Fractal geometry of the wake shed by a flapping filament in flowing soap-film

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Abstract

In this study, we illustrate the fractal nature of the wake shed by a periodically flapping filament. Such wake structure is a combination of primary vortex shedding resulting in the von Kármán vortex street, a series of concentrated vortex dipoles formed when the trailing edges of filaments reach their maximum amplitudes and small eddies form along the shear layer connected with the concentrated vortices due to the shear layer instability. The vortex dynamics of the flapping filament are visualized and imaged experimentally using a soap-film flow tunnel with a high-speed camera and a low pressure sodium lamp for the light source. The wake fractal geometry is measured using the standard box-counting method and it is shown that the fractal dimension of the soap pattern boundaries in the wake is $D = 1.38 \pm 0.05$, which agrees well with those measured for fully developed turbulences and other shear flow phenomena. The invariant of the fractality in the wake induced by the flapping filament thus provides another illustration of the geometrical self-similarity and nonlinear dynamics of chaotic fluid flows.

Keywords: Fractals; vortex pattern; wake; flapping; soap film; box-counting;

1. Introduction

The flapping of flexible bodies induced by surrounding fluid flow is a classical nonlinear dynamics problem in fluid-structure interactions and ubiquitous in nature. Examples of these interactions can be seen around us on a daily basis: flags flapping in the wind, bio-locomotion in fluids such as fish swimming or sperm making their way through embryonic fluid. These phenomena involve either active or passive flexible bodies interacting through their vortex wake as a natural mechanism to efficiently deal with the challenges of fluidic environments such as drag reduction [1].

The motivation behind this study is to gain further insight into the nonlinear wake pattern formed through complex vortex interactions due to the flapping of a thin slender object induced by a fluid flow. To have an accurate portrayal of this pattern formation, we study the phenomenon from a two-dimensional perspective and this will negate the effect of a three-dimensional flow such as in the case of a flag flapping in the wind. To this end, the use of flowing soap film is considered. It provides a simple setup of flow visualization and has long been used as a canonical experimental model to understand biological swimming and flag-in-wind problems [1,2]. The dynamic nature of instability (amplitude and frequency of oscillations) and its effects on drag reduction as well as the coupled states of two flapping filaments had been studied both numerically [3-6], and experimentally in a flowing soap film-tunnel. Focus is mainly on the flag dynamics and relatively less on the vortex wake. When objects encounter a flow some form of wake motion may ensue and the complex vortex wake structure from a flapping filament has often been qualitatively described as a von Kármán vortex street (a street of coherent strong vortices) and other fine-scale structures caused by Kelvin-Helmoholtz instability of a thin vortex layer being shed from the trailing end of the flexible filament. The pattern appears to be quite geometrically complex and the extent of geometric complexity is a related important issue in determining the nature of the flapping characteristics, such as the object's flapping modes and frequencies, as well as the effect of drag reduction [1].

Building on the previous studies, here we further elucidate the observed pattern through a different approach using fractal theory. In this work, we carry out quantitative measures of the wake structure geometry complexity by analyzing the soap film images. The wake geometry is characterized by examining the fractal dimension. Applications of fractal theory in fluid dynamics and turbulence statistics are widely studied [7-9]. The fractal dimension can be thought to provide a geometrical characterization of the degree of complexity, and hence, instability of wake structure in fluid motions.

2. Experimental Setup

Following the detailed descriptions in [2], the experimental setup consists of a two-dimensional soap film wind tunnel, having a test section of 1 m by 10 cm and capable of producing flows between 3.20 m/s to 6.50 m/s. The flow velocity is determined by measuring the position of a color dye with time. A schematic of the experimental setup is given in Fig. 1. Once the filament is placed in the soap film and oscillation ensues, we make use the optical properties of the soap film under the light source with a low pressure sodium lamp (590 nm) to track the wake structure behind the motion and capture it using a high-speed pco.1200hs camera recording at a frame rate of 800 fps. The use of the present lighting system clearly highlights the interference patterns of the soap film and illustrates how the wakes evolve in the flow.

3. Image Processing and Analysis

Fig. 2 shows time sequences of snapshots obtained from the present experiments for all three filament lengths used at a flow speed of 3.20 m/s. As illustrated in all these pictures, vortices are shed from the trailing edges of the filament in the form of primary vortex shedding with shear layer instability, i.e., the von Kármán vortex street. A concentrated vortex is formed when the trailing edges of filaments reach their maximum amplitudes and then is carried downstream by the flowing soap film. A series of small eddies also appears along the shear layer connected with the concentrated vortices due to the shear layer or Kelvin- Helmholtz instability.

