

CONCORDIA UNIVERSITY

Energy Optimization of a Cellular Network with QoS Guarantee

by

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“Anyone who stops learning is old. Whether at twenty or eighty. Anyone who keeps learning stays young. The greatest thing in life is to keep your mind young.”

Henry Ford

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Abstract

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The problem of energy optimization in a cellular network has been studied from different perspectives: sleep patterns, network interference, association of users and base stations, allocation of resources (bandwidth and power), etc. All these aspects have been discussed individually in previous works. However, none of the existing works has succeeded in proposing an accurate solution to an exact mathematical model that takes into account several of these parameters simultaneously.

In this work, we propose a modeling of several network parameters (i.e., base station transmission power and activity, user assignment, bandwidth allocation, interference, and data rate) and their interaction in order to minimize the network energy consumption. An exact solution of the proposed model, which solves the energy optimization problem in an LTE network, is presented afterward. The proposed solution guarantees to satisfy all the users with a minimum quality of service (data rate), assuming a proper call admission mechanism has been put in place. The accurate solution allows energy savings of up to 50% in a moderately loaded network, which leads to energy savings of up to twice than of the heuristic proposed by Piunti *et al.* (2015). Moreover, a strategy to solve the model using column generation method is proposed and investigated. Various numerical results are presented on hexagonal and randomly generated cellular networks.

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Abbreviations

3GPP	3rd Generation Partnership Project
BS	Base Station
CAGR	Compound Annual Growth Rate
CG	Column Generation
EOCN	Energy Optimization of a Cellular Network
GHG	GreenHouse Gases
GPS	GlobalPositioning System
ILP	Integer Linear Program
MIQP	Mixed Integer Quadratic Programming
PP	Pricing Problem
PRB	Physical Resource Block
RMP	Restricted Master Problem
QoS	Quality of Service
LP	Linear Program
LTE	Long Term Evolution
SINR	Signal to Noise plus Interference Ratio
UE	User Equipment

Chapter 1

Introduction

1.1 Background

Global mobile data traffic was increased by up to 75% in 2016 and is expected to keep growing at a Compound Annual Growth Rate (CAGR) of 42% by 2022. The number of mobile devices and connections worldwide is expected to reach 11.6 billion by 2020, which will be approximately 50% more than the world population in 2020 [1]. This includes not only the 8.2 billion personal portable or mobile devices but also 3.2 billion M2M (machine-to-machine) connections (e.g., Global Positioning System (GPS) in cars, goods tracking systems in shipping and manufacturing sectors, medical applications keeping patient records and health status). North America is experiencing the fastest growth of mobile devices and connections with a 22% CAGR between 2015 and 2020 [2]. This is an indication of the gigantic size of mobile communications industry.

This growing number of mobile devices leads to a continuous increase in cellular networks energy consumption. A typical cellular network consists of three main elements: the core network (interface to fixed network), base stations and mobile terminals. At present, most of the energy in mobile networks (up to 80%) is consumed by base stations [4]. There is an increased number of installed base stations worldwide and, as a consequence, there is a significant growth in the total energy consumed by mobile networks [5, 6]. Currently, there are more than 4 billion base stations installed worldwide, each consuming an energy equal to two average households.

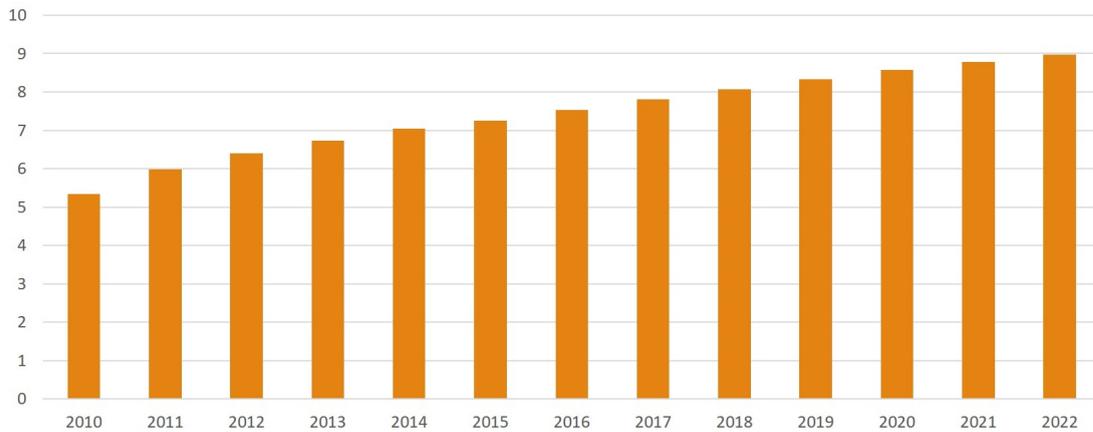


FIGURE 1.1: Total number of global mobile devices (billion). (Statistics from Ericsson mobility report, 2017 [3]).

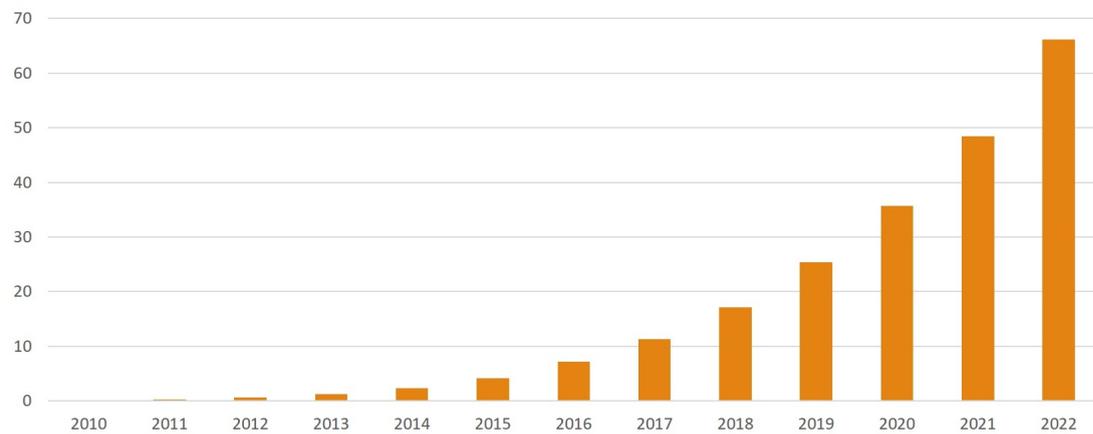


FIGURE 1.2: Global mobile data traffic (exabytes/month). (Statistics from Ericsson mobility report, 2017 [3]).

Base stations in cellular networks are often underutilized. While the load profile and network traffic exhibit large variations between peak and off-peak values (with long periods of low load), network operators plan their deployment with respect to peak traffic usage [7, 8]. Operators are more concerned about the Quality of Service (QoS) and data rates offered to the users and care less about the network energy consumption [9]. Since the major base station power consumption is not load proportional, it is difficult to achieve energy efficiency under low loads.

1.2 Sleep modes and energy optimization problem in cellular networks

Sleep mode is considered as one of the most powerful approaches to minimize the energy consumption of a cellular network [10, 11]. The proposition is to put weakly loaded base stations into sleep mode or discontinuous transmission (DTX) mode during the off-peak hours. In a sleep mode, a base station serves no user and consumes a minimum amount of energy. In other words, base station capacity should be available only when and where it is needed; it does not need to be available all the time [12].

However, switching off base stations results in increased interference as well as larger distances between users and serving base stations [13]. Base stations sleeping algorithms and strategies must consider the QoS of the users in the network. When a base station is switched off, the neighbouring base stations must provide coverage over the region which is no longer covered by the switched off base station. On the other hand, increasing the energy of neighbouring base stations results in higher interference. To summarize, interference, coverage, available channels and user assignment problems may conflict and must be studied jointly to obtain an energy/performance trade off [14]. The ultimate goal is to redirect sufficient power over an adequate number of channels so that mobile operators can always guarantee the required QoS and data rate to the users.

1.3 Contribution of this thesis

In this work, an optimization model and its *accurate* solution is proposed to minimize the power consumption of an LTE cellular network while guaranteeing a minimum data rate for each mobile user. Taking interference into consideration, the proposed optimization framework finds a solution for user - base station assignment problem in addition to power and bandwidth allocation problems, assuming a proper call admission mechanism has been put in place. The optimization model has been solved exactly for LTE networks with up to 20 base stations and 450 users. For a given traffic, it outputs the best set of active base stations to serve the users, the user assignment and the resource allocation,

while guaranteeing each user with the minimum required data rate. Moreover, applying column generation method is investigated to solve larger instances of the problem.

In addition, a paper was published and presented at the 15th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt) in March 2017 in Paris, France as a result of this thesis [15].

1.4 Thesis plan

The thesis is organized as follows. Chapter 2 reviews the problem background. Previous related studies are reviewed in Chapter 3. Chapter 4 gives the detailed statement of the problem, the newly proposed network optimization model and the solution process. Moreover, the heuristic of Piunti *et al.* (2015) [12] is briefly recalled. In Chapter 5, the use of column generation to improve the scalability of the model is investigated. Numerical results, a comparison with the heuristic of [12], and sensitivity analysis of an LTE network are presented in Chapter 6. Conclusions are drawn in the last Chapter.

Chapter 2

Background

In this Chapter, we review the background of the energy optimization problem in cellular networks, terms, and networking concepts which are necessary to proceed with the rest of this thesis. We start by an overview of cellular networks and their energy consumption problem. Next, the sleep mode strategy to overcome this issue is introduced and the associated challenges are discussed. Finally, the interference, data rate and quality of service in a mobile network is reviewed.

2.1 Cellular networks

A cellular network is a wireless communication network that provides coverage over a geographic area by dividing the land into small cells. Mobile User Equipment (UE) in each cell is covered by a fixed location transceiver known as a Base Station (BS). The most common example of a cellular network is the mobile network where a wide geographic area (e.g., a city) is divided into smaller cells so that each mobile user can receive or make a call through a base station. Dividing a huge area into small cells enables us to overcome the problem of scarce frequency resources. As a result, we can re-use the same frequency a few cells away from each base station. Thanks to the propagation properties of radio waves, the same frequency can be safely reused by different base stations without causing destructive interference if the proper distance is maintained between the transmitters that are using the same waveform.

Normally, each UE receives transmitted signal from more than one base station. In this situation the UE connects to the base station with the strongest received signal power. This identifies the coverage region of each base station, which is referred to as a cell. Since the received signal power decreases with the distance, it can be proved that the strongest signal comes from the base station with the least geographical distance. However, there exist cases (i.e., there are too many users close to a single base station) where not all the users can be covered using this strategy.

2.2 Long term evolution (LTE)

Long Term Evolution (LTE), first deployed in 2009, represents the fourth generation of cellular network technologies. It is a standard developed by 3rd Generation Partnership Project (3GPP) for high-speed communication of cellular networks. 3GPP is a collaboration between major telecommunications standard development organizations for different communication networks including cellular networks [16].

LTE employs advanced and efficient technologies to support high speed data connection both in up-link and down-link. It can provide data rates up to 75 Mbps in the up-link and 300 Mbps in the down-link [17]. Currently, less than 35% of mobile devices in the world support LTE connection [3]. However, the number of LTE users is growing and it is anticipated that LTE will become the dominant standard by the end of 2018 (Figure 2.1).

The communication channel in a cellular network (air) is shared among all users. In an LTE system, several advanced technologies are deployed to allow users to communicate over the shared channel. The main strategy is to divide resources (time and frequency) into small slots and assign each user a separate slot. That is, at a specific time slot and using a specific frequency, only a single user is communicating. Going over channel access technologies in details is beyond the scope of this work ¹.

¹In the downlink, LTE uses orthogonal frequency division multiple access (OFDMA) and in the uplink, single carrier-frequency division multiple access (SC-FDMA). Refer to [18] to read more on channel access methods in an LTE system.

