Numerical Assessment of Directional Energy Transfer for Geometric Structure

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Abstract

Numerical Assessment of Directional Energy Transfer for Geometric Structure

by Ankhy Sultana

Energy is the capacity to do work, and mechanical work is the amount of energy transferred by force. Hence, energy can be represented in the form of deformation obtained by the applied force. Energy transfer is defined in physics when the energy is moved from one place to another or energy is transferred from one form to another form. To make the energy transfer functional, energy should be moved in the right direction, at the right location, at the right time and large enough to produce a difference. If it is possible to make better use of the energy in the right direction at the right moment, the energy efficiency of the structure can be enhanced. This idea leads to the concept of directional energy transfer (DET), which refers to transferring energy from one direction to a specific direction. As force and deformation in the particular direction are responsible for the energy in that particular direction, structural properties like geometry or material can have an impact on DET property of the structure. With the recent development of additive manufacturing, complex structures can be applied to various applications to enhance performances, like a wheel and shoe mid-sole. For example, lattice structures are produced to attain lightweight, maintaining strength and specific mechanical properties. While many works are related to structural strength, there is limited research in DET. In this study, a theoretical approach is proposed to measure the DET of a structure based on the geometry of the structure, which can be used to evaluate the effectiveness of energy transfer. The purpose is to understand the energy transfer behaviour of a structure and to measure if a structure is able to transfer energy from one direction to the desired direction. The designs tested were inspired by lattice structures.

However, the tested structures do not represent optimized DET structures. The contributions in this work are developing a more generalized mathematical model to implement the mathematical model to build the concept and measure the structure-property of directional energy transfer. And also the goal includes finding out the validity of the concept from the mathematical and experimental point of view.

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DET	Directional Energy Transfer
FEA	Finite Element Analysis
AM	Additive Manufacturing
FDM	Fused Deposition Modeling

1 Introduction

In structure mechanics, optimization is an important tool in the design process to minimize the stresses or compliance for a given amount of material and boundary conditions. The method can be utilized to design engineering structures but also to tailor microstructures. However, it is possible to have a customized or optimized structure to achieve the expected mechanical property. In this analysis, the anticipated property is directional energy transfer (DET).

1.1 Motivation

Uses of more complex geometry to modify mechanical properties are becoming more and more common in advanced manufacturing technology e.g. shoe midsole in Fig. 1.1a. In the automotive sector, such kind of application is producing an airless tire (see Fig. 1.1b). The purpose was to modify the lateral stiffness while maintaining proper contact patch. Apart from automotive, the concept of energy return is implemented in sports. Energy return is a similar concept as DET, which returns the stored energy, but it does not consider the direction of the energy. It is considered that sport shoes can play an influential role in the runner's performance since running efficiency depends on the interaction between the foot and sports surface. An athlete's performance can be highly regulated by the interaction between the foot and sports surface. Hence, sports surfaces and shoes can be constructed to enhance efficiency. An athlete employs a great amount of energy during performance. While the foot strikes on the ground, energy is stored in the sport surface and can be transferred by the surface. To make the energy transfer functional, the energy transferred should be in the right direction. However, only some portions of the exerted energy are actually effective for the movement. If it is possible to make

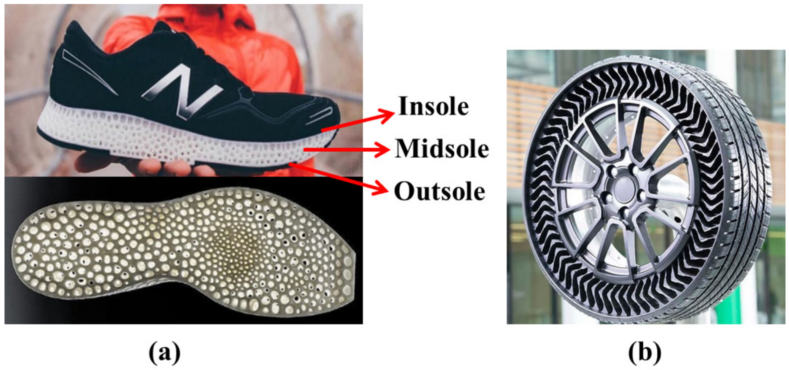


FIGURE 1.1: (a) A 3D printed midsole ©designmilk adapted with the license CC BY-SA 2.0. (b) A 3D printed airless tire.

better use of the energy in the right direction, the efficiency of the performer can be enhanced. DET is a concept that can estimate the effectiveness of energy transfer in an intended direction. With the recent development of additive manufacturing, complex structures can be applied to various applications to intensify performances, like a wheel and shoe mid-sole. While many works are related to structural strength, there is limited research in DET. In this study, to illustrate the concept, shoe midsole will be practiced as an example. Nevertheless, the objective is to attain the validity of the directional energy transfer concept for any general structure.

1.2 Concept Overview

Energy is the property to perform work on an object. Although energy is a scalar quantity, the forces that are exerted by a surface as energy are vector quantities having both magnitude and direction. Consequently, the terminology relating to energy in a direction refers to the direction of deflection. When the force towards a direction is multiplied by the amount of deflection, the result is the energy in that direction. It can be explained further through the implementation of this concept in shoe midsole. A shoe sole contains three layers: insole, midsole and outsole (see Fig. 1.1a). Insole stands directly beneath the foot, the midsole is the layer between the insole and outsole, and the layer that is in contact with the ground is called outsole. Among all these layers, the shoe midsole serves the main purpose of a shoe sole. Traditionally it provides cushioning.

While walking or running, forces applied on the ground by the foot can be decomposed into a downward and backward force. According to Newton's third law, reaction forces are applied by the ground respectively to the upward direction, helping to stand and to forward direction pushing us forward.

When the athlete contacts the sport surface, there is some work done by the athlete on the sport surface. Energy is transferred from the athlete to the surface through the foot and shoe. Sports surface deforms due to work done by the athlete and stores the potential energy. Energy can be represented in the form of deformation obtained by the applied force. Energy transfer is defined in physics when the energy is moved from one place to another or transform of energy from one form to another form. DET refers to transferring energy from one direction to a specific direction. In other words, a DET structure can transfer the 'vertical' energy into a form of horizontal force and deformation. The concept of energy return is to utilize the potential energy and get that potential energy in the form of force and deformation when the structure returns back to its initial state. Energy return is different from energy transfer as energy return occurs in later half part of the whole cycle of deformation. Energy transfer may or may not be due to energy return.

1.3 Objective

The goal of the study is to assess the concept of DET and ascertain the capability of a mechanical structure to transfer energy to different directions. With the help of three-dimensional (3D) printing technologies, complex structures with desired functionalities could be designed [1]. It could be possible to generate structures with functional increments to enhance DET. Consequently, a quantitative way to measure the efficiency of transfer is needed for the exploration of different structures. Nevertheless, the existing mathematical model does not clarify numerous inquiries regarding DET computation. For instance, Dickson's work [2] proposed energy dissipated in the vertical direction being the energy transfer in the horizontal direction, which is not surely conclusive. Furthermore, the mathematical model is structure-specific, making one set of equations invalid for another, e.g., a distinct set of equations for 1-DOF linear mechanism and another set for 1-DOF rotational mechanism. Hence, the mathematical representation of directional energy transfer was crucial to examine. The research question this study is trying to answer is: *how to mathematically quantify the DET performance of any given structures?*

1.4 Scope

The purpose of this research is to review the numerical study and propose a complete generalized mathematical model for DET. The contributions of this analysis are:

- The mathematical model of DET using spring elements is re-visited, and the necessary concepts in work done and potential energy are identified to measure DET.
- A new formulation is developed to directly calculate the energy in a particular direction, and it is concluded that there is no DET using static equilibrium analyses.
- The formulation is further extended to dynamic finite element analysis using frame elements so that the mathematical derivation and calculation is unified and general even for more complicated structures.

Experimental analysis is conducted along with the detailed numerical analysis to strengthen the DET concept following thoughtful observations from previous studies. This study assumes the structures represented by finite-elements with circular cross-section area, and the materials, as well as the geometries, are having linear properties. Further studies can be done to incorporate non-linear materials and geometries, or even dynamic structures to explore the other possibilities of DET. Since there are many different kinds of properties, the thesis only focuses on the fundamental ones with linear properties. The damping effect is also not considered to simplify the explanation.

The thesis is organized as follows. Related works are reviewed in Chapter 2. Chapter 3 explains the fundamentals of DET, investigates a case, and gives novel observations and according to the observations, the formulation is enhanced for DET in Chapter 3.3, and further generalized in Chapter 3.4. The developed analyses are done on a few examples, and the experimental results are presented in Chapter 4. The paper is concluded in Chapter 5.

2 Literature Review

2.1 Related Works

In this chapter, a thorough overview of the works related to directional energy transfer is mentioned. The previous works can be separated into different sections: Lattice structure, structure optimization, energy return and energy transfer.

2.1.1 Lattice structure

In additive manufacturing, the term infill refers to the interior structure of an object that is printed. The infill pattern and volume percentage significantly influence the printing process as well as the physical properties of the printed object [3]. Cellular materials such as foam, honeycomb, and lattice structure are used in applications due to their special mechanical properties which cannot be achieved by conventional bulk material. The lattice structure refers to a type of cellular materials [4] that have a truss-like structure with interconnected struts and nodes in a three-dimensional (3D) space. Compared to other cellular materials such as random foams and honeycombs, the lattice structure exhibits better mechanical performance [5].

Lattice structures are widely used in many engineering applications because of the ability to distribute materials at vital parts to improve specific mechanical performance, such as strength to weight ratio, heat transfer, thermal isolation, energy absorption and biocompatibility [6]. Lattice structures have many superior properties, which make it a promising solution for various applications, such as a lightweight structure due to its high specific stiffness and strength etc. [7]. Mechanical performances of lattice structures depend on various factors such as the cell topology, number of cells, geometric parameters (e.g. strut diameter and cell size), material and manufacturing process, as well as structural boundary and loading conditions [8]. By tailoring the material, the lattice structure can be optimized to satisfy specific functional requirements, which means the mechanical properties are more flexible to be controlled. designing microstructures of cellular materials with maximum bulk or shear modulus based on the different techniques [9].

The lattice structures used in tissue engineering and bone scaffolds are usually classified according to the unit cell design: CAD-based, image-based, implicit surfaces or topology optimized unit cells [10]. Recent research has been oriented towards the topology optimization(TO) of the base unit cell [11]. The effects of unit cell size on the elastic modulus, shear modulus and Poisson's ratio of triangulated lattice structures show that the elastic modulus and the shear modulus decreased as the cell size increased [12]. In a previous study [13], three lattice structures corresponding to different loading modes were designed and tested. It is found that some structures have higher energy absorption efficiency than the others. Schaedler et al. [14] have investigated different types of metallic microlattice structures for energy absorber by quasi-static compression tests. It was shown that the lattice structure offers more flexibility in tailoring the response to impulsive loads than conventional materials can. Compared to honeycombs, the lattice structure has the potential to improve compressive and shear strengths when designed to suppress buckling [15, 16].

