A Simulation Study on Control Chart Performances in Monitoring Batch Production Processes

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Abstract

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Quality control charts are graphical tools for monitoring quality characteristics of manufacturing or service systems. Control charts have been applied in many manufacturing systems such as long-run production and short-run production since 1931. It is worth noting that less research is done in applying control charts in batch production and job-shop manufacturing. In this thesis, numerical simulation is used to find appropriate control charts in batch production. For quality improvement in such or similar processes, different control charts for batch production, e.g. $\bar{x}$ chart, $\bar{x}$ chart with Western Electric Rules, Q chart, EWMA Q chart, T chart, EWMA T chart, and Schewhart sign chart are simulated and assessed. The main task of this thesis is to study control charts capabilities to detect mean shifts of the considered processes.

Key Words: statistical quality control, batch production, control charts, process mean, numerical simulation

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Contents

List of Tables

List of Figures

1 Introduction

1.1 Motivation

Product and process quality is the main issue for companies to be successful in today's business world. One of the traditional definitions of quality is based on the perspective that products and services must meet the requirements of consumers (Montgomery, 2013). However, a modern definition of quality pays more attention to the variability as an inseparable part of all manufacturing systems. Accordingly, a commonly used definition for quality is "inversely proportional to variability" (Montgomery, 2013). As one of the main techniques for quality improvement, statistical process control (SPC), a set of statistical techniques, provide powerful tools for monitoring the manufacturing process and allow high quality products to be produced (Castagliola et al, 2015). Among SPC tools, control charts are the mostly adopted tool to find the variabilities in the process parameters. Control charts demonstrate the variation of one or several key characteristics during the time period that the process is observed. The statistical control chart based on 3 times of process standard variation was first suggested by Walter A. Schewhart in 1920s and has been used in manufacturing and applicable processes ever since. However, since Schewhart control charts resort to the information contained in the current sample observation, they may not be capable of detecting small variations of process parameters (Montgomery, 2013). Therefore, when small shifts of parameters are of interest, other types of control charts such as moving average (EWMA) or cumulative sum (Cusum) charts are appropriate alternatives.

Design of experiment is an approach to study the effect of pertinent factors (possible causes) on the quality characteristics (Maynard, 2011). A designed experiment can be defined as an approach that systematically changes the controllable input factors in a manufacturing process to determine their influences on the final product. Due to the practical usage of design of experiment during development activities and at the beginning of manufacturing, it is considered as a major off-line quality control technique (Montgomery, 2013). Acceptance sampling is one of the oldest statistical techniques in quality control. Acceptance sampling is to inspect and categorize sample of units selected at random from a larger batch or lot in order to decide whether accept or dispose the lot. (Montgomery, 2013).

1.2 Batch Production

Batch production is the subset of intermittent system of operations. According to Browne et al. (1996), the main characteristic of the batch production is medium production volume and medium product variety. Indeed, batch production is to manufacture products in batches or small lots through different operations so that before starting the latest operations, previous operations had to be done. It is worth noting that in such systems machines have typically multiple functions, and workers with multiple skills may be employed.

1.3 Batch Production Control Charts

Control charts in both long-run production and short-run production have been tested in this thesis to identify effective control charts for batch production. Control charts which have better success rates (detection ability) and average times to detect, are selected in such systems. Schewhart control chart, originated in the 1920s, is the most widely used chart for monitoring the mean of a process (Castagliola, 2015). Schewhart chart is simpler in comparison to other charts. Quesensberry (1991), developed Q chart which is based on Q statistic, a standard normal variable. It can be plotted on the standard normal control chart with center line at zero and control limits at ±3 sigma. Another chart that studied in this thesis is the T control chart. This chart was considered by Zhang et al.

(2009) for more accurately estimating process standard deviation. As mentioned before, Schewhart control charts may not be capable of finding small variations in process parameters. For detecting small shifts, different EWMA charts such as EWMA $\bar{x}$, EWMA Q, and EWMA T charts are examined in this thesis. Moreover, Schewhart sign chart which is based on more realistic assumptions about the distribution of collected data after consecutive setups, will also be tested.

1.4 Objectives

The main purpose of this thesis is to compare the detection ability of different quality control charts for mean shift in batch production. More specifically, we have the following objectives:

- To review the papers related to both short-run and long run control charts and to summarize the conclusions and observations.
- To summarize the model for each available control charts that are applicable in batch production.
- To compare performances of several control charts using common parameter values.
- To test the detection ability on mean shift of several popular control charts under different conditions and observe their performances.

1.5 Methodology

The main approach used in this thesis is numerical simulation. We compare the performances and effectiveness of control charts applied to batch production. A Microsoft Excel programming is used to establish and conduct required simulation runs.

1.6 Organization

Chapter 2 reviews and summarizes the existing research on both short run and long run control charts that are applicable in batch production. In Chapter 3, models for different control charts are presented. In Chapter 4, we present simulations for control charts to find their detection ability and the average time to detect the process mean shifts. Finally, we present conclusions and future research in Chapter 5.

2 Literature Review

2.1 Non Self Starting Control Charts

In some manufacturing systems, process parameters (mean and standard deviation) are unknown, and have to be estimated. Hence, for monitoring the system by using non self-starting control charts, unbiased estimators for the process parameters should be calculated first if there values are not given as targets. For constructing non self-starting control charts usually two phases are needed. In the first phase sample measurement data from the quality characteristics are collected based on designated intervals to determine control limits and center line of the control chart. It must be assured that in the first phase all the assignable causes are removed to have accurate estimates of the process parameters before the second phase can start to monitor the system. Furthermore, effective usage of the control charts depends on verifying the control limits and the center line regularly. There should not be assignable causes in the process at the starting point of the second phase due to purifying the data from important causes of variations in the first phase. In the first phase, practitioners usually take 20 to 25 samples from production line with reasonable sub-group sizes. (Saleh et al. 2015). It is notable that the most commonly used non self-starting control charts for variables are $\bar{x}$-R charts and $\bar{x}$-S chart. R chart and S chart monitor the process standard deviation. R chart is created based on the range of the sub-groups, and S chart resorts to sample standard deviations. The combination of $\bar{x}$ chart with R chart or S chart is used to monitor the process mean and process standard deviation simultaneously. The sub-group size usually is 4 or 5.

Ma et al. (2010) improved the detection ability of S chart for different shift sizes by using two supplemental run rules. They demonstrated that applying those rules can decrease the out-of-control average run length (ARL) for detecting various shifts in the process standard deviation.

Yang et al. (2012) studied in-control and out-of-control ARL performances of $\bar{x}$ chart and individual X chart. Results demonstrated that individual X chart has always better ARL performances comparing to $\bar{x}$ control charts with larger sample sizes. Furthermore, detection ability of 3-cusum chart proposed in Reynolds and Stoumbos (2004) was compared to individual X chart. Unless the shift range is quite small, 3-cusum chart with optimal parameters has slightly better ARL performances (Yang et al. 2012). However, individual X chart is easier to use in terms of design and implementation.

Non self-starting control charts are also available for attributes such as p chart for nonconforming ratio, c chart for number of nonconformities and u chart for the proportion of the nonconformities (Montgomery, 2013).

Noskievicova et al. (2014) discussed the use of control charts for attributes for identifying very small variations in the process parameters. Conventional attribute control charts are not capable to observe very small variations (in terms of ppm) since many lots have zero defects. The cumulative count of conforming (CCC) and cumulative quantity of conforming (CQC) charts demonstrated superior enactment than the original p chart or c chart. They resorted to Matlab software in order to setup a program to construct CCC chart and CQC chart for attributes. By utilizing the software, it is easy to implement such charts in practice.

Darestani and Aminpour (2014) suggested the use of Z-MR chart for monitoring short run processes where available data may not be sufficient to set up Schewhart control charts. Z-MR chart can be used when a process has multiproducts and multi-dimensions products.

2.2 Self-Starting Control Charts

For monitoring manufacturing processes, process parameters such as process mean and process standard deviation should be known or they can be estimated. In short-run production when the process parameters are not known, there may not be sufficient data to estimate the process parameters. Even in long-run processes, at the early stage of the process, there may not be enough data to estimate the process parameters. Accordingly, a control chart can be built after knowing its process parameters. However, practitioners may need to start monitoring the system as quickly as possible. Self-starting control chart can be applied in short-run circumstances.

2.2.1 Q Charts

Quesensberry (1991) proposed the use of Q charts for monitoring short-run and long-run processes. Q statistic which is the standardized individual measurements and four different cases were presented:

- Both mean and standard deviation of the process are known;
- The mean is known and the standard deviation is unknown;
- The mean is unknown and the standard deviation is known; and
- Both mean and standard deviation are unknown.

Q statistic is a standard normal variable which is derived from t-statistic. It is possible to plot the Q statistic on a standard normal chart with center line 0 and the upper and lower control limits at +3 and -3 respectively. It is also possible for a Q chart to plot different measurements for different parts in one chart because of the standardized control limits.

Castillo and Montgomery (1994) studied the ARL performances of Q chart to verify that usually Q charts do not show appropriate ARL enactments. Therefore, they proposed four alternatives based on four different assumptions about the mean and the standard deviation. They proposed EWMA chart and adaptive Kalman filtering approach in case of known process mean and unknown process standard deviation. Furthermore, they suggested using of Kalman filtering approach with tracking signal in order to enhance the detection capability of Q charts when both mean and standard deviation of the process are unknown. One of the main concerns was to identify the mean shift as soon as possible. If a control chart is not capable to diagnose the mean shift immediately, the process will become steady at a new level and the shift may be masked (Castillo and Montgomery, 1994).

Quesenberry (1996) demonstrated that the results in some cases in Castillo and Montgomery (1994) are inaccurate. Quesenberry (1996) showed that the average run length (ARL) and standard deviation of the run length (SRL) are not proper criteria for comparing the control techniques for the situations that Castillo and Montgomery (1994) considered in their paper.

Roes et al. (1999) investigated the effect of additional run rules and tightening control limits on the performance of Q control chart compared with an EWMA chart. They developed a QR control chart based on mean moving range for estimating the standard deviation of the process. Rose et al (1999) showed that EWMA chart based on the QR statistic provided the best out-of-control ARL performances among all the combinations of the considered control charts and run rules.

Zantek (2005) compared the signal probability of each observation following a mean shift and observed that the Q chart signal probability reduced when the number of observations following the process mean shift increased. Unless an out-of-control signal is obtained quite quickly, it is likely that the run length (RL) will become quite long. By updating the sample mean and the sample standard deviation after each new observation is obtained, Q statistic tends to become closer to zero, so the shift may be masked to some extent (Zantek, 2005).

Snoussi et al. (2005) discussed the use of Q statistics in conjunction with residuals control charts for auto correlated data in short-run processes. Results showed that residuals control charts have much better shift detection capability than charts based on Q statistics. Simulation results showed that when the number of under control data set increases, the advantage of residuals charts decreases. An important advantage of using the Q statistics to the process residuals is to allow practitioners plot many quality measurements with diverse time series models in a unique control chart (Snoussi et al. 2005).

He et al. (2008) demonstrated that a control chart is biased when the out-of-control ARL is larger than the desired in-control ARL. They studied the bias problem of the Q chart before detecting the process mean shift at the start-up of the process. They suggested resorting to two schemes for reducing the bias problem of the Q chart. According to He et al. (2008), when the process variance is known Q chart is not biased.

Zhu and Zhou (2010) suggested resorting to a weighted Q control chart based on difference-declining weight parameters. Simulation study revealed that when the process parameters such as process mean and process standard deviation are not known, (ARL) performances of weighted Q chart are better than the original Q chart proposed by Quessenberry (1991).

Khoo et al. (2010) compared in-control and out-of-control average run length of Q chart for different cases regarding process mean and process standard deviation. According to Khoo et al. (2010) the case with known process mean and known process standard deviation has better ARL performances than the case with unknown parameters. They demonstrated that when c, the number of observations, increases the performance of the Q chart (when the parameters are unknown) will improve. For larger c, ARL performances of cases with at least one unknown parameter is similar to the cases when both mean and standard deviation are known. Simulation results shown that the out-of-control ARL for the cases when the process mean is known and the process standard deviation is unknown (KU) is lower than the cases when the process mean is unknown and the process standard deviation is known (UK), except for one of the western electric rules. In addition, the case when both process mean and process standard deviation are not known (UU) has the lowest sensitivity for finding the mean shifts among other cases (Khoo et al. 2010).

Lamperia and Requeijo (2012) discussed the use of Q chart and multivariate Q (MQ) control charts for monitoring vibrations of manufacturing machines. The data were not independent, but were normally distributed. Simulation results demonstrated that the proposed methodology of on-line monitoring with small samples had acceptable ARL performances.

Wen and Zhao (2012) investigated the use of a variable sampling interval Q chart in batch production. They demonstrated that Q chart in batch production has better detection ability than traditional Schewhart control charts such as $\bar{x}$-R and $\bar{x}$-S charts. Q chart does not show appropriate out-of-control ARL performance at early stages of the process. Using variable sampling interval (VSI) Q chart can boost its detection ability (Wen and Zhao, 2012).

Kawamura et al. (2013) studied the effectiveness of Q control chart using real data acquired from a horizontal low-pressure chemical vapor deposition process. They demonstrated that Q chart can plot different types of data on the same chart. Also, they indicated that Q statistics applied to the residuals of a time series model are useful alternatives for monitoring the semi-conductor manufacturing process.

Chang et al. (2013) discussed the use of a Q chart in software development processes (SDPs). Conventional control charts such as $\bar{x}$-R and $\bar{x}$-S were not suitable for the SDPs. One important issue regarding the use of conventional control charts is that they require a large amount of data from a homogeneous source of variation for calculating the control limits. However, such large data set was unattainable from the SDPs (Chang et al. 2013). They demonstrated that collecting data from different projects with the same attributes to obtain the required number of observations may lead to wide control limits when applying a conventional control chart. Q chart allows practitioners to monitor the process at the early stages of the process. In software industry experts want to start monitoring the system at very near points after start of a process to notice process shifts quickly (Chang et al. 2013). According to Chang et al. (2013) one of the benefits of using a Q chart is that it allows practitioners to plot different performance measurements on the same chart using the same control limits since Q statistics follow standard normal distribution.

Lampreia and Requeijo (2014) reviewed statistical process control (SPC) techniques for long-run productions, short-run productions and processes where the data of the process are auto correlated. They proposed a road map to control the process as “Golden Methodology for Monitoring the Manufacturing Quality”.

2.2.2 CUSUM Q Chart and EWMA Q Chart

Quesenberry (1995) studied the use of Q chart, EWMA Q chart and CUSUM Q chart for short-run processes. Simulation results demonstrated that for detecting one-step permanent shift EWMA Q chart and CUSUM Q chart have better out-of-control ARL performances than Q chart.

Zantek (2006) demonstrated that the optimal design constants for CUSUM $\bar{x}$ chart in detecting a given shifts may not be optimal for CUSUM Q chart. The reason is that the derivation of CUSUM $\bar{x}$ chart constants may not take into account the distribution of Q statistics for finding the shift. Zantek (2006) improved the design of CUSUM Q chart for finding larger range of shifts of the process mean.

Garjani et al. (2010) proposed a neural network based approach for finding trends on control charts when finding the shifts in the start-up of certain manufacturing processes. Simulation results indicated that artificial neural network-based control scheme outperforms CUSUM Q chart for finding small to moderate shifts of the considered process mean.

One important issue regarding CUSUM Q charts is that the reference value (k) is determined by the shift size of the process mean. When the shift size is unknown, it may be difficult to apply CUSUM Q chart. Li and Wang (2010) developed an adaptive cumulative sum (ACQ) control chart capable of detecting wide range of shifts in the process parameters without collecting large number of observations. In estimating the shift of the process mean, they used EWMA control chart with a reflecting boundary as a one-step-ahead forecast. Simulation results showed that the ACQ chart has better in-control ARL performance than the CUSUM $\bar{x}$ chart with known parameters. ACQ chart has also better ARL performances than CUSUM Q chart with fixed reference value and the change point method proposed in Hawkins and Dang (2010). Li and Wang (2010) demonstrated that both ACQ chart and Q chart can be applied for short-run production because both of them do not need Phase I to establish the control chart. ACQ charts are also relatively simple and easy to implement (Li and Wang, 2010).

Theroux et al. (2013) investigated the performance of individual measurement Q (IMR Q) control chart resorting to Q statistics for short-run productions in aerospace manufacturing. They compared IMR Q chart with other control charts such as CUSUM Q chart, CUSUM X chart and individual measurement X (IMR X) control chart based on success rate for finding the shifts in process parameters and average time to detect the shifts. Simulation results demonstrated that CUSUM X chart has better detection ability and average time to detect. Implementing CUSUM Q chart may not be considered in conjunction with the CUSUM X chart.

