Revisiting the Environmental Kuznets Curve in the Presence of Trade

Biva Arani Mallik

A Thesis

In the Department

of

Economics

Presented in Partial Fulfillment of the Requirements

For the Degree of

Doctor of Philosophy (Economics) at

Concordia University

Montreal, Quebec, Canada

June 2018

CONCORDIA UNIVERSITY
SCHOOL OF GRADUATE STUDIES

This is to certify that the thesis prepared

By: Biva Aranl Mallik

Entitled: Revisiting the Environmental Kuznets Curve in the Presence of Trade

and submitted in partial fulfillment of the requirements for the degree of

Doctor Of Philosophy (Economics)

complies with the regulations of the University and meets the accepted standards with respect to originality and quality.

Signed by the final examining committee:

______________________ Chair
Dr. Graham Dodds

______________________ External Examiner
Dr. Hassan Benchekroun

______________________ External to Program
Dr. Julian Schofield

______________________ Examiner
Dr. Szilvia Papai

______________________ Examiner
Dr. Dipjyoti Majumdar

______________________ Thesis Supervisor (s)
Dr. Effrosyni Diamantoudi

____

Approved by ______________________
Dr. D. Lkhagvasuren, Grad Chair of Department or Graduate Program Director

July 6, 2018
Date of Defence

____
Dr. Andre Roy Dean, Arts and Science

Abstract

Revisiting the Environmental Kuznets Curve in the presence of trade

Biva Arani Mallik, Ph.D.

Concordia University, 2018

In my research I have investigated the impact of trade on the Environmental Kuznets Curve (EKC). I investigate the impact on the level and curvature of the EKC by varying the degree of export and import.

In my first paper, I have revisited the Andreoni and Levinson (2001) model in the presence of trade. In their original work, they conclude that the desired relationship between pollution and income can be traced only when there exists increasing returns to abatement technology. I have built upon the model by introducing trade and heterogeneity in abatement technology. I conclude when pollution sources from production and countries engage in trade, increasing returns to abatement technology no longer remains a necessary condition for a country to experience the EKC. When countries experience CRS or DRS, engaging in trade with countries with IRS enables them to attain the EKC. This however is possible only in the case of transboundary pollutants.

For my second paper, I look at the impact of trade on the multi-country framework of EKC developed by Diamantoudi and Filippiadis (2010). Introducing trade shows that pollution dependence among countries can exist even when the nature of pollutants is local in addition to the case of global pollutants. Results show that the impact of trade is more favorable for local pollutants both for the shape of the EKC and the scope for pollution substitution.

My third paper is an extension of my second paper where I investigate the impact of trade on the multi-country framework when technology is heterogeneous. When countries of different levels of technology engage in trade with each other, it is

found that the relative level of technology has an impact on the scope of pollution substitution as well as the EKC. Results of this paper reveal that when the level of technology of the foreign country is better, engaging in imports, and when the level of technology of the home country is better, engaging in exports helps the scope of pollution substitution as well as the shape of EKC.

Acknowledgments

I would like to express my profound gratitude to Professor Effrosyni Diamantoudi. Her kind guidance and support, first in the capacity of a Graduate Program Director, and then in the capacity of a supervisor has propelled me through the five years journey. Her wisdom, patience and depth of knowledge has always been a constant source of motivation and encouragement for me. I would also like to thank Professor Dipjyoti Majumdar and Professor Szilvia Papai for agreeing to be a part of my PhD committee. Professor Papai has also been a source of support as the GPD for the last three years of my studies. I thank her profusely for that. My sincere thanks to Professor Benchekroun and Professor Schofield for being a part of the PhD committee and providing me with their valuable feedback. Finally, I thank Professor Leblanc for giving me the opportunity to teach at the university. The experience will help me tremendously in my future endeavors.

I would also like to extend my gratitude to the kind and compassionate staff members of the department, Mrs. Elise Melancon, Mrs. Lucy Gilson, Mrs. Lise Gosselin and Mrs. Bonnie Janicki for their constant support throughout my years at Concordia University and also for being extremely patient with me in answering countless questions.

My heartfelt gratitude goes to my family for their patience, support and prayers. Iftekhar, my pillar of strength has borne the brunt of all my ups downs through the journey with his never ending calm and endurance. Thank you for always being there for me. My two lovely daughters, Arshya and Adeena. You light up my life and give me reason to thrive! My parents. Words cannot sum up the gratitude I hold for them. Everything that I am today is a fruit of their hard work and sacrifice. My father and mother in law. These two individuals have gently pushed me on with encouragement and prayers. Finally my brother Alok, his wonderful wife Alia, and my three adoring nieces Arissa, Ayana and Aleeza for having faith in me.

I have also been blessed with a wonderful circle of friends and family in Montreal. Shafinaz, Fayzul, Shehreen, Shakib, Nourin, Amit, Tabassum and Muntasir. These people have helped me with support - from looking out for my children to providing food - when I needed it the most. They are my biggest cheerleaders and I thank them profusely for being a part of my life.

Last but most importantly, I thank God Almighty for showering my life with His many blessings. I would like to dedicate my thesis to my wonderful family.

Table of Contents:

List of Tables

List of Figures

Essay1: Revisiting "The simple analytics of the environmental Kuznets curve" - in the presence of trade

1.1 Introduction

The Environmental Kuznets Curve postulates an inverted U- relationship between pollution and per capita income.It explains that at the initial stages of a countries' development pollution increases with economic growth, where as with further development, pollution starts to go down. A similar relationship between income inequality and per capita income was identified and made popular by Kuznets (1955) which was named the Kuznets Curve. In the 1990s environmental economists replaced income inequality with environmental quality, and the term 'Environment' was added to introduce the idea of the Environmental Kuznets Curve.

Whether such a relationship exists or not is crucial for policy relevance. While opponents of the EKC look for strong policy interventions that may at times limit economic growth, proponents argue for pro-growth policies. Even if the existence of an EKC holds, it is still subject to many questions and speculations. Does the relationship hold for specific types of pollutants only, or for pollution in general? At what point of economic growth does the turning point occur? Does the environment start to revive with economic growth at some feasible level of growth, or at extremely high rates which might not be feasible for many poorer countries? Due to its strong policy relevance and implications for economic growth analysis of the EKC has gained increasing popularity since its initiation in the early 1990s.

The concept of an EKC evolved in the early 1990s through empirical observations and were documented by Grossman and Krueger (1991), Shafik and Bandyopadhyay (1992), and Panayotou (1993). These studies were based on cross country data on local air and water pollutants such as CO, NO_x, SO_x, suspended particulate matter, municupal waste and lead. They later gave economic insights behind the existence of the EKC through (i) the scale effect, (ii) the technology effect and (iii) the composition effect. With such an initiation, work on the existence, persistence and rationale for the EKC continued. Some economists focused on empirical evidence behind the existence of an EKC while others tried to model and give structure and reasoning behind the relationship[1].

Around the same time (1990s) another burgeoning debate that emerged for environmental policy makers was the issue of globalization and free trade. The debate circled over the environmental consequences of countries engaging in trade and thereby enjoying trade led growth. Clearly the two issues are intertwined. The concept of an EKC looks at the implications for environmental quality with economic growth. Trade adds a dimension to the discussion in that it hastens economic growth but at the same time it is speculated to enhance production/ consumption and thus pollution. Whether or how trade affected the existence of an EKC remained to be seen. Some light on the issue has been shed by Arrow et al (1995), Suri and Chapman (1998) and Copeland and Taylor (2001). The crux of discussion in the first two papers lie in the idea that as countries develop they switch to cleaner products, while still maintaining a demand for the dirtier products - the production responsibility of which falls into the shoulders of the poorer countries. In the latter paper, Copeland and Taylor develop a north-south model, where they show that the developed countries maintain stricter environmental regulations and therefore create an incentive for dirtier industries to relocate to the developing world - a concept

[1] A detailed discussion of the various work is included in the Literature Review.

known as the pollution haven. All the models pose a rather pessimistic picture of the impact of trade on the environment in that they showcase a pollution *transfer* rather than pollution *mitigation* as a result of trade led growth.

In this paper, I would like to look at the impact of trade on a particular model - that developed by Andreoni and Levinson (2001).The Andreoni and Levinson (2001) model show that the shape/existence of the Environmental Kuznet's Curve can be explained by a technological link between consumption of a given good and abatement of its undesirable bi-product. It is a well documented model in Environmental Economics and has been widely sighted in literature. The model specifies that the desired inverted-U relationship between pollution and per capita income exists only under the assumption of increasing returns to abatement technology.They show that when countries exhibit constant returns or decreasing returns to abatement technology the pollution-income path follows a linearly increasing or a U- shaped pattern respectively.

The existence or plausibility of an increasing returns to abatement technology has been questioned in literature. Such a form of technology suggests that doubling pollution and abatement efforts should more than double emission reduction. There is evidence of such a phenomenon in some cases, but the contrary is also true. Managi (2006) uses empirical evidence on environmental risks in the US agriculture sector since 1970 to establish the existence of increasing returns to abatement technology. In their own paper, Andreoni and Levinson (2001) provide evidence of the existence of increasing returns to abatement technology when abatement efforts involve high fixed costs with low marginal costs (example: installing scrubbers on a smoke stack). Such high cost technologies become feasible only after economies are wealthy enough and polluted enough. Such an explanation itself suggests that when countries are poor, they may be required to invest in abatement options that have lower fixed costs with higher marginal costs of operation. These economies are more likely to experience decreasing returns to abatement technology. Egli (2005) suggest that rather than IRS, countries tend to exhibit fading IRS, meaning that at initial stages, when pollution level is high, abatement exhibits the feature of increasing returns. However, as the environment gets cleaner, cleaning up the last speck becomes more resource intensive at which point the abatement efforts may exhibit constant returns to abatement technology.

In short, while increasing returns to abatement technology does exist for many countries, it does not for many others. A lot of countries exhibit constant returns or more commonly decreasing returns to abatement technology. Typically, low income countries that rely on more traditional means of production, tend to exhibit decreasing returns to abatement technology.Support of decreasing returns to abatement technology can also be found in literature. For example, in coals fly-ash landfills, in order to reduce the level of fine dust over the ground, water sprays are traditionally used. Such use of water is expensive both in terms of hiring water tanks as well as using water resources (Careddu *et al.* 2015). Amigues *et al.* (2014) write a paper assessing the optimal time of Carbon Capture Sequestration policies when abatement technologies exhibit decreasing returns to scale.

By introducing trade to the Andreoni and Levinson model in this paper, I try to identify whether engaging in trade can bring about an EKC pattern in the pollution-income path for countries that do not enjoy increasing returns to abatement technology. I modify the Andreoni and Levinson model in three ways: (i) I introduce trade to the model, (ii) I introduce heterogeneity in the abatement technology and (iii) in addition to viewing pollution sourcing from consumption I also model the case where pollution sources from production of the consumption good. The second modification is justified in that the difference in technology adds as a basis for motivation to trade. The explanation for the third modification comes from the fact that often abatement efforts are undertaken at the production stage rather than the consumption stage of a product's life cycle. Hence I look at both cases - i.e pollution sourcing from consumption as well as pollution sourcing from production. Having made these three adjustments, I take into account the existence of a pollution externality. I argue that the extent of pollution externality can vary between 0 (where pollutants are strictly local) and 1 (where pollutants are transboundary). Thus the question that I answer in this paper is : "*How does the shape of the income-pollution path of an economy change from the predictions of the Andreoni and Levinson model, when countries with different abatement technology and varying degrees of pollution externality engage in trade with each other?*"

In the paper, trade is deliberately kept exogenous because the purpose of the paper is not to explain how much or why countries should engage in trade, but rather how the model predictions change subject to varying degrees of trade. Thus to be able to vary the degree of export and imports, these variables have been kept exogenous to the model. As mentioned earlier, I have looked at two distinct sources of pollution - pollution emerging from consumption goods and pollution emerging from production of the consumption goods. When pollution sources from consumption, the model predictions of the Andreoni and Levinson model remain unchanged. Even though trade affects optimal decision with respect to consumption, optimal abatement efforts and the optimal level of pollution and the implications of abatement technology remain unchanged from the original model. Next the paper investigates the impact of trade when pollution is sourced from production of the consumption good. In this case, different results are obtained.Here, the model concludes that even when countries do not enjoy increasing returns to abatement technology, so long as they engage in trade with countries that do, the desirable inverted U pattern of a pollution-income path can be attained. This however is possible only when there exists some degree of pollution externality between the trading countries.

The rest of the paper is organized in the following manner. In section 1.2, I present a review of the existing literature. The Model is presented in section 1.3 with the results and implications for trade in section 1.4. In section 1.5 the findings of the earlier sections are demonstrated through a pragmatic selection of parameters. Finally, section 1.6 concludes the paper with some scope for further discussion.

1.2 Literature Review

The concept of EKC was initiated in the early 1990s when debate revolving around the role of economic growth on the impact of environmental quality had reached a high after continuing for over two decades. Early on, it was believed that natural resources and a clean environment could not be sustained for ever, if the economies continued to grow unabated (Georgescu-Roegen,1971). Others such as Meadows *et al. (*1972*)* argued that population growth and the use of natural resources increases exponentially whereas the extraction and discovery of natural resources occur linearly, thereby causing an eventual depletion and exhaustion of natural resources with time. In the early 1990s a group of economists observed a dynamic nature of pollution where pollution at first increases with economic growth and then eventually declines. Such an inverted U-shaped relationship was at first pointed out by empirical studies of Grossman and Krueger (1991), Shafik and Bandyopadhyay (1992), and Panayotou (1993). These studies were based on cross country data on local air and water pollutants such as CO, NO_x, SO_x,suspended particulate matter, municipal waste and lead. They later gave economic insights behind the existence of the EKC through (i) the scale effect, (ii) the technology effect and (iii) the composition effect. They explained that at the initial stages of economic development, the scale effect dominates whereby as economies grow they engage in larger scales of operation and thus increased pollution. However, with continued economic growth, countries switch to cleaner technologies or products. Thus at this stage the technology effect and the composition effect takes over, and pollution starts to go down. On a different note, Arrow *et al.* (1995) pointed out that the shape of the EKC traces the natural progression of economic growth as countries evolve from clean agrarian societies to polluting industrial economies to clean service based economies.The concept of EKC thus promoted the idea that environmental pollution should not be viewed as a cause of obstructing economic growth. In fact Beckerman (1992) writes that "*In the end the best - and probably the only- way to attain a decent environment in most counties is to become rich*".

Following this a number of economists focused attention on trying to model and thus give structure to the understanding of the EKC. One group of models were focused on the income elasticity of pollution. It is argued that the income elasticity of pollution is greater than one and thus with rising income a demand for a cleaner environment emerges. Along this line, John and Pecchenino (1994) present an overlapping generations model in which environmental quality is a stock resource that degrades with time unless maintained by investments in the environment. At the beginning an economy begins with zero pollution and sees its environment deteriorate with increased growth. After some point in time, pollution increases to the point that investment in environment becomes desirable and thus pollution starts to decline. The model thus presented by John and Pecchenino predicts an inverted V shaped relationship between pollution and income. Similary, Jaeger (1998) model that low levels of economic growth portray low levels of pollution where the consumers marginal benefit to additional environmental quality is zero. Therefore, at this stage more pollution does not lead to lower utility. With growth and further pollution, consumers become sensitive to increased pollution at which point, depending on the parameters, growth may be accompanied by improved environmental quality. Again, Jaeger's pollution-income relationship is inverse V shaped, peaking when the optimum moves from a corner solution to an interior solution.

Yet another group of economists have created models upon the explanation that with economic growth, some constraints become non-binding and when economies take advantage of these new opportunities, pollution can start to go down. For example, Jones and Manuelli (1995) posit that pollution involves externalities and appropriate internalization of these externalities is possible with advanced institutions that can be afforded only by developed economies. Along this line of explanation, Jones and Manuelli developed an overlapping generations model where economic growth is determined by market interactions and pollution is set through the collective decision making of the younger generation. The shape of the pollution-income path in this case is set by the nature of the decision making institution the shape can be an inverted U, monotonically increasing or even a "sideways mirrored S". Stokey (1998), describe a static model with a range of possible production technologies. At the initial stages of economic growth, only the dirtiest technologies are economically feasible and thus pollution increases with growth. After some threshold of income is reached, the cleaner technologies become economically feasible, adoption of which causes pollution to go down. Thus again, the anticipated pollution- income path follows an inverted V relationship.

The 1990s also witnessed the emergence of globalization and free trade. The era was thus wrought with debates over free trade and its impact on the environment. Several studies explaining how trade could explain the shape of the

EKC also emerged along this time. Arrow *et al.* (1995) use international trade to explain the shape of the EKC. They argue that as economies grow, they switch to cleaner products but there continues to be a demand for the dirty products which gets satiated by importing from the less developed economies. Along the same line, Suri and Chapman (1998) explain that as economies grow they tend to transfer the dirty industries to the developing world and thereby enjoy cleaner environment. It is also believed, that as economies grow they can afford better institutions and thus higher environmental regulations. Such restrictions create incentives for the dirtier industries to relocate to poorer economies with lower environmental regulations. Such a north- south model is developed by Copeland and Taylor (2001) to investigate the linkages between national income, pollution and international trade. All these papers provide a bleak picture where as all countries in the world grow, they will no longer have a poorer country to fall back upon! On a more positive note Dean (1999) find that trade liberalization directly aggravates environmental damage via its influence on the terms of trade, but indirectly mitigates it via its effect on income growth. Holladay (2008) look at the impact of trade liberalization on the environment. He focuses on two distinct sources of pollution - pollution from consumption and pollution from production, and concludes that pollution from consumption increases where as pollution from production decreases after trade liberalization. In all these papers, the distinction between local pollutants and transboundary pollutants is absent.

There also exists a wide pool of literature looking into the empirical evidence of the existence of the EKC. Selden and Song (1994) have confirmed the inverted U relationship between pollution and per capital GDP using a cross-national panel of data on emission of four air pollutants: suspended particulate matter, sulfur dioxide, oxides of nitrogen, and carbon monoxide. However, they predicted that emissions to decrease only in the very long run with rapid growth on global emissions to continue for several decades. Grossman and Kreuger (1995) also examine the relationship between per capita GDP and four environmental indicators and conclude in favor of the existence of the EKC. They further provide with some intuitive explanations for the downward sloping part of the EKC. They explain that as countries grow several factors come into play to cause the downward trend of pollutants. First, with growth more stringent environmental policies are adopted as with economic prosperity non economic aspects of life such as a clean environment become more desirable for economic agents. Second, as economies grow they cease to produce some of the dirtier products and start to import them instead. Finally, with time and learning from the experiences of the wealthier countries, countries may adopt cleaner technologies.

In recent times, much of the empirical work has focused on transboundary pollutants. For example, Apergis (2016) looks at the existence of the EKC in the emission of CO_2across fifteen countries and concludes that it does in twelve out of the fifteen countries. Bilgili *et al.* (2016) also look at the empirical evidence based on the emission of CO_2. However, rather than explaining the downward sloping part of the EKC with income growth, they attribute it to a shift in renewable energy with time. Jebli *et al.* (2016) also study the existence of EKC using $CO_{2.}$They examine the impact of renewable energy, non- renewable energy and trade on the shape of EKC in OECD countries.

There is vast support in literature on the existence of the EKC. This paper investigates the impact of trade on an existing model that suggests that the shape of the EKC can be explained under strict parametric restrictions. The paper attempts to see if some of the restrictions can be relaxed when trade is brought into the picture.

1.3 The Model

Consider a model with two countries $i \in \{1, 2\}$.Each country produces only one consumption good C_i for $i = 1, 2$. For simplicity assume that each of them are a single agent country. Of the consumption good C_i that is produced, each country retains a fraction of it for own consumption and exports the rest to the other country. The fraction of C_i retained by a country is τ_i. Note that τ_i kept exogenous to the model as the objective of the model is to see the impact on the predictions of Andreoni and Levinson (2001) by varying the degrees of $\tau_i$with $i = 1, 2$.

Utility in each country is a function of the consumption goods (locally produced and imported) and pollution - $U_i = U(C_1, C_2, P)$,where utility is increasing in consumption and decreasing in pollution. Utility is quasi-concave in C_1, C_2, and $-P$. In addition, each country faces a resource constraint in the form of income per capita M_i.The resource of the country is used up in the consumption of the two commodities and in abatement efforts to mitigate pollution.

The model views pollution from two sources: from consumption of $C_1$and C_2 and from production of C_1 and C_2 . The model also distinguishes between local pollutants and global pollutants. Each country engages in some abatement effort which is captured by the term E_i , $i \in \{1, 2\}$. These terms include all resources spent towards pollution mitigation. It can be considered as investing in preventive measures to arrest pollution growth or as spending in resources for cleaning up efforts. Therefore, pollution in a given country is increasing in either (i) consumption of of $C_1$and C_2, or (ii) production of $C_1$and C_2; and decreasing in abatement efforts - $P = P(C_1, C_2, E_1, E_2)$. A summary of the different types of pollution considered is presented in the table below:

Table 1.1: The different Sources of Pollution

	Local Pollution	**Trans-boundary Pollution**
Source	Consumpiton of C_1 and C_2	Consumpiton of $C_1$and C_2
	Production of $C_1$and C_2	Production of $C_1$and C_2

□When the local pollutants of every country are summed up, we get the trans-boundary pollution. The model is thus developed in two parts: Part 1 looks at the case where pollution is sourcing from consumption and Part 2 looks into the case of pollution sourcing from production.

1.3.1 Model 1 - When pollution is sourced from consumption

Consider a two country case, where the utility of each country looks as follows:

$$U_1 = \tau_1 C_1 + (1-\tau_2)C_2 - z_1 P \qquad (1a)$$

$$U_2 = (1-\tau_1)C_1 + \tau_2 C_2 - z_2 P \qquad (1b)$$

where z_1 and z_2 are the marginal disutilities associated with pollution. The resource constraint faced by the two countries are:

$$M_1 = \tau_1 C_1 + (1-\tau_2)C_2 + E_1 \qquad (2a)$$

$$M_2 = (1-\tau_1)C_1 + \tau_2 C_2 + E_2 \qquad (2b)$$

The pollution function for each country is:

$$P_1 = \tau_1 C_1 + (1-\tau_2)C_2 - (\tau_1 C_1 + (1-\tau_2)C_2)^{\alpha_1} E_1^{\beta_1} (3a)$$

$$P_2 = (1-\tau_1)C_1 + \tau_2 C_2 - ((1-\tau_1)C_1 + \tau_2 C_2)^{\alpha_2} E_2^{\beta_2} (3b)$$

P_i represents the net pollution generated in each country. In each country pollution is generated by the consumption of goods. The last term represents the abatement technology. Abatement increases in C as well as E.The i subscripts to α and β show that the abatement technology in each country is different. Thus heterogeneity is introduced to the model. In the model, transboundary pollution P is thus viewed accordingly:

$P = P_1 + lP_2$ where $l = [0, 1]$ (4)

when l takes the value of 0, it means the model views only local pollution - that generated only in country 1; and when $0 < l < 1$it means that there is some degree of permeation (but not complete) among the pollutants. Finally when l takes a value of 1, the model is viewing global pollution. Pollutant emitted in one country completely affects the other.

For simplicity allow $z_1 = z_2 = 1$.Thus plugging the value of P in the utility function $(1a)$ and substituting for the value of $(1 - \tau_2)C_2$ from the resource constraint $(2a)$ into $(1a)$, the social utility function that the planner of Country 1 seeks to optimize is:

$$U_1 = M_1 - C_1 - lC_2 - E_1 + (\tau_1 C_1 + (1 - \tau_2)C_2)^{\alpha_1} E_1^{\beta_1} + l(M_2 - E_2)^{\alpha_2} E_2^{\beta_2} \qquad (5)$$

The first order solution to the problem gives the optimal amount of consumption and abatement decision for both countries:

$C_1 = \frac{\tau_2 \alpha_1 (\alpha_2 + \beta_2) M_1 - (1 - \tau_2) \alpha_2 (\alpha_1 + \beta_1) M_2}{(\tau_1 + \tau_2 - 1)(\alpha_1 + \beta_1)(\alpha_2 + \beta_2)}$

$C_2 = \frac{\tau_1 \alpha_2 (\alpha_1 + \beta_1) M_2 - (1 - \tau_1) \alpha_1 (\alpha_2 + \beta_2) M_1}{(\tau_1 + \tau_2 - 1)(\alpha_1 + \beta_1)(\alpha_2 + \beta_2)}$

When countries engage in trade the optimal consumption decision of own commodity changes. As the fraction of goods imported from the foreign country goes up, C_i goes down. As countries choose to retain larger fractions for own consumption, C_i goes down. In other words, when they choose to export larger fractions C_i goes up.

$E_1 = \frac{\beta_1 M_1}{\alpha_1 + \beta_1}$

$E_2 = \frac{\beta_2 M_2}{\alpha_2 + \beta_2}$

The abatement effort of a country remains unchanged with a decision to engage in trade or not.

Plugging in the optimal decisions in the pollution function, the optimal pollution function for Country 1 becomes:

$P = \frac{\alpha_1 M_1}{\alpha_1 + \beta_1} - (\frac{\alpha_1}{\alpha_1 + \beta_1})^{\alpha_1} (\frac{\beta_1}{\alpha_1 + \beta_1}]^{\beta_1} M_1^{\alpha_1 + \beta_1} + l\{\frac{\alpha_2 M_2}{\alpha_2 + \beta_2} - (\frac{\alpha_2}{\alpha_2 + \beta_2})^{\alpha_2} (\frac{\beta_2}{\alpha_2 + \beta_2})^{\beta_2} M_2^{\alpha_2 + \beta_2}\}$

The slope to the Kuznets' Curve is the first derivative of the pollution function with respect to M_1.

$\frac{\partial P}{\partial M_1} = \frac{\alpha_1}{\alpha_1+\beta_1} - (\alpha_1+\beta_1)(\frac{\alpha_1}{\alpha_1+\beta_1})^{\alpha_1}(\frac{\beta_1}{\alpha_1+\beta_1}]^{\beta_1} M_1^{\alpha_1+\beta_1-1}$

Thus it is seen that the shape of the EKC is determined entirely by its own abatement technology. The technology of the second country does not affect the income pollution relationship of country 1 in any way. Engaging in trade does not affect the slope of the income-pollution path.The second order derivative to the pollution income relationship gives:

$\frac{\partial^2 P}{\partial M_1^2} = -(\alpha_1+\beta_1-1)(\alpha_1+\beta_1)(\frac{\alpha_1}{\alpha_1+\beta_1})^{\alpha_1}(\frac{\beta_1}{\alpha_1+\beta_1}]^{\beta_1} M_1^{\alpha_1+\beta_1-1}$

Therefore, the income pollution path will follow the desired inverted U-shape only in the case where countries exhibit increasing returns to abatement technology, i.e $\alpha+\beta>1$. The predictions of the Andreoni and Levinson Model remains unchanged even after I introduce trade when pollution sources from consumption of goods.

1.3.2 Model 2 - when pollution is sourced from production of the consumption goods.

In this section I look into the implications of trade by developing the model when pollution is sourced from production. This approach is quite reasonable in that the abatement technology of many pollutants are applied at the production rather than the consumption stage of a product life.

When pollutions is sourced from production of the consumption goods, the pollution functions for the two countries take the following form:

$P_1 = C_1 - C_1^{\alpha_1} E_1^{\beta_1} (6a)$

$P_2 = C_2 - C_2^{\alpha_2} E_2^{\beta_2} (6b)$

and thus the transboundary pollution faced by Country 1 becomes:

$P = P_1 + lP_2 \qquad (6)$

Again plugging in the value of P in the utility function $(1a)$ and substituting for the value of $(1-\tau_2)C_2$ from the resource constraint $(2a)$ into $(1a)$, the utility function we are seeking to optimize this time is:

$$U_1 = M_1 - C_1 - lC_2 - E_1 + C_1^{\alpha_1}E_1^{\beta_1} + lC_2^{\alpha_2}E_2^{\beta_2} \qquad (7)$$

The solution for consumpion and abaemen decisions for the two countries are as follows:

$$C_1^* = \frac{\alpha_1(\tau_2\alpha_2+\beta_2)M_1-\alpha_1\alpha_2(1-\tau_2)M_2}{(\tau_2\alpha_2+\beta_2)(\tau_1\alpha_1+\beta_1)-\alpha_1\alpha_2(1-\tau_1)(1-\tau_2)}$$

$$C_2^* = \frac{\alpha_2(\tau_1\alpha_1+\beta_1)M_2-\alpha_2\alpha_1(1-\tau_1)M_1}{(\tau_2\alpha_2+\beta_2)(\tau_1\alpha_1+\beta_1)-\alpha_1\alpha_2(1-\tau_1)(1-\tau_2)}$$

When trade is introduced, the optimal consumption decisions get changed.

$$E_1^* = \frac{\beta_1(\tau_2\alpha_2+\beta_2)M_1-\beta_1\alpha_2(1-\tau_2)M_2}{(\tau_2\alpha_2+\beta_2)(\tau_1\alpha_1+\beta_1)-\alpha_1\alpha_2(1-\tau_1)(1-\tau_2)}$$

$$E_2^* = \frac{\beta_2(\tau_1\alpha_1+\beta_1)M_2-\beta_2\alpha_1(1-\tau_1)M_1}{(\tau_2\alpha_2+\beta_2)(\tau_1\alpha_1+\beta_1)-\alpha_1\alpha_2(1-\tau_1)(1-\tau_2)}$$

When pollution is sourced from production of the consumption goods, abatement efforts are also affected by trade. How trade will affect the optimal pollution remains ambiguous.