It is possible to design material to achieve desired deformation behaviour by data-driven process [17]. Deformable objects with spatially varying elasticity is fabricated using 3D printing [18]. Also, heterogeneous objects are believed to possess superior properties in applications where multifold functional requirements are simultaneously expected. By introducing material heterogeneities into the design domain, anisotropic properties can be obtained, the different properties and advantages of various materials can be fully exploited, and traditional limitations due to material incompatibilities can be naturally alleviated with gradual material variations [19].

Lattice structures with sophisticated geometries are successfully fabricated by several manufacturing techniques. Among them, additive manufacturing (AM) is especially well suited for the fabrication of complex lattice structures. Additive manufacturing enables the fabrication of complex structures by aggregation of materials in a layer-by-layer fashion, which has unlocked the potential of lattice structures [20].

With the help of three-dimensional (3D) printing technologies, complex structures could be designed [1] to enhance the DET performance. 3D printing provides massive opportunities to fabricate parts with design complexities, thus enabling various design flexibility and application opportunities [21]. They can be used to fabricate objects with prescribed mechanical behaviour, e.g. to create anisotropic patterns with target orthotropic properties [21]. Being able to combine multiple functional materials into a single print has the potential to greatly increase the utility of printed objects [22]. Multiprocess (or hybrid) 3D printing, where complementary processes are combined to advance manufacturing by increasing the functionality of fabricated components get focused on MacDonald's study [23]

Hence, it is possible to control the energy return by using a different lattice structure to have directional energy properties. By modifying density, angle and anisotropy, it can control both the "softness" and "bending" of a sole [24]. A method is presented to precisely control the contact forces and pressure over large contact areas between the foot and a deformable shoe [25]. Another interesting structure is auxetic materials which behave unconventionally under deformation, which enhances material properties such as resistance to indentation and energy absorption. Auxetics, therefore, have the potential to enhance sporting protective equipment [26]. Lattice structures and auxetic materials possess novel properties to solid material and conventional structures. The functional flexibility of the structures motivates this paper to develop the measurement of DET so that it can be used as an objective to optimize the structures.

2.1.2 Energy return

Many papers are related to energy return showing the evolution of directional energy transfer. Basically, the energy return concept came into consideration to have possible use in sports. From previous works, the concept of energy return is implemented in sports mainly. Energy return is a similar concept as DET, and it refers to the return of the stored energy, but it does not consider the direction of the energy. It is believed that sport shoes can play an important role in the runner's performance since running efficiency depends on the interaction between the foot and sports surface. Athletic footwear has been advocated as a mechanism by which the running economy can be improved [27]. Performance enhancement is also a primary motivating reason that runners try new footwear [28]. The selection of appropriate footwear is often advocated as an essential requirement for distance running [29]. Running shoes with greater shoe cushioning, greater longitudinal shoe stiffness and greater shoe comfort were associated with improved running economy [30]. Results showed that VO_2 (volume of oxygen consumption) and respiratory exchange ratio were significantly lower, and shoe comfort was significantly greater in the footwear with energy return [31]. Wearing a running shoe is not limited to geometric changes of the foot-ground interface but may also change the stiffness of the foot-ground interface due to the deformation of the midsole [32]. Ground reaction forces and kinematic variables were found to vary with shoe hardness and shoe geometry [33], and improving forefoot push-off facilitate the augmentation of forwarding acceleration and ultimately enhances athletic performances [34, 35].

Energy considerations during athletic activity have concentrated on two major strategies to improve performance: the return of energy and reduction of loss of energy. Return of energy to improve athletic performance has been studied for sport surfaces and sports shoes [36].

The concept of energy return through sport shoe surfaces was introduced as that sports surface can store and return energy. During running, energy is transferred from the athlete to the sport surface. A portion of the conserved energy is lost due to the time-dependent material properties of the surface. Energy return is determined by the energy conserved and energy loss. Since sport surface stores and returns energy, the returned energy during take-off will contribute to athletic performance [37, 38].

The concept of energy saving is found in nature too. Energy is saved, in insect flight and mammal running, by storing elastic strain energy at one stage in the wingbeat or stride and releasing it at another[39]. Multiscale structures are characteristic for biological materials, exhibiting inherent multifunctional integration [40]. In nature, web building is an energetically costly process, making it critical that the entire web is strong enough to meet different loading demands[41].

As the athlete leaves the surface, some of this energy flows back in the opposite direction from the surface to the athlete. A compliant surface acts as a spring if the stiffness of the spring is closely tuned to the mechanical properties of the human runner; the runner's speed can be increased. Both ground contact time and step length increased on very compliant surfaces [42, 43]. The capacity to deform, and consequently to store and return elastic energy, could potentially be augmented dramatically by developing a structured sport surface.

By modifying the direction of the structural elements, differences of up to 10% in the returned energy were seen [37]. By using a discrete non-linear viscoelastic model, approximately 95% of the energy conserved in a sports surface of a shoe can be returned to the athlete. The energy return for the new structural surface/shoe combination, which was about 14 Joules, was more than 14 times higher than that of the conventional shoe/surface system [38]. The newly developed running shoes reduce the energetic cost of running by an average of 4% compared with established marathon racing shoes [44]. A highly elastic material was placed in the shoe outsole to enhance the ductility of the material before push-off, increase the energy-return effect of the material after push-off, and ultimately improve the movement characteristics during push-off [45]. Still, energy storage and recovery in the model shoe are large enough to have local effects on the energetics of the foot and lower leg, but modest when compared with passive energy transfer within and between body segments or strain energy storage and recovery in the lower limb [46].

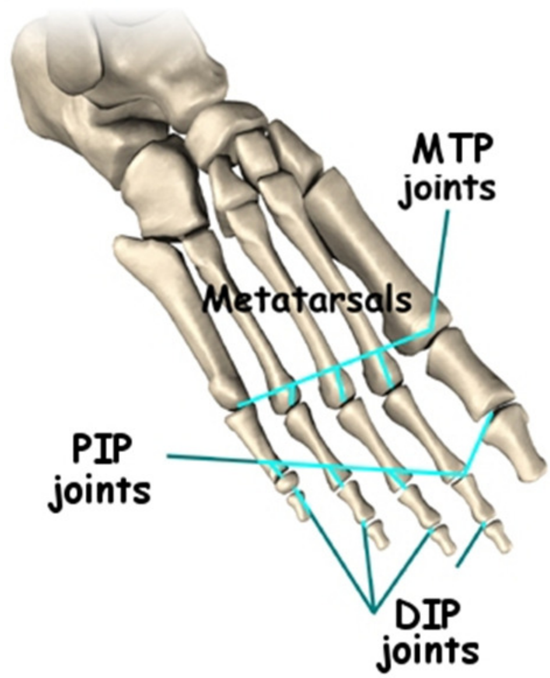


FIGURE 2.1: Foot anatomy showing MTP joint with the license © 1999-2017 Orthogate.

That is why works are being done in the second strategy: reduction of energy losses. Following this strategy, stiffening the shoe structures around the MTP (metatarsophalangeal) joint (see Fig. 2.1) caused a shift of the point of force application toward the front edge of the shoe-ground interface. Negative work was significantly reduced for the stiffest shoe condition, and at the same time, a significant increase of positive work at the MTP joint was found [47].

2.1.3 Energy transfer

The reason behind the scope of energy return being limited could be that returned energy is not being efficiently used. The concept of energy return led to the development of directional energy transfer as a critical measure of shoe efficiency. The principle of directional energy transfer is a concept developed by Fuss (2009) [2]. The efficacy of this energy return concept, according to Nigg et al. [48] relies on the energy returned at the right location, at the right time, with the right frequency. The concept of energy return led to the development of DET as a critical measure of shoe efficiency. Only a few works have been conducted in DET area previously. DET can be optimized if the bounce tubes of Adidas shoes are rotated, and the overall stiffness changed (by altering the tube length) [49]. There are a number of parameters that relate to DET. These parameters are: energy transferred (as a percentage of vertical input energy), energy returned in the horizontal direction and total system energy [50]. Dickson's thesis [2] showed how energy transfer could be determined for classified systems and a midsole can be designed, which is capable of transmitting energy from vertical direction to horizontal direction. Experimental results were used in the formulation to determine the amount of energy transfer. Although it was a big step in this area of research, more studies are needed to enhance the fundamental understanding of DET.

3 Methodology

3.1 Directional Energy Transfer (DET)

This section explains the concepts and formulations of DET following Dickson's work [2]. In this reference, a thorough survey was conducted to categorize the various theoretical models that are capable of delivering DET. The study proposed that, by utilizing the potential energy generated as force and displacement in the vertical direction, the designer is able to create displacement in the horizontal direction by using an appropriate shoe structure. After that, the study is about enhancing the sport shoe efficiency by using the concept of DET.

3.1.1 Assumptions

Without loss of generality, a midsole structure shown in Fig. 3.1b represents the midsole of Adidas Spring Blade shoe to illustrate the concept of DET. The modeling acknowledges the shape and geometry of the structure. The deformation occurs due to the load at the contact point between the foot and the sole. Additionally, the contact point does not alter due to variable loading conditions.

A lattice structure can be generated by repeating the unit cell with a direct patterning, conformal patterning, or topology optimization approach [7]. Like a lattice structure, the structure Fig. 3.1b has a repetitive unit cell in a linear pattern. Hence, inquiring about an individual unit element will be satisfactory and computationally less expensive. As mentioned earlier, shoe midsole is considered as the example case to describe the concept. Yet, a comparable phenomenon is expected for other structures while loading condition is alike.

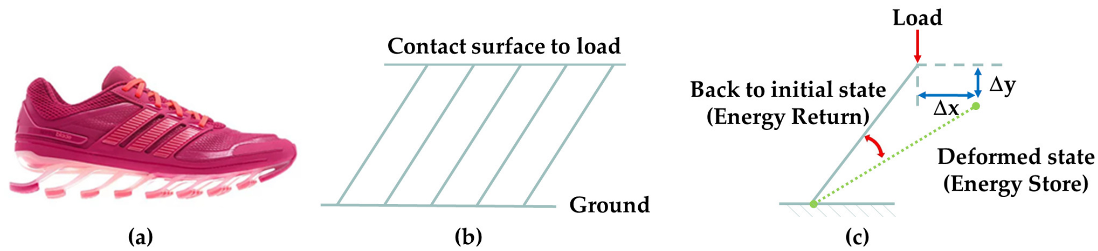


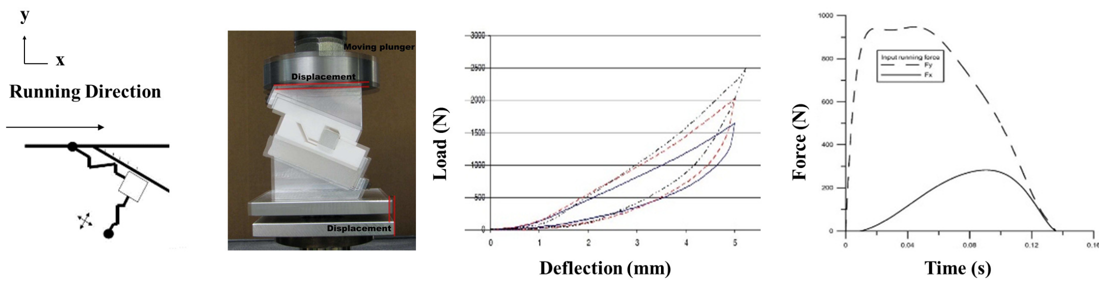
FIGURE 3.1: (a) Adidas Spring Blade shoe (b) An illustration of sample structure for analysis (c) deformation of the structure element due to loading and unloading.