Li et al. (2010) discussed the use of variable sampling interval adaptive CUSUM Q (VSIACQ) chart for the start-up of processes. CUSUM $\bar{x}$ chart has optimal ARL performances only for fixed sizes of shifts in the process mean. However, VSIACQ chart is quite robust for detecting a range of shifts in process mean. Moreover, they studied the distribution of CUSUM Q chart to solve the shift masking problem of Q chart.

Capizzi et al. (2012) proposed a new self-starting ACQSCORE control chart which uses consecutive observations to jointly update process mean and process standard deviation. Reference value of the ACQSCORE chart is updated using an adaptive EWMA control chart. Simulation results demonstrated that ACQSCORE chart has better out-of-control ARL performance for detecting small mean shifts than control charts designed to detect constant mean shifts such as EWMA or CUSUM control charts. Also, results show that ACUSCORE control chart has almost the same out-of-control ARL performance for detecting large shifts as EWMA and CUSUM charts.

2.2.3 Other CUSUM Schemes

Liu et al. (2015) proposed a self-starting sequential rank-based dual nonparametric CUSUM chart to detect shifts in the process mean. Simulation studies demonstrated that the proposed control chart not only performs well for detecting different magnitudes of shifts, but also performs robustly for different distributions. Also, they suggested a new nonparametric EWMA control chart which has more robust out-of-control ARL performance than the change-point control chart proposed in Hawkins and Deng (2010).

Amdouni et al. (2015) discussed the use of a self-starting adaptive Schewhart control chart implementing variable sample sizes. In some manufacturing processes both process mean and process standard deviation may vary, but their ratio can be constant. If the process standard deviation is the linear function of the process mean, control charts monitoring the coefficient of variation can be used effectively (Amdouni et al. 2015). The authors showed that when the process is in control and the coefficient of variation is constant ($\Upsilon = {}^{\mu}/_{\sigma}$), then assignable causes may change this ratio. Simulation results demonstrated that the proposed procedure has better ARL performances than fixed sampling rate Schewhart chart for the coefficient of variation.

Castagliola et al. (2015) proposed two separate one-sided Schewhart-type control charts monitoring the coefficient of variation in short-run production. A downward (upward) Shewhart-type chart aiming at detecting a shift decreasing (increasing) the in-control coefficient of variation when the shift size is deterministic.

2.2.4 T Charts

Zhang et al. (2009) proposed a T chart and an EWMA T chart to monitor process mean. Usually $\bar{x}$ charts are applied in processes under the well-estimated assumption of process standard deviation or stable standard deviation (Zhang et al. 2009). Simulation results showed that T chart is more robust in estimating errors and unstable process standard deviation. Furthermore, EWMA T chart is more robust than EWMA $\bar{x}$ chart in estimating errors and variations in the process standard deviation. They demonstrated that EWMA T chart is less dependent on the dispersion charts such as S chart or R Chart than EWMA $\bar{x}$ chart.

Celano et al. (2011) investigated the implementation of T chart for monitoring process mean in short-run production. They considered two initial setup conditions such as fixing the population mean at the process target with an initial setup error affecting the statistic distribution. Simulation results showed that T chart and EWMA T chart have appropriate ARL performances in short-run production.

Amin et al. (1995) discussed the use of a nonparametric procedures for the problem of detecting changes in the process median (or mean), or changes in the process standard deviation when samples are taken at regular time intervals. Their proposed procedures were based on sign-test statistics computed for each sample, and are used in Shewhart sign chart. An advantage of the non-parametric control charts is that the variance of the process does not need to be estimated in order to stablish a control chart for the mean.

Celano et al. (2015) proposed a nonparametric (distribution free) Schewhart sign (SN) control chart for monitoring the location of a process parameter in a manufacturing process. They demonstrated any model assumption about the distributions of the observations after consecutive set-ups would be true due to the nonparametric characteristic. Simulation results demonstrated that the Schewhart sign chart has better ARL performances than T chart.

2.2.5 CUSUM T Chart and EWMA T Chart

Celano et al. (2012) designed an economic CUSUM T chart for monitoring short-run processes. According to Celano et al. (2012) CUSUM T chart can be applied in the process without implementing Phase I of the control charts. Simulation results demonstrated that the CUSUM T chart has better economic performance than CUSUM $\bar{x}$ chart.

Celano et al. (2013) compared the ARL performances of T chart, EWMA T chart and CUSUM T chart with unknown shift sizes of process mean. They proposed uniform and triangular distributions to model the unknown shift sizes. Simulation results showed that both EWMA T chart and CUSUM T chart have good performances. Moreover, for sample size $n > 10$ statistical performances of EWMA T chart and CUSUM T chart are comparable to those with known distribution parameters. Practitioners should select chart design parameters according to the range of the shifts rather than simplifying the design by selecting a fixed amount of shift (Celano et al. 2013).

Castagliola et al. (2013) studied ARL performances of variable sample size T chart regarding both fixed shift sizes and unknown shift sizes. They demonstrated that when the shift size is fixed, variable sample size T chart outperforms fixed parameter T chart for medium to large shift sizes. Also, variable sample size T chart has better ARL performances than fixed parameter T chart for unknown shift sizes. They showed that T chart does not need Phase I in control chart implementation when the process mean is perfectly setup at the start point of the process, or the initial setup error is known a priory.

2.2.6 X Chart, CUSUM X Chart, and EWMA X Chart

Klein (1996) studied ARL and percentiles points of run length performances of Schewhart-EWMA chart. Simulation results indicated that Schewhart-EWMA control chart using either time-dependent or constant control limits has better ARL performances than standard Schewhart-runs rules control schemes. When the percentile points of the run length are used, constant control limits Schewhart-EWMA chart has better ARL performances than Schewhart-runs rules control schemes (Klein, 1996).

Albin et al. (1997) compared ARL performances of X chart, X &MR chart, X & EWMA chart and EWMA $\bar{x}$ chart with and without applying run rules in short-run production. Simulation results showed that X & EWMA chart without run rules has the best ARL performances.

Amin and Ethridge (1998) discussed the use of a MR chart beside an individual X chart. Simulation results demonstrated that there is no disadvantages of using MR chart beside X chart in terms of ARL performances. In some cases using an X-MR chart is better than individual X chart when estimating the process parameters in process capability analysis (Amin and Ethridge, 1998).

Liu and Tien (2011) discussed the use of a single featured EWMA-X (SFEWMA-X) to simultaneously monitor both small and large mean shifts and standard deviation shifts using only one set of statistics and control limits. Simulation results demonstrated that the proposed chart has better ARL performances than traditional control charts such as X chart and EWMA $\bar{x}$ chart. According to Liu and Tien (2011), SFEWMA-X chart is easier to interpret than the original EWMA-X chart which had two statistics along with two sets of control limits.

Sitt et al. (2014) proposed run sum T chart based on run sum control chart and T chart combination. Also, they investigated the economic design of the run sum T chart. Simulation results demonstrated that T type charts have more robust ARL and economic performances than $\bar{x}$ chart. EWMA X chart has better ARL performances than run sum T chart for small mean shifts detection.

Saleh et al. (2015) studied the standard deviation of the average run length (SDARL) of $\bar{x}$ chart and X charts by taking practitioner to practitioner variability into account. They showed that this type of variability happens because practitioners use different historical data sets for calculating the process parameters estimation. They demonstrated that taking this variability into account will result far larger samples than those proposed in Quessenberry (1995). The required number of Phase I samples based on SDARL is far larger than that based on the average of average run length (AARL). Moreover, they studied the effect of using different estimators for the process standard deviation on $\bar{x}$ chart ARL performances. They suggested that practitioners should not expect to get in-control performance obtained under known process parameters assumption.

Li et al. (2014) discussed the use of a self-starting control chart for a process producing high-dimensional products. One limitation of using Hoteling T^2 control chart for high-dimensional products is that monitoring cannot begin until after the number of sample data surpasses the dimensionality of the measurements. Accordingly, Hoteling T^2control chart cannot be applied in short-run production because the detection capability to find early shifts is decreased and monitoring the process can be continued after accumulating substantial amount of sample data. They proposed a control chart which allows monitoring with the second observation irrespective of the dimensionality of the products. Simulation results showed the effectively of the proposed control chart based on ARL performances.

2.2.7 Variable Sample Size $\bar{x}$ Chart

Jensen et al. (2008) discussed design issues of variable sample size $\bar{x}$ chart. Furthermore, they showed that how the process parameters' estimation can affect the control chart's performance. Moreover, they showed that with numerical simulation, adaptive charts should only be used for mature processes. Also, their simulation studies were based on initial state performances which assume that the process is out-of-control at the beginning of Phase II.

Castagliola et al. (2014) studied the performance of variable sample size $\bar{x}$ charts based on that the exact values of process parameters are known.

Noorossana et al. (2015) proposed using $\mathrm{VSSI_t}$ control chart instead of standard Schewhart chart for detecting small to medium shifts. They considered two sets of warning limits and adaptive sampling plan for the suggested chart. They demonstrated that proposed chart has better detection ability than variable sample size$\bar{x}$ chart, variable sampling interval and variable sample size $\bar{x}$ chart, and special variable sample size and sampling interval $\bar{x}$ control charts based on average run length and average time to signal.

Lim et al. (2015), studied the performance of the variable sample size and variable sampling interval $\bar{x}$ chart based on estimated process parameters. They demonstrated that varying both sample size and sampling interval can improve the detection ability of the control chart dramatically.

2.3 Summary

Tables 2.1 ~ 2.5 summarizes the research papers reviewed in this chapter. From the literature review, we may see that many researchers have studied various control charts for optimal ARL performances for short-run production and long-run production. Research is limited on finding suitable control charts for batch production processes which may not be considered as short-run nor long-run production due to medium production volume and medium products variety. Practitioners working with batch production processes or job-shop manufacturing should assess both long-run production and short-run production control charts to find control schemes with good ARL performances.

Table 2.1 Summary of the Literature Review part

Author	Year	Main Work	Method	Results
Non Self-Starting Control Charts				
Ma et al.	2010	Established supplemental run rules for S chart.	Markov Chain	Improved the detecting capability of s chart when the shift size is small.
Yang et al.	2012	Compared X chart and 3-CUSUM chart.	Numerical simulation	Showed the advantage of simple X chart with sample size n=1.
Noskievicova et al.	2014	CCC and CQC charts for attributes in Matlab software.	Coding in Matlab Software	Implementation of the CCC and CQC charts in practice.
Self-Starting Control Charts				
Q Chart				
Quesensberry	1991	Q chart for monitoring the short-run production	Numerical Simulation	Monitoring the start-up of the process by estimating the process parameters.
Castillo and Montgomery	1994	ARL problems of Q charts and proposed weighted moving average and an adaptive Kalman filtering method to boost the ARL performances	Numerical simulation	Proposed weighted moving average and an adaptive Kalman filtering method the methods.
Zantek	2005	Calculating signal probability of each observation following mean shift in Q chart	Analytical and numerical simulation	Solved the masking problem of the shift for Q chart.
He et al.	2008	Investigated the bias of Q charts. Proposed two alternative Q charts to decrease the bias	Analytical and numerical simulation	Solved the bias problem of Q charts.

Table 2.2 Summary of the Literature Review Part (continued)

Author	Year	Main Work	Approach	Results
Wen and Zhao	2012	Designed variable sampling Interval Q chart	Analytical and Numerical Simulation	Adaptive feature can improve the Q chart's ARL performance to some extent.
Kawamura et al.	2013	Applied Q chart to auto correlated data	Analytical	Showed the application of Q chart for auto correlated data
Roes et al.	1999	Investigated the supplemental run rules application on Q chart. Presented Q(R) chart	Numerical simulation	Efficiency of the supplemental rules for Q chart.

CUSUM Q Chart and EWMA Q Chart

Quesenberry	1995	Designed EWMA Q and CUSUM Q chart	Numerical Simulation	Appropriate ARL performances for detecting mean shift
Zantek	2006	Changed the design of CUSUM Q chart	Numerical Simulation	This approach can detect a broad range of shifts in the process parameters
Li and Wang	2010	Designed ACQ chart	Numerical Simulation	ACQ has better ARL performances than CUSUM Q chart
Li et al.	2010	Presented VSIACQ chart	Numerical Simulation	It has the capability to detect a range of shifts
Garjani et al.	2010	A neural network-based approach for finding patterns on the control charts	Genetic Algorithm	This method outperforms the CUSUM Q chart for finding small to moderate mean shifts
Capizzi and Masarotto	2012	Presented ACUSCORE control chart	Numerical Simulation	It has better ARL performances than EWMA $\bar{x}$ and CUSUM $\bar{x}$ charts

Table 2.3 Summary of the Literature Review Part (Continued)

Author	Year	Main Work	Approach	Results
Li et al.	2014	Proposed a self-starting control chart instead of Hoteling T^2chart for short-run production	Numerical Simulation	Developed a control chart that is robust for high-dimensional observations instead of Hoteling T^2 chart in short-run

Other CUSUM Schemes

Liu et al.	2015	Proposed a sequential rank-based dual nonparametric CUSUM chart and Nonparametric EWMA Chart	Numerical Simulation	Nonparametric control charts that performs robustly for different distributions.
Amdouni et al.	2015	A self-starting adaptive Schewhart control chart implementing variable sample sizes in a finite horizon process	Markov chain	Proposed chart has better ARL performances than fixed sampling rate Schewhart chart for the coefficient of variation
Castagliola et al.	2015	One-sided Shewhart-type charts for monitoring the coefficient of variation	Numerical Simulation	Schewhart-type chart for monitoring the process coefficient of variation

T Charts

Zhang et al.	2009	Proposed T chart and EWMA T chart	Numerical Simulation	T and EWMA T chart is more robust against estimation errors rather than $\bar{X}$ chart or EWMA $\bar{x}$
Celano et al.	2011	Inspected the perfect setup and imperfect setup for using T chart	Numerical Simulation	EWMA T chart is more strength than Schewhart T chart for detecting changes in the mean of the process
Celano et al.	2012	Designed Economic CUSUM T chart	Numerical Simulation	Economic loss of the CUSUM T chart is due to the imperfect setup

Table 2.4 Summary of the Literature Review Part (Continued)

Author	Year	Main Work	Approach	Results
Celano et al.	2013	Compared the statistical performance of Schewhart T chart, EWMA T chart and CUSUM T chart	Analytical	EWMA and CUSUM T charts should always be preferred to the Schewhart T chart when the shift is in a certain range
Castagliola et al.	2013	Resorting to variable sample size (VSS) T chart	Analytical and Numerical Simulation	Utilizing variable sample size T chart is more robust than fixed parameter T chart for detecting different variations in the process parameters

X Chart, EWMA X Chart and CUSUM X Chart

Klein	1996	Compared a group of Schewhart-EWMA charts with simulations	Numerical Simulation	EWMA $\bar{x}$ control chart has better ARL performances than standard Schewhart runs rules control procedures
Albin et al	1997	Compared four different chart such as X chart, X &MR chart, X and EWMA chart and EWMA $\bar{x}$ chart	Numerical Simulation	EWMA & X charts with no run rules has the best ARL performances
Amin and Ethridge	1998	Advantages and disadvantages use of an MR chart beside an X chart	Numerical Simulation	Using an X-MR is better than individual X in certain cases
Liu and Tien	2011	Single Featured EWMA-X chart instead of using two different statistics	Numerical Simulation	Utilizing SFEWMA-X is visually easier than using EWMA & X chart because of using only one statistic

Table 2.5 Summary of the Literature Review Part (Continued)

Schewhart Sign Chart

Author	Year	Main Work	Approach	Results
Amin et al.	1995	Schewhart Sign Chart	Analytical	Nonparametric control charts that performs robustly for different distributions for the collected data
Celano et al.	2015	Schewhart Sign Chart	Markov Chain and Numerical Simulation	Nonparametric control charts that performs robustly for different distributions for the collected data

Variable Sample Size $\bar{x}$ Chart

Jensen et al.	2008	Variable Sample Size $\bar{x}$ Chart	Markov Chain and Numerical Simulation	Application of variable sample size $\bar{x}$ chart with estimated parameters in mature processes
Castagliola et al.	2014	Variable Sample Size $\bar{x}$ Chart	Markov Chain	variable sample size $\bar{x}$ chart with estimated parameters
Noorossana et al.	2015	Variable Sample Size $\bar{x}$ Chart	Markov Chain and Numerical Simulation	variable sample size and sampling interval $\bar{x}$ chart
Lim et al.	2015	Variable Sample Size $\bar{x}$ Chart	Markov Chain and Numerical Simulation	variable sample size and sampling interval $\bar{x}$ chart has better performances among other adaptive charts

3 Modeling of Batch Production Control Charts

3.1 Introduction

From literature review, we see that many researchers have studied different control charts for optimal ARL performances for short-run and long-run production. Research is limited on finding control charts for processes with medium production volume and medium products variety. Practitioners working with processes of batch production or job-shop manufacturing should evaluate both long-run and short-run production control schemes for better ARL performances.

