

# **Numerical Modeling for Shallow Foundation near Slope**

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## **ABSTRACT**

Often developers build luxury buildings near shores, where foundations are built in or near slopes. Designers in this case face not only the determination of the bearing capacity and the settlement of the foundation but also the stability of a system made of foundation and slope. Design of foundation under these conditions is complex and the studies available in this regard are limited and concerned mostly about determination of the reduction of the bearing capacity coefficients associated with the presence of the slope except for Meyerhof who was a pioneer in developing a theory in 1957 to determine the ultimate bearing capacity of a foundation near a slope. However, the theory was independent of the slope inclination.

This thesis presents a numerical model using finite element technique together with RS2 9.0 software to simulate the case of a foundation near a slope, in terms of examining the bearing capacity of the foundation for given slope features, soil characteristics and geometry conditions and extends to determine the distance required to achieve reasonable factor of safety.

It's found that the theory of Meyerhof, 1957 has underestimated the effect of the distance to the slope which the bearing capacity is independent of the inclination of the slope. Design guidelines and design charts are presented for practicing purposes.

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## SYMBOLS

$\alpha$	Angle of slope, degree °
$\beta$	Slope inclination, degree °
$\gamma$	Unit weight of soil, kN/m <sup>3</sup>
$\gamma_t$	Total unit weight of soil, kN/m <sup>3</sup>
$\Theta$	Angle of radial shear zone, °
$\mu = \tan \phi$	Coefficient of internal friction
$\tau$	Shear stress on plane, kN/m <sup>2</sup>
$\sigma_f$	Normal stress on failure plane, kN/m <sup>2</sup>
$\phi$	Angle of internal friction of soil, degree °
B	Width of footing, m
c	Cohesion parameter of soil, kN/m <sup>2</sup>
$c_u$	Undrained shear strength of soil,
D	Depth of footing below ground surface, m
H	Slope Height, m
$N_c$	Bearing capacity factor due to cohesion of soil
$N_q$	Bearing capacity factor due to overburden pressure
$N_\gamma$	Bearing capacity factor due to weight of soil below the footing
$N_{cq}$	Bearing capacity factor due to effect of cohesion and overburden pressure
$N_{\gamma q}$	Bearing capacity factor due to effect of weight of soil and overburden pressure
Q	Allowable load, kN/m <sup>2</sup>
x	Distance from the edge of slope, m

# CHAPTER 1

## INTRODUCTION

Luxurious resorts built on foothills or located near shores are enjoyable and desirable by people, while it's problematic for engineers. Because in the case of building near or in slopes, the bearing capacity of the foundation is not only function in the soil condition but also in the geometry of the slope, in this case, the ultimate bearing capacity is governed by either the bearing capacity of the foundation or by the overall stability of the slope, the combination of these two factors complicate modeling of the problem. Despite the importance of the subject of building near slopes, there are limited studies available in this regard except for Meyerhof, who was prominent in developing a theory in 1957 to determine the ultimate bearing capacity for foundations built near slopes and predict the reduction in the bearing capacity coefficients associated with the presence of the slope, thus, define the distance at which the bearing capacity of the foundation is independent of the inclined ground.

In fact, the issue of building near or in slopes is of great importance in that it is often unavoidable for many reasons such as land limitation or architectural purposes, this automatically necessitates the study of several factors that may affect the safety of the building and/ or the safety of the slope and this is an inherent part of geotechnical engineering and it's frequently required from the engineers to determine the location and depth of foundation to be built near slopes.

In this matter, this research destined to trace two important and fundamental parts regarding building near slopes; examining the ultimate bearing capacity of a shallow foundation placed near slope, taking into account the inevitable reduction in the bearing capacity coefficient due to the existence of the slope. Thus, study the stability of the slope considering the presence of the foundation, the objective of the research was accomplished through two phases of the analysis in view of a typical strip foundation of equal width and depth of 1m placed near slope in different soil and geometry conditions to determine the allowable load,  $Q$  in phase I of the analysis. Then determining the minimum distance required to locate a foundation near a slope in order to maintain the stability of the whole system and achieve an acceptable factor of safety.

Numerical model was developed using Finite Element technique to simulate the problem mentioned. Parametric study was conducted to study the parameters believed to govern the problem. The software used to carry out the analysis is RS2 9.0. Design guidelines and charts were developed for practicing use.

# **CHAPTER 2**

## **LITERATURE REVIEW**

### **2.1 General**

Avalanches and landslides, earthquakes, sinkholes, volcanic eruptions, floods, tsunami etc. are natural disasters which cannot be prevented from occurrence as no one is able to stop a natural disaster to happen. What an engineer can do is to predict the happening of the disaster therefore reduce its consequences scientifically, safely and economically.

One of the most important problems in geotechnical engineering is the slope instability which broadly known as an ever-exhibit risk.

We will focus here on the problems associated with the gravity-empowered mass movement as they are important, costly and persistent wellspring of worry for geotechnical engineers and geologists around the world, particularly in topographically dynamic regions. For better understanding, we will display a brief idea about the cost and recurrence of events of slope failures of natural and of man-made slopes. Data gathered in the United States proposes that the yearly costs keep running into billions of dollars, and records of this by Schuster (1978) assess add up to misfortunes because of slope movement in California alone at about \$330 M yearly. Likewise, notwithstanding monetary expenses, there is death toll at around 25 every year on average in the United States. The Japanese are especially extremely influenced via landslide: theirs is a thickly populated arrangement of islands, susceptible to landslide and shaken by earthquakes. The loss of life there reached 171 in 1971 and 239 in the next year. Then again, in the event that we need to see the effect of those fiascos on information, we will find that the consideration was centered around the issue of the stability of tips and heaps of spoil and other wastes resulting from mining and quarrying in 1966 when a slide beginning in a colliery tip in South Wales (UK) brought about the demise of 144 inhabitants. Different mass movements originating from deficient treatment of dig waste were in charge of serious death toll at Buffalo Creek, West Virginia, USA and at Stava in northern Italy. At Jupille in Belgium, a comparative mass movement from a tip of fly-ash debris created broad death toll. The principal which apply to the stability of such waste tips are precisely the same as those applying to regular slopes or to man-made earth works.

## **2.2 History of slope stability**

The French military engineers were first to make some empirical advances in the scope of the stability of slopes (Bromhead 1992).

The initial serious work on the stability of slopes, has been done by the French canal engineer Alexander Collin (1808-1890) who has recognized the curved shape of sliding surface in both cut and fill slopes in 1846. In addition, he attempted some stability analyses.

Toward the finish of nineteenth century, British interest swung more to earth pressure than to the stability of slopes, despite early work of considerable significance. The landslide at Folkestone Warren in December 1915 had some effect on the investigation of slopes as well as the extensive persistent landslide struck Panama Canal under development begun in May 1910. The timber piles extracted after the failure occurred in quay and jetty in Gothenburg, demonstrated that the surface on which the slide occurred was approximately a circular arc in section.

Afterward, the Swedish slip circle has turned into a universal solution to the slope stability problems. Terzaghi 1925 has built up the principal of effective stress and its impact on shear strength behavior.

The program of new road construction in 1950s and 1960s enlarged the number of the stability problems. The finding of the slip surface in cut and fill slopes was the key to establish the theory of the residual strength.

## **2.3 Principles of stability analysis**

Many large rockfalls start with an initial shear failure and sliding phase before the debris entirely parts company with the parent rock mass, while many landslides remain in contact even though they move considerable distances. Simple sliding models fall into the category of limit equilibrium methods which the destabilizing force is less than or equal to the force that resists motion, the ratio between those forces gives an index of relative stability which known as the factor of safety.

The known methods for the analysis of the stability of slopes based on the limit equilibrium technique are;  $\phi=0$  method which the soil strength is assumed to be purely cohesive. Bishop's method (1955); which utilizes the method of slices to discretize the assumed slip surface into slices

and determine the factor of safety. This technique fulfills vertical force equilibrium for each slice and overall moment equilibrium about the center of the circular trial surface since horizontal forces are not considered at each slice. It also assumes zero interslice shear forces, while the complete method of Bishop considers all the components of the interslice forces. Spencer's method (1967); is an extension to the routine method of Bishop, assuming that the resultant interslice forces have constant slope throughout the sliding mass. Graphical wedge method; unlike Bishop and Spencer's technique, the graphical wedge method ignores the equilibrium of moments and concentrates on force equilibrium. There is a relinquish in accuracy and increase in uncertainty about the outcome, yet that there is a gain in calculation simplicity. Janbu's method; it was important to develop method with similar theory of the routine method of Bishop, but in meantime applicable to slides with any shapes of slip surface. Janbu's simplified method assumes that the interslice forces are horizontal an empirical correction factor is used, while Janbu's generalized assumes a line of thrust is used to define the location of the interslice normal force. Morgenstren and Price's (1965-1967); the substance of strategy is to separate the sliding mass into a generally modest number of linear areas or wedges which are vertical sided in the regular way. The states of force equilibrium can be fulfilled taking headings normal and parallel to the slip surface. The direction of the resultant interslice force is characterized using a discretionary function. The portions of the function value required for force and moment balance is processed. Maksumovic's method; this method is not perfectly suited to routine stability- calculation work, but the main advantage in this method is the possibility of including external concentrated forces. Sarma's method; which adopted a fundamentally unique approach to deal with the estimation of the stability of slopes. This technique can be embraced to find a conventional static factor of safety by simply factoring(reducing) the soil strength parameters until a zero horizontal acceleration is required for failure.

This leads us to question why engineers build on slopes. In fact, building on or near slopes is unavoidable, like what we have for highways, bridges and subways, in addition to some touristic reasons or due to land impediments. Hence, a foundation built on slope conducts a reduction in the bearing capacity of the foundation and threatening the stability of the slope itself. In that case we have two modes of failure; bearing capacity failure when the shear surface does not intersect the slopes, where bearing capacity theories for horizontal ground surface are applicable to such modes of failure. The second mode of failure is the slope failure, where the critical shear surface extends beyond the crest and therefore involves part of the slope.

## 2.4 Relevant studies

Meyerhof (1957) presented a paper to study the ultimate bearing capacity of foundation on slopes, he reasoned that the bearing capacity theory published by him in 1951 can be combined with the theory of the stability of slopes to cover that loading condition (Meyerhof 1957). Meyerhof investigated two different cases for the bearing capacity of the foundation on slope. First, bearing capacity of foundation on face of slope, and second, bearing capacity of foundation on top of slope.

According to Meyerhof, when a foundation located on the face of a slope is loaded to failure, the zone of plastic flow in the soil on the side of the slope is smaller than those of a similar foundation on level ground and the ultimate bearing capacity is correspondingly reduced. For a material, the shearing strength is given by:

$$\tau_f = c + \sigma \tan \phi \dots \dots \dots \text{Eq. 2.1}$$

The bearing capacity of a foundation on slope of inclination  $\beta$  can be presented by (Terzaghi 1943).

$$q = cN_c + P_o N_q + \gamma(BN/2) \dots \dots \dots \text{Eq. 2.2}$$

Or, more generally (Meyerhof, 1951)

$$q = cN_{cq} + \gamma(BN/2)_{\gamma q} \dots \dots \dots \text{Eq. 2.3}$$

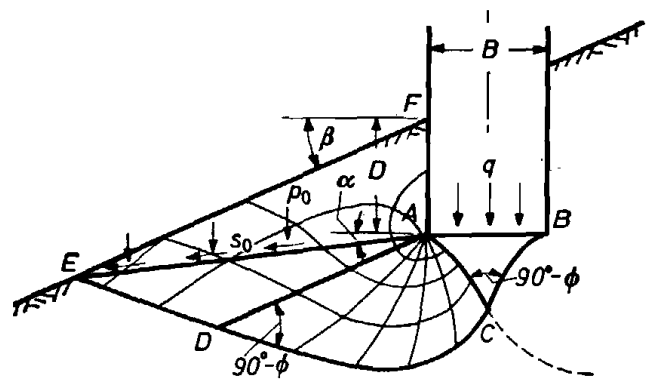


Figure 2.1 Plastic zones near rough strip foundation on face of slope (foundation failure) as Meyerhof (1957).



The study to find bearing capacity factors for foundation placed on or near slope has been done for a strip foundation in purely cohesive ( $\phi=0$ ) and cohesionless ( $c=0$ ) materials, respectively. The bearing capacity factors decrease with greater inclination of the slope to a minimum for  $\beta=90^\circ$  on purely cohesive material, and  $\beta=\phi$  for cohesionless soils. For slopes with slope angle  $\beta$  less than  $30^\circ$ , the decrease in bearing capacity is small in the case of clays but is considerable for sands and gravels because the bearing capacity of cohesionless soils decreases approximately parabolically with increase in slope angle. For the case of submerged soil, we replace  $\gamma$  by  $\gamma'$  (submerged unit weight of soil). For the case of rapid drawdown of water table, the bearing capacity can be obtained from Meyerhof's equation using a reduced angle of friction as for unloaded slopes (Terzaghi 1943)

$$\Phi' = \tan^{-1}((\gamma'/\gamma) \tan \phi) \dots\dots\dots \text{Eq. 2.4}$$

In cohesive materials with a small or no angle of shearing resistance, the bearing capacity of a foundation may be limited by the stability of the whole slope with a slip surface intersecting the toe or base of the slope. For slopes in purely cohesive soil of great depth, base failure of unloaded slope occurs along a critical mid-point circle (Fellenius 1927). So that foundations below the mid-point section increase the overall stability of the slope and vice-versa. The upper limit of the bearing capacity can then be estimated from the equation;

$$q = cN_{cq} + \gamma D \dots\dots\dots \text{Eq. 2.5}$$

In the same study Meyerhof has presented the bearing capacity of foundation on top of a slope by his well-known equation;

$$q = cN_{cq} + \gamma \left(\frac{BN}{2}\right)_{\gamma q} \dots\dots\dots \text{Eq. 2.6}$$

Where the resultant bearing capacity factors  $N_{cq}$  and  $N_{\gamma q}$  depend on the distance (b) from the edge of slope as well as  $\beta$ ,  $\phi$  and  $D/B$  of the foundation (where  $D$  &  $B$  are the embedment depth and the foundation width, respectively). Bearing capacity factors decrease with greater inclination of slope, they increase rapidly with greater distance from the foundation to the edge of the slope. Beyond a distance of about 2 to 6 times the foundation width (depending on  $\phi$  and  $D/B$ ), the

bearing capacity is independent of the inclination of the slope and becomes the same as that of a foundation on an extensive horizontal ground surface.

The bearing capacity theory of the foundation on level ground can be extended and combined with the theory of the stability of slopes to cover the stability of foundations on slopes. The bearing capacity of foundations on the face of a slope or near the top edge of a slope, decreases with greater inclination of the slope especially for cohesionless soils. At a greater distance from the edge of a slope, the bearing capacity decreases considerably with greater height of the slope and is governed by overall slope failure.

**Kusakabe et al. (1981)** approached the bearing capacity of slopes under strip loads on the top surfaces using the upper bound theorem. The lower bound analysis was also attempted.

Comparison has been made with other methods such as; the conventional circular arc methods and stress characteristics equations by Kötter. The upper bound solutions are found to be useful because of its simplicity.

The paper showed that the factor  $c/\gamma B$  (where  $c$ : cohesion of soil,  $\gamma$ : unit weight of soil and  $B$ ) is the width of the strip load) cannot be neglected when constructing the slip line, thus the results obtained by Meyerhof need some correction. They also found that in some of the previous work for others, the effect of the distance between the slope shoulder and the edge of load has been ignored.

It's been found from the study that the proposed failure mechanism by using the upper bound technique is in a good agreement with actual failure surfaces for various combinations of values of  $\beta$  and  $\alpha$  (the slope angle and the distance from the edge of a slope normalized by the width of a footing, respectively). It's also been found that the results obtained the upper bound theorem are fairly close to those obtained from lower bound, which implies that the upper bound solutions are good approximations of exact solutions. The study found that the failure mechanism and the bearing capacity factors  $N_c$  and  $N_\gamma$  vary significantly with the parameter  $C/\gamma B$ , additionally the model tests showed that the upper bound solutions underestimate the bearing capacity with average deviation of 30%.

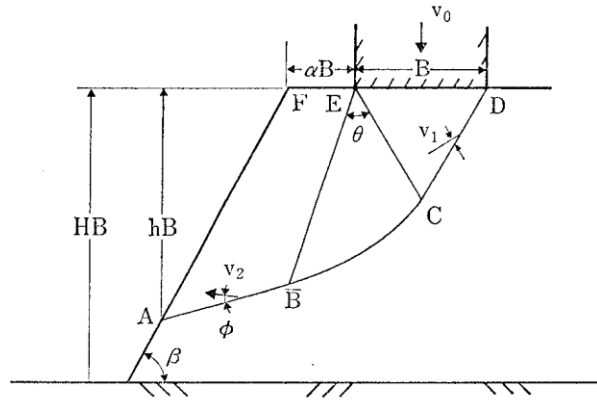


Figure 2.2 Failure mechanism as Kusakabe analysis (Kusakabe et al., 1981).

**Azzouz and Baligh (1983)** proposed a study for the loaded areas on cohesive slopes, they found that most studies for the undrained stability analysis of cohesive slopes subjected to surcharge are done on two-dimensional modes of failure, while their study showed that the magnitude of the critical surcharge load causing failure can be significantly influenced by taking into account the effect of the three dimensions of the problem. Two-dimensional (plane-strain) solutions corresponding to a strip load of infinite extent are first considered. Three-dimensional stability analyses of slopes subjected to square loaded areas are then made and the results compared to two-dimensional solutions in order to ascertain the influence of the end effects on the magnitude of the surcharge causing failure.

The paper studied the strip loads of infinite extent, it investigated the undrained bearing capacity of cohesive slope of height  $H$  and angle  $\beta$  with the horizontal on saturated clay with a total unit weight  $\gamma_t$  and undrained shear strength  $c_u$ . The slope is subjected to an infinitely long strip load of width  $B$  and intensity of  $q_0$ , this in addition to gravity forces.

Two modes of failure have been detected, first; bearing capacity failure where the shear surface does not intersect the slope (bearing capacity theories for horizontal ground surface are applicable to such modes of failure). Second; slope failure where the critical shear surface extends beyond the crest and involves part of slope.

Then, Azzouz and Baligh (1983) continued to study square loaded areas. In that part of study, the stability of slopes subjected to square loaded areas has been investigated assuming the

shear surface is of an ellipsoid attached to a cylinder whose length is assumed to be equal to that of the load  $L_p$ . The critical shear surface is found numerically.

The paper concluded the study of the undrained stability of slopes subjected to strip loads and square loaded areas in two major conclusions; the three-dimensional effect would increase the plane strain critical load causing failure by a factor of 5 to 10. The importance of this effect appears when the factor of safety of the slope due to its own weight is close to unity. Second; the strip load solutions presented in the paper were developed for different values of normalized width ratios,  $B/H$  (0.25, 0.5 and 1) which could extend the solution presented by Meyerhof (1957) which has been developed for  $B/H$  of infinity and zero, respectively.

**Graham et al. (1988)** proposed an analysis for bearing capacity of shallow foundations placed in cohesionless slopes considering the stress conditions immediately beneath the footing. The method of stress characteristics uses sand properties defined by a critical state model. This method considers different stress levels, different placement densities, the sand compressibility and the transition from strain softening to strain hardening behavior accompanies increased stresses (Graham 1986).

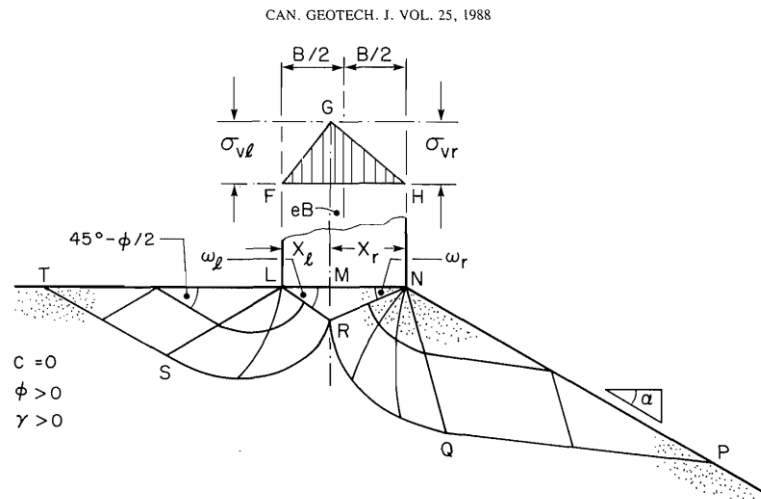


Figure 2.3 Schematic of failure zones for footing at crest of slope as Graham et al. (1988).

**Swami Saran et al. (1989)** worked out an analytical solution to obtain the ultimate bearing capacity of footings adjacent to slopes by applying limit equilibrium and limit analysis approaches.

The study assumed one-sided rupture failure on the side of slope and partial mobilization was considered on the side of flat ground.

The results have been presented in the form of non-dimensional bearing capacity parameters  $N_c$ ,  $N_q$  and  $N_\gamma$  for different values of the angle of internal friction ( $\phi$ ), slope inclination and distance of footing from slope edge.

The study showed that the two approaches (limit equilibrium and limit analysis) used to obtain the ultimate bearing capacity for shallow strip footing placed adjacent to a slope give almost same values. The minimum distance at which the bearing capacity factors become independent of slope increases with the increase in value of ( $\phi$ ) for same slope angle. Further, the results gave a good agreement with model test data, but the values obtained from the study, in most cases, are higher than those of previous investigations.

**Narita and Yamaguchi (1990)** had presented a study on finding out the bearing capacity of foundations near slopes by use of log-spiral sliding surfaces.

The study is considered an extension of the work presented by the authors earlier, for strip foundations placed on level ground using same method of analysis. They also had done comparison study with other analytical and experimental results to examine the validity of the presented technique to practical problems.

Toe and slope failure, for a strip footing of width  $B = 2b$  located at distance  $L = \lambda B$  from the edge of a slope with an inclination  $\beta$ , the sliding surface is assumed to be a log spiral.

The equation of a log spiral is written as:

$$r = r_o \exp(\mu\theta) \dots\dots\dots \text{Eq. 2.7}$$

Where  $\mu = \tan \phi$  ( $\phi$  is the angle of internal friction) and  $r_o = OA$

The log-spiral sliding surface can be defined by specifying sets of values about footing condition ( $\beta, \lambda$ ) and the location of the pole O ( $r_o, \alpha$ )

Generally, the ultimate bearing capacity of a strip footing is presented as a linear combination of cohesion of the soil  $C$ , the surcharge load  $P_o$  and the unit weight of soil  $\gamma$ .

In the log- spiral method of analysis, each component of ultimate load  $Q$  ( $Q_c$ ,  $Q_q$  and  $Q_\gamma$ ) is determined by minimizing corresponding footing load that is derived individually from the equilibrium of moments about pole  $O$ .

Base Failure, for relatively small fills of height  $H=hB$  where base failure might be dominant, the equations for toe and slope failure could be applied with some modifications.

They have concluded their work into three major points;

- The log spiral analysis tends to overestimate bearing capacity values comparing to the upper bound and simplified Bishop solutions. The involved errors are around 20% at maximum and vary with the angle of internal friction  $\phi$  and slope inclination  $\beta$ .
- For purely cohesive soils, the log spiral solution becomes close to other analytical solutions with 5% error at most.
- For cohesionless soils, both the log spiral and the upper bound solutions give successive increase in  $N_\gamma$  value as the footing distance to the edge of slope increases.

Generally, the log spiral solution gives acceptable correspondence with other analytical solutions, particularly, when base failure is expected to occur, giving lower value of the ultimate load than the upper bound solutions in a certain range of the slope height.

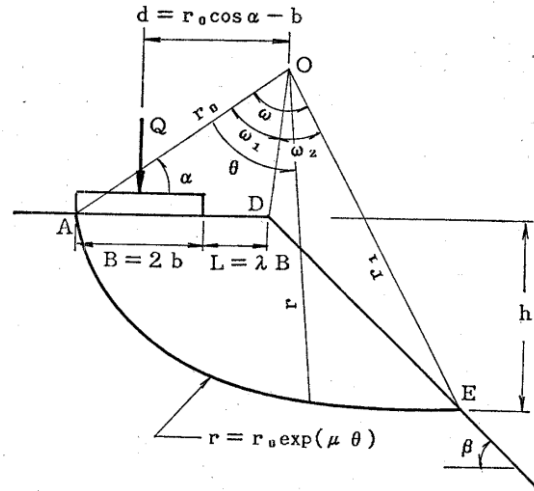


Figure 2.4 Log-spiral sliding surface as Narita and Yamaguchi (1990).

**De Buhan and Garnier (1998)** have studied the bearing capacity of a rectangular shallow foundation near a slope or an excavation using the yield design theory, considering the true three dimensions of the problem. The difficulty of solving the problem of a footing placed near a slope is coming from the impossibility of examining the bearing capacity of the foundation separately from the analysis of the stability of a slope. This kind of combination between the bearing capacity and the slope stability problem can be treated either as a classical bearing capacity problem with reduced bearing factors due to the existence of the slope, or as slope stability problem considering the load from the foundation as a surcharge acting on the top of the slope.