3.1.2 Model simplification

This phenomenon is valid for any structure irrespective of the unit element type. However, the structure was simplified as a spring (or a set of springs) to demonstrate the concept. From Fig. 3.3, it is pointed that an element can be simplified to a linear spring and a rotational spring. Linear spring can bear axial load showing linear deformation (truss-like). On the contrary, rotational spring is the element which can support transverse load performing rotational deformation (beam-like).

While running, an athlete strikes the ground with one's forefoot and applies a load to the shoe midsole. The midsole deforms due to the load at the contact point connecting the foot and the midsole structure. The midsole stores energy as a form of load and deformation (Fig. 3.1c). There are some deformations in x- and y-directions, and the structure continues deforming till it attains equilibrium. When the athlete takes off, the deformed structure tends to go back to its initial state and returns the stored potential energy as a reaction force.

For a spring, while applying a load at the contact point, it deforms with Δx and Δy storing energy. At equilibrium, the reaction force (F_s) of the spring should be equivalent to the applied force. And the total stored energy (E_{tot}) in the system is the elastic potential energy of the spring (E_{sf}). For instance, the potential energy stored in a linear spring with a spring constant k is $E_{sf} = \frac{1}{2}k\Delta s^2$ where s is the deformation of the spring.



(a) Two Inclined Linear Spring Mechanism

(b) Prototype of Two Inclined Linear Spring Mechanism

(c) Load vs deflection for the 2-DOF prototype

(d) Applied force by forefoot strike

FIGURE 3.2: Referring the work [2], (a) two linear spring mechanisms, (b) the prototype to test the mechanism, (c) the load-deflection curve of the physical test, and (d) the applied force curve used for the numerical study by the forefoot strike.

3.1.3 Energy decomposition

Recall that, DET in this case aims to transfer energy from the vertical direction to the forward direction. Therefore, in order to measure the amount of transfer in specific direction, the total energy (E_{tot}) needs to be separated into x- and y-directions: E_x & E_y. Dickson achieved this separation by decomposing the reaction force F_s into x- and y-directions, i.e., F_x and F_y, corresponding to the deformations Δx and Δy. Then, the energies are summarized as following:

$$E_{sf} = \int F_s \, ds, \qquad E_x = \int F_x \, dx, \qquad E_y = \int F_y \, dy.$$

Since the spring is at its equilibrium state and it is assumed that there is no energy converted to other forms besides deformation, the energy is conserved, i.e.,

$$E_{tot} = E_{sf} = E_x + E_y, \qquad E_{trans} = E_{tot} - E_y.$$

The energy transfer E_{trans} was defined by the total energy stored in the system minus the energy stored in the y-direction, assuming it will be the energy returned to the x-direction.

3.1.4 Mechanism with two linear springs

The basic assumptions followed in Dickson's work [2] are explained in previous section. According to the thesis [2], the simplest mechanism capable of energy transfer is believed to have only 1 or 2 degrees of freedom (DOF). Several prototypes were designed with 1-DOF (linear spring, rotational spring) and 2-DOF (linear-linear spring, linear-rotational spring, rotational-rotational spring) mechanisms. This section further explains the principles of DET using the 2-DOF mechanism with two linear springs. From the thesis, the 2-DOF linear spring mechanism is shown in Fig. 3.2a, where the two springs are inclined in 90 degree and the central node is constrained in that inclined plane. The corresponding prototype of the mechanism was designed and fabricated as shown in Fig. 3.2b.

Mathematical formulations for 2-DOF structure are provided here [2]:

$$
\begin{aligned}
F_{s1} &= k_1 \Delta l_1, & F_{s2} &= k_1 \Delta l_2, \\
F_x &= F_{s2} \sin\theta - F_{s1} \cos\theta, & F_y &= F_{s2} \cos\theta + F_{s1} \sin\theta, \\
E_x &= \int F_x dx, & E_y &= \int F_y dy, \\
E_{sf1} &= l \int F_{sf1} dl, & E_{sf2} &= l \int F_{sf2} dl, \\
E_{tot} &= E_x + E_y = E_{sf1} + E_{sf2}, & E_{trans} &= E_{sf1} + E_{sf2} - E_y.
\end{aligned}
$$

F_{s1} and F_{s2} are the spring forces of the two springs (1 & 2) obtained by their deformations Δl_1 and Δl_2. Both spring forces are used to calculate the directional forces F_x and F_y in x- and y-directions, respectively.

The energies (E_x & E_y) can then be computed by the integration of the force along each direction. Here, energy is being divided into two parts – x and y. As energy is conserved in a system, the total energy (E_{tot}) is a combination of E_x and E_y, which should also equal to the total energy computed based on each linear spring ($E_{sf1} + E_{sf2}$). Assuming there is no energy in the x-direction input to the system, the energy transfer (E_{trans}) is expressed by subtracting E_y from the total energy.

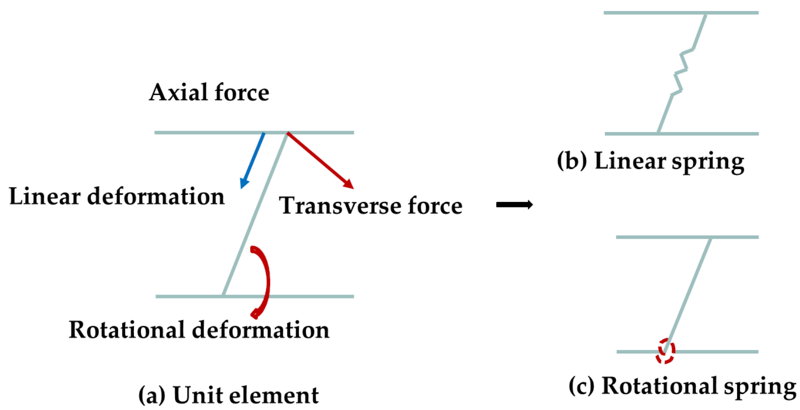


FIGURE 3.3: Model simplify (a)unit element (b) Linear and (c) Rotational spring.

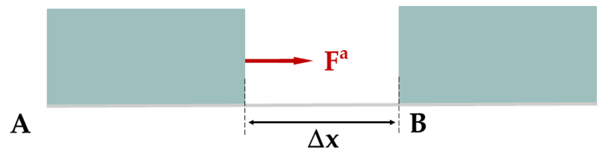
3.1.5 DET from experimental analysis

Experimental analysis was also performed on the structure to investigate if there is DET, as shown in Fig. 3.2b. The prototype was compressed to a suitable maximum. This establishes whether vertical compression results in horizontal displacement of the intermediate plate. Vertical motion causes the prototype to move horizontally and the structure moves back to the original position after the upper platen returns to its original location.

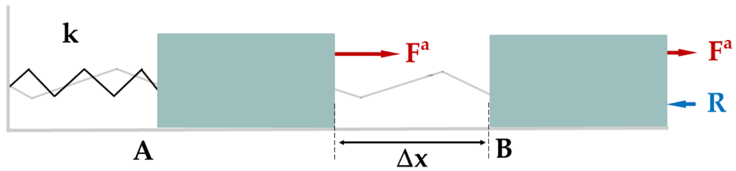
The load profiles for the vertical component against the deflection in y is shown in Fig. 3.2c. Three different loads were tested corresponding to the three curves in the chart. The energy in the y-direction was obtained by calculating the area under the curves, where the top part is the loading curve and the bottom part was the unloading curve. The area enclosed by this loop gives a measurement of the energy loss in the y-direction, which was assumed to be the energy transferred in the x-direction. The work concluded that the mechanism is capable of transferring energy from the vertical y-direction to the horizontal x-direction since they deform in horizontal direction due to compression, and thus it has DET.

3.1.6 Observations

Thanks to the thesis [2] providing a foundation for the study of DET, and a few observations were made to further enhance the fundamental understanding of the numerical



(a) Work done on a free body



(b) Work done on a body attached to spring

FIGURE 3.4: Work done by an applied force (a) on a free body and (b) on a spring.

analysis.

Firstly, the forces in x- and y-directions (F_x, F_y) are calculated by the reaction forces of the springs, and the applied forces are not taken into account. This means that the computation is based on the deformed state of the structure computing the spring energy entirely. In other words, particularly the potential energy is considered in the formulation without taking into account how the work is done on the structure. Truly, computing the energy by the area under the loading and unloading curves is basically the work done by spring:

$$WD = \int_0^{\Delta x} R(x)\, dx,$$

where $R(x)$ is the reaction force of spring, and it supposed to be $R(x) = kx$ when the spring constant k is known, but it was measured directly in the experiment. This equation is valid for the unloading curve, but there is applied force when the foot strikes on the ground, which is not accounted in the loading curve.

Regarding energy return solely shows that the structure moves horizontally due to the vertical load, although the structure requires to move in the opposite x-direction to store the energy (Fig. 3.1c). The movement in the return phase is merely compensating for the backward movement in the storage phase. Disregarding the negative deformation will overestimate the amount of energy transfer in the system, and thus we have the following remark.

Remark 1 *Both the work done and potential energy need to be analyzed along each direction to ensure if the energy in a direction is actually transferred from another direction.*

Secondly, the physical experimental validation of DET [2] assumed the amount of energy loss in the y-direction is the energy gain in the x-direction.

If we go back to work done theory again, for a constant force employed on a free body (see Fig. 3.4a), the work done is $WD = F^a \Delta x$, where F^a is the applied force. When the force is applied on a spring system (see Fig. 3.4b), at equilibrium, the elastic potential energy ($\frac{1}{2}k\Delta x^2$) is commonly used to calculate the work done, if the spring constants are known. It is okay when the energy is considered as a whole, and the system is at equilibrium.

Nevertheless, in the context of DET, the energy needs to be separately calculated in the x- and y-directions, but the spring constants of a complex system cannot be found for the x- and y-directions explicitly. However, the reaction forces in the x- and y-directions can only be computed numerically.

For the body attached to a spring in Fig. 3.4b, as spring starts to deform, a reaction force starts to work on the opposite direction of the applied force. This opposite force is the spring force which makes it more difficult to deform. It is noticeable that, at point **A**, there is no reaction force. At point **B**, the spring reaction force $R(x)$ equals to the applied force. Therefore, the spring reaction force increases gradually through the whole path. While the spring force increases, net force on the structure decreases till equilibrium is achieved. Combining both works done by the applied force and the spring (Fig. 3.4b), the net force $F^n = F^a - R(x)$ is integrated along the deformation, i.e.,

$$WD = \int_0^{\Delta x} F^a - R(x)\, dx. \tag{3.1}$$

If k is known, the work done is $WD = F^a \Delta x - \frac{1}{2}k\Delta x^2$. In this example, F^a should equal to the spring force in the equilibrium, i.e., $F^a = R(\Delta x) = k\Delta x$. Therefore, $WD = k\Delta x^2 - \frac{1}{2}k\Delta x^2 = \frac{1}{2}k\Delta x^2$, which equals to the spring potential energy. This verifies the correctness of Eq.(3.1).

