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Control Strategy Based on Fourier Transformation and Intelligent Optimization for Planar Pendubot

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Abstract

This paper presents a new control strategy based on Fourier transformation and intelligent optimization for a planar Pendubot with a passive second link, which can be treated as a second-order nonholonomic system whose control has been an open and challenging issue. A controller acting within a time corresponding to the frequency of its fundamental harmonic term is designed to realize the system control objective, which is to move the system from its initial position to the target position. By employing Fourier transformation, a general expression of the controller composed of a constant term and harmonic terms is obtained. Next, the constant term is obtained by the angular momentum theorem, and the particle swarm optimization algorithm is employed to obtain the harmonic terms of the controller. A feedback control strategy based on a nonlinear disturbance observer is then applied to overcome the uncertainties/disturbances in the system. Finally, simulation results prove the validity of this control method.

Keywords: Planar Pendubot, Nonholonomic systems, Fourier transformation, Particle swarm optimization algorithm, Nonlinear disturbance observer

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1. Introduction

When control inputs of a mechanical manipulator is less than its degree of freedom, it is considered an underactuated system [5, 10]. For underactuated manipulators, two different types have been considered: the vertical underactuated manipulators [17, 22], and the planar underactuated manipulators [2, 23]. For the former ones, gravity is considered. Thorough studies have been made on such systems and many control methods have been proposed utilizing the fact that its upright equilibrium position satisfies the approximate linearization controllable condition [4, 27, 28].

However, the planar underactuated manipulators are without gravity. Any point on the plane is its equilibrium position, and does not satisfy the approximate linearization controllable condition [21]. As a result, control methods for vertical underactuated manipulators may not work for the planar ones.

Nonholonomic systems are systems whose acceleration constraint or velocity constraint is not integrable [13, 18]. Most planar underactuated manipulators are treated as nonholonomic. The only exception is the two-link planar Acrobot with a passive first joint, which belongs to holonomic systems. [16] utilizes such holonomic characteristic, and has achieved its control objective based on the constraint between angles of the two links.

For planar underactuated manipulators with a passive first joint and with more than two links, the acceleration constraint can be integrated, but velocity constraint cannot. These are first-order nonholonomic systems. Various of proposals have been made to achieve the control objective of these manipulators effectively. Including order-reduction method [14], continuous state-feedback control [15, 30], etc.

For planar underactuated manipulators with a passive joint which is not the first one, both acceleration constraint and velocity constraint cannot be integrated, these are second-order nonholonomic systems [3, 24, 29]. No angle constraints nor angular velocity constraints can be utilized for them. As a result, it is hard to effectively control this type of nonholonomic manipulators.

[1] proposes a control strategy for planar second-order nonholonomic manipulators based on bi-direction trajectory planning. But it contains three control steps, which is complicated. And it neglects some important factors such as the value of the torque, which should be taken into consideration. A chained
35 form method is proposed to realize the control objective for planar multi-link manipulators with a passive link which is the last one [11]. However, for planar manipulators whose passive link is neither the first nor the last, an effective control strategy has not been found. To achieve their control objectives, a basic step is to first find a valid control strategy for the planar two-link Pendubot,
40 which has a passive second link. A method based on the nilpotent approximation has been suggested to control the planar Pendubot [8]. But this control method is iteratively based, which may take a long time to complete the whole control process. To conclude, finding an alternative control strategy for planar Pendubot is necessary.

45 Nowadays, with the rapid development of the intelligent control technology [12, 20], we can explore the hidden relationship between the control objective and the dynamic behavior of the system by employing intelligent optimization algorithm. Such technology allows us to discover new strategies to achieve the control objective of mechanical manipulators more effectively. Many opti-
50 mization algorithms, including the particle swarm optimization algorithm, the genetic algorithm, the differential evolution algorithm, etc. have been proposed and used widely, to improve the performance of controllers [25, 26]. On the other hand, the controller designs for mechanical manipulators usually should take into consideration of uncertainties and external disturbances, which include
55 internal friction, unwanted coupling, unmodeled dynamics etc. Thus, finding a strategy to overcome such uncertainties/disturbances should also be considered during controller design.