By plugging in the optimal decisions, pollution function can now be written as:

$$P = \frac{\alpha_1(\tau_2\alpha_2+\beta_2)M_1-\alpha_1\alpha_2(1-\tau_2)M_2}{(\tau_2\alpha_2+\beta_2)(\tau_1\alpha_1+\beta_1)-\alpha_1\alpha_2(1-\tau_1)(1-\tau_2)}$$
$$-[\frac{\alpha_1(\tau_2\alpha_2+\beta_2)M_1-\alpha_1\alpha_2(1-\tau_2)M_2}{(\tau_2\alpha_2+\beta_2)(\tau_1\alpha_1+\beta_1)-\alpha_1\alpha_2(1-\tau_1)(1-\tau_2)}]^{\alpha_1}[\frac{\beta_1(\tau_2\alpha_2+\beta_2)M_1-\beta_1\alpha_2(1-\tau_2)M_2}{(\tau_2\alpha_2+\beta_2)(\tau_1\alpha_1+\beta_1)-\alpha_1\alpha_2(1-\tau_1)(1-\tau_2)}]^{\beta_1}$$
$$l\{\frac{\alpha_2(\tau_1\alpha_1+\beta_1)M_2-\alpha_2\alpha_1(1-\tau_1)M_1}{(\tau_2\alpha_2+\beta_2)(\tau_1\alpha_1+\beta_1)-\alpha_1\alpha_2(1-\tau_1)(1-\tau_2)}$$
$$[\frac{\alpha_2(\tau_1\alpha_1+\beta_1)M_2-\alpha_2\alpha_1(1-\tau_1)M}{(\tau_2\alpha_2+\beta_2)(\tau_1\alpha_1+\beta_1)-\alpha_1\alpha_2(1-\tau_1)(1-\tau_2)}]^{\alpha_2}[\frac{\beta_2(\tau_1\alpha_1+\beta_1)M_2-\beta_2\alpha_1(1-\tau_1)M_1}{(\tau_2\alpha_2+\beta_2)(\tau_1\alpha_1+\beta_1)-\alpha_1\alpha_2(1-\tau_1)(1-\tau_2)}]^{\beta_2}\}$$

The equation above is complex to look at. To give it a meaningful perspective, we group some of the terms in the following ways:

Let, $A = (\tau_2\alpha_2 + \beta_2)(\tau_1\alpha_1 + \beta_1) - \alpha_1\alpha_2(1 - \tau_1)(1 - \tau_2)$

$B = \alpha_2(1 - \tau_2)M_2$

$C = (\tau_1\alpha_1 + \beta_1)M_2$

We can then write the pollution function in the following manner:

$$P = \frac{\alpha_1(\tau_2\alpha_2+\beta_2)M_1-\alpha_1 B}{A} - [\frac{\alpha_1}{A}]^{\alpha_1}[\frac{\beta_1}{A}]^{\beta_1}[(\tau_2\alpha_2 + \beta_2)M_1 - B]^{\alpha_1+\beta_1} +$$

$l\{\frac{\alpha_2 C - \alpha_2\alpha_1(1-\tau_1)M_1}{A} - [\frac{\alpha_2}{A}]^{\alpha_2}[\frac{\beta_2}{A}]^{\beta_2}[C - \alpha_1(1-\tau_1)M_1]^{\alpha_2+\beta_2}\}$ (8)

The pattern of the income- pollution path that this model predicts is discussed in Section 4.

1.4 Results and implications of trade

As has been shown in the previous section, when pollution sources from production, the optimal pollution function becomes as follows:

$$P = \frac{\alpha_1(\tau_2\alpha_2+\beta_2)M_1 - \alpha_1 B}{A} - [\frac{\alpha_1}{A}]^{\alpha_1}[\frac{\beta_1}{A}]^{\beta_1}[(\tau_2\alpha_2+\beta_2)M_1 - B]^{\alpha_1+\beta_1} +$$
$$l\{\frac{\alpha_2 C - \alpha_2\alpha_1(1-\tau_1)M_1}{A} - [\frac{\alpha_2}{A}]^{\alpha_2}[\frac{\beta_2}{A}]^{\beta_2}[C - \alpha_1(1-\tau_1)M_1]^{\alpha_2+\beta_2}\} \quad (8)$$

The first derivative of the equation above, with respect to the income of Country 1 will give the slope of the income-pollution path for country 1.

$$\frac{\partial P}{\partial M_1} = \frac{\alpha_1(\tau_2\alpha_2+\beta_2)}{A} - (\alpha_1+\beta_1)(\tau_2\alpha_2+\beta_2)[\frac{\alpha_1}{A}]^{\alpha_1}[\frac{\beta_1}{A}]^{\beta_1}[(\tau_2\alpha_2+\beta_2)M_1 - B]^{\alpha_1+\beta_1-1} +$$
$$l\{\frac{\alpha_1\alpha_2(1-\tau_1)}{A} - (\alpha_2+\beta_2)\alpha_1(1-\tau_1)[\frac{\alpha_2}{A}]^{\alpha_2}[\frac{\beta_2}{A}]^{\beta_2}[C - \alpha_1(1-\tau_1)M_1]^{\alpha_2+\beta_2-1}\} \quad (9)$$

Equation 9 shows that the slope of the income-pollution path depends upon:

a) abatement technology of both the home country and the foreign country,

b) the degree of trade; and

c) the degree of pollution externality.

1.4.1 Implications of abatement technology for home country and foreign country

Proposition 1 *When (i) pollution is sourced from production of a consumption good, (ii) countries engage in trade, and (iii) there exists pollution externality; increasing returns to trade no longer remains a necessary condition for a country to obtain the EKC pattern in the income pollution path when the nature of pollutant is transboundary. A sufficient condition for Country1 becomes to engage in trade with Country 2 who enjoys increasing returns to abatement technology.*

Proof. *To see what shape the income-pollution path will take, we look at the second derivative with respect to income.*

$$\frac{\partial^2 P}{\partial M_1^2} = -(\alpha_1+\beta_1-1)(\alpha_1+\beta_1)(\tau_2\alpha_2+\beta_2)^2[\tfrac{\alpha_1}{A}]^{\alpha_1}[\tfrac{\beta_1}{A}]^{\beta_1}[(\tau_2\alpha_2+\beta_2)M_1 - B]^{\alpha_1+\beta_1-2} - l\{(\alpha_2+\beta_2-1)(\alpha_2+\beta_2)\alpha_1^2(1-\tau_1)^2[\tfrac{\alpha_2}{A}]^{\alpha_2}[\tfrac{\beta_2}{A}]^{\beta_2}[C-\alpha_1(1-\tau_1)M_1]^{\alpha_2+\beta_2-2}\} \qquad (10)$$

For $\frac{\partial^2 P}{\partial M_1^2} < 0$ *either one of the following conditions have to be satisfied.*

(i) Both countries display increasing returns to abatement technology. If both $\alpha_1 + \beta_1 > 0$ *and* $\alpha_2 + \beta_2 > 0$ *then the second derivative with respect to income will be negative and income pollution path will follow the desired inverted U-pattern. This is similar to the findings of the Andreoni and Levinson Model.*

(ii) Country 1 displays increasing returns to abatement technology, but the abatement technology of Country 2 does not follow IRS. In this case the second derivative will be negative so long as:

$$(\alpha_1+\beta_1-1)(\alpha_1+\beta_1)(\tau_2\alpha_2+\beta_2)^2[\tfrac{\alpha_1}{A}]^{\alpha_1}[\tfrac{\beta_1}{A}]^{\beta_1}[(\tau_2\alpha_2+\beta_2)M_1 - B]^{\alpha_1+\beta_1-2} > -l\{(\alpha_2+\beta_2-1)(\alpha_2+\beta_2)\alpha_1^2(1-\tau_1)^2[\tfrac{\alpha_2}{A}]^{\alpha_2}[\tfrac{\beta_2}{A}]^{\beta_2}[C-\alpha_1(1-\tau_1)M_1]^{\alpha_2+\beta_2-2}\}$$

Again the condition is similar to that of the Andreoni and Levinson in that Country 1 must display IRS; but when they engage in trade with a nation that does not, an additional condition is imposed for the income-pollution path to follow the EKC pattern.

(iii) Finally, Country 1 does not need to display IRS so long as Country 2 does and

$$l\{(\alpha_2+\beta_2-1)(\alpha_2+\beta_2)\alpha_1^2(1-\tau_1)^2[\tfrac{\alpha_2}{A}]^{\alpha_2}[\tfrac{\beta_2}{A}]^{\beta_2}[C-\alpha_1(1-\tau_1)M_1]^{\alpha_2+\beta_2-2}\} > (\alpha_1+\beta_1-1)(\alpha_1+\beta_1)(\tau_2\alpha_2+\beta_2)^2[\tfrac{\alpha_1}{A}]^{\alpha_1}[\tfrac{\beta_1}{A}]^{\beta_1}[(\tau_2\alpha_2+\beta_2)M_1 - B]^{\alpha_1+\beta_1-2}$$

Thus when countries engage in international trade, when pollution sources from production of the consumption goods, IRS is no longer a necessary condition for countries to follow the EKC pattern in their income-pollution paths. The desired EKC pattern can be obtained by engaging in trade with countries that follow IRS. When Country 1 displays CRS, the second order derivative to *the income pollution path becomes:*

$$\frac{\partial P^2}{\partial M_1^2} = -l\{(\alpha_2 + \beta_2 - 1)(\alpha_2 + \beta_2)(\frac{\alpha(1-\tau_1)}{A})^2 \alpha_2^{\alpha_2} \beta_2^{\beta_2} [\frac{C-\alpha_1(1-\tau_1)M_1}{A}]^{\alpha_2+\beta_2-2}$$

This shows that when Country 1 does not engage in trade, the income pollution path will be linear. When they engage in trade with Country 2, the income pollution path will follow a quadratic pattern. So long as Country 2 displays IRS the income pollution path of Country 1 will follow the inverted U-shaped pattern. When Country 1 exhibits DRS, then the second order derivative takes the following shape:

$$\frac{\partial P^2}{\partial M_1^2} = -(\alpha_1 + \beta_1 - 1)(\alpha_1 + \beta_1)(\tau_2\alpha_2 + \beta_2)[\frac{\alpha_1}{A}]^{\alpha_1}[\frac{\beta_1}{A}]^{\beta_1}[(\tau_2\alpha_2 + \beta_2)M_1 - B]^{\alpha_1+\beta_1-2} - l\{(\alpha_2+\beta_2-1)\alpha_1(1-\tau_1)(\alpha_2+\beta_2)[\frac{\alpha_2}{A}]^{\alpha_2}[\frac{\beta_2}{A}]^{\beta_2}[C-\alpha_1(1-\tau_1)M_1]^{\alpha_2+\beta_2-2}\}$$

At lower levels of income, the value of $[(\tau_2\alpha_2 + \beta_2)M_1 - B]$ *is low and so the value of* $[C - \alpha_1(1 - \tau_1)M_1]$ *is high. Therefore, Country 1's own technology dominates the shape of the income pollution path, and with DRS, the shape follows the predicted U shape pattern. But with higher levels of income,* $[(\tau_2\alpha_2 + \beta_2)M_1 - B]$ *starts to become larger, while* $[C - \alpha_1(1 - \tau_1)M_1]$ *starts to become smaller. At this point the second part of the equation (the part defined by the abatement technology of the foreign country) starts to dominate. In that case, so long as country 2 exhibits IRS, the income pollution path of Country 1 will follow a U shape at low levels of income, but the inverted U pattern at higher levels of income. Thus in the words of Jones and Manuelli (1995), the income pollution pattern will follow that of a 'side ways mirrored S'.*

■

The proof above shows the necessary condition needed for the concavity of the income pollution path when country 1 engages in trade. To check for the sufficient condition for the inverted U shape of the income- pollution path the slope of the income pollution path needs to be checked. Equation (9) shows the general slope of the income pollution path. When country 1 exhibits Constant Returns to Abatement Technology ($\alpha_1 + \beta_1 = 1$), and country 2 exhibits Increasing Returns to Abatement Technology ($\alpha_2 + \beta_2 > 1$) equation (9) above can be re-written as follows:

$$\frac{\partial P}{\partial M_1} = \frac{\alpha_1(\tau_2\alpha_2+\beta_2)}{A} - (\tau_2\alpha_2 + \beta_2)\frac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}}{A} + l\{\frac{\alpha_1\alpha_2(1-\tau_1)}{A} - (\alpha_2+\beta_2)\alpha_1(1-\tau_1)[\frac{\alpha_2}{A}]^{\alpha_2}[\frac{\beta_2}{A}]^{\beta_2}[C-\alpha_1(1-\tau_1)M_1]^{\alpha_2+\beta_2-1}\} \quad (10)$$

Setting $\frac{\partial P}{\partial M_1} = 0$, the turning point of the EKC comes to:

$$M_1 = (\tau_1\alpha_1 + \beta_1)M_2 - A[\frac{\{(\tau_2\alpha_2+\beta_2)(\alpha_1-\alpha_1^{\alpha_1}\beta_1^{\beta_1})+l\alpha_1\alpha_2(1-\tau_1)\}}{(\alpha_2+\beta_2)\alpha_1^2(1-\tau_1)^2\alpha_2^{\alpha_2}\beta_2^{\beta_2}}]^{\frac{1}{\alpha_2+\beta_2-1}} \equiv \phi$$

where $A = (\tau_2\alpha_2 + \beta_2)(\tau_1\alpha_1 + \beta_1) - \alpha_1\alpha_2(1 - \tau_1)(1 - \tau_2)$

If $M_1 \leq \phi$, $\frac{\partial P}{\partial M_1} > 0$

$\frac{\partial P}{\partial M_1} < 0$ otherwise.

1.4.2 Implications of trade

If Country 1 chooses autarky, the optimal pollution function for Country 1 becomes:

$P_{autarky} = \frac{\alpha_1 M_1}{\alpha_1+\beta_1} - (\frac{\alpha_1}{\alpha_1+\beta_1})^{\alpha_1}(\frac{\beta 1}{\alpha_1+\beta_1})^{\beta_1}M_1^{\alpha_1+\beta_1}$

The model collapses to the Andreoni and Levinson (2001) model. Engaging in trade gives rise to the optimal pollution function as shown by equation (8). Engaging in trade affects both the level of optimal pollution as well as the shape of the income pollution path. When countries engage in trade the abatement technologies of both countries have an impact on the income pollution path of Country 1. The mechanism of the influence has already been demonstrated in Proposition 1. This part of the paper moves away from the discussion of abatement technology and focuses attention on the impact of import and export.

Proposition 2 *When (i) pollution is sourced from production of a consumption good, (ii) there exists pollution externality, and (iii) Country 2 exhibits IRS when Country 1 does not, increasing the fraction of output of Country2 imported into Country 1 decreases the optimal level of pollution, whereas, increasing the fraction of own good exported increases the level of optimal pollution. However, the impact of import and export on the slope of the income pollution path is different.*

Proof. *From Equation 8, the optimal pollution path can be written as:*

$$P = C_1 - \frac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}}{\alpha 1_1^{\alpha_1+\beta_1}} C_1^{\alpha_1+\beta_1} + l\{C_2 - \frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}}{\alpha_2^{\alpha_2+\beta_2}} C_2^{\alpha_2+\beta 2}\}$$

The proof is carried in two steps. In the first step I look at the impact of import and export on the level of pollution. Then in the second step I look at the impact of import and export on the curvature of the income- pollution path.

Step 1:

First I look at the impact of import on the level of optimal pollution.

$$\frac{\partial P}{\partial(1-\tau_2)} = \frac{\partial C_1}{\partial(1-\tau_2)}[1-\frac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}(\alpha_1+\beta_1)}{\alpha 1_1^{\alpha_1+\beta_1}} C_1^{\alpha_1+\beta_1-1}]+l\{\frac{\partial C_2}{\partial(1-\tau_2)}[1-\frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}(\alpha_2+\beta_2)}{\alpha_2^{\alpha_2+\beta_2}} C_2^{\alpha_2+\beta_2-1}]\}$$

From Proposition 1 I have established that when Country 1 exhibits CRS or DRS it is desirable to engage in trade with Country 2 who exhibits IRS. In the appendix it is shown that $\frac{\partial C_1}{\partial(1-\tau_2)} < 0$. *So long as Country 1 exhibits CRS or DRS,* $[1 - \frac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}(\alpha_1+\beta_1)}{\alpha 1_1^{\alpha_1+\beta_1}} C_1^{\alpha_1+\beta_1-1}] > 0$. $\frac{\partial C_2}{\partial(1-\tau_2)} > 0$. *Since Country 2 exhibits IRS,* $[1 - \frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}(\alpha_2+\beta_2)}{\alpha_2^{\alpha_2+\beta_2}} C_2^{\alpha_2+\beta_2-1}] < 0$. *Thus under the specifications* $\frac{\partial P}{\partial(1-\tau_2)} < 0$. *The larger the fraction of output of Country 2 imported, the lower will be the optimal level of pollution.*

Next I look at the impact of export on the optimal level of pollution.

$$\frac{\partial P}{\partial(1-\tau_1)} = \frac{\partial C_1}{\partial(1-\tau_1)}[1 - \frac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}}{\alpha 1_1^{\alpha_1+\beta_1}} C_1^{\alpha_1+\beta_1-1}] + l\{\frac{\partial C_2}{\partial(1-\tau_1)}[1 - \frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}}{\alpha_2^{\alpha_2+\beta_2}} C_2^{\alpha_2+\beta_2-1}]\}$$

So long as Country 1 exhibits CRS or DRS $[1 - \frac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}(\alpha_1+\beta_1)}{\alpha 1_1^{\alpha_1+\beta_1}} C_1^{\alpha_1+\beta_1-1}] > 0$ *and* $\frac{\partial C_1}{\partial(1-\tau_1)} > 0$. *In the appendix it is shown that* $\frac{\partial C_2}{\partial(1-\tau_1)} < 0$. *Since Country 2 exhibits IRS,* $[1 - \frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}(\alpha_2+\beta_2)}{\alpha_2^{\alpha_2+\beta_2}} C_2^{\alpha_2+\beta_2-1}] < 0$. *Therefore, the impact of export* *on the optimal level of pollution will be positive.*

In summary, engaging in imports decreases the optimal level of pollution where as engaging in exports increases the optimal level of pollution.

Step 2

In this part I look at the impact of import and export on the curvature of the income pollution path. Recall the slope of the income pollution path is as follows:

$$\frac{\partial P}{\partial M_1} = \frac{\alpha_1(\tau_2\alpha_2+\beta_2)}{A} - (\alpha_1 + \beta_1)(\tau_2\alpha_2 + \beta_2)[\frac{\alpha_1}{A}]^{\alpha_1}[\frac{\beta_1}{A}]^{\beta_1}[(\tau_2\alpha_2 + \beta_2)M_1 - B]^{\alpha_1+\beta_1-1} + l\{\frac{\alpha_1\alpha_2(1-\tau_1)}{A} - (\alpha_2+\beta_2)\alpha_1(1-\tau_1)[\frac{\alpha_2}{A}]^{\alpha_2}[\frac{\beta_2}{A}]^{\beta_2}[C-\alpha_1(1-\tau_1)M_1]^{\alpha_2+\beta_2-1}\}$$

With a little bit of manipulation, the slope of the income pollution path can be written in the following manner:

$$\frac{\partial P}{\partial M_1} = \frac{\alpha_1(\tau_2\alpha_2+\beta_2)}{A}[1-(\alpha_1+\beta_1)\frac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}}{\alpha_1^{\alpha_1+\beta_1}}C_1^{\alpha_1+\beta_1-1}] + l\{\frac{\alpha_2\alpha_1(1-\tau_1)}{A}[-1+(\alpha_2+\beta_2)\frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}}{\alpha_2^{\alpha_2+\beta_2}}C_2^{\alpha_2+\beta_2-1}]\}$$

In the appendix it is shown that the impact of import on the slope of the income pollution path is non-conclusive. However, the impact of export is more likely to be positive.The desired inverted U shape of the income pollution path is assured by the negativity of the second derivative of optimal pollution with respect to income. From Condition (iii) of Proposition 1 it can be observed that the higher the fraction of own output exported, the stronger will be the inequality. Thus the negativity of the second order derivative is guaranteed.

Proposition 2 thus concludes that the impact of import and export is distinctly different. While import and export affects the level of optimal pollution in opposite ways, export affects the pattern of the income pollution path in the desired direction. ■

An interesting observation comes forth for the case where both countries exhibit constant returns to scale in the abatement technology.When both countries exhibit CRS in abatement technology, the optimal pollution function becomes:

$P=\frac{\alpha_1(\tau_2\alpha_2+\beta_2)M_1-\alpha_1 B}{A}-[\frac{\alpha_1}{A}]^{\alpha_1}[\frac{\beta_1}{A}]^{\beta_1}[(\tau_2\alpha_2+\beta_2)M_1-B]+l\{\frac{\alpha_2 C-\alpha_2\alpha_1(1-\tau_1)M_1}{A}-$
$[\frac{\alpha_2}{A}]^{\alpha_2}[\frac{\beta_2}{A}]^{\beta_2}[C-\alpha_1(1-\tau_1)M_1]\}$ $\qquad (8a)$

With a constant slope for the pollution income path.

$\frac{\partial P}{\partial M_1}=\frac{\alpha_1(\tau_2\alpha_2+\beta_2)}{A}-\frac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}(\tau_2\alpha_2+\beta_2)}{A}+l\{\frac{\alpha_1\alpha_2(1-\tau_1)}{A}-\frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}\alpha_1(1-\tau_1)}{A}\}$

However with varying degrees of export $(1-\tau_1)$ and import $((1-\tau_2))$, we find that the magnitude of the slope can change. With no trade $t=1$ and$f=1$and $\frac{\partial P}{\partial M_1}$ in the equation above collapses to $\alpha_1-\alpha_1^{\alpha_1}\beta_1^{\beta_1}$ which is a positive constant. With no trade, we only observe a positive linear relationship between pollution and economic growth. In order to capture the impact of trade, we take the derivative of of $\frac{\partial P}{\partial M_1}$ separately with respect to τ_1 and τ_2. The derivative with respect to τ_1 gives:

$\frac{\partial P^2}{\partial M1_1\partial\tau_1}=\frac{\alpha_1(\tau_2\alpha_2+\beta_2)(\alpha_1^{\alpha_1}\beta_1^{\beta_1}-\alpha_1)}{A^2}+l\{\frac{\alpha_1^2(1-\tau_1)(\alpha_2^{\alpha_2}\beta_2^{\beta_2}-\alpha_2)}{A^2}+\frac{\alpha_1(\alpha_2^{\alpha_2}\beta_2^{\beta_2}-\alpha_2)}{A}\}<$
0

The derivative with respect to τ_2 gives:

$\frac{\partial P^2}{\partial M_1\partial\tau_2}=\frac{\alpha_2(\tau_2\alpha_2+\beta_2)(\alpha_1^{\alpha_1}\beta_1^{\beta_1}-\alpha_1)}{A^2}+\frac{\alpha_2(\alpha_1-\alpha_1^{\alpha_1}\beta_1^{\beta_1})}{A}+l\{\{\frac{\alpha_1\alpha_2(1-\tau_1)(\alpha_2^{\alpha_2}\beta_2^{\beta_2}-\alpha_2)}{A^2}\}\}\lesseqgtr$
0

This implies that as countries engage in more and more trade, the slope of the income pollution path will change. Thus with some combination of τ_1 and τ_2 it is possible to attain a negative linear relationship between pollution and economic growth.

1.4.3 The impact of pollution externality on the income-pollution path

This part of the discussion is raised to distinguish between local pollution and transboundary pollution. While some pollutants are local, others tend to infiltrate across boundaries. The degree of infiltration might vary. In some cases, pollution generated in one country permeates across to others, but the degree of permeation tends to go down with distance. In this case the degree of infiltration measured by l will be in between 0 and 1. In other cases, the pollutant generated in one country may completely carry forward to the other in which case the value of l will be 1.

I now try to look at how the difference in the degree of permeation affects the shape of the EKC in the model that I have developed. The pollution function remains the same as in equation (8):

$$P = \frac{\alpha_1(\tau_2\alpha_2+\beta_2)M_1-\alpha_1 B}{A} - [\frac{\alpha_1}{A}]^{\alpha_1}[\frac{\beta_1}{A}]^{\beta_1}[(\tau_2\alpha_2+\beta_2)M_1 - B]^{\alpha_1+\beta_1}+ l\{\frac{\alpha_2 C-\alpha_2\alpha_1(1-\tau_1)M_1}{A} - [\frac{\alpha_2}{A}]^{\alpha_2}[\frac{\beta_2}{A}]^{\beta_2}[C-\alpha_1(1-\tau_1)M_1]^{\alpha_2+\beta_2}\} \qquad (8)$$

The case of local pollutants: Clearly, when the nature of pollution is absolutely local, $l = 0$, and the second part of the equation will collapse to 0. This would mean that the shape of the EKC would be determined purely by the abatement technology of the local country. The existence of trade in this case will affect the level of pollution, but the curvature of the EKC will remain just as predicted by the Andreoni and Levinson model.

The case of transboundary pollutants: When there exists pollution externality ($0 < l \leq 1$), it holds two implications for Country1. First the level of pollution will be affected and second, there will be an impact on the income-pollution path.

The first impact that the degree of transboundariness has on pollution is its level. In order to grasp that we look at the marginal impact of the degree of captured by l in the model.

$$\frac{\partial P}{\partial l} = \frac{\alpha_2 C-\alpha_2\alpha_1(1-\tau_1)M_1}{A} - [\frac{\alpha_2}{A}]^{\alpha_2}[\frac{\beta_2}{A}]^{\beta_2}[C-\alpha_1(1-\tau_1)M_1]^{\alpha_2+\beta_2} \geq 0$$

This indicates that the higher the degree of externality, the higher will be the **level** of pollution.

The second impact that the degree of has is how **trade affects the curvature of EKC.** With higher degrees of (higher values of l) the dominance of the second part of the equation affecting the slope of EKC becomes stronger.

Proposition 3 *When pollution is sourced from the production of a consumption good, and when Country 1 engages in trade with Country 2 who exhibits increasing returns to abatement technology, the higher the extent of externality, lower is the required level of trade to bring about the inverted U pattern in income pollution path.*

Proof.

To begin with, the existence of pollution externality is necessary if the abatement technology of the foreign country is to have any impact on the slope of the income pollution path. When pollution externality is 0 in equation (9), the second part of the equation becomes zero. Thus in that case the only way that Country 1 can enjoy the EKC pattern is when they have IRS in abatement technology. Thus the model predictions collapses to that of Andreoni and Levinson. However, when pollution externality is positive, the impact of abatement technology of trading country comes into play, and IRS no longer remains a necessary condition for Country 1.

Next, the inequality in Condition (iii) of Proposition 1 must hold in order to ensure the EKC pattern in the income pollution path, when Country 2 exhibits IRS and Country 1 does not. This inequality becomes stronger the higher the value of l. Therefore, when the value of l goes up, the value of $(1-\tau_1)$ can be brought down to maintain the inequality. I therefore conclude that the higher the degree of of pollution, lower is the required level of trade in order to maintain the EKC pattern in the pollution income path.

■

1.5 Demonstration of results

From Section 1.4, I reach at three conclusions. First, when country 1 does not display IRS, by engaging in trade with countries that do, the desired EKC pattern in the income pollution path can be achieved when there exists pollution externalities. Second, increasing the fraction of Country 2's output imported affects the level of optimal pollution where as increasing the fraction of own goods exported affects the curvature of the income pollution path. Finally, the higher the degree of pollution externality, lower is the amount of trade required to achieve the desired result.

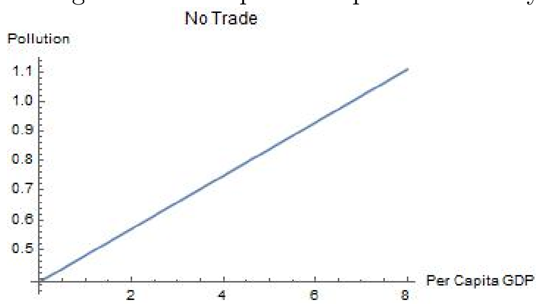
In this section, I show some simulation results that reinforce my model predictions. To maintain clarity, I highlight each of the conclusions as different cases.

Case1: Country 1 exhibits CRS but Country 2 exhibits IRS

Case 1 looks at situation where Country 1 displays CRS but engages in trade with Country 2 who displays IRS. For simulation purposes, the values assigned to the technology parameters are $\alpha_1 = 0.6; \beta_1 = 0.4; \alpha_2 = 0.7$; and $\beta_2 = 0.5$. Two of the results are highlighted here. One that engaging in trade is bringing about a change in the pattern of the income pollution path, and second that engaging in trade is bringing down the optimal level of pollution.

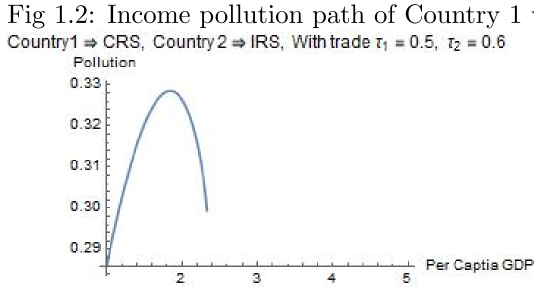
Sans trade, $\tau_1 = 1$ and $\tau_2 = 1$, $\frac{\partial P}{\partial M_1}$ becomes $(\alpha_1 - \alpha_1^{\alpha_1}\beta_1^{\beta_1})$ which is a positive constant, indicating a linear relationship between pollution and economic growth.

Fig 1.1: Income pollution path of Country 1 when it experiences CRS



With income growth and Country 1 engaging in trade, the abatement technology of Country2 starts to influence the income pollution path of Country 1 for the case of transboundary pollutants. The more country 1 engages in exports, the value of τ_1 becomes smaller, and the impact of the returns to scale to technology of Country 2 becomes more prominent. The model predicts that as long as $\alpha_2 + \beta_2 > 1$, $\frac{\partial P^2}{\partial M_1^2} < 0$ and thus the environmental Kuznets Curve is represented through trade even though Country 1 displays constant returns to scale.

Fig 1.2: Income pollution path of Country 1 when it engages in trade

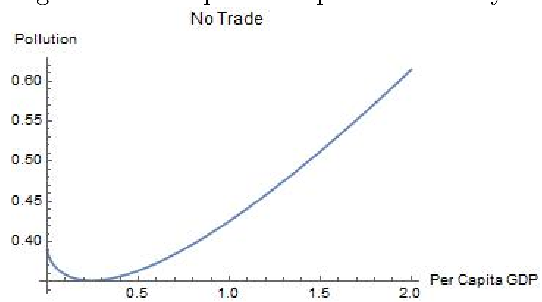


Case 2: Country 1 exhibits DRS while country 2 exhibits IRS

Case 2 portrays the situation where Country1 having DRS chooses to engage in trade with Country2 enjoying IRS. The model predicts that when Country

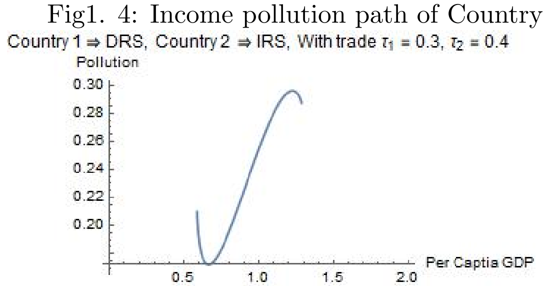
1 does not engage in trade the income pollution path will follow a U pattern as predicted by Andreon and Levinson (2001). Assuming that $\alpha_1 = 0.5$ and $\beta_1 = 0.3$, the simulation results confirm the model predictions.

1. Fig 1.3: Income pollution path of Country 1 when it experiences DRS



But as country 1 engages in exports, and income growth, the abatement technology of the trading partner starts to influence the income pollution relationship. When Country 2 enjoys IRS, let $\alpha_2 = 0.7, \beta_2 = 0.5$, the income pollution path eventually takes a downward trend. Thus, as predicted in section 1.4, the shape followed by the income pollution path resembles a sideways mirrored S.