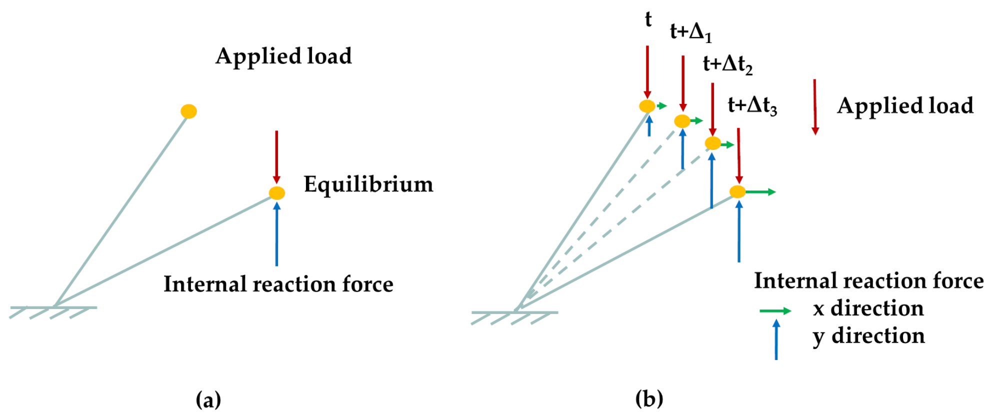


FIGURE 3.5: (a) Static vs. (b) dynamic analysis showing change in internal reaction force through time history analysis.

Hence, the applied force should be considered to find the net force for the computation of work done, and the following remark can be proposed.

Remark 2 *To find the energy for a particular direction in the storage phase (loading), it should integrate the net force between the applied force and the reaction force in that direction.*

Thirdly, the entire mathematical model of DET is based on the equilibrium state of the system, so that the total energy (E_{tot}) is defined by the total spring energy and the energy transfer (E_{trans}) is determined by the total energy minus the energy in the y-direction.

In linear static analysis, deformation is assumed to be linear, and the loading to be static – remain constant and do not change direction. Hence, reaction forces calculated using static formulation are the same as the applied forces due to equilibrium (Fig. 3.5b).

Although the static equilibrium analysis is simple and can ease the energy calculation, assuming every instant during loading and unloading is at equilibrium is not realistic. This is because besides deformation, the energy is in multiple forms like acceleration and velocity, and the whole motion is dynamic. Even the analysis is separated into different time steps.

In static analysis, it will be the same deformation for the same loads for loading and unloading. Therefore, the energy store and energy return will be the same, and thus the directional energy transfer will be zero. In real situation, when an applied load is applied, the internal reaction force increases in opposite direction resulting the net force to decrease to zero at equilibrium (see Fig. 3.5c). To truly compute the DET, the following remark can be made.

Remark 3 *DET is a dynamic process, and its study should consider the dynamics of the whole storage and return cycle.*

3.2 Synopsis

Based on the observations and remarks, the next sections present some methodologies to enhance the formulations for DET. The new formulations are developed first for both the work done and energy return, considered separately in the energy storage and return phases. Instead of subtracting the energy in the y-direction from the total energy, the energy transfer is directly computed by the energy stored and returned in the x-direction. After that, the dynamic analysis is included in the formulation, and the mathematical model is generalized using dynamic finite element analysis (FEA) with frame elements. With the new formulations, the mechanism with two linear springs are re-studied and re-analyzed for the DET.

3.3 Modified DET Formulation by Work Done

Following the Remark 1, besides the potential energy, the work done by the applied forces should be considered. To explain the concept more clearly, the whole cycle is split into two phases: energy storage and energy return. While the motion is caused by the reaction forces (potential energy) in the return phase, there are both applied and reaction forces in the storage phase. As mentioned earlier, in this analysis, it considered only the contact point for loading and assumed energy transfer would be conducted only through the contact point. Hence reaction forces are calculated only at the contact point, and other nodes were not considered.

So as stated in the Remark 2, the deformation and the net force working in the x- and y-directions are needed to be found for the storage phase.

Energy Storage Phase

Net force F^n working in any direction is the subtraction of the reaction force $R(x)$ from the applied force F^a in that direction.

In DET, if there is the net forces (F^n_x & F^n_y) and displacements (Δx & Δy) in the x-direction and the y-direction, energy in the x-direction and the y-direction of the storage phase can be determined as follows.

$$E^s_x = \int_0^{\Delta x} F^n_x \, dx, \qquad E^s_y = \int_0^{\Delta y} F^n_y \, dy. \tag{3.2}$$

The net forces in the x- and y-directions can be expressed by considering respective components of force and reaction force.

$$F^n_x = F^a_x - R_x(x), \qquad F^n_y = F^a_y - R_y(y).$$

Energy Return Phase

During unloading, the structure is returning to its initial state. Potential energy is released and the reaction force R is exerted. The energy E^r in the energy release phase can be calculated by the reaction force R:

$$E^r_x = \int_{\Delta x}^{0} R_x(x) \, dx, \qquad E^r_y = \int_{\Delta y}^{0} R_y(y) \, dy. \tag{3.3}$$

Energy Transfer

I have energy stored in the x- and y- directions. To have energy transfer, it is needed to have energy from the y-direction converted to the x-direction. The energy transfer is directly defined by the energies in the x-direction:

$$E_{trans} = E^r_x + E^s_x. \tag{3.4}$$

Here, directions of forces and deformation are important considerations. Because the different directions of deformation will result in positive and negative values in the

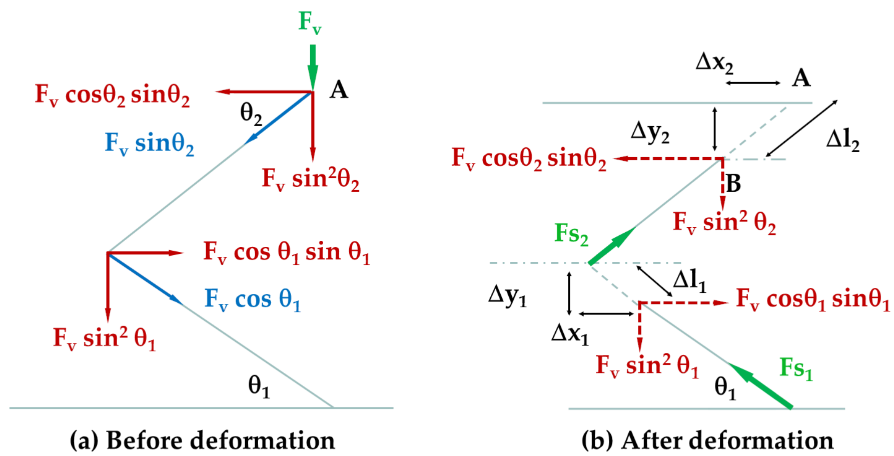


FIGURE 3.6: Vector decomposition of an applied force on two linear springs and the deformations.

energies, so the energy transferred E_{trans} is the addition of E_x^r and E_x^s. Energy transfer would be of help, if the net energy transfer is achieved towards the intended direction (running direction which is positive x-axis).

3.3.1 Re-formulate the Structure of Two Linear Springs

In this section, the two inclined linear spring structure shown in Fig. 3.2 is re-visited. The two springs are attached to each other at right angle. Let the lengths of the linear springs be l_1 and l_2 and they are inclined at angles θ_1 and θ_2 to the horizontal. A vertical load F_v is applied on the structure (Fig. 3.6a). Applied force is decomposed to axial force and transverse force (in blue) that are accounted by each of the springs.

To compute energy transfers, the axial forces are divided into x- and y-components (in red). In Fig. 3.6b, displacements are shown. By dividing axial deformations into x- and y-components, deformation in x-direction (Δx) and y-direction (Δy) are obtained. For linear spring constants of the springs are k_1 and k_2, $R_1(\Delta l_1) = F_{s1} = k_1 \Delta l_1$ and $R_2(\Delta l_2) = F_{s2} = k_2 \Delta l_2$ are the spring forces. At equilibrium,

$$k_1 \Delta l_1 - F_v \sin\theta_1 = 0, \qquad\qquad k_2 \Delta l_2 - F_v \sin\theta_2 = 0,$$

which give the deformation Δl_1 and Δl_2, and they can be used to compute the deformations in x and y (refer to Fig. 3.6b):

$$\Delta x_1 = \Delta l_1 \cos\theta_1, \qquad \Delta x_2 = \Delta l_2 \cos\theta_2,$$
$$\Delta y_1 = \Delta l_1 \sin\theta_1, \qquad \Delta y_2 = \Delta l_2 \sin\theta_2.$$

Net forces for each element in the x-direction and the y-direction are

$$F^n_{1x}(x) = F^a_{1x} - R_{1x}(x), \qquad F^n_{2x}(x) = F^a_{2x} - R_{2x}(x),$$
$$F^n_{1y}(y) = F^a_{1y} - R_{1y}(y), \qquad F^n_{2y}(y) = F^a_{2y} - R_{2y}(y),$$

where

$$F^a_{1x} = F_v \sin\theta_1 \cos\theta_1, \qquad F^a_{2x} = F_v \sin\theta_2 \cos\theta_2,$$
$$F^a_{1y} = F_v \sin^2\theta_1, \qquad F^a_{2y} = F_v \sin^2\theta_2,$$
$$R_{1x}(x) = k_1 x, \qquad R_{2x}(x) = k_2 x,$$
$$R_{1y}(y) = k_1 y, \qquad R_{2y}(y) = k_2 y.$$

Energy Storage Phase

The energy stored in the x-direction is

$$\begin{aligned} E^s_x = E^s_{1x} + E^s_{2x} &= \int_0^{\Delta x_1} F^n_{1x}\, dx + \int_0^{\Delta x_2} F^n_{2x}\, dx \\ &= \Big(F_v \sin\theta_1 \cos\theta_1 \Delta x_1 + F_v \sin\theta_2 \cos\theta_2 \Delta x_2\Big) - \Big(\frac{1}{2}k_1 \Delta x_1^2 + \frac{1}{2}k_2 \Delta x_2^2\Big) \end{aligned}$$

and the energy stored in the y-direction is

$$\begin{aligned} E^s_y = E^s_{1y} + E^s_{2y} &= \int_0^{\Delta y_1} F^n_{1y}\, dy + \int_0^{\Delta y_2} F^n_{2y}\, dy \\ &= \Big(F_v \sin^2\theta_2 \Delta y_2 + F_v \sin^2\theta_1 \Delta y_1\Big) - \Big(\frac{1}{2}k_1 \Delta y_1^2 + \frac{1}{2}k_2 \Delta y_2^2\Big) \end{aligned}$$

Energy Return Phase

The energy release is from the deformed shape back to the initial state by the reaction force, which would be like the following.

$$E_x^r = \int_{\Delta x_1}^{0} R_{1x}(x)\, dx + \int_{\Delta x_2}^{0} R_{2x}(x)\, dx = -\frac{1}{2}k_1\Delta x_1^2 - \frac{1}{2}k_2\Delta x_2^2$$
$$E_y^r = \int_{\Delta y_1}^{0} R_{1y}(y)\, dy + \int_{\Delta y_2}^{0} R_{2y}(y)\, dy = -\frac{1}{2}k_1\Delta y_1^2 - \frac{1}{2}k_2\Delta y_2^2$$

Energy Transfer

Energy transfer for two inclined linear springs is

$$E_{trans} = E_x^r + E_x^s = \Big(F_v \sin\theta_1 \cos\theta_1 \Delta x_1 + F_v \sin\theta_2 \cos\theta_2 \Delta x_2\Big) - \Big(k_1\Delta x_1^2 + k_2\Delta x_2^2\Big)$$

The same analysis has been done on other structures too using static analysis. The static analysis results are shown in the result section, and the results also show that there is no DET. Then, does it mean that DET is not possible in any structure? I argue not, and I find that the problem comes from assumptions of the static equilibrium analysis. For example, the static analysis is based on the equilibrium condition, and thus the force matrix shows the internal reaction forces at the very moment when the structure reaches at equilibrium. Hence, the applied forces are always the same as the reaction forces. However, as mentioned in Remark 3, the DET is a dynamic process, and thus the applied forces are changing and the system is not at a equilibrium state. Therefore, it is important to apply dynamic analysis to the formulation so that the DET can be truly measured.

