Numerical Modeling of Foundations on Dense Sand Overlaying Loose Sand

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ABSTRACT

Numerical modeling of foundations on dense sand overlaying loose sand

Foundation built on layered soil is often encountered in the field. Layered-soil profile could be either a natural or artificial phenomenon. For structures with large footing dimensions two layers of subsoil are usually involved in the determination of bearing capacity of soil. Also, building of heavy structures on weak soils often needs a blanket of granular material to cover the natural soil and increase the soil bearing capacity. In fact, the granular layer both distributes and alleviates the load pressure due to the infrastructure. Popular weighted average method and projected area method fail to predict accurately the soil bearing capacity when the upper layer is dense sand.

This thesis investigates the bearing capacity of strip footings on a layer of dense sand underlined by loose sand. The effects of thickness and shear strength of top layer as well as the embedment depth of footing were investigated. A numerical model was developed to predict the bearing capacity of footing on layered sand. The results showed that the bearing capacity of strip footings on dense sand (with friction angle, φ, higher than 40 degrees) with and without embedment depth is overestimated by weighted average method and is underestimated by projected area method. Comparison with the available theory and experimental result are presented. It is of interest to report herein that, in the absence of surcharge the footing bearing capacity from numerical analysis was always less than Hanna (1981). In the presence of the surcharge load due to the embedment depth of footing, the results were close to that obtained by Hanna (1981).

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LIST OF SYMBOLS

B	Width of the footing
d	Depth of the footing in the upper soil layer
D	Width of soil geometry in numerical model
L	Length of soil geometry in numerical model
H	Thickness of the upper soil layer below the footing's base
Z	Depth from the footing base
E	Young's modulus
γ_1	Unit weight of soil, upper layer
γ_2	Unit weight of soil, lower layer
c_1	Cohesion of upper soil layer
c_2	Cohesion of lower soil layer
ψ	Dilatancy angle
φ_1	Angle of shear resistance, upper layer
φ_2	Angle of shear resistance, lower layer
δ	The mobilized angle of shear resistance
P_p	The total passive earth pressure
K_p	Coefficient of passive earth pressure given by Caquot and Kerisel (1949)
q_u	Ultimate bearing capacity
N_c, N_q, N_γ	Bearing capacity factors

LIST OF ABBREVIATIONS

FEM	Finite Element Method
FDM	Finite Difference Method
BEM	Boundary Element Method
DEM	Discrete (or Distinct) Element Method
PDEs	Partial Differential Equations
WAM	Weighted Average Method
PAM	Projected Area Method
ANN	Artificial Neural Network

1. INTRODUCTION

1.1. General

The function of foundations is to transmit the load from the superstructure to the soil. Failure to do this, results in collapse of superstructure and inevitably causes heavy human and financial losses. Foundation failure often occurs when the soil bearing capacity is miscalculated. Therefore, estimation of bearing capacity of foundations is one of the most important concerns in geotechnical engineering. The common problem in estimation of soil bearing capacity is the complex behavior of the soil. The complicated behavior of soil is mostly because of the existence of distinct layers in the soil profile. Each soil layer has its own physical, chemical and mechanical characteristics. The existence of different soil layers beneath the structure might have originated from natural or artificial phenomenon.

Figure 1.1 Shanghai building collapse (Subramanian, 2016)

The age of earth is estimated to be about 4.5 billion years, and during this period several cycles of mountains, creation and erosion have occurred. For instance, glaciers have carried rock and sediments to new places. Volcanic eruptions have created new layers of soil with different properties. Continuous cycles of rising and falling of seas level has resulted in transmission of sediments and mud forming new layers of soil. Furthermore, soil profiles are always affected by other common factors such as weathering, deposition and compaction. In the weathering process rocks break into small particles by the force provided by wind, rain, flood, freezing water, etc. By deposition, the soil particles move from one place to another by means of wind, run off streams and gravity. During deposition process, heavy particles settle first and then light particles on the top of them. However, depending on the power of the factor this might be inverse and consequently heavy pieces cover small fragments. By compaction, soil particles are squeezed by the weight of upper layers. Therefore, geological history of the site mostly dictates the arrangement of soil layers. Hence, soil layers often differ in size, chemical compounds, and strength properties.

It is often an engineering problem that heavy structures such as fuel oil tanks, storage facilities and heavy traffic highways need to be constructed over a very weak deposit. In this case, an alternative might be to cover the ground surface by a layer of strong soil or to replace a certain depth of the weak deposit by granular material. In this way, top layer distributes and alleviates the stresses which will be experienced by the weak deposit. Consequently, the two-layered soil system could tolerate heavier loads. On the other hand, it is often the case that building have large shallow foundation which influence can reach a great depth on the soil. In other words, for the case of a structure with large footing a thick layer of soil below the grand surface will be affected by the weight of superstructure. The reason is that failure surface is so large that encompasses a deeper area. Therefore, shear strength properties of all layers inside the failure surface have to be considered in estimation of the bearing capacity of the soil.

The very early theories to estimate the bearing capacity of soil were developed upon limit equilibrium and limit analysis methods for homogeneous soils (Terzaghi, 1943; Meyerhof, 1951; Hansen, 1970; Vesic, 1973). Further studies have attempted to develop theories to calculate the bearing capacity of layered soil from which a simple but somehow unsafe solution is the weighted average method (PN-B-03020:1981P, 1981; JTJ 250-98, 1998; Bowles, 1988). In this method, the layered soil is assumed to be homogeneous medium with soil properties which is obtained by making weighted average of all soil layers properties. Also, the projected area method was presented (Terzaghi & Peck, 1948; Yamaguchi, 1963; Myslivec & Kysela, 1978; Baglioni, Chow, & Endley , 1982; Kraft & Helfrich, 1983; Kenny & Andrawes, 1997) in which footing load is distributed through the upper layer up to the surface of the bottom layer. Also, the theory of punching shear method has been presented to predict the bearing capacity of layered soil (Meyerhof, 1974; Hanna, 1981). In this method, the failure surface is assumed to be punching column pushes the upper layer up to the bottom. In addition, the limit analysis method has been used by many scholars to study the bearing capacity of layered soil based on satisfying requirements of compatibility and equilibrium according to mechanics of solids (Florkiewicz, 1989; Michalowski & Shi, 1995; Merifield, Sloan, & Yu, 1999; Shiau, Lyamin, & Sloan , 2003; Huang & Qin, 2009). With the growth of technology and development of computers a new area of study was emerged the so called numerical analysis. Numerical analysis consists of several techniques such as finite element method (FEM), finite difference method (FDM), boundary element method (BEM) and discrete (or distinct) element method (DEM) (Jinga & Hudsona, 2002). In fact, numerical analysis is a powerful method to simulate complicated engineering problems such as complex loading system, geometry, and material behavior that analytical solutions are almost unable to solve. For the problem of layered soil, numerical analysis has been widely used to predict the soil bearing capacity (Hanna, 1987; Szypcio & Dołżyk, 2006; Zheng, Zhou, Cheng, Liu, & Zheng, 2016; Lotfizadeh & Kamalian, 2016). Despite of different theories, the problem of footings resting on non-homogeneous soil is not fully understood and therefore it is of interest to many scholars.

1.2. Thesis objective

The objective of this thesis is to numerically model the problem of bearing capacity of strip footings on layered soil with the assumption of dense sand overlaying loose sand. To do this, the proper geometry of the soil model is investigated by finding a relationship between footing width and soil dimensions in order to avoid the effects of boundaries on the stress-strain calculations. In addition, load-settlement curves are developed to illustrate how an increase of upper layer thickness increases the bearing capacity and decreases soil settlement. A parametric study is conducted to predict the effects of upper layer thickness and shear strength of dense sand on the soil bearing capacity. In addition, the results from the numerical model are compared to the previous experimental, analytical and other numerical solutions to evaluate the acceptability of the performance of the developed numerical model.

2. LITERTURE REVIEW

2.1. General Overview

Geotechnical engineering is concerned with the issue of transferring loads from infrastructure to the soil layers such that the applied loads are less than the collapse load acting on the ground, and no damage is caused by the maximum settlement of the soil under the surcharge loads. Therefore, determining the bearing capacity of the foundations was one of the first concerns of many researchers for the past century.

In the literature, analytical methods are based on satisfying requirements of compatibility and equilibrium according to mechanics of solids. The analytical analysis contains three methods of limit equilibrium method, limit analysis method, and slip line method. All the analytical solutions are based on satisfying the two requirements of compatibility and equilibrium either simultaneously or only one of them.

The numerical analysis is totally different approach and is based on solving governing partial differential equations (PDEs) by using numerical approximations. Because it is almost either impossible or impractical to solve partial equations by means of analytical solutions, numerical analysis such as finite element and finite difference methods were developed to overcome the problem and to provide an approximate answer to the PDEs.

The early solutions to the problem of bearing capacity were presented first using analytical analysis and then by emergence of numerical approach, using finite difference and finite element methods. While, in the recent studies both numerical and analytical techniques are used frequently to predict the soil bearing capacity of layered soil.

2.2. Historical Review

Prandtl (1921) was among the pioneers in studying the bearing capacity of soil by implementation of slip line method. He considered a rigid-perfectly plastic semi-infinite two-dimensional medium which is loaded by a strip punch. He assumed soil material is homogeneous, isotropic, weightless, cohesive-frictional and behaves as a rigid body. In his study, the volume change is zero and the deformation is plastic. The failure criterion for the plastic state of soil was assumed to be Mohr-Coulomb.

He wrote two equilibrium equations in the shape of differential on the plane deformation and used the Mohr-Coulomb criterion to derive a couple of hyperbolic-type differential equations. The hyperbolic-type differential equations has been solved by using slip-line method and applying boundary conditions. The boundary condition was defined as zero stress on the surface of ground exclude under strip punch which the amount of stress was unknown. Finally, an equation proposed by Prandtl for the ultimate pressure.

Thereafter, Reissner (1924) continued the study of bearing capacity based on Prandtl theory but with two differences; soil material was cohesionless and a uniform load was applied to the ground surface surrounding the strip footing. Therefore, the hyperbolic-type differential equations was solved by the new boundary condition which was uniform pressure at the surface of ground except under strip footing and resulted into a new equation for ultimate pressure.

Terzaghi (1943) presented an approximate solution by applying global equilibrium on rigid soil blocks and also taking to account the weight of soil to investigate ultimate bearing capacity of soil. He stated the bearing capacity of soil as summation of three terms to include the effects of soil weight, surcharge load, and soil cohesion separately in each term of the equation.

The theory of bearing capacity was extended for non-homogeneous soil primary by Terzaghi & Peck (1948). They assumed that footing load distributes to the larger area on the surface of bottom layer by the slope angle of $tan^{-1}(0.5)$.

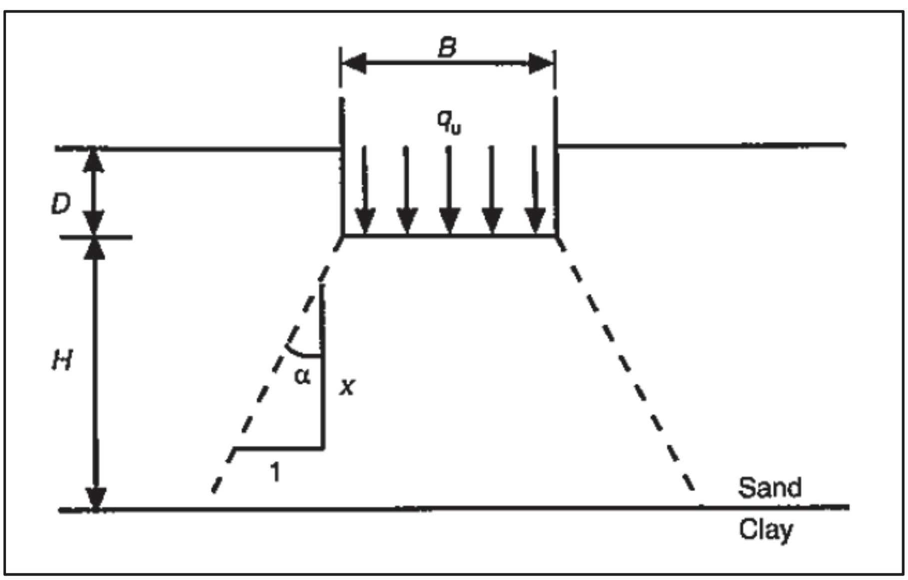

Figure 2.1 Load spreading analysis (Terzaghi & Peck, 1948)

Button (1953) proposed a solution based on a limit equilibrium for layered clay soil. He assumed the failure surface is cylindrical and both layers have almost the same degree of consolidation. The best results were obtained for soil cohesion ratio between 0.6 -1.3.

Reddy and Sirivinasan (1967) investigated the effects of non-homogeneity and anisotropy of layered clay soil with respect to the soil shear strength. The failure surface was assumed to be cylindrical. The results showed almost 7% deviation from the bearing capacity proposed by Prandtl and the deviation rises by increase of degree of non-homogeneity.

Brown and Meyerhof (1969) conducted experimental tests on two layered clay soil and proposed empirical equations based on the obtained results. They assumed the potential failure occurs as punching shape surface which starts to develop from the edge of footing extends toward the second layer.

Meyerhof (1974) proposed ultimate bearing capacity for two layered soil; loose sand on stiff clay and dense sand on soft clay. The suggested mechanism for loose sand on a stiff clay was similar to Mandel and Salencon (1972) in which it is assumed that the loose sand fails laterally by the compression caused between the footing and the stiff clay. In the case of dense sand overlying on soft clay the suggested mechanism was formation of failure surface in the shape of truncated pyramid that pushes the upper layer up to the bottom layer. He conducted laboratory tests on strip and circular footing to model the problem and compare the results with theory. The results of theory were on good agreement with model tests and also field observations.

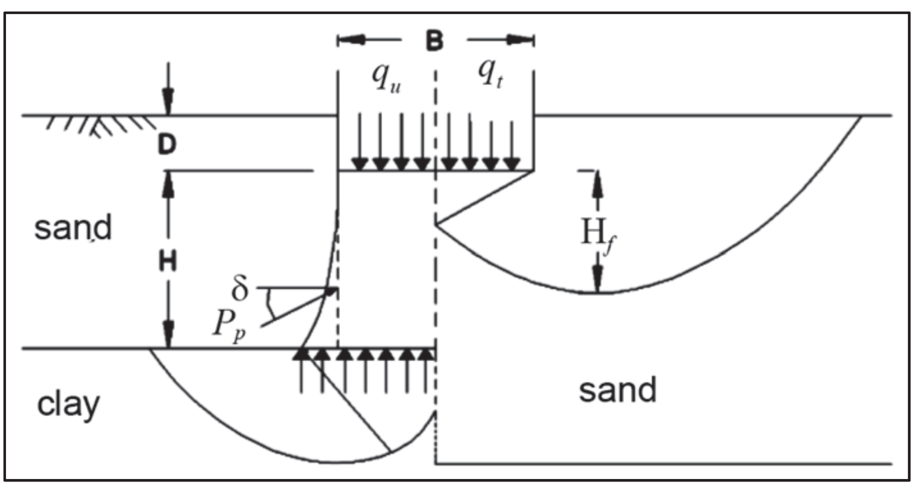

Figure 2.2. Failure mechanism of dense sand over soft clay (Meyerhof, 1974)

Purushothamaraj et al. (1974) presented a formulated method based on theorem of Drucker and Prager. They studied variation of soil cohesion, friction and unit weight. The failure surface was alike Prandtl-Terzaghi but with difference in the angle of wedge. They represented design charts for various soil cohesion and the same friction angle and unit weight. They concluded that the value of N_c decreases when $H/_B$ ratio increases for $^{c_2}/_{c_1}$ more than one. This trend is vice versa when $H/_B$ increases for the case of $^{c_2}/_{c_1}$ less than one. The failure wedge angles depends on ratios of $H/_B$ and $^{c_2}/_{c_1}$. The approximate estimation of layered soil as homogenous medium might result in 25.0 to -26.5% errors.

Meyerhof and Hanna (1978) extended the theory of Meyerhof for foundation under inclined load for footings on strong layer on weak soil and vice versa. The effects of inclined load and eccentric loads were investigated on the ultimate bearing capacity of soil. The failure mechanism for weak soil overlaying on strong soil was assumed to be as laterally failing by the act of squeezing. For the case of strong soil on top of weak soil the mechanism assumed to be truncated pyramid pushes the soil beneath. The laboratory tests were conducted to validate the theory for both strip and circular footings. The results of bearing capacity obtained from the theory were comparable to those acquired from tests model.

Hanna and Meyerhof (1979) considered a three-layer granular soil which two strong upper layer overlying a weak layer. The footing was subjected to vertical load and the results compared well to the laboratory tests for both strip and circular footing. The failure mechanisms was assumed to be the same as the previous study. The results indicated that the ultimate bearing capacity increases with the thickness of two upper layers. The results were presented in terms of design charts for engineers.

Hanna (1981) investigated the bearing capacity of strip and circular footings for the case of dense sand overlaying both loose and compacted sand. He developed the theory presented by Meyerhof (1974) by a new method to evaluate the average mobilized shear resistance angle (δ_{avg}) of upper soil. To do this, he considered different distribution (linear, circular, parabolic, etc.) for the angle of shear resistance, δ along the punching column. Then, the passive earth pressure calculated using the Equation 2.1:

$$P_p = \int_0^H \frac{\gamma_1 K_p}{\cos \delta_z}(Z + D)dz \tag{2.1}$$

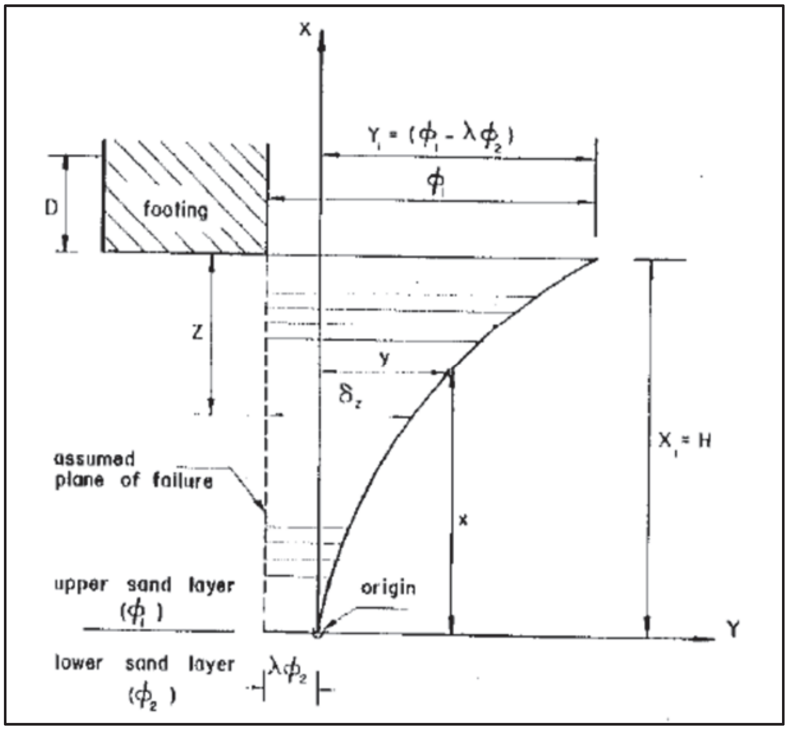

Figure 2.3. Angle of shear resistance distribution (Hanna, 1981)

On the other hand, shear resistance angle of δ is assumed and corresponding value for K_p estimated using tables presented by Caquot and Kerisel (1948). Alternative, P_p was calculated and compared to the previous obtained value. A trial and error approach was implemented until the two obtained values for P_p become equal. The δ in which this equivalency occurred was named average mobilized shear resistance angle, δ_{Avg}.

$$P_p = 0.5\,\gamma_1 H^2 \left(1 + \frac{2D}{H}\right)\frac{K_P}{\cos \delta} \tag{2.2}$$

To validate the value of δ_{Avg} from a theoretical approach, model tests carried out for both circular and strip footings rest on a two layered-soil system. The ultimate bearing capacity was investigated from test results and the following equation. The right side of the equation was found by assuming a value for δ_{Avg} and P_p from tables of Caquot and Kerisel (1948). Thereafter, a comparison was conducted between the experimental and theoretical approaches and the results were in good agreement.

