Efficient Spectrum Utilization in Large-Scale RWA and RSA Problems

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A Thesis

in

The Department

of

Computer Science and Engineering

Presented in Partial Fulfillment of the Requirements

for the Degree of

Master of Computer Science at

Concordia University

Montréal, Québec, Canada

September 2016

CONCORDIA UNIVERSITY

School of Graduate Studies

This is to certify that the thesis prepared

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Entitled: **Efficient Spectrum Utilization in Large-Scale RWA and RSA Problems**

and submitted in partial fulfillment of the requirements for the degree of

Master of Computer Science

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Abstract

Efficient Spectrum Utilization
in Large-Scale RWA and RSA Problems

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While the *Routing and Wavelength Assignment* (RWA) problem has been widely studied, very few studies attempt to solve realistic size instances, namely, with 100 wavelengths per fiber and a few hundred nodes. Indeed, state of the art is closer to around 20 nodes and 30 wavelengths. In this study, we are interested in reducing the gap between realistic data sets and testbed instances, using exact methods.

We propose different algorithms that lead to solve exactly or near exactly much larger instances than in the literature, with up to 150 wavelengths and 90 nodes. Extensive numerical experiences are conducted on both the static and the dynamic cases. For the latter, we investigate how much bandwidth is wasted when no lightpath re-arrangement is allowed, and compare it with the number of lightpath re-arrangement it requires in order to fully maximize the grade of service. Results show that the amount of lightpath re-arrangement remains very small in comparison to the amount of wasted bandwidth if not done.

The *Routing and Spectrum Assignment* (RSA) problem is a much more difficult problem than RWA, considered in elastic optical networks. Although investigated extensively, there is still a gap between the size of the instances that can be solved using the current heuristic or exact algorithms, and the size of the instances arising in the industry. As the second objective of this study, we aim to reduce the gap between the two, using a new mathematical modeling, and compare its performance with the best previous algorithms/models on realistic data instances.

Acknowledgments

I would like to express my gratitude to my supervisor professor Brigitte Jaumard, for her profound support and inspiring guidance during my studies and for being a compassionate advisor in times of difficulty. By teaching me a new attitude towards research, she has rendered me indebted to her for obtaining a lifetime asset that is systematic and practical research.

I would also like to thank my lovely husband who has been the greatest support I could have asked for. His ceaseless encouragement is my foremost motivation in pursuing academic achievements and for that I am always grateful to him.

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Chapter 1

Introduction

In the past decade, communication networks have played an undeniably crucial role in our daily life. As societies develop, the effect of this role becomes more tangible. As a vivid example, one can consider social networking services such as Facebook, Twitter and Telegram. Nowadays, they are an integrated part of entertainment, broadcasting, marketing and spreading rumors, and even have gone to the point where they can effectively enlighten (or mislead) the public about previously elite-bounded subjects like politics. In such a demanding situation, high-speed and flawless service to the users of communication networks is of vital importance. *Optical networks* have provided us with a fast and reliable transmission system by incorporating the fastest medium possible: *light*.

This study is an attempt to optimize the utilization of the resources in common optical networks, by developing new large-scale optimization algorithms that are specifically designed to deal with large size instances. Following sections state the context and description of the problems that we will handle. The objectives and contributions of the thesis will be presented and the plan of the thesis will follow.

1.1 Background

As of July, 2016, there are more than 3,424,971,237 internet users, a 7.5% growth over the last year [Worldometers.info, 2016]. Estimates suggest that annual global IP traffic will reach 1.1 ZB (1 ZB = 2^{30} TB) by the end of 2016 (first time surpassing zettabyte), and will increase to 2.3

ZB per year by 2020, at a compound annual growth rate of 22% [Cisco, 2016]. The continuous growth of traffic, nourished by the emerging rich-content, high-rate and bursty applications, such as video-on-demand, on-line gaming, high-definition television (HDTV), and cloud computing, can only be met with the abundant capacity provided by optical transport networks. Considering this rapid growth of traffic requests, researchers constantly seek to improve the underlying infrastructure and technology, along with designing algorithms to efficiently incorporate the full potentials of the networks.

Wavelength division multiplexing (WDM) networks have been serving us for a long time, being the dominant transmission systems in the industry. Their full potential has not yet been met, but the historical data suggests that we are at the edge of surpassing their capacity (see Section 2.3). As a result, network operators are now moving to *elastic optical networking* (EON), a technology that increases the capacity of the networks by providing a much more flexible pattern (see Section 2.3).

In a network, enabling a service and making it available for receiver points is called *provisioning*. In the context of optical networks, Integer Linear Programming (ILP) tools have proven themselves beneficial in finding quality solutions (optimal or near-optimal) for provisioning of the traffic requests (see Sections 2.2 and 2.3). With the current rise of bandwidth unit requests, the size of the provisioning instances has grown as well. The promising potential of large-scale optimization frameworks developed in the field of Operations Research, makes it inevitable to revisit and improve some of the previously designed algorithms.

1.2 Problems

For both classical and elastic optical networks, provisioning problems are as follows. *Routing and wavelength assignment* (RWA) is the key provisioning problem of classical WDM networks. The main challenge in elastic optical networks is optimizing further the spectrum usage through the so-called *routing and spectrum assignment* (RSA) problem.

In this thesis, three provisioning problems are revisited in order to be further optimized, with the common motivation that there is a considerable gap between the size of the instances for which the existing solution approaches are efficient and effective, and the ones that industry faces today.

The considered problems are as follows.

(1) Efficient spectrum utilization in offline large-scale RWA problems: this is the classic RWA problem with static traffic, i.e., all the bandwidth unit requests are at hand.

(2) Optimizing spectrum utilization in dynamic RWA: we consider the case where traffic is incremental, arriving at subsequent time periods, within the today's context of rapid network traffic growth.

(3) Large-scale elastic optical path networking models: in this problem we go one step further to achieve more flexibility and save on inter-carrier guard bands, see Section 2.3.

In all three cases, we are interested in designing algorithms that can meet the expectations of the industry, in terms of exact or near exact solutions of realistic network and traffic instances.

1.3 Research Objective and Contributions

In the following sections, we first describe our objectives for the problems stated in Section 1.2, then present the contributions we had in this thesis.

1.3.1 Objectives

Our main objective in this thesis is to increase the scalability of the exact algorithms for the RWA problem with static/dynamic traffic, and RSA problem with static traffic. While RWA problem was the focus of many studies in the 80's, numerous heuristics have been proposed. However, they have not been tested and compared on very large data instances except for very few papers, see, e.g., [Nagatsu et al., 1996, Noronha et al., 2006, Martins et al., 2012, Kogantia and Sidhu, 2014], and most of the time, they come without any information on the accuracy of their solutions. On the other hand, exact methods have difficulties to scale. For RWA problem with static traffic, the improvement of the algorithms can be done through the maximization of lightpaths with a route using a short path selected among all shortest paths or k-shortest paths with a small k, for every node pair, rather than using a single shortest path selected at random among all shortest paths, as

is done in many previous studies. To further enhance the algorithms, the proper strategy of path selection will be studied.

In dynamic RWA problem, our secondary objective is to revisit the problem with the goal of evaluating the minimum number of lightpath re-arrangement it requires in order to remain with an optimized RWA provisioning, using an exact solution process. We investigate how much bandwidth is wasted when no lightpath re-arrangement is allowed, and compare it with the number of lightpath re-routing it requires in order to fully maximize the grade of service. In the algorithms for this problem, we use the results obtained for the RWA problem with static traffic.

1.3.2 Contributions

The contribution of the thesis includes:

- Proposing two ε-optimal algorithms for large-scale RWA problem with static traffic. The algorithms are very efficient and capable of solving instances with up to 90 nodes and 150 wavelengths.
    - B. Jaumard, M. Daryalal. Solving very large RWA data instances. *IEEE Canadian Conference on Electrical and Computer Engineering (CCECE)*, pages xx, 2016.
- Proposing two highly efficient heuristics for RWA problem with static traffic. We showed the algorithms achieve very good solutions with remarkably high accuracy from the ε-optimal algorithms.
    - B. Jaumard, M. Daryalal. Enhanced RWA solutions for very large data instances. In *Large Scale Complex Network Analysis (LSCNA)*, pages 1-17, 2015.
- Designing an effective path selection strategy in order to further enhance the solutions of large-scale RWA problem with static traffic. With this paper, we have the $\varepsilon-$optimal algorithms (with very small ε) that solve the largest traffic/network RWA instances of the literature.
    - B. Jaumard, M. Daryalal. Efficient spectrum utilization in large scale RWA problems. (under revision) *IEEE/ACM Transactions on Networking*.

- Proposing a column generation algorithm for a large-scale RWA problem with incremental traffic, with/without lightpath re-arrangements. The results showed that when lightpath re-arrangement is possible, the algorithm is very successful in achieving the maximum GoS while re-arranging a very small percentage of lightpaths.

    - B. Jaumard, M. Daryalal. Optimizing spectrum utilization in dynamic RWA. In *IEEE International Conference on Optical Network Design and Modeling (ONDM)*, pages 1-6, 2016.

- Presenting a mathematical formulation and an ε-optimal algorithm for RSA problem, while considering guard bands. The algorithm was able to outperform all best existing solution approaches in the literature.

    - B. Jaumard, M. Daryalal. Large-scale elastic optical path networking models. *IEEE International Conference on Transparent Optical Networks (ICTON)*, pages 1-4 2016.

1.3.3 Thesis Organization

The thesis is organized as follows. Chapter 2 presents the preliminary materials and a literature review on the related subjects. We first briefly introduce the concepts of mathematical and linear programming. Then we proceed to a description of column generation and branch-and-price methods. After a short review on WDM networks and various types of multiplexing, RWA problem is stated in detail. Related works for both static and dynamic cases are comprehensively described. Finally, elastic optical networks and the notion of spectrum slot are discussed, followed by examining the works in the literature for solving the RSA problem.

Chapter 3 formally states the RWA problem, by defining the notations and parameters, and presenting the mathematical formulations for the problem with static and dynamic traffic, the latter with/without lightpath re-arrangements. New solution algorithms are provided for all three cases. For the static traffic, two ε-optimal and two heuristic algorithms are designed. The ε-optimal algorithms are improving the existing exact methods, and their enhancement lies in the use of three different mathematical formulations. Each formulation is used when it is the most useful one inside

the steps of the solution process. This contrasts with the literature works that only use one mathematical model at a time in the proposed exact methods. We conclude Chapter 3 by showing how the new solution approaches of directed RWA problem with static traffic can be modified to solve the undirected case.

Chapter 4 presents the statement for RSA problem, then proceeds with providing a new mathematical formulation, based on an original mathematical decomposition. An ε-optimal algorithm is designed to solve the new proposed formulation.

Chapter 5 conducts a comprehensive numerical study on the performance of the designed algorithms in the previous chapters, as well as characteristics of the solutions. Several comparisons between static and dynamic RWA are presented.

Chapter 6 concludes the thesis and proposes future lines of research.

Chapter 2

Literature Review and Preliminaries

This chapter provides an overview on the preliminaries and basic concepts used throughout the thesis. Section 2.1 presents a basic summary of linear programming and column generation methods, essential to our new and enhanced solution approaches. Sections 2.2 and 2.3 introduce the problems of routing and wavelength assignment, and routing and spectrum assignment respectively. A comprehensive literature review on the related works, both in the context of RWA/RSA problems and solution algorithms is provided, leading to identification of deficiencies in the existing solution approaches in the literature.

2.1 Column Generation Modeling and Algorithms

Mathematical programming is a branch of management science in which mathematical models and approaches help a decision maker in optimally allocating resources while considering a set of limitations (constraints). This optimization can be with regard to minimizing or maximizing a certain objective, and some restrictions on the possible decisions, that can be stated in terms of mathematical functions. Therefore a (deterministic) mathematical program can be written as follows:

$$\max \text{ or } \min \quad f_0(\boldsymbol{x}) \tag{1}$$

subject to:

$$f_j(\boldsymbol{x}) \leq b_j \quad j = 1, \ldots, m \tag{2}$$

$$\boldsymbol{x} \in \mathcal{X}, \tag{3}$$

where $\boldsymbol{x}$ is an n-dimensional vector. Function $f_0(\boldsymbol{x})$ is called the *objective function* and Equations (2) are *constraints*. These constraints define a numerical restriction on the functions $f_j(\boldsymbol{x}), j = 1, \ldots, m$. Equations (3), through the set $\mathcal{X}$, e.g., $\{0, 1\}^n$ or $\mathbb{Z}_+^n$, define the domain of the decision variables. A vector $\hat{\boldsymbol{x}}$ that satisfies all the constraints is *feasible*. A feasible vector $\boldsymbol{x}^\star$ at which f_0 achieves its best value (minimum or maximum) defines an *optimal vector*, and its associated value $f_0(\boldsymbol{x}^\star)$ is called the *optimum value*.

In this section, a subcategory of mathematical programs and a powerful approach for solving them are described.

2.1.1 Linear Programming

A mathematical program is *linear*, if all functions $f_j(\boldsymbol{x}), j = 0, \ldots, m$ are linear. It is common to express a linear program (LP) as follows:

$$\min \qquad \boldsymbol{c}^T \boldsymbol{x} = \sum_{i=1}^{n} c_i x_i \tag{4}$$

subject to:

$$A^T \boldsymbol{x} \geq \boldsymbol{b} \tag{5}$$

$$x_i \geq 0 \qquad i = 1, \ldots, n, \tag{6}$$

where $\boldsymbol{x} \in \mathbb{R}^n$, $A \in \mathbb{R}^{m \times n}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are m and n-dimensional real vectors, respectively. From the above definition, it can be concluded that having real vector decision variables in a mathematical program with linear objective and constraint functions is enough for having a linear program, since it can easily be transformed to the standard form stated above.

Solving a linear program has been proved to be polynomial [Khachiyan, 1980]. One of the

most practical algorithms for solving the problem (4) - (6) is *simplex* method. Although not worst-case polynomial, it is a powerful method that exploits the geometrical characteristics of feasibility spaces in linear programs to achieve optimality. This algorithm traverses the boundaries of the feasibility space in order to find the optimal solutions. Unlike the simplex method, *interior point* methods are a category of algorithms that move through the interior of the feasibility space toward the optimum point [Potra and Wright, 2000]. Karmarkar's algorithm [Karmarkar, 1984], the first efficient polynomial time method for solving a linear program, belongs to this class.

In spite of all the powerful approaches that exist for linear programs, if the number of variables and/or constraints becomes exponential, no existing algorithm is able to solve a linear program in reasonable time, even if the number of constraints remains small. Hence, researchers turned to decomposition methods for solving such linear programs. Section 2.1.2 elaborates on one of these methods that handles the problems with a large number of variables.

Integer Linear Programming

If an additional constraint of the form $\boldsymbol{x} \in \mathbb{Z}^n$ is added to program defined by (4) - (6), we have an *integer linear program* (ILP):

$$\min \qquad \boldsymbol{c}^T \boldsymbol{x} = \sum_{i=1}^{n} c_i x_i \tag{7}$$

subject to:

$$\boldsymbol{x} \in \mathcal{S} = \{\boldsymbol{x} \in \mathbb{Z}_+^n : \ A^T \boldsymbol{x} \geq \boldsymbol{b}\}, \tag{8}$$

where $\mathbb{Z}_+^n$ is the set of non-negative integer vectors. Solving this problem is much harder than LP, and is proved to be NP-complete [Papadimitriou, 1981]. There are numerous general approaches to solve an ILP, most notably *branch-and-bound algorithm*. In many of these approaches, one of the steps is solving a *relaxation* of the ILP. A relaxation of an ILP is any problem of the form:

$$\min \qquad f_{\mathrm{R}}(\boldsymbol{x}) \tag{9}$$

subject to:

$$\boldsymbol{x} \in \mathcal{S}', \tag{10}$$

such that $\mathcal{S} \subseteq \mathcal{S}'$ and $f_{\mathrm{R}}(\boldsymbol{x}) \leq \boldsymbol{c}^T\boldsymbol{x}$ for $\boldsymbol{x} \in \mathcal{S}$ [Nemhauser and Wolsey, 1988]. *LP relaxation* is a common type of relaxation, in which only the integrality constraints $\boldsymbol{x} \in \mathbb{Z}^n$ are relaxed.

2.1.2 Implicit Enumeration in Large-Scale ILP Problems

Column Generation method, first proposed by Dantzig and Wolfe [1960] and implemented by Gilmore and Gomory [1961], is a decomposition method for solving linear programs with a huge number of variables. The method is based on a concept called *duality*. With every linear problem (4) - (6), a *dual* problem is associated. It is defined as follows:

$$\max \qquad \boldsymbol{\pi}\boldsymbol{b} \tag{11}$$

subject to:

$$\boldsymbol{\pi}^T A \leq \boldsymbol{c} \tag{12}$$

$$\pi_j \geq 0 \qquad j = 1, \ldots, m. \tag{13}$$

In this context, model (4) - (6) is called the *primal* problem. A major result in linear programming is *strong duality*. It states that if a linear programming problem has an optimal solution, then so does its dual problem, and $\boldsymbol{c}^T\boldsymbol{x}^\star = \boldsymbol{\pi}^\star\boldsymbol{b}$, when $\boldsymbol{x}^\star$ and $\boldsymbol{\pi}^\star$ are optimal vectors of primal and dual problems, respectively [Chvatal, 1983, Kuhn et al., 1958]. In the following, once again we explain the duality theory using explicitly the concept of *columns*, that is defined as the columns of matrix A, i.e., the coefficient vectors of $\boldsymbol{x}$. Define by $x_{\boldsymbol{a}}$, $\boldsymbol{a} \in \mathcal{C}$, $\mathcal{C} \subset \mathbb{R}^m$, a decision variable and $\boldsymbol{a}$ its associated column, and by $c_{\boldsymbol{a}} \in \mathbb{R}$ the cost of column $\boldsymbol{a}$. Let $\boldsymbol{b} \in \mathbb{R}^m$ be the right-hand side of the constraints. The primal (P) and dual (D) problems are defined as follows:

(P)

$$\min \quad \sum_{\boldsymbol{a}\in\mathcal{C}} c_{\boldsymbol{a}} x_{\boldsymbol{a}} \tag{14}$$

subject to:

$$\sum_{\boldsymbol{a}\in\mathcal{C}} \boldsymbol{a} x_{\boldsymbol{a}} \geq \boldsymbol{b} \tag{15}$$

$$x_{\boldsymbol{a}} \geq 0, \quad \boldsymbol{a} \in \mathcal{C}, \tag{16}$$

(D)

$$\max \quad \boldsymbol{\pi b} \tag{17}$$

subject to:

$$\boldsymbol{\pi a} \leq c_{\boldsymbol{a}}, \quad \boldsymbol{a} \in \mathcal{C} \tag{18}$$

$$\boldsymbol{\pi} \geq \boldsymbol{0}, \tag{19}$$

where $\boldsymbol{\pi}$ is the vector of dual variables associated with Constraints (15). According to the strong duality theory, if a primal solution $\boldsymbol{x}^*$ is optimal for P, then $\boldsymbol{\pi}^* = \boldsymbol{c}_B B^{-1}$ is optimal for the problem D, where B is the optimal basis of problem P.

If $\mathcal{C}$ is too large, solving the problem becomes intractable. In a column generation approach, one starts with a very small subset $\mathcal{C}' \subseteq \mathcal{C}$ and builds the *Restricted Master Problem* (RMP). This is obtained by replacing the set $\mathcal{C}$ in P and D, with set $\mathcal{C}'$, which results in problems $\mathrm{P}_{\mathcal{C}'}$ and $\mathrm{D}_{\mathcal{C}'}$. The column generation method starts with $\mathrm{P}_{\mathcal{C}'}$ and obtains a primal feasible solution $\hat{\boldsymbol{x}}$. Due to the strong duality theory, $\hat{\boldsymbol{x}}$ is optimal for problem P if and only if its associated dual vector $\hat{\boldsymbol{\pi}}$ satisfies the following constraint:

$$\hat{\boldsymbol{\pi}}\boldsymbol{a} \leq c_{\boldsymbol{a}}, \quad \forall \boldsymbol{a} \in \mathcal{C}. \tag{20}$$

Consequently, the solution of the *pricing problem* (PP) allows checking the optimality of a solution $\hat{\boldsymbol{x}}$:

$$\text{(PP)} \qquad \min\{c_{\boldsymbol{a}} - \hat{\boldsymbol{\pi}}\boldsymbol{a} : \boldsymbol{a} \in \mathcal{C}\}. \tag{21}$$

The value $c_{\boldsymbol{a}} - \hat{\boldsymbol{\pi}}\boldsymbol{a}$ is called the *reduced-cost* associated with variable $x_{\boldsymbol{a}}$ and measures how much the objective function will change if variable $x_{\boldsymbol{a}}$ enters the basis (the reader who is not familiar with linear programming concepts is referred to Chvatal [1983] for detailed explanations). Hence, if the value of the reduced-cost is positive for all $\boldsymbol{a} \in \mathcal{C}$, i.e., if the optimal solution of the PP is positive, the problem (14) - (16) has been solved to optimality because no variable exists that, if added to (14) - (16), can improve its new resulting optimum value. Otherwise, the most promising variable is added to the RMP and this continues until no other promising column remains left out from the RMP. Figure 2.1 illustrates the column generation algorithm.

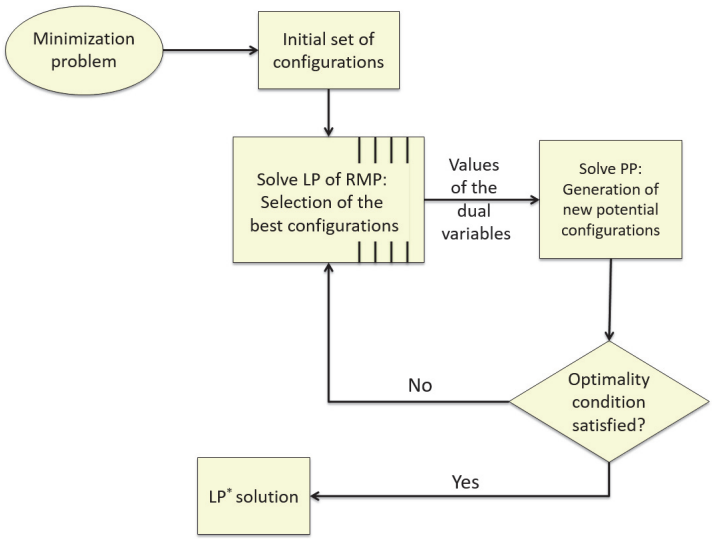


Figure 2.1: Column generation flowchart

2.1.3 Branch-and-Price Algorithm

There are two standard techniques for solving an ILP [Nemhauser and Wolsey, 1988]:

- Cutting-plane algorithm, where an LP relaxation is solved iteratively and, in each iteration, a linear constraint (*cut*) is added to the formulation such that no feasible solution is excluded, while the solution is being driven toward integrality.

- Branch-and-bound algorithm, using LP relaxation. In every iteration, the feasible space of the problem is partitioned into $\nu > 1$ sub-spaces, resulting in ν smaller subproblems (*nodes*). This is done using a pre-defined format called *branching scheme*. At every node, the local lower bound is achieved using an LP relaxation, while the local upper bound is obtained by some pre-defined heuristic. By keeping track of the previous nodes and comparing, the global lower and upper bounds are determined and potentially some unpromising nodes are exempted from investigation (*pruning*). The algorithm continues until either a certain level of precision or solution accuracy is attained, or no candidate node is left for investigation. Figure 2.2 illustrates the procedure.

Branch-and-price algorithm is a hybrid of branch-and-bound method within a column generation context. While it is conducted, in each node, for solving the LP relaxation of the ILP, a column

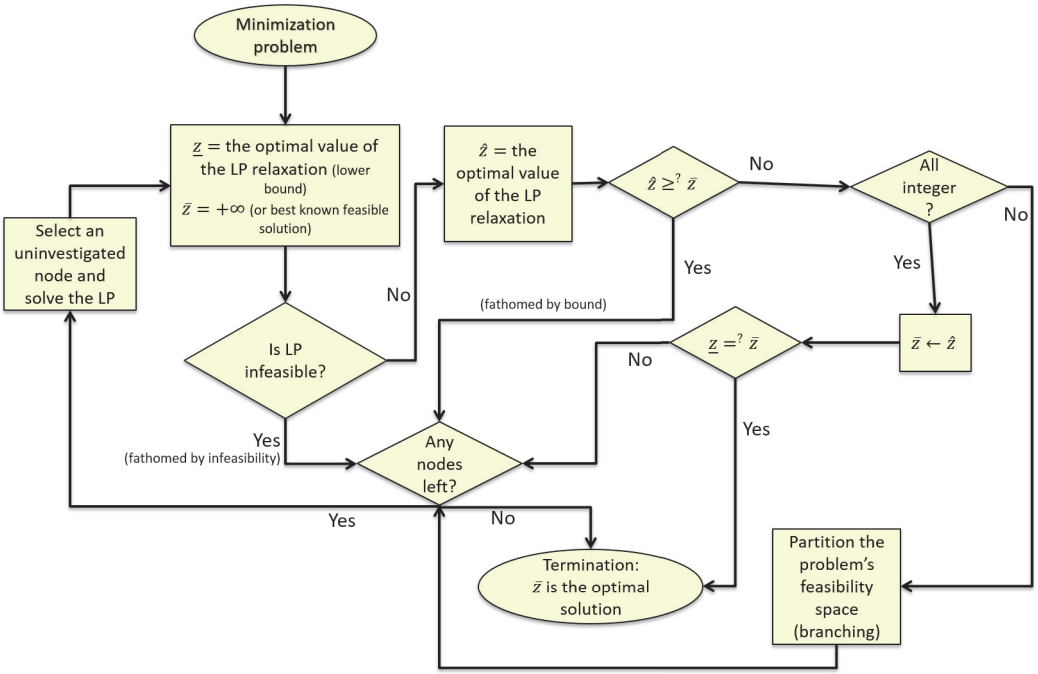


Figure 2.2: Branch-and-bound flowchart

generation approach is employed, utilizing RMP and PP formulations given before. In other words, branch-and-price integrates branch-and-bound and column generation methods in order to handle very large ILP problems. Branching occurs when the RMP is proved to have been solved to optimality, i.e., no other columns are eligible to enter. For a thorough study of this algorithm, the reader may refer to [Barnhart et al., 1998].

2.2 Routing and Wavelength Assignment Problem

Offline routing and wavelength assignment (RWA) is a problem arising in the provisioning or the dimensioning of optical networks. As stated before, today's communication industry is facing a rapid expansion of traffic requests. The following numbers provide an idea of what growth of traffic means in 2016. For instance, Global Internet Protocol (IP) traffic has increased more than fivefold in the past 5 years, and will increase nearly threefold over the next 5 years according to a Cisco study [Cisco, 2016]. A recent study [Durairajan et al., 2015] of the US long-haul fiber-optic infrastructure, based on the detailed fiber deployment maps from 5 tier-1 and 4 major cable providers (AT&T, Comcast, Cogent, EarthLink, Integra, Level3, Suddenlink, Verizon and Zayo),

suggests that the US long-haul fiber network has 273 nodes/cities, 2411 links, and 542 conduits[1] (with multiple tenants).