In order to gain the basic information into the fractal nature of the wake one can first map the contours from the appropriate photographs, then invoke a box counting algorithm in order to estimate for the fractal dimension. The image processing and box-counting analysis are performed using *ImageJ* software [10]. Fig. 3 illustrates a sequence of steps in image processing procedure for one image to identify the scalar level-set interface and to determine its fractal dimension using the box-counting method. In detail, the images that resulted from the soap-film experiments first were converted into binary images. For consistency, the conversion of the images was performed using a built-in automatic thresholding criterion in *ImageJ* based an averaging process [10]. The contour of the interface, which evolves in the wake subsequently was extracted using a built-in Sobel edge detection algorithm [10]. The fractal dimension of the developed wake contour was evaluated using the box-counting method. As a standard and efficient method in fractal analysis, the box-counting technique for fractal dimension measurement consists on applying grids or boxes, with several grids of decreasing size, to an image and counting the number of boxes required to cover an image (in our case the interface contour). For each grid size, the pixels contained in the boxes correspond to parts or details of contours features. The box-counting fractal dimension is defined as:

$$D = -\lim_{\varepsilon \to 0} \frac{\log N_{\varepsilon}}{\log \varepsilon} \qquad (1)$$

where ε is the scale - it stands for box size relative to image size - while N_{ε} is the number of boxes of relative size needed to cover the whole interface contour. *D* measures the ratio of increasing details with decreasing scale. The dimension *D* is the slope of the regression line for the *log*-*log* plot of box size (or scale) and to the number of grid boxes that contained pixels. It is important to realize that our analyses were performed on the experimental results, which are subjected to the limitation of the experimental techniques (i.e. illumination and imaging system qualities) and images interpretation. One should keep in mind that the precision of their dimension estimate is limited. Therefore, at least three images per initial condition were analyzed to have a statistically converged average. It is also worth noting that at higher flow speed, the quality of the wake image is limited by the prominent light interference on the soap film, making the image processing difficult. For example, the onset of this interference can already be observed from the experimental image with flow speed 4.72 m/s shown in Fig. 4.

Fig. 5 summarizes all fractal dimension measurement performed in this study for the 3 different filament lengths used and two flowing soap film velocities. It was observed that as the length of the filament or flow velocity changes the fractal dimension remains constant, very close to an average of 1.38. These results appear to be equivalent to that reported in a number of other studies for the fractal dimension measurement of scalar interfaces in classical shear flow, mixing layer and fully developed turbulent flows, and turbulence induced by Richtmyer-Meshkov instability [e.g., 11-20], which suggest that fractal dimension *D* develops towards a value between 1.32 - 1.4 indicating geometrical self-similarity in these types of fluid flows.

4. Concluding Remarks

In this study, we illustrate the wake flow structure behind the oscillatory motion of a thin flexible object as it encounters a two-dimensional fluid flow. The wake fractal geometry is quantified using the standard box-counting method and it is shown that the fractal dimension of the soap pattern boundaries in the wake is $D \sim 1.38 \pm 0.05$, irrespective of the filament length and flow velocity, results which agree to those measured for fully developed turbulences and other shear flow phenomena. The invariant of the fractality in the wake induced by the flapping filament thus provides another example of the geometrical self-similarity and nonlinear dynamics of chaotic fluid flows.

Acknowledgment

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Figure Captions

Fig. 1. A schematic of the experimental setup.

Fig. 2. Wake structure induced by the flapping filament with length of a) 2 cm, b) 4 cm and c) 6 cm and a mean flow velocity of 3.20 m/s.

Fig. 3. A sequence of image processing procedure and the estimation of fractal dimension for a) 2 cm and b) 4 cm filament length and a mean flow velocity of 3.20 m/s.

Fig. 4. A sample picture for the experimental result obtained using a 4 cm filament length and a flow velocity of 4.72 m/s.

Fig. 5. Fraction dimension of the wake structure estimated using the box-counting method.

