Reliable Location Allocation Routing Design under Disruption: An Improved Column Generation Decomposition Approach

Gounash Pirniya

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By: Gounash Pirniya

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Dr. Ramin Sedaghati ____________ Chair

Dr. Akif Bulgak, MIE ____________ Examiner

Dr. Ketra Schmitt, CES ____________ Examiner

Dr. Ali Akgunduz ____________ Co-Supervisor

Dr. Brigitte Jaumard ____________ Co-Supervisor

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Abstract

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Every year, various human actions (e.g., terrorist attacks, strikes, etc.) and natural disasters (e.g., earthquakes, hurricanes, and etc.) cause disruptions in supply networks, and as the result, huge financial and humanitarian loss. Not only they brought loss of services to the system, they, depending on the type, partial or complete, may result in facility failures, roads failures or both, simultaneously. Therefore, having reliable systems are essential in order to reduce risks as well as cost in case of failures. Motivated by the importance of considering the failure in design level, we, in this thesis, focused on problem of locating facilities, allocating demand points to the facilities, and defining the rout among them while considering the complete failure in the elements of the network. The Reliable Location/ Allocation/ Routing Problem (RLARP) formulation which is Mixed Integer Programming model is proposed, taking into account failures in facilities and routs in different scenarios as failure sets. Along with bringing in trustworthy systems, we also contribute an exact decomposition methodology and propose a Column Generation model to tackle the complexity. The idea is to define a supply chain network at the design level to be robust against worst case failures and disruptions scenarios. To the best of author's knowledge, the Column Generation technique has not been applied previously to solve RLARP problems in the literature. In addition, we consider the facility and transportation method failures in our model, despite the fact that mostly either facility failures or transportation failures are taken into account in the literature. Various data sets designated for validating Column Generation and RLARP formulation proposed in this thesis. Eventually, we compare the performance of CG and RLARP models over a range of instances. Results suggests that CG technique performs significantly better than solving the RLARP model with a general optimization solver (CPLEX) in terms of computational time and the size of instances that can be solved.

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Chapter 1

Introduction

In recent years, the interest in the subject of disruption in supply chain networks has a notable increase. The reason behind this fact is the financial and humanitarian loss caused by natural disasters (e.g., earthquakes, hurricane, etc.) or human actions (e.g., terrorist attacks, strikes, etc.) in the recent events. Snyder et al. (2014) introduced specific reasons behind such a growth. First, recent conspicuous incidents brought the interdiction concept into public's attention such as Japanese Tsunami in 2011, terrorist attacks of 11th September 2011, Hurricane Katrina in 2005, and the west-coast port lockout in 2002. Second, as stated in Snyder et al. (2014), experts believe that although under the normal operating conditions, philosophies such as JIT (i.e., Just In Time) and lean concept perform very well for supply chain excellence, however, they fail to sustain a reliable supply chain networks when sudden changes occur in the system. Third, today's supply chain networks are highly globalized rather than vertically integrated. Suppliers of a typical North American company such as Apple are distributed around the world; in some cases they are located in highly unstable geographies in the world. Consequently, a large body of researchers have started tackling the risk management problem for supply chain networks to address the ever increasing needs of such globalized organizations.

Mitigation techniques reduce the impacts of disruptions regardless of the kinds of them. Disruptions risks have more severe impacts on the business rather than operational risks, and that is the main reason behind such a growth in the field of mitigation techniques. To illustrate more, the operational risk is the probability of loss resulting from inadequate or failed procedures, system or policies such as employee errors, system failures, fraud or other criminal activities and any other event that disrupt the business (TechTarget). To compare operational risks with disruption risks (e.g., floods, earthquake, economic crises like changes in currency rates, strikes and machine breakdowns), the former impact operational factors rather than supply chains' component while the latter can disturb the functionality of the supply chains for an unlimited duration (Ahmadi-Javid and Seddighi, 2013; Azad et al., 2013).

It may seem that disruptions occur rarely and it is not worth investing on mitigating the risk. However, historical evidences suggest that even a small disruption on the supply chain network may result in severe and destructive impact on supply chain networks and resulting in a huge loss which its repercussions can be last for years (Hendricks and Singhal, 2003, 2005a,b). For instance, in 1998, strike on two general motors part plants resulted in closing of 100 other GM part plants, 26 assembly plants, having so many dealer lots empty for months (Snyder et al., 2014).

Due to budgetary concerns, corporations may not find a good business case to invest in fortification and mitigation strategies to minimize the risk against failures. However, there are clear evidence from the past experiences that, a reasonable investment especially at the design level results in significant savings in the long run. To further motivate, in 2011 earthquake and following tsunami in Japan caused significant disruption on Toyota's global supply chain network. As a result, Toyota's sales dropped significantly due its centralized, less diverse supply chain network (Wall Street Journal 2011). On the contrary, Ford and GM did not face such a losses as a result of having a more geographical spread supply chain networks (Snyder et al., 2014).

Disruptions on networks may cause partial or complete failures in the supply chain network. Facility failures or transportation method failures are common reasons for failure. However, occasionally, both facility and transportation method failures occur simultaneously due to the intentional or unintentional events. To protect the system and reduce risks, companies adopt fortification techniques, as mostly relocating the infrastructure is not a choice, while others invest on more reliable network at the design level which is the main scope of our research. Due to limited resources, it can be clearly stated that a moderate investment at the design level is much more economically viable than finding ad hoc solutions during the post disruptions era with significantly higher costs (Snyder, 2006).

In this thesis, we confine our attention to the subject of Reliable Location/ Allocation/ Routing Problem (RLARP), specifically at the design level of supply chain networks. We propose a mathematical model, called RLARP, to design a network which can perform efficiently and reliably, at the minimum cost in the presence of the disruptions. More specifically, the model seeks the optimal location of facilities, allocation of demands along with the determination of routes from facilities to customers by considering the failures to reach a more reliable network. The objective is to minimize the worst-case cost consisting of fixed opening cost of facilities and transportation cost which is proportional to the travelled distance and satisfied demand. Following the RLARP model, Column Generation solution methodology is proposed.

Our main contributions in the thesis are as follows: Applying a mathematical decomposition technique (Column Generation (CG)) to reformulate Reliable Location/ Allocation/ Routing Problem (RLARP) in the context of supply chain network design whereas taking into account failure in services and the network components such as routes and facilities. The Column Generation technique, lets us reformulate our model and solve it in a timely manner for medium and large instances. We generate different data sets and test the two solution methodology over a range of instances, taking into account failure sets. We use several graphs to do the performance analysis of the two techniques to figure out how beneficial is to use the CG technique. In addition, a series of experimental complexity analysis are done.

The remained of the thesis is organized as follows: We present an overall explanation regarding different mitigation strategies and more detailed review of the literature in the field of facility location. Chapter 3 describes the column generation technique which is our main contribution in this thesis. In Chapter 4, we present the problem description and formulation, while Chapter 5 is dedicated to the explanation of the data set and computational results. Finally, in Chapter 6, we discuss the conclusion of the thesis and future research directions.

Chapter 2

Literature review

In this chapter, we discuss an overview of studies related to reliable facility location problem under random disruptions. In supply chain field to mitigate the disruption different strategies will be applied as follows: Mitigating disruption through inventory, sourcing and demand flexibility, interaction with stakeholders, and facility location (Snyder et al., 2014). We explicitly review the facility location category as the subject of our studies is related to this area.

2.1. Mitigating through facility location

In this section, the literature is divided in two main categories: fortification models; and design models for reliable facility locations as described in Snyder (2006). Fortification models take into account that there exists a network in which facilities are already placed. It should be decided which infrastructures to fortify to protect them against disruptions considering the resource limitations. While design models like a classical facility location assumes that the network is built from the scratch and no facilities exist at the time. Therefore, the decision should be made to choose a set of facility locations among potential locations to perform well even at the time of disruptions. Also, it should be mentioned that these two concepts can be integrated in one model, selecting the potential facility locations as well as fortifying them, which creates at least a tri-level model. This causes solving such a model noticeably more difficult (Snyder, 2006). Also, it should be said that knowing the type of threats as well as the network that is going to be protected against the disruption could help choosing the best strategy to mitigate the system (Cappanera and Scaparra, 2011). First, we review briefly the fortification part. Then, the design models is investigated in details as our research is fit to this area.

2.1.1. Fortification models

Reliability of the network can be increased by considering disruptions in the design level. However, as relocating the infrastructure, redesigning the entire system and changing the suppliers is not an option due to its high cost, fortification strategies is a good option to protect and secure the existing network (Snyder, 2006). In this field, different models are proposed mainly based on the type of threat and the network along with the objective they are seeking, considering limitations and conditions.

Some proposed models are based on the game-theoretic or in another word defender-attacker concept (Cappanera and Scaparra, 2011; Liberatore et al., 2011; Parvaresh et al., 2014). These models are used for the case of considering the intentional attacks (planned operations) (e.g., terrorist attacks, labor strikes) rather than random unplanned disruptions (e.g., natural disasters) (Cappanera and Scaparra, 2011), (Golany et al. 2009). In defender-attacker approach, the goal of the attacker is to interdict a system in a way to cause the most harmful damage while the defender is trying to reduce the future repercussions of probable destructive attacks. That is the reason behind of seeing mostly the multi-level models in this field. However, Maria P. Scaparra (2005) and Church and Scaparra (2007) reduced the level of problem to one, by enumerating all interdiction patterns. Therefore, it became capable of solving medium-sized problems (Snyder, 2006).

Scaparra and Church (2008b) propose a P-median optimization model to minimize the worst-case effect of r intentional attacks targeted to non-fortified facilities. To do so, it should be identified which subset of q facilities have to be fortified among p located facilities in the system to face the minimum disruptive effects in the worst-case loss. r-interdiction median problem with fortification (RIMF) is first formulated as mixed integer programming (MIP) by Church and Scaparra (2007). The author reformulate MIP as a maximal covering problem (MCP) with precedence constraints. They apply a greedy heuristic and interval search to reduce the size of the problem and then solve it by general MIP solvers.

Scaparra and Church (2008a) formulate the RIMF problem as a bi-level MIP. The authors considered following assumptions in the model: the number of interdiction is known for protectors, facilities are uncapacitated and fortified facilities are immune from failures. They applied the tree search process for solving the problem. The largest size of p, r and q fortified facilities which was taken into account is 60, 8 and 7, respectively. Liberatore et al. (2011) proposed the stochastic version of bi-level RIMF as well as stochastic MCP. In their problem, the number of losses is uncertain with the specified probability. In their paper the largest size of p is 60, r takes the amount between 2 to 5 and q is the 10, 15, 20 percent of p.

Lim et al. (2010) formulate a MIP model to deal with network disruptions by considering two kinds of facilities; reliable and unreliable facilities. They found that one of the effective ways to build a reliable distribution network is fortifying some of the facilities to make them reliable while these disruptions occur randomly. They propose a Lagrangian Relaxation-based solution algorithm, seeking minimum costs and charges to solve large scale problems in reasonable computation time.

Cappanera and Scaparra (2011) develop a multi-level, defender-attacker-user, optimization model based on the Shortest-Path interdiction model of Israeli and Wood (2002) with an "additional level for modeling explicitly protection decisions". By applying a game theoretical framework, their objective is to reach the best fortification plan that in case of worst case R interdiction of unprotected links, minimize the increase in the shortest path length between the supply nodes and demand nodes. The author assumes that the only point that affects the travel cost is link status not link flow. The enumeration algorithm is used to solve the model to optimality. They propose a heuristic method for solving the lower-level interdiction problem at each node of an enumeration tree, and applied variable fixing rules to reduce the dimension of the problem. By using this methodology, they could find hardening strategies for large size networks. The size of their data sets are as follows: number of nodes and arcs are $< 51, 88 >$, $< 102, 416 >$, $< 146, 618 >$ and $< 227, 996 >$, where the first number represents the number of nodes and the second is the number of arcs in the directed graph. Maximum number of fortification and interdiction are considered (3, 5, 7) and (1, 2, 3, 4, 5), respectively. For solving the algorithm C++ was used. Also, they used CPLEX to solve the MIP problems at each node of enumeration tree.

2.1.2. Design models

It is a tradition in the facility location problem to make the leanest decision for locating the facilities (Snyder et al., 2014). However, the reality of not being able to change the facility locations at the time of the disruptions as well as the recent destructive events in the globe, highlights the essential needs for considering the interdiction in the concept of facility location far in advance in the design level. To the limits of our knowledge, for the first time, disruptions were taken into account in a facility location model in the publication of Drezner (1987). He proposed two models, the reliable version of the classical p-median with considering the probable failure of nodes as well as "(p, q)-center" problem. In the latter p facilities should be located in a way to minimize the maximum cost at the time of interdiction of at most q facilities. Neighborhood-search based heuristic was applied to solve both problems.

