The growth of fractal dimension of an interface evolution from the interaction of a shock wave with a rectangular block of SF6

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Abstract

The interface between air and a rectangular block of sulphur hexafluoride (SF6), impulsively accelerated by the passage of a planar shock wave, undergoes Richtmyer-Meshkov instability and the flow becomes turbulent. The evolution of the interface was previously simulated using a multi-component model based on a thermodynamically consistent and fully conservative formulation and results were validated against available experimental data (Bates et al. Phys Fluids 2007; 19:036101). In this study, the CFD results are analyzed using the fractal theory approach and the evolution of fractal dimension of the interface during the transition of the flow into fully developed turbulence is measured using the standard box-counting method. It is shown that as the Richtmyer-Meshkov instability on the interface develops and the flow becomes turbulent, the fractal dimension of the interface increases asymptotically toward a value close to 1.39, which agrees well to those measured for classical shear and fully developed turbulences.

Keywords: Richtmyer-Meshkov instability; Turbulence; Fractal dimension; Shock wave

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1. Introduction

Fluid interface separating two fluids (or a contact surface within a single fluid of different densities) subjected to an impulsive acceleration by the passage of a shock wave is unstable. Any perturbation will be amplified by means of baroclinic vorticity generation along the interface, which results from the misalignment of the pressure gradient of the shock and the local density gradient across the interface [1]. This phenomenon is known as the Richtmyer-Meshkov instability (RMI). This instability yields to the distortion of the interface, and the flow becomes fully-developed and turbulent that leads to strong mixing. The problem of Richtmyer-Meshkov instability is of interest to many applications – such as those related to deflagration to detonation transition [2] or astrophysics flow [3] – and serves as a useful test case for the more general studies of the transition to turbulence and mixing.

Recent advances in scientific computing now allow the development of Richtmyer–Meshkov instability to be simulated numerically with well-resolved qualitative features comparable to experimental observations. In this paper, we analyze the evolution of a rectangular block of SF6 following the interaction with a planar shock wave using the recent numerical results by Bates et al. [4]. The growth of RMI is captured by the two-dimensional numerical solution obtained using a multi-component Navier-Stokes model based on a thermodynamically consistent and fully conservative (TCFC) formulation [5, 6] and the accuracy of the CFD results was confirmed experimentally [4]. Although similar shock induced turbulent mixing experiment has also been examined through CFD by several researchers [7, 8], the resulted turbulent interface has not yet been fully investigated from the consideration of chaos and fractals. In fact, chaos and fractals theory are widely employed in fluid dynamics and related to turbulence statistics; see [9-13], etc. As an alternative step in understanding the nonlinear nature of this mixing interface and the induced turbulence from the Richtmyer–Meshkov (RM) instability, one can analyze the fractal dimension of which the evolving interface corrugates into. The fractal dimension can be considered as the geometrical signature of the phenomenon and can be used to compare or correlate with other turbulence phenomena. The fractal nature of the interface is directly related to observed flow instabilities and any growth of the fractal dimension shall be closely correlated to the degree of turbulence or mixing [10]. In this work, we conduct quantitative analysis of the time-evolution of the flow interface and the dynamics during its transition towards fully turbulent state using the classical fractal theory. The numerically generated CFD images were first processed using image processing techniques to extract the interface contour before applying the box-counting method to estimate the fractal dimension of the interface contour. The fractal dimension provides a unique geometrical characterization of the degree of complexity, and hence, instability of the interface. By obtaining the corresponding fractal dimension, attempt can be made to link the characteristic of shock-induced mixing interface with the fractal nature of other scalar interfaces in classical shear and fully developed turbulent flow (e.g., axisymmetric jet and shear mixing layer, etc.).

2. Numerical Simulation

Some key numerical parameters are given here. The basic configuration of the problem is shown in Fig.1: a rectangular block of heavy SF6 gas is held in place surrounded by air. The initial parameters for each of the two gases are provided in Table 1. A planar shock wave with Mach number 1.26 is propagating from the left of the experiment and intersects the left-most interface at a time t = 0. The same experimental configuration has been considered and investigated by

Holder et al. [14] using the AWE shock tube facility. Further numerical modeling details as well as the detailed comparison between numerical and experimental results can be found in [4, 6].

