Production Planning in Remanufacturing Systems with Uncertain Component Processing Time

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A Thesis

In

The Department

Of

Mechanical, Industrial and Aerospace Engineering (MIAE)

Presented in Partial Fulfilment of Requirements

For the Degree of Master of Applied Science at

Concordia University

Montreal, Quebec, Canada

May 2020

CONCORDIA UNIVERSITY

School of Graduate Studies

This is to certify that the thesis prepared

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Entitled: Production Planning in Remanufacturing System with Uncertain Component Processing Time

and submitted in partial fulfillment of the requirements for the degree of

Master of Applied Science (Industrial Engineering)

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Abstract

Production Planning in Remanufacturing Systems with Uncertain Component Processing Time

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Today's manufacturing industries in many countries have developed systematic product recovery, remanufacturing and recycling procedures in an environmentally supportive manner to release the regulatory pressure as well as to achieve economic benefit. This thesis presents a mixed integer programming model addressing production planning problems in hybrid system of manufacturing and remanufacturing. The objective of solving the mathematical model is to minimize the total cost based on the optimal quantity of new items to manufacture and the optimal quantity of returned products to remanufacture in each period of the planning horizon. The proposed model has a distinctive feature that considers the uncertainty of remanufacturing time for the same type of returned products. A new heuristic solution method, similar to Silver-Meal heuristic for solving traditional lot-sizing problems, is developed to solve the considered production planning problems in hybrid manufacturing-remanufacturing systems is developed in this thesis. The developed heuristic is examined using various example problems generated in three dimensions (problem size, returned products quantity and category quantity). The results show that it can generate optimal or close-to-optimal solutions for all tested example problems with much reduced computational time. The model and the solutions were analyzed and sensitive analysis is conducted to investigate the performance of the developed model and solution method.

Acknowledgements

I would first like to express my deepest gratitude to my supervisor, Professor Mingyuan Chen. He gave me a lot of guidance and support since the beginning of my thesis work. It is impossible to finish this work without his encouragement, insight and patience.

I would also like to thank my dear colleagues in the office, Cesar, Omar, Jair, Yasser, Ladan, Walla, Tiansheng, Alex, Dr. Bai, Dr. Zhao and Dr. Wei who gave me much help during my study. It is so lucky to have them as colleagues. We spent so much pleasant time together in and out of the office.

I would like to thank my dear friends Tingwen, Tong, Zenan, Pengcheng, Mengting and Zhun. I feel so lucky and blessed to meet you in this foreign country. We not only spent fun time together but also supported each other through so much trouble and tough time.

After that, I would like to profoundly thank my beloved parents， sister and Jiale. Whenever I feel lost or afraid, you are always there supporting and encouraging me.

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Chapter 1 Introduction

This chapter introduces the background of the research as well as presents a brief summary of the research work conducted in this thesis.

1.1 Introduction

Today's manufacturing industries in many countries have started to develop systematic product recovery, remanufacturing and recycling procedures in an environmentally supportive manner to release the regulatory pressure. In addition, economic benefit of remanufacturing is another important motivation. As a complete form of product reuse, it maintains much of the value assed from original manufacture to material and reprocessing conservation, leading to lower production costs and improved firm profitability (Hesse et al. 2005; Kim et al. 2006). Remanufacturing in the US is estimated to generate $100 billion of goods sold each year and employs over 500,000 people (Hagerty and Glader 2011).

1.2 Remanufacturing

The process of remanufacturing involves getting back end-of-life (EOL) products or components to upgrade or turn them into their original specifications (Li, 2007). A formal definition of remanufacturing can be found in Sundin (2002): "The process of rebuilding a product, during which: the product is cleaned, inspected, and disassembled; defective components are replaced; and the product is reassembled, tested and inspected again to ensure it meets or exceeds newly manufactured product standards". In other words, remanufacturing is a process in which a used product or parts of the product are restored to like-new conditions.

Nowadays many everyday goods are remanufactured, from expensive jet engine fans, automobile engines and medical equipment to less valued goods such as cameras, auto parts, computer equipment and machine tools (Franke, 2006).

1.3 Motivation

For remanufacturing industry, there are two primary factors driving the growth of it. The first factor is the positive impact on the environment. In the age of increasing environmental awareness, governmental and consumer pressure have induced companies to consider carefully the environmental impacts of their products as well as their process. This has become particularly evident in Europe in the form of environmental legislations. In the US, environmental regulations have put increasing pressure on industries to reduce waste disposal. Companies are increasingly being held responsible for their products throughout their life cycle.

The second reason is the cost of remanufactured goods can be much lower than the traditionally manufactured goods, so the manufacturers can gain competitiveness in the market by lowering the sale prices of their products. Remanufacturing is profitable and efficient when a large fraction of materials used in a product, and the value added to it when it is made, can be recovered at a low cost compared with that of the original manufacture.

1.4 Contributions

This research proposes a mathematical model for production planning in a hybrid system of manufacturing-remanufacturing. In the proposed model, new constraints of uncertainty regarding different remanufacturing time due to worn out conditions are considered. The objective function of the model is to minimize the total cost of production, including manufacturing cost, remanufacturing cost, set-up cost and holding cost.

A new heuristic method is introduced to solve the model for getting the optimal solution or close-to-optimal solution in shorter computational time in solving the NP-hard production planning model. The proposed heuristic is developed based on Silver-Meal heuristic. The approach is tested and validated using numerical examples with data generated in three dimensions: problem size, quantity of returned products and returned product quality levels in different product categories. The results are compared with optimal solution obtained using ILOG CPLEX and the heuristic solutions are very close to optimal solutions if they are different from the optimal ones. The proposed heuristic method provides a more efficient way to solve the model.

1.5 Outline of the Thesis

The remainder of this thesis is organized as follows. Chapter 2 categorizes and summarizes some of the relevant literatures on production planning, inventory models, and uncertainty effect of remanufacturing. In Chapter 3, we present the production planning problem of a hybrid system of manufacturing-remanufacturing with uncertain remanufacturing time and introduce a mixed integer programming model for solving the problem. A Silver-Meal based heuristic method is proposed and explained after the two versions of the developed mathematical model are presented. Chapter 4 presents numerical example problems to illustrate the developed model and solution method with computational results compared and analyzed. A sensitivity analysis is conducted and reported in this chapter. Finally, Chapter 5 concludes the study with a summary and directions for future research.

Chapter 2 Literature Review

This chapter presents a review of the literature on research in the area of production planning in manufacturing and remanufacturing systems as well as other related topics.

2.1 Introduction

In recent years, increasing number of manufacturers in developed and developing countries have taken actions in sustainable development based on public environmental awareness and recycling regulations. They have recognized the need to produce and dispose of products in an environmentally responsible manner. Remanufacturing is a life cycle strategy that allows end-of-life products to re-enter the manufacturing process to be refurbished, repaired or remanufactured to become as-good-as-new products, usable modules or components (Morgan and Gagnon 2013). It can not only help the manufacturers to meet environmental regulations but also bring enormous economic benefits to them. Research in remanufacturing and related topics has also been very active with literature abundant in the past several decades. In particular, many researchers developed various mathematical and numerical models, optimal or heuristic solution algorithms to solve different challenging problems arising from remanufacturing and related practice. In this chapter, a literature review related to remanufacturing modeling and methodologies is presented into three main sections: production planning, inventory modeling, uncertainty effect.

2.2 Production Planning

The basic problem for remanufacturing production planning and control is to determine the optimal values of a number of inter-related decision variables including how much to produce and/or order for new materials, how much and when to disassemble and to remanufacture.

2.2.1 Manufacturing

In the field of production planning of manufacturing, a classic and well-known method is Wagner-Whitin algorithm. Wagner and Whitin (1958) proposed a forward algorithm for a solution to a single-item multiple-periods economic lot sizing problem. In this problem, demands in each period, inventory holding charges and setup costs all vary over periods. Wagner-Whitin algorithm can guarantee to obtain the optimal solution for minimizing the total relevant cost. Another well-known method for solving lot sizing problem is Silver-Meal algorithm. It is an extension of Wagner-Whitin method to consider the total relevant cost per unit of time. Compared with Wagner-Whitin, Silver-Meal algorithm is much simpler in terms of user understanding and implementation and can achieve close-to-optimal solutions in solving many testing problems. In many cases, the difference is less than 1% from optimal (Silver and Meal 1973). Therefore, Silver-Meal algorithm is still being discussed and utilized in production planning modeling and solution method development today.

There is extensive research development on different extensions of Silver-Meal method after it was first introduced in Silver and Meal (1973). For example, Gaafar (2006) proposed two constructive heuristics for solving single-level uncapacitated dynamic lot-sizing problems based on a modified Silver-Meal method . The major difference between the modified heuristic method and Silver-Meal method is in the way it handles demand periods with zero demand requirements. For calculating the average period cost, the modified heuristic divides the total cost by the total number of non-zero demand periods, Silver-Meal method divides it by the total number of demand periods regardless of whether or not the demand in a particular period is zero. In that paper, the second heuristic uses an improved tie-breaking rule and a local optimal search to enhance the performance of the modified Silver-Meal method. In addition, a large-scale simulation study was performed by controlling scheduling horizon, proportion of periods with zero demand and setup cost to unit holding cost ratio. The author compared the two proposed heuristics with the original Silver-Meal method as well as 6 other constructive heuristics in the literature. The results show that the proposed heuristic could achieve better and more robust performance.

Helber, Sahling, and Schimmelpfeng (2013) presented a stochastic version of single-level, multi-product dynamic lot-sizing problem subject to a capacity constraint. In the problem, the demand of each period is random and the unmet demand can be back-ordered. The problem was formulated as a non-linear optimization model and was approximated by two separate linear programming models. In the first approximation model, the authors used a scenario analysis approach with the random samples. In the second approximation, the expected quantity of inventory level and the backlog are considered as functions of accumulated production and were approximated by piecewise linear functions. Computational results of the two different approximation models were analyzed with a numerical example. The results show that the second approximation model performed particularly well. Though the first approximation turned out to be less accurate, it was more flexible with respect to probabilistic dependencies of the demands within a single scenario.

2.2.2 Remanufacturing

Jayaraman (2006) proposed an analytical approach to solve a production planning problem of a closed-loop supply chain which includes both manufacturing and remanufacturing. The author presented a linear programming model to minimize the total cost per remanufactured item. In the model, material recovery rates, material replacement quantities, workloads and total labor hours were considered as conditional on the level of nominal quality. This model can assist decision-makers to decide the number of returned units with a nominal quality level to be disassembled, remanufactured and acquired in an intermediate to long range period. The solutions of this model also determine the inventory levels at the end of each period as well.

Torkaman et al. (2017) studied a capacitated production planning problem of closed-loop supply chain of multi-stages, for multi-products in multi-periods. In formulating and solving the problem, both the setup for changing product and the setup for changing the process were considered. The problem was formulated as a mixed-integer programming (MIP) model. The model was solved by four MIP-based heuristic algorithms. The four algorithms employ non-permutation and permutation heuristics using rolling horizon. The problem was also solved by a simulated annealing (SA) algorithm with the initial solution provided by a heuristic. Taguchi method was applied to calibrate the parameters of the SA algorithm. The result shows that compared with other heuristic methods, the SA based algorithm may solve the problem faster within reasonable computational time.

In Teunter, Bayindir, and Van Den Heuvel (2006), a dynamic lot sizing problem for systems with products returns and remanufacturing was studied. In the considered problem, the demand and return amounts were deterministic over a finite planning horizon. Demand could be met with both manufactured new items and remanufactured ones. The objective of solving the problem was to minimize the total cost composed of holding costs for returns and manufactured-remanufactured products and set up costs by determining the lot sizes for manufacturing and remanufacturing of each period. The authors discussed modifications of three well-known methods: Silver-Meal (SM), Least Unit Cost (LUC) and Part Period Balancing (PPB) heuristics. The results of an extensive numerical study showed that: (1) the SM and LUC heuristics perform much better than PPB, (2) demand predictability is more important than variation, and (3) periods with more returns than demand should, if possible, be avoided by "matching" demand and returns.

Schulz (2011) extended the Silver-Meal based approach in Teunter et al. (2006) by adding two simple improvement steps. The first improvement step was to check whether two consecutive time windows can be combined and the second step was to check whether a remanufacturing lot can be increased. With the two improvement steps, the original SM heuristic method can be improved as tested by several numerical example problems. The average gap to the optimal solution was reduced s to 2.2% when applying the improvement steps. Comparing with the heuristic method introduced in Teunter et al. (2006), the average optimality gap was reduced more than 50%.

2.3 Inventory Models of Remanufacturing

In this section, deterministic and stochastic inventory models will be discussed.

2.3.1 Deterministic Models

Deterministic inventory models can be divided into stationary demands and dynamic demands as discussed below.

2.3.1.1 Stationary Demand

In stationary demand problems, the classic Economic Order Quantity (EOQ) logic is generally used to build deterministic models for determining the optimal trade-off between setup costs and holding costs in a production system.

Among other works presented by the two authors, Dobos and Richter (2000) developed integer non-linear models to analyze EOQ repair and waste disposal problems with integer number of setups. The result from testing example problems show that "pure strategies" (total repair or total waste disposal) can lead to optimal solutions. In Dobos and Richter (2003), the authors discussed a manufacturing-remanufacturing system by assuming that there was only one recycling lot and one production lot. They proposed a mixed strategy for the cases that the pure strategy was not feasible technologically or some of the returned products cannot be remanufactured. Their work was extended to situations with multiple production lots and multiple recycling lots in Dobos and Richter (2004). The model was further extended by relaxing the assumption of perfect quality of the returned items in Dobos and Richter (2006).

Roy et al. (2009) investigated a manufacturing-remanufacturing system for a single product with constant demand. The items included the defective ones from manufacturing and recycled products from customers. The defectiveness rate of the manufacturing system was represented by a constant and a fuzzy parameter in two separate models. When precise defective rate could not be determined, optimistic and pessimistic equivalent of fuzzy objective function was obtained by using credibility measure of fuzzy event by taking fuzzy expectation. In modeling the problem, it is assumed that the remanufacturing system started from the second production cycle and after that both the manufacturing and remanufacturing processes continued simultaneously. The models were formulated for maximum total profit out of the whole system. The decision variables were the total number of cycles in the time horizon. A Genetic Algorithm based solution method was developed with Roulette wheel selection, arithmetic crossover and random mutation applied to evaluate the maximum total profit and the corresponding optimum decision variables.

Polotski, Kenne, and Gharbi (2015) addressed an optimal scheduling problem for a hybrid system of manufacturing (manufacturing mode) and remanufacturing (remanufacturing mode). The considered system has one facility and necessitates setup for switching from one production mode to another. The flow rate of returned products was a fixed percentage of the demand rate, so it was necessary to switch from one mode to another. The authors developed the feasibility conditions for such systems and categorized them for mainly manufacturing systems and for mainly remanufacturing systems. The solution of the model was to meeting customer demand with minimized manufacturing cost by determining the production and setup scheduling. First, the optimal cyclic trajectories corresponding to the production runs (for both system types) were obtained with the consideration of the setup cost and negligible setup times. Then, these results were generalized for non-zero setup times. At last, transitional trajectories corresponding to optimal policies in the vicinity of the limit cycles were described. The results of the paper can be helpful for production scheduling in companies involved in both manufacturing and remanufacturing and using the same production facility for both processes.

2.3.1.2 Dynamic Demand

Richter and Sombrutzki (2000) studied the reverse Wagner-Whitin's dynamic production planning and inventory control model and certain types of its extensions. They extended the Wagner-Whitin algorithm for a deterministic recovery system by assuming a linear cost function without backordering and with negligible lead times. They proved that in the product recovery models, the optimal solutions have the property of zero inventory. The demands of each period were fulfilled by new products or remanufactured products determined by Wagner-Whitin algorithm. In addition, the paper showed that with some combinations of the original and the reverse models, the reverse problems can be solved more efficiently. In the follow-up paper (Richter and Weber 2001), the model was extended with the consideration of variable manufacturing and remanufacturing costs. The authors proposed a model combining the classical Wagner-Whitin model and a pure reverse Wagner-Whitin model with deterministic quantity of recycling products. With the analysis of the alternate application of remanufacturing and manufacturing processes, the combined model is more suitable for practice.

Fazle Baki, Chaouch, and Abdul-Kader (2014) discussed the lot sizing problem of product returns and remanufacturing over a finite planning horizon. The problem was to determine the optimal production plan with forecasted demands and product returns to meet both demands at minimum costs. The considered total cost included the fixed setup expenses associated with manufacturing and-or remanufacturing lots and the inventory holding costs. In the paper, a heuristic method was proposed to exploit the structure of optimal solutions. The authors observed that the feasible solution to this problem can split into a sequence of blocks with a distinct structure in the way that both manufacturing and remanufacturing setups occur. Based on the observation, a heuristic method was proposed to use dynamic programming and the Wagner-Whithin algorithm to solve the problem. The results of extensive numerical testing show that the heuristic performed well in terms of percentage cost error. Moreover, since the heuristic method is effective and produce high-quality solutions, it can be embedded within CPLEX to speed up the optimization process.