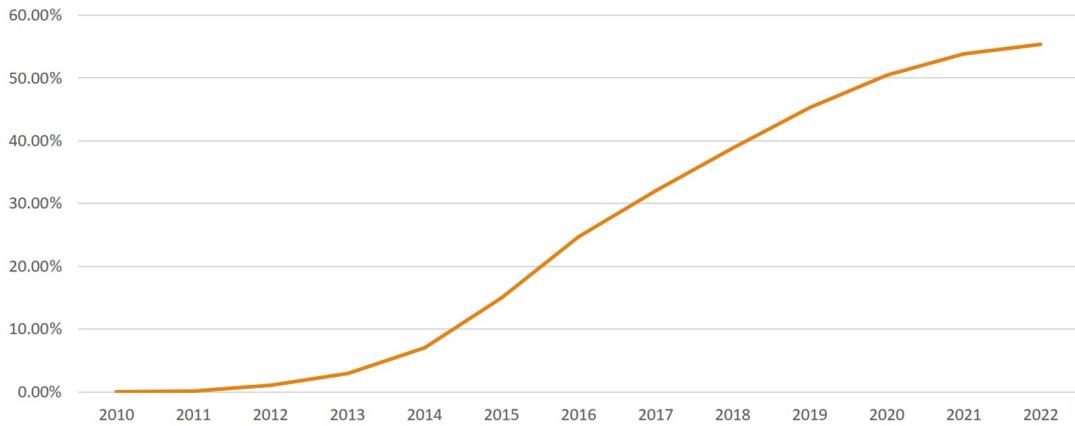


FIGURE 2.1: Global LTE subscribers portion: it is anticipated that LTE will become the dominant standard by the end of 2018. (Statistics from Ericsson mobility report, 2017 [3]).

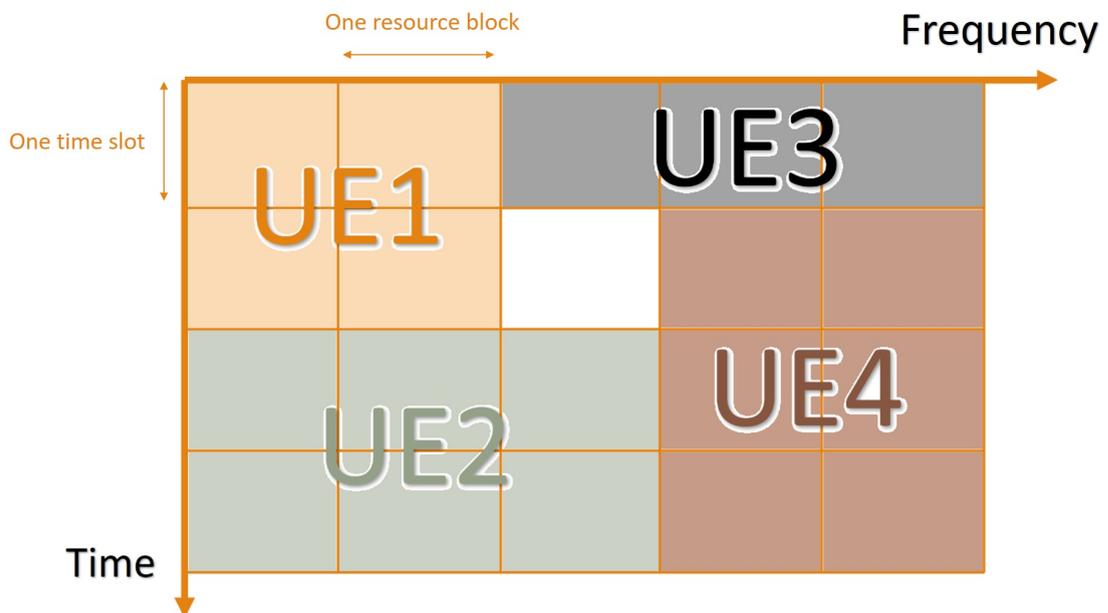


FIGURE 2.2: Physical resource block (PRB) and multiple channel access in an LTE system. A PRB cannot be shared among multiple users.

In an LTE system, the spectrum and the time is divided into smaller segments. Each segment is called a Physical Resource Block (PRB). A PRB is a resource unit having both time and frequency dimensions. This division of resources allows multiple users to communicate over a shared channel. Each user will be assigned one or more PRBs for communication, while a PRB cannot be shared among multiple users (Figure 2.2).

The number of available bandwidth blocks depends on the system bandwidth. In an LTE system, the number of PRBs varies from 6 to 100 (depending on the channel bandwidth

which varies between 1.4-20 MHz) [18].

2.3 Infrastructure energy consumption

When the first generation of wireless mobile telecommunications technology (1G) was introduced in 1981, the only purpose of a mobile network was to provide the possibility of making phone calls for the users. However, this did not last long and 3G systems, which were introduced in the early 2000, were designed to provide Internet data and web-based applications in addition to the ability of making phone calls between users. Today, the ability to access high speed Internet in a cellular network is more important than making phone calls for many of the mobile users.

As the system tries to increase the data capacity, it becomes more difficult to find enough spectrum. This problem can be overarmed by the cost of consuming more energy. In 2016, the number of global mobile devices and connections surpassed 8.0 billion, which is more than the world population [1]. Providing coverage for this growing number of mobile devices comes with the cost of enormously increasing energy consumption of mobile networks.

There are two major sources of energy consumption in a wireless network: transmission and computation. The transmitted energy depends on communication states, like QoS requirements, transmission channel characteristics, etc. The computation energy is the energy consumed by processing devices to perform signal processing and other computations. The energy consumed for computation depends on hardware, software, processing algorithms, etc. For long-distance communications (e.g., macro cell communication) the major energy consumer is transmission, while in short-distance communications (e.g., femtocell or WiFi) the major energy consumers are the processors.

In mobile networks, the energy is mostly consumed by base stations, both for communication and transmission (Figure 2.3). Since the number of available frequency blocks at each base station is limited, mobile operators have to increase the number of base stations to provide coverage for more users. If the number of users in an area is likely to increase significantly during the day (e.g., in a shopping mall or a transportation hub)

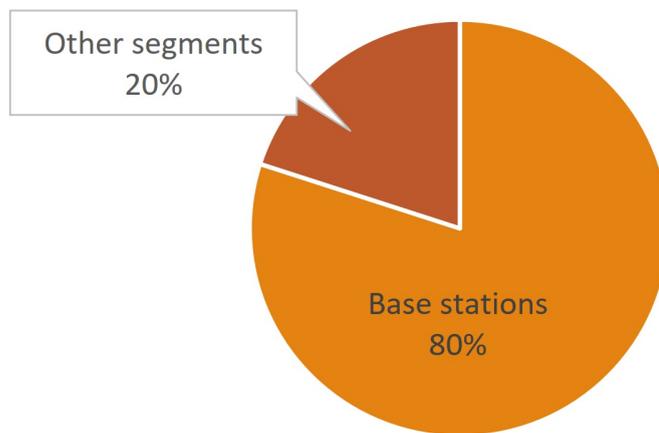


FIGURE 2.3: Energy consumption in a cellular network: up to 80% is consumed by the base stations

a usual macro cell is replaced with a number of smaller micro cells. Each microcell is serving a smaller area, and therefore, more users can be covered in total. Dividing a large cell into smaller cells does not stop at this level and in a similar way smaller cells, namely picocells and femtocells, are also used for different use-cases.

2.4 Green cellular networks

Any increase in energy consumption comes with an increase in emission of pollutants and greenhouse gasses (GHG - mainly carbon dioxide). Accumulation of GHG will speed up global warming and climate changes which cause natural calamities like typhoons, floods and changes in the sea levels. The information and communications technology (ICT) industry is responsible for about 2% of GHG emissions. China, USA and Europe union are major contributors causing almost 50% of of global GHG emissions [19]. Canada's share made up less than 2% of global GHG emissions [20]. U.S. government has recently announced its withdrawal from "Paris climate accord", which is an agreement within the United Nations framework convention on climate change dealing with greenhouse gas emissions mitigation. The U.S. government believes that this withdrawal would help American businesses and workers. This shows that a solution to GHG accumulation problem which does not cause big industries to lose their profit has a significant value.

A large number of studies in recent years have been focusing on green cellular networks. The term green in wireless networks usually refers to a network which consumes less energy and resource. Telecommunication industry has made a significant contribution to reduce CO_2 emissions in the recent years and one of the main objectives of the next generation of mobile network standard (5G) is to make the network more efficient in terms of power consumption and resource management.

2.5 Sleep modes

Current mobile networks are designed to satisfy the users needs during the peak hours and are underutilized most of the time. In order to reduce energy consumption of the network, a cell can be put into a sleep mode or discontinuous transmission (DTX) mode. While in a sleep mode, the base station covers no user and consumes less energy. In the deepest sleep mode the energy consumption is very close to zero. It is assumed that a cell can go to a sleep mode very quickly, while going back to the active mode takes a certain amount of time and energy based on the sleep mode level. The idea behind a lighter sleep mode is to deactivate only selected parts of the cell hardware. This makes the activation process faster [21]. In this thesis, we consider a single sleep mode.

Figures 2.4-2.6 illustrate the effect of base station sleeping on a cellular network. Figure 2.4 shows the UE distribution over a geographic area. This can be the center of a city with a shopping center located at the north east area. While this area is very crowded during the day, the network traffic can be very low during the night (i.e., when the shopping center is closed). By covering the whole area by only a few base stations and putting all other base stations into the sleep mode, energy consumption of network can be significantly reduced.

2.6 Base station energy consumption model

While in sleep mode the base station consumes a constant power (depending on the sleep level), its energy consumption depends on the load in the active mode. In general,

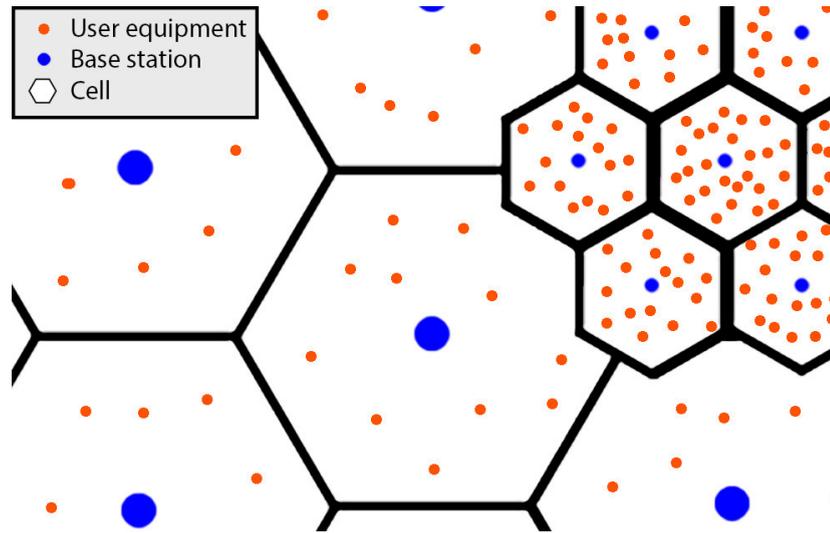


FIGURE 2.4: Sample user distribution during the peak hours

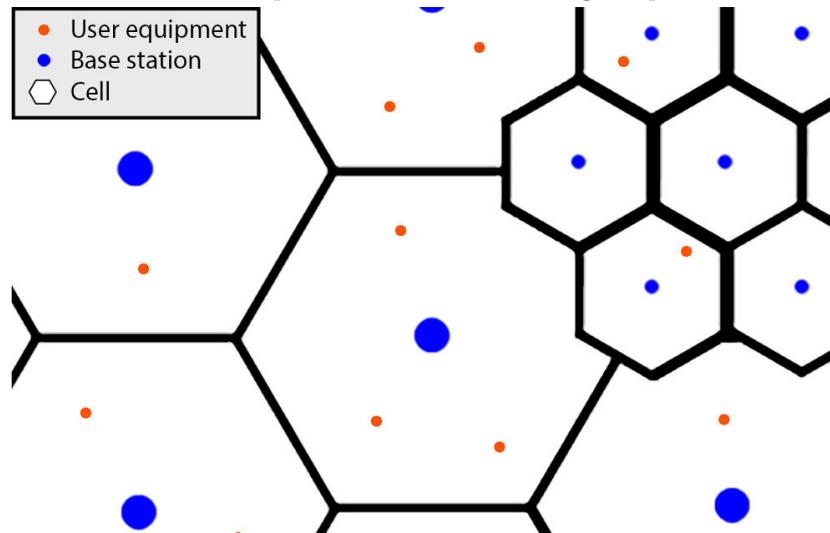


FIGURE 2.5: Sample user distribution during the off-peak hours

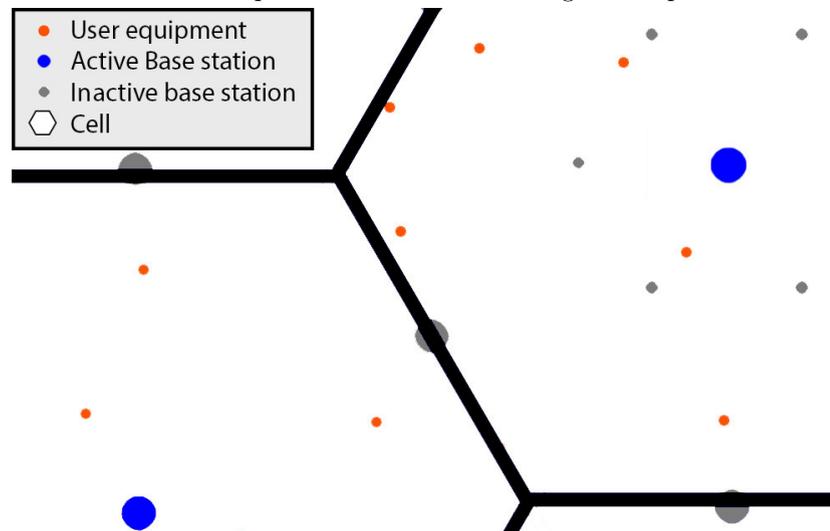


FIGURE 2.6: Turning off under-utilized base stations during the off-peak hours

energy consumption of a base station can be modelled as follows [22]:

$$P = \begin{cases} P_0 + \Delta_p \cdot P_{\text{OUT}} & 0 < P_{\text{OUT}} \leq P_{\text{max}} \\ P_{\text{SLEEP}} & P_{\text{OUT}} = 0, \end{cases} \quad (2.1)$$

where P_0 is the power consumption at the minimum non-zero output power, Δ_p is the slope of the load-dependent power consumption and P_{SLEEP} is the power consumption of the base station when in the sleep mode.