Hanna (1982) investigated the bearing capacity of loose and compact sand overlaying dense sand. The study has been carried out for both circular and strip footing. He extended the theory of bearing capacity of homogenous soil for two layered sand soil. He represented modified bearing capacity factors as a function of the relative strength of top and bottom layer and the thickness of top layer. Also, he presented the design charts.

Hardy and Townsend (1982) conducted centrifuge model tests in order to estimate the bearing capacity of circular footing on two layered soil. The upper layer was considered to be sand and the lower layer was clay. The footing width was assumed to change between rang of 0.6 to 1.5 m. The results ware compared to the Meyerhof's bearing capacity theory for layered ground. The results were in good agreement with the Meyerhof's theory and the deviation was almost 13%.

Griffiths (1982) investigated the ultimate bearing capacity of layered soil using finite element method and assuming the elastic-plastic behavior of the soil. He presented the solution for plane strain condition. In the study, soil material obeys the Mohr Coulomb criterion. Also, the non-associated flow rule (neglecting plastic volume changes) is used since the objective of the study was to investigate the formation of failure surface and ultimate loads instead of finding the settlement of footing. The ultimate bearing load of a two layered soil was calculated in which the upper layer was either dense sand or clay and the lower layer was always clay. Also, bearing capacity of a layer of clay was investigated assuming the shear strength of clay increases linearly by depth. Two type of smooth and rough footings were simulated. The loads applied to the soil by means of vertical prescribed displacements. For the case of smooth footings, the nodes which prescribed displacement applied to was allowed to move horizontally while for rough footings the horizontal displacement was set to be zero. The bearing load was acquired by calculating the average vertical stresses of points under the nodes that prescribed displacements were defined for them. An inclined load applied to the model by means of defining vertical and horizontal forces to the footing.

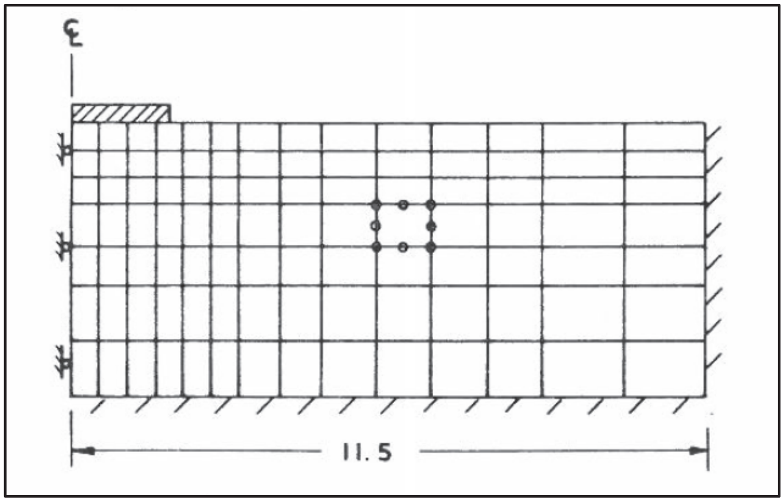

Figure 2.4. Mesh generation (Griffiths, 1982)

Griffiths simulated homogenous soil numerically and compared the three bearing capacity factors (N_c , N_γ and N_q) to the analytical solution. It was observed that by increase of friction angle the computer run time increases for the all three factors. In addition, the result of N_γ was unreasonable for the friction angle more than 35 degrees and the convergence was slow. Also, it was almost impossible to find a solution for granular soil while the applied pressure on the footing was considered to be uniform. The reason was occurrence of failure at the edge of footing because of insufficient confining pressure at the edge. In the next step, he estimated the bearing capacity of non-homogenous soils.

Griffiths found that the failure surface for weak layer of clay over strong clay layer has the tendency not to enter the lower layer and instead the soil underneath the footing tends to move laterally. For the case of stiff clay over weak clay, the displacement vectors are vertically in the top layer to have the shortest length and a large failure surface forms in the lower soil layer. For the case of dense sand over clay, friction angle assumed 40 degrees and numerical convergence was slowly. Also, he applied inclined load to the layered soil which there was a considerable different between the results of FEM analysis and other theoretical solutions. He studied the problem of one layer clay with linearly increase of strength in depth. He found the numerical calculations to obtain convergence became difficult for strength line with higher gradient and the worst condition occurred for the surface cohesion equal to zero.

Hanna (1987) utilized finite element model to estimate the bearing capacity of a strip footing on homogenous and two-layered sand soil and compared it experimental investigations and theoretical solutions. In the numerical method, he used a nonlinear relationship between stress and strain. The model was in good agreement with experimental data and theoretical solutions.

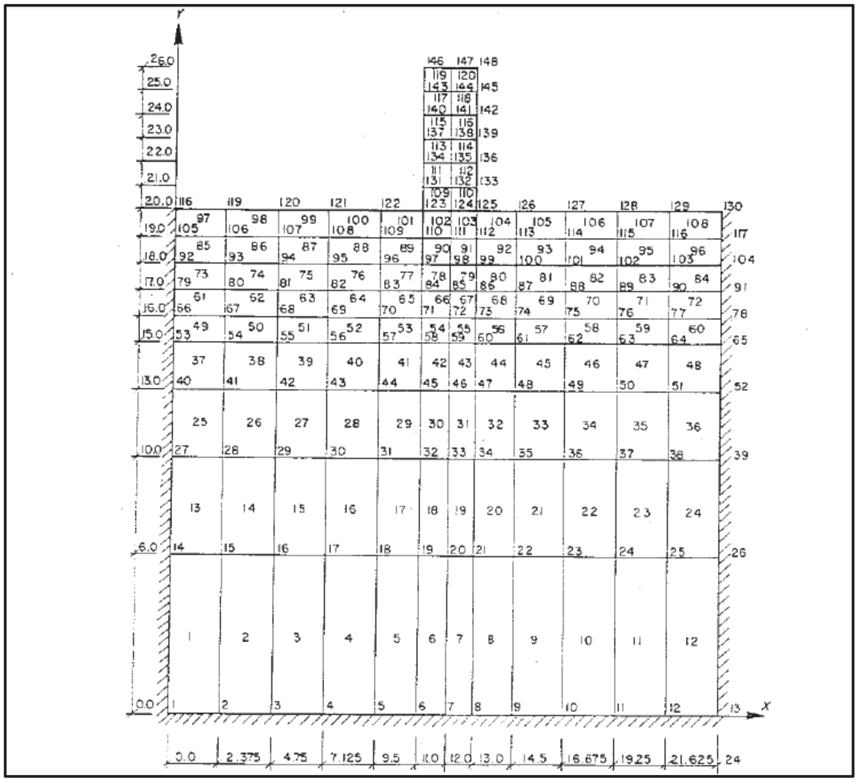

Figure 2.5 Finite element mesh (Hanna, 1987)

Das (1988) has conducted experimental study on bearing capacity of sand overlaying soft clay with and without the presence of geotextile at the interface of two layers. He stated that the bearing capacity without the presence of geotextile increases by increase of H/B ratio up to a certain value and thereafter remains constant. He claimed that the laboratory model test results are in good agreement with Meyerhof and Hanna (1978)

Bowles (1988) suggested an approximated method to estimate bearing capacity of layered soil by using conventional bearing capacity equations. He defined the effective shear depth as $0.5\ B\ tan(45 + {}^{\varphi}/_{2})$ in which layers of soil affect the bearing capacity. Thereafter, he stated the soil properties for the region can obtain by making weighted average method, c_{av}, φ_{av}. The bearing capacity is calculated for homogeneous soil with strength properties c_{av} and φ_{av} and using traditional equations.

Florkiewicz (1989) presented bearing capacity using limit analysis method and kinematic approach. He assumed a two-layered plane strain problem to calculate bearing capacity of strip footing. The analysis was conducted by assuming a non-horizontal layer in a half space medium. The model is able to analysis layered soil while only the case of determining the bearing capacity of two-layered soil was studied. The results compared with experimental data and showed well agreement with them.

Michalowski and Shi (1995) calculated bearing capacity of two-layered soil; sand underlying by clay. They used upper bound limit analysis to estimate bearing capacity of soil. The results showed that depth of collapse failure is highly depended on the shear strength of clay layer. The results presented as design charts to obtain ultimate pressure soil that can be tolerated before failure occurs, unlike traditional charts in which it needs to determine bearing capacity factors.

Burd and Frydman (1997) carried out numerical analysis to investigate the failure mechanisms and load spread angle. The numerical analysis consisted of finite element method using OXFEM and finite difference method using FLAC. The problem was a plane-strain model of a rigid footing on a two-layered soil system. It was assumed that footing is located on a sand soil overlaying clay layer. The purpose of study was to perform a parametric study and predict the mechanisms of failure. The soil model was linear elastic-perfectly plastic. The soil condition for clay was assumed to be undrained and for sand to be drained. The ratio of footing width to the upper sand layer was considered to be less than 1.5. The results of the parametric study from FEM and FDM were in excellent agreement. The results were used to predict the failure mechanisms of the soil, show the effective of sand layer in distributing the applied load, and to develop the design charts. It was observed that by increasing sand friction angle, the angle of failure surface to the vertical (α) increases. Also, α decreases by increase of shear strength ratio of two layers. Based on this finding, the authors stated that for sand layer over weak layer of clay, the sand strength properties play an important role on reduction of α. In addition, the influence of B/H (the ratio of footing width to the thickness of sand layer) was negligible on α. Also, it was observed that non-dimensional group ($c/\gamma D$) has essential impact on mechanisms of failure.

Kenny and Andrawes (1997) conducted model tests in laboratory to study the inclination of the angle of punching plane to vertical. They also investigated the stress-settlement relationship of each layer alone and the two-layered soil. They assumed two layers of soil which the top layer was sand and the bottom layer was clay. The results showed that the angle of failure surface to the vertical varies between $tan^{-1}(0.1)$ degrees to $tan^{-1}(0.36)$ degrees. The theory of inclination angle was comprehensively developed later by Okamura et al (1998).

Okamura et al. (1997) carried out a series of centrifuge model tests to evaluate the bearing capacity of dense sand overlaying soft clay and to investigate the shape of failure surface. The effects of footing width, shape and embedment depth as well as shear strength of underlying clay have been studied. They found that bearing capacity increases by thickness of overlying layer, shear strength of underlying layer and surcharge load at the footing base. They found that failure mechanism occurs as a truncated cone of sand soil blocked between footing and the lower layer and is pushed to the underlying soft clay. The punched plane angle to the vertical increase by ratio of thickness to footing width (H/B), surcharge load and the decrease of lower layer shear strength.

Okamura et al. (1998) studied soil bearing capacity of dense sand layer underlain by soft clay deposit. They investigated the effects of shape of sand block and forces acting on the surface of the block on the bearing capacity. To do this, different methods of calculating soil bearing capacity were considered to study the effect of different factors and results have compared to the centrifuge tests (Okamura et al., 1997). They concluded that the projected area method overestimates the bearing capacity of footing without embedment especially for circular footing. Also, as projected area method neglect the shear resistance of sand bock, it underestimate the bearing capacity of a footing with embedment. They stated that Meyerhof (1974) and Hanna and Meyerhof (1980) theory underestimate the bearing capacity of footing without embedment. In addition, the theory overestimate the bearing capacity for footing with embedment. This shows that the horizontal stress on the sand block is miscalculated in this theory. The horizontal stress in the case of no embedment should be more and for footing with embedment it should be less than the value is assumed in the theory. They presented a new limit equilibrium in which the angle of sand block is a function of H/B ratio, and embedment depth and shear strength of clay layer. Also, the bearing capacity presented in design charts.

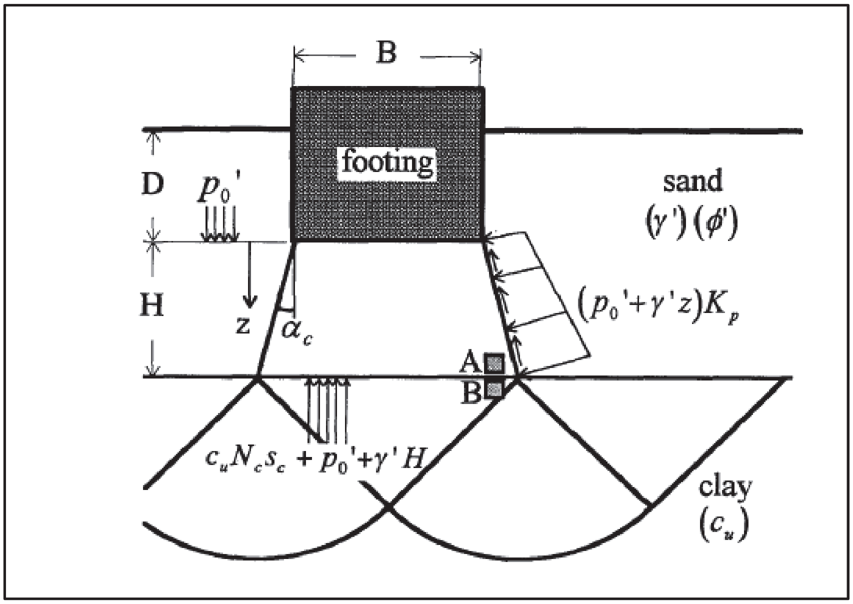

Figure 2.6 Failure mechanism by (Okamura, Takemura, & Kimura, 1998)

Merifield et al. (1999) used numerical approach in conjunction to upper and lower boundary method to model two layers of clay. The soil failure criterion was assumed to be Tresca yield surface. The obtained bearing capacity for different geometries showed a deviation up to 20% from limit analysis, empirical and semi-empirical methods.

Shiau et al. (2003) used upper and lower bound techniques for the ultimate bearing capacity of a strip footing resting on a sand layer over clay. The upper and lower bound were calculated by using finite element formulations of the classical limit analysis theorems. The results were compared with other published results and the difference was within ±10% or better.

Ghazavi and Eghbali (2008) presented a simplified limit equilibrium method to investigate the bearing capacity of strip footing on a two layered granular soil for dense sand on loose sand and vice versa. To do this, they used lateral earth pressure theory which is so called Coulomb and then associated with simple slip surface to evaluate the total pressure acts on a virtual retaining wall passing from the edge of footing. To validate the model a numerical finite element approach is followed by using a commercial well known software PLAXIS. The results of numerical and analytical approaches were in good agreement in the case both layers have near shear strength. Also, the results showed that there is a specific H/B value in which the characteristics of upper layer dictates the bearing capacity of footing. The obtained value for the ratio of the upper layer thickness to the width of footing is almost 2 which depends on the ratio of soil layer shear strength, thickness of upper layer and width of footing.

Zhu (2004), a detailed parametric study on bearing capacity of a plane strain two layered-clay soil conducted using finite element approach and commercial program ABAQUS. The strip footing was assumed to be a rough without embedment. The width of footing (B) was assumed 2 m and the length and width of the axisymmetric model were 12.5B and 7.5B, respectively. The parametric studies were conducted for H/B between 0.125 and 2 and c_1/c_2 ratio between 0.2 and 5 for nine different values. For stiff clay over soft clay the critical thickness increases by increase of c_1/c_2 and reaches 2B in the shear strength ratio of 5. However, for soft clay over stiff clay layer the critical thickness for all values of c_1/c_2 is constant and equal to 0.75B. The failure surface become large as the thickness of upper layer increases to reach the critical thickness and both layers are involved in the mechanism. When the upper layer was soft clay and the bottom layer was stiff clay, the failure surface did not change considerable and was limited to the upper layer. The results lied between lower and upper bound solutions of limit analysis methods and showed good agreement with available analytical solutions.

Abu Farah (2004) developed the theory of punching shear failure by presenting a new slip line. The soil conditions were loose sand underlying dense sand and weak clay underlying dense sand. The presented failure surface was two inclined planes passing through the edge of footing and making a Prandtl-type failure in the weak deposit. The generated bearing capacity equation is a function of shear strength of two layers, the ratio of H/B, and the angle of failure plane to the vertical. For the case of dense sand overlying loose sand the results showed a good agreement with the previous experimental data and the error increased by 17% in the case of higher H/B value and particularly in H/B equal to 4.5 and 5. The deviation theoretical and experimental solutions was between -7% to -20% for small ratios of H/B and the differences in results increased in higher ratios of H/B.

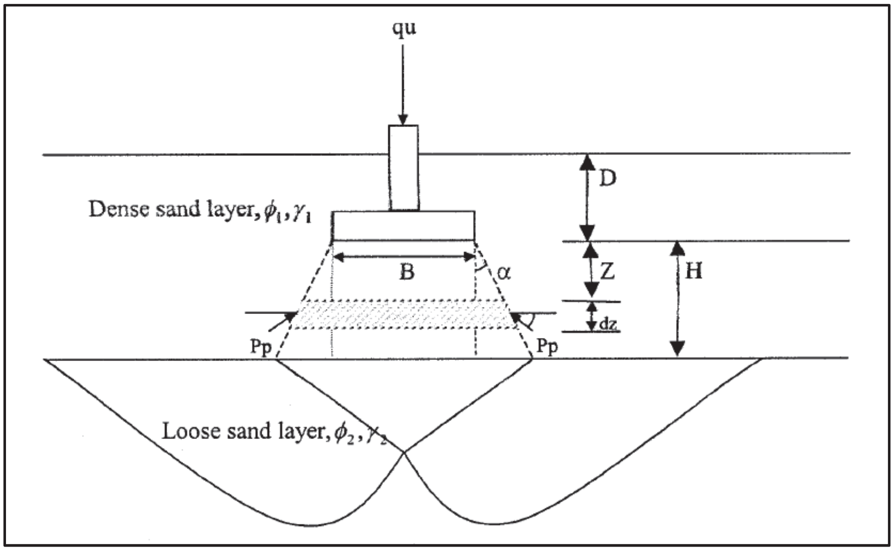

Figure 2.7 Punching shear mode (Abou Farah, 2004)

Carter (2005) carried out an experimental study on the bearing capacity of dense sand over loose sand. He has conducted seven tests using centrifuge setup at the University of New Brunswick. The bearing capacity from experiment has been compared to the theoretical values. He concluded that the punching failure occurs at the H/B ratio less than 1.5. Also, he stated that Schmertmann's method predicts settlement conservatively when the H/B ratio is less than 2. Nevertheless, he declared that more test need to be conduct to conform this difference.

Szypcio & Dołżyk (2006) studied different methods of calculation of soil bearing capacity of two layered soil and compared to the results from PLAXIS Version 8. The top soil is dense sand or stiff clay and the subsoil is loose sand or weak clay. For all cases the embedment depth of footing is 0.5B. The numerical model was conducted for both strip and square footing. The results were in good agreement with Polish Standards (PN-B-03020:1981P, 1981) for subsoil with weak cohesionless material.

Huang and Qin (2009) utilized multi-rigid-block upper-bound method and some modification of Florkiewicz (1989) to calculate the soil bearing capacity. They stated that the bearing capacity of sand layer over soft clay shows better estimation than Michalowski and Shi (1995). In addition, they claimed that the theory presented by Hanna and Meyerhof (1980) is unsafe in some cases. The calculation of bearing capacity of two layered clay improved considerably compare to Chen (1975). Also, in some cases the results are better than finite element limit analysis of Merifield et al. (1999).

