Optimum Design of Precast Bridge Systems Prestressed with Carbon Fiber Reinforced Polymers

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ABSTRACT

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The use of advanced composite materials (FRP) is rapidly growing and has exciting applications in bridge construction. These materials are light, have an excellent resistance to corrosion, and high strength. Because of these advantageous properties, structures reinforced with composite materials are lighter and have longer service life than those reinforced with steel.

On the other hand, the composite materials are very expensive as compared to steel. However, the severity of this problem can be minimized by using optimization techniques that achieve optimum design with minimum cost.

In this research, an optimization technique is used to minimize the superstructure cost of two types of bridge systems (slab-on-CPCI girder bridges and Nebraska all-precast bridge systems) reinforced or prestressed (pretensioned) with Carbon Fiber Reinforced Polymers (CFRP).

The objective of this research is to show that optimization techniques are the most efficient tools in dealing with this kind of tedious design procedure (design of prestressed concrete bridges) that is normally an iterative process based on trial and error. Furthermore, the technique adopted in this investigation can be used for minimizing the cost of structures reinforced with FRP. As a result, these tools can be used in developing design aids that can be used to achieve optimum bridge designs.

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A	Cross sectional area of concrete
A'_{ss}	Area of non-prestressed reinforcing bars in compression.
A_c	Area of the transformed composite section using $E_{ref} = E_{cg}$
A_{cg}	Cross-sectional area of the precast girder.
A_{cs}	Cross-sectional area of the deck slab supported by one girder
A_g	Area of the transformed precast girder cross-section
A_{ps}	Area of prestressing tendons.
A_{sg}	Area of non-prestressed reinforcing bars in tension.
a	Depth of equivalent rectangular stress block at ultimate.
B	First moment of area of the transformed section about an axis through O
B_c	First moment of area of the transformed composite section using $E_{ref} = E_{cg}$ about an axis through O.
B_g	First moment of area of the transformed precast girder cross-section about an axis through O
b_{eff}	Effective flange width for interior girders.
b_{fg}	Width of the top flange of the precast girder.
C_{cg}	Cost of concrete in the girder per unit volume including cost of materials, production, transportation and erection
C_{cs}	Cost of concrete in the slab per unit volume
C_{ns}	Cost of non-prestressed steel per unit mass
C_f	Coefficients calculated from equations given in Table 3.2 (OHBDC, 1991)
C_{np}	Cost of non-prestressed CFRP reinforcement per unit length
C_p	Cost of prestressed CFRP tendons per unit length
C_{ps}	Cost of prestressed steel per unit length
c	Depth of the compression zone.

D Depth of compression zone coefficients calculated from equations given in Table 3.2 (OHBDC, 1991)

$d\varepsilon/dy$ Slope of the strain diagram.

d'_{ss} Depth of the compression nonprestressing reinforcement in the deck slab from the extreme top fiber of the composite section.

DLA Dynamic load allowance to account for dynamic impact effect of vehicles given in Table 3.3 according to OHBDC (1991).

d_{ps} Depth of prestressing tendons.

d_{psc} Depth of prestressing tendon from the extreme top fiber of the composite section at the mid-span sections

d_{sg} Depth of nonprestressed reinforcement in tension.

E Modulus of elasticity of concrete or the reinforcement

E_c Elastic modulus of normal weight concrete

E_{cg} Modulus of elasticity of the concrete girder

E_{ci} Modulus of elasticity of concrete part i (precast girder or deck slab)

E_{cs} Modulus of elasticity of the deck slab

E_{ref} Reference modulus of elasticity, which could be taken as one of the concrete parts of the cross section

E_{rj} Modulus of elasticity of reinforcement layer j.

f_c' Concrete compressive strength in either girder or deck slab at 28 days.

f'_{cg} Compressive strength of concrete in girders at 28 days.

f'_{cig} Compressive strength of concrete at the time of prestress transfer

f_{cs}' Compressive strength of concrete in deck slab at 28 days.

f_s' Compressive stress in the reinforcing steel bars.

f_{pe} Effective stress in the prestressed tendons.

f_{ps} Stress in the prestressing tendons at factored flexural resistance.

f_{pu} Ultimate tensile strength of the prestressed tendons.

f_r Modulus of rupture of concrete ($= 0.6\sqrt{f'_{cg}}$)

f_s Tensile stress in the reinforcing bars.

f_y Yield strength of nonprestressed steel reinforcement.

I Second moment of area of the transformed section about an axis through O

I_c Second moment of area of the transformed composite section using $E_{ref} = E_{cg}$ about an axis through O.

I_g Second moment of area of the transformed precast girder cross-section about an axis through O.

k_p A constant calculated from Equation (3.33).

L Span length.

M Bending moment about an axis through O

M_{cr} Total moment that would be required to produce cracking

M_{dg} Moment at mid-span due to girder's self-weight

M_{ds} Moment at mid-span due to slab self-weight

M_f Factored load positive moment.

M_L Design live load moment for an interior girder of the bridge

$M_{L.\max}$ Maximum moment due to live load at mid-span section

$M_{\max}$ Maximum moment at the mid-span section

M_r Flexural moment resistance of the mid-span section.

M_{SDL} Moment at mid-span [Equation (3.4)] due to superimposed dead load

N Axial force at an axis through O

n Total number of design lanes (see Table 3.1)

n_g Number of girders

n_{np} Number of non-prestressed CFRP bars

n_p	Number of prestressed CFRP tendons
n_{ps}	Number of prestressed steel tendons
O	Reference point along the vertical axis of symmetry of the cross section.
P_e	Final effective force in the prestressed tendon
P_i	Absolute value of the initial prestressing force
S	Spacing centre-to-centre between the precast girders
S/D_d	Moment distribution factor among the bridge girders.
S'	Extension of the slab beyond the centre line of the exterior girder
t	Slab thickness.
V_{cg}	Volume of concrete in the girder
V_{cs}	Volume of concrete in the slab
W_c	Width of bridge deck
W_e	Width of design lane
W_{Girder}	Load due to girder self-weight
W_{ns}	Weight of non-prestressed steel in the girder
W_{SDL}	Superimposed dead load
W_{Slab}	Load due to weight of deck slab
X_1	Design variable assigned for the concrete compressive strength of the precast girder at age 28 days
X_2	Design variable assigned for the required amount of prestressing reinforcement in the precast girder
X_3	Design variable assigned for the eccentricity of the prestressing reinforcement in the precast girder at mid-span section
X_4	Design variable assigned for the eccentricity of the prestressing reinforcement in the precast girder at the support section
X_5	Design variable assigned for the required amounts of the non-prestressed flexural reinforcement in the deck slab and/or the precast girder
X_i	The *i*th design variable.

X_i^U Upper bound on the design variable *i*.

X_i^L Lower bound on the design variable *i*.

y Distance from reference point O to the fiber where the strain and stress are to be calculated

y_{bg} y-coordinate of the bottom fiber from the reference point O.

$y_{ps\,1\,or\,3}$ Depth of the prestressed tendon from the reference point O at the support sections

$y_{ps\,2}$ Depth of the prestressed tendon from the reference point O at the mid-span sections

y_{ts} y-coordinate of the top fiber of the slab

α_1, β_1 Coefficients for the equivalent rectangular compressive stress block at ultimate.

γ_{cg} Specific weight of concrete in the girder

γ_{cs} Specific weight of concrete in the deck slab

δ Deflection at mid-span

$\delta(x)$ Deflection at any point x along a member

$\delta_{Transfer}$ Maximum mid-span deflection constraint at prestressing transfer

$\delta_{Construction}$ Maximum mid-span deflection constraint during construction

$\delta_{Service}$ Maximum mid-span deflection constraint at service

ΔM Change in moment about O

ΔM_{cr} Additional moment to produce cracking at the critical section

ΔN Change in axial force

$\Delta \psi$ Change in curvature

$\Delta \varepsilon$ Change in strain at any fiber at coordinate y from O

$\Delta \sigma_{bot.girder}$ Changes in stress at the extreme bottom fiber of the precast girder at any of the critical sections

$\Delta \sigma_{bot.slab}$ Changes in stress at the extreme bottom fiber of the slab at any of the critical sections

$\Delta\varepsilon_O$	Change in axial strain at a reference point O
$\Delta\sigma_{top.girder}$	Changes in stress at the extreme top fiber of the precast girder at any of the critical sections
$\Delta\sigma_{top.slab}$	Changes in stress at the extreme top fiber of the slab at any of the critical sections
ε	Train stress at any distance y from reference point O
ε_O	Axial strain at a reference point O
ϕ_c	Concrete strength reduction factor.
ϕ_p	Prestressed reinforcement strength reduction factor.
ϕ_s	Nonprestressed reinforcement strength reduction factor.
ψ	Curvature
$\psi(x)$	Curvature at any point x along a member
ψ_1 and ψ_3	Curvature at each end of the span
ψ_2	Curvature at mid-span
σ	Tensile stress at any distance y from reference point O
$\sigma_{bg.total}$	Total stress at the bottom fiber of the precast girder at mid-span
σ_{bot}	Stress in concrete at the bottom fibers at any of the critical sections
$\sigma^{b}_{midspan}$	Stress at the bottom fiber of precast girder at mid-span section at transfer or during construction.
$\sigma^{b}_{Support}$	Stress at the bottom fiber of precast girder at support section at transfer or during construction.
$\sigma^{bc}_{Support}$	Stress at the bottom fiber of precast girder at support section at service
$\sigma^{bg}_{midspan}$	Stress at the bottom fiber of precast girder at mid-span section at service
$\sigma^{t}_{midspan}$	Stress at the top fiber of the precast girder at mid-span section at transfer or during construction.
$\sigma^{t}_{Support}$	Stress at the top fiber of the precast girder at support section at transfer or during construction.
$\sigma^{tc}_{midspan}$	Stress at the top fiber the composite section at mid-span section at service

$\sigma^{tc}_{Support}$ Stress at the top fiber of the composite at support section at service

$\sigma^{tg}_{midspan}$ Stress at the top fiber of the precast girder at mid-span section at service.

σ_{top} Stress in concrete at the top fibers at any of the critical sections

CHAPTER 1

INTRODUCTION

1.1 General

Use of advanced composite materials such as Glass Fiber Reinforced Polymers (GFRP) and Carbon Fiber Reinforced Polymers (CFRP) in bridges and other structures has been growing rapidly over the past few years. These materials have superior corrosion resistance and their use offers an excellent solution to overcome the corrosion problems of steel and the associated durability problems. Recently, there has been a tremendous amount of research towards the use of Fiber Reinforced Polymers (FRP) in the form of reinforcing bars, prestressed cables, plates or fabric sheets replacing conventional steel in construction of new structures or rehabilitation and strengthening of existing structures. In addition to their excellent non-corrosive characteristics, FRP reinforcement has high strength-to-weight ratio that can provide high prestressing forces without additional weight to the structure. It also has good fatigue properties and low relaxation losses, which can increase the service life and the load carrying capacity of the concrete structures.

The major disadvantages of FRP reinforcement, however, are its high cost as compared to steel and its linear elastic response up to failure, which leads to brittle failure of the concrete members reinforced with this material. One avenue to overcome these two disadvantages is to optimize the design in order to minimize the cost of the structure and to provide the appropriate amount of FRP reinforcement that would ensure a tension failure rather than a brittle compression failure. This investigation is an effort towards achieving this goal. Its emphasis is on the optimum design of precast concrete bridges reinforced or prestressed with FRP reinforcement. Particular attention is given to the minimum cost of the superstructure.

Concrete bridges can be fully or partially prestressed with pretensioned or post-tensioned tendons. This research deals with two systems of precast partially prestressed bridges. The first bridge system consists of standard precast I-girders placed apart under a cast-in-place concrete deck slab. The I-girders are designed according to the specifications of the Canadian Prestressed Concrete Institute (CPCI) and the American Association of State Highway and Transportation Officials (AASHTO) and are frequently used in North America in highway bridges of spans up to 50 m. The second bridge system consists of precast girders of shallow depth placed side by side and covered with the asphalt wearing surface to form the bridge deck. Three new cross-section shapes, which are suitable for short bridges of spans up to 15 m, have been recently proposed by the University of Nebraska-Lincoln (Kamel, 1996).

In partially prestressed concrete bridges, the prestressing reinforcement is supplemented by non-prestressed rebars for the following purposes (Gilbert and Mickleborought, 1990):

1. To increase the flexural strength and to provide additional ductility (in case of steel) in the regions where sufficient strength and ductility are not provided by the prestressing alone.
2. To control flexural cracking at service loads

3. To control shrinkage and temperature cracking in regions and directions of low (or no) prestressing.
4. To carry part of the compressive forces in heavily reinforced members or in regions where the concrete alone may not be adequate.
5. To reduce camber (upward deflection) due to prestressing in early stages, when placed in the top of the concrete member.

The structural design process usually consists of four major stages: formulation of functional requirements, the conceptual design stage, design optimization, and detailing. A significant progress has been made in the area of structural optimization as a result of development in structural analysis, digital computer, and optimization techniques. Optimization techniques are generally divided into two major categories: analytical methods and numerical techniques. In the analytical methods, the mathematical theory of calculus and variational methods are applied, whereas in the numerical techniques, computational and programming methods are employed. Much research has been done and many studies have been published on the application of numerical methods in the design of different types of concrete structures. Much of the work has been, in particular, on the design optimization of partially pretensioned and post-tensioned bridge girders reinforced with conventional prestressed and non-prestressed steel. Most of the girder sections investigated were of standard type. Virtually no research has been done on bridges prestressed or reinforced with FRP reinforcement. Therefore, there is a need to extend the research to include non-standard sections such as those proposed by Nebraska, and to consider the effects of use of FRP reinforcement on the design cost of precast girder bridges.

1.2 Objectives and Scope

The main objective of this thesis is to investigate the impact of the high cost of FRP reinforcement on the design cost of precast bridge systems prestressed and reinforced with this type of material and to provide an optimum design solution to minimize the total cost of such structures. More specifically, the objectives of the thesis are:

1. To implement an optimization technique in order to achieve the minimum design cost for bridge superstructures built up of CPCI standard precast I-girders supporting a cast-in-place slab and prestressed and reinforced with Carbon Fiber Reinforced Polymers (CFRP) reinforcement (Figure 1.1).
2. To implement an optimum design that minimizes the cost of bridge superstructure composed of the new Nebraska-proposed precast sections when reinforced and prestressed with CFRP reinforcement.
3. To compare the optimum design and minimum cost of the two bridge systems mentioned above to the case when reinforced and prestressed with conventional steel.

In order to achieve the above objective, a user-friendly and accessible optimization software tool called Excel Solver, developed by Microsoft Inc. and based on a nonlinear optimization technique known as the Generalized Reduced Gradient (GRG2) has been utilized. The software is supported by Visual Basic for Applications (VBA) that allows the user to incorporate his/her own defined functions and subroutines.

1.3 Outline of Thesis

This thesis is composed of six chapters. Chapter 2 contains a review of the literature that deals with the various economic studies related to minimum cost in the

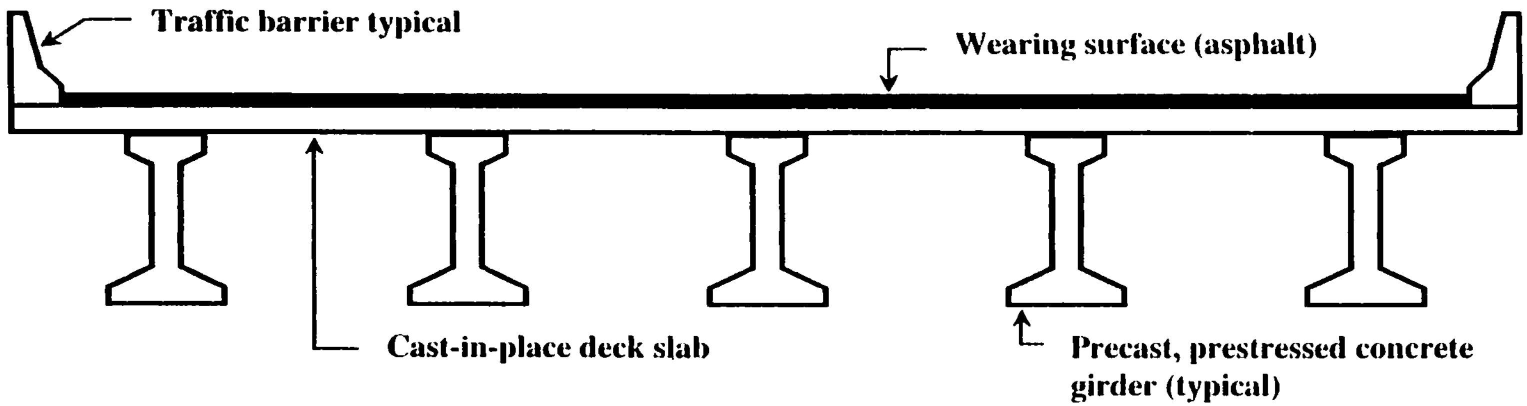

Figure 1.1 Typical Slab-on-Girder Bridge Cross Section

design of precast, prestressed concrete bridge structures. In Chapter 3, equations for determining the design moments due to live load at the critical section in simple span bridges are presented. In case of slab-on-girder bridges, an empirical method for slab design that is not based on conventional analysis is presented. The analysis and design for the requirements of serviceability and ultimate limit states are considered. Chapter 4 includes the design variables, the objective function and the design constraints required to achieve an optimum design. The mathematical formulation of a standard design optimization model is briefly presented. The Excel 97-Solver software utilized for the solution of the optimum design problem is described. In chapter 5, the findings of the optimum design and minimum cost investigation of the two bridge systems considered in this research are presented and discussed. In chapter 6, a summary and the conclusion of this study along with the recommendations for further research are listed.

CHAPTER 2

LITERATURE REVIEW

2.1 General

As mentioned in previous chapter, almost no research has been done to date on optimization of the design of concrete bridges or other structures reinforced or prestressed with FRP reinforcement. This is basically because the application of FRP as reinforcement in prestressed concrete structures is by itself a fairly new field and no standards or specifications are yet available for the design of this type of structure. Extensive efforts, however, are currently being made worldwide to develop codes and specifications for the design of FRP reinforced and prestressed concrete structures. A review of these code development efforts can be found in Gilstrap et al. (1997). Recent research has shown that the criteria and techniques currently used in the design of concrete structures reinforced with conventional steel can be also adopted, with some modifications, for the design of FRP reinforced concrete structures. Accordingly, design optimization techniques of structures reinforced with steel can also be applicable to those reinforced with FRP.

In this chapter, a review of the previous work and the different approaches to solving the optimal design of precast, prestressed concrete bridge girders is presented. The review is divided into parts: one part is on the work related to simple span bridges and the other is on the work dealing with continuous structures.

2.2 Design Optimization of Simple Span Bridges

One of the earliest work on cost optimization of structures was done by Torres et al. (1966). They presented minimum cost design of prestressed concrete highway bridges subjected to AASHTO loading by using a piecewise Linear Programming (LP) method. The independent design variables were the number and depth of girders, prestressing force, and steel tendon eccentricity. They further defined dependent design variables as the spacing of girders, the steel tendon cross-sectional area, the initial prestress, and the slab thickness and reinforcement. They claimed that their cost function includes the costs of transportation, erection, and bearings in addition to the material costs of concrete and steel. They presented results for bridges of 6.1 to 33.5 m spans and 7.6 m and 15.2 m deck widths.

Goble and Lapay (1971) minimized the cost of post-tensioned prestressed concrete T-section beams based on the ACI 318-63 Building code. They adopted the gradient projection method. Description of this method can be found in Arora (1989). Goble and Labay stated that the optimum design seems to be unaffected by changes in the cost coefficients. This conclusion was rebutted by subsequent researchers as will be discussed later in this chapter.

Naaman (1976) made a comparison between minimum cost designs with minimum weight designs for simply-supported prestressed rectangular beams and one-way slabs based on the ACI 318-71 Building code. The cost designs were optimized by a direct search technique (Siddall 1972). Naaman concluded that the minimum weight and minimum cost solutions give approximately similar results only when the ratio of cost of concrete per cubic yard to the cost of prestressing steel per pound is more than 60.

Otherwise, the minimum cost approach yields a more economical solution, and, for ratios much smaller than 60, the cost optimization approach yields substantially more economical solutions. Naaman pointed out also that, for most projects in the United States, the aforementioned ratio is less than 60.

Rabbat, Takayanagi, and Russell (1981) evaluated the design of prestressed concrete bridge girders used in the USA and determined which represented optimum designs that could be promoted as national or regional standards. However, the investigation was limited to bridges built with pre-tensioned I- and T-sections using steel tendons, for spans in excess of 24.4 m, and with concrete compressive strengths up to 48.3 MPa. They concluded that, except for the State of California, the most economical bridges for spans of approximately 21.3 to 39.6 m were constructed with pre-tensioned bridge girders. In California, cast-in-place, post-tensioned box girder bridges were most economical. The AASHTO standard bridge girders were not the most structurally efficient or cost-effective for spans of 24.4 to 42.7 m. Because of transportation restrictions, maximum spans made of single units were limited to about 42.7 m. Longer spans were possible by splicing girders. Intermediate diaphragms were not needed and end diaphragms were sufficient.

For girders with 127 mm thick webs, the most cost-effective sections were Bulb-T's. For spans varying from 24.4 to 36.6 m, Bulb-T's have 20% less in-place cost of girder and deck compared to AASHTO girders. For spans of 36.6 to 41.2 m, the cost reduction for Bulb-T's varied from 20% to 5%. The next most cost-effective sections with 127 mm thick webs were the Washington series. In most regions of the United States, it may not be easy to consolidate the concrete in girders with 5-in (127 mm) thick webs. Moreover, in these girders, steel strands must be bundled at midspan and end blocks were needed to comply with, minimum concrete cover requirements. By using girders with, 152 mm thick webs, it would be possible to consolidate the concrete in these girders in all regions of the United States. Use of 152 mm thick webs instead of 127 mm in Bulb-T's, Washington series, and Colorado G68 girders increased overall in-place cost of girder and deck by 3% to 5%. For girders with 152 mm thick webs, most cost-effective sections were the modified Bulb-T's. For spans of 24.4 to 36.6 m, Modified Bulb-T's had 17% less in-place cost of girder and deck compared to AASHTO girders. For spans of 16.6 to 42.7 m, the cost reduction varied from 17% to 2%.

Next to Modified Bulb-T's, modified Washington Series girders with 152 mm thick webs were the most cost-effective sections. For spans from 24.4 to 36.6 m, overall reduction of the in-place cost of girder and deck was 14%. For spans of 36.6 to 42.7 m, cost reduction ranged from 14-2% compared to the AASHTO girders. Reduction of top and bottom flange widths and web thickness of AASHTO Type IV, V and, VI girders by 50.8 mm reduced overall in-place cost of girders and deck by about 6%. The span capability of the modified sections was not affected by the change in width. The overall in-place cost of girders and deck was decreased substantially by placing girders at the maximum practical girder spacing. Increase of girder's concrete compressive strength from 34.5 to 48.3 MPa increased the span capability of AASHTO girders by about 15%. Bundling of steel strands at midspan in order to increase eccentricity of the prestress did not lead to any significant overall cost reduction for the girders considered.

MacRae and Cohn (1984a) considered the minimum cost design of simply-supported RC and partially or fully pretensioned and post-tensioned concrete beams of fixed cross-sectional geometry subjected to serviceability and ultimate limit states constraints. Constraints on flexural strength, deflection, ductility, fatigue, cracking, and minimum reinforcement, based on the ACI 318-77 Building code or the Canadian building code CSA A23.3-77 (1977), were included in the investigation. The feasible conjugate-direction method was adopted. A description of this method can be found in Kirsch (1993). The beam can be of any cross-sectional shape subjected to distributed and concentrated loads. For the examples considered, MacRae and Cohn concluded that, for post-tensioned members, partial prestressing appears to be more economical than full prestressing for prestressing-to-reinforcing steel cost ratio greater than 4. For pretensioned beams, on the other hand, complete prestressing seems to be the best solution. For partially prestressed concrete, they also concluded that, for prestressing-to-reinforcing steel cost ratio in the range of 0.5 to 6, the optimal solutions vary slightly.

MacRae and Cohn (1984b) further performed parametric studies on 240 simply supported reinforced, partially or fully pre- and post-tensioned prestressed concrete beams with different dimensions, depth-to-span ratios, and live load intensities. They concluded that, in general, RC beams were the most cost-effective at high depth-to-span ratios and low live load intensities. On the other hand, fully prestressed beams were the most cost-effective at low depth-to-span ratios and high live load intensities. For intermediate values, partial prestressing was the most cost-effective option.

Saouma and Murad (1984) presented minimum cost design of simply supported uniformly loaded, partially prestressed I-shaped beams with unequal flanges subjected to the constraints of the ACI 318-77 Building code. The optimization problem was formulated in terms of nine design variables: six geometrical variables plus areas of tension, compression, and prestressed steel. The constrained optimization problem was transformed to an unconstrained optimization problem using the interior penalty function method (Kirsch, 1993) and solved by the quasi-Newton method (Vanderplaats, 1984) included in the International Mathematical Statistical Library (IMSL, 1980). Saouma and Murad found the optimum solutions for several beams with spans ranging from 6 m to 42 m, assuming both cracked and uncracked sections, and reported cost reductions in the range of 5-52%. They also concluded that allowing cracking to occur did not reduce the cost by any significant measure.

Jones (1985) used integer programming to formulate minimum cost design of precast, prestressed concrete simply supported box girders used in a multi-beam highway bridge and subjected to the Standard Specifications for Highway Bridges (AASHTO, 1977) loading, assuming that the cross-sectional geometry and the gridwork of strands were given and fixed. The design variables were the concrete strength and the number, location, and draping of strands (moving the strands up at the end of the beam). The constraints used were the stresses at the time of release and at service load level, ultimate moment capacity, cracking moment, and camber.

Yu et al. (1986) presented minimum cost design of a prestressed concrete box girder bridge constructed by balanced cantilever method (the bridge consisted of two end cantilever and overhanging spans and one middle simple span) based on the British code of practice (CP110, 1976). They adopted the general geometric programming approach (Beightler and Phillips 1976; Abuyounes and Adeli 1986). The cost function included the material cost of concrete, prestressing steel, and the metal formwork. They also included the labor cost of the metal formwork as roughly equal to 1.5 times the cost of the material of the formwork. The design variables were the prestressing forces, the eccentricities, and the girder depths for both spans.

Khaleel and Itani (1993) presented minimum cost design of simply supported partially prestressed concrete unsymmetrical I-shaped girders as per ACI 318-83 Building code. The sequential quadratic programming method was used to solve the nonlinear optimization problem assuming both cracked and uncracked sections. They conclude that an increase in the concrete strength does not reduce the optimum cost significantly, and that a higher strength in the prestressing steel reduces the optimum cost to a certain extent. They stated that some amount of reinforcing steel facilitates the development of cracking in the concrete, which reduces the cost of materials and improves ductility.

Lounis and Cohn (1993b) presented a multiobjective optimization formulation for minimizing the cost and maximizing the initial camber of post-tensioned floor slabs with serviceability and ultimate limit state constraints of ACI 318-89 Building code. The cost objective function was chosen as the primary objective, and the camber objective function was transformed into a constraint with specified lower and upper bounds. The resulting single optimization problem was then solved by the projected Lagrangian method.

Han et al. (1995) discussed minimum cost design of partially prestressed concrete rectangular and T-shaped beams based on the Australian standard for concrete structures (AS 3600-1988) using the Discretized Continuum-type Optimality Criteria [DCOC] method. They concluded that, for a simply supported beam, a T-shape is more economical than a rectangular section.

Fereig (1996) presented minimum cost preliminary design of single-span bridge structures consisting of cast-in-place RC deck and girders based on the Standard Specifications for Highway Bridges (AASHTO, 1992). Fereig linearized the problem by approximating the nonlinear constraints by straight lines and solved the resulting linear problem by the simplex method. He concluded that it was always more economical to space the girder at the maximum practical spacing.

In the previous research mentioned above, initial camber and deflection constraints were considered in some of the investigations. For example, McRae and Cohn (1984) considered the constraints on flexural strength, deflection, ductility, fatigue, cracking, and minimum reinforcement. Saouma and Murad (1984) took deflection and shear into account as constraints. Jones (1985) considered the initial camber in the design constraints. Lounis and Cohn (1993) did the same, but camber, deflection and shear were checked at the final design (i.e. were not considered directly as constraints).

2.3 Design Optimization of Continuous Span Bridges

Kirsch (1972) presented minimum cost design of two-span continuous prestressed concrete beams subjected to constraints on the stresses, prestressing force, and the vertical coordinates of the tendon by linearizing the nonlinear optimization problem approximately and solving the reduced linear problem by the linear programming (LP) method.

Lounis and Cohn (1993a) presented minimum cost design of short and medium span highway bridges consisting of RC slabs on precast, post-tensioned, prestressed concrete I-girders satisfying the serviceability and ultimate limit state constraints of the

Ontario Highway Bridge Code (Ontario 1983) using a three-level optimization approach. In the first level, they dealt with optimization of the bridge components including dimensions of the girder cross-sections, slab thickness, amounts of reinforcing and prestressing steel, and tendon eccentricities. They employed the projected Lagrangian method (Haftka and Gurdal, 1992). In the second level, they considered the optimization of the longitudinal layout, such as number of spans, restraint type, span length ratios, and the transverse layout, such as number of girders and slab overhanging length. In the third level, they considered various structural systems, such as solid or voided slabs on precast I- or box-girders. They used a sieve-search technique (Kirsch, 1993) for the second and third levels of optimization. Their cost function included the material cost of concrete, reinforcement, and connections at the piers. They also included the costs of fabrication, transportation, and erection of girders, approximately assuming a constant value per length of the girder. They concluded that optimizing a complete bridge system results in a more economical structure than optimizing the individual components of the bridge. Based on their optimization studies, simply supported girders for prestressed concrete bridges of span up to 27 m, two-span continuous girders for span lengths from 28 m to 44 m, three-span continuous girders for span lengths from 55 m to 100 m, and two- or three-span continuous girders for the intermediate range of 44 m to 55 m.

Cohn and Lounis (1994) applied the above three-level cost optimization approach to multiobjective optimization of partially and fully prestressed concrete highway bridges with span lengths of 10-15 m and widths of 8-16 m. Their objective functions included the minimum superstructure cost, minimum weight of prestressing steel, minimum volume of concrete, maximum girder spacing, minimum superstructure depth, maximum span-to-depth ratio, maximum feasible span length, and minimum superstructure camber. For a four-lane 20 m length single-span bridge, they concluded that the voided slab and the precast I-girder systems are more economical than the solid slab and one- and two-cell box girders.

Lounis and Cohn (1995a) also concluded that voided slab decks were more economical than box girders for short spans (less than 20 m) and wide decks (greater than 12 m), and that single-cell box girders were more economical for medium spans (more than 20 m) and narrow decks (less than 12 m). The single-cell box girder, however, resulted in the deepest superstructure, which may be a drawback when there is restriction on the depth of the deck. Multicriteria cost optimization of bridge structures was further discussed by Lounis and Cohn (1995b, 1996). They suggested that the criteria of minimax and minimum Euclidean distance could be used by designers for selection of the best solution.

Han et al (1996) minimized the cost of continuous, partially prestressed and singly reinforced T-beams with constant cross sections within each span based on the Australian standard for concrete structures (AS 3600-1988) using the DCOC method. A three-span and a four-span continuous beam examples were presented. They considered the material cost of concrete, reinforcement, and formwork. They concluded that optimal designs using inelastic analysis result in somewhat more economical designs.

Hassanain and Loov (1997) utilized mathematical optimization techniques to study the effect of increasing the girder concrete strength on the design of slab-on-girder continuous span bridges. Preliminary results showed that with increases in concrete strength, maximum girder span limits also increase resulting in fewer piers. Alternatively, for a given design span, increasing the concrete strength resulted in an increase in girder spacing, therefore requiring fewer girders for a given structure.

CHAPTER 3

DESIGN OF INVESTIGATED BRIDGE SYSTEMS

3.1 General

Concrete members reinforced with or without prestressing are designed to comply with ultimate strength and serviceability requirements. Safety against failure can be checked by evaluating the ultimate resistance of the member and ensuring that it is greater than the applied load effects. However to satisfy the serviceability requirements, it is important to check the stresses, strains and deflection of the member at service load conditions against the maximum allowable values specified by the codes

Stresses and strains change continuously with time due to the effects of creep and shrinkage of concrete and relaxation of prestressed reinforcement. These effects cause a redistribution of stresses between various materials within a cross section and a change in the internal forces and support reactions in statically indeterminate structures. The significance of these time-dependent effects is much more important in structures constructed in stages than in those built in one operation. Examples of such structures are slab-on-girder bridges built of concrete parts, i.e. precast prestressed girders and cast-in-place concrete deck slab, which are of different ages and properties. An accurate analysis of the time-dependent effects in multi-stage structures can be found in Elbadry and Ghali (1989). This analysis is based on equilibrium of forces and compatibility of strains and does not require a prior estimate of the prestress losses.

Design codes for concrete structures reinforced or prestressed with conventional steel are well established. On the other hand, for structures reinforced or prestressed with FRP materials, analytical and experimental investigations are sufficiently complete and efforts are being made to establish recommendations for the design with this material. Experimental data on concrete members reinforced with FRP bars has indicated that the flexural capacity can be calculated based on assumptions similar to those for members reinforced steel bars (Faza and GangaRao, 1993; Nanni, 1993; and GangaRao and Vijay, 1997). It has been also shown that serviceability considerations normally control the design of FRP reinforced members.

In this chapter, the design equations for calculation of the ultimate strength, and the analysis of the stress and deformations are presented for Slab-on-CPCI bridge girders and the proposed Nebraska all-precast bridge systems partially prestressed with FRP or conventional steel reinforcement. The underlying assumptions are also presented. For simplicity, the exact analysis of the time-dependent stress and deformations as developed by Elbadry and Ghali (1989) is not adopted; instead, a lump sum of **15%** losses in prestressing due to instantaneous and long-term effects is used.