Snyder and Daskin (2005) presented reliability models based on the classical P-median problem (PMP) and the uncapacitated fixed-charge location problem (UFLP). The bi-objective formulation was used for both models, one considers the cost of location allocation without presence of failures (nominal cost) and another, responds to the expected cost after interdiction. Their objective is to choose facilities in a way to minimize the cost in case of facility failures and increase the resiliency of the system at the design time. Therefore, inexpensive and reliable facility locations would be chosen to have the less cost by considering their reliability. It is also considered that customers will be assigned to their closest non-disrupted facility. They assume the same failure probability for all the facilities except for those which cannot be failed. Lagrangian relaxation algorithm was used for solving the problem. Also, generating a trade-off curve between minimal and expected cost indicates that often by a minimal increase in the operating cost at the design level, the system reliability would improve considerably.

Relaxing the assumption that all facilities have the same disruption's probability, make solving the problem much more difficult (Snyder, 2006). To encounter such an issue, some proposed stochastic programming and enumerate either all or set of the disruption scenarios taking into accounts that the size of the problem grows exponentially by the number of facilities (Shen et al. (2011), (Snyder, 2006), and Y. Zhang et al. (2015)), while Berman et al. (2007, 2009, 2011, 2013), Zhan et al. (2008), Cui et al. (2010) and Aboolian et al. (2013) calculate the probability that a customer is assigned to its r^{th} closest facility by proposing a non-linear term.

Atoei et al. (2013) propose reliable capacitated supply chain network design (RSCND) model. The three-level of supply chain including customers, distributors and suppliers was considered. Their model is a bi-objective and also a scenario based model. They minimize the expected cost in the first level and in the second level maximize the reliability of the network. Their objective is to optimally locate the distribution centres (DCs), assign customers to DCs and DCs to suitable suppliers such that the costs are minimized and the reliability is increased. The random partial disruption can happen in the location, capacity of the DCs, and suppliers. They solve the problem by Lingo 11 but due to its limitation which cannot handle more than 50 variables, it cannot be used for solving the medium and large size instances. To be able to solve the real word instances the metaheuristic approach, Non-dominated Sorting Genetic Algorithm-II, is applied.

Azad et al. (2013) propose a capacitated supply chain network design (SCND) model. Their formal model is Mixed Integer Linear programming (MILP). The objective is to reach the optimal network design with the minimum cost, considering the constraints such as limitation in the investments. The cost is including two main categories, the reliability costs and the transportation costs. Partial random disruptions can occur in both facility and transportation. Two transportation modes are considered, safe and unsafe. In safe transportation mode, disruptions cannot occur but for sure the cost of it is higher than the unsafe transportation mode. There are two types of customers' assignments: Primary assignments is for the situations in which there are no failures. Therefore, there could be either safe or unsafe transportation mode. Secondary assignments are used at the time of existence of disruptions in which just safe transportation mode could be used. Due to the large number of variables and constraints, using a general optimization solver directly is not efficient, therefore modified Benders Decomposition (BD) algorithm was applied to solve the problem. They generated 30 random sets of data. Different ranges were taken into account as follows, customers (10-150), potential DCs (2-22), investment and transportation mode (3-5).

Ahmadi-Javid and Seddighi (2013) consider a location routing problem (LRP) under disruption in a two-echelon supply chain network consisting of producer-distributors (PDs) which produce a single commodity and distribute it among customers. The problem is formulated as a mixed-integer linear programming under three different risk management policies, moderate, cautious and pessimistic. Their goal is to locate PDs, allocate them to customers and define the routes in a way to minimize the total annual cost which includes the following components. The fixed cost of opening and operating PDs, the annual routing costs from opening PDs to customers, the annual distribution and production costs. The DCs have a limited capacity. The disruption may reduce the DCs capacity and fully interdict the vehicle depending on the type of the disruption. As the LRP is NP-hard, the two-stage heuristic based on the simulated annealing was proposed for solving the large-size instances. The largest data set they addressed in this paper had 800 scenarios, where the number of PDs was 30 and there were 35 vehicles and 200 customers.

Azad et al. (2014) later propose the stochastic version of above problem with the same objective and model with some differences. The disrupted capacity of unreliable DCs in the case of disruption is stochastic and follows a normal distribution. They apply conditional value-at-risk (CVaR) approach to control the risk of model. Two approaches are used to solve the problem. One for solving small and medium-sized problems by reformulating the problem, as a second-order cone programming model. This approach gives an exact solution to the problem. The other one uses a heuristic approach, combination of tabu search and simulated annealing, which solves large-sized instances of the problem.

Farahani et al. (2014) propose a hierarchical maximal covering location problem (HM-CLP) with considering random disruptions. A disruption can happen occasionally and randomly in any facilities regardless of their type and level. Their goal is to maximize the total demand covering. As HMCLP by adding disruptions is NP-hard, metaheuristic method is proposed for solving the large-size problems. The authors apply a hybrid artificial bee colony algorithm.

Parvaresh et al. (2014) propose a bi-level p-hub median problem under intentional disruption model. In the upper level, which fulfills the goal of leaders, there are two objectives. The former minimizes the total transportation cost in the normal situation, while the latter follows the same objective but after happening the worst-case interdiction of maximum r-hubs. In the lower level, the aim of attacker is to select r hubs among those which had been located by leaders, in a way to maximize the damages to the networks. Their objective is to design a more reliable hub network. In their publication, the capacity of hubs is considered unlimited. Also, the complete interdiction is taken into account which means in case of interdiction of a hub, it would not be functional any more. Moreover, the simultaneous failure of at most R facilities is possible which make their work different from what Berman et al. (2009) did. The problem is solved by implementing multi-objective metaheuristics based on simulated annealing and tabu search. The largest size of potential hub location, p and r is 50, 5 and 2.

Y. Zhang et al (2015), propose a two-stage stochastic mixed-integer programming in the subject of capacitated reliable location routing (RLRP) in which a set of facilities can be fully disrupted randomly. They consider a scenario-based model with the objective of minimizing the total expected cost. The most probable scenarios are taken into accounts due to the fact that the problem size will grow exponentially. In the first stage, which is independent to scenarios, they decide which depots to open and in the second stage, the allocation of customers to satisfy their demands in the presence of failure of some depots would be covered. They develop a metaheuristic solution methodology based on Simulated Annealing (SA) approach for solving their model. In addition, the authors compare the performance of their RLRP with the classical models.

L. Yun et al (2015), propose uncapacitated reliable facility location design model under imperfect information which means that customers do not have the information regarding the status of the facilities until reaching there (trial and error strategy). They consider the disruption of facilities probabilistic. They formulate their model as an integer program and their objective is to minimize the expected cost. They use a lagrangean relaxation algorithm to solve their model. They test their model in the real-world data set derived by Daskin (1995). Their data sets consist of 49 nodes, 88nodes and 150 nodes which mostly nodes are representing different states in the US.

Chapter 3

Methodology

In this chapter, we give a brief explanation regarding Column Generation (CG) technique which is our solution methodology in the thesis for solving the Reliable Location/Allocation/Routing problem. CG is a decomposition technique mostly applied for large scale problems with enormous number of variables in comparison to the number of constraints. The decision of which variable should enter the basis and whether or not we reach to an optimal solution will be done through tackling the optimization problem instead of enumeration (Nemhauser, 2012).

To our best knowledge, L. R. Ford and Fulkerson (1958) were the first people applied the notion of column generation in linear programming context (Nemhauser, 2012). Later, the linear programming columnwise was developed by Dantzig and Wolfe (1960) as a strategy in solution process. However, for the first time, Gilmore and Gomory (1961, 1963) implemented CG as part of a heuristic in an actual problem (Lubbecke and Desrosiers, 2005).

3.1. Classical Column Generation

One of the exact decomposition methods to solve large scale operations problems or NP-hard problems is Column Generation (CG). Column Generations' common strategy in solving Linear Programming (LP) and Integer Linear Programming (ILP) problems is to decompose the problem into Master and Pricing problem. The CG strategy for solving LP and ILP problems is almost the same with a slight difference, that is why we divide this section into two subsections. First, an LP Master Problem (MP) was taken into account in Subsection later, in Subsection 3.1.2, an ILP MP was considered.

Many different kinds of NP-hard problems have been defined in the operations research literature. These kinds of problems are usually impossible to solve at all or in specific optimization problems, impossible to solve within a certain/allowable amount of time. In these kinds of problems, we tackle the complicated constraints because they mostly cause the major setback while solving operations problems. They usually prevent us from reaching the solutions and best results in a timely manner. In many cases, the best way to reformulate NP-hard problems is to ignore the complicated constraint/s and enlarge the feasible solutions. Then, the rest of the problem could be categorized in either known or unknown optimization problems. If the problem is well-known, their convex hall has already been defined and they can be introduced as a set of solutions (i.e., X). For the later one, feasible solution polytope convexity should be checked so that in the decomposition process, we are able to reformulate it as a convex combination of its extreme points. Then, we will reach the Master Problem of Column Generation as a result of replacing the solutions by their convex combination of the extreme points. It also must be checked that the integrality of the solution is always held in the set of solution.

3.1.1. Having an LP Master Problem

Consider the following linear program as the Master Problem (MP):

$$Z_{MP}^{*} = \min \sum_{k \in K} c_k \lambda_k \tag{3.1}$$

Subject to:

$$\sum_{k \in K} a_k \lambda_k \geq b \tag{3.2}$$

$$\lambda_k \geq 0 \qquad\qquad k \in K \tag{3.3}$$

This is an LP problem and as mostly we are dealing with the huge number of K (columns), it may make the computation of the problem either impossible or relatively hard (impractical). To tackle this issue column generation deals with a subset of variables (K') to reduce the costly operations rather than dealing with huge number of K at once. Therefore, we solve the so called Restricted Master Problem (RMP) in CG as below:

$$Z_{RMP}^{*} = \min \sum_{k \in K} c_k \lambda_k \tag{3.4}$$

Subject to:

$$\sum_{k \in K'} a_k \lambda_k \geq b \tag{3.5}$$

$$\lambda_k \geq 0 \qquad\qquad k \in K' \tag{3.6}$$

The first columns by which we start optimizing the RMP, need to be explicitly defined. Since there is no variable in the RMP, in many cases, the initial solution will be either calculated by devising a big-M method and adding artificial variables with large cost in the objective function or implementing a heuristic. The main point that should be considered in creating an initial solution (column) is to start with a feasible solution for our RMP. In our case, we applied the first method. More variables and columns will be added when needed according to the CGs method that will be described later.

A CG iteration consists of:

1) Finding the optimal objective value (Z^*_{RMP}) and dual multipliers (π^*) associated with constraints (3.5) of the RMP,
2) Finding the minimum reduced cost value as well as a new column (i.e., configuration) that may be added to the RMP in the next iteration in pricing problem which will be explained further.

To answer the question that how the promising columns will be produced and how to be decided which one should be added to the RMP, the simplex method needs to be recalled briefly as the CG uses the same notion as the simplex method to do so. In the simplex method, in every single iteration, we are looking for a non-basic variable with the highest cost (the most negative reduced cost) to enter the basis. Therefore, we easily calculate the reduced cost by implicit enumeration from the below equation:

$$\overline{RC}_k = \overline{ReducedCost}_k = c_k - (\pi^*)^t a_k \tag{3.7}$$

where π is the dual optimal solution of the RMP in current iteration. Our goal is to fix a variable $k \in K \backslash K'$ such that its reduced cost is the minimum negative one, so its associated column (i.e., a_k) to be added to the RMP. The whole minimization process will be done through the pricing problem as follows:

$$\overline{RC}^* = \overline{ReducedCost}^* = min\{c(a) - (\pi^*)^t a \mid a \in A\} \tag{3.8}$$

Therefore, if there exists a $\overline{RC}^* < 0$, it promises that there is at least a column to be added to the current RMP associated with a new variable. The new column (i.e., configuration) will be added to the RMP and next iteration will be started. As such, column generation process will be repeated till there exists no $\overline{RC}^* < 0$. Hence, in case of non-existence of a negative reduced cost (i.e., all $\overline{RC}^* \geq 0$), there would be no improving criteria and we have reached the optimal solution (i.e., Z^*_{RMP} which is equal to Z^*_{MP}) in CG. When we reach the optimal condition (i.e., $\overline{RC}^* \geq 0$), it can be clearly observed that although we just add promising columns to our RMP, the optimal solution is equal to the case that we solve the MP with all its columns, simultaneously. However, in way less computational time.

3.1.2. Having an ILP Master Problem

In this subsection, we describe only the dissimilarity that considering ILP master problem will impose to the CG technique that already was explained in the subsection 3.1.1.

