The Effects of Object Familiarity on Fourth-Graders' Performance on Equal Sharing Fractions Problems

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Abstract

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A study was conducted that compared students' solutions to word problems that differed by the degree to which they were grounded in realistic settings. In a repeated measures design, fourth-graders (*N*=115) solved three types of equal sharing problems: grounded problems, containing familiar objects, such as brownies; semi-grounded problems with less familiar objects, such as lengths of rope measured in centimeters; and idealized problems, containing non-words (e.g., "porams"). No difference was observed between grounded and idealized problems, but both problem types yielded superior performance relative to semi-grounded problems. Prior knowledge of fractions and multiplicative structures was associated with greater performance overall. One implication is that only partial familiarity with problem contexts may activate irrelevant schemas and hinder problem-solving performance.

Chapter One: Statement of the Problem

Fractions are a notoriously difficult concept for students to understand from their introduction in the early years through high school (Empson & Levi, 2011; Gupta & Wilkerson, 2015). Part of the problem is that fractions require students to understand the relationships between numbers in a variety of different ways (Gupta & Wilkerson, 2015). For example, half of a brownie is a very different type of quantity than half the students in a classroom, yet the mathematical relationship between the two parts (i.e., one is half of two) is the same. Although challenging for many students, studies have shown the concepts can be introduced to students in the early grades (Gupta & Wilkerson, 2015).

Word problems are used frequently in classrooms to increase students' mathematical literacy (Lee, 2012), including their knowledge of fractions. However, the extent to which word problems extend students' thinking beyond the classroom is often questioned (Verschaffel, De Corte & Lasure, 1994). Word problems often provide a tenuous link between the classroom and the real world, as most students fail to consider the real-world elements of word problems; instead, they view them as mathematics puzzles to be solved with standard algorithms and very little practical knowledge (Verschaffel et al., 1994). Furthermore, Lee's study (2012) showed that not all word problems are equally accessible to students. While they are intended to serve as a link between formal structure and the mathematics grounded in real-world experience, both the structure of a word problem and the type of story chosen can alter students' ability to solve it. This encouraged me to look at the subtle nuances introduced in fractions word problems to discover what makes some problems easier than others for students to solve.

Although the research is far from definitive, much of the literature in elementary mathematics education suggests that students learn more when they are familiar with the ideas discussed in the classroom. Wason and Shapiro's (1971) landmark study showed that abstract items were more difficult for participants to comprehend than items that came from their everyday world. This suggests that familiarity with subject matter generates better solution strategies. Walkington, Petrosino, and Sherman (2013) argued that familiarity generates interest, and greater interest levels often lead to understanding. Similarly, Belenky and Shalk (2014) concluded from their review of the literature that real-world knowledge provides context and familiarity that can make a concept easier to learn. Intuitively, these theories make sense. If a student is expected to share cookies with her siblings every afternoon after school, and then is asked in class to imagine sharing thirteen cookies between four people, she should be able to share those thirteen cookies with relative ease because it is a concept she already understands through practical experience.

While familiarity may make a concept easier to learn, some theorists warn that familiarity may make transfer of knowledge more difficult (Belenky & Shalk, 2014). For this reason, some researchers see familiarity as limiting and instead advocate for the use of abstract, or formal, objects and symbols in the classroom (Goldstone & Sakamoto, 2003). They argue that using abstract concepts ensures the learning can be applied more widely and is understood at a conceptually deeper level, rather than merely superficially.

The objective of this study was to discover if students' level of familiarity with the objects in equal sharing word problems, problems involving fractional quantities, affected their solution strategies. Some research suggests that if a student is familiar with an object, he or she should find it easier to share equally. Other research suggests that abstract objects help students understand the mathematics behind a problem. However, it is important to consider prior knowledge when considering the role familiarity plays in generating solution strategies. Studies like Wason & Shapiro's (1971), which found familiarity benefited solution strategies, did not take prior knowledge into account when assessing the benefits of familiarity. Later studies that took prior knowledge into account when examining the effects of familiarity levels (e.g., Koedinger, Alibali, & Nathan, 2008) discovered that students with high prior knowledge benefited from unfamiliar problem types, whereas students with low prior knowledge benefited from familiar problem types. This variance based on prior knowledge serves to weaken the effects of familiarity by itself.

To my knowledge, none of the literature concerning equal sharing strategies considers the students' familiarity level with the object being discussed. This familiarity level may have an impact on students' understanding of the problem and thus impact the solution strategies they choose to use. If this hypothesis is supported, then greater attention to the objects chosen for equal sharing word problems is necessary in the future.

Chapter Two: Literature Review

Children's Fractions Knowledge in the Context of Equal Sharing

Fractions are a difficult subject for students to master (Tian & Siegler, 2017). Students who experience difficulties in mathematics struggle with many different aspects of fractions, including solving word problems that include fractions in the questions and answers (Tian & Siegler, 2017). Word problems are a popular tool used in mathematics classrooms to encourage strategic thinking, adaptive reasoning, and conceptual understanding (Dayal & Chandra, 2016), and are often used when teaching fractions in elementary school. One particular type that involves fractions is an equal sharing problem. Equal sharing problems involve allocating a number of items completely and equally to groups, usually people (e.g., four students want to share 14 cookies so that everyone gets the same amount with no leftovers. How many cookies will each child get?). These types of problems encourage students to make sense of mathematics because they involve items and actions that are a part of their everyday life (Carpenter, Fennema, Franke, Levi, & Empson, 2015). Therefore, fraction word problems that contain familiar items might assist students by making fraction content more accessible to them.

Using a real-world context appears to be especially beneficial to students with low prior mathematics knowledge, as it provides an entry point into mathematics that captures their interest (Walkington et al., 2013) and makes mathematics relatable (Koedinger, Alibali, & Nathan, 2008; Yang, Lai, Yao & Huang, 2014). A compelling argument for grounding fraction word problems in familiar, real-world experience is made by T. Carpenter in his introduction to Empson and Levi's (2011) book on fractions. He states that "students whose skills are perceived as deficient can actually be more successful extending the conceptual knowledge they do have than learning isolated skills" (Empson & Levi, 2011, p. xiii). Empson and Levi illustrated through example how students of a wide range of race, age, socio-economic status, and ability can solve equal sharing problems. A qualitative study with first-grade students (Empson, 1999) showed that "equal sharing tasks facilitated children's use of informal partitioning knowledge to think about fractions in mathematically viable ways" (Empson, 1999, p. 330).

To strengthen this argument, Empson (2003) conducted a qualitative study that took an in-depth look at two low-achieving students' progress in learning fractions in a classroom where students shared their intuitive strategies. In comparison to their classmates, the two students in the study knew the least about fractions both when the initial study (i.e., Empson, 1999) began and when it was finished. Empson (2003) investigated the learning of these two students in more depth to determine whether they had made *some* progress in their understanding of fractions after the study. She found that both students had a greater conceptual understanding of fractions, especially with respect to their understanding of repeated halving, after instruction. It is perhaps noteworthy these students achieved their first successes with questions involving repeated halving of highly familiar items, such as pancakes and cookies. Empson's study did not focus specifically on the level of familiarity the students had with the items they halved, but, in both cases, the fact the problems involved highly familiar items may have contributed to their success.

Although fractions are a difficult concept for students to master, introducing them through an equal sharing context is helpful because they can draw on their prior knowledge. Placing fractions in a real-world context makes them accessible for students with low prior mathematics knowledge because it gives them the opportunity to use informal solution strategies. For example, students asked to share 10 brownies between 4 students can arrive at a correct answer using informal solution strategies such as distribution and repeated halving, whereas such strategies are often not as readily apparent if the student is asked to compute $10 \div 4$. Informal strategies provide students access to fractions understanding, and with the development of conceptual knowledge, these introductory solution strategies should evolve into more abstract strategies over time.

Development of Children's Strategies for Solving Fractions Problems

Empson and Levi (2011) contended that to teach fractions through understanding, one must begin with students' intuitive understanding of sharing. Empson, Junk, Dominguez, and Turner (2005) studied the strategies that students in first-, third-, fourth-, and fifth-grades used to solve equal sharing problems. The fourth-grade students were asked to solve three equal sharing problems. One problem had 10 children sharing 3 liters of soda, one had 6 children sharing 4 candy bars, and the final one had 12 children sharing 9 cookies. Problems were read out loud to students and they could choose from concrete materials, paper and pencil, or no materials to solve the problems. They were then asked to explain their solution processes.

The authors investigated how students coordinated the information given in the problem, namely the number of people sharing and the items they shared. The strategies were then placed in three developmental categories. The first category contained strategies with no coordination between the number of people sharing and the items being shared. Sometimes the students would leave the sharing incomplete. For example, if eight children shared 18 cookies, they would answer that each student would receive two cookies, but two would be leftover. More frequently, students would share unequally – that is, if eight children shared two cookies, the student would cut the first cookie in two and the second cookie in six pieces and then announce that each child would get a "piece."

Strategies falling into the second category showed a coordination of parts. Empson et al. (2005) used the term "Coordinating parts quantities" for strategies in this category where students partition equally and share the materials by giving them out one at a time. An example of this strategy occurred when students were instructed to share 4 candy bars between 6 people and they cut each candy bar in half and shared six of the halves. Then, they might cut the last two halves into thirds, giving each person one half and one third of a half each. This method was used frequently and did not require the use of multiplication facts. However, a student with greater understanding of the relationship between sharing and multiplication might solve the problem of sharing 8 pies between 12 people by splitting the first two pies into 6 pieces and the remaining 6 pies into halves for a total of 1 sixth and 1 half for each person. The student's understanding of the math fact 6 x 2 = 12 would support this strategy.

The third category was named "Coordinating ratio quantities" because students' strategies took intuitive notions of equivalent ratios into account. The younger students used trial and error (called a "progressive" ratio strategy) to solve 15 students sharing 3 pounds of clay. To illustrate, one student started by putting 3 people next to each pound of clay. Realizing that was not enough, she tried 4, and then 5 people, finally solving the problem by saying that 5 people share one pound.

Once Empson and her colleagues reviewed the data, it became clear that students used a variety of solution strategies of varying degrees of sophistication. Some of these strategies could be used by students with limited multiplication facts (Empson et al., 2005). This lends credence to the argument that students with varying prior knowledge in mathematics can themselves design a wide range of strategies to solve equal sharing problems. Although a strategy does not need to be sophisticated to be successful, students with greater prior knowledge are able to use more sophisticated, and often more efficient, strategies, which allows them to solve a wider range of problems.

Although Empson et al. (2005) were comprehensive in detailing the various methods students use to solve equal sharing problems and explaining why some strategies are more sophisticated than others, the authors did not clarify why the same student sometimes used different solution strategies. The authors noted that students sometimes changed their solution strategies, using different methods for different questions, but the explanation the authors provided for this was that students' strategies evolved as they became more familiar with equivalent fractions. This implies that strategy variation can, at least in part, be explained by prior knowledge. The authors also acknowledged that the questions were of varying levels of difficulty, but they based difficulty level simply on the numbers in the problems without considering the actual items being shared. I argue, therefore, that the level of familiarity with the items being shared may have also contributed to the observed variation in solution strategies. Candy bars and peanut butter cookies are, arguably, easier to visualize than liters of soda, because visualizing three liters of soda requires measurement knowledge that appears to be less intuitive to children (Foster & Osana, 2017).

The belief that measurement knowledge is less intuitive for students was observed by Foster and Osana (2017) in an earlier study. In this study, 36 fourth-grade students solved eight equal sharing problems with fractional remainders. Four problems were couched in familiar contexts (e.g., slicing pizza), two in semi-familiar contexts (e.g., measurement contexts), and two in unfamiliar contexts (e.g., "wogs"). Results indicated that students' performance on the problems with the measurement context was significantly lower than performance on both the familiar and unfamiliar problems. This suggests that greater levels of object familiarity did not necessarily lead to more sophisticated solution strategies. It appeared that students' measurement schema was not fully developed, thus making sophisticated solution strategies less likely with semi-familiar problems than either familiar or unfamiliar word problems.

In sum, children's solution strategies are not static. As students' conceptual understanding of fractions deepens, their strategies become more sophisticated. This helps them solve increasingly complex problems. However, difficulty level is not simply a question of the numbers used in a problem. Foster and Osana (2017) suggested that students' solution strategies are also affected by the level of familiarity they have with the item being shared.

Effects of Grounded and Idealized Contexts for Mathematics Problems

Evidently, students use a variety of strategies for solving fractions problems. Sometimes the same student uses different strategies depending on the nature of the problem itself. Although there are a variety of reasons a student might choose to use any one strategy, a possible reason is the extent to which the context of the mathematics problem is familiar, or grounded in a real-world context.

What exactly is a real-world mathematics problem? Several authors referred to real world problems as "grounded." For the purpose of this study, I used the definition provided by Koedinger et al. (2008), who defined grounded, or real-world, word problems as those that incorporate references to physical, real-world objects that are familiar to students. An example they provided is a mathematics story problem involving how much money a waiter makes in one day (six hours) when he earns $66.00 in tips and a grand total of $81.90. The authors argued that grounded word problems like these are often associated with everyday events and places, making them easy to relate to. Evidence shows that learning and problem solving are enhanced when problems are grounded in familiar objects and events with realistic settings, or imaginary settings that students are familiar with, such as fairy tales (e.g., De Bock, Deprez, Dooren, Rolens, & Verschaffel, 2011; Koedinger & Nathan, 2004). Scholars have suggested that students' familiarity with the elements in grounded word problems activates their prior knowledge, which, in turn, assists in generating appropriate solutions (Koedinger et al., 2008).

