Methodology to Predict the Time of Blockage (ToB) for Bicycle Sharing Systems.

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ABSTRACT

Methodology to predict the Time of Blockage (ToB) for bicycle sharing systems.

Ramzi Khoury

As of 2018, Montreal's bicycle-sharing system (BSS), BIXI Montreal, is managing about 6,200 bicycles and 540 stations from mid-April till mid-November. A major problem that arises in BSS's is bicycle and/or dock availability. A more predictable system will help optimize bicycle-rebalancing and raise the customers' satisfaction of the service which, eventually, will generate incentives for mode-shifting and reducing traffic congestion. In this thesis, we used the 2017 open-source data along with basic demographic information, provided by BIXI, to create a methodology that will improve rebalancing procedures and add adjustability to a BSS. Preliminary investigation of the data shows that bicycle/dock availabilities are critical during the AM peak hours - when most users utilize the system for commute. Arriving on time in the morning is a priority for most users and providing a convenient infrastructure service for this is believed to be essential for any growing city. At the station level, we examine arrivals and departures as well as capacity and inter-station distances. At the user level, we examine gender, age, language and trip duration. The developed methods will allow bicycle sharing systems to study the variation of the Time of Blockage (ToB) as the user profile changes. This is very useful when there is interest in geographical expansion of the system. The methods will also allow for real-time monitoring of the ToB instead of using pre-set dispatch times. This will definitely optimize bicycle rebalancing operations - maximizing convenience, availability and users' satisfaction.

Acknowledgements and Dedication

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CHAPTER I: INTRODUCTION

1.1 General Background of Bicycle Sharing Systems (BSS)

In 1965, the bicycle-sharing concept made its first appearance under the name of Witte Fietsen or White Bicycles in the Dutch capital, Amsterdam. The program was considered to be a failure and had to quickly shut down due to the high rates of bicycle theft and damage. A similar situation was observed in Cambridge in the UK in 1993 where around 300 bicycles were stolen which automatically led the program to cease its existence. The system in La Rochelle, France (1974) on the other hand, proved to be successful and remains in operation today (Shaheen et al., 2010).

For nearly 30 years, and aside from La Rochelle's and Cambridge's initiatives, the world didn't witness any attempt to initiate another major bicycle-sharing program. Up until in 1995, the Danish Capital, Copenhagen, witnessed a coin-operated program that was a clear improvement to what Amsterdam had back in 1965 and introduced the second generation of bicycle-sharing programs. However, acts of vandalism and theft were still standing firm as clear major problems. Shortly after this, in 1998, the first IT-based system makes an appearance in the city of Rennes in France, finding a solution to this problem where bicycle tracking technologies were implemented and unreturned bicycles could be located. The smart-bicycle concept then grew and expanded from city to city making its way to Lyon and Paris in France as well as Barcelona in Spain, Washington D.C. in the United States, Montreal in Canada, Hangzhou in China and many other cities around the world. In 2009, BIXI Montreal is launched officially in Montreal in full-scale. A new bicycle and docking systems technology is developed that spreads to many major cities in North America, Australia and the UK. In 2010, bicycle-sharing systems are international, and major cities around the globe adopt and install these systems (Shaheen et al., 2010).

The city of Guangzhou in China integrated its program with their local Bus-Rapid Transit system in attempt to cover what is known as the "last mile" which is a term used to describe the movement of people from a transportation hub to their respective homes (The Bike-Share room, 2018). Such integration would definitely maximize connectivity and make the transportation infrastructure more convenient to the users. Many benefits encourage cities around the world to implement and install such systems. Bicycle sharing systems increase connectivity and mobility, they reduce traffic congestion and carbon dioxide emissions and they induce demand to other modes of public transit. Such systems motivate a healthier community that has greater environmental awareness, save money for users and allow lower implementation and operation costs when comparing it to other modes of transportation (Shaheen et al., 2010).

BIXI Montreal currently has 6,200 bicycles and 540 stations spread out across Montreal, Westmount and Longueuil serving greater Montreal. In 2017, the network witnessed 554,890 occasional users and 46,262 members (BIXI, 2018). The number of members increases every year and more people are becoming dependent on it to move around and commute every day to work. Managing such a big system with increasing demand is challenging especially that the network is extremely dynamic. The convenience and availability of the service, however, should not be compromised. These two properties are the most critical during the early hours of the day when people head to their work since it is rush hour and everybody wants to make it on time.

1.2 Problem Statement

Bicycle sharing systems are becoming more and more popular as the days go by. The world has witnessed an impressive decline towards bicycle use for commute over the past years and many major cities are adopting these systems and integrating them into their infrastructure. A bicycle sharing system that offers convenience, reliability, safety and availability will by default steal a share of users from other modes of transportation. This promotes healthier lifestyles, less traffic congestion and helps us achieve UN sustainable development goals that concern the environment.

Integrating a system into a city is always a challenge, but the bigger challenge lies in the management of the system which imposes many difficulties. Bicycle rebalancing and truck dispatch procedures have been headline problems for many major cities and this significantly affects the provided service. Some cities use pre-set dispatch times for rebalancing which fails to cater for the needs of the system in different occasions, other cities use maps, their own application to track station statuses and knowledge on traffic conditions instead. Also, cities experience difficulty in predicting the time when a newly installed station gets full as there is no enough data on the station and information transfer from existing stations is usually difficult as no station is fully similar to the other. Making the system as real-time as possible is very desirable, however, thinking about solutions such as integrating GPS tracking devices on bicycles is not only very expensive at the current moment, but also bicycle theft is expected to rise in case of such implementation.

1.3 Research Objective

The objective of this thesis is to make the system more available and readier to serve the public during the most critical time of the day by providing two methods: System Adjustability Method and System Improvement Method.

The System Adjustability method predicts the time of blockage (ToB) at any station as basic demographic information changes, making the system very adjustable and pro-active for sudden changes or future station planning.

The System Improvement method analyzes the behavior on a user level. It fits the travel time distribution of specific users into a mathematical model that allows predicting the arrival time based on user-specific behavior.

1.4 Research Contribution

We developed two methods in this thesis. The first method allows the operator to determine the time when the station fills up, referred to as Time of Blockage (ToB), as we change the number of trips, the origin of these trips and basic demographic information of every user making these trips. This would allow the operator to perform a series of sensitivity analyses given variations in the built environment and/or attractors within the proximity of the station of study. Also, this method is very useful in the context of station planning where station information is taken from an existing station (that has similar attributes to the new station), transferred and modified according to the expected age and gender distributions of the new station that is planned to be installed. This method also discusses adjustment factors that could be used to evaluate the performance of rebalancing procedures.

The second method allows researchers to identify users, study their behavior and their expected arrival times. This way, operators would know when to dispatch re-balancing trucks and make the system more available and convenient avoiding the situation where someone arrives at a station and finds it full (blockage).

1.5 Thesis Layout

The rest of the thesis is organized as follows: Chapter 2 provides a literature review on past work on this topic. Chapter 3 discusses the system adjustability method (impact of age and gender), analyzes data and provides results. Chapter 4 discusses the system improvement method, analyzes data and provides results. Chapter 5 concludes the thesis findings and states future-work.

CHAPTER II: LITERATURE REVIEW

Over the past years, research has inclined more towards i) finding optimal locations for stations by studying the nearby infrastructure, land-use and the built environment, ii) optimizing the route taken by rebalancing vehicles to redistribute bicycles around the network and thus reducing costs of operations, iii) analyzing and studying the effect of different rebalancing procedures and iv) determining the number of bicycles to leave at every station (inventory) after rebalancing.

2.1 Rebalancing Strategies

Fricker and Gast (2010) study the effect of users' choices on stations that have balancing problems. They quantify the influence of a stations capacity on rebalancing and they compute the optimum number of bicycles (inventory) that needs to be at any station while minimizing the total number of stations in the network that face rebalancing problems. They also compute the truck redistribution rate to insure quality service and found out that the best performance is expected to happen when a station is half full plus a few more bicycles, where the number of the extra bicycles is computed through a function of the system parameters. The authors discuss two bicycle rebalancing strategies: one happens at night when the demand is low (static repositioning) and one happens during the day when the demand is high (dynamic repositioning). Fricker and Gast (2012) also investigate in their study how the performance is affected by different user choices and different station capacities. They also compare between incentive-based strategies that induce the demand to other stations and repositioning strategies performed by trucks that incur operation costs. Chemla et al. (2012) study static repositioning and devise an algorithm that gives the minimum distance that a truck could travel to achieve given bicycle positions.

Chardon et al. (2016) provide an exploration related to time and space of bicycle sharing system rebalancing patterns of nine systems. They describe the implications for municipalities and operators and they conduct interviews and do comparisons. They discuss different rebalancing strategies. Although they evaluate rebalancing operations, they could not take into consideration the weather, operational costs and possible policy changes. In their research, the found out that 1) stations adjacent to transit hubs receive disproportionate amounts of rebalancing relative to trips, 2) rebalancing is usually in response to AM and PM peaks where the demand exceeds the capacity rather than in response to long term bicycle accumulations at stations, 3) all operators use maps and applications showing stations that are empty or full and they combine that with knowledge of traffic conditions and special events as well as historical demand statistics accumulated over time, 4) relationship between rebalancing and trips is a very complex one, 5) trips at transit hubs are very affected by the number of bicycles rebalanced, and lastly, 6) the event when a station is full or empty affects the number of trips more than expected.

Caggiani and Ottomanelli (2013) explain that BSS is used mainly for short and medium-distance trips and in many cases, one-way trips. They add that this behavior is a contributor to unbalanced distribution. The authors further add that it is essential to relocate the bicycles among the stations to increase the over-all system capacity and maximize users' satisfaction. They also present a simulation model that studies the dynamic repositioning strategy by minimizing the repositioning costs while aiming at maintaining a high users' satisfaction. They explain that the users' satisfaction increases as the probability of finding a bicycle available or a free docking spot increases. Their model explicitly considers the trucks' route choice among stations. Specifically, their work allows the determination of the optimal truck routes for repositioning as well as the number of bicycles to be repositioned.

Raviv et al. (2012) explain in their study how one of the main complaints heard from customers is not finding any bicycle at the station and even worse, the unavailability of docks upon arrival. They further add that the frequent unavailability engenders distrust and could lead to the point where users abandon the system. They discuss static repositioning strategies and how this strategy has advantage that during the night there are no parking problems and the repositioning fleet is allowed to travel the city easily with no congestion. Raviv et al. (2012) also explore static repositioning and study the optimum positioning of bicycles at the beginning of the day.

Vogel et al. (2011) analyze operational data from BSS's and derive bicycle activity patterns. Data mining was used to gain insight on activity patterns which in turn reveals imbalances in the distribution of bicycles and lead to a better understanding of the system. Their method supports planning and operating decisions for the design and management of bicycle-sharing systems.

2.2 Influence of Spatio-Temporal Factors on Demand

Faghih-Imani et al. (2014) examine the influence of weather, time of the day, land-use, built environment and bicycle infrastructure on the arrival and departure flows at the station level. They explain how these relationships will allow the identification of factors contributing to increased usage of the system. They add how this would help give insight on where to locate new stations and how big the station should be. They advise a statistical model that quantifies the influence of these elements on arrival and departure flows. The authors observe that people are more likely to use the BSS when there is good weather. They also add that the bicycle flows decrease as you go further away from the downtown and during weekends, bicycle-use decreases during the day but increases during the night. It was also observed that adding a new station has a stronger impact on bicycle flows when compared to adding capacity to a station.

Borgnat et al. (2011) model the time evolution of the dynamics of movements of the French BSS, Velo'v, and it investigates spatial patterns to visualize and understand bicycle flows in the city of Lyon. In other words, they study time and space to understand and predict how many trips are generated, where people go and the evolution in time. The authors find out that 1) many people use shared bicycles as an intermediate to get to subway or bus stations during the morning and afternoon hours for commute 2) the average cycling velocity was found to be 12-14 km/h and 3) the mean number of rented bicycles is found to non-stationary and a periodic repetition over the week has been noticed.

Yang et al. (2016) propose a spatio-temporal bicycle mobility model that is based on historical and weather data and they devise a mechanism on a station level to predict traffic. They use a probabilistic model to describe bicycle movements within the network and a random forest prediction algorithm to estimate the number of docked bicycles at a station. To the best of the authors' knowledge, they are the first to devise a traffic prediction mechanism on a per-station basis with sub-hour granularity. Their model is fine-grained and estimation results are continuously updated. The authors predict bicycle check-ins of bicycles that checkout before the time of the study, named t_now, by integrating concepts of transfer probabilities between stations i and j, departures from station i and a probability of checking-in at station j within a target period. To predict the check-ins for bicycles that departed after the time of study (t_now), they use a random forest model. They integrate online features into their study to obtain check-out times of bicycles.

Bachand-Marleau et al. (2012) conducted a survey to determine the factors that encouraged individuals to use the system and what elements influenced them to use it more. Socioeconomic and spatial factors that affect the likelihood of use were studied as well. They found out that 1) the factor having the biggest impact on likelihood was found to be the proximity of home to docking stations, 2) BSS's can maximize their potential by installing more stations in neighborhoods, 3) public transit users, multi-mode travelers and those who possess a driver's license were more likely to use bicycle-sharing systems, 4) individuals see shared bicycles as an active travel option which minimizes bicycle theft, and, 5) The better the design of the bicycle, the more people will use the system.

Zhang et al. (2016) study the trip prediction problem for BSS's using DIVVY Chicago's data. They analyze the users' behavior and they introduce a new trip destination and trip duration inference model. They performed extensive analysis about the user composition and studied the temporal and spatial usage behavior. They devise two regression based inference models allowing the prediction of the trip duration and destination for a user. The two models depend on nine features related to the user, the departure time and the station pairs.

2.3 Other Studies

Garcia-Palomares et al. (2012) propose a GIS based method to calculate geographic distribution of bicycles of the expected demand of the local population. They use the same method to locate new stations and define the demand characteristics. They showed the possibilities for integrating location-allocation models with GIS.

Faghih-Imani and Eluru (2015) examine the BSS behavior at the trip level to analyze the user preference on destination by using multinomial logit model. They also generate utility profiles allowing them to do a visual representation of the trade-offs user make in the decision process. They found out that 1) millennials are willing to drive less, which makes the BSS more appealing to them, 2) people tend to choose stations with longer cycling paths nearby, 3) users prefer choosing stations of higher capacity as their destination station, and, 4) Stations with higher job but lower population densities were given a higher priority as a destination station.

Faghih and Eluru (2016) devise a new technique to correctly study the impact of BSS infrastructure - further minimizing bias and errors. They explain that in earlier research, bicycle usage is considered as a dependent variable while BSS infrastructure is considered as an independent variable. In developed models, it is observed that factors influencing the dependent variable (usage) also strongly influence the independent variable (infrastructure) as many stations are installed based on expectation of system usage. They propose a multi-level joint econometric framework that remedies the over-estimation due to ignoring the BSS infrastructure installation decision process. They propose an equation to account for the installation process and relate it to the usage equations correcting for bias. They found out that 1) the model estimates support their hypothesis, 2) bicycle-sharing infrastructure is not randomly allocated in the urban region, and, 3) weather characteristics as well as the time of the day and weekend variables have a significant impact on BSS usage.

Researchers studied activity patterns and bicycle movements. They predicted arrivals and estimated departures according to algorithms. Other researchers characterized the system, used GIS to locate new stations, studied the effect of station capacities on demand, studied the factors affecting demand and devised methods that would allow obtaining optimized routes for rebalancing procedures. Yet, no one studied the time of blockage as an entity and how it varies as basic demographic information changes. Also, no one has contributed to the literature regarding user identification and modeling a specific user's behavior. Research was always general about users and the maximum depth investigated was gender and age. In the literature, we also do not see any methodology that pinpoints months or stations suffering from improper rebalancing procedures. All of these uninvestigated topics constitute this thesis – contributing to the literature in many areas that have one thing in common: The time of Blockage (ToB).

CHPATER THREE: BSS ADJUSTABILITY METHOD

In this study, the open-source data for the 2017 season was used along with more information (age and gender for every trip – obtained from BIXI Montreal), to associate as many attributes as possible to every bicycle check-out at any station. The radial distance between stations, temperature, precipitation and day of the week was also augmented, allowing us to filter down and study trips on a station-level as well as on a user-level.

Given the harsh winter that the city of Montreal witnesses every year, the BIXI Montreal season starts on April 15th and ends on November 15th of every year. We decided to focus our study on morning commute for two reasons: 1) we believe it's the most critical as users need to arrive on time for work and 2) the data shows higher consistency in the morning. To carry this out properly, we had to identify our morning peak interval. It is during that interval that arriving on time or finding available bicycles/docks is essential.

3.1 Peak Interval Determination and Data Filtration

In order to eliminate as much factors as we can that affecting bicycle demand (i.e. weather conditions) some data processing and cleansing was done. First, we removed rainy days (as per Environment Canada) and also weekend-days throughout the season, since these are expected to behave differently – which is supported by the literature included in chapter 2. The filtration process left us with 90 days of good weather that happen to be not a Saturday nor a Sunday. Plotting the cumulative number of departing trips (from all stations) against the time of the day for 4 different randomly-chosen days with good weather (June 22nd, July 18th, August 24th and September 22nd) throughout the summer season, helped us determine this morning interval. We tried to space these days almost a month apart so that we can capture the most variation. As per (Figure 3.1), the trend was found to be very similar on all these days during the morning hours. By studying how fast the number of trips increase per unit time, we determined our peak interval which was found to be between **7:30 and 9:00 (AM Peak).**

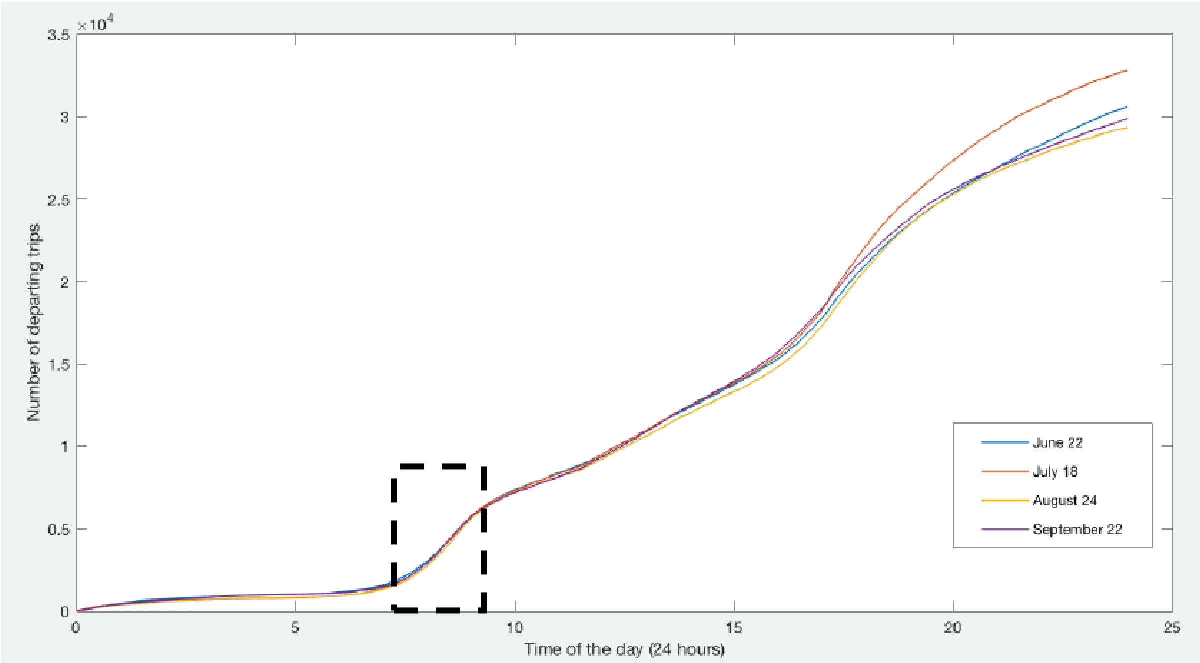


Figure 3.1 - Cumulative trips per day

All data was filtered down to trips that departed from all stations during this morning peak interval and trips that also arrived at all stations during the same interval. These trips also took place when the weather was good, no precipitation (rain), no snow and on week-days only (M, T, W, TH, F). We also add one layer of filtration which includes only the trips belonging to subscribers to BIXI Montreal with a season pass and not occasional users who will not be using the system for the same reason: having different trip purposes and different cycling speeds. To support the exclusion of occasional user, we examined all trips throughout the season during the AM peak interval and we found out that occasional trips compose **4.77%** of total trips, which is considered insignificant. Talking in numbers: 22387 occasional user trips from 469478 total trips throughout the season.

During most of the BIXI Montreal season, the busiest two stations in the AM peak interval were found to be i) Square Victoria (Viger/du Square Victoria) and ii) de Bleury/Mayor in this order. Since station Square Victoria is currently performing as a "depot" station, as in there will always be bicycles available and there is no specific capacity, station de Bleury/Mayor was chosen as our station of study. It is located in the downtown of Montreal and is within short proximity of Place-des-Arts metro station. Since this station is located in the central business district/ downtown, we expect an inflow of bicycle traffic and not an outflow. This automatically gives "finding a spot to dock the bicycle" higher priority than "finding a bicycle to check-out from that station" during the early morning.

3.2 Generic Speed Generation

The goal of this method is to be able to predict the time of blockage without having any information on real trip duration. The real trip duration is the only variable from one user to the other and it varies according to exercise habits, gender and age. We could replace this actual time by introducing a generic speed and using radial distances between stations as they are trajectory-independent. All trips heading towards station de Bleury/Mayor were studied on the trip level. The latitude and longitude coordinates of the origin and destination stations were used to get the radial distance between the two stations. This distance was divided by the actual recorded trip duration to get a generic speed for every user. Users were categorized according to age and gender into 10 categories as shown in (Table 3.1) and the generic speed for every category was calculated. Since there is no available tracking information on which route every user uses to get to the destination station, this generic speed will be used to help us estimate the time of arrival when a bicycle checks-out.

Gender			
Male		Female	
Age	Gen. Speed (m/s)	Age	Gen. Speed (m/s)
20-30	3.10	**20-30**	2.88
30-40	3.09	**30-40**	2.82
40-50	3.03	**40-50**	2.74
50-60	2.93	**50-60**	2.77
60+	2.73	**60+**	2.46

Table 3.1- Generic Speed According to Gender and Age

In other words, since we do not have the actual trajectory of the bicycles and we cannot know the distance actually crossed to reach the destination, the radial distance between the origin and destination stations is used instead. This distance is divided by the actual trip duration to obtain a generic speed for every trip. Generic speeds are then averaged for every age and gender category. Dividing any radial distance between two stations by the generic speed should give us an estimated trip duration that is close to the actual one as the actual one was used with radial distance to obtain the speeds. Different months are expected to better predict the actual durations than other months as the average generic speed per category is an underestimation for some months but an over estimation for other months. This is to be explained and described thoroughly later through the sections of this thesis.

The real data includes all departing trips (out-flow) and arriving trips (inflow) to the station of study. However, the estimated data that our method generates is only the arrivals (inflow) to the station of study obtained using the starting time from origin stations (or check-out time) with age, gender and radial distance information. To incorporate departures, we study the real-time departures from our station of study throughout the month of study and we obtain an average departure rate. Using this rate, we generate a linear departure line equation and we record the departure time once a whole number of bicycles is observed. This output of departures (outflow) is combined with our estimated arrivals (inflow) and is drawn as one plot resembling the predicted status of our station (station status = inflow-outflow).

3.3 Adjustment Time Calibration

The real data that shows all bicycles arriving and departing from station de Bleury/Mayor between 7:30 am and 9:00 am was plotted for all 90 good days in the season (Table 3.2). The time of departure of all trips that headed towards station de Bleury/Mayor from their respective stations, was used along with the distance between these two stations and the users expected generic speed (according to the category in which they belong), to estimate the time of arrival. We observed that this was always an under-estimation in terms of the number of bicycles arriving and an over-estimation for the time of arrival for all days. A clear adjustment had to be made so that we get closer to the real situation. Although it is believed that because we are using radial distance instead of actual distance (which is shorter), we should expect smaller trip durations and thus underestimating, the contrary is observed. This can be explained by the fact that once a bicycle checks out, travel doesn't necessarily start immediately. Idling time and dock searching time are expected. This, by default, increases the trip duration forcing having generic speeds slower than the real speed. Slower generic speeds predict longer trip durations, explaining the over-estimation. To correct for this, the peak interval was broken down into sub-intervals (7:30-7:45, 7:45-8:00, 8:00-8:15, 8:15-8:30, 8:30-8:45, 8:45-9:00), and the over-estimation was calculated for every recorded bicycle number and averaged within every sub-interval for every study day. The adjustment factor per interval was averaged across all days within every month to obtain adjustment factors for every interval representing every month. When we apply this method to a study day, these adjustment values are to be deducted from the predicted arrival times since our prediction before the adjustment is an over-estimation. Estimated data is then fit into a Gaussian model. The Gaussian function, named after Carl Friedrich Gauss, is a continuous function representing the probability density function of the normal distribution. It has three constants a, b and c, where a is the height of the bell curve's peak, b is the center of the peak and c is the standard deviation. This function was used as our model as it had the most significant negative log likelihood with the least RMSE. This function was found to best describe our data.

De Bleury/Mayor station has a capacity of 27 bicycles. The open-source data does not show when the station is emptied from bicycles or when it is refilled. In our Gaussian model we accumulate the bicycle arrivals and we take away departures so that we can observe how many we have at the station. The graphs we demonstrate accumulate bicycles as well as ignore the station capacity limitation. Knowing the station's capacity, we can use our model to determine at what time the station is expected to fill up (time of blockage) and knowing this time, we can schedule our rebalancing operations.

The advantage of the proposed method is that we can choose any station to study and take real data about the arrivals and departures of that station. We can perform a sensitivity analysis by changing the time of departure of the trips, choosing different origin stations and changing the age and gender for every trip in order to see how this affects the time when our station of study fills up. We can also use the developed method without using real data where we can create our own scenarios and evaluate the system. The age and gender distributions are expected to change when we witness development in the built environment and land use within proximity of the station of study and having such a method would save time given its simplicity and user friendliness. Having such a method is very useful when a new station is planned to be installed and an approximate ToB needs to be determined. Data from a station that has similar characteristics can be transferred and the age and gender distribution can be changed to suit what we expect at the new location. This method could of course be further examined to allow for prediction beyond a single year in future work.