This chapter studies several modern and traditional control charts to be assessed based on ARL performances for process mean shift using simulations. $\bar{x}$ chart is the most widely used control chart in long-run production which will be simulated in Chapter 4 to compare its detection ability with other charts in batch production. Q chart and T chart are frequently used in short-run production where there is often paucity of relevant data to estimate process parameters when production volume is low. However, both Q chart and T chart can be applied in batch production. Since Schewhart charts sometimes are less capable of finding small mean shifts, different EWMA charts such as EWMA $\bar{x}$ chart, EWMA Q chart, and EWMA T chart are presented in this chapter. EWMA charts and Western Electric Rules are applied for $\bar{x}$ chart to increase the detection ability of these charts in finding process mean shifts. Also Schewhart sign chart, a non-parametric control chart, is used in this chapter.

3.2 $\bar{x}$ Chart

One of the commonly used control charts in batch production is Schewhart $\bar{x}$-R chart. $\bar{x}$ control chart is used to monitor process mean and R chart is used to monitor process variability. The development and application of $\bar{x}$-R chart are based on normal distribution. Even if the quality characteristic follows distribution rather than normal, results with sufficient sample sizes are valid with regard to central limit theorem. Process parameters such as process mean and process standard deviation are usually unknown in practice. Often process parameters are estimated from 20 to 25 subgroups (m), usually each of them has 4 to 5 units (n) (Montgomery, 2013). Let $\bar{x}_1$, $\bar{x}_2 \ldots \bar{x}_m$ be the mean of each sub-group. According to Montgomery (2013), grand mean ($\bar{\bar{x}}$) is an unbiased estimator for the process mean.

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \cdots + \bar{x}_m}{m}$$

$\bar{\bar{x}}$ will be used as the centerline of the $\bar{x}$ control chart. In order to calculate control limits, an estimator is required for the process standard deviation. Standard deviation can be estimated using sub-groups ranges. The difference between the largest and smallest observations in a sub-group is the subgroup range. LetR_1, $R_2 \ldots R_m$ be the ranges of m samples. The mean of the sub-groups ranges is:

$$\bar{R} = \frac{R_1 + R_2 + \cdots + R_m}{m}$$

The ratio of $\frac{\bar{R}}{d_2}$ would be an unbiased estimator for the process standard deviation.

$\bar{x}$ control limits for monitoring the process mean would be:

$$\text{UCL} = \bar{\bar{x}} + A_2\, \bar{R}$$

$$\text{Center line} = \bar{\bar{x}}$$

$$\text{LCL} = \bar{\bar{x}} - A_2\, \bar{R}$$

Where $A_2 = \frac{3}{d_2\sqrt{n}}$.

Factors for constructing variables control charts such as d_2 and A_2, based on different number of observations in sub-groups are available in quality control manuals and quality control text books such as Introduction to Statistical Quality Control by Montgomery (2013).

3.3 Q Charts

Quesensberry (1991) proposed the use of Q charts for monitoring short-run and long-run processes. He presented Q statistics for four different cases:

- Both mean and standard deviation of the process are known;
- The mean of the process is known and the standard deviation is unknown;
- The mean of the process is unknown and standard deviation is known; and
- Both process mean and standard deviation are unknown.

Q statistic is a standard normal variable which is derived from t-statistic. It is possible to plot the Q statistic on the standard normal chart with center line at 0 and the upper and lower control limits at ±3. It is also possible for a Q chart to plot different measurements for different parts in one chart because of its standardized control limits, which simplifies the work for front-line workers. Let $X_1, X_2, .., X_i$ be the samples of subgroup i where i = {1, 2, …, n}. The Q statistic for monitoring the process mean are calculated by:

Case 1: $\mu = \mu_0$, $\sigma = \sigma_0$ (both known)

$$Q_i(\bar{x}_i) = \frac{(\bar{x}_i - \mu_0)\sqrt{n_i}}{\sigma_0}$$

Case 2: μ unknown and $\sigma = \sigma_0$ known

$$Q_i(\bar{x}_i) = \sqrt{\frac{n_i\,(n_1+\cdots+n_{i-1})}{n_1+\cdots+n_i}}\,(\frac{\bar{x}_i-\bar{\bar{x}}_{i-1}}{\sigma_0}) \qquad i = 2, 3, \ldots$$

Case 3: $\mu = \mu_0$ known and σ unknown

For this case put

$$s_{0,i}{}^2 = \frac{\sum_{a=1}^{i}\sum_{j=1}^{n_a}(X_{aj}-\mu_0)^2}{n_1+\cdots+n_i}$$

$$Q_i(\bar{x}_i) = \varphi^{-1}\,[G_{n_1+\cdots\,n_i}(\frac{(\bar{x}_i-\mu_0)\sqrt{n_i}}{s_{0,i}})] \qquad i = 2, 3, \ldots$$

Case 4: Both μ and σ are unknown

Put

$$W_i = \sqrt{\frac{n_i\,(n_1+n_2+\cdots n_{i-1})}{n_1+n_2+\cdots n_i}}\,(\frac{\bar{x}_i-\bar{\bar{x}}_{i-1}}{s_{p,i}})$$

$$Q_i\,(\bar{x}_i) = \delta^{-1}\,\{G_{n_1+n_2+\cdots n_{i-1}-i}(W_i)\} \quad i = 2, 3, \ldots$$

It is worth mentioning, φ^{-1} is the inverse of standard normal. For all values of t distribution function statistic, $(G_{n_1+n_2+\cdots n_{i-1}-i})$ will have (n -1)*i degrees of freedom.

Q statistics can be plotted with:

$$UCL_Q = +3$$

$$\text{Center Line} = 0$$

$$LCL_Q = -3$$

Most of the time the mean and standard deviation of the process are unknown, so case 4 often occurs.

3.4 Western Electric Rules

Western Electric Handbook was published in 1956 by Western Electric Company. When small shifts of parameters are of interest, using Western Electric Rules is useful for control charts. Resorting to Western Electric Rules may, to some extent, improve the detection ability of $\bar{x}$-R chart to find the mean shifts. Widely used Western Electric Rules include:

- One point plots outside the three-sigma control limits,
- Two out of three consecutive points plot beyond the two-sigma warning limits,
- Four out of five consecutive points plot at a distance of one-sigma or beyond from the center line.
- Eight consecutive points plot on one side of the center line.

3.5 EWMA $\bar{x}$ Chart

To detect small shifts in the process mean, EWMA $\bar{x}$ charts is a suitable alternative of Schewhart charts (Montgomery, 2013). The statistic of EWMA $\bar{x}$ chart is constructed as follows:

$$z_i = \lambda \, \bar{x}_i + (1 - \lambda) \, z_{i-1}$$

In this equation λ is a constant value between [0, 1], and the initial value for z_i is the target value of the mean. Furthermore, λ can be designed for appropriate average run length.

$$z_0 = \mu_0$$

EWMA $\bar{x}$ control limits can be defined as:

$$\text{UCL} = \mu_0 + L_z \, \sigma_{\bar{x}} \sqrt{\frac{\lambda}{2-\lambda} \left[1 - (1 - \lambda)^{2i}\right]}$$

$$\text{Center Line} = \mu_0$$

$$\text{LCL} = \mu_0 - L_z\, \sigma_{\bar{x}} \sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]}$$

$[1-(1-\lambda)^{2i}]$ tends to become 1 after some periods as i increases. Furthermore, control limits tend to approach their steady-state values at:

$$\text{UCL} = \mu_0 + L\, \sigma_{\bar{x}} \sqrt{\frac{\lambda}{2-\lambda}}$$

$$\text{LCL} = \mu_0 - L\, \sigma_{\bar{x}} \sqrt{\frac{\lambda}{2-\lambda}}$$

Similar to λ, L can be designed for suitable average run length. Since sample mean are used in EWMA $\bar{x}$ chart for plotting process measurements, $\sigma_{\bar{x}}$ is equal to $\frac{\sigma}{\sqrt{n}}$.

3.6 EWMA Q Chart

EWMA Q statistics are similar to those of Schewhart EWMA chart with small modifications. The statistic for constructing EWMA Q chart is presented below:

$$z_i = \lambda\, Q_i + (1-\lambda)\, z_{i-1}$$

Where z_0 is considered to be 0 and the control limits are:

$$\text{UCL} = +K \sqrt{\frac{\lambda}{2-\lambda}}$$

$$\text{LCL} = -K \sqrt{\frac{\lambda}{2-\lambda}}$$

K and λ are constant parameters which can be designed according to the desired average run length.

3.7 T Chart

Zhang et al. (2009) proposed a T chart to monitor process mean. According to Zhang et al. (2009) $\bar{x}$ charts are applied in processes with well-estimated process standard deviation or stable standard deviation. They showed that T chart is more robust against estimation errors and unstable process standard deviation. In this thesis, we applied T chart in batch production. To construct a T chart one needs to compute the mean and standard deviation of the sub-group data:

$$\bar{X}_i = \frac{1}{n} \sum_{j=1}^{n} X_{i,j}$$

$$S_i = \sqrt{\frac{1}{n-1} \sum_{j=1}^{n} (X_{i,j} - \bar{X}_i)^2}$$

The T statistic is:

$$T_i = \frac{\bar{X}_i - \mu_0}{\frac{S_i}{\sqrt{n}}}$$

The required control limits are:

$$UCL_t = F_t^{-1} \left(1 - \frac{\alpha}{2}, n - 1\right)$$

$$LCL_t = - UCL_t$$

$F_t^{-1}(.|n-1)$ is the inverse of t distribution with n – 1 degrees of freedom. α is the probability of Type I error equal to 0.0027.

3.8 EWMA T Chart

Zhang et al. (2009) discussed the use of an EWMA T chart to monitor the small mean shifts. EWMA T chart is robust for the standard deviation estimation errors and for the variations of process standard deviation. The required statistics for plotting EWMA T charts are:

$$Y_0 = 0$$

$$Y_i = \lambda\ T_i + (1 - \lambda)\ Y_{i-1} \qquad i = 1, 2 \ldots$$

λ is the smoothing parameter between [0, 1]. If the smoothing parameter is 1, the EWMA T chart converts to a T chart. The required control limits for EWMA T chart are:

$$UCL_t = +\ K \sqrt{\frac{\lambda}{2-\lambda}}$$

$$\text{Center Line} = 0$$

$$LCL_t = -\ K \sqrt{\frac{\lambda}{2-\lambda}}$$

In the above formula it can be assumed that K = L x σ_0 is the multiplier to the standard deviation of the process which can be determined based on the desired value of in-control average run length.

3.9 Schewhart Sign Chart

Celano et al. (2015) proposed a nonparametric (distribution free) Schewhart sign (SN) control chart for monitoring the location of a process parameter in a finite horizon manufacturing process. They demonstrated that any model assumption about the distributions of the observations after consecutive set-ups would be true due to the non-parametric characteristic. The required statistic for this chart is:

$$SN_i = \sum_{j=1}^{N} \text{Sign}\ (X_{i,j} - T_M)$$

T_M is target value of the mean or median of the collected data which is equal to μ_0. The value of the sign function, sign(X) are:

$$\text{Sign(X)} = \begin{cases} 1 & \text{if } X > 0 \\ 0 & \text{if } X = 0 \\ -1 & \text{if } X < 0 \end{cases}$$

As soon as the set-up activities are done without errors, T_M would be equal to μ_0, the process mean, and the process starts in-control. In-control distribution of the sign(X) can be attained by the following formulae:

$$SN_i = 2 \text{ x } D_i - n$$

D_i is the number of positive signs in a sub-group, and n is the number of samples that have been collected in each sub-groups. Without presence of assignable causes the in-control value for SN_i would become zero. It can be demonstrated that $P\left(X_{i,j} > T_M | T_M = \theta_0\right) = 0.5$ for all i = 1, 2 … I and j = 1, 2 …n. Accordingly, symmetric control limits around zero can be applied. Control limits for plotting Schewhart sign chart statistics would be:

$$UCL = c = n$$

$$CL = 0$$

$$LCL = - c = - n$$

3.10 Summary

In this chapter different control charts for batch production or job-shop environment are introduced to be evaluated in simulation study presented in the next chapter.

4 Simulation Study

4.1 Introduction

In this chapter, different control charts are simulated and assessed to study the detection ability in finding process mean shifts and average times to detect the mean shifts. The main objective of the work presented in this chapter is to investigate the use of different control charts in batch production and job-shop manufacturing.

$\bar{x}$ chart is the most widely used control charts in batch production which is simulated to compare its detection ability with other charts. Q chart and T chart are frequently used in short-run production. However, both Q chart and T chart can be also applied in batch production. Since Schewhart charts sometimes are less capable of finding small mean shifts, different EWMA charts such as EWMA $\bar{x}$ chart, EWMA Q chart, and EWMA T chart are tested in this chapter. Western Electric Rules are applied for $\bar{x}$ chart to increase the detection ability of this chart in finding process mean shift. We study the success rates and average times to detect mean shifts of the following charts through numerical simulation:

- $\bar{x}$ chart
- $\bar{x}$ chart with Western Electric Rules
- Q chart
- EWMA $\bar{x}$ chart
- EWMA Q chart
- T chart
- EWMA T chart
- Schewhart sign chart

4.2 Setting Simulation Experiments

All simulated experiments were run using Microsoft Excel. Success rates of control charts to find the shifts of the process mean and average times to detect the shifts in control charts based on out-of-control ARL are assessed for Schewhart charts, EWMA charts, and a nonparametric chart (Schewhart sign chart) for batch production. When a process undergoes assignable causes of variations, the mean or the standard deviation of the process will change from their target values. In order to study the control charts performances, various amounts of shifts are implanted in all tested control charts based on their applications. For example, the objective of using EWMA charts is to find smaller shifts in process parameters. However, Schewhart charts are appropriate alternatives for finding larger shifts of the process parameters. Simulation assumptions and parameter settings are explained below.

Distribution Function

We used normal distribution functions built in Microsoft Excel to simulate control charts in batch production studied in this research since it is the most commonly used distribution in studying manufacturing processes.

Number of Samples

Both short-run production and long-run production control charts were evaluated in this thesis. 45 random sub-groups each had 5 measurements following normal distribution (1, 1) were generated in each simulation run. Each experiment were replicated 40 times. There may not be sufficient data for the measured quality characteristics in batch production or job-shop manufacturing to implement long-run control charts.

Implanted Shifts

A single permanent shift would be implanted at the 5th sub-group. Any out-of-control shifts appears on the control charts after the 5th sub-group will be counted as a signal of the mean shift. It is worth noting that any shifts before the 5^{th} sub-group will be considered as false alarms.

Shift Size

Let $\mu' = \mu_0 + \delta_\mu \sigma_0$ and δ_μ is the mean shift. For all the control charts δ_μ was set to 0.6, 1.0 and 1.5 respectively. These shift sizes are the representatives of the small, medium and large shifts in the process mean.

4.3 Statistical Errors

As discussed in Montgomery (2013), there are Type I error and Type II error associated with the use of control charts. Type I error is the probability when process is actually in statistical control where the control charts show otherwise. The probability of this occurring is denoted by α. Type II error is the probability that the process is out-of-control where the control charts show otherwise. The probability of this occurring is denoted by β.

4.4 Schewhart $\bar{x}$ Chart Simulation and Results

$\bar{x}$ chart is the most widely used control charts in both long-run production and short-run production. In this thesis, $\bar{x}$ chart is simulated to compare its detection ability with other charts for batch production. $\bar{x}$ chart statistics and control limits are constructed as:

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \cdots + \bar{x}_m}{m}$$

$$\bar{R} = \frac{R_1 + R_2 + \cdots + R_m}{m}$$

$$UCL = \bar{\bar{x}} + A_2\,\bar{R}$$

$$\text{Center line} = \bar{\bar{x}}$$

$$LCL = \bar{\bar{x}} - A_2\,\bar{R}$$

$\bar{\bar{x}}$ will be used as the centerline of the $\bar{x}$ control chart. In order to calculate the control limits, an estimator is required for the process standard deviation. Standard deviation of the process can be estimated using sub-groups ranges (R). The difference between the largest and smallest observations in a sub-group is the sub-group range. The mean of the sub-groups ranges is $\bar{R}$. The ratio of $\frac{\bar{R}}{d_2}$ would be an unbiased estimator for the process standard deviation. Factor d_2 is available for different sample sizes in quality control manuals and quality control text books such as Introduction to Statistical Quality Control by Montgomery (2013).

