Assessing students' conceptual understanding of place value with the Video Evaluation Task

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ABSTRACT

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By Brittany Rappaport

This study investigates the affordances of a novel assessment of students' conceptual understanding of place value, The Video Evaluation Task, over and above two other typical measures of place value, the Place Value Word Problems task and the Place Value Relational thinking task. The Video Evaluation Task assesses a participant's evaluation of another student solving an addition problem in real-time. We individually interviewed second grade students' (N = 45) to assess their place value understanding and asked them to provide justifications for their responses. Participants' justifications and/or written work was coded and scored for evidence of place value understanding. The Video Evaluation Task was significantly correlated with the Place Value Word Problems task (r = 0.347, p = 0.02) and the Place Value Relational Thinking task (r = 0.371, p = 0.012) indicating that these measures tap into similar constructs. However, I also developed a second level of coding to determine the nuances of students' place value understanding, which allowed me to investigate the different affordances of each measure. Regardless of whether participants performed better or worst on the Video Evaluation Task as compared to the other two measures, it seems the Video Evaluation Task afforded more information about students' conceptual understanding of place value. Not only did the Video Evaluation Task provide an opportunity for participants to evaluate another student in the context of a double-digit addition problem, but it also allowed participants to demonstrate place value knowledge and/or misconceptions that may not have been revealed on the other two measures. The Video Evaluation Task is a promising measure for both researchers and teachers, in a complex mathematical domain that cuts across the elementary curriculum.

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Chapter 1: Statement of the Problem

Recent standards have outlined the importance of assessing children's conceptual understanding of mathematics (National Council of Teachers of Mathematics, 2000). Formative assessment specifically, or assessment for the purposes of teaching and advancing students' learning, is an important component of mathematics teaching practice (Ginsburg, 2009; NCTM, 2000). Because of the emphasis on conceptual understanding in teaching, it is important for researchers to develop appropriate and accurate assessments of students' conceptual understanding. This also helps them to better understand students' learning in different mathematical domains. These assessments have typically targeted students' thinking, with the objective of gaining information about their mathematical knowledge (Ginsburg, 2009).

The focus of this research is on the assessment of students' conceptual understanding in mathematics from a researcher's perspective. It is well documented that researchers experience many challenges in assessing students' conceptual understanding in mathematics. Firstly, students' thinking is not always clear and is not something that is obvious from the work shown on their worksheets (Fisher, 2007). Researchers often need to ask students to explain their reasoning, which is difficult to achieve on a paper-and-pencil test where the students' strategies might be missing. Furthermore, students may have partial knowledge of a concept and this can be difficult to pinpoint on an assessment (Hiebert & Wearne, 1992). Researchers also have difficulty knowing whether the assessments they have used are valid. It is often not clear whether assessment items are getting at the right kind of information, and if they are targeting students' conceptual understanding at all. Furthermore, using a single type of measure to assess students' conceptual understanding, which is the typical approach, can present other challenges because one type of assessment, on its own, might not provide an accurate picture of the student's knowledge and understanding (Crooks & Alibali, 2014).

I will address the above-mentioned challenges in the present study by comparing different types of assessments of students' conceptual understanding identified in the literature. Researchers have commonly used the following types of assessments: students' application of strategies, justification of students' own strategies, and evaluation (and justification) of *other* students' strategies and accompanying work, which has been used less often (Crooks & Alibali, 2014; Prather & Alibali, 2009). The literature has described the use of evaluation as a form of assessment in domains related to numeracy such as counting, equivalence, commutativity, arithmetic, and linear equations. To my knowledge, however, evaluation has not yet been used, specifically, to assess students' conceptual understanding of place value.

The first objective of the current study is to determine whether an evaluation assessment, the Video Evaluation Task (Rappaport, Blondin, Duponsel, Osana, & Proulx, 2016), assesses a similar construct to that of two other typical assessments in the domain of place value. The Video Evaluation Task is a novel assessment used in the context of a larger project on students' symbolic understanding of place value and regrouping. The task presents participants with videos of a student working out double-digit addition problems. The participant is then asked to evaluate the student's work.

The second objective of the current study is to examine the affordances of the Video Evaluation Task over and above two other more typically used measures. Because using multiple forms of assessment provides a more complete view of students' conceptual understanding of mathematics for researchers and teachers (Crooks & Alibali, 2014; NCTM, 2000), I will examine what the overall picture of students' understanding of place value and regrouping will look like by presenting examples of students' conceptual understanding across all three measures.

The current study has implications for teachers as well. Even teachers with strong practical experience and mathematical content knowledge sometimes have difficulty accessing students' mathematical thinking (Speer & Wagner, 2009). I propose that the Video Evaluation Task would provide another tool for teachers to use to gain an understanding of students' mathematical thinking and conceptual knowledge. Moreover, the current study could provide teachers with a multitude of tools for assessment and ways to go about using them together to access students' thinking. This, in turn, would be useful if one assessment, on its own, fails to provide appropriate information about the students' knowledge. Furthermore, using multiple assessments could give teachers a better idea of the student's thinking, and be better able to move his or her learning forward.

Chapter 2: Literature Review

The National Mathematics Advisory Panel (2008) stated that mathematical success in school provides individuals with better opportunities in the future, such as in university and in their subsequent careers. As Schoenfeld (1995) and Burkhardt (2007) described mathematics as the “new literacy”, individuals who are successful in mathematics can benefit by, for example, gaining access to a higher annual income and better management of work demands (National Mathematics Advisory Panel, 2008). For as long as mathematical success has been of importance from an academic perspective, researchers have attempted to find ways to best prepare and support students to learn successfully in mathematics, or in other words, to attain “mathematical proficiency.”

Kilpatrick, Swafford, and Findell (2001) defined the term mathematical proficiency as consisting of a number of different components: procedural fluency, conceptual understanding, adaptive reasoning, strategic competence, and productive disposition. Although all of these components have been studied, most attention in educational psychology research has been on two of these strands: procedural fluency and conceptual understanding. According to recent standards, conceptual understanding and procedural fluency have been outlined as key components for students to learn mathematics with understanding (NCTM, 2000). In the following sections, I will define various terms used in the literature on procedural and conceptual knowledge in mathematics.

Definitions

Concept. A mathematical concept can be understood as a web of interconnected pieces of information (Hiebert & Lefevre, 1986). By definition, a concept should not be considered individually (i.e., a concept should not be thought of as existing on its own) but rather by its relationship to other pieces of information (Hiebert & Lefevre, 1986). For example, place value concepts are related to understanding computation of whole and rational numbers.

Algorithm. An algorithm is an already known sequence of "steps or actions" to reach a goal or solve a problem (Hiebert & Lefevre, 1986; Rittle-Johnson & Schneider, 2015). For example, the algorithm to divide fractions is to invert the second fraction and then multiply the fractions ($\frac{1}{2} \div \frac{1}{3} = \frac{1}{2} \times 3$).

Strategy. A strategy is the mathematical method used to reach a goal or solve a problem. For example, when solving 38 + 26, a student might use a "combining units separately" strategy and say, "Thirty and twenty is fifty, and eight and six is fourteen. The ten from the fourteen makes sixty so it's sixty-four" (Carpenter, Franke, Jacobs, Fennema, & Empson, 1998, p. 38).

Place Value. Place value is the principle that the position of each digit in a written numeral represents a value (Burris, 2005). For example, the "2" in 23 represents "twenty" compared to the "2" in 230 which represents "two hundred."

Regrouping. Regrouping is the mathematical process of making groups of ten (Burris, 2005). Regrouping is sometimes called "exchanging" or "borrowing." For example, when adding 9 + 3, ten units can be *regrouped* into 1 ten, with 2 units remaining, written then as "12."

Conceptual and Procedural Knowledge

Conceptual understanding, or knowledge of concepts, as defined by the National Research Council, is the knowledge of "mathematical concepts, operations, and relations" (Kilpatrick et al., 2001, p. 5). After reviewing the definitions of many mathematics education researchers, Rittle-Johnson and Schneider (2015) concluded that making connections or relations between concepts is an aspect of conceptual knowledge that grows as students develop their knowledge over time. When learning place value, for example, a student who has conceptual knowledge understands the value of each place in a multi-digit number (e.g., ones and tens). For example, if such a student were to look at the number 64, he or she would know that the digit "6" in 64 represents six "tens," and the "4" represents four "ones," simply by seeing the position of the digits, indicating place value understanding (Burris, 2005). Furthermore, students who have conceptual understanding are more likely to use their place value knowledge to understand arithmetic concepts and procedures (Burris, 2005). For example, when solving an addition problem such as 12 + 79, students who have conceptual understanding of place value are more likely to connect their knowledge of regrouping to the concepts underlying the standard algorithm (i.e., by regrouping 10 ones when adding 9 + 2 into one ten, and adding this ten with the 1 ten and 7 tens already in the 12 and 79).

In contrast, procedural fluency is described as the ability to accurately execute procedures (Hiebert & Lefevre, 1986). More specifically, Byrnes and Wasik (1991) describe procedural knowledge as "knowing how" to do the mathematics. Furthermore, procedural knowledge can also be described as the knowledge a student has of the various moves necessary to complete a procedure and solve a problem (Rittle-Johnson & Schneider, 2015). For example, presented with the equation 45 + 56, students with procedural knowledge know the specific steps needed to find the answer, but may or may not understand the concepts behind them (i.e., first add up the right most column (5 and 6) together, then put a "1" for the tens, then add all the tens together, then find the answer with the remaining numbers). When researchers use the word procedure, they typically mean algorithm. Therefore, for the purposes of this study, I will use the terms interchangeably.

In comparison to procedural competence, conceptual understanding has received particularly close attention in the literature (Star, 2005). Star (2005) suggested that the importance placed on students' "deep" understanding of mathematics, which is related to conceptual understanding, could explain the reason for the focus on conceptual understanding. Additionally, conceptual understanding has been identified as a key component of students' mathematical proficiency because to be successful learners, students need to understand why what they are doing is working, or not, for a problem at hand, or why it makes sense to choose one answer over another (Crooks & Alibali, 2014). Crooks and Alibali (2014) explained that having conceptual knowledge is linked to a variety of other strengths. For example, Blöte, Van der Burg, and Klein (2001) found that conceptual instruction on two-digit addition and subtraction problems influenced second-grade students' procedural flexibility, generalization to solve novel problems successfully, and evaluation of a correct procedure on a problem that they did not solve themselves. Similarly, in the domain of addition and subtraction, Canobi (2004) found that first- and second-grade students with greater conceptual understanding also had better problem-solving skills. The NCTM (2000) also emphasized the requirement for students to learn mathematics "with understanding," conceptual knowledge being a critical component in this.

The credibility of the research on conceptual understanding is dependent on the ability to assess students' understanding. Researchers have used many different approaches to assessing students' mathematical understanding. In general, an important part of assessing students' conceptual knowledge is getting a "picture of their thinking" (Fisher, 2007). Some strategies suggested in the literature include asking students to explain, in pictures or words, what they are doing while trying to solve a problem (Fisher, 2007). For example, when asking students to compare fractional quantities and tell which one of two quantities is bigger, researchers can ask students to explain how they arrived at their answers to get at their mathematical thinking (Fisher, 2007).

Issues Related to Assessment of Conceptual Understanding

The literature has shown that researchers still have difficulty assessing students' thinking. There are several reasons why assessment for conceptual understanding is challenging. Firstly, assessing conceptual understanding is difficult because it is not something that a researcher can readily see on a student's worksheet or a paper and pencil test, for example (Fisher, 2007; Schoenfeld, 2007). Even if a paper and pencil test has a question that asks students how they arrived at their answer, or to explain their reasoning, the strategy used might be unclear and the researcher is not present to probe students' thinking to clarify. Although it is indeed challenging to assess subtle components of understanding, like "metacognition" and "strategy," Schoenfeld (2007) has stated that it is still important to do to reveal students' conceptual understanding.

Secondly, it is difficult to determine when conceptual knowledge has been achieved, as it is possible to have partial knowledge of a concept (Hiebert & Wearne, 1992). Hiebert and Wearne (1992) found that after place value instruction was given to first-grade students, their knowledge developed differently as the school year progressed. Some students' strategies showed a partial understanding of place value concepts and some connections to other concepts, whereas others had a more complete understanding. More specifically, some students who performed well on regrouping addition problems had more difficulty with regrouping in subtraction. Also, performance on written tests of place value did not always match up with students' responses during interviews, where their strategies were revealed (Hiebert & Wearne, 1992).

These two challenges reinforce the notion that a written test on its own is not an ideal form of assessment in this domain because it places students into categories of right or wrong. When looking at students' responses during interviews, it is evident that their place value understanding may be more nuanced. Therefore, in mathematical domains such as place value, assessing the presence or absence of place value knowledge may not be an appropriate approach to assessing students' conceptual understanding. Assessments conducted via interview give a more detailed picture of students' understanding in this domain.