By exploring the relationship between the control objective and the dynamic behavior of the system, this paper presents a control strategy for the planar
60 Pendubot with a second passive link using Fourier transformation and intelligent optimization to realize its control objective, which is to move the Pendubot from

any initial position to the target position. First, we design a controller by using the time required to achieve the control objective as a variable of the controller. Next, we define the above time as the period of a periodic function, whose value is the exact same as the control torque during its one period. We use the Fourier transformation on the above periodic function, and we can obtain a general form of the controller during the control process, expressed by Fourier series, which is composed of a constant term and harmonic terms. We then solve the constant term based on the angular momentum theorem. As for the solution of the harmonic terms, particle swarm optimization (PSO) algorithm is performed based on the control objective of the system. To overcome the uncertainties/disturbances in the system, a feedback control strategy is applied by employing a nonlinear disturbance observer to the system. By performing a large amount of simulation, the highest order of the harmonic terms of the controller is determined to ensure that the system control objective can be achieved effectively with high accuracy. Meantime, simulation results show that the control of the planar Pendubot can be achieved using this method, and is strong against disturbance.

2. Dynamic model of planar Pendubot

Figure 1 shows the model of a planar Pendubot, here the subscript 1 represents the actuated joint and the link attached to it, while subscript 2 represents the passive ones.

Listed below are the parameters used in Figure 1, here the subscript $i = 1, 2$.

q_i : angle of the i th link.

m_i : mass of the i th link.

L_i : length of the i th link.

L_{ci} : distance between the i th link and its center of mass.

I_i : moment of inertia of the i th link around its center of mass.

τ_i : torque applied to the i th link.

The dynamic equations related to the system can be described by Lagrange

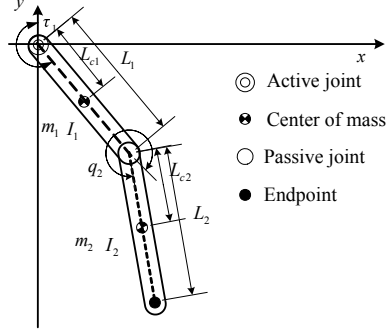


Figure 1: Planar Pendubot

equation

$$M(q)\ddot{q} + H(q, \dot{q}) = \tau + d \quad (1)$$

where $q = [q_1 \ q_2]^T$ is the angle vector of this system, \dot{q} and \ddot{q} are the angular velocity vector and the angular acceleration vector separately. $\tau = [\tau_1 \ 0]^T$ is the torque vector. $d = [d_1 \ 0]^T$ is the uncertain/disturbant component of the dynamics. $M(q)$ is the inertia matrix, which is always positive definite. And $H(q, \dot{q})$ represents the combination of the Coriolis and centrifugal forces. Their general forms can be written as

$$M(q) = \begin{bmatrix} M_{11} & M_{12} \\ M_{12} & M_{22} \end{bmatrix} \quad (2)$$

$$H(q, \dot{q}) = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix}$$

Each elements of the matrix can be given as

$$\begin{aligned} M_{11} &= a_1 + a_2 + 2a_3 \cos q_2 \\ M_{12} &= a_2 + a_3 \cos q_2 \\ M_{22} &= a_2 \\ H_1 &= -a_3 + (2\dot{q}_1 \dot{q}_2 + \dot{q}_2^2) \sin q_2 \\ H_2 &= a_3 \dot{q}_1^2 \sin q_2 \end{aligned} \quad (3)$$

where

$$\begin{aligned} a_1 &= m_1 L_{c1}^2 + m_2 L_1^2 + I_1 \\ a_2 &= m_2 L_{c2}^2 + I_2 \\ a_3 &= m_2 L_1 L_{c2} \end{aligned} \quad (4)$$

100 For a planar Pendubot, its potential energy is zero, and its total kinetic energy E is a very important variable to evaluate its dynamic characteristics

$$E = \frac{1}{2} \dot{q}^T M(q) \dot{q} \quad (5)$$

The control objective of the planar Pendubot is to move the system from its initial position to the desired final position. The position of the planar Pendubot can be described by the x and y coordinates of its endpoint in Figure 1, where

$$\begin{aligned} x &= L_1 \sin q_1 + (L_1 + L_2) \sin (q_1 + q_2) \\ y &= L_1 \cos q_1 + (L_1 + L_2) \cos (q_1 + q_2) \end{aligned} \quad (6)$$

105 For a given target position (x_d, y_d) , (6) gives two sets of solutions, and we can select one of them to be the target angles for the two links. So that we can turn the position control into angle control.