Pauls (2009), studied the problem of bearing capacity of strip footing on tow layered soil by using commercial code PLAXIS version 8.6. The upper and lower soil layers were dense and loose sand, respectively. A two-dimensional model was used. The model dimensions was assumed to be 42B in the x-direction and 36B in Y-direction. Two vertical interfaces were defined at the edge of footing inside the upper layer to investigate the shear resistance of punching column at the failure condition. The results showed that the bearing capacity increases due to increase of H/B to a certain value. It is found that the ratio of δ/φ_1 decreases along the punching column downward to the bottom of dense sand layer and reach value of zero at the interface of two layers. In addition, the ratio of δ/φ_1depended on H/B ratio, the relative shear strength of two layers and shear strength of each layer individually.

Ziaie Moayed et al. (2012) investigated the bearing capacity of ring footings on a two-layered soil by using a finite element method. The upper and lower layer were assumed to be soft clay and cohesionless soil respectively. The soil behavior was assumed to be elastic plastic model was considered to be Mohr-Coulomb as yield criterion. The effect of clay thickness and the ratio of internal radius of ring to the external radius of it on the bearing capacity was investigated. The results represented that by increasing of the radius ratio and thickness of clay layer, the bearing capacity decreases. In addition it is found that the displacement vectors by increase of clay layer thickness almost do not enter into sand layer and by decreasing the clay layer the displacement vectors are restricted to the upper load.

Dalili Shoaei et al. (2012) reviewed three most common methods to predict the bearing capacity of two layered soils. They specifically focused on sand over clay profile. The three methods consist of classical method, finite element method and artificial neural network. They studied different failure mechanism adopted by the researchers and discussed the ability of each method to predict the soil bearing capacity. They stated that too many works have been carried out using classical method, while a few studies used numerical method and artificial neural network (ANN). They declared that the problem of multi-layer soil needs more study especially in the field of finite element and ANN.

Verma et al. (2013) declared that the most accurate way to estimate the bearing capacity and settlement of shallow foundations is to study the load-settlement characteristics of soil under the footing load. To study the problem more accurately, the conducted plate load tests. The top layer was assumed to be fine gravel and the bottom layer was sand. The load was applied to the soil surface by means of a square shape steel plate. In the study, the effects of upper layer thickness, load-settlement characteristics have been studied and an equation was presented to predict the bearing capacity of two layered soil based upon plate load test data. They concluded that by increasing the size of squared footing, bearing capacity increases and settlement decreases. By increase of the thickness of upper layer, the bearing capacity increases and settlement decreases. The bearing capacity remains constant for H/B ratio more than 2.

Lotfizadeh and Kamalian, (2016) used stress characteristic lines method to estimate bearing capacity of loose sand overlaying dense sand. They proposed an algorithm to predict the soil bearing capacity and validated it to the other numerical and experimental examples. The graphs are presented for design purpose. It is observed that for the friction angle of upper and lower layer 30 and 35 degree, respectively, the H/B ratio is almost 0.76.

Zheng et al. (2016) numerically studied the problem of strip footing on sand underlined by clay. They used finite difference method to investigate the effects of footing roughness, dilation angle, surcharge on the failure mechanism and predicting the soil bearing capacity. The numerical results compared to the classical theories like weighted average method, projected area method and punching shear method. They showed that the footing roughness decreases with increase of sand friction angle. When dilation angle is small, it has a significant influence on the bearing capacity. The effect of surcharge is remarkable for soft clay. The weighted average method overestimates the baring capacity while projected area method underestimate the bearing capacity of sand. The punching shear method, provide reasonable prediction for small value of sand depth and soft clay with low strength. Nevertheless, the punching shear method might provide better results if it would take into account the strength of clay in mobilization of shear resistance.

2.3. Discussions

The problem of bearing capacity of two-layered soil has been of interest since 1940's. The studies attempt to understand the transmission of footing load from upper layer to the bottom layer and predict the bearing capacity. The different approaches to calculate the bearing capacity consist of; limit equilibrium, limit analysis and numerical methods as well as experimental studies. The current studies based upon the limit analysis methods, numerical methods and experimental data are limited to the cases which have been investigated through the studies and they do not present a general equation to calculate the bearing capacity. Whereas, the limit equilibrium analysis provides bearing capacity equations for two layered soils.

The common limit equilibrium analysis methods include weighted average method, projected area method (PAM) and punching shear method. The weighted average method assumes that layered soil is a homogenous medium in which the soil properties are estimated by making weighted average of all soil layers properties. Therefore, this method is an approximation rather than an accurate solution to the problem of bearing capacity of layered soil. The reason is that in this method the mechanism of failure as well as the presence of two soil layers are neglected. The layered soil is assumed to be homogenous with average material properties regardless of the effect of top layer thickness on failure mechanism and prediction of the bearing capacity.

In the projected area method, the load distributes with a specific angle to the bottom layer. The angle of distribution of load varies in different studies. Yamaguchi (1963) assumed the angle α is 30°; Kraft & Helfrich (1983), $tan^{-1}(1/2)$ and Myslivec & Kysela (1978), 45°. The most significant problem with this method is that the load distribution angle, α, is assumed to be constant along the thickness of upper layer. While in reality α changes along the thickness of upper layer. Also, the angle of load distribution angle α should be a function of both top and bottom layer shear strength. Nevertheless, there are numerous studies which are trying to figure out the relationship between α, thickness of upper layer and shear strength of top and bottom layer since the issue has not been fully understood up to the present time.

In the punching shear method, it is assumed that the failure surface is a vertical punching column and the shear resistance on the slip surface is not fully mobilized. The theory presented by Meyerhof (1974) and later developed by Hanna (1981). The argument about this method is how to accurately determine the amount of mobilization of shear resistance on the failure surface. Also, the failure shape is assumed to be a vertical punching column, while the actual failure shape is curved planes.

3. NUMERICAL MODEL

3.1. General

In this chapter, a numerical model is generated by using programing code PLAXIS version 8.6. The primary model simulates a strip footing rested on a homogeneous soil. The model is numerically validated to avoid boundary conditions and to study the effects of mesh element size on the stress-strain calculations. To do this, a two-dimensional plane strain model is utilized to simulate the movement of the soil particles. Also, a half of the problem geometry is modeled to reduce calculation time. The boundary conditions for the bottom of the model are fixed in both horizontal and vertical directions and for two sides are fixed in horizontal direction. The embedment depth of the soil is assumed to be zero. Since the soil model is symmetrical a half of the whole soil medium is simulated. It is assumed that the soil constitutive model is Mohr-Coulomb. Afterwards, the bearing capacity of homogenous soil is compared to the conventional bearing capacity equations presented by Meyerhof (1963) and Terzaghi (1943). The comparison is conducted for three different types of homogeneous dense, loose and medium sand soil.

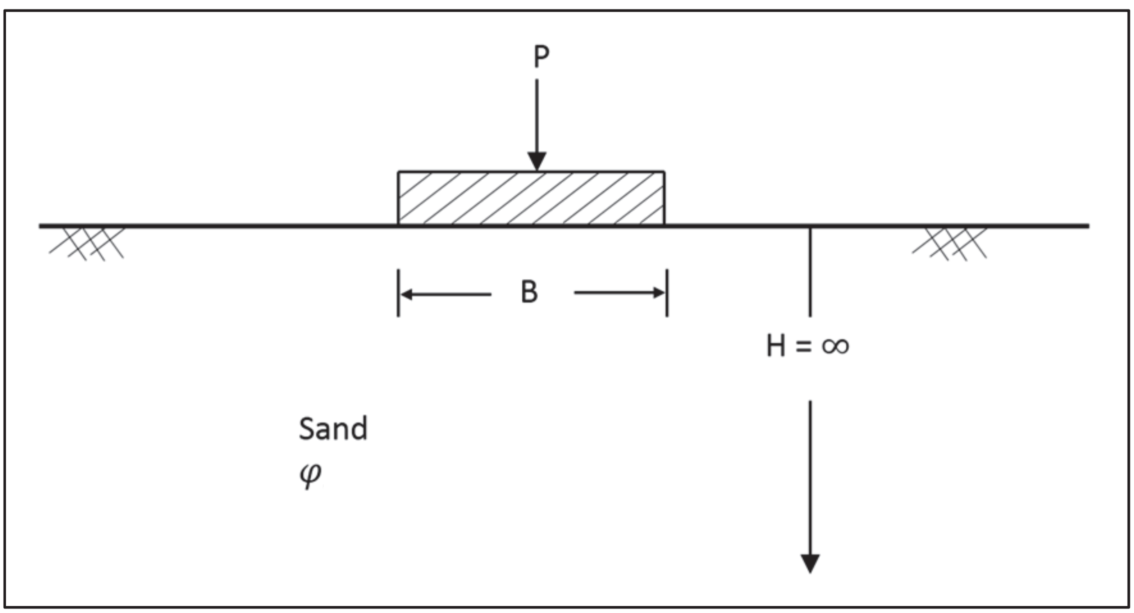

Figure 3.1 Homogeneous soil

In order to simulate the movement of soil underneath the footing, the model is assumed to be plane strain. The reason is that soil particles beneath the strip footing cannot move along the footing length. Therefore, to restrict the movement of soil particles, the soil strain along the footing length has to be equal to zero which is implemented by using a two-dimensional plane strain model. In order to reduce memory usage of computer and calculation time, the soil model is symmetrical and accordingly a half of the whole soil medium is simulated.

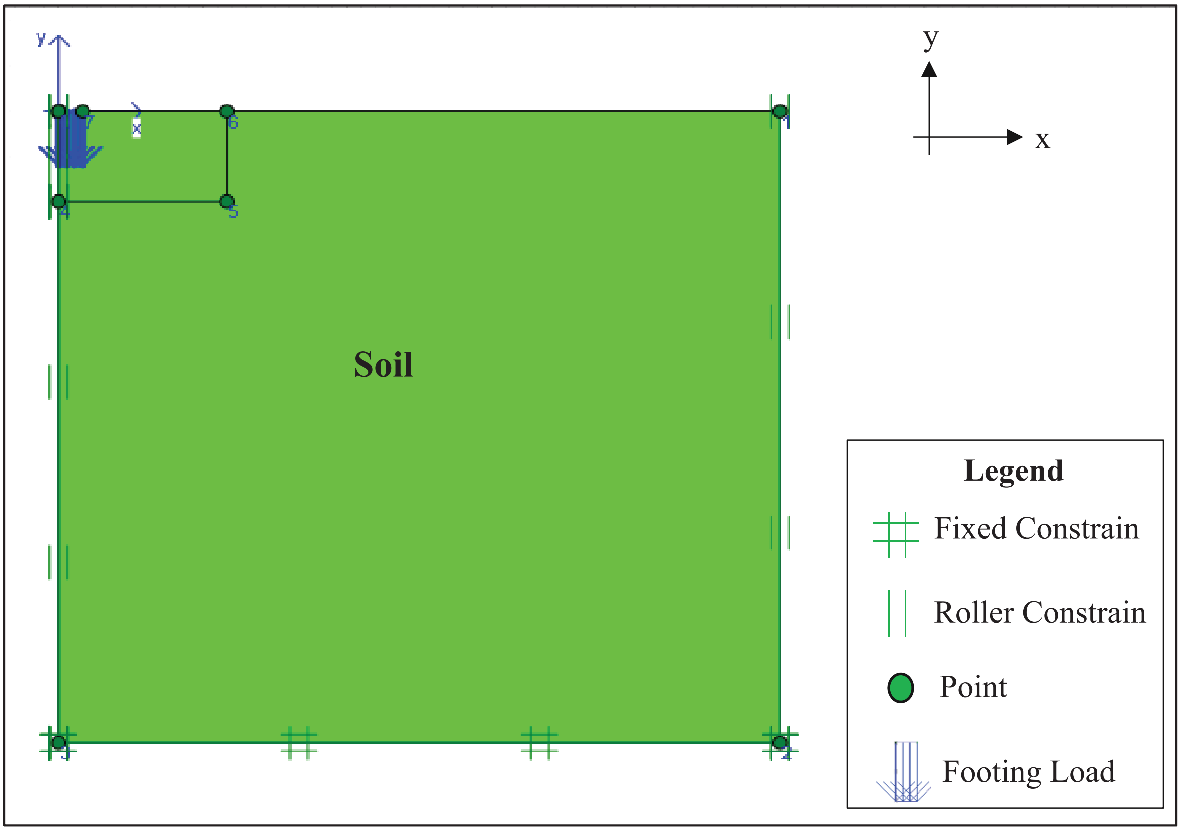

Figure 3.2 Two-dimensional axisymmetric soil model

In the finite element analysis, the elements are assumed to be 15-node triangular elements. This element type is often recommended for the case of symmetric models since it provides more accurate stress results compare to 6-node triangular elements. However, 15-node triangular elements consume more memory and needs more time for calculation. Despite this fact, since the numerical model is symmetrical, it is decided to choose 15-node triangular elements.

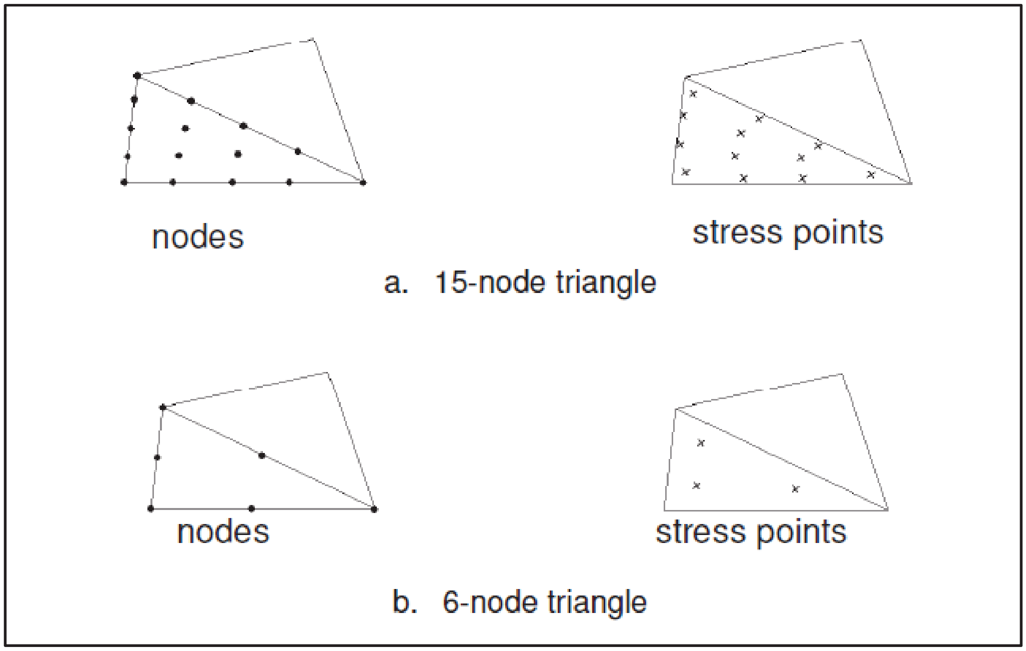

Figure 3.3 Element type: 15-node triangle and 6-node triangle

The model is fixed at the bottom in both x and y directions. The boundary conditions for two sides of the model is fixed only in horizontal directions, x. For the bottom boundary, Pinned constrains are applied to the model to prevent soil movement in both x and y directions. For two sides of the soil body, roller constrains type is used to let the soil elements to move only vertically. The vertical movement is allowed to the elements in the edges of the model since soil elements have to settle during the loading process.

The footing width is assumed to be 1 m. The footing load is applied to the soil surface by defining prescribed displacements. The prescribed displacements could be considered as a kind of boundary condition. The vertical displacement is applied to the soil elements underneath the footing. During the calculation process, the displacement of the soil elements increases gradually from zero to the vertical displacement value which is defined by the user. PLAXIS uses an incremental approach to apply the defined displacement to the elements. In each calculation step, a percent of the total displacement is applied to the elements, and this trend continues until 100 percentage of the total displacement is applied. All the soil elements subjected to the prescribed displacement boundary condition descend uniformly and simultaneously which simulates rigid footing condition. Also, the footing is assumed to be rough and as a result the soil elements underneath footing are fixed in horizontal direction.

In the numerical analysis, soil constitutive model simulates the nonlinear behavior of soil. In this study, Mohr-Coulomb soil model is used to simulate the elasto-perfectly plastic behavior of soil. The Mohr-Coulomb criteria is widely utilized in geotechnical engineering problems because it needs a few input parameters to simulate the behavior of soil. The required parameters for Mohr-Coulomb model include elastic modulus of soil E, Poisson ratio ν, cohesion c, friction angle ϕ, and dilatancy angle ψ.

3.2. Model Geometry

The geometry of a problem often plays an important role to achieve accurate results in each numerical analysis. In general, the soil body has to be not only large enough to contribute to full formation of soil failure surface but also to alleviate the effects of boundaries in calculating stresses and strains. In other words, the boundaries should be in a sufficient distance from the footing in order to prevent any effects caused by them. If the vertical boundaries in two sides of the model be close to the footing, they would keep the soil from settlement. The prevention of soil from settlement would result in an extra resistance of soil under vertical loads which eventually leads to overestimation of bearing capacity of soil. On the other hand, the horizontal boundary on the bottom of the model has to be distant from the footing. It can be said that the bottom boundary would have the same behavior as bedrock. The more the bottom boundary is near to the footing, the more vertical support it provides. Hence, determination of the length and depth of the soil model with regard to the footing width has a remarkable impact on the accuracy of the bearing capacity calculation.

The geometry of the problem of bearing capacity of strip footing can be numerically modeled using plane-strain model. In order to reduce the calculation time, half of the geometry is modeled. To study the boundary conditions effect, the length and depth of the model is increased gradually to find the optimum model size. On the one hand, by increasing the soil model length and depth the boundaries effect gradually decreases and finally disappears. On the other hand, the larger the soil geometry, the more computer attempts and the more calculation time is needed. Therefore, the optimum model size is the size in which both the boundaries effects are avoided and the computer's memory usage and calculation time is less.

In order to determine the appropriate model depth (D), the soil length (L/2) is assumed to be equal to 10B in the symmetric model and the soil depth varies between 6B to 16B.

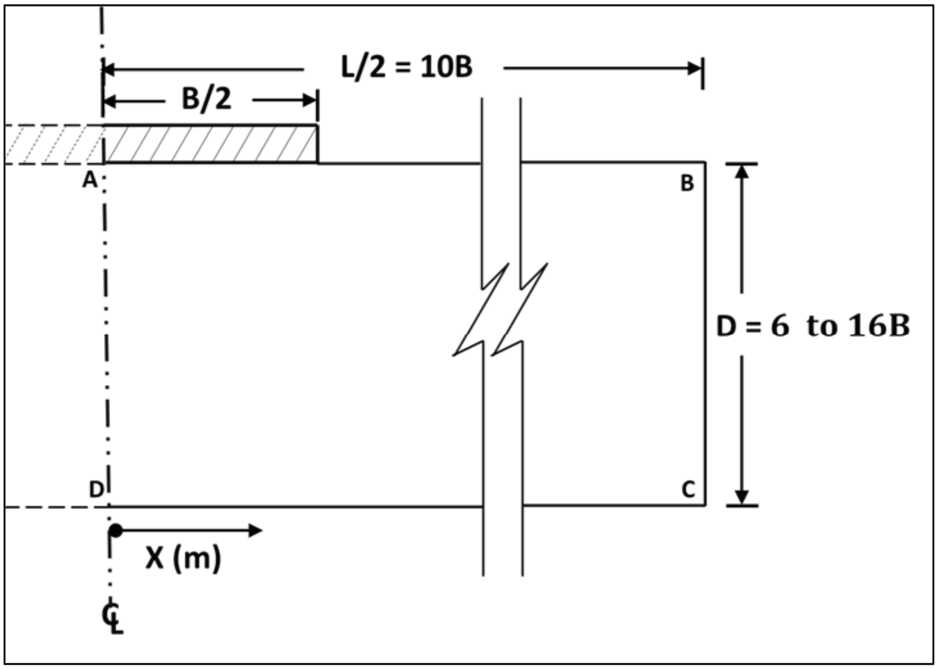

Figure 3.4 Half of the model to study boundary effect along line DC

To estimate the desire depth of the model, the changes in the vertical effective stress are studied at the bottom of the soil model (Line DC).