Thirdly, there is no certainty that any given assessment itself is valid. Researchers can use an assessment that asks students many questions, but there is no certainty as to whether these questions get at the targeted, or even useful, information (Burkhardt, 2007). Although some researchers use different measures of conceptual knowledge, and their questions are intended to be conceptual in nature, it is not clear whether their questions also tap into mathematical skill or procedural knowledge. Therefore, it is difficult to judge whether the assessment task is actually a valid measure of the construct in question.

Lastly, researchers must be cautious when relying on the presence of correlations between assessments to provide information about student understanding (Hiebert & Wearne, 1996). Sometimes researchers use one measure instead of multiple measures because of a correlation that exists among them. In the domain of conceptual understanding, there can be a disadvantage to using one measure because there are fine-grained aspects of mathematical understanding that can be overlooked by ignoring the information that can be obtained from other measures.

To get an accurate picture of a student's conceptual knowledge then, researchers have recommended using an assortment of measures to document "patterns of behavior" (Bisanz, Watchorn, Piatt, & Sherman, 2009; Crooks & Alibali, 2014). Also, researchers who use multiple assessments obtain more information about students' understanding than what is generated from only one task's specific characteristics (Schneider & Stern, 2010). For example, one assessment might provide information about students' conceptual understanding in one specific area, but this information might not be generalizable to other tasks. Furthermore, students' conceptual knowledge is seen as they draw connections, solve novel problems, and make sense of mathematics, promoting the possibility for generalizability and transfer on novel tasks (Richland, Stigler, & Holyoak, 2012). Recent reform standards have similarly stressed that students' thinking is so diverse and complex that a multifaceted view of their understanding is necessary (NCTM, 2000). This would, in turn, ensure that a teacher has a complete picture of a student's knowledge rather than a more narrow one.

When researchers use multiple forms of assessment to get at students' mathematical understanding in a particular domain, there are often different types of assessments that they can select. Each type of assessment affords a different kind of knowledge about the student's thinking and understanding (Prather & Alibali, 2009). In the literature below, I will describe different tasks that have been used to assess students' conceptual understanding in a variety of mathematical domains. These will be discussed in turn to describe the types of assessments that have commonly been used in the literature and the information that these assessments can afford about students' conceptual understanding. I will also address the use of less commonly used assessments, such as evaluation.

Assessment Tasks for Conceptual Understanding Used in the Literature

Researchers in the assessment of mathematics have categorized different ways to assess students' understanding in general (Bisanz & Lefevre, 1992) and students' conceptual understanding in particular (Crooks & Alibali, 2014; Prather & Alibali, 2009; Rittle-Johnson & Schneider, 2015). An important feature of all assessments for students' conceptual understanding is that they must be fairly novel to ensure that the student is indeed using his or her conceptual knowledge to answer the question, and not a known procedure (Rittle-Johnson & Schneider, 2015). For the purpose of this study, I am using the following categories to describe the assessment of students' conceptual understanding: application of strategies, justification of strategies, and evaluation of strategies.

Application of strategies. Assessments that rely on the application of strategies involve assessing students' conceptual understanding by looking at the way students solve a problem, or how students apply strategies to solve a problem. Bisanz and Lefevre (1992) explained that, when solving a mathematical problem, regardless of whether students obtain the right or wrong answer, if the strategy they use is consistent with the principle or concept in the specific domain, students could be assessed as having conceptual understanding. For example, when students are given an equation to solve such as 5 + 3 – 3 = ☐, those who apply an efficient strategy to solve the problem, such as realizing that + 3 and - 3 can "cancel" out, are said to have conceptual understanding by using a "shortcut" to solve a problem (Gilmore & Papadatou-Pastou, 2009). In this case, the students would have conceptual understanding of the inversion principle, namely the mathematical concept that addition and subtraction have an inverse relationship, and that inverse operations with the same values result in no change (e.g., the 3 in 5 + 3 - 3). For example, in 7 + 2 - 2 = ☐, students with an understanding of inversion would know the shortcut that + 2 and – 2 can "cancel" each other out, without having to perform any calculation to obtain the answer to the problem.

Another example is the use of the "counting-on" strategy as evidence of commutativity (Prather & Alibali, 2009). Students who solve the equation 3 + 5 by counting on from the larger number (i.e., 6, 7, 8), and know that the last number is the answer, show conceptual understanding of commutativity, the mathematical property that the placement of the addends does not affect the sum (e.g., 3 + 5 = 5 + 3). Similarly, the placement of the factors does not affect the product (e.g., 4 x 8 = 8 x 4). Some researchers have claimed that use of this strategy implies students' conceptual understanding of commutativity (Prather & Alibali, 2009).

Bisanz and Lefevre (1992) stated, however, that even if a student uses an appropriate strategy, such as a shortcut, this does not always relate to a complete understanding of the underlying concept. For example, if given an equation such as 5 + 3 – 3 = ☐, a student might immediately answer 5, appearing to demonstrate conceptual understanding. Bisanz and Lefevre (1992) explain that perhaps the student has seen this type of problem before, and if the response has typically been the first number in the equation, then the student would learn to do that for *all* other problems like it. Therefore, it is imperative not to assume conceptual understanding simply through the application of strategies, and to supplement one's knowledge of student understanding by asking students to justify their responses to provide further explanation of their understanding (Bisanz & Lefevre, 1992; Crooks & Alibali, 2014).

Justification of strategies. The justification of strategies involves assessing students' conceptual understanding through the verbal explanations of problems that they themselves solve (Bisanz & Lefevre, 1992). Researchers typically ask students a series of questions and continue probing until students' strategies are clear (Bisanz & Lefevre, 1992; Hiebert & Wearne, 1992). For example, Hiebert and Wearne (1992) gave first-grade students a series of tasks including addition and subtraction story problems, with and without regrouping, and place value questions such as identifying the value of a digit in a double digit number by asking "what is '1' worth in 16?" After these tasks, the researchers asked students to explain how they got their answers, and to talk about what numbers they were thinking of to get their answers. In the fractions domain, Reimer and Moyer (2005) studied third-grade students working with virtual manipulatives while solving addition and subtraction problems. The authors conducted interviews while students interacted with virtual manipulatives and asked students to explain what they were doing and to explain how they were using the virtual manipulatives.

Researchers have not only focused their assessment on students' justifications of their own strategies, but some have also asked students to provide explanations for their own definitions or principles of mathematical concepts. For example, Schneider and Stern (2010) gave a conceptual intervention and a procedural intervention to fifth-grade students about decimals. In the conceptual intervention, the authors asked students to provide verbal explanations to questions about the principles behind decimals. For example, the authors asked questions such as "what does it mean when a number has a comma?" or "it is better to use whole numbers or decimals in day-to-day life?" After the students provided responses to these questions, the authors asked them to provide explanations for their answers.

By examining students' justification of strategies, and by probing their thinking about the definitions of concepts, the researcher can study students' knowledge of the concept at hand rather than a memorized procedure (Bisanz & Lefevre, 1992). Crooks and Alibali (2014) explain that many researchers use both application of strategies and justification of strategies together when assessing students' conceptual understanding. For example, asking students to explain or justify problems after they have solved them allows the researcher to tap into why a student executed a certain strategy. Bisanz and Lefevre (1992) cautioned that using justification of a student's own strategy, on its own, may be misleading because a child may understand a concept but have difficulty explaining his or her own work or thinking. Conversely, children may be able to explain the principle or concept behind what they are doing, but not be able to use or apply it appropriately. Therefore, assessing both application and justification is more advantageous than with one on its own. Bisanz and Lefevre (1992) also suggested that asking students to evaluate another student's work could be helpful when assessing conceptual understanding, particularly for those who have difficulty articulating their own thinking.

Evaluation of strategies. According to Bisanz and Lefevre (1992), the evaluation of a strategy requires a student to judge, without performing it him or herself, whether a strategy is acceptable. Therefore, when a student evaluates, he or she judges another's strategy, definition, or statement of principle. For purposes of clarity, I will use "participant" to describe the student who is being assessed, and "student" to describe the fictitious student in an evaluation assessment scenario.

Crooks and Alibali (2014) stated that assessments that rely on the evaluation of strategies should incorporate more than evaluating than whether a strategy is correct or incorrect. Similar to using a combination of application of strategies and justification of procedures when assessing participants' conceptual understanding, Crooks and Alibali (2014) recommended that evaluation assessment should also include a justification component. To assess students' conceptual understanding, researchers require more information about why participants are making a specific evaluation.

A recent review of assessment literature (application, justification, and evaluation) conducted by Prather and Alibali (2009) reviewed the areas of commutativity, operations, and inversion. Based on their review, the authors concluded that researchers have not used evaluation assessment as commonly as assessments based on application and justification of procedures. Although evaluation tasks are used less often than application and justification tasks, the authors attest that they can still provide important information about a student's understanding in the context of another student's work (Prather & Alibali, 2009). Evaluation tasks have many advantages, including less cognitive demand on the student because they do not ask the student to apply or justify any of their own procedures (Rittle-Johnson & Schneider, 2015). Furthermore, evaluation as a form of assessment can potentially allow for more information about student thinking than application and justification alone. Because of their typically more open-ended nature, evaluation tasks can provide opportunities for researchers to probe more deeply into students' thinking. Finally, all of the assessments for students' conceptual understanding, together, could provide a rich profile of students' mathematical understanding.

When reviewing the literature on evaluation assessments, I have found that there are a variety of ways that researchers have asked participants to evaluate another person's strategy when solving a mathematical problem. The following section outlines different types of evaluation tasks, namely evaluation tasks that occur either as static or in real time, and evaluation tasks that involve evaluating either part or all of another student's strategy.

Static vs. real time. Researchers use evaluation assessments in different formats, one of them being a static format. For example, Fuson and Kwon (1992) used an evaluation task that showed another student's written work. The authors presented second- and third-grade participants with correct and incorrect two-digit addition and subtraction problems that another student had already solved. The authors presented the participants with problems presented vertically solved using the standard algorithm. The researchers told the participants that another student had solved the addition and subtraction problems and they wanted to know if the child solved it correctly or not. Once participants answered, the researcher asked them to explain their reasoning.

In other static evaluation assessments, researchers have asked participants to evaluate another student's definition of a mathematical principle or a statement about a mathematical principle. For example, Star and Rittle-Johnson (2009) gave fifth-grade students definitions of estimation such as “estimation is making math problems easy and quick” or “estimation is using easier numbers and getting close to the true value.” The researchers asked the participants to evaluate the definitions with ratings of not so good; kind of good; or very good. In an attempt to understand students understanding of the equal sign, McNeil and Alibali (2005) gave participants an equation solving measure (e.g., 7 + 4 + 5 = 7 + __) and another task where they asked participants to evaluate students’ definitions of the equal sign. The definitions included statements such as the equal sign means, “repeat the numbers” and “the answer to the problem.” The authors asked participants to provide a rating for their evaluation based on smartness (very smart, kind of smart, not so smart). This was done in a paper and pencil format, however, and the authors did not ask participants to provide verbal justifications afterwards.

In another study, Adrien (2012) was interested in students’ individual differences in conceptual understanding of the equal sign. After giving instruction on the equal sign, Adrien (2012) provided third and fourth-grade participants with an equation (3 + 4 + 1 = 3 + ___), with an arrow pointing to the equal sign, and read six fictitious students’ definitions of the equal sign. The author asked participants to rate the students’ definitions by circling a happy face, neutral face, or sad face, which corresponded to McNeil and Alibali (2005)’s “smartness” ratings. Similar to the McNeil and Alibali (2005) study, students were not asked to justify their answers on this task.

In contrast, some researchers have used evaluation tasks in a “real-time” format, by presenting participants with a live character (i.e., a puppet) performing a strategy or showing participants what another character did in the past. For example, Cowan et al. (1996) used a task called the Windows Task to assess children’s understanding of the order irrelevance principle, the mathematical idea that the order in which objects are counted does not affect the result of the counting process (Prather & Alibali, 2009). The researchers told three-and-a-half to five-year-old children a story about a mother who asked her children to count all the windows in their house. The researchers told the participants that one child counted the windows in one order (and showed them the order on an image of a house with windows) and the other child counted the windows in another order. The researcher asked the participants, without allowing them to count the windows themselves, if they thought that both the children would find the same number of windows. The task was designed so that participants who conclude that both children would find the same number of windows understand the principle of order irrelevance in counting.

In another study, Muldoon, Lewis, and Berridge (2007) told 3-year-old children that "Harry," a fictitious character, wanted to share ice cream cones with two other people. The researchers showed the participants different trials of how Harry decided to share the ice cream cones, some with correct equal sharing and others with incorrect equal sharing. The researchers then asked the participants, after each trial, to identify if Harry shared correctly or not. Afterwards, in instances of correct equal sharing, the researchers asked the participants to explain why the correct sharing resulted in equal sets of ice cream. The researchers also asked the participants to explain why Harry's incorrect sharing resulted in unequal sets. This has similarly been done by Canobi et al. (1998) in the domain of equivalence.