The governing equations for the flow dynamics were based on a multi-gas model formulated using a TCFC (thermodynamically consistent and fully conservative) method [5], which is shown to eliminate artificial pressure fluctuations at the material interface. By considering a flow in which all components are ideal gases and hence, have properties parameterized by the values of γ, the ratio of specific heat capacities, and M, the molecular weight, the TCFC model supplements the system of Navier-stokes equations with two conservation equations for two alternative variables $\rho\xi/M$ and ρ/M where $\xi = \gamma/(\gamma\text{-}1)$:

$$\frac{\partial}{\partial t}\left(\frac{\rho\xi}{M}\right)+\nabla\cdot\left(\frac{\rho\xi}{M}V\right)=\nabla\cdot\left(D_1\nabla\left(\frac{\rho\xi}{M}\right)\right)$$
$$\frac{\partial}{\partial t}\left(\frac{\rho}{M}\right)+\nabla\cdot\left(\frac{\rho}{M}V\right)=\nabla\cdot\left(D_2\nabla\left(\frac{\rho}{M}\right)\right)$$

where:

$$D_1=\frac{\partial Y}{\partial(\xi/M)}D=\left(\frac{\xi_{SF_6}}{M_{SF_6}}-\frac{\xi_{Air}}{M_{air}}\right)^{-1}D$$
$$D_2=\frac{\partial Y}{\partial(1/M)}D=\left(\frac{\xi_{SF_6}}{M_{SF_6}}-\frac{\xi_{Air}}{M_{air}}\right)^{-1}D$$

with Y is the mass fraction of SF_6 and D the mass diffusion coefficient $D = D_0/\rho$ with constant $D_0 = 1.6\times10^{-5}$ kg/m-s. Since the TCFC model is fully conservative, it can be solved using any available conservative solver. The numerical results are obtained using a numerical algorithm based on an operating splitting scheme using the weighted average flux (WAF) method with an approximate Riemann hyperbolic solver [4, 6, 15] and standard central finite differences for the diffusion terms. The solver is also incorporated into an Adaptive Mesh Refinement (AMR) framework for high resolution simulation. An effective resolution of 1760 × 640 cells is used and the CFL number is taken as 0.8. Results from the numerical simulations are displayed in Fig. 2.

For clear distinction of the shock-accelerated interface and the growth of RM instability along it, the tracer density ρX_{SF6}, where X_{SF6} is the mole fraction of the SF6, is plotted using a grey-scale colour map. The instability leads the flow to the transition to turbulence and strong mixing as can be observed in the last frame of Fig. 2.

3. Analysis Technique and Image Processing

The images that resulted from the numerical simulations were then analysed using the *Fraclac* toolbox [16] in the software *ImageJ* [17], which handles only binary images (black and white). Therefore the raw CFD images were first converted into binary images. The threshold level for the binary conversion is subjective; it was shown that this threshold could have an influence on the fractal dimension measurement [18]. For consistency, the conversion of the images was therefore performed using a built-in automatic thresholding criterion in *ImageJ* based on an averaging process [17]. The contour of the interface, which evolves with the growth of RM instability and the transition of the flow into turbulence, was subsequently extracted using a built-in Sobel edge detection algorithm [17].

The fractal dimension of the interface contour at each stage of its development was evaluated using the box-counting method in *Fraclac* toolbox. This method consists on applying grids or boxes, with several grids of decreasing size, to an image and counting the number of boxes required to cover an image (in our case the interface contour). For each grid size, the pixels contained in the boxes correspond to parts or details of contour features. The box-counting fractal dimension is defined as:

$$D_B = -\frac{\log N_\varepsilon}{\log \varepsilon}$$

where ε is the scale - it stands for box size relative to image size - while N_ε is the number of boxes of relative size needed to cover the whole interface contour. D_B measures the ratio of increasing details with increasing scale. The dimension D_B is the slope of the regression line for the *log-log* plot of box size (or scale) and to the number of grid boxes that contained pixels.

