Performance Analysis of Secondary Users in Cognitive Radio Networks

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Abstract

Performance Analysis of Secondary Users in Cognitive Radio Networks

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Cognitive radio technology is to improve the inefficient usage of limited spectrum resources when wireless networks coexist. Using this technology, the unlicensed users (secondary users) can opportunistically access the frequency band of licensed users (primary users). There are different kinds of cognitive radio networks. This thesis focuses on the interweave cognitive radio network, where the secondary users are only allowed to access the spectrum holes, i.e., the idle parts of the licensed spectrum band. The main task is to analyze the opportunistic dynamic spectrum allocation method based on selecting the largest available spectrum hole, where the goal is to have the maximum possible transmission rate for secondary users. The objective of this work is to give the service provider an estimation about the load of secondary users in different numbers of channels and various kinds of traffics in primary networks.

The work starts with the investigation on the spectrum allocation for secondary users in a network of single-channel primary users. In our analysis, we propose a theoretical model to calculate the probability distribution of the length of the largest available spectrum hole. The contribution of this part is the modeling and performance analysis of the existing conventional method, which

selects the largest available spectrum hole. The main contribution is the calculation of the conditional probability of having maximum consecutive idle channels under the condition of a given number of total channels and various number of busy channels. For any given number of channels, this conditional probability gives us the number of consecutive idle channels the secondary user can have, if we know the probability distribution of busy channels taken by primary users. The theoretical model works for any given number of total channels in the licensed frequency band, with numerical and simulation results confirming the precision of the proposed model.

Later, we continue our study on the spectrum allocation in a cognitive radio network of multi-channel primary users, where the secondary user temporarily takes the largest available spectrum hole. For the performance analysis, we basically need to solve two problems. First, we need to find the probability distribution of busy channels taken by primary users. Second, we need to determine the length of the largest available spectrum hole under the condition of primary users taking different channels. In the case of primary users taking multiple channels, the calculation of the conditional probability of having maximum consecutive idle channels under the condition of a given number of total channels and various number of busy channels, is approximately valid, especially in low-traffic networks. As such, the main contribution in this part is finding the probability distribution of busy channels taken by primary users. The solution scenario is based on a multidimensional Markov chain, with numerical and simulation results verifying the accuracy of the proposed model.

Finally, an approximate one-dimensional Markov chain is also proposed to simplify the complicated multidimensional solution. We provide an approximate estimation for the load of the secondary user to avoid the calculation of the complex multidimensional Markov chain. The procedure significantly decreases the complexity, although we lose some information. The main concern in the one-dimensional approximation is to find the departure rates from each state of busy channels. It is actually the main challenge of this part and by approximation we provide the solution. At the end, the performance of the proposed model was validated by numerical and simulation results.

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Dedication

To my dearest parents, Maryam and Sirous

and to my beloved wife, Nasim

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List of Abbreviations

CR Cognitive Radio

CSA Channel Selection Algorithm

MAC Medium Access Control

PU Primary User

PUEA Primary User Emulation Attack

QoS Quality of Service

SH Spectrum Hole

SU Secondary User

1 Introduction

Wireless communications is a very challenging field nowadays. Providing extra spectrum resources to meet the excessive demand comes with great costs, while even today some of available resources are underutilized. The scarcity and the inefficient usage of resources provide motivation to look for some opportunistic spectrum allocation. To deal with these challenges, cognitive radio (CR), as an emerging solution, has been proposed by Mitola [1].

With CR technology, the unlicensed users, also known as secondary users (SUs), can opportunistically access the spectrum resources of licensed users, also known as primary users (PUs). Moreover, CR technology manages the efficient spectrum allocation to fulfill the user demands. It also helps to maintain the required quality of service and provides a fair spectrum sharing among all coexisting users [2–8].

In the next section, the CR technology and its spectrum allocation procedure, as the main scenario of this thesis, will be briefly explored through some published papers.

1.1 Literature Survey

As mentioned earlier, the term "*cognitive radio*", relates the technology that improves the utilization of the spectrum by sharing the resources in a smart way. Explanation of CR technology and how it may enhance the spectrum utilization are provided in the landmark paper of Haykin [2]:

"*Cognitive radio is viewed as a novel approach for improving the utilization of a precious natural*

resource: the radio electromagnetic spectrum.

The cognitive radio, built on a software-defined radio, is defined as an intelligent wireless communication system that is aware of its environment and uses the methodology of understanding-by-building to learn from the environment and adapt to statistical variations in the input stimuli, with two primary objectives in mind:

- *highly reliable communication whenever and wherever needed;*
- *efficient utilization of the radio spectrum.*"

As mentioned before, the CR technology provides the efficient ability for sharing the spectrum resources with PUs. As such, the SUs are authorized to temporarily and opportunistically use the spectrum holes. Besides, the performance of CR networks is highly dependent on the activity of primary users. Hence, study on PUs behavior and modeling the PUs activity in CR networks is very important [9, 10].

In CR technology, SUs are allowed to sense the spectrum resources and share the unused ones without making significant interference to other users. This is called Spectrum Sensing [5]. CR technology also manages the best spectrum allocation to fulfill the user demands. This part is called Spectrum Management [6]. The CR techniques help to maintain the required quality of service (QoS) during spectrum switching. This is called Spectrum Mobility [7]. Finally, CR technology provides a fair spectrum sharing among all coexisting users which is called Spectrum Sharing [8].

There are different kinds of CR networks: in interweave CR networks, which is the focus of the thesis, SUs can access the idle channels of PUs. Accordingly, PUs and SUs in interweave CR networks do not have access to the same channels simultaneously. In this scenario, the consecutive idle channels, which can be temporarily allocated to SUs, are called spectrum holes. In other kinds of CR networks, the coexistence of PUs and SUs is allowed but with some predefined conditions [11–26].

In addition, the spectrum allocation procedure in interweave CR technology consists of four

major parts: spectrum sensing and monitoring, spectrum analysis, spectrum decision, and spectrum mobility. In spectrum sensing and monitoring, the secondary users sense the spectrum to detect the spectrum holes. Also, on a primary user appearance, this part can detect the activity to avoid making interference to the primary user. The duty of the spectrum analysis is to estimate the characteristics of the spectrum holes that are detected through spectrum sensing. In fact, it is important to define some parameters that can represent the quality of a particular spectrum band. Spectrum decision is responsible to select the most appropriate spectrum hole among the available ones. Finally, the spectrum mobility part takes care of any secondary user to vacate its spectrum hole when a primary user appears on that spectrum band [6, 27, 28]. The focus of this thesis is on the spectrum decision, with its duty being the selection of the best spectrum hole based on the given criterion.

Furthermore, lots of interesting researches have been done in the field of CR networks. For instance, a survey on spectrum sensing provided in [29] and a comprehensive survey for spectrum assignment delivered in [30]. Also, an effective channel assignment is given in [31] and a channel selection algorithm (CSA) for multichannel CR networks is provided in [32], which allows secondary users to find a vacant channel with minimal number of channel switches. The proposed method in [33] selects the spectrum hole with the maximum idle time and the presented method in [34] chooses the spectrum hole which has the minimum expected transmission time with less spectrum mobility. Further, The cooperative technique used in CR networks for spectrum sensing/sharing has been deeply studied in [35, 36].

Moreover, an overview on CR networking and communications is provided in [37] by investigating the functions of the physical, medium access control (MAC), and network layers involved in a CR design and how these layers are crossly related. Spectrum sensing is the essential component in the physical layer. The MAC layer manages the spectrum access to the identified spectrum holes, and the network layer is responsible for maintaining the required quality of service during spectrum mobility.

In addition, some researches have been done on the security of CR networks [38]. While the

CR technology performs spectrum sensing to know whether a PU is present or not, there could be some adverse SUs that can mimic the PU's signal and make some vulnerabilities to prevent proper spectrum sensing. This kind of threat is called primary user emulation attack (PUEA). The PUEA can lead to severe reduction in the availability of spectrum resources for legitimate SUs by making them vacate the spectrum bands [39]. Obviously, these threats can significantly degrade the performance of the CR networks. In order to face the PUEA, a proper model is essential to evaluate the attacks and adopt adequate mitigation strategies. As a result, many efforts have been done to analyze and combat PUEAs [40–45].

We should also mention that there are recent researches to meet the critical requirements of the fifth generation (5G) mobile networks using CR technology. A comprehensive survey of CR technology is provided in [46] which focuses on the current significant research progress in 5G. The survey is expected to help researchers develop practical applications of CR technology in the field of 5G.

Finally, based on the literature review, the lack of research for determination of the load of secondary users with different number of channels in CR networks is noticeable. To the best of our knowledge, no work has been undertaken before or is not available in the open literature. We find this topic practically significant which provides an estimation of the load of secondary users by analyzing its performance in any given number of channels. Based on that, the motivation and objectives of the thesis are explained.

1.2 Motivation and Objectives

The spectrum allocation method for SUs is a topic that directly affects the CR network capacity. Though, more specifically, how this method works with the density of the PUs and demands of the SUs, will impact the network overall performance. In addition, to understand the behavior of CR networks and for the purpose of analysis, many efforts have been done to emulate what is most

likely occurring in real situation using stochastic models. Hence, such topic should be interesting for network designers in the field of wireless networks.

This motivates us to analyze and improve the performance of CR networks as our main goal. Therefore, in this thesis, we focus on one of the major aspects of CR networks, i.e., their spectrum allocation method and the related mathematical models.

Selecting the most appropriate spectrum hole (SH), the idle parts of the licensed spectrum band, with simultaneous consideration of multiple goals, is a significant challenge. In a CR network where the spectrum holes change dynamically and their sizes are not necessarily the same, if the highest possible transmission rate is the goal, taking the largest available spectrum hole is an effective choice.

Focusing on this single-goal problem, we investigate the performance of the SU in a network of multichannel PUs. For the performance analysis, we first start with the single-channel PUs, then we continue to multichannel PUs, where we basically need to solve two problems. First, we need to find the probability distribution of busy channels taken by PUs. Second, we need to determine the length of the largest available spectrum hole under the condition of PUs taking different channels.

In summary, the objective of the thesis is the load calculation for secondary users in any given number of channels. For this purpose, development of a mathematical model for performance analysis of secondary users in primary networks is required. Based on this objective, there are three contributions in the thesis which are elaborated next.

1.3 Thesis Contributions

This thesis aims to make the utilization of wireless communication resources more efficient by proposing theoretical models for spectrum allocation. The contributions of this thesis can be summarized as follows.

The first contribution of this work is the proposal of a mathematical model to calculate the proba-

bility distribution of the length of the largest available spectrum hole in a network of single-channel primary users. The contribution is the modeling and performance analysis of the existing conventional method, which selects the largest available spectrum hole [47]. The practical significance of this work is to give the service provider an estimation of the performance of SUs in different numbers of channels and various kinds of traffics in primary networks. This important theoretical modeling and analysis has not been undertaken before and/or is not available in the open literature. We believe that the results presented here significantly advances the state-of-the-art in the CR network field.

To address the challenges, first we analyze the performance of secondary users in a network of single-channel primary users. The main contribution is to calculate the $P(x \mid n, m)$, as the conditional probability of having maximum x consecutive idle channels under the condition of n total and m busy channels. For any given number of channels, this conditional probability gives us the number of consecutive idle channels a secondary user can have if we know the probability distribution of busy channels taken by primary users. This probability distribution can be simply achieved by using the traditional $M/G/k/k$ queue system if each primary user takes only one channel by assumption.

Later, in the case of primary user taking multiple channels, the calculation of $P(x \mid n, m)$ is approximately valid, especially in low-traffic networks. As such, the main challenge is to find the probability distribution of busy channels taken by primary users. This is what we will solve as the second contribution of our work.

The mentioned challenge, as far as we know, is an open problem. In the literature, the closest case we can find is the queue with batch arrivals. However, the conventional batch arrivals in queueing theory cannot be applied here due to two reasons. First, the departures of batch arrivals are independent in queueing theory. It means that the arrivals in one batch leave independently. In our case, each arriving user occupies multiple channels simultaneously and leaves all of them later on simultaneously as well. We call this case as batch-arrival-batch-departure. Second, the

number of busy channels is important in queueing theory, but the locations of busy ones are not. In our case when primary users take consecutive channels, the locations of busy channels are also important. We need this information to find out whether a primary user will be accepted or blocked if it requests y consecutive channels when w channels are available ($w \geqslant y$).

With our second contribution, we will be able to find the probability distribution of busy channels taken by primary users and hence the performance of the SU in a network of multichannel PUs. However, if the maximum number of channels that a PU can take increases, the complexity of the solution increases accordingly. To simplify the case, we propose an approximation to find the probability distribution of busy channels taken by primary users. This approximate solution which helps us to estimate the load of SU is the third contribution of our work.

1.4 Thesis Organization

This thesis is organized as follows. Chapter 2 presents the concept of queueing theory in CR technology. It starts by an introduction to a queue system. Then it briefly outlines the required background of queueing theory needed for the subject of this work. Next, in Chapter 3 we analyze the performance of secondary users in a network of single-channel primary users. Chapter 4 targets the challenges where primary user takes a random number of channels. In the case of consecutive channels for primary users, the location tracking of busy channels is practically unfeasible. As such, as a substitute for the location tracking, we find the probability of acceptance for any primary user requesting a random number of channels. However, if the maximum number of channels that a primary user can take increases, the complexity of the solution increases accordingly. To simplify the case, in Chapter 5 we propose an approximation for the precise but complicated solution, which is followed by the conclusion and future work in Chapter 6.

2 Queueing Theory in CR Technology

Cognitive radio is the technology that provides a platform to share the spectrum between primary and secondary users. The spectrum sharing can be effectively modeled and solved by means of queueing theory. As such, queueing model is an important tool to evaluate the performance of such a system [48–50].

Therefore, we start this chapter by briefly studying the concept of a queueing system. Then we continue by providing the required background of queueing models which are required for the work of this thesis.

2.1 Queueing System

A queueing system can be described as customers arriving for service, departing the system as discouraged customers or waiting for service and leaving the system after being served [51]. Queueing theory was developed in order to provide models to predict the behavior of queueing systems especially for the ones that have randomly arriving customers. The telephone traffic congestion was one of the earliest problems studied in queueing theory, and Danish mathematician A. K. Erlang was the pioneer investigator. He observed that a telephone system was generally characterized by either [51]: "*(1) Poisson input, exponential holding (service) times, and multiple channels (servers), or (2) Poisson input, constant holding times, and a single channel.*"

An adequate representation of a queueing system needs a detailed characterization of its pro-

cesses. Some of the basic characteristics of queueing processes are arrival pattern of customers, service pattern of servers, queue discipline, and system capacity.

For describing the queueing processes a notation has been developed which is standard throughout the queueing literature. The notation for describing the queueing process is a series of symbols and slashes like $A/B/X/Y/Z$. In this case, A indicates the interarrival time distribution, B shows the service pattern which is described by the probability distribution for service time, X indicates the number of parallel service channels, Y is for the restriction on system capacity, and Z indicates the queue discipline [51].

2.2 Markovian Queueing Models

The queueing models in this thesis use the theory of birth-death processes. A birth-death process is a specific type of Markov chain which can help to have steady-state probabilities with a straightforward solution. The states in queueing theory denotes the number of customers in the system. Birth-death process consists of a set of states, for example $\{0, 1, 2, ...\}$. If the system is in state $n \geq 0$, the time until the next arrival/birth is an exponential random variable with rate λ_n. By having an arrival, the system goes to state $n+1$. On the other hand, if the system is in state $n \geq 1$, the time until the next departure/death is an exponential random variable with rate μ_n. By having a departure, the system goes from state n to state $n-1$ [51].

One of the examples of queue that can be modeled as birth-death process is $M/M/k/k$, which is the scenario of the CR network presented in this thesis. Considering the $M/M/k/k$ queueing model in the CR networks, the servers are the channels and the arriving costumers are primary users. This model assumes that interarrival times obey the exponential distribution (the average time between arriving customers is $\frac{1}{\lambda}$) or equivalently the arrival rate follows a Poisson distribution with parameter λ. There are k servers, where no line is allowed to form because maximum k customers are allowed to be in the system. Each server has an independently and identically distributed exponen-

tial service time distribution with mean $\frac{1}{\mu}$ [51]. In the following we provide more details about the $M/M/k/k$ queueing model.

2.2.1 Multi-Server, No Queue $M/M/k/k$

The $M/M/k/k$ is a multi-server, no queue system with exponential interarrival and service times. In this system, each arriving customer has its private server, unless when all the servers are occupied [51]. In that case, the customer will be lost. The following notations are important in this system.

λ: Average number of arrivals per unit of time (Arrival rate)

$\frac{1}{\lambda}$: Average time between arrivals (τ_a)

μ: Service rate

$\frac{1}{\mu}$: Average service time for each user (τ_s)

$\rho = \frac{\tau_s}{\tau_a} = \frac{\lambda}{\mu}$: Utilization ratio

k: Number of servers

n: Number of users in the system $0 \leqslant n \leqslant k$

Fig. 2.2.1 shows the Markov chain presentation of the $M/M/k/k$ system.

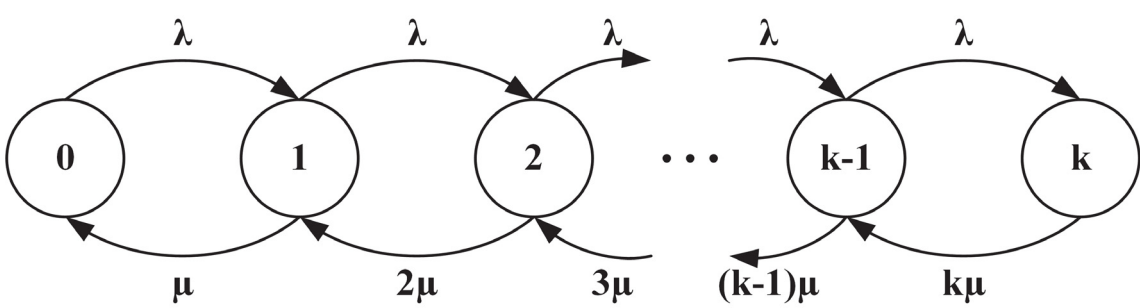

Figure 2.2.1: Queueing $M/M/k/k$ system.

Following are the corresponding equations [51, 52]

$$\begin{cases} n = 0 & \leftrightarrow \quad \lambda P_0 = \mu P_1 \\ 1 \leqslant n < k & \leftrightarrow \quad (\lambda + n\mu) P_n = \lambda P_{n-1} + (n+1) \mu P_{n+1} \\ n = k & \leftrightarrow \quad \lambda P_{k-1} = k\mu P_k \end{cases} \tag{2.2.1}$$

where the reduced equation is

$$\lambda P_{n-1} = n\mu P_n \qquad 1 \leqslant n \leqslant k \tag{2.2.2}$$

Therefore, the steady-state probabilities can be achieved, where the probability of having m customers is [51, 52]

$$P_m = \frac{\frac{\rho^m}{m!}}{\sum\limits_{n=0}^{k} \frac{\rho^n}{n!}} \qquad 0 \leq m \leq k \tag{2.2.3}$$

In this system, P_k describes the probability when all k servers are busy. This probability expression is called *Erlang's loss formula* or *Erlang's-B formula*.

$$B(k, \rho) \equiv P_k = \frac{\frac{\rho^k}{k!}}{\sum\limits_{n=0}^{k} \frac{\rho^n}{n!}} \tag{2.2.4}$$

The P_k is the steady-state probability of a full system, which is for the arriving customers who find the system full and cannot have service [51, 52].

2.2.2 Multi-Server, No Queue $M/G/k/k$

The $M/G/k/k$ is a multi-server, no queue system with exponential interarrival and general service times. It is the surprising fact that the steady-state distribution given by (2.2.3) is valid for any $M/G/k/k$ system [51]. As such, if the average service time for each user is $\frac{1}{\mu}$, the steady-state probabilities for $M/G/k/k$ queue system is known as

$$P_m = \frac{\frac{\rho^m}{m!}}{\sum_{n=0}^{k} \frac{\rho^n}{n!}} \qquad 0 \leq m \leq k \tag{2.2.5}$$

where P_m is the probability of having m customers.

2.3 Summary

Queueing models are essential tools for evaluating the performance of CR networks. Hence, in this chapter we briefly studied the concept of a queueing system. We also reviewed the theory of birth-death processes and the queueing models that we need for developing the mathematical model for performance analysis of secondary users.

3 Performance Analysis of Secondary Users in a Network of Single-Channel Primary Users

In this chapter, we propose a mathematical model to investigate the performance of SUs in a CR network of single-channel PUs. This chapter starts with Section 3.1 presenting the system model. The proposed mathematical model to calculate the load of SUs is defined in Section 3.2. Finally, some numerical and simulation results regarding the proposed model are presented in Section 3.3.

3.1 System Model

As mentioned earlier, there are two kinds of users in CR networks. Primary users are higher-priority users and their normal operation should not be harmed. Secondary users are lower-priority users, who are equipped with CR capability to opportunistically access the spectrum holes. Secondary user must vacate the spectrum hole whenever a primary user appears on any of its constituent channels. Hence, the activity of primary users is not affected by secondary users.

In this chapter, it is assumed that each primary user takes only one channel. The channel for primary user is a random and independent selection out of available idle channels. In addition, we consider the highest possible transmission rate for secondary user as its criterion for spectrum

hole selection. It means that secondary user always gets the largest available spectrum hole. Furthermore, for simplicity of analysis, we assume that secondary user has infinite data to sent and it always looks for spectrum resources.

The spectrum hole in CR networks consists of consecutive idle channels in the licensed band of primary users. An example of five available spectrum holes in a spectrum of thirty channels is shown in Fig. 3.1.1. For instance, the first spectrum hole has three channels, channel number four to six.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30

Figure 3.1.1: Example of five available spectrum holes in a frequency band of thirty channels

In this scenario, the location tracking of idle channels is practically unfeasible. Because of that, as a substitute for the location tracking, we find the probability distribution of the maximum consecutive idle channels. This important theoretical modeling, to the best of our knowledge, has not been done before in the literature.

The maximum number of consecutive idle channels, which forms the largest available spectrum hole, is a random variable, Max, from 0 to n, where n is the total number of channels in the licensed band. If r is the data transmission rate for each channel, the expected transmission rate for secondary user, which will be referred to as throughput, is

$$E[R] = r \times E[Max] \tag{3.1.1}$$

where the expected value of Max is

$$E[Max] = \sum_{x=0}^{n} x \times P(Max = x) \tag{3.1.2}$$

and $P(Max = x)$ is the probability distribution of the random variable Max. Hence, to know the throughput of secondary user, we need to calculate $E[Max]$. In addition, there is another approach

to calculate $E[Max]$. If the secondary user takes in average X_m number of channels at each state of m busy channels, the expected value for its number of channels in average can be calculated as:

$$\bar{X} = \sum_m P_m X_m \tag{3.1.3}$$

The result of (3.1.3) is exactly the same as the result of (3.1.2).

The challenge of calculating the probability distribution is addressed in Section 3.2 where we propose a mathematical model to find the probability distribution $P(Max = x)$ for any given number of channels. Before that, we provide two examples for better understanding the content.

If we are looking for the probability of $P(Max = x)$ in a total of $N = 5$ channels, x can have the values of $x = 0, 1, 2, ..., 5$. In this case, state of busy channels is $S = m$ and the probability of each state is P_m, where $m = 0, 1, 2, ..., 5$. To calculate the probability of each possible maximum length, we have:

$$P(Max = x) = \sum_{m=0}^{5} P(Max = x \mid S = m) P_m$$

where

$P(Max = 0) = P(Max = 0 \mid S = 5) P_5 = P_5$

$P(Max = 1) = P(Max = 1 \mid S = 2) P_2 + P(Max = 1 \mid S = 3) P_3 + P(Max = 1 \mid S = 4) P_4 = \left(\frac{1}{10}\right) P_2 + \left(\frac{3}{5}\right) P_3 + P_4$

$P(Max = 2) = P(Max = 2 \mid S = 1) P_1 + P(Max = 2 \mid S = 2) P_2 + P(Max = 2 \mid S = 3) P_3 = \left(\frac{1}{5}\right) P_1 + \left(\frac{3}{5}\right) P_2 + \left(\frac{2}{5}\right) P_3$

$P(Max = 3) = P(Max = 3 \mid S = 1) P_1 + P(Max = 3 \mid S = 2) P_2 = \left(\frac{2}{5}\right) P_1 + \left(\frac{3}{10}\right) P_2$

$P(Max = 4) = P(Max = 4 \mid S = 1) P_1 = \left(\frac{2}{5}\right) P_1$

$P(Max = 5) = P(Max = 5 \mid S = 0) P_0 = P_0$

Of course, sum of all the possible options for $P(Max = x)$ is one:

$$\sum_{x=0}^{5} P(Max = x) = P_0 + P_1 + P_2 + P_3 + P_4 + P_5 = 1$$

Now we can calculate the load of secondary user with (3.1.2):

$$E\left[Max\right] = 5P_0 + 3.2P_1 + 2.2P_2 + 1.4P_3 + P_4$$

In another way, by looking at each specific state of busy channels:

$m = 0$:

$x = 5 \implies P\left(Max = 5 \mid N = 5, S = 0\right) = 1 \implies X_{m=0} = 5 \times 1 = 5$

$m = 1$:

$x = 2 \implies P\left(Max = 2 \mid N = 5, S = 1\right) = \frac{1}{5}$

$x = 3 \implies P\left(Max = 3 \mid N = 5, S = 1\right) = \frac{2}{5}$

$x = 4 \implies P\left(Max = 4 \mid N = 5, S = 1\right) = \frac{2}{5}$

$\implies X_{m=1} = 2 \times \frac{1}{5} + 3 \times \frac{2}{5} + 4 \times \frac{2}{5} = 3.2$

$m = 2$:

$x = 1 \implies P\left(Max = 1 \mid N = 5, S = 2\right) = \frac{1}{10}$

$x = 2 \implies P\left(Max = 2 \mid N = 5, S = 2\right) = \frac{3}{5}$

$x = 3 \implies P\left(Max = 3 \mid N = 5, S = 2\right) = \frac{3}{10}$

$\implies X_{m=2} = 1 \times \frac{1}{10} + 2 \times \frac{3}{5} + 3 \times \frac{3}{10} = 2.2$

$m = 3$:

$x = 1 \implies P\left(Max = 1 \mid N = 5, S = 3\right) = \frac{3}{5}$

$x = 2 \implies P\left(Max = 2 \mid N = 5, S = 3\right) = \frac{2}{5}$

$\implies X_{m=3} = 1 \times \frac{3}{5} + 2 \times \frac{2}{5} = 1.4$

$m = 4$:

$$x = 1 \implies P(Max = 1 \mid N = 5, S = 4) = 1 \implies X_{m=4} = 1 \times 1 = 1$$

$m = 5$:

$$x = 0 \implies P(Max = 0 \mid N = 5, S = 5) = 1 \implies X_{m=5} = 0 \times 1 = 0$$

Then by having all the X_m, the expected value for maximum number of channels can be calculated from (3.1.3):

$$\bar{X} = 5P_0 + 3.2P_1 + 2.2P_2 + 1.4P_3 + P_4$$

It is clear that the results of (3.1.2) and (3.1.3) are the same as expected.

Now, in our system model we want to know the idle time of available spectrum hole, or in other words, what is the period of time when the secondary user can stably get access to the spectrum hole. If there are some available idle channels, the chances of arrivals to each of the idle channels are the same and these events are independent of each other.

Suppose there are x consecutive idle channels as a spectrum hole. The idle time for spectrum hole is the minimum of all the idle times of its constituent channels. So, in average, the spectrum hole with x channels will be claimed x times sooner than one idle channel which is the period of time when the secondary user can stably get access to the spectrum hole.

In addition, more number of channels together have higher data rate compared to a single channel, however, one channel lasts for a longer time. We assume in our model that whenever the secondary user has to leave the channels, it will immediately come back to take the new available channels. Hence, the secondary user does not lose any time and it always gets the largest available spectrum hole to have the highest throughput.

3.2 Proposed Mathematical Model in Cognitive Spectrum Selection

For simplicity, first we consider that only one secondary user is present in the network. Later in this section, we will extend the results to the case of multiple secondary users.

3.2.1 One Secondary User in the CR Network

If there is only one secondary user in the CR network, the question is how to calculate $P(Max = x)$ where $x = 0, 1, 2, \ldots, n$ for any given number of n. To solve this problem, first we define

$$P(Max = x \mid N = n, S = m) = P(x \mid n, m) \tag{3.2.1}$$

as the conditional probability of having maximum x consecutive idle channels under the condition of n total and m busy channels. Now, for any given number of channels, n, if we know the conditional probability distribution in (3.2.1), $P(Max = x)$ is

$$P(Max = x) = \sum_{m=0}^{n} P(Max = x \mid S = m) P_m \tag{3.2.2}$$

where $\{P_m\}$ are the steady-state probabilities of the number of busy channels used by primary users.

It is hard to find $P(x \mid n, m)$, especially when n is large. To solve this problem, we propose a theoretical model where $P(x \mid n, m)$ can be calculated in an iterative process. We start to find $P(x \mid n, m)$ for a small value of n, for example $n = 1$, in which the solution is straightforward. Once we have the solution for up to n, we use it to find the solution for $n + 1$. Therefore, the approach is able to calculate all the possibilities of $P(x \mid n + 1, m)$ by knowing all the pre-calculated values of $P(x \mid n, m)$.

