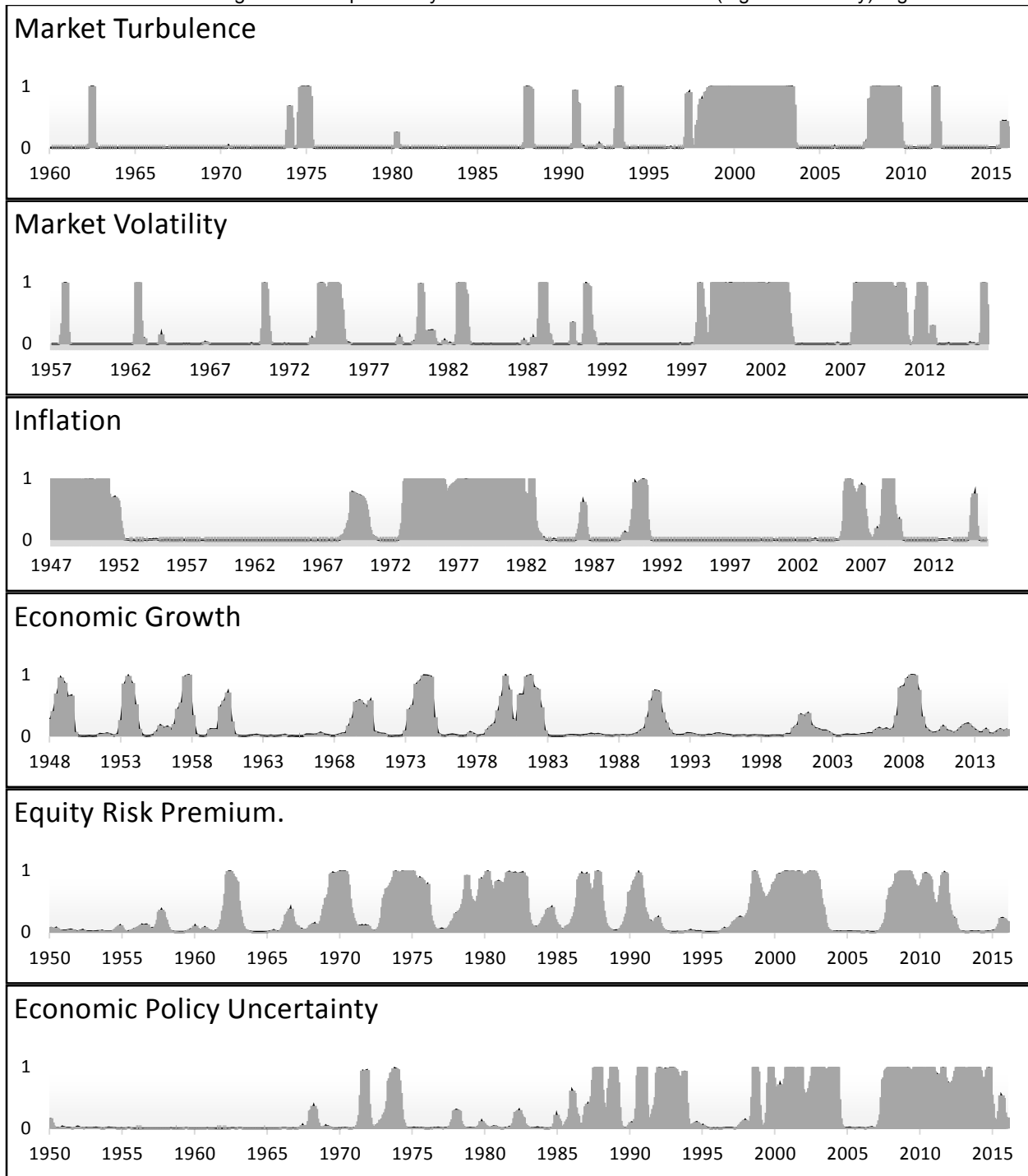
\*\*\* Significant at 1% level

\*\* Significant at 5% level

\* Significant at 10% level

### Figure 2. Historical Probabilities of the Event Regime

This figure shows the smoothed probabilities of being into an event regime for each regime variable. A probability of 1 means that we are in the event regime while a probability of 0 means we are in a normal (e.g. low-volatility) regime.



**Table 8. Performance of Smart Beta Portfolios in Each Regime**

Total return is the arithmetic mean of the in-regime monthly returns. Volatility is the standard deviation of the monthly returns. The sample is from January 1970 to December 2014 broken down using the in-sample Markov regime-switching model.