4. Fractal Dimension Results

Fig. 3 illustrates a sequence of image processing procedure for one image to identify the scalar level-set interface and to determine its fractal dimension using the box-counting method. By computing the fractal dimension D_B of a series of CFD images, Fig. 4 shows the evolution of mean fraction dimension of the interface boundary from the laminar regime to the fully developed turbulence stemmed from the growth of Richtmyer-Meshkov instability. The error bars indicate the minimum and maximum fractal dimensions obtained using different random grid orientations [17]. It is found that the fractal dimension of the evolving interface increases from an initial value about 1.10 to a maximum value of 1.39. These results are close to those reported in a number of other studies for the fractal dimension measurement of scalar interfaces in classical shear and fully developed turbulent flows. In short, these results thus suggest that the fractal dimension D develops towards a value between 1.3 and 1.4, indicating geometrical self-similarity in fully developed turbulence (e.g., axi-symmetric jet and shear mixing flow [19-23], etc.), and similar RMI experiment of shock-driven gas curtain [24].

5. Concluding Remarks

In this study, we analyze the simulation results of the evolution of a scalar interface induced by the interaction of a shock wave with a rectangular block of SF6 where the Richtmyer-Meshkov instability leads the flow to the transition to fully-developed turbulence. The high resolution of corresponding digital CFD images can meet the requirement of image processing and fractal dimension measurement using the box-counting method. The transition towards turbulence induced by the Richtmyer-Meshkov instability is depicted using the evolution of the fractal dimensions of the contours. The fractal dimension of the initial laminar evolution is about 1.10 and that of the transitional region will increase with the growth of instability. The growth of fractal dimension approaches to a value of 1.39. These values agree well with previous studies and the latter is very close to that experimentally measured for fully developed turbulence in axi-symmetric jet or turbulent shear flows. The invariant of the fractal dimension in the fully developed turbulent flow induced by the RMI thus equivalently indicates the geometrical self-similarity of the turbulent flow structure.

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Table Caption

Table 1. Properties of air and SF_6 gases used as input parameters for the multi-gas model

	Air	**Post-shock**	**SF_6**
ρ	1.153 kg/m^3	1.6672 kg/m^3	5.805 kg/m^3
γ	1.4	1.4	1.076
u	0 m/s	133.273 m/s	0 m/s
M	29	29	146
p	96856 Pa	163256 Pa	96856 Pa

Table 1.

Figure Captions

Fig. 1. Configuration for numerical simulation of the interaction of a shock wave with a rectangular block of SF_6

Fig. 2. Sequence of numerically generated frames showing the evolution of the Richtmyer-Meshkov instability on the shock-accelerated interface. Times displayed are a) 120 μs, b) 360 μs, c) 840 μs, d) 1640 μs, e) 1726 μs, f) 1960 μs, g) 2046 μs, h) 2846 μs, i) 4046 μs.

Fig. 3. Sequence of image processing a) raw CFD image b) Binary picture and c) edge detection

Fig. 4. Evolution of fraction dimension of the interface boundary estimated using the box-counting method

Figure1

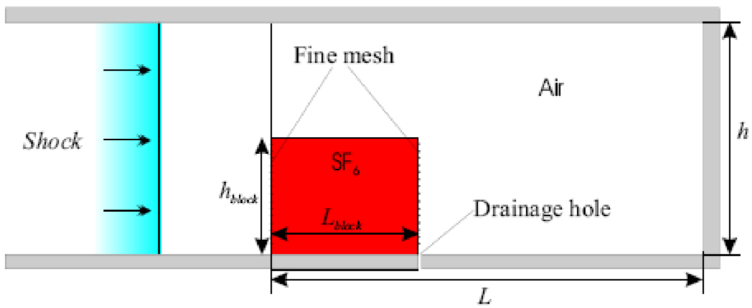

Fig. 1.