According to (1), we can obtain the constraint equation for planar Pendubot, given by

$$M_{12} \ddot{q}_1 + M_{22} \ddot{q}_2 + H_2 = 0 \quad (7)$$

110 The constraint equation (7) cannot be integrated to obtain a velocity constraint relationship, so the planar Pendubot is treated as a second-order non-holonomic system [19]. As a result, when we use conventional method to control the system to the target angles, the passive link usually ends up with a non-zero angular velocity. To solve this issue, the nilpotent approximation in [8] is proposed for the control of planar Pendubot, but this strategy is constructed iteratively, which may take a long time to achieve the control objective. An alternative effective control strategy to realize the control objective of the planar Pendubot is needed.

3. Controller design

120 In the first three subsection, we neglect the uncertain/disturbant component d in (1) and design the controller for the active link of the planar Pendubot, and we solve the parameters of the controller with angular momentum theorem and PSO algorithm. Then, in the last subsection, we take into account d , and we design a nonlinear disturbance observer to overcome such uncertainty/disturbance.

125 3.1. Controller design based on Fourier transformation

In this subsection we obtain the general form of the controller based on Fourier transformation. We define t_f as the time at which the two links of the Pendubot reach their target angles. Therefore, when the control objective is realized at $t = t_f$, the states and energy of the system can be given as follows

$$\begin{aligned} q_1(t_f) &= q_{1d}, \quad q_2(t_f) = q_{2d} \\ E(t_f) &= 0, \quad \text{which means } \dot{q}_1(t_f) = 0, \quad \dot{q}_2(t_f) = 0 \end{aligned} \quad (8)$$

130 where q_{1d} and q_{2d} are angles for the first and second link corresponding to the target position of the system. $E(t_f)$ is the total kinetic energy of the system at $t = t_f$, given by (5). Note that at $t = t_f$, the torque τ_1 that we use should be switched to zero to keep it static, which means

$$\tau_1(t) = \begin{cases} u(t) & t \in [0, t_f) \\ 0 & t \in [t_f, \infty) \end{cases} \quad (9)$$

We want to obtain a specific expression of the torque $u(t)$ that gives the inner relationship between the control torque we apply and the total control time t_f . To do so, we define t_f as the period of $u(t)$. Such definition is valid since we only care about the value of $u(t)$ during time $[0, t_f)$. Then, we use Fourier transformation [6] to expand the periodic function $u(t)$ into Fourier series

$$u(t) = \frac{1}{2}A_0 + \sum_{i=1}^{\infty} A_i \sin(i\omega t + \varphi_i) \quad (10)$$

140 where $\omega = 2\pi/t_f$. Here, the torque $u(t)$ can be divided into two parts. One is the constant term, described by a constant parameter A_0 . Another is the sum

of harmonic terms, described by harmonic parameters (ω, A_i, φ_i) ($i = 1, 2, \dots$).
 (10) gives the general form of $u(t)$ which can achieve the control objective in
 a finite time t_f . Thus, with the purpose of obtaining a controller to achieve
 145 the control objective, we need to solve the constant parameter as well as the
 harmonic parameters for (10).

3.2. Solution of the constant parameter based on angular momentum theorem

The constant term in (10) can be obtained based on the angular momentum
 theorem. Since for the planar Pendubot, the torque only applies on the first
 150 joint. We can select the first joint to be the reference point, and obtain the
 following equation based on the angular momentum theorem

$$\dot{L}(q, \dot{q}) = u(t) \quad (11)$$

where $\dot{L}(q, \dot{q})$ is the time derivative of $L(q, \dot{q})$, which is the total angular mo-
 mentum of the system relative to the first joint. By integrating both sides of
 (11) from $t = 0$ to $t = t_f$, we have

$$L(q, \dot{q})|_{t=t_f} - L(q, \dot{q})|_{t=0} = \int_0^{t_f} u(t) dt \quad (12)$$

155 The total angular momentum of the system can be given by

$$L(q, \dot{q}) = J_1(q) \dot{q}_1 + J_2(q) \dot{q}_2 \quad (13)$$

where $J_1(q)$ and $J_2(q)$ are two parameters related to the model parameters and
 angles of the two links. The system is static in both the initial and final states.
 So, according to (12) and (13), we have

$$\int_0^{t_f} u(t) dt = 0 \quad (14)$$

Substitute (10) into (14), we obtain the following result

$$A_0 = 0 \quad (15)$$

160 The equation (15) shows that the constant parameter in (10) is zero, so the
 torque of the system can be written as

$$u(t) = \sum_{i=1}^{\infty} A_i \sin(i\omega t + \varphi_i) \quad (16)$$

which means that the torque for this system only contains harmonic terms.