Consider the following ILP as the Master Problem (MP):

$$Z^*_{ILP-MP} = \min \sum_{k \in K} c_k \lambda_k \tag{3.9}$$

Subject to:

$$\sum_{k \in K} a_k \lambda_k \geq b \tag{3.10}$$

$$\lambda_k \in \{0, 1\} \qquad k \in K \tag{3.11}$$

As the CG is a technique for solving the LP problems, in case of having ILP master problem, we need to relax the constraints (3.11) to make it LP as follows:

$$Z^*_{LP-MP} = \min \sum_{k \in K} c_k \lambda_k \tag{3.12}$$

Subject to:

$$\sum_{k \in K} a_k \lambda_k \geq b \tag{3.13}$$

$$0 \leq \lambda_k \leq 1 \qquad\qquad k \in K \qquad (3.14)$$

The aforementioned LP model is considered as a MP and the rest of the procedure is done step by step based on what is former explained in Subsection 3.1.1 till reaching the optimality condition as indicated in Figure 3.1. When we reach the optimal solution (i.e., $Z_{LP-RMP}^{*} = Z_{LP-MP}^{*}$), in order to get the integer solution, either the last RMP should be solved as an ILP or the branch and price approach should be applied. Selecting the former approach, it gives us the $\tilde{Z}_{ILP}$ which is:

$$\tilde{Z}_{ILP} \neq Z_{LP-MP}^{*} \neq Z_{ILP-MP}^{*} \qquad (3.15)$$

and

$$Z_{ILP-MP}^{*} \leq Z_{ILP-MP}^{*} \leq \tilde{Z}_{ILP} \qquad (3.16)$$

Where Z_{LP-MP}^{*} and Z_{ILP-MP}^{*} are lower and upper bound for the optimal solution of the original ILP problem (i.e., Z_{ILP-MP}^{*}), respectively.

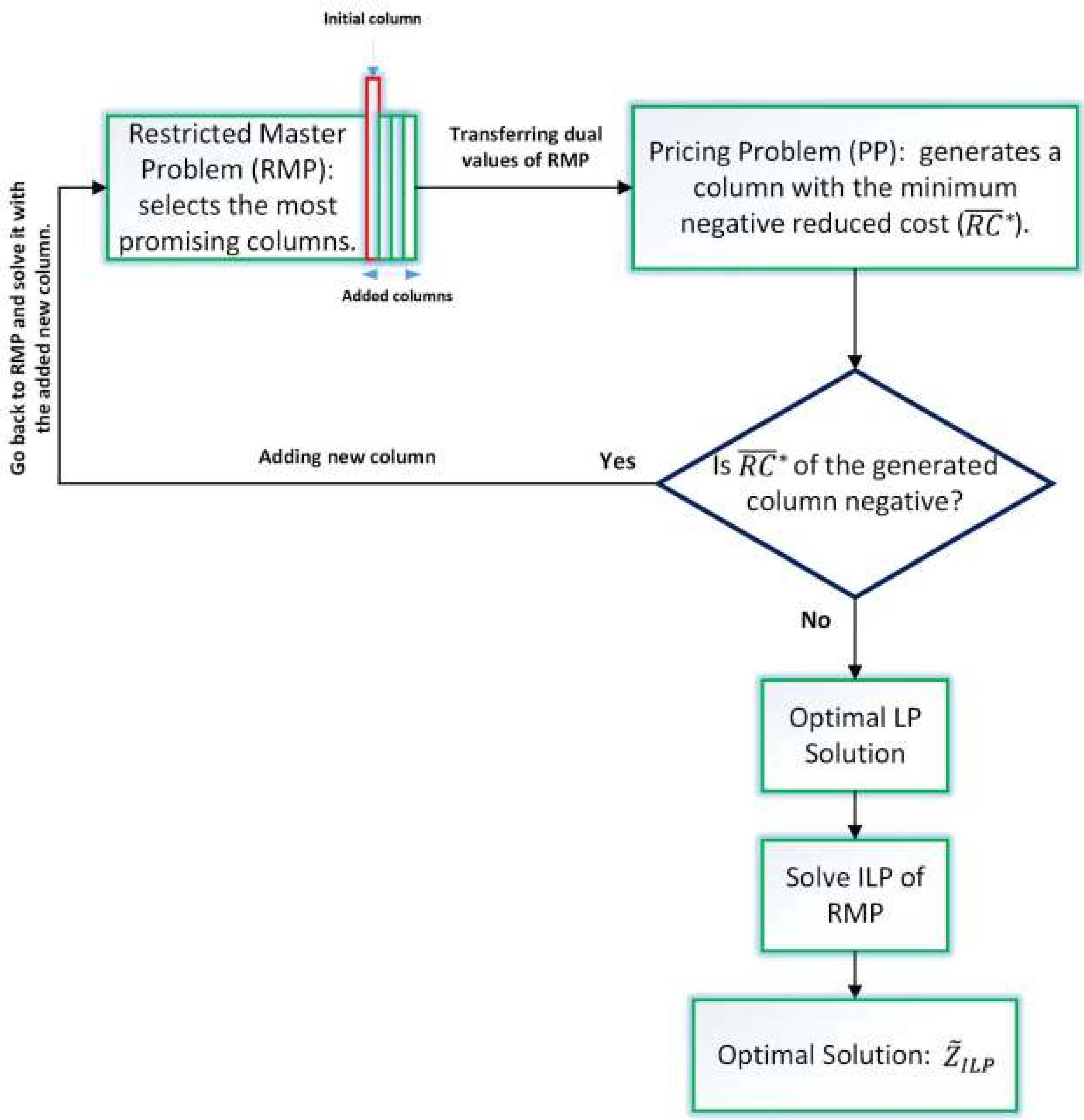

Figure 3.1: Column Generation Flowchart for ILP problem

Chapter 4

Problem description and formulation

In this chapter, we confine describe the investigated problem. Then, the proposed model formulations, regular compact model as well as column generation based model, are explained.

4.1. Problem description

In this thesis, we study Reliable Location/ Allocation/ Routing Problem in supply chain networks. Considering a network as indicated in Figure 4.1, consisting of nodes and links in which, nodes are representing either customers or facilities, and links are representing routes. More implicitly, each node is either a facility or a customer and links are the routes between the customers and facilities.

We assume that the failure sets in the problem are (pre)defined according to the most probable and disruptive interdiction in the network. Each failure set includes a combination of failed routes and facilities or nodes and links, exclusively. We also assume that if a facility is failed, it can be used as a joint node, while in case of a rout failure, it is not usable anymore and consequently that path will be unavailable completely. Because the investigated problem is a failure dependent one, decisions depend on the failure sets, in contrast to the situation in which back up facilities will be defined for each facility. Having facility and route failures into account, opening the maximum number of p facilities among potential locations in addition to allocating customers to facilities such that customers' demands are satisfied, are the challenging decisions in network design phase. Our main objective is to minimize all operational costs taking into account the worst case interdiction. The costs consist of the transportation cost, which is proportional to satisfied demand and distance, as well as the opening cost needed for building up of each facility.

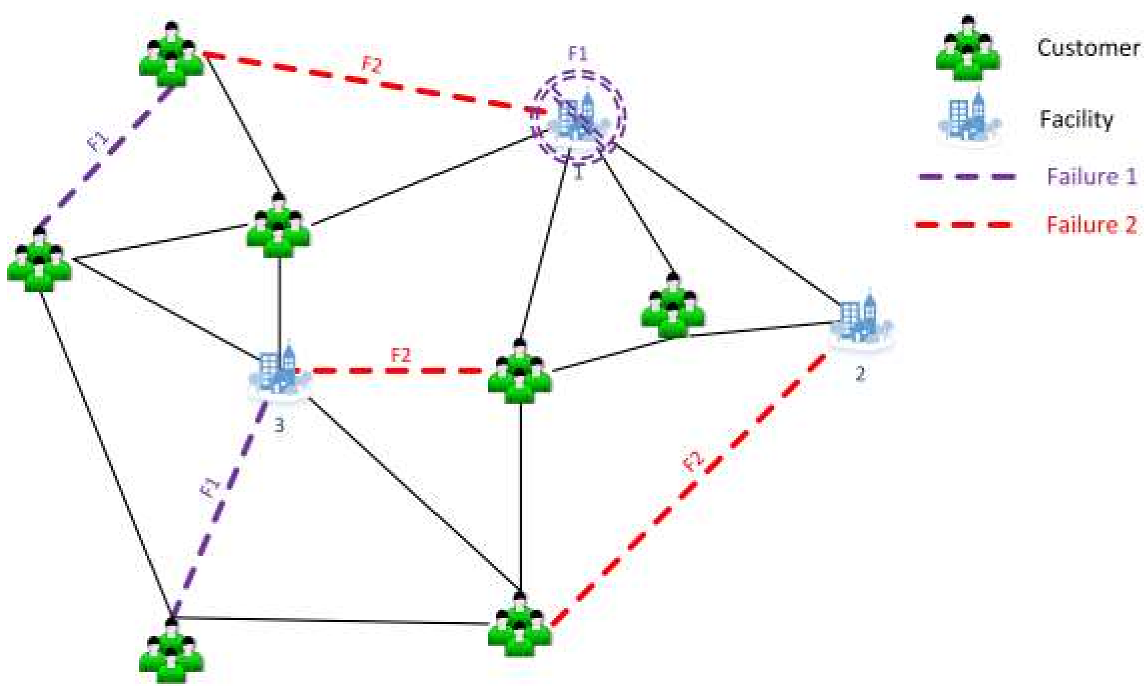

Figure 4.1: Example of the Network.

4.2. Problem formulation

In this section, we explain how the problem is formulated. Two mathematical formulations are proposed. The first one is a mixed integer programming model which is named RLARP, and the second one is the column generation based formulation. In Subsection 4.2.1 the former is discussed in detail, first notations including parameters and variables are explained and then the definition of its objective function as well as its constraints are given. The latter is formulated based on Column Generation technique which is described in Subsection 4.2.2.

4.2.1. Reliable Location/Allocation/Routing problem (RLARP) Model

We propose this model for formulating the Reliable Location/Allocation/ Routing problem (RLARP) considering different failure sets. The objective is to minimize the cost of opening a potential facility location as well as transportation cost, having the worst failure case into account. The model provides the best locations, best assignment of customers to selected potential facility locations and the best routes between customers and facilities in the presence of possible failure sets. The aforementioned model is formulated as below:

4.2.1.1. Notations

To illustrate the notations, we will first describe those defined in the model as sets, next parameters and finally variables.

Sets:

We consider a set of customers $i \in I$, potential facility locations $j \in J$ as well as failure sets $F \in \mathcal{F}$.

Parameters:

f_j = fixed opening cost of facility location j.

D_i = demand of customer i.

p = maximum number of open facilities.

$length_\ell$ = length of link ℓ.

Variables:

$g_j \in \{0, 1\}$ such that $g_j = 1$ if the facility j is open and 0 otherwise.

$x_{ij}^F \in \{0, 1\}$ to identify the customer i that is assigned to facility location j.

$\mathcal{Y}_{i\ell j}^F \in \{0, 1\}$ to identify the links which are used from a user i to facility location j.

d_{ij}^F = demand of i that is satisfied by a facility located in j.

$\mathrm{COST}_j = \max_{F \in \mathcal{F}} \sum_{i \in I} \sum_{\ell \in L} d_{ij}^F \mathcal{Y}_{i\ell j}^F length_\ell$ the cost which is proportional to the distance $\mathcal{Y}_{i\ell j}^F$ and the satisfied demand d_{ij}^F.

$m_{i\ell j}^F = d_{ij}^F \mathcal{Y}_{i\ell j}^F$ used for the linearization.

4.2.1.2. Objective

The objective function of our model is as follows:

$$\min \quad \sum_{j\in J} \mathrm{COST}_j + \sum_{j\in J} f_j g_j \tag{4.1}$$

where COST_j is the cost of satisfying the demand of those customers served by facility j under the worst failure case which is calculated by constraint (4.2).

$$\mathrm{COST}_j = \max_{F\in\mathcal{F}} \sum_{i\in I}\sum_{\ell\in L} d_{ij}^F y_{i\ell j}^F length_\ell \qquad j\in J \tag{4.2}$$

The second part of the objective function, $f_j g_j$, takes care of computing opening cost of each potential facility location j. Therefore, it can be said that our model reaches a more reliable network by minimizing the maximum cost.