2.2.1 Networking Problems

The high bandwidth requirements in present day applications makes it necessary to perform several types of optimizations, on the existing and future networks. These optimizations might occur in provisioning or dimensioning a network, assuming a static environment. It is important to setup an optimized network in order to meet the required expectations that increase daily, while remaining cost-efficient as long as possible.

Networking problems can be classified into three categories [Mukherjee, 2006]:

- Traffic engineering: traffic is routed in an environment that is assumed to be in steady-state. The routing problem together with bandwidth assignment is also known as *bandwidth provisioning* or *provisioning*. The objective is usually minimizing the blocking rate.

- Network engineering: this is a "maintenance problem". Networks are constantly facing increased traffic, thus, sooner or later there will be some congestions in parts of the network. Network engineering seeks to find and resolve these problems by measuring the *exhaustion probability* and adding additional capacity or re-routing the traffic to overcome traffic congestion.

- Network planning: a *dimensioning* problem in which, assuming a steady-state environment, an estimation of the traffic in the future is used to design a network from scratch.

In this thesis, we focus on the provisioning problems in WDM and elastic optical networks, as well as network engineering in dynamic RWA problems.

2.2.2 WDM Networks

Wide-area WDM (Wavelength Division Multiplexing) networks built on the concept of RWA are envisioned to form the backbone component of the optical network infrastructure. WDM networks

[1] A conduit is defined as a "tube" or trench specially built to house the fiber of potentially multiple providers.

meet the high bandwidth requirements, by dividing the huge transmission bandwidth of an optical fiber into multiple communication channels called *wavelengths*. In a WDM network, a *lightpath* is defined by the combination of a routing path p and a wavelength λ.

Multiplexing

In a telecommunication network, one fundamental characteristic is the ability of transmitting simultaneously multiple data and connection requests. Therefore, multiple signals need to be transmitted on a shared (expensive) medium, as obviously it is not possible to have a medium for every single request. This is where multiplexing plays a very important role. There are three common types of multiplexing [Kuri, 2003]: Time Division Multiplexing (TDM, Elliot and Schunneman [1971]), Code Division Multiplexing (CDM, Hoffmann [2004]) and Frequency Division Multiplexing (FDM, Liu [2011]). As "wavelength" and "frequency" are in a direct inverse relation (frequency $\times$ wavelength $= c$, where c is the speed of light), WDM is basically FDM. The most common type of multiplexing in networks incorporating *light* as transmission medium, i.e., *optical networks*, is WDM [Sivalingam and Subramaniam, 2000]. Width of range of frequencies that can be transmitted on a medium is called its *bandwidth*. In FDM, bandwidth is divided into non-overlapping frequency sub-bands (sub-channels). For every signal, a specific frequency acts as a carrier (modulating), called *subcarrier*. As a result, multiple signals are combined into a single one. Figure 2.3 illustrates this type of multiplexing. A simplified WDM transmission system is illustrated in Figure 2.4.

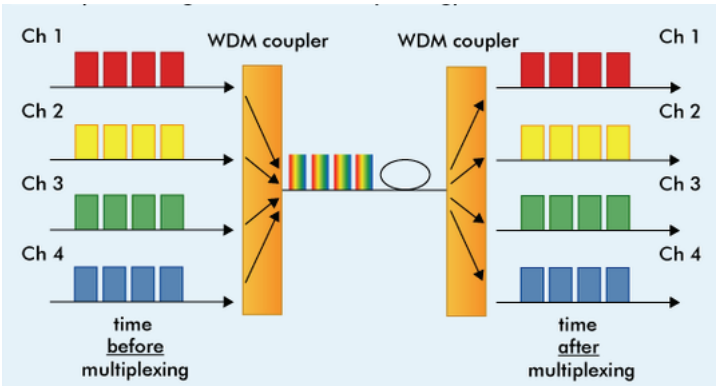


Figure 2.3: Wavelength division multiplexing [Pan-Dacom-Direkt, 2016]

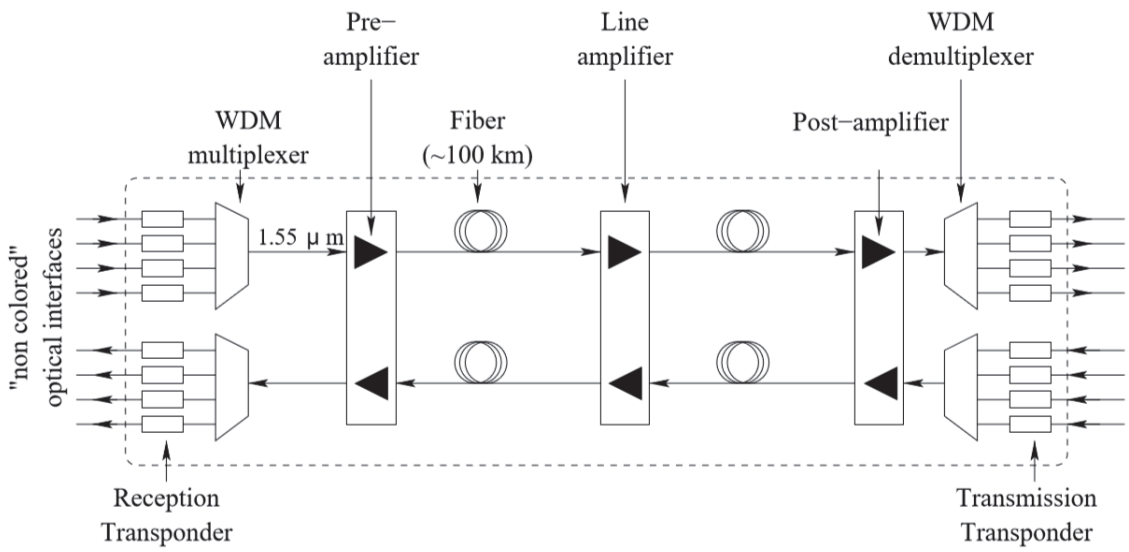


Figure 2.4: WDM transmission system [Kuri, 2003]

In FDM, in order to prevent the interference between the spectrum of subcarriers, or *inter-carrier interference*, guard bands are employed, resulting in an inefficient usage of bandwidth. Orthogonal Frequency Division Multiplexing (OFDM) [Saeki, 1999] is an FDM scheme, in which subcarriers are orthogonal to each other, hence removing the requirement for guard bands [Papadimitriou et al., 2003] (of course there is still the need to have guard bands between adjacent requests to distinguish them). Figure 2.5 shows the concept of orthogonality in subcarrier frequencies. In orthogonal subcarriers, the peak of one subcarrier corresponds to the null of the adjacent subcarrier.

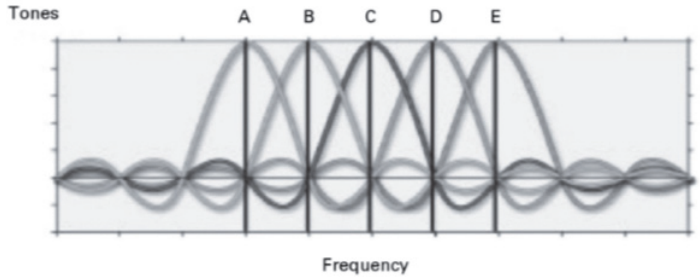


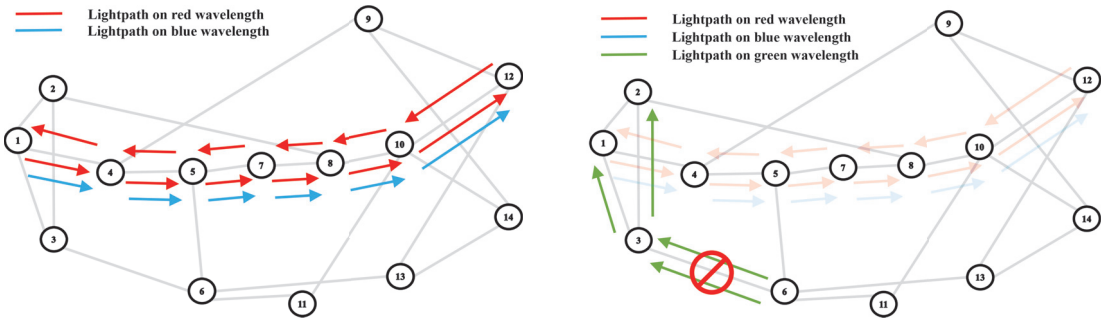
Figure 2.5: Orthogonal subcarrier frequencies [Hwang, 2009]

2.2.3 RWA Problems and Algorithms

The RWA problem consists of choosing a route and a wavelength for each connection request, so that no two connections using the same wavelength share the same fiber (for a formal definition, see Section 3). If different lightpaths traversing through the same fiber do not have distinct wavelengths, a *wavelength conflict* occurs [Jia et al., 2013]. An optical network utilizes optical connections, i.e., lightpaths, which traverse multiple fiber links and optical nodes, all with the same wavelength: this is the so-called *wavelength continuity* constraint [Ramaswami et al., 2009]. For each connection request, a lightpath is requested and provisioned. Thus, RWA problem assigns routes to the lightpath requests, and associates wavelengths to each of the links along those routes, while considering the following critical criteria:

- No wavelength conflict on links is acceptable, assuming every link contains 1 fiber.
- Wavelength continuity constraint on the route should be respected.

Figure 2.6 represents some valid and invalid lightpaths, under the assumption that every link is associated with two directional fibers, one in each direction.



(a) Valid lightpaths (b) Wavelength conflict

Figure 2.6: Lightpaths in a network

Wavelength continuity constraint can be relaxed if there are wavelength converters in the network. If not, every lightpath must utilize the same wavelength traversing on all of the fiber links

making its path [Bandyopadhyay, 2007]. Networks in which a wavelength converter is used to convert the wavelength of the data before forwarding it on the next link are referred to as wavelength-convertible networks [Ramamurthy and Mukherjee, 1998]. While there are also some works studying RWA problem in wavelength-convertible networks [Subramaniam et al., 1996, Lee and Li, 1993], it has been shown that wavelength conversion is not of great help for maximizing the grade of service (GoS) [Schupke, 2002, Jaumard et al., 2006b, Zhang et al., 2013], in spite of studies advertising some benefit of wavelength conversion, e.g., [Chu and Li, 2005].

Directed vs. Undirected Networks

In a communication network, traffic can be symmetric or asymmetric. In some cases, distinguishing between these two is not very obvious. For example, in a one-to-one video chat the traffic is symmetric, but it becomes asymmetric if there are more than two points in the conference. A study by Pesovic and Sharpe [2012] showed that, although traffic in networks is increasing both in downstream and upstream directions, the overall traffic today is asymmetric. However, most industries still have equipment suitable for symmetric traffic [Walkowiak et al., 2015]. For this reason, undirected problems are still of interest. In studying RWA problems, we will focus on directed networks and provide formulations and algorithms for them, although some modifications will be mentioned in Section 3.3.7 in order to show how to solve RWA problems on an undirected network.

Objective Function

Given a set of lightpath requests, two variants of the RWA problem have been studied in the literature:

- max-RWA: Here, the objective is to maximize the number of lightpath requests that can be routed with a given number of wavelengths, or in other words, within a given network transport capacity. This corresponds to maximizing the grade of service (GoS), equivalent to minimizing the blocking rate. max-RWA is useful for provisioning a network, while the number of available wavelengths is predetermined.

- min-RWA: Given a set of requests, the objective is to minimize the number of wavelengths

to route all the requests. min-RWA is used when planning/dimensioning of a network is considered.

In this thesis, we focus on the max-RWA problem, as provisioning is the main concern of the industry. However, most exact solution algorithms are easy to adapt for min-RWA, by modifying the objective function accordingly and adding a constraint to serve all the requests.

Static RWA Problem

With respect to traffic, RWA can be classified in two broad categories: static problems in which the set of connections are known in advance and dynamic problems in which a lightpath is set up for each connection request as it arrives, and the lightpath is released after some finite amount of time [Gerstel and Kutten, 1997]. Figure 2.7 illustrates these concepts.

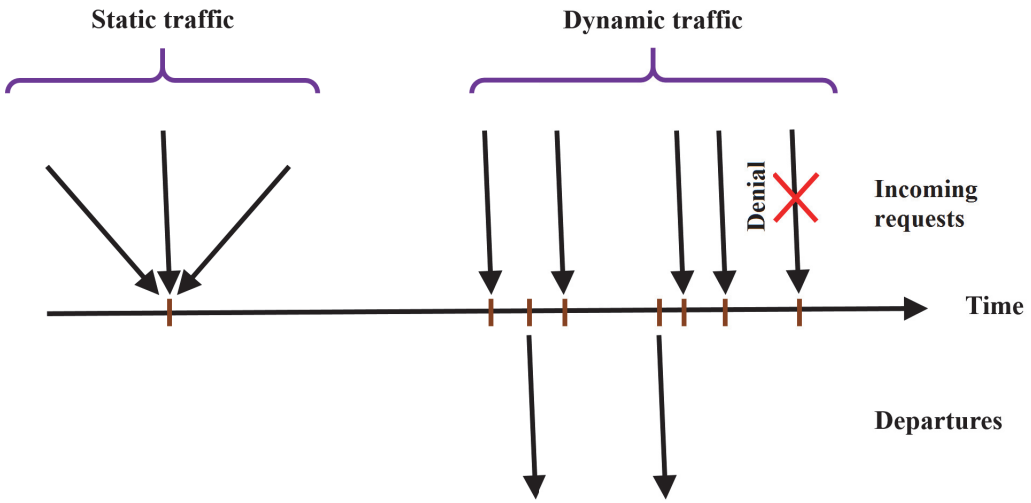


Figure 2.7: Static and dynamic traffic

RWA Literature Review: Heuristics and Meta-Heuristics

The static RWA problem is also known as the Static Lightpath Establishment (SLE) problem [Zang et al., 2000]. It is proved to be NP-complete [Ramaswami and Sivarajan, 1995] and can be divided into a routing subproblem and a wavelength assignment subproblem to become more tractable [Zang et al., 2000]. The common three approaches to tackle the routing subproblem are:

(1) Fixed routing [Chan and Yum, 1994, Birman and Kershenbaum, 1995, Subramaniam and Barry, 1997],

(2) Fixed-alternate routing [Birman and Kershenbaum, 1995, Ramamurthy and Mukherjee, 2002, Yao and Ramamurthy, 2004, Harai et al., 1997, Lin et al., 2006, Yates et al., 1997],

(3) Adaptive routing [He et al., 2007, Mokhtar and Azizoğlu, 1998, Pointurier et al., 2006, 2008, Ngo et al., 2004, Yoo et al., 2003].

The first two types are referred to as *static routing algorithms* and the third one is known as *dynamic routing algorithms* [Randhawa and Sohal, 2010]. Fixed routing is the simplest approach in which the routes are calculated offline. The main focus while using the fixed routing algorithms is on the wavelength assignment part. In fixed alternate routing approach, there is an ordered list of fixed routes to each destination node in addition to the fixed path. Alternate paths are utilized in the case that the previous fixed paths are not available. Common shortest path algorithms like Dijkstras algorithm can be used to calculate these fixed paths. In adaptive routing algorithms, each path is calculated online and based on the current network situation.

There are also different works investigating wavelength assignment subproblems. Like routing approaches, these algorithms are divided into static and dynamic categories. Static wavelength assignment problems are formulated as graph-coloring problem [Li and Simha, 2000, Kuri et al., 2003, Noronha and Ribeiro, 2006c] or bin packing problem [Skorin-Kapov, 2007]. In graph-coloring problem, nodes of a graph should be colored in such a way that no two adjacent nodes have the same color. The objective is to minimize the number of required colors. This number is called *chromatic number* of the graph. This problem is NP-complete [Pardalos et al., 1998]. Bin packing is also a classical combinatorial optimization problem in which n items with different sizes should be placed in minimum number of bins with the same capacity. This problem has also been shown to be NP-hard [Simchi-Levi, 1994]. First Fit (FF), Best Fit (BF), Best Fit Decreasing (BFD) and First Fit Decreasing (FFD) algorithms are proposed approaches based on bin packing problem for wavelength assignment problem [Pardalos et al., 1998]. The FF algorithm puts each item in the first bin with enough capacity. The BF algorithm packs each item into the bin with minimum capacity left for other items [Hsu et al., 2014]. Unlike FF and BF, the BFD and FFD algorithms are offline

algorithms. In these algorithms, due to full information about items, they are sorted nonincreasingly and the FF and BF algorithms are performed [Pardalos et al., 1998]. In order to apply FF, BF, FFD or BFD, lightpath requests represent items and duplicates of a graph represent bins such that each copy corresponds to one of the wavelengths [Hsu et al., 2014].

Krishnaswamy and Sivarajan [2001b] formulated the problem as an integer linear program (ILP). They solved the ILP for toy size networks and developed an algorithm to obtain a solution for large size problems by solving the LP relaxation of the ILP formulation. Bandyopadhyay et al. [2016] proposed six heuristics for static RWA problems and showed that these heuristics outperform the existing ones in terms of network resource conservation.

RWA Literature Review: Exact Algorithms

Among the papers that explored exact solutions, we find three classes of ILP formulations for RWA problem:

- Link-based: it is determined whether the traffic between a certain node pair is served by a link ℓ and a wavelength λ. In other words, the goal is to assign available links and wavelengths to the node pairs. In these formulations, routing is explicitly done. Wavelength continuity is guaranteed by adding appropriate constraints.

- Path-based: for each demand, a pre-defined set of all paths or a subset of them is calculated and some of them are used as serving paths, along with assigning wavelengths to them. Path-based formulations do not need the wavelength continuity constraints.

- Configuration-based: for a given wavelength, all the paths are considered, then routing is done implicitly by choosing non-overlapping paths. For every wavelength, at most one configuration is chosen. This ensures the wavelength continuity.

Comparison of link and path based formulations can be found in [Jaumard et al., 2007, 2009] with the objective of maximizing the grade of service. The authors showed that very often, but not always, the linear programming (LP) bound is equal to the ILP value, with however an LP solution that is usually with fractional values. Nevertheless, it means that the LP value can be an excellent

lower bound for the min-RWA (upper bound for the max-RWA) problem. Another comment was that, in the optimal solution, most of the chosen paths are shortest paths. These observations were made while experimenting with small/medium size instances. But as it is shown in Chapter 5, these results are challenged as we move to large size instances. It is therefore required to enhance the solution methods that rely on those observations.

The largest instances that have been solved exactly so far for mesh networks are the EON network (20 nodes, and 39 optical links) with 24 wavelengths, the Brazil network (27 nodes and 70 optical links) with 14 wavelengths, both for the GoS maximization [Jaumard et al., 2007]. In terms of heuristics, large instances have been solved by Martins et al. [2012]: 26 realistic instances with up to either 90 nodes or 175 links, where the quality of the solution is assessed by comparing with lower bounds obtained using the formulations in Jaumard et al. [2007, 2009]. Other large instances have been solved with different heuristics or meta-heuristics in [Noronha and Ribeiro, 2006a, Kogantia and Sidhu, 2014, Noronha and Ribeiro, 2006b], or with ILP models using a limited pre-computed set of paths and a rounding off technique to derive integer solutions, e.g., Banerjee and Mukherjee [1996].

Several surveys have been written on the RWA problem, where the reader can find a comprehensive survey of the various mathematical models that have been investigated, see, e.g., [Jaumard et al., 2007] for symmetrical traffic and [Jaumard et al., 2006a] for asymmetrical traffic. Other recent surveys can be found in [Miliotis et al., 2003] and [Zang et al., 2000].

There are several papers specifically using column generation technique to solve RWA problems. Here, we will have a review on some of these papers separately.

Column Generation in RWA

Lee et al. [2000] considered RWA in ring networks without wavelength conversion in order to minimize the number of wavelengths. After solving the LP relaxation of the problem by using the column generation technique, they apply branch-and-price to obtain the optimal solution. Lee et al. [2002] proposed an algorithm based on column generation that is a unified approach to routing and wavelength assignment. They implemented the algorithm on 3 networks with 18 nodes and 39 links as the largest one. Several column generation formulations for RWA problem have been reviewed in

Jaumard et al. [2009]. Vignac et al. [2009] proposed a two stage hierarchical optimization algorithm in which grooming and routing problems are solved by column generation and then wavelength assignment is performed. The objective is to minimize the number of optical hops. Colombo and Trubian [2014] used column generation to solve a multicast RWA. They proposed two exact algorithms and one tabu search heuristic to address the pricing problem. They solved 900 instances with number of nodes in $\{20, 40, 60, 80, 100\}$ and considering at most 5 wavelengths. Experimental results show that in most cases the solutions obtained from the LP relaxation are integral. Duhamel et al. [2016] proposed two heuristics based on column generation technique in order to maximize the number of connections with continuity constraints.

It should be mentioned that, employing column generation in solving RWA problems was initiated by Lee et al. [2000], then became practical in Jaumard et al. [2009]. In Lee et al. [2000], the decomposition was done with a non-scalable pricing problem, i.e., an independent set problem for the pricing where each node of the associated graph is a potential path. This results in an exponential number of variables, which leads to a non-practical pricing problem unable to generate/guarantee an optimal or ε-optimal solution. Jaumard et al. [2009] made the column generation approach practical by providing a link formulation for the pricing problem. Still, this solution method needs very long CPU times and is unable to solve very large size instances.

Column Generation Modeling vs. Various ILP Formulations

In Jaumard et al. [2007], a theoretical and experimental study is proposed for comparing the different LP bounds provided by the various existing ILP RWA models: all LP bounds are shown to be exact. Moreover, a comparison of the models is made with respect to their numbers of variables and constraints: it clearly shows that the CG model is the most economical one. We provide a synthesis of all the results discussed in Jaumard et al. [2007, 2006a, 2009] in Table 2.1 and include the CG ILP model for the $\max$-RWA problem (described in Section 3.2.2), see Appendix A for the other ILP models.

ILP models	# variables	# constraints
Link formulations, results adapted from Table 1 in Jaumard et al. [2007] using the formulations in Appendix A		
RWA_k	Link formulation #1: request indexed	
	$W\|K\|(1+\|L\|)$	$(n+2+\|L\|)W\|K\|+\|L\|W+\|K\|$
RWA_s	Link formulation #2: source indexed	
	$nW\|L\|$	$n(n-1)(W+2)+\|L\|W$
RWA_sd	Link formulation #3: node pair indexed	
	$n(n-1)W\|L\|$	$n(n-1)(n-2)W+2n(n-1)+W\|L\|$
Path formulations		
z_{PATH}	$W\|\mathcal{P}\|$	$W\|L\|+\|\mathcal{SD}\|$
Configuration (CG) formulations (see Sections 3.2.2 and 3.3 for the details)		
CG ILP$^{(\star)}$	Master Problem	
	$2\|\mathcal{SD}\|+1$	$2\|\mathcal{SD}\|+1$
	Pricing Problem (Link Formulation)	
	$\|L\|\,\|\mathcal{SD}\|$	$\|L\|+2\|\mathcal{SD}\|+(n-2)\|\mathcal{SD}\|$
	Pricing Problem (Path Formulation with One Shortest Path per Node Pair)	
	$\|\mathcal{SD}\|$	$\|L\|+\|\mathcal{SD}\|$

$n, L, W, \mathcal{SD}$: see Section 3.1 for their definition
$\mathcal{P}$ = set of all possible paths in the optical network
K = set of individual requests
$^{(\star)}$ with a column generation implementation in which we keep only the basis variables, see, e.g., Chvatal [1983]

Table 2.1: Comparison of the number of variables and constraints in the ILP RWA models

Incremental Dynamic RWA Problem

In dynamic WDM networks, connection arrivals and departures are stochastic in nature and connection provisioning is accomplished via online (or dynamic) RWA algorithms. Hence, it is likely that after some time, some of the already provisioned connections may become sub-optimal with respect to the current network provisioning. To enhance the grade of service, it may be desirable to seek a new RWA solution for all (or a subset) of the active connections. Migrating traffic from their previously optimized provisioning to a new one is referred to as *traffic migration* or *traffic defragmentation*. Other terms that are used in the literature are *re-routing* or *lightpath re-arrangement*: they refer to the action of altering the physical path and/or wavelength of an established connection.

Two types of dynamic RWA problems should be distinguished:

- Short-term dynamic or daily RWA problem, in which add/drops in connections represent the variations of traffic during a day or a week (e.g., the difference between the traffic load during the weekdays and weekends).

- Evolutionary dynamic RWA problem, following the rapid growth of the traffic, in which

incoming requests are much more significant than outgoing ones. This can be captured by *incremental* dynamic RWA problem that is only concerned with incoming requests.

While studying the dynamic traffic, our focus will be on incremental dynamic RWA problem. It should be mentioned that, the provided solution process can be adapted to daily RWA problem rather easily, but it is not done in this thesis. Therefore, we will look at lightpath re-arrangement when a new batch of connection requests comes, and minimize the number of lightpath re-arrangement so as to keep the network in a state that corresponds to the maximization of the grade of service. In other words, each new connection request or batch of connections is handled simultaneously with lightpath re-arrangement in order to maximize the GoS.

While reviewing the previous work, we will limit ourselves to the studies with no wavelength conversion. As stated before, wavelength conversion is not of great help for maximizing the GoS. Indeed, authors looking at exact methods usually conclude that wavelength conversion does not help, while authors considering heuristics have an opposite conclusion. Many studies have been conducted on re-routing or lightpath re-arrangement, all of them with heuristics. The early ones were limited to rings or torus (see, e.g., [Saengudomlert et al., 2006]). Other studies look at general mesh networks (see, e.g., [Lee and Li, 1996, Datta et al., 2003]) with many of them made before 2000, with the consideration of shortest paths only, and sometimes with a unique arbitrarily chosen shortest path for a given node pair, while, as is observed in Chapter 5, several shortest paths do exist in most networks, and even more second shortest paths that are only one hop longer than the shortest paths. More recently, some authors looked at the cases with scheduling [Koubaa and Gagnaire, 2010, Zhang et al., 2010], impairment [Amdouni et al., 2012] or grooming [Yao and Ramamurthy, 2008] considerations.

2.3 Routing and Spectrum Assignment Problem

According to Kaminow et al. [2013], today in a common 100 Gb/s fixed channel grid, wavelengths (channels) are spaced at 50 GHz, as is recommended by ITU [2012]. This scheme is unlikely to accommodate bit-rates beyond 100 Gb/s [Gerstel et al., 2012]. However, Figure 2.8 shows that, based on the historical trends, soon there will be requirements for channels with higher capacities,

beyond 400 Gb/s and 1 Tb/s [Kaminow et al., 2013]. The historical data in Figure 2.8, reveals that up to 100 Gb/s, the channel capacity has always been larger than the Ethernet port speed (the input), as it should be, since the transportation capacity has to be larger than or equal to the the amount of submitted data packets. But the trend suggests that Ethernet port speed might exceed the channel capacity after 100 Gb/s point. Note that, the single carrier bit-rate can be increased to up to 200 Gb/s using a higher order modulation, but the applicable transmission distance becomes very short [López and Velasco, 2016].