2.7 Interference

It was very easy to observe the interference in the old days when many AM radio stations were trying to attract the attention of the audience. Radio audience could sometimes hear an unwanted sound of another radio station in the background. This background sound was being sent by another radio station far from them that was inadvertently using the same frequency. Using limited resources extensively and simultaneously by many different signal transmitters naturally leads to a conflict like this. In some cases, the strength of the unwanted signal was so high that it would be possible to hear the background sound clearly, if it was alone. [18]

Interference is the power of signal received from unwanted transmitters (usually far from the user) which share the same bandwidth with the intended transmitter. Interference disrupts the receiving signal. From the receiver's point of view, interference modifies and disrupts the received signal similar to a background noise. This is a serious issue in mobile telecommunication as a huge number of base stations transmitting signal in different locations relatively close to each other. There are several techniques to overcome interference problem in a cellular network which is beyond the scope of this work. Our focus is on the effect of base station transmission power on the network interference.

2.8 Quality of service, SINR and achievable data rate

Quality of service (QoS) refers to the overall performance of the network, which depends on several characteristics, i.e., data rate, throughput, delay, etc. In this work, we focus on the data rate received by each user, as this is the only characteristic which is highly dependent on the base station transmission power. Using Shannon-Hartley theorem, maximum achievable data rate (channel capacity) can be calculated as follows [23]:

$$C = W \log_2 \left(1 + \frac{P}{WN_0} \right) \quad (2.2)$$

where $C[b/s]$ is the channel capacity (achievable data-rate), $W[Hz]$ is the bandwidth, $P[Watt]$ is the received power and $N_0[Watt/Hz]$ is noise power spectral density. Taking interference into consideration, it is concluded that

$$C = W \log_2 \left(1 + \frac{P}{WN_0 + I} \right) \quad (2.3)$$

where $I[Watt]$ is the interference.

The term $\frac{P}{WN_0 + I}$ is referred to as “Signal to Interference plus Noise Ratio”, or SINR. In wireless communications SINR is the power of received signal divided by the sum of background noise power and interference power. SINR is used to measure the quality of the connection in wireless communications. A higher value of SINR is an indication of a good quality channel with a clear transmitting signal and a weak noise and interference.

Channel capacity is the theoretical upper bound for information transmission over a communication channel. The actual data rate received by the user depends on other factors as well, most importantly the coding of information. However, sending information with transmission rates very close to the channel capacity is achievable in today’s telecommunication.

Chapter 3

Literature Review

Several studies have already looked into the issues pertaining to coverage, throughput, and energy trade-offs in cellular networks [11, 24]. Mobile networks are under utilized most of the time and the main goal of these set of studies is to take advantage of available network resources or improve energy efficiency of base stations to reduce the energy consumption of mobile networks. For instance, assigning free bandwidth blocks to the users at each base station can help reduce BS transmission power while maintaining the same data-rate for users [25].

A first set of studies relate to single cells, and investigate the radio transmission processes in order to reduce the energy consumption. Several parameters have been investigated, and numerous studies exist, see, e.g., [25–27]. More recently, studies have investigated beamforming and MIMO techniques [28, 29].

A second set of studies deal with multi-cell wireless networks, see, e.g., [30] for a rather recent survey. Green scheduling and power control for both classic and heterogeneous cellular networks are reviewed in [31]. For some recent studies related to 5G and MIMO technology, see, e.g., [29].

In most of existing studies, network parameters are investigated separately, i.e., energy consumption is optimized while the UE-BS assignment is known. In this thesis, we are interested in exact methods for energy optimization in LTE networks, taking all network parameters into consideration. While there have been many studies with decomposition

techniques for throughput and energy optimization for a single BS [32–34], there exists still no study with those techniques for LTE networks. This is explained by the fact that the problem resembles some location problems [35], for which column generation need to be combined with other techniques in order to be efficient.

Several works in wireless networks (e.g., see [33] and [34]) use column generation in energy optimization and power control problems to efficiently solve large problem instances. In these works the idea of Independent Sets (ISets) is combined with column generation to deal with SINR constraint and guarantee a minimum QoS for the users. The concept of transmission configurations (or independent sets, ISets for short) is defined as a set of links that can transmit simultaneously without violating SINR constraint. From an ISet, any combination of transmission configurations can be selected simultaneously without frustrating SINR constraint. The column generation model consists of a master problem and a pricing problem: the master problem chooses one ISet for each time frame and decides on selecting the best set of transmission configurations from the selected ISet. Since master problem chooses the configurations from an ISet, it does not need to worry about SINR constraint, as it is taken care by the pricing problem. The pricing problem generates a new ISet at each iteration. It plays the role of generator of transmission configurations such that, if added to the master problem, it improves the solution of its linear relaxation. In order to apply the idea of ISets to a cellular network problem, we need to consider possible transmission configurations instead of network links. A transmission configuration is a set of users assigned to a base station together with their transmission parameters. Then each ISet will be a set of transmission configurations that, if selected simultaneously, do not violate the SINR constraint. Therefore, the master problem can select any subset of transmission configurations from an ISet to cover all the users without being worried about the SINR constraint. However, unlike the total number of links in a wireless mesh network, which is linear to the number of nodes, the number of configurations in a cellular network is exponential to the number of users. Because of the exponential number of possible user assignments, the ISet generator (pricing problem) becomes another large scale problem and this decomposition does lead to a more efficient solution.

In this thesis, our focus is on the sleep mode, one of the classical mechanisms for energy

efficiency. Previous studies for LTE networks include the work of Zhu *et al.* (2014) [36], where a user association scheme based on cell sleeping has been proposed to reduce the energy consumption under a simplifying strategy of equal bandwidth scheduling.

In [12], Piunti *et al.* (2015) proposed an iterative heuristic method for the cellular network configuration with QoS guarantee. The proposed solution determines the user association, bandwidth allocation, identification of active base stations and their transmission power. The iterative method is based on a Mixed Integer Quadratic Programming (MIQP) model that decides on BS-UE assignment together with the base stations activity. User bandwidth, and the transmit power of each active base station are then determined using different algorithms. The algorithm iterates until all the users are satisfied with QoS requirements (see Section 4.3 for more details). However, this iterative solution does not necessarily provide an optimum solution, mainly for two reasons: first the QoS constraints are omitted from the MIQP model, and second, resources allocation is done in a second phase and MIQP does not have the flexibility to decide on resource allocation problems. The accuracy of the results provided by this solution decreases as the size and traffic of the network growth and the network interference increases, see Section 6.2.

In this thesis, we propose a model that minimizes the power consumption while taking the interference constraints accurately into consideration using a SINR modelling. The output of the model consists of the BS-UE assignment, the value of the transmission power of each base station, the selection of the base stations to be put in sleep mode and the bandwidth assigned to each user. This is the first work in the literature that finds the optimal user-base station assignment and resource allocation, while guaranteeing to satisfy each user with a minimum data rate in an LTE network.

Chapter 4

A First Model and its Solution

In this chapter, we define a mathematical model for the problem of energy optimization in an LTE network. The heuristic solution of Piunti *et al.* (2015) [12] is reviewed after. Then an exact solution is proposed to solve the proposed optimization model.

4.1 Statement of the EOCN problem

For a given LTE network, which is a given distribution of users and base stations over a geographical area, the EOCN problem aims at minimizing the total energy consumption of the network. The goal is to provide network coverage for all users but use only the minimum energy required by network infrastructures.

We have three main resources in an LTE system: energy, time and spectrum. Energy is modeled directly into the problem. Time and spectrum are modeled using PRB concept which has both dimensions (as described in Section 2.2).

The EOCN problem decides on:

- The set of active base stations. Any base station which is not active will be put on the sleep mode.
- BS-US association: assigning each user to an active base station.

- Resource allocation
 - Power assignment: determining the power transmitted to each user from its serving base station.
 - Physical resource (PRB) allocation: giving each mobile user a specific number of physical resource blocks to receive data from the base station.

In the solution output by the model, all users must be covered by a base station and satisfied with the minimum required data-rate. Moreover, allocated resources at each base station cannot exceed the maximum available limit.

4.2 EOCN optimization model

We define a new mathematical model for the EOCN problem.

Parameters

We denote by B the set of base stations, indexed by b , and U the set of users, indexed by u , assuming their locations to be distributed over a given area. Following are the sets of variables and constants used in the optimization model.

Constants

- $\sigma_{bu} \in \mathbb{R}^+$: channel gain between base station b and user u . It mainly depends on the $b - u$ distance and determines the reduction of signal strength during transmission over channel. The power received by the user is equal to the power transmitted by the base station multiplied by the channel gain between user and base station.
- $N_0[dBm/Hz]$: noise power spectral density. This is the noise power per unit of bandwidth.
- $W[Hz]$: total available bandwidth at each base station.
- $P_{\max}[W]$: base station transmission power limit.

- $r_u[\text{bit/s}]$: minimum required data rate by each user u .
- $p_{\min}[W]$: user sensitivity. It is the minimum signal power each user equipment needs to receive in order to be able to operate.
- N_{prb} : number of available PRBs at each base station. The total available PRBs at each base station is divided to N_{prb} equal resource blocks.
- $P_0[W]$: base station power consumption at the minimum non-zero output power
- $P_{\text{SLEEP}}[W]$: power consumption of the base station when in the sleep mode
- Δ_p : base station slope of the load-dependent power consumption

Decision variables

Following are the EOCN problem decision variables.

- $a_{bu} \in \{0, 1\}$: expresses the coverage association between base stations and users. $a_{bu} = 1$ if the user u is assigned to base station b and $a_{bu} = 0$ otherwise.
- w_{bu} and p_{bu} are resource allocation variables:
 - $w_{bu} \in \mathbb{Z}^+$: number of bandwidth blocks (PRBs) allocated to user u by base station b .
 - $p_{bu}[W] \in \mathbb{R}^+$: power transmitted to u by base station b .
- $I_b \in \{0, 1\}$: activity binary variable for each base station. $I_b = 1$ if base station b is active and $I_b = 0$ if b is in sleep mode.

Achievable data rate

One of the most important requirements the optimization model needs to satisfy is the data rate constraint. Using (2.3) data rate received by each user can be calculated using the following.

$$C = W \log_2 \left(1 + \frac{P}{WN_0 + I} \right). \quad (4.1)$$

We assume that the coding is efficient and the received rate is very close to the channel capacity. This is a reasonable assumption in today's mobile communication systems. Using our notation, we can rewrite the channel capacity formula to find the user data rate as the following.

$$r_u = \sum_{b \in B} a_{bu} w_{bu} \log_2(1 + \text{SINR}_{bu}),$$

where r_u is the received data rate. SINR can be calculated using:

$$\text{SINR}_{bu} = \frac{p_{bu} \sigma_{bu} a_{bu}}{\frac{w_{bu}}{W} \left(\sum_{b' \in B} P_{b'} I_{b'} \sigma_{b'u} (1 - a_{b'u}) + WN_0 \right)} \quad (4.2)$$

where $P_b = \sum_{u \in U} p_{bu}$ is the total power being transmitted by b .

Objective function

The objective function is written as follows:

$$\min \sum_{b \in B} (\Delta_p \cdot P_b + P_0) I_b + P_{\text{SLEEP}} (1 - I_b). \quad (4.3)$$

The goal is to minimize the global power consumption of the network. Δ_p , P_0 and P_{SLEEP} are base station power consumption parameters explained in Section 2.6. P_b , which is the total transmission power of base station b (the sum of p_{bu}) is defined to simplify the notation.