4.2.1.3. Constraints

Constraints of the Compact Model are written as follows:

$$x_{ij}^F \le g_j \qquad i\in I, j\in J, F\in\mathcal{F} \tag{4.3}$$

$$d_{ij}^F \leq D_i x_{ij}^F \qquad i \in I, j \in J, F \in \mathcal{F} \tag{4.4}$$

$$\sum_{j \in J} g_j \leq p \tag{4.5}$$

$$\sum_{l \in \omega^+(v) \setminus F} y_{i\ell j}^F = \sum_{l \in \omega^-(v) \setminus F} y_{i\ell j}^F \qquad v \in V \setminus \{i, j\}, i \in I, F \in \mathcal{F} \tag{4.6}$$

$$\sum_{l \in \omega^+(i) \setminus F} y_{i\ell j}^F = \sum_{l \in \omega^-(j) \setminus F} y_{i\ell j}^F = x_{ij}^F \qquad i \in I, F \in \mathcal{F} \tag{4.7}$$

$$COST_j \geq \sum_{i \in I} \sum_{\ell \in L} d_{ij}^F y_{i\ell j}^F length_\ell \qquad j \in J, F \in \mathcal{F} \tag{4.8}$$

$$g_j \in \{0, 1\} \qquad i \in I, j \in J \tag{4.9}$$

$$x_{ij}^F \in \{0, 1\} \qquad i \in I, j \in J, F \in \mathcal{F} \tag{4.10}$$

$$y_{i\ell j}^F \in \{0, 1\} \qquad i \in I, \ell \in L \setminus F, j \in J, F \in \mathcal{F} \tag{4.11}$$

$$d_{ij}^F \geq 0 \qquad i \in I, j \in J, F \in \mathcal{F} \tag{4.12}$$

$$\mathrm{COST}_j \geq 0 \qquad j \in J \tag{4.13}$$

Constraints (4.3) ensures customers will be covered just by open facilities. Constraints (4.4) make sure that either a portion or whole demand of customer i's demand may be fulfilled by facility j only if it is assigned to that facility under failure F. The customers' demand can be satisfied by either one facility or several facilities as there is no constraints restricting that. Constraints (4.6) and (4.7) take care of the flow constraints (i.e., routing) between user i and location j, under the assumption that user i is assigned to a facility located in j, under failure F. Constraints (4.8) takes care of computing the maximum transportaion cost which is proportional to satisfied demand as well as distance for facility j over all failure sets or in another word worst failure cost. Constraints (4.09), (4.10), (4.11), (4.12), and (4.13) define the domains of the variables.

As we defined the COST_j demonstrated at (4.2) in the most realistic way which is proportional to the distance and the satisfied demand, it makes constraint (4.8) a non-linear function. As the non-linearized term is made from one continuous variable and one binary variable, it can be easily linearized by adding variables $m_{i\ell j}^F \geq 0$:

$$m_{i\ell j}^F \leq y_{i\ell j}^F D_i \tag{4.14}$$

$$m_{i\ell j}^F \leq d_{ij}^F \tag{4.15}$$

$$m_{i\ell j}^F \geq d_{ij}^F + D_i\left(y_{i\ell j}^F - 1\right) \tag{4.16}$$

As a result of this linearization there would be some changes in the constraints of the model which are applied below:

$$x_{ij}^F \leq g_j \qquad\qquad i \in I, j \in J, F \in \mathcal{F} \tag{4.17}$$

$$d_{ij}^F \le D_i x_{ij}^F \qquad i \in I, j \in J, F \in \mathcal{F} \tag{4.18}$$

$$\sum_{j \in J} g_j \le p \tag{4.19}$$

$$\sum_{l \in \omega^+(v) \setminus F} y_{i\ell j}^F = \sum_{l \in \omega^-(v) \setminus F} y_{i\ell j}^F \qquad v \in V \setminus \{i, j\}, i \in I, F \in \mathcal{F} \tag{4.20}$$

$$\sum_{l \in \omega^+(i) \setminus F} y_{i\ell j}^F = \sum_{l \in \omega^-(j) \setminus F} y_{i\ell j}^F = x_{ij}^F \qquad i \in I, F \in \mathcal{F} \tag{4.21}$$

$$COST_j \ge \sum_{i \in I} \sum_{\ell \in L} m_{i\ell j}^F length_\ell \qquad j \in J, F \in \mathcal{F} \tag{4.22}$$

$$m_{i\ell j}^F \le y_{i\ell j}^F D_i \qquad i \in I, \ell \in L \setminus F, j \in J, F \in \mathcal{F} \tag{4.23}$$

$$m_{i\ell j}^F \le d_{ij}^F \qquad i \in I, \ell \in L \setminus F, j \in J, F \in \mathcal{F} \tag{4.24}$$

$$m_{i\ell j}^F \ge d_{ij}^F + D_i \left(y_{i\ell j}^F - 1 \right) \qquad i \in I, \ell \in L \setminus F, j \in J, F \in \mathcal{F} \tag{4.25}$$

$$g_j \in \{0, 1\} \qquad i \in I, j \in J \tag{4.26}$$

$$x_{ij}^F \in \{0, 1\} \qquad i \in I, j \in J, F \in \mathcal{F} \tag{4.27}$$

$$y_{i\ell j}^F \in \{0, 1\} \qquad i \in I, \ell \in L \setminus F, j \in J, F \in \mathcal{F} \tag{4.28}$$

$$d_{ij}^F \ge 0 \qquad i \in I, j \in J, F \in \mathcal{F} \tag{4.29}$$

$$\mathrm{COST}_j \geq 0 \qquad j \in J \qquad (4.30)$$

$$m_{i\ell j}^{F} \geq 0 \qquad i \in I, \ell \in L \setminus F, j \in J, F \in \mathcal{F} \qquad (4.31)$$

All constraints are the same as what we already explained except for constraints (4.23), (4.24) and (4.25) added to the model to take care of linearization of constraint (4.8). Also, constraint (4.31) define the domain of set of variables added to the model for linearization.

4.2.2. Column Generation Model

In this section the modified Column Generation (CG) technique which we name Parallel CG (P-CG) used for reformulating our RLARP. What makes the P-CG different from classical CG, already explained in Chapter 3, is that it solves a set of pricing problem (PP) in each iteration rather than one, which improves performance of P-CG considerably. To recall, CG based models require two sets of models, master problem (MP) and Pricing Problem (PP). The former is responsible for selecting the configurations such that the cost of location, allocation as well as fixed opening cost of facilities are minimized. While the latter is in charge of producing the configurations (i.e., columns) to be added to the Restricted Master Problem (RMP), to speed up reaching the optimality.

In the first place the RMP starts by an initial column. In our case a dummy column plays a role as an initial column in the RMP. Each time the RMP is solved, a set of dual variables will be generated and transferred to the pricing problem to build up reduced cost as shown in Figure 4.2. Then the PP uses those dual variables to generate a configuration for each potential facility location j. It also defines the assignment of customers to facility j under failure F in a way to facilitate reaching to a better solution for the RMP. After the qualified configurations (i.e., new columns with negative reduced cost) are added to the RMP, the RMP will be solved. This loop will continue until achieving an optimality condition which in our case is having a non-negative reduced cost (i.e., all $\overline{RC}_j^* \geq 0$).

The proposed model, RLARP and CG based Model, have a significantly different performance which will be discussed in Chapter 5. As in Column Generation technique configuration is used the notations and parameters definition will become different from RLARP.

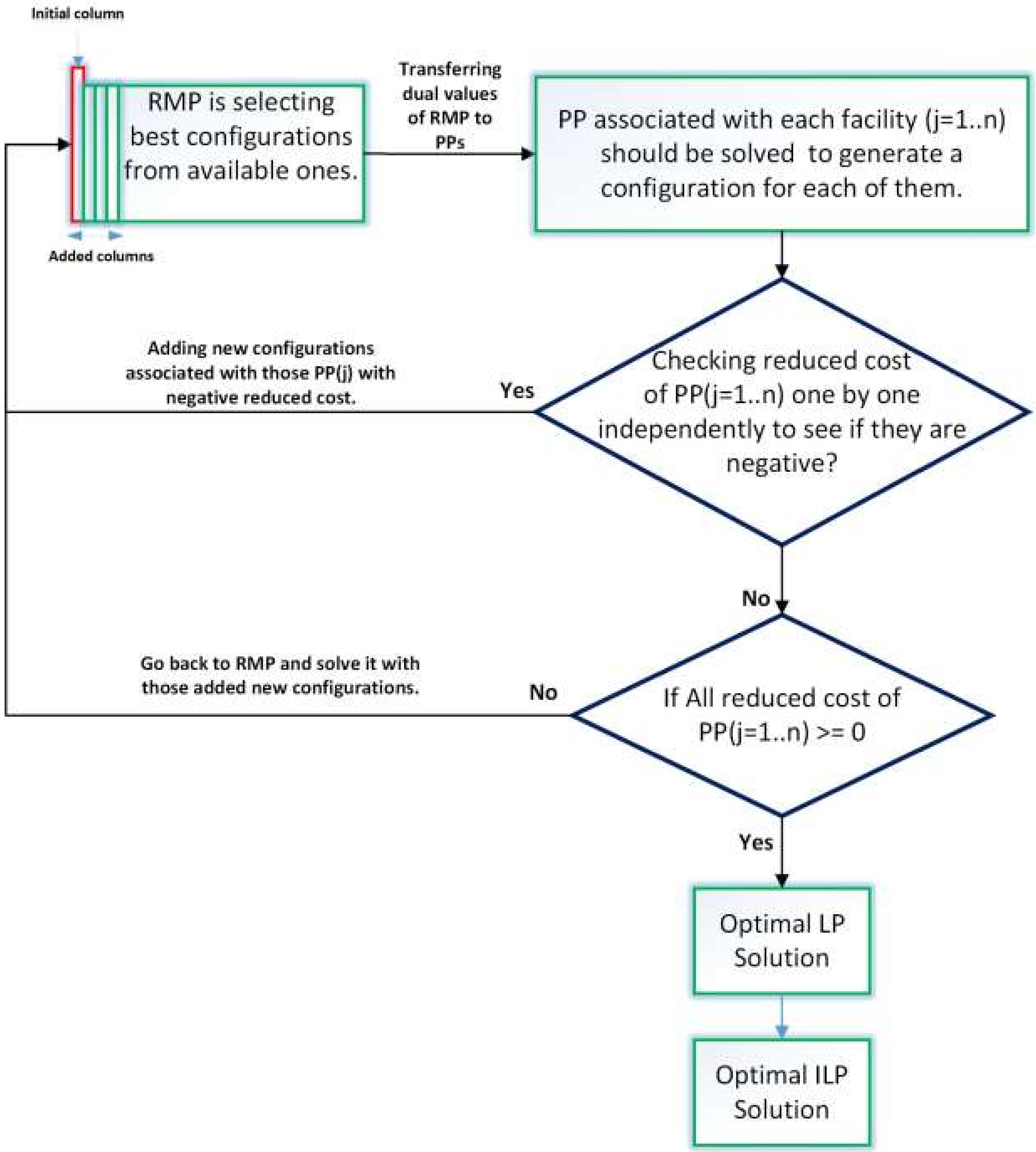

Figure 4.2: Column Generation Flowchart for RFLRP

The remainder of this Subsection would be as follows. First master problem including its notations, objective function and constraints is explained and then the same subjects are described for Pricing Problem in the same sequence.

4.2.2.1. Master Problem

The master problem aims to give us the most reliable network such that by selecting the best configurations, the cost is minimized.

4.2.2.1.1. Notations

Sets:

Let Γ_j be the set of configurations with respect to potential facility location j. Configuration $\gamma \in \Gamma_j$ is defined by a set of customers assigned to a facility located in j subject to failure F, for all $F \in \mathcal{F}$. Let

$$\Gamma = \bigcup_{j \in J} \Gamma_j$$

It is characterized by:

$d_i^{F,\gamma}$ = demand of i that is satisfied by a facility located in j when failure F occurs.

Parameters:

The rest of parameters in the master problem are as follows:

f_j = fixed opening cost of facility location j.

D_i = demand of customer i.

p = number of open facilities.

$y_{i\ell}^{F,\gamma}$ = the links in configuration γ which are used from a user i to facility location j when failure F occurs.

Variables:

The model requires two sets of decision variables:

$z_\gamma \in \{0, 1\}$ such that $z_\gamma = 1$ if configuration γ is selected.

$g_j \in \{0, 1\}$ such that $g_j = 1$ if the facility j is open and 0 otherwise.

4.2.2.1.2. Objective

The objective consists in minimizing the cost of the selected configurations and fixed opening cost:

$$\min \qquad \sum_{\gamma \in \Gamma} \mathrm{COST}_\gamma z_\gamma + \sum_{j \in J} f_j g_j \tag{4.32}$$

where

$$\mathrm{COST}_\gamma = \max_{F \in \mathcal{F}} \sum_{i \in I} d_{ij}^{F,\gamma} \sum_{\ell \in L} y_{i\ell}^{F,\gamma}\, length_\ell \tag{4.33}$$

The cost is the expense of each configuration γ which is calculated in the pricing problem. It determines the demand weighted distance under worst failure case.

4.2.2.1.3. Constraints

Constraints of the master problem are as follows:

$$\sum_{\gamma \in \Gamma_j} z_\gamma = g_j \qquad j \in J \qquad (4.34)$$

$$\sum_{\gamma \in \Gamma} d_i^{F,\gamma} z_\gamma \geq D_i \qquad i \in I, F \in \mathcal{F} \qquad (4.35)$$

$$\sum_{\gamma \in \Gamma} z_\gamma \leq p \qquad (4.36)$$

$$0 \leq z_\gamma \leq 1 \qquad \gamma \in \Gamma \qquad (4.37)$$

$$0 \leq g_j \leq 1 \qquad j \in J \qquad (4.38)$$

Constraints (4.34) select at most one configuration per potential facility location just in the case that the facility j is opened. Constraints (4.35) are demand constraints, which need to be fulfilled for any potential failure $F \in \mathcal{F}$. Constraint (4.36) ensures that no more than p facilities are opened, at any time.