	Normal Regime			Event Regime			Difference of Means		Difference of Variances	
	Total Return	Volatility	Sharpe Ratio	Total Return	Volatility	Sharpe Ratio	t-Test Value	p-value	F-Test Value	p-value
<b>Market Turbulence Regimes</b>										
S&P 500	0.0109	0.0400	0.16	0.0030	0.0634	0.00	1.25	0.21	2.51	0.00***
Equally Weighted	0.0114	0.0420	0.17	0.0044	0.0638	0.02	1.09	0.28	2.31	0.00***
Diversity Weighted	0.0110	0.0395	0.17	0.0030	0.0608	0.00	1.32	0.19	2.37	0.00***
Fundamental Weighted	0.0114	0.0369	0.19	0.0090	0.0606	0.10	0.39	0.70	2.70	0.00***
Inverse Volatility	0.0115	0.0391	0.18	0.0063	0.0596	0.06	0.87	0.38	2.32	0.00***
Minimum Variance	0.0113	0.0316	0.22	0.0042	0.0473	0.03	1.48	0.14	2.24	0.00***
Maximum Sharpe Ratio	0.0124	0.0395	0.20	0.0061	0.0607	0.05	1.04	0.30	2.36	0.00***
Risk Efficient ( $\lambda = 2$ )	0.0125	0.0406	0.20	0.0058	0.0613	0.05	1.08	0.28	2.28	0.00***
<b>Market Volatility</b>										
S&P 500	0.0106	0.0400	0.16	0.0053	0.0596	0.03	0.99	0.32	2.22	0.00***
Equally Weighted	0.0111	0.0417	0.16	0.0067	0.0605	0.05	0.80	0.43	2.10	0.00***
Diversity Weighted	0.0108	0.0394	0.16	0.0050	0.0574	0.03	1.12	0.26	2.12	0.00***
Fundamental Weighted	0.0115	0.0361	0.20	0.0092	0.0579	0.10	0.44	0.66	2.57	0.00***
Inverse Volatility	0.0112	0.0388	0.18	0.0083	0.0566	0.09	0.55	0.58	2.12	0.00***
Minimum Variance	0.0109	0.0345	0.19	0.0068	0.0446	0.08	0.99	0.32	1.67	0.00***
Maximum Sharpe Ratio	0.0122	0.0401	0.19	0.0081	0.0558	0.08	0.80	0.42	1.94	0.00***
Risk Efficient ( $\lambda = 2$ )	0.0121	0.0406	0.19	0.0081	0.0577	0.08	0.76	0.45	2.01	0.00***
<b>Inflation</b>										
S&P 500	0.0125	0.0427	0.21	0.0014	0.0521	-0.09	2.34	0.02**	1.49	0.00***
Equally Weighted	0.0132	0.0435	0.23	0.0019	0.0549	-0.08	2.31	0.02**	1.60	0.00***
Diversity Weighted	0.0126	0.0414	0.23	0.0012	0.0513	-0.09	2.48	0.01**	1.54	0.00***
Fundamental Weighted	0.0141	0.0400	0.27	0.0033	0.0482	-0.06	2.48	0.01**	1.46	0.00***
Inverse Volatility	0.0135	0.0398	0.25	0.0031	0.0526	-0.06	2.22	0.03**	1.75	0.00***
Minimum Variance	0.0118	0.0316	0.27	0.0051	0.0432	-0.02	1.75	0.08*	1.86	0.00***
Maximum Sharpe Ratio	0.0135	0.0404	0.25	0.0052	0.0533	-0.02	1.76	0.08*	1.74	0.00***
Risk Efficient ( $\lambda = 2$ )	0.0138	0.0403	0.26	0.0044	0.0562	-0.03	1.91	0.06*	1.95	0.00***
<b>Economic Growth</b>										
S&P 500	0.0118	0.0414	0.19	-0.0052	0.0641	-0.18	4.78	0.00***	2.40	0.00***
Equally Weighted	0.0126	0.0424	0.21	-0.0049	0.0672	-0.16	4.70	0.00***	2.51	0.00***
Diversity Weighted	0.0119	0.0404	0.20	-0.0050	0.0625	-0.18	4.86	0.00***	2.40	0.00***
Fundamental Weighted	0.0133	0.0382	0.25	-0.0024	0.0614	-0.14	4.65	0.00***	2.58	0.00***
Inverse Volatility	0.0127	0.0389	0.23	-0.0025	0.0653	-0.13	4.31	0.00***	2.82	0.00***
Minimum Variance	0.0114	0.0315	0.24	0.0007	0.0518	-0.10	3.79	0.00***	2.70	0.00***
Maximum Sharpe Ratio	0.0132	0.0402	0.23	-0.0009	0.0632	-0.11	4.03	0.00***	2.46	0.00***
Risk Efficient ( $\lambda = 2$ )	0.0134	0.0403	0.24	-0.0021	0.0672	-0.12	4.24	0.00***	2.77	0.00***
<b>Equity Risk Premium</b>										
S&P 500	0.0151	0.0304	0.38	0.0009	0.0605	-0.06	3.74	0.00***	3.96	0.00***
Equally Weighted	0.0156	0.0320	0.37	0.0019	0.0621	-0.05	3.50	0.00***	3.76	0.00***
Diversity Weighted	0.0151	0.0304	0.37	0.0012	0.0586	-0.06	3.75	0.00***	3.72	0.00***
Fundamental Weighted	0.0153	0.0291	0.40	0.0048	0.0563	0.00	2.95	0.00***	3.74	0.00***
Inverse Volatility	0.0154	0.0297	0.39	0.0035	0.0581	-0.02	3.24	0.00***	3.82	0.00***
Minimum Variance	0.0138	0.0247	0.41	0.0041	0.0462	-0.01	3.30	0.00***	3.50	0.00***
Maximum Sharpe Ratio	0.0166	0.0316	0.41	0.0033	0.0576	-0.03	3.61	0.00***	3.31	0.00***
Risk Efficient ( $\lambda = 2$ )	0.0166	0.0306	0.42	0.0033	0.0602	-0.02	3.50	0.00***	3.88	0.00***
<b>Economic Policy Uncertainty</b>										
S&P 500	0.0127	0.0416	0.18	0.0039	0.0514	0.01	2.41	0.02**	1.53	0.00***
Equally Weighted	0.0127	0.0432	0.17	0.0056	0.0528	0.04	1.90	0.06*	1.49	0.00***
Diversity Weighted	0.0126	0.0412	0.18	0.0043	0.0495	0.02	2.33	0.02**	1.44	0.00***
Fundamental Weighted	0.0132	0.0373	0.21	0.0074	0.0500	0.08	1.70	0.09*	1.80	0.00***
Inverse Volatility	0.0129	0.0407	0.19	0.0066	0.0488	0.07	1.78	0.07*	1.44	0.00***
Minimum Variance	0.0120	0.0346	0.19	0.0064	0.0367	0.08	2.02	0.04**	1.13	0.34
Maximum Sharpe Ratio	0.0129	0.0431	0.18	0.0083	0.0470	0.10	1.31	0.19	1.19	0.16
Risk Efficient ( $\lambda = 2$ )	0.0136	0.0437	0.19	0.0072	0.0484	0.08	1.76	0.08*	1.23	0.10