3.3. Solution of harmonic parameters by PSO

Generally speaking, when a signal expressed by Fourier series is applied to
 165 the system, people make a simplification by neglecting the high-order harmonic
 terms, leading to reduced number of parameters in the controller.

Hence, here we can define (16) as follows

$$u(t) = \sum_{i=1}^D A_i \sin(i\omega t + \varphi_i) \quad (17)$$

where parameter D is the highest order being selected. For a given control
 objective, there are a total of $2D+1$ parameters to be determined. We apply the
 170 PSO algorithm to solve these harmonic parameters (ω, A_i, φ_i) ($i = 1, 2, \dots, D$)
 based on the control objective of the system.

PSO is an optimization method where we use a population of N candidate
 solutions to navigate in a multi-dimensional space to search for a specific spot,
 which is determined based on the evaluation function [9]. The movements of
 175 these candidate solutions can be described using their positions and velocities.
 The iteration rule of the velocity and position of the k th candidate can be given
 as

$$\begin{aligned} v_{j+1}^k &= \gamma v_j^k + C_1 r_1 (b_j^k - x_j^k) + C_2 r_2 (g_j - x_j^k) \\ x_{j+1}^k &= x_j^k + v_{j+1}^k \end{aligned} \quad (18)$$

x_j^k and v_j^k are the position and velocity of the k th particle at the j th iteration
 ($j = 1, 2, \dots$ and $k = 1, 2, \dots, N$). γ is the inertia weight, C_1 and C_2 are two
 180 weighting factors, r_1 and r_2 are two random number from 0 to 1. b_j^k is the best
 position of the k th particle and g_j is the best position of the whole population,
 both at the j th generation.

The evaluation function ft for PSO is defined as follows based on the control
 objective of the system

$$ft = E + \beta_1 [q_1 - q_{1d}]^2 + \beta_2 [q_2 - q_{2d}]^2 \quad (19)$$

185 where β_1 and β_2 are positive constants. A small value of ft ensures that the system reaches the target position with high accuracy.

The process to solve the harmonic parameters based on the PSO algorithm can be described as follows:

190 *Step 1:* Initialize the candidate solutions $x_1^k = (\omega_1^k, A_{1i}^k, \varphi_{1i}^k)$ ($i = 1, 2, \dots, D$ and $k = 1, 2, \dots, N$) and set the velocity vector v_1^k to zero, subscript 1 represents the first generation.

Step 2: For the j th generation ($j = 1, 2, \dots$), based on the dynamic equation (1), calculate the angle and angular velocity of each link at time t_f . Then, calculate the corresponding evaluation function ft using (19), update b_j^k for 195 each solution and g_j for the whole population.

Step 3: If the ft corresponding to g_j is smaller than ε_0 , which is a very small positive number, the solution has been found and the whole process can stop. Otherwise, update the whole population to the $(j+1)$ th generation based on equation (18), then go to step 2.

200 The above PSO algorithm will solve parameters (ω, A_i, φ_i) of the controller given by (17). The total time for the system to achieve its control objective will be $2\pi/\omega$. Thus, by solving the parameters of the torque on the system, we obtain a controller which can achieve the control objective of the system at $t = t_f$.

205 3.4. Nonlinear disturbance observer for planar Pendubot

In the previous subsections, we have neglected d in (1) for convenience. However, for mechanical manipulators, disturbances in the system strongly affects the performance of the controller. In this subsection, we take into account the 210 disturbant components of the dynamics, and we apply a nonlinear disturbance observer to achieve a feedback control, with the purpose of compensating for the uncertainties/disturbances in the system.

The nonlinear disturbance observer that we apply provides an estimation on d , defined as $\hat{d} = [\hat{d}_1, \hat{d}_2]^T$. We assume that d varies slowly comparing with the control torque τ , and we can add a nonlinear disturbance observer according to

215 [7]. Thus, \hat{d} can be given by

$$\begin{cases} \hat{d} = z + p(\dot{q}) \\ \dot{z} = -G(q)z + G(q)(H(q, \dot{q}) - \tau - p(\dot{q})) \end{cases} \quad (20)$$

where z is an auxiliary variable vector, $G(q)$ and $p(\dot{q})$ can be given as follows

$$\begin{aligned} G(q) &= c \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} M^{-1}(q) \\ p(\dot{q}) &= c[\dot{q}_1 \quad \dot{q}_1 + \dot{q}_2]^T \end{aligned} \quad (21)$$

c is a positive constant. With the convergence discussion in [7], by setting the parameter $c > a_3 |\max(\dot{q}_2)|$ where a_3 is given by (4) and $|\max(\dot{q}_2)|$ is the maximum angular velocity of the second link, we can easily prove that this
220 nonlinear disturbance observer is convergent. This observer is designed based on a nominal model.