4.2.2.2. Pricing Problem

It is worth mentioning that pricing problem (PP) produces promising configurations to be added to restricted master problem (RMP) to accelerate reaching the optimal solution. Our PP is written for each potential facility location j. Each configuration will define the assigned customers to facility j, the amount of customers' demand that is satisfied by facility j and the the route from customer i to facility j under failure F. We now express the pricing problem associated with a configuration $\gamma \in \Gamma_j$, i.e., for a given facility location j. In order to alleviate the notations, indices γ and j will be omitted in the sequel of this section.

4.2.2.2.1. Notations

Parameters:

Let $u_j^{(4.34)}$ be unrestricted, $u_{i,F}^{(4.35)} \geq 0$ and $u^{(4.36)} \geq 0$ the values of the dual variables associated with constraints (4.34), (4.35) and (4.36) of the RMP, respectively.

$length_\ell =$ demonstrates the length of each link $\ell \in L$.

Variables:

$x_i^F \in \{0, 1\}$, to identify the customer i that are assigned to facility location j when failure F occurs.

$\mathcal{Y}_{i\ell}^F \in \{0, 1\}$ to identify the links which are used from a user i to facility location j.

$d_i^F =$ demand of i that is satisfied by a facility located in j when failure F occurs.

$\text{COST} = \max\limits_{F \in \mathcal{F}} \sum\limits_{i \in I} d_i^F \sum\limits_{\ell \in L} \mathcal{Y}_{i\ell}^F length_\ell$ the cost which is proportional to the distance $\mathcal{Y}_{i\ell}^F length_\ell$ and the satisfied demand d_i^F in the worst failure case.

4.2.2.2.2. Objective

The objective is to minimize the reduced cost which is denoted by $\overline{RC_j}$ in (4.39). To recall, in simplex method always the variable with the minimum reduced cost enters the basis in order to maximize the improvement of the objective function in each step, that is the logic behind having the reduced cost as the objective function of pricing problem.

$$[\mathrm{PP}_j] \qquad \min \qquad \overline{RC_j} \tag{4.39}$$

where

$$\overline{RC_j} = \mathrm{COST} - u_j^{(4.34)} - \sum_{i \in I} \sum_{F \in \mathcal{F}} d_i^F u_{i,F}^{(4.35)} + u^{(4.36)} \tag{4.40}$$

4.2.2.2.3. Constraints

$$d_i^F \leq D_i x_i^F \qquad i \in I, F \in \mathcal{F} \tag{4.41}$$

$$\sum_{l \in \omega^+(v) \setminus F} y_{i\ell}^F = \sum_{l \in \omega^-(v) \setminus F} y_{i\ell}^F \qquad v \in V \setminus \{i, j\}, i \in I, F \in \mathcal{F} \tag{4.42}$$

$$\sum_{l \in \omega^+(i) \setminus F} y_{i\ell}^F = \sum_{l \in \omega^-(j) \setminus F} y_{i\ell}^F = x_i^F \qquad i \in I, F \in \mathcal{F} \tag{4.43}$$

$$\text{COST} \geq \sum_{i \in I} d_i^F \sum_{\ell \in L} y_{i\ell}^F length_\ell \qquad F \in \mathcal{F} \tag{4.44}$$

$$x_i^F \in \{0, 1\} \qquad i \in I, F \in \mathcal{F} \backslash \{j\} \tag{4.45}$$

$$y_{i\ell}^F \in \{0, 1\} \qquad i \in I, \ell \in L \backslash F, F \in \mathcal{F} \tag{4.46}$$

$$d_i^F \geq 0 \qquad i \in I, F \in \mathcal{F} \tag{4.47}$$

$$\text{COST} \geq 0 \qquad F \in \mathcal{F} \tag{4.48}$$

Constraints (4.41) make the amount of d_i^F dependent to the condition that whether or not the customer i is assigned to facility j. Constraints (4.42) and (4.43) take care of the flow constraints (i.e., routing) between user i and location j, under the assumption that user i is assigned to a facility located in j, under failure F. Constraints (4.45), (4.46), (4.47) and (4.48) define the domains of the variables.

4.2.2.3. Linearized form of Pricing Problem

As the cost is a non-linear function,

$$\text{COST} = \max_{F \in \mathcal{F}} \sum_{i \in I} d_i^F \sum_{\ell \in L} y_{i\ell}^F length_\ell \tag{4.49}$$

It makes the reduced cost a nonlinear function that can be easily linearized, using variables $m_{i\ell}^F \geq 0$:

$$m_{i\ell}^F \leq y_{i\ell}^F D_i \tag{4.50}$$

$$m_{i\ell}^F \leq d_i^F \tag{4.51}$$

$$m_{i\ell}^F \geq d_i^F + D_i(y_{i\ell}^F - 1) \tag{4.52}$$

As a result of this linearization there would be some changes in the constraints of the pricing problem which are applied below:

$$d_i^F \leq D_i x_i^F \qquad i \in I, F \in \mathcal{F} \tag{4.53}$$

$$\sum_{l \in \omega^+(v) \setminus F} y_{i\ell}^F = \sum_{l \in \omega^-(v) \setminus F} y_{i\ell}^F \qquad v \in V \setminus \{i, j\}, i \in I, F \in \mathcal{F} \tag{4.54}$$

$$\sum_{l \in \omega^+(i) \setminus F} y_{i\ell}^F = \sum_{l \in \omega^-(j) \setminus F} y_{i\ell}^F = x_i^F \qquad i \in I, F \in \mathcal{F} \tag{4.55}$$

$$\text{COST} \geq \sum_{i \in I} \sum_{\ell \in L} m_{i\ell}^F length_\ell \qquad F \in \mathcal{F} \tag{4.56}$$

$$m_{i\ell}^F \leq y_{i\ell}^F D_i \qquad i \in I, \ell \in L \setminus F, F \in \mathcal{F} \tag{4.57}$$

$$m_{i\ell}^F \leq d_i^F \qquad i \in I, \ell \in L \setminus F, F \in \mathcal{F} \tag{4.58}$$

$$m_{i\ell}^F \geq d_i^F + D_i(y_{i\ell}^F - 1) \qquad i \in I, \ell \in L\backslash F, F \in \mathcal{F} \qquad (4.59)$$

$$x_i^F \in \{0,1\} \qquad i \in I, F \in \mathcal{F}\backslash\{j\} \qquad (4.60)$$

$$y_{i\ell}^F \in \{0,1\} \qquad i \in I, \ell \in L\backslash F, F \in \mathcal{F} \qquad (4.61)$$

$$d_i^F \geq 0 \qquad i \in I, F \in \mathcal{F} \qquad (4.62)$$

$$\text{COST} \geq 0 \qquad (4.63)$$

$$m_{i\ell}^F \geq 0 \qquad i \in I, \ell \in L\backslash F, F \in \mathcal{F} \qquad (4.64)$$

Here we explain the constraints which are different from those that we have already explained in 4.2.2.2. Constraints (4.57), (4.58) and (4.59) take care of the linearization while (4.63) and (4.64) define the domains of the variables.

Chapter 5

Computational Results

In this chapter, we are going to first explain the data set which is used in the thesis to validate the RLARP and the column generation based model already introduced in Chapter 4. Then, the aim is to compare the performance of the two formulations to see how beneficial is to use column generation technique. To do so, a general optimization solver CPLEX is used for solving the model with a computer having a following feature: Core i7 Q740 @ 1.73 GHZ 1.73 GHz Processor, 4 GB RAM and 64-bit Operating System, x64-based processor.

5.1. Data Set

The data set used in the thesis is generated based on the notion of the data found in Daskin (1995) as well as Snyder and Daskin (2005). Their data set consists of 49 nodes, which the nodes indicate the 49 populous cities in the United State based on the information derived from 1990 Population and Housing Census. In addition, we generate different sets of data and test our model with them to provide us with fairly enough information to be able to analyse the performance of the model and solution strategy described in Chapter 4.

Random networks were generated with 16, 19, 21, 26, and 38 nodes which present the location of either potential facility locations or customers. To build a network, we connected each node to its 2 to 5 closest nodes. The number of respective links of the nodes are 66, 74, 80, 102, and 139. The length of each link ($length_\ell$) which represents the distance between its source and destination node is generated in the interval of $[10,40]$. Also, the demand of each customer (D_i) is generated in the interval of $[30 - 100]$.

The fixed opening cost of each potential facility location depends on some factors such as price of the land, required area, facilities to be installed in the location, and cost of buildings. They are, accordingly, generated randomly in the interval of [10000-17000]. The maximum number of potential facility locations that can be opened (p) is taken into account 3. For the computational results, we consider three independent failure sets. Each set consists of multiple link failures. It is worth mentioning that our model is capable of considering facility, customer and link failures either simultaneously in one failure set or independently in different failure sets. We provide the data set associated with 16 nodes-66 links and 38 nodes-139 links in Figure 5.1and Table 5.1, respectively.

Table 5.1: Failure Sets

Failure Sets	Node Names									
Failure 1	2	3	14	17	18	29	31	33	7	22
Failure 2	2	3	7	8	17	18	22	23	34	35
Failure 3	10	14	25	29	39	43	49	51	22	7

Table 5.2: Cost of Building Facilities

Facilities	Fixed Opening Cost
3	16610
4	15040
9	17000
19	15220
20	16471
21	15792

Table 5.3: Customer's Demand

Customer	Demand
1	50
2	53
5	65
6	92
7	77
8	59
10	30
11	40
12	50
13	60
14	70
15	80
16	90
17	31
18	40
22	88
23	52
24	56
25	85
26	41
27	100
28	67
29	57
30	43
31	47
32	59

33	48
34	79
35	100
36	69
37	95
38	82

Table 5.4: Link connections and distances between customers

Start	End	Distance
1	2	27.75
1	4	33.49
1	7	23.19
1	11	27.75
2	1	27.75
2	3	22.62
2	4	23.78
2	11	33.49
3	2	22.62
3	5	28.34
3	6	31.78
3	9	13.52
3	11	23.19
3	14	21.09
4	1	33.49
4	2	23.78
4	6	30.21
4	7	14.84
4	8	12.95
5	3	28.34
5	6	31.55
5	9	21.09
6	3	31.78
6	4	30.21
6	5	31.55
6	8	16.11
7	1	23.19
7	4	14.84
7	8	32.77
7	10	33.53
7	12	28.34

Start	End	Distance
10	13	21.09
10	16	13.52
10	24	17.83
11	1	27.75
11	2	33.49
11	3	23.19
11	12	27.75
11	14	16.11
11	26	20.74
12	7	28.34
12	11	27.75
12	16	21.09
12	25	20.45
13	9	31.55
13	10	21.09
13	23	30.25
14	3	21.09
14	11	16.11
14	15	33.53
14	17	23.84
14	26	26.98
14	27	13.82
15	9	14.33
15	14	33.53
15	18	29.14
16	7	15.73
16	10	13.52
16	12	21.09
16	24	30.27
16	25	33.97
17	14	32.84

Start	End	Distance
21	22	14.87
22	9	17.65
22	21	14.87
22	23	16.04
23	13	30.25
23	22	16.04
23	24	30.84
24	10	17.83
24	16	30.27
24	23	30.84
25	12	20.45
25	16	33.97
26	11	20.74
26	14	26.98
26	28	15.72
27	14	13.82
27	28	27.65
28	26	15.72
28	27	27.65
28	29	33.43
29	17	14.5
29	28	33.43
29	30	26.12
29	38	28.47
30	29	26.12
30	31	24.47
30	34	27.89
31	30	24.47
31	33	16.12
31	38	16.39
32	33	33.62

7	16	15.73
8	4	12.95
8	6	16.11
8	7	32.77
8	9	14.33
8	10	15.73
9	3	13.52
9	5	21.09
9	8	14.33
9	10	23.06
9	13	31.55
9	15	14.33
9	22	17.65
10	7	33.53
10	8	15.73
10	9	23.06

17	18	24.62
17	29	14.5
18	15	29.14
18	17	24.62
18	19	28.24
18	20	19.85
18	33	22.3
18	36	32.94
19	18	28.24
19	20	31.67
19	21	13.31
20	18	19.85
20	19	31.67
20	36	23.01
21	19	13.31

32	34	24.75
33	18	22.3
33	31	16.12
33	32	33.62
34	30	27.89
34	32	24.75
34	37	31.51
35	37	28.85
36	18	32.94
36	20	23.01
37	34	31.51
37	35	28.85
38	29	28.47
38	31	16.39
38	40	15.96