3.2 Bridge Configuration

Only simply supported bridges with spans varying from 10 m to 30 m are considered in this investigation. Figure 3.1 shows a longitudinal section of a typical bridge. For slab-on-girder bridges, a typical cross-section is depicted in Figure 3.2, which shows the bridge components and the girder arrangement. In this type of bridge, a cast-in-situ slab is placed on top of precast girders spaced apart at a distance S centre-to-centre.

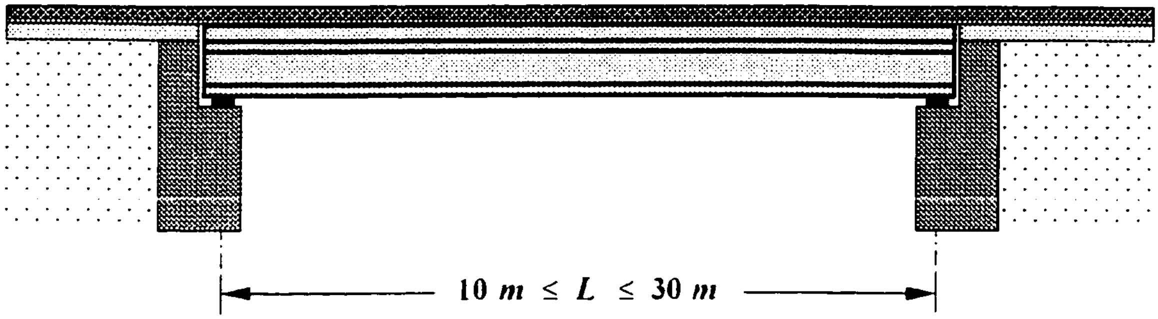

Figure 3.1 Section in Longitudinal Direction of a Simply Supported Bridge

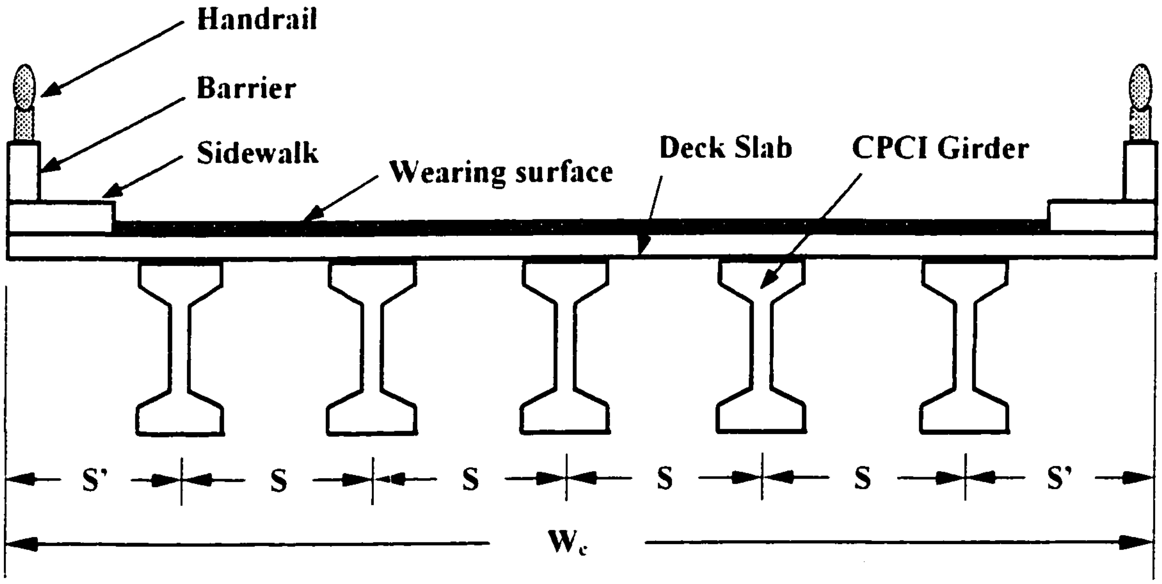

Figure 3.2 Section in Transverse Direction of a Slab-on-Girder Bridge

The total width of the bridge is W_c and the length of the slab overhanging on each side of the bridge is S'. Four CPCI girder sections are considered in this study. Their dimensions are shown in Figure 3.3.

Typical cross-sections of bridges built of proposed Nebraska all-precast girders are shown in Figure 3.4 to 3.6. Typically in this type of bridge, 6 precast units of shallow depth (610 mm) are placed side-by-side and covered with wearing surface to form the bridge deck. Three shapes of precast units are used: Bulb Tee section, Butted I-girder and Pie (π) section. Typical dimensions of the three sections are shown in Figures 3.7a, b and c, respectively.

3.3 Assumptions and Criteria

The following assumptions are made in the design and in the analysis of stresses and deformations of the bridge types considered in this investigation:

1. Plane cross-sections before deformation are assumed to remain plane after deformation. Thus, the strain in the concrete and the reinforcement is proportional to the distance from the neutral axis.
2. For slab-on-girder bridges, only unshored type of composite construction is considered. Full composite action between the cast-in-place slab and precast girders is assumed and the interface surfaces are considered intentionally roughened for proper shear transfer.
3. Prestressing tendons are assumed to be draped at two points, one fourth of the span from each support, as shown in Figure 3.8. The analysis of stresses and strains is performed at three critical sections as shown in the figure.

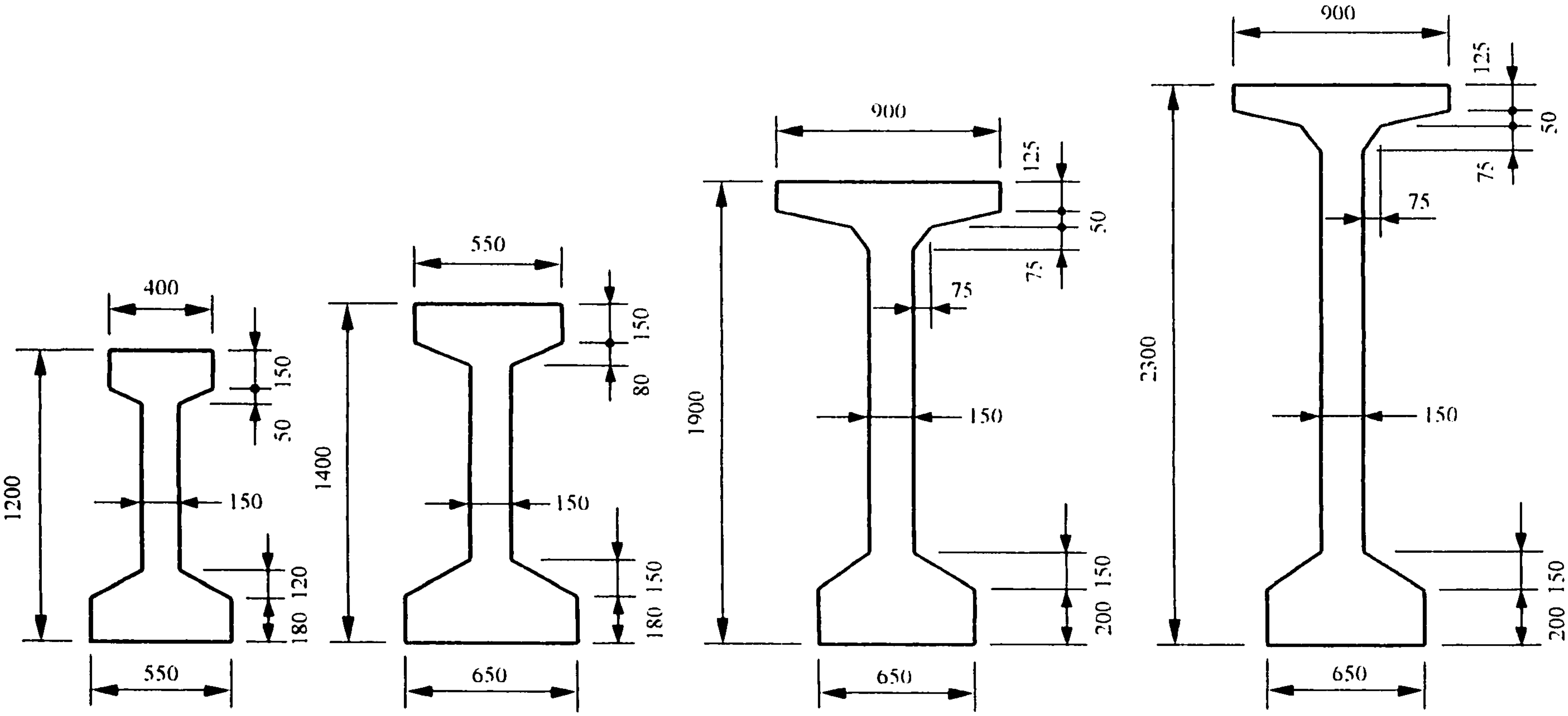

Figure 3.3 CPCI Precast Sections Considered in the Present Investigation

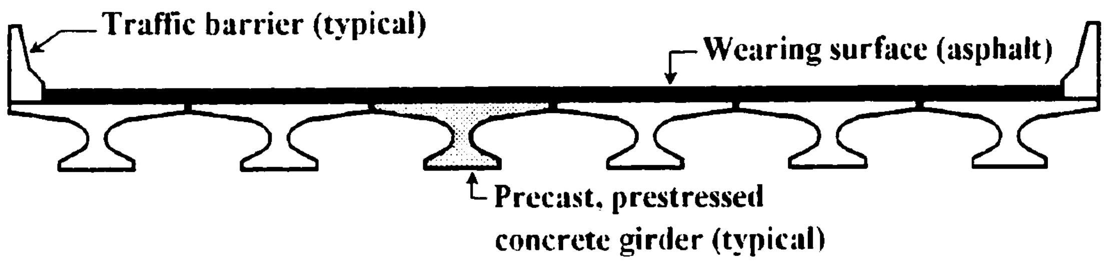

Figure 3.4 Typical Nebraska All-precast Bridge of Bulb Tee Sections

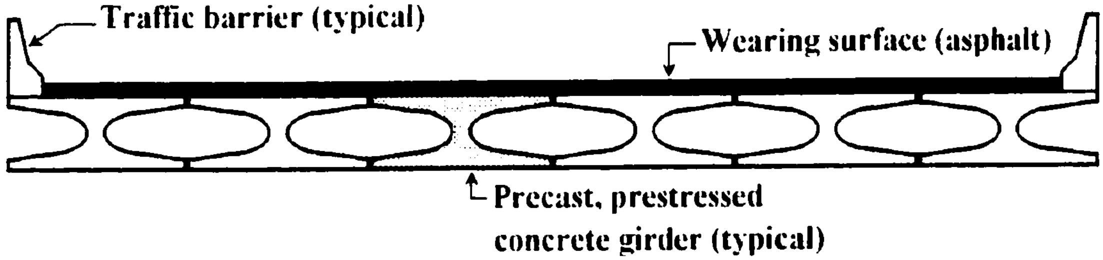

Figure 3.5 Typical Nebraska All-precast Bridge of Butted I-Girders

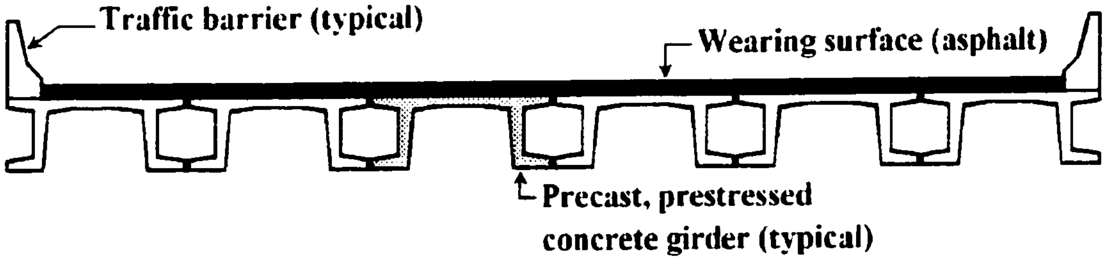

Figure 3.6 Typical Nebraska All-precast Bridge of Pie (π) Sections

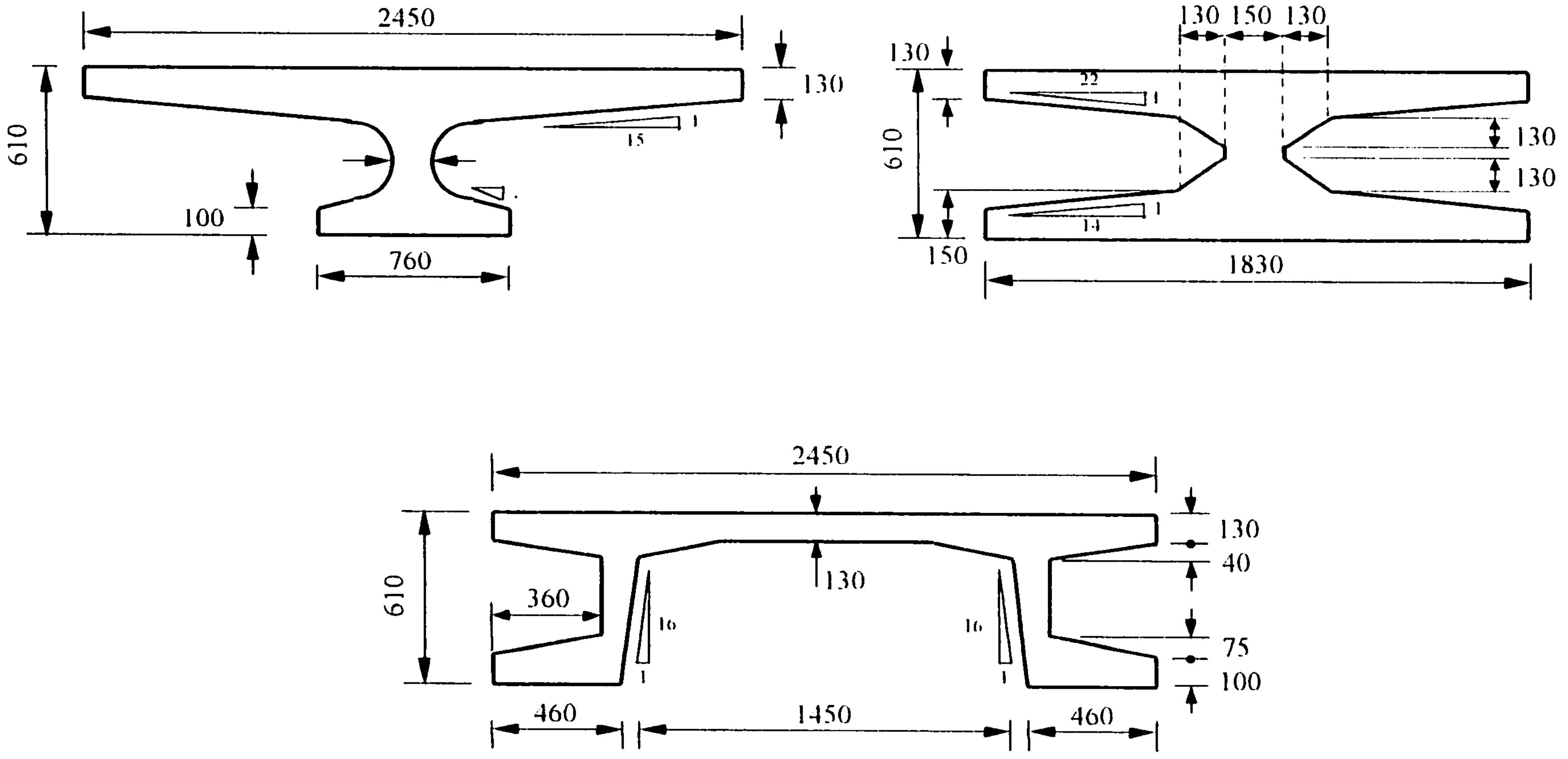

Figure 3.7 Nebraska Proposed Precast Sections Considered in the Present Investigation

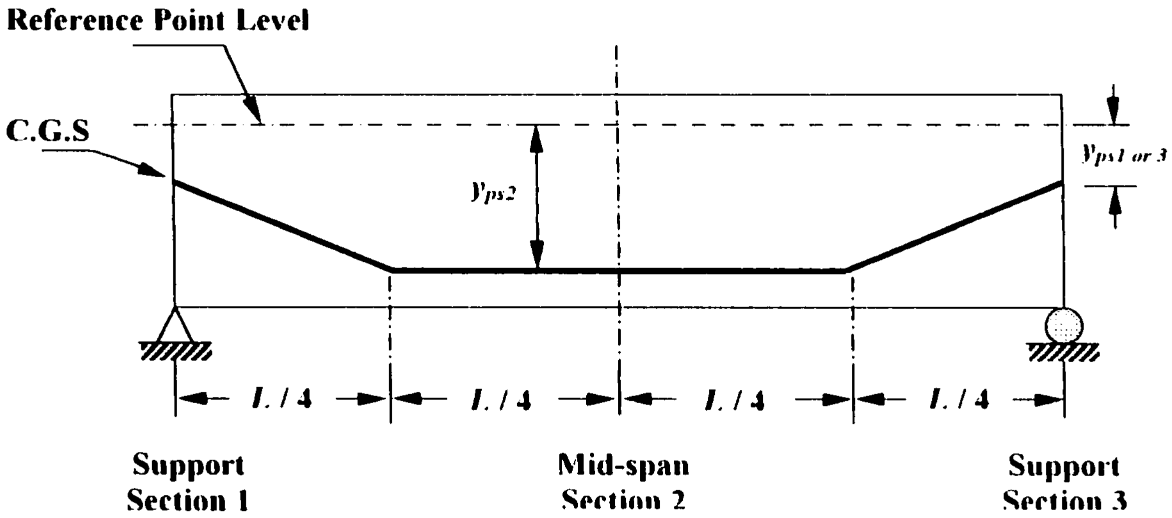

Figure 3.8 Tendons Profile and Locations of Critical Sections

4. In calculation of dead loads acting on the bridge (Figure 3.9), a 75 mm wearing surface is assumed to be placed on top of the concrete deck as shown in Figure 3.2. Each traffic barrier has a cross-sectional area of $0.3 m^2$ in accordance with OHBDC 1991 specifications. This load is assumed distributed equally among the girders.

5. The live load considered in the design consists of either a truck wheel load or a lane load acting on a traffic lane of width 3.70 m as specified by OHBDC 1991 (Figures 3.10 and 3.11). In the design of slab-on-girder bridges, **75%** of this load is used, whereas only **50%** of this load is used in the design of Nebraska proposed all-precast bridges

6. When the precast girders are prestressed with steel tendons, seven-wire, low relaxation, 15.2 mm diameter strands having a tensile strength of 1860 MPa are assumed. The effective stress in the strands is **60%** of its tensile strength. When the girders are prestressed with CFRP tendons, Leadline PC-D10 rods of 10 mm diameter and ultimate strength of 2250 MPa are used. The effective stress in the Leadline rods is limited to **55%** of the tensile strength.

7. Reinforcing steel bars with yield strength of 400 MPa and elastic modulus of 200,000 MPa are assumed. When CFRP is used, Leadline PC-D8 rods of 8 mm diameter, ultimate strength of 2250 MPa and elastic modulus of 147,000 MPa are considered.

8. The effective force in the prestressing tendons after losses is assumed as **85%** of the prestressing force at the time of transfer.

9. The concrete compressive strength at transfer is assumed **70%** of the 28-day strength. This ratio has been previously used satisfactorily in some studies (Russel Bruce W., 1994). It is also in accordance with the Canadian and American practices.

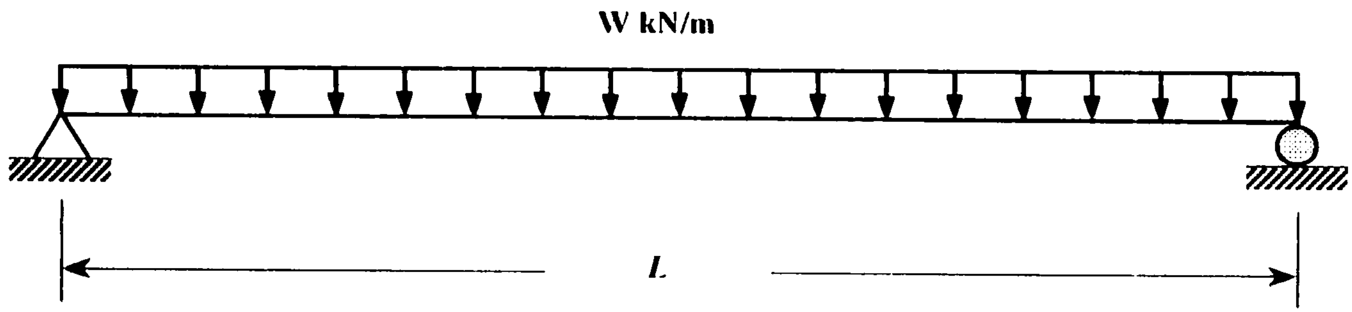

Figure 3. 9 Dead and Superimposed Load

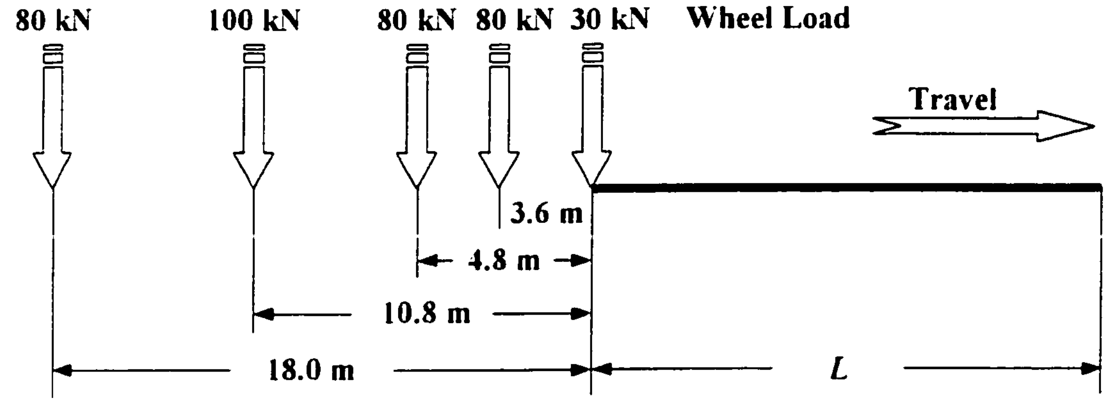

Figure 3.10 OHBD (1991) Truck - Wheel Load

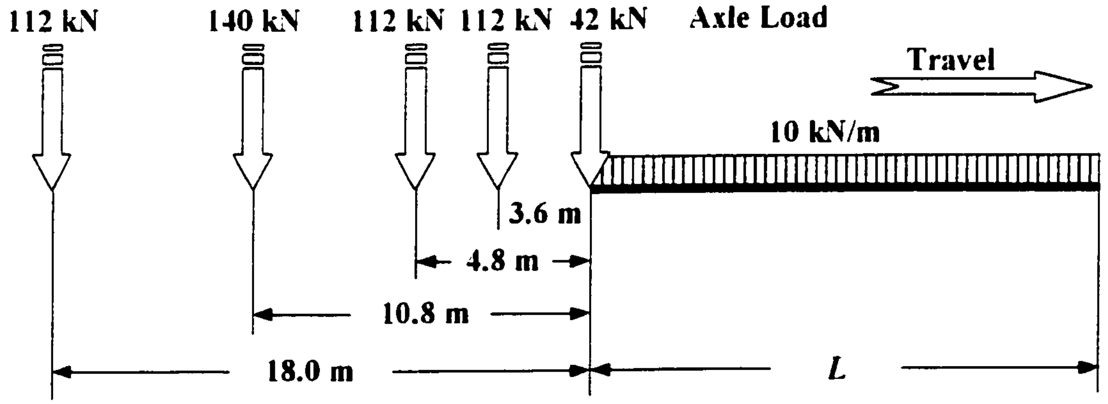

Figure 3.11 OHBD (1991) Truck - Lane Load

10. The elastic modulus of normal weight concrete, E_c, is recommended by Carrasquillo, Nelson and Slate (1981) as;

$$E_c = 3320\sqrt{f'_c} + 6900 \quad MPa \tag{3.1}$$

3.4 Empirical Design Method for Deck Slabs

The latest edition of the OHBDC (1991) permits deck slabs of slab-on-girder bridges which conform to certain conditions to be designed according to an empirical method that takes account of the internal arching action in slabs. When this method is used, it is no longer necessary to calculate and design for the transverse moments

According to the OHBDC empirical method, the slab thickness should not be less than 225 mm, steel reinforcement should be composed of two orthogonal meshes with a minimum ratio, in each direction of each mesh, of **0.3%** of the gross concrete cross sectional area. The minimum slab thickness of 225 mm is required to provide sufficient concrete cover to the reinforcement for protection against corrosion of the steel bars.

The empirical design method is applicable only when the following conditions are satisfied.

1. There are at least three girders in the bridge system.
2. The bridge has diaphragms at least at all supports. These must extend to the exterior girders.
3. The centre-to-centre spacing of girders does not exceed 3.7 m.
4. The slab extends at least 1.0 m beyond the centre line of the exterior girders, or has a curb of equivalent area of cross section.

5. The ratio of centre-to-centre spacing between girders to the slab thickness does not exceed 18.

6. Spacing of the reinforcing bars in each face does not exceed 300 mm.

3.5 Loading Cases

The following loads are considered in the analysis and design of the slab-on-CPCI girder and the Nebraska all-precast bridges during the various stages of construction and over the service life of the structure.

1. Load due to girder self-weight, $W_{Girder} = \gamma_{cg}\ A_{cg}$ (in kN/m); where γ_{cg} and A_{cg} are, respectively, the specific weight of concrete and the cross-sectional area of the precast girder

2. Load due to weight of deck slab, $W_{Slab} = \gamma_{cs}\ A_{cs}$ (in kN/m); where γ_{cs} is the specific weight of concrete in the deck slab, and $A_{cs} = S \times t$ is the cross-sectional area of the deck slab supported by one girder, with S being the spacing centre-to-centre between the precast girders and t being the slab thickness. Since unshored construction is assumed, this load is applied only on the precast girder. An exterior girder of the bridge supports a slab of width equal to $S' + S/2$, where S' is the extension of the slab beyond the centre line of the exterior girder (see Figure 3.2).

3. Superimposed dead load, which consists of the weight of sidewalk, parapet and handrails on both sides of the bridge, as well as the weight of wearing surface (asphalt). This load is distributed equally among the precast girders and its effects are applied on composite sections made up of the precast girders and the deck slab. The superimposed loads carried by each girder is given by:

$$W_{SDL} = \frac{W_{sidewalk} + W_{parapet} + W_{handrails} + W_{asphalt}}{total\ number\ of\ girders} \qquad \textbf{(kN/m)} \qquad (3.2)$$

The bending moment at any section at a distance x from the support due to any of the above distributed load is equal to;

$$M_x = \frac{W x}{2}(L - x) \qquad \textbf{(kN.m)} \qquad (3.3)$$

where $W = W_{Girder}$, W_{Slab} or W_{SDL}, and L is the span.

The maximum moment at the mid-span section is

$$M_{max} = \frac{W L^2}{8} \qquad \textbf{(kN.m)} \qquad (3.4)$$

4 Live (traffic) load, which is taken as the truck wheel load or the lane loads specified by OHBDC (1991), as shown in Figures 3.10 and 3.11, whichever gives the maximum response. For the range of spans (10 m – 30 m) considered in the present investigation, it was found that the "Lane Load" gives the maximum moment due to live load. The computer program XYmath (Taylor, 1989) has been used to perform a regression analysis for the governing case of loading to obtain the maximum moment at the mid-span section, when part or all of the wheel loads act on the span. The following relationship between the maximum moment, $M_{L,max}$ at the mid-span section and the span length, L, has been established:

$$M_{L,max} = 6.345 \times 10^{-5} L^3 - 69.558\left[\frac{1}{LOG(L)}\right]^2 + 6.157\left[e^{1/L}\right]^8 \qquad \textbf{MN.m}$$

(3.5)

This moment is distributed among the bridge girders by means of a modification or load fraction factor equals (S/D_d), where S is the spacing between girders and D_d is given by:

$$D_d = D\left[1 + \mu\left(\frac{C_f}{100}\right)\right] \tag{3.6}$$

where $$\mu = \frac{W_e - 3.3}{0.6} \leq 1.0 \tag{3.7}$$

and $$W_e = \frac{W_c}{n} \tag{3.8}$$

with W_e being the width of design lane, W_c the width of bridge deck, and n the total number of design lanes (see Table 3.1). The coefficients D and C_f are calculated from equations given in Table 3.2 (OHBDC, 1991).

A dynamic load allowance, DLA, is also used to account for dynamic impact effect of vehicles (see Table 3.3). Thus, the design live load moment for an interior girder of the bridge is given by:

$$M_L = M_{L,max}\left(1 + DLA\right)\frac{S}{D_d} \tag{3.9}$$

where $M_{L,max}$ is given by Equation (3.5). This moment has been found to be greater than that obtained from AASHTO (1994) code. Therefore, in this investigation, the traffic live load moment is reduced to **75%** for slab-on-girder bridges. For Nebraska bridge systems, the load is reduced to **50%**.

Table 3.1 Number of Design Lanes

Deck width, W_c	n
6.0 m or less	1
Over 6.0 m to 10.0 m included	2
Over 10.0 m to13.5 m included	3
Over13.5 m to17.0 m included	4
Over 17.0 m to 20.5 m included	5
Over 20.5 m to 24.0 m included	6
Over 24.0 m to 27.5 m included	7
Over 27.5 m	8

Table 3.2 Expressions for D and C_f for Longitudinal Bending Moments in Shallow Superstructures Corresponding to Ultimate and Serviceability Limit States

Type of bridge	Class of highway	No. of design lanes	External or Internal Portion	D in meters for $L \leq 10$ m but > 3 m	D in meters for $L > 10$ m	C_f in percent
Slab on Girder Bridges	A or B	1	External	2.00	2.10 - (1 / L)	5 - (12 / L)
			Internal	1.75 + (L / 40)	2.30 - (3 / L)	5 - (12 / L)
		2	External	1.90	2.00 - (1 / L)	10 - (25 / L)
			Internal	1.40 + (3 L /100)	2.10 - (4 / L)	10 - (25 / L)
		3	External	1.90	2.00 - (1 / L)	10 - (25 / L)
			Internal	1.60 + (2L / 100)	2.30 - (5 / L)	10 - (25 / L)
		4	External	1.90	2.00 - (1 / L)	10 - (25 / L)
			Internal	1.60 + (3L / 100)	2.35 - (4.5 / L)	10 - (25 / L)
	C	1	External	2.00	2.10 - (1 / L)	5 - (12 / L)
			Internal	1.75 + (L / 40)	2.30 - (3 / L)	5 - (12 / L)
		2	External	2.00	2.10 - (1 / L)	10 - (25 / L)
			Internal	1.90 - (1 / L)	2.20 - (4 / L)	10 - (25 / L)
		3	External	2.00	2.10 - (1 / L)	10 - (25 / L)
			Internal	1.80 + (L / 100)	2.40 - (5 / L)	10 - (25 / L)
		4	External	2.00	2.10 - (1 / L)	10 - (25 / L)
			Internal	1.75 + (2L / 100)	2.65 - (7 / L)	10 - (25 / L)

Table 3.3 Dynamic Load Allowance

Number of axles	*DLA*
1	0.40
2	0.30
3 or more	0.25

3.6 Serviceability Limit State

In the design of prestressed concrete members, it is necessary to check the stresses, strains and deformations at the various stages of construction and over the service live of the structure. For composite construction, the stresses in concrete need to be checked for three stages at least: 1) at transfer of prestressing to precast girder; 2) during construction at the time of casting the concrete slab; and 3) at service due to application of superimposed dead load and live loads. The equations employed for calculation the stresses and strains in concrete and the reinforcement at the various stages are given in the following subsection

3.6.1 Stresses and Strains in a Cross Section

The strains and stresses in a composite or non-composite cross-section reinforced with or without prestressing and subjected to a normal force and bending moment can be calculated from the following basic equations (Ghali and Favre, 1994)

$$\varepsilon_O = \frac{I N - B M}{E_{ref}\,(A I - B^2)} \qquad \text{and} \qquad \psi = \frac{-B N + A M}{E_{ref}\,(A I - B^2)} \tag{3.10}$$

where ε_O is the axial strain at a reference point O, and ψ is the curvature, i.e., the slope of the strain diagram ($d\varepsilon/dy$); N is an axial force at O and M is a bending moment about an axis through O; E_{ref} is a reference modulus of elasticity, which could be taken as the modulus of elasticity of one of the concrete parts of the cross section, e.g., the precast girder; A is the area of the transformed section composed of the area of each concrete part, i, multiplied by E_{ci}/E_{ref} plus the area of each reinforcement layer, j, (prestressed and non-prestressed) multiplied by E_{rj}/E_{ref}; B and I are the first and second moment of area of the transformed section about an axis through O; E_{ci} is the modulus of elasticity of concrete part i (precast girder or deck slab) and E_{rj} is the modulus of elasticity of one of the reinforcement layer j.