To introduce the model, two definitions are needed:

- $P^{'}(Max = x \mid N = n, S = m)$: Conditional probability of having maximum x consecutive idle channels under the condition of n total and m busy channels, and the last x channels being all

idle.

- $P^{''}(Max = x \mid N = n, S = m)$: Conditional probability of having maximum x consecutive idle channels under the condition of n total and m busy channels, and the last x channels not being all idle.

Obviously,

$$P(x \mid n, m) = P^{'}(x \mid n, m) + P^{''}(x \mid n, m) \tag{3.2.3}$$

Table 3.1: Definitions of the parameters

Parameters	Definitions
$P_{lcb}^{n+1,m}$	The probability that the last channel is busy, while having $n+1$ total and m busy channels
$P_{lci}^{n+1,m}$	The probability that the last channel is idle, while having $n+1$ total and m busy channels
$P_{bc-lxci}^{n+1,m}$	The probability of having a busy channel just before the last x consecutive idle channels, while having $n+1$ total and m busy channels

The calculation of the mathematical model for $P(x \mid n, m)$ starts when $n \geq m + x$, otherwise the result is zero. In addition, for any value of $n \geq 0$ if x and m are either 0 or n, the only possible probabilities are as follows.

$$\begin{aligned} P(Max = 0 \mid N = n, S = n) &= 1 \\ P(Max = n \mid N = n, S = 0) &= 1 \\ P^{'}(Max = 0 \mid N = n, S = n) &= 1 \\ P^{'}(Max = n \mid N = n, S = 0) &= 1 \end{aligned} \tag{3.2.4}$$

Now, for the other values of x and m, and with the help of the parameters defined in Table 3.1, $P(x \mid n+1, m)$ can be calculated as follows.

$$
\begin{aligned}
P(x \mid n+1, m) &= \\
P_{lcb}^{n+1,m} &\times P(x \mid n, m-1) \\
&+ \\
P_{lci}^{n+1,m} &\times [P^{'}(x-1 \mid n, m) + P^{''}(x \mid n, m)]
\end{aligned}
\tag{3.2.5}
$$

To calculate $P(x \mid n+1, m)$, it should be noted that to make $n+1$ the number of total channels, the extra added channel to the previous n channels is either busy or idle. Therefore, the calculation of $P(x \mid n+1, m)$ consists of two parts. First is the probability of the last channel being busy, i.e., channel number $n+1$, and having maximum x consecutive idle channels in the remaining n channels, $m-1$ of which being busy. The probability of this event is $P(x \mid n, m-1)$. Second is the probability of the last channel being idle and having maximum $x-1$ or x consecutive idle channels in the remaining n channels, m of which being busy.

Note that, if channel number $n+1$ is idle, the maximum x consecutive idle channels can be either the last x channels or elsewhere. The former happens when the last $x-1$ of the first n channels are idle. The probability of this event is $P^{'}(x-1 \mid n, m)$. The latter occurs by having the maximum x consecutive idle channels in the first n channels, the last x of which not being all idle. The probability of this event is $P^{''}(x \mid n, m)$. It is obvious that the last x channels cannot be all idle in n total channels, otherwise there will be maximum $x+1$ consecutive idle channels in $n+1$ total channels. In other words, these two cases are explained as follows.

1. If channel number $n+1$ is idle: The last $x-1$ of n channels are idle and it means that the last x channels of the total $n+1$ channels are idle. The probability of this event is $P^{'}(x-1 \mid n, m)$.

2. If channel number $n+1$ is idle: The $Max = x$ is anywhere in the $n+1$ channels, except in the last x channels. The probability of this event is $P^{''}(x \mid n, m)$. Note that if the last x channels are idle in the total n, we will have $Max = x+1$ in the total of $n+1$ channels, which is not the one that we are looking for.

That is why we need $P^{''}(x \mid n, m)$ and $P^{'}(x-1 \mid n, m)$. For the event related to the probability of $P^{'}(x-1 \mid n, m)$, the random variable $Max = x-1$ is located at the last $x-1$ channels, but for the event related to the probability of $P^{''}(x \mid n, m)$, the random variable $Max = x$ is located anywhere except in the last x channels. Hence, for the event related to the probability of $P^{''}(x \mid n, m)$, even if the last $x-1$ channels are idle, the random variable $Max = x$ is somewhere else, therefore, the event related to the probability of $P^{'}(x-1 \mid n, m)$ does not exist in that case. Note that even if the last $x-1$ channels are idle, they do not make the Max. Hence, the event related to the probability of $P^{'}(x-1 \mid n, m)$ does not exist.

In (3.2.5), we have

$$P_{lcb}^{n+1,m} = \frac{\binom{n}{m-1}}{\binom{n+1}{m}} = \frac{m}{n+1} \tag{3.2.6}$$

$$P_{lci}^{n+1,m} = \frac{\binom{n}{m}}{\binom{n+1}{m}} = \frac{n+1-m}{n+1} \tag{3.2.7}$$

Clearly, the addition of (3.2.6) and (3.2.7) is equal to one. To continue, $P^{'}(x \mid n+1, m)$ can be calculated as follows.

$$\begin{aligned} P^{'}(x \mid n+1, m) &= \\ P_{bc-lxci}^{n+1,m} &\times \left(\sum_{y=0}^{x} P(y \mid n-x, m-1) \right) \end{aligned} \tag{3.2.8}$$

To calculate $P^{'}(x \mid n+1, m)$, it is implicit that there is a busy channel just before the last x consecutive idle channels, otherwise there will be maximum $x+1$ consecutive idle channels. So, $P^{'}(x \mid n+1, m)$ is the probability of being a busy channel just before the last x consecutive idle channels and having 0 up to x consecutive idle channels in the remaining $n-x$ channels, $m-1$ of which being busy.

In (3.2.8),

$$P_{bc-lxci}^{n+1,m} = \frac{\binom{n-x}{m-1}}{\binom{n+1}{m}} \tag{3.2.9}$$

Therefore, $P(x \mid n+1, m)$ and $P^{'}(x \mid n+1, m)$ can be calculated by (3.2.5) and (3.2.8), respectively. It should be noted that $P(x \mid n+1, m)$ and $P^{'}(x \mid n+1, m)$ are separately calculated. Finally, $P^{''}(x \mid n+1, m)$ can be computed by (3.2.3), after $P(x \mid n+1, m)$ and $P^{'}(x \mid n+1, m)$ are calculated.

This ends the calculation of $P(x \mid n+1, m)$. Now, to calculate (3.2.2) for any given number of channels, n, the steady-state probabilities are needed. One of the models for primary usage in CR networks is the call-based model. This model utilizes two random variables. One is to identify the interarrival time between two customers and the other one is to describe the service time. As an example, the primary usage can be well modeled by $M/G/k/k$ queue system. This system has Poisson customer arrivals and a general service time distribution, where k is the number of channels [51].

The steady-state probabilities of the number of customers in the $M/G/k/k$ queue system have already been obtained [51]. If λ is the arrival rate and μ is the service rate, by defining $\rho = \frac{\lambda}{\mu}$, the probability of having m customers is:

$$P_m = \frac{\frac{\rho^m}{m!}}{\sum_{n=0}^{k} \frac{\rho^n}{n!}} \qquad 0 \leq m \leq k \tag{3.2.10}$$

Now, by having all the steady-state probabilities in the $M/G/k/k$ queue system, the probability distribution in (3.2.2) can be calculated. Having that, the load of secondary user and its throughput can be calculated by (3.1.2) and (3.1.1), respectively.

Numerical Evaluation

In this part we provide some examples to show the accuracy of the proposed mathematical method. We calculate all the available probabilities for $P(Max = x \mid N = n, S = m)$ based on the given values for n:

$N = 1$:

$P(Max = 0 \mid N = 1, S = 1) = 1$

$P^{'}(Max = 0 \mid N = 1, S = 1) = 1\,, P^{''}(Max = 0 \mid N = 1, S = 1) = 0$

$P(Max = 1 \mid N = 1, S = 0) = 1$

$P^{'}(Max = 1 \mid N = 1, S = 0) = 1\,, P^{''}(Max = 1 \mid N = 1, S = 0) = 0$

$N = 2$:

$P(Max = 0 \mid N = 2, S = 2) = 1$

$P^{'}(Max = 0 \mid N = 2, S = 2) = 1\,, P^{''}(Max = 0 \mid N = 2, S = 2) = 0$

$P(Max = 1 \mid N = 2, S = 1) = 1$

$P^{'}(Max = 1 \mid N = 2, S = 1) = 1\,, P^{''}(Max = 1 \mid N = 2, S = 1) = 0$

$P(Max = 2 \mid N = 2, S = 0) = 1$

$P^{'}(Max = 2 \mid N = 2, S = 0) = 1\,, P^{''}(Max = 2 \mid N = 2, S = 0) = 0$

$N = 3$:

$P(Max = 0 \mid N = 3, S = 3) = 1$

$P^{'}(Max = 0 \mid N = 3, S = 3) = 1\,, P^{''}(Max = 0 \mid N = 3, S = 3) = 0$

$P(Max = 1 \mid N = 3, S = 1) = \frac{1}{3}$

$P^{'}(Max = 1 \mid N = 3, S = 1) = \frac{1}{3}\,, P^{''}(Max = 1 \mid N = 3, S = 1) = 0$

$P(Max = 1 \mid N = 3, S = 2) = 1$

$P^{'}(Max = 1 \mid N = 3, S = 2) = \frac{1}{3}\,, P^{''}(Max = 1 \mid N = 3, S = 2) = \frac{2}{3}$

$P(Max = 2 \mid N = 3, S = 1) = \frac{2}{3}$

$P^{'}(Max = 2 \mid N = 3, S = 1) = \frac{1}{3}, P^{''}(Max = 2 \mid N = 3, S = 1) = \frac{1}{3}$

$P(Max = 3 \mid N = 3, S = 0) = 1$

$P^{'}(Max = 3 \mid N = 3, S = 0) = 1, P^{''}(Max = 3 \mid N = 3, S = 0) = 0$

$N = 4$:

$P(Max = 0 \mid N = 4, S = 4) = 1$

$P^{'}(Max = 0 \mid N = 4, S = 4) = 1, P^{''}(Max = 0 \mid N = 4, S = 4) = 0$

$P(Max = 1 \mid N = 4, S = 2) = \frac{1}{2}$

$P^{'}(Max = 1 \mid N = 4, S = 2) = \frac{1}{3}, P^{''}(Max = 1 \mid N = 4, S = 2) = \frac{1}{6}$

$P(Max = 1 \mid N = 4, S = 3) = 1$

$P^{'}(Max = 1 \mid N = 4, S = 3) = \frac{1}{4}, P^{''}(Max = 1 \mid N = 4, S = 3) = \frac{3}{4}$

$P(Max = 2 \mid N = 4, S = 1) = \frac{1}{2}$

$P^{'}(Max = 2 \mid N = 4, S = 1) = \frac{1}{4}, P^{''}(Max = 2 \mid N = 4, S = 1) = \frac{1}{4}$

$P(Max = 2 \mid N = 4, S = 2) = \frac{1}{2}$

$P^{'}(Max = 2 \mid N = 4, S = 2) = \frac{1}{6}, P^{''}(Max = 2 \mid N = 4, S = 2) = \frac{1}{3}$

$P(Max = 3 \mid N = 4, S = 1) = \frac{1}{2}$

$P^{'}(Max = 3 \mid N = 4, S = 1) = \frac{1}{4}, P^{''}(Max = 3 \mid N = 4, S = 1) = \frac{1}{4}$

$P(Max = 4 \mid N = 4, S = 0) = 1$

$P^{'}(Max = 4 \mid N = 4, S = 0) = 1, P^{''}(Max = 4 \mid N = 4, S = 0) = 0$

$N = 5$:

$P(Max = 0 \mid N = 5, S = 5) = 1$

$P^{'}(Max = 0 \mid N = 5, S = 5) = 1, P^{''}(Max = 0 \mid N = 5, S = 5) = 0$

$P(Max = 1 \mid N = 5, S = 2) = \frac{1}{10}$

$P^{'}(Max = 1 \mid N = 5, S = 2) = \frac{1}{10}, P^{''}(Max = 1 \mid N = 5, S = 2) = 0$

$P\,(Max = 1 \mid N = 5, S = 3) = \frac{3}{5}$

$P^{'}\,(Max = 1 \mid N = 5, S = 3) = \frac{3}{10}\,, P^{''}\,(Max = 1 \mid N = 5, S = 3) = \frac{3}{10}$

$P\,(Max = 1 \mid N = 5, S = 4) = 1$

$P^{'}\,(Max = 1 \mid N = 5, S = 4) = \frac{1}{5}\,, P^{''}\,(Max = 1 \mid N = 5, S = 4) = \frac{4}{5}$

$P\,(Max = 2 \mid N = 5, S = 1) = \frac{1}{5}$

$P^{'}\,(Max = 2 \mid N = 5, S = 1) = \frac{1}{5}\,, P^{''}\,(Max = 2 \mid N = 5, S = 1) = 0$

$P\,(Max = 2 \mid N = 5, S = 2) = \frac{3}{5}$

$P^{'}\,(Max = 2 \mid N = 5, S = 2) = \frac{1}{5}\,, P^{''}\,(Max = 2 \mid N = 5, S = 2) = \frac{2}{5}$

$P\,(Max = 2 \mid N = 5, S = 3) = \frac{2}{5}$

$P^{'}\,(Max = 2 \mid N = 5, S = 3) = \frac{1}{10}\,, P^{''}\,(Max = 2 \mid N = 5, S = 3) = \frac{3}{10}$

$P\,(Max = 3 \mid N = 5, S = 1) = \frac{2}{5}$

$P^{'}\,(Max = 3 \mid N = 5, S = 1) = \frac{1}{5}\,, P^{''}\,(Max = 3 \mid N = 5, S = 1) = \frac{1}{5}$

$P\,(Max = 3 \mid N = 5, S = 2) = \frac{3}{10}$

$P^{'}\,(Max = 3 \mid N = 5, S = 2) = \frac{1}{10}\,, P^{''}\,(Max = 3 \mid N = 5, S = 2) = \frac{2}{10}$

$P\,(Max = 4 \mid N = 5, S = 1) = \frac{2}{5}$

$P^{'}\,(Max = 4 \mid N = 5, S = 1) = \frac{1}{5}\,, P^{''}\,(Max = 4 \mid N = 5, S = 1) = \frac{1}{5}$

$P\,(Max = 5 \mid N = 5, S = 0) = 1$

$P^{'}\,(Max = 5 \mid N = 5, S = 0) = 1\,, P^{''}\,(Max = 5 \mid N = 5, S = 0) = 0$

Now we want to compare the results of the proposed approach with the provided examples. For instance, $P\,(Max = 1 \mid N = 4, S = 2) =?$

Based on the proposed method,

$P_{lcb}^{N=4,S=2} = P_{lci}^{N=4,S=2} = \frac{1}{2}$

$P\,(Max = 1 \mid N = 3, S = 1) = \frac{1}{3}$

$P^{'}\,(Max = 0 \mid N = 3, S = 2) = 0$

$P^{''}\,(Max = 1 \mid N = 3, S = 2) = \frac{2}{3}$

Therefore,

$$P\left(Max=1 \mid N=4, S=2\right)=\tfrac{1}{2}\times\tfrac{1}{3}+\tfrac{1}{2}\left(0+\tfrac{2}{3}\right)=\tfrac{1}{2}$$

which is accurate.

Now, $P^{'}\left(Max=1 \mid N=4, S=2\right)=?$

Based on (3.2.8), $P^{'}\left(Max=1 \mid N=4, S=2\right)$ can be calculated as:

$$\left(P\left(Max=0 \mid N=2, S=1\right)+P\left(Max=1 \mid N=2, S=1\right)\right)\times\frac{\binom{2}{1}}{\binom{4}{2}}=(0+1)\times\tfrac{1}{3}=\tfrac{1}{3}$$

Another example, $P\left(Max=2 \mid N=4, S=2\right)=?$

With the proposed method,

$$P_{lcb}^{N=4,S=2}=P_{lci}^{N=4,S=2}=\tfrac{1}{2}$$

$$P\left(Max=2 \mid N=3, S=1\right)=\tfrac{2}{3}$$

$$P^{'}\left(Max=1 \mid N=3, S=2\right)=\tfrac{1}{3}$$

$$P^{''}\left(Max=2 \mid N=3, S=2\right)=0$$

Hence,

$$P\left(Max=2 \mid N=4, S=2\right)=\tfrac{1}{2}\times\tfrac{2}{3}+\tfrac{1}{2}\left(\tfrac{1}{3}+0\right)=\tfrac{1}{2}$$

which is true. And based on the proposed method, the probability of $P^{'}\left(Max=2 \mid N=4, S=2\right)$ is equal to $\frac{1}{6}$ which is expected.

We continue with another example: $P\left(Max=2 \mid N=5, S=3\right)=?$

Based on the proposed method,

$$P_{lcb}^{N=5,S=3}=\tfrac{3}{5}$$

$$P_{lci}^{N=5,S=3}=\tfrac{2}{5}$$

$$P\left(Max=2 \mid N=4, S=2\right)=\tfrac{1}{2}$$

$$P^{'}\left(Max=1 \mid N=4, S=3\right)=\tfrac{1}{4}$$

$$P^{''}\left(Max=2 \mid N=4, S=3\right)=0$$

As such,

$$P\left(Max=2 \mid N=5, S=3\right)=\tfrac{3}{5}\times\tfrac{1}{2}+\tfrac{2}{5}\left(\tfrac{1}{4}+0\right)=\tfrac{2}{5}$$

which is correct. Based on the proposed method, the probability of $P^{'}(Max = 2 \mid N = 5, S = 3)$ is equal to $\frac{1}{10}$ which is the same as provided examples.

Now we want to calculate $P(Max = 2 \mid N = 6, S = 3)$. Using the traditional way, we have 12 states which are:

$(1,2,4)$, $(1,2,5)$, $(1,3,4)$, $(1,3,6)$, $(1,4,5)$, $(1,4,6)$, $(2,3,4)$, $(2,3,6)$, $(2,5,6)$, $(3,4,5)$, $(3,4,6)$, $(3,5,6)$.

Therefore, $P(Max = 2 \mid N = 6, S = 3) = \frac{12}{\binom{6}{3}} = \frac{3}{5}$.

Now, we want to calculate $P(Max = 2 \mid N = 6, S = 3)$ using the proposed method. Here we have:

$$P(x \mid n+1, m) = P(Max = x \mid N = n+1, S = m) = P(Max = 2 \mid N = 6, S = 3)$$

Therefore, we have the following:

$$P_{lcb}^{n+1,m} = \frac{\begin{pmatrix} n \\ m-1 \end{pmatrix}}{\begin{pmatrix} n+1 \\ m \end{pmatrix}} = \frac{\binom{5}{2}}{\binom{6}{3}} = \frac{1}{2}$$

$$P_{lci}^{n+1,m} = \frac{\begin{pmatrix} n \\ m \end{pmatrix}}{\begin{pmatrix} n+1 \\ m \end{pmatrix}} = \frac{\binom{5}{3}}{\binom{6}{3}} = \frac{1}{2}$$

And,

$$P(x \mid n, m-1) = P(Max = 2 \mid N = 5, S = 2) = \frac{3}{5}$$

This probability has been calculated before with the proposed method when N was 5, before it increased to 6. Therefore,

$$P^{'}(x-1 \mid n, m) = P^{'}(Max = 1 \mid N = 5, S = 3) = \frac{3}{10}$$

$$P^{''}(x \mid n, m) = P^{''}(Max = 2 \mid N = 5, S = 3) = \frac{3}{10}$$

Similarly, these probabilities have been calculated before with the proposed method when N was 5, before it increased to 6.

And now, with (3.2.5):

$$P(Max = 2 \mid N = 6, S = 3) = \frac{1}{2} \times \frac{3}{5} + \frac{1}{2} \times \left(\frac{3}{10} + \frac{3}{10}\right) = \frac{3}{5}$$

which is the same as the result of the traditional way.

In addition, it can be seen here that the event related to $P^{''}(x \mid n, m)$ does not contain the event

related to $P^{'}(x-1 \mid n, m)$. For the event of $P^{''}(x \mid n, m) = P^{''}(Max = 2 \mid N = 5, S = 3)$, there are three states, which are $(1,2,5)$, $(1,4,5)$, and $(3,4,5)$. And for the event related to $P^{'}(x-1 \mid n, m) = P^{'}(Max = 1 \mid N = 5, S = 3)$, there are three states, which are $(1,2,4)$, $(1,3,4)$, and $(2,3,4)$. It is clear that there is no overlap between the event related to $P^{''}(x \mid n, m)$ and the event related to $P^{'}(x-1 \mid n, m)$.

As a result, for any given number of $N = n$, the proposed method starts with $N = 1$ to calculate all the possible options for the event of $P(x \mid n, m)$. Then it continues with $N = 2$ and keeps up to reach the given number of $N = n$. This is the way we numerically solved the problem and as explained earlier, there is no straightforward or closed-form solution for the calculation of $P(x \mid n, m)$.

As shown above in the provided example given for $N = 6$, the algorithm calculates all the possible options for $P(Max = x \mid N = n, S = m)$ with $N = 1$, $N = 2$, $N = 3$, $N = 4$, and $N = 5$ until it can be able to calculate $P(Max = x \mid N = n, S = m)$ for $N = 6$.

This concludes the performance analysis of the case where there is only one secondary user. For the case of multiple secondary users two different approaches are provided as follows.

3.2.2 Multiple Secondary Users in the CR Network

For the case of multiple secondary users, finding the absolute, generally-applicable solution is an intractable problem. However, by approximation we can leverage the results obtained for the single secondary user case. We propose two approaches for this scenario.

3.2.2.1 Load Estimator for Multiple Secondary Users - First Approach

If there are K secondary users in the CR network, each of them in average takes $\bar{X}$ number of channels which certainly depends on the number of secondary users in the network. If one channel is claimed by a primary user, the affected secondary user leaves the network and immediately rejoins the network by taking the largest available spectrum hole. In this case, if a secondary user comes to the network at a certain time instant, $(K-1) \times \bar{X}$ of the channels have already been

occupied by other secondary users. So, if N is the total number of channels, $(K-1) \times \bar{X}$ of them are being used by other secondary users. Therefore, from one secondary user's point of view, the total number of channels in the network is $N-(K-1) \times \bar{X}$. It basically means that the effect of other secondary users is the decrease in the total number of channels potentially available for the secondary user.

Now we can approximate this scenario with the proposed theoretical model, by having one secondary user in a network of $N-(K-1) \times \bar{X}$ total channels. The steps are described by Algorithm 3.1.

Algorithm 3.1 Load Estimator for Multiple Secondary Users - First Approach

1. Assume an initial value for $\bar{X}$. It should be at most the $E[Max]$ for N channels from the proposed model.
2. Calculate $N^{'} = N-(K-1) \times E[Max]$. (Floor to the lower integer value)
3. Calculate $E[Max]$ for $N^{'}$ channels from the proposed model.
4. Repeat steps 2 and 3 until the value of $E[Max]$ converges to a number with a certain tolerance.
5. The final $E[Max]$ is the average number of channels that any of K secondary users can have.

The approximation in Algorithm 3.1 is valid for a limited number of secondary users. Note that, for a fixed λ and μ in the primary network, the number of secondary users, K, should be selected in a way to support them all ($\lambda < N^{'} \times \mu$ should be valid to support K secondary users). Hence, the parameter K should be limited by a certain admission control system to ensure the performance of the network.

Here is an example in case of $N = 50$, $\lambda = 50$, and $\mu = 2.5$, and $K = 3$:

1. Initial value, $E[Max] = 6.6420$
2. $N^{'} = 36$
3. For $N^{'}$, we have $E[Max] = 4.0455$

4. $N^{'} = 41$

5. For $N^{'}$, we have $E[Max] = 4.9840$

6. $N^{'} = 40$

7. For $N^{'}$, we have $E[Max] = 4.7976$

8. $N^{'} = 40$

9. For $N^{'}$, we have $E[Max] = 4.7976$

Therefore, for any of three secondary users, the average number of channels from numerical result is 4.7976. As a result, in case of multiple secondary users in the system, now we can calculate the expected value of the resources for secondary users.

3.2.2.2 Load Estimator for Multiple Secondary Users - Second Approach

This algorithm works based on (3.1.3), where we look at each state of m busy channels to find $\bar{X}$ as the average number of channels that any of K secondary users can have.

If there are K secondary users in the system, at the state of m busy channels, each of them in average takes X_m number of channels. Obviously, X_m is a function of K. In this scenario, the last SU is Kth SU which comes to the system when there are already $K-1$ other SUs inside. As mentioned before, each SU takes the largest available spectrum hole. Therefore, if $K = 1$, the SU takes in average X_m number of channels at each state of m busy channels. The expected value for its number of channels in average can be calculated from (3.1.3). Now if $K = 2$, we already know that at $K = 1$ the first SU got the largest spectrum hole. As such, at each state of m busy channels, the second SU sees the system as having $m^{'} = m + (K-1) \times X_m$ busy channels and gets $X_{m^{'}}$ number of channels. Hence, in average each of secondary users gets $\frac{(K-1)\times X_m + X_{m^{'}}}{K}$ number of channels at each state of m busy channels. Therefore, in this algorithm, when we have the knowledge for the state of

$K-1$, we can continue for the state of K. By having all the X_m, the expected value for maximum number of channels can be calculated from (3.1.3). The steps are described by Algorithm 3.2.

Algorithm 3.2 Load Estimator for Multiple Secondary Users - Second Approach

1. At $K=1$, calculate X_m for each state of m busy channels. It can be calculated from the proposed model.
2. Increase the value of K by one and calculate $m^{'} = m + (K-1) \times X_m$. (Round to the higher integer value)
3. Calculate $X_{m'}$ from the proposed model for each state of $m^{'}$ busy channels.
4. In average each of secondary users can have $\frac{(K-1)\times X_m + X_{m'}}{K}$ channels at state of m busy channels.
5. Set the output of step 4 as new X_m. Go to step 2 unless the desired value of K is reached.
6. The final X_m is the average number of channels that each of K secondary users can have at state of m busy channels.
7. Apply X_m in (3.1.3) to have the average number of channels that any of K secondary users can have.

Here, we do not decrease the number of available channels for the primary users. In the previous algorithm, by increasing the value of K, the number of available channels for primary users is decreasing, which is not practically correct, because secondary users should not have any influence on the performance of primary users. The second algorithm has no change on the primary network, because it considers each state of busy channels separately.

3.3 Numerical and Simulation Results

The described network is simulated using MATLAB. The arrival of primary users is a Poisson process and service time of primary users is defined based on exponential distribution. The simulation has been done for several times and the average of all the outcomes represents the simulation result.

As mentioned before, if the single goal is having the highest transmission rate, taking the largest

spectrum hole is an effective choice. The maximum length algorithm is a well known greedy method which goes for the largest spectrum hole. This chapter proposed the modeling and performance analysis of the maximum length algorithm. Numerical and simulation results are shown to evaluate the performance of the proposed theoretical model. For the case of one secondary user, we set $N = 50$, $\lambda = 50$ and $\mu = 2.5$ per second. First, we start with the results of steady-state probabilities for the number of busy channels as shown in Fig. 3.3.1.

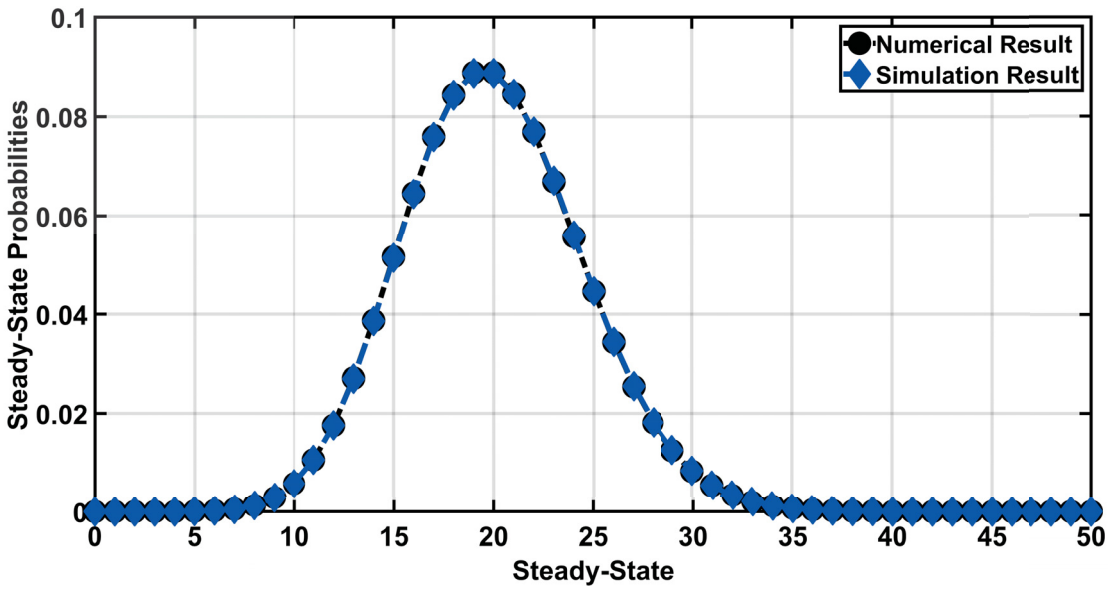

Figure 3.3.1: Steady-state probabilities for the number of busy channels

The numerical and simulation results for the probability distribution of random variable Max are shown in Fig. 3.3.2. The simulation result is almost identical to the numerical result which confirms the accuracy of the proposed mathematical model.