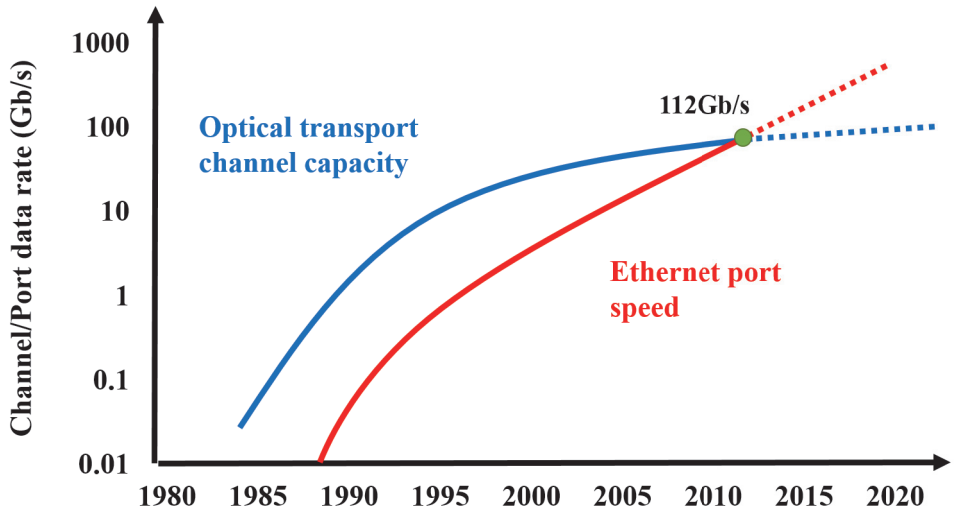


Figure 2.8: Channel capacity vs. Ethernet port speed [Kaminow et al., 2013]

In order to face the steady growth of the optical networks, network operators are now moving to flexible or elastic optical networking. In such networks, the optical spectrum is used more efficiently by allowing finer grid spacing, resulting in sub-streams, called *slots*. There is still some debates on the partitioning of the spectrum, with 6.25 GHz and 12.5 GHz being the main candidates of width slots, though 6.25 is not yet operational today [López and Velasco, 2016]. ITU [2012] recommends a frequency slot with 6.25 GHz nominal central frequency and 12.5 GHz width. Figure 2.9 illustrates the efficiency of a flexible grid over a fixed one. In a fixed grid, the entire channel capacity is spent on a single bit-rate demand. The smaller the bit-rate demand (e.g. 10 Gb/s) the more inefficient is the spectrum usage. In addition, requests are granted using adjacent slots, and the end of the granted slots is marked by a *guard band*. This mechanism allows for serving high bit-rate requests as big as 400Gb/s and 1Tb/s. Interested reader may refer to the surveys by [Talebi et al., 2014] and

[Sócrates-Dantas et al., 2014] for further discussion.

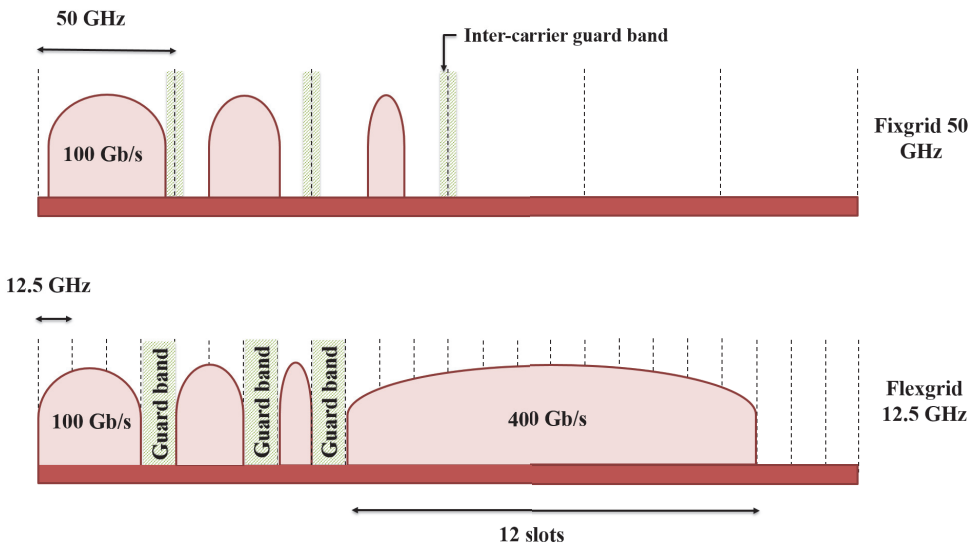


Figure 2.9: Fix and flexible frequency grids

With elastic optical networks, the challenge is optimizing the spectrum usage through the so-called routing and spectrum assignment (RSA) problem. It is a much more difficult problem than the classical routing and wavelength assignment problem. Although we will provide practical algorithms for solving realistic data instances in current wavelength-routed optical networks in Chapter 3, network utilization efficiency is limited due to their rigid nature. There are two main issues with the current WDM networks:

(1) One limitation is related to the worst case design in current wavelength-routed optical networks. In these networks, all data streams with the same rates occupy the same spectral width regardless of the path's distance. This fact will result in large transport margins at receiving end for most of the optical paths which are less than the longest path [Jinno et al., 2010].

(2) The other limitation, apart from the worst case design, comes from mismatch of granularities between the client layer and the physical wavelength layer. Network operators are very much concerned by the efficient utilization of the already deployed network capacity. Current wavelength-routed optical path networks require the full allocation of wavelength capacity to lightpaths between node pairs even when the traffic is not sufficient to fill the entire capacity of wavelength. It leads to inefficient capacity utilization, an issue expected to become even

more significant with the deployment of higher capacity WDM networks, and driven by the imminent optical capacity crunch [Chen et al., 2014].

Distance-adaptive resource allocation algorithms are the solutions to the first issue and *Orthogonal* Frequency Division Multiplexing (OFDM), also referred to as *spectrum-sliced elastic optical path network* or SLICE is the solution known to address the second issue. OFDM modulation technique allows the allocation of the bandwidth resources with granularity finer than a wavelength [Gerstel et al., 2012, Chen et al., 2014]. In other words, it can generate a large number of sub-streams (slots) by splitting a data stream and adaptively assign them to the end-to-end optical path based on the client data rate and the available spectral resources [Shieh, 2011, Jinno et al., 2009, Kozicki et al., 2009, Sone et al., 2009]. Note that, because of using the OFDM mechanism, there is no need for inter-carrier guard bands.

We consider the provisioning problem of an OFDM optical network, where connections are provisioned for their requested rate by elastically allocating spectrum using a variable number of OFDM subcarriers and choosing an appropriate modulation level taking into account the transmission distance. This corresponds to the so-called static RSA problem. Static (or offline) RSA arises when a set of traffic demands is known in advance, while in dynamic (or online) problems, optical paths are set up as needed [Talebi et al., 2014]. In RSA problem, the requested traffic is served using a number of slots that need to be *contiguous*. This requirement, referred to as *contiguity constraint*, separates the RSA problem from RWA problem, making it much more complex. Static RSA has been proved to be NP-hard in different studies [Shirazipourazad et al., 2013, Wang et al., 2011b]. It should be mentioned that, the common objective function for RSA problems is maximizing the *throughput*, unlike RWA that maximizes the GoS (see Section 4.2). Throughput is defined as the amount of bandwidth requests successfully transmitted over the network and is often measured in bits per second (b/s).

RSA has been investigated under different circumstances, hence several modeling and solution approaches have been proposed to tackle this problem. Wang et al. [2011b] formulated the RSA problem using an integer linear programming framework to solve small size instances and proposed two heuristic algorithms to obtain practical solutions. The objective function is to minimize the

maximum number of subcarriers required on any fiber of an OFDM-based spectrum-sliced elastic optical path network. Klinkowski and Walkowiak [2011] also formulated the RSA problem as an ILP using path-link approach [Pióro and Medhi, 2004]. Since the solution is not attainable for large size instances, they proposed a heuristic for these cases. Christodoulopoulos et al. [2010] presented a heuristic that served the connection requests one by one and studied the problem under two different ordering policies, along with a simulated annealing metaheuristic to improve the ordering. Takagi et al. [2011] proposed a dynamic routing and frequency slot assignment algorithm for SLICE networks that employ distance adaptive modulation.

Several attempts has been made in the literature to solve the problem exactly. Christodoulopoulos et al. [2010] presented an optimal ILP-RSA algorithm and a decomposition method in which RSA is broken into two subsequent subproblems, namely, Routing and Spectrum Allocation (R+SA). Other attempts at deigning an exact algorithm for RSA problem, mostly focus on the techniques based on column generation method. These will be investigated further in the next subsection.

Other works in the context of RSA are as follows. Patel et al. [2012] addressed the routing, wavelength assignment, and spectrum allocation problem (RWSA) with the objective of maximizing spectral efficiency. They formulated the RWSA problem using an integer linear program. After proving the NP-completeness of the RWSA problem, three efficient algorithms have been proposed. Walkowiak et al. [2014] addressed an offline problem of RSA with dedicated path protection. They applied a meta-heuristic approach and developed a Tabu Search-based algorithm (TS), and a hybrid Adaptive Frequency Assignment-TS (AFA/TS) algorithm to minimize the width of the spectrum resources.

Column Generation in RSA: Different Possible Configuration Definitions

Several authors investigated ILP formulations with and without explicit modulation concerns, however early proposed formulations are not scalable and work only on toy examples with a very limited number of slots. There are some more recent studies considering some column generation models with different decomposition schemes. Ruiz et al. [2013] used column generation technique to solve RSA, with the minimization of the number of denied demands and the amount of

unserved bit-rate. They are able to solve data instances with up to 96 slots, and an overall demand distributed over a set of 180/210 node pairs in the Spain network (21 nodes, 37 links, see Section 5.2.1). Klinkowski et al. [2014] focused on presenting a stronger formulation with valid inequalities (cuts). They developed and combined a clique cut generation procedure with a column generation technique. They evaluated the performance of the algorithm on a network with 21 nodes and 35 links and compared the results with a basic column generation algorithm. Klinkowski and Walkowiak [2015] formulated the RSA problem as a mixed-integer program and solved it using a branch-and-price algorithm. In order to enhance the performance of their algorithm, a simulated annealing-based heuristic was employed in the search for upper bound solutions. Moataz [2015] investigated a decomposition based on slot configurations, which gathers all the *slot-paths* using a given slot (with a slot-path being a path together with a slot), in an attempt to generalize the wavelength configurations used in the context of the RWA problem [Jaumard et al., 2009], with one slot acting as one wavelength in the formulation. However, results were quite disappointing, and the resulting algorithm was not scalable due to a very slow convergence of the column generation for solving the LP relaxation.

In the column generation framework of Ruiz et al. [2013], a pricing problem corresponds to a multiple slot-path. A multiple slot-path, is a path, together with a set of consecutive slots (in Ruiz et al. [2013] it is referred to as "lightpath" but we change the term to avoid confusion with the concept of lightpath in the context of RWA problem). The advantage of such formulation is that it is easy to take care of contiguity constraints, however it is applicable to one demand at a time. The main drawback is that too many columns are generated, at least $O(n^2)$, where n is the number of the nodes. Furthermore, for a given node pair, there are as many slot-paths as the number of potential paths and the possible positions in the slot grid. The strength of the algorithm by Moataz [2015], is its ability to handle more than one demand at a time, which potentially leads to less calls to the pricing problem. The major drawback is the presence of the contiguity constraints in the master problem, that are very expensive in terms of their number. As a result, the algorithm was not scalable, and had a very slow convergence of the column generation for solving the LP relaxation. We will present a more balanced decomposition that addresses the drawbacks of the two previously proposed decomposition schemes and outperforms their algorithms.

Directed vs. Undirected

As mentioned before, traffic today is considered to be asymmetric. However, the state of the art algorithms for RSA problems only have been able to tackle the instances with undirected links, and even for this type, only small size instances have been solved. In this thesis, while studying RSA problem, we will focus on undirected networks and design a solution algorithm to solve realistic instances that their size matches up to today optical networks.

Guard Band

One assumptions that has been considered widely in the studies is related to the guard band requirements. The guard bands are used in order to facilitate the signal filtering. Any two adjacent wavelength are separated by guard band frequencies [Ramaswami et al., 2009]. Chen et al. [2013] proposed a multipath RSA and showed the effect of guard band size. The results showed that multipath routing is beneficial for elastic optical networks with small guard bands. Wang et al. [2011a] showed that considering fixed size guard bands may lead to underutilization of spectrum resources. Most of the mathematical models have been formulated without guard band requirements except few studies e.g., Jaumard and Daryalal [2016], Velasco et al. [2012].

Conclusions of the Chapter

Existing exact solutions for RWA problem, in both static and dynamic cases, are not scalable for real instances in today optical networks. The gap between the size of the problems that these algorithms are able to solve and the instances required by the industry, will only grow considering the expansion in traffic requests. We will address this issue by improving the existing exact solution approaches, leading to design of highly efficient algorithms, both exact and heuristic, that are able to solve realistic size instances.

To further increase the efficiency of spectrum usage, elastic optical networks are emerging. There are three major drawbacks in the current literature works for RSA problem:

(1) Many of the proposed algorithms are heuristic. While heuristics allow faster solution of difficult combinatorial problems, they provide solutions with no estimation of their quality,

i.e., how far they are from an optimal solution.

(2) The proposed exact formulations are not scalable.

(3) Few of the studies consider the guard band requirement in their mathematical models.

In this study we will improve the scalability of exact solutions for RSA problem, alongside with considering the guard band in our formulations.

Chapter 3

Routing and Wavelength Assignment Problem

This chapter considers the routing and wavelength assignment problem. The original contribution of this chapter lies in the enhancement of the previously proposed column generation models. This includes a new solution scheme, that make use of three different ILP models at different stages of the solution process. In addition, new heuristics are proposed for (*i*) generating quickly promising wavelength configurations, (*ii*) generating sets of potential paths adapted to the length of the shortest paths and overall demand of each bandwidth request.

The plan of this chapter is as follows. Section 3.1 states the RWA problem, introducing the concepts and notations used both in static and dynamic case. Section 3.2 provides decomposition formulations for RWA problem considering both static and dynamic traffic. Sections 3.3 and 3.4 propose solution algorithms for the mathematical formulations obtained in Section 3.2, for static and dynamic RWA problems, respectively.

3.1 Problem Statement

Consider a WDM optical network represented by a multigraph $G = (V, L)$ with node set V indexed by v, where each node is associated with a node of the physical network, and with link set L indexed by ℓ where each link is associated with a fiber link of the physical network: the number

of links from v to v' is equal to the number of fibers supporting traffic from v to v'. Connections and fiber links are assumed to be directional, and the traffic to be asymmetrical. The set of available wavelengths is denoted by Λ, and is indexed by λ with $W = |\Lambda|$. The traffic is defined by a $n \times n$ matrix D where D_{sd} defines the number of requested bandwidth units (i.e., number of wavelengths) from v_s to v_d. All wavelengths are assumed to have the same transport capacity. Let $\mathcal{SD} = \{(v_s, v_d) \in V \times V : D_{sd} > 0\}$ be the set of node pairs with traffic. The set of outgoing (resp. incoming) fiber links at node v is indicated by $\omega^+(v)$ (resp. $\omega^-(v)$). We assume that the same wavelength is used from the source to the destination for all connection requests. Note that it has been shown (see Jaumard et al. [2005, 2006b]) that wavelength conversion (i.e., multiple-hop connections) does not help very much in order to reduce the blocking rate (max-RWA problem, see page 18).

The RWA problem is considered under two different traffic assumptions: static and dynamic, described in the next two subsections.

3.1.1 RWA Problem with Static Traffic

The static RWA problem applies to the case in which the set of connections is known in advance. This problem can be formally stated as follows: given a multigraph G corresponding to a WDM optical network, and a set of requested connections, find a suitable lightpath (p, λ) for each granted connection, where a lightpath is defined by the combination of a routing path p and a wavelength λ, so that no two paths sharing a fiber link of G are assigned the same wavelength. We study the objective of maximizing the number of granted connections, leading to the max-RWA problem.

3.1.2 RWA Problem with Dynamic Traffic

The dynamic RWA problem deals with the problems in which connection requests arrive dynamically and remain for some amount of time before departing. Very often, optical connections are leased for long periods of time (e.g., weeks or months), and thus new connection requests come with significant lead time to set-up. In addition, they are often configured manually. In this study, we limit ourselves to incremental traffic and will assume new connection requests come in batches.

In the context of incremental traffic, we look at the traffic increase over a set T of time periods.

The dynamic RWA problem can be formally stated as follows: at time period $t \in T$, given G defined earlier, a matrix D^{t-1} containing the traffic requests at time period $t-1$ (both granted and denied), and a set of newly requested connections indicated by a matrix D^{NEW}, assign available lightpaths to the new incoming connection requests such that the wavelength continuity constraint is respected over available wavelength resources. Note that the total amount of traffic request at time period t is obtained by the summation on the granted and denied traffic at time period $t-1$, and newly received traffic at t. We consider the objective of maximizing the GoS, i.e., dynamic max-RWA problem. Our objective is to investigate further the spectrum usage under different traffic increment rate and lightpath re-arrangement assumptions.

At each time period $t \in T$, following parameters are at hand.

- $D^t = (D^t_{sd})_{(v_s,v_d)\in\mathcal{SD}^t}$, the overall set of traffic request at time t, where the requested traffic from node v_s to v_d at time period t is defined by:

$$D^t_{sd} = D^{t-1}_{sd} + \text{new granted requests during } t + \text{new denied requests during } t, \tag{22}$$

  and the set of node pairs with traffic is $\mathcal{SD}^t = \{(v_s, v_d) : D^t_{sd} > 0\}$, i.e., the set of node pairs (v_s, v_d) for which there are some traffic requests from v_s to v_d at time t.

- The set of new requests that are described by an $n \times n$ matrix $D^{\text{NEW},t}$ where $D^{\text{NEW},t}_{sd} = D^t_{sd} \setminus \text{GoS}^{t-1}_{sd}$ defines the number of newly requested connections from v_s to v_d at time t, and GoS^{t-1}_{sd} is the set of granted requests from v_s to v_d out of D^{t-1}. Consequently, $\mathcal{SD}^{\text{NEW},t} = \{(v_s, v_d) : D^{\text{NEW},t}_{sd} > 0\}$.

- The set of legacy requests which are already provisioned, and described by an $n \times n$ matrix $D^{\text{LEG},t} = (\text{GoS}^{t-1}_{sd})_{(v_s,v_d)\in\mathcal{SD}^{(t-1)}}$.

- $\Lambda^{\text{USED}}_{t-1}$, the set of used wavelengths for provisioning the traffic up to time period $t-1$, i.e., GoS^{t-1}.

In order to alleviate the notation, and since we deal with a generic time period t, through the remainder of the thesis the index t is dropped from $D^{\text{NEW},t}$, $D^{\text{LEG},t}$ and $\mathcal{SD}^{\text{NEW},t}$. Therefore, if there is no possible confusion in time period, D^{NEW}, D^{LEG} and $\mathcal{SD}^{\text{NEW}}$ are used. In the following, we will

discuss two assumptions based on which two different formulations for the dynamic max-RWA will be given.

Incremental Traffic without Lightpath Re-arrangement

Assume that no lightpath re-arrangement of an already provisioned request is allowed. As a result, the provisioning of the new connection requests (D^{NEW}) needs to be made using lightpaths that do not conflict with those already used for the connection requests of D^{LEG}. The new requests can be provisioned either on the already used wavelengths ($\Lambda_{t-1}^{\text{USED}}$) if enough spare resources are available, or on an additional wavelength as long as we do not exceed the number of available wavelengths (W).

Incremental Traffic with Lightpath Re-arrangement

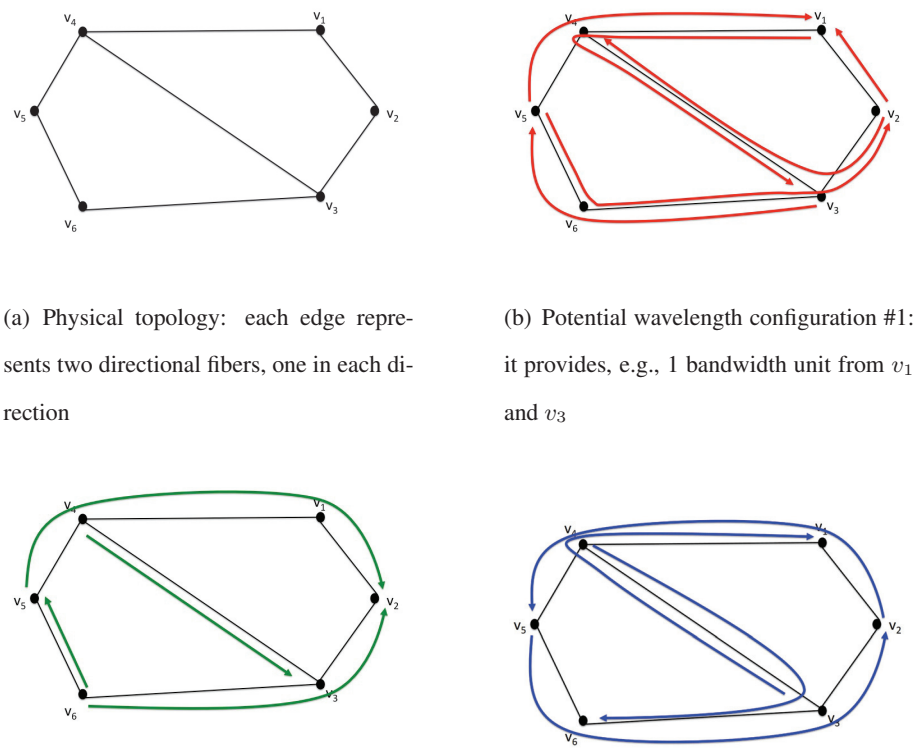
Under the scenario that lightpath re-arrangement is allowed, the objective is to provision a new batch of connection requests while allowing some minimum lightpath re-arrangement in order to maximize the GoS. Again, the new lightpaths can be defined using either the wavelengths already activated for the provisioning of D^{LEG}, or wavelengths newly made available for the new traffic. Lightpath re-arrangements consist in either modifying the wavelength of an existing lightpath, or considering a new routing and wavelength.

3.2 Column Generation Formulation for RWA Problem

As mentioned before, several authors have already investigated modeling the RWA problem with a decomposition model, within the framework of exact solution schemes, (see page 22). We revisit those models here, with the goal of enhancing them in order to solve much larger RWA instances. We first recall the decomposition optimization formulation, CG ILP model, based on maximal independent set, as initially proposed by Lee et al. [2000] and improved by Jaumard et al. [2009], and then discuss how to improve it further.

3.2.1 Wavelength Configuration

Lee et al. [2000] introduced the concept of independent routing configurations. Each configuration is associated with a set of paths that are independent from each other, so that all of them together can be used for satisfying a given fraction of the connections with the *same* wavelength. Within a wavelength configuration, routes must be pairwise link disjoint. Jaumard et al. [2009] further improved the concept by considering only maximal configurations. We call these configurations *wavelength configurations*.



(a) Physical topology: each edge represents two directional fibers, one in each direction

(b) Potential wavelength configuration #1: it provides, e.g., 1 bandwidth unit from v_1 and v_3

(c) Potential wavelength configuration #2

(d) Potential wavelength configuration #3

Figure 3.1: Potential wavelength configurations

A wavelength configuration c can be formally represented by a non-negative vector a^c such that a^c_{sd} = number of bandwidth units from v_s to v_d that are supported by configuration c. In an static RWA problem, $a^c_{sd} \leq D_{sd}$ for $(v_s, v_d) \in \mathcal{SD}$. In the case of incremental traffic we have $a^c_{sd} \leq D^t_{sd} - \text{GoS}^{t-1}_{sd}$ for $(v_s, v_d) \in \mathcal{SD}^{\text{NEW}}$ if re-arrangement is not allowed, and $a^c_{sd} \leq D^t_{sd}$

otherwise. See Figure 3.1 for an illustration of a wavelength configuration.

For static traffic, we denote by C the set of all possible wavelength configurations. In time period t of dynamic RWA problem, traffic requests of D^{t-1} are already provisioned with the wavelengths of $\Lambda_{t-1}^{\text{USED}}$. For $\lambda \in \Lambda_{t-1}^{\text{USED}}$, we define a set of configurations C_λ such that each configuration $c \in C_\lambda$ contains the legacy lightpaths associated with D^{t-1} in addition to some new lightpaths associated with $D^{\text{NEW}} = D^t \setminus \text{GoS}^{t-1}$. As we might have additional available wavelengths, in the dynamic case let C be the set of configurations associated with a generic wavelength $\lambda \in \Lambda \setminus \Lambda_{t-1}^{\text{USED}}$.

Decision Variables

We use two sets of variables. The first set of variables, $z_c \in \mathbb{Z}^+$, enables the selection of the best configurations and of their number of occurrences (i.e., to how many wavelengths they apply). Note that, in dynamic case, z_c associated with wavelength configurations for $\lambda \in \Lambda_{t-1}^{\text{USED}}$ is a binary decision variable as each set C_λ is associated with a specific wavelength.

The second set of variables, y, compute the GoS for each node pair, so that their sum provides the overall GoS. In static max-RWA, y_{sd}, determines the number of granted requests $(v_s, v_d) \in \mathcal{SD}$. With incremental traffic, we have $0 \leq y_{sd} \leq D_{sd}^t - \text{GoS}_{sd}^{t-1}$ for $(v_s, v_d) \in \mathcal{SD}^{\text{NEW}}$ if re-arrangement is not allowed, and $\text{GoS}_{sd}^{t-1} \leq y_{sd} \leq D_{sd}^t$ for $(v_s, v_d) \in \mathcal{SD}^t$ otherwise.

3.2.2 Formulation of RWA Problem with Static Traffic

The basic model of static max-RWA is written as follows. Assuming that the configurations are at hand, the model selects the best wavelength configurations to maximize the grade of service, i.e., the number of granted connections.

$$\max \sum_{(v_s,v_d)\in\mathcal{SD}} y_{sd} \tag{23}$$

subject to:

$$\sum_{c \in C} z_c \leq W \tag{24}$$

$$y_{sd} \leq \sum_{c \in C} a_{sd}^c z_c \qquad (v_s, v_d) \in \mathcal{SD} \tag{25}$$

$$y_{sd} \leq D_{sd} \qquad (v_s, v_d) \in \mathcal{SD} \tag{26}$$

$$z_c \in \mathbb{Z}^+ \qquad c \in C \tag{27}$$

$$y_{sd} \geq 0 \qquad (v_s, v_d) \in \mathcal{SD}. \tag{28}$$

Observe that, since $D_{sd} \in \mathbb{Z}^+$, we have $y_{sd} \in \mathbb{Z}^+$. However, we do not need to explicitly enforce it.

Constraints (24) ensure that we do not select more wavelength configurations than the number of available wavelengths. Constraints (25) compute the GoS for node pair (v_s, v_d): equality is enforced with the combination of (23) and (25), and we do not explicitly enforce the equality constraints. This is due to the fact that in practice, having an equality constraint is equivalent to two inequality constraints in reverse direction. Thus, solving a problem that has equality constraints is often harder. The bounds in Constraints (25) prevent the variables y_{sd} to become more than the accumulative number of granted requests over all the configurations, while the maximization objective forces them to achieve the highest possible value. Hence, there is no need for the equality constraints and we can ease the solution of the linear relaxation of (23) - (28) by avoiding them. Constraints (26) prevent from granting more connections than requested. Constraints (27) and (28) define the domains of the variables.