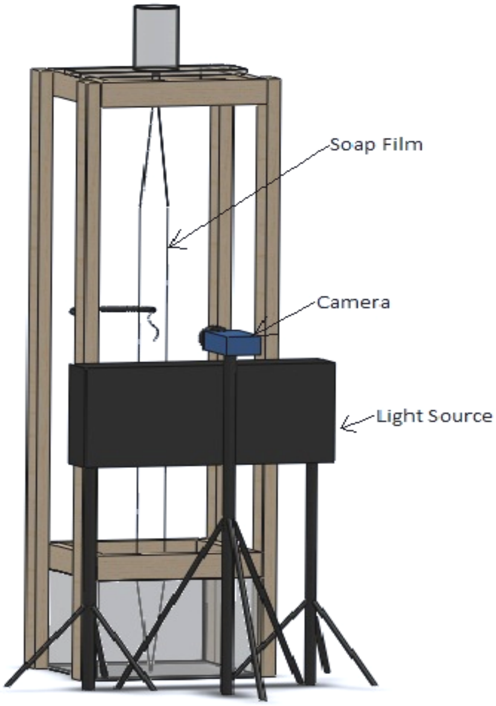
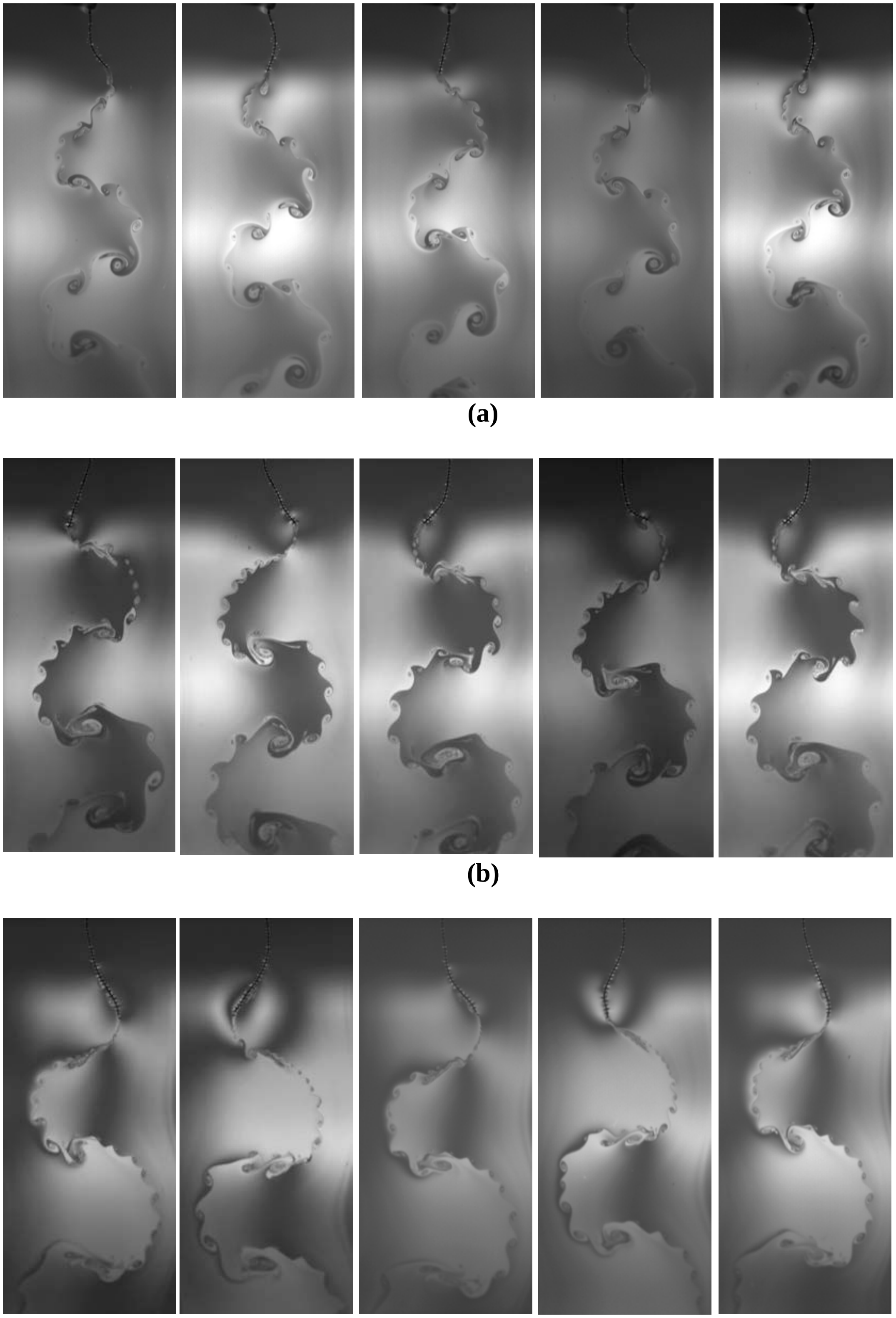