To implement a $\bar{x}$ chart in a manufacturing process, typically a two phase approach is needed. In the first phase, process parameters such as process mean and process standard deviation are estimated to build the control limits. According to Montgomery (2013), the "rule-of-thumb" is to use about 100 units of products in which 20 samples with 5 units in each sample are collected from production line. In the second phase after the control chart is established, monitoring the manufacturing system will be started. For the sake of experiment, we assume that process mean and process standard deviation are known and both are 1.0. Since n = 5, $\bar{x}$ control chart can be set as:

UCL = 2.34; Center Line = 1; LCL = - 0.34.

To investigate the effectiveness of the $\bar{x}$ chart to detect mean shifts, 3 different shift sizes are used by letting $\mu_1 = \mu_0 + 0.6\,\sigma_0$, $\mu_1 = \mu_0 + 1.0\,\sigma_0$, and $\mu_1 = \mu_0 + 1.5\,\sigma_0$ in simulations. 45 random sub-groups following N (1, 1) are generated for each replication by Microsoft Excel. 3 separate experiments are implemented to detect 0.6 σ, 1.0 σ, and 1.5 σ mean shifts. In each experiment, a mean shift is implanted at the 5th sub-group. The first effective signal will be counted if it is plotted outside the control limits after the shift is implanted. Each experiment is replicated 40 times. Results of these experiments for $\bar{x}$ chart are presented in Tables 4.1 and 4.2. Table 4.1 demonstrates alarm points for $\bar{x}$ chart. The results in Table 4.1 demonstrate that in the first simulation run for 1.0 σ shift, for example, the shift is detected at sample 9. The sign "/" in Table 4.1 indicates that the control chart was not able to detect the shift in the particular run. Table 4.2 demonstrates success rates and average times to detect 0.6 σ, 1.0 σ, and 1.5 σ mean shifts.

Table 4.1 Alarm Points for $\bar{x}$ Chart

Runs/Shifts	0.6 σ	1.0 σ	1.5 σ
1	39	9	6
2	16	5	6
3	5	7	6
4	32	5	6
5	45	6	6
6	41	5	5
7	31	7	5
8	19	13	5
9	41	18	6
10	24	11	5
11	28	6	5
12	14	5	7
13	/	11	5
14	40	8	6
15	20	9	8
16	43	14	8
17	38	6	7
18	12	16	6
19	19	5	7
20	7	7	5
21	/	9	6
22	/	14	5
23	11	5	5
24	21	11	6
25	24	8	5
26	/	7	7
27	43	5	5
28	5	5	6
29	18	8	6
30	9	5	5
31	6	6	6
32	13	13	5
33	28	7	6
34	29	10	6
35	35	11	5
36	9	6	5
37	/	6	5
38	19	6	7
39	9	12	7
40	18	12	5

Table 4.2 Simulation Results for $\bar{x}$ Chart

40 Runs	Success Rates	Average Times to Detect
0.6 σ	35 (88%)	23.17
1.0 σ	40 (100%)	8.32
1.5 σ	40 (100%)	5.82

As can be seen from the results in Table 4.2, success rates for 0.6 σ, 1.0 σ, and 1.5 σ are 88%, 100%, and 100% respectively. From Table 4.2, average times to detect the shifts decreased with the increase of the mean shifts. Accordingly, average times to detect 0.6 σ shift is 23.17 while average time to detect 1.0 σ and 1.5 σ shifts are 8.32 and 5.82 respectively. Figures 4.1 and 4.2 present graphically the data in Table 4.2.

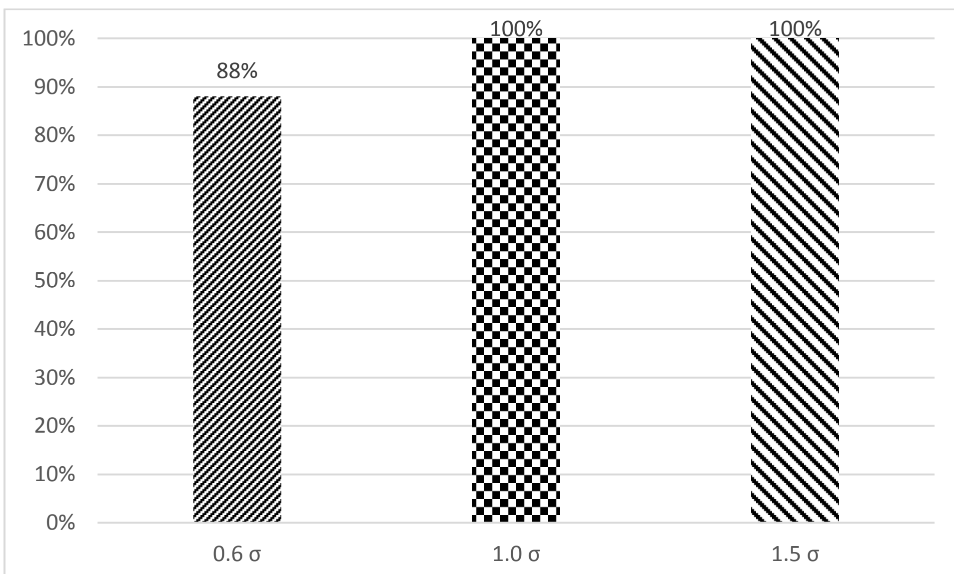

Figure 4.1 Success Rates of the $\bar{x}$ Chart

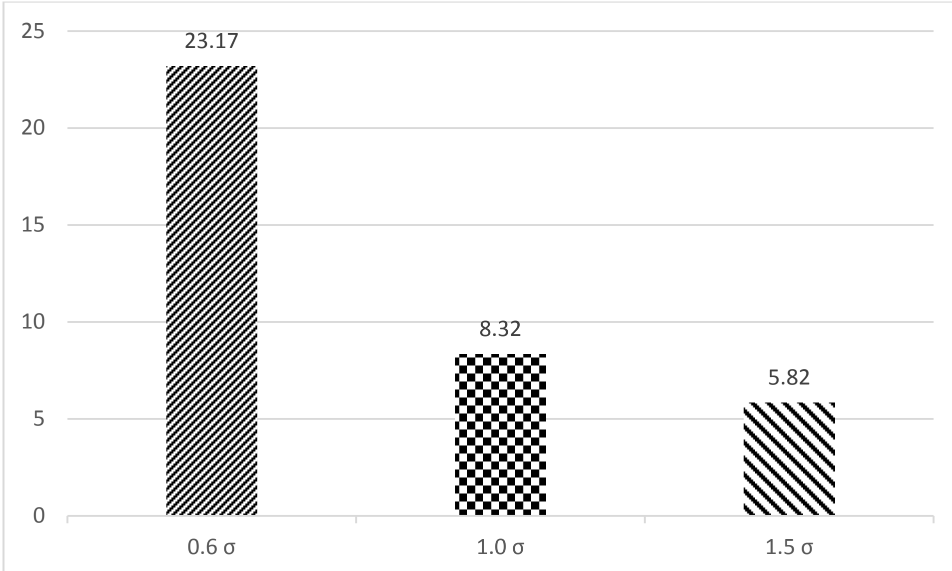

Figure 4.2 Performance of the $\bar{x}$ Chart

4.4.1 Average Run Length (ARL) for $\bar{x}$ Chart

Average run length (ARL) to detect a process mean shift using regular $\bar{x}$ chart can be calculated. Following the general approach in Montgomery (2013), we consider a $\bar{x}$ chart which has a fixed and known standard deviation σ. If the process mean shifts from its in control value μ_0 to an out-of-control value $\mu_1 = \mu_0 + k\sigma$, the probability of not detecting a shift in the process mean by next immediate sample is:

$\beta = P\,(LCL \leq \bar{x} \leq UCL \mid \mu_1 = \mu_0 + k*\sigma)$

Since $\bar{x} \sim \text{Normal}\,(\mu, \frac{\sigma^2}{n})$, and control limits which are $UCL = \mu_0 + L\,(\frac{\sigma}{\sqrt{n}})$ and $LCL = \mu_0 - L\,(\frac{\sigma}{\sqrt{n}})$ respectively, β can be calculated by:

$$\beta = \varphi\left(\frac{UCL-(\mu_0+k\sigma)}{\frac{\sigma}{\sqrt{n}}}\right) - \varphi\left(\frac{LCL-(\mu_0+k\sigma)}{\frac{\sigma}{\sqrt{n}}}\right) = \varphi\left(\frac{(\mu_0+L*\frac{\sigma}{\sqrt{n}})-(\mu_0+k\sigma)}{\frac{\sigma}{\sqrt{n}}}\right) - \varphi\left(\frac{(\mu_0-L*\frac{\sigma}{\sqrt{n}})-(\mu_0+k\sigma)}{\frac{\sigma}{\sqrt{n}}}\right) =$$

$$\Phi\,(L - k\sqrt{n}) - \Phi\,(-L - k\sqrt{n})$$

For example, we can calculate out-of-control ARL for 0.6 σ, 1.0 σ, and 1.5 σ shifts using sample size n = 5.

For 0.6 σ Shift ⟶ $\varphi\,(3 - 0.6\sqrt{5}) - \varphi\,(-3 - 0.6\sqrt{5}) = 0.95$

For 1.0 σ Shift ⟶ $\varphi\,(3 - \sqrt{5}) - \varphi\,(-3 - \sqrt{5}) = 0.77$

For 1.5 σ Shift ⟶ $\varphi\,(3 - 1.5\sqrt{5}) - \varphi\,(-3 - 1.5\sqrt{5}) = 0.36$

Then

ARL for 0.6 σ = $\frac{1}{1-\beta} = \frac{1}{1-0.95} = 20$

ARL for 1.0 σ = $\frac{1}{1-\beta} = \frac{1}{1-0.77} = 4.34$

ARL for 1.5 σ = $\frac{1}{1-\beta} = \frac{1}{1-0.36} = 1.56$

4.4.2 Comparing Analytical and Simulation Results for $\bar{x}$ Chart

We want to demonstrate that both of the analytical and simulation approaches show approximately the same performances for $\bar{x}$ chart. This validates the simulation procedure presented in this study. Out-of-control ARL is the number of samples to detect a mean shift. In the simulation approach, we implanted a shift in the 5th sub-group. Each alarm after the 4th sub-group will be considered as an effective alarm. In each run, we calculate the run length separately and these values are added up and divided by the number of runs to calculate the average run length. Table 4.3 and Table 4.4 demonstrate out-of-control ARLs from numerical simulation and those calculated in Section 4.4.2.

Table 4.3 Out-of-Control ARL for $\bar{x}$ Chart based on Simulation Approach

40 Runs	Out-of-Control ARL
0.6 σ shift	18.17
1.0 σ shift	3.32
1.5 σ shift	0.82

Table 4.4 Out-of-Control ARL for $\bar{x}$ Chart based on Analytical Approach

40 Runs	Out-of-Control ARL
0.6 σ shift	20.00
1.0 σ shift	4.34
1.5 σ shift	1.52

As can be seen from Tables 4.3 and 4.4, simulation results confirm analytical results. For the purpose of comparison, the number of runs were increased to 60, 80, and 100. The results are shown in Table 4.5.

Table 4.5 Out-of-Control ARL for $\bar{x}$ chart

Shifts/ Runs	40 Runs	60 Runs	80 Runs	100 Runs
0.6 σ	18.17	17.22	17.46	16.35
1.0 σ	3.32	3.89	3.81	3.83
1.5 σ	0.82	0.87	0.89	0.88

As can be seen from Table 4.5, out-of-control ARL performance of the $\bar{x}$ chart based on numerical simulation is similar to analytical results for mean shifts.

4.4.3 Applying Western Electric Rules on $\bar{x}$ Chart

Western Electric Rules have traditionally been used for $\bar{x}$ chart. Western Electric Rules include a set of rules to increase the detection ability of control charts. When small shifts of parameters are of interest, using Western Electric Rules can be useful for early detection. We evaluate the performance of $\bar{x}$ control chart in conjunction with some Western Electric Rules. In this section, we present simulation experiments to implement Western Electric Rules on $\bar{x}$ chart. One individual and two combinations of the following Western Electric Rules for $\bar{x}$ chart are considered in this study respectively:

(A) 1-of-1 test - signals if the last point is beyond the control limits (±3);

(B) 2-of-3 test - signals if two out of the last three points are beyond the same warning limit (±2);

(C) 4-of-5 test - signals if four of the last five points are beyond the same auxiliary limit (±1).

To investigate the effectiveness of the Western Electric Rules to detect mean shifts for $\bar{x}$ chart, a shift size of $\mu_1 = \mu_0 + 0.6\ \sigma_0$ is used in the simulation. 45 random sub-groups following N (1, 1) are generated for each replication by Microsoft Excel. 3 separate experiments are implemented to detect this mean shift. Each experiment is replicated 40 times. In each experiment, a mean shift is implanted at 5th sub-group. The first effective signal is counted if it is plotted outside the warning limit or the auxiliary limit after the shift is implanted at the 5th sub-group. For example, for applying Western Electric Rule "BC", the signal will be considered if 2 points are out of the 2.0 σ limit points or 4 points are out of the 1.0 σ limit points appear after the 4th sub-group. Table 4.6 demonstrates alarm points after applying Western Electric Rules "B", "BC" and "ABC". Table 4.6 shows, for example, at the run number 10 combination of Rules "BC" can detect the 0.6 σ shift at 5th sub-group or at run number 13 combination of Rules "ABC" can detect the 0.6 σ shift at 12th sub-group. In Table 4.6, the sign "/" indicates that control chart cannot detect the 0.6 σ shift in the particular run. For example, after applying Western Electric Rule "B", $\bar{x}$ chart cannot detect the shift at the 8th sub-group. Results of these experiments are presented in Table 4.7. Table 4.7 presents the success rates and average times to detect corresponding to some of the different individual or combinations of Western Electric Rules. In Tables 4.7, for example, “BC” means that the shift is detected by Western Electric Rule B or Rule C. “ABC” means that the shift is detected by Western Electric Rules A or B, or C.

Table 4.6 $\bar{x}$ Chart Alarm Points after Applying Western Electric Rules

Runs/Rules	B	BC	ABC
1	6	6	6
2	9	22	14
3	23	17	9
4	10	5	26
5	6	8	9
6	5	5	15
7	32	6	5
8	/	15	11
9	14	14	8
10	20	5	14
11	/	5	8
12	26	8	9
13	/	22	12
14	15	7	6
15	5	9	18
16	38	6	5
17	18	5	5
18	16	25	7
19	5	6	9
20	/	37	11
21	5	9	7
22	11	10	9
23	/	5	14
24	41	13	5
25	/	7	14
26	/	7	8
27	11	12	5
28	25	8	6
29	/	6	6
30	/	5	5
31	30	9	7
32	35	7	6
33	/	19	5
34	15	11	5
35	7	9	6
36	21	18	8
37	19	14	5
38	25	5	8
39	12	13	7
40	6	5	17

Table 4.7 Western Electric Rules Performances on $\bar{x}$ Chart for 0.6 σ Shift

40 Runs	B	BC	ABC
Success Rates	30 (75%)	40 (100%)	40 (100%)
Average time to Detect	17.03	10.87	9.00

As shown in Table 4.7, combinations of Western Electric Rules “BC” and “ABC” demonstrate highest success rates to the shift. Average times to detect the 0.6 σ shift was 23.17 before implementing Western Electric Rules. Table 4.7 shows, average times to detect the 0.6 σ shift decreased for both combinations “BC” and “ABC”. For combination “BC”, average times to detect is 10.87 while 0.6 σ shift can be detected on average at 9th sub-group if the combination of “ABC” is used. For individual Rule “B”, average times to detect 0.6 σ shift decreased compared with original experiment results in Table 4.2 where Wester Electric Rules were not used.