The problem is a slope of height  $H$  and angle  $\beta$ , subjected to a vertical load  $Q$  acting on the top of it, resulting from a rigid rectangular foundation of length  $L$  and width  $B$  located at distance  $D$  from the edge of the slope. The soil was assumed to be dry, homogeneous with parameters of  $c$ , and  $\gamma$ .

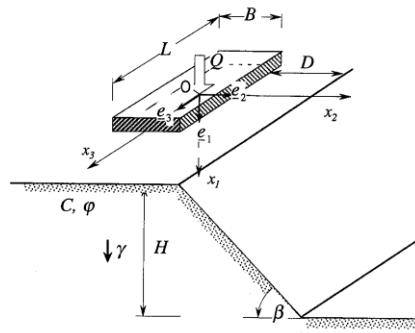


Figure 2.5 Bearing capacity of a rectangular footing acting on slope as De Buhan and Garnier (1998).

The paper showed that the yield design method is an appropriate technique to deal with a three-dimensional problem of evaluating bearing capacity of a rectangular footing located near a slope or an excavation. Implementing the upper bound kinematic approach of this theory, two kinds of failure mechanisms have been devised to **a.)** Instability failure mechanism: This failure mechanism has no deformation, only velocity discontinuity surfaces. The volume of soil mass bounded by an arc of log spiral of angle  $\varphi$  rotates about the focus of the spiral. **b.)** Punching failure mechanism: This failure mechanism is obtained from the 3D generalization of the “plane-strain” Prandtl-type mechanism used by (Kusakabe et al. 1981) in a simplified form.

**Castelli and Motta (2010)** have found that number of studies have been done of the two-dimensional case of a strip footing resting either on a horizontal or inclined slope surface. The bearing capacity of a shallow foundation in plane-strain condition is generally evaluated using the superposition method proposed by Terzaghi (1943). Further solutions of the bearing capacity were given in a more general form, considering the shape of a foundation, the shape of a load, ground inclination and depth and inclination of the bearing surface (Hansen 1970) and (Vesic 1973). Castelli and Motta studied the static and seismic bearing capacity of a strip footing located at some distance from a sloping ground considering the effect of soil inertia by using a limit equilibrium technique, assuming a circular failure mechanism. A simple equation to evaluate the seismic bearing capacity has been derived by them and also a comparison between the proposed method and available theories was presented. Castelli and Motta (2008) found that the distance ( $d$ ) of the foundation from the edge of a sloping ground plays a significant role on the bearing capacity of the soil foundation system. If the footing is far enough from the edge, then the failure mechanism



will not be affected by the slope Castelli and Motta (2008). They have conducted a parametric study in which they have concluded the following; **first for static analysis**, the ground slope factors ( $g_c$ ,  $g_q$ ,  $g_\gamma$ ) were evaluated for a distance of the footing from the edge of a slope varying from 0 to  $6D$ , where  $D$  is the embedded depth of a foundation, and for inclinations of a slope varying in the range  $0^\circ$  up to  $40^\circ$ . The maximum reduction of bearing capacity was obtained when the footing is placed at the edge of the slope. When the distance of the footing from the edge of a slope was increased, correction factors to be applied to the bearing capacity factors for not embedded footing, become closer to 1. For an embedded footing ( $H/B > 0$ ), the correction factors may become greater than unity because of the depth effect increases the limit load. When the friction angle of the soil was increased, the distance (at which the effect of the sloping ground is negligible), increased as well. There will be a distance at which the effect of the sloping ground will completely fade. They named this distance threshold distance  $d_t$ . The threshold distance is affected by the slope angle. The study also shows that an embedment ratio  $H/B=0.5$  (in which  $H$  is the embedment depth and  $B$  is the footing width) does not affect the threshold distance significantly, however it is slightly increased. **Then second for seismic analysis**, the bearing capacity of a shallow foundation should consider the load inclination and eccentricity arising from the inertia forces of the structure and the effect of the inertia of the soil (kinematic inertia) according to Eurocode 8-Part 5 (2003). The reduction in bearing capacity due to the soil inertia can be more significant than that caused by the inertia of the structure.

The study found that the threshold distance at which the sloping ground no longer affects the bearing capacity increases, first, with the increase of the friction angle, second, with the increase of the seismic coefficients and the slope angle. For the static case, the embedment depth of the footing does not affect the threshold distance significantly. However, the embedment depth may cause a considerable increase of the bearing capacity. Also, for the static case, the maximum reduction of the bearing capacity occurred when the footing is placed directly at the edge of the slope. The study found that there is no significant difference in the threshold distance when the inertia of structure or soil mass was considered. The combined effect of both inertias can be considered using the superposition principle presented by Paolucci and Pecker (1997) and Cascone et al. (2004).

It can be concluded that in the work of Castelli and Motta (2010), the effect of sloping ground near footing has been evaluated using the limit equilibrium technique. The analysis studied several distances of the footing to the edge to find what they called the threshold distance at which the effect of the sloping ground is vanishes. Their analysis showed that the threshold distance increases with the increasing of the friction angle and varies from  $d/B=1$  for  $\varphi_u=0$  to  $d/B=6$  for a value of  $\varphi'=40^\circ$ . The embedment of the footing  $H$  does not affect the normalized threshold distance  $d/B$  significantly; however, it may affect the vertical limit load of the footing. In some cases, the seismic reduction in the bearing capacity due to the soil inertia can be of the same amount of the reduction produced by the inertia of the structure. The superposition of the effects can be used to consider the reduction caused by both seismic actions, but if the reduction is great due to the high seismic coefficient, the superposition principle will lead to unconservative design.

The limit separates at which the inclining ground never again influences the bearing limit builds, in the first place, with the expansion of the grating edge, second, with the expansion of the seismic coefficients and the slant point. For the static case, the implant profundity of the balance does not influence the limit separate fundamentally. In any case, the insertion profundity may bring about a significant increment of the bearing limit. Additionally, for the static case, the greatest lessening of the bearing limit happened when the balance is put straightforwardly at the edge of the incline. The review found that there is no critical contrast in the edge remove when the inactivity of structure or soil mass was considered.

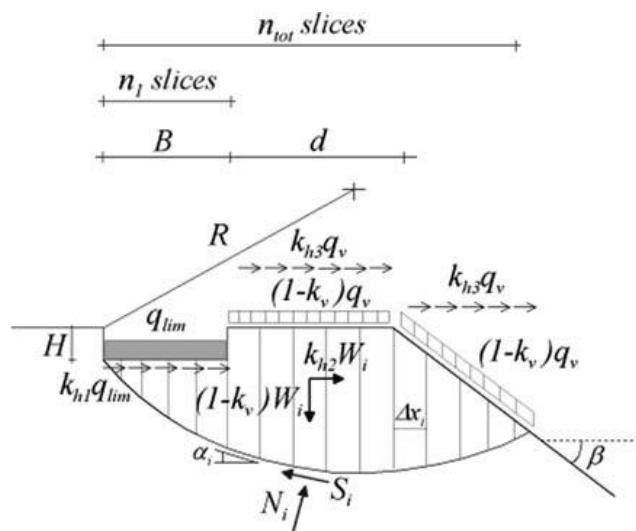


Figure 2.6 Failure mechanism and applied forces adopted in the analysis as Castelli and Motta (2010).

**Georgiadis (2010)** said that the undrained bearing capacity  $q_u$  of a foundation on level ground can be obtained from the bearing capacity equation presented by Meyerhof (1963). In contrast to the horizontal ground surface case, no exact solution is found for the bearing capacity of a foundation on a slope. The equation presented by Hansen (1961), and Vesic (1975) includes empirical factors for the case of sloping ground. There is also an empirical solution, that has been developed by Bowles (1996) considering the effect of distance from the edge of the slope but neither the slope height nor the soil properties.

In the work of Georgiadis (2010), three footing widths ( $B=1, 2$  and  $4\text{m}$ ) and three slope angles  $\beta=15^\circ, 30^\circ$  and  $45^\circ$  were considered. Different slope heights  $H$  and normalized footing distances  $\lambda=$  (footing distance/footing width) were used to evaluate the influence of these two parameters. According to the study, depending on the combination of the parameters, three failure modes have been found. The first two failure modes are bearing capacity failure, and the third one is overall slope failure.

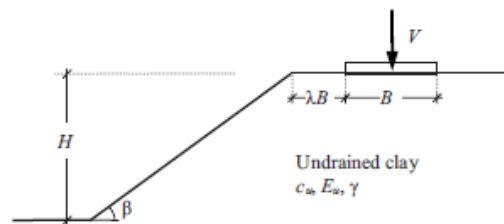


Figure 2.7 Problem definition as Georgiadis (2010).

**In summary**, Georgiadis (2010) made the following observations:

- Footing at the crest of slope

The results presented in the study were compared to solutions available in the literature for the same case, like Hansen (1961), Vesic (1975), Bowles (1996) and Kusakabe et al. (1981). All those solutions gave a linear decrease of the undrained bearing capacity factor with the increase of the slope angle. For the low  $C_u/B\gamma$  value of 0.5 it was not possible to obtain a bearing capacity type of failure; overall slope failure was always observed using finite element analysis.

- Footing at a distance from the slope

What Georgiadis (2010) did throughout his study was a comparison between his solution to the existing solutions of Meyerhof (1957), upper bound solutions by Kusakabe et al. (1981), and Azzouz and Baligh (1983).

#### **- Comparison to Meyerhof's Method**

Meyerhof (1957) presented a chart for the calculation of the undrained bearing capacity factor. For slope heights  $H$  smaller than the foundation width  $B$ , overall slope failure was assumed, and  $N_c$  was calculated as a function of the slope angle  $\beta$ , the stability number  $N_s = \gamma H / C_u$  and the ratio  $\lambda B / H$ . For  $H \geq B$ , bearing capacity failure was assumed and  $N_c$  was calculated from the slope angle  $\beta$  and the normalized footing distance  $\lambda$ . For  $H/B=1$ ,  $B=2\text{m}$ , and the distance from zero to 2m from the crest of slope, with  $\beta=30^\circ$ ,  $\gamma=20 \text{ kN/m}^3$  and  $C_u=40 \text{ kPa}$ , it's been found that Meyerhof's solution for the overall variation of  $N_c$  with  $H/B$  was unrealistic and doesn't compare well with a FE solution (FE shows initial reduction of  $N_c$  starting from horizontal ground value, followed by horizontal section of constant  $N_c$  presenting a bearing capacity failure, then final drop reflecting an overall slope failure. For  $H/B > 1$ ,  $\beta=30^\circ$  and  $C_u/\gamma B = 2.5$ , it was found that Meyerhof's solution overestimates the value of  $N_c$  by up to 6.5%, as compared to the FE results.

#### **- Comparison to Upper Bound solutions**

For the case of footing at the crest of slope ( $\lambda=0$ ), the study found that the upper bound and FE solutions are in excellent agreement. But for  $\lambda > 0$ , the upper bound solutions overestimate the value of  $N_c$  by 4.4% for  $\beta=30^\circ$  and 10% for  $\beta=45^\circ$ .

#### **- Comparison to Azzouz and Baligh's (1983) Method**

Azzouz and Baligh used a circular arc analysis. Their study found that at low values of  $\lambda$  the FE analysis predicts bearing capacity failure while for higher values, overall slope failure is predicted. The circular arc calculations by Azzouz and Baligh generally overestimate the undrained bearing capacity factor by up to 30% compared to the FE analysis. The study provided design charts, design equations and a suggested design procedure.

In conclusion, Azzouz and Baligh's results have been compared to the available solutions, and it was found that those solutions do not take into account some important parameters such as the distance of footing from the slope, the slope height and soil properties. The other solutions based on limit equilibrium and upper bound techniques, which are taking account of those

important parameters, were found to be less conservative than the FE method presented in their paper. The results in the paper showed that three failure modes identified, depending on H/B (normalized height to width ratio);

- At low H/B, the failure surface is restricted by the slope height, and the undrained bearing capacity factor  $N_c$  decreases with the increase of H/B.
- Bearing capacity failure occurs with a failure surface extending to the slope at a value of H/B between 0.25 and 0.75 depending on the slope angle.
- At high values of H/B, overall slope failure was detected, and the undrained bearing capacity factor  $N_c$  decreased in this failure mode to zero at the ultimate slope height.
- In most cases an increase of the distance of the footing from the slope increases the value of  $N_c$ .

**Shiau et al. (2011)** studied the undrained stability of footings on slope, they found that for a footing-on-slope system, the ultimate bearing capacity of the footing is governed by either the foundation bearing capacity or the overall stability of the slope. The combination of these two factors makes the problem difficult to solve.

There are various methods to solve this combined problem such as;

- Slip-line methods (Sokolovski 1960).
- Limit equilibrium techniques (Meyerhof 1957; Narita et al. 1990).
- Yield design theory.
- Upper and lower bounds (Davis and Booker 1973).
- Analytical upper bound solutions (Kusakabe et al. 1981).

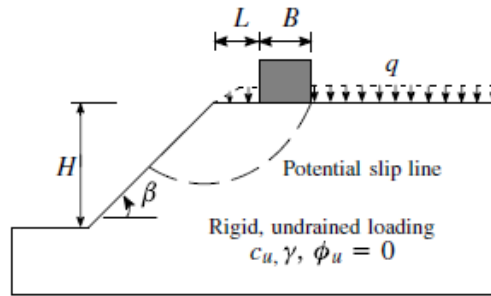


Figure 2.8 Problem notation and potential failure mechanism as Shiau et al. (2011).

**In more detail**, these methods can be characterized as follows;

The slip-line analysis is mathematically rigorous, but difficult to apply, especially for problems with complex geometry or complicated loading. While, limit equilibrium methods are less strict and can deal with a variety of complex boundary conditions, soil properties and loading conditions, but their accuracy is sometimes questioned (Shiau et al. 2011). The lower and upper bound theorems are based on statically admissible stress fields and kinematically admissible velocity fields. Most applications of limit theorems are based on the upper bound alone, because it is easier to develop a kinematically admissible failure mechanism than a statically admissible stress field. Using a very different approach, the finite element-based methods have been introduced by Sloan (1988, 1989), Sloan and Kleeman (1995) and Lyamin and Sloan (2002a, b) which allow large two-dimensional problems to be solved on a standard personal computer. This paper studied the undrained bearing capacity of a rigid strip footing of width ( $B$ ) resting near a slope on a homogeneous clay soil with a slope angle ( $\beta$ ) and slope height ( $H$ ) at distance ( $L$ ) from the edge of the slope using FE limit analysis formulations of the lower and upper bound theorems. The soil behavior is assumed to be undrained with shear strength ( $C_u$ ). For foundations near slopes, Shiau et al. (2011) found that the soil's unit weight has a great effect on the bearing capacity, conversely the bearing capacity of same foundation on level ground, where the soil unit weight has no effect on the ultimate bearing capacity. Through his study,  $H/B$  ratio was considered to be (3) at all times which caused above toe failure. In detail, the paper considered the effect of several factors on the bearing capacity of the foundation separately as follows;

### **- Effect of footing roughness**

The study found that the footing roughness may or may not be significant depending on the relative movement between the footing and the bearing soil. For the smooth case, the rigid footing tends to tilt and slide also discontinuous velocity was observed at the base of the footing. In contrast, for the rough case, both the footing and soil displace together, and little relative movement was observed at the soil/footing interface. A larger failure zone was obtained, indicating greater bearing capacity for rough footings. It was noted; however, that true footing roughness is likely to lie between the perfectly smooth and perfectly rough extremes.

### **- Effect of the dimensionless strength ratio $C_u/\gamma B$**

It was found that for  $L/B$  ranging between 0 and 6 and  $C_u/\gamma B$  up to 10, the dimensionless bearing capacity  $P/\gamma B$  decreases linearly with the strength ratio until it becomes non-linear and rapidly approaches zero at a particular value of  $C_u/\gamma B$  in which no feasible solution is available from the numerical analysis. The linear portion of the curve indicates bearing capacity failure, which is contained within the face of the slope, where the ratio  $H/B$  has no effect. The non-linear portion shows a complex interaction between the footing bearing capacity and the overall slope stability. The transition from bearing capacity failure to overall slope stability failure occurs when  $P/\gamma B=0$ .

### **- Effect of the slope angle $\beta$**

It was observed that the footing capacity decreases as the slope angle increases or the strength ratio  $C_u/\gamma B$  reduced because of reduction in  $C_u$  or an increase in  $\gamma$ .

### **- Effect of the footing distance to the crest ( $L/B$ )**

The increase in footing capacity tends to stop at certain value of  $L/B$ . For  $C_u/\gamma B =1$ , the value of  $P/\gamma B$  reaches a constant value of 5.06 (an average value between the lower and the upper bound). According to the study, for a vertical cut with  $C_u/\gamma B$  and  $L/B=5$ , the pattern of the velocity diagram is essentially identical to that for a footing placed on level ground, indicating that the vertical cut does not influence the footing performance.

### **- Effect of surcharge ( $q/\gamma B$ )**

Shiau et al. (2011) observed that the existence of surcharge on the slope surface could either increase or decrease the footing capacity depending on both  $\beta$  and  $L/B$ . An increase in  $q/\gamma B$  tends to increase the footing capacity for all ratios of  $L/B$ . On the other hand, for a vertical cut, an increase in  $q/\gamma B$  will reduce the bearing capacity for all ratios of  $L/B$ .

### **- Effect of $H/B$**

The results developed during Shiau et al.'s study (2011) were completed for a value of  $H/B=3$  to ensure that a bearing failure mechanism is developed without influence of the bottom or toe of the slope. It was found that all failure mechanisms are essentially unaffected by the toe of the slope when  $H/B \geq 3$ . For slopes in which  $\beta = 90^\circ$  and is  $L/B=1$  to 4, the location of the toe was found to affect the collapse load only very slightly. It can be concluded that the results in the study are applicable for all values of  $H/B \geq 3$ . For values of  $H/B < 3$ , further upper bound analyses were performed to investigate the effect of  $H/B$  for a range of slopes and values of  $L/B$ . It was found, provided that overall slope failure does not occur, that the normalized bearing capacity  $p/\gamma B$  will remain unchanged as  $H/B$  is decreased below 3 until reaching a critical value of  $H/B$ . This critical value of  $H/B$  was found to be a function of the problem geometry. At a very small critical value of  $H/B$ , there exists a sharp increase in the normalized bearing capacity as the value approaches that for a surface footing with no slope present. Physically, this makes perfect sense.

For very small values of  $H/B$ , the influence of the slope on the collapse mechanism becomes negligible, and, in fact, the slope becomes part of the bearing capacity failure mechanism itself.

**Chakraborty and Kumar (2013)** also provided a paper to study the bearing capacity of foundations on slopes. They found that for footings placed on level ground, the bearing capacity factor due to soil unit weight  $N_\gamma$  is affected by the footing roughness, while  $N_c$  and  $N_q$  remain invariant with respect to the roughness condition. It's not known how the footing roughness condition affects the value of bearing capacity factors  $N_c$ ,  $N_q$  and  $N_\gamma$  for foundation laid on sloping ground. Most of the available analysis for footings on sloping ground often requires assumptions related to the geometry of failure surface, except for those which using the displacement-based FE limit analyses technique. The solution determined from limit analysis in combination with finite



elements is very close to the true solution for a material following an associated flow rule. In Chakraborty and Kumar's paper, the bearing capacity factors  $N_c$ ,  $N_q$  and  $N_\gamma$  have been calculated for smooth and rough strip footings with width  $B$  placed on a sloping ground with a slope angle  $\beta$  with the horizontal, using the lower bound FE limit analysis in combination with non-linear programming. The soil was assumed to be plastic and followed the Mohr-Coulomb failure criterion and an associated flow rule.

The magnitudes of the bearing capacity factors  $N_c$ ,  $N_q$  and  $N_\gamma$  reduce with the increase of the slope angle and increase with the increasing soil friction angle. For rough footings, bearing capacity factors increase significantly compared to the smooth ones.

It's been noticed that the lower bound theorem of the limit analysis provides a true lower bound solution only if the material satisfies an associated flow rule.

The conclusions of the study are that; along sloping ground surface, the bearing capacity factors become considerably higher for rough footings compared to those with a smooth base. The extent of the plastic zone around the footing base also becomes greater for rough footings. As expected, the value of the bearing capacity factors reduces extensively with an increase in the ground inclination.

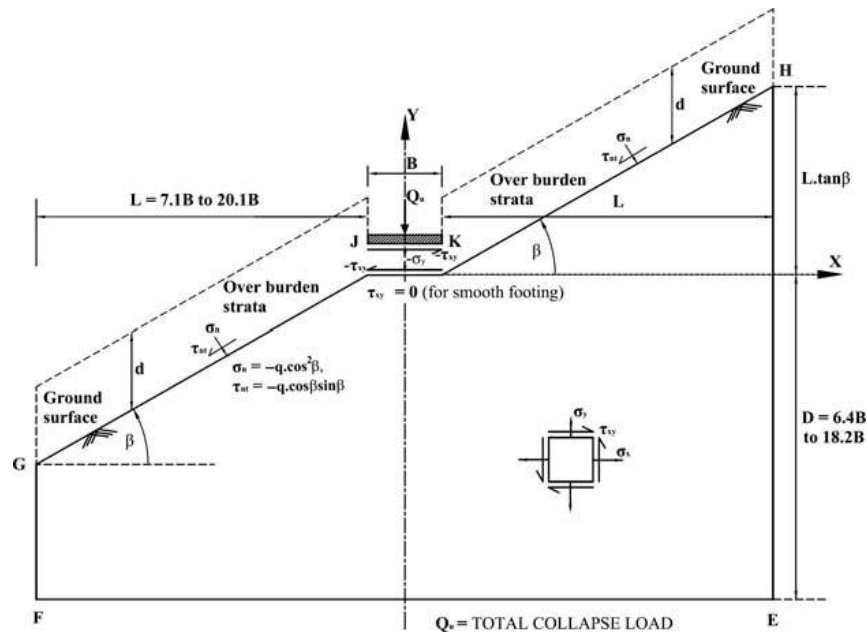


Figure 2.9 Chosen domain and the stress boundary conditions as Chakraborty and Kumar (2013)

**Leshchinsky and Xen (2017)** have found that the studies done by Meyerhof to find the ultimate bearing capacity for footings placed near or on slopes were carried out for purely cohesionless or purely cohesive soils. They found it important to consider the combination of the cohesion and internal friction of the soil. Leshchinsky and Xen have done their study by using an upper bound limit state plasticity failure discretization scheme known as Discontinuity Layout Optimization (DLO) to present revised bearing capacity coefficient for foundation adjacent to  $c'$ ,  $\phi'$  slopes by applying revised factors for footings not adjacent but in proximity of slopes.

Parametric studies of the ultimate bearing capacity for a variety of combinations of  $c'$ - $\phi'$  soils and geometric configurations for footings placed near slopes were performed. The study provides revisions to the current, conservative bearing capacity factors given by Meyerhof (1957) as used in AASHTO (2012), covering a gap in available design charts. The results are presented in the form of reduction coefficients that can be applied to the classical bearing capacity equation.

## **2.5 Shear strength reduction technique**

### **2.5.1 General**

The Finite Element Method ended up noticeably since it initially used in geotechnical engineering in 1966. In the mid 1970s, techniques for applying the FEM to slope stability analysis started appearing in geotechnical literature. They were for the most part dependent on a methodology that streams normally from the definition of slope factor of safety and is currently commonly referred to as the Shear Strength Reduction (SSR) technique. By definition, the factor of safety of a slope is the “ratio of actual soil shear strength to the minimum shear strength required to prevent failure,” or the factor by which the shear strength of the soil should be reduced to bring a slope to the verge of failure (Duncan 1996). In the SSR finite element technique, the strength of the slope material is assumed to be elasto-plastic. The material shear strengths are gradually reduced until collapse occurs.

For Mohr-Coulomb material shear strength reduced by a factor of safety (F) can be determined from the equation:

$$\tau/F = c'/F + \tan \varphi'/F \dots\dots\dots \text{Eq. 2.8}$$

Where  $\tau$ = shear stress

$c'$  = apparent cohesion

$\varphi'$  = angle of internal friction

This equation can be re-written as:

$$\tau/F = c^* + \tan \varphi^* \dots\dots\dots \text{Eq. 2.9}$$

Where  $c^*=c'/F$

$\varphi^*=\arctan(\tan\varphi'/F)$

### 2.5.2 Basic algorithm

For Mohr-Coulomb materials, the steps for systematically searching for the critical factor of safety value, F, which brings a previously stable slope to the verge of failure, are as follow

**Step 1:** Develop an FE model of a slope, using the deformation and strength properties established for the slope materials. Compute the model and record the maximum total deformation in the slope.

**Step 2:** Increase the value of F and calculate factored Mohr-Coulomb material parameters as described above. Enter the new strength properties into the slope model and re-compute. Record the maximum total deformation.

**Step 3:** Repeat Step 2, using systematic increments of F, until the FE model does not converge to a solution, i.e. continue to reduce material strength until the slope fails. The critical F value just beyond which failure occurs will be the slope factor of safety.