For planar Pendubot, the second joint does not contain an actuator, that is, $d_2 = 0$. So we should have $\hat{d}_2 = 0$. The controller for the first link after adding the nonlinear disturbance observer can be given by

$$\tau_1'(t) = \tau_1(t) - \hat{d}_1 \quad (22)$$

225 where $\tau_1(t)$ is given by (9), (17), and \hat{d}_1 is given by (20). With this feedback control strategy based on the nonlinear disturbance observer, the controller will become much stronger against disturbances in the system.

Furthermore, the term d in (1) can also represent the uncertainties in the system. As a result, we can design a nonlinear disturbance observer to overcome
230 the such uncertainties with a similar approach.

4. Simulation

A planar Pendubot model has been built with MATLAB tools. After performing simulations with different D , D is selected to be 4. Which ensures that the control objective can be achieved with high accuracy. For the PSO
235 algorithm, the parameters are selected as follows

$$\gamma = 0.5, C_1 = 2.8, C_2 = 1.3 \quad (23)$$

The parameter φ_1 of the controller is fixed to be 0 rad for simplification.

For the evaluation function given by (19). The errors of the angular velocities of the two links at the time t_f are required to be smaller than 10^{-5} rad · s⁻¹. As a result, according to (5) and (19), ε_0 is set to be 10^{-10} . Meanwhile, the errors of the angles of the two links are required to be within 10^{-4} rad. So, β_1 and β_2 are set to be 0.001, and as a result, the three terms in (19) will have the same order of magnitude when they're evaluated as a whole.

To validate the control strategy of this paper, we study two cases with different model parameters, initial positions and target positions. We then discuss another case where the uncertainties/disturbances in the system is considered.

4.1. Case A

In this case, the model parameters of the system are selected as shown in the following table

Table 1: Model parameters for planar Pendubot

Link i	m_i (kg)	L_i (m)	I_i (kg · m ²)
1	1	1	0.333
2	1	1	0.333

The initial state of the system is set to be $[q, \dot{q}] = [0, 0, 0, 0]$, with target angles $q_{1d} = 0.5$ rad, $q_{2d} = -1$ rad. PSO is performed to obtain the controller parameters

$$(\omega, A_1, A_2, A_3, A_4, \varphi_2, \varphi_3, \varphi_4) = (4.57, 3.81, 4.65, 3.84, 0, 4.16, 2.41, 5.71) \quad (24)$$

By substituting (24) into (17), we obtain the controller for this system. Simulation results in Figure 2 shows that the angles and angular velocities of both links are able to smoothly converge to their target values. The whole control process only takes 1.4 seconds. These results show that the control strategy proposed in this paper can achieve the control objective of the planar Pendubot quickly and effectively.