Note that Model (23) - (28) can be written with only one set of variables in a very compact formulation, which is less self explanatory, as follows:

$$\max \sum_{c \in C} \sum_{(v_s, v_d) \in \mathcal{SD}} a_{sd}^c z_c \tag{29}$$

subject to:

$$\sum_{c \in C} z_c \leq W \tag{30}$$

$$\sum_{c \in C} a_{sd}^c z_c \leq D_{sd} \qquad (v_s, v_d) \in \mathcal{SD} \tag{31}$$

$$z_c \in \mathbb{Z}^+ \qquad c \in C. \tag{32}$$

However, we continue with Model (23) - (28), as the description of the objective function and constraints are more clear.

3.2.3 Formulation of RWA Problem with Dynamic Traffic

In the following sections, we provide mathematical formulations for the dynamic RWA problem considering two assumptions regarding the possibility of lightpath re-arrangements.

Dynamic RWA with No Lightpath Re-arrangement

For each wavelength, the model selects at-most 1 configuration with the objective of maximizing the grade of service. The optimization model ($D^{t-1} \rightarrow D^t$) can be written as follows:

$$\max \sum_{(v_s, v_d) \in \mathcal{SD}^{\text{NEW}}} y_{sd} \tag{33}$$

subject to:

$$\sum_{c \in C_\lambda} z_c \leq 1 \qquad \lambda \in \Lambda_{t-1}^{\text{USED}} \tag{34}$$

$$\sum_{c \in C} z_c \leq W - |\Lambda_{t-1}^{\text{USED}}| \tag{35}$$

$$y_{sd} \leq \sum_{c \in C \cup \bigcup_{\lambda \in \Lambda_{t-1}^{\text{USED}}} C_\lambda} a_{sd}^c z_c \qquad (v_s, v_d) \in \mathcal{SD}^{\text{NEW}} \tag{36}$$

$$y_{sd} \leq D_{sd}^{t} - \text{GoS}_{sd}^{t-1} \qquad (v_s, v_d) \in \mathcal{SD}^{\text{NEW}} \tag{37}$$

$$z_c \in \{0, 1\} \qquad c \in C_\lambda, \lambda \in \Lambda_{t-1}^{\text{USED}} \tag{38}$$

$$z_c \in \mathbb{Z} \qquad c \in C \tag{39}$$

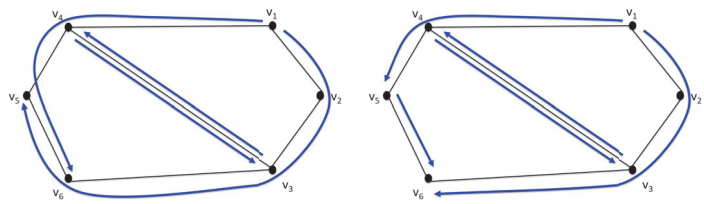
$$y_{sd} \geq 0 \qquad (v_s, v_d) \in \mathcal{SD}^{\text{NEW}}. \tag{40}$$

For every wavelength $\lambda \in \Lambda_{t-1}^{\text{USED}}$, constraints (34) restrict the number of chosen configurations to be at most 1. Constraints (35) determine the number of new required wavelengths and make sure it does not exceed the number of available ones. Constraints (36) determine the number of bandwidth units. Constraints (37) prevent the number of bandwidth units from being more than requests. Last three sets of constraints determine the domain of the variables.

Dynamic RWA with Lightpath Re-arrangement

In order to improve the grade of service in a dynamic RWA problem, one might allow some lightpath re-arrangements taking into account that new connection requests come with enough lead time to set-up. Note that, in an ideal situation, all the required resources (fibers carrying an specific wavelengths) would be available at the time of lightpath re-arrangement. This is called a *Make-Before-Break* (MBB) policy. But the actual situations might be far from ideal and establishing a new lightpath be impossible without first disrupting an old lightpath. This leads to a *Break-Before-Make* (BBM) policy. For example, in Figure 3.2, assume that a bandwidth request for node pair (v_5, v_6) is received. In order to grant this bandwidth unit using the configuration in Figure 3.2(a) and obtaining the configuration in Figure 3.2(b), we need to re-route the lightpaths for node pairs (v_1, v_5) and (v_1, v_6). But it is not possible to establish the new lightpath from v_1 to v_5 without disrupting the lightpath from v_1 to v_6, because it contains the needed fibers $< v_1, v_4 >$ and $< v_4, v_5 >$. As a result of a BBM policy, there will be some service disruptions in the network while re-arranging the lightpaths. In such a case, the objective is to minimize the number of lightpath re-arrangements, while achieving a grade of service as close as possible to the one of the static max-RWA problem.

Unlike the case, at a time period t, not only we provision new traffic requests, but we also allow some lightath re-arrangements if it helps increasing the GoS. For wavelength $\lambda \in \Lambda_{t-1}^{\text{USED}}$, denote by



(a) Old Configuration (b) New Configuration

Figure 3.2: Potential wavelength reconfigurations

γ_c, total number of re-arranged lightpaths in configuration $c \in C_\lambda$. The penalty for every unit of re-arrangement is indicated by PENAL. The resulting optimization model can be written as follows:

$$\max \sum_{(v_s,v_d)\in\mathcal{SD}^t} y_{sd} - \text{PENAL} \sum_{\lambda\in\Lambda^{\text{USED}}} \sum_{c\in C_\lambda} \gamma_c z_c \tag{41}$$

subject to:

$$(34) - (36), (38) - (40) \tag{42}$$

$$\text{GoS}_{sd}^{t-1} \leq y_{sd} \leq D_{sd}^t \qquad (v_s, v_d) \in \mathcal{SD}^t \tag{43}$$

The second term of objective function (41) is the total cost of disrupting the lightpaths. Constraints (43) bound the number of granted requests and ensure that all previously granted traffic requests are still provisioned, subject to some possible lightpath re-arrangement.

3.3 Solution Process: Static Case

3.3.1 Implicit Enumeration of Wavelength Configurations

The model proposed in 3.2.2 has an exponential number of variables, and therefore is not scalable if solved using classical ILP tools. Indeed, we need to use column generation techniques in order to manage a solution process that only requires an *implicit* enumeration of the wavelength configurations (see Section 2.1.2). Column generation method allows the exact solution of the linear relaxation of model (23) - (28), i.e., where constraints $z_c \in \mathbb{Z}^+$ are replaced by $z_c \geq 0$, for $c \in C$. It consists in solving alternatively a restricted master problem (the model of 3.2.2 with a very limited

number of columns/variables) and the pricing problem (generation of a new wavelength configuration) until the optimality condition is satisfied (i.e., no wavelength configuration with a negative reduced cost). In other words, when a new wavelength configuration is generated, it is added to the current restricted master problem only if its addition implies an improvement of the optimal value of the current restricted master problem. This condition, indeed an optimality condition, can be easily checked with the sign of the reduced cost, denoted by $\overline{\text{COST}}$, see Equation (44) for its expression of variables z_c.

Once the optimal solution of the LP relaxation ($z^\star_{\text{LP}}$) has been reached, we solve exactly the last restricted master problem, i.e., the restricted master problem (RMP) of the last iteration in the column generation solution process. It allows the generation of an ILP solution from the pool of generated columns in RMP, whose value is denoted by $\tilde{z}_{\text{ILP}}$. Note that, such a value is usually not an optimal one for the problem (23) - (28). Generation of an optimal one would require the use of a branch-and-price algorithm as explained in Chapter 2 (see Section 2.1.3). However, we can assess the accuracy (ε) of of an ε-optimal ILP solution $\tilde{z}_{\text{ILP}}$ as follows:

$$\varepsilon = \frac{z^\star_{\text{LP}} - \tilde{z}_{\text{ILP}}}{z^\star_{\text{LP}}}.$$

As we will see in the numerical results (see Chapter 5), ε is usually fairly small in practice, and the recourse to branch-and-price is not needed to reach satisfactory ε-optimal solutions. However, branch-and-price methods can be used in order to find optimal solutions, if the accuracy (ε) is not satisfactory, see, e.g., Barnhart et al. [1998], Jaumard et al. [2009].

3.3.2 ε-Optimal Algorithms: CG, CG+ and CG++

In the context of the present study, we investigated different algorithms, in which the differences lie in the generation process of new augmenting configurations, i.e., of configurations that give rise to an improvement of the value of the current restricted master problem, when solving its linear relaxation.

The first CG-ILP algorithm, denoted by CG for short, relies on a mathematical model with a link formulation, in order to generate new augmented wavelength configurations. In the context of

column generation, the configuration generator corresponds to the so-called pricing problem. The pricing problem with a link formulation will be called PP_{LINK} (see Section 3.3.4 for its detailed description), and corresponds to the algorithm used in Jaumard et al. [2009]. The flowchart of algorithm CG is represented in Figure 3.3.

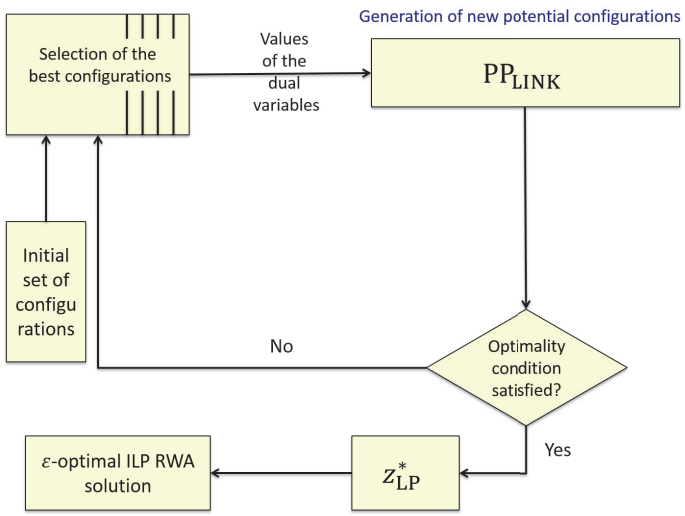


Figure 3.3: Generic flowchart for CG algorithm

Based on the observation made by several researchers, and investigated later in Chapter 5, that a very high percentage of lightpaths are supported by shortest paths, or k-shortest paths with a small k, we propose to investigate a path formulation, called PP_{PATH}, for the pricing problem (see Section 3.3.5 for its detailed description), with different strategies for selecting the paths. Since we cannot consider all possible paths, otherwise the pricing problem would not be scalable, we need to combine the use of PP_{PATH} with PP_{LINK} in order to get an ε-optimal algorithm. Indeed, when PP_{PATH} is no more able to output an augmenting wavelength configuration, we switch to PP_{LINK}, and check whether it is still possible to generate an augmented wavelength configuration using more diverse paths than those considered in PP_{PATH}. Flowchart of the corresponding algorithm is represented in Figure 3.4.

We investigated two variants for PP_{PATH}. In the first one, we consider only the shortest paths, and in the second one, we consider the shortest paths, as well as a selection of k-shortest paths (see Section 3.3.6 for how we made the selection). The resulting ε-optimal algorithms are called CG+ and CG++, respectively.

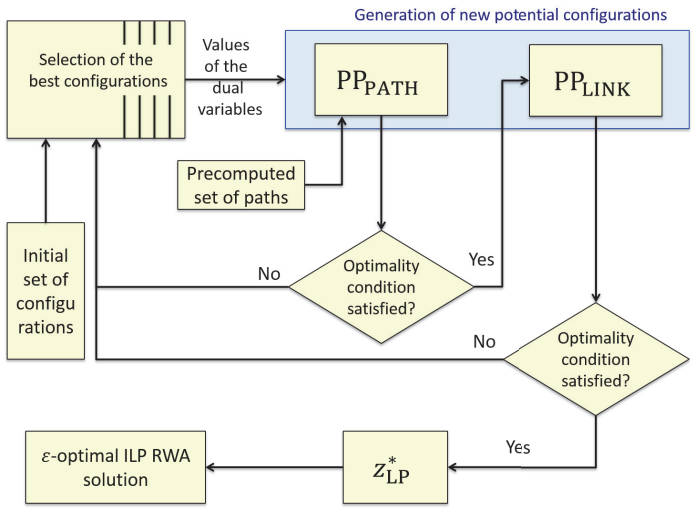


Figure 3.4: Generic flowchart for CG+ and CG++ algorithms

3.3.3 Heuristic Algorithms: CG^H+ and CG^H++

We derive two heuristic algorithms from the CG+ and CG++ algorithms, with the elimination of the recourse to PP_{LINK} in order to limit the computational times. The resulting heuristic algorithms are called CG^H+ and CG^H++ and are summarized in the flowchart represented in Figure 3.5. Both CG^H+ and CG^H++ are associated with different path selections.

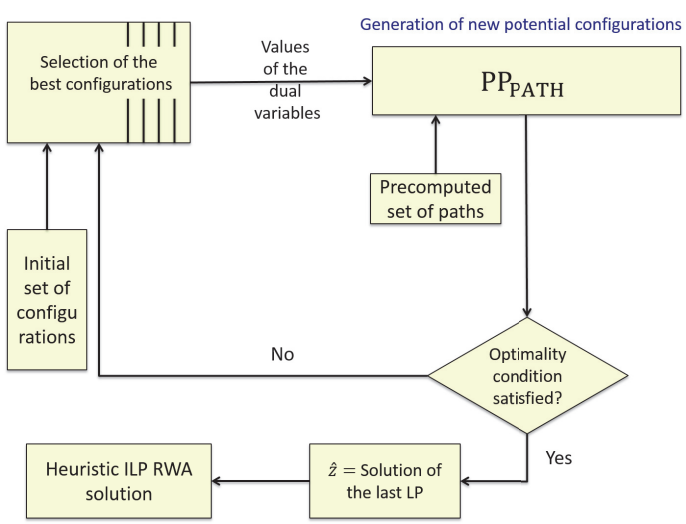


Figure 3.5: Generic flowchart for CG^H+ and CG^H++ algorithms

In the next two sections, we provide the detailed mathematical formulations of PP_{LINK} and PP_{PATH}.

3.3.4 Pricing Problem - Link Formulation

As always with the column generation method, the objective of the pricing problem (i.e., generator of new wavelength configurations) is the reduced cost ($\overline{\text{COST}}_c^{\text{LINK}}$) of variable z_c. In order to alleviate the notations, index c will be omitted in the remainder of this section.

Let $u^{(24)} \geq 0$ and $u_{sd}^{(25)} \geq 0$ be the values of the dual variables associated with constraints (24) and (25) in the optimal solution of the linear relaxation of the current restricted master problem (see the flowchart in Figure 3.4). Consider the following set of variables:

$\alpha_\ell^{sd} = 1$ if link ℓ is used in a route from v_s to v_d, 0 otherwise.

The link formulation of the pricing problem can be written as follows:

Wavelength Configuration Generator - Link Model PP_{LINK}

$$\max \quad \overline{\text{COST}}^{\text{LINK}} = -u^{(24)} + \sum_{(v_s,v_d)\in\mathcal{SD}} \sum_{\ell\in\omega^+(v_s)} \alpha_\ell^{sd} u_{sd}^{(25)} \tag{44}$$

subject to:

$$\sum_{(v_s,v_d)\in\mathcal{SD}} \alpha_\ell^{sd} \leq 1 \qquad \ell \in L \tag{45}$$

$$\sum_{\ell\in\omega^+(v)} \alpha_\ell^{sd} = \sum_{\ell\in\omega^-(v)} \alpha_\ell^{sd} \qquad (v_s, v_d) \in \mathcal{SD}, \quad v \in V \setminus \{v_s, v_d\} \tag{46}$$

$$\sum_{\ell\in\omega^+(v_s)} \alpha_\ell^{sd} \leq D_{sd} \qquad (v_s, v_d) \in \mathcal{SD} \tag{47}$$

$$\sum_{\ell\in\omega^-(v_s)} \alpha_\ell^{sd} = \sum_{\ell\in\omega^+(v_d)} \alpha_\ell^{sd} = 0 \qquad (v_s, v_d) \in \mathcal{SD} \tag{48}$$

$$\alpha_\ell^{sd} \in \{0, 1\} \qquad \ell \in L, (v_s, v_d) \in \mathcal{SD}. \tag{49}$$

Constraints (45) ensure wavelength continuity, i.e., that a link cannot be traversed by more than one route in any given wavelength configuration. Routes are established with the help of the flow conservation constraints (46): if no route is selected for node pair (v_s, v_d), then $\alpha_\ell^{sd} = 0$ for all

links $\ell \in L$, otherwise, the sum of the outgoing flow values at the source node ($\sum_{\ell \in \omega^+(v_s)} \alpha_\ell^{sd}$) gives the number of link-disjoint routes from v_s to v_d in the wavelength configuration under construction. Constraints (47) avoid exceeding the demand in terms of the number of lightpaths. Constraints (48) prevent loops around the source or the destination nodes from arising. Constraints (49) define the domain of variables α_ℓ^{sd}.

Correspondence between variables of the pricing problem and coefficients of the master problem:

$$a_{sd} = \sum_{\ell \in \omega^+(v_s)} \alpha_\ell^{sd}. \tag{50}$$

Observe that it is not forbidden to select several pairwise link-disjoint paths for a given pair (v_s, v_d) of source and destination nodes. Indeed, a_{sd} is equal to the number of link-disjoint paths from v_s to v_d in the configuration under construction.

3.3.5 Pricing Problem - Path Formulation

In the path formulation, we provide a set P_{sd} of paths for each source and destination pair of nodes, see Section 3.3.6 for the definition of P_{sd}.

The path formulation for the wavelength configuration generator is denoted by PP_{PATH}. It uses the set of decision variables:

$\beta_p^{sd} = 1$ if path p is used in the wavelength configuration under construction, 0 otherwise.

PP_{PATH} is written as follows:

Wavelength Configuration Generator - Path Model PP_{PATH}

$$\max \quad \overline{\text{COST}}^{\text{PATH}} = -u^{(24)} + \sum_{(v_s,v_d) \in \mathcal{SD}} \sum_{p \in P_{sd}} \beta_p^{sd} u_{sd}^{(25)} \tag{51}$$

subject to:

$$\sum_{(v_s,v_d)\in\mathcal{SD}} \sum_{p\in P_{sd}} \delta_\ell^p \beta_p^{sd} \leq 1 \qquad \ell \in L \tag{52}$$

$$\sum_{p\in P_{sd}} \beta_p^{sd} \leq D_{sd} \qquad (v_s, v_d) \in \mathcal{SD} \tag{53}$$

$$\beta_p^{sd} \in \{0,1\} \qquad \ell \in L, (v_s, v_d) \in \mathcal{SD}. \tag{54}$$

Pairwise link disjointness for paths is guaranteed thanks to constraints (52), in which δ_ℓ^p is a binary value representing the presence of link ℓ in path p. Constraints (53) enforce not to exceed the lightpath demand. Constraints (54) define the domain of variables β_ℓ^{sd}.

Correspondence between variables of the pricing problem and coefficients of the master problem is established using:

$$a_{sd} = \sum_{p\in P_{sd}} \beta_p^{sd}.$$

As with the link formulation, it is possible to select several pairwise link disjoint paths for a node pair (v_s, v_d).

3.3.6 Computation and Selection of k-Shortest Paths

As explained in Section 3.3.2, the difference between CG+ and CG++ (as well as CG^{H}+ and CG^{H}++) is that in the former, the pool of paths $P = \bigcup_{(v_s,v_d)\in\mathcal{SD}} P_{sd}$ contains only the shortest paths, while in the latter some k-shortest paths are also considered. In order that the proposed CG++ and CG^{H}++ solution algorithms be effective, we must carefully choose the paths to consider: a sufficiently large number so that we can maximize the GoS, but not too many in order not to increase too much the size of the constraint matrix in PP$_{\text{PATH}}$. In addition, considering paths that are much longer than the shortest paths may lead to an inefficient use of the spectrum.

We explore several strategies in which paths are first selected in the set of shortest paths, and next in the set of k-shortest paths. Very efficient algorithms already exists for enumerating such paths, see Yen [1971], Epstein [1998], and we use the open library Pavon-Marino and Izquierdo-Zaragoza [2015] that provides an implementation of Yen's algorithm Yen [1971]. We denote by

P^1_{sd} the set of all shortest paths from v_s to v_d, and then by P^k_{sd} the set of paths that their length is equal to the k shortest distinct value of length. For instance, if the ordered list according to the length of paths is: $p_1, p_2, \ldots, p_9$ of length $1, 1, 2, 2, 2, 4, 4, 4, 4$ respectively, then $\mathrm{P}^1 = \{p_1, p_2\}$, $\mathrm{P}^2 = \{p_3, p_4, p_5\}$ and $\mathrm{P}^3 = \{p_6, p_7, p_8, p_9\}$.

Strategy 1

We consider the same number of k-shortest paths, in addition to the shortest paths, for each node pair, say k^{SP}. After extensive numerical experiments on different data sets with various network topologies, the acceptable compromise we found between computing times and GoS was $k^{\mathrm{SP}} = 15$.

The drawbacks of Strategy 1 is:

- Choosing at random the paths in the last P^i set that is considered for reaching the number of selected paths, results in not considering other paths in the same set that can be more desirable.
- Selecting a number of paths that is independent of the traffic demand, leads to potentially having too few or too much potential paths for some requests.

Strategy 2

We modify the criterion for selecting the best paths in the last considered P^i as follows: Firstly, taking into account the traffic demand, we select the number of paths with the following fairness criterion:

$$\frac{D_{sd}}{\text{number of candidate paths from } v_s \text{ to } v_d} = \rho,$$

where our experiments showed that an acceptable compromise between the computing times and the GoS led to a ρ constant value equal to 0.5. Secondly, we enumerate all the paths p of P^i and order them in the increasing order of

$$\sum_{\ell \in p} \mathrm{LOAD}_\ell,$$

where LOAD_ℓ is an estimate on the number of lightpaths going through ℓ when maximizing the GoS using the following routing formulation that ignores the wavelength continuity constraints and

omits integrality requirements for the φ_ℓ^{sd} variables:

$$\max \sum_{(v_s,v_d)\in\mathcal{SD}} d_{sd} \tag{55}$$

subject to

$$\sum_{\ell\in\omega^+(v_s)} \varphi_\ell^{sd} = \sum_{\ell\in\omega^-(v_d)} \varphi_\ell^{sd} = 0 \qquad (v_s,v_d)\in\mathcal{SD} \tag{56}$$

$$\sum_{\ell\in\omega^+(v_s)} \varphi_\ell^{sd} = \sum_{\ell\in\omega^+(v_d)} \varphi_\ell^{sd} = d_{sd} \qquad (v_s,v_d)\in\mathcal{SD} \tag{57}$$

$$\sum_{\ell\in\omega^+(v)} \varphi_\ell^{sd} = \sum_{\ell\in\omega^-(v)} \varphi_\ell^{sd} \qquad v\in V\setminus\{v_s,v_d\},$$

$$(v_s,v_d)\in\mathcal{SD} \tag{58}$$

$$\sum_{(v_s,v_d)\in\mathcal{SD}} \varphi_\ell^{sd} \le W \qquad \ell\in L \tag{59}$$

$$0 \le \varphi_\ell^{sd} \qquad \ell\in L, (v_s,v_d)\in\mathcal{SD} \tag{60}$$

$$0 \le d_{sd} \le D_{sd} \qquad (v_s,v_d)\in\mathcal{SD}. \tag{61}$$

Each variable d_{sd} represents the fraction of the demand that is granted for node pair (v_s, v_d), and each variable φ_ℓ^{sd} defines the amount of traffic going through ℓ with respect to the demand of node pair (v_s, v_d). Note that Constraints (56) - (58) are multi-commodity flow constraints. Constraints (59) define the capacity of the links. As the result of the above model, the estimated load of each link is $\text{LOAD}_\ell = \sum_{sd\in\mathcal{SD}} \varphi_\ell^{sd}$.

Strategy 3

Network topologies are usually mesh topologies, and consequently, the lengths of the shortest paths vary from one node pair to the next. In addition, the traffic is not uniform. So, in this third strategy, we consider a number of paths that is larger if the traffic demand is larger, and larger as the length of the shortest path is longer: we generate the first (D_{sd} / length of shortest paths) = ($D_{sd}/\ell_{sd}^{\text{SHORT}}$) paths, where ℓ_{sd}^{SHORT} is the length of the shortest path from v_s to v_d. When $\ell_{sd}^{\text{SHORT}} = 1$,

we only select the one-hop paths made of ℓ_{sd}, the link from v_s to v_d (note that there might be more than one if there are more than one fiber link). Again, we choose the paths of the last considered P^i set according to the same criterion as in Strategy 2.

Observe that we can force the lightpath of one-hop requests to be routed on a one-hop route without loss of generality, thanks to the following result.

Theorem 1. *Consider the max-RWA problem. There is at least one optimal solution in which one hop requests are provisioned (lightpaths) on one hop path(s) if they can be granted, i.e., if $D_{sd} \leq W r_{sd}$, where r_{sd} is the number of links from v_s to v_d.*

Proof. Consider a pair $(v_s, v_d) \in \mathcal{SD}$ with $0 < D_{sd} \leq r_{sd}W$ such that v_s and v_d are connected in G by at least one link. Denote it by ℓ_{sd} one of the links connecting v_s to v_d.

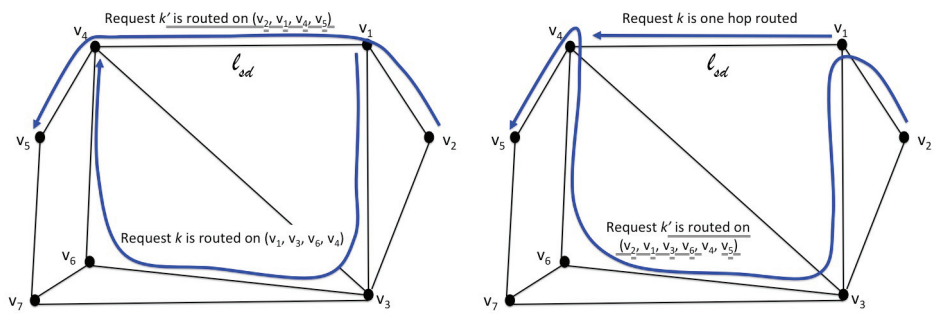
Assume that there is an optimal RWA solution such that D_{sd} is not routed completely on links from P^1_{sd}, i.e., on one-hop lightpaths. We next show that such an optimal solution can be modified in order that it satisfies the property stated in the theorem.

Four cases need to be distinguished, and the transformation described for each of them can be repeated on each link from v_s to v_d possibly several times, until the optimal solution satisfies the stated property. Indeed, each transformation reduces the number of demands that do not satisfy the property by one unit at a time.

- *There is at least one D_{sd} demand that is routed on a path with at least two hops and ℓ_{sd} has at least one available wavelength:* Rerouting that latter demand on ℓ_{sd} does not change the GoS and reduces by one unit the demand that does not satisfy the stated property.