Part of strategy (i.e., certain steps, final answer) vs. whole strategy. When reviewing the literature I have also found that researchers have designed evaluation assessments to evaluate a *part* of another student's strategy (e.g., the first step in their work to solve the problem, or just the final answer) rather than his or her *whole* strategy (e.g., the entire solution including the final answer). For example, Rittle-Johnson and Alibali (1999) asked participants to evaluate another student's strategy when solving a mathematical problem in an unfamiliar way. The unfamiliarity provided an opportunity for participants to evaluate a novel procedure, prompting the use of concepts to explain why the procedure works or does not work (Rittle-Johnson & Alibali, 1999). The researchers presented fourth-grade participants with correct and incorrectly solved equivalence problems, completed by a fictitious student. The researchers told the participants that the student who solved the mathematics problems solved it in different ways (i.e., some were unfamiliar or novel to the participant). The researchers verbally told the participants the steps that the student took to find the answer, and then asked participants to rate the participant's steps based on "smartness" (very smart, kind of smart, or not so smart). The researchers also asked participants to justify why they chose the rating they did.

In other cases, researchers asked participants to evaluate only the student's final answer. For example, Rittle-Johnson et al. (2001) studied how participants' procedural and conceptual knowledge with decimals developed after giving an intervention with number line problems and instructor feedback. Before the intervention, the researchers assessed the participants' conceptual understanding with an "Evaluation Task." The task required fifth-grade participants to evaluate other students' solutions to decimal fraction addition problems by asking them to select which solution they thought was most correct. Afterwards, the researchers asked the participants to explain why they thought the solution they chose was most correct. Therefore, in this example, researchers asked participants to evaluate just the student's final answer to the problem, without showing any of the student's work.

In another example, Schneider et al. (2011) asked participants to evaluate specific steps of a mathematical problem executed by another student. The authors investigated the relationship between conceptual and procedural understanding in the domain of linear equations by using an evaluation assessment to assess participants' conceptual understanding. After presenting 11- to 15-year-old participants with algebraic equations and a student's accompanying work and final answer, the authors asked participants to evaluate how the student went from the first step of his or her solution to the second step when solving the equation. After the students described what they thought about the student's work, the researchers asked participants to rate the way the student solved the whole equation using a scale that consisted of ratings of very good way; OK to do, but not a very good way; or not OK to do.

To summarize, I have found a variety of evaluation tasks in the literature. Most evaluation tasks have asked participants to evaluate a static format of another student's work, such as by showing his or her written work to solve a problem or stating another student's definition of a mathematical concept. If an evaluation task makes use of showing participants what another student has done in "real time", it has been with a puppet and not with the actual student solving the problem in real time. Another area of distinction between evaluation tasks is whether the evaluation task is set up for the participant to evaluate another student's entire strategy or part of strategy. In the literature, I have found that there are few evaluation tasks making use of a participant evaluating another student's entire strategy.

As mentioned earlier, Prather and Alibali (2009) reviewed the literature on evaluation assessments and found several studies conducted in numeracy domains such as commutativity and operations. I am unaware of any evaluation task, however, that focuses specifically on children's understanding of place value.

Therefore, the focus of this study is on numbers and place value, a core concept in the elementary mathematics curriculum. According to Reys et al. (2010), place value is an important component of our numeration system. The decimal numeration system also encompasses learning the base ten system, the use of zero, and the additive property. Students who at least have an implicit understanding of these components are said to be able to appropriately represent and interpret numbers (Reys et al., 2010). Therefore, place value understanding is interconnected with an understanding of the base ten system, the use of zero, and the additive property. For the purpose of this study, I will investigate children's knowledge of place value as well as the conceptual structure of the base ten system.

Present Study

I have described some of the key literature on assessing students' conceptual understanding, highlighting the importance of tapping into students' conceptual understanding of mathematics, and the challenges that researchers experience with assessment in research. Typically, researchers have assessed conceptual understanding through three main types of tasks: application of strategies, justification of strategies, and evaluation of strategies. I have described examples of researchers using application and justification of strategies as forms of assessment, but evaluation has been used less often.

When evaluation has been used, it has been used in a variety of ways, including presenting participants with a fictitious story context, another student's written work, definitions and mathematical principles, or a fictitious character solving a mathematical problem in "real-time." To my knowledge, there have not been any evaluation tasks that have asked participants to watch another student solving a problem in real time. Furthermore, typically, researchers have asked participants to evaluate part of another student's strategy rather than the whole strategy (i.e., steps and final answer).

I have described the available literature about the use of evaluation as a form of assessment in several domains related to numeracy, such as commutativity, operations, and inversion. To my knowledge, however, researchers have not yet used evaluation assessment to assess students' conceptual understanding of place value specifically.

The typical approach to assessing place value knowledge has been to give participants a problem and ask them to solve it themselves. Researchers have used participants' performance on the problem, and sometimes their justifications, as indications of the presence or absence of place value understanding. The current study proposes to give researchers another tool to measure students' understanding in a context that has not been studied before in the literature. I propose that using multiple types of assessment, together, will result in a better understanding of a student's profile of knowledge.

The present study is part of a larger project on students' symbolic understanding in mathematics. In this larger project, Rappaport et al. (2016) collected data in three schools with second-grade students. The authors individually interviewed the students during which they administered three place value measures. One of these measures, the Video Evaluation Task, was created for the purpose of the larger project. The Video Evaluation Task assesses a participant's evaluation of another student solving an addition problem in real-time. The other measures include the Place Value Word Problems task, based on Hiebert and Wearne's (1996) word problems, and the Place Value Relational Thinking task, based on Carpenter et al. (2005) and Empson, Levi, and Carpenter (2011)'s work on the development of children's understanding of the properties of arithmetic in the context of whole number operations.

In my research, I assessed students' understanding of place value with the two typical measures (Word Problems and Relational Thinking) and the new measure, the Video Evaluation Task. First, I will correlate the Video Evaluation Task with the Word Problems task and the Relational Thinking task. This correlation will provide support that the Video Evaluation Task taps into place value understanding in some way. I will then compare the relative contributions of the three measures of place value understanding. Through these comparisons, I will describe the types of knowledge that the Video Evaluation Task would provide about students' conceptual understanding of place value over and above the more typically used measures to generate a clearer picture of students' understanding and thinking.

The specific research questions of the current study are: (a) Is the Video Evaluation Task correlated with each of the two more typical place value measures (Word Problems task and Relational Thinking task)? (b) Will the Video Evaluation Task afford additional information about students' understanding of place value that the other two measures do not? If so, what is the nature of that information?

Chapter 3: Method

Participants

The present study was a part of a larger study in which participants were recruited at the end of the school year from four second-grade classrooms in three different elementary schools in the Montreal area (*N*= 61). Three classrooms had their mathematics instruction in English, and one classroom had their mathematics instruction in French. I removed 16 participants from the sample because of administrative errors during the interview. Therefore, for the present study, I used a total of 45 participants from the larger sample (*N* = 45). I only had access to the ages of the children for 28% of the sample. For this subset, the mean age was 7.23 years old, with a standard deviation of 0.43.

Design

The objective of the larger study was to assess the interaction between type of instruction and type of external representation on children's learning of place value. In the larger study, an intervention was delivered using two external representations (manipulatives and written numbers). Participants' place value understanding was assessed before and after the intervention (pretest and posttest measures). We found that all students improved from pretest to posttest, regardless of instruction type or representation type, so the data from the conditions were collapsed for the current study.

Along with two other graduate assistants, I was involved in the design of the larger study from its inception. I was also a part of a team of research assistants who collected data on students' place value understanding prior to the intervention, delivered the intervention, and assessed students' place value understanding after the intervention.

Along with trained research assistants, I conducted the posttest interviews in the language of each participant's mathematics instruction. For the present study, I analyzed the data collected at posttest on the three place value tasks.

Measures

The research assistants administered all the posttest measures during individual interviews with each participant. The posttest measures included: The Place Value Word Problems task (Hiebert & Wearne, 1996), the Place Value Relational Thinking task (Carpenter, Levi, Franke, & Koehler Zeringue; Empson et al., 2011), and the Video Evaluation Task, a task designed specifically for this study. The Video Evaluation Task was based on a task used by Osana, Adrien, and Duponsel (2017) to assess students' knowledge of regrouping.

The Place Value Word Problems task. The interview protocol for the Place Value Word Problems task is in Appendix A (Part A). Students' place value understanding involves understanding the value of the position of a digit in a multi-digit number. For example, place value understanding involves knowing how many groups of ten there are in a double-digit number. In the written numeral 52, the place of the number "5" in 52 represents 5 groups of ten. In the written numeral 502, the place of the number "5" in 502 represents 5 groups of hundred because the 5 is in a different place than in 52. In the context of a word problem, students with an understanding of place value can indicate how many groups of ten (or hundreds, thousands, etc.) there are in a multi-digit number by simply looking at the place of the digit in the number provided (see Carpenter, Fennema, Franke, & Empson, 2014). Carpenter et al. (2014) refers to this as "Direct Place Value knowledge." Students who try to calculate the answer by drawing pictures or counting on their fingers have a less developmentally sophisticated understanding of place value (Carpenter et al., 1998). For the purpose of this study, I will use the term "Clear" instead of "Direct."

The Place Value Word Problems task used in the current study is a measure based on Hiebert and Wearne (1996)'s *Teams* task, in the context of isomorphic versions of Hiebert and Wearne's (1996) word problems. The Word Problems task assesses students' place value knowledge through students' understanding of the base ten number system and grouping by 10. The task has three items. As shown in Figure 1, all of the items are measurement division problems containing a double-digit number.

The Place Value Word Problems task is administered as follows. The double-digit number in the word problem is presented to the participant on a cue card and is placed on the table in front of him or her. The research assistant reads the word problem out loud and does not say the number in the problem. Instead, the research assistant directs the participant's attention to the number on the card by circling it. For example, for item 1 in Figure 1, the research assistant says, "Look at the number on this card," and circles the number 72 on the card with her finger. Then, the research assistant continues to read the problem saying, "This is how many markers Justin has. He wants to put his markers in boxes that can fit 10 markers. How many boxes can he fill up completely with the markers he has?"

The research assistant reads the word problem as many times as necessary for the child to understand it, as described in Appendix A (Part D). If the child is having trouble understanding the problem, the interviewer may unpack the problem to help the child understand the context and question, without helping to solve the problem. The general guidelines provided in the interview protocol are in Appendix A (Part D). For example, for item 1 in Figure 1, the research assistant can tell the child that 10 stickers fill the page completely, explain that the stickers are all about the same size, and that there are only 10 spots on the page for them. Once participants give an answer, the research assistant asks them how they found the answer, and can continue to probe their thinking to understand the strategies used.

During the interviews, I did not give participants manipulatives to solve the word problems for reasons related to confounding variables in the larger study. Because the task was designed to assess students' understanding of clear place value, participants would not be able to demonstrate clear place value if they needed to use objects to model the problem. The research assistants gave participants paper and marker to use to solve the problem, but told the participants that they were not obliged to use them.

The Place Value Word Problems Task

1. Look at the number on this card *(circle the number 72 on the card with your finger)*. This is how many markers Justin has. He wants to put his markers in boxes that can fit 10 markers. How many boxes can he fill up completely with the markers he has?

2. Look at the number on this card *(circle the number 49 on the card with your finger)*. This is how many doughnuts Elsa has. She puts the doughnuts in containers that can fit 10 doughnuts. How many containers will she fill up completely with the doughnuts she has?

3. Look at the number on this card *(circle the number 84 on the card with your finger)*. This is how many goldfish Amanda has. Because the fish need space to swim, Amanda can only put 10 goldfish in each fishbowl. How many fishbowls will Amanda be able to put 10 goldfish into?

Figure 1. The Place Value Word Problems task items.

The Place Value Relational Thinking task. The interview protocol for the task is in Appendix A (Part B). The Place Value Relational Thinking task is based on Carpenter et al. (2005) and Empson et al. (2011)'s work on the development of children's understanding of the properties of whole number operations in the context of relational thinking. Relational thinking is defined as the process of relating mathematical quantities and expressions to each other to solve a variety of problems (Carpenter et al., 2005; Empson et al., 2011). For example, if the numbers 8 and 80 are in a problem, students could use relational thinking to make the quantitative relation between the "8" in 8 and "8" in 80 (i.e., the 8 in "80" is ten times larger than the "8" in 8 because of the position of the "8" in both numbers). As the previous example illustrates, students' understanding of place value can be assessed through their use of relational thinking (Carpenter et al., 1998; Carpenter et al., 2005) because they can use place value concepts, such as grouping by 10, to solve related problems. Researchers hypothesize that if students use relational thinking to solve computation questions, they have an understanding of the conceptual properties of whole numbers, including place value concepts (e.g., Empson et al., 2011; Kindrat & Osana, 2015).