The detailed methodology is included in section 3.4 and a sample calculation for July 20th is included in Appendix B.

2017							
April	**May**	**June**	**July**	**August**	**September**	**October**	**November**
18	2	7	3	1	1	2	7
20	3	8	4	3	8	3	8
24	4	9	5	7	11	5	10
27	8	12	6	8	12	6	13
	9	13	11	9	18	10	14
	10	14	12	10	19	11	15
	11	19	18	11	20	12	
	12	21	20	14	21	13	
	15	28	26	16	22	16	
	16		27	17	25	17	
	17		28	21	26	18	
	19			23	28	19	
	23			24	29	20	
				25		23	
				28		25	
				29		31	
				30			
				31			

Table 3.2 - Study Days in 2017 -Good weather & Weekdays.

3.4 Detailed Adjustment Time Calibration Methodology

Before we go into the methodology it is necessary to define the workbooks used for the database as there will be referencing throughout the detailed process. These workbooks were prepared using Microsoft Excel and then imported to PostGreSQL, which is a Structured Query Language Software for database management. Since we are dealing with a lot of data, SQL was deemed right to use.

3.4.1 Workbooks used

Workbook A – Study Days in 2017 (Table 3.2)
Good weather (according to Environment Canada) + Weekdays.

Workbook B – Departures
Retrieved from bixi.com and augmented with more data provided by bixi (Age, Gender, Language), the data in this workbook is filtered to only include good weather days, weekdays, only members and start times between and including 7:30 and 9:00.

Columns: Index, month, day, starttime, startstation, endtime, endstation, duration, age, gender, language, member.

Workbook C – Arrivals

Retrieved from bixi.com and augmented with more data provided by bixi (Age, Gender, Language), the data in this workbook is filtered to only include good weather days, weekdays, only members and end times between and including 7:30 and 9:00.

Columns: Index, month, day, starttime, startstation, startstation_latitude, startstation_longitude, endtime, endstation, endstation_latitude, endstation_longitude, duration, age, gender, language, member, radial distance, CAT, speed.

Workbook F – Generic Speeds for every Category (Table 3.1)

Category	Speed (m/s)
Male 20-30	3.1
Male 30-40	3.09
Male 40-50	3.03
Male 50-60	2.3
Male 60+	2.73
Female 20-30	2.88
Female 30-40	2.82
Female 40-50	2.74
Female 50-60	2.77
Female 60+	2.46

Workbook X – Station Capacities

Included in Appendix A.

3.4.2 Algorithm for Methodology

PROMPT
- STUDY_STATION_CODE
- MONTH
- DAY

STEP 1

OPEN **Workbook B - Departures** and FILTER table according to information retrieved from prompt.

COPY column 'D' entitled 'start time'.
ORDER ASC.
INSERT new column to the right and assign a value of '-1' in every row corresponding to every data entry.
NAME table: **TABLE 1.**

STEP 2

OPEN **Workbook C - Arrivals** and FILTER table according to information retrieved from prompt.

COPY column 'J' entitled 'end time'.
ORDER ASC.
INSERT new column to the right and assign a value of '+1' in every row corresponding to every data entry'.
NAME table: **TABLE 2.**

STEP 3

UNION **TABLE 1** and **TABLE 2.**
SELECT all items.
ORDER ASC.
INSERT new column C with header 'Cumulative'.

i.e.:

Column A	Column B	Column C
Time	**+1/-1**	**Cumulative**
7:30	+1	1
7:30	-1	0
7:31	+1	1
7:32	+1	2
…	…	…

*if there was no entry at 7:30, add one with a cumulative of 0.

DRAW Column C vs. Column A.
USE 'STUDY_STATION_CODE' to LOOKUP 'CAPACITY' FROM '**Workbook X Column E'.**

USE ‘CAPACITY’ retrieved to enter the graph from the y-axis
RECORD timestamp corresponding to point of intersection.
NAME timestamp ***ToB_real***.

<u>STEP 4</u>

OPEN **Workbook B - Departures** and FILTER table according to STUDY_STATION_CODE and MONTH information <u>ONLY</u> retrieved from prompt (study station as start station).

COPY Column ‘D’ entitled ‘Start time’.
PASTE in a new table.
SELECT all entries.
ORDER ASC.
INSERT new column to the right and assign a value of zero to the first row in the new column.
SUBTRACT every row from the one beneath it in the first column. The subtraction answer goes to the second column.

i.e.:

	Column A	**Column B**	**Column C**
#	**Start time**	**Inter-departures**	**Calculation**
1	6:39	*Leave Blank*	-
2	6:40	0:01	A2-A1
3	6:41	0:01	A3-A2
4	6:44	0:03	A4-A3
…	...	…	…

AVG Column B.

CALCULATE $\mu_{month} = \frac{1}{AVG\ (Column\ B)}$

INTRODUCE equation $n = \mu_{month} \cdot t_n$

Where n is the number of departed bicycles.
t_n is the timestamp when N bicycles depart ($7{:}30 \leq t_n \leq 9{:}00$).

SOLVE for t_n for every bicycle departure up until the last bicycle (N) in the $7{:}30 \leq t_n \leq 9{:}00$ timeframe.

i.e.:

$$\left.\begin{aligned} 1 &= \mu_{month} \cdot t_1 \\ 2 &= \mu_{month} \cdot t_2 \\ 3 &= \mu_{month} \cdot t_3 \\ 4 &= \mu_{month} \cdot t_4 \\ 5 &= \mu_{month} \cdot t_5 \\ &\ \ \dots \\ N &= \mu_{month} \cdot t_N \end{aligned}\right\} \text{Solve for } t_n \text{ for all.}$$

COPY $t_n$values and RECORD in a new table.
SELECT ALL.
ORDER ASC and assign a value of ‘-1’ to each t_n entry in a new column.

i.e.:

	Column A	Column B
1	t_1	-1
2	t_2	-1
3	t_3	-1
…	…	…
N	t_N	-1

NAME table AS ‘**TABLE A’**

STEP 5

OPEN **Workbook C - Arrivals** and FILTER table according to ALL information retrieved from prompt (study station as end station).
COPY all rows in columns: E – ‘Start time’, D – ‘CAT’ and X – ‘Radial Distance’.
CREATE a new table.
LOOKUP the speed corresponding to every CAT listed in column B using Workbook F and associate a speed to every data entry.

i.e.:

	Column A	Column B	Column C	Column D
1	Start_time_1	CAT_1	Radial_Distance_1	Speed_1
2	Start_time_2	CAT_2	Radial_Distance_2	Speed_2
3	Start_time_3	CAT_3	Radial_Distance_3	Speed_3
…	…	…	…	…

DIVIDE Column C (distance) by Column D (speed) to obtain the estimated duration for each entry and put the result in a new column.

i.e.:

	Column A	Column B	Column C	Column D	Column E
1	Start_time_1	CAT_1	Radial_Distance_1	Speed_1	Duration_1
2	Start_time_2	CAT_2	Radial_Distance_2	Speed_2	Duration_2
3	Start_time_3	CAT_3	Radial_Distance_3	Speed_3	Duration_3
…	…	…	…	…	…

ADD estimated duration to start time and put the estimated arrival time in a new column.

i.e.:

	Column A	Column B	Column C	Column D	Column E	Column F
1	Start_time_1	CAT_1	Radial_Distance_1	Speed_1	Duration_1	End_time_1
2	Start_time_2	CAT_2	Radial_Distance_2	Speed_2	Duration_2	End_time_2
3	Start_time_3	CAT_3	Radial_Distance_3	Speed_3	Duration_3	End_time_3
	…	…	…	…	…	…

*Sample: F1 = (E1/24*60*60) +A1

COPY all rows in Column F.
CREATE new table.
PASTE rows in a new table.
INSERT new column to the right and assign a value of '+1' in every row corresponding to every data entry.
NAME table AS '**TABLE B**'

i.e.:

	Column A	Column B
1	End_time_1	+1
2	End_time_2	+1
3	End_time_3	+1
…	…	…

STEP 6

UNION **TABLE A** (from step 4) and **TABLE B** (from step 5).
ORDER ASC.
INSERT column to the right and name it 'Cumulative'.
ADD a row on top for the status at time 7:30 if there is no entry at that time. Assign a value of 0 in the +1/-1 and cumulative columns.
ADD constraint to the 'Cumulative' column that the values should always be bigger than 0 an if less than 0, assign a value of 0.

i.e.:

	Column A	**Column B**	**Column C**
	Arrival time	**+1/-1**	**Cumulative**
1	7:30	0	0
2	t_1	-1	0 (=B2+C1)
3	End_time_1	+1	1 (=B3+C2)
4	t_2	-1	0 (=B4+C3)
…	…	…	…

STEP 7:

MATCH the Column C in the previous table with the Column C of the table below retrieved from Step 3 to lookup the corresponding real and estimated arrival time for each bicycle in the system. If the same number of bicycles is recorded at two different times, average them.

Table from Step 3 (Real Data):

Column A	**Column B**	**Column C**
Time	**+1/-1**	**Cumulative**
7:30	+1	1
7:30	-1	0
7:31	+1	1
7:32	+1	2
…	…	…

For the same number of bicycles in both tables, SUBTRACT the real timestamp from the estimated.

$$\Delta = t_{predicted} - t_{real} \qquad for\ all\ bike\ numbers.$$

CATEGORIZE the accumulated bicycle numbers into 15 minute-intervals depending on when (estimated) that accumulated number happened.

AVERAGE the Δ's for every 15 minute-interval obtaining 6 averaged Δ's.

i.e.:

15 minute-interval	**Accumulated bicycle numbers**	$\boldsymbol{\Delta}$	**Avg.** $\boldsymbol{\Delta}$
7:30-7:45	1	1 minute	Δ_1 = 1.5 minutes
	2	1.5 minutes	
	3	2 minutes	
7:45-8:00	4	1.25 minutes	Δ_2
	…	…	
8:00-8:15	…	…	Δ_3
8:15-8:30	…	…	Δ_4
8:30-8:45	…	…	Δ_5
8:45-9:00	…	…	Δ_6

OUTPUT the 6 Δ values as an end result for our study day in the study month at the study station.

REPEAT all 6 steps for all different days that are weekdays and of good weather for the same month of study.

STEP 8:

UNION all output tables from all study days in a month.

i.e.:

15 minute-interval	Days in a study month				Avg. Δ
	Day 1	Day2	…	Day N	
7:30-7:45	Δ_1	Δ_1	Δ_1	Δ_1	Avg. Δ_1
7:45-8:00	Δ_2	Δ_2	Δ_2	Δ_2	Avg. Δ_2
8:00-8:15	Δ_3	Δ_3	Δ_3	Δ_3	Avg. Δ_3
8:15-8:30	Δ_4	Δ_4	Δ_4	Δ_4	Avg. Δ_4
8:30-8:45	Δ_5	Δ_5	Δ_5	Δ_5	Avg. Δ_5
8:45-9:00	Δ_6	Δ_6	Δ_6	Δ_6	Avg. Δ_6

COPY first column and last column in previous table.
CREATE new table.
PASTE these two columns there.

i.e.:

15 minute-interval	Avg. Δ
7:30-7:45	Avg. Δ_1
7:45-8:00	Avg. Δ_2
8:00-8:15	Avg. Δ_3
8:15-8:30	Avg. Δ_4
8:30-8:45	Avg. Δ_5
8:45-9:00	Avg. Δ_6

Table becomes:

15 minute-interval	**Avg. $\varDelta$**
7:30-7:45	Avg. $\varDelta_1$
7:45-8:00	Avg. $\varDelta_2$
8:00-8:15	Avg. $\varDelta_3$
8:15-8:30	Avg. $\varDelta_4$
8:30-8:45	Avg. $\varDelta_5$
8:45-9:00	Avg. $\varDelta_6$

NAME table: '**Adjustment Factors – Study Month'**

<u>STEP 9:</u>

USE data of random days for validation to plot the real status at the station of study. One plot for every day (STEP 1 and 2).

RUN Steps 4 to 6 on these validation days and then USE the Adjustment Factors Table to adjust for the plot.

<u>STEP 10:</u>

FIT estimated data into a Gaussian model and obtain values for a, b and c.

$$f(x) = a \cdot e^{-\left(\frac{x-b}{c}\right)^2}$$

USE capacity retrieved from Step 3 to enter the Gaussian fit plot and read-off the timestamp.

$$Capacity\ of\ station = a \cdot e^{-\left(\frac{x-b}{c}\right)^2}$$

SOLVE for x.
NAME timestamp: ***ToB_estimated***.

*Validity of this method can be checked by comparing ToB_real with ToB_estimated. This way we compare the actual time of blockage with the expected time of blockage from our estimations.

3.5 Validation

We validated our data using 20 days throughout the season (Table 3.3). These days were selected to be either Tuesdays, Wednesdays or Thursdays because these three are believed to behave similarly, providing better consistency and more accurate inferences. The real and estimated ToB's are also tabulated for every validation day. It is important to note that the months of April and November were discarded as they do not represent whole months and the good days in these months were too few to draw conclusions.

After adjustment, we can see that now the ToB estimated from our model is generally an underestimation as it should be given that we use radial distances and not actual distances. Some days (denoted by an asterisk in table 3.3), still showed significant over-estimation. This is discussed and investigated after the plots for all the 20 days.

Day	ToB Real	ToB Model
11-May	8:47:00	8:42:59
17-May	8:23:00	8:20:07
23-May	8:27:00	8:23:39
7-Jun	8:25:00	8:21:36
14-Jun	8:14:00	8:11:26
28-Jun	8:29:00	8:26:46
4-Jul*	8:18:00	8:19:00*
11-Jul*	8:05:00	8:08:00*
18-Jul	8:07:00	8:07:35
20-Jul*	8:04:00	8:14:00*
3-Aug	8:23:00	8:23:23
10-Aug*	8:29:00	8:30:12*
30-Aug	8:35:00	8:31:03
31-Aug	8:26:00	8:23:30
19-Sep	8:29:00	8:29:11
21-Sep	8:28:30	8:24:52
28-Sep	8:28:00	8:27:01
3-Oct	8:29:00	8:29:14
11-Oct*	8:26:30	8:31:55*
18-Oct	8:48:00	8:44:36