\*\*\* Significant at 1% level  
 \*\* Significant at 5% level  
 \* Significant at 10% level

**Table 9. Return-Risk Characteristics of Dynamic Portfolios**

Total return is the arithmetic mean of the monthly returns. Volatility is the standard deviation of the monthly returns. Tracking error is the standard deviation of the active returns, which are the portfolio returns minus the benchmark returns (S&P 500). Information ratio is the expected active return divided by the tracking error. C-F 5% VaR is the Cornish-Fisher 5% Value at Risk. The sample is from January 1970 to December 2014 broken down using the Markov regime-switching model. The strategy descriptions are presented in the section in-sample performance of the article.

	Total Return	Volatility	Sharpe Ratio	Return/Risk	Tracking Error	Information Ratio	One-way Turnover	Downside Deviation	Maximum Time Under Water	Cornish-Fisher 5% VaR (Monthly)
<b>Panel A: Gross Returns</b>										
<b>S&amp;P 500</b>	0.0092	0.0458	0.11	0.20	-	-	-	0.0299	66	-
<b>Market Turbulence</b>	0.0118	0.0446	0.17	0.26	0.0235	0.11	85.83%	0.0270	47	0.0610
<b>Market Volatility</b>	0.0114	0.0453	0.16	0.25	0.0231	0.09	87.56%	0.0276	47	0.0625
<b>Inflation</b>	0.0115	0.0411	0.18	0.28	0.0186	0.12	56.87%	0.0242	44	0.0536
<b>Economic Growth</b>	0.0114	0.0407	0.18	0.28	0.0187	0.12	63.56%	0.0242	37	0.0537
<b>Equity Risk Premium</b>	0.0114	0.0383	0.19	0.30	0.0216	0.10	76.48%	0.0226	48	0.0517
<b>Economic Policy Uncertainty</b>	0.0106	0.0372	0.17	0.28	0.0206	0.06	77.15%	0.0221	39	0.0491
<b>Panel B: Returns, Net of Transaction costs</b>										
<b>S&amp;P 500</b>	0.0092	0.0458	0.11	0.20	-	-	-	0.0299	66	-
<b>Market Turbulence</b>	0.0114	0.0446	0.16	0.26	0.0235	0.09	85.83%	0.0272	48	0.0614
<b>Market Volatility</b>	0.0111	0.0453	0.15	0.24	0.0231	0.08	87.56%	0.0277	47	0.0628
<b>Inflation</b>	0.0112	0.0411	0.17	0.27	0.0187	0.11	56.87%	0.0243	44	0.0539
<b>Economic Growth</b>	0.0112	0.0406	0.17	0.27	0.0187	0.10	63.56%	0.0243	38	0.0539
<b>Equity Risk Premium</b>	0.0111	0.0382	0.18	0.29	0.0216	0.09	76.48%	0.0227	48	0.0521
<b>Economic Policy Uncertainty</b>	0.0103	0.0372	0.17	0.28	0.0207	0.05	77.15%	0.0223	39	0.0496