In contrast, problems set in abstract, or idealized, contexts can present more of a challenge because students have fewer existing knowledge representations to draw on (Fyfe, McNeil, Son, & Goldstone, 2014; Koedinger & Nathan, 2004; Weyns, Van Dooren, Dewolf, &

Verschaffel, 2016). For example, the corresponding abstract equivalent for the waiter problem above might be X*6 + 66 = 81.80 when expressed symbolically using algebraic terms. This abstract form is concise and easy to manipulate, but lacks referents to the real world, making it harder for students to visualize (Koedinger et al., 2008). Koedinger et al. (2008) noted that there are both costs and benefits of grounded problems. The context in which grounded problems are placed activate knowledge about the real-world from long term memory. In addition, the familiar contexts allow students to make sense of what is happening in the problem, which generates the use of appropriate operations consistently and reliably. Abstract problems, in contrast, alleviate cognitive load through working memory, speed, and conciseness, but if a student is not able to make sense of a problem initially, Koedinger et al. (2008) argued that there is relatively little benefit to abstract problems.

Familiarity level seems to exist on two opposite poles, with grounded problems on one pole and abstract problems on the other. These familiarity poles interact with students' prior knowledge: Those students with low prior knowledge benefit from grounded problems whereas students with high prior knowledge benefit from abstract problems (Koedinger et al., 2008; Walkington et al., 2013). Belenky and Schalk (2014) hypothesized, for example, that if fractions are couched in terms of slices of pizza, where actual photographs of pizza are shown, students may show greater performance on immediate learning tasks than they would with a slice of pizza represented by an abstract depiction of a circle with one quarter shaded. Although there are benefits to using abstract problems with students, many theorists believe it is beneficial to begin by grounding problems with familiar or real-world contexts.

Benefits of real-world problems. Koedinger and Nathan's (2004) study looked at students' answers and solution strategies when solving both real-world and symbolic problems. Their study used 247 high school participants, who answered eight questions, including four story problems (two result-unknown and two start-unknown), two word equations (one result-unknown and one start-unknown), and two symbolic equations (one result-unknown and one start-unknown). The start unknown questions were considered at a higher difficulty level than the result unknown problems.

The objective of the study was to investigate whether high school students have greater difficulty with grounded, story problems than with idealized, matched equations (structurally similar symbolic algebraic equations). The authors performed a solution strategy analysis and an error analysis to determine which strategies were most likely to lead to correct answers. For each strategy they also calculated the number of times a student attempted to answer a question using that strategy and the number of times a question was answered correctly using that strategy.

The results demonstrated that high school students' success rates (based on the number of correctly answered questions) in solving story problems were significantly higher than their success rate using matched equations. As indicated by their strategy analysis, the authors argued that the higher success rate on story problems occurred because story problems encouraged the use of a large number of solution strategies. Specifically, students were able to work backwards and use guess and check strategies with story problems, but not with symbolic equations, which they often did not even attempt to answer. Story problems then, provided students with an entry strategy and made them more willing to attempt the problems.

In addition, the authors' error analysis indicated that, at least initially, the symbolic notations of algebra led to mistakes because of the lack of meaning placed on symbols. Basically, students failed to understand what the symbols were asking them to do (Koedinger & Nathan, 2004). For example, in the question X*6 + 66 = 81.80, students often added 6 and 66, an error they did not make when the question was presented in story form because they understood the rules of the stories more comprehensively than the rules attached to the symbols. This study highlights two key reasons why real-world knowledge is beneficial for students when solving equal sharing problems: They encourage a greater number of strategies, and they minimize error that can result from algebraic language comprehension errors.

In a follow-up study, Koedinger et al. (2008) predicted that grounded representations would be advantageous for performance on simple problems (i.e., problems with one reference to the unknown quantity where working backward is a possible solution strategy), whereas abstract symbolic representations would support performance on complex problems (i.e., problems with two references to the unknown quantity where working backward is not a viable solution strategy). The study included 153 undergraduate students, 43 of which were enrolled in a basic algebra course and 110 in an intermediate level algebra course. The test contained problems couched in three representations: (a) equation form (standard algebraic notation, considered an unfamiliar form); (b) arithmetic story problem form (a familiar form where explicit arithmetic terms, like "subtracted" were used); and (c) everday story problem form (a familiar form where everyday verbs like "kept" were used). Each problem type had two difficulty levels. Three of the six problems were single referent, so considered easy in terms of problem complexity, and the other half were double referent, thus considered more difficult. Crossing problem type and difficulty level resulted in six test versions that were randomly assigned. The students' answers were coded as correct or incorrect. Those that were incorrect were further coded according to error type. The type of strategy used by each student was also coded.

The findings indicated that students were more likely to detect and correct symbolic manipulation errors they made on questions couched in familiar situations than those made on algebra problems. The authors speculated that this occurred because students could determine if their answers made sense, which is arguably harder to do using abstract symbols. As well, the authors suggested that the language of algebraic symbols was new to many of the students and still unfamiliar. They stated that symbolic language was, in many ways, like second language acquisition, and the students were not as familiar with the symbols as they were to the corresponding English phrases in the story problems. The comparison between symbolic language and second language acquisition made by Koedinger et al. (2005) suggests that it is especially important to ground learning in real-world items during the early stages of learning, as it makes new concepts feel familiar and therefore more manageable.

Another study that demonstrated the benefits of grounding learning in real-world items was Wason and Shapiro's (1971) landmark study showing that reasoning with abstract concepts is more difficult with everyday items. The goal of their study was to assess the effects of prior experience on a reasoning problem based on the rules of logic. Each of 24 undergraduates was shown four cards with either a letter or a number written on each card (D, K, 3, and 7) and was told that each card had a letter on one side and a number on the other. The participants were then asked which card to turn over to demonstrate that a specific rule was true (i.e., "every card that had a D on one side had a 3 on the other side"). The results indicated that few participants correctly chose the correct card to turn over. However, when the experiment was repeated with cards labelled with locations (e.g., "Leeds" and "Manchester") and modes of transportation (e.g., "car" and "train"), and the rule stated a fact about a journey made on four different days as indicated on the cards (e.g., every time I go to Manchester, I travel by car), the participants were statistically more likely to verify the statement correctly. This suggests that familiarity with subject matter generates better solution strategies.

Johnson-Laird, Legrenzi, and Legrenzi (1972) revised the Wason and Shapiro task and discovered that when the topic changed from transportation to the British postal system, British students outperformed American students, regardless of the fact that both groups were equally familiar with the concept of mailing letters with stamps on them. It seems the British students were more successful because of the specific prior knowledge about the British postal system they were able to draw upon. This suggests that for grounding to be most helpful, it should enable users to access relevant prior knowledge; otherwise, students may incorporate irrelevant or unrelated surface information that impedes their ability to find a solution to the problem (Belenky & Schalk, 2014).

Benefits of idealized contexts. Research suggests that as problems become more complex, idealized representations can become more advantageous (Koedinger et al., 2008). A key benefit of idealized contexts is that their concise form allows for quick processing and encoding. This makes them fast and efficient to work with and minimizes the demands placed on working memory (e.g., Koedinger et al., 2008, Osana, Blondin, Alibali, & Donovan, 2018). Kaminski, Sloutsky, and Heckler (2008) ran an experiment with undergraduate students hypothesizing that learning one example with minimal extraneous information would be more beneficial for transfer than learning from several grounded examples. The authors assigned 80 students to four groups in which they were taught a "commutative mathematical group of order three" (Kaminski, p. 454) by solving problems. One group was taught the concept by solving problems that contained a set of idealized symbols (i.e., three unrelated, abstract, geometric shapes). The other groups were given concrete, contextualized symbols, such as drawings of measuring cups and liquid, pizza slices, and tennis balls in a container. After the participants solved problems with their given representations, they were administered an immediate learning task and a transfer task in a novel context to assess which group was best able to transfer the concept they had learned. The findings showed that all four groups learned the initial task with no significant difference in learning scores or learning times, but solving the problems using the idealized symbols resulted in greater transfer of knowledge than each of the three concrete, contextualized groups.

While this experiment, along with others (see also Goldstone & Sakamoto, 2003), indicated that idealized contexts may support transfer, idealized representations did not help with initial learning. In another experiment, Kaminski et al. (2008) tested whether transfer scores would improve if undergraduate students were taught using two contextualized examples (i.e., the pizza slices and measuring cups) and specifically asked to compare them. The results from this experiment were bimodal - that is, 44% of students scored very well on a transfer task, with an average of 95%, and the rest scored very poorly, with an average of 51% on the transfer test. The authors speculated that prior knowledge may account for those bimodal findings, and suggested that perhaps explicit comparisons only helped higher learners. As my research question does not deal with transfer, however, I predict that any potential benefit of idealized contexts on immediate performance would be minimal in my study.

The Kaminski et al. (2008) study was met with fierce debate in research circles. De Bock et al. (2011), for example, questioned the validity of the study because the transfer task was set in an idealized context with no concrete referents. They argued that to make a valid comparison, the transfer tasks should have included some concrete referents, as they hypothesized that students who were initially given a concrete referent might have been successful transferring to another concrete referent. As such, De Bock et al. (2011) changed the design by introducing both concrete and abstract transfer tasks so they could measure transfer occurring from concrete to abstract, abstract to abstract, concrete to concrete, and abstract to concrete. The authors found that, similar to Kaminski et al. (2008), students who were initially given the abstract instantiation showed better results transferring this knowledge to a second abstract domain than the students who learned with concrete instantiations. However, the students who were asked to transfer their knowledge to a concrete domain performed better when their learning was also concrete and contextualized. This suggests that students transfer better using abstract examples *only* when they are transferring to another abstract example and concrete examples serve as a better transfer aid for concrete examples. This indicates a significant flaw in the Kaminski et al. (2008) study and diminishes the applicability of using abstract instantiations as supports for initial learning.

The study by Koedinger et al. (2008), discussed earlier in the literature review, showed a marked advantage to students given symbolic notation on more complex questions. This finding suggests that when problems become more difficult, symbolic notation becomes increasingly advantageous to use. However, the study used undergraduates who were solving double referent problems, which are known to be more complicated than single-referent problems because of the two unknowns they need to take into account. When these same students were given single referent problems, they performed better on real-world questions than with questions using symbolic notation. Understanding and using abstract, symbolic notation is an important objective for mathematics students but knowing when to introduce it is an important pedagogical consideration. Comparing introductory equal sharing fraction questions in grade four to undergraduate-level problem solving may be disingenuous because there is no evidence to indicate that the benefits of idealized representations in double referent problems would hold true for younger students with less prior knowledge.

Nevertheless, Goldstone and Sakamoto (2003) argued there were advantages to idealized representations for students with low prior knowledge. The objective of their fourth experiment (2003) was to see whether transfer was more likely to occur if realistic pictures (black ants with fruit) than if abstract ones (black dots representing ants and green blobs to represent food) were used. They wanted to know if making an image more realistic would make it easier for students to transfer the underlying conceptual structure to a novel task. Participants were 61 undergraduate students. They were placed in two groups. Twenty-nine students were given the idealized images and the rest were given the realistic images. The students were given 20 minutes to explore an initial simulation (with either realistic or abstract representations) and then were given a short multiple-choice quiz used as an initial learning assessment. Then they were given a transfer task in the context of a new simulation. Again, they were given 20 minutes to explore the second simulation and then they took a short multiple-choice quiz to assess transfer.

The results indicated that students using realistic representations scored significantly better on the initial learning assessment than the abstract group. However, if students were poor initial performers, they were able to transfer results better if they began with the idealized simulation. Although Goldstone and Sakamoto's (2003) results showed a benefit for abstract representations in terms of transferability for poor initial performers, when students were presented with realistic images, they outperformed the students who worked with abstract representations on immediate learning. Again, this indicates that the benefits of abstract objects lie mostly in their transferability.

Despite the fact that research indicates that abstract, unfamiliar items seem to help students most with transfer rather than initial learning, teachers often begin lessons by focusing on symbolic notations, which are unfamiliar and abstract. In his literature review, Nathan (2012) defined formalism as "specialized representations such as symbolic equations and diagrams with no inherent meaning except that which is established by convention" (p. 125). Nathan (2012) claimed that educators often place a higher value on formalism than on applied knowledge because they see symbolic reasoning in mathematics as "pure" and therefore a more "natural" way of introducing mathematics concepts to students. This claim is supported by a study conducted by Nathan and Koedinger (2000), who interviewed 105 elementary, middle, and high school teachers. These teachers, especially those in high school, held a "symbol-precedence" view. As indicated by their ranking of a set of mathematics problems, the teachers believed that story scenarios often distract and confuse students and they are unable to solve story problems unless they can first understand the symbolic notations that underpin the story problems (Nathan & Koedinger, 2000). Nathan argued that these teachers' beliefs failed to consider the key differences between someone mastering a discipline and a beginner taking his or her first steps toward understanding:

> Developing one's intellectual abilities exclusively on stripped-down formalisms without exposure to perceptually rich stimuli robs learners of opportunities to learn how to recognize deep structure and filter out irrelevancies. Learning that is steeped in the FF [Formalisms First] approach cannot develop that skill. (Nathan, 2012, p. 137)

Nathan recognized that familiarity and accessibility are key to creating a successful entry point into a new subject matter; generalization and transferability are steps taken only after initial comprehension is mastered.

In sum, the literature on grounded (i.e., real-world) and idealized (i.e., symbolic) contexts is divisive. There is a history of classroom teachers giving preference to teaching with abstract symbols (Nathan, 2012). This history is backed by studies showing that idealized contexts improve transferability (Goldstone & Sakamoto, 2003; Kaminski et al., 2008) and can help make complex problems more manageable (Koedinger et al., 2008). The concise form of idealized contexts allows students to work quickly and efficiently and it minimizes the demands placed on working memory (e.g., Koedinger et al., 2008; Osana et al., 2018). However, real-world contexts allow students greater ease of access to solutions and make them more willing to attempt to solve the presented problems, as they allow for more strategies to be used and often generate more meaningful solution strategies (Koedinger & Nathan, 2004; Wason & Shapiro, 1971). As well, at least initially, many students have a better understanding of the words in story problems than they do of the meaning of formal mathematical symbols (Koedinger et al., 2008), which helps to minimize error.