Sifaleras, Konstantaras, and Mladenović (2015) suggested a variable neighborhood search (VNS) metaheuristic algorithm to solve the economic lot sizing problem with product returns and recovery. In the problem, the dynamic demand was known for a finite planning horizon. The number of returns was given for all periods and assumed to be dynamic. The objective was to satisfy the demand of items in each period at the lowest possible total cost. For solving the problem, two novel VNS metaheuristic algorithms were proposed to employ new strategies for both the local search step and the shaking process. Several new neighborhoods were presented for this combinatorial optimization problem with an efficient local search method for exploring them. In addition, a new simple heuristic initialization method was described for this problem. Finally, a new benchmark set with 52 periods instance was developed. The results from the set show that the proposed VNS approach is quite efficient in solving large problems with small optimality errors.

2.3.2 Stochastic Models

In stochastic models, stochastic processes are employed to model demand and returns. Continuous and periodic review policies are two common approaches used in stochastic models (Ali and Gupta 2010).

2.3.2.1 Continuous Review Models

These models use continuous time axis and try to determine the optimal static control policies based on minimization of the long-run average costs per unit of time (Fleischmann et al. 1997).

Heyman (1977) presented the first study in this area by considering a continuous review strategy for a single item inventory system with remanufacturing and disposal. Queueing model was used to describe the system. When the return and demand processes follow Poisson distribution, the model can be solved to exact solutions. For more general processes, a diffusion approximation was used to the model and obtained approximate solutions. From the results of numerical examples, the diffusion approximation can provide good solutions when the parameters for Poisson processes were used. Diffusion model only uses the first two moments of the return and repair processes and such information is likely to be available in practice.

Nakashima et al. (2004) studied an optimal control problem of a remanufacturing system under stochastic demand. The system was described by a Markov decision process. It is a class of stochastic sequential processes in which the reward and transition probability depend only on the current state of the system and the current action. In the system, the actual product inventory in a factory and the virtual inventory used by a customer were considered. Both inventory levels defined the state of the remanufacturing system together. The optimal production policy minimized the expected average cost per period. Some scenarios under various conditions were also considered and an example of controlling the remanufacturing system was shown. The numerical results illustrate the property of the optimal control of the remanufacturing system. It also means that the proposed approach is applicable to different systems by choosing the parameters based on different conditions.

In Konstantaras and Skouri (2010), a manufacturing-remanufacturing inventory system was considered. The cost structure of the system included the EOQ-type setup costs, holding costs and shortage costs. The authors first studied the model with no shortage in serviceable inventory. Then they discussed the serviceable inventory shortage case. Both models were considered for the case of variable setup numbers of equal sized batches for manufacturing and remanufacturing processes. The authors proposed sufficient conditions for the model parameters to determine the class of policies in which the optimal one falls. In the end, these sufficient conditions were given based on the closed form formulae for the total cost function of the system.

2.3.2.2 Periodic Review Models

Periodic review models search for optimal policies based on the minimization of expected costs over a finite planning horizon (Fleischmann et al. 1997).

Mahadevan, Pyke, and Fleischmann (2002) studied a remanufacturing facility that received a stream of returned products according to Poisson process. The demand in the system was uncertain and followed a Poisson process. There were two problems needed to be solved: when to release the returned products to the remanufacturing line and how many new products to manufacture. In the paper, a "push" policy was employed to combine these two decisions. Moreover, bounds and heuristics were developed based on traditional approximate inventory models. Each heuristic method was based on simple, intuitive adjustments to the parameters of the traditional model. The first two approaches relied on an approximation of the manufacturing and remanufacturing sources by a single aggregate channel. The third approach explicitly considered the impact of both channels separately. All the three heuristics perform quite well on average, with average total cost errors of 3.27%, 5.96% and 0.44%, respectively.

Wang et al. (2011) discussed a single-item, dynamic lot-sizing problem for systems with remanufacturing and outsourcing. Demand and return amounts were both deterministic over a finite planning horizon. Demand could be satisfied with new manufactured items, remanufactured items and outsourcing but could not be backlogged. The objective of the study was to determine the lot sizes for manufacturing, remanufacturing and outsourcing that minimize the total cost including relevant costs. A dynamic programming approach was proposed to derive the optimal solution in the case of large quantities of returned product. The paper establishes the characteristics of single-item lot sizing with remanufacturing and outsourcing and develops a polynomial algorithm for the model.

Helmrich et al. (2014) discussed two variants of the economic lot-sizing problem with remanufacturing. The quantities of returned products were known in each period. In every period, one can choose to set up a process to manufacture new products or remanufacture returned products. These processes could have separate or joint setup costs. These two variants were discussed in the paper and they were both shown to be NP-hard. Furthermore, the authors also proposed several alternative mixed-integer programming formulations of both problems and tested their efficiency on a wide variety of test instances. The test results show that, for both problem variants, the shortest path formulation performed better than the Original and other formulations, especially in terms of the quality of the LP-relaxation.

2.4 Uncertainty Effect of Remanufacturing

One important problem related to remanufacturing is the uncertainties associated with the process of a remanufacturing system. The uncertainties in quantity, quality and the required time to process the returned products make the analysis of remanufacturing systems more complicated.

2.4.1 Static Modeling Method

Ferrer (2003) developed decision models to deal with limited information on remanufacturing yields and potentially long supplier lead time. To make a better decision, managers may attempt to identify the reparable parts early in the remanufacturing process, to develop a responsive supplier with short lead time, or to get more information about the status of the recycled machines. In the paper, the author provided a single-part lot-size decision model then analyzed the model according to each of these scenarios. Last the relative values of these alternatives were compared under a broad range of parameters. The results show that: (1) when the yield variance increases, developing early detection capability of the process yield is more important than having suppliers with short lead time; (2) when the shortage cost increases, it is better to have a responsive supplier who has a short lead time; (3) when the purchase, repair of holding cost increases, it is better to be capable to detect process yield early.

In Ketzenberg, Laan, and Teunter (2009), the value of information (VOI) was explored in the context of a firm that faces uncertainty with respect to demand, product return and product recovery. The discussion started with a single-period problem with normally distributed demands and returns. Then the problem was extended to the multi-period case. The returns in a period were correlated with demands in the previous period. The objective was to evaluate the VOI from reducing one or more uncertainties by measuring the reduction in total expected holding and shortage costs. Estimators were developed to predict the value and sensitivity of different information types. The main contribution of the paper is that the estimators can be used to determine potential gains from investing in information on demand, return and yield.

2.4.2 Dynamic Control Method

In Hilger et al. (2016), the authors considered a dynamic multi-product capacitated lot sizing problem with stochastic demand and return in remanufacturing. In the problem, the demands and return quantity were both random. Two models were proposed for this integrated stochastic production and remanufacturing problem. In the first model, the problem was transferred into a mixed-integer problem by representing the nonlinear functions with piecewise linear functions. In this way, a standard mixed-integer programming solver could be used to solve the problem. In the second model, the expected values were replaced by sample averages of independent scenarios. The two different models demonstrate that the problem considered could be well solved through a simulation model. The results show that the integrated planning approach is advantageous compared with a sequential planning approach.

Macedo et al. (2016) studied hybrid manufacturing and remanufacturing lot-sizing problems with several uncertainties. Unlike the other papers, this problem included multiple products, disposal, backlogging, and the inherent uncertainties of demands, return rates of usable products, and setup costs. The authors proposed a scenario-based two-stage stochastic programming model to solve the problem with uncertainties. In the model, production and setup costs were assumed as first-stage decision variables and inventory, disposal and backlogging were taken as second-stage decision variables. To reduce the risk associated with the dispersion of the second-stage cost, risk-averse constraints were aggregated via a mean-risk model. The results indicate that if the objective is to propose a solution with less risk, the expected total cost will increase slightly. Moreover, the combined lines of both manufacturing and remanufacturing cost less but provide more economic viability than the isolated lines of both processes.

Mukhopadhyay and Ma (2009) considered a hybrid system where both used and new parts could serve as inputs in the production process to satisfy an uncertain demand. In the system the yield of the used parts was random as well. Both the used parts and the new parts were processed in the same production line. Three different cases were discussed depending on the amount of information about the yield rate which included deterministic yield model, random yield rate model and a special case of uniform distributed yield rate and demand. In the random yield rate model, the cases of short and long delivery lead time of new parts were further discussed. In each model, the optimal procurement and production quantity were determined. Extensive numerical analyses were presented and the results of the sensitivity analyses on various problem parameters were discussed.

Assid, Gharbi, and Hajji (2019) considered a hybrid system using both raw materials and returned products in the production process with the presence of random events including facilities failures, delivery lead times of raw materials and returns. For the system, it was important to determine the appropriate storage spaces and adaptive strategies to manage the manufacturing, remanufacturing and disposal operations as well as the supply of both raw materials and returns. This paper mainly aimed to propose and efficient structure of joint control policies integrating simultaneously the production and disposal activities as well as the procurement of both return and raw material. A simulation-based optimization approach was applied of determine the optimal control parameters including the raw material supply and the storage space sizing finished products, raw materials and returns while minimizing the total incurred cost. After that, the robust behavior and the usefulness of the proposal was shown by an in-depth sensitivity analysis. The results show that the proposed control policies achieved important cost savings which varied between 6.26% and 54.14% comparing with the instances from literatures.

In Li, Li, and Saghafian (2013), some insights were generated into the acquisition management and production planning of a hybrid manufacturing and remanufacturing system with stochastic acquisition price and random yield in the remanufacturing process. In the paper, it was shown how to maximize the total expected profit by coordinating the acquisition pricing, remanufacturing and manufacturing decisions. Two different cases were considered for sequential and parallel manufacturing-remanufacturing processes. In each case, a stochastic dynamic programming was used to formulate and analyze the model, showing that the optimal policy was characterized by several critical values and functions. By comparing the two cases, it shows that the optimal acquisition price and remanufacturing quantity are both higher in the case of sequential process.

Kenné, Dejax, and Gharbi (2012) studied the production planning and control of a single product involving combined manufacturing and remanufacturing operations within a closed-loop reverse logistics network with machines subject to random failures and repairs. There were three types of inventories involved in the network. The first and second inventories stored manufactured and remanufactured items and the third inventory was for the returned products which would be then remanufactured or disposed of. A new generic model was proposed based on stochastic optimal control theory. The objective of the model was to minimize the sum of the holding and backlog costs for manufacturing and remanufacturing products. The decision variables were the production rates of the manufacturing and the remanufacturing machines. This optimal control problem was solved by a computational algorithm based on numerical methods. At last, the usefulness of the proposed approach was illustrated by a numerical example and a sensitivity analysis.

2.5 Summary

In this chapter, literature related to modeling and algorithms of remanufacturing was presented. Different aspects of the considered problem and various versions and extensions of different solution approaches were discussed. In the next chapter, a new heuristic model based on Silver-Meal method will be presented to solve a new version of lot-sizing problem in a manufacturing-remanufacturing system.

Chapter 3 Modeling

In this chapter, details of the production planning problem in a remanufacturing system studied in this thesis are presented with the development of the mathematical programming model for solving the considered problems. A new heuristic method is developed to search for optimal or near-optimal solutions with reduced computational time in solving the considered remanufacturing production planning problems.

3.1 Problem Definition

We consider that a hybrid manufacturing-remanufacturing system produces one type of products with deterministic but time-varying demands during a finite planning time horizon. This system uses both manufactured and remanufactured components to assemble products. The manufacturing process makes new components with new materials. The remanufacturing process covers inspection, disassembly, cleaning and remanufacturing to produce "as good as new" components. The two processes can be executed individually or together in the considered system.

We consider that the quantity of returned products is deterministic over the planning horizon. When a returned product is received, two options are available for the returned products: remanufacturing or disposal. The products that could be remanufactured will be inspected, disassembled, cleaned and remanufactured in the remanufacturing system. In this considered problem, we assume that inspection, disassembling and cleaning require similar amount of time for all the returned products, but the required remanufacturing time may vary depending on different levels of worn-out of each component. The different remanufacturing time required to make the returned products to become “as good as new” products affect inventory levels and production planning decisions. As presented in the literature review in the previous chapter, research in developing and solving optimization models for remanufacturing production planning problems with varying remanufacturing time is very limited. In most of the existing remanufacturing production planning models, it typically assumes that similar amount of remanufacturing time is required to process all disassembled components. In the remanufacturing production planning problems considered in this thesis, we consider that remanufacturing time may take one to several time periods depending on the quality level of the disassembled components.

In the considered problem, the returned products will be inspected and disassembled to retrieve useful components at the beginning of each period. The remanufacturing time of each component can be estimated based on their levels of worn-out. The ones that can be remanufactured within 1 time period (for example, 1 work day) are considered as category 1, between 1 and 2 time periods are considered as category 2, and so forth. To remanufacture the components in different categories is associated with different remanufacturing costs since longer remanufacturing time typically leads to higher cost.

The main problem feature considered in this research that disassembled components may require different processing time periods to remanufacture can be further illustrated in Figures 3.1 and 3.2. Figure 3.1 shows an example of the situation that all components require same remanufacturing time. The planning horizon is 5 periods and each period gets some returned products. Period 1 and period 3 are planned to remanufacture the components disassembled from returned products. All the remanufacture components can be finished in the current period. In the problem considered in this research, however, remanufacturing time can be different for the disassembled components depending on their categories as shown in Figure 3.2. For the components remanufactured in period 1, only category 1 components can satisfy the demand of period 1 since the components in categories 2 and 3 require more remanufacturing time to complete.

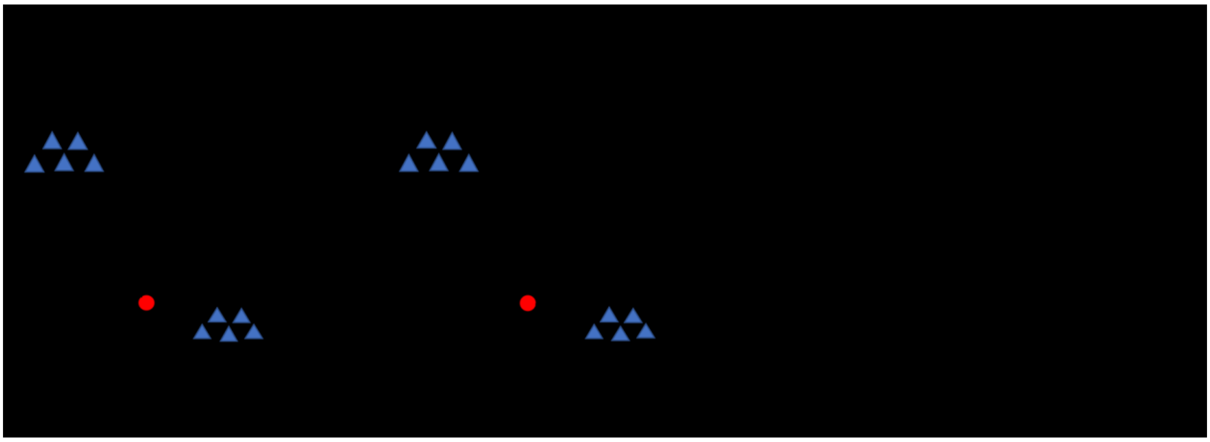

Figure 3.1 Schematic diagram of the problem in previous literature

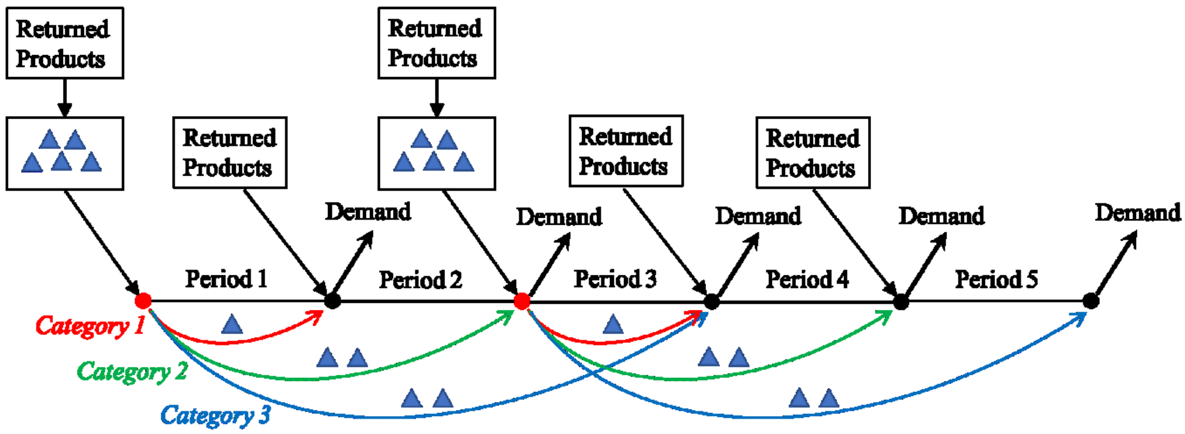

Figure 3.2 Schematic diagram of the problem in this thesis

In the hybrid manufacturing-remanufacturing system considered in this research, final products are made from either new components or recovered components. Demand for the final products does not differentiate that they are made from new or used components since recovered components have been remanufactured to the level of "as good as new". At the beginning of any time period, a manager will decide if some of the returned products should be processed including inspection, disassembling, cleaning and remanufacturing. If some of them will not be processed, they will be placed in inventory and be available for processing in the following time periods.