Figure2

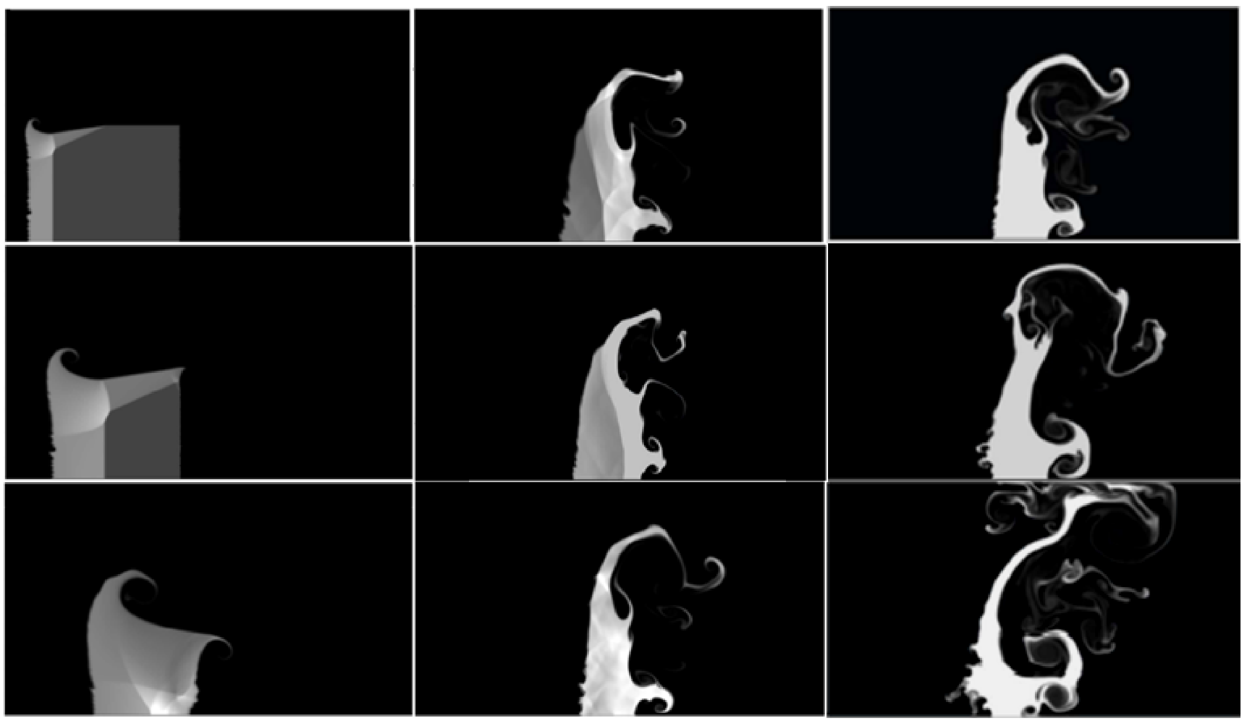

Fig. 2.

Figure3

Fig. 3.

Figure4

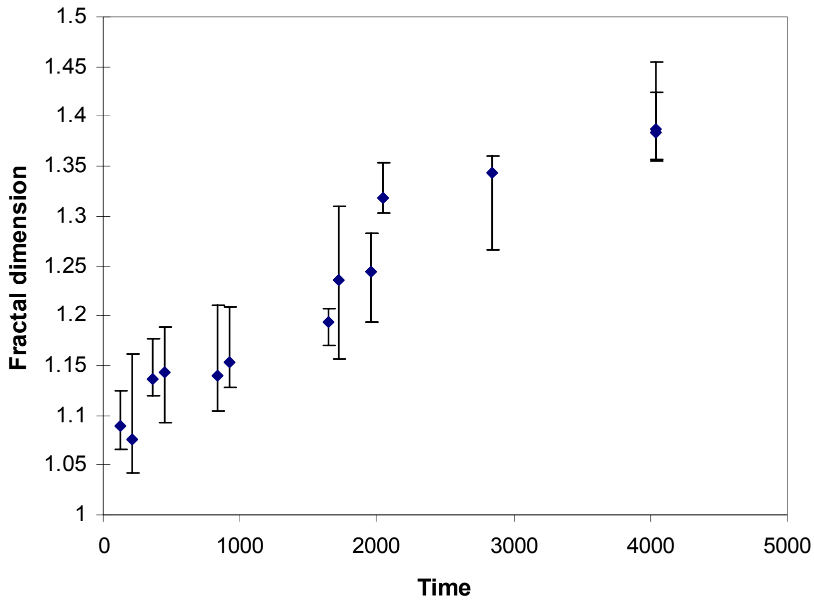

Fig. 4.

***Highlights**

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Research Highlights

We study the evolution of a scalar interface induced by the interaction of a shock wave with SF6 block

We examine the fractal dimension of the interface during the transition of the flow into turbulence

Results are example of geometrical self-similarity/nonlinear dynamics of chaotic fluid flows