Fig1. 4: Income pollution path of Country 1 when it engages in trade

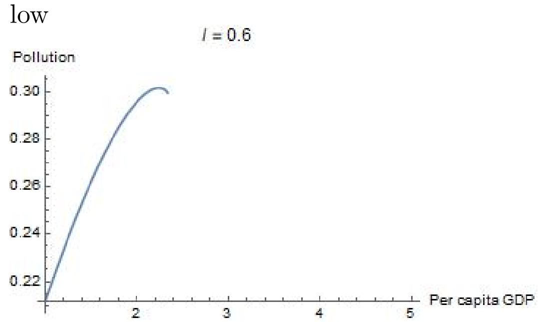


Case 3: The impact of pollution externality

In this part I discuss the final conclusion of the model developed in Section 1.2. The model predicts that abatement technology of the foreign country influences the income pollution pattern of the home country only in the case where there exists positive pollution externality of the foreign country. In other words, when Country 1 faces transboundary pollution, and Country1 engages in trade with Country2, the abatement technology of Country 2 will have an impact on the income pollution relationship of Country1. The model further concludes that the higher the extent of pollution externality, the abatement technology of Country 2 will dominate more even with lower levels of trade.

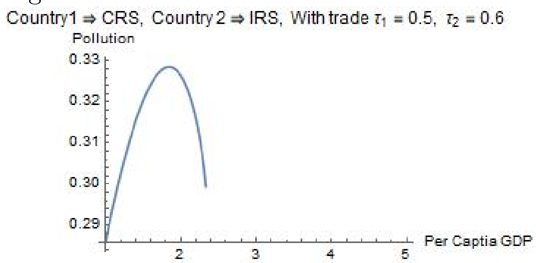
I discuss the case for when Country 1 has CRS and engages in trade with Country 2 having IRS. For example, suppose the parameter values assigned to Country 1 and 2 are $\alpha_1 = 0.6, \beta_1 = 0.4$ and $\alpha_2 = 0.7, \beta_2 = 0.5$ respectively when $l = 0.6$. With 30% exports and 30% imports, the desired shape of the EKC is not quite achieved.

Fig 1.5: Income pollution path of Country1 when pollution externality is low



But when $l = 1$, the desired shape can be attained with the same level of trade. The diagrams below help illustrate the situation:

Fig 1.6: Income pollution path of Country1 when pollution externality is high



The graphs above clearly portray both aspects of the impact of degree of on the model developed. As the degree of permeation increases, the level of pollution goes up for country 1. However, higher the degree of permeation, even if country 1 displays constant returns to scale, it can attain the desired shape of EKC by engaging in little trade. This conclusion can be extended to all the cases discussed above. But the gist of the matter remains that the higher the level of pollution permeation, smaller the degree of trade required to achieve desired results.

1.6 Conclusion

Economic growth and the environmental quality are clearly intertwined. The concept of the Environmental Kuznets Curve gives a pragmatic explanation to the relationship. To that end, Andreoni and Levinson (2001) provide a simple theoretical model where the technological link between consumption of a desirable good and abatement of its undesirable bi-product can be used to explain the existence of the EKC. However, the necessary condition of increasing returns to abatement technology that they provide has been questioned in literature. Whether IRS is always prevalent is a question of doubt. Also the implications that he draws for those that fail to display IRS is quite desolate. When countries exhibit constant returns to scale or decreasing returns to scale, the Andreoni and Levinson model predict the income pollution path to follow a linearly increasing or U-shaped pattern. Policy implications for such economies would be to restrict growth in the interest of the environment. Also, in their paper, Andreoni and Levinson do not find any distinction between local pollutants and transboundary pollutants. For both type of pollutants their model predictions remain unchanged.

In this paper, I look at the Andreoni and Levinson model in the presence of trade. Even though my model conclusions are very much in line with the findings of the Andreoni and Levinson model, engaging in trade provides with a way out for countries that fail to exhibit increasing returns to abatement technology. Along with introducing trade, I introduce heterogeneity with respect to abatement technology. I look at two possible situation. First, I look at a situation where pollution is sourced from consumption of a given good just like the original model. Even though introducing trade in this situation affects optimal consumption decision, the impact on pollution income path or even the level of pollution remain unchanged from the predictions of the original model.

As a second exercise, I introduce trade to the case where pollution is sourced from production of the consumption good. Pollution in general does in fact source from production in most cases and literature supports that. Also, abatement technology is often applied at the production phase rather than the consumption phase of a product's life. Therefore, such a deviation from the original model is completely justified. In this case I find that when nations do not exhibit IRS in their abatement technology, by engaging in trade with countries that do, it is possible to attain the desirable inverted-U-pattern in the income pollution path. However, this is possible only in the case where pollution is transboundary and not when pollution is local.

Therefore, the first contribution of my model is that introducing trade allows to draw a distinction between local pollutants and transboundary pollutants. Second, the model shows that for the case of transboundary pollutants the pattern of inverted U in the income pollution path can be attained. Thus, when countries do not enjoy increasing returns to abatement technology, engaging in trade can enable them to bring about a downward pattern in the income pollution path with economic growth when pollution is transboundary.

1.7Appendix

Appendix A

A.1 Solution to Model 3.1

For Country 1:

Utility: $U_1 = \tau_1 C_1 + (1-\tau_2)C_2 - z_1 P$ (1)

Resource Constraint: $M_1 = \tau_1 C_1 + (1-\tau_2)C_2 + E_1$ (2)

Transboundary Pollution:

$$P = C_1 + lC_2 - (\tau_1 C_1 + (1-\tau_2)C_2)^{\alpha_1} E_1^{\beta_1} - l((1-\tau_1)C_1 + \tau_2 C_2)^{\alpha_2} E_2^{\beta_2} \qquad (3)$$

The social planner will want to maximize utility subject to the resource constraint.

Let $z_1 = 1$.

Plugging in the value for $(1-\tau_2)C_2$ from Equation 2, and substituting the value for P in Equation 1, the maximization problem becomes:

$$U_1 = M_1 - C_1 - lC_2 - E_1 + (\tau_1 C_1 + (1-\tau_2)C_2)^{\alpha_1} E_1^{\beta_1} + l(M_2 - E_2)^{\alpha_2} E_2^{\beta_2}$$

The FOCs:

$$\frac{\partial P}{\partial C_1} = 1 - \alpha_1 t(\tau_1 C_1 + (1-\tau_2)C_2)^{\alpha_1 - 1} E_1^{\beta_1} = 0$$

$$\frac{\partial P}{\partial E_1} = 1 - \beta_1(1-\tau_2)(\tau_1 C_1 + (1-\tau_2)C_2)^{\alpha_1} E_1^{\beta_1 - 1} = 0$$

From the FOCS the solution for Country 1 comes to:

$C_1 = \frac{\alpha_1 M_1 - ((\alpha_1+\beta_1)(1-\tau_2)C_2}{\tau_1(\alpha_1+\beta_1)}$and $E_1 = \frac{\beta_1 M_1}{\alpha_1+\beta_1}$

Similarly, solving the models for Country 2, the solutions I get are:

$C_2 = \frac{\alpha_2 M_2 - ((\alpha_2+\beta_2)(1-\tau_1)C_1}{\tau_2(\alpha_2+\beta_2)}$and $E_2 = \frac{\beta_2 M_2}{\alpha_2+\beta_2}$

Plugging in the optimal value for C_2 in C_1, the general solutions for optimal consumption and abatement efforts are:

$C_1 = \frac{\tau_2\alpha_1(\alpha_2+\beta_2)M_1-(1-\tau_2)\alpha_2(\alpha_1+\beta_1)M_2}{(\tau_1+\tau_2-1)(\alpha_1+\beta_1)(\alpha_2+\beta_2)}$ and $E_1 = \frac{\beta_1 M_1}{\alpha_1+\beta_1}$

$C_2 = \frac{\tau_1\alpha_2(\alpha_1+\beta_1)M_2-(1-\tau_1)\alpha_1(\alpha_2+\beta_2)M_1}{(\tau_1+\tau_2-1)(\alpha_1+\beta_1)(\alpha_2+\beta_2)}$ and $E_2 = \frac{\beta_2 M_2}{\alpha_2+\beta_2}$

A.2 Solution for Model 3.2

For Country 1:

Utility: $U_1 = \tau_1 C_1 + (1-\tau_2)C_2 - z_1 P$ (1)

Resource Constraint: $M_1 = \tau_1 C_1 + (1-\tau_2)C_2 + E_1$ (2)

Transboundary Pollution: $P = C_1 + lC_2 - C_1^{\alpha_1}E_1^{\beta_1} - lC_2^{\alpha_2}E_2^{\beta_2}$ (3)

The social planner will want to maximize utility subject to the resource constraint.

Let $z_1 = 1$.

Plugging in the value for $(1-\tau_2)C_2$ from Equation 2, and substituting the value for P in Equation 1, the maximization problem becomes:

$U_1 = M_1 - C_1 - lC_2 - E_1 + C_1^{\alpha_1}E_1^{\beta_1} + lC_2^{\alpha_2}E_2^{\beta_2}$

The FOCs:

$\frac{\partial P}{\partial C_1} = 1 - \alpha_1 C_1^{\alpha_1-1}E_1^{\beta_1} = 0$

$\frac{\partial P}{\partial E_1} = 1 - \beta_1 C_1^{\alpha_1}E_1^{\beta_1-1} = 0$

The optimal decision for Consumption and Abatement Effort for Country 1 comes to:

$C_1 = \frac{\alpha_1(M_1-(1-\tau_2)C_2)}{\tau_1\alpha_1+\beta_1}$ and $E_1 = \frac{\beta_1(M_1-(1-\tau_2)C_2)}{\tau_1\alpha_1+\beta_1}$

By symmetry, the optimal decisions for consumption and abatement efforts for Country 2 will be:

$C_2 = \frac{\alpha_2(M_2-(1-\tau_1)C_1)}{\tau_2\alpha_2+\beta_2}$ and $E_1 = \frac{\beta_2(M_2-(1-\tau_1)C_1)}{\tau_2\alpha_2+\beta_2}$

Plugging in the optimal consumption decision of Country 2 in place of the optimal decisions of Country 1, the general solutions are:

$C_1^* = \frac{\alpha_1(\tau_2\alpha_2+\beta_2)M_1-\alpha_1\alpha_2(1-\tau_2)M_2}{(\tau_2\alpha_2+\beta_2)(\tau_1\alpha_1+\beta_1)-\alpha_1\alpha_2(1-\tau_1)(1-\tau_2)} \equiv \frac{B}{A}$

$C_2^* = \frac{\alpha_2(\tau_1\alpha_1+\beta_1)M_2-\alpha_2\alpha_1(1-\tau_1)M_1}{(\tau_2\alpha_2+\beta_2)(\tau_1\alpha_1+\beta_1)-\alpha_1\alpha_2(1-\tau_1)(1-\tau_2)} \equiv \frac{C}{A}$

$E_1^* = \frac{\beta_1(\tau_2\alpha_2+\beta_2)M_1-\beta_1\alpha_2(1-\tau_2)M_2}{(\tau_2\alpha_2+\beta_2)(\tau_1\alpha_1+\beta_1)-\alpha_1\alpha_2(1-\tau_1)(1-\tau_2)}$

$E_2^* = \frac{\beta_2(\tau_1\alpha_1+\beta_1)M_2-\beta_2\alpha_1(1-\tau_1)M_1}{(\tau_2\alpha_2+\beta_2)(\tau_1\alpha_1+\beta_1)-\alpha_1\alpha_2(1-\tau_1)(1-\tau_2)}$

Appendix B

Proof of Proposition 2

The optimal pollution function is:

$$P = \frac{\alpha_1(\tau_2\alpha_2+\beta_2)M_1-\alpha_1 B}{A} - [\frac{\alpha_1}{A}]^{\alpha_1}[\frac{\beta_1}{A}]^{\beta_1}[(\tau_2\alpha_2+\beta_2)M_1 - B]^{\alpha_1+\beta_1} +$$
$$l\{\frac{\alpha_2 C-\alpha_2\alpha_1(1-\tau_1)M_1}{A} - [\frac{\alpha_2}{A}]^{\alpha_2}[\frac{\beta_2}{A}]^{\beta_2}[C-\alpha_1(1-\tau_1)M_1]^{\alpha_2+\beta_2}\} \qquad (1)$$

Step 1

First I look at the impact of import and export on the optimal level of pollution.

This can be written in the following way:

$$P = C_1 - \frac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}}{\alpha 1_1^{\alpha_1+\beta_1}} C_1^{\alpha_1+\beta_1} + l\{C_2 - \frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}}{\alpha_2^{\alpha_2+\beta_2}} C_2^{\alpha_2+\beta 2}\} \qquad (2)$$

In order to look at the impact of import on the optimal level of pollution, I take the first derivative of equation (2) with respect to $(1-\tau_2)$.

$$\frac{\partial P}{\partial(1-\tau_2)} = \frac{\partial C_1}{\partial(1-\tau_2)}[1 - \frac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}(\alpha_1+\beta_1)}{\alpha_1^{\alpha_1+\beta_1}} C_1^{\alpha_1+\beta_1-1}] + l\{\frac{\partial C_2}{\partial(1-\tau_2)}[1 - \frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}(\alpha_2+\beta_2)}{\alpha_2^{\alpha_2+\beta_2}} C_2^{\alpha_2+\beta_2-1}]\}$$

$$\frac{\partial C_1}{\partial(1-\tau_2)} = \frac{-\alpha_1(\tau_2\alpha_2+\beta_2)[\alpha_2(\tau_1\alpha_1+\beta_1)M_2-\alpha_2\alpha_1(1-\tau_1)M_1]}{((\tau_2\alpha_2+\beta_2)(\tau_1\alpha_1+\beta_1)-\alpha_1\alpha_2(1-\tau_1)(1-\tau_2))^2} = \frac{-\alpha_1(\tau_2\alpha_2+\beta_2)C}{A^2}$$

$$\frac{\partial C_1}{\partial(1-\tau_2)} < 0.$$

$$\frac{\partial C_2}{\partial(1-\tau_2)} = \frac{\alpha_1\alpha_2(1-\tau_1)(\alpha_2(\tau_1\alpha_1+\beta_1)M_2-\alpha_1\alpha_2(1-\tau_1)M_1)}{((\tau_2\alpha_2+\beta_2)(\tau_1\alpha_1+\beta_1)-\alpha_1\alpha_2(1-\tau_1)(1-\tau_2))^2} = \frac{\alpha_1\alpha_2(1-t)C}{(A)^2}$$

$$\frac{\partial C_2}{\partial(1-\tau_2)} > 0.$$

Since Country1 exhibits either CRS or DRS, $[1 - \frac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}}{\alpha 1_1^{\alpha_1+\beta_1}} C_1^{\alpha_1+\beta_1-1}] > 0$.

Since Country 2 exhibits IRS, $[1 - \frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}}{\alpha_2^{\alpha_2+\beta_2}} C_2^{\alpha_2+\beta_2-1}] < 0$.

Thus I conclude that under these circumstances, $\frac{\partial P}{\partial(1-\tau_2)} < 0$. The larger the fraction of output of Country 2 imported into Country 1, the lower will be the optimal level of pollution.

Next I look at the impact on optimal pollution of export.

$\frac{\partial P}{\partial(1-\tau_1)} = \frac{\partial C_1}{\partial(1-\tau_1)}[1-\frac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}(\alpha_1+\beta_1)}{\alpha 1_1^{\alpha_1+\beta_1}}C_1^{\alpha_1+\beta_1-1}]+l\{\frac{\partial C_2}{\partial(1-\tau_1)}[1-\frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}(\alpha_2+\beta_2)}{\alpha_2^{\alpha_2+\beta_2}}C_2^{\alpha_2+\beta_2-1}]\}$

$\frac{\partial C_1}{\partial(1-\tau_1)} = \frac{\alpha_1\alpha_2(1-\tau_2)(\alpha_1(\tau_2\alpha_2+\beta_2)M_1-\alpha_1\alpha_2(1-\tau_2)M_2)}{((\tau_2\alpha_2+\beta_2)(\tau_1\alpha_1+\beta_1)-\alpha_1\alpha_2(1-\tau_1)(1-\tau_2))^2} = \frac{\alpha_1\alpha_2(1-\tau_2)B}{(A)^2}$

$\frac{\partial C_1}{\partial(1-\tau_1)} > 0.$

Since Country1 exhibits either CRS or DRS, $[1-\frac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}}{\alpha 1_1^{\alpha_1+\beta_1}}C_1^{\alpha_1+\beta_1-1}] > 0$.

$\frac{\partial C_2}{\partial(1-\tau_1)} = \frac{-\alpha_2(\tau_1\alpha_1+\beta_1)[\alpha_1(\tau_2\alpha_2+\beta_2)M_1-\alpha_1\alpha_2(1-\tau_1)M_2]}{((\tau_2\alpha_2+\beta_2)(\tau_1\alpha_1+\beta_1)-\alpha_1\alpha_2(1-\tau_1)(1-\tau_2))^2} = \frac{-\alpha_2(\tau_1\alpha_1+\beta_1)B}{(A)^2}$

$\frac{\partial C_2}{\partial(1-\tau_1)} < 0.$

Since Country 2 exhibits IRS, $[1-\frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}}{\alpha_2^{\alpha_2+\beta_2}}C_2^{\alpha_2+\beta_2-1}] < 0$.

Engaging in export will increase the optimal level of pollution.

Step 2

Next I look at the impact of export and import on the slope of the income-pollution path.

$\frac{\partial P}{\partial M_1} = \frac{\alpha_1(\tau_2\alpha_2+\beta_2)}{A}-\alpha_1^{\alpha_1}\beta_1^{\beta_1}(\alpha_1+\beta_1)\frac{(\tau_2\alpha_2+\beta_2)}{A}[\frac{(\tau_2\alpha_2+\beta_2)M_1-\alpha_2(1-\tau_2)M_2}{A}]^{\alpha_1+\beta_1-1} - l\{\frac{\alpha_1\alpha_2(1-\tau_1)}{A} - \alpha_2^{\alpha_2}\beta_2^{\beta_2}(\alpha_2+\beta_2)\frac{\alpha_1(1-\tau_1)}{A}[\frac{(\tau_1\alpha_1+\beta_1)M_2-\alpha_1(1-\tau_1)M_1}{A}]^{\alpha_2+\beta_2-1}\}$

With a little bit of manipulation, the slope of the income pollution path can be written in the following manner:

$\frac{\partial P}{\partial M_1} = \frac{\alpha_1(\tau_2\alpha_2+\beta_2)}{A}[1-(\alpha_1+\beta_1)\frac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}}{\alpha_1^{\alpha_1+\beta_1}}C_1^{\alpha_1+\beta_1-1}] + l\{\frac{\alpha_2\alpha_1(1-\tau_1)}{A}[-1+(\alpha_2+\beta_2)\frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}}{\alpha_2^{\alpha_2+\beta_2}}C_2^{\alpha_2+\beta_2-1}]\}$

First I look at the impact of import on the slope of the income pollution path

$\frac{\partial^2 P}{\partial M_1\partial(1-\tau_2)} = \alpha_1[1-(\alpha_1+\beta_1)\frac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}}{\alpha_1^{\alpha_1+\beta_1}}C_1^{\alpha_1+\beta_1-1}][[\frac{\alpha_1\alpha_2(1-\tau_1)(\tau_2\alpha_2+\beta_2)}{A^2}]]+\frac{(\tau_2\alpha_2+\beta 2)}{A}[-(\alpha_1+\beta_1-1)(\alpha_1+\beta_1)\frac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}}{\alpha_1^{\alpha_1+\beta_1-1}}C_1^{\alpha_1+\beta_1-1}\frac{\partial C_1}{\partial(1-\tau_2)}]$

$+l\{\alpha_2[-1+(\alpha_2+\beta_2)\frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}}{\alpha_2^{\alpha_2+\beta_2}}C_2^{\alpha_2+\beta_2-1}][[\frac{\alpha_1^2\alpha_2(1-\tau_1)^2}{A^2}]] + \frac{\alpha(1-\tau_1)}{A}[(\alpha_2+\beta_2-1)(\alpha_2+\beta_2)\frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}}{\alpha_2^{\alpha_2+\beta_2}}C_2^{\alpha_2+\beta_2-1}\frac{\partial C_2}{\partial(1-\tau_2)}]\}$

When Country 1 exhibits CRS or DRS $[1-(\alpha_1+\beta_1)\frac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}}{\alpha_1^{\alpha_1+\beta_1}}C_1^{\alpha_1+\beta_1-1}] > 0$. $(\alpha_1+\beta_1-1) < 0$ when Country 1 exhibits DRS. $\frac{\partial C_1}{\partial(1-\tau_2)} < 0$.Therefore, the first part to the equation above is negative. Since Country 2 represents IRS, $[1-(\alpha_2+\beta_2)\frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}}{\alpha_2^{\alpha_2+\beta_2}}C_2^{\alpha_2+\beta_2-1}] > 0$. $(\alpha_2+\beta_2-1) > 0$ and $\frac{\partial C_2}{\partial(1-\tau_2)} > 0$.Therefore $l\{.\}$ is positive. The impact of import on the slope of the income pollution path is non conclusive.

Next I look at the impact of export on the slope of the income pollution path.

$\frac{\partial^2 P}{\partial M_1\partial(1-\tau_1)} = \alpha_1[1-(\alpha_1+\beta_1)\frac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}}{\alpha_1^{\alpha_1+\beta_1}}C_1^{\alpha_1+\beta_1-1}][[\frac{\alpha_1\alpha_2(1-\tau_1)(\tau_2\alpha_2+\beta_2)}{A^2}]]+\frac{(\tau_2\alpha_2+\beta 2)}{A}[-(\alpha_1+\beta_1-1)(\alpha_1+\beta_1)\frac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}}{\alpha_1^{\alpha_1+\beta_1-1}}C_1^{\alpha_1+\beta_1-1}\frac{\partial C_1}{\partial(1-\tau_1)}]$

$+l\{\alpha_2[-1+(\alpha_2+\beta_2)\frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}}{\alpha_2^{\alpha_2+\beta_2}}C_2^{\alpha_2+\beta_2-1}][[\frac{\alpha_1^2\alpha_2(1-\tau_1)^2}{A^2}]]+\frac{\alpha(1-\tau_1)}{A}[(\alpha_2+\beta_2-1)(\alpha_2+\beta_2)\frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}}{\alpha_2^{\alpha_2+\beta_2}}C_2^{\alpha_2+\beta_2-1}\frac{\partial C_2}{\partial(1-\tau_1)}]\}$

When Country 1 exhibits CRS or DRS $[1-(\alpha_1+\beta_1)\frac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}}{\alpha_1^{\alpha_1+\beta_1}}C_1^{\alpha_1+\beta_1-1}] > 0.(\alpha_1+\beta_1-1) < 0$ and $\frac{\partial C_1}{\partial(1-\tau_1)} > 0$.Therefore the first part of the eqution is entirely positive. $\{\alpha_2[-1+(\alpha_2+\beta_2)\frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}}{\alpha_2^{\alpha_2+\beta_2}}C_2^{\alpha_2+\beta_2-1}][[\frac{\alpha_1^2\alpha_2(1-\tau_1)^2}{A^2}]]$ is positive with only $\frac{\alpha(1-\tau_1)}{A}[(\alpha_2+\beta_2-1)(\alpha_2+\beta_2)\frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}}{\alpha_2^{\alpha_2+\beta_2}}C_2^{\alpha_2+\beta_2-1}\frac{\partial C_2}{\partial(1-\tau_2)} < 0$since the derivative of $C_2$with respect to $(1-\tau_1)$ is negative. One can thus conclude that the impact of export on the slope of the income pollution path will be positive.

Essay 2: Revisiting "The Environmental Kuznets Curve in a Multi-Country Setting" in the presence of Trade - the case of homogeneous technology

2.1 Introduction

That economic growth and the environmental quality of a country are interdependent is obviously apparent. The existence of substantial environmental resources - be it in the form of clean air and water for subsistence of human life or in the form of extractable natural resources with opportunities for economic exploitation - is a critical factor ensuring economic growth. Conversely, economic activity is perhaps the most pertinent source of environmental degradation. With economic growth comes, exploitation of natural resources, increased production, consumption and hence pollution. At the same time it is also true that, economic growth itself is a panacea for environmental degradation. With economic prosperity comes a nations willingness and ability to invest in and care for environmental resources. How economic growth effects the environment is a debate that dates back to as early as the early 1970s. Meadows *et al* (1972) published a report titled "The limits to Growth" where they concluded that different natural resources would be depleted with economic growth and even went as far as to predict possible depletion dates for them. Cole (1973) vehemently criticized the report claiming that economic growth would bring forward technological innovations that would relax the constraint on economic resources. Thus while some economists focused on the constraint that economic growth posed for natural resources, others saw economic growth as a mandatory engine for the betterment of environmental quality.

Along the latter line of thought, in the early 1990s a group of economists conducted empirical studies on a number of pollutants and came up with the conclusion that while the early stages of economic growth cause pollution to rise, further economic growth causes pollution to go down (Grossman and Kreuger (1991), Shafik and Bandyopadhyay (1992), Panayotou (1993)). Such a pattern in the income-pollution path was termed as the Environmental Kuznets Curve (the EKC), named after Kuznets (1955) who observed a similar pattern in income inequality and economic growth.With the advent of the EKC, several studies to analyze the concept followed and is continuing till today. While some studies were empirical, looking for the proof of the existence of the EKC on different pollutants, others were more theoretical attempting to give structure and hindsight behind the empirical evidence. A detailed summary of the different scope and work done in the area is included in the following section, the literature review.

The discussion of the environmental quality, however, cannot be restricted within the boundaries of a given nation - simply because no boundary can be drawn upon the environment! To begin with, a lot of the resources of the world are shared property - the maintenance of which require cooperation of all parties involved. The ocean resources for example. Second, pollution generation involves externalities. Thus pollution generated in one country can permeate into the borders of the neighboring countries as well. Air pollution and water pollution are common examples. Finally, actions of individual countries can have an impact on the whole world. Example: global warming and climate change. Thus, economists today realize that there exists a certain amount of pollution interdependence among countries and a need to analyze the existence of pollution externalities or global pollution.

Some work can be found on the topic in literature. For example, Andreoni and Levinson (2001) develop a static model where they show that the existence of the EKC can be explained by a simple technological relationship between the consumption of a desirable good and abatement of its undesirable bi-product. They later expand their model to the multi-country case to account for pollution externalities. They conclude that when pollution is sourced from consumption, individual countries' optimal decision remain unchanged with the introduction of externalities. Thus, in their model, Andreoni and Levinsion find no distinction between the single-country and multi-country framework. On the contrary, Gill et al (2017) write that the pattern of the EKC is more applicable to local pollutants and not so much for transboundary pollutants. They therefore argue, that even though the advanced economies of today have followed the EKC pattern in their growth process it is a dangerous route for the less developed economies of today. If the developing countries of today continue to contribute to the global pollution irreparable damages may be caused to the environment. Developing countries of the world today should thus follow a plan other than the one proposed by the EKC.

In this paper I build upon the discussion of transboundary pollution by investigating it in the presence of trade. In their analysis of The Environmental Kuznets's Curve, Diamantoudi and Filippiadis (2012) propose a multi-country model for the analysis of the existence of the EKC in the case of global pollutants. The consideration of global pollutants, allows for capturing the interdependence in countries' pollution decision thus posing the possibility of a pollution game. They show that in the case of global pollutants there is a scope for pollution substitution among countries. In this paper, I build upon their model by introducing trade to the equation. The justification behind such an extension is that trade too allows for interdependence in pollution decision. In addition to capturing the negative impact of pollution externality (captured in a model with global pollutant) trade allows for capturing the positive impact through a wider range of consumption possibilities. I therefore argue that the pollution/production decision of one country effects others in two ways: first a positive affect by allowing for a wider range of consumption possibilities and second a negative effect through pollution generation and externality.

I have carried out a similar task in my first paper, where I introduced trade as an exogenous variable to the Andreoni and Levinson (2001) model. However, the objective of the two exercises are distinctly different. In my first paper the objective was to assess if by engaging in trade the assumption of increasing returns to abatement technology remains a necessary condition for the existence of the EKC pattern in the income-pollution path of a country. In this paper the focus is rather on the scope for pollution substitution. Thus by introducing trade, I assess the impact on the scope for pollution substitution, optimal pollution and the shape of the EKC. By the way of the model is set up, IRS is not a condition dictating the shape of the EKC in this case. For simplicity, in this paper, I maintain that the abatement technology of the two countries are homogeneous. I later relax this assumption in my third paper.

The discussion of international trade and the EKC is not new in literature. In fact, in the past theories have been posed where international trade is one of the factors causing the downward shift of the income-pollution path as proposed by the EKC. For example, Arrow *et al* (1995) postulate that as economies grow, they cease the production of certain dirty goods - the demand for which however prevails. This demand is fulfilled by importing the dirty goods from the less developed economies. Suri and Chapman (1998) suggested that as countries grow, they transfer the dirty industries to the developed world. These models are however limited to the discussion of local pollutants. There is not much literature on EKC covering the role of international trade on transboundary or global pollutants.

By introducing trade to the Diamantoudi and Filippiadis model, I also create a distinction between local pollutant and transboundary pollutant in the model. The model developed by Diamantoudi and Fillipiadis (2012) focused solely on the case of global pollutant as that was the avenue through which they created interdependence across countries. By introducing trade to the model, I have opened up a second scope of interdependence. I show that when trade exists across nations, there is still a scope of pollution substitution even when pollutants are local.

A point to note is that in the model, I have kept trade as exogenous to the model. The reason for that is that the objective of the paper is not to explain why or how much trade countries engage in. Rather the objective is to observe how varying the degree of import and export affects the optimal pollution decision and the income pollution path of a country. It also observes how engaging in export and imports creates a scope for pollution substitution among trading nations.

From the results, I find that engaging in trade does create a scope for pollution substitution for global pollutants as well as for local pollutants. Engaging in imports makes the scope for pollution substitution stronger. Engaging in exports can also make the scope for pollution substitution stronger under certain circumstances. Also, higher the degree of pollution externality, higher will be the scope for pollution substitution. For the impact on optimal pollution, the model concludes that engaging in imports can allow a country to reduce its optimal pollution. This, however, is true only for the case of local pollutants. For transboundary pollutants, engaging in imports also increases optimal pollution levels. The inverted U shape in the pollution-income path can be retained even after countries engage in trade for both local pollutants as well as global pollutants. However, for the case of local pollutants, engaging in imports shifts the EKC down, indicating that at every level of income, a country pollutes less while following the EKC pattern in their income pollution path. For the case of global pollution, the EKC shifts upwards when countries engage in imports and exports. For both local and transboundary pollution, the model finds that when countries engage in exports, the turning point of EKC shifts outwards, but the shape of the EKC is retained.