A middle ground between idealized and real-world contexts. Most of the research on the effects of external knowledge representations addresses the question, "under what conditions are abstract, or idealized, and grounded, or real-world, contexts beneficial for learning and transfer?" In asking this question, grounded and idealized contexts tend to be examined in their extremes and seen as adversaries on opposite sides of a spectrum. However, Fyfe et al. (2014) questioned this dichotomous relationship and instead advocated for abstract and concrete materials to be considered as placed on extreme ends of a continuum where optimal results are achieved by blending concrete and abstract contexts. They advocated an approach to instruction called "concreteness fading" that begins with concrete examples that gradually fade away, leading to more abstract examples. Fyfe et al.'s approach values both grounded and idealized contexts, but the authors argue that it is imperative to *begin* with grounded contexts. Fyfe et al. (2014) explained this sequence by noting that the benefits of abstract contexts are only apparent after students have a solid foundation, acquired through grounded, real-life experiences.

Fyfe et al. (2014) recognized that valuable learning opportunities occur in both concrete and abstract contexts, albeit in a specific sequence. However, this view still operates under the assumption that a context is either abstract or concrete. In their framework, one begins with a concrete context (here called "Point A") and ends with an abstract context ("Point C"). According to Fyfe et al., the middle ground is simply a place where the two different contexts exist simultaneously ("Point AC"). It fails to consider the possibility that an object can be neither entirely abstract nor entirely concrete resulting in a qualitatively different type of context ("Point B"). There is no known research on the nature and effect of a context or representation like "B," but using Fyfe et al.'s (2014) concept of concreteness fading, one might conjecture that an object that is neither entirely concrete nor entirely abstract would serve as a middle ground on a concrete-abstract spectrum. That is, representations that fall in the middle ground are potentially not as initially helpful as concrete concepts, but easier to grasp than entirely abstract ones.

Prior Knowledge and Context Effects

Empson et al. (2005) examined the methods students used to solve equal sharing problems involving fractions. They showed how important number-fact recall and multiplicative reasoning was for students to successfully solve a problem. The study revealed that students in the first-, third-, and fifth-grades used a variety of personally-constructed methods for solving equal sharing problems, including some highly sophisticated strategies, such as what was equivalent to looking for common denominators. Students who used these sophisticated strategies were more likely to have high recall scores on their multiplication facts. This indicates that students with greater prior knowledge and stronger multiplicative reasoning tend to use more sophisticated strategies for solving equal sharing problems: Clearly, a student's prior knowledge affects how he or she will respond to different types of problems.

One means of activating prior knowledge for problem solving is by presenting a subject that students are personally interested in. If students are interested in a subject, they will, presumably, have a high level of familiarity with the context and therefore be more willing to engage with the problem. Walkington et al. (2013) examined the relationship between prior knowledge and context effects with 74 students from a high school algebra class. Students were initially interviewed to discover their hobbies. After the interview, two problems about linear functions were personalized for each student to reflect their interests. As well, they were asked a third linear function problem that was general, rather than personalized to them. After answering the problems, the students were placed into a high-scoring group if they scored over 75% of their problems correctly; medium-scoring if they scored between 50 and 75%; and low-scoring if they solved less than 50% correctly. The findings showed that low-scoring and medium-scoring students were more successful on the problems that were personalized compared to high-scoring students who did significantly worse on the personalized problems. This lends credence to the argument that real-world problems are especially beneficial for low-achieving students, as greater familiarity in Walkington et al. (2013), established through context personalization, gives them more problem-solving support.

There is also evidence to suggest that prior knowledge is activated to varying degrees based on context effects. Braithwaite and Goldstone (2015) examined the effects that variation of superficially similar or varied problems had on transfer and whether prior knowledge moderated these effects. This was tested by training 131 undergraduate students how to solve "sampling with replacement" (SWR) mathematics story problems. SWR problems involve a situation where a number of selections are made from a specific set of options, such as choosing the total number of possible meal combinations for five friends when there are six meals to choose from, or determining how many different melodies can be played if there are four keys in the set and seven notes in the melody. The meal and melody questions above are superficially different, but have the same underlying mathematical structure. Following the training was a pretest, which contained four stories for each participant. In order to manipulate variation, one group received four questions in the same context (e.g., all meal problems) and the other group received questions from both contexts (e.g., both meal and melody problems).

Following the pretest, the authors assessed the students' prior knowledge by asking them if they had any experience with SWR problems prior to answering the four SWR pretest questions. They then proceeded to the posttest, which included five problems, three of which were transfer tasks. The findings indicated that participants with greater prior SWR experience improved more on the posttest in the varied condition than in the similar condition, and students with less prior knowledge showed the opposite trend. One explanation provided by authors for these results is that students with less prior knowledge have a tendency to focus strictly on surface similarities, which are often not helpful in understanding the mathematical structure of the problem, whereas students with higher prior knowledge were aided by variety because it enabled them to notice patterns in underlying structure rather than superficial similarities. Braithwaite & Goldstone (2015), concluded that the benefits of varied examples do not extend to less knowledgeable learners who achieved better results in the non-varied condition. This suggests that knowledge of the conceptual structure of SWR problems is necessary for variation to be an appropriate tool to promote learning and transfer. If a baseline level of conceptual understanding is not there initially, a large variety of questions simply serves to confuse students because the variety itself requires the construction of abstract links.

The literature reviewed in this section illustrates the importance of taking students' prior knowledge about a subject into account. Braithwaite and Goldstone (2015) concluded that "the relative benefits of varied and similar examples are not fixed but instead depend on prior knowledge, and, in particular, on sensitivity to structural features" (p. 247). Evidently, the ideal level of variation is dependent on each learner's prior knowledge. While some students benefit from a wide array of abstract concepts in word problems, others cannot handle the increase in cognitive load this presents. Braithwaite and Goldstone (2015) did not address the level of familiarity students have with the objects in each question, but it seems plausible that just as students' prior knowledge affects their sensitivity to structural features, so too will their prior knowledge create a sensitivity to object familiarity in equal sharing word problems.

Prior knowledge plays an important role in students' ability to solve problems. One should not assume, however, that students necessarily use all of their prior knowledge when solving problems. Similarly, students do not always reason about real-world constraints when solving word problems in the mathematics classroom. Verschaffel et al. (1994) argued that word problems found in many school curricula are typically designed to help students apply formal mathematics skills taught in the classroom to real world situations, but such real-world applications do not always occur. They warned of the perils of assuming that students actually use real-world logic when solving real-world word problems presented in mathematics class and designed a study to test whether fifth-graders in Belgium activated real-world knowledge when answering arithmetic word problems. In their study, 75 fifth-graders were given 10 pairs of word problems. In each set, one of the problems was a realistic word problem that could not be solved solely using arithmetic. These problems were labelled as "realistic" because the mathematically correct answer was not as appropriate as a reasonable real-world answer. For example, consider the problem: "John's best time to run 100 m is 17 sec. How long will it take him to run 1 km?" Eighty-four percent of the students produced 170 seconds as the answer, but only two of these students added that this answer was not technically correct because in the real-world, runners cannot maintain the speed at which they run 100 metres for an entire kilometer. In fact, across 10 real-world questions, students failed to give a real-world answer most of the time. The authors suggested one reason for this was that students were taught to focus on finding one correct answer in their mathematics lessons. In turning problem solving into mechanical, routine procedures, the students failed to see that real-world problems rarely have a single, arithmetic solution (Verschaffel et al., 1994).

It is noteworthy, however, that some questions administered by Verschaffel et al. (1994) elicited real-world answers more frequently than others. The authors addressed the significantly higher results for certain questions and attributed these results to the fact that in the Flemish school system, students are expected to answer word problems with a "response sentence" (i.e., "they will need _____ buses"). The authors argued that such response sentences can provide a hint that draws attention to the inadequacy of the computed answer (i.e., 12.5 buses), as students writing the answer might realize that it is impossible to have ½ of a bus (Verschaffel et al., 1994). The authors speculated that certain problem contexts turn the standard expected response into a hint, which can explain the variance in the realism of the students' responses. While this is certainly a possibility, it is also possible that the level of familiarity the students had with the items described in the questions played a significant role in their ability to answer using real-world knowledge. For example, students were more likely to consider real-world solutions for the following question: "450 soldiers must be bused to their training site. Each army bus can hold 36 soldiers. How many buses are needed?" than one where students were least likely to use real-world knowledge, (e.g., "A man wants to have a rope long enough to stretch between two poles 12 m apart, but he has only pieces of rope 1.5 m long. How many of these pieces would he need to tie together to stretch between the poles?"). Thus, an argument can be made that the level of familiarity across the questions was unequal. That is, many students actually have experience riding buses, but they are less likely to have tied pieces of rope together to stretch between two poles. The rope question is further complicated by its use of standard units of measure, which can be difficult for students to visualize (Foster & Osana, 2017). Verschaffel et al. (1994) failed to create questions that were equally concrete and realistic, so it is not surprising their participants did not consistently apply real-world reasoning to their answers. The importance of familiarity was also illustrated in Johnson-Laird et al., (1972) where British students outperformed American students on a study that used the British postal system as a point of reference. The British students were able to use specific prior knowledge that American students did not have.

Drawing from Verschaffel et al.'s work, Dewolf, Van Dooren, Hermens, and Verschaffel (2015) conducted a study using the word problems used in the Verschaffel et al. (1994) study. This time the questions included realistic, pictoral drawings of the word problems. For example, the rope question mentioned above included a picture of a man struggling to tie many small pieces of rope together. This was done in the hopes these pictures would activate students' real-world knowledge. The results indicated that the pictures were not helpful and often, the students did not even look at them.

Similarly, Weyns et al. (2016) used Verschaffel et al.'s (1994) word problems and this time included both a visual and verbal cue to encourage students to use real-world knowledge to solve them. For example, on the question involving the soldiers on buses, there was a picture of soldiers getting on a bus and a specific verbal cue, "Be aware that the last bus may not necessarily be filled completely." Although the results showed that students were more likely to use real-world knowledge after receiving a verbal cue, the overall results were still disappointing, as only approximately one third of the students accessed real-world knowledge with the verbal cues (Weyns et al., 2016). Interestingly, even with the inclusion of pictures and verbal cues, the students were most likely to use real-world knowledge on the question involving the soldiers on buses, and least likely to use real-world knowledge on the school question ("The sports centre is 17 km away from the school and the station is 8 km away from the school. How far are the sports centre and the station located from each other? *Be aware that you do not know in which direction the sports centre and the station are.").*

This finding is significant because it indicates that even with a different group of students in a different study, the bus question is still the one for which students were most likely to access real-world knowledge. In addition, the problem for which they were least likely to access real-world knowledge again involved standard units of measure. It is possible these contexts are not viewed as equally familiar by the students answering the question, suggesting that context type may play a role in students' solution strategies. It appears that some equal sharing problems could be more grounded than others, and as such, yield different levels of performance

(Carbonneau & Marley, 2015; McNeil, Uttal, Jarvin, & Sternberg, 2009).

Present Study

My work centers on testing the effects of different levels of grounding in equal sharing problems. The objective of the study was to discover if students use more developmentally sophisticated strategies to solve equal sharing word problems when they have real-world knowledge of the object being shared. I administered two measures to fourth-grade students, the Prior Knowledge Test (PKT) and the Equal Sharing Interview (ESI). The PKT evaluated students' prior-knowledge of multiplication and division facts and concepts, as well as their proportional reasoning. The ESI evaluated the strategies students used to solve equal sharing problems. The ESI included three distinct problem types: (a) problems with familiar objects to share, (b) problems with semi-familiar objects to share, and (c) problems with unfamiliar items to share.

I used equal sharing word problems during my interview because equal sharing is an accessible means for students to understand fractions (Empson et al., 2005) and word problems are often viewed as the link between classroom learning and real-world knowledge (Verschaffel et al., 1994). The concept of sharing is both relatable and familiar to students, who are asked to share everyday items frequently outside of school. My study focuses on equal sharing word problems that yield answers greater than one with fractional remainders of halves or quarters. This choice was made because repeated halving is the equal sharing action students have the most experience with (Empson & Levi, 2011). Thus, using Empson's equal sharing strategy as a starting point, I wondered how important the level of familiarity with the *object* being shared was for students' ability to develop a strategy to solve the problem.

I created three types of word problems that differed by the amount of real-world knowledge I assumed the students had about the items in each question. Some theory (i.e., Wason & Shapiro, 1971) suggests that students will answer the problems that contain familiar objects, such as brownies, with more sophisticated strategies than those that contain unfamiliar objects, such as “porams.” (a non-word used specifically in this study). Familiar, or “grounded,” objects are those that incorporate references to physical, real-world objects (Koedinger et al., 2008). Such familiar items can activate prior knowledge and generate appropriate strategies for solving problems; unfamiliar, or “idealized,” objects, such as “porams,” are not familiar, and thus will not activate prior knowledge, making it more difficult for students to produce meaningful solution strategies.

To create a mid-point on the continuum of familiarity levels, I established an item the students had some familiarity with (e.g., centimeters). Canadian students are taught metric measurement units in school, so by fourth grade they have experience measuring items, but measuring with a standard unit is less intuitive than breaking a brownie into equal pieces. Consequently, the measurement objects, meters and centimeters, were deemed moderately familiar.

In this study I predict that higher prior knowledge will result in the use of more sophisticated strategies and lower prior knowledge will result in the use of less sophisticated strategies in the equal sharing interviews, regardless of familiarity levels (Empson et al., 2005). As well, I predict an interaction between familiarity level and prior knowledge. There are two possibilities for the nature of the interaction. The first possibility is that familiarity level will have an effect on students’ ability to use sophisticated solution strategies, but this effect will differ depending on students’ prior knowledge. Specifically, students with lower prior knowledge will use the most sophisticated strategies with familiar problems, the least sophisticated solution strategies with unfamiliar problems, and mid-range sophistication strategies for semi-familiar problems (Koedinger et al., 2008). In contrast, the inverse pattern will appear for students with higher prior knowledge; they will use the most sophisticated strategies with unfamiliar problems and the least sophisticated solution strategies with familiar problems, with mid-range level of sophistication used for semi-familiar problems. This possibility is illustrated by Walkington et al. (2013), who found that low-scoring and medium-scoring students were more successful on the problems that were personalized compared to high-scoring students who did significantly worse on the personalized problems, suggesting that real-world problems are especially beneficial for low-achieving students.