Finally, to have the load of secondary user, the numerical result for the expected value of Max is 6.6418 and the simulation result is 6.6384. As expected, the results are almost the same.

Now we consider the case where the service times for the PUs are uniformly distributed. For the case of one secondary user, we set $N = 50$, $\lambda = 75$ and $\mu = 2.5$ per second. In the simulation we set the minimum and maximum service time as 0.1 and 0.7 seconds, respectively. Therefore,

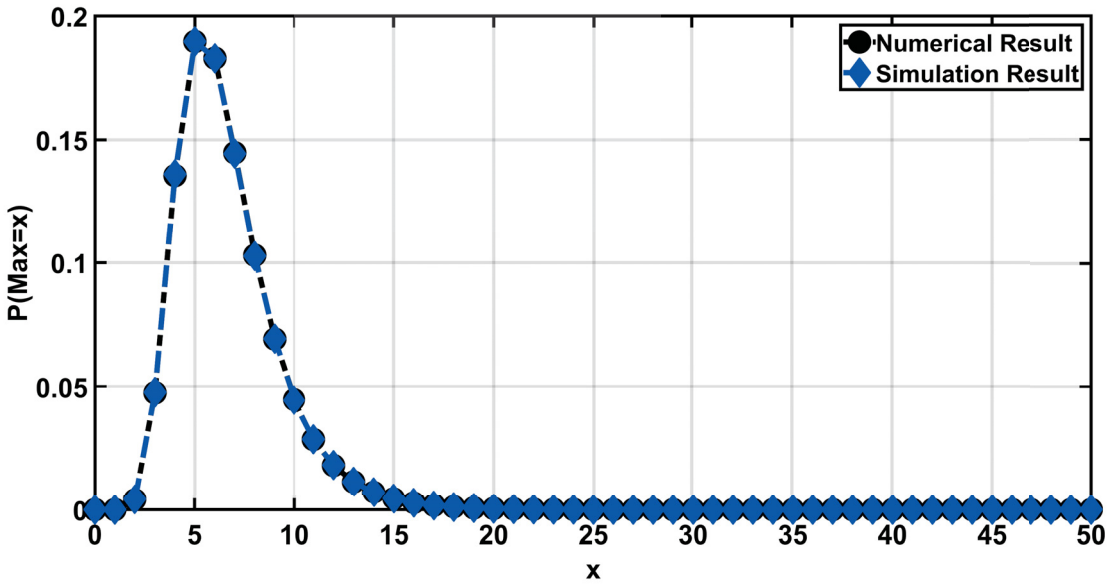

Figure 3.3.2: Probability distribution for maximum number of consecutive idle channels

the mean for the service time is 0.4 which is equal to $\frac{1}{\mu}$. We start with the results of steady-state probabilities for the number of busy channels as shown in Fig. 3.3.3.

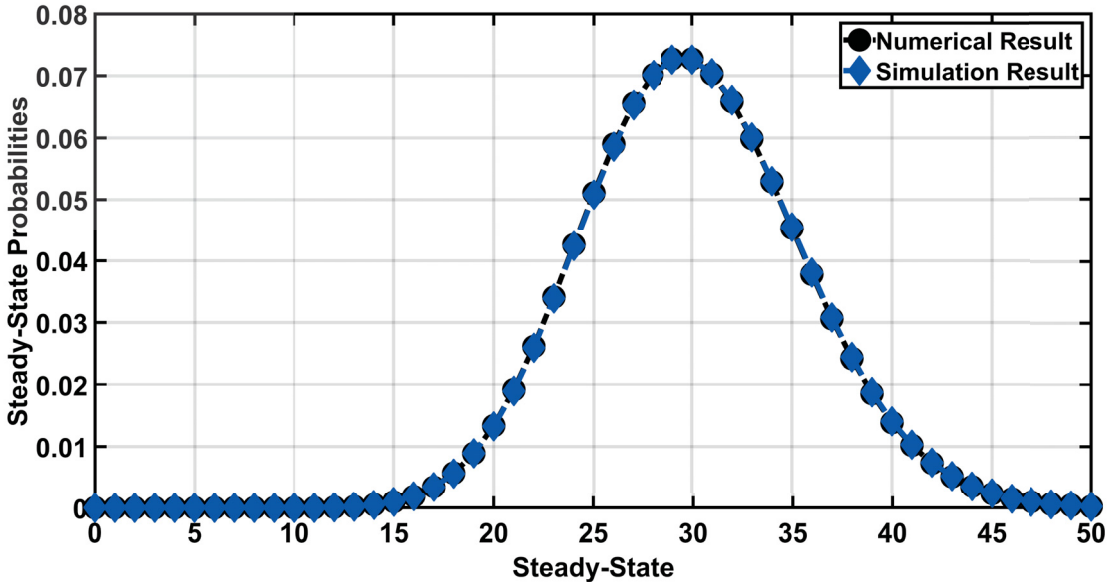

Figure 3.3.3: Steady-state probabilities for the number of busy channels

The numerical and simulation results for the probability distribution of random variable *Max*

are shown in Fig. 3.3.4. As can be seen, the simulation result is almost identical to the numerical result which confirms the accuracy of the proposed mathematical model in case of $M/G/k/k$ queue system.

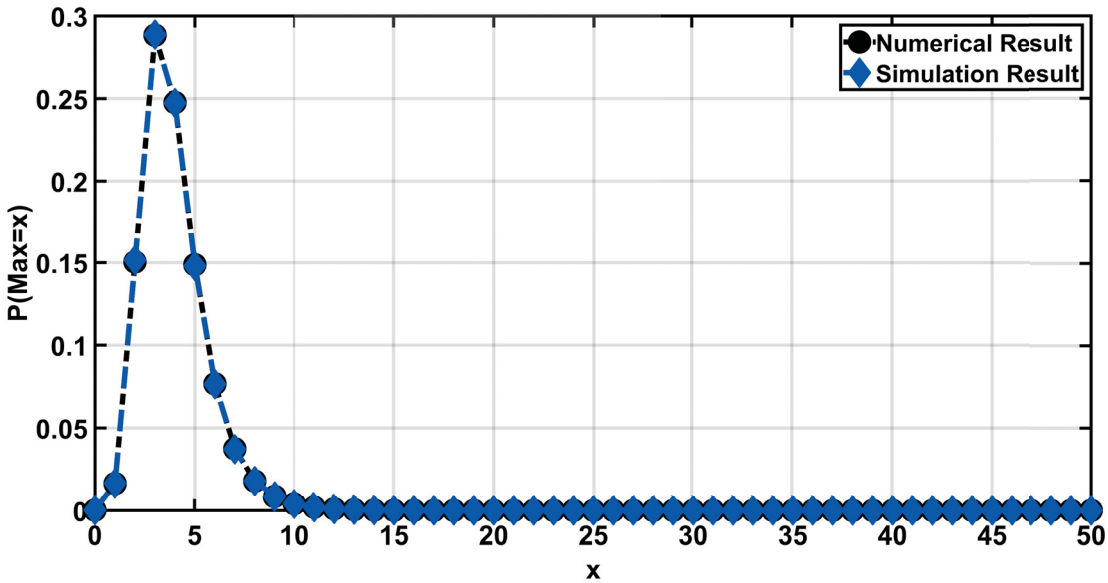

Figure 3.3.4: Probability distribution for maximum number of consecutive idle channels

At the end, the numerical result for the expected value of Max is 3.9442 and the simulation result is 3.9384. As expected, the results are almost identical.

Next, for the case of multiple secondary users, we set K from 1 to 4 and Fig. 3.3.5 shows the load of each secondary user based on numerical and simulation results from Algorithm 3.1.

At $K = 1$ the results are exactly the same, as expected from the proposed mathematical model, while the small differences between the other results are due to the approximation for calculating $N^{'}$ in Algorithm 3.1.

Finally, for different number of secondary users in CR network, Fig. 3.3.6 shows the load of each secondary user based on the results from Algorithm 3.2.

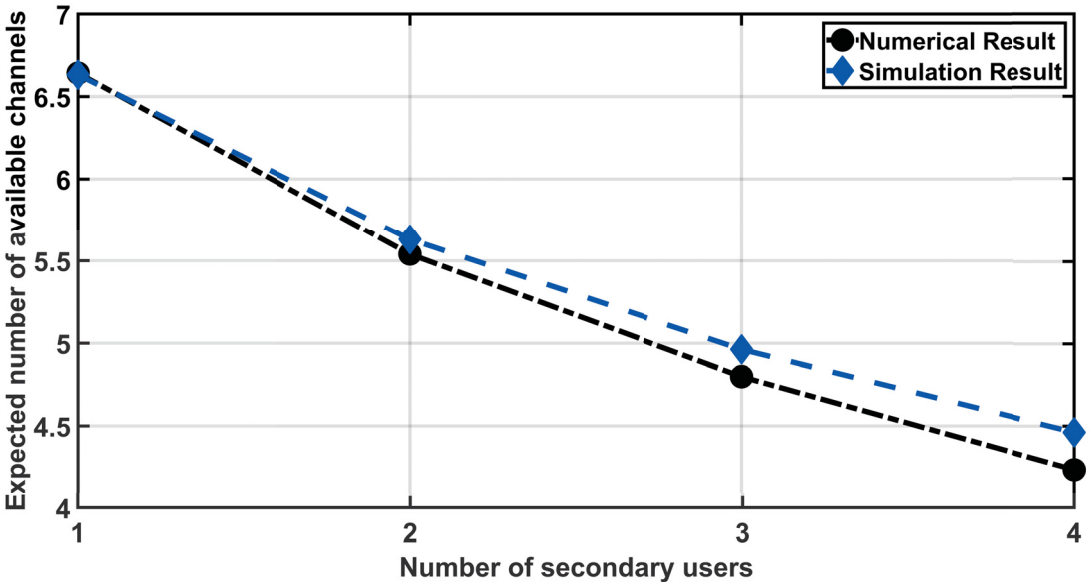

Figure 3.3.5: Expected number of available channels for each secondary user with Algorithm 3.1

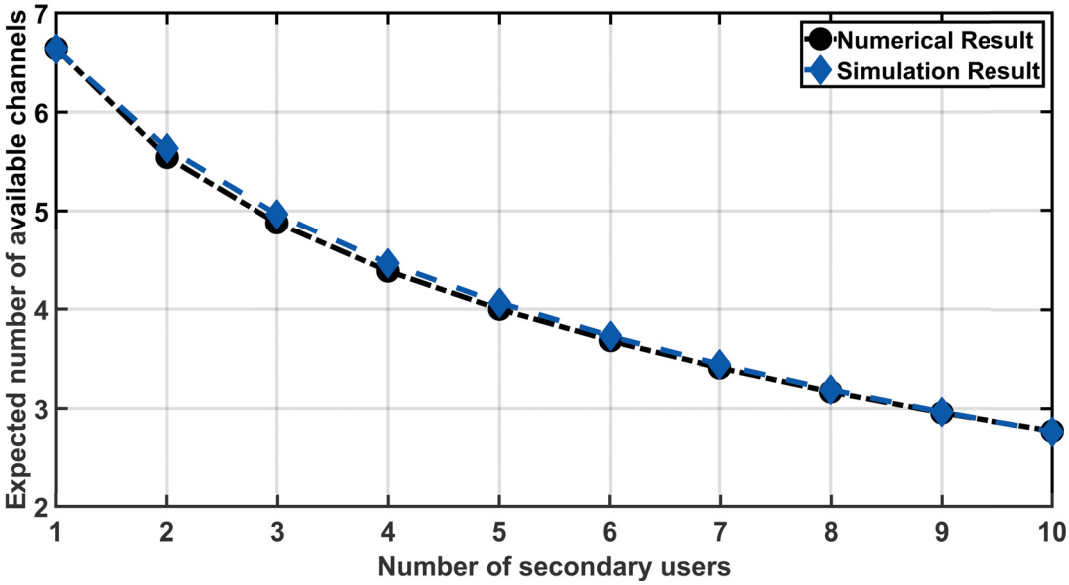

Figure 3.3.6: Expected number of available channels for each secondary user with Algorithm 3.2

3.4 Summary

This chapter proposed a theoretical model for conventional method in CR networks, which selects the maximum consecutive idle channels for the load of secondary users. The objective of the proposed model is to get an insight about the load of secondary users for the service providers. In the considered CR network, each primary user takes only one channel and secondary user gets the largest available spectrum hole. The proposed mathematical model calculates the probability distribution for the number of channels in the largest available spectrum hole, which can be used for the load calculation of secondary users. The performance of the proposed model was validated by numerical and simulation results.

4 Queueing Analysis of Multichannel Primary Users in Cognitive Radio Networks

The mathematical model for the performance of SUs in a CR network of single-channel PUs was proposed, developed, and analyzed in the previous chapter. In this chapter, we extend the proposed model for the case of PUs taking multiple channels. The main contribution in Chapter 3 was the calculation of $P(x \mid n, m)$ as the conditional probability of having maximum x consecutive idle channels under the condition of n total and m busy channels. For any given number of channels, this conditional probability gives us the number of consecutive idle channels the SU can have, if we know P_m, i.e., the probability distribution of busy channels taken by PUs. As mentioned before, P_m has been obtained in the literature using the traditional $M/G/k/k$ queue system [51].

Now, in the case of PUs taking multiple channels we basically need to solve two problems:

1. Finding the probability distribution of busy channels taken by PUs.

2. Determining the length of the largest available spectrum hole.

The first challenge, to the best of our knowledge, is an open problem in the queueing theory literature when PUs take consecutive channels. There are many works in the literature covering the theory of queues, including batch arrivals and Markov chain analysis for multichannel CR networks

[53–60]. The closest case is the queue with batch arrivals, but the conventional batch arrivals cannot be applied here due to two reasons:

1. The departures of users in the batch-arrival scenario are considered independent in queueing theory. It means that the arrivals in one batch leave independently.

2. The number of busy channels is important in queueing theory, but the locations of busy ones are not.

In our scenario:

1. Each arriving user occupies multiple channels simultaneously and leaves all of them later on simultaneously as well. We call this case the batch-arrival-batch-departure scenario.

2. The locations of busy channels are also important when PUs take consecutive channels. We need this information to find out whether a PU will be accepted or blocked if it requests y consecutive channels when w channels are available ($w \geqslant y$). For example, assume there are totally five idle channels in the network and a PU asks for five consecutive channels. In this scenario, if the idle channels are not consecutive, the PU will be rejected.

For finding the length of the largest available spectrum hole, the calculation of $P(x \mid n, m)$ in Chapter 3 is still approximately valid, especially in low-traffic networks. As such, the main challenge is finding the probability distribution of busy channels taken by PUs. The solution to this challenge is the main contribution of this chapter.

The content of this chapter is organized as follows. Section 4.1 deals with the case of batch-arrival-batch-departure when each PU takes a random number of non-consecutive channels. Section 4.2 targets the challenge in which PUs take consecutive channels. In this case, the location tracking of busy channels is practically unfeasible. As such, as a substitute for the location tracking, by approximation we calculate the probability of acceptance for any PU requesting a random number of channels. With the solutions in Section 4.1 and 4.2, we will be able to find P_m and hence the performance of the SU.

4.1 Primary Users with Non-Consecutive Channels

In this section, we study the case when each PU takes a random number of non-consecutive channels. In this scenario, we assume that the selection of the channels for each PU is random and independent of each other. To start the investigation, the arrivals and departures are modeled by a multidimensional Markov chain, and the number of dimensions is the maximum number of channels that each PU can have.

Fig. 4.1.1 shows a fraction of general case for the two-dimensional Markov chain where PUs take either one or two channels with equal probabilities. In this figure, a is the number of Type1-users, each of which takes only one channel. And b is the number of Type2-users, each of which takes two channels. The Markov chain shows the combination of different available (b,a) where P_{ba} shows the probability of state (b,a).

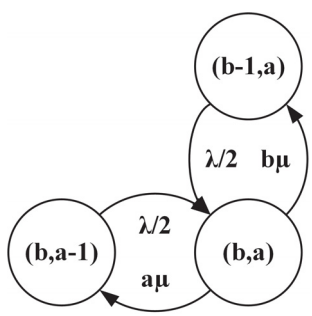

Figure 4.1.1: Fraction of two-dimensional Markov chain in case of non-consecutive channels.

Fig. 4.1.2 shows an example of two-dimensional Markov chain when the total number of channels is $N = 7$. If the probability of arrivals for both types of users are equal, then the arrival rate of each type is $\lambda/2$, where λ is the arrival rate of all incoming users. On the other hand, μ is the service rate for all types of users. In this figure, for example, $(2,3)$ shows the state of two Type2-users and three Type1-users.

The equations for Markov chain of Fig. 4.1.2 are:

$$\lambda P_{00} = \mu P_{01} + \mu P_{10}$$

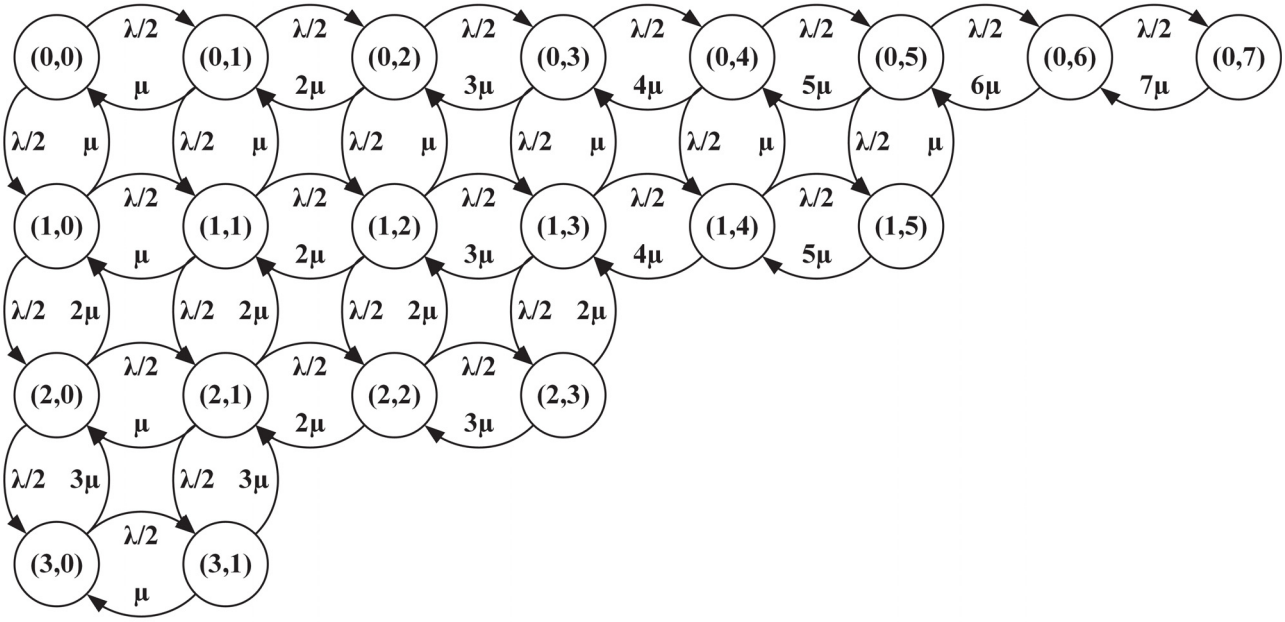

Figure 4.1.2: Example of two-dimensional Markov chain for users with non-consecutive channels.

$$(\lambda+\mu)P_{01} = \frac{\lambda}{2}P_{00} + 2\mu P_{02} + \mu P_{11}$$

$$(\lambda+2\mu)P_{02} = \frac{\lambda}{2}P_{01} + 3\mu P_{03} + \mu P_{12}$$

$$(\lambda+3\mu)P_{03} = \frac{\lambda}{2}P_{02} + 4\mu P_{04} + \mu P_{13}$$

$$(\lambda+4\mu)P_{04} = \frac{\lambda}{2}P_{03} + 5\mu P_{05} + \mu P_{14}$$

$$(\lambda+5\mu)P_{05} = \frac{\lambda}{2}P_{04} + 6\mu P_{06} + \mu P_{15}$$

$$(\frac{\lambda}{2}+6\mu)P_{06} = \frac{\lambda}{2}P_{05} + 7\mu P_{07}$$

$$7\mu P_{07} = \frac{\lambda}{2} P_{06}$$

$$(\lambda + \mu) P_{10} = \frac{\lambda}{2} P_{00} + \mu P_{11} + 2\mu P_{20}$$

$$(\lambda + 2\mu) P_{11} = \frac{\lambda}{2} P_{01} + \frac{\lambda}{2} P_{10} + 2\mu P_{12} + 2\mu P_{21}$$

$$(\lambda + 3\mu) P_{12} = \frac{\lambda}{2} P_{02} + \frac{\lambda}{2} P_{11} + 3\mu P_{13} + 2\mu P_{22}$$

$$(\lambda + 4\mu) P_{13} = \frac{\lambda}{2} P_{03} + \frac{\lambda}{2} P_{12} + 4\mu P_{14} + 2\mu P_{23}$$

$$(\frac{\lambda}{2} + 5\mu) P_{14} = \frac{\lambda}{2} P_{04} + \frac{\lambda}{2} P_{13} + 5\mu P_{15}$$

$$6\mu P_{15} = \frac{\lambda}{2} P_{05} + \frac{\lambda}{2} P_{14}$$

$$(\lambda + 2\mu) P_{20} = \frac{\lambda}{2} P_{10} + \mu P_{21} + 3\mu P_{30}$$

$$(\lambda + 3\mu) P_{21} = \frac{\lambda}{2} P_{11} + \frac{\lambda}{2} P_{20} + 2\mu P_{22} + 3\mu P_{31}$$

$$(\frac{\lambda}{2} + 4\mu) P_{22} = \frac{\lambda}{2} P_{12} + \frac{\lambda}{2} P_{21} + 3\mu P_{23}$$

$$5\mu P_{23} = \frac{\lambda}{2}P_{13} + \frac{\lambda}{2}P_{22}$$

$$(\frac{\lambda}{2} + 3\mu)P_{30} = \frac{\lambda}{2}P_{20} + \mu P_{31}$$

$$4\mu P_{31} = \frac{\lambda}{2}P_{21} + \frac{\lambda}{2}P_{30}$$

$$\sum_{i=0}^{3}\sum_{j=0}^{7} P_{ij} = 1$$

and by using the general probabilities for different kinds of users, the equations are as follows:

$$(P_{u1}\lambda + P_{u2}\lambda)P_{00} = \mu P_{01} + \mu P_{10}$$

$$(P_{u1}\lambda + P_{u2}\lambda + \mu)P_{01} = P_{u1}\lambda P_{00} + 2\mu P_{02} + \mu P_{11}$$

$$(P_{u1}\lambda + P_{u2}\lambda + 2\mu)P_{02} = P_{u1}\lambda P_{01} + 3\mu P_{03} + \mu P_{12}$$

$$(P_{u1}\lambda + P_{u2}\lambda + 3\mu)P_{03} = P_{u1}\lambda P_{02} + 4\mu P_{04} + \mu P_{13}$$

$$(P_{u1}\lambda + P_{u2}\lambda + 4\mu)P_{04} = P_{u1}\lambda P_{03} + 5\mu P_{05} + \mu P_{14}$$

$$(P_{u1}\lambda + P_{u2}\lambda + 5\mu)P_{05} = P_{u1}\lambda P_{04} + 6\mu P_{06} + \mu P_{15}$$

$$(P_{u1}\lambda + 6\mu)P_{06} = P_{u1}\lambda P_{05} + 7\mu P_{07}$$

$$7\mu P_{07} = P_{u1}\lambda P_{06}$$

$$(P_{u1}\lambda + P_{u2}\lambda + \mu)P_{10} = P_{u2}\lambda P_{00} + \mu P_{11} + 2\mu P_{20}$$

$$(P_{u1}\lambda + P_{u2}\lambda + 2\mu)P_{11} = P_{u2}\lambda P_{01} + P_{u1}\lambda P_{10} + 2\mu P_{12} + 2\mu P_{21}$$

$$(P_{u1}\lambda + P_{u2}\lambda + 3\mu)P_{12} = P_{u2}\lambda P_{02} + P_{u1}\lambda P_{11} + 3\mu P_{13} + 2\mu P_{22}$$

$$(P_{u1}\lambda + P_{u2}\lambda + 4\mu)P_{13} = P_{u2}\lambda P_{03} + P_{u1}\lambda P_{12} + 4\mu P_{14} + 2\mu P_{23}$$

$$(P_{u1}\lambda + 5\mu)P_{14} = P_{u2}\lambda P_{04} + P_{u1}\lambda P_{13} + 5\mu P_{15}$$

$$6\mu P_{15} = P_{u2}\lambda P_{05} + P_{u1}\lambda P_{14}$$

$$(P_{u1}\lambda + P_{u2}\lambda + 2\mu)P_{20} = P_{u2}\lambda P_{10} + \mu P_{21} + 3\mu P_{30}$$

$$(P_{u1}\lambda + P_{u2}\lambda + 3\mu)P_{21} = P_{u2}\lambda P_{11} + P_{u1}\lambda P_{20} + 2\mu P_{22} + 3\mu P_{31}$$

$$(P_{u1}\lambda + 4\mu)P_{22} = P_{u2}\lambda P_{12} + P_{u1}\lambda P_{21} + 3\mu P_{23}$$

$$5\mu P_{23} = P_{u2}\lambda P_{13} + P_{u1}\lambda P_{22}$$

$$(P_{u1}\lambda + 3\mu)P_{30} = P_{u2}\lambda P_{20} + \mu P_{31}$$

$$4\mu P_{31} = P_{u2}\lambda P_{21} + P_{u1}\lambda P_{30}$$

$$\sum_{i=0}^{3}\sum_{j=0}^{7} P_{ij} = 1$$

By solving all the Markov chain equations of Fig. 4.1.2, the steady-state probabilities for the number of busy channels can be calculated. If P_n indicates the steady-state probabilities for any value of n from 0 to 7, the probabilities can be calculated as follows.

$P_0 = P_{00}$

$P_1 = P_{01}$

$P_2 = P_{02} + P_{10}$

$P_3 = P_{03} + P_{11}$

$P_4 = P_{04} + P_{12} + P_{20}$

$P_5 = P_{05} + P_{13} + P_{21}$

$P_6 = P_{06} + P_{14} + P_{22} + P_{30}$

$P_7 = P_{07} + P_{15} + P_{23} + P_{31}$

Now, we provide numerical and simulation results for some examples to confirm the accuracy of the analysis. The described network is simulated using event-driven programming with MATLAB. The simulation has been done several times and the average of all the outcomes represents the simulation result.

As an example for a low-traffic network, in the case of $N = 7$, $\lambda = 2$, and $\mu = 2.5$, the steady-state

probabilities for the number of busy channels are shown in Table 4.1 and Fig. 4.1.3. The simulation results are also provided to confirm the accuracy of the numerical results for the two-dimensional Markov chain.

Table 4.1: Steady-state probabilities for two-dimensional Markov chain: Nonconsecutive channels for users, $N = 7$, $\lambda = 2$, $\mu = 2.5$.

P_n	P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Numerical	0.4499	0.1800	0.2159	0.0768	0.0509	0.0163	0.0079	0.0023
Simulation	0.4493	0.1801	0.2159	0.0771	0.0509	0.0165	0.0079	0.0023

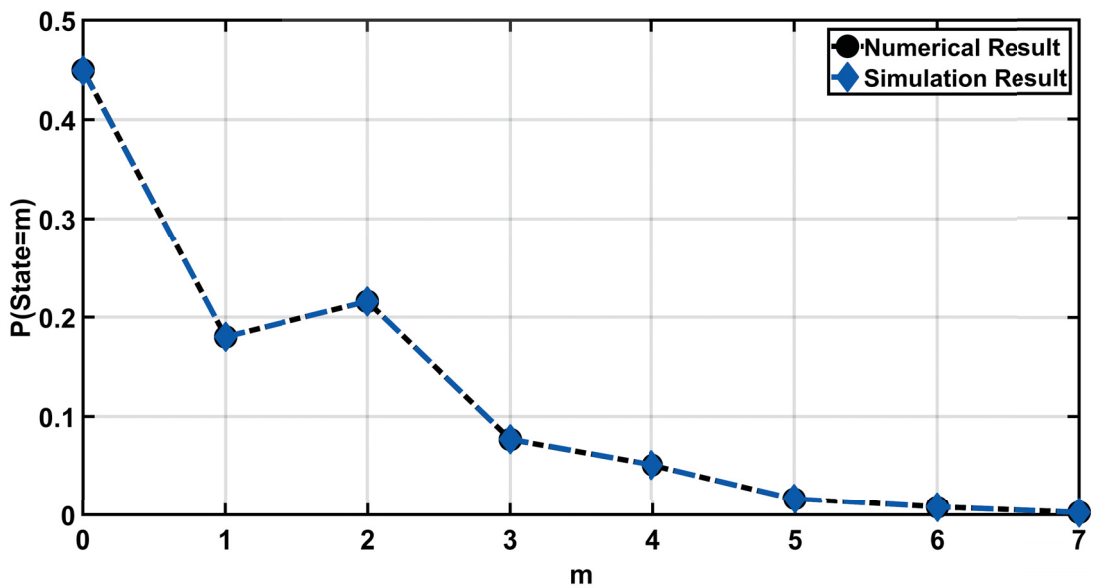

Figure 4.1.3: Steady-state probabilities for two-dimensional Markov chain: Non-consecutive channels for users, $N = 7$, $\lambda = 2$, $\mu = 2.5$.