Table 3.3 - ToB of Validation Days

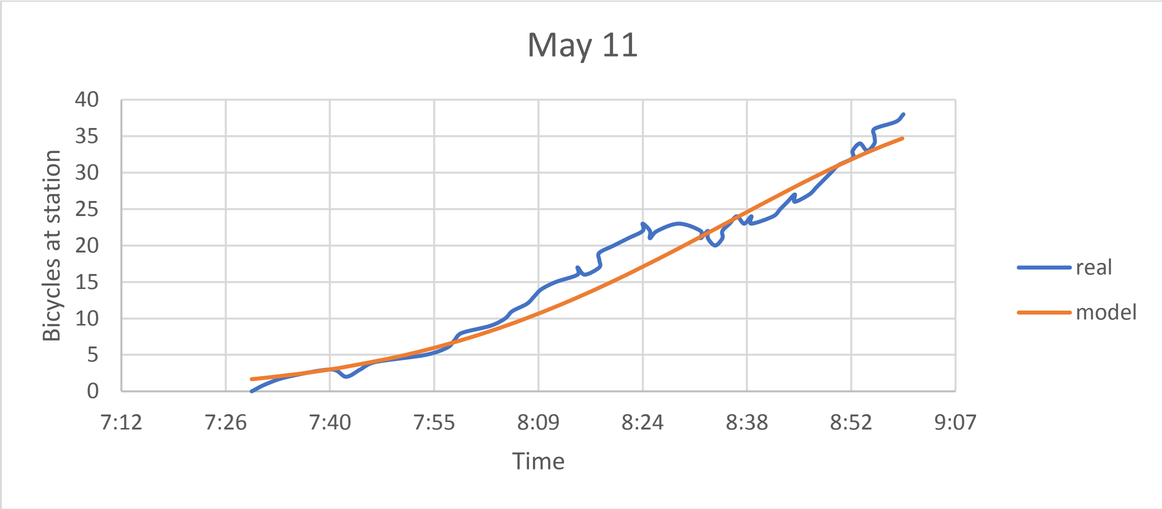


Figure 3.2 - May 11

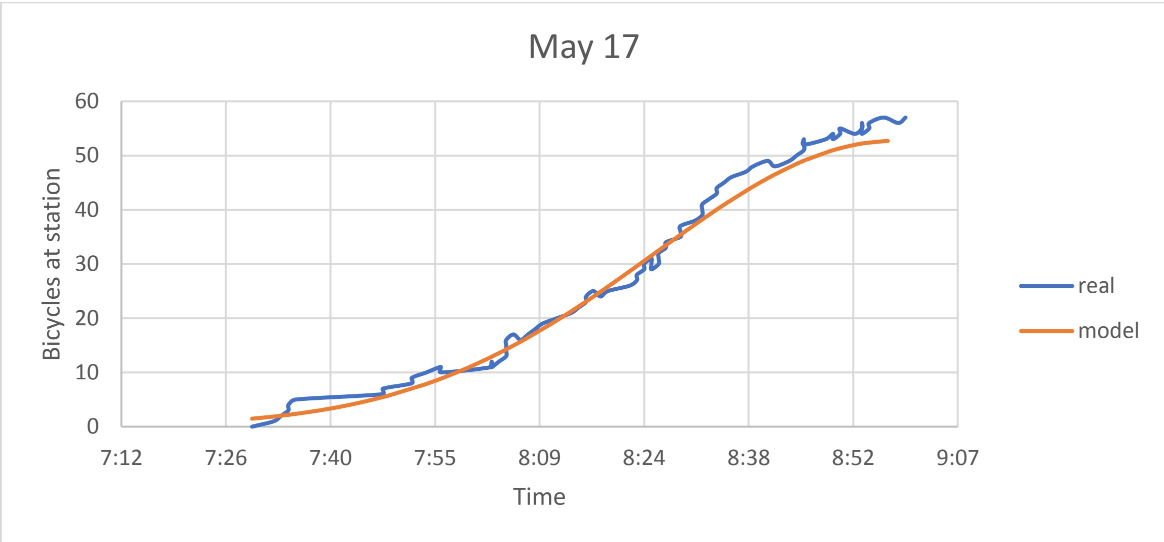


Figure 3.3 - May 17

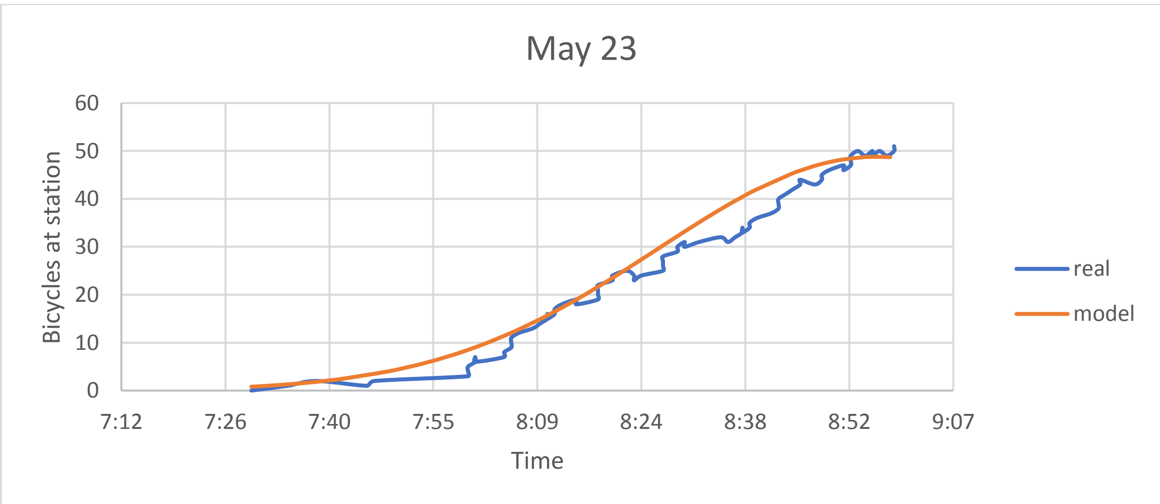


Figure 3.4 - May 23

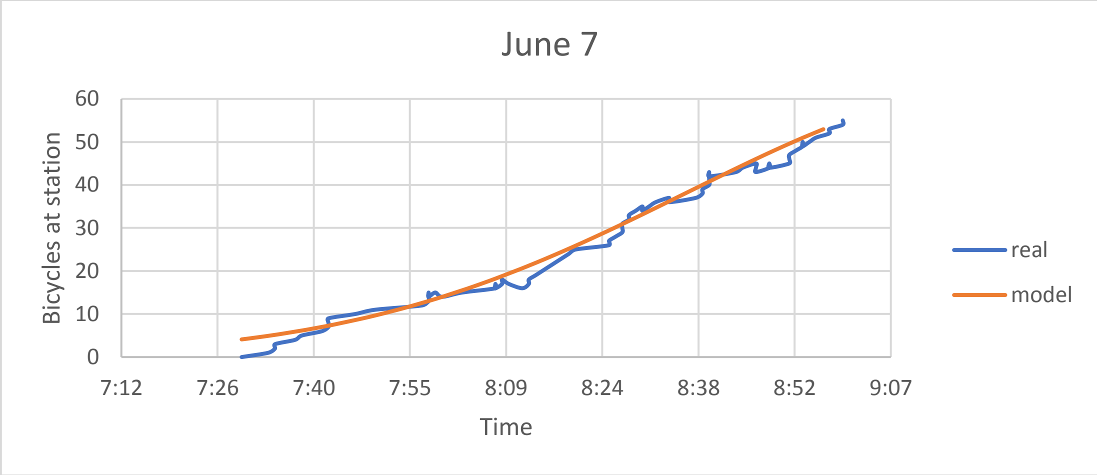


Figure 3.5 - June 7

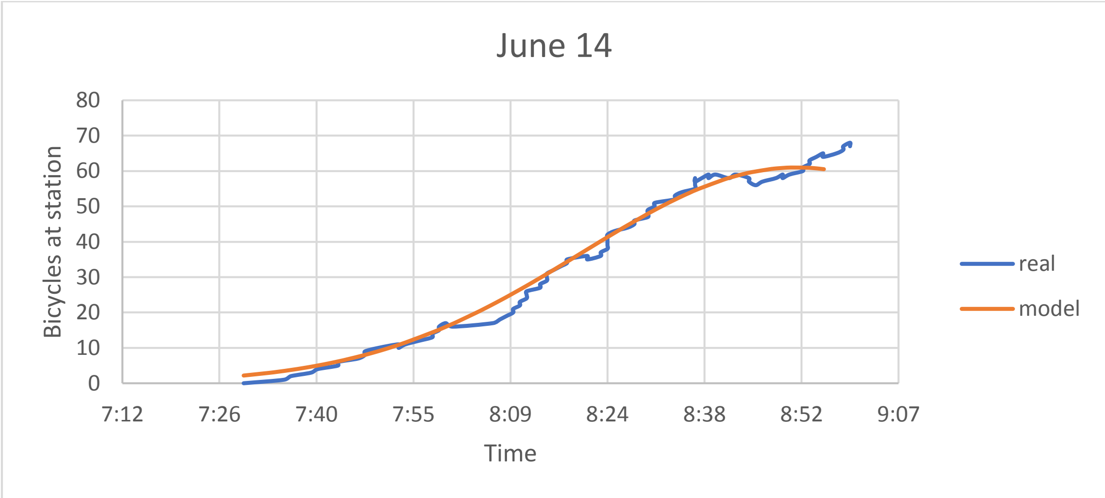


Figure 3.6 - June 14

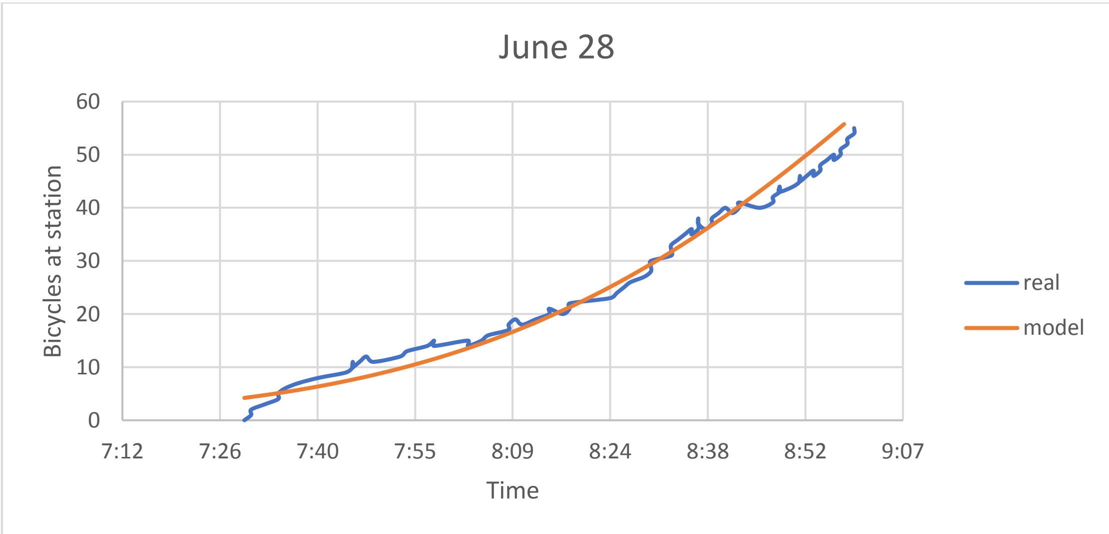


Figure 3.7 - June 28

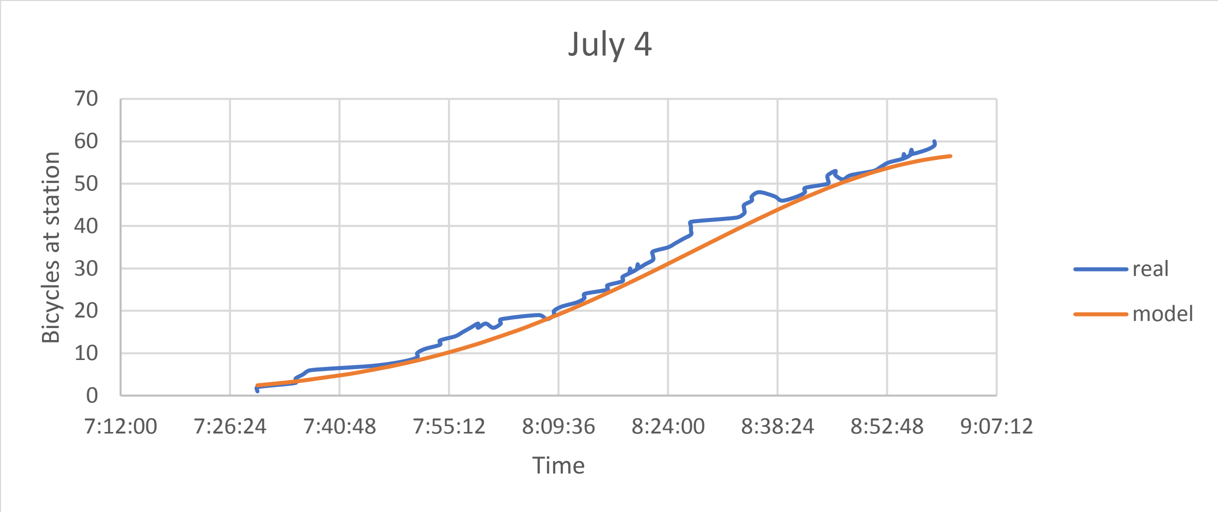


Figure 3.8 - July 4

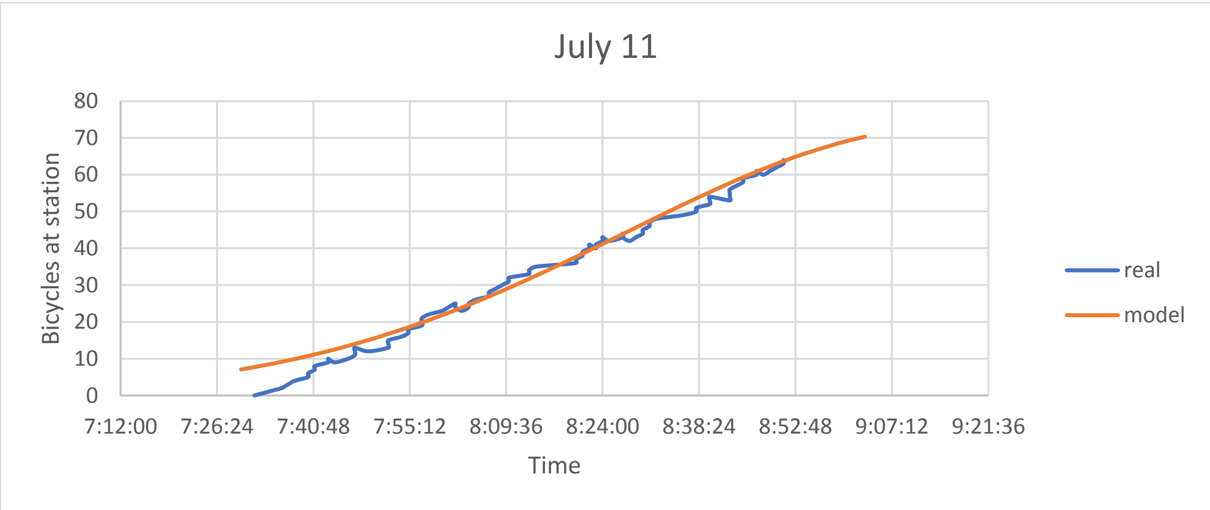


Figure 3.9 - July 11

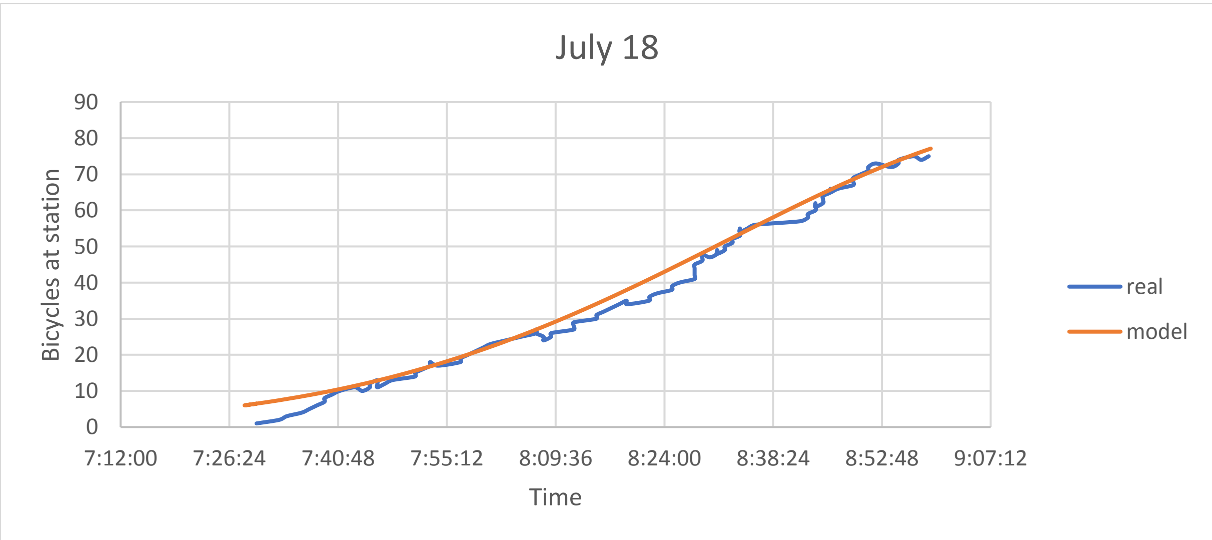


Figure 3.10 - July 18

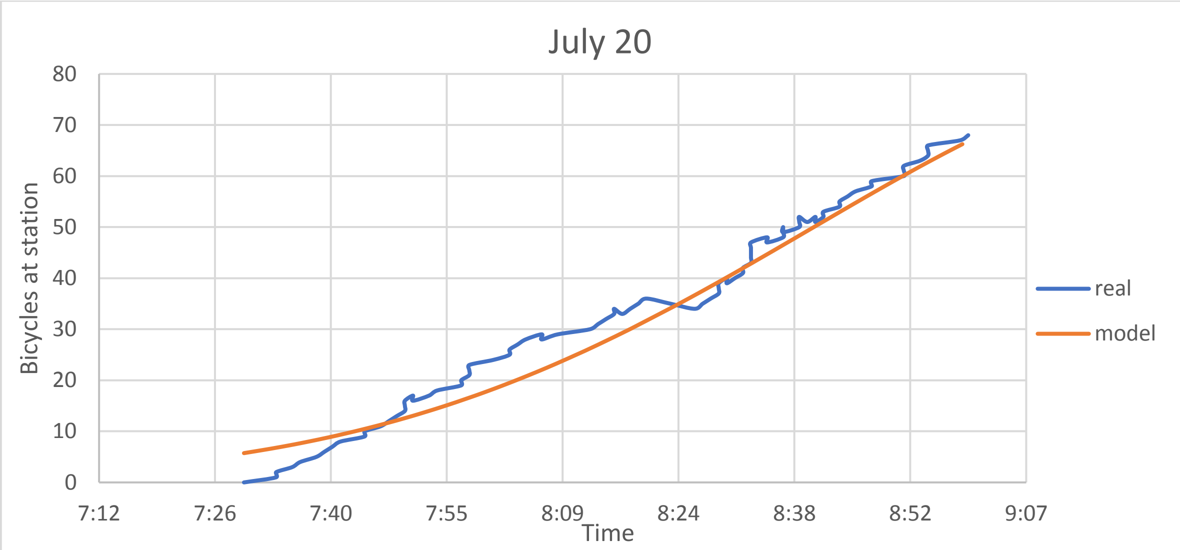


Figure 3.11 - July 20

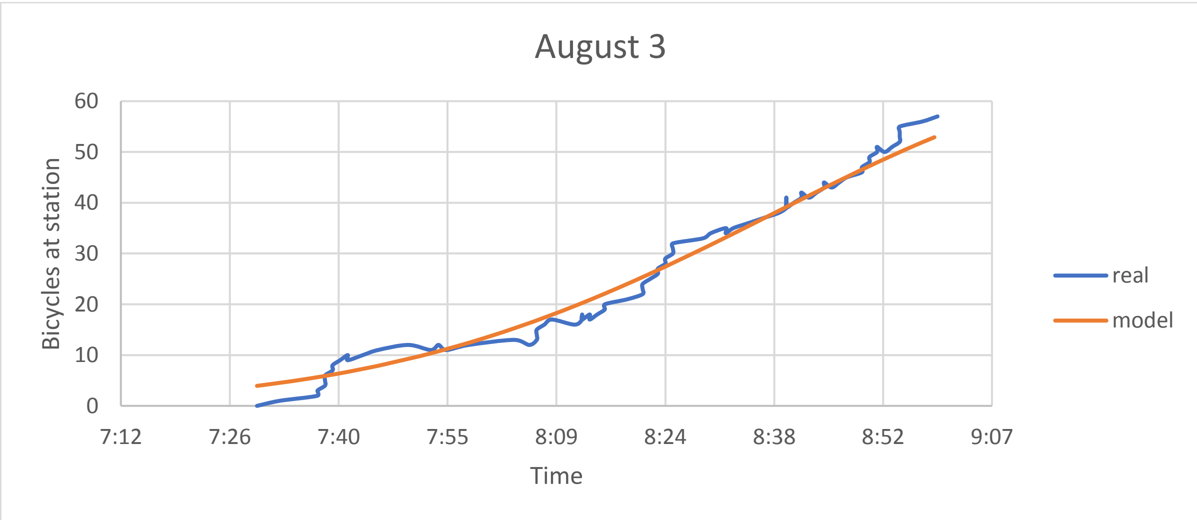


Figure 3.12 - August 3

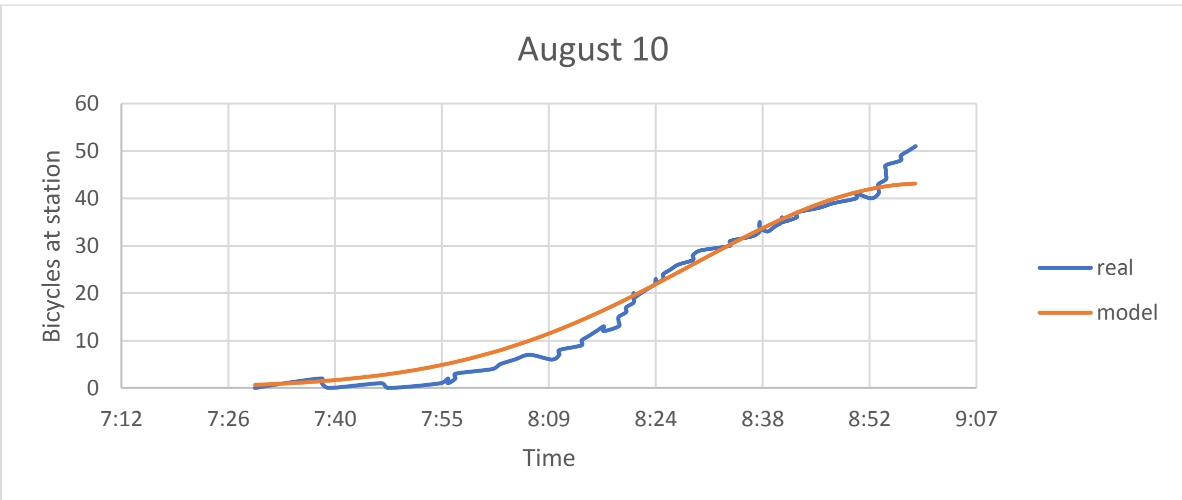


Figure 3.13 - August 10

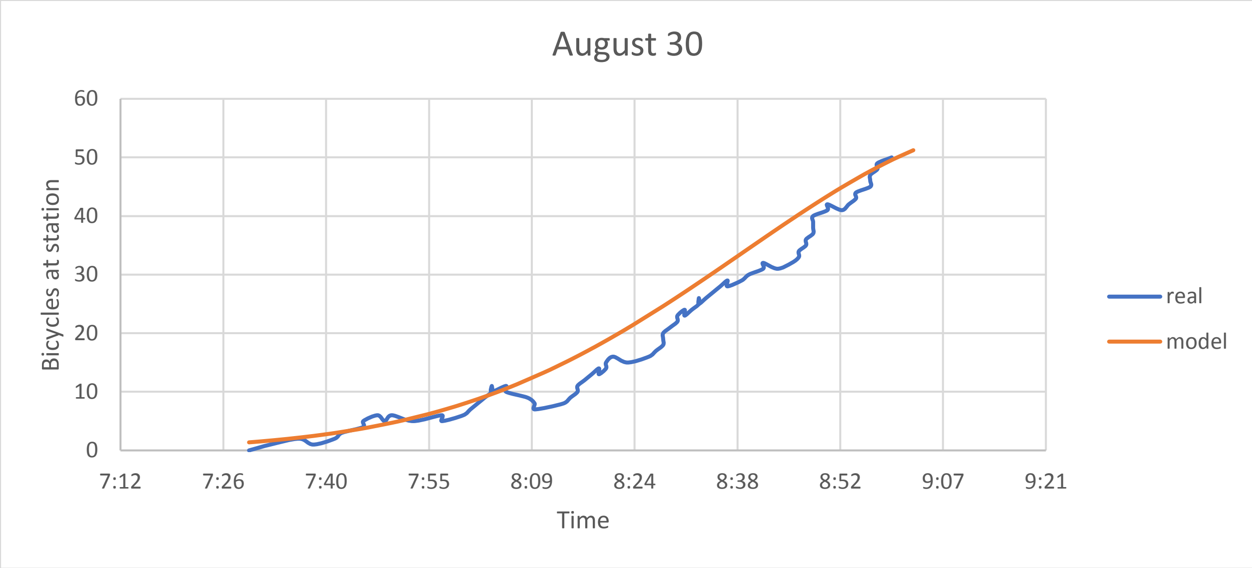


Figure 3.14 - August 30

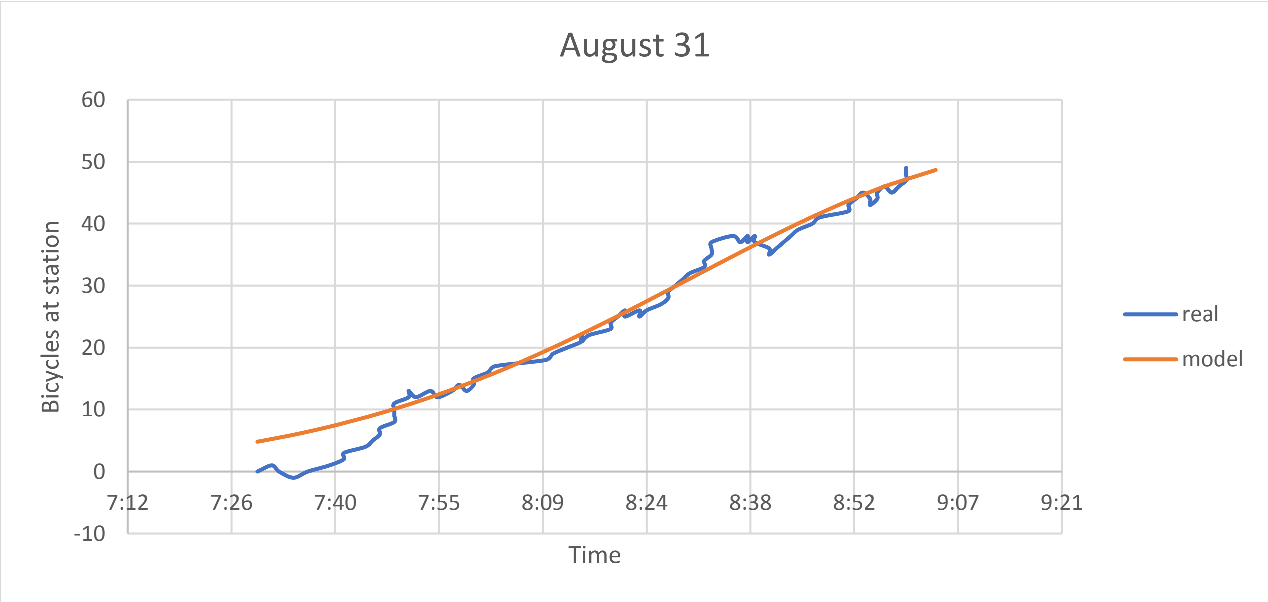


Figure 3.15 - August 31

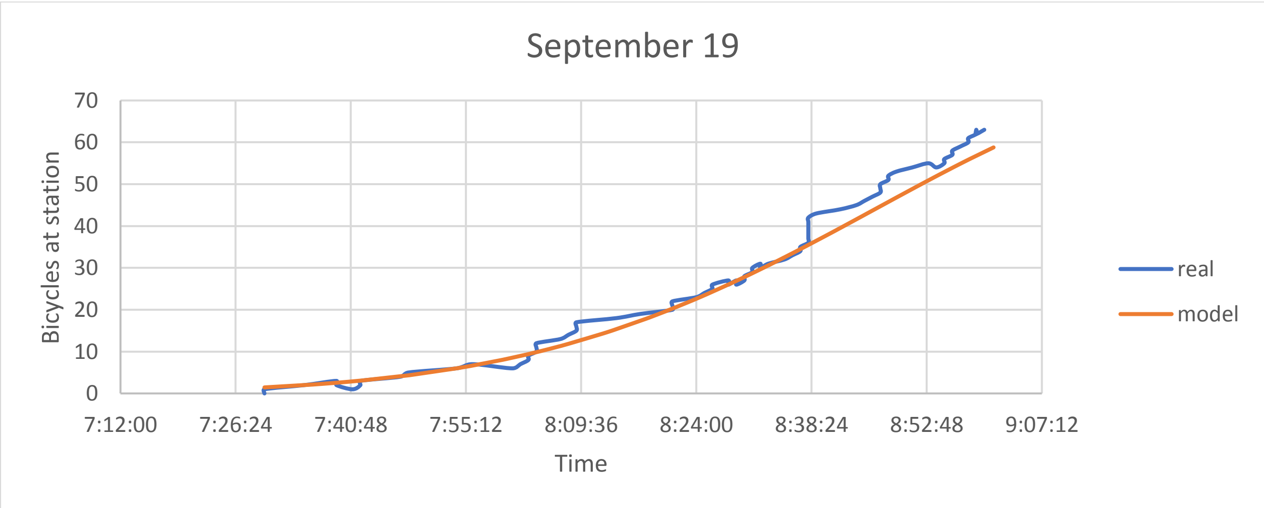


Figure 3.16 - September 19

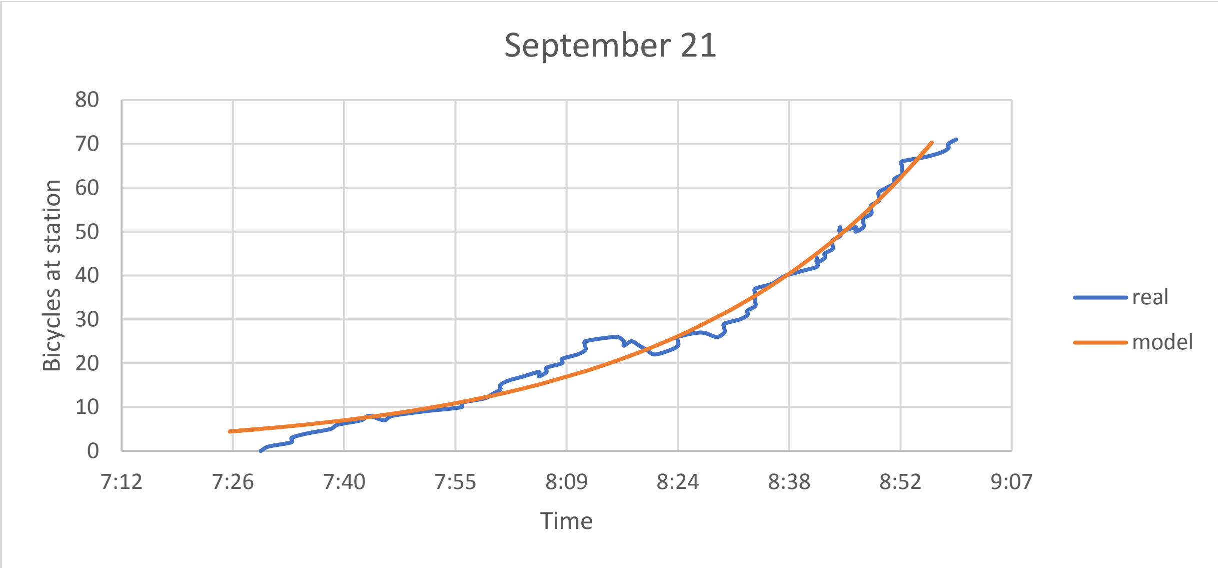


Figure 3.17 - September 21

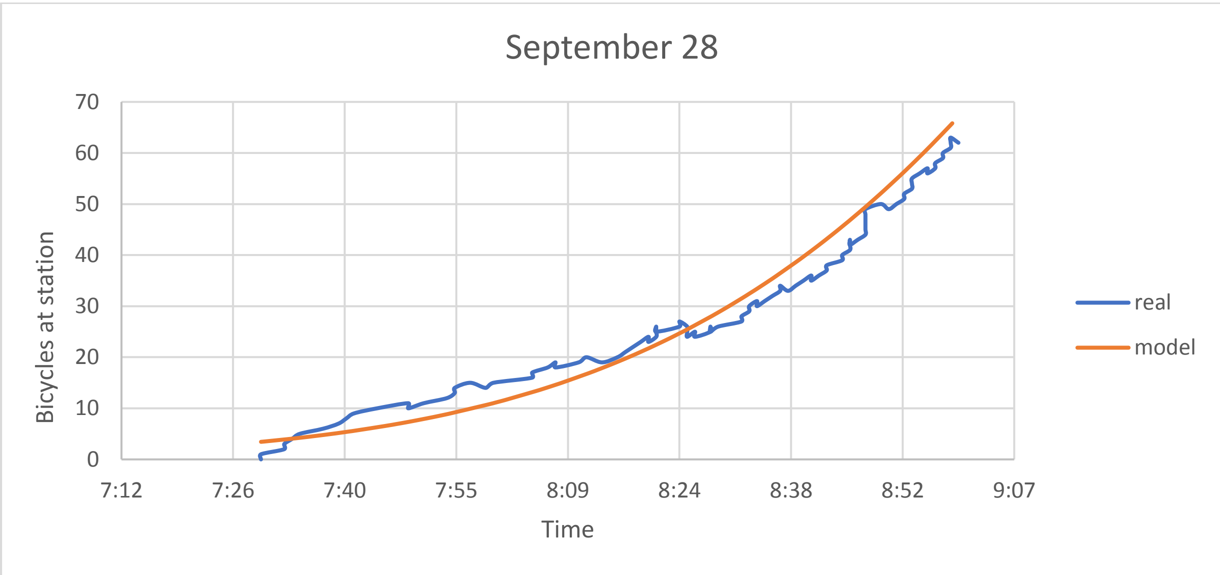


Figure 3.18 - September 28

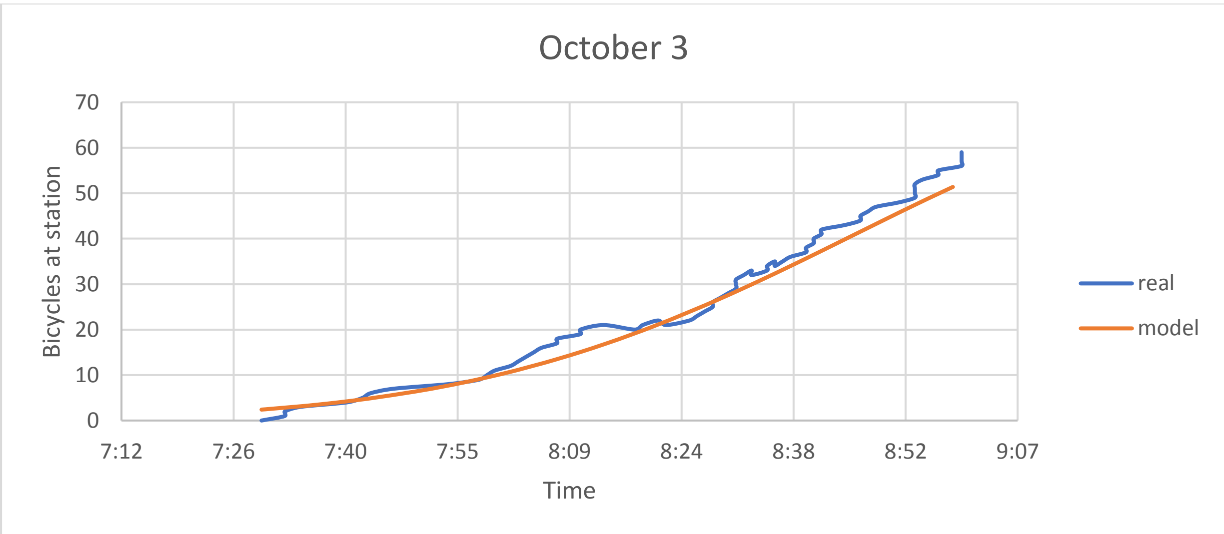


Figure 3.19 - October 3

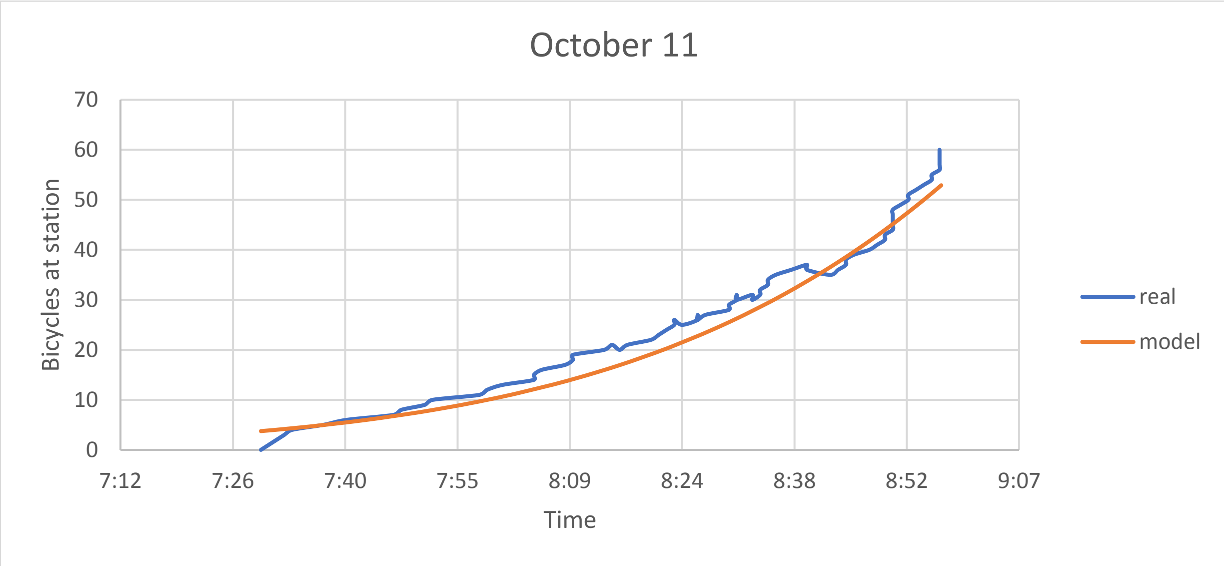


Figure 3.20 - October 11

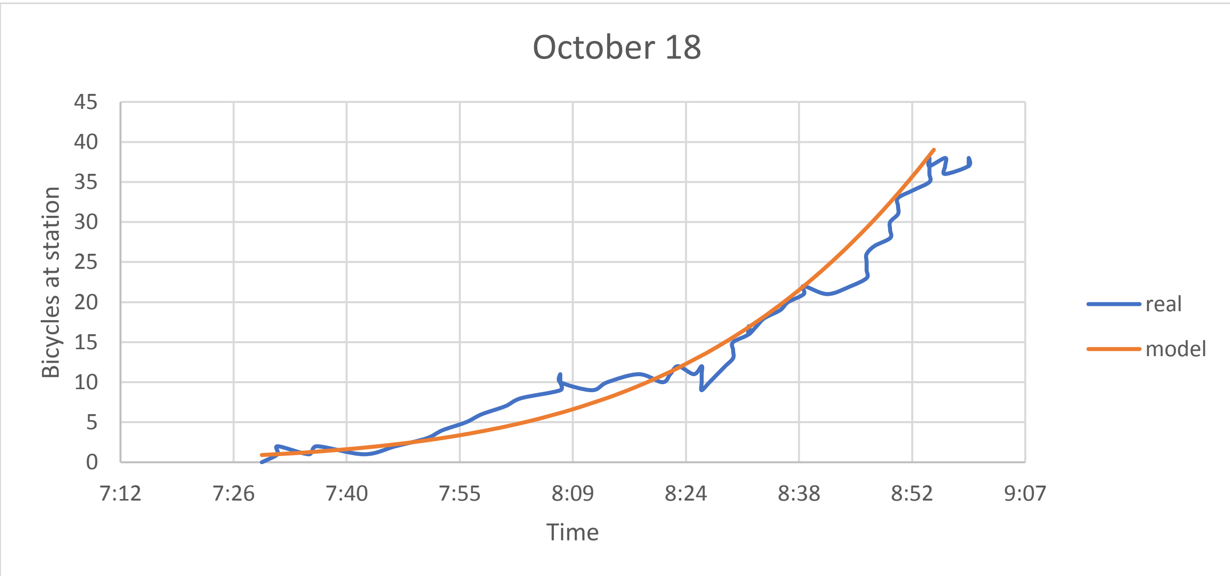


Figure 3.21 - October 18

From these results we can see that the methodology is actually working in estimating when the ToB is expected to happen and that the **advised speed for every age-gender pair is doing the job**. However, unexpected behavior was still witnessed with the days denoted by an asterisk as the estimation was still a slight over-estimate instead of an underestimate. This required further investigation as it will help shed the light on how our methodology could be improved. Details are included in the next section.

3.6 Results Justification and System Characterization

The adjustment values are tabulated below and then plotted for every month between 7:30 and 9:00.

2017 Season - 6078							
#	**Sub-interval**	**May**	**June**	**July**	**August**	**September**	**October**
1	**7:30-7:45**	03:16	02:24.4	01:44.3	02:31.4	01:50.7	02:44
2	**7:45-8:00**	05:56	07:31.7	03:10	05:41.9	05:36.4	06:30
3	**8:00-8:15**	06:58	05:23.8	03:54	07:15.9	05:39.2	06:28
4	**8:15-8:30**	05:05	03:51.0	02:59	05:17.7	03:59.4	06:25
5	**8:30-8:45**	05:06	05:16.5	01:35	03:57.8	02:06.9	04:02
6	**8:45-9:00**	03:12.1	04:38.6	02:48	02:57.7	02:04.2	04:17

Table 3.4 - Adjustment Factors

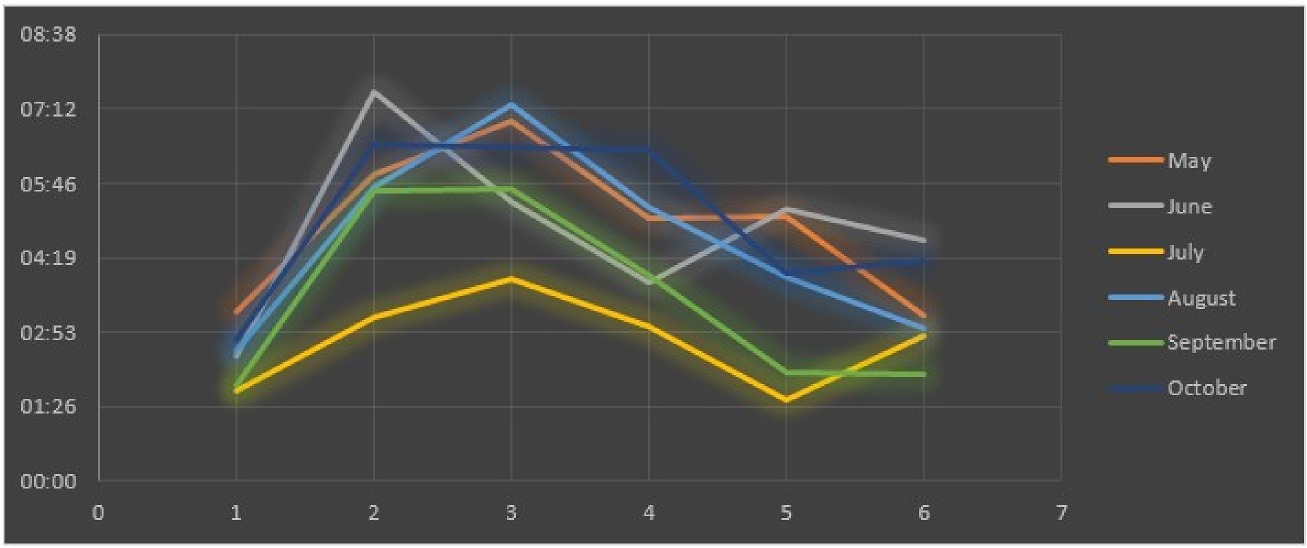


Figure 3.22 - Plot of Adjustment Values

From the above plots, it can be noticed that July is having the smallest adjustment values throughout the peak interval. July is followed by September then June then the three of August, May and October. It is also noticed that all months have a turning point at the third sub-interval (8:00-8:15), except June which has its turning point at the second sub-interval (7:45-8:00). May, August and October are noticed to behave similarly.

Given that demand is variable and that the rebalancing frequencies should keep up with the variation, the adjustment value between months shouldn't be much different. This is because, for commute, on a trip basis, the radial distance doesn't change among months and the duration of the trip also shouldn't change much. However, this is given that proper rebalancing procedures are implemented, successfully catering for the demand.

It could also be argued that a change in the cumulative arrivals adjustment factors could be observed because of extra demand on longer trips taken during peak seasons. An increase in the number of trips traveling longer distances will affect the monthly speed because as the distance of travel increases, the bigger the difference between the radial distance and the actual distance traveled to complete the trip. Longer actual travel distance brings with it longer travel time and thus dividing the radial distance by the actual trip duration would give us smaller trip speeds. Averaging all these together in a high demand month, such as July, would give us a very low monthly speed. Assigning the seasonal speed to July (as done in our calculations) would predict a trip duration that is less than that if the monthly speed was used instead (which should give adjustment values that are higher). This could be a reason to why the adjustment is so low in July. This whole argument would hold valid if the radial distances, which we use in our calculations, for the peak month are found to be greater than off-peak months. In order to answer this, the average radial distance for every month per age-gender pair is obtained and compared. The results are tabulated below in Tables 3.5 and 3.6. We can clearly see that the average radial distance doesn't witness significant increases. For 92.11% of trips (obtained from Table 3.7 – summing the percentage composition of females under the age of 50 and males under the age of 60) heading to our study station during the whole season, the average radial distance difference between the highest two months in terms of distance (which happened to mainly be June and July), for every age-gender pair, spans from 3m to 22m. This clearly shows that the change in adjustment values is mostly independent of increased demand for longer trips.

Month	Female 20-30	Female 30-40	Female 40-50	Female 50-60	Female 60+
	Distance (km)	Distance (km)	Distance (km)	Distance (km)	Distance (km)
April	1.959	2.265	2.478	2.395	2.205
May	2.087	2.298	2.504	2.479	2.431
June	2.227	2.375	2.474	2.400	2.221
July	2.243	2.389	2.545	2.542	2.133
August	2.202	2.367	2.504	2.436	2.262
September	2.052	2.312	2.553	2.271	2.182
October	2.019	2.206	2.430	2.264	2.157
November	1.783	2.005	2.120	2.034	2.086
Average	2.071	2.277	2.451	2.353	2.210

Table 3.5 - Female Distances for all Age Groups

Month	Male 20-30	Male 30-40	Male 40-50	Male 50-60	Male 60+
	Distance (km)	Distance (km)	Distance (km)	Distance (km)	Distance (km)
April	1.825	2.163	2.227	2.139	2.129
May	1.887	2.194	2.303	2.249	2.192
June	2.010	2.212	2.353	2.274	2.268
July	2.031	2.209	2.371	2.268	2.318
August	1.995	2.165	2.352	2.231	2.270
September	1.882	2.155	2.312	2.296	2.197
October	1.882	2.111	2.284	2.256	2.168
November	1.716	1.907	1.982	2.037	1.961
Average	1.903	2.139	2.273	2.219	2.188

Table 3.6 - Male Distances for all Age Groups

Age	**20-30**		**30-40**		**40-50**		**50-60**		**60+**	
Gender	**Male**	**Female**	**Male**	**Female**	**Male**	**Female**	**Male**	**Female**	**Male**	**Female**
April	924	614	1630	912	1028	492	577	358	231	69
May	4828	3242	7555	4508	4596	2395	2535	1526	834	379
June	4521	3173	6416	4072	3852	2032	2146	1230	689	346
July	5919	4559	7719	4897	4074	2226	2409	1446	716	385
August	9354	7340	13278	8369	7101	3913	4307	2416	1179	633
September	7361	5370	9612	6010	5939	3213	3250	1936	1037	542
October	7819	5454	9822	5871	6216	3405	3507	1918	1131	562
November	1329	882	2016	1078	1342	741	757	452	299	123
Total	42055	30634	58048	35717	34148	18417	19488	11282	6116	3039
Composition (%)	16.24	11.83	22.42	13.79	13.19	7.11	7.53	4.36	2.36	1.17

Table 3.7 - Percentage Composition for every Gender-Age Pair

To help us understand why each of these months is behaving this way, we broke down every sub-interval to 3 smaller intervals of 5 minutes. For every 5 minutes, the number of trips arriving at our station was recorded. This was done for every month to investigate what is happening with the demand.

	May	June	July	August	September	October	Totals
7:30-7:35	20	12	20	24	30	28	134
7:35-7:40	24	32	39	30	29	21	175
7:40-7:45	6	20	32	35	31	27	151
7:45-7:50	14	20	33	37	20	20	144
7:50-7:55	30	16	37	37	40	32	192
7:55-8:00	26	26	37	46	25	27	187
8:00-8:05	28	26	35	55	49	39	232
8:05-8:10	43	35	35	45	46	39	243
8:10-8:15	42	31	31	61	31	38	234
8:15-8:20	53	46	40	82	38	45	304
8:20-8:25	52	49	49	107	52	49	358
8:25-8:30	45	59	55	87	55	71	372
8:30-8:35	45	52	60	114	82	91	444
8:35-8:40	36	52	49	70	50	54	311
8:40-8:45	41	32	48	94	59	53	327
8:45-8:50	51	40	50	91	85	92	409
8:50-8:55	56	47	55	96	75	97	426
8:55-9:00	53	55	61	109	83	89	450
Totals	665	650	766	1220	880	912	**5093**

Table 3.8 - Trips per 5 minutes intervals