The rest of the paper is organized in the following order. Chapter 2.2 provides with the literature review. The model is presented in Chapter 2.3. The comparative static analysis is conducted in Chapter 2.4. Chapter 2.5 presents with a graphical display of the results found in Section 2.4. Finally Chapter 2.6 provides the conclusion.

2.2 Literature Review

Even though the discussion of the relationship between economic growth and the environmental quality dates back a long time, the actual advent of the concept of the EKC occurred in the early 1990s. The EKC traces out a simple and pragmatic relationship between economic growth and environmental quality, where it postulates that with the early stages of economic development pollution rises and with further economic progress pollution eventually declines. Such a relationship was first proposed by Grossman and Kreuger(1991) in his report on the potential impacts of the NAFTA and quickly strengthened by Shafik and Bandyopadhyay (1992) in their background study for the 1992 World Development Report. A similar pattern among income inequality and per capita income was observed and explained by Kuznets in 1955 who called it the Kuznets Curve. In 1993, Panayoutou picked up on this idea and suffixed the term with 'Environmental' to give rise to the term - the Environmental Kuznets Curve.

Initial work on the literature of EKC was largely empirical. Selden and Song (1994) have confirmed the inverted U pattern in the income pollution path using a cross-national panel of data on emission of four air pollutants namely suspended particulate matter, sulfur di-oxide, oxides of nitrogen, and carbon monoxide. However, they predicted emissions to decrease only in the very long run with rapid growth of global emissions to continue for several decades. Grossman and Kreuger (1995) also examine the relationship between per capita GDP and four environmental indicators and conclude in favor of the existence of the EKC. They further provide with some economic intuition for such a pattern. They explain that first, with growth more stringent environmental policies are adopted as with economic progress, non economic aspects of life such as a clean environment become more desirable for economic agents. Second, as economies grow they cease to produce some of the dirtier products and start to import them instead. Finally, with time and learning from the experiences of the wealthier countries, countries may adopt cleaner technologies.

Because of its pro-growth intuition and important policy implications the EKC gained large popularity and much work has been done towards developing models to explain the relationship. Arrow *et al.* (1995) pointed out that the shape of the EKC can be explained by the natural progression of an economy from clean agrarian society to polluting industrial economies to clean service based economies. A lot of models have been developed to explain the shape of the EKC with each making a set of assumptions. A first group of models exist that argue that economic development brings about changes in preferences and with economic prosperity comes a rise in the demand for a clean environment. Example: Lopez (1994), John and Pecchenino (1994), Jaeger (1998). Jeager(1998) shows that low levels of economic growth portray low levels of pollution at which point consumers marginal benefit to additional environmental quality is zero. Therefore, at this stage, lower pollution does not lead to reduced utility. However, with growth and increased pollution, consumers become sensitive to increased pollution.

Another group of economists pointed out that with economic development some constraints become non-binding, adoption of which lead to a decline in the level of pollution. For example Stokey (1998) model that in the beginning countries use the dirtiest technology. With economic development, when cleaner technologies become economically feasible, adoption of cleaner technologies lead to a decline in pollution. Lieb (2001) generalizes Stokey's model arguing that satiation in consumption is needed to bring about the EKC.

The models discussed above are all single agent models. In reality pollution decisions often involve more than one country. Realizing this a group of economists tried to explain the shape of the EKC through international trade. Example Arrow et al. (1995), Suri and Chapman (1998) and Copeland and Taylor (2001). The gist of argument in all these papers propose that with economic development, dirty industries get relocated to the developing world. The views presented in these papers are rather pessimistic or non sustainable in the sense that as all the economies of the world grow, there won't remain a poor country to turn to.On a more positive note Dean (1999) find that trade liberalization directly aggravates environmental damage via its influence on the terms of trade, but indirectly mitigates it via its effect on income growth. Holladay (2008) look at the impact of trade liberalization on the environment. He focuses on two distinct sources of pollution - pollution from consumption and pollution from production, and concludes that pollution from consumption increases where as pollution from production decreases after trade liberalization.

Introduction of trade opens up one aspect of interdependence among countries in the sense that pollution can be substituted by transferring pollution elsewhere. There still remains another important aspect of pollution - that of externality. Seldom, pollution generated in one location is restricted to that location alone. Pollution often infiltrates across boundaries thereby creating another source of interdependence among countries. With advancement in the literature of the EKC focus changed to incorporate transboundary pollution into the discussion.Andreoni and Levinson (2001) develop a static model where they show that the existence of the EKC can be explained by a simple technological relationship between the consumption of a desirable good and abatement of its undesirable bi-product. They later expand their model to the multi-country case to account for pollution externalities. They conclude that when pollution is sourced from consumption, individual countries' optimal decision remain unchanged with the introduction of externalities. Thus, in their model, Andreoni and Levinsion find no distinction between the single-country and multi-country framework. On the contrary, Gill et al (2017) write that the pattern of the EKC is more applicable to local pollutants and not so much for transboundary pollutants. They therefore argue, that even though the advanced economies of today have followed the EKC pattern in their growth process it is a dangerous route for the less developed economies of today. If the developing countries of today continue to contribute to the global pollution irreparable damages may be caused to the environment. Developing countries of the world today should thus follow a plan other than the one proposed by the EKC. Diamantoudi and Filippiadis (2010) extend upon the Stokey (1998) and Lieb (2004) model by introducing global pollutants. In so doing, they bring into focus the possibility of pollution substitution among countries. Filippiadis (2014) again extend upon their work by introducing a dynamic setting to the multi-country model. He questions the certainty of existence of the EKC pattern in the income pollution path and argues that such as existence depends upon the initial stock of physical capital and the total factor productivity of a country as well as the in between interaction between the two countries.

In summary, the models discussed do not confirm the existence of an EKC. All the models give a set of parameters or restrictions within which the EKC exists. In this paper I have thus selected an existing model that successfully traces the existence of the EKC under a rich set of parameters and then see how the model predictions change when introducing trade to the discussion. Two forms of inter-country interdependence has been identified in the literature of the EKC. One in the form of trade, and another in the form of pollution externality. However, the two interdependencies together is yet to be investigated. By introducing trade to the Diamantoudi and Filippiadis (2010) model, I bring the distinct discussion of interdependency through pollution externality and that of trade under one umbrella.

2.3 The Model

Analyzing the impact of trade in an n country framework is complicated. To maintain the tractability and analytical clarity of the model, I limit my discussion to a two country case. I assume there are two countries,$i \in \{1, 2\}$ each producing a single commodity. Each country retains a fraction of its production for own consumption τ_i for $i = 1, 2$, and exports the rest to the other country. The goods produced in each country generates pollution x_i as a bi-product of the production process. For each country i a social utility function is linearly separable in consumption and pollution and is given by:

$V_i = u_i(c_1, c_2) - h_i(l_1 x_1 + l_2 x_2) \qquad (1)$

where u_i is a country specific utility function, twice continuously differentiable, that is increasing and concave in consumption c_1 and $c_2 .h_i$ is also twice continuously differentiable, convex and rising x_1 and x_2 and captures the negative utility of each country associated with pollution. The term $l_i \in \{0, 1\}$ captures the extent of pollution that Country i is exposed to. For the own country the value of l_i will always be 1 as the pollution generated in own country will affect the country in question. For the foreign country, the value assigned to l_i can be 0 or 1. A value of 0 means that the pollution generated in the other country is local and a value of 1 would mean that pollution generated in the other country is global.

Let the country specific utility function take the form:

$$u_i = \begin{cases} c_i - \frac{1}{2}\beta c_i^2, if c_i \leq \frac{1}{\beta} \\ \frac{1}{2}\beta, otherwise \end{cases}$$

with $\beta > 0.\beta$ is the risk aversion indicator. The higher the value of β the more risk averse a country is.

$h_i = \frac{\rho}{2}(x_1 + l_2 x_2)^2$

with $\rho \in R_+$ is a scale parameter capturing how pollution is perceived by country i.

Output in country i is represented by the following production function:

$q_i = y_i \sigma(x_i) = y_i \frac{x_i}{\overline{x_i}}$

where y_i is the potential output of country i when the dirtiest technology is used. $\sigma(x_i)$ is a technology index that converts potential output in to quantities ready for consumption. $\overline{x_i}$ is the maximum pollution of a given country. The assumption in the model is that pollution is bounded from above by $\overline{x_i}$. x_i is the level of pollution generated by country i when producing quantity q_i.Clearly, the maximum value that $\sigma(.)$ can take is 1, that is the current level of pollution is at the maximum permissible level, and therefore y_i is the potential output when the dirtiest technology is used. Let the upper bound to pollution be an increasing function of income such that:

$\overline{x_i} = y_i^\alpha, \qquad \alpha \in (0,1)$

y_i^α represents the technology relating to the abatement of pollution. The higher the level of technology, the lower will be the value of α. One can even view this as a policy parameter where a country with more strict environmental regulations will maintain a lower value of α.

So far the model set up is quite similar to that developed by Diamantoudi and Filippiadis (2012). As a point of departure I introduce trade to the model. As explained earlier, each country retains a fraction of the output q_i for its own consumption and exports the rest to the foreign country. Given that τ_i captures the fraction of own output that each country retains for itself, the consumption set for the two countries become:

$c_1 = \tau_1 y_1 \frac{x_1}{y_1^\alpha} + (1-\tau_2) y_2 \frac{x_2}{y_2^\alpha}$ for Country 1, and

$c_2 = (1-\tau_1) y_1 \frac{x_1}{y_1^\alpha} + \tau_2 y_2 \frac{x_2}{y_2^\alpha}$ for Country 2.

Under such a framework the maximization problem faced by the social planner of Country $i = 1$ is:

$$V_1 = [\tau_1 y_1 \frac{x_1}{y_1^\alpha} + (1-\tau_2) y_2 \frac{x_2}{y_2^\alpha}] - \frac{\beta}{2}[\tau_1 y_1 \frac{x_1}{y_1^\alpha} + (1-\tau_2) y_2 \frac{x_2}{y_2^\alpha}]^2 - \frac{\rho}{2}(x_1 + l_2 x_2)^2 \qquad (2)$$

The first order derivative of equation (2) above gives the optimal pollution decision for Country 1:

$$x_1 = \begin{cases} \frac{\tau_1 y_1^{1-\alpha}}{(\beta\tau_1^2 y_1^{2-2\alpha} + \rho)} - \frac{(\beta\tau_1(1-\tau_2) y_1^{1-\alpha} y_2^{1-\alpha} + l_2\rho) x_2}{(\beta\tau_1^2 y_1^{2-2\alpha} + \rho)}, if x_2 > \phi \\ y_1^\alpha, otherwise \end{cases}$$

$$with \phi = \frac{\tau_1 y_1^{1-\alpha} - y_1^\alpha(\beta\tau_1^2 y_1^{2-2\alpha} + \rho)}{l_2\rho + \beta\tau_1(1-\tau_2) y_1^{1-\alpha} y_2^{1-\alpha}}$$

Pollution decision of Country $i = 1$ depends upon own income and parameters as well as the income and pollution decision of Country $i = 2$, and the degree of trade. In the following sections the implications of each of them are investigated more closely.

Proposition 4 *When countries do not engage in trade the nature of interdependency among pollutants can be captured only for the case of global pollutants. Introducing trade into the model allows for capture of pollution interdependence for the case of local pollutants as well as global pollutants. Engaging in imports makes the scope for pollutions substitution stronger. Engaging in exports makes the scope for pollution substitution strong under specific conditions.*

Proof. *To assess the scope for pollution substitutability I take the derivative of optimal pollution of Country $i = 1$ with respect to the pollution of Country $i = 2$.*

$$\frac{\partial x_1}{\partial x_2} = -\frac{(\beta\tau_1(1-\tau_2)y_1^{1-\alpha}y_2^{1-\alpha}+l_2\rho)}{(\beta\tau_1^2y_1^{2-2\alpha}+\rho)} < 0 \qquad (4)$$

The negative sign in the derivative suggests that there is scope for pollution substitution between nations. Note that absent trade, i.e $\tau_1 = 1$ and $\tau_2 = 1$, a scope for pollution substitution exists only when there is global pollution ($l_2 = 1$). But when countries do engage in trade, ($0 < \tau_1, \tau_2 < 1$) there is a scope for pollution substitution between trading nations even when the nature of pollutants is strictly local (i.e, $l = 0$).

I now investigate the impact of trade on the degree of pollution substitution. First the impact of imports:

$$\frac{\partial^2 x_1}{\partial x_2 \partial(1-\tau_2)} = -\frac{(\beta\tau_1 y_1^{1-\alpha}y_2^{1-\alpha}+l_2\rho)}{\beta\tau_1^2y_1^{2-2\alpha}+\rho} < 0 \qquad (5)$$

Thus the larger fraction of output of Country 2 that Country 1 imports, the stronger the scope of pollution substitution. Intuitively, rather than producing goods at home, if a country wishes to engage in imports, (the production of which generate pollution) , the level of pollution at the home country can be reduced. This conclusion holds for both the case of local pollutants as well as for global pollutants. Also, the higher the income of the foreign country, stronger will be the scope of pollution substitution through imports. Finally the higher the pollution perception at home, the weaker will be the scope of pollution substitution through imports.

The impact of exports on the possibility of pollution substitution is however less obvious. Taking the derivative with respect to τ_1 gives:

$$\frac{\partial^2 x_1}{\partial x_2 \partial \tau_1} = \frac{\beta y_1^{1-\alpha}[\beta\tau_1^2(1-\tau_2)y_1^{2-2\alpha}y_2^{1-\alpha}-\rho(1-\tau_2)(1-\tau_1)y_2^{1-\alpha}+2l_2\rho\tau_1 y_1^{1-\alpha}]}{(\beta\tau_1^2y_1^{2-2\alpha}+\rho)^2} \qquad (6)$$

When the value to equation (6) is positive, it means that the lower the value of τ_1 (the higher the value of $(1-\tau_1)$, the stronger will be the scope for pollution *substitution. This will be true for the following conditions.*

When pollutants are strictly local ($l_2 = 0$), equation (6) will be positive if

$$y_1 > [\frac{\rho(1-\tau_1)}{\beta\tau_1^2}]^{\frac{1}{2-2\alpha}} \qquad (6a)$$

Thus when pollutants are local, the higher the income of Country1, engaging in export will make the scope of pollution substitution stronger.

When pollutants are global ($l_2 = 1$) the following conditions ensure the positivity of equation 6.

$$y_2 < [\frac{2l_2\rho\tau_1 y_1^{1-\alpha}}{(1-\tau_2)(\rho(1-\tau_1)-\beta\tau_1^2 y_1^{2-2\alpha})}]^{\frac{1}{1-\alpha}} \qquad (6b)$$

When the condition above is not satisfied, equation (6) will take a negative value, meaning that the more Country 1 engages in exports, the weaker will be the scope for pollution substitution through exports ■

Also, the extent of pollution externality also has an affect on the degree of pollution substitution$\frac{\partial^2 x_1}{\partial x_2 \partial l_2} = -\frac{\rho}{\beta\tau_1^2 y_1^{2-2\alpha}+\rho} < 0$ meaning that the higher the degree of pollution externality of Country 2, the stronger is the scope for pollution substitution. The higher the income of the foreign country, the stronger will be the scope of pollution substitution.

2.4 Comparative Statics

2.4.1 The Symmetric Case

Suppose the two countries are identical with respect to income as well as other parameters. Note that under such a circumstance, the justification for engaging in trade becomes weak. However, such an exercise is necessary to set the benchmark and thereby make further investigations (for example the asymmetric case) more tractable. One can think of this as a case where there are two countries, similar in income and other parameters, but producing two different commodities. The justification of trade in this case lies in the appeal to increase the composition of consumption possibilities. Given that, the optimal pollution for the symmetric case solves as follows:

$$x^* = \frac{\tau_1 y^{1-\alpha}}{\beta\tau_1 y^{2-2\alpha}(\tau_1+(1-\tau_2))+2\rho} \qquad (7)$$

where $x^* = x_i^*$. So long as the optimal pollution $x^* < \overline{x}$, the optimal pollution-income path follows the inverted U-shape where

$$\begin{cases} \frac{\partial x^*}{\partial y} \geq 0, if y < (\frac{\beta\tau_1(\tau_1+(1-\tau_2)}{2\rho})^{-\frac{1}{2-2\alpha}}, \\ \frac{\partial x^*}{\partial y} < 0, otherwise. \end{cases} \qquad (8)$$

where the turning point of income will be at $y_{Tp} = (\frac{\beta\tau_1(\tau_1+(1-\tau_2))}{2\rho})^{-\frac{1}{2-2\alpha}}$. Trade clearly has an impact on the turning point of the EKC.

$$\frac{\partial y_{TP}}{\partial \tau_1} = -((\frac{1}{2-2\alpha}))((\frac{\beta(2\tau_1+(1-\tau_2)}{2\rho}))(\frac{\beta\tau_1(\tau_1+(1-\tau_2))}{2\rho})^{(-\frac{1}{2-2\alpha}-1)} < 0 \qquad (9)$$

Thus the higher the value of τ_1 (the lower the level of exports), the earlier will be the turning point of income. Engaging in exports, will therefore push the turning point of income outwards. This may not be entirely undesirable. For even though, the turning point occurs at a higher level of income, engaging in exports expedites income growth and may thus allow for the opportunity to reach that higher income at a faster rate.

$$\frac{\partial y_{TP}}{\partial(1-\tau_2)} = -((\frac{1}{2-2\alpha}))((\frac{\beta\tau_1}{2\rho}))(\frac{\beta(\tau_1^2+\tau_1(1-\tau_2))}{2\rho})^{(-\frac{1}{2-2\alpha}-1)} < 0 \qquad (10)$$

The higher the fraction of output imported, the lower will be the turning point. By engaging in imports, countries can bring the turning point of EKC forward. Thus it shows, that the shape of the EKC is retained even after countries engage in international trade. Engaging in imports will also reduce the optimal level of pollution as $\frac{\partial x}{\partial(1-f)} < 0$. Thus when countries engage in international trade, a combination of imports and exports will bring forth a desirable impact on the EKC of the country.

Note that the shape of the EKC is retained both in the case of local pollutants ($l = 0$) and when there exists pollution externalities ($l > 0$). However for the case of transboundary pollutants, engaging in imports will bring forward the turning point faster than the case of local pollutants.

Clearly $\frac{\partial x}{\partial \beta} < 0$and $\frac{\partial x}{\partial \rho} < 0$ stating that the more risk averse the agents of a country are and the higher the perception of pollution is respectively, the optimal level of pollution will go down.

2.4.2 The Asymmetric Case

I now check the model for the asymmetric case. As a starting point, I maintain asymmetry with respect to income . I now look at the case where there are two countries each having a different level of income. I also state that the pollution generated in the two countries can vary, i.e they can either be local or global. To maintain the tractability of the model, I maintain that the model is symmetric with respect to all other parameters, i.e, α representing technology, β representing the degree of risk averseness, and ρ capturing perception of pollution. As the technology of the two countries are assumed similar, I maintain that pollution will either be local or global. In either case $l_1 = l_2$. Solving the model in this manner gives the following optimal pollution for country 1:

$$x_1^* = \begin{cases} 0, if y_1 < \phi \\ \frac{\rho\tau_1 y_1^{1-\alpha} + \beta\tau_1\tau_2(2\tau_2 - 1)y_1^{1-\alpha}y_2^{2-2\alpha}}{\rho\beta\tau_1^2 y_1^{2-2\alpha} + \rho\beta\tau_2^2 y_2^{2-2\alpha} + \rho^2 + \beta^2\tau_1\tau_2(\tau_1 + \tau_2 - 1)y_1^{2-2\alpha}y_2^{2-2\alpha}}, if * \\ \frac{y_1^{1-\alpha}}{\rho + \beta y_1^{2-2\alpha}}, if y_2 < \psi \end{cases}$$

$*y_1 > \phi, and y_2 > \psi$

with $\phi = [\frac{l\rho}{\tau_1\rho + \beta\tau_1(\tau_2^2 + \tau_2 - 1)y_2^{2-2\alpha}}]^{\frac{1}{1-\alpha}} y_2$ and $\psi = [\frac{l\rho}{\tau_2\rho + \beta\tau_2(\tau_1^2 + \tau_1 - 1)y_1^{2-2\alpha}}]^{\frac{1}{1-\alpha}} y_1$.

When $y_1 < \phi$,optimal pollution is zero. This is because the domestic country is so poor that they are not able to engage in modern production of pollution. The same is true for the case of the foreign country when their income is below the thresh hold level. In the equation above, ϕ and ψ are the thresh hold of income of Country 1 and Country 2 respectively. Below the threshold it is not possible for a country to engage in modern production/pollution. Note that in either of these cases, countries will obviously not engage in trade, and at the interior the optimal pollution decision for domestic country will be 0 when it is the poor country and $y_1^{1-\alpha}$ when Country 2 is the poor country.

At the interior, the optimal pollution holds different implications for local pollutants and transboundary pollutants. The following sections interprets each of the scenarios separately.

2.4.2.1 The Case of Local Pollutants

In this section I look at the impact of trade on the optimal level of pollution and the shape of the income-pollution path when pollutants are strictly local.

Proposition 5 *Engaging in import can enable Country 1 to reduce the level of optimal pollution so long as* $y_1 > (\frac{2\rho\tau_2}{\beta\tau_1(1-\tau_1-2\tau_2)})^{\frac{1}{2-2\alpha}}$*,and* $(1-\tau_2) < 3/4$*. However, if Country 1 is to maintain the EKC pattern in the income pollution path, engaging in exports will raise the optimal level of pollution.*

Proof. *When the pollutant generated in both countries are strictly local, the optimal pollution decision comes to:*

$$x_1^* = \frac{\rho\tau_1 y_1^{1-\alpha} + \beta\tau_1\tau_2(2\tau_2-1)y_1^{1-\alpha}y_2^{2-2\alpha}}{\rho\beta\tau_1^2 y_1^{2-2\alpha} + \rho\beta\tau_2^2 y_2^{2-2\alpha} + \rho^2 + \beta^2\tau_1\tau_2(\tau_1+\tau_2-1)y_1^{2-2\alpha}y_2^{2-2\alpha}} \equiv \frac{A}{B} \qquad (11)$$

In order to look at the impact of trade on the optimal level of pollution, I look at the impact of import and export separately. First, the impact of imports:

$$\frac{\partial x_1^*}{\partial \tau_2} = \frac{B(\beta\tau_1(4\tau_2-1)y_1^{1-\alpha}y_2^{2-2\alpha}) - A(2\rho\beta\tau_2 y_2^{2-2\alpha} - \beta(\tau_1 - 2\tau_1\tau_2 - \tau_1^2)y_1^{2-2\alpha}y_2^{2-2\alpha})}{B^2} \qquad (12)$$

If Country 1 wishes to reduce optimal pollution through increasing the fraction of output of Country 2 imported, equation (12) must be positive. A positive value indicates that as τ_2 *goes down,* $(1-\tau_2)$ *goes up, and the optimal level of pollution goes down. Since* $x_1^* > 0$ *it must mean that* $A > 0$ *and* $B > 0$*. Therefore the derivative will be positive for* $(\beta\tau_1(4\tau_2-1)y_1^{1-\alpha}y_2^{2-2\alpha}) \geq 0$ *and* $(2\rho\beta\tau_2 y_2^{2-2\alpha} - \beta(\tau_1 - 2\tau_1\tau_2 - \tau_1^2)y_1^{2-2\alpha}y_2^{2-2\alpha}) \leq 0$*. These conditions are met when* $(1-\tau_2) \leq 3/4$ *and* $y_1 > (\frac{2\rho\tau_2}{\beta\tau_1(1-2\tau_2-\tau_1)})^{\frac{1}{2-2\alpha}}$*.When these conditions are not met, the derivative above can take a negative value indicating that increasing the fraction of output of Country 2 imported will raise the optimal level of pollution.*

Next, I look at the impact of export on the optimal level of pollution. The derivative of optimal pollution with respect to export gives:

$$\frac{\partial x_1^*}{\partial \tau_1} = \frac{B(\rho y_1^{1-\alpha}+\beta\tau_2(2\tau_2-1)y_1^{1-\alpha}y_2^{2-2\alpha})}{B^2}$$
$$-\frac{A(2\rho\beta\tau_1 y_1^{2-2\alpha}+\beta(2\tau_2 t+\tau_2^2-\tau_2)y_1^{2-2\alpha}y_2^{2-2\alpha})}{B^2} \qquad (13)$$

Again, a positive value of equation (13) would indicated that reducing the fraction of τ_1 *and thus raising the share of* $(1-\tau_1)$ *will raise the share of optimal pollution. This is possible when* $\rho y_1^{1-\alpha}+\beta\tau_2(2\tau_2-1)y_1^{1-\alpha}y_2^{2-2\alpha} \geq 0$ *and* $2\rho\beta\tau_1 y_1^{2-2\alpha}+\beta(2\tau_1\tau_2+\tau_2^2-\tau_2)y_1^{2-2\alpha}y_2^{2-2\alpha} \leq 0$*. These set of inequalities are met when* $(\frac{\rho}{\beta\tau_2(1-2\tau_2)})^{\frac{1}{2-2\alpha}} \geq y_2 \geq (\frac{2\rho\tau_1}{\beta(\tau_2-2\tau_1\tau_2-\tau_2^2)})^{\frac{1}{2-2\alpha}}$*. Such a restriction on* y_2 *will hold so long as* $\tau_1 \leq 1/4$*. For larger value of* τ_1 *the restriction will become weaker, meaning that engaging in export will in fact raise the level of optimal pollution. Thus the set of conditions under which engaging in export can reduce the level of optimal pollution is quite tight.*

Next I turn to the discussion on the shape of the income pollution path. The path will follow the desired inverted U pattern, only if $\frac{\partial^2 x_1^*}{\partial y_1^2} < 0$*. The derivation is quite complex and is included in the Appendix. but the set of conditions that ensures the negativity are* $y_2 > (\frac{\rho}{\beta\tau_2(1-2\tau_2)})^{\frac{1}{2-2\alpha}}$ *with* $(1-\tau_2) > 1/2$*. Thus we see that the condition conflicts with the condition set to ensure the positivity of the derivative of optimal pollution with respect to* τ_1*.*

I therefore conclude that when countries engage in trade in a way so that the inverted U pattern of the EKC is retained, engaging in import can reduce the optimal level of pollution, but engaging in export will raise it. ■

The conclusions of Proposition 5 are in line with the conclusions of the findings of the symmetric analysis. In the symmetric analysis we found that import is likely to reduce the optimal level of pollution and bring forward the turning point of income. Export is likely to push the turning point outward. For the case of local pollutants, I also find that it is possible to substitute optimal pollution by engaging in imports. The case for exports is a bit more complicated. Under the conditions specified, engaging in exports will raise the optimal level of pollution. However, overall, after engaging in trade the desired EKC pattern is retained when pollutants generated in both countries are strictly local.

2.4.2.2 The Case of Global Pollutants

I now turn focus on the impact of trade when the nature of pollutants are global. When both countries emit pollutants that are global, the optimal pollution decision becomes as follows:

$$x_1^* = \frac{\rho\tau_1 y_1^{1-\alpha} - \rho\tau_2 y_2^{1-\alpha} + \beta\tau_1\tau_2(2\tau_2-1)y_1^{1-\alpha}y_2^{2-2\alpha}}{\rho\beta\tau_1^2 y_1^{2-2\alpha} + \rho\beta\tau_2^2 y_2^{2-2\alpha} + \rho\beta(2\tau_1\tau_2-\tau_1-\tau_2)y_1^{1-\alpha}y_2^{1-\alpha} + \beta^2(\tau_1\tau_2^2+\tau_1^2\tau_2-\tau_1\tau_2)y_1^{2-2\alpha}y_2^{2-2\alpha}} \equiv \frac{C}{D} \qquad (14)$$

Since the optimal level of pollution is positive, it must mean that $C > 0$ and $D > 0$. However, in this case, the impact of import and export on the optimal level of trade and the shape of the income pollution path become less straight forward.

Proposition 6 *Even though the shape of the EKC can be retained when countries engage in trade, the impact of import and export on the optimal level of pollution will be positive. That is, engaging in import and export may result in higher levels of pollution even though the shape of the income pollution path follows the inverted U pattern.*

Proof. *For the shape of the income pollution path to follow the inverted U patten, the second derivative of optimal pollution with respect to income must be negative. This condition is satisfied if the following restrictions are met. The first set of restrictions are on the level of income of the two countries:* $y_1 < \left(\frac{\rho\tau_2 y_2^{1-\alpha}}{t(\beta\tau_1\tau_2(2\tau_2-1)y_2^{2-2\alpha}+\rho}\right)^{\frac{1}{1-\alpha}}$ *, and* $y_2 > \left(\frac{\rho}{2\tau_2(2\tau_2-1)}\right)^{\frac{1}{2-2\alpha}}$. *The second set of condition is on the level of trade. Compared to local pollutants, in the case of global pollutants* $(1-\tau_2)$ *should be less than* $1/2$ *in order to maintain the shape* *of the EKC. Also* $(1-\tau_1) > \tau_2$ *with* $\tau_1 < \frac{2}{2\tau_2-1}$. *Another additional requirement is that the value of* α *should be less than 1/2. This means that the level of technology should be relatively more advanced if trade among countries is to ensure the shape of the EKC. A detailed workout of the second derivative of optimal pollution with respect to income is included in Appendix.*

Next I analyze how engaging in import will affect the optimal level of pollution.The derivative of optimal pollution with respect to τ_2 *is given by:*

$$\frac{\partial x_1^*}{\partial \tau_2} = \frac{D[-\rho y_2^{1-\alpha} - \beta\tau_1(4\tau_2-1)y_1^{1-\alpha}y_2^{2-2\alpha}]}{D^2} - \frac{C\beta[2\rho\tau_2 y_2^{2-2\alpha} + \rho(2\tau_1-1)y_1^{1-\alpha}y_2^{1-\alpha} + \beta(2\tau_1\tau_2+\tau_1^2-\tau_1)y_1^{2-2\alpha}y_2^{2-2\alpha}]}{D^2} \quad (15)$$

For engagement in import to reduce the level of optimal pollution, equation (15) should be positive. This will happen when $y_1 \geq (\frac{\rho}{\beta\tau_1\tau_2(2\tau_2-1)y_2^{1-\alpha}})^{\frac{1}{1-\alpha}}$ *and* $y_2 \leq (\frac{\rho(1-2\tau_1)y_1^{1-\alpha}}{2\rho\tau_2+\beta(2\tau_1\tau_2+\tau_1^2-\tau_1)y_1^{2-2\alpha}})^{\frac{1}{1-\alpha}}$. *Comparing this to the set of restrictions previously set for the EKC pattern of the income pollution path, we see that these set of restrictions violate the ones previously set.Therefore, when countries engage in imports in ways that ensures the EKC pattern of the income pollution path, engaging in import will raise the optimal level of pollution.*

How engaging in exports will affect the optimal level of pollution is given by the derivative of optimal pollution with respect to τ_1. *This is given by:*

$$\frac{\partial x_1^*}{\partial \tau_1} = \frac{D[\rho y_1^{1-\alpha} + \beta\tau_2(2\tau_2-1)y_1^{1-\alpha}y_2^{2-2\alpha}]}{D^2} - \frac{C[2\rho\beta\tau_1 y_1^{2-2\alpha} + \rho\beta(2\tau_2-1)y_1^{1-\alpha}y_2^{1-\alpha} + \beta^2\tau_2(1-\tau_2)y_1^{2-2\alpha}y_2^{2-2\alpha}]}{D^2} \quad (16)$$

The set of conditions that will ensure the positivity of equation (16) are: $y_1 \leq (\frac{\rho(1-2\tau_2)y_2^{1-\alpha}}{\rho 2\tau_1+\beta(\tau_2^2+2\tau_1\tau_2-\tau_2)y_2^{2-2\alpha}})^{\frac{1}{1-\alpha}}$ *and* $y_2 \leq (\frac{\rho}{\beta\tau_2(1-2\tau_2)})^{\frac{1}{2-2\alpha}}$. *Again the condition placed upon the income of Country 2 that ensures the shape of EKC is violated if we seek for positivity of equation (16).*

I therefore conclude that for the case of transboundary pollutants, even though engaging in trade will still retain the shape of the EKC in the income pollution path, it will likely raise the level of optimal pollution. ■

From the conclusions of the asymmetric case, Proposition 4 and Proposition 5, I conclude that engaging in trade will retain the shape of the EKC even though the turning point may get affected. However, the impact of trade is more favorable for the case of local pollutants than for transboundary pollutants. For the local pollutants the shape of the EKC is retained. At the same time, engaging in imports can reduce the level of optimal pollution although, engaging in export is likely to raise it. For the case of transboundary pollutants, I find that even though the shape of the EKC is retained after engaging in trade, engaging in imports and exports both are likely to raise the optimal level of pollution. Therefore, even though countries face the prospect of reduced pollution with economic growth, they do it with the risk of higher pollution at any given level of income when they engage in trade.