- *There is at least one D_{sd} demand, say k, that is routed on a path with at least two hops and all wavelengths of ℓ_{sd} (as well as on all links from v_s to v_d) are used in a lightpath.* Assume that k is provisioned with lightpath (p, λ). Observe that $\ell_{sd} \notin p$ as otherwise p is not a simple path (i.e., without loop). Since all wavelengths of ℓ_{sd} are used, there exists a multi-hop lightpath (p', λ), with $\ell_{sd} \in p'$, which is used to provision a request, say k'. For example, in the left network of Figure 3.6, $k = (v_1, v_4)$ and $k' = (v_2, v_5)$. Modify the provisioning of k' so that, instead of using ℓ_{sd} with wavelength λ, it uses the multi-hop lightpath of k with wavelength λ. Re-provision k on the one-hop lightpath (ℓ_{sd}, λ). The fraction of D_{sd} that does not satisfy

the stated property, reduces by one unit. See Figure 3.6 for an example of the provisioning transformations. Wavelength continuity is respected, as both lightpaths (p, λ) and (p', λ) are using the same wavelength.



(a) Original routing of the lightpaths (b) Modified lightpaths

Figure 3.6: Lightpath modifications

- *There is at least one D_{sd} demand that is denied and ℓ_{sd} has at least one available wavelength:* Contradicts the assumption that the solution is optimal, therefore that case can be omitted.

- *There is at least one D_{sd} demand that is denied and ℓ_{sd} (as well as on all links from v_s to v_d) are used in a lightpath:* then, there exists a one multi-hop request, say k, which is routed through a link from v_s to v_d. Assume it is on ℓ_{sd}. Denying k and granting one demand unit of D_{sd} on ℓ_{sd} reduces by one unit the demand that does not satisfies the stated property.

□

3.3.7 A Note on Undirected RWA Problem with Static Traffic

In this section, we show how the model and solution approaches of RWA problem with directed links and asymmetric traffic can be changed to help solve the undirected RWA problem.

For undirected case of max-RWA, we consider the directed case and assume that all links are bidirectional and all demands are symmetric. Then in making the configurations, for every node pair $\{v_s, v_d\} \in \mathcal{SD}$, we force the opposite paths to be the same. The basic model of undirected max-RWA problem is the same as model (23)-(28). Considering our approach, the only changes are going to be in the formulations of the pricing problems defined for solution approach.

The general framework and concepts in the algorithms presented for directed RWA problem are the same for undirected case. The difference lies in the formulations of the pricing problems that generate improving columns. In an undirected network, these formulations are also responsible for forcing the notion of undirected links and symmetric traffic.

In the link formulation PP_{LINK}, adding following constraints forces the paths of node pairs (v_s, v_d) and (v_d, v_s) to be defined on the same links in opposite order and direction:

$$\alpha_{\ell}^{sd} = \alpha_{\ell'}^{ds} \qquad (v_s, v_d) \in \mathcal{SD},\ v_s \leq v_d,\ l = l'^t \in L. \tag{62}$$

Constraints (62) force the paths of reverse pairs to be the same, but in opposite directions. This constraint implements the semantics of undirected graph. Note that, because there is no loop in the paths selected by PP_{LINK} thanks to Constraints (48), just forcing the opposite links to be selected for reverse node pairs is enough for having the same paths in opposite direction, since there is only one way for them to establish a valid connected path.

In the path formulation, the job is is easier, as we are dealing with actual paths, not the constituent links. Adding the following constraints to PP_{PATH} adds the semantics of an undirected graph to the problem:

$$\beta_{p}^{sd} = \beta_{p}^{\prime ds} \quad (v_s, v_d) \in \mathcal{SD},\ v_s \leq v_d,\ p = p' \in P_{sd}. \tag{63}$$

Constraints (63) ensure that for every pair and its reverse, two paths in opposite direction are selected.

We will use this formulation in Chapter 5 to assess the solutions of RSA problem which is stated and solved for an undirected graph, and provide comparisons between RSA and RWA problems.

3.4 Solution Process: Dynamic Case

Column generation method allows the exact solution of the linear relaxation of models (33) - (40) and (41) - (43), i.e., where constraints $z_c \in \mathbb{Z}^+$ are replaced by $z_c \geq 0$, for $c \in C$, and $z_c \in \{0, 1\}$ are replaced by $0 \leq z_c \leq 1$, for $c \in C_\lambda$ for $\lambda \in \Lambda_{t-1}^{\text{USED}}$.

3.4.1 Solution Approach with No Lightpath Re-arrangement

Let $\lambda\mathcal{P}$ be the set of lightpaths used to grant connections for the legacy traffic. In every wavelength configuration $c \in C_\lambda$ with $\lambda \in \Lambda^{\text{USED}}$, the set of available links is limited to those that do not belong to any lightpath with wavelength λ in the provisioning of the legacy traffic. At time period t, the set of available links L^t_λ for wavelength λ, is:

$$L^t_\lambda = L \setminus \{\ell \in L : \exists p \text{ with } \ell \in p \text{ and } (p, \lambda) \in \lambda\mathcal{P}\}. \tag{64}$$

Considering the potentially different sets of L^t_λ for $\lambda \in \Lambda^{\text{USED}}_{t-1}$, these sets are defined for each of them, while a single set represents all the new wavelengths in $\lambda \in L \setminus \Lambda^{\text{USED}}_{t-1}$.

As before, the proposed algorithm for solving the problem generates new augmenting configurations, using a link and a path mathematical formulation that serve as configuration generators. With dynamic traffic, $\text{PP}^\lambda_{\text{LINK}}$ and $\text{PP}^\lambda_{\text{PATH}}$ are the two resulting pricing problems defined for every used wavelength, $\lambda \in \Lambda^{\text{USED}}_{t-1}$ and PP_{LINK} and PP_{PATH} for the new wavelengths. They are called in sequence as illustrated in Figure 3.7, always $\text{PP}^\lambda_{\text{LINK}}$ and $\text{PP}^\lambda_{\text{PATH}}$ for $\lambda \in \Lambda^{\text{USED}}_{t-1}$, before the generic PP_{LINK} and PP_{PATH} for the additional wavelengths, as long as they generate improving wavelength configurations.

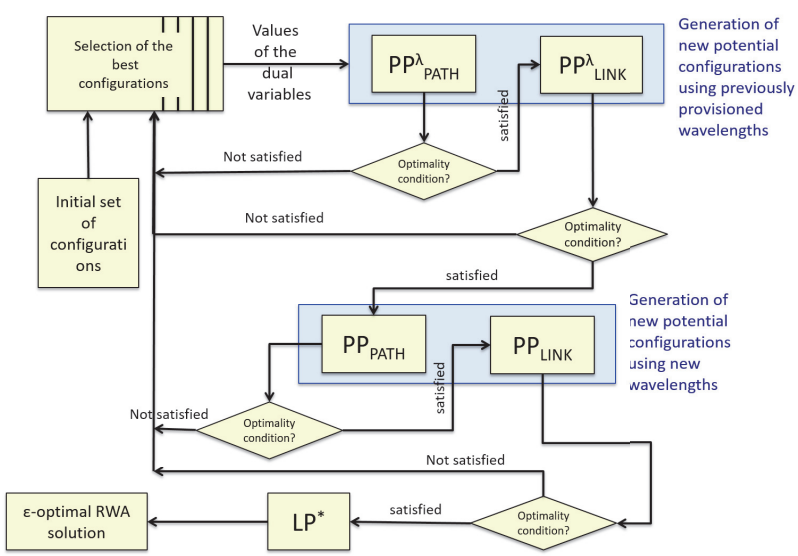


Figure 3.7: Solution approach for dynamic requests

Each PP_{PATH}/$\text{PP}^{\lambda}_{\text{PATH}}$ considers a set of restricted paths in order to generate a new improving configuration. As will be investigated in Section 5.1.2, it is worth adding few additional k-shortest paths to the set of shortest paths. Consequently, we use the pool of paths selected in 3.3.6 for CG++ in order to define the set of paths in PP_{PATH}/$\text{PP}^{\lambda}_{\text{PATH}}$.

At time period t, after the insertion of a first set of initial configurations capturing the provisioning of the legacy traffic (i.e., of D^{t-1}), the algorithm computes the sets L^t_λ for all $\lambda \in \Lambda^{\text{USED}}_{t-1}$. The column generation algorithm then alternately solves the restricted master problem (i.e., models (33) - (40) and (41) - (43) with a very small number of variables/columns) and the pricing problems PP_{PATH}/$\text{PP}^{\lambda}_{\text{PATH}}$ and PP_{LINK}/$\text{PP}^{\lambda}_{\text{LINK}}$, until the optimality condition is satisfied. In choosing the wavelength for consideration in each iteration, a round robin approach is used. The strategy is as follows. Firstly, the algorithm iterates through already provisioned wavelengths, until no other improving configurations are found. Then, it proceeds to the new wavelengths and iteratively tries to find new improving configurations. When no such configuration can be found, the algorithm goes back to the wavelengths in the set $\Lambda^{\text{USED}}_{t-1}$ and repeats until the stopping condition is fulfilled, i.e., the reduced cost of all pricing problems is positive, see again the flowchart of Figure 3.7 for an overview of the algorithm.

Pricing Problem - Link Formulation

As always with the column generation method, the objective of the pricing problem is the reduced cost ($\overline{\text{COST}}^{\text{LINK}}_c$) of variable z_c. In order to alleviate the notations, index c will be omitted in the remainder of this section.

We first describe $\text{PP}^{\lambda}_{\text{LINK}}$ for $\lambda \in \Lambda^{\text{USED}}_{t-1}$. Let $u^{(34)} \geq 0$ and $u^{(36)}_{sd} \geq 0$ be the values of the dual variables associated with constraints (34) and (36) in the optimal solution of the linear relaxation of the current restricted master problem . Consider the following set of variables:

$\alpha^{sd}_\ell = 1$ if link $\ell \in L^t_\lambda$ is used in a route from v_s to v_d, 0 otherwise.

For a wavelength $\lambda \in \Lambda^{\text{USED}}_{t-1}$, $\text{PP}^{\lambda}_{\text{LINK}}$ can be written as follows:

$$\max \quad -u^{(34)} + \sum_{(v_s,v_d)\in\mathcal{SD}^{\text{NEW}}} \sum_{\ell\in\omega^+(v_s)} \alpha^{sd}_\ell u^{(36)}_{sd} \tag{65}$$

subject to:

$$\sum_{(v_s,v_d)\in\mathcal{SD}^{\text{NEW}}} \alpha_\ell^{sd} \leq 1 \qquad \ell \in L_\lambda^t \tag{66}$$

$$\sum_{\ell\in\omega^+(v)} \alpha_\ell^{sd} = \sum_{\ell\in\omega^-(v)} \alpha_\ell^{sd} \qquad (v_s,v_d)\in\mathcal{SD}^{\text{NEW}}, \quad v \in V \setminus \{v_s, v_d\} \tag{67}$$

$$\sum_{\ell\in\omega^+(v_s)} \alpha_\ell^{sd} \leq D_{sd}^t - \text{GoS}_{sd}^{t-1} \qquad (v_s,v_d)\in\mathcal{SD}^{\text{NEW}} \tag{68}$$

$$\sum_{\ell\in\omega^-(v_s)} \alpha_\ell^{sd} = \sum_{\ell\in\omega^+(v_d)} \alpha_\ell^{sd} = 0 \qquad (v_s,v_d)\in\mathcal{SD}^{\text{NEW}} \tag{69}$$

$$\alpha_\ell^{sd} \in \{0,1\} \qquad \ell \in L_\lambda^t, (v_s,v_d)\in\mathcal{SD}^{\text{NEW}}. \tag{70}$$

Constraints (66) prevent wavelength clashes, i.e., that a link cannot be traversed by more than one route in any given wavelength configuration. Routes are established with the help of the flow conservation constraints (67): if no route is selected for node pair (v_s, v_d), then $\alpha_\ell^{sd} = 0$ for all links $\ell \in L_\lambda^t$, otherwise, the sum of the outgoing flow values at the source node ($\sum_{\ell\in\omega^+(v_s)} \alpha_\ell^{sd}$) gives the number of link-disjoint routes from v_s to v_d in the wavelength configuration under construction. Constraints (69) prevent loops around the source or the destination nodes from arising. Constraints (70) define the domain of variables α_ℓ^{sd}. Correspondence between variables of the pricing problem and coefficients of the master problem is established by:

$$a_{sd} = \sum_{\ell\in\omega^+(v_s)} \alpha_\ell^{sd}.$$

For $\lambda \in \Lambda \setminus \Lambda_{t-1}^{\text{USED}}$ PP_{LINK} is very similar to $\text{PP}_{\text{LINK}}^\lambda$: Constraints (66) and (70) are written for all $\ell \in L$, and the reduced-cost becomes as follows:

$$\max \quad -u^{(35)} + \sum_{(v_s,v_d)\in\mathcal{SD}^{\text{NEW}}} \sum_{\ell\in\omega^+(v_s)} \alpha_\ell^{sd} u_{sd}^{(36)}. \tag{71}$$

In other words, objective function (65) has to be updated in order to consider $u^{(35)} \geq 0$ rather than $u^{(34)}$.

Pricing Problem - Path formulation

In the pricing problem considering the path formulation, at time period t for every wavelength $\lambda \in \Lambda_{t-1}^{\text{USED}}$ we provide a set $P_{sd}^{\lambda,t}$ of paths for each source and destination pair of nodes, which only contain links belonging to L_λ^t. Recall the definition of P_{sd}^i from Section 3.3.6. Then the pool of considered paths for $\lambda \in \Lambda_{t-1}^{\text{USED}}$ and node pair $(v_s, v_d \in \mathcal{SD}^{\text{NEW}})$ is:

$$P_{sd}^{\lambda,t} = \bigcup_{i=1}^{k} \{p \in \mathrm{P}_{sd}^i : p \text{ only contains link(s) } \ell \in L_\lambda^t\}, \tag{72}$$

for some k (see Section 3.3.6 for more details).

$\text{PP}_{\text{PATH}}^\lambda$ for uses the set of decision variables:

$\beta_p^{sd} = 1$ if path p is used in the wavelength configuration under construction, 0 otherwise.

At time period t, the mathematical model for $\text{PP}_{\text{PATH}}^\lambda$, $\lambda \in \Lambda_{t-1}^{\text{USED}}$ is written as follows:

$$\max \quad -u^{(34)} + \sum_{(v_s,v_d)\in\mathcal{SD}^{\text{NEW}}} \sum_{p\in P_{sd}^{\lambda,t}} \beta_p^{sd} u_{sd}^{(36)} \tag{73}$$

subject to:

$$\sum_{(v_s,v_d)\in\mathcal{SD}^{\text{NEW}}} \sum_{p\in P_{sd}^{\lambda,t}} \delta_\ell^p \beta_p^{sd} \leq 1 \qquad \ell \in L_\lambda^t \tag{74}$$

$$\sum_{p\in P_{sd}^{\lambda,t}} \beta_p^{sd} \leq D_{sd}^t - \text{GoS}_{sd}^{t-1} \qquad (v_s, v_d) \in \mathcal{SD}^{\text{NEW}} \tag{75}$$

$$\beta_p^{sd} \in \{0,1\} \qquad p \in P_{sd}^{\lambda,t}, (v_s, v_d) \in \mathcal{SD}^{\text{NEW}}. \tag{76}$$

We guarantee paths that are pairwise link disjoint thanks to Constraints (74), in which δ_ℓ^p is a binary value representing the presence of link ℓ in path p. Constraints (75) enforce not to exceed the lightpath demand. Constraints (76) define the domain of variables β_p^{sd}.

Correspondence between variables of the pricing problem and coefficients of the master problem is established by:

$$a_{sd} = \sum_{p\in P_{sd}^{\lambda,t}} \beta_p^{sd}.$$

For $\lambda \in \Lambda \setminus \Lambda_{t-1}^{\text{USED}}$ PP_{PATH} is very similar to $\text{PP}_{\text{PATH}}^{\lambda}$: The pool of paths is updated to $P_{sd} = \bigcup_{i=1}^{k} \text{P}_{sd}^{i}$ in the entire model. Constraints (74) are written for all $\ell \in L$, and the reduced-cost becomes as follows:

$$\max \quad -u^{(35)} + \sum_{(v_s,v_d)\in\mathcal{SD}^{\text{NEW}}} \sum_{p\in P_{sd}^{t}} \beta_p^{sd} u_{sd}^{(36)}. \tag{77}$$

In other words, objective function (73) has to be updated in order to consider $u^{(35)} \geq 0$ rather than $u^{(34)}$.

3.4.2 Solution Approach with Lightpath Re-arrangement

Assuming re-arrangement of lightpaths is allowed and there is no sensitive traffic request with restriction on dismantling, the sets of available links and paths for all wavelengths remain intact, i.e., for all $\lambda \in \Lambda$ and time periods t, $L_{\lambda}^{t} = L$ and $P_{sd}^{\lambda,t} = P_{sd}$ for all node pairs.

The following modifications to $\text{PP}_{\text{LINK}}^{\lambda}$ provides a configuration generator for the current problem. Denote by $\bar{P}_{sd}^{\lambda,t-1}$, $(v_s, v_d) \in \mathcal{SD}^{t-1}$, the set of assigned paths for legacy pair (v_s, v_d) using wavelength $\lambda \in \Lambda_{t-1}^{\text{USED}}$. For every $p \in \bar{P}_{sd}^{\lambda,t-1}$, γ^p is a binary variable equal to 1 if path $p \in \bar{P}_{sd}^{\lambda,t-1}$ is modified in the new configuration for $\lambda \in \Lambda_{t-1}^{\text{USED}}$. The mathematical formulation of $\text{PP}_{\text{LINK}}^{\lambda}$, $\lambda \in \Lambda_{t-1}^{\text{USED}}$, is as bellow:

$$\max \quad -\text{PENAL} \sum_{(v_s,v_d)\in\mathcal{SD}^{t-1}} \sum_{p\in\bar{P}_{sd}^{\lambda,t-1}} \gamma^p - u^{(34)} + \sum_{(v_s,v_d)\in\mathcal{SD}^{t}} \sum_{\ell\in\omega^{+}(v_s)} \alpha_{\ell}^{sd} u_{sd}^{(36)} \tag{78}$$

subject to:

$$\sum_{(v_s,v_d)\in\mathcal{SD}^{t}} \alpha_{\ell}^{sd} \leq 1 \qquad \ell \in L \tag{79}$$

$$\sum_{\ell\in\omega^{+}(v)} \alpha_{\ell}^{sd} = \sum_{\ell\in\omega^{-}(v)} \alpha_{\ell}^{sd} \qquad (v_s, v_d) \in \mathcal{SD}^{t}, v \in V \setminus \{v_s, v_d\} \tag{80}$$

$$\sum_{\ell\in\omega^{+}(v_s)} \alpha_{\ell}^{sd} \leq D_{sd}^{t} \qquad (v_s, v_d) \in \mathcal{SD}^{t} \tag{81}$$

$$\sum_{\ell \in \omega^-(v_s)} \alpha_\ell^{sd} = \sum_{\ell \in \omega^+(v_d)} \alpha_\ell^{sd} = 0 \qquad (v_s, v_d) \in \mathcal{SD}^t \tag{82}$$

$$\alpha_\ell^{sd} \geq 1 - \gamma^p \qquad p \in \bar{P}_{sd}^{\lambda,t-1}, \ell \in p, (v_s, v_d) \in \mathcal{SD}^{t-1} \tag{83}$$

$$\alpha_\ell^{sd} \in \{0, 1\} \qquad \ell \in L, (v_s, v_d) \in \mathcal{SD}^t \tag{84}$$

$$\gamma^p \in \{0, 1\} \qquad p \in \bar{P}_{sd}^{\lambda,t-1}, (v_s, v_d) \in \mathcal{SD}^{t-1}. \tag{85}$$

In objective function (78), PENAL is the unit penalty for disrupting some previously established lightpaths. The first term of this objective function is the total resulting penalty. Constraints (79)-(82) and (84) are the counterparts of Constraints (45)-(48) and (70), modified to account for changes in the defining set of links and node pairs. In order to determine the number of disrupted lightpaths in $\text{PP}^\lambda_{\text{LINK}}$, Constraints (83) and (85) are added. Considering the uniqueness of paths in every configuration, Constraints (83) determine the values of γ^p, $p \in \bar{P}_{sd}^{\lambda,t-1}$, by checking whether all its consisting links contribute to the provisioning of node pair (v_s, v_d). Constraints (85) define the domain of variables γ.

Correspondence between variables of the pricing problem and the coefficients γ in the master problem becomes:

$$\gamma = \sum_{p \in \bar{P}^{\lambda,t-1}} \gamma^p,$$

where $\bar{P}^{\lambda,t-1} = \bigcup_{(v_s,v_d) \in \mathcal{SD}^{t-1}} \bar{P}_{sd}^{\lambda,t-1}$.

For $\lambda \in \Lambda \setminus \Lambda_{t-1}^{\text{USED}}$, PP_{LINK} is obtained by removing Constraints (83) and (85) from $\text{PP}^\lambda_{\text{LINK}}$ and the penalty term from the reduced-cost (78).

Similarly, $\text{PP}^\lambda_{\text{PATH}}$ is modified as follows. Let γ_{sd}, be the number of disrupted lightpaths in configuration $c \in C_\lambda$ for wavelength $\lambda \in \Lambda_{t-1}^{\text{USED}}$. The mathematical formulation of $\text{PP}^\lambda_{\text{PATH}}$ is then:

$$\max \quad -\text{PENAL} \sum_{(v_s,v_d) \in \mathcal{SD}^{t-1}} \gamma_{sd} - u^{(34)} + \sum_{(v_s,v_d) \in \mathcal{SD}^t} \sum_{\ell \in \omega^+(v_s)} \alpha_\ell^{sd} u_{sd}^{(36)} \tag{86}$$

subject to:

$$\sum_{(v_s,v_d)\in\mathcal{SD}^t}\sum_{p\in P_{sd}}\delta_\ell^p\beta_p^{sd}\leq 1 \qquad \ell\in L \tag{87}$$

$$\sum_{p\in P_{sd}}\beta_p^{sd}\leq D_{sd}^t \qquad (v_s,v_d)\in\mathcal{SD}^t \tag{88}$$

$$|\bar{P}_{sd}^{\lambda,t-1}|-\sum_{\substack{p\in\\ \bar{P}_{sd}^{\lambda,t-1}\cap P_{sd}}}\beta_p^{sd}\leq\gamma_{sd} \qquad (v_s,v_d)\in\mathcal{SD}^{t-1} \tag{89}$$

$$\beta_p^{sd}\in\{0,1\} \qquad p\in P_{sd},(v_s,v_d)\in\mathcal{SD}^t \tag{90}$$

$$\gamma_{sd}\in\mathbb{Z}^+ \qquad (v_s,v_d)\in\mathcal{SD}^{t-1}. \tag{91}$$

As with $\text{PP}_{\text{LINK}}^{\lambda}$, objective function (86) contains a term for the penalty associated with disrupting lightpaths. Constraints (87)-(88) and (90) are equivalent to Constraints (74)-(75) and (76) in the $\text{PP}_{\text{PATH}}^{\lambda}$ with no lightpath re-arrangements, except for the modified sets of links, paths, and considered node pairs. The number of re-arranged lightpaths is computed using the Constraints (89): if$(v_s,v_d)\in\mathcal{SD}^{t-1}$ the summation of variables β_p^{sd} over all the previously selected paths p from the pool of k-shortest paths P_{sd}, is less than the number of granted bandwidth units in the selected configuration of λ, the difference is the number of lightpaths that need to be re-arranged. Constraints (91) define the domain of the γ variables. In the limit of the summation in Constraints (89), the intersection $\bar{P}_{sd}^{\lambda,t-1}\cap P_{sd}$ is needed because the selected paths for wavelength λ are not necessarily from P_{sd} and some of them might have been generated using a link formulation.

Correspondence between variables of the pricing problem and the coefficients in the master problem is:

$$\gamma=\sum_{(v_s,v_d)\in\mathcal{SD}^{t-1}}\gamma_{sd}.$$

The path formulation of PP_{PATH} for new wavelengths $\lambda\in\Lambda\setminus\Lambda_{t-1}^{\text{USED}}$, can be written by removing Constraints (89) and (91) from $\text{PP}_{\text{PATH}}^{\lambda}$ and the penalty term from the reduced-cost (86).

Conclusions of the Chapter

In this chapter, we have proposed enhanced and new solution processes for solving the static and dynamic RWA problems, respectively. All proposed solutions make use of the combination of path selection heuristics and the exact solution of 3 different ILP models, in such a way that, we are able to output ε-optimal solutions with small ε, as we will see in Chapter 5.

Chapter 4

Routing and Spectrum Assignment Problem

In the previous chapter, we focused on the provisioning problem in WDM networks and designed large-scale algorithms to utilize these networks as close as possible to their maximum potential. This chapter goes one step further, and deals with the much harder problem of spectrum utilization in elastic optical network. The original contribution of this chapter is providing a new decomposition scheme and presenting a new solution algorithm for solving this decomposition that resembles the successful one of RWA problem.

This chapter is organized as follows. Section 4.1 formally states the RSA problem. Section 4.2 models the problem using a mathematical formulation and Section 4.3 provides the solution algorithm.

4.1 Statement of the RSA Problem

Let us consider an elastic (flexgrid) optical network, represented by an undirected graph $G = (V, L)$ with optical node set V (indexed by v) and fiber link (edge) set L (indexed by ℓ). Connections and fiber links are assumed to be undirected, and the traffic to be symmetrical. The bandwidth is slotted into a set S (generic index s) of spectrum slots (or slices). A guard band g (number of slots) is required between two contiguous spectrum allocations. The available bandwidth over every fiber

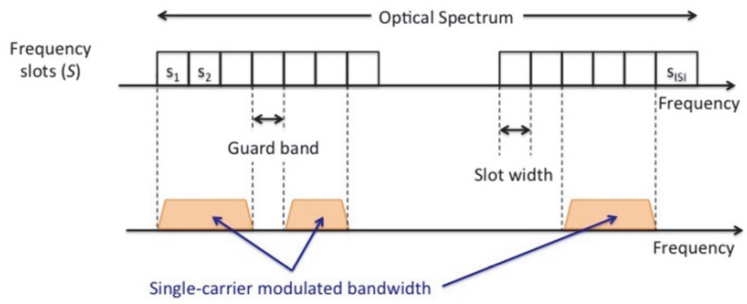


Figure 4.1: Connection requests using a group of spectrum contiguous slots

link is defined by the overall number of spectrum slots (12.5 GHz steps in this study). The traffic is defined by a set K of requests where each request $k \in K$ has a source (s_k), a destination (d_k) and a spectrum demand D_k, expressed in terms of a number of slots, requested to be contiguous. We assume that no regenerator is used, and therefore, the number of slots corresponds to the modulation that is compatible with the distance between s_k and d_k and that uses the minimum number of slots.

The RSA problem can be formally stated as follows: given a graph G corresponding to an elastic optical network, and a set of requested connections, find a path and a spectrum allocation for every request respecting the continuity and contiguity constraints. The path together with a slot is called a *slot-path*. The objective is to minimize the blocking rate, that is equivalent to maximizing the number of accepted connections, leading to the max-RSA problem.