The reference point O is arbitrarily chosen along the vertical axis of symmetry of the cross section and its location is maintained constant throughout the analysis. This is convenient in the analysis of composite sections as it eliminates the need to calculate the location of the section centroid, which changes repeatedly due to changes in the section properties with the addition of new parts of the cross section (e.g. concrete slab) during construction, due to the time-dependent effects of creep of concrete, or due to cracking. This is particularly useful in the present investigation which requires numerous changes in the amount of prestressed and non-prestressed reinforcement and in the eccentricity of the prestressing tendons in order to achieve an optimum design. In the present analysis, the reference point is chosen at the top fiber of the precast girder

The strain at any concrete fiber or reinforcement layer at a distance y from the reference point O and the corresponding stress are given by

$$\varepsilon = \varepsilon_O + \psi\, y \qquad \text{and} \qquad \sigma = E(\varepsilon_O + \psi\, y) \tag{3.11}$$

where E is the modulus of elasticity of concrete or the reinforcement.

The following sign convention is adopted in the present analysis: the normal force, N, is positive when tensile; a tensile stress, σ, and the corresponding strain, ε, are positive; a bending moment, M, is positive when producing tension at the bottom fiber; positive curvature, ψ, is associated with a positive moment; the coordinate y is positive when measured downward from the point O (Figure 3.12).

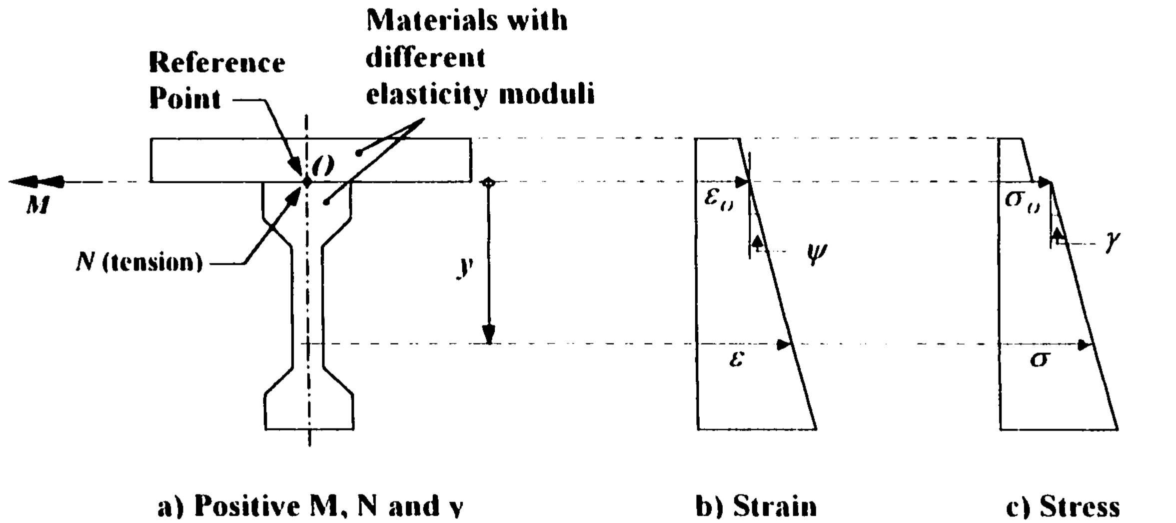

Figure 3.12 Positive Sign Convention for Moment, Normal Force, Strain and Stress

3.6.1.1 Stresses in Concrete at Transfer

At transfer of prestress, the precast girder is subjected to its self-weight and the initial prestressing force. Thus, the critical section at the support and at mid-span are subjected to the following internal forces:

At the support (Section 1 or 3),

$$N = -P_i \qquad \text{and} \qquad M = -P_i \; y_{ps1\,or\,3} \tag{3.12}$$

At the mid-span (Section 2),

$$N = -P_i \qquad \text{and} \qquad M = M_{dg} - P_i \; y_{ps2} \tag{3.13}$$

where P_i ($= 0.7 f_{pu} A_{ps}$) is the absolute value of the initial prestressing force, with f_{pu} being the ultimate tensile strength of the prestressing tendon and A_{ps} its cross-sectional area; y_{ps} is the depth of the prestressing tendon from the reference point O. M_{dg} is the moment at mid-span due to the girder's self-weight.

The axial strain, ε_O and the curvature ψ at the critical sections can be obtained from Equation (3.10) by substituting for N and M, the respective values given by Equations (3.12) and (3.13); and for A, B and I, the values A_g, B_g and I_g, the area, the first moment of area and the moment of inertia of the transformed girder cross-section, once at the support and once at mid-span.

The stress in concrete at the top and bottom fibers at any of the critical sections can be calculated from Equation (3.11) as follows:

$$\sigma_{top} = E_{cg} \, \varepsilon_O \tag{3.14a}$$

$$\sigma_{bot} = E_{cg} \, (\varepsilon_O + \psi \, y_{bg}) \tag{3.14b}$$

where E_{cg} is the modulus of elasticity of the concrete girder at the time of transfer; E_{cg} is here based on $0.7 f'_{cg}$; y_{bg} is the y-coordinate of the bottom fiber from the reference point.

3.6.1.2 Stresses in Concrete during Construction

At the time of placing the wet concrete for the deck slab on the girder, the unshored girders are not temporarily supported during construction, the girder self-weight and the weight of the wet concrete of the cast-in-place deck slab will be carried by the girder alone. This stage is completed some time after the prestress transfer. The concrete of the precast girders is assumed to have reached its full strength f'_{cg} while the force in the prestressing tendon will have a value between the initial force, P_i and the final effective force, P_e (= $0.85 P_i$). Conservatively, the force in the tendon is taken here as P_e (Collins and Mitchell, 1991).

At this stage, the critical sections of the girder are subjected to the following internal forces:

At the support (Section 1 or 3),

$$N = -P_e \qquad \text{and} \qquad M = -P_e \; y_{ps1 \, or \, 3} \tag{3.15}$$

At the mid-span (Section 2),

$$N = -P_e \qquad \text{and} \qquad M = M_{dg} + M_{ds} - P_e \; y_{ps2} \tag{3.16}$$

where M_{ds} is the moment at mid-span due to the slab self-weight.

The axial strain and curvature at the critical sections are once again calculated from Equation (3.10) using the transformed area properties A_g, B_g and I_g calculated based on E_{cg} value at the time of casting the deck slab (i.e. when the girder concrete strength is f'_{cg}). The corresponding stresses in concrete at the extreme fibers of the girder can be calculated as discussed above [Equations (3.14a and b)].

It should be noted here that in case of Nebraska all-precast bridge systems, the above stage is not considered and the stress calculations described above are not required.

3.6.1.3 Stresses in Concrete at Service

After hardening of the cast-in-place concrete slab, the composite section made of the precast girder and the deck slab carries all future loads. It is assumed here that the stresses in the girder due to prestressing, girder self-weight and slab weight are the same as those calculated during construction. The composite section resists the stresses due to the additional superimposed dead load and the live loads. The total stresses are calculated as the sum of the stresses after casting of the deck slab and the stress changes produced by the superimposed dead load and the live loads. In this stage, it is assumed that all the prestress losses have developed before the section becomes composite.

The critical sections at the supports and at mid-span are subjected to the following changes in internal forces

At the support (Section 1 or 3),

$$\Delta N = 0 \qquad \text{and} \qquad \Delta M = 0 \tag{3.17}$$

At the mid-span (Section 2),

$$\Delta N = 0 \qquad \text{and} \qquad \Delta M = M_{SDL} + M_L \tag{3.18}$$

where M_{SDL} is the moment at mid-span [Equation (3.4)] due to the superimposed dead load; M_L is the moment at the same section due to live load [Equation (3.9)].

The corresponding change in axial strain and curvature at the mid-span are given by [Equation (3.10)]

$$\Delta\varepsilon_O = \frac{-B_c \, \Delta M}{E_{cg}\,(A_c\, I_c - B_c^{\,2})} \quad \text{and} \quad \Delta\psi = \frac{A_c \, \Delta M}{E_{cg}\,(A_c\, I_c - B_c^{\,2})} \tag{3.19}$$

where A_c, B_c and I_c are the area properties of the transformed composite section using $E_{ref} = E_{cg}$

The change in strain at any fiber at coordinate y from O is given by

$$\Delta\varepsilon = \Delta\varepsilon_O + \Delta\psi \, y \tag{3.20}$$

The changes in stress at the extreme fibers of the slab and the precast girder are given by

$$\Delta\sigma_{top.slab} = E_{cs}\,(\Delta\varepsilon_O + \Delta\psi \, y_{ts}) \tag{3.21a}$$

$$\Delta\sigma_{bot.slab} = E_{cs}\,\Delta\varepsilon_O \tag{3.21b}$$

$$\Delta\sigma_{top.girder} = E_{cg}\,\Delta\varepsilon_O \tag{3.21c}$$

$$\Delta\sigma_{bot.girder} = E_{cg}\,(\Delta\varepsilon_O + \Delta\psi \, y_{bg}) \tag{3.21d}$$

where E_{cs} is the modulus of elasticity of the extreme slab, and $y_{ts} = -t$, is the y-coordinate of the top fiber of the slab, with t being the slab thickness.

It should be noted once again that in case of Nebraska all-precast system, only Equations (3.21c and d) apply, since the deck slab does not exist.

3.6.2 Deflection

Because of the lower modulus of elasticity of FRP reinforcement in comparison to steel, deflection of FRP reinforced members is larger than deflection of comparable steel reinforced members. Therefore, there is a critical need to assess deflection of FRP reinforced members more accurately than in the case of steel reinforced members.

The deflection δ at any point x along a member may be calculated by double integration of the curvature $\psi(x)$ over the member length. Assuming that deformations are small in comparison with the beam dimensions. The beam theory gives

$$\delta(x) = \iint \psi(x)\, dx\, dx \tag{3.22}$$

Equation (3.22) is general and applies to both elastic and inelastic material behavior. For a prestressed concrete beam, the curvature at any point along the span at any time after first loading can be calculated as discussed in the previous section.

Consider the simply supported beam of Figure 3.8. If the curvatures at each end of the span (ψ_1 and ψ_3) and at mid-span (ψ_2) are known and the variation of curvature along the member is assumed parabolic, then the deflection at mid-span, δ is given by (Ghali and Favre, 1994):

$$\delta = \frac{L^2}{96}(\psi_1 + 10\psi_2 + \psi_3) \tag{3.23}$$

where L is the member length.

3.6.3 Cracking Moment

The concrete is assumed to crack when the tensile stress reaches the modulus of rupture:

$$f_r = 0.6\sqrt{f'_{cg}} \tag{3.24}$$

Under service load conditions, the total stress at the bottom fiber of the precast girder at mid-span, $\sigma_{bg,total}$, is the sum of the stress calculated in Subsection 3.6.1.2 under the effects of N and M given by Equation (3.16) and the stress increment obtained from Equation (3.21d) due to superimposed dead load and live load. In order to produce cracking at this critical section, additional tensile stress must develop by an additional moment given as:

$$\Delta M_{cr} = \left[f_r - \sigma_{bg,total} \right] \frac{A_c I_c - B_c^2}{B_c + A_c y_{bg}} \tag{3.25}$$

Thus, the total moment that would be required to produce cracking is

$$M_{cr} = \Delta M_{cr} + (M_{dg} + M_{ds}) + (M_{SDL} + M_L) - P d_{psc} \tag{3.26}$$

For Nebraska bridge systems, $M_{ds} = 0$.

The moment given by Equation (3.26) is the cracking moment that can be used to check the reserved strength after cracking and the minimum reinforcement requirements (see Subsection 4.4.3.1).

3.7 Ultimate Limit State

The failure mechanism of FRP reinforced members should not be based on the formation of plastic hinges and the provisions of ductile behaviour as in the case of members reinforced with steel. This is because FRP materials process a linear-elastic behaviour up to failure. There are modes of flexural failure in reinforced or prestressed concrete members:

1. Simultaneous crushing of concrete and yielding of steel or rupture of FRP reinforcement (balanced failure).

2. Concrete crushing while the reinforcement remains in the elastic range with strain smaller than the yield strain, in case of steel, or the ultimate strain, in case of FRP reinforcement.

3. Yielding of steel, or rupture of FRP reinforcement before crushing of concrete.

Simultaneous rupture of FRP and crushing of concrete (failure mode 1) is the least desirable type of failure. Since there is no yielding of FRP, such members will fail very suddenly without prior warning. Failure due to concrete crushing is less violent and more desirable than failure due to FRP rupture. This failure is similar to that of over-reinforced concrete beams reinforced with steel.

The tension failure mode, or failure due to rupture of FRP (failure mode 3), is a sudden type of failure. It occurs when the reinforcement ratio is smaller than the balanced failure reinforcement ratio.

3.7.1 Flexural Strength

The flexural resistance of the bridge members considered in the present investigation is determined for the section of maximum moment based on procedures, which take into account equilibrium and strain compatibility. For many prestressed elements, the stress in the prestressed reinforcement at factored resistance, f_{ps} can be obtained using the approximate equations given by the code.

The effective flange width calculation is based on AASHTO 1996 limits. For interior girders, the effective flange width should be taken as the lesser of

$$b_{eff} = \begin{cases} 1/4L \\ 12t + 1/2b_{fg} \\ S \end{cases} \tag{3.27}$$

According to the OHBDC (1991), the material reduction factors ϕ_c, ϕ_s and ϕ_p are 0.60, 0.85 and 0.90 for concrete, reinforcing steel bars and prestressing steel tendons, respectively. However, the material reduction factor for both reinforcing CFRP bars and prestressing CFRP tendons is 0.75.

Figure 3.13 illustrates the strain and stress distributions, and forces at ultimate limit states. The compressive stress distribution in the concrete at ultimate is represented by an equivalent rectangular compressive stress block based on coefficients α_1 and β_1 given by

$$\alpha_1 = 0.85 - 0.0015 f_c' \geq 0.67 \tag{3.28}$$

$$\beta_1 = 0.97 - 0.0025 f_c' \geq 0.67 \tag{3.29}$$

The appropriate values of α_1 and β_1 should be evaluated with respect to the concrete compressive strength, f_c', depending on the location of the neutral axis as it is usually the case that the concrete strength of the cast-in-place slab is different from that of the precast girder.

The flexural resistance for the section shown in Figure 3.13 can be calculated from equations pertinent to rectangular cross sections and flanged sections in which the stress block lies entirely within the depth of the flange (deck-slab thickness), t. The depth of the stress block, a, can be determined from the equilibrium of forces shown in Figure 3.13

$$\mathbf{a} = \frac{\phi_p A_{ps} f_{ps} + \phi_s A_{sg} f_s - \phi_s A'_{ss} f_s'}{\alpha_1 \phi_c f'_{cs} b_{eff}} \tag{3.30}$$

from which

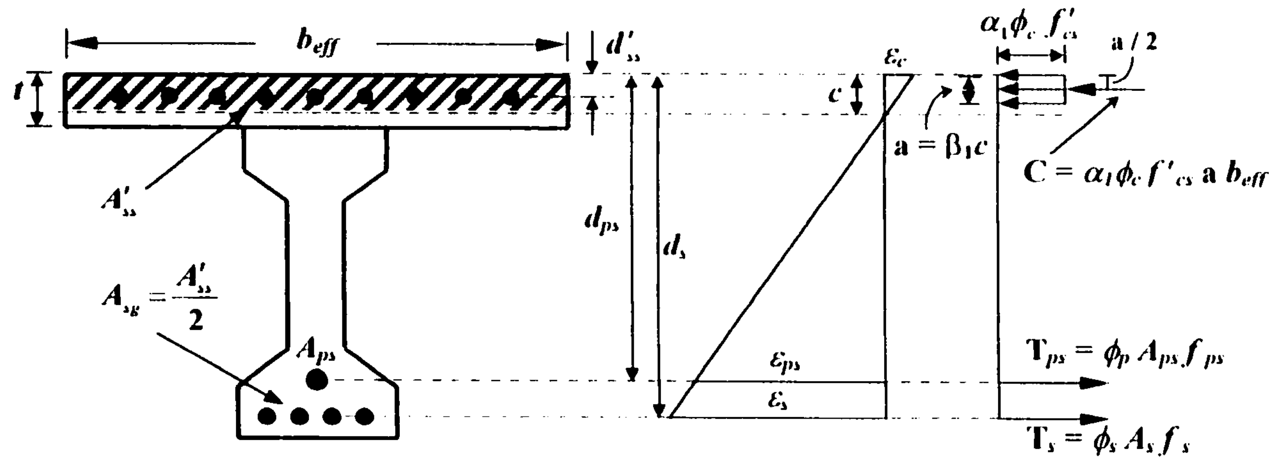

Figure 3.13 Strain and Stress Distribution, and Forces at Ultimate Limit States

$$M_r = \phi_p A_{ps} f_{ps} \left[d_{ps} - \frac{a}{2} \right] + \phi_s A_{sg} f_s \left[d_{sg} - \frac{a}{2} \right] - \phi_s A'_{ss} f'_s \left[d'_{ss} - \frac{a}{2} \right] \tag{3.31}$$

where A_{sg}, A'_{ss} are the area of the non-prestressed reinforcing bars in tension and compression, respectively, A_{ps} is the area of the prestressing tendons, f_s is the tensile stress in the reinforcing bars, f_{ps}, is the stress in the prestressing tendons at factored flexural resistance, d_{ps} is the depth of the prestressing tendons, d_{sg} is the depth of nonprestressing reinforcement in tension, and d'_{ss} is the depth of nonprestressing reinforcement in compression.

Generally, in the case when steel reinforcement is used, the stress in the nonprestressed reinforcement can be taken equal to the yield strength, f_y. However, yielding of the steel reinforcement at the ultimate condition should be verified Provided that the effective stress in the prestressed tendons, f_{pe} is not less than 50% of their ultimate strength, f_{pu} and the ratio $\frac{c}{d_{ps}}$ is not greater than 0.3, where c is the depth of the compression zone, the stress in the tendons, f_{ps}, may be found from the following approximate equation

$$f_{ps} = f_{pu} \left[1 - k_p \frac{c}{d_{ps}} \right] \tag{3.32}$$

in which

$$k_p = 3 \left[1 - \frac{f_{pu}}{f_{ps}} \right] \tag{3.33}$$

where f_{pu} is the ultimate strength of the tendons.

Otherwise, f_{ps}, should be determined by a more exact method based on strain compatibility approach.

It should be noted that the compressive strength of FRP is relatively low compared to its tensile strength FRP reinforcement can be used to support stirrups and also as reinforcement in continuous beams and slabs, however, the compressive strength of such reinforcement should not be accounted in for in the design. Therefore, the compressive reinforcement shall be ignored when calculating the flexural capacity of FRP reinforced members (Almusallam et al, 1997). Thus, in this case, the last term in Equation (3 31) is equal to zero

According to OHBDC (1991), the factored load positive moment is given by

$$M_f = 1.1 M_{dg} + 1.2 M_{ds} + 1.5 M_{SDL} + 1.6 M_L \qquad (3.34)$$

where M_{dg}, M_{ds}, M_{SDL} and M_L are the moments due to self-weight of the girder, self-weight of the deck slab, the superimposed dead load and the live load, respectively

CHAPTER 4

PROBLEM FORMULATION FOR OPTIMAL DESIGN

4.1 General

In the previous chapter, the analysis and design aspects of precast and composite concrete members reinforced with or without prestressing were explained. The ultimate flexural and serviceability requirements were particularly considered. In this investigation, the total prestress losses due to creep and shrinkage of concrete and relaxation of prestressed reinforcement were taken as 15 percent of the initial prestressing.

In the present chapter, the formulation of the optimal design problem of slab-on-CPCI girder bridges and Nebraska all-precast bridge systems is presented. The design variables considered here for statically determinate prestressed concrete bridge systems are the cross-section size, and the amount of both prestressed and non-prestressed reinforcement. The designer is also constrained by the various design requirements for the ultimate strength and serviceability limit states, which are presented later in this chapter.

Optimal design of structures is a particular combination of design variables, which satisfies all the design constraints at an objective function. The objective function and all design constraints must be function of the dependent and/or independent design variables. In structural design, minimum weight and minimum cost are the most commonly used objective functions. In the present investigation, minimum cost is selected as the objective function as it is more important for prestressed structures, particularly those reinforced with FRP material. It should be noted, however, that the cost of a particular design depends on the local conditions at time of construction. Variations in cost of materials, formwork, construction expertise, labour, plant hire, etc., can change the optimal design from one site to another and also from time to time.

The optimization process consists of cycling between two distinct phases defined as analysis and refined design in an iterative fashion until the optimum solution is reached. Use of computer is most suitable to carry out the substantial amount of repetitive calculations involved, allowing the designer to concentrate more on the creative side of the design process.

In the following sections, a description of the optimal design problem formulation for simply supported slab-on-girder bridges and Nebraska bridge systems is given. Firstly, the design variables, objective function and design constraints are identified and defined in sections 4.2 to 4.4. Secondly, the investigated design combinations are discussed in section 4.5. Finally, in section 4.6, a design optimization software package is described for the solution of the optimal design problem.

4.2 Design Variables

The design variables and the constant parameters together, entirely describe the design of a structure. The most important step in a properly formulated optimal design problem is to identify the design variables. If the design variables are not appropriately selected, the formulation will be neither correct nor feasible. Sometimes at the early stage of problem formulation, it is desirable to allocate more design variables than may be apparently needed. This makes the formulation more flexible. Later, it can be reduced in accordance with the problem requirement by transformation of any variable into a constant parameter, and thus eliminate it from the problem formulation. Another important point is that design variables should be independent of each other as far as possible. It is sometimes possible to have dependent design variables; however, this will make the formulation more complicated because of the additional constraints that have to be imposed in order to describe the relationships among the dependent variables.

In the present investigation, the cross-section dimensions of the precast girders are assumed constant. In the slab-on-girder bridges, the slab thickness is maintained constant at 225 mm, the compressive strength of the concrete slab at age 28 days is taken as $f'_{cs} = 40$ MPa. The span length varies between 10 m and 30 m. Two types of reinforcement are used in each of the bridge systems: conventional steel and carbon fiber reinforced polymer (CFRP) reinforcement. The design variables for the problem under consideration include the following (see Figure 4.1):

1. The concrete compressive strength of the precast girder at age 28 days (X_1)
2. The required amount of prestressing reinforcement in the precast girder (X_2)
3. The eccentricities of the prestressing reinforcement in the precast girder at mid-span and at the support sections (X_3 and X_4)
4. The required amounts of the non-prestressed flexural reinforcement in the deck slab and/or the precast girder (X_5). In the slab-on-girder system, when steel reinforcement is used, the amount of this reinforcement in the precast girder is taken as half the amount in the deck slab. As mentioned in chapter 3, when FRP reinforcement is used, the amount of this reinforcement in the compression zone is ignored as the compressive strength of FRP materials is very small. Thus, in the calculation of the

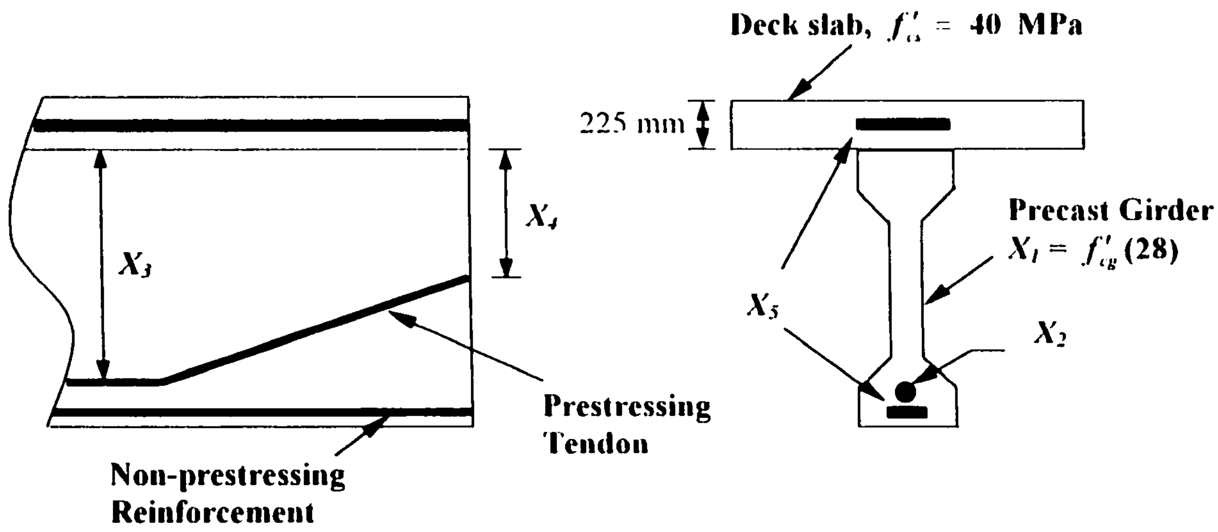

Figure 4.1 Design Variables

flexural strength of the slab-on-girder bridges, the FRP reinforcement in the deck slab is neglected and the reinforcement in the precast girder, X_5, becomes the design variable.

4.3 Objective Function

Usually there is an infinite number of feasible designs for a structure. In order to find the optimum one, it is necessary to form a function of the design variables that can be used for comparison of feasible design alternatives. Such a function is referred to as an *objective function* for the optimal design problem.

The choice of an appropriate objective function is an important part of the optimization design process. The objective function should generally represent the most important single property of a design, but it may represent also a weighted sum of a number of properties. In prestressed concrete design optimization, however, minimum weight may not be always the cheapest. Cost is of wider practical importance than weight, although it is often difficult to obtain sufficient data for the construction of a real cost function. A general cost function may include the cost of materials, fabrication, transportation, etc. In addition to the cost involved in the design and construction, other factors such as operating and maintenance cost, repair costs, etc., may be considered. However, it is not usually desirable to consider an objective function which is as general as possible. The result might be a "flat" function that is not sensitive to variations in the design variables, and, practically, the optimization process will not improve the design. Therefore, from a practical viewpoint, it is appropriate to adopt such an objective function that is both sensitive to variations in the design variables and representative of the most important cost components (Kirsch, 1993).

4.3.1 Cost Function

The objective function selected in the design of slab-on-CPCI girder bridges is the minimum superstructure cost. The only parts of bridge superstructure considered herein are the deck slab and the girders. The objective function is thus taken as the material (concrete and reinforcement) costs plus overhead and waste, in addition to the labour cost for both the deck slab and the girders. In addition, transportation and erection costs for the precast girders are also included. Slab formwork cost may vary slightly with the change in the number of girders. For example, as the number of girders decreases, the actual slab formwork area increases; however, the labour cost of placing this formwork decreases in a way that may partially or fully compensate for the increase in material cost. Therefore, it was decided to exclude the slab formwork cost from the cost function. Furthermore, costs of some other items that are relevant to the superstructure such as sidewalks, wearing surfaces (asphalt), traffic barriers, handrails and drains were not included in the cost function as their costs are not affected by the selected design variables. Therefore, these items would just be added as a common cost later, and would not influence the optimal design.

In the precast optimal design formulation, different feasible designs of the superstructure are compared in terms of their relative initial cost effectiveness on a **$/m²** of deck area basis. In other words, the objective function is defined as the minimum initial superstructure cost per deck area. For the different types of bridges considered in this investigation, the objective function is given as:

1. For slab-on-CPCI girder bridges reinforced with steel

$$\boldsymbol{Cost} = \boldsymbol{n_g}\left[\boldsymbol{C_{cs}}\,\boldsymbol{V_{cs}} + \boldsymbol{C_{cg}}\,\boldsymbol{V_{cg}} + \boldsymbol{C_{ns}}\,\boldsymbol{W_{ns}} + \boldsymbol{C_{ps}}\,\boldsymbol{n_{ps}}\,\boldsymbol{L}\right] / \boldsymbol{W_c}\,\boldsymbol{L} \qquad (4.1a)$$

2. For slab-on-CPCI girder bridges reinforced with CFRP

$$Cost = n_g \left[C_{cs} V_{cs} + C_{cg} V_{cg} + C_{np} n_{np} L + C_p n_p L \right] / W_c L \tag{4.1b}$$

3. For Nebraska all-precast girder bridge systems reinforced with steel

$$Cost = n_g \left[C_{cg} V_{cg} + C_{ns} W_{ns} + C_{ps} n_{ps} L \right] / W_c L \tag{4.1c}$$

4. For Nebraska all-precast girder bridge systems reinforced with CFRP

$$Cost = n_g \left[C_{cg} V_{cg} + C_{np} n_{np} L + C_p n_p L \right] / W_c L \tag{4.1d}$$

where n_g is the number of girders, C_{cs} is the cost of concrete in the slab per unit volume, C_{cg} is the cost of concrete in the girder per unit volume including cost of materials, production, transportation and erection, C_{ns} is the cost of non-prestressed steel per unit weight, C_{ps} is the cost of prestressed steel per unit length, C_{np} is the cost of non-prestressed CFRP reinforcement per unit length, C_p is the cost of prestressed CFRP tendons per unit length, V_{cs} is the volume of concrete in the slab, V_{cg} is the volume of concrete in the girder, W_{ns} is the weight of non-prestressed steel in the girder, n_{ps} is the number of prestressed steel tendons, n_{np} is the number of non-prestressed CFRP bars, n_p is the number of prestressed CFRP tendons, W_c is the width of the bridge, and L is the length of the bridge span.

The strength of both the precast girder concrete and the reinforcement material (steel or CFRP) can have a significant effect on the span length of the bridge and the life span of the structure. Therefore, it may be appropriate to define the objective function as the minimum initial girder cost per unit strength. In this case, the objective function will be given as the numerator of Equations (4.1a to d) divided by $(n_g \; M_r \; L)$; where M_r is the flexural moment resistance of the mid-span section.

4.4 Design Constraints

Any set of values for the design variables represents a design of the structure. Even if this design is inadequate in terms of its function or its behavior, it can still be called a design. Clearly, some designs are useful and others are not. If a design meets all the requirements placed on it, then it is a feasible design. The restrictions that must be satisfied in order to produce a feasible design are referred to as design constraints. Each design constraint must be influenced by one or more design variables. Only then is it meaningful and does it have influence on the optimal design (Kirsch, 1993 and Arora, 1989).

In structural engineering, design constraints can be classified into two types: functional constraints and practical constraints. The functional constraints are related to the compliance of the design with both serviceability and ultimate limit state requirements. The practical constraints represent some practical limits imposed on some of the design variables. Design problems may have equality (= 0 form) as well as inequality (≤ 0 form) constraints. An equality constraint may represent, for example, a required ratio between the width of a cross section and its depth. An example of an inequality constraint is that calculated stress must be less than or equal to the allowable stress for the material. A feasible design must satisfy precisely all the equality constraints. There are, however, many feasible designs with respect to an inequality constraint. It is, therefore, easier to find feasible designs for a structure having only inequality constraints (Arora, 1989).

4.4.1 Constraints at Serviceability Limit States

4.4.1.1 Stress Constraints

The stresses in the concrete at the top and bottom fibers of critical sections considered are to be calculated immediately after the application of the prestress, after the deck slab is cast, and after the additional dead load and traffic load are applied. The constraints are imposed on the stresses calculated at the three sections at the supports and at mid-span.

4.4.1.1.1 Stress Constraints at Transfer

At the time of prestress transfer, concrete stresses at the top and bottom extreme fibers of the girder due to its own weight must be within the allowable limits at all the specified sections. Thus,

At the support, the constraint on the stress at the top fiber of the precast girder due to the initial prestressing force is:

$$-0.6 f'_{cig} \leq \sigma^{t}_{Support} \leq 0.5\sqrt{f'_{cig}} \qquad (4.2a)$$

and the constraint on the stress at the bottom fiber is:

$$-0.6 f'_{cig} \leq \sigma^{b}_{Support} \leq 0.5\sqrt{f'_{cig}} \qquad (4.2b)$$

At mid-span, the constraint on the stress at the top fiber due to the initial prestressing force and the girder self-weight is:

$$-0.6 f'_{cig} \leq \sigma^{t}_{midspan} \leq 0.25\sqrt{f'_{cig}} \qquad (4.3a)$$

and the constraint on the stress at the bottom fiber is:

$$-0.6f'_{cig} \leq \sigma^{b}_{midspan} \leq 0.25\sqrt{f'_{cig}} \qquad (4.3b)$$

where f'_{cig} is the compressive strength of concrete at the time of prestress transfer and the stresses at the extreme fibers are calculated from Equations (3.14).