The soil properties are presented in Table 1. The cohesion for homogeneous sand soil is 0.01 kPa, the elastic modulus is assumed with regard to EM-1110-1904 (Army, 1904) and dilatancy factor, is assumed according to Bolton (1986).

Table 1 Soil properties (Lindeburg, 2001)

Soil Type	Unit weight, γ (kN/m^3)	Friction Angle, ϕ (degrees)	Dilatancy Factor, ψ (degrees)	Cohesion, c (kN/m^2)	Young Modulus, E (MPa)
Sand	14	34	4	0.01	10

The total vertical stress along line DC includes the vertical stress caused by footing load and the stress due to the soil weight. By increasing the soil depth from 6B to 16B, the effects of footing load decreases at the bottom line of model. Figure 3.5 shows the vertical stress of the soil elements along line DC. As can be seen, by increase of model depth the vertical stress increases which shows the effect of soil weight. In order to study the stress caused only by footing load, the vertical stress along the line DC is subtracted from the soil weight at the bottom line (γD). The results are depicted in figure 3.6. It can be understand that for depth 14B and 16B the effect of footing load is the same. The results demonstrate that by increasing the depth more than 14B the vertical stress on the bottom of the model would not change. In other words, boundary effects for depth more than 14B would be negligible.

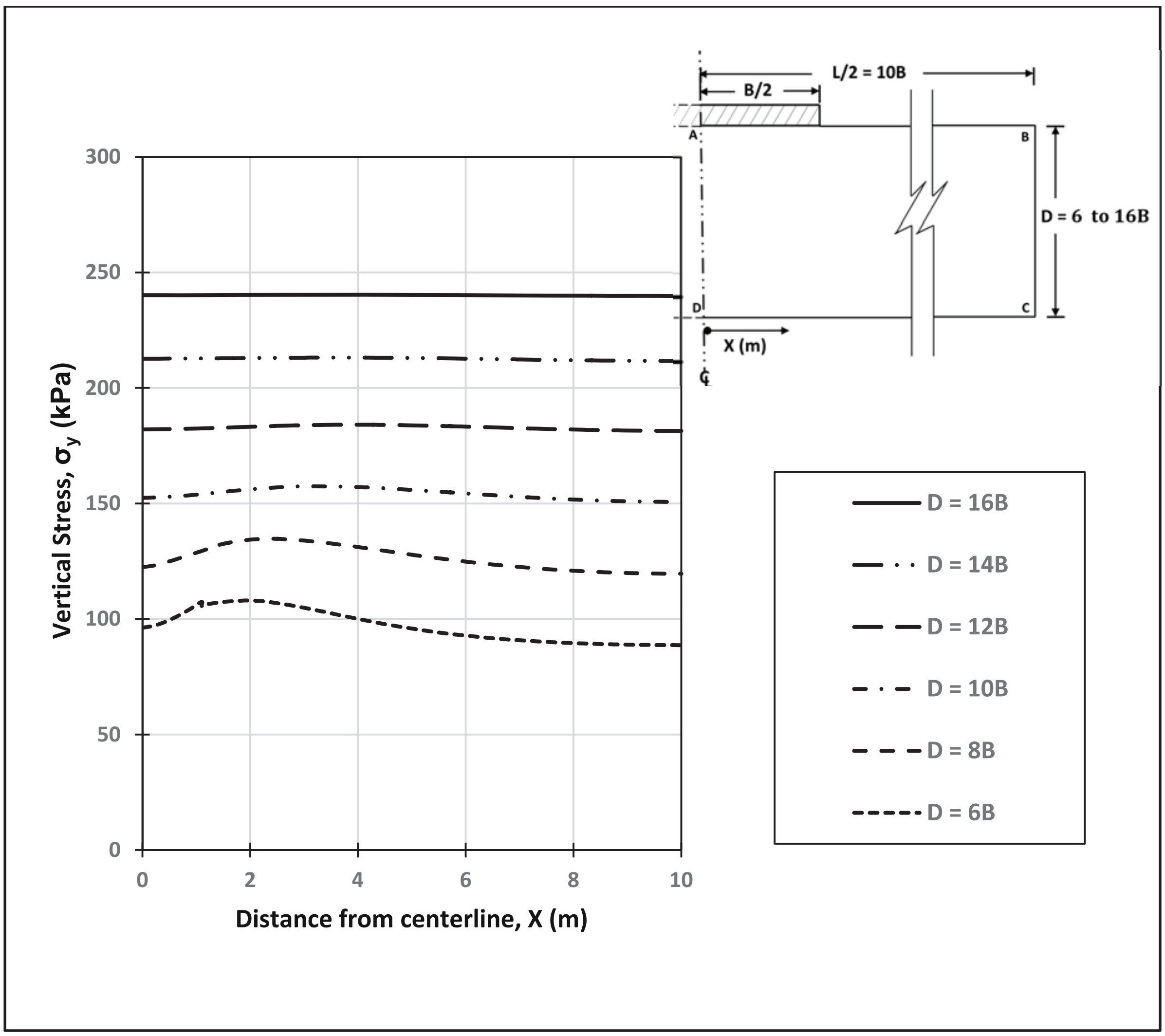

Figure 3.5 Vertical Stress along line DC

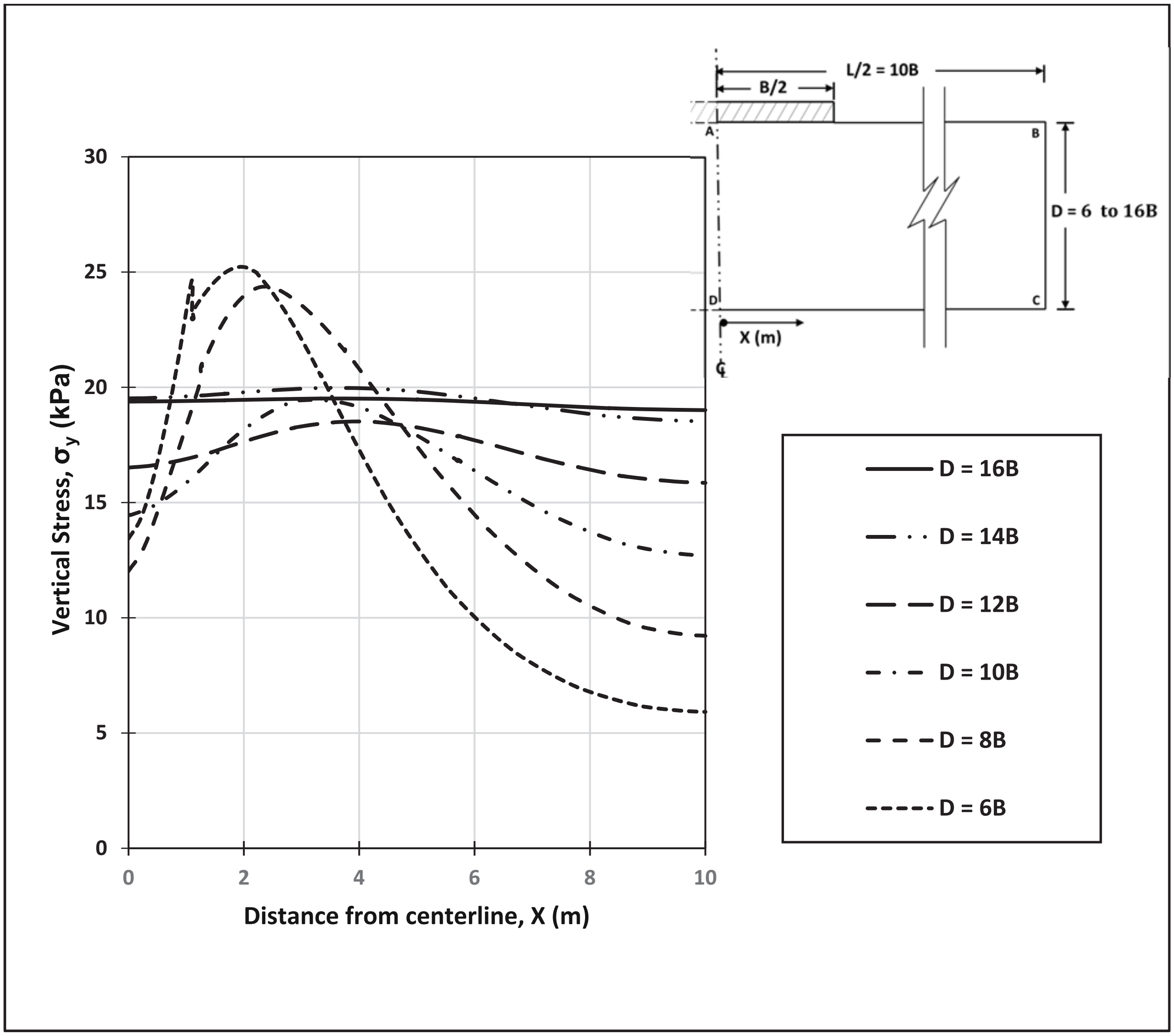

Figure 3.6 Vertical stress caused only by footing load along line DC

The same approach is followed to obtain the optimum model length. The model depth (D) is fixed to 14B and the soil length (L/2) in symmetric model increases from 7.5B to 15B.

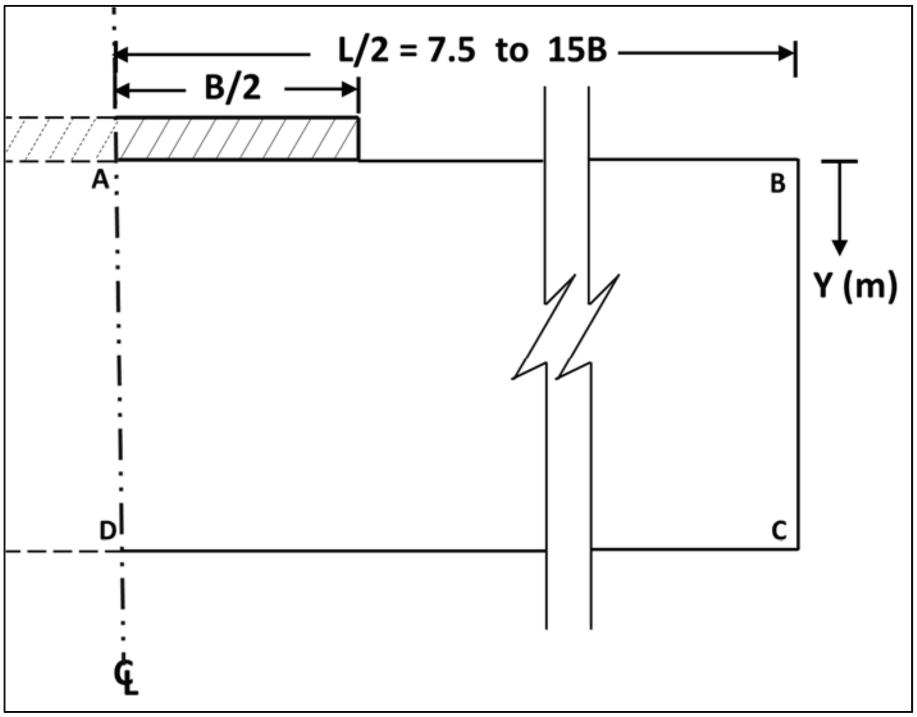

Figure 3.7 The geometry of model to study boundary effect along line BC

To determine the optimum length for the model, the soil depth (D) is fixed to 14B and soil length (L/2) in symmetric model increases from 7.5B to15B. The horizontal stress of soil elements is studied along line BC.

The changes in horizontal stress of the soil elements along the edge of model in y-direction is showed in figure 3.8. It can be seen that by increase of soil model length (L/2) more than 10B the horizontal stresses does show a significant change. Hence, the soil model size in x and y direction respectively is considered 10B and 14B for the rest of the analyses.

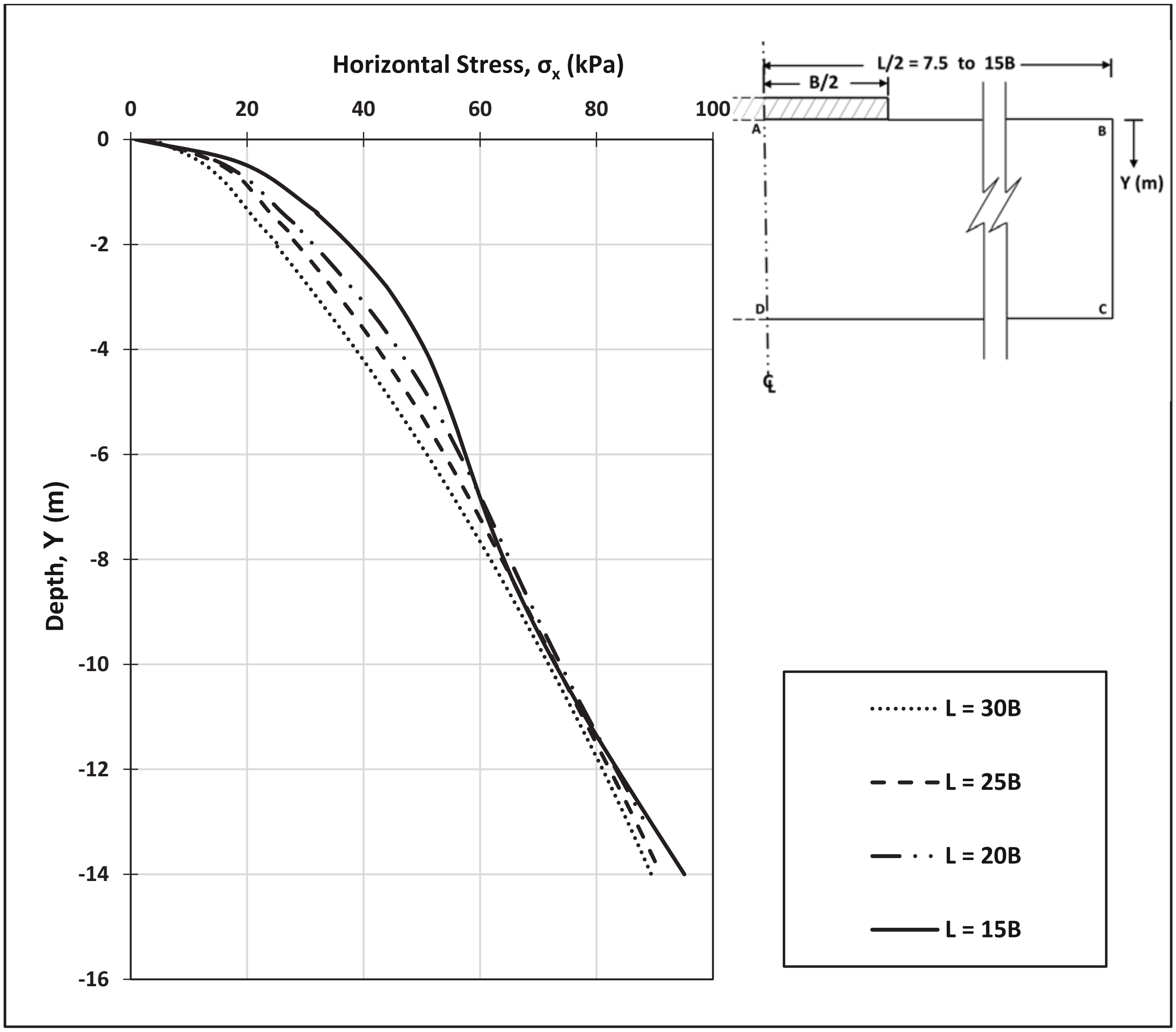

Figure 3.8 Horizontal Stress along line BC

3.3. Element Size

Since the numerical calculations are always accompanied by errors, finding the answer of the problem needs the proper usage of numerical method and the correct interpretation of the results. The FEM methodology to solve the problems is to discretize a continuum into finite volumes (elements) which are connected together by nodes. Then approximate solution is calculated for each mesh elements by use of stiffness matrix and shape functions. Therefore, the numerical analysis is basically based on the estimation of the answer to the problem and not to calculate the exact value. One factor in any numerical analysis which plays an important role to accurately predict the results is mesh element size. In this study, a sensitivity analysis is conducted on mesh element size to reduce the numerical calculation errors. By this means, the optimum average mesh size and the number of elements are evaluated both to minimize computer calculation time and to increase the accuracy of calculating the stresses and strains. However, it is often the case that the fine element size is chosen not for the whole geometry but for a small local area beneath the footing to reduce the zone of mesh refinement and also to reduce calculation time. In this study, a local area underneath the footing is defined to have more elements. To do this, three different type of homogenous soil is model. The load-settlement curves is obtained for homogenous dense, medium and loose sand soil. The element refinement for the local area has been increased until almost the same results acquired.

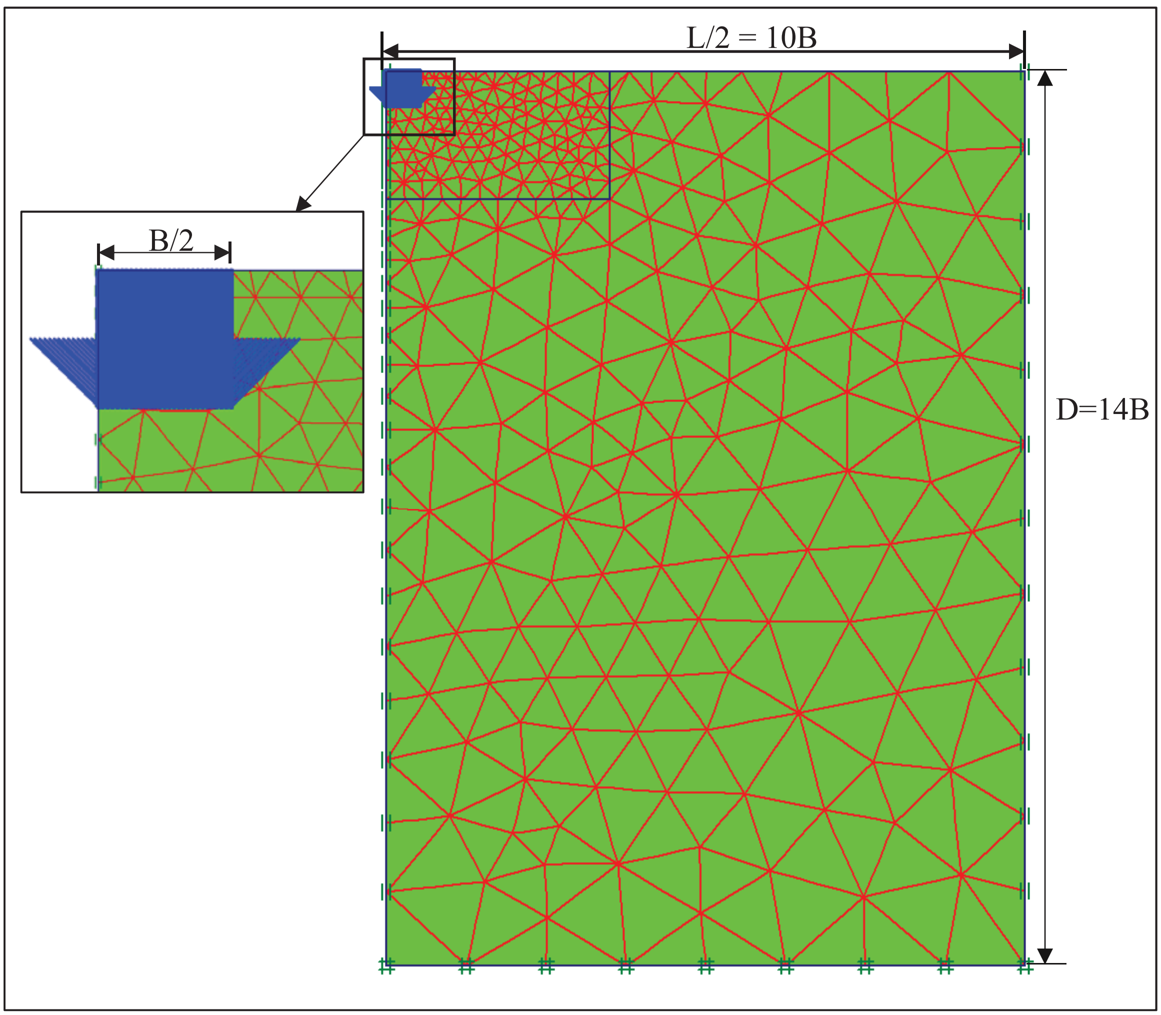

Figure 3.9 Mesh Refinement

Figure 3.10 shows a typical load-settlement curve for loose sand. The number of elements increases from 234 to 263. It can be seen that vertical stresses underneath the footing decreases when number of elements increases from 234 to 250, while refinement of the model from 250 to 263 elements doesn't affect the vertical stresses. In the present study, the mesh refinement is carried out for all the simulations to ensure the mesh element size will not influence the results.