2.5 Demonstration of Results

Section 2.4 chalks out the impact of trade on the optimal level of pollution as well as on the income pollution path of a country when pollution is local and for when it is global. In this section I carry out a simple simulation exercise to give a graphic representation to the claims of Proposition 5 and 6. The parameter values are selected as per the conditions set up by the model, the relative income of the two countries are set on the basis of the boundaries set by the proposition. The parametric restrictions set by the models are: $\alpha \in (0, 1)$; $\beta \geq 0$; and $\rho \in R_+$. Just for a quick recap, α is a parameter capturing the level of abatement technology. The lower the value of α, better is the abatement technology. β measures the risk averseness of agents, with higher values of β signifying more risk averse individuals. Finally ρ measures the pollution perception with higher values of ρ indicating a higher degree of dislike for pollution. For the simulation exercise, the parameter values that I consider are: $\alpha = 0.5$; $\beta = 2$; $\rho = 3$. The value for α is deliberately selected to measure a country with average technology. It is assumed that the agents are risk averse, and the pollution perception is quite high. With this framework, I proceed to look at the simulation results.

First I look at the impact of import and export on the optimal level of pollution when pollution is local as well as when pollution is global.

Table 2.1 : Impact of Import and Export on the Optimal Level of Pollution

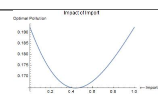
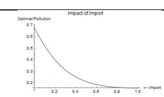
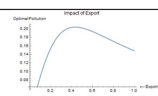
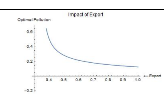
	Local Pollution	Global Pollution
Impact of Import		
Impact of Export		

Table 2.1 above showcases the claims made in Proposition 5 and 6 on the impact of export and import on the optimal level of trade. The graphs above are based on simulated values. The chosen parameter values are already stated above. The values assigned to y_1 and y_2 are 3 and 1 respectively. Again these values are within the ranged specified by proposition 5 and 6.

In proposition 5 we see that for the case of local pollutants engaging in import will reduced optimal pollution as long as $(1 - \tau_2) < 3/4$. The graph above shows that engaging in import is declining the optimal level of pollution when the level of import is lower. It is also claimed that engaging in export will reduce the optimal level of pollution so long as $\tau_1 \leq 1/4$. Again in the graph above it is shown that when the value of τ_1 is relatively high, engaging in export is increasing the optimal level of pollution. The optimal level of pollution is going down with higher values of export or lower values of τ_1. Thus it is shown that the claims made in proposition 2 hold for this set of simulation exercise.

Proposition 6 claims that when the income of the two countries are within a certain range, engaging in trade will increase the optimal level of pollution even though the shape of the EKC will be maintained. The first part of the claim is shown in Table 2.1 above where it can be seen that engaging in both exports and imports increases the optimal level of pollution. How trade will affect the shape of the income pollution path is demonstrated in Table 2.2 below.

Table 2.2: Impact of Trade on the Income-Pollution Path

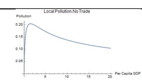
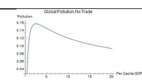
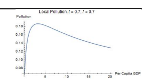
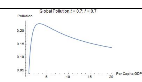
	Local Pollution	Global Pollution
No Trade		
With Trade		

Table 2.2 above shows the impact of trade on the shape of the income pollution path for both the case of local pollutants as well as for global pollutants. All of the claims made in Proposition 5 and 6 are confirmed through this simulation exercise. First of all, whether the nature of pollutant is local or global, engaging in trade allows for the income pollution path to follow the desired EKC pattern. Second, for the case of local pollutants, the shape of the EKC is retained at lower levels of pollution when countries engage in trade. For the case of global pollutants, the shape of the EKC is retained after trade, but it is so at the cost of higher levels of pollution.Engaging trade therefore is beneficial for the income pollution path. However, it is more so for the case of local pollutants than for global pollutants.

2.6 Conclusion

The concept of EKC draws out a pragmatic relationship between pollution and per capita income that poses critical guidelines for policy developments. Even though a lot of work has been done in literature, the existence of the EKC remains doubtful. Most models that successfully draw out the existence of an EKC, do so under set of parametric restrictions. In order to see the impact of trade on the existence of EKC, in this paper, I have chosen a model that sets out the existence of EKC under a rich set of parameters, and introduced trade to the picture. Trade plays an important role in the discourse of economic growth and environmental quality. To begin with, trade allows agents with a wider range of consumption possibilities. Also, trade allows for a scope for pollution substitution through imports. Finally, when the nature of pollutants are global, interdependence among countries with respect to pollution generation increase when trade is brought into the equation.

The Diamantoudi and Filippiadis (2012) model that I have chosen develop a model for global pollution in order to capture the nature of interdependence in pollution decision of countries. When introducing trade, the first conclusion I reach is that trade too allows for pollution interdependence when the nature of pollutants are global as well as when they are strictly local. The model concludes that engaging in imports makes the scope for pollution substitution stronger. Engaging in exports can also enhance the scope of pollution substitution under a certain set of circumstances.

Having introduced trade, the model has been solved for both the symmetric case as well as the asymmetric case. For the symmetric case, it is found that the shape of the EKC is retained even after introducing trade. While exports push the turning point of the EKC outwards, engaging in imports can bring forward the turning point. In addition, engaging in imports reduces the level of optimal pollution. Engaging in trade therefore has an overall desirable affect on the income pollution path.

The asymmetric analysis is carried on with respect to asymmetry in income only. The exercise is done in two parts. For the case of local pollution, it is found that when the income of a Country is more than a certain threshold, engaging in imports can reduce the optimal level of pollution. Also, when conducting trade with a second country whose income is within a certain range, the shape of the EKC in the income pollution path is retained. However, when income of the two countries are within the boundaries set, engaging in exports is likely to raise the optimal level of pollution. This conclusion is in line with the conclusions found for the symmetric analysis.

For the case of global pollutants, the findings are a bit different. Again it is found that when income of the two countries are within a set range, engaging in trade can still retain the shape of the EKC. However, these conditions imply that engaging in import and export is likely to raise the optimal level of pollution. Therefore, the model concludes that even though the shape of the EKC is retained after engaging in trade, for the case of global pollutants, it is likely to happen at a higher levels of pollution. The paper therefore concludes that trade holds beneficial implications more for the case of local pollutants than for the case of global pollutants. Trade allows for pollution substitution. When pollutants are local, engaging in trade retains the shape of the EKC in the income pollution path. When the impact of import is greater than the impact of export, engaging in trade can allow countries to obtain the EKC pattern with lower levels of pollution at every level. When pollutants are global, engaging in trade still retains the shape of the EKC but at the cost of higher levels of pollution.

2.7 Appendix

Appendix A. Derivation of the Model Solution.

For Country 1 the Social Utility Function that the social planner seeks to optimize is:

$$V_1 = [\tau_1 y_1 \frac{x_1}{y_1^\alpha} + (1-\tau_2) y_2 \frac{x_2}{y_2^\alpha}] - \frac{\beta}{2}[\tau_1 y_1 \frac{x_1}{y_1^\alpha} + (1-\tau_2) y_2 \frac{x_2}{y_2^\alpha}]^2 - \frac{\rho}{2}(x_1 + l_2 x_2)^2 \qquad (1)$$

The FOC with respect to x_1 is:

$$\frac{\partial V_1}{\partial x_1} = \tau_1 y_1^{1-\alpha} - \beta[\tau_1 y_1 \frac{x_1}{y_1^\alpha} + (1-\tau_2) y_2 \frac{x_2}{y_2^\alpha}]\tau_1 y_1^{1-\alpha} - \rho(x_1 + l_2 x_2) = 0$$

$$\Rightarrow x_1 = \frac{\tau_1 y_1^{1-\alpha}}{(\beta\tau_1^2 y_1^{2-2\alpha}+\rho)} - \frac{(\beta\tau_1(1-\tau_2) y_1^{1-\alpha} y_2^{1-\alpha} + l_2\rho) x_2}{(\beta\tau_1^2 y_1^{2-2\alpha}+\rho)} \qquad (2)$$

The upper bound to maximum pollution set within the model is y_1^α.Therefore,

$$\frac{\tau_1 y_1^{1-\alpha}}{(\beta\tau_1^2 y_1^{2-2\alpha}+\rho)} - \frac{(\beta\tau_1(1-\tau_2) y_1^{1-\alpha} y_2^{1-\alpha} + l\rho) x_2}{(\beta\tau_1^2 y_1^{2-2\alpha}+\rho)} \leq y_1^\alpha$$

$$\Rightarrow x_2 \geq \frac{\tau_1 y_1^{1-\alpha} - y_1^\alpha(\beta\tau_1^2 y_1^{2-2\alpha}+\rho)}{l_2\rho + \beta\tau_1(1-\tau_2) y_1^{1-\alpha} y_2^{1-\alpha}}$$

Solving the model for Country 2 in the same way yields the optimal level of pollution for Country 2:

$$x_2 = \frac{\tau_2 y_2^{1-\alpha}}{(\beta\tau_2^2 y_2^{2-2\alpha}+\rho)} - \frac{(\beta\tau_2(1-\tau_1) y_1^{1-\alpha} y_2^{1-\alpha} + l_1\rho) x_1}{(\beta\tau_2^2 y_2^{2-2\alpha}+\rho)} \qquad (3)$$

Substituting the value of x_2 in equation (2), at the interior, the optimal value of pollution comes to:

$$x_1 = \frac{\rho\tau_1 y_1^{1-\alpha} - \rho\tau_2 y_2^{1-\alpha} + \beta\tau_1\tau_2(2\tau_2-1) y_1^{1-\alpha} y_2^{2-2\alpha}}{\rho\beta\tau_1^2 y_1^{2-2\alpha} + \rho\beta\tau_2^2 y_2^{2-2\alpha} + \rho\beta(2\tau_1\tau_2-\tau_1-\tau_2) y_1^{1-\alpha} y_2^{1-\alpha} + \beta^2(\tau_1\tau_2^2+\tau_1^2\tau_2-\tau_1\tau_2) y_1^{2-2\alpha} y_2^{2-2\alpha}}$$

This is the solution where $l_1 = l_2 = 1$ and hence the solution for global pollutant.

For local pollutant, the model is solved in the same manner except $l_1 = l_2 = 0$. Therefore the optimal pollution decision comes to

$$x_1^* = \frac{\rho\tau_1 y_1^{1-\alpha} + \beta\tau_1\tau_2(2\tau_2-1) y_1^{1-\alpha} y_2^{2-2\alpha}}{\rho\beta\tau_1^2 y_1^{2-2\alpha} + \rho\beta\tau_2^2 y_2^{2-2\alpha} + \rho^2 + \beta^2\tau_1\tau_2(\tau_1+\tau_2-1) y_1^{2-2\alpha} y_2^{2-2\alpha}}$$

Appendix B. Proof of Propositions

B1. Proof of Proposition 2

When pollution is local, the equation representing optimal pollution is:

$x_1^* = \frac{\rho\tau_1 y_1^{1-\alpha}+\beta\tau_1\tau_2(2\tau_2-1)y_1^{1-\alpha}y_2^{2-2\alpha}}{\rho\beta\tau_1^2 y_1^{2-2\alpha}+\rho\beta\tau_2^2 y_2^{2-2\alpha}+\rho^2+\beta^2\tau_1\tau_2(\tau_1+\tau_2-1)y_1^{2-2\alpha}y_2^{2-2\alpha}} \equiv \frac{A}{B}$

To see the impact on optimal pollution with respect to import, the derivative of optimal pollution with respect to τ_2 is taken.

$\frac{\partial x_1^*}{\partial \tau_2} = \frac{B(\beta\tau_1(4\tau_2-1)y_1^{1-\alpha}y_2^{2-2\alpha})-A(2\rho\beta\tau_2 y_2^{2-2\alpha}-\beta(\tau_1-2\tau_1\tau_2-\tau_1^2)y_1^{2-2\alpha}y_2^{2-2\alpha})}{B^2}$

If engaging in import reduces the level of optimal pollution, the derivative above should take a positive value. Since pollution is positive it must be that $A > 0$and $B > 0$. Thus for the derivative to be positive it is sufficient to have the following conditions:

$(\beta\tau_1(4\tau_2-1)y_1^{1-\alpha}y_2^{2-2\alpha}) \geq 0 \Rightarrow (1-\tau_2) \leq 3/4$ and,

$2\rho\beta\tau_2 y_2^{2-2\alpha} - \beta(\tau_1 - 2\tau_1\tau_2 - \tau_1^2)y_1^{2-2\alpha}y_2^{2-2\alpha} \Rightarrow y_1 > (\frac{2\rho\tau_2}{\beta\tau_1(1-2\tau_2-\tau_1)})^{\frac{1}{2-2\alpha}}$.

To see the impact on optimal pollution with respect to export we take the derivative of optimal pollution with respect to t and get:

$\frac{\partial x_1^*}{\partial t} = \frac{B(\rho y_1^{1-\alpha}+\beta\tau_2(2\tau_2-1)y_1^{1-\alpha}y_2^{2-2\alpha})-A(2\rho\beta\tau_1 y_1^{2-2\alpha}+\beta(2\tau_1\tau_2+\tau_2^2-\tau_2)y_1^{2-2\alpha}y_2^{2-2\alpha})}{B^2}$

Again, in order to see the conditions that will allow engaging in export to reduce optimal pollution, the derivative above must be positive. This is possible under the following conditions:

$(\rho y_1^{1-\alpha} + \beta\tau_2(2\tau_2-1)y_1^{1-\alpha}y_2^{2-2\alpha}) \geq 0$ and $2\rho\beta\tau_1 y_1^{2-2\alpha} + \beta(2\tau_1\tau_2 + \tau_2^2 - \tau_1)y_1^{2-2\alpha}y_2^{2-2\alpha} \leq 0$

$\Rightarrow (\frac{\rho}{\beta\tau_2(1-2\tau_2)})^{\frac{1}{2-2\alpha}} \geq y_2 \geq (\frac{2\rho\tau_1}{\beta(\tau_2-2\tau_1\tau_2-\tau_2^2)})^{\frac{1}{2-2\alpha}}$ with $\tau_1 \leq 1/4$.

Finally I need to find the conditions that ensure that the pollution income path follows the EKC pattern.

The optimal pollution function with trade in the case of local pollutants is:

$x_{1local}^* = \frac{\rho\tau_1 y_1^{1-\alpha}+\beta\tau_1\tau_2(2\tau_2-1)y_1^{1-\alpha}y_2^{2-2\alpha}}{\rho\beta\tau_1^2 y_1^{2-2\alpha}+\rho\beta\tau_2^2 y_2^{2-2\alpha}+\rho^2+\beta^2\tau_1\tau_2(\tau_1+\tau_2-1)y_1^{2-2\alpha}y_2^{2-2\alpha}} \equiv \frac{A}{B}$

I want to see how trade among countries with different income affect the shape of the income pollution path. In order to get the desired EKC pattern, the second order derivative of optimal pollution with respect to income must be negative. The second order derivative is as follows:

$$\frac{\partial^2 x^*_{1local}}{\partial y_1^2} = \frac{2(\beta\tau_1\tau_2(2\tau_2-1)y_1^{1-\alpha}y_2^{2-2\alpha}+\rho\tau_1 y_1^{1-\alpha})(\beta^2(2-2\alpha)\tau_1\tau_2(\tau_1+\tau_2-1)y_1^{1-2\alpha}y_2^{2-2\alpha}+\beta\rho\tau_1^2(2-2\alpha)y_1^{1-2\alpha})^2}{(\rho\beta\tau_1^2 y_1^{2-2\alpha}+\rho\beta\tau_2^2 y_2^{2-2\alpha}+\rho^2+\beta^2\tau_1\tau_2(\tau_1+\tau_2-1)y_1^{2-2\alpha}y_2^{2-2\alpha})^3}$$

$$-\frac{2(\beta(1-\alpha)\tau_1\tau_2(2\tau_2-1)y_1^{-\alpha}y_2^{2-2\alpha}+\rho(1-\alpha)\tau_1 y_1^{-\alpha})(\beta^2(2-2\alpha)\tau_1\tau_2(\tau_1+\tau_2-1)y_1^{1-2\alpha}y_2^{2-2\alpha}+\beta\rho(2-2\alpha)\tau_1^2 y_1^{1-2\alpha})}{(\rho\beta\tau_1^2 y_1^{2-2\alpha}+\rho\beta\tau_2^2 y_2^{2-2\alpha}+\rho^2+\beta^2\tau_1\tau_2(\tau_1+\tau_2-1)y_1^{2-2\alpha}y_2^{2-2\alpha})^2}$$

$$-\frac{((\beta\tau_1\tau_2(2\tau_2-1)y_1^{1-\alpha}y_2^{2-2\alpha}+\rho\tau_1 y_1^{1-\alpha})(\beta^2(1-2\alpha)(2-2\alpha)\tau_1\tau_2(\tau_1+\tau_2-1)y_1^{-2\alpha}y_2^{2-2\alpha}+\beta\rho(1-2\alpha)(2-2\alpha)\tau_1^2 y_1^{-2\alpha})}{(\rho\beta\tau_1^2 y_1^{2-2\alpha}+\rho\beta\tau_2^2 y_2^{2-2\alpha}\rho^2+\beta^2\tau_1\tau_2(\tau_1+\tau_2-1)y_1^{2-2\alpha}y_2^{2-2\alpha})^2}$$

$$+\frac{-\beta\alpha(1-\alpha)\tau_1\tau_2(2\tau_2-1)y_1^{-\alpha-1}y_2^{2-2\alpha}-\rho\alpha(1-\alpha)\tau_1 y_1^{-1-\alpha}}{\rho\beta\tau_1^2 y_1^{2-2\alpha}+\rho\beta\tau_2^2 y_2^{2-2\alpha}+\rho^2+\beta^2\tau_1\tau_1(\tau_1+\tau_2-1)y_1^{2-2\alpha}y_2^{2-2\alpha}}$$

I now set out some set of conditions that is sufficient to prove the negativity of the second order derivative. Before that, I write the equation above in a simpler format.

$Let, \beta\tau_1\tau_2(2\tau_2-1)y_1^{1-\alpha}y_2^{2-2\alpha}+\rho\tau_1 y_1^{1-\alpha} = [1]$

$\beta(1-\alpha)\tau_1\tau_2(2\tau_2-1)y_1^{-\alpha}y_2^{2-2\alpha}+\rho(1-\alpha)\tau_1 y_1^{-\alpha} = [2a]$

$\beta^2(2-2\alpha)\tau_1\tau_2(\tau_1+\tau_2-1)y_1^{1-2\alpha}y_2^{2-2\alpha}+\beta\rho(2-2\alpha)\tau_1^2 y_1^{1-2\alpha} = [2b]$

$\beta\tau_1\tau_2(2\tau_2-1)y_1^{1-\alpha}y_2^{2-2\alpha}+\rho\tau_1 y_1^{1-\alpha} = [3a]$

$\beta^2(1-2\alpha)(2-2\alpha)\tau_1\tau_2(\tau_1+\tau_2-1)y_1^{-2\alpha}y_2^{2-2\alpha}+\beta\rho(1-2\alpha)(2-2\alpha)\tau_1^2 y_1^{-2\alpha} = [3b]$

$-\beta\alpha(1-\alpha)\tau_1\tau_2(2\tau_2-1)y_1^{-\alpha-1}y_2^{2-2\alpha}-\rho\alpha(1-\alpha)\tau_1 y_1^{-1-\alpha} = [4]$

The second order derivative can now be written as:

$\frac{\partial^2 x^*_{1local}}{\partial y_1^2} = \frac{2[1](.)^2}{B^3} - \frac{2[2a][2b]}{B^2} - \frac{[3a][3b]}{B^2} + \frac{[4]}{B}$

Therefore, if I can find the set of conditions for which $[1]$,$[2a]$,$[2b]$,$[3a]$,$[3b]$and$[4] < 0$, then the second derivative of optimal pollution with respect to income will be negative. In other words, these set of conditions will ensure that the shape of the EKC is maintained in the income-pollution path.

Condition1

$\beta\tau_1\tau_2(2\tau_2-1)y_1^{1-\alpha}y_2^{2-2\alpha}+\rho\tau_1 y_1^{1-\alpha} < 0$

$\Rightarrow y_2 > (\frac{\rho}{\beta\tau_2(1-2\tau_2)})^{\frac{1}{2-2\alpha}}$

Country 1 should engage in trade with Country 2 whose income is above

$(\frac{\rho}{\beta\tau_2(1-2\tau_2)})^{\frac{1}{2-2\alpha}}$. If the income of Country 2 is lower, Country 1 should reduce the level of $(1-\tau_2)$and therefore increase τ_2 to maintain the inequality. Also, $\tau_2 < 1/2$.

Condition 2a

$\beta(1-\alpha)\tau_1\tau_2(2\tau_2-1)y_1^{-\alpha}y_2^{2-2\alpha} + \rho(1-\alpha)\tau_1 y_1^{-\alpha} < 0$

Notice that Condition 2a is the first derivative of Condition 1a with respect to y_1.Therefore the same conditions that were applicable in Condition 1 will be applicable here to keep Condition 2a negative.

Condition 2b

$\beta 2(2-2\alpha)\tau_1\tau_2(\tau_1+\tau_2-1)y_1^{1-2\alpha}y_2^{2-2\alpha} + \beta\rho(2-2\alpha)\tau_1^2 y_1^{1-2\alpha} < 0$

$\Rightarrow y_2 > (\frac{\tau_1\rho}{\beta\tau_2(1-\tau_1-\tau_2)})^{\frac{1}{2-2\alpha}}$ with $\tau_1 < (1-\tau_2)$

For the inequality to hold, the income of country 2 must be bigger and the fraction of goods retained at home must be less than the fraction of country 2's output imported.

When the income of Country 2 is low, τ_1 should be reduced or $(1-\tau_1)$ raised to maintain the inequality.

Condition 3a

$\beta\tau_1\tau_2(2\tau_2-1)y_1^{1-\alpha}y_2^{2-2\alpha} + \rho\tau_1 y_1^{1-\alpha} < 0$

Condition 3a is the same as condition 1 and therefore already proved negative.

Condition 4

$-\beta\alpha(1-\alpha)\tau_1\tau_2(2\tau_2-1)y_1^{-\alpha-1}y_2^{2-2\alpha} - \rho\alpha(1-\alpha)\tau_1 y_1^{-1-\alpha} < 0$

Same as 1

In summary, the conditions that ensures the negativity of the second derivative of optimal pollution with respect to income is:

$y_2 > (\frac{\rho}{\beta\tau_2(1-2\tau_2)})^{\frac{1}{2-2\alpha}}$ with $(1-\tau_2) > 1/2$.

Thus we see that the condition conflicts with the condition set to ensure the positivity of the derivative of optimal pollution with respect to τ_1.

I therefore conclude that when pollution is local, if $y_1 > (\frac{2\rho\tau_2}{\beta\tau_1(1-2\tau_2-\tau_1)})^{\frac{1}{2-2\alpha}}$, $y_2 > (\frac{\rho}{\beta\tau_2(1-2\tau_2)})^{\frac{1}{2-2\alpha}}$ with $3/4 > (1-\tau_2) > 1/2$.engaging in import will reduce the level of pollution, engaging in export will raise the level of pollution, and the shape of the income pollution path will follow the EKC pattern.

B2. Proof of Proposition 3

When pollution is global the equation the equation representing optimal pollution is:

$$x_1 = \frac{\rho\tau_1 y_1^{1-\alpha} - \rho\tau_2 y_2^{1-\alpha} + \beta\tau_1\tau_2(2\tau_2-1)y_1^{1-\alpha}y_2^{2-2\alpha}}{\rho\beta\tau_1^2 y_1^{2-2\alpha} + \rho\beta\tau_2^2 y_2^{2-2\alpha} + \rho\beta(2\tau_1\tau_2-\tau_1-\tau_2)y_1^{1-\alpha}y_2^{1-\alpha} + \beta^2(\tau_1\tau_2^2+\tau_1^2\tau_2-\tau_1\tau_2)y_1^{2-2\alpha}y_2^{2-2\alpha}} \equiv \frac{C}{D}$$

In order to find the set of conditions that ensure that engaging in import will lead to reduced optimal pollution, I take the derivative of optimal global pollution with respect to τ_2 and set it to be positive:

$$\frac{\partial x_1^*}{\partial \tau_2} = \frac{D[-\rho y_2^{1-\alpha} - \beta\tau_1(4\tau_2-1)y_1^{1-\alpha}y_2^{2-2\alpha}] - C\beta[2\rho\tau_2 y_2^{2-2\alpha} + \rho(2\tau_1-1)y_1^{1-\alpha}y_2^{1-\alpha} + \beta(2\tau_1\tau_2+\tau_1^2-\tau_1)y_1^{2-2\alpha}y_2^{2-2\alpha}]}{D^2} > 0$$

Since pollution is positive it implies that $C > 0$ and $D > 0$. Thus sufficient condition for positivity of the derivative above is given by:

$[-\rho y_2^{1-\alpha} - \beta\tau_1(4\tau_2-1)y_1^{1-\alpha}y_2^{2-2\alpha}] \geq 0 \Rightarrow y_1 \geq (\frac{\rho}{\beta\tau_1\tau_2(2\tau_2-1)y_2^{1-\alpha}})^{\frac{1}{1-\alpha}}$ and

$2\rho\tau_2 y_2^{2-2\alpha} + \rho(2\tau_1-1)y_1^{1-\alpha}y_2^{1-\alpha} + \beta(2\tau_1\tau_2+\tau_1^2-\tau_1)y_1^{2-2\alpha}y_2^{2-2\alpha} \leq 0 \Rightarrow y_2 \leq (\frac{\rho(1-2\tau_1)y_1^{1-\alpha}}{2\rho\tau_2+\beta(2\tau_1\tau_2+\tau_1^2-\tau_1)y_1^{2-2\alpha}})^{\frac{1}{1-\alpha}}$.

To find the set of conditions that will ensure that engaging in exports will reduce the optimal level of pollution, I take the derivative of optimal pollution with respect to t and set the derivative to be positive.

$$\frac{\partial x_1^*}{\partial \tau_1} = \frac{D[\rho y_1^{1-\alpha} + \beta\tau_2(2\tau_2-1)y_1^{1-\alpha}y_2^{2-2\alpha}] - C[2\rho\beta\tau_1 y_1^{2-2\alpha} + \rho\beta(2\tau_2-1)y_1^{1-\alpha}y_2^{1-\alpha} + \beta^2\tau_2(1-\tau_2)y_1^{2-2\alpha}y_2^{2-2\alpha}]}{D^2} > 0$$

Positivity is ensured if $\rho y_1^{1-\alpha} + \beta\tau_2(2\tau_2-1)y_1^{1-\alpha}y_2^{2-2\alpha} \geq 0 \Rightarrow y_1 \leq (\frac{\rho(1-2\tau_2)y_2^{1-\alpha}}{\rho 2\tau_1+\beta(\tau_2^2+2\tau_1\tau_2-\tau_2)y_2^{2-2\alpha}})^{\frac{1}{1-\alpha}}$ and

$2\rho\beta\tau_1 y_1^{2-2\alpha} + \rho\beta(2\tau_2 - 1)y_1^{1-\alpha}y_2^{1-\alpha} + \beta^2\tau_2(1-\tau_2)y_1^{2-2\alpha}y_2^{2-2\alpha} \leq 0 \Rightarrow y_2 \leq ((\frac{\rho}{\beta\tau_2(1-2\tau_2)}))\frac{1}{2-2\alpha}$.

Finally I try to find the set of conditions that will guarantee that the income pollution path will follow the EKC pattern.