By having all the steady-state probabilities, the distribution of Max can be achieved with the proposed mathematical model in Chapter 3. The results are shown in Table 4.2 and Fig. 4.1.4.

From Table 4.2, the numerical result for the expected value of Max is 5.0568 and the simulation result is 5.0542. It is clear that the results are almost the same.

Now, for a higher traffic network, in the case of $N = 7$, $\lambda = 3$, and $\mu = 2.5$, the results of steady-state probabilities for the number of busy channels are shown in Table 4.3 and Fig. 4.1.5.

Table 4.2: Probability distribution of Max for two-dimensional Markov chain: Non-consecutive channels for users, $N = 7$, $\lambda = 2$, $\mu = 2.5$.

$P(Max = x)$	$P(Max = 0)$	$P(Max = 1)$	$P(Max = 2)$	$P(Max = 3)$	$P(Max = 4)$	$P(Max = 5)$	$P(Max = 6)$	$P(Max = 7)$
Numerical	0.0023	0.0363	0.1041	0.1518	0.1219	0.0823	0.0514	0.4499
Simulation	0.0023	0.0365	0.1041	0.1520	0.1220	0.0823	0.0515	0.4493

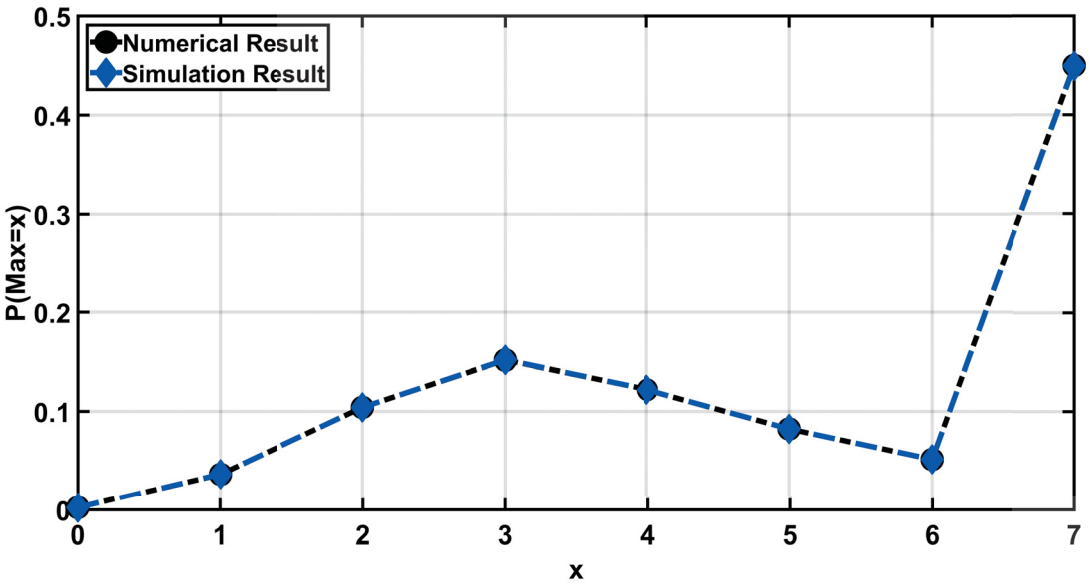

Figure 4.1.4: Probability distribution of Max for two-dimensional Markov chain: Non-consecutive channels for users, $N = 7$, $\lambda = 2$, $\mu = 2.5$.

Table 4.3: Steady-state probabilities for two-dimensional Markov chain: Nonconsecutive channels for users, $N = 7$, $\lambda = 3$, $\mu = 2.5$.

P_n	P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Numerical	0.3030	0.1818	0.2364	0.1200	0.0889	0.0395	0.0217	0.0086
Simulation	0.3022	0.1819	0.2366	0.1202	0.0890	0.0396	0.0218	0.0087

In the same way as before, the distribution of Max are provided with the help of the proposed mathematical model presented in Chapter 3. The results are shown in Table 4.4 and Fig. 4.1.6.

From Table 4.4, the numerical result for the expected value of Max is 4.3317 and the simulation result is 4.3275. We can see even for the higher traffic the results are almost the same.

Another example for two-dimensional Markov chain is for a high-traffic network, where $N = 7$,

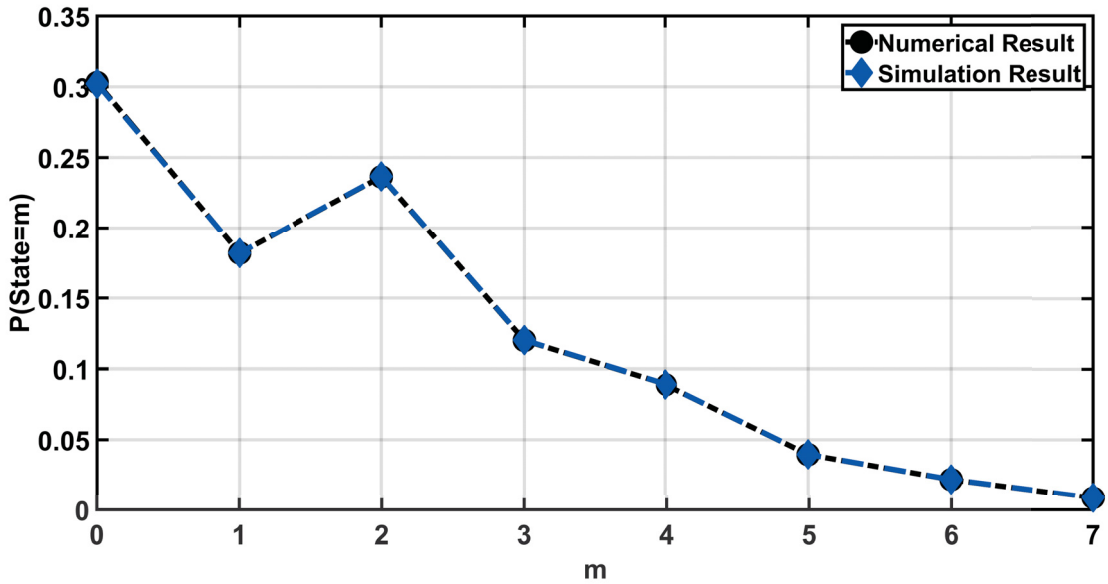

Figure 4.1.5: Steady-state probabilities for two-dimensional Markov chain: Non-consecutive channels for users, $N = 7$, $\lambda = 3$, $\mu = 2.5$.

Table 4.4: Probability distribution of Max for two-dimensional Markov chain: Non-consecutive channels for users, $N = 7$, $\lambda = 3$, $\mu = 2.5$.

$P(Max = x)$	$P(Max = 0)$	$P(Max = 1)$	$P(Max = 2)$	$P(Max = 3)$	$P(Max = 4)$	$P(Max = 5)$	$P(Max = 6)$	$P(Max = 7)$
Numerical	0.0086	0.0788	0.1576	0.1811	0.1332	0.0857	0.0520	0.3030
Simulation	0.0087	0.0790	0.1578	0.1813	0.1333	0.0858	0.0519	0.3022

$\lambda = 5$, and $\mu = 2.5$. The steady-state probabilities for the number of busy channels are provided in Table 4.5 and Fig. 4.1.7.

Table 4.5: Steady-state probabilities for two-dimensional Markov chain: Nonconsecutive channels for users, $N = 7$, $\lambda = 5$, $\mu = 2.5$.

P_n	P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Numerical	0.1408	0.1408	0.2112	0.1643	0.1467	0.0950	0.0647	0.0364
Simulation	0.1401	0.1407	0.2108	0.1643	0.1469	0.0954	0.0651	0.0367

The numerical and simulation results for the distribution of Max are depicted in Table 4.6 and Fig. 4.1.8.

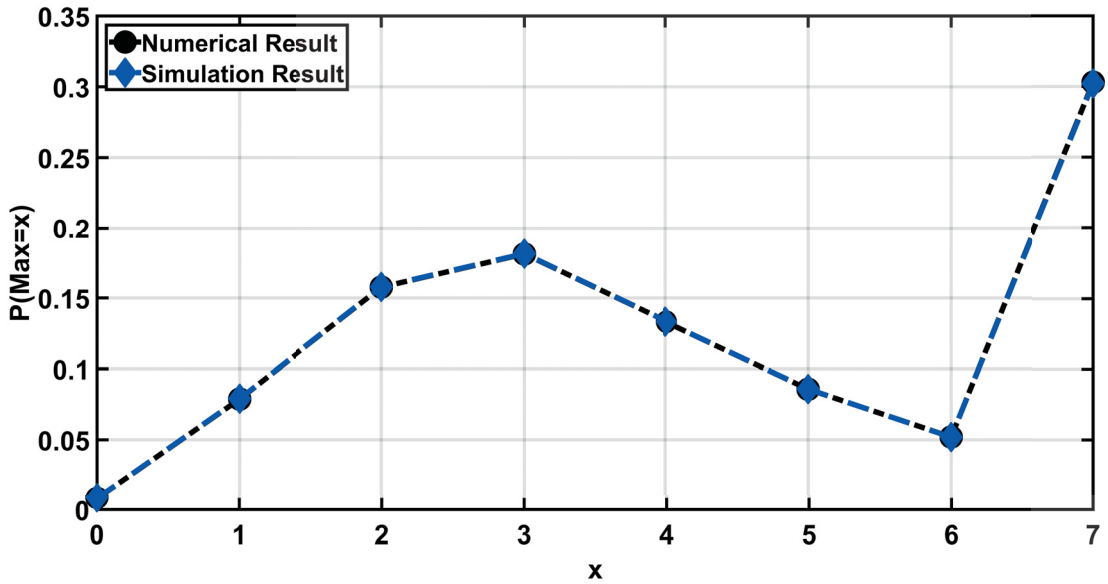

Figure 4.1.6: Probability distribution of Max for two-dimensional Markov chain: Non-consecutive channels for users, $N = 7$, $\lambda = 3$, $\mu = 2.5$.

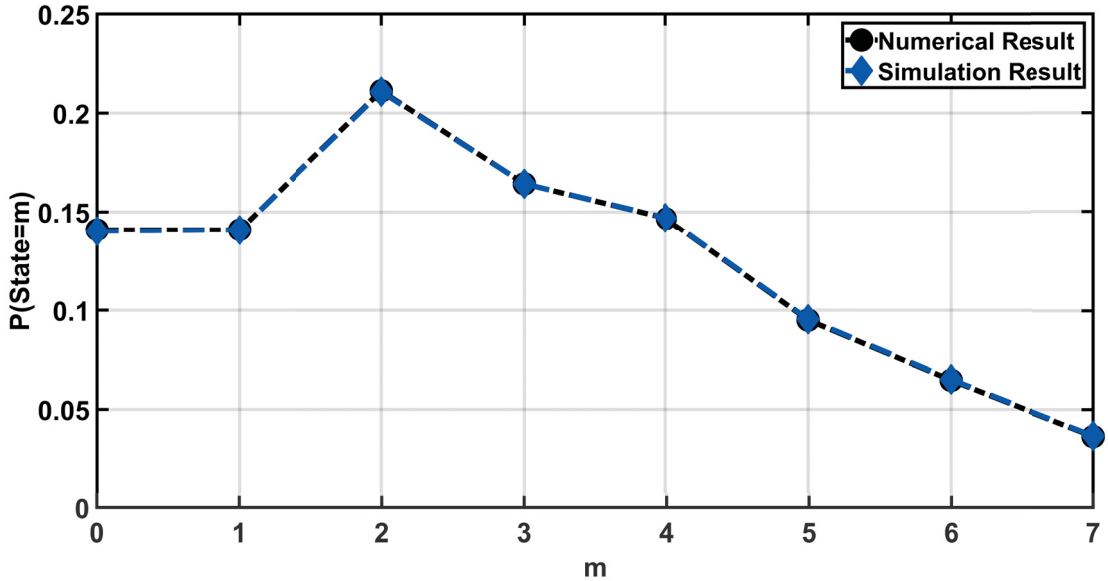

Figure 4.1.7: Steady-state probabilities for two-dimensional Markov chain: Non-consecutive channels for users, $N = 7$, $\lambda = 5$, $\mu = 2.5$.

From Table 4.6, the numerical result for the expected value of Max is 3.2508 and the simulation result is 3.2454. Therefore, the results completely show that for the two-dimensional Markov chain

Table 4.6: Probability distribution of Max for two-dimensional Markov chain: Non-consecutive channels for users, $N = 7$, $\lambda = 5$, $\mu = 2.5$.

$P(Max = x)$	$P(Max = 0)$	$P(Max = 1)$	$P(Max = 2)$	$P(Max = 3)$	$P(Max = 4)$	$P(Max = 5)$	$P(Max = 6)$	$P(Max = 7)$
Numerical	0.0364	0.1792	0.2256	0.1879	0.1194	0.0704	0.0402	0.1408
Simulation	0.0367	0.1800	0.2256	0.1878	0.1192	0.0704	0.0402	0.1401

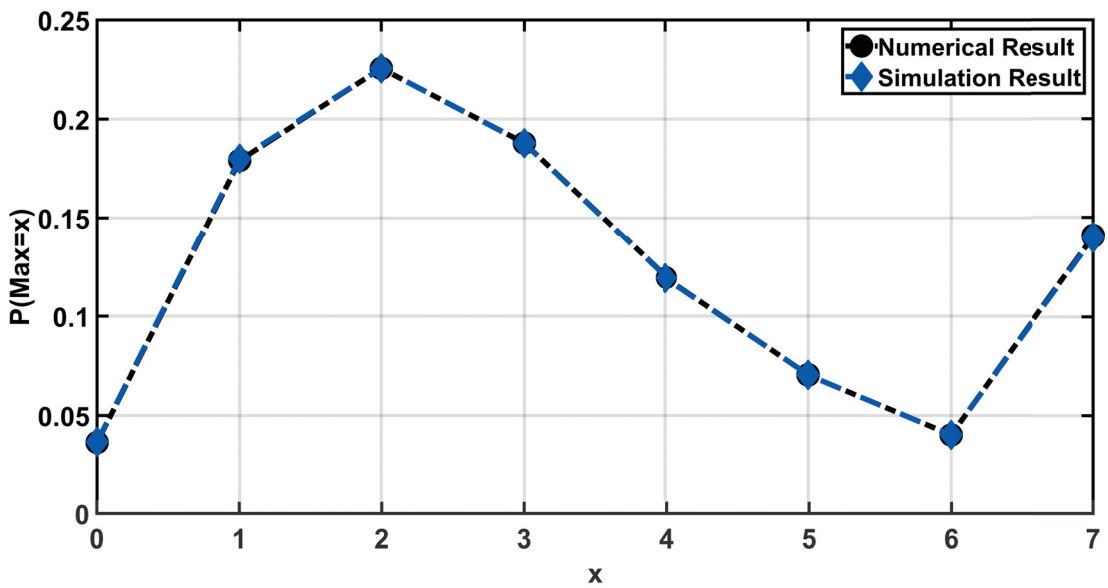

Figure 4.1.8: Probability distribution of Max for two-dimensional Markov chain: Non-consecutive channels for users, $N = 7$, $\lambda = 5$, $\mu = 2.5$.

the numerical and simulation results are almost the same.

We now continue by considering another scenario which is more complicated. Fig. 4.1.9 shows the three-dimensional Markov chain when the total number of channels is $N = 7$ and users take either one, two, or three non-consecutive channels with equal probabilities. In this figure, (c, b, a) indicates a number of Type1-users, b number of Type2-users, and c number of Type3-users. Also, P_{cba} shows the probability for the state of (c, b, a).

The equations for Markov chain of Fig. 4.1.9 are:

$$\lambda P_{000} = \mu(P_{001} + P_{010} + P_{100})$$

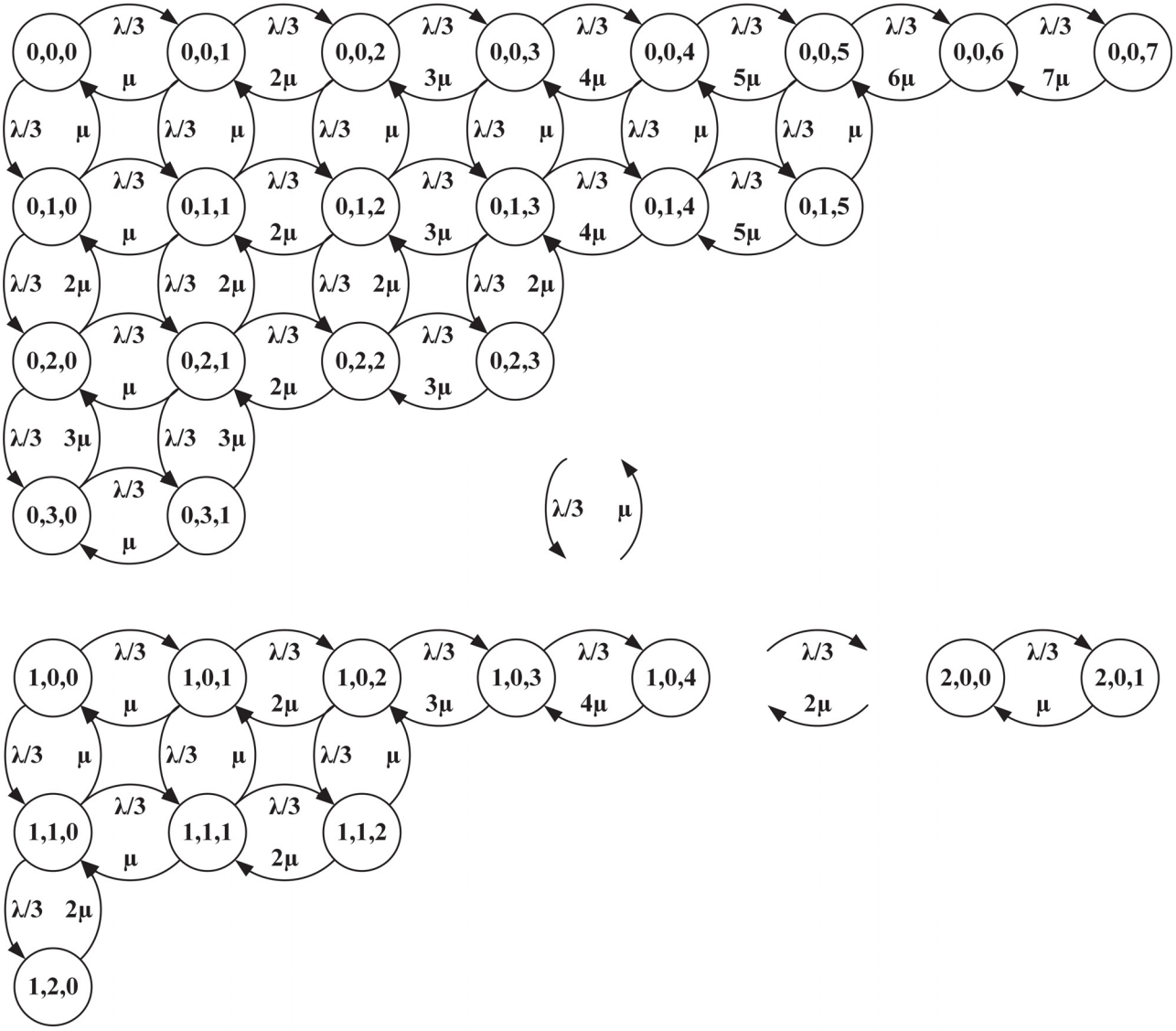

Figure 4.1.9: Example of three-dimensional Markov chain for users with non-consecutive channels.

$$(\lambda+\mu)P_{001} = \frac{\lambda}{3}P_{000} + 2\mu P_{002} + \mu P_{011} + \mu P_{101}$$

$$(\lambda+2\mu)P_{002} = \frac{\lambda}{3}P_{001} + 3\mu P_{003} + \mu P_{012} + \mu P_{102}$$

$$(\lambda+3\mu)P_{003}=\frac{\lambda}{3}P_{002}+4\mu P_{004}+\mu P_{013}+\mu P_{103}$$

$$(\lambda+4\mu)P_{004}=\frac{\lambda}{3}P_{003}+5\mu P_{005}+\mu P_{014}+\mu P_{104}$$

$$(\frac{2\lambda}{3}+5\mu)P_{005}=\frac{\lambda}{3}P_{004}+6\mu P_{006}+\mu P_{015}$$

$$(\frac{\lambda}{3}+6\mu)P_{006}=\frac{\lambda}{3}P_{005}+7\mu P_{007}$$

$$7\mu P_{007}=\frac{\lambda}{3}P_{006}$$

$$(\lambda+\mu)P_{010}=\frac{\lambda}{3}P_{000}+\mu P_{011}+2\mu P_{020}+\mu P_{110}$$

$$(\lambda+2\mu)P_{011}=\frac{\lambda}{3}P_{001}+\frac{\lambda}{3}P_{010}+2\mu P_{012}+2\mu P_{021}+\mu P_{111}$$

$$(\lambda+3\mu)P_{012}=\frac{\lambda}{3}P_{002}+\frac{\lambda}{3}P_{011}+3\mu P_{013}+2\mu P_{022}+\mu P_{112}$$

$$(\frac{2\lambda}{3}+4\mu)P_{013}=\frac{\lambda}{3}P_{003}+\frac{\lambda}{3}P_{012}+4\mu P_{014}+2\mu P_{023}$$

$$(\frac{\lambda}{3}+5\mu)P_{014}=\frac{\lambda}{3}P_{004}+\frac{\lambda}{3}P_{013}+5\mu P_{015}$$

$$6\mu P_{015} = \frac{\lambda}{3}P_{005} + \frac{\lambda}{3}P_{014}$$

$$(\lambda + 2\mu)P_{020} = \frac{\lambda}{3}P_{010} + \mu P_{021} + 3\mu P_{030} + \mu P_{120}$$

$$(\frac{2\lambda}{3} + 3\mu)P_{021} = \frac{\lambda}{3}P_{011} + \frac{\lambda}{3}P_{020} + 2\mu P_{022} + 3\mu P_{031}$$

$$(\frac{\lambda}{3} + 4\mu)P_{022} = \frac{\lambda}{3}P_{012} + \frac{\lambda}{3}P_{021} + 3\mu P_{023}$$

$$5\mu P_{023} = \frac{\lambda}{3}P_{013} + \frac{\lambda}{3}P_{022}$$

$$(\frac{\lambda}{3} + 3\mu)P_{030} = \frac{\lambda}{3}P_{020} + \mu P_{031}$$

$$4\mu P_{031} = \frac{\lambda}{3}P_{021} + \frac{\lambda}{3}P_{030}$$

$$(\lambda + \mu)P_{100} = \frac{\lambda}{3}P_{000} + \mu P_{101} + \mu P_{110} + 2\mu P_{200}$$

$$(\lambda + 2\mu)P_{101} = \frac{\lambda}{3}P_{001} + \frac{\lambda}{3}P_{100} + 2\mu P_{102} + \mu P_{111} + 2\mu P_{201}$$

$$(\frac{2\lambda}{3} + 3\mu)P_{102} = \frac{\lambda}{3}P_{002} + \frac{\lambda}{3}P_{101} + 3\mu P_{103} + \mu P_{112}$$

$$(\frac{\lambda}{3}+4\mu)P_{103}=\frac{\lambda}{3}P_{003}+\frac{\lambda}{3}P_{102}+4\mu P_{104}$$

$$5\mu P_{104}=\frac{\lambda}{3}P_{004}+\frac{\lambda}{3}P_{103}$$

$$(\frac{2\lambda}{3}+2\mu)P_{110}=\frac{\lambda}{3}P_{010}+\frac{\lambda}{3}P_{100}+\mu P_{111}+2\mu P_{120}$$

$$(\frac{\lambda}{3}+3\mu)P_{111}=\frac{\lambda}{3}P_{011}+\frac{\lambda}{3}P_{101}+\frac{\lambda}{3}P_{110}+2\mu P_{112}$$

$$4\mu P_{112}=\frac{\lambda}{3}P_{012}+\frac{\lambda}{3}P_{102}+\frac{\lambda}{3}P_{111}$$

$$3\mu P_{120}=\frac{\lambda}{3}P_{020}+\frac{\lambda}{3}P_{110}$$

$$(\frac{\lambda}{3}+2\mu)P_{200}=\frac{\lambda}{3}P_{100}+\mu P_{201}$$

$$3\mu P_{201}=\frac{\lambda}{3}P_{101}+\frac{\lambda}{3}P_{200}$$

$$\sum_{i=0}^{2}\sum_{j=0}^{3}\sum_{k=0}^{7}P_{ijk}=1$$

Again, by solving all the Markov chain equations, the steady-state probabilities for the number of busy channels can be achieved. The probabilities can be calculated as follows:

$P_0=P_{000}$

$P_1=P_{001}$

$P_2 = P_{002} + P_{010}$

$P_3 = P_{003} + P_{011} + P_{100}$

$P_4 = P_{004} + P_{012} + P_{020} + P_{101}$

$P_5 = P_{005} + P_{013} + P_{021} + P_{102} + P_{110}$

$P_6 = P_{006} + P_{014} + P_{022} + P_{030} + P_{103} + P_{111} + P_{200}$

$P_7 = P_{007} + P_{015} + P_{023} + P_{031} + P_{104} + P_{112} + P_{120} + P_{201}$

Again, we provide some examples to show the accuracy of the analysis. In a network of $N = 7$, $\lambda = 2$, and $\mu = 2.5$, the steady-state probabilities for the number of busy channels are shown in Table 4.7 and Fig. 4.1.10. The simulation results are also provided to confirm the accuracy of the three-dimensional Markov chain.

Table 4.7: Steady-state probabilities for three-dimensional Markov chain: Nonconsecutive channels for users, $N = 7$, $\lambda = 2$, $\mu = 2.5$.

P_n	P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Numerical	0.4547	0.1212	0.1374	0.1550	0.0529	0.0413	0.0272	0.0102
Simulation	0.4542	0.1213	0.1375	0.1552	0.0530	0.0413	0.0272	0.0103

In addition, the results for the distribution of Max can be achieved with the proposed mathematical model presented in Chapter 3, as shown in Table 4.8 and Fig. 4.1.11.

Table 4.8: Probability distribution of Max for three-dimensional Markov chain: Non-consecutive channels for users, $N = 7$, $\lambda = 2$, $\mu = 2.5$.

$P(Max = x)$	$P(Max = 0)$	$P(Max = 1)$	$P(Max = 2)$	$P(Max = 3)$	$P(Max = 4)$	$P(Max = 5)$	$P(Max = 6)$	$P(Max = 7)$
Numerical	0.0102	0.0763	0.1414	0.1369	0.0916	0.0543	0.0346	0.4547
Simulation	0.0103	0.0763	0.1415	0.1370	0.0917	0.0544	0.0346	0.4542

The numerical result for the expected value of Max is 4.7981 and the simulation result is 4.7964. As can be seen, the results are the same as expected.

As another example, for a higher traffic network, in the case of $N = 7$, $\lambda = 3$, and $\mu = 2.5$, the steady-state probabilities for the number of busy channels are shown in Table 4.9 and in Fig.

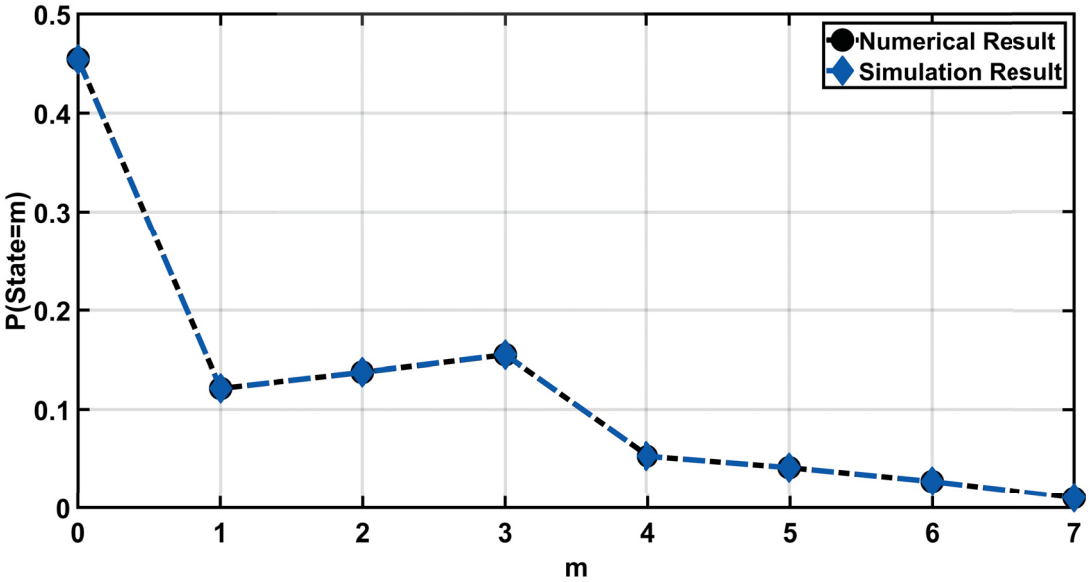

Figure 4.1.10: Steady-state probabilities for three-dimensional Markov chain: Non-consecutive channels for users, $N = 7$, $\lambda = 2$, $\mu = 2.5$.