### **2.5.3 Advantages**

The elasto-plastic SSR FE approach offers several advantages over traditional limit-equilibrium analysis. First, it eliminates the need for a prior assumption on failure mechanisms (the type, shape and location of failure surfaces) or for the inter-slice shear force distribution, as well as it is applicable to many complex conditions and can give information for stresses, movements and pore pressure which is not possible with the limit equilibrium method, and most importantly is the critical failure surface is found automatically.

Generally, in the Finite Element analysis, it is difficult to trace the failure slip surface of a slope based on the stress failure criterion. The shear strain failure criterion is employed herein. It's assumed that the failure mechanism of slope is directly related to the development of the shear strain in the shear strength reduction technique. Also, the existence of the shear strength dependency of the strain is assumed.

## **CHAPTER 3**

### **NUMERICAL MODEL**

#### **3.1 General**

The numerical model was developed using the Finite Element technique provided by the computer program Rocscience RS2 9.0. from Rocscience Inc., formerly Phase<sup>2</sup>, is a 2D finite element software for soil and rock applications founded in 1996 in Toronto, Canada. RS2 is a powerful software covers most of geotechnical engineering problems and used in 280 universities in over 120 countries (RS2 Users' manual 2019). It can simulate the geometry of the slope and foundation as well as the soil and loading conditions. The results proposed by the finite element technique are broadly acceptable by current industry.

The objective is to examine the ultimate bearing capacity of a strip foundation near a slope and study the effect of the parameters believed to govern the stability of a system consists of foundation and slope such as slope height and angle ( $H$  &  $\alpha$ ), soil properties ( $c$  &  $\phi$ ) and the distance from the foundation to the slope ( $x$ ).

#### **3.2 Problem definition**

A typical strip footing of width,  $B$  of 1m and embedment depth,  $D$  of 1m located at distance ( $x$ ) from an edge of slope has height ( $H$ ) and angle ( $\alpha$ ) (figure 3.1). A two-dimensional finite element model was used to develop the problem of a continuous foundation near slope. The slope angle in this model was assumed to be uniform across the foundation length.

The size of the model was chosen to be wide enough to ensure that there no additional stresses or displacement were developed at the boundaries. Figure 3.1 shows typical dimensions of the model. For conservative reasons, the right edge of the slope was constantly changing according to the slope angle and the location of the foundation to vary between 130 and 180 m.

To perform the finite element calculation, the model was divided into finite number of triangular elements. A 6-noded triangular elements was used in the finite element mesh, according

to the Users' manual of the software used to develop the problem, RS2, and to carry out an SSR analysis (Shear Strength Reduction), a 6-noded triangular element should be used.

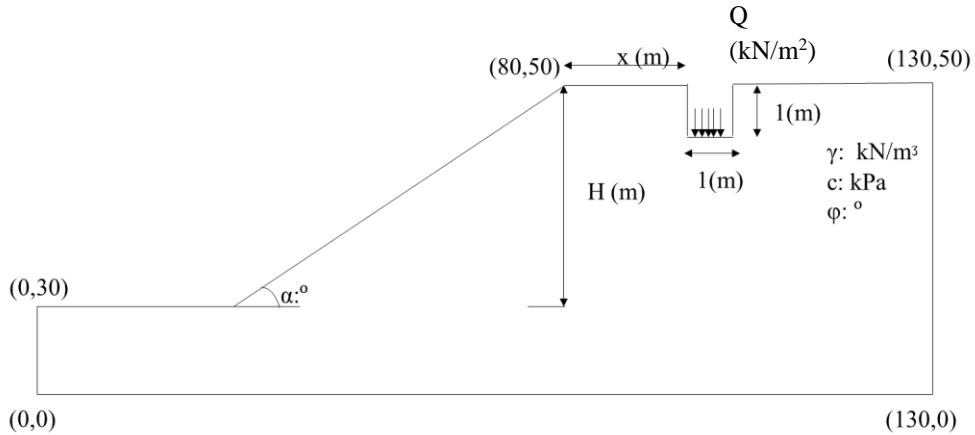


Figure 3.1 Geometry of the problem and model size.

### 3.2.1 Boundary conditions

In the model used for the analysis, the boundaries for finite element mesh are shown in figure 3.2. The boundaries to determine the slope are virtual boundaries while in reality, the ground surface extends infinitely in all directions. It necessary to be mentioned that the boundaries have been chosen to be far enough from the area where the failure would take place and the stress remained constant during loading. To simulate this case and to limit the size of the model in the finite element method, the three edges of the numerical model right, left and bottom were chosen to have movement restrain in x and y directions while the top edge of the model was always free.

The footing in this model was flexible strip footing with dimensions of 1m width, B, and 1m embedment depth, D, below ground surface.

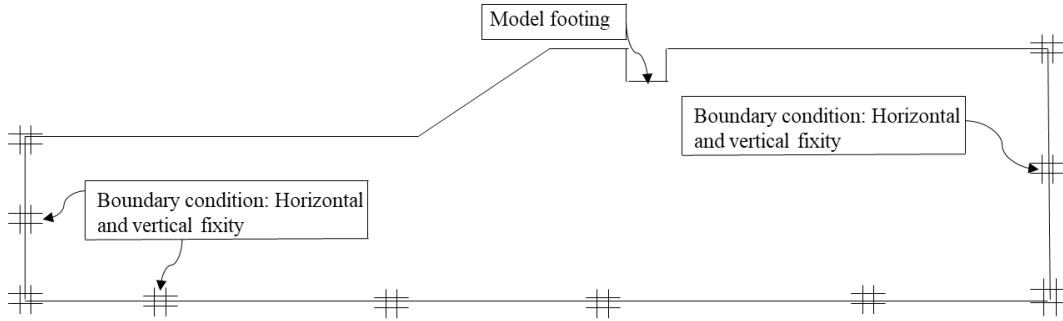


Figure 3.2 Boundary conditions.

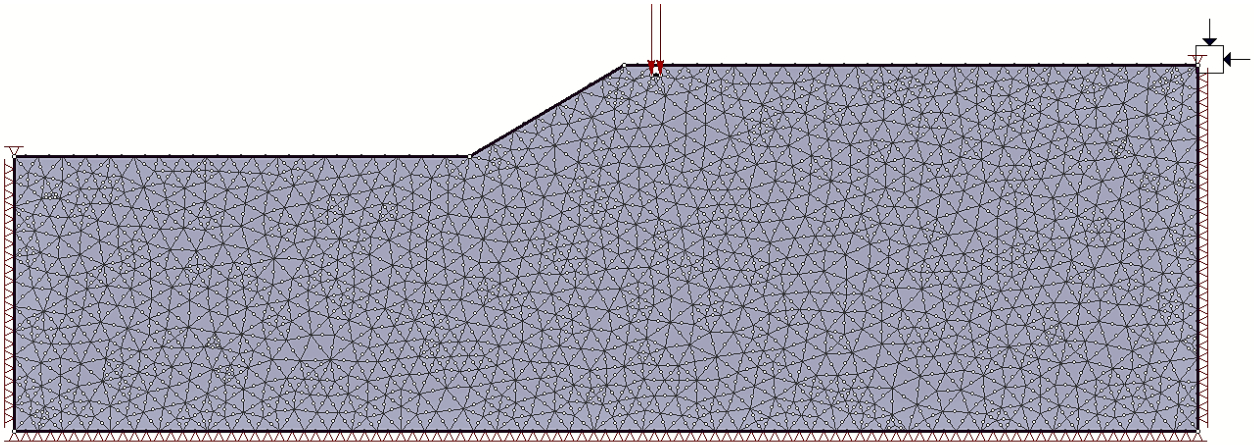


Figure 3.3 Finite element mesh.

### 3.2.2 Constitutive law

Mohr-Coulomb failure criteria has considered to model the shear strength characteristics of the soil, this model is generally used in geotechnical applications. The Mohr-Coulomb failure criterion can be written as the equation for the line that represents the failure envelope. The general equation is,

$$\tau_f = c + \sigma_f \tan \varphi \dots\dots\dots \text{Eq. 3.1}$$

Where  $\tau_f$  = shear stress on the failure plane  
 $c$  = apparent cohesion

$\sigma_f$  = normal stress on the failure plane

$\phi$  = angle of internal friction

These parameters are well known in engineering practices and obtained from lab tests.

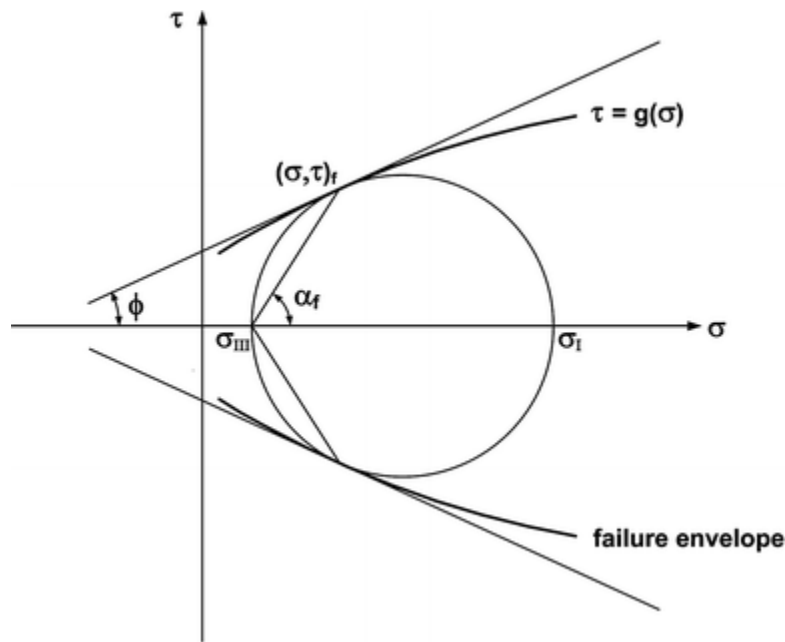


Figure 3.4 Mohr-Coulomb failure criteria as Labuz and Zang (2012).

For considering Mohr-Coulomb failure Criteria, certain assumptions have been made;

1. Soil is elasto-plastic material,
2. Young's modulus,  $E$ , does not depend on the stress level,
3. Cohesion,  $c$ , is constant with depth,
4. Soil is homogenous and isotropic.

### 3.3 Test Procedure

For one model has slope geometry of  $H$  and  $\alpha$ , soil parameters of  $c$ ,  $\phi$  and  $\gamma$ , and distance  $x$  from the foundation to the slope, around 8 different loading stages were generated by RS2 9.0 starting with no loading case then increasing the load gradually until failure occurs. The three types



of failure detected are either slope failure, bearing capacity failure or combination of both. 64 models with 8 loading stages to make a total number of runs of about 512 runs were developed. For each model, the vertical displacement corresponding to each loading stage was computed, then one load-settlement curve was drawn for each test individually. From the curve, the allowable load was obtained which corresponded to the allowable settlement of the foundation and achieved a factor of safety of at least 3 for the whole problem. The resulted load from this phase was used to perform phase II which concerned about the distance by applying the load on the foundation and change the distance, starting with the foundation sat directly at the edge of the slope with distance increasing until the factor of safety remained constant. The target  $x$  in this phase is the minimum distance to locate a foundation near a slope to maintain the stability of the whole system.

### **3.3 Monte Carlo Data Sampling**

Monte Carlo is a computational technique dependent on developing a random procedure for a problem and functioning a numerical experiment by N-fold sampling from a random sequence of numbers with a prescribed probability distribution. The technique originated from the complicated diffusion problems that were encountered in the early work on atomic bomb in the 1940's. The technique can be used for the problems that are probabilistic or deterministic. Commonly, Monte Carlo sampling is a followed technique used for modeling of complex situations where many random variables are involved and/or analytical or numerical solutions don't exist or are too difficult to implement. It is not a statistical tool, depends on repeated random sampling and provides generally approximate solutions. Generally, the technique works through certain steps;

- Determine the statistical properties of possible inputs,
- Generate many sets of possible inputs,
- Perform a deterministic calculation with these sets,
- Analyze statistically the results.

### **3.4 Parameters**

The variable parameters tested in the research are soil parameters ( $c$ ,  $\phi$  &  $\gamma$ ), slope geometry ( $H$  &  $\alpha$ ), distance from the foundation to the slope ( $x$ ) and the allowable load ( $Q$ ).

For phase I of the analysis, the variable parameters were changing constantly for each model to examine the ultimate bearing capacity of the foundation, thus, determine the allowable load. While for phase II, parametric study was conducted to test the effect of the key parameters which are the slope height and angle, the angle of internal friction of the soil and the distance from the foundation to the slope on the stability of the problem, hence, determining the minimum distance required to locate a foundation near slope in order to preserve the stability of the system. The parameters tested in phase II are these of most common for soils and slopes.

#### **3.4.1 Soil Unit weight**

The unit weight of the soil ( $\gamma$ ) varies between  $16 \text{ kN/m}^3$  for very loose sand until  $22 \text{ kN/m}^3$  for hard clay (Peck 1974).

#### **3.4.2 Soil Cohesion**

The cohesion parameter of the soil ( $c$ ) is almost zero for cohesionless soils and can increase to reach a maximum of  $100 \text{ kN/m}^2$  for hard dry clayey soils (Peck 1974).

#### **3.4.3 The angle of internal friction**

Pure clay has almost zero value of internal friction then surges with increasing of sand content and density to reach an approximate value of  $40^\circ$  for compact sandy loam soils, normally pure clay soils are not found on topsoils. For very loose sand,  $\phi$  is less than  $29^\circ$  and goes up to greater than  $41^\circ$  for very dense sand. Less than  $3^\circ$  for very soft clay and can go higher than  $25^\circ$  for hard clay soils (Peck 1974).

#### **3.4.4 Slope Height and angle**

Slopes can be classified to extra high, high, medium and low slope according to the Geological Engineering Manual (2006) and depending on the slope height. The slope can also be described as steep, medium and gentle slope depending on the slope angle according to the same

reference. Table 3.1 below shows the classification of the slope depending on the slope height and angle.

Table 3.1 Classification of slopes (Geological Engineering Manual 2006).

<b>1. Slope height</b>	
Slope type	Slope height (m)
Extra High slope	>100
High slope	30 to 100
Medium slope	10 to 30
Low slope	<10
<b>2. Slope angle</b>	
Slope type	Slope angle (degrees)
Steep slope	60-90
Medium slope	30-60
Gentle slope	0-30

### 3.4.5 Distance from the foundation to the edge of the slope

The distance range from the foundation to the edge of the slope has been guided by the distances stated in Meyerhof theory of 1957 on purely cohesive and cohesionless materials. The theory reported that beyond about 2 to 6 times the foundation width and depending on  $\phi$  and D/B (depth and width of foundation, which were always 1m in this research), the bearing capacity of the foundation is independent of the inclination of the slope, concluding that the influence of the slope on the bearing capacity coefficients of the foundation vanishes at distance 6B, thus the foundation acts as on horizontal surfaces. Upon Meyerhof's conclusion, 1957, the range of the distance was taken from 0 to 6B.

### 3.5 Summary of problem key parameters range

Table 3.2 summarizes the ranges of the parameters used to create the problem.

Table 3.2 Limits of variable parameters

Slope Geometry						Soil Parameters					
Height H (m)		Angle $\alpha^\circ$		Distance from slope x (m)		$\gamma$ (kN/m <sup>3</sup> )		$\phi^\circ$		c (kPa)	
min	max	min	max	min	max	min	max	min	max	min	max
3	30	10	70	0	6	16	22	6	42	5	60

### 3.6 Failure scheme

The study tracked two important correlated issues when dealing with a system of a foundation and slope which are the bearing capacity of the foundation and the stability of the slope. It's been found that the ultimate bearing capacity of the foundation is governed by either the bearing capacity or the overall stability of the slope, the combination of these two factors makes the problem difficult to solve (Shiau et al. 2011).

The types of failure conducted in the study either slope failure which the slope fails while the foundation hasn't reached its ultimate bearing capacity yet or bearing capacity failure where the foundation reached its ultimate bearing capacity while the slope still stable and in some cases with high factors of safety. The three primary bearing capacity failure according to Vesic (1963) are general shear failure, local shear failure and punching shear failure.

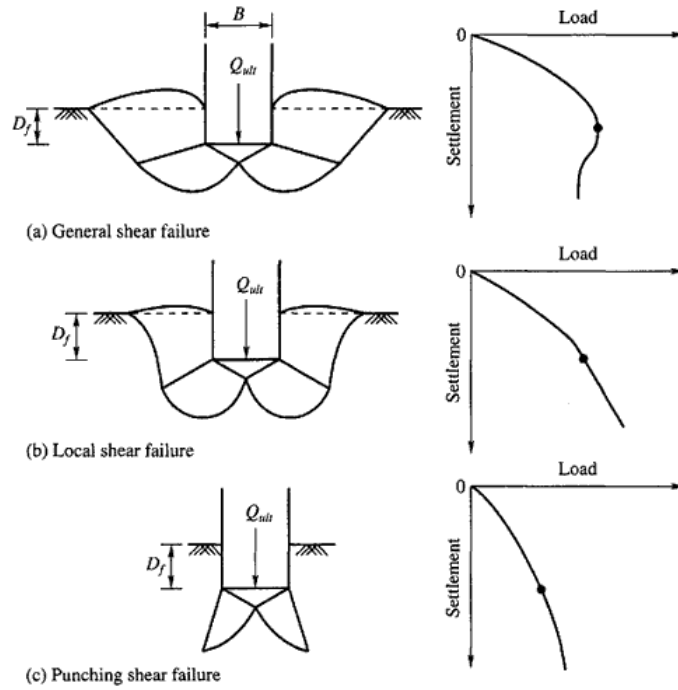


Figure 3.5 Modes of bearing capacity failure (Vesic 1963).

### 3.6.1 Allowable settlement

Settlement of shallow foundation is divided to two major categories; elastic (immediate) settlement and consolidation settlement (primary and secondary).

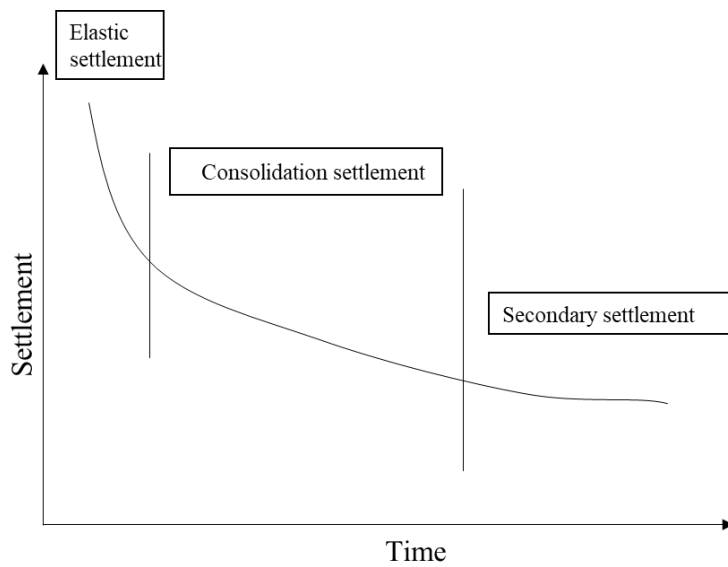


Figure 3.6 Settlement vs. time.

Skempton and MacDonald (1956) proposed limiting values for maximum settlement to be used for building purposes, maximum settlement in sand 32mm and in clay 45mm.

### 3.6.2 Allowable bearing capacity

Loads are transferred from the foundation into the ground so that the ground deforms and forms settlement. The allowable bearing capacity of shallow foundation is the maximum loading can be applied over a unit area, which the soil is able to resist without undergoing any excessive settlement and shear failure. The allowable load can be related to the allowable bearing capacity by the following relation.

$$Q_{all} = q_{all} \cdot A \dots\dots\dots \text{Eq. 3.2}$$

Where,  $Q_{all}$  = allowable load on footing,

$q_{all}$  = allowable bearing capacity

$A$  = contact area between the soil and footing.

The allowable bearing capacity can be defined in terms of the ultimate bearing capacity and the factor of safety as given below,

$$q_{all} = \frac{q_{ult}}{F.S.} \dots\dots\dots \text{Eq. 3.3}$$

Where,  $q_{ult}$  = ultimate bearing capacity,

F.S. = factor of safety.

The bearing capacity equation for shallow foundation on flat ground formulated by Terzaghi (1943).

$$Q_u = cN_c + \gamma DN_q + 0.5\gamma BN_\gamma \dots\dots\dots \text{Eq. 3.4}$$

Where,

$c$  = Cohesion of soil,

$\gamma$  = unit weight of soil

D =depth of footing,

B = width of footing,

$N_c, N_q, N_\gamma$ : Terzaghi's bearing capacity factors depend on soil friction angle,  $\phi$ .

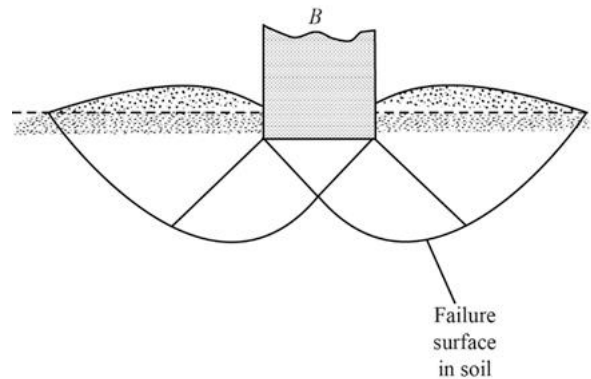


Figure 3.7 Failure plane for shallow foundation on flat ground by Terzaghi (1943).

While the bearing capacity for shallow foundation rested near slope was given by Meyerhof (1957).

$$q = cN_{cq} + \gamma \left( \frac{BN}{2} \right)_{\gamma q} \dots\dots\dots \text{Eq. 3.5}$$

Where,

c =Cohesion of soil,

$\gamma$  = unit weight of soil,

B = width of footing,

$N_{cq}, N_{\gamma q}$ : bearing capacity factors depend on the distance (b) from the edge of slope as well as Slope angle ( $\beta$ ), friction angle of the soil ( $\phi$ ) and the foundation depth and width (D/B).





Generally, with mesh density increasing, the numerical solution produced from the model will tend toward a unique value. The mesh is said to be converged when mesh refinement provides an insignificant change in the solution.

Initially, the numerical model shown in figure 3.9 was discretized by approximate number of 2438 elements increased up to 56266 elements. Table 3.3 summarizes the factors of safety and the computation time corresponding to different number of elements, it's shown from the table that the critical factor of safety was to be found 2.06 used as a referee to compute the differences in the factors of safety using the following equation.

$$F.S._{error} = (F.S. - 2.06) / 2.06 \dots\dots\dots \text{Eq. 3.6}$$

Table 3.3 The F.S. and computation time corresponding to different number of elements.

<b>Number of elements</b>	<b>F.S.</b>	<b>F.S. Differences</b>	<b>Computation time (min.)</b>	<b>Normalized computation time</b>
2438	2.1	1.94%	4.04	1.00
3872	2.07	0.48%	5.49	1.36
7824	2.06	0.00%	13.29	3.29
19902	2.06	0.00%	38.47	9.52
56266	2.06	0.00%	180.05	44.57

It's seen from the values of the factor of safety that the mesh was converged at 7824 mesh elements at which the factor of safety remained constant for higher numbers of elements and achieved an error of 0%.

### 3.8 Validation of the numerical model

#### 3.8.1 Bearing capacity

Meyerhof (1963) proposed a formula for calculation of ultimate bearing capacity like the one proposed by Terzaghi (1943) but introducing further foundation shape and depth coefficients ( $S_c, S_q, S_\gamma$  and  $D_c, D_q, D_\gamma$ ). The ultimate bearing capacity can be determined according to Meyerhof general equation given below;

$$q_u = cN_{cq} + \gamma \left(\frac{BN}{2}\right)_{\gamma q} \dots\dots\dots \text{Eq. 3.7}$$

Where,  $c$  = Cohesion of soil,

$\gamma$  = unit weight of soil,

$B$  = width of footing,

$N_{cq}, N_{\gamma q}$ : bearing capacity factors depend on the distance ( $b$ ) from the edge of slope as well as Slope angle ( $\beta$ ), friction angle of the soil ( $\phi$ ) and the foundation depth and width ( $D/B$ ) and can be determined from the table below;

Table 3.4 Meyerhof's bearing capacity coefficients.