The Relational Thinking task was created to assess students' place value understanding through relational thinking and base ten concepts. The task has three items, as shown in Figure 2. Each item has two parts that relate to each other quantitatively. The first item contains two measurement division problems, the second item contains two multiplication items, and the third item contains two partitive division problems. Both parts have the same problem structure, but involve different numbers so that they are related by a factor of 10. For example, in item 1, the number "6" is used in the first part of the question, and then the number "60" appears in the second part of the question. The correct answer to the second part can be deduced by using the answer to the first part of the item (in all items, the answer to the second part is 10 times larger than the answer to the first part). Participants are allowed to use paper and pen for the first part of the item, but are not allowed any tools for the second part (i.e., mental math question).

The task is administered as follows. First, the research assistant tells the participants that they can use a paper and marker if they wish to solve the first problem, but not the second problem. Each problem is read out loud for the participant. The problem can be read as many times as the participant needs. Once participants provide an answer for the first part of the item, the research assistant reads the second part of the item. If participants do not understand the problem, it can be explained without the research assistant helping to solve the problem. For example, for item 2 in Figure 2, the research assistant can tell participants that the cupboard only has 5 shelves in total, and that each shelf only has room for 4 books. If participants ask what size each of the books are, the research assistant can say they are all relatively the same size. Once a participant gives an answer, the research assistant asks how he or she arrived at that answer. The research assistant can continue to probe the participant's thinking to understand the strategies used.

After administering the Relational Thinking task to the participants, I decided to remove the second item of the task. The second part of this item asks participants to calculate 3 groups of 5 and then 3 groups of 50. Because many participants intuitively knew what 50 + 50 + 50 was, they did not necessarily need to use relational thinking or an invented algorithm to find the answer to the problem. Therefore, the second item of the task was not included in the analyses.

Place Value Relational Thinking Task

1. There are 6 sandwiches on the table. If you want to put 2 sandwiches in each container, how many containers will you need?

Now, if there were 60 sandwiches, how many containers of 2 sandwiches would you need?

2. There were 5 cookies in each box. If there are 3 boxes, how many cookies would there be altogether?

Now, if there were 50 cookies in each box, how many cookies would there be altogether?

3. There are 8 chocolate bars. Four friends want to share the chocolate bars so that each friend gets the same amount. How many chocolate bars should each friend get?

Now, if there were 80 chocolate bars, and the 4 friends wanted to share them equally, how many chocolate bars would each friend get?

Figure 2. The Place Value Relational Thinking task items.

The Video Evaluation Task. The Video Evaluation Task protocol is in Appendix A (Part C). Researchers have used participants' evaluations of another student's work (and their solution) as a form of assessment of children's understanding of mathematics (Bisanz & Lefevre, 1992). The Video Evaluation Task was developed based on Bisanz and Lefevre's (1992) review of evaluation as a form of assessment for students' conceptual understanding and on Osana, Adrien, and Duponsel (2017)'s Evaluation Task. Osana et al. (2017) presented participants with static images (photographs) of another student's work on a multi-digit addition problems (e.g., 44 + 7) and asked participants to evaluate the student's solution. The first image showed how a fictional student arranged his or her manipulatives at the start of their solution to the problem, and the second image was a photo of the manipulatives at the end of his or her solution. The authors then asked participants to evaluate whether the participant's solution (represented in the second image) was correct or not. Some participants had difficulty understanding the task, however, presumably because of the inherent temporal aspect (i.e., participants looked at what a student started with and ended with when solving a problem) (H. Osana, personal communication, July 4, 2017). Therefore, for the present project, Rappaport et al. (2016) modified the task by using a video, instead of images, to show participants the student's work in real time.

The Video Evaluation task was designed to assess participants' understanding of place value and regrouping. The student in the videos solves double-digit addition problems with two kinds of materials: written symbols (i.e., numbers) or manipulatives. The student's strategy of solving the problem with written symbols was purposely mimicked with the manipulatives. For example, when the student uses a standard algorithm to solve the addition problem, the student using the manipulatives would move the manipulatives in such a way to reflect the standard algorithm.

In the larger study, the research team created eight videos in total. For the purposes of the current study, I analyzed only the videos where the student solves the problem incorrectly using the standard algorithm (i.e., four videos). Preliminary analyses on “correct” videos showed that participants were just looking at the procedure and/or final answer and the videos did not prompt the participant to evaluate the student in the video.

In the context of the larger study, participants were assigned to different conditions of instruction, which varied in their use of external representation. The research assistants who administered the Video Evaluation Task showed participants videos of students using the same external representation that the participants used during instruction. Therefore, participants who were assigned to the numbers condition during instruction saw videos of a student solving the problem using written symbols, and participants assigned to the manipulatives condition watched videos of a child solving the problem with the same manipulatives used during their instruction.

Materials used in videos. The student in the video using manipulatives to solve the problem uses a novel manipulative created by the research team called “squares.” This manipulative was created specifically for the larger study to ensure that participants did not have any prior knowledge of the manipulative used both in the instruction and in the Video Evaluation Task. The squares were pieces of green foam board in the shape of squares (Figure 3). There were two sizes of squares: a large square representing 10 and the small square representing one. The dimensions of the small square is 2 cm by 2 cm and the dimensions of the large square is 6.5 cm by 6.5 cm. The squares were created in such a way that the big square was physically 10 times bigger in area than the small square.

The student in the video using written symbols to solve the problem uses paper and marker. The marker was a black permanent market and the paper was a blank piece of white paper (14” by 11”) attached to a large coiled sketchpad (as shown in Figure 4).

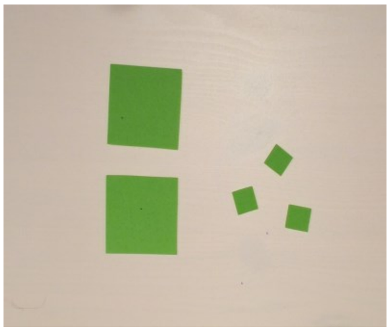

Figure 3. Squares manipulatives.

Description of videos. Each video is described below with both representations (squares and numbers).

Video 1. The student has a cue card with the written problem 25 + 17 in front of her. Using a marker, the student writes 25 + 17 vertically on her paper, as shown in Figure 4, and says, “I have 25 and 17.” The student says, “This is 12, (pointing to the 5 and the 7),” and immediately regroups the 12 ones into 1 ten (written on top of the “2” in 25), as shown in Figure 5. The student points to the “1” on top of the “2” in 25, the “2” in 25 and the “1” in 17 and says, “This is 40.” The student writes 4 in the tens position, but writes 0 in the tens position. The student says, “So my answer is 40” (Figure 6).

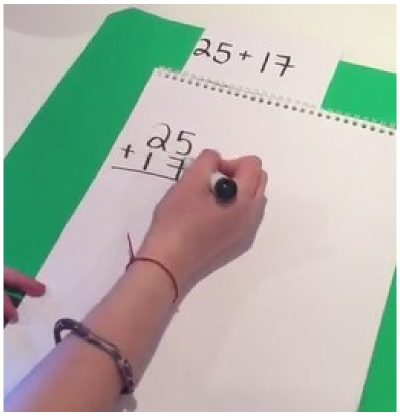

Figure 4. Picture of student writing 25 + 17 vertically.

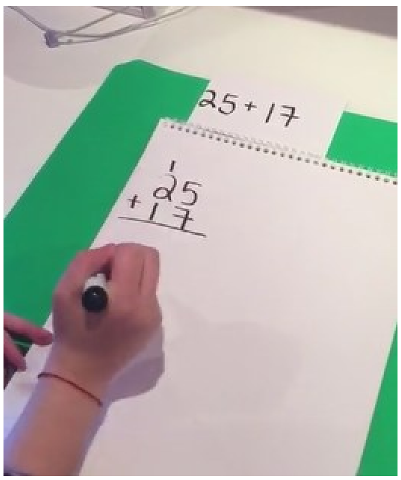

Figure 5. Picture of student replacing 12 ones into 1 ten (on top of the "2" in 25).

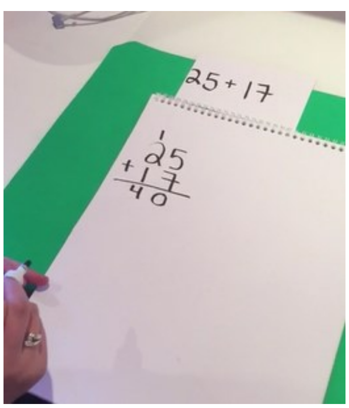

Figure 6. Picture of student's final answer for 25 + 17.

Video 2. The same set up is used for this video 2 as for video 1. The student solves the same problem (i.e., 25 + 17), with the same standard procedure and incorrect solution, but in this video, the student uses the squares to solve the problem. As shown in Figure 7, the student shows 2 big squares and 5 small squares, and 1 big square and 7 small squares and says, “I have 25 (pointing to the 2 big squares and 5 small squares) and 17 (pointing to the 1 big square and 7 small squares).” Then, the student shows the ones from both quantities together by placing 5 small squares and 7 small squares together. As shown in Figure 8, the student puts all the small squares together and says, “This is 12.” Then, the child replaces 12 small squares with 1 big square says, “I have to exchange this (holding the 12 small squares) for this (and takes a big square).” The student then points to each of the big squares, places them together, and says, “My answer is 40” (Figure 9).

Figure 7. Picture of student’s placement of squares to show “25 and 17.”

Figure 8. Picture of student’s placement of small squares to show “12.”

Figure 9. Picture of student's final answer for 25 + 17 with manipulatives.

Video 3. The student has a cue card with the written problem 18 + 36 in front of her. Using a marker, the student writes 18 + 36 vertically and says, "I have 18 and 36 (pointing to each of the quantities)" (Figure 10). The student says, "This (pointing to the 8) and this (pointing to the 6) is 14" (Figure 11). The student writes 4 in the ones position and does not write a "little one" over the 1 in the tens position. The student says, "This (pointing to the 1 in 18) and this (pointing to the 3 in 36) is 40," and writes 4 underneath the tens position. The student says, "My answer is 44" (Figure 12).

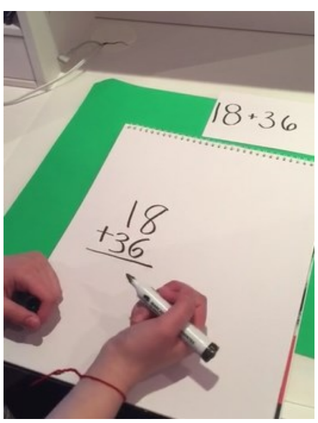

Figure 10. Picture of student writing 18 + 36 vertically.

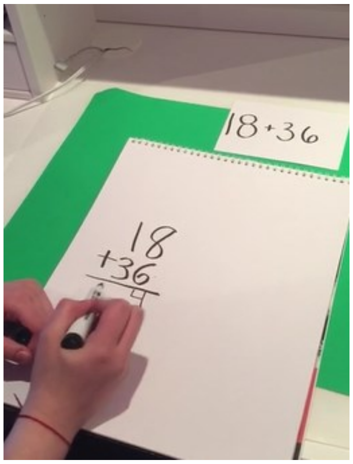

Figure 11. Picture of student writing 4 in the ones position.

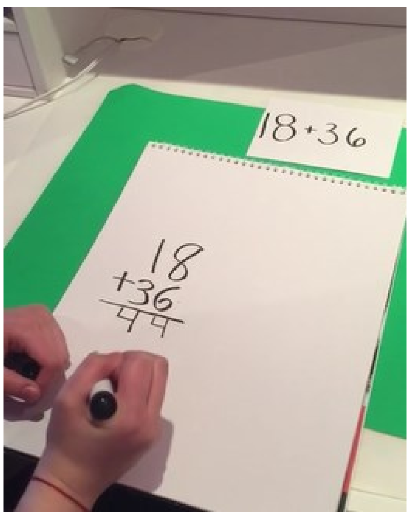

Figure 12. Picture of student's final answer for 18 + 36.

Video 4. The same set up is used for this video as for video 3. The student solves the same problem (i.e., 18 + 36), with the same standard procedure and the same incorrect solution, but with squares. The student shows 1 big square and 8 small squares, and 3 big squares and 6 small squares and says, "I have 18 (pointing to the 1 big square and 8 small squares) and 36 (pointing to the 3 big squares and 6 small squares)" (Figure 13). Then, the student places the 8 small squares and 6 small squares together. Then, the student moves ten small squares away and says, "I have to exchange this," and removes the ten small squares without replacing them with one big square (Figure 14). The student moves the remaining small squares together, counts up all the big squares by pointing to each of the big squares and says, “My answer is 44” (Figure 15).