More specifically, we consider the following problem features in the considered remanufacturing production planning problem.

1. Demands for the final products in a time horizon with multiple time periods are known.
2. The quantities of returned products from customers in each period are known as well.
3. The final products can be produced using new components from suppliers or using remanufactured components from disassembled returned products.
4. There are no differences between products made from new components or "as good as new" components.
5. After a recoverable product is disassembled, the reusable components will be sorted into different categories according to the levels of worn-out and corresponding remanufacturing time required.

6 The returned products which are not disassembled in the current period will be stored in the recoverable inventory to be processed in the remaining periods in the considered time horizon.

7 No limit on the inventory capacities of new components, returned products or final products.

In this research, we developed a mathematical programming model to solve the hybrid manufacturing-remanufacturing production planning problem. The model development is based on the above described features. The solution of the developed integer programming model is to determine the optimal values of a number of inter-related decision variables including the amount and time periods to produce new components and remanufactured components in minimizing total production cost of the system in the considered time horizon. Variable and parameter definitions used in developing the model are explained in the next section.

3.2 Definitions

Some of the terms used in developing the math model will be explained in detail.

3.2.1 Holding Cost

Holding cost is the cost to keep an item in inventory at each unit of time. In our problem, the inventory of returned products and the inventory of final products are considered separately. Since the condition and requirement of storing returned products and final products are different, the holding cost of these two inventories are considered differently as well.

3.2.2 Set-up Cost

Set-up cost is the cost incurred to get the system ready to produce components. This cost is independent of the quantity of the components processed. In the considered hybrid manufacturing-remanufacturing system, the set-up costs for both manufacturing and remanufacturing are considered, which means when the process is set up for either manufacturing or remanufacturing, the corresponding set-up cost is incurred.

3.2.3 Category of Disassembled Components

As mentioned earlier, in this research, we consider that to remanufacture the components may require different processing time based on the levels of worn-out of the component. We assume that the remanufacturing time can vary from one time period to several time periods. And define the category of disassembled components by the amount of time they need to be remanufactured. The components requiring 1 time period or less to remanufacture are considered as category 1 components; requiring 1 to 2 time periods category 2 components, and so forth. The category of components is an important feature of the problem representing one of the important aspects of remanufacturing planning considered in this research. This feature also reflects the uncertainty in remanufacturing systems and does not exist in regular manufacturing systems.

3.3 Modeling of Single-Item Production Planning Problem

We first present the model considering only one type of disassembled components. The problem is similar to single-item lot sizing problem. Specific assumptions and model notations are given below.

3.3.1 Model Assumption

1. The considered production planning time horizon has multiple time periods.
2. Demands are satisfied with final products made from either new components or remanufactured components.
3. A final product contains one type of components, manufactured or remanufactured.
4. After the returned products are disassembled, all components are inspected and categorized according to their quality levels.
5. Components in different categories require corresponding processing time to be remanufactured before they can be used to make final products.
6. Remanufacturing cost includes cost of inspection, disassembly, cleaning and remanufacturing.
7. No limit on inventory capacities of new components, recoverable products or final products.
8. Setup cost incurred for manufacturing can be different from setup cost incurred for remanufacturing.

3.3.2 Notations

Sets, parameters and variables used in the model are defined below.

Sets:

T Set of production planning periods, $t = 1..T$

J Set of component categories disassembled from the returned products, $j = 1..J$

Parameters:

D_t Demand in period t

R_t The quantity of returned products in period t

L The quantity of the component contained in the returned product

P_j The percentage of component category j disassembled from the returned products

CM Cost of manufacturing a new component

CR_j Cost of remanufacturing a component of category j

CHY_t Holding cost of returned products inventory in period t

CHZ_t Holding cost of final products inventory in period t

KM_t Set-up cost for manufacturing in period t

KR_t Set-up cost for remanufacturing in period t

M A large number

Decision variables:

m_t Quantity of component manufactured in period t

r_t Quantity of products remanufactured in period t

y_t Inventory of returned products at the end of period t

z_t Inventory of final products at the end of period t

$\theta_t^m = \begin{cases} 1, & \text{if the system is set up to manufacture product in period } t \\ 0, & \text{otherwise} \end{cases}$

$\theta_t^r = \begin{cases} 1, & \text{if the system is set up to remanufacture product in period } t \\ 0, & \text{otherwise} \end{cases}$

3.3.3 Mathematical Model

The mathematical model is presented as follows:

$Min\ \ C = \ \sum_{t=1}^{T}\sum_{j=1}^{J}(CM \times m_t + L \times P_j \times CR_j \times r_t + KM_t \times \theta_t^m + KR_t \times \theta_t^r + CHY_t \times$

$y_t + CHZ_t \times z_t)$

(1)

s.t.

$L \times P_1 \times r_t + \sum_{\sigma=1}^{t-1} L \times P_{t+1-\sigma} \times r_\sigma + L \times z_{t-1} + m_t - L \times z_t = L \times D_t \qquad \forall\ t \in T$ (2)

$y_t = y_{t-1} + R_t - r_t \qquad \forall\ t \in T$ (3)

$m_t \le M \times \theta_t^m \qquad \forall\ t \in T$ (4)

$r_t \le M \times \theta_t^r \qquad \forall\ t \in T$ (5)

$r_t \le y_{t-1} + R_t \qquad \forall\ t \in T$ (6)

$y_0 = z_0 = 0 \qquad \forall\ t \in T$ (7)

$0 \le P_j \le 1 \qquad \forall\ j \in J$ (8)

$m_t,\ r_t,\ y_t,\ z_t \ge 0 \qquad \forall\ t \in T$

(9)

$\theta_t^m, \theta_t^r = \{0, 1\} \qquad \forall\ t \in T$ (10)

The objective function (1) is the minimization of the total cost over all time periods. The objective function included six parts: cost of manufacturing new components, cost of remanufacturing components, set up cost of manufacturing, set up cost for remanufacturing, holding cost of the inventory of returned products, holding cost of the inventory of final products. Constraints (2) and (3) ensure the inventory balance in returned products and final products, respectively. Constraints (4) and (5) relates production and system setup where M is a large number. Constraint (6) limits the remanufactured products quantity in each period. Constraint (7) enforces the initial inventories to be zero. Inequality (8) ensures the percentages of component category ranges between 0 and 1. Constraints (9) and (10) are non-negativity and binary constraints.

3.4 Modeling of Multi-Item Production Planning Problem

When the product contains multiple types of components, then the problem is considered as a multi-item production planning problem. The mathematical model of multi-item production planning problem is presented below. Objective function and constraints are explained in detail as well.

3.4.1 Model Assumptions

1. The considered production planning time horizon has multiple time periods.
2. Demands are satisfied with final products made from either new components or remanufactured components.
3. A final product contains multiple types of components, manufactured or remanufactured.

4. After the returned products are disassembled, all components of each type are inspected and categorized according to their quality levels.
5. Components in different categories of each type require corresponding processing time to be remanufactured before they can be used to make final products
6. Remanufacturing cost includes cost of inspection, disassembly, cleaning and remanufacturing.
7. No limit on inventory capacities of new components, recoverable products or final products.
8. Setup cost incurred for manufacturing can be different from setup cost incurred for remanufacturing.

3.4.2 Notations

Sets, parameters and variables used in the model are defined below.

Sets:

T Set of production planning periods, $t = 1, .. T$

I Set of component types disassembled from the returned products, $i = 1..I$

I_J Set of categories of component i, $j = 1..J, i = 1, ..., I$

Parameters:

D_t Demand in period t

R_t The quantity of returned products in each period t

L_i The quantity of the component i contained in the returned product

P_{ij} The percentage of category j of component i disassembled from the returned products

CM_i Cost of manufacturing a new component i

CR_{ij} Cost of remanufacturing a component of category j of component i

CHY_t Holding cost of returned products inventory in period t

CHZ_t Holding cost of final products inventory in period t

KM_t Set-up cost for manufacturing in period t

KR_t Set-up cost for remanufacturing in period t

M A large number

Decision variables:

m_{it} Quantity of component i manufactured in period t

r_t Quantity of returned products remanufactured in period t

y_t Inventory of returned products at the end of period t

z_t Inventory of final products at the end of period t

$$\theta_t^m = \begin{cases} 1, & \text{if the system is set up to manufacture product in period } t \\ 0, & \text{otherwise} \end{cases}$$

$$\theta_t^r = \begin{cases} 1, & \text{if the system is set up to remanufacture product in period } t \\ 0, & \text{otherwise} \end{cases}$$

3.4.3 Mathematical Model

The mathematical model is presented as follows:

$$Min\ \ C = \ \sum_{i=1}^{I}\sum_{t=1}^{T}\sum_{j=1}^{J}(CM_i \times m_{it} + L_i \times P_{ij} \times CR_{ij} \times r_t + KM_t \times \theta_t^m + KR_t \times \theta_t^r + CHY_t \times y_t + CHZ_t \times z_t) \quad (1)$$

s.t.

$$L_i \times P_{i1} \times r_t + \sum_{\sigma=1}^{t-1} L_i \times P_{i(t+1-\sigma)} \times r_\sigma + L_i \times z_{t-1} + m_{it} - L_i \times z_t = L_i \times D_t$$

$\forall \ t \in T, \ \forall \ i \in I$

(2)

$$y_t = y_{t-1} + R_t - r_t \qquad \forall \ t \in T \qquad (3)$$

$$m_{it} \leq M \times \theta_t^m \qquad \forall \ t \in T \qquad (4)$$

$$r_t \leq M \times \theta_t^r \qquad \forall \ t \in T \qquad (5)$$

$$r_t \leq y_{t-1} + R_t \qquad \forall \ t \in T \qquad (6)$$

$$y_0 = z_0 = 0 \qquad \forall \ t \in T \qquad (7)$$

$$0 \leq P_{ij} \leq 1 \qquad \forall \ j \in J \qquad (8)$$

$$m_{it}, \ r_t, \ y_t, \ z_t \geq 0 \qquad \forall \ t \in T \qquad (9)$$

$$\theta_t^m, \theta_t^r = \{0, 1\} \qquad \forall \ t \in T \qquad (10)$$

The objective function (1) is the minimization of the total cost over all time periods. The objective function included six parts: cost of manufacturing new components, cost of remanufacturing components, set up cost of manufacturing, set up cost for remanufacturing, holding cost of the inventory of returned products, holding cost of the inventory of final products. Constraints (2) ensures the inventory balance in final products on component level. Constraints (3) ensures the inventory balance in returned products. Constraints (4) and (5) relates production and system setup where M is a large number. Constraint (6) limits the remanufactured products quantity in each period. Constraint (7) enforces the initial inventories to be zero. Inequality (8) ensures the percentages of component category ranges between 0 and 1. Constraints (9) and (10) are non-negativity and binary constraints.

3.5 Solution Method

The mathematical programming model for production planning for the considered manufacturing-remanufacturing systems can be solved using CPLEX or other off-shelf optimization software. Optimal solutions can be found quickly in solving the considered production planning problems if the problem sizes are not large. However, since the considered problem is NP-hard in nature, in this research, we developed a heuristic method for solving the considered problem efficiently for potentially large-scale problem solving in practical applications. The developed solution method is based on Silver-Meal (Silver and Meal, 1973) heuristic. Silver-Meal heuristic is a method that determines the optimal lot size by comparing the average cost per period. Following the same logic, we developed a heuristic that determines both the manufacturing and remanufacturing lot sizes in each period to achieve the minimum cost over a finite planning horizon. In this heuristic method, before comparing the average cost between different periods, the costs of satisfying demands with or without remanufacturing within the current period are compared first. The remanufacturing lot size in each period can then be determined and the manufacturing lot size is subsequently determined following the same logic of the Silver-Meal method.

In developing the heuristic solution method, we assume that in a time period that the remanufacturing system is set-up for production, all the returned and recoverable products will be processed. This is based on the consideration that set-up cost is one-time fixed cost and remanufacturing cost is typically lower than manufacturing cost. The total cost should be reduced when the remanufacturing process is activated to process all returned recoverable products.

In addition, for both single-item problem and multi-item problem, the calculation procedures are generally the same with difference in the calculation of manufacturing-remanufacturing cost. The multi-item problem includes more than one type of parts and each part type has its own categories and costs of remanufacturing in each category. Such differences must be included in calculating the cost. The heuristic method explained below is the general solution method which can be used to solve both single-item problem and multi-item problem.

The heuristic method contains 5 general steps.

<u>Heuristic Solution Method</u>

Step 1. Start from the initial period. Compute the costs of satisfying the demand of this period by manufacturing only and by both manufacturing and remanufacturing together. The costs include set-up cost, manufacturing-remanufacturing cost and holding cost. The quantity of remanufacturing is the total recoverable inventory.

Step 2. Compare the two cost values obtained from Step 1. If the cost with manufacturing only is lower, there is only manufacturing in this period and the products of recoverable inventory will be stored till next time period. If the cost with both manufacturing and remanufacturing is lower, the system will be setup to perform both manufacturing and remanufacturing in this period and all the products in recoverable inventory are remanufactured.

Step 3. Consider the next period together with the previous time periods. Based on the results of Step 2, compute the costs of satisfying the demand of the considered periods with and without remanufacturing in the next period. Then compute the average costs over the considered periods.

Step 4 and Step 5. Find the lowest average cost by repeating Step 3 and considering next time periods. When the lowest average cost is found, the quantities of manufacturing-remanufacturing of each period can be determined. The following period is set as initial period and the computation repeat from Step1. The algorithm stops when all of the periods are considered and all the decision variables are determined.

Following the general steps described above, the heuristic algorithm was coded in MATLAB with the pseudocode of the computational procedure presented below. A flow chart depicts the interactions of the procedure is shown in Figure 3.3.

Step 0. Initial $t = 1, \tau = 1, d = d_1$;

Step 1. Determine the cost of satisfying d only with new components. Set this cost as $cm(t)$.

Determine the cost of satisfying d with both new components and remanufactured components ($y_{t-1} + R$ is remanufactured). Set this cost as $crmm(t)$.

Set $Cmat(t) = min\{cm(t),\ crmm(t)\}$.

Step 2. If $cm(t) > crmm(t)$,

$r_t = y_{t-1} + R,$

$y_t = 0,$

Update z_t, m_{it} using Equation (2).

Else

$r_t = 0,$

$y_t = y_{t-1} + R,$

Update z_t, m_{it} using Equation (2).

End if

Step 3. Set $\tau = \tau + 1,\ d = d + d_\tau$

Determine the average period cost $AVGm$ (from period t to period τ) of satisfying d only with manufacturing in period t.

Determine the average period cost $AVGrmm$ (from period t to period τ) of satisfying d with manufacturing in period t and remanufacturing $y_{\tau-1} + R$ in period τ.

Set $Cmat(\tau) = \min\{AVGm, AVGrmm\}$.

Step 4. If $Cmat(\tau) > Cmat(\tau - 1)$,

Set $t = \tau$ and repeat from Step 1.

Else,

Go to step 5.

End if.

Step 5. If $AVGm > AVGrmm$,

$r_\tau = y_{\tau-1} + R,$

$y_\tau = 0,$

Update $z_\tau, m_{i\tau}$ using Equation (2).

Else,

$r_\tau = 0,$

$y_\tau = y_{\tau-1} + R,$

Update $z_\tau, m_{i\tau}$ using Equation (2).

End if.

Repeat Step 3 until $Cmat(\tau) > Cmat(\tau-1)$ or period τ is the last period.

End.