$\phi$	$N_c$	$N_q$	$N_\gamma$
0	5.1	1	0
5	6.5	1.6	0.1
10	8.3	2.5	0.4
15	11	3.9	1.2
20	14.9	6.4	2.9
25	20.7	10.7	6.8
30	30.1	18.4	15.1
35	46.4	33.5	34.4
40	75.3	64.1	79.4

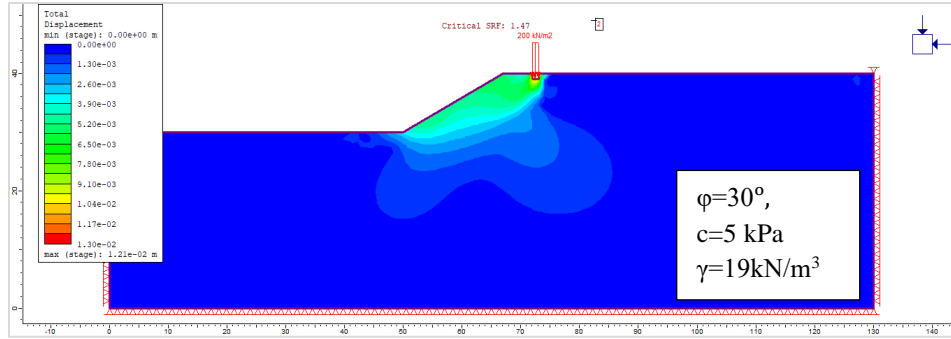


Figure 3.10 Bearing capacity deduced by the software.

Bearing capacity by software

$$q_{all} = Q_{all} / A = 200 / 1 = 200 \text{ kN/m}^2$$

$$q_u = q_{all} \times \text{F.S.} = 200 \times 1.47 = 294 \text{ kN/m}^2$$

Bearing capacity according to Meyerhof (1963)

$$q_u = 5 \times 30.1 + 0.5 \times 19 \times 15.1 = 294 \text{ kN/m}^2$$

### 3.8.2 Slope Stability

Giam and Donald (1988) presented a set of slope stability problems to test the abilities of computer programs to handle a full range of problems likely to be met in geotechnical practices, including complex soil profiles and slope geometries, pore water pressures, seismic effects, earth and rock dams, surcharges and non-circular failure surfaces. Results were compared to both Slide 5.0 limit equilibrium and referee values from published sources found in reference material such as journal and conference proceedings.

Figure 3.11 shows a verification example of a simple case of slope stability analysis in a homogenous soil. The model was initially established in the software with the geometry and soil properties indicated on the figure. The factor of safety and its corresponding failure slip line is required. The deduced results are presented in table 3.5 for comparison, as well as in figures 3.12 & 3.13 respectively. The referee F.S. is 1.00 as per Giam and Donald (1988).

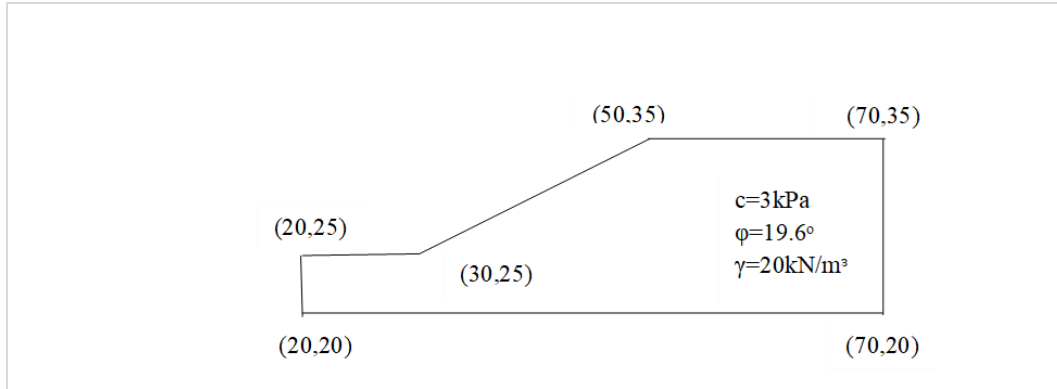


Figure 3.11 Geometry of the problem.

### 3.8.2.1 Results

Table 3.5 Results of different methods for slope stability analysis

Method	Factor of Safety
SSR (RS2)	1.01
Bishop (Slide)	0.987
Spencer (Slide)	0.986
Gle (Slide)	0.986
Janbu Corrected (Slide)	0.99

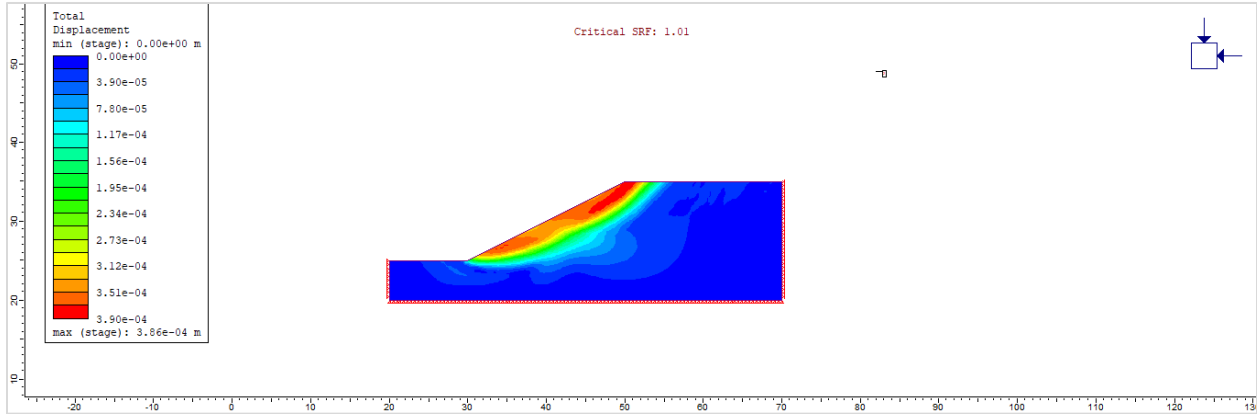


Figure 3.12 Results produced using SSR shows critical SRF 1.01.

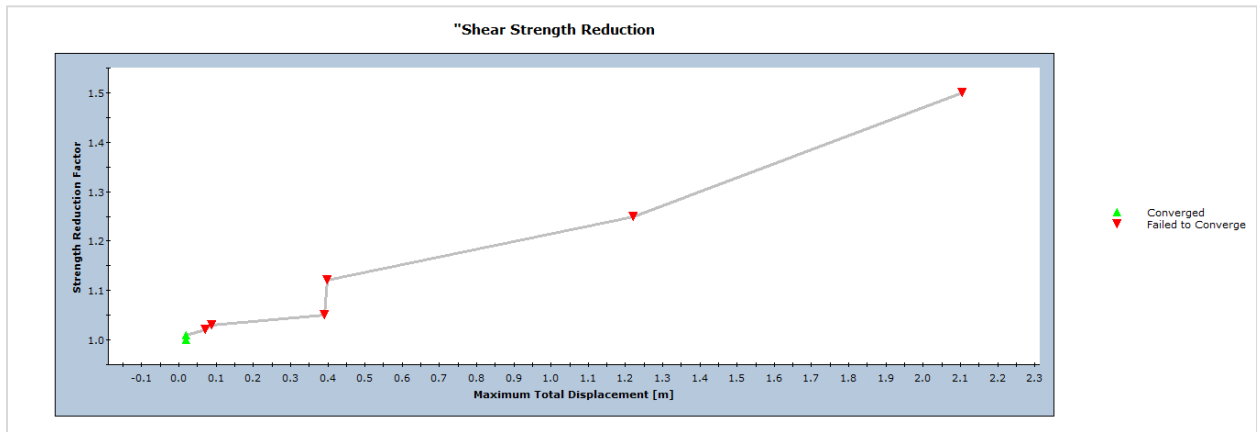


Figure 3.13 SSR convergence graph shows critical SRF 1.01 at max. total displacement 0.019m.

### 3.9 Input parameters and results

Table 3.6 shows the input parameters for 64 tests performed using RS2 9.0 software, along with the output results in terms of the allowable load corresponding to the allowable settlement for each test with the total factor of safety (F.S.) for the problem.

Test No.	Soil parameters			Slope height (H) (m)	Slope angle ( $\alpha$ ) ( $^{\circ}$ )	Distance to slope (x) (m)	Q allowable (kN/m <sup>2</sup> )	Factor of safety (F.S.)
	$\gamma$ (kN/m <sup>3</sup> )	$\varphi$ ( $^{\circ}$ )	c (kPa)					
1	20.00	26.14	26.0	20.0	12.0	0.0	200	3.1
2	20.01	40.68	18.0	22.0	40.0	5.0	200	2.98
3	19.90	35.81	33.0	20.0	27.0	5.0	200	2.43
4	20.10	24.59	31.0	25.0	47.5	2.5	200	1.05
5	20.30	37.33	15.0	26.0	45.0	2.5	200	1.35
6	19.98	13.59	45.0	13.0	20.0	1.5	100	2.67
7	20.02	32.35	15.0	11.0	28.0	3.5	50	2.16
8	19.91	31.07	12.0	21.0	52.0	4.0	Slope Failure	
9	19.89	29.78	18.0	25.0	12.0	5.0	100	3.43
10	20.03	26.14	26.0	23.0	18.0	1.0	100	2.36
11	20.10	36.87	27.0	8.0	13.0	1.0	50	5.6
12	19.95	24.07	27.0	10.0	55.0	3.5	50	1.49
13	20.00	35.81	33.0	25.0	48.0	1.0	200	1.4
14	21.04	6.00	33.8	14.0	30.0	1.5	100	1.32
15	19.00	40.00	5.0	27.0	20.0	3.0	100	1.9
16	18.30	20.80	39.0	20.0	26.0	5.5	200	1.83
17	16.00	19.10	42.0	16.0	58.0	2.0	200	1.31
18	20.50	29.40	22.0	25.0	47.0	2.3	50	1.07
19	19.66	13.00	59.2	8.0	56.5	5.0	200	1.93
20	20.64	18.00	37.5	26.0	23.0	5.0	200	1.56
21	21.20	24.10	37.0	30.0	28.0	1.5	200	1.55
22	21.80	36.90	27.0	19.0	14.5	1.0	Slope Failure	
23	22.00	22.40	33.0	21.0	33.0	1.5	200	3.84
24	20.30	16.00	49.0	14.0	38.0	2.5	200	1.6
25	19.00	35.00	5.0	18.0	31.0	2.5	200	1.63
26	18.70	27.31	14.0	23.0	13.5	2.0	Slope Failure	
27	20.60	26.14	26.0	25.0	37.0	3.3	200	1.34

Test No.	Soil parameters			Slope height (H) (m)	Slope angle ( $\alpha$ ) ( $^{\circ}$ )	Distance to slope (x) (m)	Q allowable (kN/m <sup>2</sup> )	Factor of safety (F.S.)
	$\gamma$ (kN/m <sup>3</sup> )	$\phi$ ( $^{\circ}$ )	c (kPa)					
28	22.30	27.50	26.0	12.0	21.0	5.0	50	2.6
29	20.30	14.50	39.0	8.0	55.5	4.0	200	1.46
30	20.30	29.80	18.0	16.5	31.0	0.5	200	1.55
31	20.20	37.30	15.0	14.0	13.0	2.0	50	4.36
32	20.40	23.90	14.9	11.0	18.0	2.5	50	2.32
33	20.00	25.78	40.0	14.0	19.0	5.0	200	4.68
34	19.00	35.81	33.0	25.0	64.0	1.3	Slope Failure	
35	20.00	40.68	18.0	17.0	45.0	6.0	200	1.62
36	20.02	26.14	26.0	23.0	15.0	0.0	Slope Failure	
37	20.10	36.87	27.0	27.0	66.0	2.3	Slope Failure	
38	19.99	24.07	27.0	24.0	70.0	1.8	Slope Failure	
39	20.04	25.78	40.0	23.0	45.0	4.0	200	1.31
40	20.01	41.59	15.0	8.0	22.0	4.0	200	3.47
41	19.99	26.50	16.0	15.0	10.0	6.0	200	3.08
42	20.02	14.60	48.0	6.0	39.0	1.5	200	2.03
43	20.10	9.84	44.0	4.0	28.0	4.0	100	3.35
44	20.03	26.50	16.0	17.0	41.0	3.0	100	1.18
45	20.00	5.12	43.0	14.0	33.0	6.0	200	1.14
46	21.00	27.31	14.0	18.0	35.0	4.5	200	1.24
47	19.68	9.00	36.4	20.0	29.0	5.5	Slope Failure	
48	20.64	18.00	37.5	27.0	51.0	4.0	Slope Failure	
49	19.66	13.00	59.2	20.0	18.0	5.0	200	2.12
50	20.90	12.00	46.4	8.0	51.0	3.0	200	1.55
51	21.50	16.80	47.0	12.0	17.5	2.0	50	2.74
52	21.50	19.60	44.0	12.0	26.0	2.0	Slope Failure	
53	20.50	29.40	22.0	22.0	12.0	2.0	200	3.4
54	20.00	15.50	46.0	18.0	48.0	5.0	Slope Failure	
55	20.10	19.10	42.0	15.0	64.0	0.0	0	1.36
56	20.00	22.00	32.0	7.5	36.0	6.0	200	1.46
57	19.00	20.80	39.0	26.0	41.0	2.5	200	1.39
58	19.10	20.80	32.0	16.5	50.0	3.0	Slope Failure	
59	19.00	20.00	33.0	21.0	32.0	1.0	200	1.41

Test No.	Soil parameters			Slope height (H) (m)	Slope angle ( $\alpha$ ) ( $^{\circ}$ )	Distance to slope (x) (m)	Q allowable (kN/m <sup>2</sup> )	Factor of safety (F.S.)
	$\gamma$ (kN/m <sup>3</sup> )	$\phi$ ( $^{\circ}$ )	c (kPa)					
60	19.00	30.00	5.0	22.0	28.0	1.5	200	1.69
61	20.33	15.00	36.0	12.0	19.0	3.0	100	2.18
62	19.78	26.00	23.3	6.0	64.0	1.5	200	1.21
63	19.83	21.00	33.0	24.0	26.0	4.0	200	1.56
64	20.50	26.00	46.4	7.0	28.0	5.0	100	3.55

Table 3.6 Input parameters and results

**Note:** The soil parameters, slope geometries and distances from the foundation to the slope were randomly formed using Monte Carlo sampling method, as explained in section 3.3, the random combination of the variable parameters led to overall instability of the problem in certain cases as indicated in table 3.6, achieving a factor of safety of less than 1 under the self weight of the foundation.



# CHAPTER 4

## ANALYSIS AND RESULTS

### 4.1 General

The research was accomplished through two complementary phases while each phase had different objective. The objective of phase I was examining the ultimate bearing capacity of a typical strip foundation of 1m width, B, and 1m depth, D, placed near slope with considering different soil and geometry conditions, thus, determine the allowable load, Q, bearable by the prescribed foundation in terms of the allowable settlement and the factor of safety.

Then, performing phase II of the research which aimed to study the effect of the distance between the slope and the foundation, thus, determine the minimum distance which achieves the stability of a system consists of foundation and slope.

Different soil conditions were taken into account, each soil has properties of  $c$ ,  $\phi$  and soil unit weight of  $\gamma$ . The soil parameters were obtained randomly using Monte Carlo Sampling Technique, then assigned to different slope geometries (slope heights H, and angles  $\alpha$ ) with different foundation locations from the crest of the slope.

### 4.2 Phase I Analysis: Bearing capacity

The deduced results of phase I of the analysis concerning the bearing capacity of a foundation placed at distance from slope are presented below in terms of load- settlement curves (load $\times$ 1m<sup>2</sup>), and sorted in three categories; bearing capacity failure, bearing capacity failure followed by slope failure and the third category is slope failure followed by bearing capacity failure.

#### 4.2.1 Bearing capacity failure

It is noticeable from the collection of analysis results summarized in figure 4.2 that the vertical displacement underneath the foundation increased with load increase until bearing capacity failure occurred under the foundation while the slope remained stable (F.S.>1.3). In

shallow foundations, usually, the allowable bearing capacity is governed by the settlement of the foundation. Thus, the bearing capacity failure in the following cases detected when the settlement value exceeded the allowable settlement, which according to Skempton and MacDonald (1956), was when the maximum settlement for sand is 32mm, and 45mm for clay. Figure 4.1 shows typical results characterised the mode of failure presented in the first category. Note that the slope's factor of safety is shown in each figure, further emphasizing the slope remained safe while the foundation failed.