Figure 13. Picture of student’s placement of squares to show “18 and 36.”

Figure 14. Picture of student taking ten small squares out of view.

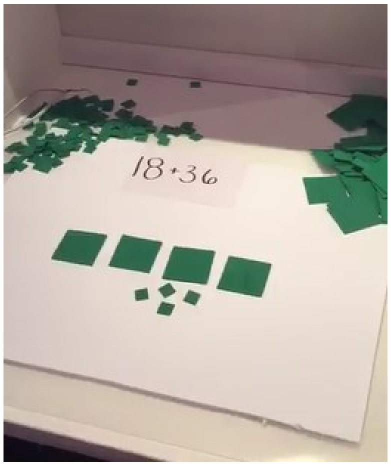

Figure 15. Picture of student's final answer for 18 + 36 with manipulatives.

Task administration. The task begins with the research assistant explaining to the participant that she gave a problem to another student and videotaped her solving the problem. The problems given in the videos are two open number sentence addition problems with double-digit numbers (i.e., 25 + 17 and 18 + 36). The research assistant places a cue card with one of the number sentences in front of the participant, tells the participant that he or she will watch a video of the student solving the problem, and then reads the problem out loud in the format "a + b = ☐." Then, the research assistant tells the student that the video will be shown twice and that he or she will be asked to talk about what the student in the video did afterwards. The participant is told to pay attention to how the student solves the problem.

After watching the video, the research assistant asks the participant to talk about what the student in the video did by saying, "Tell me what you think about how she solved it." Once the participant responds, the research assistant probes the participant's thinking further by asking questions until the participant indicates whether he or she thinks the student in the video solved the problem correctly or incorrectly and provides a justification. If necessary, the research assistant can offer the participant as many chances to watch different parts of the video again.

Once the participant's full explanation is given, the next video is shown with the second number sentence, using the same administration procedure described above.

Procedure

Several research assistants and I visited the classrooms to administer the posttest interviews with the participants. The research assistants individually interviewed students in a quiet area of the school. The three measures were administered in the following order: the Place Value Word Problems task, the Place Value Relational Thinking task, and the Video Evaluation Task. The research assistants videotaped and audio recorded the interviews, without the child's face shown in the video. During the interview, the research assistant completed the post-interview scoring sheet (Appendix B) used to record some of the participants' responses on each measure. Each post-interview took on average 39 minutes (M = 39.42, SD = 8.86) and all interviews were conducted over a 2-week period.

Chapter 4: Results and Discussion

I analyzed the data in two ways to answer my research questions. First, to see if the Video Evaluation Task was correlated with each of the two more typical measures, I coded and scored participants' justifications for each of the three measures (Level 1 coding). Then, I correlated the participants' scores on the Video Evaluation Task with the scores on the Word Problems task and the Relational Thinking task. Second, to see what kinds of information the Video Evaluation Task afforded about students' understanding of place value above the other two measures, I developed a second level coding with superordinate categories to see what each measure said about children's knowledge of place value. I created frequency tables to demonstrate where participants fell in terms of their understanding of place value across all items, on each of the three measures. Because the Level 1 and 2 coding coategories were derived from the students' justifications in the data, I describe the coding decisions in the Results section. Lastly, I provided examples of participants who demonstrated different forms of place value understanding depending on the information the measure was providing. These examples are meant to illustrate the affordances of the Video Evaluation Task relative to the other two measures.

Level 1 Coding

To answer my first research question, I watched the videotapes of the participants' interviews and coded their justifications and written work (i.e., if they used paper and marker to solve the problem) for the Place Value Word Problems task, the Place Value Relational Thinking task, and the Video Evaluation task.

Place Value Word Problems task. According to the coding and scoring rubric shown in Table 1, I coded the participants' justifications and/or written work based on evidence of place value knowledge (i.e., base ten number system and grouping by ten). If a participant immediately (i.e., within 10 seconds) provided a correct answer by looking at the double-digit number and did not use modeling (i.e., drew with paper and marker), I assigned the code of Clear Place Value. For example, in the number 72, either 7 or 8 was accepted as a correct answer. The participant may have responded with an answer of 8 if he or she thought of an additional group for the remainder.

If a participant modeled groups of 10 (e.g., drew an object to represent the group of tens and another object to represent each of the ones), regardless of his or her answer, I assigned the code of Modeling Tens. Although modeling groups of 10 does not show evidence of clear place value understanding, this strategy does provide evidence of knowledge of the base ten number system and grouping by ten. If a participant modeled with ones (e.g., drew out each item in the question and then grouped by ten), I assigned the code of Modeling Ones. These participants showed some evidence of place value and grouping patterns in the base ten number system, but did not show as sophisticated knowledge as participants who drew one object for each ten.

Lastly, I assigned a code of No Place Value to any other justifications and written work that did not fit the categories of Clear Place Value, Modeling Tens, and Modeling Ones, and did not show any evidence of place value knowledge. For example, if a participant added the digits of the number in the problem (e.g., in 54, adding 5 and 4 together to find the answer), then I assigned a code of No Place Value because there was no evidence of place value knowledge.

After coding participants' justifications and written work, I assigned scores to the codes. I assigned a score of 3 to justifications that showed evidence of clear place value, which Carpenter et al. (2014) considers the most sophisticated level of place value knowledge (i.e., "direct" place value). For example, in item 1, participants who immediately say "7 boxes" when looking at the double-digit number 72 and justify their answer by pointing to the number “7” in “72” or say, “I know the answer is 7 because there are 7 tens” or say, “there are 7 groups of ten in 72” would be coded direct value knowledge. If a participant answered either 7 or 8, both responses would be accepted and assigned a score of 3 because the participant could have put the remainder in an additional box or container. However, if a participant answered 5, the response would be assigned a score of 0 because this shows that the participant did not identify how many “tens” or “groups of 10” would be necessary to solve the problem.

I assigned a score of 2 to justifications or written work that showed evidence of modeling by tens (e.g., drawing a simple object to represent the group of ten) because this demonstrates knowledge of the base ten number system, although not clear place value knowledge. Also, I assigned this score to both correct and incorrect answers because regardless of the answer, modeling by tens demonstrates some place value knowledge through an understanding of the base-ten number system and grouping by 10.

I assigned a score of 1 to justifications or written work that showed modeling by ones (i.e., drawing each item out and then grouping by ten) because this shows some evidence of place value knowledge and grouping by ten, but is not as sophisticated as drawing by groups of ten directly.

I assigned a score of 0 to all other justifications and written work that did not show evidence of place value knowledge. The total score of the Word Problems task was the sum of all the points assigned. Because there were three items, the minimum score was 0 and the maximum score was 6.

Table 1

The Place Value Word Problems Level 1 Coding and Scoring Rubric

Code	Description	Example	Score
1. Clear Place Value	Student demonstrates clear place value knowledge by providing a correct answer (e.g., for the number 64, 6 or 7 groups of ten would be accepted) immediately (i.e., within 10 seconds), and without modeling	Student looks at number on cue card and says, “The answer is 6 because I know there are 6 tens in 64.”	3
2. Modeling Tens	Student demonstrates place value knowledge, regardless of answer, through modeling groups of 10	Student draws 5 circles and 4 lines on paper and says, “The answer is 5 because I have 5 groups of 10.”	2
3. Modeling Ones	Student demonstrates place value knowledge by modeling by ones	Student draws 54 items on paper and then circles groups of ten.	1
3. No Place Value	Student does not demonstrate evidence of place value knowledge	Student looks at number on cue card (e.g., 54) and adds both digits together (e.g., 5 + 4)	0

Place Value Relational Thinking task. I watched the videotapes of the participants' interviews and coded their responses according to the coding and scoring rubric shown in Table 2. I only coded the participants' responses for the second part of each item, as only this part was designed to assess their place value understanding. I assigned codes to the responses based on whether there was evidence of place value knowledge in their justifications, in relation to the numbers used in the problem. Participants who demonstrate evidence of place value understanding through relational thinking, without relying on modeling, and obtain a correct answer, would be coded as Clear Place Value. For example, participants who said, "I knew the answer was 80 because it's just ten times the other number (8)," demonstrate place value understanding and did not use modeling.

Participants who demonstrate evidence of place value understanding through modeling (e.g., counting by tens using fingers or tapping on the table), and have a correct answer, would be assigned the code of Modeling. For example, participants who said, "The answer is 20 because 2 tens, 2 tens, 2 tens, and 2 tens…I gave 2 tens to each (child)" show evidence of using place value understanding to find the answer, but through modeling because of counting by tens with their fingers.

Lastly, participants who do not demonstrate any evidence of place value understanding or base ten concepts, whether it was by responding with a number that did not relate to the first part of the item or not showing evidence of the use of base ten concepts, such as grouping by 10, would be assigned a code of No Place Value.

I assigned scores to the codes as shown in the scoring rubric in Table 2. If a participant used relational thinking to solve the problems on the Relational Thinking task, I assigned a score of 2 to the response because this is the most sophisticated demonstration of place value understanding in this task (Carpenter et al., 1998; Carpenter et al., 2005; Empson et al., 2011). If a participant demonstrated place value understanding through modeling, and provided a correct answer, I assigned a score of 1 to the response because he or she used modeling to find the answer, which is less sophisticated than relational thinking or an invented algorithm, but still demonstrates evidence of place value knowledge. Lastly, if a participant showed no evidence of place value or base ten knowledge, then I assigned a score of 0 to the response. The total score of the Relational Thinking task was the sum of all the points assigned. There were two items on the Relational Thinking task, so the minimum score was 0 and the maximum score was 4.

Table 2

The Place Value Relational Thinking Level 1 Coding and Scoring Rubric

Code	Description	Example	Score
1. Clear Place Value	Student provides a correct answer and demonstrates evidence of place value understanding, without relying on modeling, through relational thinking (between the two parts of the problem)	1A. Child says, "I knew the answer was 80 because it's just ten times the other number (8)."	2
2. Modeling	Student provides a correct answer and demonstrates evidence of place value understanding through modeling (e.g., counting by 10s with fingers or tapping on table by 10s)	Child says, "The answer is 20 because 2 tens (lifts a finger), 2 tens (lifts another finger), 2 tens (lifts another finger), and 2 tens (lifts another finger) for each."	1
3. No Place Value or Base Ten	Student does not demonstrate evidence of place value understanding or base ten concepts.	Child responds with a number that does not relate to the previous number (e.g., says 8 and 62) and does not show evidence of use of base ten (no grouping by 10 or use of place value).	0

Video Evaluation Task. I watched the videotapes of the participants' interviews and coded their justifications according to the coding and scoring rubric shown in Table 3. I coded the justifications based on whether students showed evidence of place value or regrouping knowledge related to the specific error that the student made in the video. Participants who demonstrate clear evidence of place value or regrouping knowledge while evaluating the student in the video, would have their strategies coded as Clear Place Value and/or Regrouping Knowledge. For example, participants who justify their evaluation by using clear place value or regrouping concepts by saying, "she exchanged 12 when she should've taken a 10. It should be exchanging 10 ones for a ten" would be coded as Clear Place Value and/or Regrouping Knowledge.

Participants who demonstrate incomplete knowledge of place value or regrouping while evaluating the student in the video, would be assigned a code of Some Place Value and/or Regrouping Knowledge to his or her strategy. For example, participants who say, "she did the 7, but then she did 1, 2, 3 but then the 4, 5 she didn't put it. It would be 42, if we made a ten. But she thought 7 plus 5...she thought it was a ten but it wasn't," would have shown some evidence of place value knowledge in their justification, but not as much for the response to be coded Clear Place Value.

Participants who evaluate the student in the video and used some place value vocabulary, but it is unclear whether they had an understanding of place value, would be assigned a code of Can't Tell. For example, students could have demonstrated some knowledge about place value or regrouping, but there was no evidence to make a determination. An example would be if participants say, "Here should be a 1 (pointing to tens place)," then it would be difficult to know whether the participant is aware that the "1" actually represents a "ten" or whether the participant is just using the terminology of a "1" without understanding that this means "ten."

Participants who demonstrate a mathematical error related to place value or numeration such as by saying, "12 ones makes a ten and there are no leftovers," would be assigned a code of Place Value and/or Numeration Error. Participants who did not offer an evaluation of the student in the video (i.e., stated what the student did but did not make a judgment), or did not show evidence of place value, or did not show evidence of place value that was not related to the problem at hand (i.e., these are the tens and these are the ones), would be assigned a code of Other.

I assigned scores to the codes as shown in the scoring rubric in Table 3. If the participant showed evidence of clear place value or regrouping knowledge, I assigned a score of 2 because this is the most sophisticated kind of strategy in the Video Evaluation Task. If a participant showed some place value or regrouping knowledge, but was less complete than in the Clear Place Value code, I assigned a score of 1 because this participant did not show a full understanding of place value in his or her justification. If I could not tell from the participant's justification whether he or she had provided evidence of place value knowledge, I assigned a score of 0 because the participant's justification did not provide evidence of place value knowledge. This does not mean that the participant did not have place value understanding, only that their justification did not show enough evidence to say that he or she had place value understanding.