In this section I look at the implications for the income pollution path when countries with different levels of income trade with each other when the nature of pollutants is global. The solution that I got for optimal pollution for the case of global pollutant is as follows:

$$x^*_{1global} = \frac{\rho\tau_1 y_1^{1-\alpha} - \rho\tau_2 y_2^{1-\alpha} + \beta\tau_1\tau_2(2\tau_2-1)y_1^{1-\alpha}y_2^{2-2\alpha}}{\rho\beta\tau_1^2 y_1^{2-2\alpha}\rho\beta\tau_2^2 y_2^{2-2\alpha} + \rho\beta(2\tau_1\tau_2 - \tau_1 - \tau_2)y_1^{1-\alpha}y_2^{1-\alpha} + \beta^2(\tau_1\tau_2^2 + \tau_1^2\tau_2 - \tau_1\tau_2)y_1^{2-2\alpha}y_2^{2-2\alpha}} = \frac{C}{D}$$

In order to obtain the desired inverted U relationship, the second order derivative to the optimal pollution function with respect to income must be negative. The second order derivative of the optimal pollution with respect to income comes to:

$$\frac{\partial^2 x^*_{1global}}{\partial y_1^2} = \frac{(2(\beta\tau_1\tau_2(2\tau_2-1)y_2^{2-2\alpha}y_1^{1-\alpha} - \rho\tau_2 y_2^{1-\alpha} + \rho\tau_1 y_1^{1-\alpha})}{(\rho\beta\tau_1^2 y_1^{2-2\alpha} + \rho\beta\tau_2^2 y_2^{2-2\alpha} + \rho\beta(2\tau_1\tau_2 - \tau_1 - \tau_2)y_1^{1-\alpha}y_2^{1-\alpha} + \beta^2(\tau_1^2\tau_2 + \tau_1\tau_2^2 - \tau_1\tau_2)y_1^{2-2\alpha}y_2^{2-2\alpha})^3} *$$

$$(\beta^2(2-2\alpha)(\tau_1^2\tau_2 + \tau_1\tau_2^2 - \tau_1\tau_2)y_1^{1-2\alpha}y_2^{2-2\alpha}$$

$$+\rho\beta\tau_1^2 y_1^{1-2\alpha} + \rho\beta(1-\alpha)(2\tau_1\tau_2 - \tau_1 - \tau_2)y_1^{-\alpha}y_2^{1-\alpha})^2$$

$$-\frac{(2(\beta(1-\alpha)\tau_1\tau_2(2\tau_2-1)y_1^{-\alpha}y_2^{2-2\alpha} + \rho(1-\alpha)\tau_1 y_1^{-\alpha})}{(\rho\beta\tau_1^2 y_1^{2-2\alpha} + \rho\beta\tau_2^2 y_2^{2-2\alpha} + \rho\beta(2\tau_1\tau_2 - \tau_1 - \tau_2)y_1^{1-\alpha}y_2^{1-\alpha} + \beta^2(\tau_1^2\tau_2 + \tau_1\tau_2^2 - \tau_1\tau_2)y_1^{2-2\alpha}y_2^{2-2\alpha})^2} *$$

$$(\beta^2(2-2\alpha)(\tau_1^2\tau_2 + \tau_1\tau_2^2 - \tau_1\tau_2)y_1^{1-2\alpha}y_2^{2-2\alpha}$$

$$+\rho\beta(2-2\alpha)\tau_1^2 y_1^{1-2\alpha} + \rho\beta(1-\alpha)(2\tau_1\tau_2 - \tau_1 - \tau_2)y_1^{-\alpha}y_2^{1-\alpha})$$

$$-\frac{((\beta\tau_1\tau_2(2\tau_2-1)y_1^{1-\alpha}y_2^{2-2\alpha} + \rho\tau_1 y_1^{1-\alpha} - \rho\tau_2 y_2^{1-\alpha})}{(\rho\beta\tau_1^2 y_1^{2-2\alpha} + \rho\beta\tau_2^2 y_2^{2-2\alpha} + \rho\beta(2\tau_1\tau_2 - \tau_1 - \tau_2)y_1^{1-\alpha}y_2^{1-\alpha} + \beta^2(\tau_1^2\tau_2 + \tau_1\tau_2^2 - \tau_1\tau_2)y_1^{2-2\alpha}y_2^{2-2\alpha})^2} *$$

$$(\beta^2(1-2\alpha)(2-2\alpha)(\tau_1^2\tau_2 + \tau_1\tau_2^2 - \tau_1\tau_2)y_1^{-2\alpha}y_2^{2-2\alpha}$$

$$+\rho\beta(1-2\alpha)(2-2\alpha)\tau_1^2 y_1^{-2\alpha} - \rho\beta\alpha(1-\alpha(2\tau_1\tau_2 - \tau_1 - \tau_2)y_1^{1-\alpha}y_2^{1-\alpha}))$$

$$+\frac{(-\beta\alpha(1-\alpha)\tau_1\tau_2(2\tau_2-1)y_1^{-1-\alpha}y_2^{2-2\alpha} - \rho\alpha\tau_1(1-\alpha)y_1^{-1-\alpha}}{(\rho\beta\tau_1^2 y_1^{2-2\alpha} + \rho\beta\tau_2^2 y_2^{2-2\alpha} + \rho\beta(2\tau_1\tau_2 - \tau_1 - \tau_2)y_1^{1-\alpha}y_2^{1-\alpha} + \beta^2(\tau_1^2\tau_2 + \tau_1\tau_2^2 - \tau_1\tau_2)y_1^{2-2\alpha}y_2^{2-2\alpha})}$$

Again I group the second derivative above to get some tractability.

Let:

$(\beta\tau_1\tau_2(2\tau_2-1)y_2^{2-2\alpha}y_1^{1-\alpha}-\rho\tau_2 y_2^{1-\alpha}+\rho\tau_1 y_1^{1-\alpha})=[1]$

$(\beta(1-\alpha)\tau_1\tau_2(2\tau_2-1)y_1^{-\alpha}y_2^{2-2\alpha}+\rho(1-\alpha)\tau_1 y_1^{-\alpha}=[2a]$

$(\beta^2(2-2\alpha)(\tau_1^2\tau_2+\tau_1\tau_2^2-\tau_1\tau_2)y_1^{1-2\alpha}y_2^{2-2\alpha}+\rho\beta(2-2\alpha)\tau_1^2 y_1^{1-2\alpha}+\rho\beta(1-\alpha)(2\tau_1\tau_2-\tau_1-\tau_2)y_1^{-\alpha}y_2^{1-\alpha})=[2b]$

$((\beta\tau_1\tau_2(2\tau_2-1)y_1^{1-\alpha}y_2^{2-2\alpha}+\rho\tau_1 y_1^{1-\alpha}-\rho\tau_2 y_2^{1-\alpha})=[3a]$

$(\beta^2(1-2\alpha)(2-2\alpha)(\tau_1^2\tau_2+\tau_1\tau_2^2-\tau_1\tau_2)y_1^{-2\alpha}y_2^{2-2\alpha}+\rho\beta(1-2\alpha)\tau_1^2 y_1^{-2\alpha}-\rho\beta\alpha(1-\alpha)(2\tau_1\tau_2-\tau_1-\tau_2)y_1^{1-\alpha}y_2^{1-\alpha}))=[3b]$

$-\beta\alpha(1-\alpha)\tau_1\tau_2(2\tau_2-1)y_1^{-1-\alpha}y_2^{2-2\alpha}-\rho\alpha\tau_1(1-\alpha)y_1^{-1-\alpha}=[4]$

Now the second derivative can be rewritten in the following format:

$\frac{\partial^2 x^*_{1global}}{\partial y_1^2}=\frac{2*[1]*(.)^2}{D^3}-\frac{2*[2a]*[2b]}{D^2}-\frac{[3a][3b]}{D^2}+\frac{[4]}{D}$

Therefore if I can show that [1],[2a],[2b],[3a],[3b] and [4]<0 then the second order condition will be negative. The sufficient conditions to ensure the negativity of the second order condition are listed below.

Condition 1

$\beta\tau_1\tau_2(2\tau_2-1)y_2^{2-2\alpha}y_1^{1-\alpha}-\rho\tau_2 y_2^{1-\alpha}+\rho\tau_1 y_1^{1-\alpha}<0$

$\Rightarrow y_1<(\frac{\rho\tau_2 y_2^{1-\alpha}}{\tau_1(\beta\tau_1\tau_2(2\tau_2-1)y_2^{2-2\alpha}+\rho)})^{\frac{1}{1-\alpha}}$ with $(1-\tau_2)<1/2$

For the case of transboundary pollution, the fraction of import is reduced and the poorer the country 1 inequality can be maintained by reducing τ_1.

Condition 2a

$\beta(1-\alpha)\tau_1\tau_2(2\tau_2-1)y_1^{-\alpha}y_2^{2-2\alpha}+\rho(1-\alpha)\tau_1 y_1^{-\alpha}<0$

$\Rightarrow y_2>(\frac{\rho}{\beta\tau_2(1-2\tau_2)})^{\frac{1}{2-2\alpha}}$

This implies that Country 1 should engage in trade with a country whose income is satisfied by the inequality above. Note that in this case to maintain the positivity of the RHS, τ_2 should be less than 1/2 which contradicts with Condition 1. However, this is a weak restriction and can be overwritten by Condition 1 which is a stronger restriction.

Condition 2b

$(\beta^2(2-2\alpha)(\tau_1^2\tau_2+\tau_1\tau_2^2-\tau_1\tau_2)y_1^{1-2\alpha}y_2^{2-2\alpha}+\rho\beta(2-2\alpha)\tau_1^2y_1^{1-2\alpha}+\rho\beta(1-\alpha)(2\tau_1\tau_2-\tau_1-\tau_2)y_1^{-\alpha}y_2^{1-\alpha})<0$

$\Rightarrow (1-\tau_1)>\tau_2$ and $\tau_1<\frac{\tau_2}{2\tau_2-1}$ with $\tau_2>1/2$

$\partial(\tau_1/(2\tau_2-1))/\partial\tau_2<0$. This means the lower the value of τ_2 the stronger the inequality.

Condition 3a

$\beta\tau_1\tau_2(2\tau_2-1)y_1^{1-\alpha}y_2^{2-2\alpha}+\rho\tau_1y_1^{1-\alpha}-\rho\tau_2y_2^{1-\alpha}<0$

Same as Condition 1 so already proved.

Condition3b

$(\beta^2(1-2\alpha)(2-2\alpha)(\tau_1^2\tau_2+\tau_1\tau_2^2-\tau_1\tau_2)y_1^{-2\alpha}y_2^{2-2\alpha}$
$+\rho\beta(1-2\alpha)(2-2\alpha)\tau_1^2y_1^{-2\alpha}-\rho\beta\alpha(1-\alpha(2\tau_1\tau_2-\tau_1-\tau_2)y_1^{1-\alpha}y_2^{1-\alpha}))<0$

this is just the first order derivative of Condition 2b with respect to y_1. Therefore, the same set of conditions apply here with the additional one that is $\alpha<1/2$.

Condition 4

$-\beta\alpha(1-\alpha)\tau_1\tau_2(2\tau_2-1)y_1^{-1-\alpha}y_2^{2-2\alpha}-\rho\alpha\tau_1(1-\alpha)y_1^{-1-\alpha}$

Same as condition 2a. Therefore the same conditions apply.

In summary the set of conditions that will ensure the negativity of the second derivative of optimal pollution with respect to income is:

$y_1<\left(\frac{\rho\tau_2y_2^{1-\alpha}}{\tau_1(\beta\tau_1\tau_2(2\tau_2-1)y_2^{2-2\alpha}+\rho}\right)^{\frac{1}{1-\alpha}}$, and $y_2>\left(\frac{\rho}{2\tau_2(2\tau_2-1)}\right)^{\frac{1}{2-2\alpha}}$, $(1-\tau_2)<1/2$, $(1-\tau_1)>\tau_2$ and $\tau_1<\frac{2\tau_2}{2\tau_2-1}$ finally $\alpha<1/2$.

Comparing the conditions on income for the shape of EKC with the conditions for positivity of the derivatives with respect to τ_1 and τ_2 it is obvious that the conditions contradict each other. Therefore, when countries engage in trade in ways that the EKC is maintained, the level of global pollution will rise.

Essay 3: Revisiting "The Environmental Kuznets Curve in a Multi-Country Setting" in the presence of Trade - the case of heterogeneous technology

3.1 Introduction

Study of the Environmental Kuznets Curve links two important determinants that are pivotal for social welfare. They are economic growth and environmental quality. The EKC postulates that even though the environmental quality deteriorates with the early stages of economic development, it improves with further economic progress. The existence of such a relationship between economic growth and the environment holds significant policy relevance and has received much attention both in empirical studies and theoretical analysis since its initiation in the early 1990s.
A review of the theoretical models explaining the shape of EKC reveal four broad avenues through which economic development eventually lead to a decline in pollution. A first group of models propose that the income elasticity of a clean environment is greater than one and therefore, with a rise in income, the demand for a clean environment rises which leads to investments towards cleaning up the environment more desirable. Examples of this kind of models include John and Pecchenino(1994), Jaeger (1998). Both of these models propose that early stages of economic development the environment is relatively clean and therefore there is no demand for investments in the environment. With economic growth, when the environment becomes sufficiently dirty, there is a demand for investment in the environment. The first model differs from the second in the way that the first is an overlapping generation model .

A second group of economists posit that at low levels of income some aspects are binding and thus not available to agents. With economic prosperity those aspects become non binding and thus enable economies to avail those aspects and reduce pollution. The work of Stokey (1998) and Jones and Manuelli (1995) fall under this line of explanation.Stokey (1998) proposes that at the early stages of economic development only the dirtiest technology is used, and so pollution rises with income. With economic development clean technology becomes economically feasible and so pollution starts to go down.

Third, another group of economists have modeled net pollution as a function of gross pollution and abatement efforts. For example Andreoni and Levinson (2001) and Smulders and Bretschger (2001). Andreoni and Levinson (2001) developed a static model in which the income pollution path could be explained by a simple relationship between consumption of a desirable good and abatement of its undesirable bi-product. They showed that net pollution is a function of gross pollution and abatement technology and that the pollution income path followed an inverted U pattern only when the abatement technology displayed increasing returns to scale. Around the same time, Smulders and Brestschger (2001) show that high levels of pollution induces policy intervention that encourages adaptation of better technology which causes a downward shift in the income pollution path.

Finally, a fourth group of economists have attributed the existence of the EKC to international trade. For example, Arrow *et al.* (1995) and Suri and Chapman (1998) explain that with economic development the dirty industries get relocated to the developing world and thereby reduce pollution in the developed economies.

In all the models mentioned above, the discussion of “technology” is explicitly or implicitly embedded. Technology clearly plays a key role in both the creation of as well as the mitigation of pollution.Thus in this paper, I would like to expand upon the discourse of technology with the presence of trade. This paper looks at the impact of trade on the possibility of pollution substitution, optimal level of pollution and the shape of the EKC when countries with different technologies trade with each other. I have conducted a similar exercise in my first paper where I have introduced trade and heterogeneity in technology to the Androni and Levinson (2001) model. However, in my first paper the focus of the discussion was centered upon the nature of scale of abatement technology and concluded that when countries engage in trade with each other, increasing returns to abatement technology no longer remains a necessary condition to achieve the EKC pattern in income-pollution path.

In my second paper I introduced trade to the multi-country model developed by Diamantoudi and Filippiadis (2012) and concluded that the exercise allows for a scope of pollution substitution in the case of global pollutants as well as local pollutants. I solved the model for both the symmetric case and asymmetric case where the asymmetry was with respect to income. In this paper I wish to introduce heterogeneity to technology. Holding income equal for the two countries, I consider distinctive technology parameters for the two countries to see how in this environment trade affects the optimal level of a countries' pollution and the shape of the EKC. By the way the model is set up, the assumption to the return to scale of technology is set to be decreasing. I further hold that pollution generated in each country can vary with respect to the intensity of externality. The externality to pollution can vary between 0 and 1, where 0 implies that pollution generated is completely local (the case of *local pollution*) and a value in between 0 and 1 suggests that part of the pollution generated permeates across the boundaries of the country (the case of *transboundary pollution*). However, the intensity of pollution goes down compared to the source of generation, perhaps due to distance. When the pollution externality takes a value of 1, it means that all of the pollution generated permeates across the borders. Such a case can be looked at as the case of *global pollution.* A point to note is that in the second paper, because I held homogeneous technology parameters, the model considered that the pollution generated was either local, or global and same for both countries. In this paper, with heterogeneity in technology, the assumption is that the pollution generated may either be local, or transboundary or global and may vary across countries. That is, while pollution of one country can be local, the pollution generated by the other country can be transboundary or even global. The original model already established the existence of the EKC. My second paper looked at the impact of trade on the shape of the EKC when countries varied with respect to income. This paper looks at the impact of trade among countries with varying degrees of technology. It shows that the relative level of the two countries holds implications for the direction of trade. Again, like my other papers, trade has been kept exogenous to the model so that the impact of import and export could be assessed by varying their intensities.

Results of this paper reveal that the better the technology of the foreign country, engaging in import makes the possibility of pollution substitution stronger. With a rise in technology of home country engaging in exports leads to stronger pollution substitution. Second, when both countries emit local pollutants, the optimal level of pollution is lower than when either or both of them emit transboundary or global pollutants. When the technology of the foreign country is better, importing a larger fraction of foreign output reduces optimal pollution and when the technology of the home country is high, exporting a larger fraction of domestically produced goods reduces optimal pollution. The shape of the EKC gets affected by the technology of both countries. When nations engage in trade, the better the technology of the foreign country, the more resemblance the income pollution path holds to the EKC. However, the impact of foreign technology on the shape of the EKC is less for local pollutants than for transboundary pollutants. Regardless, when countries engage in trade the shape of the EKC is retained. The better the technology of home country and foreign country, higher resemblance the the income pollution path takes to the desired inverted U pattern. If the technology of foreign country is better, increasing the fraction of goods imported improves the shape of the EKC. Finally, when pollution externality exists for Country 1 engaging in exports and when pollution externality exists for Country 2 engaging in higher imports ensures the desired inequalities to guarantee the shape of the EKC.

The rest of the paper is organized in the following manner. Section 3.2 provides with the literature review. The model is presented in section 3.3. Section 3.4 looks at the impact of trade on the optimal level of pollution and the shape of the income pollution path. A demonstration of the results of Section 3.4 are presented in the form of simulated graphs in Section 3.5. Finally, Section 3.6 concludes the paper.

3.2 Literature Review

That technology plays a key role in sustainable development and economic growth is manifest in any discourse on the subject. As early as in 1798, Malthus pointed out that while human population grows exponentially, food production grows at an arithmetic rate and so the growth of population would soon deem the carrying capacity of nature as unsustainable leading to a *Malthusian Catastrophe* (such as a war or famine) to bring the population back to a sustainable levels. Such pessimistic views were fortunately invalidated by the Industrial Revolution that followed soon after. However, with the onset of the industrial revolution emerged new concerns relating to the sustainability of the environment to withstand unbounded economic growth. The process of industrialization gave way to new concerns of pollution and resources depletion.

The 1970s witnessed heated debates on how economic growth would affect environmental quality. Georgescu-Roegen (1971) expressed that natural resources and clean environment cannot be sustained for ever if the economies continue to grow without limit. Around the same time Meadows *et al.* (1972) published a report titled "The limits to growth" where they concluded that different natural resources would be depleted with economic growth and even went as far as to predict possible exhaustion dates for resources such as chromium, gold and petroleum. They explained that while population and extraction of natural resources increase exponentially, the discovery and renewal of natural resources occur linearly and therefore the use or exploitation of natural resources should at some point in time lead to exhaustion of these resources. Such a view was vehemently opposed by Cole (1973) who argued that economic growth would bring about technological innovations that would relax the constraints on economic resources.

The main idea of the Environmental Kuznets Curve emerged in the early 1990s through a series of empirical observations. A number of studies were conducted on different pollutants such as CO, NOx, SOx,suspended particulate matter, municipal waste and lead by Grossman and Kreuger (1991), Shafik and Bandyopadhayay (1992) and Panayotou (1993), and it was found that while the emission of these pollutants initially increased with the growth in per capita income, with further progress of economic growth, emissions of these pollutants tended to go down. Thus emerged the idea that expedited economic growth was in fact desirable in order to protect the environment. To that end Beckerman (1992) pointed out that 'least developed countries have deficient resources for the protection of the environment and it is economic growth that can provide the resources to resolve the environmental problems'.

A similar pattern between income inequality and economic growth was observed by Kuznets in 1955 who called it the Kuznets Curve. In the early 1993 Panayotou adopted the term and suffixed it with "environmental" to give rise to the term "the Environmental Kuznets's Curve". Henceforth, the phenomenon of EKC has been widely used to describe the inverted U shaped pattern of the income pollution path.Following the empirical observations, a number of economists have sought to give structure and hindsight behind the existence of the concept. Arrow *et al.* (1995) state that the EKC pattern can be explained by the natural progression of economic development from the clean agrarian societies to polluting industrial economies to clean service based economies. Others such as Grossman and Kreuger (1991) explain that as economies grow they undergo three distinct effects (i) the scale effect - where with economic growth larger scale production and pollution occurs; (ii) the technology effect - where with economic development better technology is adopted; and finally (iii) the composition effect - where with economic prosperity cleaner composition of goods and services are adopted. They explain that while at the early stages of economic development the scale effect dominates, with further development the technology effect and the composition effect take over leading to a decline in pollution with income. John and Pecchenino (1994) present an overlapping generation model in which environmental quality is a stock resource that degrades with time unless maintained by investments in the environment. At the beginning an economy begins with zero pollution and sees its environment degrade with pollution. At low levels of income and high environmental stocks, investments in environment is not desirable. Eventually, with high income and reduced environmental stocks, investment in the environment becomes desirable and thus pollution starts to go down. They thus predict and inverse V shaped EKC. Jones and Manuelli (1995) propose that pollution involves externalities and appropriately internalizing these externalities is possible with advanced institutions that can only be afforded by developed economies. Jaeger (1998) model that low levels of economic growth are associated with low levels of pollution where the consumers marginal benefit to additional environmental quality is zero. Therefore at this stage more pollution does not lead to lower utility. With growth and further pollution, consumers become more sensitive to increased pollution at which point, depending on the parameters, growth may be accompanied by improved environmental quality. What ever the explanation, be it investment in environment or internalizing externalities the issue of technology is implicitly present in the discussion.

There also exists a wide pool of literature that focuses explicitly on the role of technology in explaining the EKC pattern in the income pollution path. Stokey (1998) developed a static model where she shows that when economies are poor, they can use only the dirtiest technology. Thus early stages of economic development are marked by rising pollution. But with economic development clean technologies become economically feasible, adoption of which lead to reduced pollution with economic growth. Andreoni and Levinson (2001) create a static model where they show that the shape of the EKC can be explained by a simple technological link between consumption of a desirable good and abatement of its undesirable bi-product - which is pollution. They state that the necessary condition for the EKC pattern to hold is maintaining increasing returns to scale in abatement technology. Following this Egli (2005) propose that rather than IRS, countries tend to exhibit fading IRS. Thus with high levels of pollution countries exhibit IRS in their abatement technology, but with cleaner environment, they start to display CRS. For this reason, even though pollution decreases with rising income, it never really goes to zero. Plassman and Khanna (2006) build upon the Andreoni and Levinson model. They show that rather than IRS the feature that explains the shape of the EKC is that if the scale of pollution is less than the scale of abatement. They therefore claim that the predictions of Andreoni and Levinson is a special case of the model that they propose.Filippiadis (2014) developed a dynamic model in which he showed that the shape of EKC depended on the relative stage of development of the two countries and their in-between interactions. In his model, the initial condition of the economy is dependent on the initial stock of physical capital as well as the total factor productivity.

The existence of EKC has also been explained by international trade in literature. Again the discussion of technology is implicitly encapsulated in the discussion. Suri and Chapman (1998) build upon the concept of a pollution haven where they explain that with economic growth environmental restrictions in the developed countries pushes the dirty industries to the developing world where environmental restrictions are more relaxed. Copeland and Taylor (2001) investigate the linkages between national income, pollution and international trade. They develop a north-south trade model to investigate the impact of trade restrictions on the environment and welfare. Dean (1999) find that trade liberalization directly aggravates environmental damage via its influence on the terms of trade, but indirectly mitigates it via its effect on income growth. Holladay (2008) investigates the impact of trade liberalization on the environment. He looks at two distinct sources of pollution - that sourcing from consumption and that sourcing from production. He concludes that when pollution is sourced from consumption, trade liberalization increases pollution and when pollution sources from production, trade liberalization reduces pollution. In the literature for international trade and the EKC, the distinction between local pollutants and global pollutants is not well covered. Ever since its initiation, research has been conducted to explain and validate the existence of the EKC by proponents of the concept. At the same time opponents of the concept has sought to invalidate the existence of the EKC. For example, Gill *et al* (2017) conclude that even though the shape of the EKC may hold for local pollutants it does not for transboundary pollutants. They further assert that the EKC may have been a basis for policy development for developed countries in the past. But now with increasing global pollution, developing countries of todays should avert from the phenomenon of the EKC in their policy development.

A lot of work has also been undertaken in looking at the empirical evidence behind the existence of the EKC. Selden and Song (1994) have confirmed the inverted U pattern in the income pollution path using a cross-national panel of data on emission of four air pollutants namely suspended particulate matter, sulfur di-oxide, oxides of nitrogen, and carbon monoxide. However, they predicted that emissions to decrease only in the very long run with rapid growth on global emissions to continue for several decades. Grossman and Kreuger (1995) also examine the relationship between per capita GDP and four environmental indicators and conclude in favor of the existence of the EKC. They further provide with some intuitive explanations for the downward sloping part of the EKC. They explain that as countries grow several factors come into play to cause the downward trend of pollutants. First, with growth more stringent environmental policies are adopted as with economic prosperity non economic aspects of life such as a clean environment become more desirable for economic agents. Second, as economies grow they cease to produce some of the dirtier products and start to import them instead. Finally, with time and learning from the experiences of the wealthier countries, countries may adopt cleaner technologies.

In recent times, much of the empirical work has focused on transboundary pollutants. For example, Apergis (2016) looks at the existence of the EKC in the emission of CO_2across fifteen countries and concludes that it does in twelve out of the fifteen countries. Bilgili *et al.* (2016) also look at the empirical evidence based on the emission of CO_2. However, rather than explaining the downward sloping part of the EKC with income growth, they attribute it to a shift in renewable energy with time. Jebli *et al.* (2016) also study the existence of EKC using $CO_{2.}$They examine the impact of renewable energy, non- renewable energy and trade on the shape of EKC in OECD countries.

In this paper I build upon the discussion of local pollutants and transboundary pollutants in the presence of trade. I also build upon the discourse of technology. I look at implications that imports and exports hold for the pollution pattern when countries with varying technology and thus varying types of pollutants engage in trade with each other.

3.3 The Model

3.3.1 The Model set up

The model considers single agent countries. There are two countries $i \in \{1, 2\}$ each producing a single commodity. To introduce trade to the model,

I consider that each country retains a fraction of the output produced for own consumption and exports the remainder to the other country. Let the fraction retained by each country be $\tau_i$with $i = 1, 2$.Therefore $(1 - \tau_i)$represent the level of export of each country.

Pollution, x_i, in each country is generated from production. The social utility function will therefore be a function of Consumption C_i and pollution x_i. Let the social utility function be additively separable and take the form:

$V_i = u_i(C_i) - h_i(x_1, x_2) \qquad (1)$

u_i represents the utility from consumption and h_i measures the disutility associated with pollution. Both functions are well behaved and follow the conventional properties of utility. That is, utility is concave, twice continuously differentiable and rising in consumption. Concavity of the functions ensures diminishing marginal utility associated with consumption. $h_i(.)$is convex, twice continuously differentiable and rising is pollution. The convexity of the curve portrays rising marginal disutility with additional units of pollution.

Let the country specific utility function take the form:

$$u_i(C_i) = \begin{cases} C_i - \frac{\beta}{2}(C_i)^2, if C_i \leq \frac{1}{\beta} \\ \frac{1}{2\beta}, otherwise. \end{cases}$$

In the equation above, β is the risk aversion indicator and takes a value greater than 0. A higher value of β represents a higher degree of risk averseness. Disutility from pollution takes the form:

$h_i = \frac{\rho}{2}(l_1x_1 + l_2x_2)), (0 \leq l_i \leq 1), \qquad i = 1, 2$

ρ is the parameter capturing pollution perception and takes a value greater than 0. l_i measures the degree of pollution externality for each country. For any given country l_i attached to its own pollution x_i will take the value of 1, meaning that the pollution generated at home will be fully exposed to the agents of that respective country. The value of $l_i$attached to the other countries' pollution can take a value equal to or in between 0 and 1. This means the extent to which the foreign country's pollution will affect the home country can vary. If it takes a value of 0, it means that pollution generated in the foreign country is strictly local and will not affect the home country at all. This is referred to as a case of *local pollution.* If it takes a value between 0 and 1, it means that some fraction of the pollution generated in the foreign country will permeate into the home country and is termed as the case of *transboundary pollution.* When the value takes a value of 1 it means that all of the pollution generated in the foreign country transgresses into the home country - the case of *global pollution.*

Output in country i depends upon income. In addition, the production of output generates pollution x_i.Pollution is bound from above by the maximum possible pollution in a given country. This maximum pollution is a function of technology. The better the technology, the lower will be the limit to maximum pollution.Therefore, output q_i takes the following functional form:

$q_i = y\frac{x_i}{\overline{x_i}} = y\frac{x_i}{y^{\alpha_i}}$

Because the model considers two countries with homogeneous income, there is no subscript attached to income y.$\overline{x_i} = y^{\alpha_i}$represents the maximum upper bound to pollution set in a country. α_i is the technology parameter taking a value between 0 and 1. The lower the value of α_i the more advanced is the technology of a given country and therefore the lower will be the value of maximum permissible pollution. At any given level of pollution max, when the dirtiest technology is used, $x_i = y^{\alpha_i}$, and thus $q_i = y$. Given the model setup, the consumption possibility of Country 1 can be written as:

$C_1 = \tau_1 y\frac{x_1}{y^{\alpha_1}} + (1-\tau_2)y\frac{x_2}{y^{\alpha_2}}$

The social utility function of Country 1 as faced by the social planner is:

$V_1 = [\tau_1 y\frac{x_1}{y^{\alpha_1}} + (1-\tau_2)y\frac{x_2}{y^{\alpha_2}}] - \frac{\beta}{2}[\tau_1 y\frac{x_1}{y^{\alpha_1}} + (1-\tau_2)y\frac{x_2}{y^{\alpha_2}}]^2 - \frac{\rho}{2}(x_1 + l_2 x_2)^2$

Taking the first order derivative with respect to x_1 gives the following solution for optimal utility for Country 1:

$x_1 = \frac{\tau_1 y^{1-\alpha_1}}{\beta\tau_1^2 y^{2-2\alpha_1}+\rho} - \frac{[\beta\tau_1(1-\tau_2)y^{2-\alpha_1-\alpha_2}+\rho l_2]x_2}{\beta\tau_1^2 y^{2-2\alpha_1}+\rho}$ (2)

The following sections investigate the impact of trade among countries with varying technology on the optimal level of pollution as well as the shape of the EKC. The analysis begins with an assessment of trade on the scope for pollution substitutability.

3.3.2 The Scope for pollution substitution

The possibility of pollution substitution can be seen by taking the derivative of optimal pollution of Country 1 with the pollution of Country2.

$$\frac{\partial x_1}{\partial x_2} = -\frac{\beta\tau_1(1-\tau_2)y^{2-\alpha_1-\alpha_2}}{\beta\tau_1^2 y^{2-2\alpha_1}+\rho} - \frac{\rho l_2}{\beta\tau_1^2 y^{2-2\alpha_1}+\rho} < 0 \qquad (3)$$

The negativity of equation (3) above ensures the possibility of pollution substitution between Country 1 and Country 2. The first term showcases the pollution substitution made possible by trade and the second part shows the pollution substitution made possible by the existence of pollution externality in Country 2. How trade affects the scope for pollution substitution has already been covered in my second paper. In this part I want to focus on how the difference in technology of the two countries affects the extent of pollution substitution when countries engage in trade.