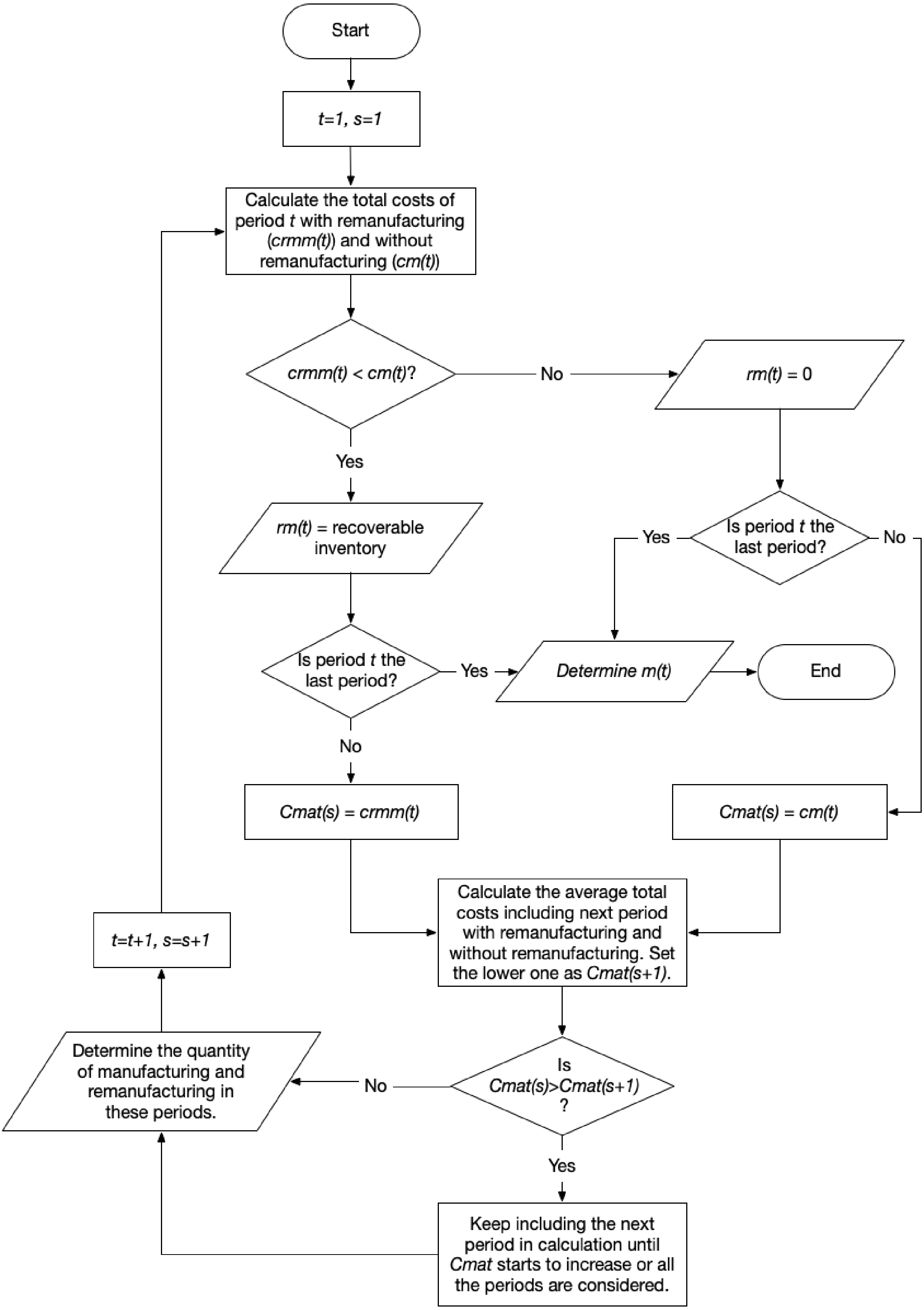

Figure 3.3 Flow Chart of Heuristic Method Solution

3.6 Summary

In this chapter, we discuss the details of the considered production planning problem in a hybrid system of manufacturing and remanufacturing. In this problem, the remanufacturing time of the components to be recovered can vary depending on their conditions. Some of them can be remanufactured within one period and some of them need longer time. A mixed integer programming model is proposed to solve the considered lot-sizing problem to minimize total cost. We also present a heuristic solution method to find close-to-optimal solutions with less computational time. In the next chapter, we will present several numerical example problems to demonstrate the developed model and solution method.

Chapter 4 Numerical Examples and Analysis

In this chapter, we present several numerical examples to illustrate the developed mathematical model and the heuristic solution method discussed in Chapter 3. Computational results are analyzed to demonstrate the effectiveness and efficiency of the developed heuristic solution method. In addition, sensitivity analysis is conducted to investigate the impact of various values of model parameters. CPLEX is used as the software to solve the problem for optimal solution. The proposed heuristic method is coded and solved in MATLAB. CPLEX and MATLAB codes are presented in Appendix A and Appendix B.

4.1 Single-item Problem

4.1.1 Example Problem and Data

An example problem is presented to test the validity and practicability of the proposed model and solution methodology presented in Chapter 3. This hypothetical example problem is based on the example given in Naeem (2013). Certain adjustments and assumption were made in the data to fit in the considered problem in this research.

The original problem is a single item dynamic lot sizing problem with manufacturing and remanufacturing provisions. The demands and returns are considered as both stochastic and deterministic. The goal of the mathematical model is to minimize the total cost, including production cost, holding cost for returns and finished goods, and backlog cost.

The example is to solve a production planning problem with 10 periods. The considered system produces one type of product from new materials or from returned products. The product contains one type of component. According to the historical data, the components disassembled from returned products could be divided into 3 different categories. Each category needs different remanufacturing time. The remanufacturing time, cost and percentage of each category are shown in Table 4.1. Table 4.2 presents the demand of product over 10 periods. In this problem, the holding cost and set-up cost are taken as: $ch_y = 3$, $ch_z = 4$, $Km = 250$, $Kr = 200$. The quantity of returned products in each period is 80. The cost of manufacturing a new component is set as cm=30. The maximum capacities of recoverable and serviceable inventories are infinite.

Table 4.1 Remanufacturing time and percentages of categories

Category (j)	Remanuf. Time	Remanuf. Cost (cr_j)	Percentage (p_j)
1	1 period	10	50%
2	2 periods	11	25%
3	3 periods	12	25%

Table 4.2 Demands of product over 10 periods

Period (t)	1	2	3	4	5	6	7	8	9	10
Demand (d_t)	184	189	169	205	190	197	210	200	195	191

4.1.2 Computational Results and Analysis

The example problem is solved by CPLEX and MATLAB 2017b. CPLEX gives the optimal solutions and the proposed heuristic in Chapter 3 is coded and solved in MATLAB.

The minimum total cost of optimal solution is 48800 and the result of heuristic is 49948. The result of heuristic is very close to optimal solution and the error is only 2.35%. In terms of time, CPLEX takes 0.1689s to get the optimal solution, and MATLAB only takes 0.0224s to complete the heuristic solution calculation, which is 13.26% of the time getting optimal solution. Though the time getting optimal solution is short in the example problem, for more complicated and more realistic problems, it may take a long time to get optimal solution. In that situation, the proposed heuristic would be more efficient with greater computational advantages.

Table 4.3 and Table 4.4 show the detailed production planning of the optimal solution and heuristic solution. They present the quantity of products that should be manufactured and remanufactured in each period. Table 4.3 illustrates that in the production planning of optimal solution, all the returned products in each period need to be remanufactured and the system has to manufacture new products during all the periods. Table 4.4 is the production planning given by heuristic method. In Tables 4.3 and 4.4, m_t and r_t are quantity of manufactured products and quantity of remanufactured products in period t, as defined in Chapter 3. The results show that in certain time periods, demand can be combined and manufactured in one period with remanufacturing the returned products in each period.

Table 4.3 Production planning of optimal solution

Periods (t)	1	2	3	4	5	6	7	8	9	10
m_t	144	129	89	125	110	117	130	120	115	111
r_t	80	80	80	80	80	80	80	80	80	80

Table 4.4 Production planning of heuristic method

Periods (t)	1	2	3	4	5	6	7	8	9	10
m_t	362	0	0	235	0	117	250	0	115	111
r_t	80	80	80	80	80	80	80	80	80	80

Table 4.5 shows the detailed costs associated with the optimal solution and the heuristic solution, including manufacturing cost for new products, remanufacturing cost for returned products, set-up cost for both manufacturing and remanufacturing, holding cost for recoverable inventory and serviceable inventory. From the table we can tell that though the optimal solution and heuristic method provide different production planning, the manufacturing and remanufacturing costs are the same. That means the total quantities of manufacturing or remanufacturing products in the 10 periods of both planning are equal. The gap is mainly caused by the balance of the set-up cost and holding cost. In the production planning of optimal solution, all the serviceable products are used to satisfy the demand of current period, so there is no holding cost. In the production planning of heuristic method, since some demands of several periods are manufactured together in one period, the set-up cost is relatively lower. But on the other hand, the holding cost is incurred because the surplus serviceable products are held to the next periods.

Table 4.5 Detailed costs of the optimal solution and heuristic

Costs	Optimal solution	Heuristic Method
Manufacturing Cost	35700	35700
Remanufacturing Cost	8600	8600
Set-up Cost	4500	3500
Holding Cost	0	2148
Total	48800	49948

4.1.3 Experimental Results

To further investigate the performance of the proposed heuristic method, 9 additional example problems are generated. The developed heuristic is tested on the instances and the results are compared with optimal solution. Same as the previous example, all the problem data are coded and solved by CPLEX to get the optimal solutions and the solution of heuristic method is coded and solved by MATLAB.

The example problems are generated in three dimensions: problem size (quantity of periods), quantity of returned products, quantity of categories. For the problem size, we consider the problems with 5 periods, 10 periods and 20 periods to represent small-size problem, medium-size problem and large-size problem respectively. In the terms of returned products, two scenarios are considered: returned products are much less than the demand and returned products are close to demand. Returned products more than demand is not considered because it is a rare situation in reality. Furthermore, quantity of categories is considered as 3 or 5 in the examples.

Table 4.6, Table 4.7 and Table 4.8 show the results of experiments. The key parameters (return quantity and category) are shown in these tables. The other parameters of each problem and production planning are presented in Appendix C and Appendix D. The results illustrate 0.25% cost gap in small-size problem, 3.94% cost gap in medium-size problem and 3.18% cost gap in large-size problem between the heuristic method and optimal solution. On average, calculation time is saved 82.5% in small-size problem, 89.3% in medium-size problem and

87.87% in large-size problem. In Example 1 and 3, the optimal solution and heuristic method obtain the same cost and production planning. That means in some cases the heuristic method can get the optimal solution. The results show that the proposed heuristic method can reach optimal or near optimal solution for the tested problems within a very short computational time.

Table 4.6 Results of Small-size Problems

Example	Parameters	Total Cost		Time(s)		Cost Gap	Time Saved
		Optimal Solution	Heuristic Method	Optimal Solution	Heuristic Method		
1	Return 100 3 Categories (20, 30, 50)	83830	83830	0.1743	0.0081	0	95.3%
2	Return 200 3 Categories (40, 60, 100)	55470	55890	0.1260	0.0139	0.75%	62.5%
3	Return 100 5 Categories (10, 20, 20, 20, 30)	87300	87300	0.1131	0.0118	0	89.6%
					Average	0.25%	82.5%

Table 4.7 Results of Medium-size Problems

Example	Parameters	Total Cost		Time		Cost Gap	Time Saved
		Optimal Solution	Heuristic Method	Optimal Solution	Heuristic Method		
4	Return 80 3 Categories (40, 20, 20)	48800	49948	0.1689	0.0224	2.35%	86.7%
5	Return 170 3 Categories (70, 50, 50)	33260	34549	0.1568	0.0150	3.88%	90.4%
6	Return 168 5 Category (50, 45, 30, 23, 20)	36885	38952	0.1718	0.0155	5.60%	90.9%
					Average	3.94%	89.3%

Table 4.8 Results of Large-size Problems

Example	Parameters	Total Cost		Time (s)		Cost Gap	Time Saved
		Optimal Solution	Heuristic Method	Optimal Solution	Heuristic Method		
7	Return 170 3 Categories (70 50 50)	189420	199114	0.2615	0.0228	5.12%	91.3%
8	Return 70 3 Categories (40 20 10)	308000	313340	0.1512	0.0236	1.73%	84.4%
9	Return 70 5 Categories (20 15 15 10 10)	312500	320940	0.1533	0.0185	2.70%	87.9%
					Average	3.18%	87.87%

Table 4.9 shows the detailed cost of the examples. It clearly shows that manufacturing and remanufacturing costs are the same for both optimal solution and heuristic method. The difference of total cost is caused by set-up cost and holding cost. More specifically, the set-up cost of heuristic, on average, is 22.75% lower than that of optimal solution. Holding cost of heuristic is much higher than that of optimal solution. As a result, the proposed heuristic method schedules the production differently from the optimal solution with the same manufacturing-remanufacturing quantity and it tends to reduce set-up cost but increase holding cost.

Table 4.9 Detailed Cost for Each Example

	Manuf. Cost		Remanf. Cost		Set-up Cost		Holding Cost		Total Cost	
Exam.	O	H	O	H	O	H	O	H	O	H
1	72000	72000	8250	8250	3500	3500	80	80	83830	83830
2	33310	33310	18400	18400	3000	1000	760	3180	55470	55890
3	72000	72000	10500	10500	4500	4500	300	300	87300	87300
4	35700	35700	8600	8600	4500	3500	0	2148	48800	49948
5	11400	11400	18500	18500	3000	2000	360	2649	33260	34549
6	15120	15120	18160	18160	3000	2000	605	3645	36885	38952
7	61000	61000	117000	117000	11100	7800	320	13314	189420	199114
8	250000	250000	46000	46000	12000	11100	0	6240	308000	313340
9	257500	257500	50000	50000	5000	4400	0	9040	312500	320940

4.2 Multiple-item Problem

4.2.1 Example 1 – Problem and Data

The difference between multiple-item problem and single-item problem (discussed in Section 4.1) is that the product considered in multiple-item problem contains more than one types of component and each type has its own categories and cost. The example of multiple-item problem is similar with single-item example, which is explained in section 4.1.1. The extra considered parameters of this problem are related to components. In this problem, the product contains 3 types of components and each type has different category quantities and remanufacturing cost. The related parameters are shown in Table 4.10.

Table 4.10 Related Parameters of Components

Component (i)	Quantity in product (l_i)	Manuf. Cost	Category (I_j)	Remanuf. Time of Each Category	Remanuf. Cost of Each Category (cr_{ij})	Percentage of Each Category (p_{ij})
1	5	20	1	1 period	5	70%
			2	2 periods	10	30%
2	8	30	1	1 period	10	50%
			2	2 periods	11	30%
			3	3 periods	12	20%
3	10	30	1	1 period	10	30%
			2	2 periods	11	30%
			3	3 periods	12	30%
			4	4 periods	13	10%

The planning horizon of this problem is 10 periods. The demands of each period are shown in Table 4.11. The holding cost and set-up cost are taken as: $ch_y = 3,\ ch_z = 5,\ Km = 250$, $Kr = 150$. The quantity of returned products in each period is 170. The maximum capacities of recoverable and serviceable inventories are infinite.

Table 4.11 The demands of product over 10 periods

Period (t)	1	2	3	4	5	6	7	8	9	10
Demand (d_t)	184	189	169	205	190	197	210	200	195	191

4.2.2 Example 1 – Computational Results and Analysis

Same as single-item problem, the example is solved by CPLEX and MATLAB 2017b. CPLEX gives the optimal solution and the proposed heuristic in Chapter 3 is coded and solved in MATLAB.

The minimum total cost of optimal solution is 637295 and the result of heuristic is 696890. The results show 9.3% gap between the developed heuristic and optimal solution. It takes 0.2075s to solve the problem with CPLEX to optimality and 0.0349s is required to solve the problem with heuristic in MATLAB, which is 83.18% shorter.

Table 4.12 and Table 4.13 show the detailed production planning of the optimal solution and heuristic solution. They present the quantity of each type of component (m_{1t}, m_{2t}, and m_{3t}) that should be manufactured and the quantity of product should be remanufactured in each period. In both of the production planning, all the returned products in each period are remanufactured. Table 4.12 shows the system should manufacture all the three component types in every period except period 3($m_{2t} = 0$, $m_{3t} = 0$). Table 4.13 illustrates the system only needs to manufacture the three component types in period 1 and 7.

Table 4.12 Production planning of optimal solution

Periods (t)	1	2	3	4	5	6	7	8	9	10
m_{1t}	325	95	0	170	100	135	200	150	125	105
m_{2t}	792	424	0	272	160	216	320	240	200	168
m_{3t}	1330	870	160	350	200	270	400	300	250	210
r_t	170	170	170	170	170	170	170	170	170	170

Table 4.13 Production planning of heuristic method

Periods (t)	1	2	3	4	5	6	7	8	9	10
m_{1t}	825	0	0	0	0	0	580	0	0	0
m_{2t}	1864	0	0	0	0	0	928	0	0	0
m_{3t}	3180	0	0	0	0	0	1160	0	0	0
r_t	170	170	170	170	170	170	170	170	170	170

Table 4.14 shows the detailed costs of optimal solution and heuristic method. Same as single-item problem, the gap between results is caused by set-up cost and holding cost. The heuristic method decreases set-up cost but increases holding cost leading to the total cost is higher than optimal solution.

Table 4.14 Detailed costs of the optimal solution and heuristic

Costs	Optimal solution	Heuristic Method
Manufacturing Cost	242060	242060
Remanufacturing Cost	391170	391170
Set-up Cost	4000	2000
Holding Cost	65	61660
Total	637295	696890

4.2.3 Example 2 – Problem and Data

The background of Example 2 is the same as Example 1. The situation is different that the set-up cost of remanufacturing is much higher than that of manufacturing and the difference between holding cost of recoverable inventory and serviceable inventory is larger. The parameters of components in Example 2 are shown in Table 4.15.

Table 4.15 Related Parameters of Components

Component (i)	Quantity in product (l_i)	Manuf. Cost	Category (l_j)	Remanuf. Time of Each Category	Remanuf. Cost of Each Category (cr_{ij})	Percentage of Each Category (p_{ij})
1	1	10	1	1 period	2	90%
			2	2 periods	3	10%
2	2	10	1	1 period	2	90%
			2	2 periods	4	10%
3	3	10	1	1 period	2	80%
			2	2 periods	3	10%
			3	3 periods	4	10%

The planning horizon of this problem is 5 periods. The demands of each period are shown in Table 4.16. The holding cost and set-up cost are taken as: $ch_y = 0.1$, $ch_z = 5$, $Km = 3000$, $Kr = 11000$. The quantity of returned products in each period is 220. The maximum capacities of recoverable and serviceable inventories are infinite.