The second possibility is based on findings from Foster and Osana, (2017). In the study, 36 fourth-grade students solved eight equal sharing problems with fractional remainders. Four problems were couched in familiar contexts (e.g., slicing pizza), two in semi-familiar contexts (e.g., measurement contexts), and two in unfamiliar contexts (e.g., nonwords such as “wogs”). Results indicated that students’ performance on the problems couched in the measurement context was significantly lower than performance on both the familiar and unfamiliar problems. As well, students performed similarly on the familiar and unfamiliar problem types. Because the pilot data suggested that the semi-familiar problems resulted in less sophisticated solution strategies, compared to familiar and unfamiliar problems, the second possibility is that this result (students using the least sophisticated solution strategies for semi-familiar problems and more sophisticated solution strategies when solving both familiar and unfamiliar problems) will be replicated in this current study. However, although Foster and Osana (2017) did not assess students’ prior knowledge of fractions concepts, the literature suggests a relationship between prior knowledge and mathematics performance. Therefore, I predicted that the higher the prior knowledge, the better performance will be on all problem types, but performance will increase more on familiar and unfamiliar problems than semi-familiar problems.

Chapter Three: Method

Participants

The data for this study were part of a larger project called TIPPS (Trauma Informed Pedagogical Practices and Strategies) that aims to help students who face behavioral, and sometimes intellectual, challenges both in and out of the classroom. As this study focuses on a specific population of students, results should take this unique population into account. In a large, multicultural, urban center in Canada, 115 grade-four students from nine classrooms in four different schools participated in the study. The oldest participant was born May 6, 2006, (10 years, 5 months) and the youngest student was born on Oct. 19, 2007 (9 years, 1 month). The study was conducted in October and November before the students had any formal fraction lessons in the classroom. In grade three students are introduced briefly to fractions by defining, representing, and comparing fractions, but the curriculum objective is to introduce the topic in grade three and then solidify it in the second term of grade four.

The schools in which the data were collected are considered inner-city schools with marginalized populations. Their “Rang Décile” or “low income cut-off” scores averaged 9.4. in 2014-15 (http://www.education.gouv.qc.ca). This score is computed based on the proportion of families with children whose income is near or below the low-income cut-off, which is defined as the level of income by which families are estimated to spend 20% more than the overall average on food, shelter, and clothing. The scale runs from 1 to 10, with 1 indicating the highest level of income and 10 indicating the lowest.

Design

The study is a fully within groups design, meaning all the students answered all three problem types that were experimentally manipulated. There were two measures, the Prior Knowledge Test (PKT) and the Equal Sharing Interview (ESI). The PKT evaluated students’ prior-knowledge of multiplication and division facts and concepts, as well as their proportional reasoning. The ESI evaluated the strategies students used to solve equal sharing problems.

The study was conducted in two phases: (a) whole class administration of the PKT, and (b) one-on-one administration of the ESI. The ESI included three distinct problem types: (a) problems with familiar objects to share, (b) problems with semi-familiar objects to share, and (c) problems with unfamiliar items to share. The interviews took place approximately one month after the prior-knowledge measure was administered.

Measures

Prior Knowledge Test (PKT). Prior literature indicates that students' strategies for solving equal sharing problems tend to be more developmentally advanced if they have prior knowledge of the relationship between multiplication, division, and ratios (Empson et al., 2005). Students who understand partitive division, know their multiplication facts, and can apply multiplicative reasoning are more likely to generate appropriate equal sharing strategies compared to those students with less prior knowledge in these domains (Empson et al., 2005). Consequently, I created the PKT, a paper-and-pencil test, to assess students' prior knowledge of these concepts (Appendix A – PKT). Figure 1 provides a breakdown of the question types seen on the PKT with specific examples.

Item Category	Item Description	Example	Number of Items	Total Score
A) Multiplication Facts and Concepts				
1) Multiplication Facts	Whole number multiplication presented symbolically	4 x 4	4	1
2) Multiple Groups	Word problems where wholes are decomposed and composed	Charlie is painting some chairs. He needs ½ a can of paint for 1 chair. How many cans of paint does he need for 6 chairs?	2	4
3) Operator	Word problems with a fraction used as an operator	Jane has 8 books. One quarter (1/4) of the books are new. How many of Jane's books are new?	2	2

Item Category	Item Description	Example	Number of Items	Total Score
B) Division Facts and Concepts				
1) Division Facts	Whole number division presented symbolically	$9 \div 3$	4	1
2) Equal Sharing – Whole Numbers	Partitive division word problems with no remainders	3 children want to share 15 cookies evenly with no leftovers. How many cookies will each student get?	2	2
3) Equal Sharing – Fractions	Partitive division word problems with fractional amounts in the answer	4 friends want to share the blueberry pie equally, so everyone gets the same amount. Divide the circle to show how much each child will get. Then write the final answer.	1	2
C) Proportional Reasoning				
1) Missing Quantity	Couched in a money context using unit price of whole numbers	3 cupcakes cost $12.00, 4 cupcakes cost _____?	1	1
2) Unit Ratio	Unit price of fractional amounts	You can get 5 apples for two dollars or 3 pears for one dollar. If you spend the whole 2 dollars, would you get more fruit if you bought apples or pears?	1	2

Figure 1. PKT – Item categories, descriptions, examples, and scoring summary

The PKT contained 17 questions in total: eight questions in the Multiplication Facts and Concepts category, seven questions in the Division Facts and Concepts category, and two questions in the Proportional Reasoning category. The Multiplication Facts and Concepts category contained three sub-categories. The first sub-category, Multiplication Facts, was designed to assess students' recall of multiplication facts. The four multiplication questions were represented symbolically. The largest factor was 8 and the largest product was 25. The Multiple Groups sub-category assessed students' ability to decompose and compose wholes with unit fractions. There were two questions in this section. The sample question in Figure 1 was designed to assess whether or not students could compose 6 halves into 3. The third sub-category, Operator, was designed to assess the students' ability to interpret fractions as an operator. Each fraction was written in the problem, so the students would be unable to answer the question unless they understood what the fraction meant and how to use it as an operator (e.g., "1/4 of the books").

There were three types of division items on the PKT. The first sub-category was designed to assess students' recall of division facts. Four division questions were represented symbolically. The largest dividend was 20, and the quotients ranged from 2 to 5. The second sub-category, Equal Sharing – Whole Numbers, was designed to assess students' ability to perform whole number partitive division when it was couched in the form of an equal sharing word problem. There were two items in this part. Students were required to write or draw the answer in a provided space below the question. In the final sub-category, Equal Sharing – Fractions, there was a partitive division word problem designed to assess students' ability to partition a circle, intended to represent the pie described in the problem, presented on the test sheet, and to name the quotient in a box provided at the bottom of the page.

The Proportional Reasoning category of the PKT contained two types of items. The first, Missing Quantity, assessed students' ability to solve a proportion problem within a money context. Students had to write the correct answer in the space provided. The second item, a unit ratio problem, assessed students' ability to determine the unit price of an object when the cost of multiple objects is given. Students had to write the answer and show how they arrived at this answer.

Scoring. The test was scored out of a total of 15 possible points; the Multiplication category had a maximum total score of seven points, the division category had a maximum total score of five points, and the proportion category had a maximum score of three points. For the Multiplication Fact questions, of which there were four, each correct answer was assigned 0.25 points and incorrect answers 0 points. This resulted in a total maximum score of 1 point and a minimum score of 0 points. Each multiplication fact assessed the same type of knowledge, (i.e., recall), so I chose to give a maximum score of 1 point for all multiplication fact items so that recall was not over-represented in the PKT score.

Responses on each of the Multiple Groups problems were assigned 2, 1, or 0 points. Two points were assigned if a student answered with the correct answer, such as, "2 sticks of clay," as this demonstrates the ability to compose unit fractions into whole numbers. One point was assigned if the students did not compose the unit fractions into wholes, such as "8 quarter sticks of clay." Zero points were assigned if the student did not provide a response or wrote an incorrect answer. Incorrect answers were based on inaccurate interpretation of the problem structure or a miscalculation. For Operator problems, correct responses were assigned one point and incorrect responses zero points.

Like the Multiplication Fact questions, each division fact question was assigned 0.25 points if answered correctly and zero points for an incorrect answer, for a total maximum

Division Fact score of 1 point. Correct responses on the Equal Sharing – Whole Number problems were assigned one point and incorrect responses zero points. On the Equal Sharing – Fractions problems, students were assigned one point for dividing the circle into four equal parts, even if the four parts were not equal. However, if they had more or fewer than four parts, they would receive zero points. Students were also assigned one point for writing the correct answer to the problem in the space provided. Correct answers included ¼, "a quarter," 25%, "1 out of 4," or another equivalent quantitative response. They would receive zero points if they wrote a non-quantitative amount, such as "a piece."

Responses on the Missing Quantity item of the Proportional Reasoning category were assigned one point for a correct answer, and an incorrect answer was assigned zero points. Responses on the Unit Ratio item was assigned 1 point for a correct answer (i.e., pears), and 0 points for an incorrect answer (i.e., apples). An additional point was awarded for providing any solution strategy that led to the correct answer. When no additional solution strategy was shown, or when an incorrect strategy was indicated, the strategy received 0 points.

All the points on the individual items were summed and converted to percent scores.

Administration. The PKT was created specifically for this study and was 30 minutes in length. On the day of the test, prior to administrating the PKT, the research assistants introduced themselves to the teachers in their classrooms, collected permission forms, and found out if any students were absent. At the start of testing in each classroom, students were asked to place a sharpened pencil and eraser on their desks. The research assistant then introduced herself to the students and explained that the class would be answering some mathematics questions independently for research on how students understand mathematics. They were reassured that it was acceptable not to answer a question and reminded not to discuss answers with their classmates. They were told they could use any representations they wished to solve the problems.

The test booklets were distributed to each student. Each question was read out loud to the entire class twice to ensure that reading ability did not play a factor in the test results. The students were told in advance of each question how much time they had to answer it and were reminded not to turn the page to the next question until they were told to do so. As well, students were permitted to ask the researcher to repeat a question as often as was needed. Teachers remained in the room during testing.

Each question had a suggested time limit ranging from 1 to 3 minutes (See Appendix B for time limits) that was told to the students before each question began. However, if the entire class finished before the time was up, or the class became loud and disruptive, the research assistants directed the class to move on to the next question. If the class was quiet and a few students needed another 30 seconds, the extra time was granted, although research assistants were not permitted to go more than 30 seconds over the allotted time for any question.

Equal Sharing Interview (ESI). Table 1 lists each equal-sharing item on the ESI and its level of familiarity. The ESI was designed to assess students' solution strategies for solving partitive division word problems with fractional remainders of ¼ or ½. Each of the questions contained the same action (sharing a number of items between a group of people), but the items in the word problems differed along dimensions of familiarity: familiar, semi-familiar, and unfamiliar (see Appendices C, D, and E for the word problems used). The word problems in the Familiar category contained the everyday items of muffins, brownies, and cookies. The semi-familiar questions contained meters of rope, centimeters of string, and centimeters of thread to be shared equally. Pilot data (Foster & Osana, 2017) suggested that the students recognized items that incorporated standard measurement units but were less familiar with them. For example, Foster and Osana (2017) observed students asking how big a centimeter was and stating that, although they remembered learning about meters in school, they had forgotten what they were.

This suggests they were less familiar with standard units than the more familiar items of brownies, cookies, and muffins. The unfamiliar questions contained “gloops,” “porams,” and “bamoes.” These are fictitious words designed specifically for this study and were designed to be entirely disconnected from the students’ previous experience, both in school and out.

Table 1

Equal Sharing Problems and their Level of Familiarity

Problem Items and Number of Groups	Familiarity Level
4 Share 10 Brownies	Familiar
4 Share 13 Cookies	Familiar
8 Share 18 Blueberry Muffins	Familiar
6 Share 9 Meters of Rope	Semi-Familiar
4 Share 9 Centimeters of Thread	Semi-Familiar
8 Share 12 Centimeters of String	Semi-Familiar
6 Share 15 Bamoes	Unfamiliar
8 Share 10 Porams	Unfamiliar
10 Share 25 Gloops	Unfamiliar

I created a scoring system to ensure that all the items in each level of familiarity on the ESI had an equal difficulty level. Creating these difficulty scores eliminated, as much as possible, the confound of problem difficulty. In the pilot study (Foster & Osana, 2017), students provided verbal explanations as to why some questions seemed more difficult than others. Many stated that the questions that contained small numbers, even numbers, and halves (rather than quarters) were easier to answer than questions that contained large numbers, odd numbers, both even and odd numbers, and quarters (rather than halves). Using this information, I created a question difficulty scale and ranked each of the nine questions individually. Each question received one point if it included small numbers, one point if it included even numbers, and one point if it involved halves. Two points were assigned to questions that involved large numbers, odd numbers, and if they involved quarters. This gave each question an individual difficulty score between three and six points. Then, the questions were placed in their respective familiarity categories and the mean difficulty score for each category was calculated. The difficulty score for each of the unfamiliar and familiar categories was 4.67, and the semi-familiar category had a mean score of 4.3.