Proposition 7 *The more advanced the technology of the foreign country, engaging in imports makes the scope for pollution substitution stronger. The more advanced the technology of domestic country, engaging in exports, makes the scope for pollution substitution stronger.*

Proof. *To see how the technology of the foreign country affects pollution substitution I look at:*

$$\frac{\partial^2 x_1}{\partial x_2 \partial \alpha_2} = \frac{(1-\tau_2)\tau_1 y^{2-\alpha_1-\alpha_2}\log(y)}{\beta\tau_1^2 y^{2-2\alpha_1}+\rho} > 0 \qquad (4)$$

The higher the value of α_2 *the weaker will be the scope for pollution substitution. A higher value of* α_2 *indicates a country with lower level of technology where the upper bound to admissible pollution will be higher. This means, the more advanced the technology of the foreign country, the lower will* *be the value of* α_2 *and thus the stronger will be the scope of pollution substitution. However, the technology of the foreign country will influence pollution substitution only when country 1 chooses to engage in imports. Thus importing from countries that are more technologically advanced, will allow a country to reduce its own pollution more.*

Next I look at the impact of own technology, α_1*, on the scope of pollution substitution.*

$$\frac{\partial^2 x_1}{\partial x_2 \partial \alpha_1} = -\frac{2\beta(1-\tau_2)\tau_1^3 y^{4-3\alpha_1-\alpha_2}\log(y)}{(\beta\tau_1^2 y^{2-2\alpha_1}+\rho)^2} + \frac{(1-\tau_2)\tau_1 y^{2-\alpha_1-\alpha_2}\log(y)}{\beta\tau_1^2 y^{2-2\alpha_1}+\rho}$$

$$-\frac{2l_2\beta\rho y^{2-2\alpha_1}\log(y)}{(\beta\tau_1^2 y^{2-2\alpha_1}+\rho)^2}\begin{cases} \leq 0 if \tau_1 > (\frac{\rho}{y^{2-2\alpha_1}})^{1/2}, \\ > 0, otherwise. \end{cases} \quad (5)$$

Own technology affects the scope for pollution substitution even when countries do not engage in trade, so long as the pollutant is transboundary or global. When it comes to local pollution, trade is the only route through which own technology will affect the scope for pollution substitution. Focusing on the first two terms of equation (5) above, the sum of the first two terms will be less than or equal to 0 so long as $\tau_1 > (\frac{\rho}{y^{2-2\alpha_1}})^{\frac{1}{2}}$*. As long as this inequality is satisfied, the second derivative above will be negative, meaning the higher value of* α_1 *stronger will be the scope for pollution substitution. This means that when home countries have less advanced technologies, the scope for them to engage in pollution substitution by keeping the fraction of own output exported or* $(1-\tau_1) < 1-(\frac{\rho}{y^{2-2\alpha_1}})\frac{1}{2}$*. If exports is more than this, then the higher the value of* α_1 *the lower will be the scope for pollution substitution.* ■

Finally, the scope for pollution substitution gets stronger with the increase in the level of pollution externality in the foreign country. This conclusion is derived by observing the second part of equation (3). However, transboundary or global pollution is affected only by exports and not by imports. Equation 3 shows that the higher the value of t lower the scope of substitution through global emissions. By increasing the fraction of own output exported countries can increase the strength of pollution substitution in the case of transboundary pollutants. The more advanced own technology is more impact fraction of output exported will have on the scope for pollution substitution. The implications of the level of home technology and foreign technology in affecting the direction of trade in order to strengthen the scope of pollution substitution is summarized in Table 3.1 below. The impact is looked at the point of view of an improvement in technology, i.e, the value of α_i.

Table 3.1: Impact on the strength of pollution substitution when countries with different technology engage in trade.

	Strength of Pollution Substitution of:	
	Local Pollutants	**Trans boundary Pollutants**
$\alpha_1 \downarrow$	Increases when $(1-\tau_1) > 1 - (\frac{\rho}{y^{2-2\alpha_1}})^{\frac{1}{2}}$	Increases with $(1-\tau_1)$
$\alpha_2 \downarrow$	Increases with $(1-\tau_2)$	No affect of trade.

3.4 Model Solution and Implications of Trade

Solving the model for both countries, the solution for optimal pollution for country 1 comes to:

$x_1^* =$

$$\frac{\rho\tau_1 y^{1-\alpha_1} - \rho l_{2\tau_2} y^{1-\alpha_2} + \beta\tau_1\tau_2(2\tau_2-1)y^{3-\alpha_1-2\alpha_2}}{\rho\beta\tau_1^2 y^{2-2\alpha_1} + \rho\beta\tau_2^2 y^{2-2\alpha_2} + \rho^2(1-l_1l_2) + \beta(\tau_1^2\tau_2 + \tau_1\tau_2^2 - \tau_1\tau_2)y^{4-2\alpha_1-2\alpha_2} - \rho\beta(l_{1\tau_1}(1-\tau_2) + l_2\tau_2(1-\tau_1))y^{2-\alpha_1-\alpha_2}}$$

$\equiv \frac{A}{B}$ (6)

I will now use this solution to carry out the rest of my analysis. First, I will investigate what happens to the level of optimal pollution when pollution is strictly local for both countries, strictly global for both countries, and finally pollution externality varies for the two countries. I will then investigate the implication for trade on optimal pollution for each of the scenarios. Finally I will look at the implication that this holds for the shape of the income pollution path. All analysis will be carried out in light of the fact that the technology of the two countries are distinctly different.

3.4.1 Impact on the optimal level of pollution

3.4.1.1 Implication for pollution externality

Proposition 8 *The higher the extent of pollution externalities of the two countries, the higher will be the optimal level of pollution.*

Proof. *Holding all other factors unchanged, if the nature of pollutants generated in both countries are strictly local,* $l_1 = l_2 = 0$,*equation (6) can be rewritten as*

$$x^*_{1local} = \frac{\rho\tau_1 y^{1-\alpha_1} + \beta\tau_1\tau_2(2\tau_2 - 1)y^{3-\alpha_1-2\alpha_2}}{\rho\beta\tau_1^2 y^{2-2\alpha_1} + \rho\beta\tau_2^2 y^{2-2\alpha_2} + \rho^2 + \beta(\tau_1^2\tau_2 + \tau_1\tau_2^2 - \tau_1\tau_2)y^{4-2\alpha_1-2\alpha_2}} \equiv \frac{A^/}{B^/} \qquad (6a)$$

Since $A^/ > A$ *and* $B^/ > B$, *but* $\triangle A < \triangle B$,*(where* $\triangle A = A^/ - A$, *and* $\triangle B = B^/ - B$*) I conclude that when pollutants generated in both countries are strictly local, the optimal level of pollution will be lower than when pollutants are transboundary or global.*

In addition, $\frac{\partial x_1^*}{\partial l_1} > 0$ *meaning that the higher the level of pollution externality generated by Country1, the higher will be the level of optimal pollution.* $\frac{\partial x_1^*}{\partial l_2} > 0$ *. I therefore conclude that when pollutants exhibit the nature of transboundary or global pollutants, be it in either one of the countries or both countries , optimal pollution will always be higher than when pollutants are strictly local.* ■

3.4.1.2 Implications of technology of home country and foreign country

When countries engage in trade the abatement technology of both countries affect the optimal pollution decision. The direction of impact is also the same, in the sense, the higher the value of $\alpha_1$or α_2 the higher will be the level of optimal pollution. What is interesting to me is to observe, when the technologies of the two countries are different, what pattern of trade will benefit in reducing the level of optimal pollution. A flavor of this discussion is provided in Section 3.2 where it can be seen that the scope of pollution substitution becomes stronger when countries engage in imports with reducing values of $\alpha_{2.}$ In other words, the better the abatement technology of the foreign country, better is the scope of pollution substitution through imports. In this section I try to elaborate on that discussion by looking at the relative impact of the two technologies on the optimal level of pollution.

Proposition 9 *When the abatement technology of the foreign country is lower, engaging in import can reduce the level of optimal pollution, and when it is higher, engaging in export can reduce the optimal level of pollution. When the abatement technology of home country is less advanced countries should engage in imports and when the abatement technology of the foreign country is less advanced, countries should engage in exports if they wish to reduced the optimal level of pollution.*

Proof. *Taking the derivative of optimal pollution with respect to* τ_2 *and* τ_1 *will give the impact on optimal pollution with respect to import and export respectively.*

$$\frac{\partial x_1^*}{\partial \tau_2} = \frac{B(-\rho l_2 y^{1-\alpha_2} + \beta(4\tau_1\tau_2 - \tau_1)y^{3-\alpha_1-2\alpha_2})}{B^2} - \frac{A(2\rho\beta\tau_2 y^{2-2\alpha_2} + \beta(\tau_1^2 + 2\tau_1\tau_2 - \tau_1)y^{4-2\alpha_1-2\alpha_2} - \rho\beta(-l_1\tau_1 + l_2(1-\tau_1))y^{2-\alpha_1-\alpha_2})}{B^2} \tag{7}$$

$$\frac{\partial x_1^*}{\partial \tau_1} = \frac{B(\rho y^{1-\alpha_1} + \beta(2\tau_2^2 - \tau_2)y^{3-\alpha_1-2\alpha_2})}{B^2} - \frac{A(\rho\beta 2\tau_1 y^{2-2\alpha_1} + \beta(2\tau_1\tau_2 + \tau_2^2 - \tau_2)y^{4-2\alpha_1-2\alpha_2} - \rho\beta(l_1(1-\tau_2) - l_2\tau_2)y^{2-\alpha_1-\alpha_2})}{B^2} \tag{8}$$

A positive value for equation (7) will ensure that the higher the value fraction of output of Country 2 imported, the lower will be the level of optimal pollution. Similarly, in order to show the conditions under which engaging in exports will *reduce the optimal level of pollution, one has to set equation (8) as positive. Since the optimal level of pollution is positive it must be that* $A > 0$*and* $B > 0$. *Therefore sufficient conditions for setting equation (7) and (8) as positive are:*

$-\rho l_2 y^{1-\alpha_2} + \beta(4\tau_1\tau_2 - \tau_1)y^{3-\alpha_1-2\alpha_2} \geq 0 \qquad (7a)$ *and*

$2\rho\beta\tau_2 y^{2-2\alpha_2} + \beta(\tau_1^2 + 2\tau_1\tau_2 - \tau_1)y^{4-2\alpha_1-2\alpha_2} - \rho\beta(-l_1\tau_1 + l_2(1-\tau_1))y^{2-\alpha_1-\alpha_2} \leq 0 \qquad (7b)$ *for equation (7) to be positive and*

$\rho y^{1-\alpha_1} + \beta(2\tau_2^2 - \tau_2)y^{3-\alpha_1-2\alpha_2} \geq 0 \qquad (8a)$*and*

$\rho\beta 2\tau_1 y^{2-2\alpha_1} + \beta(2\tau_1\tau_2 + \tau_2^2 - \tau_2)y^{4-2\alpha_1-2\alpha_2} - \rho\beta(l_1(1-\tau_2) - l_2\tau_2)y^{2-\alpha_1-\alpha_2} \leq 0 \qquad (8b)$ *for equation (8) to be positive.*

Equation (7a) implies that $y^{2-\alpha_1-\alpha_2} > ((\frac{\rho l_2}{\beta(4\tau_1\tau_2-\tau_1)}))$. *Thus the lower the value of* α_2 *the stronger the inequality. When Country 2 enjoys advanced technology, engaging in import can reduce the optimal level of pollution. Note, the higher the level of* l_2, *that is the more pollution externality there is the weaker will be the inequality. That indicates that the more pollution externality there exists, the possibility of reducing optimal pollution by engaging in imports will go down. From equation (7b) I get*

$2\rho\tau_2 y^{\alpha_1} + (\tau_1^2 + 2\tau_1\tau_2 - \tau_1)y^{2-\alpha_1} \leq \rho(-l_1\tau_1 + l_2(1-\tau_1))y^{\alpha_2}$. *The inequality will be stronger the higher the value of* α_1. *Thus when the technology of home country is poorer, engaging in import can reduce the optimal level of pollution. Equation (8a) gives* $y^{2-2\alpha_2} \leq \frac{\rho}{\beta(\tau_2-2\tau_2^2)}$. *The higher the value of* α_2 *the stronger the inequality. When the abatement technology of the foreign country is poorer, engaging in exports can reduce the optimal level of pollution. Equation (8b) gives*

$2\rho\beta\tau_1 y^{\alpha_2} + \beta(2\tau_1\tau_2 + \tau_2^2 - \tau_2)y^{2-\alpha_2} \leq \rho\beta(l_1(1-\tau_2) - l_2\tau_2)y^{\alpha_1}$. *The higher the value of* α_2*the stronger the inequality.*

Thus I conclude, when the technology of the foreign country is poorer engaging in export can reduce the optimal level of pollution and when it is better, *engaging in import can reduce the optimal level of pollution.* ■

The conclusions of this section are in line with the partial conclusions drawn in Section 3.2. When Country 2 has improved technology, by increasing the fraction of $q_2$imported in Country 1, Country 1 can seek to reduce optimal pollution. If the technology is poorer, the fraction of output imported should be reduced. When Country two has poor technology, Country 1 can reduce optimal pollution by increasing the fraction of $q_1$exported to Country 2. Clearly, a difference in abatement technology can affect the pattern and direction of trade, when the objective of trade is pollution reduction.

3.4.2 Impact on the shape of the income-pollution path

The ultimate interest of study in this paper is when countries with different technologies enter into trade with each other, what shape will the income pollution path of the country take? The obvious desired option is the EKC pattern displayed by the inverted U-shape. The necessary condition for this is for the second derivative with respect to income of the optimal pollution path to take a negative value. In this section I provide some sufficient conditions for such a path to be consistent. A glance at the equation for optimal pollution will suffice to vouch for the fact that the second derivative will be extremely lengthy and complex to look at. The full derivative and the derivation of the conditions are presented in the Appendix.

After introducing trade among countries with different technologies, the second derivative of optimal pollution with respect to income will be negative if the following conditions are satisfied.

1. Overall, the lower the value of α_2 the more technologically advanced Country 2 is, the stronger will be the inequalities to ensure the negativity of the second derivative.

2. The lower the value of α_1, the more technologically advanced Country1 is, increasing the fraction of own good exported, enhances the strength of the inequalities to ensure the negativity of the second derivative. The value of α_1 should be less than 0.5.

3. When the pollution externality of Country 2 is zero, $l_2 = 0$, the fraction of goods retained in Country 2 should be less than half. In other words, fraction of import from Country 2 should be bigger than half ($(1 - \tau_2) > 1/2$).If $\tau_2 = 1/2$,the value of α_2 should be lower than the value of α_1.

4. When both pollutants are strictly local, $l_1 = l_2 = 0$, the fraction of output of Country 2 imported by Country 1 should be greater than fraction of own goods retained at home, i.e, $(1 - \tau_2) > \tau_1$.

5. When the pollution externality in Country 1 is positive, but the pollution of Country 2 is strictly local, $l_1 > 0, l_2 = 0$, the desired inequalities will hold as long as $(1 - \tau_2) = \tau_1$or $(1 - \tau_2) > \tau_1$. For $(1 - \tau_2) < \tau_1$ the inequality will become weaker even for lower values of α_2.

6. When the pollution externality of Country 2 is positive $(1 - \tau_2)$ should be higher, and when pollution externality of Country 1 is positive, $(1 - \tau_1)$ should be higher.

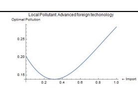
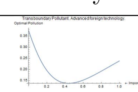
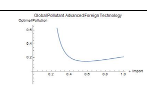
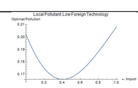
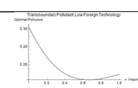
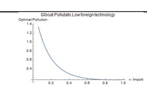
3.5 Demonstration of Results

In this section I present with some graphical representation of the results proposed in the previous section with the help of simulation. The values assigned to parameters are within the range specified by the model. In the model β measures the risk averseness of an individual and can take a value bigger than 0. For this section I assume $\beta = 2.\rho$ captures the pollution perception in the model and is assumed that $\rho \in R_+$. The value assigned to ρ in this section is 3. Given this I proceed to present with the conclusions of

Section 3.4.

Proposition 7 presents with the idea that when countries engage in imports with other countries that have better technology, the scope for pollution substitution becomes stronger. This concept is further developed in Proposition 9 where it is found that when countries engage in imports from countries that have better technology, the optimal level of pollution can be reduced. This however is dependent upon the nature of pollutant of the foreign country. The higher the level of pollution externality generated, the weaker will be the impact on optimal pollution through imports. In other words, pollution substitution through imports is strongest for the case of local pollutants and goes down for the case of transboundary or global pollutants. Table 3.2 provides a summary of this discussion below.

Table 3.2: Comparision of impact of import when technology of foreign country is different, across different types of pollutants.

	Local Pollutant	Transboundary Pollutant	Glboal Pollutant
Low α_2			
Highα_2			

For the simulation results presented in Table 3.2, the technology parameter of the domestic country is set at a mid way with $\alpha_1 = 0.5$. For the two types of technologies of Country 2, when Country 2 is assumed to have advanced technology, $\alpha_2 = 0.2$ and when Country 2 is assumed to have a poor technology, $\alpha_2 = 0.8$. When pollutants are assumed to be local, $l_1 = l_2 = 0$. For transboundary pollutants, the level of externality is set at 0.5 and finally for global pollutants, $l_1 = l_2 = 1$. The table makes it clear, that as the extent of pollution externality increases the ability to reduce the level of optimal pollution by engaging in imports goes down. However, when the technology of the foreign country is more advanced, engaging in import can still reduce the optimal level of pollution to a certain extent. The better the technology of the foreign country, the more will be impact on optimal pollution through imports for every type of pollutant.

Proposition 7 and 9 also claim that when the technology of the home country is more advanced, engaging in export can have a positive impact on the optimal level of pollution. Again, the extent of impact varies with the level of externality generated by the pollutants. Table 3.3 below presents a summary of the impact of import on the optimal level of pollution when the nature of pollutants vary.

Table 3.3: Comparision of impact of export on the optimal level of pollution when technology varies and across different types of pollutants.

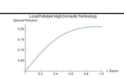
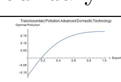
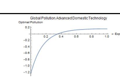
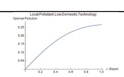
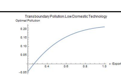
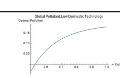
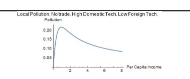
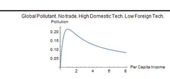
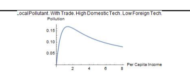
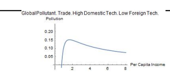
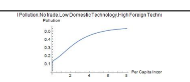
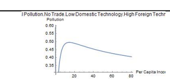
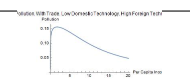
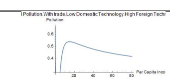
	Local Pollutant	Transboundary Pollutant	Global Pollutant
Lowα_1			
Highα_1			

Table 3.3 above summarizes the impact of export. This time, the technology of the foreign country is set at a midpoint at $\alpha_2 = 0.5$. For the case of advanced domestic technology $\alpha_1 = 0.2$ and for the case of poor domestic technology $\alpha_2 = 0.8$. For the nature of pollutants, for local pollutants, pollution externality is assumed to be zero, for transboundary, it is assumed to be 0.5 and finally for global pollutants, pollution externality is assumed to be 1. For the case of export, it is found the higher the extent of pollution externality, the lower will be the optimal level of pollution when countries engage in exports.

Also, the lower the value of α_1 that is the more advanced the technology of the home country, the greater the impact of export on reducing optimal pollution. For the last part of this section, I look at the impact of trade when technologies of the two countries vary on the income pollution path. I look at the case for local pollutants and global pollutants for two distinct cases. One for when a country with high level of technology engages in trade with a country with a low level of technology. And a second for when a country with a low level of technology engages in trade with a country with high level of technology. Table 3.4 below, summarizes the findings.

Table 3.4: Comparision of impact of trade on EKC for different technologies and across different types of pollutants.

		Local Pollutant	Global Pollutant
Lowα_1Highα_2	No trade		
	With trade		
Highα_1Lowα_2	No trade		
	With trade		

The table above demonstrates that the impact of trade on the shape of the income pollution path is more severe for the case of local pollutants than for global pollutants. Particularly when a low tech country, engages with a high tech one, it can be seen that the turning point of EKC is brought drastically forward. The last part of section 3.4 concludes that the lower the level of technologies of both countries the more likely it is that the income pollution path will follow an inverted U shape. This is also ascertained in the table above.

3.6 Conclusion

Engaging in trade does hold implications for a country's welfare. On the one hand engaging in trade increases the range of consumption goods available. On the other hand, engaging in trade also impacts pollution. How trade affects pollution is ambiguous. Engaging in trade can create a scope for pollution substitution, or it can lead to increased pollution. As a result of this, trade can also impact upon the shape of the income- pollution path of a country. How trade will affect these variables depend upon the direction of trade as well as the relative technological stand of the trading countries. In this paper I have investigated what happens to the scope of pollution substitution, the level of optimal pollution and the shape of the income pollution path, when countries with different technologies engage in trade with each other.

In this I paper I have established that when countries with different technologies engage in trade, they create a scope for pollution substitution. This can create implications for the direction of trade. The paper concludes, that the more advanced the technology of home country, by exporting a larger fraction of own output, countries can seek to reduce the level of local pollution as well as transboundary pollution. When the foreign countries are more technologically advanced, increasing the fraction of goods imported can reduce local pollution. However, the model finds that such a case does not affect the case for transboundary pollution.

The paper then explores the impact on optimal pollution when countries with different technologies trade with each other. First the model finds that when countries engage in trade with each other, the optimal level of pollution is lower when pollutants of both countries are strictly local, than when either one or both coutries emit transboundary or global pollutants. Overall it is found that when fractions of goods are imported from countries with better technology, optimal pollution at home can be reduced. When the level of technology at home is better, exporting a larger fraction of own goods can reduce the level of optimal pollution.

Finally the paper focuses on the impact of trade among countries with different technologies on the shape of the inocme pollution path. The likeliness of income pollution path to follow the desired EKC pattern increases when Country 1 imports from Country 2 who has a better technology. When the technology of Country 1 is better, the likeliness goes up with Country 1 exporting a larger fraction of own output to Country 2. The implications of trade are stronger for the case of local pollution than for transboundary or global pollution. However, even for the case of transboundary or global pollution, the shape of the EKC is retained. When the pollutant of Country 1 is transboundary, increasing the fraction of own output exported and when the pollutant of Country 2 is transboundary increasing the fraction of foreign output imported helps the shape of of EKC more.

3.7Appendix

Appendix A: Derivation of Model

A1: The case of Local Pollutants

When income in the two countries are homogeneous with different levels of technology of each country, the Social Utility Function that the social planner seeks to optimize is:

$V_1 = [\tau_1 y \frac{x_1}{y^{\alpha_1}} + (1-\tau_2) y \frac{x_2}{y^{\alpha_2}}] - \frac{\beta}{2}[\tau_1 y \frac{x_1}{y^{\alpha_1}} + (1-\tau_2) y \frac{x_2}{y^{\alpha_2}}]^2 - \frac{\rho}{2}(x_1)^2$

The FOC with respect to $x_1$gives:

$\tau_1 y \frac{1}{y^{\alpha_1}} - \beta[\tau_1 y \frac{x_1}{y^{\alpha_1}} + (1-\tau_2) y \frac{x_2}{y^{\alpha_2}}]\tau_1 y^{1-\alpha_1} - \rho(x_1) = 0$

$\Rightarrow x_1 = \frac{\tau_1 y^{1-\alpha_1}}{\beta\tau_1^2 y^{2-2\alpha_1}+\rho} - ((\frac{\beta\tau_1(1-\tau_2)y^{2-\alpha_1-\alpha_2}}{\beta\tau_1^2 y^{2-2\alpha_1}+\rho}))x_2 \qquad (1)$

The social utility function facing the social planner of Country 2 is:

$V_2 = [(1-\tau_1) y \frac{x_1}{y^{\alpha_1}} + \tau_2 y \frac{x_2}{y^{\alpha_2}}] - \frac{\beta}{2}[(1-\tau_1) y \frac{x_1}{y^{\alpha_1}} + \tau_2 y \frac{x_2}{y^{\alpha_2}}]^2 - \frac{\rho}{2}(x_2)^2$

The FOC with respect to x_2 gives:

$\tau_2 y \frac{1}{y^{\alpha_2}} - \beta[(1-\tau_1) y \frac{x_1}{y^{\alpha_1}} + \tau_2 y \frac{x_2}{y^{\alpha_2}}]\tau_2 y^{1-\alpha_2} - \rho(x_2) = 0$

$\Rightarrow x_2 = \frac{\tau_2 y^{1-\alpha_2}}{\beta\tau_2^2 y^{2-2\alpha_2}+\rho} - ((\frac{\beta\tau_2(1-\tau_1)y^{2-\alpha_1-\alpha_2}}{\beta\tau_2^2 y^{2-2\alpha_2}+\rho}))x_1 \qquad (2)$

Plugging in the optimal value of x_2 from equation (2) into equation (1) gives the optimal pollution function facing Country 1:

$x^*_{1local} = \frac{\rho\tau_1 y^{1-\alpha_1}+\beta\tau_1\tau_2(2\tau_2-1)y^{3-\alpha_1-2\alpha_2}}{\rho\beta\tau_1^2 y^{2-2\alpha_1}+\rho\beta\tau_2^2 y^{2-2\alpha_2}+\rho^2+\beta(\tau_1^2\tau_2+\tau_1\tau_2^2-\tau_1\tau_2)y^{4-2\alpha_1-2\alpha_2}}$

A2. The case of Transboundary or Global Pollutants

When income in the two countries are homogeneous with different levels of technology of each country, the Social Utility Function that the social planner seeks to optimize is:

$V_1 = [\tau_1 y \frac{x_1}{y^{\alpha_1}} + (1-\tau_2) y \frac{x_2}{y^{\alpha_2}}] - \frac{\beta}{2}[\tau_1 y \frac{x_1}{y^{\alpha_1}} + (1-\tau_2) y \frac{x_2}{y^{\alpha_2}}]^2 - \frac{\rho}{2}(x_1 + l_2 x_2)^2$

The FOC with respect to $x_1$gives:

$\tau_1 y \frac{1}{y^{\alpha_1}} - \beta[\tau_1 y \frac{x_1}{y^{\alpha_1}} + (1-\tau_2) y \frac{x_2}{y^{\alpha_2}}]\tau_1 y^{1-\alpha_1} - \rho(x_1 + l_2 x_2) = 0$

$\Rightarrow x_1 = \frac{\tau_1 y^{1-\alpha_1}}{\beta\tau_1^2 y^{2-2\alpha_1}+\rho} - ((\frac{\beta\tau_1(1-\tau_2)y^{2-\alpha_1-\alpha_2}+\rho l_2}{\beta\tau_1^2 y^{2-2\alpha_1}+\rho}))x_2 \qquad (3)$

The social utility function facing the social planner of Country 2 is:

$V_2 = [(1-\tau_1) y \frac{x_1}{y^{\alpha_1}} + \tau_2 y \frac{x_2}{y^{\alpha_2}}] - \frac{\beta}{2}[(1-\tau_1) y \frac{x_1}{y^{\alpha_1}} + \tau_2 y \frac{x_2}{y^{\alpha_2}}]^2 - \frac{\rho}{2}(l_1 x_1 + x_2)^2$

The FOC with respect to x_2 gives:

$\tau_2 y \frac{1}{y^{\alpha_2}} - \beta[(1-\tau_1) y \frac{x_1}{y^{\alpha_1}} + \tau_2 y \frac{x_2}{y^{\alpha_2}}]\tau_2 y^{1-\alpha_2} - \rho(l_1 x_1 + x_2) = 0$

$\Rightarrow x_2 = \frac{\tau_2 y^{1-\alpha_2}}{\beta\tau_2^2 y^{2-2\alpha_2}+\rho} - ((\frac{\beta\tau_2(1-\tau_1)y^{2-\alpha_1-\alpha_2}+\rho l_1}{\beta\tau_2^2 y^{2-2\alpha_2}+\rho}))x_1 \qquad (4)$

Plugging in the optimal value of x_2 from equation (4) into equation (3) gives the optimal pollution function facing Country 1:

$x^*_1 =$

$$\frac{\rho\tau_1 y^{1-\alpha_1}-\rho l_2\tau_2 y^{1-\alpha_2}+\beta\tau_1\tau_2(2\tau_2-1)y^{3-\alpha_1-2\alpha_2}}{\rho\beta\tau_1^2 y^{2-2\alpha_1}+\rho\beta\tau_2^2 y^{2-2\alpha_2}+\rho^2(1-l_1 l_2)+\beta(\tau_1^2\tau_2+\tau_1\tau_2^2-\tau_1\tau_2)y^{4-2\alpha_1-2\alpha_2}-\rho\beta(l_1\tau_1(1-\tau_2)+l_2\tau_2(1-\tau_1))y^{2-\alpha_1-\alpha_2}}$$

Appendix B: Proof of Propositions

B1. Proof of Proposition 1

The first proposition explores the scope of pollution substitution. In other words it looks at the impact of level of pollution of Country 2 on the level of pollution of Country 1 when countries with different levels of abatement technology engage in trade. To see this I take the derivative of optimal pollution in equation 3 with respect to x_2 and get:

$$\frac{\partial x_1}{\partial x_2} = -\frac{\beta\tau_1(1-\tau_2)y^{2-\alpha_1-\alpha_2}}{\beta\tau_1^2 y^{2-2\alpha_1}+\rho} - \frac{\rho l_2}{\beta\tau_1^2 y^{2-2\alpha_1}+\rho} < 0 \qquad (5)$$

The negative sign on the derivative above suggests dependence in optimal pollution decision among countries.

Proposition 1 states that when the technology of the foreign country is better the domestic country should engage in import, and when the technology level of the home country is better, the domestic country should engage in export. To see the impact of foreign country on the scope of pollution substitution I take the derivative of equation (5) with respect to α_2 and get:

$$\frac{\partial^2 x_1}{\partial x_2 \partial \alpha_2} = \frac{(1-\tau_2)\tau_1 y^{2-\alpha_1-\alpha_2}\log(y)}{\beta\tau_1^2 y^{2-2\alpha_1}+\rho} > 0$$

The derivative is positive, meaning the higher the the value of α_2 the weaker will be scope of pollution substitution. In other words, the better the level of foreign technology, the lower the value of α_2 the stronger will be the scope of pollution substitution. Note that the derivative above will have a positive value only for a positive value of $(1-\tau_2)$.Thus unless Country 1 engages in imports, the technology level of the foreign country will not have an impact on the optimal pollution of the home country. I therefore conclude that when the technology of the foreign country is better, Country 1 should engage in import in order to make the scope for pollution substitution stronger.