**Table 10. Conditional Performance Evaluation of the In-Sample Dynamic Portfolios using the Carhart Model**

The full conditional specification of the Carhart model is presented in section 5.6. The t-stat for each coefficient is presented in the parenthesis under the coefficient value. They are for each individual coefficient. The stars represent the significance for the test that the coefficient is different from zero. The Wald test tests the validity of the time varying structure of alpha by testing the joint nullity of the coefficients associated with the demeaned conditioning variable (dividend yield). The sample is from January 1970 to December 2014. The strategy descriptions are presented in the section in-sample performance of the article.

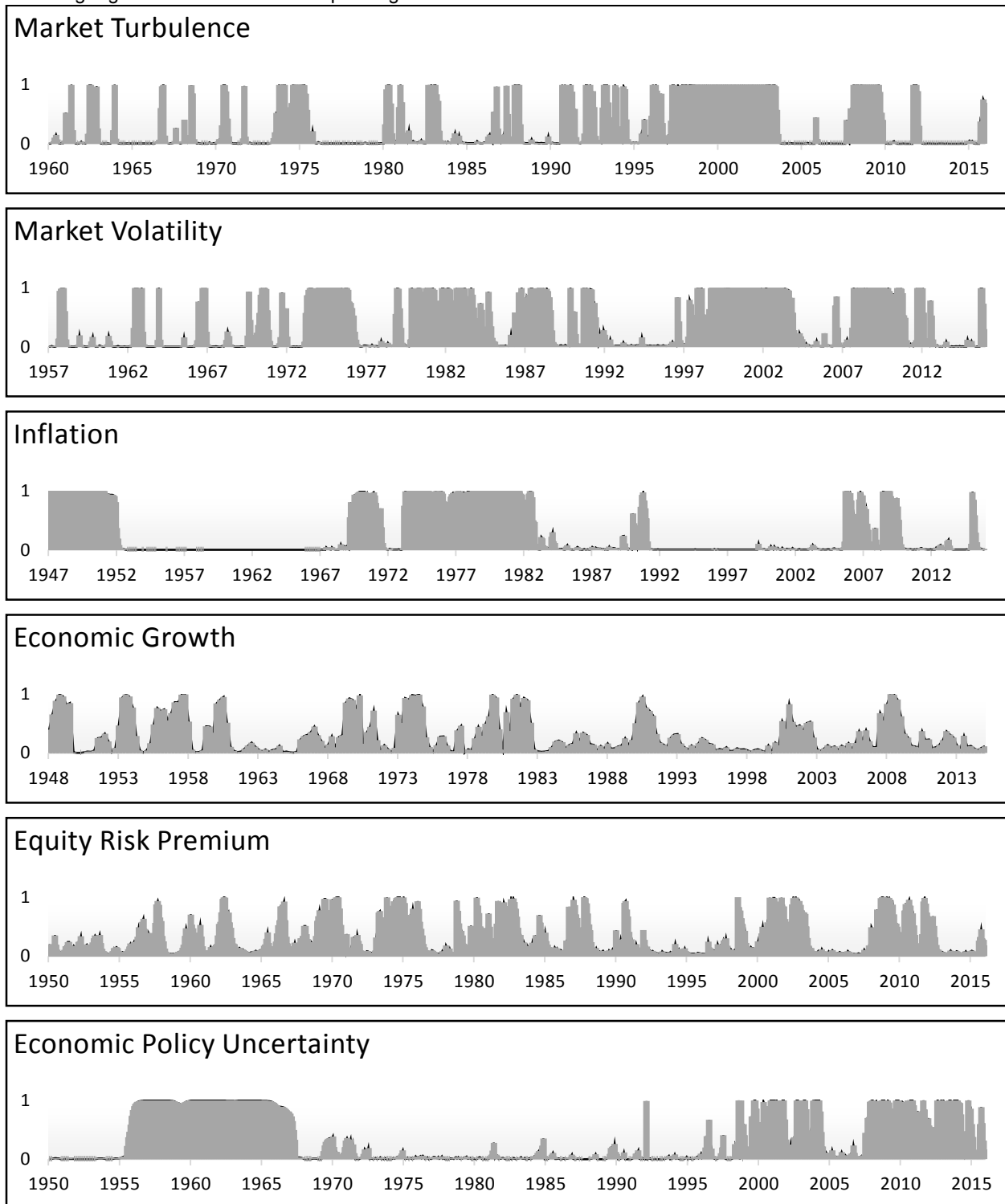
	$\alpha_{i0}$	$\alpha'_i$	$b_{i01}$	$b_{i02}$	$b_{i03}$	$b_{i04}$	$b'_{i1}$	$b'_{i2}$	$b'_{i3}$	$b'_{i4}$	Wald Test	R <sup>2</sup>
<b>Panel A: Gross Returns</b>												
<b>Market Turbulence</b>	0.002 (2.53)**	-0.112 (1.39)	0.849 (37.37)***	0.079 (2.35)**	0.174 (4.94)***	-0.036 (1.62)	2.714 (1.53)	5.120 (2.12)**	-1.149 (0.44)	1.930 (1.13)	2.87**	0.77
<b>Market Volatility</b>	0.002 (2.05)**	-0.064 (0.80)	0.870 (38.44)***	0.095 (2.83)***	0.188 (5.35)***	-0.035 (1.58)	1.539 (0.87)	3.481 (1.45)	-2.896 (1.10)	2.527 (1.49)	2.29**	0.78
<b>Inflation</b>	0.002 (3.50)***	-0.044 (0.78)	0.868 (54.93)***	-0.054 (2.32)**	0.228 (9.28)***	-0.058 (3.76)***	0.954 (0.77)	-1.554 (0.93)	-2.745 (1.50)	-0.387 (0.33)	1.26	0.87
<b>Economic Growth</b>	0.002 (2.75)***	-0.097 (1.73)*	0.867 (54.68)***	-0.042 (1.81)*	0.231 (9.38)***	0.008 (0.48)	1.350 (1.09)	-2.042 (1.21)	-2.777 (1.51)	2.646 (2.23)**	2.86**	0.87
<b>Equity Risk Premium</b>	0.002 (3.42)***	-0.063 (1.09)	0.776 (48.01)***	-0.004 (0.18)	0.208 (8.28)***	-0.008 (0.53)	4.279 (3.38)***	1.952 (1.14)	-7.821 (4.17)***	4.061 (3.35)***	16.03***	0.84
<b>Economic Policy Uncertainty</b>	0.001 (2.08)**	-0.144 (2.98)***	0.785 (57.76)***	-0.072 (3.60)***	0.241 (11.41)***	0.014 (1.07)	6.588 (6.19)***	3.318 (2.30)**	-3.198 (2.03)**	1.837 (1.80)*	19.79***	0.88