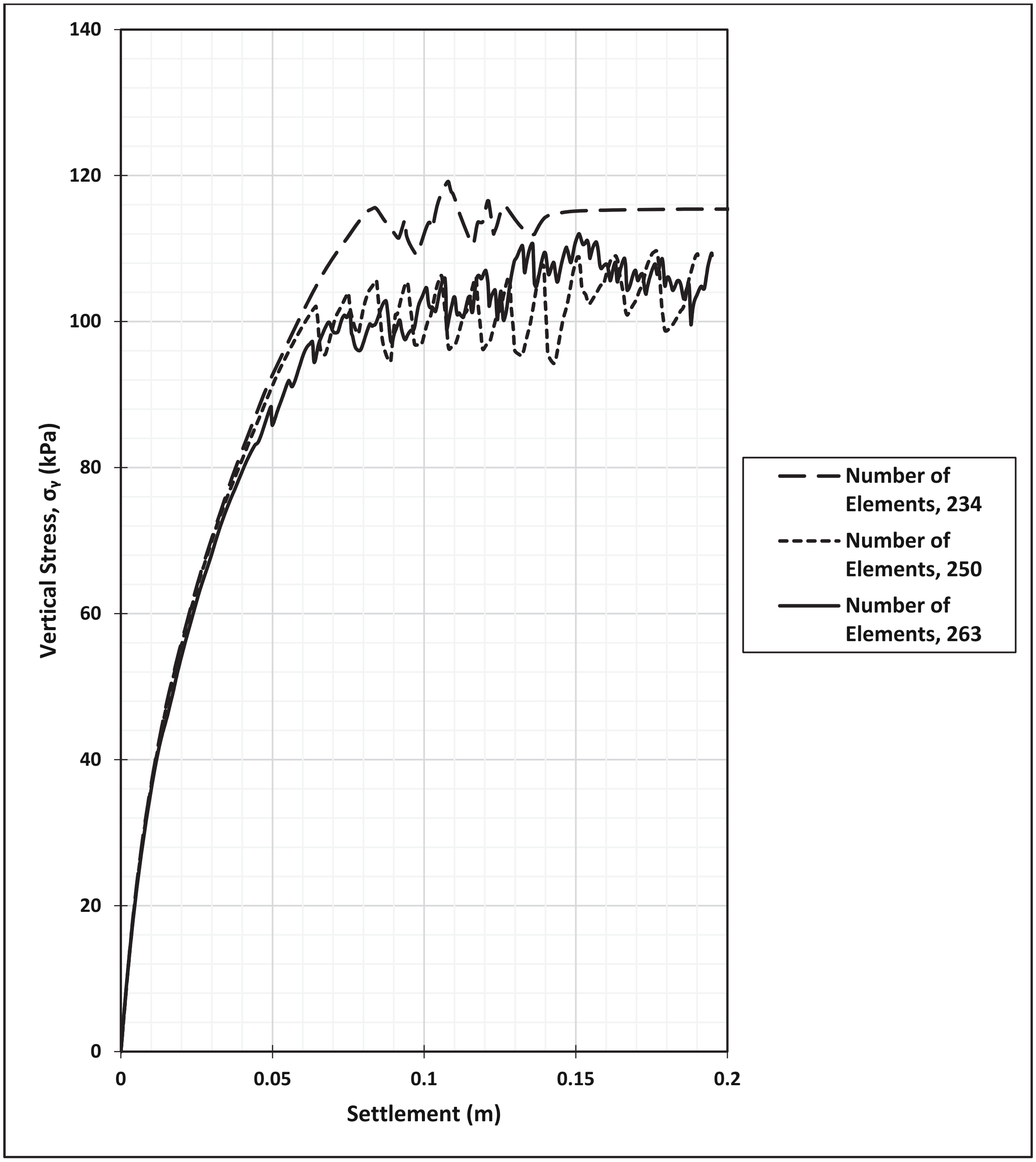

Figure 3.10 Load-settlement curve for Homogenous loose sand

Table 2 shows the properties of mesh elements for each soil type when refinement is conducted on the model.

Table 2 Mesh element properties

Soil Type	Element Properties	Model 1	Model 2	Model 3
Loose sand	No. of soil elements	234	250	263
	No. of nodes	2808	2101	2219
	Average element size (m)	0.947	0.917	0.894
Medium sand	No. of soil elements	234	250	263
	No. of nodes	2808	2101	2219
	Average element size (m)	0.947	0.917	0.894
Dense sand	No. of soil elements	147	295	331
	No. of nodes	1251	2459	2759
	Average element size (m)	1.2	0.844	0.797

Figures 3.11 to 3.13 illustrate the effect of mesh refinement on bearing capacity of homogenous loose, medium and dense sand soil.

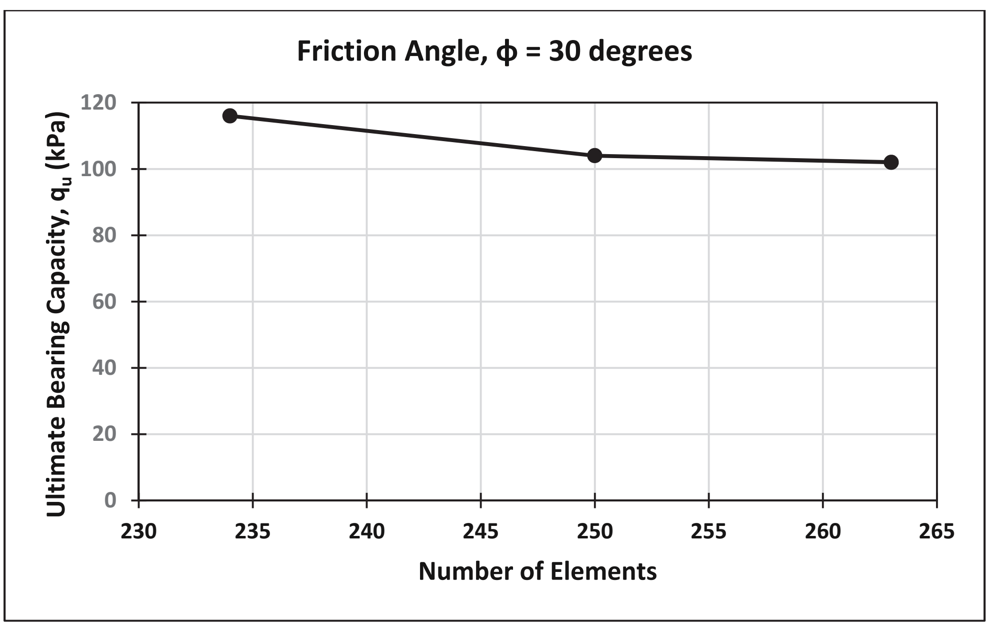

Figure 3.11 Bearing capacity vs Number of elements for loose sand

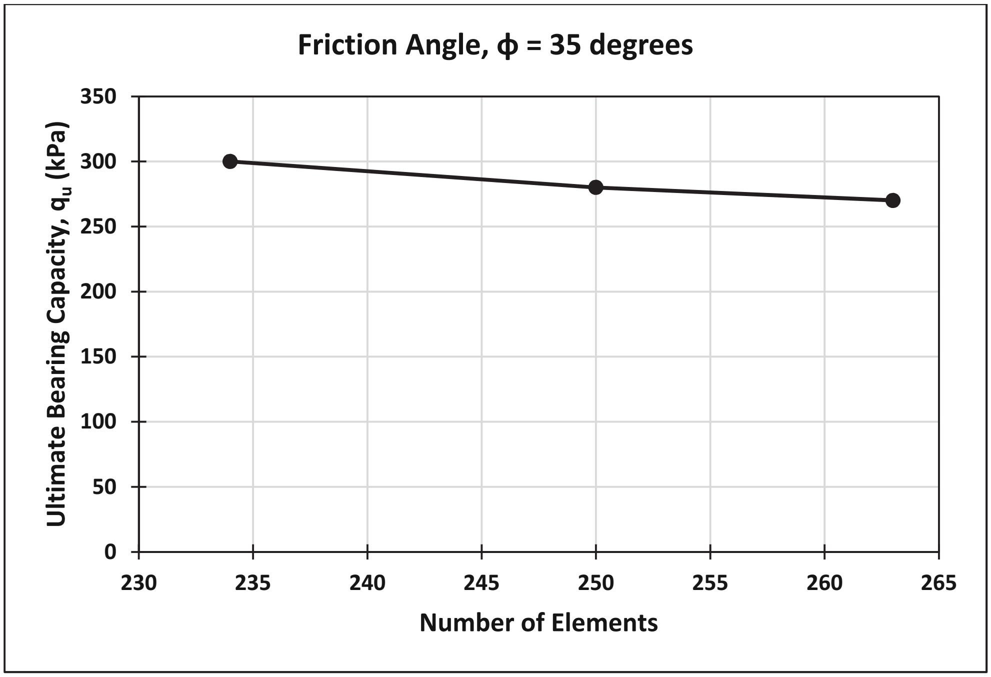

Figure 3.12 Bearing capacity vs Number of elements for medium sand

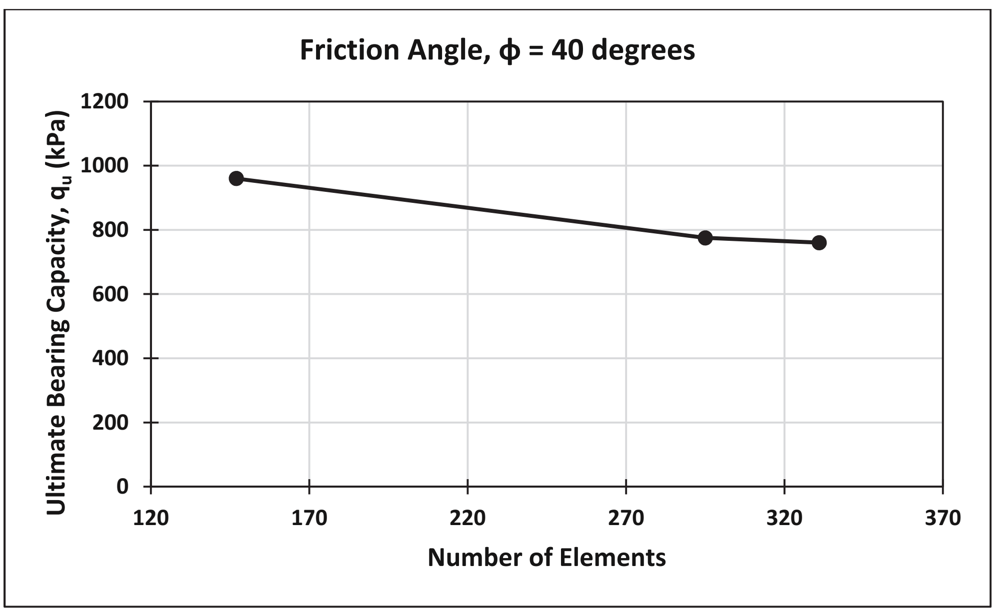

Figure 3.13 Bearing capacity vs Number of elements for dense sand

3.4. Validation of Footing on Homogenous Cohesionless Soil

The bearing capacity of homogenous loose, medium and dense sand which has been calculated in the previous section is compared to the traditional equations presented by Meyerhof (1963) and Terzaghi (1943). The soil properties for three different types of soil is showed in table 3. The cohesion of the sand soil is practically equal to zero but according to the PLAXIS manual a minimum value of 0.2 kPa is recommended (Brinkgreve, Broere, & Waterman, 2004). In this study, a minimum cohesion of 0.01 kPa is assumed in order to avoid both calculation problem and effect of cohesion on the bearing capacity. The elastic modulus is assigned to the soil model with regard to the soil type and using EM-1110-1904 (Army, 1904). The range of elastic modulus variation has been recommended to be in between 9.6 - 25 MPa and 25 - 95.8 MPa for loose and dense sand respectively. Nevertheless, elastic behavior of soil has no effect on the bearing capacity of footings since soil behavior in failure condition is completely plastic. However, elastic modulus of soil is responsible for soil deflections at the ultimate state.

Table 3 Soil properties for homogeneous sand soil (Lindeburg, 2001)

Soil Type	Unit weight, γ (kN/m^3)	Friction Angle, ϕ (degrees)	Dilatancy Factor, ψ (degrees)	Cohesion, c (kN/m^2)	Young Modulus, E (MPa)
Loose Sand	14	30	0	0.01	10
Medium Sand	16	35	5	0.01	24
Dense Sand	18	40	10	0.01	35

The dilatancy factor, ψ is assumed based on the equation represented by Bolton (1986). He established a relationship between soil friction angle and dilatancy angle which is used in this study to determine the value of ψ parameter:

$$\psi = \phi - 30^o$$

The numerical results are compared to analytical solutions and is presented in table 4. The problem is the bearing capacity of strip footing resting on a homogeneous sandy soil. The embedment depth of soil is assumed to be zero and the present numerical study is compared to the traditional bearing capacity equations presented by Terzaghi and Meyerhof.

Table 4 Comparison of present numerical results to other studies for homogenous soil

Soil Type	Dry density, γ (kN/m^3)	Friction Angle, ϕ (degrees)	$\boldsymbol{q_u}$, kPa (Meyerhof)	$\boldsymbol{q_u}$, kPa (Terzaghi)	$\boldsymbol{q_u}$, kPa (Present Study)	%Δ to Meyerhof
Loose Sand	14	30	109	138	102	6.4
Medium Sand	16	35	297	339	270	9.1
Dense Sand	18	40	843	904	760	9.8

As can be seen the numerical results are in good agreement with Meyerhof (1963). The results justify that the numerical model accurately predicts the bearing capacity for homogeneous loose, medium and dense sand.

4. RESULTS AND ANALYSIS

4.1. General

In this chapter, the bearing capacity of strip footing on two layered sand soil is investigated by developing numerical model. A parametric study has been carried out on the effect of upper layer shear strength. In addition, the thickness of the upper layered is increased from zero to the value in which the bearing capacity remains constant. Afterwards, the numerical results are compared to the experimental data, analytical solutions and numerical studies. In the present study the numerical analysis is conducted by using a 2D-model. The model size is assumed to be according to the ratios which were found in the previous chapter. This ratios are 20B and 14B for model length and depth, respectively. The embedded depth of soil is primarily zero and it changes to non-zero values.

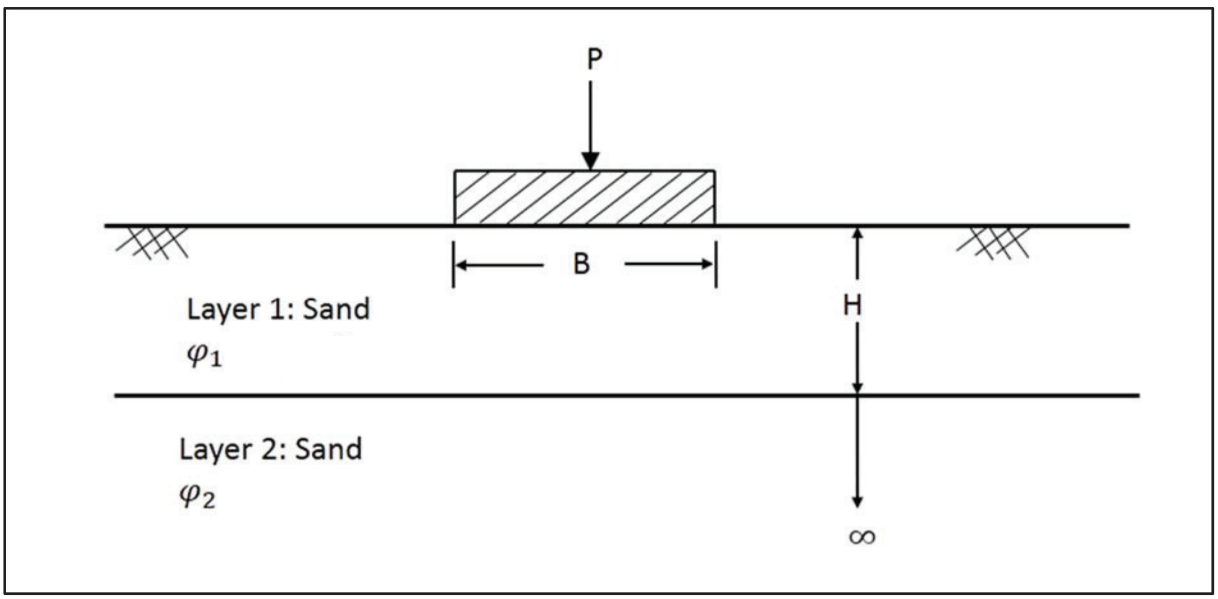

Figure 4.1 Problem definition

4.2. Test Results

The preliminary study of bearing capacity is conducted on the case wherein the upper layer is dense sand with friction angle of 40 degrees. The lower layer is loose sand with friction angle of 30 degrees. The thickness of upper layer is changed between 0 to 5B. The second case is when the friction of upper layer is changed to 35 degrees. The load-settlement curves for both cases are generated. Then, a parametric study is conducted on the shear strength of upper layer. To do this, the friction angle of upper layer is varied from 30 to 48 degrees. Thereafter, the footing is embedded in the depth equal to B. Two numerical models are developed and the results are compared.

Figure 4.2 shows the pressure-settlement curve (it is called load-settlement curve in this study) for dense sand ($\varphi = 40^{\circ}$) underlain by loose sand ($\varphi = 30^{\circ}$). As the thickness of dense layer increases, the load-settlement curves become not perfectly plastic. The reason is that for purely sandy soil without cohesion the numerical convergence is difficult especially for high values of friction angle. It is numerically troublesome to redistribute the unbalanced forces when collapse occurs and the stiffness of system turns to zero. To redistribute any unbalanced forces and obtain a perfectly plastic curve, a large number of iterations may be required which would be computationally expensive (GEO-SLOPE International Ltd, 2007).