If a participant demonstrated a place value or numeration error, or did not include an evaluation of the student in the video, I assigned a score of 0. The total score of the Video

Evaluation Task was the sum of all the points assigned. The minimum score was 0 and the maximum score was 4.

Table 3

The Video Evaluation Task Level 1 Coding and Scoring Rubric

Code	Description	Example	Score
1. Clear Place Value and/or Regrouping Knowledge	Student demonstrates clear evidence of place value and/or regrouping knowledge while evaluating the student in the video	Student says, “She exchanged 12 when she should've taken a 10. It should be exchanging 10 ones for a ten.”	2
2. Some Place Value and/or Regrouping Knowledge	Student demonstrates some evidence of place value and/or regrouping knowledge while evaluating the student in the video	Student says, “She did the 7, but then she did 1, 2, 3 but then the 4, 5 she didn’t put it. It would be 42, if we made a ten. But she thought 7 plus 5...she thought it was a ten but it wasn’t.”	1
5. Can’t Tell	Student evaluates the student in the video and uses some place value vocabulary, but it is unclear whether he or she has an understanding of place value.	Student says, “Here should be a 1 (pointing to tens place).”	0
3. Place Value and/or Numeration Error	Student demonstrates a place value and/or numeration error while evaluating the student in the video	Student says, “She took 12 ones, so she could make a ten. So she made the ten and then its 40. There’s 4 tens and there no ones.”	0
4. Other	Student: a) does not evaluate the student in video (i.e., just recites what student did in video but	4A. Student says, “First she takes the tens and then she takes the ones and she got 40.” 4B. Student says, "Because she used the numbers	0

doesn’t make a judgment about it) or, b) does not show evidence of place value c) shows evidence of place value but is not related to the problem’s context	like that.” 4C. Student says, “These are the tens and these are the ones.”

Correlation between the Video Evaluation Task, the Word Problems task, and the Relational Thinking task

To answer my first research question of whether the Video Evaluation Task is related to the two other typical measures of place value, I ran a correlation between the scores on the Video Evaluation Task and the Word Problems task, and the scores on the Video Evaluation task and the Relational Thinking task. There was a significant correlation between the Video Evaluation Task and the Word Problems task ($r = 0.347$, $p = 0.02$) and between the Video Evaluation Task and the Relational Thinking task ($r = 0.371$, $p = 0.012$). While these significant correlations indicate that these measures tap into similar constructs, there are individual cases that diminish the strength of the correlations. These cases illustrate the different nuances in students' place value understanding, which allows me to investigate the different affordances of each measure.

Affordances of the Video Evaluation Task Relative to Two Other Measures

In the following section, I will describe how I answered my second research question addressing the affordances of the Video Evaluation Task in comparison to the other two more typical measures of place value. I will describe the second level of coding that I developed to be able to compare the Video Evaluation Task with the other measures and provide descriptions of participants' justifications as examples of this coding.

Level 2 Coding. I developed a second level of coding so that each student received one code per measure to facilitate the comparison between the Video Evaluation Task and the other measures. For each participant, on each measure, I combined the codes assigned to each item and created one overall code. This was accomplished by examining the code distribution across the items. For participants with the same codes assigned on each item, I assigned the same level 1 code for that measure (or renamed it for comparison purposes). For participants who had different codes on each item, I assigned the "Developing Place Value" to the student for that measure. These participants' place value understanding was at a developing stage, reflected by the variety of different responses on the same measure.

Place Value Word Problems task. I coded all three items on the Word Problems task. The majority of participants (34 of 45, 75.5%) had the same code across all three items. For these participants, the three items collectively were assigned one Level 1 code of the Word Problems task. For example, if a participant had the Clear Place Value code assigned to all three items, then the participant's responses would be collectively coded as Clear Place Value on Level 2 Coding. To provide descriptions of participants' conceptual understanding across the three measures, I decided to combine the Modeling Ones code with the No Place Value code to highlight how Modeling Ones differs from the more sophisticated strategies, Clear Place Value and Modeling Tens. For three participants, their Modeling Ones code was replaced with the No Place Value code to simplify the coding.

Eleven participants had different codes across the three items of the Word Problems task. Ten of these participants were placed in a new category called Developing because they used a combination of strategies on the task, which could be evidence of participants' developing understanding of place value. The remaining participant, who had a combination of the codes of No Place Value and Modeling Ones, was placed in the code of No Place Value because this participant's strategies were less sophisticated compared to the participants in the Developing code. The frequencies of the final codes for the Word Problems task are shown in Table 4.

Table 4

Frequencies of Final Codes After Collapsing Items for Word Problems Task

Final codes	Frequency
Clear Place Value	25
Developing	10
Modeling	5
No Place Value	5
Total	45

Place Value Relational Thinking task. I coded the two items on the Relational Thinking task. The majority of participants (33 of 45, 73%) had the same code on both items. For the participants who had the Clear Place Value code assigned to both items, they were assigned an Clear Place Value code on the Relational Thinking task. The same coding procedure was used for participants who were assigned the Modeling Place Value on both items. However, 12 participants had different codes for each item. All of these participants were assigned a new code called Developing because these participants showed evidence of developing place value understanding reflected by the combination of strategies used across both items. The frequencies of the final codes for the Relational Thinking task are shown in Table 5.

Table 5

Frequencies of Final Codes after Collapsing Items for Relational Thinking Task

Final codes	Frequency
Clear Place Value	16
Developing	12
No Place Value	17
Total	45

Video Evaluation Task. I coded the two items of the Video Evaluation Task. First, I renamed the Level 1 code of Some Place Value to Developing Place Value, as this code included participants who had different ranges of developing place value knowledge, and also to ake the codes consistent on the other measures. The majority of participants (34 of 45, 76%) had the same code for both items on the Video Evaluation Task. These participants were assigned a Level 1 code. For example, if a participant had the Clear Place Value code assigned to both items, then the participant would receive a Clear Place Value code on the Video Evaluation Task.

Eleven of 45 participants had different codes for each item, however. These 11 participants' codes were collapsed into different codes. Participants who had the code of Other/Unrelated on one item and the Place Value Error code on the other item were assigned the Place Value Error code because they had at least one item with a place value error. Participants who had the code of Other/Unrelated on one item and the Developing code on the other item were assigned the Developing code because their place value understanding was developing. All other combinations of codes were also assigned the Developing code. For example, participants who had the code of Clear Place Value on one item and Can't Tell on the other item were assigned the Developing code. The final frequency of participants for each code is shown in Table 6.

To compare the Video Evaluation Task to the Relational Thinking task and the Word Problems task, I also collapsed the codes Place Value Error, Can't Tell, and Other/Unrelated to a code named No Place Value as shown in Table 6. All of these codes were originally assigned a score of 0 for the correlational analyses. After looking at the descriptions of the participants' justifications, I saw that it was appropriate to collapse the codes because there was no additional information provided by participants when comparing the separate codes to one another.

Table 6

Frequencies of Final Codes for Video Evaluation Task

Final codes	Frequency
Clear Place Value	22
Developing Place Value	10
No Place Value	13
Place Value Error	2
Can't Tell	2
Other/Unrelated	9
Total	45

Comparison of Video Evaluation Task to Word Problems task. As shown in Table 7, almost a third of participants (*n* = 14) had a code of Clear Place Value on the Word Problems task as well as a code of Clear Place Value on the Video Evaluation task. These 14 are represented in the top left cell of Table 7. Also, three participants had a code of No Place Value on both the Word Problems task and the Video Evaluation task. These three participants are represented in the bottom right cell of Table 7. Therefore, I conclude from this that the Video Evaluation Task did not afford more information about students' conceptual understanding of place value for these participants (*n* = 17) because they received the same code on both measures. Moreover, for these 17 participants, the Video Evaluation task allows one to conclude that students have the capacity to apply their place value knowledge to evaluate someone else's work. For example, participants coded with Clear Place Value on the Word Problems task demonstrated an understanding of the value of a place within a double-digit number (i.e., ones and tens). If these participants were also coded with Clear Place Value on the Video Evaluation Task, then they also demonstrated that the student was able to transfer or use their conceptual understanding to a different context and one that involves evaluation. The additional information provided by the Video Evaluation Task in these cases is inherent in the task's evaluation component, which is not present in the Word Problems task.

On the other hand, the Video Evaluation task did afford more information about students' conceptual understanding of place value for participants who demonstrated different levels of understanding on the Word Problems task and the Video Evaluation task (*n* = 28). These 28 participants are represented in all other cells in Table 7. Below, I present descriptions of certain participants' responses on the Word Problems task and the Video Evaluation Task to illustrate the different affordances of each task.

Table 7

Number of Participants by Coding Type across Word Problems Task and Video Evaluation Task

		Word Problems				
		Clear Place Value	Modeling	Developing	No Place Value	Total
Video Evaluation	Clear Place Value	14	3	4	1	22
	Developing Place Value	6	1	2	1	10
	No Place Value	5	1	4	3	13
	Total	25	5	10	5	45

Kailey[1]**.** This participant's strategies were coded as No Place Value on the Word Problems task because she used a modeling ones strategy (i.e., drew out all the items and then circled by groups of ten). However, on the Video Evaluation Task (25 + 17 and 18 + 36) her justifications were coded as Clear Place Value because she successfully evaluated the student in the video by identifying the error by saying, "so she forgot to leave the 2 here (pointing to the ones)" and by demonstrating evidence of place value and/or regrouping knowledge by saying, "if you have 12, you leave the 2 in the ones and you put the tens in the tens, and you count it all together in the tens section" and "8 plus 6 is 14, and she left the 4 there (points to ones) but she forgot to put the little one in the tens section."

Interestingly, although Kailey did not use a sophisticated strategy on the Word Problems task such as modeling by tens or applying clear place calue knowledge on the Video Evaluation Task, she did demonstrate knowledge of regrouping (i.e., "if you have 12, you leave the 2 in the ones" and "8 plus 6 is 14, and she left the 4 there [points to ones]") and place value (i.e., "you put the tens in the tens, and you count it all together in the tens section"). In this case, the Video Evaluation task provided an opportunity for the participant to demonstrate further place value understanding than in the Word Problems task alone. It is possible that this participant's understanding of grouping by tens is not as developed compared to her knowledge of the value of the place in a written number.

Jake. This participant's strategies were coded as No Place Value on the Video Evaluation Task because he could not find the error and evaluate the student. He also used some place value terminology, but did not present evidence of place value understanding. In contrast, on the Word Problems task, his responses were coded as Clear Place Value because he immediately identified

[1] All participants' names are pseudonyms.

how many groups of ten there were by just looking at the number in the problem. Therefore, the Video Evaluation task provided an opportunity to assess whether Jake could transfer his conceptual understanding of place value to a double-digit addition context while evaluating someone else's work. The lack of evidence of place understanding on the Video Evaluation Task highlights that the Word Problems task on its own is not sufficient to demonstrate students' complete place value understanding because of the discrepancy between Jake's evidence of understanding on each task. Although he was capable of identifying how many groups of ten there were when he looked at a number, there was no evidence of his understanding of other components of place value, such as understanding what the place of "tens" and "ones" means, and regrouping.

Sara. This participant's strategies were coded as Developing on the Video Evaluation Task because her strategies fluctuated between demonstrating some place value understanding on some items but not on others. In contrast, she was assigned Clear Place Value on the Word Problems task. Therefore, the Video Evaluation task provided some insight on the nuances of her understanding, despite the fact that she provided evidence of clear place value understanding on all three items of the Word Problems task. For example, on the Video Evaluation Task, she said, "It equals 54. I think she did it wrong, because she didn't add the 1 at the top there. She didn't get the 5 (referring to the tens)." Although Sara was able to demonstrate clear place value understanding on the Word Problems task, demonstrating her ability to identify groups of ten in a number, her conceptual understanding of place value seemed to be more tenuous as assessed by the Video Evaluation task.

George. George's strategies were coded as No Place Value on the Word Problems task because he either added the digits together to find the answer (i.e., in the number 84, did 8 + 4), drew out each item and then grouped by ten (i.e., modeled by ones), or tried to remove 10 items per box but could not find the answer in the end (i.e., wrote out 72 – 10, 62 – 10, etc.). However, on the Video Evaluation Task, George's strategies were coded as Developing because he demonstrated clear evidence of place value knowledge on one item and a justification coded as Can't Tell on the other item. On the item 18 + 36 of the Video Evaluation Task, George said, "8 plus 6 is not 4. 8 plus 6 is 14. So here is supposed to be a 4 (pointing to the ones) and here a one (pointing to the top of the tens). 3 plus 2 makes 5 so here should be a 5 (pointing to the tens). So it should be 54." On this item, George identifies the student's error, differentiates between the place of tens and ones, and provides the correct answer.