4.4.4 Calculating Type I Error with Western Electric Rules “BC” and “ABC”

According to Champ and Woodall (1989), using Western Electric Rules for synthetizing control charts will boost Type I error. In this experiment, individual Rule “B”, Combination of Rules “A”, “B”, and “C” and combination of Rules “B” and “C” were used. According to Montgomery (2013), the value of Type I error for a control chart with three-sigma limits $\alpha = 0.0027$ is the probability that a single point falls outside the limits when the process is in control. In this section, Type I errors are calculated in conjunction with the use of combination of Rules “B”, “ABC” and “BC” for $\bar{x}$ chart in three separate experiments. In these experiments, 25 random subgroups each has 5 units following N(1, 1) are generated by Microsoft Excel. No shifts are implanted. Both process mean and the standard deviation of the process are assumed to be 1.0. Each experiment is replicated 100 times. In each replication the number of false alarms are counted regarding the use of Rules “B”, “BC” and “ABC”. To calculate the Type I error, the number of false alarms in 100 replications are divided into 2500. Table 4.8 demonstrates the calculations of Type I error and in control ARL regarding the use of Rule “B” and combinations of Rules “BC” and “ABC”.

Table 4.8 Type I error and in Control ARL after implementing Runs Rules

Rules	B	BC	ABC
Number of False Alarms	7	8	17
Type I error	0.0028	0.0032	0.0068
In Control ARL	357.14	312.5	147.0

As shown in Table 4.8, in control ARL decreased for Rule “B” and both the combinations of Rules “BC” and “ABC” compared with in control ARL equal to 370.

4.5 Q Chart Simulation and Results

Quesensberry (1991) first proposed Q chart in an attempt to overcome the difficulties in estimating the process mean and standard deviation in short production runs. As explained in Quesensberry (1991) along with Castillo and Montgomery (1994), Q chart can be well used for certain types of short-run productions. In this thesis, we discussed the use of Q chart in batch production chart statistics are brought below.

$$W_i = \sqrt{\frac{n_i\,(n_1 + n_2 + \cdots n_{i-1})}{n_1 + n_2 + \cdots n_i}}\,\left(\frac{\bar{x}_i - \bar{\bar{x}}_{i-1}}{s_{p,i}}\right)$$

$$Q_i\,(x_i) = \delta^{-1}\,\{G_{n_1+n_2+\cdots n_{i-1}-i}(W_i)\}$$

According to the characteristics of Q statistics which are identically, independently and normally distributed, they can be plotted in a unique Schewhart chart with:

$$UCL_Q = +3$$

$$\text{Center Line} = 0$$

$$LCL_Q = -3$$

Q statistic is a standard normal variable which is derived from t-statistic. In a Q chart, Q statistics are plotted in a standard normal chart with center line at 0 and the upper and lower control limits at +3 and -3 respectively. To investigate the effectiveness of the Q chart to detect mean shifts, 3 different shift sizes are used by letting $\mu_1 = \mu_0 + 0.6\,\sigma_0$, $\mu_1 = \mu_0 + 1.0\,\sigma_0$, and $\mu_1 = \mu_0 + 1.5\,\sigma_0$ in simulations. 45 random sub-groups following N (1, 1) are generated for each replication by

Microsoft Excel. 3 separate experiments are implemented to detect 0.6 σ, 1.0 σ, and 1.5 σ mean shifts. In each experiment, a mean shift is implanted at the 5th sub-group. The first effective signal will be counted if it is plotted outside the control limits after the shift is implanted at the 5th sub-group. Each experiment is replicated 40 times. Results of these experiments for Q chart are presented in Tables 4.9 and 4.10. Table 4.9 demonstrate alarm points for Q chart. The results in Table 4.9 demonstrate that in the 16th simulation run for 1.5 σ shift, for example, the shift is detected at sample 5. The sign "/" in Table 4.9 indicates that the control chart was not able to detect the shift in the particular run. Table 4.10 demonstrates success rates and average times to detect 0.6 σ, 1.0 σ, and 1.5 σ mean shifts.

Table 4.9 Alarm Points for Q Chart

Runs/Shifts	0.6 σ	1.0 σ	1.5 σ
1	/	/	5
2	21	/	6
3	37	20	5
4	5	/	/
5	/	14	5
6	8	6	/
7	/	6	5
8	5	7	7
9	38	22	5
10	6	7	7
11	/	5	37
12	41	/	5
13	41	/	/
14	30	/	5
15	10	/	10
16	30	5	5
17	10	/	6
18	5	29	/
19	6	10	5
20	/	7	6
21	8	5	6
22	/	9	/
23	/	8	5
24	6	5	8
25	/	5	/
26	11	10	5
27	5	10	7
28	24	13	5
29	15	5	5
30	16	5	6
31	/	/	/
32	23	10	8
33	/	5	/
34	36	/	5
35	/	23	27
36	/	12	5
37	5	23	5
38	/	/	5
39	/	5	6
40	/	22	16

Table 4.10 Simulation Results for Q Chart

40 Runs	Success Rate	Average Time to Detect
0.6 σ	25 (63%)	17.68
1.0 σ	30 (75%)	10.8
1.5 σ	32 (80%)	7.75

As shown in Table 4.10, success rates of the Q chart increased with the increase of the mean shifts. As can be seen from the results in Table 4.10, success rates for 0.6 σ, 1.0 σ, and 1.5 σ are 63%, 75%, and 80% respectively. As shown in Table 4.10, average times to detect the shifts decreased with the increase of the mean shifts. Accordingly, average times to detect 0.6 σ shift is 17.68 while average time to detect 1.0 σ and 1.5 σ shifts are 10.80 and 7.75 respectively. Figures 4.3 and 4.4 present graphically the data in Table 4.10.

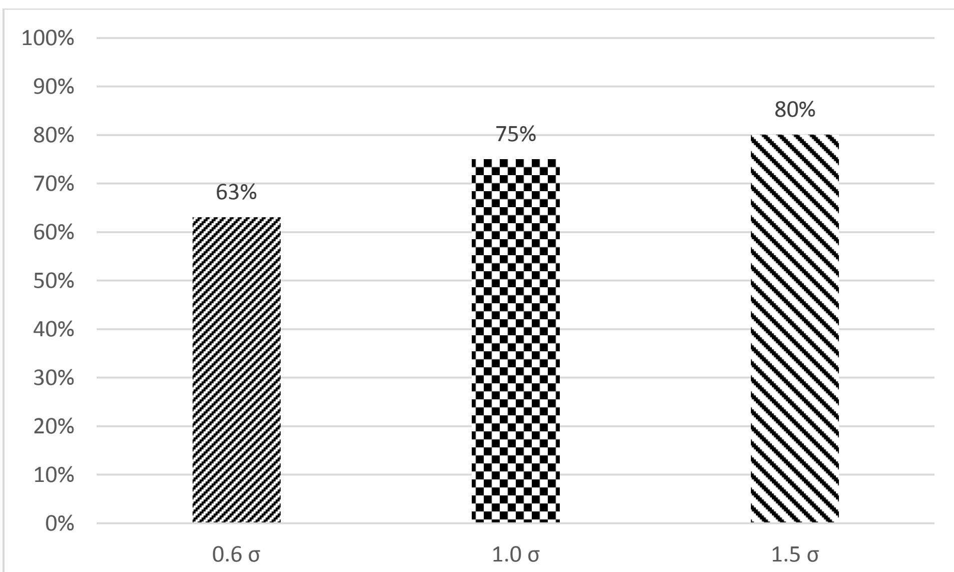

Figure 4.3 Success Rates of the Q Chart

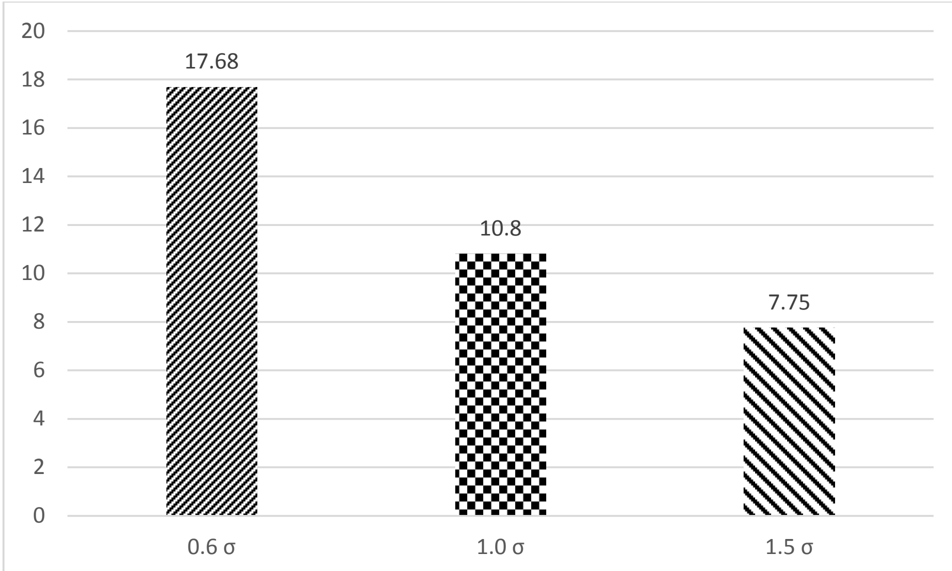

Figure 4.4 Performance of the Q Chart

4.6 EWMA x̄ Chart Simulation and Results

In order to detect small shifts in process mean and process standard deviation, EWMA charts may be useful. EWMA charts is a suitable alternative of the Schewhart charts when small shifts of process parameters are of interest.

$$z_i = \lambda\, \bar{x}_i + (1 - \lambda)\, z_{i-1}$$

$$\text{UCL} = \mu_0 + L_z\, \sigma_{\bar{x}} \sqrt{\frac{\lambda}{2-\lambda}\,[1-(1-\lambda)^{2i}]}$$

$$\text{Center Line} = \mu_0$$

$$\text{LCL} = \mu_0 - L_z\, \sigma_{\bar{x}} \sqrt{\frac{\lambda}{2-\lambda}\,[1-(1-\lambda)^{2i}]}$$

After some periods control limits tend to their steady formulas:

$$UCL = \mu_0 + L\,\sigma_{\bar{x}}\sqrt{\frac{\lambda}{2-\lambda}}$$

$$LCL = \mu_0 - L\,\sigma_{\bar{x}}\sqrt{\frac{\lambda}{2-\lambda}}$$

In the above equations, Z_i is the exponentially moving average and λ is the smoothing parameter that has value between 0 and 1 usually. In this thesis, we use the Crowder (1989) to identify optimal parameters and control limits for EWMA $\bar{x}$ chart. Optimal parameters from Crowder (1989) are brought in Table 4.11 as follows:

Table 4.11 Optimal Parameters from Crowder (1989) for EWMA $\bar{x}$ Chart

Shifts	Optimal Parameters
0.6 σ	λ = 0.05, K = 2.5
1.0 σ	λ = 0.14, K = 2.8
1.5 σ	λ = 0.25, K = 2.9

To study the effectiveness of the EWMA $\bar{x}$ chart to detect mean shifts, 3 different shift sizes are implanted by letting $\mu_1 = \mu_0 + 0.6\,\sigma_0$, $\mu_1 = \mu_0 + 1.0\,\sigma_0$, and $\mu_1 = \mu_0 + 1.5\,\sigma_0$ in the simulation. 45 random sub-groups following N (1, 1) are generated for each replication by Microsoft Excel. 3 separate experiments are implemented to detect 0.6 σ, 1.0 σ, and 1.5 σ mean shifts. In each experiment, a mean shift is implanted at the 5th subgroup. The first effective signal will be counted if it is plotted outside the control limits after the shift is implanted. Each experiment is replicated 40 times. Results of these experiments for EWMA $\bar{x}$ chart are presented in Tables 4.12 and 4.13. Table 4.12 demonstrates alarm points for EWMA $\bar{x}$ chart. The sign "/" in Table 4.12 indicates that the control chart was not able to detect the shift in a particular run. Table 4.13 demonstrates success rate and average times to detect 0.6 σ, 1.0 σ, and 1.5 σ mean shifts.

Table 4.12 Alarm Points for EWMA $\bar{x}$ Chart

Runs/Shifts	0.6 σ	1.0 σ	1.5 σ
1	9	5	6
2	7	6	6
3	14	7	6
4	11	6	6
5	9	9	7
6	8	7	6
7	8	6	6
8	7	7	6
9	10	7	6
10	9	6	7
11	6	6	6
12	14	7	6
13	10	9	6
14	18	7	5
15	9	8	5
16	15	7	7
17	7	9	6
18	8	5	6
19	10	6	6
20	7	7	6
21	8	7	6
22	8	7	6
23	10	7	7
24	13	11	6
25	12	7	6
26	10	9	6
27	10	8	6
28	8	8	6
29	6	8	6
30	11	7	5
31	17	5	6
32	6	7	6
33	12	7	6
34	9	9	6
35	7	9	7
36	7	6	6
37	8	7	6
38	8	7	6
39	10	9	5
40	10	7	6

Table 4.13 Simulation Results for EWMA $\bar{x}$ Chart

40 Runs	Success Rate	Average Time to Detect
0.6 σ	40 (100%)	9.65
1.0 σ	40 (100%)	7.22
1.5 σ	40 (100%)	6.02

As shown in Table 4.13, success rates of the EWMA $\bar{x}$ chart increased with the increase of the mean shifts. It also shows that, success rates for 0.6 σ, 1.0 σ, and 1.5 σ are all 100% respectively and average times to detect the shifts decreased with the increase of the mean shifts. Average times to detect 0.6 σ shift is 9.65 while average time to detect 1.0 σ and 1.5 σ shifts are 7.22 and 6.02 respectively. Figure 4.5 presents graphically the data in Table 4.13.

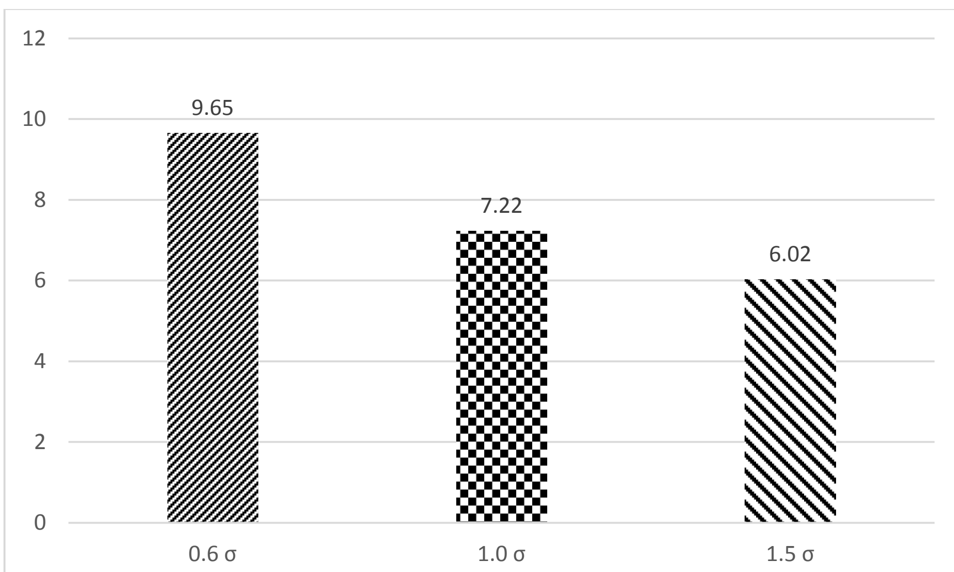

Figure 4.5 Performance of the EWMA $\bar{x}$ Chart

4.7 EWMA Q Chart Simulation and Results

Sub-groups sample means ($\bar{x}_i$) are used in EWMA $\bar{x}$ chart statistics to plot process measurements. EWMA Q chart uses Q statistics calculated from each sub-group (Q_i) instead of sample means to plot process measurements. It is worth noting that when the small shifts of parameters are of interest, using EWMA Q chart may be useful to monitor the process mean. EWMA Q chart statistics are constructed as:

$$Z_i = \lambda\, Q_i + (1 - \lambda)\, Z_{i-1} \qquad i = 1, 2\ldots$$

$$\text{UCL} = +\text{K}\sqrt{\frac{\lambda}{2-\lambda}}$$

$$\text{CL} = 0$$

$$\text{LCL} = -\,\text{K}\sqrt{\frac{\lambda}{2-\lambda}}$$

Quessenberry (1993) first used exponentially weighted moving average (EWMA) Q chart to detect small shifts of process mean and process standard deviation. Crowder (1989), calculated optimal smoothing parameters and control limit constants to make the design of EWMA charts easier. In this thesis, we use the results in Crowder (1989) to identify optimal parameters and control limits for EWMA Q chart. Optimal smoothing parameters from Crowder (1989) are shown in Table 4.14:

Table 4.14 Optimal Parameters from Crowder (1989) for EWMA Q Chart

Shifts	Optimal Parameters
0.6 σ	λ = 0.05, K = 2.5
1.0 σ	λ = 0.14, K = 2.8
1.5 σ	λ = 0.25, K = 2.9

To study the usefulness of the EWMA Q chart to detect mean shifts, 3 different shift sizes are implanted by letting $\mu_1 = \mu_0 + 0.6\,\sigma_0$, $\mu_1 = \mu_0 + 1.0\,\sigma_0$, and $\mu_1 = \mu_0 + 1.5\,\sigma_0$ in our simulations. 45 random sub-groups following N (1, 1) are generated for each replication by Microsoft Excel. 3 separate experiments are implemented to detect 0.6 σ, 1.0 σ, and 1.5 σ mean shifts. In each experiment, a mean shift is implanted at the 5th sub-group. The first effective signal will be counted if it is plotted outside the control limits after the shift is implanted. Each experiment is replicated 40 times. Results of these experiments for EWMA Q chart are presented in Tables 4.15 and 4.16. Table 4.15 demonstrates alarm points for EWMA Q chart. The sign "/" in Table 4.15 indicates that the control chart was not able to detect the shift in the particular run. Table 4.16 demonstrates success rates and average times to detect 0.6 σ, 1.0 σ, and 1.5 σ mean shifts.