<b>Panel B: Returns, Net of Transaction costs</b>												
<b>Market Turbulence</b>	0.002 (2.18)**	-0.108 (1.33)	0.850 (37.30)***	0.074 (2.21)**	0.172 (4.85)***	-0.035 (1.56)	2.678 (1.50)	4.989 (2.06)**	-1.358 (0.51)	1.883 (1.10)	2.82**	0.77
<b>Market Volatility</b>	0.002 (1.73)*	-0.064 (0.79)	0.869 (38.26)***	0.092 (2.75)***	0.186 (5.28)***	-0.036 (1.60)	1.486 (0.84)	3.267 (1.36)	-3.213 (1.22)	2.417 (1.42)	2.25**	0.78
<b>Inflation</b>	0.002 (3.16)***	-0.039 (0.70)	0.869 (54.89)***	-0.056 (2.39)**	0.228 (9.25)***	-0.057 (3.63)***	0.782 (0.63)	-1.569 (0.93)	-2.957 (1.61)	-0.448 (0.38)	1.26	0.87
<b>Economic Growth</b>	0.002 (2.41)**	-0.096 (1.72)*	0.866 (54.72)***	-0.044 (1.90)*	0.229 (9.31)***	0.009 (0.59)	1.075 (0.87)	-1.974 (1.18)	-2.984 (1.63)	2.498 (2.11)**	2.77**	0.87
<b>Equity Risk Premium</b>	0.002 (2.99)***	-0.059 (1.03)	0.777 (48.05)***	-0.008 (0.35)	0.209 (8.30)***	-0.006 (0.36)	4.095 (3.23)***	1.650 (0.96)	-8.138 (4.34)***	4.088 (3.37)***	15.94***	0.84
<b>Economic Policy Uncertainty</b>	0.001 (1.61)	-0.137 (2.84)***	0.785 (57.56)***	-0.074 (3.69)***	0.241 (11.36)***	0.015 (1.12)	6.479 (6.06)***	3.059 (2.11)**	-3.455 (2.18)**	1.958 (1.92)*	19.36***	0.88

\*\*\* Significant at 1% level  
 \*\* Significant at 5% level  
 \* Significant at 10% level



### Figure 3 One-Step Ahead Probabilities of the Event Regime

This figure shows the one-step ahead forecasted probabilities of being in an event regime. A probability of 1 means that we are in the event regime while a probability of 0 means we are in a normal regime. The data are generated using a Markov regime-switching regression model with an expanding window.



**Table 11. Performance of Smart Beta Portfolios in Each Regimes Estimated Out-of-Sample**

Total return is the arithmetic mean of the in-regime monthly returns. Volatility is the standard deviation of the monthly returns. The sample is from January 1970 to December 2014 broken down using the out of sample Markov regime-switching model.