On the item 25 + 17, however, George answers differently than he did on the 18 + 36 item. George said "12 plus 1 is not 40... 12 plus 1 is 13. She said now that's 12 and plus 1 is 40... it's not 40 (counts). Here it should be a 3 (points to ones in the sum). 5 plus 7 is 12. So here it should be a 2 (points to ones in the sum). The answer would be 32...42….uh…." On this item, it seems that George might have a misconception about the place of the tens and ones when he says "12 plus 1 is not 40…12 plus 1 is 13," but the rest of his response is difficult to determine place value understanding. In this case, the Video Evaluation Task afforded more information by revealing some place value understanding, but that it is still developing, as shown in George's response on the second item. Although George did not show evidence of a sophisticated strategy on the Word Problems Task, he was able to show some place value knowledge on the Video Evaluation Task.

Comparison of Video Evaluation Task to Place Value Relational Thinking task. As shown in Table 8, almost a fourth of participants (11 of 45, 24%) had a code of Clear Place Value on the Relational Thinking task and Clear Place Value on the Video Evaluation task.

Additionally, seven participants (7 of 45, 15.5%) had a code of No Place Value on both the Relational Thinking task and the Video Evaluation task. Therefore, the Video Evaluation Task does not appear to afford more information about students' conceptual understanding of place value for these participants (*n* = 18) because they received the same code on both measures. However, as with the Word Problems task, for these participants, the Video Evaluation task did provide additional information about a participant's application of his or her place value knowledge in an evaluation context. For example, a participant who is coded with Clear Place Value on the Relational Thinking task demonstrates an understanding of place value through relational thinking. If this participant is also coded with Clear Place Value on the Video Evaluation Task, then this participant also demonstrates an understanding of base ten concepts and/or place value, but through the context of a double-digit addition problem solved by someone else.

The Video Evaluation task appeared to afford more information about students' conceptual understanding of place value for those participants who demonstrated variability in codes on both the Relational Thinking task and the Video Evaluation task (*n* = 27). For example, this could be a participant who received a code of Developing on both of the tasks (i.e., demonstrating different levels of place value understanding on each measure), or a Clear Place Value code on the Relational Thinking task and a No Place Value code on the Video Evaluation Task. Below are some examples of participants' justifications on the Relational Thinking task and the Video Evaluation Task to compare the affordances of each measure.

Table 8

Number of Participants by Coding Type Across Relational Thinking Task and Video Evaluation Task

		Relational Thinking			
		Clear Place Value	Developing	No Place Value	Total
Video Evaluation	Clear Place Value	11	6	5	22
	Developing	3	2	5	10
	No Place Value	2	4	7	13
	Total	16	12	17	45

Holly. Holly's strategies were coded as No Place Value on the Relational Thinking task because she did not demonstrate any evidence of place value knowledge either through relational thinking or an invented algorithm. However, on the Video Evaluation Task, she demonstrated clear evidence of place value knowledge. This shows that even if a participant does not demonstrate evidence of relational thinking, the Video Evaluation Task provides an opportunity for her to exhibit place value understanding in the context of a double-digit addition problem. For example, on the Video Evaluation task's item 18 + 36, she said, "she's wrong because 14 (pointing to the 8 and the 6) you are supposed to add a ten there (pointing to the top of the 1 in the tens) and she didn't," but on the Relational Thinking task said that the answer would be the same for the second part of the item (i.e., did not use a different strategy even when the problem had different numbers). It is possible that Holly did not yet develop the capacity for relational thinking or think about solving the problem with an invented algorithm, yet understood that the place of a digit a number has a specific value. It is also possible that this student may have answered differently on a task she's never seen before (i.e., prior exposure). This student may have also not been accustomed to using prior problems to think about subsequent ones (see Richland et al., 2012).

John. John did not show evidence of place value understanding on the Video Evaluation Task, but did show evidence of clear place value on the Relational Thinking task. Although he technically did not show evidence of place value on the Video Evaluation task, the task still afforded more information than the Relational Thinking task on its own because it provided the opportunity for John to demonstrate the nuances of his developing understanding. For example, on the Video Evaluation task, for the item 18 + 36, he said, "she did it wrong because 8 plus 6 does not equal 4…it equals (8, 9, 10, 11, 12, 13, 14...counts by ones) 14, but this is right (pointing to tens in the addition)…yes the tens….so it would be I think 56? 58?" Here, it is difficult to tell where John's place value understanding lies because his statement about 8 + 6 being equal to 14 does not connect to the statement that the tens in the addition is correct, when in fact there was an error in the tens place.

On the item 25 + 17, John said, ""I think the 12 part is wrong. She put the 2 beside the 1 (pointing to the tens in the addition). She should put the 2 together with the 7. The answer should be 69." On this item, John starts off by saying that the 12 (5 + 7) part is wrong, which is accurate because the student doesn't write the 2 ones for the 12. Afterwards, however, when John says, "she should put the 2 together with the 7. The answer should be 69," it is difficult to know exactly what he means and where his place value understanding lies. The Video Evaluation task, then, provides evidence that John is developing his knowledge of ones and tens in the context of an addition problem. On the Relational Thinking task however, John showed some understanding at an abstract level because he used an invented algorithm on both items of the task. Interestingly, it is difficult to decipher where John stands in terms of place value understanding on the Video Evaluation task, but it is evident that he has the capacity to apply place value concepts as evidenced by his response on the Relational Thinking task.

Karina. This participant did not show evidence of place value understanding on the Relational Thinking task, but did show developing understanding of place value on the Video Evaluation Task. For example, on the Video Evaluation Task, with the item 25 + 17, she said, "Oh it's 42. The mistake is in the 0 (points to the ones). She should have put a 2 there." And on the 18 + 36 item said, "She didn't put the one on top (points to the tens)." Although Karina may not have named the ones and tens appropriately, she demonstrated evidence of place value understanding by explaining the error correctly and appropriately identifying where the ones and tens should be. Therefore, seeing as Karina was not able to answer the items on the Relational Thinking task, the Video Evaluation Task provided more information about her place value understanding.

Chapter 5: Conclusions

This study showed that The Video Evaluation Task provides a novel way to assess students' conceptual understanding of place value. I found that the Video Evaluation Task assessed a similar construct to other more typical place value assessments because it was correlated with the Word Problems task and the Relational Thinking task. I am not arguing, however, that the Video Evaluation Task can be used in absence of the other measures if the objective is to get a complete picture of students' place value understanding. In fact, this study provided evidence that if one measure is used on its own, it is possible that some evidence of students' place value understanding may be missed because of the nuanced information each type of measure affords.

To illustrate, when I compared the Word Problems task with the Video Evaluation Task, I found that some participants' grouping by tens knowledge was not as developed as compared to their knowledge of the value of the tens and ones places in a number. I found that for the students who were coded as Clear Place Value on both the Video Evaluation Task and the Word Problems task, the Video Evaluation Task afforded more information. I argue that this is not necessarily because the participants in fact used "clearer strategies." Rather, the Video Evaluation Task afforded more information in reality because of the measure's ability to demonstrate more developing kinds of place value.

Furthermore, when participants did not show as much evidence of place value knowledge on the Video Evaluation Task as compared to the Word Problems task, The Video Evaluation Task still afforded more information because it showed that the Word Problems task on its own was not sufficient to demonstrate participants' complete understanding of place value. For example, on the Video Evaluation Task, some participants demonstrated discrepancies of other place value aspects, such as the value of the place of tens and ones and regrouping, which was revealed through their justifications.

When I compared the Relational Thinking task with the Video Evaluation Task, I found that participants' relational thinking was indeed related to their place value understanding assessed in an evaluation context. Importantly, however, there were some participants who showed evidence of relational thinking but were not necessarily able to evaluate another student's thinking, and other participants who were capable of evaluating, but did not show evidence of relational thinking in their strategies. This point highlights that further research is needed to understand what kinds of place value understanding are required for both relational thinking and the evaluation of another student's strategy, and how these concepts transfer to other aspects of place value understanding.

Therefore, the Video Evaluation Task allowed me to conclude that, in place value, students seem to have clearer understanding in certain contexts, but then do not show the same understanding in another. I would argue, then, that this study provides some evidence that place value is not "all or nothing": Students can have connected or disconnected place value knowledge. This shows that indeed, according to Hiebert and Lefevre's (1986) definition of conceptual knowledge, stronger conceptual knowledge is that which is more connected. For example, participants who demonstrated clear place value knowledge on the Word Problems task, but then did not show place value knowledge on the Video Evaluation task, led me to conclude that students may be able to identify "tens" and "ones" in a double digit number, or even groups of ten in a word problem, but might not have a deeper understanding of the meaning of place value beyond this (see Osana & Blondin, 2017), such as regrouping or identifying "tens" and "ones" in an addition context. Furthermore, there are several components of place value knowledge that are important to assess when trying to evaluate students' place value understanding as a whole. Therefore, using multiple measures that provide different contexts in which to assess students' place value knowledge, and perhaps other mathematical concepts more generally, is vital.

I argue that using multiple measures is especially important in the place value domain because there are other areas of mathematical understanding that may or may not relate to place value understanding, such as relational thinking. Although this study has shown that relational thinking and place value understanding are indeed related, how this develops in combination with students' place value understanding is still uncertain and warrants further investigation.

Theoretical and Practical Implications

This study has several theoretical and practical implications. Firstly, this study confirms Bisanz and Lefevre (1992) and Crooks and Alibali's (2014) claim that evaluation can serve as an informative assessment of students' conceptual understanding by examining their comments on another student's entire strategy (i.e., steps and final answer). As Bisanz and Lefevre (1992) and Crooks and Alibali (2014) described, evaluation provides another level of assessment of conceptual understanding, which could be of interest to teachers' in their practice. The authors also maintained that students are successful with an evaluation assessment if they have already mastered application and justification types of assessment. With this in mind, teachers, then, could use this kind of information to better situate their students' understanding and conceptual difficulties to better move their learning forward.

Indeed, this study has shown that it is possible that some students who have the capacity to evaluate another student's work may have a deeper understanding of place value. For example, as Bisanz and Lefevre (1992) and Crooks and Alibali (2014) suggest, a participant who showed evidence of clear place value on the Word Problems task and clear place value on the Video Evaluation Task showed evidence of place value through all three assessment types: application, justification, and evaluation. The results of this study, however, indicated that, at least in place value, it is not necessary for students to completely master and application and justification to be successful on evaluation tasks, and being successful on evaluation does not imply that they are able to apply and justify their knowledge. In fact, my data show that there are many components to place value knowledge and that they can be detected through any number of assessment approaches. More precisely, some students in my study were able to apply and justify their place value knowledge without being able to evaluate, and in other cases, students were able to evaluate without evidence of application or justification.

The Video Evaluation Task is a suitable assessment to reveal students' developing place value understanding. Even in the cases where the students were not able to demonstrate deep place value knowledge on the Video Evaluation Task, the measure still provided crucial information about possible misconceptions of place value and developing understanding of certain aspects of related concepts. This kind of task may be helpful for teachers who are interested in gaining a clearer picture of their students' conceptual understanding, which can be missed by other, more typical, classroom assessments.

The Video Evaluation Task could also be relevant for teachers' practices by providing a context for students to talk about a mathematical problem in real time. Teachers have struggled to assess students' strategies and their mathematical understanding (Even & Wallach, 2004; Wallach & Even, 2005). Despite these struggles, teachers who incorporate their students' thinking within their instructional practices are well suited to guide their students' knowledge growth. Indeed, teacher education programs are moving towards training teachers to better understand their students' thinking and integrate this information into their instruction (e.g., Cognitively Guided Instruction, Carpenter, Fennema, & Franke, 1996). Wallach and Even (2005) have shown, however, that discrepancies still exist between what students say and do when approached with a mathematical problem, and what teachers hear. The Video Evaluation Task, then, could provide an opportunity for the student to talk more clearly by commenting on someone else's work rather than their own, and for the teacher to better understand students' conceptual understanding of place value within a set scenario. More specifically, teachers have struggled with assessing students' thinking because asking the right kind of questions, at the right time, has still proved to be challenging (Franke et al., 2009). The Video Evaluation Task could place teachers in a good position to ask targeted questions because of the assessment's evaluation nature, leaving less ambiguity about students' understanding.

The Video Evaluation Task provides an assessment for students' conceptual understanding in a domain that is particularly useful for teachers because place value is a core concept that cuts across the entire elementary school curriculum. Although I found several evaluation assessments in the literature that exist in different domains, to my knowledge, there was no such task for place value, a topic that concerns elementary teachers in the classroom everyday. The Video Evaluation Task certainly could be adapted to suit other mathematical domains. Given that assessing place value is particularly challenging, though, the Video Evaluation Task would allow teachers to gain insight into students' thinking about a domain that would affect their later learning in other areas, such as operations.