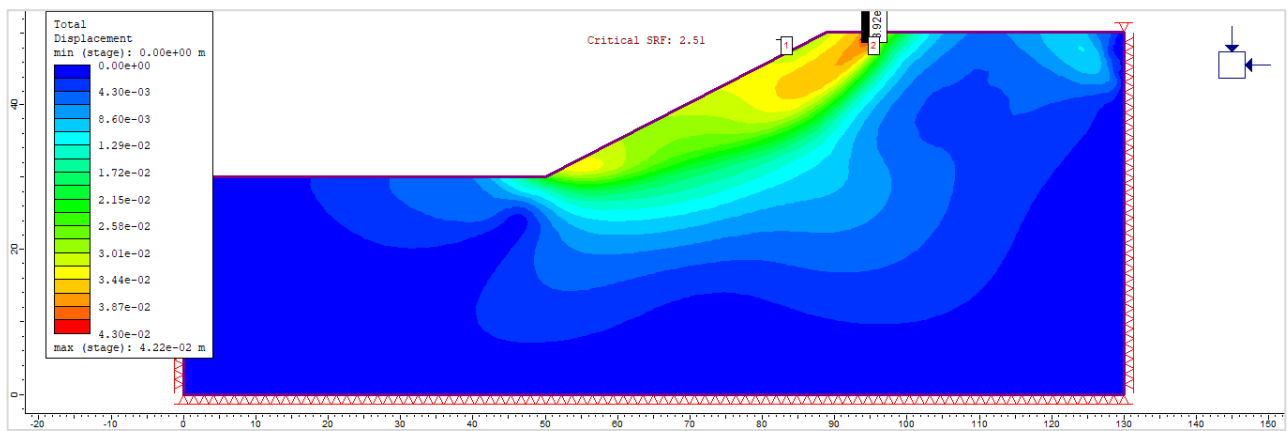
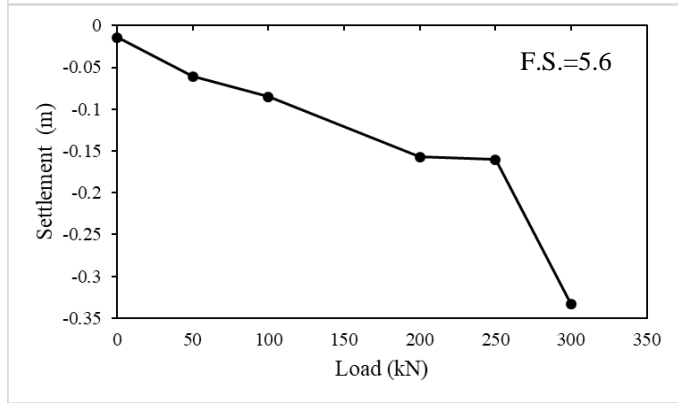
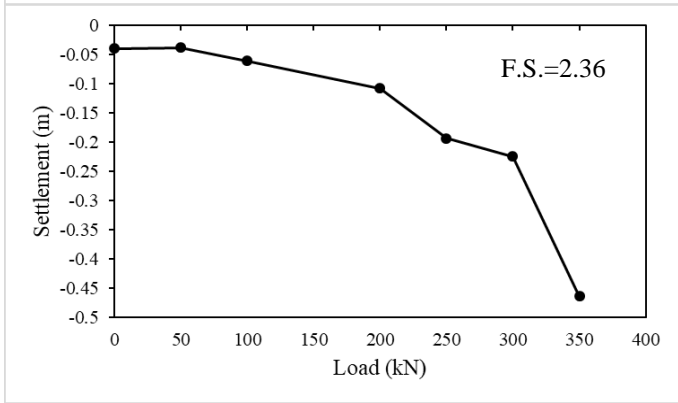
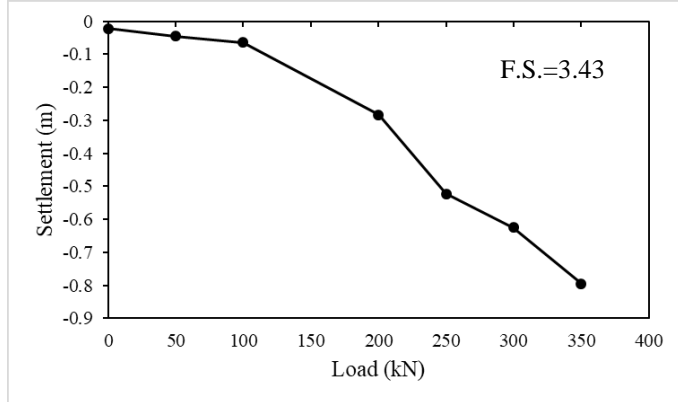
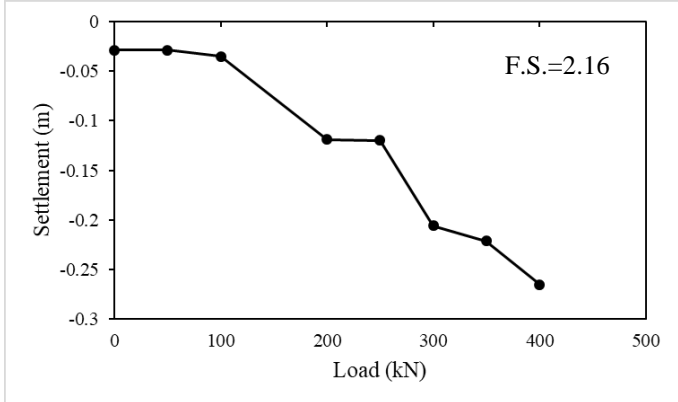
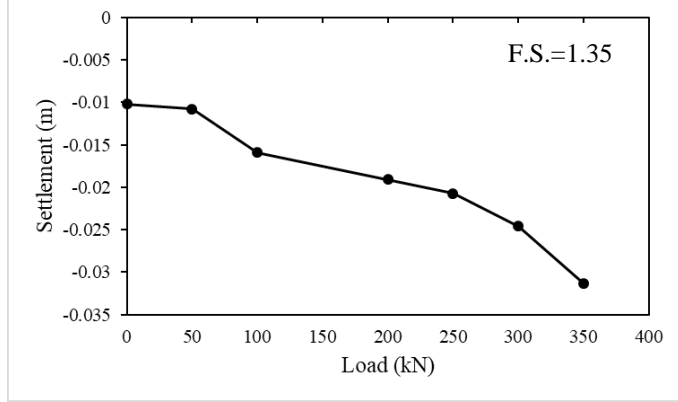
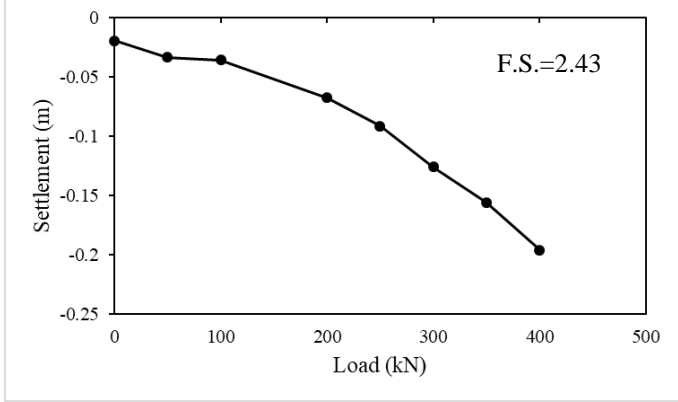
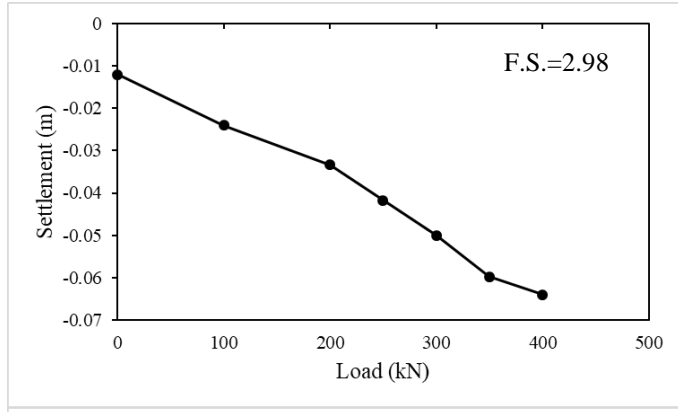
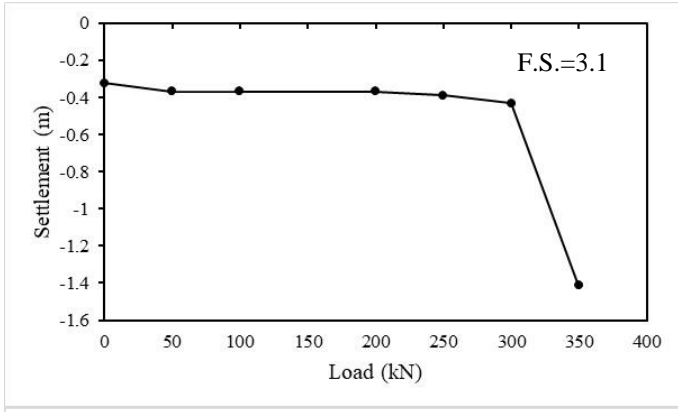
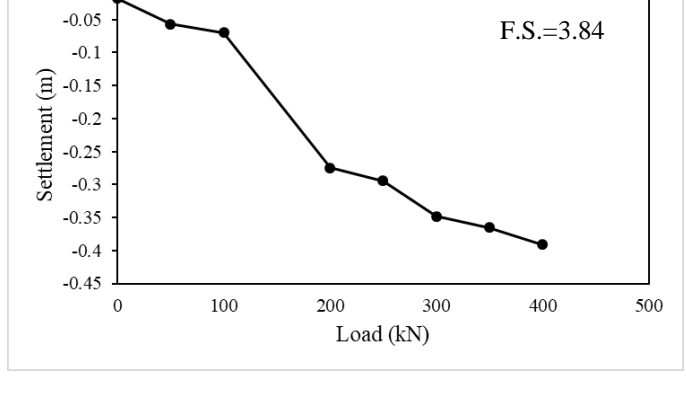
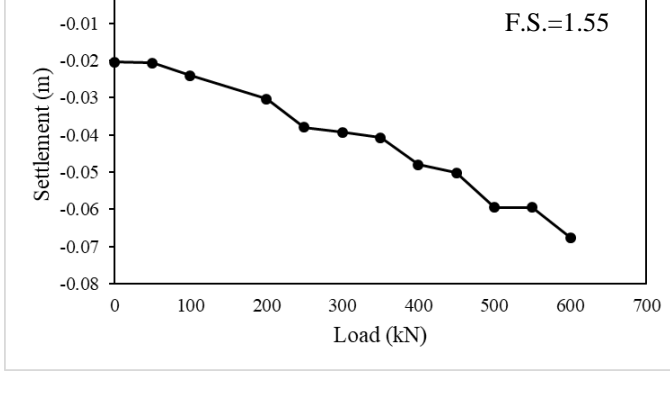
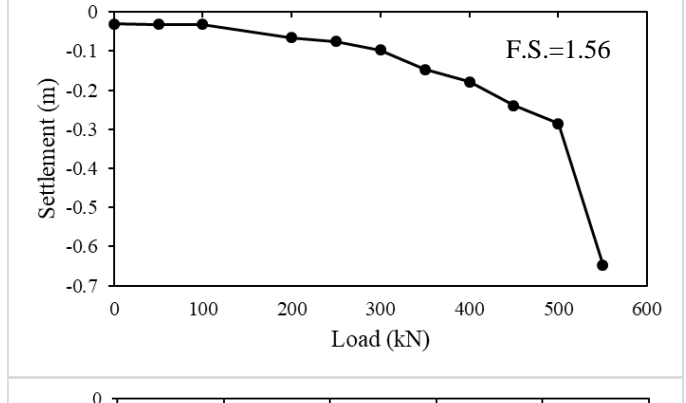
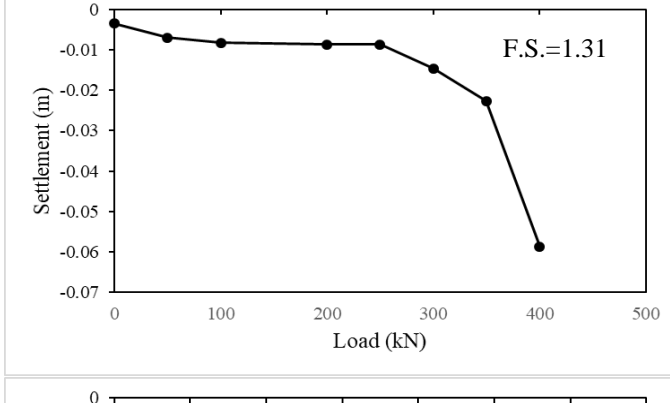
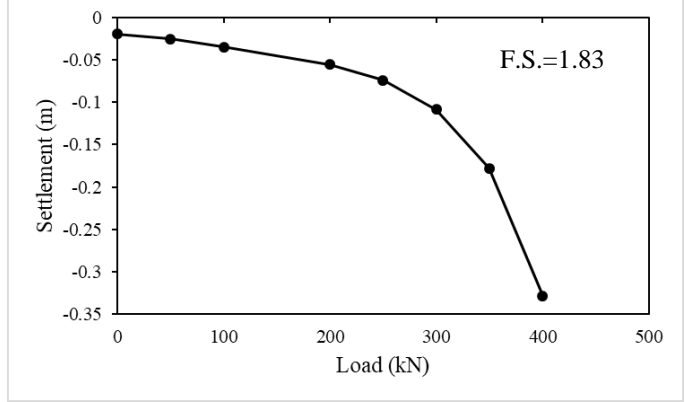
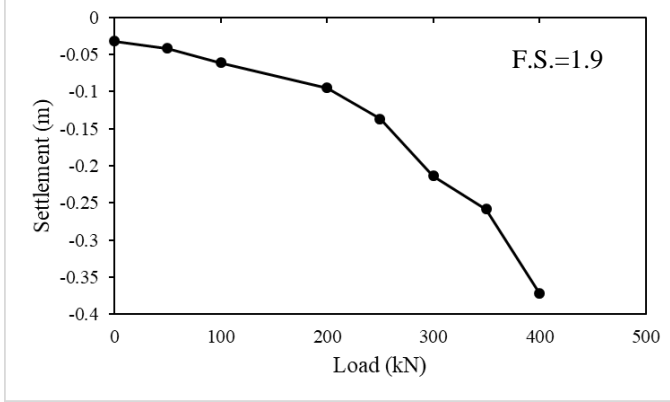
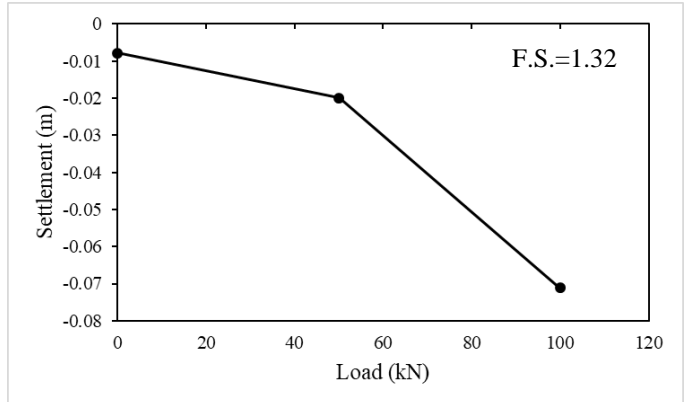
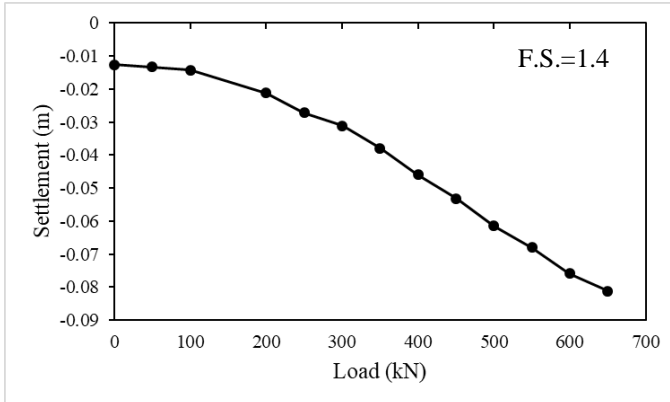
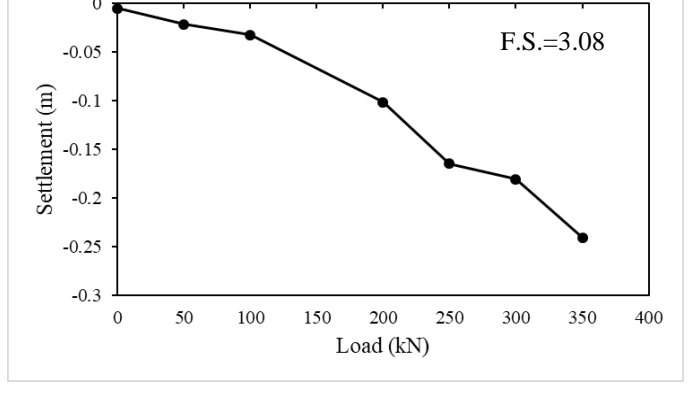
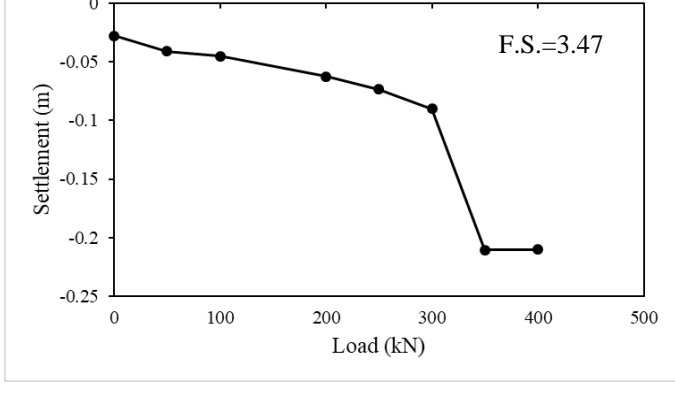
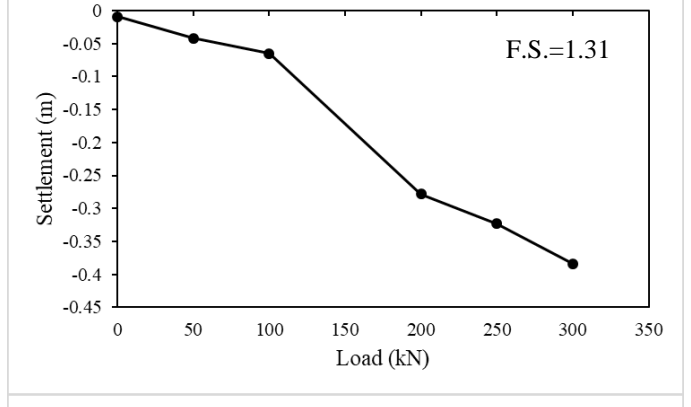
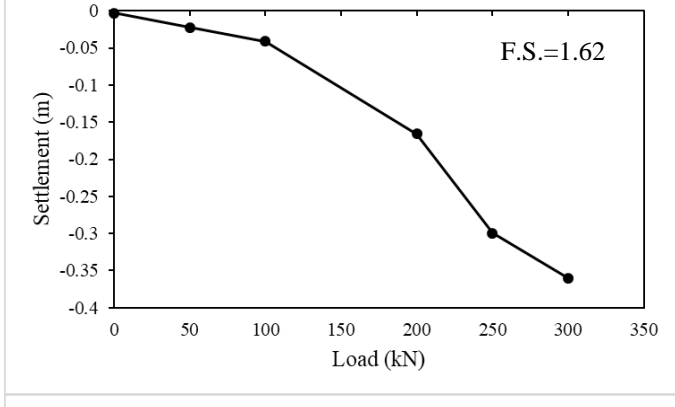
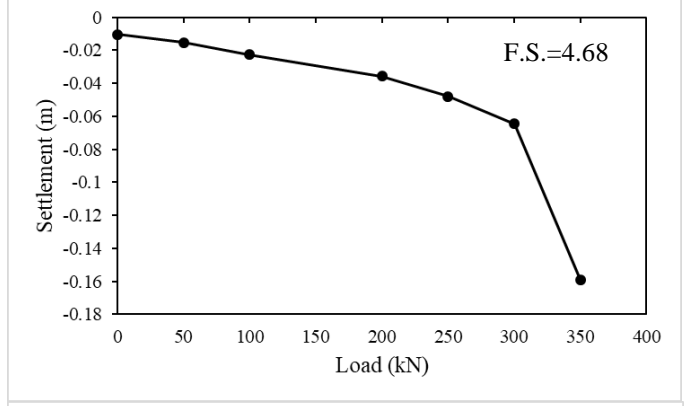
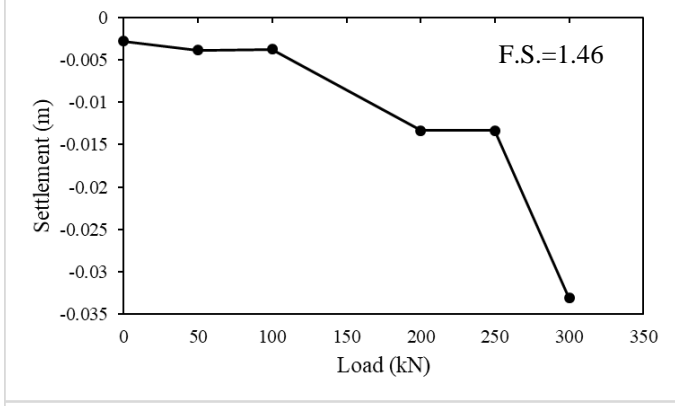
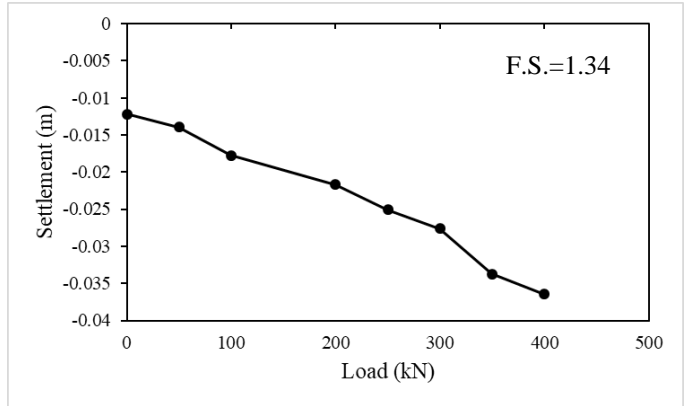
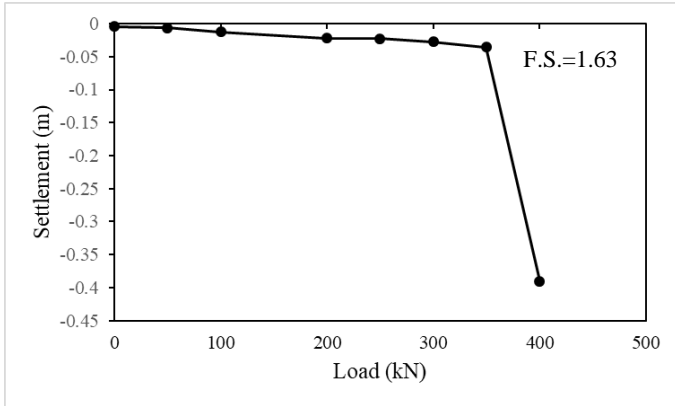
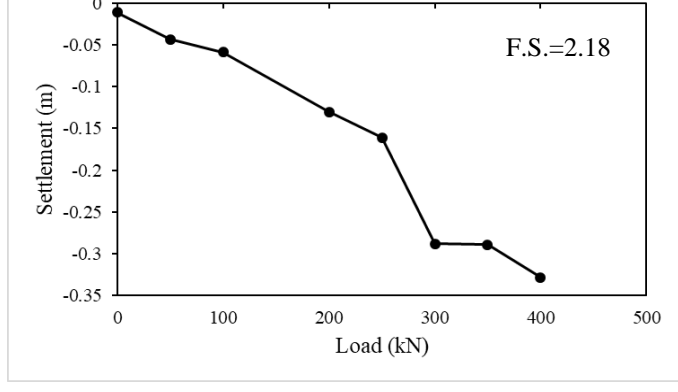
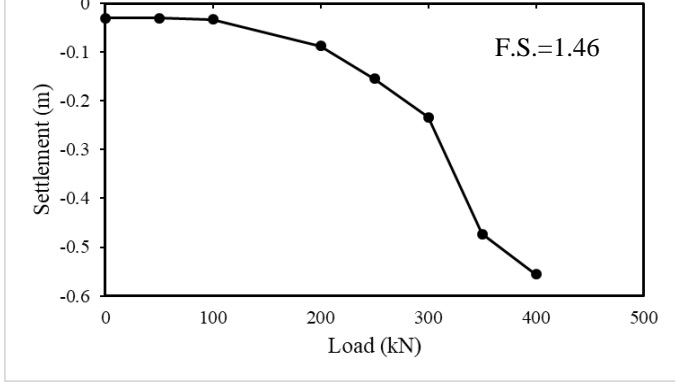
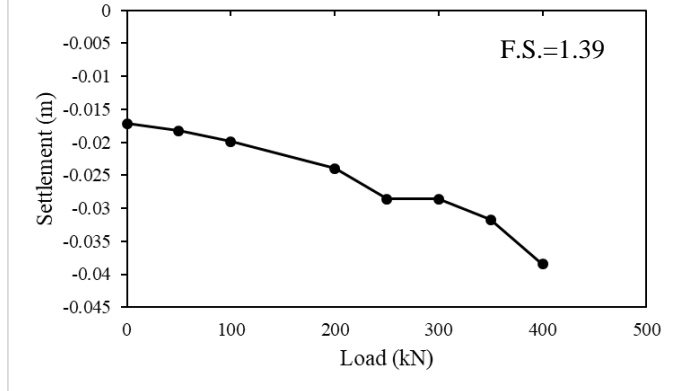
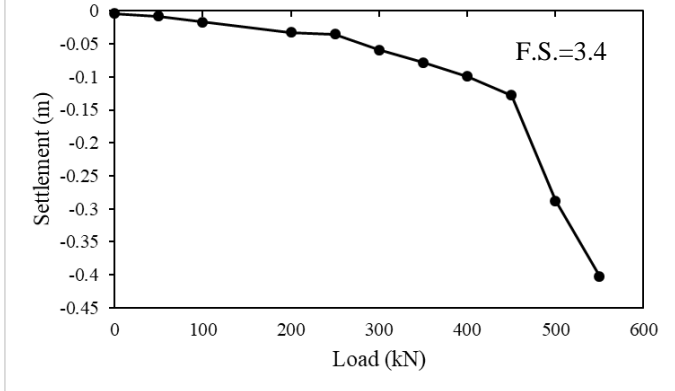
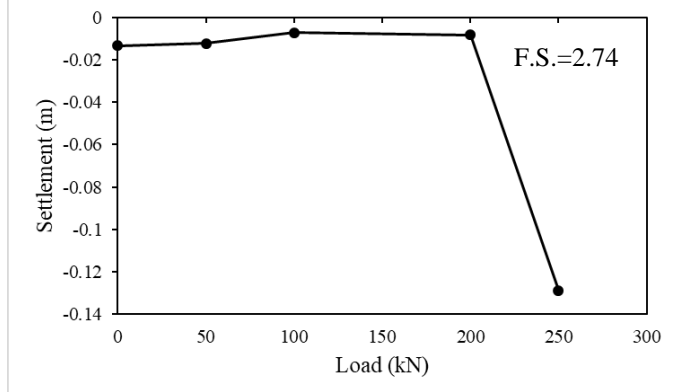
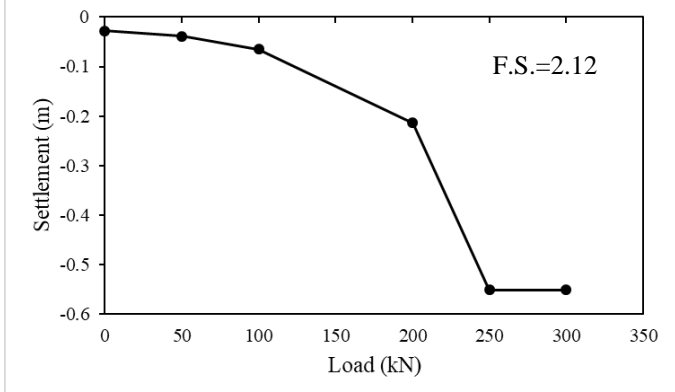
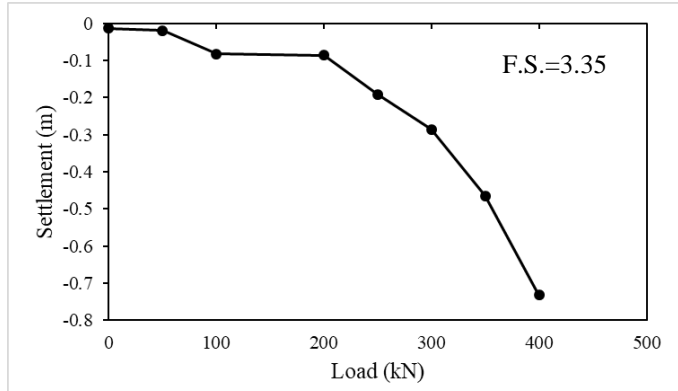
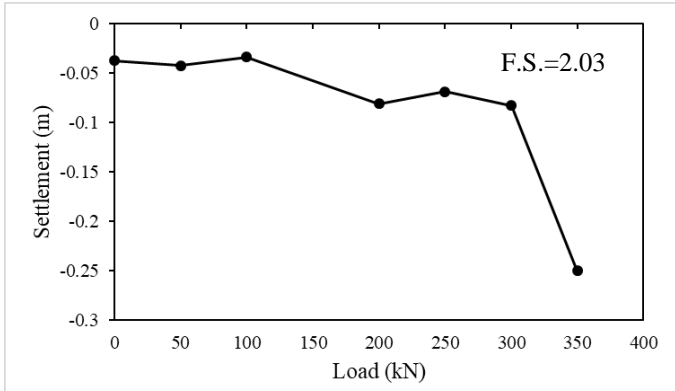


Figure 4.1 Typical results show bearing capacity failure presented in 1<sup>st</sup> category









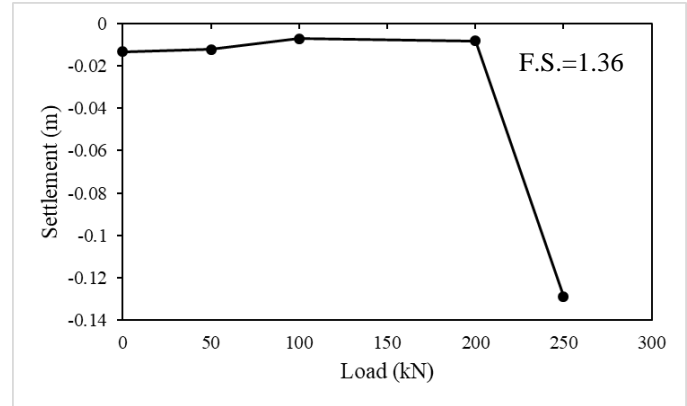
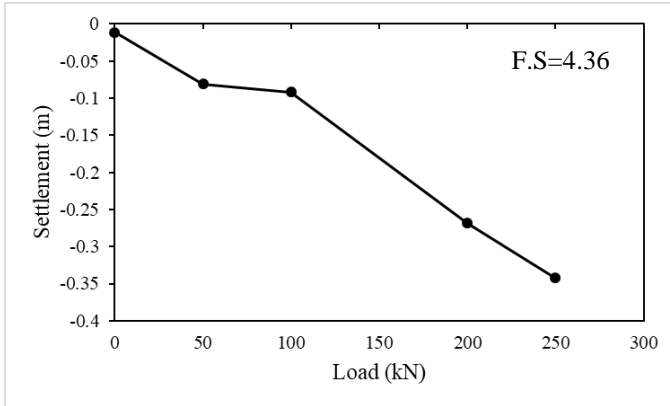
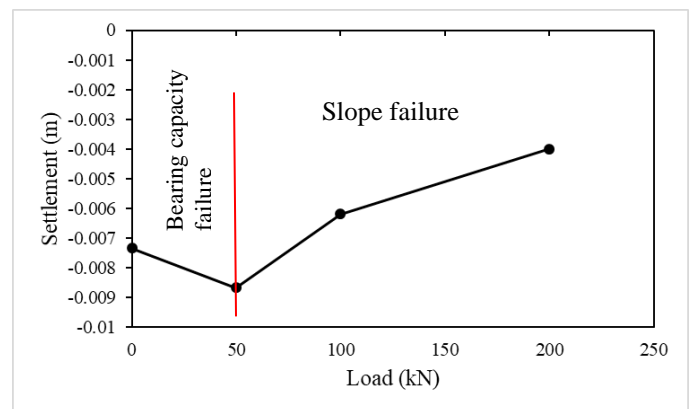
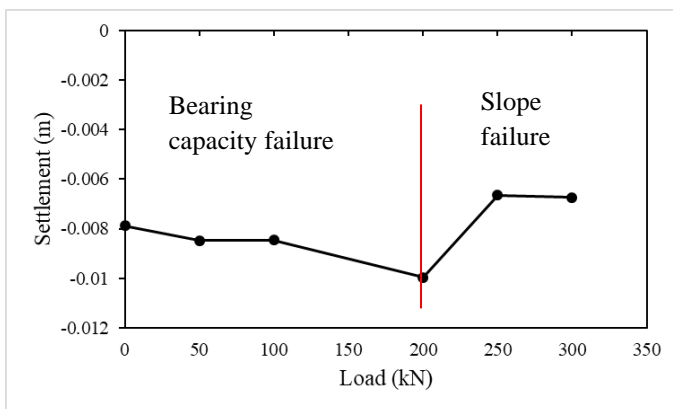


Figure 4.2 Bearing capacity failure (Pahse I analysis)

#### 4.2.2 Bearing capacity failure followed by slope failure

The following set of charts represent the second mode of failure which reported a case of bearing capacity failure followed by slope failure, which is seen from the figures the bearing capacity of the foundation has reached its ultimate value, in terms of the vertical displacement of the soil underneath the foundation, while the slope still stable and achieving the acceptable factor of safety of at least 1.3 for designing purposes. With load increasing, the soil underneath the foundation tended to move horizontally towards the slope which explains the decrease in the vertical displacement values with load increase as the slope started to slide until it reached the least values of it when slope failure occurred. Specifically, for the first two charts in figure 4.3, it's seen that the slope started to fail before the foundation was completely settled, no transition zone occurred for these two models as with the increasing load, the slope failed.