	Normal Regime			Event Regime			Difference of Means		Difference of Variances	
	Total Return	Volatility	Sharpe Ratio	Total Return	Volatility	Sharpe Ratio	t-Test Value	p-value	F-Test Value	p-value
<b>Market Turbulence Regimes</b>										
Equally Weighted	0.0106	0.0418	0.15	0.0086	0.0573	0.09	0.40	0.69	1.88	0.00***
Diversity Weighted	0.0104	0.0393	0.15	0.0071	0.0545	0.06	0.70	0.48	1.92	0.00***
Fundamental Weighting	0.0108	0.0365	0.18	0.0096	0.0538	0.11	0.26	0.79	2.17	0.00***
Inverse Volatility	0.0105	0.0387	0.16	0.0102	0.0538	0.12	0.07	0.95	1.93	0.00***
Minimum Variance	0.0104	0.0310	0.20	0.0086	0.0435	0.11	0.50	0.62	1.97	0.00***
Maximum Sharpe Ratio	0.0116	0.0392	0.19	0.0101	0.0545	0.12	0.32	0.75	1.93	0.00***
Risk Efficient ( $\lambda = 2$ )	0.0117	0.0402	0.18	0.0099	0.0555	0.11	0.39	0.70	1.90	0.00***
<b>Market Volatility</b>										
Equally Weighted	0.0113	0.0367	0.20	0.0083	0.0572	0.07	0.71	0.48	2.43	0.00***
Diversity Weighted	0.0113	0.0351	0.21	0.0070	0.0537	0.05	1.07	0.28	2.35	0.00***
Fundamental Weighting	0.0113	0.0332	0.23	0.0105	0.0518	0.12	0.22	0.83	2.43	0.00***
Inverse Volatility	0.0112	0.0344	0.21	0.0096	0.0532	0.10	0.42	0.68	2.38	0.00***
Minimum Variance	0.0110	0.0345	0.21	0.0084	0.0421	0.09	0.78	0.44	1.50	0.00***
Maximum Sharpe Ratio	0.0122	0.0363	0.23	0.0098	0.0527	0.10	0.60	0.55	2.11	0.00***
Risk Efficient ( $\lambda = 2$ )	0.0121	0.0354	0.23	0.0100	0.0552	0.10	0.51	0.61	2.43	0.00***
<b>Inflation</b>										
Equally Weighted	0.0106	0.0426	0.17	0.0083	0.0567	0.04	0.47	0.64	1.77	0.00***
Diversity Weighted	0.0105	0.0409	0.17	0.0067	0.0526	0.02	0.82	0.42	1.66	0.00***
Fundamental Weighting	0.0118	0.0389	0.22	0.0089	0.0507	0.06	0.65	0.52	1.71	0.00***
Inverse Volatility	0.0109	0.0390	0.19	0.0094	0.0541	0.07	0.31	0.76	1.92	0.00***
Minimum Variance	0.0103	0.0316	0.22	0.0087	0.0431	0.07	0.42	0.67	1.86	0.00***
Maximum Sharpe Ratio	0.0121	0.0402	0.22	0.0088	0.0536	0.06	0.69	0.49	1.78	0.00***
Risk Efficient ( $\lambda = 2$ )	0.0118	0.0396	0.21	0.0095	0.0572	0.06	0.48	0.63	2.08	0.00***
<b>Economic Growth</b>										
Equally Weighted	0.0135	0.0404	0.24	-0.0024	0.0648	-0.12	2.55	0.01**	2.57	0.00***
Diversity Weighted	0.0129	0.0387	0.24	-0.0032	0.0599	-0.15	3.18	0.00***	2.40	0.00***
Fundamental Weighting	0.0138	0.0361	0.28	0.0009	0.0597	-0.08	2.56	0.01**	2.74	0.00***
Inverse Volatility	0.0134	0.0371	0.26	0.0002	0.0618	-0.09	2.54	0.01**	2.77	0.00***
Minimum Variance	0.0122	0.0305	0.28	0.0014	0.0482	-0.08	2.64	0.01**	2.49	0.00***
Maximum Sharpe Ratio	0.0141	0.0389	0.27	0.0007	0.0598	-0.08	2.64	0.01**	2.36	0.00***
Risk Efficient ( $\lambda = 2$ )	0.0144	0.0386	0.28	-0.0003	0.0634	-0.09	2.76	0.01**	2.70	0.00***
<b>Equity Risk Premium</b>										
Equally Weighted	0.0147	0.0338	0.31	-0.0023	0.0698	-0.09	2.86	0.00***	4.25	0.00***
Diversity Weighted	0.0140	0.0319	0.31	-0.0027	0.0660	-0.11	2.98	0.00***	4.29	0.00***
Fundamental Weighting	0.0144	0.0302	0.34	0.0021	0.0641	-0.03	2.26	0.02**	4.51	0.00***
Inverse Volatility	0.0140	0.0316	0.31	0.0013	0.0655	-0.05	2.28	0.02**	4.30	0.00***
Minimum Variance	0.0127	0.0266	0.32	0.0024	0.0512	-0.04	2.35	0.02**	3.71	0.00***
Maximum Sharpe Ratio	0.0151	0.0345	0.32	0.0008	0.0628	-0.05	2.63	0.01**	3.32	0.00***
Risk Efficient ( $\lambda = 2$ )	0.0154	0.0330	0.34	0.0000	0.0671	-0.06	2.70	0.01**	4.14	0.00***
<b>Economic Policy Uncertainty</b>										
Equally Weighted	0.0103	0.0449	0.13	0.0087	0.0559	0.12	0.28	0.78	1.55	0.00***
Diversity Weighted	0.0099	0.0425	0.12	0.0069	0.0526	0.09	0.55	0.58	1.53	0.00***
Fundamental Weighting	0.0109	0.0390	0.16	0.0110	0.0554	0.16	0.01	0.99	2.02	0.00***
Inverse Volatility	0.0108	0.0425	0.14	0.0091	0.0502	0.14	0.32	0.75	1.40	0.02**
Minimum Variance	0.0103	0.0354	0.16	0.0079	0.0358	0.16	0.64	0.52	1.03	0.84
Maximum Sharpe Ratio	0.0113	0.0441	0.15	0.0105	0.0471	0.18	0.15	0.88	1.14	0.35
Risk Efficient ( $\lambda = 2$ )	0.0114	0.0443	0.15	0.0101	0.0506	0.16	0.23	0.82	1.30	0.07*