Table 4.16 The demands of product over 5 periods

Period (t)	1	2	3	4	5
Demand (d_t)	220	280	360	140	270

4.2.4 Example 2 – Computational Results and Analysis

Example 2 is solved by CPLEX and MATLAB 2017b as well. CPLEX gives the optimal solution and the proposed heuristic in Chapter 3 is coded and solved in MATLAB.

The minimum total cost of optimal solution is 88093 and the result of heuristic is 89302. The results show 1.4% gap between the developed heuristic and optimal solution. It takes 0.2030s to solve the problem with CPLEX to optimality and 0.0028s is required to solve the problem with heuristic in MATLAB, which is 98.62% shorter.

Table 4.17 and Table 4.18 show the detailed production planning of the optimal solution and heuristic solution. Unlike Example 1, in Example 2, the optimal solution and heuristic method obtain different remanufacturing production planning over the horizon. Table 4.17 shows the system should remanufacture 310 returned products in period 2 and 130 returned products in period 4. Table 4.18 illustrates the system only needs to remanufacture 440 returned products in period 2.

Table 4.17 Production planning of optimal solution

Periods (t)	1	2	3	4	5
m_{1t}	221	0	352	0	257
m_{2t}	442	0	704	0	514
m_{3t}	756	0	1002	0	771
r_t	0	310	0	130	0

Table 4.18 Production planning of heuristic method

Periods (t)	1	2	3	4	5
m_{1t}	220	0	340	0	270
m_{2t}	440	0	680	0	540
m_{3t}	660	0	1020	0	810
r_t	0	440	0	0	0

Table 4.19 shows the detailed costs of optimal solution and heuristic method in Example 2. The manufacturing cost in optimal solution is slightly higher than that of heuristic method. That is because in optimal solution, among the 130 remanufactured in period 4, the third category of component 3 are not available in period 5 so the inadequate components need to be manufactured. The general trend is same with the previous examples that the heuristic method decreases set-up cost but increases holding cost leading to the total cost is higher than optimal solution.

Table 4.19 Detailed costs of the optimal solution and heuristic

Costs	Optimal solution	Heuristic Method
Manufacturing Cost	50190	49800
Remanufacturing Cost	5808	5808
Set-up Cost	31000	20000
Holding Cost	1095	13694
Total	88093	89302

4.2.5 Experimental Results

To validate the proposed heuristic method for multiple-item problem, 9 example problems are generated. All the problems are coded and solved by CPLEX to get the optimal solutions and the solution of heuristic method is coded and solved by MATLAB.

Similar with single-item problem, the multiple-item example problems are generated in three dimensions as well: problem size (quantity of periods), quantity of returned products and quantity of component types. Small-size problem, medium-size problem and large-size problem are still considered with 5 periods, 10 periods and 20 periods, respectively. In the terms of returned products, two scenarios are considered: returned products are much less than the demand and returned products are close to demand. For the component types, 3 types and 5 types are set as two different situations in the examples. The category quantity of each component type is assumed randomly.

Table 4.20, Table 4.21 and Table 4.22 show the experiment results of multi-item problems. The key parameters (return quantity and category) are shown in these tables. The other parameters of each problem and production planning are presented in Appendix C and Appendix D. The results illustrate 1.57% cost gap in small-size problem, 3.80% cost gap in medium-size problem and 1.10% cost gap in large-size problem between the heuristic method and optimal solution. On average, calculation time is saved 96.23% in small-size problem, 84.73% in medium-size problem and 87.90% in large-size problem. The heuristic method gets the optimal solution and minimize cost in Example 14.

Table 4.20 Results of Small-size Problems

Example	Parameters	Total Cost		Time(s)		Cost Gap	Time Saved
		Optimal Solution	Heuristic Method	Optimal Solution	Heuristic Method		
10	Return 100 3 Component Types	76800	77220	0.1356	0.0073	0.5%	94.6%
11	Return 200 3 Component Types	71100	73180	0.1247	0.0017	2.9%	98.6%
12	Return 100 5 Component Types	333675	338115	0.1159	0.0052	1.3%	95.5%
					Average	1.57%	96.23%

Table 4.21 Results of Medium-size Problems

Example	Parameters	Total Cost		Time (s)		Cost Gap	Time Saved
		Optimal Solution	Heuristic Method	Optimal Solution	Heuristic Method		
13	Return 170 3 Component Types	637295	696890	0.2075	0.0349	9.3%	83.2%
14	Return 80 3 Component Types	538800	538800	0.1866	0.0499	0	73.3%
15	Return 200 5 Component Types	799035	815910	0.1536	0.0036	2.1%	97.7%
					Average	3.80%	84.73%

Table 4.22 Results of Large-size Problems

Example	Parameters	Total Cost		Time (s)		Cost Gap	Time Saved
		Optimal Solution	Heuristic Method	Optimal Solution	Heuristic Method		
16	Return 170 3 Component Types	626856	632592	0.1972	0.0278	0.9%	85.9%
17	Return 70 3 Component Types	1111770	1126514	0.1555	0.0050	1.3%	96.8%
18	Return 70 5 Component Types	4009309	4054858	0.1111	0.0211	1.1%	81.0%
					Average	1.10%	87.90%

Table 4.23 shows the detailed cost of the multiple-item examples. Same as single-item problems, it indicates that manufacturing and remanufacturing costs are the same for both optimal solution and heuristic method. The proposed heuristic method tends to reduce set-up cost but increase holding cost. On average, the set-up cost of heuristic is 17.1% lower than optimal solution. Whereas holding cost of heuristic is much higher than optimal solution.

Table 4.23 Detailed Cost for Each Example

	Manuf. Cost		Remanf. Cost		Set-up Cost		Holding Cost		Total Cost	
Exam.	O	H	O	H	O	H	O	H	O	H
10	49600	49600	14700	14700	12500	11000	0	1920	76800	77220
11	19480	19480	32340	32340	17000	14000	2280	7360	71100	73180
12	242775	242775	83400	83400	7500	6500	0	5440	333675	338115
13	242060	242060	391170	391170	4000	2000	65	61660	637295	696890
14	371840	371840	152960	152960	14000	14000	0	0	538800	538800
15	89655	89655	594400	594400	12300	7600	102680	124255	799035	815910
16	210280	210280	293760	293760	80000	75000	42816	53552	626856	632592
17	770370	770370	281400	281400	60000	56000	0	18744	1111770	1126514
18	2865470	2865470	1074039	1074039	60000	54000	9800	61349	4009309	4054858

4.3 Sensitivity Analysis

We take the single-item problem example described in section 4.1.1 to perform a sensitivity analysis to investigate the impact of various value of model parameters to the total cost. Figure

4.1 to 4.5 show the relationship between total cost and each parameter with the other parameters being fixed.

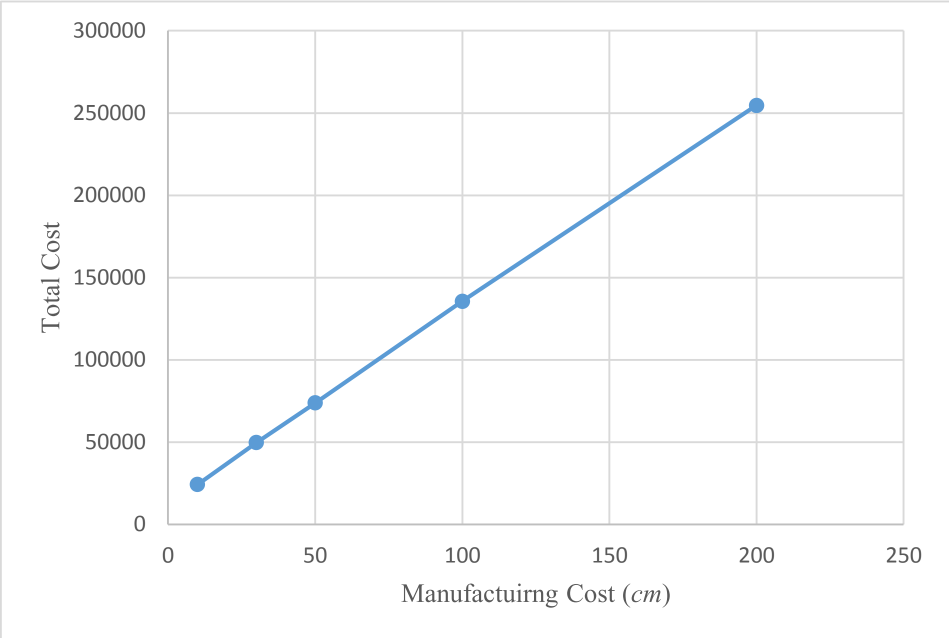

Figure 4.1 Total Cost versus Manufacturing Cost

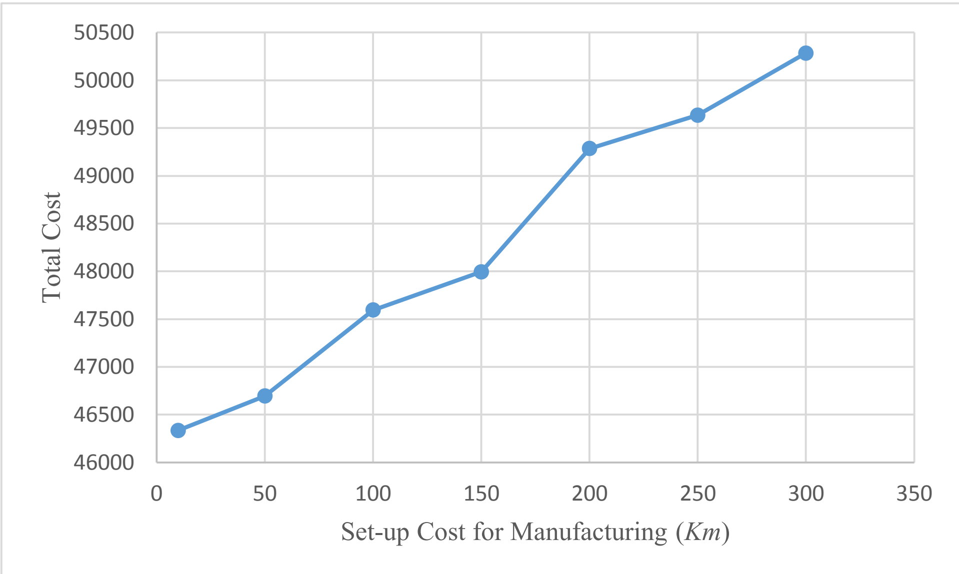

Figure 4.2 Total Cost versus Set-up Cost for Manufacturing

Figure 4.1 and Figure 4.2 show the total cost is positively correlated with manufacturing cost and manufacturing set-up cost. Because the returned product quantity (80) is less than the demand (average 193), manufacturing has to occur in every period. These two parameters do not influence the production planning, so when they increase, the total cost increase accordingly.

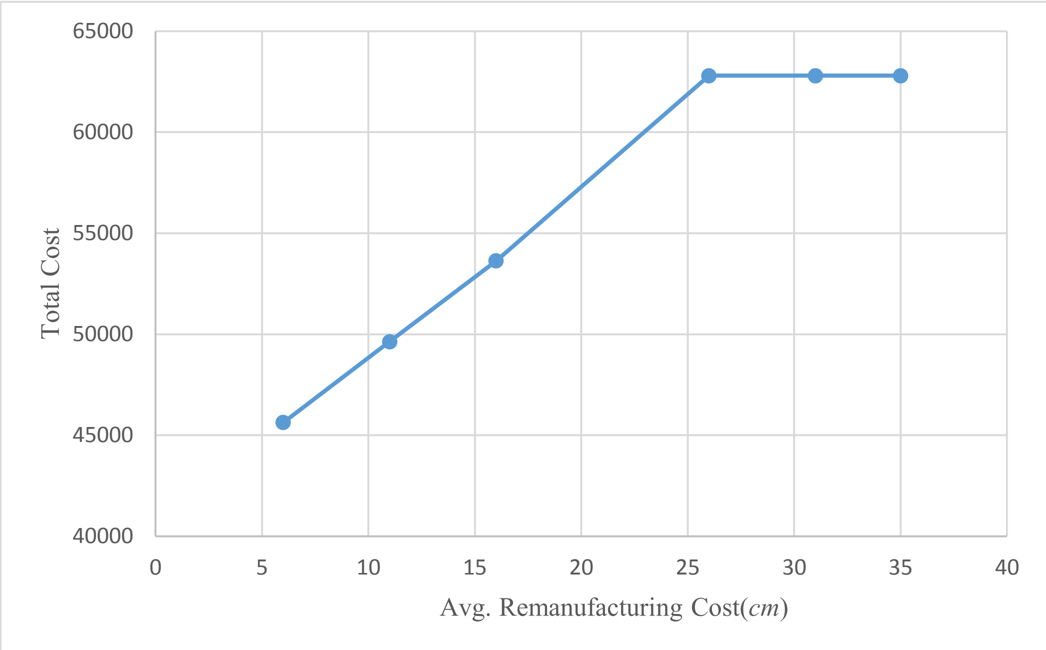

Figure 4.3 Total Cost versus Avg. Remanufacturing Cost

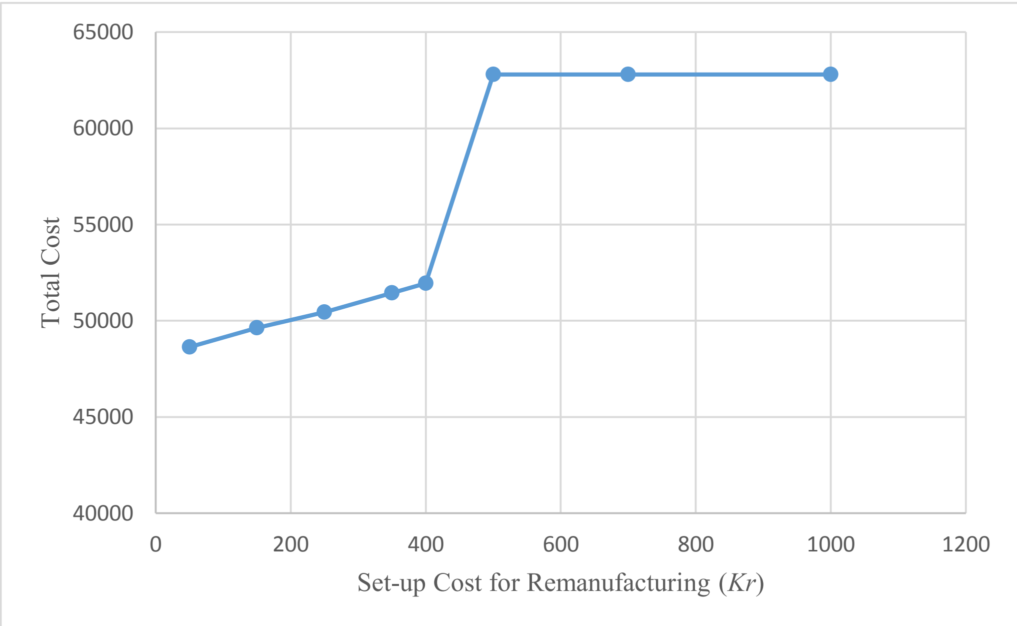

Figure 4.4 Total Cost versus Set-up cost for Remanufacturing

Figure 4.3 and Figure 4.4 present the change of total cost when the average remanufacturing cost of three categories or the remanufacturing set-up cost increasing. They both show that within certain ranges, total cost increases when average remanufacturing cost or set-up cost grows. Out of the ranges, total cost keeps stable regardless of remanufacturing cost and set-up cost. The reason is that when remanufacturing cost or set-up cost is large enough, the economic advantage of remanufacturing will disappear, the system will satisfy demands only with manufacturing. In that case, the remanufacturing quantity will be zero and its cost and set-up cost cannot affect the total cost.

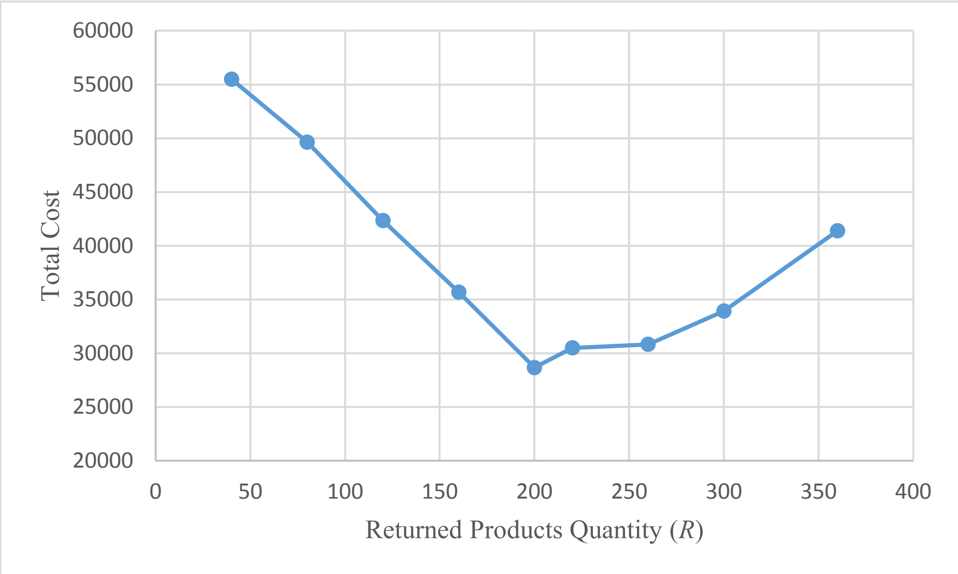

Figure 4.5 Total Cost versus Returned Products Quantity

Figure 4.5 shows that with returned products quantity raising, the total cost decreases first and then increases. The remanufacturing cost is lower than manufacturing cost, so with more returned products, the system will remanufacture more parts and the total cost will be lower. But when there are too much returned products (above 200 in this example), a part of them can be remanufactured to satisfy all the demands. The leftover has to be held in the inventory, which increases the holding cost and leads the total cost to increase as a result.