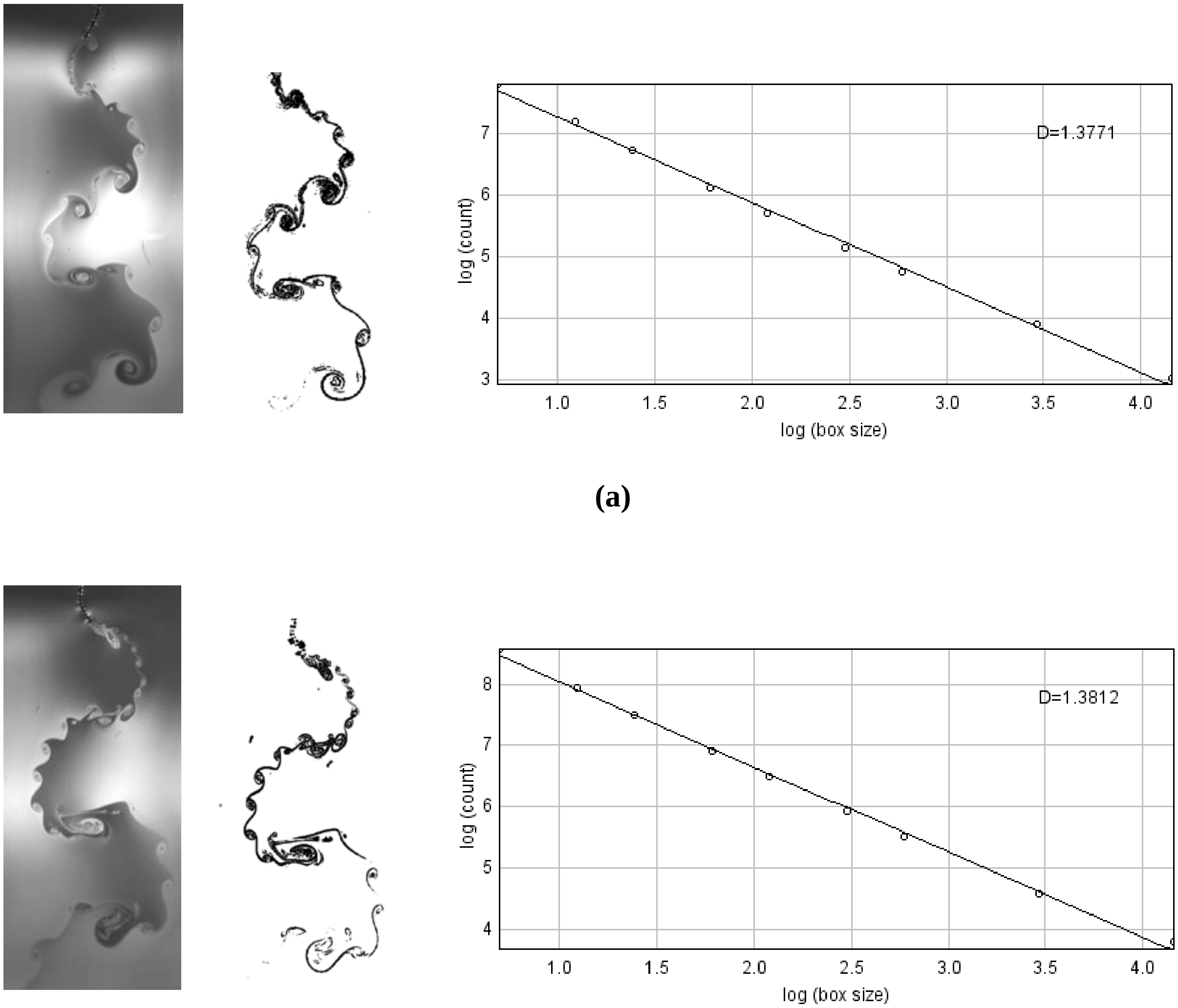
Fig. 1.