\*\*\* Significant at 1% level  
 \*\* Significant at 5% level  
 \* Significant at 10% level

**Table 12. Return-Risk Characteristics of Dynamic Portfolios**

Total return is the arithmetic mean of the monthly returns. Volatility is the standard deviation of the monthly returns. Tracking error is the standard deviation of the active returns, which are the portfolio returns minus the benchmark returns (S&P 500). Information ratio is the expected active return divided by the tracking error. C-F 5% VaR is the Cornish-Fisher 5% Value at Risk. The sample is from January 1970 to December 2014. The strategy descriptions are presented in the section in-sample performance of the article. One-way transaction costs are 17 and 35 basis point when in the high volatility and normal regimes, respectively.

	Total Return	Volatility	Sharpe Ratio	Return/Risk	Tracking Error	Information Ratio	One-way Turnover	Downside Deviation	Maximum Time Under Water	Cornish-Fisher 5% VaR (Monthly)
<b>Panel A: Gross Returns</b>										
<b>S&amp;P 500</b>	0.0092	0.0458	0.11	0.20	-	-	-	0.0299	66	0.0686
<b>Market Turbulence</b>	0.0115	0.0444	0.17	0.26	0.0226	0.10	160.00%	0.0270	48	0.0609
<b>Market Volatility</b>	0.0114	0.0441	0.17	0.26	0.0218	0.10	159.60%	0.0265	39	0.0596
<b>Inflation</b>	0.0109	0.0402	0.17	0.27	0.0192	0.08	68.29%	0.0242	44	0.0536
<b>Economic Growth</b>	0.0110	0.0394	0.18	0.28	0.0180	0.10	68.14%	0.0239	37	0.0540
<b>Equity Risk Premium</b>	0.0118	0.0393	0.19	0.30	0.0203	0.12	138.23%	0.0230	39	0.0531
<b>Economic Policy Uncertainty</b>	0.0106	0.0427	0.15	0.25	0.0203	0.07	131.20%	0.0266	48	0.0604
<b>Panel B: Returns, Net of Transaction costs</b>										
<b>S&amp;P 500</b>	0.0092	0.0458	0.11	0.20	-	-	-	0.0299	66	0.0686
<b>Market Turbulence</b>	0.0109	0.0445	0.15	0.25	0.0227	0.07	160.00%	0.0274	64	0.0616
<b>Market Volatility</b>	0.0109	0.0442	0.15	0.25	0.0220	0.08	159.60%	0.0268	40	0.0603
<b>Inflation</b>	0.0106	0.0402	0.16	0.26	0.0192	0.07	68.29%	0.0243	61	0.0538
<b>Economic Growth</b>	0.0108	0.0394	0.17	0.27	0.0181	0.08	68.14%	0.0240	38	0.0544
<b>Equity Risk Premium</b>	0.0112	0.0395	0.18	0.28	0.0202	0.10	138.23%	0.0235	48	0.0543
<b>Economic Policy Uncertainty</b>	0.0101	0.0428	0.14	0.24	0.0202	0.04	131.20%	0.0269	63	0.0611

**Table 13. Conditional Performance Evaluation of the Out-of-Sample Dynamic Portfolios using the Carhart Model**

The full conditional specification of the Carhart model is presented in section 5.6. The t-stat for each coefficient is presented in the parenthesis under the coefficient value. They are for each individual coefficient. The stars represent significance for the test that the coefficient is different from zero. The Wald test tests the validity of the time varying structure of alpha by testing the joint nullity of the coefficients associated with the demeaned conditioning variable. The sample is from January 1970 to December 2014. The strategy descriptions are presented in the section in-sample performance of the thesis.