It is worth noting that, if there are too many small requests, close to the full bandwidth of a single wavelength, RSA is not of interest over RWA, because of its requirement for guard band. Figure 4.2 illustrates this with an example. In this figure, assume the same amount of available spectrum on both grids. A fixgrid today usually has fibers that carry 50 GHz wavelengths with bandwidths up to 100 Gb/s. Each of these wavelengths would be equal to four 12.5 GHz slots. A 100 GB/s request can be transmitted on a single wavelength in a fixgrid, and on three slots in a flexgrid. But in elastic optical networks, we need guard bands between requests in order to distinguish them. Therefore, if we assume a single slot guard band, there is no gain in RSA over RWA. So, although we do not aggregate the traffics for every node pair, we know that, in order for the RSA to be of interest, we cannot have many requests, especially if all the node pairs (or a high percentage of them) have traffic.

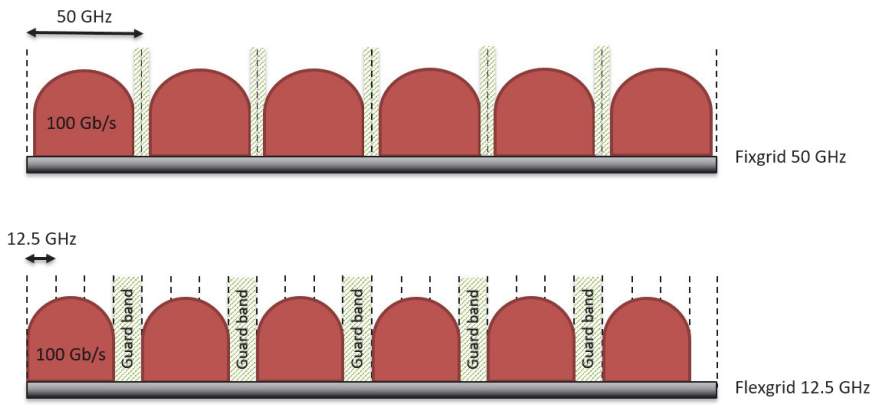


Figure 4.2: 100 Gb/s connection requests on RSA vs. RWA

4.2 Configuration Optimization Model

We propose a new decomposition scheme that relies on slot-path configurations. In the following, we first introduce the notion of slot-path configurations, then provide the mathematical formulation.

4.2.1 Slot-path Configuration

Our new decomposition scheme takes advantage of *slot-path configurations*, such that, for each configuration, blocks of consecutive slots are all starting at the same slot, see Figure 4.3 for an illustration of three slot-paths in the Spain network represented in Figure 4.4. For example, the blue slot-path in Figure 4.4, serving the requests between v_2 and v_{10}, consists of 9 slots (as is seen in Figure 4.3) and goes through links ℓ_2, ℓ_8 and ℓ_{11}. The slots are contiguous, and the same slots are reserved for this particular slot-path on all its constituent links. Slot-paths blue, green and orange, do not share a link and each have different number of slots, but in all of them the starting slot is the same. This makes a valid configuration. Consequently, each configuration is indexed by s, the starting slot, and contains a set of slot-paths using one or more slots, but such that the first slot of each slot-path is s indexed. Whenever a request is granted in a configuration, it is for its overall bandwidth/slot requirement, with taking care of the slot contiguity constraint.

A configuration $c \in C = \bigcup_{s \in S} C_s$ is characterized by: a_k^c that is equal to 1 if request k is provisioned in c, 0 otherwise, and $a_{k,\ell}^{s,c}$ that is equal to one if k is provisioned with a slot-path going

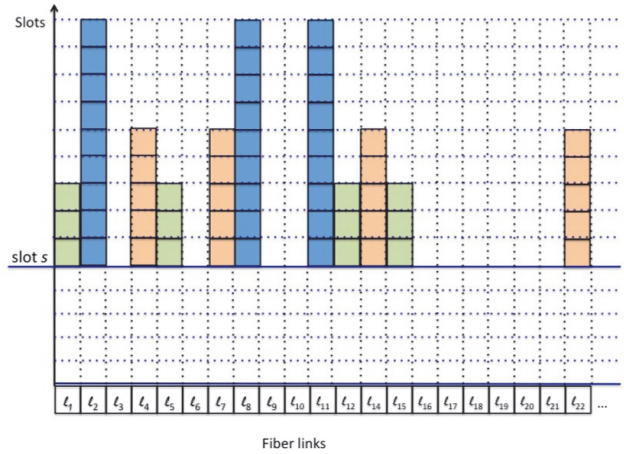


Figure 4.3: A slotpath configuration example

through link ℓ and slot s, 0 otherwise.

4.2.2 Formulation of RSA Problem

The objective of RSA problem is maximizing the throughput, unlike the RWA problem which considered the GoS. The reason is that we want to measure the spectrum efficiency more precisely. Companies today, charge the customers not only on the number of the lightpath/slot-paths, but also on the provided speed (Gb/s). In industry, RWA is still evaluated with "older" criteria, while RSA (which is more data-driven) is evaluated based on throughput. Considering the concept of slot-paths, the model of RSA problem is written as follows:

$$\max \quad \sum_{c \in C} \left(\sum_{k \in K} D_k \, a_k^c \right) z_c \tag{92}$$

subject to:

$$\sum_{c \in C_s} z_c \leq 1 \qquad s \in S \tag{93}$$

$$\sum_{c \in C} a_k^c z_c \leq 1 \qquad k \in K \tag{94}$$

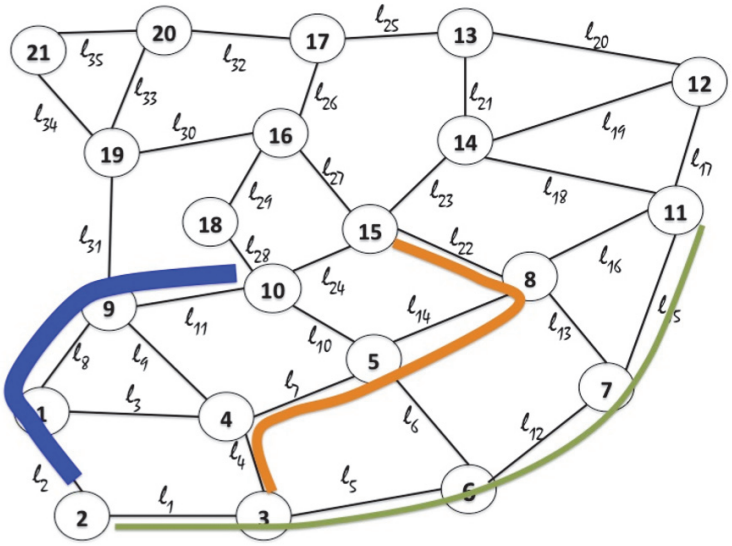


Figure 4.4: Three slot-paths on Spain network

$$\sum_{c \in C} \sum_{k \in K} a_{k,\ell}^{s,c} z_c \leq 1 \qquad s \in S, \ell \in L \tag{95}$$

$$z_c \in \{0,1\} \qquad c \in C. \tag{96}$$

The objective function (92) measures the total throughput. Constraints (93) allow at most one configuration per "starting" slot. Constraints (94) ensure that each request is accepted at most once. Constraints (95) do not allow more than one slot-path going through a given link (ℓ) for a given slot (s). Constraints (96) determine the domain of variables z_c.

4.3 Solution Scheme

As the number of variables of model (92) - (96) is exponential, we need to, again, as for RWA, recourse to column generation techniques for solving the linear relaxation as in Ruiz et al. [2013]. Consequently, we define the corresponding slot-path configuration generator problem (called pricing problem in the column generation literature). The flowchart of the solution scheme is depicted in Figure 4.5. In order to ease the solution of the configuration generator, we consider two formulations, a link one that can check thoroughly for new improving configurations, but that is computationally expensive, and a path one, that is more scalable, with a set of pre-computed paths, but

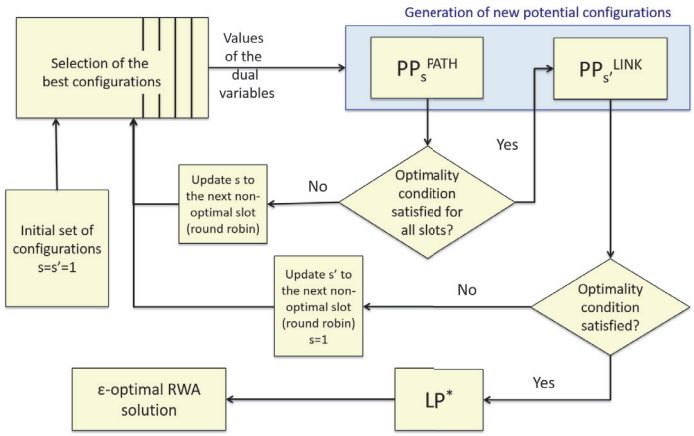


Figure 4.5: Flowchart of column generation algorithm for RSA problem

which do not guarantee to find an improving configuration when one exists. The role and order of the two pricing problems is depicted in the flowchart of Figure 4.5 in order to guarantee an optimal solution of the linear relaxation of (92) - (96).

Our model considers having the guard bands between requests. Roughly speaking, we are going to increase the slot requirement by one unit on the right endpoint, except when the last required slot reaches the right boundary of the spectrum. Although the idea is simple, the formulations become more complex. Thus, to ease the understanding, in mathematical formulations for the pricing problems, we are going to first present the model without the guard band, then explain the needed constraints for guard bands.

4.3.1 Pricing Problem - Link Formulation

As always with the column generation method, the objective of the pricing problem (i.e., generator of new configurations) is the reduced cost ($\overline{\text{COST}}_c^{\text{LINK}}$) of variable z_c. In order to alleviate the notations, index c will be omitted in the remainder of this section.

Let $u_s^{(93)} \geq 0$, $u_k^{(94)} \geq 0$ and $u_{s\ell}^{(95)} \geq 0$ be the values of the dual variables associated with constraints (93), (94) and (95) in the optimal solution of the linear relaxation of the current restricted master problem (see the flowchart in Figure 4.5).

Let K_s be the set of requests that have the potential to be provisioned in $\text{PP}^s_{\text{LINK}}$:

$$K_s = \{k \in K : s + D_k - 1 \leq |S|\}.$$

In other words, for $s \in S$, K_s contains all the requests that, if started from s, can respect the contiguity constraint: there is enough consecutive slots after s that can carry the requested bandwidth. In order to reduce the number of different notations, in the model, we have used the same notation for parameter of the master problem and decision variables of the pricing problem, dropping the index c. Therefore, with some abuse of notation, the following are the sets of variables:

$a^{k,s}_\ell = 1$ if link ℓ is used in a route of width D_k slots from s_k to d_k such that the index of the lower slot is s, 0 otherwise.

$a_k = 1$ if request k is granted in the configuration under construction, 0 otherwise.

For slot $s \in S$, the link formulation of the Slot-path Configuration Generator, called $\text{PP}^s_{\text{LINK}}$, can be written as follows:

$$\max \quad \overline{\text{COST}}^{\text{LINK}} = \sum_{k \in K_s} D_k a_k \; - \; u^{(93)}_s - \sum_{k \in K_s} u^{(94)}_k a_k - \sum_{k \in K_s} \sum_{s'=s}^{s+D_k-1} \sum_{\ell \in L} u^{(95)}_{s'\ell} \, a^{k,s'}_\ell \tag{97}$$

subject to:

$$\sum_{k \in K_s} a^{k,s}_\ell \leq 1 \qquad \ell \in L \tag{98}$$

$$\sum_{\ell \in \omega(s_k)} a^{k,s}_\ell = \sum_{\ell \in \omega(d_k)} a^{k,s}_\ell = a_k \qquad k \in K_s \tag{99}$$

$$\sum_{\ell \in \omega(v)} a^{k,s}_\ell \leq 2\, a_k \qquad k \in K_s, v \in V \setminus \{s_k, d_k\} \tag{100}$$

$$\sum_{\ell \in \omega(v) \setminus \{\ell'\}} a^{k,s}_\ell \geq a^{k,s}_{\ell'} \qquad \ell' \in \omega(v), v \in V \setminus \{s_k, d_k\}, \quad k \in K_s \tag{101}$$

$$\sum_{i=1}^{D_k-1} a^{k,s+i}_\ell = a^{k,s}_\ell \, (D_k - 1) \qquad \ell \in L, k \in K_s \tag{102}$$

$$a_\ell^{k,s'} \in \{0,1\} \qquad s' \in \{s, s+1, \ldots, s+D_k-1\}$$

$$k \in K_s, \ell \in L \tag{103}$$

$$a_k \in \{0,1\} \qquad k \in K_s. \tag{104}$$

Constraints (98) are the continuity constraints, i.e., a link cannot be traversed by more than one route in any given slot-path configuration. Routes are established with the help of constraints (100) and (101): if no route is selected for request k, then $a_\ell^{k,s} = 0$ for all links $\ell \in L$, otherwise, $a_k = 1$ and intermediate nodes of the single path from s_k to d_k are identified. Constraints (102) take care of the slot contiguity constraints, and reserve adjacent slots in order to fulfil the slot demand of granted requests. Constraints (103) define the domain of variables $a_\ell^{k,s'}$, and constraints (104) define the domain of variables a_k.

We now provide the formulation, considering the guard band. Guard band is one slot, therefore, in order to enforce it, every request requires one more slot (rightmost one), unless the last slot granted to a request to fulfil its spectrum demand, is the last slot available in the spectrum. The link formulation considering the guard band is as follows:

$$\begin{aligned}\max \quad \overline{\text{COST}}^{\text{LINK}} = & \sum_{k \in K_s} D_k a_k - u_s^{(93)} - \sum_{k \in K_s} u_k^{(94)} a_k \\ & - \sum_{k \in K_s : s+D_k-1<|S|} \sum_{s'=s}^{s+D_k} \sum_{\ell \in L} u_{s'\ell}^{(95)} a_\ell^{k,s'} \\ & - \sum_{k \in K_s : s+D_k-1=|S|} \sum_{s'=s}^{s+D_k-1} \sum_{\ell \in L} u_{s'\ell}^{(95)} a_\ell^{k,s'}\end{aligned} \tag{105}$$

subject to:

(98), (99), (100), (101), (104)

$$\sum_{i=1}^{D_k} a_\ell^{k,s+i} = a_\ell^{k,s} D_k \qquad \ell \in L, \quad k \in K_s : s+D_k-1 < |S| \tag{106}$$

$$\sum_{i=1}^{D_k-1} a_\ell^{k,s+i} = a_\ell^{k,s}(D_k-1) \qquad \ell \in L, \quad k \in K_s : s+D_k-1 = |S| \tag{107}$$

$$a_\ell^{k,s'} \in \{0,1\} \qquad s' \in \{s, s+1, \ldots, s+D_k-1\}, \quad k \in K_s, \ell \in L$$
$$p \in P_k, r \in K_s : s + D_k - 1 < |S| \tag{108}$$
$$a_\ell^{k,s'} \in \{0,1\} \qquad s' \in \{s, s+1, \ldots, s+D_k-1\}, \quad k \in K_s, \ell \in L$$
$$p \in P_k, k \in K_s : s + D_k - 1 = |S| \tag{109}$$

Constraints (98), (99), (100), (101) and (104) do not need any change when guard band is considered. However, Constraints (102) need to be splitted in to two categories: if for a request $k \in K_s$, $s \in S$, the spectrum demand needs all the remaining slots in the spectrum after s, then there is no need for a guard band and the contiguity constraints are guaranteed thanks to Constraints (107), otherwise a guard band is needed and Constraints (106) ensure that the assigned slots for a request and the associated guard band are next to each other. In the objective function (105), the term associated with the dual values of Constraints (95), is splitted in to two terms the same way as in Constraints (106) and (107). Constraints (108) and (109) are the modified versions of Constraints (103) for RSA problem with guard bands.

4.3.2 Pricing Problem - Path Formulation

In the path formulation, we provide a pre-computed set P_{sd} of paths for each source and destination pair (v_s, v_d) of nodes, which will be used for provisioning requests between v_s and v_d. We denote by P_k the set of paths associated with requests k. Selection of paths (how many for each node pair) is made as in the algorithms we developed for the Routing and Wavelength Assignment (RWA) problem, see Jaumard and Daryalal [2015]. The path formulation for the wavelength configuration generator is denoted by $\text{PP}^s_{\text{PATH}}$. It uses the set of decision variables: $\beta_p^{k,s}$ = 1 if path p is used in the wavelength configuration under construction for provisioning request k, 0 otherwise.

For slot $s \in S$, the path formulation of the Slot-path Configuration Generator, called $\text{PP}^s_{\text{PATH}}$, can be written as follows:

$$\max \quad \overline{\text{COST}}^{\text{PATH}} = \sum_{k \in K_s} D_k a_k \; - \; u_s^{(93)} - \sum_{k \in K_s} u_k^{(94)} a_k - \sum_{k \in K_s} \sum_{p \in P_k} \sum_{s'=s}^{s+D_k-1} u_{s',\ell}^{(95)} \delta_p^\ell \beta_p^{k,s'} \tag{110}$$

subject to:

$$\sum_{k \in K_s} \sum_{p \in P_k} \beta_p^{k,s} \delta_p^{\ell} \leq 1 \qquad \ell \in L \tag{111}$$

$$\sum_{p \in P_k} \beta_p^{k,s} = a_k \qquad k \in K_s \tag{112}$$

$$\sum_{i=1}^{D_k-1} \beta_p^{k,s+i} = \beta_p^{k,s} \left(D_k - 1\right) \qquad p \in P_k, k \in K_s \tag{113}$$

$$\beta_p^{k,s'} \in \{0,1\} \qquad s' \in \{s, \ldots, s + D_k - 1\}$$

$$p \in P_k, k \in K_s \tag{114}$$

$$a_k \in \{0,1\} \qquad k \in K_s. \tag{115}$$

Pairwise link disjointness for paths is guaranteed thanks to constraints (111), in which δ_p^{ℓ} is a binary parameter indicating whether link ℓ belongs or not to path p. Constraints (112) enforces to choose only one slot-path per granted request. Constraints (113) ensure the slot contiguity requirement for each request. Constraints (114) and (115) define the domain of the variables.

Correspondence between variables of the pricing problem and coefficients of the master problem:

$$a_{k,\ell}^s = \sum_{p \in P_k} \sum_{s'=s}^{s+D_k-1} \delta_p^{\ell} \beta_p^{k,s'}.$$

As for $\text{PP}^s_{\text{LINK}}$, considering the guard band changes the contiguity constraints. The path formulation considering the guard band is as follows:

$$\begin{aligned} \max \quad \overline{\text{COST}}^{\text{PATH}} = & \sum_{k \in K_s} D_k a_k \; - \; u_s^{(93)} - \sum_{k \in K_s} u_k^{(94)} a_k \\ & - \sum_{k \in K_s : s+D_k-1<|S|} \sum_{p \in P_k} \sum_{s'=s}^{s+D_k} u_{s',\ell}^{(95)} \delta_p^{\ell} \beta_p^{k,s'} \\ & - \sum_{k \in K_s : s+D_k-1=|S|} \sum_{p \in P_k} \sum_{s'=s}^{s+D_k-1} u_{s',\ell}^{(95)} \delta_p^{\ell} \beta_p^{k,s'} \end{aligned} \tag{116}$$

subject to:

$$(111), (112), (115)$$

$$\sum_{i=1}^{D_k} \beta_p^{k,s+i} = \beta_p^{k,s}\, D_k \qquad p \in P_k, k \in K_s : \; s + D_k - 1 < |S| \tag{117}$$

$$\sum_{i=1}^{D_k-1} \beta_p^{k,s+i} = \beta_p^{k,s}(D_k - 1) \qquad p \in P_k, k \in K_s : \; s + D_k - 1 = |S| \tag{118}$$

$$\beta_p^{k,s'} \in \{0,1\} \qquad s' \in \{s, \ldots, s + D_k\}, p \in P_k$$

$$k \in K_s : s + D_k - 1 < |S| \tag{119}$$

$$\beta_p^{k,s'} \in \{0,1\} \qquad s' \in \{s, \ldots, s + D_k - 1\}, p \in P_k$$

$$k \in K_s : s + D_k - 1 = |S|. \tag{120}$$

In the above formulation, the objective function, contiguity and domain constraints differ from their previous form. For $s \in S$, Constraints (118) take care of the contiguity restrictions for the requests, that if granted in this configuration, there is no slots left in the spectrum after s, while Constraints (117) concerns all the remaining requests in K_s. Constraints (119) and (120) define the domain of the variables. The objective function (116) is defined the same way as in equation (105).

Conclusions of the Chapter

In this chapter, we presented a new decomposition scheme for solving max-RSA problem with static traffic. Further, a new algorithm is provided to solve this new decomposition. The algorithm takes advantage of path and link formulations to improve the solutions and at the end ensures their optimality. Numerical results in Chapter 5 show that the algorithm is successful in solving instances based on a typical network example in the literature, and surpasses far beyond other existing solution algorithm by solving instances with up to 400 slots, which was at most 96 in Ruiz et al. [2013].

Chapter 5

Numerical Results

The original contribution of this chapter is a very thorough set of experiments for the static RWA/RSA and dynamic RWA problems. Results show that we manage to reach our goal of solving both RWA and RSA data instances whose sizes match those of today real optical networks.

The chapter is organized as follows. In the first part, Section 5.1, we examine our proposed solution algorithms for static and dynamic RWA problem, and assess their performances in comparison to a well-known existing algorithm in the literature. In the second part, Section 5.2, several instances are being solved using the solution algorithm provided to solve the RSA problem, and the results are discussed

5.1 Large-Scale RWA Problem

We present here the numerical experiments that were conducted in order to validate and test the performance of the proposed algorithms: the three ε-optimal algorithms, CG, CG+, and CG++, as well as the two heuristic algorithms, CG^H+ and CG^H++. After describing the data sets (Section 5.1.1), we discuss the results for the static case (Section 5.1.2), starting with the quality of the solutions (Section 5.1.2), exact vs. heuristic solutions (Section 5.1.2), and then the impact of the path selection on the GoS (Section 5.1.2), as well as the usage of the bandwidth spectrum from one link to the next (Section 5.1.2). The remaining results refer to the dynamic case (Section 5.1.3) where we investigate the bandwidth spectrum waste when no re-arrangements are performed.

All computational results have been obtained with running the programs on a server with the help of CPLEX [Cplex, 2014] (version V12.6.2) for solving the (integer) linear programs. Several fine tuning of cplex parameters are required for solving the data sets as efficiently as possible. These include switching off the presolve operations, restricting the number of threads to 1, solving the problems using Barrier algorithms, setting the emphasis of the pricing problem on feasibility rather than optimality, and switching back the solver of pricing problems to traditional branch-and-bound, rather than dynamic search. Programs never used more than 2Gb memory and 2 CPUs.

5.1.1 Data Sets

We run experiments on six different networks: NSFNET [Orlowski et al., 2007], USANET [Batayneh et al., 2011], GERMANY [Orlowski et al., 2007], NTT [Vega-Rodriguez and Rubio-Largo], ATT [Martins et al., 2012], and BRAZIL [Jaumard et al., 2006b], whose characteristics are reported in Table 5.1. Column entitled ”deg.” is the average nodal degree, in order to measure the network connectivities. Last two columns report on the traffic distribution thanks to the mean and the variance on the number of requests per node pair with traffic, as identified by μ and σ, respectively. Topologies are reproduced in Figures 5.1-5.6 where undirected lines represent bidirectional links.

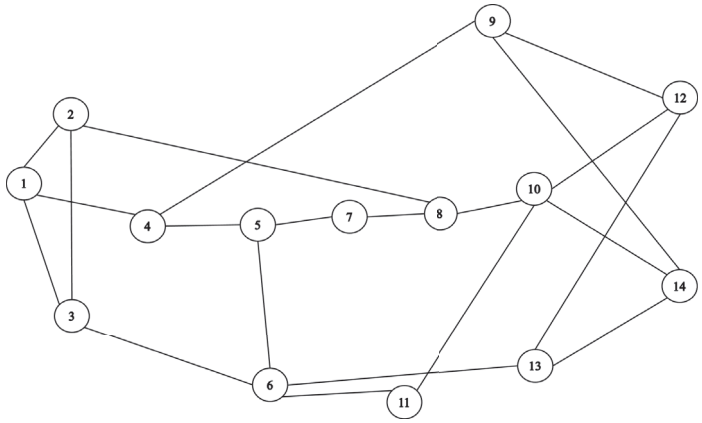


Figure 5.1: Network topologies: NSF

For each network topology, we consider various traffic instances with up to 150 wavelengths. For the first traffic instances of the NSF and USA topologies, the directed traffic demand matrix $T = [T_{sd}]$ is generated by drawing integer traffic demands (with one unit being the transport capacity

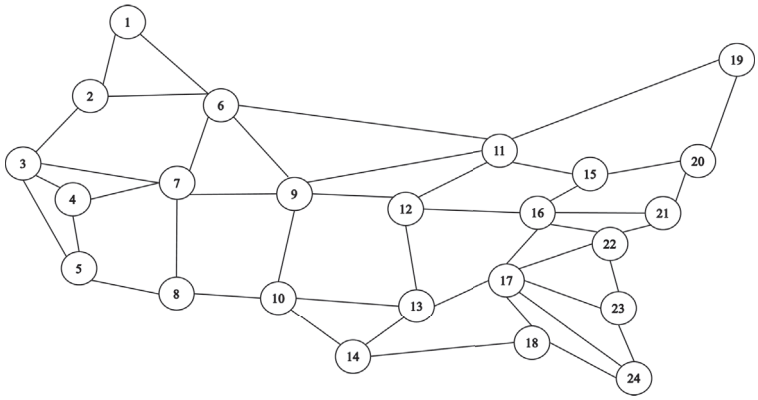


Figure 5.2: Network topologies: USA

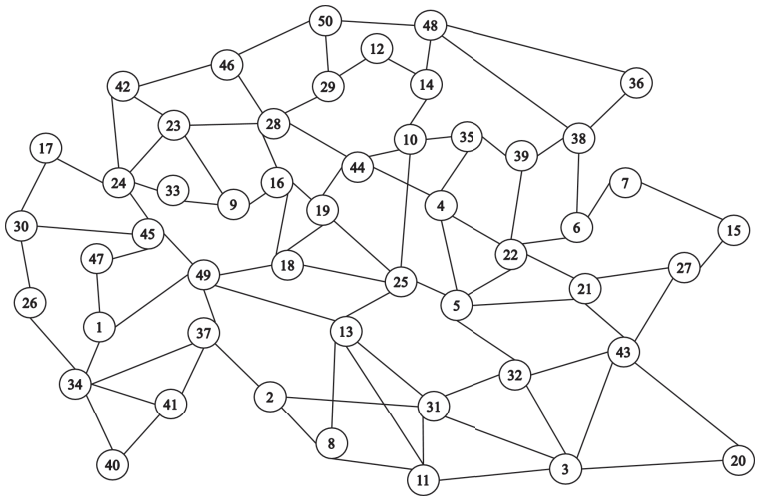


Figure 5.3: Network topologies: GER

of one wavelength) uniformly at random in $\{0, 1, 2, 3, 4, 5\}$. For the GERMANY topology, the first traffic instance (i.e., GER_{100}) comes from the snd·lib library [Orlowski et al., 2007].