Table 4.15 Alarm Points for EWMA Q Chart

Runs/Shifts	0.6 σ	1.0 σ	1.5 σ
1	10	8	6
2	8	/	8
3	6	7	7
4	25	21	6
5	16	/	6
6	/	9	9
7	6	18	6
8	11	6	12
9	41	10	7
10	6	10	6
11	/	6	7
12	6	6	7
13	37	6	8
14	/	6	6
15	32	7	6
16	14	6	7
17	/	37	7
18	/	8	6
19	/	6	6
20	27	21	6
21	11	/	/
22	23	8	9
23	33	7	6
24	6	6	6
25	24	6	6
26	8	28	15
27	30	9	6
28	13	/	7
29	14	/	6
30	6	6	6
31	7	/	/
32	/	6	7
33	15	19	6
34	10	/	6
35	6	6	/
36	6	6	/
37	12	26	7
38	14	6	6
39	12	6	6
40	/	6	6

Table 4.16 Simulation Results for EWMA Q Chart

40 Runs	Success Rate	Average Time to Detect
0.6 σ	32 (80%)	15.46
1.0 σ	34 (85%)	8.17
1.5 σ	36 (90%)	6.94

From Table 4.16, success rates of the EWMA Q chart increased with the increase of the mean shifts. As can be seen from the results in Table 4.16, success rates for 0.6 σ, 1.0 σ, and 1.5 σ are 80%, 85%, and 90% respectively. From Table 4.16, average times to detect the shifts decreased with the increase of the mean shifts. Accordingly, average times to detect 0.6 σ shift is 15.46 while average times to detect 1.0 σ and 1.5 σ shifts are 8.17 and 6.94 respectively. Figures 4.6 and 4.7 present graphically the data in Table 4.16.

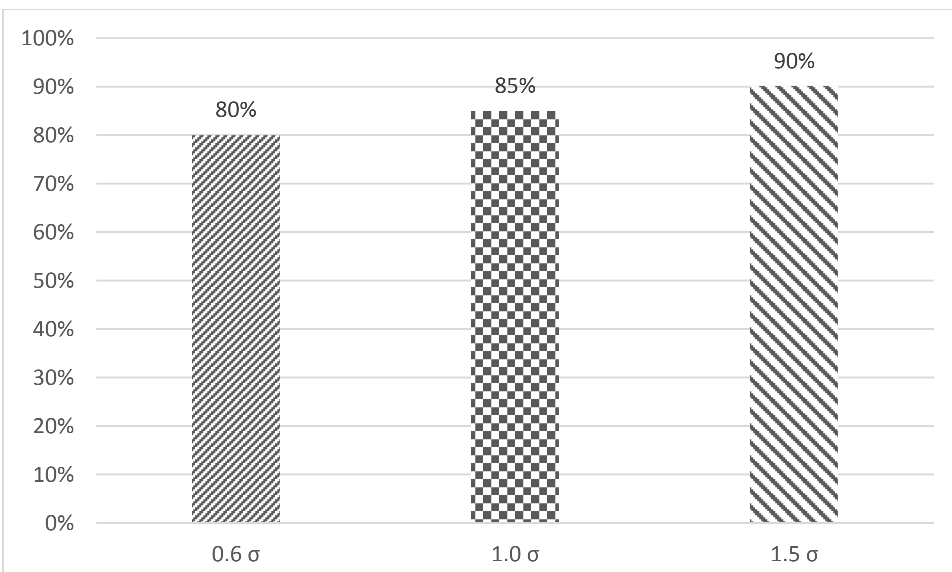

Figure 4.6 Success Rates of the EWMA Q Chart

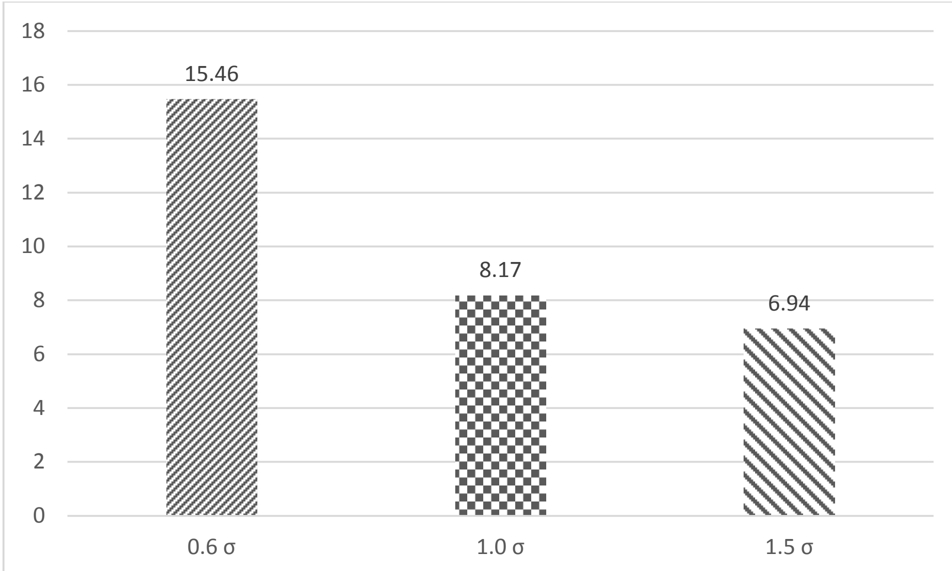

Figure 4.7 Performance of the EWMA Q Chart

4.8 T Chart Simulation and Results

Zhang et al. (2009) showed that T chart is more robust against estimation errors and unstable process standard deviation. To construct a T chart one needs to compute the mean and standard deviation of the sub grouped data. In this thesis we applied T chart in batch production.

$$\overline{X}_i = \frac{1}{n}\sum_{j=1}^{n} X_{i,j}$$

$$S_i = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(X_{i,j} - \overline{X}_i)^2}$$

$$T_i = \frac{\overline{X}_i - \mu_0}{\frac{S_i}{\sqrt{n}}}$$

$$UCL_t = F_t^{-1}\left(1 - \frac{\alpha}{2}, n - 1\right)$$

$$LCL_t = - UCL_t$$

There are two primary reasons that T chart may be implemented to monitor batch production. First, the use of T chart does not need Phase I sample collection to stablish μ_0 and σ_0. The second reason is that T chart is easy to implement. In conducting our simulation, we assume that process mean is known at 1.0. Initial setup assume that the process is in control with the population mean μ_0 at the target M. Since n = 5, T control chart can be set as:

UCL = 6.62; 1; LCL = - 6.62.

Initially, we simulated T chart so as to calculate its in control ARL. Accordingly, 45 random subg-roups following N (1, 1) are generated each had 5 units for each replication by Microsoft Excel. No shifts are implanted at a particular run. This experiment was replicated 100 times. The result indicates that T chart monitoring the process mean gives false alarm after producing 375 units. To study the usefulness of the T chart in detecting mean shifts, 3 different shift sizes are implanted by letting $\mu_1 = \mu_0 + 0.6\ \sigma_0$, $\mu_1 = \mu_0 + 1.0\ \sigma_0$, and $\mu_1 = \mu_0 + 1.5\ \sigma_0$ in simulations. 45 random sub-groups following N (1, 1) are generated for each replication by Microsoft Excel. 3 separate experiments are implemented to detect 0.6 σ, 1.0 σ, and 1.5 σ mean shifts. In each experiment, a mean shift is implanted at the 5th sub-group. The first effective signal will be counted if it is plotted outside the control limits after the shift is implanted. Each experiment is replicated 40 times. Results of these experiments are presented in Tables 4.17 and 4.18. Table 4.17 demonstrates alarm points for T chart. The sign "/" in Table 4.17 indicates that the control chart was not able to detect the shift in a particular run. Table 4.18 demonstrates success rates and average times to detect 0.6 σ, 1.0 σ, and 1.5 σ mean shifts.

Table 4.17 Alarm Points for T Chart

Runs/Shifts	0.6 σ	1.0 σ	1.5 σ
1	25	6	13
2	/	/	7
3	/	16	6
4	20	15	7
5	/	36	6
6	/	/	/
7	/	40	10
8	7	11	6
9	15	13	9
10	7	14	11
11	33	/	7
12	/	8	21
13	/	40	16
14	37	6	6
15	31	12	7
16	/	20	10
17	/	13	6
18	/	/	24
19	/	6	9
20	29	8	9
21	/	25	13
22	/	13	27
23	42	22	19
24	14	14	25
25	24	15	19
26	9	8	12
27	/	35	17
28	/	/	12
29	36	18	6
30	38	7	13
31	/	26	6
32	39	9	16
33	29	12	6
34	28	30	7
35	/	28	7
36	/	34	6
37	32	10	9
38	45	29	7
39	44	/	13
40	/	/	9

Table 4.18 Simulation Results for T Chart

40 Runs	Success Rate	Average Time to Detect
0.6 σ	21 (53%)	27.8
1.0 σ	33 (83%)	18.15
1.5 σ	39 (98%)	15.13

As can be seen from the results in Tables 4.18, the T chart can detect 0.6, 1.0 σ and 1.5 σ shifts with success rates of 53 %, 83 %, and 98 % respectively. Table 4.18 also shown that average times to detect 0.6 σ shift is 27.80 while average times to detect 1.0 σ and 1.5 σ shifts are 18.15 and 15.13 respectively. Figures 4.8 and 4.9 present graphically the data in Table 4.18.

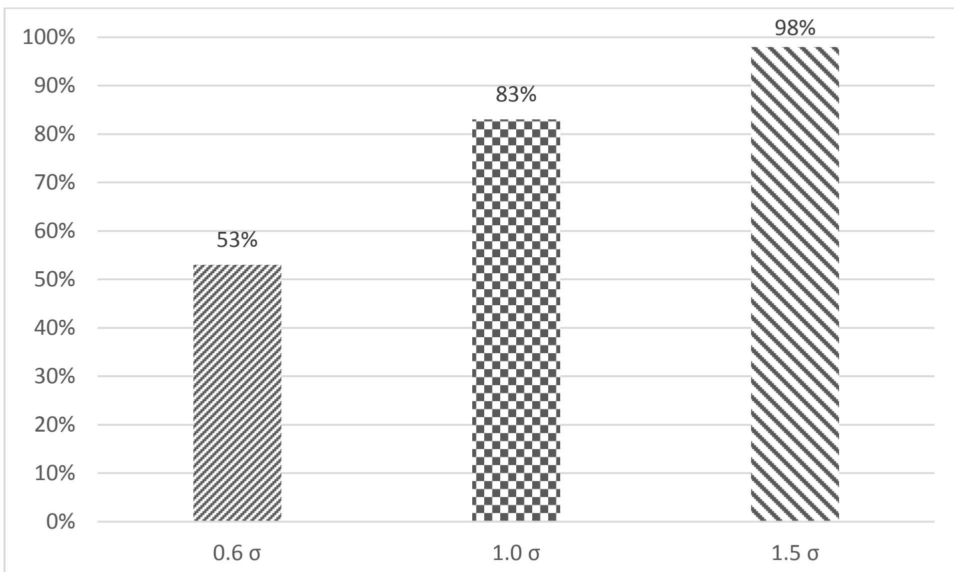

Figure 4.8 Success Rates of the T Chart

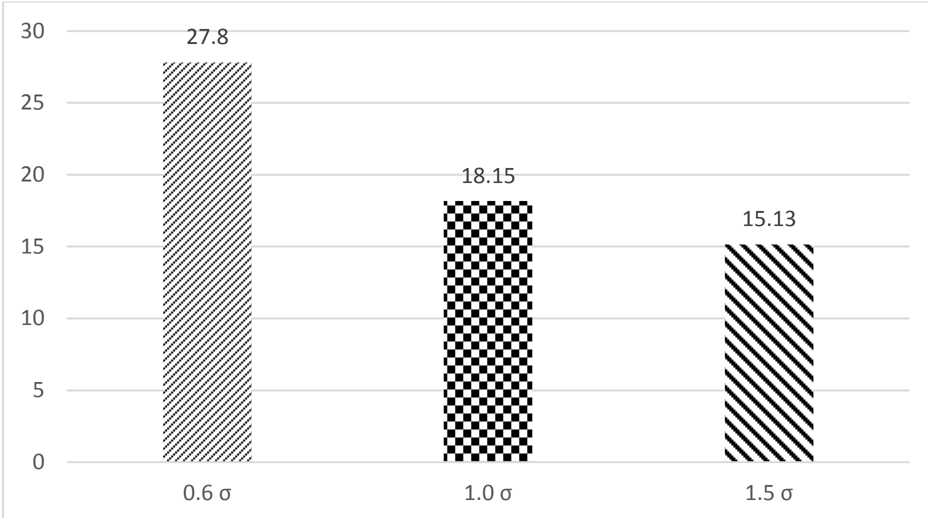

Figure 4.9 Performances of the T Chart

4.9 EWMA T Chart Simulation and Results

When the small shifts of process parameters such as process mean and process standard deviation are of interest, resorting to an EWMA charts may improve the detection ability of the control procedures. EWMA T chart uses T statistics to plot the process measurements. However, EWMA Q chart uses Q statistics and EWMA $\bar{x}$ chart uses sub-groups sample means for plotting process measurements.

$$Y_i = \lambda\, T_i + (1 - \lambda)\, Y_{i-1}$$

$$Y_0 = 0$$

$$UCL_t = +\,K \sqrt{\frac{\lambda}{2-\lambda}}$$

$$LCL_t = -\,K \sqrt{\frac{\lambda}{2-\lambda}}$$

$$K = L * \sigma_0$$

In the above equation λ is the smoothing parameter and T_i is the exponentially moving average. The lower and upper control limits of EWMA T chart satisfy $LCLt = -UCLt$ and the center line is 0. The control limits are $\pm K\sqrt{\frac{\lambda}{2-\lambda}}$ where K=L x σ_0. L is the constant parameter.

We use optimal parameters in Zhang et al. (2009) in which they calculated optimal parameters for EWMA T Chart with the use of markov chain approach. Table 4.19 demonstrates the optimal parameters for EWMA T chart as follows:

Table 4.19 Optimal Parameters for EWMA T Chart

Shifts	Optimal Parameters
0.6 σ	λ = 0.09, K = 3.9
1.0 σ	λ = 0.14, K = 4.5
1.5 σ	λ = 0.25, K = 4.8

To study the usefulness of the EWMA T chart to detect mean shifts, 3 different shift sizes are implanted by letting $\mu_1 = \mu_0 + 0.6\,\sigma_0$, $\mu_1 = \mu_0 + 1.0\,\sigma_0$, and $\mu_1 = \mu_0 + 1.5\,\sigma_0$ in simulations. 45 random subgroups following N (1, 1) are generated for each replication by Microsoft Excel. 3 separate experiments are implemented to detect 0.6 σ, 1.0 σ, and 1.5 σ mean shifts. In each experiments, a mean shift is implanted at the 5^{th} sub-group. The first effective signal will be counted if it is plotted outside the control limits after the shift is implanted. Each experiment is replicated 40 times. Results of these experiments for EWMA T chart are presented in Tables 4.20 and 4.21. Table 4.20 demonstrates alarm points for EWMA T chart. The sign "/" in Table 4.20 indicates that the control chart was not able to detect the shift in the particular run. Table 4.21 demonstrates success rates and average times to detect 0.6 σ, 1.0 σ, and 1.5 σ mean shifts.