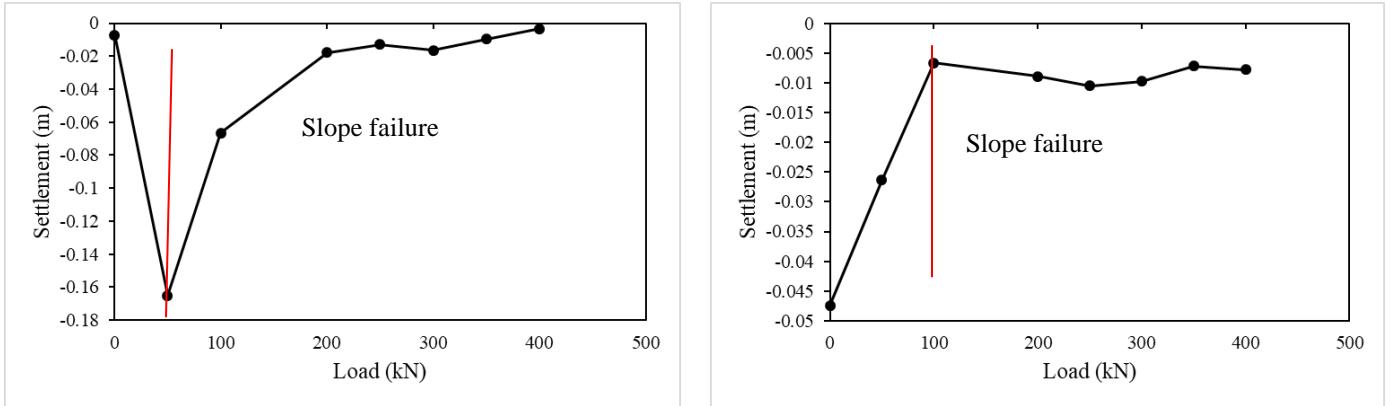


Figure 4.3 Bearing capacity failure followed by slope failure (Phase I analysis).

#### 4.2.3 Slope failure followed by bearing capacity failure

The third category of the charts represented a case of slope failure followed by bearing capacity failure with increasing loads. In geotechnical engineering, the acceptable factor of safety of slope is at least 1.3 then it is considered failed for less factors of safety. While for the software, the failure is detected when the total factor of safety of the problem is less than 1. That explains the capability of the program for analyzing models with increased loads although the engineering failure criterion has been met. Figure 4.4 presents a sketch of the location of the foundation for the slip line of the slope for the third set of charts which show that the foundation is located completely outside the critical circular failure.

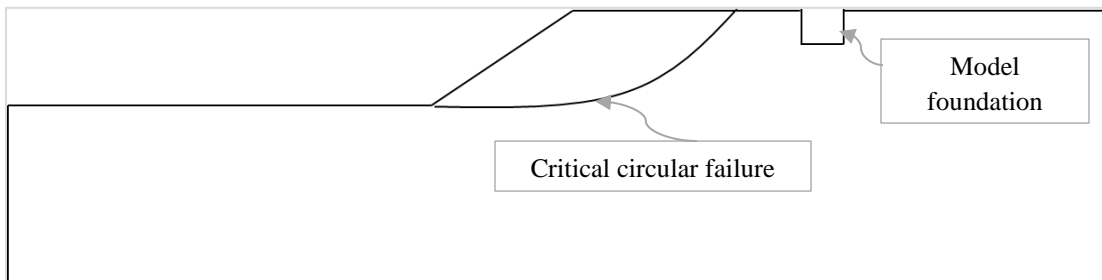
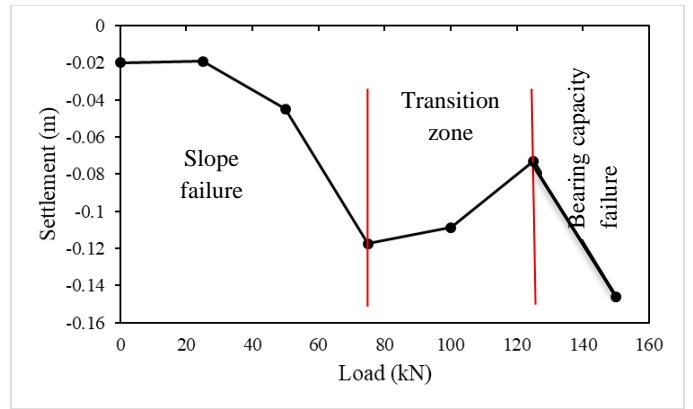
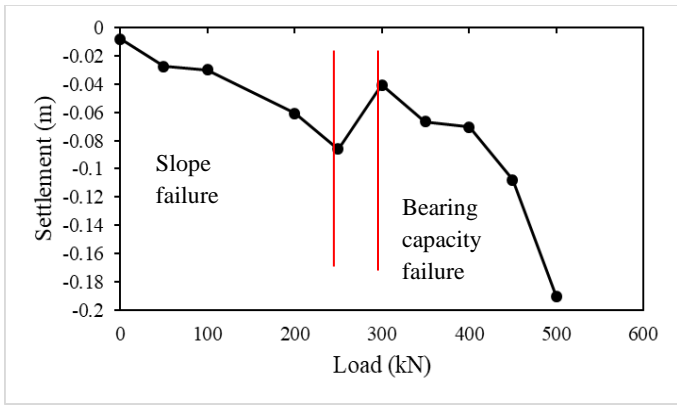
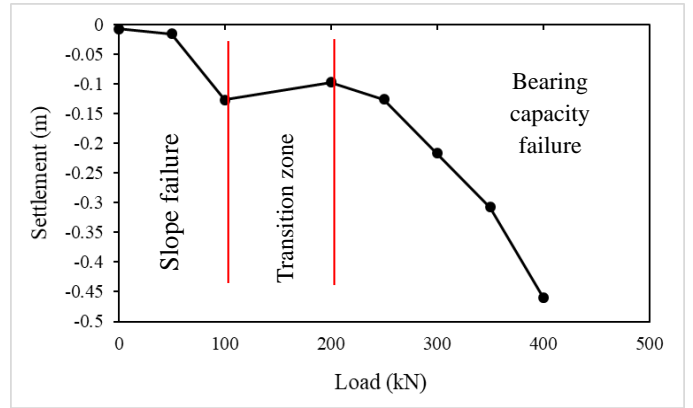
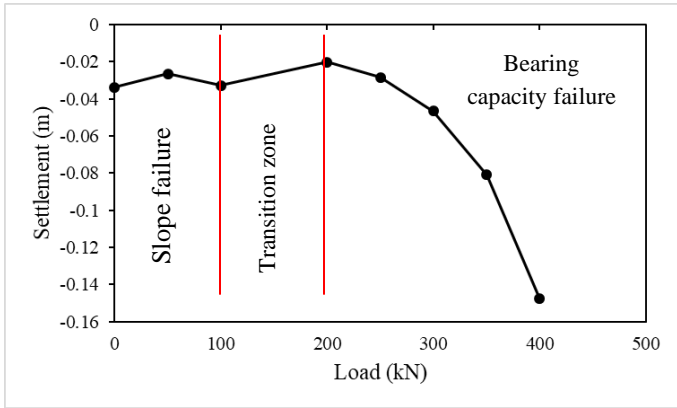
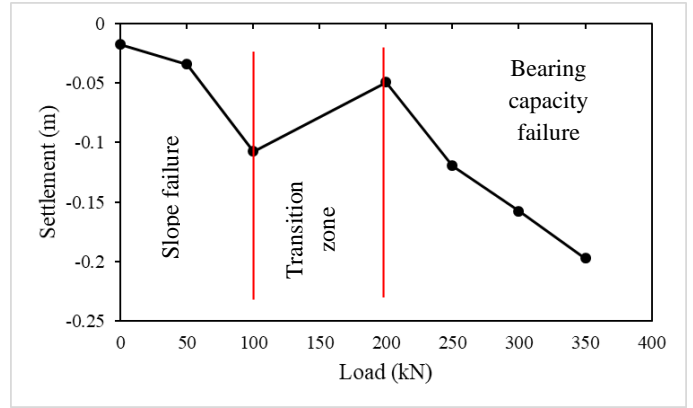
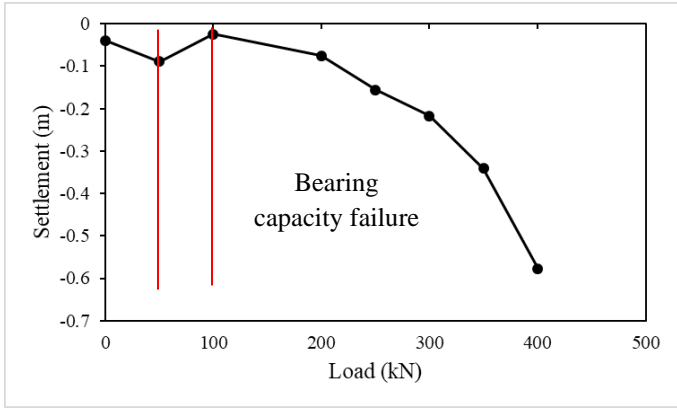


Figure 4.4 Sketch for the location of the foundation to the failure surface.





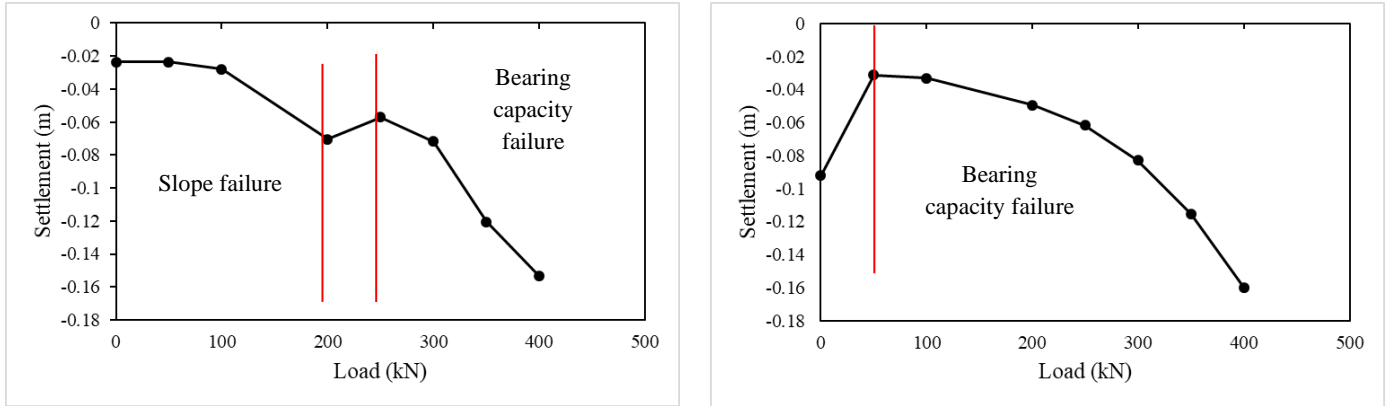


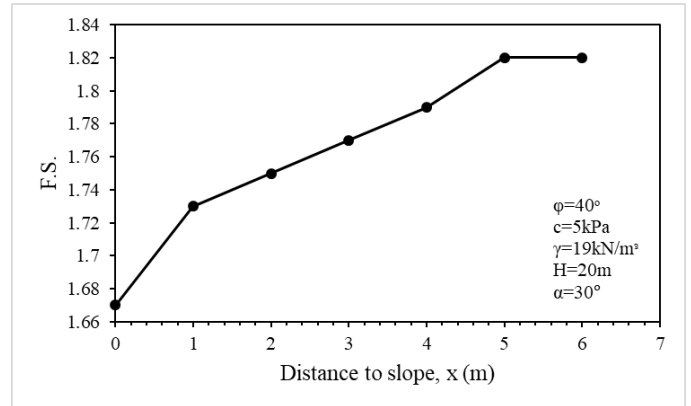
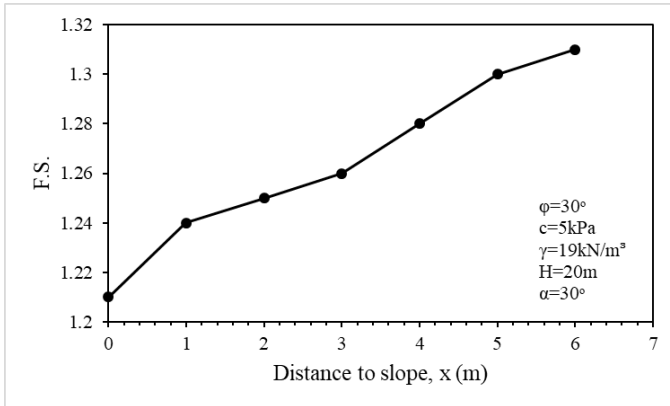
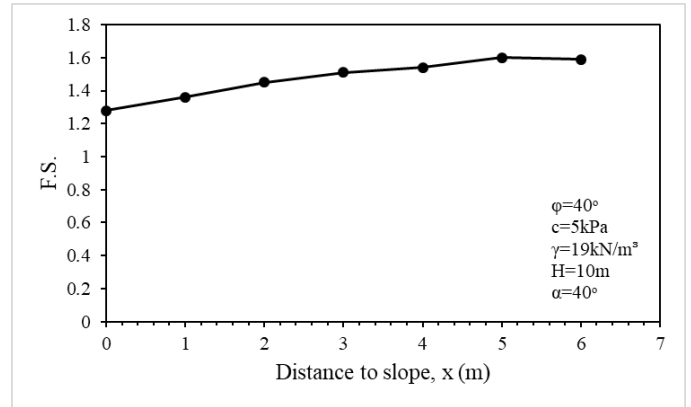
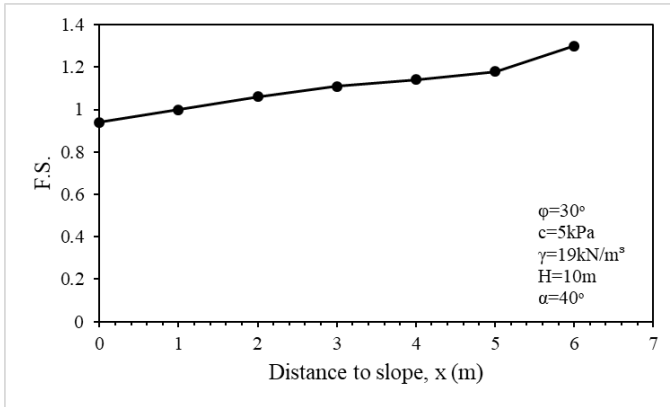
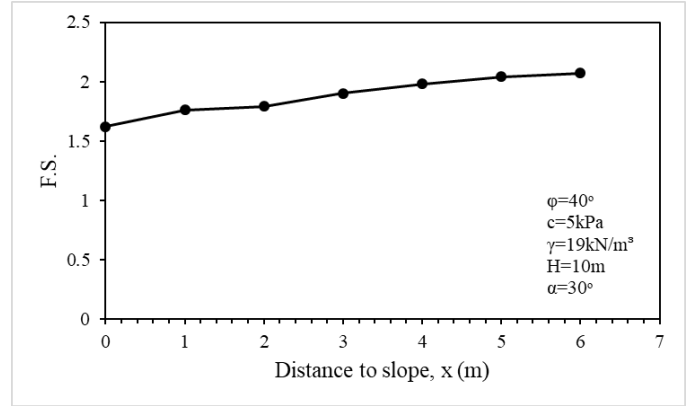
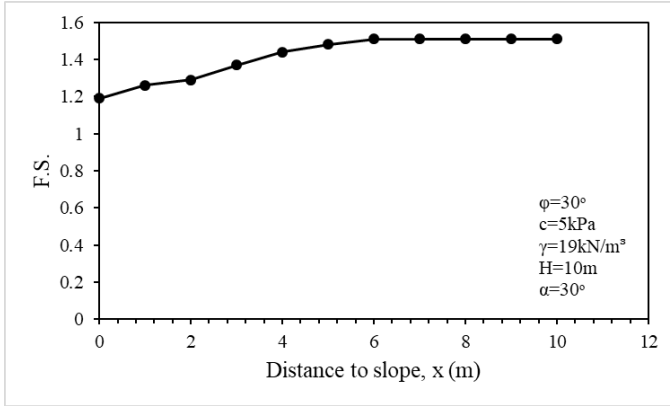
Figure 4.5 Slope failure followed by bearing capacity failure (Phase I analysis).

### 4.3 Allowable Load

The allowable load is the maximum load can be applied on a foundation before failure or uncontrolled deformation occur. As the settlement usually controls the allowable bearing capacity of shallow foundations, the allowable load was measured as the corresponding load to the allowable settlement reached by the foundation during loading and achieved a factor of safety of at least 3. From the analysis of phase I and after discarding the results with F.S. less than 3, the allowable load for a strip footing of depth and width of 1m was found to be 200kN/m<sup>2</sup>.

### 4.4 Phase II analysis: Slope stability

Figures below are the curve form of the tests performed in phase II of the analysis which concerned about examining the effect of the slope height and angle, the angle of internal friction of the soil and finally the distance from the foundation to the slope on the stability of the slope and foundation. It demonstrates the distance (x) of the foundation to the slope in meters on the horizontal axis with the corresponding factor of safety for each distance on the vertical axis. For all tests, the value of the factor of safety augmented as expected by increasing the distance from the foundation to the slope until it remained constant at the distances stated in the theory of Meyerhof (1957), which indicates the agreement between the theory and the research in terms of the factor of safety of the slope.



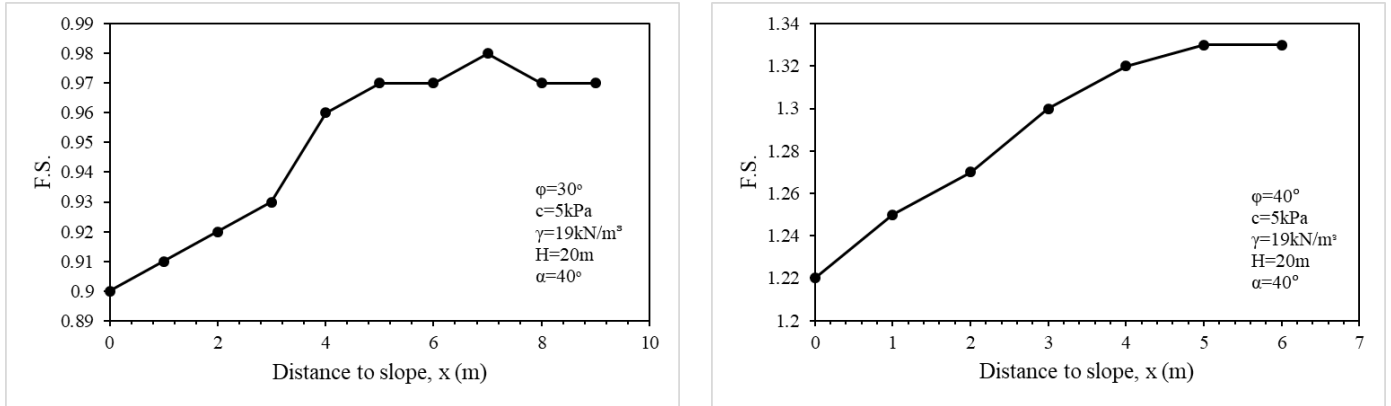


Figure 4.6 Slope stability (Phase II analysis).

## 4.5 Parametric study

From the values of the ultimate bearing capacity, displacements and factor of safeties measured during testing, it was found that the  $x/B$  ratio, the slope height and angle  $\alpha$  are the most significant parameters affecting the stability of a system of a foundation and slope.

For the cases of the foundation placed at greater distance from the slope ( $>6B$ ), the bearing capacity becomes independent of these parameters. While changing the shearing resistance of the soil by increasing or decreasing affected the stability of the slope regardless the foundation.

### 4.5.1 Distance to the slope (x)

Meyerhof (1975) determined the reduction in the bearing capacity coefficients of the foundation associated with existence of a slope, as well as defining the distance at which the bearing capacity of the foundation is independent of the inclination of the slope. According to Meyerhof, the foundation becomes independent of the slope at distance from 2 to  $6B$  depending on  $\phi$  and  $D/B$  (where  $D$  &  $B$  are the foundation depth and width, respectively). As expected, the factor of safety increased with distance increase from the slope until it remained constant at distance  $6B$  and beyond. For cases that the foundation placed close to the crest of the slope, the displacement of the soil under the footing concentrates more on the side of the slope than on the side of the horizontal surface. From the analysis and results, the validation of Meyerhof's theory is detected in figure 4.7.

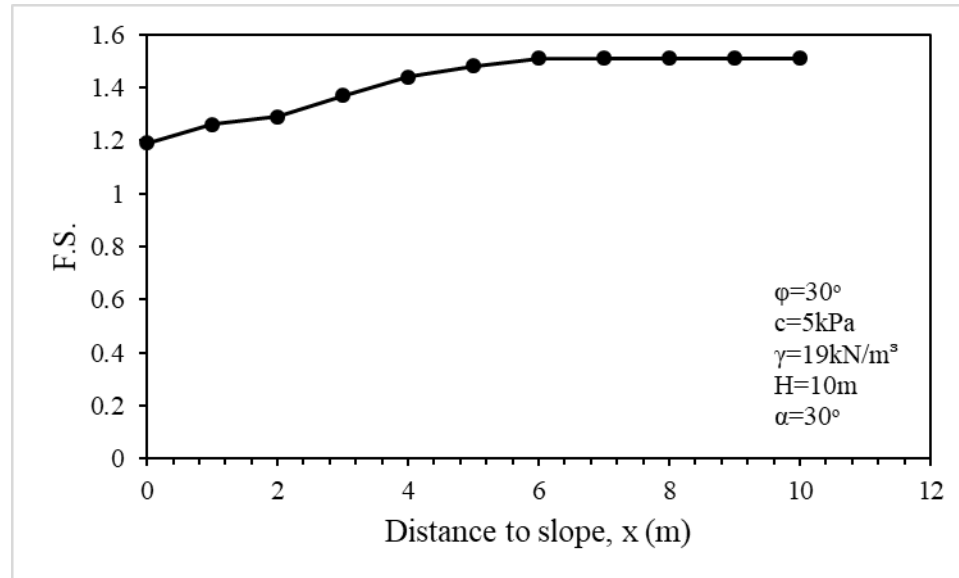


Figure 4.7 Constant F.S. at 6B and beyond.

#### 4.5.2 The slope angle ( $\alpha$ ) and the angle of internal friction ( $\phi$ )

It's expected that the bearing capacity of the foundation reduces with the increase in the slope inclination and the decrease in the angle of shearing resistance of soil. From investigating and comparing the results, it's found that for medium slopes the effect of increasing the slope angle is almost as significant as the effect of decreasing the angle of internal friction of the soil on the stability of the slope. As seen from figures below for two slopes of same height of 10m, decreasing the angle of shearing resistance by 25% caused reduction in the factor of safety by around 25% for the same ratio of x/B and  $\alpha$ , then increasing the slope angle by 33% reduced the factor of safety by around 22% for same ratio of x/B and  $\phi$  figure 4.6.

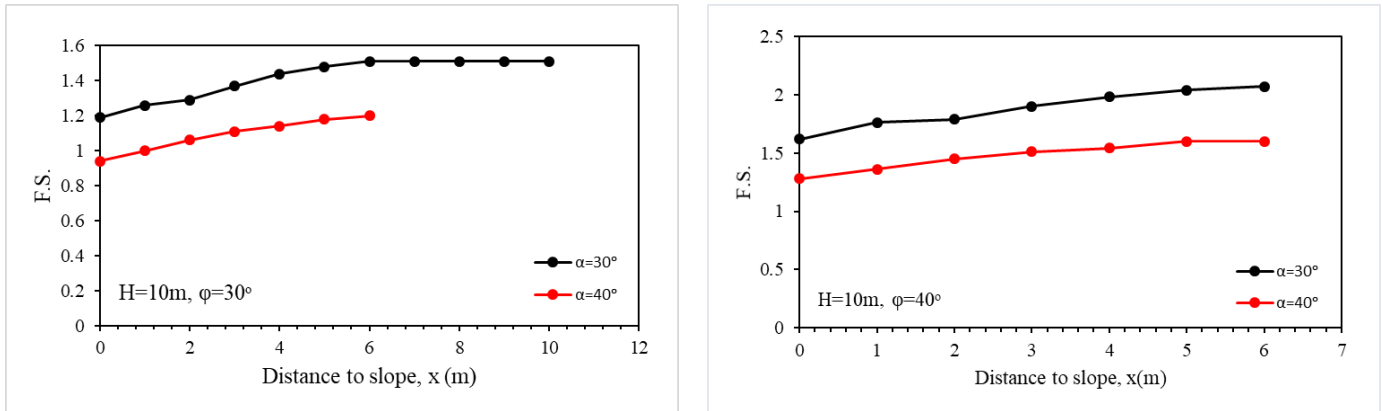


Figure 4.8 Effect of slope angle and friction angle.