For the NTT topology, the first two traffic instances (i.e., NTT_{42} and NTT_{50}) are from [Vega-Rodriguez and Rubio-Largo]. The next augmented traffic instances correspond to incremental traffic: NSF/USA/GER/NTT_{λ} $\subseteq$ NSF/USA/GER/$\text{NTT}_{\lambda'}$ where NSF/USA/$\text{GER}_{\lambda'}$ are built upon NSF/USA/GER_{λ} by REPEAT times randomly adding ALEA more requests for each pair of nodes. ALEA is taken uniformly at random from $\{1, 2, 3, 4, 5\}$ for NSF/USA and from $\{0, 1, 2, 3\}$ for GERMANY. For NSF, REPEAT $= 10, 19$, for USA, REPEAT $= 4, 8$ and for GERMANY, REPEAT $= 1, 5$. NTT_{150} is built directly upon NTT_{50}, considering a random additional number of requests from $\{1, \ldots, 25\}$, which leads to a less uniform traffic instance, as shown by the variance indicator in

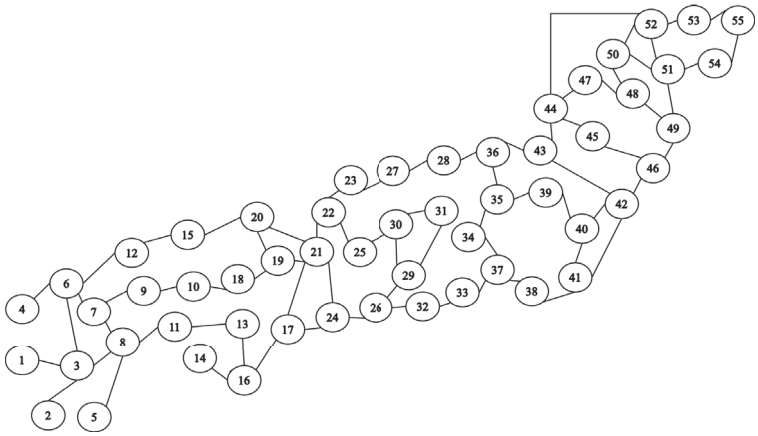


Figure 5.4: Network topologies: NTT

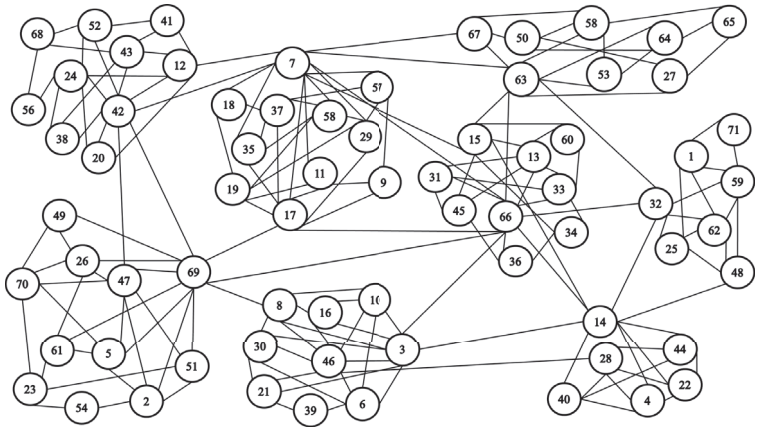


Figure 5.5: Network topologies: ATT

the last column of Table 5.1. Traffic matrix of BRAZIL comes from Jaumard et al. [2006b]. We selected a limited number of traffic instances when reporting on the numerical experiments, their characteristics are described in Table 5.1, the index of the instance names refer to the number of wavelengths.

5.1.2 Static Case: Algorithm Comparative Performances

Solution Accuracies and Algorithm Efficiencies

In Tables 5.2 and 5.3, we compare the performances of the four new proposed algorithms and the algorithm of Jaumard et al. [2009], using Strategy 3 for selecting the paths.

In Table 5.2, we focus on the comparison of the maximum GoS, and the number of generated

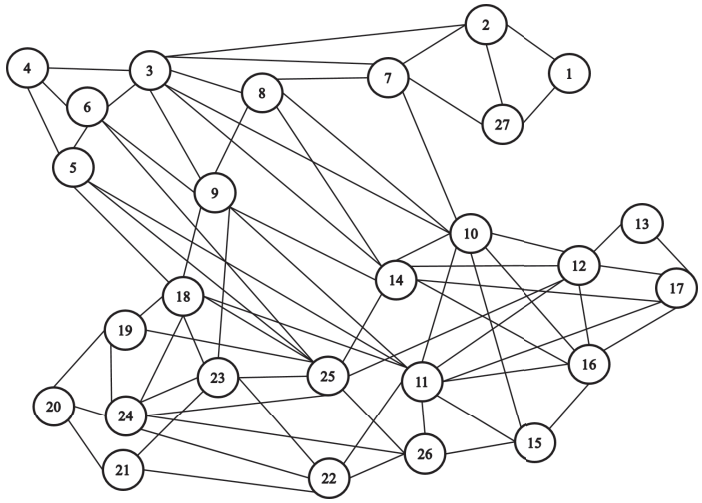


Figure 5.6: Network topologies: BRAZIL

Table 5.1: Characteristics of the datasets

Data instances	$\|V\|$	$\|L\|$	deg.	W	$\|\mathcal{SD}\|$	$\sum D_{sd}$	Traffic distribution μ	Traffic distribution σ
NSF_{30}				30	141	436	3.1	1.4
NSF_{75}	14	40	3.0	75	182	1,371	7.5	2.4
NSF_{115}				115	182	2,194	12.1	2.7
USA_{75}				75	455	1,336	2.9	2.4
USA_{125}	24	88	3.7	125	541	2,422	4.5	1.9
USA_{150}				150	552	3,509	6.4	2.2
GER_{100}				100		2,365	3.6	6.2
GER_{130}	50	176	3.5	130	660	3,041	6.2	4.6
GER_{150}				150		4,989	8.6	6.3
NTT_{42}				42	338	1,038	3.1	1.4
NTT_{50}	55	144	2.6	50	452	1,362	3.0	1.4
NTT_{150}				150	452	5,684	12.6	7.4
ATT_{20}	90	274	3.0	20	272	359	1.3	0.7
ATT_{113}	71	350	4.9	113	2,869	2,918	1.0	0.7
$BRAZIL_{48}$	27	140	5.2	48	549	1,370	2.5	1.1

wavelength configurations, using either PP_{PATH} or PP_{LINK}. While there is no dominance in terms of best generated ε-optimal solutions between CG and CG+, algorithm CG++ dominates both CG and CG+ for all data sets, increasing the number of granted demand from 2 (NSF_{30}) to 71 (NTT_{150}) lightpaths.

In Table 5.3, we report the number of columns in the last iteration of the column generation algorithms, and the computational times (in seconds). Reported computational times (in columns entitled CPU) do not include the preprocessing operations, which is reported in the columns entitled PRE_CPU. While in most cases CG+ is much faster than CG, it requires more iterations. However, as the pricing problem restricts the search of wavelength configurations using pre-computed shortest

Table 5.2: Static case: comparative performance of the algorithms

Data instances	$z^{\star}_{\mathrm{LP}}$	ε-optimal algorithms											Heuristic algorithms					
		CG [Jaumard et al., 2009]			CG+				CG++				$\mathrm{CG^H}$+			$\mathrm{CG^H}$++		
		$\tilde{z}_{\mathrm{ILP}}$	ε	$\#\mathrm{PP}_{\mathrm{LINK}}$	$\tilde{z}_{\mathrm{ILP}}$	ε	#PP_iter PATH	#PP_iter LINK	$\tilde{z}_{\mathrm{ILP}}$	ε	#PP_iter PATH	#PP_iter LINK	$\tilde{z}_{\mathrm{ILP}}$	ε	$\#\mathrm{PP}_{\mathrm{PATH}}$	$\tilde{z}_{\mathrm{ILP}}$	ε	$\#\mathrm{PP}_{\mathrm{PATH}}$
NSF_{30}	430	412	4.2	85	419	2.6	158	6	421	2.1	140	3	410	4.7	88	411	4.4	100
NSF_{75}	1,242	1,218	1.9	109	1,221	1.7	189	7	1,230	1.0	217	3	1,198	3.5	126	1,222	1.6	108
NSF_{115}	1,924	1,898	1.4	92	1,900	1.2	167	6	1,909	0.8	217	2	1,854	3.6	104	1,904	1.0	108
USA_{75}	1,281	1,229	4.1	249	1,227	4.2	378	2	1,241	3.1	402	2	1,227	4.2	345	1,234	3.7	253
USA_{125}	2,255	2,190	2.9	324	2,160	4.2	315	1	2,201	2.4	458	2	2,160	4.2	315	2,160	4.2	299
USA_{150}	3,029	2,914	3.8	252	2,885	4.8	255	1	2,975	1.8	508	2	2,885	4.8	255	2,885	4.8	258
GER_{100}	2,306	2,185	5.2	349	2,197	4.7	2,850	42	2,245	2.7	537	3	2,099	9.0	236	2,206	4.3	336
GER_{130}	2,960	2,791	5.7	437	2,840	4.1	3,543	104	2,889	2.4	639	3	2,658	10.2	192	2,798	5.5	414
GER_{150}	4,663	4,472	4.1	731	4,502	3.5	6,253	119	4,519	3.1	937	6	4,219	9.5	336	4,466	4.2	711
NTT_{42}	1,038	1,031	0.7	16	1,024	2.3	75	2	1,038	0.0	31	2	1,022	1.5	57	1,025	1.3	20
NTT_{50}	1,362	1,356	0.4	32	1,310	3.8	287	5	1,362	0.0	59	2	1,308	4.0	112	1,318	3.2	55
NTT_{150}	5,553	5,431	2.2	314	5,372	3.3	2,292	55	5,502	0.9	867	21	5,204	3.6	277	5,431	2.2	311
ATT_{20}	359	328	8.6	118	328	8.6	2,571	62	354	1.4	380	7	272	24.2	113	321	10.6	71
ATT_{113}	2,918	2,856	2.1	319	2,884	1.2	7,303	112	2,902	0.5	397	6	2,740	6.1	1,182	2,841	2.6	211
BRAZIL_{48}	1,370	1,297	5.3	168	1,295	5.5	895	17	1,317	3.9	313	2	1,273	7.0	195	1,301	5.0	156

Table 5.3: Static case: computational times (seconds) of the algorithms

Data instances	CG				ε-optimal algorithms						Heuristic algorithms			
					CG+			CG++			CG^H+		CG^H++	
	$\tilde{z}_{ILP}$	GoS	CPU	# cols.	PRE_CPU	CPU	# cols.	PRE_CPU	CPU	# cols.	CPU	# cols.	CPU	# cols.
NSF_{30}	412	94.5	20	84	0	3	157	2	7	139	1	87	6	99
NSF_{75}	1,218	88.7	26	108	0	3	188	3	7	216	2	125	5	107
NSF_{115}	1,898	86.5	24	91	0	3	166	3	7	216	1	103	5	107
USA_{75}	1,229	92.0	663	248	1	34	377	12	129	401	27	344	70	252
USA_{125}	2,190	90.4	1,211	323	1	21	314	17	138	457	21	314	106	298
USA_{150}	2,914	83.0	890	251	1	17	254	21	155	507	15	254	87	257
GER_{100}	2,185	92.4	6,045	348	2	885	2,849	47	427	536	13	235	161	335
GER_{130}	2,791	91.8	7,050	436	2	2,111	3,542	54	467	638	12	191	108	413
GER_{150}	4,472	78.9	16,863	730	2	3,448	6,252	70	1,747	936	23	335	283	710
NTT_{42}	1,031	99.3	49	15	0	4	74	25	5	30	1	56	5	19
NTT_{50}	1,356	99.6	161	31	1	22	286	30	7	58	2	111	8	54
NTT_{150}	5,431	95.5	2,403	313	1	686	2,291	48	183	866	10	276	81	310
ATT_{20}	328	91.3	1,644	117	1	1,917	2,570	76	1,473	379	8	112	131	70
ATT_{113}	2,877	97.9	26,035	318	9	3,807	7,302	714	893	396	1,018	1,181	273	210
$BRAZIL_{48}$	1,297	94.7	1,145	167	1	1,191	894	23	563	312	195	181	44	155

paths most of the time (indeed, 97% of the time on average), then computational times can be significantly reduced.

CG++ requires more computational time than CG+, in all smaller data sets even if there are less wavelength configurations generated. This is due to the fact that PP_{PATH} contains more paths and therefore requires longer computational times for its solution. However, the number of calls to PP_{LINK} is less for CG++ than for CG+. This is as expected since PP_{PATH} is richer in terms of the number of paths that are considered. The effect of this smaller number of calls to PP_{LINK} is revealed in the largest data sets, i.e., GER network, NTT_{150}, ATT and BRAZIL. In Table 5.2, it can be seen that in the latter data sets, the number of calls to PP_{LINK} is much smaller for CG++ than for CG+ ; it results in significantly smaller computational times.

Comparison of CG, CG+, and CG++

We observe in Table 5.2 that the number of wavelength configuration generations with the link formulation, i.e., PP_{LINK}, which is more computationally expensive than PP_{PATH}, decreases, as we move from CG to CG+, and then to CG++. Except for NTT_{150}, the number of PP_{LINK} solutions is very small for CG++, contributing significantly to the overall reduction of the computational effort for CG++.

As the size of the network topology increases, i.e., moving from NSF to USA topology, while CG performs better than CG+ in terms of GoS, the differences between the computational times widen: on average, computational times for CG are 43 times longer than for CG+. It is interesting to observe that except for USA_{75}, we recourse to PP_{LINK} only one time, meaning that the solution provided by the wavelength configurations of PP_{PATH} account for most of the computational times, and explain the reduction of it in comparison with algorithm CG. CG++ improves on CG+, while requiring to generate less wavelength configurations for USA_{75} and USA_{130}, but more wavelength configurations for the largest USA data set USA_{150}. This can be explained by the larger number of considered k-shortest paths.

In GERMANY network, except for one instance, CG+ gets better solutions than CG, with an increased number of lightpaths ranging from 12 (in addition to 2,185 in GER_{75}) to 49 (in addition to 2,791 in GER_{130}). Computational times of CG in most cases are higher than those of CG+, i.e., on

average 5 times larger. As for previous networks, CG++ always returns better GoS results than both CG and CG+. It is interesting to see that CG++ succeeds in obtaining better results in considerably shorter time than CG+, which itself performs dramatically faster than CG. In fact, the computational times of CG are on average 13 times longer than that of CG++, which is a significant improvement. This can be explained by the low number of PP_{LINK} solutions in CG++ (from 3 to at most 21), that can make up for the larger computing times of its PP_{PATH} problem in comparison to that of CG+.

NTT network has an interesting feature and that is the degree of its nodes, which in most cases is equal to 2. In two out of three data sets, CG++ reaches 100% GoS. Using Strategy 3 for NTT_{150}, the number of pre-computed paths is 1,568. Table 5.3 shows that this set of paths is rich enough to reduce the number of calls to PP_{LINK}, while it is not too big, which otherwise would increase the computing times of PP_{PATH}.

Exact vs. Heuristic

When comparing with the exact methods, the heuristic algorithms fulfil their purpose: they provide very good solutions with small computational times. Indeed, differences between the accuracies of the exact and heuristic algorithms is, in most cases, quite low. For CG^H+ the difference between returned GoS and the corresponding ε-optimal GoS from CG+ varies between 0 lightpaths (0% in all USA instances) to 283 (6.3% in GER_{130}). The performance of the algorithm CG^H++ is even better, with the difference between its solution and that of CG++ lying in the range of 5 lightpaths (0.3% in NSF_{115}) to 91 (3.1% GER_{130}).

Heuristic CG^H++ always obtains better heuristic solutions than CG^H+. Improvement can lead to up to 247 more granted lightpaths, see, e.g., data set GER_{150}, 4,219 granted lighpaths with CG^H+ vs. 4,466 with CG^H++, while the lower bound is equal to 4,663. The resulting accuracy is $\varepsilon = 4\%$, which is excellent for this large data set with 50 nodes, 176 links, 5666 lightpath requests, and 150 wavelengths. Computational time of CG^H++ is usually larger than CG^H+, due to the larger number of considered paths in the wavelength generator problems (i.e., the pricing problems).

Table 5.4: Comparison of various strategies for selecting the set of paths in CG++

	CG algorithm		CG++ algorithm											
Data			Strategy 1				Strategy 2				Strategy 3			
Instances	$z^{\star}_{\text{LP}}$	$\tilde{z}_{\text{ILP}}$	$\tilde{z}_{\text{ILP}}$	#		CPU	$\tilde{z}_{\text{ILP}}$	#		CPU	$\tilde{z}_{\text{ILP}}$	#		CPU
				PP_{PATH}	PP_{LINK}			PP_{PATH}	PP_{LINK}			PP_{PATH}	PP_{LINK}	
NSF_{30}	430	412	415	127	2	29	415	110	1	14	421	140	3	7
NSF_{75}	1,242	1,218	1,222	108	1	3	1,222	127	1	55	1,230	217	3	7
NSF_{115}	1,924	1,898	1,907	145	2	2	1,907	114	1	61	1,909	217	2	7
USA_{75}	1,281	1,229	1,234	253	1	67	1,237	238	1	259	1,241	402	2	129
USA_{125}	2,255	2,190	2,191	361	3	92	2,195	311	1	737	2,201	458	2	138
USA_{150}	3,029	2,914	2,948	406	3	80	2,961	292	1	797	2,975	508	2	155
GER_{100}	2,306	2,185	2,206	336	1	82	2,226	317	1	358	2,245	537	3	427
GER_{130}	2,960	2,791	2,861	432	2	107	2,874	399	1	802	2,889	639	3	467
GER_{150}	4,663	4,472	4,507	1,443	11	808	4,507	692	2	3,354	4,519	937	6	1,747
NTT_{42}	1,038	1,031	1,038	28	2	6	1,038	11	1	19	1,038	31	2	5
NTT_{50}	1,362	1,356	1,362	103	3	5	1,362	29	1	19	1,362	59	2	7
NTT_{150}	5,553	5,431	5,431	311	1	75	5,491	273	1	1,672	5,502	867	21	183
ATT_{20}	359	330	330	544	3	1,906	335	512	1	3,493	354	380	7	1,473
ATT_{113}	2,918	2,877	2,884	419	9	2,806	2,889	286	2	4,688	2,902	397	6	893
BRAZIL_{48}	1,370	1,297	1,303	157	1	1,235	1,300	142	1	1,709	1,317	313	2	563

Path Selection

It is of interest to notice that, while the number of shortest paths for a given node pair is usually very small, although it increases with the length of the shortest path, the number of k-shortest paths in P^2 can increase sharply. For instance, the number of 2nd shortest paths can reach 52 and 70 for some node pairs in USA and GER networks, respectively.

Table 5.5: Characteristics of the selected paths in ε-optimal solutions of CG++

Data	SP^1		SP^2		SP^3		SP^4		$SP^{\geq 5}$	
instances	#	%	#	%	#	%	#	%	#	%
NSF_{30}	391	83.2	66	14.0	11	2.3	1	0.2	1	0.2
NSF_{75}	1,116	87.9	136	10.7	18	1.4	0	0.0	0	0.0
NSF_{115}	1,808	89.2	193	9.5	24	1.2	2	0.1	0	0.0
USA_{75}	914	61.0	418	27.9	154	10.3	10	0.7	2	0.1
USA_{125}	1,599	62.0	670	26.0	278	10.8	26	1.0	7	0.3
USA_{150}	2,108	64.5	831	25.4	299	9.1	24	0.7	6	0.2
GER_{100}	1,321	49.8	841	31.7	383	14.5	78	2.9	27	1.0
GER_{130}	1,642	49.5	923	27.8	559	16.8	153	4.6	42	1.3
GER_{150}	3,221	64.9	1,002	20.2	742	14.9	0	0.0	0	0.0
NTT_{42}	1,304	81.2	173	10.8	86	5.4	25	1.6	17	1.1
NTT_{50}	1,409	79.9	202	11.5	94	5.3	37	2.1	22	1.2
NTT_{150}	4,633	77.8	900	15.1	298	5.0	84	1.4	37	0.6
ATT20	205	51.5	49	12.3	58	14.6	26	6.5	60	15.1
ATT_{113}	1,195	40.2	1,264	42.5	500	16.8	7	0.2	5	0.2
$BRAZIL_{48}$	739	46.7	640	40.5	201	12.7	1	0.1	0	0.0

In Table 5.4, we compare the three strategies for selecting the paths in CG++ and this table is a summary of the extensive computational experiments we conducted. For Strategy 1, we found out that a good value was $k^{\text{SP}} = 15$, and in Strategy 2, a good constant value $\rho = 0.5$. Both correspond to a very good compromise between the computational times and the maximum GoS

value. Computational times in columns entitled CPU are in seconds and do not include the time needed for preprocessing.

Results of Table 5.4 show that CG++ using Strategy 1 mostly has the smallest computational times, while Strategy 3 results in improved GoS. In all data sets, Strategy 2 is at least as good as Strategy 1, and, in many cases, improves it. However, the computational time is increased drastically, due to a significantly larger number of considered paths, while not all of them are very useful. Strategy 3 improves the GoS of almost all the data sets, while reducing the computational time compared to Strategy 2. For example, employing Strategy 3 in solving NTT_{150} grants 71 and 11 more requests than Strategy 1 and 2 respectively, while the computational times are still in a reasonable range. In conclusion, algorithm CG++ with Strategy 3 provides the best GoS within reasonable computational times.

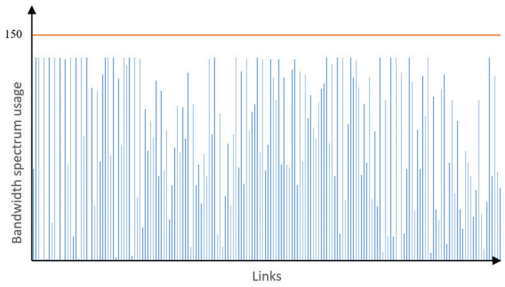
Characteristics of the selected paths in CG++ are described in Table 5.5. We observe that, while shortest paths play an important role in the ε-optimal solution of CG++, 2nd and 3rd shortest paths also have a significant share in the final set of chosen paths. It is interesting to see that, for ATT_{113}, the share of the 2nd shortest paths is even more than the 1st shortest paths. This justifies the enlargement of the initial set of pre-computed paths to take into account the paths beyond shortest paths.

Spectrum Utilization

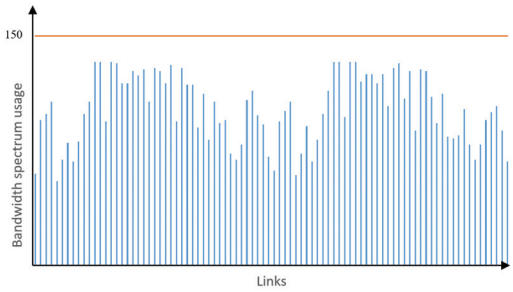
Lastly, we investigate the bandwidth spectrum usage for 3 instances, in the ε-optimal solutions of CG++. In Table 5.6, we report the average, min and max link spectrum usage for the ε-optimal solutions of Algorithm CG++, while in Figure 5.7, we display the spectrum link usage for 3 different instances.

Table 5.6: Average, min and max link spectrum usage (%)

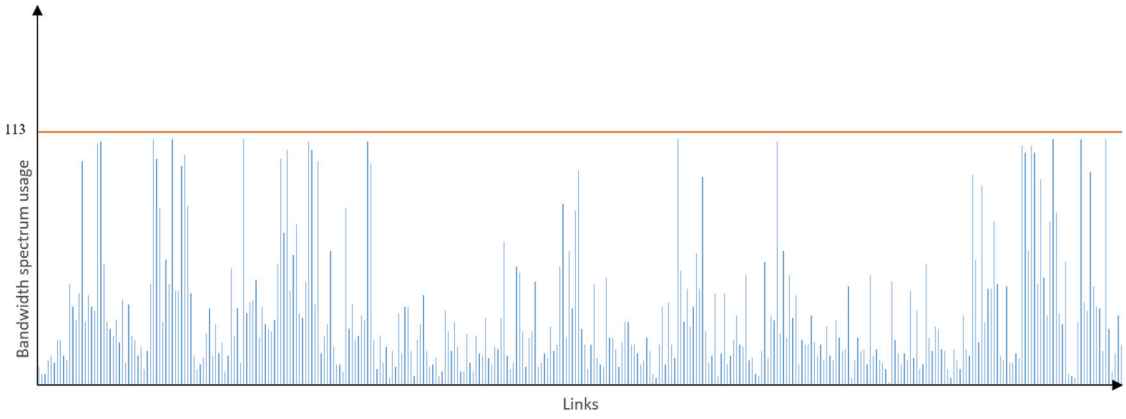
Data instances	Min	Average	σ	Max
NSF_{30}	50.0	84.4	13.0	100.0
NSF_{75}	44.0	62.0	7.6	69.3
NSF_{115}	50.4	71.7	7.7	80.0
USA_{75}	29.3	71.8	16.4	96.0
USA_{125}	32.8	63.8	14.3	84.8
USA_{150}	36.7	63.2	14.7	86.7
GER_{100}	0.0	53.5	28.6	97.0
GER_{130}	0.0	51.7	28.0	93.8
GER_{150}	0.0	55.3	29.2	90.0
NTT_{42}	0.0	19.2	9.1	40.8
NTT_{50}	0.0	31.7	14.2	58.0
NTT_{150}	0.0	62.8	20.8	88.7
ATT_{20}	0.0	41.0	31.0	100.0
ATT_{113}	0.0	24.9	23.6	97.3
$BRAZIL_{48}$	37.5	72.0	15.0	100.0



(a) GER_{150}



(b) USA_{150}



(c) ATT_{113}

Figure 5.7: Bandwidth spectrum usage with CG++

For both GER_{150} and the ATT_{113} instances, the variance in the spectrum usage of links is quite high, while in USA_{150}, it is smaller. It can be explained by the characteristics of the topologies: USA is closer to a grid like topology in which nearly all links participate equally in the overall set of k-shortest paths, while GER and ATT topologies are more irregular. It may mean that if traffic patterns deviate further from rather homogeneously distributed traffic patterns (see the mean and the variance of the current traffic patterns in Table 5.1), then, we might observe even larger variances in the spectrum usage of the links.