4.4.1.1.2 Stress Constraints during Construction

The stress in concrete at the top and bottom fibers of the precast girder after casting of the concrete slab must be within the allowable limits at all the specified sections. These stresses are calculated based on the properties of the precast girder as discussed in Subsection 3.6.1.2. Thus,

At the support, the constraint on the stress at the top fiber of the girder due to the effective prestressing force is:

$$-0.45f'_{cg} \leq \sigma^{t}_{Support} \leq 0.25\sqrt{f'_{cg}} \qquad (4.4a)$$

and the constraint on the stress at the bottom fiber of the girder is:

$$-0.45f'_{cg} \leq \sigma^{b}_{Support} \leq 0.25\sqrt{f'_{cg}} \qquad (4.4b)$$

At mid-span, the constraint on the stress at the top fiber of the girder due to the effective prestressing force, the girder self-weight and deck slab weight is:

$$-0.45f'_{cg} \leq \sigma^{t}_{midspan} \leq 0.25\sqrt{f'_{cg}} \qquad (4.5a)$$

and the constraint on the stress at the bottom fiber of the girder is:

$$-0.45f'_{cg} \leq \sigma^{b}_{midspan} \leq 0.25\sqrt{f'_{cg}} \qquad (4.5b)$$

4.4.1.1.3 Stress Constraints at Service

At the time of application of superimposed dead load (sidewalks, traffic barriers, handrails and asphalt) and live load (traffic load), concrete stresses at the top and bottom extreme fibers of the composite section due to girder self-weight, deck-slab weight, superimposed dead and live loads must be within the allowable limits at all the specified critical sections. These stresses are calculated using the geometric properties of the transformed composite section as discussed in Subsection 3.6.1.3. Thus,

At the support, the constraint on the stress at the top fiber of the composite section due to the effective prestressing force is:

$$-0.45 f'_{cs} \leq \sigma^{tc}_{Support} \leq 0.25\sqrt{f'_{cs}} \tag{4.6a}$$

and the constraint on the stress at the bottom fiber of the precast girder is:

$$-0.45 f'_{cg} \leq \sigma^{bc}_{Support} \leq 0.25\sqrt{f'_{cg}} \tag{4.6b}$$

At mid-span, the constraint on the stress at the top fiber of the composite section due to the superimposed and live loads, is:

$$-0.45 f' \leq \sigma^{tc}_{midspan} \leq 0.25\sqrt{f'_{cs}} \tag{4.7a}$$

and the constraint on the stress at the top fiber of the precast girder is:

$$-0.45 f'_{cg} \leq \sigma^{tg}_{midspan} \leq 0.25\sqrt{f'_{cg}} \tag{4.7b}$$

and the constraint on the stress at the bottom fiber of the precast girder is:

$$-0.45 f'_{cg} \leq \sigma^{bg}_{midspan} \leq 0.25\sqrt{f'_{cg}} \tag{4.7c}$$

4.4.1.2 Deflection Constraints

The deflection constraints are defined by the following equations for three consecutive stages:

Maximum mid-span deflection constraint at prestressing transfer (Stage #1) is

$$-\frac{L}{800} \leq \delta_{Transfer} \leq \frac{L}{800} \tag{4.8a}$$

Maximum mid-span deflection constraint during construction (Stage #2) is

$$-\frac{L}{800} \leq \delta_{Construction} \leq \frac{L}{800} \tag{4.8b}$$

Maximum mid-span deflection constraint at service (Stage #3) is

$$-\frac{L}{800} \leq \delta_{Service} \leq \frac{L}{800} \tag{4.8c}$$

4.4.2 Constraints at Ultimate Limit States

4.4.2.1 Ultimate Moment Resistance

At ultimate limit states, a structural component should be designed so that the factored flexural resistance, M_r is equal to or greater than the factored load moment, M_f, that is,

$$M_r \geq M_f \tag{4.9}$$

where M_f is given by Equation (3.34) and M_r is calculated by Equations (3.30) and (3.31).

4.4.3 Practical Constraints

4.4.3.1 Constraint on the Minimum Amount of Flexural Reinforcement

It is desirable that girders contain sufficient flexural reinforcement at the critical sections to ensure that a reserve of strength exists after initial cracking. If the girders do not contain enough reinforcement, they may fail abruptly with rupturing of the reinforcement immediately after cracking. According to CSA-A23.3-94, the minimum amount of flexural reinforcement in steel reinforced members should be controlled by:

$$M_r \geq 1.20\, M_{cr} \tag{4.10a}$$

where M_{cr} is given by Equation (3.26). The requirement given by Equation (4.10a) can be waived when the factored moment of resistance, M_r is not least 33 percent greater than the moment due to factored loads, M_f, i.e. when

$$M_r \geq 1.33\, M_f \tag{4.10b}$$

In case of FRP reinforced concrete members, ISIS Canada Design Manual (1999) suggests that the moment of resistance, M_r, be at least 50 percent greater than the cracking moment, M_{cr}, in order to avoid failure immediately after cracking. Thus,

$$M_r \geq 1.50\, M_{cr} \tag{4.11a}$$

The draft manual also suggests that if an FRP reinforced member is failing in tension before reaching the maximum compressive strain in concrete, the moment of resistance, M_r, should be at least 50 percent greater than the factored load moment, M_f, that is

$$M_r \geq 1.50\, M_{cr} \tag{4.11b}$$

4.4.3.2 Constraint on the Ratio of Girder Spacing to Slab Thickness

According to the empirical design method for deck slabs allowed by the OHBDC (1991), the ratio of centre-to-centre girder spacing, S to the thickness of the deck slab, t, shall not exceed 18.0. Thus,

$$S \leq 18t \tag{4.12}$$

For the deck slab thickness $t = 225\,mm$ selected in this investigation, the maximum spacing between girders, $S_{max} = 4050\,mm$.

4.4.3.3 Upper and Lower Bounds on Design Variables

There are some practical constraints that represent explicit lower and upper bounds on some of the design variables, which might reflect minimum practical dimensions for construction, maximum dimensions for transportation, architectural considerations, code restrictions, or desired relationships between design variables. These constraints are expressed as:

$$X_i^L \leq X_i \leq X_i^U \qquad i = 1 \text{ to } n \tag{4.13}$$

where X_i, X_i^U, X_i^L represent the ith design variable, the upper bound and the lower bound, respectively and n is the number of variables.

4.5 The Investigated Design Combinations

The above optimization model has been applied to produce optimal bridge designs for the selected precast prestressed concrete bridge girders (CPCI1200, CPCI1400, CPCI1900, CPCI2300, Bulb T, Butted I, and Pie sections) using different combinations of concrete strengths, deck widths and number of girders as shown in Tables 4.1 and 4.2.

Table 4.1: CPCI Girders Reinforced with Steel or CFRP.

Section Type	W_c m	f'_{cg} MPa	Number of Girders					
			3	4	5	6	7	8
CPCI1200	8	40	×	×				
CPCI1400		50	√	×				
CPCI1900								
CPCI2300		60	×	×				
		70	×	×				
		80	×	×				
		90	×	×				
		100	×	×				
CPCI1200	12	40		×	×	×		
CPCI1400		50		√	×	×		
CPCI1900								
CPCI2300		60		×	×	×		
		70		×	×	×		
		80		×	×	×		
		90		×	×	×		
		100		×	×	×		
CPCI1200	16	40			×	×	×	×
CPCI1400		50			√	×	×	×
CPCI1900								
CPCI2300		60			×	×	×	×
		70			×	×	×	×
		80			×	×	×	×
		90			×	×	×	×
		100			×	×	×	×

Table 4.2: Nebraska All-precast Girders Reinforced with Steel or CFRP.

Section Type	W_c m	f'_{cg} MPa	Number of Girders 3	4	5	6	7	8
Bulb T	14.70	40				×		
		50				√		
		60				×		
		70				×		
		80				×		
		90				×		
		100				×		
Butted I	14.64	40						×
		50						√
		60						×
		70						×
		80						×
		90						×
		100						×
Pie	14.70	40				×		
		50				√		
		60				×		
		70				×		
		80				×		
		90				×		
		100				×		

However, due to the large amount of output, only the results of the combinations marked with (√) have been considered for discussion. The results and discussion are shown in the following chapter.

For all designs the following data are adopted as constant parameters:

- Span length: varying from 10 to 30 m
- Traffic loading corresponding to class A highway
- Slab thickness: $t = 225\,mm$, which is the minimum thickness according to OHBDC (1991)
- Asphalt pavement thickness = 75 mm
- Precast girders are not shored during construction
- Concrete Strength of deck-slab is, $f'_{cs} = 40\,MPa$
- Concrete Strength of precast girder is f'_{cig} = 70% of f'_{cg} . f'_{cg} = value varying from 40 to 100 MPa
- Strength of prestressed steel tendons is $f_{pu} = 1860\,MPa$ and Elasticity Modulus is $E_p = 190\,GPa$
- Strength of non-prestressed steel bars is $f_y = 400\,MPa$ and Elasticity Modulus is $E_{np} = 200\,GPa$
- Strength of prestressed CFRP tendons is $f_{pu} = 2250\,MPa$ and Elasticity Modulus is $E_p = 147\,GPa$
- Strength of non-prestressed CFRP bars is $f_y = 1575\,MPa$ and Elasticity Modulus is $E_p = 147\,GPa$

- Unit costs expressed in Canadian dollars as follows:

For concrete:

The unit cost of concrete was adopted from data available in the literature (Hassanain, 1998). It is normalized to the cost of $40\,MPa$ concrete mix, which has been assumed to be $\$95/m^3$ including an overhead rate of 18%. In addition to the cost of mix and overhead charges, labour and curing cost is estimated at $\$34/m^3$. Using regression analysis, the following relationship for the total cost of concrete as a function of the concrete strength was established:

$$Concrete\ Cost\,(f_c') = 95\left(0.939 + \left\{\frac{f_c'}{100}\right\}^3\right) + 34 \qquad \$/m^3 \qquad (4.14)$$

For conventional steel:

Seven-wire, 15.2-mm-diameter prestressing strands cost $\$1.78/m$ for material and labour including a wastage rate of 10% and an overhead rate of 18%. Epoxy-coated reinforcing steel bars cost $\$1.68/kg$ for material and labour including a wastage rate of 5% and an overhead rate of 15% (Hassanain, 1998).

For Carbon Fiber Reinforced Polymer (CFRP):

LEADLINE PC-D8, prestressed tendon costs $13/***m*** (including transportation).

LEADLINE PC-D10, non-prestressed bar costs $19/***m*** (including transportation).

The optimum design of the bridge systems considered in this investigation gives the following values for a specific compressive strength of the precast girders:

- The minimum span length for each cross section shape and dimensions.
- The amount of prestressed reinforcement required for the girder.

- The eccentricity of the prestressing tendons at mid-span and at the supports.
- The amount of non-prestressed reinforcement in the girder and the slab.
- Minimum cost of superstructure/deck area (excluding the fixed cost of sidewalk, barriers, handrails, asphalt and girder end bearing).
- Minimum cost of girder/unit strength.

4.6 Design Optimization Software (EXCEL 97 – Solver)

Microsoft Excel 97 Solver incorporates a nonlinear optimization code based on the Generalized Reduced Gradient (GRG2) technique which was developed by Leon Lasdon, University of Texas at Austin, and Allan Waren, Cleveland State University (Microsoft, 1997). This tool is readily available and easy to use, and supports Visual Basic programming that enables the user to design his own defined functions for a lengthy and intensive calculation process. Because of these advantages, this tool is employed in the present investigation to solve the optimal design problem. In this section, a description of this software package is presented and instructions on its use for the solution of optimization problems is given.

The main purpose of the Solver is to find a *solution*, that is, values for the *design variables*, which satisfy the *constraints* and maximize or minimize the *objective function*. These variables are included in changing cells in the Excel 97 spreadsheet. Figure 4.2 shows where the solver can be found and accessed.

4.6.1 Parameters of the Model

The input values may be fixed numbers associated with the problem, which will be referred to here as *parameters* of the model. These parameters are entered in input

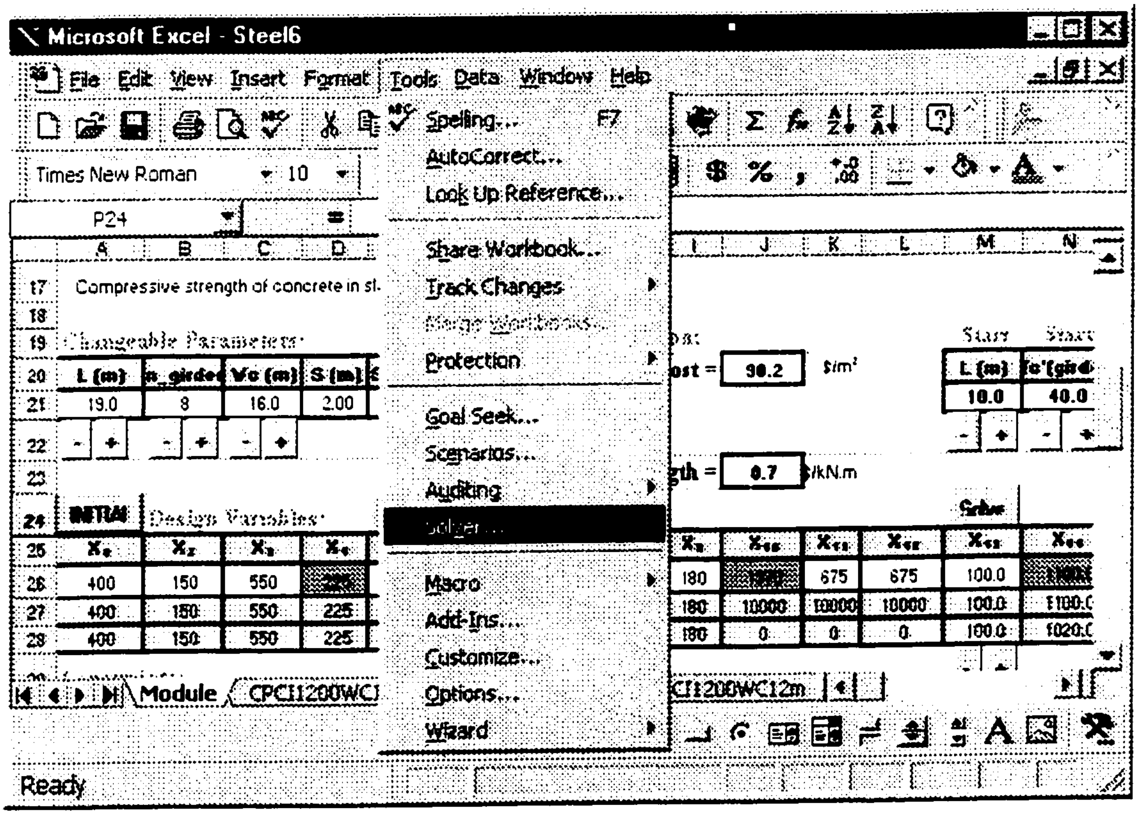

Figure 4.2 Microsoft Excel 97 – Solver Access

cells and used in the calculations of the objective function and constraints. The parameters are constant in the Solver problem. Although their values can be changed by user, the *Solver* will never change them automatically. The user will often have several "cases" or variations of the same problem to solve, and the parameter values will change in each problem variation.

4.6.2 Decision Variables (Changing Cells)

The input parameters to the optimization problem can be quantities which are variable, or under the control of the user (decision maker). These parameters are referred to as *decision variables* and appear in the spreadsheet in changing cells. These are the cells that the Solver will change automatically in order to maximize or minimize the objective or target cell. These cells are listed in the Changing Cells edit box of the Solver Parameters dialog.

4.6.3 The Objective Function (Target Cell)

The quantity that is aimed to maximize or minimize is called the *objective function* or target cell. This cell is listed in the Set Cell edit box of the Solver Parameters dialog (Figure 4.3). In case of a Solver model that has nothing to maximize or minimize, the Set Cell edit box will be empty. In this situation the Solver will simply find a solution which satisfies the constraints.

The standard spreadsheet Solvers also allow entering a specific value, which must be achieved by the objective function or Set Cell. This feature was included for compatibility with the spreadsheet's Goalseek or Backsolver command menu, which allows seeking a specific value for a cell by adjusting the value of one other cell on which it depends. It should be noted that, entering a specific value for the Solver's Set Cell is

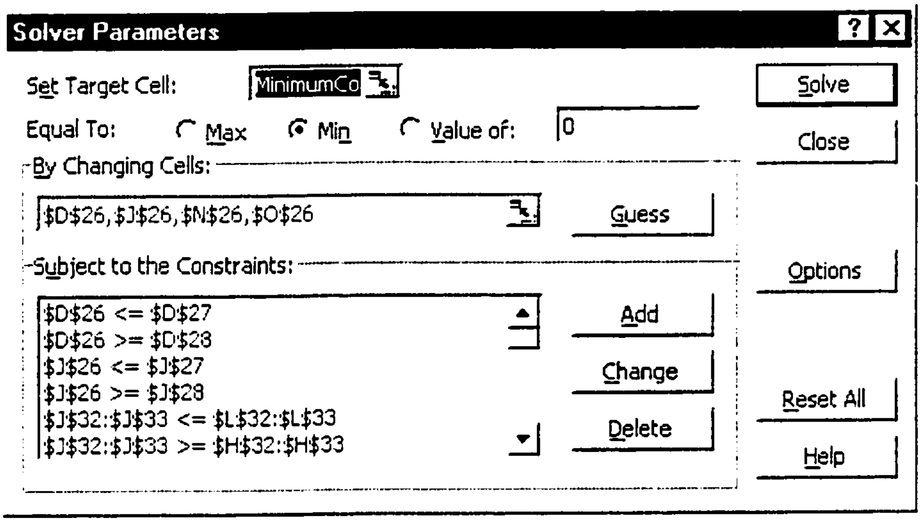

Figure 4.3 Microsoft Excel 97 - Solver Parameters

exactly the same as leaving the Set Cell blank and entering an equality constraint for the Set Cell in the Constraint List Box.

There is rarely a good reason to use the Set Cell Value of edit box in the Solver Parameters dialog. If the problem requires only a single Set Cell value and a single variable or Changing Cell, it is recommended to use the Goalseek. command. If there is nothing to maximize or minimize, it is recommended to leave the Set Cell blank and entering any constraints needed in the Constraint List Box.

4.6.4 Constraints

Constraints are relations such as **A1** $\geq$ **0**. A constraint is *satisfied* if the condition it specifies is true *within a small tolerance*. But with the default Solver Precision setting, the constraint would be satisfied. Because of the numerical methods used to find solutions to Solver models and the finite precision of computer arithmetic, it would be unrealistic to require that constraints like **A1** $\geq$ **0** be satisfied exactly -- such solutions would rarely be found. The values of tolerance and solver precision and the numerical method to be used in the solution are specified in the Solver Options window (Figure 4.4).

Constraints are specified by giving a cell reference such as **A1** (the "left hand side"), a relation ($\leq$, = or $\geq$), and an expression for the "right hand side." The left hand side may, and often will be a *range* of cells such as (**A1:A10**). Although the Excel Solver allows entering any numeric expression on the right hand side, it is strongly encouraged to use only *constants*, or references to cells, which contain *constant values* on the right hand side. A constant value to the Solver is any value, which does not depend on any of

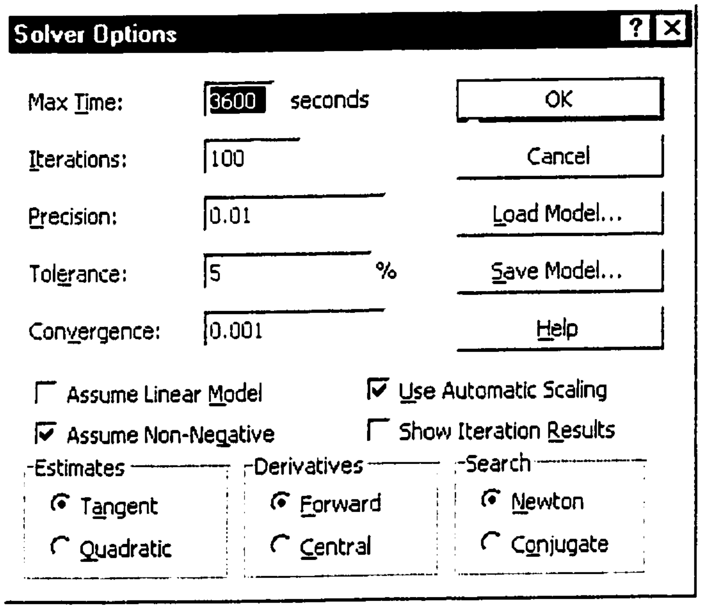

Figure 4.4 Microsoft Excel 97 - Solver Options

the decision variables. Using constant right hand sides in constraints will simplify the model, and is essential to obtain the benefits of *fast problem setup*.

4.6.5 Feasible and Optimal Solutions

A solution (values for the decision variables) for which all of the constraints in the Solver model are satisfied is called a *feasible solution*. The Solver proceeds by first finding a feasible solution, and then seeking to improve upon it by changing the decision variables to move from one feasible solution to another feasible solution until the objective function reaches its maximum or minimum. This is called an *optimal solution*.

4.6.6 User-defined functions

The user-defined functions are functions written by the user in Visual Basic Editor environment to achieve his needs and can be inserted in the spreadsheet and used in the same way as built-in functions. Figure 4.5 shows how to get into the Visual Basic Editor.

The Function procedure is a series of Visual Basic statements enclosed by the Function and End Function statements. A Function procedure is similar to a Sub procedure, but a function can also return a value. A Function procedure can take arguments, such as constants, variables, or expressions that are passed to it by a calling procedure.

Examples of user defined functions for calculating the Stress and strain are given below. When the function is inserted in a cell on spreadsheet, the values of variables are passed to the function through its arguments. The result obtained by this function is returned and displayed into the same cell.

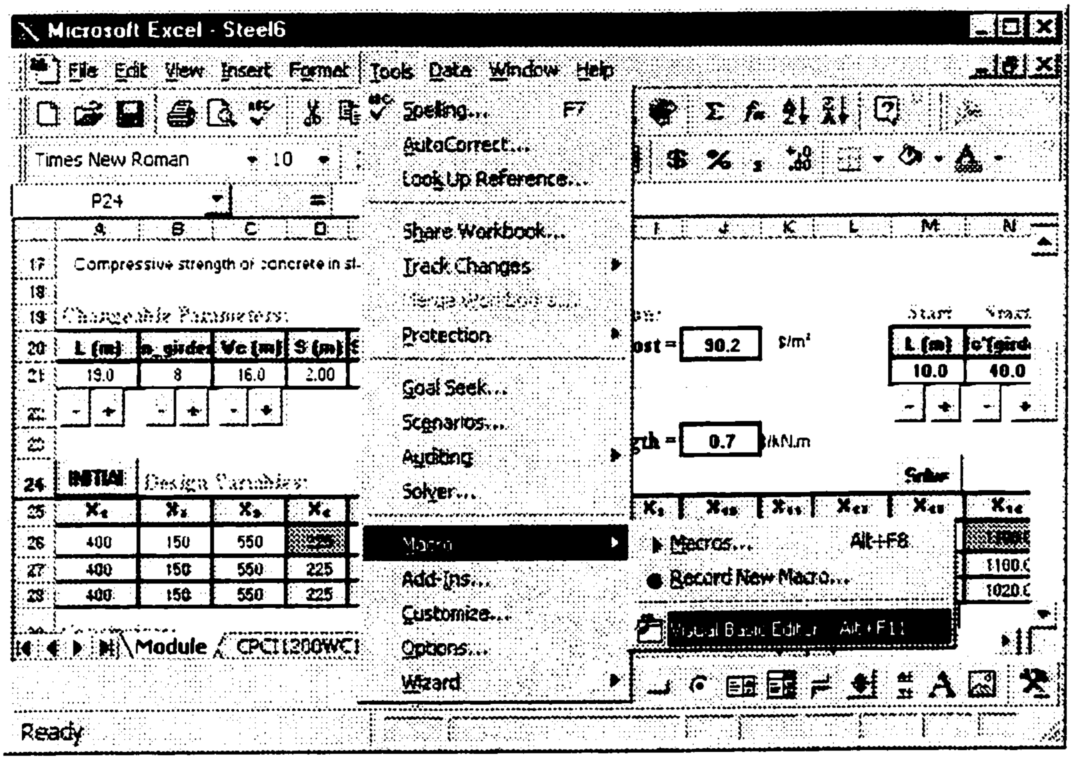

Figure 4.5 Microsoft Excel 97 – Access to Visual Basic Editor

```
' Function to calculate the axial strain at any layer.
Function STRAIN(AA, BB, II, Eref, AN, AM, Y)
  ' AXIAL STRAIN AT REFERENCE POINT O.
  EPSLONO = STRAINO(AA, BB, II, Eref, AN, AM)
  ' CURVATURE.
  PSY = CURVATURE(AA, BB, II, Eref, AN, AM)
  ' AXIAL STRAIN AT ANY LAYER.
  STRAIN = EPSLONO + PSY * (Y) / 1000
End Function

' Function to calculate the stress at any layer.
Function STRESS(AA, BB, II, Eref, E, AN, AM, Y)
  ' AXIAL STRAIN AT REFERENCE POINT O.
  EPSLONO = STRAINO(AA, BB, II, Eref, AN, AM)
  ' CURVATURE.
  PSY = CURVATURE(AA, BB, II, Eref, AN, AM)
  ' AXIAL STRAIN AT ANY LAYER.
  STRESS = E * (EPSLONO + PSY * Y / 1000)
End Function

' Axial Strain at Reference Point O.
Function STRAINO(AA, BB, II, Eref, AN, AM)
  DET = AA * II - BB ^ 2
  ' AXIAL STRAIN AT O.
  STRAINO = (II * AN - BB * AM * 1000) * 1000 / (Eref * DET)
End Function

' Curvature.
Function CURVATURE(AA, BB, II, Eref, AN, AM)
  DET = AA * II - BB ^ 2
  ' CURVATURE.
  CURVATURE = (-BB * AN + AA * AM * 1000) * 1000 / (Eref * DET) * 1000
End Function
```

Figure 4.6 shows the Visual Basic Editor environment, where the user defined functions and subroutines to be written.

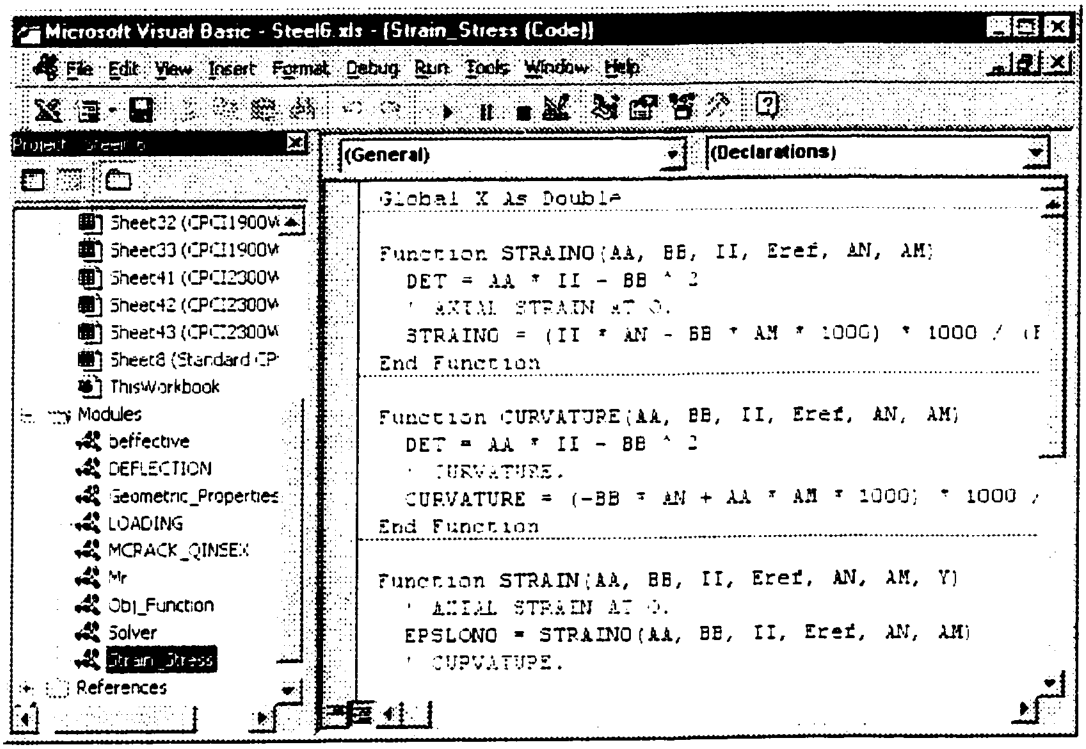

Figure 4.6 Microsoft Excel 97 – User-defined Functions in Visual Basic Editor

CHAPTER 5

RESULTS AND DISCUSSION

5.1 General

In the previous two chapters, the methods of design and analysis of the two bridge systems investigated in this thesis and the design optimization model were described. The major objective of the present chapter is to present the results of the optimum design of the bridge systems. These results include the relationships between the minimum superstructure cost per deck area and span length as well as the girder cost per unit strength versus the span length. The results also include the constraints relationships, from which the active constraint can be clearly identified. However, as indicated in the previous chapters, the results presented are only for selected number of design combinations (see Tables 4.1 and 4.2).

5.2 Optimum Design Results for CPCI Girders

The optimum design results shown below are for the design combinations selected from Table 4.1.

5.2.1 Bridges Reinforced with Steel

5.2.1.1 Superstructure Cost per Deck Area

The optimum design results obtained for bridge decks of 12 m width supported by 4 girders of each of the selected CPCI sections, CPCI1200, CPCI1400, CPCI1900, CPCI2300 (Figure 3.3) with concrete strength of 50 MPa are summarized in Table 5.1 and depicted graphically in Figure 5.1.

As shown in Figure 5.1, the optimization results indicate that the maximum span length that can be obtained using CPCI1200 is 20 m. Within the range of 10 to 20 m, the use of mentioned section leads to the lowest superstructure cost compared to the other sections. In terms of the maximum span length that can be obtained ($20 \text{ m} < L \leq 30 \text{ m}$), both CPCI1900 and CPCI2300 are comparable. However, in terms of superstructure cost, the former section is more competitive than the latter.

Figure 5.2 depicts the effect of the deck width on the cost of superstructure. Bridge decks of widths W_c = 8, 12 and 16 m built up of CPCI 1400 girders of the strength f'_{cg} = 50 MPa are investigated. The figure indicates that the 16 m wide bridge decks attain the lowest superstructure cost per deck area.

Figures 5.3 and 5.4 show the effect of the girder concrete strength on the superstructure cost. A bridge composed of 4 CPCI 1400 girders supporting a 12 m wide deck slab is selected for this investigation. As one may expect, Figure 5.3 shows that the increase in the girder concrete strength can lead to an increase in the bridge span. The two figures also indicate that the superstructure cost per deck area is directly proportional to the concrete strength and increases with the increase in the strength. The use of girders with concrete strength of 40 MPa has led to the lowest cost.

Table 5.1: Superstructure cost per deck area and girder cost per unit strength for 12 m wide bridge decks supported by 4 CPCI girders of strength 50 MPa and reinforced with steel.

Span Length m	SECTION TYPE							
	CPCI 1200		CPCI 1400		CPCI 1900		CPCI 2300	
	$/m²	$/kN.m	$/m²	$/kN.m	$/m²	$/kN.m	$/m²	$/kN.m
10	59.41	0.6	62.46	0.9	69.20	0.8	71.90	0.7
11	59.78	0.7	62.82	1.0	69.56	0.9	72.26	0.7
12	59.99	0.7	63.63	0.9	69.78	0.9	72.48	0.8
13	61.17	0.7	64.22	0.9	69.78	1.0	72.48	0.9
14	61.76	0.6	64.80	0.8	70.37	1.0	72.48	1.0
15	62.34	0.6	65.98	0.7	70.96	0.9	73.07	1.0
16	62.93	0.6	66.57	0.7	71.55	0.9	73.66	0.9
17	64.10	0.5	67.16	0.7	72.13	0.9	73.66	0.9
18	64.69	0.5	68.33	0.7	72.72	0.8	74.24	0.8
19	65.87	0.5	68.33	0.6	73.30	0.8	74.83	0.9
20	67.04	0.5	69.50	0.6	73.89	0.8	75.42	0.8
21			70.09	0.6	74.48	0.7	76.01	0.8
22			71.26	0.6	75.06	0.7	76.60	0.8
23			72.44	0.5	76.24	0.7	77.18	0.7
24			74.20	0.5	76.83	0.7	77.77	0.7
25					78.00	0.7	78.35	0.7
26					78.00	0.7	79.53	0.7
27					79.17	0.6	79.53	0.7
28					80.35	0.6	80.70	0.6
29					80.93	0.6	81.88	0.6
30					82.11	0.6	82.46	0.6

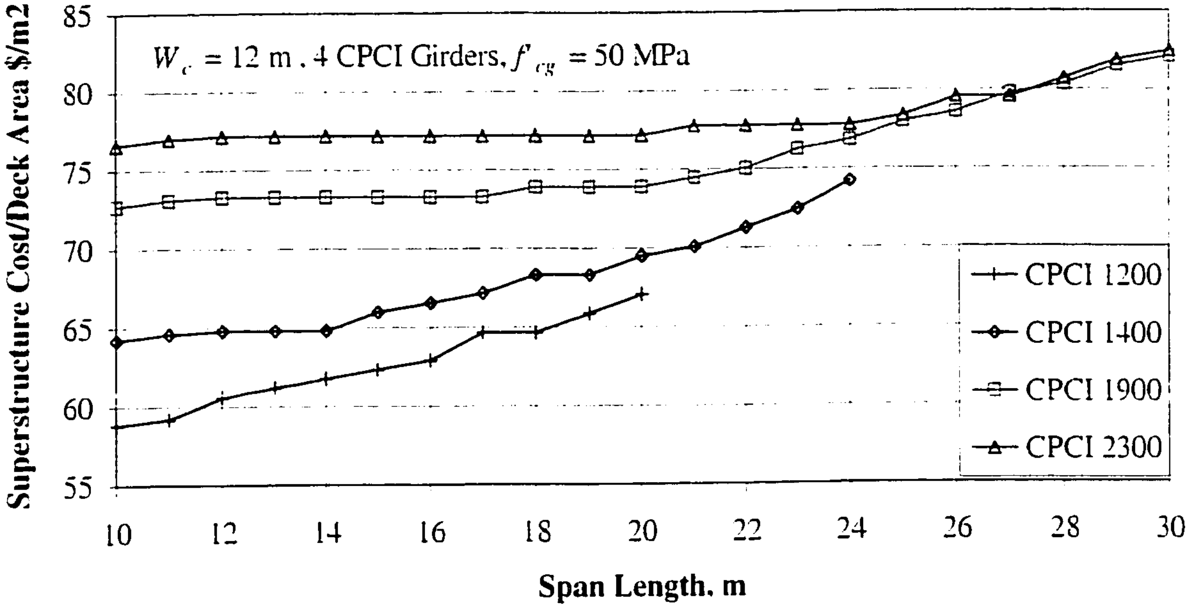

Figure 5.1 Superstructure Cost per Deck Area - CPCI Sections for Bridges Reinforced with Steel.