After breaking down the number of trips, we normalized the values over the number of days used in our analysis for every month so that we can allow comparison. The study days for every month are tabulated below.

Month	Days
May	13
June	9
July	11
August	18
September	13
October	16

Table 3.9 - Days Studied per Month

The following normalized (per day) trip breakdown for every 5 minutes was obtained:

#	Sub-intervals	May	June	July	August	September	October	Total
1	7:30-7:35	1.54	1.33	1.82	1.33	2.31	1.75	10.08
2	7:35-7:40	1.85	3.56	3.55	1.67	2.23	1.31	14.16
3	7:40-7:45	0.46	2.22	2.91	1.94	2.38	1.69	11.61
4	7:45-7:50	1.08	2.22	3.00	2.06	1.54	1.25	11.14
5	7:50-7:55	2.31	1.78	3.36	2.06	3.08	2.00	14.58
6	7:55-8:00	2.00	2.89	3.36	2.56	1.92	1.69	14.42
7	8:00-8:05	2.15	2.89	3.18	3.06	3.77	2.44	17.49
8	8:05-8:10	3.31	3.89	3.18	2.50	3.54	2.44	18.85
9	8:10-8:15	3.23	3.44	2.82	3.39	2.38	2.38	17.64
10	8:15-8:20	4.08	5.11	3.64	4.56	2.92	2.81	23.12
11	8:20-8:25	4.00	5.44	4.45	5.94	4.00	3.06	26.91
12	8:25-8:30	3.46	6.56	5.00	4.83	4.23	4.44	28.52
13	8:30-8:35	3.46	5.78	5.45	6.33	6.31	5.69	33.02
14	8:35-8:40	2.77	5.78	4.45	3.89	3.85	3.38	24.11
15	8:40-8:45	3.15	3.56	4.36	5.22	4.54	3.31	24.15
16	8:45-8:50	3.92	4.44	4.55	5.06	6.54	5.75	30.26
17	8:50-8:55	4.31	5.22	5.00	5.33	5.77	6.06	31.69
18	8:55-9:00	4.08	6.11	5.55	6.06	6.38	5.56	33.74
-	Total	51.15	72.22	69.64	67.78	67.69	57.00	**385.48**

Table 3.10 - Normalized trips per day

Studying and plotting the number of trips recorded for each of these 5-minute intervals shows that during the AM peak interval, we witness two peaks. We denote the first as P1 and the second as P2 (shown on Figure 3.24). We can also visibly see that before 8:00-8:05 (sub-interval 7), July had the highest demand. Real-life ToB, including having a specific number of bikes already docked at 7:30 am, is believed to happen around Sub-interval 7. Monthly demand ranking would be best done taking into consideration sub-intervals 1-7 as after sub-interval 7 many bicycles would reroute to different stations as our station of study is expected to be full.

July has the highest demand meaning that if rebalancing is not properly done, the recorded travel time increases (more time wasted as users wait for rebalancing to happen or till someone checks-out a bicycle for them to dock) which will result in a slower generic speed than other months where rebalancing is sufficient. Since we are using a generic speed that is an average to all months in our calculations, and by looking at our plot, it is believed that the seasonal generic speed used is higher than the July generic speed (indicating rebalancing not meeting demand). Having a higher generic speed will give a smaller difference between the predicted and real time for every recorded bicycle and thus smaller adjustment factors as we can clearly see on Figure 3.22. Less difference means less adjustment which would justify the over-estimation of ToB.

August and May have very similar behaviors although August has a higher demand. This is explained by the fact that rebalancing frequency is not the same in these two months. In august we have more demand, but rebalancing is more frequent. In May we have less demand, but re-balancing is less frequent. As a package, they tend to behave the same as the change in demand and re-balancing frequency is roughly proportional.

October comes in the 5th place when it comes to demand. October shows the least fluctuation and the peaks are clearly visible. Rebalancing frequencies are believed to be adjusted for the end of the season. Lower demand and reduced rebalancing frequencies maintain the adjustment values high. Mentioning all of this, it is believed that the adjustment values, if proper rebalancing happens, falls in the whereabouts of the values from October, May and August.

September is the second highest with most demand during the early minutes. Being second highest in demand means more waiting time if rebalancing wasn't done properly. September falling below May, August and October is an indication that rebalancing is not meeting the demand and hence, less adjustment values as the speed used in our calculations is more than the one specific to the month of September. The situation is like July, however, the adjustment is not as low as July's due to the difference in demand between both. According to this data, rebalancing frequencies are not believed to change much from July to September. It could be the case that the August frequency is the maximum frequency achievable by BIXI and this frequency is not able to cater properly for September and July as the adjustment factors are seen to drop in both of these months.

June comes third place in demand after July and September. However, as BIXI transitions from low frequency rebalancing season to high frequency rebalancing season, June operations are trapped in between. June experiences a high demand with lower frequency of rebalancing causing the station to fill up at the earliest time recorded. As we can see on the plot, after the 7:45-8:00 interval, June's adjustment drops significantly indicating major delays due to blockage (big demand for very low rebalancing frequencies). As June users see a blocked station and they wait, their time of travel increases, meaning their June generic speed will be lower. The seasonal generic speed is higher than that of June. This is clearly shown as the adjustment drops significantly at a rate steeper than others and with later recovery – explaining lower rebalancing frequencies.

It could be concluded that the average adjustment factors should be near the values of the months August, May and October as in these three months, both, the demand and frequency, are believed to keep up. Any plot different from that is an indication of different degrees of the demand exceeding the rebalancing frequency. In other words, this adjustment calculation method can be used as a measure to determine the efficiency and measure the performance of rebalancing procedures. More can be elaborated on this in future work.

The demand variation for different months is plotted on the next page along with the first and second peaks. The red dashed line shows the point before which July had the largest daily demand among all.

It is very important to note that in our analysis we assume that we have zero bicycles at the beginning of the peak interval. Due to limited access to data, we do not know when rebalancing takes place, how many bicycles are docked at 7:30 and how frequent rebalancing happens. The bicycle status at a station should drop to a specific pre-determined number after every rebalancing. Since we do not have such information, we assume an indefinite capacity, accumulate bicycles and use the station capacity value to determine ToB. Since the initial number of bicycles at the station is larger than zero, all obtained ToB values are believed to take place earlier than expected.

We could roughly determine the actual rebalancing times by looking at Figure 3.22. Whenever we see a turning point, rebalancing procedures are expected to have happened. Another way to study the time of rebalancing procedures is to plot the generic speed variations versus the time and deduce patterns. Whenever we observe declining speeds, blockage can be assumed to have happened.

The below plot shows the speed variation versus time for July 11, 18 and 20 as an example. We can clearly observe a trend, indicating rebalancing. The trends in the middle are steeper as these sub-intervals observe more demand per minute which leads to faster blockage and reduction in speeds.

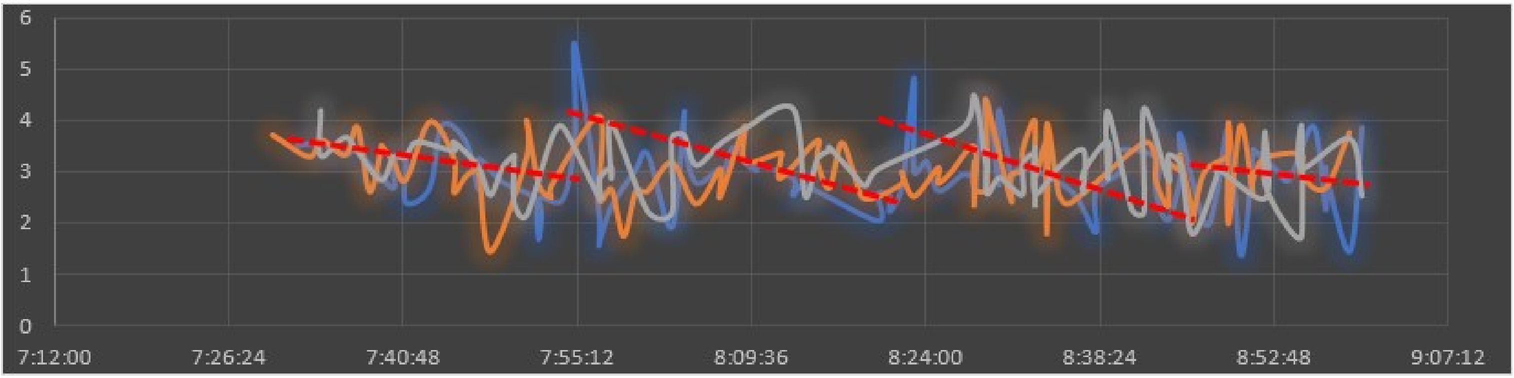


Figure 3.23 – Speed Variation versus Time

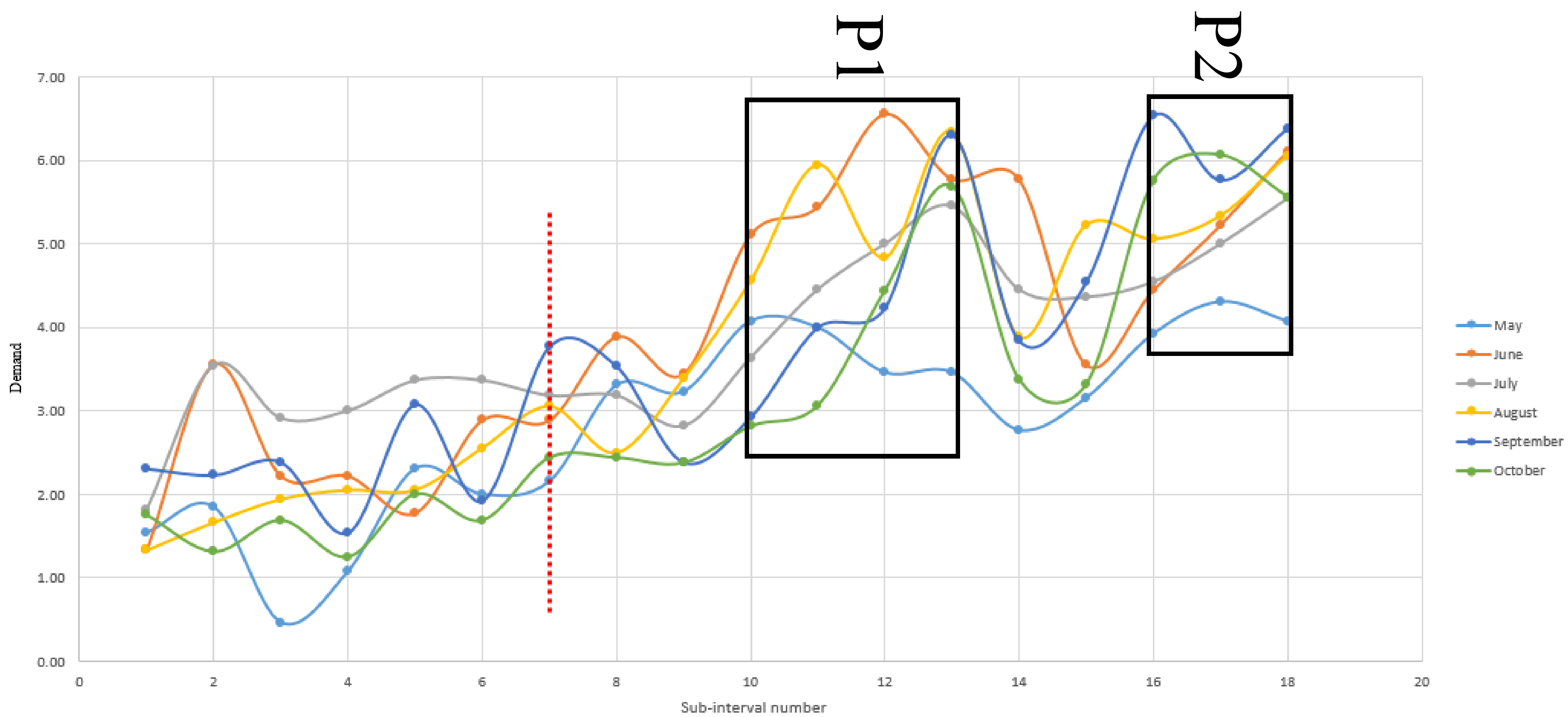


Figure 3.24 - Plot of Demand versus Time

CHPATER IV: BSS IMPROVEMENT METHOD

4.1 User-Identification System and User-Level Analysis

In order to make ToB as real-time as possible, we need to take this work down to the user-level. The records of every member with BIXI had to be obtained so that we can study the user behavior. Due to privacy policy, this data was not available. Which took us to innovate and create our own user-identification system.