	$\alpha_{i0}$	$\alpha'_i$	$b_{i01}$	$b_{i02}$	$b_{i03}$	$b_{i04}$	$b'_{i1}$	$b'_{i2}$	$b'_{i3}$	$b'_{i4}$	Wald Test	R <sup>2</sup>
<b>Panel A: Gross Returns</b>												
<b>Market Turbulence</b>	0.002 (2.28)**	-0.141 (1.84)*	0.859 (39.60)***	0.064 (2.00)**	0.181 (5.36)***	-0.036 (1.68)*	2.418 (1.42)	5.653 (2.45)**	-0.993 (0.39)	1.681 (1.03)	3.25***	0.79
<b>Market Volatility</b>	0.002 (2.19)**	-0.164 (2.21)**	0.874 (41.86)***	0.049 (1.58)	0.222 (6.84)***	-0.039 (1.90)*	3.394 (2.08)**	0.561 (0.25)	0.100 (0.04)	2.418 (1.55)	2.14*	0.80
<b>Inflation</b>	0.002 (2.24)**	-0.088 (1.52)	0.851 (51.90)***	-0.058 (2.39)**	0.212 (8.34)***	0.000 (0.02)	0.284 (0.22)	-1.039 (0.60)	-2.084 (1.10)	2.098 (1.71)*	1.52	0.85
<b>Economic Growth</b>	0.002 (2.47)**	-0.091 (1.80)*	0.848 (59.39)***	-0.043 (2.04)**	0.234 (10.54)***	0.016 (1.14)	2.202 (1.97)**	-1.383 (0.91)	-2.272 (1.37)	2.903 (2.71)***	3.88***	0.88
<b>Equity Risk Premium</b>	0.003 (3.54)***	-0.012 (0.19)	0.793 (47.39)***	0.041 (1.68)*	0.148 (5.69)***	0.031 (1.89)*	3.884 (2.96)***	0.964 (0.54)	-6.959 (3.59)***	2.251 (1.80)*	10.18***	0.84
<b>Economic Policy Uncertainty</b>	0.001 (1.50)	-0.061 (1.01)	0.840 (49.54)***	0.075 (3.01)***	0.137 (5.20)***	0.017 (1.04)	8.068 (6.08)***	4.936 (2.74)***	-8.184 (4.16)***	2.791 (2.20)**	28.26***	0.86
<b>Panel B: Returns, Net of Transaction costs</b>												
<b>Market Turbulence</b>	0.002 (1.63)	-0.145 (1.87)*	0.862 (39.39)***	0.061 (1.89)*	0.182 (5.36)***	-0.034 (1.58)	2.381 (1.39)	5.340 (2.30)**	-0.957 (0.38)	1.661 (1.01)	3.01**	0.79
<b>Market Volatility</b>	0.001 (1.61)	-0.166 (2.22)**	0.874 (41.53)***	0.050 (1.61)	0.221 (6.75)***	-0.039 (1.89)*	3.210 (1.95)*	0.562 (0.25)	-0.245 (0.10)	2.181 (1.38)	2.06*	0.80
<b>Inflation</b>	0.001 (1.87)*	-0.089 (1.54)	0.851 (52.12)***	-0.061 (2.51)**	0.212 (8.36)***	0.002 (0.13)	0.183 (0.14)	-1.196 (0.69)	-2.246 (1.19)	2.062 (1.69)*	1.59	0.85
<b>Economic Growth</b>	0.001 (1.98)**	-0.082 (1.62)	0.849 (59.48)***	-0.044 (2.08)**	0.236 (10.64)***	0.019 (1.36)	2.051 (1.83)*	-1.401 (0.92)	-2.600 (1.57)	2.740 (2.56)**	3.77***	0.88
<b>Equity Risk Premium</b>	0.002 (2.74)***	-0.011 (0.19)	0.800 (47.74)***	0.037 (1.50)	0.149 (5.72)***	0.034 (2.09)**	3.576 (2.73)***	0.867 (0.49)	-7.038 (3.62)***	2.497 (1.99)**	9.82***	0.84
<b>Economic Policy Uncertainty</b>	0.001 (0.74)	-0.020 (0.34)	0.842 (49.64)***	0.071 (2.85)***	0.133 (5.03)***	0.017 (1.05)	7.659 (5.77)***	4.718 (2.62)***	-8.527 (4.34)***	2.879 (2.27)**	27.57***	0.86

\*\*\* Significant at 1% level

\*\* Significant at 5% level

\* Significant at 10% level