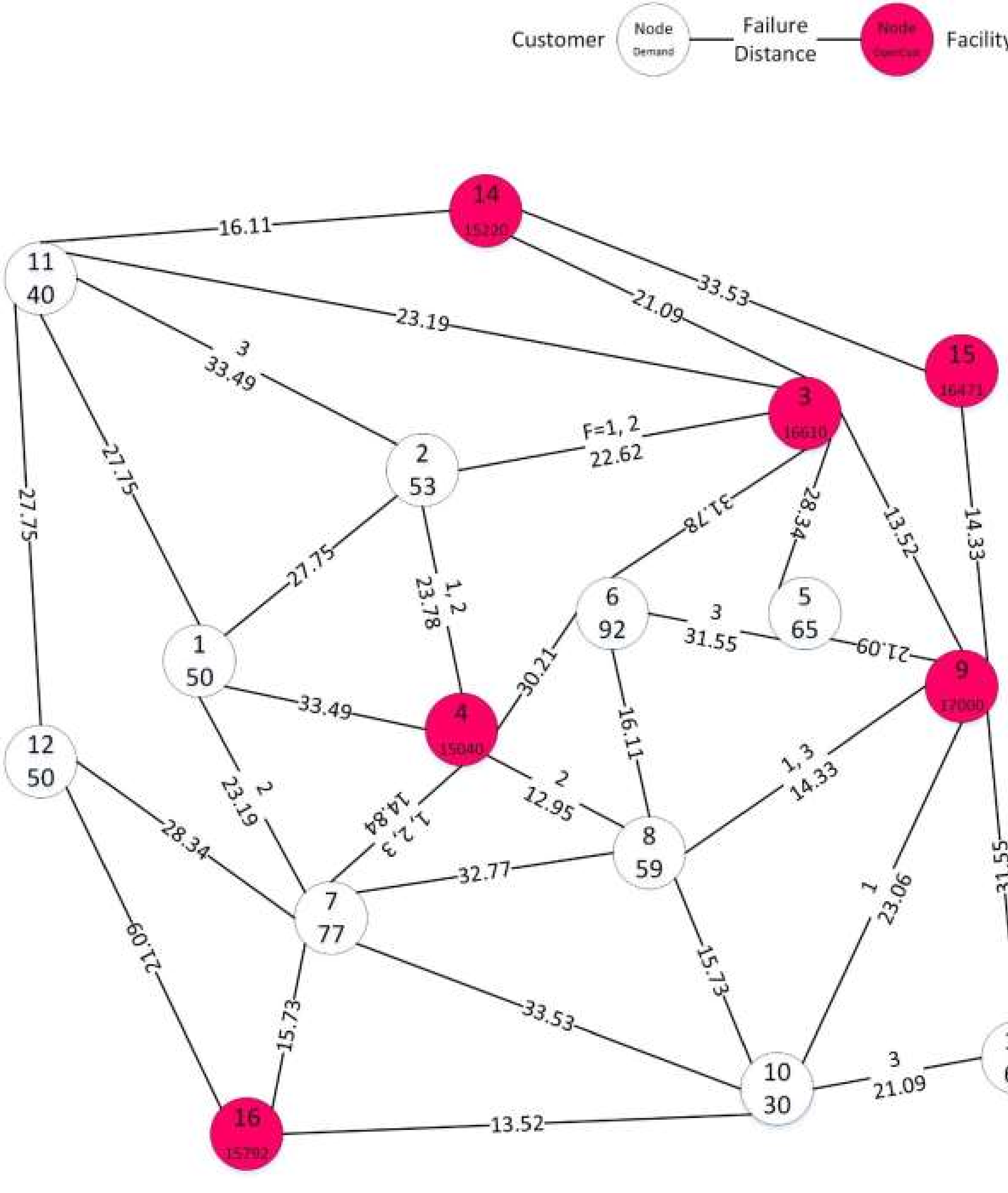

Figure 5.1: 16 Nodes-66 Links Network.

5.2. Result

In this section, we present our computational results for the proposed model and its reformulation, RLARP and Column Generation (CG), in Chapter 4. The models are solved with data sets which are different from each other in terms of at least one of the following criteria: number of nodes (n), links (l), facilities, and customers. In what follows, for each model, in addition to demonstrating the computational results, some analytical and complexity analysis are proposed. At the end, in 5.2.3 we investigate the differences in the performance of the RLARP model and CG based formulation.

5.2.1. Results of RLARP Model

To recall, the RLARP seeks the optimal location of facilities, allocation of customers to them as well as the routes between them while minimizing the worst failure case cost along with fixed opening cost of facilities. We solve our model for different networks as it can be seen in the Table 5.5 and Table 5.7 from smaller network to the bigger one. In this section the computational results corresponding to the RLARP model (i.e., Uncapacitated RLARP model), described in Chapter 4, and Capacitated RLARP model are provided. Besides, the comparison of the aforementioned models is explained, eventually.

5.2.1.1. Result of Uncapacitated RLARP Model

For each network, we consider two different size of potential facilities, 4 and 6 to see how the model performs while increasing the number of facilities and keeping the size of the network as it is. Also, it should be mentioned that a node is considered either a facility or customer and no capacity constraints are taken into accounts for facilities. As it is demonstrated in Table 5.5, when the size of the network grows the computational time is increased exponentially as well, for both cases of 4 and 6 potential facilities. Therefore, the observed increase in computational time is attributed to the hike in number of nodes and links.

Table 5.5: Results for Uncapacitated RLARP Model.

Nodes	Links	Customers	Facilities	P	Best Bound (LP*)	Best Integer (ILP~)	Obj (ILP*)	Time (Second)	Gap (%)
11	44	7	4	3	-	-	35,063.70	10.11	
11	44	5	6	3	-	-	30,320.57	10.10	
13	52	9	4	3	-	-	41,660.11	22.01	
13	52	7	6	3	-	-	37,369.01	35.79	
16	66	12	4	3	-	-	55,789.11	248.26	
16	66	10	6	3	-	-	40,333.93	375.26	
21	66	17	4	3	67,944.31	75,493.68		2,410.13	10
21	80	15	6	3	60,666.53	67,407.25		3,589.42	10

5.2.1.1.1. 4 vs. 6 Potential Facilities

To better understand the impact of decreasing the number of potential facilities in an identical network, Figure 5.2 is used to demonstrate the performance of the model. It can be observed that, in general, the computational time for the case of having 4 potential facilities is less than the 6 potential facilities. Considering the observations, one possible explanation for such a trend is that in case of considering the unlimited capacity, number of facilities have brought more difficulties for solving the model than the number of customers in the same network.

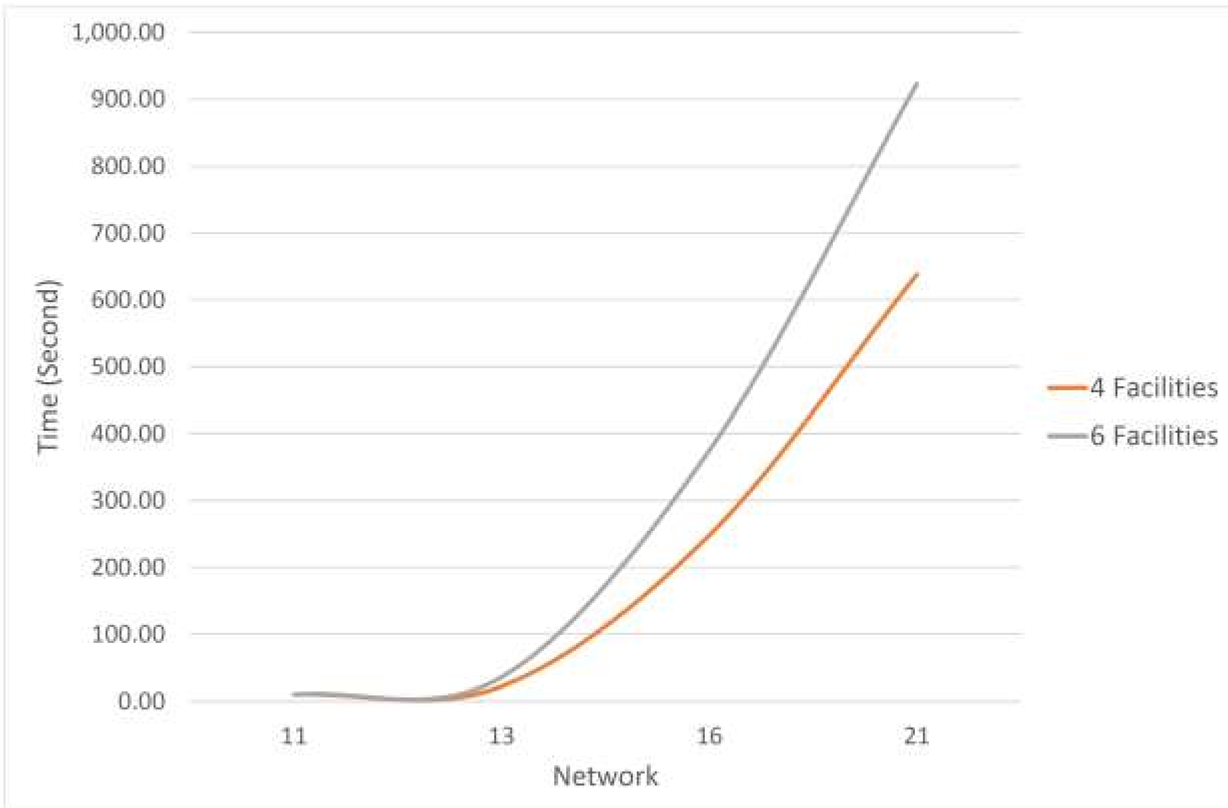

Figure 5.2: Performance Comparison of 4 VS. 6 Potential Facilities in Uncapacitated RLARP Model.

5.2.1.1.2. Experimental Complexity Analysis

We present a brief experimental complexity analysis for the 21 nodes-80 links network as CPLEX cannot reach the solution after 3,804.15 seconds for this specific instance. To do so, we fix the gap to 30%, 20% and 10% and solve the uncapacitated RLARP model, accordingly. As it can be observed in Table 5.6 in terms of CPU time reaching the solution in the case of considering 10% gap is way more time consuming than either 30% or 20%. However, in all three gaps we get the same solution regarding the selection of facilities.

Therefore, it is high probable to say that even by considering 30% gap, we may reach the optimal solution in a timely manner.

Table 5.6: Uncapacitated RLARP Model Experimental Complexity Analysis for 21 nodes-80 links

Gap (%)	Lower Bound (LP*)	Upper Bound (ILP~)	Objective Value	Open Facilities	Time (Second)
30	40,435.65	61,312.35	61,312.35	Facility 3	386.11
20	40,069.93	61,312.35	61,312.35	Facility 3	511.03
10	51,666.53	57,407.25	57,407.25	Facility 3	3,804.15

5.2.1.2. Result of Capacitated RLARP Model

In this part, first we explain what changes are made to our uncapacitated RLARP model to create a capacitated version of it. Then proposed the results in Table 5.7. Later, we argue how the network size as well as number of potential facilities have an impact on the computational time. Finally, a complexity analysis of the model is discussed.

To analyze how adding capacity constraints will have an effect on the performance of the RLARP model, we add the following constraint to the model, already explained in Section 4.2.1 as below:

$$\sum_{i \in I} d_{ij}^F \leq Q_j \qquad j \in J, F \in \mathcal{F} \qquad (5.1)$$

We calculate a specific capacity for each facility (Q_j) in order not to let one facility to give an unlimited service to all customers while others are idle. To balance the amount of services given to the customers among facilities, the capacity associated with each of them is generated in the interval of $[2\overline{D} - 2.3\overline{D}]$, where $\overline{D}$ can be explained as an average demand and is calculated as follows:

$$\bar{D} = \sum_{i \in I} D_i/p \tag{5.2}$$

Table 5.7: Results for Capacitated RLARP Model.

Nodes	Links	Customers	Facilities	P	Best Bound (LP*)	Best Integer (ILP~)	Obj (ILP*)	Time (Second)	Gap (%)
11	44	7	4	3	-	-	43,260.04	16,221.49	-
11	44	5	6	3	-	-	40,624.94	742.84	-
13	52	9	4	3	48,333.07	50,876.92	-	25,371.70	5%
13	52	7	6	3	44,500.02	46,842.12	-	4,724.00	5%
16	66	12	4	3	55,106.72	58,007.07	-	385,643.62	5%
16	66	10	6	3	47,612.48	50,118.40	-	257,879.86	5%

In Table 5.7 the results associated with capacitated version of RLARP model is demonstrated. We solve the model for the same data sets which are used for uncapacitated RLARP model to be able to compare their performance later in 5.2.1.3. It can be observed in Table 5.7, when the network enlarges in terms of nodes and links the computational time raises. Except for the case of having 11 nodes-44 links, for the rest of instances the program could not reach the optimal solution. To clarify more, for the instance of 16 nodes-66 links even after running the program for 3 days we do not get a better solution than 5% gap. Therefore, to be able to compare their performance, we fix the gap to 5% for the instances that could not reach to the optimal solution either in a timely manner or due to the RAM limitations. The possible explanation for such results is that considering the capacity constraints makes the problem more complicated (i.e., increase the complexity of the problem) to be solved.