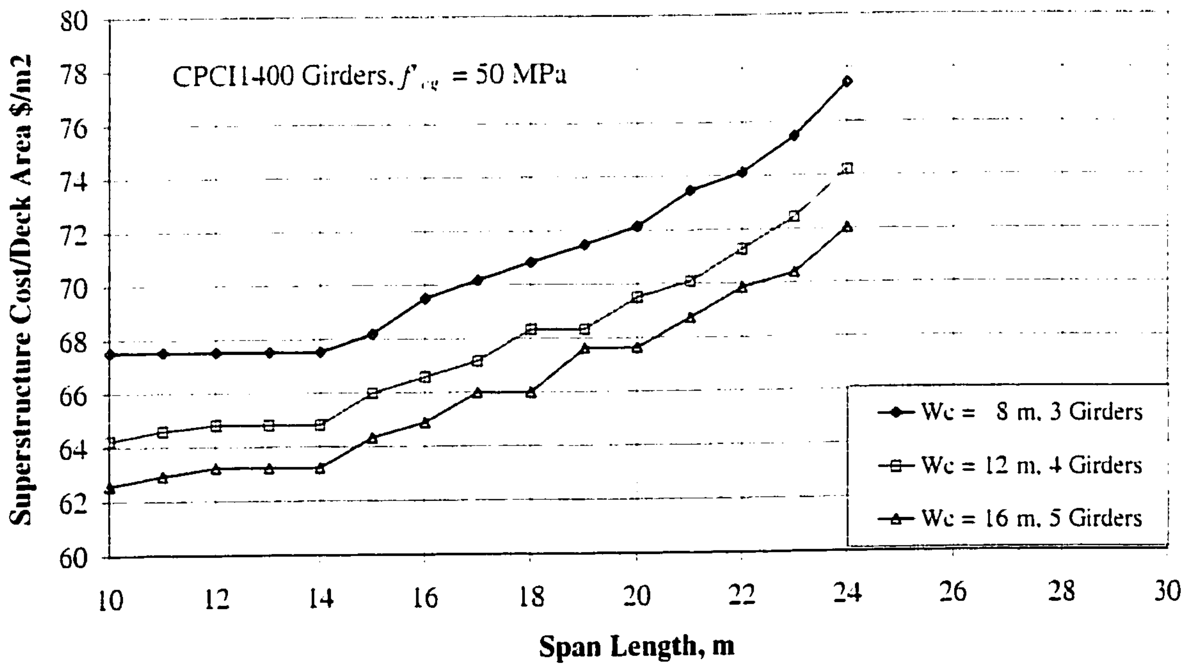

Figure 5.2 Effect of Deck Width on Superstructure Cost per Deck Area for Bridges Reinforced with Steel.

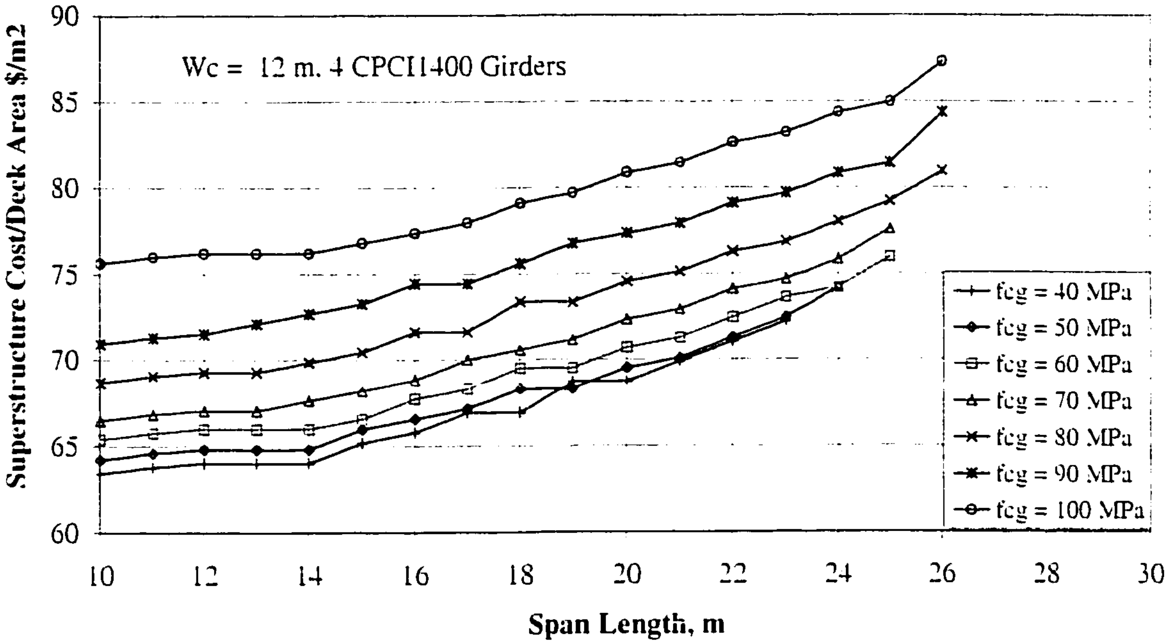

Figure 5.3 Effect of Girder Concrete Strength on Superstructure Cost per Deck Area for Bridges Reinforced with Steel

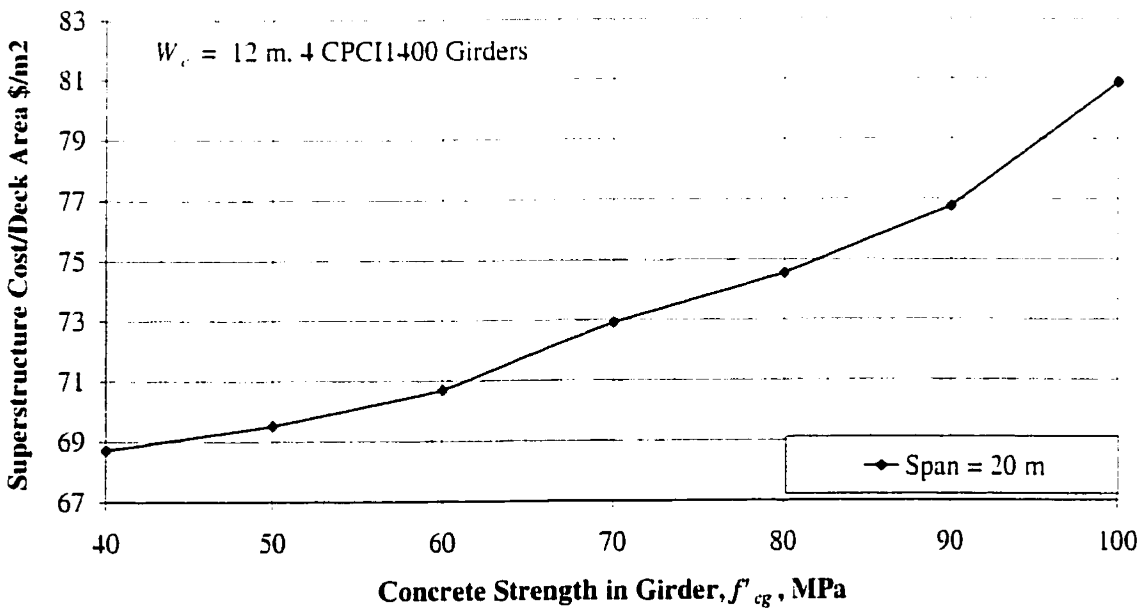

Figure 5.4 Variation of Superstructure Cost per Deck Area with the Girder Concrete Strength for Bridges Reinforced with Steel

5.2.1.2 Girder Cost per Unit Strength

For comparison between the selected sections in terms of girder cost per unit strength, Table 5.1 and Figure 5.5 show the relationship between the girder cost per unit strength and the span length for a deck width of 12 m and girder concrete strength of 50 MPa. As can be seen, the relationships for all the sections show the same behavior pattern. However, CPCI2300 is the most competitive in terms of both girder cost per unit strength and span length.

5.2.2 Bridges Reinforced with CFRP

5.2.2.1 Superstructure Cost per Deck Area

The optimum design results obtained for bridge decks of 12 m width supported by 4 girders of concrete strength 50 MPa of each of the selected CPCI sections, CPCI1200, CPCI1400, CPCI1900, and CPCI2300 (Figure 3.3) with concrete strength of 50 MPa are summarized in Table 5.2 and represented graphically in Figure 5.6

As can be seen, the relationships for all the sections show the same behavior pattern. For span length $10 \leq L \leq 30$ m, CPCI2300 is considered to be the most competitive among the selected girders in terms of superstructure cost per deck area.

Figure 5.7 shows the effect of width of bridge deck on the superstructure cost. Once again, bridge deck widths, W_c = 8, 12 and 16 m and CPCI 1400 girders of f'_{cg} = 50 MPa are considered. Similar to the case when steel is used as reinforcement, the 16 m wide bridge decks have the lowest superstructure cost per deck area.

Figure 5.8 shows the effect of the girder concrete strength on the superstructure cost. As the figure indicates, the superstructure cost per deck area is not significantly affected by the change in the girder concrete strength. This is attributed to the fact that the

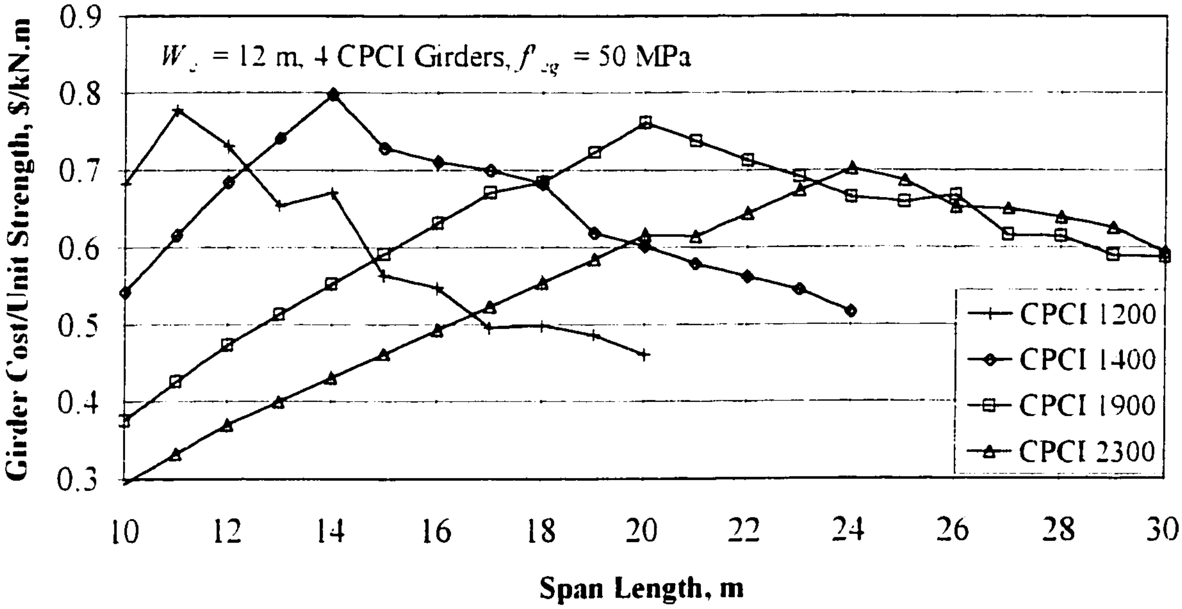

Figure 5.5 Girder Cost per Unit Strength - CPCI Sections for Bridges Reinforced with Steel.

Table 5.2: Superstructure cost per deck area and girder cost per unit strength for 12 m wide bridge decks supported by 4 CPCI girders of strength 50 MPa and reinforced CFRP.

Span Length m	SECTION TYPE							
	CPCI 1200		CPCI 1400		CPCI 1900		CPCI 2300	
	$/m²	$/kN.m	$/m²	$/kN.m	$/m²	$/kN.m	$/m²	$/kN.m
10	432.54	2.7	423.77	3.1	417.51	3.0	402.89	2.8
11	450.70	2.9	437.60	3.4	427.01	3.3	416.72	3.0
12	465.69	3.0	452.59	3.7	437.67	3.5	431.71	3.1
13	478.69	3.2	474.25	3.4	450.66	3.6	440.37	3.2
14	496.01	3.3	478.58	3.6	463.66	3.7	449.04	3.4
15	509.01	3.5	495.90	3.7	472.32	3.8	457.70	3.5
16	517.67	3.6	504.57	3.8	480.99	4.0	466.36	3.6
17	543.66	3.7	526.22	3.9	493.98	4.1	475.02	3.7
18	552.32	3.9	530.55	4.0	502.64	4.2	483.69	3.8
19	578.31	4.0	556.54	4.0	519.97	4.2	496.68	3.8
20			565.20	4.2	528.63	4.4	501.01	4.0
21			582.53	4.4	537.29	4.5	522.67	4.0
22			599.85	4.5	550.28	4.7	527.00	4.1
23			612.85	4.7	567.61	4.9	539.99	4.3
24			647.50	4.9	580.60	5.0	552.99	4.4
25					597.93	4.7	565.98	4.5
26					610.92	4.9	574.64	4.6
27					628.25	4.9	591.97	4.5
28					641.24	5.0	609.29	4.5
29					662.90	5.0	617.95	4.6
30					675.89	5.1	639.61	4.6

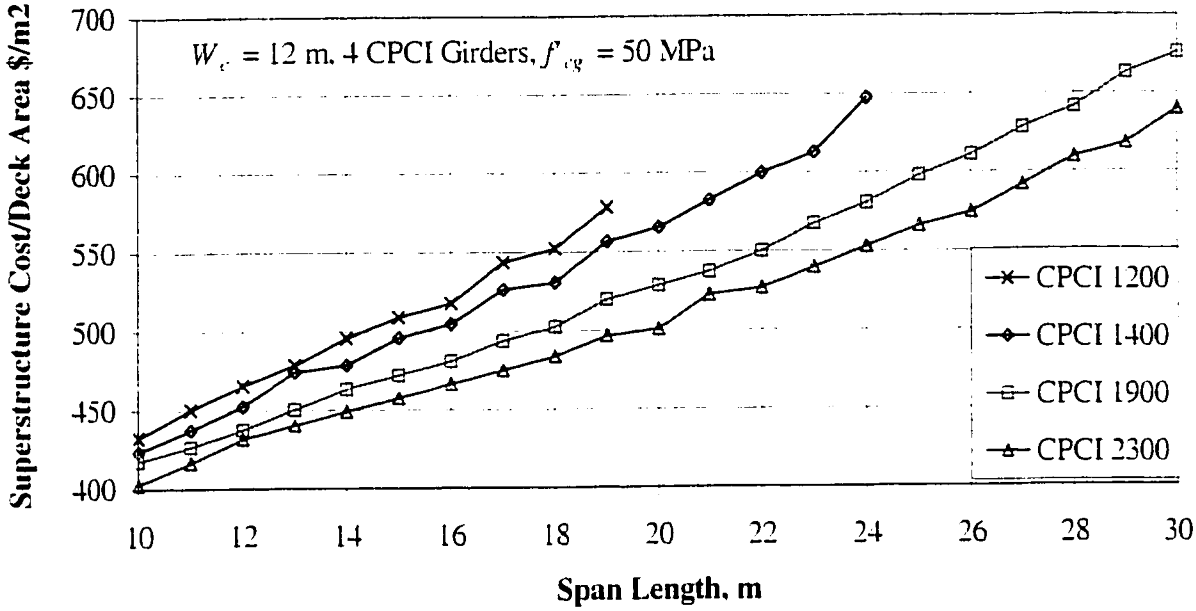

Figure 5.6 Superstructure Cost per Deck Area - CPCI Sections for Bridges Reinforced with CFRP.

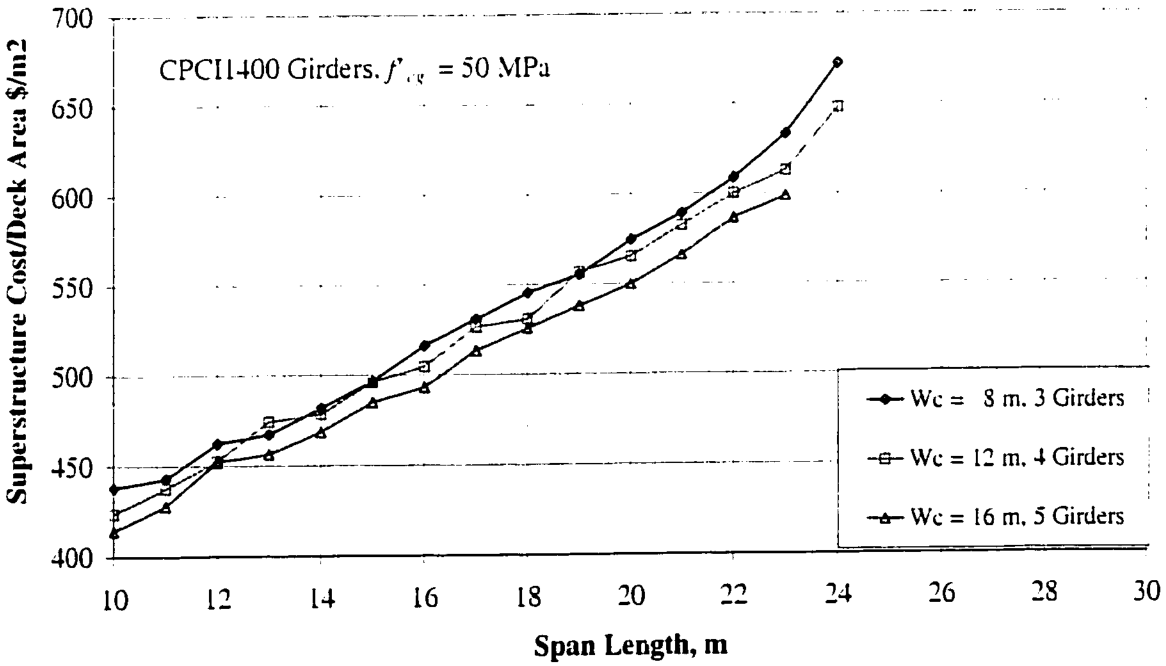

Figure 5.7 Effect of Deck Width on Superstructure Cost per Deck Area for Bridges Reinforced with CFRP.

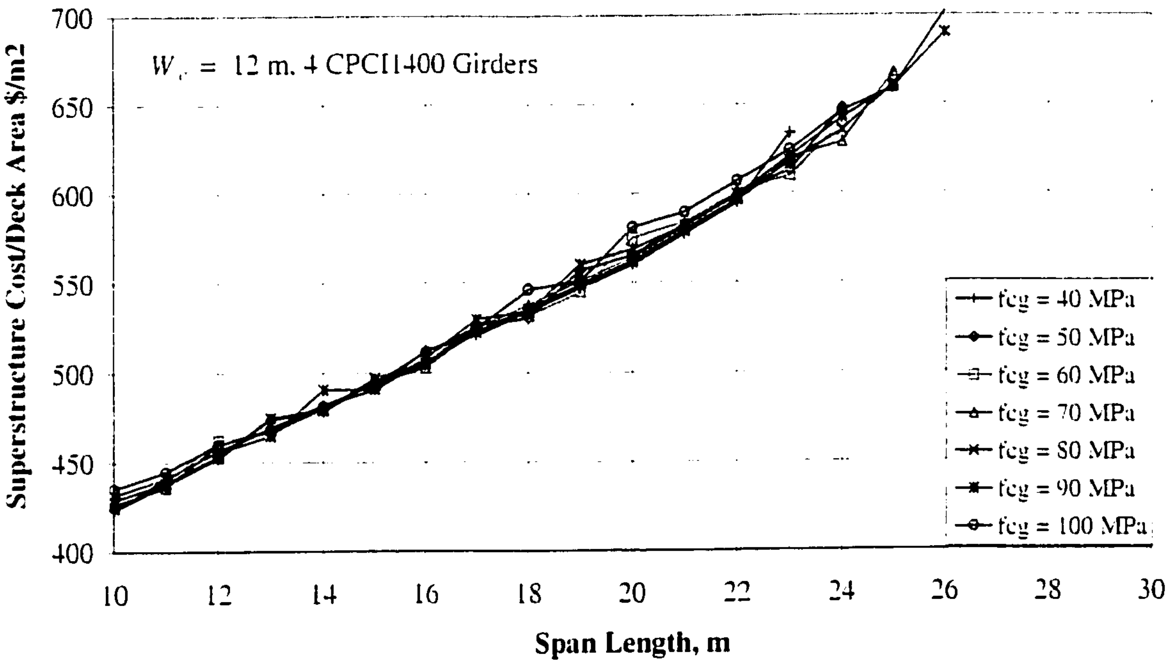

Figure 5.8 Effect of Girder Concrete Strength on Superstructure Cost per Deck Area for Bridges Reinforced with CFRP

cost of CFRP reinforcement is very high in comparison with the cost due to the change in concrete strength. The cost of CFRP thus becomes the predominant portion of the total cost of the superstructure.

5.2.2.2 Girder Cost per Unit Strength

Figure 5.9 shows a comparison of the relationship between the girder cost per unit strength for the different CPCI girders of Figure 3.3, reinforced and prestressed with CFRP material. As can be seen, the relationships for all the sections show the same behaviour pattern. The CPCI 2300 section showed the lowest girder cost per unit strength.

5.3 Optimum Design Results for Nebraska Girders

The optimum design results shown below are for the design combinations selected from Table 4.2.

5.3.1 Bridges Reinforced with Steel

5.3.1.1 Superstructure Cost per Deck Area

The optimum design results obtained for bridge decks of 14.70 m width built up of Nebraska proposed all-precast girders, Bulb T, Butted I, and Pie (Figure 3.7) with concrete strength of 50 MPa are summarized in Table 5.3 and depicted graphically in Figure 5.10.

As shown in this Figure, the optimization results indicate that for span length between 10 m and 13 m, the Pie section is the most cost effective. The Butted I-section is the only section that can allow for spans greater than 13 m and up to 17 m.

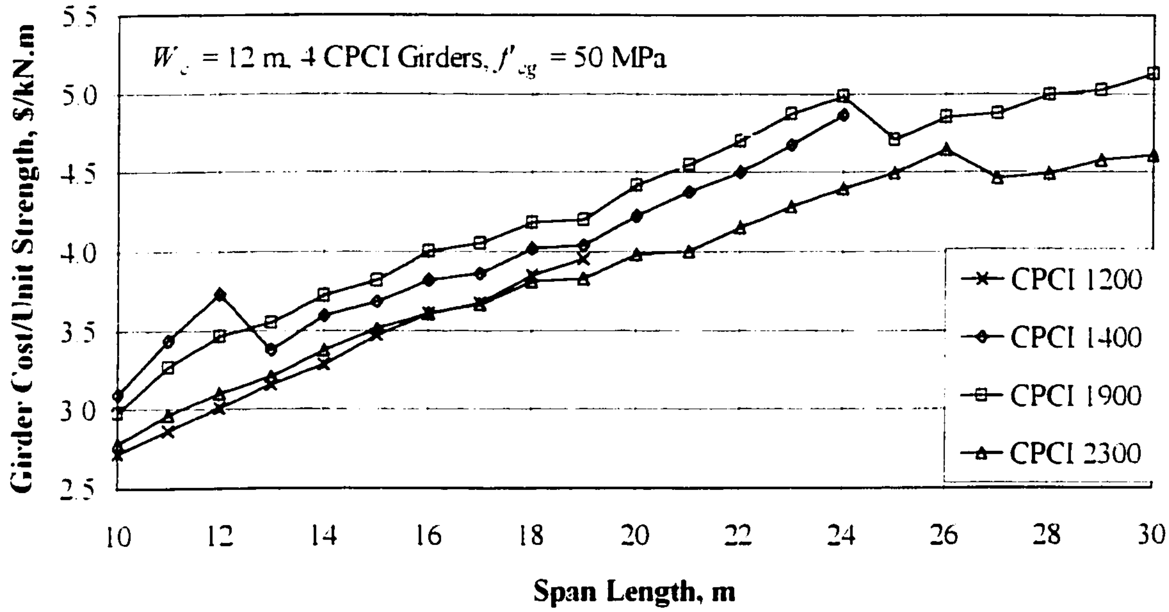

Figure 5.9 Girder Cost per Unit Strength - CPCI Sections for Bridges Reinforced with CFRP.

Table 5.3: Superstructure cost per deck area and girder cost per unit strength for 14.70 m wide bridges built up of Nebraska precast girders of strength 50 MPa and reinforced with steel.

Span Length m	SECTION TYPE					
	Bulb T		Butted I		Pie	
	$/m²	$/kN.m	$/m²	$/kN.m	$/m²	$/kN.m
10	38.21	0.9	65.24	0.9	38.79	0.8
11	38.93	1.0	65.24	1.0	38.79	0.9
12	40.37	0.9	69.09	0.8	40.23	0.8
13			70.05	0.9	41.67	0.8
14			72.94	0.8		
15			75.82	0.8		
16			77.75	0.8		
17			85.45	0.7		
18						
19						
20						

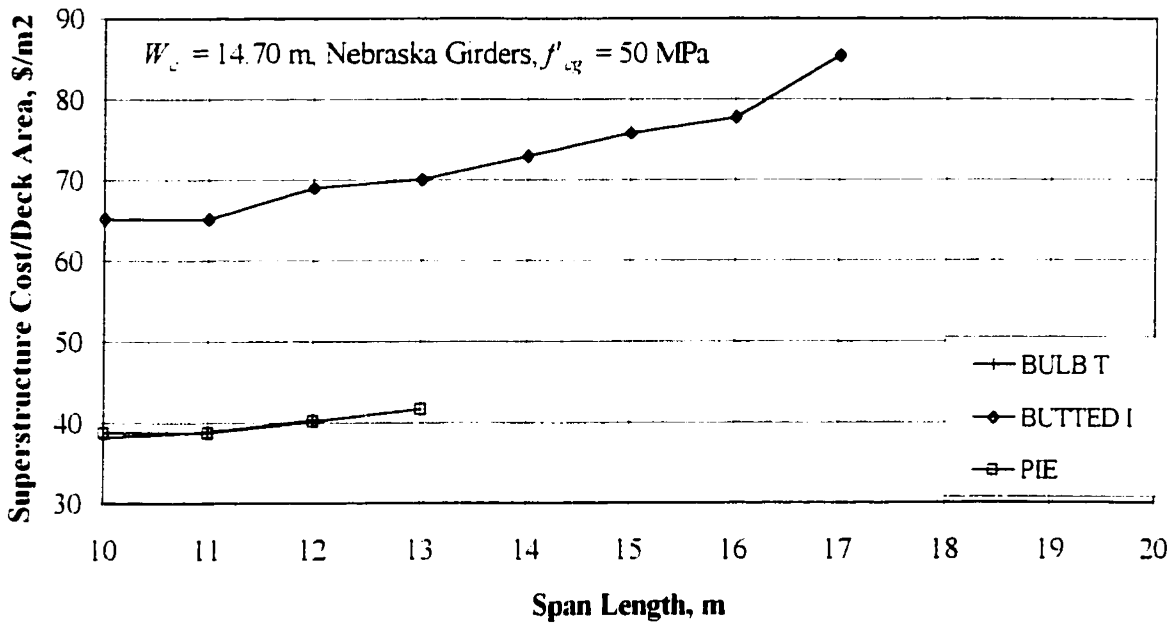

Figure 5.10 Superstructure Cost per Deck Area - Nebraska Proposed Sections Reinforced with Steel.

The effect of the girder's concrete strength on the superstructure cost is shown in Figure 5.11. The design is made for a bridge built up of 8 Butted I girders. Once again, as in the case of CPCI girders, the superstructure cost increases with the increase in the concrete strength. The lowest cost was achieved for 40 MPa concrete strength. But it should be noted that the increase in the concrete strength allows for an increase in the bridge span.

5.3.1.2 Girder Cost per Unit Strength

Figure 5.12 depicts the relationship between the girder cost per unit strength and span length. As can be seen, the relationship for Butted I and Pie sections show the same behavior pattern, with the Pie section giving the lowest cost per unit strength for spans up to 13 m, where as the Butted I-section can allow for increase in the span up to 17 m.

5.3.2 Bridges Reinforced with CFRP

5.3.2.1 Superstructure Cost per Deck Area

The optimum design results obtained for Bridge decks of 14.70 m width made up of Nebraska precast girders with concrete strength of 50 MPa are summarized in Table 5.4 and shown graphically in Figures 5.13. Similar to the girders reinforced with steel, the Pie section leads to the lowest cost in comparison to the other two sections and Butted I-section has led to the longer span (17 m) among the three sections.

Figure 5.14, compares the effect of the girder's concrete strength on the cost of the superstructure for different spans. Similar to the slab-on-CPCI girder bridges reinforced with CFRP, the concrete strength of the girder has a smaller effect on the cost of the superstructure in comparison with the cost of the CFRP reinforcement.

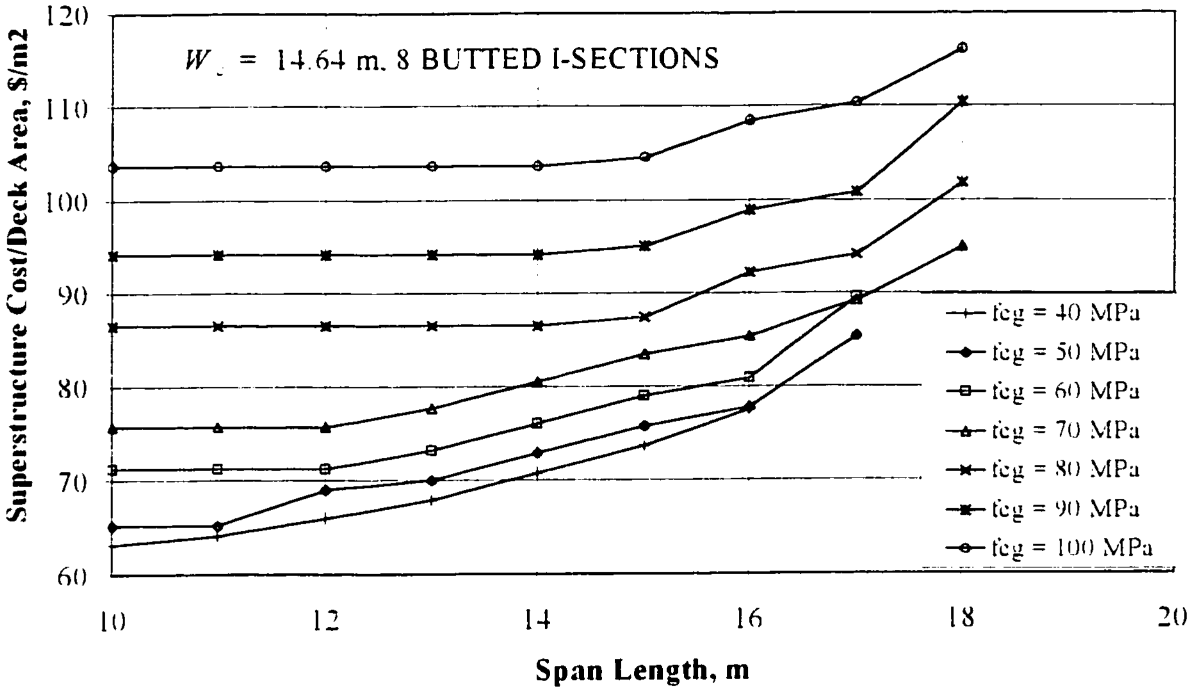

Figure 5.11 Effect of Girder's Concrete Strength on the Superstructure Cost per Deck Area – Butted I Sections Reinforced with Steel.

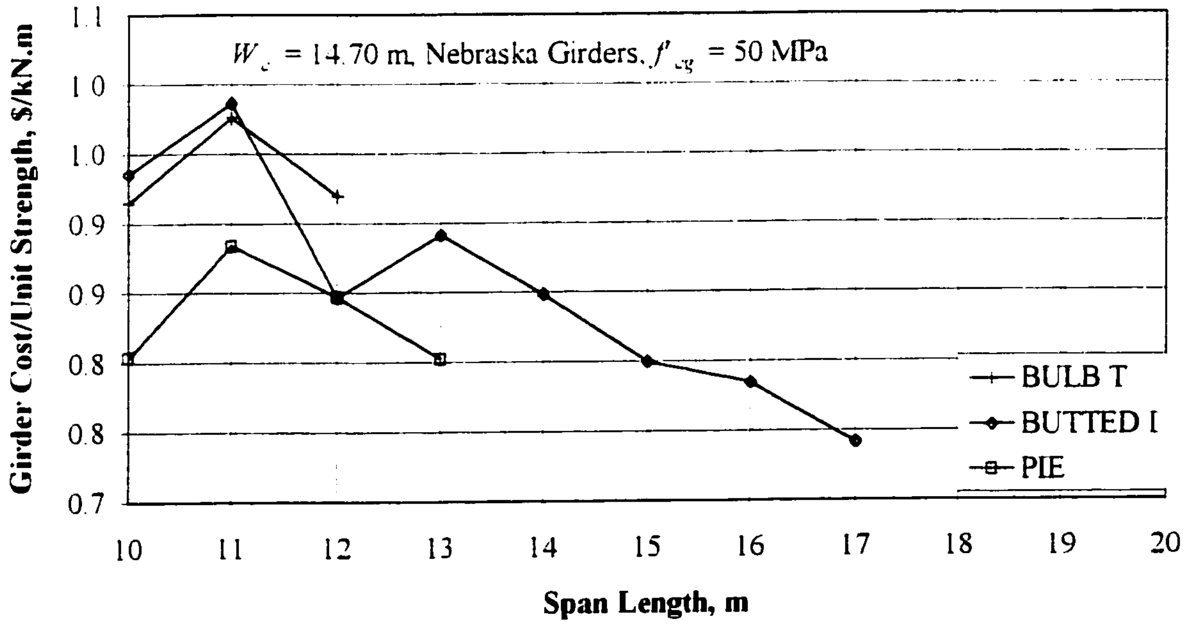

Figure 5.12 Girder Cost per Unit Strength - Nebraska Proposed Sections Reinforced with Steel.