Future Directions

Although this is the first study to investigate the affordances of the Video Evaluation Task, the descriptive comparison of the task to other place value tasks has acted as a stepping stone for future studies. Firstly, the Video Evaluation Task yields data that are richer and more informative than data from more typical assessments, such as those that involve application and justification without evaluation. This would mean, then, that researchers using the Video Evaluation Task have the challenge of trying to extract meaning from this kind of data. For example, a student may show many different aspects of his or her place value understanding, such as being able to compare numbers represented symbolically, but is not necessarily able to indicate how many groups of ten there are in a number. In a situation like this, it would be challenging for a researcher to draw conclusions about this student's place value understanding. Therefore, it might be useful to more deeply investigate the nature of a student's mathematical understanding with fragmented place value understanding.

Another related area for future research is determining the nature of place value understanding that is required to be successful on the Video Evaluation Task. The results of this study have shown that some participants had the capacity to evaluate, whereas others were less able to do so. It would be important to know what kinds of factors would be predictive of success on the Video Evaluation Task as compared to other place value assessments, and how this might be related to students' ability to transfer their place value knowledge to other problems.

Secondly, it is possible that the participants in this study were not accustomed to discussing student thinking in the classroom. Teachers who are procedurally oriented typically show students the correct procedures leading to correct answers. Also, teachers who have a procedural style of teaching may not talk about students' strategies in the classroom, let alone other students' strategies. Therefore, participants in this study may have had difficulty on the Video Evaluation Task if they were not accustomed to a task like this. In the future, it would be useful to design an evaluation task with multiple items to ensure that participants have ample opportunity to demonstrate their ability to evaluate and justify their responses.

Further, participants could have had difficulty with the concept of finding an error in another student's problem. For example, some participants did not initially find the error in the first item of the Video Evaluation Task, but then were able to find it on their own on the second item. Some participants may not have been used to looking for, or talking about, strategies that have errors. Therefore, it would be worthwhile to design the Evaluation Task with items involving both correct and incorrect strategies so that they are made aware of the possibility that some strategies may be incorrect.

Moreover, I have not found any classroom- or research-based assessments in the domain of place value that have asked students to talk about another's errors. This could be an interesting venue for future assessments of students' conceptual understanding for several reasons. Firstly, the Video Evaluation Task places students in a novel situation, which lowers the chances that prior exposure would influence their responses on the assessment. Furthermore, the Video Evaluation Task provides the opportunity for an assessment to isolate conceptual understanding of place value without a focus on procedural knowledge, which is a primary consideration in the design of effective learning environments in mathematics.

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Appendix A

Post-Interview Protocol

RYT

Post-Intervention Interview Protocol

Set-up and Introduction

List of materials:	
- Cards (Deck A, B, D) - Scoring sheets - Markers - Blank paper	- Camera - Microphone - Tripod - iPad
General Set-up and instructions:	
Place the camera so that the child's hands and the area in which they will work are clearly visible. The child's face must NOT be visible. *Make sure to put the child's* ***<u>participant ID on EVERY sheet</u>*** *related to that child (including the child's work on the blank pages).*	
Introducing the interview:	
Do you remember when we did some math activities with you in front of the camera last time? We are going to do some math activities again today. Like last time, I would like to know how you think while you do these activities, so I will ask you questions to help me understand what you are thinking. Don't worry if you don't know if you're right or not, what is important to me is that you try your best and that I understand what you are thinking. **Do you have any questions before we start?**	

Part A: Place Value Word Problems

<u>Materials:</u>	
- Cards: Deck B - Marker	- Blank paper
<u>Set-up:</u>	
Place the marker and blank paper in front of the child. Read each problem twice to the child.	
<u>Introduction to the task:</u>	
I'm going to ask you some questions now. I will read each question twice, but you can ask me to read it again at any time if you need me to. **Here you have a marker and paper. If you want, you can use them when you try to figure out the answer, but you don't have to if you don't want to.**	
<u>The task:</u>	
Place the corresponding card for the given problem in front of the child. Read each problem twice to the child. *Once the child has given an answer, ask:* **How did you figure that out?**	

Repeat the above for each of the problems. **Be sure to indicate on the child's work which problem it corresponds with.*
<u>Word problems:</u>
Look at the number on this card *(circle the number 72 on the card with your finger)*. **This is how many markers Justin has. He wants to put his markers in boxes that can fit 10 markers. How many boxes can he fill up completely with the markers he has?** **Look at the number on this card** *(circle the number 49 on the card with your finger)*. **This is how many doughnuts Elsa has. She puts the doughnuts in containers that can fit 10 doughnuts. How many containers will she fill up completely with the doughnuts she has?** **Look at the number on this card** *(circle the number 84 on the card with your finger)***. This is how many goldfish Amanda has. Because the fish need space to swim, Amanda can only put 10 goldfish in each fishbowl. How many fishbowls will Amanda be able to put 10 goldfish into?**

Part B: Place Value Relational Thinking

<u>Materials:</u>	
- Marker	- Blank paper
<u>Set-up:</u>	
Place the marker and blank paper in front of the child. Read each problem twice to the child.	
<u>Introduction to the task:</u>	
We are going to do some more questions now. I will read each question twice, but you can ask me to read it again at any time if you need me to. **Some times I'll let you use the marker and paper, but not all the time. I'll tell you when I don't want you to use it. But like last time, you don't have to use it if you don't want to.**	
<u>The task:</u>	
<u>**ITEM 1** - ***WITH*** *marker and paper:*</u> **There are 6 sandwiches on the table. If you want to 2 sandwiches in each container, how many containers will you need?** *Once the child has given an answer, ask:* **How did you figure that out?** *If the child had not written down their answer, ask them to write it down on the paper in front of them (they do not have to show their work, just write down the answer).* <u>***WITHOUT*** *marker:*</u> **Now, if there were 60 sandwiches, how many containers of 2 sandwiches would you need?** *Once the child has given an answer, ask:* **How did you figure that out?** *Remove the worksheet from in front of the child and replace it with the next worksheet.* **Be sure to indicate on the child's work which problem it corresponds with.*	
<u>**ITEM 2** - ***WITH*** *marker and paper:*</u> **There were 5 cookies in each box. If there are 3 boxes, how many cookies would there be altogether?**	

Once the child has given an answer, ask:

How did you figure that out?

If the child had not written down their answer, ask them to write it down on the paper in front of them (they do not have to show their work, just write down the answer).

***WITHOUT** marker:*

Now, if there were 50 cookies in each box, how many cookies would there be altogether?

Once the child has given an answer, ask:

How did you figure that out?

Remove the worksheet from in front of the child and replace it with the next worksheet.

ITEM 3 - ***WITH** marker and paper:*

There are 8 chocolate bars. Four friends want to share the chocolate bars so that each friend gets the same amount. How many chocolate bars should each friend get?

Once the child has given an answer, ask:

How did you figure that out?

If the child had not written down their answer, ask them to write it down on the paper in front of them (they do not have to show their work, just write down the answer).

***WITHOUT** marker:*

Now, if there were 80 chocolate bars, and the 4 friends wanted to share them equally, how many chocolate bars would each friend get?

Once the child has given an answer, ask:

How did you figure that out?

Part C: Video Evaluation Task

Materials:
Cards: Deck D iPad
Set-up:
Place the iPad in front of the child. Before each video, place the corresponding card from Deck D next to the iPad.
Introduction to the task:
We gave this problem to Anna and videotaped her solving the problem *(show the cue card to the child)*. **The problem is # + # = blank. I'm going to show you the video and afterwards we will talk about what Anna did.** **Let's watch how she solves the problem. If you need to pause or stop the video, you just have to ask me. First, I will show you the video twice.**
The task:
Show the video twice to the child. *(If child speaks during video, pause video and acknowledge their comment and continue)* *After video was shown twice, ask:* **Tell me what you think about how she solved it?** *If child seems hesitant, we can ask them if they need to see the video again* *If the child hasn't referred to whether Anna solved the problem correctly or not by the end of their explanation, ask:* **Did the child in the video get it right or wrong?** **Why do you think she got it right/wrong?** *Probe until you have a good understanding of the child's thinking/reasoning.* **If the child seems stuck, you may offer the child to watch the video again.* <u>Examples of other probes</u> What did she do to get it right/wrong? Can you tell me more about that? What is it in the video that makes you think that? What do you mean by […] (eg. "the right order")? What is it about that order that's not right? Why? What do you mean by (their answer) **(Make sure the probes always focus on what is going on in the video and not on not what the child you are interviewing would do)**
If the child <u>gets it right away</u> (that the child in the video is correct or incorrect and explains their thinking completely), then do not show the video again and <u>move on to the</u>

next item.
If the child doesn't get it right away, continue to probe their thinking and offer to show the video as many times as he or she needs. *Once the child has completed his or her answer/reasoning, and the student has indicated whether the child in the video got it correct/incorrect when it's really the opposite, say:* **Well, actually, it turns out that she got the right answer (idiosyncratic) OR the wrong answer (standard). Can you explain to me now what she did?** *If child is still insisting, do not argue. Just ask them why, and move on to the next item.* *Repeat the above for each video.*

Part D: General Guidelines

Unpacking:
It is ok to unpack a problem for a child so that they understand the situation in the problem. We want to avoid giving them any hints as to how to solve the problem, however. We can also answer children's questions about a given problem as long as we are not providing them with clues as to how to solve the problem. For example, consider the following problem: *This is how many stickers Mary has (54). She pastes them in her sticker book so that there are 10 stickers on each page. How many pages can she fill up completely?* A child may ask how many stickers are needed to fill a page. It is fine to tell them that 10 stickers fill the page completely. A child may also ask how big the stickers are. Again, it is ok to explain that the stickers are all about the same size and that there are only 10 spots on the page for them. Children often ask what they need to do to answer the problem. For example, they may ask if they need to subtract to find the answer. We do not want to give them any indication of what to do, so we need to answer this type of question as neutrally as possible. In response to this question we can simply answer, "I don't know, what do you think you need to do?" If a child seems completely lost as to what to do and they have not yet attempted to solve the problem through drawing, it is ok to suggest to them to try to solve the problem using drawings. Do NOT, however, tell them how to draw it (e.g., do NOT say, "Try drawing the 54 stickers and see what you can do from there").
Encouragement:
Generally when giving encouragement we want to avoid giving any indication as to whether the child is getting the write answer or not. Therefore, we want to encourage the child's effort in general and not their specific solution strategy. Things to say: I can see you are trying very hard. I like that you are trying so hard. I can see that you're thinking really hard about this. Thank you for working so hard on this. Thank you for your explanation, it made it much easier for me to understand what you were thinking.

Things NOT to say: That's right. Good thinking. Good job. If a child asks if their answer is correct or not, remind them that you are more interested in what they are thinking than if they are right or not. Emphasize that we want them to try their best to get the correct answer but that the most important thing to us is understanding what they are thinking.
Eliciting:
When trying to get a child to explain their thinking we need to do so while avoiding calling specific attention to what they are doing incorrectly (even if we do not say it is incorrect) or using questioning to funnel their thinking in any direction. For example, one could say: "Tell me how you did that?" or "How did you think about that to get that answer?" Do NOT say: "Why did you get that answer here but something different there?" (This may cause them to question their solution)

Appendix B

Post-Interview Scoring Sheet

Scoring Sheet

Part A: Word Problems

Item 1: 72 markers: ___________ Used worksheet: Y / N
Item 2: 49 doughnuts: ___________ Used worksheet: Y / N
Item 3: 84 goldfish: ___________ Used worksheet: Y / N

Part B: Relational Thinking

Item 1: 6 sandwiches: ___________
60 sandwiches: ___________
Item 2: 5 cookies, 3 boxes: ___________
50 cookies, 3 boxes: ___________
Item 3: 4 share 8 chocolate bars: ___________
4 share 80 chocolate bars: ___________

Part C: Video Evaluation Task

Video 1:

17 + 18
Anna is
Correct

Child INITIALLY assesses video as
Correct______Incorrect_______

If **INCORRECT**: Child's second assessment of the video is
Correct______Incorrect_______

Video 2:

25 + 17
Anna is
Incorrect

Child INITIALLY assesses video as
Correct______Incorrect_______

If **CORRECT**: Child's second assessment of the video is
Correct______Incorrect_______

Video 3:

18 + 36
Anna is
Incorrect

Child INITIALLY assesses video as
Correct______Incorrect_______

If **CORRECT**: Child's second assessment of the video is
Correct______Incorrect_______

Video 4:

19 + 12
Anna is
Correct

Child INITIALLY assesses video as
Correct______Incorrect_______

If **INCORRECT**: Child's second assessment of the video is
Correct______Incorrect_______