### 4.5.3 The slope height (H)

The analysis gave an expected result regarding the slope height in terms of the factor of safety of slope, where the safety factor gave lower values at higher altitudes as it is shown in the figures below the effect of increasing the height of slope with fixing the friction angle  $\phi$  for two sets of slope angles, from the first set of data and at the same slope angle  $\alpha=30^\circ$  and friction of soil  $\phi=30^\circ$  the factor of safety decreases linearly with the increase of slope height. For slope angle of  $\alpha=40^\circ$  with increasing the slope height by 100% the factor of safety is less than 1 in case of the presence of foundation at 9m and it equals exactly 1 in case of no foundation, which concluded that for this parameters combination it's not recommended to have a foundation at any distance from the slope. The key factor here is the slope height and neither is the slope angle nor the friction angle since for the slope of height 10m and angle  $\alpha=40^\circ$  and friction angle of  $30^\circ$  it was safe to have foundation adjacent to the slope ( $x=0$ ). So, it is noticeable that generally with increasing the slope angle and fixing the shearing resistance angle, it was not possible to increase the slope height and have a foundation at the same time.

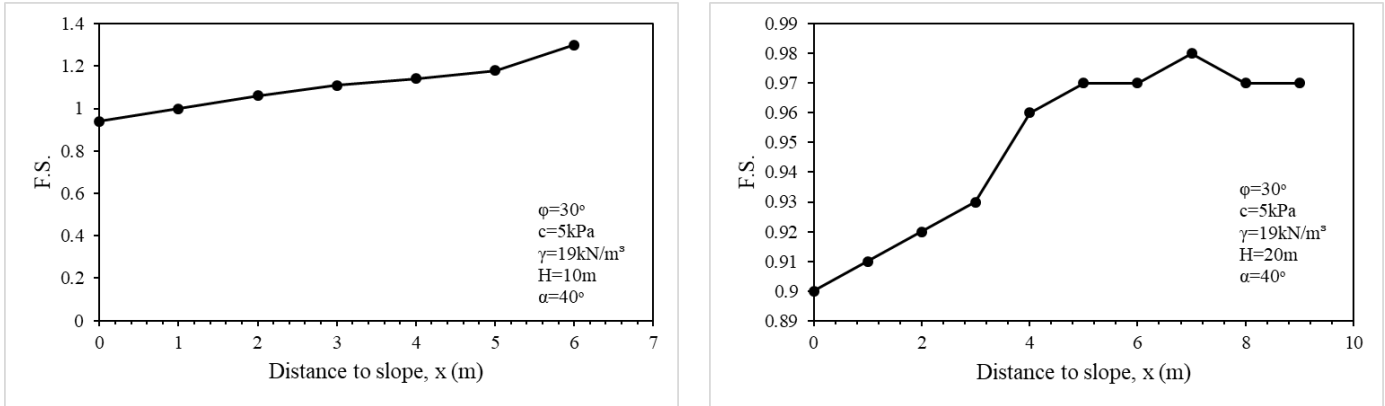


Figure 4.9 Effect of slope height.

## 4.6 Output

The analysis was performed through two phases. The objective of phase I was to examine the ultimate bearing capacity of a strip foundation of width, B and depth, D of 1m placed at distance from slope, thus, determine the allowable load for the given footing. Then a parametric study was proceeded in phase II of the analysis to study the key factors affecting the stability of the system of foundation and slope such as (H,  $\alpha$ ,  $\phi$  and x).

From the figures below, the influence of the presence of the slope on the bearing capacity of the foundation is seen through the size and shape of the failure plane below the foundation. To explain the difference in the failure plane due to the effect of the slope angle, slope height, x/B ratio (in this case, x/B is the distance to the slope as B=1m) as well as the soil properties. The cases of soil with  $\phi=30^\circ$  &  $40^\circ$ ,  $\alpha=30^\circ$  &  $40^\circ$  and slope height H=10 & 20m were chosen for comparison. It is noticeable that there no displacements were initiated on or near the boundaries of the model, and no additional stresses were generated which confirms that the boundaries were suitable for all geometry conditions. It's noted that when the foundation was placed directly at or close to the edge of the slope (x/B=0, 1), the soil under the foundation tended to move horizontally toward the slope since it has less shearing resistance. With increasing the distance of the foundation to 2B, the ultimate bearing capacity increased as expected and part of the stresses due to the presence of the foundation began to be governed by soil on the side of the ground surface. With locating the foundation farther, the slip line of the slope passed the farther edge of the foundation to make the

foundation located completely inside the slope failure plane, the slope at this case experienced case of toe failure and this explains the constancy of the factor of safety at distance of 6B and beyond to be equal to the case of no foundation as so is the foundation is governed by the overall stability of the slope, and not by the ultimate bearing capacity. For case of  $\alpha^\circ > \varphi^\circ$  and for H=10m, the slope experienced case of face failure as shown in figure 4.10.

For higher and steeper slopes, the effect of the distance of the foundation became more significant, and the foundation is governed by the ultimate bearing capacity for cases of  $\alpha^\circ = \varphi^\circ$ . While for case that  $\alpha^\circ > \varphi^\circ$ , slope sliding occurred on the slope surface to produce case of toe failure for all distances to the foundation. It can be concluded that for such steep and high slopes, the foundation is governed by the slope failure. So, the increase in the slope geometry requires increase in the distance of the foundation where the bearing capacity is independent of the slope.

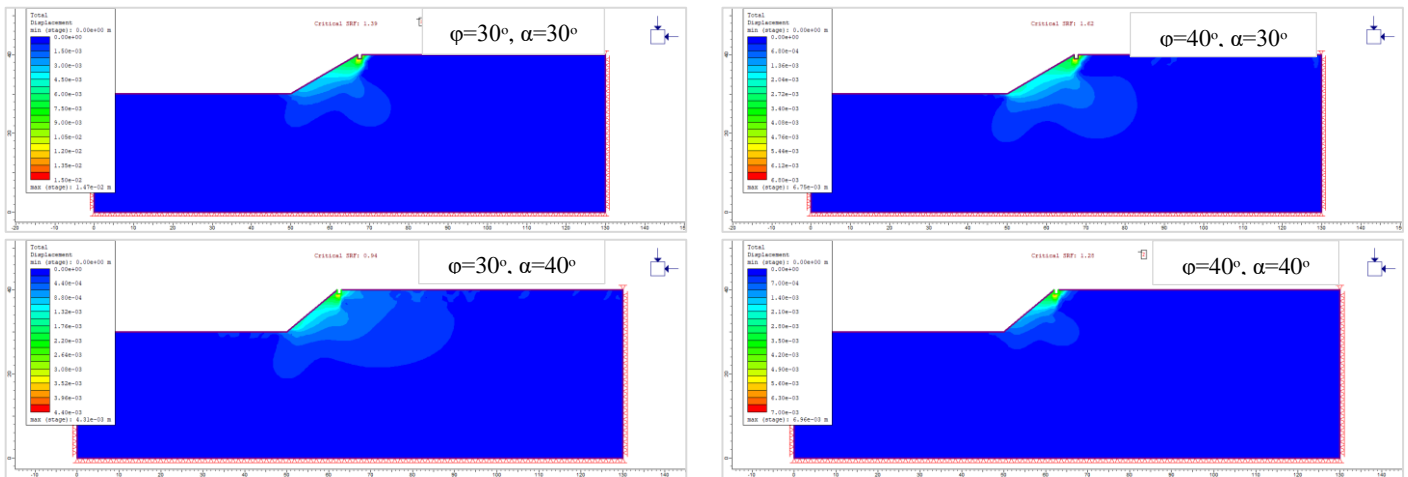


Figure 4.10 Output for H=10m, D/B=1 & x/B=0

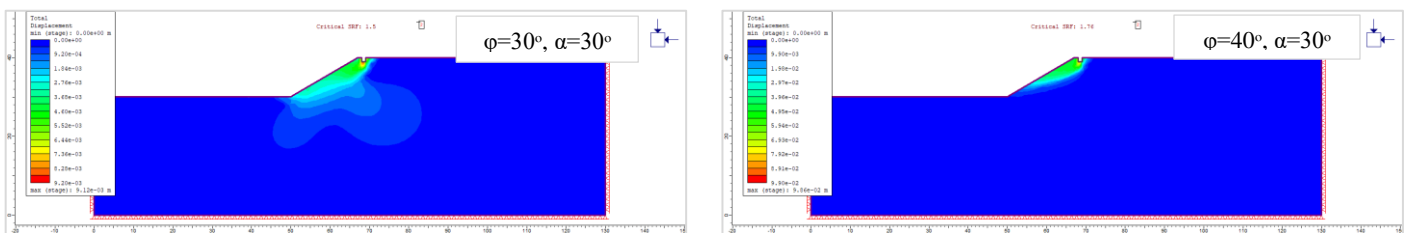


Figure 4.11 Output for H=10m, D/B=1 & x/B=1.



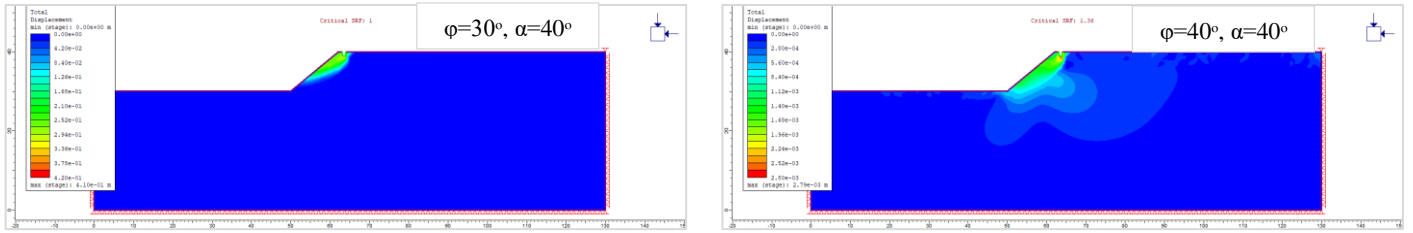


Figure 4.11 Output for H=10m, D/B=1 & x/B=1.

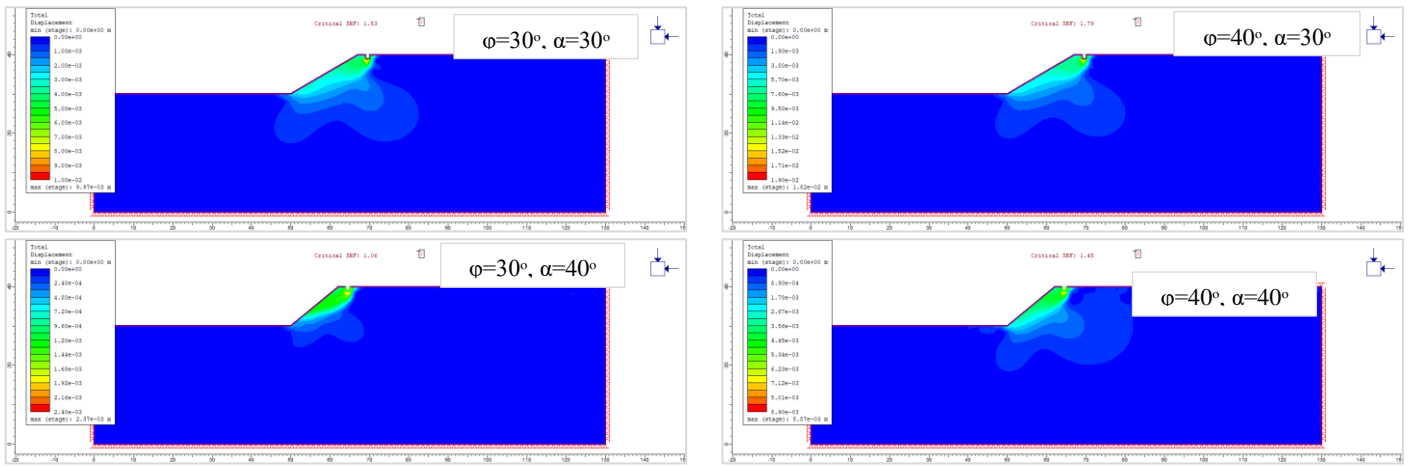


Figure 4.12 Output for H=10m, D/B=1 & x/B=2.

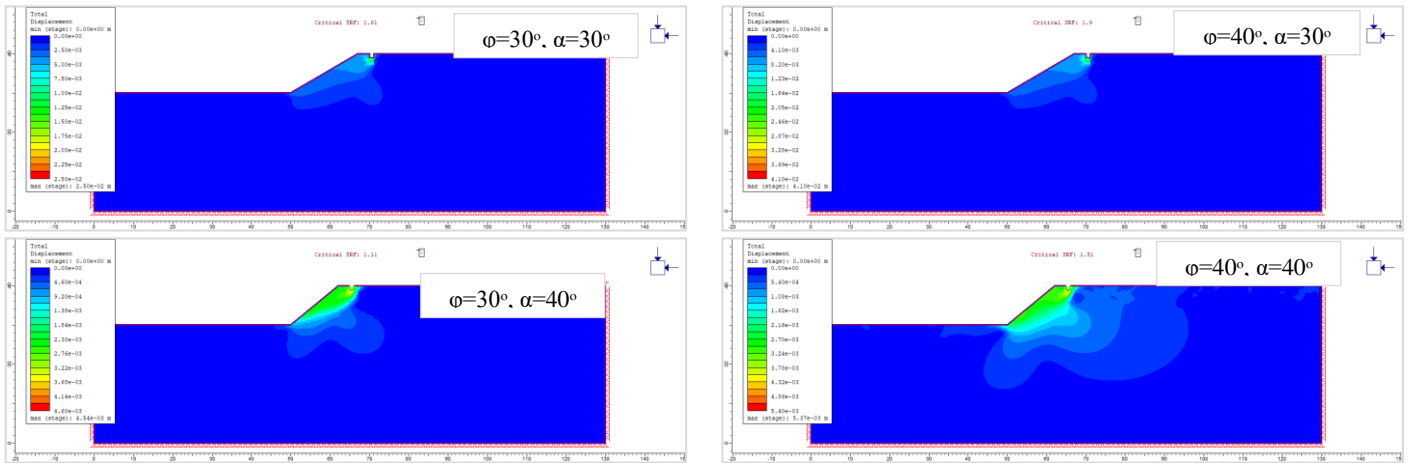


Figure 4.13 Output for H=10m, D/B=1 & x/B=3.

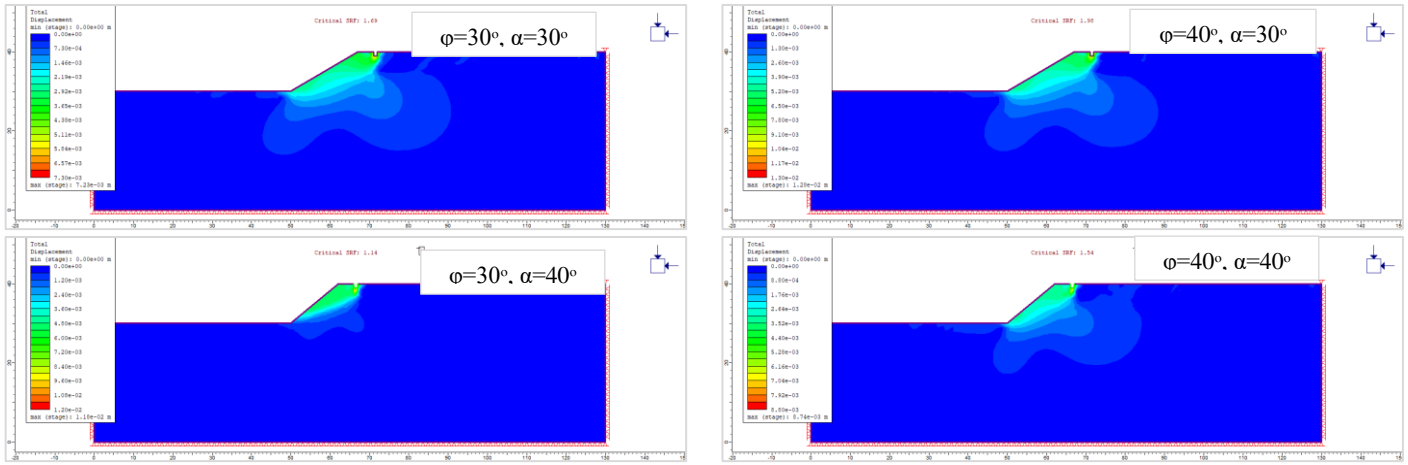


Figure 4.14 Output for H=10m, D/B=1 & x/B=4.

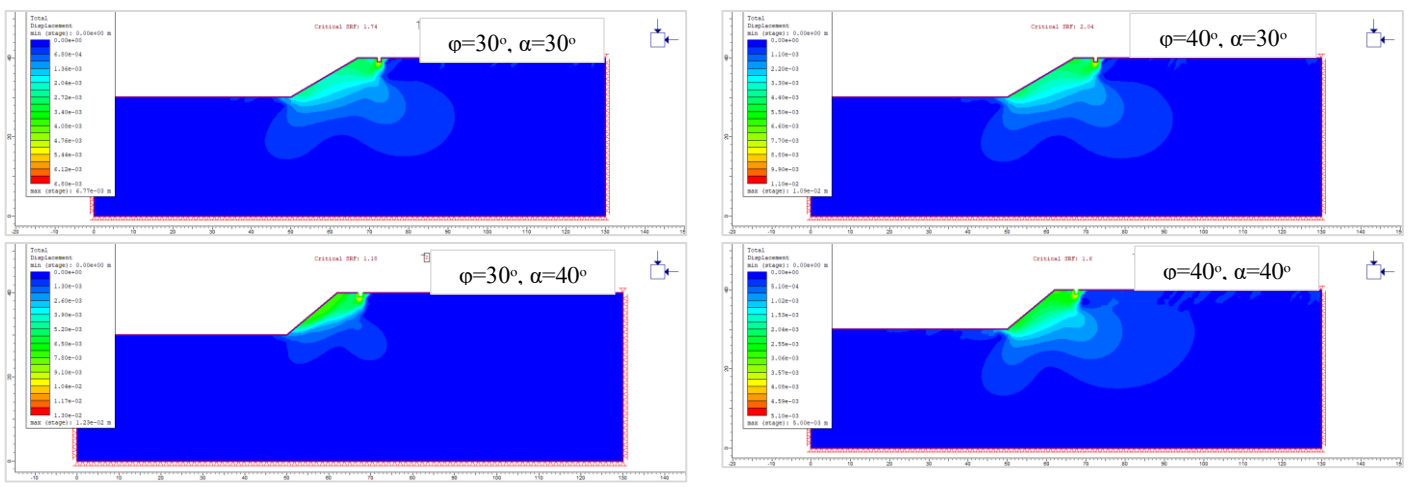


Figure 4.15 Output for H=10m, D/B=1 & x/B=5.

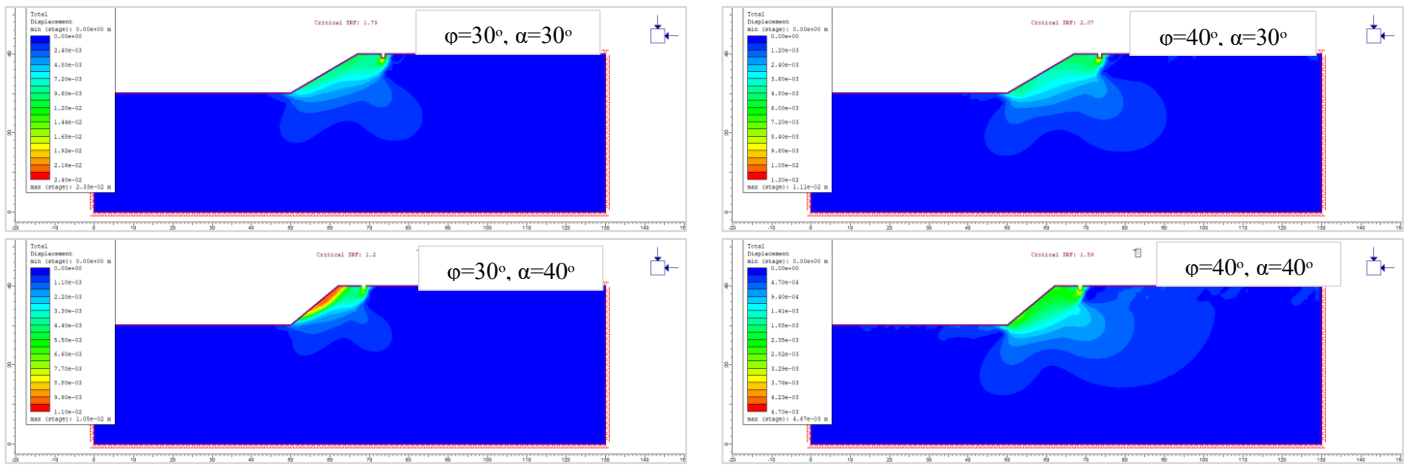


Figure 4.16 Output for H=10m, D/B=1 & x/B=6.

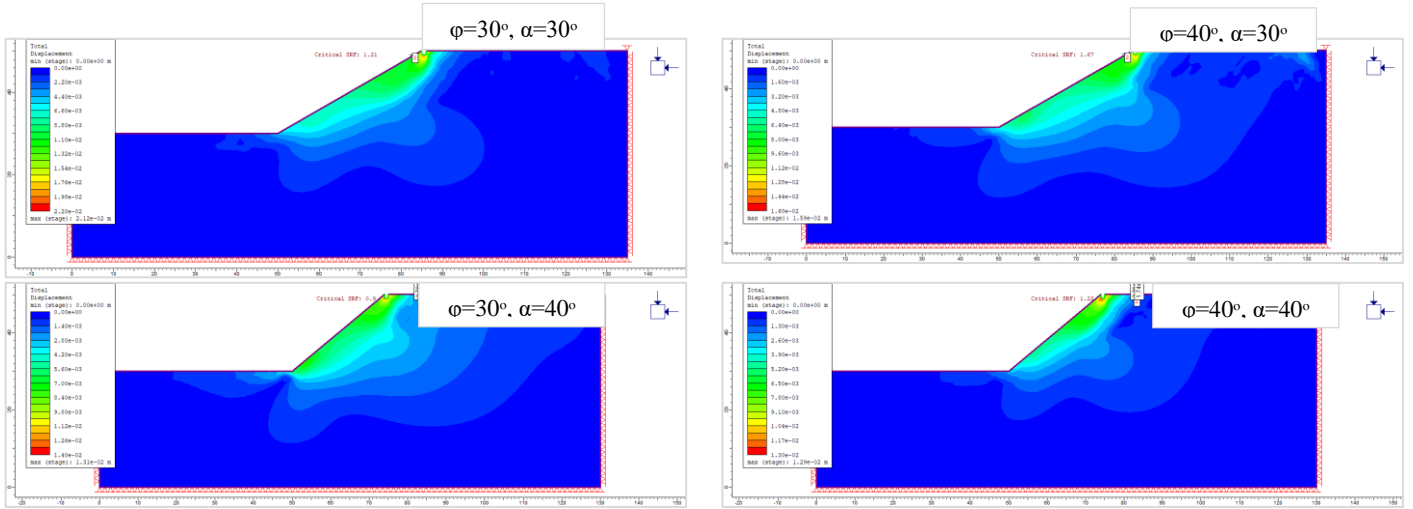


Figure 4.17 Output for  $H=20\text{m}$ ,  $D/B=1$  &  $x/B=0$ .