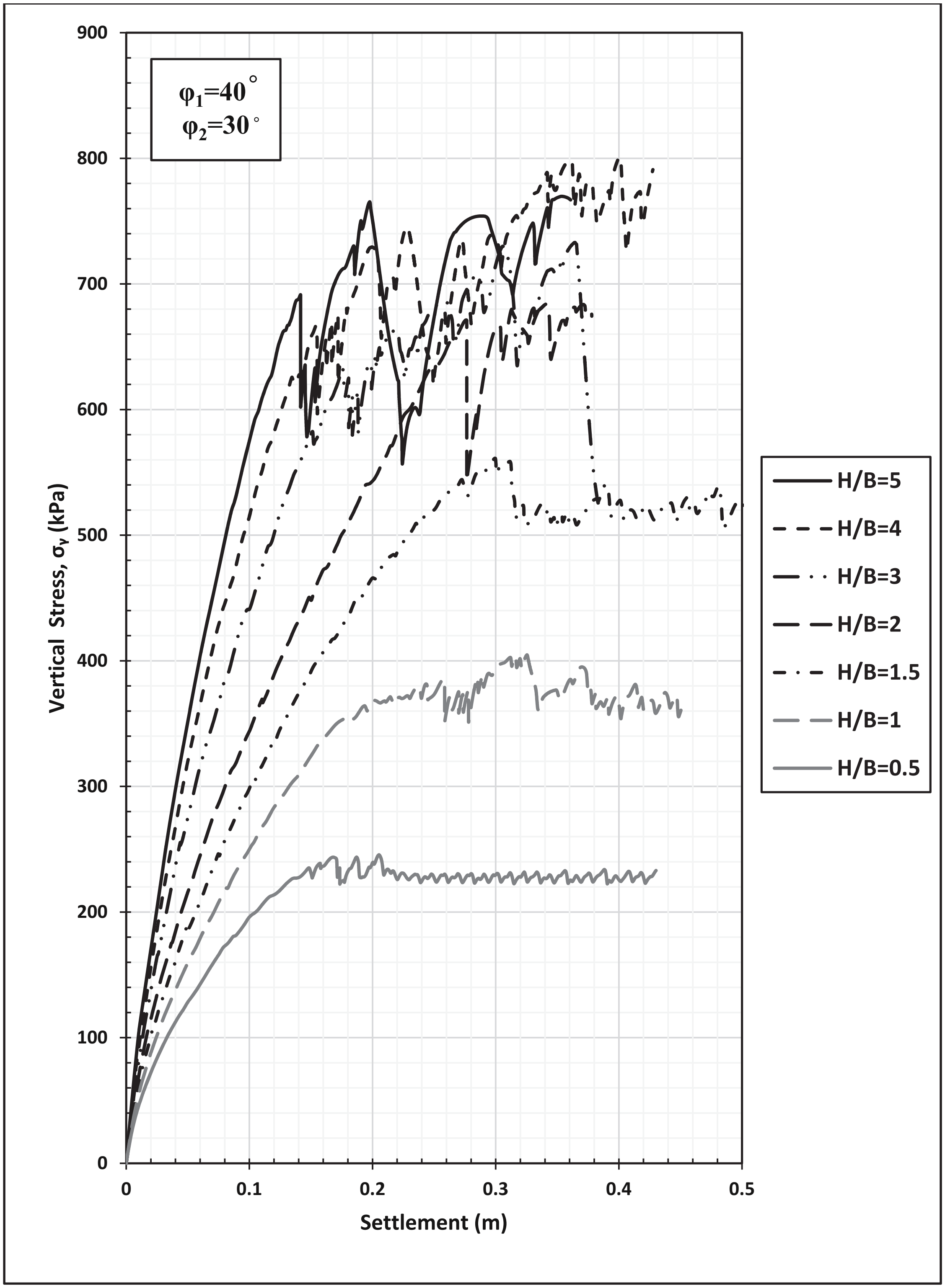

Figure 4.2 The load-settlement curve for $\varphi_1 = 40^{\circ}$

According to the Figure 4.2, by increase the upper layer thickness, the bearing capacity increases. The critical thickness in which increase of upper layer thickness does not affect the bearing capacity is almost 3. In other words, for H/B more than 3 the soil bearing capacity is the same as homogeneous dense sand. According to the load-settlement curves, it can be understood that the relation between increase of H/B and bearing capacity is not linear. To clarify, the bearing capacity of homogeneous loose sand is 102 kPa and the bearing capacity of layered soil with H/B equal to 0.5 is 245 kPa. It states that the bearing capacity of loose sand increases by 140 percent when a layer of dense sand with the thickness of 0.5B covers the loose sand surface. While by increasing H/B ratio from 1.5 to 2 the bearing capacity of soil increases by only 20 percent. By increase of H/B ratio the stiffness of soil increases. In other words, the more the thickness of upper layer the less settlement occurs when soil collapses. To clarify the concept, assume that the allowable settlement is restricted to 4 cm. In this case, the corresponding allowable bearing capacity for the curve with H/B ratio equal to 5 according to Figure 4.2 is 300 kPa while for H/B equal to 0.5 is 100 kPa. It can be understood that, the allowable bearing capacity of two layered soil with H/B equal to 5 is far more than H/B equal to 0.5, and it is almost three times of it. For engineering purpose, both criteria of ultimate bearing capacity and allowable settlement should be satisfied.

For medium sand ($\varphi = 35^{\circ}$) overlaying loose sand ($\varphi = 30^{\circ}$) the load settlement curves are presented in Figure 4.3. A similar trend can be noticed for the load-settlement curves of medium sand overlaying loose sand. By increasing the upper layer thickness, the bearing capacity increases. The soil settlement decreases by increase of H/B ratio. The critical thickness is 2B wherein the bearing capacity remains constant and equal to the bearing capacity of homogeneous medium sand.

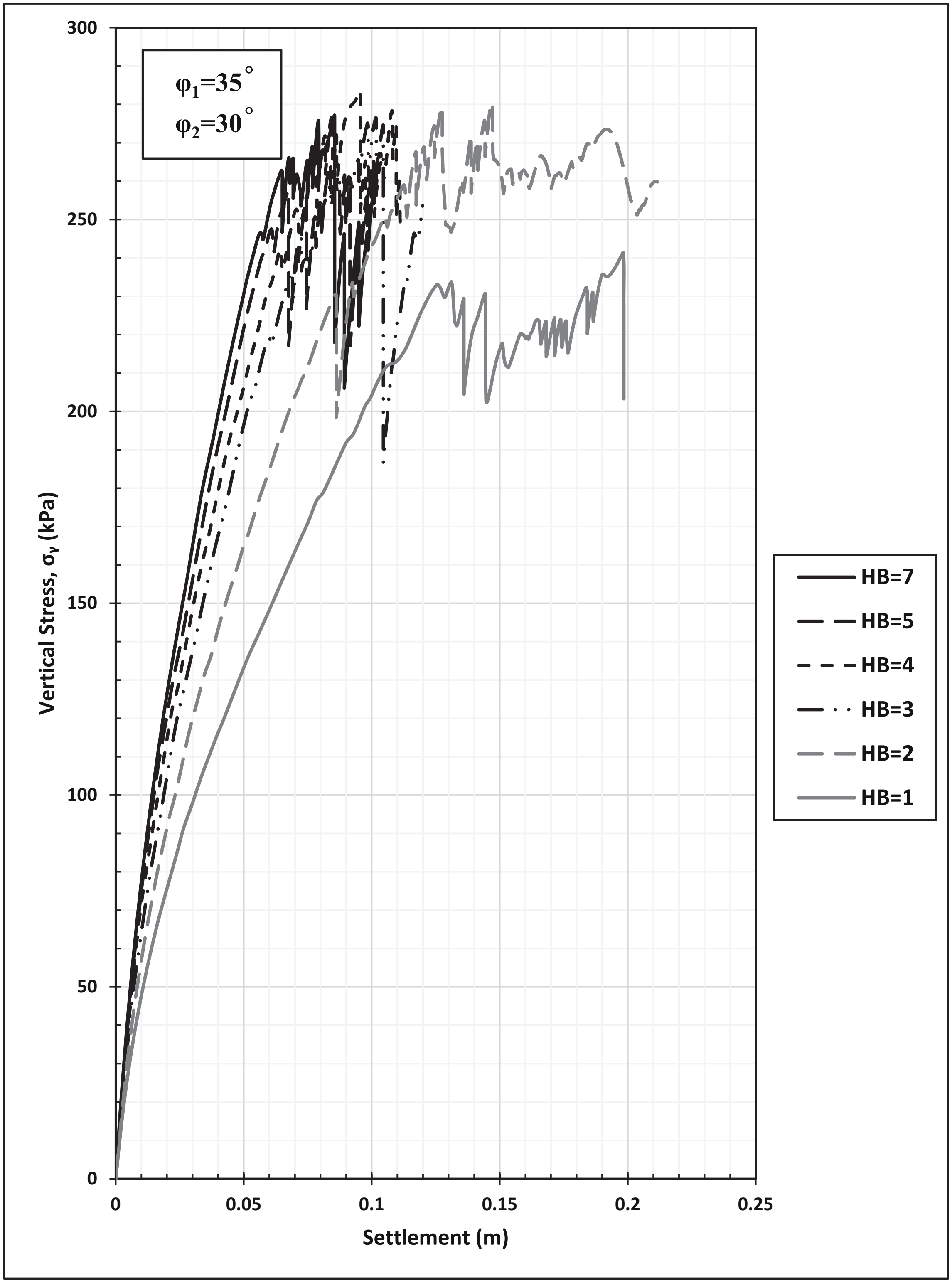

Figure 4.3 The load-settlement curve for $\varphi_1 = 35^{\circ}$

By comparing the two load-settlement curves, it can be concluded that bearing capacity of soil is highly affected by shear strength of upper layer soil. For instance, the bearing capacity of dense sand over loose sand in H/B ratio equal to 1 is almost 170 percent of bearing capacity of medium sand over loose sand with the same thickness. Therefore, by increasing the friction angle of upper layer from 35 to 40 degrees the bearing capacity increases by 170.

To estimate the effective thickness of upper layer with different shear strength, a parametric study has been conducted. The friction angle of upper sand layer is changed between 30 to 48 degrees and the friction angle of the lower layer is always 30 degrees. The results are presented in Figure 4.4.

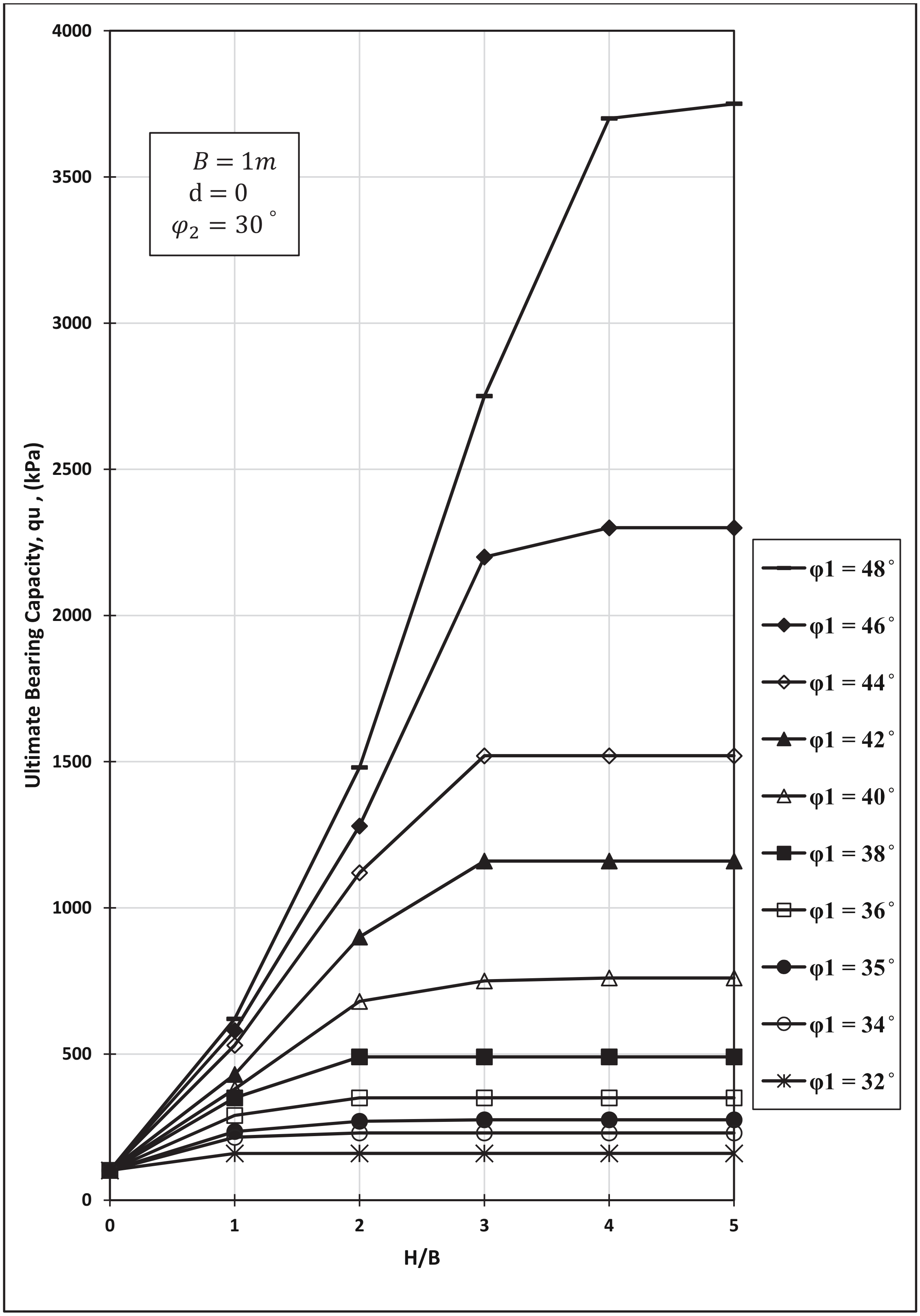

Figure 4.4 The bearing capacity vs H/B when d/B=0

The parametric study shows that by increasing the friction angle of upper layer, the bearing capacity increases. Also, by increasing the thickness of upper layer the bearing capacity increases. In H/B ratio of 1, the bearing capacity of upper sand with $\varphi = 48^{\circ}$ is almost three times of upper sand with $\varphi = 32^{\circ}$. While, in H/B ratio of 4 the bearing capacity of upper layer with friction angle of 48 degrees is around 22 times of upper layer with friction angle 32 degrees. The relation between increase of friction angle and increase of bearing capacity is not linear and it depends on the thickness of upper layer. For instance, for H/B equal to 1 when friction angle increase from 32° to 34° the bearing capacity increases by 46%. In the same H/B ratio by increase of friction angle from 46° to 48° the bearing capacity increases by 7%. While, For H/B equal to 4 when friction angle increase from 32° to 34° the bearing capacity increases by 44%. In the same H/B ratio by increase of friction angle from 46° to 48° the bearing capacity increases by 60%. It can be concluded that when shear strength of upper layer is in between loose and medium sand, the increase of friction angle by 2 number noticeably increase the bearing capacity of two layered soil with low H/B ratio. The effective thickness of upper layer increases from 1B to 4B by increase of upper layer shear strength from loose sand ($\varphi = 30^{\circ}$) to very dense sand ($\varphi = 48^{\circ}$). It can be said that the effective thickness of upper loose to medium sand ($\varphi = 30^{\circ}\ to\ 35^{\circ}$) is almost 1.5 and for upper medium to dense sand ($\varphi = 35^{\circ}\ to\ 40^{\circ}$) is almost 2. For dense sand with friction angle in between ($\varphi = 40^{\circ}\ to\ 44^{\circ}$) is approximately 3B and for very dense sand ($\varphi = 46^{\circ}\ to\ 48^{\circ}$) is 4B.

To study the effect of embedment depth on the bearing capacity, two numerical models are developed and the results are compared to each other. In the first numerical model, the whole combination of wall and concrete footing is simulated and the load is applied to the top of the wall as prescribed displacement elements. In the second model, the footing load is applied to the model as prescribed displacement as illustrated in Figure 4.5 and 4.6.