BIXI provided us with the following for every trip recorded:

1) Start timestamp
2) Start station code
3) Latitude and Longitude of Start Station
4) End timestamp
5) End station code
6) Latitude and Longitude of End Station
7) Duration
8) Age
9) Gender
10) Language
11) Membership

Restricting the arrival time between 7:30am and 9:00am (as we are studying the morning peak), we created an index for every trip which allowed us to identify users. This index is placed in a column of its own corresponding to every trip recorded and is a concatenation of the start station code, end station code, age, gender and language. Every distinct index is then coupled with a "count" function to get the number of times that specific index appeared throughout the whole season. All indices are then ordered from the most occurring to the least occurring. All those occurring more than once on any study day during the peak hour, are automatically eliminated as they resemble multiple users and could be highly misrepresentative. The most occurring index from different age-gender groups is taken and studied, and thus we introduce the follows personas:

- Celeste (28, Female, 64 out of 90 days) – index: 6918_6032_28_F_fr
- Nicholas (27, Male, 70 out of 90 days) – index: 6744_6725_27_M_fr
- Amandine (38, Female, 73 out of 90 days) – index: 6182_6061_38_F_fr
- Jacques (36, Male, 70 out of 90 days) – index: 6060_6221_36_M_fr
- Jennifer (47, Female, 70 out of 90 days) – index: 6314_6078_47_F_en
- Stephane (46, Male, 76 out of 90 days) – index: 6116_6060_46_M_fr
- Jeanne (50, Female, 72 out of 90) – index: 6174_6034_50_F_fr
- Alain (51, Male, 73 out of 90) – index: 6138_6411_51_M_fr

These personas showed consistency in their behavior - departing around the same time every day for commute.

The data for every persona was studied and distribution fitting was performed using different distributions. The most appropriate distribution that described the behavior of all users was the Burr distribution.

The Burr type XII distribution has three parameters a, c and k and is on the positive real line. Parameter 'a' is the scale parameter and 'c' and 'k' are the shape parameters. The Burr distribution is used in various fields, some of which are finance and hydrology. Some of the examples of use are: household income, crop prices, insurance risk, flood levels and travel time.

Burr Distribution Equation:

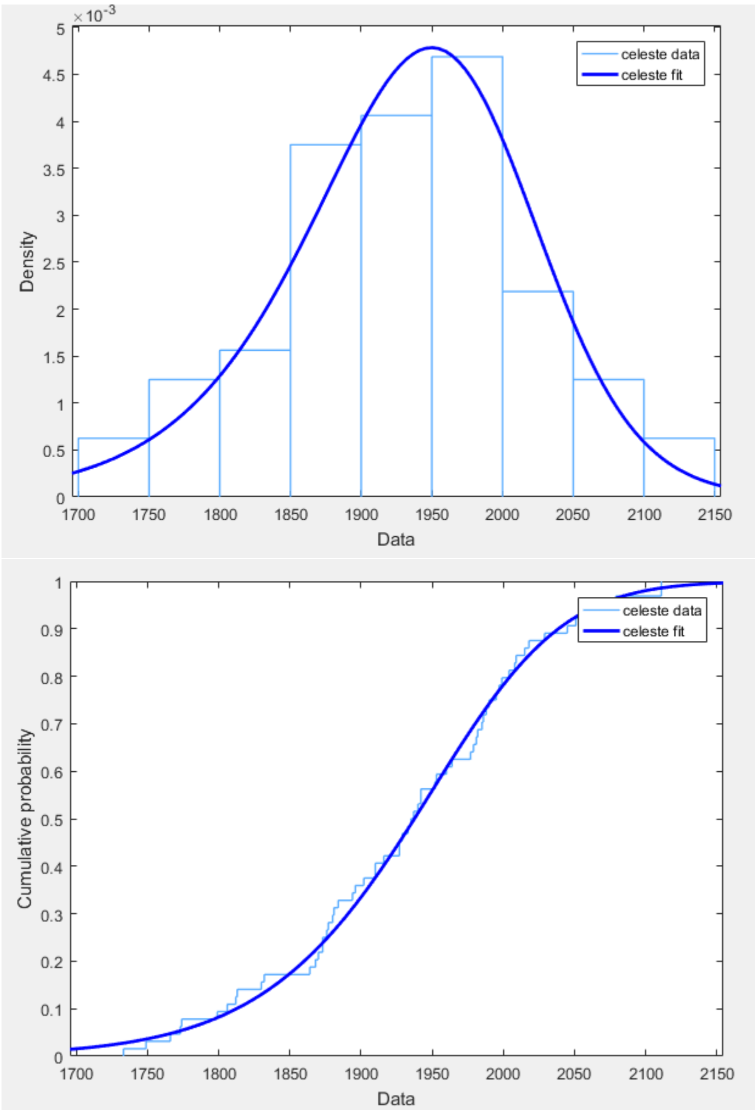
$$f(x|a,c,k) = \frac{\frac{kc}{\alpha}\cdot\left(\frac{x}{\alpha}\right)^{c-1}}{\left(1+\left(\frac{x}{\alpha}\right)^{c}\right)^{k+1}} \quad \text{x} > 0,\ \alpha > 0, c > 0, k > 0.$$

$$F(x|a,c,k) = \frac{1}{\left(1+\left(\frac{x}{\alpha}\right)^{c}\right)^{k}} \quad \text{x} > 0,\ \alpha > 0, c > 0, k > 0$$

This Burr distribution is fitted and values for a, c and k are obtained for each user. These parameters differentiate users and the mathematical model can be used to identify the 95th percentile duration when a bicycle checks out from the usual departure station for daily commute.

The data and the fitting for every persona is included on the next pages.

Celeste – 28 – Female – 64/90 – 71.11%
Index: 6918_6032_28_F_fr
95th percentile: 2066.90 seconds



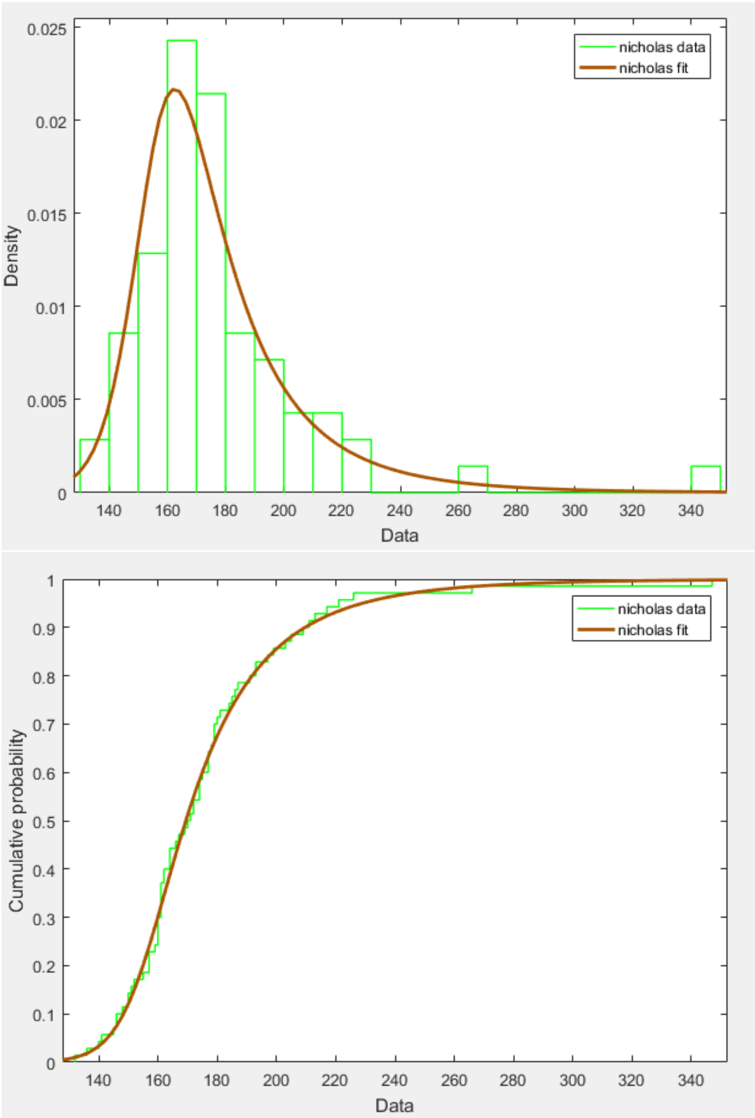

```
Distribution:     Burr
Log likelihood:   -377.88
Domain:           0 < y < Inf
Mean:             1930.33
Variance:         8407.91

Parameter  Estimate    Std. Err.
alpha      2019.37     86.7717
c          29.9222     5.50261
k          2.72238      2.4668

Estimated covariance of parameter estimates:
          alpha       c           k
alpha      7529.33   -412.078    211.917
c         -412.078    30.2787   -11.4067
k          211.917   -11.4067    6.08509
```

Figure 4.1 - Celeste

Nicholas – 27 – Male – 70/90 – 77.78%
Index: 6744_6725_27_M_fr
95th percentile: 228.97 seconds



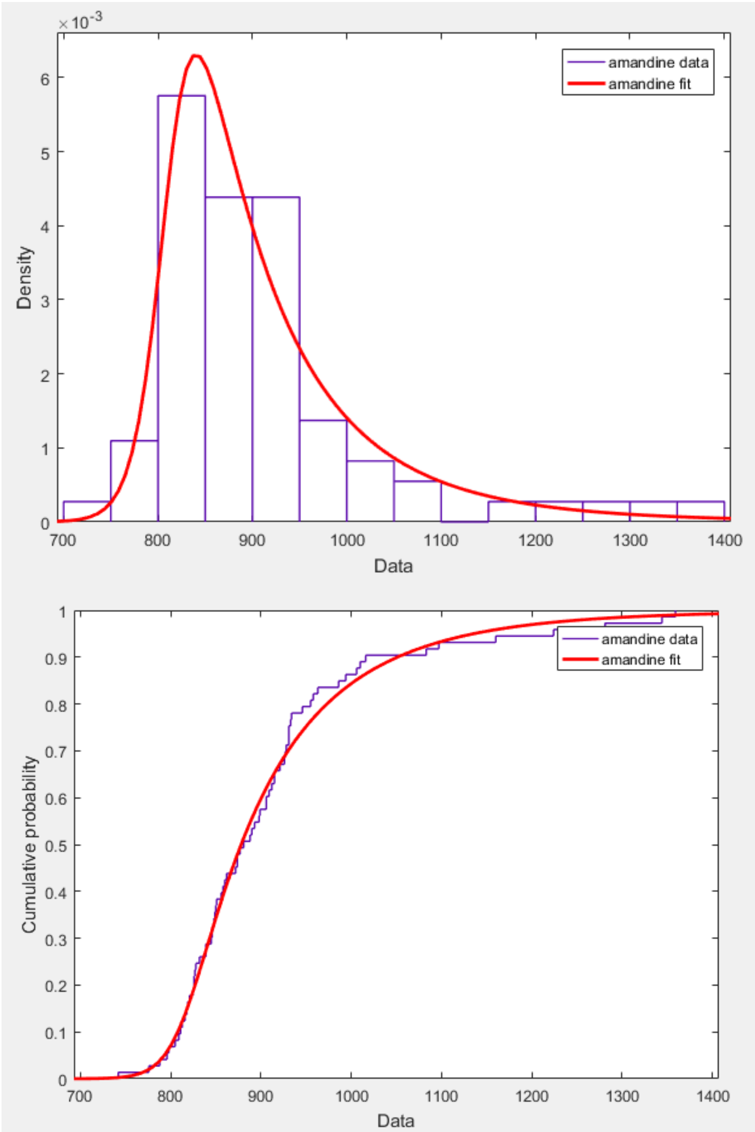

```
Distribution:     Burr
Log likelihood:   -320.482
Domain:           0 < y < Inf
Mean:             175.631
Variance:         857.099

Parameter  Estimate  Std. Err.
alpha       156.282  4.25893
c           21.3284  4.77871
k          0.367764  0.12702

Estimated covariance of parameter estimates:
       alpha      c          k
alpha   18.1385   -14.2001   0.457935
c      -14.2001     22.836  -0.534418
k      0.457935  -0.534418   0.016134
```

Figure 4.2 - Nicholas

Amandine – 38 – Female – 73/90 – 81.11%
Index: 6182_6061_38_F_fr
95th percentile: 1135.50 seconds



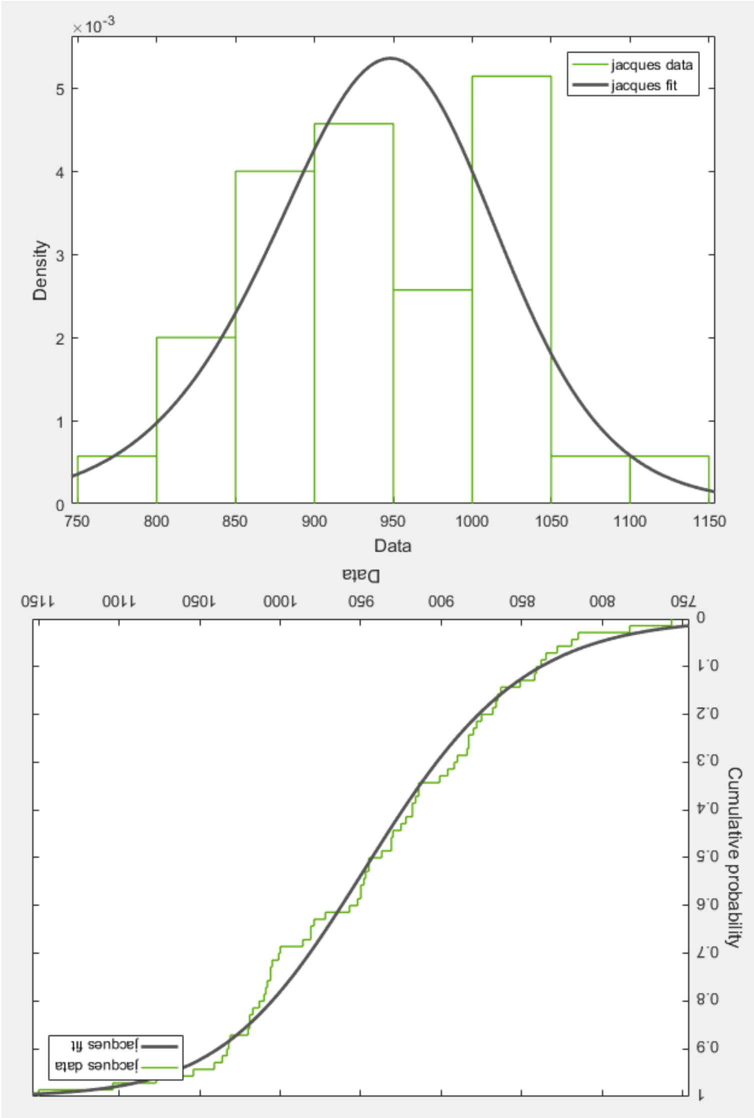

```
Distribution:      Burr
Log likelihood:    -430.16
Domain:            0 < y < Inf
Mean:              910.478
Variance:          14352.2

Parameter  Estimate   Std. Err.
alpha       813.302    10.7158
c            46.904    13.6283
k          0.191375   0.070822

Estimated covariance of parameter estimates:
       alpha      c          k
alpha   114.829   -96.5383     0.57971
c      -96.5383     185.73   -0.894792
k       0.57971  -0.894792  0.00501575
```

Figure 4.3 - Amandine

Jacques – 36 – Male – 70/90 – 77.78%
Index: 6060_6221_36_M_fr
95th percentile: 1068.10 seconds



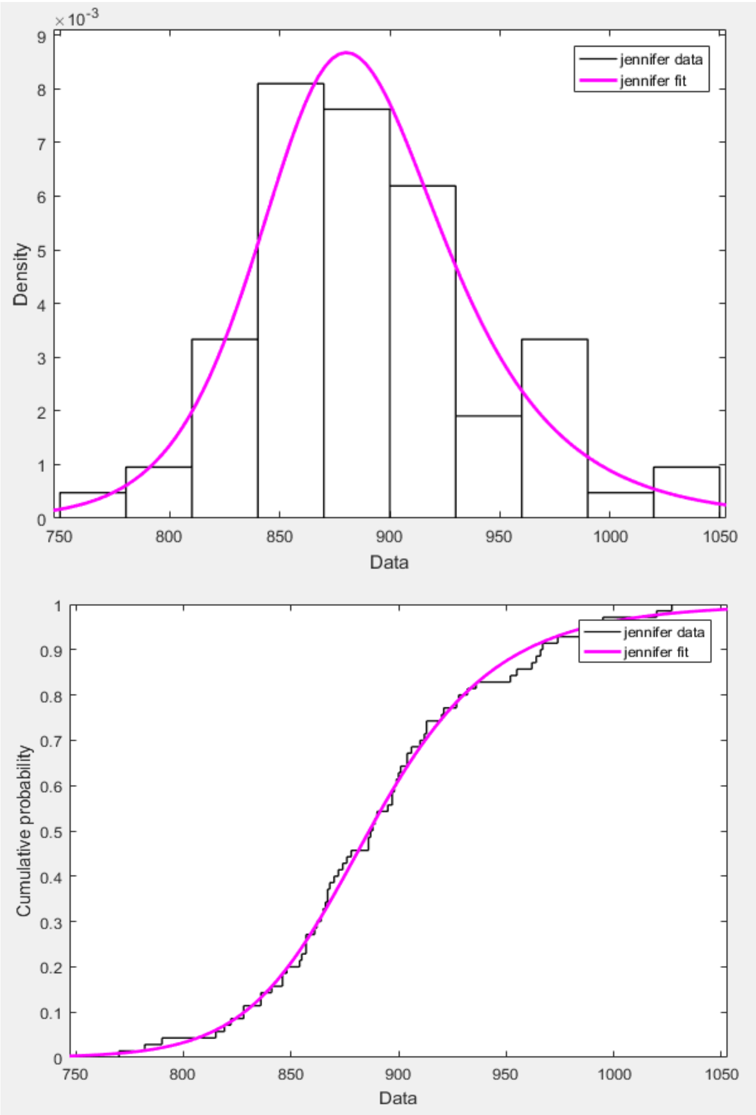

```
Distribution:     Burr
Log likelihood:   -405.288
Domain:           0 < y < Inf
Mean:             940.469
Variance:         6570.85

Parameter  Estimate  Std. Err.
alpha      989.773   59.8445
c          17.3215   3.12082
k          1.92587    1.3601

Estimated covariance of parameter estimates:
       alpha      c          k
alpha   3581.36  -160.084   80.2192
c      -160.084   9.73955  -3.59814
k        80.2192 -3.59814   1.84988
```

Figure 4.4 - Jacques

Jennifer – 47 – Female – 70/90 – 77.78%
Index: 6314_6078_47_F_en
95th percentile: 987.87 seconds



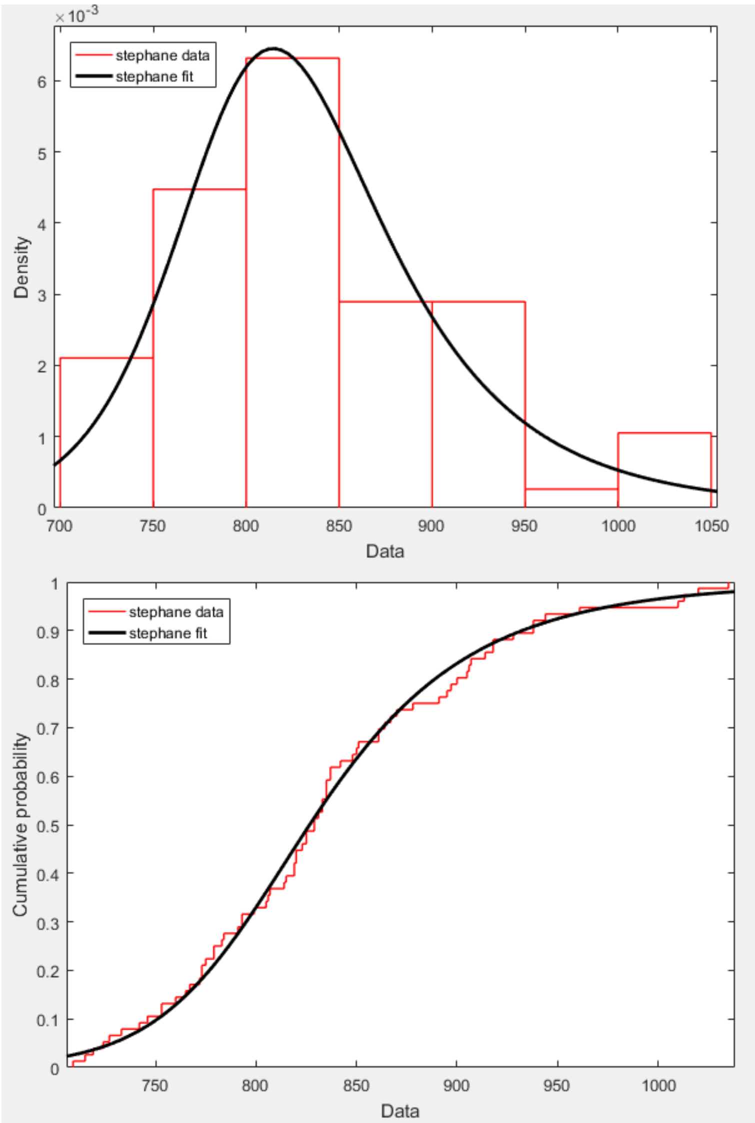

```
Distribution:     Burr
Log likelihood:   -377.176
Domain:           0 < y < Inf
Mean:             891.085
Variance:         3079.39

Parameter  Estimate  Std. Err.
alpha       872.775   16.8565
c           34.6172   7.27045
k          0.696388  0.289546

Estimated covariance of parameter estimates:
              alpha       c          k
alpha      284.143  -98.3308    4.56647
c         -98.3308   52.8594   -1.83778
k          4.56647  -1.83778   0.083837
```

Figure 4.5 - Jennifer

Stephane – 46 – Male – 76/90 – 84.44%
Index: 6116_6060_46_M_fr
95th percentile: 976.68 seconds



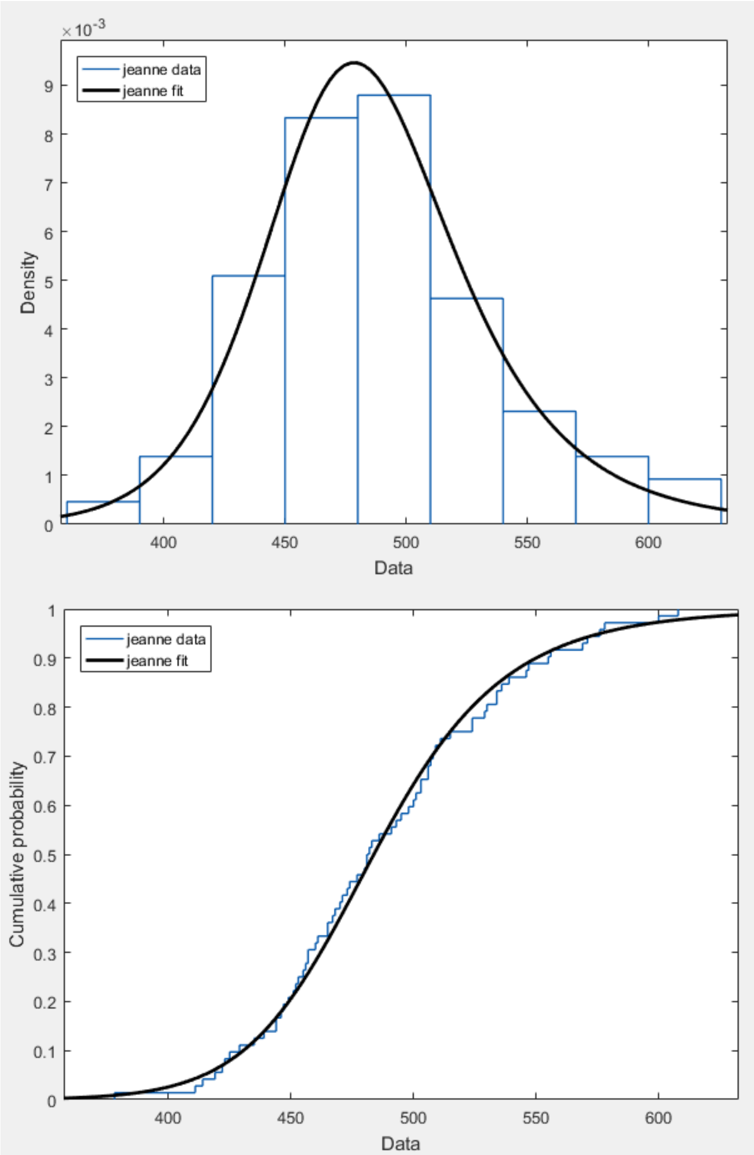

```
Distribution:     Burr
Log likelihood:   -432.939
Domain:           0 < y < Inf
Mean:             836.383
Variance:         6202.25

Parameter  Estimate   Std. Err.
alpha       801.422    21.0663
c           25.3538    5.41327
k          0.596654   0.241701

Estimated covariance of parameter estimates:
          alpha       c          k
alpha    443.788   -92.0068    4.73911
c       -92.0068    29.3035   -1.16137
k        4.73911   -1.16137  0.0584194
```

Figure 4.6 - Stephane

Jeanne – 50 – Female – 72/90 – 80.00%
Index: 6174_6034_50_F_fr
95th percentile: 575.96 seconds



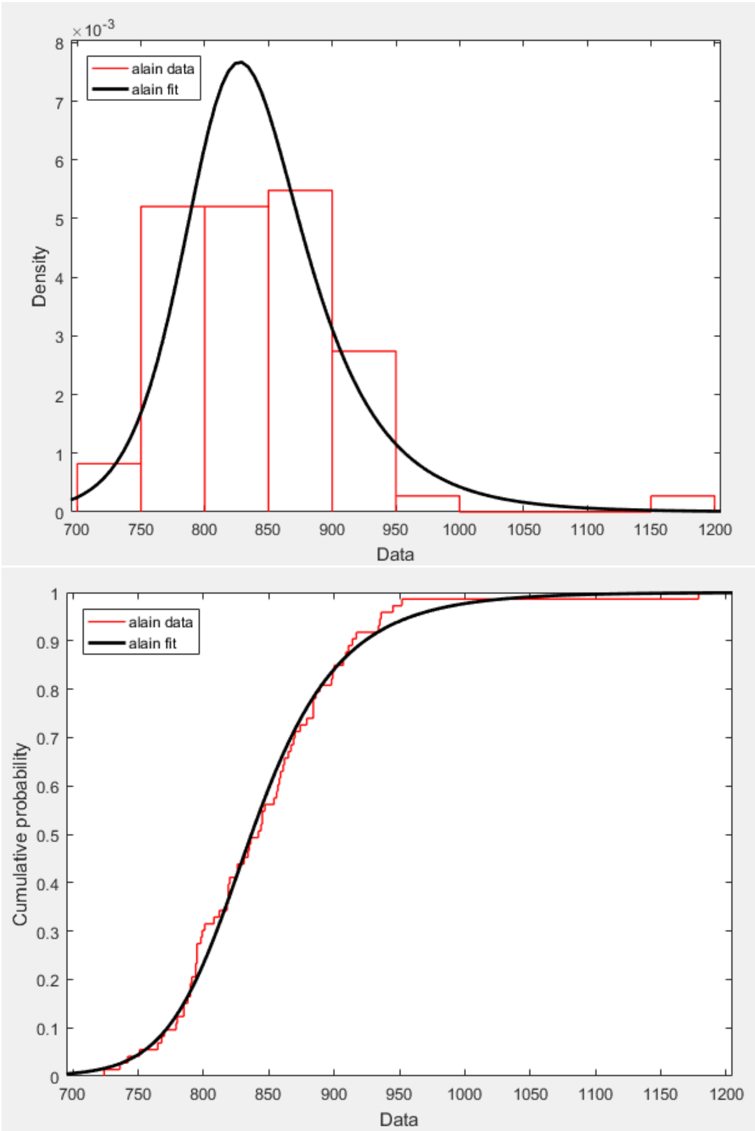

```
Distribution:     Burr
Log likelihood:   -380.352
Domain:           0 < y < Inf
Mean:             488.086
Variance:         2563.88

Parameter  Estimate  Std. Err.
alpha       475.691   20.3511
c           19.6095   4.60261
k          0.793816  0.411333

Estimated covariance of parameter estimates:
         alpha       c          k
alpha   414.168   -81.4417   8.06651
c      -81.4417     21.184  -1.71751
k       8.06651   -1.71751  0.169195
```

Figure 4.7 - Jeanne

Alain – 51 – Male – 73/90 – 81.11%
Index: 6138_6411_51_M_fr
95th percentile: 939.00 seconds




```
Distribution:     Burr
Log likelihood:   -402.599
Domain:           0 < y < Inf
Mean:             843.903
Variance:         4217.53

Parameter  Estimate   Std. Err.
alpha       816.677    19.5385
c           30.1843     6.6941
k          0.616791   0.268668

Estimated covariance of parameter estimates:
       alpha      c         k
alpha   381.753  -107.997    4.93455
c      -107.997    44.811   -1.60915
k         4.93455 -1.60915  0.0721825
```

Figure 4.8 - Alain

4.2 Goodness of fit test (validation)

To validate our claim that the Burr distribution is a good fit, the Chi-Square test was used.