Problem order. The ESI consisted of 14 questions: nine equal sharing, four filler questions, and one final item that asked the students which question they found the most challenging. The test contained three blocks of questions with three questions in each block. Three test versions (A, B, C) were created by varying the order of the problems in each block. Students were randomly assigned to each test version to ensure that question order played no relevance in their performance. Each block contained one equal sharing problem from each familiarity category. The order for version A was familiar, unfamiliar, semi-familiar. The order for version B was unfamiliar, semi-familiar, familiar. The order for version C was semi-familiar, familiar, unfamiliar.

The three blocks were separated by two filler questions designed to minimize participant fatigue. They also provided some differentiation in the types of questions the participants answered, making it less likely they would recognize the similarity between the equal sharing questions. The filler questions included multidigit addition and subtraction computations with one double digit number and one single digit number (14 + 3 & 17 – 2) and two comparison questions (“which number is larger 52 or 41?” and “which number is smaller 36 or 63?”). The four filler questions were identical in each test version. The final question on each version of the ESI required students to comment on which problem they found the most difficult.

Procedure. The interview was conducted individually with each student in a quiet room in his or her school and video recorded, with the camera angled toward the materials in front of the students and their hands. Students were given a pad of paper, two markers, and unifix cubes, and were told they could use any of them to solve the equal sharing problems if they wished. The unifix cubes were not available for the filler questions. At the start of the interview, the research assistant (RA) introduced herself, and explained to the student that she was interested in knowing how the student thinks while solving the problems rather than his or her final answers.

The RA read each question out loud to the student. Each question was repeated as many times as needed. A question was repeated when a student asked or when the RA ascertained that it might be beneficial, such as if the student was silent and unmoving, showing no indication he or she understood the question. The identical wording was used if the question was repeated. If the student said he or she did not understand the question, the RA asked which part was confusing and then answered the student's specific question based on the list of answers from the Student Question/RA Answer Sheet (see Appendix G). Once the RA was satisfied with the student's response to the item, once the student stated he or she could go no further, or if the child became agitated or frustrated, the research assistant moved onto the next item. The students had approximately four minutes to answer each item on the ESI, with the exception of the filler questions and the final question, for which the student was given approximately one minute to answer each of these. The goal was to keep each interview under 45 minutes.

The answers the research assistants could give to participant questions were tightly scripted to ensure no leading information was provided to the students. They were based on the types of questions students asked during the pilot study (Foster & Osana, 2017). Examples of questions students could ask, and scripted responses for the RAs, can be seen in Table 2 (for full list of possible student questions and RA responses, please see Appendix G).

Table 2

Some Examples of Student Questions and Research Assistants' Scripted Responses

Possible Student Questions	Scripted Responses
What is a Gloop?	"I am not sure." If student is unable to proceed and needs a further prompt you should say, "Can you answer the question without knowing?" If the word prevents them from working through the problem, stop and move to the next question.
Student gives a remainder as an answer (i.e., students get 2 brownies each and 2 leftover)	"Leftovers are not allowed." If they still do not partition, then move to the next question. If the student says, "throw those pieces in the garbage" or "give them to someone else," stop and move on to the next question.
Student gives a final answer that includes the words "a piece" (e.g., each student would get 3 cookies and a piece of another one)	"Can you tell me more about that piece/those pieces?" Once they answer, do not ask more questions.
Student states he/she cannot answer the meter question because a meter will not fit on the page	"You can pretend a meter is a smaller size than it really is, if it makes it easier for you."

Research assistants offered encouragement based on effort ("I can see that you are trying hard") but did not indicate if the student was thinking or answering correctly or incorrectly (e.g., "good job"). Similarly, answers to previous items were not discussed ("why did you get that answer here but something different in the previous question?") to minimize attention to the variance in the strategies they used throughout the interview. Once the interview was completed, all worksheets used by the student were collected and assigned page numbers, and the student's name, school, and teacher were written on the back of each page. Data on the memory card in the video camera were downloaded onto a computer within 24 hours of the interview.

Coding and scoring. For each of the equal sharing problems, students received a code to capture the type of strategy used to solve the problem. Each code was then assigned a score to reflect the developmental level as described by Empson (1999). The individual ESI score was converted to a percentage score. This was determined by converting the raw score (scored out of 9, with 0 being the lowest possible score and 9 being the highest possible score, per student, per familiarity level) into a percent. The students' strategies were coded and scored using the rubric based on Foster and Osana (2017) presented in Table 3.

Table 3

Code Descriptions and Scores for Equal Sharing Items on the ESI

Code	Description	Example	Score
A	Problem structure difficulty – Includes guessing, adding the numbers given in the problem.	4 students share 10 brownies, so each student gets 14 because 4 + 10 is 14	0
B	Equal amounts are distributed, but no partitioning of remaining wholes into parts	4 students share 10 brownies, so each student gets 2 brownies with 2 brownies left over	1
C	Partitioning and or distributing incorrectly	4 students share 10 brownies, so each student gets 2 whole brownies, and one more bite	2
D	Correct symbolic representation (could be personally constructed symbols)	4 students share 10 brownies, so each student gets 2 whole brownies and 1 half brownie	3

Strategies given a Code A received a score of 0 because these students were unable to share items equally between sharers. In some cases, they may have assigned unequal parts to sharers, giving more items to a particular person (i.e., "my Dad eats the most in our house, so I will give him six cookies and the rest of us will have two"). Sometimes, they did not understand what the question was asking so they would add the numbers together, or sometimes they would be unable to provide a response. Strategies given a Code B would receive 1 point because they demonstrated that the student could share whole numbers of items but was unable to partition wholes into parts. For instance, if 10 students were sharing 25 "gloops," they would give each child 2 (i.e., equal groups) but then leave the other 5 whole "gloops" as a remainder, calling them "leftovers" or stating they should be thrown out, or stating it is impossible to separate the last ones. This was the key difference between Code B and Code C, for in Code C, students knew the wholes should be partitioned into parts, and they attempted to do so, but were unable to do so correctly. Perhaps with 2 cookies left between 4 people, they would state that each person gets "a bite," or "a third," or "a piece." Code C responses received a score of 2. Finally, Code D responses received a score of 3 because of a correct symbolic representation. For example, a student might have answered that each child gets "one shaded piece of the cookie" when the answer is ¼. If this child drew a picture of a cookie with ¼ shaded in, this strategy would be considered a correct symbolic representation. For example, a student might have answered that each child gets "one shaded piece of the cookie" when the answer is ¼. If this child drew a picture of a cookie with ¼ shaded in, this strategy would be considered a correct symbolic representation.

Inter-rater reliability was obtained on portions of the pilot data (Foster & Osana, 2017). A second rater was trained on the rubric in Table 2 and was then given a portion of the pilot data to code independently. Discrepancies between my codes and those of the second raters were discussed and resolved to clarify and refine the rubric. Then, another portion of the pilot data, specifically 20 questions chosen at random (7% of the data) were coded independently by the second rater. Agreement was 95%. The same second rater then coded the current data set. She was blind to the study's goals and hypotheses.

Training of Research Assistants

I trained a team of research assistants to administer the Prior Knowledge Test in the students' classrooms (See Appendix B for PKT Protocol). They were blind to the purpose of the study to minimize bias. The research assistants were presented with the PKT Protocol a few days before attending a training session. During the training session, they had the opportunity to re-read the protocol as a group, while I read through it with them, making sure they understood each section and addressing any of their questions or concerns. Then, they practiced administering portions of the test to each other in pairs or small groups while I listened. The training session lasted approximately one hour, and all the research assistants indicated to me that they understood how to administer the test after the meeting. They were instructed to contact me immediately if they experienced any problems during data collection, so I could make any necessary revisions to the protocol.

Two separate training sessions were held for the ESI. Six research assistants were hired for the Equal Sharing Interviews (ESI). They were blind to the purpose of the study to minimize bias. Prior to the first training on the ESI, I sent participants, via e-mail, the protocol instruction sheets and I asked them to read them on their own before the first session. These instruction sheets included Version A, B, and C of the ESI, the scripted responses that research assistants were permitted for specific student questions, and the Interview Protocol.

For each ESI version (A, B and C), the Research Assistant was provided with an interview protocol sheet detailing the procedures for conducting the interview (see Appendix F for the interview protocol). This protocol included how to use the video recording equipment, how to introduce themselves and the materials to the students, the duration of each question, and how to deal with unexpected comments from the students. They were told that each interview should last approximately 45 minutes.

Within seven days of sending the protocol, I ran the first session, which was a face-to-face meeting with all six research assistants to answer any questions they might have had and to ensure they were using the same pronunciation of the unfamiliar items (i.e., gloops, bamoes, porams). Within five days of this session, the second session took place. The research assistants were tested by me individually in a mutually acceptable location, either in person or via Skype, to run through the interview items and to make sure their answers corresponded to the script. In this meeting, I pretended to be a grade four student and each research assistant conducted the interview with me. The focus was on their ability to use the scripted responses confidently, correctly, and fluently. The interview lasted between 15 and 75 minutes, depending on the RA's performance. Once I determined that the research assistants were prepared to administer the interview questions, by using the scripted responses accurately and comfortably (i.e., without lengthy pauses), they were deemed ready to conduct the individual interviews. If during data collection, students asked questions that were not on the list, the research assistants were asked to report these to me immediately, so I could send out a new scripted response if necessary.

Analysis and Hypotheses

The statistical test I ran was a 3 x 4 mixed design ANOVA. The dependent variable was the strategy score on the ESI. The prior knowledge and familiarity level were the independent variables, with prior knowledge as the between-group variable and familiarity as the within-group factor. I predicted a main effect of prior knowledge on strategy sophistication. Specifically, I predicted that students with higher prior knowledge would use more sophisticated solution strategies than students with low prior knowledge. In addition, I predicted an interaction between familiarity level and prior knowledge. There were two possibilities for the nature of this interaction.

The first possibility was that familiarity level would have an effect on students' ability to use sophisticated solution strategies, but this effect would differ depending on students' prior knowledge. Specifically, students with higher prior knowledge would use more sophisticated strategies on unfamiliar problems and less sophisticated strategies on familiar problems. Students with lower prior knowledge would use more sophisticated strategies on familiar problems and less sophisticated strategies on unfamiliar problems. For semi-familiar problems, there would be no difference in sophistication level, but performance would be higher for lower knowledge students than on unfamiliar problems but lower than unfamiliar problems for high knowledge students. In this possibility, familiarity would have an effect only combined with prior knowledge. In particular, I predicted an interaction between prior knowledge and familiarity level.

The second possibility was based on findings from Foster and Osana (2017). Results from that study indicated that students' performance on the problems couched in the measurement context was significantly lower than performance on both the familiar and unfamiliar problems. As well, students performed similarly on the familiar and unfamiliar problem types. Because the pilot data suggested that the semi-familiar problems resulted in less sophisticated solution strategies than both the familiar and unfamiliar problems, the second possibility was that students would use the least sophisticated solution strategies for semi-familiar problems than both familiar and unfamiliar. However, because of the literature suggesting a relationship between prior knowledge and mathematics performance, the higher the prior knowledge, the better performance would be on all problem types, but performance would increase more on familiar and unfamiliar problems than semi-familiar problems.

Chapter Four: Results

Prior Knowledge Groups

I had 121 participants in my initial sample, but I removed six participants from the analyses. One participant was removed because he did not write the PKT. The other five were removed because they were missing at least two questions from one or more problem types on the ESI. This resulted in 115 participants in the sample for the analyses.

The mean PKT score across all levels and problem types was *.53 (SD = .25).*

Using the distribution of score I created four knowledge groups: low (more than one standard deviation below the mean), low-medium (within and including one standard deviation below the mean), medium-high (within one standard deviation above the mean, including 0 and 1 standard deviation), and high (more than one standard deviation above the mean). Means and standard deviations for performance on the PKT as a function of prior knowledge group are presented in Table 4. The distribution of the PKT scores indicate a normal distribution, with 60.87% of the students falling within one standard deviation above and below the mean and 39.13% falling beyond two standard deviations above and below the mean.

Table 4

Means and Standard Deviation of Prior Knowledge Test Scores by Four Knowledge Level Groups

Prior Knowledge Level	*N*	M	SD
Low	23	.18	.08
Low-Medium	33	.40	.07
Medium-High	37	.63	.07
High	22	.91	.06
Total	115	.53	.25

Effects of Problem Type and Prior Knowledge

The means and standard deviations of scores on the ESI as a function of problem type and prior knowledge group are presented in Table 5.

Table 5

Means and SD of ESI scores as a Function of Prior Knowledge and Problem Type

Problem Type	Prior Knowledge Category	*M*	*SD*
Familiar	Low	.30	.28
	Low-Medium	.36	.30
	Medium-High	.56	.31
	High	.67	.32
	Total	.47	.33
Semi-Familiar	Low	.24	.25
	Low-Medium	.24	.27
	Medium-High	.47	.36
	High	.60	.34
	Total	.38	.34
Unfamiliar	Low	.27	.27
	Low-Medium	.33	.29
	Medium-High	.50	.33
	High	.67	.31
	Total	.44	.33

I conducted a 3 (problem type) x 4 (prior knowledge) ANOVA to test the effects of problem type and prior knowledge on strategy score[1]. The within group factor was problem type (familiar, semi-familiar, or unfamiliar) and the between groups factor was prior knowledge (low, low-medium, medium-high, and high). The dependent variable was ESI score. I found a main effect of problem type, $F(2,222) = 10.397$, $p < .001$, partial $\eta^2 = .09$. Post-hoc comparisons with Bonferroni corrections indicated there was no significant difference in ESI performance on familiar ($M = .47$, $SD = .33$) and unfamiliar ($M = .44$, $SD = .33$) problem types, although both were significantly higher than semi-familiar problem types ($M = .38$, $SD = .34$) across all prior knowledge groups ($p = .001$, $p = .014$, respectively). The results indicated that semi-familiar problems produced the least sophisticated solution strategies across all levels of prior knowledge.