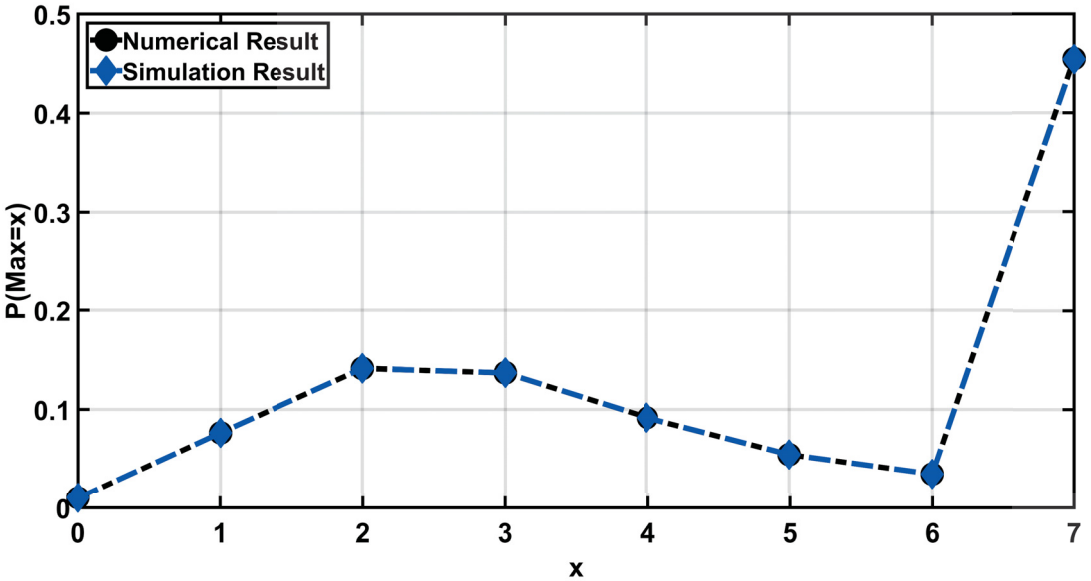

Figure 4.1.11: Probability distribution of Max for three-dimensional Markov chain: Non-consecutive channels for users, $N = 7$, $\lambda = 2$, $\mu = 2.5$.

4.1.12. It can be seen that the simulation result is almost identical to the numerical result which confirms the accuracy of the analysis.

Table 4.9: Steady-state probabilities for three-dimensional Markov chain: Nonconsecutive channels for users, $N = 7$, $\lambda = 3$, $\mu = 2.5$.

P_n	P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Numerical	0.3125	0.1250	0.1500	0.1783	0.0853	0.0714	0.0518	0.0257
Simulation	0.3120	0.1250	0.1501	0.1783	0.0854	0.0715	0.0520	0.0258

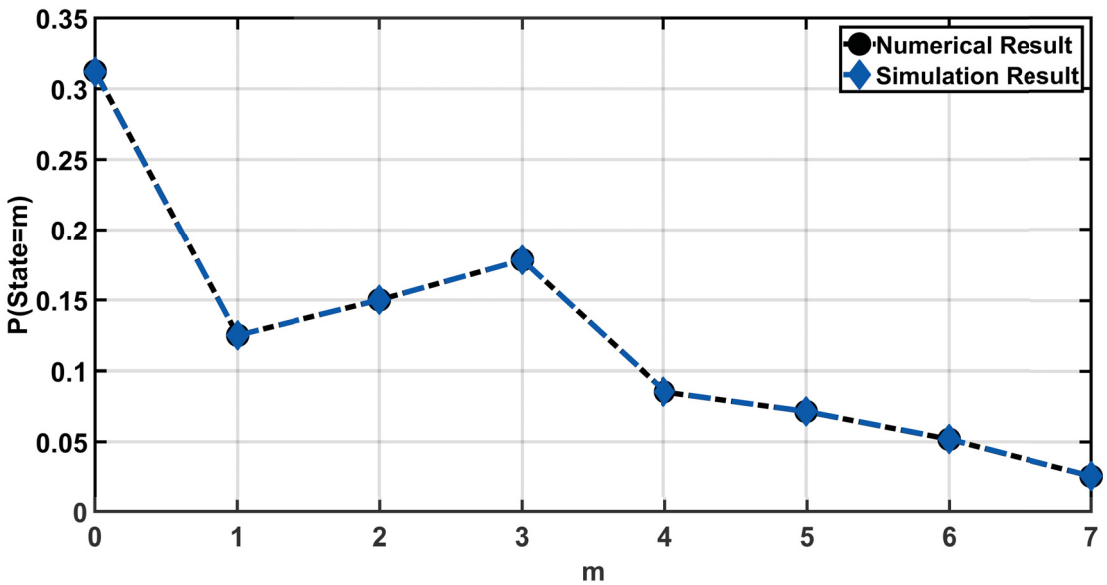

Figure 4.1.12: Steady-state probabilities for three-dimensional Markov chain: Non-consecutive channels for users, $N = 7$, $\lambda = 3$, $\mu = 2.5$.

As mentioned before, the final goal is the load calculation for the SU. As such, by having all the steady-state probabilities, the probability distribution for maximum number of consecutive idle channels can be achieved by our proposed method in Chapter 3. The results are shown in Table 4.10 and in Fig. 4.1.12.

Table 4.10: Probability distribution of Max for three-dimensional Markov chain: Non-consecutive channels for users, $N = 7$, $\lambda = 3$, $\mu = 2.5$.

$P(Max = x)$	$P(Max = 0)$	$P(Max = 1)$	$P(Max = 2)$	$P(Max = 3)$	$P(Max = 4)$	$P(Max = 5)$	$P(Max = 6)$	$P(Max = 7)$
Numerical	0.0257	0.1322	0.1823	0.1555	0.0989	0.0571	0.0357	0.3125
Simulation	0.0258	0.1326	0.1824	0.1554	0.0990	0.0572	0.0356	0.3120

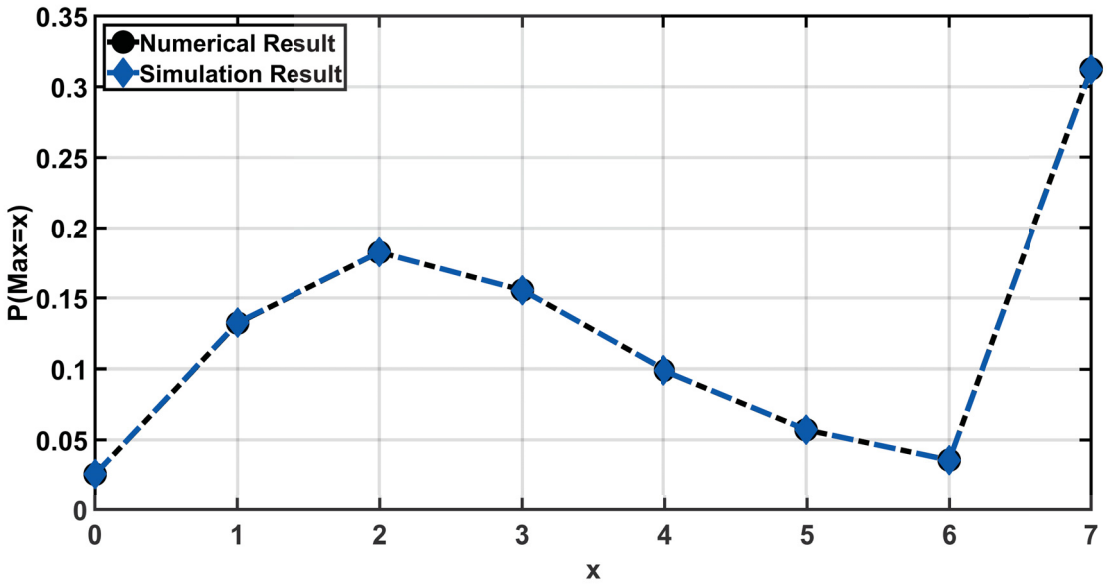

Figure 4.1.13: Probability distribution of Max for three-dimensional Markov chain: Non-consecutive channels for users, $N = 7$, $\lambda = 3$, $\mu = 2.5$.

Based on the results in Table 4.10, the expected value of Max, which is the load of the SU, can be achieved. The numerical result for the expected value of Max is 4.0463 and the simulation result is 4.0432. As can be seen, the results are exactly the same, which proves the accuracy of the analysis for the case of primary users with non-consecutive channels. As such, this section concludes the investigation for the case of PUs with non-consecutive channels. The challenge of PUs taking consecutive channels is addressed in the next section.

4.2 Primary Users with Consecutive Channels

In this section, we investigate the case when each PU, if possible, take a random number of consecutive channels. For the selection of the channels in this scenario, the locations of busy channels are important. Clearly, as explained earlier, based on the location of busy channels we can determine whether a request can be accepted or not. But the location tracking of busy channels is practically unfeasible. Instead of the location tracking, by approximation, our solution is to find the probability

of acceptance for each PU which requests a random number of consecutive channels. As before, to start the investigation, the arrivals and the departures of PUs are modeled by a multidimensional Markov chain. The number of dimensions is at most the maximum number of channels that each PU can have.

Now, as a similar example presented in the previous section, Fig. 4.2.1 shows a two-dimensional Markov chain, for users with consecutive channels, when the total number of channels is $N = 7$. With the same probability of arrivals for both types of users, the arrival rate of each type is $\lambda/2$, where λ is the arrival rate of all incoming users. Besides, μ is the service rate for all types of users.

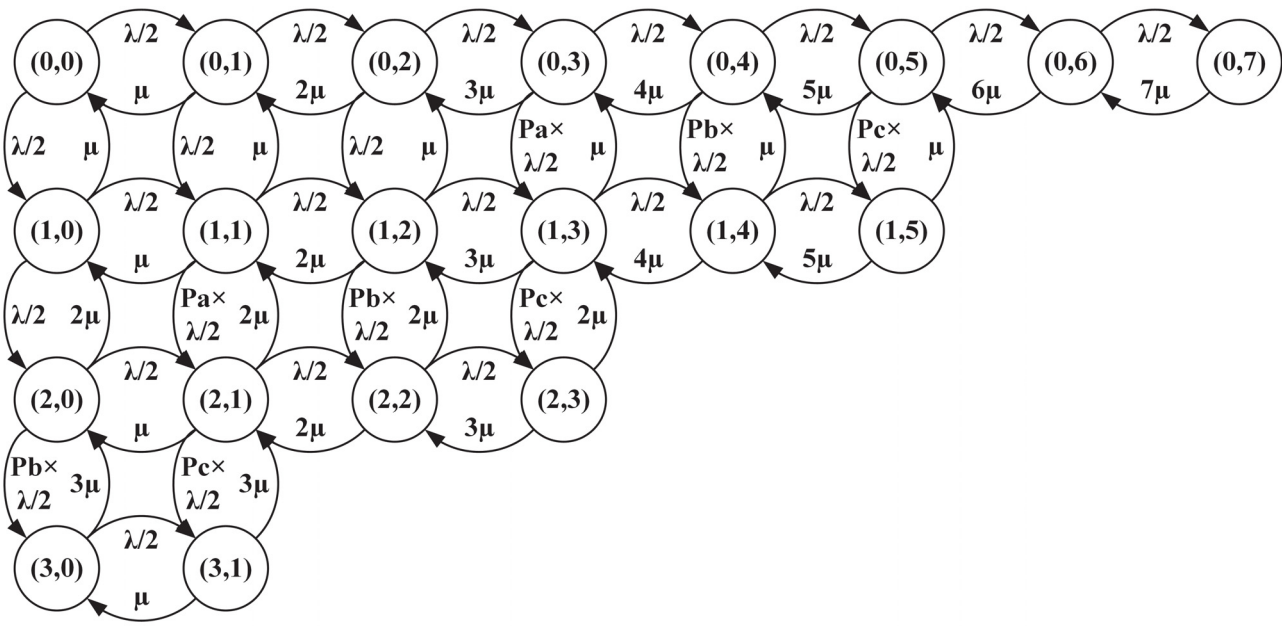

Figure 4.2.1: Example of two-dimensional Markov chain for users with consecutive channels.

In this case, for example if three channels are busy, it is not guaranteed for an arriving Type2-user to get the service. It may get the channels by approximation with probability of P_a as shown in the figure, where $P_a = P(Max \geq 2 \mid S_3)$. This shows the acceptance probability for Type2-user when there are three busy channels in the network. Moreover, when there are four or five busy channels, Type2-user may still be accepted. The acceptance possibilities are shown in the figure as P_b and P_c, where $P_b = P(Max \geq 2 \mid S_4)$ and $P_c = P(Max \geq 2 \mid S_5)$. All of the acceptance probabilities

can be calculated from our proposed method in Chapter 3. For example, if we have three busy channels in a total of seven, the length of maximum possible consecutive idle channels is four. Therefore, $P(Max \geq 2 \mid S_3)$ can be calculated as the sum of $P(Max = 2 \mid S_3)$, $P(Max = 3 \mid S_3)$, and $P(Max = 4 \mid S_3)$, where all the conditional probabilities can be calculated by our proposed mathematical model. Accordingly, the values for acceptance probabilities are $P_a = 0.9714$, $P_b = 0.7143$, and $P_c = 0.2857$. Note that, the acceptance probability for Type2-user in state of less than three busy channels is one. On the other hand, for more than five busy channels there is no possibility for Type2-user to have service.

Now, the same as before, the steady-state probabilities for the number of busy channels can be calculated by solving the Markov chain equations. The equations for Markov chain of Fig. 4.2.1 are as follows:

$$\lambda P_{00} = \mu P_{01} + \mu P_{10}$$

$$(\lambda + \mu)P_{01} = \frac{\lambda}{2}P_{00} + 2\mu P_{02} + \mu P_{11}$$

$$(\lambda + 2\mu)P_{02} = \frac{\lambda}{2}P_{01} + 3\mu P_{03} + \mu P_{12}$$

$$(\frac{\lambda}{2} + \frac{\lambda}{2}P_a + 3\mu)P_{03} = \frac{\lambda}{2}P_{02} + 4\mu P_{04} + \mu P_{13}$$

$$(\frac{\lambda}{2} + \frac{\lambda}{2}P_b + 4\mu)P_{04} = \frac{\lambda}{2}P_{03} + 5\mu P_{05} + \mu P_{14}$$

$$(\frac{\lambda}{2} + \frac{\lambda}{2}P_c + 5\mu)P_{05} = \frac{\lambda}{2}P_{04} + 6\mu P_{06} + \mu P_{15}$$

$$(\frac{\lambda}{2}+6\mu)P_{06}=\frac{\lambda}{2}P_{05}+7\mu P_{07}$$

$$7\mu P_{07}=\frac{\lambda}{2}P_{06}$$

$$(\lambda+\mu)P_{10}=\frac{\lambda}{2}P_{00}+\mu P_{11}+2\mu P_{20}$$

$$(\frac{\lambda}{2}+\frac{\lambda}{2}P_a+2\mu)P_{11}=\frac{\lambda}{2}P_{01}+\frac{\lambda}{2}P_{10}+2\mu P_{12}+2\mu P_{21}$$

$$(\frac{\lambda}{2}+\frac{\lambda}{2}P_b+3\mu)P_{12}=\frac{\lambda}{2}P_{02}+\frac{\lambda}{2}P_{11}+3\mu P_{13}+2\mu P_{22}$$

$$(\frac{\lambda}{2}+\frac{\lambda}{2}P_c+4\mu)P_{13}=\frac{\lambda}{2}P_aP_{03}+\frac{\lambda}{2}P_{12}+4\mu P_{14}+2\mu P_{23}$$

$$(\frac{\lambda}{2}+5\mu)P_{14}=\frac{\lambda}{2}P_bP_{04}+\frac{\lambda}{2}P_{13}+5\mu P_{15}$$

$$6\mu P_{15}=\frac{\lambda}{2}P_cP_{05}+\frac{\lambda}{2}P_{14}$$

$$(\frac{\lambda}{2}+\frac{\lambda}{2}P_b+2\mu)P_{20}=\frac{\lambda}{2}P_{10}+\mu P_{21}+3\mu P_{30}$$

$$(\frac{\lambda}{2}+\frac{\lambda}{2}P_c+3\mu)P_{21}=\frac{\lambda}{2}P_aP_{11}+\frac{\lambda}{2}P_{20}+2\mu P_{22}+3\mu P_{31}$$

$$(\frac{\lambda}{2}+4\mu)P_{22}=\frac{\lambda}{2}P_bP_{12}+\frac{\lambda}{2}P_{21}+3\mu P_{23}$$

$$5\mu P_{23}=\frac{\lambda}{2}P_cP_{13}+\frac{\lambda}{2}P_{22}$$

$$(\frac{\lambda}{2}+3\mu)P_{30}=\frac{\lambda}{2}P_bP_{20}+\mu P_{31}$$

$$4\mu P_{31}=\frac{\lambda}{2}P_cP_{21}+\frac{\lambda}{2}P_{30}$$

and by using the general probabilities of different kinds of users, the equations are like:

$$(P_{u1}\lambda+P_{u2}\lambda)P_{00}=\mu P_{01}+\mu P_{10}$$

$$(P_{u1}\lambda+P_{u2}\lambda+\mu)P_{01}=P_{u1}\lambda P_{00}+2\mu P_{02}+\mu P_{11}$$

$$(P_{u1}\lambda+P_{u2}\lambda+2\mu)P_{02}=P_{u1}\lambda P_{01}+3\mu P_{03}+\mu P_{12}$$

$$(P_{u1}\lambda+P_aP_{u2}\lambda+3\mu)P_{03}=P_{u1}\lambda P_{02}+4\mu P_{04}+\mu P_{13}$$

$$(P_{u1}\lambda+P_bP_{u2}\lambda+4\mu)P_{04}=P_{u1}\lambda P_{03}+5\mu P_{05}+\mu P_{14}$$

$$(P_{u1}\lambda+P_cP_{u2}\lambda+5\mu)P_{05}=P_{u1}\lambda P_{04}+6\mu P_{06}+\mu P_{15}$$

$$(P_{u1}\lambda + 6\mu)P_{06} = P_{u1}\lambda P_{05} + 7\mu P_{07}$$

$$7\mu P_{07} = P_{u1}\lambda P_{06}$$

$$(P_{u1}\lambda + P_{u2}\lambda + \mu)P_{10} = P_{u2}\lambda P_{00} + \mu P_{11} + 2\mu P_{20}$$

$$(P_{u1}\lambda + P_a P_{u2}\lambda + 2\mu)P_{11} = P_{u2}\lambda P_{01} + P_{u1}\lambda P_{10} + 2\mu P_{12} + 2\mu P_{21}$$

$$(P_{u1}\lambda + P_b P_{u2}\lambda + 3\mu)P_{12} = P_{u2}\lambda P_{02} + P_{u1}\lambda P_{11} + 3\mu P_{13} + 2\mu P_{22}$$

$$(P_{u1}\lambda + P_c P_{u2}\lambda + 4\mu)P_{13} = P_a P_{u2}\lambda P_{03} + P_{u1}\lambda P_{12} + 4\mu P_{14} + 2\mu P_{23}$$

$$(P_{u1}\lambda + 5\mu)P_{14} = P_b P_{u2}\lambda P_{04} + P_{u1}\lambda P_{13} + 5\mu P_{15}$$

$$6\mu P_{15} = P_c P_{u2}\lambda P_{05} + P_{u1}\lambda P_{14}$$

$$(P_{u1}\lambda + P_b P_{u2}\lambda + 2\mu)P_{20} = P_{u2}\lambda P_{10} + \mu P_{21} + 3\mu P_{30}$$

$$(P_{u1}\lambda + P_c P_{u2}\lambda + 3\mu)P_{21} = P_a P_{u2}\lambda P_{11} + P_{u1}\lambda P_{20} + 2\mu P_{22} + 3\mu P_{31}$$

$$(P_{u1}\lambda + 4\mu)P_{22} = P_b P_{u2}\lambda P_{12} + P_{u1}\lambda P_{21} + 3\mu P_{23}$$

$$5\mu P_{23} = P_c P_{u2} \lambda P_{13} + P_{u1} \lambda P_{22}$$

$$(P_{u1}\lambda + 3\mu) P_{30} = P_b P_{u2} \lambda P_{20} + \mu P_{31}$$

$$4\mu P_{31} = P_c P_{u2} \lambda P_{21} + P_{u1} \lambda P_{30}$$

Solving the above equations leads to steady-state probabilities for the number of busy channels, as follows:

$P_0 = P_{00}$

$P_1 = P_{01}$

$P_2 = P_{02} + P_{10}$

$P_3 = P_{03} + P_{11}$

$P_4 = P_{04} + P_{12} + P_{20}$

$P_5 = P_{05} + P_{13} + P_{21}$

$P_6 = P_{06} + P_{14} + P_{22} + P_{30}$

$P_7 = P_{07} + P_{15} + P_{23} + P_{31}$

Now, as an example for a low-traffic network of $N = 7$, $\lambda = 2$, and $\mu = 2.5$, the steady-state probabilities for the number of busy channels are shown in Table 4.11 and Fig. 4.2.2. The simulation results are also provided to confirm the accuracy of the numerical result.

Table 4.11: Steady-state probabilities for two-dimensional Markov chain: Consecutive channels for users, $N = 7$, $\lambda = 2$, $\mu = 2.5$.

P_n	P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Numerical	0.4514	0.1810	0.2165	0.0773	0.0509	0.0161	0.0058	0.0010
Simulation	0.4499	0.1806	0.2165	0.0774	0.0510	0.0165	0.0068	0.0012

As explained earlier, by having all the steady-state probabilities, the distribution of Max can be

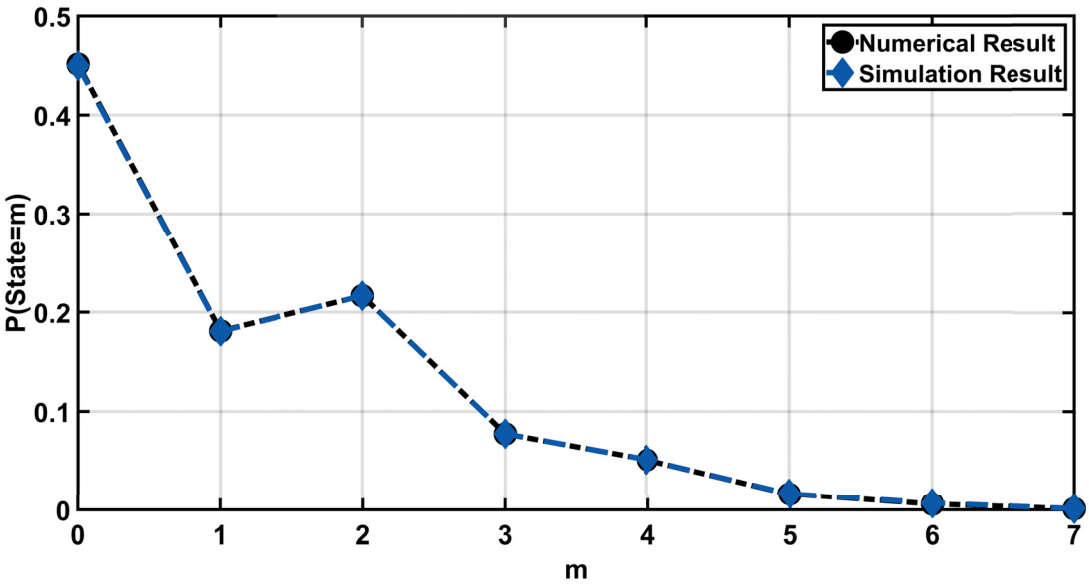

Figure 4.2.2: Steady-state probabilities for two-dimensional Markov chain: Consecutive channels for users, $N = 7$, $\lambda = 2$, $\mu = 2.5$.

achieved with the proposed mathematical model presented in Chapter 3. The results are shown in Table 4.12 and Fig. 4.2.3.

Table 4.12: Probability distribution of Max for two-dimensional Markov chain: Consecutive channels for users, $N = 7$, $\lambda = 2$, $\mu = 2.5$.

$P(Max = x)$	$P(Max = 0)$	$P(Max = 1)$	$P(Max = 2)$	$P(Max = 3)$	$P(Max = 4)$	$P(Max = 5)$	$P(Max = 6)$	$P(Max = 7)$
Numerical	0.0010	0.0341	0.1044	0.1524	0.1224	0.0826	0.0517	0.4514
Simulation	0.0012	0.0243	0.0717	0.1444	0.1358	0.1195	0.0532	0.4499

Finally, from Table 4.12, the numerical result for the expected value of Max is 5.0730 and the simulation result is 5.2100. Although, we see almost similar results for the load of the SU, the small difference is due to the results of the probability distribution for maximum number of consecutive idle channels, which are not very much the same. As expressed before, our proposed method for the calculation of $P(x \mid n, m)$ in Chapter 3 is for fair scheduling policy, which is not valid in the case of PUs with consecutive channels. However, for the low-traffic case of PUs, which is the case in

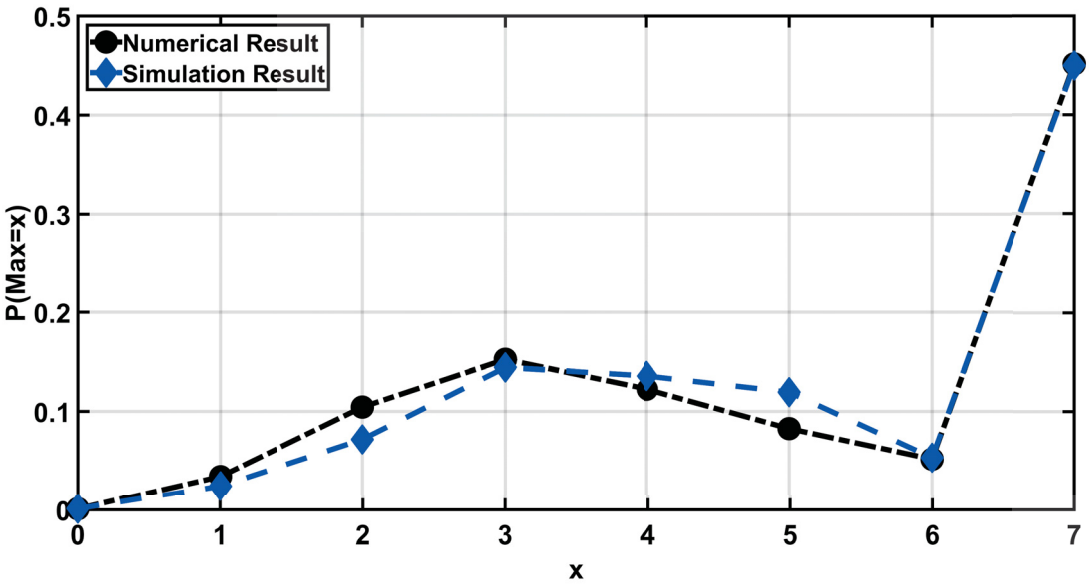

Figure 4.2.3: Probability distribution of Max for two-dimensional Markov chain: Consecutive channels for users, $N = 7$, $\lambda = 2$, $\mu = 2.5$.

interweave CR networks, the scheduling could be considered as almost fair. As such, the calculation of $P(x \mid n, m)$, e.g., P_a, in Fig. 4.2.1, where we used the probabilities of acceptance in the case of consecutive channels, is a valid approximation. Therefore, the numerical and simulation results in Fig. 4.2.4 are slightly different, but the small differences are not discernible. Now, after having P_m from Fig. 4.2.4, we apply the result to (3.2.2) to calculate $P(Max = x)$. But the calculation of $P(Max = x \mid S = m)$ in (3.2.2) is again based on fair scheduling policy, which is not valid here. This is the reason for the differences between the results in Fig. 4.2.5 and accordingly for the small difference in the results of the $E[Max]$. To be more clear, if we apply the simulation results for steady-state probabilities in the proposed method in Chapter 3, we will have $E[Max] = 5.0625$ which is very similar to the numerical results, as expected. But the scheduling is not fair, as such, the results for probability distribution of Max are not exactly the same, but they are good enough to have the load estimation.

As another numerical example, in case of $N = 7$, $\lambda = 3$, and $\mu = 2.5$, the steady-state probabilities are shown in Table 4.13 and in Fig. 4.2.4. It can be seen that the numerical and simulation

results are almost the same, as expected.

Table 4.13: Steady-state probabilities for two-dimensional Markov chain: Consecutive channels for users, $N = 7$, $\lambda = 3$, $\mu = 2.5$.