Constraints can be expressed as follows:

$$\sum_{b \in B} a_{bu} = 1 \quad u \in U \quad (4.4)$$

$$\sum_{b \in B} a_{bu} w_{bu} \log(1 + \text{SINR}_{bu}) \geq r_u \quad u \in U \quad (4.5)$$

$$a_{bu} \leq \frac{p_{bu} \sigma_{bu}}{p_u^{\min}} \quad u \in U, b \in B \quad (4.6)$$

$$\sum_{u \in U} p_{bu} \leq P_{\max} \quad b \in B \quad (4.7)$$

$$\sum_{u \in U} a_{bu} w_{bu} \leq N_{prb} \quad b \in B \quad (4.8)$$

$$\varepsilon \sum_{u \in U} a_{bu} \leq I_b \leq \sum_{u \in U} a_{bu} \quad b \in B \quad (4.9)$$

$$p_{bu} \geq 0, a_{bu} \in \{0, 1\}, w_{bu} \in \{1..N_{bb}\} \quad u \in U, b \in B \quad (4.10)$$

$$I_b \in \{0, 1\} \quad b \in B. \quad (4.11)$$

All users must be covered and satisfied. Constraint (4.4) forces each user to be assigned to one, and exactly one, base station. Constraint (4.5) is the bit-rate constraint: it guarantees that each user is satisfied with its minimum required data rate. Note that the minimum required data rate, r_u , depends on each user, so that different users can be provided with different data rates. Constraint (4.6) provides each user with a minimum power, which should be greater than or equal to the user sensitivity. Constraint (4.7) is the upper bound constraint for the transmission power of each base station. Constraint (4.8) enforces a limitation on the maximum number of bandwidth blocks each base station can provide. Constraint (4.9) determines the activity of the base stations: each base station is active if it is serving at least one user. If no user is assigned to a base station, it will be in sleep mode. Note that ε has to be small enough to limit the value of $\varepsilon \sum_{u \in U} a_{bu}$ to a value always smaller than 1. This constraint forces the activity of base station b if the value of $\sum_{u \in U} a_{bu}$ is not zero.

4.3 Heuristic solution of Piunti *et al.* (2015)

In this section, we briefly recall the model and the `MinPowerQos` algorithm proposed by Piunti *et al.* [12]. Therein, the authors propose a heuristic to solve the EOCN problem. An iterative algorithm is used to deal with the non-linear bitrate constraint and to find a solution to the EOCN problem, see [12] for more details. We will compare our exact solution with the heuristic of [12] in Section 6.2.

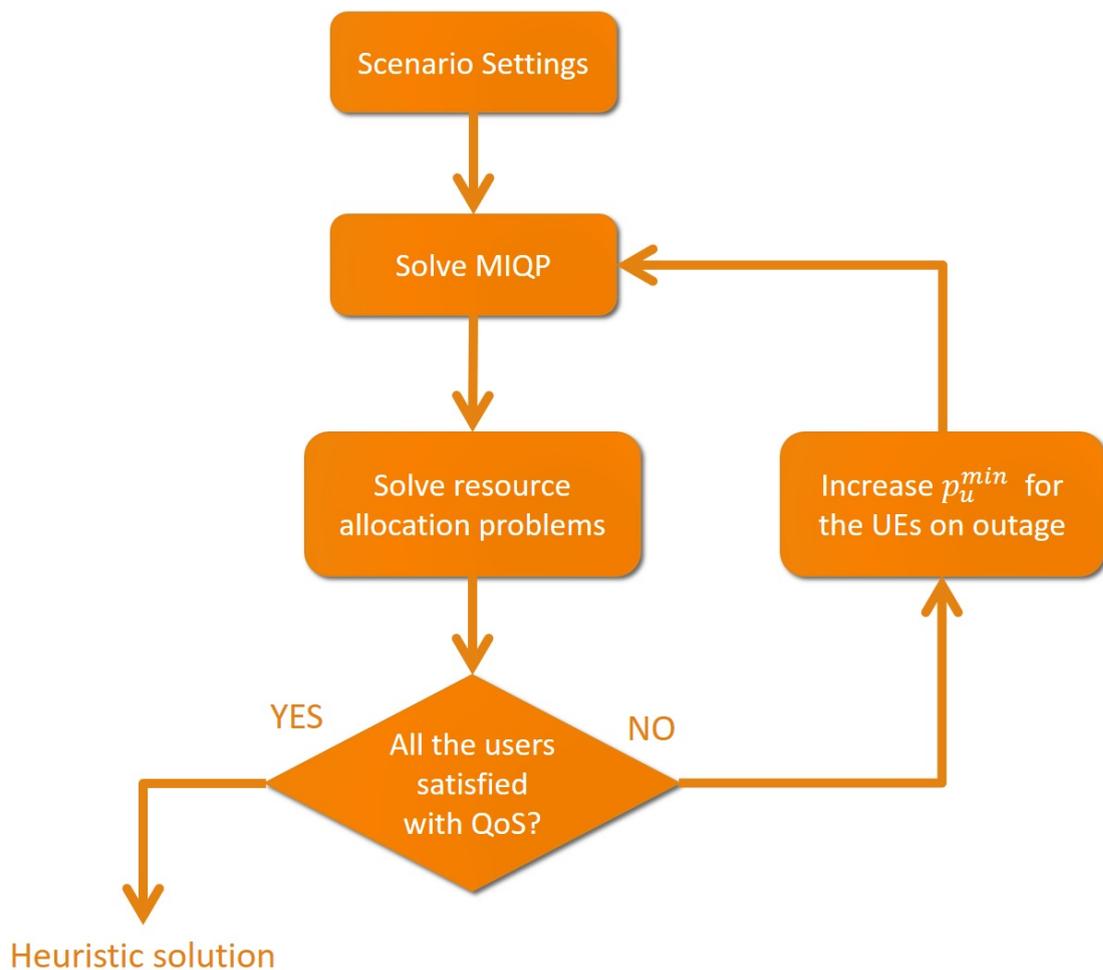
In the solution of Piunti *et al.*, a Mixed Integer Quadratic Program (MIQP) is defined initially which is a simplified version of the EOCN model. The bandwidth assignment and bit rate constraints from the EOCN optimization model are removed, i.e., constraints (4.5) and (4.10) of EOCN model are omitted and constraints (4.8) are replaced by the following ones:

$$\sum_{u \in U} a_{bu} \leq N_{prb} \quad b \in B. \quad (4.12)$$

This MIQP decides on BS-UE assignment only. Since the MIQP does not include the non-linear constraint, i.e., constraint (4.5), it can be solved using an MIQP solver. The provided solution determines the BS-UE association.

The bandwidth allocation and power assignment is not taken care by the MIQP model. Therefore, it must be done in a second step. Several heuristics are suggested in [12] for the bandwidth assignment. For example, the equal bandwidth assignment can be used: it divides the available bandwidth blocks at each base station equally among the assigned users. The next step is to solve the power allocation problem. Given a BS-UE association and a bandwidth assignment, the power control algorithm suggested in [12] provides the optimum BS transmission power. After applying power control to determine the transmitting power, the received users' data rate is calculated. For any user who is not satisfied with the minimum data rate, its minimum required power is increased and the MIQP is solved again. The algorithm iterates until all users are satisfied or there is no feasible solution to the MIQP.

Note that `MinPowerQos` does not necessarily provide an optimal solution. User-base station assignment and resource allocation are done in different steps and the assignment

FIGURE 4.1: MinPowerQos heuristic of Piunti *et al.*, 2015

```

while not all the users are satisfied do
  Solve MIQP and get a UE-BS association;
  Do the bandwidth and power allocation based on the UE-BS association;
  forall users do
    Calculate receiving data rate;
    if the user is not satisfied then
      | increase the minimum power required by the not satisfied user,  $P_u^{min}$ ;
    end
  end
end
Return UE-BS association, power and bandwidth allocated to each user.
  
```

Algorithm 1: MinPowerQos heuristic

step does not take the bandwidth limitations, interference and bit-rate constraint into consideration. Moreover, finding a feasible solution by the `MinPowerQos` heuristic is not guaranteed and the algorithm may fail to find a feasible solution (as the MIQP can become infeasible after a few iterations). It can be seen (Section 6.2) that the gap between the solution provided by the exact solution of EOCN model (see Section 4.4) and the solution of this heuristic iterative process is small when the network load is low. However, the gap increases as the network load and interference increases.

4.4 Exact solution of the EOCN problem

The bit-rate constraint (4.5) of the EOCN optimization model described in Section 4.2 is logarithmic and therefore is not linear. In order to deal with non-linearity of (4.5) and solve the EOCN model, an exact linearization is proposed in this section.

In order to perform the linearization, we define a set of constants $T^{r,w}$: for each data rate r and bandwidth w , $T^{r,w}$ is the minimum SINR required for a user who is assigned bandwidth w , and who requires data rate r .

Value of $T^{r,w}$ is given by the following formula:

$$T^{r,w} = 2^{\frac{r}{w}} - 1.$$

Following is the proof. For each user, if the received SINR is greater than $T^{r,w}$:

$$\begin{aligned} \text{SINR}_{bu} &\geq T^{r,w} = 2^{\frac{r}{w}} - 1 \\ \implies w \log_2(\text{SINR}_{bu} + 1) &\geq r \\ \implies r_u &\geq r, \end{aligned}$$

which shows that the bit-rate received by the user is greater than the required rate, r .

A new 0 – 1 variable x_{ub}^w is defined in order to select the right threshold for each user:

$x_{bu}^w = 1$ if user u is assigned the bandwidth w by base station b , 0 otherwise.

Using variable x_{ub}^w and constant $T^{r,w}$, constraints (4.5) can be equivalently rewritten:

$$\sum_{b \in B} a_{bu} \text{SINR}_{bu} \geq \sum_{b \in B} \sum_{w \in W} x_{bu}^w T^{r,w} \quad u \in U. \quad (4.13)$$

For each user, the left-hand side of (4.13) is the SINR that the user receives and the right-hand side is the minimum required SINR threshold.

Combining (4.2) with (4.13), and after performing some algebraic manipulations (manipulations can be found in the Appendix), constraint (4.13) can be equivalently rewritten:

$$\sum_{b \in B} \sum_{w \in W} \frac{x_{bu}^w \sigma_{bu} p_{bu} W}{T^{r,w}} - P_b \sigma_{bu} (1 - a_{bu}) \geq W N_0 \quad u \in U, \quad (4.14)$$

where $P_b = \sum_{u \in U} p_{bu}$ is the total power that base station b consumes.

The new variable x_{ub}^w needs to also satisfy the next two set of constraints:

$$\sum_{w \in W} x_{bu}^w \leq 1 \quad u \in U, b \in B \quad (4.15)$$

$$\sum_{u \in U} \sum_{w \in W} x_{bu}^w \times w \leq W \quad b \in B, \quad (4.16)$$

where constraints (4.15) force each user to be assigned at most one specific number of bandwidth blocks only, and constraints (4.16) guarantee that the total number of assigned bandwidth blocks at each base station does not exceed the number of available bandwidth blocks.

In other words, the logarithmic constraint (4.5) of the optimization model (4.4)-(4.11) is replaced by the above linear constraints (4.14), (4.15) and (4.16). Note that the proposed linearization is exact and does not alter the model constraints and limitations. The solution of the resulting model is therefore exact and the algorithm will be referred to as `OptPowQoS` in the sequel.

4.5 Concluding remarks

In this section, we defined an optimization model to minimize energy consumption of an LTE network. An exact solution is then proposed by linearizing the EOCN model. Solution obtained by solving the EOCN model includes network energy consumption, user base station assignment and resource allocation.

Chapter 5

A Column Generation Approach

5.1 Introduction

Column generation is a powerful tool for solving linear programs with a huge number of variables [37, 38]. In this chapter, we investigate on using column generation to overcome the scalability problem of the `OptPowerQos` solution suggested in Chapter 4. In this chapter, the EOCN problem is reformulated and a column generation solution is investigated to solve the EOCN problem.

We first review a related problem in operations research. The configuration concept is defined after, which is necessary to reformulate the EOCN problem. Then a new formulation of the EOCN problem is proposed and its solution using column generation technique is presented.

5.2 Capacitated p -median: a related problem

Given a set of potential locations for a set of facility centers and a set of demand points that must be serviced, the Capacitated p -Median Problem (CPMP) aims at locating p facilities at potential locations to minimize the sum of the distances from each demand point to its assigned facility center. In the capacitated p -median problem, each facility location has a limited capacity to service the demands.