Next I investigate the impact of domestic technology on the scope of pollution substitution. For that, I take the derivative of equation 5 with respect to α_1. This gives:

$$\frac{\partial^2 x_1}{\partial x_2 \partial \alpha_1} = -\frac{2\beta(1-\tau_2)\tau_1^3 y^{4-3\alpha_1-\alpha_2}\log(y)}{(\beta\tau_1^2 y^{2-2\alpha_1}+\rho)^2} + \frac{(1-\tau_2)\tau_1 y^{2-\alpha_1-\alpha_2}\log(y)}{\beta\tau_1^2 y^{2-2\alpha_1}+\rho} - \frac{2l_2\beta\rho y^{2-2\alpha_1}\log(y)}{(\beta\tau_1^2 y^{2-2\alpha_1}+\rho)^2}$$

The sign attached to the derivative above is uncertain. If I set the derivative to negative, it will mean that the higher the value of α_1, the lower the level of domestic technology, the stronger will be the scope of pollution substitution.

$$-\frac{2\beta(1-\tau_2)\tau_1^3 y^{4-3\alpha_1-\alpha_2}\log(y)}{(\beta\tau_1^2 y^{2-2\alpha_1}+\rho)^2} + \frac{(1-\tau_2)\tau_1 y^{2-\alpha_1-\alpha_2}\log(y)}{\beta\tau_1^2 y^{2-2\alpha_1}+\rho} \leq 0 \Rightarrow \tau_1 > \left(\frac{\rho}{y^{2-2\alpha_1}}\right)^{1/2} .$$

Thus for higher values of α_1, the negativity of the derivative will be guaranteed by keeping low levels of exports. When the domestic technology is more advanced, one can raise the level of exports and still maintain the negativity of the derivative.

B2. Proof of Proposition 2

The optimal pollution function when both countries emit pollutants with externality is:

$$x_1^* = \frac{\rho\tau_1 y^{1-\alpha_1} - \rho l_2\tau_2 y^{1-\alpha_2} + \beta\tau_1\tau_2(2\tau_1-1)y^{3-\alpha_1-2\alpha_2}}{\rho\beta\tau_1^2 y^{2-2\alpha_1} + \rho\beta\tau_2^2 y^{2-2\alpha_2} + \rho^2(1-l_1 l_2) + \beta(\tau_1^2\tau_2+\tau_1\tau_2^2-\tau_1\tau_2)y^{4-2\alpha_1-2\alpha_2} - \rho\beta(l_1\tau_1(1-\tau_2)+l_2\tau_2(1-\tau_1))y^{2-\alpha_1-\alpha_2}} \equiv \frac{A}{B}$$

$$\frac{\partial x_1^*}{\partial l_1} = \frac{A(\rho^2 l_2 + \rho\beta\tau_1(1-\tau_2)y^{2-\alpha_1-\alpha_2})}{B^2} \geq 0$$

Since the optimal level of pollution is positive, it must mean that $A > 0$ and $B > 0$. If the pollution externality of Country 2 is 0 and Country 1 does not engage in any import, the term in the bracket will be equal to 0. Otherwise it will be positive. I therefore conclude that the higher the level of pollution externality of Country 1 the higher will be the optimal level of pollution.

$$\frac{\partial x_1^*}{\partial l_2} = \frac{-B(\rho\tau_2 y^{1-\alpha_2}) + A(\rho^2 l_1 + \rho\beta\tau_2(1-\tau_1)y^{2-\alpha_1-\alpha_2})}{B^2}$$

$\Rightarrow$

$$\frac{\beta\rho\tau_2^3 t(1-2\tau_1)y^{5-2\alpha_1-3\alpha_2} + \rho^2\beta\tau_1\tau_2(1-2\tau_1)y^{3-2\alpha_1-2\alpha_2} - \rho^2\tau_2^2 y^{1-\alpha_2}(\beta\tau_2 y^{2-2\alpha_2}-\rho) + \rho^2\beta l_1\tau_1\tau_2((1-\tau_1)y^{3-\alpha_1-2\alpha_2}}{B^2}$$

The derivative above will be positive so long as $\tau_1 \geq 1/2$. Therefore, so long as Country 1 retains half or more of its own output, the higher the level of pollution externality of Country 2 the higher will be the optimal level of pollution.

I therefore conclude that the higher the level of pollution externality of both countries, the higher will be the optimal level of pollution.

B3. Conditions under which the second derivative of optimal pollution with respect to income will be negative.

The optimal pollution function is:

$$x_1^* = \frac{\rho\tau_1 y^{1-\alpha_1} - \rho l_2 \tau_2 y^{1-\alpha_2} + \beta\tau_1\tau_2(2\tau_1 - 1)y^{3-\alpha_1-2\alpha_2}}{\rho\beta\tau_1^2 y^{2-2\alpha_1} + \rho\beta\tau_2^2 y^{2-2\alpha_2} + \rho^2(1 - l_1 l_2) + \beta(\tau_1^2\tau_2 + \tau_1\tau_2^2 - \tau_1\tau_2)y^{4-2\alpha_1-2\alpha_2} - \rho\beta(l_1\tau_1(1-\tau_2) + l_2\tau_2(1-}$$

The second order derivative of optimal pollution with respect to income.

$$\frac{\partial^2 x}{\partial y^2} = \frac{(2(\beta\tau_1\tau_2(2\tau_2-1)y^{3-\alpha_1-2\alpha_2} + \rho\tau_1 y^{1-\alpha_1} - \rho\tau_2 l_2 y^{1-\alpha_2})\{.\}^2)}{B^3}$$

$$- \frac{(2(\beta(3-\alpha_1-2\alpha_2)\tau_1\tau_2(2\tau_2-1)y^{3-\alpha_1-2\alpha_2} + (1-\alpha_1)\rho\tau_1 y^{-\alpha_1} - (1-\alpha_2)\rho\tau_2 l_2 y^{1-\alpha_2})*}{B^2}$$

$$\{\beta(4 - 2\alpha_1 - 2\alpha_2)(\tau_1^2\tau_2 + \tau_1\tau_2^2 - \tau_1\tau_2)y^{3-2\alpha_1-2\alpha_2} + \beta\rho((2 - 2\alpha_2)\tau_2^2 y^{1-2\alpha_2}$$

$$+\beta\rho(2 - 2\alpha_1)\tau_1^2 y^{1-2\alpha_1} - \beta\rho(2 - \alpha_1 - \alpha_2)(\tau_2 l_2(1 - \tau_1) + \tau_1 l_1(1 - \tau_2))y^{1-\alpha_1-\alpha_2}\}$$

$$- \frac{(\beta\tau_1\tau_2(2\tau_2-1)y^{3-\alpha_1-2\alpha_2} + \rho\tau_1 y^{1-\alpha_1} - \rho\tau_2 l_2 y^{1-\alpha_2})*}{B^2}$$

$$\{\beta(3 - 2\alpha_1 - 2\alpha_2)(4 - 2\alpha_1 - 2\alpha_2)(\tau_1^2\tau_2 + \tau_1\tau_2^2 - \tau_1\tau_2)y^{2-2\alpha_1-2\alpha_2} +$$

$$\beta\rho(1 - 2\alpha_1)(2 - 2\alpha_2)\tau_2^2 y^{1-2\alpha_2} + \beta\rho(1 - 2\alpha_1)(2 - 2\alpha_1)\tau_1^2 y^{1-2\alpha_1}$$

$$-\beta\rho(1 - \alpha_1 - \alpha_2)(2 - \alpha_1 - \alpha_2)(\tau_2 l_2(1 - \tau_1) + \tau_1 l_1(1 - \tau_2))y^{1-\alpha_1-\alpha_2}\}$$

$$+ \frac{\beta(2-\alpha_1-2\alpha_2)(3-\alpha_1-2\alpha_2)\tau_1\tau_2(2\tau_2-1)y^{1-\alpha_1-2\alpha_2} - \rho\alpha_1(1-\alpha_1)\tau_1 y^{1-\alpha_1} + \rho\alpha_2(1-\alpha_2)\tau_2 l_2 y^{1-\alpha_2}}{B}$$

To give it a meaningful frame, I group the equation in the following way.

Let $[1] = \beta\tau_1\tau_2(2\tau_2 - 1)y^{3-\alpha_1-2\alpha_2} + \rho\tau_1 y^{1-\alpha_1} - \rho\tau_2 l_2 y^{1-\alpha_2}$,

$[2a] =$

$\beta(3-\alpha_1-2\alpha_2)\tau_1\tau_2(2\tau_2-1)y^{3-\alpha_1-2\alpha_2}+(1-\alpha_1)\rho\tau_1 y^{-\alpha_1}-(1-\alpha_2)\rho\tau_2 l_2 y^{1-\alpha_2}$,

$[2b] = \{\beta(4-2\alpha_1-2\alpha_2)(\tau_1^2\tau_2+\tau_1\tau_2^2-\tau_1\tau_2)y^{3-2\alpha_1-2\alpha_2}+\beta\rho((2-2\alpha_2)\tau_2^2 y^{1-2\alpha_2}$

$+\beta\rho(2-2\alpha_1)\tau_1^2 y^{1-2\alpha_1}-\beta\rho(2-\alpha_1-\alpha_2)(\tau_2 l_2(1-\tau_1)+\tau_1 l_1(1-\tau_2))y^{1-\alpha_1-\alpha_2}\}$,

$[3a] = \beta\tau_1\tau_2(2\tau_2-1)y^{3-\alpha_1-2\alpha_2}+\rho\tau_1 y^{1-\alpha_1}-\rho\tau_2 l_2 y^{1-\alpha_2}$,

$[3b] = \{\beta(3-2\alpha_1-2\alpha_2)(4-2\alpha_1-2\alpha_2)(\tau_1^2\tau_2+\tau_1\tau_2^2-\tau_1\tau_2)y^{2-2\alpha_1-2\alpha_2}+$

$\beta\rho(1-2\alpha_1)(2-2\alpha_2)\tau_2^2 y^{1-2\alpha_2}+\beta\rho(1-2\alpha_1)(2-2\alpha_1)\tau_1^2 y^{1-2\alpha_1}$

$-\beta\rho(1-\alpha_1-\alpha_2)(2-\alpha_1-\alpha_2)(\tau_2 l_2(1-\tau_1)+\tau_1 l_1(1-\tau_2))y^{1-\alpha_1-\alpha_2}\}$, and finally

$[4] = \beta(2-\alpha_1-2\alpha_2)(3-\alpha_1-2\alpha_2)\tau_1\tau_2(2\tau_2-1)y^{1-\alpha_1-2\alpha_2}-\rho\alpha_1(1-$

$\alpha_1)\tau_1 y^{1-\alpha_1}+\rho\alpha_2(1-\alpha_2)\tau_2 l_2 y^{1-\alpha_2}$

To write the derivative in its simplified form I can write it in the following manner.

$\frac{\partial^2 x_1^*}{\partial y^2} = \frac{2*[1](.)^2}{B^3} - \frac{2[2a][2b]}{B^2} - \frac{[3a][3b]}{B^2} + \frac{[4]}{B}$

It is already established that $B > 0$. If I can show that $[1]$, $[2a]$, $[2b]$, $[3a]$, $[3b]$, and $[4] < 0$, then the second order derivative of optimal pollution with respect to income will be negative.

I now list some sufficient conditions that ensure the negativity of the second order derivative.

Condition 1

$\beta\tau_1\tau_2(2\tau_2-1)y^{3-\alpha_1-2\alpha_2}+\rho\tau_1 y^{1-\alpha_1}-\rho\tau_2 l_2 y^{1-\alpha_2} < 0$

When $l_2 = 0 \Longrightarrow \beta\tau_1\tau_2(1-2\tau_2)y^{3-\alpha_1-2\alpha_2} > \rho\tau_1 y^{1-\alpha_1}$

This is possible as long as $\tau_2 < 1/2$. Lower the value of α_2 stronger will be the inequality.

When $l_2 > 0 \Longrightarrow \beta\tau_1\tau_2(2\tau_2-1)y^{3-\alpha_1-2\alpha_2}+\rho\tau_1 y^{1-\alpha_1} < \rho\tau_2 l_2 y^{1-\alpha_2}$

$\Longrightarrow < \tau_1 \frac{\rho\tau_2 l_2 y^{1-\alpha_2}}{y^{1-\alpha_1}(\beta\tau_2(2\tau_2-1)y^{2-2\alpha_2}+\rho)}$

Lower the value of $\alpha_1$smaller the value of RHS. τ_1 should be reduced to to maintain inequality. This implies technology of country 1 goes up, fraction exported should go up for the case of positive pollution externality.

——————————————x——————————————

Condition 2a

$\beta(3-\alpha_1-2\alpha_2)\tau_1\tau_2(2\tau_2-1)y^{3-\alpha_1-2\alpha_2}+(1-\alpha_1)\rho\tau_1 y^{-\alpha_1}-(1-\alpha_2)\rho\tau_2 l_2 y^{1-\alpha_2}<0$

Notice Condition 2a is the first derivative of Condition 1 with respect to y.

Thus under the same restrictions as Condition 1, Condition 2a will also hold.

When $l_2 = 0 \Longrightarrow \beta(3-\alpha_1-2\alpha_2)\tau_1\tau_2(1-2\tau_2)y^{2-\alpha_1-2\alpha_2} > (1-\alpha_1)\rho\tau_1 y^{-\alpha_1}$

Again, this is possible as long as $\tau_2 < 1/2$. Lower the value of α_2 stronger will be the inequality.

When $l_2 > 0 \Longrightarrow \beta(3-\alpha_1-2\alpha_2)\tau_1\tau_2(2\tau_2-1)y^{2-\alpha_1-2\alpha_2} + (1-\alpha_1)\rho\tau_1 y^{-\alpha_1} < (1-\alpha_2)\rho\tau_2 l_2 y^{-\alpha_2}$

$\Longrightarrow \tau_1 < \frac{(1-\alpha_2)\rho\tau_2 l_2 y^{-\alpha_2}}{y^{-\alpha_1}(\beta\tau_2(2\tau_2-1)(3-\alpha_1-2\alpha_2)y^{2-2\alpha_2}+(1-\alpha_1)\rho)}$

Again, lower the value of $\alpha_1$smaller will be the right hand side. In that case τ_1 has to be reduced to maintain inequality. When $\alpha_1$is lower, and there exists pollution externality, increasing the size of $(1-\tau_1)$ will make the inequality stronger.

——————————————x——————————————-

Condition 2b

$\{+\beta(4-2\alpha_1-2\alpha_2)(\tau_1^2\tau_2+\tau_1\tau_2^2-\tau_1\tau_2)y^{3-2\alpha_1-2\alpha_2}+\beta\rho((2-2\alpha_2)\tau_2^2 y^{1-2\alpha_2}+\beta\rho(2-2\alpha_1)\tau_1^2 y^{1-2\alpha_1} - \beta\rho(2-\alpha_1-\alpha_2)(\tau_2 l_2(1-\tau_1)+\tau_1 l_1(1-\tau_2))y^{1-\alpha_1-\alpha_2}\} < 0$

2bi:$l_1 = l_2 = 0$

$\beta(4-2\alpha_1-2\alpha_2)(-\tau_1^2\tau_2-\tau_1\tau_2^2+\tau_1\tau_2)y^{3-2\alpha_1-2\alpha_2} > \beta\rho((2-2\alpha_1)y^{1-2\alpha_1}(\tau_1^2+\tau_2^2)$

For the inequality to hold, $(1-\tau_2) > \tau_1$. The lower the value of α_2 the stronger the inequality.

When pollution is strictly local, importing a larger fraction from Country 2 will make the inequality stronger.

2bii:$l_1 > 0; l_2 = 0$

$\{\beta(4-2\alpha_1-2\alpha_2)(\tau_1^2\tau_2+\tau_1\tau_2^2-\tau_1\tau_2)y^{3-2\alpha_1-2\alpha_2}+\beta\rho((2-2\alpha_2)\tau_2^2y^{1-2\alpha_2}$

$+\beta\rho(2-2\alpha_1)\tau_1^2y^{1-2\alpha_1}-\beta\rho(2-\alpha_1-\alpha_2)(\tau_2l_2(1-\tau_1)+\tau_1l_1(1-\tau_2))y^{1-\alpha_1-\alpha_2}<0$

$\implies \beta(4-2\alpha_1-2\alpha_2)(-\tau_1^2\tau_2-\tau_1\tau_2^2+\tau_1\tau_2)y^{3-2\alpha_1-2\alpha_2}+\beta\rho(2-\alpha_1-$

$\alpha_2)(\tau_1l_1(1-\tau_2))y^{1-\alpha_1-\alpha_2}>\beta\rho((2-2\alpha_1)y^{1-2\alpha_1}(\tau_1^2+\tau_2^2)$

1. if $(1-\tau_2)=\tau_1$

$(2-\alpha_1-\alpha_2)(\tau_1l_1(1-\tau_2))y^{1-\alpha_1-\alpha_2}>((2-2\alpha_1)y^{1-2\alpha_1}(\tau_1^2+\tau_2^2)$ Inequality will hold stronger with lower value of α_2.

2. if $(1-\tau_2)<\tau_1 \implies$

$-\beta(4-2\alpha_1-2\alpha_2)(-\tau_1^2\tau_2-\tau_1\tau_2^2+\tau_1\tau_2)y^{3-2\alpha_1-2\alpha_2}+\beta\rho(2-\alpha_1-\alpha_2)(\tau_1l_1(1-\tau_2))y^{1-\alpha_1-\alpha_2}>\beta\rho((2-2\alpha_1)y^{1-2\alpha_1}(\tau_1^2+\tau_2^2)$

Inequality will become weaker. lower the value of α_2 weaker the inequality.

3. if $(1-\tau_2)>\tau_1 \implies$

$+\beta(4-2\alpha_1-2\alpha_2)(-\tau_1^2\tau_2-\tau_1\tau_2^2+\tau_1\tau_2)y^{3-2\alpha_1-2\alpha_2}+\beta\rho(2-\alpha_1-\alpha_2)(\tau_1l_1(1-\tau_2))y^{1-\alpha_1-\alpha_2}>\beta\rho((2-2\alpha_1)y^{1-2\alpha_1}(\tau_1^2+\tau_2^2)$

Inequality will become stronger. Lower the value of α_2 stronger will be the inequality.

4. Overall higher the level of pollution externality of country 1 stronger the inequality.

2biii:$l_1 = 0; l_2 > 0$

$\{\beta(4-2\alpha_1-2\alpha_2)(\tau_1^2\tau_2+\tau_1\tau_2^2-\tau_1\tau_2)y^{3-2\alpha_1-2\alpha_2}+\beta\rho((2-2\alpha_2)\tau_2^2y^{1-2\alpha_2}$

$+\beta\rho(2-2\alpha_1)\tau_1^2y^{1-2\alpha_1}-\beta\rho(2-\alpha_1-\alpha_2)(\tau_2l_2(1-\tau_1)+\tau_1l_1(1-\tau_2))y^{1-\alpha_1-\alpha_2}<0$

$\implies \beta(4-2\alpha_1-2\alpha_2)(-\tau_1^2\tau_2-\tau_1\tau_2^2+\tau_1\tau_2)y^{3-2\alpha_1-2\alpha_2}+\beta\rho(2-\alpha_1-$

$\alpha_2)(\tau_2 l_2(1-\tau_1))y^{1-\alpha_1-\alpha_2} > \beta\rho((2-2\alpha_1)y^{1-2\alpha_1}(\tau_1^2+\tau_2^2)$

1. Same as 2bii.

2. Bigger the value of $(1-\tau_1)$ stronger the inequality in the case where pollution externality exists for Country2.

2biv: $l_1 > 0; l_2 > 0$

$\beta(4-2\alpha_1-2\alpha_2)(\tau_1^2\tau_2+\tau_1\tau_2^2-\tau_1\tau_2)y^{3-2\alpha_1-2\alpha_2} + \beta\rho((2-2\alpha_1)\tau_2^2 y^{1-2\alpha_1} +$

$\beta\rho(2-2\alpha_1)\tau_1^2 y^{1-2\alpha_1} - \beta\rho(2-\alpha_1-\alpha_2)(\tau_2 l_2(1-\tau_1)+\tau_1 l_1(1-\tau_2))y^{1-\alpha_1-\alpha_2} < 0$

$\implies \beta(4-2\alpha_1-2\alpha_2)(-\tau_1^2\tau_2-\tau_1\tau_2^2+\tau_1\tau_2)y^{3-2\alpha_1-2\alpha_2} + \beta\rho(2-\alpha_1-$

$\alpha_2)(\tau_1 l_1(1-\tau_2)+\tau_2 l_2(1-\tau_1))y^{1-\alpha_1-\alpha_2} > \beta\rho((2-2\alpha_1)y^{1-2\alpha_1}(\tau_1^2+\tau_2^2)$

1. Same as 2bii.

2. When country 1 has pollution externality, importing helps the inequality and when country2 has externality exporting helps the inequality.

x

Condition 3a

$\beta\tau_1\tau_2(2\tau_2-1)y^{3-\alpha_1-2\alpha_2} + \rho\tau_1 y^{1-\alpha_1} - \rho\tau_2 l_2 y^{1-\alpha_2} < 0$

Condition 3a is the same as Condition 1 and so it is negative under the same set of conditions.

x

Condition 3b

$\{\beta(3-2\alpha_1-2\alpha_2)(4-2\alpha_1-2\alpha_2)(\tau_1^2\tau_2+\tau_1\tau_2^2-\tau_1\tau_2)y^{2-2\alpha_1-2\alpha_2}+$

$\beta\rho(1-2\alpha_1)(2-2\alpha_2)\tau_2^2 y^{1-2\alpha_2} + \beta\rho(1-2\alpha_1)(2-2\alpha_1)\tau_1^2 y^{1-2\alpha_1}$

$-\beta\rho(1-\alpha_1-\alpha_2)(2-\alpha_1-\alpha_2)(\tau_2 l_2(1-\tau_1)+\tau_1 l_1(1-\tau_2))y^{1-\alpha_1-\alpha_2}\} < 0$

Notice that Condition 3b is the first order derivative of Condition 2b with respect to y. Therefore the conditions of here will also be the same.

One added condition: $\alpha_1 < 0.5$

———————————x———————————

Condition 4

$\beta(2-\alpha_1-2\alpha_2)(3-\alpha_1-2\alpha_2)\tau_1\tau_2(2\tau_2-1)y^{1-\alpha_1-2\alpha_2}-\rho\alpha_1(1-\alpha_1)\tau_1 y^{1-\alpha_1}+\rho\alpha_2(1-\alpha_2)\tau_2 l_2 y^{1-\alpha_2}<0$

$\Longrightarrow \beta(2-\alpha_1-2\alpha_2)(3-\alpha_1-2\alpha_2)\tau_1\tau_2(2\tau_2-1)y^{1-\alpha_1-2\alpha_2}+\alpha_2(1-\alpha_2)\rho\tau_2 l_2 y^{1-\alpha_2}<\alpha_1(1-\alpha_1)\rho\tau_1 y^{1-\alpha_1}$

1. If $\tau_2=1/2, \Longrightarrow \tau_2\alpha_2(1-\alpha_2)\rho l_2 y^{1-\alpha_2}<\alpha_1(1-\alpha_1)\rho\tau_1 y^{1-\alpha_1}$

If there exists pollution externality in Country2 then inequality will hold for $\alpha_2<\alpha_1$ or $\tau_2 l_2<\tau_1$.

2. If $\tau_2>1/2 \Longrightarrow \beta(2-\alpha_1-2\alpha_2)(3-\alpha_1-2\alpha_2)\tau_1\tau_2(2\tau_2-1)y^{1-\alpha_1-2\alpha_2}+\alpha_2(1-\alpha_2)\rho\tau_2 l_2 y^{1-\alpha_2}<\alpha_1(1-\alpha_1)\rho\tau_1 y^{1-\alpha_1}$

Inequality will be weak. It will be stronger for smaller values of α_2.

3. if $\tau_2<1/2 \Longrightarrow -\beta(2-\alpha_1-2\alpha_2)(3-\alpha_1-2\alpha_2)\tau_1\tau_2(2\tau_2-1)y^{1-\alpha_1-2\alpha_2}+\alpha_2(1-\alpha_2)\rho\tau_2 l_2 y^{1-\alpha_2}<\alpha_1(1-\alpha_1)\rho\tau_1 y^{1-\alpha_1}$

Inequality is stronger.

Again when l_2 is positive increasing $(1-\tau_2)$ helps the inequality.

Bibliography

1. Andreoni, J., Levinson, A., 2001. The simple analytics of the Environmnetal Kuznets Curve. *Journal of Public Economics,* 80 (2),269-286.

2. Amigues, J., Lafforgue, G., Moreaux, M., 2014. Optimal timing for CCS policies under decreasing returns to scale. Working paper TSE 529.

3. Antweiler,W., Copeland, B.R.,Taylor, M.S.,2001.Is free trade good for the environment? *American Economic Review,* 91 (4),877-908.

4. Apergis,N., 2016. Environmental Kuznets curves: New evidence on both panel and country-level CO_2 emissions. *Energy Economics, Elsevier,* 54(C), 263 - 271.

5. Arrow,K., Bolin,B., Costanza,R., Folke, C., Holling, C.S., Janson, B., Levin,S., Maler,K.,Perrings, C.,Pimental, D., 1995. Economic growth,carrying capacity, and the envrironment. *Science, 15,* 91-95.

6. Beckerman, W., 1992. Economic growth and the environment: Whose growth? whose environment? *World Development,* 20, 481-496.

7. Bilgili, F., Kocak, E., Bulut, U., 2016. The dynamic impact of renewable energy consumption on CO_2 emissions: A revisited environmental Kuznets curve approach. *Renewable and Sustainable Energy Reviews,* 54 (C), 838 - 845.

8. Careddu, N., Medda, P., Sarritzu, C., Grasso, F., 2015. Particulate matter in fly-ash landfills: an abatement technology using anionic flocculant. *Journal of Cleaner Production,* 102, 477-484.

9. Copeland, B.R., Taylor, M.S.,1995. Trade and transboundary pollution. *American Economic Review,* 85, 716-737.

10. Dean,J.M.,2002. Does trade liberalization harm the environment? A new test. *Canadian Journal of Economics,* 35, 819-842.

11. Diamantoudi,F.,Filippiadis,E.,2012.

12. Egli,H., 2005. A new approach to pollution modelling in models of the environmental kuznets curve. Working Paper 05/39.

13. GeorgescuRoegen,N.,1971. The entropy law and the economic process. Harvard University Press, Cambridge, Massachusetts.

14. Gill, A.R., Viswanathan, K.K., Hassan, S., 2017. Is environmental Kuznetes curve still relevant? *International Journal of Energy Economics and Policy,* 7, 156-165.

15. Grossman,G.M.,Krueger, A.B.,1991. Environmental impacts of the North American Free Trade Agreement. NBER Working paper 3914.

16. Grossman, G.M., Krueger, A.B., 1993. Environmental impacts of the North American Free Trade Agreement. In: Garber, P. (Ed.), The U.S. - Mexico Free Trade Agreement. MIT Press, Cambridge, pp. 13-56.

17. Grossman, G.M., Krueger, A.B., 1995. Economic growth and the environment. *Quarterly Journal of Economics,* 110 (2), 353-377.

18. Holladay, S., 2008. Pollution from consumption and the trade and environmnet debate. JEL Codes: F18, Q5.

19. Jebli, M. B., Youssef, S. B., Ozturk, I., 2016. Testing environmental Kuznets curve hypothesis: The role of renewable and non-renewable energy consumption and trade in OECD countries. *Ecological Indicators.* 60, 824-831.

20. John, A., Pecchenino, R., 1994. An overlapping generations Model of Growth and the environment. *Economic Journal,* 104, 1393- 1410.

21. Jones, L., Manuelli, R.,1995. Growth and the effects of inflation. *Journal of Economic Dymanics and Control,* 19, 1405 - 1428.

22. Khanna, N., Plassman, F., 2006. Total factor productivity and the environmnetal Kuznets curve: A comment and some intution. Department of Economics Working Paper 0518.

23. Kuznets, S., 1955. Economic growth and income inequality. *American Economic Review,* 45, 1-28.

24. Lieb,C.M., 2002. The Environmental Kuznets Curve and satiation: A simple static model. *Environment and Development Economics,* 7, 429-448.

25. Lopez,R., 1994. The environment as a factor of production: the effects of economic growth and trade liberalization. *Journal of Environmental Economics and Management.* 27, 163-184.

26. Lopez, R., 1994. The environment as a factor of production: the effects of economic growth and trade liberalization. *Journal of Environmental Economics and management,* 27, 163- 184.

27. Magnani, E., 2000. The Environmental Kuznets Curve, environmental policy and income distribution. *Ecological Economics,* 32, 431-443.

28. Managi, S.,2006. Are there increasing returns to pollution abatement? Empirical analytics of the Environmental Kuznets Curve in pesticides. *Ecological Economics,* 58, 617-636.

29. Meadows, D.H., Meadows, D. L., Renders,J., BehrensIII, W.W., 1972. The limits to growth. A report on THE CLUB OF ROME'S project on the Predicament of mankind. Universe.

30. Panayotou, T., 1993. Empirical tests and policy analysis of environmental degradation at different stages of economic development, ILO, Technology and Employment Programme, Geneva.

31. Pfaff, A.S.P., Chaudhuri, S., Nye, H.L.M., 2004. Household Production and Environmental Kuznets Curves. Examining the desirability and feasibility of substitution. *Environmental and Resource Economics,* 27, 187-200.

32. Plassmann, F., Khanna, N., 2006. A note on 'the simple analytics of the Environmnetal Kuznets Curve'. *Environmental and Developmental Economics,* 11, 697-707.

33. Selden, T., Song, D., 1994. Environmental quality and development: is there a Kuznets Curve for air pollution emissions? *Journal of Environmental Economics,* 6, 359-369.

34. Smulders, S., Bretschger, L., 2000. Explaining environmental Kuznets curves: How pollution induces policy and new technologies. JEL Codes: O41,Q20.

35. Stokey,N.L., 1998. Are there limits to growth? *International Economic Review,* 39 (1), 1-31.

36. Suri, V., Chapman, D.,1998. Economic growth, trade and the environment: implications for the environmental Kuznets curve. *Ecological Economics,* 25, 195-208.

37. Stern, D.I., 2004. The rise and fall of the environmental Kuznets curve. *World Development* , 34, 1419 - 1439.

38. World Bank, 1992. World Development Report. Oxford Univ. Press, New York.