As explained by Mann, in his book "*Introductory Statistics*", the Chi-Square test has only one parameter which is the degrees of freedom. At smaller degrees of freedom, the shape of a chi-square distribution curve is skewed to the right and as the degrees of freedom increase, the distribution curve becomes symmetrical. The distribution lies to the right of the y-axis and above the x-axis and assumes only non-negative values that are denoted by χ^2 – read as "Chi-Square".

This test can be used to determine the goodness of a fit by summing the squared difference between the expected and observed frequency, normalized by the expected frequency for every data point. Then, using n-1 degrees of freedom, where n is the number of data points, and α, the probability of a larger value of χ^2, we can obtain a value for χ^2 from the table below. This "Chi-Square" value is then compared to the value obtained from the summation. If the summed value is smaller than the χ^2, the model distribution curve is said to be a good fit.

Percentage Points of the Chi-Square Distribution

Degrees of Freedom	Probability of a larger value of x^2								
	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
1	0.000	0.004	0.016	0.102	0.455	1.32	2.71	3.84	6.63
2	0.020	0.103	0.211	0.575	1.386	2.77	4.61	5.99	9.21
3	0.115	0.352	0.584	1.212	2.366	4.11	6.25	7.81	11.34
4	0.297	0.711	1.064	1.923	3.357	5.39	7.78	9.49	13.28
5	0.554	1.145	1.610	2.675	4.351	6.63	9.24	11.07	15.09
6	0.872	1.635	2.204	3.455	5.348	7.84	10.64	12.59	16.81
7	1.239	2.167	2.833	4.255	6.346	9.04	12.02	14.07	18.48
8	1.647	2.733	3.490	5.071	7.344	10.22	13.36	15.51	20.09
9	2.088	3.325	4.168	5.899	8.343	11.39	14.68	16.92	21.67
10	2.558	3.940	4.865	6.737	9.342	12.55	15.99	18.31	23.21
11	3.053	4.575	5.578	7.584	10.341	13.70	17.28	19.68	24.72
12	3.571	5.226	6.304	8.438	11.340	14.85	18.55	21.03	26.22
13	4.107	5.892	7.042	9.299	12.340	15.98	19.81	22.36	27.69
14	4.660	6.571	7.790	10.165	13.339	17.12	21.06	23.68	29.14
15	5.229	7.261	8.547	11.037	14.339	18.25	22.31	25.00	30.58
16	5.812	7.962	9.312	11.912	15.338	19.37	23.54	26.30	32.00
17	6.408	8.672	10.085	12.792	16.338	20.49	24.77	27.59	33.41
18	7.015	9.390	10.865	13.675	17.338	21.60	25.99	28.87	34.80
19	7.633	10.117	11.651	14.562	18.338	22.72	27.20	30.14	36.19
20	8.260	10.851	12.443	15.452	19.337	23.83	28.41	31.41	37.57
22	9.542	12.338	14.041	17.240	21.337	26.04	30.81	33.92	40.29
24	10.856	13.848	15.659	19.037	23.337	28.24	33.20	36.42	42.98
26	12.198	15.379	17.292	20.843	25.336	30.43	35.56	38.89	45.64
28	13.565	16.928	18.939	22.657	27.336	32.62	37.92	41.34	48.28
30	14.953	18.493	20.599	24.478	29.336	34.80	40.26	43.77	50.89
40	22.164	26.509	29.051	33.660	39.335	45.62	51.80	55.76	63.69
50	27.707	34.764	37.689	42.942	49.335	56.33	63.17	67.50	76.15
60	37.485	43.188	46.459	52.294	59.335	66.98	74.40	79.08	88.38

Table 4.1 – Percentage Points of the Chi-Square Distribution

In order to do that, we had to create a fit that best described the real data. After that, this fit was compared to our Burr distribution fit to test for goodness. The script below was input to MATLAB and the software generated the number of points that best represent the real data.

```
function [N_ C_] = createFit(y)
y = y(:);
t_ = ~isnan(y);
Data_ = y(t_);
[F_,X_] = ecdf(Data_,'Function','cdf');  % compute empirical cdf
Bin_.rule = 1;
[C_,E_] = dfswitchyard('dfhistbins',Data_,[],[],Bin_,F_,X_);
[N_,C_] = ecdfhist(F_,X_,'edges',E_); % empirical pdf from cdf
```

```
>>[y1 x] = createFit(data);
```

The output x and y1 values were the trip duration and probability density respectively. These were taken for every user and the Burr distribution probability density was calculated for every trip duration generated (x). The probability density was cumulated and then normalized with the highest value for both the expected (Burr) and observed (real fit). The difference was recorded for every data point, squared and then normalized by the expected value. All values were summed up and then the answer was compared to the Chi-Square value obtained from the table at n-1 degrees of freedom and 0.05 significance.

The following tables show the Chi-Square test for all 8 personas:

Celeste									
Original				**Fit**					
Time	**Frequency**	**Cum. Proba.**	**Normalized Cum. Probability**	**Frequency**	**Cum. Proba.**	**Normalized Cum. Proba.**	$E-O$	$(E-O)^2$	$\frac{(E-O)^2}{E}$
1725	0.00063	0.00063	0.03125	0.00041	0.00041	0.02086	0.0104	0.00011	0.0051808
1775	0.00125	0.00188	0.09375	0.00089	0.00130	0.06644	0.0273	0.00075	0.011226708
1825	0.00156	0.00344	0.17188	0.00181	0.00311	0.15871	0.0132	0.00017	0.001092641
1875	0.00375	0.00719	0.35938	0.00321	0.00633	0.32247	0.0369	0.00136	0.004223077
1925	0.00406	0.01125	0.56250	0.00455	0.01088	0.55444	0.0081	0.00007	0.000117285
1975	0.00469	0.01594	0.79688	0.00453	0.01541	0.78511	0.0118	0.00014	0.000176316
2025	0.00219	0.01813	0.90625	0.00283	0.01824	0.92932	-0.0231	0.00053	0.000572604
2075	0.00125	0.01938	0.96875	0.00110	0.01933	0.98520	-0.0164	0.00027	0.000274523
2125	0.00063	0.02000	1.00000	0.00029	0.01962	1.00000	0.0000	0.00000	0
n = 9								Sum	0.022863954
								df	8
								χ^2 (0.05)	15.507
								PASS	

Table 4.2 – Chi Square Celeste

Alain									
Original				Fit					
Time	Frequency	Cum. Proba.	Normalized Cum. Probability	Frequency	Cum. Proba.	Normalized Cum. Proba.	$E-O$	$(E-O)^2$	$\frac{(E-O)^2}{E}$
725	0.00082	0.00082	0.04110	0.00068	0.00068	0.03414	0.0070	0.00005	0.001416785
775	0.00521	0.00603	0.30137	0.00367	0.00435	0.21859	0.0828	0.00685	0.031351862
825	0.00521	0.01123	0.56164	0.00767	0.01202	0.60437	-0.0427	0.00183	0.003020815
875	0.00548	0.01671	0.83562	0.00486	0.01689	0.84883	-0.0132	0.00017	0.000205605
925	0.00274	0.01945	0.97260	0.00190	0.01878	0.94410	0.0285	0.00081	0.000860593
975	0.00027	0.01973	0.98630	0.00069	0.01947	0.97887	0.0074	0.00006	5.64797E-05
1025	0.00000	0.01973	0.98630	0.00026	0.01973	0.99192	-0.0056	0.00003	3.17935E-05
1075	0.00000	0.01973	0.98630	0.00010	0.01983	0.99703	-0.0107	0.00012	0.000115401
1125	0.00000	0.01973	0.98630	0.00004	0.01987	0.99911	-0.0128	0.00016	0.000164321
1175	0.00027	0.02000	1.00000	0.00002	0.01989	1.00000	0.0000	0.00000	0
								Sum	0.037223656
								df	9
								χ^2 (0.05)	16.919
								PASS	

Table 4.3 – Chi Square Alain

Stephane									
Original				Fit					
Time	Frequency	Cum. Proba.	Normalized Cum. Probability	Frequency	Cum. Proba.	Normalized Cum. Proba.	$E-O$	$(E-O)^2$	$\frac{(E-O)^2}{E}$
725	0.00211	0.00211	0.10526	0.0015	0.00146	0.07573	0.0295	0.00087	0.011517413
775	0.00447	0.00658	0.32895	0.0047	0.00621	0.32116	0.0078	0.00006	0.000188602
825	0.00632	0.01289	0.64474	0.0063	0.01255	0.64885	-0.0041	0.00002	2.60221E-05
875	0.00289	0.01579	0.78947	0.0039	0.01643	0.84938	-0.0599	0.00359	0.004225432
925	0.00289	0.01868	0.93421	0.0018	0.01821	0.94145	-0.0072	0.00005	5.55993E-05
975	0.00026	0.01895	0.94737	0.0008	0.01899	0.98188	-0.0345	0.00119	0.001213074
1025	0.00105	0.02000	1.00000	0.0004	0.01934	1.00000	0.0000	0.00000	0
n=7								Sum	0.017226144
								df	6
								χ^2 (0.05)	12.592
								PASS	

Table 4.4 – Chi Square Stephane

Nicholas									
Original				Fit			$E-O$	$(E-O)^2$	$\frac{(E-O)^2}{E}$
Time	Frequency	Cum. Proba.	Normalized Cum. Probability	Frequency	Cum. Proba.	Normalized Cum. Proba.			
135	0.00286	0.00286	0.02857	0.0024	0.00243	0.02448	0.0041	0.00002	0.000683178
145	0.00857	0.01143	0.11429	0.0086	0.01098	0.11074	0.0035	0.00001	0.000113533
155	0.01286	0.02429	0.24286	0.0185	0.02953	0.29770	-0.0548	0.00301	0.010102666
165	0.02429	0.04857	0.48571	0.0214	0.05096	0.51381	-0.0281	0.00079	0.001536387
175	0.02143	0.07000	0.70000	0.0164	0.06738	0.67937	0.0206	0.00043	0.000626742
185	0.00857	0.07857	0.78571	0.0109	0.07824	0.78884	-0.0031	0.00001	1.2351E-05
195	0.00714	0.08571	0.85714	0.0070	0.08521	0.85911	-0.0020	0.00000	4.49635E-06
205	0.00429	0.09000	0.90000	0.0045	0.08971	0.90451	-0.0045	0.00002	2.25264E-05
215	0.00429	0.09429	0.94286	0.0030	0.09267	0.93432	0.0085	0.00007	7.7961E-05
225	0.00286	0.09714	0.97143	0.0020	0.09464	0.95424	0.0172	0.00030	0.000309701
235	0.00000	0.09714	0.97143	0.0013	0.09598	0.96777	0.0037	0.00001	1.38235E-05
245	0.00000	0.09714	0.97143	0.0009	0.09691	0.97711	-0.0057	0.00003	3.30923E-05
255	0.00000	0.09714	0.97143	0.0006	0.09756	0.98366	-0.0122	0.00015	0.000152149
265	0.00143	0.09857	0.98571	0.0005	0.09802	0.98831	-0.0026	0.00001	6.83336E-06
275	0.00000	0.09857	0.98571	0.0003	0.09835	0.99166	-0.0059	0.00004	3.56333E-05
285	0.00000	0.09857	0.98571	0.0002	0.09859	0.99409	-0.0084	0.00007	7.06365E-05
295	0.00000	0.09857	0.98571	0.0002	0.09877	0.99589	-0.0102	0.00010	0.000103894
305	0.00000	0.09857	0.98571	0.0001	0.09890	0.99722	-0.0115	0.00013	0.000132716
315	0.00000	0.09857	0.98571	0.0001	0.09900	0.99822	-0.0125	0.00016	0.000156638
325	0.00000	0.09857	0.98571	0.0001	0.09908	0.99898	-0.0133	0.00018	0.000176056
335	0.00000	0.09857	0.98571	0.0001	0.09914	0.99955	-0.0138	0.00019	0.00019164
345	0.00143	0.10000	1.00000	0.0000	0.09918	1.00000	0.0000	0.00000	0
n=22								Sum	0.014562654
								df	21
								χ^2 (0.05)	32.671
								PASS	

Table 4.5 – Chi Square Nicholas

Amandine									
Original				Fit			$E-O$	$(E-O)^2$	$\frac{(E-O)^2}{E}$
Time	Frequency	Cum. Proba.	Normalized Cum. Probability	Frequency	Cum. Proba.	Normalized Cum. Proba.			
725	0.00063	0.00063	0.02703	0.0001	0.00006	0.00280	0.0242	0.00059	0.209413202
775	0.00125	0.00188	0.08108	0.0011	0.00112	0.05627	0.0248	0.00062	0.010936785
825	0.00156	0.00344	0.14865	0.0058	0.00694	0.34855	-0.1999	0.03996	0.114648467
875	0.00375	0.00719	0.31081	0.0051	0.01205	0.60537	-0.2946	0.08677	0.143328829
925	0.00406	0.01125	0.48649	0.0031	0.01510	0.75873	-0.2722	0.07411	0.097683243
975	0.00469	0.01594	0.68919	0.0018	0.01691	0.84998	-0.1608	0.02585	0.030415438
1025	0.00219	0.01813	0.78378	0.0011	0.01802	0.90557	-0.1218	0.01483	0.016379625
1075	0.00125	0.01938	0.83784	0.0007	0.01871	0.94025	-0.1024	0.01049	0.011155466
1125	0.00063	0.02000	0.86486	0.0004	0.01915	0.96235	-0.0975	0.00950	0.009875746
1175	0.00063	0.02063	0.89189	0.0003	0.01943	0.97671	-0.0848	0.00719	0.007366487
1225	0.00063	0.02125	0.91892	0.0002	0.01962	0.98622	-0.0673	0.00453	0.0045924
1275	0.00063	0.02188	0.94595	0.0001	0.01975	0.99261	-0.0467	0.00218	0.002193716
1325	0.00063	0.02250	0.97297	0.0001	0.01984	0.99698	-0.0240	0.00058	0.000577874
1375	0.00063	0.02313	1.00000	0.0001	0.01990	1.00000	0.0000	0.00000	0
n= 14								Sum	0.658567279
								df	13
								χ^2 (0.05)	22.362
								PASS	

Table 4.6 – Chi Square Amandine

Jacques									
Original				Fit			$E-O$	$(E-O)^2$	$\frac{(E-O)^2}{E}$
Time	Frequency	Cum. Proba.	Normalized Cum. Probability	Frequency	Cum. Proba.	Normalized Cum. Proba.			
775	0.00057	0.00057	0.02857	0.0006	0.00060	0.03210	-0.0035	0.00001	0.000386935
825	0.00200	0.00257	0.12857	0.0058	0.00641	0.34432	-0.2157	0.04655	0.135183207
875	0.00400	0.00657	0.32857	0.0051	0.01152	0.61866	-0.2901	0.08415	0.136024465
925	0.00457	0.01114	0.55714	0.0031	0.01458	0.78248	-0.2253	0.05078	0.064892162
975	0.00257	0.01371	0.68571	0.0018	0.01639	0.87996	-0.1942	0.03773	0.042876607
1025	0.00514	0.01886	0.94286	0.0011	0.01750	0.93935	0.0035	0.00001	1.31151E-05
1075	0.00057	0.01943	0.97143	0.0007	0.01819	0.97639	-0.0050	0.00002	2.52362E-05
1125	0.00057	0.02000	1.00000	0.0004	0.01863	1.00000	0.0000	0.00000	0
n=8								Sum	0.379401727
								df	7
								χ^2 (0.05)	14.067
								PASS	

Table 4.7 – Chi Square Jacques

Jeanne									
Original				Fit					
Time	Frequency	Cum. Proba.	Normalized Cum. Probability	Frequency	Cum. Proba.	Normalized Cum. Proba.	$E-O$	$(E-O)^2$	$\frac{(E-O)^2}{E}$
375	0.00046	0.00046	0.01389	0.0004	0.00039	0.01181	0.0021	0.00000	0.00036575
405	0.00139	0.00185	0.05556	0.0015	0.00192	0.05847	-0.0029	0.00001	0.000145631
435	0.00509	0.00694	0.20833	0.0047	0.00660	0.20114	0.0072	0.00005	0.000257202
465	0.00833	0.01528	0.45833	0.0089	0.01547	0.47109	-0.0128	0.00016	0.000345642
495	0.00880	0.02407	0.72222	0.0086	0.02408	0.73327	-0.0110	0.00012	0.000166369
525	0.00463	0.02870	0.86111	0.0050	0.02907	0.88515	-0.0240	0.00058	0.000652852
555	0.00231	0.03102	0.93056	0.0023	0.03138	0.95549	-0.0249	0.00062	0.000650656
585	0.00139	0.03241	0.97222	0.0010	0.03239	0.98635	-0.0141	0.00020	0.000202244
615	0.00093	0.03333	1.00000	0.0004	0.03284	1.00000	0.0000	0.00000	0
n=9								Sum	0.002786345
								df	8
								χ^2 (0.05)	15.507
								PASS	

Table 4.8 – Chi Square Jeanne

Jennifer									
Original				Fit					
Time	Frequency	Cum. Proba.	Normalized Cum. Probability	Frequency	Cum. Proba.	Normalized Cum. Proba.	$E-O$	$(E-O)^2$	$\frac{(E-O)^2}{E}$
765	0.00048	0.00048	0.01429	0.0003	0.00032	0.00988	0.0044	0.00002	0.001963599
795	0.00095	0.00143	0.04286	0.0011	0.00145	0.04420	-0.0013	0.00000	4.06382E-05
825	0.00333	0.00476	0.14286	0.0033	0.00479	0.14574	-0.0029	0.00001	5.70406E-05
855	0.00810	0.01286	0.38571	0.0071	0.01184	0.36045	0.0253	0.00064	0.001770914
885	0.00762	0.02048	0.61429	0.0086	0.02047	0.62309	-0.0088	0.00008	0.000124481
915	0.00619	0.02667	0.80000	0.0062	0.02670	0.81269	-0.0127	0.00016	0.00019813
945	0.00190	0.02857	0.85714	0.0034	0.03006	0.91504	-0.0579	0.00335	0.00366291
975	0.00333	0.03190	0.95714	0.0016	0.03170	0.96486	-0.0077	0.00006	6.17338E-05
1005	0.00048	0.03238	0.97143	0.0008	0.03248	0.98860	-0.0172	0.00029	0.000298399
1035	0.00095	0.03333	1.00000	0.0004	0.03285	1.00000	0.0000	0.00000	0
n=10								Sum	0.008177845
								df	9
								χ^2 (0.05)	16.919
								PASS	

Table 4.9 – Chi Square Jennifer

After making sure that the Burr distribution is an actual fit to all of our personas, we continued to obtain the 95th percentile duration as it is the duration that we are interested in. This duration was obtained from the cumulative distribution function for every persona using MATLAB by inputting this script:

```
>> pd = fitdist(persona_name, 'Burr')
>> x = icdf(pd,0.95)
```

Summary of findings:

Persona	**Age**	**Gender**	**a**	**c**	**k**	**95th Percentile Duration (s)**
Celeste	28	Female	2019.37	29.92	2.72	2066.90
Nicholas	27	Male	156.28	21.33	0.37	228.97
Amandine	38	Female	813.30	46.90	0.19	1135.50
Jacques	36	Male	989.77	17.32	1.93	1068.10
Jennifer	47	Female	872.78	34.62	0.70	987.87
Stephane	46	Male	801.42	25.35	0.60	976.68
Jeanne	50	Female	475.69	19.61	0.80	575.96
Alain	51	Male	816.68	30.18	0.62	939.00

Table 4.10 – Summary

After successfully fitting our user data into a Burr distribution, the 95th percentile duration can be calculated for every user. This method can be used with all members in a BSS. Whenever a bicycle checks out, the system would look up all trip information related to their user ID. The system would collect the real data and fit it into a Burr distribution. The distribution can then be used to obtain the 95th percentile duration. This duration is then added to the starting time to obtain the arrival time. This will allow monitoring the ToB in real-time and study its variation over the days.

CHAPTER V: CONCLUSION AND FUTURE WORK

5.1 Summary and Concluding Remarks

The analysis of the data identified that the city of Montreal witnesses an AM demand for BIXI services that peaks between 7:30 and 9:00 am. In addition, it was determined that during the AM peak interval the majority of the trips are performed by the BIXI members (i.e. 95% of the trips) while a small fraction (i.e. 5% of trips) are made by occasional users. There was also a clear general pattern of decrease in the users' speed as the age increases. As expected, male users were observed to travel similar distances in less time than female users. The highest percentage composition of an age-gender pair was for males in their 30's at a value of 22.42% during the AM peak interval. The highest number of trips recorded during AM peak interval commute corresponded to those who are of 28 years of age, traveling a radial distance between 600 and 800 meters.

In this study, we propose two methodologies. The System Adjustability Method can be applied to predict the time of blockage (ToB) at any station based on the advised generic speeds (specific to the city of Montreal) and the radial distance between the origin and destination station as we vary basic demographic information. The same methodology is suitable for any other bike sharing system around the world, however, different generic speeds should be obtained for different systems, to ensure capturing the demographics of the study site and ensure the accuracy of the analysis.

This methodology allows performing a sensitivity analysis based on the gender and age distributions of the users to observe the impact on the expected time of blockage. A sensitivity analysis can be necessary, for example, whenever we expect special events occurring during the morning hours that attract a specific group of users or when we have new facilities or institutions operating within the proximity of our study station. This methodology can be also applied when the ToB has to be determined for any new station (i.e. often times new stations are deployed from one season to another, or during any given season, to accommodate increase of changes in demand for this service). Our model was calibrated and validated and the predicted results were observed to be very close to the actual time when the station fills up - which validates the performance of our model. The obtained adjustment factors, could be plotted and compared to evaluate the performance of rebalancing between months.

The other method, The System Improvement Method, can be applied to all users when the operator has the trip-log data for every user. It was found that the user trip duration distribution follows a Burr distribution. Modeling the behavior of a user as a Burr distribution allows us to obtain the expected user trip duration with 95% probability of occurrence. The Burr fitting was validated by performing the chi-square test.

5.2 Future Work

There is definitely more room for improvement to the proposed system assessment methods. For the System Adjustability Method, investigating various methods where actual bicycle routes can be obtained would contribute to increasing the accuracy of speed values used in the methodology and would make the ToB prediction more accurate. Using actual traveled distances instead of radial distances would also minimize the error related to distance and would result in more accurate adjustment factors that are only dependent on rebalancing procedures and their associated delays. This could definitely be implemented and analyzed in the future, by using the date from a system that a tracking technology on their bicycles.

For the System Improvement Method, not much data was available for us and this was a limitation. Should additional information be provided, we would like to test the robustness of the method that we proposed to infer the missing data.