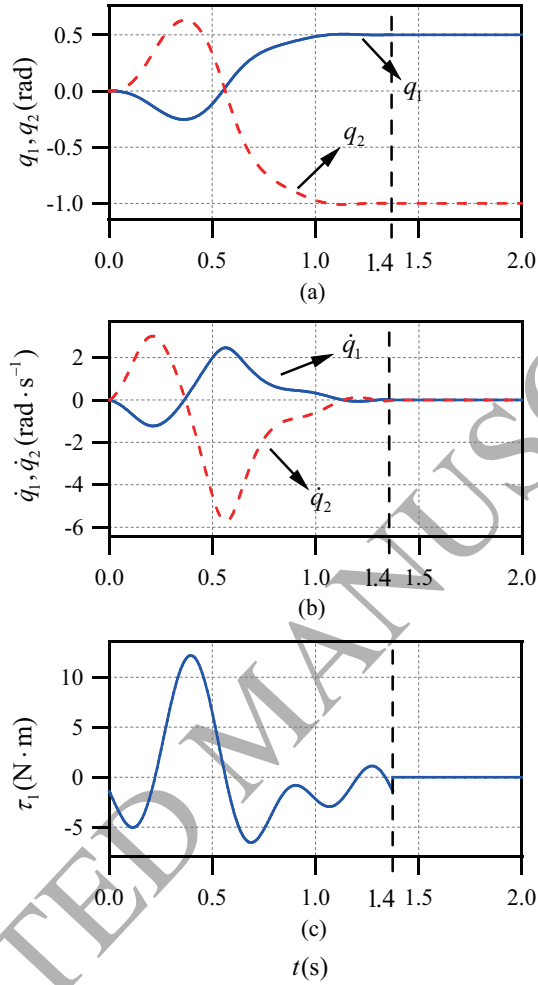


Figure 2: Simulation results for case A

4.2. Case B

To further illustrate the effectiveness of the control strategy proposed in this paper, we compare with the nilpotent approximation method for planar Pendubot proposed in [8]. We select the model parameters, initial state and target angles of the system to be the same as in [8]. According to parameters

in [8], we obtain a_1 , a_2 and a_3 to be

$$a_1 = 0.447, a_2 = 0.420, a_3 = 0.195 \quad (25)$$

The initial states of the two links are $[q, \dot{q}] = [1.291, 1.588, 0, 0]$, their target angles are $q_{1d} = 0$ rad, $q_{2d} = 0.785$ rad. Using PSO, parameters for the controller can be obtained

$$(\omega, A_1, A_2, A_3, A_4, \varphi_2, \varphi_3, \varphi_4) = (0.78, 0.10, 0, 0, 2.20, 2.51, 2.80, 4.29) \quad (26)$$

The simulation results are shown in Figure 3. The torque applied in this case has a maximum value $2.3 \text{ N} \cdot \text{m}$, smaller than the maximum torque value shown in Figure 8 of [8], which is $5.0 \text{ N} \cdot \text{m}$. The system can achieve its control objective in about 8.0 seconds, much faster than the total control time shown in Figure 5 of [8], which takes more than 50 seconds.

4.3. Case C

In this case, we verify the validity of the feedback control strategy based on the nonlinear disturbance observer when uncertainties/disturbances are taken into consideration. The model parameters, initial state and target angles are selected to be the same as in case B. We apply a disturbance d_1 on the first torque, given by

$$d_1 = A_d \sin(\omega_d t) \quad (27)$$

We choose $A_d = 0.2 \text{ N} \cdot \text{m}$ which is about 10% of the maximum torque in case B, and $\omega_d = 0.4 \text{ rad} \cdot \text{s}^{-1}$, which is about half of the frequency ω in case B. Parameter c in (21) is selected to be 100.

Figure 4 shows the simulation results when there is disturbance without its observer, the angles of the two links will be divergent, which means that they cannot converge to the target angles. Figure 5 shows the simulation results for the system with the aid of the nonlinear disturbance observer. Figure 5(a) and Figure 5(b) show that the system can overcome the disturbance, and the angles and angular velocities of the system both converge to the target values smoothly. To conclude, the feedback control strategy based on the nonlinear

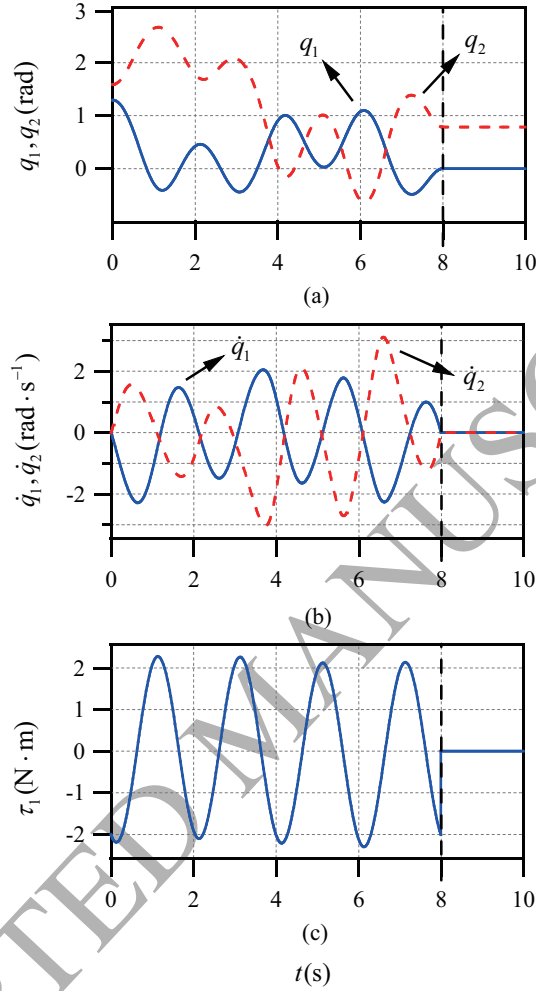


Figure 3: Simulation results for case B

disturbance observer can effectively overcome the uncertainties/disturbances in the system.