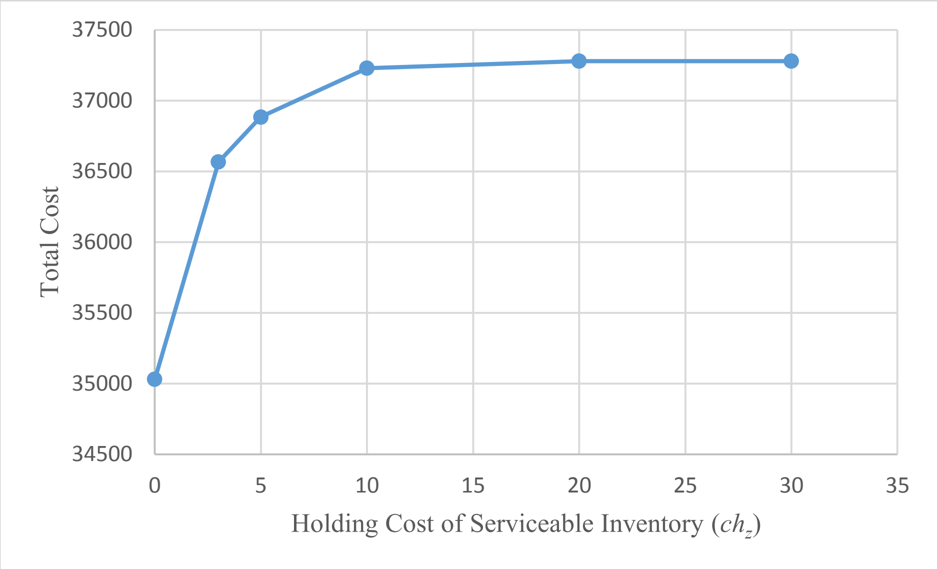

Figure 4.6 Total Cost versus Holding Cost of Serviceable Inventory

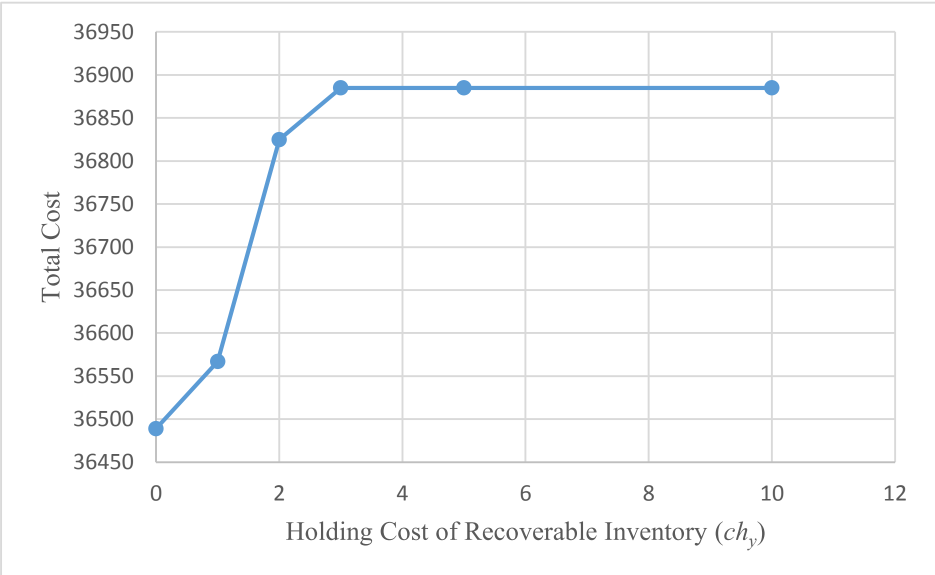

Figure 4.7 Total Cost versus Holding Cost of Recoverable Inventory

Figure 4.6 and Figure 4.7 show that with the holding cost of serviceable inventory or recoverable inventory increasing, total cost increases at first and then remains stable. When the holding cost is relatively low, the system tends to produce products of several periods to avoid occurring more set-up cost. When the holding cost is high, the system will product products in each period to avoid holding cost so the total cost stays stable.

4.4 Summary

In this chapter, for testing the developed model and the proposed heuristic method, we generated 9 examples for single-item problem and 9 examples for multi-item problem. These examples are generated in three dimensions: problem size, quantity of returned products and quantity of categories. On average, the results from the heuristic show that there is 2.46% gap between heuristic and optimal solution in single-item problem and 2.16% gap in multi-item problem. The results of both single-item problems and multi-item problems show that the heuristic method tends to obtain a production planning with lower set-up cost and higher holding cost compared with the optimal solutions. At last, sensitivity analysis is performed to investigate the impact of various value of model parameters to the total cost.

Chapter 5 Conclusion and Future Research

In this chapter we present a summary of the research carried out in this thesis. It also includes several concluding remarks based on the problem modeling. Future research directions are discussed as well.

5.1 Conclusion

Production planning problem in hybrid manufacturing remanufacturing system with uncertain remanufacturing time is studied in this thesis. The objective of solving the mathematical model is to minimize the total cost based on the optimal quantity of new items to manufacture and the optimal quantity of returned products to remanufacture in each period of the planning horizon. The considered problem has a distinctive feature that considers the uncertainty of remanufacturing time for the same type of returned products. The definition of category is introduced to differentiate components with different remanufacturing time. The percentages and remanufacturing costs of categories are different as well. Both single-item and multiple-item problems are studied in the thesis.

A mixed integer programming is developed to obtain optimal solution of the considered problem. The objective of the model is to minimize the total cost of production, including manufacturing cost, remanufacturing cost, set-up cost and holding cost. To solve the MIP model efficiently, a heuristic solution procedure based on Silver-Meal heuristic is developed.

To validate the proposed heuristic method, 18 example instances of single-item problem and multiple-item problem are tested and the results are compared with optimal solution obtained using ILOG CPLEX. It shows that the optimal solution or close-to-optimal solution can be obtained with the proposed heuristic in a relatively short time. On average, the results from the heuristic show that there is 2.46% gap between heuristic and optimal solution in single-item problem and 2.16% gap in multi-item problem. The results of both single-item problems and multi-item problems show that the heuristic method tends to obtain a production planning with lower set-up cost and higher holding cost compared with the optimal solutions. Following the numerical experiments, a sensitivity analysis is performed to investigate the impact of various value of model parameters to the total cost.

The main contributions of this research are the consideration of uncertain remanufacturing time in the model and the Silver-Meal based heuristic method. Several numerical example problems and different instances are used to test the developed model and heuristic method extensively with results showing the advantages of the development made in this thesis. The proposed heuristic method provides a more efficient way to solve the model. The model may be used as a framework in development more formal systems for production planning, inventory control and end-of-life product recovery. The model is made for general hybrid system of manufacturing and remanufacturing; therefore, its use is not limited to specific area in the remanufacturing industry.

5.2 Future Work

There are several options to extend the research presented in this thesis. Our suggestions for future research in this area are:

- Considering dynamic demands for new products and dynamic returned product quantity.
- Considering more detailed inventory control strategies with limited capacity and back orders.
- The uncertain parameters can be modeled with possibility distributions.
- Considering the combined assembly of new products from new and remanufactured components at the same time.

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Appendix A

Codes of single-item example problem in CPLEX:

```
range T=1..10;
range H=0..10;
int M=10000;
int d[1..10]=[184, 189, 169, 205, 190, 197,210, 200, 195, 191];
int N=80;
range J=1..3;
int rm[1..3]=[40, 20, 20];
int cm=30;
int cr[1..3]=[10, 11, 12];
int chy=3;
int chz=4;
int km=250;
int kr=200;

dvar int+ m[T];
dvar int+ r[T];
dvar int+ y[H];
dvar int+ z[H];
dvar boolean bm[T];
dvar boolean br[T];

dexpr float C=sum(t in T) (m[t]*cm+br[t]*sum(j in J)(rm[j]*cr[j])+ km*bm[t]+kr*br[t]+chy*y[t]+
chz*z[t]);

minimize C;

subject to{
        z[0]+m[1]+rm[1]*br[1]-d[1]==z[1];
        y[1]==y[0]+N-r[1];
        m[1]<=M*bm[1];
        r[1]<=M*br[1];
        r[1]<=y[0]+N;
        y[0]==0;
        z[0]==0;

        sum (t in T) br[t]==sum(t in T)(r[t]/N);//in case of r[t]=0, br[t]=1 which satisfy
r[t]<=M*br[t] but C will be incorrect

        forall(t in 2..10){
            y[t]==y[t-1]+N-r[t];
```

```
        r[t]<=M*br[t];
        r[t]<=y[t-1]+N;
        m[t]<=M*bm[t];
    }

    br[2]*rm[1]+br[1]*rm[2]+z[1]+m[2]-z[2]==d[2];
    br[1]*rm[3]+br[2]*rm[2]+br[3]*rm[1]+z[2]+m[3]-z[3]==d[3];

    forall(t in 4..10){
        br[t-2]*rm[3]+br[t-1]*rm[2]+br[t]*rm[1]+z[t-1]+m[t]-z[t]==d[t];
    }

}
```

Codes of multi-item example 1 problem in CPLEX:

```
range T=1..10;
range H=0..10;
int M=10000;
int d[1..10]=[184, 189, 169, 205, 190, 197,210, 200, 195, 191];
int N=170;
range J=1..4;//the most categories
range P=1..3;//number of part types in the product
int npp[1..3]=[5,8,10];//number of each type of part in the product
int rm[1..3][1..4]=[
[595,255,0,0],
[680,408,272,0],
[510,510,510,170]];
int cm[1..3]=[20,30,30];
int cr[1..3][1..4]=[
[5,10,0,0],
[10,11,12,0],
[10,11,12,13]];
int chy=3;
int chz=5;
int km=250;
int kr=150;

dvar int+ m[P][T];
dvar int+ r[P][T];
dvar int+ y[P][H];
dvar int+ z[P][H];
dvar boolean bm[T];
```

```
dvar boolean br[T];

dexpr float C=sum(p in P)sum(t in T) (m[p][t]*cm[p]+br[t]*sum(j in
J)(rm[p][j]*cr[p][j])+chy*y[p][t]+chz*z[p][t])+sum(t in T)(km*bm[t]+kr*br[t]);

minimize C;

subject to{
	forall(p in P){
		z[p][0]+m[p][1]+rm[p][1]*br[1]-d[1]*npp[p]==z[p][1];
		y[p][1]==y[p][0]+N*npp[p]-r[p][1];
		m[p][1]<=M*bm[1];
		r[p][1]<=M*br[1];
		r[p][1]<=y[p][0]+N*npp[p];
		y[p][0]==0;
		z[p][0]==0;
	}
	sum (t in T) br[t]==sum(t in T)(r[1][t]/(N*npp[1]));//in case of r[t]=0, br[t]=1 which satisfy
r[t]<=M*br[t] but C will be incorrect
	forall(t in 2..10, p in P){
		y[p][t]==y[p][t-1]+N*npp[p]-r[p][t];
		r[p][t]<=M*br[t];
		r[p][t]<=y[p][t-1]+N*npp[p];
		m[p][t]<=M*bm[t];
	}

	//for p=1,the first part with 2 categories
		br[2]*rm[1][1]+br[1]*rm[1][2]+z[1][1]+m[1][2]-z[1][2]==d[2]*npp[1];

		forall(t in 3..10){
			br[t-1]*rm[1][2]+br[t]*rm[1][1]+z[1][t-1]+m[1][t]-z[1][t]==d[t]*npp[1];
		}

	//for p=2, the second part with 3 categories
		br[2]*rm[2][1]+br[1]*rm[2][2]+z[2][1]+m[2][2]-z[2][2]==d[2]*npp[2];
		br[1]*rm[2][3]+br[2]*rm[2][2]+br[3]*rm[2][1]+z[2][2]+m[2][3]-z[2][3]==d[3]*npp[2];

		forall(t in 4..10){
			br[t-2]*rm[2][3]+br[t-1]*rm[2][2]+br[t]*rm[2][1]+z[2][t-1]+m[2][t]-
z[2][t]==d[t]*npp[2];
		}

	//for p=3, the third part with 4 categories
		br[2]*rm[3][1]+br[1]*rm[3][2]+z[3][1]+m[3][2]-z[3][2]==d[2]*npp[3];
```

```
    br[1]*rm[3][3]+br[2]*rm[3][2]+br[3]*rm[3][1]+z[3][2]+m[3][3]-z[3][3]==d[3]*npp[3];
    br[1]*rm[3][4]+br[2]*rm[3][3]+br[3]*rm[3][2]+br[4]*rm[3][1]+z[3][3]+m[3][4]-
z[3][4]==d[4]*npp[3];

    forall(t in 5..10){
        br[t-3]*rm[3][4]+br[t-2]*rm[3][3]+br[t-1]*rm[3][2]+br[t]*rm[3][1]+z[3][t-1]+m[3][t]-
z[3][t]==d[t]*npp[3];
    }

}
```

Appendix B

Codes of heuristic method of single-item example problem in MATLAB 2017b:

```
function [z, x, Cmat] = silver_meal(d, K, c, h)
tic
d = [184 189 169 205 190 197 210 200 195 191];     %demand
k = 250;                                                   %manufacturing setup cost
rk = 200;                                                  %remanu setup cost
h = 4;                                                     %holding cost of part per period
rh = 3;                                            %holding cost of returned product    per period
rm = [40 20 20];                                   %remanu quantity of 3 catagories in each period
rn = 80;                                                   %returned product in each period
rc = [10 11 12];                                   %remanu cost for 3 catagories
c = 30;                                            %manu cost
J = length(rm);                                    % number of catergories of remanu
p= [0.5 0.25 0.25];                            %percentages of categories

%-----------------------

n                  = length(d);
Cmat               = zeros(1, n);
crmm               = zeros(1, n);
cm = zeros(1, n);
x                  = zeros(1, n);
xr                 = zeros(1, n);
flg                = zeros(1, n);
flgrm       = zeros(1,n);
drm = zeros(1,n);
r                  = 1;
s                  = 1;
t                  = 0;
A = 0;                                             % for calculating the demand when produce the
demand for several periods together
crema = 0;                                         %sum of the remanu cost
irema = 0;                                         % inventory of remanufactured products
irecy = 0;                                         % inventory of returned products

% ----------------------------start iteration-------------------------

% r is for exploring the production period. s and t are loop variables.

while s < (n + 1)
       if (s-r) == 0                                       %if =0, this is a production period
```

```
        flg(r) = 1;                                     %indicate this is a manu period
        crmm(s) = k+(d(s)-rm(1)-irema)*c+rk;          %cost of setup+manu
        for j=1:J
            crmm(s)=crmm(s)+rc(j)*(rn+irecy)*p(j); %plus cost of remanu
        end
        cm(s) = k+(d(s)-irema)*c + rh* rn;              %cost of manu, no remanu

        if crmm(s) < cm(s)
            Cmat(s) = crmm(s);                          %for the comparision with next period
            flgrm(r) = 1;                               %indicate this is a remanu period
            x(r)=d(s)-rm(1)-irema;
            A=A+1;
            irema=0;
                if A<J
                    for a=1:A
                        irema = irema+rm(a+1)*flgrm(A+1-a);
                    end
                else
                    for a = 1:(J-1)
                        irema=irema+rm(a+1)*flgrm(A+1-a);
                    end
          end
          xr(r)=rn+irecy;          %update the remanu quantity
          irecy = 0;
        else
            Cmat(s) = cm(s);
            x(r)=d(s)-irema;
            irema = 0;
            irecy = irecy+rn;                           %updte inventory of recycled products
            flgrm(r) = 0;                               %indicate this is not a remanu period
        end

        s=s+1;
%=====================finish updating remanu inventory========================
    else
        drm(s) = d(s)-irema;
        for j=1:J
            crema=crema+rc(j)*(rn+irecy)*p(j);                 % cost of remanu
        end
       crmm(s) = (Cmat(s-1)*(s - t - 1) + (drm(s)-rm(1))*c+rk+crema+(drm(s)-rm(1)) * h * (s - t -
1))/(s-t);
        cm(s) = (Cmat(s-1)*(s - t - 1) + drm(s)*c+ drm(s)*h * (s - t - 1)+(irecy+rn)*rh)/(s-t);
        if crmm(s)<cm(s)
            Cmat(s) = crmm(s);
```

```
        crema = 0;
else
        Cmat(s) = cm(s);
        crema = 0;
end
if Cmat(s)>Cmat(s-1)
        r = s;
        t= s-1;
        xr(r)=rn+irecy;
else
        if crmm(s)<cm(s)
                flgrm(s)=1;                     %indicate this is a remanu period
                xr(s)=rn+irecy;
                x(r)=x(r)+d(s)-rm(1)-irema;
                A = A+1;
                irema=0;
                if A<J
                        for a=1:A
                                irema = irema+rm(a+1)*flgrm(A+1-a);
                        end
                else
                        for a = 1:(J-1)
                                irema=irema+rm(a+1)*flgrm(A+1-a);
                        end
                end
                 irecy = 0;
        else
                flgrm(s)=0;                     %indicate this is not a remanu period
                xr(s)=0;
                x(r)=x(r)+d(s)-irema;
                A = A+1;
                irema=0;
                if A<J
                        for a=1:A
                                irema = irema+rm(a+1)*flgrm(A+1-a);
                        end
                else
                        for a = 1:(J-1)
                                irema=irema+rm(a+1)*flgrm(A+1-a);
                        end
                end

                irecy = irecy+rn;               %updte inventory of recycled products
        end
```

```
            flg(r) = flg(r)+1;
            s = s+1;
        end
    end
end
T = flg(flg ~= 0);                      % period numbers for each produciton
end_T= cumsum(flg(flg ~= 0));           % the period before each production
avgC = Cmat(end_T);                     % average cost
z = dot(T, avgC);                       % total cost
toc
```