The p -median problem is a well-known and well-investigated problem in operations research. Several heuristics and exact solutions have been proposed in the literature to solve the CPMP. Lorena and Senne (2003) [35] suggested a column generation approach to the capacitated p -median problem. The authors show that Lagrangean/surrogate relaxation can accelerate the column generation process by generating new productive set of columns. In [35], the authors identify Lagrangean/surrogate relaxation directly from the master problem dual. It is shown that the Lagrangean/surrogate relaxation is able to identify very good lower bounds and contributes with new columns that accelerate the column generation process. Please refer to [35] for more details.

The EOCN problem can be compared with the CPMP as each base station can be mapped to a facility center and each user can be mapped to a demand point. Although EOCN is more constrained, both problems seek to locate facility centers (base stations) close to the users. The major difference between the two problems is the complicated data rate constraint (4.5) of EOCN problem which depends on all other base stations and highly limits the feasibility area of the problem. Moreover, the CPMP objective is to minimize the sum of the distances between demand points and facility centers, while in EOCN problem the objective is to minimize the total cost associated with base stations. Despite the differences, CPMP solution ideas can be useful to find a solution to the EOCN problem.

In this chapter, we investigate on solving EOCN problem using the basic application of column generation. Improving the quality of generated columns by applying Lagrangean relaxation is beyond the scope of this thesis.

5.3 Configuration concept

In this section we define the concept of transmission configuration, which is needed to reformulate the EOCN problem. For a given base station b , a transmission configuration, denoted by γ , is a potential set of UEs assigned to b , together with their transmission

parameters. Let Γ denote the overall set of configurations:

$$\Gamma = \bigcup_{b \in B} \Gamma_b,$$

where Γ_b denotes the set of configurations associated with b .

Each configuration γ is associated with a base station b , and is characterized by:

- $a_u^\gamma \in \{0, 1\}$ with $a_u^\gamma = 1$ if u is assigned to b , and $a_u^\gamma = 0$ otherwise.
- $w_u^\gamma =$ bandwidth assigned to u by b .
- $p_u^\gamma =$ power transmitted to u from b .

5.4 Objective

In this section, we define a new mathematical model, for the EOCN problem. The EOCN_CG_ILP problem seeks to select a set of configurations consuming the minimum total energy so that the selected configurations provide all the users with network coverage having all constraints satisfied. We define a decision variable $z_\gamma = 1$ if configuration γ is selected (0 otherwise). At most 1 configuration will be selected from each base station, i.e., from each set Γ_b . If no configuration is selected from Γ_b , then b will be put to sleep mode.

Each configuration γ consumes the total power P_γ and each BS in sleep mode consumes P_{SLEEP} energy amount.

The objective of EOCN_CG_ILP optimization model can be written as follows.

$$\min \quad \sum_{\gamma \in \Gamma} P_\gamma z_\gamma + \sum_{b \in B} P_{\text{SLEEP}} (1 - \sum_{\gamma \in \Gamma_b} z_\gamma) \quad (5.1)$$

where:

$$P_\gamma = \Delta_p \sum_{u \in U} p_u^\gamma + P_0$$

is the total power consumed by configuration γ .

Constraints:

$$\sum_{\gamma \in \Gamma} a_u^\gamma z_\gamma = 1 \quad u \in U \quad (5.2)$$

$$\sum_{\gamma \in \Gamma_b} z_\gamma \leq 1 \quad b \in B \quad (5.3)$$

$$\sum_{\gamma \in \Gamma} a_u^\gamma w_u^\gamma \log(1 + \text{SINR}_u^\gamma) z_\gamma \geq r_u \quad u \in U \quad (5.4)$$

$$z_\gamma \in \{0, 1\} \quad \gamma \in \Gamma. \quad (5.5)$$

Constraints (5.2) guarantee that each user will be served by exactly one base station. With (5.3), at most one configuration will be assigned to a base station and constraints (5.4) guarantee the minimum bit-rate constraints for each user.

The set of all possible transmission configurations, Γ , grows exponentially with number of users. Therefore, the column generation model, EOCN_CG_ILP, is a large-scale model containing an exponential number of variables, z_γ . It is evident that complete enumeration of all these variables is not an option. We will explain in Section 5.6 how to solve this large scale model using an implicit enumeration of the variables.

5.5 Bit-rate constraint

Using the configuration notation, SINR calculation formula (5.4) can be re-written as:

$$\text{SINR}_u^\gamma = \frac{W \sigma_{bu} \frac{p_u^\gamma}{w_u^\gamma}}{\sum_{\gamma' \in \Gamma} P^\gamma \sigma_{b'u} (1 - a_u^{\gamma'}) z_{\gamma'} + W N_0}. \quad (5.6)$$

We again linearize constraint (5.6) the same way that we linearized the bit rate constraint (4.5) in Chapter 4. After performing the similar algebraic manipulations, we have the following.

$$\sum_{\gamma \in \Gamma} \left(\sum_{w \in W} \frac{b_{w,u}^{\gamma} \sigma_{bu} p_u^{\gamma} W}{T^{r,w}} - P^{\gamma} \sigma_{bu} (1 - a_u^{\gamma}) \right) z_{\gamma} \geq W N_0 \quad u \in U, \quad (5.7)$$

where $P^{\gamma} = \sum_{u \in U} p_u^{\gamma}$ is the total power that the configuration γ consumes. We then replace constraint (5.4) with (5.7).

5.6 Solving the LP relaxation with column generation

Solution of the model described in previous section by enumerating all potential configurations would lead to a non scalable model. However, only a few configurations will be selected in the final solution. Following the results obtained by linear programming theory, this case can be efficiently handled by column generation. The idea of using column generation is to start from a small set of configurations and then, generate only new configurations that will improve the value of the objective function. Generating new configurations stops when we can not find any new configuration that can improve the objective function value. We can then find the optimal LP solution considering only a limited set of configurations generated so far [37].

The problem being solved is split into two problems: a Restricted Master Problem (RMP) and a Pricing Problem (PP). The restricted master problem is a restricted version of the original problem considering only a limited set of configurations (referred to as columns). Pricing problem is then responsible for providing proper columns to be added to the restricted master problem pool of columns and can be viewed as a column (configuration) generator.

For any variable in a linear program, the reduced cost of the variable is the cost for increasing the value of it by a small amount. Formally, the reduced cost value for each variable in a linear program is defined as the amount by which the objective function coefficient on the variable need to improve so that the value of the variable becomes positive in the optimal solution. In a minimization problem, adding a variable which

has a negative reduced cost to the problem will result in a decrease (improve) of objective function value.

Pricing problem: configuration generator

In a column generation model, the pricing problem tries to generate new columns with a negative (positive if a maximization problem) reduced cost. Indeed, if no column can be generated with a negative (positive if a maximization problem) reduced cost then the latest solution obtained by solving the restricted master problem is the optimal solution.

At each step, and among the set of all potential configurations to be added to the set of current configurations, the one with the minimum reduced cost will improve the objective function value the most. The pricing problem therefore is to find a configuration with the minimum reduced cost.

We have N_b different pricing problems, each associated with one base station. The configuration generation iteration stops when none of the pricing problems can generate a new column with a negative reduced cost.

In order to write the reduced cost, we rewrite the master problem's objective function as follows.

$$\min \quad N_{BS}P_{\text{SLEEP}} + \sum_{\gamma \in \Gamma} (\Delta_p P^\gamma + B') z_\delta$$

where B' is $P_0 - P_{\text{SLEEP}}$. We now write the pricing problem for a given base station b , and therefore will omit the b index in the sequel of this paragraph in order to alleviate the notations:

$$\begin{aligned} \overline{\text{COST}} = & \Delta_p \sum_{u \in U} p_u + B' - \sum_{u \in U} v_u^{(5.2)} a_u - \sum_{u \in U} v_u^{(5.3)} \\ & - \sum_{u \in U} v_u^{(5.7)} \left(\sum_{w \in W} \frac{b_{w,u} \sigma_{bu} p_u W}{Tr,w} - P \sigma_{bu} (1 - a_u) \right). \end{aligned} \quad (5.8)$$

We have N_{BS} pricing problems where N_{BS} is the number of base stations. For a given base station $b \in B$, the set of constraints of the pricing problem (PP_b) is next described.

$$a_u \leq \frac{p_u \sigma_{bu}}{P_u^{\min}} \quad u \in U, \quad (5.9)$$

$$\sum_{u \in U} p_u \leq P_{MAX} \quad (5.10)$$

$$\sum_{w \in W} b_{w,u} \leq 1 \quad u \in U \quad (5.11)$$

$$\sum_{u \in U} \sum_{w \in W} b_{w,u} w \leq W \quad (5.12)$$

$$ma_u \leq \sum_{w \in W} b_{w,u} \leq Ma_u \quad u \in U \quad (5.13)$$

$$ma_u \leq w_u \leq Ma_u \quad u \in U \quad (5.14)$$

$$ma_u \leq p_u \leq Ma_u \quad u \in U \quad (5.15)$$

$$b_{w,u} \in \{0, 1\} \quad w \in W, u \in U \quad (5.16)$$

$$a_u \in \{0, 1\} \quad u \in U \quad (5.17)$$

$$p_u \in R^+ \quad u \in U. \quad (5.18)$$

Constraints (5.9) impose that u is mapped to b only if the power received by u is greater than the required minimum power threshold (P_u^{\min}). Constraint (5.10) is an upper bound on transmission power of each base station.

5.7 Obtaining an ε -optimal integer solution

Reduced cost can be only obtained from a linear program (LP) and not an integer linear program (ILP). However, the decision variable of EOCN_CG_ILP model, z_γ is an integer. Therefore, column generation can not be applied directly to the EOCN_CG_ILP model. As a result we apply the column generation method to the LP relaxation of EOCN_CG_ILP. The linear relaxation, referred to as EOCN_CG_LP, is obtained by replacing the binary decision variable of EOCN_CG_ILP with the following.

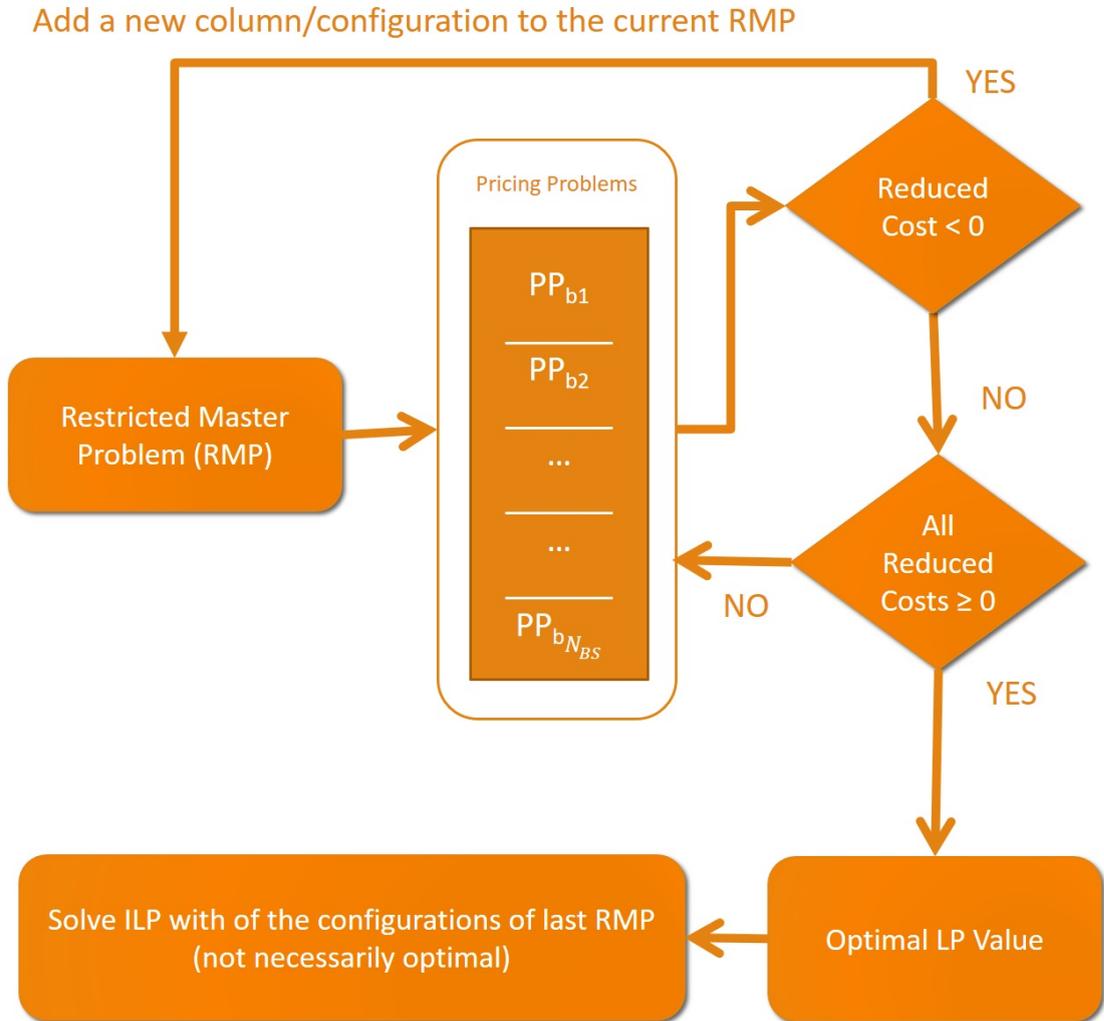


FIGURE 5.1: Obtaining an ILP solution using column generation for a problem with many pricing problems

$$0 \leq z_\gamma \leq 1 \quad \gamma \in \Gamma \quad (5.19)$$

$$z_\gamma \in R. \quad (5.20)$$

We solve the EOCN_CG_LP model using column generation and when the column generation process terminates, we solve the EOCN_CG_ILP model with the same columns generated for the EOCN_CG_LP. Although the solution obtained for the EOCN_CG_LP model is optimum, the solution of EOCN_CG_ILP model is an ε -optimal solution (Figure 5.1). Defining the following notations

- Z_{LP}^* : Optimal solution of the LP relaxation obtained by column generation
- Z_{ILP}^* : Optimal ILP solution of the problem
- Z_{ILP}^- : A solution to the ILP model obtained by solving it using the columns of the last RMP of the column generation solution

we can conclude that $Z_{LP}^* \leq Z_{ILP}^* \leq Z_{ILP}^-$. Therefore, $\varepsilon = Z_{ILP}^- - Z_{LP}^*$ estimates an upper bound to the accuracy of the ILP solution obtained by column generation.