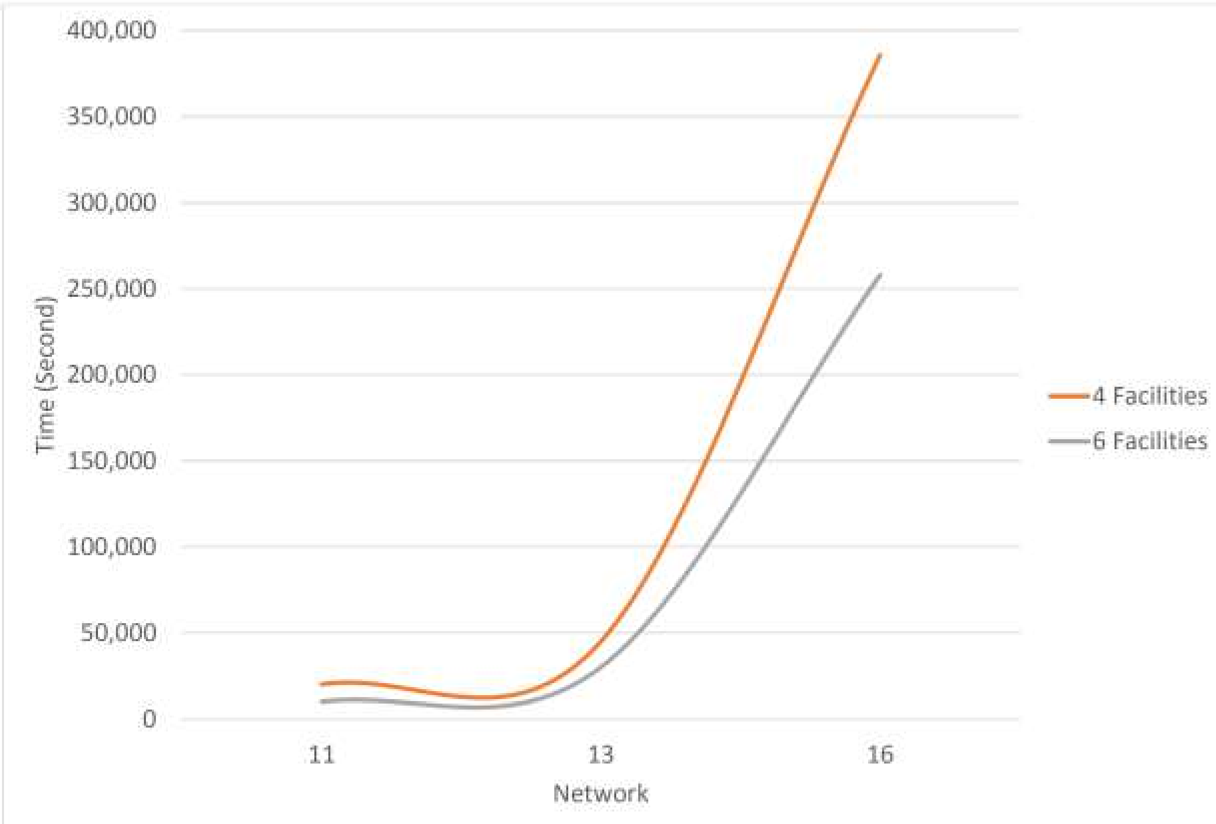

Figure 5.3: Comparison of 4 vs. 6 Potential Facilities in Capacitated RLARP Model.

5.2.1.2.1. 4 vs. 6 Potential Facilities

To compare having 6 potential facilities with 4, as it can be seen in Figure 5.3, unlike Uncapacitated RLARP, solving the network with 6 potential facilities is less time consuming than solving the problem for the same network with considering 4 potential facilities. A probable explanation for this might be that when considering the capacity for the network with the same number of nodes and links, the more customers means the more complexity. To clarify it more, when considering 4 potential facilities, the number of customers are two more than the time of having 6 potential facilities. Thus, satisfying the demand of 2 more customers while taking care of capacity limitations seems to be likely the reason behind such a discrepancy. Therefore, we conclude that raising the number of potential facilities, which result in decreasing the number of customers, for the identical network lessen the computational time.

5.2.1.2.1. Experimental Complexity Analysis

In capacitated RLARP model, for the instances larger than 11 nodes-44 links, CPLEX cannot reach the optimal solution as indicated in Table 5.7. Therefore, we propose an experimental complexity analysis considering 30%, 20%, and 5% gap for the two instance of 13 nodes-52 links and 16 nodes-88 links. As indicated in Table 5.8 and Table 5.9, solving the model with 5% gap is far more time taking than considering 20% and 30% gap. Also, as with different gaps we are getting different solutions, there is not enough ground to state that whether or not the final solution is the optimal one. However, there could be a possibility to reach optimal solution in 5% gap. In many cases, it was observed that LP^* was changing, while having no changes in the amount of $\widetilde{ILP}$. Besides, the $\widetilde{ILP}$ is a feasible solution for the RLARP model. Hence, there is a chance of having the same optimal solution as the $\widetilde{ILP}$ over a range of experiments.

Table 5.8: Capacitated RLARP Model Experimental Complexity Analysis for 13 nodes-52 links

Gap (%)	Lower Bound (LP^*)	Upper Bound ($\widetilde{ILP}$)	Objective Value	Open Facilities	Time (Second)
30	42,697.06	61,052.06	61,052.06	Facility 8, 9	39.58
20	43,637.78	54,547.22	54,547.22	Facility 9, 11	66.89
5	44,712.92	46,842.12	46,842.12	Facility 9, 11	4,724.00

Table 5.9: Capacitated RLARP Model Experimental Complexity Analysis for 16 nodes-88 links

Gap (%)	Lower Bound (LP*)	Upper Bound (ILP~)	Objective Value	Open Facilities	Time (Second)
30	37,487.24	61,256.21	61,256.21	Facility 9, 14	174.67
20	44,211.13	55,263.91	55,263.91	Facility 3, 9	66.89
5	47,612.48	50,118.40	50,118.40	Facility 4, 9	257,879.86

5.2.1.3. Uncapacitated vs. Capacitated

In this part, we want to analyse the performance of the RLARP model with and without the presence of the capacity constraints. To do so, we use Figure 5.4 and Figure 5.5 which the former belongs to the case of having 6 potential facilities and the latter is for 4 potential facilities.

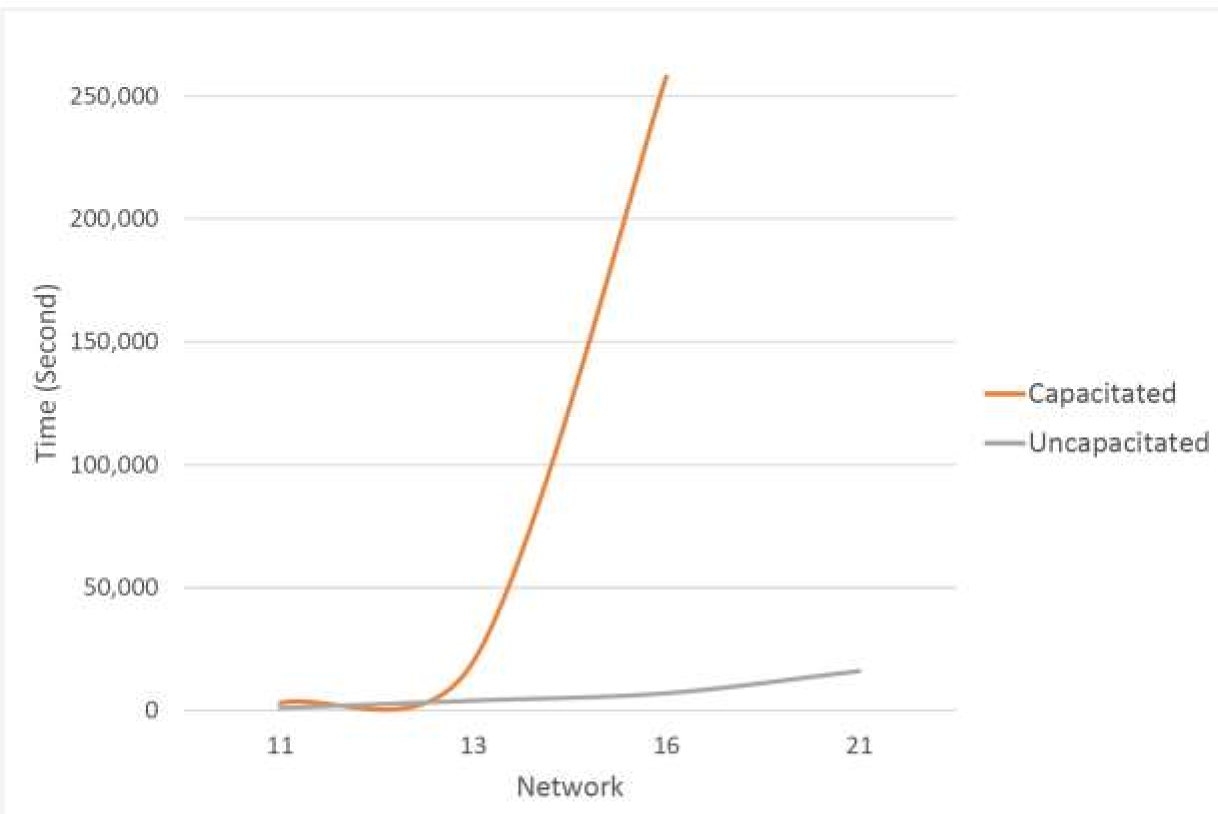

Figure 5.4: Performance Comparison of Capacitated vs. Uncapacitated Considering 6 Potential Facilities in RLARP Model.

Both graphs show that there has been a sharp rise for capacitated RLARP model when enlarging the network more than 13 nodes-52 links, while uncapacitade RLARP model has a steady increase. Also, it can be observed that in both cases the ucapacitated RLARP model reaches the solution in a way less computational time especially for larger instances than 13 nodes.

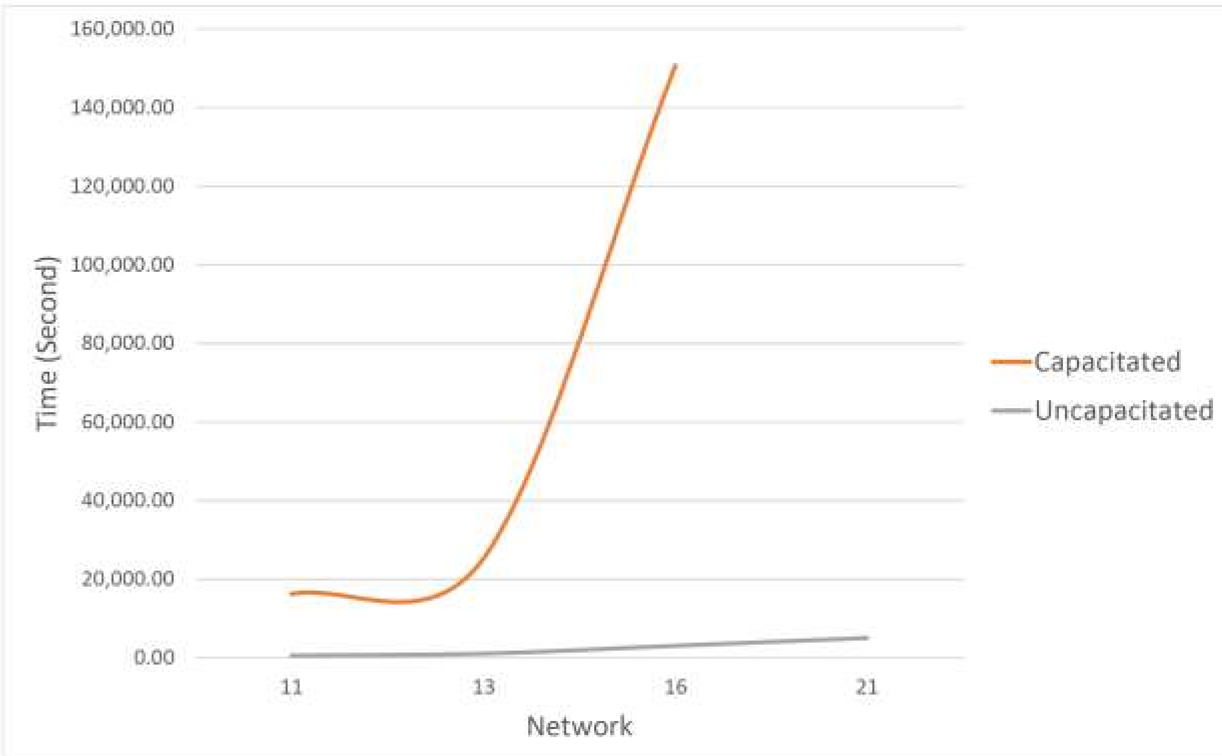

Figure 5.5: Performance Comparison of Capacitated vs. Uncapacitated Considering 4 Potential Facilities in RLARP Model.