Table 5.4: Superstructure cost per deck area and girder cost per unit strength for 14.70 m wide bridges built up of Nebraska precast girders of strength 50 MPa and reinforced with CFRP.

Span Length m	SECTION TYPE					
	Bulb T		Butted I		Pie	
	$/m²	$/kN.m	$/m²	$/kN.m	$/m²	$/kN.m
10	257.76	5.3	310.18	5.7	213.26	5.6
11	257.76	5.8	324.39	6.1	229.17	5.9
12	268.36	6.4	388.29	6.3	255.69	6.3
13			402.49	6.8	282.21	6.7
14			452.19	7.1		
15			516.10	7.5		
16			551.60	7.9		
17			672.30	8.0		
18						
19						
20						

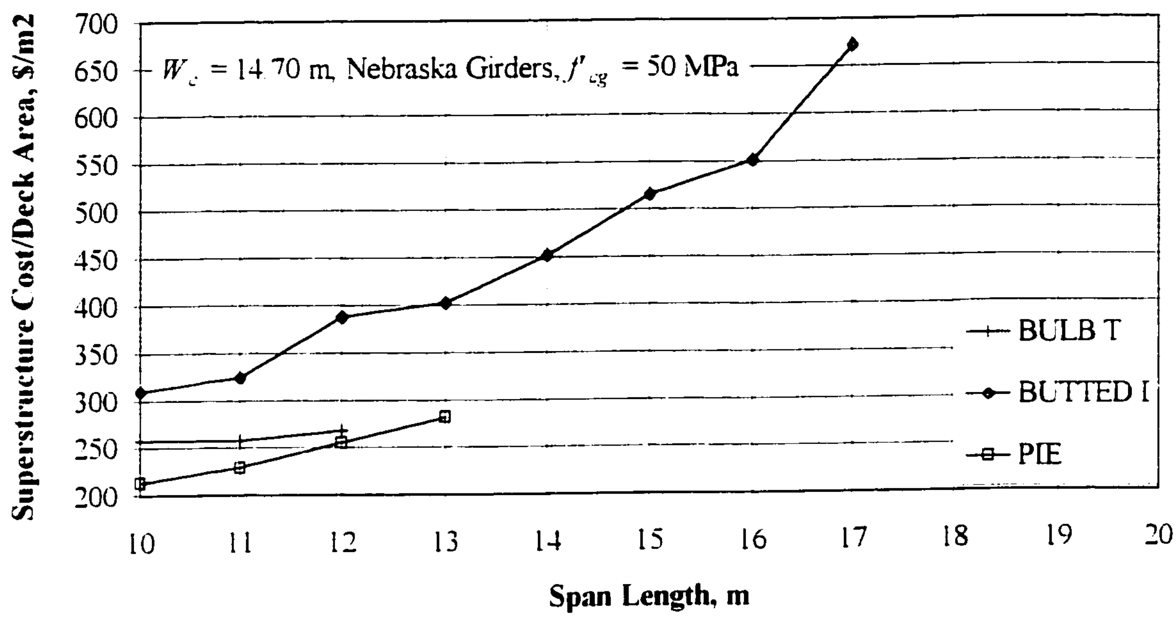

Figure 5.13 Superstructure Cost per Deck Area - Nebraska Proposed Sections Reinforced with CFRP.

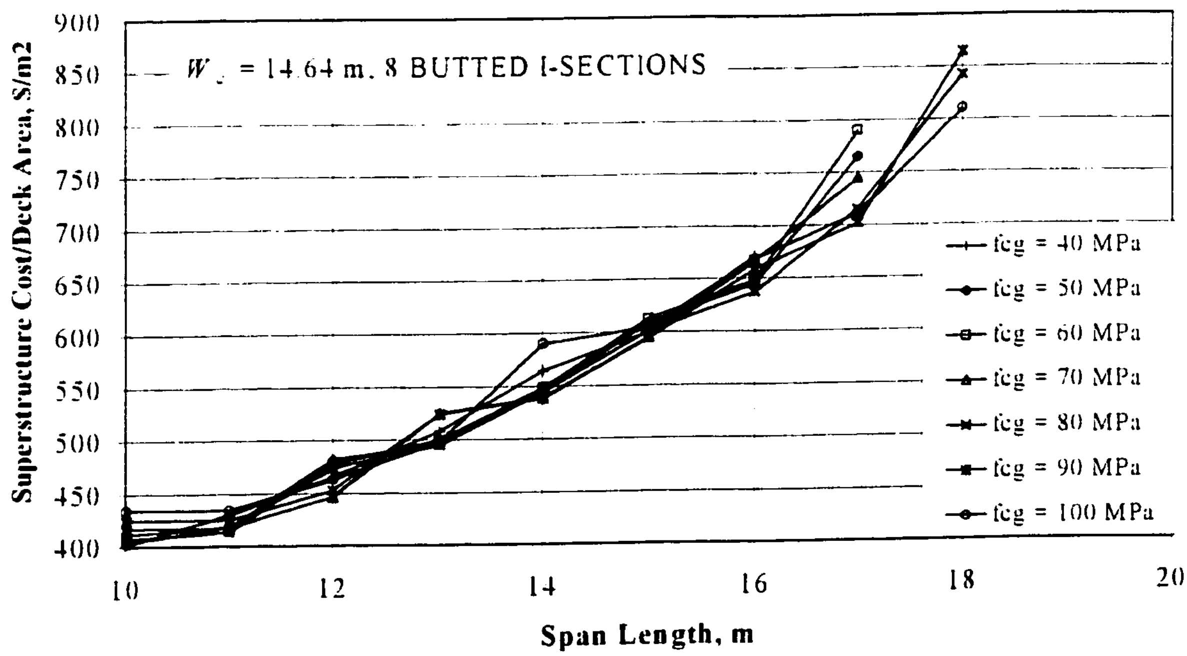

Figure 5.14 Effect of Girder's Concrete Strength on the Superstructure Cost per Deck Area – Butted I Sections Reinforced with CFRP.

5.3.2.2 Girder Cost per Unit Strength

Figure 5.15, depicts the relationship between the girder cost per unit strength and the span of Nebraska precast bridge systems reinforced with CFRP. The figure indicates that the Butted I-section leads to the length span and is cost effective.

5.4 Constraint Activity

The optimum solution is usually governed by the constraints. A constraint is said to be active if it is binding to one of the imposed limits. In other words, an active constraint belongs to a critical design requirement that controls the optimum solution. For present investigation, the active constraints for the selected design combinations have been identified as follows:

- Figures 5.16 to 5.25 represent graphically the constraints related to the CPCI sections reinforced with steel. Examination of these figures reveals that the bottom fiber stress at service stage at the mid-span section [Equations (3.2) and (4.7c)] is the only active constraint.
- Figures 5.26 to 5.35 represent graphically the constraints related to the CPCI sections reinforced with CFRP. The figures indicate that the bottom fiber stress at service stage [Equations (3.2) and (4.7c)] and deflection [Equations (3.22) and (4.8a-c)] at the mid-span section are the only active constraints.
- Figures 5.36 to 5.42 represent graphically the constraints related to the Nebraska sections reinforced with steel. The figures show that the bottom fiber stress at service stage [Equations (3.2) and (4.7c)] and deflection [Equations (3.22) and (4.8a-c)] at the mid-span section are the only active constraints.

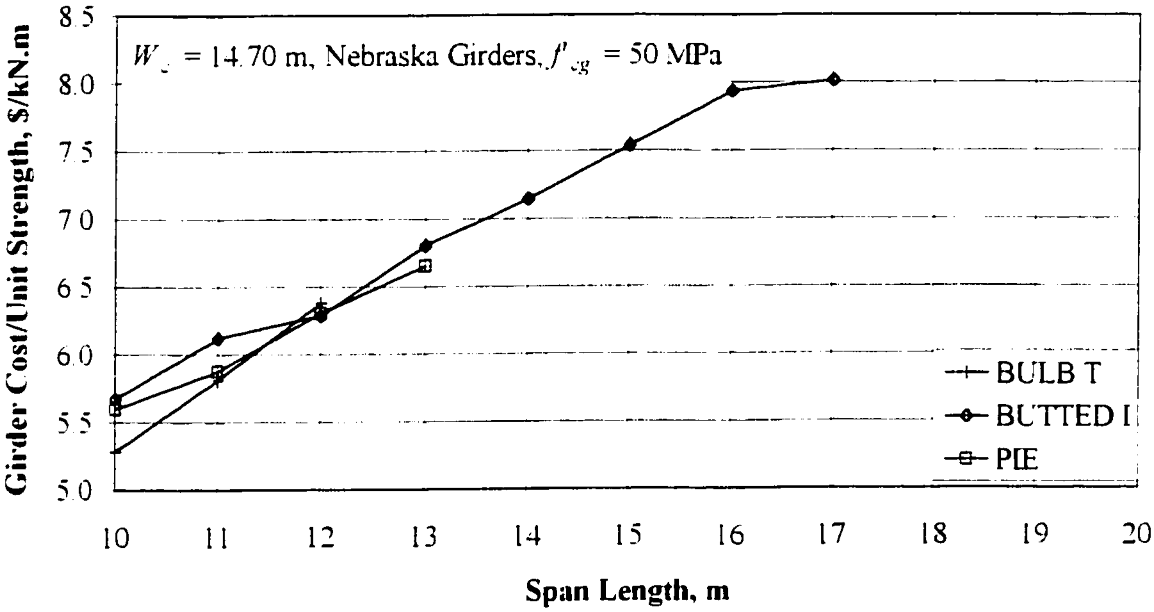

Figure 5.15 Girder Cost per Unit Strength - Nebraska Proposed Sections Reinforced with CFRP.

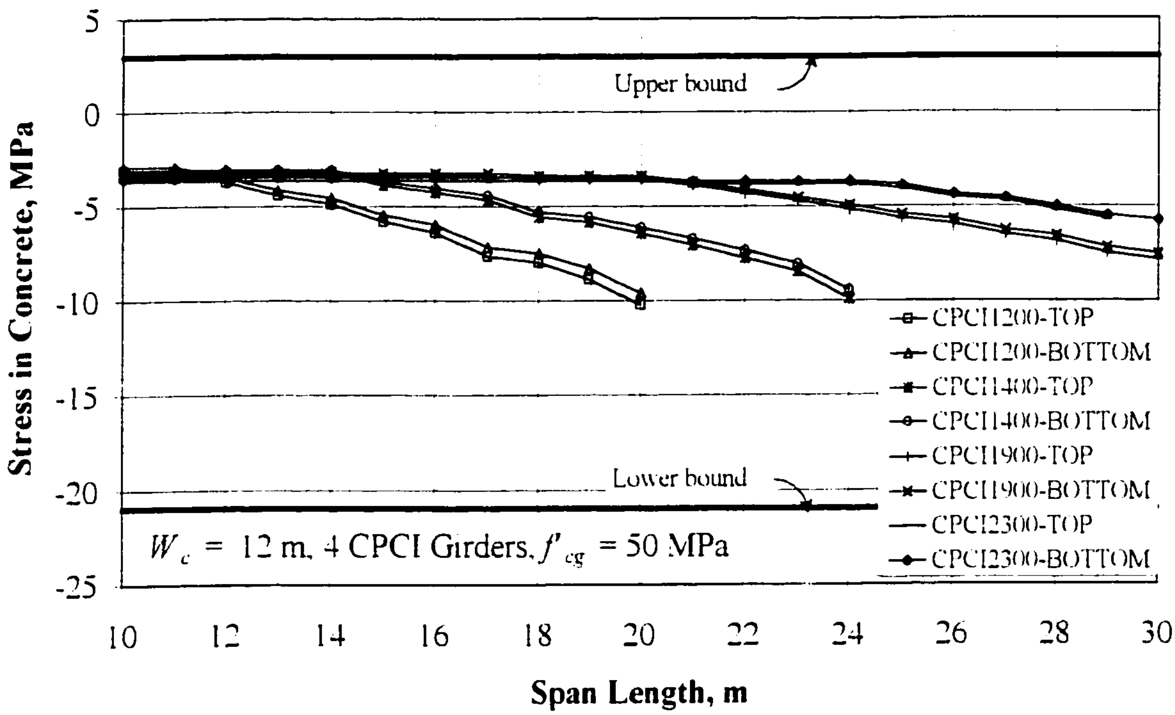

Figure 5.16 Support Section Stress Constraints at Transfer - CPCI Sections Reinforced with Steel.

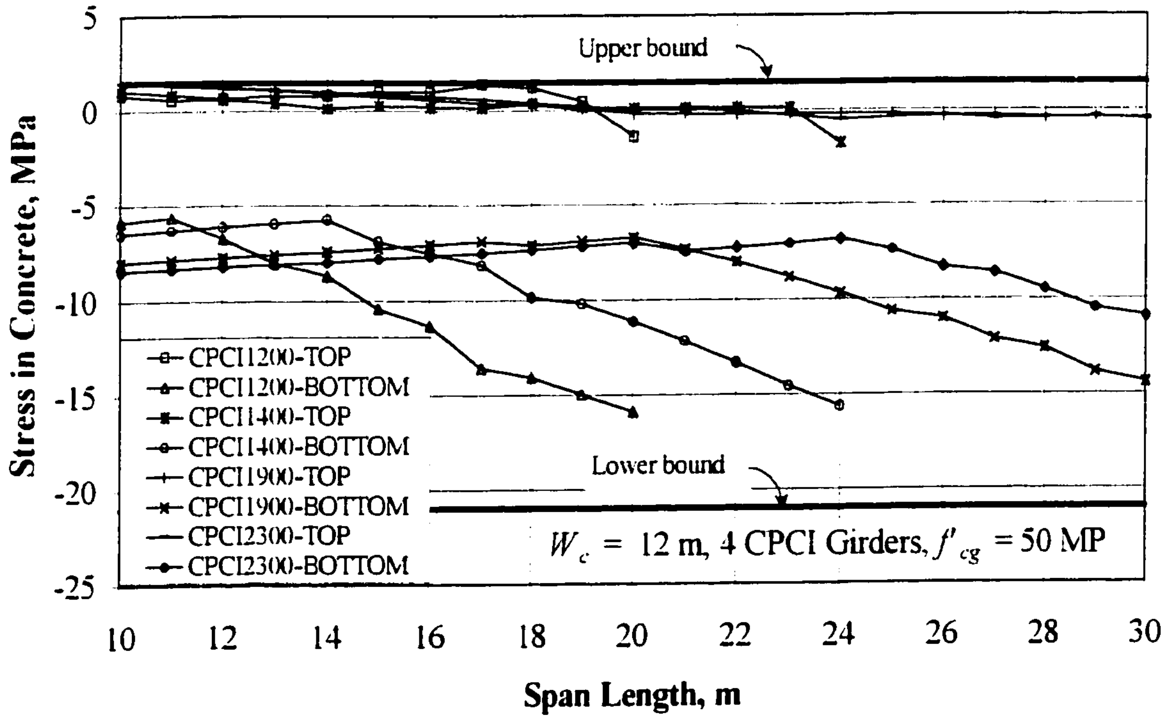

Figure 5.17 Mid-span Section Stress Constraints at Transfer - CPCI Sections Reinforced with Steel.

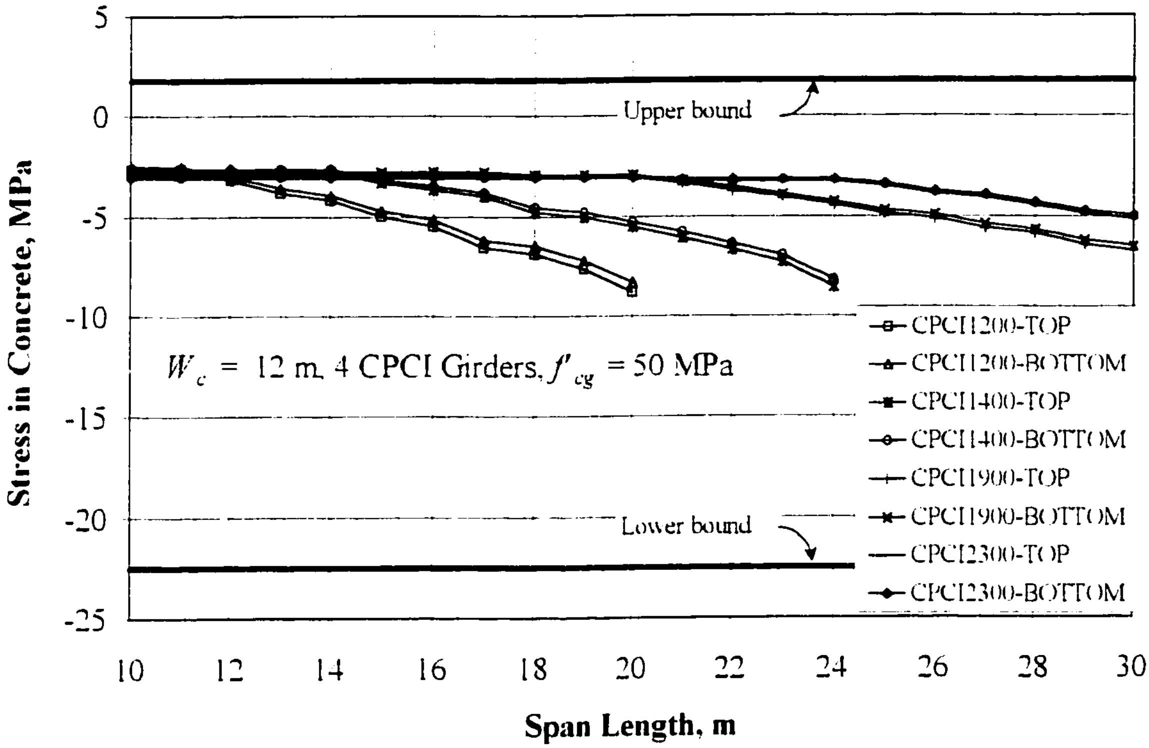

Figure 5.18 Support Section Stress Constraints during Construction - CPCI Sections Reinforced with Steel

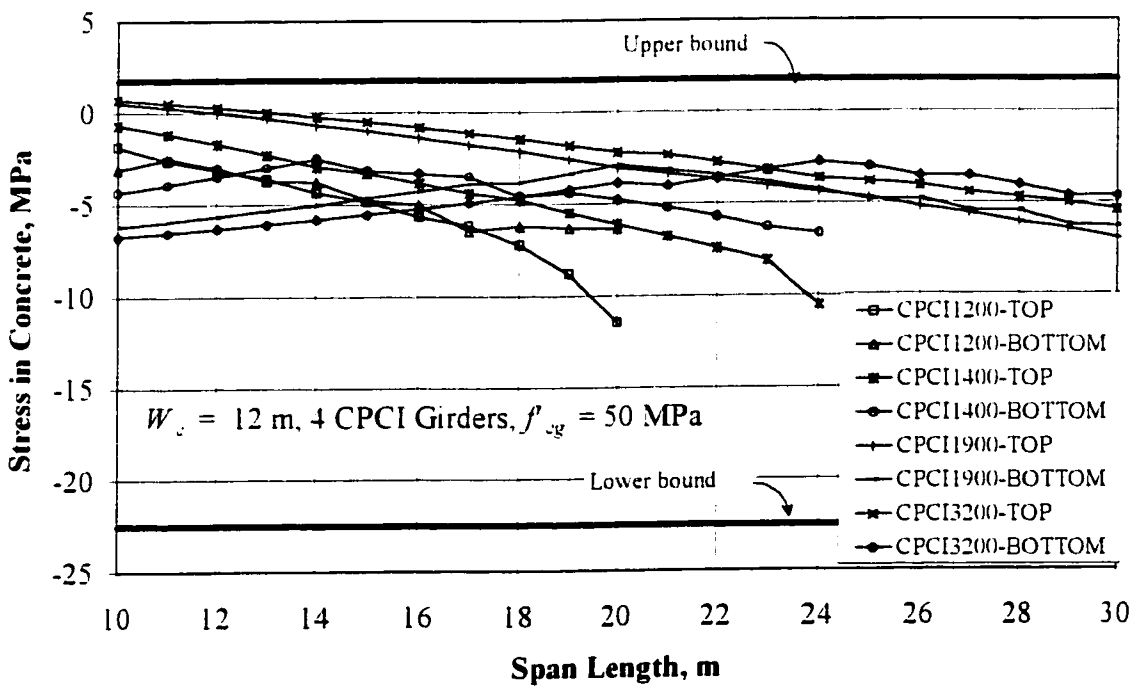

Figure 5.19 Mid-span Section Stress Constraints during Construction - CPCI Sections Reinforced with Steel.

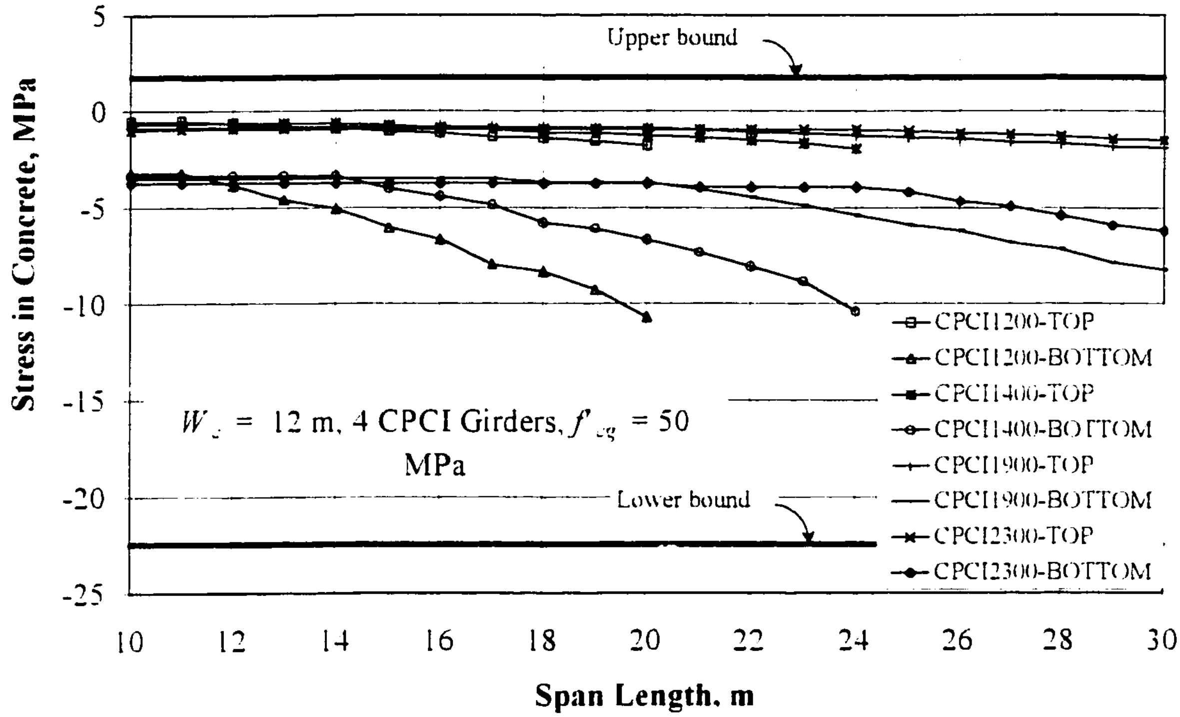

Figure 5.20 Support Section Stress Constraints at Service - CPCI Sections Reinforced with Steel.

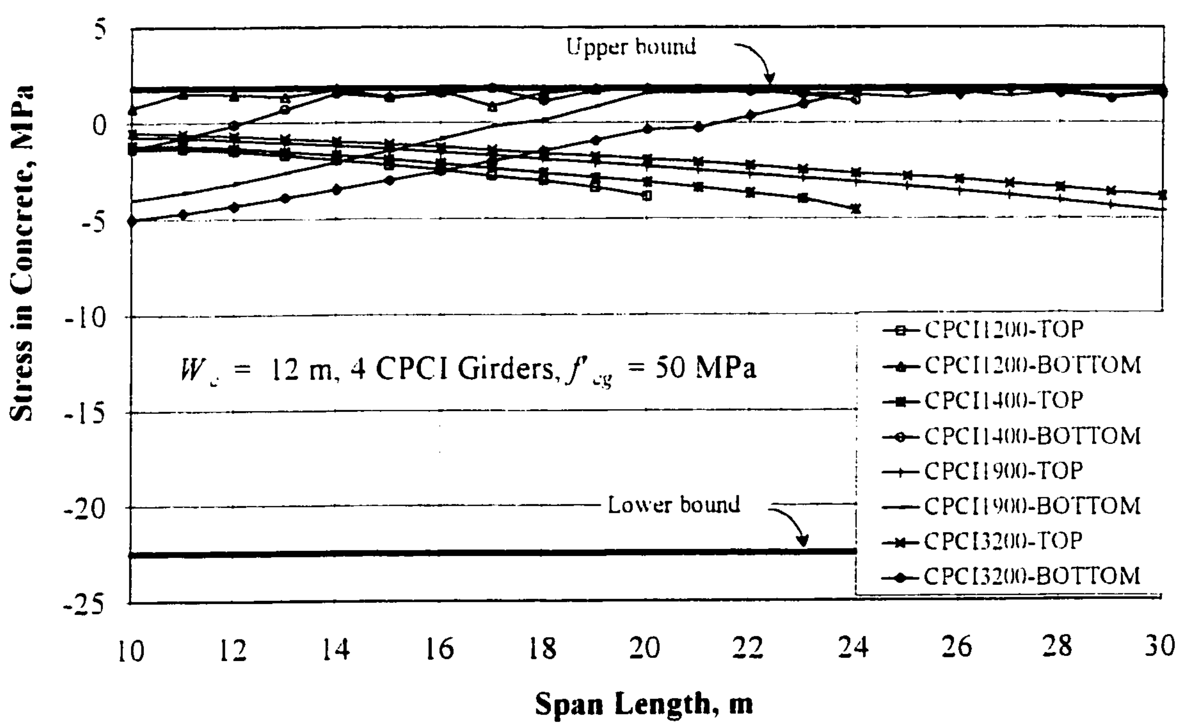

Figure 5.21 Mid-span Section Stress Constraints at Service - CPCI Sections Reinforced with Steel.

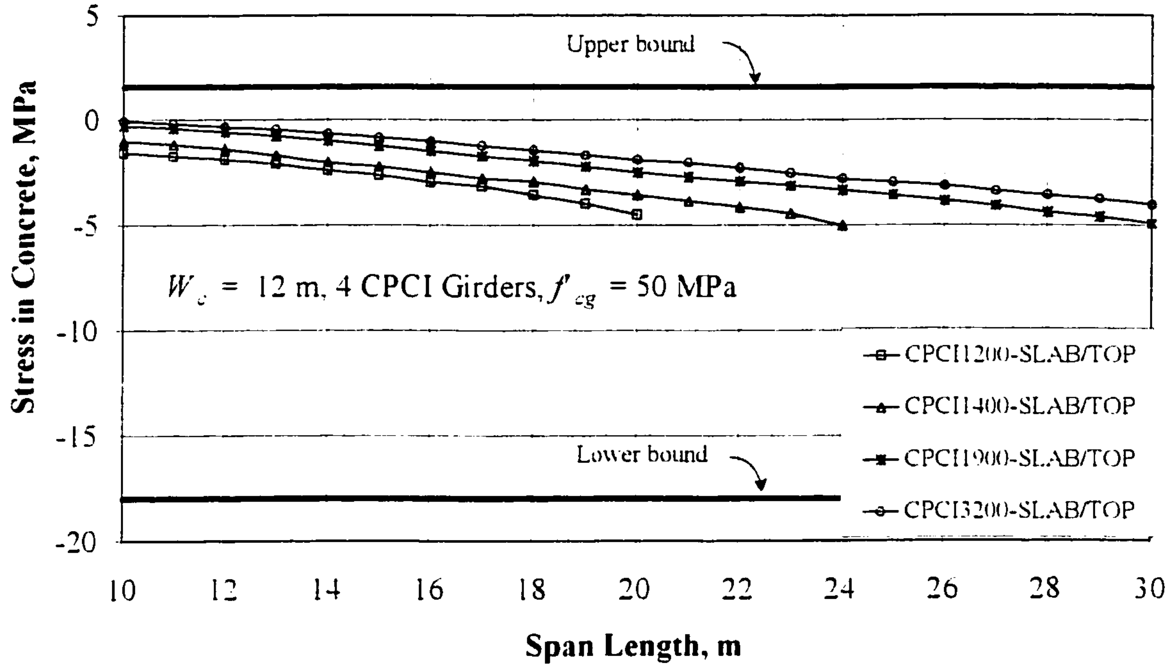

Figure 5.22 Mid-span Section Stress in Slab Constraints at Service - CPCI Sections Reinforced with Steel.

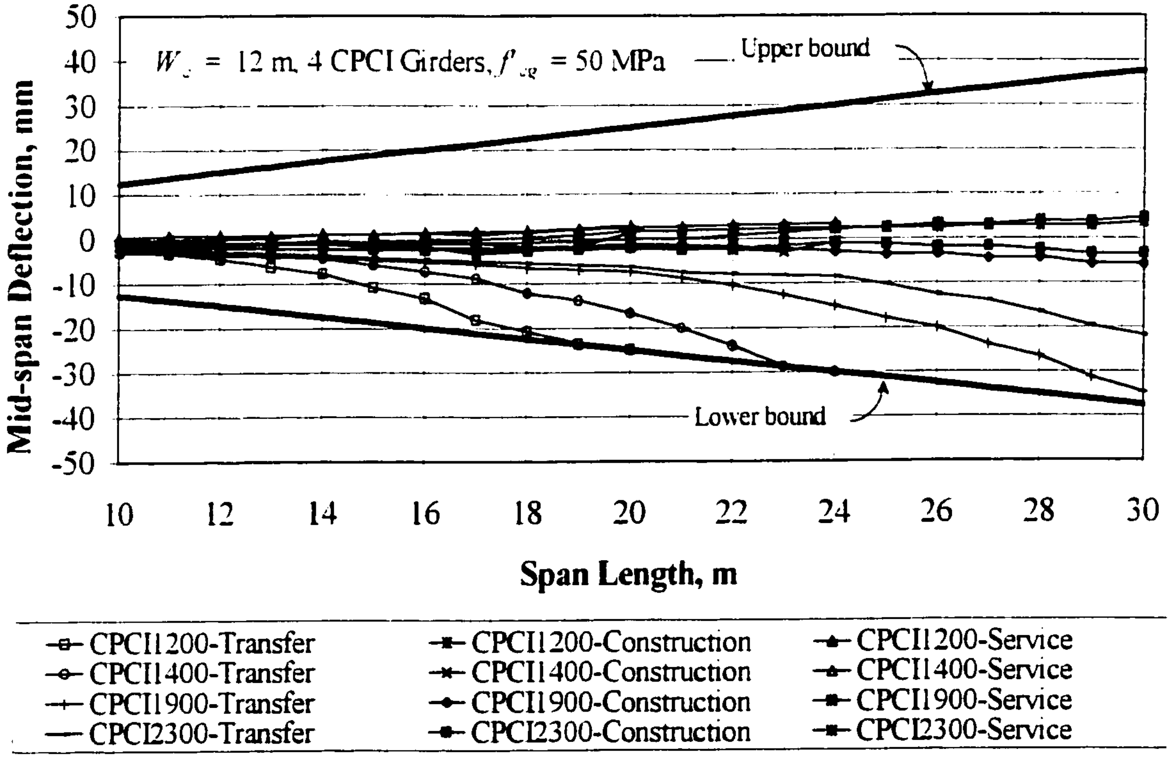

Figure 5.23 Mid-span Deflection Constraint - CPCI Sections Reinforced with Steel.

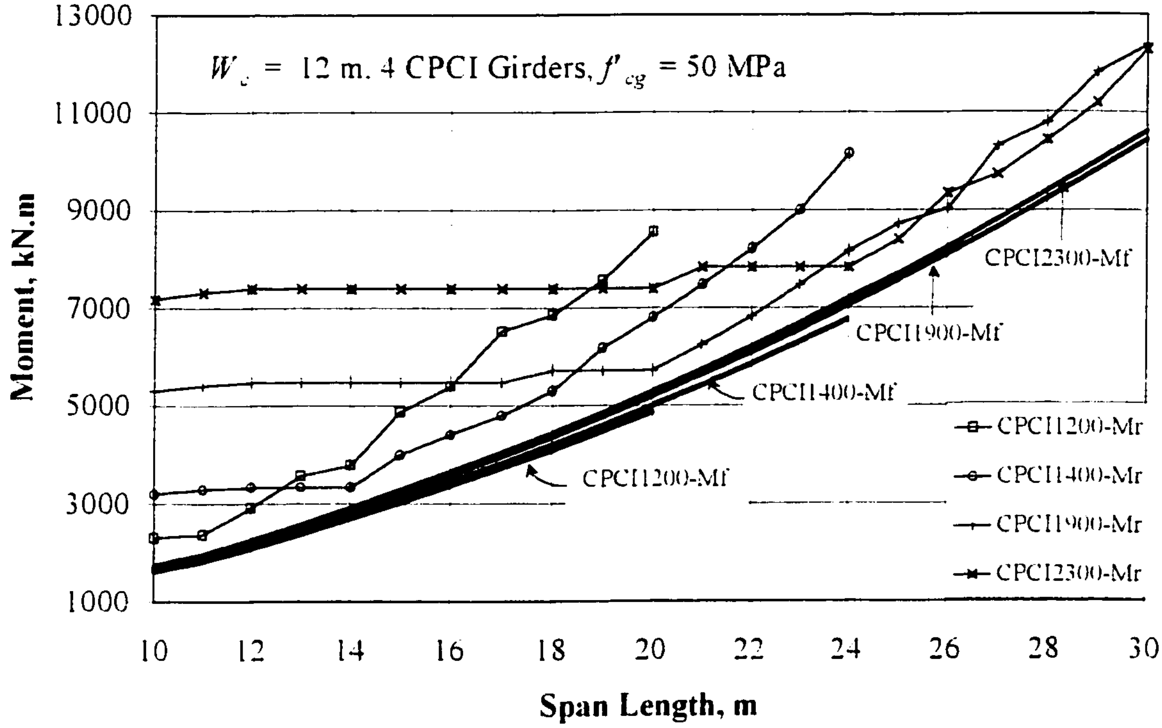

Figure 5.24 Flexural Resistance Constraint - CPCI Sections Reinforced with Steel.