3.4 Generalized Dynamic Formulation

To study the DET behaviour more accurately, the model was extended using implicit dynamic FEA. The spring element is also extended to frame element. A frame element can model a straight bar of an arbitrary cross-section, which can deform in the axial and perpendicular direction to the axis of the bar. A frame is capable of carrying both axial and transverse forces, as well as moments. The major difference from the previous formulation is that while the spring element is described by a spring constant k, the frame element is described by a stiffness matrix K. The dynamic force equilibrium at the nodes of a system of structural elements at any time can be expressed as

$$F^i(t) + F^d(t) + F^e(t) = F(t) \tag{3.5}$$

where F^i = inertia force vector, F^d = damping force vector, F^e = internal resisting force vector and F = vector of externally applied force. These force vectors are related to the physical properties of the structure elements, i.e., $F^i = M\ddot{u}$, $F^d = C\dot{u}$ and $F^e = Ku$ for linear systems. Hence, a structural system subjected to dynamic forces is modelled by the following expression:

$$M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = F(t), \tag{3.6}$$

where M is mass matrix, C is damping matrix, K is stiffness matrix, $\dot{u}$ and $\ddot{u}$ are time derivatives of deformation u. For time history analysis for the derivatives, direct integration method is used so that no transformation of the equations into different forms is carried out. A popular implicit method for numerical integration called 'Newmark Method' is used here. This method is unconditionally stable and has no restriction on the time step size. Although damping is highly related to vibration in reality, it requires a set of experiments to determine damping ratios of the structure at two separate frequencies, so the damping effect is not considered here for the sake of simplicity. Here, the frame is assumed to have a uniform cross-sectional area A with length L. Material is linear with elastic modulus E and geometry is linear. The mass matrix M and the stiffness matrix K are expressed as

$$M = \frac{\rho A L}{420}\begin{bmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ 0 & 156 & 22L & 0 & 54 & -13L \\ 0 & 22L & 4L^2 & 0 & 13L & -3L^2 \\ 70 & 0 & 0 & 140 & 0 & 0 \\ 0 & 54 & 13L & 0 & 56 & -22L \\ 0 & -13L & -3L^2 & 0 & -22L & 4L^2 \end{bmatrix}, \tag{3.7}$$

$$K = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI_z}{L^3} & \frac{6EI_z}{L^2} & 0 & -\frac{12EI_z}{L^3} & \frac{6EI_z}{L^2} \\ 0 & \frac{6EI_z}{L^2} & \frac{4EI_z}{L} & 0 & -\frac{6EI_z}{L^2} & \frac{2EI_z}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI_z}{L^3} & -\frac{6EI_z}{L^2} & 0 & \frac{12EI_z}{L^3} & -\frac{6EI_z}{L^2} \\ 0 & \frac{6EI_z}{L^2} & \frac{2EI_z}{L} & 0 & -\frac{6EI_z}{L^2} & \frac{4EI_z}{L} \end{bmatrix} \tag{3.8}$$

For a frame element inclined at an angle θ with the x-axis, a transformation matrix is required to include the orientation of the element, and the equation becomes:

$$T^\top M T \begin{Bmatrix} \Delta\ddot{x}_1 \\ \Delta\ddot{y}_1 \\ \Delta\ddot{\theta}_1 \\ \Delta\ddot{x}_2 \\ \Delta\ddot{y}_2 \\ \Delta\ddot{\theta}_2 \end{Bmatrix} + T^\top K T \begin{Bmatrix} \Delta x_1 \\ \Delta y_1 \\ \Delta\theta_1 \\ \Delta x_2 \\ \Delta y_2 \\ \Delta\theta_2 \end{Bmatrix} = \begin{Bmatrix} F_{1x} \\ F_{1y} \\ M_1 \\ F_{2x} \\ F_{2y} \\ M_2 \end{Bmatrix}, \tag{3.9}$$

with

$$T = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta & \sin\theta & 0 \\ 0 & 0 & 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.10}$$

With the given applied load $F^a = \{F_{1x}, F_{1y}, M_1\}$, the deformations in x- and y-directions $(\Delta x_1, \Delta y_1, \Delta\theta_1)$ at different time instants during the motion can be computed by Eq.(3.6). To obtain the reaction forces (R) at the loading point at a deformation instant, Eq.(3.10) can be used as well by dropping the mass term $(M\ddot{u}(t))$ and inputting the deformations $(u(t))$, and then the solved forces (F_{1x}, F_{1y}) are the reaction forces due to the potential energy stored in the system.

Note that, although Eq.(3.10) is just for one single element, the matrix can be easily extended when there are more elements, like the finite element method (FEM). The mechanism with two inclined frames is also tested and reported in the result section. Moreover, the benefits of using frame elements over spring elements are that they can be used to model the widest variety of components, more related to the real design resulting in a better result.

If we re-think about the structure with two linear springs, the mechanism is actually a simplified representation of the actual design considering only the axial loads. The purpose is to apply the frame elements to develop a more general method to deal with other different structures irrespective of complexity. In fact, the stiffness of a particular structure could be complex, which would make the discrete analysis more complicated. Since the FEM represents the small blocks of materials and connects them to other elements, they can be shaped to almost any geometry subjected to any loading.

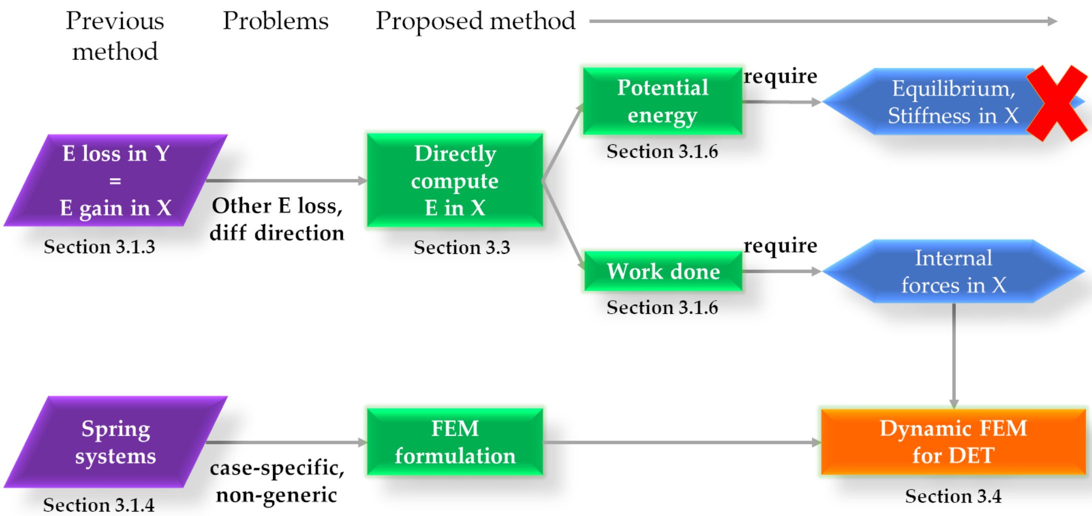


FIGURE 3.7: Flow chart for DET.

3.5 Synopsis

If the process from problem identification to find the answer is reviewed, it could be displayed as the flowchart in (Fig. 3.7). Two main intricacies were distinguished from Dickson's work[2]. One of the challenges was energy loss in y-direction was regarded as an energy gain in the x-direction (Section 3.1.3). Another challenge was, the mathematical models were entirely case-specific (Section 3.1.4). To resolve the first query, energy should be assessed in x-direction directly since the purpose is to gain energy transfer in x-direction (Section 3.3). Momentarily, potential energy and work done can be considered to determine the energy. However, potential energy requires directional stiffness which might be tricky or complicated to manage sometimes. Work done method is more compatible in this situation as force and deformation are simply achievable. The liable force for deformation is mentioned as a net force in this investigation. Net force requires internal force which can be solved by mathematical or numerical analysis (Section 3.1.6).

On the other hand, finite element analysis is utilized in this analysis to answer the second problem because of its generic features. However, static analysis has some shortcomings to satisfy the necessities in this analysis. Therefore, dynamic analysis is employed to conduct the analysis and to determine the internal force and deformation of the structure (Section 3.4).

4 Results

4.1 Numerical analysis

Following the previous study [2], the shoe midsole structure is being focused, and the loading curve is shown in Fig. 3.2d is also applied here to find out how the structures respond to the loading and unloading cycle.

The loading curve was based on the entire weight of a body, but it should be distributed uniformly on the contact surface, so a fraction of the load is used, i.e., $\frac{1}{10}$th arbitrarily. With the applied forces in different time instants, the deformations and reaction forces, as well as the energies, can be computed in the x- and y-directions, respectively.

The energies stored are shown as positive, and the energies returned are shown as negative. After that, the DET can be determined. Since the development changes the static analysis to dynamic as well as spring element to frame element, the frame element was tested first with static analysis to make sure it is actually the dynamic analysis that matters to the DET. Also, to mention, applying frame elements to the model helps build a unified framework to be extended to any cases.

4.1.1 Analysis using the new formulation for two spring elements

Initial considerations

Referring to the formulations in Section 3.3.1, the following values are used for the calculation: stiffness of spring 1, $k_1 = 12000\text{N/mm}$, stiffness of spring 2, $k_2 = 9200\text{N/mm}$, inclination angle of spring 2, $\theta_1 = 60^\circ$, inclination angle of spring 2, $\theta_2 = 30^\circ$, and length of both springs, $l_1 = l_2 = 30\text{mm}$.

Calculation

With an applied load $F_v = -822.15$N in the vertical direction, the deformations for spring 1 and spring 2 are $\Delta l_1 = 0.059$mm and $\Delta l_2 = 0.0447$mm. The total energy stored in the springs can be computed as $E_{tot} = \frac{1}{2}k_1\Delta l_1^2 + \frac{1}{2}k_2\Delta l_2^2 = 30.307$J. The decomposed displacements in the x- and y-directions for the two springs are: $\Delta x_1 = -0.0297$mm, $\Delta x_2 = 0.0387$mm, $\Delta y_1 = -0.051$mm, and $\Delta y_2 = -0.022$mm. The energies stored can then be computed: $E_x^s = 12.169$J and $E_y^s = 18.138$J.

Validation

The sum of them is $E_{tot}^s = E_x^s + E_y^s = 30.307$J, which is the same as the E_{tot} and it shows that the formulations for the energy storage phase are correct.