(a)

(b)

(c)

Fig. 2.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log (count)
log (box size)
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5
6
7
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1.5
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3.5
4.0
D=1.3812
log (count)
log (box size)
4
5
6
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8
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1.5
2.0
2.5
3.0
3.5
4.0

(a)

(b)

Fig. 3.

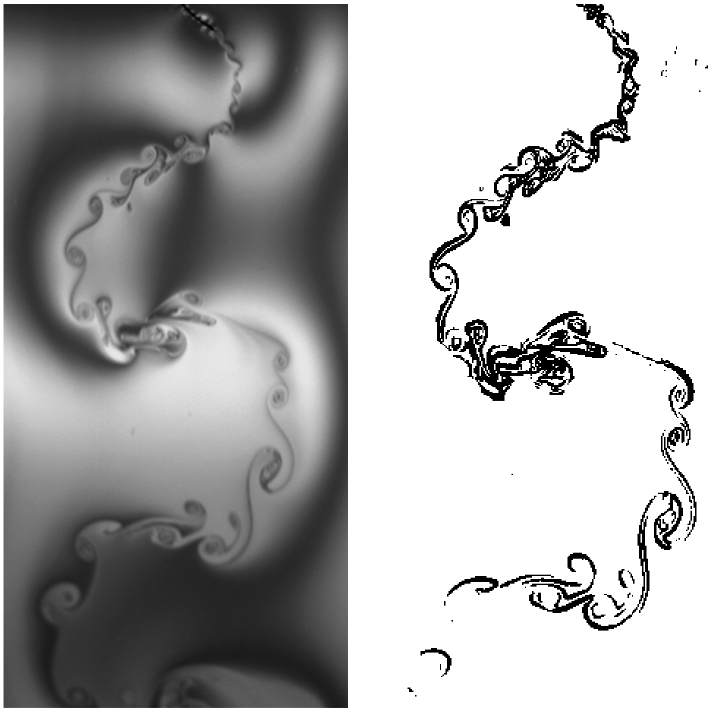

Fig. 4.

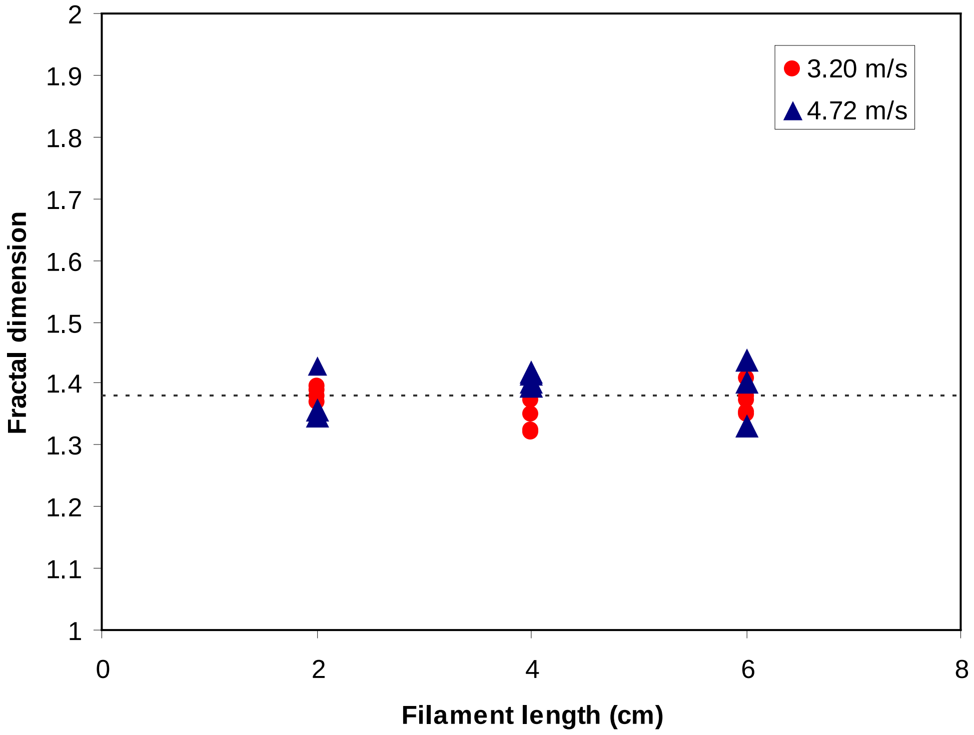
Fractal dimension
2
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
0
2
4
6
8
Filament length (cm)
● 3.20 m/s
▲ 4.72 m/s

Fig. 5.