P_n	P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Numerical	0.3060	0.1847	0.2386	0.1221	0.0896	0.0391	0.0162	0.0037
Simulation	0.3042	0.1839	0.2378	0.1216	0.0894	0.0399	0.0186	0.0046

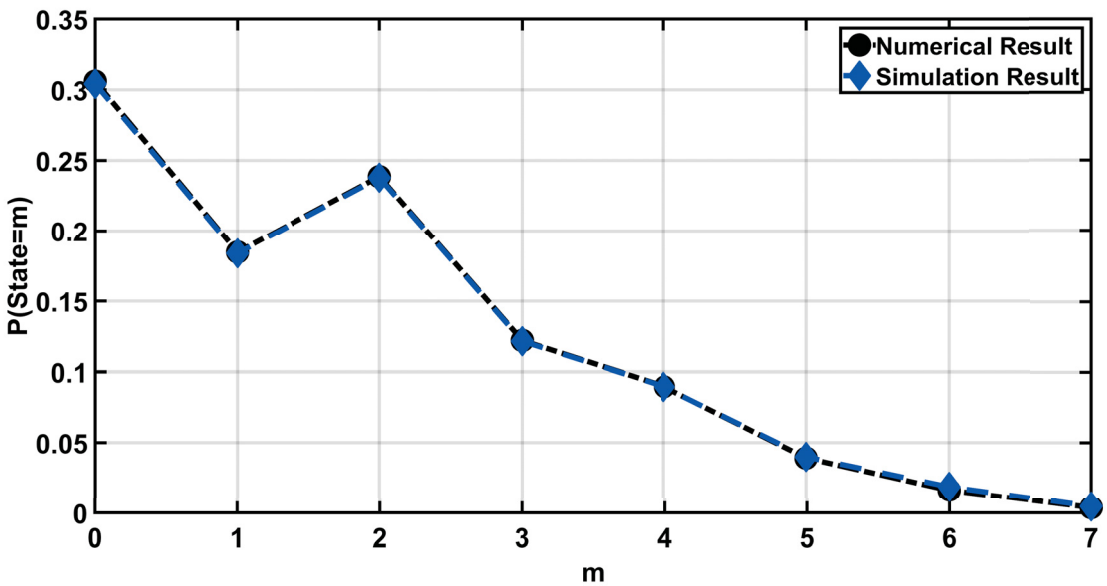

Figure 4.2.4: Steady-state probabilities for two-dimensional Markov chain: Consecutive channels for users, $N = 7$, $\lambda = 3$, $\mu = 2.5$.

To continue, with our proposed method in Chapter 3, the probability distribution for maximum number of consecutive idle channels can be achieved when we have all the steady-state probabilities. The results are shown in both Table 4.14 and Fig. 4.2.5.

Finally, the numerical result for the expected value of Max is 4.3741 and the simulation result is 4.5333. The same as previous example, we see almost similar results which shows that the analysis

Table 4.14: Probability distribution of Max for two-dimensional Markov chain: Consecutive channels for users, $N = 7$, $\lambda = 3$, $\mu = 2.5$.

$P(Max = x)$	$P(Max = 0)$	$P(Max = 1)$	$P(Max = 2)$	$P(Max = 3)$	$P(Max = 4)$	$P(Max = 5)$	$P(Max = 6)$	$P(Max = 7)$
Numerical	0.0037	0.0732	0.1592	0.1833	0.1349	0.0868	0.0528	0.3060
Simulation	0.0046	0.0567	0.1224	0.1806	0.1516	0.1248	0.0550	0.3042

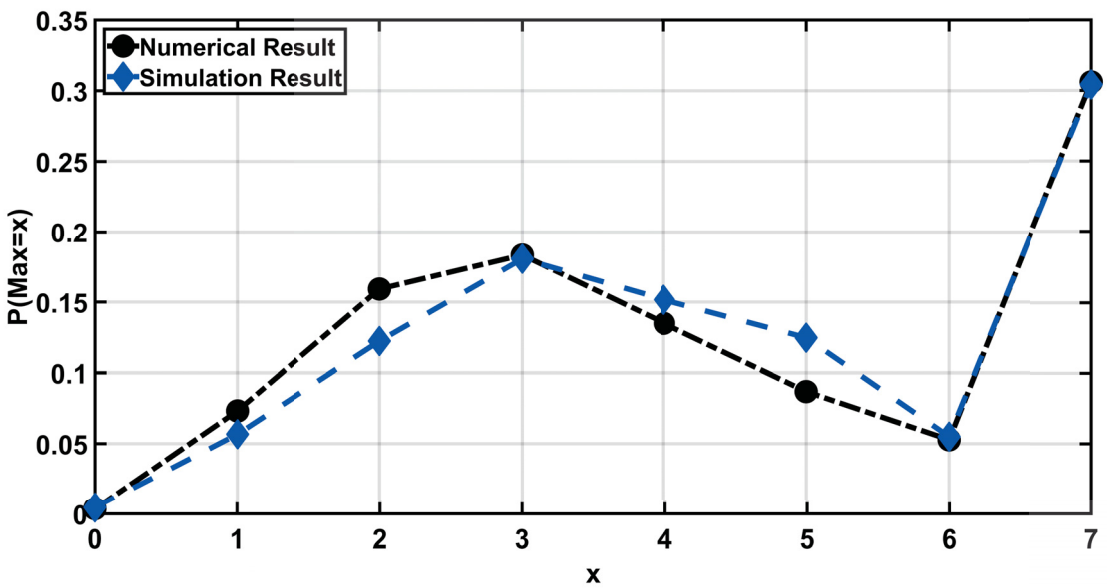

Figure 4.2.5: Probability distribution of Max for two-dimensional Markov chain: Consecutive channels for users, $N = 7$, $\lambda = 3$, $\mu = 2.5$.

works fine and the results can be used by the service provider to have a good insight for the load of the secondary user.

To continue, we consider a more complicated scenario. Fig. 4.2.6 shows the three-dimensional system when the total number of channels is $N = 7$ and users can take either one, two, or three consecutive channels with equal probabilities.

In this case, the probabilities of acceptance are shown in Fig. 4.2.6 as $P_a = P(Max \geq 2 \mid S_3)$, $P_b = P(Max \geq 2 \mid S_4)$, $P_c = P(Max \geq 2 \mid S_5)$, $P_d = P(Max \geq 3 \mid S_2)$, $P_e = P(Max \geq 3 \mid S_3)$, and $P_f = P(Max \geq 3 \mid S_4)$. Again, these probabilities can be achieved by our proposed method. The results are $P_a = 0.9714$, $P_b = 0.7143$, $P_c = 0.2857$, $P_d = 0.8571$, $P_e = 0.4571$, and $P_f = 0.1429$. Here are the equations,

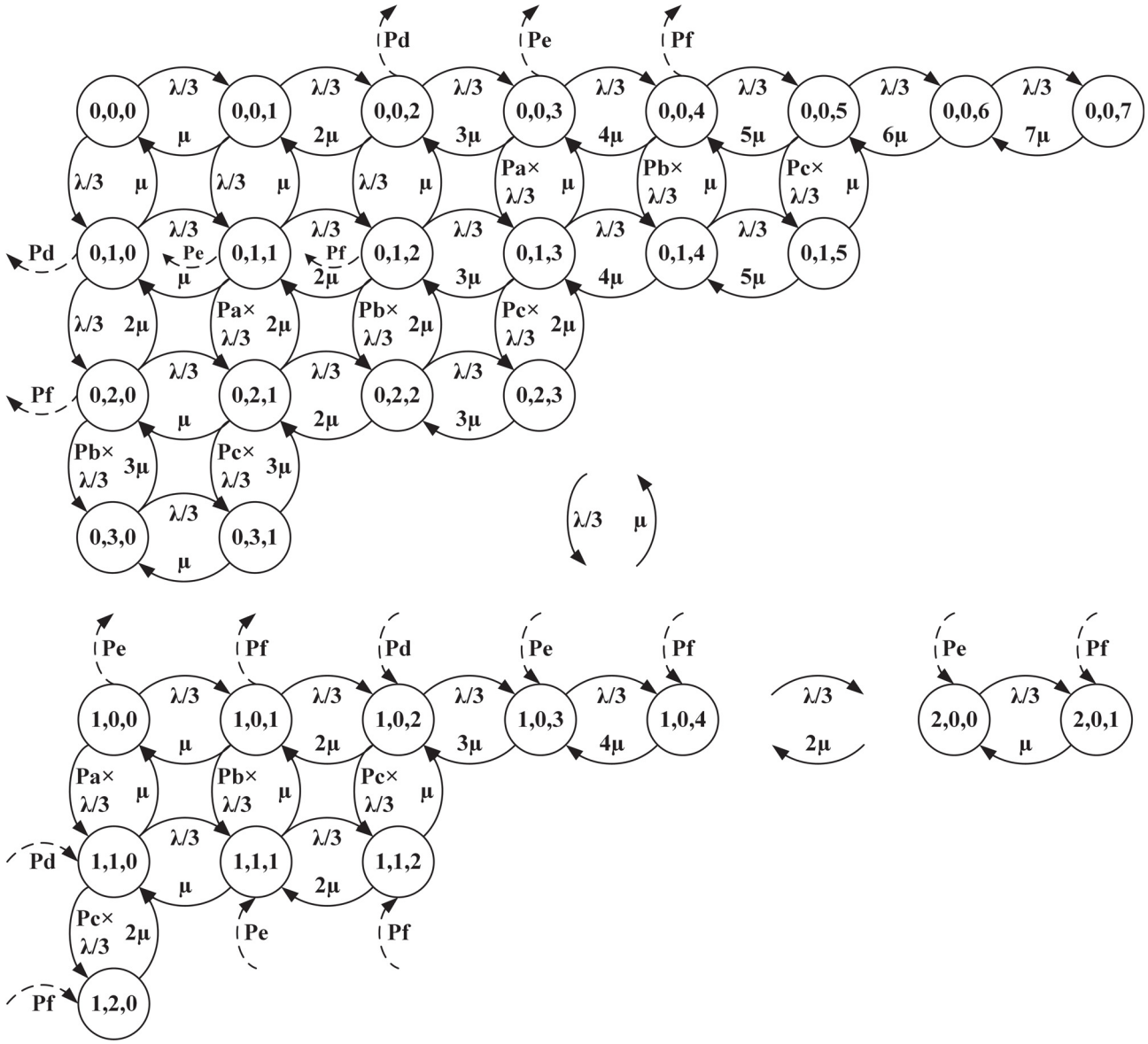

Figure 4.2.6: Example of three-dimensional Markov chain for users with consecutive channels.

$$\lambda P_{000} = \mu P_{001} + \mu P_{010} + \mu P_{100}$$

$$(\lambda + \mu) P_{001} = \frac{\lambda}{3} P_{000} + 2\mu P_{002} + \mu P_{011} + \mu P_{101}$$

$$(\frac{2\lambda}{3}+\frac{\lambda}{3}P_d+2\mu)P_{002}=\frac{\lambda}{3}P_{001}+3\mu P_{003}+\mu P_{012}+\mu P_{102}$$

$$(\frac{\lambda}{3}+\frac{\lambda}{3}P_a+\frac{\lambda}{3}P_e+3\mu)P_{003}=\frac{\lambda}{3}P_{002}+4\mu P_{004}+\mu P_{013}+\mu P_{103}$$

$$(\frac{\lambda}{3}+\frac{\lambda}{3}P_b+\frac{\lambda}{3}P_f+4\mu)P_{004}=\frac{\lambda}{3}P_{003}+5\mu P_{005}+\mu P_{014}+\mu P_{104}$$

$$(\frac{\lambda}{3}+\frac{\lambda}{3}P_c+5\mu)P_{005}=\frac{\lambda}{3}P_{004}+6\mu P_{006}+\mu P_{015}$$

$$(\frac{\lambda}{3}+6\mu)P_{006}=\frac{\lambda}{3}P_{005}+7\mu P_{007}$$

$$7\mu P_{007}=\frac{\lambda}{3}P_{006}$$

$$(\frac{2\lambda}{3}+\frac{\lambda}{3}P_d+\mu)P_{010}=\frac{\lambda}{3}P_{000}+\mu P_{011}+2\mu P_{020}+\mu P_{110}$$

$$(\frac{\lambda}{3}+\frac{\lambda}{3}P_a+\frac{\lambda}{3}P_e+2\mu)P_{011}=\frac{\lambda}{3}P_{001}+\frac{\lambda}{3}P_{010}+2\mu P_{012}+2\mu P_{021}+\mu P_{111}$$

$$(\frac{\lambda}{3}+\frac{\lambda}{3}P_b+\frac{\lambda}{3}P_f+3\mu)P_{012}=\frac{\lambda}{3}P_{002}+\frac{\lambda}{3}P_{011}+3\mu P_{013}+2\mu P_{022}+\mu P_{112}$$

$$(\frac{\lambda}{3}+\frac{\lambda}{3}P_c+4\mu)P_{013}=\frac{\lambda}{3}P_aP_{003}+\frac{\lambda}{3}P_{012}+4\mu P_{014}+2\mu P_{023}$$

$$(\frac{\lambda}{3}+5\mu)P_{014}=\frac{\lambda}{3}P_bP_{004}+\frac{\lambda}{3}P_{013}+5\mu P_{015}$$

$$6\mu P_{015}=\frac{\lambda}{3}P_cP_{005}+\frac{\lambda}{3}P_{014}$$

$$(\frac{\lambda}{3}+\frac{\lambda}{3}P_b+\frac{\lambda}{3}P_f+2\mu)P_{020}=\frac{\lambda}{3}P_{010}+\mu P_{021}+3\mu P_{030}+\mu P_{120}$$

$$(\frac{\lambda}{3}+\frac{\lambda}{3}P_c+3\mu)P_{021}=\frac{\lambda}{3}P_aP_{011}+\frac{\lambda}{3}P_{020}+2\mu P_{022}+3\mu P_{031}$$

$$(\frac{\lambda}{3}+4\mu)P_{022}=\frac{\lambda}{3}P_bP_{012}+\frac{\lambda}{3}P_{021}+3\mu P_{023}$$

$$5\mu P_{023}=\frac{\lambda}{3}P_cP_{013}+\frac{\lambda}{3}P_{022}$$

$$(\frac{\lambda}{3}+3\mu)P_{030}=\frac{\lambda}{3}P_bP_{020}+\mu P_{031}$$

$$4\mu P_{031}=\frac{\lambda}{3}P_cP_{021}+\frac{\lambda}{3}P_{030}$$

$$(\frac{\lambda}{3}+\frac{\lambda}{3}P_a+\frac{\lambda}{3}P_e+\mu)P_{100}=\frac{\lambda}{3}P_{000}+\mu P_{101}+\mu P_{110}+2\mu P_{200}$$

$$(\frac{\lambda}{3}+\frac{\lambda}{3}P_b+\frac{\lambda}{3}P_f+2\mu)P_{101}=\frac{\lambda}{3}P_{001}+\frac{\lambda}{3}P_{100}+2\mu P_{102}+\mu P_{111}+2\mu P_{201}$$

$$(\frac{\lambda}{3}+\frac{\lambda}{3}P_c+3\mu)P_{102}=\frac{\lambda}{3}P_dP_{002}+\frac{\lambda}{3}P_{101}+3\mu P_{103}+\mu P_{112}$$

$$(\frac{\lambda}{3}+4\mu)P_{103}=\frac{\lambda}{3}P_eP_{003}+\frac{\lambda}{3}P_{102}+4\mu P_{104}$$

$$5\mu P_{104}=\frac{\lambda}{3}P_fP_{004}+\frac{\lambda}{3}P_{103}$$

$$(\frac{\lambda}{3}+\frac{\lambda}{3}P_c+2\mu)P_{110}=\frac{\lambda}{3}P_dP_{010}+\frac{\lambda}{3}P_aP_{100}+\mu P_{111}+2\mu P_{120}$$

$$(\frac{\lambda}{3}+3\mu)P_{111}=\frac{\lambda}{3}P_eP_{011}+\frac{\lambda}{3}P_bP_{101}+\frac{\lambda}{3}P_{110}+2\mu P_{112}$$

$$4\mu P_{112}=\frac{\lambda}{3}P_fP_{012}+\frac{\lambda}{3}P_cP_{102}+\frac{\lambda}{3}P_{111}$$

$$3\mu P_{120}=\frac{\lambda}{3}P_fP_{020}+\frac{\lambda}{3}P_cP_{110}$$

$$(\frac{\lambda}{3}+2\mu)P_{200}=\frac{\lambda}{3}P_eP_{100}+\mu P_{201}$$

$$3\mu P_{201}=\frac{\lambda}{3}P_fP_{101}+\frac{\lambda}{3}P_{200}$$

$$\sum_{i=0}^{2}\sum_{j=0}^{3}\sum_{k=0}^{7}P_{ijk}=1$$

Solving the above equations leads to steady-state probabilities for the number of busy channels:

$P_0=P_{000}$

$P_1 = P_{001}$

$P_2 = P_{002} + P_{010}$

$P_3 = P_{003} + P_{011} + P_{100}$

$P_4 = P_{004} + P_{012} + P_{020} + P_{101}$

$P_5 = P_{005} + P_{013} + P_{021} + P_{102} + P_{110}$

$P_6 = P_{006} + P_{014} + P_{022} + P_{030} + P_{103} + P_{111} + P_{200}$

$P_7 = P_{007} + P_{015} + P_{023} + P_{031} + P_{104} + P_{112} + P_{120} + P_{201}$

Now, in the case of $N = 7$, $\lambda = 2$, and $\mu = 2.5$, the steady-state probabilities for the number of busy channels are shown in Table 4.15 and Fig. 4.2.7.

Table 4.15: Steady-state probabilities for three-dimensional Markov chain: Consecutive channels for users, $N = 7$, $\lambda = 2$, $\mu = 2.5$.

P_n	P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Numerical	0.4649	0.1252	0.1425	0.1573	0.0546	0.0381	0.0148	0.0027
Simulation	0.4590	0.1236	0.1395	0.1564	0.0541	0.0408	0.0218	0.0049

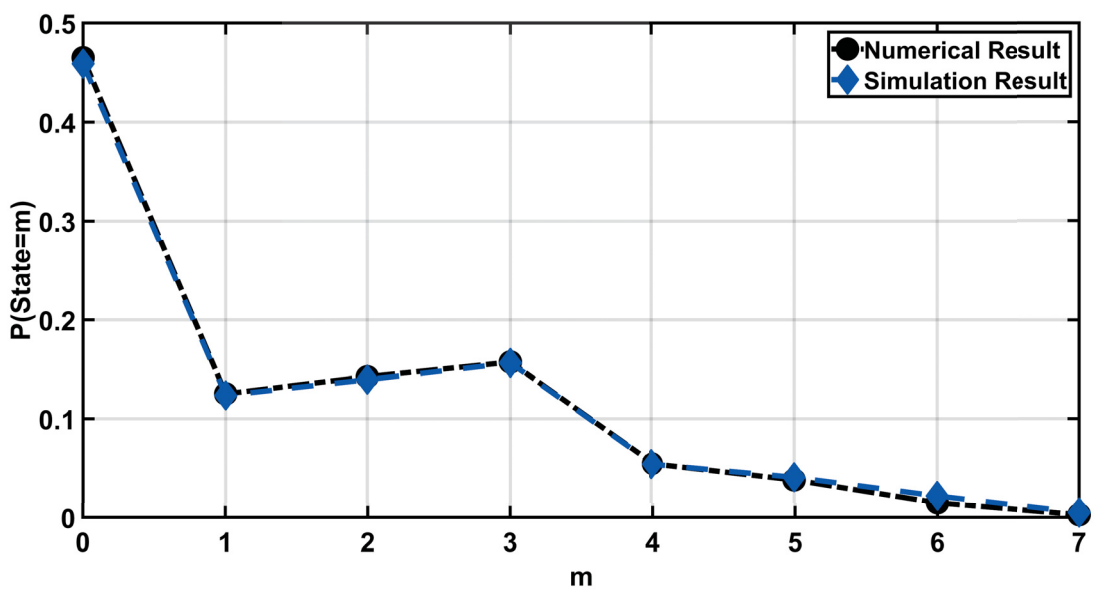

Figure 4.2.7: Steady-state probabilities for three-dimensional Markov chain: Consecutive channels for users, $N = 7$, $\lambda = 2$, $\mu = 2.5$.

Again, with our proposed method in Chapter 3, the probability distribution for maximum number of consecutive idle channels can be achieved. The results are shown in both Table 4.15 and Fig. 4.15.

Table 4.16: Probability distribution of Max for three-dimensional Markov chain: Consecutive channels for users, $N = 7$, $\lambda = 2$, $\mu = 2.5$.

$P(Max = x)$	$P(Max = 0)$	$P(Max = 1)$	$P(Max = 2)$	$P(Max = 3)$	$P(Max = 4)$	$P(Max = 5)$	$P(Max = 6)$	$P(Max = 7)$
Numerical	0.0027	0.0621	0.1433	0.1407	0.0945	0.0561	0.0358	0.4649
Simulation	0.0049	0.0484	0.0901	0.1378	0.1398	0.0822	0.0378	0.4590

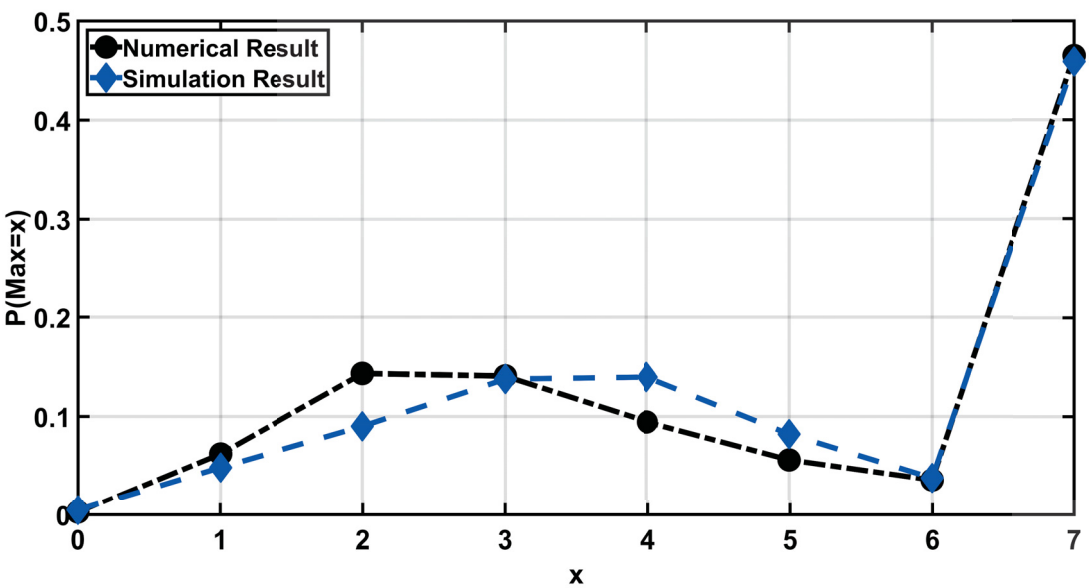

Figure 4.2.8: Probability distribution of Max for three-dimensional Markov chain: Consecutive channels for users, $N = 7$, $\lambda = 2$, $\mu = 2.5$.

From Table 4.16, the expected value of maximum for numerical result is 4.8981 and for simulation result is 5.0520. Again, in this case, we see almost the same results for the steady-state probabilities, but the results for the load of the SU are not exactly the same. Again, for this example, if we apply the steady-state probabilities from the simulation results in our proposed method in Chapter 3, we will have $E[Max] = 4.8464$ which is very similar to the numerical results.

The last example is for the case of $N = 7$, $\lambda = 3$, and $\mu = 2.5$, where the steady-state probabilities for the number of busy channels are shown in Table 4.17 and Fig. 4.2.9.

Table 4.17: Steady-state probabilities for three-dimensional Markov chain: Consecutive channels for users, $N = 7$, $\lambda = 3$, $\mu = 2.5$.

P_n	P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Numerical	0.3265	0.1332	0.1601	0.1853	0.0906	0.0672	0.0299	0.0073
Simulation	0.3184	0.1298	0.1551	0.1822	0.0889	0.0710	0.0418	0.0127

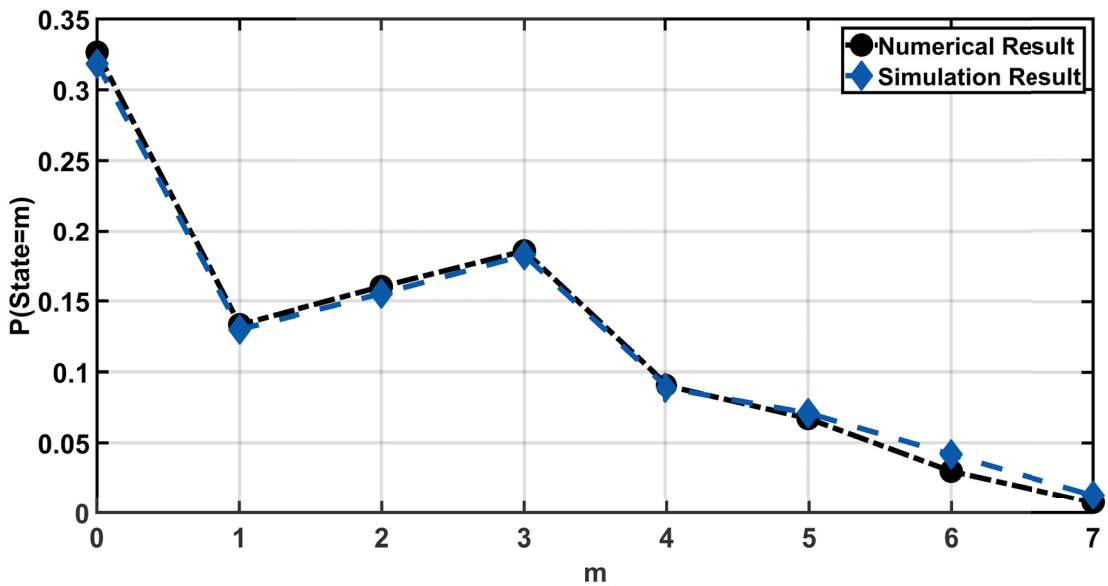

Table 4.18: Steady-state probabilities for three-dimensional Markov chain: Consecutive channels for users, $N = 7$, $\lambda = 3$, $\mu = 2.5$.

The results for the probability distribution of *Max* are shown in both Table 4.19 and Fig. 4.2.9.

From Table 4.19, the expected value of *Max* for numerical result is 4.2180 and for simulation result is 4.3780. Again, in this case, we see almost similar results for the load of the SU.

Table 4.19: Probability distribution of Max for three-dimensional Markov chain: Consecutive channels for users, $N = 7$, $\lambda = 3$, $\mu = 2.5$.

$P(Max = x)$	$P(Max = 0)$	$P(Max = 1)$	$P(Max = 2)$	$P(Max = 3)$	$P(Max = 4)$	$P(Max = 5)$	$P(Max = 6)$	$P(Max = 7)$
Numerical	0.0073	0.1090	0.1891	0.1641	0.1050	0.0609	0.0381	0.3265
Simulation	0.0127	0.0884	0.1336	0.1640	0.1535	0.0887	0.0407	0.3184

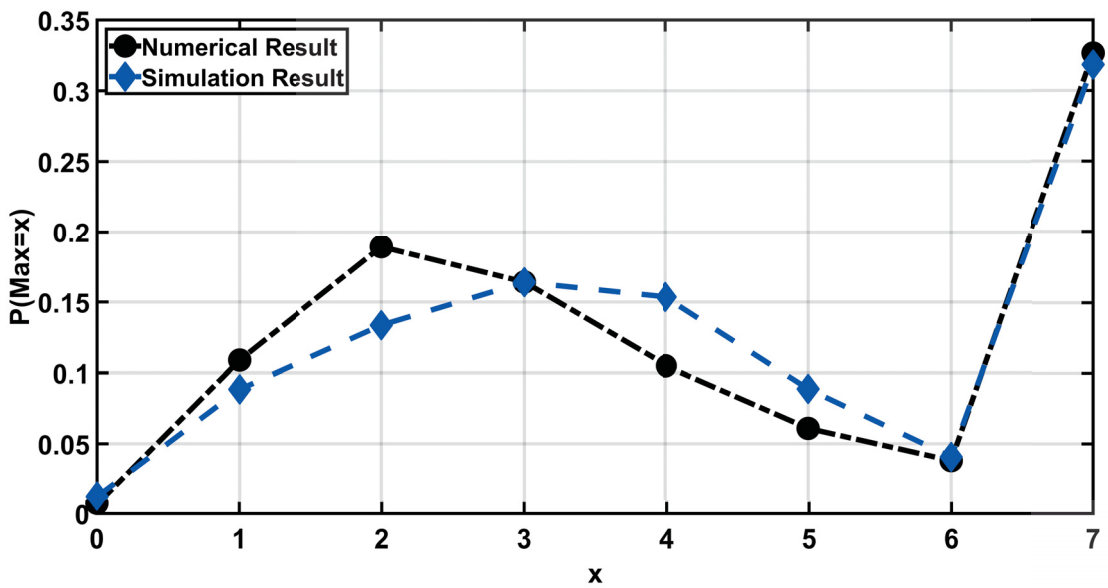

Figure 4.2.9: Probability distribution of Max for three-dimensional Markov chain: Consecutive channels for users, $N = 7$, $\lambda = 3$, $\mu = 2.5$.

At the end, if the variety of users are more, we face multidimensional cases which are very complicated and difficult to solve. As such, we propose a one-dimensional Markov chain approximation to deal with the excessive complexity. Next section covers this approximation.

4.3 Summary

This chapter proposed a mathematical model for conventional method in CR networks, which selects the largest spectrum hole for the load of the secondary user. In the considered CR network, each primary user takes a random number of channels and the secondary user gets the largest available spectrum hole. The objective of the proposed model is giving the service providers an insight

about the load of the secondary user. The proposed mathematical model calculates the load of the secondary user, where the main contribution is the calculation of the probability distribution of busy channels taken by primary users. The solution is based on the multidimensional Markov chain. At the end, the performance of the proposed model was validated by numerical and simulation results.