Similarly, the energies in the return phase can be computed: $E_x^r = -12.168$J and $E_y^r = -18.138$J (negative for storage). The absolute value of their sum is $E_{tot}^r = 30.307$J, which equals to E_{tot}, demonstrating that the formulations for the return phase are also valid.

DET calculation

The energy transfer is thus $E_{trans} = E_x^r + E_x^s = 0$, which shows no DET in this loading case. From the calculation, E_{trans} is always zero, meaning that this structure is not capable of transferring any energy.

Note that, the previous study [2] considered only the energy in the x-direction in the return phase, which is $E_x^r = -12.168$J computed in this study, and reported this is the energy transferred. However, if the energy stored in the storage phase is considered, it is exactly the same amount as in the return phase. In other words, during the return phase, the structure is just moving back to compensate for the forward movement done in the storage phase, which cannot be counted as energy transferred.

4.1.2 Static analysis with frame element

The frame element is a straight bar that can deform both in the axial direction and perpendicular to the axis of the bar. The bar is capable of carrying both axial and transverse forces, as well as moments. Hence the frame element was analyzed separately as a truss and a beam element. Since the frame element can carry the axial and transverse load, energy for the truss element and the beam element were determined independently. Energy from the truss element and the beam element should be equivalent to the energy from the frame element. However, the bending moment is prominent over axial loading to create deformation and displays the major share of the total energy of the element (Fig. 3.3).

Initial considerations

An inclined frame element is considered with $A = 95$ mm^2, $E = 2900$J, $L = 30$mm, inclined angle, $\theta = 45°$ and used FEA to calculate deformation for applied vertical and horizontal forces at the contact point of shoe-sole and foot. The contact point of the element and outsole/ground was set as boundary conditions. Also, reaction force were calculated at equilibrium using Eq.(3.10). Let the reaction force at equilibrium be R. In this particular case, as deformation is linear due to loading, the reaction force will increase linearly from zero to R.

Calculation

For the case when $t = 0.01$s with $F_v = -822.15$N:

Bending

Bending deformation, $\Delta l^{beam} = 3.33$mm, resulting in x and y-deformation from bending are respectively $\Delta x_1^{beam} = 2.355$mm and $\Delta y_1^{beam} = -2.355$mm. Bending force coming from F_v is 581.6N resulting in $F_{1x}^{a-beam} = 411.253$N and $F_{1y}^{a-beam} = -411.253$N and reaction forces at equilibrium $R_{1x}^{beam} = -411.253$N and $R_{1y}^{beam} = 411.253$N. Therefore, the stored energy would be $E_x^{s-beam} = 484.183$J. Similarly, $E_y^{s-beam} = 484.183$J and $E_{tot}^{s-beam} = 968.365$J. Energy return would be $E_x^{r-beam} = -484.183$J and $E_y^{r-beam} = -484.183$J. Therefore, $E_{trans}^{beam} = 0$.

Truss

Axial deformation, $\Delta l^{truss} = 0.073$mm, resulting in x and y-deformation are respectively $\Delta x_1^{truss} = 0.052$mm and $\Delta y_1^{truss} = -0.052$mm. Axial force coming from F_v is 581.6N resulting in $F_{1x}^{a-truss} = -411.253$N and $F_{1y}^{a-truss} = -411.253$N and reaction forces at equilibrium $R_{1x}^{truss} = 411.253$N and $R_{1y}^{truss} = 411.253$N. Therefore, the stored energy

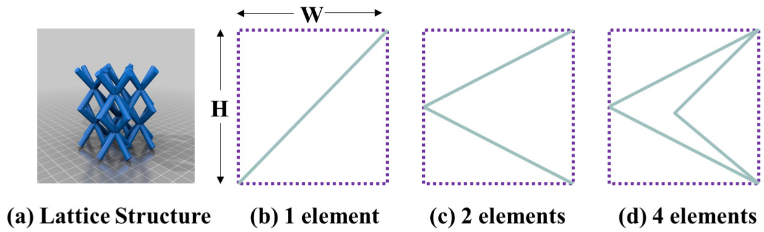


FIGURE 4.1: (a) One of the common lattice structures for 3D printing, and it is modified for testing the dynamic analysis: (b) 1-element, (c) 2-element and (d) 4-element.

would be $E_x^{s-truss} = 10.7$J. Similarly, $E_y^{s-truss} = 10.7$J and $E_{tot}^{s-truss} = 21.4$J. Energy return would be $E_x^{r-truss} = -10.7$J and $E_y^{r-truss} = -10.7$J. Therefore, $E_{trans}^{truss} = 0$.

Validation

For frame element, x and y-deformation are respectively $\Delta x_1^{frame} = 2.303$mm and $\Delta y_1^{frame} = -2.408$mm. Strain energy is 989.957J which is sum of $E_{tot}^{s-beam} = 968.365$J and $E_{tot}^{s-truss} = 21.4$J which shows the validity of this procedure.

Observations

In summary, vertical load was applied, which provided energy in both directions, x and y, which means some portion of energy from the y-direction is being converted to the energy in the x-direction. However, that stored energy in the x-direction is being nullified by the same amount of energy in the x-direction during the energy return phase.

In other words, the energy return and the energy stored is precisely corresponding but opposite in direction. The structures are indeed moving in a horizontal direction following the vertical load, but to gain stored energy, an equal amount of energy is being invested in the opposite direction making this situation obsolete. Therefore, it can be concluded that, for any linear geometry with linear material properties, they do not show any energy transfer while using static analysis.

4.1.3 Dynamic analysis

The static analysis does not provide any information other than the equilibrium state. To investigate a more pragmatic loading situation, some predefined cases are going to be reported to find out the effects of impact loading using dynamic FEA.

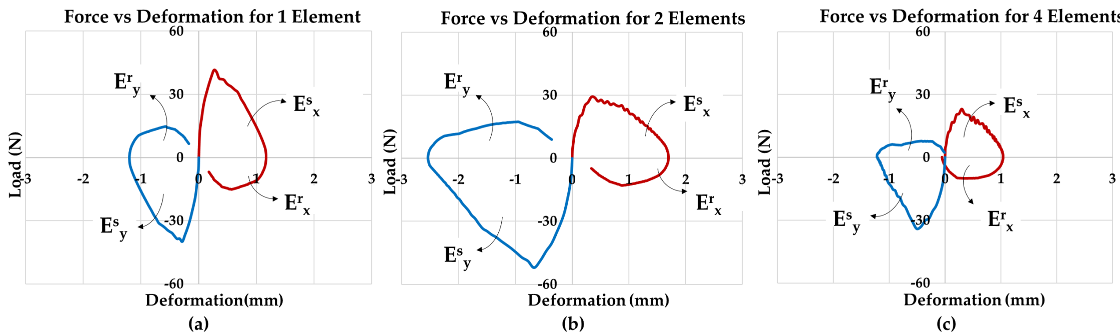


FIGURE 4.2: Net force vs. deformation curves for ABS with volume 3000 mm^3 to obtain the energy by the area under the curves of (a) 1-element, (b) 2-element and (c) 4-element structures.

Motivation: Lattice structure

The study here is motivated by the lattice structures used in 3D-printed parts, one of them is shown in Fig. 4.1a. However, to have DET behaviour, the structure shouldn't be symmetric so that deformation in the x-direction is possible, even only a vertical load in the y-direction is applied. To perform the analysis, the lattice structure was modified and obtained three different structures as shown in Fig. 4.1, including a 1-element bar inclined to the right side (inspired by Adidas Airblade Shoe), a 2-element spring (inspired by 3D printed airless tire) and a 4-element structure (stiffen spring). The goal is to find a lattice structure that has DET behaviour.

Considerations

To facilitate the observation, the same material volume was used along with the same surface boundary for planer frame elements. Likewise, the objective is to observe if there is any energy transfer, and if the structure will be strong enough to stand the loads.

Two quite different materials were used to demonstrate the performance: ABS (Acrylonitrile butadiene styrene) and Aluminum. For surface boundary box, height and width were assumed to be 30mm and 30mm, respectively. This considers the average dimension of a shoe sole so that the structure can reach the midsole height and also can be repetitively used along the midsole length and width. The energy transfer behaviour was observed for two different volumes (3000 mm^3 and 6000 mm^3) with a circular cross-section area. The results are summarized in Table 4.1.

Forces

Unlike previous cases, the applied load has both x and y components as a real-life loading situation is used. Loading curve is mentioned in 3.2 (d). In the beginning, the internal reaction force is zero, so the net force equals the applied force. As the structure starts to deform, it builds an internal reaction force. The net force is calculated by both the applied force and the internal reaction force vectors.

It is noted that, throughout the cycle, the net force starts to increase, reaches a peak, and then decreases due to higher reaction force formation. At a point, the net force becomes zero and turns to grow in the opposite direction. At this transition point, the structure starts to deform in the opposite direction until it reaches its initial state, which is the unloading.

By plotting the net force against the deformation for the whole cycle, the curves in Fig. 4.2 show that they indeed have different values.

Energy: Area under curve

Energy is defined by the integration of net force along with the deformation, which is the area under the curves in Fig. 4.2. Energy towards the x-direction is the intention of the study, which is exhibited in the right section (horizontal axis > 0) of all the plots (since every structure deform to the right first and then come back). The upper-division (vertical axis > 0) is the loading phase, and the lower-division is the unloading phase. Therefore, the area under the curve in the upper-division is the energy stored (E_x^s). Similarly, the area under the curve in the lower-division is the energy return (E_x^r).

In this case, the areas under the curve for energy store and return are on the opposite side, hence resulting in positive and negative energy for loading and unloading, respectively. The energy transfer is the net energy of the two areas under the loading and unloading curves, and the values are provided with Table 4.1.

Energy transfer

At this circumstance, recall the concept of energy transfer from the y-direction to the x-direction. In previous cases, there was deformation in the horizontal direction due to vertical load, which means that it did convert the y-direction energy to the x-direction

Properties			Unit type	1 element		2 elements		4 elements	
Boundary Box: Height, H= 30 Width, W= 30 t = 0, 0.001,.., 0.14s			Length	L=42.4264		$L_1, L_2 = 30$		$L_1, L_3 = 21.21$, $L_2, L_4 = 30$	
			Angle	45°		$\theta_1 = 150°$, $\theta_2 = 30°$		$\theta_1 = 135°$, $\theta_2 = 150°$, $\theta_1 = 45°$, $\theta_2 = 30°$	
			Volume	3000	6000	3000	6000	3000	6000
ABS	Y	2000	E_{trans}	19.3851	4.9282	20.2029	5.0075	6.5838	1.6023
	σ_{max}	40.7	σ	-4.7601	-1.5915	-72.5574	-29.7903	-70.3442	-32.8264
	τ_{max}	28.49	τ	1.4921	0.833	-3.7967	-1.8675	-5.1042	-2.5391
Al	Y	70000	E_{trans}	0.111	0.0145	0.2136	0.0299	0.0934	0.0136
	σ_{max}	276	σ	-1.3672	-0.6677	-98.2314	-35.4728	-121.504	-44.8398
	τ_{max}	193.2	τ	1.8593	0.9337	-3.6503	-1.8226	-4.9133	-2.4464

TABLE 4.1: Results of dynamic analysis on various structures and materials. (Y = elastic modulus, σ_{max} = yield strength, τ_{max} = shear strength, σ = max normal stress and τ = max shear stress obtained from the analysis. The units for length, volume and stresses are mm, mm^3 and N/mm^2, respectively.)

energy. However, it was not beneficial due to energy compensation between energy storage and return.