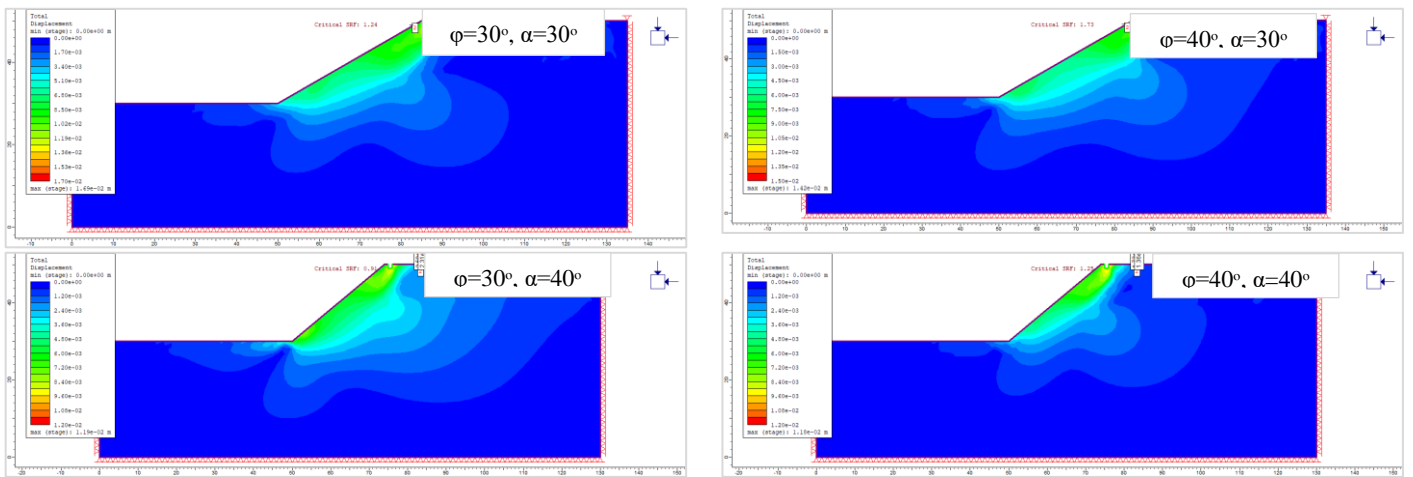


Figure 4.18 Output for  $H=20\text{m}$ ,  $D/B=1$  &  $x/B=1$ .

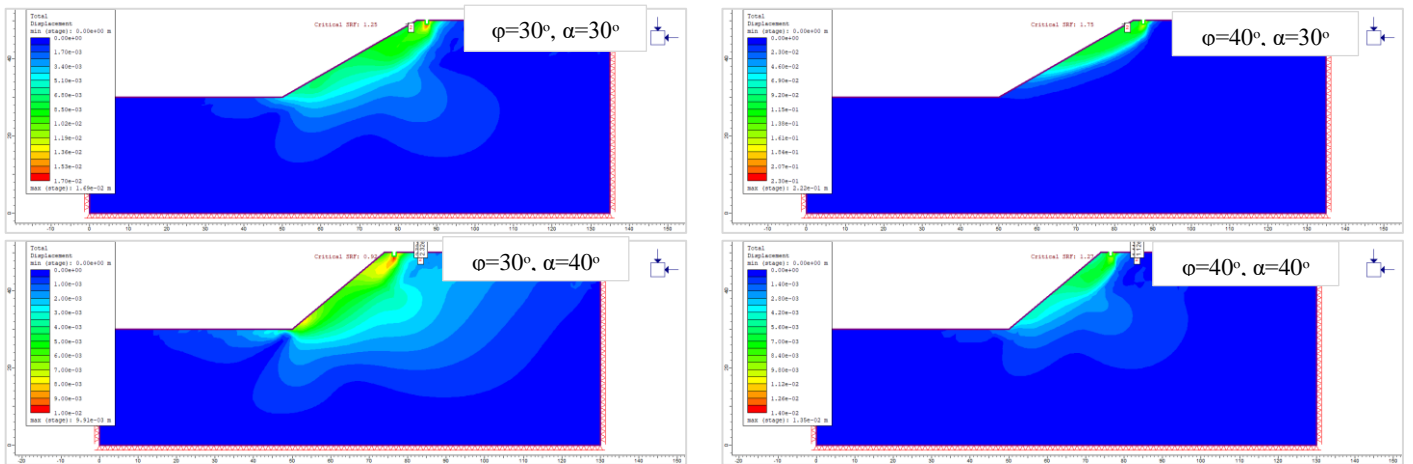


Figure 4.19 Output for  $H=20\text{m}$ ,  $D/B=1$  &  $x/B=2$ .

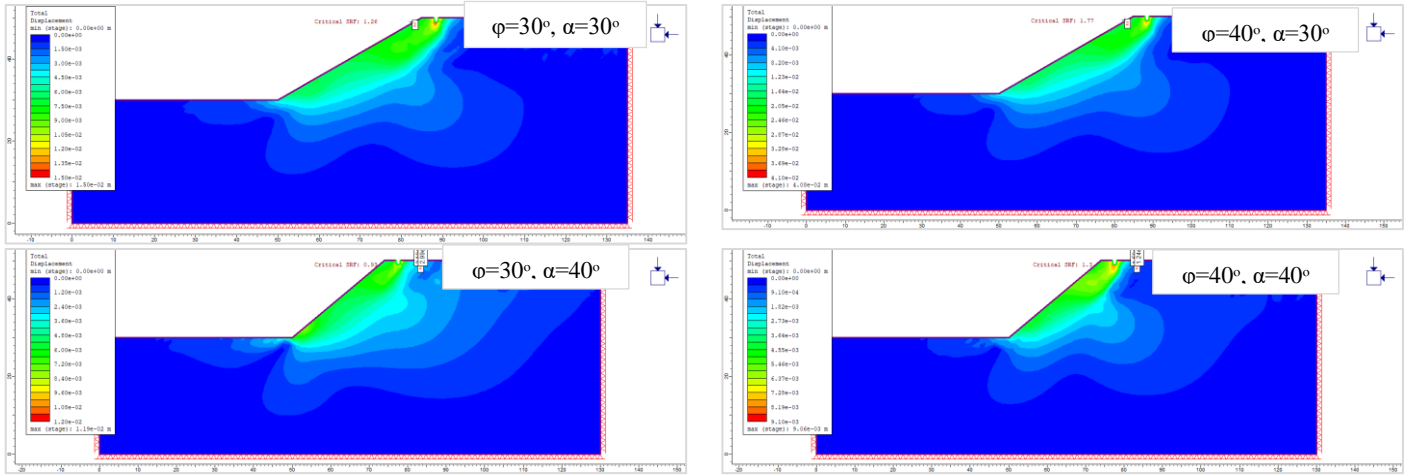


Figure 4.20 Output for H=20m, D/B=1 & x/B=3.

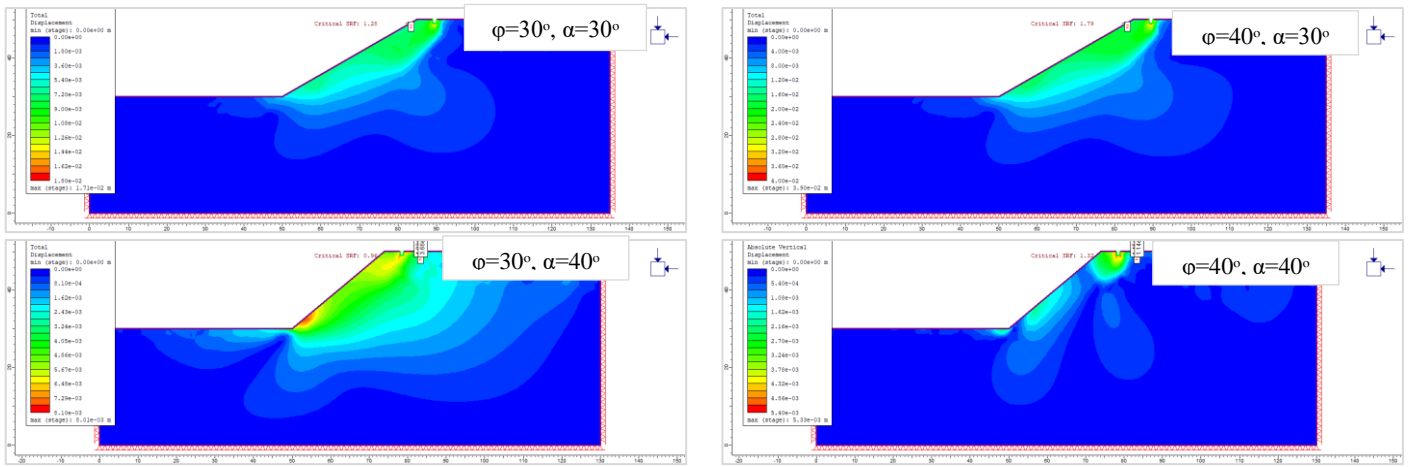


Figure 4.21 Output for H=20m, D/B=1 & x/B=4.

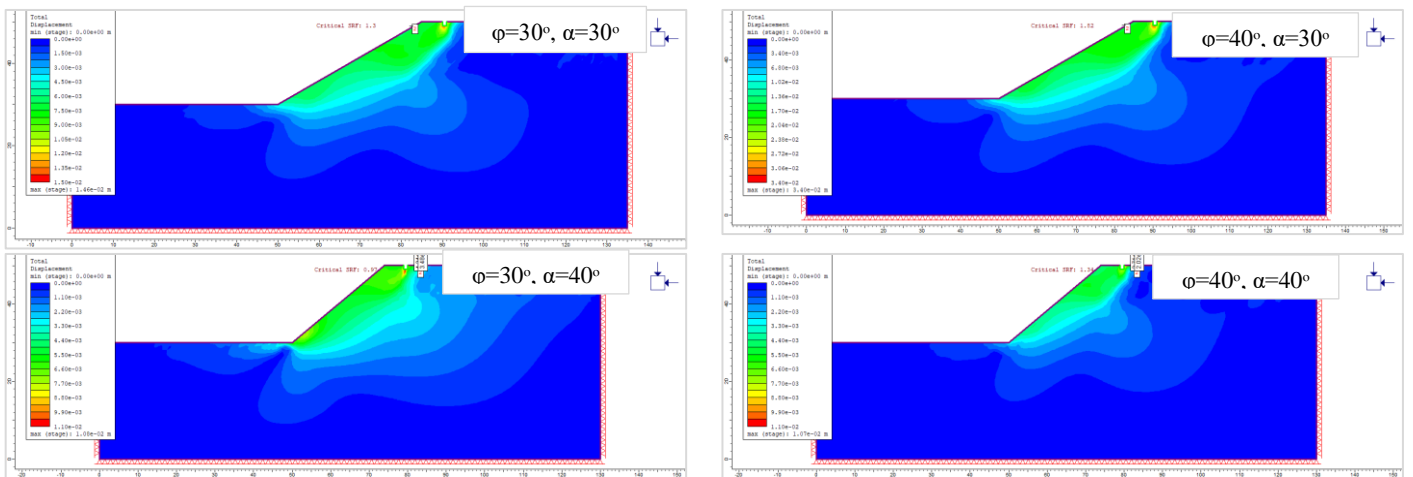


Figure 4.22 Output for H=20m, D/B=1 & x/B=5.

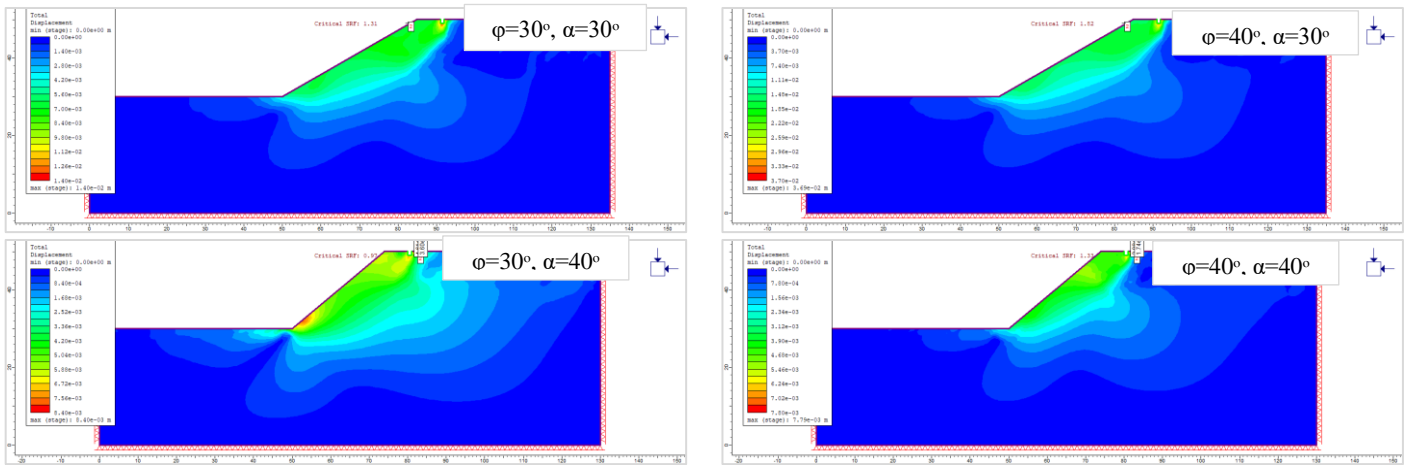


Figure 4.23 Output for  $H=20\text{m}$ ,  $D/B=1$  &  $x/B=6$ .

## 4.7 Design charts

The design charts represent a relationship between the slope height ( $H$ ) and angle ( $\alpha$ ), the angle of internal friction of the soil ( $\phi$ ) and distance from the foundation to the edge of the slope ( $x$ ) based on factor of safety from 1.3 to 1.7. These charts are limited to cohesionless soils and to a ratio of  $D/B=1$  (where  $D$  &  $B$  are the embedment depth and the width of the foundation, respectively). If higher factor of safety is required, it's recommended to place the foundation at farther distance from slope.

Table 4.1 summarises the values used to create the design charts. The target distance is the minimum distance at which the slope reached a factor of safety from 1.3 to 1.7.

Table 4.1 Design charts values

Point	Soil parameters			Slope height (H) (m)	Slope angle ( $\alpha$ ) ( $^{\circ}$ )	Distance to slope (x) (m)	Q allowable (kN/m <sup>2</sup> )	Factor of Safety (F.S.)
	$\gamma$ (kN/m <sup>3</sup> )	$\phi$ ( $^{\circ}$ )	c (kPa)					
1	19.00	30.00	5.0	10.0	30.0	3.0	200	1.37
2	19.00	30.00	5.0	20.0	30.0	6.0	200	1.31
3	19.00	30.00	5.0	30.0	30.0	8.0	200	1.4
4	19.00	30.00	5.0	40.0	30.0	9.0	200	1.42
5	19.00	30.00	5.0	10.0	40.0			
6	19.00	30.00	5.0	20.0	40.0			
7	19.00	30.00	5.0	30.0	40.0			
8	19.00	30.00	5.0	40.0	40.0			
9	19.00	35.00	5.0	10.0	30.0	2.0	200	1.31
10	19.00	35.00	5.0	20.0	30.0	3.0	200	1.3
11	19.00	35.00	5.0	30.0	30.0	5.0	200	1.4
12	19.00	35.00	5.0	40.0	30.0	6.0	200	1.41
13	19.00	35.00	5.0	10.0	40.0	3.0	200	1.32
14	19.00	35.00	5.0	20.0	40.0	5.0	200	1.45
15	19.00	35.00	5.0	30.0	40.0	7.0	200	1.41
16	19.00	35.00	5.0	40.0	40.0	8.0	200	1.5
17	19.00	40.00	5.0	10.0	30.0	0.0	200	1.62
18	19.00	40.00	5.0	20.0	30.0	1.0	200	1.73
19	19.00	40.00	5.0	30.0	30.0	3.0	200	1.31
20	19.00	40.00	5.0	40.0	30.0	6.0	200	1.4
21	19.00	40.00	5.0	10.0	40.0	2.0	200	1.45
22	19.00	40.00	5.0	20.0	40.0	3.0	200	1.3
23	19.00	40.00	5.0	30.0	40.0	5.0	200	1.45
24	19.00	40.00	5.0	40.0	40.0	7.0	200	1.5

**Note:** Certain combination of soil characteristics and slope geometries caused an overall instability for the slope as it achieved a factor of safety of exactly 1 only for the case of no foundation. Consequently, for these soil and slope parameters combination it is not recommended to have foundation at any distance from the slope.

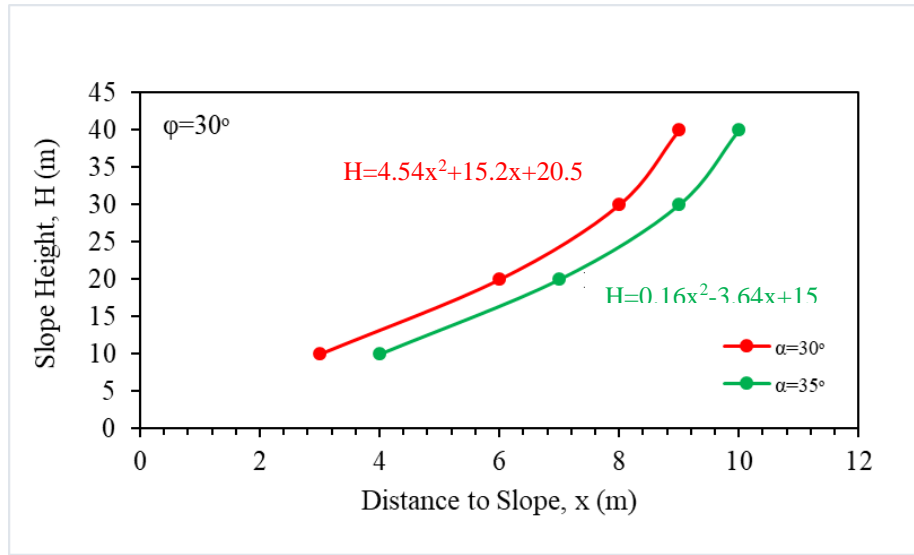


Figure 4.24 Design chart,  $\phi=30^\circ$ .

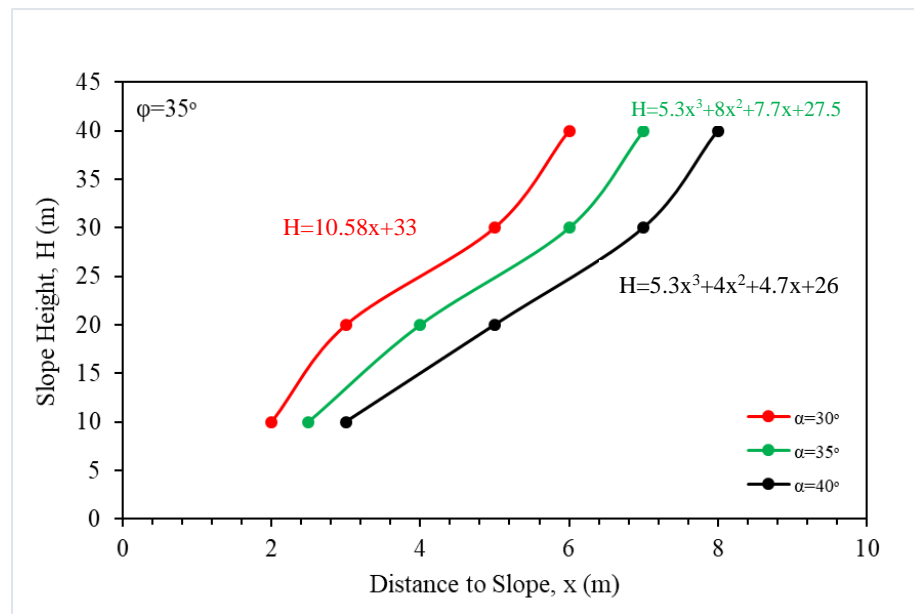


Figure 4.25 Design chart,  $\phi=35^\circ$ .

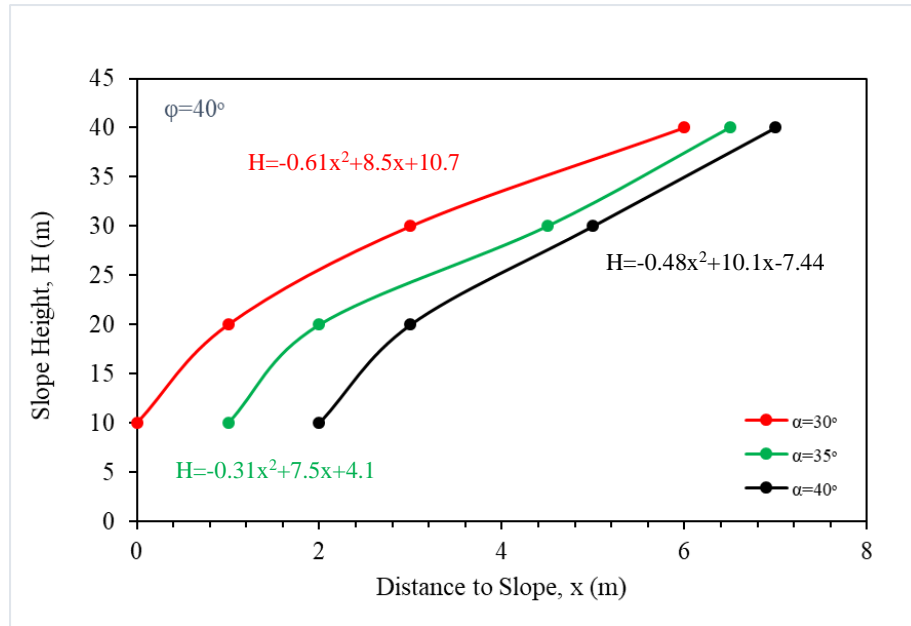


Figure 4.26 Design chart,  $\phi=40^\circ$ .

#### 4.8 Design example

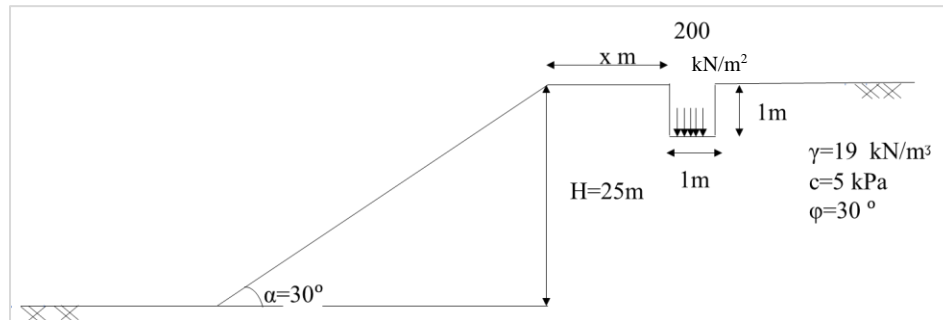


Figure 4.27 Design example

From the given information on the figure,

medium slope of  $H=25\text{m}$ , slope angle  $\alpha=30^\circ$

soil friction angle  $\phi=30^\circ$  from chart figure 4.24  $\therefore x=7\text{m}$



#### **4.9 Design guideline:**

- 1- Obtain soil parameters from one of the lab tests which determine the shearing strength parameters of the soil depending on the soil type (direct shear, triaxial test...etc.).
- 2- Determine the load from the building on the foundation, according to this research for a strip footing of width of 1m and embedment depth of 1m and placed at a distance from 0 to 6B (B: width of foundation), the allowable load which maintains the stability of the slope and foundation with respecting the allowable settlement for the foundation in cohesionless soil can not exceed 200 kN/m<sup>2</sup>.
- 3- Know the slope height and angle.
- 4- Determine the slope type from table 3.1.
- 5- Giving (H,  $\alpha$ ,  $\phi$ ), the design charts can be used to predict the minimum distance (x) from the foundation to the slope to achieve a factor of safety of at least 1.3.
- 6- In case of instability of the problem, it's recommended to increase the embedment depth of the foundation and/or increase the distance between the foundation and the edge of the slope.

## CHAPTER 5

### CONCLUSION

Numerical model was developed using finite element technique to examine the parameters governing the bearing capacity of shallow foundation near slope. Parametric study was conducted to examine the effect of the slope height (H), slope angle ( $\alpha$ ), the angle of the internal friction of the soil ( $\phi$ ) and the distance from the foundation to the edge of the slope (x). Design procedures and charts were developed to assist designers to predict the location of the foundation for given slope geometries and soil conditions in order to maintain the stability of both foundation and slope. Design example was presented. The following can be concluded:

- 1- The bearing capacity of the foundation placed near slope decreases with the increase of the slope angle and height while increases with the increase of the distance to the slope and/or the angle of internal friction of the soil.
- 2- The theory of Meyerhof, 1957 underestimated the effect of the distance from the foundation to the slope. The distance at which the bearing capacity of the foundation is independent of the inclination of the slope was found to be greater than that mentioned in the theory, while it was in full agreement with the results in terms of the factor of safety of the slope.
- 3- For higher angles of shearing resistance and in case of relatively medium slopes, the stability of slope is less affected by the presence of the foundation.
- 4- For medium slopes, the effect of the slope angle ( $\alpha$ ) on the stability of the slope has almost the same significance as the effect of the shearing resistance angle of the soil ( $\phi$ ).
- 5- At certain combination of soil parameters and slope geometries (presented in table 4.1), it's not recommended to have a foundation at any distance from the slope as the factor of safety was found to be exactly 1 only in case of no foundation.
- 6- For medium slopes and for  $\alpha > \phi$ , the stability of a system of foundation and slope is governed by the slope height.

## **RECOMMENDATIONS**

- 1- The presented study could be extended to consider the true three dimensions of the problem.
- 2- Different types and dimensions of foundations can be considered in further researches.
- 3- The presented study can be extended to consider layered soils.
- 4- The presented study can be performed for case of presence of ground water table.

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