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APPENDIX A – Station Capacities

Code	Name	Capacity
4000	Jeanne-d'Arc / Ontario	27
4001	Parc Plage	27
4002	Métro Place-des-Arts (de Maisonneuve / de Bleury)	19
5002	Montarville / St-Charles	15
5003	Place Longueuil	15
5004	St-Charles / Grant	11
5005	St-Charles / St-Sylvestre	15
5006	Collège Édouard-Montpetit (de Gentilly / de Normandie)	27
5007	Métro Longueuil - Université de Sherbrooke	31
6001	Métro Champ-de-Mars (Sanguinet / Viger)	33
6002	Ste-Catherine / Dézéry	23
6003	Clark / Evans	19
6004	Hôtel-de-Ville (du Champs-de-Mars / Gosford)	35
6005	de la Cathédrale / René-Lévesque	39
6006	18e avenue / Rosemont	23
6007	de l'Hôtel-de-Ville / Ste-Catherine	23
6008	Sanguinet / Ste-Catherine	27
6009	Ste-Catherine / Labelle	15
6011	St-André / St-Antoine	27
6012	Métro St-Laurent (de Maisonneuve / St-Laurent)	49
6013	Sanguinet / de Maisonneuve	31
6014	Métro Berri-UQAM (St-Denis / de Maisonneuve)	35
6015	BAnQ (Berri / de Maisonneuve)	31
6016	Jacques-Le Ber / de la Pointe Nord	19
6017	du Square Ahmerst / Wolfe	19
6018	St-André / Ontario	19
6019	Métro Sherbrooke (de Rigaud / Berri)	23
6020	Sanguinet / Ontario	19
6021	de l'Hôtel-de-Ville / Sherbrooke	23
6022	Molson / William-Tremblay	23
6023	de la Commune / Berri	51
6024	Parthenais / Sherbrooke	15
6025	Notre-Dame / St-Gabriel	15
6026	de la Commune / Place Jacques-Cartier	73
6027	de Maisonneuve / Mansfield (ouest)	19
6029	Bel Air / St-Antoine	19
6031	St-Antoine / St-François-Xavier	15
6032	Métro Place-d'Armes (Viger / St-Urbain)	49
6033	15e avenue / Beaubien	23
6034	St-Urbain / René-Lévesque	15
6035	Viger / Chenneville	23

6036	de la Commune / St-Sulpice	43
6037	St-Nicolas / Place d'Youville	15
6038	McGill / Place d'Youville	23
6039	McGill / des Récollets	23
6040	St-Jacques / St-Pierre	23
6041	St-Jacques / Gauvin	31
6042	de la Gauchetière / Robert-Bourassa	27
6043	Square Victoria (Viger / du Square-Victoria)	19
6044	Roy / St-Hubert	27
6046	Métro Bonaventure (de la Gauchetière / Mansfield)	31
6047	University / Prince-Arthur	39
6048	Queen / Ottawa	19
6049	Queen / Wellington	47
6050	de la Commune / McGill	31
6051	de Gaspé / Marmier	22
6052	de la Commune / King	65
6053	Belmont / du Beaver Hall	23
6057	Peel / avenue des Canadiens de Montréal	23
6058	Cypress / Peel	23
6059	Mansfield / René-Lévesque	23
6060	Stanley / du Docteur-Penfield	15
6061	McGill College / Ste-Catherine	23
6062	Drummond / Ste-Catherine	35
6063	Drummond / de Maisonneuve	31
6064	Métro Peel (de Maisonneuve / Stanley)	57
6065	de la Montagne / Sherbrooke	23
6066	McTavish / des Pins	23
6067	de Maisonneuve / Robert-Bourassa	27
6068	Mansfield / Sherbrooke	27
6070	Milton / University	39
6072	Metcalfe / de Maisonneuve	23
6073	de Maisonneuve / Aylmer	23
6075	University / des Pins	31
6076	Ville-Marie / Ste-Catherine	15
6078	de Bleury / Mayor	27
6079	Hutchison/ Prince-Arthur	23
6080	Marlowe / de Maisonneuve	23
6081	St-Alexandre / Ste-Catherine	31
6082	5e avenue / Rosemont	19
6083	Square Phillips	19
6084	Duke / Brennan	15
6085	Notre-Dame / Peel	43
6086	Murray / William	15
6087	Notre-Dame / de la Montagne	31
6088	Guy / Notre-Dame	31

6089	Henri-Julien / du Carmel	19
6090	Métro Lucien-L'Allier (Argyle / Lucien-l'Allier)	17
6091	Joseph-Manceau / René-Lévesque	15
6092	Crescent / René-Lévesque	23
6093	Atwater / Sherbrooke	23
6094	de Gaspé / Fairmount	23
6095	Chomedey / de Maisonneuve	19
6096	Lucien L'Allier / St-Jacques	19
6097	Ste-Catherine / St-Marc	27
6098	Bishop / Ste-Catherine	31
6099	Crescent / de Maisonneuve	19
6100	Mackay / de Maisonneuve	55
6101	Métro Villa-Maria (Décarie / de Monkland)	17
6102	Lincoln / du Fort	15
6103	Lespérance / de Rouen	19
6104	Wolfe / René-Lévesque	43
6105	Plessis / René-Lévesque	19
6106	Papineau / René-Lévesque	25
6107	St-Mathieu /Ste-Catherine	31
6108	Logan / Fullum	19
6109	Quai de la navette fluviale	19
6110	Poupart / Ste-Catherine	19
6111	Parthenais / Ste-Catherine	19
6112	Montcalm / de Maisonneuve	19
6113	Alexandre-DeSève / de Maisonneuve	15
6114	Métro Papineau (Cartier / Ste-Catherine)	25
6115	Benny / de Monkland	27
6116	Plessis / Ontario	19
6117	Robin / de la Visitation	15
6118	de Champlain / Ontario	19
6119	Dorion / Ontario	31
6120	Métro Frontenac (Ontario / du Havre)	15
6121	du Havre / de Rouen	27
6122	Logan / d'Iberville	15
6123	Alexandre-DeSève / la Fontaine	15
6124	Poupart / Ontario	19
6125	Ellendale / de la Côte-des-Neiges	27
6126	Rouen / Fullum	15
6127	Henri-Julien / de Castelnau	23
6128	Hogan / Ontario	11
6129	de Bordeaux / Sherbrooke	15
6130	Sherbrooke / Frontenac	15
6131	Fullum / Sherbrooke	31
6132	Larivière / de Lorimier	23
6133	Terrasse Mercure / Fullum	19

6134	Gascon / Rachel	15
6136	Métro Laurier (Rivard / Laurier)	19
6137	Gauthier / Papineau	23
6138	Gauthier / de Lorimier	11
6139	Gauthier / Parthenais	15
6140	Marquette / Rachel	15
6141	de Bordeaux / Rachel	23
6142	Calixa-Lavallée / Rachel	27
6143	Rachel / de Brébeuf	14
6145	du Parc-La Fontaine / Duluth	19
6146	du Parc-La Fontaine / Roy	15
6147	Calixa-Lavallée / Sherbrooke	39
6148	Émile-Duployé / Sherbrooke	31
6149	Chapleau / du Mont-Royal	23
6150	Messier / du Mont-Royal	19
6151	Parthenais / du Mont-Royal	19
6152	Chabot / du Mont-Royal	19
6153	Cartier / Marie-Anne	23
6154	Marquette / du Mont-Royal	19
6155	Garnier / du Mont-Royal	31
6156	Marie-Anne / de la Roche	19
6157	de la Roche / du Mont-Royal	23
6158	Gilford / de Brébeuf	19
6159	Ann / William	15
6160	Garnier / St-Joseph	23
6161	Cartier / St-Joseph	7
6162	Fullum / St-Joseph	15
6163	Marquette / Laurier	27
6164	Chambord / Laurier	15
6165	de Brébeuf / Laurier	27
6166	de Brébeuf / St-Grégoire	31
6167	Marquette / St-Grégoire	15
6168	Marmier / St-Denis	15
6169	Boyer / du Mont-Royal	27
6170	de Mentana / Laurier	23
6171	Wolfe / Robin	23
6173	Berri / Cherrier	27
6174	Roy / St-Denis	23
6175	St-André / Cherrier	23
6176	de Mentana / Rachel	15
6177	St-Hubert / Duluth	11
6178	Rivard / Rachel	19
6179	Duluth / St-Denis	11
6180	St-Dominique / René-Lévesque	11
6181	Clark / Rachel	31

6182	de Bullion / du Mont-Royal	19
6183	Laval / du Mont-Royal	27
6184	Métro Mont-Royal (Rivard / du Mont-Royal)	39
6185	des Érables / Bélanger	15
6186	St-André / Laurier	19
6187	Resther / du Mont-Royal	23
6188	de Mentana / Marie-Anne	23
6189	Chabot / Everett	15
6190	Pontiac / Gilford	23
6191	St-Zotique / Clark	15
6192	Berri / St-Grégoire	27
6193	de l'Esplanade / Fairmount	15
6194	Métro Atwater (Atwater / Ste-Catherine)	27
6195	de Bullion / St-Joseph	27
6196	Villeneuve / St-Laurent	19
6197	de Bordeaux / Masson	19
6198	Hélène-Baillargeon / St-Denis	23
6199	St-Viateur / St-Laurent	23
6200	Maguire / St-Laurent	23
6201	Villeneuve / de l'Hôtel-de-Ville	23
6202	Ste-Famille / Sherbrooke	23
6203	Hutchison / Sherbrooke	23
6204	Milton / Durocher	31
6205	Milton / du Parc	31
6206	Prince-Arthur / du Parc	35
6207	Ste-Famille / des Pins	23
6208	Hutchison / des Pins	15
6209	Milton / Clark	23
6210	Métro Sauvé (Berri / Sauvé)	26
6211	Roy / St-Laurent	27
6212	de l'Esplanade / Duluth	15
6213	Duluth / St-Laurent	15
6214	Square St-Louis (du Square St-Louis / Laval)	27
6215	St-Cuthbert / St-Urbain	19
6216	Parc Jeanne Mance (monument à sir George-Étienne Cartier)	35
6217	Vallières / St-Laurent	23
6218	Prince-Arthur / St-Urbain	27
6219	de l'Hôtel-de-Ville / Roy	23
6220	Laval / Duluth	27
6221	du Mont-Royal / Clark	23
6222	Jeanne Mance / du Mont-Royal	23
6223	du Mont-Royal / du Parc	33
6224	Villeneuve / du Parc	15
6225	Villeneuve / St-Urbain	19
6226	Hôpital Maisonneuve-Rosemont (Rosemont / Chatelain)	49

6227	de l'Esplanade / Laurier	27
6228	Waverly / Van Horne	15
6229	Coloniale / du Mont-Royal	15
6230	Waverly / St-Viateur	19
6231	Jeanne-Mance / St-Viateur	23
6232	Hutchison / Van Horne	19
6233	Bernard / Jeanne-Mance	27
6234	Bernard / Clark	19
6235	St-Dominique / St-Viateur	31
6236	Laurier / de Bordeaux	27
6237	Gilford / de Lanaudière	15
6240	Parc Kent (de Kent / Hudson)	27
6241	Hutchison / Fairmount	15
6243	Bloomfield / Bernard	19
6245	Bloomfield / Van Horne	19
6246	Métro Outremont (Wiseman / Van Horne)	20
6247	St-Dominique / St-Zotique	23
6248	St-Dominique / Rachel	23
6249	Bélanger / St-Denis	15
6250	Marché Jean-Talon (Henri-Julien / Jean-Talon)	23
6251	de Gaspé / Dante	19
6252	Mozart / St-Laurent	15
6253	Berri / Jean-Talon	19
6254	Boyer / Bélanger	19
6255	Boyer / St-Zotique	19
6257	de St-Vallier / St-Zotique	15
6258	Parc Père-Marquette (Chambord / Rosemont)	35
6259	Dandurand / de Lorimier	23
6260	Dandurand / Papineau	27
6261	Louis Hémon / Rosemont	19
6262	de la Roche / de Bellechasse	15
6263	de Hampton / de Monkland	15
6264	Chabot / de Bellechasse	15
6265	Parthenais / Laurier	19
6266	Louis-Hébert / Beaubien	19
6267	Chabot / Beaubien	15
6268	Chambord / Beaubien	11
6269	Wurtele / Rouen	19
6270	Fabre / St-Zotique	19
6271	Casgrain / de Bellechasse	23
6272	de Bordeaux / St-Zotique	19
6273	Cartier / Bélanger	19
6274	de la Roche / St-Joseph	19
6275	Chambord / Jean-Talon	15
6276	de Normanville / Bélanger	19

6277	Louis-Hébert / de Bellechasse	15
6278	Louis-Hébert / St-Zotique	19
6279	Louis-Hébert / Bélanger	15
6280	Fairmount / St-Dominique	15
6281	Resther / St-Joseph	15
6301	Parc Outremont (Bloomfield / Elmwood)	15
6302	Stuart / de la Côte-Ste-Catherine	23
6303	Dunlop / Van Horne	11
6304	Rockland / Lajoie	19
6305	Davaar / de la Côte-Ste-Catherine	19
6306	Métro Édouard-Montpetit (du Mont-Royal / Vincent-d'Indy)	39
6307	Laval / Rachel	15
6309	4e avenue / de Verdun	15
6310	de Darlington / de la Côte-Ste-Catherine	31
6311	Drolet / St-Zotique	15
6312	de Kent / de la Côte-des-Neiges	31
6313	Palm / St-Remi	23
6314	de Lanaudière / Marie-Anne	19
6315	Métro Côte-des-Neiges (Jean-Brillant / de la Côte-des-Neiges)	11
6316	Swail / Decelles	27
6321	Gary-Carter / St-Laurent	23
6322	St-Dominique / Gounod	19
6323	Guizot / St-Laurent	15
6324	de Liège / Lajeunesse	15
6327	Drolet / Faillon	19
6328	Henri-Julien / Villeray	15
6329	Drolet / Gounod	19
6330	de Gaspé / Jarry	15
6331	Guizot / St-Denis	23
6332	de Gaspé / de Liège	15
6333	Leman / de Chateaubriand	15
6334	Lajeunesse / Jarry	23
6335	du Rosaire / St-Hubert	15
6336	Faillon / St-Hubert	15
6338	Boyer / Jarry	23
6339	d'Oxford / de Monkland	15
6340	de la Roche / Everett	19
6341	Regina / de Verdun	15
6343	Marquette / Villeray	15
6344	St-Dominique / Jean-Talon	31
6345	Louis-Hémon / Villeray	19
6346	Nicolet / Sherbrooke	19
6347	Métro St-Michel (Shaughnessy / St-Michel)	19
6349	Ryde / Charlevoix	23
6350	Island / Centre	23

6354	Marcil / Sherbrooke	23
6355	Ontario / Sicard	15
6356	de Monkland / Girouard	15
6357	12e avenue / St-Zotique	15
6358	1ère avenue / St-Zotique	15
6359	Ste-Catherine / Clark	15
6360	8e avenue / Beaubien	15
6361	Molson / Beaubien	15
6362	1ère avenue / Rosemont	19
6363	de la Côte St-Antoine / Royal	15
6364	de Chambly / Rachel	19
6366	Wilderton / Van Horne	15
6367	3e avenue / Dandurand	15
6368	10e avenue / Masson	19
6369	U. Concordia - Campus Loyola (Sherbrooke / West Broadway)	27
6370	d'Orléans / Masson	15
6371	4e avenue / Masson	31
6372	1ère avenue / Masson	27
6373	7e avenue / St-Joseph	19
6374	Laurier / 15e avenue	27
6375	Métro Place St-Henri (St-Ferdinand / St-Jacques)	19
6376	16e avenue / St-Joseph	23
6377	Grand Trunk / Hibernia	23
6379	de l'Église / Bannantyne	15
6380	Parc J.-Arthur-Champagne (de Chambly / du Mont-Royal)	19
6381	Omer-Lavallée / du Midway	19
6383	Bourbonnière / du Mont-Royal	27
6384	Darling / Sherbrooke	15
6385	de Bordeaux / Gilford	19
6386	Métro Préfontaine (Moreau / Hochelaga)	23
6387	Métro Joliette (Joliette / Hochelaga)	19
6388	d'Orléans / Hochelaga	15
6389	Boyer / Jean-Talon	15
6391	Aylwin / Ontario	15
6393	Dézéry / Ontario	23
6394	Valois / Ontario	23
6395	Métro Viau (Pierre-de-Coubertin / Sicard)	49
6396	Métro Pie-IX (Pierre-de-Coubertin / Pie-IX)	33
6397	Marché Maisonneuve	19
6398	Desjardins / Ontario	19
6401	des Seigneurs / Notre-Dame	19
6402	Square Sir-Georges-Etienne-Cartier / Ste-Émilie	23
6403	Georges-Vanier / Notre-Dame	23
6404	Quesnel / Vinet	27
6405	Duvernay / Charlevoix	19

6406	Marché Atwater	33
6407	Charlevoix / Lionel-Groulx	23
6408	Métro Georges-Vanier (St-Antoine / Canning)	17
6409	Lionel-Groulx / George-Vanier	19
6410	Métro Crémazie (Crémazie / Lajeunesse)	15
6411	Clark / Prince-Arthur	23
6412	Complexe sportif Claude-Robillard	35
6413	Cathcart / Union	39
6414	Laporte / St-Antoine	15
6415	Wilson / Sherbrooke	15
6416	Notre-Dame-de-Grâce / Décarie	19
6417	Desjardins / Hochelaga	15
6418	de Vendôme / de Maisonneuve	15
6419	Beaucourt / de la Côte-Ste-Catherine	23
6420	Métro Snowdon (de Westbury / Queen-Mary)	27
6421	Cartier / Rosemont	19
6422	Fleury / Lajeunesse	15
6423	Hôpital général juif (de la Côte Ste-Catherine / Légaré)	19
6424	du Président-Kennedy / McGill College	15
6425	Ross / de l'Église	15
6426	Métro Verdun (Willibrord / de Verdun)	19
6427	Métro Lasalle (de Rushbrooke / Caisse)	19
6428	Berlioz / de l'Île des Soeurs	23
6429	Place du Commerce	33
6432	de Maisonneuve / Greene	15
6433	Hillside / Ste-Catherine	15
6434	Victoria / de Maisonneuve	15
6435	Victoria Hall	15
6436	Argyle / Sherbrooke	19
6501	Parc Jean-Drapeau	39
6502	Casino de Montréal	19
6503	Métro Parc (Ogilvy / Hutchison)	33
6504	La Ronde	23
6700	de la Salle / Ste-Catherine	19
6701	Centre Pierre-Charbonneau	19
6702	Chauveau / de l'Assomption	15
6703	Jardin Botanique (Pie-IX / Sherbrooke)	15
6704	19e avenue / St-Zotique	23
6705	5e avenue / Bannantyne	15
6706	Beatty / de Verdun	15
6707	Métro Jolicoeur (Drake / de Sève)	19
6708	Place Jean-Paul Riopelle (Viger / de Bleury)	27
6709	Le Caron / Marc-Sauvalle	23
6710	Georges-Baril / Fleury	15
6711	Alexandra / Waverly	11

6712	LaSalle / Crawford	23
6713	30e avenue / St-Zotique	15
6714	LaSalle / Sénécal	31
6715	Natatorium (LaSalle / Rolland)	19
6716	Francis / Fleury	15
6717	de Kent / Victoria	19
6718	Grand Boulevard / Sherbrooke	15
6719	Park Row O / Sherbrooke	15
6720	Ontario / Viau	23
6721	Métro Cadillac (Sherbrooke / de Cadillac)	19
6722	Pierre-de-Coubertin / Louis-Veuillot	19
6723	26e avenue / Beaubien	15
6724	Square Sir-Georges-Étienne-Cartier / St-Ambroise	23
6725	Métro Monk (Allard / Beaulieu)	15
6726	Hamel / Sauvé	15
6727	Richardson / de Montmorency	23
6728	d'Outremont / Ogilvy	15
6729	St-André / Ste-Catherine	15
6730	35e avenue / Beaubien	19
6731	28e avenue / Rosemont	19
6732	Fortune / Wellington	27
6733	de Maisonneuve/ Mansfield (est)	19
6734	Lajeunesse / Villeray (place Tapéo)	15
6735	François-Perrault / L.-O.-David	15
6736	Basile-Routhier / Gouin	19
6737	Jacques-Casault / Christophe-Colomb	19
6738	Union / René-Lévesque	31
6739	de Repentigny / Sherbrooke	19
6741	Canning / Notre-Dame	19
6742	Briand / le Caron	23
6743	St-Marc / Sherbrooke	23
6744	Hamilton / Jolicoeur	15
6745	de Maisonneuve / de Bleury	15
6746	Métro Acadie (de l'Acadie / Beaumont)	11
6747	Waverly / St-Zotique	15
6748	Young / Wellington	15
6749	St-Jacques / St-Laurent	15
6750	des Érables / Rachel	7
6752	Hutchison / Beaubien	27
6753	Centre ÉPIC (St-Zotique / 40e avenue)	27
6754	Eadie / Dubois	15
6901	Gare d'autocars de Montréal (Berri / Ontario)	15
6902	Montcalm / Ontario	15
6903	Napoléon / St-Dominique	23
6904	Fabre / Beaubien	15

6905	Parc Rosemont (Dandurand / d'Iberville)	19
6906	Métro Rosemont (Rosemont / de St-Vallier)	26
6907	Boyer / Rosemont	23
6908	de Bellechasse / de St-Vallier	15
6910	Boyer / Beaubien	19
6912	Métro Beaubien (de Chateaubriand / Beaubien)	23
6913	Drolet / Beaubien	19
6915	Alma / Beaubien	19
6916	Parc du Pélican (2e avenue / St-Joseph)	23
6917	Basile-Routhier / Chabanel	15
6918	Marquette / des Carrières	19
6919	Bibliothèque de Rosemont (9e avenue / Rosemont)	15
6921	Augustin-Cantin / Shearer	31
6923	Marquette / Jean-Talon	19
6924	de Bordeaux / Jean-Talon	15
6925	des Écores / Jean-Talon	15
6926	Marie-Anne / St-Hubert	19
6927	Édouard-Montpetit / de Stirling	19
6928	Jean-Brillant / McKenna	11
6929	St-André / St-Grégoire	15
6930	Paul Boutet / des Regrattiers	35
7001	Ball / Querbes	19
7002	Tolhurst / Fleury	15
7003	George-Baril / Sauvé	19
7004	Émile-Journault / de Chateaubriand	15
7005	Marquette / Fleury	15
7006	Clark / Fleury	15
7007	Gare Canora (Jean-Talon / Canora)	14
7008	Rousselot / Jarry	15
7009	CHSLD Benjamin-Victor-Rousselot (Dickson / Sherbrooke)	15
7010	de Mayfair / Monkland	15
7011	Girouard / de Terrebonne	15
7012	Louis-Colin / McKenna	23
7013	Benny / Sherbrooke	15
7014	Métro Université de Montréal	23
7015	Parc des Rapides (LaSalle / 6e avenue)	23
7016	Métro Langelier (Sherbrooke / Langelier)	19
7017	Bennett / Ste-Catherine	15
7018	Joliette / Ste-Catherine	15
7019	Casgrain / St-Viateur	23
7020	St-Germain / Hochelaga	15
7021	Dollard / Van Horne	23
7022	Durocher / Bernard	19
7023	CHSLD St-Michel (Jarry / 8e avenue)	15
7024	Berri / Gilford	15

7025	St-Dominique / Bernard	23
7026	Maguire / Henri-Julien	23
7027	Terrasse Guindon / Fullum	15
7028	de Gaspé / St-Viateur	23
7029	Cartier / Masson	15
7030	de Bordeaux / Marie-Anne	11
7031	Berri / Rachel	15
7032	Drolet / Laurier	27
7033	Aylmer / Prince-Arthur	19
7034	Atwater / Greene	25
7035	Fullum / Gilford	15
7036	Hutchison / Edouard Charles	11
7037	Prince-Arthur / Ste-Famille	15
7038	Guilbault / Clark	19
7039	du Mont-Royal / Augustin-Frigon	15
7040	St-Urbain / Beaubien	15
7041	15e avenue / Masson	15
7042	Alexandra / Jean-Talon	15
7043	Ernest-Gendreau / du Mont-Royal	15
7044	Hôpital Santa Cabrini (St-Zotique / Jeanne-Jugan)	15
7045	Casgrain / Mozart	15
7046	15e avenue / Rosemont	15
7047	Ottawa / Peel	27
7048	Métro Angrignon	31
7049	Ottawa / St-Thomas	27
7050	Ottawa / William	15
7051	Jogues / Allard	19
7052	Shearer / Centre	15
7053	Ropery / Augustin-Cantin	23
7054	Cote St-Paul / St-Ambroise	35
7055	Greene / Workman	15
7056	Bibliothèque de Verdun (Brown / Bannantyne)	15
7057	2e avenue / Wellington	19
7058	Gordon / Wellington	15
7059	Argyle / Bannantyne	15
7060	de l'Église / de Verdun	15
7061	Lajeunesse / de Castelnau	19
7062	Wellington / Robert-Bourassa	11
7063	Drolet / Jarry	23
7064	Clark / de Liège	19
7065	de Lanaudière / Bélanger	23
7066	St-Urbain / de la Gauchetière	31
7067	City Councillors / du President-Kennedy	31
7068	Basin / Richmond	19
7069	Union / du Président-Kennedy	31

7070	Bourgeoys / Favard	27
7071	St-Mathieu / Sherbrooke	19
7072	Beaudry / Sherbrooke	15
7073	Logan / de Champlain	15
7074	St-André / Robin	19
7075	CHSLD Éloria-Lepage (de la Pépinière / de Marseille)	23
7076	Tupper / du Fort	15
7077	Jean Langlois / Fullum	27
7078	Hochelaga / Chapleau	15
7079	Gauvin / Notre-Dame	27
7080	du President-Kennedy / Robert Bourassa	23
7081	Lincoln / Lambert Closse	19
7082	Métro Lionel-Groulx (Atwater / Lionel-Groulx)	19
7083	Parc de Bullion (de Bullion / Prince-Arthur)	19
7084	McTavish / Sherbrooke	15
10002	Métro Charlevoix (Centre / Charlevoix)	23

APPENDIX B – Sample Calculation

Sample calculation (steps 1 through 7) for July 20th, 2017.