On the other hand, static analysis is invalid as it does not provide adequate information about the change in net force. Dynamic analysis considering the time history helps in this circumstance. From the dynamic analysis, different sets of x and y reaction forces from the applied loads are found, which means if there is a difference between net forces of the loading and unloading in the x-direction, the structure might have DET.

If we recall, energy transfer is the net energy of stored and returned in a particular direction. The net energy should be positive in the expected direction (running in the right direction). It can be realized from the curve that the energy storage for loading is higher than the energy return.

As damping is not considered in this analysis, the energy difference is only because of the inertia due to mass. Hence, energy transfer is obtained in the x-direction due to the geometric effect.

Stress

To find the maximum normal and shear stress, the largest stress was obtained for each element from the whole cycle. Maximum stress was found by comparing all the stresses at every element and the element subjected to higher stress was located. Obtained maximum normal stress was compared with yield strength σ_{max}, and maximum shear stress was compared with shear strength τ_{max} of the material. As no plastic deformation is expected, the yield strength of the material worked as a benchmark for this analysis. All stress values are provided in Table 4.1.

Comparison among different structures

Comparison among 1-element, 2-elements and 4-elements based on strength, volume and Elastic Modulus is mentioned below:

DET

From the results shown in the Table 4.1, the 2-element structure is showing a higher energy transfer than the 1-element, whereas the 4-element structure has the lowest energy transfer for any material irrespective of volume.

Stress

In terms of the normal and shear stresses, the 2-element and the 4-elements both have higher stress values than the 1-element. Since 2-element and the 4-elements have shallower cross-section area, higher bending stress is generated.

Volume

Moreover, the use of a smaller volume of material with smaller yield stress (e.g. ABS) causes yield or failure of the material more easily. The aforementioned reason eliminates the 2-element structure (although it has a higher DET), making the 1-element be a better choice when less volume is needed.

However, the use of a higher volume of material causes less stress concentration (for 6000mm^3, normal stress is $-29.7903\text{N}/\text{mm}^2$) for the 2-element. In this case, the 2-element is more preferable for two reasons. First, the structure is remaining below the yield strength point. Second, the 2-element structure has a higher energy transfer.

Elastic modulus

The use of Aluminum instead of ABS shows that the 2-element structure is preferable over the 1-element when the material has higher elastic modulus and strength.

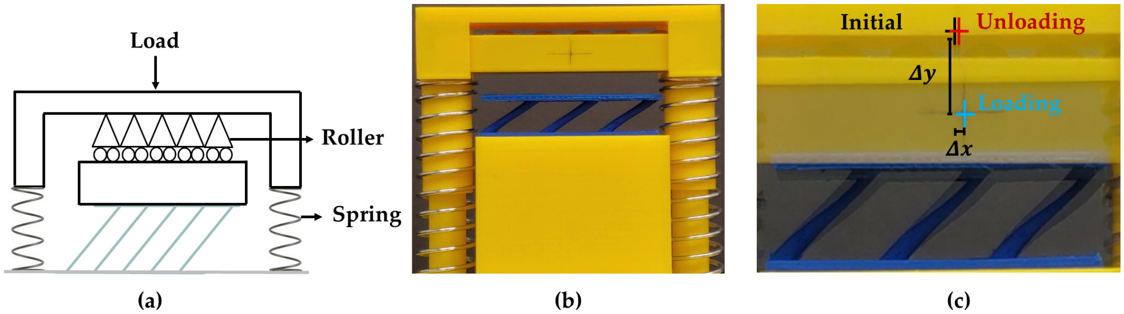


FIGURE 4.3: (a) Schematic diagram of experimental setup (b) actual experimental setup (c) deformation from experimental analysis after loading and unloading.

4.1.4 Physical test

A physical experiment is also conducted to verify the numerical analysis. An experimental setup was made to show deformation behaviour due to the vertical load applied.

Experimental setup

The experimental setup is shown in Fig. 4.3a. It is a device that allows a load to be applied at the top and deforms the testing structure put under it. Roller support is added in-between the structure and loading mechanism, such that the component below the rollers will follow the structure to move in the x-direction, but follow the device to move in the y-direction. The purpose of the spring is to mimic the unloading condition returning to the original height, as soon as the load is released. The structure was fabricated using fused deposition modeling (FDM) 3D printing method and the material was Thermoplastic polyurethane (TPU). An elastomer is chosen here to maximize the deformation for visual compassion since the deformation of ABS was too small to see.

Procedure

The vertical load was applied on the top and deformation of the cross-marked point is observed. From the numerical analysis, it is clear that internal reaction forces generated in the structure are one of the main driving contributions to obtain directional energy transfer. However, it is challenging to apply load exactly as the loading profile in the numerical analysis, and the current device cannot obtain the corresponding internal reaction force. Fortunately, the numerical analysis showed that there is a displacement in x after the loading and unloading cycle, and if this phenomenon is also observed in

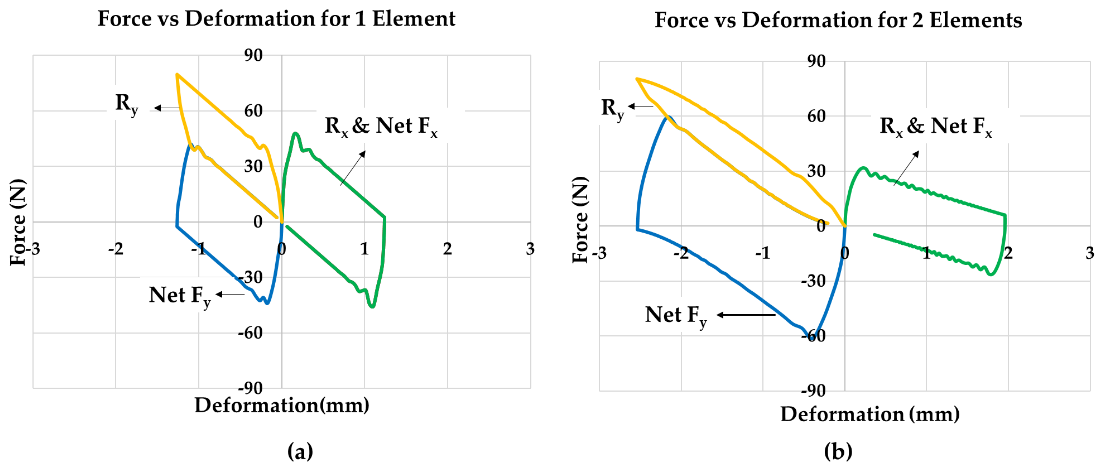


FIGURE 4.4: Different Forces vs. deformation.

the physical experiment, it can be safely said that the numerical analysis is verified.

Results

The experiment result shown in Fig. 4.3b, where the images of three stages are overlaid: initial, loaded, and unloaded. A particular point was selected on the roller support and followed the deformation of the point. A maximum deformation in x-direction and y-direction was observed 3.214mm and 9.643mm, respectively. The difference between the initial position before loading and the end position after unloading is 0.67mm. As a result, there is indeed a deformation in x-direction with a load only in the y-direction.

Conclusion

The vertical load can generate horizontal deformation is visible from the experimental analysis. Moreover, there appears a deformation difference between the initial position and the end position. This means that the deformations during energy storage and energy return are different. From the experiment, the deformation is attainable; however, applied load and internal reaction force are not achievable. Hence, energy transfer is not countable from this experimental setup.

4.2 Validation of Mathematical Model

Admittedly, energy loss in y-direction does not necessarily confirm energy is transferred in the x-direction. Hence, the proposed method to calculate DET is to determine energy in x-direction directly. However, why DET calculated in x-direction directly is the energy transferred from y-direction is explained in this section.

4.2.1 Physical interpretation

Mathematically shown earlier, dynamic analysis shows the structure has energy transfer which static analysis can not. Yet a question arises what happens to a structure radically to show DET merely by changing the numerical analysis method.

In two linear springs equations, mass of the spring or damping effect was not recognized. In static analysis, a similar thing resembles; mass and damping effects are ignored. These parameters are influencing DET a great value.

While reflecting dynamic analysis, mass, velocity, acceleration, and damping comes into deliberation. Inertia generated due to mass, also damping effect causes energy loss affecting internal force. In Fig. 4.4a, the internal force generation is shown for the entire period. In Fig. 4.4b, net forces and internal forces are displayed. Internal forces are separate for loading and unloading. This difference indicates that the internal force generated during loading deviates from that of unloading.

Hence, in summary, the directional energy transfer is reliant on internal forces over the loading-unloading path, the geometry of the structure, mass moment of inertia, velocity, acceleration, and damping.

4.2.2 Loading types

To verify that energy in x-direction is due to only vertical load, two types of loading are presented: variable y-loading and constant y-loading.

Constant loading

A fixed load is applied for half of the cycle and withdrawn following the previous half. The load is increased in several rests to lessen vibration. Following this, the load remains consistent for the half of the time. Likewise, the load was lowered in multiple runs at the end of the first half cycle.

Variable loading

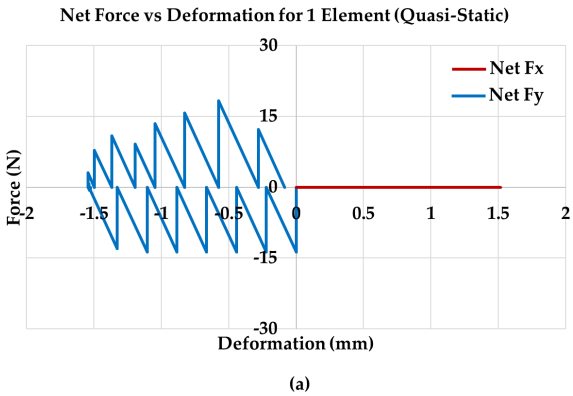
Considering real forefoot strike loading data from previous work, the loading shown in Fig.3.2d is mentioned as variable loading. The entire cycle is 0.14s starting from the initial contact between the foot and the midsole to the endpoint while foot loses connection with the midsole. Following this point, the foot and midsole stays in the air until the next cycle commences when the foot and shoe midsole regains the contact due to the ground strike. If the sequence is considered repetitively, the structure gets some relaxation time to go back to its initial stage.

4.2.3 Direct X-energy as DET

To obtain DET, force and deformation should be obtained in the intended direction as output due to the input force and deformation. However, directional energy transfer does not necessarily have to be from y-direction to x-direction. Energy transfer could be from any direction to the expected predetermined direction. In this study, the applied load is working only in the vertical direction, but the deformation is obtained in the horizontal direction as output. Hence, there must be some force working in x-direction which came from y-direction.