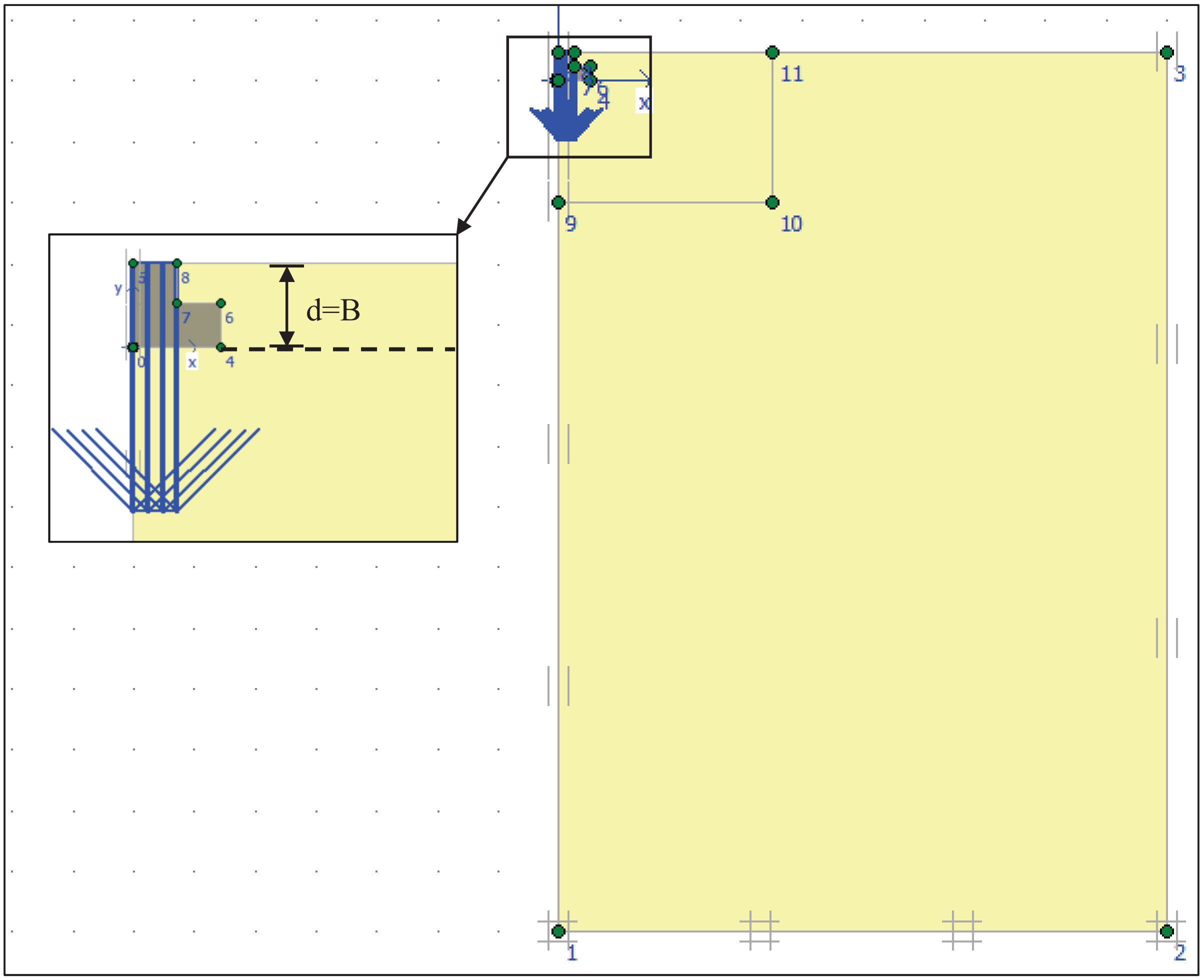

Figure 4.5 Model with concrete footing

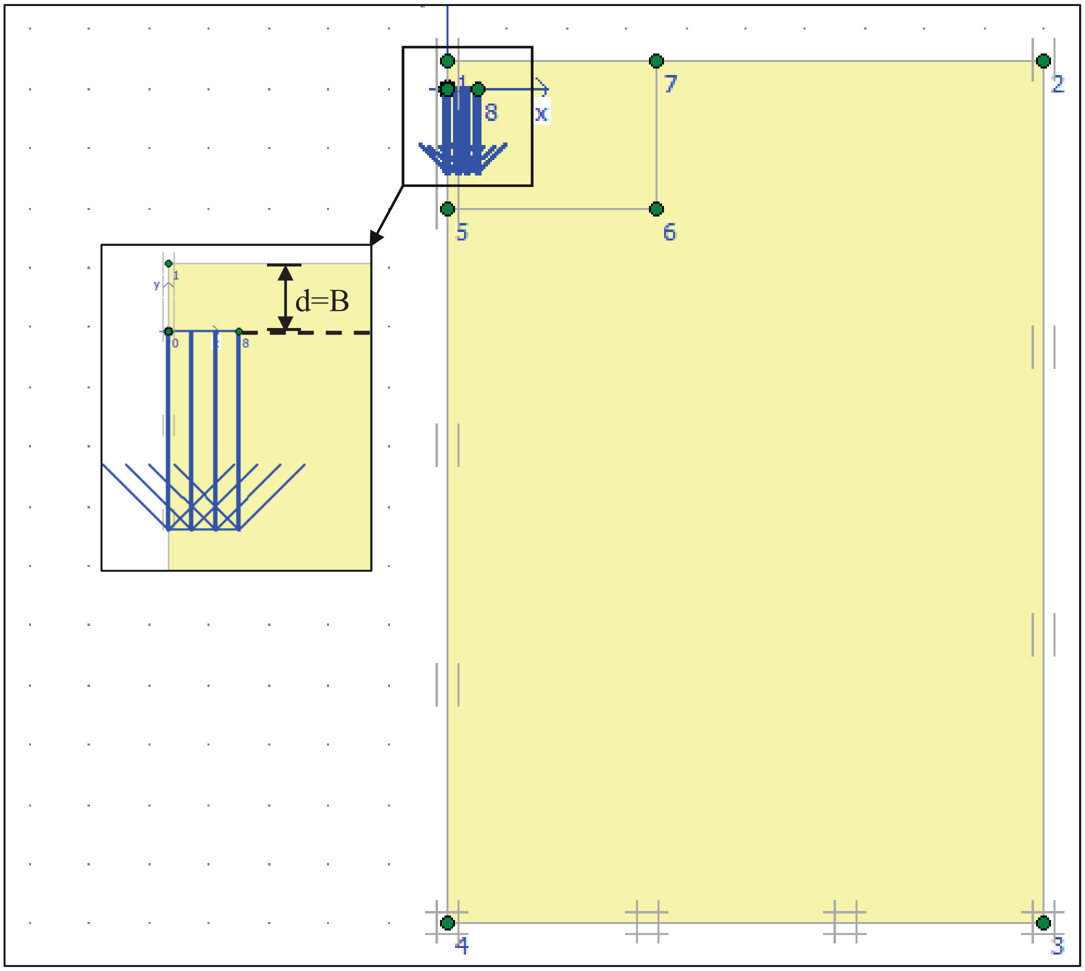

Figure 4.6 Model without concrete footing

The load-settlement curves for homogenous soil from both models are compared in Figure 4.7. It can be concluded that the two model predicts the same bearing capacity. For the rest of the study, the prescribed displacement is used as footing load.

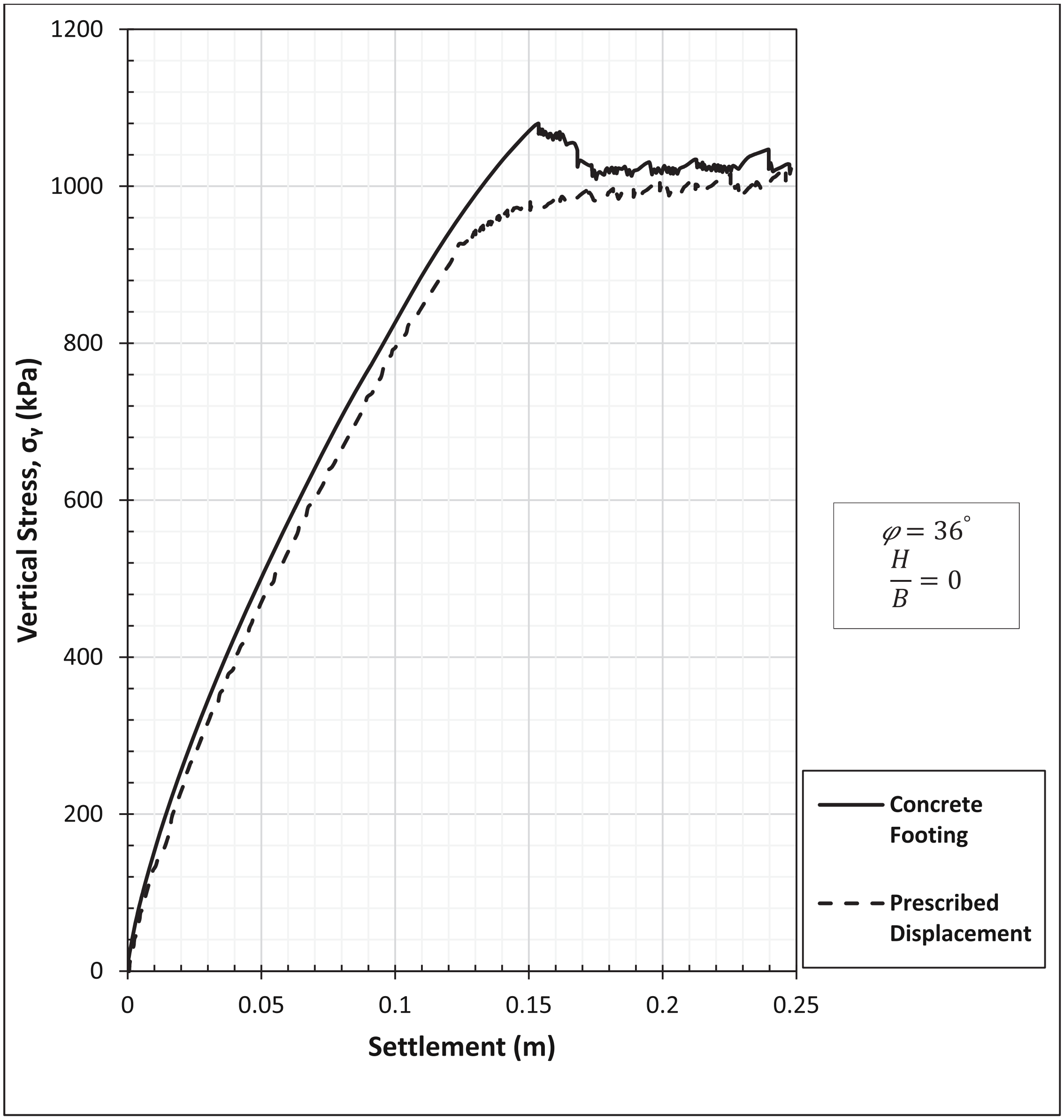

Figure 4.7 Load-settlement curves for model with and whithout concrete footing

The effective thickness of upper layer for different soil shear strength is investigated when the embedment depth is equal to B. To do this, the friction angle of upper sand various in between 30 to 48 degrees. Figure 4.8 shows the results.

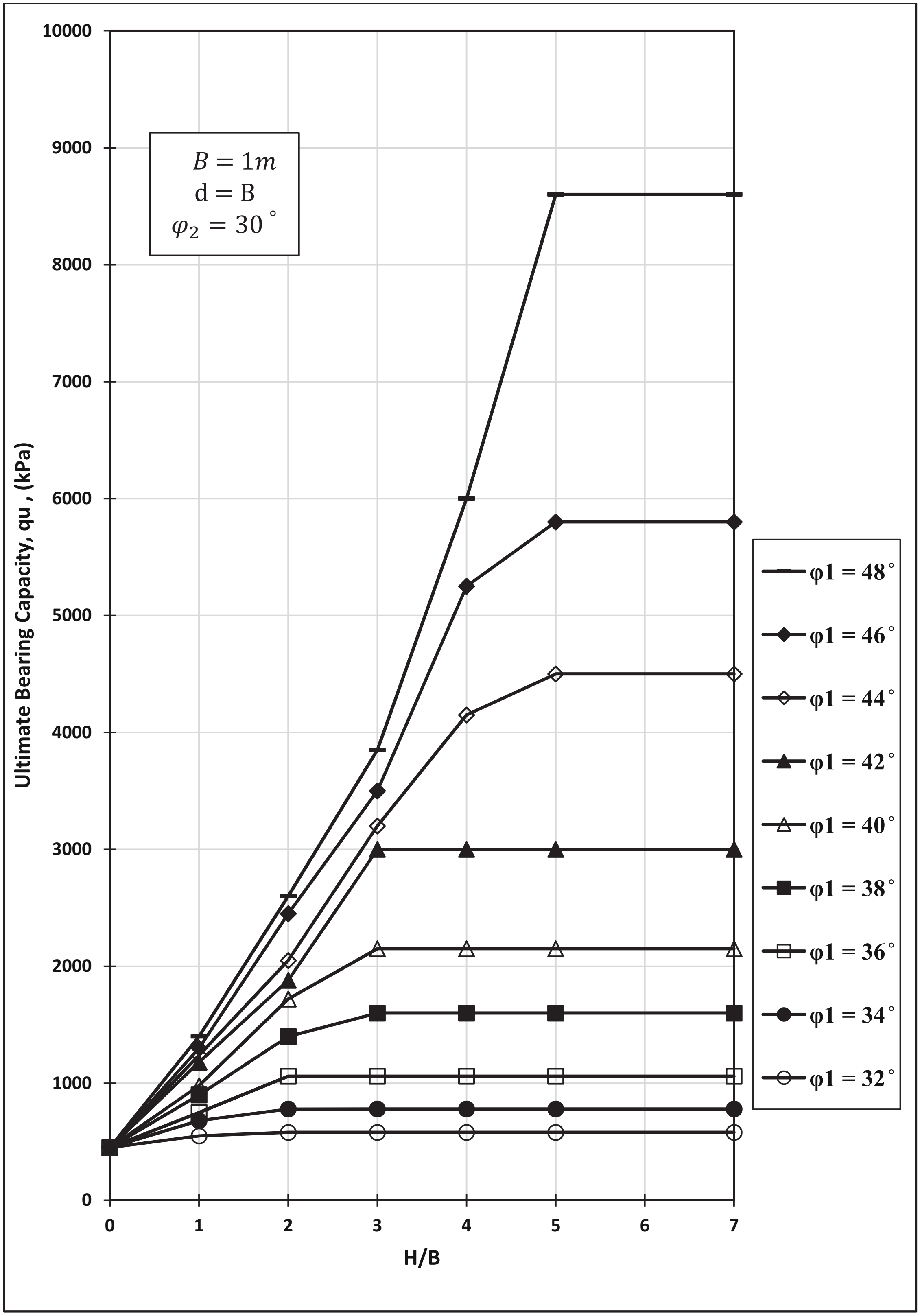

Figure 4.8 The bearing capacity vs H/B when d/B=1

4.3. Comparison with Experimental Studies

The bearing capacity from numerical model is compared to the experimental studies conducted by Hanna (1981) and Carter (2005). Hanna (1981) carried out an experimental study to investigated the bearing capacity of dense sand ($\varphi_1 = 47.7^{\circ}$) underlines by loose sand ($\varphi_2 = 34^{\circ}$). Figure 4.9 displays the differences between the results of the current numerical model and the experiment.

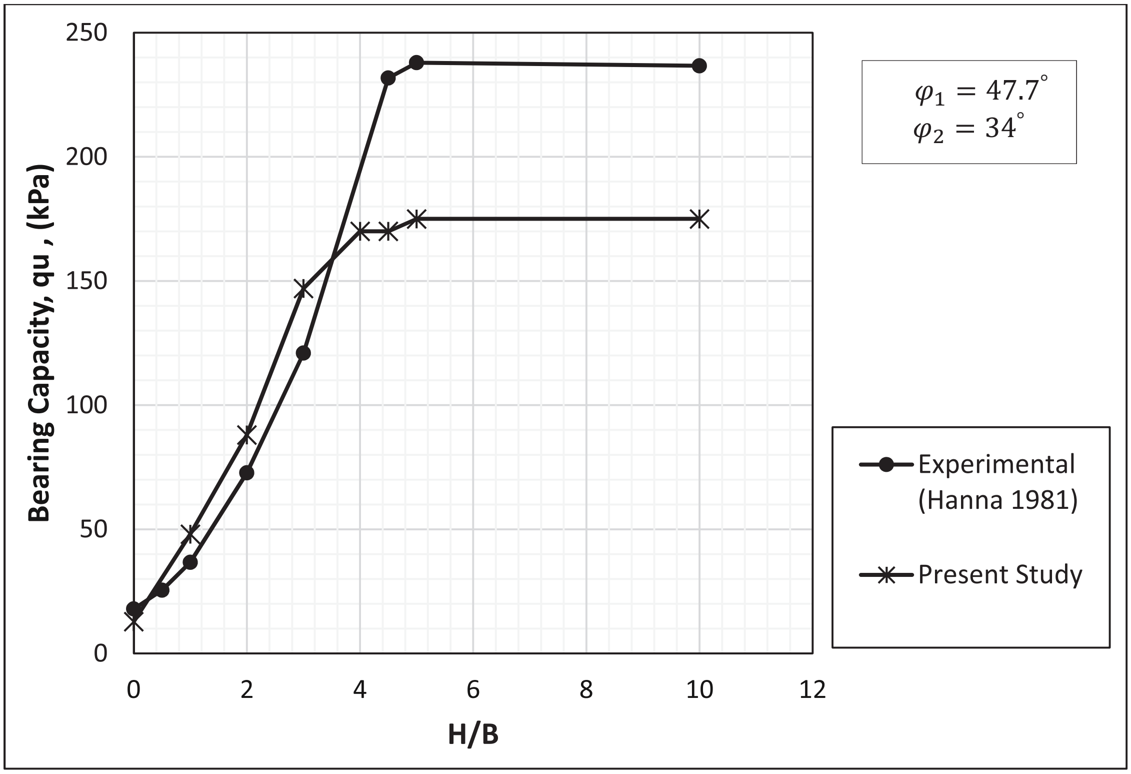

Figure 4.9. Test results vs experimental study conducted by (Hanna, 1981)

It can be seen that for two layered soil the obtained bearing capacity from numerical analysis is more than experimental data, when the thickness of dense sand is less than 3.6 B. In addition, general shear failure occurs at H/B ratio more than almost 4. The numerical model underestimates the bearing capacity when general shear failure occurs. The reason might be the fact that for friction angle almost equal to 48 degrees the numerical convergence is very difficult. In other words, recognizing the soil bearing capacity from load-settlement curve is difficult because the curve is not perfectly plastic.

Figure 4.10 shows the comparison of numerical model result with experimental study of Carter (2005). The friction angle of top and bottom layer are 35° and 28.3°, respectively. The numerical model is in good agreement with the experiment and the maximum difference is almost 13%.

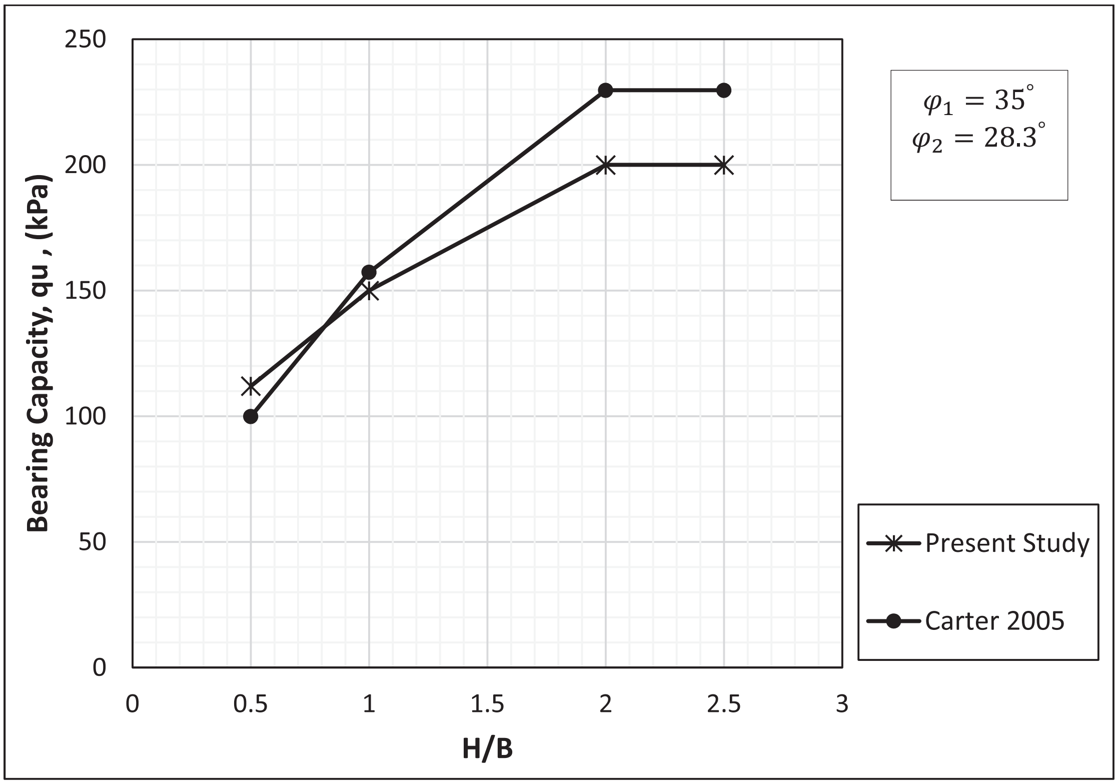

Figure 4.10 Test results vs experimental study conducted (Carter, 2005)

4.4. Comparison with Analytical Solutions

The selected analytical solutions for comparison purpose are based upon limit analysis and limit equilibrium methods. In the first step, the numerical model is compared to the limit analysis solution presented by Florkiewicz (1989). Afterwards, the numerical result is compared to the limit equilibrium methods. As discussed in the literature, the most common classical equations of calculating the bearing capacity of two layered soil which are based on limit equilibrium method consist of projected area method (PAM), weighted average method and punching shear method. The selected studies for comparison purpose based on PAM (projected area method) are

Yamaguchi (1963), Kraft & Helfrich (1983) and Myslivec & Kysela (1978). The numerical results are also compared to punching shear method by Hana (1981), weighted average method by Bowles (1988) and a simple limit equilibrium solution by Ghazavi & Eghbal (2008).

Florkiewicz (1989) studied the bearing capacity of dense over loose sand using upper bound solutions. According to figure 4.11, the numerical results are always less than the upper bound results. The reason is that upper bound solutions predict the ultimate load more than the real load which causes the failure. Hence, the values predicted by Florkiewicz (1989) overestimates the bearing capacity.