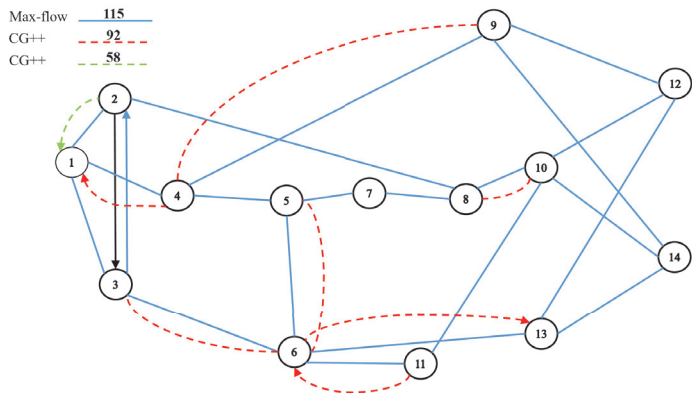


Figure 5.8: Comparison between the highly loaded links in the solutions of model (55) - (61) vs. CG++ model: NSF_{115}

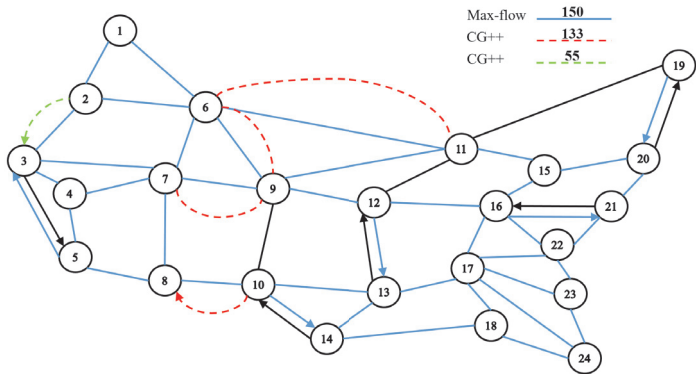


Figure 5.9: Comparison between the highly loaded links in the solutions of model (55) - (61) vs. CG++ model: USA_{150}

In Figures 5.8 - 5.12, we have indicated in red the links with the highest number of lightpath traversals, and in green the links with the small number. Blue links represent the most loaded links obtained by solving the max-flow formulation (55) - (60). Comparison between the sets of blue and

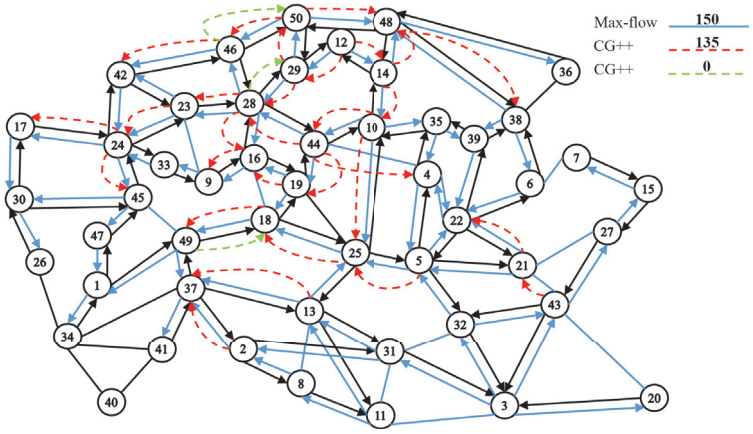


Figure 5.10: Comparison between the highly loaded links in the solutions of model (55) - (61) vs. CG++ model: GER_{150}

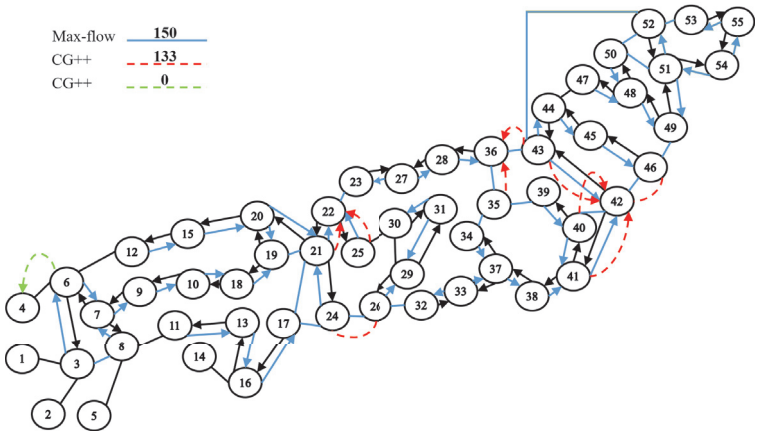


Figure 5.11: Comparison between the highly loaded links in the solutions of model (55) - (61) vs. CG++ model: NTT_{150}

red links shows that the max-flow model is successful in identifying the potentially bottleneck links. This leads to a smarter approach in selecting the paths used in the wavelength generator problem PP_{PATH} of the CG++ algorithm with Strategy 3. It also results in a more load balanced solution that has less highly loaded links. In NTT_{150}, link $\ell_{(6,4)}$ is the least loaded link after solving the problem with CG++ and has a 0% spectrum usage. Since the number of traffic requests in NTT_{150} between node v_6 and v_4 is 0, the result is justified. GER_{150} has three links with 0% spectrum usage: $\ell_{(28,29)}, \ell_{(46,71)}, \ell_{(49,18)}$, meaning that those links do not belong to any path of interest for provisioning the demand.

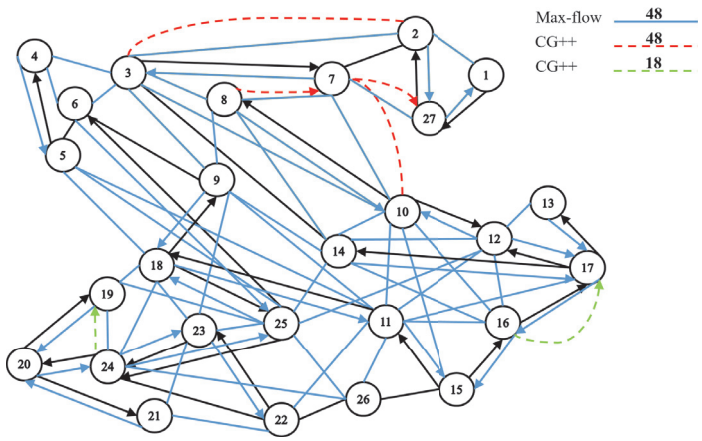


Figure 5.12: Comparison between the highly loaded links in the solutions of model (55) - (61) vs. CG++ model: BRAZIL_{48}

5.1.3 Dynamic Case: Algorithm Performances

In order to simulate the incremental traffic, the connection requests of every data set are divided into smaller sets, such that $\mathcal{SD}^{t-1} \subseteq \mathcal{SD}^t$ and for every pair $(v_s, v_d) \in \mathcal{SD}^t$, $D_{sd}^{t-1} \leq D_{sd}^t$. This is done in consecutive steps of size δ: in each time period, δ traffic requests are selected randomly, from the original demand set $\mathcal{SD}$ at time $t = 0$. The requests are selected one unit at a time.

Wasted Spectrum Utilization

We use four different values of δ, 10, 50, 200 and 500, and report the results in Table 5.7 for the first dynamic RWA model, with no lightpath re-arrangement. We report the maximum GoS for each strategy and dataset. The results for the static case are obtained by considering all the traffic requests at once at time period $t = 1$ and solving the problem using algorithm CG++ with strategy 3. After each addition of δ unit requests, we optimize their provisioning while preserving the provisioning of the legacy requests. We keep adding batches of δ requests until all requests have been added.

We observe that, when there is no lightpath re-arrangement, GoS with dynamic traffic is much less than the optimum GoS of the static case. The difference varies with the step size δ. In every data set, up to a turning point, a greater step size affects more the set of available lightpaths and causes the GoS to drop. From that turning point on, sets of new traffic requests become big enough and GoS behavior is closer to the one of the static traffic. For instance, in GER_{130}, GoS decreases

Table 5.7: Dynamic RWA with no lightpath re-arrangement

Data instances	GoS				
	Static	Dynamic			
		$\delta = 10$	$\delta = 50$	$\delta = 200$	$\delta = 500$
NSF_{30}	96.6	83.7	71.6	74.1	96.6
NSF_{75}	89.6	71.3	62.3	56.6	63.0
NSF_{115}	87.0	70.6	61.3	52.8	59.4
USA_{75}	92.9	87.0	78.6	71.2	78.5
USA_{125}	90.9	80.6	76.5	66.3	68.7
USA_{150}	84.8	73.7	65.8	57.6	59.3
GER_{100}	94.9	82.7	75.7	68.9	76.3
GER_{130}	95.0	81.8	71.6	67.9	71.7
GER_{150}	79.8	60.7	55.0	44.7	48.5
NTT_{42}	100.0	98.1	96.3	93.3	96.6
NTT_{50}	100.0	90.1	89.4	85.2	89.7
NTT_{150}	96.8	76.4	73.9	59.9	63.5
$ATT1_{20}$	98.6	81.3	75.5	86.1	98.6
$ATT2_{113}$	99.5	84.5	80.0	85.5	95.5

as the step size grows from 50 to 200. After this point, with increasing the step size, GoS starts to improve.

Minimizing the Lightpath Re-arrangements

We summarize in Table 5.8 the results obtained when considering some lightpath re-arrangement. In these experiments, the value for PENAL is 0.1. For each problem instance, we report the maximum GoS (absolute and percentage values), the cumulative number of re-arranged lightpaths (absolute and percentage values), and then the average number of lightpath re-arrangement per time period. As can be observed, this last number is quite small, i.e., always smaller than 8%. This shows that, by allowing the re-arrangement of lightpaths, one can achieve a GoS very close to that of the static traffic, while paying the cost of re-arranging a very small percentage of already established lightpaths.

5.2 Large-Scale RSA Problem

We implemented the algorithms proposed in the previous section and tested them on the Spain network (21 nodes, 35 edges) as in [Ruiz et al., 2013], see Figure 5.13.

Table 5.8: Dynamic RWA with minimum lightpath re-arrangement

Data instances	δ	GoS		Cumulative disruption		Average disruption
		#	%	#	%	%
NSF_{30}	50	415	95.2	78	18.8	3.7
NSF_{75}	200	1,222	89.0	215	17.6	1.9
NSF_{115}	500	1,920	87.5	361	18.8	4.3
USA_{75}	50	1,234	92.4	351	28.4	1.1
USA_{75}	200	1,234	92.4	314	25.4	4.1
USA_{125}	100	2,160	89.2	573	26.5	1.4
USA_{125}	500	2,158	89.1	447	20.7	5.6
USA_{150}	200	2,855	81.4	802	28.1	1.9
USA_{150}	500	2,841	81.0	695	24.5	3.1
GER_{100}	250	2,206	93.3	505	22.9	2.9
GER_{100}	500	2,201	93.1	392	17.8	4.7
GER_{130}	250	2,861	94.1	856	29.9	2.8
GER_{130}	500	2,848	93.7	670	23.5	4.4
GER_{150}	250	4,507	79.5	1,712	38.0	1.9
GER_{150}	500	4,483	79.1	1,303	29.1	3.0
NTT_{42}	200	1,038	100.0	9	0.9	0.2
NTT_{50}	200	1,362	100.0	24	1.8	0.8
NTT_{150}	500	5,431	95.5	111	2.0	0.2
ATT1	50	359	100.0	160	44.6	7.3
ATT2	250	2,898	99.3	840	29.0	3.3

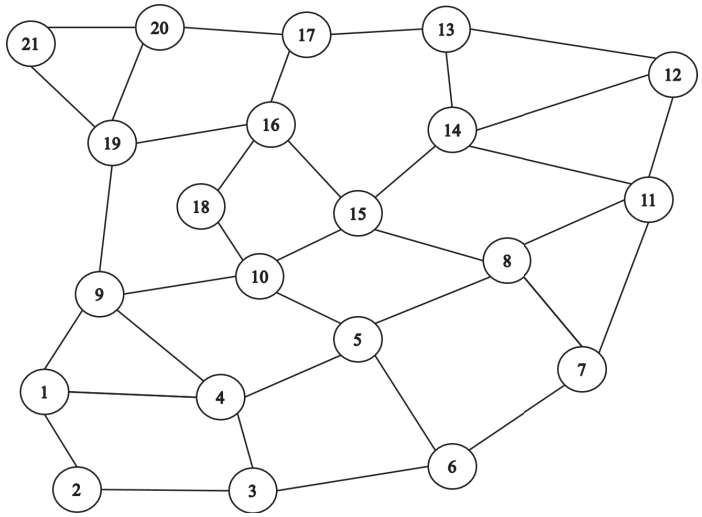


Figure 5.13: Spain network topology

All computational results have been obtained with running the programs on a 1.9-2.5GHz Core i5 machine with 4GB RAM running Windows 10, with the help of CPLEX (version V12.6.2) for solving the (integer) linear programs.

5.2.1 Data Sets

We conducted our experiments on several randomly generated data instances. Each instance is characterized by:

- A random selection of K source/destination pairs,
- A traffic profile in which each demand has a number of slots randomly generated in $\{1, 2, \ldots, 8\}$ or $\{2, 4, \ldots, 16\}$.

For computing the associated bandwidth, we assumed a slot width of 12.5 GHz and a spectral efficiency of 25 Gb/s per slot as in [Ruiz et al., 2013].

5.2.2 Algorithm Performance

While the largest instances that Ruiz et al. [2013] were able to solve had up to 180 demands and 96 available slots, we were able to solve instances with 180 requests and 400 available slots on the

Spain network. The heuristic rule that we borrowed from Jaumard and Daryalal [2015] works well as it allowed us to reach the optimal LP relaxation value in much less time than Ruiz et al. [2013].

We observe that for all data sets, the optimal LP value is indeed equal to the input load, and can be reached fairly easily. Therefore, a challenge is to generate sufficiently good columns (i.e., lightpath configurations) in order to get good integer solutions, with a reasonable integrality gap (ε). While ε is not very small, it is comparable or smaller than what has been obtained by Ruiz et al. [2013].

Table 5.9: Performance of the algorithm for RSA problem

Traffic demand			# precomputed k-shortest paths	ε-optimal solution							load per link (%)		# cols	CPU time (sec.)
				Weighted GoS (Tbps)				Unweighted GoS		average				
Total load (in Tbps)	$\lvert D\rvert$	$\lvert S\rvert$		GoS_{LB}	GoS_{ILP}	%	ε	# granted requests	%	# lightpaths per config.	μ	σ		
Randomly generated (node pair) requests with a random number of slots drawn in $\{2, 4, \ldots, 16\}$ slots														
7.45	35	80	639	7.45	6.70	90%	10%	30	86%	6.2	40.0	15.9	331	134
9.76	45	110	791	9.76	8.80	90%	10%	38	84%	6.7	35.5	10.7	365	177
10.70	60	156	812	10.70	9.45	88%	12%	51	85%	9.8	22.3	6.3	375	261
15.50	64	170	847	15.50	12.95	84%	16%	52	81%	7.7	28.9	9.2	477	630
15.10	70	236	956	15.10	13.10	87%	13%	59	84%	6.9	21.8	7.4	519	1,342
16.85	80	256	971	16.85	14.45	86%	14%	68	85%	7.7	23.1	5.8	622	1,419
Randomly generated (node pair) requests with a random number of slots drawn in $\{1, 2, \ldots, 8\}$ slots														
3.67	35	50	451	3.67	3.17	86%	14%	30	86%	5.8	27.9	12.3	208	50
4.75	45	60	498	4.75	4.15	87%	13%	41	91%	4.8	34.3	12.4	326	86
6.77	60	75	904	6.77	5.75	85%	15%	50	83%	4.5	35.1	12.1	654	147
7.45	64	85	717	7.45	6.00	81%	19%	51	80%	7.4	31.7	11.3	380	176
7.37	70	100	729	7.37	6.17	84%	16%	57	81%	6.1	24.9	12.6	305	263
9.67	80	120	1,014	9.67	8.15	84%	16%	68	85%	7.2	29.0	8.7	356	323
11.95	112	150	1,301	11.95	10.22	86%	14%	98	88%	7.6	29.1	7.1	480	417
20.52	180	330	2,202	20.52	16.85	82%	18%	148	82%	7.1	20.8	4.6	635	1,606
20.52	180	400	2,202	20.52	18.27	92%	8%	160	89%	8.9	19.0	3.9	840	2,760

Chapter 6

Conclusion and Future Works

We studied the problem of Routing and Wavelength Assignment (RWA) in WDM networks, when the size of the problem is big, either due to the structure of the network itself, or the number of incoming requests and available wavelengths. We provided exact solutions for both static and dynamic traffic.

In a static state, we proposed very efficient algorithms for solving large scale RWA problems, which allow the exact solutions of data sets with up to 90 nodes and 150 wavelengths. In addition, we also provided two heuristics that provide very good solutions, in very short computing times, while their accuracy can be assessed. It appears that the concept of limiting the search of wavelength configurations using shortest paths, or a limited number of k-shortest paths is very promising in terms of enhancing the performance of the ε-optimal or heuristic algorithms. Under dynamic traffic, we found out that the level of re-arrangement that is required for maintaining a network in the state of maximum GoS, is indeed very small (always smaller than 8%).

Future work should account for multi-rate lightpaths, and the minimum number of required regenerators in the static case. Taking into account that the traffic can be divided into sensitive and non-sensitive one, under dynamic traffic one can explore whether the re-arrangement level remains small, when restricted to non sensitive lightpaths.

We also examined the Routing and Spectrum Assignment problem (RSA) and enhanced state of the art results when big instances are concerned. We provided a new decomposition model for the RSA problem which outperformed the performances of the decomposition used in Ruiz et al.

[2013]. Improvements are still needed in order to solve realistic sized data instances.

Future work will include improving the scalability of the proposed solution scheme, and the selection of the modulation. We might also consider the use of regenerators to enhance further the GoS at the expense of an additional cost.

Appendix A

Existing ILP max-RWA Models

We provide here a review on the ILP max-RWA formulations of the literature. There are three general classes of ILP formulations for max-RWA problem. The first one corresponds to *link formulations* and uses variables indexed with links. The second class of formulations relies on paths, resulting in *path formulations*. The third one uses configurations and decomposition schemes and is reviewed in the core part of the paper. General definitions and notations are the same as the ones in Section 3.1.

A.1 Link Formulations

There are mainly three types of link formulations, characterized by the way the connection requests are considered: individually (model RWA_k), grouped with respect to their source/destination nodes (model RWA_sd), grouped with respect to their source nodes (model RWA_s).

A.1.1 RWA_k Request Indexed Model

Many authors used that formulation, it is the most used exact ILP one for the max RWA problem, see, e.g., Krishnaswamy [1998]. Every connection request $k \in K$ is characterized by its source v_s^k and its destination v_d^k. Variables are defined as follows:

$x_k^\lambda = 1$ if request k is granted by wavelength λ, 0 otherwise.

$x_{k\ell}^\lambda = 1$ if request k is granted by wavelength λ on link ℓ, 0 otherwise.

The mathematical formulation is as follows:

$$\max \quad z_{\text{RWA\_}k}(x) = \sum_{k \in K} \sum_{\lambda \in \Lambda} x_k^\lambda \tag{121}$$

subject to:

$$\sum_{\ell \in \omega^+(v)} x_{k\ell}^\lambda = \sum_{\ell \in \omega^-(v)} x_{k\ell}^\lambda \qquad k \in K, \lambda \in \Lambda, v \in V \setminus \{v_s^k, v_d^k\} \tag{122}$$

$$\sum_{\ell \in \omega^+(v_s^k)} x_{k\ell}^\lambda = \sum_{\ell \in \omega^-(v_d^k)} x_{k\ell}^\lambda = x_k^\lambda \qquad k \in K, \lambda \in \Lambda \tag{123}$$

$$\sum_{\ell \in \omega^-(v_s^k)} x_{k\ell}^\lambda = \sum_{\ell \in \omega^+(v_d^k)} x_{k\ell}^\lambda = 0 \qquad k \in K, \lambda \in \Lambda \tag{124}$$

$$\sum_{k \in K} x_{k\ell}^\lambda \leq 1 \qquad \ell \in L, \lambda \in \Lambda \tag{125}$$

$$\sum_{\lambda \in \Lambda} x_k^\lambda \leq 1 \qquad k \in K \tag{126}$$

$$x_{k\ell}^\lambda \leq x_k^\lambda \qquad k \in K, \ell \in L, \lambda \in \Lambda \tag{127}$$

$$x_k^\lambda, x_{k\ell}^\lambda, \in \{0, 1\} \qquad k \in K, \ell \in L, \lambda \in \Lambda. \tag{128}$$

The objective function (121) maximizes the number of granted requests. Constraints (122) and (123) establish the routes while respecting the wavelength continuity constraints. Loops at the source and destination nodes are eliminated by Constraints (124). Constraints (125) prevent the wavelength conflict: on each link, each wavelength belongs to at most one lightpath. Constraints (126) make sure at most one wavelength is used for every request. Consistency between variables is guaranteed thanks to Constraints (127). Constraints (128) define the domain of the variables.

A.1.2 RWA_s Source Indexed Model

The second model is a source indexed formulation in which requests are grouped based on their source nodes. This type of modelling was studied in Coudert and Rivano [2002], Tornatore et al. [2002], Krishnaswamy and Sivarajan [2001a] and improved in Jaumard et al. [2007]. Let K_s denote the set of connection requests starting from node v_s, while V^{D_s} contains their destination nodes.

Define by $D_s = \sum_{v \in V^{D_s}} D_{sd}$, the aggregated traffic request from node v_s. Consider the following variables:

$y^{\lambda}_{s\ell} = 1$ if a connection request from v_s is granted by wavelength λ on link ℓ, 0 otherwise.

The mathematical formulation is as follows:

$$\max \quad z_{\text{RWA\_}s}(y) = \sum_{(v_s,v_d)\in\mathcal{SD}} \left(\sum_{\lambda\in\Lambda} \sum_{\ell\in\omega^-(v_d)} y^{\lambda}_{s\ell} - \sum_{\lambda\in\Lambda} \sum_{\ell\in\omega^+(v_d)} y^{\lambda}_{s\ell} \right) \tag{129}$$

subject to:

$$\sum_{\ell\in\omega^+(v)} y^{\lambda}_{s\ell} - \sum_{\ell\in\omega^-(v)} y^{\lambda}_{s\ell} = 0 \qquad \lambda \in \Lambda, v_s \in V : D_s > 0, v \in V \setminus (V^{D_s} \cup \{v_s\}) \tag{130}$$

$$\sum_{\lambda\in\Lambda} \sum_{\ell\in\omega^-(v_d)} y^{\lambda}_{s\ell} - \sum_{\lambda\in\Lambda} \sum_{\ell\in\omega^+(v_d)} y^{\lambda}_{s\ell} \leq D_{sd} \qquad v_s \in V : D_s > 0, v_d \in V^{D_s} \tag{131}$$

$$\sum_{\ell\in\omega^+(v_d)} y^{\lambda}_{s\ell} - \sum_{\ell\in\omega^-(v_d)} y^{\lambda}_{s\ell} \leq 0 \qquad \lambda \in \Lambda, v_s \in V : D_s > 0, v_d \in V^{D_s} \tag{132}$$

$$\sum_{v_s\in V:D_s>0} y^{\lambda}_{s\ell} \leq 1 \qquad \ell \in L, \lambda \in \Lambda \tag{133}$$

$$y^{\lambda}_{s\ell} \in \{0,1\} \qquad \ell \in L, \lambda \in \Lambda, v_s \in V : D_s > 0. \tag{134}$$

Again, the objective (129) maximizes the grade of service. Constraints (130) - (132) are flow conservation constraints and take care of wavelength continuity requirements. Constraints (133) guarantee that there is at most one connection request per wavelength on each link. Constraints (134) define the domains of the variables.

A.1.3 RWA_sd Node Pair Indexed Model

Link model RWA_sd aggregates the requests according to their source and destination nodes. It uses the following variables:

$y^{\lambda}_{sd\ell} = 1$ if a connection request from v_s to v_d is granted by wavelength λ on link ℓ, 0 otherwise.

The mathematical formulation is as follows:

$$\max \quad z_{\text{RWA\_}sd}(y) = \sum_{\lambda \in \Lambda} \sum_{(v_s,v_d) \in \mathcal{SD}} \sum_{\ell \in \omega^+(v_s)} y^{\lambda}_{sd\ell} \tag{135}$$

subject to:

$$\sum_{\ell \in \omega^+(v)} y^{\lambda}_{sd\ell} = \sum_{e \in \omega^-(v)} y^{\lambda}_{sd\ell} \qquad \lambda \in \Lambda, (v_s, v_d) \in \mathcal{SD}, v \in V \setminus \{v_s, v_d\} \tag{136}$$

$$\sum_{\lambda \in \Lambda} \sum_{\ell \in \omega^+(v_s)} y^{\lambda}_{sd\ell} = \sum_{\lambda \in \Lambda} \sum_{\ell \in \omega^-(v_d)} y^{\lambda}_{sd\ell} \leq D_{sd} \qquad (v_s, v_d) \in \mathcal{SD} \tag{137}$$

$$\sum_{\lambda \in \Lambda} \sum_{\ell \in \omega^-(v_s)} y^{\lambda}_{sd\ell} = \sum_{\lambda \in \Lambda} \sum_{\ell \in \omega^+(v_d)} y^{\lambda}_{sd\ell} = 0 \qquad (v_s, v_d) \in \mathcal{SD} \tag{138}$$

$$\sum_{(v_s,v_d) \in \mathcal{SD}} y^{\lambda}_{sd\ell} \leq 1 \qquad \ell \in L, \lambda \in \Lambda \tag{139}$$

$$y^{\lambda}_{sd\ell} \in \{0, 1\} \qquad \ell \in L, \lambda \in \Lambda, (v_s, v_d) \in \mathcal{SD}. \tag{140}$$

The objective (135) maximizes the grade of service. Constraints (136) and (137) are the continuity constraints. Constraints (138) prevent the loops around source and destination nodes. Wavelength conflict is avoided thanks to Constraints (139). Constraints (140) define the domain of the variables.

A.2 Path Formulation

This section discusses the ILP model that uses path variables. Such a model was studied in e.g., Lee et al. [2002], Saad and Luo [2002]. Note that it is an exact model only if all paths are considered, and therefore not scalable as soon as we consider topologies with more than 10 nodes.

Let $\mathcal{P} = \bigcup_{sd \in \mathcal{SD}} P_{sd}$ be the overall set of simple (i.e., loopless) paths indexed by p, and δ^p_ℓ a parameter that is equal to 1 if link ℓ is in path p, 0 otherwise. Consider the following variables:

$x^{\lambda}_p = 1$ if a lightpath is established using path p and wavelength λ, 0 otherwise.

The mathematical formulation is as follows:

$$\max \quad z_{\text{PATH}}(x) = \sum_{\lambda \in \Lambda} \sum_{p \in \mathcal{P}} x^{\lambda}_p \tag{141}$$

subject to:

$$\sum_{p \in \mathcal{P}} \delta_\ell^p x_p^\lambda \leq 1 \qquad \ell \in L, \lambda \in \Lambda \tag{142}$$

$$\sum_{\lambda \in \Lambda} \sum_{p \in P_{sd}} x_p^\lambda \leq D_{sd} \qquad (v_s, v_d) \in \mathcal{SD} \tag{143}$$

$$x_p^\lambda \in \{0, 1\} \qquad p \in \mathcal{P}, \lambda \in \Lambda. \tag{144}$$

Again, the objective (141) maximizes the grade of service. Constraints (142) make sure that only one lightpath is going through every pair of wavelengths and links. Constraints (143) force the model to grant at most D_{sd} lightpaths to every pair (v_s, v_d). Constraints (144) define the domains of the variables.

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