Beam and truss separation of frame element

A structure can divide the applied load as components in x and y-directions as mentioned in two linear spring mechanisms (Fig. 3.2). Frame element which is considered for the analysis is a combination of a beam and a truss element, hence force can be divided for beam and truss. The beam element carries the transverse load whereas the



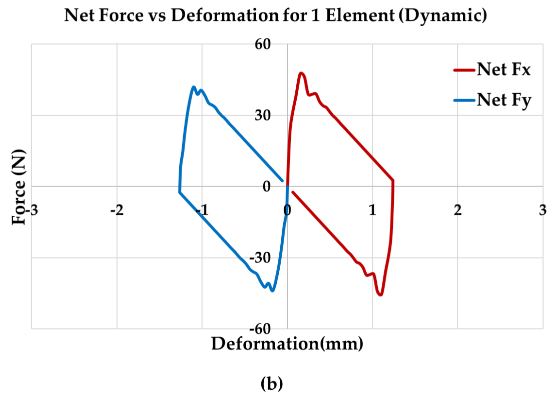


FIGURE 4.5: Quasi-static vs dynamic analysis for 1 element.

truss element carries the axial load. X and y-components for a frame element can be obtained separately from the truss and beam element. In the bigger picture, if no load is applied in x-direction, x-components from truss and beam nullify each other. However, it is possible to determine energy separately for truss and beam elements in x and y-direction (Section 4.1.2).

Yet, it is complicated to do separate calculations for beam and truss elements for multiple elements. Therefore, separation of beam and truss element is not performed for intricate structures.

Frame element

The frame element is applied in this analysis for suitability. As mentioned earlier, horizontal movement is attainable for vertical load. As there is no x-load applied, it is a challenge to answer how energy is possible to achieve in x-direction. To answer the question, the concept of internal force needs to be recalled. An internal force is generated in the horizontal direction due to the applied load in the vertical direction. The aforementioned internal force in x-direction is accountable for energy in x-direction. If 'net energy' is obtained in the intended direction, we can conclude there is energy transfer.

Static vs. Dynamic

In Fig. 4.5, the impact of quasi-static and dynamic analysis in graphical analysis is shown. Recall, static analysis is only concerned about the equilibrium state of the structure, net force at equilibrium is always zero. There is no information available on how the internal force changes from the initial point to the endpoint. Hence, for both energy storage and energy return, internal force is assumed to evolve linearly from zero to applied force. Due to this assumption, the corresponding structure as the dynamic analysis does not exhibit any energy transfer. In dynamic analysis, internal forces are different for loading and unloading. The reason behind this is mainly acknowledging the energy loss due to inertia or damping. Furthermore, in static analysis velocity and acceleration are ignored which has very crucial interference to internal force especially for impact loading.

In Fig. 4.5a, quasi-static analysis is presented unlike static analysis. Quasi-static analysis can manage additional force applied slowly (without considering time history). In quasi-static analysis, the energy return is only visible in the y-direction. Hence, the variable load factor is not sufficient to confirm energy transfer in the x-direction.

4.2.4 Effects of various parameters

The effects of the changing numerous parameters will be mentioned here. Those parameters are: variable load, constant load, and inclination angle.

Constant vs. variable loading: only Y

Loading condition serves as a critical feature to achieve DET as it has a direct impact on the internal force. The variable loading over time (only Y load) explains the phenomenon where structure deforms due to uneven loading (heel strike).

Net forces in x and y-directions are shown by red and blue curves respectively. The area under curves are used to measure the energy for energy store and energy return (For example, from Fig. 4.6b, Quadrant I: energy store-x, Quadrant II: energy return-y,

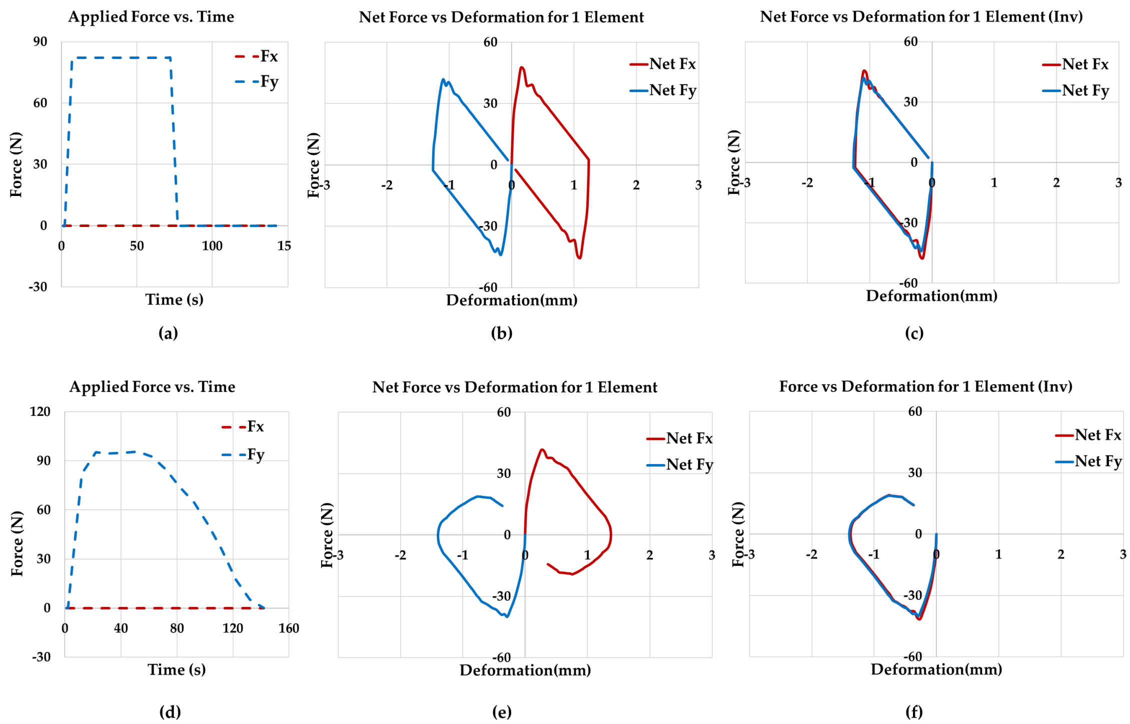


FIGURE 4.6: Net force vs deformation curves for constant and variable forces for structure (45°) and inverse structure (135°).

Quadrant III: energy store-y, Quadrant IV: energy return-x). The curve is generated due to load and cross the x-axis when the structure starts to move in the opposite direction.

If the intended direction is the positive x-axis, the expectation would be to have net energy in the positive x-axis (hence, energy due to deformation in positive x-direction). However, as loading and unloading phases both need to be considered, energy in positive x-direction should be higher than energy in negative x-direction (as intended energy transfer direction is positive x-direction).

Now, Fig. 4.6b, shows the net force curves for Fig. 3.1b which is inclined at 45°. The structure deforms towards the intended direction first, hence energy store contributes in the intended direction. Consequently, the energy return is contributing in the opposite direction. However, if the structure is in the reverse direction (135°), it deforms in negative x-direction first (energy store). In simple words, energy is being transferred to negative x-direction instead of positive x-direction (the intended direction). Hence, the geometric properties structure has a direct correlation along with the expected direction of energy transfer. It is understandable that just by reversing the structure direction the

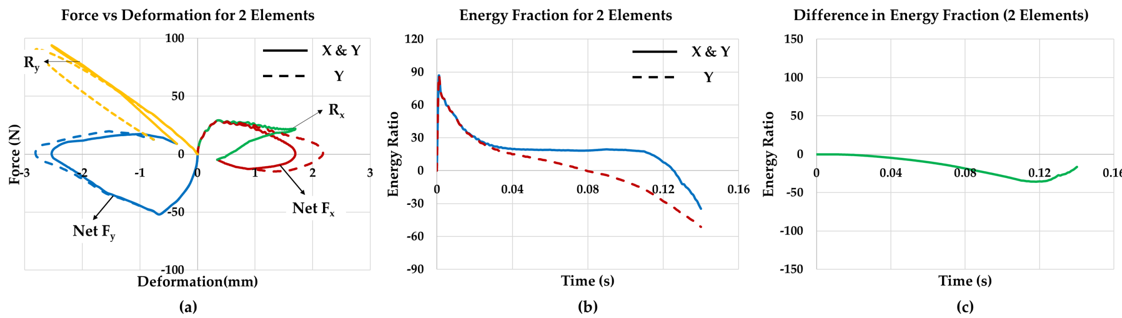


FIGURE 4.7: (a) Force vs deformation, (b) Energy fraction and (c) difference between energy fractions for y-load and xy-load for two frame elements.

structure is not being able to have DET in expected direction anymore. Energy summary is mentioned here.

For constant loading (only Y), the structure at 45°, showed DET ($E_{trans} = 2.886$J) at positive x-direction due to energy store. (Fig. 4.6b) and when the structure is reversed at 135°, DET is obtained ($E_{trans} = -2.886$J) at negative x-direction due to energy store (Fig. 4.6c). For variable load (only Y) in (Fig. 4.6d), the structure at 45° shows DET ($E_{trans} = 20.778$J) at positive x-direction due to energy store, whereas the inverse structure at 135° showed DET ($E_{trans} = -20.778$J) at negative x-direction due to energy store (Fig. 4.6e).

Variable load: both X and Y

Previously, the graphs were shown based on the load in the vertical direction solely. However, in reality, the casual loading would have both x and y-components. Hence, applied x-load would directly affect DET. As mentioned formerly, DET is the transfer of energy from one direction to another, input direction not necessarily has to be the vertical direction. The applied load was taken in a vertical direction as reference. Similarly, output direction not necessarily has to be x-direction. The main concept is to find that how load applied in a different direction is affecting the energy in the intended direction.

When y-load is applied solely, the energy in x-directly purely comes from y-loading. However, applied load in x-direction would affect the energy in x-direction depending on the direction of the applied load. In Fig. 4.7a, force curves are shown and compared between ’only y-load’ and ’xy-load’. As a measurement to show the difference, energy fraction for both (y-load and xy-load) is calculated where,

Energy fraction $= \frac{E_x}{|E_x|+|E_y|} * 100\%$.

The difference between the energy fraction for ’y-load’ and ’xy-load’ indicates indirectly that some energy in x-direction is coming due to load in y-direction. When additional x-load is applied, it is contributing for energy in x-direction hence, energy fraction for ’xy-load’ differs from ’y-load’ as shown in Fig.4.7b and c.

4.3 Synopsis

The purpose of the result sections is to show how actually the theoretical equations were implemented, and ultimately directional energy transfer was calculated. Further optimization analysis can be performed to show the best structure with the highest energy transfer. This study can be the basis to develop a DET lattice structure, and directional energy transfer property can be manipulated by generating more complicated but functional design. Meta-materials can be possible scopes to be analyzed in the future, including other desired properties of the structure.

5 Conclusion

In this paper, a modified mathematical approach for DET is proposed. The revised model includes considering the net force and loading paths. The formulation in estimating the energies in the x- and y-directions are verified by the static equilibrium analysis, for both the storage and return phases. From the study, it can be concluded that simple linear structures with linear materials were able to show DET using energy in the storage phase. This mathematical model serves as a handy computational tool for generative design. Dynamic analysis was performed to have a better understanding of the DET.

However, the investigation has some limitations. For example, it did not regard damping, and the specific orientations of elements were analyzed that satisfy the boundary conditions. The research did not contemplate the non-linear behaviour effect of geometry change on the stiffness matrix, considering the circular cross-section area only. Hence, the results may differ depending on the physical properties of the structure, cross-section area or higher load, causing failure of the structure. More analyses should be done considering optimum structure design and material selection in different applications. To explore further scopes of DET, more extensive analysis is needed with more complex structure, non-linear materials, and/or dynamic structure.

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