5 One-Dimensional Markov Chain Approximation

As explained in the previous chapter, if the maximum number of channels that a PU can take increases, the complexity of the multidimensional Markov chain solution increases accordingly. For this scenario, in this chapter, we provide an approximate estimation for the load of the SU. We propose a one-dimensional approximation to avoid the calculation of the complicated multi-dimensional Markov chain. The procedure significantly decreases the complexity of the solution, however, we lose some information.

This chapter is organized as follows. Section 5.1 presents the proposed one-dimensional approximation, which is followed by the numerical and simulation results in Section 5.2.

5.1 Proposed One-Dimensional Approximation

The main concern in the one-dimensional approximation is to find the departure rates from each state. In multidimensional scenario in any state of busy channels we know exactly how many of each type of users are available. But in one-dimensional approximation we do not know the number of users, all we now is the number of busy channels. Therefore, we need to find out the probability for each type of users in a given m busy channels. With this information, we can find out the departure rates from each state, which is actually the main challenge of this section.

To solve the problem, it is clear that by assuming the steady-state in the network, the probability of type i users which are the users with i number of channels, can be achieved with dividing the average number of type i users by the average number of total users. Therefore, we need to find out the average number for each type of users in the network. The following explains the steps.

We first find the average number for each type of users in the system. To reach that, we can use the Little's formula, where the average number of users $L = E[N]$ in the system is:

$$L = \lambda_L W \tag{5.1.1}$$

and λ_L is the average rate of users entering the queueing system and $W = E[T]$ is the average waiting time in the system [51]. Therefore, we first need the effective arrival rate (arrival rate of users entering the system) for each type of users. In general, if λ is the arrival rate of all incoming users and P_i shows the probability of type i users, then the arrival rate for type i users is:

$$\lambda_i = P_i \times \lambda \tag{5.1.2}$$

Now, if m is the number of busy channels in a total of n channels where there are l types of users available, the effective arrival rate for type i user, $i = 1, 2, \ldots, l$, can be calculated as follow.

$$\lambda_i^{'} = \sum_{m=0}^{n-i} P_m \times P(Max \geq i \mid S_m) \times \lambda_i \tag{5.1.3}$$

where P_m is the probability of state m, and $P(Max \geq i \mid S_m)$ is the acceptance probability for type i user at the state of m busy channels, which can be calculated from our proposed method in Chapter 3.

When we have the effective arrival rate, with Little's formula we can calculate the average number of users for each type of users in the system. The Little's formula states that the average number of customers in a steady-state system is equal to the average effective arrival rate multiplied by the average time that a customer spends in the system [51]. Therefore, the average number for each

type of users is:

$$E[N_i] = \lambda_i^{'} \times E[T] \tag{5.1.4}$$

consequently, the percentage q_i for each type of users is:

$$q_i = \frac{E[N_i]}{\sum_{j=1}^{l} E\left[N_j\right]} \tag{5.1.5}$$

Now to calculate the departure rates from each state of busy channels, we should calculate the average number for each type of users at each state. A random user in the system takes M channels, where

$$M = \sum_{i=1}^{l} i \times q_i \tag{5.1.6}$$

and at each state of m busy channels, the number of users on average is $\frac{m}{M}$. Therefore, the number of type i users at state of m busy channels is

$$N_i^m = q_i \times \frac{m}{M} \tag{5.1.7}$$

Now, we can calculate the departure rates from state m to state $m^{'} = m - i$ as

$$D_{mm'} = N_i^m \times \frac{1}{E\left[T\right]} \tag{5.1.8}$$

For a better understanding, we will explain the concept of the process in another way through an example. To have the one-dimensional approximation, we need to merge all the states that have the total of m busy channels in a single state of m busy channels. If we randomly pick up an admitted user from the system, its type is i with the probability of q_i. However, the exact number of type i users is not clear at state of m busy channels. This number is an integer random variable which is shown by X_i^m as follows.

We provide an example to make it clear. Assume, the users take either one, two, or three chan-

nels, then at state of m busy channels we have,

$$q_1 + q_2 + q_3 = 1$$

$$X_1^m + 2X_2^m + 3X_3^m = m$$

And the joint probability mass function is,

$$P(X_1^m = x_1, X_2^m = x_2, X_3^m = x_3)$$

Now, if we want to merge all the states that have $m = 5$ busy channels in a single state, these are all the possible options:

$$\begin{cases} X_1^5 = 5, X_2^5 = 0, X_3^5 = 0 \\ X_1^5 = 3, X_2^5 = 1, X_3^5 = 0 \\ X_1^5 = 1, X_2^5 = 2, X_3^5 = 0 \\ X_1^5 = 2, X_2^5 = 0, X_3^5 = 1 \\ X_1^5 = 0, X_2^5 = 1, X_3^5 = 1 \end{cases}$$

The probability for each of the above possibilities can be calculated as presented in Chapter 4, where basically, each of them is one of the states in the original multidimensional Markov chain.

For the one-dimensional approximation we need to know the departure rates. For example, for departure from $m = 5$ to $m = 4$, only Type1-users are included and we need to know the number of them. As shown above, all the possibilities for X_1^5 are $\{0, 1, 2, 3, 5\}$. Hence, the departure rates could be μ, 2μ, 3μ, or 5μ. Therefore, the departure rate from $m = 5$ to $m = 4$, which is shown by D_{54}, is:

$$D_{54} = \mu P(X_1^5 = 1, X_2^5 = 2, X_3^5 = 0) + 2\mu P(X_1^5 = 2, X_2^5 = 0, X_3^5 = 1)$$
$$+3\mu P(X_1^5 = 3, X_2^5 = 1, X_3^5 = 0) + 5\mu P(X_1^5 = 5, X_2^5 = 0, X_3^5 = 0)$$

more specifically,

$$D_{54} = \sum x_1 \mu P(X_1^5 = x_1, X_2^5 = x_2, X_3^5 = x_3) = \mu E[X_1^5]$$

In general, if m is the number of busy channels in a total of n channels where there are l types of users available, the departure rate for type i user, $i = 1, 2, \ldots, l$, from state m to state $m^{'} = m - i$ is

$$D_{mm'} = \sum x_i \mu P(X_1^m = x_1, X_2^m = x_2, \ldots, X_l^m = x_l) = \mu E[X_i^m]$$

As a result, we do not need to find the joint probability mass function to calculate the departure rate of type i user at state of m busy channels. We just need the expected value for the number of type i user, which is $E[X_i^m]$. To reach that, at state of m busy channels we have

$$E[X_1^m] + 2E[X_2^m] + \ldots + lE[X_l^m] = m$$

If X^m is the total number of users at state of m busy channels,

$$X^m = X_1^m + X_2^m + \ldots + X_l^m$$

therefore,

$$E[X^m] = E[X_1^m] + E[X_2^m] \ldots + E[X_l^m]$$

and by knowing the probability for each type of users,

$$E[X_i^m] = q_i \times E[X^m]$$

then,

$$(q_1 + 2q_2 + ... + lq_l)E[X^m] = m$$

So, if we define $M = q_1 + 2q_2 + ... + lq_l = \sum_{i=1}^{l} i \times q_i$, therefore,

$$E[X^m] = \frac{m}{M}$$

which means any admitted user in the system takes M channels in average. The expected number of type i users at state of m busy channels is

$$E[X_i^m] = q_i \times E[X^m] = q_i \times \frac{m}{M}$$

We actually use the expected value instead of each random variable to make our approximation. We basically merged all the states that have m busy channels into a single state of m busy channels.

5.2 Numerical and Simulation Results

To clarify the procedure, we provide an example where there are only two kinds of users available. Fig. 5.2.1 is the one-dimensional approximation of the two-dimensional Markov chain presented in Fig. 4.2.1, where users either need one or two channels.

In Fig. 5.2.1, for each type of users $\lambda_1 = \lambda_2 = \lambda/2$. To calculate the effective arrival rates, it is clear that except the last state where all the channels are busy, any arriving Type1-user gets the service in any other states. Therefore, the effective arrival rate for Type1-users, $\lambda_1^{'}$, can be calculated by (5.1.3) as below.

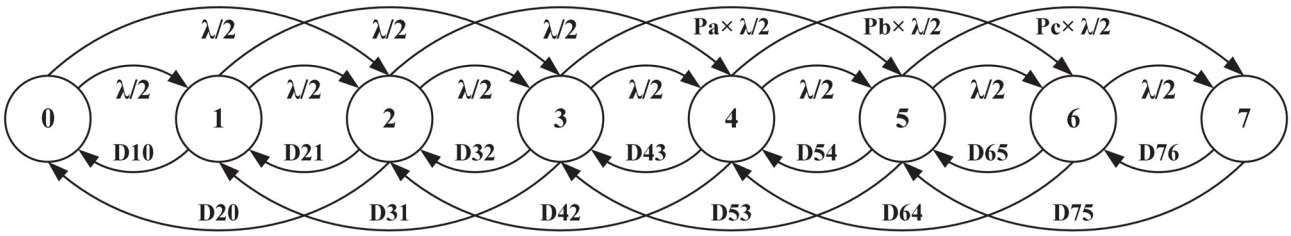

Figure 5.2.1: Approximation of the two-dimensional case: one-dimensional Markov chain for users with consecutive channels.

$$\lambda_1^{'} = (P_0 + P_1 + P_2 + P_3 + P_4 + P_5 + P_6) \times \lambda_1 \tag{5.2.1}$$

For Type2-users, the acceptance probability is one until the state of three busy channels, but it is different for the other states as shown in Fig. 5.2.1. So, the effective arrival rate for Type2-users, $\lambda_2^{'}$, is calculated by (5.1.3) as follow.

$$\lambda_2^{'} = (P_0 + P_1 + P_2 + P_3 \times P_a + P_4 \times P_b + P_5 \times P_c) \times \lambda_2 \tag{5.2.2}$$

If μ is the service rate for all types of users, from (5.1.4) we have:

$$E[N_1] = \lambda_1^{'} \times E[T] = \lambda_1^{'} \times \frac{1}{\mu} \tag{5.2.3}$$

$$E[N_2] = \lambda_2^{'} \times E[T] = \lambda_2^{'} \times \frac{1}{\mu} \tag{5.2.4}$$

Now, when we know the average number for each type of users, we can achieve their percentages with (5.1.5) as follows:

$$q_1 = \frac{E[N_1]}{E[N_1] + E[N_2]} \tag{5.2.5}$$

$$q_2 = \frac{E[N_2]}{E[N_1]+E[N_2]} \tag{5.2.6}$$

Hence, based on (5.1.6), a random user takes $M = q_1 + 2q_2$ channels. And as an example, at state of $m = 5$, the number of Type1-users and Type2-users can be calculated by (5.1.7). And finally with (5.1.8), the departure rates from state $m = 5$ are:

$$\begin{aligned} D_{53} &= \tfrac{5q_2}{M}\mu \\ D_{54} &= \tfrac{5q_1}{M}\mu \end{aligned} \tag{5.2.7}$$

where D_{53} is the departure rate from state $m = 5$ for Type2-users, which clearly goes to state $m = 3$. In the same way, D_{54} is the departure rate from state $m = 5$ for Type1-users, which goes to state $m = 4$.

The whole departure rates at Fig. 5.2.1 are as follows. $D_{10} = \frac{q_1}{q_1}\mu = \mu$, $D_{21} = \frac{2q_1}{q_1+2q_2}\mu$, $D_{32} = \frac{3q_1}{q_1+2q_2}\mu$, $D_{43} = \frac{4q_1}{q_1+2q_2}\mu$, $D_{54} = \frac{5q_1}{q_1+2q_2}\mu$, $D_{65} = \frac{6q_1}{q_1+2q_2}\mu$, $D_{76} = \frac{7q_1}{q_1+2q_2}\mu$. Also, $D_{20} = \frac{2q_2}{q_1+2q_2}\mu$, $D_{31} = \frac{3q_2}{q_1+2q_2}\mu$, $D_{42} = \frac{4q_2}{q_1+2q_2}\mu$, $D_{53} = \frac{5q_2}{q_1+2q_2}\mu$, $D_{64} = \frac{6q_2}{q_1+2q_2}\mu$, and $D_{75} = \frac{7q_2}{q_1+2q_2}\mu$.

To have the numerical results for the steady-state probabilities, we do an iterative process to reach the convergence to have the final result. We assume uniform distribution to start the process. The results are shown in Table 5.1 for a network of $N = 7$, $\lambda = 2$, and $\mu = 2.5$. Moreover, Fig. 5.2.2 compares the results of the steady-state probabilities between two-dimensional Markov chain and its one-dimensional approximation.

In addition, the results for the probability distribution of Max with the approximate approach in the case of $N = 7$, $\lambda = 2$, and $\mu = 2.5$ are shown in Table 5.2, which are compared with the results of two-dimensional Markov chain in Fig. 5.2.3.

From Table 5.2, the expected value of maximum is 5.0487 which is very similar to the numerical result of the related two-dimensional Markov chain presented in Chapter 4, which was 5.0730.

As an another example, for a network of $N = 7$, $\lambda = 3$, $\mu = 2.5$, the results for the approxi-

Table 5.1: Approximate steady-state probabilities for two-dimensional Markov chain: Consecutive channels for users, $N = 7$, $\lambda = 2$, $\mu = 2.5$.

Iteration	P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7
1	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250
2	0.4245	0.2269	0.1920	0.0909	0.0441	0.0163	0.0046	0.0007
3	0.4315	0.2183	0.1923	0.0905	0.0450	0.0168	0.0049	0.0008
4	0.4314	0.2184	0.1923	0.0905	0.0450	0.0168	0.0049	0.0008
5	0.4314	0.2184	0.1923	0.0905	0.0450	0.0168	0.0049	0.0008

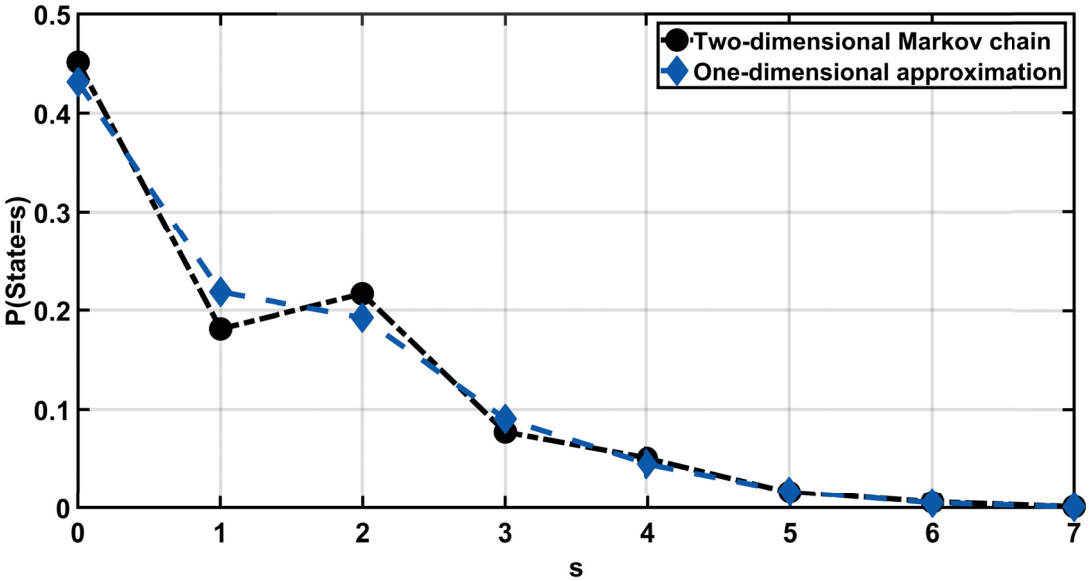

Figure 5.2.2: Comparison between steady-state probabilities: $N = 7$, $\lambda = 2$, $\mu = 2.5$.

Table 5.2: Approximate probability distribution of Max for two-dimensional Markov chain: Consecutive channels for users, $N = 7$, $\lambda = 2$, $\mu = 2.5$.

$P(Max = x)$	$P(Max = 0)$	$P(Max = 1)$	$P(Max = 2)$	$P(Max = 3)$	$P(Max = 4)$	$P(Max = 5)$	$P(Max = 6)$	$P(Max = 7)$
Approximation	0.0008	0.0323	0.1045	0.1510	0.1277	0.0899	0.0624	0.4314

mate steady-state probabilities are shown in Table 5.3. Also, Fig. 5.2.4 compares the results of the steady-state probabilities between two-dimensional Markov chain and its one-dimensional approximation.

As explained before, by knowing the steady-state probabilities, we can calculate the probability distribution for maximum number of consecutive idle channels. The results for the probability

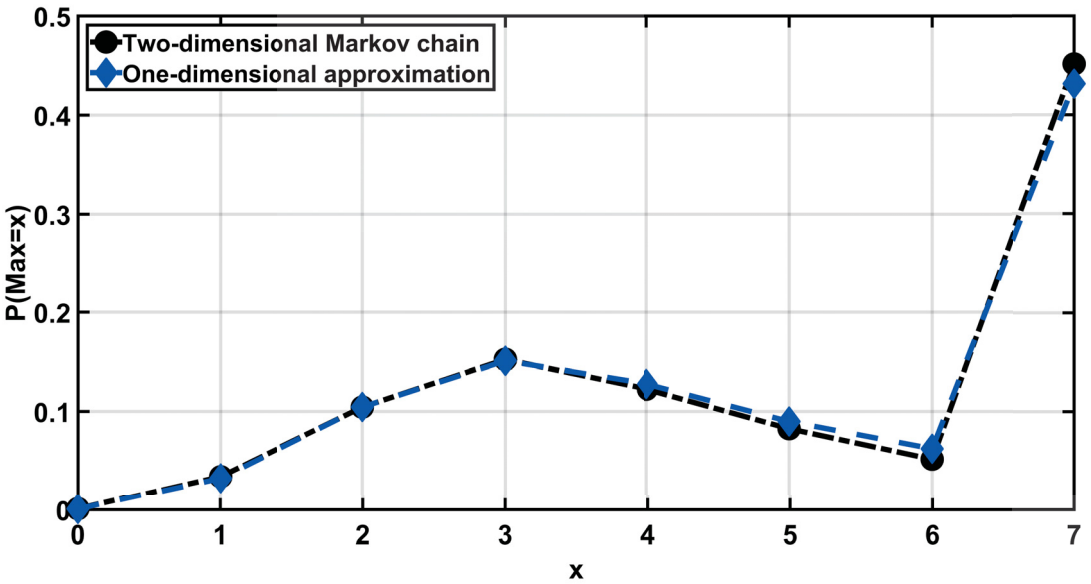

Figure 5.2.3: Comparison between probability distributions of Max: $N = 7$, $\lambda = 2$, $\mu = 2.5$.

Table 5.3: Approximate steady-state probabilities for two-dimensional Markov chain: Consecutive channels for users, $N = 7$, $\lambda = 3$, $\mu = 2.5$.

Iteration	P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7
1	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250
2	0.2865	0.2156	0.2185	0.1385	0.0835	0.0401	0.0142	0.0031
3	0.2927	0.2098	0.2175	0.1372	0.0839	0.0407	0.0148	0.0034
4	0.2927	0.2098	0.2175	0.1372	0.0839	0.0407	0.0148	0.0034

distribution of $P(Max = x)$ are shown in Table 5.4 and are compared with the results of the related two-dimensional Markov chain in Fig. 5.2.5.

Table 5.4: Approximate probability distribution of Max for two-dimensional Markov chain: Consecutive channels for users, $N = 7$, $\lambda = 3$, $\mu = 2.5$.

$P(Max = x)$	$P(Max = 0)$	$P(Max = 1)$	$P(Max = 2)$	$P(Max = 3)$	$P(Max = 4)$	$P(Max = 5)$	$P(Max = 6)$	$P(Max = 7)$
Approximation	0.0034	0.0718	0.1612	0.1822	0.1378	0.0910	0.0600	0.2927

At the end, the load of the SU or the expected value of Max for one-dimensional approximation is 4.3556. Again, it can be seen that this result is very similar to the numerical result of the related two-dimensional Markov chain presented in Chapter 4, which was 4.3741.

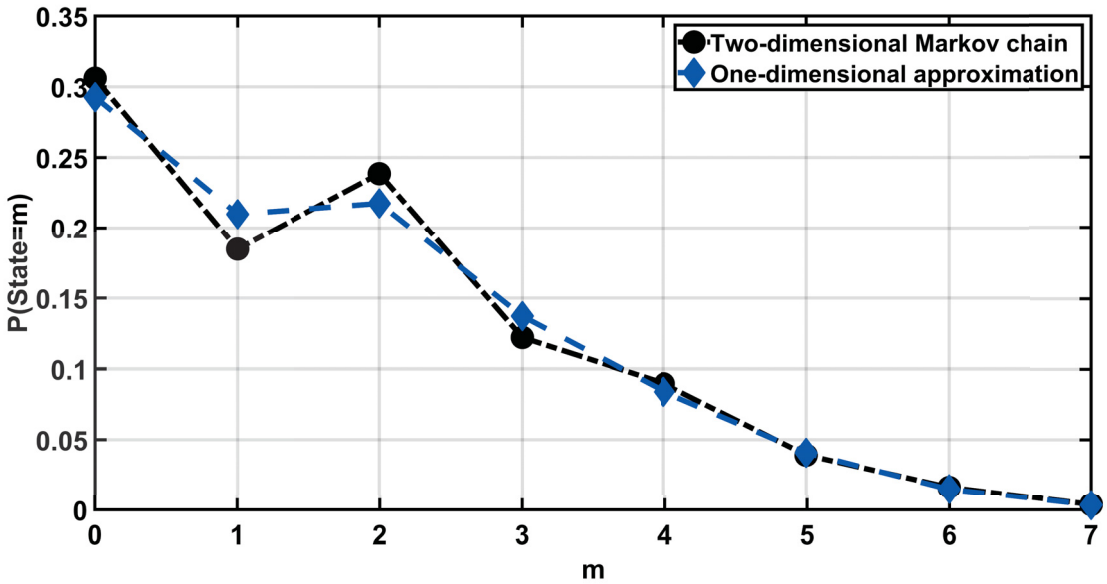

Figure 5.2.4: Comparison between steady-state probabilities: $N = 7$, $\lambda = 3$, $\mu = 2.5$.

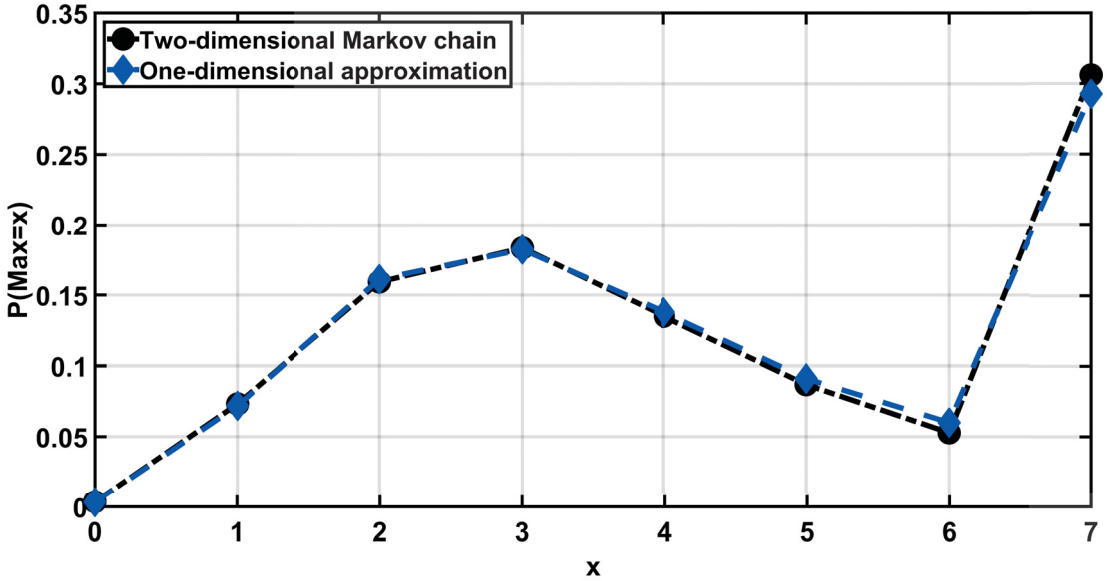

Figure 5.2.5: Comparison between probability distributions of Max: $N = 7$, $\lambda = 3$, $\mu = 2.5$.

Moreover, to show more complicated results, Fig. 5.2.6 depicts the approximation process for three-dimensional case, as previously discussed in Fig. 4.2.6.

As explained earlier, $M = q_1 + 2q_2 + 3q_3$ and for example, the departure rates from state $m = 5$

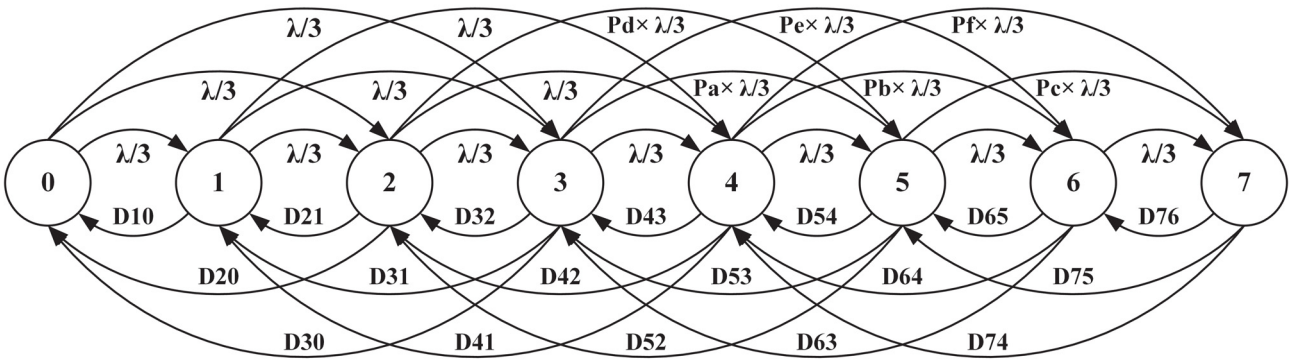

Figure 5.2.6: Approximation of the three-dimensional case: one-dimensional Markov chain for users with consecutive channels.

busy channels can be calculated as:

$$D_{52} = \frac{5q_3}{M}\mu$$

$$D_{53} = \frac{5q_2}{M}\mu \tag{5.2.8}$$

$$D_{54} = \frac{5q_1}{M}\mu$$

where D_{52} is for Type3-users, D_{53} is for Type2-users, and D_{54} is for Type1-users. The whole departure rates at Fig. 5.2.6 are as follows: $D_{10} = \frac{q_1}{q_1}\mu = \mu$, $D_{21} = \frac{2q_1}{q_1+2q_2}\mu$, $D_{32} = \frac{3q_1}{q_1+2q_2+3q_3}\mu$, $D_{43} = \frac{4q_1}{q_1+2q_2+3q_3}\mu$, $D_{54} = \frac{5q_1}{q_1+2q_2+3q_3}\mu$, $D_{65} = \frac{6q_1}{q_1+2q_2+3q_3}\mu$, $D_{76} = \frac{7q_1}{q_1+2q_2+3q_3}\mu$. Also, $D_{20} = \frac{2q_2}{q_1+2q_2}\mu$, $D_{31} = \frac{3q_2}{q_1+2q_2+3q_3}\mu$, $D_{42} = \frac{4q_2}{q_1+2q_2+3q_3}\mu$, $D_{53} = \frac{5q_2}{q_1+2q_2+3q_3}\mu$, $D_{64} = \frac{6q_2}{q_1+2q_2+3q_3}\mu$, $D_{75} = \frac{7q_2}{q_1+2q_2+3q_3}\mu$. Finally, $D_{30} = \frac{3q_3}{q_1+2q_2+3q_3}\mu$, $D_{41} = \frac{4q_3}{q_1+2q_2+3q_3}\mu$, $D_{52} = \frac{5q_3}{q_1+2q_2+3q_3}\mu$, $D_{63} = \frac{6q_3}{q_1+2q_2+3q_3}\mu$, and $D_{74} = \frac{7q_3}{q_1+2q_2+3q_3}\mu$.

To have the numerical results for the steady-state probabilities, as explained earlier, we do an iterative process to reach the final result. We start by assuming uniform distribution as the initial probability distribution. The results are shown in Table 5.5 for a network of $N = 7$, $\lambda = 2$, and $\mu = 2.5$.

Table 5.5: Approximate steady-state probabilities for three-dimensional Markov chain: Consecutive channels for users, $N = 7$, $\lambda = 2$, $\mu = 2.5$.

Iteration	P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7
1	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250
2	0.4118	0.1898	0.1526	0.1319	0.0668	0.0338	0.0113	0.0021
3	0.4224	0.1812	0.1472	0.1329	0.0668	0.0350	0.0122	0.0023
4	0.4223	0.1813	0.1472	0.1329	0.0668	0.0350	0.0122	0.0023
5	0.4223	0.1813	0.1472	0.1329	0.0668	0.0350	0.0122	0.0023

Fig. 5.2.7 compares the results of the steady-state probabilities between three-dimensional Markov chain and its one-dimensional approximation.