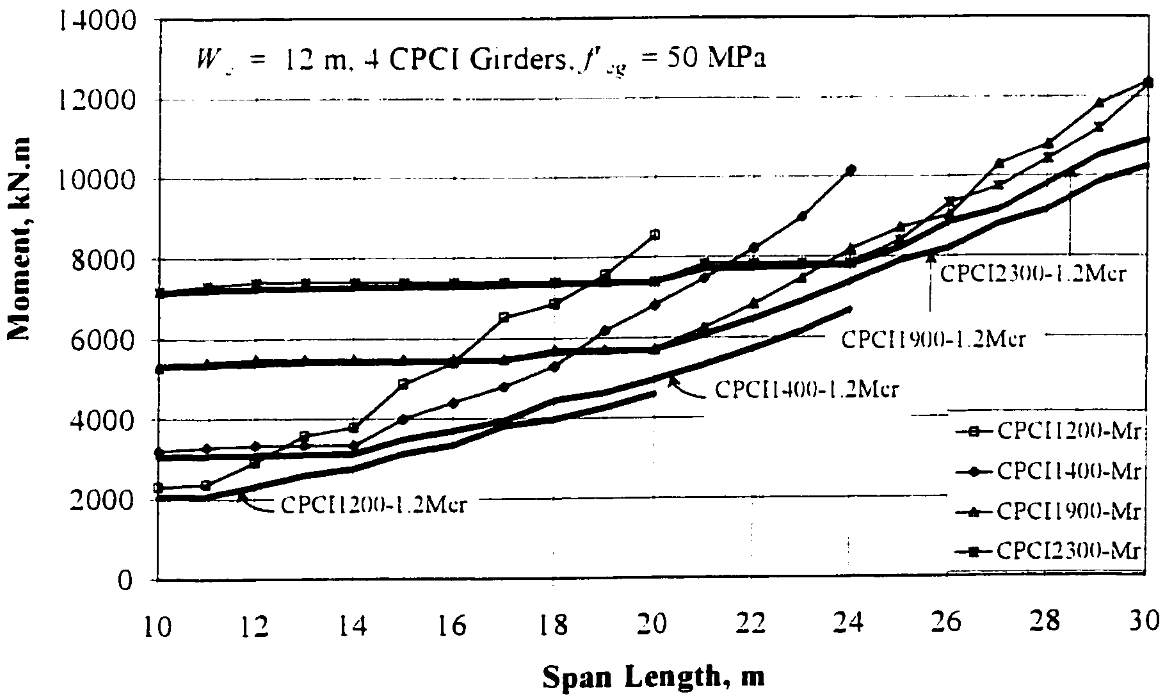

Figure 5.25 Minimum Reinforcement Constraint - CPCI Sections Reinforced with Steel.

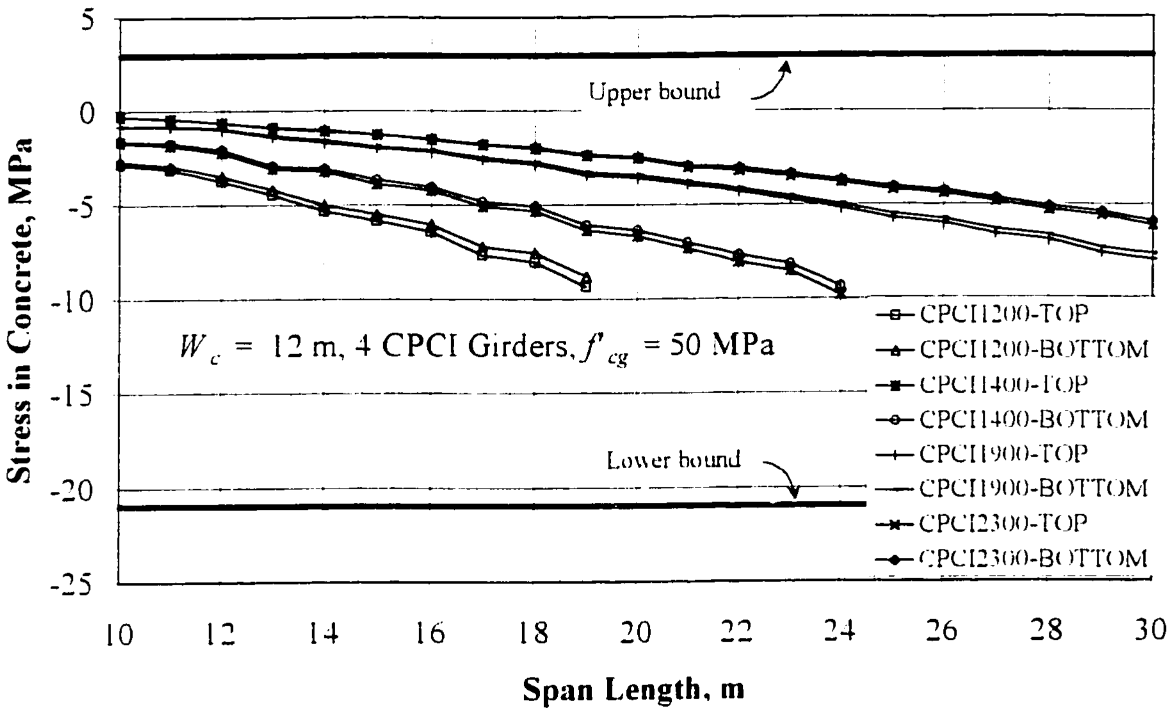

Figure 5.26 Support Section Stress Constraints at Transfer - CPCI Sections Reinforced with CFRP.

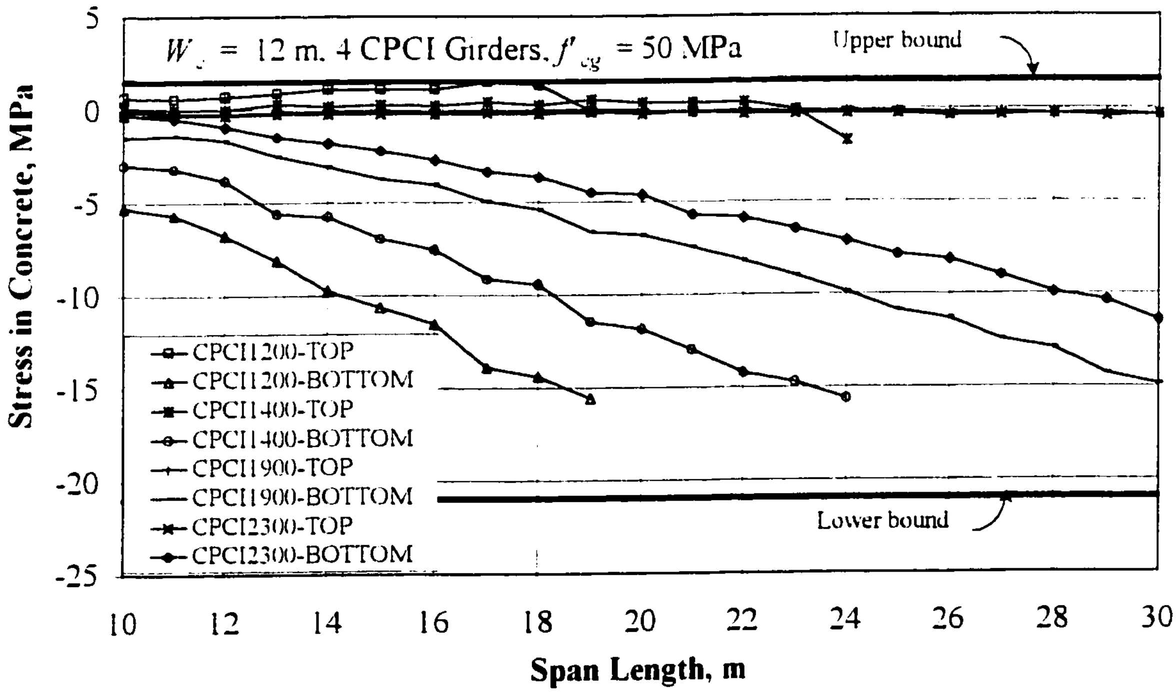

Figure 5.27 Mid-span Section Stress Constraints at Transfer - CPCI Sections Reinforced with CFRP.

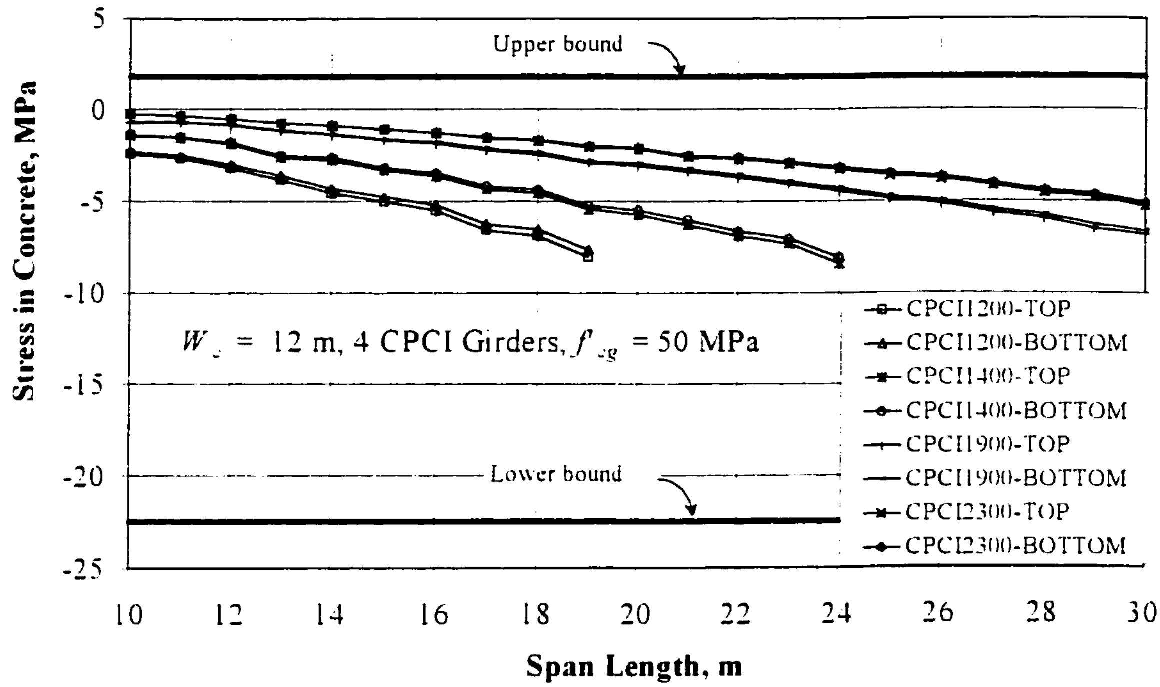

Figure 5.28 Support Section Stress Constraints during Construction - CPCI Sections Reinforced with CFRP.

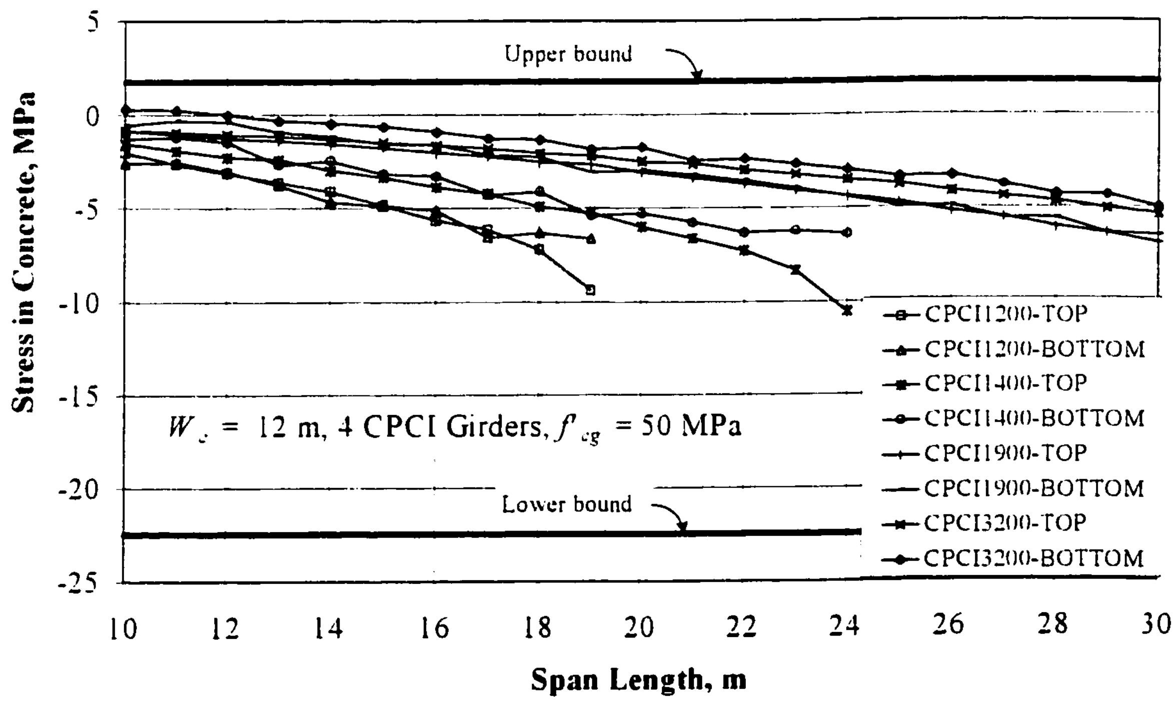

Figure 5.29 Mid-span Section Stress Constraints during Construction - CPCI Sections Reinforced with CFRP.

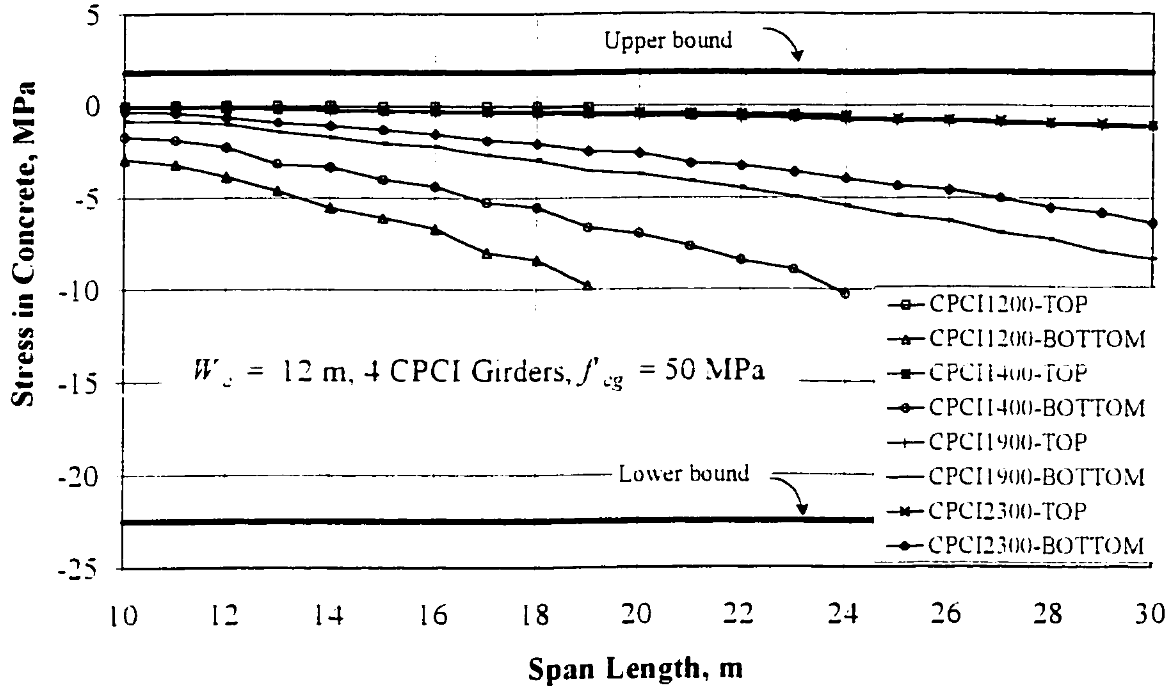

Figure 5.30 Support section stress constraints at service - CPCI sections Reinforced with CFRP.

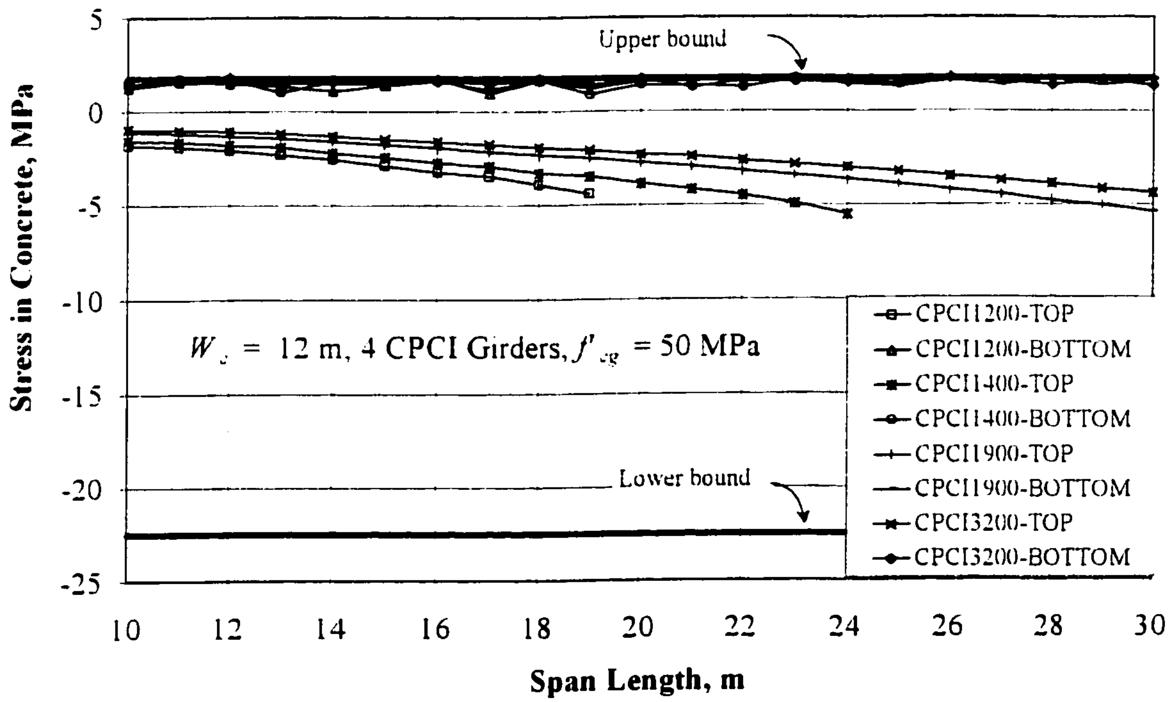

Figure 5.31 Mid-span Section Stress Constraints at Service - CPCI Sections Reinforced with CFRP.

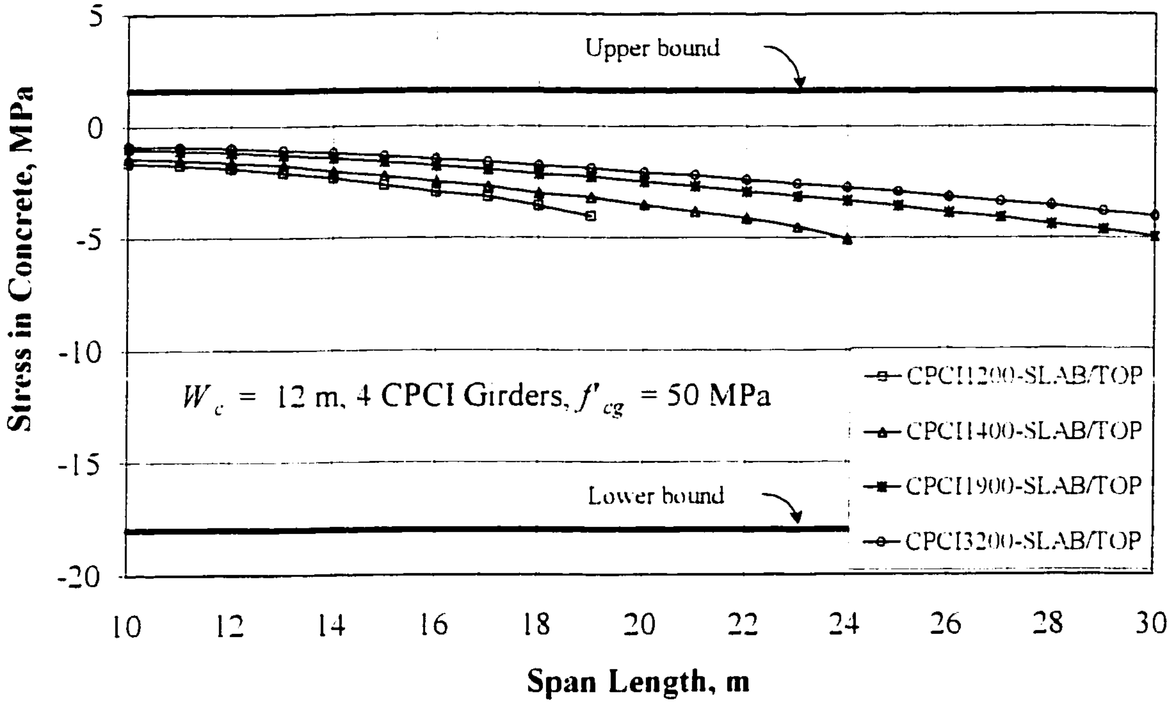

Figure 5.32 Mid-span Section stress in Slab Constraints at Service - CPCI Sections Reinforced with CFRP.

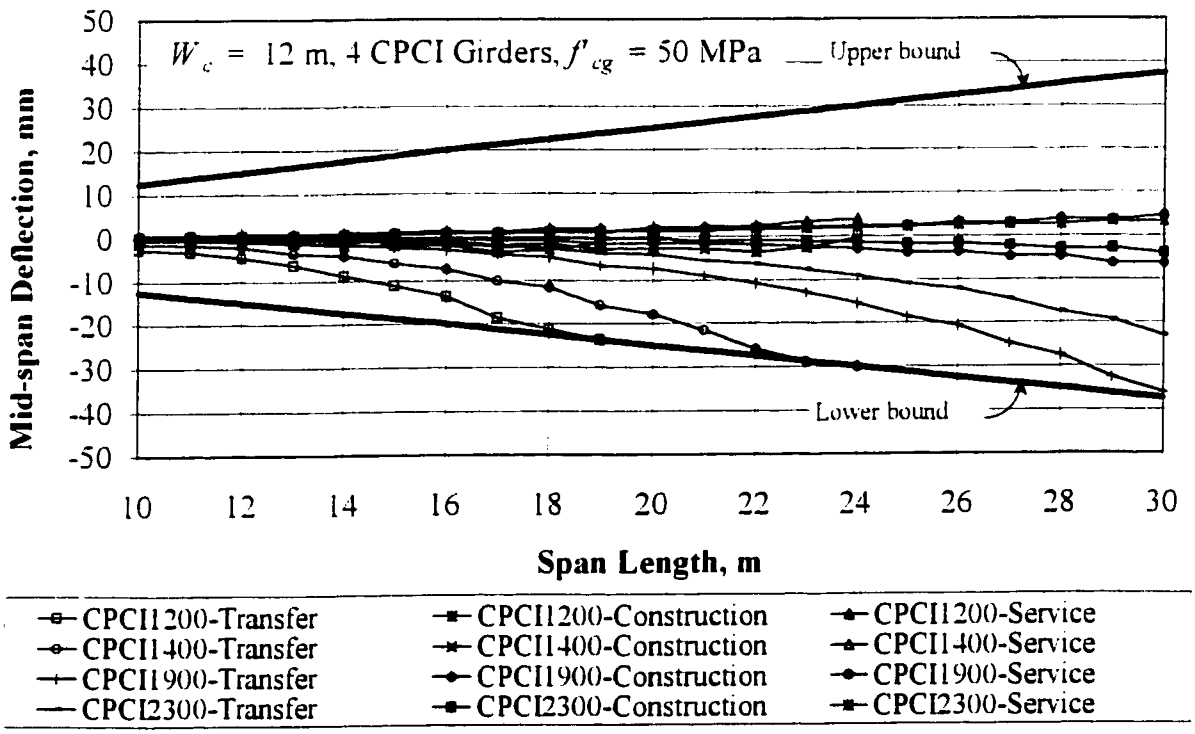

Figure 5.33 Mid-span Deflection Constraint - CPCI Sections Reinforced with CFRP.

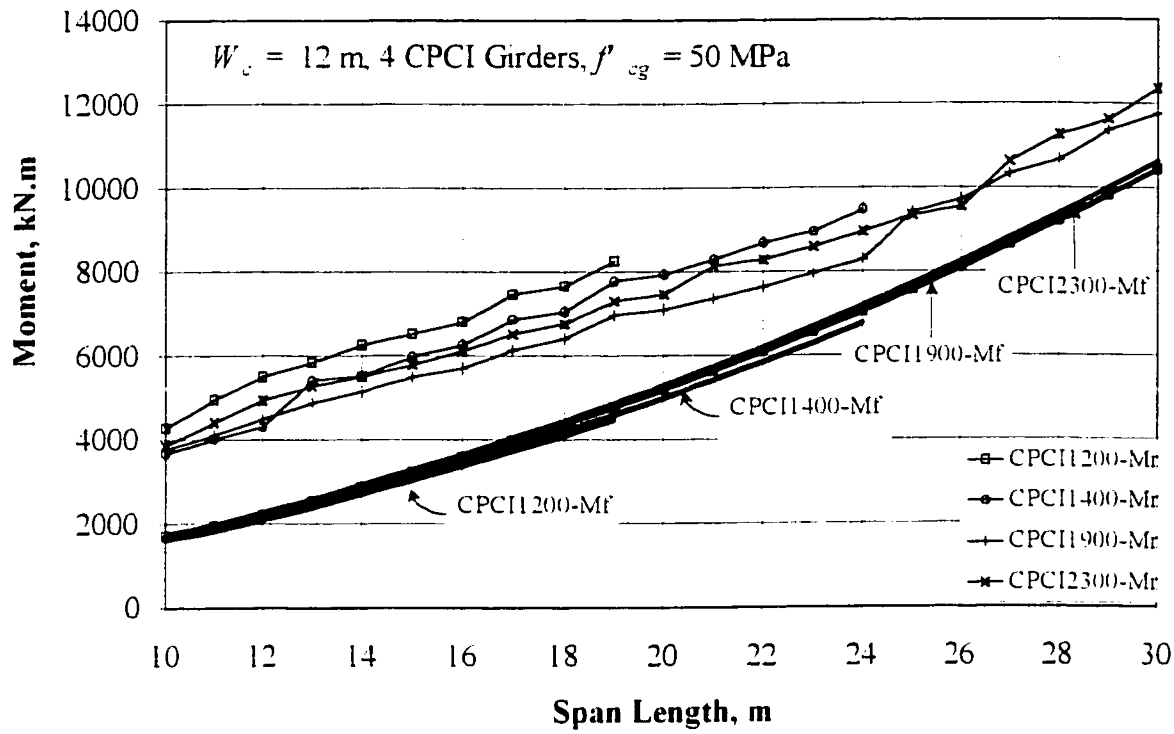

Figure 5.34 Flexural Resistance Constraint - CPCI Sections Reinforced with CFRP.

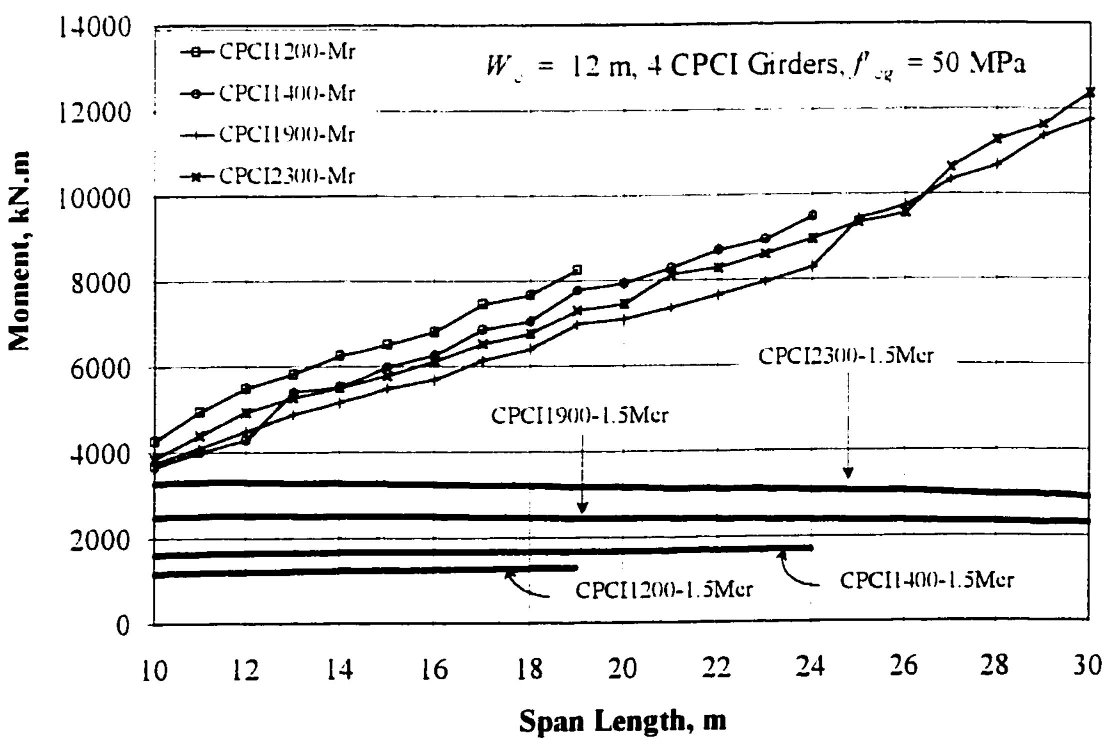

Figure 5.35 Minimum Reinforcement Constraint - CPCI Sections Reinforced with CFRP.

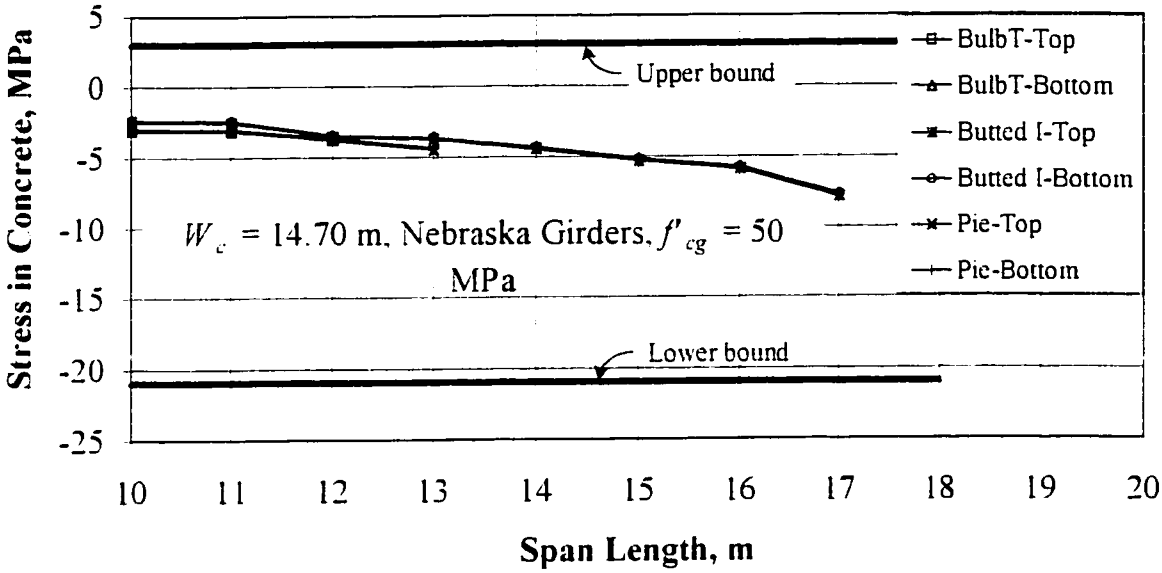

Figure 5.36 Support Section Stress Constraints at Transfer - Nebraska Sections Reinforced with Steel.