5. Conclusion

This paper proposes a new control strategy for the planar Pendubot based on Fourier transformation and intelligent optimization. We design a controller

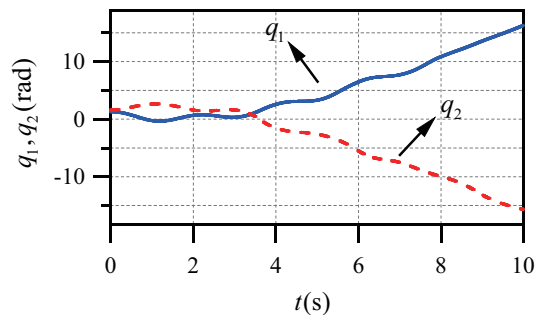


Figure 4: Simulation results without disturbance observer

which uses the total control time as a variable of the controller. Its general form is obtained by expressing it in Fourier series using Fourier transformation. We obtain the constant term of Fourier series based on the angular momentum theorem. Meanwhile, we employ the PSO algorithm to solve the harmonic terms of the Fourier series according to the control objective of the system. We then use a feedback control strategy based on a nonlinear disturbance controller to overcome the uncertainties/disturbances in the system. The simulation results show that this control strategy can achieve the control objective of the planar Pendubot effectively, with the ability to overcome uncertain/disturbant components of the dynamics.

It is worth mentioning that such control strategy for planar Pendubot can be easily extended to planar APAⁿ ($n \geq 1$) manipulators with one passive link which is the second link. For this type of systems, we can first apply the method proposed in this paper to achieve the control objective of the first link, so that the whole multi-link system can be reduced to a PAⁿ system. Then we can apply the control method for the holonomic system ($n = 1$) or the first-order nonholonomic system ($n > 1$) to control the rest of the links, so that the control objective for the whole system can be achieved. However, this paper focuses on showing the controller design strategy based on Fourier transformation and intelligent optimization in a simple setting that reveals its essential features.

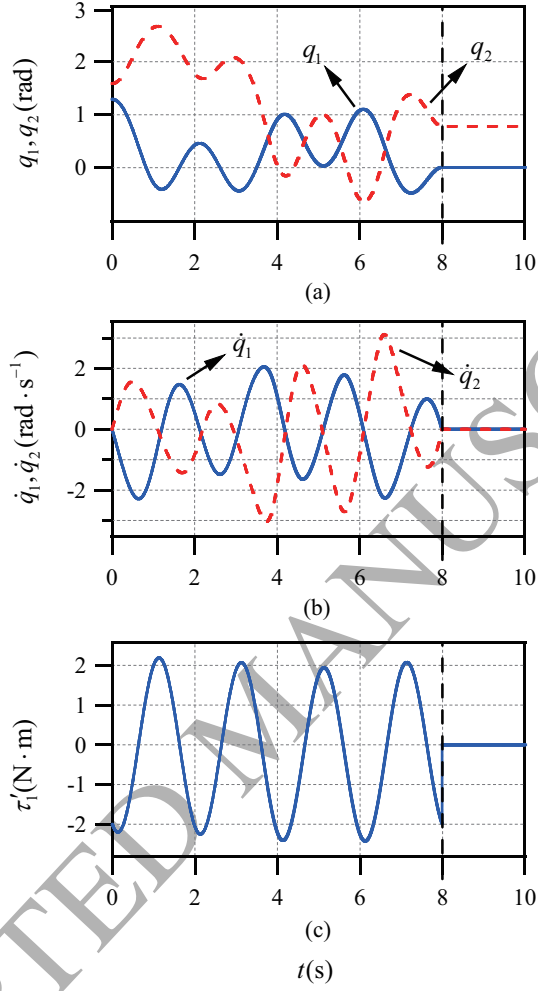


Figure 5: Simulation results with disturbance observer

This is the reason for simply discussing the control of a planar Pendubot with a second passive link.

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