Codes of heuristic method of multi-item example 1 problem in MATLAB 2017b:

```
function [z, x, Cmat] = silver_meal(d, K, c, h)
d = [184 189 169 205 190 197 210 200 195 191];   %demand
k = 250;                                          %manufacturing setup cost
rk = 150;                                         %remanu setup cost
h = 5;                                           %holding cost of part per period
rh = 3;                                          %holding cost of returned product   per period
rn = 170;                                         %returned product in each period
npp =[5 8 10];                                   %number of each type of part in the product
np = length(npp);                                %number of part types in the product
rnp = zeros(1,np);                               %returned parts quantity
rc = [5 10 0 0; 10 11 12 0;10 11 12 13 ];         %remanu cost of all the parts catagories
c = [20 30 30];                                   %manu cost
p= [0.7 0.3 0 0;0.5 0.3 0.2 0; 0.3 0.3 0.3 0.1];  %category percentages of all the parts
J = 4;                                           % the column quantity from p.
%-----------------------
n = length(d);
dp= zeros(np,n);                                 %demand of parts
Cmat = zeros(1, n);
crmm = zeros(1, n);
cm = zeros(1, n);
x = zeros(np, n);
xr = zeros(np, n);
flg = zeros(1, n);
flgrm = zeros(1,n);
drm = zeros(np,n);
r = 1;
s = 1;
t = 0;
A = 0;          % for calculating the demand when produce the demand for several periods together
crema = zeros(np,n);                           %sum of the remanu cost
irema = zeros(1,np);                           % inventory of remanufactured parts
irecy = zeros(1,np);                           % inventory of returned products
% ----------------------------start iteration--------------------------
tic
% r is for exploring the production period. s and t are loop variables.
for i=1:np
      dp(i,:)=d*npp(i);                          %calculate the demands of all the parts
end
for i=1:np
      rnp(1,i)=rn*npp(i);                        %calculate the returned quantity of all the parts
end
while s < (n + 1)
```

```
if (s-r) == 0                                   %if =0, this is a production period
    flg(r) = 1;                                 %indicate this is a manu period
    crmm(s) = k+rk;                             %setup cost of manu and remanu
    for i = 1:np
        crmm(s)=crmm(s)+(dp(i,s)-p(i,1)*rnp(1,i)-irema(1,i))*c(i);          %plus manu cost
    end
    for i =1:np                                 %loop of parts
        for j=1:J                               %loop of categories
            crmm(s)=crmm(s)+rc(i,j)*(rnp(1,i)+irecy(1,i))*p(i,j);       %plus remanu cost
        end
    end                                         %(remanu+manu) cost
    cm(s) = k;                                  %setup cost
    for i=1:np
        cm(s)=cm(s)+(dp(i,s)-irema(1,i))*c(i);          %plus manu cost
    end
    irecyrn=0;
    for i = 1:np
        irecyrn=irecyrn+irecy(1,i)+rnp(1,i);            %calculate the recovery inventory
    end
    cm(s)=cm(s)+rh*irecyrn;
    if crmm(s) < cm(s)
        Cmat(s) = crmm(s);                              %for the comparision with next period
        flgrm(r) = 1;                                   %%indicate this is a remanu period
        for i=1:np
            x(i,r) = dp(i,s)-p(i,1)*rnp(1,i)-irema(1,i);        %manu quantity of all the parts
        end
        A=A+1;
        for i =1:np
            irema(1,i)=0;
            if A<J
                for a=1:A
                    irema(1,i) = irema(1,i)+p(i,a+1)*rnp(1,i)*flgrm(A+1-a);
                end
            else
                for a = 1:(J-1)
                    irema(1,i) = irema(1,i)+p(i,a+1)*rnp(1,i)*flgrm(A+1-a);
                end
            end
        end
        for i =1:np
            xr(i,r) = rnp(1,i)+irecy(1,i);              %update the remanu quantity
            irecy(1,i) = 0;
        end
    else
```

```
            Cmat(s) = cm(s);
             for i=1:np
                x(i,r) = dp(i,s)-irema(1,i);          %manu quantity of all the parts
                irema(1,i) = 0;
                irecy(1,i) = irecy(1,i)+rnp(1,i);       %updte inventory of returned products
            end
            flgrm(r) = 0;                              %indicate this is not a remanu period
        end
        s=s+1;
%=====================finish updating remanu inventory=========================
    else
        for i=1:np
            drm(i,s) = dp(i,s)-irema(1,i);
        end
        for i=1:np
            for j=1:J
                crema(i,s)=crema(i,s)+rc(i,j)*(rnp(1,i)+irecy(1,i))*p(i,j);          % remanu cost
            end
                                                       %remanu cost of all the parts
        end
       cdrm = 0;
       ccrema=0;
       cdrmrm=0;
       cdrmcm=0;
       drmrm=0;
       irecyrn=0;
       for i = 1:np
           cdrm = cdrm+(drm(i,s)-p(i,1)*rnp(1,i))*c(1,i);
           ccrema=ccrema+crema(i,s);
           cdrmrm=cdrmrm+drm(i,s)-p(i,1)*rnp(1,i);
           cdrmcm=cdrmcm+drm(i,s)*c(1,i);
           drmrm=drmrm+drm(i,s);
           irecyrn=irecyrn+irecy(1,i)+rnp(1,i);
       end
       crmm(s) = (Cmat(s-1)*(s - t - 1) + cdrm+rk+ccrema+cdrmrm * h * (s - t - 1))/(s-t);
        cm(s) = (Cmat(s-1)*(s - t - 1) + cdrmcm+ drmrm*h * (s - t - 1)+irecyrn*rh)/(s-t);
        if crmm(s)<cm(s)
            Cmat(s) = crmm(s);
            crema(:,s) = 0;
        else
            Cmat(s) = cm(s);
            crema(:,s) = 0;
        end
        if Cmat(s)>Cmat(s-1)
```

```
        r = s;
        t= s-1;
        for i=1:np
            xr(i,r) = rnp(1,i)+irecy(1,i);
        end
    else
        if crmm(s)<cm(s)
            flgrm(s)=1;                    %indicate this is a remanu period
            for i=1:np
                xr(i,s)=rnp(1,i)+irecy(1,i);
                x(i,r) = x(i,r)+dp(i,s)-p(i,1)*rnp(1,i)-irema(1,i);
            end
        A=A+1;
        for i =1:np
            irema(1,i)=0;
            if A<J
                for a=1:A
                   irema(1,i) = irema(1,i)+p(i,a+1)*rnp(1,i)*flgrm(A+1-a);
                end
            else
                for a = 1:(J-1)
                    irema(1,i) = irema(1,i)+p(i,a+1)*rnp(1,i)*flgrm(A+1-a);
                end
            end
            irecy(1,i) = 0;
        end
        else
            flgrm(s)=0;                    %indicate this is not a remanu period
            for i=1:np
                xr(i,s)=0;
                x(i,r)=x(i,r)+dp(i,s)-irema(1,i);
            end
             A=A+1;
        for i =1:np
            irema(1,i)=0;
            if A<J
                for a=1:A
                   irema(1,i) = irema(1,i)+p(i,a+1)*rnp(1,i)*flgrm(A+1-a);
                end
            else
                for a = 1:(J-1)
                    irema(1,i) = irema(1,i)+p(i,a+1)*rnp(1,i)*flgrm(A+1-a);
                end
            end
```

```
                irecy(1,i) = irecy(1,i)+rnp(1,i);     %updte inventory of returned products
            end
            end
            flg(r) = flg(r)+1;
            s = s+1;
        end
    end
end
T = flg(flg ~= 0);                                   % period numbers for each produciton
end_T = cumsum(flg(flg ~= 0));                      % the period before each production
avgC = Cmat(end_T);                                 % average cost
z = dot(T, avgC);                                    % total cost
toc
```

Appendix C

Single-item example problems parameters:

Table C.1 The demands of product over 5 periods for Example 1-3

Period (t)	**1**	**2**	**3**	**4**	**5**
Demand (d_t)	220	280	360	140	270

Table C.2 The demands of product over 10 periods for Example 4-6

Period (t)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Demand (d_t)	184	189	169	205	190	197	210	200	195	191

Table C.3 The demands of product over 10 periods for Example 7-9

Period (t)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Demand (d_t)	184	189	169	205	190	197	210	184	189	169
Period (t)	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Demand (d_t)	200	195	191	205	190	197	210	200	195	191

Table C.4 Parameters of Example 1

Category (j)	**Remanuf. Time**	**Remanuf. Cost (cr_j)**	**Percentage (p_j)**
1	1 period	10	20%
2	2 periods	15	30%
3	3 periods	20	50%
Returned Quantity	**Manufacturing Cost**	ch_y	ch_z
100	80	6	2
Km	Kr		
250	500		

Table C.5 Parameters of Example 2

Category (j)	**Remanuf. Time**	**Remanuf. Cost (cr_j)**	**Percentage (p_j)**
1	1 period	10	20%
2	2 periods	20	30%
3	3 periods	30	50%
Returned Quantity	**Manufacturing Cost**	ch_y	ch_z
200	80	2	6
Km	Kr		
250	500		

Table C.6 Parameters of Example 3

Category (j)	**Remanuf. Time**	**Remanuf. Cost (cr_j)**	**Percentage (p_j)**
1	1 period	10	40%
2	2 periods	20	20%
3	3 periods	30	20%
4	4 periods	30	10%
5	5 periods	40	10%
Returned Quantity	**Manufacturing Cost**	ch_y	ch_z
100	80	2	6
Km	Kr		
500	500		

Table C.7 Parameters of Example 4

Category (j)	**Remanuf. Time**	**Remanuf. Cost (cr_j)**	**Percentage (p_j)**
1	1 period	10	50%
2	2 periods	11	25%
3	3 periods	12	25%
Returned Quantity	**Manufacturing Cost**	ch_y	ch_z
80	30	3	4
Km	Kr		
250	200		

Table C.8 Parameters of Example 5

Category (j)	**Remanuf. Time**	**Remanuf. Cost (cr_j)**	**Percentage (p_j)**
1	1 period	10	41.2%
2	2 periods	11	29.4%
3	3 periods	12	29.4%
Returned Quantity	**Manufacturing Cost**	ch_y	ch_z
170	30	3	5
Km	Kr		
250	150		

Table C.9 Parameters of Example 6

Category (j)	**Remanuf. Time**	**Remanuf. Cost (cr_j)**	**Percentage (p_j)**
1	1 period	10	29.8%
2	2 periods	10	26.8%
3	3 periods	11	17.9%
4	4 periods	12	13.7%
5	5 periods	13	11.8%
Returned Quantity	**Manufacturing Cost**	$\boldsymbol{ch_y}$	$\boldsymbol{ch_z}$
168	30	2	5
$\boldsymbol{Km}$	$\boldsymbol{Kr}$		
250	150		

Table C.10 Parameters of Example 7

Category (j)	**Remanuf. Time**	**Remanuf. Cost (cr_j)**	**Percentage (p_j)**
1	1 period	30	41.2%
2	2 periods	35	29.4%
3	3 periods	40	29.4%
Returned Quantity	**Manufacturing Cost**	$\boldsymbol{ch_y}$	$\boldsymbol{ch_z}$
170	100	15	20
$\boldsymbol{Km}$	$\boldsymbol{Kr}$		
300	300		

Table C.11 Parameters of Example 8

Category (j)	**Remanuf. Time**	**Remanuf. Cost (cr_j)**	**Percentage (p_j)**
1	1 period	30	57.1%
2	2 periods	35	28.6%
3	3 periods	40	14.3%
Returned Quantity	**Manufacturing Cost**	ch_y	ch_z
70	100	15	20
Km	Kr		
300	300		

Table C.12 Parameters of Example 9

Category (j)	**Remanuf. Time**	**Remanuf. Cost (cr_j)**	**Percentage (p_j)**
1	1 period	30	28.6%
2	2 periods	35	21.4%
3	3 periods	35	21.4%
4	4 periods	40	14.3%
5	5 periods	45	14.3%
Returned Quantity	**Manufacturing Cost**	ch_y	ch_z
70	100	15	20
Km	Kr		
150	100		

Multi-item example problems parameters:

Table C.13 The demands of product over 5 periods for Example 10-12

Period (t)	**1**	**2**	**3**	**4**	**5**
Demand (d_t)	220	280	360	140	270

Table C.14 The demands of product over 10 periods for Example 13-15

Period (t)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Demand (d_t)	184	189	169	205	190	197	210	200	195	191

Table C.15 The demands of product over 10 periods for Example 16-18

Period (t)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Demand (d_t)	184	189	169	205	190	197	210	184	189	169
Period (t)	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Demand (d_t)	200	195	191	205	190	197	210	200	195	191

Table C.16 Parameters of Example 10

Component (i)	Quantity in product (l_i)	Manuf. Cost	Category (I_j)	Remanuf. Time of Each Category	Remanuf. Cost of Each Category (cr_{ij})	Percentage of Each Category (p_{ij})
1	1	10	1	1 period	3	80%
			2	2 periods	5	20%
2	2	10	1	1 period	3	60%
			2	2 periods	7	40%
3	3	10	1	1 period	4	50%
			2	2 periods	6	20%
			3	3 periods	8	30%
Returned Quantity		ch_y		ch_z	Km	Kr
100		5		8	1500	1000

Table C.17 Parameters of Example 11

Component (i)	Quantity in product (l_i)	Manuf. Cost	Category (I_j)	Remanuf. Time of Each Category	Remanuf. Cost of Each Category (cr_{ij})	Percentage of Each Category (p_{ij})
1	1	10	1	1 period	3	80%
			2	2 periods	5	20%
2	2	10	1	1 period	3	60%
			2	2 periods	7	40%
3	3	10	1	1 period	4	50%
			2	2 periods	6	20%
			3	3 periods	8	30%
Returned Quantity		ch_y		ch_z	Km	Kr
220		1		8	3000	10000

Table C.18 Parameters of Example 12

Component (i)	**Quantity in product (l_i)**	**Manuf. Cost**	**Category (I_j)**	**Remanuf. Time of Each Category**	**Remanuf. Cost of Each Category (cr_{ij})**	**Percentage of Each Category (p_{ij})**
1	3	10	1	1 period	4	60%
			2	2 periods	6	40%
2	5	15	1	1 period	8	25%
			2	2 periods	8	25%
			3	3 periods	10	30%
			4	4 periods	12	20%
3	4	15	1	1 period	5	50%
			2	2 periods	9	50%
4	2	20	1	1 period	12	60%
			2	2 periods	15	40%
5	3	25	1	1 period	10	10%
			2	2 periods	15	40%
			3	3 periods	20	50%
Returned Quantity		ch_y		ch_z	Km	Kr
100		5		8	1000	500

Table C.19 Parameters of Example 13

Component (i)	**Quantity in product (l_i)**	**Manuf. Cost**	**Category (I_j)**	**Remanuf. Time of Each Category**	**Remanuf. Cost of Each Category (cr_{ij})**	**Percentage of Each Category (p_{ij})**
1	5	20	1	1 period	5	70%
			2	2 periods	10	30%
2	8	30	1	1 period	10	50%
			2	2 periods	11	30%
			3	3 periods	12	20%
3	10	30	1	1 period	10	30%
			2	2 periods	11	30%
			3	3 periods	12	30%
			4	4 periods	13	10%
Returned Quantity		ch_y		ch_z	Km	Kr
170		3		5	250	150

Table C.20 Parameters of Example 14

Component (i)	**Quantity in product (l_i)**	**Manuf. Cost**	**Category (I_j)**	**Remanuf. Time of Each Category**	**Remanuf. Cost of Each Category (cr_{ij})**	**Percentage of Each Category (p_{ij})**
1	4	40	1	1 period	20	80%
			2	2 periods	25	20%
2	4	30	1	1 period	18	50%
			2	2 periods	22	50%
3	2	20	1	1 period	10	40%
			2	2 periods	15	30%
			3	3 periods	17	30%
Returned Quantity		ch_y		ch_z	Km	Kr
80		10		15	1000	400