I also found a main effect of prior knowledge, $F(3, 111) = 9.37$, $p < .001$, partial $\eta^2 = .20$. The performance on the ESI in the low group was significantly lower than in the medium-high group ($p = .012$ and the high group ($p < .001$), but it was not significantly different from the performance in the low-medium group ($p = .99$). The performance on the ESI in the low-medium group was lower than the medium-high group ($p = .027$) and the high group ($p < .001$). Finally, the performance on the ESI in the medium-high group was not significantly lower than in the high group ($p = .513$).

There was no statistically significant interaction between prior knowledge and familiarity level, $F(6, 222) =.44$, $p = .85$.

[1] There were four outliers in the PKT data as assessed by inspecting the boxplot for values greater than 1.5 box lengths from the top and bottom whiskers. I ran the ANOVA twice, once including the outliers and once excluding the outliers. The analyses revealed the same effects, so I report the analysis that included the outliers.

Chapter Five: Discussion

The objective of this study was to discover if students' level of familiarity with the objects in equal sharing word problems affected their solution strategies. Some research suggests that if a student is familiar with an object, he or she should find it easier to share equally (Walkington et al., 2013). Other research suggests that abstract objects help students understand the mathematics behind a problem whereas objects grounded in real-world contexts only serve to shift attention away from the concepts needed to solve the problem (Braithwaite & Goldstone, 2015). However, it is important to consider prior knowledge when considering the role familiarity plays in generating solution strategies. Studies that took prior knowledge into account when examining the effects of familiarity levels (e.g., Koedinger, et al., 2008) discovered that students with high prior knowledge benefited from problems with unfamiliar representations, whereas students with low prior knowledge benefited from familiar representations. This interaction based on prior knowledge serves to weaken the effects of familiarity by itself.

To achieve my objective, I administered the Prior Knowledge Test (PKT) and the Equal Sharing Interview (ESI) to fourth-grade students. The PKT evaluated students' prior knowledge of multiplication and division facts and concepts, as well as their proportional reasoning. The ESI evaluated the strategies students used to solve equal sharing problems and included three distinct problem types: (a) problems with familiar objects to share (e.g., brownies), (b) problems with semi-familiar objects to share (e.g., centimeters of thread), and (c) problems with unfamiliar items to share (e.g., "gloops").

In this study, I predicted that students with higher prior knowledge would use more sophisticated strategies and students with lower prior knowledge would use less sophisticated strategies for solving equal sharing problems, regardless of familiarity levels. This prediction was confirmed insofar as students with lower prior knowledge scores used less sophisticated solution strategies than students with higher prior knowledge scores. This result replicates results found in a variety of studies, including Empson et al. (2005), who showed how important number-fact recall and multiplicative reasoning was for students to successfully solve equal sharing word problems. As well, Johnson-Laird et al. (1972) illustrated the importance of having access to specific prior knowledge when British students outperformed American students on a study that used the British postal system as a point of reference. Finally, Braithwaite and Goldstone (2015) concluded that the benefits of varied examples on sampling with replacement problems were dependent on one's prior knowledge of the subject matter. Thus, the correlation I found between prior knowledge and strategy use is in line with previous research that highlights the importance of taking students' prior knowledge into account when examining their reasoning and problem solving.

As well, I predicted an interaction between familiarity level and prior knowledge. There were two possibilities for the nature of the interaction. The first possibility was based on the existing literature, namely that familiarity level would have an effect on students' ability to use sophisticated solution strategies, but this effect would differ depending on students' prior knowledge. Specifically, students with higher prior knowledge would use more sophisticated strategies on unfamiliar problems and less sophisticated strategies on familiar problems. Students with lower prior knowledge, on the other hand, would use more sophisticated strategies on familiar problems and less sophisticated strategies on unfamiliar problems (Koedinger et al., 2008). For semi-familiar problems, there would be no difference in sophistication level regardless of prior knowledge level, but performance would be higher than lower knowledge students on unfamiliar problems but lower on unfamiliar problems than high knowledge students. This possibility was supported by Walkington et al. (2013), who found that low-scoring and medium-scoring students were more successful on the problems that were personalized compared to high-scoring students, who did significantly worse on the personalized problems, suggesting that real-world problems are especially beneficial for low-achieving students.

The second possibility was based on findings from Foster and Osana, 2017. The pilot study was conducted at the same schools as the current study with a different group of students. In the study, 36 fourth-grade students solved eight equal sharing problems with fractional remainders. Four problems were couched in familiar contexts (e.g., slicing pizza), two in semi-familiar contexts (e.g., measurement contexts), and two in unfamiliar contexts (e.g., nonwords such as “wogs”). Results indicated that students’ performance on the problems couched in the measurement context was significantly lower than performance on both the familiar and unfamiliar problems. As well, students performed similarly on the familiar and unfamiliar problem types. Because the pilot data suggested that the semi-familiar problems resulted in less sophisticated solution strategies than on both the familiar and unfamiliar problems, the second possibility for the outcome of the present study was that students would use the least sophisticated solution strategies for semi-familiar problems relative to both familiar and unfamiliar. However, because of the literature suggesting a relationship between prior knowledge and mathematics performance, the higher the prior knowledge, the better performance would be on each of the problem types, but that performance would be higher on familiar and unfamiliar than semi-familiar problems as prior knowledge increased.

The data from the current study partially supported the second possibility. There was a main effect of problem type, regardless of prior knowledge, but there was no interaction between familiarity level and prior knowledge. All students used more sophisticated solution strategies with familiar problem types than they did with semi-familiar problem types. As well, they used more sophisticated solution strategies with unfamiliar problem types than they did with semi-familiar problem types, with no statistically significant difference in performance between familiar and unfamiliar problems.

One explanation for my results differing from the prediction coming out of previous research is in the inconsistent interpretations of the term “unfamiliar.” My definition of a “familiar” mathematics problem was taken directly from Koedinger et al. (2008), who defined grounded word problems as those that incorporate references to physical, real-world objects that are recognizable to students. This definition has been used in other research (e.g., De Bock et al., 2011; Koedinger & Nathan, 2004) and appears to be generally accepted. I adopted the definition of an unfamiliar item as one that is abstract or idealized (Fyfe et al., 2014; Koedinger & Nathan, 2004; Weyns et al., 2016), but I operationalized abstract or idealized to mean an unfamiliar item, or fictitious object, that has no connection to a real-world object, or one that does not activate existing knowledge about objects in the world. Other researchers (e.g., Koedinger et al., 2008) operationalized this definition through the use of algebraic symbols. There are key differences between a fictitious object and an algebraic symbol. While students may not understand or remember the intended meaning of a particular symbol in a mathematics equation, they typically know there are rules that must be employed when using symbols to solve an algebraic equation. These rules may be misunderstood or forgotten, but I maintain that the existence of these rules prevents students from giving algebraic symbols any meaning of their choosing. This differs from the unfamiliar objects in my study, where students used the very ambiguity of the object to their advantage. I speculate that the ambiguity allowed students to create their own meaning, or pretend the unknown object was familiar, and treat it as such. This gave students the freedom and flexibility to mold the unfamiliar object into something they would more easily understand. In this respect, there was a qualitative difference between the way I operationalized unfamiliarity compared to other researchers’, which may account for the different results. It also suggests that familiarity levels are not the only dimension involved when assessing difficulty levels for problems, and perhaps a second dimension also needs to be taken into account. This possibility is discussed in more detail below.

Contribution to the Literature

Lee's study (2012) showed that not all word problems are equally accessible to students, as both the structure of a word problem and the type of story chosen can alter students' methods and accuracy rates. My study contributes to this literature in that it provides additional evidence that students find some word problems more accessible than others. One of my findings is that the semi-familiar word problems proved to be the least accessible to students, regardless of the students' level of prior knowledge. There was something about the semi-familiar word problems that prevented students from using as sophisticated solution strategies as on the other problems.

My study showed that real-world, or grounded, contexts were beneficial in comparison to semi-familiar contexts, which is in line with previous research (Walkington et al., 2013). In addition, however, I also found no difference between a real-world context and an unfamiliar, or idealized, context. These findings suggest that object familiarity in and of itself is not solely responsible for the schemas activated by students when solving equal sharing problems. The issue is more complex than simply a question of familiarity.

Bransford and Johnson (1972) argued that certain phrases or words activate particular schemas that are used by students while trying to remember and comprehend a story. Their research suggests that appropriate schemas are often a necessary pre-condition for meaningful processing to occur. Applying this theory to my study, one might expect that a completely unfamiliar object would fail to activate a useful schema for solving a problem. On the contrary, however, the unfamiliar object appeared to provide an opportunity for the students to pick an existing schema and apply it to the unfamiliar object. For example, in Foster and Osana (2017), because "wogs" were unknown, students pretended a "wog" was a known object, like a log, and then imagined partitioning a log. It was precisely the lack of information about the unfamiliar object that gave the students the freedom to use a familiar schema to solve the problem, allowing them to capitalize on the object's unfamiliarity. To contrast, I speculate that students' familiarity with centimeters created an obstacle for them, because it activated a procedural memory that led them away from partitioning and equal sharing. For this reason, I argue that prior exposure to an object creates familiarity, but this familiarity can either help or hinder students.

This finding allows me to speculate that the information in semi-familiar problems that the students had some prior experience with (e.g., specific units of measurement, like centimeters) could account for their poor performance. In Foster and Osana (2017), students were frequently confused by the measurement component of the semi-familiar problems. For example, when students heard the word "centimeter," they would often ask exactly how big a centimeter was because they could not remember, and felt they needed to know this to answer the problem. "Centimeter" was a word students had encountered before, and to some extent, they understood it to be a fixed unit of measure, but they did not appear to know what to do with this information in the context of an equal sharing problem. Their partial knowledge of fixed units of measure did not include how to partition a centimeter into a fractional quantity.

An added complication for the semi-familiar problems is that students had to know how to partition equally *and* how to use a specific unit of measure to quantify the size of the piece of each partition. Making this connection appeared to be difficult for students precisely because they did not see a connection between partitioning a quantity and measuring the size of a quantity, perhaps because these concepts are often taught separately. In other words, students interpret partitioning an unmeasured quantity into equal pieces (e.g., dividing a rope into four equal pieces) and partitioning a quantified object (e.g., sharing a rope with a length of 32cm equally between four people) as different problems. It appeared that the students' inability to establish a conceptual connection between partitioning quantified and unquantified objects led to less sophisticated solution strategies. Specifically, it seemed like students' understanding of the concept of measurement (e.g., choosing a unit and iterating it) was not well developed, resulting in the activation of a more familiar, but less useful, schema for these problems. I speculate that they became focused on trying to remember a procedure shown by teachers for using standard units of measure (e.g., measuring lengths with rulers) rather than on partitioning lengths. Their weak conceptual knowledge of measurement allowed them only one "entry" into the problem, which was the activation of a "how to measure" schema that was not useful for solving these semi-familiar equal sharing problems and, at the same time, blocked their partitioning knowledge. Perhaps if they had a conceptual understanding of centimeters that included an abstract knowledge of how centimeters can be partitioned into fractions, students would have been able to use more sophisticated solution strategies on the semi-familiar problems (Richland et al., 2012).

The attribute being measured in the semi-familiar problems may offer another explanation as to why the students used less sophisticated solution strategies when solving these problems. Objects can have many different attributes including length, weight, and volume. However, students are generally taught fractions using the attribute of area. For example, sharing a pie into four equal pieces (i.e., in terms of area) is a typical fraction question in the textbook used in the classrooms from my study. Solving equal sharing problems using area or length involves the same fundamental partitioning process, but the students in my study likely had more exposure to the area model in the classroom and this may explain why they found sharing a length, like string, more challenging. This suggests that partitioning challenges are about more than object familiarity; students are likely equally familiar with brownies and rope, but less confident partitioning rope because of the length attribute that is being measured. Altogether, these explanations suggest there are numerous issues to consider when defining and operationalizing object familiarity. Because of this, placing familiarity on a linear continuum is questionable. Instead, one should consider familiarity levels using a more nuanced model that includes a variety of factors simultaneously.

An additional contribution of the present study is in an increased specificity of the grounded-abstract framework that already exist in the literature on external knowledge representations. Fyfe et al. (2014) questioned the dichotomous relationship between abstract and concrete materials and advocated instead for a “concreteness fading” approach for instruction that begins with concrete examples that gradually fade away, leading to more abstract examples. My study also questions the standard practice of studying grounded and idealized contexts as adversaries on opposite sides of a spectrum, as my results indicated that the relationship between grounded and idealized contexts is not a dichotomous one. Rather, the equal sharing questions containing unfamiliar and familiar objects were answered with a similar level of sophistication, regardless of familiarity level. It seems that the conceptualization of grounded and idealized as extreme ends of a continuum may not be useful for the purpose of my research; if it were, the familiar questions would have produced more sophisticated solution strategies than the idealized problems, to the extent that one can consider nonwords idealized and semi-familiar items more “grounded”. Thus, I concur with Fyfe et al. (2014) that pitting abstract and concrete examples against each other is unproductive, because the students in Foster and Osana (2017) and in the present study were able to answer questions with familiar and unfamiliar objects using equally sophisticated solution strategies, but questions with semi-familiar objects did not fit in the middle but rather used the least sophisticated solution strategies. Thus, in terms of object familiarity, it appears that possessing a little bit of knowledge was less beneficial than having none at all.

Limitations of the Study

There are two key limitations of this study. The first is that the validity of the measure of prior knowledge, which was designed specifically for this study, has not been established. The relationship I found between prior knowledge and solution strategy is aligned with previous research (Empson et. al., 2005), showing that students with strong multiplication facts and strong multiplicative reasoning skills use more sophisticated solution strategies, which provides some evidence of the measure's validity. Nevertheless, without further study, I cannot guarantee the test I created was a valid assessment of students' prior knowledge.