5.2.2. Result of CG based model

To have a recall, CG models consists of two sets of problem, master problem and pricing problem. As it was explained before in Chapter 4, in each iteration the restricted master problem and a set of pricing problems (i.e., one per facility location) are solved as we used a parallel CG strategy. It should be mentioned that in CG model which is the reformulation of the RLARP model by CG technique, we consider the same assumptions as RLARP model which in this chapter is named Uncapacitated RLARP. Therefore, in CG model we do not take into account the capacity constraint for each potential facility. It is worth mentioning that we programme the CG approach in JAVA and use CPLEX as the popular optimization solver.

To be able to evaluate the CG model which is our main contribution in the thesis, we solve it with different data sets from 11 nodes-44 links to 38 nodes-139 links as indicated in Table 5.3. In addition, for each network we consider 4 and 6 potential facilities to be able to later analyse the effect of such a change in the performance of the model.

Table 5.10: Results for CG based Model.

Nodes	Links	Customers	Facilities	P	Obj (ILP*)	Time (Second)
11	44	7	4	3	35,063.70	3.79
11	44	5	6	3	30,320.57	3.50
13	52	9	4	3	45,154.85	6.48
13	52	7	6	3	37,438.25	6.78
16	66	12	4	3	55,789.11	12.08
16	66	10	6	3	40,333.93	21.27
21	80	17	4	3	73,589.73	55.97
21	80	15	6	3	57,407.25	219.27
26	102	22	4	3	85,830.38	154.75
26	102	20	6	3	74,845.21	326.54
38	139	34	4	3	184,699.09	1,145.12
38	139	32	6	3	150,461.37	2,652.85

Looking at the table, it can be clearly observed that, by increasing the size of the network (i.e., number of nodes and links) the computational time raises for both cases of 4 and 6 potential facilities. As a result, we conclude that the size of the network has a direct impact on the computational time and increasing it results in the hike of computational time.

5.2.2.1. 4 vs. 6 Potential Facilities

Here we want to figure out how the number of potential facilities have an impact on the performance of the model. To do so, Figure 5.6 indicates the computational time trend for the case of having 4 and 6 potential facilities independently. As it can be clearly observed, the computational time in case of considering 4 potential facilities is less than 6 potential facilities which is the same as what was observed in Uncapacitated RLARP model. Therefore, one possible explanation behind such a behaviour is that in the CG model for each potential facility we need to solve one more pricing problem in each iteration. By taking into account that in our problem solving one PP is more time consuming than solving a RMP, it justifies why increasing the number of potential facilities (6 potential facilities-10 customers) for an identical network (16 nodes-66 links) has raised the computational time more than increasing the number of customers (4 potential facilities-12 customers). For instance, in a network of 16 nodes-66 links, in case of having 6 potential facilities-10 customers the computational time is 21.27 seconds, while in 4 potential facilities-12 customers the solving time is 12.08 seconds which is taking almost half as long as 6 potential facility's case.

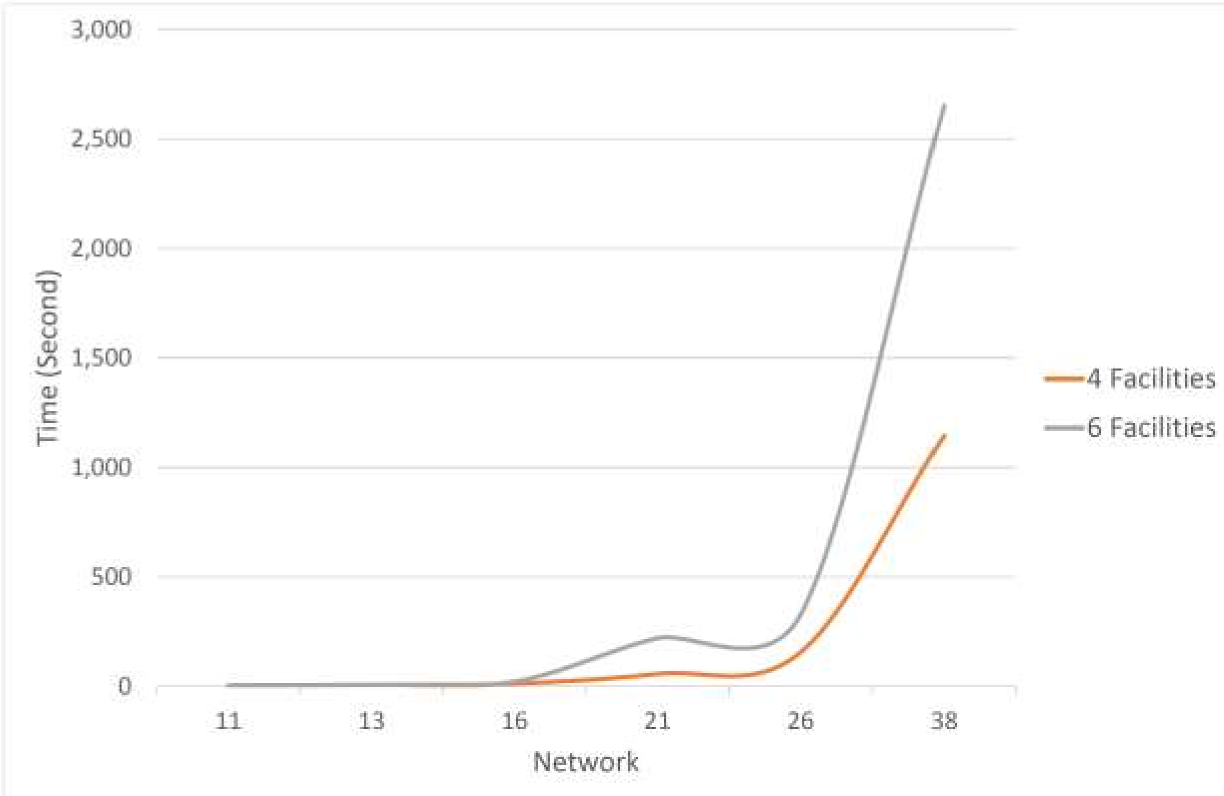

Figure 5.6: Comparison of 4 VS. 6 Potential Facilities in CG Model.

5.2.3. Comparison of CG vs. RLARP Model

The following part of this thesis moves on to compare the performance of the CG and RLARP model. To do so, Figure 5.7 demonstrates the performance of the RLARP and CG model in terms of computational time over different size of networks which are denoted in the figure by the number of their nodes. As it is expressed in Figure 5.7, in general the time required to reach a solution in CG model is much less than RLARP model. It is worth mentioning that in RLARP model as indicated in Table 5.5 when we go above 16 nodes-66 links the program cannot reach the optimal solution, in contrast to the CG model that is capable of reaching optimal solution for the network of 38 nodes-139 links. Also, the figure reveals that there has been a sharp increase in computational time of RLARP strategy after passing 16 nodes-66 links, while CG based model faces the hike after 26 nodes-102 links. It seems that these aforementioned results are mainly due to the capability of CG technique in which not all variables and columns will be taken into account simultaneously. Therefore, in comparison with RLARP model, it has the ability to handle larger instances with better performance.

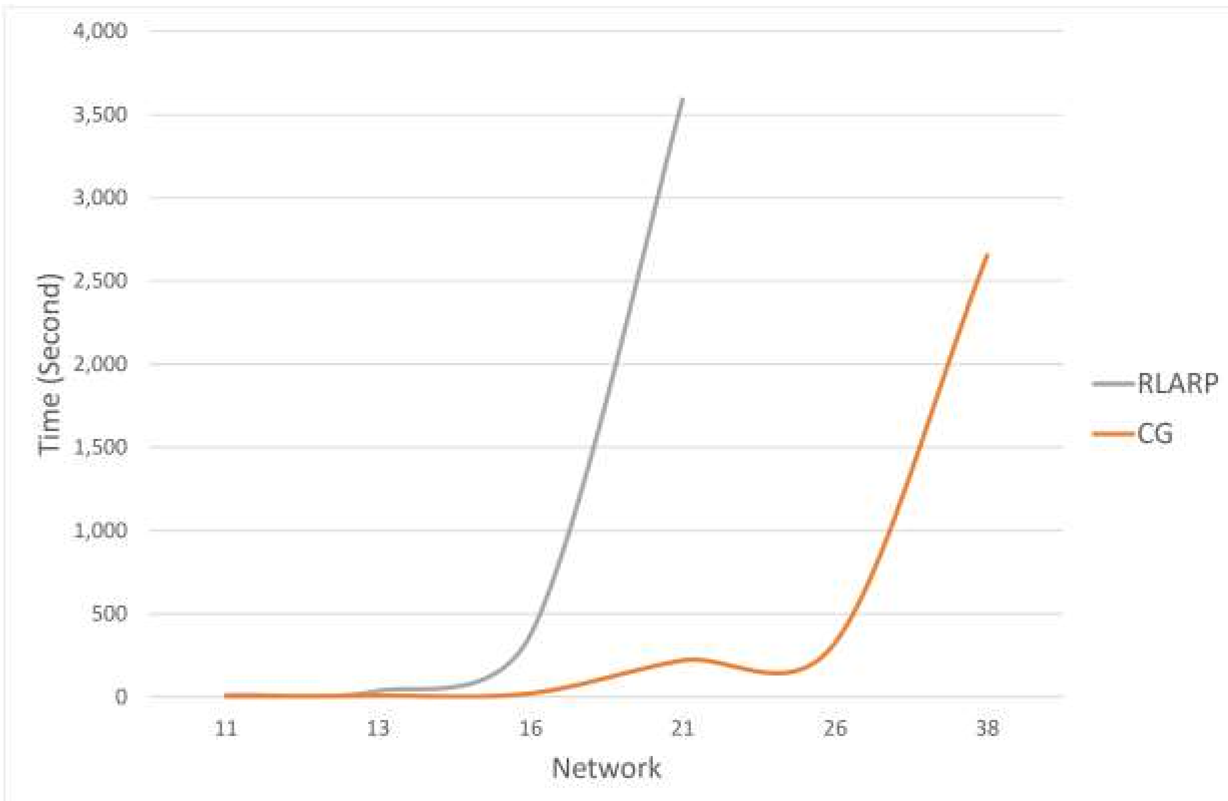

Figure 5.7: Performance Comparison of RLARP and CG based Model.

Chapter 6

Conclusion and future research

In the next chapter, we will first present the principal findings of the thesis in Section 6.1 and later discuss the possible future research that can be done in Section 6.2.

6.1. Conclusion

To recall, this thesis studies on the area of reliable network by taking into account the interdiction. As it was mentioned earlier the interest in the aforementioned area has achieved a lot of attentions in the recent years. The main reasons behind such an increase can be summarized as below:

- Recent conspicuous destructive incident
- Failure of a lean concept at the time of unpredictable changes

In this thesis, we focus on the Reliable Location/Allocation/Routing Problem in the design level and proposed a RLARP model with the objective of minimizing the worst interdiction cost as well as fixed opening cost of potential facilities. As it was reported in

Chapter 5 we tested the model with different size of data sets from 11 nodes-44 links to 21 nodes-80 links.

According to our observations we can conclude that due to the complexity of the model, it cannot perform well when enlarging the network size more than 16 nodes-66 links. Therefore, to tackle this issue, we used column generation decomposition technique as a solution methodology to reformulate the RLARP model as the Column Generation (CG) model. It is worth mentioning that the two formulation, RLARP and CG, are identical, however with a significantly different performance in terms of computational time and the size of the problems that they can solve.

Considering all proposed results and analysis, we conclude that CG technique reduces the complexity of the model. The logical explanation behind this is CG technique decomposes the model into two parts and add the promising configurations (i.e., columns) in each iteration rather than considering all at once in one model. That is the reason why the CG model has the capability of solving the instances twice as large as what RLARP model can handle. In addition, it should be mentioned that the Column Generation model reaches the optimal solution significantly faster in all instances.

6.2. Future Work

The following is a description of the future research that can be done as the extension of this thesis:

- Partial interdiction should be considered in the network. As it was mentioned earlier in other chapters, failures can be either complete or partial. It should be mentioned that the network does not always face a complete interdiction. To clarify more, one part of a facility may fail while others work which means they cannot work with their full capacity.
- Capacity could be taken into account in our CG model as it is a very critical constraint.
- Better validating the model with a real case study.
- Taking into account stochastic failure sets in the model.
- Fortification can be taken into consideration in this model. As explained before, one way of tackling interdictions in the supply chain networks is to fortify the elements of the network. This strategy is an option mostly for the built networks' elements as relocation of them is almost impossible considering the cost of it. However, depending on the budgetary limitation as well as the area of studies, fortification of the network can be taken into consideration in the design level.

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