Table C.21 Parameters of Example 15

Component (i)	**Quantity in product (l_i)**	**Manuf. Cost**	**Category (I_j)**	**Remanuf. Time of Each Category**	**Remanuf. Cost of Each Category (cr_{ij})**	**Percentage of Each Category (p_{ij})**
1	2	30	1	1 period	15	50%
			2	2 periods	20	50%
2	4	30	1	1 period	10	60%
			2	2 periods	17	30%
			3	3 periods	20	10%
3	5	20	1	1 period	10	50%
			2	2 periods	12	10%
			3	3 periods	14	40%
4	7	15	1	1 period	8	20%
			2	2 periods	10	20%
			3	3 periods	13	60%
5	10	15	1	1 period	5	50%
			2	2 periods	9	20%
			3	3 periods	9	20%
			4	4 periods	10	10%
Returned Quantity		ch_y		ch_z	Km	Kr
200		15		20	900	600

Table C.22 Parameters of Example 16

Component (i)	**Quantity in product (l_i)**	**Manuf. Cost**	**Category (I_j)**	**Remanuf. Time of Each Category**	**Remanuf. Cost of Each Category (cr_{ij})**	**Percentage of Each Category (p_{ij})**
1	3	10	1	1 period	5	20%
			2	2 periods	8	60%
			3	3 periods	8	20%
2	5	20	1	1 period	5	60%
			2	2 periods	7	20%
			3	3 periods	7	20%
3	8	25	1	1 period	5	30%
			2	2 periods	5	70%
Returned Quantity		ch_y		ch_z	Km	Kr
170		5		8	5000	2000

Table C.23 Parameters of Example 17

Component (i)	**Quantity in product (l_i)**	**Manuf. Cost**	**Category (I_j)**	**Remanuf. Time of Each Category**	**Remanuf. Cost of Each Category (cr_{ij})**	**Percentage of Each Category (p_{ij})**
1	1	50	1	1 period	30	90%
			2	2 periods	40	10%
2	4	20	1	1 period	10	80%
			2	2 periods	15	20%
3	6	30	1	1 period	20	70%
			2	2 periods	20	10%
			3	3 periods	25	20%
Returned Quantity		ch_y		ch_z	Km	Kr
70		5		8	2000	1000

Table C.24 Parameters of Example 18

Component (i)	**Quantity in product (l_i)**	**Manuf. Cost**	**Category (I_j)**	**Remanuf. Time of Each Category**	**Remanuf. Cost of Each Category (cr_{ij})**	**Percentage of Each Category (p_{ij})**
1	2	20	1	1 period	12	50%
			2	2 periods	16	50%
2	3	20	1	1 period	10	70%
			2	2 periods	12	30%
3	5	50	1	1 period	25	60%
			2	2 periods	30	10%
			3	3 periods	35	30%
4	8	60	1	1 period	30	50%
			2	2 periods	30	30%
			3	3 periods	40	20%
5	10	60	1	1 period	30	50%
			2	2 periods	35	20%
			3	3 periods	40	10%
			4	4 periods	45	20%

Returned Quantity	ch_y	ch_z	Km	Kr
70	5	8	2000	1000

Appendix D

Production planning of optimal solution and heuristic method of single-item example problems:

Table D.1 Production planning of optimal solution of Example 1

Periods (t)	1	2	3	4	5
m_t	200	230	300	0	170
r_t	100	100	100	100	100

Table D.2 Production planning of heuristic method of Example 1

Periods (t)	1	2	3	4	5
m_t	200	230	300	0	170
r_t	100	100	100	100	100

Table D.3 Production planning of optimal solution of Example 2

Periods (t)	1	2	3	4	5
m_t	180	180	160	0	50
r_t	200	200	200	200	0

Table D.4 Production planning of heuristic method of Example 2

Periods (t)	1	2	3	4	5
m_t	180	340	0	0	50
r_t	200	0	400	200	0

Table D.5 Production planning of optimal solution of Example 3

Periods (t)	1	2	3	4	5
m_t	180	220	330	0	170
r_t	100	100	100	100	100

Table D.6 Production planning of heuristic method of Example 3

Periods (t)	1	2	3	4	5
m_t	180	220	330	0	170
r_t	100	100	100	100	100

Table D.7 Production planning of optimal solution of Example 4

Periods (t)	1	2	3	4	5	6	7	8	9	10
m_t	144	129	89	125	110	117	130	120	115	111
r_t	80	80	80	80	80	80	80	80	80	80

Table D.8 Production planning of heuristic method of Example 4

Periods (t)	1	2	3	4	5	6	7	8	9	10
m_t	362	0	0	235	0	117	250	0	115	111
r_t	80	80	80	80	80	80	80	80	80	80

Table D.9 Production planning of optimal solution of Example 5

Periods (t)	1	2	3	4	5	6	7	8	9	10
m_t	134	138	0	79	0	29	74	0	50	0
r_t	168	168	168	168	168	168	168	168	168	168

Table D.10 Production planning of heuristic method of Example 5

Periods (t)	1	2	3	4	5	6	7	8	9	10
m_t	380	0	0	0	0	0	124	0	0	0
r_t	168	168	168	168	168	168	168	168	168	168

Table D.11 Production planning of optimal solution of Example 6

Periods (t)	1	2	3	4	5	6	7	8	9	10
m_t	114	69	0	54	0	27	70	0	46	0
r_t	170	170	170	170	170	170	170	170	170	170

Table D.12 Production planning of heuristic method of Example 6

Periods (t)	1	2	3	4	5	6	7	8	9	10
m_t	264	0	0	0	0	0	116	0	0	0
r_t	170	170	170	170	170	170	170	170	170	170

Table D.13 Production planning of optimal solution of Example 7

Periods (t)	1	2	3	4	5	6	7	8	9	10
m_t	114	69	0	34	20	27	54	0	19	0
r_t	170	170	170	170	170	170	170	170	170	170
Periods (t)	11	12	13	14	15	16	17	18	19	20
m_t	29	25	21	35	20	27	40	30	25	21
r_t	170	170	170	170	170	170	170	170	170	170

Table D.14 Production planning of heuristic method of Example 7

Periods (t)	1	2	3	4	5	6	7	8	9	10
m_t	264	0	0	0	0	0	72	0	0	0
r_t	170	170	170	170	170	170	170	170	170	170
Periods (t)	11	12	13	14	15	16	17	18	19	20
m_t	76	0	0	55	0	27	116	0	0	0
r_t	170	170	170	170	170	170	170	170	170	170

Table D.15 Production planning of optimal solution of Example 8

Periods (t)	1	2	3	4	5	6	7	8	9	10
m_t	144	129	99	135	120	127	140	114	119	99
r_t	70	70	70	70	70	70	70	70	70	70
Periods (t)	11	12	13	14	15	16	17	18	19	20
m_t	130	125	121	135	120	127	140	130	125	121
r_t	70	70	70	70	70	70	70	70	70	70

Table D.16 Production planning of heuristic method of Example 8

Periods (t)	1	2	3	4	5	6	7	8	9	10
m_t	144	228	0	135	120	127	254	0	218	0
r_t	70	70	70	70	70	70	70	70	70	70
Periods (t)	11	12	13	14	15	16	17	18	19	20
m_t	130	125	121	135	120	127	140	130	125	121
r_t	70	70	70	70	70	70	70	70	70	70

Table D.17 Production planning of optimal solution of Example 9

Periods (t)	1	2	3	4	5	6	7	8	9	10
m_t	164	154	119	145	120	127	140	114	119	99
r_t	70	70	70	70	70	70	70	70	70	70
Periods (t)	11	12	13	14	15	16	17	18	19	20
m_t	130	125	121	135	120	127	140	130	125	121
r_t	70	70	70	70	70	70	70	70	70	70

Table D.18 Production planning of heuristic method of Example 9

Periods (t)	1	2	3	4	5	6	7	8	9	10
m_t	164	273	0	265	0	127	254	0	218	0
r_t	70	70	70	70	70	70	70	70	70	70
Periods (t)	11	12	13	14	15	16	17	18	19	20
m_t	130	125	121	135	120	127	140	130	125	121
r_t	70	70	70	70	70	70	70	70	70	70

Production planning of optimal solution and heuristic method of multi-item example problems:

Table D.19 Production planning of optimal solution of Example 10

Periods (t)	1	2	3	4	5
m_{1t}	140	180	260	40	170
m_{2t}	320	360	520	80	340
m_{3t}	510	630	780	120	510
r_t	100	100	100	100	100

Table D.20 Production planning of heuristic method of Example 10

Periods (t)	1	2	3	4	5
m_{1t}	140	180	300	0	170
m_{2t}	320	360	600	0	340
m_{3t}	510	630	900	0	510
r_t	100	100	100	100	100

Table D.21 Production planning of optimal solution of Example 11

Periods (t)	1	2	3	4	5
m_{1t}	44	60	140	0	0
m_{2t}	176	120	280	0	0
m_{3t}	330	378	420	0	0
r_t	220	220	220	220	220

Table D.22 Production planning of heuristic method of Example 11

Periods (t)	1	2	3	4	5
m_{1t}	44	60	60	0	50
m_{2t}	176	120	120	0	100
m_{3t}	330	378	180	0	150
r_t	220	220	220	220	220

Table D.23 Production planning of optimal solution of Example 12

Periods (t)	1	2	3	4	5
m_{1t}	480	540	780	120	510
m_{2t}	975	1150	1400	200	850
m_{3t}	680	720	1040	160	680
m_{4t}	320	360	520	80	340
m_{5t}	630	690	780	120	510
r_t	100	100	100	100	100

Table D.24 Production planning of heuristic method of Example 12

Periods (t)	1	2	3	4	5
m_{1t}	480	540	900	0	510
m_{2t}	975	1150	1600	0	850
m_{3t}	680	720	1200	0	680
m_{4t}	320	360	600	0	340
m_{5t}	630	690	900	0	510
r_t	100	100	100	100	100

Table D.25 Production planning of optimal solution of Example 13

Periods (t)	1	2	3	4	5	6	7	8	9	10
m_{1t}	325	95	0	170	100	135	200	150	125	105
m_{2t}	792	424	0	272	160	216	320	240	200	168
m_{3t}	1330	870	160	350	200	270	400	300	250	210
r_t	170	170	170	170	170	170	170	170	170	170

Table D.26 Production planning of heuristic method of Example 13

Periods (t)	1	2	3	4	5	6	7	8	9	10
m_{1t}	825	0	0	0	0	0	580	0	0	0
m_{2t}	1864	0	0	0	0	0	928	0	0	0
m_{3t}	3180	0	0	0	0	0	1160	0	0	0
r_t	170	170	170	170	170	170	170	170	170	170

Table D.27 Production planning of optimal solution of Example 14

Periods (t)	1	2	3	4	5	6	7	8	9	10
m_{1t}	480	436	356	500	440	468	520	480	460	444
m_{2t}	576	436	356	500	440	468	520	480	460	444
m_{3t}	304	266	178	250	220	234	260	240	230	222
r_t	80	80	80	80	80	80	80	80	80	80

Table D.28 Production planning of heuristic method of Example 14

Periods (t)	1	2	3	4	5	6	7	8	9	10
m_{1t}	480	436	356	500	440	468	520	480	460	444
m_{2t}	576	436	356	500	440	468	520	480	460	444
m_{3t}	304	266	178	250	220	234	260	240	230	222
r_t	80	80	80	80	80	80	80	80	80	80

Table D.29 Production planning of optimal solution of Example 15

Periods (t)	1	2	3	4	5	6	7	8	9	10
m_{1t}	168	0	0	0	0	0	0	0	0	0
m_{2t}	256	36	0	96	0	68	120	80	104	0
m_{3t}	420	345	0	0	0	0	0	0	0	0
m_{4t}	1008	763	0	0	0	0	0	0	0	0
m_{5t}	840	490	0	0	0	0	0	0	0	0
r_t	200	200	200	200	200	200	200	200	200	200

Table D.30 Production planning of heuristic method of Example 15

Periods (t)	1	2	3	4	5	6	7	8	9	10
m_{1t}	168	0	0	0	0	0	0	0	0	0
m_{2t}	256	36	0	96	0	68	120	80	104	0
m_{3t}	420	345	0	0	0	0	0	0	0	0
m_{4t}	1008	763	0	0	0	0	0	0	0	0
m_{5t}	840	490	0	0	0	0	0	0	0	0
r_t	200	200	200	200	200	200	200	200	200	200

Table D.31 Production planning of optimal solution of Example 16

Periods (t)	1	2	3	4	5	6	7	8	9	10
m_{1t}	450	360	0	369	0	183	366	0	258	0
m_{2t}	410	265	0	270	0	135	270	0	95	0
m_{3t}	1064	152	0	432	0	216	432	0	152	0
r_t	170	170	170	170	170	170	170	170	170	170
Periods (t)	11	12	13	14	15	16	17	18	19	20
m_{1t}	192	342	0	369	0	183	414	0	342	0
m_{2t}	145	230	0	275	0	135	350	0	230	0
m_{3t}	232	368	0	440	0	216	560	0	368	0
r_t	170	170	170	170	170	170	170	170	170	170

Table D.32 Production planning of heuristic method of Example 16

Periods (t)	1	2	3	4	5	6	7	8	9	10
m_{1t}	810	0	0	369	0	183	366	0	258	0
m_{2t}	670	0	0	275	0	135	270	0	90	0
m_{3t}	1208	0	0	440	0	216	432	0	144	0
r_t	170	170	170	170	170	170	170	170	170	170
Periods (t)	11	12	13	14	15	16	17	18	19	20
m_{1t}	369	0	165	369	0	183	591	0	0	165
m_{2t}	275	0	105	275	0	135	475	0	0	105
m_{3t}	440	0	168	440	0	216	760	0	0	168
r_t	170	170	170	170	170	170	170	170	170	170

Table D.33 Production planning of optimal solution of Example 17

Periods (t)	1	2	3	4	5	6	7	8	9	10
m_{1t}	121	119	99	135	120	127	140	114	119	99
m_{2t}	512	476	396	540	480	508	560	456	476	396
m_{3t}	810	798	594	810	720	762	840	684	714	594
r_t	70	70	70	70	70	70	70	70	70	70
Periods (t)	11	12	13	14	15	16	17	18	19	20
m_{1t}	130	125	121	135	120	127	140	130	125	121
m_{2t}	520	500	484	540	480	508	560	520	500	484
m_{3t}	780	750	726	810	720	762	840	780	750	726
r_t	70	70	70	70	70	70	70	70	70	70

Table D.34 Production planning of heuristic method of Example 17

Periods (t)	1	2	3	4	5	6	7	8	9	10
m_{1t}	121	218	0	135	120	127	254	0	119	99
m_{2t}	512	872	0	540	480	508	1016	0	476	396
m_{3t}	810	1392	0	810	720	762	1524	0	714	594
r_t	70	70	70	70	70	70	70	70	70	70
Periods (t)	11	12	13	14	15	16	17	18	19	20
m_{1t}	130	125	121	135	120	127	140	130	125	121
m_{2t}	520	500	484	540	480	508	560	520	500	484
m_{3t}	780	750	726	810	720	762	840	780	750	726
r_t	70	70	70	70	70	70	70	70	70	70

Table D.35 Production planning of optimal solution of Example 18

Periods (t)	1	2	3	4	5	6	7	8	9	10
m_{1t}	298	238	198	270	240	254	280	228	238	198
m_{2t}	405	357	297	405	360	381	420	342	357	297
m_{3t}	710	700	495	675	600	635	700	570	595	495
m_{4t}	1192	1064	792	1080	960	1016	1120	912	952	792
m_{5t}	1490	1400	1130	1350	1200	1270	1400	1140	1190	990
r_t	70	70	70	70	70	70	70	70	70	70
Periods (t)	11	12	13	14	15	16	17	18	19	20
m_{1t}	260	250	242	270	240	254	280	260	250	312
m_{2t}	390	375	363	405	360	381	420	390	375	510
m_{3t}	650	625	605	675	600	635	700	650	625	815
m_{4t}	1040	1000	968	1080	960	1016	1120	1040	1000	1248
m_{5t}	0	0	0	0	0	0	0	0	0	0
r_t	70	70	70	70	70	70	70	70	70	70

Table D.36 Production planning of heuristic method of Example 18

Periods (t)	1	2	3	4	5	6	7	8	9	10
m_{1t}	298	436	0	270	240	254	508	0	436	0
m_{2t}	405	654	0	405	360	381	762	0	654	0
m_{3t}	710	1195	0	675	600	635	1270	0	1090	0
m_{4t}	1192	1856	0	1080	960	1016	2032	0	1744	0
m_{5t}	1490	2530	0	1350	1200	1270	2540	0	2180	0
r_t	70	70	70	70	70	70	70	70	70	70
Periods (t)	11	12	13	14	15	16	17	18	19	20
m_{1t}	260	250	242	270	240	254	280	260	250	242
m_{2t}	390	375	363	405	360	381	420	390	375	363
m_{3t}	650	625	605	675	600	635	700	650	625	605
m_{4t}	1040	1000	968	1080	960	1016	1120	1040	1000	968
m_{5t}	1300	1250	1210	1350	1200	1270	1400	1300	1250	1210
r_t	70	70	70	70	70	70	70	70	70	70