The second limitation involves the difficulty in defining a "semi-familiar" object and choosing an appropriate framework for the conceptualization of familiarity. Some scholars may object to the semi-familiar objects chosen in my study because these objects were continuous, whereas the familiar objects could be considered discrete. However, defining an object as discrete or continuous has proven difficult, because the definitions for these words are not standardized in the literature. For example, Kornilaki and Nunes (2005) argued that a "fishcake" is a continuous item, and that individual fishes are discrete items that cannot be partitioned, but rather are objects that can be shared in a "one for you, one for me" style. I would argue, however, that any item can be partitioned into pieces that can be shared in "a piece for you, and a piece for me" manner. As such, I view both a fish and a fishcake as continuous.

Interpreting objects as discrete or continuous types of quantities can create complications, not least of which is related to students' and teachers' interpretations of partitioning these quantities, which appears to be subject to change depending on the situation. For example, Verschaffel et al. (1994) noted that some students viewed a school bus as a discrete object and others viewed it as a continuous object. When asked how many buses were required to take students to school, some students answered 12.5 buses, as they believed the bus was a symbolic object and their teacher required an exact numerical answer, whereas other students answered 13 buses, as they viewed the bus as a concrete object which cannot be split in half. The context in which an object is presented, and the individual's interpretation of the problem, often create differences of opinion as to whether an object is continuous or discrete. This may lead to confusion and creates a circumstance where there exists more than one correct answer to a mathematical problem. Thus, individual, idiosyncratic interpretations add to the difficulty in agreeing on what discrete and continuous mean. Therefore, I discount the argument that a confound existed.

This leads, however, to a related limitation, which is the difficulty in establishing an objective framework that comprehensively captures the familiarity level of all objects. It is not sufficient to use the definition of Koedinger et al. (2008), who defined grounded word problems as those that incorporate references to physical, real-world objects that are recognizable to students as a framework for testing familiarity. One must also account for different measurement attributes, such as area and length, among others. This implies that a useful familiarity framework is one that exists on multiple dimensions that take the nuances involved in defining familiarity into account. However, one problem I encountered in my study was the type of instruction the children have had in the past. It seemed the students in my study had been exposed to area model instruction, but had limited exposure to number line instruction, so what presented as a familiarity issue may have been an instructional issue. This potential confound should be explored in further studies.

Future Research: What New Questions Have Been Generated?

It remains uncertain what the specific difficulties were for the students solving the semi-familiar problems. It could be attributed to the manner in which I operationalized unfamiliar items, or students' weak knowledge of standard units of measurement, or the procedural measurement schema that some students may have employed to solve the problem. Further research should attempt to tease apart and isolate the effects of these factors to determine how each contributes to solution strategy difficulties.

In addition, further research should question what conditions allow for the activation of useful knowledge and what conditions activate knowledge that impedes students from solving equal sharing problems. Familiarity is not the only factor to consider when examining students' solution strategies. There are certain kinds of prior experience that seem to aid performance and other types which impede it. In my study, students solved the familiar and unfamiliar problems with more sophisticated solution strategies than the semi-familiar contexts. Arguably, practical knowledge (i.e., an understanding of how to share brownies based upon previous experience) may have helped students answer the familiar questions, but this does not explain the success students had with unfamiliar contexts. Students had no practical experience with "gloops," but this lack of experience created an opportunity for students to imagine a "gloop" was something convenient to partition. In contrast, the students had enough knowledge of the semi-familiar items that they could not turn them into something convenient to partition. However, the students' prior knowledge about standard units of measure appeared incomplete and this led to less sophisticated solution strategies. Perhaps, if students were told they had to share ten "figlias" of rope between four people rather than ten centimeters of rope, this might have been easier for them, as they would not have become frustrated or confused by their limited understanding of standard units of measurement.

Often, we assume that greater prior knowledge about an object necessarily corresponds to greater familiarity about partitioning this object, which then leads to more sophisticated solution strategies, but this process appears to be more complex. In my study, sophisticated solution strategies were as likely to be used with familiar objects, with which students had a great deal of practical knowledge, and unfamiliar objects, of which students had no practical knowledge.

Having *some* practical knowledge about an object (for example, knowing what a centimeter is but only knowing how to use it in the context of a "how to measure" schema) created the least sophisticated solution strategies. This suggests that a little knowledge is sometimes worse than none at all. Future research needs to untangle this complex issue by identifying the types of familiarity that help or hinder students' solving of equal sharing problems.

Implications for Practice

Importantly, this study indicated that "real world" knowledge is not the only factor involved in students' ability to use appropriate strategies for solving equal sharing problems. This finding implies that teachers should regularly use unfamiliar objects when creating word problems, as unfamiliar problem types often lead to greater transfer (Braithwaite & Goldstone, 2015). As well, we should question the instructional practice of moving in a single direction from familiar to unfamiliar objects when creating mathematics problems, as this study showed that abstract questions can be answered by students at the same level of sophistication as familiar questions, suggesting that moving in the reverse order may also be effective. At the same time, teachers should not assume that all realistic contexts are "equal." Items for which students only had limited knowledge (i.e., standard units of measurement and partitioning length) may have impeded the students' reasoning by activating irrelevant schemas, such as the "how to measure" schema (Nokes & Belenky, 2011). Perhaps then, teachers need to pay greater attention to the objects chosen for equal sharing word problems while recognizing that object familiarity is only a piece of the puzzle when assessing problem complexity, as increasing levels of familiarity does not necessarily make a question easier for students to solve.

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Appendix

Appendix A – Prior Knowledge Test

Name: ______________________________

DON’T TURN THE PAGE YET

School:

Teacher:

Date:

Time:

RA:

T1 T2

____

____

____

____

____

3 children want to share 15 cookies equally with no leftovers.

How much will each child get?

Each child would get [ ] cookies.

DON'T TURN THE PAGE YET

Answer the multiplication questions below:

3 x 6 =

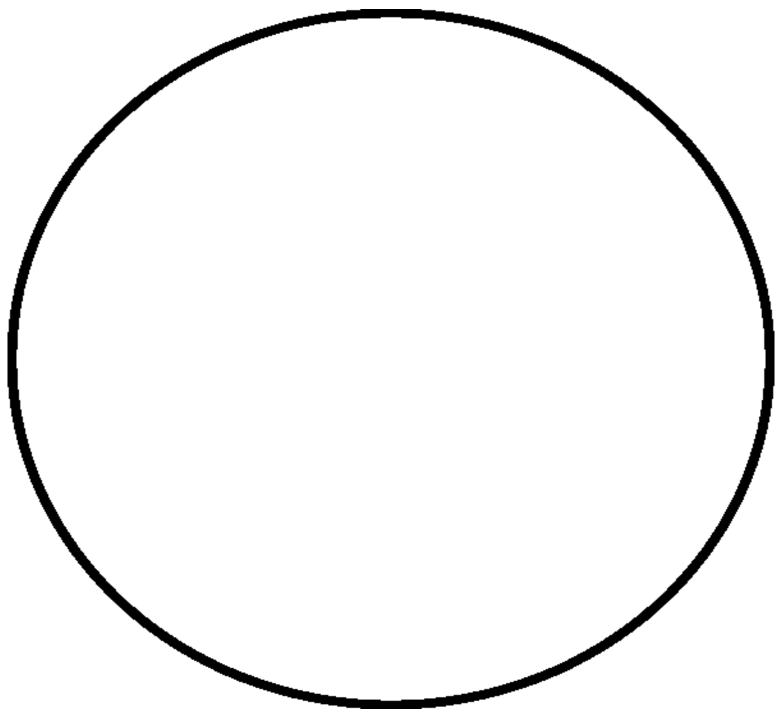
5 x 5 =

4 x 4 =

2 x 8 =

DON’T TURN THE PAGE YET

4 friends want to share the blueberry pie below equally, so everyone gets the same amount. Divide the circle below to show how much each child will get, then write the final answer in the space below.



Each friend would get ______ of a blueberry pie.

DON’T TURN THE PAGE YET

Charlie is painting some chairs. He needs $\frac{1}{2}$ a can of paint for one chair. How much paint will he need for 6 chairs?

Charlie will need ☐ cans of paint.

STOP

DON’T TURN THE PAGE YET

Sarah has 8 friends at her birthday party. She wants to give each friend $\frac{1}{4}$ of a stick of clay in their treat bag. How many sticks of clay will she need to buy altogether?

Sarah will need ______ sticks of clay.

DON’T TURN THE PAGE YET

Answer the following questions:

$8 \div 2 =$

$9 \div 3 =$

$12 \div 6 =$

$20 \div 4 =$

DON’T TURN THE PAGE YET

4 children want to share 12 chocolate bars equally with no leftovers.

How much can each child get?

Each child would get [ ] chocolate bars.

STOP

DON’T TURN THE PAGE YET

3 Cupcakes Cost $12.00

$12.00

4 Cupcakes Cost $____________

$_______?

DON’T TURN THE PAGE YET

Jane has 8 books. One quarter (1/4) of the books are new. How many of Jane’s books are new?

Jane has _____________________ new books.

DON'T TURN THE PAGE YET

Sam has 14 stickers. He gave half (1/2) of them to his best friend. How many stickers did Sam give away?

Sam gave away _________________ stickers.

DON'T TURN THE PAGE YET

You have 2 dollars to spend on fruit. You can get 5 apples for two dollars or 3 pears for one dollar. If you spend the whole 2 dollars, would you get more fruit if you bought apples or pears? Show your work.

Appendix B – Prior Knowledge Test Protocol

TIPPS

Prior Knowledge Test Protocol

Prior to beginning the test:

Introduce yourself to the teacher and let them know you are representing the Concordia TIPPS project. Let them know that last year the test was most successful when their teachers remained in the classroom to monitor student behaviour. The test should take a maximum of 30 minutes. Before beginning the test, make sure to ask the teachers if any students are absent. If they are, record their name. As well, collect all permission forms that have come in and encourage the teachers to have their students bring them in – It is a wonderful opportunity for the students to practice their math skills individually. Ask teacher to ensure each student has a sharpened pencil and an eraser on his/her desk.

Note:
The times given for each question are a guideline. If the entire class is finished before the time is up, move on to the next question. If there are still a few people writing but the class is starting to become loud and disruptive move on to the next question. If the class is quiet and a few students need another 30 seconds that is also acceptable. Do not, however, go more than 30 seconds over the allotted time.

Introduction

Hi,

My name is ___. I am from Concordia University, and I do research on how students like you understand math. Today, I am going to ask your class to answer some math questions. Don't worry if you don't know the answers to some of the questions, I just want to know how you are thinking about these questions. I want you to answer the questions yourself, so please don't talk to your friends.

I will tell you how much time you have to answer each question, and I am going to read each question out loud two times. Remember, even if you're not sure how to answer the question, try your best and write down what you think. You can also use drawings to solve the problems or show your answer if you don't know how to write it. I will also tell you when to turn the page, so don't turn the page before I tell you to.

Pass out booklets and ask students to write their first and last name. Tell them not to open the booklet before you tell them to.

*The **bolded** text is the part that is read twice. Start the timer after you have read the instructions twice.*

3 Share 15 Cookies

You will have 3 minutes to answer this question. **3 children want to share 15 cookies equally with no leftovers. How much will each child get?**

You can draw pictures to help you figure out the answer. There is also a place to put your answer at the bottom of the page *(point to the answer area)*. 🕒🕒🕒 There are 30 seconds left. 🕒 Stop writing. Turn the page.

Fluency (Multiplication)

After you turn the page, you will have **1 minute** to answer the questions on the following page. You don't have to answer all the questions in order. Answer as many as you can. 🕒🕒🕒 There are 30 seconds left. 🕒 Stop writing. Turn the page.

(Note: For fluency questions you do not need to read out each multiplication question. If someone needs help, you can read the question exactly as written (i.e. 3 x 6 would read three times six). Students and/or their teachers may ask if they can use tables to assist them. They need to try and answer this question without any assistance. Encourage them to do what they can and not to worry if they don't have time to finish).

Blueberry Pie

You will have 2 minutes to answer this question. **4 friends want to share the blueberry pie below equally, so everyone gets the same amount. Divide the circle below to show how much each child will get, then write the final answer in the space below.** *(Point to the answer area)*. You can write your final answer using words or numbers 🕒🕒🕒. There are 30 seconds left. 🕒 Stop writing. Turn the page.

Paint for Chairs

You will have 3 minutes to answer this question. Charlie is painting some chairs. He needs $\frac{1}{2}$ a can of paint for one chair. How much paint will he need for 6 chairs? **You can draw pictures to help you figure out the answer. There is also a place to put your answer at the bottom of the page** *(point to the answer area)*. 🕒🕒🕒 There are 30 seconds left. 🕒 Stop writing. Turn the page.

Clay for Birthday Party

You will have 3 minutes to answer this question. Sarah has 8 friends at her birthday party. She wants to give each friend $\frac{1}{4}$ of a stick of clay in their treat bag. How many sticks of clay will she need to buy altogether? **You can draw pictures to help you figure out the answer. There is also a place to put your answer at the bottom of the page** *(point to the answer area)*. 🕒🕒🕒 There are 30 seconds left. 🕒 Stop writing. Turn the page.

Fluency (Division)

After you turn the page, you will have **1 minute** to answer the questions on the following page. You don't have to answer all the questions in order. Answer as many as you can. 🕒🕒🕒 There are 30 seconds left. 🕒 Stop writing. Turn the page.

(Note: For fluency section, you do not need to read out each division question. If someone needs help, you can read the question exactly as written (i.e. eight divided by two). Students may say they don't know what divide means. Tell them if they do not know that is ok. Encourage them to do what they can and not to worry if they don't have time to finish or don't know how to answer the question).

4 Share 12

You will have 3 minutes to answer this question. 4 children want to share 12 chocolate bars equally with no leftovers. How much will each child get? **You can draw pictures to help you figure out the answer. There is also a place to put your answer at the bottom of the page** *(point to the answer area)*. 🕐🕐🕐 There are 30 seconds left. 🕐 Stop writing. Turn the page.