STEP 1

7:51:00	-1
8:07:00	-1
8:17:00	-1
8:23:00	-1
8:26:00	-1
8:30:00	-1
8:35:00	-1
8:37:00	-1
8:40:00	-1
8:41:00	-1
8:49:00	-1

STEP 2

7:34:00	1	8:03:00	1	8:33:00	1
7:34:00	1	8:04:00	1	8:35:00	1
7:36:00	1	8:05:00	1	8:37:00	1
7:37:00	1	8:07:00	1	8:37:00	1
7:39:00	1	8:09:00	1	8:37:00	1
7:40:00	1	8:13:00	1	8:39:00	1
7:41:00	1	8:14:00	1	8:39:00	1
7:42:00	1	8:15:00	1	8:39:00	1
7:45:00	1	8:16:00	1	8:41:00	1
7:45:00	1	8:16:00	1	8:42:00	1
7:47:00	1	8:18:00	1	8:42:00	1
7:48:00	1	8:19:00	1	8:44:00	1
7:49:00	1	8:20:00	1	8:44:00	1
7:50:00	1	8:27:00	1	8:45:00	1
7:50:00	1	8:28:00	1	8:46:00	1
7:50:00	1	8:29:00	1	8:48:00	1
7:51:00	1	8:29:00	1	8:48:00	1
7:53:00	1	8:29:00	1	8:52:00	1
7:54:00	1	8:30:00	1	8:52:00	1
7:57:00	1	8:31:00	1	8:52:00	1
7:57:00	1	8:32:00	1	8:54:00	1
7:58:00	1	8:32:00	1	8:55:00	1
7:58:00	1	8:33:00	1	8:55:00	1
7:58:00	1	8:33:00	1	8:55:00	1
8:01:00	1	8:33:00	1	8:59:00	1
8:03:00	1	8:33:00	1	9:00:00	1

STEP 3

Time	+1/-1	Cumulative	Time	+1/-1	Cumulative	Time	+1/-1	Cumulative
7:30:00	0	0	8:05:00	1	28	8:35:00	-1	46
7:34:00	1	1	8:07:00	-1	27	8:35:00	1	47
7:34:00	1	2	8:07:00	1	28	8:37:00	-1	46
7:36:00	1	3	8:09:00	1	29	8:37:00	1	47
7:37:00	1	4	8:13:00	1	30	8:37:00	1	48
7:39:00	1	5	8:14:00	1	31	8:37:00	1	49
7:40:00	1	6	8:15:00	1	32	8:39:00	1	50
7:41:00	1	7	8:16:00	1	33	8:39:00	1	51
7:42:00	1	8	8:16:00	1	34	8:39:00	1	52
7:45:00	1	9	8:17:00	-1	33	8:40:00	-1	51
7:45:00	1	10	8:18:00	1	34	8:41:00	-1	50
7:47:00	1	11	8:19:00	1	35	8:41:00	1	51
7:48:00	1	12	8:20:00	1	36	8:42:00	1	52
7:49:00	1	13	8:23:00	-1	35	8:42:00	1	53
7:50:00	1	14	8:26:00	-1	34	8:44:00	1	54
7:50:00	1	15	8:27:00	1	35	8:44:00	1	55
7:50:00	1	16	8:28:00	1	36	8:45:00	1	56
7:51:00	-1	15	8:29:00	1	37	8:46:00	1	57
7:51:00	1	16	8:29:00	1	38	8:48:00	1	58
7:53:00	1	17	8:29:00	1	39	8:48:00	1	59
7:54:00	1	18	8:30:00	-1	38	8:49:00	-1	58
7:57:00	1	19	8:30:00	1	39	8:52:00	1	59
7:57:00	1	20	8:31:00	1	40	8:52:00	1	60
7:58:00	1	21	8:32:00	1	41	8:52:00	1	61
7:58:00	1	22	8:32:00	1	42	8:54:00	1	62
7:58:00	1	23	8:33:00	1	43	8:55:00	1	63
8:01:00	1	24	8:33:00	1	44	8:55:00	1	64
8:03:00	1	25	8:33:00	1	45	8:55:00	1	65
8:03:00	1	26	8:33:00	1	46	8:59:00	1	66
8:04:00	1	27	8:33:00	1	47	9:00:00	1	67

STEP 4

Month	Day	Start time	Inter-departure time
7	3	7:48:00	
7	3	8:15:00	0:27:00
7	3	8:29:00	0:14:00
7	3	8:55:00	0:26:00
7	4	7:48:00	
7	4	7:59:00	0:11:00
7	4	8:01:00	0:02:00
7	4	8:08:00	0:07:00
7	4	8:19:00	0:11:00
7	4	8:20:00	0:01:00
7	4	8:38:00	0:18:00
7	4	8:39:00	0:01:00
7	4	8:46:00	0:07:00
7	4	8:47:00	0:01:00
7	4	8:55:00	0:08:00
7	4	8:56:00	0:01:00
7	4	9:00:00	0:04:00
7	5	7:51:00	
7	5	7:59:00	0:08:00
7	5	8:02:00	0:03:00
7	5	8:02:00	0:00:00
7	5	8:02:00	0:00:00
7	5	8:07:00	0:05:00
7	5	8:10:00	0:03:00
7	5	8:28:00	0:18:00
7	5	8:28:00	0:00:00
7	5	8:37:00	0:09:00
7	5	8:40:00	0:03:00
7	5	8:48:00	0:08:00
7	5	8:48:00	0:00:00
7	5	8:57:00	0:09:00
7	6	7:45:00	
7	6	7:58:00	0:13:00
7	6	8:01:00	0:03:00
7	6	8:06:00	0:05:00

7	6	8:09:00	0:03:00
7	6	8:24:00	0:15:00
7	6	8:28:00	0:04:00
7	6	8:30:00	0:02:00
7	6	8:31:00	0:01:00
7	6	8:34:00	0:03:00
7	6	8:40:00	0:06:00
7	6	8:48:00	0:08:00
7	6	8:58:00	0:10:00
7	11	7:43:00	
7	11	7:47:00	0:04:00
7	11	8:02:00	0:15:00
7	11	8:02:00	0:00:00
7	11	8:22:00	0:20:00
7	11	8:24:00	0:02:00
7	11	8:27:00	0:03:00
7	11	8:27:00	0:00:00
7	11	8:40:00	0:13:00
7	11	8:47:00	0:07:00
7	11	8:51:00	0:04:00
7	11	8:52:00	0:01:00
7	11	8:56:00	0:04:00
7	11	8:58:00	0:02:00
7	11	9:00:00	0:02:00
7	12	7:31:00	
7	12	7:36:00	0:05:00
7	12	7:46:00	0:10:00
7	12	7:51:00	0:05:00
7	12	7:55:00	0:04:00
7	12	8:01:00	0:06:00
7	12	8:10:00	0:09:00
7	12	8:27:00	0:17:00
7	12	8:29:00	0:02:00
7	12	8:35:00	0:06:00
7	12	8:43:00	0:08:00
7	12	8:52:00	0:09:00
7	18	7:44:00	
7	18	7:46:00	0:02:00
7	18	7:46:00	0:00:00

7	18	7:54:00	0:08:00
7	18	8:07:00	0:13:00
7	18	8:08:00	0:01:00
7	18	8:08:00	0:00:00
7	18	8:19:00	0:11:00
7	18	8:30:00	0:11:00
7	18	8:31:00	0:01:00
7	18	8:34:00	0:03:00
7	18	8:44:00	0:10:00
7	18	8:46:00	0:02:00
7	18	8:54:00	0:08:00
7	18	8:58:00	0:04:00
7	20	7:51:00	
7	20	8:07:00	0:16:00
7	20	8:17:00	0:10:00
7	20	8:23:00	0:06:00
7	20	8:26:00	0:03:00
7	20	8:30:00	0:04:00
7	20	8:35:00	0:05:00
7	20	8:37:00	0:02:00
7	20	8:40:00	0:03:00
7	20	8:41:00	0:01:00
7	20	8:49:00	0:08:00
7	26	7:51:00	
7	26	7:54:00	0:03:00
7	26	8:06:00	0:12:00
7	26	8:24:00	0:18:00
7	26	8:24:00	0:00:00
7	26	8:25:00	0:01:00
7	26	8:32:00	0:07:00
7	26	8:33:00	0:01:00
7	26	8:44:00	0:11:00
7	26	8:53:00	0:09:00
7	26	8:54:00	0:01:00
7	26	8:55:00	0:01:00
7	26	8:55:00	0:00:00
7	27	8:30:00	
7	27	8:30:00	0:00:00
7	27	8:30:00	0:00:00

7	27	8:32:00	0:02:00
7	27	8:33:00	0:01:00
7	27	8:34:00	0:01:00
7	27	8:36:00	0:02:00
7	27	8:37:00	0:01:00
7	27	8:46:00	0:09:00
7	27	8:56:00	0:10:00
7	27	8:58:00	0:02:00
7	28	7:37:00	
7	28	8:05:00	0:28:00
7	28	8:05:00	0:00:00
7	28	8:09:00	0:04:00
7	28	8:12:00	0:03:00
7	28	8:15:00	0:03:00
7	28	8:17:00	0:02:00
7	28	8:25:00	0:08:00
7	28	8:27:00	0:02:00
7	28	8:28:00	0:01:00
7	28	8:28:00	0:00:00
7	28	8:40:00	0:12:00
7	28	8:49:00	0:09:00

Average inter-departure time for the month of July was found to be **5 minutes and 57 seconds**, thus the departure rate μ is **0.1681** bicycles/minute.

TABLE A		
t1	7:35	-1
t2	7:41	-1
t3	7:47	-1
t4	7:53	-1
t5	7:59	-1
t6	8:05	-1
t7	8:11	-1
t8	8:17	-1
t9	8:23	-1
t10	8:29	-1
t11	8:35	-1
t12	8:41	-1
t13	8:47	-1
t14	8:53	-1
t15	8:59	-1

STEP 5

Start time	CAT	Radial distance	Speed	Estimated duration	ETA
7:16:00	M45	4.492979616	3.03167271	1482.013412	7:40:42
7:18:00	M34	6.326736564	3.08619477	2050.012081	7:52:10
7:20:00	M45	5.131886251	3.03167271	1692.757346	7:48:13
7:21:00	F23	3.180661334	2.8807218	1104.119577	7:39:24
7:25:00	M23	1.868950129	3.100364	602.8163561	7:35:03
7:25:00	M23	2.966196929	3.100364	956.725381	7:40:57
7:29:00	M34	1.71002023	3.08619477	554.0869444	7:38:14
7:30:00	F34	2.773417342	2.817764722	984.2615037	7:46:24
7:31:00	M45	2.965907518	3.03167271	978.3072916	7:47:18
7:31:00	M23	1.153571343	3.100364	372.0760991	7:37:12
7:32:00	M23	1.996950265	3.100364	644.1018748	7:42:44
7:33:00	M34	4.869080459	3.08619477	1577.697074	7:59:18
7:33:00	F34	1.554771247	2.817764722	551.7746869	7:42:12
7:33:00	F34	2.87368508	2.817764722	1019.845645	7:50:00
7:36:00	M45	2.767139572	3.03167271	912.7435039	7:51:13
7:39:00	F56	1.54875432	2.770198053	559.0771094	7:48:19
7:39:00	M23	1.381011405	3.100364	445.4352473	7:46:25
7:42:00	M23	2.972224305	3.100364	958.6694675	7:57:59
7:42:00	M34	4.818770518	3.08619477	1561.395465	8:08:01
7:42:00	M45	2.162072081	3.03167271	713.1614418	7:53:53
7:44:00	M45	1.990480316	3.03167271	656.5617421	7:54:57
7:45:00	M34	2.303020766	3.08619477	746.2331245	7:57:26
7:45:00	F34	2.263063873	2.817764722	803.1415311	7:58:23
7:46:00	M56	3.496904527	2.926828885	1194.77587	8:05:55
7:47:00	M45	0.49042702	3.03167271	161.7677985	7:49:42
7:49:00	F34	2.303020766	2.817764722	817.3218822	8:02:37
7:52:00	M34	0.849810662	3.08619477	275.3587266	7:56:35
7:52:00	M45	2.263063873	3.03167271	746.4736762	8:04:26
7:53:00	M6+	1.081242771	2.72943115	396.1421672	7:59:36
7:53:00	F45	2.915125827	2.737980783	1064.699155	8:10:45
7:59:00	F6+	3.263364835	2.457594341	1327.869608	8:21:08
8:01:00	F23	3.15734283	2.8807218	1096.024903	8:19:16
8:01:00	F23	3.136551737	2.8807218	1088.807582	8:19:09
8:01:00	M23	2.95560322	3.100364	953.3084567	8:16:53
8:01:00	M23	4.959434364	3.100364	1599.629709	8:27:40
8:03:00	M23	2.578444839	3.100364	831.6587468	8:16:52
8:04:00	M45	5.970783723	3.03167271	1969.46844	8:36:49
8:04:00	M45	2.759087801	3.03167271	910.0876199	8:19:10

8:06:00	M23	1.245556549	3.100364	401.7452625	8:12:42
8:06:00	M56	1.733389711	2.926828885	592.2415622	8:15:52
8:06:00	M34	0.694410823	3.08619477	225.0055083	8:09:45
8:10:00	F56	4.563352155	2.770198053	1647.30177	8:37:27
8:11:00	M34	3.585848841	3.08619477	1161.899721	8:30:22
8:11:00	F34	3.047489282	2.817764722	1081.527233	8:29:02
8:12:00	M34	5.373450392	3.08619477	1741.124845	8:41:01
8:13:00	M34	4.980772861	3.08619477	1613.888051	8:39:54
8:13:00	F34	6.00253065	2.817764722	2130.245511	8:48:30
8:15:00	M45	2.759087801	3.03167271	910.0876199	8:30:10
8:15:00	F34	3.822133257	2.817764722	1356.441589	8:37:36
8:16:00	M34	4.980772861	3.08619477	1613.888051	8:42:54
8:16:00	F45	2.548240227	2.737980783	930.7005523	8:31:31
8:16:00	F23	1.959995263	2.8807218	680.3833897	8:27:20
8:17:00	M45	2.416132272	3.03167271	796.9634268	8:30:17
8:17:00	F23	3.75075401	2.8807218	1302.018824	8:38:42
8:17:00	M23	2.965907518	3.100364	956.6320335	8:32:57
8:18:00	M34	2.215067931	3.08619477	717.7343285	8:29:58
8:19:00	F34	5.038929289	2.817764722	1788.271835	8:48:48
8:20:00	M23	2.215067931	3.100364	714.4541515	8:31:54
8:20:00	M23	2.804317823	3.100364	904.5124453	8:35:05
8:21:00	M34	1.615372509	3.08619477	523.4188473	8:29:43
8:22:00	F34	4.489606559	2.817764722	1593.322013	8:48:33
8:22:00	M34	2.35471885	3.08619477	762.9845248	8:34:43
8:22:00	F34	1.868950129	2.817764722	663.2740181	8:33:03
8:24:00	F34	3.897330673	2.817764722	1383.128493	8:47:03
8:28:00	M56	1.670549854	2.926828885	570.7712749	8:37:31
8:31:00	F34	3.143268408	2.817764722	1115.518405	8:49:36
8:32:00	M23	1.341290335	3.100364	432.6235032	8:39:13
8:34:00	M23	2.030215454	3.100364	654.8313211	8:44:55
8:38:00	F45	0.905035868	2.737980783	330.548656	8:43:31
8:38:00	F34	3.298824245	2.817764722	1170.723808	8:57:31
8:41:00	M34	0.677532916	3.08619477	219.5366678	8:44:40
8:46:00	M45	1.389134735	3.03167271	458.2073554	8:53:38
8:47:00	M23	0.819745827	3.100364	264.4030917	8:51:24
8:48:00	M23	1.608747065	3.100364	518.8897384	8:56:39
8:49:00	M23	0.422100079	3.100364	136.1453297	8:51:16
8:50:00	F56	1.013204155	2.770198053	365.7515223	8:56:06
8:51:00	M56	0.401038556	2.926828885	137.0215246	8:53:17
8:54:00	M23	1.081242771	3.100364	348.747041	8:59:49

TABLE B			
7:35	1	8:27	1
7:37	1	8:27	1
7:38	1	8:29	1
7:39	1	8:29	1
7:40	1	8:29	1
7:40	1	8:30	1
7:42	1	8:30	1
7:42	1	8:30	1
7:46	1	8:31	1
7:46	1	8:31	1
7:47	1	8:32	1
7:48	1	8:33	1
7:48	1	8:34	1
7:49	1	8:35	1
7:50	1	8:36	1
7:51	1	8:37	1
7:52	1	8:37	1
7:53	1	8:37	1
7:54	1	8:38	1
7:56	1	8:39	1
7:57	1	8:39	1
7:57	1	8:41	1
7:58	1	8:42	1
7:59	1	8:43	1
7:59	1	8:44	1
8:02	1	8:44	1
8:04	1	8:47	1
8:05	1	8:48	1
8:08	1	8:48	1
8:09	1	8:48	1
8:10	1	8:49	1
8:12	1	8:51	1
8:15	1	8:51	1
8:16	1	8:53	1
8:16	1	8:53	1
8:19	1	8:56	1
8:19	1	8:56	1
8:19	1	8:57	1
8:21	1	8:59	1

STEP 6

Arrival time	+1/-1	Cumulative
7:30	0	0
7:35	1	1
7:35	-1	0
7:37	1	1
7:38	1	2
7:39	1	3
7:40	1	4
7:40	1	5
7:41	-1	4
7:42	1	5
7:42	1	6
7:46	1	7
7:46	1	8
7:47	1	9
7:47	-1	8
7:48	1	9
7:48	1	10
7:49	1	11
7:50	1	12
7:51	1	13
7:52	1	14
7:53	-1	13
7:53	1	14
7:54	1	15
7:56	1	16
7:57	1	17
7:57	1	18
7:58	1	19
7:59	1	20
7:59	1	21
7:59	-1	20
8:02	1	21
8:04	1	22
8:05	-1	21
8:05	1	22
8:08	1	23
8:09	1	24
8:10	1	25

8:11	-1	24
8:12	1	25
8:15	1	26
8:16	1	27
8:16	1	28
8:17	-1	27
8:19	1	28
8:19	1	29
8:19	1	30
8:21	1	31
8:23	-1	30
8:27	1	31
8:27	1	32
8:29	1	33
8:29	-1	32
8:29	1	33
8:29	1	34
8:30	1	35
8:30	1	36
8:30	1	37
8:31	1	38
8:31	1	39
8:32	1	40
8:33	1	41
8:34	1	42
8:35	1	43
8:35	-1	42
8:36	1	43
8:37	1	44
8:37	1	45
8:37	1	46
8:38	1	47
8:39	1	48
8:39	1	49
8:41	1	50
8:41	-1	49
8:42	1	50
8:43	1	51
8:44	1	52
8:44	1	53
8:47	1	54
8:47	-1	53

8:48	1	54
8:48	1	55
8:48	1	56
8:49	1	57
8:51	1	58
8:51	1	59
8:53	1	60
8:53	-1	59
8:53	1	60
8:56	1	61
8:56	1	62
8:57	1	63
8:59	-1	62
8:59	1	63

STEP 7

From step 6	
ETA	Cumulative
7:30	0
7:35	1
7:35	0
7:37	1
7:38	2
7:39	3
7:40	4
7:40	5
7:41	4
7:42	5
7:42	6
7:46	7
7:46	8
7:47	9
7:47	8
7:48	9
7:48	10
7:49	11
7:50	12
7:51	13
7:52	14
7:53	13

From step 3	
Real arrival time	Cumulative
7:30	0
7:34:00	1
7:34:00	2
7:36:00	3
7:37:00	4
7:39:00	5
7:40:00	6
7:41:00	7
7:42:00	8
7:45:00	9
7:45:00	10
7:47:00	11
7:48:00	12
7:49:00	13
7:50:00	14
7:50:00	15
7:50:00	16
7:51:00	15
7:51:00	16
7:53:00	17
7:54:00	18
7:57:00	19

7:53	14
7:54	15
7:56	16
7:57	17
7:57	18
7:58	19
7:59	20
7:59	21
7:59	20
8:02	21
8:04	22
8:05	21
8:05	22
8:08	23
8:09	24
8:10	25
8:11	24
8:12	25
8:15	26
8:16	27
8:16	28
8:17	27
8:19	28
8:19	29
8:19	30
8:21	31
8:23	30
8:27	31
8:27	32
8:29	33
8:29	32
8:29	33
8:29	34
8:30	35
8:30	36
8:30	37
8:31	38
8:31	39
8:32	40
8:33	41
8:34	42
8:35	43

7:57:00	20
7:58:00	21
7:58:00	22
7:58:00	23
8:01:00	24
8:03:00	25
8:03:00	26
8:04:00	27
8:05:00	28
8:07:00	27
8:07:00	28
8:09:00	29
8:13:00	30
8:14:00	31
8:15:00	32
8:16:00	33
8:16:00	34
8:17:00	33
8:18:00	34
8:19:00	35
8:20:00	36
8:23:00	35
8:26:00	34
8:27:00	35
8:28:00	36
8:29:00	37
8:29:00	38
8:29:00	39
8:30:00	38
8:30:00	39
8:31:00	40
8:32:00	41
8:32:00	42
8:33:00	43
8:33:00	44
8:33:00	45
8:33:00	46
8:33:00	47
8:35:00	46
8:35:00	47
8:37:00	46
8:37:00	47

8:35	42
8:36	43
8:37	44
8:37	45
8:37	46
8:38	47
8:39	48
8:39	49
8:41	50
8:41	49
8:42	50
8:43	51
8:44	52
8:44	53
8:47	54
8:47	53
8:48	54
8:48	55
8:48	56
8:49	57
8:51	58
8:51	59
8:53	60
8:53	59
8:53	60
8:56	61
8:56	62
8:57	63
8:59	62
8:59	63

8:37:00	48
8:37:00	49
8:39:00	50
8:39:00	51
8:39:00	52
8:40:00	51
8:41:00	50
8:41:00	51
8:42:00	52
8:42:00	53
8:44:00	54
8:44:00	55
8:45:00	56
8:46:00	57
8:48:00	58
8:48:00	59
8:49:00	58
8:52:00	59
8:52:00	60
8:52:00	61
8:54:00	62
8:55:00	63
8:55:00	64
8:55:00	65
8:59:00	66
9:00:00	67

	Bicycles	Predicted	Real	Adjustment
1	0	7:30:00	7:30	**0:00:00**
	1	7:36:07	7:34:00	**0:02:07**
	2	7:38:14	7:34:00	**0:04:14**
	3	7:39:24	7:36:00	**0:03:24**
	4	7:41:18	7:37:00	**0:04:18**
	5	7:41:34	7:39:00	**0:02:34**
	6	7:42:44	7:40:00	**0:02:44**
2	7	7:46:24	7:41:00	**0:05:24**
	8	7:47:08	7:42:00	**0:05:08**
	9	7:47:46	7:45:00	**0:02:46**
	10	7:48:19	7:45:00	**0:03:19**
	11	7:49:42	7:47:00	**0:02:42**
	12	7:50:00	7:48:00	**0:02:00**
	13	7:52:31	7:49:00	**0:03:31**
	14	7:53:02	7:50:00	**0:03:02**
	15	7:54:57	7:50:30	**0:04:27**
	16	7:56:35	7:50:30	**0:06:05**
	17	7:57:26	7:53:00	**0:04:26**
	18	7:57:59	7:54:00	**0:03:59**
	19	7:58:23	7:57:00	**0:01:23**
	20	7:59:32	7:57:00	**0:02:32**
3	21	8:02:39	7:58:00	**0:04:39**
	22	8:05:11	7:58:00	**0:07:11**
	23	8:08:01	7:58:00	**0:10:01**
	24	8:10:42	8:01:00	**0:09:42**
	25	8:11:43	8:03:00	**0:08:43**
4	26	8:15:52	8:03:00	**0:12:52**
	27	8:17:14	8:05:30	**0:11:44**
	28	8:18:01	8:06:00	**0:12:01**
	29	8:19:10	8:09:00	**0:10:10**
	30	8:21:25	8:13:00	**0:08:25**
	31	8:24:14	8:14:00	**0:10:14**
	32	8:28:35	8:15:00	**0:13:35**
	33	8:29:22	8:16:30	**0:12:52**
	34	8:29:58	8:20:00	**0:09:58**
5	35	8:30:10	8:23:00	**0:07:10**
	36	8:30:17	8:24:00	**0:06:17**
	37	8:30:22	8:29:00	**0:01:22**
	38	8:31:31	8:29:30	**0:02:01**
	39	8:31:54	8:29:30	**0:02:24**

	40	8:32:57	8:31:00	**0:01:57**
	41	8:33:03	8:32:00	**0:01:03**
	42	8:35:05	8:32:00	**0:03:05**
	43	8:35:57	8:33:00	**0:02:57**
	44	8:37:27	8:33:00	**0:04:27**
	45	8:37:31	8:33:00	**0:04:31**
	46	8:37:36	8:35:00	**0:02:36**
	47	8:38:42	8:35:00	**0:03:42**
	48	8:39:13	8:37:00	**0:02:13**
	49	8:40:39	8:37:00	**0:03:39**
	50	8:41:58	8:40:00	**0:01:58**
	51	8:43:31	8:40:00	**0:03:31**
	52	8:44:40	8:40:30	**0:04:10**
6	53	8:46:08	8:42:00	**0:04:08**
	54	8:47:47	8:44:00	**0:03:47**
	55	8:48:33	8:44:00	**0:04:33**
	56	8:48:48	8:45:00	**0:03:48**
	57	8:49:36	8:46:00	**0:03:36**
	58	8:51:16	8:48:30	**0:02:46**
	59	8:52:22	8:50:00	**0:02:22**
	60	8:53:28	8:52:00	**0:01:28**
	61	8:56:06	8:52:00	**0:04:06**
	62	8:57:57	8:54:00	**0:03:57**
	63	8:58:40	8:55:00	**0:03:40**
	64		8:55:00	
	65		8:55:00	
	66		8:59:00	
	67		9:00:00	

OUTPUT	**Adjustment Factors**	
	7:30-7:45	0:02:46
	7:45-8:00	0:03:37
	8:00-8:15	0:08:03
	8:15-8:30	0:11:19
	8:30-8:45	0:03:17
	8:45-9:00	0:03:28