5.8 Branch and price and obtaining an optimal integer solution

In the ε -optimal solution methodology described in previous section, there is no guarantee that the value of ε is small and the returned ILP solution is close to the optimal solution. For example, if the gap between the optimal solution of an ILP problem and its LP relaxation is big, then the generated columns for the LP model may not be efficient for the ILP model. Branch and price is a method to find an optimal solution to an integer linear problem using branch and bound and column generation. Branch and price is in fact branch and bound method in which the LP relaxation is solved using column generation method. At each node in the branch and price tree, the LP relaxation of the problem is solved using column generation. If the obtained solution is not integer, then a branching occurs. At each node, the LP and ILP solutions provide upper and lower bounds of the optimal ILP solution to be used by the branch and price algorithm. Similar to branch and bound, the branch and price continues the tree traversal until the optimal ILP solution is obtained [38].

5.9 Concluding remarks

In this section a column generation model is proposed to solve the EOCN problem. The problem is reformulated using the configuration concept and is broken down into a master and a pricing problem. The column generation method is then applied to the

LP relaxation of the original problem. The generated columns can finally be used to derive an ε -optimal solution to the ILP. The column generation method is known as a powerful tool to solve large-scale problems efficiently.

Chapter 6

Numerical Results

The algorithms described in this work to solve the EOCN problem are implemented and the solutions are presented in this Chapter. Section 6.1 defines simulation parameters and settings. Results of the exact solution of OptPowQoS (described in Section 4.4) and the heuristic of Piunti *et al.* (described in section 4.3) and the comparisons between the two solutions is presented in Section 6.2. Section 6.3 includes the results obtained by applying the column generation solution (described in Chapter 5). Sensitivity analysis results are presented in Section 6.4.

IBM ILOG CPLEX Optimization Studio 12.6.0.0, which is a powerful optimization software package, is used to solve the optimization models.

6.1 Simulation settings

Network model

The network is modeled as a set of base stations and a set of users distributed in a geographic area. The users are distributed uniformly in a circle of radius $R = 1100$ meters. We consider two different cell configuration networks: a hexagonal grid and a randomly generated network. The hexagonal grid consists of 19 base stations distributed on a hexagonal grid with an inter-site distance of 500 meters. The randomly generated

network consists of 20 base stations uniformly distributed in a circle with a radius of 1,000 meters with no two base stations closer than 300 meters to each other.

Solving the EOCN problem on the whole network (e.g., a city or a country) is not an option. In this case, the network can be broken down into smaller areas, each with a reasonable problem size for the problem solver algorithm. Since the interference caused by the base stations far away from each cell is negligible, this is a reasonable solution methodology. In this Chapter, we solve the EOCN problem on a circle of radius $R = 1100$ meters.

Base station parameters

For each base station, power consumption constants are: $P_0 = 130$, $\Delta_p = 4.7$ and $P_{\text{SLEEP}} = 13$. The maximum base station transmission power is limited to 20 watts [22]. Each base station has a total bandwidth of 5 MHz divided into 25 bandwidth blocks (each 0.180 MHz) [18].

User equipment parameters

Users data rate demands are modelled using an exponential distribution with an average of 64 kbps and a maximum of 8 Mbps (Any demand greater than 8 Mbps is satisfied with 8Mbps). The average data rate required by LTE users in a mobile network is less than 64kbps in practice, even during the peak hours [3]. The maximum of 8 Mbps is based on streaming a High Definition (HD) video which needs a bandwidth of 5-8 Mbps.

3GPP Typical Urban model is used as a channel gain (signal attenuation) model between the users and base stations:

$$\sigma = \frac{1}{10^{-0.1L}}$$

where σ is the channel gain (signal attenuation factor) and

$$L = 15.3 + 37.6 \log d$$

d is the distance between the user and the base station. User sensitivity is -90 dBm and noise PSD is -174 dBm/Hz [12].

Static traffic vs dynamic traffic

Note that we solve the EOCN problem on a static problem instance, which is a snapshot of the network. In other words, we propose an assignment for each network snapshot and find the energy saving that can be achieved by following the proposed assignment. However, changing the network structure and turning base stations on/off comes with costs in practice which is managed at a higher level in the network. When a new network configuration is suggested by solving the EOCN problem on a snapshot, a higher level management decides whether or not to follow the new proposed assignment. This decision is based on the cost associated with changing the network configuration, statistical data and prediction of the network traffic for the next few hours, and the amount of energy saving which can be achieved by following the new configuration [39]. This is a different problem which is not in the scope of this thesis.

All reported results correspond to an average over 5 traffic instances. We observed that the difference between solution values in different problem instances of a same size is negligible, and therefore, averaging over 5 traffic instances provides us with numerical results acceptable for our experiments.

The simulation parameters defined in this section are valid for all the experiments presented in this section, unless it is explicitly mentioned.

6.2 Solution results with the first model

In this section, we present the simulation results of the solutions described in Chapter 4. With the analysis of the solutions on various data sets, we evaluate the power consumption savings obtained by the base station sleep mode and the transmission power adaptation.

Each data set is solved using 4 different algorithms described below:

- **OptPowQoS**: the exact solution of EOCN problem using the algorithm proposed in section 4.4.
- **MinPowerQoS**: the heuristic of Piunti *et al.* described in section 4.3
- **ClosestBSMapping**: the solution of **MinPowerQoS** but instead of solving the MIQP to get the user association, assign each user to its closest base station. [12].
- **MinPower**: the solution of **MinPowerQoS** but accept the solution and terminate at the end of the first iteration [12].

ClosestBSMapping simply assigns each user to its closest base station. Comparison results of **OptPowQoS** with **ClosestBSMapping** gives us an indication of the amount of energy saving we obtain by solving the EOCN problem accurately.

MinPower covers all the users, but does not necessarily satisfy them with the minimum data rate they require. It terminates at the first iteration of **MinPowerQoS** without checking the QoS of the users. This is in fact a relaxation of the EOCN problem. The difference between the exact solution of EOCN and the solution provided by **MinPower** shows the cost we need to pay, in terms of consumed energy, to satisfy covered users with the required QoS.

6.2.1 Power consumption savings

In this section we evaluate achievable power saving by solving the EOCN problem using **OptPowQoS** solution. Results are compared with the results of **ClosestBSMapping** and **MinPower**. The network used for this set of experiments is the hexagonal grid network.

Figure 6.1 and 6.2, respectively, show power consumption and number of active base stations based on the number of users in the network. Simulation results show that in a network with a moderate load, more than 50% of energy saving can be achieved by deploying the base station sleeping strategy and doing the proper assignment and resource allocation. While by assigning each user to its closest base station, energy consumption of the network grows very fast (exponentially), it grows almost linearly with the number of users when using the solution provided by **OptPowQoS**. Moreover,

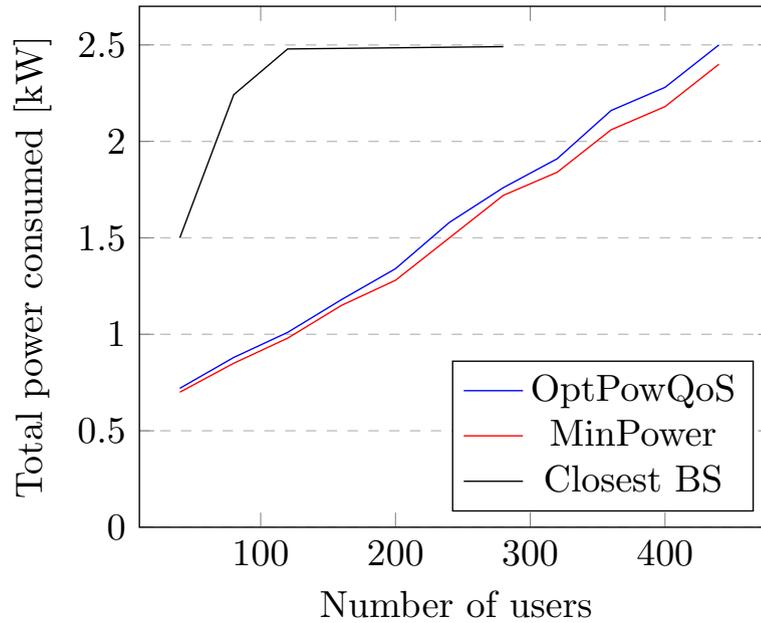


FIGURE 6.1: Total power consumption in LTE hexagonal grid networks

the model becomes infeasible almost 2 times faster by assigning each user to the closest base station. For a specific solution methodology, a network is infeasible if its provided solution can't satisfy all the users with their requirement. As the number of available bandwidth blocks and amount of energy at each base station is limited, having many users around any base station makes it infeasible for the `ClosestBSMapping` to cover all users. However, this case can be managed properly by solving the problem using `OptPowQoS`. The total number of bandwidth blocks in the simulated network is 475 and `OptPowQoS` allows satisfying up to 440 users.

6.2.2 Comparison results with the heuristic of Piunti *et al.* (2015)

In this section we compare the results obtained by `OptPowQoS` with the results obtained by the heuristic of [12] described in Section 4.3 (`MinPowerQoS`). Figures 6.3 to 6.6 provide the simulation results of the two different solution schemes on different networks. The solutions are compared with `ClosestBSMapping` and `MinPower`

Figure 6.3 and 6.4 show the power consumption and the number of active base stations of the LTE hexagonal network based on the number of users. Figures 6.5 and 6.6 show the total power consumed and number of active base stations based on the number of users for the randomly generated networks.