Cupcake Question

You will have 1 and half minutes to answer this question. **3 cupcakes cost $12.00. How much do 4 cupcakes cost? There is a place to show your work at the bottom of the page** *(point to area)*. 🕐🕐🕐 There are 30 seconds left. 🕐 Stop writing. Turn the page.

Book Question

You will have 1 and a half minutes to answer this question. Jane has 8 books. One quarter (1/4) of the books are new. How many of Jane's books are new? **You can draw pictures to help you figure out the answer. There is also a place to put your answer at the bottom of the page** *(point to the answer area)*. 🕐🕐🕐 There are 30 seconds left. 🕐 Stop writing. Turn the page.

Sticker Question

You will have 1 and a half minutes to answer this question. Sam has 14 stickers. He gave half (1/2) of them to his best friend. How many stickers did Sam give away? **You can draw pictures to help you figure out the answer. There is also a place to put your answer at the bottom of the page** *(point to the answer area)*. 🕐🕐🕐 There are 30 seconds left. 🕐 Stop writing. Turn the page.

Fruit Question

You will have 2 minutes to answer this question. You have 2 dollars to spend on fruit. You can get 5 apples for $2.00 or 3 pears for $1.00. If you spend the whole 2 dollars, would you get more fruit if you bought apples or pears? You must clearly show your work to answer this questions. Simply writing “apples” or “pears” is unacceptable as I need to know how you made your decision. **You can draw pictures to help you figure out the answer. There is also a place to put your answer at the bottom of the page** *(point to the answer area)*. 🕐🕐🕐 There are 30 seconds left. 🕐 Stop writing. Turn the page.

Appendix C – Equal Sharing Interview Questions – Version A

Version A - TIPPS Interview Questions

*After each question the RA should ask ONE TIME, **"How did you figure that out?"** or **"Can you explain to me how you got your answer?"***

*If the student spontaneously (i.e. without prompting) provides a clear explanation the RA should say, **"You explained yourself clearly. I understand how you figured that out."***

1. **4 students want to share 10 brownies equally. How many brownies will each student get? 2.5**
2. **6 students want to share 15 Bamoes equally. How many Bamoes will each student get? 2.5**
3. **6 people want to share 9 metres of rope equally. How many metres of rope should each person get? 1.5**

*RA says, **"For the next two questions, I would like you to answer them without using the unifix cubes".***

4. **Which number is larger: 52 or 41?**
5. **What is 14 + 3?**

*RA says, **"For the next set of questions you can use the paper and sharpie, the unifix cubes, or neither, whichever you prefer."***

6. **4 students want to share 13 cookies equally. How many cookies will each student get? 3.25**
7. **8 people want to share 10 Porams equally. How many Porams will each person get? 1.25**
8. **4 people want to share 9 centimetres of thread equally. How many centimetres of thread will each person get? 2.25**

*RA says, **"For the next two questions, I would like you to answer them without using the unifix cubes".***

9. **Which number is smaller: 36 or 63?**
10. **What is 17 – 2?**

*RA says, **"For the next set of questions you can use the paper and sharpie, the unifix cubes, or neither, whichever you prefer."***

11. **8 students want to share 18 blueberry muffins equally. How many blueberry muffins will each student get? 2.25**
12. **10 students want to share 25 Gloops equally. How many Gloops will each student get? 2.5**
13. **8 people want to share 12 centimetres of string equally. How many centimetres of string will each person get? 1.5**
14. **What was the hardest problem that you had to answer today? Why?**

Appendix D – Equal Sharing Interview Questions – Version B

Version B - TIPPS Interview Questions

*After each question the RA should ask ONE TIME, **"How did you figure that out?"** or **"Can you explain to me how you got your answer?"***

*If the student spontaneously (i.e. without prompting) provides a clear explanation the RA should say, **"You explained yourself clearly. I understand how you figured that out."***

1. **6 students want to share 15 Bamoes equally. How many Bamoes will each student get?**
2. **6 people want to share 9 metres of rope equally. How many metres of rope should each person get?**
3. **4 students want to share 10 brownies equally. How many brownies will each student get?**

*RA says, **"For the next two questions, I would like you to answer them without using the unifix cubes".***

4. **Which number is larger 52 or 41?**
5. **What is 14 + 3?**

*RA says, **"For the next set of questions you can use the paper and sharpie, the unifix cubes, or neither, whichever you prefer."***

6. **10 students want to share 25 Gloops equally. How many Gloops will each student get?**
7. **4 people want to share 9 centimetres of thread equally. How many centimetres of thread will each person get?**
8. **8 students want to share 18 blueberry muffins equally. How many blueberry muffins will each student get?**

*RA says, **"For the next two questions, I would like you to answer them without using the unifix cubes".***

9. **Which number is smaller 36 or 63?**
10. **What is 17 – 2?**

*RA says, **"For the next set of questions you can use the paper and sharpie, the unifix cubes, or neither, whichever you prefer."***

11. **8 people want to share 10 Porams equally. How many Porams will each person get?**
12. **8 people want to share 12 centimetres of string equally. How many centimetres of string will each person get?**
13. **4 students want to share 13 cookies equally. How many cookies will each student get?**
14. **What was the hardest problem that you had to answer today? Why?**

Appendix E – Interview Questions – Version C

Version C - TIPPS Interview Questions

*After each question the RA should ask ONE TIME, **"How did you figure that out?"** or **"Can you explain to me how you got your answer?"***

*If the student spontaneously (i.e. without prompting) provides a clear explanation the RA should say, **"You explained yourself clearly. I understand how you figured that out."***

1. **6 people want to share 9 metres of rope equally. How many metres of rope should each person get?**
2. **4 students want to share 10 brownies equally. How many brownies will each student get?**
3. **6 students want to share 15 Bamoes equally. How many Bamoes will each student get?**

*RA says, **"For the next two questions, I would like you to answer them without using the unifix cubes".***

4. **Which number is larger 52 or 41?**
5. **What is 14 + 3?**

*RA says, **"For the next set of questions you can use the paper and sharpie, the unifix cubes, or neither, whichever you prefer."***

6. **4 people want to share 9 centimetres of thread equally. How many centimetres of thread will each person get?**
7. **8 students want to share 18 blueberry muffins equally. How many blueberry muffins will each student get?**
8. **10 students want to share 25 Gloops equally. How many Gloops will each student get?**

*RA says, **"For the next two questions, I would like you to answer them without using the unifix cubes".***

9. **Which number is smaller 36 or 63?**
10. **What is 17 – 2?**

*RA says, **"For the next set of questions you can use the paper and sharpie, the unifix cubes, or neither, whichever you prefer."***

11. **8 people want to share 12 centimetres of string equally. How many centimetres of string will each person get?**
12. **4 students want to share 13 cookies equally. How many cookies will each student get?**
13. **8 people want to share 10 Porams equally. How many Porams will each person get?**
14. **What was the hardest problem that you had to answer today? Why?**

Appendix F – Equal Sharing Interview Questions Protocol – Version A

VERSION A
TIPPS - Grade Four Interview Protocol

Set-up and Introduction

List of materials:
- Pad of Paper - 2 working markers - Pencil and paper - Unifix cubes - Child assent form - Camera with SD card - Tripod - Charger
General Set-up:
Paper, marker, and unifix cubes are placed on the table within reach of the child at all times. However, for questions 4, 5, 9 & 10 students should **not** use the unifix cubes. Make sure the microphone is turned on and the video camera is clearly recording the student's hands and paper before beginning. Take note of how much time is on the memory card before beginning.
Child Assent:
Read through the child assent form with the child. Have the child respond to the questions on the form.
Introducing the Interview:
Say, "I am going to give you some problems. I would like to know how you think while you solve these problems. I will ask you questions to help me understand what you are thinking. For example, if you say something that I don't understand, I will ask you questions about it. I'm not interested in knowing if you are right or wrong, I want to know what you're thinking. So don't worry if you don't think you have the right answer, all that is important to me is that you try your best and that I understand what you are thinking." "If you want, we have some paper and pencils, and some cubes here that you can use any time to help you figure out the problems. You don't have to use them, but you can if you want." "Do you have any questions before we start?"

Set-up:
Paper, marker, and unifix cubes are placed on the table within reach of the child. However, for questions 4, 5, 9 & 10 students should **not** use the unifix cubes.
Instructions:
Note: Proceed through the problems IN THE ORDER GIVEN. Research will be compromised if you switch the order of the questions. Introduce the task to the child using the introduction below. Read each problem to the child once. If they need to hear it again, you can repeat the question as often as necessary but do NOT reword the question. If they inform you they don't understand the question, ask which part is confusing and then answer their specific question based on the list of answers from the Student Question/RA Answer sheet (legal sized sheet). Move on to the next problem once you are satisfied with the child's explanation or once the child states he/she can go no further or if the child becomes overly agitated or frustrated.
Introduction:
RA says, **"We're going to do some problems. I will read the problem out loud, but if you need me to read it again, just tell me, and I'll read it again, as many times as you need. I want to make sure that you understand what is happening in each problem, so if you don't understand, ask, and I will try to help you if I can."**
Problems: Use Problem Sheet VERSION A

General Guidelines

Each student has approximately four minutes to answer each question, (with the exception of questions 4, 5, 9, 10 & 14 which each should take them one minute to answer). If they go a little bit over on one question, that is acceptable. Our goal is to keep each interview under 45 minutes. We want to avoid giving them any hints as to how to solve the problem. On the legal sized paper, you will find a list of possible questions and answers students may give along with the replies the RA should give. **<u>These need to be reviewed carefully prior to conducting any interviews.</u>** Should any questions come up during the interview that you are unsure how to answer, please report them to Aryann or Katie immediately and we will send out a scripted response to the RA team.
Encouragement:
Generally, when giving encouragement we want to avoid giving any indication as to whether the child is getting the correct answer or not. Therefore, we want to encourage the child's effort in general and not their specific solution strategy. Things to say: I can see you are trying very hard. I like that you are trying so hard. I can see that you're thinking really hard about this. Thank you for working so hard on this. Things NOT to say: That's right. Good thinking. Good job.

If a child asks if their answer is correct or not, remind them that you are more interested in what they are thinking than if they are correct. Emphasize that we want them to try their best to get the correct answer but that the most important thing to us is understanding what they are thinking.
Eliciting:
When trying to get a child to explain their thinking we need to do so while avoiding calling specific attention to what they are doing incorrectly (even if we do not say it is incorrect) or using questioning to funnel their thinking in any direction. For this reason, we never discuss their past answers. For example, we never ask, **"Why did you get that answer here but something different in the previous question?"** (This may cause students to question their solution). It is very important that you stick to the scripted RA questions.
Conclusion:
Once the interview is finished, collect all the worksheets that were used by the student and write his/her name, school and teacher on the back of each page along with the page numbers (Mary Smith, Riverview Elementary, Ms. Jade's Class, p. 2 of 7). Bring the memory card back to Concordia within 24 hours and transfer it onto the computer.

Appendix G – List of Possible Student Questions and Scripted Responses

List of Possible Questions Students May Ask and RA Answers

	Student Question
1	Student asks, "What is a Gloop?" (or a Poram or a Bamose)
2	Student gives a remainder as an answer (i.e. when asked to share 10 brownies between 4 students, they answer each gets 2 and then there are 2 leftover)
3	Student gives a final answer that includes the word “a piece” (i.e. each student would get 3 cookies and a piece of another)
4	Student gives a verbal answer with no written work
5	Student gives a partial answer (i.e. the answer is 2 ½ but they lose track of the question and state that each child gets ½ because they forget about the whole numbers they initially counted).
6	Correct answer is 1 1/4. Student tells you that each person gets 2 pieces
7	Student does a drawing of four very uneven pieces
8	Student tries to answer question by drawing a circle and can’t split it
9	Student cuts an object into four pieces and then gives an answer of a half.
10	Student is using cubes to solve the problem and announces because the cubes cannot be cut there are some cubes leftover.
11	Student says he/she cannot divide the item (i.e. a Gloop) because he/she does not know what it is
12	"I can't answer this question"
13	Never ask leading questions (i.e. would you call something that you cut into four pieces a quarter?)

14 Student asks, "What's a centimetre?" (or a metre)

15 Student says, "I don't know how big a centimetre (or metre) is"

16 Student states he/she can't answer the metre question because a metre won't fit on the page.

17 Student asks, "How big are the brownies, cookies etc."

RA Answer

1. "I am not sure". If they are unable to proceed and need a further prompt you should say, "Can you answer the question without knowing?"

If the word prevents them from working through the problem, stop and move to the next question.

An RA Should NOT tell them to pretend or imagine it is something else.

2. "Leftovers are not allowed". If they still don't partition, then move to the next question.

If the student says, "throw those pieces in the garbage" or "give them to someone else", stop and move on to the next question.

3. "Can you tell me more about that piece/ those pieces?" Once they answer, do not ask any more questions.

4. "Can you explain how you got that answer?" or "Could you draw a picture to show me how much each person gets?"

5. "So how much does each person get?" Once they answer, do not ask any more questions.

6. Can you tell me more about that piece/those pieces?" Once they answer, do not ask any more questions.

7. RA points to each piece once and asks, "can you tell me about the size of these pieces?", then moves on regardless of answer

8. RA should NOT say "try drawing a rectangle instead". The same applies to any shape a student is drawing and having problems working with.

9. RA should say "Can you show me the halves?" RA should not say "Do you mean these quarters here?"

Generally speaking, never give the student a term (like ¼ or ½) let them use and find their own terminology for the fractions they are working with.

10. "Could you use a pencil and paper to show me how you could cut the cubes?"

11. RA says, "I understand" and then moves onto the next question. RA NEVER says “just pretend it is something else"

12. The RA asks, “why not?” Then, regardless of answer, RA moves onto the next question

13. Instead ask "What would you call that?"

14. "It is a way to measure how long something is"

15. "It doesn't have to be exact. You can estimate your answer".

16. "You can pretend a metre is a smaller size than it really is, if it makes it easier for you."

17. “I’m not sure. You can decide.”