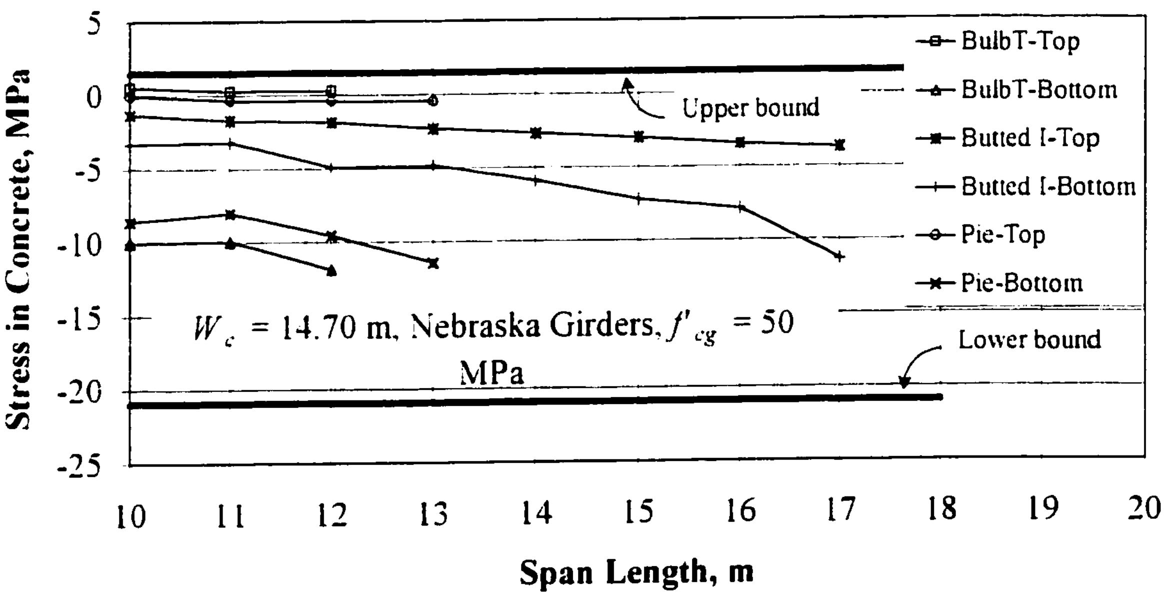

Figure 5.37 Support Section Stress Constraints at Transfer – Nebraska Sections Reinforced with Steel.

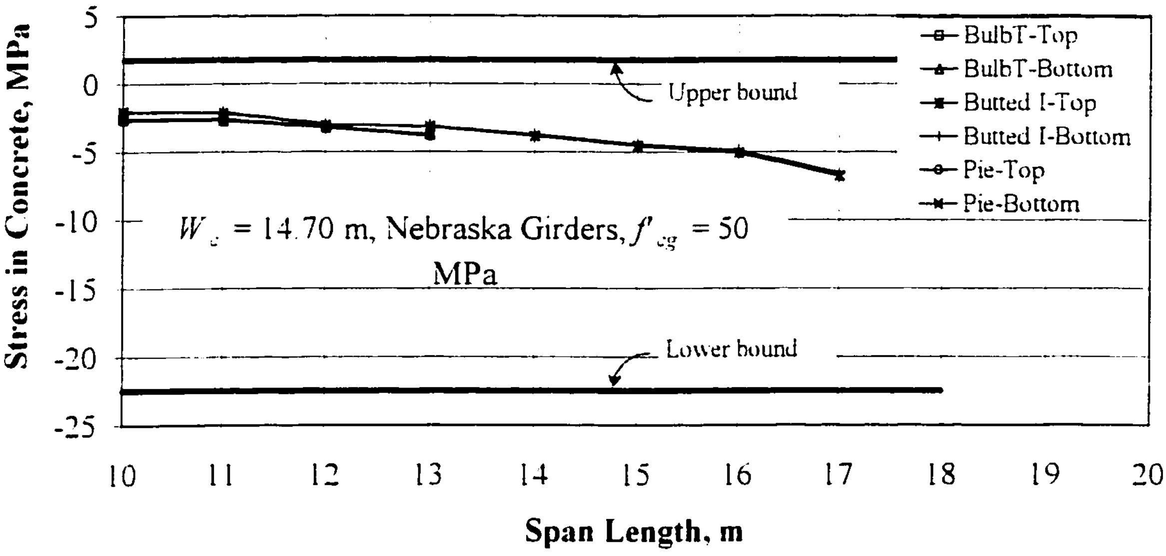

Figure 5.38 Support Section Stress Constraints at Service - Nebraska Sections Reinforced with Steel.

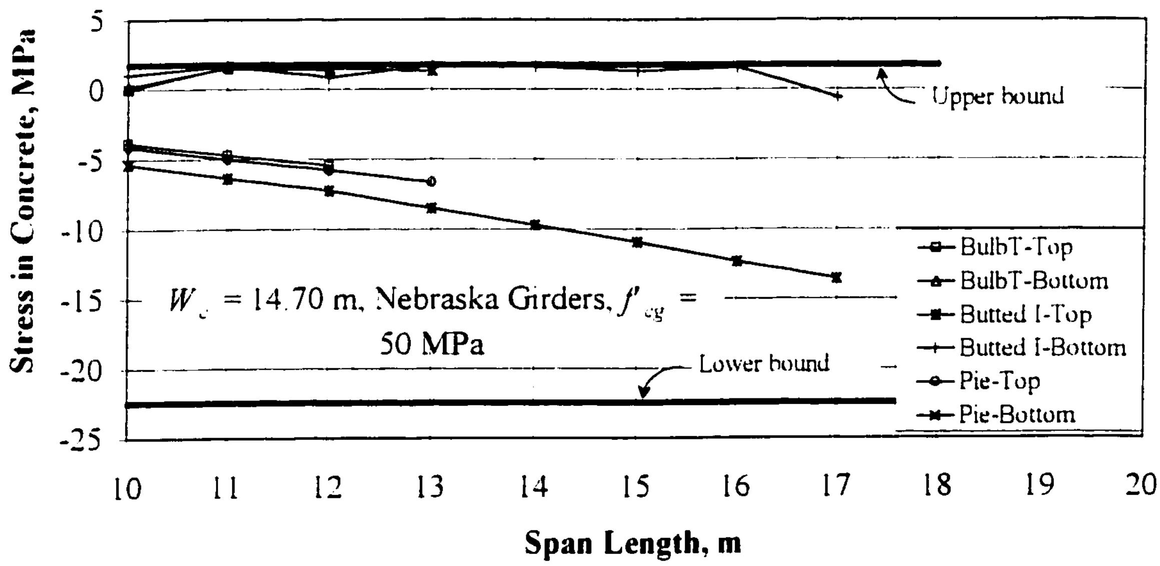

Figure 5.39 Mid-span Section Stress Constraints at Service - Nebraska Sections Reinforced with Steel.

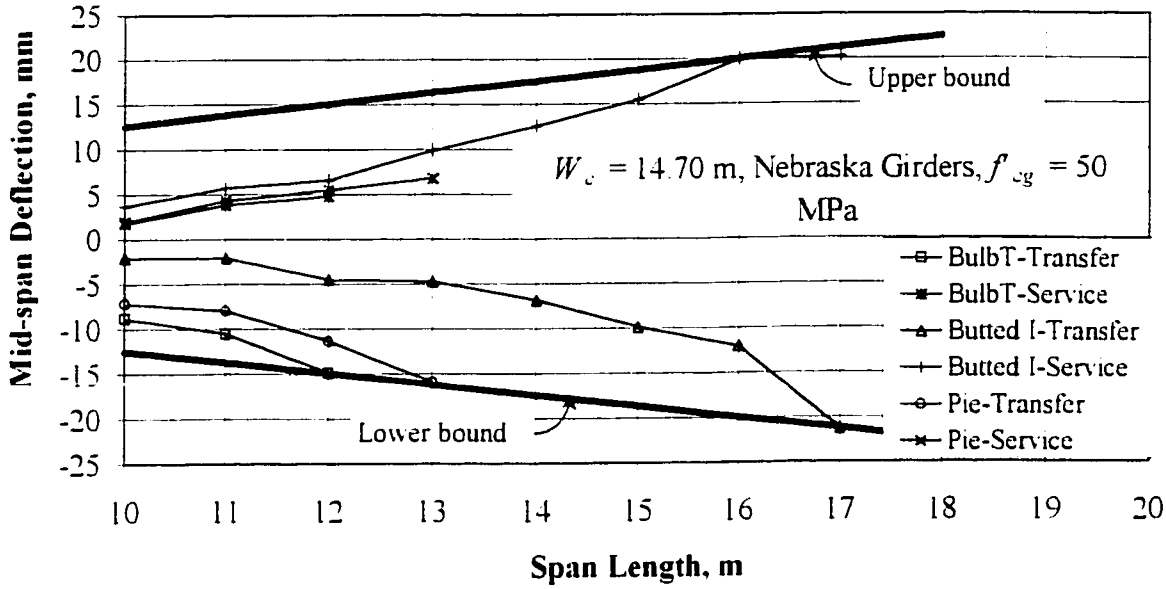

Figure 5.40 Mid-span Deflection Constraint – Nebraska Sections Reinforced with Steel

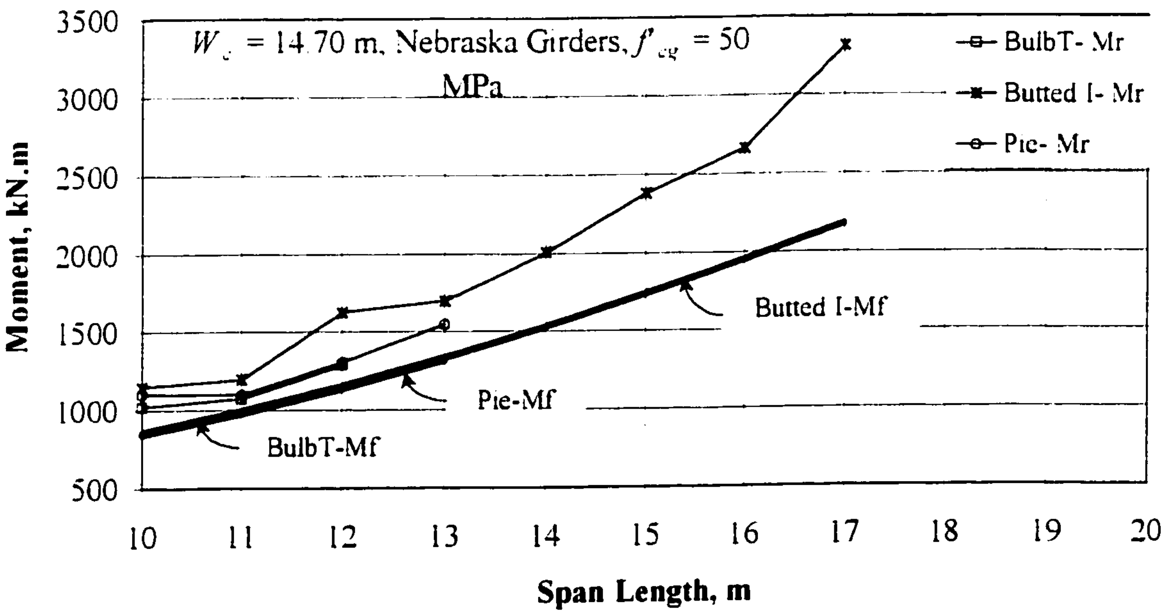

Figure 5.41 Flexural Resistance Constraint - Nebraska Sections Reinforced with Steel

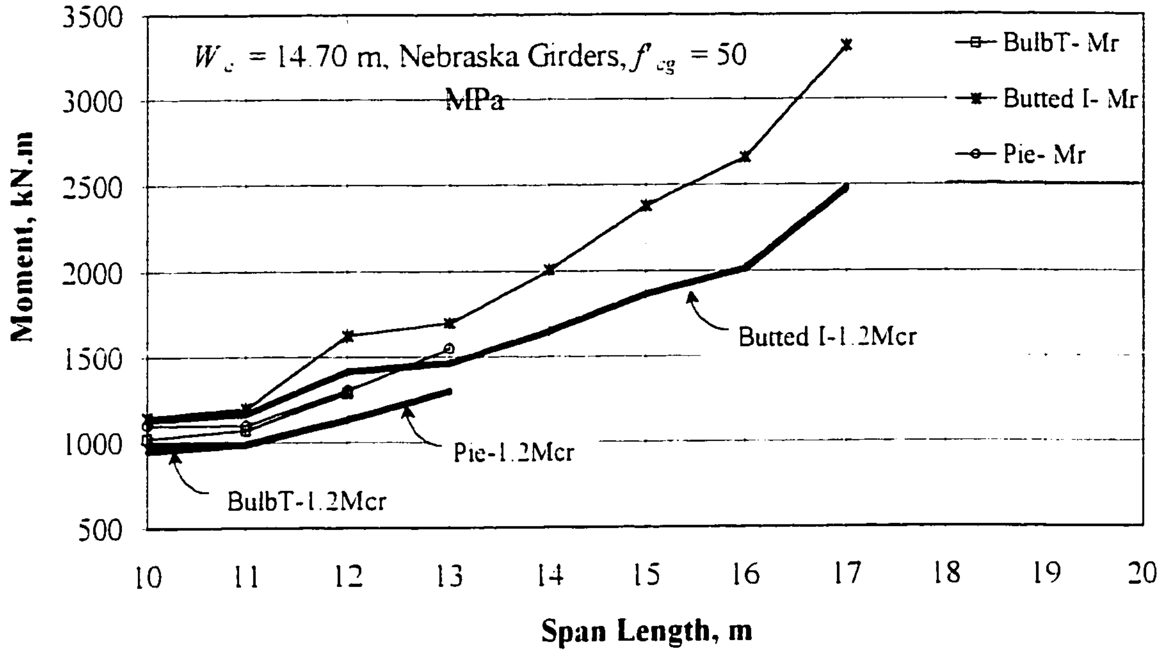

Figure 5.42 Minimum Reinforcement Constraint - Nebraska Sections Reinforced with Steel

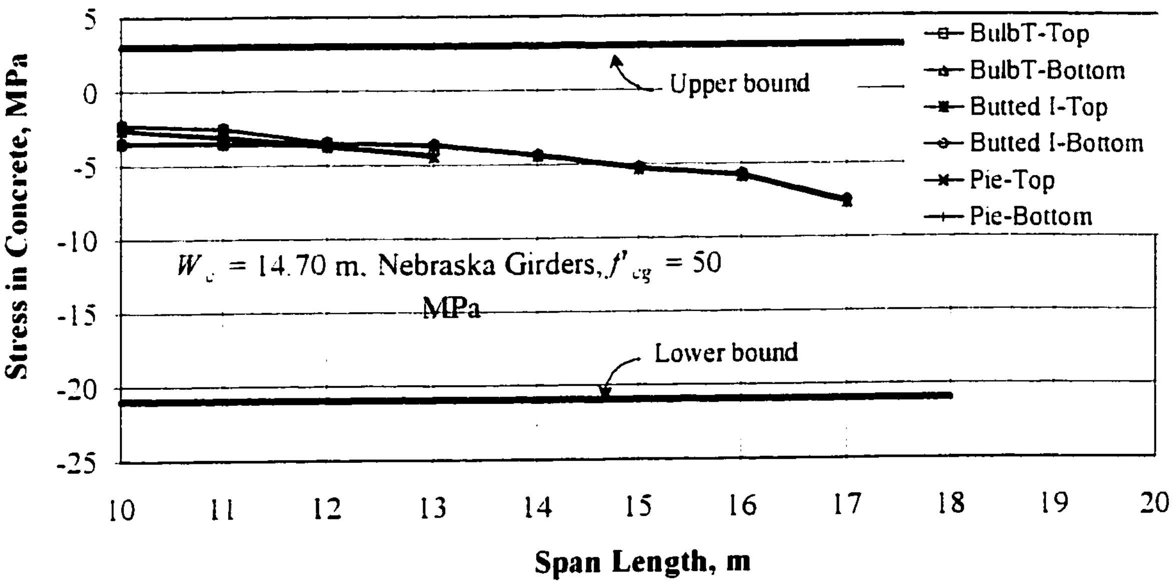

Figure 5.43 Support Section Stress Constraints at Transfer - Nebraska Sections Reinforced with CFRP

- Figures 5.43 to 5.49 represent graphically the constraints related to the Nebraska sections reinforced with CFRP. These figures indicate that the bottom fiber stress at service stage [Equations (3.2) and (4.7c)] and deflection [Equations (3.22) and (4.8a-c)] at the mid-span section are the only active constraints.

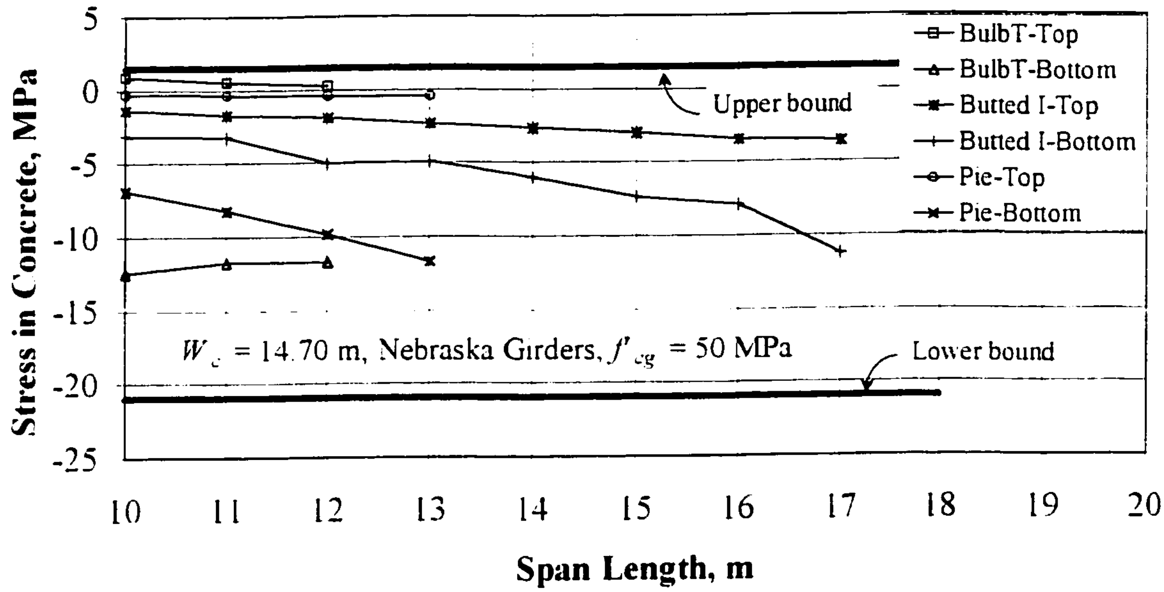

Figure 5.44 Mid-span Section Stress Constraints at Transfer – Nebraska Sections Reinforced with CFRP.

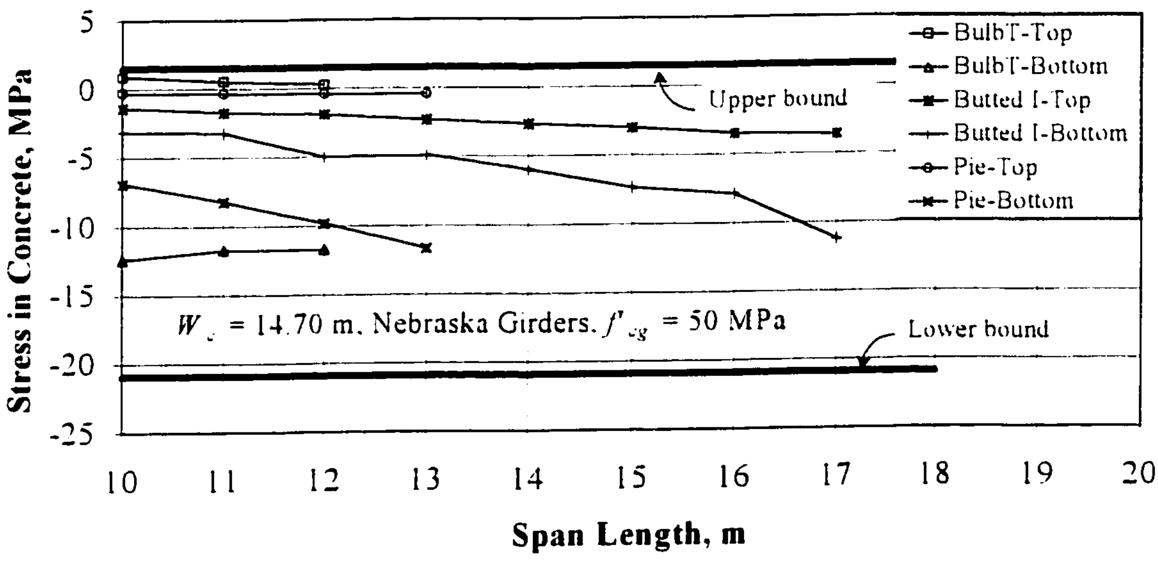

Figure 5.45 Support Section Stress Constraints at Service - Nebraska Sections Reinforced with CFRP.

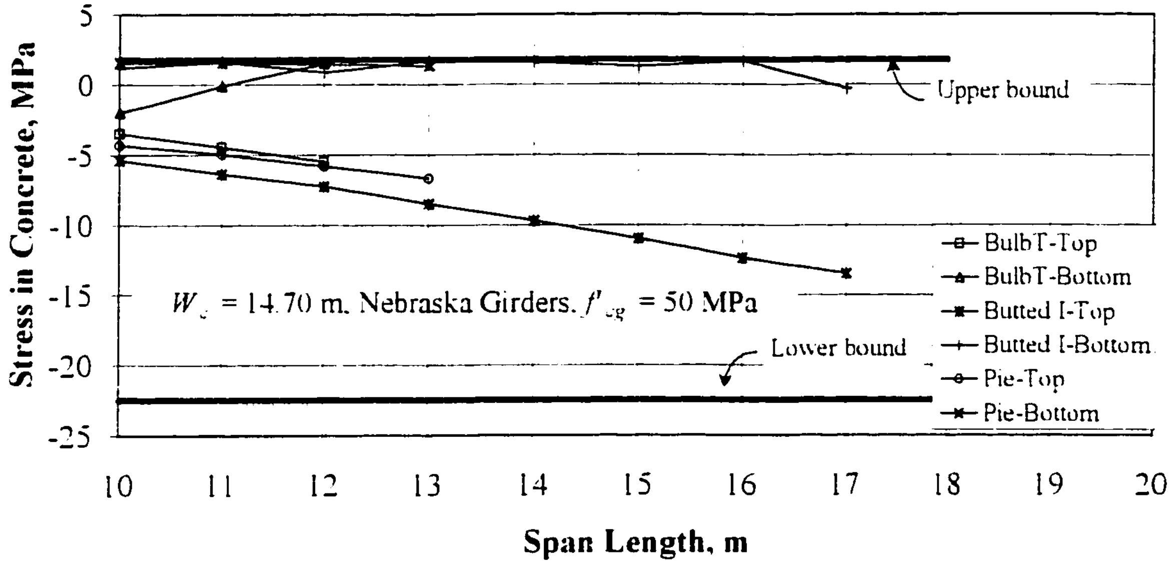

Figure 5.46 Mid-span Section Stress Constraints at Service- Nebraska Sections Reinforced with CFRP.

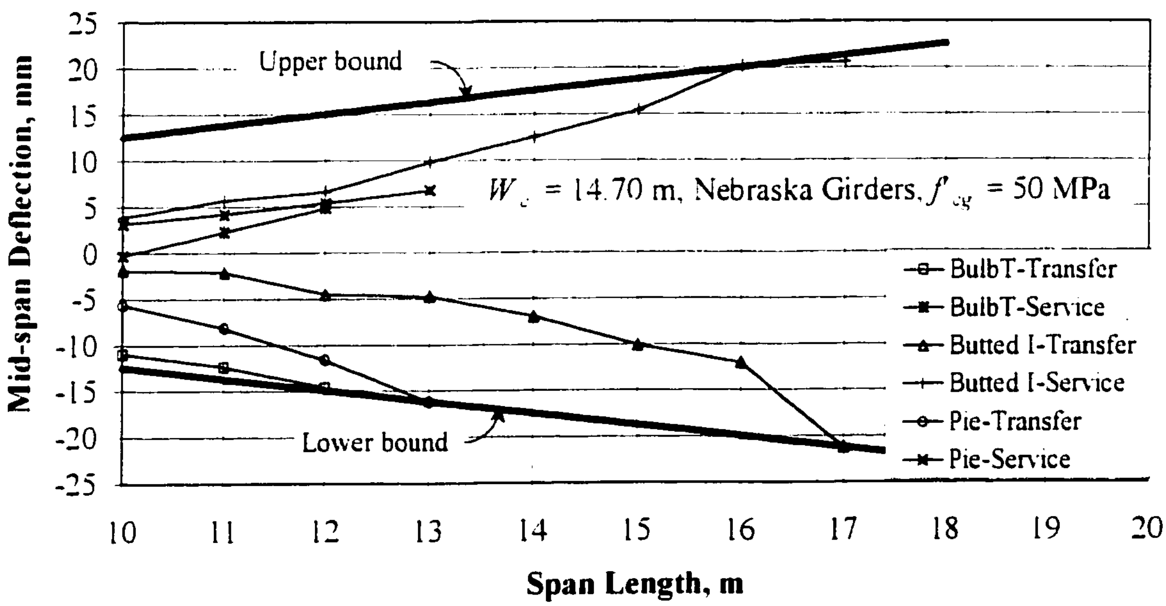

Figure 5.47 Mid-span Deflection Constraint – Nebraska Sections Reinforced with CFRP.

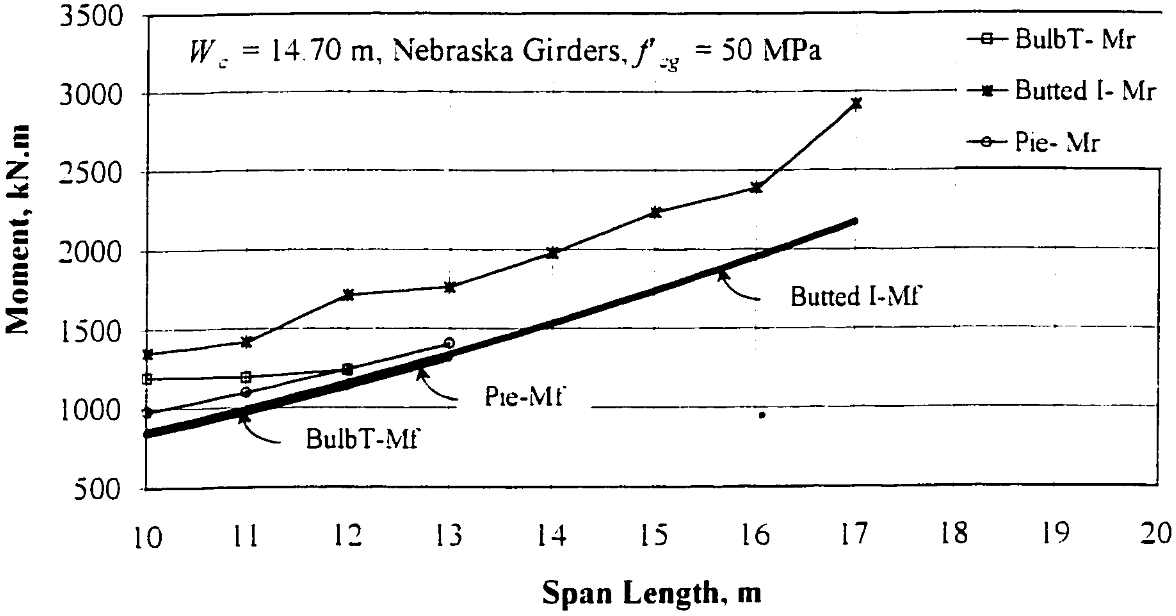

Figure 5.48 Flexural Resistance Constraint - Nebraska Sections Reinforced with CFRP

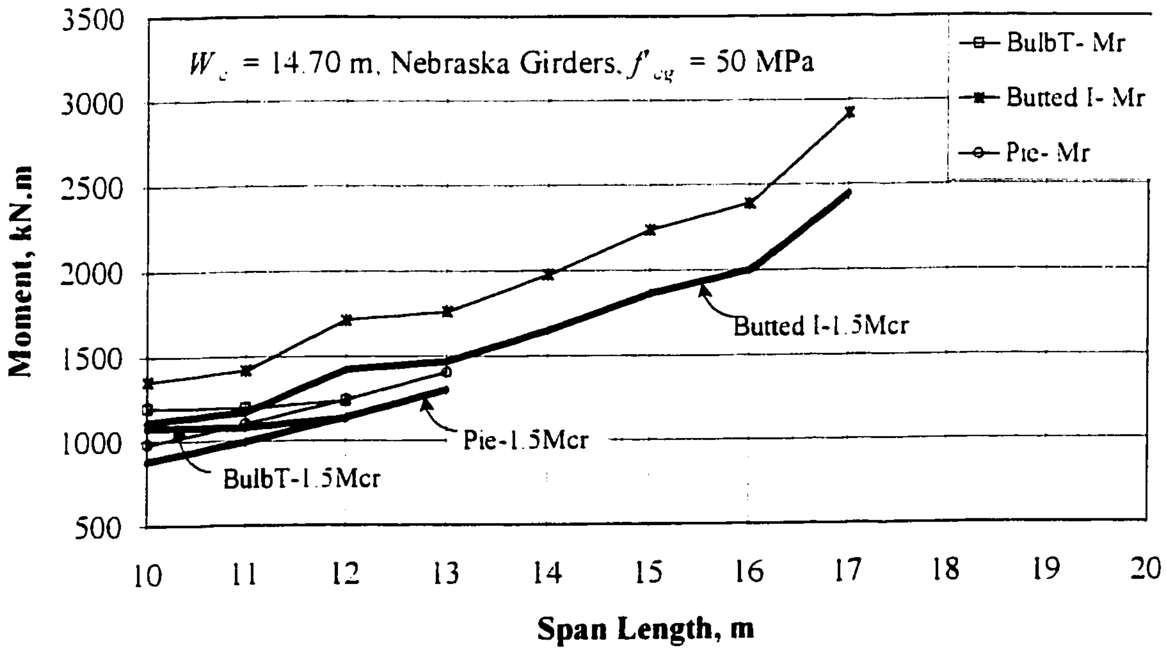

Figure 5.49 Minimum Reinforcement Constraint - Nebraska Sections Reinforced with CFRP

CHAPTER 6

SUMMARY AND CONCLUSIONS

6.1 Summary

This thesis investigates two types of simply supported bridges; the first is built up of CPCI standard precast girders supporting a cast-in-place deck slab, and the second is composed of all-precast sections proposed by the University of Nebraska - Lincoln. The bridge systems considered are partially prestressed, once with conventional steel and another with Carbon Fiber Reinforced Polymers (CFRP).

Investigations were conducted on the use of four standard precast girders CPCI1200, CPCI1400, CPCI1900 and CPCI2300. In each bridge, the girder concrete strength was varied; concrete strengths of 40, 50, 60, 70, 80, 90 and 100 MPa were used. Designs were also made for spans varying between 10 and 30 m and for bridge widths W_c = 8, 12 and 16 m. In addition, the Nebraska proposed sections (Bulb T, Butted I and Pie) with deck width of 14.70 m were studied in the same way as the CPCI standard girders.

An optimization technique was used in this study employing the Excel 97 Solver. User-defined functions were used to calculate loading, ultimate flexural strength, concrete stresses during construction and in service, as well as deflections. A cost–based objective function for each design system was identified and used to investigate two optimization design alternatives:

1) Minimum superstructure cost per deck area ($/m^2) and girder cost per unit strength ($/kN.m) for the design systems using conventional steel;

2) Minimum superstructure cost ($/m^2) and girder cost per unit strength ($/kN.m) for the design systems using CFRP.

6.2 Conclusions

The main conclusions of this thesis are:

- CPCI1200 reinforced with steel can be considered the most competitive in terms of superstructure cost per deck area for span length between 10 m and 20 m.
- CPCI1400 reinforced with steel can be considered the most competitive in terms of superstructure cost per deck area for span length between 20 m and 24 m.
- CPCI1900 reinforced with steel can be considered the most competitive in terms of superstructure cost per deck area for span length between 24 m and 30 m.
- CPCI2300 reinforced with steel can be considered the most competitive in terms of girder cost per unit strength.
- Among the selected bridge widths, the slab-on-girder bridge systems of 16 m width built up of CPCI1400 girders of 50 MPa concrete strength and reinforced with steel can be considered the most competitive in terms of superstructure cost per deck area.
- The slab-on-girder bridge systems of 12 m width built up of four CPCI1400 girders of 60 MPa concrete strength and reinforced with steel can be considered the most competitive in terms of superstructure cost per deck area.
- Among the bridge systems reinforced with CFRP, the bridges made up of CPCI2300 girders can be considered the most competitive in terms of costs and span length.

- Among the selected bridge widths, the slab-on-girder bridge systems of 16 m width supported with CPCI1400 of 50 MPa concrete strength and reinforced with CFRP can be considered the most competitive in terms of superstructure cost per deck area.
- Bridge decks of 12 m width supported on four CPCI1400 girders reinforced with CFRP give no significant change in superstructure cost per deck area. This is attributed to the very high cost of CFRP reinforcement in comparison to the cost of increase in the concrete strength.
- Nebraska Pie sections reinforced with steel or CFRP can be considered most competitive in terms of superstructure cost per deck area for span length up to 13 m.
- Butted I-section reinforced with steel or CFRP can attain the longest span length compared to the other Nebraska sections.
- The bottom fiber stresses at service stage and deflection at the mid-span section are the most active constraints.
- From material cost point of view, the use of conventional steel reinforcement is still much cheaper than that of CFRP.
- Using Excel solver as a software tool for solving optimization problems is much easier and more accessible than the other software tools (e.g., GAMS, IDESIGN, LINGO, etc.) used by other researchers.

6.3 Recommendations for Further Research

- One of the limitations of this study is the use of conventional design and analysis methods, which consider a lump sum loss in prestressing due to time dependent effects. Further research is needed that takes into consideration the time dependent effects more accurately, for example, by using the procedure developed by Elbadry and Ghali (1989).

- In this research, only the material cost was considered in the objective functions. More research is needed that takes into account other aspects of cost in addition to material cost.
- In this research, the bottom fiber concrete stress at service is generally the governing constraint in order to achieve a design that does not allow for cracking of concrete. In order to achieve more economical structures, a design based on criteria that allow cracking of concrete while controlling the crack width and deflection is needed. This may require nonlinear analysis and design procedures.
- The present work should be extended to include shear strength in the design of the bridge girders. The work should also consider continuous bridge superstructures.

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