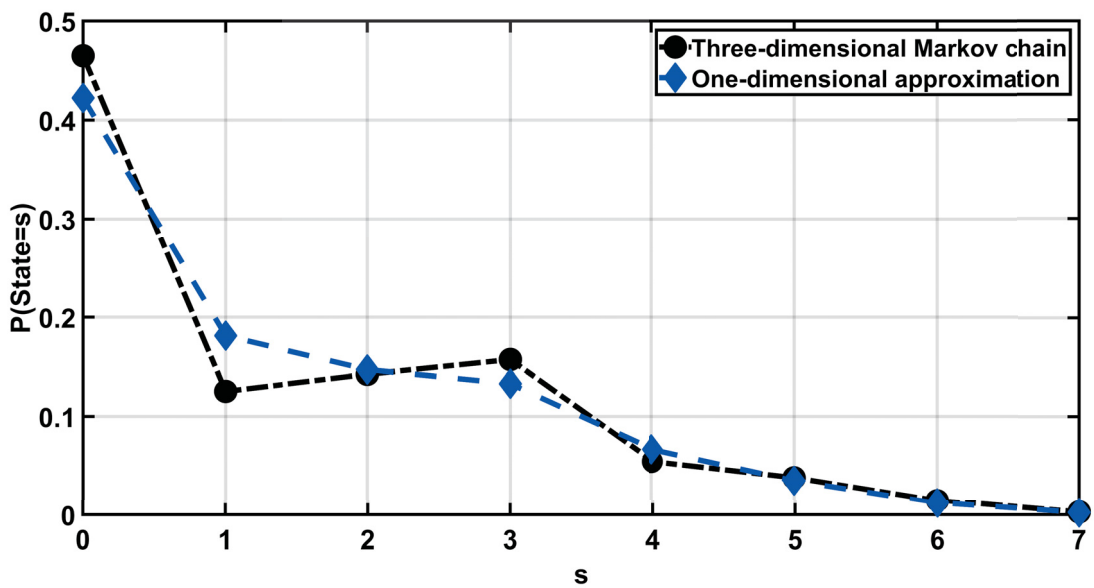

Figure 5.2.7: Comparison between steady-state probabilities: $N = 7$, $\lambda = 2$, $\mu = 2.5$.

Moreover, the results for the probability distribution of Max with the approximate approach in the case of $N = 7$, $\lambda = 2$, and $\mu = 2.5$ are shown in Table 5.6, which are compared with the results of the related three-dimensional Markov chain in Fig. 5.2.8.

From Table 5.6, the expected value of maximum is 4.8345. It can be seen that this result is very similar to the numerical result of the related three-dimensional Markov chain presented in Chapter 4, which was 4.8981.

Table 5.6: Approximate probability distribution of Max for two-dimensional Markov chain: Consecutive channels for users, $N = 7$, $\lambda = 2$, $\mu = 2.5$.

$P(Max = x)$	$P(Max = 0)$	$P(Max = 1)$	$P(Max = 2)$	$P(Max = 3)$	$P(Max = 4)$	$P(Max = 5)$	$P(Max = 6)$	$P(Max = 7)$
Approximation	0.0023	0.0601	0.1375	0.1441	0.1090	0.0728	0.0518	0.4223

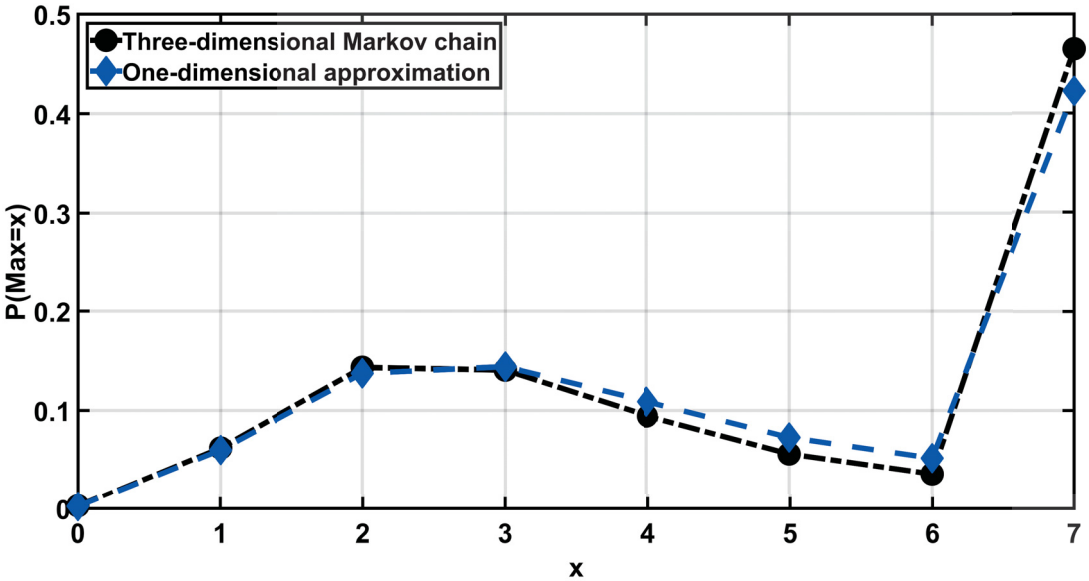

Figure 5.2.8: Comparison between probability distributions of Max: $N = 7$, $\lambda = 2$, $\mu = 2.5$.

Now, for a higher traffic situation, the results for the steady-state probabilities with the approximation approach in the case of $N = 7$, $\lambda = 3$, and $\mu = 2.5$ are shown in Table 5.7. In addition, Fig. 5.2.9 compares the results of the steady-state probabilities between one-dimensional approximation and its related three-dimensional Markov chain.

Table 5.7: Approximate steady-state probabilities for three-dimensional Markov chain: Consecutive channels for users, $N = 7$, $\lambda = 3$, $\mu = 2.5$.

Iteration	P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7
1	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250
2	0.2850	0.1805	0.1684	0.1649	0.1065	0.0633	0.0255	0.0059
3	0.2930	0.1754	0.1644	0.1643	0.1057	0.0642	0.0266	0.0065
4	0.2930	0.1754	0.1644	0.1643	0.1057	0.0642	0.0266	0.0065

Moreover, the results for the probability distribution of Max with the approximate approach in

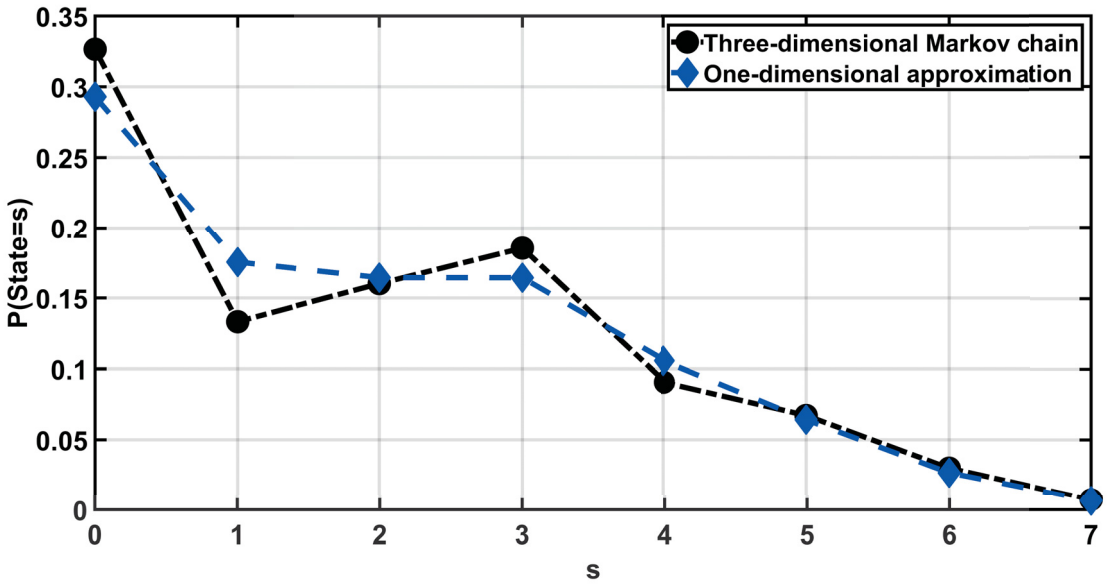

Figure 5.2.9: Comparison between steady-state probabilities: $N = 7$, $\lambda = 3$, $\mu = 2.5$.

the case of $N = 7$, $\lambda = 3$, and $\mu = 2.5$ are shown in Table 5.8, which are compared with the results of the related three-dimensional Markov chain.

Table 5.8: Approximate maximum-size probabilities, $N = 7$, $\lambda = 3$, $\mu = 2.5$, Maximum batch-size = 3

$P(Max = x)$	$P(Max = 0)$	$P(Max = 1)$	$P(Max = 2)$	$P(Max = 3)$	$P(Max = 4)$	$P(Max = 5)$	$P(Max = 6)$	$P(Max = 7)$
Approximation	0.0065	0.1073	0.1867	0.1669	0.1158	0.0736	0.0501	0.2930

From Table 5.8, the expected value of maximum is 4.1645. It can be seen that this result is very similar to the numerical result of the related three-dimensional Markov chain presented in Chapter 4, which was 4.2180.

At the end, it is very complicated to have the results from the multidimensional Markov chain in higher number of channels and multiple types of users. But we still can use the one-dimensional approximation to estimate the load of the SU.

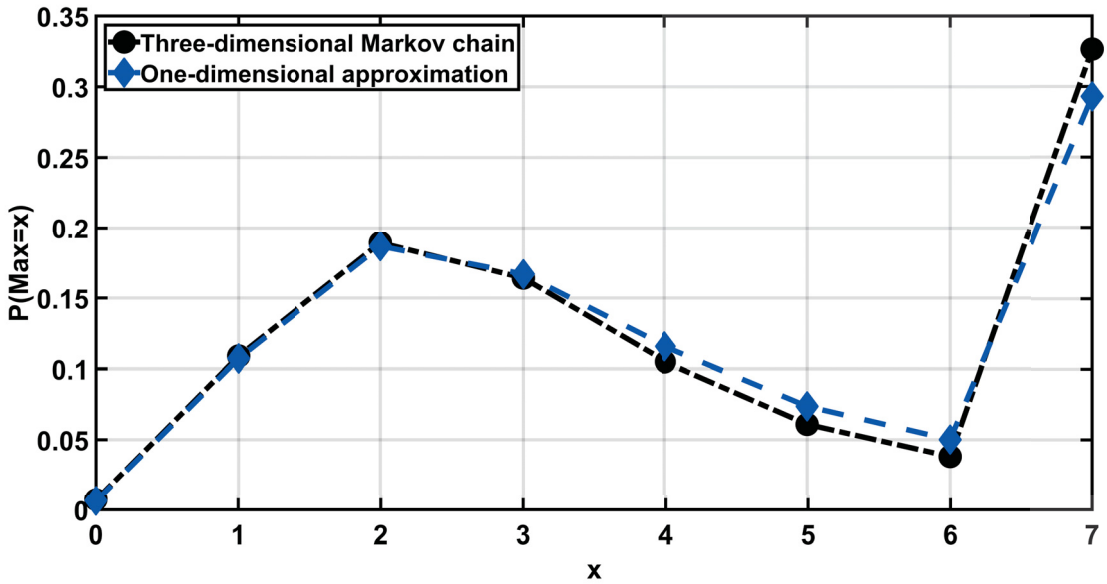

Figure 5.2.10: Comparison between probability distributions of Max: $N = 7$, $\lambda = 2$, $\mu = 2.5$.

5.3 Summary

This chapter proposed an approximate one-dimensional Markov chain to facilitate the load estimation of the secondary user in a CR network of multichannel PUs. In one-dimensional approximation, we do not know exactly how many of each type of users are available in any state of busy channels, all we now is the number of busy channels. Although we lost some information, the proposed procedure avoids the precise calculation of the complicated multidimensional Markov chain. The performance of the proposed model was validated by numerical and simulation results.

6 Conclusions and Future Work

6.1 Conclusions

Providing extra spectrum resources to meet the endless demand in wireless communications is a very challenging problem nowadays, specially while some of the available resources are underutilized. The bandwidth scarcity and the inefficient usage of spectrum resources provide motivation to look for some opportunistic spectrum allocation, which leads to proposing the cognitive radio (CR) technology, as an emerging solution. Allowing unlicensed users to cognitively operate in a licensed spectrum is a promising approach to improve the utilization of the spectrum resources in wireless networks.

In this thesis, we have done a major analysis in the CR networks with dynamic spectrum allocation, that has not been undertaken before or is not available in the open literature. We believe that the results presented here significantly advances the state-of-the-art in the CR network field.

In Chapter 3, we investigated the spectrum allocation for secondary users in a network of single-channel primary users. In the considered network, each primary user takes only one channel and secondary user gets the largest available spectrum hole. The contribution was the modeling and performance analysis of the existing conventional method, which selects the maximum consecutive idle channels for spectrum allocation. The objective of this part was to give the service provider an insight about the performance of secondary users for different numbers of total channels in primary networks. The main contribution was the calculation of the conditional probability of

having maximum x consecutive idle channels under the condition of n total and m busy channels. If we know the probability distribution of busy channels taken by primary users, this conditional probability provides the number of consecutive idle channels the secondary user can have. In other words, the proposed mathematical model calculates the probability distribution for the number of channels in the largest available spectrum hole. The theoretical model works for any given number of total channels in the licensed frequency band. Finally, at the end of Chapter 3, the performance of the proposed model was validated by numerical and simulation results.

In Chapter 4, we studied the spectrum allocation for a secondary user in the network of multi-channel primary users. Each primary user in the considered network takes a random number of channels, while the secondary user gets the largest available spectrum hole. For the performance analysis, we needed to find the probability distribution of busy channels taken by primary users, before determination of the length of the largest available spectrum hole under the condition of primary users taking different channels. The calculation of the conditional probability of having maximum x consecutive idle channels under the condition of n total and m busy channels in Chapter 3, is still approximately valid for the scenario of Chapter 4, especially in low-traffic networks. As such, the proposed mathematical model calculated the load of the secondary user, where the main contribution was the derivation of the probability distribution of busy channels taken by primary users. The solution scenario was based on the multidimensional Markov chain, with numerical and simulation results verifying the accuracy of the proposed model.

In Chapter 5, an approximate one-dimensional Markov chain was proposed to avoid the precise calculation of the complicated multidimensional Markov chain, which is to facilitate the performance estimation of the secondary user in cognitive radio networks. Although we lost some information, the procedure has absolutely decreased the complexity of the solution. The main concern in Chapter 5 was to find the departure rates from each state of busy channels, because in multidimensional scenario, we know exactly how many of each type of users are available in any state of busy channels. But in one-dimensional approximation, all we now is the number of busy channels,

and we do not know the number of users. Therefore, we calculated the probability for each type of users in a given m busy channels as the main challenge, which helped us to find the departure rates from each state. At the end, the performance of the proposed model was validated by numerical and simulation results.

6.2 Future Work

In this thesis, we have provided two algorithms for the case of multiple secondary users in the CR network of single-channel primary users. It is possible to have another algorithm which is more accurate but complicated. The total number of channels can be divided in three parts, busy with PUs, busy with SUs, and idle ones. The main challenge is to find the probability distribution of SUs, where the probability distribution of PUs is known.

We can also extend our investigation to the case of multiple secondary users in a CR network of multichannel primary users. The procedure might be similar to the algorithms that we have provided for the case of multiple secondary users in a CR network of single-channel primary users.

In the case of multichannel primary users, we have studied two separate cases, consecutive and non-consecutive channels. As another work we can investigate the case when primary users are allowed to have both options together. A practical assumption is that primary users always get the consecutive channels unless they have to take non-consecutive channels to avoid having no services.

Furthermore, in the proposed one-dimensional Markov chain approximation, we have calculated the percentage for each type of users in the whole system. To make the one-dimensional approximation more accurate, the percentage for each type of users can be calculated for each specific state of busy channels.

Bibliography

[1] J. Mitola and G. Q. Maguire, "Cognitive radio: making software radios more personal," *Personal Communications, IEEE*, vol. 6, no. 4, pp. 13–18, Aug 1999.

[2] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *Selected Areas in Communications, IEEE Journal on*, vol. 23, no. 2, pp. 201–220, Feb 2005.

[3] G. I. Tsiropoulos, O. A. Dobre, M. H. Ahmed, and K. E. Baddour, "Radio resource allocation techniques for efficient spectrum access in cognitive radio networks," *IEEE Communications Surveys Tutorials*, vol. 18, no. 1, pp. 824–847, Firstquarter 2016.

[4] G. Ozcan, M. C. Gursoy, and J. Tang, "Spectral and energy efficiency in cognitive radio systems with unslotted primary users and sensing uncertainty," *IEEE Transactions on Communications*, vol. 65, no. 10, pp. 4138–4151, Oct 2017.

[5] T. Yucek and H. Arslan, "A survey of spectrum sensing algorithms for cognitive radio applications," *IEEE Communications Surveys Tutorials*, vol. 11, no. 1, pp. 116–130, First 2009.

[6] I. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty, "A survey on spectrum management in cognitive radio networks," *Communications Magazine, IEEE*, vol. 46, no. 4, pp. 40–48, April 2008.

[7] Y. Zeng, Y.-C. Liang, A. T. Hoang, and R. Zhang, "A review on spectrum sensing for cognitive

radio: Challenges and solutions," *EURASIP J. Adv. Signal Process*, vol. 2010, pp. 2:2–2:2, Jan. 2010.

[8] J. Wang, M. Ghosh, and K. Challapali, "Emerging cognitive radio applications: A survey," *IEEE Communications Magazine*, vol. 49, no. 3, pp. 74–81, March 2011.

[9] Y. Saleem and M. H. Rehmani, "Primary radio user activity models for cognitive radio networks: A survey," *Journal of Network and Computer Applications*, vol. 43, pp. 1 – 16, 2014.

[10] D. Willkomm, S. Machiraju, J. Bolot, and A. Wolisz, "Primary user behavior in cellular networks and implications for dynamic spectrum access," *Communications Magazine, IEEE*, vol. 47, no. 3, pp. 88–95, March 2009.

[11] S. MacDonald, D. C. Popescu, and O. Popescu, "Analyzing the performance of spectrum sensing in cognitive radio systems with dynamic pu activity," *IEEE Communications Letters*, vol. 21, no. 9, pp. 2037–2040, Sept 2017.

[12] T. W. Ban, W. Choi, B. C. Jung, and D. K. Sung, "Multi-user diversity in a spectrum sharing system," *IEEE Transactions on Wireless Communications*, vol. 8, no. 1, pp. 102–106, Jan 2009.

[13] L. M. Lopez-Ramos, A. G. Marques, and J. Ramos, "Jointly optimal sensing and resource allocation for multiuser interweave cognitive radios," *IEEE Transactions on Wireless Communications*, vol. 13, no. 11, pp. 5954–5967, Nov 2014.

[14] N. Devroye, P. Mitran, and V. Tarokh, "Achievable rates in cognitive radio channels," *IEEE Transactions on Information Theory*, vol. 52, no. 5, pp. 1813–1827, May 2006.

[15] M. E. Tanab and W. Hamouda, "Resource allocation for underlay cognitive radio networks: A survey," *IEEE Communications Surveys Tutorials*, vol. 19, no. 2, pp. 1249–1276, Secondquarter 2017.

[16] A. Jovicic and P. Viswanath, “Cognitive radio: An information-theoretic perspective,” *IEEE Transactions on Information Theory*, vol. 55, no. 9, pp. 3945–3958, Sept 2009.

[17] W. Liang, S. X. Ng, and L. Hanzo, “Cooperative overlay spectrum access in cognitive radio networks,” *IEEE Communications Surveys Tutorials*, vol. 19, no. 3, pp. 1924–1944, thirdquarter 2017.

[18] M. T. Masonta, M. Mzyece, and N. Ntlatlapa, “Spectrum decision in cognitive radio networks: A survey,” *IEEE Communications Surveys Tutorials*, vol. 15, no. 3, pp. 1088–1107, Third 2013.

[19] W. Lee, “Resource allocation for multi-channel underlay cognitive radio network based on deep neural network,” *IEEE Communications Letters*, vol. 22, no. 9, pp. 1942–1945, Sept 2018.

[20] J. Park, C. Jang, and J. H. Lee, “Outage analysis of underlay cognitive radio networks with multihop primary transmission,” *IEEE Communications Letters*, vol. 20, no. 4, pp. 800–803, April 2016.

[21] L. Sboui, Z. Rezki, and M. Alouini, “Energy-efficient power allocation for underlay cognitive radio systems,” *IEEE Transactions on Cognitive Communications and Networking*, vol. 1, no. 3, pp. 273–283, Sept 2015.

[22] F. Mehmeti and T. Spyropoulos, “Performance analysis, comparison, and optimization of interweave and underlay spectrum access in cognitive radio networks,” *IEEE Transactions on Vehicular Technology*, vol. 67, no. 8, pp. 7143–7157, Aug 2018.

[23] M. Usman and I. Koo, “Access strategy for hybrid underlay-overlay cognitive radios with energy harvesting,” *IEEE Sensors Journal*, vol. 14, no. 9, pp. 3164–3173, Sept 2014.

[24] T. M. C. Chu and H. Zepernick, "Optimal power allocation for hybrid cognitive cooperative radio networks with imperfect spectrum sensing," *IEEE Access*, vol. 6, pp. 10 365–10 380, 2018.

[25] ——, "Performance optimization for hybrid two-way cognitive cooperative radio networks with imperfect spectrum sensing," *IEEE Access*, vol. 6, pp. 70 582–70 596, 2018.

[26] T. M. C. Chu, H. Phan, and H. Zepernick, "Hybrid interweave-underlay spectrum access for cognitive cooperative radio networks," *IEEE Transactions on Communications*, vol. 62, no. 7, pp. 2183–2197, July 2014.

[27] A. Ali and W. Hamouda, "Advances on spectrum sensing for cognitive radio networks: Theory and applications," *IEEE Communications Surveys Tutorials*, vol. 19, no. 2, pp. 1277–1304, Secondquarter 2017.

[28] B. Wang and K. Liu, "Advances in cognitive radio networks: A survey," *Selected Topics in Signal Processing, IEEE Journal of*, vol. 5, no. 1, pp. 5–23, Feb 2011.

[29] H. Sun, A. Nallanathan, C. Wang, and Y. Chen, "Wideband spectrum sensing for cognitive radio networks: a survey," *IEEE Wireless Communications*, vol. 20, no. 2, pp. 74–81, April 2013.

[30] E. Z. Tragos, S. Zeadally, A. G. Fragkiadakis, and V. A. Siris, "Spectrum assignment in cognitive radio networks: A comprehensive survey," *IEEE Communications Surveys Tutorials*, vol. 15, no. 3, pp. 1108–1135, Third 2013.

[31] J. Wu, Y. Dai, and Y. Zhao, "Effective channel assignments in cognitive radio networks," *Computer Communications*, vol. 36, no. 4, pp. 411 – 420, 2013.

[32] S. Sengottuvelan, J. Ansari, P. Mähönen, T. G. Venkatesh, and M. Petrova, "Channel selection

algorithm for cognitive radio networks with heavy-tailed idle times," *IEEE Transactions on Mobile Computing*, vol. 16, no. 5, pp. 1258–1271, May 2017.

[33] M. Hoyhtya, S. Pollin, and A. Mammela, "Performance improvement with predictive channel selection for cognitive radios," in *Cognitive Radio and Advanced Spectrum Management, 2008. CogART 2008. First International Workshop on*, Feb 2008, pp. 1–5.

[34] Y. S. Chen, C. H. Cho, I. You, and H. C. Chao, "A cross-layer protocol of spectrum mobility and handover in cognitive LTE networks," *Simulation Modelling Practice and Theory*, vol. 19, no. 8, pp. 1723 – 1744, 2011.

[35] G. Ganesan and Y. Li, "Cooperative spectrum sensing in cognitive radio, part i: Two user networks," *Wireless Communications, IEEE Transactions on*, vol. 6, no. 6, pp. 2204–2213, June 2007.

[36] ——, "Cooperative spectrum sensing in cognitive radio, part ii: Multiuser networks," *Wireless Communications, IEEE Transactions on*, vol. 6, no. 6, pp. 2214–2222, June 2007.

[37] Y. Liang, K. Chen, G. Y. Li, and P. Mahonen, "Cognitive radio networking and communications: an overview," *IEEE Transactions on Vehicular Technology*, vol. 60, no. 7, pp. 3386–3407, Sep. 2011.

[38] S. Parvin, F. K. Hussain, O. K. Hussain, S. Han, B. Tian, and E. Chang, "Cognitive radio network security: A survey," *Journal of Network and Computer Applications*, vol. 35, no. 6, pp. 1691 – 1708, 2012.

[39] R. Chen, J.-M. Park, and J. Reed, "Defense against primary user emulation attacks in cognitive radio networks," *Selected Areas in Communications, IEEE Journal on*, vol. 26, no. 1, pp. 25–37, Jan 2008.

[40] H. Li and Z. Han, "Dogfight in spectrum: Combating primary user emulation attacks in cognitive radio systems, part i: Known channel statistics," *IEEE Transactions on Wireless Communications*, vol. 9, no. 11, pp. 3566–3577, November 2010.

[41] ——, "Dogfight in spectrum: Combating primary user emulation attacks in cognitive radio systems, part ii: Unknown channel statistics," *IEEE Transactions on Wireless Communications*, vol. 10, no. 1, pp. 274–283, January 2011.

[42] C. Chen, H. Cheng, and Y. Yao, "Cooperative spectrum sensing in cognitive radio networks in the presence of the primary user emulation attack," *IEEE Transactions on Wireless Communications*, vol. 10, no. 7, pp. 2135–2141, July 2011.

[43] M. Haghighat and S. M. S. Sadough, "Cooperative spectrum sensing for cognitive radio networks in the presence of smart malicious users," *{AEU} - International Journal of Electronics and Communications*, vol. 68, no. 6, pp. 520 – 527, 2014.

[44] R. Yu, Y. Zhang, Y. Liu, S. Gjessing, and M. Guizani, "Securing cognitive radio networks against primary user emulation attacks," *IEEE Network*, vol. 30, no. 6, pp. 62–69, November 2016.

[45] O. Leon, J. Hernandez-Serrano, and M. Soriano, "Cooperative detection of primary user emulation attacks in crns," *Computer Networks*, vol. 56, no. 14, pp. 3374 – 3384, 2012.

[46] F. Hu, B. Chen, and K. Zhu, "Full spectrum sharing in cognitive radio networks toward 5g: A survey," *IEEE Access*, vol. 6, pp. 15 754–15 776, 2018.

[47] S. Khodadadi, D. Qiu, and Y. R. Shayan, "Performance analysis of secondary users in cognitive radio networks with dynamic spectrum allocation," *IEEE Communications Letters*, vol. 22, no. 8, pp. 1684–1687, Aug 2018.

[48] V. Kumar, S. Minz, and V. Kumar, "Performance analysis of cognitive radio networks under spectrum sharing using queuing approach," *Computers & Electrical Engineering*, vol. 52, pp. 390 – 398, 2016.

[49] A. N. Dudin, M. H. Lee, O. Dudina, and S. K. Lee, "Analysis of priority retrial queue with many types of customers and servers reservation as a model of cognitive radio system," *IEEE Transactions on Communications*, vol. 65, no. 1, pp. 186–199, Jan 2017.

[50] M. M. Rashid, M. J. Hossain, E. Hossain, and V. K. Bhargava, "Opportunistic spectrum scheduling for multiuser cognitive radio: a queueing analysis," *IEEE Transactions on Wireless Communications*, vol. 8, no. 10, pp. 5259–5269, October 2009.

[51] D. Gross, J. F. Shortle, J. M. Thompson, and C. M. Harris, *Fundamentals of Queueing Theory*, 4th ed. New York, NY, USA: Wiley-Interscience, 2008.

[52] L. Kleinrock, *Queueing Systems - Volume I: Theory*. Wiley-Interscience, 1975.

[53] F. Paluncic, A. S. Alfa, B. T. Maharaj, and H. M. Tsimba, "Queueing models for cognitive radio networks: A survey," *IEEE Access*, vol. 6, pp. 50 801–50 823, 2018.

[54] L. Akter and B. Natarajan, "Modeling and forecasting secondary user activity considering bulk arrival and bulk departure traffic model," in *2009 IEEE 70th Vehicular Technology Conference Fall*, Sept 2009, pp. 1–5.

[55] J. Wang, A. Huang, L. Cai, and W. Wang, "On the queue dynamics of multiuser multichannel cognitive radio networks," *IEEE Transactions on Vehicular Technology*, vol. 62, no. 3, pp. 1314–1328, March 2013.

[56] N. Tadayon and S. Aissa, "Multi-channel cognitive radio networks: Modeling, analysis and synthesis," *IEEE Journal on Selected Areas in Communications*, vol. 32, no. 11, pp. 2065–2074, November 2014.

[57] N. Tadayon and S. Aïssa, "Modeling and analysis framework for multi-interface multi-channel cognitive radio networks," *IEEE Transactions on Wireless Communications*, vol. 14, no. 2, pp. 935–947, Feb 2015.

[58] A. Abhishek, M. Boon, O. Boxma, R. Núñez-Queija *et al.*, "A single-server queue with batch arrivals and semi-markov services," *Queueing Systems*, vol. 86, 2017.

[59] W. Jiao, M. Sheng, K. Lui, and G. Liu, "Queue performance of cognitive radio networks with general primary user activity model," *IET Communications*, vol. 9, no. 15, pp. 1821–1828, 2015.

[60] M. R. Amini, M. Mahdavi, and M. J. Omidi, "Discrete-time markov chain analysis of energy efficiency in a cr network regarding primary and secondary traffic with primary user returns," *IEEE Access*, vol. 6, pp. 22 305–22 323, 2018.