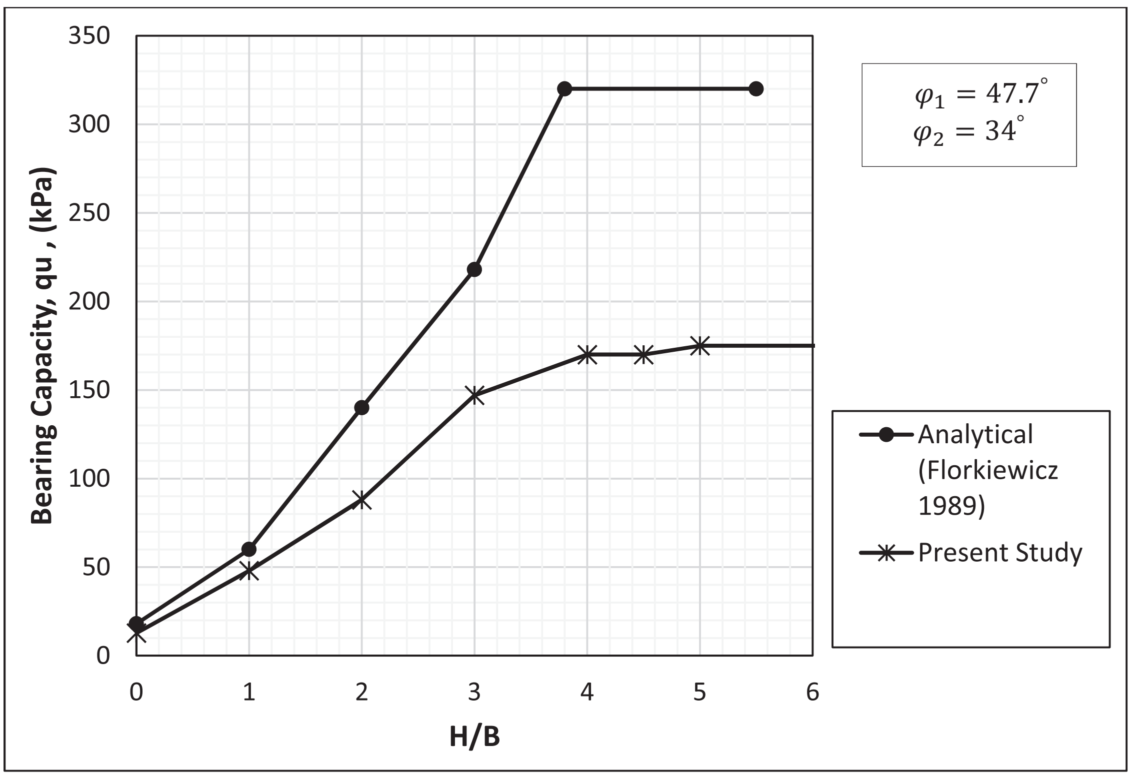

Figure 4.11 Bearing capacity of strip footing, Analytical Study by (Florkiewicz, 1989)

Tables 5 to 8 display the results of numerical calculation and analytical prediction of the bearing capacity of two layered soil with and without considering embedment depth of footing (d). The lower sand is loose with friction angle of 30 degrees. The friction angle of top layer increases from 32 to 48 degrees. Table 5 and 6 show the bearing capacity by assuming that the embedment depth of footing is zero.

Table 5 Comparison of bearing capacity for friction angle of dense sand between 32 to 38, d=0

H/B	φ_1	φ_2	q_{ult} (kPa)						
			Present Study	Yamaguchi (1963)	Myslivec & Kysela (1978)	Kraft & Helfrich (1983)	Bowles (1988)	Hanna (1981)	Ghazavi & Eghbal (2008)
1	32	30	160	163	163	163	163	163	170
2			160	163	163	163	163	163	197
3			160	163	163	163	163	163	197
4			160	163	163	163	163	163	197
5			160	163	163	163	163	163	197
1	34	30	215	236	243	219	243	243	204
2			230	243	243	243	243	243	277
3			230	243	243	243	243	243	277
4			230	243	243	243	243	243	277
5			230	243	243	243	243	243	277
1	35	30	235	236	297	219	297	297	223
2			270	297	297	297	297	297	331
3			275	297	297	297	297	297	331
4			275	297	297	297	297	297	331
5			275	297	297	297	297	297	331
1	36	30	290	236	329	219	364	364	244
2			350	363	364	329	364	364	395
3			350	364	364	364	364	364	395
4			350	364	364	364	364	364	395
5			350	364	364	364	364	364	395
1	38	30	350	236	329	219	435	483	294
2			490	363	548	329	551	551	570
3			490	490	551	439	551	551	570
4			490	551	551	548	551	551	570
5			490	551	551	551	551	551	570

Table 6 Comparison of bearing capacity for friction angle of dense sand between 40 to 48, d=0

H/B	φ_1	φ_2	q_{ult} (kPa)						
			Present Study	Yamaguchi (1963)	Myslivec & Kysela (1978)	Kraft & Helfrich (1983)	Bowles (1988)	Hanna (1981)	Ghazavi & Eghbal (2008)
1	40	30	380	236	329	219	580	506	356
2			680	363	548	329	843	843	837
3			750	490	768	439	843	843	837
4			760	616	843	548	843	843	837
5			760	743	843	658	843	843	837
1	42	30	430	236	329	219	755	528	419
2			900	363	548	329	1254	1158	1147
3			1160	490	768	439	1254	1254	1203
4			1160	616	987	548	1254	1254	1203
5			1160	743	1206	658	1254	1254	1203
1	44	30	530	236	329	219	959	547	506
2			1120	363	548	329	1913	1229	1548
3			1520	490	768	439	1913	1913	1795
4			1520	616	987	548	1913	1913	1795
5			1520	743	1206	658	1913	1913	1795
1	46	30	580	236	329	219	1189	568	620
2			1280	363	548	329	2991	1308	2084
3			2200	490	768	439	2991	2332	2723
4			2300	616	987	548	2991	2991	2723
5			2300	743	1206	658	2991	2991	2723
1	48	30	620	236	329	219	1433	649	765
2			1480	363	548	329	4791	1632	2782
3			2750	490	768	439	4791	3061	4171
4			3700	616	987	548	4791	4791	4171
5			3700	743	1206	658	4791	4791	4171

According to Table 5 and 6 the following can be concluded:

- The bearing capacity calculation for homogeneous soil by Ghazavi & Eghbal (2008) for friction angle less than 40 degrees is more than the estimated bearing capacity by present study, Yamaguchi (1963), Myslivec & Kysela (1978), Kraft & Helfrich (1983), Bowles (1988) and Hanna (1981).
- The present study shows bearing capacity values lower than Hanna (1981) for all thicknesses and for all values of top layer friction angle between 32 to 48 degrees.
- The weighted average method presented by Bowles (1988) estimates the bearing capacity almost the same as Hanna (1981) for upper layer friction angle less than 40 degrees. Although, the method overestimates the bearing capacity when friction angle is more than 40 degrees.
- The projected area method underestimates the bearing capacity when the friction angle of upper layer is more than 40 degrees.

To study the effect of embedment depth, it is assumed that the footing is embedded in a depth of equal to the footing width, B. The footing width in the numerical model is assumed to be 1. Table 7 and 8 display the results for the same soil properties as the previous tables.

Table 7 Comparison of bearing capacity for friction angle of dense sand between 32 to 38, d=B

H/B	φ_1	φ_2	q_{ult} (kPa)						
			Present Study	Yamaguchi (1963)	Myslivec & Kysela (1978)	Kraft & Helfrich (1983)	Bowles (1988)	Hanna (1981)	Ghazavi & Eghbal (2008)
1	32	30	550	506	506	506	506	506	403
2			580	506	506	506	506	506	451
3			580	506	506	506	506	506	451
4			580	506	506	506	506	506	451
5			580	506	506	506	506	506	451
1	34	30	680	702	702	702	702	702	487
2			780	702	702	702	702	702	616
3			780	702	702	702	702	702	616
4			780	702	702	702	702	702	616
5			780	702	702	702	702	702	616
1	36	30	750	887	983	823	983	893	590
2			1060	983	983	983	983	983	852
3			1060	983	983	983	983	983	852
4			1060	983	983	983	983	983	852
5			1060	983	983	983	983	983	852
1	38	30	900	918	1279	852	1255	947	719
2			1400	1393	1393	1279	1393	1393	1195
3			1600	1393	1393	1393	1393	1393	1195
4			1600	1393	1393	1393	1393	1393	1195
5			1600	1393	1393	1393	1393	1393	1195

Table 8 Comparison of bearing capacity for friction angle of dense sand between 40 to 48, d=B

H/B	φ_1	φ_2	q_{ult} (kPa)						
			Present Study	Yamaguchi (1963)	Myslivec & Kysela (1978)	Kraft & Helfrich (1983)	Bowles (1988)	Hanna (1981)	Ghazavi & Eghbal (2008)
1	40	30	980	950	1323	882	1633	1003	884
2			1720	1459	1999	1323	1999	1732	1708
3			2150	1968	1999	1764	1999	1999	1708
4			2150	1999	1999	1999	1999	1999	1708
5			2150	1999	1999	1999	1999	1999	1708
1	42	30	1180	950	1323	882	2027	1070	1057
2			1880	1459	2204	1323	2791	1910	2305
3			3000	1968	2791	1764	2791	2791	2305
4			3000	2477	2791	2204	2791	2791	2305
5			3000	2791	2791	2645	2791	2791	2305
1	44	30	1240	954	1328	885	2479	1125	1301
2			2050	1465	2214	1328	4000	2051	3091
3			3200	1976	3099	1771	4000	3223	3479
4			4150	2488	3985	2214	4000	4000	3479
5			4500	2999	4000	2656	4000	4000	3479
1	46	30	1300	958	1334	889	2972	1185	1623
2			2450	1471	2223	1334	5876	2209	4145
3			3500	1985	3112	1778	5876	3515	5143
4			5250	2498	4001	2223	5876	5104	5143
5			5800	3011	4890	2667	5876	5876	5143
1	48	30	1400	958	1334	889	3474	1428	2042
2			2600	1471	2223	1334	8837	2857	5523
3			3850	1985	3112	1778	8837	4730	7674
4			6000	2498	4001	2223	8837	7048	7674
5			8600	3011	4890	2667	8837	8837	7674

In the existence of surcharge, γB, the present study does not follow a regular pattern as in tables 5 and 6. Nevertheless, the bearing capacity values are in a good comparison with Hanna (1981) for most cases with deviation less than 10% and a few other cases less than 15%. The weighted average method (Bowles 1988) overestimate the bearing capacity for friction angle more than 40 degrees, while projected area method underestimates the bearing capacity. The bearing capacity calculated based on Ghazavi & Eghbal (2008) does not show regular pattern. The results are more or less than Hanna (1981) when friction angle and thickness of upper layer changes. This method provides unsafe bearing capacity estimation for some cases in which the bearing capacity is almost twice the bearing capacity calculated from Hanna's equation.

4.5. Comparison with Numerical Studies

The numerical results are compared to Szypcio & Dolžyk (2006) and Ghazavi & Eghbal (2008). Figure 4.12 presents the difference between the present numerical estimation of the bearing capacity and Szypcio & Dolžyk (2006). The friction angle of upper and lower layer is 32 and 29.5, respectively. The present study gives the bearing capacity by maximum discrepancy of 13%.

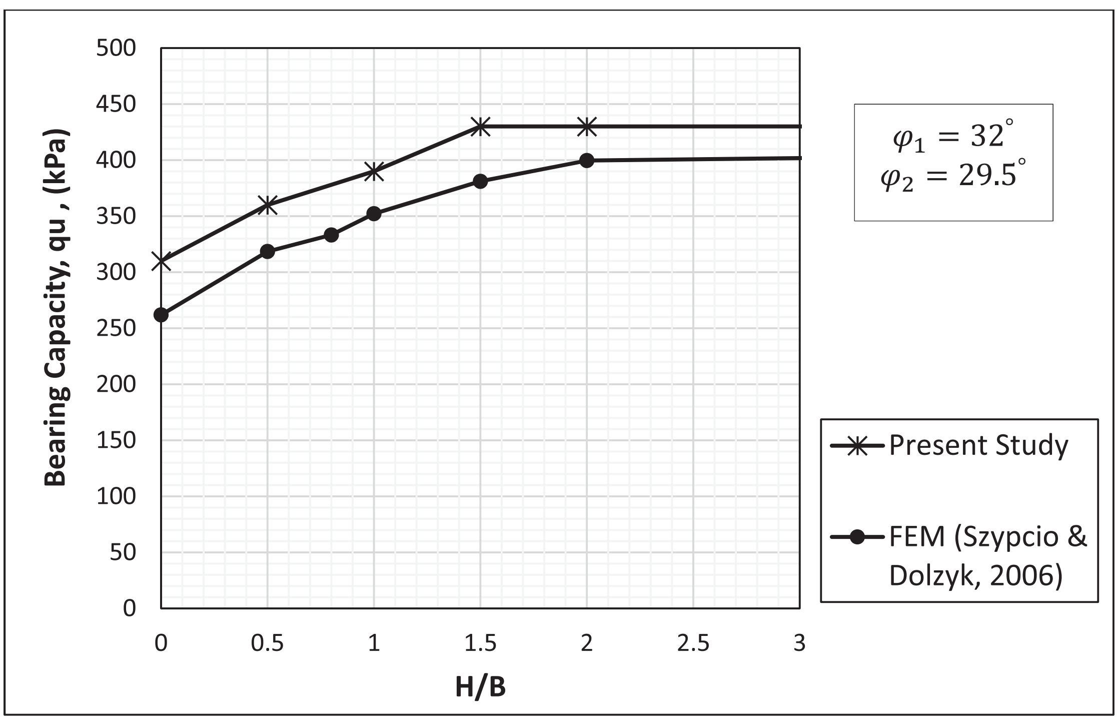

Figure 4.12 Test results vs numerical study conducted by (Szypcio & Dolžyk, 2006)

Ghazavi & Eghbal (2008) presented the bearing capacity of two layered soil when the footing width changes from 1m to 3m. The comparison between the current numerical model and Ghazavi & Eghbal (2008) are displayed in Table 9.

The bearing capacity increases by increase of upper layer thickness from 0.25B to 1.75B. The bearing capacity from the present study is more than Ghazavi & Eghbal (2008) for all soil shear strength combinations, top layer thicknesses and footing widths. The reason might be the fact that they applied the load due to the embedment depth of footing as surcharge load on the surface of soil. While, in this study the footing is embedded in the depth of d and the whole soil body is modeled. Nonetheless, the maximum difference between the results is almost 17%. They predicted the effective thickness of upper layer to be 1.75B, while in the present study it is 1B.

Table 9 Bearing capacity of strip footing, Numerical Study by (Szypcio & Dołżyk, 2006)

H/B	B (m)	q_{ult} (kPa)			
		$\varphi_1 = 39, \quad \varphi_2 = 36$		$\varphi_1 = 34, \quad \varphi_2 = 31$	
		Ghazavi & Eghbal (2008)	**Present Study**	**Ghazavi & Eghbal (2008)**	**Present Study**
0.25	1	1012	1000	507	500
0.5		1092	1150	538	560
0.75		1118	1180	576	580
1		1143	1340	592	630
1.5		1295	1420	649	720
1.75		1298	1420	-	720
0.25	2	1734	1900	869	900
0.5		1801	2100	902	960
0.75		1968	2200	1008	1040
1		2013	2300	1036	1160
1.5		2155	2500	1108	1320
1.75		2176	2500	-	1320
0.25	3	2558	2800	1240	1380
0.5		2714	2950	1339	1500
0.75		2804	3200	1460	1600
1		2915	3400	1545	1640
1.5		3230	3900	1648	1900
1.75		3310	3900	-	-

5. CONCLUSION

5.1. General

The bearing capacity of strip footing on strong sand overlying loose sand was investigated numerically. The effective length and depth of numerical model were presented as a function of footing width. A parametric study was performed to investigate the effects of shear strength of dense sand and thickness of upper layer on the bearing capacity of the system. The presented numerical model results were compared to various experimental, analytical and other numerical studies. The followings were concluded:

1. Bearing capacity of footing on dense sand overlying loose increases with the increase of the upper layer thickness up to a maximum of the bearing capacity of the homogeneous upper layer sand. Also it increases with the increase of the upper layer shear strength.
2. Based on the analysis of the boundary, of the numerical model, the appropriate values for the length and depth of the mesh were taken as 20B and 14B respectively.
3. The bearing capacity of footing on dense sand overlying loose sand increases with the increase of H/B ratio in a non-linear manner.
4. Increasing the H/B ratio will increase the stiffness of soil system. In other words, the higher the thickness of upper layer the less settlement occurs at failure. This fact is an advantage when the allowable bearing capacity is calculated based on settlement restriction.
5. It can be said that when d/B is zero, the effective thickness of upper sand layer with $\varphi = 30^{\circ}\ to\ 35^{\circ}$ is almost 1.5B and with $\varphi = 35^{\circ}\ to\ 40^{\circ}$ is almost 2B, for dense sand with friction angle in between $40^{\circ}\ to\ 44^{\circ}$ is approximately 3B and for $\varphi = 46^{\circ}\ to\ 48^{\circ}$ is 4B.
6. It can be said that when d/B is 1, the effective thickness of upper layer with friction angle between $30^{\circ}\ to\ 36^{\circ}$ is almost 2 B, for friction angle between $36°\ to\ 42°$ is 3B, and for friction angle between $42°\ to\ 48°$ is 5B.
7. The numerical model generally predicts the bearing capacity which is in good agreement with other methods. Nevertheless, for top layer with friction angle more than 46 degrees, the numerical convergence is difficult. It is numerically troublesome to redistribute the unbalanced forces when failure occurs. Hence, the load-settlement curve is not fully plastic and recognizing the soil bearing capacity from the curve is difficult and accompanied by error.

8. For embedded footings of d/B=1, the numerical model shows a very good agreement with Hanna's equation such that for most cases the difference is less than 10%. For d/B=0, the numerical model predicts the bearing capacity values less than Hanna (1981) for all thicknesses and for all values of upper layer friction angle between 32 to 48 degrees. Although, the surface footing (d/B=0) is rarely used in engineering practice.
9. When the friction angle of upper layer is more than 40 degrees, the weighted average method presented by Bowles (1988) overestimates the bearing capacity and the projected area methods (Yamaguchi, 1963; Myslivec & Kysela, 1978; Kraft & Helfrich, 1983) underestimates it.
10. The equation presented by Ghazavi & Eghbal (2008) for some cases provides unsafe bearing capacity estimation in which the calculated bearing capacity is almost twice the bearing capacity from Hanna's equation.
11. The bearing capacity prediction from the present study is slightly more than the results of numerical analysis by Szypcio & Dolžyk (2006) and Ghazavi & Eghbal (2008). The reason might be the fact that they applied a surcharge load to the model as the weight of soil due to the embedment depth of footing. While, in this study the footing was embedded in the depth of d.
12. It can be said that although the numerical model provides good results in comparison to the other studies, the process of numerically modeling the problem and interpreting the results might be slightly difficult. Hence for engineering purpose, it is recommended to use the equation proposed by Hanna (1981) to calculate the bearing capacity of dense sand over loose sand. Because the equation provides good results among the other bearing capacity equations which have been developed up to the present time.

5.2. Recommendations for Future Work

1. Modeling the footing by defining concrete material and interface material between the base of concrete and the soil surface. Thereafter, conducting a sensitivity analysis on the effect of interface material in predicting the bearing capacity.
2. Studying the effect of shear strength of lower layer on the bearing capacity calculations.
3. Studying the effect of presence of water table and the depth of it in calculating the bearing capacity of layered soil.
4. Extend the two layered soil model for all combination of loose or dense sand stiff or soft clay.

5. Extend the model to calculate the bearing capacity of multilayered soil.
6. Extend the model to consider the effects of eccentric loads applied to circular or square shape footings.

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