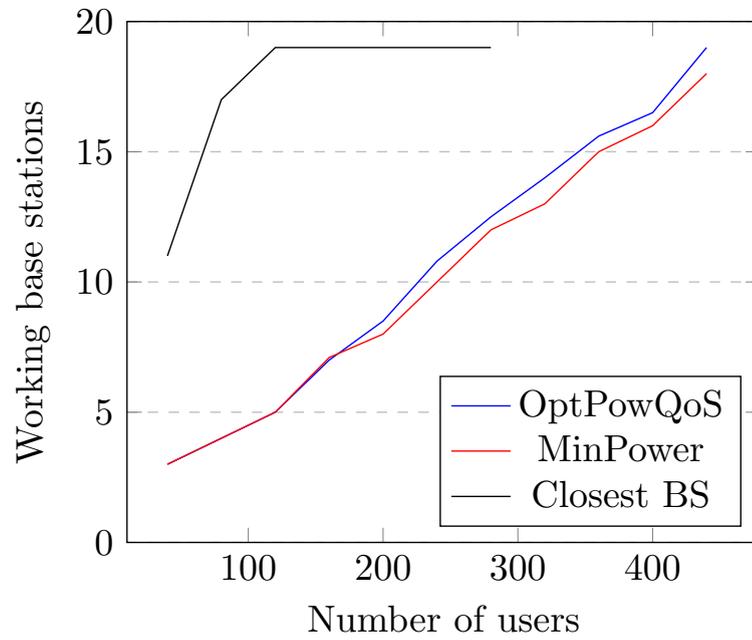


FIGURE 6.2: Active base stations in LTE hexagonal grid networks

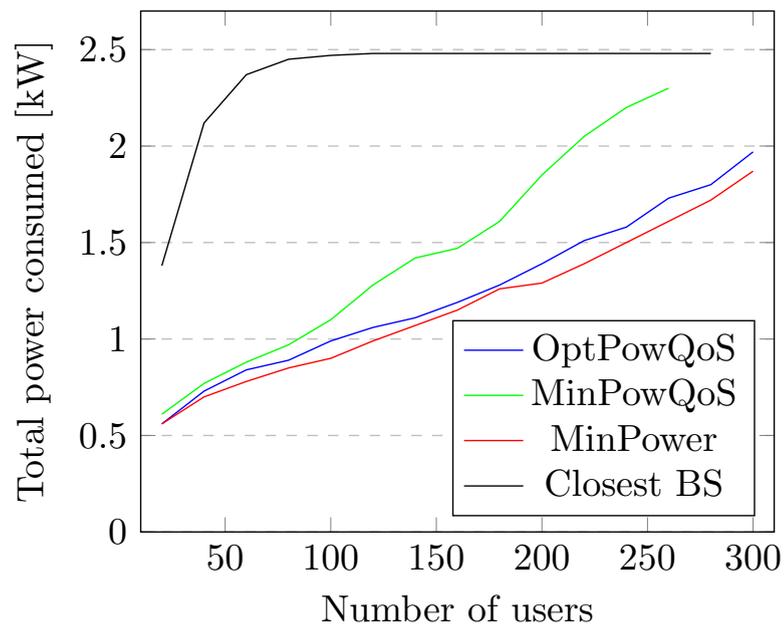


FIGURE 6.3: Total power consumption in LTE hexagonal grid networks

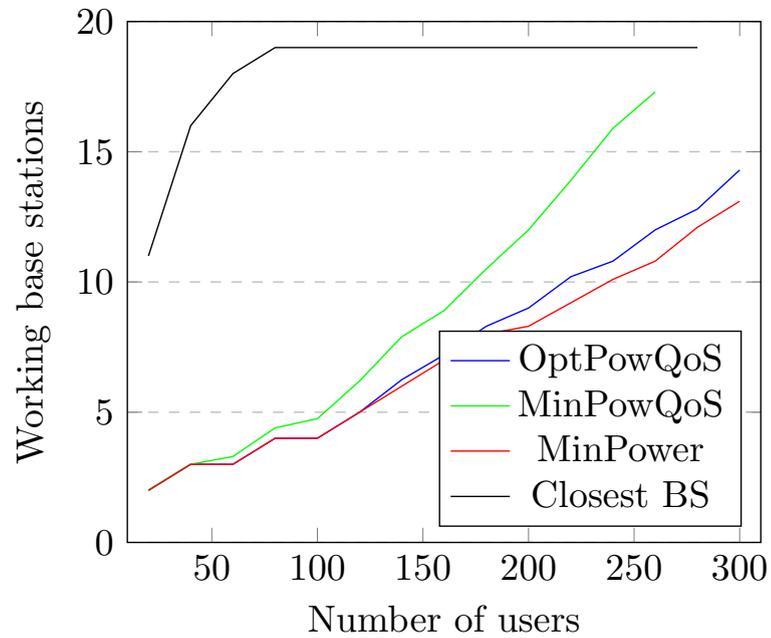


FIGURE 6.4: Number of active base stations in LTE hexagonal grid networks

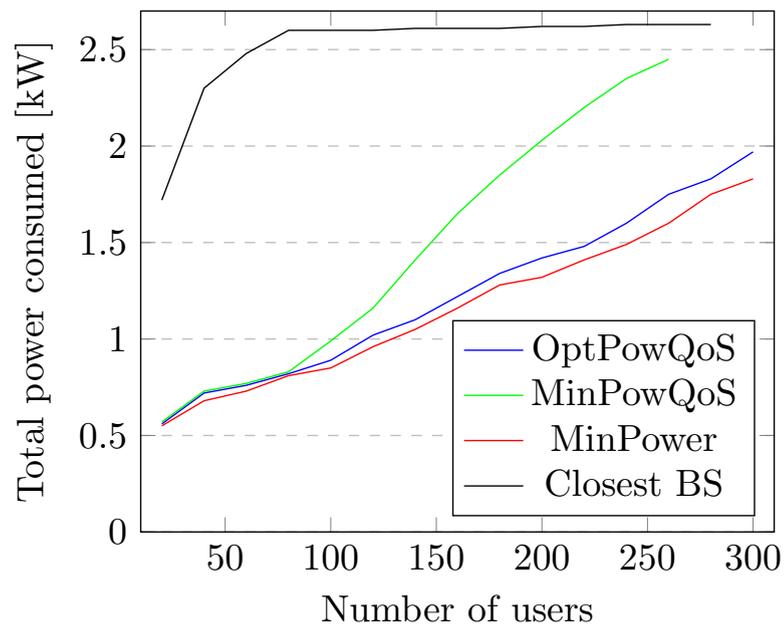


FIGURE 6.5: Total power consumption in randomly generated networks

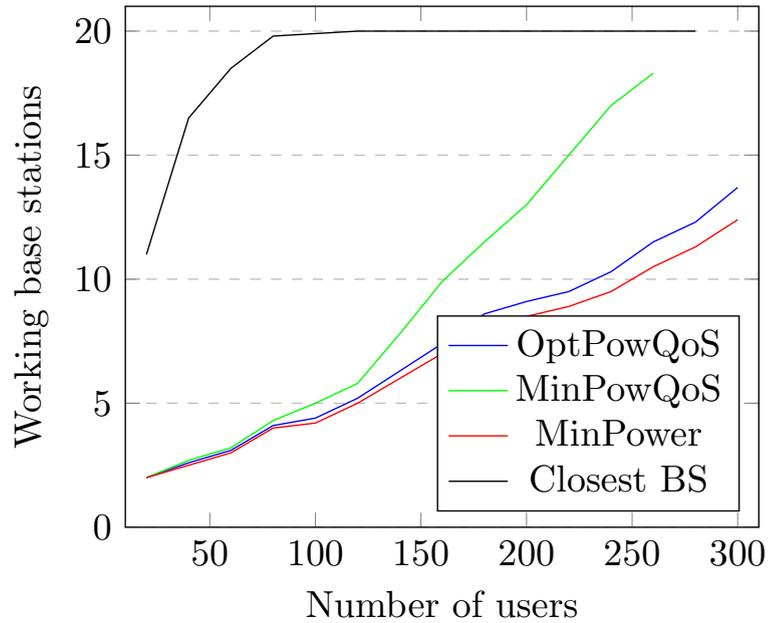


FIGURE 6.6: Number of active base stations in randomly generated networks

We observe that for low traffic load scenarios, the solution provided by `MinPowerQoS` is very close to the optimum solution of `OptPowQoS`. However, for a busy and high loaded network, the solution provided by the iterative heuristic algorithm becomes less accurate. This is due to the increased interference in the more crowded network. For a network with 200 users, the heuristic solution is up to %35 off the optimal solution. Moreover, the `MinPowerQoS` algorithm fails to find the solution when the network traffic increases to around 300 users. In the optimal solution, energy consumption increases linearly with the network traffic. Note that the `MinPower` solution does not take interference into consideration, while the exact solution guarantees the required bit rate for each user, taking interference into account.

Comparing `MinPower` and `OptPowerQoS`, we notice that the cost of satisfying users with the minimum bit rate is not high in terms of the energy consumption. In the simulated networks, the exact solution is always less than only %10 higher than the solution provided by the `MinPower` algorithm.

6.3 Solution with column generation

We have applied the suggested column generation solution (CG) to the example networks. The simulation settings are the same as the hexagonal grid network with 19 base stations in the previous section.

We define the following notations.

- E_{ILP}^* : the optimal solution of EOCN obtained by solving the problem using OptPowerQoS.
- $E_{ILP_CG}^*$: the integer solution obtained by solving EOCN problem using column generation method (EOCN_CG_ILP).
- $E_{LP_CG}^*$: the solution obtained by solving linear relaxation of EOCN problem using column generation method (EOCN_CG_LP).

Simulation results shows that the value of $E_{LP_CG}^*$ is very close to the optimal solution, E_{ILP}^* . However, when the columns are used to solve the ILP our results shows an infeasible ILP solution most of the time. The columns generated for LP do not let the data rate constraint to be satisfied in ILP and the column generation method fails to return $E_{ILP_CG}^*$ value.

We further investigated the solution by adding a set of dummy initial columns. Each dummy column represents a base station located very close to a user, transmitting only to that specific user and creating zero interference for other users. Adding these dummy columns makes sure that the ILP model is feasible. However, the cost associated with these columns is set to a very big number so that the ILP solver does not select it, if possible.

Figures 6.7 and 6.8 represent the obtained column generation solutions by using a set of dummy columns as an initial solution. We consider each dummy column used by the final solution as an uncovered user. For a network with only 100 users, it can be observed that more than 25 users are not covered. This shows that the basic application of column generation does not provide the desired solution, which is to cover all the users.

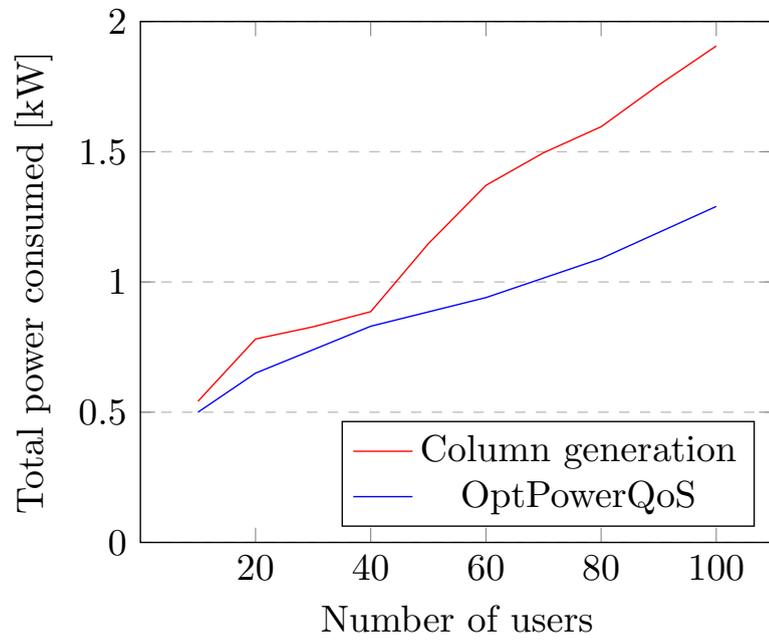


FIGURE 6.7: Column generation solution vs OptPowerQoS

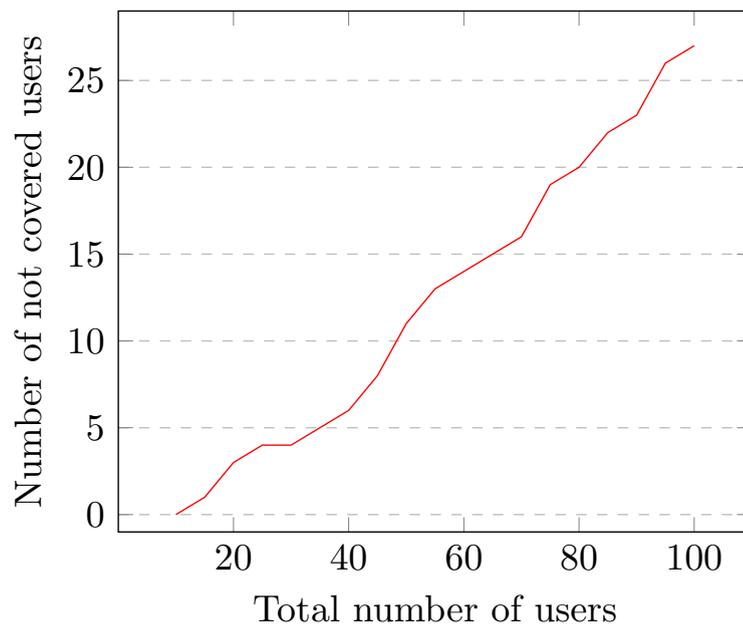


FIGURE 6.8: Unserved users in the solution by column generation

By solving the ILP model using the solution proposed in chapter 4 we can confirm that the gap between the ILP and LP solution is not high. However, the columns generated by LP relaxation are not suitable for the ILP solution. This is because of the high dependency of the LP solution to the rational numbers. In the LP solution, columns are picked partially and mixed together to make the final feasible solution. It can be seen that sometimes even a very small fraction of a dummy column is also used to satisfy the SINR constraint, in the case where a user needs to receive only a very small amount of additional power to be satisfied. The ILP model does not have this flexibility and this highly influences the obtained solution.