Table 4.20 Alarm Points for EWMA T Chart

Runs/Shifts	0.6 σ	1.0 σ	1.5 σ
1	8	9	6
2	12	9	9
3	9	7	9
4	15	7	7
5	12	8	7
6	13	11	7
7	11	7	7
8	11	7	6
9	17	8	7
10	5	8	6
11	21	15	7
12	9	9	9
13	16	9	8
14	8	6	9
15	14	7	8
16	13	7	6
17	9	7	5
18	19	9	7
19	7	14	6
20	19	8	8
21	10	11	7
22	22	6	5
23	10	11	7
24	12	7	7
25	15	8	7
26	15	10	6
27	10	9	7
28	7	10	8
29	10	7	6
30	11	9	6
31	21	8	6
32	13	8	7
33	9	7	7
34	12	8	6
35	15	6	8
36	9	7	7
37	10	7	7
38	11	7	7
39	12	8	7
40	12	5	7

Table 4.21 Simulation Results for EWMA T Chart

40 Runs	Success Rate	Average Time to Detect
0.6 σ	40 (100%)	12.35
1.0 σ	40 (100%)	8.27
1.5 σ	40 (100%)	6.97

As can be seen from the results in Tables 4.21, EWMA T chart can detect 0.6, 1.0 σ and 1.5 σ shifts with the success rates of 100%. According to Table 4.21, average times to detect 0.6 σ shift is 12.35 while average times to detect the 1.0 σ and 1.5 σ shifts are 8.27 and 6.97 respectively. Figure 4.10 presents graphically the data in Table 4.21.

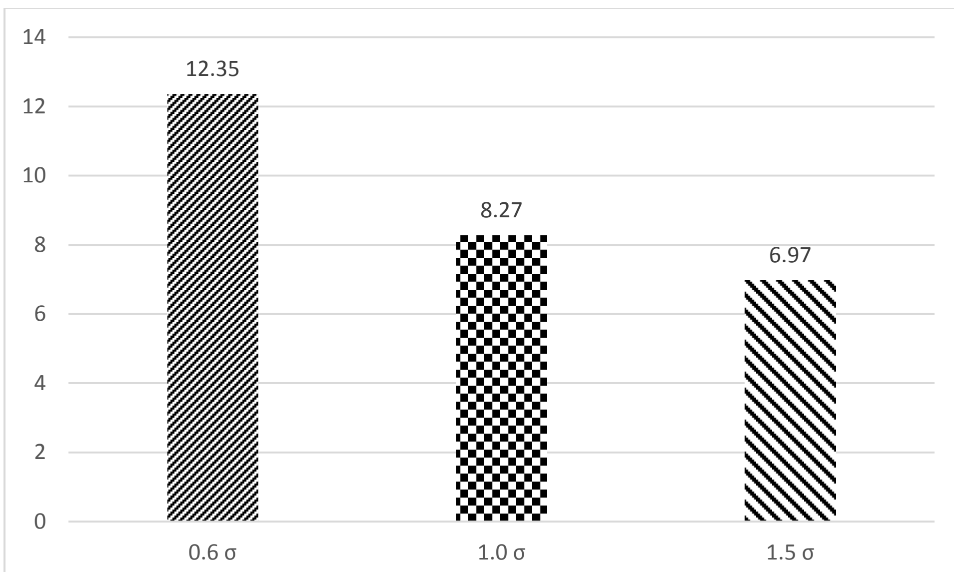

Figure 4.10 Performance of EWMA T Chart

4.10 Schewhart Sign Chart Simulation and Results

Schewhart sign chart is based on the assumption that the distribution of collected data is nonparametric. The previous experiments of control charts are based on the assumption that the process characteristics follow normal distribution. In this section, we also assume normal distribution to test Schewhart sign chart.

$$SN_i = \sum_{j=1}^{N} \text{Sign}\ (X_{i,j} - T_M)$$

$$SN_i = 2^*D_i - n$$

$$UCL = c = n$$

$$CL = 0$$

$$LCL = -c = -n$$

In the above equations, D_i is the number of positive signs in a sub-group, and n is the number of samples that have been collected in each sub-groups. Without presence of assignable causes in-control value for SN_i would become zero. D_i is a random variable which has binomial distribution($D_i \sim B\ (n, p_0)$). The constant c is computed as c = 2 × d − n. T_M is target value of the mean or median of the collected data which is equal to μ_0 (process mean). As soon as the set-up activities are done without errors, T_M would be equal to μ_0.

To investigate the effectiveness of the Schewhart sign chart to detect mean shifts, 3 different shift sizes are used by letting $\mu_1 = \mu_0 + 0.6\ \sigma_0$, $\mu_1 = \mu_0 + 1.0\ \sigma_0$, and $\mu_1 = \mu_0 + 1.5\ \sigma_0$ in simulations. 45 random sub-groups following N (1, 1) are generated for each replication by Microsoft Excel. 3 separate experiments are implemented to detect 0.6 σ, 1.0 σ, and 1.5 σ mean shifts. In each experiment, a mean shift is implanted at the 5th sub-group. The first effective signal will be counted if it is plotted outside the control limits after the shift is implanted. We assume that T_M is equal to 1. Each experiment is replicated 40 times. Results of these experiments for Schewhart sign chart are presented in Tables 4.22 and 4.23. Table 4.22 demonstrates alarm points for Schewhart sign chart. The sign "/" in Table 4.22 indicates that the control chart was not able to detect the shift in the particular run. Table 4.23 demonstrates success rates and average times to detect 0.6 σ, 1.0 σ, and 1.5 σ mean shifts.

Table 4.22 Alarm Points for Schewhart Sign Chart

Runs/Shifts	0.6 σ	1.0 σ	1.5 σ
1	9	6	5
2	29	6	5
3	16	10	5
4	6	5	5
5	5	5	6
6	5	5	5
7	6	10	5
8	7	5	5
9	11	5	6
10	9	5	5
11	5	6	5
12	11	5	5
13	6	5	5
14	22	7	6
15	7	5	6
16	6	5	6
17	14	5	7
18	6	7	6
19	9	7	6
20	6	7	5
21	16	6	5
22	5	7	5
23	7	6	5
24	8	6	6
25	19	10	5
26	17	8	8
27	7	6	6
28	8	7	6
29	7	8	5
30	7	7	5
31	11	5	6
32	5	7	5
33	12	5	5
34	15	6	6
35	13	8	5
36	5	6	5
37	6	9	5
38	18	5	5
39	8	5	5
40	5	7	5

Table 4.23 Simulation Results for Schewhart Sign Chart

40 Run	Success Rate	Average Time to Detect
0.6 σ	40 (100%)	9.85
1.0 σ	40 (100%)	6.37
1.5 σ	40 (100%)	5.43

From Table 4.23, success rates of the Schewhart sign chart increased with the increase of the mean shifts. As can be seen from the results in Table 4.23, success rates for 0.6 σ, 1.0 σ, and 1.5 σ are all 100%. From Table 4.23, average times to detect the shifts decreased with the increase of the mean shifts. Accordingly, average times to detect 0.6 σ shift is 9.85 while average times to detect 1.0 σ and 1.5 σ shifts are 6.37 and 5.43 respectively. Figure 4.11 presents graphically the data in Table 4.23.

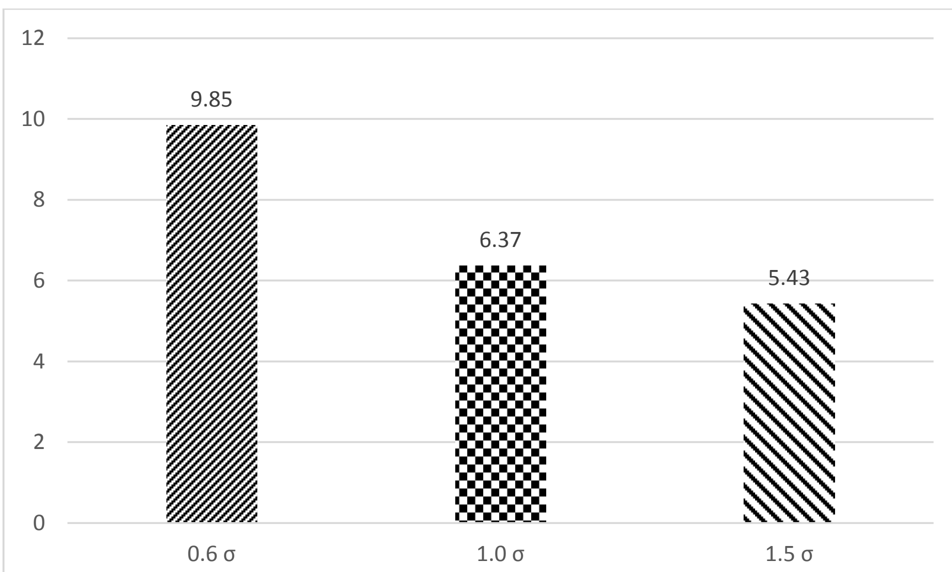

Figure 4.11 Performance of the Schewhart Sign Chart

4.11 Comparison of Tested Control Charts

Control charts simulated in this chapter are compared in this section based on success rates and average times to detect. Success rates indicate the number of successes (detecting a mean shift) in 40 runs for all simulation experiments and average times to detect is the out-of-control ARL. The difference between the point where the shift is detected at a particular sub-group and the implanted shift at the 5th sub-group is used to calculate out-of-control ARL. Various control charts were tested in this thesis to investigate their capabilities to detect mean shifts in batch production. $\bar{x}$ chart, $\bar{x}$ chart with Western Electric Rules, Q chart, T chart, EWMA $\bar{x}$ chart, EWMA Q chart, EWMA T chart and Schewhart sign chart were simulated in this chapter to identify appropriate control charts for batch production. To study the usefulness of control charts in detecting mean shifts, 3 different shift sizes were implanted by letting $\mu_1 = \mu_0 + 0.6\ \sigma_0$, $\mu_1 = \mu_0 + 1.0\ \sigma_0$, and $\mu_1 = \mu_0 + 1.5\ \sigma_0$ in simulations. 45 random sub-groups following N (1, 1) are generated for each replication by Microsoft Excel. Figure 4.12 demonstrates control charts success rates in detecting 0.6 σ mean shift and Figure 4.13 shows control charts average times to detect.

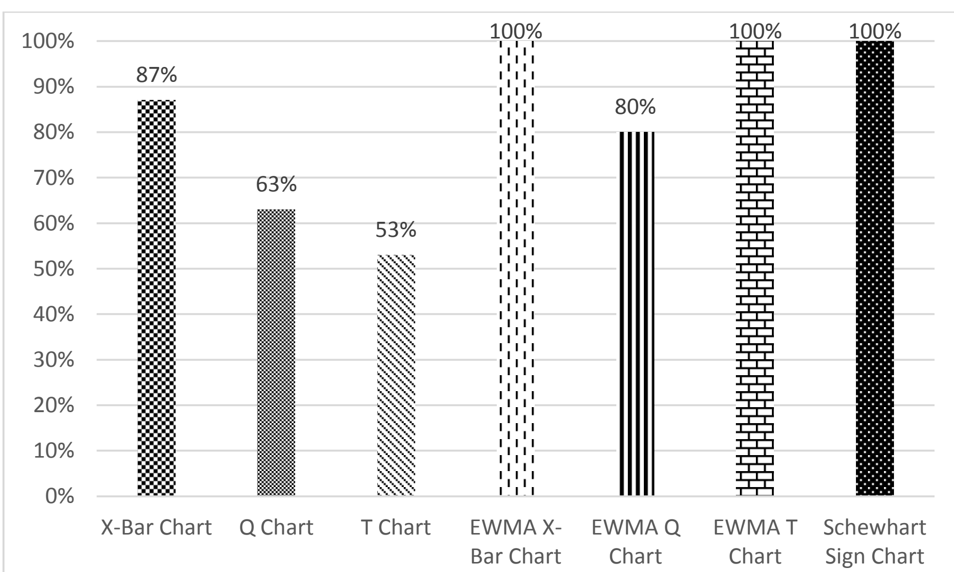

Figure 4.12 Success Rates of the Different Charts to Detect 0.6 σ Mean Shift

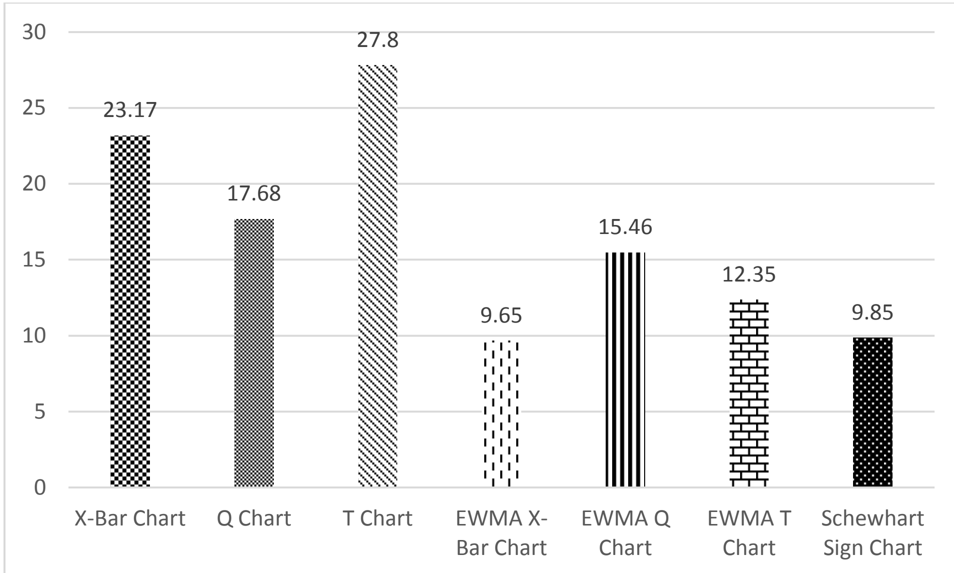

Figure 4.13 Performances of the Different Charts to Detect 0.6 σ Mean Shift

As can be seen from Figures 4.12 and 4.13, EWMA $\bar{x}$ chart and Schewhart sign chart has better success rates and ARL performances in detecting 0.6 σ mean shift in comparison to other charts. EWMA T chart could be another appropriate alternative in detecting 0.6 σ mean shift. As shown in Figures 4.12 and 4.13, EWMA $\bar{x}$ chart, Schewhart sign chart, and EWMA T chart are 100 % successful to detect 0.6 σ shift, but EWMA $\bar{x}$ chart and Schewhart sign chart are able to detect the 0.6 σ mean shift earlier than EWMA T chart. From Figure 4.13, average times to detect 0.6 σ for EWMA $\bar{x}$ chart and Schewhart sign chart are 9.65 and 9.85 respectively while average times to detect 0.6 σ for EWMA T chart is 12.35. Figures 4.12 and 4.13 also shown that T chart is not capable enough to detect 0.6 σ mean shift in comparison with other charts. Also, Q chart does not have appropriate success rates and ARL performances in comparison to other charts except T chart. As shown in Table 4.7 in Section 4.3.4, performances of the $\bar{x}$ chart was improved by applying Western Electric Rules. Figures 4.14 and 4.15 demonstrate performances of the $\bar{x}$ chart to detect 0.6 σ mean shift.

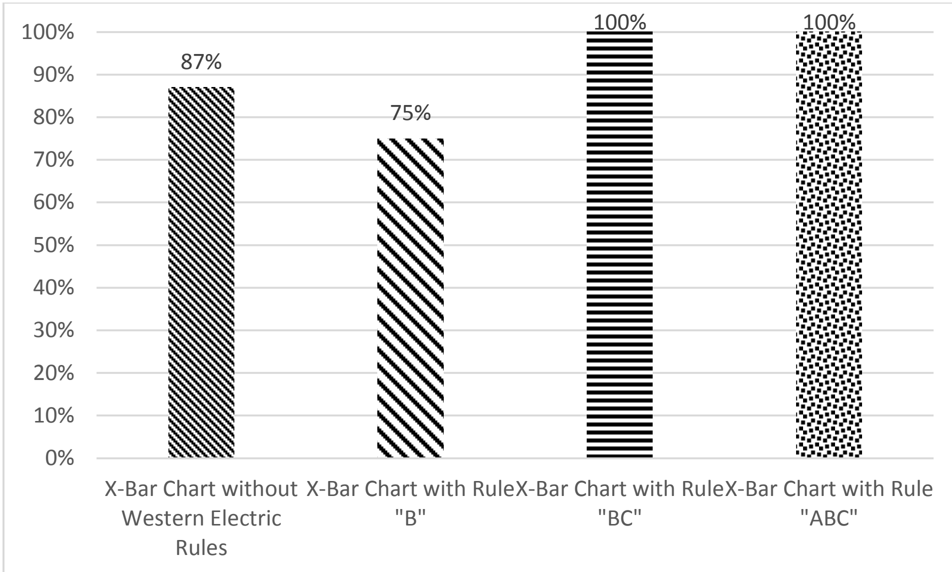

Figure 4.14 Success Rates for $\bar{x}$ Chart before and after Applying Western Electric Rules

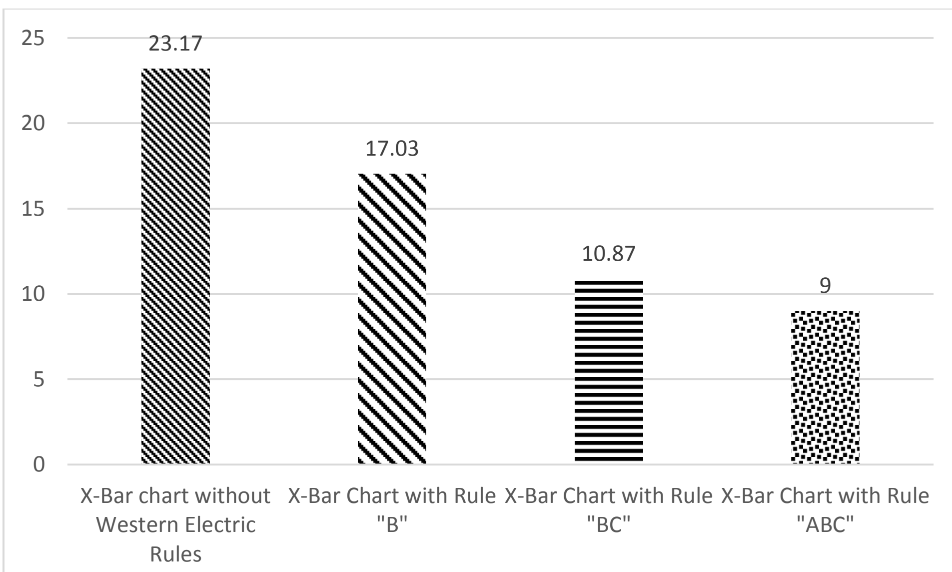

Figure 4.15 Performances of $\bar{x}$ Chart before and after Applying Western Electric Rules

From Figures 4.14 and 4.15, the performances of $\bar{x}$ chart improves for all the Western Electric Rules However, average times to detect decreases by applying Western Electric Rule "B" from 23.17 to 17.03 for $\bar{x}$ chart. As was mentioned in Section 4.4.5 resorting to Western Electric Rules increases the probability of Type I error occurrence. Practitioners should consider Type I error augments which cause the number of false alarms to increase.

Figure 4.16 demonstrates control charts success rates to detect 1.0 σ mean shift and Figure 4.17 shows control charts average time to detect for 1.0 σ mean shift.

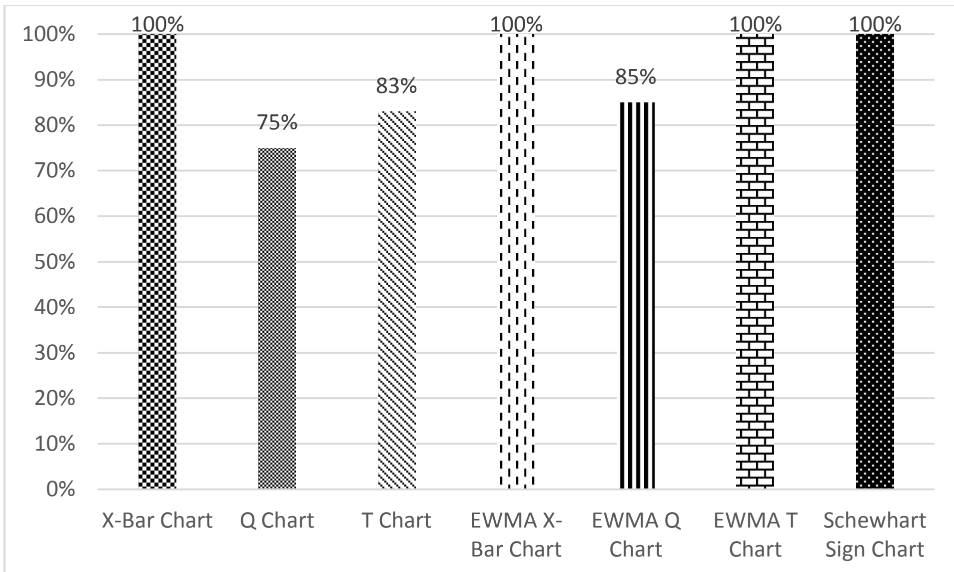

Figure 4.16 Success Rates of the Different Charts to Detect 1.0 σ Mean Shift

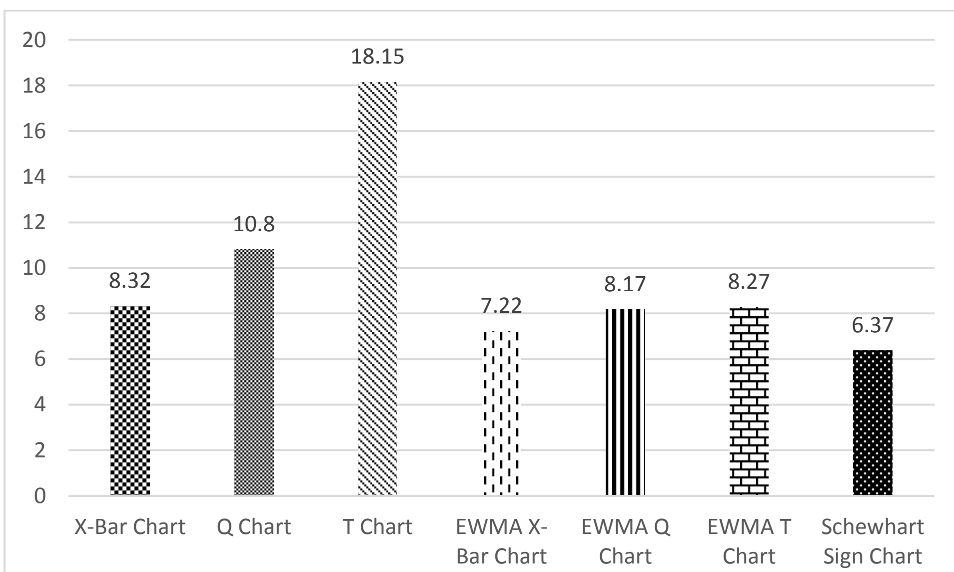

Figure 4.17 Performances of the Different Charts to Detect 1.0 σ Mean Shift

As can be seen from Figures 4.16 and 4.17, Schewhart sign chart has better success rates and ARL performances in comparison to other charts. Accordingly, Schewhart sign chart is 100 % successful in detecting 1.0 σ shift and average times to detect 1.0 σ mean shift is 6.32. From Figures 4.16 and 4.17, EWMA $\bar{x}$ chart, $\bar{x}$ chart, and EWMA T chart are 100 % successful in detecting 1.0 σ shift while average times to detect for EWMA $\bar{x}$ chart, $\bar{x}$ chart, and EWMA T chart are 7.22, 8.32, and 8.27 respectively. Although EWMA Q chart is 85 % successful in detecting 1.0 σ shift, EWMA Q chart has better average times to detect in comparison with $\bar{x}$ chart and EWMA T chart. Q chart has appropriate ARL performance which is 10.80. However, success rates for the Q chart is 75 % which is the minimum among the tested charts. Q chart has better out-of-control ARL performances in comparison to T chart. As shown in Figure 4.17, average times to detect for T chart is 18.15 which is the maximum time to detect 1.0 σ mean shift in comparison to other charts.

Figure 4.18 demonstrates control charts success rates to detect 1.5 σ mean shift and Figure 4.19 shows control charts average time to detect for 1.5 σ mean shift.

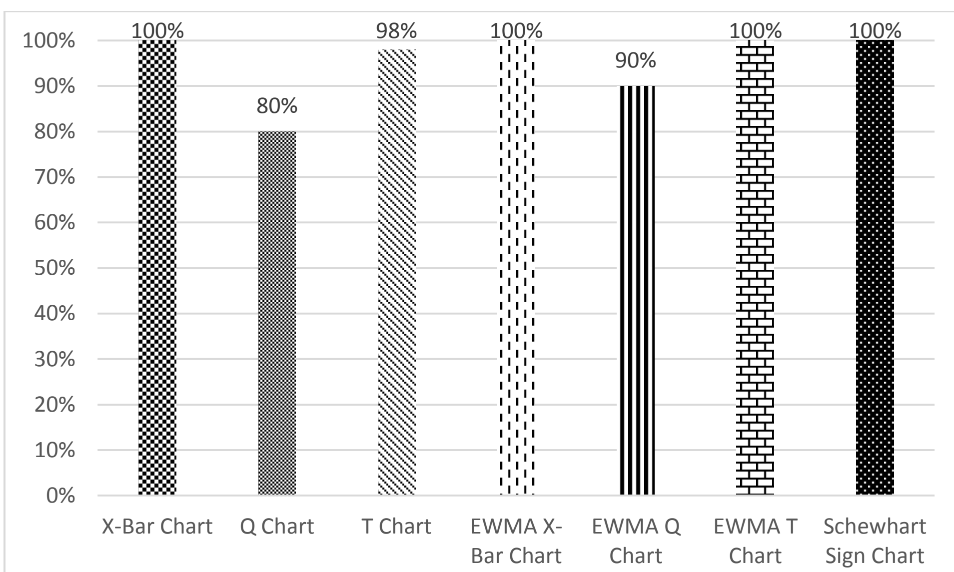

Figure 4.18 Success Rates of the Different Charts to Detect 1.5 σ Mean Shift

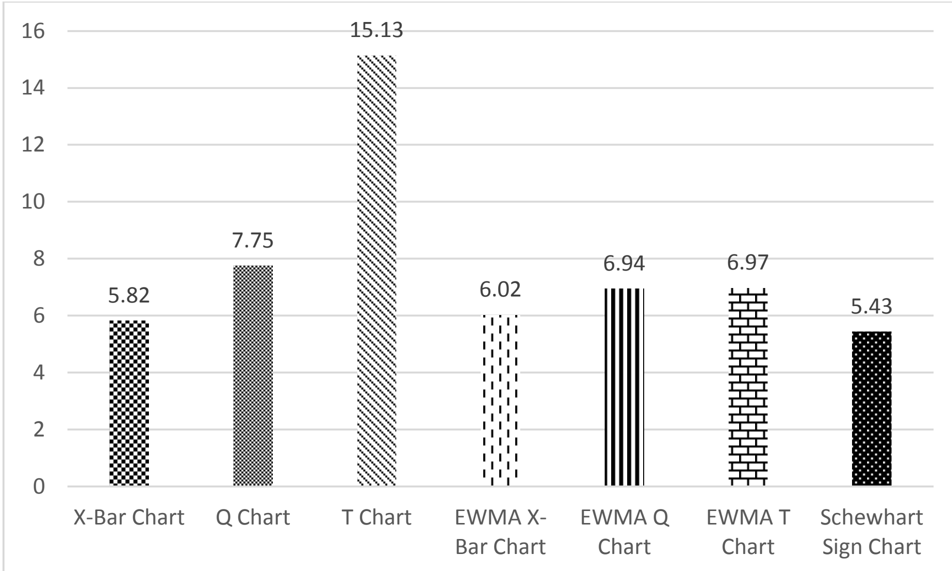

Figure 4.19 Performances of the Different Charts to Detect 1.5 σ Mean Shift

From Figures 4.18 and 4.19, Schewhart sign chart and $\bar{x}$ chart has better success rates and ARL performances in comparison with other charts. Accordingly, both of the Schewhart sign chart and $\bar{x}$ chart are 100 % successful and average times to detect for these chart are 5.43 and 5.82 respectively. As shown in Figures 4.18 and 4.19, EWMA $\bar{x}$ chart has nearly the same performances as Schewhart sign chart and $\bar{x}$ chart. Figures 4.18 and 4.19 show that although EWMA T chart has better success rates than EWMA Q chart to detect 1.5 σ shift, both of them have the same average times to detect which is 6.9 approximately. From Figures 4.18 and 4.19, T chart has better success rates than Q chart, but Q chart can detect 1.5 σ mean shift earlier than T chart. Figure 4.19 shows that average times to detect 1.5 σ mean shift is 7.75 for Q chart while average times to detect 1.5 σ mean shift is 15.13 for T chart.

In conclusions, different control charts are suggested for batch production and job-shop manufacturing as shown in Table 4.24 based on their success rates and average times to detect for different sizes of process mean shifts.

Table 4.24 Suggested Control Charts for Batch Production

0.6 σ (Small Mean Shift)	EWMA X-Bar Chart, EWMA T Chart, Schewhart Sign Chart
1.0 σ (Medium Mean Shift)	EWMA X-Bar Chart, X-Bar Chart EWMA T Chart, Schewhart Sign Chart
1.5 σ (Large Mean Shift)	EWMA X-Bar Chart, X-Bar Chart EWMA T Chart, EWMA Q Chart Schewhart Sign Chart

4.12 Summary

In this chapter, various control charts were simulated and assessed to investigate their capabilities in finding small, medium and larger shifts in process mean. The main objective of this chapter was to investigate the usefulness of different control charts in batch production and job-shop manufacturing. In the first part of Chapter 4, analytical approach to calculate out-of-control ARL for $\bar{x}$ chart was presented. The results from analytical and simulation were compared to investigate conformity between the results of the two different approaches. Western Electric Rules were applied for $\bar{x}$ chart and it was shown that resorting to Western Electric Rules increases the probability of Type I error and decreases in control ARL which gives higher false alarms rates.

In the second part of Chapter 4, all the simulation experiments and results were brought to compare different control charts for batch production and job-shop manufacturing. In order to detect 0.6 σ mean shift (small mean shift), Schewhart sign chart, EWMA $\bar{x}$ chart and EWMA T chart are more effective. For detecting 1.0 σ mean shift (medium mean shift), EWMA $\bar{x}$ chart, EWMA T chart, $\bar{x}$ chart and Schewhart sign chart are preferred. To detect 1.5 σ mean shift (large mean shift), EWMA T chart, EWMA Q chart, Schewhart sign chart, EWMA $\bar{x}$ chart, and $\bar{x}$ chart have good performances.

5 Conclusions and Future Research

5.1 Summary

The main objective of this thesis is to compare different quality control charts for on-line monitoring of manufacturing processes of batch production. Monitoring such processes is a challenging issue when processes should be frequently reconfigured to produce different parts and maintain small inventory levels. In Chapter 2, different control charts and quality control procedures were reviewed. In Chapter 3, models and formulas of control charts were presented for monitoring batch production. Those control charts were tested by simulation and were evaluated in terms of success rates and average times to detect the mean shifts in Chapter 4.

5.2 Contributions of the Thesis

In this thesis, detection abilities of various quality control charts for batch production and job-shop manufacturing were tested using simulation. The main intention of using simulation in this thesis is its simplicity to study the control charts performances. Furthermore, all companies should have access to Microsoft Excel to model and study their manufacturing processes.

Production volume is typically larger in batch production comparing to job-shop production and is smaller comparing to mass production. Quality control charts more suitable for batch production are simulated in this thesis for production of medium volume and medium variety of products. Researchers have studied different control charts for optimal ARL performances for short-run and long-run production. Research is limited on finding control charts for processes with medium production volume and medium products variety.

From simulation results, we have the following general observations:

Various control charts are recommended in batch production which were less studied in the literature. $\bar{x}$ chart is applied to batch production and its detection ability is compared with other charts. Both Q chart and T chart are applied in batch production in this thesis to check their detection abilities to find various sizes of the process mean shifts. EWMA $\bar{x}$ chart, EWMA Q chart, and EWMA T chart are tested in this thesis to compare their detection abilities with regular Schewhart charts. Also Schewhart sign chart was tested in this thesis. Western Electric Rules were applied for $\bar{x}$ chart to increase the detection ability of this charts in finding small process mean shift.

5.3 Future Research

Future research can deal with statistical measure of performance which uses Markov chains to calculate average run length (ARL), average number of sample size before shift (ANOS) or average time to signal (ATS). Furthermore, cost analysis is another interesting topic which deals with operation research techniques to design economic control charts. Studying adaptive control charts such as variable sample size (VSS) charts, variable sampling interval (VSI) charts, variable sample size and variable sampling interval (VSSI) charts and variable parameters (VP) charts are other topics which can be followed in the future research.

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