There exist problems where the columns generated by the direct application of column generation to solve the LP relaxation are less helpful for the ILP model. Our results shows that EOCN is one of those problems. In [35], methods to combine Lagrangean/-surrogate relaxation with column generation to generate more effective columns are discussed for a CPMP.

Concluding remarks

Simulation results show that the EOCN_CG_ILP fails to provide a solution for the EOCN problem. This happens because the generated columns for LP relaxation of the suggested column generation model are not useful for the ILP model. In this respect, future work includes investigations on generating columns which can effectively improve the solution value of ILP model.

6.4 Sensitivity analysis

6.4.1 Data traffic impact

In this section, we investigate the impact of mobile data traffic generated by the users on energy consumption of the network.

Figure 6.9 compares the energy consumption of the network based on the number of users for four different average data rates: 20, 60, 100 and 140 Kbps. As mentioned

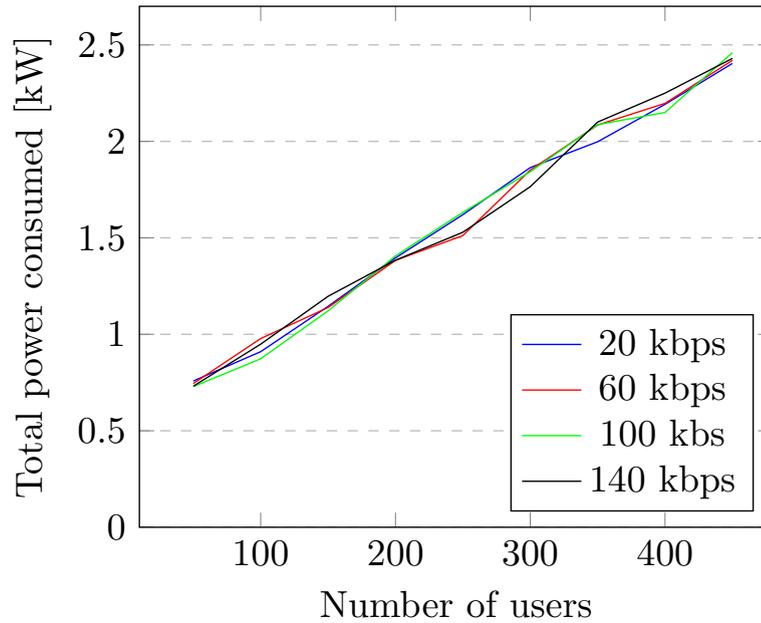


FIGURE 6.9: Data traffic impact

earlier in this Chapter, the current average data rate traffic per user in a mobile network is less than 64 Kbps. Data rate of 20 Kbps in this set of experiments represents a not busy network, while the data rate of 140 Kbps represents a busy network. It is observed that the energy consumption is almost independent of the traffic generated by the users. This is because of the high energy consumption of base stations compared to their transmitting power. The increased transmitting energy to provide users with faster data rates is negligible, compared the the total energy consumption of base station.

This explains the small distance between the red and blue lines in Figures 6.1-6.6. The red graph shows the solution of the network without considering the data rate constraint. The graphs show that the energy required to provide a covered user with the data rate constraint as well is very low. This is again because of the small portion of base station energy which is transmitted to the users.

6.4.2 Number of PRB impact

In an LTE system, the number of PRBs varies from 6 to 100 (depending on the channel bandwidth which varies between 1.4-20 MHz) [18]. In this section, we investigate the impact of the number of available PRBs at each base station to the amount of energy consumed by the network. The experiment is performed for a fixed number of users and

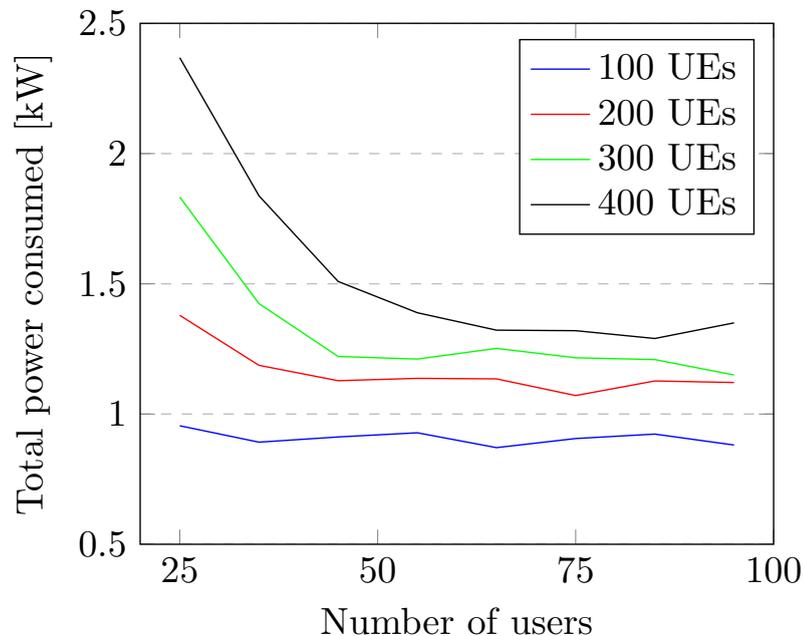


FIGURE 6.10: Number of PRBs impact

increasing the number of PRBs from 25 to 100. We do not consider networks with than 25 available PRBs as it can cover only a limited number of users.

Figure 6.10 shows the impact of the number of available PRBs on network energy consumption for different number of users. Increasing the number of resource blocks in the network helps to reduce the network energy consumption exponentially. We have this exponential impact in a networks if the number of PRBs is the bottleneck constraint of the optimization model. Clearly, after some threshold and when there are enough number of available PRBs, it is the available energy which limits the solution and increasing the number of PRBs will not help to get a different solution for the network.

6.4.3 Concluding remarks

Sensitivity analysis shows that the current LTE networks are not limited by the available resources (power and bandwidth). In fact, the resources are under-utilized most of the time. Allocating more/less resources to the network does not have a big impact on the network energy consumption, or even providing a better QoS for the users. However, adding the flexibility to put some base stations on the sleep mode has a big impact on the network energy consumption.

Chapter 7

Conclusions of the Thesis

7.1 Critical summary

In this thesis, we designed and solved a novel optimization model that minimizes the power consumption with guaranteed QoS for the users in a cellular network. While previous heuristic solutions showed that in moderate load scenarios, by putting the cells in sleep mode, up to 25% power savings can be achieved with respect to the basic scheme, exact solution shows that additional power saving can be obtained, e.g., up to 50% in a moderate load scenario when the network is half-loaded. In addition, more users (up to 30%) can be covered and satisfied using the optimal solution in comparison with the usual closest base station mapping strategy. The optimal solution provided by the exact optimization model finds the maximum achievable energy saving in a cellular network and evaluates the results quality of any other proposed heuristic.

In addition, we applied column generation technique to find a scalable solution for the optimization model. However, the numerical results showed that direct application of column generation to solve LP relaxation of the model does not find satisfactory results. The provided solution is not feasible and some of the users are not satisfied, specially for more crowded networks.

7.2 Future work

Future work will include increasing the scalability of the exact model and solution in order to be able to solve larger data instances. Most likely, it means using a decomposition model and the use of large-scale optimization algorithms, while for real time algorithms, we need to recourse to heuristics.

Chapter 8

Appendix

Algebraic Manipulations to obtain (4.14)

In this section, the detailed algebraic manipulations are presented to combine (4.2) and (4.13), simplify the inequality and obtain (4.14).

We start from (4.2):

$$\sum_{b \in B} a_{bu} \text{SINR}_{bu} \geq \sum_{b \in B} \sum_{w \in W} c_{bu}^w T^{r,w} \quad u \in U \quad (8.1)$$

where as of (4.13):

$$\text{SINR}_{bu} = \frac{\sigma_{bu} p_{bu} W}{w_{bu} \left(\sum_{b' \in B} P^{b'} \sigma_{b'u} (1 - a_{b'u}) + W N_0 \right)} \quad (8.2)$$

Combining (8.1) and (8.2):

$$\sum_{b \in B} a_{bu} \frac{\sigma_{bu} p_{bu} W}{w_{bu} \left(\sum_{b' \in B} P^{b'} \sigma_{bu} (1 - a_{b'u}) + W N_0 \right)} \geq \sum_{b \in B} \sum_{w \in W} a_{bu} c_{bu}^w T^{r,w} \quad u \in U \quad (8.3)$$

Selecting a term:

$$\sum_{b \in B} a_{bu} \frac{\sigma_{bu} p_{bu} W}{w_{bu} \left(\sum_{b' \in B} P^{b'} \sigma_{bu} (1 - a_{b'u}) z_{b'} + W N_0 \right)} \geq \sum_{b \in B} \sum_{w \in W} a_{bu} c_{bu}^w T^{r,w} \quad u \in U \quad (8.4)$$

The highlighted term does not have any dependency to the outer summation and selected b . Therefore:

$$\sum_{b \in B} a_{bu} \frac{\sigma_{bu} p_{bu} W}{w_{bu}} \geq \left(\sum_{b' \in B} P^{b'} \sigma_{bu} (1 - a_{b'u}) + W N_0 \right) \sum_{b \in B} \sum_{w \in W} a_{bu} c_{bu}^w T^{r,w} \quad u \in U \quad (8.5)$$

next:

$$\frac{\sum_{b \in B} a_{bu} \frac{\sigma_{bu} p_{bu} W}{w_{bu}}}{\sum_{b \in B} \sum_{w \in W} a_{bu} c_{bu}^w T^{r,w}} \geq \sum_{b \in B} P^b \sigma_{bu} (1 - a_{bu}) + W N_0 \quad (8.6)$$

Highlight:

$$\frac{\sum_{b \in B} a_{bu} \frac{\sigma_{bu} p_{bu} W}{w_{bu}}}{\sum_{b \in B} \sum_{w \in W} a_{bu} c_{bu}^w T^{r,w}} \geq \sum_{b \in B} P^b \sigma_{bu} (1 - a_{bu}) + W N_0 \quad (8.7)$$

Since $\sum_{b \in B} a_{bu} = 1$ for every user, we have:

$$(a_{bu})(a_{b'u}) = 0 \quad b \neq b' \quad (8.8)$$

knowing (8.8), we can write (8.7) as follow:

$$\sum_{b \in B} a_{bu} \frac{\sigma_{bu} p_{bu} W}{\sum_{w \in W} c_{bu}^w T^{r,w}} \geq \sum_{b \in B} P^b \sigma_{bu} (1 - a_{bu}) W N_0 \quad (8.9)$$

then:

$$\sum_{b \in B} a_{bu} \frac{\sigma_{bu} p_{bu} W}{w_{bu} \sum_{w \in W} c_{bu}^w T^{r,w}} \geq \sum_{b \in B} P^b \sigma_{bu} (1 - a_{bu}) + W N_0 \quad (8.10)$$

We now do another variable change that can help us simplify our model. Since $T^{r,w}$ depends on w , we can replace $w_{bu} T^{r,w}$ with a new constant $T^{*r,w}$

$$\sum_{b \in B} a_{bu} \frac{\sigma_{bu} p_{bu} W}{\sum_{w \in W} c_{bu}^w T^{*r,w}} \geq \sum_{b \in B} P^b \sigma_{bu} (1 - a_{bu}) + W N_0 \quad (8.11)$$

We now need to eliminate $b_{w,u}^b$ in the denominator. Since $b_{w,u}^b$ is a 0 – 1 taking care of selection the correct $T^{*r,w}$, we can move it to the nominator. It doesn't change anything:

$$\sum_{b \in B} a_{bu} \sum_{w \in W} \frac{c_{bu}^w \sigma_{bu} p_{bu} W}{T^{*r,w}} \geq \sum_{b \in B} P^b \sigma_{bu} (1 - a_{bu}) + W N_0 \quad (8.12)$$

We can also remove a_u^b . Actually, p_{bu} takes care of it. (With adding the following constraint to the PP:)

$$ma_u \leq p_u \leq Ma_u$$

Now, we move all the variables into the left hand side:

$$\sum_{b \in B} \left(\sum_{w \in W} \frac{c_{bu}^w \sigma_{bu} p_{bu} W}{T^{*r,w}} - P^b \sigma_{bu} (1 - a_{bu}) \right) \geq WN_0 \quad (8.13)$$

(8.13) is what we were looking for.

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